9

Some Applications of Trigonometry

Checkpoint (Page 173)

1. If $\sec A = \frac{41}{9}$, find $\cot A$ when $0° < A < 90°$.

Sol. We have

$$\sec A = \frac{41}{9}$$

$$\therefore \quad \sec^2 A = \frac{41^2}{9^2} = \frac{1681}{81}$$

$$\Rightarrow \quad \tan^2 A + 1 = \frac{1681}{81}$$

$$\Rightarrow \quad \tan^2 A = \frac{1681}{81} - 1 = \frac{1600}{81}$$

$$\therefore \quad \tan A = \sqrt{\frac{1600}{81}} = \frac{40}{9}$$

$$\therefore \quad \cot A = \frac{1}{\tan A} = \frac{9}{40}$$

which is the required value.

2. Express cosec θ in terms of sec θ where $0° < \theta < 90°$.

Sol. We have

$$\operatorname{cosec}\theta = \sqrt{1 + \cot^2\theta}$$

$$= \sqrt{1 + \frac{1}{\tan^2\theta}}$$

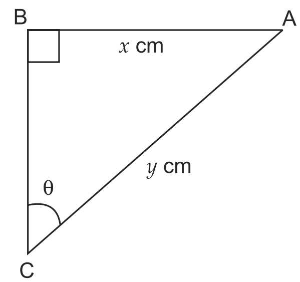
$$= \sqrt{1 + \frac{1}{\sec^2\theta - 1}}$$

$$= \sqrt{\frac{\sec^2\theta}{\sec^2\theta - 1}}$$

$$= \frac{\sec\theta}{\sqrt{\sec^2\theta - 1}}$$

which is the required expression.

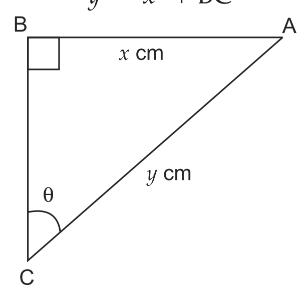
3. In the following figure, find all trigonometric ratios of the angle θ of ΔABC, where ∠B = 90°, AB = x cm, AC = y cm and ∠ACB = θ.



Sol. From ΔABC, we have by Pythagoras' theorem,

$$AC^2 = AB^2 + BC^2$$

$$\Rightarrow \quad y^2 = x^2 + BC^2$$



$$\therefore \quad BC = \sqrt{y^2 - x^2}$$

$$\therefore \quad \sin A = \sin \angle BCA = \frac{AB}{AC} = \frac{x}{y}$$

$$\cos\theta = \frac{BC}{AC} = \frac{\sqrt{y^2 - x^2}}{y}$$

$$\tan\theta = \frac{AB}{BC} = \frac{x}{\sqrt{y^2 - x^2}}$$

$$\therefore \quad \cot\theta = \frac{1}{\tan\theta} = \frac{\sqrt{y^2 - x^2}}{x}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{y}{\sqrt{y^2-x^2}}$$

$$\operatorname{cosec}\theta = \frac{1}{\sin\theta} = \frac{y}{x}$$

Hence, the required values of sin θ, cos θ, tan θ, cot θ, sec θ, and cosec θ are

$\frac{x}{y}, \frac{\sqrt{y^2-x^2}}{y}, \frac{x}{\sqrt{y^2-x^2}}, \frac{\sqrt{y^2-x^2}}{x}, \frac{y}{\sqrt{y^2-x^2}}$

and $\frac{y}{x}$ respectively.

4. If $\sin A = \frac{3}{5}$, find the values of all other trigonometric ratios of angle A.

Sol. We have $\sin A = \frac{3}{5}$

$$\therefore \quad \cos A = \sqrt{1-\sin^2 A} = \sqrt{1-\frac{9}{25}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$$

$$\therefore \quad \tan A = \frac{\sin A}{\cos A} = \frac{3}{4}$$

$$\cot A = \frac{1}{\tan A} = \frac{4}{3}$$

$$\sec A = \frac{1}{\cos A} = \frac{5}{4}$$

and $\operatorname{cosec} A = \frac{1}{\sin A} = \frac{5}{3}$

Hence, the required values of cos A, tan A, cot A, sec A and cosec A are $\frac{4}{5}, \frac{3}{4}, \frac{4}{3}, \frac{5}{4}$ and $\frac{5}{3}$ respectively.

5. If $\tan A = \sqrt{2}-1$, show that $\frac{\tan A}{1+\tan^2 A} = \frac{\sqrt{2}}{4}$.

Sol. We have

$$\tan A = \sqrt{2} - 1$$

$$\therefore \quad \tan^2 A = \left(\sqrt{2}-1\right)^2 = 2 + 1 - 2\sqrt{2} = 3 - 2\sqrt{2}$$

$$\therefore \quad \text{LHS} = \frac{\tan A}{1+\tan^2 A} = \frac{\sqrt{2}-1}{3-2\sqrt{2}+1} = \frac{\sqrt{2}-1}{4-2\sqrt{2}} = \frac{\sqrt{2}-1}{2\left(2-\sqrt{2}\right)} = \frac{\left(\sqrt{2}-1\right)\left(2+\sqrt{2}\right)}{2\left(2^2-2\right)} = \frac{2\sqrt{2}+2-2-\sqrt{2}}{2(2)} = \frac{\sqrt{2}}{4} = \text{RHS}$$

6. If $\cot B = \frac{12}{5}$, show that $\tan^2 B - \sin^2 B = \sin^2 B \tan^2 B$.

Sol. We have, $\cot B = \frac{12}{15}$

$$\therefore \quad \tan B = \frac{1}{\cot B} = \frac{5}{12}$$

$$\therefore \quad \tan^2 B = \frac{25}{144} \qquad \ldots(1)$$

$$\therefore \quad \cot^2 B = \frac{144}{25}$$

$$\therefore \quad \operatorname{cosec}^2 B = 1 + \cot^2 B = 1 + \frac{144}{25} = \frac{169}{25}$$

$$\therefore \quad \sin^2 B = \frac{1}{\operatorname{cosec}^2 B} = \frac{25}{169} \qquad \ldots(2)$$

Now, $\text{LHS} = \tan^2 B - \sin^2 B$

$$= \frac{25}{144} - \frac{25}{169} = 25 \times \frac{169-144}{144 \times 169} = \frac{25 \times 25}{144 \times 169} = \frac{25}{144} \times \frac{25}{169} = \tan^2 B \,.\, \sin^2 B$$

[From (1) and (2)]

$= \text{RHS}$

Hence, proved.

7. If $\tan\theta = 2$, evaluate $\sin\theta\sec\theta + \tan^2\theta - \operatorname{cosec}\theta$.

Sol. We have

$$\tan\theta = 2$$

$$\therefore \quad \sec^2\theta = 1 + \tan^2\theta = 1 + (2)^2$$

$$= 1 + 4 = 5$$

$$\therefore \quad \cos^2\theta = \frac{1}{\sec^2\theta} = \frac{1}{5}$$

$$\therefore \quad \sin^2\theta = 1 - \cos^2\theta$$

$$= 1 - \frac{1}{5} = \frac{4}{5}$$

$$\therefore \quad \text{cosec}^2\theta = \frac{1}{\sin^2\theta} = \frac{5}{4}$$

$$\therefore \quad \text{cosec}\,\theta = \frac{\sqrt{5}}{2}$$

Also, $\sec\theta = \sqrt{5}$, $\sin\theta = \sqrt{\frac{4}{5}}$

Now, $\sin\theta\sec\theta + \tan^2\theta - \text{cosec}\,\theta$

$$= \sqrt{\frac{4}{5}} \times \sqrt{5} + 4 - \frac{\sqrt{5}}{2}$$

$$= \frac{2}{\sqrt{5}} \times \sqrt{5} + 4 - \frac{\sqrt{5}}{2}$$

$$= 6 - \frac{\sqrt{5}}{2}$$

$$= \frac{12 - \sqrt{5}}{2}$$

which is the required value.

8. If $\tan\theta = \frac{1}{\sqrt{7}}$, show that

$$\frac{\text{cosec}^2\theta - \sec^2\theta}{\text{cosec}^2\theta + \sec^2\theta} = \frac{3}{4}$$

Sol. We have

$$\tan\theta = \frac{1}{\sqrt{7}}$$

$$\therefore \quad \sec^2\theta = 1 + \tan^2\theta$$

$$= 1 + \frac{1}{7} = \frac{8}{7}$$

$$\Rightarrow \quad \cot\theta = \frac{1}{\tan\theta} = \sqrt{7}$$

$$\therefore \quad \text{cosec}^2 = 1 + \cot^2\theta$$

$$= 1 + 7 = 8$$

$$\therefore \quad \text{LHS} = \frac{\text{cosec}^2\theta - \sec^2\theta}{\text{cosec}^2\theta + \sec^2\theta}$$

$$= \frac{8 - \frac{8}{7}}{8 + \frac{8}{7}}$$

$$= \frac{48}{64}$$

$$= \frac{3}{4} = \text{RHS}$$

Hence, proved.

9. Find the numerical value of the expression:

$$\frac{4\sin^2 30° + 5\cos^2 45° - 6\tan^2 60°}{\text{cosec}^2 60° \cot^2 30° + \sec^2 45°}$$

Sol. We have,

$$\frac{4\sin^2 30° + 5\cos^2 45° - 6\tan^2 60°}{\text{cosec}^2 60° \cot^2 30° + \sec^2 45°}$$

$$= \frac{4 \times \left(\frac{1}{2}\right)^2 + 5 \times \left(\frac{1}{\sqrt{2}}\right)^2 - 6 \times \left(\sqrt{3}\right)^2}{\left(\frac{2}{\sqrt{3}}\right)^2 \times \left(\sqrt{3}\right)^2 + \left(\sqrt{2}\right)^2}$$

$$= \frac{1 + \frac{5}{2} - 18}{4 + 2}$$

$$= \frac{-\frac{29}{2}}{6}$$

$$= -\frac{29}{12}$$

which is the required value of the given expression.

Check Your Progress

(Page 180)

Multiple-Choice Questions

1. The angle formed by the line of sight with the horizontal when the point being viewed lies above the horizontal level, is called

(*a*) vertical angle

(*b*) angle of depression

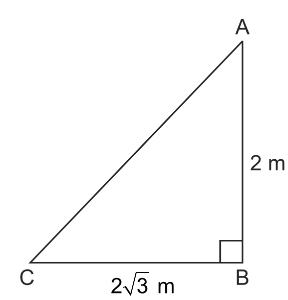
(*c*) angle of elevation

(*d*) obtuse angle **[CBSE SP 2012]**

Sol. (*c*) angle of elevation

Clearly, it is called the angle of elevation.

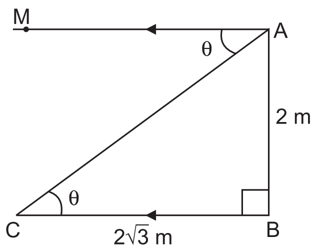
2. The given figure shows the observation of the point C from the point A. The angle of depression from A is



(*a*) 60° (*b*) 30°
(*c*) 45° (*d*) 75° **[CBSE SP 2012]**

Sol. (*b*) 30°

We draw AM as a horizontal ray through A. Hence, AM ∥ BC and AB ⊥ BC.



$$\therefore \quad \angle MAC = \text{angle of depression}$$
$$= \angle ACB \qquad [\because AM \| BC]$$
$$= \theta$$

Now, in ΔABC, we have

$$\tan\theta = \tan\angle ACB = \frac{AB}{BC} = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}} = \tan 30°$$

$$\therefore \quad \theta = 30°$$

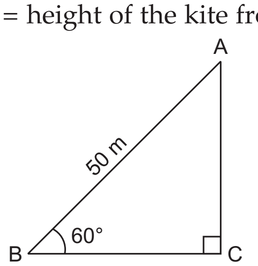
3. The string of a kite in air is 50 m long and it makes an angle of 60° with the horizontal. Assuming the string to be straight, the height of the kite from the ground is

(*a*) $50\sqrt{3}$ m (*b*) $\frac{100}{\sqrt{3}}$ m
(*c*) $\frac{50}{\sqrt{3}}$ m (*d*) $25\sqrt{3}$ m

[CBSE 2023 Basic]

Sol. (*d*) $25\sqrt{3}$ m

AB = length of the string
= 50 m
AC = height of the kite from the ground

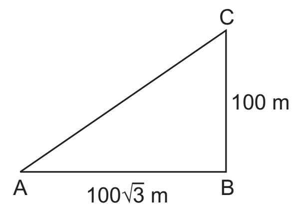


In ΔABC,

$$\sin 60° = \frac{AC}{AB} = \frac{AC}{50\text{ m}}$$

$$\Rightarrow \quad AC = \sin 60° \,.\, 50\text{ m}$$
$$AC = \frac{\sqrt{3}}{2} \times 50\text{ m}$$
$$\therefore \quad AC = 25\sqrt{3}\text{ m}$$

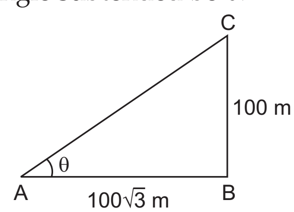
4. The angle subtended by a vertical pole of height 100 m at a point on the ground $100\sqrt{3}$ m from the base, has measure of



(*a*) 90° (*b*) 60°
(*c*) 45° (*d*) 30°

Sol. (*d*) 30°

Let the angle subtended be θ.



$$\text{Then} \quad \tan\theta = \frac{BC}{AB}$$
$$\Rightarrow \quad \tan\theta = \frac{100}{100\sqrt{3}}$$
$$\Rightarrow \quad \tan\theta = \frac{1}{\sqrt{3}}$$
$$\Rightarrow \quad \tan\theta = \tan 30°$$
$$\therefore \quad \theta = 30°$$

Very Short Answer Type Questions

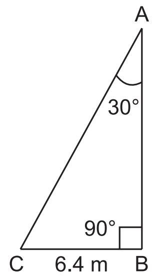
5. The angle between a ladder and a vertical wall, when the ladder is leaning against the wall, is 30° and the foot of the ladder is 6.4 m away from the wall. Find the length of the ladder.

Sol. Let AB be the vertical wall, BC, the horizontal ground and AC, the ladder such that ∠CAB = 30°. Clearly, ∠ABC = 90°.

It is given that CB = 6.4 m.

