4

Quadratic Equations

Checkpoint (Page 53)

1. Which of the following is a polynomial?

(*a*) $3x^2 + \frac{1}{x} - 5$

(*b*) $-2x^2 + 5\sqrt{x} + 8$

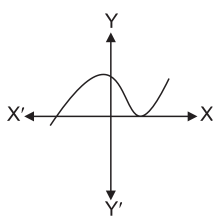
(*c*) $\sqrt{2}\,x^3 + \sqrt{3}\,x^2 + \sqrt{5}\,x - 3$

(*d*) $\frac{3}{x^3} + 4x^2 - 5x + \frac{1}{3}$

Sol. (*c*) $\sqrt{2}x^3 + \sqrt{3}x^2 + \sqrt{5}x - 3$

We know that a polynomial is an algebraic expression in which the exponents of x in each term is a non-negative integer. Only the expression given in option (*c*) of the choices is such an expression.

2. The graph of $y = p(x)$ is given. The number of zeroes of $p(x)$ are:



(*a*) 0
(*b*) 3
(*c*) 2
(*d*) 4

[CBSE SP 2011]

Sol. (*c*) 2

The given graph cuts and touches the x-axis only at two points. Hence, the required number of zeroes of $p(x)$ is 2.

3. If a pair of equations is consistent, then the lines will be

(*a*) always intersecting.
(*b*) always coincident.
(*c*) intersecting or coincident.
(*d*) parallel.

Sol. (*c*) intersecting or coincident.

We know that a pair of equations is consistent when one or more points of the graphs of these equations is or are common. In this case, the lines will be either intersecting or coincident.

4. Verify that 3 is a zero of polynomial $x^3 - 3x^2 - x + 3$.

Sol. Putting $x = 3$ in the polynomial
$p(x) = x^3 - 3x^2 - x + 3$, we get

$$\begin{aligned} p(3) &= 3^3 - 3 \times 3^2 - 3 + 3 \\ &= 27 - 27 - 3 + 3 \\ &= 0 \end{aligned}$$

Hence, 3 is a zero of $p(x)$.

5. If α, β and γ are zeroes of cubic polynomial $x^3 + px^2 + qx + 2$ such that $\alpha\beta + 1 = 0$. Find the value of $2p + q + 5$.

Sol. We have,

$$\alpha + \beta + \gamma = -p \qquad \ldots(1)$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = q \qquad \ldots(2)$$

and $$\alpha\beta\gamma = -2 \qquad \ldots(3)$$

Also, $$\alpha\beta = -1 \qquad \text{[Given]}\ldots(4)$$

$\therefore$ From (3) and (4), we have

$$\gamma = 2 \qquad \ldots(5)$$

$\therefore$ From (1) and (5),

$$\alpha + \beta = -p - 2 \qquad \ldots(6)$$

$\therefore$ From (2), (4) and (6), we have

$$-1 + \gamma(\alpha + \beta) = q$$

$$\Rightarrow \quad -1 + 2(-p - 2) = q$$

$$\Rightarrow \quad 2p + q + 5 = 0$$

6. If α, β and γ are the zeroes of the polynomial $x^3 + 3x^2 + 10x - 24$ then find the value of $\frac{1}{\gamma} + \frac{1}{\alpha} + \frac{1}{\beta}$.

Sol. We have

$$\alpha\beta + \alpha\gamma + \beta\gamma = 10 \quad \ldots(1)$$

and
$$\alpha\beta\gamma = 24 \quad \ldots(2)$$

$\therefore$ We have

$$\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = \frac{\beta\gamma + \gamma\alpha + \alpha\beta}{\alpha\beta\gamma} = \frac{10}{24}$$

[From (1) and (2)]

$$= \frac{5}{12}$$

which is the required value.

7. If $\frac{x}{3} + \frac{y}{4} = 6$ and $\frac{x}{6} + \frac{y}{2} = 6$, find the value of $3y - 2x$ and $\frac{x}{y} + \frac{1}{2}$.

Sol. We have $\frac{x}{3} + \frac{y}{4} = 6$

$$\Rightarrow \frac{4x + 3y}{12} = 6$$

$$\Rightarrow 4x + 3y = 72 \quad \ldots(1)$$

and
$$\frac{x}{6} + \frac{y}{2} = 6$$

$$\Rightarrow \frac{x + 3y}{6} = 6$$

$$\Rightarrow x + 3y = 36 \quad \ldots(2)$$

Multiplying (2) by 4, we get

$$4x + 12y = 144 \quad \ldots(3)$$

$\therefore$ Subtracting (1) from (3), we get

$$9y = 144 - 72 = 72$$

$$\Rightarrow y = \frac{72}{9} = 8$$

$\therefore$ From (2), we get

$$x = 36 - 3 \times 8 = 12$$

Now, putting the values of x and y in the given expressions, we get

$$3y - 2x = 3 \times 8 - 2 \times 12$$
$$= 24 - 24 = 0$$

and
$$\frac{x}{y} + \frac{1}{2} = \frac{12}{8} + \frac{1}{2}$$
$$= \frac{3}{2} + \frac{1}{2}$$
$$= \frac{3+1}{2}$$
$$= \frac{4}{2} = 2$$

Hence, the required values of $3y - 2x$ and $\frac{x}{y} + \frac{1}{2}$ are 0 and 2 respectively.

8. Solve for x and y: $149x - 330y = -511$ and $-330x + 149y = -32$

Sol. We have

$$149x - 330y = -511 \quad \ldots(1)$$

and
$$-330x + 149y = -32 \quad \ldots(2)$$

Adding (1) and (2), we get

$$149(x + y) - 330(x + y) = -543$$

$$\Rightarrow (149 - 330)(x + y) = -543$$

$$\Rightarrow -181(x + y) = -543$$

$$\Rightarrow x + y = \frac{543}{181} = 3 \quad \ldots(3)$$

Again, subtracting (2) from (1), we get

$$149(x - y) + 330(x - y) = -511 + 32$$

$$\Rightarrow (149 + 330)(x - y) = -479$$

$$\Rightarrow 479(x - y) = -479$$

$$\Rightarrow x - y = -1 \quad \ldots(4)$$

Adding (3) and (4), we get

$$2x = 3 - 1 = 2$$

$$\Rightarrow x = 1$$

$\therefore$ From (3), $y = 3 - x$
$$= 3 - 1 = 2$$

$\therefore$ The required solution is $x = 1$, $y = 2$.

9. Five years ago, A was thrice as old as B, and ten years later, A shall be twice as old as B. What are the present ages of A and B?

Sol. Let the present ages of A and B be x years and y years respectively.

$\therefore$ According to the problem, we have

$$x - 5 = 3(y - 5)$$

$$\Rightarrow x - 3y + 10 = 0 \quad \ldots(1)$$

and
$$x + 10 = 2(y + 10)$$

$$\Rightarrow x - 2y - 10 = 0 \quad \ldots(2)$$

Subtracting (2) from (1), we get

$$-y + 20 = 0$$

$$\Rightarrow y = 20$$

$\therefore$ From (1), $x = 3y - 10$
$$= 3 \times 20 - 10$$
$$= 50$$

$\therefore$ The required present ages of A and B are 50 years and 20 years respectively.

10. Divide 100 into two parts such that the sum of their reciprocals is $\frac{1}{24}$.

Sol. Let one of the two parts of the number be x. Then the other part is $100 - x$.

$\therefore$ According to the problem,

$$\frac{1}{x} + \frac{1}{100 - x} = \frac{1}{24}$$

$$\frac{100 - x + x}{x(100 - x)} = \frac{1}{24}$$

$\Rightarrow$ $x(100 - x) = 2400$

$\Rightarrow$ $x^2 - 100x + 2400 = 0$

$\Rightarrow$ $x^2 - 60x - 40x + 2400 = 0$

$\Rightarrow$ $x(x - 60) - 40(x - 60) = 0$

$\Rightarrow$ $(x - 60)(x - 40) = 0$

$\therefore$ Either $x - 60 = 0$ ...(1)

or $x - 40 = 0$ ...(2)

From (1), $x = 60$ and from (2), $x = 40$.

Hence, the required two parts are 60 and 40.

