10

Circles

Checkpoint (Page 189)

1. The distance of a chord of a circle of radius 5 cm from its centre is 3 cm. What is the length of the chord?

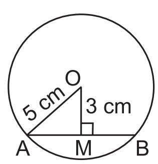
Sol. Let AB be the chord of a circle with centre at O.

$\because$ OM $\perp$ AB

$\therefore$ M is the mid-point of AB. OA is the radius, OA = 5 cm.

In ΔOAM, by using Pythagoras' theorem, we have

$$OA^2 = OM^2 + AM^2$$
$$\Rightarrow (5)^2 = (3)^2 + AM^2$$
$$\Rightarrow AM^2 = 25 \text{ cm}^2 - 9 \text{ cm}^2$$
$$\Rightarrow AM^2 = 16 \text{ cm}^2$$
$$\Rightarrow AM = 4 \text{ cm}$$
$$\therefore AB = 2AM = 2 \times 4 \text{ cm} = 8 \text{ cm}$$

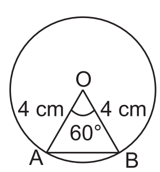


Hence, the required length of the chord is 8 cm.

2. A chord of a circle subtends an angle of 60° at the centre of the circle. If the radius of the circle is 4 cm, find the area of the triangle formed by the chord and the two radii of the circle.

Sol. Let the chord AB subtend an angle of 60° at the centre O, i.e. $\angle AOB = 60°$

$$\text{Radius} = OA = OB = 4 \text{ cm}$$
$$\therefore \angle A = \angle B = 60°$$

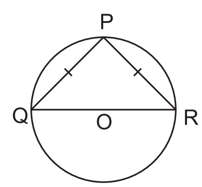


$\therefore$ Area of the equilateral triangle OAB

$$= \frac{\sqrt{3}}{4} \times 4^2 \text{ cm}^2$$
$$= 4\sqrt{3} \text{ cm}^2$$

3. P is a point on the circumference of a circle with diameter QR such that PQ = PR. Then find the angles of the triangle PQR.

Sol. Let the diameter QR through the centre O of the circle form a ΔPQR at a point P on the circle such that PQ = PR.



$\therefore \angle QPR = 90°$ [Angle in a semicircle]

$\because PQ = PR$

$\therefore \angle PQR = \angle PRQ = 45°$

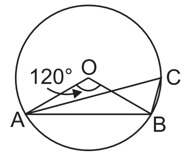
Hence, the angles of the triangle PQR are $\angle P = 90°$, $\angle Q = 45°$ and $\angle R = 45°$.

4. The arc AB of a circle with centre at O subtends an angle 120° at O. If C be a point on the remaining part of the circumference, what is the measure of $\angle ACB$?

Sol. We have $\angle AOB = 120°$

$$\angle AOB = 2\angle ACB$$

[$\because$ The angle subtended by an arc of a circle at the centre is double the angle subtended by it at any point on the remaining part of the circle.]



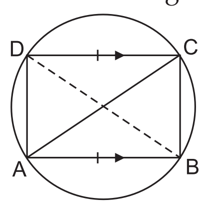
$\Rightarrow \qquad 120° = 2\,\angle ACB$

$\Rightarrow \qquad \angle ACB = \dfrac{120°}{2} = 60°$

$\therefore$ The measure of $\angle ACB$ is 60°.

5. AB and CD are two opposite sides of a cyclic quadrilateral such that AB = CD and AB ∥ CD. If one diagonal of the quadrilateral is of length 5 cm, what is the length of the other diagonal? Give reasons.

Sol. Given that AB and CD are two opposite sides of a cyclic quadrilateral such that AB = CD. Let AC and BD be two diagonals of ABCD and AC = 5 cm. To find the length of BD.



We have

$\angle B + \angle D = 180°$

[∵ Opposite ∠s of cyclic quadrilateral] …(1)

$\angle B + \angle C = 180°$

[∵ AB ∥ CD, Interior ∠s] …(2)

From (1) and (2), we have

$\angle B + \angle D = \angle B + \angle C$

$\Rightarrow \qquad \angle D = \angle C$

$\Rightarrow \qquad \angle ADC = \angle BCD$ …(3)

In ΔADC and ΔBCD

$\angle ADC = \angle BCD$ [From (3)]

$\angle DAC = \angle CBD$

[Angles in the same segment of a circle are equal]

DC = CD [Common]

$\therefore \qquad \Delta ADC \cong \Delta BCD$

[By AAS criterion of similarity]

$\therefore \qquad AC = BD$ [By CPCT]

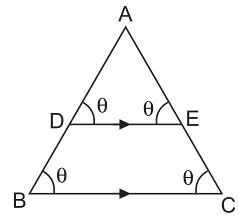
$\Rightarrow \qquad BD = 5$ cm

$\therefore$ The required length is 5 cm.

6. If a line is drawn parallel to the base of an isosceles triangle to intersect its equal sides, prove that the quadrilateral so formed is cyclic.

Sol. Let ΔABC be an isosceles triangle with AB = AC. D and E are two points on AB and AC respectively such that DE ∥ BC.

To prove that the quadrilateral BCED is cyclic.



We have

$\angle ABC = \angle ADE$ [∵ DE ∥ BC]

$= \theta$, say

$\therefore \qquad \angle ACB = \angle ABC = \theta$ …(1)

But $\qquad \angle ACB = \angle AED$ [∵ DE ∥ BC]

$\therefore \qquad \angle AED = \theta$

$\Rightarrow \qquad \angle DEC = 180° - \theta$ …(2)

Now, $\angle DBC + \angle DEC$

$= \angle ABC + \angle DEC$

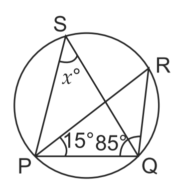
$= \theta + 180° - \theta$ [From (1) and (2)]

$= 180°$

$\therefore$ DBCE is a cyclic quadrilateral, since, a pair of opposite angles is supplementary.

Hence, proved.

7. In the given figure, find the value of x, if $\angle RPQ = 15°$ and $\angle PQR = 85°$.



Sol. Given that PQ is a chord of a circle and R, S are two points on the circle such that $\angle PSQ = x°$, $\angle PQR = 85°$ and $\angle RPQ = 15°$. To find the value of x.

In ΔPQR, we have

$\angle PRQ + \angle PQR + \angle RPQ = 180°$

[Angle sum property of a triangle]

$\angle PRQ = 180° - (\angle PQR + \angle RPQ)$

$= 180° - (85° + 15°)$

$= 180° - 100°$

$= 80°$

Now, since $\angle PRQ$ and $\angle PSQ$ stand on the same chord PQ of the circle.

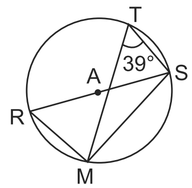
$\therefore \quad \angle PSQ = \angle PRQ$

$\Rightarrow \quad x° = 80°$

$\Rightarrow \quad x = 80$

Hence, the required value of x is 80.

8. In the given figure, A is the centre of a circle and $\angle MTS = 39°$. What is the measure of $\angle MSR$?



Sol. Given that A is the centre of a circle and RAS is a diameter of the circle. M is a point on the circle, such that $\angle MTS = 39°$, where T is a point on the circle on the other side of the diameter RAS.

To find $\angle MSR$.

We have

$\angle RMS = 90° \quad \ldots(1)$

[$\because$ RS is a diameter and angle in a semi-circle is 90°]

Now, $\angle MRS$ and $\angle MTS$ stand on the same chord MS of the circle, hence, we have

$\angle MRS = \angle MTS = 39° \quad \ldots(2)$

Now, in ΔMSR, we have

$\angle RMS + \angle MRS + \angle MSR = 180°$

[Angle sum property of a triangle]

$\Rightarrow \quad 90° + 39° + \angle MSR = 180°$

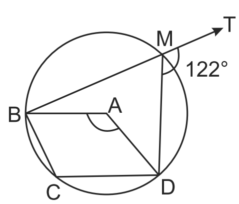
[From (1) and (2)]

$\Rightarrow \quad \angle MSR = 180° - 129°$

$= 51°$

$\therefore$ The measure of $\angle MSR$ is 51°.

9. In the given figure, A is the centre of a circle and $\angle DMT = 122°$. Find the measures of $\angle BAD$ and $\angle BCD$.



Sol. Given that B, C, D and M are points on a circle of centre at A such that ABCD is a quadrilateral. BM is produced to T such that $\angle TMD = 122°$. To find $\angle BAD$ and $\angle BCD$.

We have

$\because$ BCDM is a cyclic quadrilateral,

$\therefore \quad \angle BCD + \angle BMD = 180° \quad \ldots(1)$

Also, $\quad \angle BMD + \angle TMD = 180°$

[Linear pair]…(2)

$\therefore$ From (1) and (2), we have

$\angle BCD = \angle TMD$

$= 122°$

$\therefore \quad \angle BMD = 180° - 122°$

[From (1)]

$= 58° \quad \ldots(3)$

Now, $\quad \angle BAD = 2\angle BMD$

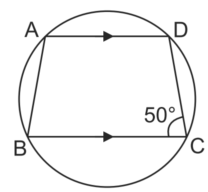
[$\because$ Angle at the centre of a circle is twice the angle on the circumference on the other side of the arc BD]

$= 2 \times 58° \quad$ [From (3)]

$= 116°$

Hence, the required measures of $\angle BAD$ and $\angle BCD$ are 116° and 122° respectively.

10. ABCD is a cyclic quadrilateral with AD $\parallel$ BC. If $\angle C = 50°$, determine the remaining angles.



Sol. Given that ABCD is a cyclic quadrilateral with AD $\parallel$ BC and $\angle C = 50°$. To find $\angle B$, $\angle D$ and $\angle A$.

We have

$\angle C + \angle D = 180°$

[$\because$ AD $\parallel$ BC and CD is a transversal]

$\Rightarrow \quad 50° + \angle D = 180°$

$\Rightarrow \quad \angle D = 180° - 50° = 130°$

Again, $\quad \angle C + \angle A = 180°$

[$\because$ ABCD is a cyclic quadrilateral]

$\Rightarrow \quad 50° + \angle A = 180°$

$\therefore \quad \angle A = 180° - 50° = 130°$

Also, $\quad \angle B + \angle D = 180°$

[$\because$ ABCD is a cyclic quadrilateral]

$\Rightarrow \quad \angle B + 130° = 180°$

$\Rightarrow \quad \angle B = 180° - 130° = 50°$

Hence, the required measures of $\angle A$, $\angle B$ and $\angle D$ are 130°, 50° and 130° respectively.

Check Your Progress

(Page 193)

Multiple-Choice Questions

1. How many tangents can be drawn to a circle from a point on it?

(*a*) One (*b*) Two

(*c*) Infinite (*d*) Zero

[CBSE 2023 Basic]

Sol. (*a*) One

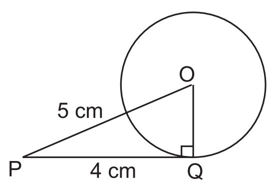
2. The length of a tangent PQ from an external point P is 4 cm. If the distance of P from the centre is 5 cm, then the diameter of the circle is

(*a*) $\sqrt{7}$ cm (*b*) 3 cm

(*c*) 6 cm (*d*) $2\sqrt{2}$ cm

Sol. (*c*) 6 cm

Given that PQ is a tangent to a circle with centre O at a point Q.



Also, OP = 5 cm and PQ = 4 cm.

To find the diameter of the circle.

We have

PQ is a tangent at a point Q on the circle and OQ is a radius, hence, $\angle OQP = 90°$.

$\therefore$ In ΔOPQ, by using Pythagoras' theorem, we have

$$OP^2 = OQ^2 + PQ^2$$

$$\Rightarrow \quad OQ^2 = OP^2 - PQ^2$$

$$= 5^2 - 4^2$$

$$= 25 - 16$$

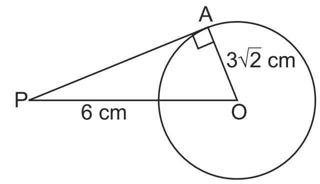
$$= 9$$

$$\therefore \quad OQ = 3 \text{ cm}$$

$$\therefore \quad 2OQ = 2 \times 3 = 6 \text{ cm}$$

Hence, the required length of the diameter of the circle is 6 cm.

3. A tangent PA is drawn from an external point P to a circle of radius $3\sqrt{2}$ cm such that the distance of the point P from the centre at O is 6 cm as shown in the figure.



The value of $\angle APO$ is

(*a*) 45° (*b*) 50°

(*c*) 60° (*d*) 30° **[CBSE SP 2012]**

Sol. (*a*) 45°

Since PA is a tangent to a circle with centre at O and radius, OA = $3\sqrt{2}$ cm such that PO = 6 cm and $\angle PAO = 90°$

Now, in ΔAPO, we have

$$\sin \angle APO = \frac{OA}{PO}$$

$$= \frac{3\sqrt{2}}{6}$$

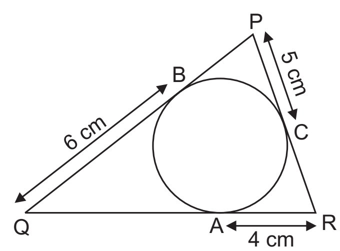
$$= \frac{\sqrt{2}}{2}$$

$$= \frac{1}{\sqrt{2}}$$

$$= \sin 45°$$

$$\therefore \quad \angle APO = 45°$$

4. In the given figure, the perimeter of ΔPQR is



(*a*) 30 cm (*b*) 15 cm

(*c*) 45 cm (*d*) 60 cm

Sol. (*a*) 30 cm

We know that the length of tangents drawn from an external point to a circle are equal.

