7

Coordinate Geometry

Checkpoint (Page 128)

1. In which quadrant does point (–3, 5) lie?
   (*a*) first quadrant (*b*) second quadrant
   (*c*) third quadrant (*d*) fourth quadrant

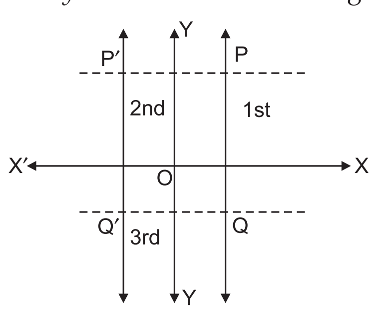
Sol. (*b*) second quadrant

Since x-coordinate is negative and y-coordinate is positive, hence, the given point lies in the second quadrant.

2. Two points having same abscissae but different ordinates lie on
   (*a*) x-axis
   (*b*) y-axis
   (*c*) a line parallel to y-axis
   (*d*) a line parallel to x-axis

Sol. (*c*) a line parallel to y-axis

Two such points, P and Q, must lie either in the first and fourth quadrants or, in the second and third quadrants as shown in the figure. Hence, these two points, P and Q must lie on a line parallel to y-axis as shown in the figure.



3. Which of the following is a solution set of the equation $3x + 2y = 10$?
   (*a*) (1, 3) (*b*) (2, 2)
   (*c*) (2, 3) (*d*) (3, 2)

Sol. (*b*) (2, 2)

We see that $x = 2$ and $y = 2$ satisfies the given equation $3x + 2y = 10$.

$$\begin{aligned} \text{LHS} &= 3 \times 2 + 2 \times 2 \\ &= 6 + 4 \\ &= 10 \\ &= \text{RHS} \end{aligned}$$

4. Find the area of the triangle ABC, formed joining the vertices A(0, 1), B(0, 5) and C(3, 4).
   (*a*) 4 sq units (*b*) 5 sq units
   (*c*) 6 sq units (*d*) 8 sq units

Sol. (*c*) 6 sq units

We know that the area of ΔABC with vertices A(x_1, y_1), B(x_2, y_2), and C(x_3, y_3) is

$$\frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right]$$

$$= \left|\frac{1}{2}\left[0\times(5-4)+0\times(4-1)+3\times(1-5)\right]\right| \text{ sq units}$$

$$= \left|\frac{1}{2} \times 3 \times -4\right| \text{ sq units}$$

$$= |3 \times -2| \text{ sq units}$$

$$= |-6|$$

$$= 6$$

Hence, the required area of the triangle ABC is 6 sq units.

5. The distance of point P(4, 3) from origin is
   (*a*) 7 units (*b*) 5 units
   (*c*) 8 units (*d*) 10 units

Sol. (*b*) 5 units

The required distance is $\sqrt{3^2 + 4^2}$ units, i.e. 5 units.

6. On which axes do the following points lie?

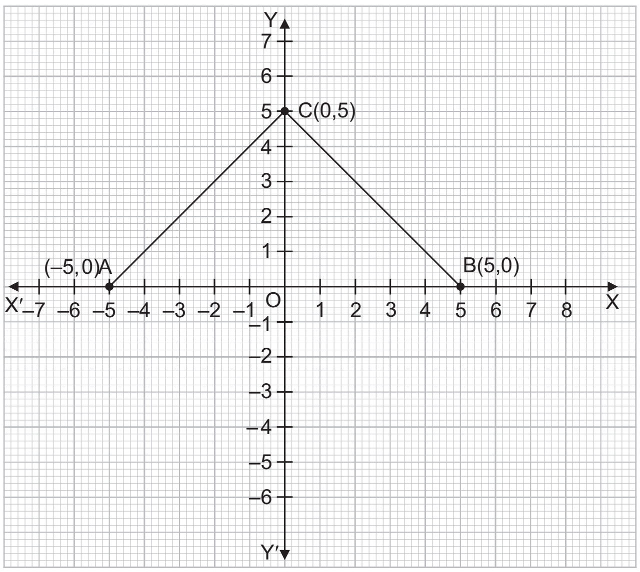
(*a*) (0, 1) (*b*) (–3, 0)

Sol. (*a*) Since x-coordinate is 0, hence, the point (0, 1) lies on the y-axis.

(*b*) Since y-coordinate is 0, hence, the point (–3, 0) lies on the x-axis.

7. Plot the points (–5, 0), (0, 5) and (5, 0) in rectangular coordinate system. Join them and name the type of the triangle formed.

Sol. We plot the points A(–5, 0), B(5, 0) and C(0, 5) on the graph paper and join AC and BC.



Let O be the origin.

We see that OA = OB = OC = 5 units.

$\therefore$ In ΔAOC,

$$\angle OAC = \angle OCA = 45°$$

and in ΔOBC,

$$OB = OC = 5 \text{ units}$$

$$\therefore \quad \angle OBC = \angle OCB = 45°$$

$$\therefore \quad \angle ACB = \angle ACO + \angle BCO$$

$$= 45° + 45°$$

$$= 90°$$

$\therefore$ In ΔABC,

$$AC = \sqrt{OA^2 + OC^2}$$

$$= \sqrt{5^2 + 5^2}$$

$$= \sqrt{50}$$

$$= 5\sqrt{2}$$

$$BC = \sqrt{OB^2 + OC^2}$$

$$= \sqrt{5^2 + 5^2}$$

$$= \sqrt{50}$$

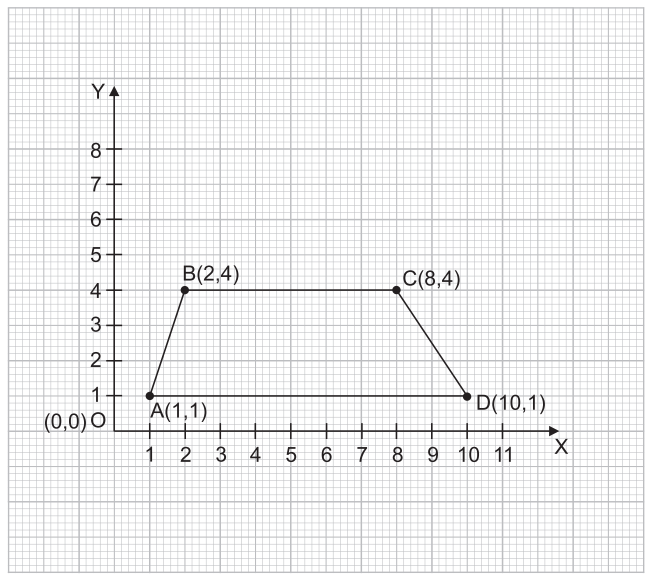
$$= 5\sqrt{2}$$

$\therefore$ In ΔABC, AC = BC and $\angle$ACB = 90°.

$\therefore$ ΔABC is a right-angled isosceles triangle.

8. Draw the quadrilateral whose vertices are A(1, 1), B(2, 4), C(8, 4) and D(10, 1). Name the type of quadrilateral formed.

Sol. We plot the points A(1, 1), B(2, 4), C(8, 4) and D (10, 1) on a graph paper with a suitable scale. We join the points A, B, C, and D by 4 line segments AB, BC, CD and DA to form a quadrilateral ABCD. Since, B and C are at a distance of 4 units and A and C are at a distance of 1 unit from the x-axis, hence, BC ∥ AD. But AB is not parallel to CD. Hence, the quadrilateral ABCD is a trapezium.



9. Write the equation of

(*a*) x-axis

(*b*) y-axis

(*c*) a line parallel to x-axis

Sol. (*a*) On x-axis, the y-coordinate of every point is zero.

$\therefore$ The required equation of x-axis is $y = 0$.

(*b*) On the y-axis, the x-coordinate of every point is zero.

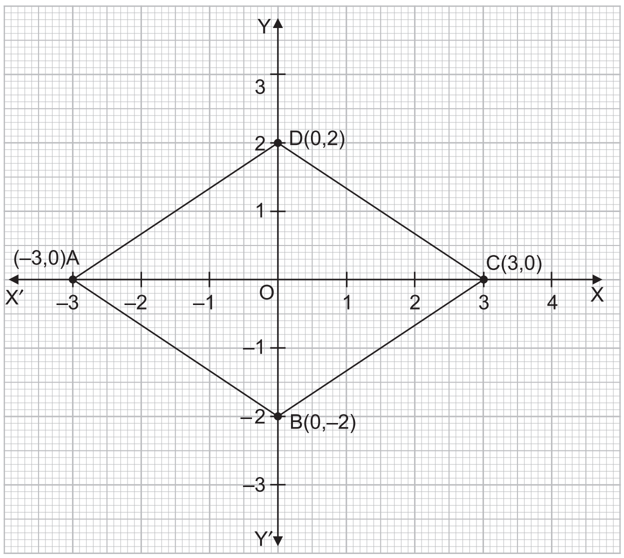
Hence, the required equation of y-axis is $x = 0$.

(*c*) Every point on the line parallel to x-axis is at a constant distance, say c, from the x-axis. Hence, the required equation of such a line is $y = c$, where c is a constant.

10. If coordinates of three vertices of a rhombus are (–3, 0), (0, –2) and (3, 0), then find the coordinates of the fourth vertex.

Sol. Let A (–3, 0), B(0, –2), C(3, 0) be the three coordinate of the three of the three vertices of a rhombus ABCD. We know that the two diagonals AC and BD of a rhombus bisect each other at right-angle at the origin 0, so that OA = OC – 3 units and OB = OD. But OB = 2 units.

$\therefore$ OD = 2 units



$\therefore$ The required coordinates of the fourth vertex are (0, 2)

Check Your Progress 1

(Page 133)

Multiple-Choice Questions

1. The distance between two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is given by

(*a*) $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

(*b*) $\sqrt{(x_1+x_2)^2+(y_1+y_2)^2}$

(*c*) $\sqrt{(x_1+y_1)^2+(x_2+y_2)^2}$

(*d*) $\sqrt{(x_1-y_1)^2+(x_2-y_2)^2}$

Sol. (*a*) $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

We know the well-known formula for distance between two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$.

2. The distance between the points (–1, 6) and (2, 2) is

(*a*) 5 units (*b*) 8 units

(*c*) 4 units (*d*) – 5 units

Sol. (*a*) 5 units

The required distance = $\sqrt{(2+1)^2+(2-6)^2}$ units

$= \sqrt{9+16}$ units

$= \sqrt{25}$ units

= 5 units

3. The distance of the point (5, 0) from the origin is

(*a*) 0 (*b*) 5

(*c*) $\sqrt{5}$ (*d*) 5^2 **[CBSE 2023 Basic]**

Sol. (*b*) 5

Distance of a point $P(x, y)$ from the origin (0, 0) is given by

$$OP = \sqrt{x^2+y^2}$$

$$= \sqrt{5^2} = 5$$

4. The distance between the points $(0, 2\sqrt{5})$ and $(-2\sqrt{5}, 0)$ is

(*a*) $2\sqrt{10}$ units

(*b*) $4\sqrt{10}$ units

(*c*) $2\sqrt{20}$ units

(*d*) 0 **[CBSE 2023 Standard]**

Sol. (*a*) $2\sqrt{10}$ units

$$P(x_1, y_1) = (0, 2\sqrt{5})$$

$$Q(x_2, y_2) = (-2\sqrt{5}, 0)$$

The distance between points P and Q is given by

$$PQ = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$= \sqrt{(-2\sqrt{5}-0)^2+(0-2\sqrt{5})^2}$$

$$= \sqrt{4\times5+4\times5} = \sqrt{20+20}$$

$$= \sqrt{40}$$

$$= 2\sqrt{10} \text{ units}$$

5. The coordinates of the vertex A of a rectangle ABCD whose three vertices are given as B(0, 0), C(3, 0) and D(0, 4) are

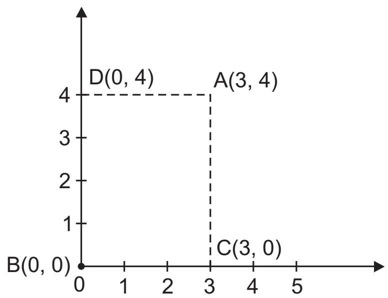
(*a*) (4, 0)

(*b*) (0, 3)

(*c*) (3, 4)

(*d*) (4, 3) **[CBSE 2023 Standard]**

Sol. (*c*) (3, 4)



ABCD is a rectangle.

$$AC = BD \text{ and}$$
$$AD = BC.$$

6. If the distance between the points (3, –5) and (x, –5) is 15 units, then the values of x are

(a) 12, –18 (b) –12, 18

(c) 18, 5 (d) –9, –12

[CBSE 2024 Standard]

Sol. (b) –12, 18

Distance between two points (3, –5) and (x, –5) is 15 units.

$$\therefore \quad \sqrt{(x-3)^2+(-5+5)^2} = 15$$
$$\Rightarrow \quad \sqrt{(x-3)^2} = 15$$
$$\Rightarrow \quad (x-3)^2 = (15)^2$$
$$\Rightarrow \quad x^2 - 6x + 9 = 225$$
$$\Rightarrow \quad x^2 - 6x - 216 = 0$$
$$\Rightarrow \quad x^2 - 18x + 12x - 216 = 0$$
$$\Rightarrow \quad x(x-18) + 12(x-18) = 0$$
$$\Rightarrow \quad (x-18)(x+12) = 0$$
$$\therefore \quad x = 18 \text{ or } x = -12$$

Very Short Answer Type Questions

7. Find the distance of the point (3, –4) from origin.

Sol. The required distance $= \sqrt{(3-0)^2+(-4-0)^2}$ units

$$= \sqrt{9+16} \text{ units}$$
$$= \sqrt{25} \text{ units} = 5 \text{ units}$$

8. On which axis do the point A(0, –5) lie?

Sol. Since the abscissa of the point A(0, –5) is 0, hence, A lies on the y-axis.

9. For what value(s) of y is the distance between the points A(3, –1) and B(11, y) is 10 units?

[CBSE 2023 Basic]

Sol. Here, A(3, –1) and B(11, y) be the given points $x_1 = 3, y_1 = -1$ and $x_2 = 11, y_2 = y$

Given AB = 10 units

By distance formula,

$$AB = \sqrt{[11-(3)]^2+[y-(-1)]^2}$$
$$\Rightarrow \quad 10 = \sqrt{(11-3)^2+(y+1)^2}$$
$$\Rightarrow \quad \sqrt{64+y^2+2y+1} = 10$$
$$\Rightarrow \quad \sqrt{y^2+2y+65} = 10$$

Squaring both sides,

$$y^2 + 2y + 65 = 100$$
$$\Rightarrow \quad y^2 + 2y - 35 = 0$$
$$\Rightarrow \quad y^2 + 7y - 5y - 35 = 0$$
$$\Rightarrow \quad y(y+7) - 5(y+7) = 0$$
$$\Rightarrow \quad (y+7)(y-5) = 0$$
$$\Rightarrow \quad y = -7 \text{ or } y = 5$$

Hence, the required value of y is –7 or 5.

10. Find a point on the x-axis which is equidistant from the points A(5, 2) and B(1, –2).

Sol. Let any point on the x-axis be P(x_1, 0).

According to the problem, we have PA = PB, where A(5, 2) and B(1, –2).

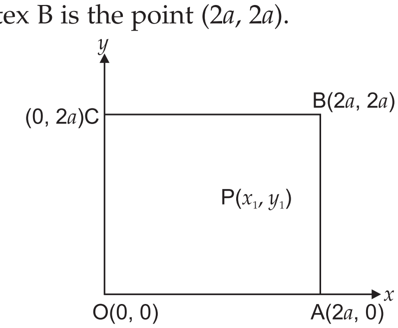
$\therefore$ By distance formula

$$\sqrt{(x_1-5)^2+(0-2)^2} = \sqrt{(x_1-1)^2+(0+2)^2}$$
$$\Rightarrow \quad x_1^2 - 10x_1 + 25 + 4 = x_1^2 - 2x_1 + 1 + 4$$
$$\Rightarrow \quad -10x_1 + 29 = -2x_1 + 5$$
$$\Rightarrow \quad 8x_1 + 5 - 29 = 0$$
$$\Rightarrow \quad 8x_1 = 24$$
$$\Rightarrow \quad x_1 = \frac{24}{8} = 3$$

$\therefore$ The required point is (3, 0).

