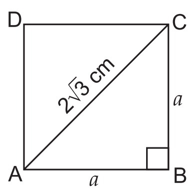
12

Surface Areas and Volumes

Checkpoint (Page 221)

1. The length of the diagonal of a face of a cube is $2\sqrt{3}$ cm. What are the total surface area and the volume of the cube?

Sol. Let a be the side of the cube with a diagonal AC = $2\sqrt{3}$ cm.



Hence, by using Pythagoras' Theorem, we have

$$a^2 + a^2 = \left(2\sqrt{3}\text{ cm}\right)^2 = 12\text{ cm}^2$$

$\Rightarrow$ $2a^2 = 12\text{ cm}^2$

$\Rightarrow$ $a^2 = 6\text{ cm}^2$

$\Rightarrow$ $a = \sqrt{6}$ cm

$\therefore$ Total surface area of the cube

$= 6a^2$

$= 6 \times 6\text{ cm}^2$

$= 36\text{ cm}^2$

and the total volume

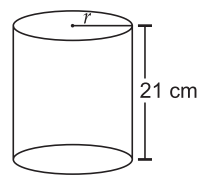
$= a^3$

$= \left(\sqrt{6}\right)^3\text{ cm}^3$

$= 6\sqrt{6}\text{ cm}^3$

2. The curved surface area of an open right circular cylinder is 924 cm^2. If the height of the cylinder is 21 cm, what is its volume?

Sol. Let r be the radius of the base of the cylinder and h be its height. Then $h = 21$ cm.



Now, curved surface area of the cylinder

$= 2\pi rh$

$= 2 \times \frac{22}{7} \times r \times 21$ cm

$= 132r$ cm

$\therefore$ $132r\text{ cm} = 924\text{ cm}^2$

$\Rightarrow$ $r = \frac{924}{132}$ cm

$\Rightarrow$ $r = 7$ cm

$\therefore$ Required volume $= \pi r^2 h$

$= \frac{22}{7} \times 7 \times 7 \times 21\text{ cm}^3$

$= 22 \times 7 \times 21\text{ cm}^3$

$= 3234\text{ cm}^3$

Hence, the required volume is 3234 cm^3.

3. The volume of a cube is 1728 cm^3. Find its total surface area.

Sol. Let a be the side of the cube.

Then $a^3 = 1728\text{ cm}^3$

$= 4^3 \times 3^3\text{ cm}^3$

$\Rightarrow$ $a = 4 \times 3\text{ cm} = 12$ cm

$\therefore$ Total surface area of the cube

$= 6a^2$

$= 6 \times 12^2\text{ cm}^2$

$= 864\text{ cm}^2$

Hence, the required total surface area is 864 cm^2.

4. The length, breadth and height of a rectangular solid object are in the ratio 2 : 3 : 1. Find the length, breadth and height of the solid if the total surface area of the solid is 88 cm^2.

Sol. Let the length, $l = 2x$, breadth, $b = 3x$ and height, $h = x$, where x is a non-zero positive number.

Then the total surface area of the solid

$$= 2(lb + lh + bh)$$
$$= 2(2x \times 3x + 2x \times x + 3x \times x)$$
$$= 2(6x^2 + 2x^2 + 3x^2)$$
$$= 2 \times 11x^2$$
$$= 22x^2$$

$\therefore \quad 22x^2 = 88 \text{ cm}^2$

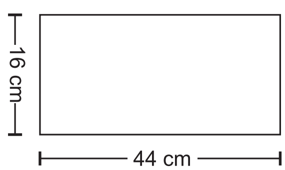
$\Rightarrow \quad x^2 = 4 \text{ cm}^2$

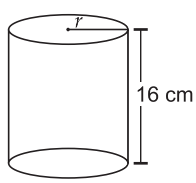
$\Rightarrow \quad x = 2 \text{ cm}$

Hence, the required length = 2 × 2 cm = 4 cm, breadth = 3 × 2 cm = 6 cm and height = 2 cm.

5. A rectangular piece of paper is 44 cm long and 16 cm wide. A cylinder is formed by rolling this rectangular piece of paper along its length. Find the volume of the cylinder so formed.

Sol. Let l be the length of the rectangular paper and b be its breadth. Then the length of the circumference of the cylinder = l and height of the cylinder = b. Let r be the radius of the base of the cylinder.





Then, $\quad l = 2\pi r$

$\Rightarrow \quad 44 \text{ cm} = 2 \times \frac{22}{7} \times r$

$\Rightarrow \quad r = \frac{44 \times 7}{2 \times 22} \text{ cm}$

$\Rightarrow \quad r = 7 \text{ cm}$

$\therefore$ Volume of the cylinder

$$= \pi r^2 b$$
$$= \frac{22}{7} \times 7 \times 7 \times 16 \text{ cm}^3$$
$$= 22 \times 7 \times 16 \text{ cm}^3$$
$$= 2464 \text{ cm}^3$$

Hence, the required volume of the cylinder is 2464 cm^3.

6. The diameter of a garden roller is 1.4 m and it is 2.5 m long. How much area will it cover in 5 revolutions?

Sol. Given, diameter = 1.4 m

$\therefore \quad$ Radius, $r = \frac{1.4}{2} \text{ m} = 0.7 \text{ m}$

Height, $h = 2.5$ m

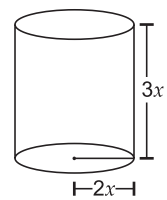
Curved surface area of garden roller

$$= 2\pi rh$$
$$= 2 \times \frac{22}{7} \times 0.7 \times 2.5 \text{ m}^2$$
$$= 11 \text{ m}^2$$

Hence, area covered by garden roller in 5 revolutions = 5 × 11 m^2 = 55 m^2.

7. The radius of the base and the height of a solid right circular cylinder are in the ratio 2 : 3 and its volume is 1617 cm^3. Find the total surface area of the cylinder.

Sol. Let r be the radius of the base and h be the height of the cylinder. Then, $r : h = 2 : 3$



Let $r = 2x$ and $h = 3x$, where x is a non-zero positive number.

Now, volume of the cylinder

$$= \pi r^2 h$$
$$= \frac{22}{7} \times 4x^2 \times 3x$$

$\therefore \quad \frac{22}{7} \times 4x^2 \times 3x = 1617 \text{ cm}^3$

$\Rightarrow \quad x^3 = \frac{1617 \times 7}{22 \times 4 \times 3} \text{ cm}^3$

$$= \left(\frac{7}{2}\text{cm}\right)^3$$

$\therefore \quad x = \frac{7}{2} \text{ cm}$

$\therefore \quad r = 2 \times \frac{7}{2} \text{ cm} = 7 \text{ cm}$

$h = 3 \times \frac{7}{2} \text{ cm} = \frac{21}{2} \text{ cm}$

$\therefore$ Total surface area of the cylinder

$$= 2\pi rh + 2\pi r^2$$
$$= 2\pi r(h + r)$$

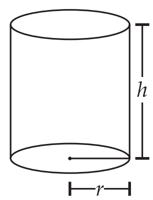
$$= 2 \times \frac{22}{7} \times 7 \times \left(\frac{21}{2} + 7\right) \text{ cm}^2$$

$$= 44 \times \frac{35}{2} \text{ cm}^2 = 770 \text{ cm}^2$$

Hence, the required total surface area of the cylinder is 770 cm^2.

8. The total surface area of a solid cylinder is 231 cm^2. If the curved surface area of this cylinder is $\frac{2}{3}$ of its total surface area, then find its radius and height.

Sol. Let r be the radius of the base of the cylinder and h be its height. Then the total surface area of the solid cylinder = $2\pi rh + 2\pi r^2$ and the curved surface area = $2\pi rh$.



It is given that

$$2\pi rh = \frac{2}{3} \times (2\pi rh + 2\pi r^2) \qquad \ldots(1)$$

and $\quad 2\pi rh + 2\pi r^2 = 231 \text{ cm}^2 \qquad \ldots(2)$

From (1), we have

$$h = \frac{2}{3}h + \frac{2}{3}r$$

$$\Rightarrow \quad \frac{h}{3} = \frac{2}{3}r$$

$$\Rightarrow \quad h = 2r \qquad \ldots(3)$$

$\therefore$ From (2)

$$2\pi r(h + r) = 231 \text{ cm}^2$$

$$\Rightarrow \quad 2 \times \frac{22}{7} \times r \times 3r = 231 \text{ cm}^2$$

$$\Rightarrow \quad r^2 = \frac{49}{4} \text{ cm}^2$$

$$\therefore \quad r = \frac{7}{2} \text{ cm} = 3.5 \text{ cm}$$

From (3), $\qquad h = 2 \times 3.5 \text{ cm} = 7 \text{ cm}$

Hence, the required radius is 3.5 cm and the height is 7 cm.

9. Find the radius of a sphere whose surface area is numerically equal to its volume.

Sol. If r be the radius of the sphere.

Then its volume = $\frac{4}{3}\pi r^3$

and the surface area = $4\pi r^2$

It is given that

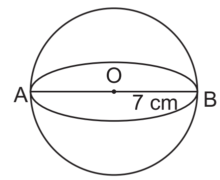
$$4\pi r^2 = \frac{4}{3}\pi r^3$$

$$\Rightarrow \quad r = 3$$

Hence, the required radius is 3 units.

10. A circle of radius 7 cm is rotated about its diameter. Find the surface area of the solid thus generated.

Sol. The circle of radius, r = 7 cm will generate a solid sphere with radius, r = 7 cm when rotated about its diameter AB.



$\therefore$ Surface area of the sphere, thus generated

$$= 4\pi r^2$$

$$= 4 \times \frac{22}{7} \times 7 \times 7 \text{ cm}^2$$

$$= 616 \text{ cm}^2$$

Hence, the required surface area of the solid thus generated is 616 cm^2.

Check Your Progress

(Page 225)

Multiple-Choice Questions

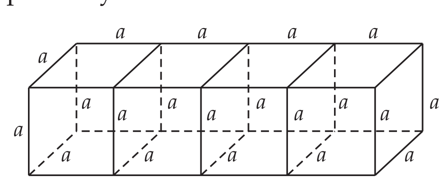
1. If four cubes each of side a units are joined together end to end to form a cuboid, then the ratio of the volume of the cuboid to its surface area is

(*a*) 9 : 2a (*b*) a : 18

(*c*) 18 : a (*d*) 2a : 9

Sol. (*d*) 2a : 19

Let a be the side of each cube. Then the four cubes when joined together end to end as in the figure to form a cuboid, then the length, breadth and height of the cuboid will be 4a cm, a cm and a cm respectively.



$\therefore$ Volume of the cuboid $= 4a \times a \times a = 4a^3$

and surface area of the cuboid

$$= 2(4a \times a + 4a \times a + a \times a)$$
$$= 18a^2$$

$\therefore$ Required ratio $= 4a^3 : 18a^2 = 2a : 9$

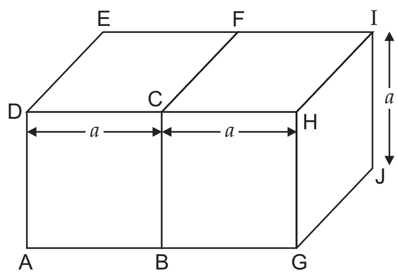
2. Two identical solid cubes are joined by a side to form a cuboid. What fraction of the surface area of the two cubes is the surface area of the cuboid?

