# 8

# Introduction to Trigonometry

## Checkpoint (Page 149)

**1.** Can a triangle have two obtuse angles? Why?

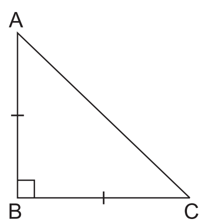
**Sol.** No; since in this case, the sum of two angles of the triangle will be greater than 180°. This is impossible, since, we know that the sum of three angles of any triangle is 180°.

**2.** Is it possible to construct a triangle such that each of three angles is less than 60°? Why?

**Sol.** No; since in this case, the sum of three angles of the triangle will be less than 180°. This is impossible.

**3.** The length of the hypotenuse of an isosceles right-angled triangle is 2.88 m. What are the lengths of the three sides of this triangle in cm?

**Sol.** Let ΔABC be an isosceles right-angled triangle with ∠ABC = 90° and AB = BC and the hypotenuse AC = 2.88 m = 288 cm.




Let $a$ cm be the length of the sides BC and AB.

Then, since ∠ABC = 90°, hence, by Pythagoras' theorem, we have

$$AC^2 = BC^2 + AB^2$$

$$\Rightarrow \quad 288^2 = a^2 + a^2 = 2a^2$$

$$\Rightarrow \quad a^2 = \frac{288 \times 288}{2}$$

$$= 144 \times 288$$

$$= 144 \times 2 \times 144$$

$$\therefore \quad a = 144\sqrt{2}$$

Hence, the required sides of the triangle are $144\sqrt{2}$ cm, $144\sqrt{2}$ cm and 288 cm.

**4.** If $x$ cm, $(x + 1)$ cm and $(x + 2)$ cm are the lengths of the three sides of a right-angled triangle, then find the value of $x$.

**Sol.** The lengths of the three sides of the right-angled triangle are $x$ cm, $(x + 1)$ cm and $(x + 2)$ cm. Clearly, $(x + 2)$ cm is the greatest side and so the length of the hypotenuse is $(x + 2)$ cm.

∴ By Pythagoras' theorem, we have

$$(x + 2)^2 = x^2 + (x + 1)^2$$

$$\Rightarrow \quad x^2 + 4x + 4 = x^2 + x^2 + 2x + 1$$

$$\Rightarrow \quad x^2 - 2x - 3 = 0$$

$$\Rightarrow \quad x^2 + x - 3x - 3 = 0$$

$$\Rightarrow \quad x(x + 1) - 3(x + 1) = 0$$

$$\Rightarrow \quad (x + 1)(x - 3) = 0$$

$$\therefore \text{ Either} \quad x + 1 = 0$$

$$\Rightarrow \quad x = -1$$

which is neglected, since $x$ cannot be negative.

$$\text{or,} \quad x - 3 = 0$$

$$\Rightarrow \quad x = 3$$

Hence, the required value of $x$ is 3.

**5.** If the area of a triangle whose all three sides are of equal length is $4\sqrt{3}$ cm$^2$, what is the length of its each side?

**Sol.** Clearly, the triangle is an equilateral triangle. Hence, the area of the equilateral triangle of side $a$ cm is $\frac{\sqrt{3}}{4}a^2$ cm$^2$.

$\therefore \quad \frac{\sqrt{3}}{4}a^2 = 4\sqrt{3}$

$\Rightarrow \quad a^2 = 16$

$\Rightarrow \quad a = 4$

[Taking positive value only, since the length of a side cannot be negative]

Hence, the required length is 4 cm.

**6.** In which of the following cases where the lengths of three sides are given, can you construct a triangle?

(*a*) 3 cm, 5 cm and 9 cm

(*b*) 7 cm, 6 cm and 8 cm

(*c*) 12 cm, 12 cm and 13 cm

**Sol.** We know that the sum of any two sides of any triangle is greater than its third side.

(*a*) We see that $3 + 5 < 9$.

Hence, we cannot construct any triangle in this case.

(*b*) We see that $7 + 6 > 8$, $7 + 8 > 6$ and $6 + 8 > 7$

So, we can construct a triangle in this case.

(*c*) We see that $12 + 12 > 13$, $12 + 13 > 12$ and so, we can construct a triangle in this case also.

So, we can construct a triangle only in cases (*b*) and (*c*), but not in case of (*a*).

**7.** Three angles 90°, 45° and 45° are given. How many different triangles can you construct with these three angles?

**Sol.** In this case, we can draw infinitely many triangles with different lengths of sides but all the triangles will be similar.

**8.** State two points by which an equation can be distinguished from an identity.

**Sol.** (*i*) An equation is a statement which is true for a limited number of values of the variable(s) involved, but an identity is a statement which is true for all arbitrary values of the variable(s) involved.

(*ii*) We solve an equation, whereas we prove an identity.

# (I) Trigonometric Ratios

## Check Your Progress 1

**(Page 152)**

### Multiple-Choice Questions

**1.** If the value of $\sec A = \frac{5}{4}$, then the value of $\cot A$ is equal to

(*a*) $\frac{5}{3}$ (*b*) $\frac{3}{4}$

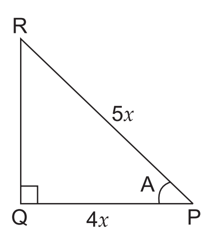
(*c*) $\frac{3}{5}$ (*d*) $\frac{4}{3}$

**Sol.** (*d*) $\frac{4}{3}$

Let ΔPQR be a right-angled triangle where ∠Q = 90° and ∠QPR = A. Clearly, QP is the base, QR is the perpendicular and RP is the hypotenuse of the triangle.

Now, we have $\sec A = \frac{5}{4}$.

Let $QP = 4x$ units and $RP = 5x$ units, where $x$ is a non-zero positive number. Then by Pythagoras' theorem, we have




$$RP^2 = QP^2 + QR^2$$

$$\Rightarrow \quad (5x)^2 = (4x)^2 + QR^2$$

$$\Rightarrow \quad QR^2 = 25x^2 - 16x^2 = 9x^2$$

$$\therefore \quad QR = 3x$$

$$\therefore \quad \cot A = \frac{QP}{QR} = \frac{4x}{3x} = \frac{4}{3}$$

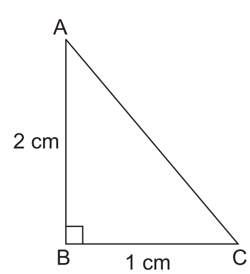
$\therefore$ The value of $\cot A$ is $\frac{4}{3}$.

**2.** In a triangle ABC, if ∠B = 90°, AB = 2 cm and BC = 1 cm, then the value of cosec A is equal to

(*a*) $\sqrt{3}$ (*b*) $\sqrt{5}$

(*c*) $\frac{1}{2}$ (*d*) 2

**Sol.** (*b*) $\sqrt{5}$



From $\Delta ABC$, we have by Pythagoras' theorem,

$$AC = \sqrt{BC^2 + AB^2}$$
$$= \sqrt{1^2 + 2^2} \text{ cm}$$
$$= \sqrt{5} \text{ cm}$$

$$\therefore \quad \text{cosec } A = \frac{AC}{BC} = \frac{\sqrt{5}}{1} = \sqrt{5}$$

$\therefore$ The value of cosec A is $\sqrt{5}$ .

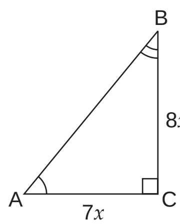
**3.** In $\Delta ABC$ right-angled at C, if $\tan A = \frac{8}{7}$, then the value of cot B is

(*a*) $\frac{7}{8}$ (*b*) $\frac{8}{7}$

(*c*) $\frac{7}{\sqrt{113}}$ (*d*) $\frac{8}{\sqrt{113}}$

**[CBSE 2023 Basic]**

**Sol.** (*b*) $\frac{8}{7}$




$$\tan A = \frac{8}{7}$$
$$\Rightarrow \quad BC = 8x \text{ units}$$
$$AC = 7x \text{ units}$$
$$\cot B = \frac{BC}{AC}$$
$$\Rightarrow \quad \cot B = \frac{8x}{7x} = \frac{8}{7}$$

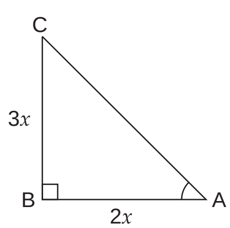
**4.** If 2 tan A = 3, then the value of $\frac{4 \sin A + 3 \cos A}{4 \sin A - 3 \cos A}$ is

(*a*) $\frac{7}{\sqrt{13}}$ (*b*) $\frac{1}{\sqrt{13}}$

(*c*) 3 (*d*) does not exist

**[CBSE 2023 Standard]**

**Sol.** (*c*) 3




$$2 \tan A = 3$$
$$\Rightarrow \quad \tan A = \frac{3}{2}$$
$$AB = 2x \text{ units}$$
$$BC = 3x \text{ units}$$
$$\therefore \quad AC = \sqrt{AB^2 + BC^2}$$
$$= \sqrt{4x^2 + 9x^2}$$
$$\Rightarrow \quad AC = \sqrt{13}x \text{ units}$$
$$\therefore \quad \sin A = \frac{3x}{\sqrt{13}x} = \frac{3}{\sqrt{13}}$$
$$\cos A = \frac{2x}{\sqrt{13}x} = \frac{2}{\sqrt{13}}$$

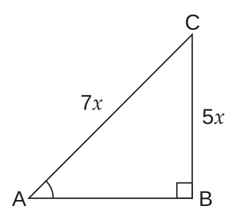
$$\frac{4 \sin A + 3 \cos A}{4 \sin A - 3 \cos A} = \frac{4 \times \frac{3}{\sqrt{13}} + 3 \times \frac{2}{\sqrt{13}}}{4 \times \frac{3}{\sqrt{13}} - 3 \times \frac{2}{\sqrt{13}}}$$
$$= \frac{12 + 6}{12 - 6}$$
$$= \frac{18}{6} = 3$$

**5.** If $\text{cosec } A = \frac{7}{5}$, then value of tan A . cos A is

(*a*) $\frac{7}{5}$ (*b*) $\frac{2\sqrt{6}}{5}$

(*c*) $\frac{24}{49}$ (*d*) $\frac{5}{7}$ **[CBSE 2023C Basic]**

**Sol.** (*d*) $\frac{5}{7}$




$$\text{cosec } A = \frac{7}{5}$$
$$\Rightarrow \quad AC = 7x \text{ units}$$
$$BC = 5x \text{ units}$$
$$AC^2 = AB^2 + BC^2$$
$$\Rightarrow \quad AB^2 = AC^2 - BC^2$$
$$\Rightarrow \quad AB = \sqrt{AC^2 - BC^2}$$
$$= \sqrt{49x^2 - 25x^2} = \sqrt{24x^2}$$
$$\Rightarrow \quad AB = 2\sqrt{6}x$$
$$\tan A = \frac{5x}{2\sqrt{6}x} = \frac{5}{2\sqrt{6}}$$

$$\cos A = \frac{2\sqrt{6}}{7}$$

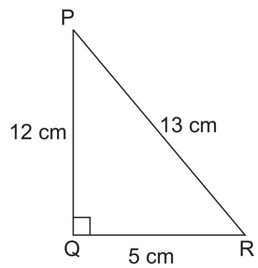
$$\therefore \quad \tan A \cdot \cos A = \frac{5}{2\sqrt{6}} \times \frac{2\sqrt{6}}{7} = \frac{5}{7}$$

### Very Short Answer Type Questions

**6.** In $\Delta PQR$, $\angle Q = 90°$. If PQ = 12 cm, QR = 5 cm and PR = 13 cm, what is the value of $\sec \angle R$?

**Sol.** We have

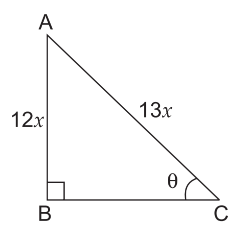
$$\sec \angle R = \frac{PR}{QR} = \frac{13}{5}$$




$\therefore$ The value of $\sec \angle R$ is $\frac{13}{5}$.

**7.** If $\sin \theta = \frac{12}{13}$, what is the value of $\cos \theta$?

**Sol.** Let ABC be a right-angled triangle with BC as base, AB as perpendicular, $\angle B = 90°$ and AC as the hypotenuse.




Let AC = $13x$ and AB = $12x$, where $x$ is a non-zero positive number.

$\therefore$ By Pythagoras' theorem, we have

$$AC^2 = AB^2 + BC^2$$

$$\Rightarrow \quad (13x)^2 = (12x)^2 + BC^2$$

$$\Rightarrow \quad 169x^2 = 144x^2 + BC^2$$

$$\Rightarrow \quad BC^2 = 169x^2 - 144x^2$$

$$= 25x^2$$

$$\therefore \quad BC = 5x$$

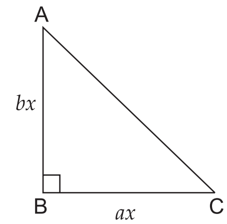
$$\therefore \quad \cos \theta = \frac{BC}{AC} = \frac{5x}{13x} = \frac{5}{13}$$

$\therefore$ The value of $\cos \theta$ is $\frac{5}{13}$.

**8.** If $\cot A = \frac{b}{a}$, prove that $\frac{2 \sec A + 1}{\cos A + 2} = \frac{\sqrt{a^2 + b^2}}{b}$.

**[CBSE SP 2011]**

**Sol.** Let ABC be a right-angled triangle with $\angle ABC = 90°$, BC = $ax$ units, AB = $bx$ units, where $x$ is a non-zero positive constant.




$\therefore$ By Pythagoras' theorem, we have

$$AC^2 = AB^2 + BC^2 = b^2x^2 + a^2x^2$$

$$= (a^2 + b^2)x^2$$

$$\therefore \quad AC = \sqrt{(a^2 + b^2)}\, x$$

$$\therefore \quad \sec A = \frac{AC}{AB} = \frac{\sqrt{a^2 + b^2}\, x}{bx} = \frac{\sqrt{a^2 + b^2}}{b}$$

$$\cos A = \frac{AB}{AC} = \frac{bx}{\sqrt{a^2 + b^2}\, x} = \frac{b}{\sqrt{a^2 + b^2}}$$

$$\therefore \quad LHS = \frac{2 \sec A + 1}{\cos A + 2}$$

$$= \frac{2 \times \frac{\sqrt{a^2 + b^2}}{b} + 1}{\frac{b}{\sqrt{a^2 + b^2}} + 2}$$

$$= \frac{2\sqrt{a^2 + b^2} + b}{b} \times \frac{\sqrt{a^2 + b^2}}{b + 2\sqrt{a^2 + b^2}}$$

$$= \frac{\sqrt{a^2 + b^2}}{b}$$

$$= RHS$$

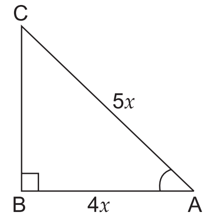
Hence, proved.

**9.** If $\sec A = \frac{5}{4}$, prove that $\tan A + \frac{1}{\cos A} = 2$.

**Sol.** Let ABC be a right-angled triangle with $\angle B = 90°$, AB = $4x$ units and AC = $5x$ units, where $x$ is a non-zero positive number.

Now, by Pythagoras' theorem, we have

$$AC^2 = AB^2 + BC^2$$




$\Rightarrow \quad (5x)^2 = (4x)^2 + BC^2$

$\Rightarrow \quad 25x^2 = 16x^2 + BC^2$

$\Rightarrow \quad BC^2 = 25x^2 - 16x^2$

$\Rightarrow \quad BC^2 = 9x^2$

$\Rightarrow \quad BC = 3x$ units

$\therefore \quad \tan A = \dfrac{BC}{AB} = \dfrac{3x}{4x} = \dfrac{3}{4}$

and $\quad \cos A = \dfrac{AB}{AC} = \dfrac{4x}{5x} = \dfrac{4}{5}$

$\therefore \quad \text{LHS} = \tan A + \dfrac{1}{\cos A}$

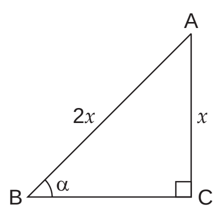
$= \dfrac{3}{4} + \dfrac{5}{4}$

$= \dfrac{8}{4} = 2 = \text{RHS}$

Hence, proved.

**10.** If $\sin\alpha = \dfrac{1}{2}$, then find the value of $(3\cos\alpha - 4\cos^3\alpha)$. **[CBSE 2023 Basic]**

**Sol.** $\sin\alpha = \dfrac{1}{2}$




$AC = x$ units

$AB = 2x$ units

$AB^2 = AC^2 + BC^2$

$\Rightarrow \quad BC^2 = AB^2 - AC^2$

$BC = \sqrt{AB^2 - AC^2}$

$= \sqrt{(2x)^2 - (x)^2} = \sqrt{4x^2 - x^2}$

$\Rightarrow \quad BC = \sqrt{3}x$

$\cos\alpha = \dfrac{BC}{AB} = \dfrac{\sqrt{3}x}{2x} = \dfrac{\sqrt{3}}{2}$

$\therefore \quad 3\cos\alpha - 4\cos^3\alpha = 3 \times \dfrac{\sqrt{3}}{2} - 4\left(\dfrac{\sqrt{3}}{2}\right)^3$

$= \dfrac{3\sqrt{3}}{2} - 4 \times \dfrac{3\sqrt{3}}{8}$

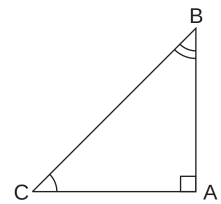
$= \dfrac{3\sqrt{3}}{2} - \dfrac{3\sqrt{3}}{2}$

$= 0$

Hence, $3\cos\alpha - 4\cos^3\alpha = 0$.

