SUPPLEMENT

Based on latest CBSE Syllabus 2025–26

STELLAR LEARNING

READING MATERIAL

Mathematics

On Board! BOOKS

An imprint of Ratna Sagar P. Ltd.

9

Contents

3

Coordinate Geometry

Knowledge Digest

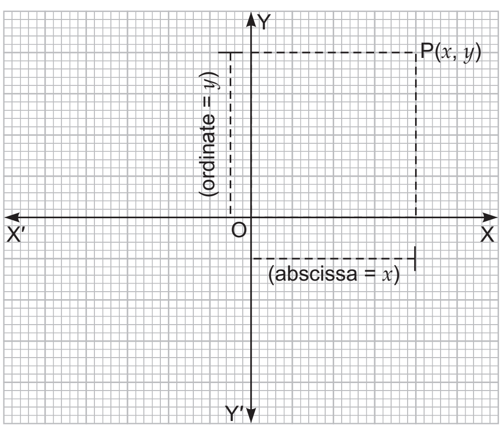
Cartesian System

Coordinate Axes

Coordinate axes consists of two mutually perpendicular axes that intersect at the origin (0, 0). The horizontal axis is the x-axis. The vertical axis is the y-axis.

Cartesian Coordinates of a Point

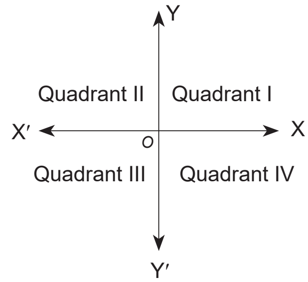
- The x-coordinate of a point is its perpendicular distance from y-axis. The x-coordinate is called the abscissa.



- The y-coordinate of a point is its perpendicular distance from the x-axis. The y-coordinate is called the ordinate.

Note: In stating the coordinates of a point in the coordinate plane, we write x-coordinate first and then the y-coordinate.

Quadrants



The axes divide the plane into four parts XOY, X′OY, X′OY′ and XOY. These four parts are called Quadrants, numbered as Quadrant I, Quadrant II, Quadrant III, Quadrant IV, anticlockwise from OX.

We call this plane the Cartesian plane or Coordinate plane or xy-plane.

Convention of Signs

The four quadrants are characterised by the following signs of abscissa and ordinate.

Region	XOY	YOX′	X′OY′	Y′OX
Quadrant	I	II	III	IV
Nature of x, y	$x > 0$, $y > 0$	$x < 0$, $y > 0$	$x < 0$, $y < 0$	$x > 0$, $y < 0$
Signs of coordinates	(+, +)	(–, +)	(–, –)	(+, –)

We shall consider the following type of questions:

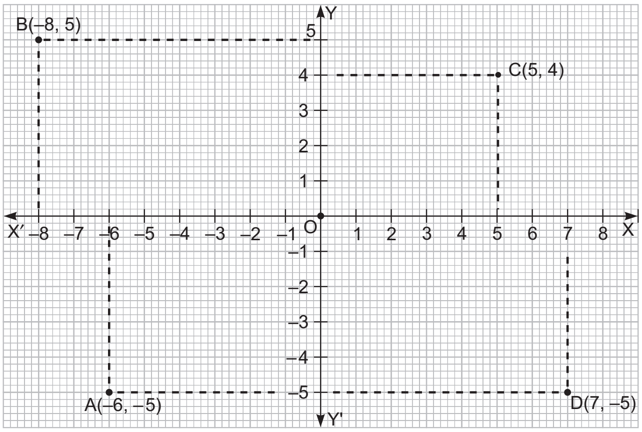
Type 1: To plot the points in the Cartesian system, observe the figure made by joining them and find its area

Solved Examples

Example 1: *Plot the following points on the graph paper.*

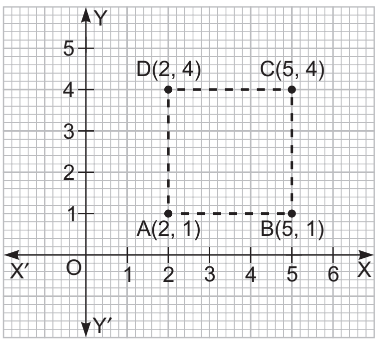
(a) A(–6, –5)   (b) B(–8, 5)   (c) C(5, 4)   (d) D(7, –5)

Solution:



Example 2: *Three vertices of a square are A(2, 1), B(5, 1), C(5, 4). Plot these points and find the coordinates of the fourth vertex.*

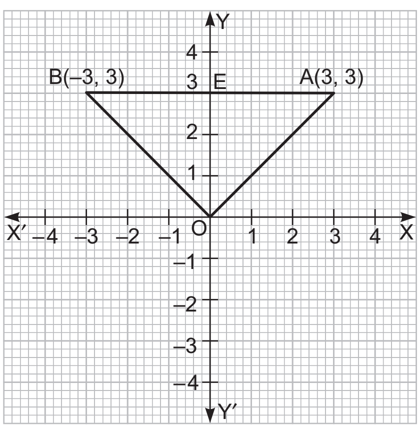
Solution:



The coordinates of the fourth vertex are (2, 4).

Example 3: *Plot the points A(3, 3) and B(–3, 3) and join A and B with origin. What figure can you obtain? Find its area.*

Solution:



The figure obtained is an isosceles triangle AOB.

$$ar(\Delta AOB) = \frac{1}{2} \times BA \times OE$$

$$= \frac{1}{2} \times 6 \times 3$$

$$= 9 \text{ sq units}$$

Check Your Progress

Very Short Answer Type Questions

1. Plot the points P(8, 0), Q(–1, 2), R(–5, –5) and S(4, –8) on graph paper.
2. Plot the points (0, 0), (0, 4), (4, 4), (4, 0). Name the figure obtained on joining them and find its area.

Short Answer Type Questions

3. Find the area of the triangle whose vertices are (0, 4), (0, 0), (2, 0) by plotting them on graph.
4. Draw a rectangle PQRS with vertices P(5, 0), Q(5, 3), R(0, 3) and S(0, 0).
5. Plot the points (x, y) given in the following table by using a graph paper.

x	–2	0	–1	5	3
y	7	–4.5	–4	5	–2

6. Plot the points A(1, 3), B(1, –1), C(6, –1), D(6, 3). Join them respectively and name the type of figure so formed and find its area.

Long Answer Type Questions

7. Plot the points P(1, 0), Q(4, 0) and S(1, 3). Find the coordinates of the point R such that PQRS is a square.
8. Plot the points A(1, 3), B(1, –1), C(7, –1) and D(7, 3) in the cartesian plane. Join them in order and name the figure so obtained.
9. Draw the quadrilateral with vertices (–3, 3), (–1, 0), (–3, –3) and (–5, 0). Also, name the type of quadrilateral formed and find its area.
10. Plot the points (3, 4), (3, –2) and (0, 0). Check whether they are collinear or not. If not, find the area of the figure.

4

Linear Equations in Two Variables

Knowledge Digest

Graph of a Linear Equation in two Variables

Graph of $ax + by + c = 0$

(*i*) Express y in terms of x.

(*ii*) Assign two or more suitable values to x and determine the corresponding value of y.

(*iii*) Tabulate the values of x and the corresponding values of y.

(*iv*) Plot these points (x, y) on a graph paper.

(*v*) Draw a line passing through these plotted points.

All the points representing different solutions of the linear equation in two variables lie on the same straight line, that is the points are collinear.

Thus, a linear equation in two variables is represented by a line whose points make up the collection of solutions of the equation. This is called the **graph of a linear equation**.

- The graph of every linear equation in two variables is a straight line.

Note: The graph of the equation of the form $y = kx$ is a line which always passes through the origin.

- Every point on the graph of linear equation in two variables is a solution of the equation.
- Geometrically, every solution of the linear equation in two variables represents a point on the graph of the equation.
- If a linear equation contains one variable, say x or y, then it will represent a line parallel to any coordinate axes, i.e. either x-axis or y-axis.

We shall consider the following types of questions:

Type 1: Graphs of linear equation in two variables or one variable and intersection of two graphs

Type 2: Application of linear equations in the formation and solution of a few daily life problems

Solved Examples

Example 1: *Draw the graph of the following linear equation in two variables.*

$$2x - 5y = 8$$

Solution: The given equation can be rewritten as

$$5y = 2x - 8$$

$$\Rightarrow \quad y = \frac{2x-8}{5}$$

Taking $x = -1$, we get

$$y = \frac{2(-1)-8}{5} = \frac{-2-8}{5} = \frac{-10}{5} = -2$$

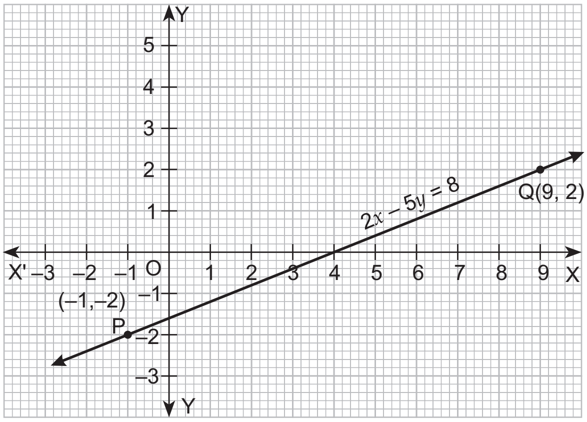
Taking $x = 9$, we get

$$y = \frac{2\times 9-8}{5} = \frac{10}{5} = 2$$

We have the following table for $2x - 5y = 8$.

x	–1	9
y	–2	2

With suitable scales on both x and y-axes, plot the points P(–1, –2), and Q(9, 2) on a graph paper.



Draw a line passing through the points P and Q. Thus, the line PQ is the graph of the equation

$$2x - 5y = 8$$

Example 2: *From the choices given below, choose the equations whose graphs are given in figure* (*a*) *and figure* (*b*).

For figure (*a*)	*For figure* (*b*)
(*a*) $y = x$	(*a*) $y = x + 2$
(*b*) $y + x = 0$	(*b*) $y = x - 2$
(*c*) $y = 2x$	(*c*) $y = -x + 2$
(*d*) $2 + 3y = 7x$	(*d*) $x + 2y = 6$

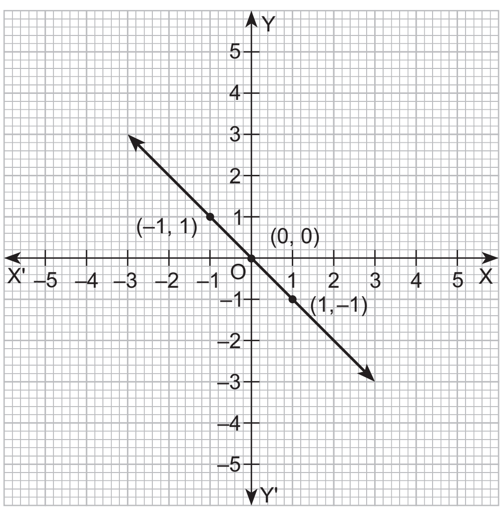


Fig. (*a*)

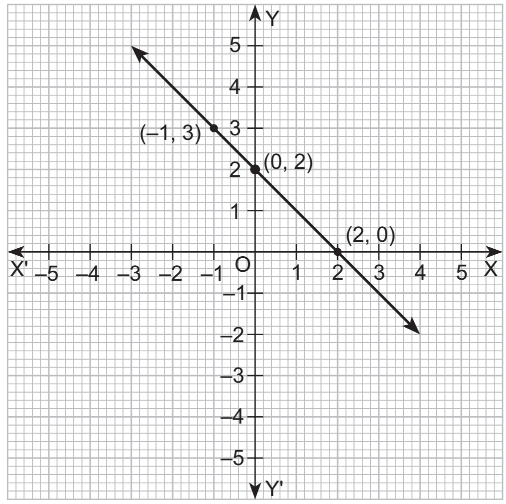


Fig. (*b*)

[CBSE SP 2011]

Solution: For figure (*a*):

The points on the graph line are

(1, –1), (0, 0) and (–1, 1)

For point (–1, 1): (–1) + 1 = 0

For point (0, 0): 0 + 0 = 0

For point (1, –1): 1 + (–1) = 0

By inspection, $x + y = 0$ is the equation corresponding to the given graph.

For figure (*b*):

The points on the graph line are (–1, 3), (0, 2) and (2, 0)

For point (–1, 3): (–1) + 3 = 2

For point (0, 2): 0 + 2 = 2

For point (2, 0): 2 + 0 = 2

By inspection $x + y = 2$ or $y = -x + 2$ is the equation corresponding to the given graph.

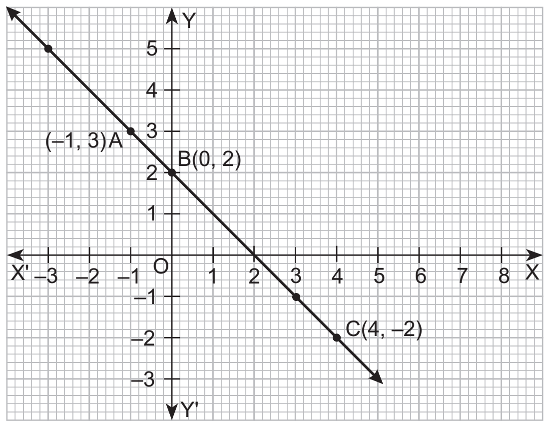
Example 3: *Use the following table to draw the graph.*

x	4	3	0	q	–1
y	–2	p	2	5	3

From the graph, find the values of p and q. State a linear relationship between the variables x and y.

Solution: Plot the points A(–1, 3), B(0, 2) and C(4, –2) on the graph paper. Draw a straight line passing through A, B and C.

The y-coordinate corresponding to the x-coordinate 3 is –1. Hence, $p = -1$.



Also, the x-coordinate corresponding to the y-coordinate 5 is –3. Hence, $q = -3$.

Now, to find a relationship between x and y, we see from the given table of values of x and y that $x = 0$ when $y = 2$. Hence, x must be of the form

$x = k(y - 2)$, where k is constant.

To find k, put $x = 4$ and $y = -2$ in this equation since this equation is satisfied by these values of x and y.

$\therefore \quad 4 = k(-2 - 2)$

$\Rightarrow \quad 4 = k(-4)$

$\Rightarrow \quad k = -1$

$\therefore \quad x = -y + 2$

$\Rightarrow \quad x + y = 2$

which is the required linear relationship between x and y.

Example 4: *Draw the graph of the equation $4x + 3y = 12$. Hence, find the area of the triangle formed by this line and the coordinate axes.*

Solution: The given equation can be rewritten as

$$y = \frac{12 - 4x}{3}$$

$$\therefore \quad x = 3 \Rightarrow y = \frac{12 - 4(3)}{3} = \frac{0}{3} = 0$$

$$x = 0 \Rightarrow y = \frac{12 - 4(0)}{3} = \frac{12}{3} = 4$$

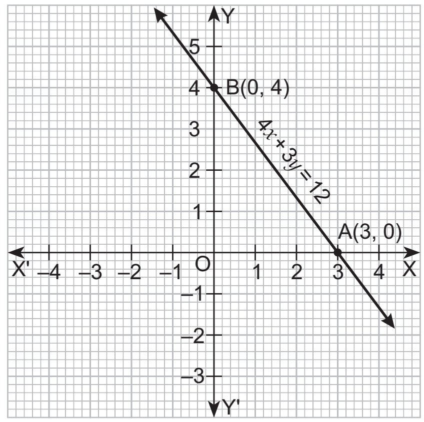
Thus, we have the following table for $4x + 3y = 12$

x	3	0
y	0	4

Plot the points A(3, 0) and B(0, 4) on a graph paper with some suitable scales on both x and y-axes.

Draw a line passing through the points A and B. Thus, the line AB represents the graph of $4x + 3y = 12$.

We see from the graph that the line AB forms a right-angled triangle AOB with x and y-axes where $\angle AOB = 90°$.



Take OA as the base of the triangle, then OB will be its height.

The required area of $\Delta AOB = \frac{1}{2} \times \text{base} \times \text{altitude}$

$$= \frac{1}{2} \times OA \times OB$$

$$= \frac{1}{2} \times 3 \times 4 \text{ sq units}$$

$$= 6 \text{ sq units}$$

Hence, area of triangle formed by line and the coordinate axes is 6 sq units.

Example 5: *Draw the graphs of $2x - y = 1$ and $x + 2y = 3$. What is the point of intersection of the two lines representing the above equations?*

Solution: Graph of the equation $2x - y = 1$

$2x - y = 1$

$\Rightarrow \quad y = 2x - 1$

$\therefore \quad x = 0 \quad \Rightarrow \quad y = 2(0) - 1 = -1$

$x = -1 \quad \Rightarrow \quad y = 2(-1) - 1 = -3$

Thus, we have the following table for $2x - y = 1$.

x	0	–1
y	–1	–3

Plot the points A(0, –1) and B(–1, –3) on a graph paper.

Draw a line passing through the points A and B. Then, the line AB is the graph of $2x - y = 1$.

Graph of the equation $x + 2y = 3$

$$x + 2y = 3$$

$$\Rightarrow \quad y = \frac{3-x}{2}$$

$$\therefore \quad x = 3 \quad \Rightarrow \quad y = \frac{3-3}{2} = 0$$

$$x = -1 \quad \Rightarrow \quad y = \frac{3-(-1)}{2} = 2$$

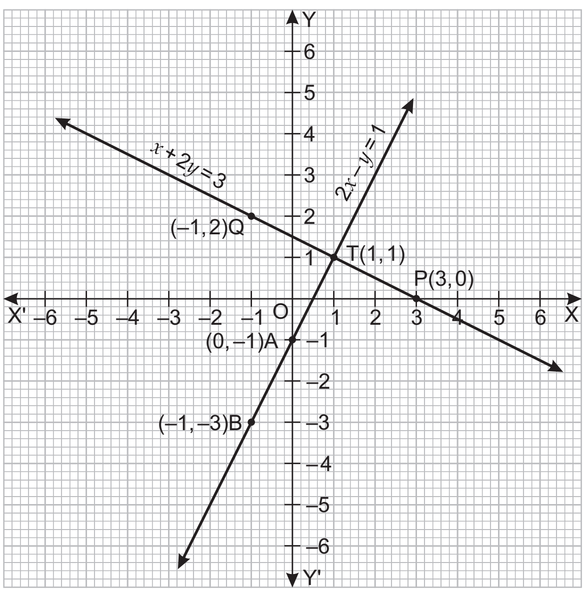
Thus, we have the following table for $x + 2y = 3$.

x	3	–1
y	0	2

Plot the points P(3, 0) and Q(–1, 2) on the same graph paper.

