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Arithmetic Progressions

Checkpoint (Page 82)

1. Which of the following is (or are) a linear equation (or linear equations) in one variable?

(*a*) $ax^2 + bx + c = 0$ (*b*) $\sqrt{2}x - 3 = 6$

(*c*) $2x - y = 3$ (*d*) $x^2 + y^2 = a^2$

(*e*) $-\frac{2}{x} + 5x + 7 = 0$ (*f*) $n(n+1) - \frac{2n+1}{6} = 3$

Sol. (*b*) $\sqrt{2}x - 3 = 6$

Only in (*b*), we see that there is only one variable x with exponent 1.

2. If $2y - 3 = -5y + 11$, then the solution of this equation is

(*a*) $y = -2$ (*b*) $y = 3$

(*c*) $y = -3$ (*d*) $y = 2$

Sol. (*d*) $y = 2$

From the given equation, we see that

$$2y + 5y = 11 + 3$$

$$\Rightarrow \quad 7y = 14$$

$$\Rightarrow \quad y = \frac{14}{7} = 2$$

3. Find the correct answer from the following. The solution of the inequation $-3x - 2 > -6 + x$ is

(*a*) $x > 1$ (*b*) $x < -1$

(*c*) $x < 1$ (*d*) $-1 < x < 1$

Sol. (*c*) $x < 1$

We have $-3x - x > -6 + 2$

$$\Rightarrow \quad -4x > -4$$

$$\Rightarrow \quad x < 1$$

4. If $t_n = (-1)^n\, n + 5$, find the values of

(*a*) t_4 (*b*) $t_7 t_9$ (*c*) $\frac{t_3}{t_2}$

Sol. (*a*) For $n = 4$,

$$t_4 = (-1)^4 \times 4 + 5 = 4 + 5 = 9$$

(*b*) For $n = 7$,

$$t_7 = (-1)^7 \times 7 + 5 = -7 + 5 = -2$$

and for $n = 9$,

$$t_9 = (-1)^9 \times 9 + 5 = -9 + 5 = -4$$

$$\therefore \quad t_7 t_9 = (-2) \times (-4) = 8$$

which is the required value.

(*c*) For $n = 3$, $t_3 = (-1)^3 \times 3 + 5 = -3 + 5 = 2$

and for $n = 2$,

$$t_2 = (-1)^2 \times 2 + 5 = 2 + 5 = 7$$

$$\therefore \quad \frac{t_3}{t_2} = \frac{2}{7}$$

which is the required value.

5. If $t_n = \frac{2n+1}{n+1}$, find the value of t_{n-1} in terms of n.

Sol. Replacing n by $n - 1$, we get

$$t_{n-1} = \frac{2(n-1)+1}{n-1+1} = \frac{2n - 2 + 1}{n} = \frac{2n-1}{n} = 2 - \frac{1}{n}$$

6. If $(x + y)^2 = 4xy$, prove that $x = y$.

Sol. We have $(x + y)^2 - 4xy = 0$

$\Rightarrow \quad x^2 + y^2 + 2xy - 4xy = 0$

$\Rightarrow \quad x^2 + y^2 - 2xy = 0$

$\Rightarrow \quad (x - y)^2 = 0$

$\Rightarrow \quad x = y$

Hence, proved.

7. Find three consecutive positive integers whose sum is 219.

Sol. Let three consecutive positive integers be x, x+1 and $x + 2$. According to the problem, we have

$x + x + 1 + x + 2 = 219$

$\Rightarrow \quad 3x + 3 = 219$

$\Rightarrow \quad 3x = 219 - 3 = 216$

$\Rightarrow \quad x = \frac{216}{3} = 72$

$\therefore$ The required numbers are 72, 73 and 74.

8. The angles of a triangle are $3x°$, $(2x + 20)°$ and $(5x - 40)°$. Find the angles.

Sol. Using angle sum property of a triangle, we have

$3x° + (2x + 20)° + (5x - 40)° = 180°$

$\Rightarrow \quad 10x° = 180° + 20°$

$= 200°$

$\therefore \quad x = \frac{200°}{10}$

$= 20°$

$\therefore$ The required angles are $3 \times 20°$, $(2 \times 20 + 20)°$ and $(5 \times 20 - 40)°$, i.e. 60°, 60° and 60°, i.e. 60° each.

Check Your Progress 1

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Multiple-Choice Questions

1. The 15th term of the AP $-5, \frac{-5}{2}, 0, \frac{5}{2}, \ldots$ is

(*a*) –30 (*b*) 20

(*c*) 30 (*d*) –20

Sol. (*c*) 30

For the given AP, first term of $a = -5$, common difference, $d = -\frac{5}{2} - (-5) = \frac{-5}{2} + 5 = \frac{5}{2}$.

Let us denote the nth term of the AP by a_n.

$\therefore \quad a_n = a + (n - 1)d$

$= -5 + (n - 1) \times \frac{5}{2}$

$= -5 + \frac{5n}{2} - \frac{5}{2}$

$\Rightarrow \quad a_n = -\frac{15}{2} + \frac{5n}{2}$

$\therefore \quad a_{15} = -\frac{15}{2} + \frac{5}{2} \times 15$

$= \frac{75 - 15}{2}$

$= \frac{60}{2} = 30$

2. The 10th term from the end of the AP 7, 11, 15, 19, …, 103 is

(*a*) 64 (*b*) 67

(*c*) 70 (*d*) 54

Sol. (*b*) 67

Here, first term, $a = 7$, common difference, $d = 11 - 7 = 4$. Let n be the total number of terms of the AP and a_n be the nth term of the AP.

$\therefore \quad a_n = 103$

$\Rightarrow \quad a + (n - 1)d = 103$

$\Rightarrow \quad 7 + (n - 1)4 = 103$

$\Rightarrow \quad n - 1 = \frac{103 - 7}{4}$

$= \frac{96}{4} = 24$

$\therefore \quad n = 24 + 1$

$= 25$

Now, 10th term from the end = (25 – 10 + 1)th = 16th term from the beginning.

$\therefore \quad a_{16} = 7 + (16 - 1) \times 4$

$= 7 + 15 \times 4 = 67$

3. The 8th term of an AP whose first two terms are 5 and 11 is

(*a*) 40 (*b*) 46

(*c*) 35 (*d*) 47

Sol. (*d*) 47

Here, the first term, $a = 5$, common difference, $d = 11 - 5 = 6$.

$\therefore \quad a_8 = a + (8 - 1)d$

$= 5 + 7 \times 6$

$= 5 + 42$

$= 47$

4. The seventh term of an AP, whose first term is 28 and common difference –4, is

(*a*) 0 (*b*) 4

(*c*) 52 (*d*) 56 **[CBSE 2023 Basic]**

Sol. (*b*) 4

First term = 28

Common difference, $d = -4$

Seventh term, $a_7 = a + 6d$

$= 28 + 6(-4)$

$= 28 - 24$

$\therefore \quad a_7 = 4$

5. If $-5, x, 3$ are three consecutive terms of an AP, then the value of x is

(*a*) -2

(*b*) 2

(*c*) 1

(*d*) -1 **[CBSE 2023 Basic]**

Sol. (*d*) -1

$-5, x, 3$ are three consecutive terms of an AP

$$\begin{aligned} \therefore \quad x - (-5) &= 3 - x \\ \Rightarrow \quad x + 5 &= 3 - x \\ \Rightarrow \quad 2x &= -2 \\ \Rightarrow \quad x &= -1 \end{aligned}$$

6. If $p - 1$, $p + 1$ and $2p + 3$ are in AP, then the value of p is

(*a*) -2 (*b*) 0

(*c*) 1 (*d*) 2

[CBSE 2023 Standard]

Sol. (*b*) 0

$p - 1$, $p + 1$ and $2p + 3$ are in AP

$$\begin{aligned} \therefore \quad p + 1 - (p - 1) &= 2p + 3 - (p + 1) \\ \Rightarrow \quad p + 1 - p + 1 &= 2p + 3 - p - 1 \\ \Rightarrow \quad 2 &= p + 2 \\ \Rightarrow \quad p &= 0 \end{aligned}$$

7. The common difference of the AP whose nth term is given by $a_n = 3n + 7$, is

(*a*) 7 (*b*) 3

(*c*) $3n$ (*d*) 1

[CBSE 2023 Standard]

Sol. (*b*) 3

nth term is given by $a_n = 3n + 7$

$a_1 = 3 \times 1 + 7 = 10$

$a_2 = 3 \times 2 + 7 = 13$

Common difference $= a_2 - a_1 = 13 - 10 = 3$

8. In an AP, if $a = 8$ and $a_{10} = -19$, then the value of d is

(*a*) 3 (*b*) $\frac{-11}{9}$

(*c*) $\frac{-27}{10}$ (*d*) -3 **[CBSE 2024 Basic]**

Sol. (*d*) -3

$a = 8, a_{10} = -19$ [Given]

$$\begin{aligned} a_{10} &= a + 9d = -19 \\ \Rightarrow \quad 8 + 9d &= -19 \\ \Rightarrow \quad 9d &= -19 - 8 = -27 \\ \Rightarrow \quad d &= \frac{-27}{9} = -3 \end{aligned}$$

Very Short Answer Type Questions

9. If d be the common difference of an AP, and if each term of the AP is decreased by 4, what is the common difference of this new AP?

Sol. Let the original AP be $a, a + d, a + 2d, \dots a + (n - 1)d$.

Then the new AP is $a - 4, a + d - 4, a + 2d - 4, \dots a + (n - 1)d - 4$.

If D be the new common difference for the new AP, then

$$D = (a + d - 4) - (a - 4) = d$$

Hence, the required common difference of the new AP is also d.

10. If each term of an AP is multiplied by 3, will the resulting sequence be an AP? If so, what will be its common difference?

Sol. Let the original AP be a, $a + d$, $a + 2d$, ... , $a + (n - 1)d$.

Then the new sequence will be $3a$, $3(a + d)$, $3(a + 2d)$, ... , $3\{a + (n - 1)d\}$

Here $a_n = 3\{a + (n - 1)d\}$

$\therefore \quad a_{n-1} = 3\{a + (n - 2)d\}$

$\therefore$ If D be the common difference of this new sequence, then

$$\begin{aligned} D &= a_n - a_{n-1} \\ &= 3a + 3(n - 1)d - 3a - 3(n - 2)d \\ &= 3nd - 3d - 3nd + 6d \\ &= 3d \end{aligned}$$

which is independent of n.

$\because$ D is a constant

$\therefore$ Yes, the new sequence will also be an AP with common difference equal to 3 times the common difference of the original AP.

11. There are 1000 terms in an AP. If $a_{107} = a_{106} + 3a + b$ where $a > 0$ and $b > 0$ and the successive terms of the AP are in increasing order, what is its common difference?

Sol. Successive terms of the given AP are a_{106} and a_{107}. We have

$$a_{107} - a_{106} = 3a + b$$

$$a + 106d - a - 105d = 3a + b$$
$$d = 3a + b$$

Hence, the required common difference is $3a + b$.

12. If there are 60 terms in an AP, what will be the position of a certain term of the AP from the beginning, if its position from the end is 19th?

Sol. 19th term from the end of the given AP is $(60 - 19 + 1)$th term, i.e. 42nd term from the beginning. Hence, the required position is 42nd term from the beginning.

13. If the simple interest rate on ₹200 is 5% every year, then what will be its total interest at the end of 4th year? Are the total interests at the end of 1st year, 2nd year, 3rd year, etc. in AP? If so, what is the common difference of this AP?

Sol. We see that simple interest for 1 year on ₹200 is ₹ $\frac{200 \times 5 \times 1}{100}$, i.e. ₹ $\frac{2 \times 5}{1}$, i.e. 10

$\therefore$ Interests at the end of 1st year, 2nd year, 3rd year, 4th year, etc. will be ₹10, ₹20, ₹30, ₹40, … etc respectively, which is clearly an AP with common difference of ₹10. Also, the required total interest at the end of 4 years is ₹40.

14. Insert a number between 2 and 3 such that these three numbers are in AP.

Sol. Let the three numbers in AP be $2, 2 + d, 3$, where d is the common difference.

$$\therefore \quad 2 + d - 2 = 3 - 2 - d$$
$$= 1 - d$$
$$\Rightarrow \quad 2d = 1$$
$$\Rightarrow \quad d = \frac{1}{2}$$

$\therefore$ The required middle term between 2 and 3 is $2 + d$, i.e. $2 + \frac{1}{2}$, i.e. $\frac{5}{2}$.

15. Find the sequence whose nth term is $\frac{n^2}{n+3}$.

Sol. If a_n be the nth term of the sequence, then

$$a_n = \frac{n^2}{n + 3}$$
$$\therefore \quad a_1 = \frac{1}{1 + 3} = \frac{1}{4},$$
$$a_2 = \frac{2^2}{2 + 3} = \frac{4}{5},$$
$$a_3 = \frac{3^2}{3 + 3} = \frac{9}{6} = \frac{3}{2},$$
$$a_4 = \frac{4^2}{4 + 3} = \frac{16}{7}, \text{ etc.}$$

Hence, the required sequence is $\frac{1}{4}, \frac{4}{5}, \frac{3}{2}, \frac{16}{7}, \ldots$

16. Find the 7th term of the AP 1, 6, 11, 16,…

Sol. Here, first term, $a = 1$, common difference, $d = 6 - 1 = 5$.

$\therefore$ 7th term, $a_7 = a + (7 - 1)d = 1 + 6 \times 5 = 31$

$\therefore$ The required 7th term is 31.

17. Find the 10th term of the AP $\sqrt{2}, \sqrt{8}, \sqrt{18}, \ldots$ **[CBSE 2015]**

Sol. Here, first term, $a = \sqrt{2}$, common difference, $d = \sqrt{8} - \sqrt{2} = 2\sqrt{2} - \sqrt{2} = \sqrt{2}$

$$\therefore \quad a_{10} = a + (10 - 1)d$$
$$= \sqrt{2} + 9 \times \sqrt{2}$$
$$= 10\sqrt{2}$$
$$= \sqrt{200}$$

$\therefore$ The required 10th term is $\sqrt{200}$.

18. Find the 25th term of the AP $-5, -\frac{5}{2}, 0, \frac{5}{2}, \ldots$ **[CBSE 2015]**

Sol. Here, first term, $a = -5$, common difference, $d = -\frac{5}{2} + 5 = \frac{5}{2}$.

$$\therefore \quad a_{25} = a + (25 - 1)d$$
$$= -5 + 24 \times \frac{5}{2}$$
$$= 60 - 5$$
$$= 55$$

$\therefore$ The required 25th term is 55.

19. The nth term of an AP is $7 - 4n$. Find its common difference. **[CBSE 2008]**

Sol. We have $a_n = 7 - 4n$

$$\therefore \quad a_{n-1} = 7 - 4(n - 1)$$
$$= 7 - 4n + 4$$
$$= 11 - 4n$$
$$\therefore \quad a_n - a_{n-1} = 7 - 4n - (11 - 4n)$$
$$= -4$$

$\therefore$ The required common difference is -4.

Short Answer Type Questions

20. Which term of the AP 21, 18, 15, … is 0? **[CBSE SP 2012]**

Sol. Here, first term $a = 21$, common difference, $d = 18 - 21 = -3$. If a_n be 0, then

$$a_n = a + (n - 1)d = 0$$
$$\Rightarrow \quad 21 - 3(n - 1) = 0$$
$$\Rightarrow \quad n - 1 = \frac{21}{3} = 7$$

$$\Rightarrow \quad n = 7 + 1 = 8$$

Hence, the required term is 8th term.