**11.** In a $\Delta ABC$, $\angle A = 90°$. If $\tan C = \sqrt{3}$, then find the value of $\sin B + \cos C - \cos^2 B$. **[CBSE 2024 Basic]**

**Sol.** $\tan C = \sqrt{3}$




$\tan C = \dfrac{AB}{AC}$

$AB = \sqrt{3}x$ units

$AC = x$ units

$BC^2 = AB^2 + AC^2$

$BC = \sqrt{AB^2 + AC^2}$

$= \sqrt{(\sqrt{3}x)^2 + x^2}$

$= \sqrt{3x^2 + x^2}$

$= \sqrt{4x^2} = 2x$

$\therefore \quad BC = 2x$ units

$\sin B = \dfrac{AC}{BC} = \dfrac{x}{2x} = \dfrac{1}{2}$

$\cos C = \dfrac{AC}{BC} = \dfrac{x}{2x} = \dfrac{1}{2}$

$\cos B = \dfrac{AB}{BC} = \dfrac{\sqrt{3}x}{2x} = \dfrac{\sqrt{3}}{2}$

$\sin B + \cos C - \cos^2 B = \dfrac{1}{2} + \dfrac{1}{2} - \left(\dfrac{\sqrt{3}}{2}\right)^2$

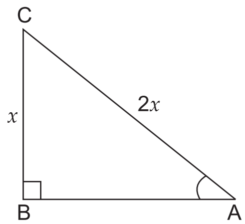
$= 1 - \dfrac{3}{4} = \dfrac{1}{4}$

$\therefore \sin B + \cos C - \cos^2 B = \dfrac{1}{4}$

## Short Answer Type Questions

**12.** If cosec A = 2, find the value of $\dfrac{1}{\tan A} + \dfrac{\sin A}{1 + \cos A}$. **[CBSE SP 2011]**

**Sol.** Let $\Delta ABC$ be a right-angled triangle with $\angle B = 90°$, $BC = x$ units and $AC = 2x$ units, where $x$ is a non-zero positive number.




$\therefore$ By Pythagoras' theorem, we have

$$AC^2 = AB^2 + BC^2$$

$$\Rightarrow \quad (2x)^2 = AB^2 + x^2$$

$$\Rightarrow \quad AB^2 = 4x^2 - x^2 = 3x^2$$

$$\therefore \quad AB = \sqrt{3}\,x$$

$$\therefore \quad \tan A = \frac{BC}{AB} = \frac{x}{\sqrt{3}x} = \frac{1}{\sqrt{3}},$$

$$\sin A = \frac{BC}{AC} = \frac{x}{2x} = \frac{1}{2}$$

and $\cos A = \dfrac{AB}{AC} = \dfrac{\sqrt{3}x}{2x} = \dfrac{\sqrt{3}}{2}$

$$\therefore \quad \frac{1}{\tan A} + \frac{\sin A}{1 + \cos A} = \sqrt{3} + \frac{\frac{1}{2}}{1 + \frac{\sqrt{3}}{2}}$$

$$= \sqrt{3} + \frac{1}{2 + \sqrt{3}}$$

$$= \frac{2\sqrt{3} + 3 + 1}{2 + \sqrt{3}}$$

$$= \frac{2\sqrt{3} + 4}{2 + \sqrt{3}}$$

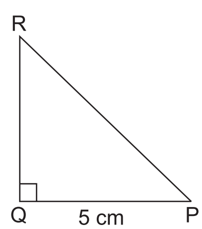
$$= \frac{2\left(2 + \sqrt{3}\right)}{2 + \sqrt{3}} = 2$$

Hence, the required value is 2.

**13.** In $\Delta PQR$, right angled at Q, PR + QR = 25 cm and PQ = 5 cm. Determine the values of sin P, cos P and tan P. **[CBSE SP 2011]**

**Sol.** In $\Delta PQR$, we have

$\angle Q = 90°$, PQ = 5 cm




Let QR = $x$ cm

$$\because \quad PR + QR = 25 \text{ cm}$$

$$\therefore \quad PR = 25 \text{ cm} - x \text{ cm}$$

$$\Rightarrow \quad PR = (25 - x) \text{ cm} \qquad \ldots(1)$$

Now, by Pythagoras' theorem, we have

$$PR^2 = QR^2 + PQ^2 = x^2 + 5^2$$

$$\Rightarrow \quad (25 - x)^2 = x^2 + 25 \qquad \text{[From (1)]}$$

$$\Rightarrow \quad 625 + x^2 - 50x = x^2 + 25$$

$$\Rightarrow \quad 50x = 600$$

$$\Rightarrow \quad x = 12$$

$$\therefore \quad QR = 12 \text{ cm}$$

and $\quad PR = (25 - 12)$ cm

$$= 13 \text{ cm} \qquad \text{[From (1)]}$$

Now, $\quad \sin P = \dfrac{QR}{PR} = \dfrac{12}{13}$

$$\cos P = \frac{PQ}{PR} = \frac{5}{13}$$

and $\quad \tan P = \dfrac{QR}{PQ} = \dfrac{12}{5}$

Hence, the required values of sin P, cos P and tan P are $\dfrac{12}{13}$, $\dfrac{5}{13}$ and $\dfrac{12}{5}$ respectively.

## Long Answer Type Questions

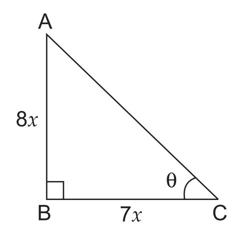
**14.** If $\cot \theta = \dfrac{7}{8}$, evaluate

(*a*) $\dfrac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$

(*b*) $\cot^2 \theta$ **[CBSE SP 2010, 2011]**

(*c*) $\dfrac{\operatorname{cosec}^2\theta - \cot^2\theta + \sec\theta}{\sec^2\theta - \tan^2\theta + \operatorname{cosec}\theta}$

**Sol.** Let $\Delta ABC$ be a right-angled triangle with $\angle B = 90°$, BC = $7x$ units and AB = $8x$ units, where $x$ is a non-zero positive number.




$\therefore$ By Pythagoras' theorem, we have

$$AC^2 = AB^2 + BC^2$$

$$= (8x)^2 + (7x)^2 = 113x^2$$

$$\therefore \quad AC = \sqrt{113}\,x \text{ units}$$

(*a*) Let $\quad \angle ACB = \theta$

$$\therefore \quad \sin\theta = \frac{AB}{AC} = \frac{8x}{\sqrt{113}x} = \frac{8}{\sqrt{113}}$$

$$\cos\theta = \frac{BC}{AC} = \frac{7x}{\sqrt{113}x} = \frac{7}{\sqrt{113}}$$

$$\therefore \quad \frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$$

$$= \frac{\left(1+\frac{8}{\sqrt{113}}\right)\left(1-\frac{8}{\sqrt{113}}\right)}{\left(1+\frac{7}{\sqrt{113}}\right)\left(1-\frac{7}{\sqrt{113}}\right)}$$

$$= \frac{1^2-\left(\frac{8}{\sqrt{113}}\right)^2}{1^2-\left(\frac{7}{\sqrt{113}}\right)^2}$$

$$= \frac{1-\frac{64}{113}}{1-\frac{49}{113}}$$

$$= \frac{49}{113} \times \frac{113}{64}$$

$$= \frac{49}{64}$$

$\therefore$ The required value is $\frac{49}{64}$.

(*b*) We have

$$\cot\theta = \frac{7}{8}$$

On squaring both sides,

$$\cot^2\theta = \left(\frac{7}{8}\right)^2 = \frac{49}{64}$$

$\therefore$ The required value is $\frac{49}{64}$.

(*c*) We have

$$\operatorname{cosec}\theta = \frac{AC}{AB} = \frac{\sqrt{113}}{8},\ \sec\theta = \frac{AC}{BC} = \frac{\sqrt{113}}{7}$$

and $\tan\theta = \frac{8}{7}$

$$\therefore \quad \frac{\operatorname{cosec}^2\theta - \cot^2\theta + \sec\theta}{\sec^2\theta - \tan^2\theta + \operatorname{cosec}\theta}$$

$$= \frac{\frac{113}{64}-\frac{49}{64}+\frac{\sqrt{113}}{7}}{\frac{113}{49}-\frac{64}{49}+\frac{\sqrt{113}}{8}}$$

$$= \frac{\frac{64}{64}+\frac{\sqrt{113}}{7}}{\frac{49}{49}+\frac{\sqrt{113}}{8}}$$

$$= \frac{1+\frac{\sqrt{113}}{7}}{1+\frac{\sqrt{113}}{8}}$$

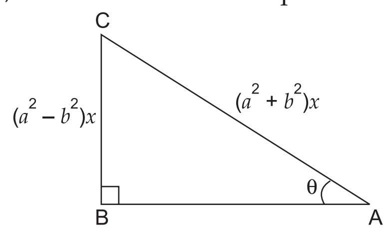
$$= \frac{8\left(7+\sqrt{113}\right)}{7\left(8+\sqrt{113}\right)}$$

$\therefore$ The required value is $\frac{8\left(7+\sqrt{113}\right)}{7\left(8+\sqrt{113}\right)}$.

**15.** If $\sin\theta = \frac{a^2-b^2}{a^2+b^2}$, show that

$$\sin\theta\cos\theta\tan\theta = \left(\frac{a^2-b^2}{a^2+b^2}\right)^2$$

**Sol.** Let $\Delta ABC$ be a right-angled triangle with $\angle B = 90°$, $BC = (a^2-b^2)x$ units and $AC = (a^2+b^2)x$ units, where $x$ is a non-zero positive number.




Let $\angle BAC = \theta$

$\therefore$ By Pythagoras' theorem, we have

$$AC^2 = BC^2 + AB^2$$

$$\Rightarrow \quad (a^2+b^2)^2x^2 = (a^2-b^2)^2x^2 + AB^2$$

$$\Rightarrow \quad AB^2 = \{(a^2+b^2)^2 - (a^2-b^2)^2\}x^2$$

$$= 4a^2b^2x^2$$

$$\therefore \quad AB = 2abx$$

$$\therefore \quad LHS = \sin\theta\cos\theta\tan\theta$$

$$= \frac{BC}{AC} \times \frac{AB}{AC} \times \frac{BC}{AB}$$

$$= \frac{\left(a^2-b^2\right)x}{\left(a^2+b^2\right)x} \times \frac{2abx}{\left(a^2+b^2\right)x} \times \frac{\left(a^2-b^2\right)x}{2abx}$$

$$= \left(\frac{a^2-b^2}{a^2+b^2}\right)^2$$

$$= RHS$$

Hence, proved.

## Check Your Progress 2

**(Page 156)**

### Multiple-Choice Questions

**1.** The value of $\sin^2 30° - \cos^2 30°$ is equal to

(*a*) $-\frac{1}{2}$ (*b*) $\frac{\sqrt{3}}{2}$

(*c*) $\frac{3}{2}$ (*d*) $\frac{2}{3}$ **[CBSE SP 2011]**

**Sol.** (*a*) $-\frac{1}{2}$

We have

$$\sin^2 30° - \cos^2 30° = \left(\frac{1}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2$$
$$= \frac{1}{4} - \frac{3}{4}$$
$$= -\frac{2}{4} = \frac{-1}{2}$$

**2.** If $\sin A = \frac{1}{2}$ and $\cos B = \frac{1}{2}$, then the value of A + B is equal to

(*a*) 0° (*b*) 60°

(*c*) 90° (*d*) 30° **[CBSE SP 2011]**

**Sol.** (*c*) 90°

We have $\sin A = \frac{1}{2} = \sin 30°$

$\Rightarrow$ $A = 30°$

and $\cos B = \frac{1}{2} = \cos 60°$

$\Rightarrow$ $B = 60°$

$\therefore$ $A + B = 30° + 60°$

$= 90°$

**3.** The value of $2\sin^2 30° + 3\tan^2 60° - \cos^2 45°$ is

(*a*) $3\sqrt{3}$ (*b*) $\frac{19}{2}$

(*c*) $\frac{9}{4}$ (*d*) 9 **[CBSE 2023 Basic]**

**Sol.** (*d*) 9

$2\sin^2 30° + 3\tan^2 60° - \cos^2 45°$

$$= 2 \times \left(\frac{1}{2}\right)^2 + 3 \times \left(\sqrt{3}\right)^2 - \left(\frac{1}{\sqrt{2}}\right)^2$$
$$= 2 \times \frac{1}{4} + 3 \times 3 - \frac{1}{2}$$
$$= \frac{1}{2} + 9 - \frac{1}{2}$$
$$= 9$$

$\therefore 2\sin^2 30° + 3\tan^2 60° - \cos^2 45° = 9$

**4.** For what value of θ, $\sin^2\theta + \sin\theta + \cos^2\theta$ is equal to 2?

(*a*) 45° (*b*) 0°

(*c*) 90° (*d*) 30° **[CBSE 2024 Basic]**

**Sol.** (*c*) 90°

$\sin^2\theta + \sin\theta + \cos^2\theta = 2$

$\Rightarrow$ $1 + \sin\theta = 2$ $[\because \sin^2\theta + \cos^2\theta = 1]$

$\Rightarrow$ $\sin\theta = 1$

$\theta = 90°$

**5.** If $\cos(\alpha + \beta) = 0$, then value of $\cos\left(\frac{\alpha+\beta}{2}\right)$ is equal to

(*a*) $\frac{1}{\sqrt{2}}$ (*b*) $\frac{1}{2}$

(*c*) 0 (*d*) $\sqrt{2}$

**[CBSE 2024 Standard]**

**Sol.** (*a*) $\frac{1}{\sqrt{2}}$

$\cos(\alpha + \beta) = 0 = \cos 90°$

$\Rightarrow$ $\alpha + \beta = 90°$

$$\frac{\alpha+\beta}{2} = 45°$$
$$\cos\left(\frac{\alpha+\beta}{2}\right) = \cos 45° = \frac{1}{\sqrt{2}}$$

### Very Short Answer Type Questions

**6.** What is the value of $4\cos^3\theta - 3\cos\theta$ when $\theta = 30°$? Is it equal to $\cos 3\theta$?

**Sol.** We have

$$4\cos^3\theta - 3\cos\theta = 4\cos^3 30° - 3\cos 30°$$
$$= 4 \times \left(\frac{\sqrt{3}}{2}\right)^3 - 3 \times \frac{\sqrt{3}}{2}$$
$$= 4 \times \frac{3}{4}\frac{\sqrt{3}}{2} - \frac{3}{2}\sqrt{3}$$
$$= \frac{3\sqrt{3}}{2} - \frac{3\sqrt{3}}{2}$$
$$= 0$$

Also, $\cos 3\theta = \cos(3 \times 30°)$

$= \cos 90° = 0$

Hence, $4\cos^3\theta - 3\cos\theta = \cos 3\theta$ when $\theta = 30°$.

$\therefore$ Yes, the given expression is equal to $\cos 3\theta$.

**7.** You are given an identity:

$\sin(A - B) = \sin A \cos B - \cos A \sin B$

Is it possible to find the value of sin 15° by putting A = 45° and B = 30° in the above formula? If so, what is the value of sin 15°?

**Sol.** We have

$\sin(A - B) = \sin(45° - 30°) = \sin 15°$

Also, $\sin A \cos B - \cos A \sin B$

$$= \sin 45° \cos 30° - \cos 45° \sin 30°$$
$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2}$$
$$= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}$$

$$= \frac{\sqrt{3}-1}{2\sqrt{2}}$$

$$\Rightarrow \quad \sin 15° = \frac{\sqrt{3}-1}{2\sqrt{2}}$$

∴ Yes, it is possible to find the value of sin 15° by putting A = 45° and B = 30° in the given formula. Also, the value of sin 15° is $\frac{\sqrt{3}-1}{2\sqrt{2}}$.

**8.** If cos ($x$ + 40°) = sin 30°, find the value of $x$ if $0° \le x \le 90°$.

**Sol.** We have

$$\cos(x + 40°) = \sin 30°$$
$$= \frac{1}{2} = \cos 60°$$
$$\therefore \quad x + 40° = 60°$$
$$\Rightarrow \quad x = 60° - 40°$$
$$= 20°$$

Hence, the required value of $x$ is 20°.

**9.** Verify the identity $\cos A = \frac{1}{\sqrt{1+\tan^2 A}}$ by taking A = 30°.

**Sol.** We have, for A = 30°

$$\text{LHS} = \cos A = \cos 30° = \frac{\sqrt{3}}{2}$$

$$\text{RHS} = \frac{1}{\sqrt{1+\tan^2 A}}$$
$$= \frac{1}{\sqrt{1+\tan^2 30°}}$$
$$= \frac{1}{\sqrt{1+\frac{1}{3}}}$$
$$= \frac{\sqrt{3}}{2}$$
$$= \text{LHS}$$

Hence, proved.

