11

Surface Areas and Volumes

Checkpoint (Page 263)

1. Find the lateral surface area and the total surface area of a cuboid whose length = 12 cm, breadth = 10 cm and height = 14 cm.

Sol. Let l, b and h be the length, breadth and height of the cuboid.

Then l = 12 cm, b =10 cm and h = 14 cm.

$$\therefore \text{ Lateral surface area} = 2h(l + b)$$
$$= 2 \times 14 \times (12 + 10) \text{ cm}^2$$
$$= 28 \times 22 \text{ cm}^2$$
$$= 616 \text{ cm}^2$$

$$\text{Total surface area} = 2(lb + lh + bh)$$
$$= 2(12 \times 10 + 12 \times 14 + 10 \times 14) \text{ cm}^2$$
$$= 2(120 + 168 + 140) \text{ cm}^2$$
$$= 2 \times 428 \text{ cm}^2$$
$$= 856 \text{ cm}^2$$

2. Find the edge of a cube whose surface area is 26.46 cm^2.

Sol. Let a be the side of a cube.

Then $6a^2 = 26.46 \text{ cm}^2$

$$\Rightarrow \quad a^2 = \frac{26.46}{6} \text{ cm}^2$$
$$= 4.41 \text{ cm}^2$$
$$\therefore \quad a = \sqrt{4.41} \text{ cm} = 2.1 \text{ cm}$$

Hence, the required edge of the cube is 2.1 cm.

3. The perimeter of each face of a cube is 20 cm. Find its lateral surface area.

Sol. Let a be the length of each square face of the cube.

$$\therefore \quad 4a = 20 \text{ cm}$$
$$\Rightarrow \quad a = 5 \text{ cm}$$
$$\therefore \text{ Lateral surface area} = 4a^2$$
$$= 4 \times 5^2 \text{ cm}^2$$
$$= 100 \text{ cm}^2$$

4. The surface area of a cuboid is 2350 cm^2. If its length and breadth are 25 cm and 20 cm respectively, find its height.

Sol. Let l, b and h be the length, breadth and height of the cuboid.

$\therefore$ l = 25 cm, b = 20 cm

Then its surface area

$$= 2(lb + lh + bh)$$
$$= [2(25 \times 20) + 2h(l + b)] \text{ cm}^2$$
$$= [2 \times 500 + 2h(25 + 20)] \text{ cm}^2$$
$$= (1000 + 90h) \text{ cm}^2$$
$$\therefore \quad 90h + 1000 = 2350$$
$$\Rightarrow \quad 90h = 1350$$
$$\therefore \quad h = \frac{1350}{90} = 15$$

Hence, the required height is 15 cm.

5. The curved surface area of a right circular cylinder of height 14 cm is 924 cm^2. Find the radius of its base.

Sol. Let r and h be the radius of the base and the height of the cylinder respectively.

Then h = 14 cm

$$\therefore \quad 2\pi rh = 924$$
$$\Rightarrow \quad 2 \times \frac{22}{7} \times r \times 14 = 924$$
$$\Rightarrow \quad r = \frac{924}{88} = 10.5$$

Hence, the required radius of the base is 10.5 cm.

6. The volume of a cuboid is 528 cm^2 and the area of its base is 88 cm^2. Find its height.

Sol. Let l, b and h be the length, breadth and height of the cuboid respectively.

Then its volume $= l \times b \times h = 88h$

$$88h = 528$$

$$\therefore \quad h = \frac{528}{88} = 6$$

Hence, the required height is 6 cm.

7. Find the height of a cylinder whose volume is 3.08 m^3 and diameter of the base is 140 cm.

Sol. Let r be the radius of the base and h be the height of the cylinder.

Then $r = \frac{140}{2}$ cm = 70 cm

Volume of the cylinder $= \pi r^2 h$

$$= \frac{22}{7} \times 70^2 \times h$$

$$\Rightarrow \quad 3.08 \times 100 \times 100 \times 100 = \frac{22}{7} \times 70^2 \times h$$

$$\Rightarrow \quad h = \frac{30800 \times 7 \times 100}{22 \times 70 \times 70} = 200$$

Hence, the required height is 200 cm or 2 m.

8. The circumference of the base of a cylindrical vessel is 132 cm and its height is 25 cm. How many litres of water can it hold?

Sol. Let r and h be the radius of the base and height of the cylinder respectively.

Then $2\pi r = 132$ cm

$$\Rightarrow \quad r = 66 \times \frac{7}{22} \text{ cm} = 21 \text{ cm}$$

Also, $h = 25$ cm

$\therefore$ Volume of a cylinder $= \pi r^2 h$

$$= \frac{22}{7} \times 21 \times 21 \times 25 \text{ cm}^3$$

$$= 66 \times 525 \text{ cm}^3$$

$$= 34650 \text{ cm}^3$$

$$= 34.65 \text{ litres}$$

9. A road roller is cylindrical in shape. Its circular end has a diameter of 280 cm and its width is 2 m. Find the least number of revolutions that the roller must make in order to level a playground 220 × 50 m.

Sol. Let r and h be the radius of the base and the height of the cylinder respectively.

Then $r = 140$ cm $= 1.4$ m and $h = 2$ m.

In 1 revolution, the roller levels a ground of surface area

$$= 2\pi rh$$

$$= 2 \times \pi \times 1.4 \times 2 \text{ m}^2$$

$$= 5.6\pi \text{ m}^2$$

$$\therefore \text{ Required number of revolution} = \frac{220 \times 50}{5.6 \times \frac{22}{7}}$$

$$= \frac{220 \times 50}{17.6}$$

$$= 625$$

10. A river 2.5 m deep and 30 m wide is flowing at the rate of 2 km/h. How much water will fall into the sea in a minute?

Sol. Speed of the river = 2 km/h

$$= \frac{2000}{60} \text{ m/minute}$$

$$= \frac{100}{3} \text{ m/minute}$$

The shape of the river is cuboid of length l, breadth = b and depth = h.

We have

$$h = 2.5 \text{ m}, b = 30 \text{ m}, l = \frac{100}{3} \text{ m}$$

$\therefore$ Required volume of water

$$= l \times b \times h$$

$$= \frac{100}{3} \times 30 \times 2.5 \text{ m}^3$$

$$= 2500 \text{ m}^3$$

Check Your Progress 1

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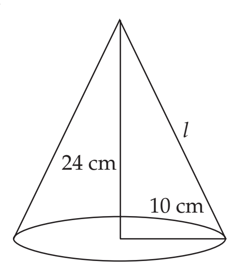
Multiple-Choice Questions

1. The curved surface area of a right circular cone with base diameter 20 cm and height 24 cm is

(*a*) 200π cm^2 (*b*) 240π cm^2

(*c*) 260π cm^2 (*d*) None of these

Sol. (*c*) 260π cm^2



Slant height of the right circular cone, l

$$= \sqrt{(24\text{ cm})^2 + (10\text{ cm})^2}$$

$$= \sqrt{576 + 100}\text{ cm}$$

$$= \sqrt{676}\text{ cm}$$

$$= 26\text{ cm}$$

Curved surface area of the cone = πrl

$$= \pi \times 10\text{ cm} \times 26\text{ cm}$$

$$= 260\pi\text{ cm}^2$$

2. The total surface area of a cone whose radius is $\frac{r}{2}$ and slant height $2l$ is

(*a*) $2\pi r(l + r)$ (*b*) $\pi r\left(l + \frac{r}{4}\right)$

(*c*) $\pi r(l + r)$ (*d*) $2\pi rl$

Sol. (*b*) $\pi r\left(l + \frac{r}{4}\right)$

Total surface area of the cone = πR (L + R), where L is slant height and R is radius.

$$\therefore \quad \text{TSA} = \pi \times \frac{r}{2}\left(2l + \frac{r}{2}\right)$$

$$= \pi r\left(l + \frac{r}{4}\right)$$

3. A heap of paddy seeds is in the form of a cone. The radius of the heap is 4.2 m and height is 5.6 m. To protect heap from rain, it is to covered with waterproof tarpaulin sheet. If the rate of tarpaulin sheet is ₹ 5 per m^2, what is the total cost of the sheet needed to cover the heap?

(*a*) ₹ 92.4 (*b*) ₹ 462

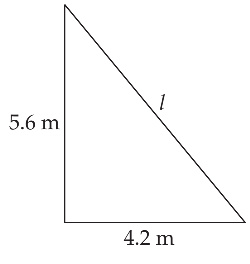
(*c*) ₹ 924 (*d*) ₹ 1386

Sol. (*b*) ₹ 462

Radius of the heap = 4.2 m

Height of the heap = 5.6 m

Slant height of the heap = l



$$l = \sqrt{(4.2)^2 + (5.6)^2}\text{ m}$$

$$= \sqrt{17.64 + 31.36}\text{ m}$$

$$= \sqrt{49}\text{ m} = 7\text{ m}$$

Area of tarpaulin sheet needed = Curved surface area of the heap

$$= \pi rl$$

$$= \frac{22}{7} \times 4.2\text{ m} \times 7\text{ m} = 92.4\text{ m}^2$$

Total cost of the sheet needed = ₹ 5 × 92.4

= ₹ 462

4. If the surface areas of two spheres are in the ratio 4 : 9, then their radii are in the ratio

(*a*) 4 : 9 (*b*) 3 : 2

(*c*) 2 : 3 (*d*) None of these

Sol. (*c*) 2 : 3

Surface area of the sphere = $4\pi r^2$

Let surface area of the first sphere of radius r_1 be s_1 and surface area of the second sphere of radius r_2 be s_2. Then,

$$\frac{s_1}{s_2} = \frac{4\pi r_1^2}{4\pi r_2^2} = \frac{4}{9}$$

$$\Rightarrow \quad \frac{r_1^2}{r_2^2} = \frac{4}{9}$$

$$\Rightarrow \quad \frac{r_1}{r_2} = \frac{2}{3}$$

5. A hemispherical bowl made of iron has inner radius of 6.3 cm. The inner surface of the bowl is to be nickel-plated. Find the cost of nickel-plating at the rate of ₹ 5 per cm^2.

