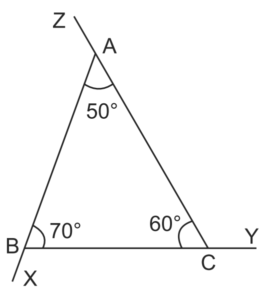
6

Triangles

Checkpoint (Page 107)

1. Name the interior angles of the triangle ABC given below. Also, write the measurements of the interior angles.



Sol. The interior angles of the triangle are

$\angle ABC = 70°$, $\angle BCA = 60°$ and $\angle CAB = 50°$.

2. What is the sum of

(*a*) the three interior angles of a triangle?

(*b*) the three exterior angles of a triangle?

Sol. (*a*) The sum of the three interior angles of a triangle is 180°, by angle sum property of the triangle.

(*b*) The sum of the three exterior angles of a triangle is 360°.

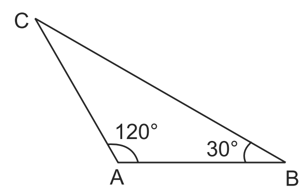
3. Can you draw a triangle ABC with sides AB = 3 cm, BC = 4 cm and CA = 8 cm? If not, explain why.

Sol. No, since in this case AB + BC = (3 + 4) cm = 7 cm < CA. This is because the sum of any two sides of a triangle is greater than its third side.

4. What is the relation between the three sides AB, BC and CA of an equilateral triangle ABC?

Sol. For an equilateral triangle, all the three sides are of equal length, hence, AB = BC = CA.

5. In ΔABC, if $\angle CAB = 120°$ and $\angle ABC = 30°$,



(*a*) what is the measurement of $\angle ACB$?

(*b*) what is the relation between the sides AB and AC?

(*c*) what kind of triangle is it – (*i*) an equilateral triangle, (*ii*) an isosceles triangle, (*iii*) a scalene triangle, (*iv*) an obtuse-angled triangle, (*v*) an acute-angled triangle or (*vi*) a right-angled triangle?

Sol. (*a*) By angle sum property of a triangle, we know that

$$\angle ABC + \angle CAB + \angle ACB = 180°$$
$$\Rightarrow \quad 30° + 120° + \angle ACB = 180°$$
$$\Rightarrow \quad \angle ACB = 180° - 150° = 30°$$

which is the required measurement of $\angle ACB$.

(*b*) Since $\angle ABC = \angle ACB = 30°$

$$\therefore \quad AB = AC$$

which defines their relation.

(*c*) (*i*) No, since for an equilateral triangle, all the three sides are of equal length.

(*ii*) Yes, since the two base angles are equal, hence it is an isosceles triangle.

(*iii*) No, since the length of three sides are unequal.

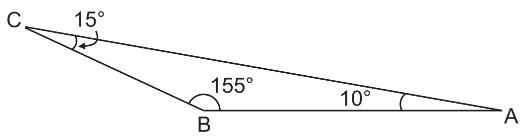
(*iv*) Yes, since one angle is 120° which is obtuse, hence it is an obtuse-angled triangle.

(*v*) No, since all the angles of this triangle are not acute.

(*vi*) No, since no angle of this triangle is of 90°.

6. If in ΔABC, $\angle BAC = 10°$ and $\angle ACB = 15°$, what is the relation between the sides AB, BC and AC of ΔABC?

Sol.



In ΔABC,

$$\angle BAC + \angle ACB + \angle ABC = 180°$$

[By angle sum property of a triangle]

$$\Rightarrow \quad 10° + 15° + \angle ABC = 180°$$

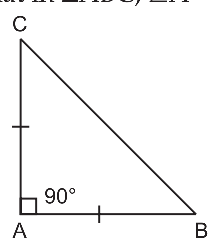
$$\Rightarrow \quad \angle ABC = 180° - 25° = 155°$$

Since $\angle BAC < \angle ACB < \angle ABC$,

$\therefore$ BC < AB < AC [since the side opposite to the smaller angle of a triangle is less than the side opposite to the greater angle]

7. In a right triangle ABC, $\angle A = 90°$ and ΔABC is also an isosceles triangle with AB = AC, what are the measures of $\angle ABC$ and $\angle ACB$?

Sol. It is given that in ΔABC, $\angle A = 90°$ and AB = AC.



$$\therefore \quad \angle ABC = \angle ACB = \theta \text{ (say)}$$

Since $\angle A + \angle B + \angle C = 180°$

[By angle sum property of a triangle]

$$\Rightarrow \quad 90° + \theta + \theta = 180°$$

$$\Rightarrow \quad 2\theta = 180° - 90° = 90°$$

$$\Rightarrow \quad \theta = \frac{90°}{2} = 45°$$

$$\therefore \quad \angle ABC = \angle ACB = 45°$$

are the required measures of the angles.

8. Can you construct a unique triangle ABC with

(*a*) $\angle ABC = 95°$, $\angle ACB = 69°$ and $\angle BAC = 16°$?

(*b*) $\angle ABC = 70°$, $\angle ACB = 80°$ and $\angle BAC = 40°$?

Explain why.

Sol. (*a*) No, since when three angles are given, we can draw infinite number of similar triangles, although the sum of three angles in each case is 180°.

(*b*) No, since $\angle ABC + \angle ACB + \angle BAC = 70° + 80° + 40° = 190° > 180°$.

Hence, in this case, not a single triangle can be constructed.

9. In each of the following problems, the measures of the three angles of a triangle are given. In each case, identify the type of triangle on the basis of three angles.

(*a*) 60°, 60°, 60° (*b*) 75°, 15°, 90°

(*c*) 130°, 20°, 30° (*d*) 45°, 95°, 40°

Sol. (*a*) Since each angle is acute being less than 90°, hence, the triangle is an acute-angled triangle.

(*b*) In this case, one of the angles is 90° which is a right angle. Hence, the triangle is a right-angled triangle.

(*c*) In this case, one of the angles is 130° which is obtuse, since it greater than 90°. Hence, the triangle is obtuse-angled triangle.

(*d*) In this case, one of the angle is 95° which is also obtuse, since it is greater than 90°. Hence, the triangle is obtuse-angled triangle.

10. Find the angles of a triangle, which are in the ratio 3 : 5 : 10.

Sol. Let the three angles be $(3k)°$, $(5k)°$ and $(10k)°$ where k is a non-zero positive constant.

$\therefore$ By angle sum property of a triangle, we have

$$3k + 5k + 10k = 180$$

$$\Rightarrow \quad 18k = 180$$

$$\Rightarrow \quad k = \frac{180}{18} = 10$$

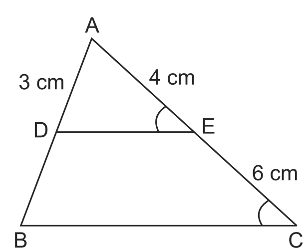
Hence, the required angles are $(3 \times 10)°$, $(5 \times 10)°$ and $(10 \times 10)°$, i.e. 30°, 50° and 100°.

Check Your Progress 1

(Page 112)

Multiple-Choice Questions

1. In the given figure, if $\angle AED = \angle ACB$, then DB is equal to



(*a*) 4.5 cm (*b*) 4 cm
(*c*) 3 cm (*d*) 3.5 cm

Sol. (*a*) 4.5 cm

Given that D and E are two points on AB and AC of ΔABC, respectively such that ∠AED = ∠ACB.

Also, AD = 3 cm, AE = 4 cm, EC = 6 cm.

To find DB.

Given that ∠AED = ∠ACB. But these two are corresponding angles with respect to line segments DE and BC and their transversal AC. Hence, DE ∥ BC.

By the basic proportionality theorem, we have

$$\frac{AD}{DB} = \frac{AE}{EC}$$

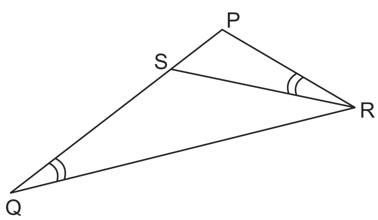
$$\Rightarrow \quad \frac{3}{DB} = \frac{4}{6} = \frac{2}{3}$$

$$\Rightarrow \quad 2DB = 9$$

$$\Rightarrow \quad DB = \frac{9}{2} = 4.5$$

Hence, the required length of DB is 4.5 cm.

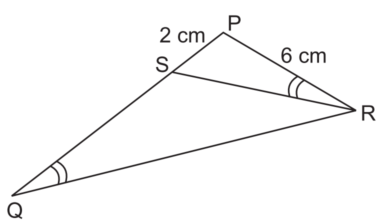
2. In the given figure, ∠PQR = ∠PRS. If PR = 6 cm and PS = 2 cm, then PQ is equal to



(*a*) 12 cm (*b*) 16 cm
(*c*) 18 cm (*d*) 20 cm

Sol. (*c*) 18 cm

Given that S is a point on the side PQ of a triangle PQR such that ∠PRS = ∠PQR.



Also, PR = 6 cm and PS = 2 cm.

To find the length of PQ.

In ΔPQR and ΔPRS, we have

∠PQR = ∠PRS [Given]

∠QPR = ∠RPS [Common]

By AA criterion of similarity, we have

∴ ΔPQR ~ ΔPRS

$$\Rightarrow \quad \frac{PR}{PS} = \frac{QR}{RS} = \frac{PQ}{PR} \quad \text{[By BPT]...(1)}$$

Now, from (1), we get

$$\frac{PR}{PS} = \frac{PQ}{PR}$$

$$\Rightarrow \quad \frac{6}{2} = \frac{PQ}{6}$$

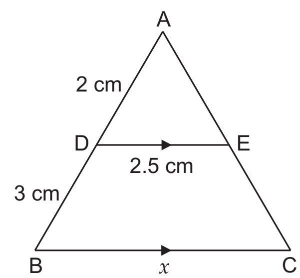
$$\Rightarrow \quad 2PQ = 36$$

$$\Rightarrow \quad PQ = \frac{36}{2}$$

$$= 18$$

∴ Required length of PQ is 18 cm.

3. In the given figure, AD = 2 cm, DB = 3 cm, DE = 2.5 cm and DE ∥ BC. The value of x is



(*a*) 6 cm (*b*) 3.75 cm
(*c*) 6.25 cm (*d*) 7.5 cm

[CBSE 2023 Basic]

Sol. (*c*) 6.25 cm

In ΔADE and ΔABC,

∠ADE = ∠ABC [Corresponding angles]

∠AED = ∠ACB [Corresponding angles]

∴ By AA criterion of similarity,

ΔADE ~ ΔABC

Corresponding sides of similar triangles are proportional,

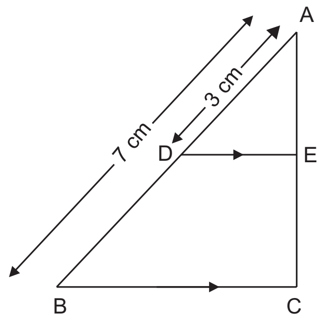
$$\frac{AD}{AB} = \frac{DE}{BC} = \frac{AE}{AC}$$

$$\frac{2}{2+3} = \frac{2.5}{BC}$$

$$\Rightarrow \quad x = 2.5 \times \frac{5}{2}$$

$$= 6.25 \text{ cm}$$

4. In the given figure, DE ∥ BC. If AD = 3 cm, AB = 7 cm and EC = 3 cm, then the length of AE is



(*a*) 2 cm (*b*) 2.25 cm

(*c*) 3.5 cm (*d*) 4 cm

[CBSE 2023 Standard]

Sol. (*b*) 2.25 cm

In ΔABC

$$AD = 3 \text{ cm}$$

$$AB = 7 \text{ cm}$$

$$\therefore \quad BD = AB - AD = 7 \text{ cm} - 3 \text{ cm} = 4 \text{ cm}$$

Let $AE = x$

By Basic Proportionality Theorem,

$$\frac{AD}{DB} = \frac{AE}{EC}$$

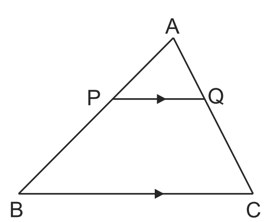
$$\Rightarrow \quad \frac{3}{4} = \frac{x}{3}$$

$$\Rightarrow \quad x = \frac{3}{4} \times 3$$

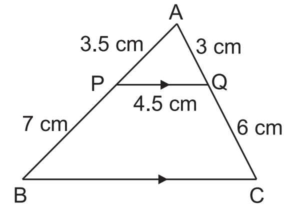
$$\Rightarrow \quad x = 2.25 \text{ cm}$$

Very Short Answer Type Questions

5. In the given figure, P and Q are points on the sides AB and AC respectively of ΔABC such that AP = 3.5 cm, PB = 7 cm, AQ = 3 cm, QC = 6 cm and PQ = 4.5 cm, find BC. **[CBSE 2008]**



Sol. Given that P and Q are two points on the sides AB and AC respectively of ΔABC. Also, AP = 3.5 cm, PB = 7 cm, AQ = 3 cm, QC = 6 cm and PQ = 4.5 cm.



To find the length of BC.

We see that $\dfrac{AP}{PB} = \dfrac{3.5}{7} = \dfrac{1}{2}$

and $\dfrac{AB}{QC} = \dfrac{3}{6} = \dfrac{1}{2}$

$$\therefore \quad \frac{AP}{PB} = \frac{AQ}{QC}$$

∴ By the converse of basic proportionality theorem, we have PQ ∥ BC.