21. How many terms are there in an AP whose first and fifth terms are –14 and 2, respectively and the last term is 62. **[CBSE 2023 Standard]**

Sol.

$$a = -14$$
$$a_5 = 2$$
$$a_n = 62$$
$$a_5 = a + 4d = 2$$
$$\Rightarrow \quad -14 + 4d = 2$$
$$\Rightarrow \quad 4d = 2 + 14 = 16$$
$$\Rightarrow \quad d = \frac{16}{4} = 4$$
$$a_n = a + (n-1)d = 62$$
$$\Rightarrow \quad -14 + (n-1) \times 4 = 62$$
$$\Rightarrow \quad -14 + 4n - 4 = 62$$
$$\Rightarrow \quad 4n = 62 + 18$$
$$\Rightarrow \quad 4n = 80$$
$$\Rightarrow \quad n = 20$$
$$\therefore \quad \text{Number of terms} = 20$$

22. Which term of the AP 3, 10, 17, 24,… will be 84 more than its 13th term? **[CBSE 2004]**

Sol. Here, first term, $a = 3$, common difference, $d = 10 - 3 = 7$.

$$\therefore \quad a_{13} = a + (13-1)d$$
$$= 3 + 12 \times 7$$
$$= 3 + 84$$
$$= 87$$

Let nth term, a_n be 84 + 87 = 171

$$\therefore \quad a_n = 171$$
$$\Rightarrow \quad a + (n-1)d = 171$$
$$\Rightarrow \quad 3 + (n-1) \times 7 = 171$$
$$\Rightarrow \quad (n-1) \times 7 = 171 - 3 = 168$$
$$\Rightarrow \quad n - 1 = \frac{168}{7} = 24$$
$$\Rightarrow \quad n = 24 + 1 = 25$$

$\therefore$ The required term is 25th term.

23. The 7th term of an AP is – 4 and its 13th term is –16. Find the AP. **[CBSE 2004, SP 2011]**

Sol. Let a be the 1st term and d be the common difference of the AP.

Then according to the problem, we have

$$a_7 = -4$$
$$\Rightarrow \quad a + (7-1)d = -4$$
$$\Rightarrow \quad a + 6d + 4 = 0 \qquad \ldots(1)$$

and $\quad a_{13} = -16$

$$\Rightarrow \quad a + (13-1)d = -16$$
$$\Rightarrow \quad a + 12d + 16 = 0 \qquad \ldots(2)$$

Subtracting (1) from (2), we get

$$6d = -12$$
$$\Rightarrow \quad d = -\frac{12}{6} = -2$$

$\therefore$ From (1),

$$a = -4 - 6d$$
$$= -4 - 6 \times (-2)$$
$$= -4 + 12$$
$$= 8$$

Hence, the required AP is 8, 6, 4, 2, 0, –2, …

24. The 9th term of an AP is –32 and the sum of its 11th and 13th terms is –94. Find the common difference of the AP. **[CBSE 2013]**

Sol. Let a be the 1st term, d be the common difference and let a_n be the nth term. Now, according to the problem, we have

$$a_9 = -32$$
$$\Rightarrow \quad a + (9-1)d = -32$$
$$\Rightarrow \quad a + 8d = -32 \qquad \ldots(1)$$
$$a_{11} + a_{13} = -94$$
$$\Rightarrow \quad a + (11-1)d + a + (13-1)d = -94$$
$$\Rightarrow \quad 2a + 22d = -94$$
$$\Rightarrow \quad a + 11d = -47 \qquad \ldots(2)$$

$\therefore$ Subtracting (2) from (1), we get

$$-3d = 47 - 32 = 15$$
$$\Rightarrow \quad d = \frac{15}{-3} = -5$$

Hence, the required common difference is –5.

25. The angles of a quadrilateral are in AP. If the least angle is 60°, find the other angles.

Sol. The least angle of a quadrilateral = 60° [Given]

Let the other angles of the quadrilateral be $60° + d°$, $60° + 2d°$ and $60° + 3d°$.

$$\therefore \quad 60° + 60° + d° + 60° + 2d° + 60° + 3d° = 360°$$

[By angle sum property of a quadrilateral]

$$\Rightarrow \quad 6d° = 360° - 240° = 120°$$
$$\therefore \quad d° = \frac{120°}{6} = 20°$$

$\therefore$ The required angles of the quadrilateral are $60° + 20°$, $60° + 40°$, $60° + 60°$, i.e. 80°, 100° and 120°.

26. If pth term of an AP is q and qth term is p, then prove that its nth term is $(p + q - n)$.

[CBSE 2023 Standard]

Sol. Let a be the first term and the common difference be d.

pth term is q

$$\Rightarrow \quad a + (p-1)d = q \qquad \ldots(1)$$

qth term is p

$$\Rightarrow \quad a + (q-1)d = p \qquad \ldots(2)$$

Subtracting equation (2) from (1), we get

$$\begin{aligned} (p-1)d - (q-1)d &= q - p \\ \Rightarrow \quad pd - d - qd + d &= q - p \\ \Rightarrow \quad pd - qd &= q - p \\ \Rightarrow \quad d(p-q) &= -1(p-q) \\ \Rightarrow \quad d &= -1 \end{aligned}$$

Putting $d = -1$ in (1), we get

$$\begin{aligned} a + (p-1) \times -1 &= q \\ \Rightarrow \quad a - p + 1 &= q \\ \Rightarrow \quad a &= p + q - 1 \qquad \ldots(3) \end{aligned}$$

nth term, $a_n = a + (n-1)d$

$$\begin{aligned} a_n &= p + q - 1 + (n-1) \times -1 \\ &= p + q - 1 - n + 1 \\ \therefore \quad a_n &= p + q - n \end{aligned}$$

Long Answer Type Questions

27. Each of the AP's 2, 4, 6, 8,… and 3, 6, 9, 12, … is continued to 200 terms. How many terms of these two AP's are identical?

Sol. We see that for the 1st AP, the first term, $a = 2$, common difference, $d = 4 - 2 = 2$. If a_n denote the nth term of this AP, then

$$\begin{aligned} a_{200} &= a + (n-1)d \\ &= 2 + 199 \times 2 \\ &= 2 \times 200 \\ &= 400 \end{aligned}$$

Similarly, for the 2nd AP, $a' = 3$, $d' = 6 - 3 = 3$

$$\begin{aligned} \therefore \quad a'_{200} &= a' + (200-1)d' \\ &= 3 + 199 \times 3 \\ &= 3 \times 200 \\ &= 600 \end{aligned}$$

$$\because \quad 400 < 600$$

$\therefore$ The identical terms in the two AP's will be upto 400 only.

Now, the terms in the 1st AP are multiples of 2 and those in the 2nd AP are multiples of 3. Hence, the identical terms of the two AP's will be multiples of $3 \times 2 = 6$. Hence, the terms which are identical to the two AP's are 6, 12, 18, 24, … which is again another AP with $a = 6$ and $d = 12 - 6 = 6$. Let the mth term of this AP be 400.

$$\begin{aligned} \therefore \quad a_m &= 6 + (m-1)6 \le 400 \\ \Rightarrow \quad 6m &\le 400 \\ \Rightarrow \quad m &\le 66\frac{2}{3} \end{aligned}$$

Since m is a positive integer, hence $m = 66$ which is the required number of terms.

28. If $\frac{b+c-a}{a}$, $\frac{c+a-b}{b}$ and $\frac{a+b-c}{c}$ are in AP and $a + b + c \neq 0$, then show that $\frac{1}{a}$, $\frac{1}{b}$ and $\frac{1}{c}$ are in AP.

Sol. Since, $\frac{b+c-a}{a}$, $\frac{c+a-b}{b}$ and $\frac{a+b-c}{c}$ are in AP

$$\therefore \quad \frac{2(c+a-b)}{b} + 2 = \frac{b+c-a}{a} + 1 + \frac{a+b-c}{c} + 1$$

$$\frac{2(a+c)}{b} - \frac{2b}{b} + 2 = \frac{b+c}{a} - \frac{a}{a} + 1 + \frac{a+b}{c} - \frac{c}{c} + 1$$

$$\frac{2(a+c)}{b} - 2 + 2 = \frac{b+c}{a} - 1 + 1 + \frac{a+b}{c} - 1 + 1$$

$$\frac{2(a+c)}{b} = \frac{b+c}{a} + \frac{a+b}{c}$$

$$\Rightarrow \quad \frac{2(a+c)}{b} + 2 = \left(\frac{b+c}{a} + 1\right) + \left(\frac{a+b}{c} + 1\right)$$

$$\Rightarrow \quad \frac{2(a+b+c)}{b} = \frac{a+b+c}{b} + \frac{a+b+c}{c}$$

$$\Rightarrow \quad \frac{2}{b} = \frac{1}{a} + \frac{1}{c}$$

[Dividing both sides by $a + b + c$, where $a + b + c \neq 0$]

Hence, $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in AP.

29. If $l^2(m + n)$, $m^2(n + l)$ and $n^2(l + m)$ are in AP, prove that either l, m, n are in AP or $lm + mn + nl = 0$.

Sol. Given that $l^2(m + n)$, $m^2(n + l)$ and $n^2(l + m)$ are in AP

$$\therefore \quad 2m^2(n+l) = l^2(m+n) + n^2(l+m)$$

$$\Rightarrow \quad 2m^2n + 2m^2l - l^2m - l^2n - n^2l - n^2m = 0 \qquad \ldots(1)$$

Now, l, m, n will be in AP if $2m = l + n$ $\quad \ldots(2)$

$\therefore$ l, m, n will be in AP or, $lm + mn + nl = 0$ $\quad \ldots(3)$

If either (2) is true or (3) is true.

i.e. if $(2m - l - n)(lm + mn + nl) = 0$

i.e. if $2m^2l + 2m^2n + 2mnl - l^2m - lmn - nl^2 - nlm - mn^2 - n^2l = 0$

i.e. if $2m^2n + 2m^2l - l^2m - l^2n - n^2l - n^2m = 0$

which is true. [From (1)]

Hence, proved.

30. Divide 30 into four parts which are in AP such that the ratio of the product of the first and the fourth terms and the product of the second and third terms is 2 : 3.

Sol. Let a, $a + d$, $a + 2d$, $a + 3d$ be four terms of an AP

Given that

$$a + a + d + a + 2d + a + 3d = 4a + 6d = 30$$

$$\Rightarrow \quad 2a + 3d = 15$$

$$\Rightarrow \quad a = \frac{15 - 3d}{2} \quad \ldots(1)$$

Also, $$\frac{a(a + 3d)}{(a + d)(a + 2d)} = \frac{2}{3}$$

$$\Rightarrow \quad 3a^2 + 9ad = 2a^2 + 6ad + 4d^2$$

$$\Rightarrow \quad a^2 + 3ad - 4d^2 = 0$$

$$\Rightarrow \quad \frac{(15 - 3d)^2}{4} + 3d \times \frac{15 - 3d}{2} - 4d^2 = 0$$

[From (1)]

$$\Rightarrow \quad \frac{225 + 9d^2 - 90d}{4} + \frac{45d - 9d^2}{2} - 4d^2 = 0$$

$$\Rightarrow \quad 225 + 9d^2 - 90d + 90d - 18d^2 - 16d^2 = 0$$

$$\Rightarrow \quad 25d^2 = 225$$

$$\Rightarrow \quad d^2 = 9$$

$$\Rightarrow \quad d = \pm 3$$

...(2)

When $d = 3$, then from (1), $a = \dfrac{15 - 3 \times 3}{2} = 3$

When $d = -3$, then from (1), $a = \dfrac{15 + 3 \times 3}{2} = 12$

Hence, the required terms are either 3, 6, 9, 12 or 12, 9, 6, 3.

31. If the pth terms of the progressions 2, $3\frac{11}{18}$, $5\frac{2}{9}$, $6\frac{5}{6}$, ... and 94, $91\frac{7}{9}$, $89\frac{5}{9}$, $87\frac{1}{3}$, ... be the same, find the value of p and the value of the term.

Sol. For the 1st AP, the first term, $a = 2$ and the common difference, $d = 3\frac{11}{18} - 2 = \frac{65}{18} - 2$

$= \frac{65 - 36}{18} = \frac{29}{18}$, and for the 2nd AP, $a' = 94$ and

$$d' = 91\frac{7}{9} - 94 = \frac{826 - 846}{9} = -\frac{20}{9}$$

Given, the pth terms for both the AP's are same.

Now, for the 1st AP, $a_p = 2 + (p - 1)\frac{29}{18}$

and for the 2nd AP, $a'_p = 94 - (p - 1)\frac{20}{9}$

$$\therefore \quad 2 + (p - 1)\frac{29}{18} = 94 - (p - 1)\frac{20}{9}$$

$$\Rightarrow \quad (p - 1)\left(\frac{29}{18} + \frac{20}{9}\right) = 94 - 2$$

$$\Rightarrow \quad (p - 1)\left(\frac{29 + 40}{18}\right) = 92$$

$$\Rightarrow \quad p - 1 = 92 \times \frac{18}{69}$$

$$\Rightarrow \quad p - 1 = 4 \times 6$$

$$\Rightarrow \quad p - 1 = 24$$

$$\therefore \quad p = 25$$

Now, $$a_{25} = 2 + 24 \times \frac{29}{18} = 2 + 4 \times \frac{29}{3} = 2 + \frac{116}{3} = \frac{6 + 116}{3} = \frac{122}{3} = 40\frac{2}{3}$$

Hence, the required value of p is 25 and the required 25th term is $40\frac{2}{3}$.

Check Your Progress 2

(Page 93)

Multiple-Choice Questions

1. The nth term of an AP whose sum of n terms is S_n, is

(*a*) $S_n - S_{n+1}$ (*b*) $S_n + S_{n-1}$

(*c*) $S_n + S_{n+1}$ (*d*) $S_n - S_{n-1}$

Sol. (*d*) $S_n - S_{n-1}$

We have $$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$= na + \frac{n(n-1)d}{2}$$

and $$S_{n-1} = \frac{n-1}{2}[2a + (n-2)d]$$
$$= (n-1)a + \frac{(n-1)(n-2)d}{2}$$

$\therefore$ $S_n - S_{n-1}$
$$= na + \frac{n(n-1)d}{2} - (n-1)a - \frac{(n-1)(n-2)d}{2}$$
$$= na - na + a + \frac{d(n^2 - n - n^2 + 3n - 2)}{2}$$
$$= a + \frac{d(2n-2)}{2}$$
$$= a + (n-1)d$$
$$= a_n$$

$\therefore$ The nth term of an AP whose sum of n terms is S_n, is $S_n - S_{n-1}$.

2. The nth term of an AP whose sum is given by $S_n = \frac{5n^2}{2} + \frac{3n}{2}$, will be

(*a*) $7n - 1$ (*b*) $5n + 1$
(*c*) $6n - 1$ (*d*) $5n - 1$

Sol. (*d*) $5n - 1$

We have
$$a_n = S_n - S_{n-1}$$
$$= \frac{5n^2}{2} + \frac{3n}{2} - \frac{5(n-1)^2}{2} - \frac{3(n-1)}{2}$$
$$= \frac{5n^2 - 5(n-1)^2}{2} + \frac{3n}{2} - \frac{3n}{2} + \frac{3}{2}$$
$$= \frac{5(n+n-1)(n-n+1)}{2} + \frac{3}{2}$$
$$= \frac{5(2n-1)+3}{2}$$
$$= \frac{10n-2}{2}$$
$$= 5n - 1$$

$\therefore$ The required nth term of an AP is $5n - 1$.