(*a*) ₹ 12.474 (*b*) ₹ 124.74

(*c*) ₹ 1247.4 (*d*) ₹ 12474

Sol. (*c*) ₹ 1247.4

Inner surface area of the bowl = $2\pi r^2$

$$= 2 \times \frac{22}{7} \times 6.3\text{ cm} \times 6.3\text{ cm}$$

$$= 249.48\text{ cm}^2$$

Cost of nickel-plating = ₹ 5 × 249.48

= ₹ 1247.4

Very Short Answer Type Questions

6. Find the curved surface area and the total surface area of a solid cone whose base radius and height are 3 cm and 4 cm respectively. [Take π = 3.14]

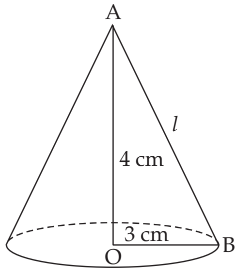
Sol. Slant height of the cone, $l = \sqrt{3^2 + 4^2}$ cm

$$l = 5\text{ cm}$$

Curved surface area = πrl

$$= 3.14 \times 3 \times 5\text{ cm}^2$$

$$= 47.1\text{ cm}^2$$



Total surface area $= \pi r(l + r)$

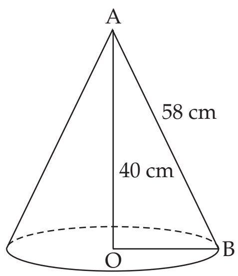
$= 3.14 \times 3 \times (5 + 3)\ cm^2$

$= 3.14 \times 3 \times 8\ cm^2 = 75.36\ cm^2$

7. Find the curved surface area and the total surface area of a solid cone whose slant height and height are respectively 58 cm and 40 cm.

Sol. Slant height = 58 cm

Height = 40 cm



Radius, OB = r

$$r = \sqrt{l^2 - h^2}$$

$$= \sqrt{(58)^2 - (40)^2}\ \text{cm}$$

$$= \sqrt{(58+40)(58-40)}\ \text{cm}$$

$$= \sqrt{98 \times 18}\ \text{cm}$$

$$= \sqrt{7 \times 7 \times 2 \times 2 \times 9}\ \text{cm}$$

$$= 7 \times 2 \times 3\ \text{cm}$$

$$= 42\ \text{cm}$$

$\Rightarrow$ $r = 42$ cm

Curved surface area of the solid cone $= \pi rl$

$$= \frac{22}{7} \times 42 \times 58\ \text{cm}^2$$

$$= 7656\ \text{cm}^2$$

Total surface area of the solid cone

$$= \pi r(l + r)$$

$$= \frac{22}{7} \times 42 \times (58 + 42)\ \text{cm}^2$$

$$= 22 \times 6 \times 100\ \text{cm}^2$$

$$= 13200\ \text{cm}^2$$

8. The circumference of the base of a right circular cylinder is 44 cm. If its whole surface area is 968 cm^2, then what is the sum of its height and the radius of its base?

Sol. Let r and h be the radius of the base and the height of the cylinder respectively.

Then $2\pi r = 44$ cm

$\Rightarrow$ $r = \dfrac{44 \times 7}{2 \times 22}$ cm = 7 cm

Now, whole surface area $= 2\pi r(h + r) = 968$

$\Rightarrow$ $2 \times \dfrac{22}{7} \times 7 \times (h + r) = 968$

$\Rightarrow$ $h + r = \dfrac{968}{44} = 22$

Hence, the required sum of radius and height is 22 cm.

9. If the radius of the base of a hemisphere is doubled, then find the ratio of the total surface area of the new hemisphere to that of the original hemisphere.

Sol. Let r be the radius of the original hemisphere and R be the radius of the new hemisphere.

Then $R = 2r$

Total surface area of the original hemisphere

$$= 3\pi r^2$$

and that of the new hemisphere

$$= 3\pi R^2$$

$$= 3\pi(2r)^2$$

$$= 12\pi r^2$$

$\therefore$ Required ratio $= \dfrac{12\pi r^2}{3\pi r^2} = \dfrac{4}{1} = 4 : 1$

10. If the total surface area of a hemisphere is 462 cm^2, find its diameter.

Sol. If r be the radius of the hemisphere, then its total surface area $= 3\pi r^2$

$\therefore$ $3\pi r^2 = 462\ cm^2$

$\Rightarrow$ $r^2 = \dfrac{462 \times 7}{3 \times 22}\ cm^2 = 49\ cm^2$

$\therefore$ $r = 7$ cm

$\therefore$ Required diameter $= 2r = 2 \times 7$ cm = 14 cm

11. The curved surface area of a cone is 4070 cm^2 and its diameter is 70 cm. What is its slant height?

Sol. Let r be the radius of the base and l be the slant height of the cone.

Then $r = \dfrac{70}{2}$ cm = 35 cm

$\therefore$ Curved surface area of the cone $= \pi rl$

Then $\pi rl = 4070$

$$\Rightarrow \quad \frac{22}{7} \times 35 \times l = 4070$$

$$\Rightarrow \quad l = \frac{4070}{22 \times 5} = 37$$

Hence, the required slant height is 37 cm.

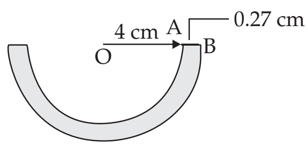
12. A hemispherical bowl is 0.27 cm thick. If its inner radius is 4 cm, find its outer surface area.

Sol. Inner radius = 4 cm

Thickness = 0.27 cm

Outer radius = (4 + 0.27) cm

= 4.27 cm



Outer surface area $= 2\pi r^2$

$$= 2 \times \frac{22}{7} \times 4.27 \times 4.27 \text{ cm}^2$$

$= 114.6 \text{ cm}^2$ (approx.)

Short Answer Type Questions

13. How many metres of cloth 1.1 m wide will be required to make a conical tent, whose vertical height is 12 m and base radius is 16 m? Also find the cost of cloth used at the rate of ₹ 14 per metre.

Sol. Let r and h be the radius of the base and the vertical height of a cone respectively and l be the slant height of the cone. Then $h = 12$ m, $r = 16$ m.

$$\therefore \quad l = \sqrt{h^2 + r^2}$$

$$= \sqrt{12^2 + 16^2} \text{ m}$$

$$= \sqrt{144 + 256} \text{ m}$$

$$= \sqrt{400} \text{ m}$$

$$= 20 \text{ m}$$

$\therefore$ Curved surface area of the cone

$$= \pi r l$$

$$= \frac{22}{7} \times 16 \times 20 \text{ m}^2$$

$$= \frac{352 \times 20}{7} \text{ m}^2$$

Area of the rectangular cloth = 1.1 L

where L is the length of the cloth.

$$\therefore \quad 1.1 \text{ L} = \frac{352 \times 20}{7}$$

$$\Rightarrow \quad \text{L} = \frac{352 \times 200}{7 \times 11}$$

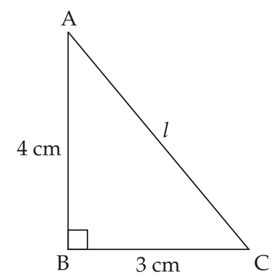
$$= \frac{6400}{7}$$

Hence, the required length of the cloth is $\frac{6400}{7}$ m.

Also, required cost of the cloth is ₹ 14 × $\frac{6400}{7}$, i.e. ₹ 12800.

14. A right-angled triangle in which the sides containing the right angle are 3 cm and 4 cm in length. It is turned around the longer side. Find the curved surface area of the solid, thus formed.

Sol.



$$AC^2 = AB^2 + BC^2$$

$$\Rightarrow \quad AC^2 = 16 \text{ cm}^2 + 9 \text{ cm}^2 = 25 \text{ cm}^2$$

$$\Rightarrow \quad AC = 5 \text{ cm}$$

Side AC is the slant height of the cone.

$$\therefore \quad l = 5 \text{ cm}$$

$$r = 3 \text{ cm}$$

Curved surface area $= \pi r l$

$$= \frac{22}{7} \times 3 \text{ cm} \times 5 \text{ cm}$$

$= 47.14 \text{ cm}^2$ (approx.)

15. If the total surface area of a solid hemisphere is equal to the surface area of a sphere, find the ratio of the square of the radius of the sphere and the square of the radius of the hemisphere.

Sol. Let the radius of the the solid hemisphere $= r_1$

and the radius of the sphere $= r_2$

Total surface area of the hemisphere $= 3\pi r_1^2$

Surface area of the sphere $= 4\pi r_2^2$

$4\pi r_2^2 = 3\pi r_1^2$ [Given]

$$\Rightarrow \quad \frac{r_2^2}{r_1^2} = \frac{3}{4}$$

16. Find the amount of water displaced by a solid spherical ball of diameter 5.6 cm when it is completely immersed in water.

Sol. Amount of water displaced = Volume of the spherical ball

Volume of the spherical ball $= \frac{4}{3} \pi r^3$

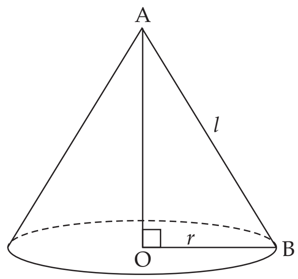
$$= \frac{4}{3}\times\frac{22}{7} \times 2.8 \times 2.8 \times 2.8 \text{ cm}^3$$

$$= 91.99 \text{ cm}^3 \text{ (approx.)}$$

Long Answer Type Questions

17. The radius and slant height of a cone are in ratio 4 : 7. Find its radius and slant height, if its surface area is 5632 cm^2.

Sol. Let radius of the cone = $4x$ and slant height = $7x$.



Given surface area = 5632 cm^2

$$\pi rl = 5632 \text{ cm}^2$$

$$\Rightarrow \frac{22}{7} \times 4x \times 7x = 5632 \text{ cm}^2$$

$$\Rightarrow x^2 = \frac{5632}{22\times 4} \text{ cm}^2 = 64 \text{ cm}^2$$

$$\Rightarrow x = 8 \text{ cm}$$

$\therefore$ Radius = 4 × 8 cm = 32 cm

Slant height = 7 × 8 cm = 56 cm

18. There are two cones. The curved surface area of one is twice that of the other. The slant height of the latter is twice that of the former. Find the ratio of their radii.