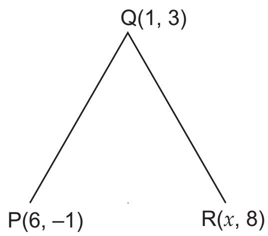
11. Find the vertices of the square of side '$2a$' with one vertex as origin such that other two vertices lie on the positive x and y axes.

Sol. If one vertex O(0, 0) lie at the origin and the other two vertices A and C lie on the positive x and y-axes respectively, then the coordinates of A are ($2a$, 0) and those of C are (0, $2a$). Hence, the fourth vertex B is the point ($2a$, $2a$).



12. Find the value(s) of x, so that PQ = QR, where the coordinates of P, Q and R are (6, –1), (1, 3) and (x, 8) respectively. **[CBSE 2023 Basic]**

Sol. Distance between P(6, –1) and Q(1, 3)



Q(1, 3) and R(x, 8)

Given that PQ = QR

$$\sqrt{(1-6)^2+[3-(-1)]^2} = \sqrt{(x-1)^2+(8-3)^2}$$

$$\sqrt{(-5)^2+(4)^2} = \sqrt{(x-1)^2+(5)^2}$$

Squaring both sides, we get

$\Rightarrow \quad 5^2 + 4^2 = (x-1)^2 + 5^2$

$\Rightarrow \quad x^2 - 2x + 1 = 16$

$\Rightarrow \quad x^2 - 2x - 15 = 0$

$\Rightarrow \quad x^2 - 5x + 3x - 15 = 0$

$\Rightarrow \quad x(x-5) + 3(x-5) = 0$

$\Rightarrow \quad (x-5)(x+3) = 0$

$\therefore \quad x = 5$ or $x = -3$

13. If the points A(2, 3), B(–5, 6), C(6, 7) and D(p, 4) are the vertices of a parallelogram ABCD, find the value of p? **[CBSE 2023 Basic]**

Sol. ABCD is a parallelogram, where the coordinates of the vertices are A(2, 3), B(–5, 6), C(6, 7) and D(p, 4).

We know that diagonals of a parallelogram bisect each other.

$\therefore$ Coordinates of mid-point of line segment joining points A and C = Coordinates of mid-point of line segment joining points B and D.

$$\therefore \quad \left[\frac{2+6}{2}, \frac{3+7}{2}\right] = \left[\frac{-5+p}{2}, \frac{6+4}{2}\right]$$

$$\Rightarrow \quad (4, 5) = \left(\frac{-5+p}{2}, 5\right)$$

$$\Rightarrow \quad \frac{-5+p}{2} = 4$$

$\Rightarrow \quad -5 + p = 8$

$\therefore \quad p = 13$

Short Answer Type Questions

14. If the point P(x, y) is equidistant from A(3, 6) and B(–3, 4), prove that $3x + y - 5 = 0$. **[CBSE 2008]**

Sol. We have

$$PA = PB$$

$\therefore$ By distance formula,

$$\sqrt{(x-3)^2+(y-6)^2} = \sqrt{(x+3)^2+(y-4)^2}$$

On squaring both sides,

$$(x-3)^2 + (y-6)^2 = (x+3)^2 + (y-4)^2$$

$\Rightarrow \quad x^2 + y^2 - 6x - 12y + 9 + 36$

$\qquad = x^2 + y^2 + 6x - 8y + 9 + 16$

$\Rightarrow \quad 12x + 4y + 25 - 45 = 0$

$\Rightarrow \quad 12x + 4y - 20 = 0$

$\Rightarrow \quad 3x + y - 5 = 0$

Hence, proved.

15. Find a point which is equidistant from points (7, –6), (–1, 0), and (–2, –3).

Sol. Let the point P(x, y) be equidistant from the points A(7, –6), B(–1, 0) and C(–2, –3).

$\therefore \quad PA^2 = PB^2 = PC^2$

$\Rightarrow \quad (7-x)^2 + (-6-y)^2 = (-1-x)^2 + y^2$

$\qquad = (-2-x)^2 + (-3-y)^2$

$\Rightarrow \quad x^2 + y^2 - 14x + 12y + 49 + 36$

$\qquad = x^2 + y^2 + 2x + 1$

$\qquad = x^2 + y^2 + 4x + 6y + 4 + 9$

$\Rightarrow \quad -14x + 12y + 85 = 2x + 1 = 4x + 6y + 13$

$\therefore$ From the first and the middle expressions, we have

$14x + 2x - 12y + 1 - 85 = 0$

$\Rightarrow \quad 16x - 12y - 84 = 0$

$\Rightarrow \quad 4x - 3y - 21 = 0 \qquad \ldots(1)$

Again, from the first and the last expressions, we have

$-14x + 12y + 85 = 4x + 6y + 13$

$\Rightarrow \quad 18x - 6y - 72 = 0$

$\Rightarrow \quad 3x - y - 12 = 0 \qquad \ldots(2)$

From (2), $\quad y = 3x - 12 \qquad \ldots(3)$

$\therefore$ From (1),

$4x - 3(3x - 12) - 21 = 0$

$\Rightarrow \quad 4x - 9x + 36 - 21 = 0$

$\Rightarrow \quad 5x = 15$

$$\Rightarrow \quad x = \frac{15}{5} = 3$$

$\therefore$ From (3), $\quad y = 3 \times 3 - 12 = -3$

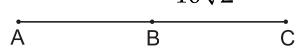
$\therefore$ The required point is (3, –3).

16. Using distance formula, show that the points (–4, –1), (0, 3) and (6, 9) are collinear.

Sol. Let A, B and C be the points (–4, –1), (0, 3) and (6, 9) respectively.

By distance formula,

$$AB = \sqrt{(0+4)^2 + (3+1)^2}$$
$$= \sqrt{4^2 + 4^2}$$
$$= \sqrt{16+16}$$
$$= \sqrt{32}$$
$$= 4\sqrt{2}$$
$$BC = \sqrt{(6-0)^2 (9-3)^2}$$
$$= \sqrt{6^2 + 6^2}$$
$$= \sqrt{36+36}$$
$$= \sqrt{72}$$
$$= 6\sqrt{2}$$
$$AC = \sqrt{(6+4)^2 + (9+1)^2}$$
$$= \sqrt{10^2 + 10^2}$$
$$= \sqrt{100+100}$$
$$= \sqrt{200}$$
$$= 10\sqrt{2}$$



$\therefore$ We see that

$$AB + BC = 4\sqrt{2} + 6\sqrt{2}$$
$$= 10\sqrt{2} = AC$$

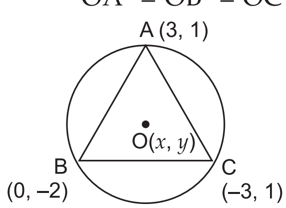
Hence, A, B, C are collinear.

17. Find the circumcentre of the triangle whose vertices are (3, 1), (0, –2) and (–3, 1).

Sol. Let A, B, C be the points (3, 1), (0, –2) and (–3, 1) respectively and let O(x, y) be the circumcentre of this triangle.

Then $\qquad OA = OB = OC$

$\Rightarrow \qquad OA^2 = OB^2 = OC^2$



By distance formula

$$(3-x)^2 + (1-y)^2 = (0-x)^2 + (-2-y)^2$$
$$= (-3-x)^2 + (1-y)^2$$

$\Rightarrow \; x^2 + y^2 - 6x - 2y + 9 + 1$

$$= x^2 + y^2 + 4y + 4$$
$$= x^2 + y^2 + 6x - 2y + 9 + 1$$

$\Rightarrow \qquad -6x - 2y + 10 = 4y + 4$

$$= 6x - 2y + 10$$

From the first and the second expressions, we have

$6x + 4y + 2y + 4 - 10 = 0$

$\Rightarrow \qquad 6x + 6y - 6 = 0$

$\Rightarrow \qquad x + y - 1 = 0 \qquad \ldots(1)$

From the first and the last expressions, we have

$$12x = 0$$

$\Rightarrow \qquad x = 0$

$\therefore$ From (1), $\qquad y = 1$

$\therefore$ The required coordinates of the circumcentre are (0, 1).

18. Find the distance between the points

(a) (a, b) and (–a, –b)

(b) (p sin 55°, 0) and (0, p sin 35°)

Sol. (a) The distance between the points (a, b) and (–a, –b)

$$= \sqrt{(-a-a)^2 + (-b-b)^2}$$
$$= \sqrt{(-2a)^2 + (-2b)^2}$$
$$= \sqrt{4a^2 + 4b^2}$$
$$= 2\sqrt{a^2 + b^2}$$

$\therefore$ The required distance is $2\sqrt{a^2 + b^2}$.

(b) We have sin 35° = cos (90° – 35°) = cos 55°

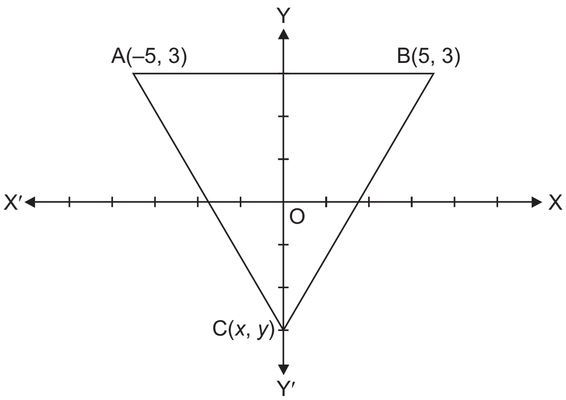
$\therefore$ The distance between the points (p sin 55°, 0) and (0, p cos 55°)

$$= \sqrt{(0 - p\sin 55°)^2 + (p\cos 55° - 0)^2}$$
$$= \sqrt{(-p\sin 55°)^2 + (p\cos 55°)^2}$$
$$= \sqrt{p^2(\sin^2 55° + \cos^2 55°)}$$
$$= \sqrt{p^2} = p$$

$\therefore$ The required distance is p.

19. If (–5, 3) and (5, 3) are two vertices of an equilateral triangle, then find coordinates of the third vertex, given that origin lies inside the triangle. (Take $\sqrt{3}$ = 1.7) **[CBSE 2023 Standard]**

Sol. Let the coordinates of the third vertex be C(x, y). The sides of an equilateral triangle are equal.



$\therefore$ Distance between points A and B

$$= \sqrt{[5-(-5)]^2 + (3-3)^2} = \sqrt{(10)^2} = 10 \quad ...(1)$$

Distance between points A and C

$$= \sqrt{[x-(-5)]^2 + (y-3)^2}$$

$$= \sqrt{(x+5)^2 + (y-3)^2} \quad ...(2)$$

Distance between points B and C

$$= \sqrt{(x-5)^2 + (y-3)^2} \quad ...(3)$$

Equating equations (2) and (3), we get

$$\sqrt{(x+5)^2 + (y-3)^2} = \sqrt{(x-5)^2 + (y-3)^2}$$

$$\Rightarrow x^2 + 10x + 25 + y^2 - 6y + 9 = x^2 - 10x + 25 + y^2 - 6y + 9 = 0$$

$$\Rightarrow 20x = 0$$

$$\Rightarrow x = 0$$

Equating equation (2) and (1), we get

$$\sqrt{(x+5)^2 + (y-3)^2} = 10$$

Putting $x = 0$ in the above equation, we get

$$\sqrt{5^2 + (y-3)^2} = 10$$

$$\Rightarrow 25 + (y-3)^2 = 100$$

$$\Rightarrow (y-3)^2 = 75$$

$$\Rightarrow (y-3)^2 = (5\sqrt{3})^2$$

$$\Rightarrow y - 3 = \pm 5\sqrt{3}$$

$$\Rightarrow y = 3 \pm 5\sqrt{3}$$

Since origin lies inside the triangle.

$$\therefore y = 3 - 5\sqrt{3}$$

Therefore, coordinates of the third vertex is $(0, 3 - 5\sqrt{3})$.

20. Show that the points A(1, 7), B(4, 2), C(–1, –1) and D(–4, 4) are vertices of the square ABCD.

[CBSE 2023 Basic]

Sol. In a square, all sides are of equal length and the length of the diagonals are also equal.

$$AB = \sqrt{(4-1)^2 + (2-7)^2} = \sqrt{(3)^2 + (-5)^2}$$

$$= \sqrt{9+25}$$

$$= \sqrt{34} \text{ units}$$

$$BC = \sqrt{[-1-4]^2 + [-1-2]^2} = \sqrt{(-5)^2 + (-3)^2}$$

$$= \sqrt{25+9}$$

$$= \sqrt{34} \text{ units}$$

$$CD = \sqrt{[-4+1]^2 + [4+1]^2} = \sqrt{(-3)^2 + (5)^2}$$

$$= \sqrt{9+25}$$

$$= \sqrt{34} \text{ units}$$

$$AD = \sqrt{[-4-1]^2 + (4-7)^2} = \sqrt{(-5)^2 + (-3)^2}$$

$$= \sqrt{25+9}$$

$$= \sqrt{34} \text{ units}$$

$\therefore$ AB = BC = CD = AD

$\therefore$ Sides are equal

$$AC = \sqrt{(-1-1)^2 + (-1-7)^2} = \sqrt{(-2)^2 + (-8)^2}$$

$$= \sqrt{4+64}$$

$$= \sqrt{68} \text{ units}$$

$$BD = \sqrt{(-4-4)^2 + (4-2)^2} = \sqrt{(-8)^2 + (2)^2}$$

$$= \sqrt{64+4}$$

$$= \sqrt{68} \text{ units}$$

$\therefore$ AC = BD

$\Rightarrow$ Diagonals are equal.

$\therefore$ ABCD is a square.

Long Answer Type Questions

21. If the points (3, 7) and (x, y) are equidistant from point (9, 10), prove $x^2 + y^2 - 18x - 20y + 136 = 0$.

Sol. Let A, B and C be the points (3, 7), (x, y) and (9, 10) respectively. It is given that A and B are equidistant from the point C.

i.e. AC = BC

$\therefore$ By distance formula,

$$\sqrt{(9-3)^2 + (10-7)^2} = \sqrt{(9-x)^2 + (10-y)^2}$$

$$\Rightarrow \sqrt{6^2 + 3^2} = \sqrt{(9-x)^2 + (10-y)^2}$$

$$\Rightarrow \sqrt{36+9} = \sqrt{x^2 + y^2 - 18x - 20y + 81 + 100}$$

On squaring both sides,

$\Rightarrow\ 45 = x^2 + y^2 - 18x - 20y + 181$

$\Rightarrow\ x^2 + y^2 - 18x - 20y + 181 - 45 = 0$

$\Rightarrow\ x^2 + y^2 - 18x - 20y + 136 = 0$

Hence, proved.

22. Show (5, 1) is the centre of circle circumscribing the triangle whose vertices are (3, 3), (7, 3) and (3, –1).

Sol. Let O be the point (5, 1) and A(3, 3), B(7, 3) and C(3, –1) be the vertices of ΔABC.

$\therefore$ By distance formula,

$$OA = \sqrt{(3-5)^2 + (3-1)^2} = \sqrt{(-2)^2 + 2^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$$

$$OB = \sqrt{(7-5)^2 + (3-1)^2} = \sqrt{2^2 + 2^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$$

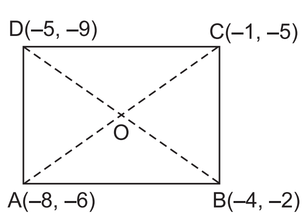
$$OC = \sqrt{(3-5)^2 + (-1-1)^2} = \sqrt{(-2)^2 + (-2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$$

$\therefore$ OA = OB = OC

$\therefore$ O(5, 1) is the centre of the circle circumscribing the triangle whose vertices are (3, 3), (7, 3) and (–3, –1). Hence, proved.

23. Show that points (–8, –6), (–4, –2), (–1, –5) and (–5, –9) are vertices of a rectangle.

Sol. Let A(–8, –6), B(–4, –2), C(–1, –5) and D(–5, –9) be the vertices of a quadrilateral ABCD. To prove that ABCD is a rectangle. We join AC and BD.