3. The sum of the first 21 terms of an AP 16, 12, 8, 4, … is

(*a*) –480 (*b*) –504
(*c*) 1176 (*d*) –484

[CBSE 2023 Basic]

Sol. (*b*) –504

The sum of n terms is given by,
$$S_n = \frac{n}{2}[2a + (n-1)d]$$

Here $n = 21$
$a = 16$
$d = 12 - 16 = -4$

$$\therefore \quad S_{21} = \frac{21}{2}[2 \times 16 + (21-1)(-4)]$$
$$= \frac{21}{2}[32 - 80]$$
$$= \frac{21}{2} \times -48 = -21 \times 24$$
$$= -504$$

4. If the nth term of an AP is $(2n + 1)$, then the sum of the first n terms of the AP is

(*a*) $n(n + 2)$ (*b*) $n(n - 2)$
(*c*) $n(n + 1)$ (*d*) $n(n - 1)$

Sol. (*a*) $n(n + 2)$

We have
$$a_p = 2p + 1$$
$$\therefore \quad S_n = \sum_{p=1}^{n} a_p$$
$$= 2\sum_{p=1}^{n} p + \sum_{p=1}^{n} 1$$
$$= 2(1 + 2 + 3 + \ldots + n) + n$$
$$= 2 \times \frac{n(n+1)}{2} + n$$
$$= n(n + 1 + 1)$$
$$= n(n + 2)$$

which is the required sum.

5. The sum of first seven terms of an AP is 49 and that of 17 terms is 289. The sum of first n terms is

(*a*) $2n$ (*b*) $\frac{n(n+1)}{2}$
(*c*) n^2 (*d*) $\frac{n^2+1}{2}$

Sol. (*c*) n^2

Let a be the 1st term, d be the common difference and S_n be the sum of n terms of an AP

Then $$S_7 = \frac{7}{2}\{2a + (7-1)d\} = 49$$
$\Rightarrow$ $a + 3d = 7$ …(1)

and $$S_{17} = \frac{17}{2}\{2a + (17-1)d\}$$
$$= 289$$
$\Rightarrow$ $a + 8d = 17$ …(2)

Subtracting (1) from (2), we have
$$5d = 10$$

$$\Rightarrow \quad d = \frac{10}{5} = 2$$

$\therefore$ From (1), $\quad a = 7 - 3 \times 2 = 1$

Now, $\quad S_n = \frac{n}{2}\{2a + (n-1)d\}$

$$= \frac{n}{2}\{2 + (n-1)2\}$$

$$= n^2$$

$\therefore$ The sum of first n terms is n^2.

6. In an AP, if the first term $a = 7$, nth term $a_n = 84$ and the sum of first n terms $S_n = \frac{2093}{2}$, then n is equal to

(*a*) 22 (*b*) 24

(*c*) 23 (*d*) 26

[CBSE 2024 Standard]

Sol. (*c*) 23

We have,

$$a = 7$$

$$a_n = 84$$

$$S_n = \frac{2093}{2}$$

$$a_n = a + (n-1)d$$

$$84 = 7 + (n-1)d$$

$$\Rightarrow \quad (n-1)d = 84 - 7$$

$$\Rightarrow \quad (n-1)d = 77 \quad \ldots(1)$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\Rightarrow \quad \frac{n}{2}[2a + (n-1)d] = \frac{2093}{2}$$

$$\Rightarrow \quad \frac{n}{2}[2 \times 7 + 77] = \frac{2093}{2} \quad \text{[From (1)]}$$

$$\Rightarrow \quad \frac{n}{2}[91] = \frac{2093}{2}$$

$$\Rightarrow \quad n = \frac{2093}{2} \times \frac{2}{91} = 23$$

Very Short Answer Type Questions

7. If all the terms of a sequence are equal, can it be an AP? If so, what is its common difference?

Sol. Let the sequence be $a, a, a, a, \ldots$

Here, we see that the common difference d is $a - a = 0$ which is a constant. Hence, it can be called an AP with common difference 0.

8. If the first terms and the common differences of two AP's are the same. Will they have the same number of terms? Give an example.

Sol. Not necessarily, because the total number of terms of the two AP's may be different. For example:

Two AP's are

(*i*) $a, a + d, a + 2d, \ldots a + 100d$ and

(*ii*) $a, a + d, a + 2d, \ldots a + 100d, a + 101d$

They have 101 and 102 terms respectively, and so they are different AP's.

9. If the sum of the first four terms of an AP is 19 and that of first three terms is 16, what is its fourth term?

Sol. Let a be the 1st term and d be the common difference.

Then, we have

$$a + (a + d) + (a + 2d) + (a + 3d) = 19$$

$$\Rightarrow \quad 4a + 6d = 19 \quad \ldots(1)$$

Also, $\quad a + (a + d) + (a + 2d) = 16$

$$\Rightarrow \quad 3a + 3d = 16$$

$$\Rightarrow \quad 6a + 6d = 32 \quad \ldots(2)$$

Subtracting (1) from (2), we get

$$2a = 32 - 19 = 13$$

$$\Rightarrow \quad a = \frac{13}{2}$$

$\therefore$ From (1), $\quad 6d = 19 - 4a$

$$= 19 - 4 \times \frac{13}{2}$$

$$= 19 - 13 \times 2$$

$$= 19 - 26$$

$$= -7$$

$$\therefore \quad d = \frac{-7}{6}$$

$\therefore$ The fourth term of the AP is

$$a + 3d = \frac{13}{2} - 3 \times \frac{7}{6}$$

$$= \frac{13}{2} - \frac{7}{2}$$

$$= \frac{6}{2} = 3$$

10. Find the sum of the AP $x + y, x - y, x - 3y, \ldots$ to 20 terms.

Sol. The first term of an AP, $a = x + y$ and the common difference, $d = x - y - x - y = -2y$.

We have $\quad S_{20} = \frac{20}{2}(2a + (20-1)d)$

$$S_{20} = \frac{20}{2}\{2(x + y) + 19 \times (-2y)\}$$

$$= 10\{2(x + y) + 19 \times (-2y)\}$$

$$= 20(x + y) + 190 \times (-2y)$$

$= 20x + 20y - 380y$

$= 20x - 360y$

$\therefore$ The required sum is $20x - 360y$.

11. Find the sum of the first 24 terms of the AP 99, 96, 93, 90, …

Sol. The first term, $a = 99$ and the common difference, $d = 96 - 99 = -3$. Let S_n denote the sum to n terms of the AP.

$$\therefore \quad S_{24} = \frac{24}{2}\{2a + (24-1)d\}$$
$$= \frac{24}{2}\{2 \times 99 + 23 \times (-3)\}$$
$$= 24 \times 99 - 12 \times 23 \times 3$$
$$= 2376 - 828 = 1548$$

$\therefore$ The required sum is 1548.

12. If the sum of n terms of an AP is given by $S_n = \frac{n(5n+7)}{12}$, find its a_n.

Sol. We know that the nth term, a_n is given by

$$a_n = S_n - S_{n-1}$$
$$= \frac{n(5n+7)}{12} - \frac{(n-1)[5(n-1)+7]}{12}$$
$$= \frac{n(5n+7)}{12} - \frac{(n-1)(5n+2)}{12}$$
$$= \frac{5n^2 + 7n - 5n^2 - 2n + 5n + 2}{12}$$
$$= \frac{10n+2}{12}$$
$$= \frac{5n+1}{6}$$

$\therefore$ The value of a_n is $\frac{5n+1}{6}$.

13. Find the sum of all even numbers between 9 and 99.

Sol. All even numbers between 9 and 99 are 10, 12, 14,… 98 which is in AP with 1st term, $a = 10$ and common difference, $d = 12 - 10 = 2$.

Let S_n denote the sum of all the terms of this AP. If a_n denote the nth term, where n is the number of terms of this AP, then

$$a_n = 98$$
$$\Rightarrow \quad 98 = a + (n-1)d$$
$$\Rightarrow \quad 98 = 10 + (n-1) \times 2$$
$$\Rightarrow \quad 98 = 10 + 2n - 2$$
$$\Rightarrow \quad 90 = 2n$$
$$\Rightarrow \quad n = \frac{90}{2} = 45$$

$$\therefore \quad S_{45} = \frac{45}{2}\{2a + (45-1)d\}$$
$$= \frac{45}{2}(2 \times 10 + 44 \times 2)$$
$$= 450 + 45 \times 44$$
$$= 450 + 1980$$
$$= 2430$$

$\therefore$ The required sum is 2430.

14. Find the sum of all numbers between 100 and 200 which are multiples of 5.

Sol. Numbers between 100 and 200 which are multiples of 5 are 105, 110, 115, …195, which is in AP.

If a is the 1st term, d is the common difference and n is the number of terms of this AP, then $a = 105$, $d = 110 - 105 = 5$

$$a_n = 195$$
$$\Rightarrow \quad a + (n-1)d = 195$$
$$\Rightarrow \quad 105 + (n-1)5 = 195$$
$$\Rightarrow \quad (n-1)5 = 90$$
$$\Rightarrow \quad n - 1 = 18$$
$$\Rightarrow \quad n = 19$$

$$\therefore \quad S_{19} = \frac{19}{2}\{2a + (19-1)d\}$$
$$= \frac{19}{2}(2 \times 105 + 18 \times 5)$$
$$= 105 \times 19 + 19 \times 45$$
$$= 19 \times (105 + 45)$$
$$= 19 \times 150$$
$$= 2850$$

$\therefore$ The required sum is 2850.

Short Answer Type Questions

15. Find the sum of $\left(1-\frac{1}{n}\right)+\left(1-\frac{2}{n}\right)+\left(1-\frac{3}{n}\right)+\ldots$ upto n terms.

Sol. We have

$$\left(1-\frac{1}{n}\right)+\left(1-\frac{2}{n}\right)+\left(1-\frac{3}{n}\right)+\ldots \text{ upto } n \text{ terms}$$
$$= (1 + 1 + \ldots \text{ upto } n \text{ terms}) - \frac{1}{n}(1 + 2 + 3 + \ldots + n)$$
$$= n - \frac{1}{n} \times \frac{n(n+1)}{2}$$
$$= \frac{2n - n - 1}{2}$$
$$= \frac{n-1}{2}$$

$\therefore$ The required sum is $\frac{n-1}{2}$.

16. Find the sum of the first 54 terms of the AP whose third term is −103 and the 7th term is −63.

Sol. Let a be the 1st term and d be the common difference of the AP. We denote nth term by a_n and sum to n terms of the AP by S_n.

Then, we have $a_3 = -103$

$\Rightarrow \quad a + (3-1)d = -103$

$\Rightarrow \quad a + 2d = -103 \quad \ldots(1)$

and $a_7 = -63$

$\Rightarrow \quad a + (7-1)d = -63$

$\Rightarrow \quad a + 6d = -63 \quad \ldots(2)$

Subtracting (1) from (2), we get

$$4d = 40$$

$$\Rightarrow \quad d = \frac{40}{4} = 10$$

$\therefore$ From (1),

$$\begin{aligned} a &= -103 - 2d \\ &= -103 - 2 \times 10 \\ &= -103 - 20 \\ &= -123 \end{aligned}$$

$$\begin{aligned} \therefore \quad S_{54} &= \frac{54}{2}\{2a + (54-1)d\} \\ &= 27(2 \times (-123) + 53 \times 10) \\ &= 27 \times (-246 + 530) \\ &= 27 \times 284 = 7668 \end{aligned}$$

$\therefore$ The required sum is 7668.

17. In an AP, if the 12th term is −13 and the sum of its first 4 terms is 24, find the sum of its first 10 terms. **[CBSE 2015]**

Sol. Let a be the 1st term and d be the common difference of the AP. Let a_n denote the nth term and S_n denote the sum of the n terms of the AP. Then,

$$a_{12} = -13$$

$\Rightarrow \quad a + (12-1)d = -13$

$\Rightarrow \quad a + 11d = -13 \quad \ldots(1)$

Also, $S_4 = \frac{4}{2}\{2a + (4-1)d\}$

$$24 = 2(2a + 3d)$$

$\Rightarrow \quad 2a + 3d = 12 \quad \ldots(2)$

Multiplying (1) by 2 and subtracting it from (2), we get

$$-19d = 38$$

$$\Rightarrow \quad d = -2$$

$\therefore$ From (1),

$$\begin{aligned} a &= -13 - 11 \times (-2) \\ &= -13 + 22 \\ &= 9 \end{aligned}$$

$$\begin{aligned} \therefore \quad S_{10} &= \frac{10}{2}\{2a + (10-1)d\} \\ &= 5\{2 \times 9 + 9 \times (-2)\} \\ &= 5(18 - 18) \\ &= 5 \times 0 = 0 \end{aligned}$$

$\therefore$ The required sum is 0.

18. Find the sum of n terms of the sequence 1, −3, 5, −7, 9, −11, … when (a) n is even and (b) n is odd.

Sol. (a) Let $n = 2m$, where m is any positive integer.

Now, taking only positive terms in the odd positions only of the given sequence, we have the sequence 1, 5, 9, … which is an AP with 1st term, $a = 1$ and the common difference, $d = 5 - 1 = 4$. If a_m denote the mth term of this AP, then $a_m = a + (m-1)d = 1 + (m-1)4 = 4m - 3$ and if S_m denote the sum of m terms of this AP, then

$$\begin{aligned} S_m &= \frac{m}{2}\{2a + (m-1)d\} \\ &= ma + \frac{m(m-1)d}{2} \\ &= m + \frac{m(m-1) \times 4}{2} \\ &= m + 2m(m-1) \\ &= m + 2m^2 - 2m \\ &= 2m^2 - m \end{aligned}$$

Again, taking negative terms in the even positions of the given sequence, we have the sequence −3, −7, −11, … which is another AP with $a = -3$, $d = -7 + 3 = -4$.

$\therefore$ Sum S'_m of m terms of this AP is given by

$$\begin{aligned} S'_m &= \frac{m}{2}\left\{2 \times (-3) + (m-1)(-4)\right\} \\ &= -3m - 2m(m-1) \\ &= -3m - 2m^2 + 2m \\ &= -2m^2 - m \end{aligned}$$

$\therefore$ Sum of $2m$ terms of the given sequence

$$\begin{aligned} &= S_m + S'_m \\ &= 2m^2 - m - 2m^2 - m \\ &= -2m \\ &= -n \end{aligned}$$

Hence, the required sum is $-n$, when n is even.

(b) Let $n = 2m + 1$, where m is any positive integer.

We see that each number in the odd position of the given sequence is a positive number of the AP 1, 5, 9, 13, …

$\therefore$ Sum of $2m + 1$ terms of the given sequence

$$= S_m + \frac{2}{45} + a_{m+1}$$
$$= -2m + 4(m + 1) - 3 \qquad \text{[From part (}a\text{)]}$$
$$= -2m + 4m + 4 - 3$$
$$= 2m + 1 = n$$

$\therefore$ The required sum is n, when n is odd.

19. Find the sum of the integers between 1 and 499 which are multiples of 3 and 5.

Sol. Numbers between 1 to 499 which are multiples of 3 and 5, i.e. $3 \times 5 = 15$ are 15, 30, 45, … which is in AP with 1st term, $a = 15$ and common difference, $d = 30 - 15 = 15$.

If n be the number of terms of this AP, then $a_n < 499$

i.e. $\quad a + (n - 1)d < 499$

$\Rightarrow \quad 15 + (n - 1)15 < 499$

$\Rightarrow \quad 15n < 499$

$\Rightarrow \quad n < \frac{499}{15} = 33\frac{4}{15}$

$\therefore$ $n = 33$ which is a positive integer.

$$\therefore \quad S_{33} = \frac{33}{2}[2 \times 15 + (33 - 1) \times 15]$$
$$= \frac{33}{2} \times 15 \times (2 + 32)$$
$$= 33 \times 15 \times 17$$
$$= 8415$$

$\therefore$ The required sum is 8415.

20. A manager was appointed in an office at a salary of ₹15000 per month. It was decided that his annual increment will be ₹500. Find his salary in the 15th year and also his total salary after 15 years of service.