Now, in ΔABC, we have

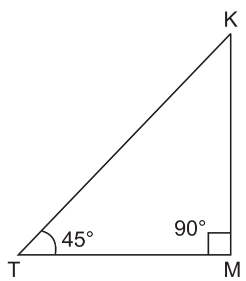
$$\sin 30° = \sin(\angle CAB) = \frac{CB}{AC}$$
$$\Rightarrow \quad \frac{1}{2} = \frac{6.4}{AC}$$
$$\Rightarrow \quad AC = 6.4 \times 2 = 12.8\text{ m}$$



Hence, the required length of the ladder is 12.8 m.

6. A kite is flying with a thread 300 m long. If the thread is assumed to be stretched straight and make an angle of 45° with the horizontal, find the height of the kite above the ground.

Sol. Let K be the position of the kite in air, KT be the thread 300 m long and TM be the horizontal ground. Clearly, KM is the height of the ground so that $\angle KMT = 90°$. Given that $\angle KTM$ = angle of elevation of the kite K from the ground T = 45°.



Now, in ΔKTM, we have

$$\sin 45° = \sin(\angle KTM) = \frac{KM}{KT}$$

$$\Rightarrow \quad \frac{1}{\sqrt{2}} = \frac{KM}{300}$$

$$\Rightarrow \quad KM = \frac{300}{\sqrt{2}} = \frac{300 \times \sqrt{2}}{2} = 150\sqrt{2}$$

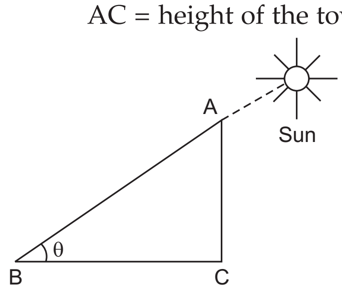
$\therefore$ The required height of the kite above the ground is $150\sqrt{2}$ m.

7. The length of the shadow of a tower on the plane ground is $\sqrt{3}$ times the height of the tower. Find the angle of elevation of the Sun.

[CBSE 2023 Standard]

Sol. Let the angle of elevation of the Sun be θ.

AC = height of the tower



Given,

Shadow of the tower = BC = $\sqrt{3}$ AC

In ΔABC,

$$\tan\theta = \frac{AC}{BC} = \frac{AC}{\sqrt{3}\,AC}$$

$$\Rightarrow \quad \tan\theta = \frac{1}{\sqrt{3}}$$

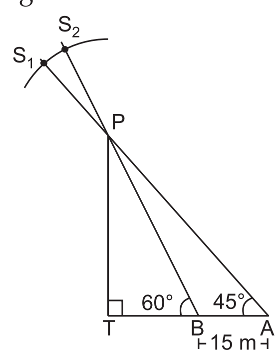
$$\Rightarrow \quad \tan\theta = \tan 30°$$

$$\therefore \quad \theta = 30°$$

Short Answer Type Questions

8. The shadow of a vertical tower on level ground decreases by 15 m when the altitude of the Sun changes from angle of elevation of 45° to 60°. Find the height of the tower.

Sol. Let PT be the vertical tower and AT be the horizontal ground so that $\angle PTA = 90°$.



Let TA be the length of the shadow of the tower when the position of the sun is S_1 such that S_1PA is a straight line and $\angle PAT = 45°$. Let TB be the length of the shadow when the position of the sun is at S_2 such that S_2PB is a straight line and $\angle PBT = 60°$.

It is given that AB = 15 m.

To find the height PT of the tower.

Now, in ΔPTA, we have

$$\tan(\angle PAT) = \frac{PT}{AT}$$

$$\Rightarrow \quad \tan 45° = \frac{PT}{AT}$$

$$\Rightarrow \quad 1 = \frac{PT}{AT}$$

$$\Rightarrow \quad \frac{AT}{PT} = 1 \qquad \ldots(1)$$

Again, in ΔPTB, we have

$$\tan(\angle PBT) = \frac{PT}{TB}$$

$$\Rightarrow \quad \tan 60° = \frac{PT}{TB}$$

$$\Rightarrow \quad \sqrt{3} = \frac{PT}{TB}$$

$$\Rightarrow \quad \frac{TB}{PT} = \frac{1}{\sqrt{3}} \qquad \ldots(2)$$

Subtracting (2) from (1), we get

$$\frac{AT - TB}{PT} = 1 - \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad \frac{AB}{PT} = \frac{\sqrt{3} - 1}{\sqrt{3}}$$

$$\Rightarrow \quad \frac{15}{PT} = \frac{\sqrt{3} - 1}{\sqrt{3}}$$

$$\Rightarrow \quad PT = \frac{15\sqrt{3}}{\sqrt{3} - 1}$$

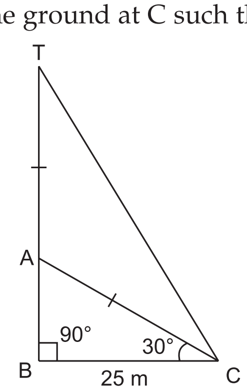
$$= \frac{15\sqrt{3}\left(\sqrt{3} + 1\right)}{\left(\sqrt{3}\right)^2 - 1^2}$$

$$= \frac{15\left(3 + \sqrt{3}\right)}{2}$$

Hence, the required height of the tower is $\frac{15\left(3 + \sqrt{3}\right)}{2}$ m.

9. The upper part of a tree, broken by the wind makes an angle of 30° with the ground and the horizontal distance from the root of the tree to the point where the top of the tree meets the ground is 25 m. Find the height of the tree before it was broken. [Use $\sqrt{3} = 1.73$] **[CBSE SP 2012]**

Sol. Let BT be the height of the tree where T is its top and B, the foot is on the horizontal ground BC. Let A be the point on the upper part of the tree where the tree broke and the top T of the tree has touched the ground at C such that ∠ACB = 30°.



Clearly, AC = AT and ∠ABC = 90°.

We join TC.

Given that BC = 25 m.

Now, in ΔABC, we have

$$\tan(\angle ACB) = \frac{AB}{BC}$$

$$\Rightarrow \quad \tan 30° = \frac{AB}{25}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{AB}{25}$$

$$\therefore \quad AB = \frac{25}{\sqrt{3}}$$

Now, since ∠ABC = 90°.

∴ By Pythagoras' theorem, we have

$$AC^2 = AB^2 + BC^2$$

$$= \left(\frac{25}{\sqrt{3}}\right)^2 + 25^2$$

$$= 25^2\left(1 + \frac{1}{3}\right)$$

$$= 25^2 \times \frac{4}{3}$$

$$\therefore \quad AC = \frac{50}{\sqrt{3}}$$

∴ Required height of BT = AB + AT

$$= AB + AC$$

$$= \frac{25}{\sqrt{3}} + \frac{50}{\sqrt{3}}$$

$$= \frac{75}{\sqrt{3}}$$

$$= \frac{75\sqrt{3}}{3}$$

$$= 25\sqrt{3}$$

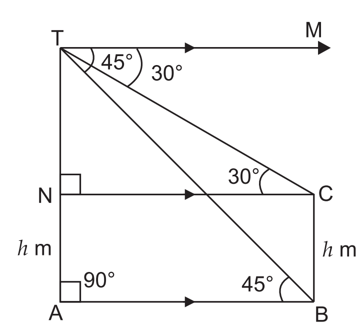
$$= 25 \times 1.73$$

$$= 43.3$$

Hence, the required height of the tree is 43.3 m.

10. From the top of a 50 m high tower, the angles of depression of the top and bottom of a pole are observed to be 30° and 45°, respectively. Find the height of the pole. [Use $\sqrt{3} = 1.73$] **[CBSE SP 2011]**

Sol. Let T be the top of the vertical tower AT and A be the foot of the tower on the horizontal ground AB so that ∠TAB = 90°.



Let C be the top and B be the bottom of a vertical pole, so that $\angle ABC = 90°$.

Let TM be a horizontal through T. The angle of depression of C from T = 30°.

$\therefore \quad \angle MTC = 30°$

Also, the angle of depression of B from T = 45°

$\therefore \quad \angle MTB = 45°$

$\therefore \quad \angle ABT = \angle MTB = 45°$

and $\quad \angle TCN = \angle MTC = 30°$

where CN is drawn horizontal and N is a point on AT.

Given that AT = 50 m. Let BC = h m be the height of the pole.

Now, in ΔTAB, we have

$$\tan(\angle ABT) = \frac{AT}{AB}$$

$$\Rightarrow \quad \tan 45° = \frac{50}{AB}$$

$$\Rightarrow \quad 1 = \frac{50}{AB}$$

$$\Rightarrow \quad AB = 50 \qquad \ldots(1).$$

Now, in ΔTNC, we have

$$\tan(\angle TCN) = \frac{TN}{CN}$$

$$\Rightarrow \quad \tan 30° = \frac{AT - AN}{CN}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{50 - h}{50}$$

[$\because$ ABCN is a rectangle, $\therefore$ AN = BC = h and CN = AB = 50 [From (1)]]

$$\Rightarrow \quad (50 - h)\sqrt{3} = 50$$

$$\Rightarrow \quad h\sqrt{3} = 50\sqrt{3} - 50$$

$$= 50(\sqrt{3} - 1)$$

$$\Rightarrow \quad h = \frac{50(\sqrt{3} - 1)}{\sqrt{3}}$$

$$= \frac{50\sqrt{3}(\sqrt{3} - 1)}{3}$$

$$= \frac{50(3 - \sqrt{3})}{3}$$

$$= \frac{50 \times (3 - 1.73)}{3}$$

$$= \frac{50 \times 1.27}{3}$$

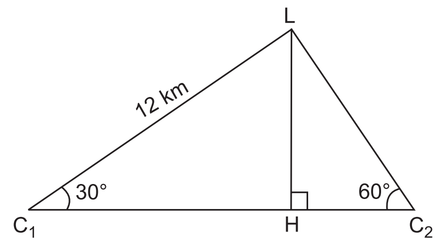
$$= 21.2 \text{ (approx.)}$$

Hence, the height of the pole is 21.2 m (approx.)

11. The angles of elevation of the top of a hill at the city centres of two towns on either side of the hill are observed to be 30° and 60°. If the distance uphill from the first city centre is 12 km, then find (in km) the distance uphill from the other city centre correct upto two decimal places.

[Use $\sqrt{3} = 1.732$]

Sol. Let L be the top and H be the bottom of a hill standing on the horizontal ground $C_1 C_2$ where C_1 and C_2 are two city centres such that $\angle LC_1H = 30°$ and $\angle LC_2H = 60°$. It is given that C_1L = 12 km.



To find C_2L.

Now, in ΔLC_1H, since $\angle LHC_2 = 90°$, we have

$$\sin(\angle LC_1H) = \frac{LH}{C_1L}$$

$$\Rightarrow \quad \sin 30° = \frac{LH}{12}$$

From ΔLC_2H, we have

$$\Rightarrow \quad \frac{1}{2} = \frac{LH}{12}$$

$$\therefore \quad LH = 6 \qquad \ldots(1)$$

$$\sin(\angle LC_2H) = \frac{LH}{C_2L}$$

$$\Rightarrow \quad \sin 60° = \frac{6}{C_2L} \qquad \text{[From (1)]}$$

$$\Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{6}{C_2L}$$

$$\Rightarrow \quad C_2L = \frac{12}{\sqrt{3}}$$

$$= \frac{12\sqrt{3}}{3} = 4\sqrt{3}$$

$$= 4 \times 1.732$$

$$= 6.93 \text{ (approx.)}$$

Hence, the required distance is 6.93 km (approx.)

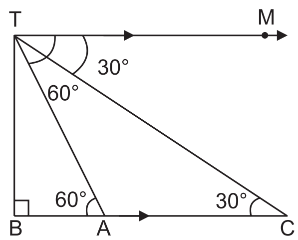
Long Answer Type Questions

12. A man on the top of a vertical tower observes a car coming towards the tower moving with a uniform speed. If it takes 20 minutes for the car to change the angle of depression from 30° to 60°, how soon after this will the car reach the foot of the tower?

Sol. Let T be the top and B be the bottom of the vertical tower TB standing on a horizontal ground BAC so that $\angle TBA = 90°$. Let TM be the horizontal ray through T.

$\therefore$ TM ∥ BC.

The car comes from C to A on the ground. When the car is at C, its angle of depression = $\angle MTC = \angle TCB = 30°$ and when it is at A, then $\angle MTA = \angle TAB = 60°$.



Let v m/min be the uniform speed of the car.

Then AC = $20v$ metres.

$\because$ It takes 20 minutes for the car to come from C to A.

Now, in ΔTBC, we have

$$\tan(\angle TCB) = \frac{TB}{BC}$$

$$\Rightarrow \quad \tan 30° = \frac{TB}{BC}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{TB}{BC}$$

$$\Rightarrow \quad BC = \sqrt{3}\,TB \qquad \ldots(1)$$

Again, in ΔTBA, we have

$$\tan(\angle TAB) = \frac{TB}{AB}$$

$$\Rightarrow \quad \tan 60° = \frac{TB}{AB}$$

$$\Rightarrow \quad AB = \frac{TB}{\sqrt{3}} \qquad \ldots(2)$$

From (1) and (2), we have

$$\frac{AB}{BC} = \frac{1}{\sqrt{3}} \times \frac{1}{\sqrt{3}} = \frac{1}{3}$$

$$\therefore \quad BC = 3AB$$

$$\therefore \quad AB = BC - AC = 3AB - AC$$

$$\Rightarrow \quad 2AB = AC = 20v$$

$$\therefore \quad AB = 10v$$

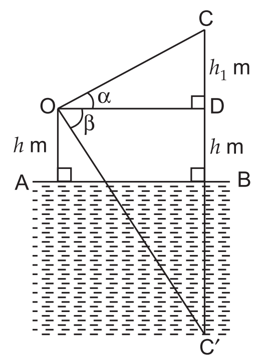
$\therefore$ Required time from A to B = $\dfrac{AB}{v} = \dfrac{10v}{v}$ min.

= 10 min.

13. If the angle of elevation of a cloud from a point h metres above a lake is α and the angle of depression of its reflection in the lake is β, prove that the distance of the cloud from the point of observation is $\dfrac{2h\sec\alpha}{\tan\beta - \tan\alpha}$ metres.

[CBSE 2004, CBSE SP 2003, 2011]

Sol. Let AB be the horizontal surface of the lake, AO be the vertical tower, O being the top of the tower, from where a man observes the cloud C in the sky at an angle of elevation $\angle COD = \alpha$ where OD is horizontal. Let C′ be the reflection of the cloud in the lake so that BC = BC′ and $\angle CBA = 90°$, AB ∥ OD.



Angle of depression of C′ in $\angle C'OD = \beta$.