Check Your Progress 1

(Page 56)

Multiple-Choice Questions

1. Which one of the following is not a quadratic equation?

(*a*) $3(x - 1)^2 = 4x^2 - 5x + 3$

(*b*) $5x - x^2 = 2x^2 + 3$

(*c*) $(\sqrt{3}x + \sqrt{2})^2 - x^2 = 2x^2 + 2x$

(*d*) $3(x - 1)^2 = 5x^2 - 2x + 1$

Sol. (*c*) $\left(\sqrt{3}x + \sqrt{2}\right)^2 - x^2 = 2x^2 + 2x$

When an algebraic expression can be simplified in the form $ax^2 + bx + c$ where a, b and c are constants and $a \neq 0$ is called a quadratic expression and the corresponding equation $ax^2 + bx + c = 0$ is called a quadratic equation. We see that out of four expressions in (*a*), (*b*), (*c*) and (*d*), only the equation in (*c*) cannot be simplified in the form $ax^2 + bx + c = 0$ where $a \neq 0$, since the second degree terms from both sides of the equation cancel each other giving a linear equation.

2. If $\frac{1}{2}$ is a root of the equation $x^2 + kx - \frac{5}{4} = 0$, then the value of k is

(*a*) $\frac{1}{2}$ (*b*) -2

(*c*) $\frac{1}{4}$ (*d*) 2

Sol. (*d*) 2

Putting $x = \frac{1}{2}$ in the equation $x^2 + kx - \frac{5}{4} = 0$, we get

$$\left(\frac{1}{2}\right)^2 + \frac{k}{2} - \frac{5}{4} = 0$$

$\Rightarrow$ $\frac{1}{4} - \frac{5}{4} + \frac{k}{2} = 0$

$\Rightarrow$ $\frac{k}{2} = 1$

$\therefore$ $k = 2$

Very Short Answer Type Questions

3. Check whether the following equations are quadratic or not.

(*a*) $3x^2 = x + 4$

(*b*) $x^2 + x - 12 = 0$

(*c*) $x^3 - 4x^2 - 7x + 3 = 0$

(*d*) $(5x + 1)(2x + 3) = (10x + 1)(x + 2)$

Sol. (*a*) The given equation can be written as $3x^2 - x - 4 = 0$, which is of the form $ax^2 + bx + c = 0$ where $a \neq 0$.

$\therefore$ This equation is quadratic.

(*b*) The given equation being of the form $ax^2 + bx + c = 0$, $a \neq 0$, is quadratic.

(*c*) The highest exponent of x on the left side of the given equation is 3 and not 2. So, it is not quadratic.

(*d*) The given equation can be simplified as

$$10x^2 + 17x + 3 = 10x^2 + 21x + 2$$

$\Rightarrow$ $4x - 1 = 0$

which is a linear equation and so it is not quadratic.

4. Represent the following situations in the form of quadratic equations:

(*a*) The sum of two numbers is 18. The sum of their reciprocals is $\frac{1}{4}$. We need to find the numbers.

(*b*) The product of the ages (in years) of two sisters is 238. The difference in their ages (in years) is 3. We would like to find their present ages.

Sol. (*a*) Let one of the numbers be x. Then the other number is $18 - x$.

$\therefore$ According to the problem,

$$\frac{1}{x} + \frac{1}{18-x} = \frac{1}{4}$$

$$\Rightarrow \quad \frac{18-x+x}{x(18-x)} = \frac{1}{4}$$

$$\Rightarrow \quad x(18-x) = 72$$

$$\Rightarrow \quad x^2 - 18x + 72 = 0$$

which is the required quadratic equation.

(*b*) Let x years be the age of one of the sisters (say, elder sister)

Then the age of another sister is $\frac{238}{x}$ years.

According to the problem, $x - \frac{238}{x} = 3$

$$\Rightarrow \quad x^2 - 3x - 238 = 0$$

which is the required equation.

5. For the quadratic equation $x^2 + 2x - 15 = 0$, determine which of the following are solution(s):

(*a*) $x = -5$ (*b*) $x = -3$ (*c*) $x = 3$

Sol. (*a*) Putting $x = -5$ in the given equation, we see that

$$\begin{aligned} \text{LHS} &= (-5)^2 + 2(-5) - 15 \\ &= 25 - 10 - 15 \\ &= 0 \\ &= \text{RHS} \end{aligned}$$

$\therefore$ This is a solution of the given equation.

(*b*) For $x = -3$, we have

$$\begin{aligned} \text{LHS} &= (-3)^2 + 2(-3) - 15 \\ &= 9 - 6 - 15 \\ &= -12 \\ &\neq \text{RHS} \end{aligned}$$

$\therefore$ This is not a solution of the given equation.

(*c*) For $x = 3$, we have

$$\begin{aligned} \text{LHS} &= 3^2 + 2 \times 3 - 15 \\ &= 9 + 6 - 15 \\ &= 15 - 15 \\ &= 0 \\ &= \text{RHS} \end{aligned}$$

$\therefore$ This is a solution of the given equation.

6. In each of the following, determine whether the given values of x are solutions of the given equation or not.

(*a*) $2x^2 + 5x - 25 = 0$; $x = -5, x = \frac{5}{2}$

(*b*) $24x^2 - 2x - 15 = 0$; $x = \frac{5}{6}, x = -\frac{3}{4}$

(*c*) $mnx^2 + (m^2 + n^2)x + mn = 0$; $x = -\frac{m}{n}$, $x = -\frac{n}{m}$

(*d*) $(2x + 3)(3x - 2) = 0$; $x = -\frac{3}{2}, x = -\frac{2}{3}$

Sol. (*a*) For $x = -5$,

$$\begin{aligned} \text{LHS} &= 2 \times (-5)^2 + 5 \times (-5) - 25 \\ &= 50 - 25 - 25 \\ &= 0 \\ &= \text{RHS} \end{aligned}$$

$\therefore$ $x = -5$ is a solution.

Again, for $x = \frac{5}{2}$,

$$\begin{aligned} \text{LHS} &= 2 \times \left(\frac{5}{2}\right)^2 + 5\left(\frac{5}{2}\right) - 25 \\ &= \frac{25}{2} + \frac{25}{2} - 25 \\ &= 25 - 25 \\ &= 0 \\ &= \text{RHS} \end{aligned}$$

$\therefore$ $x = \frac{5}{2}$ is also a solution.

(*b*) For $x = \frac{5}{6}$,

$$\begin{aligned} \text{LHS} &= 24 \times \left(\frac{5}{6}\right)^2 - 2 \times \frac{5}{6} - 15 \\ &= 24 \times \frac{25}{36} - \frac{5}{3} - 15 \\ &= \frac{2 \times 25}{3} - \frac{5}{3} - 15 \\ &= \frac{50-5}{3} - 15 \\ &= \frac{45}{3} - 15 \\ &= 15 - 15 \\ &= 0 \\ &= \text{RHS} \end{aligned}$$

$\therefore$ $x = \frac{5}{6}$ is a solution.

Again, for $x = -\frac{3}{4}$, we have

$$\begin{aligned} \text{LHS} &= 24 \times \left(-\frac{3}{4}\right)^2 - 2 \times \left(-\frac{3}{4}\right) - 15 \\ &= \frac{24 \times 9}{16} + \frac{3}{2} - 15 \end{aligned}$$

$$= \frac{27}{2} + \frac{3}{2} - 15$$
$$= \frac{27+3}{2} - 15$$
$$= 15 - 15$$
$$= 0$$
$$= \text{RHS}$$

$\therefore \quad x = -\frac{3}{4}$ is also a solution.

(*c*) For $x = -\frac{m}{n}$, we have

$$\text{LHS} = mn\left(-\frac{m}{n}\right)^2 + (m^2+n^2)\left(-\frac{m}{n}\right) + mn$$
$$= mn \times \frac{m^2}{n^2} - \frac{m^3}{n} - mn + mn$$
$$= \frac{m^3}{n} - \frac{m^3}{n} - mn + mn$$
$$= 0 + 0$$
$$= 0$$
$$= \text{RHS}$$

$\therefore \quad x = -\frac{m}{n}$ is a solution.