Sol. Here, we shall find the 15th term of an AP with the 1st term $a = 15000$ and the common difference, $d = 500$.

If a_n denote the nth term of this AP, then

$$a_{15} = 15000 + (15 - 1) \times 500$$
$$= 15000 + 14 \times 500$$
$$= 15000 + 7000$$
$$= 22000$$

Hence, his required salary in the 15th year will be ₹22000.

His yearly salary = ₹15000 × 12 = ₹180000. Yearly increment is ₹500. Hence, his total salary after 15 years = Sum of 15 terms of the AP 180000, 180500, 181000, … to 15th term

$$\therefore \quad S_{15} = \frac{15}{2}\{2 \times 180000 + (15 - 1) \times 500\}$$
$$= \frac{15}{2}\{360000 + 14 \times 500\}$$
$$= \frac{15}{2}\{360000 + 7000\}$$
$$= \frac{15}{2}\{367000\} = 15 \times 183500$$
$$= 2752500$$

Hence, his total salary after 15 years will be ₹2752500.

Long Answer Type Questions

21. Find the maximum sum of the terms of the AP 148, 138, 128, 118, …

Sol. We see that the 1st term of the AP, $a = 148$ and the common difference, $d = 138 - 148 = -10$. The terms of the AP gradually decreases, so, the sum of the terms of the AP will be maximum only when all the terms are positive. Let n be the number of terms of the AP till the last term is near 0.

Now,

$$a_n = a + (n - 1)d$$
$$= 148 + (n - 1) \times (-10)$$
$$= 148 - (n - 1) \times 10$$

We see that $\quad a_n \geq 0$

$\Rightarrow \quad 148 - (n - 1) \times 10 \geq 0$

$\Rightarrow \quad (n - 1) \leq 14.8$

$\Rightarrow \quad n \leq 15.8$

$\because$ n is an integer, we have

$$n = 15$$

$\therefore$ The required maximum sum is given by

$$S_{15} = \frac{15}{2}[2 \times 148 - 14 \times 10]$$
$$= 15 \times 148 - 15 \times 70$$
$$= 15 \times (148 - 70)$$
$$= 15 \times 78$$
$$= 1170$$

22. If the sum of the first 7 terms of an AP is –14 and that of 11 terms is –55, then find the sum of its first 'n' terms. **[CBSE 2023 Basic]**

Sol. Let a and d be the first term and common difference of the AP.

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$\Rightarrow \quad S_7 = \frac{7}{2}[2a + (7-1)d] = -14$

$\Rightarrow \quad \frac{7}{2}[2a + 6d] = -14$

$\Rightarrow \quad 2a + 6d = \frac{-\cancel{14}^{2}}{\cancel{7}} \times 2 = -4$

$\Rightarrow \quad 2a = -4 - 6d \quad \ldots(1)$

Now, $\quad S_{11} = \frac{11}{2}[2a + (11-1)d]$

$\Rightarrow \quad \frac{11}{2}[2a + 10d] = -55$

$\Rightarrow \quad [-4 - 6d + 10d] = \frac{-55 \times 2}{11} \quad$ [From (1)]

$\Rightarrow \quad -4 + 4d = -10$

$\Rightarrow \quad 4d = -10 + 4$

$\Rightarrow \quad 4d = -6$

$\Rightarrow \quad d = \frac{-6}{4} = \frac{-3}{2}$

From (1)

$2a = -4 - 6d$

$= -4 - 6 \times \left(\frac{-3}{2}\right)$

$= -4 + 9$

$= 5$

$\Rightarrow \quad a = \frac{5}{2}$

$\therefore \quad S_n = \frac{n}{2}\left[2 \times \frac{5}{2} + (n-1) \times \left(\frac{-3}{2}\right)\right]$

$= \frac{n}{2}\left[5 + \frac{3}{2} - \frac{3}{2}n\right]$

$= \frac{n}{4}[10 + 3 - 3n]$

$S_n = \frac{n}{4}[13 - 3n]$

23. In an AP, the sum of the first 'n' terms is $3n^2 + n$. Find the first term and the common difference of the AP. Hence, find its 15th term.

[CBSE 2023 Basic]

Sol. Sum of first 'n' terms $= 3n^2 + n$

$\Rightarrow \quad S_n = n(3n + 1)$

$\therefore \quad S_1 = 1(3 + 1)$

$\Rightarrow \quad S_1 = 4$

$\therefore$ First term, $\quad a = 4 \quad \ldots(1)$

$S_2 = 2(3 \times 2 + 1)$

$\Rightarrow \quad S_2 = 14$

But $\quad S_2 = 2a + d$

$\Rightarrow \quad 2a + d = 14$

$\Rightarrow \quad 2 \times 4 + d = 14$

$\Rightarrow \quad d = 14 - 8 = 6$

$\Rightarrow \quad d = 6$

$T_n = a + (n-1)d$

$\Rightarrow \quad T_{15} = 4 + (15-1) \times 6$

$= 4 + 84$

$\Rightarrow \quad T_{15} = 88$

24. If the ratio of the sums of the first m and n terms of an AP is $m^2 : n^2$, show that the ratio of its mth and nth terms is $(2m - 1) : (2n - 1)$.

[CBSE 2016, 2017]

Sol. We have

$$\frac{S_m}{S_n} = \frac{m^2}{n^2}$$

Let $S_m = km^2$ and $S_n = kn^2$, where k is a non-zero constant.

Now, $\quad a_m = S_m - S_{m-1}$

$= k\{m^2 - (m-1)^2\}$

$= k(m + m - 1)(m - m + 1)$

$= k(2m - 1)$

and $\quad a_n = S_n - S_{n-1}$

$= k\{n^2 - (n-1)^2\}$

$= k(n + n - 1)(n - n + 1)$

$= k(2n - 1)$

$\therefore \quad \frac{a_m}{a_n} = \frac{(2m-1)k}{(2n-1)k}$

$= \frac{2m-1}{2n-1}$

Hence, proved.

25. The ratio of the 11th term to the 18th term of an AP is 2 : 3. Find the ratio of the 5th term to the 21st term. Also, find the ratio of the sum of first 5 terms to the sum of first 21 terms.

[CBSE 2023 Standard]

Sol. Let a and d be first term and common difference of the AP respectively.

Given $\quad \frac{a_{11}}{a_{18}} = \frac{2}{3}$

$\Rightarrow \quad \frac{a + 10d}{a + 17d} = \frac{2}{3}$

$\Rightarrow \quad 3a + 30d = 2a + 34d$

$\Rightarrow \quad a = 4d \quad \ldots(1)$

$a_5 = a + 4d = 4d + 4d = 8d$

$a_{21} = 4 + 20d = 4d + 20d = 24d$

$$\therefore \quad \frac{a_5}{a_{21}} = \frac{8d}{24d} = \frac{1}{3}$$

$$\Rightarrow \quad a_5 : a_{21} = 1 : 3$$

$$S_5 = \frac{5}{2}\,[2a + (5-1)d]$$

$$= \frac{5}{2}\,[2 \times 4d + 4d]$$

$$= \frac{5}{2}\,[8d + 4d] = \frac{5}{2} \times 12d$$

$$= 30d$$

$$S_{21} = \frac{21}{2}\,[2a + (21-1)d]$$

$$= \frac{21}{2}\,[2 \times 4d + 20d]$$

$$= \frac{21}{2}\,[8d + 20d]$$

$$= \frac{21}{2} \times 28d$$

$$= 294d$$

$$\therefore \quad S_5 : S_{21} = 30d : 294d$$

$$= 5 : 49$$

$$\Rightarrow \quad S_5 : S_{21} = 5 : 49$$

26. The sum of three numbers in AP is 12 and the sum of their cubes is 288. Find the numbers. **[CBSE 2016]**

Sol. Let the three numbers in AP be $\alpha - d$, α and $\alpha + d$.

Then, according to the problem, we have

$$\alpha - d + \alpha + \alpha + d = 12$$

$$\Rightarrow \quad 3\alpha = 12$$

$$\Rightarrow \quad \alpha = 4$$

Also, $\quad (\alpha - d)^3 + \alpha^3 + (\alpha + d)^3 = 288$

$$\Rightarrow \quad (4-d)^3 + 4^3 + (4+d)^3 = 288$$

$$\Rightarrow \quad (4-d)^3 + (4+d)^3 = 288 - 64 = 224$$

$$\Rightarrow \quad (4 - d + 4 + d)\,\{(4-d)^2 - (4-d)(4+d) + (4+d)^2\} = 224$$

$$\Rightarrow \quad 8\{2\,(4^2 + d^2) - (16 - d^2)\} = 224$$

$$\Rightarrow \quad 8\{32 + 2d^2 - 16 + d^2\} = 224$$

$$\Rightarrow \quad 32 + 2d^2 - 16 + d^2 = \frac{224}{8} = 28$$

$$\Rightarrow \quad 3d^2 = 28 - 16 = 12$$

$$\Rightarrow \quad d^2 = 4$$

$$\Rightarrow \quad d = \pm 2$$

$\therefore$ The three numbers are $4 - 2, 4, 4 + 2$, when $d = 2$, i.e. 2, 4, 6 or, $4 + 2, 4, 4 - 2$, when $d = -2$, i.e. 6, 4, 2.

Hence, the required numbers are 2, 4 and 6.

27. If the sums of n terms of two AP's are in the ratio $(3n - 13) : (5n + 21)$, find the ratio of their 24th terms.

Sol. Let a and a' be the 1st term and d and d' be the common difference of the two AP's. Let S and S' be the respective sums of the two AP's.

Then $\quad S = \frac{n}{2}\,[2a + (n-1)d] \quad \ldots(1)$

and $\quad S' = \frac{n}{2}\,[2a' + (n-1)d'] \quad \ldots(2)$

From (1) and (2), we have

$$\frac{S}{S'} = \frac{2a + (n-1)d}{2a' + (n-1)d'}$$

$$= \frac{3n - 13}{5n + 21} \quad \text{[Given]} \ldots(3)$$

Now, the ratio of the 24th terms of the two AP's is

$$\frac{a + (24-1)d}{a' + (24-1)d'} = \frac{a + 23d}{a' + 23d'}$$

$$= \frac{2a + 46d}{2a' + 46d'} \quad \ldots(4)$$

Comparing (3) and (4), i.e. $\frac{2a + (n-1)d}{2a' + (n-1)d'}$ and $\frac{2a + 46d}{2a' + 46d'}$

We see that $\quad n - 1 = 46$

$$\Rightarrow \quad n = 46 + 1 = 47$$

Putting $n = 47$ in (3), we get

$$\frac{2a + 46d}{2a' + 46d'} = \frac{3 \times 47 - 13}{5 \times 47 + 21}$$

$$= \frac{141 - 13}{235 + 21}$$

$$= \frac{128}{256}$$

$$= \frac{1}{2}$$

$\therefore$ The required ratio is 1 : 2.

28. The interior angles of a polygon are in AP. The smallest angle is 120° and the common difference is 5°. Find the number of sides of the polygon.

Sol. The first term of the given AP will be $a = 120°$ and the common difference, $d = 5°$. Let n be the number of sides of the polygon. Then the sum of all interior angles of the polygon is $(n - 2) \times 180°$.

$$\therefore \quad (n-2) \times 180° = 120° + 125° + 130° + \ldots \text{ to } n \text{ terms}$$

$$\Rightarrow \quad (n-2) \times 180° = \frac{n}{2}\,[2 \times 120° + (n-1) \times 5°]$$

$$\Rightarrow \quad (n-2) \times 36 = \frac{n}{2}[2 \times 24 + n - 1]$$

$$\Rightarrow \quad (n-2) \times 36 = \frac{n}{2}(48 + n - 1)$$

$$\Rightarrow \quad (n-2) \times 36 = \frac{n}{2}(47 + n)$$

$$\Rightarrow \quad 72n - 144 - 47n - n^2 = 0$$

$$\Rightarrow \quad n^2 - 25n + 144 = 0$$

$$\Rightarrow \quad n^2 - 16n - 9n + 144 = 0$$

$$\Rightarrow \quad n(n-16) - 9(n-16) = 0$$

$$\Rightarrow \quad (n-16)(n-9) = 0$$

$\therefore$ Either $n = 16$ or $n = 9$.

But $n = 16$ is not possible, since in this case the 16th angle of the polygon, i.e. $a_{16} = 120° + 15 \times 5° = 120° + 75° = 195° > 180°$ which is absurd.

Hence, the required number of sides is 9.

29. In an AP, if $S_n = 4n^2 - n$, then

(*a*) find the first term and common difference.

(*b*) write the AP.

(*c*) which term of the AP is 107?

[CBSE 2024 Basic]

Sol. (*a*) Let a and d be the first term and common difference of the AP.

$$S_n = 4n^2 - n$$

First term $= a = S_1 = 4(1)^2 - 1 = 4 - 1 = 3$

$$\Rightarrow \quad a = 3$$

$$S_2 = 4(2)^2 - 2 = 4 \times 4 - 2 = 16 - 2 = 14$$

But $\quad S_2 = 2a + d = 14$

$$\Rightarrow \quad 2a + d = 14$$

$$\Rightarrow \quad 2 \times 3 + d = 14$$

$$\Rightarrow \quad d = 14 - 6 = 8$$

$$\Rightarrow \quad d = 8$$

(*b*) $\quad \text{AP} = a, a + d, a + 2d, a + 3d, \ldots$

$$= 3, 3 + 8, 3 + 2 \times 8, 3 + 3 \times 8, \ldots$$

$$= 3, 11, 19, 27, \ldots$$

(*c*) Let nth term of the AP = 107

$$a_n = a + (n-1)d$$

$$a + (n-1)d = 107$$

$$3 + (n-1) \times 8 = 107$$

$$\Rightarrow \quad (n-1) \times 8 = 104$$

$$\Rightarrow \quad n - 1 = \frac{104}{8} = 13$$

$$\Rightarrow \quad n = 13 + 1 = 14$$

$$\therefore \quad n = 14$$

30. How many terms of the AP 27, 24, 21, … must be taken so that their sum is 105? Which term of the AP is zero? **[CBSE 2024 Basic]**

Sol. $a = 27$

$d = 24 - 27 = -3$

Let n terms should be taken so that their sum is 105.

$$105 = \frac{n}{2}[2a + (n-1)d]$$

$$105 = \frac{n}{2}[2 \times 27 + (n-1)(-3)]$$

$$105 = \frac{n}{2}[54 + 3 - 3n]$$

$$210 = n[57 - 3n]$$

$$\Rightarrow \quad 210 = 57n - 3n^2$$

$$\Rightarrow \quad 3n^2 - 57n + 210 = 0$$

$$\Rightarrow \quad n^2 - 19n + 70 = 0$$

$$\Rightarrow \quad n^2 - 5n - 14n + 70 = 0$$

$$\Rightarrow \quad n(n-5) - 14(n-5) = 0$$

$$\Rightarrow \quad (n-5)(n-14) = 0$$

$$\Rightarrow \quad n = 5 \text{ or } n = 14$$

$\therefore$ 5 or 14 terms of the given AP must be taken so that the sum is 105.

Let the nth term be 0.

$$\Rightarrow \quad a_n = 0$$

$$\Rightarrow \quad a + (n-1)d = 0$$

$$\Rightarrow \quad 27 + (n-1) \times (-3) = 0$$

$$\Rightarrow \quad 27 - 3n + 3 = 0$$

$$\Rightarrow \quad 30 - 3n = 0$$

$$\Rightarrow \quad 3n = 30$$

$$\Rightarrow \quad n = 10$$

$\therefore$ 10th term of AP is zero.