(*a*) $\frac{11}{12}$ (*b*) $\frac{5}{6}$ (*c*) 1 (*d*) 2

Sol. (*b*) $\frac{5}{6}$

Surface ares of the two cubes

$$= 6a^2 + 6a^2 = 12a^2 = S_1$$



Surface area of the cuboid formed

$$= 2[2a \times a + 2a \times a] + [2a \times a]$$
$$= 2\left[2a^2 + 2a^2\right] + 2a^2$$
$$= 8a^2 + 2a^2$$
$$= 10a^2 = S_2$$
$$\frac{S_2}{S_1} = \frac{10a^2}{12a^2}$$
$$\therefore \quad S_2 = \frac{5}{6} S_1$$

3. The curved surface area of a right circular cylinder of height 5 cm is 110 cm^2. The diameter of its circular base is

(*a*) 4 cm (*b*) 5 cm
(*c*) 7 cm (*d*) 10 cm

Sol. (*c*) 7 cm

Curved surface area = 110 cm^2

$$2\pi rh = 110 \text{ cm}^2$$
$$\Rightarrow \quad r = \frac{1}{2\pi h} \times 110 \text{ cm}^2$$
$$= \frac{1 \times 7 \times 110 \text{ cm}^2}{2 \times 22 \times 5 \text{ cm}} = \frac{7}{2} \text{ cm}$$

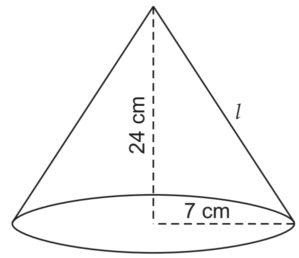
$\therefore$ Diameter $= 2r = 2 \times \frac{7}{2}$ cm $= 7$ cm

4. The curved surface area of a cone having height 24 cm and radius 7 cm, is

(*a*) 528 cm^2 (*b*) 1056 cm^2
(*c*) 550 cm^2 (*d*) 500 cm^2

[CBSE 2023 Standard]

Sol. (*c*) 550 cm^2



Slant height of the cone,

$$l = \sqrt{(24)^2 + (7)^2} \text{ cm}$$
$$l = \sqrt{576 + 49} \text{ cm}$$
$$l = \sqrt{625} \text{ cm}$$
$$\therefore \quad l = 25 \text{ cm}$$

Curved surface area of the cone

$$= \pi rl$$
$$= \frac{22}{7} \times 7 \text{ cm} \times 25 \text{ cm} = 550 \text{ cm}^2$$

5. The curved surface area of a right circular cone of radius 7 cm is 550 sq cm. The slant height of the cone is

(*a*) 24 cm (*b*) 25 cm
(*c*) 22 cm (*d*) 20 cm

[CBSE 2024 Basic]

Sol. (*b*) 25 cm

Curved surface area = 550 cm^2

$$\pi rl = 550 \text{ cm}^2$$
$$\Rightarrow \quad l = \frac{1}{\pi r} \times 550 \text{ cm}^2$$
$$\Rightarrow \quad l = \frac{1 \times 7 \times 550 \text{ cm}^2}{22 \times 7 \text{ cm}}$$
$$\therefore \quad l = 25 \text{ cm}$$

6. The volume of the largest circular cone that can be carved out from a solid cube of edge 2 cm is

(*a*) $\frac{4\pi}{3}$ cu cm (*b*) $\frac{5\pi}{3}$ cu cm
(*c*) $\frac{8\pi}{3}$ cu cm (*d*) $\frac{2\pi}{3}$ cu cm

[CBSE 2024 Standard]

Sol. (*d*) $\frac{2\pi}{3}$ cu cm

The height of the cone = 2 cm

Diameter of the cone = 2 cm

$\therefore$ Radius = 1 cm

Volume of the cone = $\frac{1}{3} \times \pi \times 1 \text{ cm} \times 1 \text{ cm} \times 2 \text{ cm}$

$= \frac{2}{3}\pi$ cu cm

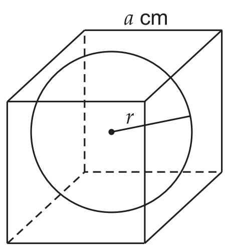
7. The volume of the largest sphere that can be carved out of a cube of side 21 cm is

(*a*) 4410 cm^3 (*b*) 4851 cm^3

(*c*) 6615 cm^3 (*d*) 5292 cm^3

Sol. (*b*) 4851 cm^3

Let a be the side of the cube and r be the radius of the sphere. The largest sphere will clearly touch the four square faces of the cube. Hence, the diameter of the sphere will be equal to the side of the cube in this case.



$\therefore$ $a = 2r$

$\Rightarrow$ $21 \text{ cm} = 2r$

$\Rightarrow$ $r = \frac{21}{2}$ cm

$= 10.5$ cm

Hence, the radius of the sphere is 10.5 cm.

$\therefore$ Volume of the sphere

$= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times 10.5 \times 10.5 \times 10.5 \text{ cm}^3$

$= 11 \times 42 \times 10.5 \text{ cm}^3$

$= 4851 \text{ cm}^3$

Hence, the required volume is 4851 cm^3.

8. A hemispherical bowl is made of steel of thickness 1 cm. The inner radius of the bowl is 5 cm. The volume of the steel used (in cm^3) is

(*a*) 182π (*b*) $\frac{182\pi}{3}$

(*c*) $\frac{682\pi}{3}$ (*d*) $\frac{364\pi}{3}$

[CBSE 2023C Basic]

Sol. (*b*) $\frac{182\pi}{3}$

Volume of hemisphere

$= \frac{2}{3}\pi r^3$

Volume of steel used

$= \frac{2}{3}\pi(R^3 - r^3)$

$= \frac{2}{3}\pi(6^3 - 5^3) \text{ cm}^3$

$= \frac{2}{3}\pi \times 91 \text{ cm}^3$

$= \frac{182}{3}\pi \text{ cm}^3$

9. A solid sphere is cut into two hemispheres. The ratio of the surface areas of sphere to that of two hemispheres taken together is

(*a*) 1 : 1 (*b*) 1 : 4

(*c*) 2 : 3 (*d*) 3 : 2

[CBSE 2024 Standard]

Sol. (*c*) 2 : 3

Surface area, S_1, of the solid sphere = $4\pi r^2$

Surface area, S_2, of the two solid hemispheres

$= 3\pi r^2 + 3\pi r^2$

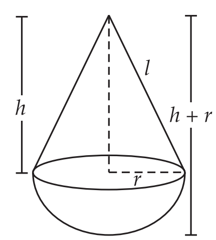
$= 6\pi r^2$

$\therefore$ $\frac{S_1}{S_2} = \frac{4\pi r^2}{6\pi r^2} = \frac{2}{3}$

Very Short Answer Type Questions

10. A solid is hemispherical at the bottom and conical above. If the volumes of the two parts are equal, then find the ratio of the height of the combined solid to the radius of the hemisphere.

Sol. Let r be the common radius of the base of the cone and the hemisphere and let h be the height of the cone. It is given that their volumes are equal to each other.



Now, the volume of the cone = $\frac{1}{3}\pi r^2 h$ and the volume of the hemisphere = $\frac{2}{3}\pi r^3$.

$\therefore \qquad \frac{1}{3}\pi r^2 h = \frac{2}{3}\pi r^3$

$\Rightarrow \qquad h = 2r \qquad \ldots(1)$

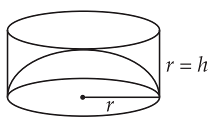
Now, the height of the combined solid $= h + r$

$\therefore \qquad \frac{h+r}{r} = \frac{2r+r}{r} \qquad$ [From (1)]

Hence, the required ratio is 3 : 1

11. A hemisphere and a cylinder stand together on the same common circular base on the ground such that their heights are the same. What is the relation between the volume of the cylinder and that of the hemisphere?

Sol. Let r be the common radius of the cylinder and the hemisphere.



Then the height, h of the cylinder be r. If V_1 and V_2 be volumes of the cylinder and the hemisphere respectively, then

$$V_1 = \pi r^2 h = \pi r^2 \times r = \pi r^3$$

and $\qquad V_2 = \frac{2}{3}\pi r^3$

$\therefore \qquad \frac{V_1}{V_2} = \frac{\pi r^3}{\frac{2\pi r^3}{3}} = \frac{3}{2}$

$\therefore \qquad V_2 = \frac{2}{3} V_1$

Hence, the required relation is that the volume of the hemisphere is equal to $\frac{2}{3}$ rd of the volume of the cylinder.

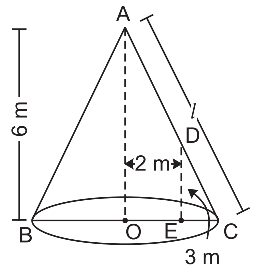
12. Find the area of the canvas required for a 6 m high conical tent in which an object of height 3 m may just be kept at a distance of 2 m from the centre of the base of the tent.

Sol. Let r be the radius of the base of the cone ABC and h be its height. Let l be the slant height of the cone. Let DE be an object standing vertically on the base BOC of the cone such that OE = 2 m, DE = 3 m, O being the centre of the circular base of the cone. Then, in ΔDEC and ΔAOC, we see that DE $\parallel$ AO.

$\therefore \qquad \frac{DE}{AO} = \frac{EC}{OC}$

$\Rightarrow \qquad \frac{3}{6} = \frac{OC - OE}{OC} = \frac{r-2}{r}$

$\Rightarrow \qquad \frac{1}{2} = \frac{r-2}{r}$



$\Rightarrow \qquad r = 2r - 4$

$\Rightarrow \qquad r = 4 \text{ m} \qquad \ldots(1)$

Also, $\qquad h = AO = 6$ m

$\therefore$ In ΔAOC, by Pythagoras' Theorem

$$AC = \sqrt{AO^2 + OC^2}$$

$\Rightarrow \qquad l = \sqrt{36 + r^2}$

$= \sqrt{36 + 16} \qquad$ [From (1)]

$= \sqrt{52}$

$= 2\sqrt{13} \qquad \ldots(2)$

Hence, the curved surface area of the cone

$= \pi r l$

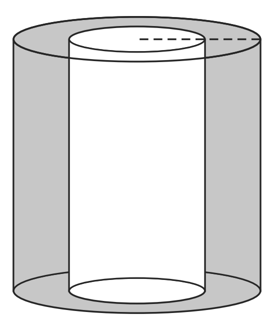
$= \frac{22}{7} \times 4 \times 2\sqrt{13}$ m^2

[From (1) and (2)]

$= \frac{176\sqrt{3}}{7}$ m^2

Hence, the area of the canvas required to cover the tent is $\frac{176\sqrt{3}}{7}$ m^2.

13. Two cylinders of the same height are standing on the same base as shown in the figure such that the radius of the base of the inner cylinder is half that of the outer cylinder. Find the ratio of the volume of the shaded portion to the volume of the outer cylinder.



Sol. Let r and R be the radii of the bases of the two cylinders, where R > r and let h be the common height of the two cylinders. Let V_1 and V_2 be the volumes of the two cylinders, where $V_2 > V_1$.

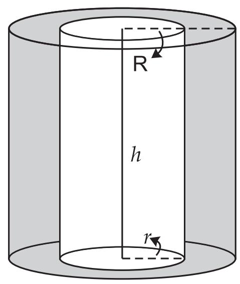
Then $V_1 = \pi r^2 h$, $V_2 = \pi R^2 h = \pi(2r)^2 h = 4\pi r^2 h$

[$\because$ R = 2r, given]

$\therefore V_2 - V_1$ = Volume of the shaded portion

$= 4\pi r^2 h - \pi r^2 h$

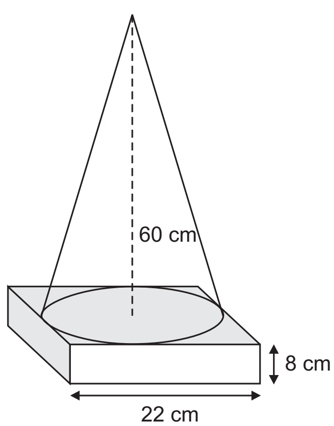
$= 3\pi r^2 h$



$\therefore$ Required ratio $= \dfrac{V_2 - V_1}{V_2}$

$= \dfrac{3\pi r^2 h}{4\pi r^2 h} = \dfrac{3}{4} = 3 : 4$

14. A traffic cone for road safety is mounted on a cuboidal base. The exterior of the traffic cone, excluding the base is to be painted red. Find the surface area of the traffic cone that is to be painted red. (Take π = 3.14)



Sol. Slant height, l, of the traffic cone

$= \sqrt{(60)^2 + (11)^2}$ cm

$l = \sqrt{3600 + 121}$ cm

$l = \sqrt{3721}$ cm

$\therefore \quad l = 61$ cm

Surface area to be painted red

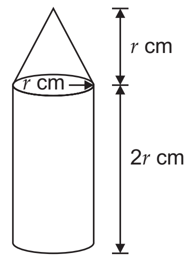
$= \dfrac{22}{7} \times 11 \text{ cm} \times 61 \text{ cm} + 4 \times 22 \text{ cm} \times 8 \text{ cm}$

$+ \left[22 \text{ cm} \times 22 \text{ cm} - \dfrac{22}{7} \times 11 \text{ cm} \times 11 \text{ cm}\right]$

$= 2108.9 \text{ cm}^2 + 704 \text{ cm}^2 + \left[484 \text{ cm}^2 - 380.3 \text{ cm}^2\right]$

$= 2916.6 \text{ cm}^2$ (approx.)