**10.** Evaluate: $\tan^2 60° - 2\cos^2 30° - 2\tan^2 30°$

**[CBSE 2023 Basic]**

**Sol.**

$$\tan 60° = \sqrt{3}$$
$$\cos 30° = \frac{\sqrt{3}}{2}$$
$$\tan 30° = \frac{1}{\sqrt{3}}$$

$$\tan^2 60° - 2\cos^2 30° - 2\tan^2 30°$$
$$= (\sqrt{3})^2 - 2\times\left(\frac{\sqrt{3}}{2}\right)^2 - 2\times\left(\frac{1}{\sqrt{3}}\right)^2$$
$$= 3 - 2\times\frac{3}{4} - 2\times\frac{1}{3}$$
$$= 3 - \frac{3}{2} - \frac{2}{3}$$
$$= \frac{18-9-4}{6}$$
$$= \frac{5}{6}$$

$$\therefore \tan^2 60° - 2\cos^2 30° - 2\tan^2 30° = \frac{5}{6}$$

**11.** If $\cot^2 45° - \sec^2 60° + \sin^2 60° + p = \frac{3}{4}$, then find the value of $p$. **[CBSE 2023 Standard]**

**Sol.**

$$\cot 45° = 1$$
$$\sec 60° = 2$$
$$\sin 60° = \frac{\sqrt{3}}{2}$$

$$\cot^2 45° - \sec^2 60° + \sin^2 60° + p = \frac{3}{4}$$
$$\Rightarrow \quad (1)^2 - (2)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 + p = \frac{3}{4}$$
$$\Rightarrow \quad 1 - 4 + \frac{3}{4} + p = \frac{3}{4}$$
$$\Rightarrow \quad \frac{4-16+3}{4} + p = \frac{3}{4}$$
$$\Rightarrow \quad \frac{-9}{4} + p = \frac{3}{4}$$
$$\Rightarrow \quad p = \frac{3}{4} + \frac{9}{4} = \frac{12}{4} = 3$$
$$\Rightarrow \quad p = 3$$

**12.** If A and B are acute angles such that sin(A – B) = 0 and 2 cos (A + B) – 1 = 0, then find angles A and B. **[CBSE 2023 Standard]**

**Sol.**

$$\sin(A - B) = 0$$
$$\sin(A - B) = \sin 0°$$
$$\Rightarrow \quad A - B = 0° \qquad \ldots(1)$$
$$2\cos(A + B) - 1 = 0$$
$$\Rightarrow \quad \cos(A + B) = \frac{1}{2}$$
$$\Rightarrow \quad \cos(A + B) = \cos 60°$$
$$\Rightarrow \quad A + B = 60° \qquad \ldots(2)$$

Adding (1) and (2)

$$A - B = 0°$$
$$A + B = 60°$$
$$\overline{2A = 60°}$$
$$\therefore \quad A = 30°$$
$$\Rightarrow \quad B = 60° - 30° = 30°$$

**13.** If $\sin A = \frac{1}{2}$ and $\cos B = \frac{1}{\sqrt{2}}$, then find the value of sin A sin B + cos A cos B. **[CBSE 2024 Basic]**

**Sol.** $\sin A = \frac{1}{2}$

$\sin A = \sin 30°$

$\Rightarrow \quad A = 30° \quad \ldots(1)$

$\cos B = \frac{1}{\sqrt{2}}$

$\cos B = \cos 45°$

$\Rightarrow \quad B = 45° \quad \ldots(2)$

sin A sin B + cos A cos B

$= \sin 30° \sin 45° + \cos 30° \cos 45°$

$= \frac{1}{2} \times \frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}}$

$= \frac{1}{2\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{2}}$

$= \frac{\sqrt{3}+1}{2\sqrt{2}}$

$\therefore$ $\sin A \sin B + \cos A \cos B = \frac{\sqrt{3}+1}{2\sqrt{2}}$

Hence, $\sin A \sin B + \cos A \cos B = \frac{\sqrt{3}+1}{2\sqrt{2}}$

**14.** If A = 60° and B = 30°, verify that sin(A + B)= sin A cos B + cos A sin B.

**[CBSE 2024 Standard]**

**Sol.** $\sin (A + B) = \sin (60° + 30°) = \sin 90° = 1 \quad \ldots(1)$

$\sin A . \cos B = \sin 60° . \cos 30°$

$= \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$

$= \frac{3}{4}$

$\therefore \quad \sin A . \cos B = \frac{3}{4} \quad \ldots(2)$

$\cos A . \sin B = \cos 60° \times \sin 30°$

$= \frac{1}{2} \times \frac{1}{2}$

$= \frac{1}{4}$

$\cos A . \sin B = \frac{1}{4} \quad \ldots(3)$

From (2) and (3),

$\sin A . \cos B + \cos A . \sin B = \frac{3}{4} + \frac{1}{4} = \frac{4}{4} = 1 \ldots(4)$

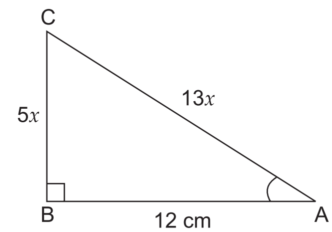
From (1) and (4),

sin (A + B) = sin A . cos B + cos A sin B

### Short Answer Type Questions

**15.** In ΔABC, ∠ABC = 90°. If AB = 12 cm and $\sin A = \frac{5}{13}$, find the lengths of the hypotenuse and the other side of the triangle. Also, find tan C.

**Sol.** Let BC = $5x$ cm and AC = $13x$ cm, where $x$ is a non-zero positive number.




Now, ∵ ∠B = 90°,

∴ By Pythagoras' theorem, we have

$AC^2 = AB^2 + BC^2$

$\Rightarrow \quad (13x)^2 = 12^2 + (5x)^2$

$\Rightarrow \quad 169x^2 = 144 + 25x^2$

$\Rightarrow \quad (169 - 25)x^2 = 144$

$\Rightarrow \quad 144x^2 = 144$

$\Rightarrow \quad x^2 = 1$

$\Rightarrow \quad x = 1$

[Taking positive value only since the length of a side cannot be negative]

$\therefore \quad BC = 5$ cm

and $\quad AC = 13$ cm

$\therefore \quad \tan C = \frac{AB}{BC} = \frac{12}{5}$

Hence, the required length of the hypotenuse is 13 cm and the other side is 5 cm. Also, the value of tan C is $\frac{12}{5}$.

**16.** If $\tan (A - B) = \frac{1}{\sqrt{3}}$ and $\sin (A + B) = \frac{\sqrt{3}}{2}$, where $0° \le A + B \le 90°$, then find A and B. Also, find the value of $\sin^2 (A + B) - \cos^2 (A - B)$.

**[CBSE SP 2011]**

**Sol.** We have

$\tan (A - B) = \frac{1}{\sqrt{3}} = \tan 30°$

$\therefore \quad A - B = 30° \quad \ldots(1)$

[∵ 0° < A – B < 90°]

Also, $\sin(A+B) = \frac{\sqrt{3}}{2} = \sin 60°$

$\therefore \quad A + B = 60° \quad \ldots(2)$

$[\because 0° < A + B < 90°]$

Adding (1) and (2), we get

$$2A = 30° + 60° = 90°$$

$$\Rightarrow \quad A = \frac{90°}{2} = 45°$$

Subtracting (1) from (2), we get

$$2B = 60° - 30°$$

$$\Rightarrow \quad B = \frac{30°}{2} = 15°$$

Hence, the required value of A and B are 45° and 15° respectively.

Now, to find the value of $\sin^2(A+B) - \cos^2(A-B)$,

We have $\quad A + B = 45° + 15° = 60°$

and $\quad A - B = 45° - 15° = 30°$

$\therefore \quad \sin^2(A+B) - \cos^2(A-B)$

$$= \sin^2 60° - \cos^2 30°$$

$$= \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2 = 0$$

Hence, the required value of the given expression is 0.

### Long Answer Type Questions

**17.** In $\Delta ABC$, $\angle ABC = 90°$, $\angle BAC = 60°$ and BC = 7 cm. Find the lengths of the hypotenuse, the other side and also all trigonometric ratios of the remaining third angle.

**Sol.** We have

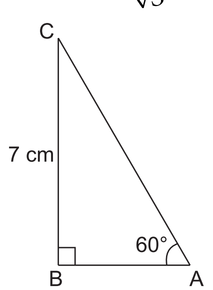
$$\sin 60° = \sin A$$

$$= \frac{BC}{AC}$$

$$= \frac{7}{AC}$$

$$\Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{7}{AC}$$

$$\Rightarrow \quad AC = \frac{14}{\sqrt{3}}$$




$\therefore$ The length of the hypotenuse AC is $\frac{14}{\sqrt{3}}$ cm.

Also, $\quad \cos 60° = \cos A = \frac{AB}{AC}$

$$\Rightarrow \quad \frac{1}{2} = \frac{AB\sqrt{3}}{14}$$

$$\Rightarrow \quad AB = \frac{7}{\sqrt{3}}$$

$\therefore$ The length of the other side AB is $\frac{7}{\sqrt{3}}$ cm.

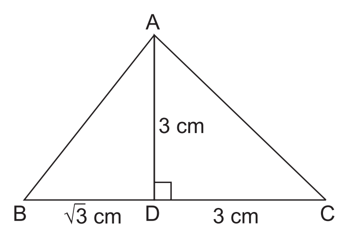
Now, since the third angle of the triangle is $180° - (60° + 90°) = 30°$.

$\therefore \quad \sin 30° = \frac{1}{2}$, $\cos 30° = \frac{\sqrt{3}}{2}$, $\tan 30° = \frac{1}{\sqrt{3}}$,

$\cot 30° = \sqrt{3}$, $\sec 30° = \frac{2}{\sqrt{3}}$ and $\operatorname{cosec} 30° = 2$.

**18.** In $\Delta ABC$, AD is the altitude of $\Delta ABC$ where D is a point on BC such that AD = 3 cm, BD = $\sqrt{3}$ cm and DC = 3 cm. Determine $\angle BAC$ and $\sin\angle DAC + \tan\angle ACD + \cos\angle ABD$.

**Sol.** Given that AD is the altitude of $\Delta ABC$ from A, where D is a point on BC such that BD = $\sqrt{3}$ cm, AD = 3 cm and DC = 3 cm.




Now, in $\Delta ABD$,

$\because \quad \angle ADB = 90°$,

$$\therefore \quad \tan B = \frac{AD}{BD}$$

$$= \frac{3}{\sqrt{3}}$$

$$= \sqrt{3}$$

$$= \tan 60°$$

$\therefore \quad B = 60° \quad \ldots(1)$

Again, in $\Delta ADC$,

$\because \quad \angle ADC = 90°$,

$$\therefore \quad \tan C = \frac{AD}{DC}$$

$$= \frac{3}{3} = 1$$

$$= \tan 45°$$

$\therefore \quad C = 45°$ ...(2)

$\therefore \quad \angle BAC = 180° - (\angle B + \angle C)$

[Angle sum property of a triangle]

$= 180° - (60° + 45°)$

$= 75°$

Hence, the required value of ∠BAC is 75°.

Now, in ΔDAC, we have

$\angle DAC + \angle ADC + \angle ACD = 180°$

[Angle sum property of a triangle]

$\Rightarrow \quad \angle DAC + 90° + 45° = 180°$

$\therefore \quad \angle DAC = 180° - 135° = 45°$ ...(3)

Hence, we have

$\sin \angle DAC + \tan \angle ACD + \cos \angle ABD$

$= \sin 45° + \tan 45° + \cos 60°$

[From (1), (2) and (3)]

$$= \frac{1}{\sqrt{2}} + 1 + \frac{1}{2}$$

$$= \frac{2 + 2\sqrt{2} + \sqrt{2}}{2\sqrt{2}}$$

$$= \frac{3\sqrt{2} + 2}{2\sqrt{2}}$$

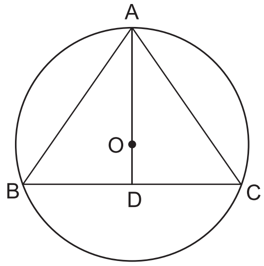
Hence, the required value of the given expression is $\frac{3\sqrt{2} + 2}{2\sqrt{2}}$.

## Higher Order Thinking Skills (HOTS) Questions

**(Page 157)**

1. An equilateral triangle is inscribed in a circle of radius 4 cm. Find its side.

**Sol.** Let O be the centre of the circle circumscribing ΔABC. Then O is the circumcentre of the equilateral triangle ABC. We draw AOD ⊥ BC. Then AD is a median of ΔABC, D being the mid-point of BC. For an equilateral triangle, the centroid and the circumcentre are identical point. Hence, O is the centroid of ΔABC dividing the median AD in the ratio AO : OD = 2 : 1.




$\therefore \quad AO = \frac{2}{3} AD$

$\Rightarrow \quad 4 \text{ cm} = \frac{2}{3} AD$

$\Rightarrow \quad AD = \frac{3 \times 4}{2} \text{ cm} = 6 \text{ cm}$

Hence, the length of the median AD is 6 cm.

Now, in ΔABD, since ∠ADB = 90°

$\therefore \quad \tan \angle ABD = \frac{AD}{BD} = \frac{AD}{\frac{1}{2}BC}$

$\Rightarrow \quad \frac{2AD}{BC} = \frac{2 \times 6}{BC} = \frac{12}{BC}$

$\Rightarrow \quad \tan 60° = \frac{12}{BC}$

[∵ In an equilateral triangle, each angle is 60°]

$\Rightarrow \quad \sqrt{3} = \frac{12}{BC}$

$\Rightarrow \quad BC = \frac{12}{\sqrt{3}} = \frac{12\sqrt{3}}{3} = 4\sqrt{3}$

$\Rightarrow \quad BC = 4\sqrt{3} \text{ cm}$

Hence, the required length of each side of ΔABC is $4\sqrt{3}$ cm.

2. Given that $\cos\frac{\theta}{2} + \sin\frac{\theta}{2} = \sqrt{1+\sin\theta}$ and $\cos\frac{\theta}{2} - \sin\frac{\theta}{2} = \sqrt{1-\sin\theta}$. Using these formulae, find the values of cos 15° and sin 15°.

**Sol.** Given that

$$\cos\frac{\theta}{2} + \sin\frac{\theta}{2} = \sqrt{1 + \sin\theta} \quad \text{...(1)}$$

and $$\cos\frac{\theta}{2} - \sin\frac{\theta}{2} = \sqrt{1 - \sin\theta} \quad \text{...(2)}$$

Adding equation (1) and (2), we get

$$2\cos\frac{\theta}{2} = \sqrt{1 + \sin\theta} + \sqrt{1 - \sin\theta} \quad \text{...(3)}$$

Subtracting equation (2) from (1), we get

$$2\sin\frac{\theta}{2} = \sqrt{1 + \sin\theta} - \sqrt{1 - \sin\theta} \quad \text{...(4)}$$

Putting θ = 30° on both sides of each of (3) and (4), we get

$$2 \cos 15° = \sqrt{1 + \sin 30°} + \sqrt{1 - \sin 30°}$$

$$= \sqrt{1 + \frac{1}{2}} + \sqrt{1 - \frac{1}{2}}$$

$$= \frac{\sqrt{3}}{\sqrt{2}} + \frac{1}{\sqrt{2}}$$

$$= \frac{\sqrt{3} + 1}{\sqrt{2}}$$

$$\Rightarrow \quad \cos 15° = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

and $$2 \sin 15° = \sqrt{1+\sin 30°} - \sqrt{1-\sin 30°}$$

$$= \frac{\sqrt{3}}{\sqrt{2}} - \frac{1}{\sqrt{2}}$$

$$= \frac{\sqrt{3}-1}{\sqrt{2}}$$

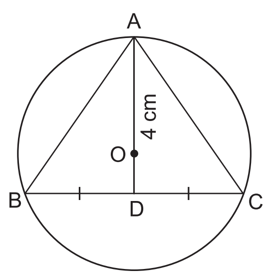
$$\Rightarrow \quad \sin 15° = \frac{\sqrt{3}-1}{2\sqrt{2}}$$

Hence, the required values of cos 15° and sin 15° are $\frac{\sqrt{3}+1}{2\sqrt{2}}$ and $\frac{\sqrt{3}-1}{2\sqrt{2}}$ respectively.

**3.** If O is the circumcentre of a circle circumscribing an equilateral triangle ABC and AOD is an altitude of ΔABC through A on the side BC, then find the length of OD, if the radius of the circumscribed circle is 4 cm.

**Sol.** Given that OA = circumradius of the equilateral triangle ABC = 4 cm.

We know that for an equilateral triangle, the circumcentre is identical with the centroid of ΔABC.




∴ O is the centroid and if AOD ⊥ BC, then AOD is a median of ΔABC.

$$\therefore \quad OA : OD = 2 : 1$$

$$\therefore \quad OA = \frac{2}{3} AD$$

$$\Rightarrow \quad 4 \text{ cm} = \frac{2}{3} AD$$

$$\Rightarrow \quad AD = \frac{3 \times 4}{2}$$

$$= 6 \text{ cm}$$

$$\therefore \quad OD = \frac{1}{3} AD$$

$$= \frac{1}{3} \times 6 \text{ cm}$$

$$= 2 \text{ cm}$$

Hence, the required length of OD is 2 cm.

## Self-Assessment

**(Page 158)**

### Multiple-Choice Questions

**1.** Which of the following is not defined?

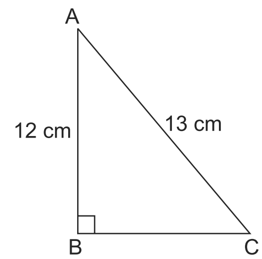
(*a*) cos 0° (*b*) tan 45°
(*c*) sec 90° (*d*) sin 90°

**[CBSE SP 2011]**

**Sol.** (*c*) sec 90°

We know that out of cos 0°, tan 45°, sin 90°, and sec 90°, only sec 90° is not defined.

**2.** In the given figure, tan A – cot C is equal to




(*a*) $\frac{7}{13}$ (*b*) $\frac{-7}{13}$
(*c*) $\frac{5}{12}$ (*d*) 0 **[CBSE SP 2011]**

**Sol.** (*d*) 0

In Δ ABC, ∠B = 90°, AB = 12 cm and AC = 13 cm.