Again, for $x = -\frac{n}{m}$, we have

$$\text{LHS} = mn \times \left(-\frac{n}{m}\right)^2 + (m^2+n^2)\left(-\frac{n}{m}\right) + mn$$
$$= mn \times \frac{n^2}{m^2} - \frac{nm^2}{m} - \frac{n^3}{m} + mn$$
$$= \frac{n^3}{m} - nm - \frac{n^3}{m} + nm$$
$$= 0 + 0$$
$$= 0$$
$$= \text{RHS}$$

$\therefore \quad x = -\frac{n}{m}$ is also a solution.

(*d*) For $x = -\frac{3}{2}$,

$$\text{LHS} = \left(-2 \times \frac{3}{2} + 3\right)\left(-3 \times \frac{3}{2} - 2\right)$$
$$= 0 \times \left(-\frac{9}{2} - 2\right)$$
$$= 0$$
$$= \text{RHS}$$

$\therefore \quad x = -\frac{3}{2}$ is a solution.

Again, for $x = -\frac{2}{3}$, we have

$$\text{LHS} = \left(-2 \times \frac{2}{3} + 3\right)\left(-3 \times \frac{2}{3} - 2\right)$$
$$= \left(-\frac{4}{3} + 3\right)(-4)$$
$$= \frac{5}{3} \times (-4)$$
$$\neq \text{RHS}$$

$\therefore \quad x = -\frac{2}{3}$ is not a solution.

Short Answer Type Questions

7. In each of the following, find the value of k for which the given value of x is a solution of the given equation.

(*a*) $kx^2 - x - 2 = 0;\ x = \frac{2}{3}$

(*b*) $\sqrt{5}x^2 + kx + 4\sqrt{5} = 0;\ x = -\sqrt{5}$

Sol. (*a*) For $x = \frac{2}{3}$, we have from the given equation

$$k\left(\frac{2}{3}\right)^2 - \frac{2}{3} - 2 = 0$$
$$\Rightarrow \quad \frac{4k}{9} - \frac{8}{3} = 0$$
$$\Rightarrow \quad \frac{4k}{9} = \frac{8}{3}$$
$$\Rightarrow \quad k = \frac{8}{3} \times \frac{9}{4} = 6$$

which is the required value of k.

(*b*) For $x = -\sqrt{5}$, we have from the given equation

$$\sqrt{5}\left(-\sqrt{5}\right)^2 - \sqrt{5}k + 4\sqrt{5} = 0$$
$$\Rightarrow \quad 5\sqrt{5} - \sqrt{5}k + 4\sqrt{5} = 0$$
$$\Rightarrow \quad 5 - k + 4 = 0$$

$\Rightarrow$ $k = 9$ which is the required value of k.

8. If one root of the quadratic equation $2x^2 + px + 4 = 0$ is 2, find the other root. Also find the value of p.

Sol. Let α be the other root.

Then, sum of the roots $= \alpha + 2 = -\frac{p}{2}$ …(1)

and the product of the roots $= 2\alpha = \frac{4}{2} = 2$

$\therefore \quad \alpha = 1$ …(2)

$\therefore$ From (1) and (2),

$$1 + 2 = -\frac{p}{2}$$
$$\Rightarrow \quad p = 3 \times -2$$
$$\Rightarrow \quad p = -6$$

$\therefore$ The other root is 1 and the required value of p is –6.

9. Find the value of λ for which $x = 2$ is a root of the equation $(2\lambda + 1)\, x^2 + 2x - 3 = 0$.

Sol. Putting $x = 2$ in the given equation, we get

$$(2\lambda + 1)2^2 + 2 \times 2 - 3 = 0$$

$$\Rightarrow \quad (2\lambda + 1)4 + 4 - 3 = 0$$

$$\Rightarrow \quad 8\lambda + 4 + 4 - 3 = 0$$

$$\Rightarrow \quad 8\lambda = -5$$

$$\therefore \quad \lambda = -\frac{5}{8}$$

which is the required value of λ.

Long Answer Type Questions

10. Find the values of p and q for which $x = \frac{1}{2}$ and $x = \frac{3}{4}$ are the roots of the equation $px^2 + qx - 3 = 0$.

Sol. We have

$$\text{Sum of the roots} = \frac{1}{2} + \frac{3}{4} = -\frac{q}{p}$$

$$\Rightarrow \quad \frac{q}{p} = -\left(\frac{2+3}{4}\right) = -\frac{5}{4}$$

$$\Rightarrow \quad q = -\frac{5p}{4} \qquad \ldots(1)$$

$$\text{Product of the roots} = \frac{1}{2} \times \frac{3}{4} = -\frac{3}{p}$$

$$\Rightarrow \quad -\frac{3}{p} = \frac{3}{8}$$

$$\therefore \quad p = -8 \qquad \ldots(2)$$

From (1) and (2),

$$q = -\frac{5}{4} \times (-8)$$

$$= 5 \times 2$$

$$= 10$$

$\therefore$ The required values of p and q are –8 and 10 respectively.

11. Find the values of m and n for which $x = -3$ and $x = \frac{2}{3}$ are the roots of the equation $mx^2 + 7x + n = 0$.

Sol. $$\text{Sum of the roots} = -3 + \frac{2}{3} = -\frac{7}{m}$$

$$\Rightarrow \quad \frac{-7}{3} = -\frac{7}{m}$$

$$\Rightarrow \quad m = 3 \qquad \ldots(1)$$

$$\text{Product of the roots} = -3 \times \frac{2}{3} = \frac{n}{m}$$

$$\Rightarrow \quad \frac{n}{3} = -2 \qquad \text{[From (1)]}$$

$$\Rightarrow \quad n = -6.$$

$\therefore$ The required values of m and n are 3 and –6 respectively.

12. If $x = \frac{2}{3}$ and $x = -3$ are the roots of the equation $ax^2 + 7x + b = 0$, find the values of a and b.

Sol. Sum of the roots

$$= \frac{2}{3} - 3 = -\frac{7}{a}$$

$$\Rightarrow \quad \frac{7}{a} = \frac{7}{3}$$

$$\Rightarrow \quad a = 3 \qquad \ldots(1)$$

$$\text{Product of the roots} = -3 \times \frac{2}{3} = \frac{b}{a}$$

$$\Rightarrow \quad \frac{b}{3} = -2 \qquad \text{[From (1)]}$$

$$\Rightarrow \quad b = -6$$

$\therefore$ The required values of a and b are 3 and –6 respectively.

Check Your Progress 2

(Page 62)

Multiple-Choice Questions

1. The roots of the quadratic equation $2x^2 - x - 6 = 0$ are

(*a*) $-2, \frac{3}{2}$ (*b*) $2, \frac{-3}{2}$

(*c*) $-2, \frac{-3}{2}$ (*d*) $2, \frac{3}{2}$ **[CBSE SP 2012]**

Sol. (*b*) $2, -\frac{3}{2}$

We have

$$2x^2 - x - 6 = 0$$

$$\Rightarrow \quad 2x^2 + 3x - 4x - 6 = 0$$

$$\Rightarrow \quad x(2x + 3) - 2\,(2x + 3) = 0$$

$$\Rightarrow \quad (x - 2)\,(2x + 3) = 0$$

$$\therefore \text{ Either } \quad x - 2 = 0 \qquad \ldots(1)$$

$$\text{or} \quad 2x + 3 = 0 \qquad \ldots(2)$$

From (1) and (2), we get $x = 2$ and $x = -\frac{3}{2}$ which are the required roots.

2. The roots of the quadratic equation $x^2 - 3x - m(m+3) = 0$, where m is a constant, are

(a) $m, m+3$ (b) $-m, m+3$

(c) $m, -(m+3)$ (d) $-m, -(m+3)$

[CBSE 2011]

Sol. (b) $-m, m+3$

We have,

$$x^2 - 3x - m(m+3) = 0$$

$$\Rightarrow \quad x^2 - 3x - m^2 - 3m = 0$$

$$\Rightarrow \quad x^2 - m^2 - 3(x+m) = 0$$

$$\Rightarrow \quad (x+m)(x-m) - 3(x+m) = 0$$

$$\Rightarrow \quad (x+m)(x-m-3) = 0$$

$\therefore$ Either $x + m = 0$ …(1)

or $x - (m+3) = 0$ …(2)

From (1) & (2), we have $x = -m$ and $x = m + 3$ which are the required roots of the given equation.