Sol. Let the surface area of one cone = s_1

Radius = r_1

Slant height = l_1

and

the surface area of the other cone = s_2

Radius = r_2

Slant height = l_2

$s_1 = 2s_2$ [Given] ...(1)

$l_2 = 2l_1$ [Given] ...(2)

$s_1 = \pi r_1 l_1$ ...(3)

$s_2 = \pi r_2 l_2 = \pi r_2 \times 2l_1$ ...(4)

From (1), (3) and (4)

$$\pi r_1 l_1 = 2 \times \pi r_2 2l_1$$

$$r_1 = 4r_2$$

$$\frac{r_1}{r_2} = \frac{4}{1}$$

19. The radius of a spherical balloon increases from 14 cm to 21 cm, as air is pumped into it. Find the ratio of the surface areas of the balloon in the two cases.

Sol. Initial radius of the spherical balloon = 14 cm

Surface area, $s_1 = 4\pi\ (14 \text{ cm})^2$ ...(1)

Increased radius of the spherical balloon = 21 cm

Surface area, $s_2 = 4\pi\ (21 \text{ cm})^2$ ...(2)

Ratio of the two surface areas

$$\frac{s_1}{s_2} = \frac{4\pi\left(14 \text{ cm}\right)^2}{4\pi\left(21 \text{ cm}\right)^2}$$

$$\frac{s_1}{s_2} = \frac{4}{9}$$

20. If the radius of a sphere is twice that of another sphere, find the ratio of the surface of the two spheres.

Sol. Let the radius of first sphere = r_1

Radius of another sphere = r_2

Given,

$r_1 = 2r_2$ ...(1)

$s_1 = 4\pi r_1^2$

$\Rightarrow$ $s_1 = 4 \times \pi \times (2r_2)^2 = 4 \times 4 \times \pi \times r_2^2$ ...(2)

$s_2 = 4\pi r_2^2$ ...(3)

$$\frac{s_1}{s_2} = \frac{4\times 4\times \pi\times r_2^2}{4\times \pi\times r_2^2}$$

[From (2) and (3)]

$$\Rightarrow \frac{s_1}{s_2} = \frac{4}{1}$$

21. If the ratio of the surface area of the earth and that of the moon is 16 : 1, find the ratio of the radius of the earth and that of the moon.

Sol. Let the radius of the earth and the moon be R_E and r_m respectively.

$$S_E = 4\pi R_E^2$$

$$S_m = 4\pi r_m^2$$

$$\frac{S_E}{S_m} = \frac{16}{1} \quad \text{[Given]}$$

$$\Rightarrow \frac{4\pi R_E^2}{4\pi r_m^2} = \frac{16}{1}$$

$$\Rightarrow \quad \frac{R_E^2}{r_m^2} = \frac{16}{1}$$

$$\Rightarrow \quad \frac{R_E}{r_m} = \frac{4}{1}$$

Check Your Progress 2

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Multiple-Choice Questions

1. The volume of a sphere is 288π cm^3. Then, its surface area is

(*a*) 72 cm^2 (*b*) 144π cm^2

(*c*) 288π cm^2 (*d*) None of these

Sol. (*b*) 144π cm^2

Given, volume of the sphere = 288π cm^3

$$\frac{4}{3}\pi r^3 = 288\pi \text{ cm}^3$$

[r = radius of the sphere]

$$\Rightarrow \quad r^3 = \frac{288 \times 3}{4} \text{ cm}^3$$

$$= 72 \times 3 \text{ cm}^3$$

$$= 3 \times 3 \times 2 \times 2 \times 2 \times 3 \text{ cm}^3$$

$$\Rightarrow \quad r = 6 \text{ cm}$$

$$\text{Surface area} = 4\pi r^2$$

$$= 4 \times \pi \times (6 \text{ cm})^2$$

$$= 144\pi \text{ cm}^2$$

2. The radius of a sphere is $3r$, then its volume will be

(*a*) $\frac{4}{3}\pi r^3$ (*b*) $12\pi r^3$

(*c*) $4\pi r^3$ (*d*) $36\pi r^3$

Sol. (*d*) $36\pi r^3$

Radius of the sphere = $3r$

$$\text{Volume of the sphere} = \frac{4}{3}\pi (3r)^3$$

$$= \frac{4}{3}\pi \times 3r \times 3r \times 3r$$

$$= 36\pi r^3$$

3. A solid cone with base radius of 2.8 cm and height 11.2 cm is melted. It is recast into a sphere. Find the radius of the sphere.

(*a*) 5.6 cm (*b*) 2.8 cm

(*c*) 1.4 cm (*d*) 11.2 cm

Sol. (*b*) 2.8 cm

Volume of the solid cone = Volume of the sphere

Let radius of sphere = r

$$\frac{4}{3}\pi r^3 = \frac{1}{3} \times \pi \times 2.8 \text{ cm} \times 2.8 \text{ cm} \times 11.2 \text{ cm}$$

$$\Rightarrow \quad r^3 = \frac{2.8 \text{ cm} \times 2.8 \text{ cm} \times 11.2 \text{ cm}}{4}$$

$$\Rightarrow \quad r^3 = 2.8 \text{ cm} \times 2.8 \text{ cm} \times 2.8 \text{ cm}$$

$$\Rightarrow \quad r = 2.8 \text{ cm}$$

4. If the ratio of the diameters of two spheres is 1 : 4, find the ratio of their volumes.

(*a*) 1 : 4 (*b*) 1 : 8

(*c*) 1 : 16 (*d*) 1 : 64

Sol. (*d*) 1 : 64

Let the diameter and radius of two spheres be d_1, r_1 and d_2, r_2 respectively.

$$\frac{d_1}{d_2} = \frac{1}{4} \quad \text{[Given]}$$

$$\frac{d_1/2}{d_2/2} = \frac{1}{4}$$

$$\Rightarrow \quad \frac{r_1}{r_2} = \frac{1}{4}$$

$$r_2 = 4r_1 \quad \text{...(1)}$$

Ratio of their volumes,

$$\frac{V_1}{V_2} = \frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \frac{r_1^3}{r_2^3} = \frac{r_1^3}{(4r_1)^3} = \frac{1}{64}$$

$$\Rightarrow \quad \frac{V_1}{V_2} = \frac{1}{64}$$

or, $\quad V_1 : V_2 = 1 : 64$

5. A hemispherical dome of a tomb needs to be painted. The circumference of the base of the dome is 17.6 cm. If the cost of painting is ₹ 7 per cm^2, what is the cost, rounded to the nearest rupee, to paint the dome? $\left(\text{Use } \pi = \frac{22}{7}\right)$

(*a*) ₹ 345 (*b*) ₹ 390

(*c*) ₹ 571 (*d*) ₹ 690

Sol. (*a*) ₹ 345

Let the radius of the dome = r

$$2\pi r = 17.6 \text{ cm}$$

$$\Rightarrow \quad r = 17.6 \times \frac{1}{2} \times \frac{7}{22} \text{ cm}$$

$$\Rightarrow \quad r = 2.8 \text{ cm}$$

$$\text{Area to be painted} = 2\pi r^2$$

$$= 2 \times \pi \times (2.8)^2 \text{ cm}^2$$

$$\text{Cost of painting} = ₹\, 7 \times 2 \times \frac{22}{7} \times (2.8)^2$$
$$= ₹\, 344.96$$
$$= ₹\, 345 \text{ (approx.)}$$

Very Short Answer Type Questions

6. If the volume of a sphere is numerically equal to its surface area then what is the diameter of the sphere?

Sol. If r units be the radius of the sphere, then

$$\frac{4}{3}\pi r^3 = 4\pi r^2$$
$$\Rightarrow \quad r = 3$$
$$\therefore \quad 2r = 6$$

Hence, the required diameter of the sphere is 6 units.

7. The radii of a solid sphere and a solid cone are respectively 3 cm and 2 cm. How many such spheres can be moulded to form such a cone, if the height of the cone is 54 cm?

Sol. Let r and R be the radii of the sphere and the cone respectively and let h be the height of the cone.

Then r = 3 cm, R = 2 cm and h = 54 cm.

Then the volume of the cone V_1

$$= \frac{1}{3}\pi R^2 h$$
$$= \frac{1}{3}\pi \times 4 \times 54 \text{ cm}^3$$
$$= 72\pi \text{ cm}^3$$

Also, the volume of each sphere $= \frac{4}{3}\pi r^3$

$$= \frac{4}{3} \times \pi \times 3^3 \text{ cm}^3$$
$$= 36\pi \text{ cm}^3$$

$\therefore$ Required number of spheres $= \frac{72\pi}{36\pi} = 2$

8. Find the mass of a metallic ball of diameter 6.3 cm, if the density of the metal is 8.5 g per cm^3.

Sol. Diameter of the metallic ball = 6.3 cm

Radius of the metallic ball $= \frac{6.3}{2}$ cm

Volume of the metallic ball $= \frac{4}{3}\pi r^3$

$$= \frac{4}{3}\pi\left(\frac{6.3}{2}\text{ cm}\right)^3$$

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

$$\Rightarrow \quad \text{Mass} = \text{Density} \times \text{Volume}$$

$$= 8.5 \text{ g/cm}^3 \times \frac{4}{3}\pi\left(\frac{6.3}{2}\text{ cm}\right)^3$$
$$= 8.5 \times \frac{4}{3} \times \frac{22}{7} \times \frac{6.3}{2} \times \frac{6.3}{2} \times \frac{6.3}{2} \text{ g}$$
$$= 1113.3 \text{ g (approx.)}$$

9. The ratio of the volumes of two spheres is 1 : 125. Find the ratio of their radii.

Sol. Let the radius of the spheres be r_1 and r_2 respectively.

$$\frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \frac{1}{125}$$

$$\Rightarrow \quad \frac{r_1^3}{r_2^3} = \frac{(1)^3}{(5)^3}$$

$$\Rightarrow \quad \frac{r_1}{r_2} = \frac{1}{5}$$

10. A girl playing with clay forms a spherical ball of radius 6 cm. After some time, she recasts the spherical ball into a cone of the same radius. Find the height of the new cone formed.