∴ By Pythagoras' theorem, we have

$$AC^2 = AB^2 + BC^2$$

$$\Rightarrow \quad (13 \text{ cm})^2 = (12 \text{ cm})^2 + BC^2$$

$$\Rightarrow \quad BC^2 = 169 \text{ cm}^2 - 144 \text{ cm}^2$$

$$= 25 \text{ cm}^2$$

$$\therefore \quad BC^2 = 25 \text{ cm}^2$$

$$\Rightarrow \quad BC = 5 \text{ cm}$$

$$\therefore \quad \tan A - \cot C = \frac{BC}{AB} - \frac{BC}{AB}$$

$$= \frac{5}{12} - \frac{5}{12} = 0$$

**3.** If 2 cos θ = 1, then the value of θ is

(*a*) 45° (*b*) 60°
(*c*) 30° (*d*) 90° **[CBSE 2023 Basic]**

**Sol.** (*b*) 60°

$$2 \cos \theta = 1$$

$$\Rightarrow \quad \cos \theta = \frac{1}{2}$$

$$\Rightarrow \quad \cos \theta = \cos 60°$$

$$\Rightarrow \quad \theta = 60°$$

**4.** The value of $5 \sin^2 90° - 2 \cos^2 0°$ is

(*a*) –2 (*b*) 5

(*c*) 3 (*d*) –3 **[CBSE 2023 Basic]**

**Sol.** (*c*) 3

$$5 \sin^2 90° - 2 \cos^2 0° = 5 \times (1)^2 - 2(1)^2$$
$$= 5 - 2$$
$$= 3$$

$\therefore$ $5 \sin^2 90° - 2 \cos^2 0° = 3$

**5.** $\left[\frac{3}{4}\tan^2 30° - \sec^2 45° + \sin^2 60°\right]$ is equal to

(*a*) –1 (*b*) $\frac{5}{6}$

(*c*) $\frac{-3}{2}$ (*d*) $\frac{1}{6}$

**[CBSE 2023 Standard]**

**Sol.** (*a*) –1

$$\sin 60° = \frac{\sqrt{3}}{2}$$
$$\sec 45° = \sqrt{2}$$
$$\tan 30° = \frac{1}{\sqrt{3}}$$

$$\left[\frac{3}{4}\tan^2 30° - \sec^2 45° + \sin^2 60°\right]$$
$$= \left[\frac{3}{4}\times\left(\frac{1}{\sqrt{3}}\right)^2 - \left(\sqrt{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2\right]$$
$$= \left[\frac{3}{4}\times\frac{1}{3} - 2 + \frac{3}{4}\right]$$
$$= \frac{1}{4} - 2 + \frac{3}{4}$$
$$= 1 - 2$$
$$= -1$$

$\therefore$ $\left[\frac{3}{4}\tan^2 30° - \sec^2 45° + \sin^2 60°\right] = -1$

**6.** For $\theta = 30°$, the value of $(2 \sin\theta \cos\theta)$ is

(*a*) 1 (*b*) $\frac{\sqrt{3}}{2}$

(*c*) $\frac{\sqrt{3}}{4}$ (*d*) $\frac{3}{2}$

**[CBSE 2024 Standard]**

**Sol.** (*b*)

$$2 \sin\theta \cos\theta = 2 \sin 30° \cos 30°$$
$$= 2 \times \frac{1}{2}\times\frac{\sqrt{3}}{2}$$
$$= \frac{\sqrt{3}}{2}$$

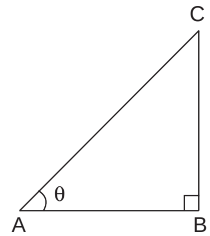
$\therefore$ $2 \sin\theta \cos\theta = \frac{\sqrt{3}}{2}$ for $\theta = 30°$

## Fill in the Blanks

**7.** If $\sec\theta = \frac{5}{3}$, then the value of $\sin\theta + \tan\theta$ is $\mathbf{\frac{32}{15}}$.

**Sol.** Draw a right $\Delta$ABC in which

$\angle B = 90°$, $\angle CAB = \theta$,




such that $\sec\theta = \frac{5}{3}$

$\Rightarrow$ $\frac{AC}{AB} = \frac{5}{3}$

Let AC = $5k$. Then, AB = $3k$

In right $\Delta$ABC, we have

$$AC^2 = AB^2 + BC^2$$
$$\Rightarrow (5k)^2 = (3k)^2 + BC^2$$
$$\Rightarrow BC^2 = 25k^2 - 9k^2 = 16k^2$$
$$\Rightarrow BC = 4k$$
$$\sin\theta = \frac{BC}{AC} = \frac{4k}{5k} = \frac{4}{5}$$

and $\tan\theta = \frac{BC}{AB} = \frac{4k}{3k} = \frac{4}{3}$

$\therefore$ $\sin\theta + \tan\theta = \frac{4}{5} + \frac{4}{3} = \frac{12 + 20}{15} = \frac{32}{15}$

**8.** The value of cosec 30° + tan 45° is **3**.

**Sol.** cosec 30° + tan 45° = 2 + 1 = 3

**9.** If $2 \cos 3A = 1$, then the value of A is **20°**.

**Sol.**
$$2 \cos 3A = 1$$
$$\Rightarrow \cos 3A = \frac{1}{2}$$
$$\Rightarrow \cos 3A = \cos 60°$$
$$\Rightarrow 3A = 60°$$
$$\Rightarrow A = 20°$$

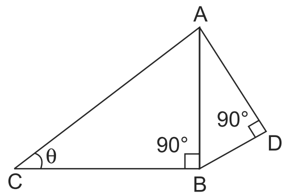
**10.** The value of $3 \sin^2 30° + 2 \tan^2 60° - 5 \cos^2 45°$ is $\mathbf{\frac{17}{4}}$.

**Sol.** $3 \sin^2 30° + 2 \tan^2 60° - 5 \cos^2 45°$

$$= 3\left(\frac{1}{2}\right)^2 + 2\left(\sqrt{3}\right)^2 - 5\left(\frac{1}{\sqrt{2}}\right)^2$$
$$= \frac{3}{4} + 6 - \frac{5}{2}$$
$$= \frac{3 + 24 - 10}{4}$$
$$= \frac{17}{4}$$

## Very Short Answer Type Questions

**11.** In the given figure, AD = 4 cm, BD = 3 cm and CB = 12 cm. Then what is the value of cot θ?




**[CBSE 2008, CBSE SP 2010, 2011]**

**Sol.** In ΔADB,

By Pythagoras' theorem, we have

$$AB^2 = AD^2 + BD^2$$

$$\Rightarrow \quad AB^2 = (4 \text{ cm})^2 + (3 \text{ cm})^2$$

$$= 16 \text{ cm}^2 + 9 \text{ cm}^2$$

$$= 25 \text{ cm}^2$$

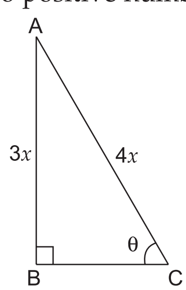
$$\Rightarrow \quad AB = 5 \text{ cm}$$

∴ From ΔABC, $\cot\theta = \cot C = \dfrac{CB}{AB} = \dfrac{12}{5}$

∴ The required value of cot θ is $\dfrac{12}{5}$.

**12.** If $\operatorname{cosec}\theta = \dfrac{4}{3}$, what is the value of $\dfrac{1-\tan^2\theta}{1+\tan^2\theta}$?

**Sol.** Let ΔABC be a right-angled triangle where ∠B = 90°, AC = 4$x$ units and AB = 3$x$ units, where $x$ is a non-zero positive number.




Then, by Pythagoras' theorem, we have

$$AC^2 = AB^2 + BC^2$$

$$\Rightarrow \quad (4x)^2 = (3x)^2 + BC^2$$

$$\Rightarrow \quad BC^2 = 16x^2 - 9x^2$$

$$\Rightarrow \quad BC^2 = 7x^2$$

$$\Rightarrow \quad BC = \sqrt{7}x$$

If ∠ACB = θ, then

$$\frac{1-\tan^2\theta}{1+\tan^2\theta} = \frac{1-\dfrac{AB^2}{BC^2}}{1+\dfrac{AB^2}{BC^2}}$$

$$= \frac{BC^2 - AB^2}{AB^2 + BC^2}$$

$$= \frac{7x^2 - 9x^2}{9x^2 + 7x^2}$$

$$= \frac{-2x^2}{16x^2}$$

$$= \frac{-2}{16}$$

$$= -\frac{1}{8}$$

**13.** Solve the equation (sec A – 2) (tan 3A – 1) = 0, when 0° < A < 90°.

**Sol.** We have

(sec A – 2) (tan 3A – 1) = 0

∴ Either sec A – 2 = 0 …(1)

or, tan 3A – 1 = 0 …(2)

From (1), sec A = 2 = sec 60°

∴ A = 60°

From (2), tan 3A = 1 = tan 45°

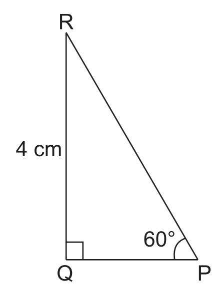
∴ 3A = 45°

$$\Rightarrow \quad A = \frac{45^\circ}{3} = 15^\circ$$

∴ The required solution is A = 60° or 15°.

**14.** In a right triangle PQR, ∠Q = 90°, ∠QPR = 60° and QR = 4 cm. Find the lengths of PQ and PR.

**Sol.** In ΔPQR, we have




$$\sin P = \frac{QR}{PR}$$

$$\Rightarrow \quad \sin 60^\circ = \frac{4}{PR} \text{ cm}$$

$$\Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{4}{PR} \text{ cm}$$

$$\therefore \quad PR = \frac{8}{\sqrt{3}} \text{ cm}$$

Again, $\tan P = \dfrac{QR}{PQ}$

$$\Rightarrow \quad \tan 60^\circ = \frac{4}{PQ} \text{ cm}$$

$$\Rightarrow \quad \sqrt{3} = \frac{4}{PQ} \text{ cm}$$

$$\Rightarrow \quad PQ = \frac{4}{\sqrt{3}} \text{ cm}$$

Hence, the required lengths of PQ and PR are $\dfrac{4}{\sqrt{3}}$ cm and $\dfrac{8}{\sqrt{3}}$ cm respectively.

**15.** For A = 30° and B = 60°, verify that
sin(A + B) = sin A cos B + cos A sin B.

**[CBSE 2023 Basic]**

**Sol.** sin (A + B) = sin (30° + 60°) = sin 90° = 1

$\therefore \quad \sin(A+B) = 1 \quad \ldots(1)$

$$\sin A \cos B = \sin 30° \cos 60° = \frac{1}{2}\times\frac{1}{2} = \frac{1}{4}$$

$\therefore \quad \sin A \cos B = \frac{1}{4} \quad \ldots(2)$

$$\cos A \,.\, \sin B = \cos 30° \,.\, \sin 60° = \frac{\sqrt{3}}{2}\times\frac{\sqrt{3}}{2} = \frac{3}{4}$$

$\therefore \quad \cos A \,.\, \sin B = \frac{3}{4} \quad \ldots(3)$

From (2) and (3),

$$\sin A \cos B + \cos A \,.\, \sin B = \frac{1}{4}+\frac{3}{4} = \frac{4}{4} = 1 \quad \ldots(4)$$

From (1) and (4),

sin (A + B) = sin A cos B + cos A . sin B

**16.** Evaluate: $\dfrac{5\cos^2 60° + 4\sec^2 30° - \tan^2 45°}{\sin^2 30° + \cos^2 30°}$

**[CBSE 2023 Standard]**

**Sol.** $$\frac{5\cos^2 60° + 4\sec^2 30° - \tan^2 45°}{\sin^2 30° + \cos^2 30°}$$

$$= \frac{5\times\left(\frac{1}{2}\right)^2 + 4\times\left(\frac{2}{\sqrt{3}}\right)^2 - (1)^2}{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= \frac{5\times\frac{1}{4} + 4\times\frac{4}{3} - 1}{\frac{1}{4}+\frac{3}{4}}$$

$$= \frac{\frac{5}{4}+\frac{16}{3}-1}{1}$$

$$= \frac{15+64-12}{12} = \frac{67}{12}$$

**17.** Evaluate: $2\sqrt{2}\cos 45° \sin 30° + 2\sqrt{3}\cos 30°$

**[CBSE 2024 Standard]**

**Sol.** $2\sqrt{2}\cos 45° \sin 30° + 2\sqrt{3}\cos 30°$

$$= 2\sqrt{2}\times\frac{1}{\sqrt{2}}\times\frac{1}{2} + 2\sqrt{3}\times\frac{\sqrt{3}}{2}$$

$$= 1 + 3 = 4$$

$\therefore \; 2\sqrt{2}\cos 45° \sin 30° + 2\sqrt{3}\cos 30° = 4$

## Short Answer Type Questions

**18.** Find the value of the expression

$$\frac{23\operatorname{cosec}^3 30° - 47\sec^2 45° + 57\cot^2 60°}{31\sec^4 0° + 59\tan^2 60° - 136\cos^3 60°}$$

**Sol.** We have

$$\frac{23\operatorname{cosec}^3 30° - 47\sec^2 45° + 57\cot^2 60°}{31\sec^4 0° + 59\tan^2 60° - 136\cos^3 60°}$$

$$= \frac{23\times 2^3 - 47\times\left(\sqrt{2}\right)^2 + 57\times\left(\frac{1}{\sqrt{3}}\right)^2}{31\times 1^4 + 59\times\left(\sqrt{3}\right)^2 - 136\times\left(\frac{1}{2}\right)^3}$$

$$= \frac{23\times 8 - 47\times 2 + 57\times\frac{1}{3}}{31 + 59\times 3 - 136\times\frac{1}{8}}$$

$$= \frac{184 - 94 + 19}{31 + 177 - 17}$$

$$= \frac{203 - 94}{208 - 17}$$

$$= \frac{109}{191}$$

$\therefore$ The required value of the expression is $\frac{109}{191}$.

**19.** Verify the following for θ = 60° and θ = 45° separately:

$$\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} - \sec\theta = \sec\theta - \sqrt{\frac{1-\sin\theta}{1+\sin\theta}}.$$

**Sol.** For θ = 60°, we have

$$\text{LHS} = \sqrt{\frac{1+\sin\theta}{1-\sin\theta}} - \sec\theta$$

$$= \sqrt{\frac{1+\sin 60°}{1-\sin 60°}} - \sec 60°$$

$$= \sqrt{\frac{1+\frac{\sqrt{3}}{2}}{1-\frac{\sqrt{3}}{2}}} - 2$$

$$= \sqrt{\frac{2+\sqrt{3}}{2-\sqrt{3}}} - 2$$

$$= \frac{\sqrt{\left(2+\sqrt{3}\right)^2}}{\sqrt{4-3}} - 2$$

$$= 2 + \sqrt{3} - 2 = \sqrt{3}$$

$$\text{RHS} = \sec\theta - \sqrt{\frac{1-\sin\theta}{1+\sin\theta}}$$

$$= \sec 60° - \sqrt{\frac{1-\sin 60°}{1+\sin 60°}}$$

$$= 2 - \sqrt{\frac{1-\frac{\sqrt{3}}{2}}{1+\frac{\sqrt{3}}{2}}}$$

$$= 2 - \sqrt{\frac{2-\sqrt{3}}{2+\sqrt{3}}}$$

$$= 2 - \frac{\sqrt{(2-\sqrt{3})^2}}{\sqrt{4-3}}$$

$$= 2 - 2 + \sqrt{3}$$

$$= \sqrt{3}$$

$\therefore$ LHS = RHS

Again, for $\theta = 45°$, we have

$$\text{LHS} = \sqrt{\frac{1+\sin\theta}{1-\sin\theta}} - \sec\theta$$

$$= \sqrt{\frac{1+\sin 45°}{1-\sin 45°}} - \sec 45°$$

$$= \sqrt{\frac{1+\frac{1}{\sqrt{2}}}{1-\frac{1}{\sqrt{2}}}} - \sqrt{2}$$

$$= \sqrt{\frac{\sqrt{2}+1}{\sqrt{2}-1}} - \sqrt{2}$$

$$= \frac{\sqrt{(\sqrt{2}+1)^2}}{\sqrt{(\sqrt{2})^2-1}} - \sqrt{2}$$

$$= \sqrt{2} + 1 - \sqrt{2}$$

$$= 1$$

$$\text{RHS} = \sec\theta - \sqrt{\frac{1-\sin\theta}{1+\sin\theta}}$$

$$= \sec 45° - \sqrt{\frac{1-\sin 45°}{1+\sin 45°}}$$

$$= \sqrt{2} - \sqrt{\frac{1-\frac{1}{\sqrt{2}}}{1+\frac{1}{\sqrt{2}}}}$$

$$= \sqrt{2} - \sqrt{\frac{\sqrt{2}-1}{\sqrt{2}+1}}$$

$$= \sqrt{2} - \frac{\sqrt{(\sqrt{2}-1)^2}}{\sqrt{(\sqrt{2})^2-1}}$$

$$= \sqrt{2} - \sqrt{2} + 1$$

$$= 1$$

$\therefore$ LHS = RHS

Hence, proved.

### Long Answer Type Questions

**20.** (*a*) Find the values of A and B if $\sin(A + 2B) = \frac{\sqrt{3}}{2}$, $\cos(A + 4B) = 0$ and $0° < A, B < 90°$.

(*b*) Verify the relation, $\sin 2x = \frac{2\cot x}{1+\cot^2 x}$ where $x = A + B$, where A and B are obtained in (*a*).

**Sol.** (*a*) We have

$$\sin(A + 2B) = \frac{\sqrt{3}}{2} = \sin 60°$$

$\therefore$ $A + 2B = 60°$ ...(1)

Again,

$$\cos(A + 4B) = 0 = \cos 90°$$

$\therefore$ $A + 4B = 90°$ ...(2)

Subtracting (1) from (2), we get

$$2B = 90° - 60°$$
$$= 30°$$

$\Rightarrow$ $B = 15°$

$\therefore$ From (1), $A = 60° - 2 \times 15°$
$$= 60° - 30°$$
$$= 30°$$

$\therefore$ The required values of A and B are 30° and 15° respectively.

(*b*) We have

$A = 30°$ and $B = 15°$

$\therefore$ $A + B = 30° + 15° = 45°$

$\therefore$ $x = 45°$

Now, LHS = $\sin 2x$
$$= \sin 2 \times 45°$$
$$= \sin 90°$$
$$= 1$$

$$\text{RHS} = \frac{2 \times \cot x}{1+\cot^2 x}$$
$$= \frac{2 \times \cot 45°}{1+\cot^2 45°}$$
$$= \frac{2 \times 1}{1+1}$$
$$= \frac{2}{2}$$
$$= 1$$

$\therefore$ LHS = RHS

Hence, proved.

**21.** Using the identities:

$\cos\theta = \sqrt{\dfrac{1+\cos 2\theta}{2}}$ and $\sin\theta = \sqrt{\dfrac{1-\cos 2\theta}{2}}$,

find the value of $\cos 22\dfrac{1°}{2} \times \sin 22\dfrac{1°}{2}$.

**Sol.** Given that

$$\cos\theta = \sqrt{\frac{1+\cos 2\theta}{2}} \quad \ldots(1)$$

and
$$\sin\theta = \sqrt{\frac{1-\cos 2\theta}{2}} \quad \ldots(2)$$

Putting $\theta = 22\dfrac{1°}{2}$ so that $2\theta = 22\dfrac{1°}{2} \times 2 = 45°$ in equation (1) and (2), we get

$$\cos 22\frac{1°}{2} = \sqrt{\frac{1+\cos 45°}{2}}$$

$$= \sqrt{\frac{1+\frac{1}{\sqrt{2}}}{2}} = \sqrt{\frac{\sqrt{2}+1}{2\sqrt{2}}}$$

and
$$\sin 22\frac{1°}{2} = \sqrt{\frac{1-\cos 45°}{2}}$$

$$= \sqrt{\frac{1-\frac{1}{\sqrt{2}}}{2}} = \sqrt{\frac{\sqrt{2}-1}{2\sqrt{2}}}$$

$$\therefore \cos 22\frac{1°}{2} \times \sin 22\frac{1°}{2} = \frac{\sqrt{\sqrt{2}+1}}{\sqrt{2\sqrt{2}}} \times \frac{\sqrt{\sqrt{2}-1}}{\sqrt{2\sqrt{2}}}$$

$$= \frac{\sqrt{(\sqrt{2}+1)(\sqrt{2}-1)}}{2\sqrt{2}}$$

$$= \frac{\sqrt{2-1}}{2\sqrt{2}} = \frac{1}{2\sqrt{2}}$$

$\therefore$ The required value of the expression is $\dfrac{1}{2\sqrt{2}}$.