Draw a line passing through the points P and Q. Then, the line PQ is the graph of $x + 2y = 3$.

The graph lines AB and PQ intersect at T(1, 1). The point of intersection of two lines representing the given equations is T(1, 1).



Example 6: *The ratio of girls and boys in a class is 1 : 3. Set up an equation between the students of a class and boys and then draw its graph and find the number of boys in a class of 40 students.* **[CBSE SP 2011]**

Solution: Let the number of boys be y and the total number of the students (i.e. both boys and girls) be x.

Then the number of girls = $x - y$

It is given that the ratio of girls and boys in a class is 1 : 3.

$$\therefore \quad \frac{x-y}{y} = \frac{1}{3}$$

$$\Rightarrow \quad 3x - 3y = y$$

$$\Rightarrow \quad 3x = 3y + y = 4y$$

$$\Rightarrow \quad y = \frac{3x}{4} \qquad ...(1)$$

This is the required equation between the students numbering x and boys numbering y.

Now, the graph of $y = \frac{3x}{4}$

$$y = \frac{3x}{4}$$

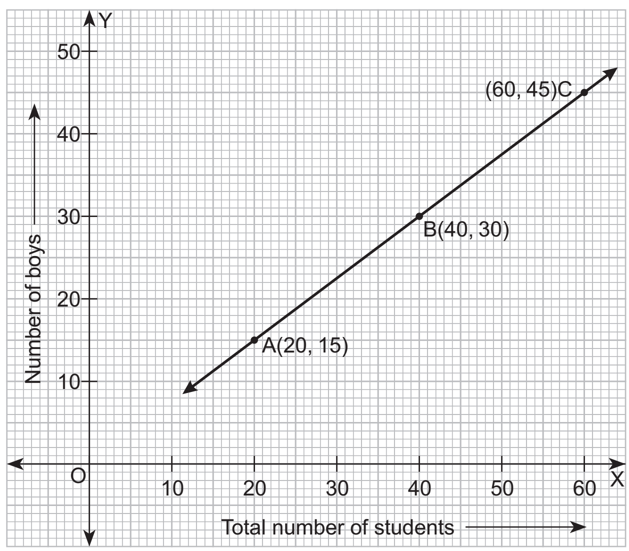
$$\therefore \quad x = 20 \quad \Rightarrow \quad y = \frac{3(20)}{4} = 15$$

$$x = 40 \quad \Rightarrow \quad y = \frac{3(40)}{4} = 30$$

$$x = 60 \quad \Rightarrow \quad y = \frac{3(60)}{4} = 45$$

Thus, we have following table for $y = \frac{3x}{4}$

x	20	40	60
y	15	30	45



Plot the total number of students along x-axis and the total number of boys along y-axis on the graph paper.

Choose the following scales on the graph paper: Along x-axis: 1 unit of the graph paper = 10 students

Along y-axis: 1 unit of the graph paper = 10 boys.

Now plot the points A(20, 15), B(40, 30) and C(60, 45) on the graph paper.

Draw a line passing through the points A and B.

From the graph we see that when the total number of students = 40, i.e. when $x = 40$, then the ordinate is 30. Hence, we conclude that out of 40 students, the number of boys is 30.

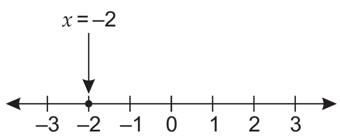
Example 7: *Give the geometric representations of $x + 2 = 0$ as an equation*

(a) in one variable *(b) in two variables*

Solution: (*a*) $x + 2 = 0$

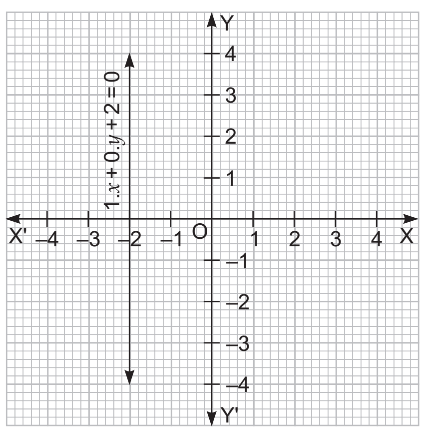
$$x = -2$$

The representation of the solution on the number line is shown in the figure given below, where $x = -2$ is treated as equation in one variable.



(*b*) $x + 2 = 0$

$\Rightarrow$ $1 \cdot x + 0 \cdot y + 2 = 0$ which is a linear equation in two variables.



We see that for all values of y, $x = -2$.

Thus, the graph line of the equation $1 \cdot x + 0 \cdot y + 2 = 0$ is a line parallel to the y-axis at a distance of 2 units to the left of it.

Check Your Progress

Very Short Answer Type Questions

1. Show that the point (3, –1) does not lie on the line $3x - 5y = 9$.

Short Answer Type Questions

2. Draw the graph of equation $5x - 2y = 1$ and from the graph, determine whether (*a*) $x = 1$, $y = 2$ and (*b*) $x = -1$, $y = 3$ are solutions or not.
3. Draw the graph of equation $5x + 6y = 30$ and hence find the area of the triangle formed by this line and the coordinate axes.
4. Draw the graphs of $3x + 2y = 0$ and $2x - 3y = 0$. What is the point of intersection of the two lines representing the above equations. **[CBSE SP 2013]**
5. Draw the graph of two lines whose equations are $3x - 2y + 6 = 0$ and $x + 2y - 6 = 0$ on the same graph paper. Find the area of the triangle formed by the two lines and the x-axis. **[CBSE SP 2011]**

Long Answer Type Questions

6. In an election 60% of the voters cast their votes. Form an equation and draw the graph with this data. From the graph, find
   (*a*) the total number of voters if 2,100 voters cast their votes and
   (*b*) the number of votes cast if the total number of voters are 10,000. **[CBSE SP 2012]**
7. The population of a small city is 12,000. The ratio of the female and male is 5 : 7. Set up an equation between the population and females. Then, draw the graph with the help of this equation. By reading the graph, find the number of females in the city. **[CBSE SP 2012]**

6

Lines and Angles

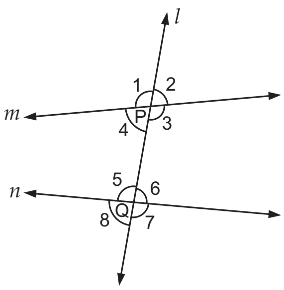
Knowledge Digest

Parallel Lines and a Transversal

Transversal: A straight line that cuts two or more lines at distinct points is called a *transversal.*

Angles Made by a Transversal with Two Lines

In the given figure, line *l* intersects lines *m* and *n* at P and Q respectively. Therefore line *l* is the transversal for lines *m* and *n*. The following pairs of angles are formed.



(*a*) **Corresponding angles** (abbreviation: corr. ∠s)

(*i*) ∠1 and ∠5 (*ii*) ∠2 and ∠6

(*iii*) ∠4 and ∠8 (*iv*) ∠3 and ∠7

(*b*) **Alternate interior angles** (abbreviation: alt. ∠*s*)

(*i*) ∠4 and ∠6 (*ii*) ∠3 and ∠5

(*c*) **Alternate exterior angles**

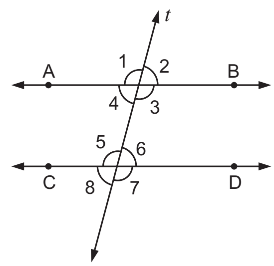
(*i*) ∠1 and ∠7 (*ii*) ∠2 and ∠8

(*d*) **Interior angles on the same side of the transversal**

(Consecutive interior angles or cointerior angles)

(*i*) ∠4 and ∠5 (*ii*) ∠3 and ∠6

Axiom 6.3: Corresponding angles axiom: *If a transversal intersects two parallel lines, then each pair of corresponding angles is equal.*

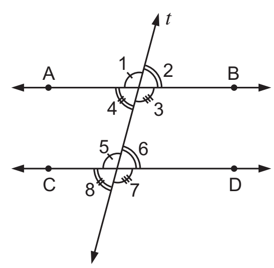


In the given figure, AB || CD.

Transversal *t* cuts AB and CD.

Then, ∠1 = ∠ 5, ∠2 = ∠6, ∠4 = ∠8 and ∠3 = ∠7.

Axiom 6.4: *If a transversal intersects two lines such that a pair of corresponding angles is equal, then the two lines are parallel to each other.*



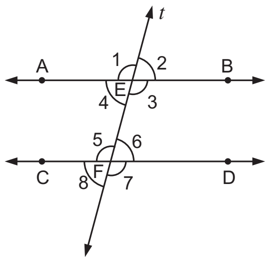
In the given figure, transversal *t* cuts lines AB and CD such that

∠1 = ∠ 5 or ∠2 = ∠6 or ∠4 = ∠8 or ∠3 = ∠7

Then, AB || CD.

Corresponding angles axiom can be used to find out the relation between the alternate interior angles formed when a transversal intersects two parallel lines.

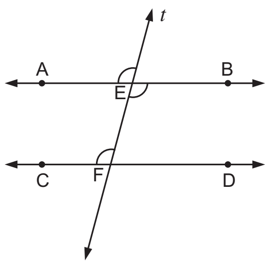
Theorem 6.2: *If a transversal intersects two parallel lines, then each pair of alternate interior angles is equal.*



In the given figure, AB || CD. Transversal t intersects AB at E and CD at F, forming two pairs of alternate interior angles, namely (∠3, ∠5) and (∠4, ∠6).

∴ $\angle 3 = \angle 5$ and $\angle 4 = \angle 6$

Theorem 6.3: *If a transversal intersects two lines such that a pair of alternate interior angles is equal, then the two lines are parallel.*

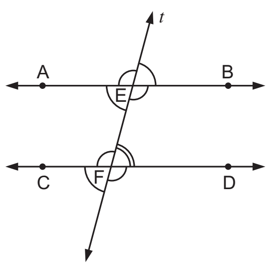


In the given figure, transversal t intersects two lines AB and CD at E and F respectively such that

$$\angle BEF = \angle EFC$$

∴ AB || CD

Theorem 6.4: *If a transversal intersects two parallel lines then each pair of interior angles on the same side of the transversal is supplementary.*

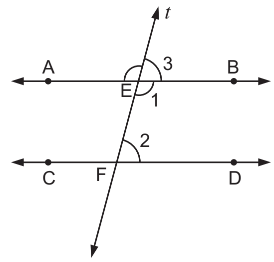


In the given figure, AB || CD. Transversal t intersects AB at E and CD at F, forming two pairs of consecutive interior angles namely (∠BEF, ∠EFD) and (∠AEF, ∠EFC)

∴ $\angle BEF + \angle EFD = 180°$

and $\angle AEF + \angle EFC = 180°$

Theorem 6.5: *If a transversal intersects two lines such that a pair of interior angles on the same side of the transversal is supplementary, then the two lines are parallel.*

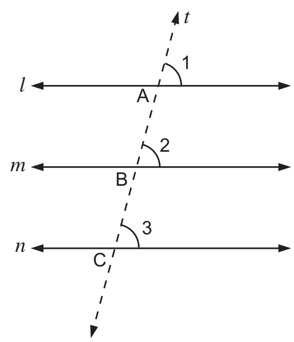


In the given figure, a transversal t intersects two lines AB and CD at E and F respectively such that

$$\angle BEF + \angle EFD = 180°$$

∴ AB || CD

Theorem 6.6: *Lines which are parallel to the same line are parallel to each other.*



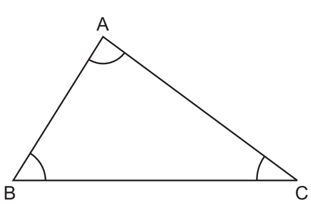
In the given figure, the lines l, m and n are such that $l \parallel n$ and $m \parallel n$.

∴ $l \parallel m$

Remark. The above property can be extended to more than two lines also.

Angle Sum Property of a Triangle

Theorem 6.7: *The sum of the angles of a triangle is 180°.*

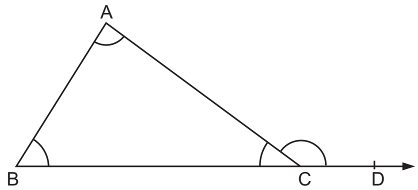


Here, a triangle ABC is given

∴ $\angle A + \angle B + \angle C = 180°$

Exterior Angles of a Triangle

Theorem 6.8: *If a side of a triangle is produced, then the exterior angle so formed is equal to the sum of the two interior opposite angles.*



In the given figure, side BC of the triangle ABC has been produced to D forming exterior angle ACD.

$\therefore \quad \angle ACD = \angle CAB + \angle ABC$

We shall consider the following types of questions:

Type 1: Questions of determination of numerical values of certain angles formed when a transversal intersects two parallel lines

Type 2: Questions based on angle sum property of a triangle

Type 3: Questions based on exterior angle theorem

Type 4: Miscellaneous questions

Solved Examples

Example 1: *If the angles of a triangle are in the ratio 3 : 2 : 5, find the measure of each angle of the triangle.*

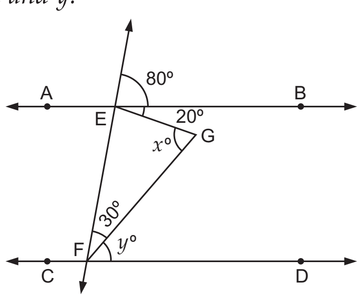
Solution: Let the angles of the triangle be $(3x)°$, $(2x)°$ and $(5x)°$.

By using angle sum property of a triangle, we get

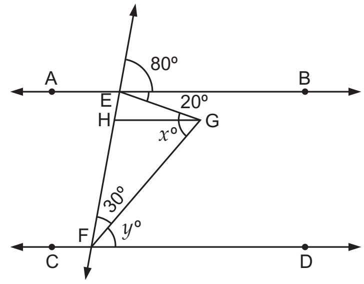
$$3x + 2x + 5x = 180$$
$$10x = 180$$
$$x = \frac{180}{10}$$
$$x = 18$$

Hence, the angles of the triangle are $(3 \times 18°)$, $(2 \times 18)°$, $(5 \times 18)°$, i.e. 54°, 36° and 90°.

Example 2: *In the given figure, AB || CD. Find the values of x and y.*



Solution: Draw GH parallel to AB or CD intersecting EF at H.



Now, $\angle GEF = 180° - 80° - 20°$

$= 180° - 100° = 80°$

Also, from ΔEGF

$\angle GEF + \angle GFE + \angle EGF = 180°$

[Angle sum property of a triangle]

$\Rightarrow \quad 80° + 30° + x° = 180°$

$\Rightarrow \quad x° = 180° - 110° = 70°$

$\therefore \quad x° = 70°$

Now, $\angle EGH$ = alternate $\angle BEG = 20°$

and

$\angle HGF$ = alternate $\angle GFD = y°$

$\therefore \quad x° = \angle EGF = \angle EGH + \angle HGF$

$= 20° + y°$

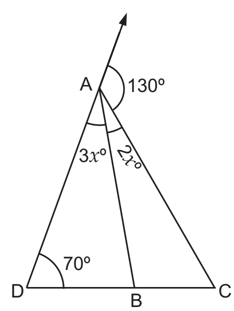
$\Rightarrow \quad 70° = 20° + y°$

$\Rightarrow \quad y° = 70° - 20° = 50°$

$y° = 50$

Thus, the values of x and y are 70 and 50 respectively.

Example 3: *In the given figure, AB divides $\angle DAC$ in the ratio $\angle DAB : \angle BAC = 3 : 2$. Find $\angle ACB$ and $\angle ABC$.*



Solution: Let $\angle DAB = 3x°$ and $\angle BAC = 2x°$.

Now, $\angle 130° + \angle DAC = 180°$ [Linear pair]

$\Rightarrow \quad \angle DAC = 180° - 130° = 50°$

$\therefore \quad 3x° + 2x° = 50°$

$\Rightarrow \quad 5x° = 50°$

$\Rightarrow \quad x = 10°$

Now, by angle sum property of a triangle we have

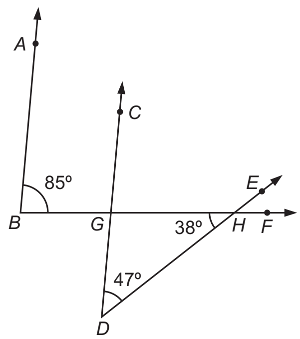
$$\angle ADB + \angle DAC + \angle ACB = 180°$$
$$\Rightarrow \quad 70° + 50° + \angle ACB = 180°$$
$$\Rightarrow \quad \angle ACB = 180° - 120° = 60°$$

Now, $\angle ABC = 70° + 3x° = 70° + 3 \times 10°$

$= 70° + 30° = 100°$

Hence, $\angle ACB = 60°$ and $\angle ABC = 100°$.

