11

Areas Related to Circles

Checkpoint (Page 206)

1. What is the circumference of a circle of radius 7 cm?

Sol. Circumference = $2\pi r = 2 \times \frac{22}{7} \times 7$ cm = 44 cm

Hence, the required circumference of a circle is 44 cm.

2. The perimeter of a square of side 11 cm is equal to the circumference of a circle of radius r. What is the area of this circle?

Sol. The perimeter of the square = 4 × 11 cm = 44 cm

The circumference of the circle = $2\pi r$

$\therefore$ According to the problem, we have

$$2\pi r = 44 \text{ cm}$$

$$\Rightarrow \quad 2 \times \frac{22}{7} \times r = 44 \text{ cm}$$

$$\Rightarrow \quad r = \frac{44 \times 7}{2 \times 22} \text{ cm}$$

$$= \frac{44 \times 7}{44} \text{ cm} = 7 \text{ cm}$$

$\therefore$ Area of the circle = πr^2

$$= \frac{22}{7} \times 7 \times 7 \text{ cm}^2$$

$$= 154 \text{ cm}^2$$

Hence, the required area of the circle is 154 cm^2.

3. A wheel rotates 50000 times to cover a distance of 176 km. Find the radius of the wheel in cm.

Sol. If r cm be the radius of the wheel, then

$$2\pi r \times 50000 = 176 \times 1000 \times 100 \text{ cm}$$

$$\Rightarrow \quad r = \frac{17600000}{100000} \times \frac{7}{22} \text{ cm}$$

$$\Rightarrow \quad r = 8 \times 7 \text{ cm}$$

$$= 56 \text{ cm}$$

Hence, the required radius of the wheel is 56 cm.

4. What is the perimeter of a semicircular wire of radius 14 cm?

Sol. Perimeter of a semicircle

= Circumference of a semicircle + Diameter

$= \pi r + 2r$

$= r(\pi + 2)$

$= 14 \times \left(\frac{22}{7} + 2\right)$ cm

$= 14 \times \frac{36}{7}$ cm

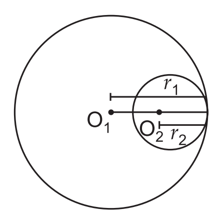
= 2 × 36 cm

= 72 cm

Hence, the required perimeter of a semicircular wire is 72 cm.

5. Two circles of radii 8 cm and 6 cm have centres at O_1 and O_2 respectively. If O_1O_2 = 2 cm, will the two circles touch each other internally or externally?

Sol. If r_1 cm and r_2 cm be the radii of two circles with centres at O_1 and O_2 respectively, then r_1 = 8 cm, r_2 = 6 cm.

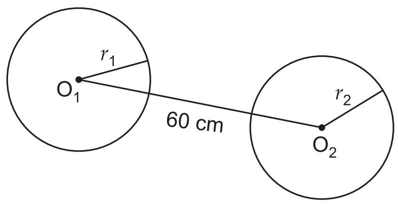


Now, $\quad O_1O_2 = 2 \text{ cm} = r_1 - r_2$

So, the two circles touch each other internally.

6. Two circles with centres at O_1 and O_2 have radii 25 cm and 31 cm respectively. If O_1O_2 = 60 cm, can the two circles intersect each other or not?

Sol. If r_1 cm and r_2 cm be the radii of two circles with centres at O_1 and O_2 respectively, then r_1 = 25 cm, r_2 = 31 cm.



Now, $O_1O_2 = 60$ cm

and $r_1 + r_2 = (25 + 31)$ cm = 56 cm

$\therefore$ $r_1 + r_2 < O_1O_2$

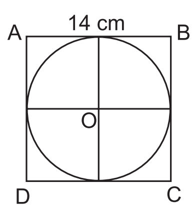
Hence, the two circles will not intersect each other.

7. In the above problem what is the area of the common region between the two circles?

Sol. Since the two circles do not intersect each other at all, there is no common area between the two circles. Hence, the common area is zero.

8. A circle is drawn inside a square of side 14 cm such that it touches all the sides of the square. What is the area of the circle?

Sol. If a circle with centre at O is inscribed within a square of side 14 cm, then the radius r of the circle is $\frac{15}{3}$ cm = 7 cm.



$\therefore$ Area of the circle $= \pi r^2$

$$= \frac{22}{7} \times 7 \times 7 \text{ cm}^2$$

$$= 154 \text{ cm}^2$$

Hence, the required area of the circle is 154 cm^2.

9. A circular wire of diameter 56 cm is cut into 4 equal pieces and a square of maximum area is formed with these 4 pieces. What is the area of this square?

Sol. Circumference of the circle $= 2\pi r$

$$= 2 \times \frac{22}{7} \times \frac{56}{2} \text{ cm}$$

$$= 22 \times 8 \text{ cm}$$

$$= 176 \text{ cm}$$

Length of each piece of the wire

$$= \frac{176}{4} \text{ cm}$$

$$= 44 \text{ cm}$$

$\therefore$ Maximum area of the square $= 44 \times 44$ cm^2

$$= 1936 \text{ cm}^2$$

Hence, the required area of the square is 1936 cm^2.

10. What is the maximum length of a rod that can be kept inside a circular enclosure of area 346.5 m^2? [Use $\pi = \frac{22}{7}$]

Sol. If r m be the radius of the circle, then

$$\pi r^2 = 346.5 \text{ m}^2$$

$$\Rightarrow \quad \frac{22}{7} \times r^2 = 346.5 \text{ m}^2$$

$$\Rightarrow \quad r^2 = \frac{346.5 \times 7}{22} \text{ m}^2$$

$$= \frac{2425.5}{22} \text{ m}^2$$

$$= 110.25 \text{ m}^2$$

$$\Rightarrow \quad r = \sqrt{110.25} \text{ m}$$

$$= 10.5 \text{ m}$$

$\therefore$ Radius of the circle is 10.5 m.

$\therefore$ The maximum length of the rod

= diameter of the circle

$= 2r$

$= 10.5 \times 2$ m

= 21 m

Hence, the maximum length of a rod is 21 m.

Check Your Progress

(Page 210)

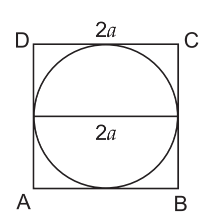
Multiple-Choice Questions

1. The perimeter (in cm) of a square circumscribing a circle of radius a cm is

(*a*) $8a$ (*b*) $4a$

(*c*) $2a$ (*d*) $16a$ **[CBSE 2011]**

Sol. (*a*) $8a$



Side of the square = diameter of the circle

$= 2a$ cm

$\therefore$ Perimeter of the square $= 4 \times 2a$ cm $= 8a$ cm

2. If the circumference of a circle increases from 4π cm to 5π cm, then the area of the smaller circle is increased by

(*a*) 50% (*b*) 56%

(*c*) 56.25% (*d*) 52%

Sol. (*c*) 56.25%

Let the radii of the original circle and the increased circle be r cm and R cm respectively.

Then $2\pi r = 4\pi$ cm

$\Rightarrow$ $r = 2$ cm

and $2\pi R = 5\pi$ cm

$\Rightarrow$ $R = \frac{5}{2}$ cm

$\therefore$ The radii of the original circle and the increased circle are 2 cm and $\frac{5}{2}$ cm respectively.

Then the area of the original circle $= \pi r^2 = 4\pi$ cm^2 and the area of the increased circle $= \pi R^2 = \pi \frac{60}{4}$ cm^2.

$\therefore$ Increase in area $= \left(\frac{25\pi}{4} - 4\pi\right) = \frac{9\pi}{4}$ cm^2

$\therefore$ Percentage increase in the area

$$= \left(\frac{9\pi}{4} \times \frac{1}{4\pi}\right) \times 100\%$$

$$= \frac{9 \times 100}{16}\%$$

$$= \frac{225}{4}\% = 56.25\%$$

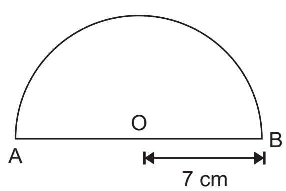
3. In the radius of a semi-circular protractor is 7 cm, then its perimeter is

(*a*) 11 cm (*b*) 14 cm

(*c*) 22 cm (*d*) 36 cm

[CBSE 2023 Basic]

Sol. (*d*) 36 cm



Perimeter of the semi-circular protractor

$$= \frac{22}{7} \times 7 \text{ cm} + 2 \times 7 \text{ cm}$$

$= 22$ cm $+ 14$ cm

$= 36$ cm

4. The length of the arc of a circle of radius 21 cm which subtends an angle of 120° at the centre of the circle is

(*a*) $\frac{44}{3}$ cm (*b*) 44 cm

(*c*) 308 cm (*d*) $\frac{308}{3}$ cm

Sol. (*b*) 44 cm

Length of an arc of a circle, subtending an angle θ at the centre $= 2\pi r \times \frac{\theta}{360}$

Here $r = 21$ cm

$\theta = 120°$

$\therefore$ Length of the arc $= 2 \times \frac{22}{7} \times 21 \text{ cm} \times \frac{120}{360}$

$= 2 \times 22$ cm

$= 44$ cm

5. The hour hand of a clock is 6 cm long. The angle swept by it between 7 : 20 am and 7 : 55 am is

(*a*) $\left(\frac{35}{4}\right)^\circ$ (*b*) $\left(\frac{35}{2}\right)^\circ$

(*c*) 35° (*d*) 70°

[CBSE 2023 Standard]

Sol. (*b*) $\left(\frac{35}{2}\right)^\circ$

In 12 hours, the hour hand moves 360°.

12 hours $= 12 \times 60 = 720$ minutes

In 720 minutes, the hour hand moves 360°

In 1 minute, the hour hand moves $\frac{360°}{720}$

Time difference = 7 : 55 am – 7 : 20 am

= 35 minutes

$\therefore$ Angle swept $= \frac{360°}{720} \times 35 = \left(\frac{35}{2}\right)^\circ$

6. The circumferences of two circles are in the ratio 3 : 4. What is the ratio of their radii?

(*a*) 9 : 16 (*b*) $\sqrt{3} : 2$

(*c*) 3 : 4 (*d*) 16 : 9

Sol. (*c*) 3 : 4

Let r_1 and r_2 be the radii of the two circles.

$$\frac{2\pi r_1}{2\pi r_2} = \frac{3}{4}$$

$$\Rightarrow \quad \frac{r_1}{r_2} = \frac{3}{4}$$

7. The radius of a circle is same as the side of square. Their perimeters are in the ratio

(*a*) 1 : 1 (*b*) 2 : π

(*c*) π : 2 (*d*) $\sqrt{\pi} : 2$

[CBSE SP 2024 Basic]

Sol. (*c*) π : 2

Let r be the radius of the circle.

Perimeter of the circle = $2\pi r$

Side of the square = r

Perimeter of the square = $4r$

$$\frac{\text{Perimeter of the circle}}{\text{Perimeter of the square}} = \frac{2\pi r}{4r} = \frac{\pi}{2}$$

8. If the radii of two circles are in the ratio of 5 : 6, then their areas are in the ratio of

(*a*) 5 : 6 (*b*) 6 : 5

(*c*) 25 : 36 (*d*) 36 : 25

Sol. (*c*) 25 : 36

The radii of the two circles are $5r$ and $6r$.

Let the area of the two circles be A_1 and A_2.

$$\frac{A_1}{A_2} = \frac{\pi(5r)^2}{\pi(6r)^2} = \frac{\pi \times 25r^2}{\pi \times 36r^2} = \frac{25}{36}$$

$$\Rightarrow \quad \frac{A_1}{A_2} = \frac{25}{36}$$

Very Short Answer Type Questions

9. Find the diameter of a circle whose area is equal to the sum of the areas of two circles of radii 40 cm and 9 cm. **[CBSE SP 2024 Basic]**

Sol. Area of circle of radius 40 cm

$$= A_1 = \pi \times 40 \times 40 \text{ cm}^2$$

$$A_1 = 1600\pi \text{ cm}^2$$

Area of circle of radius 9 cm

$$= A_2 = \pi \times 9 \times 9 \text{ cm}^2$$

$$A_2 = 81\pi \text{ cm}^2$$

Let r be the radius of the bigger circle.

$$\pi r^2 = A_1 + A_2$$

$$= 1600\pi \text{ cm}^2 + 81\pi \text{ cm}^2$$

$$\Rightarrow \quad r^2 = 1681 \text{ cm}^2$$

$$\Rightarrow \quad r = \sqrt{1681} \text{ cm}$$

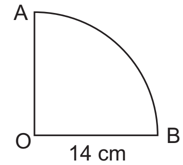
$$\Rightarrow \quad r = 41 \text{ cm}$$

∴ Diameter of the circle

$$= 2r = 2 \times 41 \text{ cm} = 82 \text{ cm}$$

10. Find the perimeter of a quadrant of a circle of radius 14 cm. **[CBSE SP 2023 Basic]**

Sol. Perimeter of the quadrant = $2r + \frac{1}{4} \times 2\pi r$

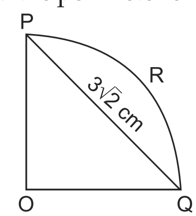


$$= 2 \times 14 \text{ cm} + \frac{1}{4} \times 2 \times \frac{22}{7} \times 14 \text{ cm}$$

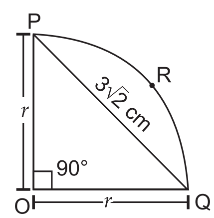
$$= 28 \text{ cm} + 22 \text{ cm}$$

$$= 50 \text{ cm}$$

11. In the given figure, the length of the chord PQ is $3\sqrt{2}$ cm. Find the perimeter of the quadrant OPRQ.



Sol. Let r be the radius of the quadrant of the circle, OPRQ.