15. The volume of the solid toy as shown below is 198 cm^3. Find the radius of the solid.



Sol. Volume of cylindrical $= \pi r^2 h$

$= \pi r^2 \times 2r$

$= 2\pi r^3$

Volume of conical part $= \dfrac{1}{3}\pi r^2 h$

$= \dfrac{1}{3}\pi r^2 \times r$

$= \dfrac{1}{3}\pi r^3$

Volume of the solid toy $= 2\pi r^3 + \dfrac{1}{3}\pi r^3$

$= \dfrac{7}{3}\pi r^3$

According to the question,

$\dfrac{7}{3}\pi r^3 = 198 \text{ cm}^3$

$r^3 = \dfrac{3}{7} \times \dfrac{7}{22} \times 198 \text{ cm}^3$

$\Rightarrow \quad r^3 = 27 \text{ cm}^3$

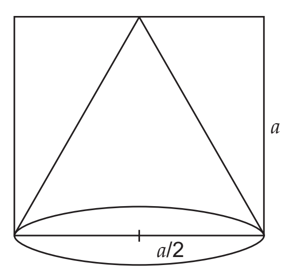
$\Rightarrow \quad r = 3$ cm

$\therefore$ Radius of the solid toy = 3 cm.

Short Answer Type Questions

16. Determine the ratio of the volume of a cube to the right circular cone that fits exactly inside the cube.

Sol. Let the side of the cube be a.



Then, volume of the cube = a^3.

Volume of the right circular cone

$$= \frac{1}{3} \times \frac{22}{7} \times \frac{a}{2} \times \frac{a}{2} \times a$$

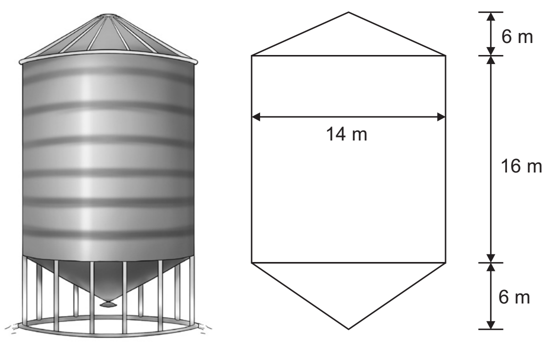
$$= \frac{11}{42} a^3$$

Volume of cube : Volume of right circular cone

$$= \frac{a^3}{\frac{11}{42} a^3}$$

$$= \frac{42}{11} = 42 : 11$$

17. The Food Corporation of India (FCI) stores wheat in silos. The typical silo is given below. The silo is of cylindrical shape with two cones on its circular bases. If the height of the wheat stored in the silo is 16 m, what fraction of the silo's volume is filled with wheat.

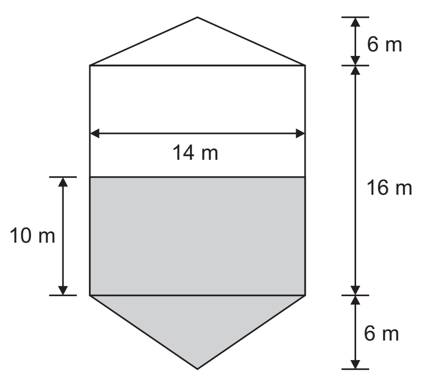


Sol. Total volume of the silo

$$= 2 \times \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 6 \text{ m}^3 + \frac{22}{7} \times 7 \text{ m} \times 7 \text{ m} \times 16 \text{ m}$$

$$= 616 \text{ m}^3 + 2464 \text{ m}^3$$

$$= 3080 \text{ m}^3$$



Volume of wheat stored in the silo

$$= \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 6 \text{ m}^3 + \frac{22}{7} \times 7 \text{ m} \times 7 \text{ m} \times 10 \text{ m}$$

$$= 308 \text{ m}^3 + 1540 \text{ m}^3$$

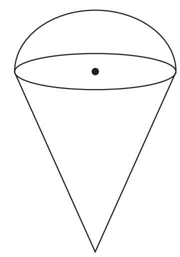
$$= 1848 \text{ m}^3$$

Fraction of silo's volume filled with wheat

$$= \frac{1848}{3080}$$

$$= \frac{3}{5}$$

18. An empty cone is of radius 3 cm and height 12 cm. Ice cream is filled in it so that lower part of the cone which is $\left(\frac{1}{6}\right)$th of the volume of the cone is unfilled but hemisphere is formed on the top. Find volume of the ice cream. (Take $\pi = 3.14$)



[CBSE 2023 Standard]

Sol. Volume of the cone

$$= \frac{1}{3} \times \pi \times r^2 \times h$$

$$= \frac{1}{3} \times \frac{22}{7} \times 3 \text{ cm} \times 3 \text{ cm} \times 12 \text{ cm}$$

$$= 113.14 \text{ cm}^3$$

Volume of the unfilled portion of the cone

$$= \frac{1}{6} \times 113.14 \text{ cm}^3 = 18.86 \text{ cm}^3$$

Volume of the hemisphere

$$= \frac{2}{3} \pi r^3$$

$$= \frac{2}{3} \times \frac{22}{7} \times 3 \text{ cm} \times 3 \text{ cm} \times 3 \text{ cm}$$

$$= 56.57 \text{ cm}^3$$

$\therefore$ Volume of the ice-cream

$$= 113.14 \text{ cm}^3 - 18.86 \text{ cm}^3 + 56.57 \text{ cm}^3$$

$$= 150.85 \text{ cm}^3 \text{ (approx.)}$$

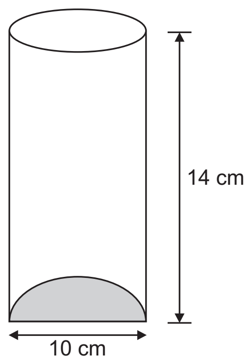
19. A juice glass is cylindrical in shape with hemispherical raised up portion at the bottom. The inner diameter of glass is 10 cm and its height is 14 cm. Find the capacity of the glass. [Use $\pi = 3.14$] **[CBSE 2024 Basic]**

Sol. Volume of the hemispherical raised up portion

$$= \frac{2}{3} \pi r^3$$

$= \frac{2}{3}\pi\ (5\text{ cm})^3$

$= \frac{250}{3}\pi\text{ cm}^3$



Volume of the cylindrical part

$= \pi r^2 h$

$= \pi \times 5\text{ cm} \times 5\text{ cm} \times 14\text{ cm}$

$= 350\pi\text{ cm}^3$

$\therefore$ Capacity of the glass

$= 350\pi\text{ cm}^3 - \frac{250}{3}\pi\text{ cm}^3$

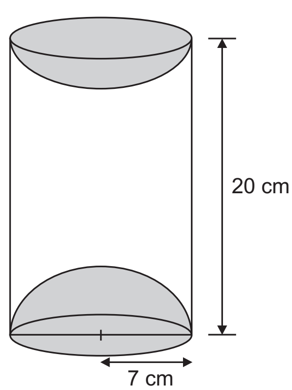
$= \frac{800}{3}\pi\text{ cm}^3$

$= 837.3\text{ cm}^3$

20. A wooden toy is made by scooping out a hemisphere of same radius as of cylinder, from each end of a wooden solid cylinder. If the height of the cylinder is 20 cm and its base is of radius 7 cm, find the total surface area of the toy. **[CBSE 2024 Standard]**

Sol. The total surface area of the toy

= Curved surface area of the cylinder + 2 × inner surface area of a hemisphere



$= 2\pi rh + 2 \times 2\pi r^2$

$= 2 \times \frac{22}{7} \times 7\text{ cm} \times 20\text{ cm} + 2 \times 2 \times \frac{22}{7} \times 7\text{ cm} \times 7\text{ cm}$

$= 880\text{ cm}^2 + 616\text{ cm}^2$

$= 1496\text{ cm}^2$

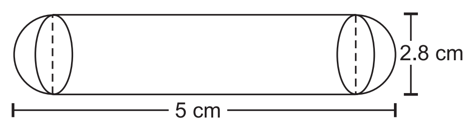
$\therefore$ Total surface area of the toy = 1496 cm^2.

21. A gulab jamun, when ready for eating, contains sugar syrup of about 30% of its volume. Find approximately how much syrup would be found in 45 such gulab jamuns, each shaped like a cylinder with two hemispherical ends, if the complete length of each of them is 5 cm and its diameter is 2.8 cm. **[CBSE 2008, SP 2011]**

Sol. Let r be the radius of each hemispherical ends.

Then, $r = \frac{2.8}{2}$ cm

$= 1.4$ cm



Let h be the length of the cylindrical part of the gulab jamun.

Then, the total length of the gulab jamun

$= r + r + h$

$= 2r + h$

$\therefore\quad 2r + h = 5$ cm [Given]

$\Rightarrow\quad 2 \times 1.4\text{ cm} + h = 5$ cm

$\Rightarrow\quad h = (5 - 2.8)$ cm

$= 2.2$ cm

Hence, the height of the cylinder is 2.2 cm.

$\therefore$ Volume of the each gulab jamun = Volume of the cylinder + Sum of the volumes of two hemispheres

$= \pi r^2 h + \frac{2}{3}\pi r^3 \times 2$

$= \pi r^2 h + \frac{4}{3}\pi r^3$

$= \frac{22}{7} \times [(1.4)^2 \times 2.2 + \frac{4}{3} \times (1.4)^3]\text{ cm}^3$

$\therefore$ Volumes of 45 gulab jamuns

$= 45 \times \frac{22}{7} \times 1.4 \times 1.4 \times (2.2 + \frac{4 \times 1.4}{3})\text{ cm}^3$

$= 45 \times \frac{22}{7} \times \frac{14 \times 14}{100} \times \frac{6.6 + 5.6}{3}\text{ cm}^3$

$= 45 \times \frac{22}{7} \times \frac{14 \times 14}{100} \times \frac{12.2}{3}\text{ cm}^3$

$= \frac{15 \times 22 \times 2 \times 14 \times 122}{1000}\text{ cm}^3$

$= \frac{1127280}{1000}\text{ cm}^3 = 1127.28\text{ cm}^3$

$\therefore$ Volume of syrup = 30% of 1127.28 cm^3

$$= \frac{1127.28 \times 30}{100} \text{ cm}^3$$

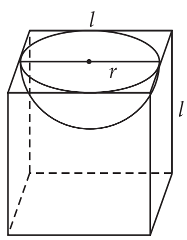
$$= 338.18 \text{ cm}^3$$

$$= 338 \text{ cm}^3 \text{ (approx.)}$$

Hence, the required volume of the syrup is 338 cm^3 (approx.)

22. A hemispherical depression is cut out from one face of a cubical wooden block such that the diameter l of the hemisphere is equal to the edge of the cube. Determine the surface area of the remaining solid. **[CBSE 2012, SP 2011]**

Sol. Let r be the radius of the base of the hemisphere. Then $2r = l$ = edge of the cube.