Sol. Volume of the clay in the form of a spherical ball

$$= \frac{4}{3}\pi r^3$$
$$= \frac{4}{3}\pi (6)^3 \text{ cm}^3$$

Let the height of the cone be h

Volume of the cone $= \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3}\pi (6)^2 \text{ cm}^2 \times h$$

Volume of the cone = Volume of the sphere

$$\frac{1}{3}\pi (6)^2 \text{ cm}^2 \times h = \frac{4}{3}\pi (6)^3 \text{ cm}^3$$

$$\Rightarrow \quad h = 4 \times 6 \text{ cm}$$
$$\Rightarrow \quad h = 24 \text{ cm}$$

Short Answer Type Questions

11. A shopkeeper has one spherical ladoo of radius 10 cm. With the same amount of material, how many ladoos of radius 2 cm can be made?

Sol. Volume of ladoo of radius 10 cm $= \frac{4}{3}\pi (10 \text{ cm})^3$

Volume of ladoo of raidus 2 cm $= \frac{4}{3}\pi (2 \text{ cm})^3$

Let the number of ladoos made be n

$\therefore$ n × Volume of each ladoo of radius 2 cm

= Volume of ladoo of radius 10 cm

$$\Rightarrow n \times \frac{4}{3}\pi \times (2 \text{ cm})^3 = \frac{4}{3}\pi\,(10 \text{ cm})^3$$

$$\Rightarrow \qquad n = \frac{10\times10\times10}{2\times2\times2}$$

$$\therefore \qquad n = 125$$

12. The surface area of a sphere is 221.76 cm^2. Find the base radius and volume of the sphere.

Sol. Let 'r' be the radius of the sphere.

$$4\pi r^2 = 221.76 \text{ cm}^2$$

$$\Rightarrow \qquad r^2 = \frac{1}{4}\times\frac{7}{22} \times 221.76 \text{ cm}^2$$

$$\Rightarrow \qquad r^2 = 17.64 \text{ cm}^2$$

$$\therefore \qquad r = 4.2 \text{ cm}$$

Volume of the sphere

$$= \frac{4}{3} \times \pi \times r^3$$

$$= \frac{4}{3}\times\frac{22}{7} \times 4.2 \times 4.2 \times 4.2 \text{ cm}^3$$

$\therefore$ Volume of the sphere = 310.5 cm^3 (approx.)

13. A conical vessel of base radius 9 cm and height 20 cm is full of water. A part of this water is now poured into a hollow sphere, till the sphere is completely filled with water. If the radius of the sphere is 4.2 cm, find the volume of water which is left in the cone. (Take $\pi = 22/7$)

Sol. Let R and r be the radii of the bases of the cone and sphere respectively and H be the corresponding vertical height of the cone.

Then R = 9 cm, H = 20 cm and r = 4.2 cm

$$\therefore \text{ Volume } V_1 \text{ of the cone} = \frac{1}{3}\pi R^2 H$$

$$= \frac{1}{3} \times \pi \times 9^2 \times 20 \text{ cm}^3$$

$$= 540\pi \text{ cm}^3$$

$$\text{Volume } V_2 \text{ of the sphere} = \frac{4}{3}\pi r^3$$

$$= \frac{4}{3}\pi \times (4.2)^3 \text{ cm}^3$$

$$= 98.8\pi \text{ cm}^3 \text{ (approx.)}$$

$\therefore$ Required volume of water left in the cone

$$= V_1 - V_2$$

$$= (540 - 98.8)\pi \text{ cm}^3$$

$$= 441.2 \times \frac{22}{7} \text{ cm}^3$$

$$= 1386.6 \text{ cm}^3 \text{ (approx.)}$$

14. The mass of a spherical ball of radius 2 cm is 8 kg. Find the mass of a spherical shell of the same material whose inner and outer radii are 4 cm and 5 cm respectively.

Sol. Let r, r_1, and r_2 be the radii of the spherical ball, the inner shell and the outer shell respectively, where $r_2 > r_1$. Then r = 2 cm, r_1 = 4 cm and r_2 = 5 cm.

$$\text{Then the volume of the sphere} = \frac{4}{3}\pi r^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times 8 \text{ cm}^3$$

Hence, the density, d of the ball

$$= \frac{\text{mass}}{\text{volume}}$$

$$= \frac{8000 \times 3 \times 7}{4 \times 22 \times 8} \text{ g/cm}^3$$

$$= \frac{21 \times 125}{11} \text{ g/cm}^3$$

Now, volume of the spherical shell

$$= \frac{4}{3}\pi\left(r_2^3 - r_1^3\right)$$

$$= \frac{4}{3} \times \frac{22}{7} \times (5^3 - 4^3) \text{ cm}^3$$

$$= \frac{4 \times 22 \times (125 - 64)}{21} \text{ cm}^3$$

$$= \frac{88 \times 61}{21} \text{ cm}^3$$

$\therefore$ Required mass of the spherical shell

= Its volume × Its density

$$= \frac{88 \times 61}{21} \times \frac{21 \times 125}{11} \text{ g}$$

= 61000 g = 61 kg

Long Answer Type Questions

15. The inner part of the hemispherical dome of a building is whitewashed at a total cost of ₹ 3929.31 and at the rate of ₹ 15.75 per m^2. Find the inner radius of the dome and the volume of air inside the dome.

Sol. Let 'r' be the inner radius of the dome.

Then inner surface area to be whitewashed

$$= 2\pi r^2$$

Total cost of whitewashing

= Area × Rate

= $2\pi r^2$ × ₹ 15.75 per m^2

But, total cost = ₹ 3929.31

$\therefore$ $2\pi r^2 \times 15.75/\text{m}^2$ = ₹ 3929.31

$$\Rightarrow \qquad r^2 = 3929.31 \times \frac{1}{2}\times\frac{7}{22}\times\frac{1}{15.75} \text{ m}^2$$

$$\Rightarrow \qquad r^2 = 39.69 \text{ m}^2$$

$\therefore \quad r = 6.3$ m

Volume of air inside the dome

$= $ Volume of the dome

$$= \frac{2}{3}\pi r^3$$

$$= \frac{2}{3}\times\frac{22}{7} \times 6.3 \times 6.3 \times 6.3 \text{ m}^3$$

$= 523.9$ m^3 (approx.)

16. A sphere and a right circular cylinder of the same radius have equal volumes. By what percentage does the diameter of the cylinder exceed its height?

Sol. Let 'r' be the radius of the sphere and the cylinder. Let 'H' be the height of the cylinder.

Volume of the sphere = Volume of the cylinder

$$\frac{4}{3}\pi r^3 = \pi r^2 \text{H}$$

$$\Rightarrow \quad \text{H} = \frac{4}{3}r$$

Diameter of the cylinder = $2r$

Difference between Diameter and Height

$$= 2r - \frac{4}{3}r$$

$$= \frac{6r - 4r}{3} = \frac{2r}{3}$$

Percentage by which diameter exceeds the height

$$= \frac{\frac{2r}{3}}{2r}\times 100\%$$

$= 33.33\%$

17. The volume of two spheres are in the ratio 8 : 27. Find the ratio of their surface areas.

Sol. Let r_1 and r_2 be the radii of the two spheres respectively. Let V_1 and V_2 be their volumes

$$V_1 = \frac{4}{3}\pi r_1^3$$

$$V_2 = \frac{4}{3}\pi r_2^3$$

Then, according to the question,

$$\frac{V_1}{V_2} = \frac{8}{27}$$

$$\Rightarrow \quad \frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \frac{8}{27}$$

$$\Rightarrow \quad \frac{r_1^3}{r_2^3} = \frac{8}{27}$$

$$\Rightarrow \quad \frac{r_1}{r_2} = \frac{2}{3}$$

Let $r_1 = 2k$ and $r_2 = 3k$

Ratio of their surface areas, s_1 and s_2

$$\frac{s_1}{s_2} = \frac{4\pi r_1^2}{4\pi r_2^2} = \frac{r_1^2}{r_2^2} = \frac{4k^2}{9k^2}$$

$$\Rightarrow \quad \frac{s_1}{s_2} = \frac{4}{9}$$

18. 64 solid iron spheres each of radius r and surface area S are melted to form a new sphere with surface area S′ and radius R. Show that

(*a*) R = $4r$, and (*b*) S′ = 16S.

Sol. (*a*) Volume of the sphere with radius r

$$= \frac{4}{3}\pi r^3$$

Volume of 64 such iron spheres

$$= 64 \times \frac{4}{3}\pi r^3$$

Volume of the new sphere $= \frac{4}{3}\pi \text{R}^3$

$$\frac{4}{3}\pi \text{R}^3 = 64 \times \frac{4}{3}\pi r^3$$

$$\Rightarrow \quad \text{R}^3 = 64r^3$$

$$\therefore \quad \text{R} = 4r$$

(*b*) Surface area of the new sphere = S′

$$\text{S}' = 4\pi \text{R}^2$$

$$\Rightarrow \quad = 4\pi (4r)^2$$

$$= 4 \times 16\pi r^2$$

$$\Rightarrow \quad \text{S}' = 16 \times 4\pi r^2$$

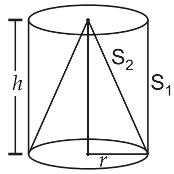
$$\therefore \quad \text{S}' = 16\text{S} \qquad [\text{S} = 4\pi r^2]$$

Higher Order Thinking Skills (HOTS) Questions

(Page 162)

1. A right circular cylinder and a right circular cone have equal bases and equal heights. If their curved surfaces are in the ratio 8 : 5, show that the ratio of the base to the height is 3 : 4 for each of them.

Sol. Let r and h be the common radius of the bases and the height respectively of the cylinder and the cone and let S_1 and S_2 be the curved surface areas of the cylinder and the cone respectively. Let l be the slant height of the cone.