Then since ∠POQ = 90°.

∴ By using Pythagoras' Theorem in ΔPOQ, we have

$$OP^2 + OQ^2 = PQ^2$$

$$2r^2 = \left(3\sqrt{2} \text{ cm}\right)^2$$

$$\Rightarrow \quad 2r^2 = 18 \text{ cm}^2$$

$$\Rightarrow \quad r^2 = 9 \text{ cm}^2$$

$$\Rightarrow \quad r = 3 \text{ cm}$$

∴ Radius of the quadrant = 3 cm

∴ Perimeter of the quadrant OPRQ

$$= 2r + \text{arc PRQ}$$

$$= \left(2 \times 3 + \frac{1}{4} \times 2\pi \times 3\right) \text{cm}$$

$$= \left(6 + \frac{1}{2} \times \frac{22}{7} \times 3\right) \text{ cm}$$

$$= \left(6 + \frac{33}{7}\right) \text{ cm}$$

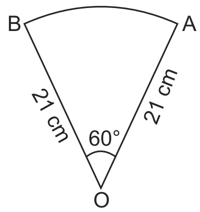
$$= \frac{42 + 33}{7} \text{ cm}$$

$$= \frac{75}{7} \text{ cm}$$

$$= 10.71 \text{ cm (approx.)}$$

Hence, the perimeter of the quadrant OPRQ is 10.71 cm (approx.)

12. Find the perimeter of the sector OAB as shown in the figure. [Use $\pi = \frac{22}{7}$]



Sol. Radius of the sector, r = OA = OB = 21 cm

$\therefore$ Perimeter of the sector OAB

$$= 2\pi r \times \frac{60°}{360°} + 2r$$

$$= \left(2 \times \frac{22}{7} \times 21 \times \frac{1}{6} + 2 \times 21\right) \text{cm}$$

$$= (22 + 42) \text{ cm}$$

$$= 64 \text{ cm}$$

Hence, the perimeter of the sector OAB is 64 cm.

13. If the area of a circle is numerically equal to twice its circumference, then what is the diameter of the circle? **[CBSE 2011]**

Sol. Let r be the radius of the circle.

Then according to the problem, we have

$$\pi r^2 = 2 \times 2\pi r$$

$$= 4\pi r$$

$$\Rightarrow \quad r = 4 \text{ units}$$

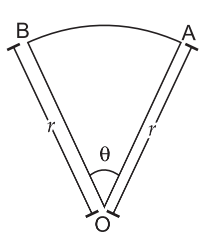
$$\therefore \quad \text{Diameter} = 2r = 2 \times 4 \text{ units}$$

$$= 8 \text{ units}$$

Hence, the required diameter of the circle is 8 units.

14. If the area of a sector of angle θ of a circle is $\frac{5}{18}$ times the area of the whole circle, then find the value of θ.

Sol. Let r be the radius of the sector.



Then area of the sector $= \pi r^2 \times \frac{\theta}{360°}$

$\therefore$ According to the problem, we have

$$\pi r^2 \times \frac{\theta}{360°} = \frac{5}{18} \times \pi r^2$$

$$\Rightarrow \quad \theta = \frac{5}{18} \times 360°$$

$$= 5 \times 20°$$

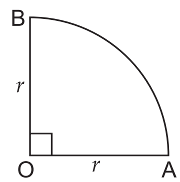
$$= 100°$$

Hence, the required value of θ is 100°.

15. If the area of a quadrant of a circle is $\frac{77}{8}$ cm^2, what is its perimeter?

Sol. Let r be the radius of the quadrant of the circle.

Then $\qquad$ OA = OB = r



$\therefore$ According to the problem, we have

$$\frac{1}{2}\pi r^2 = \frac{77}{8} \text{ cm}^2$$

$$\Rightarrow \quad \frac{1}{4} \times \frac{22}{7} \times r^2 = \frac{77}{8} \text{ cm}^2$$

$$\Rightarrow \quad r^2 = \frac{77 \times 7 \times 4}{22 \times 8} \text{ cm}^2$$

$$\Rightarrow \quad r^2 = \frac{49}{4} \text{ cm}^2$$

$$\Rightarrow \quad r^2 = \left(\frac{7}{2} \text{ cm}\right)^2$$

$$\Rightarrow \quad r = \frac{7}{2} \text{ cm}$$

$\therefore$ Radius of the quadrant of the circle $= \frac{7}{2}$ cm

$\therefore$ Perimeter of the quadrant

$$= \frac{1}{4} \times 2\pi r + 2r$$

$$= r\left(\frac{\pi}{2} + 2\right)$$

$$= \frac{7}{2} \times \left(\frac{22}{7 \times 2} + 2\right) \text{ cm}$$

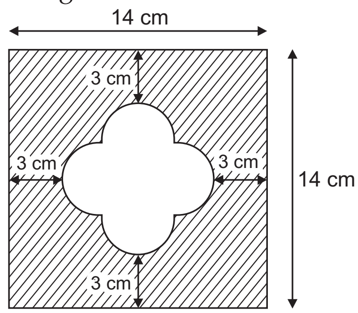
$$= \left(\frac{11}{2} + 7\right) \text{ cm}$$

$$= \frac{25}{2} \text{ cm}$$

$$= 12.5 \text{ cm}$$

Hence, the required perimeter is 12.5 cm.

16. Find the area of the unshaded region shown in the given figure.



[CBSE SP 2024 Standard]

Sol. Let the radius of each semicircle be r cm.

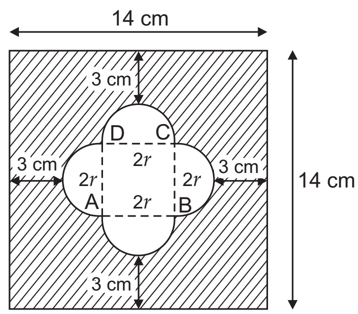
Then,

$3 \text{ cm} + r + 2r + r + 3 \text{ cm} = 14 \text{ cm}$

$\Rightarrow \quad 4r = 14 \text{ cm} - 6 \text{ cm}$

$\Rightarrow \quad 4r = 8 \text{ cm}$

$\Rightarrow \quad r = 2 \text{ cm}$



Area of the unshaded region

= Area of four semicircles + Area of square ABCD

$= 4 \times \frac{1}{2} \times \frac{22}{7} \times 2 \text{ cm} \times 2 \text{ cm} + 4 \text{ cm} \times 4 \text{ cm}$

$= \frac{176}{7} \text{ cm}^2 + 16 \text{ cm}^2$

$= \frac{288}{7} \text{ cm}^2$

$= 41.14 \text{ cm}^2$ (approx.)

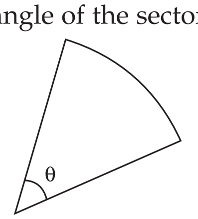
Short Answer Type Questions

17. If the perimeter of a sector of a circle of radius 5.6 cm is 27.2 cm, find the area of the sector.

[CBSE SP 2011]

Sol. Let r be the radius of the sector. Then $r = 5.6$ cm.

Let θ be the angle of the sector.



$\therefore$ Perimeter of the sector

$$= 2r + 2\pi r \times \frac{\theta}{360°}$$

$$\Rightarrow \quad 2 \times 5.6 + 11.2 \times \frac{\pi\theta}{360°} = 27.2 \qquad \text{[Given]}$$

$$\Rightarrow \quad 11.2 \times \frac{\pi\theta}{360°} = 27.2 - 11.2 = 16$$

$$\Rightarrow \quad \frac{\pi\theta}{360°} = \frac{16}{11.2} = \frac{10}{7} \qquad \ldots(1)$$

$$\therefore \text{ Area of the sector} = \pi r^2 \times \frac{\theta}{360°} \text{ cm}^2$$

$$= 5.6^2 \times \frac{\pi\theta}{360°} \text{ cm}^2$$

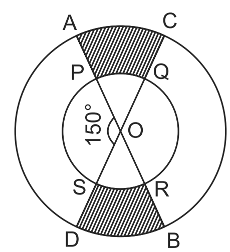
$$= 5.6^2 \times \frac{10}{7} \text{ cm}^2 \qquad \text{[From (1)]}$$

$$= \frac{5.6 \times 56}{7} \text{ cm}^2$$

$$= 44.8 \text{ cm}^2$$

Hence, the required area of the sector is 44.8 cm^2.

18. In the given figure, O is the centre of two concentric circles. The radius of the smaller circle is half the radius of the bigger circle. If OP = the radius of the smaller circle = 7 cm, $\angle AOD = 150°$ where AOB and COD are diameters of the bigger circle. Calculate the area of the shaded region.



Sol. Given that $\quad \angle AOD = 150°$

$\therefore \quad \angle AOD = \angle BOC = 150°$ [Vertically opposite $\angle$s]

Now $\quad \angle AOD + \angle BOD = 180°$ [Linear pair]

$\Rightarrow \quad 150° + \angle BOD = 180°$

$\Rightarrow \quad \angle BOD = 180° - 150° = 30°$

$\therefore \quad \angle BOD = \angle AOC = 30°$ [Vertically opposite $\angle$s]

Also, OP = 7 cm and OA = 14 cm.

$$\therefore \text{ Area of the sector OAC} = \pi \times OA^2 \times \frac{\theta}{360°}$$

$$= \pi \times 14^2 \times \frac{30°}{360°} \text{ cm}^2$$

$$= \frac{14^2\pi}{12} \text{ cm}^2$$

$$\text{Area of the sector OPQ} = \pi \times OP^2 \times \frac{\theta}{360°}$$

$$= \pi \times 7^2 \times \frac{30°}{360°}\ \text{cm}^2$$

$$= \frac{7^2\pi}{12}\ \text{cm}^2$$

$\therefore$ Area of the shaded region APQC

$$= \pi \times \left(\frac{14^2}{12} - \frac{7^2}{12}\right) \text{cm}^2$$

$$= \pi \times \left(\frac{14^2 - 7^2}{12}\right) \text{cm}^2$$

$$= \frac{22}{7} \times \frac{(14+7)(14-7)}{12}\ \text{cm}^2$$

$$= \frac{22}{7} \times \frac{21 \times 7}{12}\ \text{cm}^2$$

$$= \frac{77}{2}\ \text{cm}^2$$

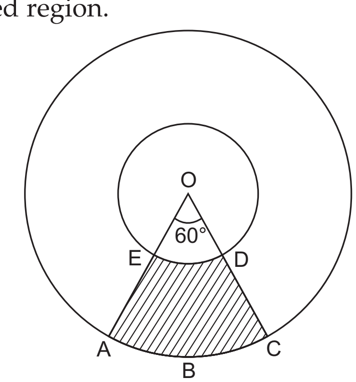
= Area of the shaded region BDSR.

$\therefore$ Area of the sum of the two shaded regions

$$= \frac{77}{2} \times 2\ \text{cm}^2 = 77\ \text{cm}^2$$

Hence, the required area of the shaded region is 77 cm^2.

19. In the given figure, two concentric circles with centre O are shown. Radii of the circles are 2 cm and 5 cm respectively. Find the area of the shaded region.



[CBSE 2023 Basic]

Sol. Area of the shaded region

= Area of sector OAC

– Area of sector OED

$$= \frac{\theta}{360} \pi R^2 - \frac{\theta}{360} \pi r^2$$

$$= \frac{60}{360} \times \pi(5\ \text{cm})^2 - \frac{60}{360} \times \pi\ (2\ \text{cm})^2$$

$$= \frac{1}{6}\pi\ [(5\ \text{cm})^2 - (2\ \text{cm})^2]$$

$$= \frac{1}{6}\pi \times 21\ \text{cm}^2$$

$$= \frac{1}{6} \times \frac{22}{7} \times 21\ \text{cm}^2$$

$$= 11\ \text{cm}^2$$

$\therefore$ Area of shaded region = 11 cm^2.

20. A car has two vipers which do not overlap. Each wiper has a blade of length 21 cm sweeping through an angle of 120°. Find the total area cleaned at each sweep of the two blades.