After the hemispherical depression is cut out, the surface area of the remaining solid

= Total surface area of the cubical block – Area of the base of the hemisphere + Curved surface area of the hemispherical depression

$$= 6l^2 - \pi r^2 + 2\pi r^2$$

$$= 6l^2 + \pi r^2 = 6l^2 + \frac{\pi l^2}{4}$$

$$= \frac{l^2}{4}(24 + \pi)$$

Hence, the required surface area of the remaining solid is $\frac{l^2}{4}(24 + \pi)$ sq units.

Long Answer Type Questions

23. A right triangle with sides 3 cm and 4 cm is revolved around its hypotenuse. Find the volume of the double cone thus generated. **[CBSE SP 2011]**

Sol. Let ΔBAC be a right-angled triangle with AB = 3 cm, AC = 4 cm and $\angle$BAC = 90°.

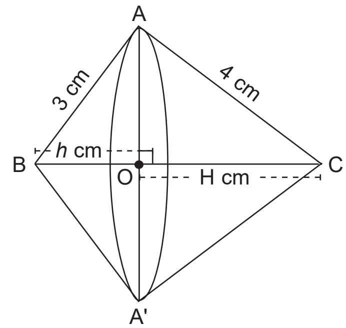
$\therefore$ From ΔBAC, by using Pythagoras' Theorem, we have

$$BC^2 = AB^2 + AC^2$$

$$= (3 \text{ cm})^2 + (4 \text{ cm})^2$$

$$= 25 \text{ cm}^2$$

$$\therefore \quad BC = 5 \text{ cm}$$



$\therefore$ Length of the hypotenuse is 5 cm.

This triangle ABC is now revolved around its hypotenuse BC. As a result, two cones ABA′OA and ACA′OA with O as the centre of the common base circle of the two cones. Here, O lies on BC and AOA′ $\perp$ BC. Let the common radius of the base of two cones be r and the heights BO and CO of the two cones be h and H respectively.

$$\therefore \quad h + H = BC = 5$$

$$\Rightarrow \quad H = 5 - h \qquad \text{...(1)}$$

$\therefore$ From ΔABO and ΔACO, by using Pythagoras' Theorem, we have

$$3^2 = h^2 + r^2$$

$$\Rightarrow \quad h^2 + r^2 = 9 \qquad \text{...(2)}$$

and $$4^2 = H^2 + r^2$$

$$\Rightarrow \quad H^2 + r^2 = 16 \qquad \text{...(3)}$$

From (1) and (3), we get

$$(5 - h)^2 + r^2 = 16$$

$$\Rightarrow \quad 25 - 10h + h^2 + r^2 - 16 = 0$$

$$\Rightarrow \quad 9 - 10h + 9 = 0 \qquad \text{[From (2)]}$$

$$\Rightarrow \quad h = \frac{18}{10} = \frac{9}{5} \qquad \text{...(4)}$$

$\therefore$ From (2), $$r = \sqrt{9 - h^2}$$

$$= \sqrt{9 - \frac{81}{25}} \qquad \text{[From (4)]}$$

$$= \sqrt{\frac{225 - 81}{25}}$$

$$= \frac{12}{5} \qquad \text{...(5)}$$

From (1) and (4),

$$H = 5 - \frac{9}{5} = \frac{16}{5} \qquad \text{...(6)}$$

Now, volume of the cone ABA′OA

$$= \frac{1}{3}\pi r^2 h$$

$$= \frac{\pi}{3} \times \frac{144}{25} \times \frac{9}{5} \text{ cm}^3$$

[From (4) and (5)]

and volume of the cone ACA′OA

$$= \frac{1}{3}\pi r^2 H$$

$$= \frac{\pi}{3} \times \frac{144}{25} \times \frac{16}{5}\ \text{cm}^3$$

[From (5) and (6)]

$\therefore$ The required sum of the volumes of the above two cones

$$= \frac{\pi}{3} \times \left[\frac{144 \times 9}{125} + \frac{144 \times 16}{125}\right] \text{cm}^3$$

$$= \frac{22}{21} \times \frac{144}{125} \times 25\ \text{cm}^3$$

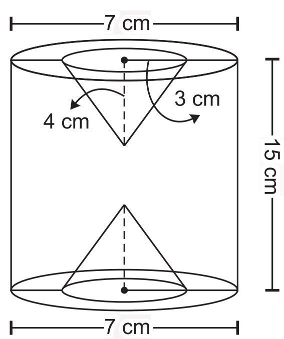
$$= \frac{22 \times 48}{35}\ \text{cm}^3$$

$$= \frac{1056}{35}\ \text{cm}^3$$

24. The height of a solid cylinder is 15 cm and its diameter is 7 cm. Two equal conical holes, each of radius 3 cm and height 4 cm, are cut off. Find the surface area of the solid. **[CBSE 2015]**

Sol. Let r be the radius of the base of the cylinder and h be its height. Also, let the radius of the base of each cone at the two ends of the solid cylinder be r_1 and the vertical height be h_1. If l_1 be the slant height of each cone, then

$$l_1 = \sqrt{r_1^2 + h_1^2}$$



Here, $r = \frac{7}{2}$ cm, $h = 15$ cm, $r_1 = 3$ cm, $h_1 = 4$ cm

and $l_1 = \sqrt{3^2 + 4^2} = 5$ cm

Now, the total surface area of the cylinder

$$= 2\pi rh + 2\pi r^2$$

$$= 2\pi r(r + h)$$

$$= 2\pi \times \frac{7}{2} \times \left(\frac{7}{2} + 15\right) \text{cm}^2$$

$$= \frac{259}{2}\pi\ \text{cm}^2 \qquad \ldots(1)$$

Sum of the areas of the circular bases of two cones

$$= 2\pi r_1^2$$

$$= 2\pi \times 9\ \text{cm}^2$$

$$= 18\pi\ \text{cm}^2 \qquad \ldots(2)$$

Sum of the curved surface areas of the two cones

$$= 2\pi r_1 l_1$$

$$= 2\pi \times 3 \times 5\ \text{cm}^2$$

$$= 30\pi\ \text{cm}^2 \qquad \ldots(3)$$

$\therefore$ Surface area of the remaining solid

= Total surface area of the solid cylinder – Sum of the areas of the circular bases of two cones + Sum of the curved surface areas of the two cones

$$= \left(\frac{259}{2} - 18 + 30\right)\pi\ \text{cm}^2$$

[From (1), (2) and (3)]

$$= \frac{22}{7} \times \frac{259 + 60 - 36}{2}\ \text{cm}^2$$

$$= \frac{11 \times 283}{7}\ \text{cm}^2$$

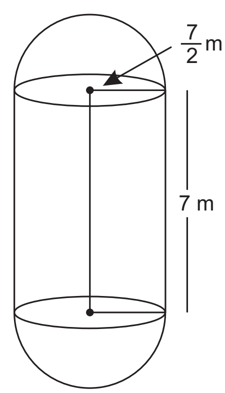
$$= \frac{3113}{7}\ \text{cm}^2$$

$$= 444.7\ \text{cm}^2 \text{ (approx.)}$$

Hence, the required surface area of the solid is 444.7 cm^2 (approx.)

25. The boilers are used in thermal power plants to store water and then used to produce steam. One such boiler consists of a cylindrical part in middle and two hemispherical parts at its both ends.

Length of the cylindrical part is 7 m and radius of cylindrical part is $\frac{7}{2}$ m.



Find the total surface area and the volume of the boiler. Also, find the ratio of the volume of cylindrical part to the volume of one hemispherical part. **[CBSE 2023 Basic]**

Sol. Total surface area = Curved surface area of the cylinder + 2 × surface area of a hemisphere

$= 2\pi rh + 2 \times 2\pi r^2$

$= 2 \times \frac{22}{7} \times \frac{7}{2}\text{ m} \times 7\text{ m} + 2 \times 2 \times \frac{22}{7} \times \frac{7}{2}\text{ m} \times \frac{7}{2}\text{ m}$

$= 154\text{ m}^2 + 154\text{ m}^2$

$= 308\text{ m}^2$

Total surface area $= 308\text{ m}^2$.

Volume of the boiler = Volume of the cylindrical part + 2 × Volume of a hemispherical part

$= \pi r^2 h + 2 \times \frac{2}{3}\pi r^3$

$= \frac{22}{7} \times \frac{7}{2}\text{ m} \times \frac{7}{2}\text{ m} \times 7\text{ m} + 2 \times \frac{2}{3} \times \frac{22}{7} \times \frac{7}{2}\text{ m} \times \frac{7}{2}\text{ m} \times \frac{7}{2}\text{ m}$

$= 269.5\text{ m}^3 + 179.67\text{ m}^3$

$= 449.17\text{ m}^3$

Volume of the boiler $= 449.17\text{ m}^3$

Volume of the cylindrical part : Volume of the hemispherical part

$$= \frac{\pi r^2 h}{\frac{2}{3}\pi r^3} = \frac{3}{2}\frac{h}{r}$$

$$= \frac{3}{2} \times \frac{7\text{ m}}{\frac{7}{2}\text{ m}} = 3 : 1$$

26. A tent is in the shape of a right circular cylinder up to a height of 3 m and then a right circular cone with a maximum height of 13.5 m above the ground. Calculate the cost of painting the inner side of the tent at the rate of ₹ 2 per square metre, if the radius of the base is 14 m.

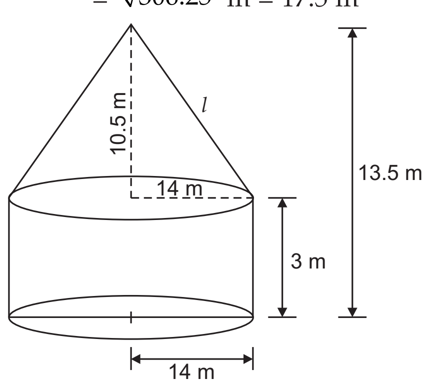
[CBSE 2023C Standard]

Sol. Slant height, l of the conical part

$$= \sqrt{(10.5)^2 + (14)^2}\text{ m}$$

$$= \sqrt{110.25 + 196}\text{ m}$$

$$= \sqrt{306.25}\text{ m} = 17.5\text{ m}$$



Total surface to be painted

= Curved surface of the conical part + Curved surface of the cylindrical part

$= \pi rl + 2\pi rh$

$= \frac{22}{7} \times 14\text{ m} \times 17.5\text{ m} + 2 \times \frac{22}{7} \times 14\text{ m} \times 3\text{ m}$

$= 770\text{ m}^2 + 264\text{ m}^2$

$= 1034\text{ m}^2$

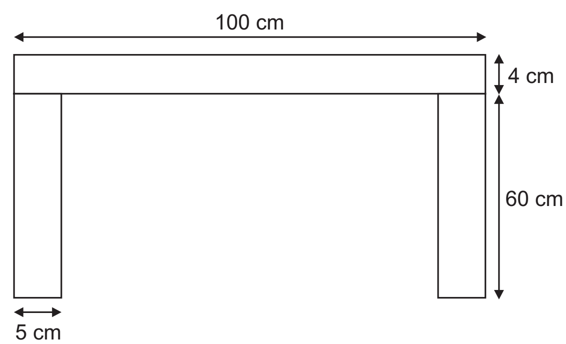
∴ Cost of painting the inner side of the tent

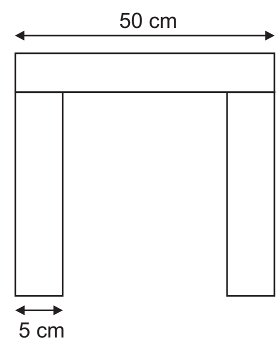
= ₹2 × 1034 = ₹2068.

Higher Order Thinking Skills (HOTS) Questions

(Page 228)

1. Ramesh makes a wooden table with four legs as per the design given below. Once the table gets completed, the table is to be polished. Only the part which touches the floor is not to be polished. Find the area of the table that is to be polished. Find the cost of polishing if the rate is ₹5 per sq cm.