Example 4: *In the given figure, BF intersects DC and DE at G and H respectively. Prove that BA || DC.*



Solution: We have from ΔDGH

$$\angle GDH + \angle DHG + \angle DGH = 180°$$

[Angle sum property of a triangle]

$\Rightarrow \quad 47° + 38° + \angle DGH = 180°$

[From the given angles]

$\Rightarrow \quad \angle DGH = 180° - 47° - 38° = 95°$

Now, $\angle DGH + \angle CGH = 180°$ [Linear pair]

$\Rightarrow \quad 95° + \angle CGH = 180°$

$\Rightarrow \quad \angle CGG = 180° - 95° = 85°$

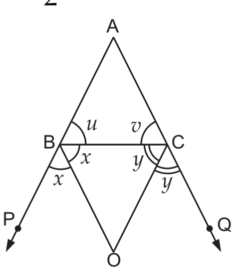
Also, $\angle ABG = 85°$ [Given]

$\therefore \quad \angle ABG = \angle CGH$

But these are corresponding angles on the same side of the transversal BF.

Hence, BA || DC.

Example 5: *In the given figure, the sides AB and AC of a triangle ABC are produced to P and Q respectively. The bisectors of ∠PBC and ∠QCB intersect at O. Prove that* $\angle BOC = 90° - \frac{1}{2} \angle BAC$. **[CBSE SP 2013]**



Solution: Let $\angle PBO = \angle OBC = x$,

[∵ OB is a bisector ∠PBC]

$\angle QCO = \angle OCB = y$

[∵ OC is a bisector of ∠BCQ].

Now, $\angle BOC + x + y = 180°$

$\therefore \quad \angle BOC = 180° - (x + y)$

[Angle sum property of a triangle]

$= 180° - \frac{1}{2}(2x + 2y)$

$= 180° - \frac{1}{2} \times 2x - \frac{1}{2} \times 2y$

$= 180° - \frac{1}{2}(180° - u) - \frac{1}{2}(180° - v)$

[∵ ∠PBC and ∠CBA are linear pairs. Similarly, ∠QCB and ∠BCA are linear pairs]

$= 180° - 90° + \frac{1}{2}u - 90° + \frac{1}{2}v$

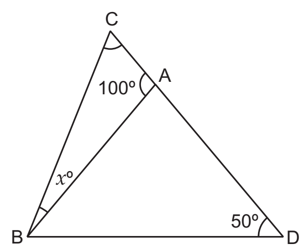
$= \frac{1}{2}(u + v) = \frac{1}{2}[180° - \angle BAC]$

[Angle sum property of a triangle, $u + v + \angle BAC = 180°$]

$= 90° - \frac{1}{2}\angle BAC$

Hence, proved.

Example 6: *In the given figure, ∠C = 3∠ABC, find ∠ABD, ∠ABC and ∠C.*



Solution: We have

$$\angle BAC = \angle ABD + \angle ADB$$

[By exterior angle theorem]

$\Rightarrow \quad 100° = \angle ABD + 50°$

[From given angles]

$\Rightarrow \quad \angle ABD = 100° - 50° = 50°$

Again, let $\angle ABC = x°$, Then, $\angle C = 3x°$

[From the given condition]

$\therefore \quad x° + 3x° + 100° = 180°$

[Angle sum property of a triangle]

$\Rightarrow \quad 4x° = 180° – 100° = 80°$

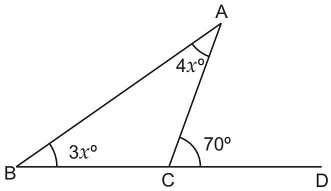
$\Rightarrow \quad x° = \dfrac{80°}{4} = 20°$

$\therefore \quad \angle ABC = 20°$

and $\quad \angle ACB = \angle C = 3x° = 3 \times 20° = 60°$

Thus, ∠ABD = 50°, ∠ABC = 20° and ∠C = 60°

Example 7: *One of the exterior angles of a triangle is 70° and the interior opposite angles are in the ratio 3 : 4. Find all the angles of the triangle.*



Solution: Let ABC be a triangle, ∠ACD = 70° is one of its exterior angle.

Let the interior opposite angles ∠B and ∠A be 3x° and 4x° respectively.

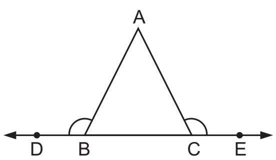
Now, $\quad 70° = 3x° + 4x°$

$\Rightarrow \quad 7x° = 70°$

$\Rightarrow \quad x° = \dfrac{70°}{7} = 10°$

Hence, ∠B = 3 × 10° = 30°, ∠A = 4 × 10° = 40° and ∠ACB = 180° – 70° = 110°.

Example 8: *The side BC of a ΔABC is produced on both sides. Show that the sum of the exterior angles so formed exceeds ∠A by two right angles.*



Solution: Let the side BC of ΔABC be produced on both sides upto D and E as shown in the figure.

To prove that ∠ABD + ∠ACE – ∠A = 180°

Proof: We know that

$\angle ACE = \angle A + \angle ABC \quad \ldots(1)$

[By exterior angle theorem]

and $\quad \angle ABD = \angle A + \angle ACB \quad \ldots(2)$

[By exterior angle theorem]

Adding (1) and (2), we get

$\angle ACE + \angle ABD = (\angle A + \angle ABC + \angle ACB) + \angle A$

$= 180° + \angle A$

[Angle sum property of a triangle]

$\Rightarrow \quad \angle ABD + \angle ACE – \angle A = 180° = 2 \times 90°$

Hence, proved.

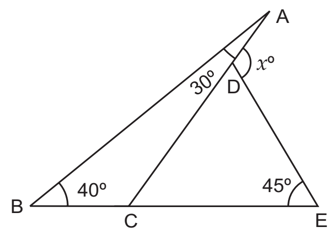
Check Your Progress

Multiple-Choice Question

1. The angles of triangle are in the ratio 4 : 5 : 9. The triangle is
   (*a*) an isosceles triangle.
   (*b*) an obtuse-angled triangle.
   (*c*) an acute-angled triangle.
   (*d*) a right triangle.

Very Short Answer Type Questions

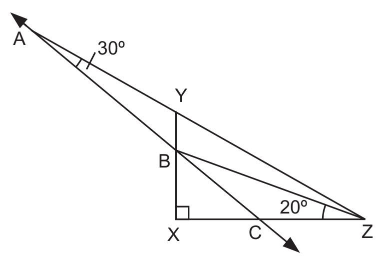
2. An exterior angle of a triangle is 130° and its two opposite interior angles are equal. Then what is the measure of each of these two equal angles?
3. If the measure of each of two equal base angles of an isosceles triangle is 56°, what is the measure of its vertical angle?
4. Find the value of *x* in the given figure.



[CBSE SP 2011]

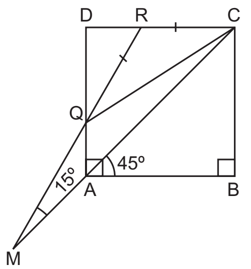
Short Answer Type Questions

5. In ΔYXZ, ∠X = 90°, and YX = XZ. A is a point on ZY produced and the line ABC cuts XY at B and XZ at C. If ∠BAY = 30° and ∠CZB = 20°, find the measures of ∠CBZ and ∠BCZ.



6. In the figure given below, ABCD is a square. R is a point on CD and Q is a point on AD such that CR = QR. CA and RQ produced meet at

M such that ∠AMQ = 15°. Find the measures of ∠ACQ and ∠QCR. Also find the relation between QC and QM.

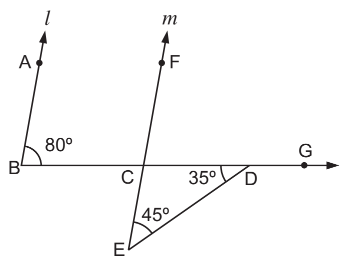


Long Answer Type Questions

7. The four internal bisectors of the angles of a parallelogram form a quadrilateral within this parallelogram. Prove that each angle of this quadrilateral is 90°.

8. The bisectors OA and OC of two acute angles of an obtuse-angled triangle ABC with ∠B = 130° meet at O. Prove that ΔOAC is an obtuse-angled triangle and find the measure of this obtuse angle.

9. In the given figure, show that $l \parallel m$.



[CBSE SP 2013]

Self-Assessment

Multiple-Choice Question

1. If an exterior angle of a triangle is acute, then the triangle must be

(*a*) a right-angled triangle

(*b*) an obtuse-angled triangle

(*c*) an acute-angled triangle

(*d*) an equilateral triangle

Assertion-Reason Type Questions

Directions (Q. Nos. 2 to 3): Each of these questions contains an assertion followed by reason. Read them carefully, and answer the question on the basis of the following options, select the one that best describes the two statements.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

2. **Assertion (A):** Three collinear points will always form a triangle.

Reason (R): A triangle needs at least 3 non-collinear points.

3. **Assertion (A):** Three lines are concurrent if they form a triangle.

Reason (R): Concurrent lines have only one common point.

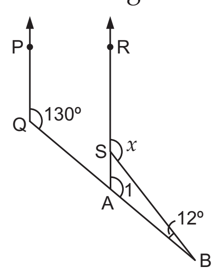
Very Short Answer Type Questions

4. If the vertex angle of an isosceles triangle is 70°, then what is the measure of the exterior angle to one of the base angles of this triangle?

Short Answer Type Questions

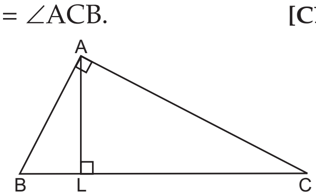
5. In ΔPQR, ∠P + ∠Q = 120°, ∠Q + ∠R = 140°. Find ∠P, ∠Q and ∠R.

6. In the given figure, PQ || RS. Find the values of x if RSA is the straight line.



Long Answer Type Questions

7. In ΔABC, ∠A = 90°. AL is drawn perpendicular to the hypotenuse BC. Prove that ∠BAL = ∠ACB. **[CBSE SP 2010]**



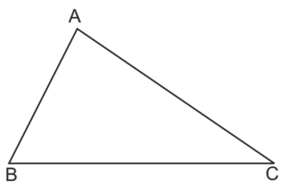
7

Triangles

Knowledge Digest

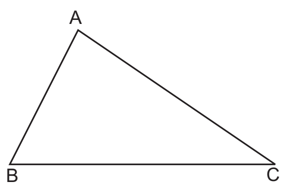
Inequalities in a Triangle

Theorem 7.6: *If two sides of a triangle are unequal, the angle opposite to the longer side is larger (or greater).*



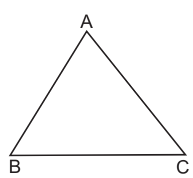
In the given figure, if AC > AB then ∠ABC > ∠ACB.

Theorem 7.7: *In any triangle, the side opposite to the larger (greater) angle is longer.*



In the given figure, if ∠ABC > ∠ACB then AC > AB.

Theorem 7.8: *The sum of any two sides of a triangle is greater than the third side.*



In the given figure, AB + AC > BC, AB + BC > AC and BC + AC > AB.

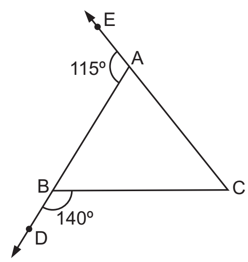
We shall consider the following types of questions:

Type 1: Questions involving the shorter and longer side of a triangle

Type 2: Questions based on the property that the sum of any two sides of a triangle is greater than the third side

Solved Examples

Example 1: *In ΔABC, the exterior angles, ∠BAE = 115° and ∠CBD = 140°. Prove that AB > BC.*



Solution: Given that in ΔABC, the exterior angles ∠BAE = 115° and ∠CBD = 140°.

To prove that AB > BC.

We have

∠BAC + ∠BAE = 180° [∵ Linear pair]

∠BAC = 180° – ∠BAE

= 180° – 115° = 65° ... (1)

∠ABC + ∠CBD = 180° [∵ Linear pair]

∠ABC = 180° – ∠CBD

= 180° – 140° = 40° ... (2)

In ΔABC,

∠ABC + ∠BAC + ∠ACB = 180°

[Angle sum property of a triangle]

$\Rightarrow \quad \angle ACB = 180° - (\angle BAC + \angle ABC)$

$= 180° - (65° + 40°)$

[From (1) and (2)]

$= 180° - 105° = 75°$

Now, $\quad AB$ = side opposite to $\angle ACB = 75°$

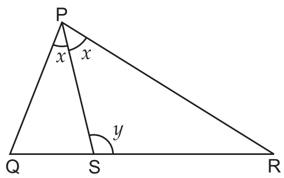
BC = side opposite to $\angle BAC = 65°$

$\because \quad 75° > 65° \qquad \therefore \; AB > BC$

Hence, proved.

Example 2: *In ∠PQR, PR > PQ and PS bisects ∠QPR. Prove that ∠PSR > ∠PSQ.*

Solution: Given that in ΔPQR, PR > PQ and PS bisects ∠QPR.



To prove that ∠PSR > ∠PSQ.

Let $\quad \angle QPS = \angle RPS = x$ and $\angle PSR = y$

Then $\quad y = x + \angle PQS$

[Property of exterior angle of a triangle]

$\therefore \quad \angle PQS = y - x \quad \ldots (1)$

In ΔPSR, we have

$\angle PSR + \angle PRS + \angle RPS = 180°$

[Angle sum property of a triangle]

$y + \angle PRS + x = 180°$

$\angle PRS = 180° - (x + y) \quad \ldots (2)$

Now, given that

$PR > PQ$

$\Rightarrow \quad \angle PQS > \angle PRS$

[Angle opposite to longer side is greater]

$\Rightarrow \quad y - x > 180° - x - y \quad$ [From (1) and (2)]

$\Rightarrow \quad 2y > 180°$

$\Rightarrow \quad y > 90°$

$\Rightarrow \quad \angle PSR > 90° \quad \ldots (3)$

$\therefore \quad -y < -90°$

$\Rightarrow \quad 180° - y < 180° - 90° = 90°$

$\Rightarrow \quad \angle PSQ < 90° \quad \ldots (4)$

[∠PSQ and ∠PSR = y are linear pair]

From (3) and (4), we get

$\angle PSR > \angle PSQ$

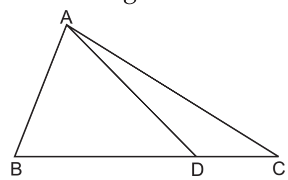
Example 3: *If D is any point on the base BC of a triangle ABC, prove that*

AB + BC + AC > 2 AD

Solution: In ΔABD, we have

$AB + BD > AD \quad \ldots (1)$

[∵ Sum of any two sides of a triangle is greater than the third side]



In ΔADC, we have

$AC + CD > AD \quad \ldots (2)$

[∵ Sum of any two sides of a triangle is greater than the third side]

Adding (1) and (2), we get

$AB + (BD + CD) + AC > AD + AD$

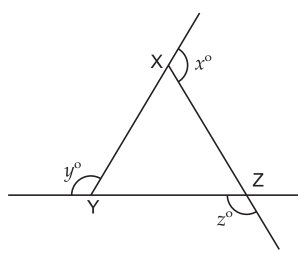
$\Rightarrow \quad AB + BC + AC > 2\,AD \quad [\because BD + CD = BC]$

Hence, proved.

Check Your Progress

Multiple-Choice Questions

1. In ΔPQR if ∠R > ∠Q, then
   (*a*) QR > PR
   (*b*) PQ > PR
   (*c*) PQ < PR
   (*d*) QR < PR
2. In ΔXYZ, the exterior angles at X, Y and Z are of measures $x°$, $y°$ and $z°$ respectively as shown in the given figure.

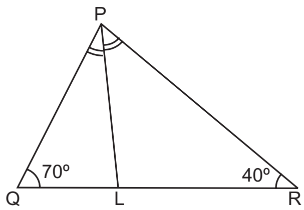


If XY = 8 cm, YZ = 5.5 cm and ZX = 6.8 cm, then

(*a*) $x < z < y$ (*b*) $y < x < z$
(*c*) $x < y < z$ (*d*) $z < y < x$

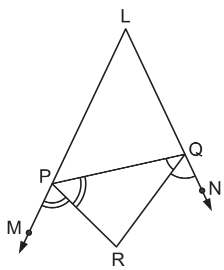
Very Short Answer Type Questions

3. In ΔABC, if $\angle A = 40°$ and $\angle B = 80°$ then arrange the sides of the triangle in ascending order.
4. Which side of ΔPQR will be least if $\angle P = 120°$ and $\angle Q = 25°$?
5. In ΔXYZ, $\angle X = 65°$ and $\angle Z = 70°$. Then find the shortest and longest sides of the triangle.
6. In the given figure, PL bisects $\angle QPR$. If $\angle PQL = 70°$ and $\angle PRL = 40°$, arrange PR, LR and PL in descending order.



Short Answer Type Questions

7. The sides LP and LQ of ΔLPQ are produced to M and N respectively and the bisectors of $\angle MPQ$ and $\angle NQP$ meet at R. If LP > LQ, prove that RQ > RP.



8. Show that the difference of any two sides of a triangle is less than the third side.

Long Answer Type Questions

9. In the given figure, $PM \perp QR$.

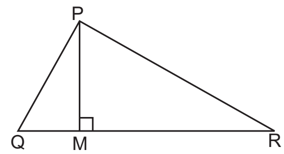
   Prove that

   (*a*) PQ > QM

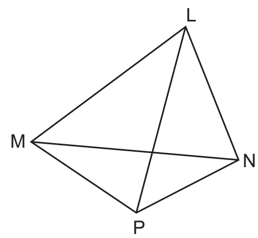
   (*b*) PR > MR

   (*c*) PQ + PR > QR

   (*d*) PQ + PR > 2 PM



10. In the given figure, prove that



   (*a*) PN + NL + LM > MP

   (*b*) PN + NL + LM + MP > 2LP

   (*c*) PN + NL + LM + MP > 2MN

   (*d*) PN + NL + LM + MP > LP + MN

A

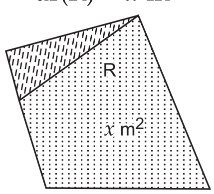
Areas of Parallelograms and Triangles

Knowledge Digest

Area Axioms

1. Every polygonal region R (union of a polygon and its interior) has an area, measured in square units and is denoted by ar(R), i.e. if ar(R) in square metres is x, then we write

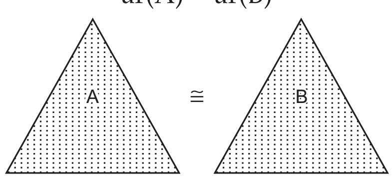
$$ar(R) = x \text{ m}^2$$



ar(R) = x m^2

2. Two congruent regions have equal areas. If A and B are congruent figures then

$$ar(A) = ar(B)$$

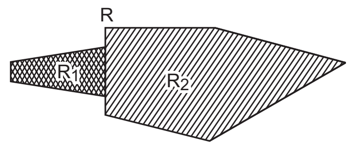


ar(A) = ar(B)

However, the converse of this property is not true. In other words, two figures having equal areas need not be congruent.