[CBSE 2023 Standard]

Sol. Length of the blade = 21 cm

$\therefore$ Radius = 21 cm

Sweeping angle = 120°

Area cleaned by the two vipers

= 2 × area cleaned by one viper

= 2 × area of sector with angle 120°

$$= 2 \times \frac{\theta}{360} \times \pi r^2$$

$$= 2 \times \frac{120}{360} \times \frac{22}{7} \times 21\ \text{cm} \times 21\ \text{cm}$$

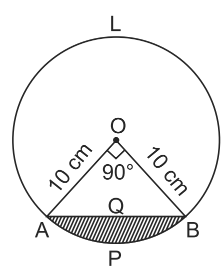
$$= 2 \times 22 \times 21\ \text{cm}^2$$

$$= 924\ \text{cm}^2$$

$\therefore$ Total area cleaned by each sweep with two blades = 924 cm^2.

Long Answer Type Questions

21. In the given figure, AB is a chord of a circle with centre O and radius 10 cm, that subtends a right angle at the centre of the circle. Find the area of the minor segment AQBP. Hence, find the area of the major segment ALBQA. [Use π = 3.14]



[CBSE 2016]

Sol. Area of the sector OAB

$$= \pi r^2 \times \frac{\theta}{360°}$$

$$= \pi \times 10^2 \times \frac{90°}{360°}\ \text{cm}^2$$

$$= 3.14 \times 100 \times \frac{1}{4}\ \text{cm}^2$$

$$= 3.14 \times 25\ \text{cm}^2$$

$$= 78.5\ \text{cm}^2$$

Area of $\Delta OAB = \frac{8}{3} \times 10 \times 10 \text{ cm}^2 = 50 \text{ cm}^2$

$\therefore$ Area of the minor segment AQBP

$(78.5 - 50) \text{ cm}^2 = 28.5 \text{ cm}^2$

Area of the entire circle $= \pi r^2$

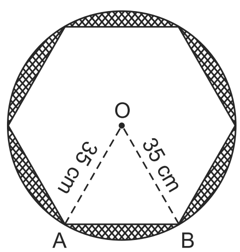
$$= 3.14 \times 10^2 \text{ cm}^2$$
$$= 3.14 \times 100 \text{ cm}^2$$
$$= 314 \text{ cm}^2$$

$\therefore$ Area of the major segment ALBQA

$$= (314 - 28.5) \text{ cm}^2$$
$$= 285.5 \text{ cm}^2$$

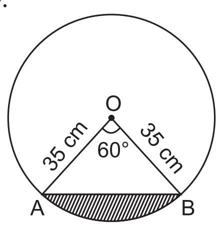
Hence, the required area of the minor segment AQBP is 28.5 cm^2 and the area of the major segment is 285.5 cm^2.

22. A round table cover has six equal designs as shown in the figure below. If the radius of the cover is 35 cm, then find the total area of the design. $\left[\text{Use } \pi = 3.14 \text{ and } \sqrt{3} = 1.732\right]$



[CBSE 2009]

Sol. Each design is in the shape of a minor segment of a circle subtending an angle of $\frac{360°}{6} = 60°$ at the centre O.



$\therefore$ Area of each segment

= Area of the sector OAB – area of the equilateral triangle OAB

$$= \left(\pi \times 35^2 \times \frac{60°}{360°} - \frac{\sqrt{3}}{4} \times 35^2\right) \text{ cm}^2$$

$$= 35^2 \times \left(\frac{3.14}{6} - \frac{1.732}{4}\right) \text{ cm}^2$$

$$= 35^2 \times \frac{6.28 - 5.196}{12} \text{ cm}^2$$

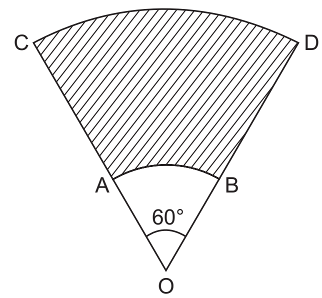
$$= \frac{35^2 \times 1.084}{12} \text{ cm}^2$$

$\therefore$ Area of the six designs

$$= \frac{35^2 \times 1.084}{12} \times 6 \text{ cm}^2$$
$$= 35^2 \times 0.542 \text{ cm}^2$$
$$= 663.95 \text{ cm}^2 \text{ (approx.)}$$

Hence, the required total area of the design is 663.95 cm^2 (approx.)

23. AB and CD are arcs of two concentric circles of radii 3.5 cm and 10.5 cm respectively and centred at O. Find the area of the shaded region if $\angle AOB = 60°$. Also, find the length of arc CD.



[CBSE 2023 Basic]

Sol. Area of the shaded region

= Area of sector OCD – Area of sector OAB

$$= \frac{\theta}{360} \times \pi R^2 - \frac{\theta}{360} \times \pi r^2$$

$$= \frac{60}{360} \times \pi \times (10.5 \text{ cm})^2 - \frac{60}{360} \times \pi \times (3.5 \text{ cm})^2$$

$$= \frac{1}{6} \times \pi \left[(10.5 \text{ cm})^2 - (3.5 \text{ cm})^2\right]$$

$$= \frac{1}{6} \times \pi \times [(10.5 \text{ cm} + 3.5 \text{ cm}) - (10.5 \text{ cm} - 3.5 \text{ cm})]$$

$$= \frac{1}{6} \times \frac{22}{7} \times 14 \text{ cm} \times 7 \text{ cm}$$

$$= \frac{154}{3} \text{ cm}^2$$

$$= 51.33 \text{ cm}^2 \text{ (approx.)}$$

Length of the arc CD

$$= \frac{\theta}{360} \times 2\pi R$$

$$= \frac{60}{360} \times 2 \times \frac{22}{7} \times 10.5 \text{ cm}$$

$$= \frac{1}{6} \times 2 \times \frac{22}{7} \times 10.5 \text{ cm}$$

$$= 11 \text{ cm}$$

Length of the arc CD = 11 cm.

24. A horse is tied to a peg at one corner of a square shaped grass field of side 15 m by means of a 5 m long rope. Find the area of that part of the field in which the horse can graze. Also, find the increase in grazing area if length of rope is increased to 10 m. [Use $\pi = 3.14$] **[CBSE 2023 Standard]**

Sol. Area of the part of the field in which the horse can graze $= \text{Area of sector DEF}$

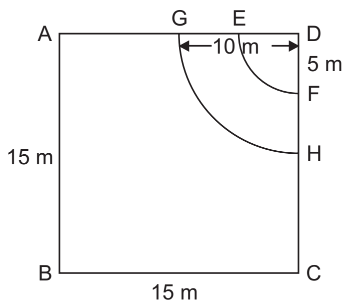
$$= \frac{\theta}{360} \times \pi r^2$$

$$= \frac{90}{360} \times 3.14 \times 5 \times 5 \text{ m}^2$$

$$= 19.625 \text{ m}^2$$

Area of sector DGH $= \frac{90}{360} \times 3.14 \times 10 \times 10 \text{ m}^2$

$$= 78.5 \text{ m}^2$$



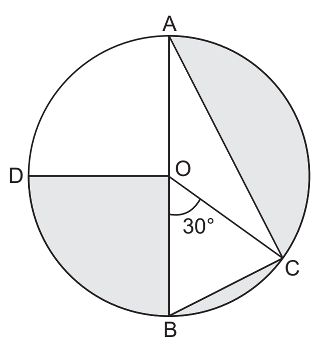
Increasing in grazing area

$= \text{Area of sector DGH} - \text{Area of sector DEF}$

$= 78.5 \text{ m}^2 - 19.625 \text{ m}^2$

$= 58.875 \text{ m}^2$

25. O is the centre of the circle. If AC = 28 cm, BC = 21 cm. $\angle BOD = 90°$ and $\angle BOC = 30°$, then find the area of the shaded region given in the figure.



[CBSE 2024 Basic]

Sol. In ΔABC,

$\angle ACB = 90°$ [Angle in a semicircle]

$\therefore \quad AB^2 = AC^2 + BC^2$

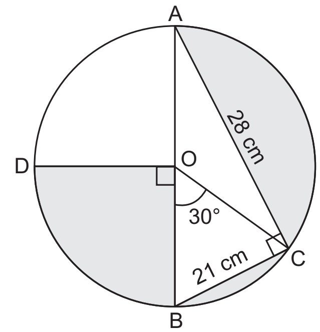
$= (28 \text{ cm})^2 + (21 \text{ cm})^2$

$\Rightarrow \quad AB = \sqrt{784 + 441} \text{ cm}$

$= \sqrt{1225} \text{ cm}$

$\Rightarrow \quad AB = 35 \text{ cm}$

$\therefore \quad \text{Radius} = AO = \frac{35}{2} \text{ cm}$



Area of the shaded region

= Area of circle – Area of quadrant AOD – Area of ΔABC

$$= \frac{22}{7} \times \frac{35}{2} \times \frac{35}{2} \text{ cm}^2 - \frac{1}{4} \times \frac{22}{7} \times \frac{35}{2} \times \frac{35}{2} \text{ cm}^2 - \frac{1}{2} \times 28 \text{ cm} \times 21 \text{ cm}$$

$= 962.5 \text{ cm}^2 - 240.625 \text{ cm}^2 - 294 \text{ cm}^2$

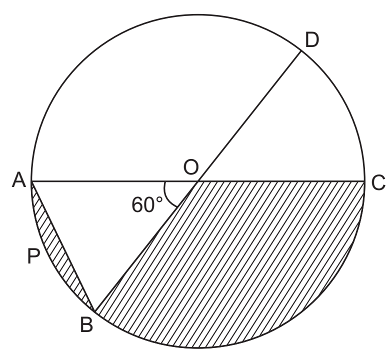
$= 427.875 \text{ cm}^2$

$\therefore$ Area of the shaded region $= 427.875 \text{ cm}^2$

26. In the given figure, diameters AC and BD of the circle intersect at O. If $\angle AOB = 60°$ and OA = 10 cm, then

(*a*) find the length of the chord AB.

(*b*) find the area of shaded region. (Take $\pi = 3.14$ and $\sqrt{3} = 1.73$)



[CBSE 2024 Standard]

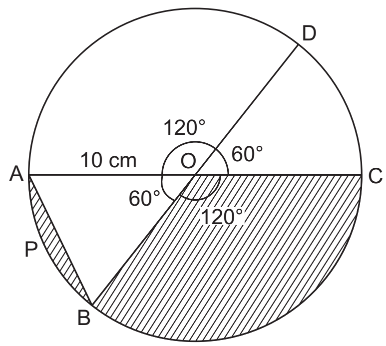
Sol. (*a*) In ΔAOB,

$AO = OB$ [radii of the circle]

$\therefore \quad \angle OAB = \angle OBA = 60°$

ΔAOB is an equilateral triangle.

$\therefore \quad AB = 10 \text{ cm}$



(*b*) Area of the shaded region

= Area of the circle – Area of ΔAOB – Area of sector DOC – Area of sector AOD

$$= \pi \times (10 \text{ cm})^2 - \frac{\sqrt{3}}{4} \times 10 \text{ cm} \times 10 \text{ cm} - \frac{60}{360} \times \pi \times 10 \text{ cm} \times 10 \text{ cm} - \frac{120}{360} \times \pi \times 10 \text{ cm} \times 10 \text{ cm}$$

$$= 3.14 \times 100 \text{ cm}^2 - 25\sqrt{3} \text{ cm}^2 - \frac{1}{3} \times 3.14 \times 50 \text{ cm}^2 - \frac{1}{3} \times 3.14 \times 100 \text{ cm}^2$$

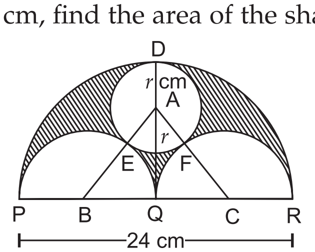
$$= 314 \text{ cm}^2 - 43.25 \text{ cm}^2 - 52.33 \text{ cm}^2 - 104.67 \text{ cm}^2$$

$= 113.75 \text{ cm}^2$ (approx.)

Higher Order Thinking Skills (HOTS) Questions

(Page 212)

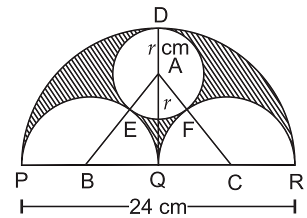
1. In the given figure, a semicircle is drawn with line segment PR as a diameter. Q is the midpoint of line segment PR. Two semicircles with line segments PQ and QR as diameters are drawn. A circle is drawn which touches the three semicircles. If PR = 24 cm, find the area of the shaded region.



Sol. Radius of the complete semicircle PDR $= \frac{24}{2}$ cm

$= 12$ cm

Let r be the radius of the complete circle with centre at A and passing through the points E and F on the two semicircles, PEQ and QFR.