By distance formula,

$$AB = \sqrt{(-4+8)^2 + (-2+6)^2} = \sqrt{4^2 + 4^2} = \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$$

$$BC = \sqrt{(-1+4)^2 + (-5+2)^2} = \sqrt{3^2 + (-3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$$

$$CD = \sqrt{(-5+1)^2 + (-9+5)^2} = \sqrt{(-4)^2 + (-4)^2} = \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$$

$$DA = \sqrt{(-8+5)^2 + (-6+9)^2} = \sqrt{(-3)^2 + 3^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$$

$$BD = \sqrt{(-5+4)^2 + (-9+2)^2} = \sqrt{(-1)^2 + (-7)^2} = \sqrt{1+49} = \sqrt{50} = 5\sqrt{2}$$

$$AC = \sqrt{(-1+8)^2 + (-5+6)^2} = \sqrt{7^2 + 1^2} = \sqrt{49+1} = \sqrt{50} = 5\sqrt{2}$$

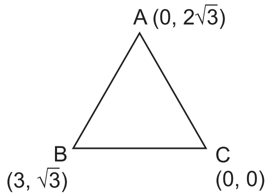
$\therefore$ AB = CD and BC = DA

i.e. each pair of opposite sides of the quadrilateral is equal. Also, diagonals AC and BD are equal to each other. Hence, the quadrilateral ABCD is a rectangle.

Hence, proved.

24. Show that points $\left(0, 2\sqrt{3}\right)$, $\left(3, \sqrt{3}\right)$ and (0, 0) are vertices of an equilateral triangle.

Sol. Let $A\left(0, 2\sqrt{3}\right)$, $B\left(3, \sqrt{3}\right)$ and C(0, 0) be the vertices of a triangle ABC. To prove that ΔABC is an equilateral triangle.



We have by distance formula

$$AB = \sqrt{(3-a)^2 + \left(\sqrt{3} - 2\sqrt{3}\right)^2}$$
$$= \sqrt{3^2 + \left(-\sqrt{3}\right)^2}$$
$$= \sqrt{9+3}$$
$$= \sqrt{12}$$
$$= 2\sqrt{3}$$
$$AC = \sqrt{0^2 + \left(2\sqrt{3}\right)^2}$$
$$= \sqrt{12}$$
$$= 2\sqrt{3}$$

and $$BC = \sqrt{3^2 + \left(\sqrt{3}\right)^2}$$
$$= \sqrt{12}$$
$$= 2\sqrt{3}$$

$\therefore$ AB = BC = AC

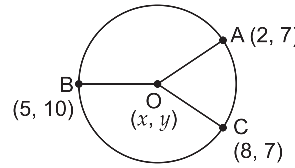
$\therefore$ ΔABC is an equilateral triangle.

$\therefore$ The points (0, $2\sqrt{3}$), $\left(3, \sqrt{3}\right)$ and (0, 0) are vertices of an equilateral triangle.

Hence, proved.

25. Find the coordinates of the centre of circle passing through points (2, 7), (5, 10) and (8, 7). Also find its radius.

Sol. Let the coordinate of the centre O of the circle be (x, y) and be A(2, 7), B(5, 10) and C(8, 7) be three given points. A circle passes through the points A, B and C. To find the centre (x, y) and the radius OA of the circle.



We see that OA = OB = OC = radius of the circle.

$\therefore$ By distance formula

$$OA = \sqrt{(2-x)^2 + (7-y)^2}$$
$$= \sqrt{x^2 + y^2 + 4 + 49 - 4x - 14y}$$
$$= \sqrt{x^2 + y^2 - 4x - 14y + 53} \quad \ldots(1)$$
$$OB = \sqrt{(5-x)^2 + (10-y)^2}$$
$$= \sqrt{x^2 + y^2 + 25 + 100 - 10x - 20y}$$
$$= \sqrt{x^2 + y^2 - 10x - 20y + 125}$$
$$OC = \sqrt{(8-x)^2 + (7-y)^2}$$
$$= \sqrt{x^2 + y^2 + 64 + 49 - 16x - 14y}$$
$$= \sqrt{x^2 + y^2 - 16x - 14y + 113}$$

$\therefore \quad OA^2 = OB^2 = OC^2$

$\therefore \; x^2 + y^2 - 4x - 14y + 53$
$$= x^2 + y^2 - 10x - 20y + 125$$
$$= x^2 + y^2 - 16x + 14y + 113$$

$\Rightarrow \quad -4x - 14y + 53 = -10x - 20y + 125$
$$= -16x - 14y + 113$$

Hence, from the first and the second expressions, we have

$$-4x + 10x - 14y + 20y + 53 - 125 = 0$$
$\Rightarrow \quad 6x + 6y - 72 = 0$

$\Rightarrow \quad x + y - 12 = 0 \quad \ldots(2)$

Also, from the first and the last expressions, we have

$$-4x - 14y + 53 + 16x + 14y - 113 = 0$$
$\Rightarrow \quad 12x - 60 = 0$

$\Rightarrow \quad x = \dfrac{60}{12} = 5$

$\therefore$ From (2), $\quad y = 12 - x$
$$= 12 - 5$$
$$= 7$$

Hence, the required coordinates of the centre O of the circle are (5, 7).

Also, radius = OA = $\sqrt{x^2 + y^2 - 4x - 14y + 53}$

[From (1)]

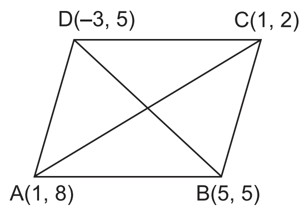
$$= \sqrt{25 + 49 - 4 \times 5 - 14 \times 7 + 53}$$
$$= \sqrt{74 - 20 - 98 + 53}$$
$$= \sqrt{127 - 118}$$
$$= \sqrt{9} = 3$$

Hence, the required coordinates are (5, 7) and the required radius is 3 units.

26. If A(1, 8), B(5, 5), C(1, 2), and D(–3, 5) are the four points in a plane. Show that quadrilateral formed by joining them, ABCD is a rhombus but not a square. Also find its area.

Sol. Here A(1, 8), B(5, 5), C(1, 2) and D(–3, 5) are four points in a plane forming a quadrilateral ABCD.

To prove that ABCD is a rhombus.



We have, by distance formula

$$AB = \sqrt{(5-1)^2 + (5-8)^2}$$
$$= \sqrt{4^2 + (-3)^2}$$
$$= \sqrt{16 + 9}$$
$$= \sqrt{25}$$
$$= 5$$
$$BC = \sqrt{(1-5)^2 + (2-5)^2}$$
$$= \sqrt{(-4)^2 + (-3)^2}$$
$$= \sqrt{16 + 9}$$
$$= \sqrt{25}$$
$$= 5$$
$$CD = \sqrt{(-3-1)^2 + (5-2)^2}$$
$$= \sqrt{(-4)^2 + 3^2}$$
$$= \sqrt{16 + 9}$$
$$= \sqrt{25}$$
$$= 5$$
$$DA = \sqrt{(1+3)^2 + (8-5)^2}$$
$$= \sqrt{4^2 + 3^2}$$
$$= \sqrt{16 + 9}$$
$$= \sqrt{25}$$
$$= 5$$
$$AC = \sqrt{(1-1)^2 + (2-8)^2}$$
$$= \sqrt{0^2 + (-6)^2}$$
$$= \sqrt{36}$$
$$= 6$$
$$BD = \sqrt{(-3-5)^2 + (5-5)^2}$$
$$= \sqrt{(-8)^2 + 0^2}$$
$$= \sqrt{64}$$
$$= 8$$

$\therefore$ We see that AB = BC = CD = DA.

But the diagonals AC and BD are not equal.

Hence, the quadrilateral ABCD is a rhombus and not a square.

Also, the area of the rhombus

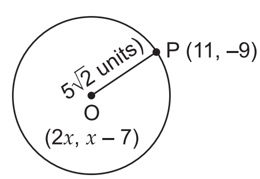
$$= \frac{1}{2} \times AC \times BD$$
$$= \frac{1}{2} \times 6 \times 8 \text{ sq units}$$
$$= 24 \text{ sq units}$$

$\therefore$ The required area is 24 sq units.

27. The radius of a circle passing through (11, –9) is $5\sqrt{2}$ units. Find x if the centre of circle is $(2x, x - 7)$.

Sol. Let P(11, –9) be any point on the circle with centre at O$(2x, x - 7)$ and radius OP = $5\sqrt{2}$ units.

To find x.



We have, by distance formula

$$OP = \sqrt{(11 - 2x)^2 + (-9 - x + 7)^2}$$
$$\Rightarrow \quad 5\sqrt{2} = \sqrt{(11 - 2x)^2 + (-2 - x)^2}$$
$$= \sqrt{4x^2 + 11^2 - 44x + 4x + 4 + x^2}$$
$$\Rightarrow \quad 5\sqrt{2} = \sqrt{5x^2 - 40x + 125}$$

On squaring both sides

$$5x^2 - 40x + 125 = \left(5\sqrt{2}\right)^2 = 50$$

$$\Rightarrow \quad 5x^2 - 40x + 75 = 0$$

$$\Rightarrow \quad x^2 - 8x + 15 = 0$$

$$\Rightarrow \quad x^2 - 3x - 5x + 15 = 0$$

$$\Rightarrow \quad x(x - 3) - 5(x - 3) = 0$$

$$\Rightarrow \quad (x - 3)(x - 5) = 0$$

$\therefore$ Either $\quad x - 3 = 0 \quad \Rightarrow \quad x = 3$

or, $\quad x - 5 = 0 \quad \Rightarrow \quad x = 5$

Hence, the required values of x is 3 or 5.

Check Your Progress 2

(Page 139)

Multiple-Choice Questions

1. The mid-point of the line segment joining points A(–2, 8) and B(–6, –4) is

(*a*) (–4, –6) (*b*) (–4, 2)

(*c*) (2, 6) (*d*) (4, 2)

Sol. (*b*) (–4, 2)

If $P(x_1, y_1)$ be the mid-point of AB, then

$$x_1 = \frac{-2-6}{2} = -4, \quad y_1 = \frac{8-4}{2} = 2$$

$\therefore$ (–4, 2) is the required mid-point.

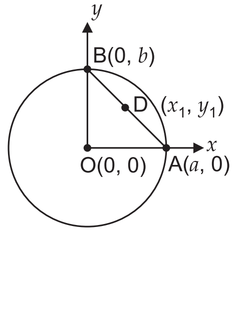
2. The coordinates of the circumcentre of the triangle formed by the points O(0, 0), A(a, 0) and B(0, b) are

(*a*) (a, b) (*b*) $\left(\frac{a}{2}, \frac{b}{2}\right)$

(*c*) $\left(\frac{b}{2}, \frac{a}{2}\right)$ (*d*) (b, a)

Sol. (*b*) $\left(\frac{a}{2}, \frac{b}{2}\right)$

Clearly, O is the origin. A(a, 0) is a point on the x-axis and B(0, b) is a point on the y-axis. A circle with centre at O passes through A and B as shown is the figure. Then $\angle AOB = 90°$.



$\therefore$ AB is the hypotenuse of the right-angled triangle AOB. Hence, the circumcentre of ΔAOB will be the mid-point of the hypotenuse AB. If $D(x_1, y_1)$ be the mid-point of AB, then

$$x_1 = \frac{1}{2}(0 + a) = \frac{a}{2} \text{ and } y_1 = \frac{1}{2}(b + 0) = \frac{b}{2}.$$

$\therefore$ The required coordinates of the circumcentre are $\left(\frac{a}{2}, \frac{b}{2}\right)$.

3. In what ratio, does x-axis divide the line segment joining the points A(3, 6) and B(–12, –3)?

(*a*) 1 : 2 (*b*) 1 : 4

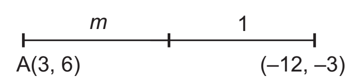
(*c*) 4 : 1 (*d*) 2 : 1

[CBSE Standard 2023]

Sol. (*d*) 2 : 1

Let the x-axis divided the line segment in the ratio m : 1.

Let the coordinates of the point of intersection be (x, 0).



$$y = \frac{-3m + 1(6)}{m + 1}$$

$$\Rightarrow \quad 0 = \frac{-3m + 6}{m + 1}$$

$$\Rightarrow \quad -3m + 6 = 0$$

$$\Rightarrow \quad -3m = -6$$

$$\Rightarrow \quad m = 2$$

$\therefore \quad$ Ratio = 2 : 1

Very Short Answer Type Questions

4. Find the coordinates of the centroid of a triangle whose vertices are (0, 12), (8, 6) and (7, 0).

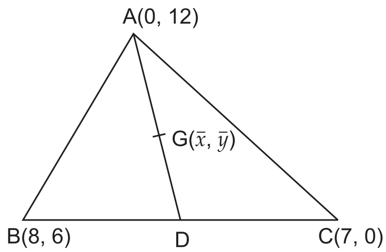
Sol. Let $G(\bar{x}, \bar{y})$ be the centroid of ΔABC where A(0, 12), B(8,6) and C(7, 0) are the vertices of ΔABC.

Then, $\quad \bar{x} = \dfrac{0 + 8 + 7}{3}$

$$= \frac{15}{3} = 5$$

$$\bar{y} = \frac{12 + 6 + 0}{3}$$

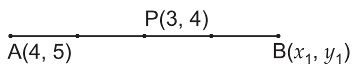
$$= \frac{18}{3} = 6$$



$\therefore$ The required coordinates of the centroid are (5, 6).

5. If line segment AB is bisected at P(3, 4) where coordinates of point A are (4, 5). Then find the coordinates of point B.

Sol. Let the coordinates of B be (x_1, y_1). Then, since P(3, 4) is the mid-point of AB.



$\therefore$ By mid-point formula,

$$3 = \frac{4 + x_1}{2} \quad \Rightarrow \quad x_1 = 6 - 4 = 2$$

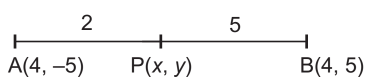
$$4 = \frac{5 + y_1}{2} \quad \Rightarrow \quad y_1 = 8 - 5 = 3$$

$\therefore$ The required coordinates of B are (2, 3).

6. The line segment joining the points A(4, –5) and B(4, 5) is divided by the point P such that AP : PB = 2 : 5. Find the coordinates of P.

[CBSE 2023 Standard]

Sol. Let coordinates of point P be (x, y)



$$x = \frac{2\times4+5\times4}{2+5} = \frac{8+20}{7} = \frac{28}{7} = 4$$

$$y = \frac{2\times5+5\times(-5)}{2+5} = \frac{10-25}{7} = \frac{-15}{7}$$

$\therefore$ Coordinates of point P = $\left(4, \dfrac{-15}{7}\right)$

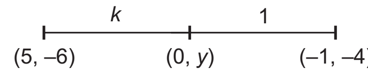
7. Find the ratio in which y-axis divides the line segment joining the points (5, –6) and (–1, –4).

[CBSE 2023 Standard]

Sol. On y-axis, x-coordinate is 0.

Let y-axis divides the line segment in the ratio k : 1.

Let the coordinates of the point of intersection be (0, y).



$$0 = \frac{-1\times k + 1\times 5}{k+1}$$

$\Rightarrow \quad -k + 5 = 0$

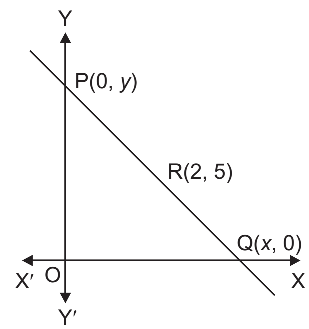
$\Rightarrow \quad -k = -5$

$\Rightarrow \quad k = 5$

$\therefore$ The ratio in which y-axis divides the line segment is 5 : 1.

8. A line intersects y-axis and x-axis at point P and Q respectively. If R(2, 5) is the mid-point of the line segment PQ, then find the coordinates of P and Q. **[CBSE 2023 Standard]**

Sol. Let coordinates of P and Q be (0, y) and (x, 0) respectively.



R is the mid-point of PQ.

$\Rightarrow \quad 2 = \dfrac{0 + x}{2}$

$\therefore \quad x = 4$

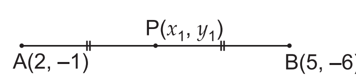
$\Rightarrow \quad 5 = \dfrac{y + 0}{2}$

$\therefore \quad y = 10$

$\therefore$ Coordinates of P and Q are (0, 10) and (4, 0) respectively.

9. The line joining the points (2, –1) and (5, –6) is bisected at P. If P lies on the line $2x + 4y + k = 0$, find the value of k.

Sol. Let A(2, –1) and B(5, –6) be the given points. Let $P(x_1, y_1)$ be the mid-point of AB.