Then $l = \sqrt{r^2 + h^2}$, $S_1 = 2\pi rh$

and $S_2 = \pi rl = \pi r\sqrt{r^2 + h^2}$

It is given that

$$\frac{S_1}{S_2} = \frac{8}{5}$$

$$\therefore \quad \frac{2\pi rh}{\pi r\sqrt{r^2 + h^2}} = \frac{8}{5}$$

$$\Rightarrow \quad \frac{h}{\sqrt{r^2 + h^2}} = \frac{4}{5}$$

$$\Rightarrow \quad \frac{h^2}{r^2 + h^2} = \frac{16}{25}$$

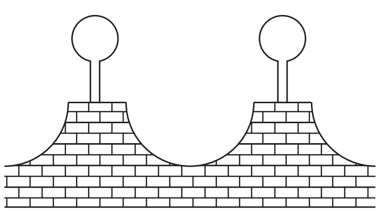
$$\Rightarrow \quad 25h^2 = 16r^2 + 16h^2$$

$$\Rightarrow \quad 9h^2 = 16r^2$$

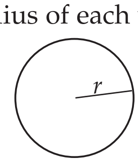
$$\Rightarrow \quad \frac{r}{h} = \frac{3}{4}$$

Hence, the ratio of the base to the height is 3 : 4.

2. The front compound wall of a house is decorated by wooden spheres of diameter 21 cm, placed on small supports as shown in the following figure. Eight such spheres are used for this purpose and are to be painted silver. Find the cost of paint required if silver paint costs 25 paise per cm^2.



Sol. Let r be the radius of each wooden sphere.



$\therefore$ Surface area of each sphere

$$= 4\pi r^2 \text{ cm}^2$$

$$= 4 \times \frac{22}{7} \times (10.5 \text{ cm})^2$$

$$= 1386 \text{ cm}^2$$

$\therefore$ Total surface area of 8 such spheres

$$= 8 \times 1386 \text{ cm}^2$$

$$= 11088 \text{ cm}^2$$

$\therefore$ The cost of the silver paint

$$= ₹\,0.25 \times 11088$$

$$= ₹\,2772$$

Self-Assessment

(Page 162)

Multiple-Choice Questions

1. The total surface area of a cone whose radius is $\frac{r}{2}$ and slant height is $2l$ is

(*a*) $2\pi rl$ (*b*) $\pi r(l + r)$

(*c*) $\pi r\left(l + \frac{r}{4}\right)$ (*d*) $2\pi r(l + r)$

Sol. (*c*) $\pi r\left(l + \frac{r}{4}\right)$

Let R be the radius of the base of the cone and L be its slant height. Then $R = \frac{r}{2}$ and $L = 2l$.

$\therefore$ Total surface area of the cone

$$= \pi R^2 + \pi RL$$

$$= \pi \times \frac{r^2}{4} + \pi \times \frac{r}{2} \times 2l$$

$$= \pi r\left(l + \frac{r}{4}\right)$$

2. If the slant height and the vertical height of a cone are 5 cm and 4 cm respectively, then the volume of the cone is

(*a*) 12π cm^3 (*b*) 13π cm^3

(*c*) 14π cm^3 (*d*) 15π cm^3

Sol. (*a*) 12π cm^3

Let r be the radius of the base, h be the vertical height and l, the slant height of the cone.

Then $l = 5$ cm and $h = 4$ cm.

$$\text{Then} \quad l = \sqrt{r^2 + h^2} = \sqrt{r^2 + 16}$$

$$\Rightarrow \quad 5^2 = r^2 + 16$$

$$\Rightarrow \quad r^2 = 9$$

$$\Rightarrow \quad r = 3$$

$\therefore$ Required volume of the cone $= \frac{h}{3}\pi r^2$

$$= \frac{4}{3} \times 9\pi \text{ cm}^3$$

$$= 12\pi \text{ cm}^3$$

3. Two right circular cones C_1 and C_2 have the same volume. If their base radii are in the ratio 4 : 3, then the heights of C_1 and C_2 are in the ratio

(*a*) 9 : 16 (*b*) 16 : 9

(*c*) 3 : 4 (*d*) 4 : 3

Sol. (*a*) 9 : 16

Let h_1 and h_2 be the heights of the cones C_1 and C_2 respectively.

Let, the radii of cones C_1 and C_2 are $4x$ and $3x$ respectively.

$$\text{Volume of } C_1 = \frac{1}{3}\pi r_1^2 h_1 = \frac{1}{3}\pi(4x)^2 h_1$$

$$\text{Volume of } C_2 = \frac{1}{3}\pi r_2^2 h_2 = \frac{1}{3}\pi(3x)^2 h_2$$

According to the question,

Volume of C_1 = Volume of C_2

$$\Rightarrow \quad \frac{1}{3}\pi(4x)^2 h_1 = \frac{1}{3}\pi(3x)^2 h_2$$

$$\therefore \quad \frac{h_1}{h_2} = \frac{9}{16}$$

4. The radius of a hemispherical balloon increases from 6 cm to 12 cm as air is being pumped into it. The ratio of the surface areas of the balloon in the two cases is

(*a*) 1 : 3 (*b*) 1 : 4

(*c*) 2 : 3 (*d*) 2 : 1

Sol. (*b*) 1 : 4

Surface area of the hemispherical balloon = $4\pi r^2$

Let surface area of the balloon in the two cases be S_6 and S_{12} respectively.

$$S_6 = 4\pi r^2 = 4\pi \times 6 \text{ cm} \times 6 \text{ cm}$$

$$S_{12} = 4\pi R^2 = 4\pi \times 12 \text{ cm} \times 12 \text{ cm}$$

$$\frac{S_6}{S_{12}} = \frac{4\pi \times 6 \text{ cm} \times 6 \text{ cm}}{4\pi \times 12 \text{ cm} \times 12 \text{ cm}}$$

$$\therefore \quad \frac{S_6}{S_{12}} = \frac{1}{4}$$

5. Given that the surface area of a spherical shot-put is 616 cm^2, its diameter is

(*a*) 12 cm (*b*) 14 cm

(*c*) 16 cm (*d*) 18 cm

Sol. (*b*) 14 cm

Surface area of shot-put = 616 cm^2

Let r be the radius of the shot-put.

$$4\pi r^2 = 616 \text{ cm}^2$$

$$\Rightarrow \quad r^2 = \frac{1}{4} \times \frac{7}{22} \times 616 \text{ cm}^2$$

$$\Rightarrow \quad r^2 = 7 \times 7 \text{ cm}$$

$$\therefore \quad r = 7 \text{ cm}$$

Diameter of the shot-put = $2r$ = 14 cm.

Fill in the Blanks

6. The surface area of a sphere of radius 3.5 cm is **154 cm²**.

Sol. S.A. of a sphere = $4\pi r^2$

$$= 4 \times \frac{22}{7} \times 3.5 \times 3.5$$

$$= 4 \times 22 \times 0.5 \times 3.5$$

$$= 154 \text{ cm}^2$$

7. The total surface area of a cone of radius $2r$ and slant height $\frac{1}{2}$ is $\boldsymbol{\pi r(l + 4r)}$.

Sol. Let $R = 2r$ and $L = \frac{l}{2}$

$$\text{T.S.A. of a cone} = \pi R(L + R) = \pi(2r)\left(\frac{l}{2} + 2r\right) = 2\pi r\left(\frac{l + 4r}{2}\right) = \pi r(l + 4r)$$

8. The volume of a right circular cone is equal to $\frac{1}{3}$ of the volume of a **cylinder**.

9. If V_1 be the volume of a cone whose height is equal to its base diameter and V_2 be the volume of a hemisphere whose base radius is equal to that of the cone, then $V_1 : V_2$ = **1 : 1**.

Sol. Let r and h be the radius and height of the cone respectively.

Then $\quad h = 2r \quad$ ...(1)

Volume V_1 of the cone = $\frac{1}{3}\pi r^2 h$

$$\Rightarrow \quad V_1 = \frac{1}{3}\pi r^2 \times 2r \quad \text{[From (1)]}$$

$$\Rightarrow \quad V_1 = \frac{2}{3}\pi r^3 \quad ...(2)$$

Volume of the hemisphere = V_2

$$V_2 = \frac{2}{3}\pi r^3 \quad ...(3)$$

$$\frac{V_1}{V_2} = \frac{\frac{2}{3}\pi r^3}{\frac{2}{3}\pi r^3} \quad \text{[From (2) and (3)]}$$

$$\therefore \quad V_1 : V_2 = 1 : 1$$

Assertion-Reason Type Questions

Directions (Q. Nos. 10 to 13): Each of these questions contains an assertion followed by reason. Read them carefully, and answer the question on the basis of the following options, select the one that best describes the two statements.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

10. Assertion (A): A sphere is cut into two halves and volume of each will be exactly half of the original.

Reason (R): Total volume of the sphere remains same.

Sol. (*a*)

Both assertion and reason are correct and reason is correct explanation of assertion.

11. Assertion (A): The volume of a cone can be found by using the formula, $V = \frac{1}{3}\pi r^2 h$, where r is the base radius and h is the height of the cone.

Reason (R): The volume of a cone is one-third of the volume of the cylinder.

Sol. (*a*)

Both assertion and reason are true and reason is the correct explanation of assertion.

12. Assertion (A): For any right-circular cone of radius r, height h and slant height l, the slant height l is always the largest numerically.

Reason (R): The slant height l of a right-circular cone can be expressed in terms of height, h and the base radius, r as $l = \sqrt{h^2 + r^2}$.

Sol. (*a*)

Both assertion and reason are true and reason is the correct explanation of assertion.

13. Assertion (A): The surface area of a sphere is doubled if its radius is doubled.

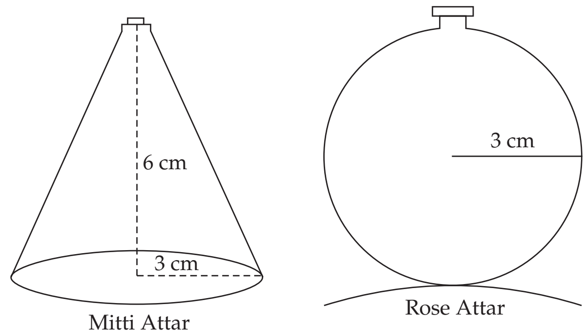
Reason (R): The surface area of the sphere is $4\pi r^2$.

Sol. (*d*)

Assertion is not correct since if its radius is doubled, then surface area will become four times, but reason is correct.

Case Study Based Questions

14. Kannauj is also known as the perfume capital of India. The city has been producing attar for over 400 years. The attar is packed in glass bottles of different shapes. Altmas Perfumery sells its two known brands of attar in two different types of glass bottles. The cost of the bottle is based on the quantity and type of attar it contains.