∴ ∠AQP = corresponding ∠ACB

and ∠APQ = corresponding ∠ABC

∴ $\Delta APQ \sim \Delta ABC$ [By AA similarity]

$$\Rightarrow \quad \frac{AP}{AB} = \frac{AQ}{AC} = \frac{PQ}{BC} \quad \text{[By BPT]}$$

$$\Rightarrow \quad \frac{3.5}{3.5+7} = \frac{3}{3+6} = \frac{4.5}{BC}$$

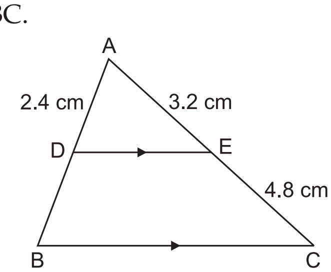
$$\Rightarrow \quad \frac{1}{3} = \frac{4.5}{BC}$$

$$\Rightarrow \quad BC = 4.5 \times 3 = 13.5$$

Hence, the required length of BC is 13.5 cm.

6. In ΔABC, D and E are the points on the sides AB and AC respectively such that DE ∥ BC. If AD = 2.4 cm, AE = 3.2 cm and CE = 4.8 cm, find BD. **[CBSE SP 2011]**

Sol. Given that D and E are two points on the sides AB and AC respectively of ΔABC such that DE ∥ BC.



∴ By basic proportionality theorem, we have

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\Rightarrow \quad \frac{2.4}{DB} = \frac{3.2}{4.8}$$

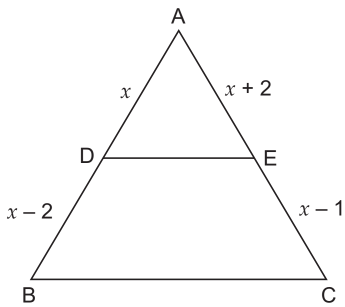
$$\Rightarrow \quad \frac{2.4}{DB} = \frac{2}{3}$$

$\Rightarrow \quad 2DB = 2.4 \times 3 = 7.2$

$\Rightarrow \quad DB = \dfrac{7.2}{2} = 3.6$

Hence, the required length of DB is 3.6 cm.

7. In the given figure, ABC is a triangle in which DE ∥ BC. If AD = x, DB = $x - 2$, AE = $x + 2$ and EC = $x - 1$, then find the value of x.



[CBSE 2023 Standard]

Sol. DE ∥ BC [Given]

By Basin Proportionality Theorem,

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\frac{x}{x-2} = \frac{x+2}{x-1}$$

$$x(x-1) = (x-2)(x+2)$$

$$x^2 - x = x^2 - 4$$

$$-x = -4$$

$$x = 4$$

8. In ΔABC, D and E are the points on the sides AB and AC respectively such that AD × EC = AE × DB. Prove that DE ∥ BC.

[CBSE SP 2010, 2011]

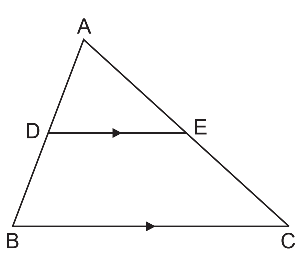
Sol. Given that D and E are the points on the sides AB and AC respectively such that AD × EC = AE × DB.

To prove that DE ∥ BC.

We have

$$AD \times EC = AE \times DB$$

$$\Rightarrow \quad \frac{AD}{DB} = \frac{AE}{EC}$$



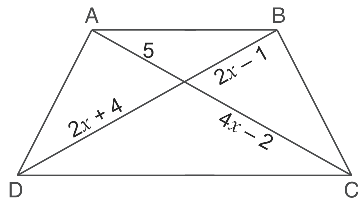
$\therefore$ By the converse of basic proportionality theorem, we get

$$DE \parallel BC$$

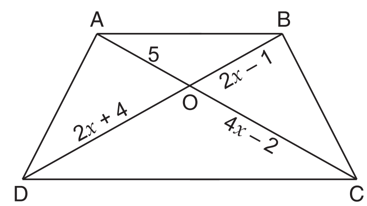
Hence, proved.

9. In the given figure, AB ∥ DC. Find the value of x.

[CBSE SP 2010]



Sol. Given that ABCD is a trapezium in which AB ∥ DC. Let the two diagonals AC and DB intersect each other at O such that AO = 5, CO = $4x - 2$, BO = $2x - 1$ and DO = $2x + 4$. To find the value of x.



In ΔAOB and ΔCOD, we have

$\angle BAO = \angle DCO$ [Alternate angles, $\because$ AB ∥ DC]

$\angle ABO = \angle CDO$ [Alternate $\angle$s]

$\therefore \quad \Delta AOB \sim \Delta COD$ [By AA similarity]

$$\Rightarrow \quad \frac{AO}{CO} = \frac{BO}{DO}$$

$$\Rightarrow \quad \frac{5}{4x-2} = \frac{2x-1}{2x+4}$$

$$\Rightarrow \quad \frac{5}{2(2x-1)} = \frac{2x-1}{2(x+2)}$$

$$\Rightarrow \quad \frac{5}{2x-1} = \frac{2x-1}{x+2}$$

$$\Rightarrow \quad 5x + 10 = (2x-1)^2 = 4x^2 - 4x + 1$$

$$\Rightarrow \quad 4x^2 - 9x - 9 = 0$$

$$\Rightarrow \quad 4x^2 + 3x - 12x - 9 = 0$$

$$\Rightarrow \quad x(4x+3) - 3(4x+3) = 0$$

$$\Rightarrow \quad (x-3)(4x+3) = 0$$

$\therefore$ Either $\quad x - 3 = 0$

$\Rightarrow \quad x = 3$

or $\quad 4x + 3 = 0$

$\Rightarrow \quad x = -\dfrac{3}{4}$

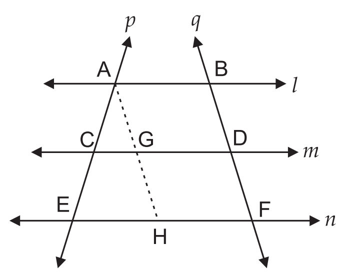
Now, when $x = -\frac{3}{4}$, OB = $2x - 1$ becomes negative which is not possible.

$\therefore$ $x = 3$ which is the required value.

Short Answer Type Questions

10. If three or more parallel lines are intersected by two transversals, prove that the intercepts made by them on the transversals are proportional.

Sol. Given that l, m, n are three lines parallel to each other and these lines are intersected by two transversals p and q at A, C, E and B, D, F respectively forming the intercepts AC, CE, BD and DF.



To prove that

$$\frac{AC}{CE} = \frac{BD}{DF}$$

Construction: Draw the line segment AH parallel to the line q meeting the lines m and n at G and H respectively.

$\because$ AB $\parallel$ GD [Given]

and AG $\parallel$ BD [By construction]

Hence, the figure ABDG is a parallelogram.

$\therefore$ AG = BD ...(1)

Again, GD $\parallel$ HF [Given]

and GH $\parallel$ DF [By construction]

$\therefore$ GDFH is a parallelogram.

$\therefore$ GH = DF ...(2)

Now, in ΔAEH, CG $\parallel$ EH

$\therefore$ By basic proportionality theorem, we have

$$\frac{AC}{CE} = \frac{AG}{GH} = \frac{BD}{DF}$$

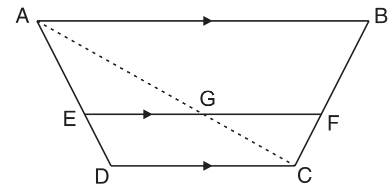
[From (1) and (2)]

$$\Rightarrow \quad \frac{AC}{CE} = \frac{BD}{DF}$$

Hence, proved.

11. Prove that any line parallel to parallel sides of a trapezium divides the non-parallel sides proportionally. **[CBSE SP 2003, 2006, 2010]**

Sol. Given that ABCD is a trapezium with AB $\parallel$ DC. EF is a line segment parallel to AB and DC. To prove that $\frac{AE}{ED} = \frac{BF}{FC}$.



Construction: We join AC intersecting EF at G.

In ΔADC, we have

EG $\parallel$ DC

$\therefore$ By basic proportionality theorem, we have

$$\frac{AE}{ED} = \frac{AG}{GC} \quad ...(1)$$

Again, in ΔACB, we have

GF $\parallel$ AB

$\therefore$ By basic proportionality theorem, we have

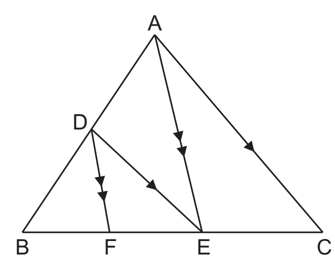
$$\frac{AG}{GC} = \frac{BF}{FC} \quad ...(2)$$

$\therefore$ From (1) and (2), we have

$$\frac{AE}{ED} = \frac{BF}{FC}$$

Hence, proved.

12. In the given figure, DE $\parallel$ AC and DF $\parallel$ AE.



Prove that $\frac{BF}{FE} = \frac{BE}{EC}$ **[CBSE 2023 Basic]**

Sol. In ΔABC,

AC $\parallel$ DE [Given]

By Basic Proportionality Theorem,

$$\frac{AD}{DB} = \frac{EC}{BE} \quad ...(1)$$

In ΔABE,

AE $\parallel$ DF [Given]

By Basin Proportional Theorem,

$$\frac{AD}{DB} = \frac{FE}{BF} \quad ...(2)$$

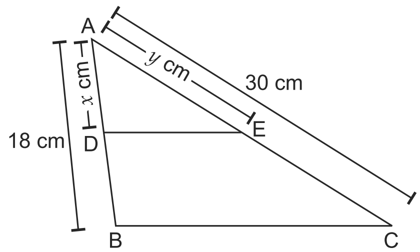
From (1) and (2)

$$\frac{FE}{BF} = \frac{EC}{BE}$$

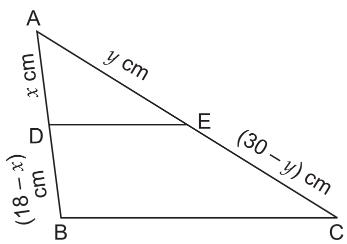
$$\Rightarrow \quad \frac{BF}{FE} = \frac{BE}{EC}$$

Long Answer Type Questions

13. In the given figure, D is a point on AB and E is a point on AC of a triangle ABC such that DE ∥ BC. If AD : DB = 1 : 2, AB = 18 cm and AC = 30 cm, find AE, EC, AD and DB.



Sol. Given that D and E are points on the sides AB and AC respectively of a triangle ABC such that DE ∥ BC.



Also, $\frac{AD}{DB} = \frac{1}{2}$, AB = 18 cm and AC = 30 cm.

Let AD = x cm and AE = y cm

Then from the figure,

$$DB = (18 - x) \text{ cm}$$

and $$EC = (30 - y) \text{ cm}$$

To find AE, EC, AD and DB.

In ΔABC, DE ∥ BC [Given]

∴ By basic proportionality theorem, we have

$$\frac{AE}{EC} = \frac{AD}{DB} = \frac{1}{2} \quad \text{[Given]}$$

$$\Rightarrow \quad \frac{y}{30-y} = \frac{x}{18-x} = \frac{1}{2} \quad \ldots(1)$$

∴ From (1), we have

$$30 - y = 2y$$

$$\Rightarrow \quad 3y = 30$$

$$\Rightarrow \quad y = \frac{30}{3} = 10$$

and $$18 - x = 2x$$

$$\Rightarrow \quad 3x = 18$$

$$\Rightarrow \quad x = \frac{18}{3} = 6$$

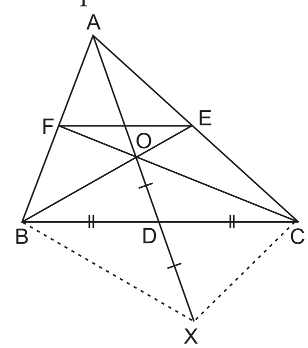
Hence, AE = y cm = 10 cm, EC = $(30 - y)$ cm = (30 – 10) cm = 20 cm, AD = x cm = 6 cm and DB = $(18 - x)$ cm = (18 – 6) cm = 12 cm.

∴ Required length of AE, EC, AD and DB are 10 cm, 20 cm, 6 cm and 12 cm respectively.

14. The side BC of ΔABC is bisected at D. O is any point on AD. BO and CO are produced to meet AC and AB at E and F respectively and AD is produced to X so that D is the mid-point of OX. Prove that AO : AX = AF : AB = AE : AC and show that EF ∥ BC.

Sol. Given that ΔABC is a triangle, D is a point on BC such that BD = DC. O is any point on the median AD.

BO produced meets AC at E and CO produced meets AB at F as shown in the figure. AD is produced to a point X such that OD = DX.



To prove that

(*i*) $\frac{AO}{AX} = \frac{AF}{AB} = \frac{AE}{AC}$ and (*ii*) EF ∥ BC.

Construction: We join FE, BX and CX.

(*i*) We see that in the quadrilateral OBXC, OD = DX and BD = DC, i.e. the two diagonals OX and BC bisect each other at D. Hence, the quadrilateral OBXC is a parallelogram.

∴ OC ∥ BX

i.e., OF ∥ BX

and OB ∥ CX

i.e., OE ∥ CX

Now, in ΔAFO and ΔABX,

$$\angle AFO = \angle ABX$$

[Corresponding ∠s, FO ∥ BX]

$$\angle AOF = \angle AXB$$

[Corresponding ∠s, FO ∥ BX]

∴ By AA criterion of similarity,

$$\Delta AFO \sim \Delta ABX$$

Since, corresponding sides of similar triangles are proportional,

$$\Rightarrow \quad \frac{AF}{AB} = \frac{AO}{AX} \quad \ldots(1)$$

Similarly, $\Delta AEO \sim \angle ACX$

$$\Rightarrow \quad \frac{AE}{AC} = \frac{AO}{AX} \quad \text{...(2)}$$

From (1) and (2)

$$\Rightarrow \quad \frac{AF}{AB} = \frac{AE}{AC} = \frac{AO}{AX}$$

Hence, proved.