Then $\quad x_1 = \dfrac{2+5}{2} = \dfrac{7}{2}$ $\quad$ …(1)

and $\quad y_1 = \dfrac{-1-6}{2} = -\dfrac{7}{2}$ $\quad$ …(2)

$\because \quad (x_1, y_1)$ lies on the line $2x + 4y + k = 0$

$\therefore \quad 2x_1 + 4y_1 + k = 0$

$\Rightarrow \quad 2 \times \frac{7}{2} - 4 \times \frac{7}{2} + k = 0$ [From (1) and (2)]

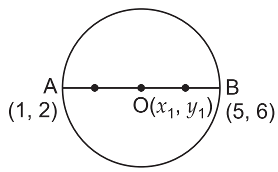
$\Rightarrow \quad 7 - 14 + k = 0$

$\Rightarrow \quad k = 7$

$\therefore$ The required value of k is 7.

10. If the coordinates of the end points of a diameter of a circle are A(1, 2) and B(5, 6), find the coordinates of the centre of circle.

Sol. Let AB be a diameter of the circle with centre at O on AB. Let (x_1, y_1) be the coordinate of the centre O of the circle.



$\because$ (x_1, y_1) is the mid-point of AB.

$\therefore$ By mid-point formula,

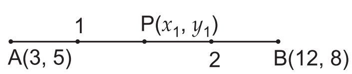
$$x_1 = \frac{1+5}{2} = \frac{6}{2} = 3,\ y_1 = \frac{2+6}{2} = \frac{8}{2} = 4$$

$\therefore$ The required coordinates of the centre O of the circle are (3, 4).

Short Answer Type Questions

11. Find the coordinates of the points P which divides the line segment joining points A(3, 5) and B(12, 8) internally in the ratio 1 : 2.

Sol. Let P be the point (x_1, y_1). Now, P divides AB in the ratio 1 : 2.



$\therefore$ PA : PB = 1 : 2

$\therefore$ By section formula,

$$x_1 = \frac{12 \times 1 + 3 \times 2}{1+2} = \frac{12+6}{3} = \frac{18}{3} = 6$$

$$y_1 = \frac{1 \times 8 + 5 \times 2}{2} = \frac{8 \times 10}{3} = \frac{18}{3} = 6$$

$\therefore$ The required coordinates of P are (6, 6).

12. In what ratio does the point P(–7, –5) divides the line segment joining points A(–8, –3) and B(–5, –9).

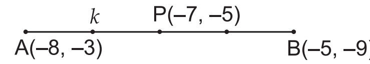
Sol. Let P divide AB in the ratio AP : PB = k : 1 where k is same non-zero constant. Then by section formula, we have

$$-7 = \frac{-5 \times k - 8 \times 1}{k+1} = \frac{-5k - 8}{k+1}$$

$\Rightarrow \quad 7k + 7 = 5k + 8$

$\Rightarrow \quad 2k = 1$

$\Rightarrow \quad k = \frac{1}{2}$

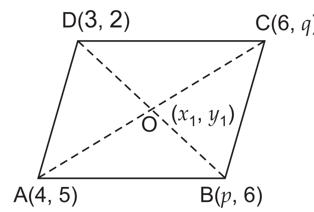


$\therefore$ P divides AB in the ratio $\frac{1}{2} : 1$, i.e. 1 : 2.

$\therefore$ The required ratio is 1 : 2.

13. If A(4, 5), B(p, 6), C(6, q) and D(3, 2) are the vertices of a parallelogram, find the value of p and q.

Sol. Let O be the point of intersection of the two diagonals AC and BD of the parallelogram ABCD. Then O will be the common middle point of the two diagonals AC and BD. Let (x_1, y_1) be the coordinates of O.



Since, O(x_1, y_1) is the mid-point of BD

$\therefore \quad x_1 = \frac{3+p}{2}$

$\Rightarrow \quad 3 + p = 2x_1$ ...(1)

and $\quad y_1 = \frac{2+6}{2} = \frac{8}{2} = 4$ ...(2)

Again, since O(x_1, y_1) is the mid-point of AC

$\therefore \quad x_1 = \frac{4+6}{2} = \frac{10}{2} = 5$ ...(3)

and $\quad y_1 = \frac{5+q}{2}$

$\Rightarrow \quad 5 + q = 2y_1$ ...(4)

Now, from (1) and (2), we have

$3 + p = 2 \times 5$

$\Rightarrow \quad p = 10 - 3 = 7$

Also, from (2) and (4), we have

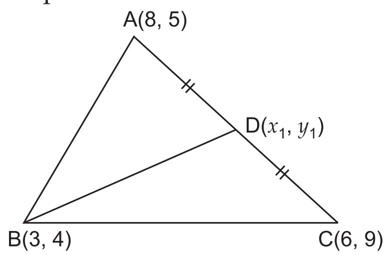
$5 + q = 2 \times 4$

$\Rightarrow \quad q = 8 - 5 = 3$

$\therefore$ The required values of p and q are 7 and 3 respectively.

14. Find the lengths of medians of the triangle through vertex B to AC whose vertices are A(8, 5), B(3, 4) and C(6, 9).

Sol. Let BD be a median of ΔABC, where D (x_1, y_1) is the mid-point of AC.



$\therefore$ By mid-point formula,

$$x_1 = \frac{1}{2}(8 + 6) = \frac{14}{2} = 7$$

$$y_1 = \frac{1}{2}(9 + 5) = \frac{14}{2} = 7$$

$\therefore$ D is the point (7, 7).

$\therefore$ The required length of the median BD

$$= \sqrt{(7-3)^2 + (7-4)^2} \text{ units}$$

$$= \sqrt{4^2 + 3^2} \text{ units}$$

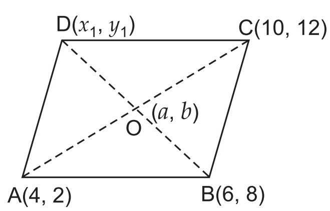
$$= \sqrt{16 + 9} \text{ units}$$

$$= \sqrt{25} \text{ units}$$

$$= 5 \text{ units.}$$

15. Three consecutive vertices of a parallelogram ABCD are A(4, 2), B(6, 8) and C(10, 12). Find the fourth vertex D.

Sol. Let (x_1, y_1) be the coordinates of the fourth vertex D of the parallelogram and let (a, b) be the coordinates of the point O, where two diagonals. AC and BD intersect each other. Since, the diagonals of a parallelogram bisect each other at O.



$\because$ (a, b) is the mid-point at DB

$\therefore \quad a = \frac{1}{2}(x_1 + 6)$

$\Rightarrow \quad x_1 + 6 = 2a \quad \ldots(1)$

$b = \frac{1}{2}(y_1 + 8)$

$\Rightarrow \quad y_1 + 8 = 2b \quad \ldots(2)$

Also, (a, b) is the mid-point of AC

$\therefore$ By mid-point formula,

$$a = \frac{1}{2}(4 + 10) = \frac{14}{2} = 7 \quad \ldots(3)$$

$$b = \frac{1}{2}(2 + 12) = \frac{14}{2} = 7 \quad \ldots(4)$$

$\therefore$ From (1) and (3), we have

$$x_1 = 2a - 6$$
$$= 2 \times 7 - 6$$
$$= 14 - 6$$
$$= 8$$

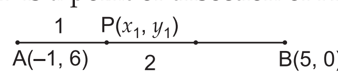
and from (2) and (4), we have

$$y_1 = 2b - 8$$
$$= 2 \times 7 - 8$$
$$= 14 - 8$$
$$= 6$$

$\therefore$ The required coordinates of D are (8, 6).

16. The line joining points A(–1, 6) and B(5, 0) is trisected at points P and Q. If point P also lies on the line $x + y - k = 0$, find the value of k.

Sol. Since, P is a point of trisection of AB



$\therefore$ AP : PB = 1 : 2

If P is the point (x_1, y_1), then by section formula,

$$x_1 = \frac{1 \times 5 - 2 \times 1}{2 + 1}$$

$$= \frac{5 - 2}{3}$$

$$= \frac{3}{3} = 1$$

$$y_1 = \frac{1 \times 0 + 2 \times 6}{2 + 1}$$

$$= \frac{12}{3} = 4$$

Now, (x_1, y_1) i.e. (1, 4) lies on the line $x + y - k = 0$

$\therefore \quad x_1 + y_1 - k = 0$

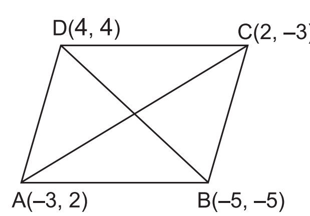
$\Rightarrow \quad 1 + 4 - k = 0$

$\Rightarrow \quad k = 5$

$\therefore$ The required value of k is 5.

17. Show that A(–3, 2), B(–5, –5), C(2, –3) and D(4, 4) are the vertices of a rhombus.

Sol.



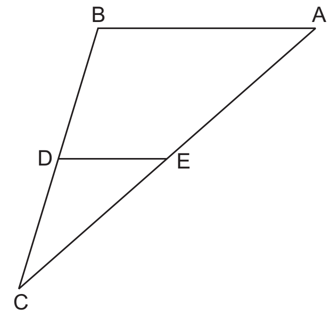
We have

$$AB = \sqrt{(-5+3)^2 + (-5-2)^2}$$
$$= \sqrt{(-2)^2 + (-7)^2}$$
$$= \sqrt{4+49}$$
$$= \sqrt{53}$$
$$BC = \sqrt{(2+5)^2 + (-3+5)^2}$$
$$= \sqrt{7^2 + 2^2}$$
$$= \sqrt{49+4}$$
$$= \sqrt{53}$$
$$CD = \sqrt{(4-2)^2 + (4+3)^2}$$
$$= \sqrt{2^2 + 7^2}$$
$$= \sqrt{4+49}$$
$$= \sqrt{53}$$
$$DA = \sqrt{(-3-4)^2 + (2-4)^2}$$
$$= \sqrt{(-7)^2 + (-2)^2}$$
$$= \sqrt{49+4}$$
$$= \sqrt{53}$$
$$AC = \sqrt{(2+3)^2 + (-3-2)^2}$$
$$= \sqrt{5^2 + (-5)^2}$$
$$= \sqrt{25+25}$$
$$= \sqrt{50}$$
$$= 5\sqrt{2}$$
$$BD = \sqrt{(4+5)^2 + (4+5)^2}$$
$$= \sqrt{81+81}$$
$$= 9\sqrt{2}$$

$\therefore$ AB = BC = CD = DA, but AC $\neq$ BD.

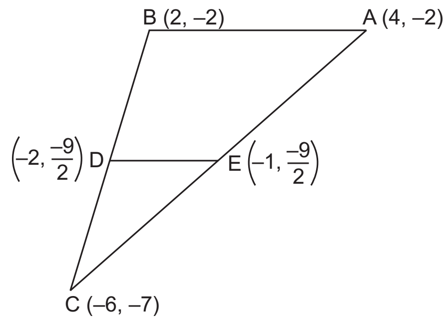
i.e. all sides of a quadrilateral ABCD are of equal length but the two diagonals are not of equal length. Hence, ABCD is a rhombus i.e. the given points are the vertices of a rhombus.

18. In the given figure in ΔABC, points D and E are mid-points of sides BC and AC respectively. If given vertices are A(4, –2), B(2, –2) and C(–6, –7), then verify the result DE = $\frac{1}{2}$ AB.



[CBSE 2023 Basic]

Sol.



E is the mid-point of AC.

$$\text{Coordinates of E} = \left(\frac{4-6}{2}, \frac{-2-7}{2}\right)$$
$$E = \left(-1, \frac{-9}{2}\right)$$

D is the mid-point of BC.

$$\text{Coordinates of D} = \left(\frac{2-6}{2}, \frac{-2-7}{2}\right)$$
$$E = \left(-2, \frac{-9}{2}\right)$$

Now,
$$DE = \sqrt{(-1+2)^2 + \left(\frac{-9}{2}+\frac{9}{2}\right)^2}$$
$$= \sqrt{(1)^2} = 1 \qquad \text{...(1)}$$
$$AB = \sqrt{(4-2)^2 + (-2+2)^2}$$
$$= \sqrt{(2)^2} = 2 \qquad \text{...(2)}$$

From (1) and (2)

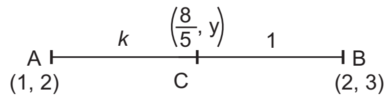
$$DE = \frac{1}{2} AB$$

19. Find the ratio in which the point $\left(\frac{8}{5}, y\right)$ divides the line segment joining the points (1, 2) and (2, 3). Also find the value of y.

[CBSE 2024 Standard]

Sol. Let the point $\left(\frac{8}{5}, y\right)$ divides the line segment in the ratio $k : 1$.

Coordinates of the points are (1, 2) and (2, 3).



$$\frac{8}{5} = \frac{2k+1}{k+1} \quad \text{...(1)}$$

$$y = \frac{3k+2}{k+1} \quad \text{...(2)}$$

Solving equation (1),

$$\frac{8}{5} = \frac{2k+1}{k+1}$$

$$8(k+1) = 5(2k+1)$$

$$8k + 8 = 10k + 5$$

$\Rightarrow \quad 2k = 3$

$\Rightarrow \quad k = \frac{3}{2}$

Putting $k = \frac{3}{2}$, in equation (2), we get

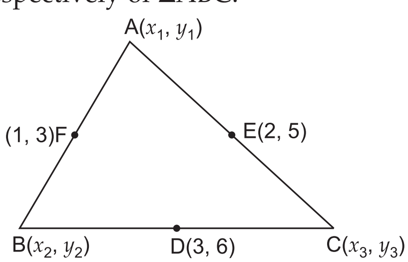
$$y = \frac{3 \times \frac{3}{2} + 2}{\frac{3}{2} + 1} = \frac{9+4}{3+2} = \frac{13}{5}$$

Therefore, the ratio is 3 : 2 and $y = \frac{13}{5}$.

Long Answer Type Questions

20. If the mid-points of sides of ΔABC are (3, 6), (2, 5) and (1, 3), then find the coordinates of three vertices of ΔABC.

Sol. Let D(3, 6), E(2, 5) and F(1, 3) be the coordinates of the mid-points of the sides BC, CA and AB respectively of ΔABC. Let (x_1, y_1), (x_2, y_2) and (x_3, y_3) be the coordinates of the vertices A, B and C respectively of ΔABC.



Since, D is the mid-point of BC.

$\therefore$ By mid-point formula,

$$3 = \frac{x_2 + x_3}{2} \quad \Rightarrow \quad x_2 + x_3 = 6 \quad \text{…(1)}$$

and $$6 = \frac{y_2 + y_3}{2} \quad \Rightarrow \quad y_2 + y_3 = 12 \quad \text{…(2)}$$

E is the mid-point of AC.

$\therefore$ By mid-point formula,

$$2 = \frac{x_1 + x_3}{2} \quad \Rightarrow \quad x_1 + x_3 = 4 \quad \text{…(3)}$$

and $$5 = \frac{y_1 + y_3}{2} \quad \Rightarrow \quad y_1 + y_3 = 10 \quad \text{…(4)}$$

Lastly, F is the mid-point of AB.

$\therefore$ By mid-point formula,

$$1 = \frac{x_1 + x_2}{2} \quad \Rightarrow \quad x_1 + x_2 = 2 \quad \text{…(5)}$$

and $$3 = \frac{y_1 + y_2}{2} \quad \Rightarrow \quad y + y_2 = 6 \quad \text{…(6)}$$

Adding (1), (3) and (5), we have

$$2(x_1 + x_2 + x_3) = 6 + 4 + 2 = 12$$

$\Rightarrow \quad x_1 + x_2 + x_3 = 6 \quad \text{…(7)}$

Subtracting (1), (3) and (5) successively from (7), we get $x_1 = 0$, $x_2 = 2$ and $x_3 = 4$

Again, adding (2), (4) and (6), we get

$$2(y_1 + y_2 + y_3) = 28$$

$\Rightarrow \quad y_1 + y_2 + y_3 = 14 \quad \text{…(8)}$

Subtracting (2), (4) and (6) successively from (8), we get $y_1 = 2$, $y_2 = 4$ and $y_3 = 8$.

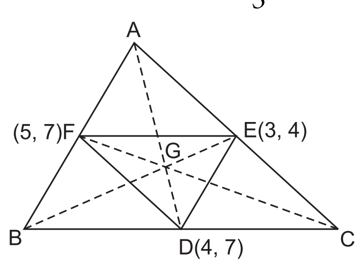
$\therefore$ The required coordinates of A, B and C are (0, 2), (2, 4) and (4, 8) respectively.