$\therefore$ QA = QB, AR = CR and PB = PC

Therefore, perimeter of the circle

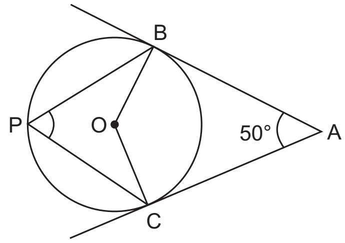
= PB + BQ + QA + AR + RC + PC

= 5 cm + 6 cm + 6 cm + 4 cm + 4 cm + 5 cm

= 30 cm

Perimeter of ΔPQR = 30 cm

5. From an external point A, two tangents AB and AC are drawn to a circle with centre at O. P is a point on the major arc BC of the circle. If ∠BAC = 50°, then ∠BPC is equal to



(*a*) 50° (*b*) 130°
(*c*) 65° (*d*) 90°

Sol. (*c*) 65°

In quadrilateral OCAB,

∠BAC = 50°

∠ABO = ∠ACO = 90°

∴ ∠BOC = 360° – 50° – 90° – 90°

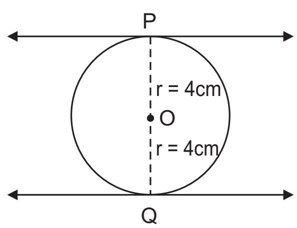
⇒ ∠BOC = 130°

∴ $\angle BPC = \frac{1}{2}\angle BOC = \frac{1}{2} \times 130° = 65°$

6. A circle is of radius 4 cm. The distance between two of its parallel tangents is

(*a*) 12 cm (*b*) 8 cm
(*c*) 4 cm (*d*) 6 cm

Sol. (*b*) 8 cm



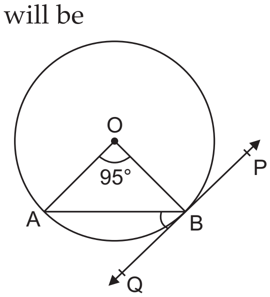
Distance between the parallel tangents

= OP + OQ

= 4 cm + 4 cm

= 8 cm

7. In the given figure, PQ is tangent to the circle centred at O. If ∠AOB = 95°, then the measure of ∠ABQ will be



(*a*) 47.5° (*b*) 42.5°
(*c*) 85° (*d*) 95°

[CBSE 2023 Standard]

Sol. (*a*) 47.5°

In ΔAOB,

OA = OB [Radius of the circle]

∴ ∠OAB = ∠OBA = x (say)

∠AOB + ∠OAB + ∠OBA = 180°

95° + x + x = 180°

⇒ $2x$ = 85°

⇒ x = 42.5°

We know that the tangent to any point on a circle is perpendicular to the radius through the point of contact,

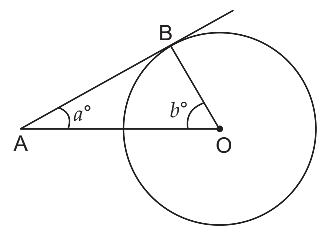
∴ ∠QBO = 90°

∠ABQ = ∠QBO – ∠ABO

= 90° – 42.5°

∠ABQ = 47.5°

8. In the given figure, AB is a tangent to the circle whose centre is at O. Given ∠BAO = a° and ∠BOA = b°. Then a° + b° is equal to



(*a*) 45° (*b*) 60°
(*c*) 90° (*d*) 120°

Sol. (*c*) 90°

In ΔAOB,

∠ABO + ∠BAO + ∠AOB = 180°

90° + a° + b° = 180°

⇒ a° + b° = 90°

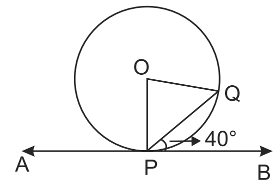
Very Short Answer Type Questions

9. How many parallel tangents can a circle have?

[CBSE SP 2012]

Sol. We know each pair of tangents at the two ends of any diameter of a circle is parallel to each other. Now, a circle has infinite number of diameters. Hence, infinite number of parallel tangents can be drawn to a circle.

10. In the given figure, APB is a tangent to a circle with centre O at a point P. If ∠QPB = 40°, what is the measure of ∠POQ?



Sol. Given that APB is a tangent to a circle with centre O at a point P on it and PQ is a chord of the circle such that $\angle QPB = 40°$.

To find $\angle POQ$.

We have

$\because$ OP is a radius at a point P on the tangent APB,

$\therefore \quad \angle OPB = 90°$

$\therefore \quad \angle OPQ = \angle OPB - \angle QPB$

$= 90° - 40° = 50°$

Now, in ΔOPQ, we have

$OP = OQ$ [Radius of the circle]

$\therefore \quad \angle OPQ = \angle OQP = 50°$

$\therefore$ In ΔOPQ, we have

$\angle OPQ + \angle OQP + \angle POQ = 180°$

[Angle sum property of a triangle]

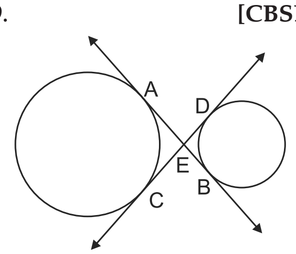
$\Rightarrow \quad 50° + 50° + \angle POQ = 180°$

$\Rightarrow \quad 100° + \angle POQ = 180°$

$\Rightarrow \quad \angle POQ = 180° - 100° = 80°$

Hence, the required measure of $\angle POQ$ is 80°.

11. In the given figure, common tangents AB and CD to the two circles intersect at E. Prove that AB = CD. **[CBSE 2014, SP 2016]**



Sol. Given that AB and CD are two common tangents to the two circles and these two tangents intersect each other at a point E.

To prove that AB = CD

We have

$\because$ E is an external point. Also EA and EC are two tangents to a circle from the point E,

$\therefore \quad EA = EC \quad \ldots(1)$

Similarly, from the second circle,

$EB = ED \quad \ldots(2)$

Adding (1) and (2), we get

$EA + EB = EC + ED$

$\Rightarrow \quad AB = CD$

Hence, proved.

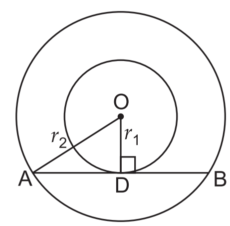
12. If r_1 and r_2 where $r_2 > r_1$ be the radii of two concentric circles and if p be the length of a chord of the bigger circle touching the smaller circle at a point, prove that $4r_2^2 = p^2 + 4r_1^2$.

Sol. Given that r_1 and r_2 ($r_2 > r_1$) are the radii of two concentric circles with common centre O and AB be a chord of the bigger circle touching the smaller circle at a point D. OD and OA are joined. Then $OD = r_1$, $OA = r_2$ and $AB = p$. To prove that $4r_2^2 = p^2 + 4r_1^2$

We have

$\because$ OD is the radius of the smaller circle and ADB is a tangent to this circle at a point D on it.

$\therefore \quad OD \perp AB$



Now, in ΔOAD, by using Pythagoras' theorem, we have

$$OA^2 = OD^2 + AD^2$$

$$\Rightarrow \quad r_2^2 = r_1^2 + \left(\frac{1}{2}AB\right)^2$$

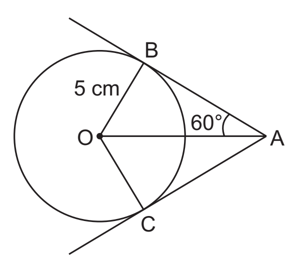
[Perpendicular from the centre of the circle to the chord bisects the chord]

$$\Rightarrow \quad r_2^2 = r_1^2 + \frac{1}{4}AB^2$$

$$\Rightarrow \quad 4r_2^2 = p^2 + 4r_1^2$$

Hence, proved.

13. In the given figure, tangents AB and AC are drawn to a circle centred at O. If $\angle OAB = 60°$ and OB = 5 cm, find lengths OA and AC.



[CBSE 2023 Basic]

Sol. In ΔABO,

$\angle ABO = 90^\circ$ [Tangent is perpendicular to the radius at the point of contact]

In right-angled ΔABO,

$$\sin 60^\circ = \frac{OB}{OA}$$

$$OA = \frac{OB}{\sin 60^\circ} = \frac{5\text{ cm} \times 2}{\sqrt{3}}$$

$$\Rightarrow \quad OA = \frac{10}{\sqrt{3}}\text{ cm}$$

$$\tan 60^\circ = \frac{OB}{AB}$$

$$\Rightarrow \quad AB = \frac{OB}{\tan 60^\circ} = \frac{5}{\sqrt{3}}\text{ cm}$$

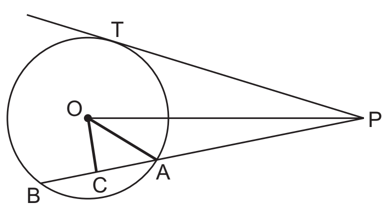
Since tangents drawn from an external point to a circle are equal,

$$\therefore \quad AB = AC$$

$$\Rightarrow \quad AC = \frac{5}{\sqrt{3}}\text{ cm}$$

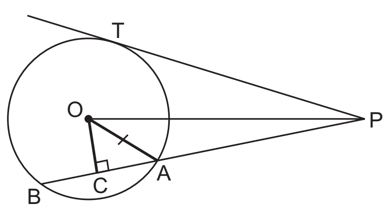
14. In the given figure, PT is a tangent to the circle at O. OC is perpendicular to chord AB. Prove that

$$PA \cdot PB = PC^2 - AC^2$$



[CBSE 2023 Standard]

Sol.



$$OC \perp AB,$$

$$\therefore \quad BC = CA \quad \ldots(1)$$

LHS, $PA \cdot PB = (PC - AC)(PC + BC)$

$= (PC - AC)(PC - AC)$ [From (1)]

$= PC^2 - AC^2$

$= RHS$

Hence proved.

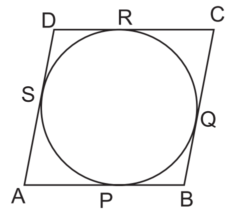
Short Answer Type Questions

15. Prove that the parallelogram circumscribing a circle is a rhombus.

[CBSE 2001, 2002C, 2008, 2009, 2012, SP 2011, 2023 Basic]

Sol. Given that ABCD is a parallelogram circumscribing a circle. To prove that ABCD is a rhombus.

We know that the lengths of tangents drawn from an external point to a circle are equal.



$\because$ A is an external point to the circle and AS and AP touch the circle at S and P respectively, hence

$AP = AS$ [Tangents from A] …(1)

Similarly, since BP and BQ are two tangents to the circle,

$\therefore$ $BP = BQ$ [Tangents from B] …(2)

Similarly, since CR and CQ are two tangents to the circle,

$\therefore$ $CQ = CR$ [Tangents from C] …(3)

Finally, DS and DR are two tangents to the circle,

$\therefore$ $DR = DS$ [Tangents from D] …(4)

Adding (1) and (2), we get

$AP + BP = AS + BQ$

$\Rightarrow$ $AB = AS + BQ$ …(5)

Similarly, adding (3) and (4), we get

$DR + CR = CQ + DS$

$\Rightarrow$ $DC = CQ + DS$ …(6)

Now, adding (5) and (6), we get

$AB + DC = (AS + DS) + (BQ + CQ)$

$\Rightarrow$ $AB + DC = AD + BC$

$\Rightarrow$ $2AB = 2AD$

[$\because$ ABCD is a parallelogram, $\therefore$ AB = DC and AD = BC]

$\Rightarrow$ $AB = AD$

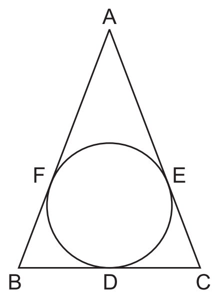
Hence, two adjacent sides AB and AD of the parallelogram ABCD are equal. Hence, ABCD is a rhombus.

Hence, proved.

16. ABC is an isosceles triangle in which AB = AC, circumscribed about a circle. Show that BC is bisected at the point of contact.

[CBSE 2008, 2012, 2023 Basic]

Sol. Given that ABC is an isosceles triangle with AB = AC. A circle is inscribed within this triangle touching the sides BC, CA and AB at D, E and F respectively.



To prove that BC is bisected at D.

We know that the lengths of tangents drawn from an external point to a circle are equal.

$\because$ A is an external point and AF and AE are two tangents to the circle at F and E respectively,

$\therefore$ AF = AE [Tangents from A] …(1)

Also, AB = AC [Given] …(2)

$\therefore$ Subtracting (1) from (2), we get

AB – AF = AC – AE

$\Rightarrow$ BF = CE …(3)

Again, BF and BD are tangents at F and D respectively.

$\therefore$ BF = BD [Tangents from B] …(4)

Also, CD and CE are tangents at D and E respectively.