Let CD = h_1 m. Then BD = AO = h m

Now, $\quad BC' = BC = (h + h_1)$ m

[$\because$ ABDO is a rectangle]

Now in ΔOCD, we have

$$\tan(\angle COD) = \frac{CD}{OD}$$

$$\Rightarrow \quad \tan\alpha = \frac{h_1}{OD}$$

$$\Rightarrow \quad OD\tan\alpha = h_1$$

$$\therefore \quad OD = h_1\frac{1}{\tan\alpha} = h_1\cot\alpha \qquad \ldots(1)$$

In $\Delta OC'D$, we have

$$\tan(\angle C'OD) = \frac{C'D}{OD}$$

$$\Rightarrow \quad \tan\beta = \frac{C'B + BD}{OD}$$

$$= \frac{CB + BD}{OD}$$

$$= \frac{h + h_1 + h}{h_1\cot + \alpha} \qquad \text{[From (1)]}$$

$$= \frac{2h + h_1}{h_1\cot a} \qquad \ldots(2)$$

Now, in ΔOCD, we have

$$\sin(\angle COD) = \frac{CD}{OC}$$

$$\Rightarrow \quad \sin\alpha = \frac{h_1}{OC}$$

$$\therefore \quad OC = \frac{h_1}{\sin\alpha} = h_1 \text{cosec}\,\alpha \quad \ldots(3)$$

From (2), we have

$$2h + h_1 = h_1 \cot\alpha \tan\beta$$

$$\Rightarrow \quad 2h = h_1\left(\frac{\tan\beta}{\tan\alpha} - 1\right)$$

$$= h_1 \frac{\tan\beta - \tan\alpha}{\tan\alpha}$$

$$\therefore \quad h_1 = \frac{2h\tan\alpha}{\tan\beta - \tan\alpha}$$

$\therefore$ From (3),

$$OC = \frac{h_1}{\sin\alpha}$$

$$= \frac{2h\tan\alpha}{\tan\beta - \tan\alpha} \times \frac{1}{\sin\alpha}$$

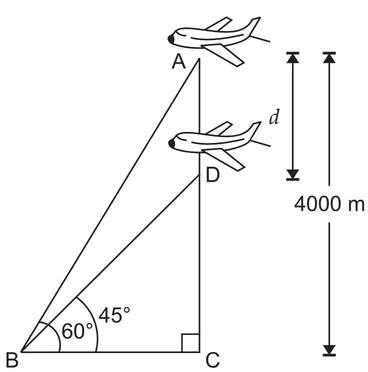
$$= 2h\frac{\sin\alpha}{\cos\alpha} \times \frac{1}{\tan\beta - \tan\alpha} \times \frac{1}{\sin.\alpha}$$

$$= \frac{2h\sec\alpha}{\tan\beta - \tan\alpha}$$

$\therefore$ The required distance of the cloud is $\frac{2h\sec\alpha}{\tan\beta - \tan\alpha}$ m.

14. An aeroplane is flying at a height of 4000 m from the ground. It passes vertically above another aeroplane. At that particular instant, the angles of elevation of the two planes from the same point on the ground are 60° and 45° respectively. Find the vertical distance between the two aeroplanes at that instant. [Use $\sqrt{3}$ = 1.73]

Sol. Let the vertical distance between the two planes be d.



In ΔABC,

$$\tan 60° = \frac{AC}{BC}$$

$$\Rightarrow \quad BC = \frac{AC}{\tan 60°} = \frac{4000}{\sqrt{3}} \quad \ldots(1)$$

In ΔDBC

$$\tan 45° = \frac{CD}{BC}$$

$$\Rightarrow \quad BC = \frac{CD}{\tan 45°} = \frac{4000 - d}{1}$$

$$\Rightarrow \quad BC = 4000 - d$$

But $\quad BC = \frac{4000}{\sqrt{3}}$ from (1)

$$\Rightarrow \quad \frac{4000}{\sqrt{3}} = 4000 - d$$

$$\Rightarrow \quad 4000 = 4000\sqrt{3} - \sqrt{3}d$$

$$\Rightarrow \quad \sqrt{3}d = 4000\sqrt{3} - 4000$$

$$\Rightarrow \quad \sqrt{3}d = 4000[1.73 - 1]$$

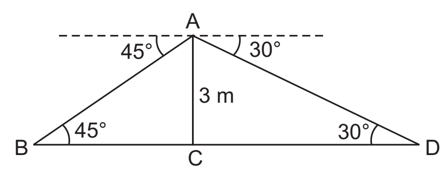
$$\Rightarrow \quad d = \frac{4000 \times 0.73}{\sqrt{3}} = \frac{4000 \times 0.73}{1.73}$$

$$d = 1687.9 \text{ m approx.}$$

$\therefore$ Distance between the two planes is 1687.9 m.

15. From a point on a bridge across a river, the angles of depression of the banks on opposite sides of a river are 30° and 45° respectively. If the bridge is at a height of 3 m from the banks, find the width of the river. [Use $\sqrt{3}$ = 1.73] **[CBSE 2023 Basic]**

Sol. B and D are the points on opposite sides of the river. BD is the width of the river.



In ΔABC,

$$\tan 45° = \frac{AC}{BC}$$

$$\Rightarrow \quad 1 = \frac{3}{BC}$$

$$\Rightarrow \quad BC = 3 \text{ m} \quad \ldots(1)$$

In ΔACD,

$$\tan 30° = \frac{AC}{CD}$$

$$\Rightarrow \quad CD = 3\sqrt{3} \text{ m} \quad \ldots(2)$$

$$BD = BC + CD$$

$$= 3 \text{ m} + 3\sqrt{3} \text{ m}$$

[From (1) and (2)]

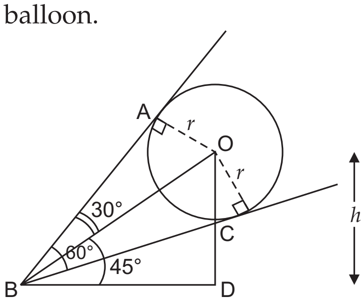
$$= 3(1+\sqrt{3})\text{ m} = 3 \times 2.73 \text{ m}$$

$$= 8.19 \text{ m}$$

$\therefore$ Width of the river is 8.19 m.

16. A spherical balloon of radius r subtends an angle of 60° at the eye of an observer. If the angle of elevation of its centre is 45° from the same point, then prove that height of the centre of the balloon is $\sqrt{2}$ times its radius. **[CBSE 2023 Standard]**

Sol. Let r and h be the radius and height of the centre of the balloon.



In ΔBOD,

$$\sin 45° = \frac{OD}{OB}$$

$$\Rightarrow \quad \frac{1}{\sqrt{2}} = \frac{OD}{OB}$$

$$\Rightarrow \quad OB = \sqrt{2}\,h \quad ...(1)$$

In ΔAOB,

$$\sin 30° = \frac{r}{OB}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{r}{OB}$$

$$\Rightarrow \quad OB = 2r$$

Putting the value of OB from (1), we get

$$\sqrt{2}h = 2r$$

Hence, proved $h = \sqrt{2}r$

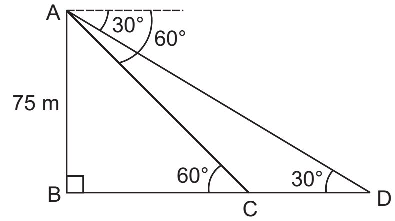
17. As observed from the top of a 75 m high lighthouse from the sea-level, the angles of depression of two ships are 30° and 60°. If one ship is exactly behind the other on the same side of the lighthouse, find the distance between the two ships. (Use $\sqrt{3}$ = 1.73) **[CBSE 2023 Standard]**

Sol. AB is the lighthouse and the two ships are at points C and D respectively.

In ΔABC,

$$\tan 60° = \frac{AB}{BC}$$

$$\Rightarrow \quad BC = \frac{AB}{\tan 60°} = \frac{75}{\sqrt{3}} \quad ...(1)$$



In ΔABD,

$$\tan 30° = \frac{AB}{BD}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{75}{BD}$$

$$\Rightarrow \quad BD = 75\sqrt{3} \quad ...(2)$$

$$BD = BC + CD$$

$$\Rightarrow \quad CD = BD - BC$$

$$= 75\sqrt{3} - \frac{75}{\sqrt{3}} = 75\left(\frac{3-1}{\sqrt{3}}\right)$$

$$= 75 \times \frac{2}{\sqrt{3}} = \frac{25 \times 3 \times 2}{\sqrt{3}}$$

$$CD = 50\sqrt{3}$$

$$\Rightarrow \quad CD = 50 \times 1.73 \text{ m}$$

$$\therefore \quad CD = 86.5 \text{ m}$$

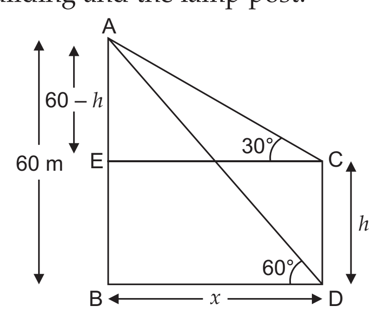
18. From the top of a building 60 m high, the angles of depression of the top and bottom of the vertical lamp post are observed to be 30° and 60° respectively.

(*a*) Find the horizontal distance between the building and the lamp post.

(*b*) Find the distance between the tops of the building and the lamp post.

[CBSE 2024 Standard]

Sol. (*a*) Let x be the horizontal distance between the building and the lamp post.



In ΔABD,

$$\tan 60° = \frac{AB}{BD}$$

$$\Rightarrow \quad \sqrt{3} = \frac{60}{x}$$

$$\Rightarrow \qquad x = \frac{60}{\sqrt{3}} = \frac{3 \times 20}{\sqrt{3}} = 20\sqrt{3}$$

$$= 20 \times 1.73 = 34.6 \text{ m}$$

$\therefore$ The horizontal distance between the building and the lamp post is 34.6 m.

(*b*) Let h be the height of the lamp post.

In ΔAEC,

$$\tan 30^\circ = \frac{AE}{EC}$$

$$\Rightarrow \qquad \frac{1}{\sqrt{3}} = \frac{60 - h}{20\sqrt{3}} \qquad [EC = BD]$$

$$\Rightarrow \qquad 1 = \frac{60 - h}{20}$$

$$\Rightarrow \qquad 60 - h = 20$$

$$\Rightarrow \qquad h = 40 \text{ m}$$

$$\Rightarrow \qquad AE = (60 - 40) \text{ m} = 20 \text{ m}$$

$\therefore$ Distance between the tops of the building and the lamp post = 20 m.

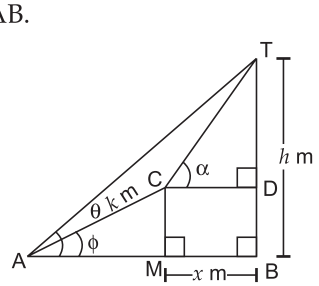
Higher Order Thinking Skills (HOTS) Questions

(Page 182)

1. The angle of elevation of a cliff from a fixed point is θ. After going up a distance of k metres towards the top of the cliff at an angle of inclination ϕ, it is found that the angle of elevation is α. Show that the height of the cliff, in metres, is

$$\frac{k(\cos\phi - \sin\phi\cot\alpha)}{\cot\theta - \cot\alpha}$$

Sol. Let T be the top and B be the bottom of a cliff and A is a fixed point on the ground AB such that $\angle$TAB = angle of elevation of T = θ. AC = k m such that $\angle$CAB = ϕ. We draw CM $\perp$ AB.



From C, the angle of elevation of T is α so that $\angle$TCD = α where CD is horizontal and D is a point on TB.

We have $\angle$TDC = $\angle$TBA = $\angle$CMB = 90°.

Let BT = h m, MB = CD = x m. It is given that AC = k m.

Now, in $\angle$CAM we have, $\frac{CM}{AC} = \sin\phi$

$$\Rightarrow \qquad \frac{CM}{k} = \sin\phi$$

$$\Rightarrow \qquad CM = k\sin\phi \qquad \ldots(1)$$

Similarity, $\frac{AM}{AC} = \cos\phi$

$$\Rightarrow \qquad \frac{AM}{k} = \cos\phi$$

$$\Rightarrow \qquad AM = k\cos\phi \qquad \ldots(2)$$

Now, ΔCDT, we have

$$\tan\alpha = \frac{TD}{CD}$$

$$= \frac{BT - BD}{CD}$$

$$= \frac{BT - CM}{CD}$$

$$= \frac{h - k\sin\phi}{MB} \qquad [\text{From (1)}]$$

$$= \frac{h - k\sin\phi}{AB - AM}$$

$$= \frac{h - k\sin\phi}{AB - k\cos\phi} \quad [\text{From (2)}] \ldots(3)$$

Now, in ΔABT, we have

$$\tan\theta = \tan(\angle TAB) = \frac{TB}{AB} = \frac{h}{AB}$$

$$\therefore \qquad AB = \frac{h}{\tan\theta} = h\cot\theta \qquad \ldots(4)$$

$\therefore$ From (3) and (4), we have

$$\tan\alpha = \frac{h - k\sin\phi}{h\cot\theta - k\cos\phi}$$

$$\Rightarrow h\cot\theta\tan\alpha - k\cos\phi\tan\alpha = h - k\sin\phi$$

$$\Rightarrow h(\cot\theta\tan\alpha - 1) = k(\cos\phi\tan\alpha - \sin\phi)$$

$$\Rightarrow \quad h\left[\frac{\cot\theta}{\cot\alpha} - 1\right] = k[\cos\phi\tan\alpha - \sin\phi]$$

$$\Rightarrow h(\cot\theta - \cot\alpha) = k[\cos\phi\tan\alpha - \sin\phi] \times \frac{\cos\alpha}{\sin\alpha}$$

$$= k\left[\cos\phi\frac{\sin\alpha}{\cos\alpha} - \sin\phi\right] \times \frac{\cos\alpha}{\sin\alpha}$$

$$= k[\cos\phi - \sin\phi\cot\alpha]$$

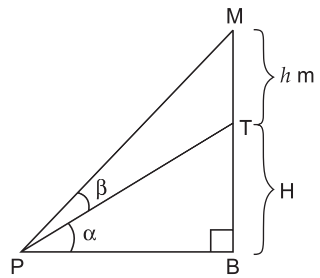
$$\Rightarrow \quad h = \frac{k[\cos\phi - \sin\phi\cot\alpha]}{\cot\theta - \cot\alpha}$$

Hence, proved.

2. At a point on a level plane, a tower subtends an angle α and a man who is h m tall standing on its top subtends an angle β at the same point. Prove

that the height of the tower is $\dfrac{h\tan\alpha}{\tan(\alpha+\beta)-\tan\alpha}$ in metres.