(*ii*) In ΔABX, $\quad FO \parallel BX$

$$\Rightarrow \quad \frac{AF}{FB} = \frac{AO}{OX} \quad \text{[By BPT] ...(3)}$$

In ΔACX, $\quad EO \parallel CX$

$$\Rightarrow \quad \frac{AE}{EC} = \frac{AO}{OX} \quad \text{[By BPT] ...(4)}$$

From (3) and (4),

$$\frac{AF}{FB} = \frac{AE}{EC}$$

$\Rightarrow$ In ΔABC, FE divides AB and AC in same ratio.

$\therefore$ By converse of basic proportionality theorem,

$$FE \parallel BC$$

Hence, proved.

Check Your Progress 2

(Page 117)

Multiple-Choice Questions

1. In ΔABC and ΔPQR, $\angle A = \angle P$ and $\dfrac{AB}{PQ} = \dfrac{AC}{PR} = \dfrac{2}{3}$, then BC : QR is equal to

(*a*) 3 : 2 (*b*) 1 : 2

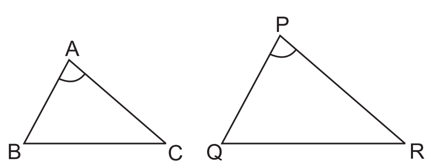
(*c*) 2 : 1 (*d*) 2 : 3

Sol. (*d*) 2 : 3

Given that in ΔABC and ΔPQR,

$$\angle A = \angle P$$

and $$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{2}{3}$$



$\therefore$ By SAS similarity criterion, we have

$$\Delta ABC \sim \Delta PQR$$

$$\Rightarrow \quad \frac{BC}{QR} = \frac{AB}{PQ} = \frac{AC}{PR} = \frac{2}{3}$$

$\therefore \quad BC : QR = 2 : 3$

2. In ΔABD, $\angle DAB = 90°$. C is a point on the hypotenuse BD such that $AC \perp BD$. Then AB^2 is equal to

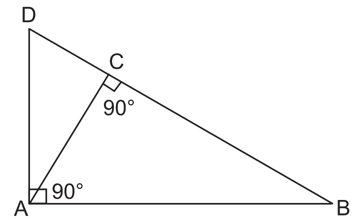
(*a*) $BC \times DC$ (*b*) $BD \times CD$

(*c*) $BC \times BD$ (*d*) $AD \times AC$

Sol. (*c*) $BC \times BD$

Given that ΔDAB is a right-angled triangle, with $\angle DAB = 90°$. C is a point on the hypotenuse BD such that $AC \perp DB$.

To find AB^2 in terms of other two sides of the triangles.



In ΔDAB and ΔACB, we have

$$\angle DAB = \angle ACB = 90°$$

$$\angle ABD = \angle CBA \quad \text{[Common]}$$

$\therefore$ By AA similarity criterion, we have

$$\Delta DAB \sim \Delta ACB$$

$$\Rightarrow \quad \frac{DA}{AC} = \frac{DB}{AB} = \frac{AB}{CB} \quad \text{...(1)}$$

$\therefore$ From (1), we have

$$AB^2 = DB \times CB$$

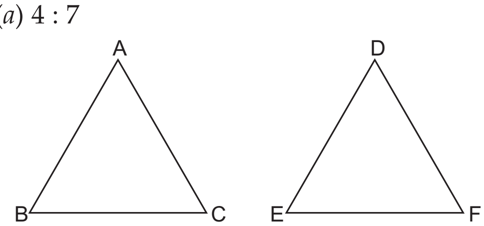
i.e., $$AB^2 = BC \times BD$$

3. The sides of two similar triangles are in the ratio 4 : 7. The ratio of their perimeters is

(*a*) 4 : 7 (*b*) 12 : 21

(*c*) 16 : 49 (*d*) 7 : 4 **[CBSE 2023 Basic]**

Sol. (*a*) 4 : 7



$$\Delta ABC \sim \Delta DEF \quad \text{[Given]}$$

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = \frac{4}{7} \quad \text{[Given]}$$

Let, $\quad AB = 4x, \quad DE = 7x$

$\quad BC = 4y, \quad EF = 7y$

$\quad AC = 4z, \quad DF = 7z$

$$\frac{\text{Perimeter of } \Delta ABC}{\text{Perimeter of } \Delta DEF} = \frac{AB + BC + AC}{DE + EF + DF}$$

$$= \frac{4x + 4y + 4z}{7x + 7y + 7z}$$

$$= \frac{4\,(x + y + z)}{7\,(x \;\; y + z)} = \frac{4}{7}$$

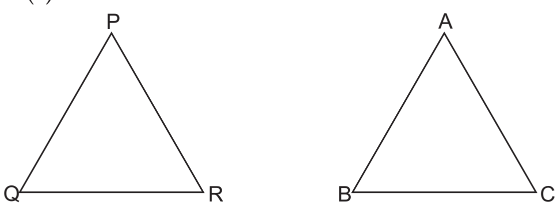
4. In two triangles ΔPQR and ΔABC, it is given that $\frac{AB}{BC} = \frac{PQ}{PR}$. For these two triangles to be similar, which of the following should be true?

(*a*) $\angle A = \angle P$ (*b*) $\angle B = \angle Q$

(*c*) $\angle B = \angle P$ (*d*) $CA = QR$

[CBSE 2023 Basic]

Sol. (*c*) $\angle B = \angle P$



In ΔPQR and ΔABC,

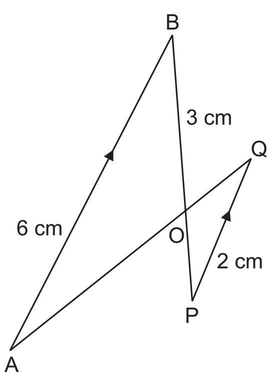
$$\frac{AB}{BC} = \frac{PQ}{PR} \quad \text{[Given]}$$

$$\Rightarrow \quad \frac{AB}{PQ} = \frac{BC}{PR}$$

i.e., $$\frac{BA}{PQ} = \frac{BC}{PR}$$

For $\Delta PQR \sim \Delta ABC$, the included angles should also be equal, that is, $\angle B = \angle P$.

5. In the given figure, $AB \parallel PQ$. If $AB = 6$ cm, $PQ = 2$ cm and $OB = 3$ cm, then the length of OP is



(*a*) 9 cm (*b*) 3 cm

(*c*) 4 cm (*d*) 1 cm

[CBSE 2023 Standard]

Sol. (*d*) 1 cm

In ΔABO and ΔQPO,

$\angle ABO = \angle QPO$ [Alternate angles]

$\angle BAO = \angle PQO$ [Alternate angles]

$\angle AOB = \angle QOP$ [Vert. opposite angles]

By AAA criterion of similarity,

$$\Delta ABO \sim \Delta QPO$$

$$\Rightarrow \quad \frac{AB}{PQ} = \frac{BO}{OP}$$

$$\Rightarrow \quad OP = \frac{BO}{AB} \times PQ$$

$$= \frac{3}{6} \times 2$$

$$= 1 \text{ cm}$$

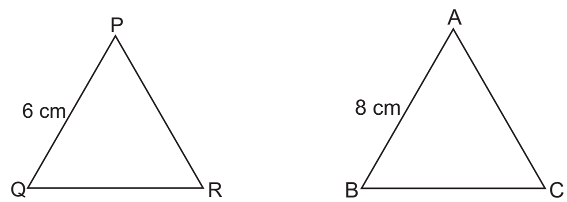
6. If $\Delta PQR \sim \Delta ABC$; $PQ = 6$ cm, $AB = 8$ cm and the perimeter of ΔABC is 36 cm, the perimeter of ΔPQR is

(*a*) 20.25 cm (*b*) 27 cm

(*c*) 48 cm (*d*) 64 cm

[CBSE 2023 Standard]

Sol. (*b*) 27 cm



$AB = 8$ cm [Given]

$PQ = 6$ cm [Given]

$\Delta PQR \sim \Delta ABC$ [Given]

Ratio of side = Ratio of perimeter

$$\frac{PQ}{AB} = \frac{\text{Perimeter of } \Delta PQR}{\text{Perimeter of } \Delta ABC}$$

$$\frac{6}{8} = \frac{\text{Perimeter of } \Delta PQR}{36 \text{ cm}}$$

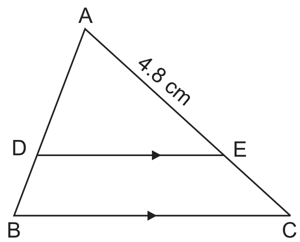
$$\text{Perimeter of } \Delta PQR = \frac{6}{8} \times 36 \text{ cm}$$

$$= 27 \text{ cm}$$

Very Short Answer Type Questions

7. In ΔABC, D and E are points on the sides AB and AC respectively such that $DE \parallel BC$. If $\frac{AD}{DB} = \frac{3}{2}$ and $AE = 4.8$ cm, find EC. **[CBSE SP 2011]**

Sol. Given that D and E are two points on the sides AB and AC respectively of ΔABC such that $DE \parallel BC$, $\frac{AD}{DB} = \frac{3}{2}$ and $AE = 4.8$ cm.



To find the length of EC.

Since DE $\parallel$ BC, hence by the basic proportionality theorem, we have

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\Rightarrow \quad \frac{3}{2} = \frac{4.8}{EC}$$

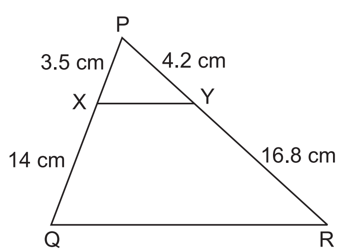
$$\Rightarrow \quad 3\,EC = 4.8 \times 2$$

$$\Rightarrow \quad EC = \frac{9.6}{3} = 3.2$$

Hence, the required length of EC is 3.2 cm.

8. X and Y are points on the sides PQ and PR respectively of a ΔPQR such that PX = 3.5 cm, XQ = 14 cm, PY = 4.2 cm and YR = 16.8 cm, then state if XY $\parallel$ QR or not.

Sol. Given that X and Y are two points on the sides PQ and PR respectively of a triangle PQR such that PX = 3.5 cm, XQ = 14 cm, PY = 4.2 cm and YR = 16.8 cm.



To test whether XY $\parallel$ QR, or not.

We have $\quad \dfrac{PX}{XQ} = \dfrac{3.5}{14} = \dfrac{35}{140} = \dfrac{1}{4}$

and $\quad \dfrac{PY}{YR} = \dfrac{4.2}{16.8} = \dfrac{42}{168} = \dfrac{1}{4}$

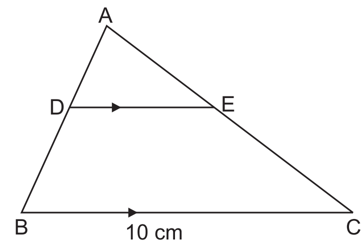
$\therefore \quad \dfrac{PX}{XQ} = \dfrac{PY}{YR}$

$\therefore$ By the converse of the basic proportionality theorem, we get

$$XY \parallel QR$$

9. ABC is an equilateral triangle with points D and E on sides AB and AC respectively such that DE $\parallel$ BC. If BC = 10 cm and $\dfrac{AE}{EC} = \dfrac{1}{3} = \dfrac{AD}{DB}$, then find the length of AE.

Sol. Given that ΔABC is an equilateral triangle with the side BC = 10 cm. Also, D and E are two points on the sides AB and AC respectively of ΔABC such the DE $\parallel$ BC.



Also, $\quad \dfrac{AE}{EC} = \dfrac{1}{3} = \dfrac{AD}{DB}.$

To find the length of AE.

Since, $\quad \dfrac{AE}{EC} = \dfrac{1}{3} \quad$ [Given]

$\Rightarrow \quad EC = 3\,AE$

$\Rightarrow \quad AC - AE = 3\,AE$

$\Rightarrow \quad AC = 3\,AE + AE$

$\Rightarrow \quad 10 = 4\,AE$

[$\because$ AC = BC = 10 cm]

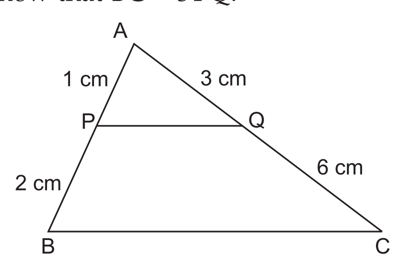
$\Rightarrow \quad AE = \dfrac{10}{4}$

$\Rightarrow \quad AE = 2.5$ cm

Hence, the required length of AE is 2.5 cm.

10. P and Q are points on sides AB and AC respectively of ΔABC. If AP = 1 cm, PB = 2 cm, AQ = 3 cm and QC = 6 cm, show that BC = 3 PQ. **[CBSE SP 2011]**

Sol. Given that P and Q are two points on the sides AB and AC respectively of ΔABC such that AP = 1 cm, PB = 2 cm, AQ = 3 cm and QC = 6 cm. To show that BC = 3PQ.



Since, $\dfrac{AP}{PB} = \dfrac{1}{2}$ and $\dfrac{AQ}{QC} = \dfrac{3}{6} = \dfrac{1}{2}$

$\Rightarrow \quad \dfrac{AP}{PB} = \dfrac{AQ}{QC}$

$\therefore$ PQ $\parallel$ BC, by the converse of the basic proportionality theorem.