$\therefore$ CE = CD [Tangents from C] …(5)

$\therefore$ From (3) and (4), we have

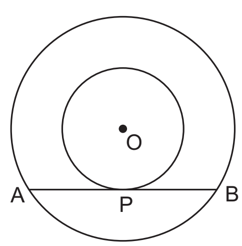
BD = CE …(6)

$\therefore$ From (5) and (6), we get

BD = CD, i.e., D is the mid-point of the tangent BC. Hence, BC is bisected at D.

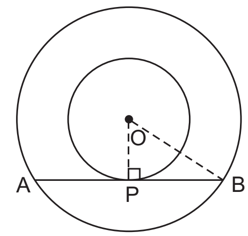
Hence, proved.

17. The concentric circles with centre O are of radii 3 cm and 5 cm. Find the length of the chord AB of the larger circle which touches the smaller circle at P.



Sol. OP = 3 cm

OB = 5 cm



In ΔOPB,

$$OP^2 + PB^2 = OB^2$$

$$\Rightarrow \quad PB^2 = OB^2 - OP^2$$

$$\Rightarrow \quad PB = \sqrt{OB^2 - OP^2}$$

$$= \sqrt{25 - 9} \text{ cm}$$

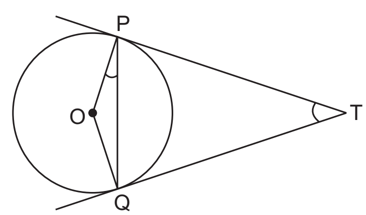
$$= \sqrt{16} \text{ cm}$$

$$\therefore \quad PB = 4 \text{ cm}$$

$$AP = PB$$

$$\Rightarrow \quad AB = 4 \text{ cm} + 4 \text{ cm} = 8 \text{ cm}$$

18. Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that $\angle PTQ = 2\angle OPQ$.



[CBSE 2023 Standard]

Sol. In ΔTPQ

TP = TQ [Length of tangents from an external point to a circle are equal] …(1)

$\therefore$ $\angle TPQ = \angle PQT$ [Angles opposite to equal sides are equal] …(2)

$\angle OPQ + \angle TPQ = 90°$

$\angle TPQ = 90° - \angle OPQ$ …(3)

In ΔTPQ,

$\angle PTQ + \angle TPQ + \angle TQP = 180°$

$\Rightarrow$ $\angle PTQ + 2\angle TPQ = 180°$ [From (2)]

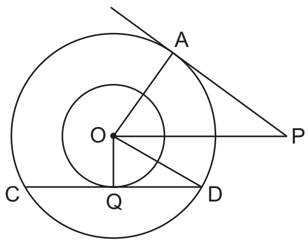
$\Rightarrow$ $\angle PTQ + 2(90° - \angle OPQ) = 180°$

$\Rightarrow$ $\angle PTQ + 180° - 2\angle OPQ = 180°$

$\Rightarrow$ $\angle PTQ = 2\angle OPQ$

Hence, proved.

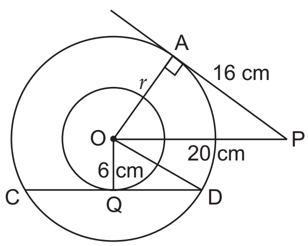
19. In two concentric circles, the radii are OA = r cm and OQ = 6 cm, as shown in the figure. Chord CD of larger circle is a tangent to smaller circle at Q. PA is tangent to larger circle. If PA = 16 cm and OP = 20 cm, find the length CD.



[CBSE 2024 Basic]

Sol. In ΔOAP

$\angle OAP = 90°$ [Radius is perpendicular to tangent at point of contact]

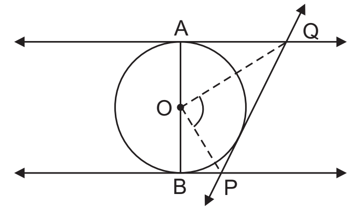


$$OA^2 + AP^2 = OP^2$$
$$\Rightarrow \quad OA^2 = (20\text{ cm})^2 - (16\text{ cm})^2$$
$$\Rightarrow \quad OA^2 = 400\text{ cm}^2 - 256\text{ cm}^2$$
$$\Rightarrow \quad OA^2 = 144\text{ cm}^2$$
$$\Rightarrow \quad OA = 12\text{ cm}$$

In ΔQOD

$$\angle OQD = 90°$$
$$OD = OA = 12\text{ cm}$$
$$OQ = 6\text{ cm}$$
$$OQ^2 + QD^2 = OD^2$$
$$\Rightarrow \quad QD^2 = OD^2 - OQ^2$$
$$\Rightarrow \quad QD^2 = 144\text{ cm}^2 - 36\text{ cm}^2$$
$$\Rightarrow \quad QD^2 = 108\text{ cm}^2$$
$$\Rightarrow \quad QD = 6\sqrt{3}\text{ cm}$$
$$QD = CQ$$
$$\therefore \quad CD = 2 \times 6\sqrt{3}\text{ cm}$$
$$\Rightarrow \quad CD = 12\sqrt{3}\text{ cm}$$

20. In the given figure, AB is a diameter of the circle with centre O. AQ, BP and PQ are tangents to the circle. Prove that $\angle POQ = 90°$.



[CBSE 2024 Standard]

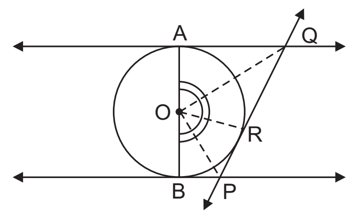
Sol. Join OR. [Construction]

In ΔBPO and ΔRPO,

BP = RP [Tangents from an external point to a circle are equal]

OB = OR [Radius of the circle]

OP = OP [Common]



$\therefore$ By SSS criterion of congruency,

$$\Delta BPO \cong \Delta RPO$$

$\therefore$ By CPCT,

$$\angle BOP = \angle ROP \quad \ldots(1)$$

In ΔROQ and ΔAOQ

AQ = RQ [Tangents from an external point to a circle are equal]

OQ = OQ [Common]

OA = OR [Radius of the circle]

$\therefore$ By SSS criterion of congruency,

$$\Delta ROQ \cong \Delta AOQ$$
$$\therefore \quad \angle AOQ = \angle ROQ \quad \ldots(2)$$
$$\therefore \quad \angle AOQ + \angle ROQ + \angle BOP + \angle ROP = 180°$$

[AB is a straight line]

$$2\angle ROQ + 2\angle ROP = 180° \quad \text{[From (1) and (2)]}$$
$$\Rightarrow \quad \angle ROQ + \angle ROP = 90°$$
$$\Rightarrow \quad \angle POQ = 90°$$

$[\angle POQ = \angle ROQ + \angle ROP]$

Hence, proved.

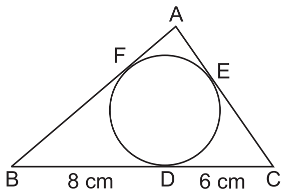
Long Answer Type Questions

21. ΔABC is drawn to circumscribe a circle of radius 4 cm such that segments BD and DC into which BC is divided by the point of contact are of

lengths 8 cm and 6 cm respectively. If the area of ΔABC is 84 cm^2, find the sides AB and AC.

[CBSE SP 2012]

Sol. Let a circle is inscribed in a ΔABC whose sides BC, CA and AB touch the circle at the points D, E and F respectively, such that BD = 8 cm, DC = 6 cm and ar (ΔABC) = 84 cm^2.



To find the lengths of the sides AB and AC.

Let BC = a cm, CA = b cm and AB = c cm.

Then $a = BC = BD + DC$

$= (8 + 6)$ cm

$= 14$ cm ...(1)

We know that the lengths of tangents, drawn from an external point to a circle are equal.

$\therefore$ BF = BD = 8 cm

$\therefore$ AF = AB – BF

$= (c - 8)$ cm

Also, CE = CD = 6 cm

$\therefore$ AE = AC – CE = $(b - 6)$ cm

Now, since AF = AE

$\therefore$ $c - 8 = b - 6$

$\Rightarrow$ $c = b + 2$ ...(2)

Now, by Heron's formula, we have

$$\text{ar}(\Delta ABC) = \sqrt{s(s-a)(s-b)(s-c)}$$

where $s = \frac{1}{2}(a + b + c)$

$\therefore$ $s = \frac{1}{2}(14 + b + 2 + b)$

[From (1) and (2)]

$\Rightarrow$ $s = 8 + b$ ...(3)

$\therefore$ $s - a = 8 + b - 14$

$= b - 6$

$s - b = 8 + b - b = 8$

$s - c = 8 + b - 2 - b = 6$

$\therefore$ $\text{ar}(\Delta ABC) = \sqrt{s(s-a)(s-b)(s-c)}$

$\Rightarrow$ $\text{ar}(\Delta ABC) = \sqrt{(8+b)(b-6)(8 \times 6)}$

$\Rightarrow$ $\text{ar}(\Delta ABC) = \sqrt{48(b^2 + 2b - 48)}$

$\Rightarrow$ $84 \times 84 = 48(b^2 + 2b - 48)$

[$\because$ ar(ΔABC) = 84 cm^2]

$\Rightarrow$ $b^2 + 2b - 48 = \frac{84 \times 84}{48} = 147$

$\Rightarrow$ $b^2 + 2b - 48 = 147$

$\Rightarrow$ $b^2 + 2b - 195 = 0$

$\Rightarrow$ $b^2 + 15b - 13b - 195 = 0$

$\Rightarrow$ $b(b + 15) - 13(b + 15) = 0$

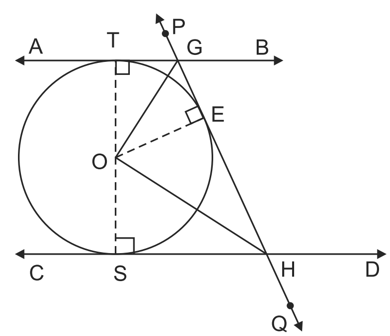
$\Rightarrow$ $(b + 15)(b - 13) = 0$

$\therefore$ $b = 13$ [$\because b \neq -15$]

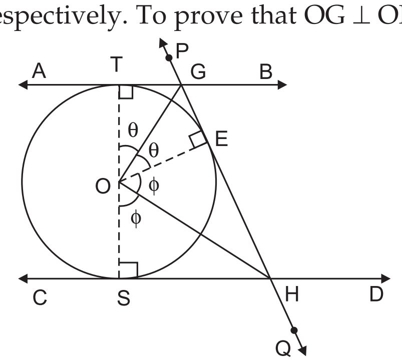
Hence, from (2), $c = 13 + 2 = 15$

Hence, the required lengths of AB and AC are 15 cm and 13 cm respectively.

22. In the given figure, AB and CD are two parallel tangents to a circle with centre O. Another tangent PQ touching the circle E intersects the tangents AB and CD at G and H respectively. Prove that OG $\perp$ OH.



Sol. TOS is a diameter of a circle with centre at O. AB and CD are two parallel tangents at T and S respectively. PQ is another tangent to the circle at E intersecting the tangents AB and CD at G and H respectively. To prove that OG $\perp$ OH.



We know that two tangents GT and GE of equal lengths subtend equal angle, say θ, at the centre O. Similarly, two tangents HE and HS of equal length subtend equal angle, say ϕ, at the centre O.

$\therefore$ $\angle TOG = \angle GOE = \theta$

and $\angle SOH = \angle HOE = \phi$

$\therefore \quad \angle GOE + \angle HOE = \theta + \phi$

$\because$ TS is a diameter of the circle,

$\therefore \quad \angle TOS = 180°$

$\Rightarrow \quad 2(\theta + \phi) = 180°$

$\Rightarrow \quad \theta + \phi = 90°$

$\therefore \quad \angle GOE + \angle HOE = \angle GOH = 90°$

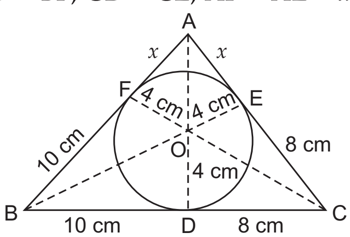
$\therefore \quad OG \perp OH$

Hence, proved.