(*a*) Find the volume of mitti attar in the bottle.

Ans. Volume of mitti attar in the bottle

$$= \frac{1}{3} \times \pi \times r^2 \times h$$

$$= \frac{1}{3} \times \frac{22}{7} \times 3 \text{ cm} \times 3 \text{ cm} \times 6 \text{ cm}$$

$$= 56.57 \text{ cm}^3 \text{ (approx.)}$$

(*b*) Find the volume of rose attar in the bottle.

Ans. Volume of rose attar in the bottle

$$= \frac{4}{3}\pi \times r^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times 6 \text{ cm} \times 6 \text{ cm} \times 6 \text{ cm}$$

$$= 905.14 \text{ cm}^3 \text{ (approx.)}$$

(*c*) (*i*) If both the bottles cost the same, which fragrance is the most expensive per unit volume?

Ans. Mitti attar, as the volume is less.

or

(*ii*) Which bottle has a larger surface area?

Ans. Surface area of mitti attar bottle

$$= \pi r l + \pi r^2$$

$$l = \sqrt{(6)^2 + (3)^2} \text{ cm}$$

$$l = \sqrt{36 + 9} \text{ cm}$$
$$\Rightarrow \quad l = \sqrt{45} \text{ cm}$$
$$\Rightarrow \quad l = 3\sqrt{5} \text{ cm}$$
$$\Rightarrow \quad l = 3 \times 2.24 \text{ cm} = 6.72 \text{ cm}$$

$\therefore$ Surface area of mitti attar bottle

$$= \pi \times 3 \times 6.72 \text{ cm}^2 + \pi \times 3 \times 3 \text{ cm}^2$$
$$= \pi\,(20.16 + 9) \text{ cm}^2 = 29.16\pi \text{ cm}^2 \quad ...(1)$$

Surface area of rose attar bottle

$$= 4\pi r^2$$
$$= 4 \times \pi \times 3 \text{ cm} \times 3 \text{ cm}$$
$$= 36\pi \text{ cm}^2 \quad ...(2)$$

Therefore, from (1) and (2) surface area of rose attar bottle is greater.

15. On a hot summer day, Sheila set-up a stall near her house in the village under the shade of a tree. She took cold water in a completely full spherical matka (an earthen pot) of radius 39 cm and served this water to thirsty people passing by in conical glasses each of radius 4 cm and height 13 cm.

Based on the above information, answer the following questions.

(*a*) What is the volume of the spherical matka?

Ans. $79092\pi \text{ cm}^3$

(*b*) What is the surface area of the matka?

Ans. $6084\pi \text{ cm}^2$

(*c*) (*i*) What is the volume of each conical glass?

Ans. $\frac{208}{3}\pi \text{ cm}^3$

or

(*ii*) If she fills each glass up to three-fourths of its height with water, find the number of people she can give water to.

Ans. 1521

Very Short Answer Type Questions

16. The circumference of the base of a 27 m high solid cone is 44 m. Find the volume of the cone.

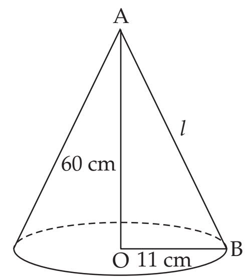
Sol. Let r be the radius of the base of the cone and h be its vertical height.

$$\therefore \quad h = 27 \text{ m}$$
$$\text{Also,} \quad 2\pi r = 44 \text{ m}$$
$$\Rightarrow \quad r = \frac{22}{\pi} \text{ m} \quad \ldots(1)$$

$$\therefore \text{ Required volume} = \frac{1}{3}\pi r^2 h$$
$$= \frac{1}{3} \times \pi \times \frac{22 \times 22}{\pi \times \pi} \times 27 \text{ m}^3$$
$$= \frac{22 \times 22 \times 9 \times 7}{22} \text{ m}^3$$
$$= 1386 \text{ m}^3$$

17. Find the curved surface area and the total surface area of a solid cone whose base diameter is 22 cm and height is 60 cm.

Sol. Let l be the slant height of the cone.



$$l = \sqrt{h^2 + r^2}$$
$$= \sqrt{(60)^2 + (11)^2} \text{ cm}$$
$$= \sqrt{3721} \text{ cm}$$
$$= 61 \text{ cm}$$
$$\Rightarrow \quad l = 61 \text{ cm}$$

Curved surface area of the cone

$$= \pi r l$$
$$= \pi \times 11 \text{ cm} \times 61 \text{ cm}$$
$$= 671\pi \text{ cm}^2$$

Area of the base $= \pi r^2$

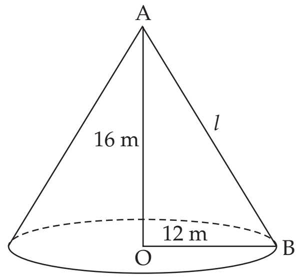
$$= \pi \times 11 \times 11 \text{ cm}^2$$
$$= \pi \times 121 \text{ cm}^2$$
$$= 121\pi \text{ cm}^2$$

Total surface area of the cone

$$= 671\pi \text{ cm}^2 + 121\pi \text{ cm}^2$$
$$= 792\pi \text{ cm}^2$$

18. A tent is in the shape of a cone with a base radius of 12 m and height 16 m. Find the cost of canvas required to make the tent, if the cost of canvas is ₹ 100 per m^2.

Sol.



Slant height of the conical tent, l

$$= \sqrt{(16)^2 + (12)^2} \text{ m}$$

$$= \sqrt{400} \text{ m}$$

$\Rightarrow \quad l = 20$ m

Canvas required = Curved surface area of the tent

$= \pi rl$

$= \pi \times 12 \text{ m} \times 20 \text{ m}$

$= 240\pi \text{ m}^2$

Cost of canvas $=$ ₹ $100 \times 240 \times 3.14$

$=$ ₹ 75360

19. The diameters of two cones are in ratio 5 : 4. If their slant heights are in ratio 3 : 8, find the ratio of their curved surfaces.

Sol. Let diameters of the two cones be $5d$ and $4d$ and slant heights be $3l$ and $8l$ respectively.

Let c_1 and c_2 be the curved surfaces of the two cones.

$$\frac{c_1}{c_2} = \frac{\pi \frac{5d}{2} \times 3l}{\pi \frac{4d}{2} \times 8l} = \frac{5 \times 3}{4 \times 8} = \frac{15}{32}$$

$\therefore \quad c_1 : c_2 = 15 : 32$

20. In a conical tent of base radius 7 m and height 12 m, how many bags of wheat can be emptied if volume of each bag is 3.5 m^3.

Sol. Volume of the conical tent

$$= \frac{1}{3} \pi \times r^2 \times h$$

$$= \frac{1}{3} \times \frac{22}{7} \times 7 \text{ m} \times 7 \text{ m} \times 12 \text{ m}$$

$$= 22 \times 7 \times 4 \text{ m}^3$$

Number of bags that can be emptied

$$= \frac{22 \times 7 \times 4}{3.5}$$

$$= 176$$

21. The height of a cone is 30 cm. Find the diameter of the base of the cone, it its volume is 1540 cm^3.

Sol. Height of the cone = 30 cm

Let radius of the cone = r

Volume of the cone $= \frac{1}{3} \times \pi \times r^2 \times h$

$$\Rightarrow \quad \frac{1}{3} \times \pi \times r^2 \times h = 1540 \text{ cm}^3$$

$$\Rightarrow \quad r^2 = 1540 \times 3 \times \frac{7}{22} \times \frac{1}{30} \text{ cm}^2$$

$$r^2 = 7 \text{ cm} \times 7 \text{ cm}$$

$$\Rightarrow \quad r = 7 \text{ cm}$$

$\therefore$ Diameter of the base = $2r = 2 \times 7$ cm = 14 cm.

22. The circumference of the edge of a hemispherical bowl is 132 cm. Find the volume of the bowl.

Sol. Circumference = 132 cm

Let r be the radius of the hemispherical bowl.

$$2\pi r = 132 \text{ cm}$$

$$\Rightarrow \quad r = \frac{1}{2} \times \frac{7}{22} \times 132 \text{ cm}$$

$$\Rightarrow \quad r = 21 \text{ cm}$$

Volume of the bowl

$$= \frac{2}{3} \pi r^3$$

$$= \frac{2}{3} \times \frac{22}{7} \times 21 \text{ cm} \times 21 \text{ cm} \times 21 \text{ cm}$$

$$= 19404 \text{ cm}^3$$

23. The volumes and diameters of a sphere and a cone are equal. Prove that the diameter of the sphere is half the height of the cone.

Sol. Let d be the diameters of the sphere and the cone respectively and H be the height of the cone.

Volume of the sphere $= \frac{4}{3} \pi r^3$

$$= \frac{4}{3} \pi \left(\frac{d}{2}\right)^3$$

Volume of the cone $= \frac{1}{3} \pi r^2 \text{ H}$

$$= \frac{1}{3} \pi \left(\frac{d}{2}\right)^2 \text{H}$$

According to the question,

Volume of the sphere = Volume of the cone

$$\Rightarrow \quad \frac{4}{3} \pi \left(\frac{d}{2}\right)^3 = \frac{1}{3} \pi \left(\frac{d}{2}\right)^2 \text{H}$$

$$\Rightarrow \quad 4 \times \frac{d}{2} = \text{H}$$

$$\Rightarrow \quad 2d = \text{H}$$

$$\therefore \quad d = \frac{\text{H}}{2}$$

Hence, proved.

Short Answer Type Questions

24. If the height of a right circular cone is double its diameter, then what is the ratio of its slant height to its radius?

Sol. Let r be the radius of the base and h be the vertical height of the cone. Given that $h = 4r$.

Let l be the slant height of the cone.

Then $\quad l = \sqrt{h^2 + r^2}$

$= \sqrt{16r^2 + r^2}$

$= \sqrt{17}\, r$

$\therefore \; l : r = \sqrt{17} : 1$ which is the required ratio.

25. Find the volume of a sphere whose surface area is 616 cm^2.

Sol. Let r be the radius of the sphere.