In ΔAPQ and ΔABC, we have

$$\angle APQ = \angle ABC \quad \text{[Corresponding } \angle s]$$

$$\angle AQP = \angle ACB \quad \text{[Corresponding } \angle s]$$

$$\angle PAQ = \angle BAC \quad \text{[Common]}$$

$\therefore$ By AAA similarity criterion, we have

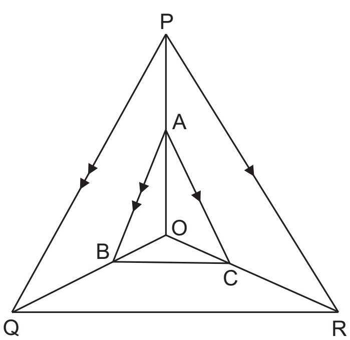
$$\Delta APQ \sim \Delta ABC$$

$$\Rightarrow \quad \frac{PQ}{BC} = \frac{AP}{AB} = \frac{1}{1+2} = \frac{1}{3}$$

$$\therefore \quad BC = 3PQ$$

Hence, proved.

11. In the given figure, A, B and C are points on OP, OQ and OR respectively such that AB || PQ and AC || PR. Show that BC || QR.



[CBSE 2023 Basic]

Sol. In ΔOPQ and ΔOAB,

$$PQ \parallel AB \quad \text{[Given]}$$

By Basic Proportionality Theorem,

$$\frac{OA}{AP} = \frac{OB}{BQ} \quad \ldots(1)$$

In ΔOPR and ΔOAC

$$PR \parallel AC \quad \text{[Given]}$$

By Basic Proportionality Theorem,

$$\frac{OA}{AP} = \frac{OC}{CR} \quad \ldots(2)$$

From (1) and (2)

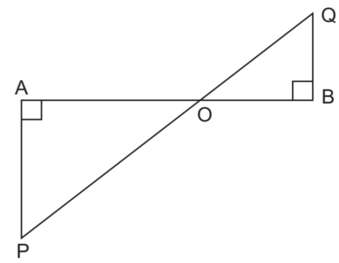
$$\frac{OB}{BQ} = \frac{OC}{CR}$$

By converse of Basic Proportionality Theorem, in ΔOQR and ΔOBC

$$BC \parallel QR$$

Hence, provide.

12. In the given figure, $AP \perp AB$ and $BQ \perp AB$. If OA = 15 cm, BO = 12 cm and AP = 10 cm, then find the length of BQ.



[CBSE 2024 Basic]

Sol. In ΔAOP and ΔBOQ,

$$\angle OAP = \angle OBQ = 90°$$

$$\angle AOP = \angle BOQ$$

[vert. opposite angles]

$\therefore$ By AA similarity criterion,

$$\Delta AOP \sim \Delta BOQ$$

$$\Rightarrow \quad \frac{AP}{BQ} = \frac{AO}{BO}$$

$$\Rightarrow \quad BQ = \frac{AP \times BO}{AO} \times \frac{10 \times 12}{15} \text{ cm}$$

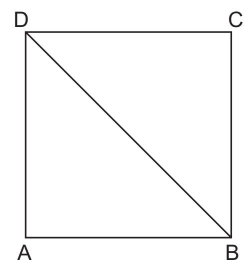
$$= 8 \text{ cm}$$

$$\therefore \quad BQ = 8 \text{ cm}$$

13. In the given figure, ABCD is a quadrilateral. Diagonal BD bisects $\angle B$ and $\angle D$ both. Prove that

(*a*) $\Delta ABD \sim \Delta CBD$

(*b*) AB = BC



[CBSE 2024 Standard]

Sol. (*a*) In ΔABD and ΔCBD,

$$\angle ADB = \angle CDB \quad \text{[BD bisects } \angle D]$$

$$\therefore \quad \angle ABD = \angle CBD \quad \text{[BD bisects } \angle B]$$

By AA similarity criterion,

$$\Delta ABD \sim \Delta CBD$$

(*b*) $\Delta ABD \sim \Delta CBD$

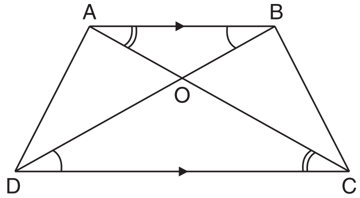
$$\Rightarrow \quad \frac{AB}{CB} = \frac{DB}{DB}$$

$$\Rightarrow \quad AB = BC$$

Short Answer Type Questions

14. Prove that the diagonals of a trapezium divide each other proportionally.

Sol. Given that ABCD is a trapezium such that AB $\parallel$ DC and the two diagonals AC and BD intersect each other at a point O.



To prove that

$$\frac{AO}{CO} = \frac{BO}{DO}$$

In ΔAOB and ΔCOD, we have

$$\angle AOB = \angle COD$$

[Vertically opposite angles]

$$\angle ABO = \angle CDO$$

[Alternate angles, $\because$ AB $\parallel$ DC]

$$\angle BAO = \angle DCO \text{ [Alternate angles]}$$

$\therefore$ By AAA similarity criterion, we have

$$\Delta AOB \sim \Delta COD$$

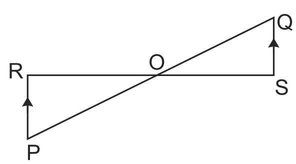
$$\Rightarrow \quad \frac{AO}{CO} = \frac{BO}{DO}$$

Hence, proved.

15. In the given figure, prove that

(*a*) $\Delta PRO \sim \Delta QSO$

(*b*) $\frac{PO}{RO} = \frac{QO}{SO}$

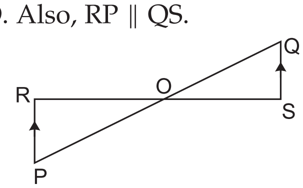


Sol. Given that two line segments RS and PQ intersect each other at a point O. Also, RP $\parallel$ QS.

To prove that

(*a*) $\Delta PRO \sim \Delta QSO$

(*b*) $\frac{PO}{RO} = \frac{QO}{SO}$



(*a*) In ΔPRO and ΔQSO, we have

$$\angle PRO = \angle QSO$$

[Alternate angles, $\because$ RP $\parallel$ QS]

$$\angle ROP = \angle SOQ$$

[Vertically opposite angles]

$\therefore$ By AA similarity criterion, we have

$$\Delta PRO \sim \Delta QSO$$

(*b*) From (*a*), we have

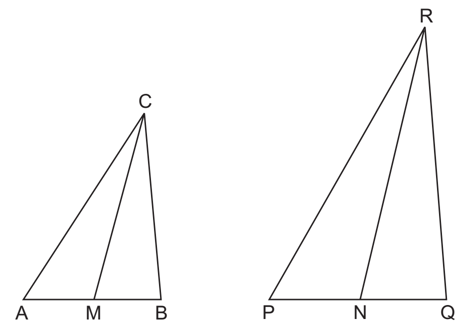
$$\frac{PO}{QO} = \frac{RO}{SO}$$

$$\Rightarrow \quad \frac{PO}{RO} = \frac{QO}{SO}$$

Hence, proved.

16. In the given figure, CM and RN are respectively the medians of ΔABC and ΔPQR. If $\Delta ABC \sim \Delta PQR$, then prove that $\Delta AMC \sim \Delta PNR$.

[CBSE 2023 Basic]



Sol. $\Delta ABC \sim \Delta PQR$ [Given]

$AM = MB$

[CM is the median] ...(1)

$PN = NQ$

[RN is the median] ...(2)

$\angle A = \angle P$ ...(3)

[$\Delta ABC \sim \Delta PQR$]

$$\Rightarrow \quad \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$$

$$\Rightarrow \quad \frac{AB}{PQ} = \frac{AC}{PR}$$

$$\Rightarrow \quad \frac{AM + MB}{PN + NQ} = \frac{AC}{PR}$$

$$\Rightarrow \quad \frac{2AM}{2PM} = \frac{AC}{PR} \quad \text{[From (1) and (2)]}$$

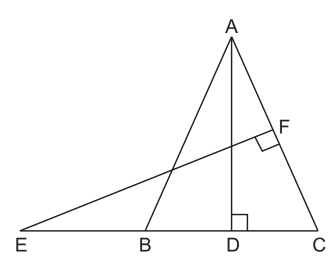
$$\Rightarrow \quad \frac{AM}{PN} = \frac{AC}{PR} \quad ...(4)$$

By SAS similarity criterion,

$\Delta AMC \sim \Delta PNR$ [From (3) and (4)]

Hence, proved.

17. In the given figure, E is a point on the side CB produced of an isosceles triangle ABC with AB = AC. If AD $\perp$ BC and EF $\perp$ AC, then prove that $\Delta ABD \sim \Delta ECF$.



[CBSE 2023 Standard]

Sol. In ΔABD and ΔECF

$$\angle ADB = \angle EFC = 90°$$

$$\angle ECF = \angle ABD \qquad [AB = AC]$$

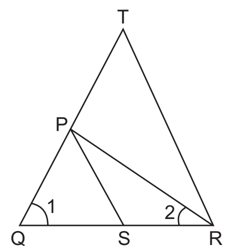
By AA similarity criterion,

$$\Delta ABD \sim \Delta ECF$$

Hence, proved.

Long Answer Type Questions

18. In the given figure, $\frac{QR}{QS} = \frac{QT}{PR}$ and $\angle 1 = \angle 2$. Prove that $\Delta PQS \sim \Delta TQR$



[CBSE 2023 Basic]

Sol. In ΔPQR,

$$\angle 1 = \angle 2$$

$\Rightarrow \Delta PQR$ is an isosceles triangle.

$\therefore \qquad PQ = PR \qquad \ldots(1)$

$$\frac{QR}{QS} = \frac{QT}{PR}$$

$\Rightarrow \qquad \frac{QR}{QS} = \frac{QT}{QP} \qquad$ [From (1)] …(2)

In ΔPQS and ΔTQR

$$\frac{QR}{QS} = \frac{QT}{QP} \qquad \text{[From (2)]}$$

$$\angle Q = \angle Q \qquad \text{[Common]}$$

$\therefore$ By SAS similarity criterion,

$$\Delta PQS \sim \Delta TQR$$

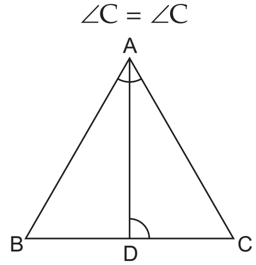
Hence, proved.

19. D is a point on the side BC of a triangle ABC such that $\angle ADC = \angle BAC$, prove that $CA^2 = CB \cdot CD$ **[CBSE 2023 Standard]**

Sol. In ΔABC and ΔDAC,

$$\angle BAC = \angle ADC \qquad \text{[Given]}$$

$$\angle C = \angle C \qquad \text{[Common]}$$



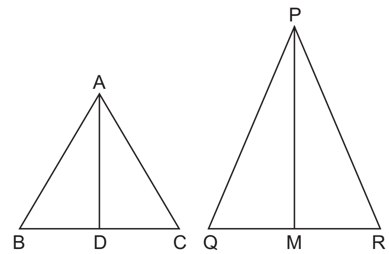
$\therefore$ By AA similarity criterion,

$$\Delta ABC \sim \Delta DAC$$

$$\Rightarrow \qquad \frac{CB}{CA} = \frac{CA}{CD}$$

$$\Rightarrow \qquad CA^2 = CB \cdot CD$$

20. Sides AB and AC and median AD of a ΔABC are respectively proportional to sides PQ and PR and median PM of ΔPQR. Show that $\Delta ABC \sim \Delta PQR$

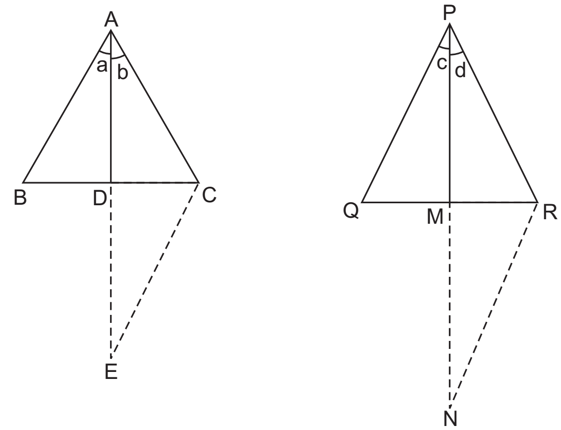


[CBSE 2024 Basic]

Sol. $\qquad \frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM} \qquad$ [Given]

Produce AD to E such that AD = DE. Join EC.

Similarly, produce PM to N such that PM = MN. Join NR.



In ΔABD and ΔCDE,

$$\angle ADB = \angle EDC$$

[Vertically opposite angles]

$BD = CD \qquad$ [AD is the median]

$AD = DE \qquad$ [By construction]

$\therefore$ By SAS congruency criterion,

$$\Delta ABD \cong \Delta ECD$$

$AB = EC \qquad$ [By CPCT] …(1)

In the same manner, we can prove that

$$\Delta PQM \cong \Delta NRM$$

$PQ = NR \qquad \ldots(2)$

Now we have,

$$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}$$

$$\Rightarrow \quad \frac{EC}{NR} = \frac{AC}{PR} = \frac{AD}{PM}$$

[From (1) and (2)]

$$\Rightarrow \quad \frac{EC}{NR} = \frac{AC}{PR} = \frac{2AD}{2PM}$$

$$\Rightarrow \quad \frac{EC}{NR} = \frac{AC}{PR} = \frac{AE}{PN}$$

$\therefore$ By SSS similarity criterion,

$$\Delta ACE \sim \Delta PRN$$

$$\Rightarrow \quad \angle b = \angle d$$

Similarly, we can prove that

$$\angle a = \angle c$$

$$\therefore \quad \angle a + \angle b = \angle c + \angle d$$

$$\Rightarrow \quad \angle BAC = \angle QPR \qquad \ldots(3)$$

In ΔABC and ΔPQR

$$\frac{AB}{PQ} = \frac{AC}{PR} \qquad \text{[Given]}$$

$$\angle BAC = \angle QPR \qquad \text{[From (3)]}$$

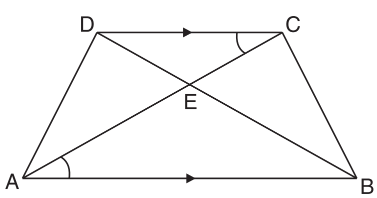
$$\therefore \quad \Delta ABC \sim \Delta PQR$$

[By SAS similarity criterion]

21. ABCD is a trapezium in which AB $\parallel$ CD. If diagonals AC and BD intersect each other at E, and if $\Delta AED \sim \Delta BEC$, prove that BD = AC and AD = BC.