Sol. Let T be the top and B, the bottom of the tower, standing vertically at B on a horizontal ground PB. TM is the man of height h m standing at T such that MTB is a straight line and $\angle$MBP = 90°, where P is a point on the ground PB such that $\angle$TPB = α and $\angle$MPT = β.



To find the height BT of the tower. Let H m be the height of the tower.

Now, in ΔPMB, we have

$$\tan(\angle MPB) = \frac{MB}{PB}$$

$$\Rightarrow \quad \tan(\alpha+\beta) = \frac{H+h}{PB}$$

$$\Rightarrow \quad PB = \frac{H+h}{\tan(\alpha+\beta)} \quad \ldots(1)$$

Again, in ΔTPB, we have

$$\tan(\Delta TPB) = \frac{BT}{PB}$$

$$\Rightarrow \quad \tan\alpha = \frac{H\tan(\alpha+\beta)}{H+h} \quad \text{[From (2)]}$$

$$\Rightarrow \quad (H+h)\tan\alpha = H\tan(\alpha+\beta)$$

$$\Rightarrow \quad H[\tan(\alpha+\beta)-\tan\alpha] = h\tan\alpha$$

$$\Rightarrow \quad H = \frac{h\tan\alpha}{\tan(\alpha+\beta)-\tan\alpha}$$

Hence, the required height of the tower is $\dfrac{h\tan\alpha}{\tan(\alpha+\beta)-\tan\alpha}$ m.

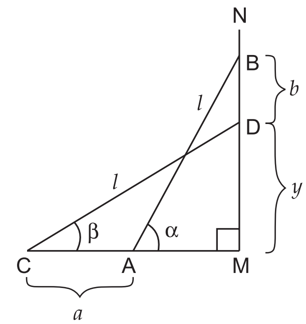
3. A ladder rests against a wall at an angle α to the horizontal. Its foot is pulled away from the wall through a distance a so that it slides a distance b down the wall making an angle β with the horizontal. Show that $\dfrac{a}{b} = \dfrac{\cos\alpha-\cos\beta}{\sin\beta-\sin\alpha}$.

Sol. Let MN be the vertical wall standing on a horizontal ground CM, M being on the ground so that $\angle$NMC = 90°.

Let AB or CD be the ladder of length l m, so that AB = CD = l m.

Given that $\angle$BAM = α and $\angle$DCM = β,

BD = b m and CA = a m.



Let AM = x m and MD = y m.

In ΔABM, we have

$$\sin(\angle BAM) = \frac{BM}{AB}$$

$$\Rightarrow \quad \sin\alpha = \frac{b+y}{l}$$

$$\Rightarrow \quad b+y = l\sin\alpha \quad \ldots(1)$$

$$\text{and} \quad \cos\alpha = \frac{x}{l} \quad \ldots(2)$$

Also, in ΔDCM, we have

$$\cos(\angle DCM) = \frac{CM}{CD}$$

$$\Rightarrow \quad \cos\beta = \frac{a+x}{l}$$

$$\Rightarrow \quad a+x = l\cos\beta \quad \ldots(3)$$

$$\text{and} \quad \sin\beta = \frac{y}{l} \quad \ldots(4)$$

From (1) and (4), we have

$$b + l\sin\beta = l\sin\alpha$$

$$\Rightarrow \quad b = l(\sin\alpha - \sin\beta) \quad \ldots(5)$$

From (2) and (3), we have

$$a + l\cos\alpha = l\cos\beta$$

$$\Rightarrow \quad a = l(\cos\beta - \cos\alpha) \quad \ldots(6)$$

Dividing (6) by (5), we get

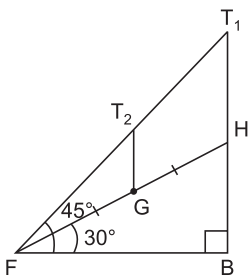
$$\frac{a}{b} = \frac{\cos\beta-\cos\alpha}{\sin\alpha-\sin\beta} = \frac{\cos\alpha-\cos\beta}{\sin\beta-\sin\alpha}$$

Hence, proved.

4. The line joining the top of a hill to the foot of the hill makes an angle of 30° with the horizontal through the foot of the hill. There is one temple at the top of the hill and a guest house halfway

from the foot to the top of the hill. The top of the temple and the top of the guest house both make an elevation of 45° at the foot of the hill. If the guest house is 100 m away from the foot of the hill, show that the height of the guest house and the height of the temple from the surface of the hill are respectively $50\left(\sqrt{3}-1\right)$ m and $100\left(\sqrt{3}-1\right)$ m.

Sol. Let H be the top and B be the bottom of the hill HB standing vertically on the horizontal ground FB, where F is a point on the ground such that $\angle HFB = 30°$.



Let HT_1 be the vertical temple at the top H of the hill such that T_1HB is a straight line and $\angle T_1BF = 90°$. Let G T_2 be the vertical guesthouse at the mid-point G of FH, the line segment joining the foot and the top of the hill. Hence, $GT_2 \parallel HT_1$. Given that $\angle T_1FB = 45°$ and FG = GH = 100 m.

We see by the basic proportionality theorem in ΔT_1FH, T_2 is the mid-point of FT_1 and $HT_1 = 2GT_2$.

Now, in ΔFBT_1, we have

$$\tan(\angle T_1FB) = \frac{T_1B}{FB}$$

$$\Rightarrow \quad \tan 45° = \frac{T_1B}{FB}$$

$$\Rightarrow \quad 1 = \frac{T_1B}{FB}$$

$$\Rightarrow \quad T_1B = FB \qquad \ldots(1)$$

Now, in ΔFBH, we have

$$\tan(\angle HFB) = \frac{BH}{FB}$$

$$\Rightarrow \quad \tan 30° = \frac{BH}{FB}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{BH}{T_1B} \qquad \text{[From (1)]}$$

$$\Rightarrow \quad T_1B = \sqrt{3}\ BH$$

$$\Rightarrow \quad T_1H + BH = \sqrt{3}\ BH$$

$$\Rightarrow \quad T_1H = (\sqrt{3}-1)\ BH \qquad \ldots(2)$$

Now, in ΔFBH

$$\sin(\angle HFB) = \frac{BH}{FH}$$

$$\Rightarrow \quad \sin 30° = \frac{BH}{200}$$

$$[\because FH = FG + GH = (100 + 100)\text{ m} = 200\text{ m}]$$

$$\Rightarrow \quad \frac{1}{2} = \frac{BH}{200}$$

$$\Rightarrow \quad BH = 100$$

$\therefore$ From (2), we have

$$HT_1 = (\sqrt{3}-1)\ 100\text{ m}$$

and $$GT_2 = \frac{1}{2} HT_1 = (\sqrt{3}-1)\text{ m}$$

Hence, the required height of the guesthouse and the temple are respectively $50\left(\sqrt{3}-1\right)$ m and $100\left(\sqrt{3}-1\right)$ m.

Self-Assessment

(Page 182)

Multiple-Choice Questions

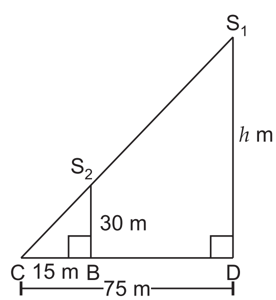
1. A vertical stick, 30 m long casts a shadow 15 m long on the ground. At the same time, a tower casts a shadow 75 m long on the ground. Then the height of the tower is

(*a*) 150 m (*b*) 100 m

(*c*) 25 m (*d*) 200 m **[CBSE SP 2012]**

Sol. (*a*) 150 m

Let S_1D and S_2B be the vertical tower and vertical stick respectively standing on the horizontal ground CD



so that $\angle CBS_2 = \angle CDS_1 = 90°$

The sun rays fall on the ground at C along the line S_1S_2 C forming shadows CB of length 15 m and CD of length 75 m. Let h m be the height of the tower. Given that $BS_2 = 30$ m.

Now, $\Delta S_2CB \sim S_1CD$.

$\therefore$ We have $\dfrac{30}{h} = \dfrac{CB}{CD}$

$\Rightarrow \quad \dfrac{30}{h} = \dfrac{15}{75}$

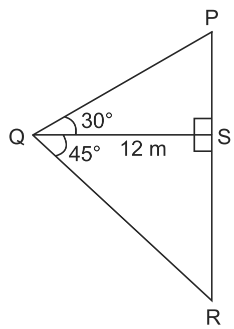
$\Rightarrow \quad \dfrac{30}{h} = \dfrac{1}{5}$

$\Rightarrow \quad h = 30 \times 5$

$\Rightarrow \quad h = 150$

Hence, the height of the tower is 150 m.

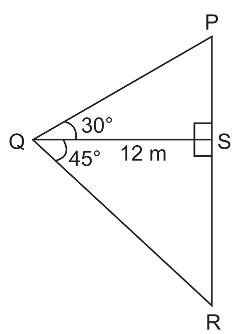
2. In the figure given below the perimeter of ΔPQR where $QS \perp PR$, $QS = 12$ m, $\angle PQS = 30°$ and $\angle RQS = 45°$ is



(*a*) $12\left(\sqrt{3}+\sqrt{2}-1\right)$ m (*b*) $12\left(\sqrt{3}-\sqrt{2}+1\right)$ m

(*c*) $12\left(\sqrt{3}+\sqrt{2}+1\right)$ m (*d*) $12\left(\sqrt{3}-\sqrt{2}-1\right)$ m

Sol. (*c*) $12\ (\sqrt{3} + \sqrt{2} + 1)$ m



In ΔPQS, we have

$$\cos(\angle PQS) = \frac{QS}{PQ}$$

$$\cos 30° = \frac{12}{PQ}$$

$\Rightarrow \quad \dfrac{\sqrt{3}}{2} = \dfrac{12}{PQ}$

$\Rightarrow \quad PQ = \dfrac{24}{\sqrt{3}}$

$$= \frac{24\sqrt{3}}{3}$$

$\Rightarrow \quad PQ = 8\sqrt{3}$ ...(1)

Also, $\sin(\angle PQS) = \dfrac{PS}{PQ}$

$\Rightarrow \quad \sin 30° = \dfrac{PS}{8\sqrt{3}}$ [From (1)]

$\Rightarrow \quad \dfrac{1}{2} = \dfrac{PS}{8\sqrt{3}}$

$\Rightarrow \quad PS = 4\sqrt{3}$ ...(2)

In ΔQRS, we have

$$\cos(\angle RQS) = \frac{QS}{QR}$$

$\Rightarrow \quad \cos 45° = \dfrac{12}{QR}$

$\Rightarrow \quad \dfrac{1}{\sqrt{2}} = \dfrac{12}{QR}$

$\Rightarrow \quad QR = 12\sqrt{2}$

Also, $\sin(\angle RQS) = \dfrac{RS}{QR}$

$\Rightarrow \quad \sin 45° = \dfrac{RS}{12\sqrt{2}}$ [From (3)]

$\Rightarrow \quad \dfrac{1}{\sqrt{2}} = \dfrac{RS}{12\sqrt{2}}$

$\Rightarrow \quad RS = 12$ ...(4)

From (1), (2), (3) and (4),

Perimeter of $\Delta PQR = PQ + PS + SR + QR$

$$= (8\sqrt{3} + 4\sqrt{3} + 12 + 12\sqrt{2})\ \text{m}$$

$$= 12\ (\sqrt{3} + \sqrt{2} + 1)\ \text{m}$$

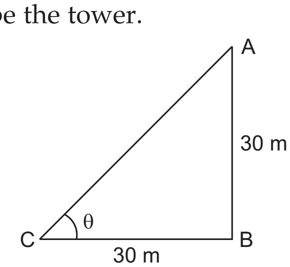
3. The angle of elevation of the top of a 30 m high tower at a point 30 m away from the base of the tower is

(*a*) 30° (*b*) 45°

(*c*) 60° (*d*) 90° **[CBSE 2023 Basic]**

Sol. (*b*) 45°

Let AB be the tower.



$$\tan\theta = \frac{AB}{BC}$$

$\Rightarrow \quad \tan\theta = \dfrac{30\ \text{m}}{30\ \text{m}}$

$\Rightarrow \quad \tan\theta = 1$

$$\Rightarrow \qquad \tan\theta = \tan 45°$$

$$\therefore \qquad \theta = 45°$$

4. If a pole of 6 m high casts a shadow $2\sqrt{3}$ m long on the ground, then Sun's elevation is

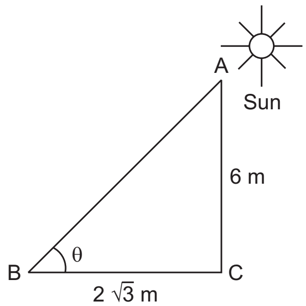
(*a*) 60° (*b*) 45°

(*c*) 30° (*d*) 90°

[CBSE 2023 Standard]

Sol. (*a*) 60°

Let the sun makes an angle θ with the ground.



$$\Rightarrow \qquad \tan\theta = \frac{AC}{BC}$$

$$\Rightarrow \qquad \tan\theta = \frac{6\text{ m}}{2\sqrt{3}\text{ m}}$$

$$\Rightarrow \qquad \tan\theta = \frac{3}{\sqrt{3}}$$

$$\Rightarrow \qquad \tan\theta = \sqrt{3}$$

$$\Rightarrow \qquad \tan\theta = \tan 60°$$

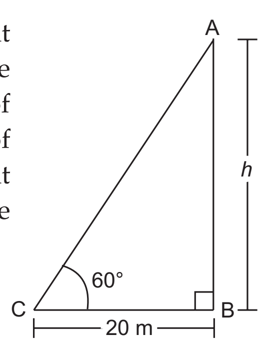
$$\therefore \qquad \theta = 60°$$

Fill in the Blanks

5. The angle of **elevation** of an object viewed, is the angle formed by the line of sight with the horizontal when it is above the horizontal level.

6. The angle of elevation of the top of a tower from a point on the ground, 20 m away from the foot of the tower is 60°. Then, the height of the tower is **20 $\sqrt{3}$ m**.

Sol. Let AB (= h) be the height of the tower and CB be the distance from the point of observation. The angle of elevation of A at the point C is 60°. In right ΔABC, we have



$$\tan 60° = \frac{AB}{BC}$$

$$\Rightarrow \qquad \sqrt{3} = \frac{h}{20\text{ m}}$$

$$\Rightarrow \qquad h = 20\sqrt{3}\text{ m}$$

7. The **line of sight** is the line drawn from the eye of an observer to the point in the object viewed by the observer.

8. If the length of the shadow of a vertical pole is equal to its height, the angle of elevation of Sun's altitude is **45°**.