Sol. Area of the table top = 100 cm × 50 cm = 5000 cm^2

Area of the bottom of the table top excluding the leg surface

$= 100\text{ cm} \times 50\text{ cm} - 4 \times 5\text{ cm} \times 5\text{ cm}$

$= 5000\text{ cm}^2 - 100\text{ cm}^2$

$= 4900\text{ cm}^2$

Area of the edges of the table

$= 2 \times 100 \text{ cm} \times 4 \text{ cm} + 2 \times 50 \text{ cm} \times 4 \text{ cm}$

$= 800 \text{ cm}^2 + 400 \text{ cm}^2$

$= 1200 \text{ cm}^2$

Area of the four legs

$= 4 \times 4 \times 60 \text{ cm} \times 5 \text{ cm}$

$= 4800 \text{ cm}^2$

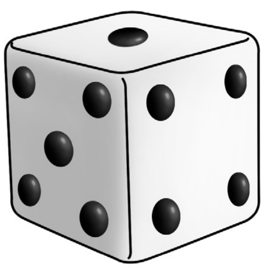
Total area of the table to be polished

$= 5000 \text{ cm}^2 + 4900 \text{ cm}^2 + 1200 \text{ cm}^2 + 4800 \text{ cm}^2$

$= 15900 \text{ cm}^2$

Cost of polishing = ₹5 × 15900 = ₹79500.

2. A cube-shaped regular die is made up of wood. The numbers on the side of the die are represented by the number of hemispherical indents on each side as shown in the figure. Find the number of hemispherical indents that has to be scooped. If the radius of each hemispherical indent is 4 cm and the edge of the die is 30 cm long, find the volume of the remaining wood in the die.



Sol. The number of hemispherical indents that has to be scooped

$$= 1 + 2 + 3 + 4 + 5 + 6 = 21$$

Volume of one hemispherical indent

$$= \frac{2}{3}\pi r^3$$

$$= \frac{2}{3} \times \frac{22}{7} \times 4 \text{ cm} \times 4 \text{ cm} \times 4 \text{ cm}$$

$$= \frac{2816}{21} \text{ cm}^3$$

Volume of 21 hemispherical indents

$$= 21 \times \frac{2816}{21} \text{ cm}^3 = 2816 \text{ cm}^3$$

Volume of the die

$$= 30 \text{ cm} \times 30 \text{ cm} \times 30 \text{ cm}$$

$$= 27000 \text{ cm}^3$$

∴ Volume of remaining wood in the die

$$= 27000 \text{ cm}^3 - 2816 \text{ cm}^3$$

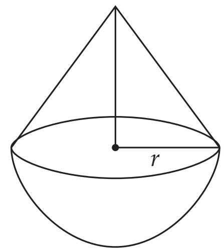
$$= 24184 \text{ cm}^3$$

Self-Assessment

(Page 229)

Multiple-Choice Questions

1. A solid is of the form of cone of radius 'r' surmounted on a hemisphere of the same radius. If the height of the cone is the same as the diameter of its base, then the volume of the solid is



(*a*) πr^3 (*b*) $\frac{4}{3}\pi r^3$

(*c*) $3\pi r^3$ (*d*) $\frac{2}{3}\pi r^3$

[CBSE 2023 Basic]

Sol. (*b*) $\frac{4}{3}\pi r^3$

Volume of the hemispherical part = $\frac{2}{3}\pi r^3$

Volume of the conical part $= \frac{1}{3} \times \pi \times r^2 \times 2r$

$$= \frac{2}{3}\pi r^3$$

∴ Volume of the solid = $\frac{2}{3}\pi r^3 + \frac{2}{3}\pi r^3 = \frac{4}{3}\pi r^3$

2. What is the total surface area of a solid hemisphere of diameter 'd'?

(*a*) $3\pi d^2$ (*b*) $2\pi d^2$

(*c*) $\frac{1}{2}\pi d^2$ (*d*) $\frac{3}{4}\pi d^2$

[CBSE 2023 Standard]

Sol. (*d*) $\frac{3}{4}\pi d^2$

$$= 2\pi \times \frac{d}{2} \times \frac{d}{2} + \pi \times \frac{d}{2} \times \frac{d}{2}$$

$$= \pi\frac{d^2}{2} + \pi\frac{d^2}{4} = \frac{3}{4}\pi d^2$$

3. If the volumes of two spheres are in the ratio 125 : 64, then the ratio of their surface areas is

(*a*) 5 : 4 (*b*) 4 : 5

(*c*) 16 : 25 (*d*) 25 : 16

[CBSE 2023C Basic]

Sol. (*d*) 25 : 16

Let the radius of two spheres be r_1 and r_2.

$$\frac{V_1}{V_2} = \frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \frac{125}{64}$$

$$\frac{r_1^3}{r_2^3} = \left(\frac{5}{4}\right)^3$$

$$\Rightarrow \quad \frac{r_1}{r_2} = \frac{5}{4}$$

$$\Rightarrow \quad r_1 = \frac{5}{4}r_2$$

Let A_1 and A_2 be their surface areas.

$$\frac{A_1}{A_2} = \frac{4\pi r_1^2}{4\pi r_2^2} = \frac{r_1^2}{r_2^2}$$

$$= \frac{\left(\frac{5}{4}r_2\right)^2}{r_2^2}$$

$$= \frac{25}{16}$$

$$\therefore \quad \frac{A_1}{A_2} = \frac{25}{16}$$

4. Outer surface area of a cylindrical juice glass with radius 7 cm and height 10 cm, is

(*a*) 440 sq cm (*b*) 594 sq cm
(*c*) 748 sq cm (*d*) 1540 sq cm

[CBSE 2024 Basic]

Sol. (*b*) 594 sq cm

Outer surface area of the juice glass

$$= 2\pi rh + \pi r^2$$

$$= 2 \times \frac{22}{7} \times 7 \text{ cm} \times 10 \text{ cm} + \frac{22}{7} \times 7 \text{ cm} \times 7 \text{ cm}$$

$$= 440 \text{ cm}^2 + 154 \text{ cm}^2$$

$$= 594 \text{ cm}^2 \quad \text{or} \quad 594 \text{ sq cm}$$

5. A tent is in the shape of a right circular cylinder up to a height of 4 m and conical above it. The total height of the tent is 16 m and radius of the base is 5 m. Then, its total surface area is

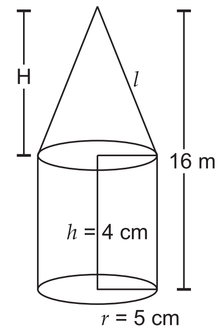
(*a*) 100π m^2 (*b*) 110π m^2
(*c*) 105π m^2 (*d*) 210π m^2

Sol. (*c*) 105π m^2

Let r be the radius of the base of the cylinder and h be its height.

Then, $r = 5$ m, $h = 4$ m.

Height of the cone, $H = 16$ m $- 4$ m $= 12$ m



If l be the slant height of the cone, then

$$l = \sqrt{H^2 + r^2}$$

$$= \sqrt{12^2 + 5^2} \text{ m}$$

$$= \sqrt{169} \text{ m} = 13 \text{ m}$$

$\therefore$ Surface area of the cone

$$= \pi rl$$

$$= \pi \times 5 \times 13 \text{ m}^2$$

$$= 65\pi \text{ m}^2$$

Curved surface area of the cylinder

$$= 2\pi rh$$

$$= 2\pi \times 5 \times 4 \text{ m}^2$$

$$= 40\pi \text{ m}^2$$

$\therefore$ Total surface area of the cylinder and the cone

$$= (65\pi + 40\pi) \text{ m}^2$$

$$= 105\pi \text{ m}^2$$

Hence, the required total surface area is 105π m^2.

Fill in the Blanks

6. The edge of a cube whose volume is $8x^3$ is $\mathbf{2x}$.

Sol. $a^3 = (2x)^3 = 8x^3$

$\Rightarrow \quad a = 2x$

7. Total surface area of a cube is 216 cm^2, its volume is **216 cm^3**.

Sol. $6a^2 = 216$ cm^2

$\Rightarrow \quad a = 6$ cm

$\Rightarrow \quad V = a^3 = (6)^3 = 216$ cm^3

8. If the surface area of a sphere is 144π, then its radius is **6 cm**.

Sol. $4\pi r^2 = 144\pi$

$\Rightarrow \quad r^2 = \frac{144}{4} = 36$

$\Rightarrow \quad r = 6$ cm

9. The curved surface area of one cone is twice that of the other cone. If the slant height of the latter is twice that of the former, then the ratio of their radii is **4 : 1**.

Sol. $\pi r_1 l_1 = 2\,(\pi r_2 l_2)$

$\Rightarrow \quad \pi r_1 l_1 = 2\,(\pi r_2 \times 2l_1)$

$\Rightarrow \quad \dfrac{r_1}{r_2} = \dfrac{4}{1} = 4 : 1$

Assertion-Reason Type Questions

Directions (Q. Nos. 10 to 13): Each of these questions contains an assertion followed by reason. Read them carefully, and answer the question on the basis of the following options, select the one that best describes the two statements.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

10. **Assertion (A):** A cuboid and a cube of volume 10 units and 15 units are glued together, the new volume would be 25 units.

Reason (R): Volume of combined solids is equal to the sum of individual solids.

Sol. Volume of combined solids is equal to sum of individual solids. Thus, the combined volume will be 10 + 15 = 25 units. Thus reason is a correct explanation of the assertion.

The answer is (*a*).

11. **Assertion (A):** When two cubes are joined together by glue, the total surface area increases.

Reason (R): The number of faces decreases.

Sol. When two cubes are glued, two of the faces are reduced, so total surface area will be less. Thus, assertion is wrong but reason is correct.

The answer is (*d*).

12. **Assertion (A):** A medicine capsule can be considered as combination of a cylinder bounded by two hemispheres. The total surface area of the capsule will be the exact sum of the individual surface areas.

Reason (R): Joined faces will not contribute to the total surface area.

Sol. When two solids are joined, two of the faces are reduced, so total surface area will be less. Thus, assertion is wrong but reason is correct.

The answer is (*d*).

13. **Assertion (A):** The surface area of the cuboid formed by joining two cubes of sides 4 cm each, end to end, is 160 cm^2.

Reason (R): Surface area of a cuboid of dimensions $l \times b \times h$ is $(lb + bh + hl)$.

[CBSE 2023C Standard]

Sol. Assertion (A) is true but Reason (R) is false.

The answer is (*c*).

Case Study Based Questions

14. The great Stupa at Sanchi is one of the oldest stone structures in India which was originally made by Emperor Ashoka. It is basically a big hemispherical dome with a cuboidal structure mounted on it.

Based on the above, answer the following questions:

(*a*) What is the volume of the hemispherical dome, if its height is 21 m? (use $\pi = \dfrac{22}{7}$)

Ans. Height of the hemispherical dome

= radius of the hemispherical dome

Volume of the dome $= \dfrac{2}{3}\pi r^3$

$= \dfrac{2}{3} \times \dfrac{22}{7} \times 21\text{ m} \times 21\text{ m} \times 21\text{ m}$

$= 19404\text{ m}^3$

(*b*) What is the area of plastic cloth required to cover the hemispherical dome, if radius of its base is 14 m?

Ans. Radius of the base = 14 m

Area of plastic cloth required

= surface area of the dome

$= 2\pi r^2$

$= 2 \times \dfrac{22}{7} \times 14\text{ m} \times 14\text{ m}$

$= 1232\text{ m}^2$

$\therefore$ Area of plastic cloth required = 1232 m^2.

(*c*) (*i*) If the dimensions of the cuboidal top are 8 m × 6 m × 4 m, then what is the surface area of this cuboidal top?

Ans. Surface area of the cuboidal top

$= 2[8\text{ m} \times 6\text{ m} + 6\text{ m} \times 4\text{ m} + 8\text{ m} \times 4\text{ m}]$

$= 2[48\text{ m}^2 + 24\text{ m}^2 + 32\text{ m}^2]$

$= 2 \times 104\text{ m}^2$

$= 208\text{ m}^2$

or

(*ii*) What is the volume of the cuboidal top of dimensions given in part (*iii*)?