3. If one root of the equation $2x^2 + kx - 6 = 0$ is 2, then the value of $k + 1$ is

(a) 1 (b) -1

(c) 0 (d) -2

Sol. (c) 0

Putting $x = 2$ in the given equation, we get

$$2 \times 2^2 + 2k - 6 = 0$$

$$\Rightarrow \quad 8 - 6 + 2k = 0$$

$$\Rightarrow \quad k + 1 = 0$$

which is the required value of $k + 1$.

4. The quadratic equation $2y^2 - \sqrt{3}y + 1 = 0$ has

(a) more than two real roots

(b) two equal real roots

(c) no real roots

(d) two distinct real roots

Sol. (c) no real roots

We have, the discriminant,

$$D = \left(-\sqrt{3}\right)^2 - 4 \times 2 \times 1$$

$$= 3 - 8$$

$$= -5 < 0$$

Since, $D < 0$, hence, the given equation has no real roots.

5. Which one of the following equations has two distinct roots?

(a) $x^2 + 2x - 7 = 0$ (b) $3y^2 - 3\sqrt{3}y + \frac{9}{4} = 0$

(c) $x^2 + 2x + 2\sqrt{3} = 0$ (d) $6x^2 - 3x + 1 = 0$

Sol. (a) $x^2 + 2x - 7 = 0$

We first calculate the discriminant D of each of the given quadratic equations.

For (a), $D = 2^2 + 4 \times 7 > 0$

Hence, this equation has two real distinct roots.

For (b), $D = \left(-3\sqrt{3}\right)^2 - 4 \times 3 \times \frac{9}{4}$

$= 27 - 27 = 0$

Hence, this equation has two equal real roots.

For (c), $D = 2^2 - 4 \times 1 \times 2\sqrt{3}$

$= 4 - 8\sqrt{3} < 0$

Hence, this equation has no real roots at all.

For (d), $D = (-3)^2 - 4 \times 6 \times 1$

$= 3 - 24$

$= -21 < 0$

Hence, this equation has no real roots at all.

$\therefore$ (a) is the only choice, since this equation has two distinct roots.

6. Which one of the following equations has no real roots?

(a) $x^2 - 2x - 2\sqrt{3} = 0$

(b) $x^2 - 4x + 4\sqrt{2} = 0$

(c) $3x^2 + 4\sqrt{3}x + 3 = 0$

(d) $x^2 + 4x - 2\sqrt{2} = 0$

Sol. (b) $x^2 - 4x + 4\sqrt{2} = 0$

For (a), $D = (-2)^2 - 4 \times 1 \times \left(-2\sqrt{3}\right)$

$= 4 + 8\sqrt{3} > 0$

Hence, this equation has two distinct real roots.

For (b), $D = (-4)^2 - 4 \times 1 \times 4\sqrt{2}$

$= 16 - 16\sqrt{2}$

$= 16\left(1 - \sqrt{2}\right) < 0$

$\therefore$ This equation has no real roots at all.

For (c), $D = \left(4\sqrt{3}\right)^2 - 4 \times 3 \times 3$

$= 48 - 36$

$= 12 > 0$

$\therefore$ This equation has two distinct real roots.

For (d), $D = 4^2 - 4 \times 1 \times \left(-2\sqrt{2}\right)$

$= 16 + 8\sqrt{2} > 0$

$\therefore$ This equation has two distinct real roots.

$\therefore$ (*b*) is the only choice, since this equation does not have any real roots.

7. $(x^2 + 2)^2 - x^2 = 0$ has

(*a*) four real roots (*b*) two real roots

(*c*) one real root (*d*) no real roots

Sol. (*d*) no real roots

From the given equation, we have

$$(x^2 + 2 + x)(x^2 + 2 - x) = 0$$

$\therefore$ Either $x^2 + x + 2 = 0$ ...(1)

or, $x^2 - x + 2 = 0$ ...(2)

We see that both (1) and (2) are quadratic equations and the discriminant D for both the quadratic equations is given by

$$\begin{aligned} D &= (\pm 1)^2 - 4 \times 1 \times 2 \\ &= 1 - 8 = -7 < 0 \end{aligned}$$

$\therefore$ Both the roots of each of the two equations (1) and (2) are imaginary. In other words, the given equation does not have any real roots at all.

8. If the equation $x^2 + 4x + k = 0$ has real and distinct roots, then

(*a*) $k \leq 4$ (*b*) $k < 4$

(*c*) $k > 4$ (*d*) $k \geq 4$

Sol. (*b*) $k < 4$

The discriminant D for the given quadratic equation is given by

$$\begin{aligned} D &= 4^2 - 4 \times 1 \times k \\ &= 16 - 4k \end{aligned}$$

Now, the given equation will have two real and distinct roots only if

$$D > 0$$

$\Rightarrow$ $16 - 4k > 0$

$\Rightarrow$ $k < 4$

9. If the equation $25x^2 - kx + 9 = 0$ has equal roots, then

(*a*) $k = \pm 30$ (*b*) $k = \pm 25$

(*c*) $k = \pm 9$ (*d*) $k = \pm 34$

Sol. (*a*) $k = \pm 30$

We have, the discriminant D for the given equation is given by

$$\begin{aligned} D &= (-k)^2 - 4 \times 25 \times 9 \\ &= k^2 - 900 \end{aligned}$$

$\therefore$ The given equation will have two real equal roots if D = 0

i.e. $k^2 - 900 = 0$

$\Rightarrow$ $k = \pm\sqrt{900} = \pm 30$

10. If the roots of the equation $x^2 - 2x(1 + 3k) + 7(3 + 2k) = 0$ are real and equal, then

(*a*) $k = 2, \frac{-10}{9}$ (*b*) $k = -2, \frac{10}{9}$

(*c*) $k = 9, \frac{1}{10}$ (*d*) $k = -9, \frac{-1}{10}$

Sol. (*a*) $k = 2, -\frac{10}{9}$

The discriminant D for the given equation is given by

$$\begin{aligned} D &= \{-2(1 + 3k)\}^2 - 4 \times 1 \times 7(3 + 2k) \\ &= 4(1 + 3k)^2 - 28(3 + 2k) \\ &= 4(1 + 9k^2 + 6k) - 84 - 56k \\ &= 4 + 36k^2 + 24k - 84 - 56k \\ &= 36k^2 - 32k - 80 \\ &= 4(9k^2 - 8k - 20) \end{aligned}$$

$\therefore$ The given equation will have two real and equal roots if D = 0.

i.e. $9k^2 - 8k - 20 = 0$

$\Rightarrow$ $9k^2 + 10k - 18k - 20 = 0$

$\Rightarrow$ $k(9k + 10) - 2(9k + 10) = 0$

$\Rightarrow$ $(9k + 10)(k - 2) = 0$

$\therefore$ Either $9k + 10 = 0$ ...(1)

or, $k - 2 = 0$ ...(2)

From (1), we have $k = -\frac{10}{9}$ and from (2), $k = 2$.

$\therefore$ The required values of k are 2 and $-\frac{10}{9}$.

11. The two roots of the equation $3x^2 - 2\sqrt{6}x + 2 = 0$ are

(*a*) real and distinct. (*b*) not real.

(*c*) real and equal. (*d*) rational.

[CBSE 2023 Basic]

Sol. (*c*) real and unequal

We have, $D = (-2\sqrt{6})^2 - 4 \times 3 \times 2$

$= 24 - 24 = 0$

$\therefore$ The given equation will have real and equal roots.

12. The least value of k, for which the quadratic equation $2x^2 + kx - 4 = 0$ has rational roots, is

(*a*) $\pm 2\sqrt{2}$ (*b*) 2

(*c*) ± 2 (*d*) $\sqrt{2}$

[CBSE 2023 Standard]

Sol. (*b*) 2

We have, $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

For rational roots, $b^2 - 4ac$ should be perfect square.