## Let's Compete

**(Page 159)**

### Multiple-Choice Questions

**1.** If $\sin(2\theta - \phi) = 1$ and $\cos(\theta + \phi) = \dfrac{1}{2}$, $\theta$ and $\phi$ being positive acute angles, then $\theta$ and $\phi$ are respectively equal to

(*a*) 20° and 70° (*b*) 70° and 20°

(*c*) 10° and 50° (*d*) 50° and 10°

**Sol.** (*d*) 50° and 10°

We have

$$\sin(2\theta - \phi) = 1 = \sin 90°$$

$$\therefore \quad 2\theta - \phi = 90° \quad \ldots(1)$$

and
$$\cos(\theta + \phi) = \frac{1}{2} = \cos 60°$$

$$\theta + \phi = 60° \quad \ldots(2)$$

Adding (1) and (2), we get

$$3\theta = 90° + 60° = 150°$$

$$\therefore \quad \theta = \frac{150°}{3} = 50°$$

$\therefore$ From (2), we get

$$\phi = 60° - \theta$$
$$= 60° - 50°$$
$$= 10°$$

$\therefore$ The required values of $\theta$ and $\phi$ are 50° and 10° respectively.

**2.** Using the identity: $\cot(A + B) = \dfrac{\cot A \cot B - 1}{\cot B + \cot A}$ and choosing suitable standard values of A and B, we can find cot 75° as

(*a*) $2+\sqrt{3}$ (*b*) $2-\sqrt{3}$

(*c*) $3-\sqrt{2}$ (*d*) $3+\sqrt{2}$

**Sol.** (*b*) $2-\sqrt{3}$

Putting A = 30° and B = 45°, so that

$$A + B = 30° + 45° = 75°$$

We have

$$\cot(A+B) = \frac{\cot A \cot B - 1}{\cot B + \cot A}$$

$$\Rightarrow \quad \cot(30° + 45°) = \frac{\cot 30° \cot 45° - 1}{\cot 45° + \cot 30°}$$

$$\Rightarrow \quad \cot 75° = \frac{\sqrt{3}\times 1 - 1}{1+\sqrt{3}}$$

$$= \frac{\sqrt{3}-1}{\sqrt{3}+1}$$

$$= \frac{(\sqrt{3}-1)^2}{(\sqrt{3})^2 - 1^2}$$

$$= \frac{3+1-2\sqrt{3}}{2}$$

$$= \frac{4-2\sqrt{3}}{2}$$

$$= 2-\sqrt{3}$$

**3.** The value of $\dfrac{\tan 60° - \tan 30°}{1+\tan 60° \tan 30°}$ is equal to

(*a*) $\dfrac{\sin 60°}{\cos 30°}$ (*b*) tan 45° tan 60°

(*c*) $\dfrac{\tan 30°}{\sin 90°}$ (*d*) sin 60° cosec 30°

**Sol.** (*c*) $\dfrac{\tan 30^\circ}{\sin 90^\circ}$

We have

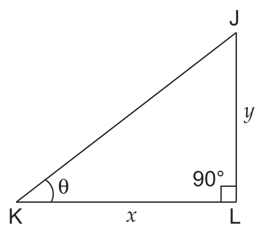
$$\frac{\tan 60^\circ - \tan 30^\circ}{1 + \tan 60^\circ \tan 30^\circ} = \frac{\sqrt{3} - \frac{1}{\sqrt{3}}}{1 + \sqrt{3} \times \frac{1}{\sqrt{3}}}$$

$$= \frac{3 - 1}{(1+1)\sqrt{3}}$$

$$= \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$= \frac{\tan 30^\circ}{\sin 90^\circ}$$

**4.** In the adjoining figure, if $\angle JKL = \theta$, KL = $x$ and JL = $y$, then



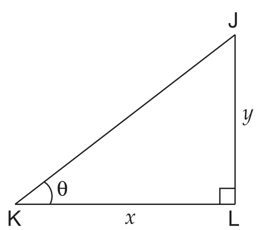

(*a*) $\cot \theta < \operatorname{cosec} \theta$

(*b*) $\cot \theta > \operatorname{cosec} \theta$

(*c*) $\cot \theta = \operatorname{cosec} \theta$

(*d*) $\cot \theta$ and $\operatorname{cosec} \theta$ are both undefined

**Sol.** (*a*) $\cot \theta < \operatorname{cosec} \theta$




In $\Delta JKL$,

$\because$ $\angle L = 90^\circ$

$\therefore$ By Pythagoras' theorem, we have

$$JK^2 = KL^2 + JL^2$$

$$\Rightarrow \quad JK^2 = x^2 + y^2$$

$$\therefore \quad JK = \sqrt{x^2 + y^2}$$

Now, in $\Delta JKL$,

$$\cot \theta = \frac{KL}{JL} = \frac{x}{y} \quad \ldots(1)$$

and $$\operatorname{cosec} \theta = \frac{JK}{JL}$$

$$= \frac{\sqrt{x^2 + y^2}}{y} \quad \ldots(2)$$

Now, $\quad x < \sqrt{x^2 + y^2}$

[$\because$ Length of the hypotenuse is always greater than the other two sides]

$$\Rightarrow \quad \frac{x}{y} < \frac{\sqrt{x^2 + y^2}}{y}$$

$$\Rightarrow \quad \cot \theta < \operatorname{cosec} \theta \quad \text{[From (1) and (2)]}$$

**5.** In $\Delta ABC$, $\angle A = 90^\circ$ and $\cot B = \dfrac{12}{5}$. Then $\tan^2 B - \sin^2 B$ is equal to

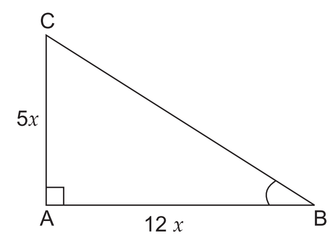
(*a*) $\sin^2 B + \tan^2 B$ (*b*) $\dfrac{\sin^2 B}{\tan^2 B}$

(*c*) $\dfrac{\tan^2 B}{\sin^2 B}$ (*d*) $\sin^2 B \tan^2 B$

**Sol.** (*d*) $\sin^2 B \tan^2 B$

In $\Delta ABC$,

Let AB = $12x$ units and AC = $5x$ units, where $x$ is a non-zero positive number.




Now, by Pythagoras' theorem, we have

$$BC^2 = AB^2 + AC^2$$

$$= (12x)^2 + (5x)^2$$

$$= 169x^2$$

Now, $$\tan^2 B - \sin^2 B = \frac{AC^2}{AB^2} - \frac{AC^2}{BC^2}$$

$$= AC^2 \times \frac{BC^2 - AB^2}{AB^2 \times BC^2}$$

$$= AC^2 \times \frac{AC^2}{AB^2 \times BC^2}$$

[$\because BC^2 = AC^2 + AB^2$]

$$= \frac{AC^2}{AB^2} \times \frac{AC^2}{BC^2}$$

$$= \tan^2 B \times \sin^2 B$$

$$= \sin^2 B \tan^2 B$$

**6.** In a right triangle ABC, $\angle C = 90°$, $\tan A = \frac{1}{\sqrt{3}}$, $\tan B = \sqrt{3}$. Then the value of $\cos A \cos B - \sin A \sin B$ is equal to

(*a*) 1 (*b*) 0

(*c*) –1 (*d*) 2

**Sol.** (*b*) 0

We have

$$\tan A = \frac{1}{\sqrt{3}} = \tan 30°$$

$$\therefore \quad A = 30° \quad \ldots(1)$$

$$\tan B = \sqrt{3} = \tan 60°$$

$$\therefore \quad B = 60° \quad \ldots(2)$$

$\therefore$ $\cos A \cos B - \sin A \sin B$

$$= \cos 30° \cos 60° - \sin 30° \sin 60°$$

$$= \frac{\sqrt{3}}{2} \times \frac{1}{2} - \frac{1}{2} \times \frac{\sqrt{3}}{2}$$

$$= \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = 0$$

**7.** If $45° < \theta < 90°$, then

(*a*) $\cos\theta > \sin\theta$ (*b*) $\sin\theta > \tan\theta$

(*c*) $\sin\theta > \cos\theta$ (*d*) $\sin\theta = \tan\theta$

**Sol.** (*c*) $\sin\theta > \cos\theta$

We see that $\cos 45° = \frac{1}{\sqrt{2}}$, $\cos 60° = \frac{1}{2}$ and $\cos 90° = 0$ and $\sin 45° = \frac{1}{\sqrt{2}}$, $\sin 60° = \frac{\sqrt{3}}{2}$ and $\sin 90° = 1$.

So, when $\theta$ increases from 45° to 90°, then $\cos\theta$ decreases from $\frac{1}{\sqrt{2}}$ to 0, but $\sin\theta$ increases from $\frac{1}{\sqrt{2}}$ to 1. Hence, $\sin\theta > \cos\theta$ when $45° < \theta < 90°$.

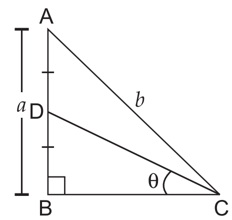
**8.** ABC is a right triangle such that $\angle ABC = 90°$, AB = $a$ units, D is the mid-point of AB, AC = $b$ units and $\angle DCB = \theta$. Then the value of $\tan\theta$ is equal to

(*a*) $\frac{a}{\sqrt{4b^2 - 3a^2}}$ (*b*) $\frac{2\sqrt{b^2 - a^2}}{\sqrt{4b^2 - 3a^2}}$

(*c*) $\frac{\sqrt{4b^2 - 3a^2}}{2\sqrt{b^2 - a^2}}$ (*d*) $\frac{a}{2\sqrt{b^2 - a^2}}$

**Sol.** (*d*) $\frac{a}{2\sqrt{b^2 - a^2}}$

In $\Delta$ABC, $\angle B = 90°$, D is the mid-point of AB, AB = $a$ units, AC = $b$ units and $\angle DCB = \theta$.




From right-angled triangle DBC,

$$DB = \frac{1}{2} AB = \frac{a}{2} \quad \ldots(1)$$

$\therefore$ In $\Delta$ABC, by Pythagoras' theorem, we have

$$AC^2 = AB^2 + BC^2$$

$$\Rightarrow \quad b^2 = a^2 + BC^2$$

$$\therefore \quad BC = \sqrt{b^2 - a^2} \quad \ldots(2)$$

Now, in $\Delta$DBC, we have

$$\tan\theta = \tan \angle DCB$$

$$= \frac{BD}{BC}$$

$$= \frac{\frac{a}{2}}{\sqrt{b^2 - a^2}}$$

[From (1) and (2)]

$$= \frac{a}{2\sqrt{b^2 - a^2}}$$

$\therefore$ The required value of $\tan\theta$ is $\frac{a}{2\sqrt{b^2 - a^2}}$.

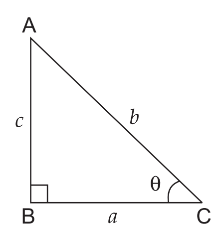
**9.** If $5\sin\theta - 3\cos\theta = \tan\theta\cos\theta$, then the value of $\tan\theta$ is equal to

(*a*) $\frac{3}{4}$ (*b*) $\frac{4}{3}$

(*c*) $\frac{5}{3}$ (*d*) $\frac{3}{5}$

**Sol.** (*a*) $\frac{3}{4}$

Let ABC be a right-angled triangle, where $\angle B = 90°$.




Let $\angle ACB = \theta$, BC = $a$ units, AC = $b$ units and AB = $c$ units

Now, from the given equation

$5 \sin \theta - 3 \cos \theta = \tan \theta \cos \theta$, we have

$$5 \times \frac{c}{b} - 3 \times \frac{a}{b} = \frac{c}{a} \times \frac{a}{b}$$

$$\Rightarrow \quad \frac{5c}{b} - \frac{3a}{b} = \frac{c}{b}$$

$$\Rightarrow \quad 5c - 3a = c$$

$$\Rightarrow \quad 4c = 3a$$

$$\Rightarrow \quad \frac{c}{a} = \frac{3}{4}$$

$$\Rightarrow \quad \tan \theta = \frac{3}{4}$$

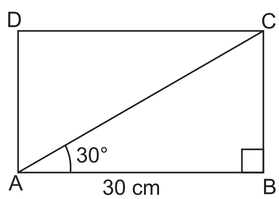
$\therefore$ The required value of $\tan \theta$ is $\frac{3}{4}$.

**10.** In a rectangle ABCD, AB = 30 cm and $\angle BAC = 30°$ where AC is a diagonal of the rectangle. Then the area of the rectangle ABCD is

(*a*) $300 \text{ cm}^2$ (*b*) $300\sqrt{3} \text{ cm}^2$

(*c*) $310\sqrt{3} \text{ cm}^2$ (*d*) $300\sqrt{2} \text{ cm}^2$

**Sol.** (*b*) $300\sqrt{3} \text{ cm}^2$




In $\Delta ABC$, we have

$$\tan 30° = \tan \angle BAC = \frac{BC}{AB}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{BC}{30}$$

$$\Rightarrow \quad BC = \frac{30}{\sqrt{3}} = \frac{30\sqrt{3}}{3} = 10\sqrt{3}$$

$\therefore$ Area of rectangle ABCD

$$= AB \times BC = 30 \times 10\sqrt{3} \text{ cm}^2 = 300\sqrt{3} \text{ cm}^2$$

## Life Skills

**(Page 159)**

**1.** Two poor young brothers in a village were living separately with their respective families in their paternal property after the death of their parents. They had a common orchard where four vertical fruit trees of different heights were present. Their parents told them before their death that their younger son will be the owner of those trees whose heights were less than or equal to 10 m and the elder son will be the owner of trees whose heights were more than 10 m. But they could not measure the heights of the trees due to the fact that they were not climbable. So, there was frequent quarrel between the two brothers about the ownership of the fruit trees. There was a surveyor in the village who had an instrument known as Sextant to measure the angle of elevation of the top of any object (i.e. the angle made by an object) at a known point. He agreed to solve the problem of measuring the heights of the four trees with the help of his Sextant instrument. He measured the angles of elevation of the top of the trees from their feet say A, B, C and D, from four suitable different points in the orchard, say P, Q, R and S respectively. These angles were 30°, 45°, 60° and 30° respectively. If PA = 12 m, QB = 11 m, RC = 5 m and SD = 24 m, find the trees which would be owned by the younger brother and the elder brother. Find their heights also.

**Sol.** (*a*)

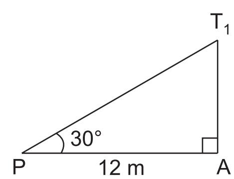


Fig. (*i*)

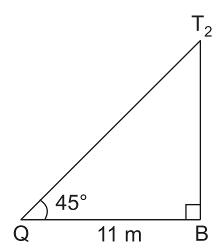


Fig. (*ii*)

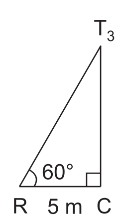


Fig. (*iii*)

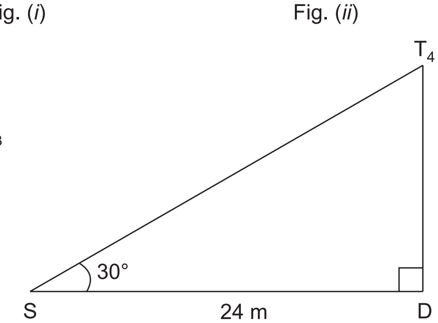


Fig. (*iv*)

Let $T_1$, $T_2$, $T_3$ and $T_4$ be the trees at A, B, C and D respectively. $\angle T_1AP = \angle T_2BQ = \angle T_3CR = \angle T_4DS = 90°$

$\therefore$ From Fig. (*i*) to (*iv*), we have

$$\tan 30° = \frac{AT_1}{PA} = \frac{AT_1}{12}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{AT_1}{12}$$

$$\Rightarrow \quad AT_1 = \frac{12}{\sqrt{3}} = 4\sqrt{3}$$

$$= 6.92 \text{ m (approx.)}$$

$$\tan 45° = \frac{BT_2}{QB} = \frac{BT_2}{11}$$

$$\Rightarrow \quad 1 = \frac{BT_2}{11}$$

$$\Rightarrow \quad BT_2 = 11 \text{ m}$$

$$\tan 60° = \frac{CT_3}{RC} = \frac{CT_3}{5}$$

$$\Rightarrow \quad \sqrt{3} = \frac{CT_3}{5}$$

$$\Rightarrow \quad CT_3 = 5\sqrt{3}$$

$$= 8.65 \text{ m (approx.)}$$

$$\tan 30° = \frac{DT_4}{SD} = \frac{DT_4}{24}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{DT_4}{24}$$

$$\Rightarrow \quad DT_4 = \frac{24}{\sqrt{3}}$$

$$= \frac{24\sqrt{3}}{3} = 8\sqrt{3}$$

$$= 13.85 \text{ m (approx.)}$$

Hence, trees with feet A and C of heights 6.92 m and 8.65 m (approx.) respectively will be owned by the younger brother and those with feet B and D of heights 11 m and 13.85 m (approx.) respectively will be owned by the elder brother.