[CBSE 2023C Basic]

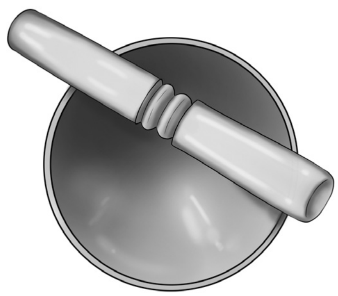
Ans. Volume of the cuboidal top

$= l \times b \times h$

$= 8 \text{ m} \times 6 \text{ m} \times 4 \text{ m}$

$= 192 \text{ m}^3$

15. Singing bowls (hemispherical in shape) are commonly used in sound healing practices. Mallet (cylindrical in shape) is used to strike the bowl in a sequence to produce sound and vibration.



One such bowl is shown here whose dimensions are as follows:

Hemispherical bowl has outer radius 6 cm and inner radius 5 cm. Mallet has height of 10 cm and radius 2 cm.

Based on the above, answer the following questions:

(*a*) What is the volume of the material used in making the mallet?

Ans. Volume of the material used in making of mallet

$= \pi r^2 h$

$= \dfrac{22}{7} \times 2 \text{ cm} \times 2 \text{ cm} \times 10 \text{ cm}$

$= 125.71 \text{ cm}^3$ (approx.)

(*b*) The bowl is to be polished from inside. Find the inner surface area of the bowl.

Ans. Inner surface area of the bowl

$= 2\pi r^2$

$= 2 \times \dfrac{22}{7} \times 5 \text{ cm} \times 5 \text{ cm}$

$= 157.14 \text{ cm}^2$ (approx.)

(*c*) (*i*) Find the volume of metal used to make the bowl.

Ans. Volume of metal used to make the bowl

$= \dfrac{2}{3}\pi (6 \text{ cm})^3 - \dfrac{2}{3}\pi (5 \text{ cm})^3$

$= \dfrac{2}{3}\pi (216 \text{ cm}^3 - 125 \text{ cm}^3)$

$= \dfrac{2}{3} \times \dfrac{22}{7} \times 91 \text{ cm}^3$

$= 190.67 \text{ cm}^3$ (approx.)

or

(*ii*) Find total surface area of the mallet. (Use $\pi = 3.14$) **[CBSE 2023 Basic]**

Ans. Total surface area of the mallet

$= 2\pi rh + 2\pi r^2$

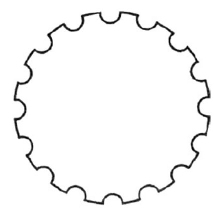
$= 2 \times 3.14 \times 2 \text{ cm} \times 10 \text{ cm}$

$+ 2 \times 3.14 \times 2 \text{ cm} \times 2 \text{ cm}$

$= 125.6 \text{ cm}^2 + 25.12 \text{ cm}^2$

$= 150.72 \text{ cm}^2$

16. A golf ball is spherical with about 300 – 500 dimples that help increase its velocity while in play. Golf balls are traditionally white but available in colours also. In the given figure, a golf ball has diameter 4.2 cm and the surface has 315 dimples (hemispherical) of radius 2 mm.



Based on the above, answer the following questions:

(*a*) Find the surface area of one such dimple.

Ans. Surface area of one such dimple

$= 2\pi r^2$

$= 2 \times 3.14 \times 2 \text{ mm} \times 2 \text{ mm}$

$= 25.12 \text{ mm}^2$

$= 0.2512 \text{ cm}^2 \quad [\because \ 1 \text{ cm}^2 = 100 \text{ mm}^2]$

(*b*) Find the volume of the material dug out to make one dimple.

Ans. Volume of material dug out to make one dimple

$= \dfrac{2}{3}\pi r^3$

$= \dfrac{2}{3} \times 3.14 \times \dfrac{2}{10} \text{ cm} \times \dfrac{2}{10} \text{ cm} \times \dfrac{2}{10} \text{ cm}$

$= 0.0167 \text{ cm}^3$ (approx.)

(c) (i) Find the total surface area exposed to the surroundings.

Ans. Total surface area exposed to the surroundings

$= 4 \times 3.14 \times 2.1 \text{ cm} \times 2.1 \text{ cm} + 315 \times 0.2512 \text{ cm}^2$

$- 315 \times 3.14 \times \frac{2}{10} \text{ cm} \times \frac{2}{10} \text{ cm}$

$= 55.3896 \text{ cm}^2 + 79.128 \text{ cm}^2 - 39.564 \text{ cm}^2$

$= 94.9536 \text{ cm}^2$

or

(ii) Find the volume of the golf ball.

[CBSE 2023 Standard]

Ans. Volume of the golf ball

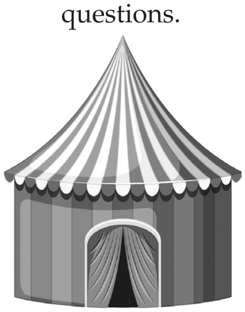
$= \frac{4}{3} \times \frac{22}{7} \times 2.1 \text{ cm} \times 2.1 \text{ cm} \times 2.1 \text{ cm}$

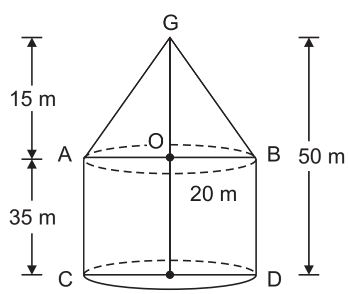
$- 315 \times 0.0167$

$= 38.808 \text{ cm}^3 - 5.2605 \text{ cm}^3$

$= 33.5475 \text{ cm}^3$ (approx.)

17. The owner of the circus rented the ground in a certain city. A big tent was put up there. A circus tent of total height 50 metres is to be made in the form of a right circular cylinder surmounted by a right circular cone. If the height and radius of the conical portion of the tent are 15 metres and 20 metres respectively, answer the following questions.





(a) What is the slant height of the conical part of the tent?

Ans. 25 m

(b) What is the curved surface area of the conical part of the tent?

Ans. 1571.43 m^2

(c) (i) What is the curved surface area of the cylindrical part of the tent?

Ans. 4400 m^2

or

(ii) Find the cost of the cloth required, at the rate of ₹14 per square metre to make the tent. (Note that the base of the tent will not be covered with canvas.)

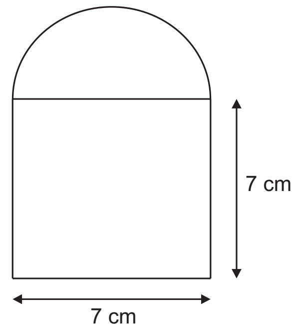
Ans. ₹83600 (approx.)

Very Short Answer Type Questions

18. A wooden paperweight is made such that the top is a hemisphere and the bottom is a cube. The diameter of the hemisphere is equal to the edge of the cube. The entire surface of the paperweight is to be painted green. If edge of the cube is of length 7 cm, find the total surface area of the paperweight that is to be painted. $\left[\text{Take } \pi = \frac{22}{7}\right]$

Ans. Total surface area of the paper weight

$= 2\pi r^2 + 5a^2$



$= 2 \times \frac{22}{7} \times \frac{7}{2} \text{ cm} \times \frac{7}{2} \text{ cm} + 5 \times 7 \text{ cm} \times 7 \text{ cm}$

$= 77 \text{ cm}^2 + 245 \text{ cm}^2$

$= 322 \text{ cm}^2$

19. In a coffee shop, coffee is served in two types of cups. One cup is in the shape of hemisphere with diameter 21 cm. The other cup is cylindrical in shape with diameter 7 cm and height 14 cm. Which cup will hold greater volume of coffee?

Ans. Diameter of the hemispherical cup = 21 cm

Volume of the hemispherical cup $= \frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \frac{22}{7} \times \frac{21}{2} \text{ cm} \times \frac{21}{2} \text{ cm} \times \frac{21}{2} \text{ cm}$

$= 2425.5 \text{ cm}^3$

Diameter of the cylindrical cup = 7 cm

Height of the cylindrical cup = 14 cm

Volume of the cylindrical cup $= \pi r^2 h$

$= \frac{22}{7} \times \frac{7}{2} \text{ cm} \times \frac{7}{2} \text{ cm} \times 14 \text{ cm}$

$= 539 \text{ cm}^3$

$\therefore$ The hemispherical cup will hold greater volume of coffee.

20. A cone and a sphere have the same radius and volume. Find the ratio of the radius of the cone to its height.

Ans. Let r be the radius of the cone and the sphere and h be the height of the cone.

Volume of the cone = Volume of the sphere

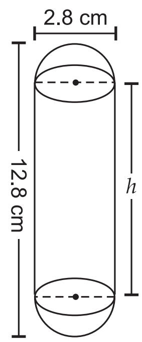
$\Rightarrow \quad \frac{1}{3}\pi r^2 h = \frac{4}{3}\pi r^3$

$\Rightarrow \quad h = 4r$

$\therefore$ Ratio of radius of the cone to its height = 1 : 4.

21. Find the volume of a solid in the form of a right circular cylinder with hemispherical ends whose total length is 12.8 cm. The diameter of each hemispherical end is 2.8 cm.

Sol. Let h be the height of the cylinder and r be the radius of the base of each hemisphere.



Then $\quad r = \frac{2.8}{2}$ cm = 1.4 cm

$h = (12.8 - 2.8)$ cm = 10 cm

Volume of the cylinder = $\pi r^2 h$

$= \pi \times (1.4)^2 \times 10 \text{ cm}^3$

Sum of the volumes of two hemispheres

$= 2 \times \frac{2}{3}\pi \times (1.4)^3 \text{ cm}^3$

$= \pi \times (1.4)^2 \left(10 + \frac{56}{30}\right) \text{ cm}^3$

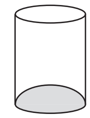
$= \frac{22}{7} \times (1.4)^2 \times \frac{356}{30} \text{ cm}^3$

$= 73.10 \text{ cm}^3$ (approx.)

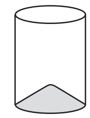
Hence, the required volume of a solid is 73.10 cm^3 (approx.)

22. Isha is 10 years old girl. On the result day, Isha and her father Suresh were very happy as she got first position in the class. While coming back to their home, Isha asked for a treat from her father as a reward for her success. They went to a juice shop and asked for two glasses of juice.

Aisha, a juice seller, was serving juice to her customers in two types of glasses. Both the glasses had inner radius 3 cm. The height of both the glasses was 10 cm.



First type: A glass with hemispherical raised bottom.



Second type: A glass with conical raised bottom of height 1.5 cm.

Isha insisted to have the juice in first type of glassand her father decided to have the juice in second type of glass. Out of the two, Isha or her father Suresh, who got more quantity of juice to drink and by how much? **[CBSE SP(Standard) 2019]**

Sol. Capacity of first glass

$= \pi r^2 H - \frac{2}{3}\pi r^3$

$= \pi \times 9(10 - 2) = \pi \times 9(8)$

$= 72\,\pi \text{ cm}^3$

Capacity of second glass

$= \pi r^2 H - \frac{1}{3}\pi r^2 h$

$= \pi \times 3 \times 3(10 - 0.5) = \pi \times 9\,(9.5)$

$= 85.5\,\pi \text{ cm}^3$

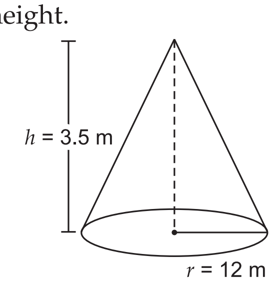
$\therefore$ Suresh got more quantity of juice.

$\therefore$ Suresh got [$(85.5\,\pi - 72\,\pi) \text{ cm}^3 =$] $13.5\,\pi \text{ cm}^3$ more quantity of juice than Isha.

Short Answer Type Questions

23. A heap of rice is in the form of a cone of base diameter 24 m and height 3.5 m. Find the volume of the rice. How much canvas cloth is required to just cover the heap? **[CBSE 2018]**

Sol. Let r and h be the radius of the base and the vertical height respectively of the cone and l be its slant height.