Then since $\angle AQB = 90°$,

$AQ = DQ - DA = (12 - r)$ cm

and $AB = BE + EA = (6 + r)$ cm,

$\therefore$ By using Pythagoras' Theorem in ΔABQ, we have

$$AB^2 = AQ^2 + BQ^2$$

$$\Rightarrow \quad (6 + r)^2 = (12 - r)^2 + 36$$

$$\Rightarrow \quad 36 + r^2 + 12r - 144 - r^2 + 24r = 36$$

$$\Rightarrow \quad 36r = 144$$

$$\Rightarrow \quad r = \frac{144}{36} = 4$$

$\therefore$ The radius of the circle with centre A is 4 cm.

$\therefore$ Its area $= \pi \times 4^2 \text{ cm}^2 = 16\pi \text{ cm}^2$ ...(1)

Sum of the areas of the two semicircles PEQ and QFR

$$= \frac{\pi \times 6^2}{2} \times 2 \text{ cm}^2 = 36\pi \text{ cm}^2 \quad ...(2)$$

Area of the complete semicircle PDR

$$= \frac{\pi \times 12^2}{2} \text{ cm}^2 = 72\pi \text{ cm}^2 \quad ...(3)$$

$\therefore$ Area of the shaded region

$= (72\pi - 36\pi - 16\pi) \text{ cm}^2$ [From (1), (2) and (3)]

$= (72 - 52)\pi \text{ cm}^2$

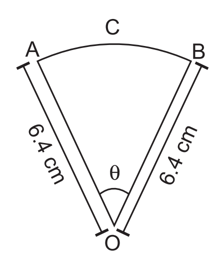
$= 20 \times \frac{22}{7} \text{ cm}^2 = \frac{440}{7} \text{ cm}^2$

$= 62.86 \text{ cm}^2$ (approx.)

Hence, the required area of the shaded region is 62.86 cm^2 (approx.)

2. The perimeter of a sector of a circle of radius 6.4 cm is 18.8 cm. Find the area of the sector.

Sol. Let the arc ACB of the sector OACD makes an angle θ at the centre O. Here radius r of the sector is 6.4 cm.



$\therefore$ Perimeter of the sector

$$= 2\pi r \times \frac{\theta}{360°} + 2r$$

$$= 2\pi \times 6.4 \times \frac{\theta}{360°} + 2 \times 6.4$$

$\therefore$ According to the problem, we have

$$\frac{\pi\theta}{360°} \times 12.8 + 12.8 = 18.8$$

$$\Rightarrow \quad \frac{\pi\theta}{360°} = \frac{18.8 - 12.8}{12.8} = \frac{6}{12.8} = \frac{60}{128} = \frac{15}{32} \quad \ldots(1)$$

$\therefore$ Area of the sector OACB

$$= \pi r^2 \frac{\theta}{360°}$$

$$= \frac{\pi\theta}{360°} \times 6.4 \times 6.4 \text{ cm}^2$$

$$= \frac{15}{32} \times 6.4 \times 6.4 \text{ cm}^2 \quad \text{[From (1)]}$$

$$= \frac{15}{32} \times 40.96 \text{ cm}^2$$

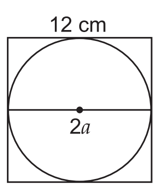
$$= \frac{61.44}{32} \text{ cm}^2$$

$$= 19.2 \text{ cm}^2$$

Hence, the required area of the sector is 19.2 cm^2.

3. Find the radius of the greatest circle (i.e. the circle with the greatest area) which can be inscribed within a square of side 12 cm.

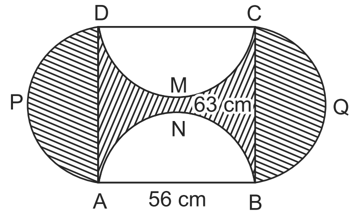
Sol. Let a be the radius of the circle. For the area of the circle to be greatest, $2a$ = 12 cm.



$$\Rightarrow \quad a = 6 \text{ cm}$$

Hence, the required radius of the circle is 6 cm.

4. Find the area of the shaded region in the given figure where ABCD is a rectangle with AB = 56 cm, BC = 63 cm, $\overparen{DMC}$, $\overparen{ANB}$, $\overparen{BQC}$ and $\overparen{APD}$ are semicircles on DC, AB, BC and AD respectively.



Sol. Area of the shaded region

= Sum of the areas of two semicircles APD and BQD, each with radius $\frac{63}{2}$ cm + Area of the rectangle ABCD – Sum of the areas of two semicircles ANB and DMC, each with radius 28 cm

$$= \left[\pi \times \left(\frac{63}{2}\right)^2 \times \frac{1}{2} \times 2 + 56 \times 63 - \frac{1}{2} \times \pi \times 28^2 \times 2\right] \text{cm}^2$$

$$= \left(\frac{22}{7} \times \frac{63 \times 63}{4} + 3528 - \frac{22}{7} \times 28 \times 28\right) \text{cm}^2$$

$$= \left(\frac{11 \times 9 \times 63}{2} + 3528 - 22 \times 4 \times 28\right) \text{cm}^2$$

$$= \left(\frac{6237}{2} + 3528 - 2464\right) \text{cm}^2$$

$$= (3118.5 + 1064) \text{ cm}^2$$

$$= 4182.5 \text{ cm}^2$$

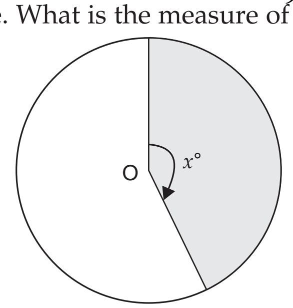
Hence, the required area of the shaded region is 4182.5 cm^2.

Self-Assessment

(Page 213)

Multiple-Choice Questions

1. In the given figure, O is the centre of the circle. The area of the shaded region is $\frac{4}{9}$ of the area of the circle. What is the measure of angle x?



(*a*) 110° (*b*) 120°

(*c*) 160° (*d*) 180°

Sol. (*c*) 160°

Let r be the radius of the circle.

$$\frac{x}{360°} \times \pi r^2 = \frac{4}{9} \times \pi r^2$$

$$\Rightarrow \qquad x = \frac{4}{9} \times 360°$$

$$\Rightarrow \qquad x = 160°$$

2. A park which is circular in shape is to be fenced using barbed wires. One-third of the park has been fenced. The cost of fencing is ₹1000 at the rate of ₹40 per metre. Find the remaining length of the park that needs to be fenced.

(*a*) 20 m (*b*) 30 m
(*c*) 40 m (*d*) 50 m

Sol. (*d*) 50 m

Let r be the radius of the park.

$$\frac{1}{3} \times 2\pi r \times ₹40 = ₹1000$$

$$\Rightarrow \qquad 2\pi r = \frac{1000 \times 3}{40}$$

$$\Rightarrow \qquad 2\pi r = 75 \text{ m}$$

Two-thirds of the park is to be fenced

$$= \frac{2}{3} \times 2\pi r = \frac{2}{3} \times 75 \text{ m}$$

$$= 50 \text{ m}$$

3. The area of a sector of angle α (in degrees) of a circle with radius R is

(*a*) $\frac{\alpha}{180} \times 2\pi R$ (*b*) $\frac{\alpha}{360} \times 2\pi R$

(*c*) $\frac{\alpha}{180} \times \pi R^2$ (*d*) $\frac{\alpha}{360} \times \pi R^2$

[CBSE 2023 Basic]

Sol. (*d*) $\frac{\alpha}{360} \times \pi R^2$

4. What is the area of semicircle of diameter '*d*'?

(*a*) $\frac{1}{16} \pi d^2$ (*b*) $\frac{1}{4} \pi d^2$

(*c*) $\frac{1}{8} \pi d^2$ (*d*) $\frac{1}{2} \pi d^2$

[CBSE 2023 Standard]

Sol. (*c*) $\frac{1}{8} \pi d^2$

$$\text{Area of the semicircle} = \frac{1}{2} \times \pi r^2$$

$$= \frac{1}{2} \times \pi \times \left(\frac{d}{2}\right)^2$$

$$= \frac{1}{2} \times \pi \times \frac{d^2}{4}$$

$$= \frac{1}{8} \times \pi \times d^2$$

5. If the perimeter and the area of a circle are numerically equal, then the radius of the circle is,

(*a*) 2 units (*b*) π units
(*c*) 4 units (*d*) 7 units

[CBSE 2024 Standard]

Sol. (*a*) 2 units

Let r be the radius of the circle.

Perimeter of the circle = $2\pi r$

Area of the circle = πr^2

According to the question,

$$\pi r^2 = 2\pi r$$

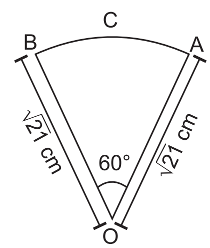
$$\Rightarrow \qquad r = 2 \text{ units}$$

6. The minute hand of a clock is $\sqrt{21}$ cm long. Then the area described by the minute hand on the face of the clock between 7 : 00 am and 7 : 10 am is

(*a*) 11.5 cm^2 (*b*) 10.5 cm^2
(*c*) 10 cm^2 (*d*) 11 cm^2

Sol. (*d*) 11 cm^2

We know that the angle described by a minute hand of a clock in 1 minute = $\frac{360°}{60} = 6°$



∴ In 10 minutes from 7 : 00 a.m. to 7 : 10 am, the minute hand OA described an angle of 6° × 10 = 60°. Hence, in 10 minutes the minute hand describes an area equal to the area of the sector OACB of radius $\sqrt{21}$ cm.

$$\therefore \qquad \text{Area} = \pi\left(\sqrt{21}\right)^2 \times \frac{60°}{360°} \text{ cm}^2$$

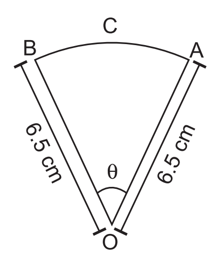
$$= \frac{22}{7} \times 21 \times \frac{1}{6} \text{ cm}^2 = 11 \text{ cm}^2$$

Hence, the required area is 11 cm^2.

7. If the perimeter of a sector of a circle of radius 6.5 cm is 21 cm, then the area of the sector is

(*a*) 52 cm^2 (*b*) 26 cm^2
(*c*) 25 cm^2 (*d*) 56 cm^2

Sol. (*b*) 26 cm^2



The perimeter of the sector OACB with $\angle AOB = \theta$

$$= \left(2 \times 6.5 + \frac{\pi\theta}{360^\circ} \times 2 \times 6.5\right) \text{ cm}$$

$$= 13 + \frac{13\pi\theta}{360}$$

$\therefore$ According to the problem, we have

$$13 + \frac{13\pi\theta}{360} = 21$$

$$\Rightarrow \quad \frac{\pi\theta}{360^\circ} = \frac{21 - 13}{13} = \frac{8}{13} \quad \ldots(1)$$

$\therefore$ Area of the sector OACB

$$= \pi \times 6.5^2 \times \frac{\theta}{360^\circ} \text{ cm}^2$$

$$= 6.5 \times 6.5 \times \frac{8}{13} \text{ cm}^2 \quad \text{[From (1)]}$$

$$= 6.5 \times 0.5 \times 8 \text{ cm}^2$$

$$= 26 \text{ cm}^2$$

Hence, the required area of the sector is 26 cm^2.

8. To increase the vegetation cover, the RWA decided to build a new circular park equal in area to the sum of areas of two existing circular parks of diameters 16 m and 12 m. The diameter of the new park is

(*a*) 20 m (*b*) 30 m
(*c*) 40 m (*d*) 48 m

Sol. (*a*) 20 m

Area of the two parks $= \pi(8 \text{ m})^2 + \pi(6 \text{ m})^2$

$$= 64\pi \text{ m}^2 + 36\pi \text{ m}^2$$

$$= 100\pi \text{ m}^2$$

Let the radius of the new park be R

$$\therefore \quad \pi R^2 = 100\pi \text{ m}^2$$

$$\Rightarrow \quad R = 10 \text{ m}$$

$\therefore$ Diameter of the new park = 20 m.

9. If the perimeter of a circle is equal to that of a square, then the ratio of their areas is

(*a*) 22 : 7 (*b*) 14 : 11
(*c*) 7 : 22 (*d*) 11 : 14

[CBSE SP 2023 Basic]

Sol. (*b*) 14 : 11

Let r and a be the radius of the circle and side of the square respectively.