21. If the coordinates of the mid-points of the sides of the triangle are (4, 7), (3, 4) and (5, 7), find its centroid.

Sol. Let D(4, 7), E(3, 4) and F(5, 7) be the mid-points of the sides BC, CA and AB respectively of ΔABC. Then we know that the coordinates of the centroid of ΔABC will be identical with the coordinates of the centroid of ΔDEF. If $(\bar{x}, \bar{y})$ be the centroid of ΔDEF, then

$$\bar{x} = \frac{4+3+5}{3} = \frac{12}{3} = 4$$

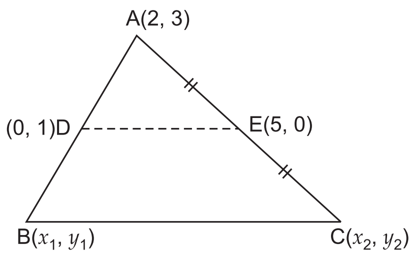
$$\bar{y} = \frac{7+4+7}{3} = \frac{18}{3} = 6$$



Hence, the required coordinates of the centroid of ΔABC are (4, 6).

22. If a vertex of a triangle is (2, 3) and the middle points of the sides through it are (0, 1) and (5, 0), find other vertices.

Sol. Let A(2, 3) be the vertex of ΔABC and D and E be the mid-points of AB and AC respectively such that D(0, 1) and E(5, 0). Let (x_1, y_1) and (x_2, y_2) be respectively the coordinates of the two other vertices B and C of ΔABC.



Then since (0, 1) is the mid-point of AB and (5, 0) is the mid-point of AC.

$\therefore$ By mid-point formula,

$$\frac{2+x_1}{2} = 0$$

$$\Rightarrow \quad x_1 = -2$$

$$\text{and} \quad \frac{y_1+3}{2} = 1$$

$$\Rightarrow \quad y_1 = 2 - 3 = -1$$

$\therefore$ The coordinates of B are (–2, –1)

$$\text{Again,} \quad \frac{2+x_2}{2} = 5$$

$$\Rightarrow \quad x_2 = 10 - 2 = 8$$

$$\text{and} \quad \frac{y_2+3}{2} = 0$$

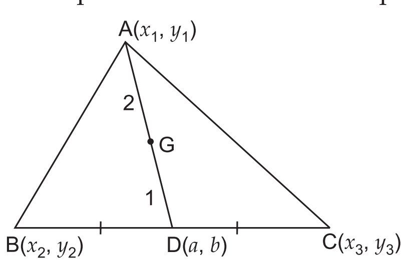
$$\Rightarrow \quad y_2 = -3$$

$\therefore$ The coordinates of C are (8, –3).

Hence, the required coordinates of other vertices are (–2, –1) and (8, –3).

23. Prove that coordinates of the centroid of a triangle whose vertices are (x_1, y_1), (x_2, y_2) and (x_3, y_3) are given by $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$.

Sol. Let $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ be the vertices of ΔABC and AD be a median of ΔABC, where D is the mid-point of BC. Let D be the point (a, b).



$\therefore$ By mid-point formula,

$$a = \frac{x_2+x_3}{2} \quad \Rightarrow \quad x_2 + x_3 = 2a \quad \ldots(1)$$

$$\text{and} \quad b = \frac{y_2+y_3}{2} \quad \Rightarrow \quad y_2 + y_3 = 2b \quad \ldots(2)$$

Now, let the centroid be $G(\bar{x}, \bar{y})$. Since G divides the median AD is the ratio AG : GD = 2 : 1.

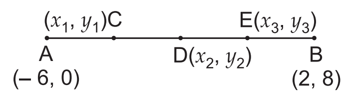
$$\therefore \quad \bar{x} = \frac{2a + 1 \times x_1}{2+1} = \frac{x_2+x_3+x_1}{3} \quad \text{[From (1)]}$$

$$\text{and} \quad \bar{y} = \frac{2b + y_1}{3} = \frac{y_2+y_3+y_1}{3} \quad \text{[From (2)]}$$

Hence, the required coordinates of the centroid are $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$.

24. Find the coordinates of the points which divides the line segment joining A(–6, 0) and B(2, 8) in four equal parts.

Sol. Let $C(x_1, y_1)$, $D(x_2, y_2)$ and $E(x_3, y_3)$ divide AB into four equal parts so that AC = CD = DE = EB.



Clearly, C divides AB in the ratio AC : CB = 1 : 3

$$\therefore \quad x_1 = \frac{1 \times 2 + 3 \times (-6)}{1+3}$$

$$= \frac{2-18}{4}$$

$$= \frac{-16}{4}$$

$$= -4$$

$$y_1 = \frac{1 \times 8 + 3 \times 0}{1+3}$$

$$= \frac{8}{4}$$

$$= 2$$

Finally, E divides AB in the ratio AE : EB = 3 : 1

$$\therefore \quad x_3 = \frac{3 \times 2 + 1 \times (-6)}{3+1}$$

$$= \frac{6-6}{4} = 0$$

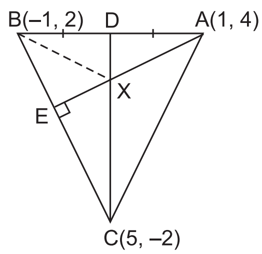
$$y_3 = \frac{3 \times 8 + 1 \times 0}{3+1}$$

$$= \frac{24}{4} = 6$$

$\therefore$ The required coordinates are (–4, 2), (–2, 4) and (0, 6).

25. A(1, 4), B(–1, 2) and C(5, –2) are the vertices of a ΔABC. Find the coordinates of the point where the right bisector of BC intersects the median through C.

Sol.



We have

$$AC = \sqrt{(5-1)^2 + (-2-4)^2}$$
$$= \sqrt{4^2 + (-6)^2}$$
$$= \sqrt{16 + 36}$$
$$= \sqrt{52}$$
$$= 3\sqrt{6}$$
$$BC = \sqrt{(5+1)^2 + (-2-2)^2}$$
$$= \sqrt{6^2 + (-4)^2}$$
$$= \sqrt{36 + 16}$$
$$= \sqrt{52}$$
$$= 3\sqrt{6}$$
$$BA = \sqrt{(1+1)^2 + (4-2)^2}$$
$$= \sqrt{2^2 + 2^2}$$
$$= \sqrt{4 + 4}$$
$$= \sqrt{8}$$
$$= 2\sqrt{2}$$

$\therefore$ AC = BC $\neq$ BA

$\therefore$ ΔABC is an isosceles triangle with D and E as the mid-points of AB and BC respectively.

$\therefore$ $\angle CDA = \angle CDB = 90°$.

$\therefore$ CD is the perpendicular bisector of AB. Let the perpendicular bisector of CB meet that of AB at X. To find the coordinates of X. Let (x_1, y_1) be the coordinates of x. Clearly X will be the circumcentre of ΔABC.

$\therefore$ AX = BX = CX

$$\Rightarrow (x_1 - 1)^2 + (y_1 - 4)^2$$
$$= (x_1 + 1)^2 + (y_1 - 2)^2$$
$$= (x_1 - 5)^2 + (y_1 + 2)^2$$

$$\Rightarrow x_1^2 + y_1^2 - 2x_1 - 8y_1 + 17$$
$$= x_1^2 + y_1^2 - 2x_1 - 4y_1 + 5$$
$$= x_1^2 + y_1^2 - 10x_1 + 4y_1 + 29$$

$$\Rightarrow -2x_1 - 8y_1 + 17 = 2x_1 - 4y_1 + 5$$
$$= -10x_1 + 4y_1 + 29$$

From the first and the second expressions, we have

$$4x_1 + 4y_1 - 12 = 0$$
$$\Rightarrow x_1 + y_1 - 3 = 0$$
$$\Rightarrow y_1 = 3 - x_1 \quad \ldots(1)$$

Also, from the first and the last expressions, we have

$$8x_1 - 12y_1 - 12 = 0$$
$$\Rightarrow 2x_1 - 3y_1 - 3 = 0$$
$$\Rightarrow 2x_1 - 3(3 - x_1) - 3 = 0 \quad \text{[From (1)]}$$
$$\Rightarrow 2x_1 - 9 + 3x_1 - 3 = 0$$
$$\Rightarrow 5x_1 = 12$$
$$\Rightarrow x_1 = \frac{12}{5}$$

$\therefore$ From (1), we have

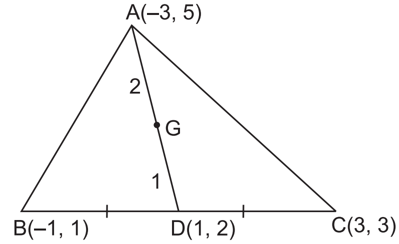
$$y_1 = 3 - \frac{12}{5} = \frac{15 - 12}{5} = \frac{3}{5}$$

$\therefore$ The required coordinates of X are $\left(\frac{12}{5}, \frac{3}{5}\right)$.

26. If A(–3, 5), B(–1, 1), C(3, 3) are the vertices of triangle ΔABC, find the length of median AD. Also find the coordinates of the point which divides the median in the ratio 2 : 1. **(CBSE 2013)**

Sol. Since D is the mid-point of BC, if (x_1, y_1) are the coordinates of D then by mid-point formula,

$$x_1 = \frac{1}{2}(-1 + 3) = \frac{2}{2} = 1,$$
$$y_1 = \frac{1}{2}(1 + 3) = \frac{4}{2} = 2$$



$\therefore$ D is the point (1, 2).

Let G be a point on AD such that

AG : GD = 2 : 1.

If $(\bar{x}, \bar{y})$ be the coordinates of G, then

$$\bar{x} = \frac{2 \times 1 + 1 \times (-3)}{2 + 1}$$

$$= \frac{2 - 3}{3} = \frac{-1}{3}$$

$$\bar{y} = \frac{2 \times 2 + 1 \times 5}{2 + 1}$$

$$= \frac{4 + 5}{3} = \frac{9}{3} = 3$$

$\therefore$ The required coordinates of G are $\left(-\frac{1}{3}, 3\right)$ and length of median AD is

$$\sqrt{(1+3)^2 + (2-5)^2} \text{ units} = \sqrt{4^2 + (-3)^2} \text{ units}$$

$$= \sqrt{16 + 9} \text{ units}$$

$$= \sqrt{25} \text{ units}$$

$$= 5 \text{ units}$$

Higher Order Thinking Skills (HOTS) Questions

(Page 141)

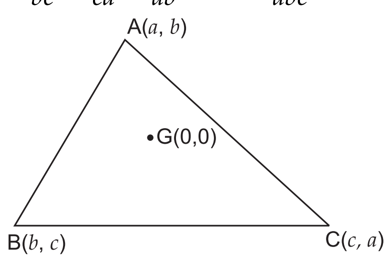
1. If the centroid of the triangle formed by A(a, b), B(b, c) and C(c, a) is at the origin, what is the value of $\frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab}$.

Sol. If G(0, 0) be the centroid of ΔABC, then

$$0 = \frac{a + b + c}{3}$$

$$\Rightarrow \quad a + b + c = 0 \quad \ldots(1)$$

$$\text{Now,} \quad \frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab} = \frac{a^3 + b^3 + c^3}{abc} \quad \ldots(2)$$



Now, if $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$ …(3)

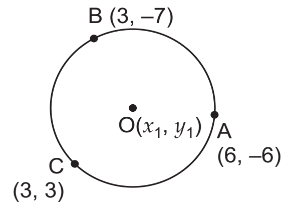
$\therefore$ From (2) and (3), we get

$$\frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab} = \frac{3abc}{abc} = 3$$

$\therefore$ The required value of $\frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab}$ is 3.

2. Find the centre of a circle passing through the points (6, – 6), (3, – 7) and (3, 3).

Sol. Let O(x_1, y_1) be the centre of the circle passing through the points A(6, –6), B(3, –7) and C(3, 3).



Then $AO^2 = BO^2 = CO^2$

$$\Rightarrow \quad (x_1 - 6)^2 + (y_1 + 6)^2 = (x_1 - 3)^2 + (y_1 + 7)^2$$

$$= (x_1 - 3)^2 + (y_1 - 3)^2$$

$$\Rightarrow \quad x_1^2 + y_1^2 - 12x_1 + 12y_1 + 72$$

$$= x_1^2 + y_1^2 - 6x_1 + 14y_1 + 58$$

$$= x_1^2 + y_1^2 - 6x_1 - 6y_1 + 18$$

$$\Rightarrow \quad -12x_1 + 12y_1 + 72 = -6x_1 + 14y_1 + 58$$

$$= -6x_1 - 6y_1 + 18$$

From the first and the last expressions, we have

$$6x_1 - 18y - 54 = 0$$

$$\Rightarrow \quad x_1 - 3y_1 - 9 = 0$$

$$\Rightarrow \quad x_1 = 3y_1 + 9 \quad \ldots(1)$$

From the first and the second expressions, we have

$$6x_1 + 2y_1 - 14 = 0$$

$$\Rightarrow \quad 3x_1 + y_1 - 7 = 0$$

$$\Rightarrow \quad 3(3y_1 + 9) + y_1 - 7 = 0 \quad \text{[From (1)]}$$

$$\Rightarrow \quad 10y_1 + 20 = 0$$

$$\Rightarrow \quad y_1 = -2$$

$$\therefore \text{ From (1),} \quad x_1 = -3 \times 2 + 9$$

$$= -6 + 9$$

$$= 3$$

$\therefore$ The required centre of the circle is (3, –2).

3. ABCD is a rectangle formed by A(–1, – 1), B(–1, 4), C(5, 4) and D(5, –1). P, Q, R and S are the mid-points of AB, BC, CD, DA respectively. Is the quadrilateral PQRS a square, a rectangle or a rhombus? Justify your answer.

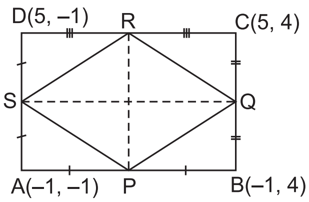
Sol. Since P, Q, R and S are the mid-point of AB, BC, CD and DA respectively, hence the coordinates of P, Q, R and S are

$$P : \left(\frac{-1-1}{2}, \frac{-1+4}{2}\right) = \left(-1, \frac{3}{2}\right)$$

$$Q : \left(\frac{5-1}{2}, \frac{4+4}{2}\right) = (2, 4)$$

R : $\left(\frac{5+5}{2}, \frac{-1+4}{2}\right) = \left(5, \frac{3}{2}\right)$

S : $\left(\frac{5-1}{2}, \frac{-1-1}{2}\right) = (2, -1)$



Now $\quad PQ = \sqrt{(2+1)^2 + \left(4 - \frac{3}{2}\right)^2}$

$= \sqrt{3^2 + \left(\frac{5}{2}\right)^2}$

$= \sqrt{9 + \frac{25}{4}}$

$= \sqrt{\frac{36+25}{4}}$

$= \sqrt{\frac{61}{2}}$

$QR = \sqrt{(2-5)^2 + \left(4 - \frac{3}{2}\right)^2}$

$= \sqrt{3^2 + \left(\frac{5}{2}\right)^2}$

$= \sqrt{9 + \frac{25}{4}}$

$= \sqrt{\frac{36+25}{4}}$

$= \sqrt{\frac{61}{2}}$

$RS = \sqrt{(5-2)^2 + \left(\frac{3}{2} + 1\right)^2}$

$= \sqrt{3^2 + \left(\frac{5}{2}\right)^2}$

$= \sqrt{9 + \frac{25}{4}}$

$= \sqrt{\frac{36+25}{4}}$

$= \sqrt{\frac{61}{2}}$

$SP = \sqrt{(-1-2)^2 + \left(\frac{3}{2} + 1\right)^2}$

$= \sqrt{3^2 + \left(\frac{5}{2}\right)^2}$

$= \sqrt{9 + \frac{25}{4}}$

$= \sqrt{\frac{36+25}{4}}$

$= \sqrt{\frac{61}{2}}$

Also, $\quad SQ = \sqrt{(2-2)^2 + (4+1)^2}$

$= \sqrt{0 + 5^2}$

$= \sqrt{25}$

$= 5$

and $\quad PR = \sqrt{(5+1)^2 + \left(\frac{3}{2} - \frac{3}{2}\right)^2}$

$= \sqrt{6^2 + 0}$

$= \sqrt{36}$

$= 6$

$\therefore$ PQ = QR = RS = SP, but diagonals SQ $\neq$ PR

Hence, the quadrilateral PQRS is a rhombus and not a square.