23. A triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC are of lengths 10 cm and 8 cm respectively. Find the lengths of sides AB and AC, if it is given that $\Delta ABC = 90 \text{ cm}^2$. **[CBSE 2023 Standard]**

Sol. We know the lengths of tangents drawn from an external point a circle are equal,

$\therefore \quad BD = BF, CD = CE, AF = AE = x$ (say)



Area of ΔABC = Area of ΔBOC + Area of ΔAOC + Area of ΔAOB

$= \frac{1}{2} \times (10 + 8) \text{ cm} \times 4 \text{ cm} + \frac{1}{2} \times (10 + x) \text{ cm} \times 4 \text{ cm} + \frac{1}{2} \times (8 + x) \text{ cm} \times 4 \text{ cm}$

But area of $\Delta ABC = 90 \text{ cm}^2$

$$90 \text{ cm}^2 = \frac{1}{2} \times 18 \times 4 \text{ cm}^2 + \frac{1}{2} \times (10 + x) \times 4 \text{ cm}^2 + \frac{1}{2} \times (8 + x) \times 4 \text{ cm}^2$$

$$= \frac{1}{2} \times 4 \, [18 + 10 + x + 8 + x] \text{ cm}^2$$

$$\Rightarrow 18 + 10 + x + 8 + x = \frac{90 \times 2}{4}$$

$\Rightarrow \quad 36 + 2x = 45$

$\Rightarrow \quad 2x = 45 - 36$

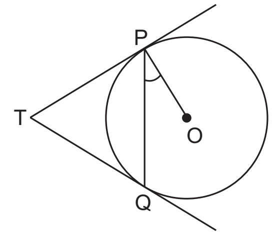
$\Rightarrow \quad 2x = 9$

$\Rightarrow \quad x = 4.5 \text{ cm}$

$\therefore \quad AB = 10 \text{ cm} + 4.5 \text{ cm} = 14.5 \text{ cm}$

$AC = 8 \text{ cm} + 4.5 \text{ cm} = 12.5 \text{ cm}$

24. Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that $\angle PTQ = 2 \angle OPQ$.



[CBSE 2023 Standard]

Sol. $TP = TQ$ [Lengths of tangents drawn from an external point to a circle are equal]

$\therefore \quad \angle TPQ = \angle TQP$ [Angles opposite to equal sides] ...(1)

$\angle OPT = 90°$

$\Rightarrow \quad \angle OPQ + \angle TPQ = 90°$

$\Rightarrow \quad \angle TPQ = 90° - \angle OPQ$ ...(2)

In ΔTPQ,

$\angle PTQ + \angle TPQ + \angle TQP = 180°$

From (1), we have

$\angle PTQ + \angle TPQ + \angle TPQ = 180°$

$\Rightarrow \quad \angle PTQ + 2\angle TPQ = 180°$

$\Rightarrow \quad \angle PTQ + 2(90° - \angle OPQ) = 180°$

$\Rightarrow \quad \angle PTQ + 180° - 2\angle OPQ = 180°$

$\Rightarrow \quad \angle PTQ = 2\angle OPQ$

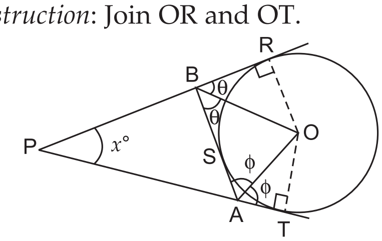
Higher Order Thinking Skills (HOTS) Questions

(Page 197)

1. Two tangents PT and PR are drawn from a point P outside a circle with centre O touching it at T and R respectively. A third tangent is drawn at a point S on the minor arc TR of the circle cutting PT and PR at A and B respectively such that OA and OB bisect $\angle TAB$ and $\angle RBA$ respectively. Show that

$\angle AOB = 90° - \frac{x°}{2}$, where $\angle RPT = x°$.

Sol. *Construction*: Join OR and OT.



$\therefore \quad \angle ORP = \angle OTP = 90°$

Let $\quad \angle OBR = \angle OBA = \theta$

and $\angle OAT = \angle OAB = \phi$

$\therefore$ $\angle ROB = 90° - \theta$

and $\angle AOT = 90° - \phi$

Now, in ΔAOB, we have

$\angle OBA + \angle OAB + \angle AOB = 180°$

[Angle sum property of a triangle]

$\Rightarrow$ $\theta + \phi + \angle AOB = 180°$

$\therefore$ $\angle AOB = 180° - (\theta + \phi)$ ...(1)

Now, since $\angle PRO + \angle PTO = 90° + 90° = 180°$,

$\therefore$ PTOR is a cyclic quadrilateral.

$\therefore$ $\angle ROT + \angle RPT = 180°$

$\Rightarrow$ $\angle AOB + \angle AOT + \angle BOR + \angle RPT = 180°$

$\Rightarrow$ $180° - (\theta + \phi) + 90° - \phi + 90° - \theta + x° = 180°$

$\Rightarrow$ $180° - 2(\theta + \phi) + x° = 0$

$\Rightarrow$ $\theta + \phi = 90° + \frac{x°}{2}$ ...(2)

$\therefore$ From (1) and (2), we get

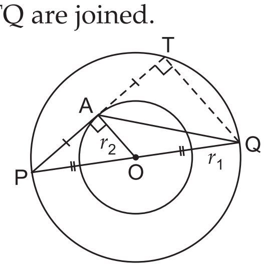
$$\angle AOB = 180° - 90° - \frac{x°}{2}$$

$$= 90° - \frac{x°}{2}$$

Hence, proved.

2. The radii of two concentric circles are r_1 cm and r_2 cm ($r_1 > r_2$). PQ is a diameter of the bigger circle and PA is a tangent to the smaller circle touching it at A. Show that the length of QA is $\sqrt{r_1^2 + 3r_2^2}$ cm.

Sol. Given that POQ is the diameter of the bigger of two concentric circles with common centre at O and radii r_1 cm and r_2 cm where $r_1 > r_2$. A and T are two points on the smaller and bigger circles respectively such that PAT is a tangent to the smaller circle and a chord of the bigger circle. AQ and TQ are joined.



To prove that QA = $\sqrt{r_1^2 + 3r_2^2}$ cm.

Since PQ is a diameter of the bigger circle,

$\therefore$ $\angle PTQ = 90°$

$\therefore$ In ΔATQ, by Pythagoras' theorem, we have

$AQ^2 = AT^2 + TQ^2$ ...(1)

Now, $OP = OQ = r_1$

Also, $OA \perp PT$

$\therefore$ $\angle PAO = \angle PTQ = 90°$

$\therefore$ $AO \parallel TQ$

$\therefore$ $PA : AT = PO : OQ = 1 : 1$

$\therefore$ A is the mid-point of PT and $AO = \frac{1}{2} TQ$

i.e. $TQ = 2AO = 2r_2$ ...(2)

and $AT = PA = \sqrt{OP^2 - r_2^2}$

[From right-angled triangle PAO]

$= \sqrt{r_1^2 - r_2^2}$ ...(3)

$\therefore$ From (1), $QA = \sqrt{AT^2 + TQ^2}$

$= \sqrt{r_1^2 - r_2^2 + 4r_2^2}$

[From (2) and (3)]

$\Rightarrow$ $QA = \sqrt{r_1^2 + 3r_2^2}$

Hence, the length of QA is $\sqrt{r_1^2 + 3r_2^2}$ cm.

Hence, proved.

Self-Assessment

(Page 197)

Multiple-Choice Questions

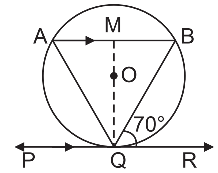
1. PQR is a tangent to a circle at Q, whose centre is at O. AB is a chord parallel to PR and $\angle BQR = 70°$, then $\angle AQB$ is equal to

(*a*) 20° (*b*) 40°

(*c*) 35° (*d*) 45° **[CBSE SP 2012]**

Sol. (*b*) 40°

Given that PQR is a tangent to the circle with centre at O.



OQ is joined and OM is drawn perpendicular to the chord AB which is parallel to the tangent PQR. Given that $\angle BQR = 70°$. To find $\angle AQB$.

Since OQ is a radius of the circle and PQR is a tangent at Q,

$\angle MQR = 90°$

$\therefore$ $\angle MQB = 90° - \angle BQR$

$= 90° - 70°$

$= 20°$

Now, since QOM ⊥ AB,

∴ M is the mid-point of AB.

∴ In ΔQMA and QMB, we have

AM = BM [M is the mid-point of AB]

MQ = MQ [Common]

∠QMA = ∠QMB [Each equal to 90°]

∴ ΔQMA ≅ ΔQMB [By SAS similarity criterion]

∴ ∠MQA = ∠MQB [By CPCT]

$= 20°$

∴ ∠AQB = 2∠MQA

$= 2 \times 20° = 40°$

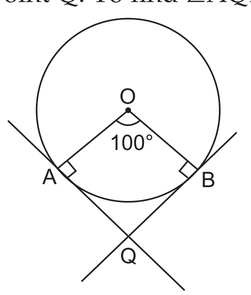
2. If the angle between the radii of a circle is 100°, then the angle between the tangents at the ends of those two radii is

(*a*) 50° (*b*) 60°

(*c*) 80° (*d*) 90° **[CBSE 2012]**

Sol. (*c*) 80°

Given that A and B are two points on a circle with centre at O such that ∠AOB = 100°. AQ and BQ are tangents to the circle at A and B, meeting each other at a point Q. To find ∠AQB.



Since AQ and BQ are tangents to the circle at A and B respectively and OA and OB are the radii of the circle,

∴ ∠OAQ = ∠OBQ = 90°

∴ ∠OAQ + ∠OBQ = 90° + 90° = 180°

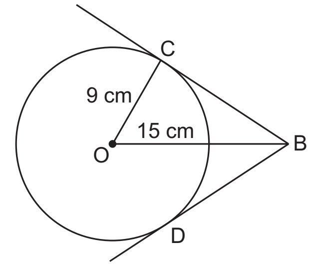
Hence, two opposite angles of the quadrilateral AQBO are supplementary. Hence, the quadrilateral AQBO is cyclic.

∴ ∠AQB + ∠AOB = 180°

⇒ ∠AQB + 100° = 180°

⇒ ∠AQB = 180° – 100° = 80°

3. In the given figure, BC and BD are tangents to the circle with centre O and radius 9 cm. If OB = 15 cm, then the length (BC + BD) is



(*a*) 18 cm (*b*) 12 cm

(*c*) 24 cm (*d*) 36 cm

[CBSE 2023 Basic]

Sol. (*c*) 24 cm

In ΔOBC, ∠BCO = 90°

$OB^2 = OC^2 + BC^2$

⇒ $BC^2 = OB^2 - OC^2$

$BC^2 = (15 \text{ cm})^2 - (9 \text{ cm})^2$

$BC^2 = 225 \text{ cm}^2 - 81 \text{ cm}^2$

⇒ $BC^2 = 144 \text{ cm}^2$

⇒ BC = 12 cm

But BC = BD

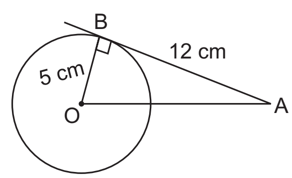
∴ BC + BD = 12 cm + 12 cm = 24 cm

4. The length of the tangent from an external point A to a circle, of radius 5 cm, is 12 cm. The distance of A from the centre of the circle is

(*a*) 17 cm (*b*) 13 cm

(*c*) $\sqrt{13}$ cm (*d*) 169 cm

Sol. (*b*) 13 cm



AB = 12 cm [Given]

OB = 5 cm [Given]

In ΔABO,

$OA^2 = AB^2 + OB^2$

$OA = \sqrt{AB^2 + OB^2}$

$= \sqrt{(12 \text{ cm})^2 + (5 \text{ cm})^2}$

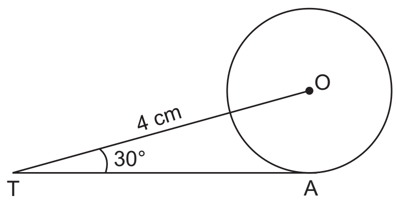
$= \sqrt{144 + 25}$ cm

$= \sqrt{169}$ cm

⇒ OA = 13 cm

$\therefore$ The distance of point A from the centre of the circle is 13 cm.

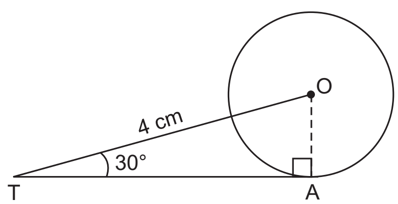
5. In the given figure, TA is a tangent to the circle with centre O such that OT = 4 cm, $\angle OTA = 30°$, then the length of TA is



(*a*) $2\sqrt{3}$ cm (*b*) 2 cm
(*c*) $2\sqrt{2}$ cm (*d*) $\sqrt{3}$ cm

[CBSE 2023 Standard]

Sol. (*a*) $2\sqrt{3}$ cm



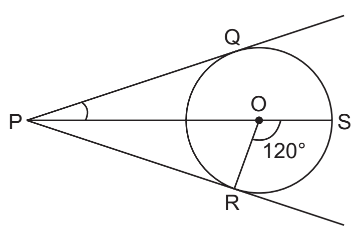
$$\cos 30° = \frac{TA}{OT}$$

$$\Rightarrow \quad TA = OT \cos 30°$$

$$= 4 \text{ cm} \times \frac{\sqrt{3}}{2}$$

$$\Rightarrow \quad TA = 2\sqrt{3} \text{ cm}$$

6. In the given figure, PQ and PR are two tangents to a circle with centre O. If $\angle ROS = 120°$, the $\angle QPO$ is equal to



(*a*) 30° (*b*) 45°
(*c*) 60° (*d*) 90°

Sol. (*a*) 30°

$$\angle POR = 180° - 120° = 60°$$

$$\angle PRO = 90°$$

In ΔPOR,

$$\angle POR + \angle PRO + \angle OPR = 180°$$

$$60° + 90° + \angle OPR = 180°$$

$$\Rightarrow \quad \angle OPR = 180° - 150°$$

$$\Rightarrow \quad \angle OPR = 30°$$

Since, $\angle OPR = \angle QPO$

$$\therefore \quad \angle QPO = 30°$$

Fill in the Blanks

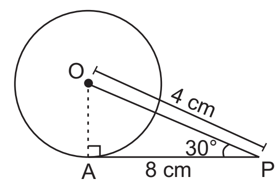
7. The distance between two parallel tangents of a circle of radius 3 cm is **6 cm.**

Sol. Tangents at the end of a diameter of a circle are parallel. So, the distance between them is equal to the diameter or 2*r*.