Sol. Given that ABCD is a trapezium in which AB $\parallel$ CD. The two diagonals AC and BD intersect each other at a point E. It is given that $\Delta AED \sim \Delta BEC$.

To prove that BD = AC and AD = BC.



We have

$$\Delta AED \sim \Delta BEC \qquad \text{[Given]}$$

$$\frac{AE}{BE} = \frac{AD}{BC} = \frac{ED}{EC} \qquad \ldots(1)$$

Also, since $\angle AEB = \angle CED$

[Vertically opposite angles]

$$\angle BAE = \angle DCE$$

[Alternate angles, since AB $\parallel$ CD]

$\therefore$ By AA similarity criterion, we have

$$\Delta AEB \sim \Delta CED$$

$$\Rightarrow \quad \frac{AE}{CE} = \frac{AB}{CD} = \frac{EB}{ED} \qquad \ldots(2)$$

From (1), we have

$$\frac{AE}{BE} = \frac{ED}{EC} \qquad \ldots(3)$$

From (2), we have

$$\frac{AE}{CE} = \frac{BE}{DE}$$

$$\frac{AE}{BE} = \frac{EC}{ED} \qquad \ldots(4)$$

$\therefore$ From (3) and (4), we have

$$\frac{ED}{EC} = \frac{EC}{ED}$$

$$\Rightarrow \quad EC^2 = ED^2$$

$$\Rightarrow \quad EC = ED \qquad \ldots(5)$$

$\therefore$ From (3), we have

$$\frac{AE}{BE} = 1 \quad \left[\because \frac{EC}{ED} = 1, \text{ from (5)}\right]$$

$$\Rightarrow \quad AE = BE \qquad \ldots(6)$$

Now, $\quad AC = AE + EC$

$$= BE + ED$$

[From (5) and (6)]

$$= BD$$

Hence, $\quad AC = BD.$

$\therefore$ From (1), $\quad \dfrac{AD}{BC} = \dfrac{AE}{BE} = 1$

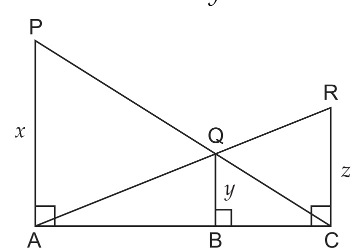
[$\because$ From (6), AE = BE]

$$\therefore \quad AD = BC$$

Hence, proved.

22. In the given figure, PA, QB and RC are perpendicular to AC and if PA = x, BQ = y and CR = z, prove that

$$\frac{1}{x} + \frac{1}{z} = \frac{1}{y}$$



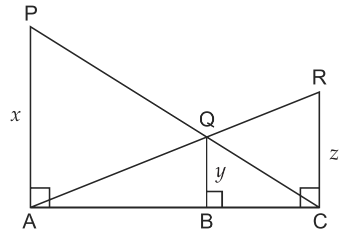
[CBSE 2024 Standard]

Sol. Given that PA, QB and RC are perpendiculars to the line segment ABC at the points A, B and C respectively. Also, PA = x, QB = z and RC = y.

In ΔBCQ and ΔACP,

$$QB \parallel PA$$

$$\therefore \quad \Delta BCQ \sim \Delta ACP$$



$$\Rightarrow \quad \frac{QB}{PA} = \frac{BC}{AC}$$

$$\Rightarrow \quad \frac{y}{x} = \frac{BC}{AC} \quad \ldots(1)$$

Also, in ΔBAQ and ΔCAR,

$$QB \parallel RC$$

$$\therefore \quad \Delta BAQ \sim \Delta CAR$$

$$\Rightarrow \quad \frac{QB}{RC} = \frac{AB}{AC}$$

$$\Rightarrow \quad \frac{y}{z} = \frac{AB}{AC}$$

$$= \frac{AC - BC}{AC} = 1 - \frac{BC}{AC}$$

$$\Rightarrow \quad 1 - \frac{y}{z} = \frac{BC}{AC} \quad \ldots(2)$$

$\therefore$ From (1) and (2), we have

$$\frac{y}{x} = 1 - \frac{y}{z}$$

$$\Rightarrow \quad \frac{1}{x} = \frac{1}{y} - \frac{1}{z}$$

$$\Rightarrow \quad \frac{1}{y} = \frac{1}{x} + \frac{1}{z}$$

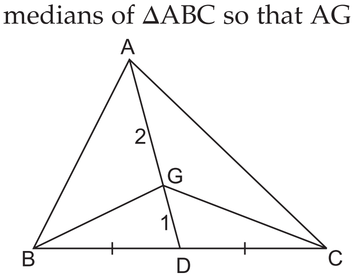
Hence, proved.

Higher Order Thinking Skills (HOTS) Questions

(Page 120)

1. If G be the centroid of ΔABC, prove that $AB^2 + BC^2 + CA^2 = 3(GA^2 + GB^2 + GC^2)$.

Sol. Given that G is the centroid of ΔABC, AD is one of the medians of ΔABC so that AG : GD = 2 : 1.



To prove that

$$AB^2 + BC^2 + CA^2 = 3(GA^2 + GB^2 + GC^2)$$

Construction: Join GB and GC.

In ΔABC, since AD is a median, hence, we have

$$AB^2 + AC^2 = 2BD^2 + 2AD^2 \quad \ldots(1)$$

and in ΔGBC, since GD is a median, hence, we have

$$GB^2 + GC^2 = 2BD^2 + 2DG^2 \quad \ldots(2)$$

Now, since G is the centroid of ΔABC,

$$\therefore GA = \frac{2}{3} AD \text{ and } GD = \frac{1}{3} AD \quad \ldots(3)$$

Now,

$$AB^2 + AC^2 + BC^2 = 2BD^2 + 2AD^2 + 4BD^2$$

$$[\because BC = 2BD \text{ and from (1)}]$$

$$= 6BD^2 + 2AD^2 \quad \ldots(4)$$

Also, $GB^2 + GC^2 + GA^2$

$$= 2BD^2 + 2DG^2 + \left(\frac{2}{3}AD^2\right)$$

[From (2) and (3)]

$$= 2BD^2 + 2 \times \frac{1}{9}AD^2 + \frac{4}{9}AD^2 \quad \text{[From (3)]}$$

$$= 2BD^2 + \frac{2}{3}AD^2$$

$$\Rightarrow \quad 3(GA^2 + GB^2 + GC^2) = 6BD^2 + 2AD^2 \quad \ldots(5)$$

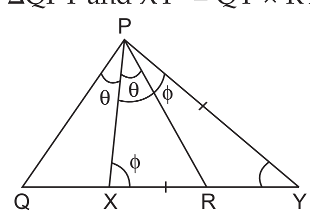
$\therefore$ From (4) and (5), we have

$$AB^2 + AC^2 + BC^2 = 3(GA^2 + GB^2 + GC^2)$$

Hence, proved.

2. In ΔPQR, the angle P is bisected by PX which meets QR in X. Y is a point in QR produced such that PY = XY. Prove that $\Delta PRY \sim \Delta QPY$ and hence show that XY is a mean proportional between QY and RY, i.e. show that $XY^2 = QY \times RY$.

Sol. Given that X is a point on the side QR of ΔPQR such that PX is the bisector of $\angle QPR$. QR is produced to Y such that PY = XY. To prove that $\Delta PRY \sim \Delta QPY$ and $XY^2 = QY \times RY$



Let $\angle QPX = \angle RPX = \theta$ and $\angle XPY = \phi$

Since $\quad PY = XY$

$\therefore \quad \angle PXY = \angle XPY = \phi$

$\therefore$ In Δ PXR,

$$\text{exterior } \angle PRY = \theta + \phi = \angle QPY \quad \ldots(1)$$

Now, in ΔPRY and ΔQPY, we have

$$\angle PRY = \angle QPY \quad \text{[From (1)]}$$

$$\angle PYR = \angle QYP \quad \text{[Common]}$$

$\therefore$ By AA similarity criterion, we have

$$\Delta PRY \sim \Delta QPY$$

$$\therefore \quad \frac{PR}{QP} = \frac{PY}{QY} = \frac{RY}{PY} \quad \ldots(2)$$

$\therefore$ From (2), we have

$$\frac{PY}{QY} = \frac{RY}{PY}$$

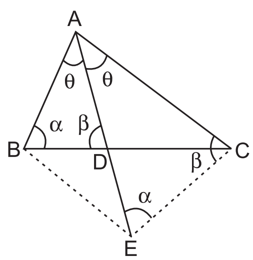
$$\Rightarrow \quad PY^2 = QY \times RY$$

$$\Rightarrow \quad XY^2 = QY \times RY \quad [\because PY = XY]$$

Hence, proved.

3. D is a point on the side BC of ΔABC such that $\frac{BD}{DC} = \frac{AB}{AC}$. AD is produced to E so that $\frac{AB}{AE} = \frac{AD}{AC}$. Prove that ΔBEC is isosceles.

Sol. Given that D is a point on the side BC of ΔABC such that $\frac{BD}{DC} = \frac{AB}{AC}$. AD is produced to E so that $\frac{AB}{AE} = \frac{AD}{AC}$.



To prove that ΔBEC is an isosceles triangle.

Construction: Join EB and EC.

Since $\frac{BD}{DC} = \frac{AB}{AC}$, hence, by the converse of the theorem of internal bisector of an angle, we have AD is the internal bisector of $\angle BAC$.

Let $\angle BAD = \angle EAC = \theta$

Now, in ΔBAD and ΔEAC, we have

$$\angle BAD = \angle EAC = \theta \text{ and } \frac{AB}{AE} = \frac{AD}{AC}$$

Hence, by SAS similarity criterion, we have

$$\Delta BAD \sim \Delta EAC$$

$$\therefore \quad \angle BDA = \angle ECA = \beta, \text{ say} \quad \ldots(1)$$

$$\text{and} \quad \angle ABD = \angle AEC = \alpha, \text{ say} \quad \ldots(2)$$

Again, in ΔCAD and ΔEAB, we have

$$\angle CAD = \angle EAB = \theta$$

$$\text{and} \quad \frac{AC}{AE} = \frac{AD}{AB}$$

Hence, by SAS similarity criterion, we have

$$\Delta CAD \sim \Delta EAB$$

$$\therefore \quad \angle CDA = \angle EBA$$

$$\text{But} \quad \angle CDA = \theta + \alpha$$

$$[\because \text{In } \Delta ABD, \text{ exterior angle } CDA = \theta + \alpha]$$

$$\therefore \quad \angle CDA = \angle EBA = \theta + \alpha \quad \ldots(3)$$

$$\text{But since} \quad \angle CDA = 180° - \beta$$

$$\therefore \quad \angle CDA = \angle EBA = 180° - \beta \quad \ldots(4)$$

$$\text{Also,} \quad \angle ADC = \angle ABE$$

$$= \theta + \alpha = 180° - \beta \quad \ldots(5)$$

Now, in ΔEBC

$$\angle EBC = \angle ABE - \angle ABD$$

$$= \theta + \alpha - \alpha$$

$$\text{[From (5) and (2)]}$$

$$= \theta \quad \ldots(6)$$

$$\text{Also,} \quad \angle ECB = \angle ECA - \angle ACB$$

$$\text{Now,} \quad \angle ACB = 180° - 2\theta - \alpha$$

$$\text{and} \quad \angle ECA = \beta \quad \text{[From (1)]}$$

$$\therefore \quad \angle ECB = \beta - 180° + 2\theta + \alpha$$

$$= -\theta - \alpha + 2\theta + \alpha$$

$$\text{[From (5)]}$$

$$\therefore \quad \angle EBC = \angle ECB = \theta$$

$$\text{[From (5) and (7)]}$$

$\therefore$ ΔBEC is an isosceles.

Self-Assessment

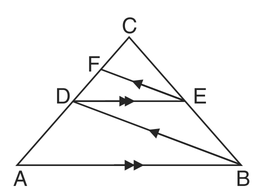
(Page 120)

Multiple-Choice Questions

1. In the given figure, AB ∥ DE and BD ∥ EF. Then

(*a*) $BC^2 = AB \times CE$ (*b*) $AB^2 = AC \times DE$

(*c*) $AC^2 = BC \times DC$ (*d*) $DC^2 = CF \times AC$



Sol. (*d*) $DC^2 = CF \times AC$

In ΔABC, we have

$$DE \parallel AB$$

$\therefore$ By the basic proportionality theorem, we have

$$\frac{AD}{CD} = \frac{BE}{EC} \quad \ldots(1)$$

In ΔBDC, $BD \parallel CF$

$\therefore$ By the basic proportionality theorem, we have

$$\frac{DF}{FC} = \frac{BE}{EC} \quad \ldots(2)$$

From (1) and (2), we have

$$\frac{AD}{CD} = \frac{DF}{FC}$$

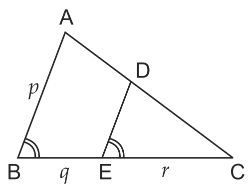
$$\Rightarrow \quad \frac{AC - CD}{CD} = \frac{CD - FC}{FC}$$

$$\Rightarrow \quad \frac{AC}{CD} - 1 = \frac{CD}{FC} - 1$$

$$\Rightarrow \quad \frac{AC}{CD} = \frac{CD}{FC}$$

$$\Rightarrow \quad CD^2 = AC \times CF$$

2. In the given figure, DE is equal to

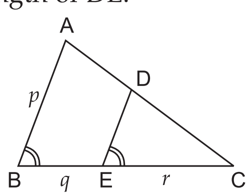


(*a*) $\frac{pq}{p+q}$ (*b*) $\frac{pr}{q+r}$

(*c*) $\frac{qr}{p+r}$ (*d*) $\frac{qr}{q+r}$

Sol. (*b*) $\frac{pr}{q + r}$

Given that in ΔABC, D and E are points on AC and BC respectively such that $\angle ABC = \angle DEC$, $EC = r$ units, $BE = q$ units and $AB = p$ units. To find the length of DE.