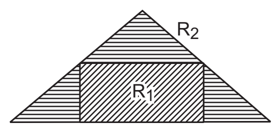
3. If a planar region formed by figure R is made up of two non-overlapping planar regions formed by figures R_1 and R_2, then

$$ar(R) = ar(R_1) + ar(R_2)$$



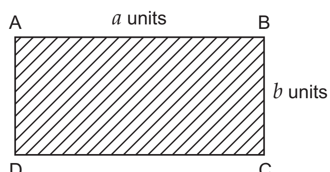
ar(R) = ar(R_1) + ar(R_2)

4. If R_1 and R_2 are two polygonal regions such that R_1 is a part of R_2, then ar(R_1) < ar(R_2).



ar(R_1) < ar(R_2)

5. If ABCD is a rectangular region with AB = a units and BC = b units, then ar(rect ABCD) = ab sq units.

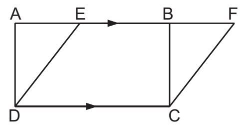
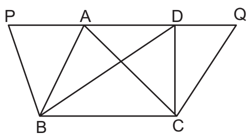
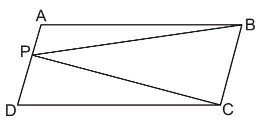


ar(ABCD) = ab sq units

Figures on the Same Base and Between the Same Parallels

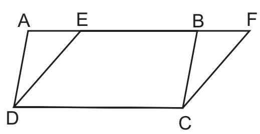
Two figures are said to be on the same base and between the same parallel if they have a common base (side) and the vertices (or the vertex) opposite to the common base of each figure lie on a line parallel to the base.

Illustration

Figure	*Common base*	*Between parallels*
 Rectangle ABCD and Parallelogram EFCD	Base DC	Between parallels DC and AF
 Triangle ABC, Triangle DBC, Trapezium PBCQ	Base BC	Between parallels BC and PQ
 Parallelogram ABCD and Triangle PBC	Base BC	Between parallels BC and AD

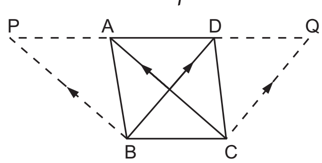
Parallelograms on the Same Base and Between the Same Parallels

Theorem 9.1: *Parallelograms on the same base and between the same parallels are equal in area.*



In the given figure, two parallelograms ABCD and EFCD are on the same base DC and between the same parallels AF and DC.
Then, ar(∥gm ABCD) = ar(∥gm EFCD)

Theorem 9.2: *Two triangles on the same base (or equal bases) and between the same parallels are equal in area.*



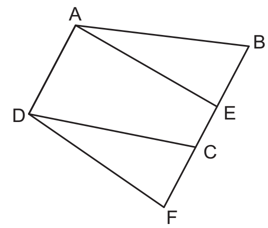
In the given figure, two triangles ABC and DBC are on the same base BC and between the same parallels BC and AD.

Then, ar(ΔABC) = ar(ΔDBC)

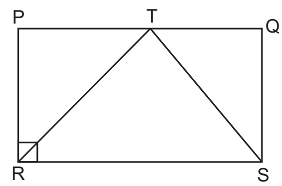
Solved Examples

Example 1: *Which of the following figures lie on the same base and between the same parallels? In such a case, write the common base and the two parallels.*

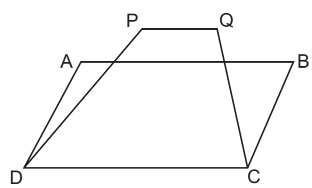
(*a*)



(*b*)



(*c*)



Solution:

(*a*) Parallelograms ABCD and AEFD are on the same base AD and between the same parallels AD and BF.

(*b*) Rectangle PRSQ and ΔTRS are on the same base RS and between the same parallels RS and PQ.

(*c*) Parallelogram ABCD and trapezium PQCD are on the same base CD but not on the same parallels. Thus, they don't have common parallels.

B

Constructions

Knowledge Digest

In this section, we should remember certain results given below:

(*i*) The sum of three angles of a triangle is 180°.

(*ii*) The sum of any two sides of a triangle is greater than its third side.

(*iii*) The difference of any two sides of a triangle is less than its third side.

(*iv*) The hypotenuse is the greatest side of a right-angled triangle.

(*v*) A triangle cannot have more than one obtuse angle.

(*vi*) Each angle of an acute-angled triangle cannot be less than 60°.

(*vii*) Each angle of an equilateral triangle is 60°.

(*viii*) The two angles of an isosceles triangle are equal.

We shall consider the following types of questions:

Type 1: Questions concerning construction of bisectors of angles of measure 30°, 60°, 45°, 90°, 120° etc.

Type 2: Construction of triangles when its different components are given

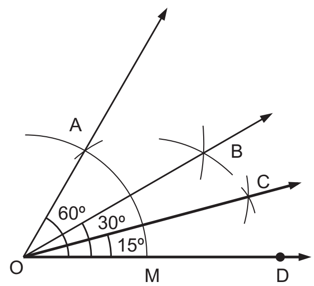
Type 3: Construction of a triangle when the sum or difference of other two sides is given

Solved Examples

Example 1: *Construct an angle of 15° with the help of a pencil, ruler and the compass only.*

Solution:

Steps of Construction:



1. Draw a ray OD. With O as centre and any suitable length as radius, draw an arc cutting OD at a point M.
2. With M as centre and the same length as in step 1 as radius, draw another arc cutting the previous arc at A.
3. Join OA, then ∠AOM = 60°.

   Now, bisect ∠AOD to get an angle of 30°.
4. With M as centre and a suitable length as radius, draw an arc.
5. With A as centre and the same length as in step 4 as radius, draw another arc to cut the previous arc at B.
6. Join OB, then ∠BOD = 30°.
7. Similarly bisect ∠BOD by OC to get an angle of 15°.

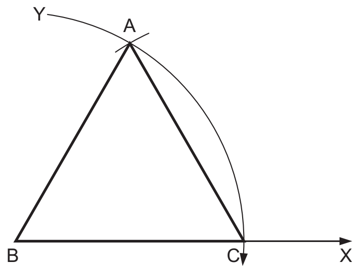
   Then, ∠COD is the required angle of 15°.

Example 2: *Construct an equilateral triangle ABC in which AB = 4 cm. Justify your construction.*

Solution:

Steps of Construction:

1. Draw a ray BX with initial point B.



2. With centre B and radius equal to 4 cm, draw an arc CY, cutting the ray BX at C.
3. With centre C and the same radius, draw an arc cutting the arc CY at A.
4. Join AB and AC.

Then, ΔABC is the required equilateral triangle.

Justification: In ΔABC, we have

BC = 4 cm [Radius BC = 4 cm]

∠ABC = 60° [By construction] …(1)

CA = 4 cm [Radius CA = 4 cm]

∠BAC = 60° …(2)

[Angles opposite to equal sides of a triangle]

∠ACB = 60° …(3)

[Sum of angles of a triangle is 180°]

Hence, ΔABC is the required equilateral triangle.

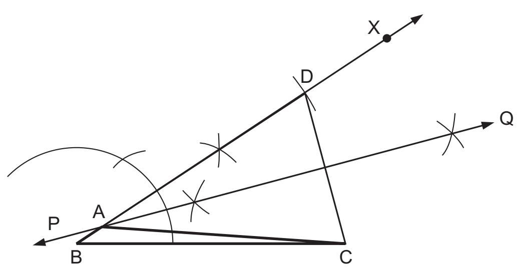
[From (1), (2) and (3)]

Example 3: *Construct a triangle ABC in which the base BC = 5 cm, ∠B = 30° and AB + AC = 5.2 cm. Write the steps of construction and justification of construction.*

Solution:

Steps of Construction:

1. Draw a line segment BC = 5 cm.
2. At B, construct ∠XBC = 30°.



3. Taking B as centre and radius equal 5.2 cm, draw an arc cutting BX at D.
4. Join DC.
5. Draw a perpendicular bisector PQ of the line segment DC, cutting BD at A.
6. Join AC.

Then, ΔABC is the required triangle.

Justification: Since A lies on the perpendicular bisector of DC, hence, it is equidistant from D and C

∴ AD = AC …(1)

Now, AB + AC = AB + AD [From (1)]

= BD = 5.2 cm

[By construction]

Hence, ΔABC is the required triangle.

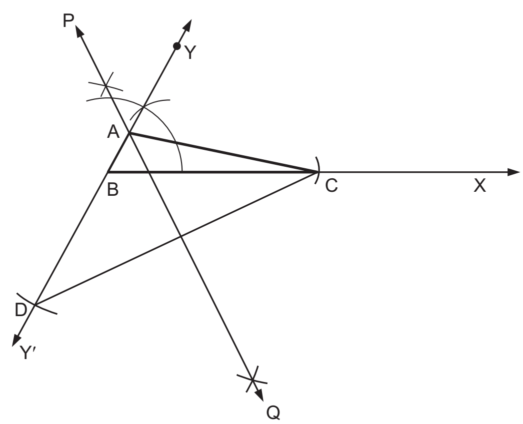
Example 4: *Construct a triangle with base of length 5 cm, one base angle is 60° and the difference of the other two sides is 3.8 cm.*

Solution:

Steps of Construction:

1. Draw a ray BX and cut off a line segment BC = 5 cm from it.
2. Draw a ray BY making an angle 60° with BC and produce YB to form a line YBY′.
3. Cut off a line segment BD = 3.8 cm from BY.
4. Join CD.
5. Draw the perpendicular bisector PQ of DC meeting BY at A.
6. Join AC.

Then, ΔABC is the required triangle.



Justification: Since A lies on the perpendicular bisector of DC,

∴ AC = AD = AB + BD

⇒ AC – AB = BD = 3.8 cm [From (1)]

Hence, ΔABC is the required triangle.

Check Your Progress

Multiple-Choice Questions

1. With the help of a ruler, compass and pencil only, it is possible to construct an angle of
(*a*) 8.5° (*b*) 37.5°
(*c*) 55° (*d*) 70°

2. We cannot construct any triangle ABC when BC = 3 cm, ∠B = 30°, if (AB – AC) is equal to
(*a*) 2.5 cm (*b*) 1 cm
(*c*) 1.8 cm (*d*) 3.5 cm

Very Short Answer Type Questions

3. Can you construct a triangle with sides of length 5 cm, 7 cm and 8 cm? State the reason.
4. How many triangles can you draw through three collinear points? Explain the reason.
5. Using a ruler, a pencil and a compass, draw an angle of 7.5°.

Short Answer Type Questions

6. Construct a right-angled triangle whose hypotenuse measures 10 cm and the length of one of its side containing the right angle measures 8 cm. Measure the third side of the triangle and verify it using Pythagoras' theorem.

Long Answer Type Question

7. Construct a triangle PQR in which QR = 9 cm, ∠Q = 45° and PQ – PR = 3 cm. Write the steps of construction and justification.

Higher Order Thinking Skills (HOTS) Questions

1. Construct a triangle ABC such that the ratio of its two sides is 1 : 3. The difference of these two sides is 10 cm and one base angle between these two sides is 45°. Write the steps of construction and proof.
2. Consider a triangle ABC with sides BC = *a*, CA = *b* and AB = *c*. We denote the angles of the triangle ABC by the corresponding capital letters. State in which of the following cases, you can construct a unique triangle, two triangles and no triangle at all.
(*a*) *b* = 3 cm, *c* = 8 cm and ∠B = 30°
(*b*) *b* = 4 cm, *c* = 8 cm and ∠B = 30°
(*c*) *b* = 7 cm and *c* = 4 cm and ∠B is any acute angle
(*d*) *b* = *c* = 8 cm and ∠B is any acute angle
(*e*) *b* = 3 cm, *c* = 4 cm and ∠B = 30°

Self-Assessment

Multiple-Choice Questions

1. The angle which cannot be constructed using a ruler and compass only is
(*a*) 7.5° (*b*) 44°
(*c*) 75° (*d*) 105°

2. We cannot construct any ΔABC when AB = 4 cm, ∠B = 40°, if (BC – AC) is equal to
(*a*) 3.5 cm (*b*) 3 cm
(*c*) 4 cm (*d*) 2.5 cm

Very Short Answer Type Questions

3. With the help of a pencil, a ruler and a compass only, it is not possible to construct an angle of 35°. Is this statement true or false?
4. How many distinct triangles can be drawn through four non-collinear points? Name them.
5. Construct an angle of $22\frac{1}{2}^\circ$ by using a pencil, a ruler and a compass only.
6. Construct an equilateral triangle of side 3.5 cm.

Short Answer Type Questions

7. Construct a triangle ABC is which AB = 6 cm, ∠A = 45° and AC – BC = 2 cm. **[CBSE SP 2011]**
8. Construct a triangle ABC in which BC = 8 cm, ∠B = 30° and AB – AC = 3.5 cm **[CBSE SP 2011]**

Long Answer Type Question

9. The sides of a triangle are in the ratio 2 : 3 : 4. If the perimeter of the triangle is 12.6 cm, construct the triangle by taking help of the lengths of the three sides only.

10

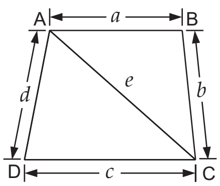
Heron's Formula

Knowledge Digest

Application of Heron's Formula for Finding Areas of Quadrilaterals

Heron's formula can be used to find the areas of quadrilaterals by dividing them into triangular parts. The formula is then applied to find the areas of the triangles formed.

Quadrilateral whose all Sides and one Diagonal are given



Let ABCD be quadrilateral in which a, b, c and d are the measures of the sides and e be the length of one diagonal. Then,

Area of quadrilateral ABCD = Area of ΔABC + Area of ΔADC

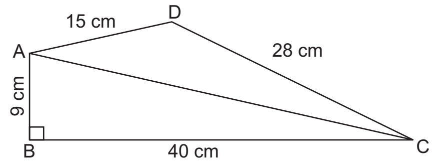
We shall consider the following type of questions:

Type 1: Finding areas of various quadrilaterals by dividing them into triangles and then using Heron's formula

Solved Examples

Example 1: *Find the area of a quadrilateral ABCD whose sides are AB = 9 cm, BC = 40 cm, CD = 28 cm, AD = 15 cm and ∠ABC = 90°.*

Solution: In right ΔABC, we have



$$(AC)^2 = (AB)^2 + (BC)^2$$ [By Pythagoras' Theorem]

$$\Rightarrow \quad AC = \sqrt{(AB)^2 + (BC)^2}$$

$$\Rightarrow \quad AC = \sqrt{(9)^2 + (40)^2} \text{ cm}$$

$$= \sqrt{1681} \text{ cm}$$

$$\Rightarrow \quad AC = 41 \text{ cm}$$

Area of right ΔABC

$$= \frac{1}{2} \times BC \times AB$$

$$= \left(\frac{1}{2} \times 40 \times 9\right) \text{cm}^2 = 180 \text{ cm}^2 \quad \ldots(1)$$

In ΔACD, we have

a = AC = 41 cm, b = CD = 28 cm, c = AD = 15 cm

$$\therefore \quad s = \frac{a+b+c}{2} = \left(\frac{41+28+15}{2}\right) \text{ cm}$$

$$= \frac{84}{2} \text{ cm} = 42 \text{ cm}$$

$$\text{Area of } \Delta ACD = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{42(42-41)(42-28)(42-15)} \text{ cm}^2$$

$$= \sqrt{42 \times 1 \times 14 \times 27} \text{ cm}^2$$

$$= \sqrt{7 \times 3 \times 2 \times 7 \times 2 \times 3 \times 3 \times 3} \text{ cm}^2$$

$$= 7 \times 3 \times 2 \times 3 \text{ cm}^2$$

$$= 126 \text{ cm}^2 \quad \ldots(2)$$

Area of quadrilateral ABCD

= Area of right ΔABC + Area of ΔACD

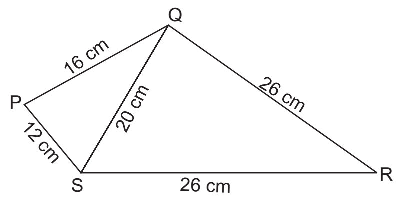
= (180 + 126) cm^2 [From (1) and (2)]

= 306 cm^2.

Hence, the area of the quadrilateral ABCD is 306 cm^2.

Example 2: *Find the area of a quadrilateral PQRS in which PQ = 16 cm, QR = RS = 26 cm and PS = 12 cm and the diagonal QS = 20 cm.*

Solution: We shall first find the area of ΔQRS.



In ΔQRS, let a = 26 cm, b = 26 cm and c = 20 cm.

Let s be the semi-perimeter of ΔQRS.

$$\therefore \quad s = \frac{1}{2}(26 + 26 + 20) \text{ cm} = \frac{72}{2} \text{ cm} = 36 \text{ cm}$$

Area of ΔQRS = $\sqrt{s(s-a)(s-b)(s-c)}$ [By Heron's formula]

$= \sqrt{36(36-26)(36-26)(36-20)}$ cm^2

$= \sqrt{36 \times 10 \times 10 \times 16}$ cm^2

= 6 × 10 × 4 cm^2 = 240 cm^2

Again, we shall now find the area of ΔPQS.