Hence, distance = 2 × 3 cm = 6 cm

8. AP is a tangent to the circle with centre O such that OP = 4 cm and $\angle OPA = 30°$. Then, AP is equal to **$2\sqrt{3}$ cm.**

Sol. $\because$ The tangent to a circle at any point is perpendicular to the radius through the point of contact



$$\therefore \quad OA \perp AP$$

$$\Rightarrow \quad \angle OAP = 90°$$

In right ΔOAP, we have

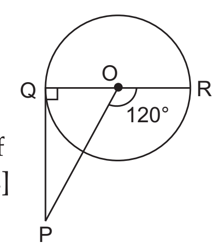
$$\cos 30° = \frac{AP}{OP}$$

$$\Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{AP}{4 \text{ cm}}$$

$$\Rightarrow \quad AP = 2\sqrt{3} \text{ cm}$$

9. PQ is a tangent drawn from a point P to a circle with centre O and QOR is a diameter of the circle such that $\angle POR = 120°$, then $\angle OPQ$ is **30°**.

Sol.



$$OQ \perp QP$$

$$\Rightarrow \quad \angle OQP = 90°$$

$$\angle OQP + \angle OPQ = 120°$$

[Exterior angle = Sum of interior opposite angles]

$$\Rightarrow \quad 90° + \angle OPQ = 120°$$

$$\Rightarrow \quad \angle OPQ = 120° - 90°$$

$$\Rightarrow \quad \angle OPQ = 30°$$

10. The maximum number of common tangents that can be drawn to two circles intersecting at two distinct points is **2**.

Assertion-Reason Type Questions

Directions (Q. Nos. 11 to 16): Each of these questions contains an assertion followed by reason. Read them carefully, and answer the question on the basis of the following options, select the one that best describes the two statements.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

11. Assertion (A): From an external point P, two tangents are drawn to a circle with points of contact A and B, then PA = 2 PB.

Reason (R): The lengths of tangents drawn from an external point to a circle are equal.

Sol. The correct answer is (*d*).

The lengths of tangents drawn from an external point to a circle are equal.

Thus, PA = PB.

∴ Assertion is wrong but reason is correct.

12. Assertion (A): Diameter of a circle represents the largest secant inside the circle.

Reason (R): Two ends of a diameter are the farthest apart pair of points in a circle.

Sol. The correct answer is (*a*).

Two ends of a diameter are the farthest apart pair of points in a circle. Thus diameter represents the largest possible secant inside a circle.

Thus reason is the correct explanation of the assertion.

13. Assertion (A): A normal to a circle is perpendicular to the tangent at that point.

Reason (R): Normal is the line containing the radius through the point of contact of a tangent in a circle.

Sol. The correct answer is (*a*).

The line containing the radius through the point of contact is also sometimes called the normal to the circle at the point.

Thus reason is a correct explanation of the assertion.

14. Assertion (A): A tangent to a circle is perpendicular to the radius through the point of contact.

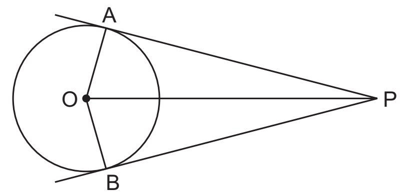
Reason (R): The lengths of tangents drawn from the external point to a circle are equal.

[CBSE 2023 Basic]

Sol. The correct answer is (*b*).

Both assertion and reason are true but reason is not the correct explanation of assertion.

15.



Assertion (A): If PA and PB are tangents drawn to a circle with centre O from an external point P, then the quadrilateral OAPB is a cyclic quadrilateral.

Reason (R): In a cyclic quadrilateral, opposite angles are equal.

Sol. The correct answer is (*c*).

Quadrilateral OAPB is a cyclic quadrilateral, because $\angle OAP + \angle OBP = 180°$

$\angle AOB + \angle APB = 180°$

The sum of opposite angles is supplementary, i.e. 180°.

∴ Assertion is true but reason is false.

16. Assertion (A): The tangents drawn at the end points of a diameter of a circle, are parallel.

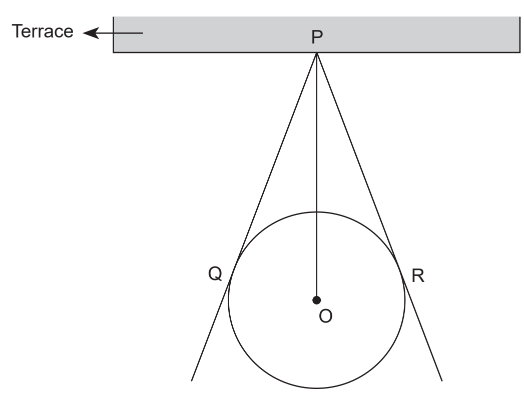
Reason (R): Diameter of a circle is the longest chord. **[CBSE 2024 Standard]**

Sol. The correct answer is (*b*).

Both assertion and reason are true but reason is not the correct explanation of assertion.

Case Study Based Questions

17. Rain water harvesting is the collection and storage of rain water, rather than allowing it to run off. To save rain water, Sushant presented a project for rain water harvesting. Diagrammatic representation of the project is given below.



Through pipes PQ and PR, each of 15 m long are bringing water from the terrace of a building (as shown in the figure). Based on the above information, answer the following questions.

(*a*) If the radius of circle is 8 m, then find the length of OP.

Ans. 17 m

(*b*) If two circles intersect each other at two points, then what will be the number of common tangents?

Ans. 2

(*c*) (*i*) If a line OP makes an angle 60° from one of the tangent line, then find the length of OP.

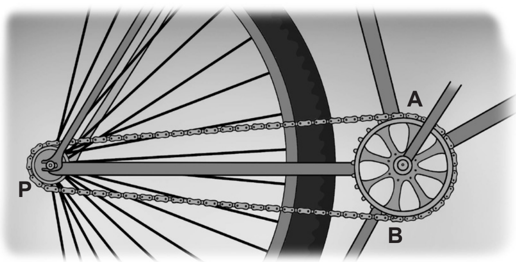
Ans. 30 m

or

(*ii*) Find the length QR.

Ans. $15\sqrt{3}$ m

18. The chain and gears of bicycles or motorcycles or belt around two pulleys are some real-life examples of tangents and circles.



In the given figure, the big gear represents the circle while the smaller one represents the exterior point P of the intersection of tangent lines. PA and PB are two tangents intersecting outside the circle at the point P.

Based on the above situation, answer the following questions.

(*a*) If PB is equal to 16 inches, then find the measure of PA.

Ans. 16 inches.

(*b*) If a point is inside the circle, how many tangents can be drawn from that point?

Ans. 0

(*c*) (*i*) If the angle made by the tangents PA and PB at the centre O of the circle is 120°, then find the measure of ∠APB.

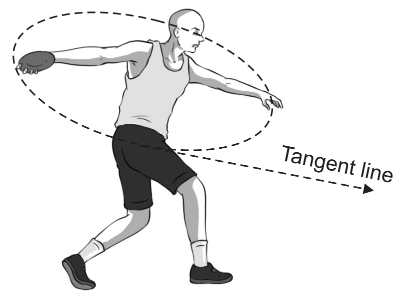
Ans. 60°

or

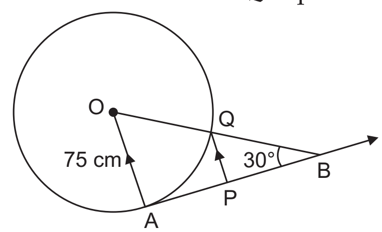
(*ii*) If PA is equal to 12 inches and radius of bigger circle is 5 inches, then find the distance of P from the centre O of larger circle.

Ans. 13 inches.

19. The discus throw is an event in which an athlete attempts to throw a discus. The athlete spins anticlockwise around one and a half times through a circle, then releases the throw. When released, the discus travels along tangent to the circular spin orbit.



In the given figure, AB is one such tangent to a circle of radius 75 cm. Point O is centre of the circle and ∠ABO = 30°. PQ is parallel to OA.



Based on above information.

(*a*) Find the length of AB.

Ans. In ΔOAB,

$$\tan 30° = \frac{OA}{AB}$$

$$\Rightarrow \quad AB = \frac{OA}{\tan 30°} = 75\sqrt{3} \text{ cm}$$

(*b*) Find the length of OB.

Ans. In ΔOAB,

$$\sin 30° = \frac{OA}{OB}$$

$$\Rightarrow \quad OB = \frac{OA}{\sin 30°}$$

$$= 75 \times 2 = 150 \text{ cm}$$

(*c*) (*i*) Find the length of AP.

Ans. In ΔOAB and ΔQPB,

OA || QP

∴ ∠BQP = ∠BOA [Corresponding angles]

∠B = ∠B [Common]

∴ By AA criterion of similarity,

ΔOAB ~ ΔQPB

$$\therefore \quad \frac{BO}{BQ} = \frac{AO}{PQ} = \frac{BA}{BP}$$

$$\Rightarrow \quad \frac{BO}{BO - OQ} = \frac{BA}{BA - AP}$$

$$\Rightarrow \quad \frac{150}{150 - 75} = \frac{75\sqrt{3}}{75\sqrt{3} - AP} \quad [\because \ OQ = OA]$$

$$\Rightarrow \quad \frac{150}{75} = \frac{75\sqrt{3}}{75\sqrt{3} - AP}$$

$$\Rightarrow \quad 75\sqrt{3} - AP = \frac{75\sqrt{3}}{2}$$

$$\Rightarrow \quad AP = \frac{75\sqrt{3}}{2} \text{ cm}$$

or

(*ii*) Find the length of PQ.

[CBSE 2023 Standard]

Ans. Since ΔOAB ~ ΔQPB

$$\therefore \quad \frac{BO}{BQ} = \frac{AO}{PQ}$$

$$\Rightarrow \quad \frac{BO}{BO - OQ} = \frac{75}{PQ}$$

$$\Rightarrow \quad \frac{150}{150 - 75} = \frac{75}{PQ} \quad [\because \ OQ = OA]$$

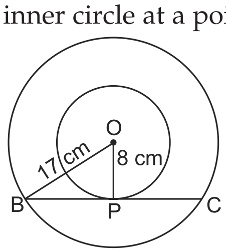
$$\Rightarrow \quad \frac{150}{75} = \frac{75}{PQ}$$

$$\Rightarrow \quad PQ = \frac{75}{2}$$

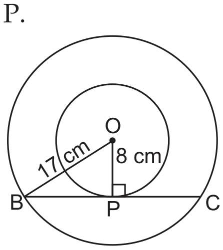
$$\therefore \quad PQ = 37.5 \text{ cm}$$

Very Short Answer Type Questions

20. Two concentric circles of radii 17 cm and 8 cm are given. What is the length of the chord BC which touches the inner circle at a point P?



Sol. Given that B and C are two points on a circle with centre at O and radius 17 cm such that the chord BC of this circle is a tangent to a smaller concentric circle of centre at O and radius 8 cm at the point P.



To find the length of the chord BC.

Since BPC is a tangent to the smaller circle at P,

$\therefore \quad \angle OPB = 90°$

Also, OP = 8 cm and OB = 17 cm.

∴ In ΔOBP, by using Pythagoras' theorem, we have

$$OB^2 = OP^2 + BP^2$$

$$\Rightarrow \quad 17^2 = 8^2 + BP^2$$

$$\Rightarrow \quad BP = \sqrt{17^2 - 8^2}$$

$$= \sqrt{(17+8)(17-8)}$$

$$= \sqrt{25 \times 9}$$

$$= 5 \times 3 = 15$$

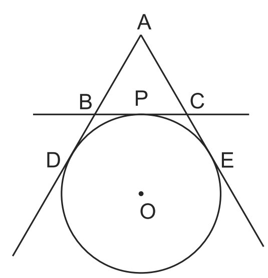
Now, since BC is a chord of the bigger circle and OP ⊥ BC,

∴ P is the mid-point of BC.

∴ BC = 2 BP = 2 × 15 = 30

Hence, the required length of the chord BC is 30 cm.

21. In the given figure, the side BC of ΔABC touches the circle with centre O, at the point P. If AB and AC produced touch the same circle at D and E respectively and if AD = 10.5 cm, then what is the perimeter of ΔABC?



Sol. Given that ABC is a triangle. Also, AB and AC produced touch a circle with centre at O at the points D and E respectively. Let BC touch the circle at P. It is given that AD = 10.5 cm. To find the perimeter of ΔABC.