According to the question,

$$2\pi r = 4a$$

$$\Rightarrow \quad a = \frac{\pi}{2} r \quad \ldots(1)$$

Ratio of their areas

$$= \frac{\pi r^2}{a^2}$$

$$= \frac{\pi r^2}{\left(\frac{\pi}{2} r\right)^2} = \frac{\pi r^2}{\frac{\pi^2}{4} r^2}$$

$$= \frac{4}{\pi} = \frac{4 \times 7}{22} = \frac{14}{11}$$

10. The area of the circle that can be inscribed in a square of 6 cm is

(*a*) 36π cm^2 (*b*) 18π cm^2
(*c*) 12π cm^2 (*d*) 9π cm^2

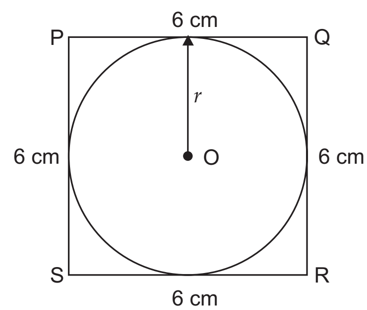
[CBSE SP 2023 Standard]

Sol. (*d*) 9π cm^2

Diameter of the circle = side of the square

$$\Rightarrow \quad 2r = 6 \text{ cm}$$

$$\Rightarrow \quad r = 3 \text{ cm}$$



$\therefore$ Area of the circle $= \pi r^2$

$$= \pi(3 \text{ cm})^2 = 9\pi \text{ cm}^2$$

Fill in the Blanks

11. If the circumference of a circle exceeds its diameter by 16.8 cm, then the radius of the circle is **3.92 cm**.

Sol.

$$2\pi r = 2r + 16.8$$

$$44r = 14r + 16.8 \times 7$$

$$30r = 117.6$$

$$r = \frac{117.6}{30} = 3.92 \text{ cm}$$

12. If the radius of a circle is 3.5 cm, then the perimeter of the semicircle is **18 cm**.

Sol. Perimeter of the semicircle

$$= \left(\frac{1}{2} \times 2 \times \frac{22}{7} \times 3.5 + 2 \times 3.5\right) \text{ cm}$$

$$= (22 \times 0.5 + 2 \times 3.5) \text{ cm}$$

$$= (11 + 7) \text{ cm}$$

$$= 18 \text{ cm}$$

13. If the areas of two circles are in the ratio 9 : 16, then the ratio of the perimeters of the circles is **3 : 4**.

Sol. r_1 and r_2 being the radius of two circles

$$\frac{\pi r_1^2}{\pi r_2^2} = \frac{9}{16}, \frac{r_1}{r_2} = \frac{3}{4}$$

$$\therefore \quad \frac{2\pi r_1}{2\pi r_2} = \frac{3}{4} = 3 : 4$$

14. If the area of a circle is equal to the sum of areas of circles of diameters 10 cm and 24 cm, then the diameter of the larger circle (in cm) is **26 cm**.

Sol.
$$\pi r^2 = \pi(5)^2 + \pi(12)^2$$
$$\pi r^2 = \pi(25 + 144)$$
$$r^2 = 169$$
$$r = 13$$
$$\therefore \quad \text{Diameter} = 2 \times 13 \text{ cm} = 26 \text{ cm}$$

Assertion-Reason Type Questions

Directions (Q. Nos. 15 to 17): Each of these questions contains an assertion followed by reason. Read them carefully, and answer the question on the basis of the following options, select the one that best describes the two statements.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

15. **Assertion (A):** The circumference of a circle is an exact multiple of its diameter.

Reason (R): The ratio of circumference and diameter of a circle is always π.

Sol. (*d*) The ratio of circumference and diameter of a circle is π which is non-recurring non-terminating decimal. Thus circumference is not an exact multiple of diameter.

16. **Assertion (A):** If the angle of a sector is doubled, its area will be doubled.

Reason (R): Area of a sector is directly proportional to the angle.

Sol. (*a*) Both the statements are correct. Since the area of sector is directly proportional to its angle, hence doubling the angle will make the area double. Thus Reason is proper explanation of the Assertion.

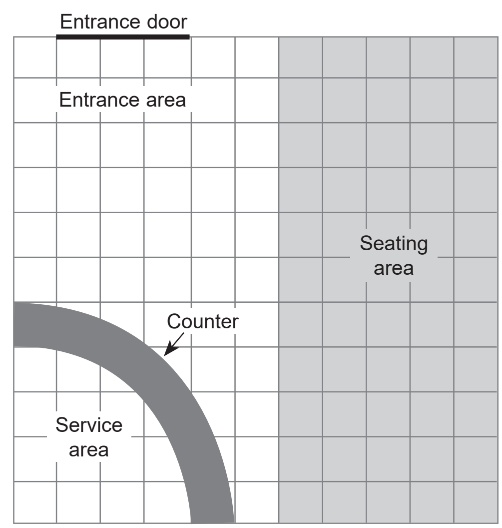
17. **Assertion (A):** A cake is cut into two halves along the diameter. The two segments thus formed are equal.

Reason (R): Diameter divides the circle into two equal halves.

Sol. (*a*) Both statements are correct. Reason is correct explanation of the Assertion.

Case Study Based Questions

18. Sanjana plans to open a coffee shop. So, she has created a floor plan for it. Floor plan for Sanjana's coffee shop is given below on the grid. The service area is surrounded by the service counter. Each square on the grid represents 1 metre × 1 metre.



Based on the above situation, answer the following questions.

(*a*) What is the radius of the inner edge of the counter?

Ans. 4 m

(*b*) Find the area of the outer edge of the counter.

Ans. $\frac{275}{14}$ m^2

(*c*) (*i*) What is the area of the counter?

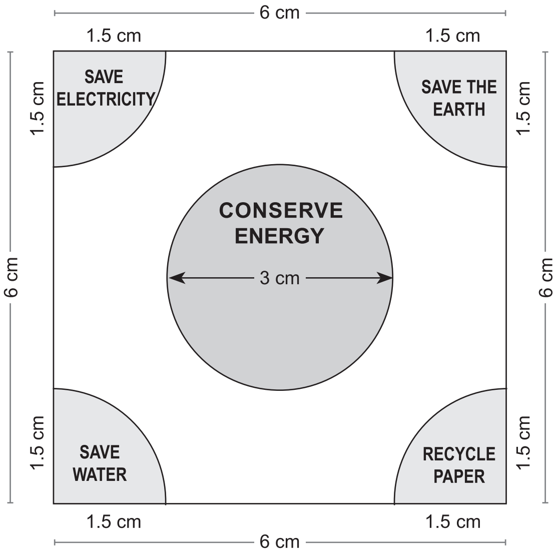
Ans. $\frac{153}{7}$ m^2

or

(*ii*) What is the area of the floor of the coffee shop excluding the counter area and the service area?

Ans. $\frac{1419}{14}$ m^2

19. The Principal of a school asked each student to prepare a badge to be worn on every Friday of the week, so as to spread awareness about 'conserving energy'. Each badge has to be made on a square cardboard of side 6 cm with a quadrant of a circle of radius 1.5 cm drawn at each vertex of the square and a circle of diameter 3 cm drawn in its centre with the slogan 'CONSERVE ENERGY' written on it. Also, write the message on 4 quadrants as shown in the figure below.



Based on the given situation, answer the following questions.

(*a*) What is the area of each quadrant?

Ans. $\frac{99}{56}$ cm^2

(*b*) Find the area of the square cardboard.

Ans. 36 cm^2

(*c*) (*i*) What is the area of the circle?

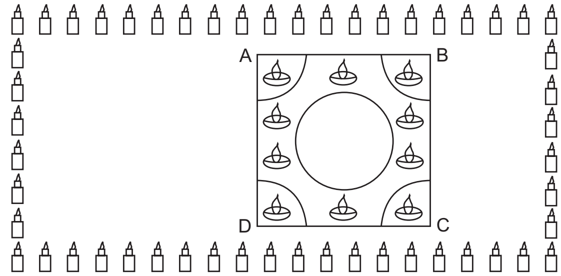
Ans. $\frac{99}{14}$ cm^2

or

(*ii*) What is the area of the remaining portion of the badge?

Ans. $\frac{153}{7}$ cm^2

20. Interschool Rangoli Competition was organized by one of the reputed schools of Odisha. The theme of the Rangoli Competition was Diwali celebrations where students were supposed to make mathematical designs. Students from various schools participated and made beautiful Rangoli designs. One such design is given below.



Rangoli is in the shape of square marked as ABCD, side of square being 40 cm. At each corner of a square, a quadrant of circle of radius 10 cm is drawn (in which diyas are kept). Also a circle of diameter 20 cm is drawn inside the square.

Based on the above, answer the following questions:

(*a*) What is the area of square ABCD?

Sol. Area of square ABCD = 40 cm × 40 cm

$= 1600 \text{ cm}^2$

(*b*) Find the area of the circle.

Sol. Area of the circle = $\pi r^2 = \pi \times (20 \text{ cm})^2$

$= 400\pi \text{ cm}^2$

(*c*) (*i*) If the circle and the four quadrants are cut off from the square ABCD and removed, then find the area of remaining portion of square ABCD.

Sol. Area of the four quadrants

$$= 4 \times \frac{1}{4} \times \pi \times 10 \text{ cm} \times 10 \text{ cm}$$

$$= 100\pi \text{ cm}^2$$

Area of the remaining portion of square ABCD

= Area of square ABCD – Area of circle – Area of four quadrants

$= 1600 \text{ cm}^2 - 400 \times 3.14 \text{ cm}^2 - 100 \times 3.14 \text{ cm}^2$

$= 1600 \text{ cm}^2 - 1256 \text{ cm}^2 - 314 \text{ cm}^2$

$= 30 \text{ cm}^2$

or

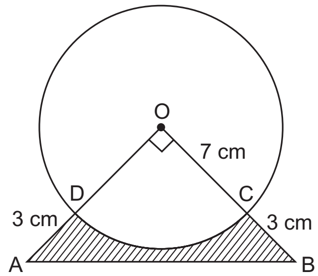
(*ii*) Find the combined area of 4 quadrants and the circle, removed. **[CBSE 2023 Basic]**

Sol. Combined area of 4 quadrants and the circle removed

$$= 314 \text{ cm}^2 + 1256 \text{ cm}^2$$

$$= 1570 \text{ cm}^2$$

21. In an annual day function of a school, the organizers wanted to give a cash prize along with a memento to their best students. Each memento is made as shown in the figure and its base ABCD is shown from the front side. The rate of silver plating is ₹ 20 per cm^2.



Based on the above, answer the following questions:

(*a*) What is the area of the quadrant ODCO?

Sol. Area of quadrant ODCO

$$= \frac{90}{360} \times \frac{22}{7} \times 7 \text{ cm} \times 7 \text{ cm}$$

$$= \frac{1}{4} \times 22 \times 7 \text{ cm}^2 = 38.5 \text{ cm}^2$$

(*b*) Find the area of ΔAOB.

Sol. ΔAOB is a right angled triangle.

$$\text{Area of } \Delta AOB = \frac{1}{2} \times 10 \text{ cm} \times 10 \text{ cm}$$

$$= 50 \text{ cm}^2$$

(*c*) (*i*) What is the total cost of silver plating the shaded part ABCD?

Sol. Area of the shaded part

= Area of the triangle

– Area of the quadrant ODCO

$= 50 \text{ cm}^2 - 38.5 \text{ cm}^2$

$= 11.5 \text{ cm}^2$

Cost of silver plating the shaded part ABCD

= ₹20 × 11.5 = ₹230

or

(*ii*) What is the length of arc CD?

[CBSE 2023 Standard]

Sol. Length of the arc CD $= \frac{\theta}{360} \times 2\pi r$

$$= \frac{90}{360} \times 2 \times \frac{22}{7} \times 7 \text{ cm}$$

$= 11$ cm

Very Short Answer Type Questions

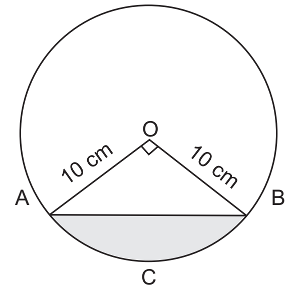
22. A chord of radius 10 cm subtends a right angle at the centre. Find the area of minor segment. [Use $\pi = 3.14$] **[CBSE SP 2024 Basic]**

Sol. Area of the sector OACB

$$= \frac{\theta}{360} \times \pi r^2$$

$$= \frac{90}{360} \times 3.14 \times 10 \times 10 \text{ cm}^2$$

$= 78.5 \text{ cm}^2$



$$\text{Area of } \Delta OAB = \frac{1}{2} \times 10 \text{ cm} \times 10 \text{ cm}$$

$= 50 \text{ cm}^2$

Area of the minor segment

= Area of the sector OACB

– Area of ΔOAB

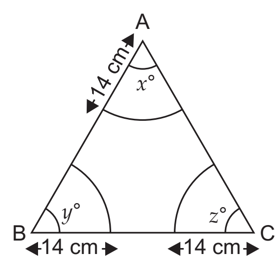
$= 78.5 \text{ cm}^2 - 50 \text{ cm}^2$

$= 28.5 \text{ cm}^2$

23. With vertices A, B and C of ΔABC as centres, arcs are drawn with radii 14 cm and the three portions of the triangle so obtained are removed. Find the total area removed from the triangle.