Then $r = 12$ m, $h = 3.5$ m

$\therefore$ Volume of the rice = $\frac{\pi}{3} r^2 h$

$$= \frac{\pi}{3} \times 12^2 \times 3.5 \text{ m}^3$$

$$= \frac{22}{7 \times 3} \times 12 \times 12 \times \frac{35}{10} \text{ m}^3$$

$$= 22 \times 2 \times 12 \text{ m}^3$$

$$= 528 \text{ m}^3$$

Also, $l = \sqrt{r^2 + h^2}$

$$= \sqrt{12^2 + (3.5)^2} \text{ m}$$

$$= \sqrt{144 + \frac{1225}{100}} \text{ m}$$

$$= \frac{\sqrt{14400 + 1225}}{10} \text{ m}$$

$$= \frac{\sqrt{15625}}{10} \text{ m}$$

$$= \frac{125}{10} \text{ m}$$

$$= 12.5 \text{ m}$$

$\therefore$ Curved surface area of the cone $= \pi rl$

$$= \frac{22}{7} \times 12 \times 12.5 \text{ m}^2$$

$$= \frac{3300}{7} \text{ m}^2$$

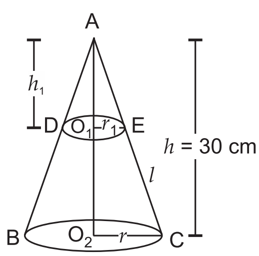
$$= 471.4 \text{ m}^2 \text{ (approx.)}$$

Hence, the required canvas cloth is 471.4 m^2 (approx.)

24. The height of a cone is 30 cm. A small cone is cut off at the top by a plane parallel to the base. If its volume is $\frac{1}{27}$ of the volume of the given cone, at what height above the base is the section made?

[CBSE 2017, 2005 C, SP 2011]

Sol. Let r, h and V be the radius of the base, vertical height and the volume respectively of the entire cone ABC and let r_1, h_1 and V_1 be the radius of the base, vertical height and the volume respectively of the smaller cone ADE which is cut off.



Then, $h = 30$ cm.

Let O_1 and O_2 be the centres of the circular bases of the smaller and bigger cones respectively.

Then from similar triangles AO_1E and AO_2C, we have

$$\frac{r_1}{r} = \frac{h_1}{h} = \frac{h_1}{30}$$

$$\therefore \quad r_1 = \frac{r^2 h_1^2}{900} \qquad \ldots(1)$$

$$V_1 = \frac{\pi}{3} r_1^2 h_1$$

and $$V = \frac{\pi}{3} r^2 h = 10\pi r^2 \qquad \ldots(2)$$

Given that $$V_1 = \frac{V}{27}$$

$$\therefore \quad \frac{\pi}{3} r_1^2 h_1 = \frac{1}{27} \times 10\pi r^2 \qquad \text{[From (2)]}$$

$$\Rightarrow \quad 9 r_1^2 h_1 = 10 r^2$$

$$\Rightarrow \quad 9 \times \frac{r^2 h_1^2}{900} \times h_1 = 10 r^2 \qquad \text{[From (1)]}$$

$$\Rightarrow \quad h_1^3 = 1000$$

$$\Rightarrow \quad h_1 = 10$$

$\therefore$ $h - h_1 = O_1O_2 = (30 - 10)$ cm $= 20$ cm

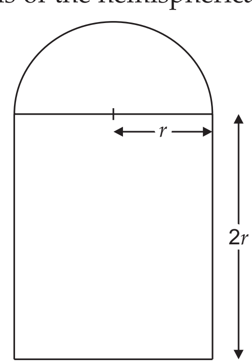
Hence, the required height above the base of the bigger cone, where the smaller cone was cut off is 20 cm.

25. A room is in the form of cylinder surmounted by a hemispherical dome. The base radius of hemisphere is one-half the height of cylindrical part. Find total height of the room if it contains $\left(\frac{1408}{21}\right)$ m^3 of air. $\left(\text{Take } \pi = \frac{22}{7}\right)$

[CBSE 2023C Basic]

Ans. Let height of cylindrical part $= 2r$

$\therefore$ Radius of the hemispherical part $= r$



Volume of the room

$$= \frac{2}{3}\pi r^3 + \pi r^2 \times 2r$$

$$= \frac{2}{3}\pi r^3 + 2\pi r^3$$

$$= \frac{8}{3}\pi r^3$$

According to the question,

$$\frac{8}{3}\pi r^3 = \frac{1408}{21}\text{ m}^3$$

$$\Rightarrow \quad r^3 = \frac{3}{8}\times\frac{7}{22}\times\frac{1408}{21}\text{ m}^3$$

$$\Rightarrow \quad r^3 = 8\text{ m}^3$$

$$\Rightarrow \quad r = 2\text{ m}$$

$\therefore$ Total height of the room $= 2r + r$

$= 3r$

$= 3 \times 2\text{ m} = 6\text{ m}$

26. The difference between the outer and inner radii of a hollow right circular cylinder of length 14 cm is 1 cm. If the volume of the metal used in making the cylinder is 176 cm^3, find the outer and inner radii of the cylinder. **[CBSE 2024 Standard]**

Ans. Let r be the inner radius of the cylinder.

$\therefore$ Outer radius of the cylinder $= r + 1$.

Volume of the metal used in making of cylinder

$= \pi(r+1)^2 h - \pi r^2 h$

$= \pi h[(r+1)^2 - r^2]$

$= \pi h(2r + 1)$

According to the question,

$$\pi h(2r+1) = 176$$

$$\Rightarrow \quad 2r + 1 = \frac{7}{22}\times\frac{1}{14}\times 176$$

$$\Rightarrow \quad 2r + 1 = 4$$

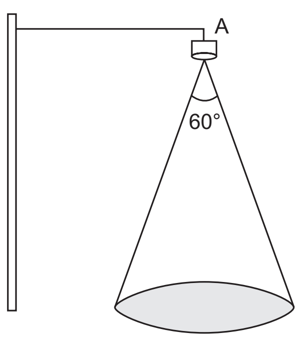
$$\Rightarrow \quad 2r = 3$$

$$\Rightarrow \quad r = 1.5\text{ cm}$$

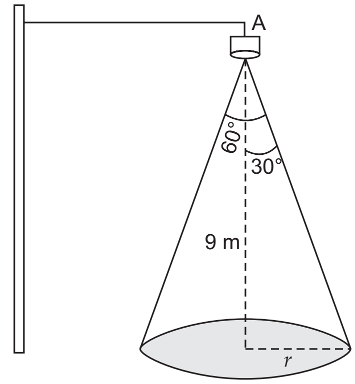
$\therefore$ Inner radius = 1.5 cm

Outer radius = 2.5 cm

27. In an amusement park, a spotlight is fixed at a height of 9 m from the surface. The light from A forms a right circular cone and forms an illuminated circular region on the ground. Find the area of the illuminated circular region on the ground, in terms of π.



Ans. Let the radius of the illuminated circular region be r m.



$$\tan 30^\circ = \frac{r}{9\text{ m}}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{r}{9\text{ m}}$$

$$\Rightarrow \quad r = \frac{9}{\sqrt{3}}\text{ m}$$

$$\Rightarrow \quad r = 3\sqrt{3}\text{ m}$$

Area of the illuminated circular region

$= \pi r^2$

$= \pi \times (3\sqrt{3})^2\text{ m}^2$

$= 27\pi\text{ m}^2$

Long Answer Type Questions

28. A vessel is in the form of a hemispherical bowl surmounted by a hollow cylinder of same diameter. The diameter of the hemispherical bowl is 14 cm and the total height of the vessel is 13 cm. Find the inner surface area of the vessel. Also, find the volume of the vessel.

[CBSE 2024 Standard]

Ans. Inner surface area of the vessel

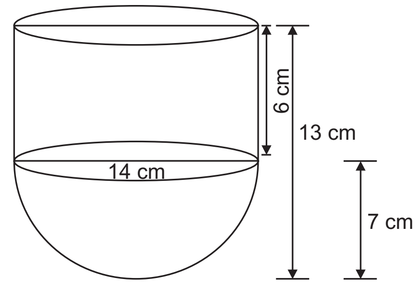
$= 2\pi rh + 2\pi r^2$

$= 2 \times \frac{22}{7} \times 7\text{ cm} \times 6\text{ cm}$

$+ 2 \times \frac{22}{7} \times 7\text{ cm} \times 7\text{ cm}$

$= 264\text{ cm}^2 + 308\text{ cm}^2$

$= 572\text{ cm}^2$



Volume of the vessel

$= \pi r^2 h + \frac{2}{3}\pi r^3$

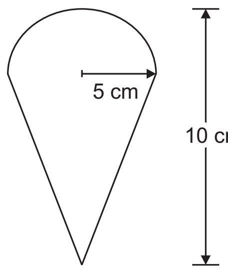
$= \frac{22}{7} \times 7 \text{ cm} \times 7 \text{ cm} \times 6 \text{ cm}$

$+ \frac{2}{3} \times \frac{22}{7} \times 7 \text{ cm} \times 7 \text{ cm} \times 7 \text{ cm}$

$= 924 \text{ cm}^3 + 718.67 \text{ cm}^3$

$= 1642.67 \text{ cm}^3$ (approx.)

29. An ice cream filled cone having radius 5 cm and height 10 cm is as shown in the figure. Find the volume of the ice cream in 7 such cones.



[CBSE 2023 Basic]

Ans. Radius of the conical part = 5 cm

Height of the conical part = 10 cm – 5 cm = 5 cm

Volume of ice cream in one cone

$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$

$= \frac{1}{3} \times \pi \times 5 \text{ cm} \times 5 \text{ cm} \times 5 \text{ cm} + \frac{2}{3}\pi$

$\times 5 \text{ cm} \times 5 \text{ cm} \times 5 \text{ cm}$

$= \frac{1}{3}\pi \times 125 \text{ cm}^3 + \frac{2}{3}\pi \times 125 \text{ cm}^3$

$= \pi \times 125 \text{ cm}^3 = \frac{22}{7} \times 125 \text{ cm}^3$

$\therefore$ Volume of ice cream in 7 such cones

$= 7 \times \frac{22}{7} \times 125 \text{ cm}^3$

$= 2750 \text{ cm}^3$

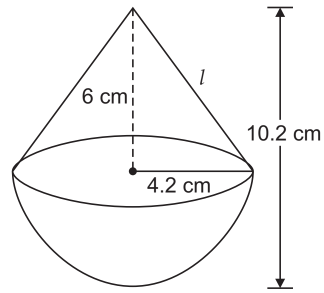
30. A solid wooden toy is in the shape of a right circular cone mounted on a hemisphere of same radius. If the radius of the hemisphere is 4.2 cm and the total height of the toy is 10.2 cm, find the volume of the wooden toy. Also find the total surface area of the toy.

[CBSE 2023C Standard]

Ans. Height of the conical part of the toy

$= 10.2 \text{ cm} - 4.2 \text{ cm}$

$= 6 \text{ cm}$



Volume of the wooden toy

$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$

$= \frac{1}{3} \times \frac{22}{7} \times 4.2 \text{ cm} \times 4.2 \text{ cm} \times 6 \text{ cm}$

$+ \frac{2}{3} \times \frac{22}{7} \times (4.2 \text{ cm})^3$

$= 110.88 \text{ cm}^3 + 155.232 \text{ cm}^3$

$= 266.112 \text{ cm}^3$

Slant height of the cone

$= l = \sqrt{(4.2)^2 + (6)^2} \text{ cm} = 7.32 \text{ cm}$

(approx.)

Total surface area of the toy

$= \pi r l + 2\pi r^2$

$= \frac{22}{7} \times 4.2 \text{ cm} \times 7.32 \text{ cm} + 2 \times \frac{22}{7}$

$\times 4.2 \text{ cm} \times 4.2 \text{ cm}$

$= 96.624 \text{ cm}^2 + 110.88 \text{ cm}^2$

$= 207.504 \text{ cm}^2$ (approx.)

Let's Compete

(Page 232)

Multiple-Choice Questions

1. The height and base radius of a cone, each is increased by 50%. The ratio between the volume of the new cone and the given cone is

(*a*) 3 : 2 (*b*) 9 : 4 (*c*) 8 : 27 (*d*) 27 : 8

Sol. (*d*) 27 : 8

Let r and h be the radius and height of the given cone.