Since A is an external point to the circle, Also, AD and AE are two tangents from A,

$\therefore \quad AD = AE \quad \ldots(1)$

Now, AB + AC = AD – BD + AE – CE

= AD – BP + AD – CP

[∵ BD = BP and CE = CP]

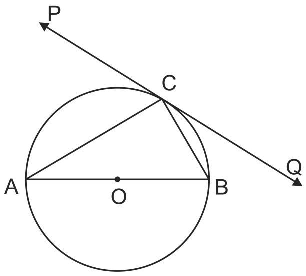
= 2 AD – (BP + CP)

= 2 × 10.5 – BC

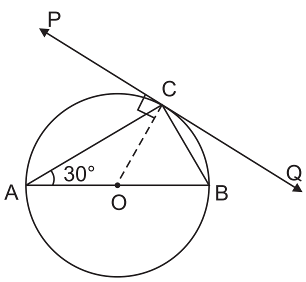
⇒ AB + AC + BC = 21

Hence, the required perimeter is 21 cm.

22. In the given figure, PQ is a tangent at a point C to a circle with centre O. If AB is a diameter and $\angle CAB = 30°$, find $\angle PCA$. **[CBSE 2016]**



Sol. Given that AOB is a diameter of a circle with centre at O and C is a point on the circle such that $\angle CAB = 30°$.



PCQ is a tangent to the circle at C.

To find $\angle PCA$.

Construction: Join OC.

Since OC is a radius of the circle and PCQ is a tangent at C,

$\therefore \quad \angle OCP = 90°$

Since AB is a diameter of the circle,

$\therefore \quad \angle ACB = 90°$

$\therefore \quad \angle PCA = \angle OCP - \angle OCA$

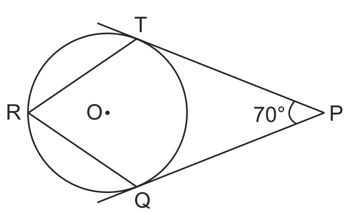
$= 90° - 30°$

[$\because$ OA = OC = radius of the same circle, $\therefore \angle OCA = \angle OAC = 30°$]

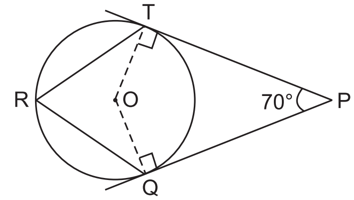
$= 60°$

Hence, the required angle is 60°.

23. In the given figure, O is the centre of the circle. PT and PQ are tangents to the circle from an external point P. If $\angle TPQ = 70°$, find $\angle TRQ$. **[CBSE 2015]**



Sol. *Construction*: Join OT and OQ. To find $\angle TRQ$.



Since OT, OQ are the radii of the same circle and TP, PQ are tangents to the circle, hence, we have

$\angle OTP = \angle OQP = 90°$

$\therefore \quad \angle OTP + \angle OQP = 90° + 90° = 180°$

$\therefore$ TPQO is cyclic quadrilateral.

$\therefore \quad \angle TOQ + \angle TPQ = 180°$

$\Rightarrow \quad \angle TOQ = 180° - \angle TPQ$

$= 180° - 70°$

$= 110°$

But $\quad \angle TOQ = 2\angle TRQ$

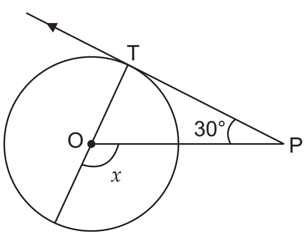
[Angle subtended by the arc TQ at O]

$\Rightarrow \quad 110° = 2\angle TRQ$

$\Rightarrow \quad \angle TRQ = \frac{110°}{2} = 55°$

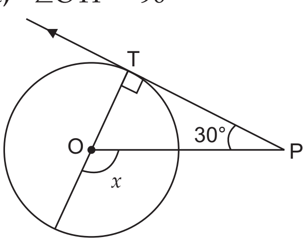
$\therefore$ The required angle is 55°.

24. In the given figure, PT is a tangent at T to the circle with centre O. If $\angle TPO = 30°$, find the value of x.



[CBSE 2023 Basic]

Sol. We have, $\angle OTP = 90°$



In ΔPTO,

$\angle OTP + \angle TOP + \angle TPO = 180°$

$\Rightarrow \quad 90° + \angle TOP + 30° = 180°$

$\Rightarrow \quad \angle TOP = 180° - 120° = 60°$

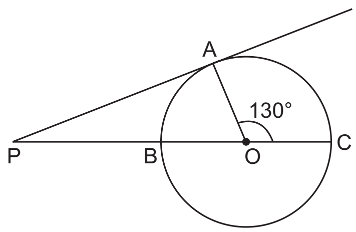
$\angle TOP + x = 180°$ [TO is a straight line]

$\Rightarrow \quad x = 180° - \angle TOP$

$\Rightarrow \quad x = 180° - 60° = 120°$

$\therefore \quad x = 120°$

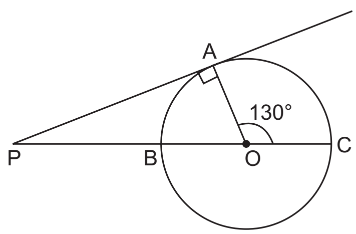
25. In the given figure, PA is a tangent to the circle drawn from the external point P and PBC is a secant to the circle with BC as diameter. If $\angle AOC = 130°$, then find the measure of $\angle APB$ where O is the centre of the circle.



[CBSE 2023 Standard]

Sol. $\angle AOP = 180° - 130° = 50°$

$\angle OAP = 90°$



In ΔAPQ,

$$\angle AOP + \angle APB + \angle OAP = 180°$$

$$\Rightarrow \quad 50° + \angle APB + 90° = 180°$$

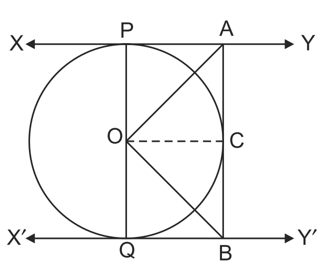
$$\Rightarrow \quad \angle APB = 180° - 90° - 50°$$

$$\therefore \quad \angle APB = 40°$$

Short Answer Type Questions

26. In the given figure, XY and X′ Y′ are two parallel tangents to a circle with centre O and another tangent AB, with point of contact C intersects XY at A and X′Y′ at B. Prove that $\angle AOB = 90°$.

[CBSE SP 2012, 2013, 2016, 2017]



Sol. Given that XY and X′Y′ are two parallel tangents to a circle with centre at O. Clearly, POQ is a diameter of the circle, C is another point on the circle such that ACB is a tangent to the circle where A and B are points on XY and X′Y′ respectively.

To prove that $\angle AOB = 90°$.

We have AP = AC and BQ = BC. Hence, AP and AC will subtend equal angles at O. Similarly, BQ and BC will subtend another pair of equal angles at O.

Let $\angle AOP = \angle AOC = \theta$ and $\angle BOQ = \angle BOC = \phi$.

$$\therefore \quad \angle POC = \angle AOP + \angle AOC = 2\theta$$

$$\text{and} \quad \angle COQ = \angle BOQ + \angle BOC = 2\phi$$

$$\therefore \quad \angle POC + \angle COQ = 2(\theta + \phi)$$

$$\Rightarrow \quad \angle POQ = 2(\theta + \phi)$$

$$\Rightarrow \quad 180° = 2(\theta + \phi)$$

[$\because$ PQ is a diameter]

$$\Rightarrow \quad \theta + \phi = \frac{180°}{2} = 90° \qquad \ldots(1)$$

$$\text{Now,} \quad \angle AOB = \angle AOC + \angle COB = \theta + \phi = 90° \qquad \text{[From (1)]}$$

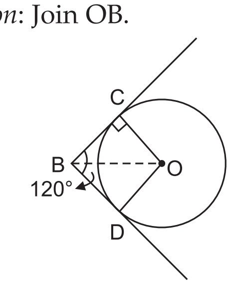
Hence, proved.

27. From an external point B of a circle with centre at O, two tangents BC and BD are drawn to the circle. If $\angle DBC = 120°$, prove that OB = 2BC.

Sol. Given that B is an external point to a circle with centre at O and two tangents BC and BD touching the circle at C and D respectively such that $\angle DBC = 120°$.

To prove that OB = 2BC.

Construction: Join OB.



We know that OB is the bisector of $\angle DBC$.

$$\therefore \quad \angle OBC = \frac{120°}{2} = 60°$$

Now, in ΔOBC,

since $\angle OCB = 90°$,

$$\therefore \quad \cos \angle OBC = \frac{BC}{OB}$$

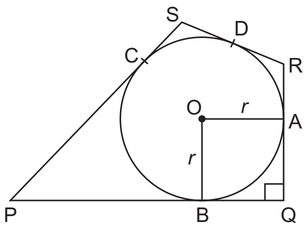
$$\Rightarrow \quad \cos 60° = \frac{BC}{OB}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{BC}{OB}$$

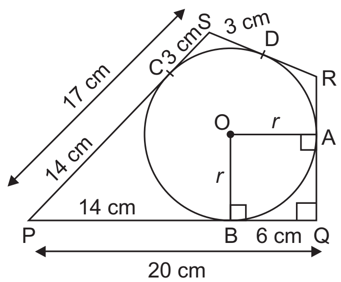
$$\Rightarrow \quad OB = 2BC$$

Hence, proved.

28. In the given figure, a circle is inscribed in a quadrilateral PQRS in which ∠Q = 90°. If PS = 17 cm, PQ = 20 cm and DS = 3 cm, then find the radius of the circle.



Sol.



DS = 3 cm

∴ CS = 3 cm

PS = PC + CS

⇒ PC = PS – CS

= 17 cm – 3 cm

⇒ PC = 14 cm

∴ PB = 14 cm

PQ = PB + BQ

⇒ BQ = PQ – PB

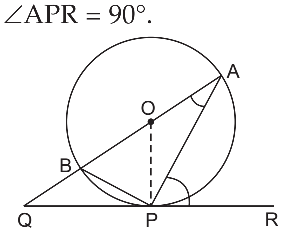
= 20 cm – 14 cm

⇒ BQ = 6 cm

∴ OA = 6 cm [∵ BQ = OA]

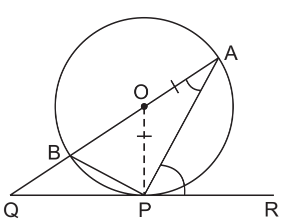
Radius of the circle = 6 cm

29. In the given figure, O is the centre of the circle and QPR is a tangent to it at P. Prove that ∠QAP + ∠APR = 90°.



[CBSE 2023 Standard]

Sol. In ΔOPA,



∠QAP = ∠OPA ...(1)

[Angles opposite to equal sides are equal]

∠OPR = 90° [The tangent is perpendicular to the radius through the point of contact]

∠OPR = ∠OPA + ∠APR

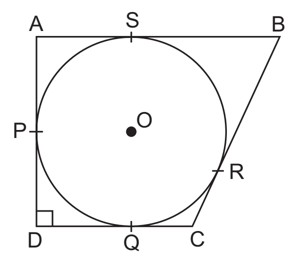
∠OPA + ∠APR = 90°

∠QAP + ∠APR = 90° [From (1)]

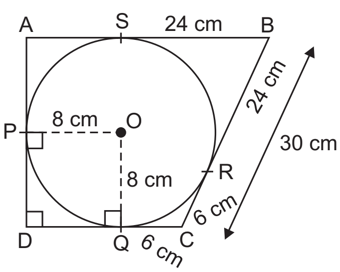
Hence, proved.

30. A circle with centre O and radius 8 cm is inscribed in a quadrilateral ABCD in which P, Q, R and S are the point of contact as shown. If AD is perpendicular to DC, BC = 30 cm and BS = 24 cm, then find the length DC.

[CBSE 2024 Standard]



Sol.



BS = 24 cm [Given]

∴ BR = 24 cm

CR = BC – BR

⇒ CR = 30 cm – 24 cm = 6 cm

∴ QC = 6 cm

Quadrilateral OQDP is a square

∴ DQ = 8 cm

DC = DQ + QC

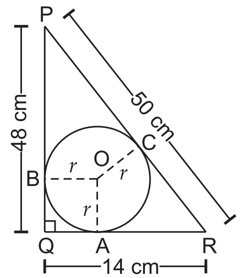
= 8 cm + 6 cm

∴ DC = 14 cm

Long Answer Type Questions

31. PQR is a right triangle with PQ = 48 cm, QR = 14 cm and PR = 50 cm. A circle with centre at O and radius r is inscribed in ΔPQR. Find the value of r.

Sol. Given that PQR is a right-angled triangle with $\angle PQR = 90°$. A circle with centre at O is inscribed within this triangle touching the sides QR, PQ and PR at the points A, B and C respectively. It is given that PQ = 48 cm, QR = 14 cm and PR = 50 cm. Let r cm be the radius of the circle. To find the value of r.



Construction: Join OA, OB and OC.

We know that the length of tangents drawn from an external point to a circle are equal.

Since, OA = OB = r and OB ⊥ PQ, hence OBQA is a square of side r cm.