Then its surface area $= 4\pi r^2$

$\therefore \quad 4\pi r^2 = 616 \text{ cm}^2$ [Given]

$\Rightarrow \quad r^2 = \dfrac{616 \times 7}{4 \times 22} \text{ cm}^2 = 49 \text{ cm}^2$

$\therefore \quad r = 7$ cm

$\therefore$ Required volume of the sphere

$$= \frac{4}{3}\pi r^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times 7 \times 7 \times 7 \text{ cm}^3$$

$$= \frac{4312}{3} \text{ cm}^3$$

26. Two solid spheres made of the same metal have weights 5920 g and 740 g respectively. Find the radius of the larger sphere, if the diameter of the smaller one is 5 cm.

Sol. Let r be the radius of larger sphere.

Since both the solid spheres are made of the same metal, their density will be same.

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

Volume of the larger sphere $= \dfrac{4}{3}\pi r^3$

$$\text{Density} = \frac{5920 \text{ g}}{\frac{4}{3}\pi r^3} \qquad \text{...(1)}$$

Volume of the smaller sphere

$$= \frac{4}{3}\pi \times \left(\frac{5}{2}\right)^3 \text{ cm}^3$$

$$\text{Density} = \frac{740 \text{ g}}{\frac{4}{3}\pi \times \left(\frac{5}{2}\right)^3 \text{ cm}^3} \qquad \text{...(2)}$$

From (1) and (2)

$$\frac{740}{\frac{4}{3}\pi \times \left(\frac{5}{2}\right)^3} = \frac{5920}{\frac{4}{3}\pi r^3}$$

$\Rightarrow \quad r^3 = \dfrac{5920}{740} \times \left(\dfrac{5}{2}\right)^3$

$\Rightarrow \quad r^3 = 8 \times \left(\dfrac{5}{2}\right)^3$

$\Rightarrow \quad r = 2 \times \dfrac{5}{2}$

$\therefore \quad r = 5$ cm

27. How many spherical bullets can be made out of a solid cuboid of lead of volume 1584 cm^3 each bullet being 2 cm in diameter?

Sol. Volume of the solid cuboid = 1584 cm^3

Volume of each spherical bullet $= \dfrac{4}{3} \times \pi \times 1 \text{ cm}^3$

Let the number of bullets made be n.

Then,

$n \times \dfrac{4}{3} \times \pi \times 1 \text{ cm}^3 = 1584 \text{ cm}^3$

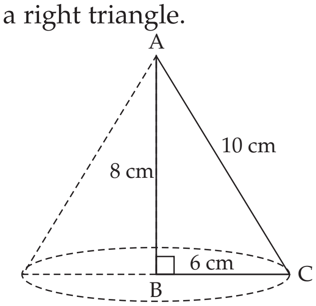
$\Rightarrow \quad n = \dfrac{3}{4} \times \dfrac{7}{22} \times 1584$

$\Rightarrow \quad n = 378$

$\therefore$ 378 bullets can be made out of the solid cuboid.

28. A right triangle with sides 6 cm, 8 cm and 10 cm is revolved about the side 8 cm. Find the volume and the curved surface of the solid so formed.

Sol. ABC is a right triangle.



AB = 8 cm

BC = 6 cm

AC = 10 cm

The right triangle is revolved about the side 8 cm. The resulting figure formed will be a cone of radius 6 cm, height 8 cm and slant height 10 cm.

Curved surface area $= \pi r l$

$= \dfrac{22}{7} \times 6 \text{ cm} \times 10 \text{ cm}$

$= 188.6 \text{ cm}^2$ (approx.)

$$\text{Volume} = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3} \times \frac{22}{7} \times 6\text{ cm} \times 6\text{ cm} \times 8\text{ cm}$$

$$= 301.7\text{ cm}^3 \text{ (approx.)}$$

Long Answer Type Questions

29. Find the mass of a hollow sphere of metal having internal and external diameters of 20 cm and 22 cm respectively, given 1 cm^3 of metal has a mass of 21 g.

Sol. Let R and r be the external and internal radii of the hollow sphere. Then R = 11 cm and r = 10 cm.

$\therefore$ Volume of the sphere

$$= \frac{4}{3}\pi(R^3 - r^3)$$

$$= \frac{4}{3} \times \frac{22}{7}(11^3 - 10^3)\text{ cm}^3$$

$$= \frac{88}{21} \times (1331 - 1000)\text{ cm}^3$$

$$= \frac{88}{21} \times 331\text{ cm}^3$$

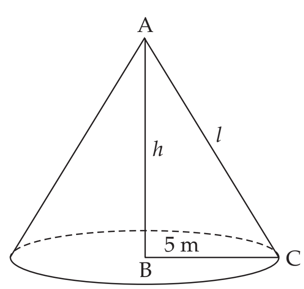
$$\therefore \text{ Required mass} = \frac{88}{21} \times 331 \times 21\text{ g}$$

$$= 29128\text{ g}$$

$$= 29.128\text{ kg}$$

30. A tent, in the form of a right circular cone, has a radius of 5 m. A cloth having an area of 65π m^2 is required to build the tent. Find how many persons can sit in the tent, if a person, on an average, occupies $\frac{5}{7}$ m^2 on the ground. Also find the volume of air inside the tent.

Sol. Let l and h be the slant height and height of the tent.



Cloth required = Curved surface area

$$\Rightarrow \quad \pi rl = 65\pi\text{ m}^2$$

$$\Rightarrow \quad l = \frac{65}{5}\text{ m}$$

$$\Rightarrow \quad l = 13\text{ m}$$

$$l^2 = h^2 + r^2$$

$$\Rightarrow \quad h^2 = l^2 - r^2$$

$$= 169\text{ m}^2 - 25\text{ m}^2 = 144\text{ m}^2$$

$$\Rightarrow \quad h = 12\text{ m}$$

Area of the base of the tent $= \pi r^2$

$$= \pi \times 5\text{ m} \times 5\text{ m}$$

Number of persons that can sit in the tent

$$= \frac{\pi \times 5\text{ m} \times 5\text{ m}}{\frac{5}{7}\text{ m}^2}$$

$$= \frac{22}{7} \times 5\text{ m} \times 5\text{ m} \times \frac{7}{5\text{ m}^2}$$

$$= 22 \times 5$$

$$= 110$$

Volume of air in the tent = Volume of the tent

$$= \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3} \times \frac{22}{7} \times 5\text{ m} \times 5\text{ m} \times 12\text{ m}$$

$$= 314.3\text{ m}^3 \text{ (approx.)}$$

Let's Compete

(Page 164)

Multiple-Choice Questions

1. If the radius, r of a sphere is reduced to its half, then the new volume would be

(*a*) $\frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$ (*b*) $\frac{4\pi}{3}\left(\frac{r^3}{8}\right)$

(*c*) $\frac{4}{3}\pi\frac{r^3}{2}$ (*d*) $\frac{4}{6}\pi\frac{r^3}{8}$

Sol. (*b*) $\frac{4\pi}{3}\left(\frac{r^3}{8}\right)$

If V and V′ be the volumes of the original sphere and the reduced sphere respectively, then

$$V = \frac{4}{3}\pi r^3 \text{ and } V' = \frac{4}{3}\pi\left(\frac{r}{2}\right)^3 = \frac{4}{3}\pi\left(\frac{r^3}{8}\right)$$

2. If the ratio of the radii of bases of two cones is 3 : 1 and the ratio of their heights is 1 : 3, then the ratio of their volume is

(*a*) 1 : 2 (*b*) 2 : 1

(*c*) 1 : 3 (*d*) 3 : 1

Sol. (*d*) 3 : 1

Since radii of bases are in ratio 3 : 1, the radii of the two cones are 3r and r respectively.

Similarly heights are h and 3h respectively.

Ratio of their volumes

$$= \frac{\frac{1}{3}\pi \times 3r \times 3r \times h}{\frac{1}{3}\pi \times r \times r \times 3h} = \frac{3}{1}$$

3. If the circumference of the base of a 18 m high conical tent is 66 m, then the volume of air contained in it is

(*a*) 2079 m^3 (*b*) 2541 m^3

(*c*) 3969 m^3 (*d*) 4851 m^3

Sol. (*a*) 2079 m^3

Circumference of the base = 66 m.

Let r be the radius of the conical tent.

$$2\pi r = 66 \text{ m}$$

$$\Rightarrow \quad r = \frac{1}{2} \times \frac{7}{22} \times 66 \text{ m}$$

$$\Rightarrow \quad r = \frac{21}{2} \text{ m}$$

Volume of air contained in the tent

$$= \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3} \times \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} \times 18 \text{ m}^3$$

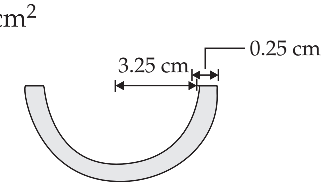
$$= 2079 \text{ m}^3$$

4. The thickness of a hemispherical bowl made of steel is 0.25 cm. If the inner radius of the bowl is 3.25 cm, then the outer curved surface area of the bowl is

(*a*) 38.5 cm^2 (*b*) 77 cm^2

(*c*) 115.5 cm^2 (*d*) 154 cm^2

Sol. (*b*) 77 cm^2



Inner radius of the bowl = 3.25 cm

Thickness of the bowl = 0.25 cm

Outer radius of the bowl

$$= 3.25 \text{ cm} + 0.25 \text{ cm}$$

$$= 3.5 \text{ cm}$$

Outer curved surface area

$$= 2\pi r^2$$

$$= 2 \times \frac{22}{7} \times 3.5 \text{ cm} \times 3.5 \text{ cm}$$

$$= 77 \text{ cm}^2$$

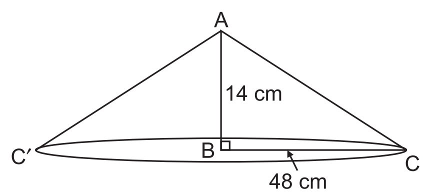
5. ABC is a right-angled triangle with AB = 14 cm, BC = 48 cm and ∠ABC = 90°. This triangle is turned around the shorter side so as to form a cone of volume V_1 cm^3. The triangle is now turned around the longer side to form another cone of volume V_2 cm^3. Then the $V_1 : V_2$ is equal to

(*a*) 1 : 1 (*b*) 24 : 7

(*c*) 7 : 24 (*d*) 12 : 7

Sol. (*b*) 24 : 7

When the triangle ABC is revolved about the shorter side AB, then the cone ACC′ is generated. Let r_1 and h_1 be the radius of the base and the vertical height of this cone respectively, then r_1 = 48 cm, h_1 = 14 cm.