In ΔPQS, let a_1 = 12 cm, b_1 = 20 cm, c_1 = 16 cm.

Let s_1 be the semi-perimeter of ΔPQS.

$$\therefore \quad s_1 = \frac{1}{2}(12 + 20 + 16) \text{ cm} = \frac{48}{2} = 24 \text{ cm}$$

Area of ΔPQS = $\sqrt{s_1(s_1-a_1)(s_1-b_1)(s_1-c_1)}$ [By Heron's formula]

$= \sqrt{24(24-12)(24-20)(24-16)}$ cm^2

$= \sqrt{24 \times 12 \times 4 \times 8}$ cm^2

$= \sqrt{2 \times 12 \times 12 \times 4 \times 4 \times 2}$ cm^2

= 2 × 12 × 4 cm^2 = 96 cm^2

Diagonal QS divides the quadrilateral into two triangles QRS and PQS.

∴ Area of quadrilateral PQRS

= Area of ΔQRS + area of ΔPQS

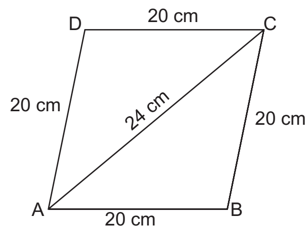
= (240 + 96) cm^2

= 336 cm^2

Hence, the area of the quadrilateral PQRS is 336 cm^2.

Example 3: *Find the area of a rhombus, one side of which measures 20 cm and one of whose diagonals is 24 cm.* **[CBSE 2013]**

Solution: Let ABCD represent the given rhombus of side 20 cm and one diagonal AC = 24 cm.



In ΔABC, we have

a = AB = 20 cm, b = BC = 20 cm, c = AC = 24 cm

$$\therefore s = \frac{a+b+c}{2} = \frac{20+20+24}{2} \text{ cm}$$

$$= \frac{64}{2} \text{ cm} = 32 \text{ cm}$$

Area of ΔABC = $\sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{32(32-20)(32-20)(32-24)}$ cm^2

$= \sqrt{32(12)(12)(8)}$ cm^2

$= \sqrt{2 \times 2 \times 8 \times 12 \times 12 \times 8}$ cm^2

= 2 × 8 × 12 cm^2

= 192 cm^2 ...(1)

We know that the diagonal of rhombus divides it into two congruent triangles.

∴ ΔABC ≅ ΔADC

⇒ Area of ΔABC = Area of ΔADC ...(2)

Area of rhombus ABCD

= Area of ΔABC + Area of ΔADC

= 2 × Area of ΔABC [From (2)]

= 2 × 192 cm^2 [From (1)]

= 384 cm^2

Hence, the area of the rhombus is 384 cm^2.

11

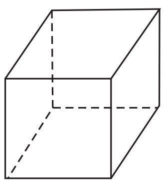
Surface Areas and Volumes

Knowledge Digest 1

Surface Area of a Cuboid and a Cube

Solid: Any object that occupies space is called a solid.

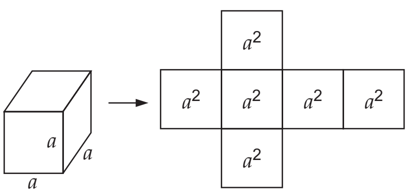
Cube: A solid bounded by six squares is called a cube.



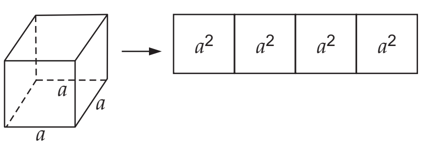
For example, ice cubes, sugar cubes and dice.

Consider a cube of edge = a

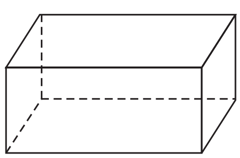
$\therefore$ Total surface area of the cube = $6a^2$



Lateral surface area of the cube = $4a^2$



Cuboid: A solid bounded by six rectangular faces is called a cuboid.

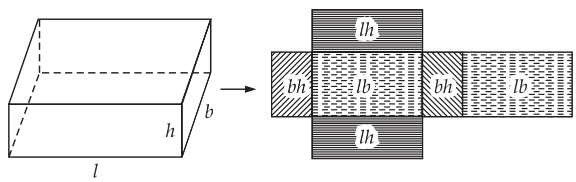


For example, books, matchboxes and bricks.

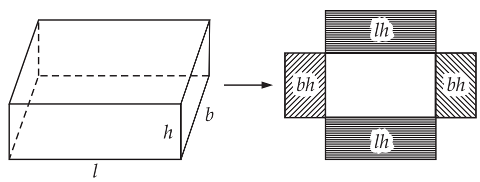
Consider a cuboid of length l, breadth b and height h.

$\therefore$ Total surface area of the cuboid

$= 2(lb + bh + hl)$



Lateral surface area of the cuboid = $2(l + b)h$



Surface Area of a Right Circular Cylinder

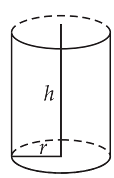
Cylinder: A solid obtained by revolving a rectangle about its one side as its axis is called a right circular cylinder.

Curved surface area of the cylinder = $2\pi rh$

Total surface area of the cylinder

= Curved surface area + Area of two circular ends

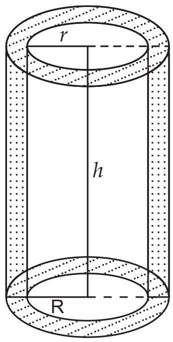
$= 2\pi rh + 2\pi r^2$



Total surface area of the cylinder = $2\pi r(h + r)$

Hollow Cylinder: A solid bounded by two coaxial cylinders of the same height and different radii is called a hollow cylinder. For example, rubber tubes, capillary tubes and iron pipes.

Consider a hollow cylinder with external radius R, internal radius r and height h.



$\therefore$ Thickness of the cylinder = $R - r$

Area of the cross section = $\pi(R^2 - r^2)$

External curved surface area = $2\pi Rh$

Internal curved surface area = $2\pi rh$

Curved surface area

= External surface area + Internal surface area

= $2\pi Rh + 2\pi rh = 2\pi(R + r)h$

Total surface area

= External surface area + Internal surface area + Area of 2 base rings

= $2\pi Rh + 2\pi rh + 2\pi(R^2 - r^2)$

= $2\pi[Rh + rh + (R^2 - r^2)]$

We shall consider the following types of questions:

Type 1: Questions involving surface areas of cuboids and cubes

Type 2: Questions involving surface areas of a cylinder

Solved Examples

Example 1: *Wall paper 260 m long and 30 cm broad is required to cover the walls of a room. The length of the room is 7 m and its breadth is twice its height. Determine the height of the room.*

Solution: Let the length, breadth and height of the room be l, b and h respectively.

Then, $l = 7$ m and $b = 2h$ m

$\therefore$ The area of the four walls

= lateral surface area of the cuboidal room

= $2(l + b)h = 2(7 + 2h)h$ m^2

Since, lateral surface area of room

= Area of the wallpaper

$\therefore \quad 2(7 + 2h)h = 260 \times 0.30 \text{ m}^2 = 78 \text{ m}^2$

$\Rightarrow \quad 7h + 2h^2 = 39$

$\Rightarrow \quad 2h^2 + 7h - 39 = 0$

$$\Rightarrow \quad h = \frac{-7 \pm \sqrt{49 + 8 \times 39}}{4} = \frac{-7 \pm \sqrt{361}}{4}$$

$$= \frac{-7 \pm 19}{4}$$

= 3, neglecting negative value of h which is impossible.

Hence, the height of the room is 3 m.

Example 2: *The diameter of a cylindrical garden roller is 1.4 m and its length is 2 m. How much area will it cover in 5 revolutions?*

Solution: Let r be the radius and h the height of the cylindrical roller.

Then, $r = \frac{1.4}{2}$ m = 0.7 m and $h = 2$ m

Area covered by the roller is one revolution

= Curved surface area of the cylindrical roller

= $2\pi rh$

= $2 \times \frac{22}{7} \times 0.7 \times 2$ m^2

= $2 \times 22 \times 0.1 \times 2$ m^2

= 8.8 m^2

$\therefore$ Area covered by the roller in 5 revolutions

= 8.8×5 m^2 = 44 m^2

Hence, the area covered in 5 revolutions is 44 m^2.

Example 3: *The total surface area of a hollow metallic cylinder open at both ends of external radius 8 cm and height 10 cm is 338π cm^2. Taking r to be inner radius, find the thickness of the metal in the cylinder.*

Solution: Let R be the external radius of the cylinder and h be the height of the cylinder.

Then, $R = 8$ cm and $h = 10$ cm

$\therefore$ The total surface area of hollow cylinder

= $2\pi Rh + 2\pi rh + 2\pi R^2 - 2\pi r^2$

= $2\pi(Rh + rh + R^2 - r^2)$

Given, total surface area of a hollow metallic cylinder = 338π cm^2

$\therefore \quad 2\pi(Rh + rh + R^2 - r^2) = 338\pi$

$\Rightarrow \quad (10 \times 8 + 10r + 64 - r^2) = 169$

$\Rightarrow \quad 80 + 10r + 64 - r^2 = 169$

$\Rightarrow \quad r^2 - 10r + 169 - 144 = 0$

$\Rightarrow \quad r^2 - 10r + 25 = 0$

$\Rightarrow \quad (r - 5)^2 = 0$

$\Rightarrow \quad r = 5$

$\therefore$ The thickness $= R - r = (8 - 5)$ cm $= 3$ cm

Hence, the thickness of the metal in the cylinder is 3 cm.

Example 4: *The radii of two right circular cylinders are in the ratio 2 : 3. If their heights are in the ratio 5 : 4, find the ratio of their curved surface areas.*

Solution: Let the radii of the two cylinders be r and R.

Then $r = 2x$ and $R = 3x$

and heights be $h = 5y$ and $H = 4y$,

where x and y are positive real numbers.

$\therefore$ Their curved surfaces areas are

$$S_1 = 2\pi rh = 2\pi \times 2x \times 5y = 20\pi xy$$

and $$S_2 = 2\pi RH = 2\pi \times 3x \times 4y = 24\pi xy$$

$$\frac{S_1}{S_2} = \frac{20\pi xy}{24\pi xy} = \frac{5}{6}$$

$\therefore \quad S_1 : S_2 = 5 : 6$

Hence, the ratio of their curved surface area is 5 : 6.

Check Your Progress 1

Multiple-Choice Questions

1. The difference between the total surface area of a cube of side 6 cm and its lateral surface area is

   (*a*) 144 cm^2 (*b*) 72 cm^2
   (*c*) 36 cm^2 (*d*) 100 cm^2

2. The curved surface area of a cylinder is 132 cm^2 and its height is 14 cm. Then its base radius is

   (*a*) 2.5 cm (*b*) 1 cm
   (*c*) 2 cm (*d*) 1.5 cm

Short Answer Type Question

3. The diameter of a roller, 1 m 40 cm long, is 80 cm. If it takes 600 complete revolutions to level a playground, find the cost of levelling the ground at 75 paise per sq metre.

Long Answer Type Questions

4. A metal pipe is 77 cm long. The inner diameter of the cross section is 4 cm, the outer diameter being 4.4 cm. Find its
   (*a*) inner curved surface area
   (*b*) outer curved surface area
   (*c*) total surface area.

5. The curved surface area of a 15 cm high cylinder is 2310 cm^2. A wire of diameter 6 mm is wound around it so as to cover it completely. Find the length of the wire.

6. Four equal cubes are placed side by side in a row to form a cuboid. Find the ratio of the total surface area of the cuboid to the total surface area of the four cubes.

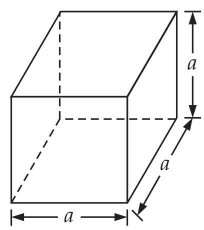
Knowledge Digest 2

Volume of a Cube

Consider a cube of edge $= a$

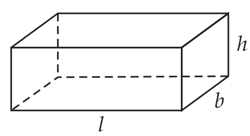
Volume of the cube = edge × edge × edge $= a^3$

Diagonal of the cube $= \sqrt{3}a$



Volume of Cuboid

Consider a cuboid of length l, breadth b and height h.



Then, Volume of the cuboid
= base area × height
= length × breadth × height $= lbh$

Diagonal of the cuboid $= \sqrt{l^2 + b^2 + h^2}$

Volume of a Cylinder

Consider a cylinder whose height is h and the radius of whose base is r. Then,

Volume of the cylinder
= area of the cylinder base × height $= \pi r^2 h$

$\Rightarrow$ Volume of cylinder $= \pi r^2 h$

We shall consider the following types of questions:

Type 1: Questions involving volume of a cuboid, cube and length of the diagonal of a cuboid or a cube

Type 2: Questions involving volume of a cylinder

Solved Examples

Example 5: *The length of a cold storage is double its breadth. Its height is 3 m. The area of its four walls (including doors) is 108 m². Find its volume.*

Solution: Let the length, breadth and height of the cold storage be l, b and h respectively.

Then, $l = 2b$

$h = 3$ m

$\therefore$ The area of the four walls

$= 2(l + b)h = 2(2b + b)\ 3$ m

$= 18b$ m

Given, area of the four walls = 108 m²

$\therefore$ $18b \text{ m} = 108 \text{ m}^2$

$\Rightarrow$ $b = \dfrac{108 \text{ m}^2}{18 \text{ m}} = 6$ m

$\therefore$ Breadth = 6 m,

Length = 2 × 6 m = 12 m

$\therefore$ Hence, the volume of the cold storage

$= l \times b \times h$

$= (12 \times 6 \times 3) \text{ m}^3$

$= 216 \text{ m}^3$

Hence, the volume of the storage is 216 m³.

Example 6: *Find the length of the longest rod that can be placed in a room 12 m long, 9 m broad and 8 m high.*

Solution: Let the length, breadth and height of the rod be l, b and h respectively. Then,

$l = 12$ m,

$b = 9$ m,

and $h = 8$ m

Length of the longest rod which can be placed in a room = length of the diagonal of cuboid

$= \sqrt{l^2 + b^2 + h^2}$

$= \sqrt{12^2 + 9^2 + 8^2}$ m

$= \sqrt{144 + 81 + 64}$ m

$= \sqrt{289}$ m = 17 m

Hence, the length of the longest rod that can be placed in a room is 17 m.

Example 7: *Water flows in a tank 150 m × 100 m at the base, through a pipe whose cross section is 2 dm by 1.5 dm at the speed of 15 km/h. In what time will the water be 3 m deep?* **[CBSE SP 2011]**

Solution: Suppose the water will be 3 m deep in x hours.

Area of cross section of the pipe

$= \dfrac{2}{10} \text{ m} \times \dfrac{1.5}{10} \text{ m}$

$= 0.2 \text{ m} \times 0.15 \text{ m}$ ...(1)

Water flows through the pipe at the rate of 15 km/h.

$\therefore$ Length of water column flowing through the pipe in

1 hour = 15 km = 15000 m ...(2)

$\Rightarrow$ Length of water column flowing through the pipe in

x hours = $15000x$ m [Using (1) and (2)]

$\Rightarrow$ Volume of water flowing through the pipe in

x hours = $0.2 \times 0.15 \times 15000x$ m³ ...(3)

$\Rightarrow$ Volume of water collected in the tank in x hours

$= 150 \times 100 \times 3$ m³ ...(4)

[Volume of water flowing through the pipe in x minutes] = [Volume of water collected in the tank in x minutes]

[Using (3) and (4)]

$0.2 \times 0.15 \times 15000x = 150 \times 100 \times 3$

$x = \dfrac{150 \times 100 \times 3}{0.2 \times 0.15 \times 15000} = 100$

Hence, the water will be 3 m deep in 100 hours.

Example 8: *A metal pipe has a bore with internal diameter 5 cm. The pipe is 2 mm thick all around. Find the mass of 3 m long pipe if 1 cm³ of the metal has a mass of 7 g.*

Solution: The internal radius of the cylinder,

$r = \dfrac{5}{2}$ cm = 2.5 cm

The external radius of the cylinder,

R = internal radius + thickness

= 2.5 cm + 2 mm

= (2.5 + 0.2) cm

= 2.7 cm

The height or length of the pipe,

$$h = 3 \text{ m} = 300 \text{ cm}$$

Hence, the volume of the cylindrical pipe

$$= \pi(R^2 - r^2)h$$

$$= \frac{22}{7} \times (2.7^2 - 2.5^2) \times 300 \text{ cm}^3$$

$$= \frac{22}{7} \times (2.7 + 2.5)(2.7 - 2.5) \times 300 \text{ cm}^3$$

$$= \frac{22}{7} \times 5.2 \times 0.2 \times 300 \text{ cm}^3$$

$$= \frac{22 \times 52 \times 2 \times 3}{7} \text{ cm}^3 = \frac{6864}{7} \text{ cm}^3$$

Given, mass of 1 cm^3 of metal = 7 g

$\therefore$ The mass of $\frac{6864}{7}$ cm^3 of metal pipe will have a mass

$$= \frac{6864}{7} \times \frac{7}{1000} \text{ kg}$$

$$= \frac{6864}{1000} \text{ kg} = 6.864 \text{ kg}$$

Hence, the mass of 3 m long pipe is 6.864 kg.

Example 9: *A cylinder whose height is two third of its diameter, has the same volume as a sphere of radius 4 cm. Calculate the radius of the base of the cylinder.*

Solution: Let the radius of the base of the cylinder be r and height be h.