$\therefore$ QA = QB = r

$\therefore$ PB = $(48 - r)$ cm = PC

$\Rightarrow$ PC = $(48 - r)$ cm ...(1)

and AR = $(14 - r)$ cm = CR

$\Rightarrow$ CR = $(14 - r)$ cm ...(2)

Now, PC + CR = PR = 50 cm

$\Rightarrow$ $(48 - r)$ cm + $(14 - r)$ cm = 50 cm [From (1) and (2)]

$\Rightarrow$ 62 cm − $2r$ = 50 cm

$\Rightarrow$ $2r$ = 12 cm

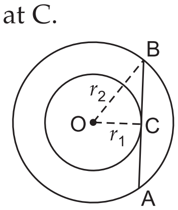
$\Rightarrow$ $r = \frac{12}{2}$ cm

$\Rightarrow$ r = 6 cm

Hence, the required length of r is 6 cm.

32. Two concentric circles have radii $a^2 + b^2$ and $a^2 - b^2$. Show that the length of the chord of the larger circle which touches the smaller circle is $4ab$.

Sol. Let r_1 cm and r_2 cm where $r_2 > r_1$ be the radii of two concentric circles with common centre O and let C be a point on the smaller circle such that the chord AB of the bigger circle is a tangent to the smaller circle at C.



It is given that $r_1 = a^2 - b^2$ and $r_2 = a^2 + b^2$.

To prove that AB = $4ab$.

Construction: Join OC and OB.

We have, $\angle OCB = 90°$, OB = $r_2 = a^2 + b^2$ and OC = $r_1 = a^2 - b^2$

$\therefore$ In ΔOCB, by using Pythagoras' theorem, we have

$$BC^2 + OC^2 = OB^2$$

$$\Rightarrow \quad BC^2 + r_1^2 = r_2^2$$

$$\Rightarrow \quad BC = \sqrt{r_2^2 - r_1^2}$$

$$= \sqrt{\left(a^2 + b^2\right)^2 - \left(a^2 - b^2\right)^2}$$

$$= \sqrt{4a^2b^2} = 2ab$$

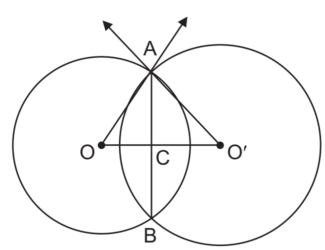
$$\therefore \quad AB = 2BC$$

$$= 2 \times 2ab$$

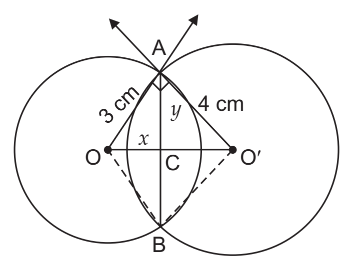
$$= 4ab$$

Hence, proved.

33. Two circles with centres O and O′ of radii 3 cm and 4 cm, respectively intersect at two points A and B. OA and O′A are tangents to the two circles, as shown in the figure. Find the length of the chord AB.



Sol.



In $\Delta OO'A$

$$OO'^2 = AO^2 + AO'^2$$

$$= (3 \text{ cm})^2 + (4 \text{ cm})^2$$

$$= 9 \text{ cm}^2 + 16 \text{ cm}^2$$

$$= 25 \text{ cm}^2$$

$$\Rightarrow \quad OO' = \sqrt{25} \text{ cm}$$

$$\Rightarrow \quad OO' = 5 \text{ cm} \qquad \text{...(1)}$$

We know that line joining the centres of two intersecting circles is perpendicular bisector of their common chord,

$\therefore$ OC $\perp$ AB and AC = BC

Let OC = x and AC = y

In ΔAOC,

$$OC^2 + AC^2 = AO^2$$

$$x^2 + y^2 = (3)^2 = 9 \quad \ldots(2)$$

In ΔACO′,

$$CO' = 5 - x$$
$$AC = y$$
$$AO' = 4 \text{ cm}$$
$$(CO')^2 + (AC)^2 = (4)^2$$

$\Rightarrow$ $(5 - x)^2 + y^2 = 16$

$\Rightarrow$ $25 + x^2 - 10x + y^2 = 16$

$\Rightarrow$ $25 + 9 - 10x = 16$

[$x^2 + y^2 = 9$ from (2)]

$\Rightarrow$ $-10x = 16 - 34$

$\Rightarrow$ $-10x = -18$

$\Rightarrow$ $x = 1.8$ cm

Putting value of $x = 1.8$ cm in (2), we get

$$(1.8)^2 + y^2 = 9$$

$\Rightarrow$ $3.24 + y^2 = 9$

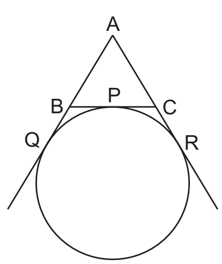
$\Rightarrow$ $y^2 = 9 - 3.24 = 5.76$

$\Rightarrow$ $y = \sqrt{5.76}$

$\therefore$ $y = 2.4$ cm

$\therefore$ $AB = 2y = 2 \times 2.4 \text{ cm} = 4.8 \text{ cm}$

34. A circle touches the side BC of a ΔABC at a point P and touches AB and AC when produced at Q and R respectively. Show that AQ = $\frac{1}{2}$ (Perimeter of ΔABC).



[CBSE 2023 Standard]

Sol. Perimeter of ΔABC = AB + BC + CA

= AB + BP + PC + CA

= AB + BQ + CR + CA

[Length of tangents from an external point to a circle are equal, $\therefore$ BP = BQ and PC = CR]

= AQ + AR

= 2AQ [Length of tangents from an external point are equal]

$\therefore$ AQ = $\frac{1}{2}$ (Perimeter of ΔABC)

Let's Compete

(Page 201)

Multiple-Choice Questions

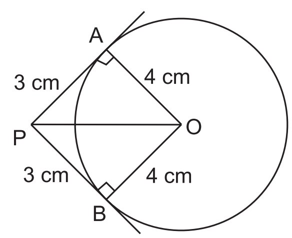
1. The area of the quadrilateral formed by two tangents PA and PB each of length 3 cm, drawn from an external point P to a circle with centre at O and two radii, each of length 4 cm, is

(*a*) 6 cm^2 (*b*) 10 cm^2

(*c*) 15 cm^2 (*d*) 12 cm^2

Sol. (*d*) 12 cm^2

Let P be an external point to a circle with centre at O and radius 4 cm. A and B are two points on the circle such that PA and PB are two tangents to the circle at A and B respectively such that PA = PB = 3 cm.



To find the area of the quadrilateral PAOB.

Construction: Join PO, OA and OB.

We see that

ΔPAO $\cong$ ΔPBO, by SSS congruence property, since PA = PB, OA = OB and OP is common.

$\therefore$ ar(ΔPAO) = ar(ΔPBO)

$= \frac{1}{2} \times AP \times AO$

$= \frac{1}{2} \times 3 \times 4 \text{ cm}^2$

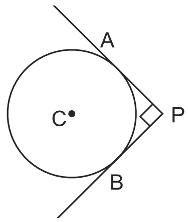
$= 6 \text{ cm}^2$

$\therefore$ Area of quadrilateral APBO

$= 2 \times 6 \text{ cm}^2$

$= 12 \text{ cm}^2$

2. In the given figure, PA and PB are two tangents drawn from an external point to a circle with centre C, and radius 4 cm. If PA $\perp$ PB, then the length of each tangent is **[CBSE SP 2013]**



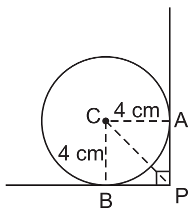
(*a*) 3 cm (*b*) 4 cm
(*c*) 5 cm (*d*) 6 cm

Sol. (*b*) 4 cm

Given that PA and PB are two tangents drawn from an external point P to a circle with centre at C such that ∠APB = 90°.

To find the length of PA or PB.

Construction: Join CA, CB and CP.



We have

$$\angle CAP = 90^\circ = \angle APB$$

Also, $\angle CBP = 90^\circ = \angle APB$

Hence, APBC is a square.

$\therefore$ PA = PB = AC = 4 cm

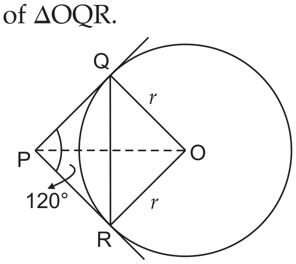
$\therefore$ The required length of each tangent is 4 cm.

3. If PQ and PR are tangents drawn from an external point P to the circle with centre O and radius *r* cm at the points of contact Q and R such that ∠QPR = 120°, then the area of the triangle OQR is

(*a*) $\frac{\sqrt{3}}{4}r^2$ cm² (*b*) $\frac{\sqrt{3}}{2}r^2$ cm²
(*c*) $\sqrt{3}r^2$ cm² (*d*) $\frac{4r^2}{\sqrt{3}}$ cm²

Sol. (*a*) $\frac{\sqrt{3}}{4}r^2$ cm²

Given that PQ and PR are two tangents drawn from an external point P to a circle with centre at O such that ∠QPR = 120°. QR, OQ and OR are joined. Let *r* cm be the radius of the circle. To find the area of ΔOQR.



Since $\angle OQP = \angle ORP = 90^\circ$,

$\therefore \angle OQP + \angle ORP = 90^\circ + 90^\circ = 180^\circ$

$\therefore$ The quadrilateral PQOR is cyclic.

$\therefore \angle QPR + \angle QOR = 180^\circ$

$\Rightarrow 120^\circ + \angle QOR = 180^\circ$

$\Rightarrow \angle QOR = 180^\circ - 120^\circ = 60^\circ$

Now, in ΔOQR, OQ = OR = *r* cm

$\therefore \angle OQR = \angle ORQ$

$\therefore \angle OQR + \angle ORQ + \angle QOR = 180^\circ$

[Angle sum property of a triangle]

$\Rightarrow \angle OQR + \angle ORQ + 60^\circ = 180^\circ$

$\Rightarrow 2\angle OQR = 180^\circ - 60^\circ$

$\Rightarrow 2\angle OQR = 120^\circ$

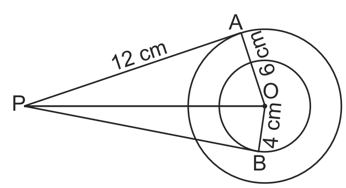
$\Rightarrow \angle OQR = 60^\circ$

$\Rightarrow \angle OQR = \angle ORQ = \angle QOR = 60^\circ$

$\therefore$ ΔOQR is an equilateral triangle of side *r* cm.

$\therefore$ Area of ΔOQR is $\frac{\sqrt{3}}{4}r^2$ cm².

4. Two concentric circles with centre O are of radii 6 cm and 4 cm. From an external point P, tangents PA and PB are drawn to these two circles as shown in the figure. If AP = 12 cm, then BP is equal to



(*a*) 12 cm (*b*) $2\sqrt{41}$ cm
(*c*) $4\sqrt{7}$ cm (*d*) 11 cm

Sol. (*b*) $2\sqrt{41}$ cm

Given that A and B are two points on two concentric circles with centre at O and radii 6 cm and 4 cm respectively. P is an external point to both the circles and PA and PB are tangents to the two circles from P at the points A and B respectively such that AP = 12 cm. To find the length of PB.

We have

In ΔPAO,

$\because \angle PAO = 90^\circ$

$\therefore$ By using Pythagoras' theorem, we have

$$OP^2 = AP^2 + AO^2$$

$$= 12^2 + 6^2$$
$$= 144 + 36$$
$$= 180$$

$\therefore \quad OP^2 = 180 \quad \ldots(1)$

Now, in ΔPBO, since $\angle PBO = 90°$,

$\therefore$ By using Pythagoras' theorem, we have

$$BP = \sqrt{OP^2 - OB^2} = \sqrt{180 - 16}$$
$$= \sqrt{164}$$

[$\because$ From (1), $OP^2 = 180$ and $OB = 4$]

$$= \sqrt{4 \times 41}$$
$$= 2\sqrt{41}$$

Hence, the required length of BP is $2\sqrt{41}$ cm.

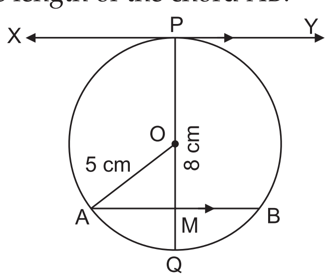
5. At one end of a diameter PQ of a circle of radius 5 cm, a tangent XPY is drawn to the circle. Then the length of the chord AB parallel to XY and at a distance of 8 cm from P is

(*a*) 8 cm (*b*) 10 cm

(*c*) 11 cm (*d*) 12 cm

Sol. (*a*) 8 cm

Let PQ be a diameter of a circle with centre at O and radius 5 cm. XY is a tangent to the circle at P and AB is a chord of the circle parallel to XY such that PM = 8 cm where PM is drawn through O perpendicular to AB and M is a point on AB. To find the length of the chord AB.



Construction: Join OA.

Since $OM \perp AB$,

$\therefore$ M is the mid-point of AB.