We have,

$$a = 2$$
$$b = k$$
$$c = -4$$
$$b^2 - 4ac = k^2 - 4 \times 2 \times (-4)$$
$$= k^2 + 32$$

For $k = 2$, $b^2 - 4ac = 4 + 32 = 36$, which is a perfect square.

13. A quadratic equation whose roots are $(2+\sqrt{3})$ and $(2-\sqrt{3})$ is

(*a*) $x^2 - 4x + 1 = 0$ (*b*) $x^2 + 4x + 1 = 0$

(*c*) $4x^2 - 3 = 0$ (*d*) $x^2 - 1 = 0$

[CBSE 2023 Standard]

Sol. (*a*) $x^2 - 4x + 1 = 0$

Roots are $(2+\sqrt{3})$ and $(2-\sqrt{3})$

Sum of the roots $= 2 + \sqrt{3} + 2 - \sqrt{3} = 4$

Product of the roots $= (2+\sqrt{3})(2-\sqrt{3})$

$= 4 - 3 = 1$

The quadratic equation is

x^2 – (sum of the roots) x + product of roots = 0

$\Rightarrow \quad x^2 - 4x + 1 = 0$

14. If the roots of quadratic equation $4x^2 - 5x + k = 0$ are real and equal, then k is equal to

(*a*) $\dfrac{5}{4}$ (*b*) $\dfrac{25}{16}$

(*c*) $\dfrac{-5}{4}$ (*d*) $\dfrac{-25}{16}$

[CBSE 2024 Standard]

Sol. (*b*) $\dfrac{25}{16}$

The given quadratic equation is,

$$4x^2 - 5x + k = 0$$
$$a = 4, b = -5, c = k$$

For real and equal roots

$$b^2 - 4ac = 0$$
$$\Rightarrow \quad (-5)^2 - 4(4)(k) = 0$$
$$\Rightarrow \quad 25 - 16k = 0$$
$$\Rightarrow \quad k = \frac{25}{16}$$

Very Short Answer Type Questions

Solve:

15. $(x-5)(x+4) = 0$

Sol. From the given equation, we have

$$(x-5)(x+4) = 0$$

$\therefore$ Either $x - 5 = 0$ …(1)

or, $x + 4 = 0$ …(2)

From (1) & (2), we have, $x = 5$, $x = -4$

$\therefore$ Required roots are 5 and -4.

16. $(2x+3)(3x+2) = 0$

Sol. From the given equation, we have

$$(2x+3)(3x+2) = 0$$

$\therefore$ Either $2x + 3 = 0$ …(1)

or, $3x + 2 = 0$ …(2)

From (1), $x = -\dfrac{3}{2}$ and from (2), we have, $x = -\dfrac{2}{3}$

$\therefore$ Required roots are $-\dfrac{3}{2}$ and $-\dfrac{2}{3}$.

17. $\left(\dfrac{x}{2} - 1\right)\left(\dfrac{x}{3} + 5\right) = 0$

Sol. From the given equation, we have

$$\left(\frac{x}{2} - 1\right)\left(\frac{x}{3} + 5\right) = 0$$

$\therefore$ Either $\dfrac{x}{2} - 1 = 0$ …(1)

or $\dfrac{x}{3} + 5 = 0$ …(2)

From (1) and (2), we have, $x = 2$, $x = -15$.

$\therefore$ Required roots are 2 and -15.

18. $7x^2 + 3x = 0$

Sol. From the given equation, we have

$$7x^2 + 3x = 0$$
$$\Rightarrow \quad x(7x+3) = 0$$

$\therefore$ Either $x = 0$ …(1)

or $7x + 3 = 0$

$\Rightarrow \quad x = -\dfrac{3}{7}$ …(2)

From (1) and (2), required roots are 0, $-\dfrac{3}{7}$.

Using factorization, solve each of the following quadratic equations (Q. 19 to Q. 22):

19. $16x^2 - 49 = 0$

Sol. We have $16x^2 - 49 = 0$

$\Rightarrow \quad (4x)^2 - 7^2 = 0$

$\Rightarrow \quad (4x + 7)(4x - 7) = 0$

$\therefore$ Either $4x + 7 = 0 \quad \ldots(1)$

or $4x - 7 = 0 \quad \ldots(2)$

$\therefore$ From (1) and (2), we have,

$$x = -\frac{7}{4}, \frac{7}{4}$$

which are the required roots.

20. $18x^2 - 50 = 0$

Sol. We have $18x^2 - 50 = 0$

$\Rightarrow \quad 2(9x^2 - 25) = 0$

$\Rightarrow \quad (3x)^2 - 5^2 = 0$

$\Rightarrow \quad (3x + 5)(3x - 5) = 0$

$\therefore$ Either $3x + 5 = 0 \quad \ldots(1)$

or $3x - 5 = 0 \quad \ldots(2)$

From (1) and (2), we have $x = -\frac{5}{3}, \frac{5}{3}$ which are the required roots.

21. $(y - 2)^2 - 9 = 0$

Sol. We have $(y - 2)^2 - 9 = 0$

$\Rightarrow \quad (y - 2)^2 = 9$

$\Rightarrow \quad y - 2 = \pm 3$

$\therefore \quad y = 2 \pm 3 = 5, -1$

Hence, required roots are 5, –1.

22. $(x + 3)^2 - 36 = 0$

Sol. We have $(x + 3)^2 - 36 = 0$

$\Rightarrow \quad (x + 3)^2 = 36$

$\Rightarrow \quad x + 3 = \pm 6$

$\therefore$ $x = -3 \pm 6 = 3, -9$ which are the required roots.

23. Write the discriminant of each of the following quadratic equations.

(*a*) $x^2 - 6x + 8 = 0$

(*b*) $3\sqrt{3}x^2 + 10x + \sqrt{3} = 0$ **[CBSE 2009]**

Sol. (*a*) We have, discriminant,

$D = (-6)^2 - 4 \times 1 \times 8 = 36 - 32 = 4$

which is the required discriminant.

(*b*) We have, discriminant,

$$D = 10^2 - 4 \times 3\sqrt{3} \times \sqrt{3}$$
$$= 100 - 36 = 64$$

which is the required value of the discriminant.

24. Examine which of the following quadratic equations have real roots.

(*a*) $x^2 + 5x + 5 = 0$

(*b*) $2y - 1 - \dfrac{2}{y - 2} = 3$

Sol. (*a*) Here discriminant,

$$D = 5^2 - 4 \times 1 \times 5$$
$$= 25 - 20 = 5 > 0$$

$\therefore$ This equation has two real and distinct roots.

(*b*) We have

$$\frac{(2y - 1)(y - 2) - 2}{y - 2} = 3$$

$\Rightarrow \quad 2y^2 - 5y + 2 - 2 = 3y - 6$

$\Rightarrow \quad 2y^2 - 8y + 6 = 0$

$\Rightarrow \quad y^2 - 4y + 3 = 0$

$\therefore$ Discriminant, $D = (-4)^2 - 4 \times 1 \times 3$

$= 16 - 12$

$= 4 > 0.$

$\therefore$ This equation has also two distinct real roots.

25. Which of the following equations have real roots?

(*a*) $4x^2 + 7x + 2 = 0$

(*b*) $x^2 + x + 1 = 0$

Sol. (*a*) We have discriminant,

$$D = 7^2 - 4 \times 4 \times 2$$
$$= 49 - 32$$
$$= 17 > 0$$

$\therefore$ This equation has two real distinct roots.

(*b*) Here, $D = 1^2 - 4 \times 1 \times 1 = 1 - 4 = -3 < 0$

$\therefore$ This equation has no real roots at all.

26. What is the nature of the roots of the following quadratic equations?

(*a*) $x^2 - 2\sqrt{3}x + 3 = 0$ **[CBSE 2008]**

(*b*) $4x^2 - 12x - 9 = 0$ **[CBSE SP 2012]**

Sol. (*a*) The discriminant,

$$D = \left(-2\sqrt{3}\right)^2 - 4 \times 1 \times 3$$
$$= 12 - 12 = 0$$

$\therefore$ This equation has two real and equal roots.

(*b*) Here, the discriminant,

$$D = (-12)^2 - 4 \times 4 \times (-9)$$
$$= 144 + 144 = 288 > 0$$

$\therefore$ This equation has two real and unequal roots.