Then $$h = \frac{2}{3} \times 2r = \frac{4r}{3} \text{ cm} \quad \ldots (1)$$

Volume of the sphere of radius 4 cm

$$= \frac{4}{3}\pi(\text{radius})^3$$

$$= \frac{4}{3}\pi \times 4^3 \text{ cm}^3$$

Volume of the cylinder = $\pi r^2 h$

$$= \pi r^2 \times \frac{4r}{3} \text{ cm}^3 \text{ [From (1)]}$$

$$= \frac{4\pi r^3}{3} \text{ cm}^3$$

Given, volume of sphere = Volume of cylinder

$$\therefore \quad \frac{4\pi}{3} \times 4^3 = \frac{4\pi r^3}{3}$$

$$\Rightarrow \quad r^3 = 4^3$$

$$\Rightarrow \quad r = 4$$

Hence, the radius of the base of cylinder is 4 cm.

Example 10: *The diameters of the internal and external surfaces of a hollow hemispherical shell are 6 cm and 10 cm respectively. If it is melted and recast into a solid cylinder of diameter 14 cm, find the height of the cylinder.*

Solution: Internal radius of the hollow hemisphere,

$$r = \frac{6}{2} \text{ cm} = 3 \text{ cm}$$

External radius of the hollow hemisphere,

$$R = \frac{10}{2} \text{ cm} = 5 \text{ cm}$$

$\therefore$ Volume of the hemispherical shell

$$= \frac{2}{3}\pi(R^3 - r^3)$$

$$= \frac{2}{3}\pi(5^3 - 3^3) \text{ cm}^3$$

$$= \frac{2\pi}{3}(125 - 27) \text{ cm}^3$$

$$= \frac{2\pi}{3} \times 98 \text{ cm}^3$$

Let x be the radius of solid cylinder and h be its height. Then, $x = \frac{14}{2}$ cm = 7 cm

$\therefore$ Volume of the solid cylinder = $\pi \times x^2 \times h$

$$= \pi \times 7^2 \times h \text{ cm}^3$$

Volume of solid cylinder

= Volume of hollow hemisphere

$$\therefore \quad 7^2\pi h = \frac{2\pi}{3} \times 98$$

$$\Rightarrow \quad h = \frac{2 \times 98}{3 \times 49} = \frac{4}{3} \text{ cm}$$

Hence, the height of the cylinder is $\frac{4}{3}$ cm.

Check Your Progress 2

Multiple-Choice Questions

1. The total surface area of a cube is 150 cm^2. Then the volume of the cube is

(*a*) 100 cm^3 (*b*) 125 cm^3

(*c*) 96 cm^3 (*d*) 150 cm^3

2. A cylindrical piece of maximum volume has to be cut out of an iron cube of edge 6 cm. Then the maximum volume of the iron cylinder is

(*a*) 36π cm^3 (*b*) 81π cm^3

(*c*) 27π cm^3 (*d*) 54π cm^3

Very Short Answer Type Questions

3. A cube and a sphere have equal surface areas. Prove that their volumes are in the ratio $1 : \sqrt{\frac{\pi}{6}}$.
4. The diameter of a copper sphere is 6 cm. The sphere is melted and drawn into a long wire of uniform thickness. If the length of the wire is 36 cm, find its radius.
5. A hollow cylinder open at one end, with base radius r and height h contains the same volume of water which is three times the volume of a hollow cone of the same base radius and same height. Is this statement true or false? State with reason.
6. The ratio of the heights of a cone and a cylinder is 2 : 1 and the ratio of the radii of their bases is 3 : 1 respectively. Find the ratio of their volumes.
7. A cone, a hemisphere and a cylinder stand on equal bases and have the same height equal to the radius of the bases of these solids. Show that their volumes are in the ratio 1 : 2 : 3.

Short Answer Type Questions

8. A conical vessel of base radius 9 cm and height 20 cm is full of water. A part of this water is now poured into a hollow cylinder, closed at one end, till the cylinder is completely filled with water. If the base radius and the height of the cylinder are 6 cm and 10 cm respectively, find the volume of water which is left in the cone. (Take $\pi = 3.14$)
9. Read the following passage and answer the questions that follows:

   On a hot summer day in Rajasthan, a poor woman named Aasha set-up a stall near her house in the village under the shade of a big Banyan tree. She bought seven earthen pots, each of radius 40 cm. She took cold water in all the seven earthen pots and served this water to thirsty people passing by in cylindrical glasses each of radius 5 cm and height 14 cm. Each earthen pot was full up to three-fourths of its height.

   (*a*) If Aasha fills each glass completely with water, find the number of people she can give water to.

   (*b*) What is the volume of each cylindrical glasses?

Long Answer Type Questions

10. A circular well of diameter 5.6 m and depth 25 m is to be dug out and the earth so dug is spread on a rectangular plot of length 21 m and breadth 13 m. Find the volume of the earth dug out, area of the rectangular plot and the height of the platform formed by spreading the earth on the rectangular plot.
11. The difference between the outside and the inside surfaces of a cylindrical metallic pipe 21 cm long is 66 cm^2. If the pipe is made of 214.5 cm^3 of metal, find the inner and the outer radii of the pipe.

12

Statistics

Knowledge Digest 1

Collection of Data

Statistical data are of two types:

(*i*) **Primary Data:** The data collected by the investigator herself or himself for a specific purpose is known as primary data. As the primary data is original in character, it is highly reliable.

(*ii*) **Secondary Data:** The data collected by someone, other than the investigator, is known as secondary data. This data should be used after evaluating the credibility of the source of data and the methods used to collect it.

Presentation of Data

Data is often in an unorganised and random manner. Organising the collected data by using variety of tools such as distribution tables, pie charts, graphs, etc. to provide information about its salient features is known as presentation of data.

Some Terms Related to Data

Raw Data: The numerical data recorded in its original form is called raw (or ungrouped) data.

Variables or Variates: The quantity measured in an experiment is called a variable. For example, number of workers in a factory, heights, ages and weights of people.

Variables are of two types:

(*i*) continuous

(*ii*) discontinuous or discrete

A variable which can take any value between two given values is called a continuous variable. For example, ages, weights and heights of people are continuous variables.

A variable which cannot take all possible values between two given values is called a discontinuous or discrete variable. It can take only a finite set of values. For example, number of students in a school, number of members in different families.

Array: The raw data when arranged in an ascending or descending order of magnitude is called an array.

Range: The difference between the highest and lowest values of the variate in the data is called the range of the data.

Grouped Data: The data presented by condensing into classes or groups is known as grouped data.

Frequency: The number of times an observation occurs in the given data, is called the frequency of the observation.

Ungrouped or Discrete Frequency Distribution: A tabular arrangement of given numerical data displaying the frequency of each observation, is called an ungrouped or discrete frequency distribution and the table itself is called an ungrouped or discrete frequency distribution table.

Construction of an Ungrouped or Discrete Frequency Distribution Table:

(*i*) Obtain the given raw data.

(*ii*) Draw three columns first for variates, second for tally marks and third for total of tally marks representing corresponding frequency to each variate.

(*iii*) For each observation given in the raw data, put a bar opposite to it in the second column.

Record these tally marks (bars) in bunches of five, with the fifth one crossing the other four diagonally.

(*iv*) Count the number of tally marks for each variate and write in the third column of frequency.

Frequency Distribution of Grouped Data

If the number of observations in a data is large and the difference between the highest and the least observation is large, then the data is condensed into convenient number of groups called classes.

A tabular arrangement of given numerical data showing the classes together with corresponding frequencies is called a grouped frequency distribution and the table itself is called a grouped frequency distribution table.

Some Terms Related to Grouped Frequency Distribution

Class Intervals: Groups into which all the observations in the given data are condensed, are known as class intervals.

Class Limits: The end numbers of a class interval are called the class limits. The smaller number is the lower class limit and the larger number is the upper class limit. For example, in the class interval 10 – 20, 10 is the lower class limit and 20 is the upper class limit.

Types of Grouped Frequency Distribution

(*i*) **Exclusive Form (or Continuous Form).** The grouped frequency distribution in which the upper limit of one class is the lower limit of the next class is said to be an exclusive form of grouped frequency distribution. In this type of distribution, upper limit of each class is excluded and the lower limit is included.

Example: Suppose the marks obtained by some students are tabulated into classes 0 – 10, 10 – 20, 20 – 30 and 30 – 40. Then, in class 0 – 10, 0 is included but 10 is excluded. In class 10 – 20, 10 is included and 20 is excluded and so on.

(*ii*) **Inclusive Form (or Discontinuous Form).** The grouped frequency distribution in which the upper limit as well as the lower limit are included in the class, is said to be an inclusive form of grouped frequency distribution.

Example: Suppose the ages (in years) of students are tabulated into classes 9 – 11, 12 – 14, 15 – 17 and so on. Then, in class 9 – 11, both 9 and 11 are included. In class 12 – 14, both 12 and 14 are included and so on.

Class Boundaries or True Class Limits

(*i*) In exclusive form of grouped frequency distribution, the upper and lower limits of a class are respectively called the true upper limit and true lower limit.

For example, consider the classes 10 – 20 and 20 – 30.

Then, 10 is the true lower limit and 20 is the true upper limit for class interval 10 – 20.

Also, 20 is the true lower limit and 30 is the true upper limit of the class interval 20 – 30.

(*ii*) In an inclusive form of grouped frequency distribution, the difference between the lower limit of a class and the upper limit of its preceeding class has to be found out. Half of this difference is added to each of the upper limits to obtain the true upper limits and subtracted from each of the lower limits to get the true lower limits.

For example, consider the classes 51 – 55 and 56 – 60.

The lower limit of 56 – 60 is 56. The upper limit of 51 – 55 is 55.

The difference = 56 – 55 = 1

Half the difference = $\frac{1}{2}$ = 0.5

So the new class interval formed from 51 – 55 is (51 – 0.5) – (55 + 0.5), i.e. 50.5 – 55.5.

In this manner, the continuous classes bounded by true lower limits and true upper limits are formed.

Note that by convention 55.5 is considered in the class 55.5 – 60.5 and not in 50.5 – 55.5.

Class Size (or Width)): The difference between the true upper limit and the true lower limit of a class is known as its class size.

Class Mark: The class mark is the mid-point of the class interval.

$$\text{Class mark} = \frac{\text{Upper limit} + \text{Lower limit}}{2}$$

Note that the difference between two successive class marks gives the class size provided the class size is equal for all the classes.

Construction of Grouped Frequency Distribution Table

(*i*) Determine the maximum and minimum values of the variate and find their difference to compute the range.

(*ii*) Decide upon the class size (if it is not given).

(*iii*) Divide the range by the chosen (or given) class size, so as to determine the number of classes.

(*iv*) Decide the individual class limits and choose a suitable starting point of the first class which may be less than or equal to the minimum value.

(*v*) For each observation, put a tally mark against the class to which this observation belongs. If the tally marks are more than 4 then record them in bunches of five, with the fifth one crossing the other four diagonally.

(*vi*) Determine the total number of tally marks in each class and record the number as the frequency of the corresponding class.

(*vii*) The total of all frequencies should be equal to the total number of observations.

Cumulative Frequency and Cumulative Frequency Table

In an ungrouped or discrete frequency distribution, the cumulative frequency of a particular value of the variable is the total of all the frequencies up to and including that value.

In grouped frequency distribution, the cumulative frequency corresponding to a class is the sum total of all frequencies up to and including that class.

A table which shows the distribution of cumulative frequencies over various classes is called a cumulative frequency distribution table.

We shall consider the following types of questions.

Type 1: Questions involving presentation of different kind of data, etc.

Solved Examples

Example 1: *Form a discrete frequency distribution from the following scores:*

15, 18, 16, 20, 25, 24, 25, 20, 16, 15, 18, 18, 16, 24, 15, 20, 28, 30, 27, 16, 24, 25, 20, 18, 28, 27, 25, 24, 24, 18, 18, 25, 20, 16, 15, 20, 27, 28, 29, 16

Find also the most frequently and least occurring scores.

Solution: The frequency distribution table for the given data is given below:

Scores	*Tally marks*	*Frequency*
15	||||	4
16	~~||||~~ |	6
18	~~||||~~ |	6
20	~~||||~~ |	6
24	~~||||~~	5
25	~~||||~~	5
27	|||	3
28	|||	3
29	|	1
30	|	1
Total		40

We see from the above table that the most frequently occurring scores are 16, 18 and 20 and the least frequently occurring scores are 29 and 30.

Example 2: *The number of books in different shelves in a library is as follows:*

34, 23, 27, 18, 43, 29, 21, 33, 22, 36, 33, 27, 39, 28, 20, 13, 32, 39, 29, 26, 23, 35, 32, 27, 16, 20, 27, 25, 30, 37, 31, 27, 19, 32, 36, 40, 41, 33, 15, 26, 30, 28, 22, 17, 42, 34, 28, 21, 30, 28, 25, 38, 31, 26, 29, 23, 48, 31, 25, 24

Prepare a grouped frequency distribution table for above data using class intervals 10 – 15, 15 – 20, 20 – 25 and so on.

Solution: Here, the minimum and the maximum observations are 13 and 48 respectively.

$\therefore$ Range = 48 – 13 = 35

Since the class size is 15 – 10 = 5, hence, the number of class intervals = $\frac{35}{5}$ = 7.

Hence, the classes are 10 – 15, 15 – 20, 20 – 25, 25 – 30, 30 – 35, 35 – 40 and 40 – 45.

Since the last class 40 – 45 does not include the maximum observation 48, we take another class 45 – 50.

Hence, we obtain the following grouped frequency distribution table:

Class intervals	*Tally marks*	*Frequency*
10 – 15	|	1
15 – 20	||||/	5
20 – 25	||||/ ||||/	10
25 – 30	||||/ ||||/ ||||/ |||	18
30 – 35	||||/ ||||/ ||||	14
35 – 40	||||/ ||	7
40 – 45	||||	4
45 – 50	|	1
Total		60

Example 3: *The class marks of a distribution are 25, 31, 37, 43, 49, 55, 61, 67, 73, and 79. Determine the class size, class limits and the true class limits, if the classes are exclusive.*

Solution: Since the class marks are uniformly spaced, therefore, the class size is the difference between the consecutive class marks.

∴ Class size = 31 – 25 = 6

If x be the class mark of a class interval and h be the class size, then the lower and upper limits of class interval are

$x-\frac{h}{2}$ and $x+\frac{h}{2}$ respectively, where $h = 6$

∴ The lower limit of the first class interval is

$$25-\frac{h}{2} = 25 - 3 = 22$$

and the upper limit of the first class interval is

$$25+\frac{h}{2} = 25 + 3 = 28$$

∴ The first class interval is 22 – 28.

Similarly, we get the other class intervals as follows:

28 – 34, 34 – 40, 40 – 46, 46 – 52, 52 – 58, 58 – 64, 64 – 70, 70 – 76, 76 – 82.

Example 4: *Find the unknown entries (i.e. a, b, c, d, e, f and g) from the following frequency distribution of the heights of 40 students in a class.*

Classes (heights in cm)	*Frequency*	*Cumulative frequency (c.f.)*
150 – 155	*10*	*a*
155 – 160	*b*	*21*
160 – 165	*8*	*c*
165 – 170	*d*	*33*
170 – 175	*5*	*e*
175 – 180	*f*	*40*
Total	*g*	

Solution: The value of g is clearly, equal to the total number of students which is 40.

Hence, $g = 40$

From the table, $a = 10$

$b + a = 21$

⇒ $b = 21 - a = 21 - 10 = 11$

Also, $8 + 21 = c$

⇒ $c = 29$

Again, $d + c = 33$

⇒ $d + 29 = 33$

⇒ $d = 4$

Also, $5 + 33 = e$

⇒ $e = 38$

Finally, $f + e = 40$

⇒ $f + 38 = 40$

⇒ $f = 40 - 38 = 2$

Hence, the required values of the unknown quantities are as follows:

$a = 10,\quad b = 11,\quad c = 29,\quad d = 4,$

$e = 38,\quad f = 2$ and $g = 40$

Check Your Progress 1

Multiple-Choice Questions

1. An observation is such that its maximum value is 83 and the range is 30. Then its minimum value is

   (*a*) 50 (*b*) 53

   (*c*) 55 (*d*) 113

2. One of the sides of a frequency polygon is

   (*a*) both the coordinate axes

(*b*) neither of the coordinate axes
(*c*) x-axis only
(*d*) y-axis only

Very Short Answer Type Questions

3. Find any class with class size 5 and class mark 53.5.
4. If 40 is subtracted from the sum of 25 observations, then the result is 260. What is the mean of these observations?

Short Answer Type Questions

5. Prepare a discrete frequency distribution table for the following monthly wages in ₹ of 15 workers in a factory.

   2000, 1900, 1200, 2500, 1900, 2000, 3000, 1500, 2500, 1900, 1200, 1900, 1500, 1900, 1500

   Answer the following questions from the table:

   (*a*) What is the range of the wages?
   (*b*) What are the monthly wages of maximum number of workers and how many workers get these wages?
   (*c*) What are the monthly wages of minimum number of workers(s) and how many workers(s) gets/get these wages?
   (*d*) Find the number of workers whose monthly wages are
   (*i*) less than ₹ 2000
   (*ii*) more than ₹ 2000
   (*iii*) lie between ₹ 1000 and ₹ 2100.

6. The mean of five observations was found to be 36. Later on, it was detected that one observation 15 was misread is 50. Find the correct mean.

Knowledge Digest 2

Measures of Central Tendency

The commonly used measures of central tendency (or averages) are as follows:

(*i*) Arithmetic mean (or mean)
(*ii*) Geometric mean
(*iii*) Harmonic mean
(*iv*) Median
(*v*) Mode

In this section, we shall be dealing with three most commonly used averages: the mean, the median and the mode.

Arithmetic Mean

Arithmetic mean (or simply mean) of a set of numbers is the average value of the numbers.

$$\text{Mean} = \frac{\text{Sum of all observations}}{\text{Number of observations}}$$

Arithmetic Mean of Ungrouped Data

The mean of n observations (variates) $x_1, x_2, x_3, \ldots, x_n$ is given by

$$\bar{x} = \frac{(x_1 + x_2 + x_3 + \ldots + x_n)}{n} = \frac{\sum_{i=1}^{n} x_i}{n}$$

where the Greek letter Σ (read as sigma) represents the sum

i.e. $(x_1 + x_2 + x_3 + \ldots + x_n) = \sum_{i=1}^{n} x_i$

Properties of Arithmetic Mean

Let the mean of observations $x_1, x_2, x_3 \ldots x_n$ be $\bar{x}$.