Now, in ΔAOM, we have

$$AO = 5 \text{ cm}$$

and $OM = PM - PO$

$$= (8 - 5)\text{cm}$$
$$= 3 \text{ cm}$$

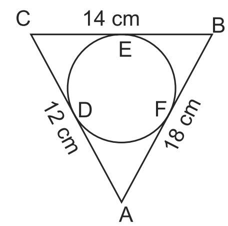
$\therefore$ By using Pythagoras' theorem, we have

$$AM = \sqrt{AO^2 - OM^2}$$
$$= \sqrt{5^2 - 3^2} \text{ cm}$$
$$= \sqrt{16} \text{ cm}$$
$$= 4 \text{ cm}$$

$\therefore \quad AB = 2\,AM$

$$= 2 \times 4 \text{ cm}$$
$$= 8 \text{ cm}$$

6. A circle is inscribed in a triangle ABC having sides BC = 14 cm, AB = 18 cm and AC = 12 cm as shown in the figure. If AC, CB and AB touch the circle at the points D, E and F respectively, then



(*a*) AD = 8 cm and BE = 4 cm

(*b*) AD = 10 cm and BE = 8 cm

(*c*) AD = 8 cm and BE = 10 cm

(*d*) AD = 10 cm and BE = 4 cm

Sol. (*c*) AD = 8 cm and BE = 10 cm

Given that ABC is a triangle of sides AB = 18 cm, BC = 14 cm and AC = 12 cm. A circle is inscribed within this triangle touching the sides AC, CB and AB at the point D, E and F respectively. To find the lengths of AD and BE.

Let $AD = AF = x$ cm

and $BE = BF = y$ cm

Now, $CD = CE$

$\Rightarrow \quad AC - AD = CB - BE$

$\Rightarrow \quad 12 - x = 14 - y$

$\Rightarrow \quad y - x = 14 - 12 = 2$

$\Rightarrow \quad y - x = 2 \quad \ldots(1)$

Also, $AF + FB = 18$

$\Rightarrow \quad x + y = 18 \quad \ldots(2)$

Adding (1) and (2), we get

$$2y = 20$$

$\Rightarrow \quad y = 10$

$\therefore$ From (1), $x = 10 - 2 = 8$

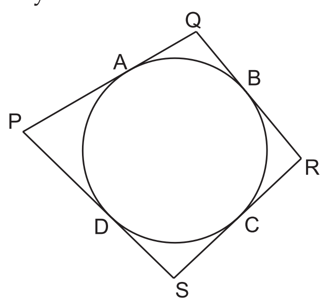
Hence, AD = 8 cm and BE = 10 cm

7. A quadrilateral PQRS circumscribes a circle such that the sides PQ, QR, RS and SP touch the circle at A, B, C and D respectively. Then PD + QB is equal to (*a*) PS (*b*) QR

(*c*) RS (*d*) PQ

Sol. (*d*) PQ

Let PQRS be a quadrilateral and a circle is inscribed within this quadrilateral touching the sides PQ, QR, RS and SP at the points A, B, C and D respectively.



To find PD + QB

Let PD = x and QB = y

We know that the lengths of tangents drawn from an external point to a circle are equal.

$$PA = PD = x \quad \text{[Tangents from P]}$$

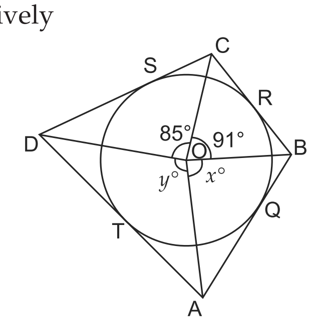
and $$QA = QB = y \quad \text{[Tangents from Q]}$$

$\therefore$ $$PQ = PA + QA = x + y$$

$\Rightarrow$ $$PQ = x + y \quad \ldots(1)$$

Hence, $PD + QB = x + y = PQ$ [From (1)]

8. A quadrilateral ABCD circumscribes a circle with centre O as shown in the figure. The sides AB, BC, CD and DA of the quadrilateral touch the circle at the points Q, R, S and T respectively. If $\angle DOC = 85°$ and $\angle BOC = 91°$, $\angle AOB = x°$ and $\angle AOD = y°$, then the values of x and y are respectively



(*a*) 95° and 89° (*b*) 89° and 95°

(*c*) 97° and 79° (*d*) 97° and 89°

Sol. (*a*) 95° and 89°

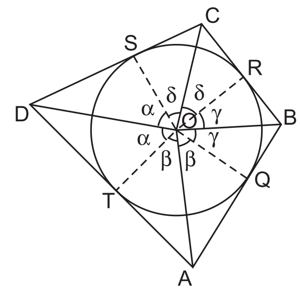
Given that ABCD is a quadrilateral which circumscribes a circle with centre at O such that

$\angle DOC = 85°$, $\angle COB = 91°$, $\angle AOB = x°$ and $\angle AOD = y°$.

To find the values of $x°$ and $y°$.

Construction: Join OQ, OR OS and OT.

We know that two equal tangents drawn from an external point to a circle subtend equal angles at the centre of the circle.



$\therefore$ $$\angle TOD = \angle SOD = \alpha, \text{ say}$$

$$\angle TOA = \angle QOA = \beta, \text{ say}$$

$$\angle QOB = \angle ROB = \gamma, \text{ say}$$

and $$\angle ROC = \angle SOC = \delta, \text{ say}$$

$\therefore$ $$2\alpha + 2\beta + 2\gamma + 2\delta = 360°$$

$\Rightarrow$ $$\alpha + \beta + \gamma + \delta = \frac{360°}{2} = 180° \quad \ldots(1)$$

Now, given that

$$\angle DOC = 85°$$

$\Rightarrow$ $$\alpha + \delta = 85° \quad \ldots(2)$$

$$\angle BOC = 91°$$

$\Rightarrow$ $$\gamma + \delta = 91° \quad \ldots(3)$$

$$\angle AOB = x°$$

$$\beta + \gamma = x° \quad \ldots(4)$$

and $$\angle AOD = y°$$

$\Rightarrow$ $$\alpha + \beta = y° \quad \ldots(5)$$

From (2) and (3), we have

$$\gamma - \alpha = 91° - 85° = 6° \quad \ldots(6)$$

Also, from (4) and (5),

$$\gamma - \alpha = x° - y° \quad \ldots(7)$$

From (6) and (7),

$$x° - y° = 6° \quad \ldots(8)$$

Adding (2), (3), (4) and (5), we get

$$2(\alpha + \beta + \gamma + \delta) = 85° + 91° + x° + y°$$

$\Rightarrow$ $$2 \times 180° - 176° = x° + y° \quad \text{[From (1)]}$$

$\Rightarrow$ $$360° - 176° = x° + y°$$

$\Rightarrow$ $$x° + y° = 184° \quad \ldots(9)$$

Adding (8) and (9), we get

$$2x° = 190°$$

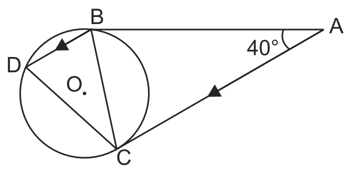
$\Rightarrow$ $$x° = \frac{190°}{2} = 95°$$

$\therefore$ From (8), we have

$$y° = 95° - 6° = 89°$$

Hence, the required values of $x°$ and $y°$ are 95° and 89° respectively.

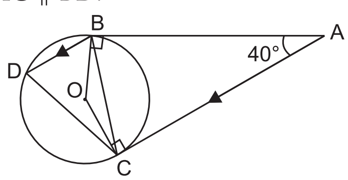
9. In the given figure, AB and AC are tangents to a circle with centre O from an external point A. If $\angle BAC = 40°$, AC || BD where D is a point on the circle and B and C are the points of contact of the tangents with the circle, then the measures of $\angle DBC$, $\angle BDC$ and $\angle BCD$ of ΔBDC are respectively



(*a*) 40°, 70° and 70°
(*b*) 75°, 40° and 65°
(*c*) 65° 40° and 75°
(*d*) 70°, 70° and 40°

Sol. (*d*) 70°, 70° and 40°

Given that B and D are two points on a circle with center at O. C is another point on the circle such that tangents AB and AC at B and C meet each other at an external point A and $\angle BAC = 40°$. Also, AC ∥ BD.



BC is joined. To find the angles of ΔBDC.

Construction: Join OB and OC.

$\because$ $OB \perp AB$ and $OC \perp AC$,

$\therefore$ OBAC is a cyclic quadrilateral.

$\therefore$ $\angle BOC + \angle BAC = 180°$

$\Rightarrow$ $\angle BOC + 40° = 180°$

$\Rightarrow$ $\angle BOC = 180° - 40° = 140°$

But $\angle BOC = 2\angle BDC$

$\Rightarrow$ $2\angle BDC = 140°$

$\Rightarrow$ $\angle BDC = \dfrac{140°}{2} = 70°$

Since AB = AC,

$\therefore$ $\angle ABC = \angle ACB$

$= \dfrac{180° - 40°}{2}$

$= \dfrac{140°}{2} = 70°$

$\therefore$ $\angle DBC = \angle DBA - \angle ABC$

$= (180° - 40°) - 70°$

[$\because$ AC ∥ BD,

$\therefore$ $\angle DBA + \angle BAC = 180°$

$\Rightarrow$ $\angle DBA = 180° - 40°$]

$= 140 - 70° = 70°$

$\therefore$ $\angle DCB = 180° - (70° + 70°)$

$= 40°$

$\therefore$ $\angle DBC = \angle BDC$

$= 70°$

and $\angle DCB = 40°$

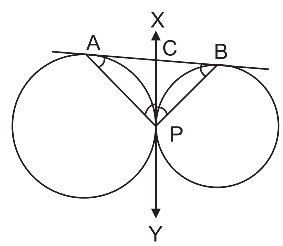
$\therefore$ The required angles are 70°, 70° and 40° respectively.

10. Two circles touch each other externally at P. AB is a common tangent to the circles touching them at A and B. Then the value of $\angle APB$ is

(*a*) 90° (*b*) 30°
(*c*) 45° (*d*) 60° **[CBSE 2014]**

Sol. (*a*) 90°

Draw XY the common tangent at C to the externally touching circles and let it intersect AB at P.



Since the lengths of tangents drawn from an external point to a circle are equal

$\therefore$ CA = CP and CB = CP

$\therefore$ $\angle CPA = \angle CAP = x$, say

and $\angle CPB = \angle CBP = y$, say ...(1)

[Angles opposite to equal sides]

In ΔAPB, we have

$\angle BAP + \angle APB + \angle ABP = 180°$

[Angle sum property of a triangle]

$\Rightarrow$ $\angle CAP + (\angle CPA + \angle CPB) + \angle CBP = 180°$

$\Rightarrow$ $x + (x + y) + y = 180°$ [From (1)]

$\Rightarrow$ $2x + 2y = 180°$

$\Rightarrow$ $x + y = 90°$

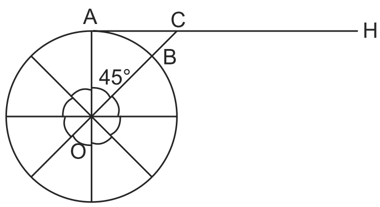
$\Rightarrow$ $\angle CPA + \angle CPB = 90°$

$\Rightarrow$ $\angle APB = 90°$

Life Skills

(Page 202)

1. A circular park has 8 radial roads equally inclined to each other. A girl decides to take her grandfather on a wheel chair to the centre O of the park. She starts from her house at H and follows the road along HA which is a tangent to the circular park at the point A and then takes a turn at A to reach the centre O. On her return trip she opts to go along the radial road OB and continues along the path BC in the same direction to be able to reach the point C on the road AH. She then follows the path CH to reach home.



If the radius of the circular park is $16\sqrt{2}$ m, find the length of the path BC.

Sol. Angle between any two consecutive radii of the circle = $\frac{360°}{8}$ = 45°

∴ In ΔAOC,

Since ∠AOC = 45° and ∠OAC = 90°,

∴ ∠OCA = 90° – 45° = 45°

∴ AC = AO = $16\sqrt{2}$ m

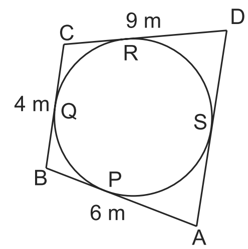
∴ By using Pythagoras' theorem in ΔOAC, we have

$$OC = \sqrt{OA^2+AC^2}$$
$$= \sqrt{2OA^2}$$
$$= \sqrt{2}\ OA$$
$$= 16\sqrt{2} \times \sqrt{2}\ m$$
$$= 32\ m$$

∴ BC = OC – OB = $(32 - 16\sqrt{2})$ m

which is the required length.

2. Mr Joshi constructed a circular pond for the ducks somewhere within his rectangular garden. He also got a fencing made in the form of a quadrilateral ABCD (as shown in the figure) whose sides AB, BC, CD and DA touch the circular pond at points P, Q, R and S respectively.



If AB = 6 m, BC = 4 m and CD = 9 m, find the length of the side AD.

Sol. Let DR = DS = x m

∴ AS = (AD – x) m …(1)

CR = CQ [Tangents from C]

⇒ 9 m – x m = 4 m – BQ

⇒ 9 m – x m = 4 m – BP [∵ BQ = BP]

⇒ 9 m – x m = 4 m – (6 m – AP)

⇒ 9 m – x m = –2 m + AP

= –2 m + AS

= –2 m + AD – x m [From (1)]

⇒ 9 m + 2 m = AD

⇒ AD = 11 m

Hence, the required length of AB is 11 m.