$\because$ $\angle DEC = \angle ABC$ [Corresponding $\angle$s]

$\therefore$ $AB \parallel DE$

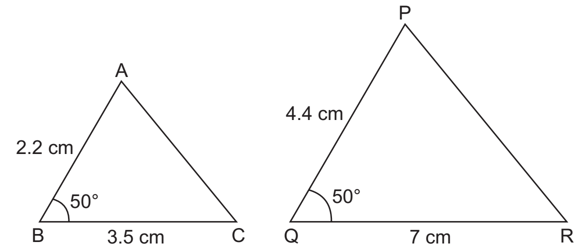
$\therefore$ $\Delta DEC \sim \Delta ABC$

$$\therefore \quad \frac{BC}{EC} = \frac{AB}{DE}$$

$$\Rightarrow \quad \frac{q + r}{r} = \frac{p}{DE}$$

$\therefore$ Required length of DE is $\frac{pr}{q + r}$ units.

3. In the given figure, the criterion of similarity by which $\Delta ABC \sim \Delta PQR$ is



(*a*) SSA (Side – Side – Angle) Similarity

(*b*) ASA (Angle – Side – Angle) Similarity

(*c*) SAS (Side – Angle – Side) Similarity

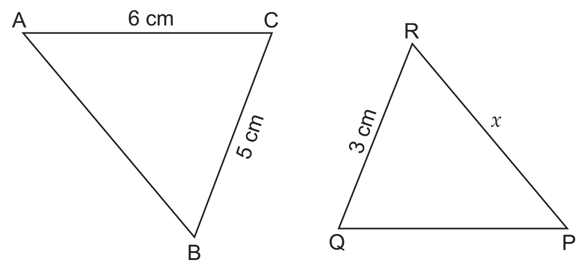
(*d*) AA (Angle – Angle) Similarity

[CBSE 2023 Basic]

Sol. (*c*) SAS (Side – Angle – Side) Similarity

4. In the given figure $\Delta ABC \sim \Delta QPR$.

If $AC = 6$ cm, $BC = 5$ cm, $QR = 3$ cm and $PR = x$; then the value of x is



(*a*) 3.6 cm (*b*) 2.5 cm

(*c*) 10 cm (*d*) 3.2 cm

[CBSE 2023 Standard]

Sol. (*b*) 2.5 cm

$$\Delta ABC \sim \Delta QPR$$

$$\Rightarrow \quad \frac{AB}{QP} = \frac{BC}{PR} = \frac{AC}{QR}$$

$$\frac{BC}{PR} = \frac{AC}{QR}$$

$$\frac{5 \text{ cm}}{x} = \frac{6 \text{ cm}}{3 \text{ cm}}$$

$$\Rightarrow \quad x = \frac{5 \times 3}{6} \text{ cm}$$

$$\Rightarrow \quad x = 2.5 \text{ cm}$$

Fill in the Blanks

5. The perimeters of two similar triangles are 25 cm and 15 cm respectively. If one side of the first triangle is 9 cm, then the corresponding side of the second triangle is **<u>5.4 cm</u>**.

6. All squares are **similar** (congruent/similar).

7. All concentric circles are **similar** to each other.

Assertion-Reason Type Questions

Directions (Q. Nos. 8 to 10): Each of these questions contains an assertion followed by reason. Read them carefully, and answer the question on the basis of the following options, select the one that best describes the two statements.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

8. **Assertion (A):** Two similar triangles will have same size.

Reason (R): Similar triangles have the same measurement of angles.

Sol. The correct answer is (*d*) as similar triangles have same measurement of angles but not sides. The Assertion is wrong but Reason is correct.

9. **Assertion (A):** A square and rectangle are not similar.

Reason (R): Ratio of sides of a square and rectangle are not same.

Sol. The correct answer is (*a*).

Both the statements are correct. Rectangles have different size of length and breadth. Hence their ratios are not same, so they are not similar. Thus both statements are correct and Reason is correct explanation of the Assertion.

10. **Assertion (A):** All right-angled isosceles triangles are similar.

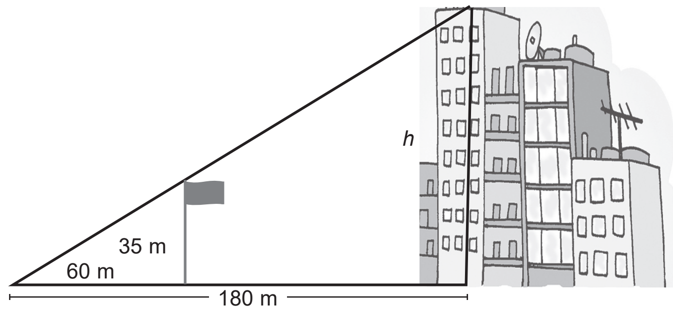
Reason (R): All isosceles triangles have all angles equal.

Sol. The correct answer is (*c*).

Assertion is correct as all right-angled isosceles triangles have the same angles of 90, 45 and 45 degrees. Thus Assertion is correct but Reason is wrong.

Case Study Based Questions

11. On the occasion of 15th August, lots of programmes are scheduled to be held. A flagpole stand is made outside the apartment. Nidhi looks up from the ground which is 60 m away from the flagpole. She looks up to the top of the building and the top of the flagpole in the same line up. If the flagpole is 35 m tall, and Nidhi is 180 m away from the building. Based on this given situation, answer the following questions.



(*a*) Find the height of the building.

Ans. 105 m

(*b*) You are standing at the same distance 180 m away from the building and place the flagpole 85 m away from you, what will be the distance of the flagpole from the building?

Ans. 95 m

(*c*) (*i*) If the height of the flagpole is reduced by 3 m, then what is the height of the building in line?

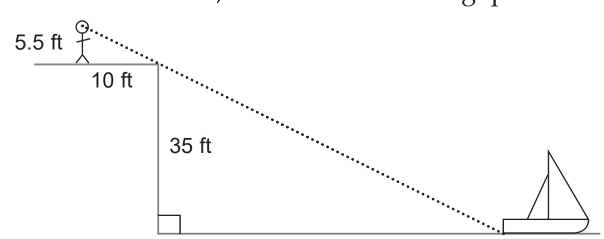
Ans. 96 m

or

(*ii*) If the height of the flagpole is 30 m, then what is the height of the building in line?

Ans. 90 m

12. A teacher of secondary school took 10 students of class 10 to Nongpoh, Meghalaya in summer vacation. One of the students, Amit was standing on a cliff. The cliff was 35 ft above the lake. Amit's height was 5.5 ft. Amit was standing 10 ft away from the edge of the cliff. Amit could visually align the top of the cliff with the water at the back of the boat. The situation was drawn and levelled. Based on this situation, answer the following questions.



(*a*) Find the distance of the boat from the cliff.

Ans. 63.64 ft

(*b*) If Amit stands 15 ft away from the cliff, what will be the distance of the boat from the cliff?

Ans. 95.45 ft

(*c*) (*i*) A girl 160 cm tall, stands 360 cm from a lamp post at night. Her shadow from the light is 90 cm long. What is the height of the lamp post?

Ans. 800 cm

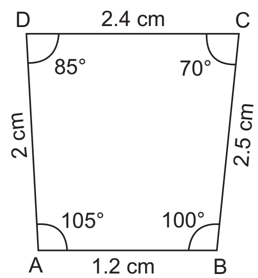
or

(*ii*) A tower casts a shadow 7 m long. A vertical stick casts a shadow 0.6 m long. If the stick is 1.2 m high, what is the height of the tower?

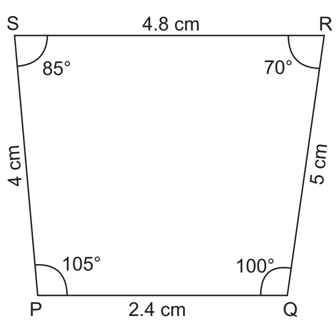
Ans. 14 m

13. Observe the figures given below carefully and answer the questions:

Figure A

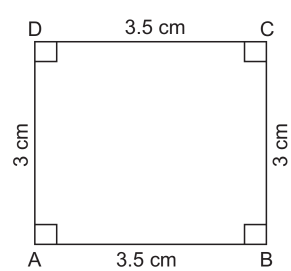


A (*i*)

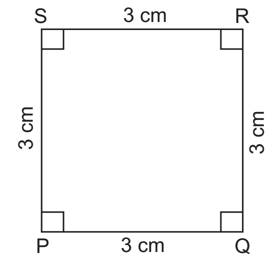


A (*ii*)

Figure B

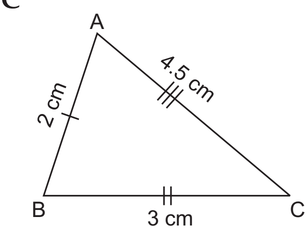


B (*iii*)

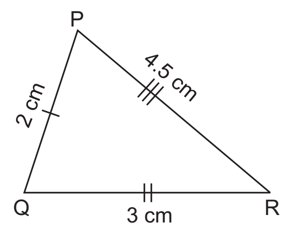


B (*iv*)

Figure C



C (*v*)



C (*vi*)

(*a*) Name the figure(s) wherein two figures are similar.

Sol. Figures A and C are similar.

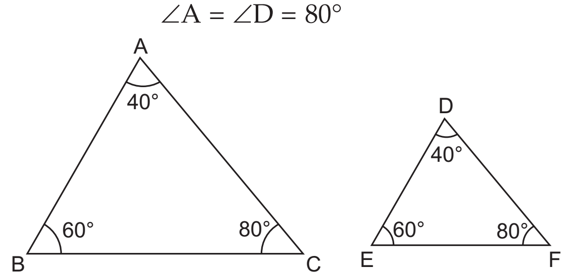
(*b*) Name the figure(s) wherein the figures are congruent.

Sol. Figure C is congruent

(*c*) (*i*) Prove that congruent triangles are also similar but not the converse.

Sol. In ΔABC and ΔDEF

$$\angle A = \angle D = 40°$$
$$\angle A = \angle D = 60°$$
$$\angle A = \angle D = 80°$$



By AA similarity criterion,

$$\Delta ABC \sim \Delta DEF$$

But, ΔABC and ΔDEF are not congruent because corresponding sides of two triangles are not equal.

or

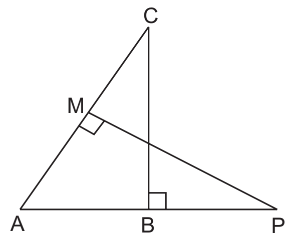
(*ii*) What more is least needed for two similar triangles to be congruent?

[CBSE 2023 Basic]

Sol. For the similar triangles to be congruent, the length of the corresponding sides must be equal.

Very Short Answer Type Questions

14. In the given figure, ABC and AMP are two right triangles, right-angled at B and M respectively. Prove that $\Delta ABC \sim \Delta AMP$.



[CBSE 2023 Basic]

Sol. In ΔABC and ΔAMP,

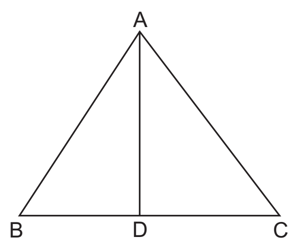
$$\angle ABC = \angle AMP = 90°$$

$$\angle A = \angle A \quad \text{[Common]}$$

$\therefore$ By AA similarity criterion,

$$\Delta ABC \sim \Delta AMP$$

15. D is a point on the side BC of ΔABC such that $\angle ADC = \angle BAC$. Show that $AC^2 = BC \times DC$



[CBSE 2024 Basic]

Sol. In ΔABC and ΔADC

$$\angle ADC = \angle BAC \quad \text{[Given]}$$

$$\angle C = \angle C \quad \text{[Common]}$$

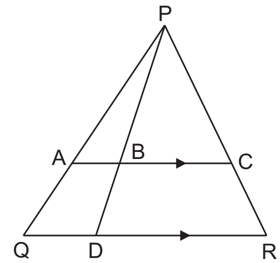
By AA similarity criterion,

$$\Delta ABC \sim \Delta ADC$$

$$\therefore \quad \frac{AC}{DC} = \frac{BC}{AC}$$

$$\Rightarrow \quad AC^2 = BC \times DC$$

16. In the given figure given, AC is drawn parallel to QR. If DR = 5 cm, QD = 3 cm, CR = 2 cm and PC = 3 cm, then find the length of AB.