# (II) Trigonometric Identities

## Check Your Progress

**(Page 166)**

### Multiple-Choice Questions

**1.** sin θ (cosec θ – sin θ) is equal to

(*a*) $\sin^2 \theta$ (*b*) $\sin \theta \cos \theta$

(*c*) $\cos^2 \theta$ (*d*) $\cos \theta - \sin^2 \theta$

**Sol.** (*c*) $\cos^2 \theta$

We have

$$\sin \theta (\operatorname{cosec} \theta - \sin \theta) = \sin \theta \times \frac{1}{\sin \theta} - \sin^2 \theta$$

$$= 1 - \sin^2 \theta = \cos^2 \theta$$

**2.** The value of $\sec A - \dfrac{\tan^2 A}{1 + \sec A}$ is equal to

(*a*) 1 (*b*) 2

(*c*) $\sec^2 A$ (*d*) $\tan^2 A$

**Sol.** (*a*) 1

We have

$$\sec A - \frac{\tan^2 A}{1 + \sec A} = \frac{1}{\cos A} - \frac{\frac{\sin^2 A}{\cos^2 A}}{1 + \frac{1}{\cos A}}$$

$$= \frac{1}{\cos A} - \frac{\sin^2 A}{\cos^2 A} \times \frac{\cos A}{1 + \cos A}$$

$$= \frac{1}{\cos A} - \frac{\sin^2 A}{\cos A(1 + \cos A)}$$

$$= \frac{1 + \cos A - \sin^2 A}{\cos A(1 + \cos A)}$$

$$= \frac{(1 - \sin^2 A) + \cos A}{\cos A(1 + \cos A)}$$

$$= \frac{\cos^2 A + \cos A}{\cos^2 A + \cos A} = 1$$

**3.** $9 \sec^2 A - 9 \tan^2 A$ is equal to

(*a*) 9

(*b*) 0

(*c*) 8

(*d*) $\frac{1}{9}$ **[CBSE 2023 Basic]**

**Sol.** (*a*) 9

$$9 \sec^2 A - 9 \tan^2 A = 9[1 + \tan^2 A] - 9 \tan^2 A$$

$$= 9 + 9 \tan^2 A - 9 \tan^2 A$$

$$= 9$$

$$\Rightarrow \quad 9 \sec^2 A - 9 \tan^2 A = 9$$

**4.** If $\tan A = \frac{2}{5}$, then the value of $\frac{1-\cos^2 A}{1-\sin^2 A}$ is

(*a*) $\frac{25}{4}$ (*b*) $\frac{4}{25}$

(*c*) $\frac{4}{5}$ (*d*) $\frac{5}{4}$

**Sol.** (*b*) $\frac{4}{25}$

$$\frac{1-\cos^2 A}{1-\sin^2 A} = \frac{\sin^2 A + \cos^2 A - \cos^2 A}{\sin^2 A + \cos^2 A - \sin^2 A}$$

$$= \frac{\sin^2 A}{\cos^2 A}$$

$$= \tan^2 A$$

$$= \left(\frac{2}{5}\right)^2 = \frac{4}{25}$$

$$\therefore \quad \frac{1-\cos^2 A}{1-\sin^2 A} = \frac{4}{25}$$

**5.** If $\sin\theta = \frac{3}{4}$, then $\frac{(\sec^2\theta - 1)\cos^2\theta}{\sin\theta}$ equals

(*a*) $\frac{3}{5}$ (*b*) $\frac{3}{4}$

(*c*) $\frac{4}{3}$ (*d*) $\frac{9}{16}$

**Sol.** (*b*) $\frac{3}{4}$

$$\frac{(\sec^2\theta - 1)\cos^2\theta}{\sin\theta} = \frac{(1+\tan^2\theta - 1)\cos^2\theta}{\sin\theta}$$

$$= \frac{\tan^2\theta \,.\, \cos^2\theta}{\sin\theta}$$

$$= \frac{\sin^2\theta \,.\, \cos^2\theta}{\cos^2\theta \,.\, \sin\theta}$$

$$= \sin\theta$$

$$= \frac{3}{4}$$

$$\therefore \quad \frac{(\sec^2\theta - 1)\cos^2\theta}{\sin\theta} = \frac{3}{4}$$

**6.** If $\sec\theta - \tan\theta = m$, then the value of $\sec\theta + \tan\theta$ is

(*a*) $1 - \frac{1}{m}$ (*b*) $m^2 - 1$

(*c*) $\frac{1}{m}$ (*d*) $-m$

**[CBSE 2024 Standard]**

**Sol.** (*c*) $\frac{1}{m}$

Given, $\sec\theta - \tan\theta = m$

we have, $\sec^2\theta - \tan^2\theta = 1$

$\Rightarrow$ $(\sec\theta + \tan\theta)(\sec\theta - \tan\theta) = 1$

$$\Rightarrow \quad \sec\theta + \tan\theta = \frac{1}{\sec\theta - \tan\theta} = \frac{1}{m}$$

$$\Rightarrow \quad \sec\theta + \tan\theta = \frac{1}{m}$$

### Very Short Answer Type Questions

**7.** What is the constant value of the expression $2\cos^2\theta\,(1+\tan^2\theta)$?

**Sol.** We have

$$2\cos^2\theta\,(1+\tan^2\theta) = 2\cos^2\theta \times \sec^2\theta$$

$$= 2 \times \cos^2\theta \times \frac{1}{\cos^2\theta} = 2$$

which is the required value.

**8.** What is the result of elimination of θ between the equations $x = a\cos\theta$ and $y = a\sin\theta$?

**Sol.** We have

$$x^2 + y^2 = a^2\cos^2\theta + a^2\sin^2\theta$$

$$= a^2(\cos^2\theta + \sin^2\theta)$$

$$= a^2 \times 1 = a^2$$

∴ The required result of elimination of θ is $x^2 + y^2 = a^2$.

**9.** Prove that $\frac{\sec^2\theta - \sin^2\theta}{\tan^2\theta} = \text{cosec}^2\theta - \cos^2\theta$

($0° < \theta < 90°$).

**Sol.** We have

$$\text{LHS} = \frac{\sec^2\theta - \sin^2\theta}{\tan^2\theta}$$

$$= \frac{\frac{1}{\cos^2\theta} - \sin^2\theta}{\frac{\sin^2\theta}{\cos^2\theta}}$$

$$= \frac{1-\cos^2\theta\sin^2\theta}{\cos^2\theta} \times \frac{\cos^2\theta}{\sin^2\theta}$$

$$= \frac{1-\cos^2\theta\sin^2\theta}{\sin^2\theta}$$

$$= \frac{1}{\sin^2\theta} - \cos^2\theta$$

$$= \text{cosec}^2\theta - \cos^2\theta = \text{RHS}$$

Hence, proved.

**10.** Prove that $\frac{\cos A}{1-\tan A} - \frac{\sin^2 A}{\cos A - \sin A} = \cos A + \sin A$.

($A \neq 45°$)

**Sol.** $$\text{LHS} = \frac{\cos A}{1-\tan A} - \frac{\sin^2 A}{\cos A - \sin A}$$

$$= \frac{\cos A}{1-\frac{\sin A}{\cos A}} - \frac{\sin^2 A}{\cos A - \sin A}$$

$$= \frac{\cos^2 A}{\cos A - \sin A} - \frac{\sin^2 A}{\cos A - \sin A}$$

$$= \frac{(\cos A + \sin A)(\cos A - \sin A)}{\cos A - \sin A}$$

$$= \cos A + \sin A \qquad [\because A \neq 45°]$$

$$= \text{RHS}$$

Hence, proved.

**11.** If $\sin\theta + \cos\theta = \sqrt{3}$, then find the value of $\sin\theta \cdot \cos\theta$. **[CBSE 2023 Standard]**

**Sol.** $$\sin\theta + \cos\theta = \sqrt{3}$$

Squaring both sides, we get

$$(\sin\theta + \cos\theta)^2 = (\sqrt{3})^2$$

$$\Rightarrow \sin^2\theta + \cos^2\theta + 2\sin\theta \cdot \cos\theta = 3$$

$$\Rightarrow 1 + 2\sin\theta \cdot \cos\theta = 3 \quad [\because \sin^2\theta + \cos^2\theta = 1]$$

$$\Rightarrow 2\sin\theta \cdot \cos\theta = 2$$

$$\Rightarrow \sin\theta \cdot \cos\theta = 1$$

**12.** If $\cos A + \cos^2 A = 1$, then find the value of $\sin^2 A + \sin^4 A$.

**Sol.** Given, $\cos A + \cos^2 A = 1$

$$\Rightarrow \cos A = 1 - \cos^2 A \qquad \ldots(1)$$

$$\sin^2 A + \sin^4 A = \sin^2 A + \sin^2 A \cdot \sin^2 A$$

$$= \sin^2 A + (1-\cos^2 A)(1-\cos^2 A)$$

$$= \sin^2 A + \cos A \cdot \cos A$$

$$= \sin^2 A + \cos^2 A$$

$$= 1$$

$$\therefore \sin^2 A + \sin^4 A = 1$$

### Short Answer Type Questions

**13.** If $x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$, and $x\sin\theta = y\cos\theta$, show that $x^2 + y^2 = 1$.

**Sol.** We have $x\sin\theta = y\cos\theta$

$$\Rightarrow \frac{\sin\theta}{y} = \frac{\cos\theta}{x}$$

Squaring both sides

$$\Rightarrow \frac{\sin^2\theta}{y^2} = \frac{\cos^2\theta}{x^2}$$

$$= \frac{\sin^2\theta + \cos^2\theta}{x^2 + y^2}$$

$$= \frac{1}{x^2 + y^2}$$

$$\therefore \sin^2\theta = \frac{y^2}{x^2+y^2} \qquad \ldots(1)$$

and $$\cos^2\theta = \frac{x^2}{x^2+y^2} \qquad \ldots(2)$$

Now, from the first given equation, we have

$$\frac{x\sin^3\theta}{\sin\theta\cos\theta} + \frac{y\cos^3\theta}{\sin\theta\cos\theta} = 1$$

$$\Rightarrow \frac{x\sin^2\theta}{\cos\theta} + \frac{y\cos^2\theta}{\sin\theta} = 1$$

$$\Rightarrow x \times \frac{y^2}{x^2+y^2} \times \frac{\sqrt{x^2+y^2}}{x} + y \times \frac{x^2}{x^2+y^2} \times \frac{\sqrt{x^2+y^2}}{y} = 1$$

[From (1) and (2)]

$$\Rightarrow \frac{y^2}{\sqrt{x^2+y^2}} + \frac{x^2}{\sqrt{x^2+y^2}} = 1$$

$$\Rightarrow \frac{x^2+y^2}{\sqrt{x^2+y^2}} = 1$$

$$\Rightarrow \sqrt{x^2+y^2} = 1$$

$$\Rightarrow x^2 + y^2 = 1$$

Hence, proved.

**14.** If $\cot\theta + \tan\theta = m$ and $\operatorname{cosec}\theta - \sin\theta = n$, show that $(m^2 n)^{\frac{2}{3}} - (mn^2)^{\frac{2}{3}} = 1$.

**Sol.** We have

$$\cot\theta + \tan\theta = m$$

$$\Rightarrow \frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta} = m$$

$$\Rightarrow \frac{\cos^2\theta + \sin^2\theta}{\sin\theta\cos\theta} = m$$

$$\Rightarrow m = \frac{1}{\sin\theta\cos\theta} \qquad \ldots(1)$$

Again, $\operatorname{cosec}\theta - \sin\theta = n$

$$\Rightarrow \frac{1}{\sin\theta} - \sin\theta = n$$

$$\Rightarrow \frac{1-\sin^2\theta}{\sin\theta} = n$$

$$\Rightarrow n = \frac{\cos^2\theta}{\sin\theta} \qquad \ldots(2)$$

Now, LHS = $\left(m^2n\right)^{\frac{2}{3}} - \left(mn^2\right)^{\frac{2}{3}}$

$= m^{\frac{4}{3}}n^{\frac{2}{3}} - m^{\frac{2}{3}}n^{\frac{4}{3}}$

$$= \frac{1}{(\sin\theta)^{\frac{4}{3}}(\cos\theta)^{\frac{4}{3}}} \times \frac{(\cos\theta)^{\frac{4}{3}}}{(\sin\theta)^{\frac{2}{3}}} - \frac{1}{(\sin\theta)^{\frac{2}{3}}(\cos\theta)^{\frac{2}{3}}} \times \frac{(\cos\theta)^{\frac{8}{3}}}{(\sin\theta)^{\frac{4}{3}}}$$

$$= \frac{1}{\sin^2\theta} - \frac{\cos^2\theta}{\sin^2\theta}$$

$$= \frac{1-\cos^2\theta}{\sin^2\theta}$$

$$= \frac{\sin^2\theta}{\sin^2\theta}$$

= 1 = RHS

Hence, proved.

**15.** Prove that

$\sec\theta\,(1 - \sin\theta)\,(\sec\theta + \tan\theta) = 1.$

**[CBSE 2023 Basic]**

**Sol.** LHS = $\sec\theta\,(1 - \sin\theta)\,(\sec\theta + \tan\theta)$

$$= \sec\theta\,(1-\sin\theta)\left(\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}\right)$$

$$= \sec\theta\,(1-\sin\theta)\left(\frac{1+\sin\theta}{\cos\theta}\right)$$

$$= \sec\theta\,\frac{(1-\sin^2\theta)}{\cos\theta}$$

$$= \frac{1}{\cos\theta} \times \frac{\cos^2\theta}{\cos\theta}$$

$[\because \sin^2\theta + \cos^2\theta = 1]$

= 1

= RHS

**16.** Prove that

$\sin^6\theta + \cos^6\theta + 3\sin^2\theta\cos^2\theta = 1.$

**[CBSE 2024 Basic]**

**Sol.** LHS = $\sin^6\theta + \cos^6\theta + 3\sin^2\theta\cos^2\theta$

$= (\sin^2\theta)^3 + (\cos^2\theta)^3 + 3\sin^2\theta\cos^2\theta$

$= (\sin^2\theta + \cos^2\theta)(\sin^4\theta + \cos^4\theta - \sin^2\theta\cos^2\theta) + 3\sin^2\theta\cos^2\theta$

$= 1[(\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta - \sin^2\theta\cdot\cos^2\theta] + 3\sin^2\theta\cos^2\theta$

$= [1 - 3\sin^2\theta\cos^2\theta] + 3\sin^2\theta\cos^2\theta$

$= 1 - 3\sin^2\theta\cos^2\theta + 3\sin^2\theta\cos^2\theta$

= 1

= RHS

## Long Answer Type Questions

**17.** If $\sec\theta = x + \frac{1}{4x}$, prove that

$\sec\theta + \tan\theta = 2x$ or $\frac{1}{2x}$. **[CBSE 2001]**

**Sol.** We have $\quad \sec\theta = x + \frac{1}{4x}$ ...(1)

Squaring both sides, we get

$$\therefore \quad \sec^2\theta = \left(x + \frac{1}{4x}\right)^2$$

$$= x^2 + \frac{1}{16x^2} + 2x \times \frac{1}{4x}$$

$$= x^2 + \frac{1}{16x^2} + \frac{1}{2}$$

$$\therefore \quad 1 + \tan^2\theta = x^2 + \frac{1}{16x^2} + \frac{1}{2}$$

$$\Rightarrow \quad \tan^2\theta = x^2 + \frac{1}{16x^2} - \frac{1}{2}$$

$$= \left(x - \frac{1}{4x}\right)^2$$

$\therefore \quad \tan\theta = \pm\left(x - \frac{1}{4x}\right)$ ...(2)

$\therefore$ Adding (1) and (2), we get

$$\sec\theta + \tan\theta = x + \frac{1}{4x} \pm \left(x - \frac{1}{4x}\right)$$

$$= 2x \text{ or } \frac{1}{2x}$$

Hence, proved.

**18.** Prove that

$$\frac{\cos x}{\sin x + \cos y} + \frac{\cos y}{\sin y - \cos x} = \frac{\cos x}{\sin x - \cos y} + \frac{\cos y}{\sin y + \cos x}$$

**Sol.** We have

$$\frac{\cos x}{\sin x + \cos y} - \frac{\cos x}{\sin x - \cos y}$$

$$= \frac{\cos x(\sin x - \cos y) - \cos x(\sin x + \cos y)}{\sin^2 x - \cos^2 y}$$

$= \frac{-2\cos x\cos y}{\sin^2 x - \cos^2 y}$ ...(1)

Also,

$$\frac{\cos y}{\sin y + \cos x} - \frac{\cos y}{\sin y - \cos x}$$

$$= \frac{\cos y(\sin y - \cos x) - \cos y(\sin y + \cos x)}{\sin^2 y - \cos^2 x}$$

$$= \frac{-2\cos x \cos y}{1 - \cos^2 y - 1 + \sin^2 x}$$

$$= \frac{-2\cos x \cos y}{\sin^2 x - \cos^2 y} \quad \ldots(2)$$

From (1) and (2), we have

$$\frac{\cos x}{\sin x + \cos y} - \frac{\cos x}{\sin x - \cos y}$$

$$= \frac{\cos y}{\sin y + \cos x} - \frac{\cos y}{\sin y - \cos x}$$

$$\Rightarrow \quad \frac{\cos x}{\sin x + \cos y} + \frac{\cos y}{\sin y - \cos x}$$

$$= \frac{\cos x}{\sin x - \cos y} + \frac{\cos y}{\sin y + \cos x}$$

$\therefore$ LHS = RHS

Hence, proved.

## Higher Order Thinking Skills (HOTS) Questions

**(Page 166)**

**1.** Prove that

$$1 + \tan\theta + \sec\theta = \frac{2}{1 + \cot\theta - \operatorname{cosec}\theta}$$

**Sol.** We have

$(1 + \tan\theta + \sec\theta)(1 + \cot\theta - \operatorname{cosec}\theta)$

$$= \left(1 + \frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta}\right)\left(1 + \frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta}\right)$$

$$= \frac{\cos\theta + \sin\theta + 1}{\cos\theta} \times \frac{\sin\theta + \cos\theta - 1}{\sin\theta}$$

$$= \frac{(\sin\theta + \cos\theta)^2 - 1^2}{\cos\theta\sin\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta - 1}{\cos\theta\sin\theta}$$

$$= \frac{1 - 1 + 2\sin\theta\cos\theta}{\cos\theta\sin\theta} = 2$$

Hence, we have

$$1 + \tan\theta + \sec\theta = \frac{2}{1 + \cot\theta - \operatorname{cosec}\theta}$$

Hence, proved.

**2.** If $x$ and $y$ are two unequal real numbers, show that the equations

(*a*) $\sin^2\theta = \dfrac{(x+y)^2}{4xy}$ and

(*b*) $\cos\theta = x + \dfrac{1}{x}$ are both impossible.

**Sol.** (*a*) We have

$$(x + y)^2 - 4xy = (x - y)^2 > 0, \text{ since } x \neq y$$

$$\therefore \quad (x + y)^2 > 4xy$$

$$\Rightarrow \quad \frac{(x + y)^2}{4xy} > 1$$

$$\Rightarrow \quad \sin^2\theta > 1$$

which is impossible, since we know that $0 \leq \sin^2\theta \leq 1$.

(*b*) We have

$$x + \frac{1}{x} = \frac{x^2 + 1}{x}$$

Now, $\quad x^2 + 1 - x = x^2 - 2 \times \frac{1}{2} \times x + \frac{1}{4} + \frac{3}{4}$

$$= \left(x - \frac{1}{2}\right)^2 + \frac{3}{4} > 0$$

$$\therefore \quad x^2 + 1 > x$$

$$\Rightarrow \quad \frac{x^2 + 1}{x} > 1$$

$$\Rightarrow \quad x + \frac{1}{x} > 1$$

$$\Rightarrow \quad \cos\theta > 1$$

which is also impossible, since we know that $-1 \leq \cos\theta \leq 1$.

Hence, both (*a*) and (*b*) are impossible.