Check Your Progress 3

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Very Short Answer Type Questions

1. A sum of ₹500 is invested at 6% simple interest per annum. Will the interests at the end of first year, second year, third year, fourth year, … form an AP? If so, what is the common difference of the AP ?

Sol. Interest at the end of 1st year

$$= ₹\,\frac{500 \times 6}{100} = ₹\,30$$

We see that this interest on each year will be the same, i.e. ₹30

$\therefore$ Total interest at the end of 2nd year

$$= ₹30 + ₹30$$
$$= ₹60$$

Total interest at the end of 3rd year

$$= (60 + 30)$$
$$= 90 \text{ and so on.}$$

$\therefore$ Total interests at the end of 1st year, 2nd year, 3rd year, 4th year, and so on are respectively ₹30, ₹60, ₹90, ₹120, and so on.

The sequence of numbers 30, 60, 90, 120, … forms an AP with common difference, $d = 60 - 30 = 30$.

2. In the month of July 2017, the total number of visitors in a zoo was 1550. If the number of visitors to the zoo decreased daily by 6 from 1st July to 31st July, 2017, then find the number of visitors to the zoo on 1st July, 2017.

Sol. The sum of the number of all daily visitors in the zoo for 31 days is given to be 1550. The number of daily visitors to the zoo form an AP with first term, a and the common difference, $d = -6$.

If S_{31} is the sum of the numbers of all daily visitors, then

$$S_{31} = \frac{31}{2}[2a - 30 \times 6]$$

$$\Rightarrow \quad 1550 = \frac{31}{2}[2a - 180]$$

$$\Rightarrow \quad 1550 = 31a - 2790$$

$$\Rightarrow \quad 4340 = 31a$$

$$\Rightarrow \quad a = \frac{4340}{31} = 140$$

$\therefore$ The number of visitors to the zoo on 1st July, 2017 was 140.

3. A manufacturer of TV sets produced 600 units in the third year and 700 units in the seventh year. Assuming that the production increases uniformly by a fixed number of units every year, find

(*a*) the production in the first year.

(*b*) the production in the 10th year.

(*c*) the total production in 7 years.

[CBSE SP 2016]

Sol. (*a*) Let the initial production in the 1st year be a units, and let d be the increase in production every year.

We see that the numbers of yearly production form an AP with 1st term, a and common difference, d.

According to the problem,

a_3 = production in the third year = 600

$$\Rightarrow \quad a + 2d = 600 \quad \ldots(1)$$

and a_7 = production in the seventh year = 700

$$\Rightarrow \quad a + 6d = 700 \quad \ldots(2)$$

Subtracting (1) from (2), we get

$$4d = 700 - 600 = 100$$

$$\Rightarrow \quad d = \frac{100}{4} = 25$$

$\therefore$ From (1), $a = 600 - 2 \times 25$

$$= 600 - 50 = 550$$

Hence, the required production in the first year is 550 units.

(*b*) Now, production in the 10th year = a_{10}

$$= a + 9d$$
$$= 550 + 9 \times 25$$
$$= 550 + 225$$
$$= 775$$

Hence, the required production in the 10th year is 775 units.

(*c*) Finally, if S_7 denote the total production in 7 years, then

$$S_7 = \frac{7}{2}\{2a + (7-1)d\}$$
$$= \frac{7}{2}\{2 \times 550 + 6 \times 25\}$$
$$= 7 \times 550 + 75 \times 7$$
$$= 7 \times (550 + 75)$$
$$= 7 \times 625$$
$$= 4375$$

Hence, the required total production in 7 years is 4375 units.

4. You are saving ₹1 today, ₹2 the next day and ₹3 the third day and so on and your friend is saving ₹2, ₹6, ₹10, ₹14 … successively in every alternate day in the month of February, 2018, then who will save more and by how much?

Sol. We see that the month of February, 2018 contains 28 days.

You are saving ₹(1 + 2 + 3 + … + 28)

$$= ₹\,\frac{28 \times 29}{2} = 406$$

and your friend are saving ₹(2 + 6 + 10 + 14 + …) for 14 alternate days in February, 2018. Now the sequence of numbers 2, 6, 10, 14, … form an AP with first term, $a = 2$ and the common difference, $d = 6 - 2 = 4$.

∴ Total amount of your friend's saving in February 2018 is

$$S_{14} = ₹\ \frac{14}{2}\{2a + (14-1) \times d\}$$
$$= ₹7 \times (2 \times 2 + 13 \times 4)$$
$$= ₹7 \times (4 + 52)$$
$$= ₹7 \times 56$$
$$= ₹392$$

∴ You are saving ₹(406 – 392) = ₹14 more than your friend in February, 2018.

5. A man buys infrastructure bonds every year. The value of these bonds exceeds the previous year purchase value by ₹200. After 15 years, the total value of the infrastructure bonds purchased by him is ₹96000. Find the face value of the bonds bought by him in the first year.

Sol. Let the face value of the bonds in the 1st year be ₹a. Then the value of the bonds increases by ₹200 every year for 15 years and the total value of the bonds in 15 years is ₹96000. We see that the values of the bonds every year for 15 years form an AP with 1st term, a and the common difference, $d = 200$.

If S_{15} is the sum of 15 terms of this AP,

then $$S_{15} = \frac{15}{2}\{2a + (15-1) \times 200\}$$

$$\Rightarrow \quad 96000 = \frac{15}{2}\{2a + 14 \times 200\}$$
$$\Rightarrow \quad 96000 = 15a + 15 \times 14 \times 100$$
$$\Rightarrow \quad 96000 = 15a + 21000$$
$$\Rightarrow \quad 75000 = 15a$$
$$\Rightarrow \quad a = \frac{75000}{15}$$
$$= 5000$$

Hence, the required face value of the bonds in the 1st year is ₹5000.

6. A man takes up a job of ₹60000 per month with annual increment of ₹2000. How much will he earn in 10 years?

Sol. We see that the earnings of the man each year for 10 years form an AP with 1st term, a = ₹60000 × 12 = ₹720000 and common difference, d = ₹2000. Let S_{10} denote his total earning in 10 years. Then S_{10} is the sum of 10 terms of this AP.

$$\therefore \quad S_{10} = ₹\ \frac{10}{2}\{2a + (10-1)d\}$$
$$= ₹5 \times (2 \times 720000 + 9 \times 2000)$$
$$= ₹5 \times (1440000 + 18000)$$
$$= ₹5 \times 1458000$$
$$= ₹7290000$$

∴ A man will earn ₹7290000 in 10 years.

Short Answer Type Questions

7. The minimum age of children to participate in a painting competition is 8 years. It is observed that the age of the youngest boy was 8 years and the ages of the rest of the participants are having a common difference of 4 months. If the sum of the ages of all participants is 168 years, find the age of the eldest participant in the painting competition. **[CBSE SP 2015]**

Sol. We see that the ages of different boys form an AP with 1st term, $a = 8$ years and the common difference, $d = 4$ months $= \frac{1}{3}$ year.

The sum S_n of the ages of all participants is given by

$$S_n = 168 \qquad \text{[Given]}$$
$$\Rightarrow \quad \frac{n}{2}\left[2a + \frac{n-1}{3}\right] = 168$$
$$\Rightarrow \quad \frac{n}{2}\left[2 \times 8 + \frac{n-1}{3}\right] = 168$$
$$\Rightarrow \quad 8n + \frac{n(n-1)}{6} = 168$$
$$\Rightarrow \quad \frac{n^2 - n + 48n}{6} = 168$$
$$\Rightarrow \quad n^2 + 47n - 1008 = 0$$
$$\Rightarrow \quad n^2 + 63n - 16n - 1008 = 0$$
$$\Rightarrow \quad n(n + 63) - 16\,(n + 63) = 0$$
$$\Rightarrow \quad (n + 63)\,(n - 16) = 0$$

∴ Either $n + 63 = 0$

$$\Rightarrow \quad n = -63$$

which is rejected, since n cannot be negative.

or, $n - 16 = 0$

$$\Rightarrow \quad n = 16$$

which is accepted.

Hence, the total number of participants is 16 and the age of the eldest participant will be the 16th term, a_{16}, of the AP.

Now, $$a_{16} = 8 + (16-1) \times \frac{1}{3} = 13$$

Hence, the required age of the eldest participant is 13 years.

8. Jaipal Singh repays the total loan of ₹118000 by paying every month starting with the first instalment of ₹1000. If he increases the instalment by ₹100 every month, what amount will be paid by him in 30th instalment? What amount of loan does he still have to pay after 30th instalment?

[CBSE SP 2012]

Sol. We see that the amounts of different monthly instalments form an AP with 1st term, a = ₹1000 and the common difference, d = ₹100. If a_{30} represents the amount in the 30th instalment and S_{30} represents the total amount of loan paid by Jaipal singh in 30 instalments, then

$$a_{30} = a + (30 - 1)d$$
$$= (1000 + 29 \times 100)$$
$$= ₹3900$$

and
$$S_{30} = ₹\ \frac{30}{2}(2 \times 1000 + 29 \times 100)$$
$$= ₹15 \times (2000 + 2900)$$
$$= ₹15 \times 4900$$
$$= ₹73500$$

∵ Total amount of loan is ₹118000.

∴ Balance amount of loan, he will still have to pay after 30th instalment is (₹118000 – ₹73500), i.e ₹44500.

9. Two cyclists C_1 and C_2 start together in the same direction from the same place. The cyclist C_1 cycles at a uniform speed of $11\frac{1}{4}$ km/h and the cyclist C_2 cycles at a uniform speed of $9\frac{3}{4}$ km/h in the first hour and then increases his speed by $\frac{1}{4}$ km/h in each succeeding hour. After how many hour(s) will the cyclist C_2 overtake C_1, if both the cyclists cycle non-stop?

Sol. Let the two cyclists meet together after t hours.

Now, in t hours, the cyclist C_1 travels a distance of

$$11\frac{1}{4} \times t \text{ km} = \frac{45t}{4} \text{ km.}$$

We see that the distances travelled by the cyclist C_2 in different hours form an AP with the 1st term, $a = 9\frac{3}{4}$ km $= \frac{39}{4}$ km and the common difference, $d = \frac{1}{4}$ km. Hence, the total distance travelled by the cyclist C_2 in t hours will be the sum of t terms, S_t of this AP.

$$\therefore \quad S_t = \frac{t}{2}[2a + (t-1)d]$$
$$= \frac{t}{2}\left[2 \times \frac{39}{4} + (t-1) \times \frac{1}{4}\right] \text{ km}$$
$$= \left[\frac{39}{4}t + \frac{t(t-1)}{8}\right] \text{ km}$$
$$= \frac{78t + t^2 - t}{8} \text{ km}$$
$$= \frac{77t + t^2}{8} \text{ km}$$

We have

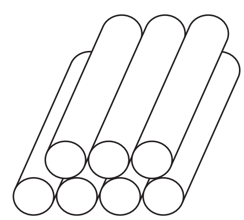
$$\frac{77t + t^2}{8} = \frac{45t}{4}$$
$$\Rightarrow \quad \frac{77 + t}{8} = \frac{45}{4}$$
$$\Rightarrow \quad 77 + t = 90$$
$$\Rightarrow \quad t = 90 - 77 = 13$$

Hence, the required time is 13 hours.

Long Answer Type Questions

10. 250 logs are stacked in the following manner:

22 logs in the bottom row, 21 in the next row, 20 in the row next to it and so on (as shown by an example). In how many rows, are the 250 logs placed and how many logs are there in the top row?



(Example)

[CBSE 2023 Standard]

Sol.
$$a = 22$$
$$d = 21 - 22 = -1$$

Let the number of rows be 'n'.

$$S_n = 250$$
$$\Rightarrow \quad \frac{n}{2}[2a + (n-1)d] = 250$$
$$\Rightarrow \quad n[2 \times 22 + (n-1)(-1)] = 250 \times 2$$
$$\Rightarrow \quad n[44 - n + 1] = 500$$
$$\Rightarrow \quad 45n - n^2 = 500$$
$$\Rightarrow \quad n^2 - 45n + 500 = 0$$
$$\Rightarrow \quad n^2 - 20n - 25n + 500 = 0$$
$$\Rightarrow \quad n(n-20) - 25(n-20) = 0$$
$$\Rightarrow \quad (n-20)(n-25) = 0$$
$$\Rightarrow \quad n = 20 \quad \text{or} \quad n = 25$$

There are 22 logs in the first row, therefore $n \neq 25$, as the number of logs will be negative in the topmost row.

$$\therefore \quad n = 20$$

Number of logs in the 20th row

$$a_n = a + (n-1)d$$
$$a_{20} = 22 + (20-1)(-1)$$
$$= 22 + 19(-1)$$
$$= 22 - 19$$
$$\Rightarrow \quad a_{20} = 3$$

$\therefore$ Number of logs in the topmost row = 3.

11. A polygon has 41 sides, the length of which, starting from the smallest, are in AP. If the perimeter of the polygon is 902 cm and the length of the largest side is 21 times the smallest, find the length of the smallest side and the common difference of the AP.

Sol. Let the smallest side of the polygon be of length a cm. Then the largest side is of length $21a$ cm.

Clearly, the different lengths of the sides of the polygon form an AP with 1st term be a cm and the last or 41st term be $21a$ cm.

Let d cm be the common difference of this AP.

Then $a_{41} = a + 40d = 21a$

$$\Rightarrow \quad 40d = 20a$$
$$\Rightarrow \quad d = \frac{a}{2} \quad \ldots(1)$$

The perimeter of the polygon will be the sum of all 41 terms of the AP, i.e. S_{41}.

Now, $S_{41} = \frac{41}{2}(2a + 40d)$

$$= \frac{41}{2}\left(2a + 40 \times \frac{a}{2}\right) \text{ cm}$$

[From (1)]

$$= \frac{41}{2}(2a + 20a) \text{ cm}$$
$$= 41 \times 11a \text{ cm}$$
$$= 451a \text{ cm}$$

$\therefore$ According to the problem, we have

$$451a = 902$$
$$\Rightarrow \quad a = \frac{902}{451}$$
$$\Rightarrow \quad a = 2 \quad \ldots(2)$$

Now, $d = \frac{2}{2}$ [From (2)]

$$\Rightarrow \quad d = 1$$

Hence, the required smallest side is 2 cm and common difference, d is 1 cm.

12. A thief, after committing a theft, runs at a uniform speed of 50 m/min. After 2 minutes, a policeman runs to catch him. He goes 60 m in the first minute and increases his speed by 5 m/min every succeeding minute. After how many minutes will the policeman catch the thief? **[CBSE 2016]**

Sol. When the police starts running, the thief is 100 m apart.

Speed for 1st minute is 60 m/minute and increases by 5 m/minute.

AP: 10, 15 …

a = 10 m/minute (distance reduced in 1st minute)

$$d = 5$$
$$S_n = 100$$
$$S_n = \frac{n}{2}[2a + (n-1)d]$$
$$\Rightarrow \quad 100 = \frac{n}{2}[20 + (n-1)5]$$
$$\Rightarrow \quad 200 = n[20 + 5n - 5]$$
$$\Rightarrow \quad 200 = n[15 + 5n]$$
$$\Rightarrow \quad 40 = n[3 + n]$$
$$\Rightarrow \quad 40 = 3n + n^2$$
$$\Rightarrow \quad n^2 + 3n - 40 = 0$$
$$\Rightarrow \quad n^2 + 8n - 5n - 40 = 0$$
$$\Rightarrow \quad n(n+8) - 5(n+8) = 0$$
$$\Rightarrow \quad (n+8)(n-5) = 0$$

$\therefore$ Either $n = -8$

or $n = 5$

Since, time cannot be negative, hence we will reject –8.

$\therefore$ Policeman will catch the thief in 5 minutes.