Sol. Let AB be the length of the vertical pole and BC be the length of its shadow.

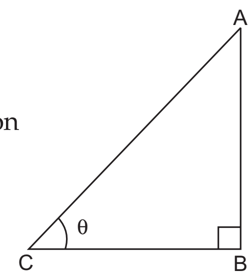
Then, AB = BC

and ∠ABC = 90°

Let θ be the angle of elevation of the Sun's altitude.

Thus,

∠ACB = θ



In right ΔABC, we have

$$\tan\theta = \frac{AB}{BC}$$

$$\Rightarrow \qquad \tan\theta = 1 \qquad [\because AB = BC]$$

$$\Rightarrow \qquad \tan\theta = \tan 45°$$

$$\Rightarrow \qquad \theta = 45°$$

Assertion-Reason Type Questions

Directions (Q. Nos. 9 to 11): Each of these questions contains an assertion followed by reason. Read them carefully, and answer the question on the basis of the following options, select the one that best describes the two statements.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

9. **Assertion (A):** If angle of elevation is 60°, distance from base is equal to the height.

Reason (R): The value of tan 45° is 1.

Sol. The correct answer is (*d*).

If tan 45° is 1 which means if angle of elevation is 45°, then distance from base is equal to height. Thus, Reason is correct but Assertion is wrong.

10. Assertion (A): The height of a pole is 6 m, then angle of depression from the top will be 45° at a distance of 6 m from the base.

Reason (R): The angle formed by the line of sight with the vertical is called the angle of depression.

Sol. The correct answer is (*b*).

Both statements are correct. However, the Reason does not tell anything about measurement.

Thus it is not a proper explanation of Assertion.

11. Assertion (A): As the distance from the base increases, the angle of elevation decreases.

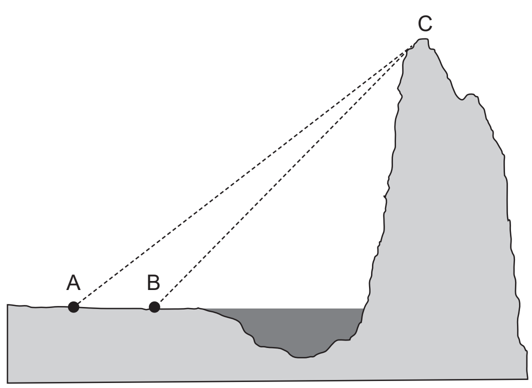
Reason (R): Angle of elevation does not depend on distance from base.

Sol. The correct answer is (*c*).

Assertion is correct, however reason is wrong as one moves away, the angle of elevation will keep decreasing.

Case Study Based Questions

12. A team went to Nainital to survey mountains. The team members A and B were standing on the ground and wanted to find the height of the mountain some distance away from the other side of the lake. One team member was standing on the top of the mountain. The angle between the horizontal ground at A and the line of sight to the top of the mountain to be 30°. The angle between the horizontal ground at B and the line of sight to the top of the mountain be 60°. The distance between A and B is 400 m. Based on the given situation, answer the following questions.



(*a*) Find the horizontal distance from B to the base of the mountain.

Ans. 200 m

(*b*) Find the height of the mountain.

Ans. 346 m

(*c*) (*i*) If the person moves to a point D, between A and B such that the angle of elevation to the top of the mountain is 45°. Find the distance of D from A.

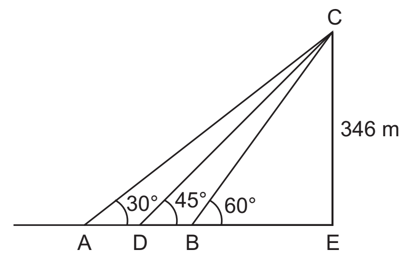
Ans. In ΔCDE,

$$\tan 45° = \frac{CE}{DE}$$

$$\Rightarrow \quad 1 = \frac{CE}{DE}$$

$$\Rightarrow \quad DE = CE$$

$$\Rightarrow \quad DE = 346 \text{ m}$$



In ΔACE,

$$\tan 30° = \frac{CE}{AE}$$

$$\Rightarrow \quad AE = \frac{346 \text{ m}}{\tan 30°} = \frac{346 \text{ m}}{1/\sqrt{3}}$$

$$\Rightarrow \quad AE = 346\sqrt{3} \text{ m} = 346 \times 1.73 \text{ m}$$

$$= 598.58 \text{ m}$$

$$\text{Distance AD} = AE - DE = 598.58 \text{ m} - 346 \text{ m}$$

$$= 252.58 \text{ m}$$

or

(*ii*) Find the distance AC.

Ans. In ΔACE,

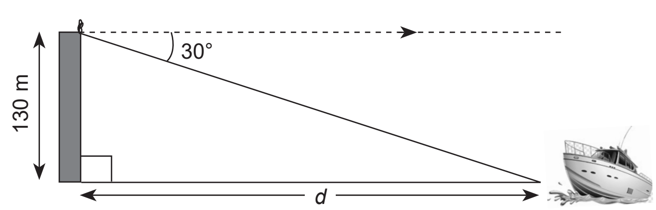
$$\cos 30° = \frac{AE}{AC}$$

$$\Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{346\sqrt{3}}{AC} \text{ m}$$

$$\Rightarrow \quad AC = 346 \times 2 \text{ m}$$

$$\Rightarrow \quad AC = 692 \text{ m}$$

13. Rishi was sitting on a roof from a height of 130 m above sea level. He saw a motor boat at sea at an angle of depression of 30°. Based on the above situation, answer the following questions.



(*a*) What is the angle made if a person sitting on boat and looking at Rishi?

Ans. 30°

(*b*) What is the horizontal distance from boat to the roof?

Ans. 224.9 m

(*c*) (*i*) What is the angle made by the roof and the sea level?

Ans. 90°

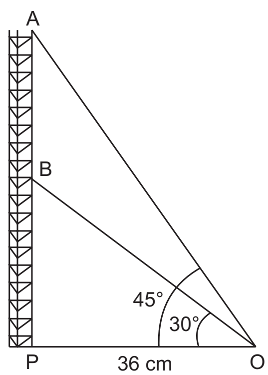
or

(*ii*) If Rishi was standing on the roof and he is 1.74 m tall, then what is the total height from where Rishi was looking at the boat?

Ans. 131.74 m

14. Radio towers are used for transmitting a range of communication services including radio and television. The tower will either act as an antenna itself or support one or more antennas on its structure. On a similar concept, a radio station tower was built in two Sections A and B. Tower is supported by wires from a point O.

Distance between the base of the tower and point O is 36 cm. From point O, the angle of elevation of the top of the Section B is 30° and the angle of elevation of the top of Section A is 45°.



Based on the above information, answer the following questions:

(*a*) Find the length of the wire from the point O to the top of Section B.

Ans. In ΔBPO,

$$\cos 30° = \frac{OP}{BO}$$

$$\Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{36 \text{ cm}}{BO}$$

$$\Rightarrow \quad BO = \frac{2}{\sqrt{3}} \times 36 \text{ cm}$$

$$= \frac{2 \times 12 \times 3}{\sqrt{3}} \text{ cm}$$

$$BO = 24\sqrt{3} \text{ cm}$$

∴ Length of wire from point O to the top of section B = $24\sqrt{3}$ cm.

(*b*) (*i*) Find the distance AB.

Ans. In ΔBPO,

$$\tan 30° = \frac{BP}{PO}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{BP}{36 \text{ cm}}$$

$$\Rightarrow \quad BP = \frac{36}{\sqrt{3}} \text{ cm} = 12\sqrt{3} \text{ cm}$$

$$\therefore \quad BP = 20.76 \text{ cm}$$

In ΔAPO,

$$\tan 45° = \frac{AP}{PO}$$

$$\Rightarrow \quad 1 = \frac{AP}{36 \text{ cm}}$$

$$\Rightarrow \quad AP = 36 \text{ cm}$$

$$AB = AP - BP$$

$$\Rightarrow \quad AB = 36 \text{ cm} - 20.76 \text{ cm}$$

$$AB = 15.24 \text{ cm}$$

or

(*ii*) Find the area of ΔOPB.

Ans. Area of ΔBPO $= \frac{1}{2} \times OP \times BP$

$$= \frac{1}{2} \times 36 \text{ cm} \times 20.76 \text{ cm}$$

$$= 373.68 \text{ cm}^2$$

(*c*) Find the height of the Section A from the base of the tower. **[CBSE 2023 Standard]**

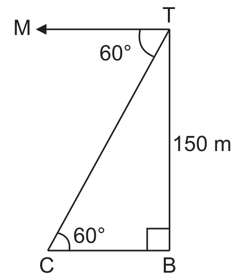
Ans. Height of the Section A from the base of the tower = AP = 36 cm.

Very Short Answer Type Questions

15. What is the distance of a car which is parked on the road, from a tower 150 m high, if the angle of depression of the car from the top of the tower is 60°?

Sol. Let TB be the tower standing vertically at B of the horizontal ground CB. C is the position of the car. TM is drawn horizontal through T so that

TM $\parallel$ CB. Also, $\angle TBC = 90°$, $\angle MTC = \angle TCB = 60°$ and BT = 150 m.



Now, in ΔCBT, we have

$$\tan(\angle TCB) = \frac{TB}{CB}$$

$$\Rightarrow \quad \tan 60° = \frac{150}{CB}$$

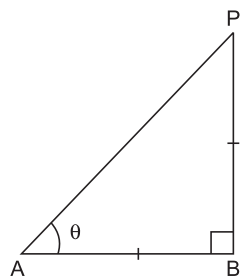
$$\Rightarrow \quad \sqrt{3} = \frac{150}{CB}$$

$$\therefore \quad CB = \frac{150}{\sqrt{3}} = \frac{150\sqrt{3}}{3} = 50\sqrt{3}.$$

$\therefore$ The required distance = $50\sqrt{3}$ m.

16. What is the angle of elevation of the Sun's altitude if the length of the shadow of a vertical pole is equal to its height?

Sol. Let PB be the vertical pole standing on the horizontal ground AB. The sun rays are falling along PA so that AB is the length of the shadow of the pole such that AB = BP. Let θ be required angle of elevation of the Sun's altitude so that $\angle PAB = \theta$.



In ΔPAB, we have

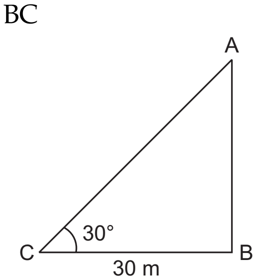
$$\tan\theta = \frac{PB}{AB} = \frac{PB}{PB} = 1 = \tan 45°$$

$$\therefore \quad \theta = 45°$$

Hence, the required angle of elevation is 45°.

17. The angle of elevation of the top of a tower from a point on the ground which is 30 m away from the foot of the tower, is 30°. Find the height of the tower.

Sol. $\tan 30° = \dfrac{AB}{BC}$



$$\Rightarrow \quad AB = \tan 30° \times BC$$

$$= \frac{1}{\sqrt{3}} \times 30 \text{ m}$$

$$= \frac{3 \times 10}{\sqrt{3}} \text{ m}$$

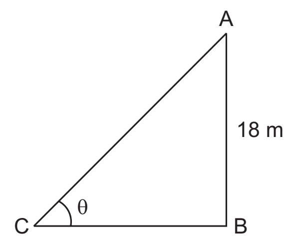
$$\Rightarrow \quad AB = 10\sqrt{3} \text{ m}$$

$\therefore$ Height of the tower = $10\sqrt{3}$ m.

18. Find the length of the shadow on the ground of a pole of height 18 m when angle of elevation θ of the Sun is such that $\tan\theta = \dfrac{6}{7}$.

[CBSE 2023 Standard]

Sol. $\tan\theta = \dfrac{AB}{BC}$



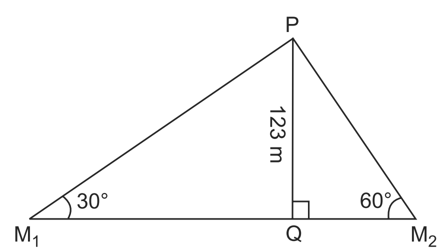
$$\Rightarrow \quad \frac{6}{7} = \frac{18\,\text{m}}{BC}$$

$$\Rightarrow \quad BC = \frac{18 \times 7}{6} \text{ m} = 21 \text{ m}$$

$\therefore$ Length of the shadow = 21 m.

Short Answer Type Questions

19. In the given figure, two men M_1 and M_2 are standing on opposite sides of a tower PQ of height 123 m. Find the distance between the men.



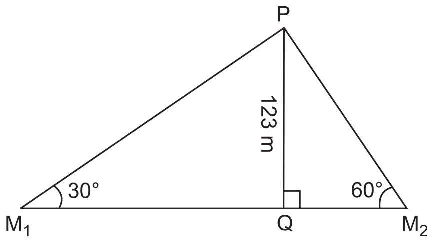
Sol. In PM_1Q, we have

$$\tan(\angle PM_1Q) = \frac{PQ}{M_1Q}$$

$$\Rightarrow \quad \tan 30° = \frac{123}{M_1Q}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{123}{M_1Q}$$

$$\Rightarrow \quad M_1Q = 123\sqrt{3} \qquad \ldots(1)$$



In, ΔPQM_2, we have

$$\tan(\angle PM_2Q) = \frac{PQ}{M_2Q}$$

$$\Rightarrow \quad \tan 60° = \frac{123}{M_2Q}$$

$$\Rightarrow \quad \sqrt{3} = \frac{123}{M_2Q}$$

$$\Rightarrow \quad M_2Q = \frac{123}{\sqrt{3}} = \frac{123\sqrt{3}}{3}$$

$$= 41\sqrt{3} \qquad \ldots(2)$$

Hence, $M_1M_2 = M_1Q + M_2Q$

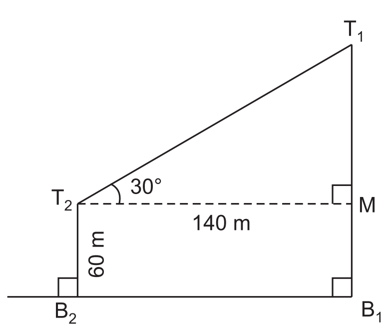
$$= 123\sqrt{3} + 41\sqrt{3}$$

$$= 164\sqrt{3}$$

Hence, the required distance between the men is $164\sqrt{3}$ m.