**3.** If $\dfrac{\cos^4 x}{\cos^2 y} + \dfrac{\sin^4 x}{\sin^2 y} = 1$, then show that

$$\frac{\cos^4 y}{\cos^2 x} + \frac{\sin^4 y}{\sin^2 x} = 1.$$

**Sol.** From the given equation, we have

$\cos^4 x \sin^2 y + \sin^4 x \cos^2 y = \cos^2 y \sin^2 y$

$\Rightarrow \cos^4 x(1 - \cos^2 y) + (1 - \cos^2 x)^2 \cos^2 y - \cos^2 y(1 - \cos^2 y) = 0$

$\Rightarrow \cos^4 x - \cos^4 x \cos^2 y + \cos^2 y + \cos^4 x \cos^2 y - 2\cos^2 x \cos^2 y - \cos^2 y + \cos^4 y = 0$

$$\Rightarrow \quad \cos^4 x + \cos^4 y - 2\cos^2 x \cos^2 y = 0$$

$$\Rightarrow \quad (\cos^2 x - \cos^2 y)^2 = 0$$

$$\Rightarrow \quad \cos^2 x = \cos^2 y \quad \ldots(1)$$

$$\Rightarrow \quad 1 - \sin^2 x = 1 - \sin^2 y$$

$$\Rightarrow \quad \sin^2 x = \sin^2 y \quad \ldots(2)$$

Now, $$\text{LHS} = \frac{\cos^4 y}{\cos^2 x} + \frac{\sin^4 y}{\sin^2 x}$$

$$= \frac{\cos^4 y}{\cos^2 y} + \frac{\sin^4 y}{\sin^2 y}$$

$$= \cos^2 y + \sin^2 y$$

$$= 1 = \text{RHS}$$

Hence, proved.

**4.** If $\sec x = \sec y \sec z + \tan y \tan z$, then show that $\sec y = \sec z \sec x \pm \tan z \tan x$.

**Sol.** We have

$$(\sec x - \sec y \sec z)^2 - (\sec y - \sec z \sec x)^2$$

$$= \sec^2 x + \sec^2 y \sec^2 z - 2 \sec x \sec y \sec z - (\sec^2 y + \sec^2 z \sec^2 x - 2 \sec x \sec y \sec z)$$

$$= \sec^2 x + \sec^2 y \sec^2 z - \sec^2 y - \sec^2 z \sec^2 x$$

$$= -\sec^2 x (\sec^2 z - 1) + \sec^2 y (\sec^2 z - 1)$$

$$= -\sec^2 x \tan^2 z + \sec^2 y \tan^2 z$$

$$= -\tan^2 z (1 + \tan^2 x) + \tan^2 z (1 + \tan^2 y)$$

$$= \tan^2 z \tan^2 y - \tan^2 z \tan^2 x \quad \ldots(1)$$

Now, given that $\sec x = \sec y \sec z + \tan y \tan z$

$$\therefore \quad (\sec x - \sec y \sec z)^2 = \tan^2 y \tan^2 z \quad \ldots(2)$$

From (1) and (2), we have

$$\tan^2 y \tan^2 z - (\sec y - \sec z \sec x)^2 = \tan^2 z \tan^2 y - \tan^2 z \tan^2 x$$

$$\Rightarrow \quad (\sec y - \sec z \sec x)^2 = \tan^2 z \tan^2 x$$

$$\Rightarrow \quad \sec y - \sec z \sec x = \pm \tan z \tan x$$

$$\Rightarrow \quad \sec y = \sec z \sec x \pm \tan z \tan x$$

Hence, proved.

## Self-Assessment

**(Page 167)**

### Multiple-Choice Questions

**1.** The value of $5 \tan^2 \theta - 5 \sec^2 \theta$ is

(*a*) 1

(*b*) 5

(*c*) 0

(*d*) –5 **[CBSE SP 2011]**

**Sol.** (*d*) –5

We have

$$5 \tan^2 \theta - 5 \sec^2 \theta = 5 \tan^2 \theta - 5(1 + \tan^2 \theta)$$

$$= -5$$

**2.** If $0° < \theta < 90°$, then $\sqrt{\frac{1+\cos\theta}{1-\cos\theta}}$ is equal to

(*a*) $\text{cosec}^2 \theta + \cot^2 \theta$ (*b*) $\text{cosec}\, \theta + \cot \theta$

(*c*) $\cot \theta - \text{cosec}\, \theta$ (*d*) $\text{cosec}^2 \theta - \cot^2 \theta$

**Sol.** (*b*) $\text{cosec}\, \theta + \cot \theta$

We have $$\sqrt{\frac{1+\cos\theta}{1-\cos\theta}} = \frac{\sqrt{1+\cos\theta}}{\sqrt{1-\cos\theta}} \times \frac{\sqrt{1+\cos\theta}}{\sqrt{1+\cos\theta}}$$

$$= \frac{\sqrt{(1+\cos\theta)^2}}{\sqrt{1-\cos^2\theta}}$$

$$= \frac{1+\cos\theta}{\sqrt{\sin^2\theta}}$$

$$= \frac{1+\cos\theta}{\sin\theta}$$

$$= \frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}$$

$$= \text{cosec}\, \theta + \cot \theta$$

**3.** $\frac{\cos^2 \theta}{\sin^2 \theta} - \frac{1}{\sin^2 \theta}$ in simplified form is

(*a*) $\tan^2 \theta$ (*b*) $\sec^2 \theta$

(*c*) 1 (*d*) –1

**[CBSE 2023 Standard]**

**Sol.** (*d*) –1

$$\frac{\cos^2 \theta}{\sin^2 \theta} - \frac{1}{\sin^2 \theta} = \frac{\cos^2 \theta - 1}{\sin^2 \theta}$$

$$= -\left[\frac{1-\cos^2\theta}{\sin^2\theta}\right]$$

$$= -\left[\frac{\sin^2\theta}{\sin^2\theta}\right]$$

$$= -1$$

**4.** If $x = a \sin \theta$ and $y = b \cos \theta$, then $b^2x^2 + a^2y^2$ is equal to

(*a*) 1 (*b*) $a^2b^2$

(*c*) $\frac{a^2 + b^2}{a^2b^2}$ (*d*) $a^2 + b^2$

**Sol.** (*b*) $a^2b^2$

Given: $$x = a \sin \theta$$

$$y = b \cos \theta$$

$$b^2x^2 + a^2y^2 = b^2 (a \sin \theta)^2 + a^2 (b \cos \theta)^2$$

$$= b^2a^2 \sin^2 \theta + a^2b^2 \cos^2 \theta$$

$$= a^2b^2 (\sin^2 \theta + \cos^2 \theta)$$

$$= a^2b^2$$

### Fill in the Blanks

**5.** If $\sin\theta = \frac{p}{q}$, then the value of $\tan\theta + \sec\theta$ is $\sqrt{\frac{p+q}{q-p}}$.

**Sol.** $\tan\theta + \sec\theta = \frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta}$

$$= \frac{\sin\theta + 1}{\cos\theta} = \frac{\sin\theta + 1}{\sqrt{1-\sin^2\theta}}$$

$$= \frac{\frac{p}{q}+1}{\sqrt{1-\frac{p^2}{q^2}}} \qquad \left[\because \sin\theta = \frac{p}{q}, \text{ given}\right]$$

$$= \frac{p+q}{\sqrt{q^2-p^2}} = \frac{\sqrt{p+q}\,\sqrt{p+q}}{\sqrt{q-p}\,\sqrt{q+p}}$$

$$= \frac{\sqrt{p+q}}{\sqrt{q-p}}$$

**6.** If $x = 3\sec^2\theta - 1$ and $y = 3\tan^2\theta - 2$, then $x - y$ is equal to **4**.

**Sol.** $x - y = 3\sec^2\theta - 1 - 3\tan^2\theta + 2$

$= 3(\sec^2\theta - \tan^2\theta) + 1$

$= 3(1) + 1$

$= 4$

**7.** If $x = m\sin\theta$ and $y = n\cos\theta$, then the value of $n^2x^2 + m^2y^2$ is $\boldsymbol{m^2n^2}$.

**Sol.** $n^2x^2 + m^2y^2 = n^2(m\sin\theta)^2 + m^2(n\cos\theta)^2$

$= n^2m^2\sin^2\theta + m^2n^2\cos^2\theta$

$= m^2n^2(\sin^2\theta + \cos^2\theta)$

$= m^2n^2$

**8.** If $\cos\theta + \cos^2\theta = 1$, then the value of $\sin^2\theta + \sin^4\theta$ is **1**.

**Sol.** $\cos\theta + \cos^2\theta = 1$

$\cos\theta = 1 - \cos^2\theta$

$\cos\theta = \sin^2\theta$

$\sin^2\theta + \sin^4\theta = \sin^2\theta(1 + \sin^2\theta)$

$= \cos\theta(1 + \cos\theta)$

$= \cos\theta + \cos^2\theta$

$= 1$ $[\because \cos\theta + \cos^2\theta = 1, \text{ given}]$

### Assertion-Reason Type Questions

**Directions** (Q. Nos. 9 to 12): Each of these questions contains an assertion followed by reason. Read them carefully, and answer the question on the basis of the following options, select the one that best describes the two statements.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

**9.** **Assertion (A):** The value of sin A is always greater than 1.

**Reason (R):** Hypotenuse is the longest side in a right triangle.

**Sol.** The correct answer is (*d*).

Since, sin A is the ratio of Perpendicular and Hypotenuse and, Hypotenuse is the longest side in a right-angled triangle, hence sin A will always be less than or equal to 1.

Thus, assertion is wrong and reason is correct.

**10.** **Assertion (A):** cot 0° is not defined.

**Reason (R):** tan 0° is not defined and cot is inverse of tan.

**Sol.** The correct answer is (*c*).

tan 0° is 0, cot is inverse of tan so cot 0° will have division by 0 which is not defined.

Thus, reason is wrong.

**11** **Assertion (A):**
$(\sin A - \cos A)^2 + 2\sin A\cos A = 1$

**Reason (R):** $\sin^2 A + \cos^2 A = 1$

**Sol.** The correct answer is (*a*).

$(\sin A - \cos A)^2 + 2\sin A\cos A$

$= \sin^2 A + \cos^2 A - 2\sin A\cos A + 2\sin A\cos A$

$= 1$

Thus, both statements are correct and reason is correct explanation of the assertion.

**12.** **Assertion (A):** For $0 < \theta < 90°$, $\operatorname{cosec}\theta - \cot\theta$ and $\operatorname{cosec}\theta + \cot\theta$ are reciprocal of each other.

**Reason (R):** $\operatorname{cosec}^2\theta - \cot^2\theta = 1$.

**[CBSE 2023 Standard]**

**Sol.** The correct answer is (*a*).

We have, $\operatorname{cosec}^2\theta = 1 + \cot^2\theta$

$\operatorname{cosec}^2\theta - \cot^2\theta = 1$

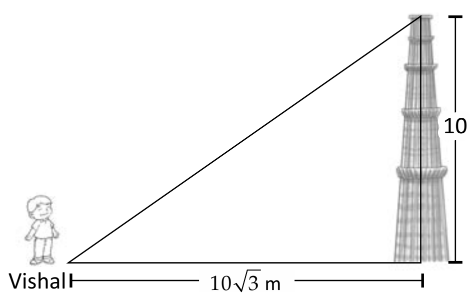
(cosec θ – cot θ) (cosec θ + cot θ) = 1

$$\text{cosec}\,\theta - \cot\theta = \frac{1}{\text{cosec}\,\theta + \cot\theta}$$

∴ cosec θ – cot θ and cosec θ + cot θ are reciprocal of each other.

### Case Study Based Questions

**13.** Vishal went to Delhi with his parents during his summer vacation. One day, they visited a Minar. They took photographs of this place. The height of the Minar was found to be 10 m. They were standing at a distance of $10\sqrt{3}$ m from the base of the Minar. Based on the above information, answer the following questions.




(*a*) What will be the tangent of the angle made by the line segment connecting top of the minar and position of Vishal with the base line?

**Sol.** $\frac{1}{\sqrt{3}}$

P = 10 m, B = $10\sqrt{3}$ m

$$\tan\theta = \frac{P}{B} = \frac{10}{10\sqrt{3}} = \frac{1}{\sqrt{3}}$$

(*b*) What will be the cotangent of the angle made by the line segment connecting top of the minar and position of Vishal with the base line?

**Sol.** $\sqrt{3}$

$$\cot\theta = \frac{B}{P} = \frac{10\sqrt{3}}{10} = \sqrt{3}$$

(*c*) (*i*) What will be the secant of the angle made by the line segment connecting top of the minar and position of Vishal with the base line?

**Sol.** $\frac{2}{\sqrt{3}}$

$$\sec\theta = \frac{H}{B}$$

$$H = \sqrt{P^2 + B^2}$$

$$= \sqrt{(10)^2 + (10\sqrt{3})^2}$$

$$= \sqrt{100 + 100 \times 3} = \sqrt{400}$$

$$H = 20$$

$$\therefore \quad \sec\theta = \frac{H}{B} = \frac{20}{10\sqrt{3}} = \frac{2}{\sqrt{3}}$$

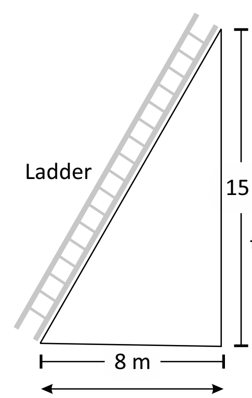
*or*

(*ii*) What will be the cosine of the angle made by the line segment connecting top of the minar and position of Vishal with the base line?

**Sol.** $\frac{\sqrt{3}}{2}$

$$\cos\theta = \frac{B}{H} = \frac{10\sqrt{3}}{20} = \frac{\sqrt{3}}{2}$$

**14.** The roof of a house was made up of galvanized steel sheets. One day due to a heavy storm, some sheets were damaged. To repair the sheets, the workers climbed the roof using a ladder. The foot of the ladder was at a distance of 8 m from the wall of the house and the top of the ladder was at a height of 15 m from the ground. Based on the above situation, answer the following questions.




(*a*) What was the length of the ladder?

**Sol.** 17 m

P = 15 m, B = 8 m, H = ?

$$H = \sqrt{(P)^2 + (B)^2}$$

$$= \sqrt{(15)^2 + (8)^2}$$

$$= \sqrt{225 + 64}$$

$$= \sqrt{289}$$

$$H = 17 \text{ m}$$

(*b*) If θ be the angle made by the ladder with the ground, then what is the value of the sine of the angle made by the ladder?

**Sol.** $\frac{15}{17}$

$$\sin\theta = \frac{P}{H} = \frac{15}{17}$$

(*c*) (*i*) If θ be the angle made by the ladder with the ground, then what is the value of the cosine of the angle made by the ladder with the base level?

**Sol.** $\frac{8}{17}$

$$\cos\theta = \frac{B}{H} = \frac{8}{17}$$

*or*

(*ii*) What is the value of the $\sin^2\theta - \cos^2\theta$?

**Sol.** $\frac{161}{289}$

$$\sin^2\theta - \cos^2\theta$$

$$= \left(\frac{P}{H}\right)^2 - \left(\frac{B}{H}\right)^2 = \left(\frac{15}{17}\right)^2 - \left(\frac{8}{17}\right)^2$$

$$= \frac{225-64}{289} = \frac{161}{289}$$

## Very Short Answer Type Questions

**15.** What is the value of $\frac{1-\tan^2\theta}{1+\tan^2\theta}$ in terms of $\sin^2\theta$ and $\cos^2\theta$?

**Sol.** We have

$$\frac{1-\tan^2\theta}{1+\tan^2\theta} = \frac{1-\frac{\sin^2\theta}{\cos^2\theta}}{\sec^2\theta}$$

$$= \frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta \times \frac{1}{\cos^2\theta}}$$

$$= \cos^2\theta - \sin^2\theta$$

which is the required value.

**16.** If $\sin A + \sin^2 A = 1$, then show that $\cos^2 A + \cos^4 A = 1$.

**Sol.** We have

$$\sin A = 1 - \sin^2 A = \cos^2 A \quad \ldots(1)$$

$$\therefore \quad \cos^2 A + \cos^4 A = \cos^2 A + \sin^2 A \quad \text{[From (1)]}$$

$$= 1$$

Hence, proved.

**17.** Prove that

$$\cos^2\theta + \frac{1}{1+\cot^2\theta} = 1 \qquad (0° < \theta < 90°)$$

**Sol.** We have

$$\text{LHS} = \cos^2\theta + \frac{1}{1+\cot^2\theta}$$

$$= \cos^2\theta + \frac{1}{\text{cosec}^2\theta}$$

$$= \cos^2\theta + \sin^2\theta$$

$$= 1 = \text{RHS}$$

Hence, proved.

**18.** Prove that

$$\sin^2\theta(1+\cot^2\theta) = 1 \qquad (0° < \theta < 90°)$$

**Sol.**

$$\text{LHS} = \sin^2\theta\,(1+\cot^2\theta)$$

$$= \sin^2\theta \times \text{cosec}^2\theta$$

$$= \sin^2\theta \times \frac{1}{\sin^2\theta}$$

$$= 1 = \text{RHS}$$

Hence, proved.

**19.** If $a\cos\theta + b\sin\theta = m$ and $a\sin\theta - b\cos\theta = n$, then prove that $a^2 + b^2 = m^2 + n^2$.

**[CBSE 2023 Standard]**

**Sol.** Given: $a\cos\theta + b\sin\theta = m$ and …(1)

$a\sin\theta - b\cos\theta = n$ …(2)

Squaring (1), we get

$$(a\cos\theta + b\sin\theta)^2 = m^2$$

$$\Rightarrow a^2\cos^2\theta + b^2\sin^2\theta + 2ab\sin\theta\cos\theta = m^2 \quad \ldots(3)$$

Squaring (2), we get

$$(a\sin\theta - b\cos\theta)^2 = n^2$$

$$\Rightarrow a^2\sin^2\theta + b^2\cos^2\theta - 2ab\sin\theta\cos\theta = n^2 \quad \ldots(4)$$

Adding (3) and (4), we get

$$a^2\cos^2\theta + b^2\sin^2\theta + 2ab\sin\theta\cos\theta + a^2\sin^2\theta + b^2\cos^2\theta - 2ab\sin\theta\cos\theta = m^2 + n^2$$

$$\Rightarrow a^2(\cos^2\theta + \sin^2\theta) + b^2(\sin^2\theta + \cos^2\theta) = m^2 + n^2$$

$$\Rightarrow \quad a^2 + b^2 = m^2 + n^2$$

## Short Answer Type Questions

**20.** Prove that

$$(1 + \cot\theta - \text{cosec}\,\theta)(1 + \tan\theta + \sec\theta) = 2$$

**[CBSE 2000]**

**Sol.**

$$\text{LHS} = \left(1 + \frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta}\right)\left(1 + \frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta}\right)$$

$$= \frac{\sin\theta + \cos\theta - 1}{\sin\theta} \times \frac{\cos\theta + \sin\theta + 1}{\cos\theta}$$

$$= \frac{(\sin\theta + \cos\theta)^2 - 1^2}{\sin\theta\cos\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta - 1 + 2\sin\theta\cos\theta}{\sin\theta\cos\theta}$$

$$= \frac{1 - 1 + 2\sin\theta\cos\theta}{\sin\theta\cos\theta}$$

$$= \frac{2\sin\theta\cos\theta}{\sin\theta\cos\theta}$$

$$= 2$$

Hence, proved.