27. Which of the following equations have both roots equal?

(*a*) $x^2 + 10x - 39 = 0$

(*b*) $4x^2 - 12\sqrt{3}\,x + 27 = 0$

(*c*) $4x^2 - 5x + \frac{25}{16} = 0$

Sol. (*a*) The discriminant,

$$D = 10^2 - 4 \times 1 \times (-39)$$
$$= 100 + 156 = 256 > 0.$$

$\therefore$ Both the roots of this equation are real but unequal.

(*b*) Here, the discriminant,

$$D = \left(-12\sqrt{3}\right)^2 - 4 \times 4 \times 27$$
$$= 432 - 432 = 0.$$

$\therefore$ Both the roots are real and equal.

(*c*) Here the discriminant,

$$D = (-5)^2 - 4 \times 4 \times \frac{25}{16}$$
$$= 25 - 25 = 0$$

$\therefore$ Both the roots are real and equal.

Using factorization, solve each of the following quadratic equations (Q. 28 to Q. 32):

28. $x^2 + 11x - 152 = 0$

Sol. We have

$$x^2 + 11x - 152 = 0$$
$$\Rightarrow \quad x^2 + 19x - 8x - 152 = 0$$
$$\Rightarrow \quad x(x + 19) - 8(x + 19) = 0$$
$$\Rightarrow \quad (x + 19)(x - 8) = 0$$

$\therefore$ Either $x + 19 = 0$ …(1)

or $x - 8 = 0$ …(2)

$\therefore$ From (1) & (2), we have $x = -19, 8$ which are the required roots.

29. $x^2 - 7x - 44 = 0$

Sol. We have

$$x^2 - 7x - 44 = 0$$
$$\Rightarrow \quad x^2 + 4x - 11x - 44 = 0$$
$$\Rightarrow \quad x(x + 4) - 11(x + 4) = 0$$
$$\Rightarrow \quad (x + 4)(x - 11) = 0$$

$\therefore$ Either $x + 4 = 0$ …(1)

or $x - 11 = 0$ …(2)

$\therefore$ From (1) and (2), $x = -4, 11$ which are the required roots.

30. $12x^2 - x - 1 = 0$

Sol. We have

$$12x^2 - x - 1 = 0$$
$$\Rightarrow \quad 12x^2 + 3x - 4x - 1 = 0$$
$$\Rightarrow \quad 3x(4x + 1) - 1(4x + 1) = 0$$
$$\Rightarrow \quad (4x + 1)(3x - 1) = 0$$

$\therefore$ Either $4x + 1 = 0$ …(1)

or $3x - 1 = 0$ …(2)

From (1) and (2), we have $x = -\frac{1}{4}, \frac{1}{3}$ which are the required roots.

31. $\frac{2}{5}x^2 - x - \frac{3}{5} = 0$ **[CBSE SP 2011]**

Sol. We have

$$\frac{2}{5}x^2 - x - \frac{3}{5} = 0$$
$$\Rightarrow \quad 2x^2 - 5x - 3 = 0$$
$$\Rightarrow \quad 2x^2 + x - 6x - 3 = 0$$
$$\Rightarrow \quad x(2x + 1) - 3(2x + 1) = 0$$
$$\Rightarrow \quad (2x + 1)(x - 3) = 0$$

$\therefore$ Either $2x + 1 = 0$ …(1)

or $x - 3 = 0$ …(2)

From (1) and (2), we have $x = -\frac{1}{2}, 3$ which are the required roots.

32. $x^2 - (\sqrt{3} + 1)x + \sqrt{3} = 0$ **[CBSE 2015]**

Sol. We have

$$x^2 - \left(\sqrt{3} + 1\right)x + \sqrt{3} = 0$$
$$\Rightarrow \quad x^2 - \sqrt{3}\,x - x + \sqrt{3} = 0$$
$$\Rightarrow \quad x\left(x - \sqrt{3}\right) - 1\left(x - \sqrt{3}\right) = 0$$
$$\Rightarrow \quad \left(x - \sqrt{3}\right)(x - 1) = 0$$

$\therefore$ Either $x - \sqrt{3} = 0$ …(1)

or $x - 1 = 0$ …(2)

$\therefore$ From (1) and (2), we have $x = \sqrt{3}, 1$ which are the required roots.

33. In the following, determine whether the given quadratic equations have real roots and if so, find the roots.

(*a*) $3x^2 - 6x + 5 = 0$

(*b*) $4\sqrt{3}\,x^2 + 5x - 2\sqrt{3} = 0$ **[CBSE 2013]**

Sol. (*a*) We have the discriminant,

$$D = (-6)^2 - 4 \times 3 \times 5$$
$$= 36 - 60$$
$$= -24 < 0$$

$\therefore$ This equation has no real roots.

(b) We have discriminant,

$$D = 5^2 - 4 \times 4\sqrt{3} \times \left(-2\sqrt{3}\right)$$
$$= 25 + 32 \times 3$$
$$= 25 + 96$$
$$= 121 > 0.$$

$\therefore$ This equation has two real and unequal roots.

Now, we have

$$4\sqrt{3}\, x^2 + 5x - 2\sqrt{3} = 0$$
$$\Rightarrow \quad 4\sqrt{3}\, x^2 + 8x - 3x - 2\sqrt{3} = 0$$
$$\Rightarrow 4x\left(\sqrt{3}x + 2\right) - \sqrt{3}\left(\sqrt{3}x + 2\right) = 0$$
$$\Rightarrow \quad \left(4x - \sqrt{3}\right)\left(\sqrt{3}x + 2\right) = 0$$

$\therefore$ Either $\quad 4x - \sqrt{3} = 0 \quad \ldots(1)$

or $\quad \sqrt{3}x + 2 = 0 \quad \ldots(2)$

$\therefore$ From (1) and (2), we have $x = \dfrac{\sqrt{3}}{4}, -\dfrac{2}{\sqrt{3}}$

which are the required roots.

34. Solve each of the following equations by using the quadratic formula.

(a) $x^2 + 2x - 4 = 0$

(b) $3x^2 - 32x + 12 = 0$

(c) $4x^2 + 4ax + (a^2 - b^2) = 0$

(d) $ab^2x\left(\dfrac{a}{d}x + 2\dfrac{c}{b}\right) + c^2d = 0$

Sol. (a) We have,

$$x^2 + 2x - 4 = 0$$

Comparing this equation with the standard quadratic equation $ax^2 + bx + c = 0$, we have $a = 1$, $b = 2$ and $c = -4$.

$$\therefore \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-2 \pm \sqrt{4 + 4 \times 4}}{2}$$
$$= \frac{-2 \pm 2\sqrt{5}}{2} = -1 \pm \sqrt{5}$$

$\therefore$ Required roots are $-1 + \sqrt{5},\ -1 - \sqrt{5}$

(b) Comparing the given equation $3x^2 - 32x + 12 = 0$ with the equation $ax^2 + bx + c = 0$, we get $a = 3$, $b = -32$ and $c = 12$.

$$\therefore \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{32 \pm \sqrt{1024 - 4 \times 3 \times 12}}{2 \times 3}$$
$$= \frac{32 \pm \sqrt{1024 - 144}}{6}$$
$$= \frac{32 \pm \sqrt{880}}{6}$$
$$= \frac{32 \pm 4\sqrt{55}}{6}$$
$$= \frac{16 \pm 2\sqrt{55}}{3}$$

$\therefore$ Required roots are $\dfrac{16 + 2\sqrt{55}}{3}$ and $\dfrac{16 - 2\sqrt{55}}{3}$.

(c) Comparing the given equation $4x^2 + 4ax + (a^2 - b^2) = 0$ with the equation $Ax^2 + Bx + C = 0$, we get $A = 4$, $B = 4a$, $C = a^2 - b^2$.

$$\therefore \quad x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$
$$= \frac{-4a \pm \sqrt{16a^2 - 4 \times 4(a^2 - b^2)}}{2 \times 4}$$
$$= \frac{-4a \pm \sqrt{16a^2 - 16a^2 + 16b^2}}{8}$$
$$= \frac{-4a \pm 4b}{8}$$
$$= \frac{-(a \pm b)}{2}$$

$\therefore$ Required roots are $\dfrac{-(a+b)}{2}$ and $\dfrac{b-a}{2}$.