20. The horizontal distance between two towers of different heights is 140 m. The angle of elevation of the top of the first tower when seen from the top of the second tower is 30°. If the height of the second tower is 60 m, show that the height of the first tower is $\frac{20(7\sqrt{3}+9)}{3}$ m. **[CBSE SP 2011]**

Sol. Let T_1B_1 and T_2B_2 be two vertical towers standing on the horizontal ground B_2B_1 at B_1 and B_2 respectively so that $\angle T_2B_2B_1 = \angle T_1B_1B_2 = 90°$.



We draw T_2M horizontal, where M is a point on T_1B_1.

$\therefore \quad T_2M \parallel B_2B_1$.

It is given that

$$\angle T_1T_2M = 30°,$$

$$T_2M = 140 \text{ m}$$

and $\quad T_2B_2 = 60$ m $\qquad \ldots(1)$

Since, $\quad T_1MT_2 = 90°$,

$\therefore$ In ΔT_1T_2M, we have

$$\tan(T_1T_2M) = \frac{T_1M}{T_2M}$$

$$\Rightarrow \quad \tan 30° = \frac{T_1M}{140}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{T_1M}{140}$$

$$\therefore \quad T_1M = \frac{140}{\sqrt{3}} \qquad \ldots(2)$$

$\therefore$ Required height of the tower T_1B_1 is

$$T_1B_1 = T_1M + MB_1$$

$$= T_1M + T_2B_2$$

[$\because$ $T_2MB_1B_2$ is a rectangle]

$$= \left(\frac{140}{\sqrt{3}} + 60\right) \text{m}$$

[From (1) and (2)]

$$= \frac{140 + 60\sqrt{3}}{\sqrt{3}} \text{ m}$$

$$= \frac{\left(140 + 60\sqrt{3}\right)\sqrt{3}}{3} \text{ m}$$

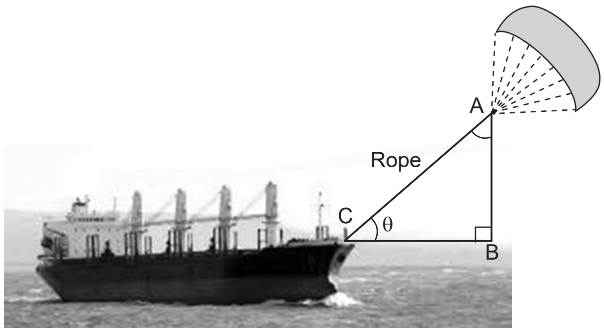
$$= \frac{140\sqrt{3} + 180}{3} \text{ m}$$

$$= \frac{20\left(7\sqrt{3} + 9\right)}{3} \text{ m}$$

Hence, the height of the first tower is $\frac{20(7\sqrt{3}+9)}{3}$ m.

21. 'Skysails' is that genre of engineering science that uses extensive utilization of wind energy to move a vessel in the sea water. The 'Skysails' technology allows the towing kite to gain a height of anything between 100 metres – 300 metres. The sailing kite is made in such a way that it can be raised to its proper elevation and then brought back with the help of a 'telescopic mast' that enables the kite to be raised properly and effectively.

Based on the following figure related to sky sailing, answer the questions:



(*a*) In the given figure, if $\sin\theta = \cos(3\theta - 30°)$, where θ and $3\theta - 30°$ are acute angles, then find the value of θ.

(*b*) What should be the length of the rope of the kite sail in order to pull the ship at the angle θ (calculated above) and be at a vertical height of 200 m? **[CBSE SP(Standard) 2019]**

Sol. (*a*) $\sin\theta = \cos(3\theta - 30°)$

$\Rightarrow \quad \cos(90° - \theta) = \cos(3\theta - 30°)$

$[\because \sin\theta = \cos(90° - \theta)]$

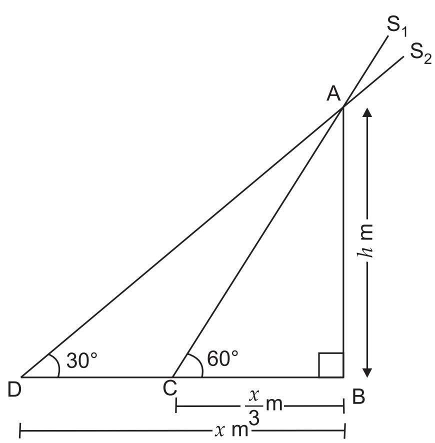
$\Rightarrow \quad 90° - \theta = 3\theta - 30° \Rightarrow \theta = 30°$

(*b*) $\frac{AB}{AC} = \sin 30°$

$\therefore$ Length of rope = AC = 400 m

22. The length of the shadow of a tower at a particular time is one-third of its shadow, when the Sun's rays meet the ground at an angle of 30°. Find the angle between the Sun's rays and the ground at the time of shorter shadow.

Sol. Let AB be the vertical tower of height h m, BC is the length of the shadow when the position of the Sun is at S_1 and the Sun's ray marks an angle θ with the horizontal ground CB. BD is the length of the shadow when the position of the Sun is at S_2 and the Sun's ray AD makes an angle 30° with the ground DB.



Hence, $\angle ADB = 30°$ and $\angle ACB = \theta$.

Also, $\angle ABC = \angle ABD = 90°$. Let BD = x m.

Then BC = $\frac{x}{3}$ m.

Now, in ΔACB, we have

$$\tan(\angle ACB) = \frac{AB}{BC}$$

$$\Rightarrow \quad \tan\theta = \frac{h}{\frac{x}{3}} = \frac{3h}{x} \quad \ldots(1)$$

Also, in ΔADB, we have,

$$\tan(\angle ADB) = \frac{AB}{DB}$$

$$\Rightarrow \quad \tan 30° = \frac{h}{x}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{x} \quad \ldots(2)$$

$\therefore$ From (1) and (2), we have

$$\tan\theta = \frac{3}{\sqrt{3}} = \sqrt{3} = \tan 60°$$

$$\therefore \quad \theta = 60°$$

Hence, the required angle is 60°.

23. A man on the roof of a house which is 10 m high observes the angle of elevation of the top of a building as 45° and the angle of depression of the base of the building as 30°. Find the height of the building and its distance from the house. **[CBSE 2000]**

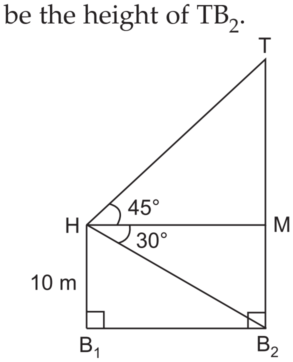
Sol. Let HB_1 be the house standing vertically on the horizontal ground B_1B_2 at the point B_1 such that

$$HB_1B_2 = 90°$$

and $\quad HB_1 = 10$ m

Let TB_2 be the building vertically on the ground at the point B_2 such that $\angle TB_2B_1 = 90°$, $\angle THM = 45°$ $\angle MHB_2 = 30°$, where HM is horizontal through H and M is a point on TB_2.

Let h m be the height of TB_2.



Now, in ΔTHM, we have

$$\tan(\angle THM) = \frac{TM}{HM}$$

$$\Rightarrow \quad \tan 45° = \frac{TM}{HM}$$

$$\Rightarrow \quad 1 = \frac{TM}{B_1B_2}$$

[$\because$ B_1B_2MH is a rectangle]

$$\therefore \quad B_1B_2 = TM \qquad \ldots(1)$$

Again, in HMB_2, we have

$$\tan(\angle MHB_2) = \frac{MB_2}{HM}$$

$$\Rightarrow \quad \tan 30° = \frac{MB_2}{B_1B_2}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{MB_2}{B_1B_2} = \frac{MB_2}{TM} \qquad \text{[From (1)]}$$

$$= \frac{10}{TM} \qquad [\because MB_2 = HB_1 = 10]$$

Hence, TM = $10\sqrt{3}$.

$$\Rightarrow \quad B_1B_2 = TM = 10\sqrt{3}$$

Now, $\quad TB_2 = TM + MB_2 = TM + HB_1$

$$= 10\sqrt{3} + 10$$

$$= 10\left(\sqrt{3}+1\right)$$

Hence, the required height of the building is $10\left(\sqrt{3}+1\right)$ m and its distance from the house is $10\sqrt{3}$ m.

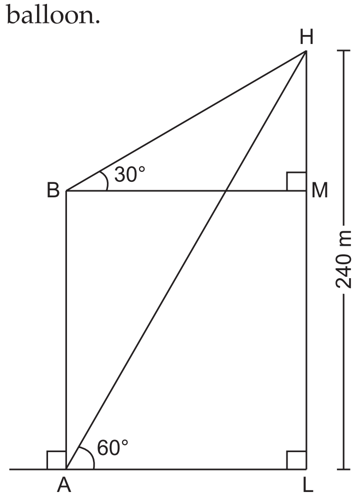
Long Answer Type Questions

24. The height of a hill is 240 m above the level of a horizontal plane. From a point A on this plane, the angular elevation of the top of the hill is 60°. A balloon rises from A and ascends vertically upwards at a uniform rate; after $2\frac{1}{12}$ minutes, the angular elevation of the top of the hill to an observer in the balloon is 30°. Find the speed of ascent of the balloon in m/s.

Sol. Let HL be the vertical hill standing on the horizontal ground AL such that $\angle HAL = 60°$ and $\angle ALH = 90°$.

A balloon ascends vertically upwards along AB at a uniform speed and comes to B after $2\frac{1}{12}$ minutes where $\angle BAL = 90°$. We draw BM horizontal through B where M is a point on HL. Given that $\angle HBM = 30°$, HL = 240 m.

Let HM = h m and v m/min be the uniform speed of the balloon.



Since, ABML is a rectangle, hence, ML = AB = $2\frac{1}{12}$ metres.

Now, in ΔHBM, we have,

$$\tan(\angle HBM) = \frac{HM}{BM}$$

$$\Rightarrow \quad \tan 30° = \frac{h}{BM}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{BM}$$

$$\therefore \quad BM = h\sqrt{3} \qquad \ldots(1)$$

Also, in ΔHAL, we have

$$\tan(\angle HAL) = \frac{HL}{AL}$$

$$\Rightarrow \quad \tan 60° = \frac{HL}{BM} \qquad [\because AL = BM]$$

$$\Rightarrow \quad \sqrt{3} = \frac{HL}{h\sqrt{3}} \qquad \text{[From (1)]}$$

$$\therefore \quad HL = 3h$$

$$\Rightarrow \quad 240 = 3h$$

$$\Rightarrow \quad h = 80 \qquad \ldots(2)$$

$$\therefore \quad AB = ML$$

$$= HL - HM$$

$$= 240 - h$$

$$= 240 - 80 = 160 \qquad \text{[From (2)]}$$

$$\therefore \quad v = \frac{AB}{2\frac{1}{12} \times 60} \text{ m/s}$$

$$= \frac{160}{\frac{25}{12} \times 60} \text{ m/s}$$

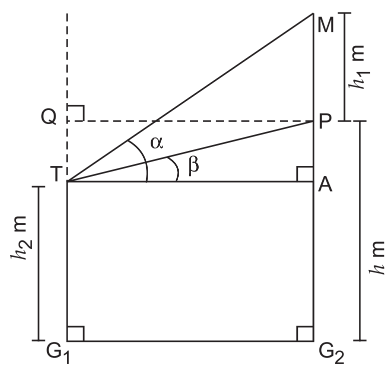
$$= \frac{160 \times 12}{25 \times 60} \text{ m/s}$$

$$= 1.28 \text{ m/s}$$

Hence, the required speed is 1.28 m/s.

25. A marble statue of height h_1 metres is mounted on a pedestal. The angles of elevation of the top and bottom of the statue from a point h_2 metres above the ground level are α and β respectively. Show that the height of the pedestal in metres is $\dfrac{(h_1 - h_2)\tan\beta + h_2\tan\alpha}{\tan\alpha - \tan\beta}$.

Sol. Let PQ be the horizontal pedestal and PM be the marble statue of height h_1 on it so that PM = h_1 m.



M is the top and P is the bottom of the statue. G_1G_2 is the horizontal ground and $TG_1 \perp G_1G_2$ and $TG_1 = h_2$ m. Given that the angles of elevation of M and P of the temple from the point T are α and β respectively so that ∠MTA = α and ∠PTA = β where $MPAG_2$ is a straight line and $MG_2 \perp G_1G_2$. Also, ∠PAT = 90°. Now let the height of the pedestal from the ground be h m so that $PG_2 = h$ m.

Now, since G_1G_2 AT is a rectangle,

$\therefore \quad AG_2 = TG_1 = h_2$

$\therefore \quad AP = PG_2 - AG_2 = h - h_2$

$$MA = MP + AP$$
$$= h_1 + h - h_2$$
$$= (h_1 - h_2) + h$$

Now, in ΔMTA, we have

$$\tan(\angle MTA) = \frac{MA}{AT}$$

$$\Rightarrow \quad \tan\alpha = \frac{(h_1 - h_2) + h}{AT} \quad \ldots(1)$$

Also, in ΔPTA, we have $\tan(\angle PTA) = \dfrac{PA}{AT}$

$$\Rightarrow \quad \tan\beta = \frac{h - h_2}{AT} \quad \ldots(2)$$

∴ Dividing (1) by (2), we get

$$\frac{\tan\alpha}{\tan\beta} = \frac{(h_1 - h_2) + h}{h - h_2}$$

$\Rightarrow h\tan\alpha - h_2\tan\alpha = (h_1 - h_2)\tan\beta + h\tan\beta$

$\Rightarrow h(\tan\alpha - \tan\beta) = h_2\tan\alpha + (h_1 - h_2)\tan\beta$

$$\Rightarrow h = \frac{h_2\tan\alpha + (h_1 - h_2)\tan\beta}{\tan\alpha - \tan\beta}$$

Hence, proved.

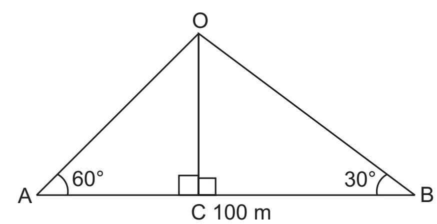
26. One observer estimates the angle of elevation to the basket of a hot air balloon to be 60°, while another observer 100 m away estimates the angle of elevation to be 30°. Find

(*a*) the height of the basket from the ground.

(*b*) the distance of the basket from the first observer's eye.

(*c*) the horizontal distance of the second observer from the basket. **[CBSE 2023 Standard]**

Sol. (*a*) Let the basket of the hot air balloon be at point O.