[CBSE SP 2024 Standard]

Sol. Let the measures of ∠A, ∠B and ∠C be $x°$, $y°$ and $z°$ respectively.



Total area removed from the triangle

$$= \frac{x}{360} \times \pi \times (14 \text{ cm})^2 + \frac{y}{360} \times \pi \times (14 \text{ cm})^2 + \frac{z}{360} \times \pi \times (14 \text{ cm})^2$$

$$= \pi \times (14 \text{ cm})^2 \left[\frac{x}{360} + \frac{y}{360} + \frac{z}{360}\right]$$

$$= \pi \times (14 \text{ cm})^2 \left[\frac{x+y+z}{360}\right]$$

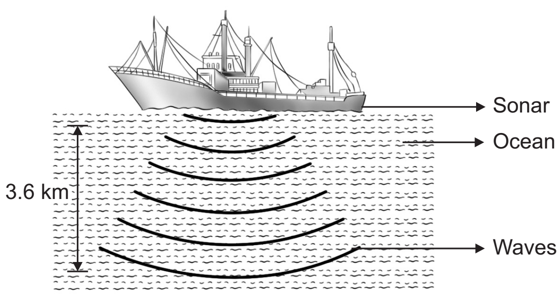
$$= \frac{22}{7} \times 14 \times 14 \times \left[\frac{180}{360}\right] \text{cm}^2$$

[Sum of angles in a triangle is 180°]

$$= \frac{22}{7} \times 14 \times 14 \times \frac{1}{2} \text{ cm}^2$$

$$= 308 \text{ cm}^2$$

24. Sonar systems are extensively used for underwater surveillance and monitoring. It protects warships by detecting enemy weapon and submerged vehicles. A sonar device is attached to the bottom-side of a ship. It emits ultrasound. A particular sonar covers a sector with a central angle of 120°. It has a maximum surveillance range of 3.6 kilometres under water. How much area is covered by the sonar during the surveillance period.



Sol. Area covered by the sonar during the surveillance period

$$= \frac{\theta}{360} \times \pi r^2$$

$$= \frac{120}{360} \times \frac{22}{7} \times 3.6 \times 3.6 \text{ km}^2$$

$$= 13.6 \text{ km}^2 \text{ (approx.)}$$

25. The length of the minute hand of a clock is 6 cm. Find the area swept by it when it moves from 6 : 05 pm to 6 : 40 pm.

Sol. The minute hand moves 360° in 60 minutes.

In one minute, it will move $\frac{360°}{60°} = 6°$.

Time duration between 6:05 to 6:40 pm

= 35 minutes

In 35 minutes, it will move through

= 35 × 6° = 210°

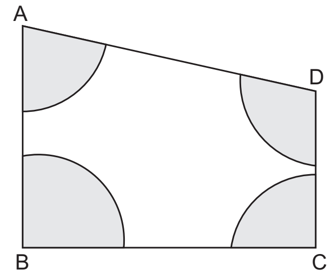
Area swept by the minute hand

$$= \frac{210}{360} \times \pi r^2$$

$$= \frac{210}{360} \times \frac{22}{7} \times 6 \text{ cm} \times 6 \text{ cm}$$

$$= 66 \text{ cm}^2$$

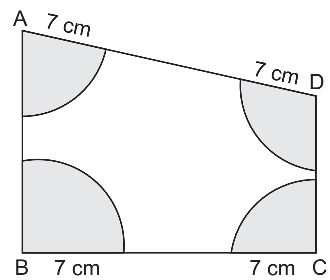
26. In the given figure arcs have been draw of radius 7 cm each with vertices A, B, C and D of quadrilateral ABCD as centres. Find the area of the shaded region.



[CBSE SP 2023 Standard]

Sol. Area of the shaded region

$$= \frac{\angle A}{360°} \times \pi \times (7 \text{ cm})^2 + \frac{\angle B}{360°} \times \pi \times (7 \text{ cm})^2 + \frac{\angle C}{360°} \times \pi \times (7 \text{ cm})^2 + \frac{\angle D}{360°} \times \pi \times (7 \text{ cm})^2$$



$$= \pi \times 49 \text{ cm}^2 \left[\frac{\angle A}{360°} + \frac{\angle B}{360°} + \frac{\angle C}{360°} + \frac{\angle D}{360°}\right]$$

$$= \frac{22}{7} \times 49 \text{ cm}^2 \left[\frac{\angle A + \angle B + \angle C + \angle D}{360°}\right]$$

$$= 22 \times 7 \text{ cm}^2 \left[\frac{360°}{360°}\right]$$

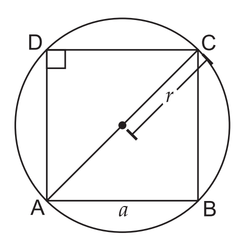
$$= 154 \text{ cm}^2$$

Area of the shaded region = 154 cm^2.

27. A circle has a perimeter of 660 m. A square is inscribed within the circle such that the four vertices of the square lie on the circumference of the circle. Find the area of the square.

[CBSE SP 2011]

Sol. Let r be the radius of the circle.



Then $2\pi r = 660$ m [Given]

$\therefore \quad 2 \times \frac{22}{7} \times r = 600$ m

$\Rightarrow \quad r = \frac{660 \times 7}{2 \times 22}$ m

$$\Rightarrow \qquad r = 105 \text{ m}$$

$\therefore$ Radius of the circle is 105 m.

If a be the side of the square, then by Pythagoras' Theorem, we have

$$a^2 + a^2 = 4r^2$$

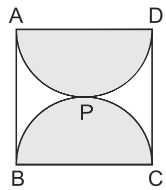
$$\Rightarrow \qquad a^2 = 2 \times 105 \times 105 \text{ m}^2$$

$$= 22050 \text{ m}^2$$

$\therefore$ Area of the square is 22050 m^2.

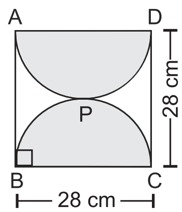
Hence, the required area of the square is 22050 m^2.

28. Find the area of the shaded region in the given figure, if ABCD is a square of side 28 cm and APD and BPC are semicircles.



[CBSE SP 2012]

Sol.



Total area of the two semicircles

$$\text{BPC and APD} = \frac{1}{2}\pi r^2 \times 2$$

$$= \frac{22}{7} \times 14^2 \text{ cm}^2$$

$$= 616 \text{ cm}^2$$

$\therefore$ Area of the shaded region

$$= 616 \text{ cm}^2$$

Hence, the required area of the shaded region is 616 cm^2.

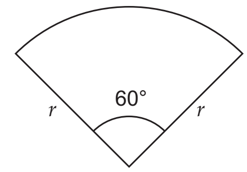
Short Answer Type Questions

29. A piece of wire 20 cm long is bent into the form of an arc of a circle subtending an angle of 60° at the centre. Find the radius of the circle.

[NCERT Exemplar]

Sol. Let r be the radius of the circle.

$$\text{Perimeter of the arc} = \frac{60}{360} \times 2\pi r$$



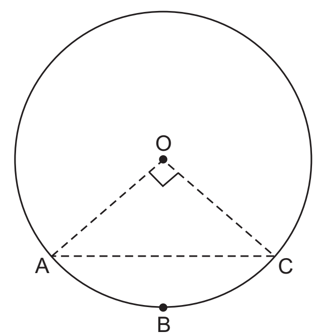
Length of the wire = 20 cm

$$\frac{60}{360} \times 2\pi r = 20 \text{ cm}$$

$$\Rightarrow \qquad \frac{1}{3}\pi r = 20 \text{ cm}$$

$$\Rightarrow \qquad r = \frac{60}{\pi} \text{ cm}$$

30. In the given figure, the length of the arc ABC is 22 cm. It subtends an angle of 90° at the centre. A triangle AOC is cut along the dotted lines. Find the area of the remaining circle. [Take $\pi = \frac{22}{7}$]



Sol. Let r be the radius of the circle.

$$\frac{90}{360} \times 2\pi r = 22 \text{ cm}$$

$$\Rightarrow \qquad \frac{1}{4} \times 2 \times \frac{22}{7} \times r = 22 \text{ cm}$$

$$\Rightarrow \qquad r = \frac{22}{11} \times 7 \text{ cm}$$

$$\Rightarrow \qquad r = 14 \text{ cm}$$

$$\text{Area of the circle} = \pi r^2 = \frac{22}{7} \times 14 \text{ cm} \times 14 \text{ cm}$$

$$= 616 \text{ cm}^2$$

$$\text{Area of } \Delta\text{AOC} = \frac{1}{2} \times 14 \text{ cm} \times 14 \text{ cm} = 98 \text{ cm}^2$$

Area of the remaining circle

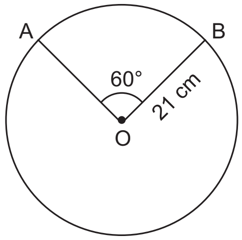
$$= \text{Area of the circle} - \text{Area of } \Delta\text{AOC}$$

$$= 616 \text{ cm}^2 - 98 \text{ cm}^2$$

$$= 518 \text{ cm}^2$$

31. In a circle of radius 21 cm, an arc subtends an angle of 60° at the centre. Find the area of the sector formed by the arc. Also, find the length of the arc. **[CBSE 2023 Standard]**

Sol.



Area of the sector formed by the arc

$$= \frac{\theta}{360} \times \pi r^2$$

$$= \frac{60}{360} \times \frac{22}{7} \times 21 \text{ cm} \times 21 \text{ cm}$$

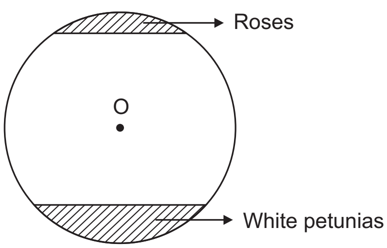
$$= 231 \text{ cm}^2$$

Length of the arc $= \frac{\theta}{360} \times 2\pi r$

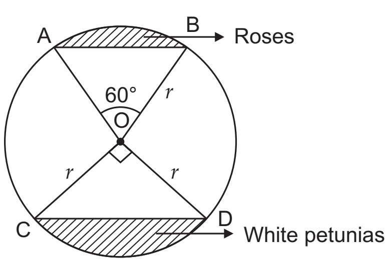
$$= \frac{60}{360} \times 2 \times \frac{22}{7} \times 21 \text{ cm}$$

$$= 22 \text{ cm}$$

32. In a circular park of radius 'r' m, there are two segments with flowers. One segment is full of white petunias and it subtends an angle of 90° at the centre. The other segment with central angle of 60° is full of roses. The combined area of two segments of flowers is $256\frac{2}{3}$ m^2. Find the area of the segments with petunias and roses.



Sol.



Area of sector OAB

$$= \frac{\theta}{360} \times \pi r^2$$

$$= \frac{60}{360} \times \pi r^2$$

$$= \frac{1}{6} \times \pi r^2$$

Area of ΔABO $= \frac{\sqrt{3}}{4} \times r^2$

[$\because$ ΔABO is an equilateral triangle]

Area of segment with roses $= \frac{1}{6}\pi r^2 - \frac{\sqrt{3}}{4} r^2$ ...(i)

Area of sector OCD $= \frac{\theta}{360} \times \pi r^2$

$$= \frac{90}{360} \times \pi r^2 = \frac{1}{4}\pi r^2$$

Area of ΔOCD $= \frac{1}{2} \times r \times r = \frac{1}{2} r^2$

$\therefore$ Area of segment with petunias $= \frac{1}{4}\pi r^2 - \frac{1}{2} r^2$ ...(ii)

According to the question,

$$\frac{1}{6}\pi r^2 - \frac{\sqrt{3}}{4} r^2 + \frac{1}{4}\pi r^2 - \frac{1}{2} r^2 = 256\frac{2}{3}$$

$$\Rightarrow \quad r^2\left[\frac{\pi}{6} - \frac{\sqrt{3}}{4} + \frac{\pi}{4} - \frac{1}{2}\right] = \frac{770}{3}$$

$$\Rightarrow \quad r^2\left[\frac{22}{7 \times 6} - \frac{1.732}{4} + \frac{22}{7 \times 4} - \frac{1}{2}\right] = \frac{770}{3}$$

$$\Rightarrow \quad r^2 [0.524 - 0.433 + 0.786 - 0.5] = \frac{770}{3}$$

$$\Rightarrow \quad r^2 \times 0.377 = \frac{770}{3}$$

$$\Rightarrow \quad r^2 = \frac{770}{3 \times 0.377}$$

$$\Rightarrow \quad r = 26 \text{ m (approx.)}$$

Area of segment with roses

$$= \frac{1}{6}\pi r^2 - \frac{\sqrt{3}}{4} r^2$$

$$= \frac{1}{6}\pi \times 26 \text{ m} \times 26 \text{ m} - \frac{\sqrt{3}}{4} \times 26 \text{ m} \times 26 \text{ m}$$

$$= \frac{1}{6} \times \frac{22}{7} \times 26 \times 26 \text{ m}^2 - \frac{\sqrt{3}}{4} \times 26 \times 26 \text{ m}^2$$

$$= 354.1 \text{ m}^2 - 292.71 \text{ m}^2 = 61.39 \text{ m}^2 \text{ (approx.)}$$

Area of segment with petunias

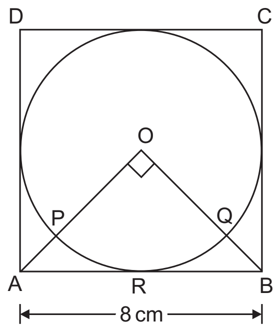
$$= \frac{1}{4}\pi r^2 - \frac{1}{2} r^2$$

$$= \frac{1}{4} \times \frac{22}{7} \times 26 \text{ m} \times 26 \text{ m} - \frac{1}{2} \times 26 \text{ m} \times 26 \text{ m}$$

$$= 531.14 \text{ m}^2 - 338 \text{ m}^2$$

$$= 193.14 \text{ m}^2 \text{ (approx.)}$$

33. O is the centre of the circle and $\angle$AOB = 90°. Find the area of sector OPRQO. Also find area of remaining part of square ABCD when area of circle is excluded.