**21.** If $\operatorname{cosec}\theta = \frac{\sqrt{10}}{3}$, find the value of $\frac{1}{\sec\theta + \tan\theta} - \frac{1}{\cos\theta}$ analytically.

**Sol.** We have $\operatorname{cosec}\theta = \frac{\sqrt{10}}{3}$

$$\therefore \quad \sin\theta = \frac{3}{\sqrt{10}} \qquad \ldots(1)$$

$$\therefore \quad \cos\theta = \sqrt{1 - \sin^2\theta}$$

$$= \sqrt{1 - \frac{9}{10}}$$

$$= \frac{1}{\sqrt{10}} \qquad \ldots(2)$$

$$\therefore \quad \frac{1}{\sec\theta + \tan\theta} - \frac{1}{\cos\theta} = \frac{1}{\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}} - \frac{1}{\cos\theta}$$

$$= \frac{\cos\theta}{1 + \sin\theta} - \frac{1}{\cos\theta}$$

$$= \frac{\frac{1}{\sqrt{10}}}{1 + \frac{3}{\sqrt{10}}} - \sqrt{10}$$

[From (1) and (2)]

$$= \frac{1}{\sqrt{10} + 3} - \sqrt{10}$$

$$= \frac{\sqrt{10} - 3}{10 - 9} - \sqrt{10}$$

$$= \sqrt{10} - 3 - \sqrt{10}$$

$$= -3$$

which is the required value.

**22.** Prove that $\frac{1 - \cos\theta}{1 + \cos\theta} = (\operatorname{cosec}\theta - \cot\theta)^2$

**[CBSE 2023 Basic]**

**Sol.** RHS $= (\operatorname{cosec}\theta - \cot\theta)^2$

$$= \operatorname{cosec}^2\theta + \cot^2\theta - 2\operatorname{cosec}\theta \cdot \cot\theta$$

$$= \frac{1}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} - 2 \times \frac{1}{\sin\theta} \times \frac{\cos\theta}{\sin\theta}$$

$$= \frac{(1 + \cos^2\theta)}{\sin^2\theta} - \frac{2\cos\theta}{\sin^2\theta}$$

$$= \frac{1 + \cos^2\theta - 2\cos\theta}{\sin^2\theta}$$

$$= \frac{(1 - \cos\theta)^2}{(1 - \cos^2\theta)}$$

$$= \frac{(1 - \cos\theta)^2}{(1 + \cos\theta)(1 - \cos\theta)}$$

$$= \frac{1 - \cos\theta}{1 + \cos\theta} = \text{LHS}$$

Hence proved.

**23.** Prove that

sec A (1 – sin A) (sec A + tan A) = 1

**[CBSE 2023 Standard]**

**Sol.** LHS = sec A (1 – sin A) (sec A + tan A)

$$= \frac{1}{\cos A}(1 - \sin A)\left(\frac{1}{\cos A} + \frac{\sin A}{\cos A}\right)$$

$$= \frac{1}{\cos A}(1 - \sin A)\frac{(1 + \sin A)}{\cos A}$$

$$= \frac{1 - \sin^2 A}{\cos^2 A}$$

$$= \frac{\cos^2 A}{\cos^2 A}$$

$$= 1$$

$$= \text{RHS}$$

Hence proved.

**24.** Prove that $(\cot\theta - \operatorname{cosec}\theta)^2 = \frac{1 - \cos\theta}{1 + \cos\theta}$

**[CBSE 2024 Basic]**

**Sol.** LHS $= (\cot\theta - \operatorname{cosec}\theta)^2$

$$= \cot^2\theta + \operatorname{cosec}^2\theta - 2\cot\theta \,.\, \operatorname{cosec}\theta$$

$$= \frac{\cos^2\theta}{\sin^2\theta} + \frac{1}{\sin^2\theta} - 2\frac{\cos\theta}{\sin\theta} \cdot \frac{1}{\sin\theta}$$

$$= \frac{1 + \cos^2\theta - 2\cos\theta}{\sin^2\theta}$$

$$= \frac{(1 - \cos\theta)^2}{\sin^2\theta}$$

$$= \frac{(1 - \cos\theta)^2}{(1 - \cos^2\theta)}$$

$$= \frac{(1-\cos\theta)^2}{(1+\cos\theta)(1-\cos\theta)}$$

$$= \frac{1-\cos\theta}{1+\cos\theta}$$

$$= \text{RHS}$$

Hence proved.

### Long Answer Type Questions

**25.** If $\sin^2\theta - \cos^2\theta = \cot^2\phi$, prove that

(*a*) $\sin^2\phi - \cos^2\phi = \cot^2\theta$ and

(*b*) $\sqrt{2}\sin\theta\sin\phi = 1$

**Sol.** (*a*) We have

$$\sin^2\theta - \cos^2\theta = \cot^2\phi$$

$$\Rightarrow \quad 1 - 2\cos^2\theta = \cot^2\phi$$

$$\Rightarrow \quad 2\cos^2\theta = 1 - \cot^2\phi$$

$$\Rightarrow \quad \frac{2}{\sec^2\theta} = 1 - \cot^2\phi$$

$$\Rightarrow \quad \sec^2\theta = \frac{2}{1-\cot^2\phi}$$

$$\Rightarrow \quad 1 + \tan^2\theta = \frac{2}{1-\cot^2\phi}$$

$$\Rightarrow \quad \tan^2\theta = \frac{2}{1-\cot^2\phi} - 1$$

$$= \frac{2-1+\cot^2\phi}{1-\cot^2\phi}$$

$$= \frac{1+\cot^2\phi}{1-\cot^2\phi}$$

$$\Rightarrow \quad \cot^2\theta = \frac{1-\cot^2\phi}{1+\cot^2\phi}$$

$$= \frac{1-\dfrac{\cos^2\phi}{\sin^2\phi}}{1+\dfrac{\cos^2\phi}{\sin^2\phi}}$$

$$= \frac{\sin^2\phi - \cos^2\phi}{\sin^2\phi + \cos^2\phi}$$

$$= \sin^2\phi - \cos^2\phi$$

Hence, proved.

(*b*) From the given equation, we have

$$\sin^2\theta - \cos^2\theta = \cot^2\phi = \frac{\cos^2\phi}{\sin^2\phi}$$

$$\Rightarrow \quad \sin^2\theta - 1 + \sin^2\theta = \frac{1-\sin^2\phi}{\sin^2\phi}$$

$$\Rightarrow \quad (2\sin^2\theta - 1)\sin^2\phi = 1 - \sin^2\phi$$

$$\Rightarrow \quad 2\sin^2\theta\sin^2\phi = 1$$

$$\Rightarrow \quad \sqrt{2}\sin\theta\sin\phi = 1$$

Hence, proved.

**26.** If $\tan^2\theta = 1 - e^2$, show that

$\sec\theta + \tan^3\theta\,\text{cosec}\,\theta = (2-e^2)^{\frac{3}{2}}$.

**Sol.** We have $\quad \tan^2\theta = 1 - e^2$

$$\Rightarrow \quad 1 + \tan^2\theta = 2 - e^2$$

$$\Rightarrow \quad \sec^2\theta = 2 - e^2$$

$$\Rightarrow \quad \sec\theta = \left(2-e^2\right)^{\frac{1}{2}}$$

$$\Rightarrow \quad \sec^3\theta = \left(2-e^2\right)^{\frac{3}{2}} \qquad \ldots(1)$$

Now, $\sec\theta + \tan^3\theta\,\text{cosec}\,\theta$

$$= \frac{1}{\cos\theta} + \frac{\sin^3\theta}{\cos^3\theta}\cdot\frac{1}{\sin\theta}$$

$$= \frac{1}{\cos\theta} + \frac{\sin^2\theta}{\cos^3\theta}$$

$$= \frac{\cos^2\theta + \sin^2\theta}{\cos^3\theta}$$

$$= \frac{1}{\cos^3\theta}$$

$$= \sec^3\theta$$

$$= \left(2-e^2\right)^{\frac{3}{2}} \qquad \text{[From (1)]}$$

Hence, proved.

## Let's Compete

**(Page 169)**

### Multiple-Choice Questions

**1.** If $0° < \theta < 90°$, then $\tan^2\theta + \cot^2\theta + 2$ is equal to

(*a*) $\dfrac{\sec^2\theta}{\text{cosec}^2\theta}$ (*b*) $\dfrac{\text{cosec}^2\theta}{\sec^2\theta}$

(*c*) $\sec^2\theta - \text{cosec}^2\theta$ (*d*) $\sec^2\theta + \text{cosec}^2\theta$

**Sol.** (*d*) $\sec^2\theta + \text{cosec}^2\theta$

We have

$\tan^2\theta + \cot^2\theta + 2$

$$= \frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} + 2$$

$$= \frac{\sin^4\theta + \cos^4\theta + 2\sin^2\theta\cos^2\theta}{\sin^2\theta\cos^2\theta}$$

$$= \frac{(\sin^2\theta + \cos^2\theta)^2}{\sin^2\theta\cos^2\theta}$$

$$= \frac{1}{\sin^2\theta\cos^2\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\sin^2\theta\cos^2\theta}$$

$$= \frac{1}{\cos^2\theta} + \frac{1}{\sin^2\theta}$$

$$= \sec^2\theta + \mathrm{cosec}^2\theta$$

**2.** If $\sin A + \sin^2 A = 1$, then the value of $\cos^{12} A + 3\cos^{10} A + 3\cos^8 A + \cos^6 A - 1$ is equal to

(*a*) 2 (*b*) $-1$

(*c*) 1 (*d*) 0

**Sol.** (*d*) 0

We have $\sin A = 1 - \sin^2 A = \cos^2 A$

$\therefore$ $\cos^{12} A = \sin^6 A$, $\cos^{10} A = \sin^5 A$,

$\cos^8 A = \sin^4 A$, $\cos^6 A = \sin^3 A$

$\therefore$ $\cos^{12} A + 3\cos^{10} A + 3\cos^8 A + \cos^6 A - 1$

$= \sin^6 A + 3\sin^5 A + 3\sin^4 A + \sin^3 A - 1$

$= (\sin^2 A + \sin A)^3 - 1$

$= 1^3 - 1 = 0$ [$\because$ $\sin A + \sin^2 A = 1$]

**3.** If $7\sin^2\theta + 3\cos^2\theta = 4$, then $\tan\theta$ is equal to

(*a*) $\pm\frac{1}{2}$ (*b*) $\pm\frac{1}{\sqrt{3}}$

(*c*) $\pm\frac{1}{\sqrt{2}}$ (*d*) $\pm\frac{1}{3}$

**Sol.** (*b*) $\pm\frac{1}{\sqrt{3}}$

We have

$7\sin^2\theta + 3\cos^2\theta = 4$

$\Rightarrow$ $7\tan^2\theta + 3 = 4\sec^2\theta$

[Dividing both sides by $\cos^2\theta$, since $\cos\theta \neq 0$]

$\Rightarrow$ $7\tan^2\theta + 3 - 4(1 + \tan^2\theta) = 0$

$\Rightarrow$ $7\tan^2\theta + 3 - 4 - 4\tan^2\theta = 0$

$\Rightarrow$ $3\tan^2\theta = 1$

$\Rightarrow$ $\tan^2\theta = \frac{1}{3}$

$\Rightarrow$ $\tan\theta = \pm\frac{1}{\sqrt{3}}$

**4.** If $\tan\theta + \sec\theta = x$, then $\sin\theta$ is equal to

(*a*) $\frac{x^2-1}{x^2+1}$ (*b*) $\frac{x^2+1}{x^2-1}$

(*c*) $x^2 - 1$ (*d*) $x - 1$

**Sol.** (*a*) $\frac{x^2-1}{x^2+1}$

We have

$\tan\theta + \sec\theta = x$

$\Rightarrow$ $(\tan\theta + \sec\theta)^2 = x^2$

$\Rightarrow$ $\tan^2\theta + \sec^2\theta + 2\tan\theta\sec\theta = x^2$

$\Rightarrow$ $\frac{\sin^2\theta}{\cos^2\theta} + \frac{1}{\cos^2\theta} + 2 \times \frac{\sin\theta}{\cos\theta} \times \frac{1}{\cos\theta} = x^2$

$\Rightarrow$ $\frac{\sin^2\theta + 1 + 2\sin\theta}{\cos^2\theta} = x^2$

$\Rightarrow$ $\frac{(\sin\theta + 1)^2}{\cos^2\theta} = x^2$

$\Rightarrow$ $\frac{(1 + \sin\theta)^2}{1 - \sin^2\theta} = x^2$

$\Rightarrow$ $\frac{1 + \sin\theta}{1 - \sin\theta} = x^2$

$\Rightarrow$ $\frac{x^2+1}{x^2-1} = \frac{1 + \sin\theta + 1 - \sin\theta}{1 + \sin\theta - 1 + \sin\theta}$

[By componendo and dividendo]

$= \frac{2}{2\sin\theta} = \frac{1}{\sin\theta}$

$\therefore$ $\sin\theta = \frac{x^2-1}{x^2+1}$

**5.** If $\tan\theta = \frac{a}{b}$, then the value of $\frac{a^2\sin\theta - b^2\cos\theta}{a^2\sin\theta + b^2\cos\theta}$ is equal to

(*a*) $\frac{a^2-b^2}{a^2+b^2}$ (*b*) $\frac{a^3-b^3}{a^3+b^3}$

(*c*) $\frac{a-b}{a+b}$ (*d*) $\frac{a-b}{a^2+b^2}$

**Sol.** (*b*) $\frac{a^3-b^3}{a^3+b^3}$

We have $\tan\theta = \frac{a}{b}$

$\frac{\sin\theta}{\cos\theta} = \frac{a}{b}$

$\Rightarrow$ $\frac{a^2\sin\theta}{b^2\cos\theta} = \frac{a^3}{b^3}$

[Multiplying both sides by $\frac{a^2}{b^2}$]

$$\Rightarrow \quad \frac{a^2 \sin\theta - b^2 \cos\theta}{a^2 \sin\theta + b^2 \cos\theta} = \frac{a^3 - b^3}{a^3 + b^3}$$

[By componendo and dividendo]

**6.** The result of elimination of $\theta$ between the equations $x = c(\text{cosec } \theta + \cot \theta)$ and $y = c(\text{cosec } \theta - \cot \theta)$ is

(*a*) $xy = c$ (*b*) $x + y = c$

(*c*) $x + y = c^2$ (*d*) $xy = c^2$

**Sol.** (*d*) $xy = c^2$

We have

$$x = c(\text{cosec } \theta + \cot \theta) \quad \ldots(1)$$

and $$y = c(\text{cosec } \theta - \cot \theta) \quad \ldots(2)$$

Multiplying (1) by (2), we get

$$xy = c^2(\text{cosec}^2\theta - \cot^2\theta)$$
$$= c^2 \times 1$$
$$= c^2$$

**7.** If $0° < \theta < 90°$, then the value of

$\dfrac{(\sin\theta + \cos\theta)^2}{\sin\theta\cos\theta}$ will

(*a*) lie between –1 and 2

(*b*) be less than 2

(*c*) be greater than 2

(*d*) be equal to 2

**Sol.** (*c*) be greater than 2

We have

$$\frac{(\sin\theta + \cos\theta)^2}{\sin\theta\cos\theta} = \frac{\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta}{\sin\theta\cos\theta}$$

$$= 2 + \frac{1}{\sin\theta\cos\theta} > 2 \qquad [\because 0° < \theta < 90°]$$

**8.** If $\sec^2 A \tan^2 A = 1$, then the value of $\sec^4 A - \sec^2 A$ is equal to

(*a*) 1 (*b*) 0

(*c*) –1 (*d*) 2

**Sol.** (*a*) 1

We have

$$\sec^4 A - \sec^2 A = \sec^2 A\,(\sec^2 A - 1)$$
$$= \sec^2 A \tan^2 A = 1 \quad \text{[Given]}$$

**9.** If $0° < \theta < 90°$, then the value of

$\dfrac{1}{\cos^2 A} - \dfrac{1}{\text{cosec}^2 A - 1}$ is equal to

(*a*) 2 (*b*) 1

(*c*) –1 (*d*) 0

**Sol.** (*b*) 1

We have

$$\frac{1}{\cos^2 A} - \frac{1}{\text{cosec}^2 A - 1} = \frac{1}{\cos^2 A} - \frac{1}{\frac{1}{\sin^2 A} - 1}$$

$$= \frac{1}{\cos^2 A} - \frac{\sin^2 A}{1 - \sin^2 A}$$

$$= \frac{1}{\cos^2 A} - \frac{\sin^2 A}{\cos^2 A}$$

$$= \frac{1 - \sin^2 A}{\cos^2 A}$$

$$= \frac{\cos^2 A}{\cos^2 A}$$

$$= 1$$

**10.** $\dfrac{\sec\theta - 1}{\sec\theta + 1}$ is equal to

(*a*) $\dfrac{\tan\theta - 1}{\tan\theta + 1}$ (*b*) $\dfrac{1 - \tan\theta}{1 + \tan\theta}$

(*c*) $\dfrac{1 - \sin\theta}{1 + \sin\theta}$ (*d*) $\dfrac{1 - \cos\theta}{1 + \cos\theta}$

**Sol.** (*d*) $\dfrac{1 - \cos\theta}{1 + \cos\theta}$

We have

$$\frac{\sec\theta - 1}{\sec\theta + 1} = \frac{\frac{1}{\cos\theta} - 1}{\frac{1}{\cos\theta} + 1}$$

$$= \frac{1 - \cos\theta}{1 + \cos\theta}$$