1.	When each observation is increased by 'a' Observations: $x_1 + a$, $x_2 + a, x_3 + a, \ldots, x_n + a$	Mean increases by a Mean: $\bar{x} + a$
2.	When each observation is decreased by 'a' Observations: $x_1 - a$, $x_2 - a, x_3 - a, \ldots, x_n - a$	Mean decreases by a Mean: $\bar{x} - a$
3.	When each observation is multiplied by non-zero number 'a' Observations: $ax_1, ax_2, ax_3, \ldots, ax_n$	Mean gets multiplied by a Mean: $a\bar{x}$
4.	When each observation is divided by non-zero number 'a' observations: $\frac{x_1}{a}, \frac{x_2}{a}, \frac{x_3}{a}, \ldots, \frac{x_n}{a}$	Mean gets divided by a Mean: $\frac{\bar{x}}{a}$
5.	The algebraic sum of deviations from the mean is zero. i.e. $(x_1 - \bar{x}) + (x_2 - \bar{x}) + (x_3 - \bar{x}) + \ldots + (x_n - \bar{x}) = 0$	

Mean for an Ungrouped Frequency Distribution

1. Direct Method

If $x_1, x_2, x_3, ..., x_n$ be n observations with frequency $f_1, f_2, f_3, ..., f_n$ respectively, then the mean $\overline{x}$ of these observations is given by

$$\overline{x} = \frac{f_1x_1 + f_2x_2 + f_3x_3 + ... + f_nx_n}{f_1 + f_2 + f_3 + ... + f_n} = \frac{\sum_{i=1}^{n} f_ix_i}{\sum_{i=1}^{n} f_i}$$

2. Assumed Mean Method (Optional)

When the numerical values of x_i and f_i are large, then the calculation of arithmetic mean by direct method becomes lengthy, cumbersome, tedious and time consuming, as the product $f_i\,x_i$ involves large numbers, and we resort to a short cut method, also called the 'Assumed Mean Method', in the following way:

(*i*) Choose one among the x_is as assumed mean and denote it be a.

Take a to be that x_i which lies in the centre of $x_1, x_2, x_3, ..., x_n$. In case there are two middle terms, then choose the one with greater frequency.

(*ii*) Find the difference d_i between a and each of the x_is_i that is, the deviation of 'a' from each of the x_is. i.e. $d_i = x_i - a$

(*iii*) Calculate the product d_i with corresponding frequency f_i and find the sum of all the f_id_i's.

(*iv*) Calculate the mean by using the formula:

$$\text{Mean} = a + \frac{\sum f_id_i}{\sum f_i}$$

3. Step-Deviation Method (Optional)

In cases where the deviations ($d_i = x_i - a$) are found to be divisible by a common number h (say), the computation is further simplified by putting

$$u_i = \frac{x_i - a}{h}$$

Then, mean is calculated by using the formula

$$\text{Mean} = a + h\left(\frac{\sum f_iu_i}{\sum f_i}\right)$$

MEDIAN

A median is the middle score for a set of observations arranged in numerical order.

Median of an Ungrouped Data

Method (*i*) Arrange the given data in an increasing or decreasing order of magnitude.

(*ii*) Let n be the number of observations in the given data.

(*iii*) If n is odd, then median

$$= \text{value of } \left(\frac{n+1}{2}\right) \text{th observation.}$$

(*iv*) If n is even, then median

$$= \text{mean of } \left(\frac{n}{2}\right) \text{th and } \left(\frac{n}{2}+1\right) \text{th observations.}$$

MODE

The mode is the score that appears most often in a given set of observations. It is that value of the observation which occurs most frequently.

Remark: The empirical relationship between three measures of central tendency, namely mean, median and mode is as follows

3 Median = Mode + 2 Mean

We shall consider the following types of questions:

Type 1: Questions based on mean

Type 2: Questions based on mean, median and mode

Solved Examples

Example 5: *A girl got 94, 92 and 97 marks in three successive mathematics tests. How many marks must she obtain in the fourth test to have an average of exactly 95 in the four tests?*

Solution: Let the girl obtain x marks in the fourth test, so as to have an average of exactly 95.

Then, mean marks = 95

$$\text{Mean} = \frac{\text{Sum of marks obtained in the tests}}{\text{Number of tests}}$$

$$\Rightarrow \quad 95 = \frac{94 + 92 + 97 + x}{4}$$

$$\Rightarrow \quad 95 \times 4 = 283 + x$$

$$\Rightarrow \quad 380 = 283 + x$$

$$\Rightarrow \quad x = 97$$

Hence, the marks in her fourth test should be 97.

Example 6: *Find the mean salary of 60 workers of a factory from the following table.*

Salary (in ₹)	*Number of workers*
3000	16
4000	12
5000	10
6000	8
7000	6
8000	4
9000	3
10000	1

Solution: We apply the step-deviation method to find the mean.

Here, $h = 1000$. Let assumed mean, $a = 5000$

We now form the following table:

Salary (in ₹) (x_i)	*Number of workers* (f_i)	$d_i = x_i - a$ $(a = 5000)$	$u_i = \dfrac{x_i - a}{h}$	$f_i u_i$
3000	16	– 2000	– 2	–32 } – 44
4000	12	– 1000	– 1	–12
5000	10	0	0	0
6000	8	1000	1	8 } + 49
7000	6	2000	2	12
8000	4	3000	3	12
9000	3	4000	4	12
10000	1	5000	5	5
	$\sum f_i = 60$			$\sum f_i u_i = 5$

$$\therefore \quad \text{Mean} = \bar{x} = a + h\left(\frac{\sum f_i u_i}{\sum f_i}\right)$$

$$= 5000 + 1000\left(\frac{5}{60}\right)$$

$$= 5000 + \frac{250}{3}$$

$$= \frac{15000 + 250}{3}$$

$$= \frac{15250}{3}$$

Hence, the mean salary of 60 workers is ₹ $\dfrac{15250}{3}$

Example 7: *Find the missing frequencies in the following frequency distribution if it is known that the mean of the distribution is 43.8 and total frequency is 100.*

x_i	15	25	35	45	55	65
f_i	8	f_1	18	24	f_2	16

Solution: We form the following table:

x_i	f_i	$f_i x_i$
15	8	120
25	f_1	$25 f_1$
35	18	630
45	24	1080
55	f_2	$55 f_2$
65	16	1040
Total	$\Sigma f_i = 66 + f_1 + f_2 = 100$	$\sum f_i x_i = 2870 + 25 f_1 + 55 f_2$

$$\text{Mean} = \bar{x} = \frac{\sum_{i=1}^{n} f_i x_i}{\sum_{i=1}^{n} f_i}$$

$$\Rightarrow \quad 43.8 = \frac{2870 + 25 f_1 + 55 f_2}{100}$$

[∵ Total frequency = 100]

$\Rightarrow \quad 4380 = 2870 + 25f_1 + 55f_2$

$\Rightarrow \quad 4380 - 2870 = 25f_1 + 55f_2$

$\Rightarrow \quad 1510 = 25f_1 + 55f_2$

$\Rightarrow \quad 302 = 5f_1 + 11f_2$ ...(1)

Also, $66 + f_1 + f_2 = 100$

$\Rightarrow \quad f_1 + f_2 = 34$ ...(2)

From (2), $f_2 = 34 - f_1$ ...(3)

∴ Putting value of f_2 in (1), we have

$5f_1 + 11(34 - f_1) = 302$

$\Rightarrow \quad 5f_1 + 374 - 11f_1 = 302$

$\Rightarrow \quad -6f_1 + 374 = 302$

$\Rightarrow \quad 6f_1 = 72$

$\Rightarrow \quad f_1 = 12$

$\therefore$ From (3), $f_2 = 34 - 12 = 22$

Hence, the missing frequencies are 12 and 22.

Example 8: *The mean weight of 40 students of a class is 49 kg. If the mean weight of 15 students of this class is 53 kg, find the mean weight of the remaining 25 students of the class.*

Solution: Let n_1 and n_2 be the numbers of two groups of student in the class so the $n_1 = 15$ and $n_2 = 25$. Let the means of the weights of these two groups of students be denoted by $\bar{x}_1$ and $\bar{x}_2$ (in kg) respectively. If $\bar{x}$ be the mean weight of all the 40 students of the class, then the total weight

$$= (n_1 + n_2)\ \bar{x} = n_1\bar{x}_1 + n_2\bar{x}_2$$

$$\Rightarrow \quad 40 \times 49 = 15 \times 53 + 25\ \bar{x}_2$$

$$1960 - 795 = 25\,\bar{x}_2$$

$$\Rightarrow \quad \bar{x}_2 = \frac{1165}{25} = 46.6$$

Hence, the mean weight of the remaining 25 students of the class is 46.6 kg.

Example 9: *Find the median of the data 24, 36, 15, 54, 75, 18, 62, 32 and 45. If the observation 24 is removed from the data, find the new median.*

Solution: We first arrange the given data in increasing (or ascending) order of magnitude as follows

15, 18, 24, 32, 36, 45, 54, 62, and 75 ... (1)

The number of observations is 9 which is odd.

Hence, $\frac{9+1}{2}$ th or 5th observation is the median.

Now, the 5th observation in (1) is 36, which is, therefore, the required median.

If the observation 24 is removed from (1), then the total number of observation, becomes 8 which is even. Hence, the average of $\frac{8}{2}$ th and $\left(\frac{8}{2}+1\right)$ th i.e. 4th and 5th observations is the required new median in this case. In (1), when the observation 24 is removed, then the new observations in increasing order are

15, 18, 32, 36, 45, 54, 62 and 75, ...(2)

The 4th and 5th observations in (2), are 36 and 45 respectively. The average of 36 and 45

$$= \frac{36+45}{2} = \frac{81}{2} = 40.5$$

which is the required new median.

Example 10: *Find the mode of the following data:*
3.7, 2.3 2.5, 1.9, 4.5, 2.5, 4.5, 1.3, 3.7, 4.5, 1.3, 3.7

Solution: We prepare a frequency distribution table from the given raw data as follows:

Observation	*Tally marks*	*Frequency*
1.3	||	2
1.9	|	1
2.3	|	1
2.5	||	2
3.7	|||	3
4.5	|||	3

We see from the above table that each of the two observations 3.7 and 4.5 has the same maximum frequency, i.e. 3. Hence, there are two modes here, i.e. 3.7 and 4.5.

Hence, the modes are 3.7 and 4.5.

Example 11: *Find the mode of the following data, if it exists:*

15, 18, 15, 12, 23, 18, 25, 12, 23, 22, 9, 25, 22, 9

Solution: We first prepare a frequency distribution table as follows:

Observations	*Tally marks*	*Frequency*
9	||	2
12	||	2
15	||	2
18	||	2
22	||	2
23	||	2
25	||	2

We observe in the above table that each observation, has the same frequency, i.e. 2. Hence, in this case, we say that the mode does not exist.

Hence, the given data do not have any mode at all.

Check Your Progress 2

Multiple-Choice Questions

1. The value of p for which the mode of the following data

12, 15, 11, 12, 18, 15, 9, p, 12, 16, 17, 15, 18, 11, is 12 is

(*a*) 11 (*b*) 18
(*c*) 15 (*d*) 12

2. The median of all prime numbers between 70 and 100 is

(*a*) 86 (*b*) 81
(*c*) 89 (*d*) 83

Very Short Answer Type Questions

3. The mean of 8 numbers is 25. If each number is multiplied by 3, what will be the new mean?

4. The percentage of marks obtained by students of a class in mathematics are 64, 36, 37, 23, 0, 19, 82, 91, 72, 31, 10, 5. Find the median.

5. What is the mean of four consecutive prime numbers the greatest of which is 67?

5. Find the mean of all factors of 210.

6. The following observations have been arranged in ascending order of magnitude. If the median of the data is 72, find the value of x.

26, 38, 42, 55, x, $x + 4$, 79, 85, 91, 97

8. The mean of six numbers is 20. If one number is deleted, their mean is 15. Find the deleted number.

9. The numbers 7, 6, 5, 5, $2x + 1$, 4, 4, 3, 2, are written in descending order. If their median is 5, find x and hence the mode.

Short Answer Type Questions

10. For the data: 3, 9, $x + 6$, $2x + 3$, 4, 10 and 5, if the mean is 7, find the value of x.
Using this value of x, find the mode of the data.

11. Numbers 50, 42, 35, $2x + 10$, $2x - 8$, 12, 11, 8 and 6 are written in descending order. Prove that the value of x does not exist, if the median of these data is 25. What is the maximum value of x so that the given numbers are in decreasing order and that the median exists? Find the median of the numbers, when the median exists.

Long Answer Type Questions

12. The mean marks scored by 40 students was found to be 65. Later on, it was discovered that a score of 80 was misread as 50. Find the correct mean.

13. Find the values of f_1 and f_2 from the following data, if its mean is 25.25:

x_i	12	18	25	$f_1 + 8$	40
f_i	3	f_2	5	6	2

Total frequency = 20.

14. Using the assumed mean method, find the mean daily earnings of a worker from the following frequency distribution table:

Daily earnings (in ₹)	25	75	125	175	225	275
Number of workers	8	15	32	26	12	7

C

Probability

Knowledge Digest

Probability – an Experimental Approach

In this section, we shall consider problems on finding the empirical probability of an event. You should remember the following results:

(*i*) The empirical probability (also called experimental probability) of an event E, written as P(E), is defined as

$$P(E) = \frac{\text{Number of trials in which event E happened}}{\text{The total number of trials}}$$

(*ii*) The sum of the probabilities of all the elementary events of an experiment is 1.

$$\sum_{i=1}^{n} P(E_i) = 1$$

(*iii*) For an event E,

$$P(\bar{E}) = P(\text{not } E) = 1 - P(E)$$

(*iv*) **Probability of an Impossible Event:** The probability of an event which is impossible to occur is 0. Such an event is called an impossible event.

$$P(E) = 0$$

(*v*) **Probability of a Sure Event or Certain Event:** The probability of an event which is sure or certain to occur is 1. Such an event is called a sure event or a certain event.

$$P(E) = 1$$

(*vi*) From (*iv*) and (*v*),

$$0 \leq P(E) \leq 1$$

We shall consider the following types of questions:

Type 1: Questions involving throwing of a coin or a die or simultaneous throwing of more than one coin or die

Type 2: Miscellaneous types of questions

Solved Examples

Example 1: *A coin is tossed 150 times. Find the probabilities of getting 60 heads and the remaining number of tails.*

Solution: Let E be the event of getting a head and E′ be the event of getting a tail in the throwing of a coin once.

Then the number of outcomes in favour of the event E in 150 trials = 60 and

the total number of outcomes = 150

Hence, P(getting a head) = P(E)

$$= \frac{\text{Number of trials in which the event E happens}}{\text{Total number of trials}}$$

$$= \frac{60}{150} = \frac{2}{5} = 0.4$$

Again, in 150 trials, we get 150 – 60 = 90 tails.

Hence, the number of outcomes in favour of E′ = 90

Hence, P(getting a tail) = P(E′)

$$= \frac{\text{Number of trials in which the event E}' \text{ happens}}{\text{Total number of trials}}$$

$$= \frac{90}{150} = \frac{3}{5} = 0.6$$

Note that P(E) + P(E′) = 0.4 + 0.6 = 1 and E and E′ are the only two possible outcomes of each trials.

Example 2: *Three coins are tossed once. Find the probability of getting*

(a) 3 heads

(b) exactly two heads

(c) *at least 2 heads*

(d) *at most 2 heads*

(e) *no heads*

Solution: Let us denote the event of getting one head and two tails by HTT, two heads and one tail by HHT and so on in the three coins. Now, in the case of simultaneous throwing of three coins, the following outcomes are possible:

HTT, HTH, HHT, THH, THT, TTH, HHH, TTT

Hence we see the total number of outcomes = 8

(*a*) Let E_1 be the event of getting 3 heads.

∴ Number of cases in favour of E_1 = 1, since HHH occurs once only. Hence,

$$P(E_1) = \frac{\text{Number of cases in favour of } E_1}{\text{Total number of outcomes}} = \frac{1}{8}$$

(*b*) Let E_2 be the event of getting exactly, 2 heads. Then the number of cases in favour of E_2 = 3, since exactly two heads occur in HTH, HHT and THH only.

Hence by definition,

$$P(E_2) = \frac{3}{8}$$

(*c*) In this case, there may be either two heads or three heads. These outcomes occur in HHT, HTH, THH and HHH.

If E_3 be the event of getting at least two heads, then number of cases in favour of E_3 = 4.

Hence by definition,

$$P(E_3) = \frac{4}{8} = \frac{1}{2}$$

(*d*) In this case, there may be two heads, one head or no head (i.e. all tails).

If E_4 be the event of getting at most two heads, then number of cases in favour of E_4 = 7 (i.e. except HHH).

Hence by definition, $P(E_4) = \frac{7}{8}$

(*e*) In this case, let E_5 be the event of getting no heads, then the number of outcomes in favour of E_5 = 1, i.e. TTT.

Hence by definition, $P(E_5) = \frac{1}{8}$

Example 3: *A die is thrown once. What is the probablity of getting a number which is*

(a) *less than 5?*

(b) *more than 3?*

(c) *prime?*

(d) *multiple of 3?*

(e) *neither prime nor composite?*

Solution:

(*a*) Let E_1 be the event of 'getting a number less than 5'. Then, the number of outcomes in favour of E_1 is 4, since the possible outcomes here are 1, 2, 3 and 4.

Also, the total number of outcomes = 6, i.e. 1, 2, 3, 4, 5 and 6.

Hence, by definition,

$$P(E_1) = \frac{4}{6} = \frac{2}{3}$$

(*b*) Let E_2 be the required event in this case.