(d) We have

$$ab^2x \times \frac{a}{d}x + \frac{2c}{b} \times ab^2x + c^2d = 0$$
$$\Rightarrow \quad \frac{a^2b^2x^2}{d} + 2abcx + c^2d = 0$$
$$\Rightarrow \quad a^2b^2x^2 + 2abc\,dx + c^2d^2 = 0$$

Comparing this equation with the equation $Ax^2 + Bx + C = 0$, we get $A = a^2b^2$, $B = 2abcd$ and $C = c^2d^2$, we get

$$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$
$$= \frac{-2abcd \pm \sqrt{4a^2b^2c^2d^2 - 4a^2b^2c^2d^2}}{2a^2b^2}$$
$$= \frac{-2abcd}{2a^2b^2}$$
$$= \frac{-cd}{ab}$$

$\therefore$ The two required roots are $\dfrac{-cd}{ab}, -\dfrac{cd}{ab}$.

35. In each of the following, determine value(s) of p for which the given quadratic equation has real roots:

(a) $px(x-2)+6=0$ **[CBSE SP 2012]**

(b) $x^2+6x+2p+1=0$

Sol. (a) We have

$$px(x-2)+6=0$$
$$\Rightarrow \quad px^2-2px+6=0$$
$$\therefore \quad \text{Discriminant, D} = (-2p)^2-4p\times 6$$
$$= 4p^2-24p$$

For real roots, $\quad D \geq 0$

$$\Rightarrow \quad 4p^2-24p \geq 0$$
$$\Rightarrow \quad p^2-6p \geq 0$$
$$\Rightarrow \quad p-6 \geq 0 \qquad [\because p \neq 0]$$
$$\Rightarrow \quad p \geq 6$$

which are the required value(s) of p.

(b) We have

$$x^2+6x+2p+1=0$$
$$\therefore \text{Discriminant, D} = 6^2-4\times 1\,(2p+1)$$
$$= 36-8p-4$$
$$= 32-8p$$
$$= 8(4-p)$$

For real roots, $\quad D \geq 0$

$$\Rightarrow \quad 8(4-p) \geq 0$$
$$\Rightarrow \quad 4-p \geq 0$$
$$\Rightarrow \quad p \leq 4$$

which are the required value(s) of p.

36. Find the values of k for which the roots are real and equal in each of the following equations:

(a) $5x^2-10x+k=0$ **[CBSE 2023 Basic]**

(b) $(k+4)x^2+(k+1)x+1=0$

[CBSE 2002 C, SP 2011]

(c) $x^2-2(1+3k)x+7(3+2k)=0$

(d) $(k+1)x^2-2(k-1)x+1=0$ **[CBSE 2002]**

Sol. (a) We have, $5x^2-10x+k=0$

$$\therefore \quad \text{Discriminant, D} = (-10)^2-4\times 5\times k$$
$$= 100-20k$$

Now, for real and equal roots, D = 0.

$$\therefore \quad 100-20k=0$$
$$\Rightarrow \quad k = \frac{100}{20} = 5$$

$\therefore$ $k=5$ which is the required value of k.

(b) We have $(k+4)x^2+(k+1)x+1=0$

$$\therefore \quad \text{Discriminant, D} = (k+1)^2-4(k+4)\times 1$$
$$= k^2+2k+1-4k-16$$
$$= k^2-2k-15$$

For real and equal roots, we have D = 0.

$$\Rightarrow \quad k^2-2k-15=0$$
$$\Rightarrow \quad k^2+3k-5k-15=0$$
$$\Rightarrow k(k+3)-5(k+3)=0$$
$$\Rightarrow \quad (k+3)(k-5)=0$$

$\therefore$ Either $\quad k-5=0 \qquad \ldots(1)$

or $\quad k+3=0 \qquad \ldots(2)$

From (1) and (2), we have $k=5, -3$ which are the required values of k.

(c) We have $x^2-2(1+3k)x+7(3+2k)=0$

$$\therefore \quad \text{Discriminant, D} = \{-2(1+3k)\}^2-4\times 1 \times 7\,(3+2k)$$
$$= 4(1+3k)^2-28\,(3+2k)$$
$$= 4(1+9k^2+6k)-84-56k$$
$$= 36k^2+24k-56k+4-84$$
$$= 36k^2-32k-80$$
$$= 4(9k^2-8k-20)$$

Now, for real and equal roots, D = 0.

$$\therefore \quad 9k^2-8k-20=0$$
$$\Rightarrow \quad 9k^2+10k-18k-20=0$$
$$\Rightarrow \quad k(9k+10)-2(9k+10)=0$$
$$\Rightarrow \quad (9k+10)(k-2)=0$$

$\therefore$ Either $\quad 9k+10=0 \qquad \ldots(1)$

or $\quad k-2=0 \qquad \ldots(2)$

From (1) and (2), we have $k = \frac{-10}{9}$, 2 which are the required values of k.

(d) We have $(k+1)x^2-2\,(k-1)x+1=0$

$$\therefore \text{Discriminant, D} = \{-2(k-1)\}^2-4(k+1)\times 1$$
$$= 4(k-1)^2-4(k+1)$$
$$= 4k^2-8k+4-4k-4$$
$$= 4k^2-12k$$

For real and equal roots, we have D = 0.

$$\therefore \quad 4k^2-12k=0$$
$$\Rightarrow \quad 4k(k-3)=0$$
$$\Rightarrow \quad k=0, 3$$

which are the required values of k.

37. If one root of the quadratic equation $3x^2-8x-(2k+1)=0$ is seven times the other, then find the value of k.

[CBSE 2023 Basic]

Sol. Let the two roots be α and β.

Then, according to the question,

$$\alpha = 7\beta \qquad \ldots(1)$$

The quadratic equation is $3x^2-8x-(2k+1)=0$.

Here, $a=3$, $b=-8$, $c=-(2k+1)$

Sum of the roots $= \frac{-b}{a} = \frac{8}{3} \qquad \ldots(2)$

$$\alpha + \beta = \frac{8}{3}$$

$$\Rightarrow \quad 7\beta + \beta = \frac{8}{3} \quad \text{[From (1)]}$$

$$\Rightarrow \quad 8\beta = \frac{8}{3}$$

$$\Rightarrow \quad \beta = \frac{1}{3} \quad \ldots(3)$$

Product of the roots $= \frac{c}{a}$

$$= \frac{-(2k+1)}{3}$$

$$\alpha\beta = \frac{-(2k+1)}{3}$$

$$\Rightarrow \quad \frac{7}{3} \times \frac{1}{3} = \frac{-(2k+1)}{3}$$

$$\Rightarrow \quad \frac{7}{9} = \frac{-(2k+1)}{3}$$

$$\Rightarrow \quad 7 = -6k - 3$$

$$\Rightarrow \quad -6k = 10$$

$$\Rightarrow \quad k = \frac{-10}{6}$$

$$\Rightarrow \quad k = \frac{-5}{3}$$

38. Find the discriminant of the quadratic equation $4x^2 - 5 = 0$ and hence comment on the nature of roots of the equation. **[CBSE 2023 Standard]**

Sol. $4x^2 - 5 = 0$

$a = 4, b = 0, c = -5$

Discriminant, $D = b^2 - 4ac$

$\Rightarrow \quad D = 0 - 4\,(4)\,(-5)$

$\Rightarrow \quad D = 80$

Since $D > 0$, therefore roots are real and distinct.

Short Answer Type Questions

Using factorization, solve each of the following quadratic equations (Q. 39 to Q. 43):

39. $3\sqrt{5}\,x^2 + 2x - \sqrt{5} = 0$

Sol. We have

$$3\sqrt{5}x^2 + 2x - \sqrt{5} = 0$$

$$\Rightarrow \quad 3\sqrt{5}x^2 + 5x - 3x - \sqrt{5} = 0$$

$$\Rightarrow \quad \sqrt{5}x\left(3x + \sqrt{5}\right) - 1\left(3x + \sqrt{5}\right) = 0$$

$$\Rightarrow \quad \left(3x + \sqrt{5}\right)\left(\sqrt{5}x - 1\right) = 0$$

$\therefore$ Either $\quad 3x + \sqrt{5} = 0 \quad \ldots(1)$

or $\quad \sqrt{5}x - 1 = 0 \quad \ldots(2)$

$\therefore$ From (1) and (2), we have $x = \frac{-\sqrt{5}}{3}, \frac{1}{\sqrt{5}}$ which are the required roots.