Sol. In ΔPBC and ΔPDR,

$$\angle BPC = \angle DPR \quad \text{[Common]}$$

$$\angle PBC = \angle PDR$$

[Corresponding angles]

By AA similarity criterion,

$$\Delta PBC \sim \Delta PDR$$

$$\therefore \quad \frac{DR}{BC} = \frac{PR}{PC}$$

$$\Rightarrow \quad \frac{5\text{ cm}}{BC} = \frac{PC + CR}{PC}$$

$$\Rightarrow \quad \frac{5\text{ cm}}{BC} = \frac{5\text{ cm}}{3\text{ cm}}$$

$$\Rightarrow \quad BC = \frac{5 \times 3}{5}\text{ cm}$$

$$\Rightarrow \quad BC = 3\text{ cm} \quad \ldots(1)$$

In ΔPAC and ΔPQR,

$$\angle P = \angle P \quad \text{[Common]}$$

$$\angle PAC = \angle PQR$$

[Corresponding angles]

By AA similarity criterion,

$$\Delta PAC \sim \Delta PQR$$

$$\Rightarrow \quad \frac{QR}{AC} = \frac{PR}{PC}$$

$$\Rightarrow \quad \frac{QD + DR}{AB + BC} = \frac{PC + CR}{PC}$$

$$\Rightarrow \quad \frac{(3+5)\text{ cm}}{AB + 3\text{ cm}} = \frac{3\text{ cm} + 2\text{ cm}}{3\text{ cm}}$$

$$\Rightarrow \quad \frac{8\text{ cm}}{AB + 3\text{ cm}} = \frac{5\text{ cm}}{3\text{ cm}}$$

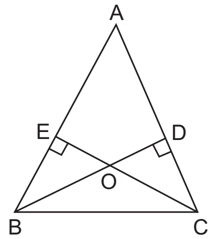
$$\Rightarrow \quad AB + 3\text{ cm} = \frac{8 \times 3}{5}\text{ cm}$$

$$\Rightarrow \quad AB = 4.8\text{ cm} - 3\text{ cm}$$

$$\Rightarrow \quad AB = 1.8\text{ cm}$$

17. In a ΔABC, BD and CE are perpendiculars to sides AC and AB respectively. Show that

$$AE \times BD = AD \times CE$$



Sol. In ΔAEC and ΔADB,

$$\angle A = \angle A \qquad \text{[Common]}$$

$$\angle AEC = \angle ADB = 90°$$

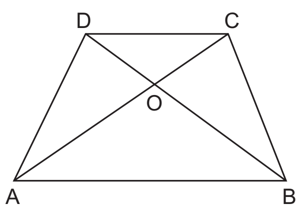
$\therefore$ By AA similarity criterion,

$$\Delta AEC \sim \Delta ADB$$

$$\therefore \quad \frac{AE}{AD} = \frac{CE}{BD}$$

$$\Rightarrow \quad AE \times BD = AD \times CE$$

18. Diagonals AC and BD of a trapezium ABCD intersect at O, where AB $\parallel$ DC. If $\frac{DO}{OB} = \frac{1}{2}$, then show that AB = 2CD.



[CBSE 2024 Standard]

Sol. In ΔAOB and ΔCOD,

$$\angle AOB = \angle COD \qquad \text{[Vertically opposite angles]}$$

$$\angle OAB = \angle OCD \qquad \text{[Alternate angles]}$$

$\therefore$ By AA similarity criterion,

$$\Delta AOB \sim \Delta COD$$

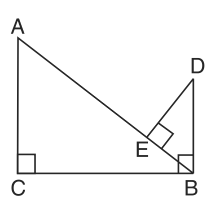
$$\therefore \quad \frac{OB}{OD} = \frac{AB}{CD}$$

$$\Rightarrow \quad 2 = \frac{AB}{CD} \qquad \left[\because \frac{DO}{OB} = \frac{1}{2}\right]$$

$$\Rightarrow \quad AB = 2CD$$

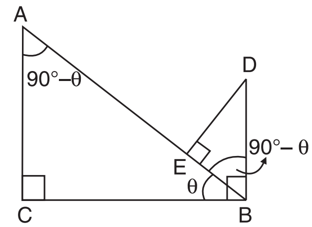
Short Answer Type Questions

19. In the given figure, $DB \perp BC$, $AC \perp BC$ and $DE \perp AB$. Prove that $\frac{BE}{DE} = \frac{AC}{BC}$. **[CBSE 2008]**



Sol. Given that ΔABC and ΔBED, where E is a point on AB, are two triangle such that

$$\angle ACB = \angle BED = \angle DBC = 90°$$



To prove that $\frac{BE}{DE} = \frac{AC}{BC}$

Let $\quad \angle ABC = \theta$

$$\therefore \quad \angle DBE = \angle DBC - \angle ABC$$

$$= 90° - \theta \qquad \ldots(1)$$

and $\quad \angle CAB = 90° - \theta \qquad \ldots(2)$

Now, in ΔACB and ΔBED, we have

$$\angle ACB = \angle BED = 90° \qquad \text{[Given]}$$

$$\angle CAB = \angle EBD = 90° - \theta$$

[From (1) and (2)]

$\therefore$ By AA similarity criterion, we have

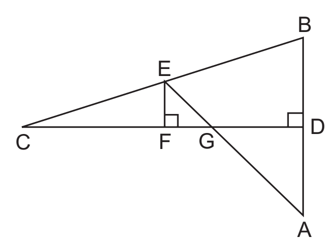
$$\Delta ACB \sim \Delta BED$$

$$\therefore \quad \frac{AC}{BE} = \frac{BC}{DE}$$

$$\Rightarrow \quad \frac{BE}{DE} = \frac{AC}{BC}$$

Hence, proved.

20. In the given figure, CD is the perpendicular bisector of AB. EF is perpendicular to CD. AE intersects CD at G. Prove that $\frac{CF}{CD} = \frac{FG}{DG}$.



[CBSE 2023 Standard]

Sol. In ΔCFE and CDB,

$$\angle C = \angle C \qquad \text{[Common]}$$

$$\angle CFE = \angle FDB = 90°$$

By AA similarity criterion,

$$\Delta CFE \sim \Delta CDB$$

$$\therefore \quad \frac{CF}{CD} = \frac{FE}{DB} \qquad \ldots(1)$$

In ΔFGE and ΔDGA

$$\angle FGE = \angle DGA \quad \text{[Vertically opposite angles]}$$

$$\angle EFG = \angle ADG = 90°$$

By AA similarity criterion,

$$\Delta FGF \sim \Delta DGA$$

$$\therefore \quad \frac{FG}{DG} = \frac{EF}{AD}$$

But $\quad AD = DB$

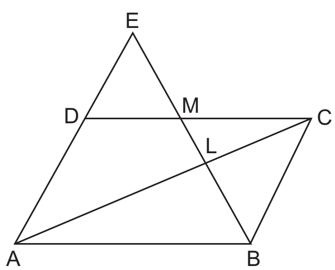
$$\Rightarrow \quad \frac{FG}{DG} = \frac{EF}{DB} \quad \ldots(2)$$

From equations (1) and (2), we have

$$\frac{CF}{CD} = \frac{FG}{DG}$$

Hence, proved.

21. In the given figure, ABCD is a parallelogram BE bisects CD at M and intersects AC at L. Prove that EL = 2BL.



Sol. In ΔEDM and ΔBCM,

$$MD = MC \quad \text{[M is the mid-point of CD]}$$

$$\angle DME = \angle CMB \quad \text{[Vertically opposite angles]}$$

$$\angle EDM = \angle BCM \quad \text{[Alternate angles]}$$

$$\therefore \quad \Delta EDM \cong \Delta BCM \quad \text{[ASA cong.]}$$

$$\Rightarrow \quad BC = ED \quad \ldots(1)$$

But $\quad BC = AD \quad [\because \text{ ABCD is a } \| \text{gm}] \ldots(2)$

$$\Rightarrow \quad AD + DE = BC + BC$$

$$\Rightarrow \quad AE = 2BC \quad \ldots(3)$$

In ΔALE and $\angle CLB$,

$$\angle ALE = \angle CLB \quad \text{[Vertically opposite angles]}$$

$$\angle EAL = \angle CBL \quad \text{[Alternate angles]}$$

$\therefore$ By AA similarly criterion,

$$\Delta ALE \sim \Delta CLB$$

$$\therefore \quad \frac{AE}{CB} = \frac{EL}{BL}$$

From (3), $\quad AE = 2BC$

$$\therefore \quad \frac{2BC}{BC} = \frac{EL}{BL}$$

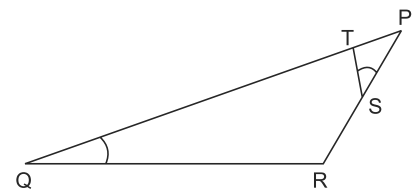
$$\Rightarrow \quad 2BL = EL$$

$$\therefore \quad EL = 2BL$$

Hence, proved.

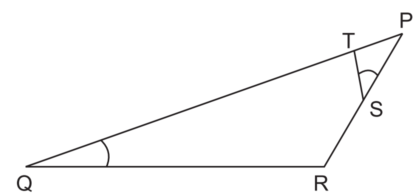
Long Answer Type Questions

22. In the given figure, $\angle PST = \angle PQR$. Prove that $\Delta PQR \sim \Delta PST$.



Hence, show that $PQ \times ST = PS \times QR$ and $PT \times QR = ST \times PR$.

Sol. Given that PQR is a triangle and S, T are points on RP and QP respectively such that TSP is a triangle with $\angle PST = \angle PQR$.



To prove that $\Delta PQR \sim \Delta PST$ and $PQ \times ST = PS \times QR$, $PT \times QR = ST \times PR$.

In ΔPQR and ΔPST, we have

$$\angle PQR = \angle PST \quad \text{[Given]}$$

$$\angle QPR = \angle SPT \quad \text{[Common]}$$

$\therefore$ By AA similarity criterion, we have

$$\Delta PQR \sim \Delta PST$$

$$\Rightarrow \quad \frac{PQ}{PS} = \frac{PR}{PT} = \frac{QR}{ST} \quad \ldots(1)$$

From (1), we have

$$\frac{PQ}{PS} = \frac{QR}{ST}$$

$$PQ \times ST = PS \times QR$$

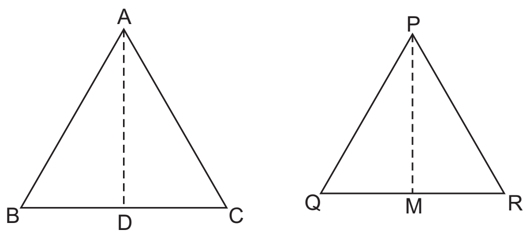
Again, from (1), we have

$$PT \times QR = ST \times PR$$

Hence, proved.

23. If AD and PM are medians of triangles ABC and PQR respectively where $\Delta ABC \sim \Delta PQR$, prove that $\frac{AB}{PQ} = \frac{AD}{PM}$. **[CBSE 2023 Standard]**

Sol.



$\Delta ABC \sim \Delta PQR$ [Given]

$\Rightarrow \quad \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$ ...(1)

$\angle A = \angle P, \angle B = \angle Q, \angle C = \angle R$ ...(2)

In ΔABC,

$BD = CD$ [D is the mid-point of BC]

$\Rightarrow \quad BC = 2BD$ ...(3)

In ΔPQR,

$QM = MR$ [M is the mid-point of QR]

$\Rightarrow \quad QR = 2QM$ ...(4)

In ΔABD and ΔPQM,

$\angle B = \angle Q$ [From (2)]

From (1), we have

$$\frac{AB}{PQ} = \frac{BC}{QR}$$

$\Rightarrow \quad \frac{AB}{PQ} = \frac{2BD}{2QM}$ [From (3) and (4)]

$\Rightarrow \quad \frac{AB}{PQ} = \frac{BD}{QM}$

$\therefore$ By SAS similarity criterion,

$\Delta ABD \sim \Delta PQM$

$\Rightarrow \quad \frac{AB}{PQ} = \frac{AD}{PM}$

Let's Compete

(Page 124)

Multiple-Choice Questions

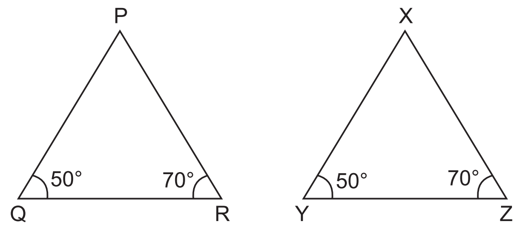
1. If $\Delta PQR \sim \Delta XYZ$, $\angle Q = 50°$ and $\angle R = 70°$, then $\angle X + \angle Y$ is equal to

(*a*) 70° (*b*) 110°

(*c*) 120° (*d*) 50°

Sol. (*b*) 110°

Given that $\Delta PQR \sim \Delta XYZ$, where $\angle Q = 50°$, $\angle R = 70°$.



To find $\angle X + \angle Y$.

In ΔPQR, we have

$\angle Q = 50°, \angle R = 70°$

$\therefore \quad \angle P = 180° - (50° + 70°)$

[By angle sum property of a triangle]

$\because \quad \Delta PQR \sim \Delta XYZ$

$\therefore \quad \angle X = \angle P = 60°, \angle Y = \angle Q = 50°$

$\therefore \quad \angle X + \angle Y = 60° + 50° = 110°$

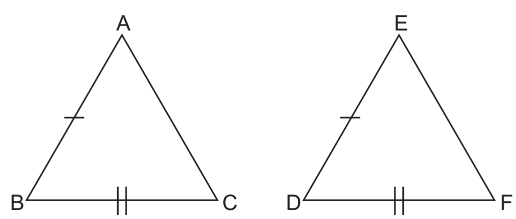
2. If in ΔABC and ΔDEF, $\frac{AB}{DE} = \frac{BC}{FD}$, then they will be similar when

(*a*) $\angle A = \angle F$ (*b*) $\angle A = \angle D$

(*c*) $\angle B = \angle D$ (*d*) $\angle B = \angle E$

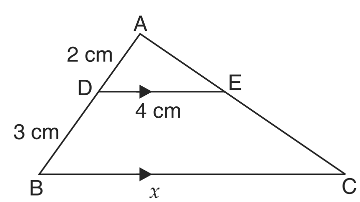
Sol. (*c*) $\angle B = \angle D$

Given that $\frac{AB}{DE} = \frac{BC}{FD}$ in ΔABC and ΔDEF.