Sol. Let r be the radius of the circle.

$$2r = 8 \text{ cm}$$

$$\Rightarrow \qquad r = 4 \text{ cm}$$

Area of sector OPRQO

$$= \frac{\theta}{360} \times \pi r^2$$

$$= \frac{90}{360} \times \frac{22}{7} \times 4 \text{ cm} \times 4 \text{ cm}$$

$$= 12.6 \text{ cm}^2 \text{ (approx.)}$$

Area of the circle $= \pi r^2 = \frac{22}{7} \times 4 \text{ cm} \times 4 \text{ cm}$

$$= 50.3 \text{ cm}^2 \text{ (approx.)}$$

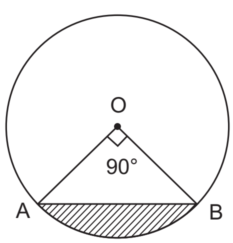
Area of square ABCD = 8 cm × 8 cm = 64 cm^2

$\therefore$ Area of the remaining part of square ABCD when area of circle is excluded

$$= 64 \text{ cm}^2 - 50.3 \text{ cm}^2 = 13.7 \text{ cm}^2$$

Long Answer Type Questions

34. In the given figure, AB is a chord of a circle of radius 7 cm and centred at O. Find the area of the shaded region if $\angle AOB = 90°$. Also, find length of minor arc AB.



[CBSE 2023 Basic]

Sol. Area of sector OAB $= \frac{90}{360} \times \frac{22}{7} \times 7 \text{ cm} \times 7 \text{ cm}$

$$= 38.5 \text{ cm}^2$$

Area of $\Delta AOB = \frac{1}{2} \times 7 \text{ cm} \times 7 \text{ cm}$

$$= 24.5 \text{ cm}^2$$

Area of the shaded region

= Area of sector OAB

– Area of ΔAOB

$$= 38.5 \text{ cm}^2 - 24.5 \text{ cm}^2$$

$$= 14 \text{ cm}^2$$

Length of minor arc AB $= \frac{90}{360} \times 2 \times \frac{22}{7} \times 7 \text{ cm}$

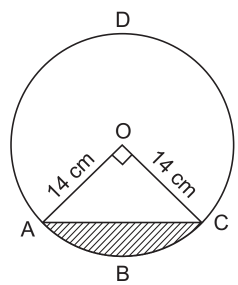
$$= 11 \text{ cm}$$

35. A chord of a circle of radius 14 cm subtends an angle of 90° at the centre. Find the area of the corresponding minor and major segments of the circle.

Sol. Area of sector OABC $= \frac{\theta}{360} \times \pi r^2$

$$= \frac{90}{360} \times \frac{22}{7} \times 14 \text{ cm} \times 14 \text{ cm}$$

$$= 154 \text{ cm}^2$$



Area of $\Delta AOC = \frac{1}{2} \times 14 \text{ cm} \times 14 \text{ cm}$

$$= 98 \text{ cm}^2$$

Area of minor segment ABC

= Area of sector OABC – Area of ΔAOC

$= 154 \text{ cm}^2 - 98 \text{ cm}^2$

$= 56 \text{ cm}^2$

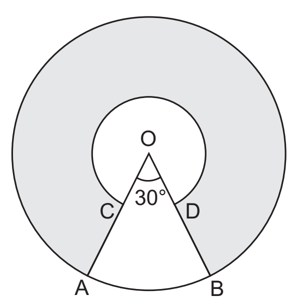
Area of the major segment ADC

= Area of circle – Area of minor segment ABC

$= \frac{22}{7} \times 14 \text{ cm} \times 14 \text{ cm} - 56 \text{ cm}^2$

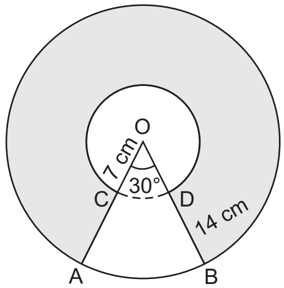
$= 616 \text{ cm}^2 - 56 \text{ cm}^2 = 560 \text{ cm}^2$

36. In the given figure, two concentric circles with centre O have radii 14 cm and 7 cm. If $\angle AOB = 30°$, find the area of the shaded region.



[CBSE 2024 Basic]

Sol.



Area of region ABDC

$= \text{Area of sector OAB} - \text{Area of sector OCD}$

$= \frac{30}{360} \times \frac{22}{7} \times (14 \text{ cm})^2 - \frac{30}{360} \times \frac{22}{7} \times (7 \text{ cm})^2$

$= \frac{1}{12} \times \frac{22}{7} [(14 \text{ cm} + 7 \text{ cm})(14 \text{ cm} - 7 \text{ cm})]$

$= \frac{1}{12} \times \frac{22}{7} \times 21 \text{ cm} \times 7 \text{ cm}$

$= 38.5 \text{ cm}^2$

Area of the circular ring

$= \frac{22}{7} \times 14 \text{ cm} \times 14 \text{ cm} - \frac{22}{7} \times 7 \text{ cm} \times 7 \text{ cm}$

$= \frac{22}{7} \times (14 \text{ cm} + 7 \text{ cm})(14 \text{ cm} - 7 \text{ cm})$

$= \frac{22}{7} \times 21 \text{ cm} \times 7 \text{ cm} = 462 \text{ cm}^2$

Area of the shaded region

$= 462 \text{ cm}^2 - 38.5 \text{ cm}^2$

$= 423.5 \text{ cm}^2$

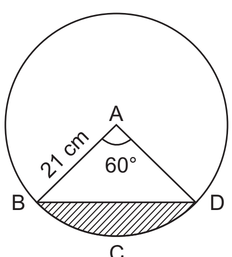
37. An arc of a circle of radius 21 cm subtends an angle of 60° at the centre. Find

(*a*) length of the arc.

(*b*) the area of the minor segment of the circle made by the corresponding chord.

[CBSE 2024 Standard]

Sol.



(*a*) Length of the arc

$= \frac{60}{360} \times 2 \times \frac{22}{7} \times 21 \text{ cm}$

$= 22 \text{ cm}$

(*b*) Area of sector ABCD

$= \frac{60}{360} \times \frac{22}{7} \times 21 \text{ cm} \times 21 \text{ cm}$

$= 231 \text{ cm}^2$

Area of ΔABD $= \frac{\sqrt{3}}{4} \times 21 \text{ cm} \times 21 \text{ cm}$

$= 190.7 \text{ cm}^2$ (approx.)

∴ Area of the minor segment

$= 231 \text{ cm}^2 - 190.7 \text{ cm}^2$

$= 40.3 \text{ cm}^2$ (approx.)

Let's Compete

(Page 217)

Multiple-Choice Questions

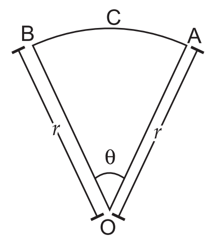
1. The area of a sector of a circle bounded by an arc of length 12π cm is equal to 48π cm². Then the radius of the circle is

(*a*) 16 cm (*b*) 8 cm

(*c*) 12 cm (*d*) 10 cm

Sol. (*b*) 8 cm

Let r be the radius of the sector and θ be the ∠AOB of the sector.



Given that

$$\pi r^2 \times \frac{\theta}{360°} = 48\pi \text{ cm}^2$$

$$\Rightarrow \quad \frac{\theta r^2}{360°} = 48 \text{ cm}^2 \quad \ldots(1)$$

Also, $$2\pi r \frac{\theta}{360°} = 12\pi \text{ cm}$$

$$\Rightarrow \quad \frac{\theta r}{360°} = 6 \text{ cm} \quad \ldots(2)$$

Dividing (1) by (2), we have

$$r = \frac{48}{6} \text{ cm} = 8 \text{ cm}$$

Hence, the required radius of the circle is 8 cm.

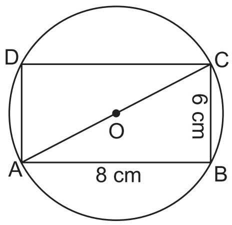
2. A rectangle whose length is $\frac{4}{3}$ times its breadth is inscribed inside a circle with centre at O. If the breadth of the rectangle is 6 cm, then the ratio of the area of the circle to the area of the rectangle is

(*a*) 163 : 270 (*b*) 168 : 275

(*c*) 275 : 168 (*d*) 270 : 163

Sol. (*c*) 275 : 168.

Let ABCD be a rectangle inscribed in a circle with centre at O and radius = r.



Now, breadth BC = AD of the rectangle is 6 cm and so the length AB = DC of the rectangle is $\frac{4 \times 6}{3}$ cm, i.e. 8 cm.

Now, $\quad AC^2 = AD^2 + DC^2$

[By using Pythagoras' Theorem in ΔADC, $\because \angle ADC = 90°$]

$$\Rightarrow \quad 4r^2 = (6 \text{ cm})^2 + (8 \text{ cm})^2$$
$$= 36 \text{ cm}^2 + 64 \text{ cm}^2 = 100 \text{ cm}^2$$
$$\Rightarrow \quad r^2 = 25 \text{ cm}^2$$
$$\Rightarrow \quad r = 5 \text{ cm}$$

$$\therefore \frac{\text{Area of the circle}}{\text{Area of the rectangle}} = \frac{\pi \times 25}{8 \times 6}$$
$$= \frac{22}{7} \times \frac{25}{48}$$
$$= \frac{275}{168}$$

$\therefore$ Required ratio = 275 : 168

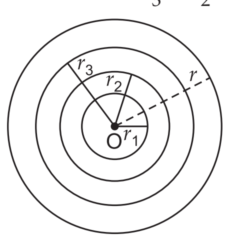
3. The radius of a circle is 20 cm. It is divided into four parts of equal area by drawing three concentric circles inside it. Then the radius of the largest of the three concentric circles drawn is

(*a*) $10\sqrt{3}$ cm (*b*) $10\sqrt{5}$ cm

(*c*) 10 cm (*d*) 20 cm

Sol. (*a*) $10\sqrt{3}$ cm

Let r_1, r_2 and r_3 be the radii of concentric circles of common centre O and r be the radius of the original circle such that $r_3 > r_2 > r_1$ and $r = 20$ cm.



According to the problem, we have

Area of each of three concentric circles

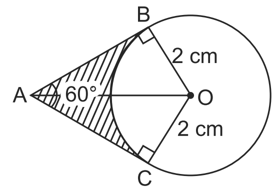
$$= \frac{1}{4} \times \pi \times 20^2 \text{ cm}^2 = 100 \text{ cm}^2$$

$$\therefore \quad \pi r_1^2 = 100\,\pi,\ \pi r_2^2 = 200\pi,\ \pi r_3^2 = 300\pi$$

$$\therefore \quad r_3^2 = 300$$
$$\therefore \quad r_3 = \sqrt{300} = 10\sqrt{3}$$

Hence, the required radius of the largest concentric circle is $10\sqrt{3}$ cm.