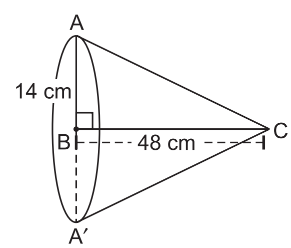


Let V_1 be the volume of this cone.

$$\text{Then} \quad V_1 = \frac{1}{3}\pi r_1^2 h_1$$

$$= \frac{1}{3}\pi \times 48^2 \times 14 \text{ cm}^3$$

When the triangle ABC is revolved about the longer side BC then the cone ACA′ is generated. Let r and h be the radius of the base and the vertical height of this cone respectively.



Then r = 14 cm, h = 48 cm.

Let V_2 be the volume of this cone.

$$\text{Then} \quad V_2 = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\pi \times 14^2 \times 48 \text{ cm}^3$$

$$\therefore \quad \frac{V_1}{V_2} = \frac{\frac{1}{3}\pi \times 48 \times 48 \times 4}{\frac{1}{3}\pi \times 14 \times 14 \times 48} = \frac{24}{7}$$

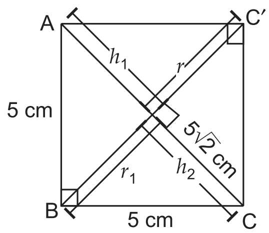
$$\therefore \quad V_1 : V_2 = 24 : 7$$

6. An isosceles right-angled triangle, each equal side of length 5 cm, is turned around the hypotenuse so as to form a double cone. The volume of this combined cone is (*a*) $\frac{125\sqrt{2}}{3}$ cm^3 (*b*) $\frac{125\sqrt{2}\pi}{3}$ cm^3

(*c*) $\frac{125\sqrt{2}}{6}$ cm^3 (*d*) $\frac{125\sqrt{2}\pi}{6}$ cm^3

Sol. (*d*) $\frac{125\sqrt{2}\pi}{6}$ cm^3

Let ABC be an isosceles right-angled triangle with AB = BC = 5 cm and ∠ABC = 90°.



When it is revolved about the hypotenuse AC, double cones AC′B and CC′B arc generated where BC′ ⊥ AC.

Let r be the common radius of the base of the two cones and let h_1 and h_2 be the vertical heights of the cones ABC′ and CBC′ respectively.

Then $h_1 = h_2 = \frac{1}{2}\text{AC} = \frac{5\sqrt{2}}{2} = \frac{5}{\sqrt{2}}$

If O is the mid-point of AC, then AO = OC = $\frac{5}{\sqrt{2}}$.

$\because$ ∠AOB = 90°

$\therefore$ By Pythagoras' Theorem, we have

$$\text{AO}^2 + \text{OB}^2 = \text{AB}^2$$

$$\Rightarrow \frac{25}{2} + r^2 = 5^2 = 25$$

$$\therefore r^2 = 25 - \frac{25}{2}$$

$$= \frac{50-25}{2} = \frac{25}{2}$$

$$\therefore r = \frac{5}{\sqrt{2}}$$

$\therefore$ Volume of each cone = $\frac{1}{3}\pi r^2 h_1$

$$= \frac{1}{3} \times \frac{25}{2} \times \frac{5}{\sqrt{2}} \text{ cm}^3$$

$$= \frac{125}{6\sqrt{2}}\pi \text{ cm}^3$$

$\therefore$ The required volume of the combined cone

$$= 2 \times \frac{125\pi}{6\sqrt{2}}$$

$$= \frac{125\sqrt{2}\pi}{6} \text{ cm}^3$$

7. If the radius and the slant height of a cone are in the ratio 7 : 13 and its curved surface area is 286 cm^2, then its radius is

(*a*) 7 cm (*b*) 7.5 cm

(*c*) 10 cm (*d*) 10.5 cm

Sol. (*a*) 7 cm

The radius and the slant height of a cone are in the ratio 7 : 13.

Let radius be $7r$ and slant height be $13r$.

Curved surface area = 286 cm^2

$$\pi rl = 286 \text{ cm}^2$$

$$\Rightarrow \frac{22}{7} \times 7r \times 13r = 286 \text{ cm}^2$$

$$\Rightarrow 13r^2 = \frac{286}{22} \text{ cm}^2$$

$$\Rightarrow 13r^2 = 13 \text{ cm}^2$$

$$\Rightarrow r^2 = 1 \text{ cm}^2$$

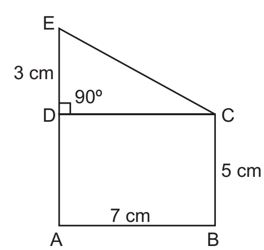
$$\Rightarrow r = 1 \text{ cm}$$

$\therefore$ Radius of the cone $= 7r$

$= 7 \times 1$ cm

$= 7$ cm

8. ABCD is a rectangle with AB = 7 cm and BC = 5 cm and ΔEDC is a right-angled triangle where E is a point on AD produced such that DE = 3 cm and ∠EDC = 90° as shown in the figure. The combined plane figure ABCE is now revolved around AE so as to form a combined solid figure. The volume of this combined solid is

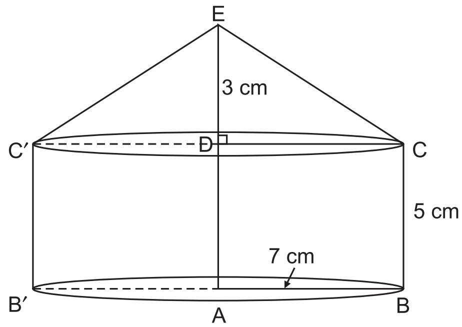


(*a*) 850 cm^3 (*b*) 924 cm^3

(*c*) 900 cm^3 (*d*) 890 cm^3

Sol. (*b*) 924 cm^3

Let ABCD be the rectangle with AB = 7 cm, BC = 5 cm and E is a point on AD produced such that ED = 3 cm and ∠EDC = 90°.



If the combined figure ABCE is revolved about AE, then one cone ECC′ and one cylinder CB B′C′ are generated. It r and R be the radii of this cone and this cylinder respectively, h and H be the vertical height of the cone and the cylinder respectively.

Then $r = 7$ cm, R = 7 cm, $h = 3$ cm and H = 5 cm

$\therefore$ Volume of the cone

$$= \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\pi \times 7^2 \times 3 \text{ cm}^3$$

$$= \frac{1}{3} \times \frac{22}{7} \times 7^2 \times 3 \text{ cm}^3$$

$$= 154 \text{ cm}^3$$

and volume of the cylinder

$$= \pi R^2 H$$

$$= \frac{22}{7} \times 7 \times 7 \times 5 \text{ cm}^3$$

$$= 770 \text{ cm}^3$$

$\therefore$ Required volume of the combined solid

$$= (154 + 770) \text{ cm}^3$$

$$= 924 \text{ cm}^3$$

9. If each bag containing wheat occupies 2.2 m^3 of space, then the number of full bags which can be emptied into a conical drum of radius 6 m and height 3.5 m is

(*a*) 90 (*b*) 70

(*c*) 80 (*d*) 60

Sol. (*d*) 60

Let r and h be the radius and the height of the conical drum. Then $r = 6$ m and $h = 3.5$ m.

$\therefore$ Volume of the conical drum

$$= \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3} \times \frac{22}{7} \times 6 \times 6 \times 3.5 \text{ m}^3$$

$$= 132 \text{ m}^3$$

Volume of wheat in each bag = 2.2 m^3

$\therefore$ Required number of bags $= \frac{132}{2.2} = 60$

10. A conical pandal 350 m in radius and 120 m high is made of cloth which is 220 m wide. Then the length of the cloth used to make the pandal is

(*a*) 1850 m (*b*) 1800 m

(*c*) 1875 m (*d*) 1900 m

Sol. (*a*) 1850 m

Let r and h be the radius of the base and the vertical height of the cone respectively. Let l be its slant height.

Then $r = 350$ m, $h = 120$ m.

$$\therefore \quad l = \sqrt{r^2 + h^2}$$

$$= \sqrt{350^2 + 120^2} \text{ m}$$

$$= \sqrt{136900} \text{ m}$$

$$= 370 \text{ m}$$

$\therefore$ Curved surface area of the cone

$$= \pi r l$$

$$= \frac{22}{7} \times 350 \times 370 \text{ m}^2$$

$$= 22 \times 50 \times 370 \text{ m}^2$$

If L be the length of the cloth, then

$$L \times 220 = 22 \times 50 \times 370$$

$$\therefore \quad L = \frac{22 \times 50 \times 370}{220} = 1850$$

Hence, the required length is 1850 m

Life Skills

(Page 165)

1. A conical tent was set-up to accommodate 30 students for a summer camp in which students participated in activities like planting saplings, yoga, cleaning lakes, testing the water for contaminants and pollutant levels and desilt the lake bed and also using the silt to strengthen bunds under the guidance of two teachers who accompanied them, but were staying in a nearby guest house. Find the height of the tent if each student must have 4 m^2 of the space on the ground and 20 m^3 of air to breathe.

Sol. Let r and h be the radius of the base and the vertical height of the conical tent.

For 30 students, the area of the space needed

$$= 30 \times 4 \text{ m}^2$$

$$= 120 \text{ m}^2$$

$$\therefore \quad \pi r^2 = 120 \text{ m}^2$$

$$\Rightarrow \quad r^2 = \frac{120 \times 7}{22} \text{ m}^2 = \frac{420}{11} \text{ m}^2 \quad \ldots(1)$$

Now, the volume of the conical tent

$$= \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3} \times \frac{22}{7} \times \frac{420}{11} \times h \text{ m}^3 \quad \text{[From (1)]}$$

$$= 40h \text{ m}^3$$

$$\therefore \quad 40h = 20 \times 30 \quad \text{[Given]}$$

$$\therefore \quad h = \frac{600}{40} = 15$$

Hence, the required height of the tent is 15 m.