Now, $\Delta ABC \sim \Delta EDF$ only if $\angle B = \angle D$ by SAS similarity criterion, since only $\angle B$ corresponds to $\angle D$.

3. In the given figure, DE ∥ BC. Then x equals to



(*a*) 6 cm (*b*) 8 cm

(*c*) 12 cm (*d*) 10 cm

Sol. (*d*) 10 cm

Given that D and E are two points on the sides AB and AC respectively of ΔABC such that DE ∥ BC. Also, given that AD = 2 cm, DB = 3 cm, DE = 4 cm and BC = x.

To find the value of x.

$\because \quad DE \parallel BC$

$\therefore \quad \Delta ADE \sim \Delta ABC$

$$\Rightarrow \quad \frac{AB}{AD} = \frac{BC}{DE}$$

$$\Rightarrow \quad \frac{2+3}{2} = \frac{x}{4}$$

$$\Rightarrow \quad \frac{5}{2} \times 4 = x$$

$$\Rightarrow \quad x = 10$$

Hence, the value of x is 10 cm.

4. P and Q are two points on the sides AB and AC respectively of a triangle ABC such that PQ $\parallel$ BC, AP = 3.5 cm, PB = 7 cm, AQ = 3 cm, QC = 6 cm and PQ = 4.5 cm. Then the measure of BC is equal to

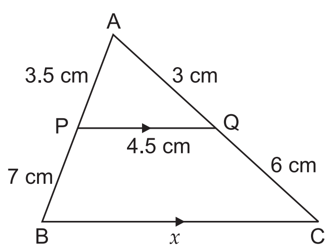
(*a*) 13.5 cm (*b*) 9 cm

(*c*) 12.5 cm (*d*) 15 cm

Sol. (*a*) 13.5 cm

Given that P and Q are two points on the sides AB and AC respectively of ΔABC, such that PQ $\parallel$ BC. Also, given that AP = 3.5 cm, PB = 7 cm, AQ = 3 cm, QC = 6 cm and PQ = 4.5 cm.

To find the length of BC.



Let BC = x

Since PQ $\parallel$ BC

$\therefore \quad \Delta APQ \sim \Delta ABC$

$$\Rightarrow \quad \frac{PQ}{BC} = \frac{AQ}{AC}$$

$$\Rightarrow \quad \frac{4.5}{x} = \frac{3}{3+6} = \frac{3}{9} = \frac{1}{3}$$

$$\Rightarrow \quad x = 4.5 \times 3 = 13.5$$

Hence, the required length of BC is 13.5 cm.

5. A vertical stick 30 m long casts a shadow 15 m long on the ground. At the same time, a tower casts a shadow 75 m long on the ground. Then the height of the tower is

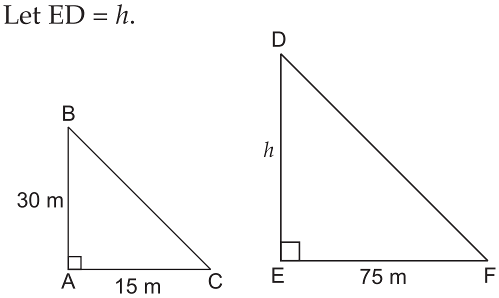
(*a*) 150 m (*b*) 100 m

(*c*) 25 m (*d*) 200 m

Sol. (*a*) 150 m

Let AB be the vertical stick and AC be its shadow on the horizontal ground. Given that AB = 30 m and AC = 15 m. Let DE be the vertical tower and EF = 75 m its shadow on the horizontal ground. To find the height of the tower ED.

Let ED = h.



Since the shadows are formed by the same sun, hence, $\angle ACB = \angle EFD$.

Also, $\angle BAC = \angle DEF = 90°$

$\therefore$ By AA similarity criterion, we have

$$\Delta ABC \sim \Delta EDF$$

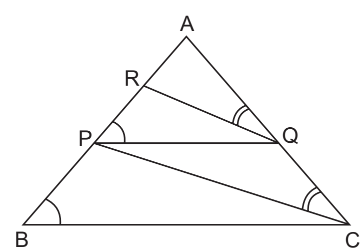
$$\Rightarrow \quad \frac{BA}{ED} = \frac{AC}{EF}$$

$$\Rightarrow \quad \frac{30}{h} = \frac{15}{75} = \frac{1}{5}$$

$$\Rightarrow \quad h = 30 \times 5 = 150$$

Hence, the required height of the tower is 150 m.

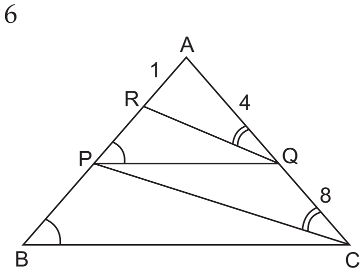
6. In the given figure, $\angle APQ = \angle ABC$, $\angle AQR = \angle ACP$. Given that AR = 1 unit, AQ = 4 units and QC = 8 units, then AR : PB equals



(*a*) 1 : 2 (*b*) 1 : 3

(*c*) 1 : 6 (*d*) 1 : 8

Sol. (*c*) 1 : 6



In ΔARQ and ΔAPC

$\angle A = \angle A$ [Common]

$\angle AQR = \angle ACP$ [Given]

By AA similarly criterion,

$$\Delta ARQ \sim \Delta APC$$

$$\Rightarrow \quad \frac{AR}{AP} = \frac{AQ}{AC}$$

$$\Rightarrow \quad \frac{1}{AP} = \frac{4}{12}$$

$$\Rightarrow \quad AP = 3 \text{ units} \quad \ldots(1)$$

In ΔAPQ and ΔABC,

$$\angle A = \angle A \quad \text{[Common]}$$

$$\angle APQ = \angle ABC \quad \text{[Given]}$$

$\therefore$ By AA similarity criterion,

$$\Delta APQ \sim \Delta ABC$$

$$\therefore \quad \frac{AP}{AB} = \frac{AQ}{AC}$$

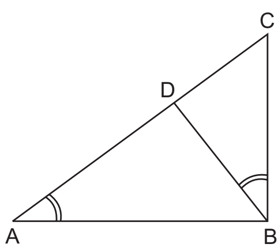
$$\Rightarrow \quad \frac{3}{AB} = \frac{4}{12}$$

$$\Rightarrow \quad AB = 9 \text{ units} \quad \ldots(2)$$

$$PB = 9 - 3 = 6 \text{ units}$$

$$\therefore \quad \frac{AR}{PB} = \frac{1}{6}$$

7. In the given figure, $\angle BAC = \angle CBD$. If BC = 16 cm, CD = 8 cm, then AC is equal to



(*a*) 12 cm (*b*) 16 cm
(*c*) 32 cm (*d*) 48 cm

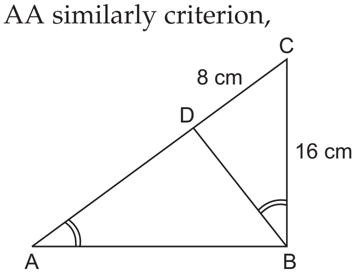
Sol. (*c*) 32 m

In ΔABC and ΔBDC,

$$\angle C = \angle C \quad \text{[Common]}$$

$$\angle BAC = \angle DBC \quad \text{[Given]}$$

$\therefore$ By AA similarly criterion,



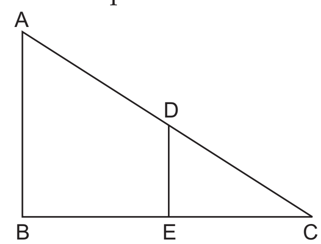
$$\Delta ABC \sim \Delta BDC$$

$$\therefore \quad \frac{BC}{CD} = \frac{AC}{BC}$$

$$\Rightarrow \quad AC = \frac{BC \times BC}{CD} = \frac{16 \times 16}{8} \text{ cm}$$

$$\Rightarrow \quad AC = 32 \text{ cm}$$

8. In ΔABC, DE $\parallel$ AB. If AB = a, DE = x, BE = b and EC = c, then x expressed in terms of a, b and c is



(*a*) $\frac{ac}{b}$ (*b*) $\frac{ac}{b+c}$
(*c*) $\frac{ab}{c}$ (*d*) $\frac{ab}{b+c}$ **[CBSE SP 2024]**

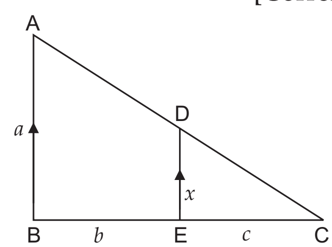
Sol. (*b*) $\frac{ac}{b+c}$

In ΔABC and ΔCDE,

$$\angle C = \angle C \quad \text{[Common]}$$

$$\angle ABC = \angle DEC$$

[Corresponding $\angle$s]



$\therefore$ By AA similarity criterion,

$$\frac{AB}{DE} = \frac{BC}{EC}$$

$$\Rightarrow \quad \frac{AB}{DE} = \frac{BE + EC}{EC}$$

$$\Rightarrow \quad \frac{a}{x} = \frac{b+c}{c}$$

$$\Rightarrow \quad x = \frac{ac}{b+c}$$

9. Which of the following may be similar to each other?

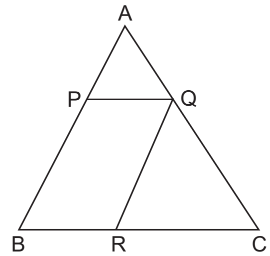
(*i*) Any two circles
(*ii*) Any two rhombuses
(*iii*) Any two regular hexagons.

Choose the correct option:

(*a*) only (*ii*) (*b*) only (*i*) and (*ii*)
(*c*) only (*i*) and (*iii*) (*d*) All

Sol. (*d*) All

10. In the given figure, AP : PB = 1 : 4, AQ : QC = 1 : 4 and BR : RC = 1 : 5, then



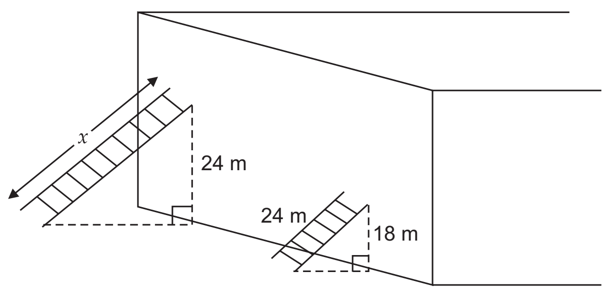
(*a*) PQ || BC (*b*) AP || RC
(*c*) AQ || PC (*d*) CQ || BP

Sol. (*a*) PQ || BC

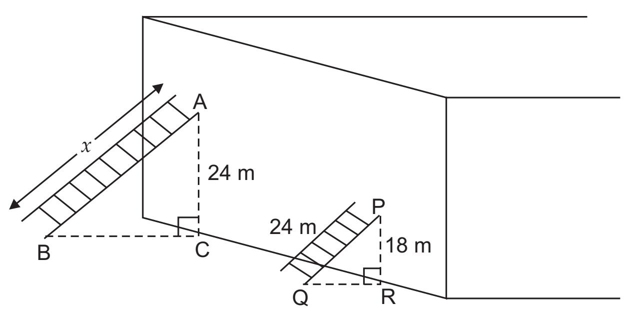
Life Skills

(Page 125)

1. Two ladders were kept leaning against a wall at the same angle as shown in the figure. Roshan was asked to determine the length of the bigger ladder. What is the length of the bigger ladder?



Sol.



In ΔABC and ΔPQR

$$\angle ABC = \angle PQR$$

[Both ladders are inclined to the wall at the same angle]

$$\angle ACB = \angle PRQ = 90°$$

$\therefore$ By AA similarity criterion,

$$\Delta ABC \sim \Delta PQR$$

$$\therefore \quad \frac{AC}{PR} = \frac{AB}{PQ}$$

$$\Rightarrow \quad \frac{24}{18} = \frac{x}{24} \text{ m}$$

$$\Rightarrow \quad x = \frac{24 \times 24}{18} \text{ m}$$

$$\Rightarrow \quad x = 32 \text{ m}$$

2. During the summer holidays, Radhika usually visited her ancestral village. The village was situated on the banks of a river. To measure the width of the river, she uses a banyan tree situated on the opposite bank as a reference point. She marks the position of the tree by placing a wooden pole. She walks 10 m down the bank and places another wooden pole at that point. She then further moves 5 m down the bank. At this point she takes a left turn and walks out in a perpendicular direction to the bank. She stops at a point where she is able to see both the tree and the wooden pole in a straight line. If she walks 15 m from the bank, find the width of the river.

Sol. AB = width of the river.

In ΔABC and ΔCDE

$\angle ACB = \angle ECD$ [Vertically opposite angles]

$\angle ABC = \angle EDC = 90°$

By AA similarity criterion,

$$\Delta ABC \sim \Delta EDC$$

$$\therefore \quad \frac{BC}{DC} = \frac{AB}{ED}$$

$$\Rightarrow \quad AB = \frac{BC \times ED}{DC} = \frac{10 \text{ m} \times 15 \text{ m}}{5 \text{ m}} = 30 \text{ m}$$