Self-Assessment

(Page 141)

Multiple-Choice Questions

1. The distance of a point P(–6, 8) from origin is

(*a*) 6 units (*b*) –6 units

(*c*) 8 units (*d*) 10 units

[CBSE 2023 Standard]

Sol. (*d*) 10 units

The required distance $= \sqrt{(-6)^2 + 8^2}$ units

$= \sqrt{36+64}$ units

$= \sqrt{100}$ units

$= 10$ units

2. The value of *a* if A(0, 0), B(3, $\sqrt{3}$) and C(3, *a*) form an equilateral triangle is

(*a*) 2 (*b*) –3

(*c*) –4 (*d*) $-\sqrt{3}$

Sol. (*d*) $-\sqrt{3}$

We have $\quad AB^2 = 3^2 + \left(\sqrt{3}\right)^2$

$= 9 + 3$

$= 12$

$AC^2 = 3^2 + a^2$

and $\quad BC^2 = (3-3)^2 + (a - \sqrt{3})^2$

$= 0 + 3 + a^2 - 2\sqrt{3}\,a$

$\because \quad AB^2 = AC^2 = BC^2$

$\therefore \quad 9 + a^2 = 12$

$\Rightarrow \quad a = \pm\sqrt{3}$

When $a = \sqrt{3}$, B and C become identical point.

$a \neq \sqrt{3}$

$\therefore \quad a = -\sqrt{3}$

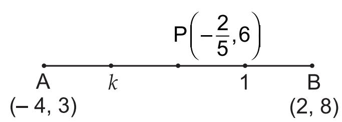
3. The ratio in which the point $P\left(\frac{-2}{5}, 6\right)$ divides the line segment joining the points A(–4, 3) and B(2, 8) is

(*a*) 2 : 3 (*b*) 1 : 3

(*c*) 3 : 2 (*d*) 3 : 1

Sol. (*c*) 3 : 2

Let P divide AB is the ratio PA : PB = k : 1 where k is a positive constant.



$$\therefore \quad -\frac{2}{5} = \frac{2 \times k - 4 \times 1}{k+1} = \frac{2k-4}{k+1}$$

$\Rightarrow \quad -2k - 2 = 10\,k - 20$

$\Rightarrow \quad 12k = 18$

$$\Rightarrow \quad k = \frac{18}{12} = \frac{3}{2}$$

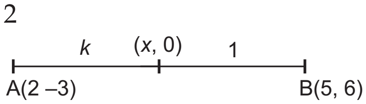
$$\therefore \quad k : 1 = \frac{3}{2} : 1 = 3 : 2$$

4. In what ratio does x-axis divide the line segment joining the points A(2, –3) and B(5, 6)?

(*a*) 2 : 3 (*b*) 2 : 1

(*c*) 3 : 4 (*d*) 1 : 2 **[CBSE 2023 Basic]**

Sol. (*d*) 1 : 2



On the x-axis, y-coordinate is zero.

Let x-axis divide the line segment AB in the ratio k : 1.

$$0 = \frac{6k + 1(-3)}{k+1}$$

$\Rightarrow \quad 6k - 3 = 0$

$\Rightarrow \quad 6k = 3$

$$\Rightarrow \quad k = \frac{1}{2}$$

$\therefore$ The ratio is 1 : 2.

5. The distance of the point (–6, 8) from the origin is

(*a*) 6 units (*b*) –6 units

(*c*) 8 units (*d*) 10 units

[CBSE 2023 Standard]

Sol. (*d*) 10 units

Distance of the point (–6, 8) from the origin

$= \sqrt{(-6)^2 + (8)^2}$ units

$= \sqrt{36 + 64}$ units

$= \sqrt{100}$ units

$= 10$ units

6. The distance between the point (2, –3) and (–2, 3) is

(*a*) $2\sqrt{13}$ units (*b*) 5 units

(*c*) $13\sqrt{2}$ units (*d*) 10 units

[CBSE 2024 Basic]

Sol. (*a*) $2\sqrt{13}$ units

Distance $= \sqrt{(-2-2)^2 + (3+3)^2}$ units

$= \sqrt{(-4)^2 + (6)^2}$ units

$= \sqrt{16 + 36}$ units

$= \sqrt{52}$ units

$= 2\sqrt{13}$ units

7. The mid-point of the line segment joining the points (–1, 3) and $\left(8, \frac{3}{2}\right)$ is

(*a*) $\left(\frac{7}{2}, \frac{-3}{4}\right)$ (*b*) $\left(\frac{7}{2}, \frac{9}{2}\right)$

(*c*) $\left(\frac{9}{2}, \frac{-3}{4}\right)$ (*d*) $\left(\frac{7}{2}, \frac{9}{4}\right)$

[CBSE 2024 Basic]

Sol. (*d*) $\left(\frac{7}{2}, \frac{9}{4}\right)$

$$\text{Coordinates of the mid-point} = \left(\frac{-1+8}{2}, \frac{3+\frac{3}{2}}{2}\right)$$

$$= \left(\frac{7}{2}, \frac{9}{4}\right)$$

Fill in the Blanks

8. The distance between the points P(6, 0) and Q(–2, 0) is **8 units**.

Sol. $$PQ = \sqrt{(-2-6)^2 + (0-0)^2}$$
$$= \sqrt{(-8)^2} = 8 \text{ units}$$

9. The coordinates of a point P dividing the line segment joining the points A(1, 3) and B(4, 6) in the ratio 2 : 1 are **(3, 5)**.

Sol. Let the coordinates of the point P be (x, y). As the point P divides the line segment AB in the ratio 2 : 1.

Then, $$x = \frac{1 \times 1 + 2 \times 4}{1+2} = \frac{1+8}{3} = 3$$

and $$y = \frac{1 \times 3 + 2 \times 6}{1+2} = \frac{3+12}{3} = 5$$

Hence, the coordinates of the point P are (3, 5).

10. The ratio in which the line segment joining A(6, 3) and B(–2, –5) is divided by the x–axis is **3 : 5**.

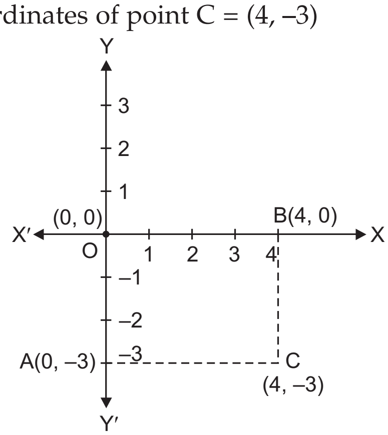
Sol. Let P(x, 0) be the coordinates of x-axis. The point P divides the line segment A(6, 3) and B(–2, –5) in the ratio k : 1. By using section formula,

$$0 = \frac{k \times (-5) + 1 \times 3}{k+1}$$
$$\Rightarrow \quad 5k = 3$$
$$\Rightarrow \quad k = \frac{3}{5}$$

Hence, the ratio is 3 : 5.

11. AOBC is a rectangle whose three vertices are A(0, –3), O(0, 0) and B(4, 0). The length of its diagonal is **5 units**.

Sol. Coordinates of point C = (4, –3)



Length of the diagonal

$$= \sqrt{(4)^2 + (-3)^2} \text{ units}$$
$$= \sqrt{16+9} \text{ units}$$
$$= \sqrt{25} \text{ units}$$
$$= 5 \text{ units}$$

Assertion-Reason Type Questions

Directions (Q. Nos. 12 to 14): Each of these questions contains an assertion followed by reason. Read them carefully, and answer the question on the basis of the following options, select the one that best describes the two statements.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

12. **Assertion (A):** Distance of the point (1, 1) from the origin is 2 units.

Reason (R): Distance between two points is given by root of sum of square of the differences in coordinates.

Sol. The correct answer is (d) as the Assertion is wrong as the correct distance is $\sqrt{(1-0)^2 + (1-0)^2}$ $= \sqrt{2}$.

As per distance formula, distance between two points is given by root of sum of square of the differences in coordinates.

13. **Assertion (A):** Points (4, 5), (4, 7) and (4, 9) cannot form a triangle.

Reason (R): They fall on a single line.

Sol. The correct answer is (a).

Points (4, 5), (4, 7) and (4, 9) cannot form a triangle as they fall in a straight line. Thus both statements are correct and reason is correct explanation of the assertion.

14. **Assertion (A):** Area of a triangle whose vertices are (1, –1), (–4, 6) and (–3, –5) is 24 square units.

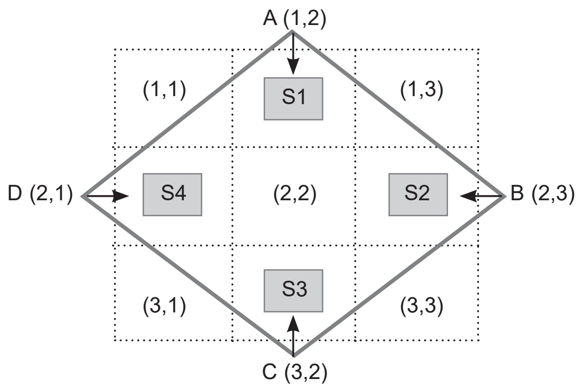
Reason (R): Area is given by half of the sum of the product of x-coordinate and difference of y-coordinates.

Sol. The correct answer is (a).

According to the formula of area of triangle, for vertices (1, –1), (–4, 6) and (–3, –5), area = 24 sq units. Thus both statements are correct and reason is correct explanation of the assertion.

Case Study Based Questions

15. Nikhil and Ayesha are playing a game on a coordinate grid. Nikhil has written some coordinates on some cells of the grid. Nikhil told Ayesha that the coordinates are written in a



pattern. Some cells are written as S1, S2, S3 and S4. Looking at the given pattern, Ayesha should fill the cells. Ayesha enjoyed and filled the cells with correct coordinates. Ayesha joined the cells and find the shape formed. Based on this situation, answer the following questions.

(*a*) Name the shape formed by joining the points.

Ans. Square

(*b*) Join A and C, B and D. What is the relation between two diagonals AC and BD?

Ans. AC = BD

(*c*) (*i*) Find the coordinates of the point where the two diagonals meet.

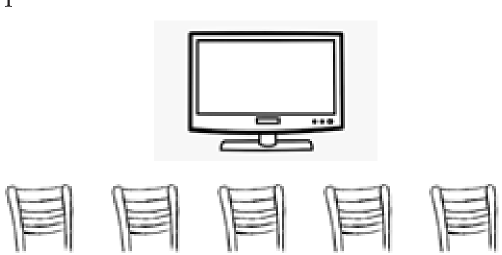
Ans. (2, 2)

or

(*ii*) Find the side of the square.

Ans. 1.41 units (approx.)

16. Five friends are watching TV in a room. In a row, there are five chairs. The first chair's position is (3, 5). The last chair's position is (3, 9). The students sitting on the first chair and the last chair have some difficulty in watching TV. Based on the given situation, answer the following questions.



(3, 5) (3, 9)

(*a*) What is the formula for finding the position of the chair which is placed in the middle?

Ans. $\left(\dfrac{x_1+x_2}{2}, \dfrac{y_1+y_2}{2}\right)$

(*b*) All the chairs are in a row. What will be the coordinates of the middle position of the row?

Ans. (3, 7)

(*c*) (*i*) If the mid-point of the line segment joining the points A(2, 3) and B(a, 5) is P(x, y) and $x - y = 6$, find the value of a.

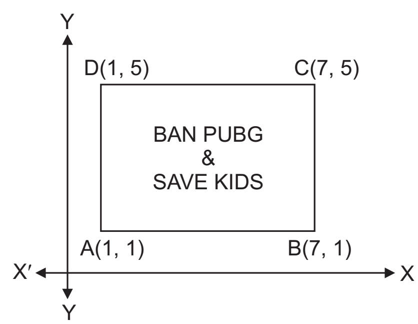
Ans. 18

or

(*ii*) Is the point P(–4, 6) lies on the line segment joining the points A(–5, 3) and B(–3, 9)?

Ans. Yes

17. Use the mobile screen for long hours makes your eye sight weak and give you headaches. Children who are addicted to play "PUBG" can get easily stressed out. To raise social awareness about ill effects of playing PUBG, a school decided to start 'BAN PUBG' campaign, in which students are asked to prepare campaign board in the shape of rectangle. One such campaign board made by class X students of the school is shown in the figure.



Based on the above information, answer the following questions:

(*a*) Find the coordinates of the point of intersection of diagonals AC and BD.

Sol. Coordinates of the point of intersection of diagonals AC and BD $= \left(\dfrac{1+7}{2}, \dfrac{5+1}{2}\right)$

$$= (4, 3)$$

(*b*) Find the length of the diagonal AC.

Sol. Length of the diagonal

$$AC = \sqrt{(7-1)^2 + (5-1)^2} \text{ units}$$

$$= \sqrt{(6)^2 + (4)^2} \text{ units}$$

$= \sqrt{36+16}$ units

$= \sqrt{52}$ units

$= 2\sqrt{13}$ units

(*c*) (*i*) Find the area of the campaign Board ABCD.

or

Sol. Length of side AB $= \sqrt{(7-1)^2 + (1-1)^2}$ units

$= \sqrt{(6)^2}$ units

= 6 units

Length of side BC $= \sqrt{(7-7)^2 + (5-1)^2}$ units

$= \sqrt{(4)^2}$ units

= 4 units

Area of Campaign Board ABCD

= 6 units × 4 units

= 24 sq units

or

(*ii*) Find the ratio of the length of side AB to the length of the diagonal AC.

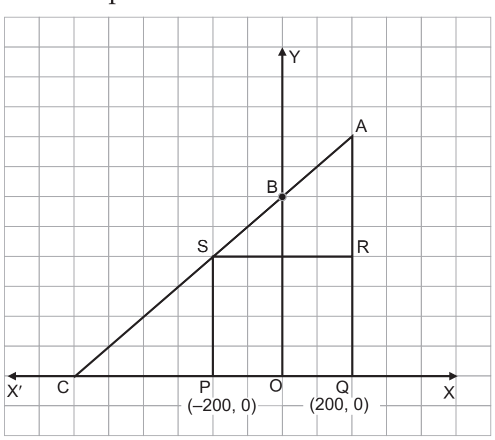
[CBSE 2023 Basic]

Sol. Length of side AB : Length of diagonal AC

$= 6 : 2\sqrt{13}$

$= 3 : \sqrt{13}$

18. Jagdish has a field which is in the shape of a right angled triangle AQC. He wants to leave a space in the form of a square PQRS inside the field for growing wheat and the remaining for growing vegetables (as shown in the figure). In the field, there is a pole marked as O.



Based on the above information, answer the following questions:

(*a*) Taking O as origin, coordinates of P are (–200, 0) and of Q are (200, 0). PQRS being a square, what are the coordinates of R and S?

Sol. Coordinates of R = (200, 400)

Coordinates of S = (–200, 400)

(*b*) (*i*) What is the area of square PQRS?

Sol. Length of side PQ

$= \sqrt{(200+200)^2 + (0-0)^2}$ units

$= \sqrt{(400)^2}$ units

= 400 units

Area of square PQRS = (side)^2

= 400 × 400 sq units

= 160000 sq units

or

(*ii*) What is the length of diagonal PR in square PQRS?

Sol. Length of diagonal PR

$= \sqrt{(200+200)^2 + (400)^2}$ units

$= \sqrt{(400)^2 + (400)^2}$ units

$= \sqrt{1600+1600}$ units

$= \sqrt{3200}$ units

$= \sqrt{2 \times 1600}$ units

$= 40\sqrt{2}$ units

(*c*) If S divides CA in the ratio *k* : 1, what is the value of *k*, where point A is (200, 800)?