13. The houses of a row are numbered consecutively from 1 to 49. Show that there is a value of x such that the sum of the numbers of the houses preceding the house numbered x is equal to the sum of the numbers of the houses following it. Find the value of x. **[CBSE 2016]**

Sol. Let us first calculate the sum of houses before x.

$a = 1$, $d = 1$, $l = x - 1$, $n = x - 1$

$$S_n = \frac{n}{2}[a + l]$$
$$S_x = \frac{x-1}{2}[1 + x - 1]$$
$$S_x = \frac{x(x-1)}{2} \quad \ldots(1)$$

Now, we will calculate the sum of houses after x.

$a = x + 1, d = 1, l = 49, n = 49 - x$

$$S_x = \frac{49 - x}{2}[x + 1 + 49]$$

$$= \frac{(49 - x)}{2}(x + 50) \quad \ldots(2)$$

We will now equate (1) and (2), we get

$$\frac{x(x-1)}{2} = \frac{(49 - x)(x + 50)}{2}$$

$\Rightarrow \quad x^2 - x = 49x + 2450 - x^2 - 50x$

$\Rightarrow \quad 2x^2 - x = -x + 2450$

$\Rightarrow \quad 2x^2 = 2450$

$\Rightarrow \quad x^2 = 1225$

$\Rightarrow \quad x = \pm 35$

Since, house number cannot be negative, hence we will reject –35.

$\therefore$ The house number is 35.

Higher Order Thinking Skills (HOTS) Questions

(Page 101)

1. If p^2, q^2 and r^2 are in AP, prove that $\frac{1}{q+r}$, $\frac{1}{r+p}$ and $\frac{1}{p+q}$ are also in AP.

Sol. Since p^2, q^2 and r^2 are in AP,

$\therefore \quad 2q^2 = p^2 + r^2 \quad \ldots(1)$

Now, $\frac{1}{q+r}$, $\frac{1}{r+p}$ and $\frac{1}{p+q}$ will be in AP if

$$\frac{2}{r+p} = \frac{1}{q+r} + \frac{1}{p+q}$$

$$= \frac{p + q + q + r}{(q + r)(p + q)}$$

$$= \frac{p + 2q + r}{(q + r)(p + q)}$$

i.e. if $(r + p)(p + 2q + r) = 2(q + r)(p + q)$

i.e. if $rp + 2rq + r^2 + p^2 + 2pq + pr$
$= 2pq + 2q^2 + 2rp + 2rq$

i.e. if $r^2 + p^2 = 2q^2$ which is true by (1).

Hence, proved

2. Find the first negative term of the AP 2000, 1990, 1980, 1970, …

Sol. Here, the 1st term, $a = 2000$, common difference, $d = 1990 - 2000 = -10$.

$\therefore$ If a_n be the nth term, then

$$a_n = a + (n - 1)d$$
$$= 2000 - (n - 1) \times 10 \quad \ldots(1)$$

We shall find the maximum value of n for which $a_n \geq 0$.

i.e. $\quad 2000 - (n - 1) \times 10 \geq 0 \quad$ [From (1)]

$\Rightarrow \quad n - 1 \leq 200$

$\Rightarrow \quad n \leq 201$

$\therefore$ Maximum value of n for which n is positive is $n = 201$.

$\therefore$ The 1st negative term will be negative when $n = 202$.

Hence, the required term is 202nd term.

Also, $\quad a_{202} = 2000 - (202 - 1) \times 10$ [From (1)]

$= 2000 - 2010$

$= -10$

Hence, the required 1st negative term is –10.

3. How many terms are identical in the two AP's 3, 6, 9, 12, … upto 300 terms and 5, 10, 15, 20, … upto 200 terms?

Sol. In the two AP's, the successive terms are multiples of 3 and 5 respectively. Hence, the identical terms of the resulting AP will be those which are multiples of 3 as well as 5, i.e. multiples of 3 × 5 = 15. Now, in the 1st AP, the last term

$$a_{300} = 3 + 299 \times 3$$
$$= 3 + 897$$
$$= 900$$

Also, in the second AP, the last term

$$a_{200} = 5 + 199 \times 5$$
$$= 5 \times (199 + 1)$$
$$= 5 \times 200$$
$$= 1000$$

Hence, the terms of the resulting AP which are multiples of 15 will be upto 900.

$\therefore$ If a_n be the nth term, then

$$a_n \leq 900$$

$\Rightarrow \quad 15 + (n - 1) \times 15 \leq 900$

$\Rightarrow \quad 15n \leq 900$

$\Rightarrow \quad n \leq \frac{900}{15} = 60$

$\therefore$ The maximum value of n is 60.

Hence, there are 60 identical terms in the two given AP's.

4. The 9th term of an AP is 11 times the first term. Prove that the 17th term is 6 times as great as the third term.

Sol. Let a be the 1st term and d be the common difference of the AP. If a_n denotes its nth term, then

$$a_n = a + (n - 1)d$$

Given that $a_9 = 11a$

$\Rightarrow \quad a + (9 - 1)d = 11a$

$\Rightarrow \quad 8d = 10a$

$\Rightarrow \quad d = \dfrac{5a}{4} \qquad \ldots(1)$

We shall now prove that

$$a_{17} = 6a_3$$

i.e. $\quad \dfrac{a_{17}}{a_3} = 6$

Now, $\quad \dfrac{2}{45} = \dfrac{a + 16d}{a + 2d}$

$$= \frac{a + 16 \times \frac{5a}{4}}{a + 2 \times \frac{5a}{4}} \quad \text{[From (1)]}$$

$$= \frac{4a + 16 \times 5a}{4a + 2 \times 5a}$$

$$= \frac{4a + 80a}{4a + 10a} = \frac{84a}{14a}$$

$\therefore \quad \dfrac{a_{17}}{a_3} = 6$

$\Rightarrow \quad a_{17} = 6a_3$

Hence, proved.

5. Find the sum of integers between 1 and 100 which are divisible by 2 or 5.

Sol. Numbers between 1 and 100, which are divisible by 2 are 2, 4, 6, 8, …, 98. If S be the sum of these numbers, then

$$S = 2 + 4 + 6 + 8 + \ldots + 98$$

$$= 2(1 + 2 + 3 + 4 + \ldots + 49)$$

$$= 2 \times \frac{49 \times 50}{2}$$

$\therefore \quad S = 2450 \qquad \ldots(1)$

Also, numbers between 1 and 100, which are divisible by 5 are 5, 10, 15, 25, … , 95. If S' be the sum of these numbers, then

$$S' = 5 + 10 + 15 + 20 + 25 + \ldots + 95$$

$$= 5\,(1 + 2 + 3 + 4 + 5 + \ldots + 19)$$

$$= 5 \times \frac{19 \times 20}{2}$$

$\therefore \quad S' = 950 \qquad \ldots(2)$

Among all these numbers which are divisible by 2 or 5, there are some common numbers, which are divisible by both 2 and 5, i.e. by $2 \times 5 = 10$.

These numbers are 10, 20, 30, 40, … 90.

If S'' be the sum of all these numbers, then

$$S'' = 10 + 20 + 30 + \ldots + 90$$

$$= 10(1 + 2 + 3 + \ldots + 9)$$

$$= 10 \times \frac{9 \times 10}{2}$$

$\therefore \quad S'' = 450 \qquad \ldots(3)$

Hence, the required sum

$$= S + S' - S''$$

$$= 2450 + 950 - 450$$

[From (1), (2) and (3)]

$$= 2450 + 500$$

$$= 2950$$

$\therefore$ The required sum is 2950.

6. Find the sum of 11 terms of an AP whose middle term is 11.

Sol. Let a be the 1st term and d be the common difference of the AP. If n be the nth term of the AP, then

$$a_n = a + (n - 1)d \qquad \ldots(1)$$

We know that the middle term is $a_{\frac{n+1}{2}}$ if n is odd.

Here, $n = 11$ which is odd.

$\therefore \quad$ Middle term $= a_{\frac{11+1}{2}} = a_6$

$= a + 5d \qquad$ [From (1)]

It is given that $\quad a_6 = 11$

$\therefore \quad a + 5d = 11 \qquad \ldots(2)$

Also, $\quad S_{11} = \dfrac{11}{2}[2a + (11 - 1)d]$

$= 11(a + 5d)$

$= 11 \times 11 \qquad$ [From (2)]

$= 121$

$\therefore \quad$ The required sum is 121.

Self-Assessment

(Page 101)

Multiple-Choice Questions

1. How many terms are there in the AP given below?

14, 19, 24, 29, …., 119

(*a*) 18 (*b*) 14

(*c*) 22 (*d*) 21 **[CBSE 2023 Basic]**

Sol. (*c*) 22

If l is the last term, then

$$l = a + (n - 1)d$$

Here, $a = 14$

$$d = 19 - 14 = 5$$

$$l = 119$$

$\therefore$ $119 = 14 + (n - 1) \times 5$

$\Rightarrow$ $(n - 1) \times 5 = 119 - 14 = 105$

$\Rightarrow$ $n - 1 = \dfrac{105}{5} = 21$

$\therefore$ $n = 21 + 1 = 22$

2. The 8th term of an AP is 17 and its 14th term is 29. The common difference of this AP is

(*a*) 3 (*b*) 2

(*c*) 5 (*d*) –2 **[CBSE 2023 Basic]**

Sol. (*b*) 2

Let a and d be the first term and common difference of the AP.

$$a_8 = 17$$

$\Rightarrow$ $a + 7d = 17$

$\Rightarrow$ $a = 17 - 7d$ …(1)

$$a_{14} = 29$$

$\Rightarrow$ $a + 13d = 29$

$\Rightarrow$ $17 - 7d + 13d = 29$

[Putting the value of a from (1)]

$\Rightarrow$ $17 + 6d = 29$

$\Rightarrow$ $6d = 29 - 17 = 12$

$\therefore$ $d = \dfrac{12}{6} = 2$

3. The next term of the AP : $\sqrt{6}, \sqrt{24}, \sqrt{54}$ is

(*a*) $\sqrt{60}$ (*b*) $\sqrt{96}$

(*c*) $\sqrt{72}$ (*d*) $\sqrt{216}$

[CBSE 2023 Standard]

Sol. (*b*) $\sqrt{96}$

$$a = \sqrt{6}$$

$$d = \sqrt{24} - \sqrt{6} = 2\sqrt{6} - \sqrt{6} = \sqrt{6}$$

$\therefore$ Next term $= \sqrt{54} + \sqrt{6} = 3\sqrt{6} + \sqrt{6} = 4\sqrt{6}$

$$= \sqrt{96}$$

4. nth term of an AP is $7n + 4$. The common difference is

(*a*) $7n$ (*b*) 4

(*c*) 7 (*d*) 1

[CBSE 2024 Standard]

Sol. (*c*) 7

$$a_n = 7n + 4$$

$$a_1 = 7 \times 1 + 4 = 7 + 4 = 11$$

$$a_2 = 7 \times 2 + 4 = 14 + 4 = 18$$

$$d = a_2 - a_1 = 18 - 11 = 7$$

5. If 2 + 4 + 6 + 8 + … to nth term = 110, then the value of n is equal to

(*a*) 11 (*b*) 12

(*c*) 10 (*d*) 9

Sol. (*c*) 10

We have

2 + 4 + 6 + … to nth term

$= 2(1 + 2 + 3 + \ldots$ to n terms)

$= 2 \times \dfrac{n(n+1)}{2}$

$= n(n + 1)$

$\therefore$ $n(n + 1) = 110$ [Given]

$\Rightarrow$ $n^2 + n - 110 = 0$

$\Rightarrow$ $n^2 + 11n - 10n - 110 = 0$

$\Rightarrow$ $n(n + 11) - 10(n + 11) = 0$

$\Rightarrow$ $(n + 11)(n - 10) = 0$

$\therefore$ Either $n + 11 = 0$

$\Rightarrow$ $n = -11$

which is rejected, since n cannot be negative.

or $n - 10 = 0$

$\Rightarrow$ $n = 10$

which is accepted.

$\therefore$ The value of n is 10.

6. If the sum of three numbers in AP is 18 and their product is 120, then the numbers are

(*a*) 9, 6, 3 (*b*) 5, 6, 7

(*c*) 1, 6, 11 (*d*) 2, 6, 10

Sol. (*d*) 2, 6, 10

Let $\alpha - d$, α and $\alpha + d$ be any three terms of an AP

$\therefore$ According to the problem,

$$\alpha - d + \alpha + \alpha + d = 18$$

$\Rightarrow$ $3\alpha = 18$

$\Rightarrow$ $\alpha = 6$ …(1)

Also, $(\alpha - d)\,\alpha\,(\alpha + d) = 120$

$$6(\alpha^2 - d^2) = 120$$

$$\Rightarrow \quad \alpha^2 - d^2 = 20$$

$$\Rightarrow \quad 36 - d^2 = 20 \quad \text{[From (1)]}$$

$$\Rightarrow \quad d^2 = 16$$

$$\Rightarrow \quad d = \pm 4$$

$\therefore$ The required AP is 6 + 4, 6 and 6 – 4, i.e. 10, 6 and 2 or 6 – 4, 6, 6 + 4, i.e. 2, 6, 10.

$\therefore$ The required numbers are 2, 6, 10.

Fill in the Blanks

7. The tenth term of the AP: 2.5, 4.5, 6.5, … is **20.5**.

Sol.

$$a_n = a + (n-1)d$$

$$a_{10} = 2.5 + (10 - 1)(2)$$

$$= 2.5 + 9(2)$$

$$= 2.5 + 18$$

$$= 20.5$$

8. If $a_{20} - a_{12} = -32$, then the common difference of the AP is **–4**.

Sol.

$$a_{20} - a_{12} = -32$$

$$\Rightarrow \quad a + 19d - a - 11d = -32$$

$$\Rightarrow \quad 8d = -32$$

$$\Rightarrow \quad d = -4$$

9. If k, $2k - 1$ and $2k + 1$ are three consecutive terms of an AP, the value of k is **3**.

Sol.

$$d = 2k - 1 - k$$

$$= k - 1$$

$$d = 2k + 1 - 2k + 1$$

$$= 2$$

On comparing, we have

$$k - 1 = 2$$

$$\Rightarrow \quad k = 3$$

10. The sum of first five terms of the AP: 3, 7, 11, 15, … is **55**.

Sol. Here,

$$d = 7 - 3 = 4$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_5 = \frac{5}{2}[2(3) + (5-1)(4)]$$

$$= \frac{5}{2}[6 + 4(4)]$$

$$= \frac{5}{2}(6 + 16) = \frac{5}{2} \times 22$$

$$= 5 \times 11 = 55$$

Assertion-Reason Type Questions

Directions (Q. Nos. 11 to 14): Each of these questions contains an assertion followed by a reason. Read them carefully, and answer the question on the basis of the following options, select the one that best describes the two statements.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

11. Assertion (A): The 10th term in the series 2, 4, 6, 8,… is 20.

Reason (R): The series is an arithmetic progression with common difference of 3.

Sol. The correct answer is (*c*).

The Assertion is correct but the Reason is wrong as common difference is 2 and not 3.

12. Assertion (A): The sum of first 8 terms in the series 45, 43, 41, 39,… is 304.

Reason (R): The first term is 45 and the common difference is – 2.

Sol. The correct answer is (*a*).

The first term is 45 and the common difference is –2.

So, sum of 8 terms is 304.

Thus, both Assertion and Reason are correct and Reason is the correct explanation of Assertion.

13. Assertion (A): 89 is not a term of the list of numbers 5, 11, 17, 23,…

Reason (R): The first term is 5 and common difference is 6.

Sol. The correct answer is (*d*).

In the series, the common difference is 6.

So, 15th term will be 89.

Thus, the Assertion is wrong but Reason is correct.