Radius of the new cone

$= r + \frac{50}{100} r = r + \frac{r}{2} = \frac{3r}{2}$

Height of the new cone

$= h + \frac{50}{100} h = h + \frac{h}{2} = \frac{3h}{2}$

$$\therefore \quad \frac{\text{Volume of new cone}}{\text{Volume of the given cone}}$$

$$= \frac{\frac{1}{3}\times\pi\times\frac{3r}{2}\times\frac{3r}{2}\times\frac{3h}{2}}{\frac{1}{3}\times\pi\times r\times r\times h}$$

$$= \frac{3}{2}\times\frac{3}{2}\times\frac{3}{2}$$

$$= \frac{27}{8}$$

2. An ice cream brick is in the shape of cuboid of dimensions 33 cm × 25 cm × 20 cm. The ice cream is to be distributed among children by filling ice cream cones of diameter 6 cm and height 7 cm. Find how many children will get the ice cream cones?

(*a*) 248 (*b*) 260 (*c*) 250 (*d*) 255

Sol. (*c*) 250

Volume of ice cream brick

$= 33 \text{ cm} \times 25 \text{ cm} \times 20 \text{ cm}$

Volume of one icre cream cone

$$= \frac{1}{3}\times\frac{22}{7}\times 3 \text{ cm} \times 3 \text{ cm} \times 7 \text{ cm}$$

$= 22 \times 3 \text{ cm}^3$

Number of children who will get the ice cream

$$= \frac{33 \text{ cm}\times 25 \text{ cm}\times 20 \text{ cm}}{22\times 3 \text{ cm}^3} = 250$$

3. The radius of the base and height of a cone are 4 cm and 9 cm respectively. If its height is decreased and base radius is increased each by 2 cm, then the ratio of the volume of the new cone to that of the original cone is

(*a*) 9 : 2 (*b*) 5 : 2

(*c*) 7 : 4 (*d*) 8 : 3

Sol. (*c*) 7 : 4

Radius of the original cone = 4 cm

Height of the original cone = 9 cm

Radius of the new cone = 4 cm + 2 cm = 6 cm

Height of the new cone = 9 cm – 2 cm = 7 cm

$$\frac{\text{Volume of new cone}}{\text{Volume of original cone}}$$

$$= \frac{\frac{1}{3}\times\pi\times 6 \text{ cm}\times 6 \text{ cm}\times 7 \text{ cm}}{\frac{1}{3}\times\pi\times 4 \text{ cm}\times 4 \text{ cm}\times 9 \text{ cm}}$$

$$= \frac{6\times 6\times 7}{4\times 4\times 9} = \frac{7}{4}$$

4. If the total surface area of a cube is 216 cm^2, then its volume is

(*a*) 144 cm^3 (*b*) 196 cm^3

(*c*) 212 cm^3 (*d*) 216 cm^3

[CBSE SP 2012]

Sol. (*d*) 216 cm^3

Let a be the side of the cube.

Then $6a^2 = 216 \text{ cm}^2$

$\Rightarrow \quad a^2 = 36 \text{ cm}^2$

$\Rightarrow \quad a = 6 \text{ cm}$

$\therefore$ Required volume $= a^3 = 6^3 \text{ cm}^3 = 216 \text{ cm}^3$

5. The number of circular plates each of radius 7 cm and thickness 0.5 cm that should be placed one above the other to form a solid right circular cylinder of volume 7700 cm^3 is

(*a*) 70 (*b*) 100

(*c*) 200 (*d*) 150

Sol. (*b*) 100

Volume of each circular plate of radius 7 cm and thickness 0.5 cm $= \pi \times 7^2 \times 0.5 \text{ cm}^3$

$$= \frac{22}{7} \times 49 \times 0.5 \text{ cm}^3$$

$= 77 \text{ cm}^3$

$\therefore$ Required number of circular plates $= \frac{7700}{77}$

$= 100$

6. Volume of a cylindrical wire of radius 1 cm is 220 cm^3. It is cut into three unequal segments. If the lengths of two cut segments are 10 cm and 15 cm, then the length of the third segment is

(*a*) 45 cm (*b*) 50 cm

(*c*) 40 cm (*d*) 55 cm

Sol. (*a*) 45 cm

Let r be the radius and h be the length of the cylindrical wire

Then, its volume $= \pi r^2 h = \pi \times h$

Given, volume $= 220 \text{ cm}^3$

$$\therefore \quad \frac{22}{7} \times h = 220$$

$\Rightarrow \quad h = 70$

$\therefore$ Total length of the wire = 70 cm

Total length of two segment of wire

$= (10 + 15) \text{ cm} = 25 \text{ cm}$

$\therefore$ Required length of the third segment

$= (70 – 25) \text{ cm} = 45 \text{ cm}$

7. If the total surface area of a hemisphere is 9 cm^2, then its volume is

(*a*) $\sqrt{\frac{\pi}{3}}$ cm^3 (*b*) $2\sqrt{\frac{\pi}{3}}$ cm^3

(*c*) $\sqrt{\frac{3}{\pi}}$ cm^3 (*d*) $2\sqrt{\frac{3}{\pi}}$ cm^3

Sol. (*d*) $2\sqrt{\frac{3}{\pi}}$ cm^3

We have $3\pi r^2 = 9$ where r is the radius of the hemisphere.

$$\therefore \quad r = \left(\frac{3}{\pi}\right)^{1/2} \qquad \ldots(1)$$

$\therefore$ Required volume of the hemisphere

$$= \frac{2}{3}\pi r^3$$

$$= \frac{2}{3} \times \pi \times \left(\frac{3}{\pi}\right)^{3/2} \text{cm}^3 \qquad \text{[From (1)]}$$

$$= 2 \times \frac{3^{3/2-1}}{\pi^{3/2-1}} \text{cm}^3$$

$$= 2\sqrt{\frac{3}{\pi}} \text{cm}^3$$

8. The ratio of the volumes of two cones is 2 : 5. If the ratio of their diameters is 4 : 5, then the ratio of their heights is

(*a*) 25 : 64 (*b*) 16 : 25

(*c*) 5 : 8 (*d*) 4 : 25

Sol. (*c*) 5 : 8

Let r_1 and r_2 be the radii of the bases of two cones, h_1 and h_2 be their respective vertical heights and let V_1 units and V_2 be their respective volumes.

Then, $V_1 = \frac{\pi}{3} r_1^2 h_1$ and $V_2 = \frac{\pi}{3} r_2^2 h_2$

$$\therefore \quad \frac{V_1}{V_2} = \frac{r_1^2 h_1}{r_2^2 h_2}$$

$$\Rightarrow \quad \frac{2}{5} = \frac{r_1^2}{r_2^2} \times \frac{h_1}{h_2} = \frac{4^2}{5^2} \times \frac{h_1}{h_2}$$

$$\Rightarrow \quad \frac{h_1}{h_2} = \frac{2}{5} \times \frac{25}{16} = \frac{5}{8}$$

$$\therefore \quad h_1 : h_2 = 5 : 8$$

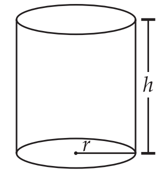
9. A cuboid and a right circular cylinder have equal volumes. Their heights are also equal. If r and h are respectively the radius of the base and the height of the cylinder, then the area of the bottom of the cuboid is

(*a*) πhr^2 (*b*) πr^2

(*c*) πh^2 (*d*) πrh

Sol. (*b*) πr^2

The radius of the base and the height of the cylinder are r and h respectively.



$\therefore$ Volume of the cylinder = $\pi r^2 h$

Height of the cuboid = h

Let a and b be the length and breadth of the cuboid.

$\therefore$ Volume of the cuboid = abh

According to the problem, we have

$$\therefore \quad abh = \pi r^2 h$$

$$\Rightarrow \quad ab = \pi r^2$$

$\therefore$ Area of the bottom of the cuboid = $ab = \pi r^2$

10. The volume of the largest sphere that can be cut from a cylindrical log of wood of base radius 1 m and height 4 m is

(*a*) $\frac{4}{3}\pi$ m^3 (*b*) $\frac{16\pi}{3}$ m^3

(*c*) $\frac{10}{3}$ m^3 (*d*) $\frac{8\pi}{3}$ m^3

[CBSE SP 2012]

Sol. (*a*) $\frac{4}{3}\pi$ m^3

We see that the volume of the sphere will be largest when the diameter of the sphere = the diameter of the base of the cylinder, i.e. their radii are equal.

$\therefore$ The radius of the sphere will be 1 m.

Hence, the required volume of the sphere

$$= \frac{4}{3}\pi \times 1^3 \text{ m}^3$$

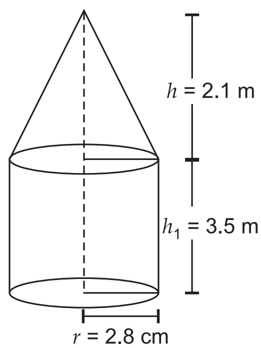
$$= \frac{4}{3}\pi \text{ m}^3$$

Life Skills

(Page 233)

1. Thousands of people were rendered homeless due to heavy floods in a state. 50 schools collectively offered to the State Government to provide place and the canvas for 1500 tents to be fixed by the Government and decided to share the whole expenditure equally. The lower part of each tent is cylindrical of base radius 2.8 m and height 3.5 m, with conical upper part of the same base radius but of height 2.1 m. If the canvas used to make the tents costs ₹120 per sq m, find the amount shared by each school to set-up the tents. $\left[\text{Use } \pi = \frac{22}{7}\right]$ **[CBSE 2016]**

Sol. Let r be the common radius of the base of the cylinder and the cone. Then r = 2.8 m. Let h and h_1 be the vertical height of the cone and the cylinder respectively.



Then h = 2.1 m and h_1 = 3.5 m. If l be the slant height of the cone, then

$$l = \sqrt{h^2 + r^2}$$
$$= \sqrt{(2.1)^2 + (2.8)^2} \text{ m}$$
$$= \sqrt{4.41 + 7.84} \text{ m}$$
$$= \sqrt{12.25} \text{ m} = 3.5 \text{ m}$$

∴ Curved surface area of the cone

$$= \pi rl$$
$$= \frac{22}{7} \times 2.8 \times 3.5 \text{ m}^2$$
$$= 30.8 \text{ m}^2 \quad \ldots(1)$$

Also, curved surface area of the cylinder

$$= 2\pi rh_1$$
$$= 2 \times \frac{22}{7} \times 2.8 \times 3.5 \text{ m}^2$$
$$= 61.6 \text{ m}^2 \quad \ldots(2)$$

∴ Total surface area of the cone and the cylinder

$$= (30.8 + 61.6) \text{ m}^2$$
$$= 92.4 \text{ m}^2 \quad \text{[From (1) and (2)]}$$

∴ Total surface area of the canvas required for each tent = 92.4 m^2

∴ Total surface area of 1500 tents

$$= 92.4 \times 1500 \text{ m}^2$$
$$= 138600 \text{ m}^2$$

∴ Total cost required for each of 50 schools is ₹ $\frac{138600 \times 120}{50}$, i.e. ₹332640

2. Milk in a container which is in the form of a cylinder of height 28 cm and radius 10 cm respectively, is to be distributed in a camp for flood victims. If this milk is available at the rate of ₹35 per litre and 880 litres of milk is needed daily for a camp, find how many such containers of milk are needed for a camp and what cost will it put on the donor agency for this.

Sol. Given, r = 10 cm

h = 28 cm

Volume of the milk in a container

$$= \pi r^2 h$$
$$= \frac{22}{7} \times 10 \text{ cm} \times 10 \text{ cm} \times 28 \text{ cm}$$
$$= 8800 \text{ cm}^3 = 8.8 \text{ litres}$$

∴ Required number of containers

$$= 880 \div 8.8 = 100$$

Also, the total cost of milk = ₹35 × 880 = ₹30800