[CBSE 2023 Standard]

Sol. Coordinates of point A = (200, 800)

Coordinates of point C = (–600, 0)

Coordinates of point S = (–200, 400)

S divides CA in the ratio *k* : 1.

$$-200 = \frac{200k - 600}{k+1}$$

$\Rightarrow \quad -200k - 200 = 200k - 600$

$\Rightarrow \quad 400k = 400$

$\therefore \quad k = 1$

Very Short Answer Type Questions

19. Find the distance 2AB, if A and B are points (–4, 2) and (–8, –1) respectively.

Sol. The required distance

$= 2\sqrt{(-8+4)^2 + (-1-2)^2}$ units

$= 2\sqrt{(-4)^2 + (-3)^2}$

$= 2\sqrt{16+9}$ units

$= 2 \times 5$ units

$= 10$ units

20. Find x so that the point $(3, x)$ lies on the line represented by $2x - 3y + 5 = 0$.

Sol. If the point $(3, x)$ lies on the line $2x - 3y + 5 = 0$, then $x = 3$ and $y = x$ will satisfy this equation.

$\therefore \quad 2 \times 3 - 3 \times x + 5 = 0$

$\Rightarrow \quad 6 - 3x + 5 = 0$

$\Rightarrow \quad 3x = 11$

$\Rightarrow \quad x = \dfrac{11}{3}$

21. If the mid-point of a line segment joining $A\left(\dfrac{x}{2}, \dfrac{y+1}{2}\right)$ and $B(x + 1, y - 3)$ is $C(5, -2)$. Find x and y.

Sol. We have

$$\frac{\frac{x}{2} + x + 1}{2} = 5 \qquad \ldots(1)$$

and
$$\frac{\frac{y+1}{2} + y - 3}{2} = -2 \qquad \ldots(2)$$

From (1), $\dfrac{3x}{2} + 1 = 10$

$\Rightarrow \quad \dfrac{3x}{2} = 10 - 1 = 9$

$\therefore \quad x = 9 \times \dfrac{2}{3} = 6$

From (2) $\dfrac{3y+1}{2} - 3 = -4$

$\Rightarrow \quad \dfrac{3y+1}{2} = -1$

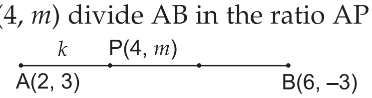
$\Rightarrow \quad 3y = -2 - 1 = -3$

$\therefore \quad y = -1$

$\therefore$ The required values of x and y are 6 and -1 respectively.

22. Find the ratio in which $P(4, m)$ divides the line segment joining the points $A(2, 3)$ and $B(6, -3)$. Hence, find m. **[CBSE 2018]**

Sol. Let $P(4, m)$ divide AB in the ratio $AP : PB = k : 1$



$\therefore \quad 4 = \dfrac{6k + 1 \times 2}{k + 1}$

$\Rightarrow \quad 4k + 4 = 6k + 2$

$\Rightarrow \quad 2k = 2$

$\Rightarrow \quad k = 1$

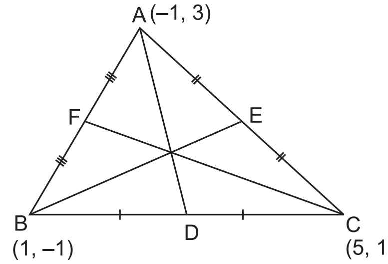
$\therefore$ Required ratio is $k : 1$, i.e. $1 : 1$.

Also, $m = \dfrac{1}{2}(3 - 3) = 0$ which is the required value of m.

Short Answer Type Questions

23. Find the length of median of a triangle whose vertices are $A(-1, 3)$, $B(1, -1)$ and $C(5, 1)$.

Sol. Let D, E and F be the mid-points of the sides BC, CA and AB respectively.



Then, $D : \left(\dfrac{5+1}{2}, \dfrac{-1+1}{2}\right) = (3, 0)$

$\therefore$ Length of the median AD

$= \sqrt{(3+1)^2 + (0-3)^2}$ units

$= \sqrt{16 + 9}$ units

$= 5$ units

$E : \left(\dfrac{-1+5}{2}, \dfrac{3+1}{2}\right) = (2, 2)$

$\therefore$ Length of the median BE

$= \sqrt{(2-1)^2 + (2+1)^2}$ units

$= \sqrt{1 + 9}$ units $= \sqrt{10}$ units

$F : \left(\dfrac{1-1}{2}, \dfrac{-1+3}{2}\right) = (0, 1)$

$\therefore$ Length of the median CF

$= \sqrt{(5-0)^2 + (1-1)^2}$ units

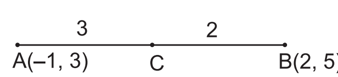
$= \sqrt{25}$ units

$= 5$ units.

Hence, the required length of the three medians are AD = 5 units, BE = $\sqrt{10}$ units and CF = 5 units respectively.

24. Find the coordinates of the point C on the line segment joining points $A(-1, 3)$ and $B(2, 5)$ such that $AC = \dfrac{3}{5}AB$.

Sol.



We have

$$\frac{AC}{AB} = \frac{3}{5}$$

$$\Rightarrow \quad \frac{AB}{AC} = \frac{5}{3}$$

$$\Rightarrow \quad \frac{AC + CB}{AC} = \frac{5}{3}$$

$$\Rightarrow \quad 1 + \frac{CB}{AC} = \frac{5}{3}$$

$$\Rightarrow \quad \frac{BC}{AC} = \frac{5}{3} - 1 = \frac{2}{3}$$

$$\Rightarrow \quad AC : CB = 3 : 2$$

Let C be the point (x_1, y_1)

$$\therefore \quad x_1 = \frac{3 \times 2 + 2 \times (-1)}{3 + 2}$$

$$= \frac{6 - 2}{5} = \frac{4}{5}$$

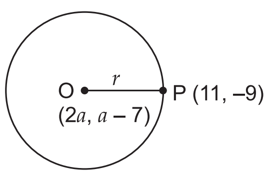
$$y_1 = \frac{3 \times 5 + 2 \times 3}{3 + 2}$$

$$= \frac{15 + 6}{5} = \frac{21}{5}$$

Hence, the required coordinates of C are $\left(\frac{4}{5}, \frac{21}{5}\right)$.

25. The centre of the circle is $(2a, a - 7)$. Find the value of a of the circle which passes through the point $(11, -9)$ has diameter $10\sqrt{2}$ units.

Sol. Let O be the centre, P(11, –9) be a point on the circle. Let r be its radius.



Then $\quad r = OP$

$$= \sqrt{(2a - 11)^2 + (a - 7 + 9)^2}$$

$$= \sqrt{(2a - 11)^2 + (a + 2)^2}$$

But $\quad r = \dfrac{10\sqrt{2}}{2} = 5\sqrt{2}$

$\therefore$ We have

$$(2a - 11)^2 + (a + 2)^2 = (5\sqrt{2})^2 = 50$$

$$\Rightarrow \quad 4a^2 + 121 - 44a + a^2 + 4a + 4 = 50$$

$$\Rightarrow \quad 5a^2 - 40a + 75 = 0$$

$$\Rightarrow \quad a^2 - 8a + 15 = 0$$

$$\Rightarrow \quad (a - 5)(a - 3) = 0$$

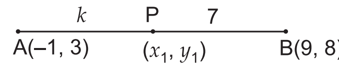
$\therefore$ Either $\quad a - 5 = 0 \Rightarrow a = 5$

or $\quad a - 3 = 0 \Rightarrow a = 3$

Hence, the required value of a is 5 or 3.

26. Point P divides the line segment joining the points A(–1, 3) and B(9, 8) such that $\frac{AP}{PB} = \frac{k}{7}$. If P lies on line $x - y + 2 = 0$, find the value of k.

Sol. Let P be the point (x_1, y_1) and P divides AB in the ratio AP : PB = k : 7.



$$\therefore \quad x_1 = \frac{9k - 7}{k + 7}$$

and $\quad y_1 = \dfrac{8k + 7 \times 3}{k + 7} = \dfrac{8k + 21}{k + 7}$

But $P(x_1, y_1)$ lies on the line $x - y + 2 = 0$.

$$\therefore \quad x_1 - y_1 + 2 = 0$$

$$\Rightarrow \quad \frac{9k - 7}{k + 7} - \frac{8k + 21}{k + 7} + 2 = 0$$

$$\Rightarrow \quad 9k - 7 - 8k - 21 + 2(k + 7) = 0$$

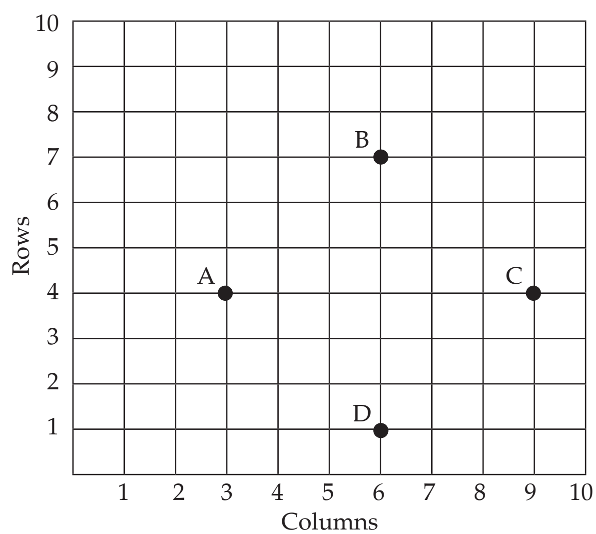
$$\Rightarrow \quad k - 28 + 2k + 14 = 0$$

$$\Rightarrow \quad 3k = 14$$

$\Rightarrow \quad k = \dfrac{14}{3}$ which is the required value of k.

27. Read the following passage and answer the questions that follows:

In a classroom, four students Sita, Gita, Rita and Anita are sitting at A(3,4), B(6,7), C(9,4), D(6,1) respectively. Then a new student Anjali joins the class.



(*a*) Teacher tells Anjali to sit in the middle of the four students. Find the coordinates of the position where she can sit.

(*b*) Calculate the distance between Sita and Anita.

(*c*) Which two students are equidistant from Gita. **[CBSE SP(Basic) 2019]**

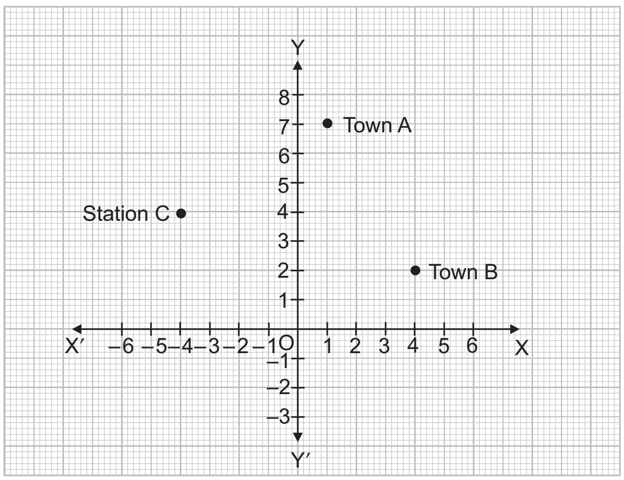
Sol. (*a*) (6, 4)

(*b*) $AD = \sqrt{(6-3)^2 + (1-4)^2}$

$= \sqrt{(3)^2 + (-3)^2} = \sqrt{9+9} = \sqrt{18}$

(*c*) Sita and Rita

28. Two friends Seema and Aditya work in the same office at Delhi. In the Christmas vacations, both decided to go to their hometowns represented by Town A and Town B respectively in the figure given below. Town A and Town B are connected by trains from the same station C (in the given figure) in Delhi. Based on the given situation, answer the following questions:



(*a*) Who will travel more distance, Seema or Aditya, to reach to their hometown?

(*b*) Seema and Aditya planned to meet at a location D situated at a point D represented by the mid-point of the line joining the points represented by Town A and Town B. Find the coordinates of the point represented by the point D. **[CBSE SP (Standard) 2019]**

Sol. (*a*) A(1, 7), B(4, 2), C(–4, 4)

Distance travelled by Seema

$= AC = \sqrt{(-4-1)^2 + (4-7)^2}$

$= \sqrt{34}$ units

Distance travelled by Aditya

$= BC = \sqrt{(-4-4)^2 + (4-2)^2}$

$= \sqrt{68}$ units

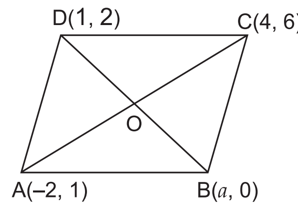
$\therefore$ Aditya travels more distance

(*b*) Coordinates of D are $\left(\dfrac{1+4}{2}, \dfrac{7+2}{2}\right) = \left(\dfrac{5}{2}, \dfrac{9}{2}\right)$

Long Answer Type Questions

29. If A(–2, 1), B(*a*, 0), C(4, *b*) and D(1, 2) are the vertices of a parallelogram ABCD, find the values of *a* and *b*. Hence find the lengths of its sides. **[CBSE 2018]**

Sol. Let O be the point of intersection of the two diagonals AC and BD of the parallelogram. Then O is the mid-point of both BD and AC. Now, the mid-point of BD is the point $\left(\dfrac{a+1}{2}, \dfrac{2}{2}\right) = \left(\dfrac{a+1}{2}, 1\right)$ and the mid-point of AC is the point $\left(\dfrac{4-2}{2}, \dfrac{b+2}{2}\right) = \left(1, \dfrac{b+1}{2}\right)$.



$\therefore \quad \dfrac{a+1}{2} = 1$

$\Rightarrow \quad a = 1$

and $\quad \dfrac{b+1}{2} = 2$

$\Rightarrow \quad b = 1$

$\therefore$ AB = distance between (–2, 1) and (1, 0)

$= \sqrt{(1+2)^2 + 1^2}$ units

$= \sqrt{10}$ units

DC = distance between (1, 2) and (4, 1)

$= \sqrt{3^2 + 1^2}$ units

$= \sqrt{10}$ units

Also, BC = distance between (1, 0) and (4, 1)

$= \sqrt{(4-1)^2 + 1^2}$ units

$= \sqrt{10}$ units

AD = distance between (–2, 1) and (1, 2)

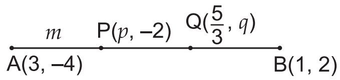
$= \sqrt{(1+2)^2 + (2-1)^2}$ units

$= \sqrt{10}$ units

$\therefore$ Each side is of length $\sqrt{10}$ units and $a = 1$, $b = 1$.

30. The line segment joining the points (3, –4) and (1, 2) is trisected at the points P and Q. If the coordinates of P and Q are $(p, -2)$ and $\left(\frac{5}{3}, q\right)$ respectively, find the value of p. **[CBSE 2005]**

Sol.



P divides AB in the ratio AP : PB = 1 : 2

and Q divides AB is the ratio AQ : QB = 2 : 1.

$\therefore \quad p = \dfrac{1 \times 1 + 2 \times 3}{2+1} = \dfrac{7}{3}$

$q = \dfrac{2 \times 2 - 4 \times 1}{2+1} = \dfrac{0}{3} = 0$

$\therefore$ The required values of p and q are $\frac{7}{3}$ and 0 respectively.

31. Find the circumcentre of the triangle whose vertices are (–2, –3), (–1, 0) and (7, –6).

Sol. Let $P(x_1, y_1)$ be the coordinates of the circumcentre of the triangle. Let A, B and C be the points (–2, –3), (–1, 0) and (7, –6) respectively. Then, PA = PB = PC

$\Rightarrow \quad PA^2 = PB^2 = PC^2$

$\Rightarrow \quad (x_1+2)^2 + (y_1+3)^2 = (x_1+1)^2 + y_1^2$

$= (x_1 - 7)^2 + (y_1 + 6)^2$

$\Rightarrow \quad x_1^2 + y_1^2 + 4x_1 + 6y_1 + 13 = x_1^2 + y_1^2 + 2x_1 + 1$

$= x_1^2 + y_1^2 - 14x_1 + 12y_1 + 85$

$\Rightarrow \quad 4x_1 + 6y_1 + 13 = 2x_1 + 1 = 12y_1 - 14x_1 + 85.$

$\therefore$ We have

$4x_1 + 6y_1 + 13 = 2x_1 + 1$

$\Rightarrow \quad 2x_1 + 6y_1 + 12 = 0$

$\Rightarrow \quad x_1 + 3y_1 + 6 = 0 \quad \ldots(1)$

and $\quad 12y_1 - 14x_1 + 85 - 2x_1 - 1 = 0$

$\Rightarrow \quad -16x_1 + 12y_1 + 84 = 0$

$\Rightarrow \quad 4x_1 - 3y_1 - 21 = 0 \quad \ldots(2)$

Adding (1) and (2), we get

$5x_1 - 15 = 0$

$\Rightarrow \quad x_1 = 3$

$\therefore$ From (1), $\quad 3 + 3y_1 + 6 = 0$

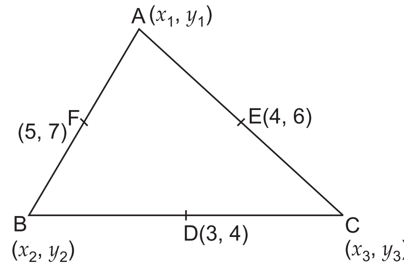
$\Rightarrow \quad 3y_1 = -9$

$\Rightarrow \quad y_1 = -3$

$\therefore$ The required coordinates of the circumcentre are (3, –3).