14. Assertion (A): a, b, c are in AP if and only if $2b = a + c$.

Reason (R): The sum of first n odd natural numbers is n^2. **[CBSE 2023 Standard]**

Sol. The correct answer is (*b*).

a, b and c are in AP.

$\Rightarrow \quad b - a = c - b$

$\Rightarrow \quad 2b = a + c$

$\therefore$ Assertion is correct.

The first n odd natural numbers are as follows:

1, 3, 5, 7, 9, 11, …., $(2n - 1)$

Here, $a = 1$

$d = 3 - 1 = 2$

$a_n = 2n - 1$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{n}{2}[2 \times 1 + (n-1) \times 2]$$

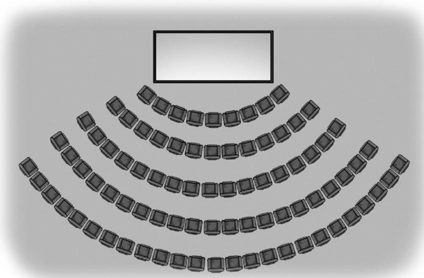
$$= \frac{n}{2}[2 + 2n - 2]$$

$$= \frac{n}{2} \times 2n = n^2$$

$\therefore$ Reason is also true.

Case Study Based Questions

15. In the school auditorium, annual day function is being organised. Organisers observe that, 9 guests occupied the 1st row, 13 guests occupied the 2nd row, 17 guests occupied the 3rd row and so on. There were 25 guests in the last row.



Read the above situation and answer the following questions.

(*a*) Write a series of number of guests based on their seating arrangement.

Ans. 9, 13, 17,…, 25

(*b*) How many rows are in the auditorium?

Ans. 5

(*c*) (*i*) Write the number of guests seated in 4th row.

Ans. 21

or

(*ii*) Write the total number of guests seated in the auditorium.

Ans. 85

16. In an energy drink factory, manager observes that, in the 3rd year, the factory manufactured 3000 drinks and in the 6th year, the factory manufactured 6000 drinks (assuming that the production increases uniformly by a fixed number every year). Read the above information and answer the following questions:

(*a*) What is the production of drinks in the 1st year?

Ans. 1000

(*b*) What is the production increase uniformly by a fixed number every year?

Ans. 1000

(*c*) (*i*) What is the production of drinks in the 4th year?

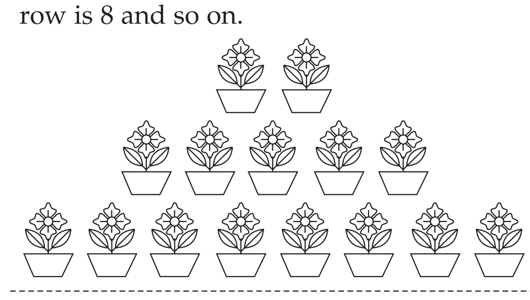
Ans. 4000

or

(*ii*) What is the total production of energy drinks in first 6 years?

Ans. 21000

17. Aahana being a plant lover decides to convert her balcony into beautiful garden full of plants. She bought few plants with pots for her balcony. She placed the pots in such a way that number of pots in the first row is 2, second row is 5, third row is 8 and so on.



Based on the above information, answer the following questions:

(*a*) Find the number of pots placed in the 10th row.

Ans. $a = 2$

$$d = 5 - 2 = 3$$
$$a_{10} = a + (n-1)d$$
$$= 2 + (10-1) \times 3$$
$$= 2 + 9 \times 3$$
$$= 29$$
$$\therefore \quad a_{10} = 29$$

(*b*) Find the difference in the number of pots placed in 5th row and 2nd row.

Ans. $a_5 = a + (5-1)d = 2 + 4 \times 3 = 14$

$a_2 = 2 + (2-1)(3) = 2 + 3 = 5$

$a_5 - a_2 = 14 - 5 = 9$

(*c*) (*i*) If Aahana wants to place 100 pots in total, then find the total number of rows formed in the arrangement.

Ans. Total number of pots = 100

Let total number of rows = n

We have,

$$S_n = \frac{n}{2}[2a + (n-1)d]$$
$$100 = \frac{n}{2}[2 \times 2 + (n-1) \times 3]$$
$$\Rightarrow \quad 100 = \frac{n}{2}[4 + 3n - 3]$$
$$\Rightarrow \quad 200 = n[3n + 1]$$
$$\Rightarrow \quad 3n^2 + n - 200 = 0$$
$$\therefore \quad n = \frac{-1 \pm \sqrt{1 + 4 \times 3 \times 200}}{2 \times 3}$$
$$= \frac{-1 \pm \sqrt{1 + 2400}}{6}$$
$$= \frac{-1 \pm \sqrt{2401}}{6}$$
$$= \frac{-1 \pm 49}{6}$$
$$\therefore \quad n = \frac{-1-49}{6} \text{ or } \frac{-1+49}{6}$$
$$n = \frac{-50}{6} \text{ or } \frac{48}{6}$$
$$n = \frac{-25}{3} \text{ or } 8$$

Since number of rows cannot be negative.

$$\therefore \quad n = 8$$

Total number of rows formed = 8.

or

(*ii*) If Aahana has sufficient space for 12 rows, then how many total number of pots are placed by her with the same arrangement.

[CBSE 2023 Basic]

Ans. Number of rows = 12

$$a = 2$$
$$d = 3$$

Let total number of pots be S_n

$$S_n = \frac{n}{2}[2a + (n-1)d]$$
$$= \frac{12}{2}[2 \times 2 + (12-1)3]$$
$$= \frac{12}{2}[4 + 11 \times 3]$$
$$= 6[4 + 33] = 6 \times 37$$
$$S_n = 222$$

Very Short Answer Type Questions

18. Are the numbers $\sqrt{18}$, $\sqrt{50}$, $\sqrt{98}$, … in AP? If so, what is its fourth term?

Sol. We have
$$\sqrt{18} = \sqrt{9 \times 2} = 3\sqrt{2},$$
$$\sqrt{50} = \sqrt{25 \times 2} = 5\sqrt{2},$$
$$\sqrt{98} = \sqrt{49 \times 2} = 7\sqrt{2}, \text{ etc.}$$

Hence, the numbers $\sqrt{18}$, $\sqrt{50}$, $\sqrt{98}$, …, i.e. $3\sqrt{2}$, $5\sqrt{2}$, $7\sqrt{2}$, … form an AP with common difference, $d = 5\sqrt{2} - 3\sqrt{2} = 2\sqrt{2}$

Here, the 1st term, $a = 3\sqrt{2}$. If a_n denotes the nth term of this AP, then $a_n = a + (n-1)d$

$$\therefore \quad a_4 = a + 3d$$
$$= 3\sqrt{2} + 3 \times 2\sqrt{2}$$
$$= 9\sqrt{2}$$
$$= \sqrt{9^2 \times 2}$$
$$= \sqrt{162}$$

$\therefore$ Its fourth term is $\sqrt{162}$.

19. If the 10th term of an AP with common difference –3 is –20, what is its first term?

Sol. Let a be the 1st term and the common difference, $d = -3$. If a_n be the nth term of this AP, then

$$a_n = a + (n-1)d$$
$$\therefore \quad a_{10} = a + 9d = -20 \quad \text{[Given]}$$
$$\Rightarrow \quad a - 9 \times 3 = -20$$
$$\Rightarrow \quad a = 27 - 20 = 7$$

$\therefore$ Its first term is 7.

20. Find the sum of two middle-most terms of the AP $-\frac{4}{3}, -1, -\frac{2}{3}, \ldots, 4\frac{1}{3}$. **[CBSE 2014]**

Sol. For the given AP, 1st term, $a = -\frac{4}{3}$ and the common difference, $d = -1 + \frac{4}{3} = \frac{1}{3}$

If a_n denote the nth term of the AP and n be the total number of terms of the given AP, then

$$a_n = a + (n-1)d$$

$$\Rightarrow \quad a_n = -\frac{4}{3} + \frac{n-1}{3}$$

$$\Rightarrow \quad \frac{13}{3} = \frac{-4}{3} + \frac{n-1}{3}$$

$$\Rightarrow \quad \frac{n-1}{3} = \frac{13+4}{3} = \frac{17}{3}$$

$$\therefore \quad n = 17 + 1 = 18$$

$\therefore$ The total number of terms of the given AP is 18, which is even.

$\therefore$ There are two middle terms of this AP which are $a_{\frac{18}{2}}$ and $a_{\frac{18}{2}+1}$, i.e. a_9 and a_{10}.

Now,
$$a_9 = -\frac{4}{3} + \frac{9-1}{3} = \frac{-4}{3} + \frac{8}{3} = \frac{-4+8}{3} = \frac{4}{3}$$

$$a_{10} = a_9 + d = \frac{4}{3} + \frac{1}{3} = \frac{5}{3}$$

$$\therefore \quad a_9 + a_{10} = \frac{4}{3} + \frac{5}{3} = \frac{9}{3} = 3$$

$\therefore$ The sum of two middle most terms of the AP is 3.

21. In an AP, the sum of n terms is $\frac{3n^2}{2} + \frac{13n}{2}$. Find its 25th term. **[CBSE 2015, 2006C]**

Sol. We have

$$S_n = \frac{3n^2}{2} + \frac{13n}{2}$$

where S_n denotes the sum of n terms of the AP. Let a_n denote the nth terms of the AP.

Then $a_n = S_n - S_{n-1}$

$$= \frac{3n^2}{2} + \frac{13n}{2} - \frac{3(n-1)^2}{2} - \frac{13(n-1)}{2}$$

$$= \frac{3n^2 + 13n - 3n^2 - 3 + 6n - 13n + 13}{2}$$

$$= \frac{6n - 3 + 13}{2}$$

$$= \frac{6n+10}{2}$$

$$= 3n + 5$$

$$\therefore \quad a_n = 3n + 5$$

$$\Rightarrow \quad a_{25} = 3 \times 25 + 5 = 75 + 5 = 80$$

$\therefore$ Its 25th term is 80.

Short Answer Type Questions

22. The fourth term of an AP is zero. Prove that the 25th term of the AP is three times its 11th term. **[CBSE 2016]**

Sol. Let a be the 1st term and d be its common difference. If a_n denote the nth term of the AP, then

$$a_n = a + (n-1)d$$

It is given that $a_4 = 0$

$$\Rightarrow \quad a + 3d = 0$$

$$\therefore \quad a = -3d \qquad \ldots(1)$$

Now, $a_{25} = a + 24d$

$= -3d + 24d$ [From (1)]

$= 21d$

and $a_{11} = a + 10d = -3d + 10d = 7d$

$$\therefore \quad \frac{a_{25}}{a_{11}} = \frac{21d}{7d} = 3$$

$$\therefore \quad a_{25} = 3a_{11}$$

Hence, proved.

23. Find the sum of all possible integral multiples of 3, which are less than 200.

Sol. Numbers which are less than 200 but multiples of 3 are 3, 6, 9, 12, … 198

If S be their sum, then

$$S = 3 + 6 + 9 + \ldots + 198$$
$$= 3(1 + 2 + 3 + \ldots + 66)$$
$$= 3 \times \frac{66 \times 67}{2}$$
$$= 99 \times 67$$
$$= 6633$$

$\therefore$ The required sum is 6633.

24. How many terms of the AP 1, 4, 7, … are needed to give the sum 715?

Sol. Let a be the 1st term and d be the common difference of the given AP. Then $a = 1$ and

$d = 4 - 1 = 3$. If S_n denote the sum of n terms of the AP, then

$$S_n = \frac{n}{2}[2a + (n-1)d]$$
$$= na + \frac{n(n-1)d}{2}$$

When $S_n = 715$, then

$$S_n = 715 = n + \frac{n(3n-3)}{2}$$

$$\Rightarrow \quad 715 = \frac{2n + 3n^2 - 3n}{2}$$

$$\Rightarrow \quad 3n^2 - n - 1430 = 0$$

$$\therefore \quad n = \frac{1 \pm \sqrt{(-1)^2 + 4 \times 3 \times 1430}}{6}$$
$$= \frac{1 \pm \sqrt{17161}}{6}$$
$$= \frac{1 \pm 131}{6}$$
$$= \frac{132}{6} \text{ or } \frac{-130}{6}$$
$$= 22 \quad \text{or} \quad \frac{-65}{3}$$

We reject the negative value since number of terms cannot be negative.

Hence, the required term is 22.

25. Which term of the AP 65, 61, 57, 53, … is the first negative term. **[CBSE 2023 Standard]**

Sol. First term, $a = 65$

Common difference, $d = 61 - 65 = -4$

Let the nth term of the AP be the first negative term

$$\Rightarrow \quad a_n < 0$$
$$a_n = a + (n-1)d$$
$$a + (n-1)\, d < 0$$
$$\Rightarrow \quad 65 + (n-1)(-4) < 0$$
$$\Rightarrow \quad 65 - 4n + 4 < 0$$
$$\Rightarrow \quad 69 - 4n < 0$$
$$\Rightarrow \quad 4n > 69$$
$$\Rightarrow \quad n > \frac{69}{4}$$
$$\Rightarrow \quad n > 17.25$$
$$\Rightarrow \quad n = 18$$

$\therefore$ 18th term must be the first negative term.

Long Answer Type Questions

26. In an AP, if $S_n = 3n^2 + 5n$ and $a_k = 164$, find the value of k. **[CBSE 2023 Basic]**

Sol. Let a_n be the nth term of the AP.

$$a_n = S_n - S_{n-1}$$
$$= 3n^2 + 5n - 3(n-1)^2 - 5(n-1)$$
$$= 3n^2 + 5n - 3[n^2 - 2n + 1] - 5(n-1)$$
$$= 3n^2 + 5n - 3n^2 + 6n - 3 - 5n + 5$$
$$= 6n + 2$$
$$\therefore \quad a_n = 6n + 2$$
$$\therefore \quad a_k = 6k + 2$$
$$\text{Given} \quad a_k = 164$$
$$\Rightarrow \quad 6k + 2 = 164$$
$$\Rightarrow \quad 6k = 164 - 2 = 162$$
$$\Rightarrow \quad k = \frac{162}{6} = 27$$
$$\therefore \quad k = 27$$

27. How many terms of the arithmetic progression 45, 39, 33, … must be taken so that their sum is 180? Explain the double answer. **[CBSE 2023 Standard]**

Sol. $a = 45$

$d = 39 - 45 = -6$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$
$$\Rightarrow \quad 180 = \frac{n}{2}[2 \times 45 + (n-1)(-6)]$$
$$\Rightarrow \quad 360 = n[90 - 6n + 6]$$
$$\Rightarrow \quad 360 = n[96 - 6n]$$
$$\Rightarrow \quad 60 = n[16 - n]$$
$$\Rightarrow \quad 60 = 16n - n^2$$
$$\Rightarrow \quad n^2 - 16n + 60 = 0$$
$$\Rightarrow \quad n^2 - 6n - 10n + 60 = 0$$
$$\Rightarrow \quad n(n-6) - 10(n-6) = 0$$
$$\Rightarrow \quad (n-6)(n-10) = 0$$
$$\Rightarrow \quad n = 6 \quad \text{or} \quad n = 10$$

As the common difference is negative, therefore some of the terms will be negative. The negative terms will cancel out the positive terms. Therefore, for two values of n, sum of terms will be the same.