In ΔOAC,

$$\tan 60° = \frac{OC}{AC}$$

$$\Rightarrow \quad AC = \frac{OC}{\tan 60°} = \frac{OC}{\sqrt{3}}$$

$$\Rightarrow \quad AC = \frac{OC}{\sqrt{3}} \quad \ldots(1)$$

In ΔOBC,

$$\tan 30° = \frac{OC}{BC}$$

$$\Rightarrow \quad BC = \frac{OC}{\tan 30°}$$

$$\Rightarrow \quad BC = \sqrt{3}\,OC \quad \ldots(2)$$

Adding equations (1) and (2), we get

$$AC + BC = \frac{OC}{\sqrt{3}} + \sqrt{3}\,OC$$

$$\Rightarrow \quad 100 \text{ m} = OC\left[\frac{1}{\sqrt{3}} + \sqrt{3}\right]$$

[AC + BC = AB = 100 m]

$$100 \text{ m} = OC \times \frac{4}{\sqrt{3}}$$

$$\Rightarrow \quad OC = \frac{100 \times \sqrt{3}}{4} \text{ m} = 25\sqrt{3} \text{ m}$$

$\therefore$ Height of the basket from the ground

$$= 25\sqrt{3} \text{ m}$$

(*b*) In ΔOAC,

$$\sin 60° = \frac{OC}{OA}$$

$$\Rightarrow \quad OA = OC \times \frac{2}{\sqrt{3}} = 25\sqrt{3} \times \frac{2}{\sqrt{3}}$$

$$= 50 \text{ m}$$

$\therefore$ Distance of the basket from the first observer's eye = 50 m.

(*c*) In ΔOCB,

$$\tan 30° = \frac{OC}{BC}$$

$$\Rightarrow \quad BC = OC \times \sqrt{3}$$

$$\Rightarrow \quad BC = 25\sqrt{3} \times \sqrt{3} \text{ m}$$

$$\Rightarrow \quad BC = 75 \text{ m}$$

$\therefore$ The horizontal distance of the second observer form the basket = 75 m.

Let's Compete

(Page 185)

Multiple-Choice Questions

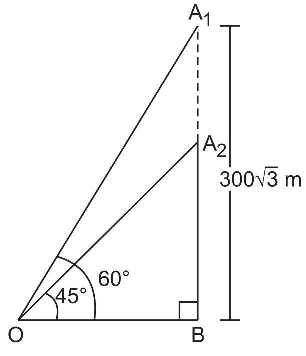
1. An aeroplane when $300\sqrt{3}$ m high passes vertically above another aeroplane at an instant when the angles of elevation of the two aeroplanes from the same point on the ground are 60° and 45° respectively. Then the vertical distance between the two aeroplanes in metres is

(*a*) $300(\sqrt{3}-1)$ m (*b*) $300(\sqrt{2}-1)$ m

(*c*) $200\sqrt{3}$ m (*d*) $200(\sqrt{3}-1)$ m

Sol. (*a*) $300\left(\sqrt{3}-1\right)$ m

Let A_1 and A_2 be the positions of two aeroplanes, A_1 being vertically above A_2 and the straight line A_1A_2B is perpendicular to OB, the horizontal ground.



It is given that $A_1B = 300\sqrt{3}$ m, $\angle A_1OB = 60°$ and $\angle A_2OB = 45°$.

Now, in ΔA_1OB, we have

$$\tan(\angle A_1OB) = \frac{A_1B}{OB}$$

$$\Rightarrow \quad \tan 60° = \frac{300\sqrt{3}}{OB}$$

$$\Rightarrow \quad \sqrt{3} = \frac{300\sqrt{3}}{OB}$$

$$\Rightarrow \quad OB = 300 \quad \ldots(1)$$

Now, in ΔA_2OB, we have

$$\tan(\angle A_2OB) = \frac{A_2B}{OB} = \frac{A_2B}{300} \quad \text{[From (1)]}$$

$$\Rightarrow \quad \tan 45° = \frac{A_2B}{300}$$

$$\Rightarrow \quad 1 = \frac{A_2B}{300}$$

$$\therefore \quad A_2B = 300 \quad \ldots(2)$$

Now, $A_1A_2 = A_1B - A_2B$

$$= \left(300\sqrt{3} - 300\right) \text{ m} \quad \text{[From (2)]}$$

$$= 300\left(\sqrt{3}-1\right) \text{ m}$$

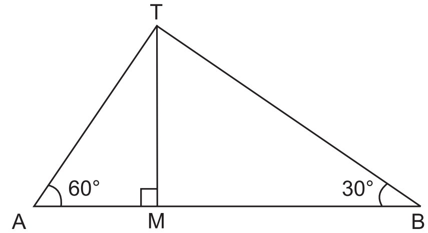
2. There is a small island in the middle of a river, *h* m wide. A tall tree stands on the island. A and B are points directly opposite to each other on the two banks, and in line with the tree. If the angles of elevation of the top of the tree from A and B are 60° and 30° respectively, then the height of the tree in metres is

(*a*) $\frac{2}{\sqrt{3}}h$ (*b*) $\frac{4}{\sqrt{3}}h$

(*c*) $\frac{\sqrt{3}}{4}h$ (*d*) $\frac{\sqrt{2}}{4}h$

Sol. (*c*) $\frac{\sqrt{3}}{4}h$

Let A and B be two points on the same line directly opposite to each other on the two banks of the river such that AB = *h* m. TM is the vertical tree on an island in the river such that $\angle TAM = 60°$ and $\angle TBM = 30°$.



Now, in ΔATM, we have

$$\tan(\angle TAM) = \frac{TM}{AM}$$

$$\Rightarrow \qquad \tan 60° = \frac{TM}{AM}$$

$$\Rightarrow \qquad \sqrt{3} = \frac{TM}{AM}$$

$$\Rightarrow \qquad AM = \frac{TM}{\sqrt{3}} \qquad \ldots(1)$$

In ΔBTM, we have

$$\tan(\angle TBM) = \frac{TM}{BM}$$

$$\Rightarrow \qquad \tan 30° = \frac{TM}{BM}$$

$$\Rightarrow \qquad \frac{1}{\sqrt{3}} = \frac{TM}{BM}$$

$$\therefore \qquad BM = TM\sqrt{3} \qquad \ldots(2)$$

Adding (1) and (2) we get

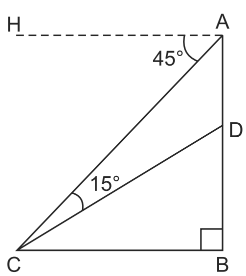
$$AM + BM = TM\left(\frac{1}{\sqrt{3}} + \sqrt{3}\right) = \frac{4TM}{\sqrt{3}}$$

$$\Rightarrow \qquad AB = h = \frac{4TM}{\sqrt{3}}$$

$$\Rightarrow \qquad TM = \frac{\sqrt{3}h}{4}$$

Hence, the height of the tree in $\frac{\sqrt{3}h}{4}$ m.

3. In the given figure, if DB = $\sqrt{3}$ m, then the measure of BC and the angle of depression of the point C when observed from the point D are respectively



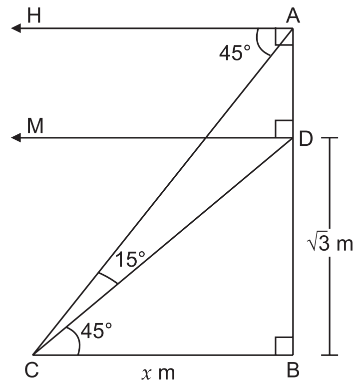
(*a*) $\sqrt{3}$ m, 45° (*b*) 3 m, 30°

(*c*) $3\sqrt{3}$ m, 30° (*d*) $\frac{\sqrt{3}}{3}$ m, 45°

Sol. (*b*) 3 m, 30°

We draw DM horizontal through D, a point on AB such that DB = $\sqrt{3}$ m.

$\therefore$ DM $\parallel$ AH $\parallel$ BC.



The angle of depression of the point C when observed from D = $\angle$MDC = $\angle$DCB

$$= \angle ACB - \angle ACD$$

$$= 45° - 15° = 30°$$

Let $\qquad$ BC = x m

Now, in ΔDCB, we have

$$\tan(\angle DCB) = \frac{DB}{BC}$$

$$\Rightarrow \qquad \tan 30° = \frac{\sqrt{3}}{x}$$

$$\Rightarrow \qquad \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{x}$$

$$\Rightarrow \qquad x = 3$$

$\therefore$ BC = 3 m and angle of depression = 30°.

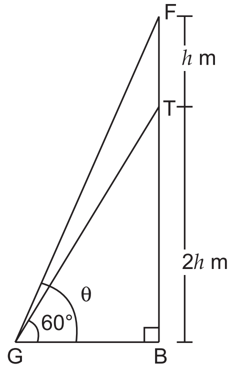
4. If the height of a flagstaff is half the height of the tower on which it is fixed and the angle of elevation of the top of the tower as seen from a point on the ground is 60°, then the angle of elevation θ of the top of the flagstaff as seen from the same point is given by the equation cosec $\theta = a$ where a is equal to

(*a*) $\frac{3\sqrt{3}}{\sqrt{31}}$ (*b*) $\sqrt{\frac{3}{31}}$

(*c*) $\frac{31}{3\sqrt{3}}$ (*d*) $\frac{\sqrt{31}}{3\sqrt{3}}$

Sol. (*d*) $\frac{\sqrt{31}}{3\sqrt{3}}$

Let TB be the vertical tower of height $2h$ m standing at a point B on the horizontal ground GB so that $\angle$TBG = 90°.



TF is the flagstaff on the top of the tower of height h m such that FTB is a straight line.

Given that $\angle TGB = 60°$.

Let $\angle FGB = \theta$.

Now, in ΔTGB, we have

$$\tan(\angle TGB) = \frac{TB}{GB}$$

$$\Rightarrow \quad \tan 60° = \frac{2h}{GB}$$

$$\Rightarrow \quad \sqrt{3} = \frac{2h}{GB} \quad \ldots(1)$$

Also, in ΔFGB, we have

$$\tan(\angle FGB) = \frac{FB}{GB}$$

$$\Rightarrow \quad \tan\theta = \frac{3h}{GB} \quad \ldots(2)$$

Dividing (1) by (2), we get

$$\frac{\sqrt{3}}{\tan\theta} = \frac{2}{3}$$

$$\Rightarrow \quad \tan\theta = \frac{3\sqrt{3}}{2}$$

$$\therefore \quad \cot\theta = \frac{2}{3\sqrt{3}} \quad \ldots(3)$$

$$\therefore \quad \operatorname{cosec}\theta = \sqrt{1+\cot^2\theta}$$

$$= \sqrt{1+\frac{4}{27}} \quad \text{[From (3)]}$$

$$= \sqrt{\frac{31}{27}}$$

$$= \frac{\sqrt{31}}{3\sqrt{3}}$$

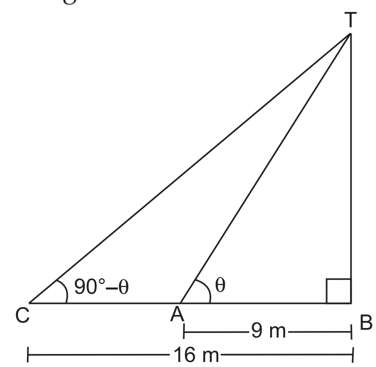
$$\therefore \quad a = \frac{\sqrt{31}}{3\sqrt{3}} \quad [\because a = \operatorname{cosec}\theta]$$

5. If the angles of elevation of the top of a tower from two points at distances of 9 m and 16 m from the base of the tower and in the same line and in the same direction of the tower, are complementary, then the height of the tower is

(*a*) 15 m (*b*) 14 m
(*c*) 12 m (*d*) 20 m

Sol. (*c*) 12 m

Let TB be the vertical tower standing on the horizontal ground CAB at B such that $\angle CBT = 90°$.



A and B are two points on the ground lying on the same line CAB such that $\angle TAB = \theta$ and $\angle TCB = 90° - \theta$.

Given that AB = 9 m and BC = 16 m.

In ΔTAB, we have

$$\tan(\angle TAB) = \frac{TB}{AB}$$

$$\Rightarrow \quad \tan\theta = \frac{TB}{9} \quad \ldots(1)$$

and in ΔTCB, we have

$$\tan(\angle TCB) = \frac{TB}{BC}$$

$$\Rightarrow \quad \tan(90° - \theta) = \frac{TB}{16}$$

$$\Rightarrow \quad \cot\theta = \frac{TB}{16}$$

$$\Rightarrow \quad \frac{1}{\tan\theta} = \frac{TB}{16} \quad \ldots(2)$$

Multiplying (1) and (2), we get

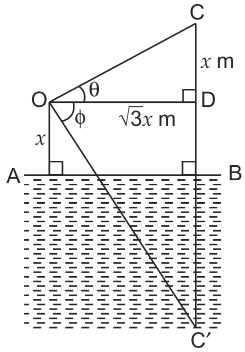
$$1 = \frac{TB^2}{16 \times 9}$$

$$\Rightarrow \quad TB = \sqrt{16 \times 9}$$

$$= 4 \times 3 = 12$$

$\therefore$ The height of the tower is 12 m.

6. In the figure given below, C′ is reflection of the cloud C in the lake with surface of water along the horizontal ground AB. If the angle of elevation of C and the angle of depression of the point C′ from the same point of observation O be θ and ϕ respectively, and if CD = x m, OD = $\sqrt{3}\,x$ m, then $\theta + \phi$ is equal to



(*a*) 45° (*b*) 60°
(*c*) 30° (*d*) 90°

Sol. (*d*) 90°

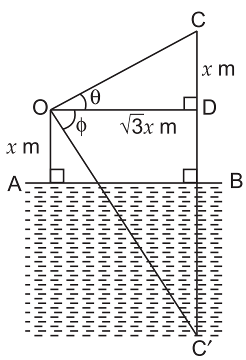
In ΔCOD, we have

$$\tan(\angle COD) = \frac{CD}{OD}$$

$$\Rightarrow \quad \tan\theta = \frac{x}{\sqrt{3}x}$$

$$= \frac{1}{\sqrt{3}} = \tan 30^\circ$$

$$\therefore \quad \theta = 30^\circ \qquad \ldots(1)$$



In ΔOC′D, we have

$$\tan(\angle DOC') = \frac{DC'}{OD} = \frac{DC'}{\sqrt{3}x} \qquad \ldots(2)$$

Now, $DC' = CC' - CD$

$= 2BC - CD$

$= 2 \times 2x - x$

$= 3x$

∴ From (2),

$$\tan\phi = \frac{3x}{\sqrt{3}x} = \sqrt{3} = \tan 60^\circ$$

$$\therefore \quad \phi = 60^\circ$$

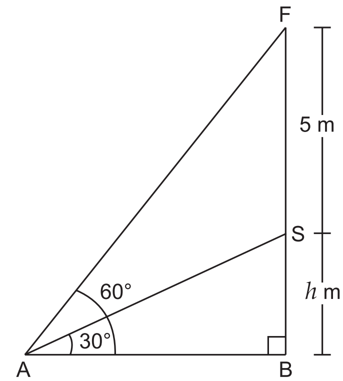
$$\therefore \quad \theta + \phi = 30^\circ + 60^\circ = 90^\circ$$

7. A vertical steel rod stands on a horizontal plane and is surmounted by a vertical flagstaff of height 5 m. At a point on the plane the angles of elevation of the bottom and the top of the flagstaff are 30° and 60° respectively. Then the height of the steel rod is

(*a*) 1.7 m (*b*) 2.5 m
(*c*) 2 m (*d*) 1.5 m

Sol. (*b*) 2.5 m

Let h m be the height of the vertical steel rod BS standing on the horizontal ground AB at the point B so that ∠ABF = 90°.