Then the outcomes in favour of E_2 are 4, 5 and 6.

Hence, number of outcomes in favour of E_2 = 3

Hence, $P(E_2) = \frac{3}{6} = \frac{1}{2}$

(*c*) Let E_3 be the required event in this case.

Prime numbers between 1 and 6 are 2, 3 and 5.

Hence, number of outcomes in favour of E_3 = 3

Hence, $P(E_3) = \frac{3}{6} = \frac{1}{2}$

(*d*) Let E_4 be the required event in this case.

Numbers which are multiple of 3 are 3 and 6.

Hence, number of outcomes in favour of E_4 = 2

Hence, $P(E_4) = \frac{2}{6} = \frac{1}{3}$

(*e*) Let E_5 be the required event in this case. We know that 1 is the only number which is neither prime nor composite. Hence, number of outcomes in favour of E_5 = 1.

Hence, $P(E_5) = \frac{1}{6}$

Example 4: *An experiment results in four possible outcomes with respective probabilities p_1, p_2, p_3 and p_4. State which of the following probability assignments are not possible. Give reasons for your answers.*

(a) $p_1 = 0.1, p_2 = 0.5, p_3 = 0.8, p_4 = 0.9$

(b) $p_1 = 0.37, p_2 = 0.89, p_3 = 0.99, p_4 = 0.01$

(c) $p_1 = 0.3, p_2 = 0.2, p_3 = 0.8, p_4 = 0.5$

(d) $p_1 = 1.00, p_2 = 1.1, p_3 = 0.95, p_4 = 0.09$

(e) $p_1 = 0.0, p_2 = 0.23, p_3 = -0.25, p_4 = 0.25$

Solution: In the above cases, we see that only (*d*) and (*e*) are not possible and (*a*), (*b*) and (*c*) are possible, since we know that $0 \leq P(A) \leq 1$. In (*d*) we see the $p_2 > 1$ and in (*e*), $p_3 < 0$ and in other cases, $0 \leq P(A) \leq 1$.

Example 5: *A vessel contains 9 red, 8 white and 3 black balls. A ball is drawn at random. What is the probability that the ball drawn will be*

(a) black?

(b) white?

(c) red?

(d) not black?

Solution: Total number of balls

= 9 + 8 + 3 = 20

= Total number of outcomes

(*a*) If E_1 be the event of getting a black ball, then number of outcomes in favour of $E_1 = 3$, since there are only 3 black balls.

Hence,

$$P(E_1) = \frac{\text{Number of cases in favour of } E_1}{\text{Total number of outcomes}} = \frac{3}{20}$$

(*b*) If E_2 be the event of getting a white ball, then number of outcomes in favour of $E_2 = 8$, since there are 8 white balls.

Hence, $P(E_2) = \frac{8}{20} = \frac{2}{5}$

(*c*) Let E_3 be the required event. Then, number of outcomes in favour of $E_3 = 9$, since there are 9 red balls.

Hence, $P(E_3) = \frac{9}{20}$

(*d*) Let E_4 be the required event. Then number of outcomes in favour of $E_4 = 9 + 8 = 17$, since "no black balls" means all red and white balls which are 17 in number.

Hence, $P(E_4) = \frac{17}{20}$

Example 6: *What is the probability that two girls Mira and Sita will have*

(a) different birthdays

(b) same birthday in a non-leap year?

Solution: Let us assume that Mira's birthday be any day of a non-leap year consisting of 365 days. Then Sita's birthday can also be any day of 365 days in a year.

(*a*) If Mira's birthday is different from Sita's birthday then number of favourable outcomes for her birthday = 365 – 1 = 364

P(Mira's birthday is different from Sita's birthday) = $\frac{364}{365}$

(*b*) P(Mira's birthday is the same as Sita's birthday) = 1 – P (they have different birthdays)

$$= 1 - \frac{364}{365} = \frac{1}{365}$$

Example 7: *In a survey, 1000 families with two children were selected at random and the following data were recorded.*

Number of girls in a family	*2*	*1*	*0*
Number of families	*320*	*460*	*220*

Find the probability of a family, chosen at random, having (a) 2 girls (b) 1 girl (c) less than 1 girl.

[CBSE 2013]

Solution: Let E_1, E_2 and E_3 be the events that the family has 2 girls, 1 girl and less than 1 girl respectively.

Also, total number of outcomes

= total number of families = 1000

(*a*) In this case,

Number of outcomes in favour of E_1= 320, since number of families having 2 girls = 320.

Hence, $P(E_1) = \frac{320}{1000} = \frac{8}{25}$

(*b*) In this case,

Number of outcomes in favour of E_2 = 460, since, number of families having 1 girl = 460.

Hence, $P(E_2) = \frac{460}{1000} = \frac{23}{50}$

(*c*) In this case,

Number of outcomes in favour of E_3 = 220, since number of families having no girl = 220

Hence, $P(E_3) = \frac{220}{1000} = \frac{11}{50}$

Example 8: *A die is thrown once. Find all the probabilities p_1, p_2, p_3, p_4, p_5 and p_6 of getting 1 dot, 2 dots, 3 dots, ..., 6 dots respectively and show that* $\sum_{i=1}^{6} p_i = 1$.

Solution: We have

$$p_1 = \frac{1}{6}, p_2 = \frac{1}{6}, p_3 = \frac{1}{6}, ..., p_6 = \frac{1}{6}.$$

Hence, $p_1 + p_2 + p_3 + p_4 + p_5 + p_6 = 6 \times \frac{1}{6} = 1$

Hence, proved.

Example 9: *The following frequency distribution gives the weight of 40 students of a class:*

Weight (in kg)	*31–35*	*35–40*	*41–45*	*46–50*	*51–55*	*56–60*
Number of students	*6*	*9*	*14*	*6*	*4*	*1*

Find the probability that weight of a student in the class is

(a) at most 50 kg

(b) at least 41 kg

(c) not more than 40 kg.

Solution: Total number of students = 40

(*a*) Number of students whose weight is at most 50 kg = 6 + 9 + 14 + 6 = 35

P(weight of a student is at most 50 kg)

$$= \frac{35}{40} = \frac{7}{8}$$

(*b*) Number of students whose weight is at least 41 kg = 14 + 6 + 4 + 1 = 25

P(weight of a student is at least 41 kg)

$$= \frac{25}{40} = \frac{5}{8}$$

(*c*) Number of students whose weight is not more than 40 kg = 6 + 9 = 15

P(weight of a student is not more than 40 kg)

$$= \frac{15}{40} = \frac{3}{8}$$

Checck Your Progress

Multiple-Choice Questions

1. In a sample study of 300 students, it was found that 240 students were interested in mathematics. If a student is selected at random, then the probability that the student is not interested in mathematics is

(*a*) 0.5 (*b*) 0.8

(*c*) 0.2 (*d*) 0.4

2. Two coins are tossed 50 times and the outcomes are recorded as follows:

Number of heads	2	1	0
Frequency	20	25	5

Then the probability of at least one head is

(*a*) 0.25 (*b*) 0.9

(*c*) 0.20 (*d*) 0.1

Very Short Answer Type Questions

3. Stating reason, which of the following cannot be the probability of any event:

$\frac{4}{9}, \frac{5}{4}, \frac{3}{7}, 0.05, -\frac{2}{3}, 5.5$

4. What is the probability of getting an odd number when a die is rolled once?

5. Weather forecast from a news channel shows that out of past 200 consecutive days, its weather forecast was correct 180 times.

What is the probability that on a given day the weather forecast

(*a*) was correct (*b*) was not correct.

6. Marks obtained by students of two sections of a class in a school in mathematics out of 100 marks are given in the following table:

Marks (in %)	0–20	20–40	40–60	60–70	70–80	80–100
Number of students	5	7	4	30	2	2

Find the probability that a student obtained

(*a*) less than 60% marks

(*b*) more than or equal to 70% marks.

Short Answer Type Questions

7. The following frequency distribution gives the weights of 30 students of a class:

Weight (in kg)	*Number of students*
30 – 35	3
35 – 40	4
40 – 45	6
45 – 50	10
50 – 55	3
55 – 60	2
60 – 65	1
65 – 70	1
Total	30

Find the probability that the weight of a student is at most 60 kg.

8. The distances (in km) of 30 employees from their places of residences to their places of work are as follows:

2	5	3	7	5	9	11	20	8	6
18	16	20	3	2	8	14	13	12	9
23	7	6	5	2	10	4	8	10	3

Find the probability that an employee lives

(*a*) less than or equal to 5 km

(*b*) more than 10 km

(*c*) within 1 km

(*d*) more than 25 km

from his/her place of work

Long Answer Type Questions

9. The number of family members in 30 families were recorded as follows:

2, 1, 3, 4, 5, 2, 3, 1, 6, 2, 3, 2, 4, 3, 5, 6, 7, 2, 3, 4, 5, 4, 2, 2, 3, 5, 6, 4, 3, 2

Find, in percentage, the probability that a randomly chosen family has

(*a*) less than 6 members

(*b*) more than or equal to 4 members

(*c*) only 1 member

(*d*) more than 7 members.

10. There are 150 telephone numbers on one page of a telephone directory. The frequency distribution of their unit's digit is given below:

Unit's digit	0	1	2	3	4	5	6	7	8	9	Total
Number of telephones	17	13	7	15	27	38	10	11	9	3	150

One of the numbers is chosen at random from this page. What is the probability that the unit's digit of the chosen number is

(*a*) less than or equal to 3

(*b*) greater than or equal to 7.

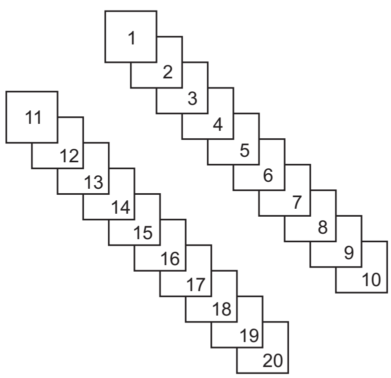
Enrichment Activity

Objective: To get familiar with the idea of finding the probability through a simple activity.

Materials Required: 20 square pieces of cardboard, sketch pen, an empty box, pen/pencil, ruler

Procedure:

1. Take 20 small pieces of cardboard and mark one number on each card starting from 1 to 20.
2. Put all the 20 cards in an empty box.
3. To find the required probability of an event, take out each card one by one without replacement and fill the observation table by putting (✓) on favourable outcomes and (✗) otherwise.
4. Count the total number of possible outcomes.
5. Count the total number of favourable outcomes for each event.



Observations and Calculations:

S.No.	*Possible outcome*	*Prime number*	*Composite number*	*Neither prime nor composite*
1	13	✓	✗	✗
2	6	✗	✓	✓
3	7	✓	✗	✗
4	1	✗	✗	✓
5				
6				

7				
8				
9				
10				
11				
12				
13				
14				
15				
16				
17				
18				
19				
20				
Total				

Fill in the following results:

(*a*) Total number of possible outcomes = ______

(*b*) Total number of favourable outcomes (Prime number) = ______

(*c*) Total number of favourable outcomes (Composite number) = ______

(*d*) Total number of favourable outcomes (neither prime nor composite) = ______

Now, using formula

$$P(E) = \frac{\text{Total number of favourable outcomes to E}}{\text{Total number of possible outcomes}}$$

find out the probability of all the events.

Result: From the above activity, we have learnt the concept of finding the probability of events.

Higher Order Thinking Skills (HOTS) Questions

1. A bag contains 3 red balls each bearing one of the numbers 1, 2 and 3; and 2 black balls each bearing one of the numbers 4 and 5. A ball is drawn, its number is noted and the ball is replaced in the bag. Then another ball is drawn and its number is noted.

Find the probability of drawing

(*a*) a number less than or equal to 4 on the first draw and 5 on the second draw

(*b*) a total of 7.

2. Two dice are thrown simultaneously.

Find the probability of getting

(*a*) an odd number on the first die and an even number on the second die.

(*b*) a number greater than or equal to 5 on each die.

(*c*) a total of 10.

(*d*) a total greater than or equal to 4 but less than 7.

Self-Assessment

Multiple-Choice Questions

1. The sum of the probabilities of all events of a trial is

(*a*) less than 1

(*b*) equal to 1

(*c*) greater than 1

(*d*) equal a number lying between 0 and 1.

2. A fair coin is tossed 50 times and the tail occurs 37 times. Then the probability of getting a head is

(*a*) 0.6 (*b*) 0.5

(*c*) 0.37 (*d*) 0.26

Fill in the Blanks

3. The probability of an impossible event is __________.

4. The probability of a sure event is __________.

5. In n trials of a random experiment, if an event E happens m times, then P(E) is equal to __________.

6. The probability of an event of a trial is always __________.

Assertion-Reason Type Questions

Directions (Q. Nos. 7 to 10): Each of these questions contains an assertion followed by reason. Read them carefully, and answer the question on the basis of the following options, select the one that best describes the two statements.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

7. Assertion (A): Coin tossing is a random experiment.

Reason (R): The result of coin tossing is not fixed and cannot be predicted.

8. Assertion (A): Probability of a sure event is 1 and that of an impossible event is –1.

Reason (R): The probability of an impossible event is 0.

9. Assertion (A): When a dice is thrown, probability of getting an even number is 0.5.

Reason (R): When a dice is thrown, out of 6 possible outcomes, 3 are even numbers.

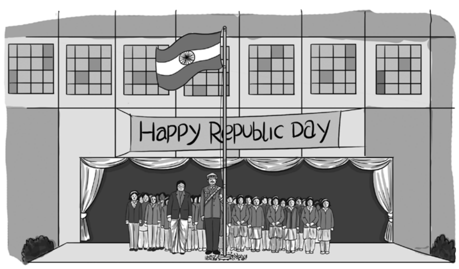
10. Assertion (A): A coin has the heavier head side and that makes probability of head as 0.8, then probability of tail is 0.2.

Reason (R): Total probability of all outcomes is 1.

Case Study Based Questions

11. Republic Day is celebrated on 26th January in every school. It is celebrated to mark the day when the Constitution of India came into effect.

In a school, Republic day was celebrated. Out of 1200 students, 480 students took part in this celebration. Based on the above information, answer the following questions.



(*a*) What is the probability of the number of students who participated in the celebration?

(*b*) What is the probability of the number of students who did not participate in the celebration?

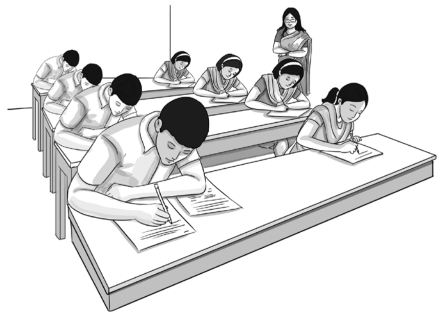
(*c*) (*i*) What is the sum of the probabilities of all events of a trial?

or

(*ii*) What is the probability of the students participated if 880 students took part in the celebration out of 1200 students?

12. A mathematics teacher has recently joined a certain school. She wanted to analyse the performance of the class. So she decided to take a test and recorded the distribution of marks into the following table:

Marks	0–20	20–40	40–60	60–80	80–100
Number of students	7	13	11	10	9



Based on the above information, answer the following questions.

(*a*) What is the number of students who appeared for the test?

(*b*) What is the probability that a student obtained less than 40 marks in the test?

(*c*) (*i*) What is the probability that a student obtained 40 or more marks?

or

(*ii*) What is the probability that a student obtained 60 or more marks?

Very Short Answer Type Questions

13. A die is thrown 200 times and the odd numbers are obtained 150 times. Then what is the probability of getting an even number?

14. Two coins are tossed simultaneously 100 times. Either 1 head or 2 heads occur 25 times. What is the probability of getting no head?

15. Following is the distribution of marks of 50 students in a certain school test:

Marks below	10	20	30	40	50	60	70	80	90	100
Number of students	2	3	5	7	11	13	27	42	48	50

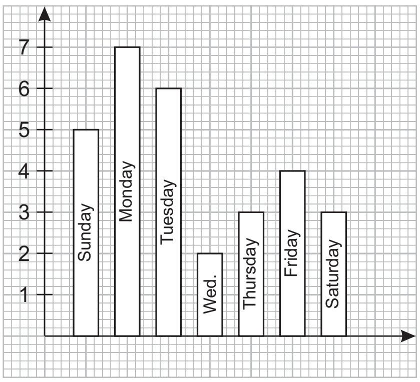
Find the probability that a student scores

(*a*) less than 20 marks

(*b*) marks less than 50 but greater than or equal to 20

(*c*) marks more than 90.

16. The given bar graph provides information on the day of birth of 30 students. Read the bar and find the probability that a student of the class was born on Thursday.



Short Answer Type Questions

17. A die is thrown 20 times and the outcomes are noted as given below:

Outcome	1	2	3	4	5	6
Number of times	3	4	1	3	5	4

Find the probability of happening of each outcome and hence find the sum of all these probabilities.

18. In a cricket match, a batsman hits a boundary 4 times out of 30 balls he plays. Find the probability that he did not hit a boundary. **[CBSE SP 2012]**

19. In a survey of 250 students conducted by a school, it was found that 160 students like to participate in outdoor games, yoga and jogging while the remaining students like to watch TV instead. Based on the given situation, answer the following questions:

(*a*) Find the probability that a student chosen at random likes outdoor games, yoga and jogging.

(*b*) Find the probability that a student chosen at random likes to watch TV.

Long Answer Type Questions

20. In a single throw of two dice, what is the probability of getting a prime number on each die?

21. The table given below shows the weekly pocket money of 50 students:

Pocket money (in ₹)	*Number of students*
115	10
125	10
140	8
150	5
175	7
200	6
250	4
Total	50

Find the probability that the weekly pocket money is

(*a*) ₹ 250

(*b*) less than ₹ 200

(*c*) less than ₹ 150

(*d*) ₹ 100

(*e*) more than ₹ 175.