28. In an AP of 40 terms, the sum of first 9 terms is 153 and the sum of last 6 terms is 687. Determine the first term and common difference of AP. Also, find the sum of all the terms of the AP. **[CBSE 2024 Standard]**

Sol. Let a and d be the first term and common difference of the AP.

$$S_9 = 153$$

$$\Rightarrow \quad S_9 = \frac{9}{2}[2a + (9-1)d]$$

$$\Rightarrow \quad 153 = \frac{9}{2}[2a + 8d]$$

$$\Rightarrow \quad 34 = 2a + 8d$$

$$\Rightarrow \quad a + 4d = 17 \quad \ldots(1)$$

Considering the last term as the first term, then d will be $-d$.

$$S_{\text{last }6} = \frac{6}{2}[2l - (6-1)d]$$

$$= \frac{6}{2}[2(a + 39d) - 5d]$$

$$687 = 3[2a + 78d - 5d]$$

$$\Rightarrow \quad 2a + 73d = 229 \quad \ldots(2)$$

Solving equations (1) and (2)

$$2a + 8d = 34$$
$$2a + 73d = 229$$
$$- \quad - \quad\quad -$$

$$-65d = -195$$

$$\Rightarrow \quad d = \frac{195}{65} = 3$$

From (1)

$$a + 4d = 17$$

$$\Rightarrow \quad a = 17 - 4d = 17 - 4 \times 3 = 17 - 12$$

$$\Rightarrow \quad a = 5$$

Sum of all the terms of the AP = S_{40}

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{40} = \frac{40}{2}[2 \times 5 + (40-1)(3)]$$

$$= 20[10 + 39 \times 3]$$
$$= 20[10 + 117]$$
$$= 20 \times 127$$
$$= 2540$$

$$\therefore \quad S_{40} = 2540$$

29. A gentleman buys every year bank's certificates of value exceeding the last year's purchase by ₹250. After 20 years, he finds that the total value of the bank certificates purchased by him is ₹72500. Find the value of the certificates purchased by him in

(*a*) the first year and (*b*) the 13th year.

Sol. (*a*) We see that the values of the certificates in successive years form an AP with the 1st term, a and the common difference, d = ₹250. We denote the sum of n terms of this AP by S_n.

Then $\quad S_n = \frac{n}{2}[2a + (n-1)d]$

Given that $\quad S_{20}$ = ₹72500

$$\therefore \quad \frac{20}{2}[2a + (20-1) \times 250] = 72500$$

$$\Rightarrow \quad 20a + 190 \times 250 = 72500$$

$$\Rightarrow \quad 20a + 47500 = 72500$$

$$\Rightarrow \quad 20a = 25000$$

$$\Rightarrow \quad a = \frac{25000}{20}$$

$$= 1250$$

$\therefore$ The required value of the certificate in the 1st year is ₹1250.

(*b*) Here, we shall have to find a_{13}.

We have $\quad a_{13} = a + (13-1)d$

$$= 1250 + 12 \times 250$$
$$= 1250 + 3000$$
$$= 4250$$

$\therefore$ The required value of the certificate in the 13th year is ₹4250.

Let's Compete

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Multiple-Choice Questions

1. The 10th term of the AP –1.0, –1.5, –2.0, … is

(*a*) 3.5 (*b*) –5.5

(*c*) 5.5 (*d*) –6.5 **[CBSE SP 2012]**

Sol. (*b*) –5.5

Here, the 1st term, $a = -1$, common difference, $d = -1.5 + 1.0 = -0.5$

Let a_n denote the nth term.

Then $\quad a_n = a + (n-1)d$

$$\therefore \quad a_{10} = a + 9d$$
$$= -1 - 9 \times 0.5$$
$$= -1 - 4.5$$
$$= -5.5$$

2. In an AP, if the common difference, $d = -\frac{1}{2}$, the number of terms, $n = 37$ and the sum to 37 terms is $425\frac{1}{2}$, then the first term is equal to

(a) $\frac{41}{2}$ (b) $\frac{1}{2}$

(c) $-\frac{1}{2}$ (d) $-\frac{41}{2}$

Sol. (a) $\frac{41}{2}$

Let a be the 1st term.

If S_n denote the sum of n terms, then

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$= an + \frac{n(n-1)d}{2} \quad \ldots(1)$$

It is given that $S_{37} = 425\frac{1}{2} = \frac{851}{2}$

$$\therefore \quad \frac{851}{2} = 37a + \frac{37 \times 36}{2} \times \frac{-1}{2} \quad \text{[From (1)]}$$

$$= 37a - \frac{37 \times 36}{4}$$

$$= 37a - \frac{1332}{4}$$

$$= 37a - 333$$

$$\Rightarrow \quad 37a = \frac{851}{2} + 333$$

$$= \frac{851 + 666}{2}$$

$$= \frac{1517}{2}$$

$$\therefore \quad a = \frac{1517}{2 \times 37}$$

$$= \frac{1517}{74} = \frac{41}{2}$$

3. If $3k$, $8k - 1$ and $5k + 1$ are three consecutive terms of an AP, then the value of k is

(a) $\frac{1}{4}$ (b) $\frac{2}{5}$

(c) $\frac{3}{8}$ (d) $\frac{3}{4}$

Sol. (c) $\frac{3}{8}$

Clearly, the common difference of any two successive terms will be constant.

$$\therefore \quad 8k - 1 - 3k = 5k + 1 - (8k - 1)$$

$$\Rightarrow \quad 8k - 1 - 3k = 5k + 1 - 8k + 1$$

$$\Rightarrow \quad 5k - 1 = -3k + 2$$

$$\Rightarrow \quad 8k = 3$$

$$\Rightarrow \quad k = \frac{3}{8}$$

4. If 3 times the 3rd term of an AP is equal to 4 times its 4th term, then its 7th term is equal to

(a) 18 (b) 11

(c) 7 (d) 0

Sol. (d) 0

Let a be the 1st term and d be the common difference of the AP. Let a_n be the nth term of the AP.

Given that $\quad 3a_3 = a_4$

$$\therefore \quad 3(a + 2d) = 4(a + 3d)$$

$$\Rightarrow \quad a + 6d = 0 \quad \ldots(1)$$

$$\therefore \quad a_7 = a + 6d = 0 \quad \text{[From (1)]}$$

5. The sum of $1^2 - 2^2 + 3^2 - 4^2 + 5^2 - 6^2 + \ldots$ to 50 terms is equal to

(a) –1175 (b) –1275

(c) –1375 (d) –125

Sol. (b) –1275

We have, $S = 1^2 - 2^2 + 3^2 - 4^2 + \ldots$ to 50 terms.

Now, writing two consecutive terms in a bracket to form a group, we have

$S = (1^2 - 2^2) + (3^2 - 4^2) + (5^2 - 6^2) + \ldots$ to 25 groups.

$= \{(1-2)(1+2)\} + \{(3-4)(3+4)\} + \{(5-6)(5+6)\} + \ldots$ to 25 groups

$= -1 \times (3 + 7 + 11 + \ldots$ to 25 terms$)$

$= -1 \times \frac{25}{2}(2 \times 3 + 24 \times 4)$

$= -1 \times (75 + 1200)$

$= -1275$

6. If the sum of the first p even numbers is equal to q times the sum of the first p odd numbers, then q is equal to

(a) $1 - \frac{1}{1+p}$ (b) $1 + \frac{1}{1+p}$

(c) $1 - \frac{1}{p}$ (d) $1 + \frac{1}{p}$

Sol. (d) $1 + \frac{1}{p}$

It is given that

$2 + 4 + 6 + 8 + \ldots$ to p terms

$= q(1 + 3 + 5 + 7 + \ldots$ to p terms$) \quad \ldots(1)$

Now, for the 1st AP, the 1st term = 2 and the common difference = 4 – 2 = 2 and for the 2nd AP, the 1st term = 1 and the common difference = 3 – 1 = 2.

Also, for each AP, the total number of terms = p.

$\therefore$ From (1), we have

$$\frac{p}{2}\{2 \times 2 + (p-1)2\} = q\{2 \times 1 + (p-1)2\} \times \frac{p}{2}$$

$$\Rightarrow \quad 4 + 2p - 2 = 2q + 2pq - 2q$$
$$\Rightarrow \quad 2 + 2p = 2pq$$
$$\Rightarrow \quad 1 + p = pq$$
$$\Rightarrow \quad q = \frac{1+p}{p}$$
$$\Rightarrow \quad q = \frac{1}{p} + 1$$
$$\therefore \quad q = 1 + \frac{1}{p}$$

7. If S_2 and S_3 be the sums of two and three terms respectively of an AP with the first term a, then

(*a*) $3S_2 - S_3 = 3a$ (*b*) $S_3 - 3S_2 = 3a$
(*c*) $3S_2 + S_3 = 3a$ (*d*) $S_3 + 3S_2 = 5a$

Sol. (*a*) $3S_2 - S_3 = 3a$

Let a be the 1st term, d be the common difference and S_n be the sum of n terms of the AP.

Then $$S_n = \frac{n}{2}\{2a + (n-1)d\}$$
$$\therefore \quad S_2 = \frac{2}{2}(2a + d)$$
$$\Rightarrow \quad S_2 = 2a + d \qquad \ldots(1)$$
and $$S_3 = \frac{3}{2}(2a + 2d)$$
$$\Rightarrow \quad S_3 = 3a + 3d \qquad \ldots(2)$$

Multiplying (1) by 3, we get
$$3S_2 = 6a + 3d \qquad \ldots(3)$$

Subtracting (2) from (3) to eliminate d, we get
$$3S_2 - S_3 = 3a$$

8. The sum of three consecutive terms of an increasing AP is 36 and the product of its first and third terms is 23. Then its third term is

(*a*) 1 (*b*) 12
(*c*) 11 (*d*) 23

Sol. (*d*) 23

Let $\alpha - d$, α and $\alpha + d$, where $d > 0$, be three consecutive terms of an increasing AP. Then, according to the problem, we have
$$(\alpha - d) + \alpha + (\alpha + d) = 36$$
$$\Rightarrow \quad 3\alpha = 36$$
$$\Rightarrow \quad \alpha = 12$$
Also, $$(\alpha - d)(\alpha + d) = 23 \qquad \text{[Given]}$$
$$\Rightarrow \quad \alpha^2 - d^2 = 23$$
$$\Rightarrow \quad 12^2 - d^2 = 23$$
$$\Rightarrow \quad d^2 = 144 - 23$$
$$\Rightarrow \quad d^2 = 121$$
$$\Rightarrow \quad d = \pm 11$$

Hence, the required 3rd term is $\alpha + d = 12 + 11 = 23$

9. The sum of all two-digit odd positive numbers is

(*a*) 2250 (*b*) 2475
(*c*) 2275 (*d*) 2450

Sol. (*b*) 2475

We see that the two-digit odd numbers are 11, 13, 15, 17, … 99 which form an AP with the 1st term, $a = 11$ and the common difference, $d = 13 - 11 = 2$.

Let n denote the nth term of the AP. If n be the total number of terms of this AP, then
$$a_n = 99$$
$$\Rightarrow \quad a + (n-1)d = 99$$
$$\Rightarrow \quad 11 + (n-1)2 = 99$$
$$\Rightarrow \quad n - 1 = \frac{99 - 11}{2} = 44$$
$$\therefore \quad n = 44 + 1 = 45$$

Hence, the total number of terms of this AP is 45.

Now, 11 + 13 + 15 + 17 +… to 45 terms
$$S_n = \frac{45}{2} \times \{2a + (45-1)d\}$$
$$= \frac{45}{2}\{2 \times 11 + 44 \times 2\}$$
$$= 45 \times 11 + 45 \times 22 \times 2$$
$$= 45(11 + 44)$$
$$= 45 \times 55$$
$$= 2475$$

10. If the ratio of the 16th term to the 4th term of an AP is 1 : 4, then the ratio of the 72nd term to its 84th term is

(*a*) 16 : 13 (*b*) 4 : 5
(*c*) 13 : 16 (*d*) 5 : 4

Sol. (*c*) 13 : 16

Let a be the 1st term, d be the common difference of the AP, and a_n denote the nth term of the AP.

Then $$a_n = a + (n-1)d$$
Given that $$\frac{a_{16}}{a_4} = \frac{1}{4}$$
$$\therefore \quad \frac{a + 15d}{a + 3d} = \frac{1}{4}$$
$$\Rightarrow \quad 4a + 60d = a + 3d$$
$$\Rightarrow \quad 3a = -57d$$
$$\therefore \quad a = -\frac{57}{3}d = -19d \qquad \ldots(1)$$
$$\therefore \quad \frac{a_{72}}{a_{84}} = \frac{a + 71d}{a + 83d}$$

$$= \frac{-19d + 71d}{-19d + 83d} \quad \text{[From (1)]}$$

$$= \frac{52}{64} = \frac{13}{16}$$

$\therefore \quad a_{72} : a_{84} = 13 : 16$

Life Skills

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1. Yasmeen saves ₹32 during the first month, ₹36 in the second month and ₹40 in the third month. If she continues to save in this manner, in how many months she will save ₹2000, which she has intended to give for the college fee of her maid's daughter. **[CBSE 2014]**

Sol. Let n be the required number of months.

It is given that

₹32 + ₹36 + ₹40 + … to n term = ₹2000

Now, numbers 32, 36, 40, … form an AP with the 1st term, $a = 32$ and the common difference, $d = 36 - 32 = 4$

$\therefore$ The sum $S_n = 32 + 36 + 40 + \ldots$ to n terms

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\therefore \quad 2000 = na + \frac{n(n-1)d}{2}$$

$$\Rightarrow \quad 2000 = 32n + 2n(n-1)$$

$$\Rightarrow \quad 2000 = 2n^2 + 30n$$

$$\Rightarrow \quad 2n^2 + 30n - 2000 = 0$$

$$\Rightarrow \quad n^2 + 15n - 1000 = 0$$

$$\Rightarrow \quad n^2 + 40n - 25n - 1000 = 0$$

$$\Rightarrow \quad n(n+40) - 25(n+40) = 0$$

$$\Rightarrow \quad (n-25)(n+40) = 0$$

$$\therefore \quad n = 25 \quad [\because n > 0]$$

Hence, the required number of months is 25.

2. Reshma wanted to save at least ₹6500 for sending her daughter to school next year (after 12 months). She saved ₹450 in the 1st month and raised her saving by ₹20 every next month. How much will she be able to save in the next 12 months? Will she be able to send her daughter to the school next year?

Sol. Savings in the 1st month, 2nd month, 3rd month, 4th month, … are ₹450, ₹470, ₹490, ₹510, … respectively which are in AP with 1st term, a = ₹450 and the common difference, d = ₹20, If S_n denote the sum of n terms of this AP, then

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

We now find S_{12}.

Now, $$S_{12} = \frac{12}{2}[2 \times 450 + 11 \times 20]$$

$$= 12 \times 450 + 66 \times 20$$

$$= 5400 + 1320 = 6720$$

$\therefore \quad S_{12}$ = ₹6720

which is greater than ₹6500.

Hence, she will be able to send her daughter to a school.

3. The sports teacher of a school wants the students to make a pyramid formation for one of the items for the sports day. She wants all 184 students of the school to participate in this item. She makes the first row of 16 students, the row above it of 14 students and the next higher row of 12 students and so on. Will she be able to involve all the 184 students? If yes, then how many rows will the pyramid consist of?

Sol. We find the sum 16 + 14 + 12 + 10 + … + 2

Now, 16, 14, 12, 10, … , 2 form an AP with 1st term, $a = 16$, common difference, $d = 14 - 16 = -2$.

Let a_n be its nth term and S_n be the sum to n terms.

Then, $$a_n = a + (n-1)d$$

$$= 16 - (n-1)2 = 18 - 2n$$

When $a_n = 2$, then

$$18 - 2n = 2$$

$$\Rightarrow \quad n = 8$$

$$\therefore \quad S_8 = \frac{8}{2}[2 \times 16 - 7 \times 2]$$

$$= 8 \times 16 - 8 \times 7$$

$$= 128 - 56$$

$$= 72$$

$\therefore$ 72 students are needed to form the pyramid. So, with 184 students, it is possible to form a pyramid which will consist of 8 rows.