Let SF be the height of the flagstaff so that SF = 5 m and FSB is a straight line. A is a point on the ground such that ∠SAB = 30° and ∠FAB = 60°.

Now, in ΔSAB, we have

$$\tan(\angle SAB) = \frac{SB}{AB}$$

$$\Rightarrow \quad \tan 30^\circ = \frac{h}{AB}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{AB}$$

$$\therefore \quad AB = h\sqrt{3} \qquad \ldots(1)$$

Again, in ΔFAB, we have

$$\tan(\angle FAB) = \frac{BF}{AB}$$

$$\Rightarrow \quad \tan 60^\circ = \frac{5+h}{\sqrt{3}h} \qquad \text{[From (1)]}$$

$$\Rightarrow \quad \sqrt{3} = \frac{5+h}{\sqrt{3}h}$$

$$\Rightarrow \quad 3h = 5 + h$$

$$\Rightarrow \quad 2h = 5$$

$$\Rightarrow \quad h = \frac{5}{2} = 2.5$$

Hence, the required height of the steel rod is 2.5 m.

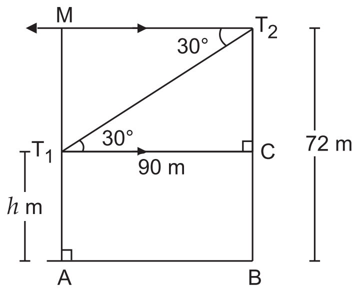
8. The horizontal distance between two trees of different heights is 90 m. The angle of depression of the top of the first tree when seen from the top of the second tree is 30°. If the height of the second tree is 72 m, then the height of the first tree will be

(*a*) $\left(72-30\sqrt{3}\right)$ m (*b*) 30 m

(*c*) $\frac{30}{\sqrt{3}}$ m (*d*) $25\sqrt{3}$ m

Sol. (*a*) $\left(72-30\sqrt{3}\right)$ m

Let T_1A be the first and T_2B be the second of the two vertical trees standing on the horizontal ground AB at A and B respectively. T_1C is drawn horizontal so that $T_1C \parallel AB$, C being a point on BT_2.



Clearly, $\angle ABC = \angle T_1CT_2 = \angle T_1AB$

$= \angle MT_1C = 90°$

where T_2M is drawn parallel to AB.

$\Rightarrow$ $T_2M \parallel CT_1 \parallel AB$.

Given that $T_1C = 90$ m, $T_2B = 72$ m,

$\angle MT_2T_1 = \angle T_2T_1C = 30°$.

Let $T_1A = h$ m.

Now, in ΔT_1T_2C, we have

$$\tan(\angle T_2T_1C) = \frac{T_2C}{T_1C}$$

$$\Rightarrow \quad \tan 30° = \frac{72-h}{90}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{72-h}{90}$$

$$\Rightarrow \quad 72\sqrt{3} - h\sqrt{3} = 90$$

$$\Rightarrow \quad h = \frac{72\sqrt{3}-90}{\sqrt{3}}$$

$$= 72 - \frac{90\sqrt{3}}{3}$$

$$= 72 - 30\sqrt{3}$$

Hence, the height of the first tree is $\left(72-30\sqrt{3}\right)$ m.

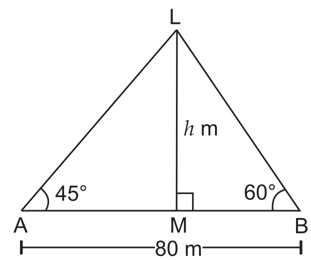
9. Two boats approach a lighthouse in mid sea from opposite directions. The angles of elevation of the top of the lighthouse from two boats are 45° and 60° respectively. If the distance between the two boats is 80 m, then the height of the lighthouse is

(*a*) 41.7 m (*b*) 40 m

(*c*) $40\left(3+\sqrt{3}\right)$ m (*d*) $40\left(3-\sqrt{3}\right)$ m

Sol. (*d*) $40(3-\sqrt{3})$ m

Let LM be the vertical lighthouse in mid sea and A, B are two boats on the opposite sides of the lighthouse. The two boats are on the two opposite sides of the lighthouse and these two boats are approaching towards the lighthouse so that A, M and B may lie on the same straight line. Given that AB = 80 m, $\angle LAM = 45°$ and $\angle LBM = 60°$. Let h m be the height of the lighthouse. We have $\angle LMB = \angle LMA = 90°$.



Now, in ΔLAM, we have

$$\tan(\angle LAM) = \frac{LM}{AM}$$

$$\Rightarrow \quad \tan 45° = \frac{h}{AM}$$

$$\Rightarrow \quad 1 = \frac{h}{AM}$$

$$\therefore \quad h = AM \quad \text{...(1)}$$

Again, in ΔLMB, we have

$$\tan(\angle LMB) = \frac{LM}{MB}$$

$$\Rightarrow \quad \tan 60° = \frac{h}{MB}$$

$$\Rightarrow \quad \sqrt{3} = \frac{h}{MB}$$

$$\Rightarrow \quad MB = \frac{h}{\sqrt{3}} \quad \text{...(2)}$$

$\therefore$ From (1) and (2) we have

$$AM + MB = AB = 80$$

$$\Rightarrow \quad h + \frac{h}{\sqrt{3}} = 80$$

$$\Rightarrow \quad h.\frac{1+\sqrt{3}}{\sqrt{3}} = 80$$

$$\Rightarrow \quad h = \frac{80\sqrt{3}}{\sqrt{3}+1}$$

$$= \frac{80\sqrt{3}\left(\sqrt{3}-1\right)}{3-1}$$

$$= 40\sqrt{3}\left(\sqrt{3}-1\right)$$

$$= 40\left(3-\sqrt{3}\right)$$

$\therefore$ The required height of the lighthouse is $40\left(3-\sqrt{3}\right)$ m.

10. Two poles of equal heights are standing opposite to each other on either side of a road which is 100 m wide. From a point between them on the road, the angles of elevation of their tops are 30° and 60°. Then the height of each pole is

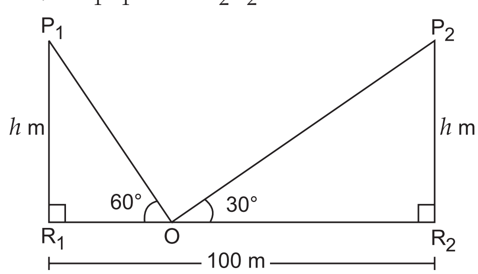
(*a*) $\left(25-\sqrt{3}\right)$ m (*b*) $\left(25+\sqrt{3}\right)$ m

(*c*) $25\sqrt{3}$ m (*d*) $\frac{25}{\sqrt{3}}$ m

Sol. (*c*) $25\sqrt{3}$ m

Let R_1R_2 be the width of the horizontal road such that $R_1R_2 = 100$ m.

R_1P_1 and R_2P_2 are two vertical poles of equal height *h* m standing opposite to each other on the two sides of the road. Let O be a point on R_1R_2 such that $\angle P_1OR_1 = 60°$ and $\angle P_2OR_2 = 30°$. Also, $\angle P_1R_1O = \angle P_2R_2O = 90°$.



Now, in ΔP_1R_1O, we have

$$\tan(\angle P_1OR_1) = \frac{P_1R_1}{R_1O}$$

$$\Rightarrow \quad \tan 60° = \frac{h}{R_1O}$$

$$\Rightarrow \quad \sqrt{3} = \frac{h}{R_1O}$$

$$\Rightarrow \quad R_1O = \frac{h}{\sqrt{3}} \quad \ldots(1)$$

Again, in ΔP_2R_2O, we have

$$\tan(\angle P_2OR_2) = \frac{P_2R_2}{OR_2}$$

$$\Rightarrow \quad \tan 30° = \frac{h}{OR_2}$$

$$\Rightarrow \quad OR_2 = h\sqrt{3} \quad \ldots(2)$$

Adding (1) and (2), we get

$$R_1O + OR_2 = R_1R_2 = 100$$

$$\Rightarrow \quad \frac{h}{\sqrt{3}} + h\sqrt{3} = 100$$

$$\Rightarrow \quad h\left(\frac{1+3}{\sqrt{3}}\right) = 100$$

$$\Rightarrow \quad h = \frac{100\sqrt{3}}{4}$$

$$= 25\sqrt{3}$$

Hence, the required height of each pole is $25\sqrt{3}$ m.

Life Skills

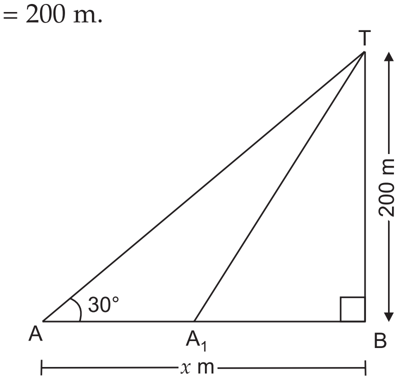
(Page 187)

1. A guard observes an enemy boat from the top of a 200 m high observation tower. He finds the angle of depression of the boat to be 30°.

(*a*) Calculate the distance of the boat from the foot of the observation tower.

(*b*) He raises an alarm when the distance of the enemy boat from the foot of the tower reduces by $200\left(\sqrt{3}-1\right)$ m. What is the new angle of depression of the boat from the top of the observation tower?

Sol. (*a*) Let BT be the vertical observation tower in the mid sea and AB is the horizontal surface of the sea. Let A be the initial position of the enemy boat such that $\angle TAB = 30°$. Also, $\angle TBA = 90°$ and TB = 200 m.



Let AB = *x* m.

In ΔTAB, we have

$$\tan(\angle TAB) = \frac{TB}{AB}$$

$$\Rightarrow \quad \tan 30° = \frac{TB}{x}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{TB}{x}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{200}{x}$$

$$\Rightarrow \qquad x = 200\sqrt{3}$$

Hence, the required distance of the boat A from B is $200\sqrt{3}$ m.

(*b*) The guard raises an alarm, when the boat comes to A_1 when $AA_1 = \left(\sqrt{3}-1\right)$ m

So that $\qquad A_1B = AB - AA_1$

$$= \left\{200\sqrt{3} - 200\left(\sqrt{3}-1\right)\right\} \text{ m}$$

$$= 200 \text{ m} \qquad \ldots(1)$$

$\therefore$ In ΔTA_1B, we have

$$\tan(\angle TA_1B) = \frac{TB}{A_1B} = \frac{200}{200} \qquad \text{[From (1)]}$$

$$= 1 = \tan 45°$$

$$\therefore \qquad \angle TA_1B = 45°$$

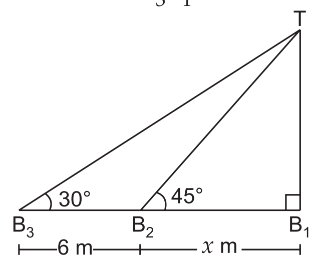
Hence, the required new angle of depression of the boat from the top of the observation tower is 45°.

2. Two villages were connected by a small canal full of water. There was only one school in one of the villages. The children of another village could not attend this school, because there was no bridge over the canal. So, the poor children were facing tremendous difficulties, to get education. An NGO, after knowing about this difficulty, came to visit the place. They decided to construct a temporary bridge at a cheap rate with bamboo sticks of shortest possible lengths for the convenience of the children. So, they wanted to find the breadth of the canal cleverly. An expert in the NGO observed the top of a tall tree just on the opposite bank of the canal from one side of the canal where all other people were standing. He measured the angle of elevation of the top of the tree with his sextant instrument to be 45°. On receding 6 m from the bank, perpendicular to its edge, he found the angle of elevation of the top of the same tree to be 30°. Finally, he found the breadth of the canal and hence, the minimum length of each bamboo pole to build the temporary bridge. Finally, the NGO was successful in constructing the bridge at a minimum cost.

Find the breadth of the river and hence the minimum length of each bamboo pole.

Sol. Let B_1 and B_2 be two points on two opposite sides of the canal such that B_1B_2 is perpendicular to the flow of water in the canal. Let B_1T be the vertical height of a tree on one side of the bank. B_2 is a point on just opposite side of the canal such that $\angle TB_2B_1 = 45°$. B_3 is a point on B_1B_2 produced such that $B_2B_3 = 6$ m. Let B_1B_2 = breadth of the river = x m.

Also, given that $\angle TB_3B_1 = 30°$.



(*a*) Now, in ΔTB_1B_2, we have

$$\tan(\angle TB_2B_1) = \frac{TB_1}{B_1B_2}$$

$$\Rightarrow \qquad \tan 45° = \frac{TB_1}{x}$$

$$\Rightarrow \qquad 1 = \frac{TB_1}{x}$$

$$\therefore \qquad TB_1 = x \qquad \ldots(1)$$

In ΔTB_3B_1, we have

$$\tan(\angle TB_3B_1) = \frac{TB_1}{B_3B_1}$$

$$\Rightarrow \qquad \tan 30° = \frac{x}{x+6}$$

$$\Rightarrow \qquad \frac{1}{\sqrt{3}} = \frac{x}{x+6}$$

$$\Rightarrow \qquad x + 6 = \sqrt{3}\,x$$

$$\therefore \qquad x = \frac{6}{\sqrt{3}-1}$$

$$= \frac{6\left(\sqrt{3}+1\right)}{3-1}$$

$$= 3\left(\sqrt{3}+1\right)$$

$$= 3(1.73 + 1)$$

$$= 3 \times 2.73 = 8.19 \text{ (approx.)}$$

$\therefore$ The required breadth of the river

= 8.19 m (approx.)

$\therefore$ The minimum length of each bamboo pole

= 8.19 m(approx.)