32. If the coordinates of the mid-points of the sides of a triangle are (3, 4), (4, 6) and (5, 7), find its vertices. **[CBSE 2008]**

Sol. Let (x_1, y_1), (x_2, y_2) and (x_3, y_3) be the coordinates of the vertices A, B and C of ΔABC respectively and let D, E and F be the mid-points of BC, CA and AB respectively with coordinates.



D : (3, 4), E : (4, 6) and F : (5, 7)

$\therefore \quad 3 = \dfrac{x_2 + x_3}{2}$

$\Rightarrow \quad x_2 + x_3 = 6 \quad \ldots(1)$

Similarly, $\quad 4 = \dfrac{y_2 + y_3}{2}$

$\Rightarrow \quad y_2 + y_3 = 8 \quad \ldots(2)$

Similarly, $\quad x_3 + x_1 = 8 \quad \ldots(3)$

$y_3 + y_1 = 12 \quad \ldots(4)$

$x_1 + x_2 = 10 \quad \ldots(5)$

$y_1 + y_2 = 14 \quad \ldots(6)$

Adding (1), (3) and (5), we get $2(x_1 + x_2 + x_3) = 24$

Subtracting (1), (3) and (5) successively from (7), we get $x_1 = 6$, $x_2 = 4$ and $x_3 = 2$.

Similarly adding (2), (4) and (6), we get

$2(y_1 + y_2 + y_3) = 34$

$\Rightarrow \quad y_1 + y_2 + y_3 = 17 \quad \ldots(8)$

Subtracting (2), (4) and (6) successively from (8), we get $y_1 = 9$, $y_2 = 5$, $y_3 = 3$.

Hence, the required coordinates of D, E and F are respectively (6, 9), (4, 5) and (2, 3).

Let's Compete

(Page 145)

Multiple-Choice Questions

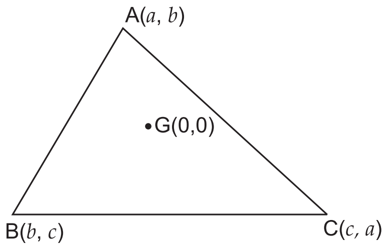
1. If centroid of a triangle formed by points (a, b), (b, c) and (c, a) is at (0, 0), then $a^3 + b^3 + c^3$ equals to

(*a*) abc (*b*) $a + b + c$

(*c*) $3abc$ (*d*) 0

Sol. (*c*) $3abc$

The centroid of ΔABC is $\left(\frac{a+b+c}{3}, \frac{b+c+a}{3}\right)$



$\therefore \quad a + b + c = 0$

$\therefore \quad a^3 + b^3 + c^3 - 3abc$

$= (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

$= 0$

$\therefore \quad a^3 + b^3 + c^3 = 3abc$

$\therefore$ (c) is correct.

2. The point which lies on circle of radius 6 cm and centre (3, 5) is

(a) (2, 4) (b) (3, 4)

(c) (–3, 5) (d) (–2, 4)

Sol. (c) (–3, 5)

If (x, y) be any points on the circle, then

$$(x - 3)^2 + (y - 5)^2 = 6^2$$

which is clearly satisfied by $x = -3$, $y = 5$

$\therefore$ (c) is the correct solution.

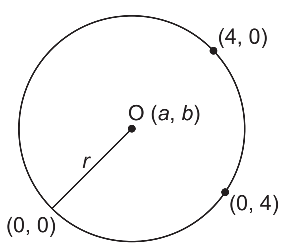
3. Find the radius of the circle which passes through the origin, (0, 4) and (4, 0).

(a) 2 (b) $4\sqrt{2}$

(c) $\sqrt{8}$ (d) $3\sqrt{2}$

Sol. (c) $\sqrt{8}$

Let coordinates of the centre of the circle be (a, b) and r be the radius of the circle.



$$\sqrt{(a-0)^2 + (b-0)^2} = r$$

$$\Rightarrow \quad \sqrt{a^2 + b^2} = r$$

$$\Rightarrow \quad a^2 + b^2 = r^2 \quad \text{...(1)}$$

Distance from (a, b) to (4, 0)

$$\sqrt{(4-a)^2 + (0-b)^2} = r$$

$$\sqrt{(4-a)^2 + b^2} = r$$

$$\Rightarrow \quad (4 - a)^2 + b^2 = r^2$$

$$\Rightarrow \quad 16 - 8a + a^2 + b^2 = r^2$$

$$\Rightarrow \quad 16 - 8a + a^2 + b^2 = r^2$$

$$\Rightarrow \quad 16 - 8a + r^2 = r^2 \quad \text{[From (1)]}$$

$$\Rightarrow \quad 8a = 16$$

$$\Rightarrow \quad a = 2$$

Distance from (a, b) to (0, 4)

$$\Rightarrow \quad \sqrt{(0-a)^2 + (4-b)^2} = r$$

$$\Rightarrow \quad \sqrt{a^2 + (4-b)^2} = r$$

$$\Rightarrow \quad a^2 + (4 - b) = r^2$$

$$\Rightarrow \quad a^2 + 16 - 8b + b^2 = r^2$$

$$\Rightarrow \quad a^2 + b^2 + 16 - 8b = r^2$$

$$\Rightarrow \quad r^2 + 16 - 8b = r^2$$

$$\Rightarrow \quad 8b = 16$$

$$\Rightarrow \quad b = 2$$

$\therefore$ Radius of the circle $= \sqrt{a^2 + b^2}$ units

$= \sqrt{4 + 4}$ units

$= \sqrt{8}$ units

4. If $k\left(\frac{a}{3}, 4\right)$ is the mid-point of the line segment joining the points Q(–6, 5) and R(–2, 3), then the value of a is

(a) –8 (b) –12

(c) 12 (d) –4

Sol. (b) –12

We have

$$\frac{a}{3} = \frac{-6-2}{2} = -4$$

$\therefore \quad a = -12$

5. The ratio in which the line segment joining the points A(a_1, b_1) and B(a_2, b_2) is divided by y-axis is

(a) $-b_1 : b_2$ (b) $b_1 : b_2$

(c) $a_1 : a_2$ (d) $-a_1 : a_2$

Sol. (d) $-a_1 : a_2$

The coordinates of the point which divides. AB in the ratio $k : 1$ are $\left(\frac{ka_2 + a_1}{k+1}, \frac{kb_2 + b_1}{k+1}\right)$.

$\because$ This points lies on the y-axis its x-coordinate $= 0$

$$ka_2 + a_1 = 0$$

$\Rightarrow \quad k = \dfrac{-a_1}{a_2}$

$\therefore \quad k : 1 = -a_1 : a_2$

$\therefore$ The required ratio is $-a_1 : a_2$

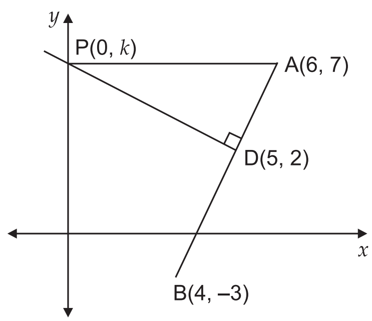
6. The coordinates of a point on y-axis which lies on the perpendicular bisector of line segment joining the points (6, 7) and (4, –3) are

(*a*) (0, 3) (*b*) (0, –3)

(*c*) (0, 2) (*d*) (0, –2)

Sol. (*a*) (0, 3)

Let A (6, 7) and B(4, –3) be the given points and P(0, k) be a point on the y-axis such that PD is the perpendicular bisector of AB. Hence, D is a point on AB such that AD = BD and ∠PDA = 90°.



Now, D is the mid-point of AB.

$\therefore$ The coordinates of D are $\left(\dfrac{6+4}{2}, \dfrac{7-3}{2}\right) = (5, 2)$

Now, from ΔPDA,

$\because \quad \angle PDA = 90°$

$\therefore$ By Pythagoras' theorem, we have

$$PA^2 = PD^2 + AD^2$$

$\Rightarrow \quad 6^2 + (7-k)^2 = 5^2 + (2-k)^2 + (5-6)^2 + (2-7)^2$

$\Rightarrow \quad 36 + 49 + k^2 - 14k = 25 + 4 + k^2 - 4k + 1 + 25$

$\Rightarrow \quad 10k + 55 - 85 = 0$

$\Rightarrow \quad 10k - 30 = 0$

$\Rightarrow \quad k = 3$

$\therefore$ The required coordinates are (0, 3).

7. The opposite vertices of a square are (–1, 2) and (3, 2), then the coordinates of other two vertices are

(*a*) (0, 1) and (1, 4) (*b*) (1, 0) and (1, 4)

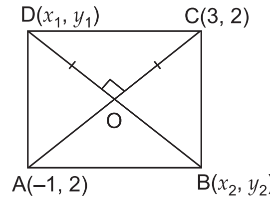
(*c*) (0, 1) and (4, 1) (*d*) (1, 0) and (4, 1)

Sol. (*b*) (1, 0) and (1, 4)

Let A(–1, 2) and C(3, 2) be the opposite vertices of the square ABCD. Then the two diagonals AC and BD will bisect each other at a point, say O. Then the coordinates of O are $\left(\dfrac{1}{2}(3-1), \dfrac{1}{2}(2+2)\right)$

$= (1, 2)$

Let the coordinates of the other two vertices be D(x_1, y_1) and B(x_2, y_2).



Then $\quad \dfrac{1}{2}(x_1 + x_2) = 1$

$\Rightarrow \quad x_1 + x_2 = 2 \quad \ldots(1)$

Also, $\quad CD^2 = AD^2$

$\Rightarrow \quad (x_1 - 3)^2 + (y_1 - 2)^2 = (x_1 + 1)^2 + (y_1 - 2)^2$

$\Rightarrow \quad x_1^2 - 6x_1 + 9 = x_1^2 + 2x_1 + 1$

$\Rightarrow \quad 8x_1 - 8 = 0$

$\Rightarrow \quad x_1 = 1 \quad \ldots(2)$

$\therefore$ From (2), $\quad x_2 = 2 - 1 = 1 \quad \ldots(3)$

Now $\quad OC^2 = OD^2$

$\Rightarrow \quad (3-1)^2 + (2-2)^2 = (x_1 - 1)^2 + (y_1 - 2)^2$

$\Rightarrow \quad 4 = (y_1 - 2)^2 \quad$ [From (2)]

$\therefore \quad y_1 - 2 = \pm 2$

$\Rightarrow \quad y_1 = 2 \pm 2 = 4, 0$

$\therefore \quad y_1 = 4, y_2 = 0$

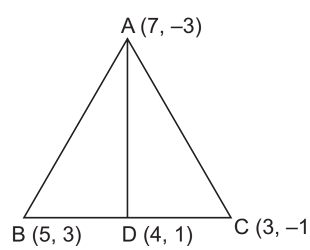
$\therefore$ The required points are (1, 4) and (1, 0).

8. The length of the median AD of a triangle formed by A(7, –3), B(5, 3) and C(3, –1) is

(*a*) 3 units (*b*) 7 units

(*c*) 5 units (*d*) 10 units

Sol. (*c*) 5 units



D is the mid-point of BC.

Coordinates of point D $= \left(\dfrac{5+3}{2}, \dfrac{3-1}{2}\right)$

$= (4, 1)$

Length of the median

$$AD = \sqrt{(7-4)^2 + (-3-1)^2} \text{ units}$$
$$= \sqrt{(3)^2 + (-4)^2} \text{ units}$$
$$= \sqrt{9+16} \text{ units}$$
$$= \sqrt{25} \text{ units}$$
$$= 5 \text{ units}$$

9. The point A which divides the line segment joining the points P(1, –4) and Q(3, 5) in ratio 3 : 2 internally lies in the

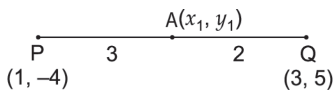
(*a*) I quadrant (*b*) II quadrant
(*c*) III quadrant (*d*) IV quadrant

Sol. (*a*) I quadrant

Since A divides PQ internally in the ratio. PA : AQ = 3 : 2, hence if (x_1, y_1) be the coordinates of A,

then $x_1 = \dfrac{3 \times 3 + 2 \times 1}{3+2} = \dfrac{9+2}{5} = \dfrac{11}{5}$

and $y_1 = \dfrac{3 \times 5 - 4 \times 2}{3+2} = \dfrac{15-8}{5} = \dfrac{7}{5}$



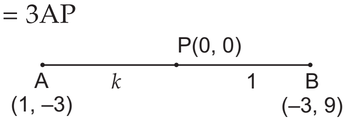
$\therefore$ The point $A\left(\dfrac{11}{5}, \dfrac{7}{5}\right)$ lies in the first quadrant.

10. If the point P(0, 0) lies on the line segment joining the points A(1, −3) and B(−3, 9), then

(*a*) $PB = \dfrac{1}{3}AP$ (*b*) PB = 2AP

(*c*) $PB = \dfrac{1}{2}AP$ (*d*) PB = 3AP

Sol. (*d*) PB = 3AP



Let P(0, 0) divides AB in the ratio AP : PB = k : 1

$\therefore \quad 0 = \dfrac{-3k+1}{k+1}$

$\Rightarrow \quad k = \dfrac{1}{3}$

$\therefore \quad AP : PB = \dfrac{1}{3} : 1 = 1 : 3$

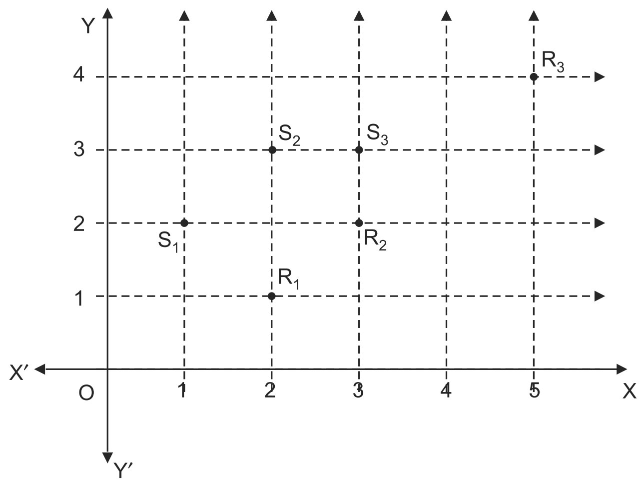
$\therefore \quad \dfrac{AP}{PB} = \dfrac{1}{3}$

$\Rightarrow \quad PB = 3AP$

Life Skills

(Page 145)

1. Riya and Sohan planted some trees in their garden as shown in the figure and both arguing that they planted them in a straight line.



Find who is correct? 'R' stands for Riya, 'S' for Sohan.

Sol. We see that the coordinates of R_1, R_2 and R_3 are (2, 1), (3, 2) and (5, 4) respectively and the coordinates of S_1, S_2 and S_3 are (1, 2), (2, 3) and (3, 3) respectively.

$\therefore$ Area of $\Delta R_1R_2R_3$

$$= \left|\frac{1}{2}\left[2(2-4) + 3(4-1) + 5(1-2)\right]\right| \text{ sq units}$$
$$= \left|\frac{1}{2}(-4+9-5)\right| \text{ sq units}$$
$$= 0 \text{ sq units}$$

and Area of $\Delta S_1S_2S_3$

$$= \left|\frac{1}{2}\left[1(3-3) + 2(3-2) + 3(2-3)\right]\right| \text{ sq units}$$
$$= \left|\frac{1}{2}(2-3)\right| \text{ sq units}$$
$$= \frac{1}{2} \text{ sq units}$$

$\therefore$ R_1, R_2 and R_3 are in a straight line, but S_1, S_2 and S_3 are not in a straight line.

Hence, only Riya planted the trees in a straight line, but not Sohan.