4. In the given figure, if the radius of the circle is 2 cm and $\angle A = 60°$, then the area of the shaded region is



(*a*) $4\left(\sqrt{3} + \frac{\pi}{3}\right) \text{cm}^2$ (*b*) $4\left(\frac{\pi}{\sqrt{3}} + 3\right) \text{cm}^2$

(*c*) $4\left(\frac{\pi}{\sqrt{3}} - 3\right) \text{cm}^2$ (*d*) $4\left(\sqrt{3} - \frac{\pi}{3}\right) \text{cm}^2$

Sol. (*d*) $4\left(\sqrt{3} - \frac{\pi}{3}\right)$ cm^2

We see that ABOC is a cyclic quadrilateral,

$$\therefore \quad \angle COB = 180° - 60° = 120°$$

$\therefore$ Area of the sector BOC

$$= \pi \times 2^2 \times \frac{120°}{360°} \text{ cm}^2$$
$$= \frac{1}{2}\pi \text{ cm}^2 \qquad \text{...(1)}$$

In ΔAOB,

$$\because \quad \angle ABO = 90°,$$
$$\therefore \quad \frac{2}{AB} = \tan \angle OAB$$
$$= \tan 30° = \frac{1}{\sqrt{3}}$$
$$\therefore \quad AB = 2\sqrt{3}$$

$\therefore$ Length of AB = $2\sqrt{3}$ cm

$$\therefore \text{ Area of } \Delta AOB = \frac{1}{2} \times 2 \times 2\sqrt{3} \text{ cm}^2$$
$$= 2\sqrt{3} \text{ cm}^2$$

Area of the quadrilateral ABOC

$$= 2 \times \text{area of } \Delta AOB$$
$$= 4\sqrt{3} \text{ cm}^2 \qquad \text{...(2)}$$

$\therefore$ Area of the shaded region

= Area of quadrilateral ABOC

– Area of the sector BOC

$$= \left(4\sqrt{3} - \frac{4\pi}{3}\right) \text{cm}^2$$

[From (1) and (2)]

$$= 4\left(\sqrt{3} - \frac{\pi}{3}\right) \text{cm}^2$$

5. The area of the largest triangle, i.e. triangle of the greatest area, that can be inscribed in a semicircle of radius 4 cm is

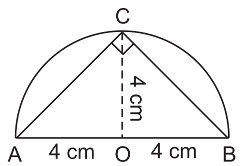
(*a*) 8 cm^2 (*b*) 16 cm^2

(*c*) 4 cm^2 (*d*) 2 cm^2

Sol. (*b*) 16 cm^2

Area of the largest triangle

= Area of ΔABC, where $OC \perp AB$ and O is the centre of the circle.



Now, $AC = BC$

$$= \sqrt{AO^2 + CO^2}$$

$$= \sqrt{4^2 + 4^2} \text{ cm}$$

$$= \sqrt{32} \text{ cm}$$

$\therefore$ Required area $= \frac{1}{2} \times \sqrt{32} \times \sqrt{32}$ cm^2

$= 16$ cm^2

6. A pendulum swings through an angle of 36° and describes an arc of length 13.2 cm. Then the length of the pendulum sweeps an area equal to

(*a*) 135.8 cm^2 (*b*) 130.6 cm^2

(*c*) 138.6 cm^2 (*d*) 136 cm^2

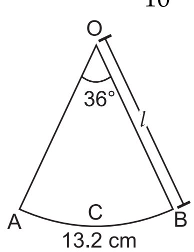
Sol. (*c*) 138.6 cm^2

The pendulum describes an arc ACB of a sector OAB of a circle with centre at O and radius = l, (say).

Length of arc ACB $= \frac{36}{360°} \times 2\pi l$

$$= \frac{36}{360°} \times 2 \times \frac{22}{7} \times l$$

$$= \frac{1}{10} \times \frac{1}{2} \times 22\, l = \frac{22l}{35}$$



$\therefore$ $\frac{22l}{35} = 13.2$ cm

$\Rightarrow$ $l = \frac{13.2 \times 35}{22}$ cm = 21 cm

$\therefore$ Radius of the sector = 21 cm

$\therefore$ Area of the sector $= \frac{36}{360°} \times \pi l^2$

$$= \frac{1}{10} \times \frac{22}{7} \times 21 \times 21 \text{ cm}^2$$

$$= \frac{1}{10} \times 22 \times 3 \times 21 \text{ cm}^2$$

$$= 138.6 \text{ cm}^2$$

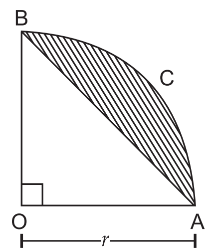
7. An arc ACB of a circle forms an angle of 90° at the centre O of the circle. Then the ratio of the area of the sector OAB to the area of the minor segment ACBA is

(*a*) $\pi : (\pi - 2)$ (*b*) $\pi : (\pi - 1)$

(*c*) $(\pi + 1) : (\pi - 1)$ (*d*) $(\pi + 1) : \pi$

Sol. (*a*) $\pi : (\pi - 2)$

Let r be the radius of the sector.



Then area of sector OAB $= \frac{1}{4}\pi r^2$ cm^2 ...(1)

$\therefore$ Area of the minor segment ACBA

$$= \left(\frac{\pi r^2}{4} - \frac{1}{2}r^2\right) \text{cm}^2$$

$$= \frac{(\pi - 2)r^2}{4} \quad \text{...(2)}$$

$\therefore$ Required ratio $= \frac{\pi}{\pi - 2}$ [From (1) and (2)]

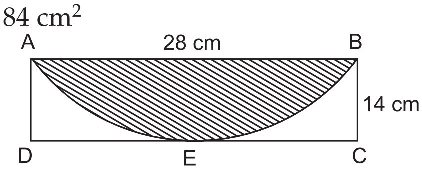
$= \pi : (\pi - 2)$

8. A semicircle of maximum area is cut off from a rectangular piece of paper of length 28 cm and breadth 14 cm. Then the area of the remaining portion of the rectangle is

(*a*) 96 cm^2 (*b*) 90 cm^2

(*c*) 100 cm^2 (*d*) 84 cm^2

Sol. (*d*) 84 cm^2



Area of the rectangle ABCD

$= 28 \times 14$ cm^2 = 392 cm^2 ...(1)

Area of the semicircle of maximum area

= Area of the semicircle AEB

$$= \frac{\pi \times 14^2}{2} \text{ cm}^2$$

$$= \frac{22}{14} \times 14 \times 14 \text{ cm}^2$$

$$= 308 \text{ cm}^2 \quad \ldots(2)$$

$\therefore$ Required area of the remaining portion

$$= (392 - 308) \text{ cm}^2 \quad \text{[From (1) and (2)]}$$

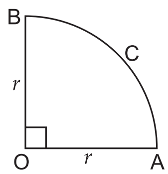
$$= 84 \text{ cm}^2$$

9. If the perimeter of a quadrant of a circle is 25 cm, then the area of the quadrant is

(*a*) 77 cm^2 (*b*) 38.5 cm^2

(*c*) 12.5 cm^2 (*d*) 20 cm^2

Sol. (*b*) 38.5 cm^2



If r be the radius of the quadrant OACB of a circle, then

$$2r + \frac{2\pi r}{4} = 25 \text{ cm}$$

$$\Rightarrow \quad 2r\left(1 + \frac{22}{7} \times \frac{1}{4}\right) = 25 \text{ cm}$$

$$\Rightarrow \quad 2r\left(1 + \frac{11}{14}\right) = 25 \text{ cm}$$

$$\Rightarrow \quad 2r \times \frac{25}{14} = 25 \text{ cm}$$

$$\Rightarrow \quad r = \frac{25 \times 14}{25 \times 2} \text{ cm}$$

$$\Rightarrow \quad r = 7 \text{ cm}$$

$$\therefore \text{ Required area} = \frac{\pi r^2}{4}$$

$$= \frac{22}{7} \times \frac{1}{4} \times 7 \times 7 \text{ cm}^2$$

$$= 38.5 \text{ cm}^2$$

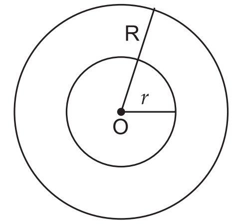
10. If the areas of two concentric circles are in the ratio 9 : 16, then the ratio of the perimeter of the ring formed by the two concentric circles and the difference of the perimeters of the two circles is

(*a*) 2 : 7 (*b*) 7 : 2

(*c*) 7 : 1 (*d*) 1 : 7

Sol. (*c*) 7 : 1

Let r and R be the radii of two concentric circles with centre at O, where R > r.



$$\text{Given that} \quad \frac{\pi r^2}{\pi R^2} = \frac{9}{16}$$

$$\Rightarrow \quad \frac{r}{R} = \frac{3}{4} \quad \ldots(1)$$

Now, perimeter of the ring = $2\pi(r + R)$ ...(2)

Difference of the perimeter of two circles

$$= 2\pi(R - r) \quad \ldots(3)$$

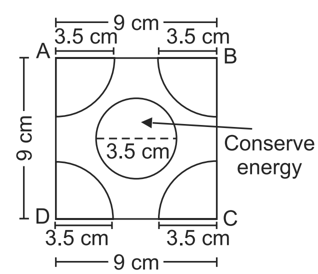
$$\therefore \text{ Required ratio} = \frac{R + r}{R - r} = \frac{7}{1} \quad \text{[From (1)]}$$

$$= 7 : 1$$

Life Skills

(Page 218)

1. The Principal of a school asked each student to prepare a badge to be worn on every Monday of the week, so as to spread awareness about 'Conserving energy'. Each badge had to be made on a square cardboard of side 9 cm with a quadrant of a circle of radius 3.5 cm drawn at each vertex of the square and a circle of diameter 3.5 cm drawn in its centre with the slogan 'Conserve energy' written on it.



(*a*) Find the area of the remaining portion of the badge.

(*b*) Suggest the writings which students can provide in the four quadrants.

Sol. Sum of the area of 4 quadrants at the four corners of the square ABCD

= Area of the circle with radius 3.5 cm

$$= \pi \times 3.5^2 \text{ cm}^2$$

$$= \frac{22}{7} \times 3.5 \times 3.5 \text{ cm}^2$$

$$= 38.5 \text{ cm}^2 \quad \ldots(1)$$

Area of the circle of radius $\frac{3.5}{2}$ cm, at the centre of the square

$$= \pi \times \frac{3.5^2}{4} \text{ cm}^2$$

$$= \frac{1}{4} \times 38.5 \text{ cm}^2 \qquad \text{[From (1)]}$$

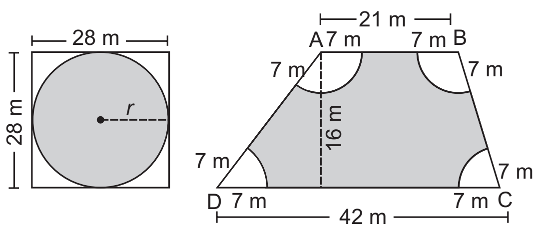
$$= 9.625 \text{ cm}^2 \qquad \ldots(2)$$

$\therefore$ Area of the square = 9×9 cm^2 = 81 cm^2 …(3)

(*a*) Area of the remaining portion of the badge
= (81 – 38.5 – 9.625) cm^2 [From (1), (2) and (3)]
= (81 – 48.125) cm^2
= 32.875 cm^2

(*b*) Save water, make car pools, switch off electrical appliances when not in use, repair leaking taps, use LED, use solar energy, etc. are some phrases which the students can write in the quadrants.

2. Farmer A owns a square field whereas a farmer B holds a field in the form of a trapezium as shown below:



On Van Mahotsav Day, farmers A and B plant trees in the shaded regions of their respective fields.
Which of them uses more area to plant trees and by how much?

Sol. From the square field, for farmer A

$$2r = 28 \text{ m}$$

$$\Rightarrow \qquad r = 14 \text{ m}$$

$\therefore$ Area of the circle inscribed in the square

$$= \pi \times 14^2 \text{ m}^2$$

$$= \frac{22}{7} \times 14 \times 14 \text{ m}^2$$

$$= 22 \times 2 \times 14 \text{ m}^2$$

$$= 616 \text{ m}^2 \qquad \ldots(1)$$

Now,
Area of the trapezium ABCD

$$= \frac{1}{2}(21 + 42) \times 16 \text{ m}^2$$

$$= 8 \times 63 \text{ m}^2$$

$$= 504 \text{ m}^2 \qquad \ldots(2)$$

Let θ_1, θ_2, θ_3 and θ_4 be the angles of arcs at A, B, C and D respectively.

Then $\theta_1 + \theta_2 + \theta_3 + \theta_4 = 360°$ …(3)

Sum of the areas of the four sectors of circles with centres at A, B, C and D

$$= \frac{\pi 7^2}{360°}(\theta_1 + \theta_2 + \theta_3 + \theta_4) \text{ m}^2$$

$$= \frac{22}{7} \times 49 \times \frac{36°}{360°} \text{ m}^2 \qquad \text{[From (3)]}$$

$$= 154 \text{ m}^2 \qquad \ldots(4)$$

$\therefore$ Area of the shaded region in trapezium ABCD

$$= (504 - 154) \text{ m}^2 \quad \text{[From (2) and (4)]}$$

$$= 350 \text{ m}^2 \qquad \ldots(5)$$

From (1) and (5), we see that
Farmer A uses (616 – 350) m^2, i.e.
266 m^2 more area than B.