40. $9x + \frac{1}{x} = 6$

Sol. We have $\quad 9x + \frac{1}{x} = 6$

$$\Rightarrow \quad 9x^2 + 1 = 6x$$

$$\Rightarrow \quad 9x^2 - 6x + 1 = 0$$

$$\Rightarrow \quad (3x)^2 - 2 \times 3x + 1^2 = 0$$

$$\Rightarrow \quad (3x - 1)^2 = 0$$

$\therefore$ $x = \frac{1}{3}, \frac{1}{3}$ which are the required roots.

41. $\frac{3}{x^2} + \frac{14}{x} + 8 = 0$

Sol. We have $\quad \frac{3}{x^2} + \frac{14}{x} + 8 = 0$

$$\Rightarrow \quad 3 + 14x + 8x^2 = 0$$

$$\Rightarrow \quad 8x^2 + 14x + 3 = 0$$

$$\Rightarrow \quad 8x^2 + 2x + 12x + 3 = 0$$

$$\Rightarrow \quad 2x(4x + 1) + 3\,(4x + 1) = 0$$

$$\Rightarrow \quad (4x + 1)\,(2x + 3) = 0$$

$\therefore$ Either $\quad 4x + 1 = 0 \quad \ldots(1)$

or $\quad 2x + 3 = 0 \quad \ldots(2)$

$\therefore$ From (1) and (2), we have $x = -\frac{1}{4}, -\frac{3}{2}$ which are the required roots.

42. $\frac{4}{9}x^2 - \frac{4}{3}x + 1 = 0$

Sol. We have $\quad \frac{4}{9}x^2 - \frac{4}{3} + 1 = 0$

$$\Rightarrow \quad 4x^2 - \frac{4}{3} \times 9x + 9 = 0$$

$$\Rightarrow \quad 4x^2 - 12x + 9 = 0$$

$$\Rightarrow \quad (2x)^2 - 2 \times 2x \times 3 + 3^2 = 0$$

$$\Rightarrow \quad (2x - 3)^2 = 0$$

$\Rightarrow$ $x = \frac{3}{2}, \frac{3}{2}$ which are the required roots.

43. $a(x^2 + 1) + (a^2 + 1)x = 0$

Sol. We have $\quad a(x^2 + 1) + (a^2 + 1)x = 0$

$$\Rightarrow \quad ax^2 + (a^2 + 1)x + a = 0$$

$$\Rightarrow \quad ax^2 + a^2x + x + a = 0$$

$$\Rightarrow \quad ax(x + a) + 1\,(x + a) = 0$$

$$\Rightarrow \quad (x + a)\,(ax + 1) = 0$$

$\therefore$ Either $\quad x + a = 0 \quad \ldots(1)$

or $\quad ax + 1 = 0 \quad \ldots(2)$

From (1) and (2), we have $x = -a, -\frac{1}{a}$ which are the required roots.

44. Solve:

(*a*) $\frac{6}{x} - \frac{2}{x-1} = \frac{1}{x+2}$; $x \neq 0, 1, -2$ **[CBSE SP 2006]**

(*b*) $\frac{a}{x-b} + \frac{b}{x-a} = 2$; $(x \neq a, b)$ **[CBSE 2016]**

Sol. (*a*) We have

$$\frac{6}{x} - \frac{2}{x-1} = \frac{1}{x+2}$$

$$\Rightarrow \quad \frac{6(x-1)-2x}{x(x-1)} = \frac{1}{x+2}$$

$$\Rightarrow \quad \frac{6x-6-2x}{x^2-x} = \frac{1}{x+2}$$

$$\Rightarrow \quad (4x-6)(x+2) = x^2 - x$$

$$\Rightarrow \quad 4x^2 + 2x - 12 - x^2 + x = 0$$

$$\Rightarrow \quad 3x^2 + 3x - 12 = 0$$

$$\Rightarrow \quad x^2 + x - 4 = 0$$

$$\therefore \quad x = \frac{-1 \pm \sqrt{1 - 4 \times (-4)}}{2}$$

$$= \frac{-1 \pm \sqrt{17}}{2}$$

$\therefore$ The required roots are $\frac{-1 \pm \sqrt{17}}{2}$ and $\frac{-1 - \sqrt{17}}{2}$.

(*b*) We have

$$\frac{a}{x-b} + \frac{b}{x-a} = 2$$

$$\Rightarrow \quad \frac{a(x-a)+b(x-b)}{(x-b)(x-a)} = 2$$

$$\Rightarrow \quad \frac{(a+b)x - a^2 - b^2}{x^2 - (a+b)x + ab} = 2$$

$$\Rightarrow \quad 2x^2 - 2(a+b)x + 2ab = (a+b)x - (a^2+b^2)$$

$$\Rightarrow \quad 2x^2 - 3(a+b)x + a^2 + b^2 + 2ab = 0$$

$$\Rightarrow \quad 2x^2 - 3(a+b)x + (a+b)^2 = 0$$

$$x = \frac{+3(a+b) \pm \sqrt{9(a+b)^2 - 8(a+b)^2}}{4}$$

$$= \frac{+3(a+b) \pm (a+b)}{4} = (a+b),\ \frac{a+b}{2}$$

which are the required roots.

45. Determine k so that the equation $x^2 - 4x + k = 0$ has

(*a*) two distinct real roots

(*b*) coincident roots

Sol. We have discriminant, $D = (-4)^2 - 4k$
$= 16 - 4k$
$= 4(4 - k)$

(*a*) For distinct real roots, we have $D > 0$

$\Rightarrow \quad 4 - k > 0$

$\Rightarrow \quad k < 4$

which are the required values of k.

(*b*) For coincident roots, $D = 0$

$\Rightarrow \quad 4 - k = 0$

$\Rightarrow \quad k = 4$

which is the required value of k.

46. For what values of m will the equation $2mx^2 - 2(1 + 2m)x + (3 + 2m) = 0$ have real but distinct roots? When will the roots be equal?

Sol. We have discriminant,

$$D = \{-2(1+2m)\}^2 - 4 \times 2m\,(3+2m)$$

$$= 4(1+2m)^2 - 8m\,(3+2m)$$

$$= 4 + 16m^2 + 16m - 24m - 16m^2$$

$$= 4 - 8m$$

$$= 4(1 - 2m)$$

$\therefore$ For real distinct roots, $D > 0$

$\Rightarrow \quad 1 - 2m > 0$

$\Rightarrow \quad m < \frac{1}{2}$

For real and equal roots, $D = 0$

$\Rightarrow \quad 1 - 2m = 0$

$\Rightarrow \quad m = \frac{1}{2}$

which are the required values of m.

Long Answer Type Questions

Using factorization, solve each of the following quadratic equations (Q. 47 to Q. 53):

47. $\frac{1}{x-5} - \frac{1}{x+1} = \frac{6}{7}$

Sol. We have

$$\frac{1}{x-5} - \frac{1}{x+1} = \frac{6}{7}$$

$$\Rightarrow \quad \frac{x+1-x+5}{(x-5)(x+1)} = \frac{6}{7}$$

$$\Rightarrow \quad \frac{6}{x^2-4x-5} = \frac{6}{7}$$

$$\Rightarrow \quad x^2 - 4x - 5 = 7$$

$$\Rightarrow \quad x^2 - 4x - 12 = 0$$

$$\Rightarrow \quad x^2 + 2x - 6x - 12 = 0$$

$$\Rightarrow \quad x(x+2) - 6(x+2) = 0$$

$$\Rightarrow \quad (x+2)(x-6) = 0$$

$\therefore$ Either $\quad x + 2 = 0 \quad \ldots(1)$

or, $\quad x - 6 = 0 \quad \ldots(2)$

$\therefore$ From (1) and (2), we have $x = -2, 6$ which are the required roots.

48. $\dfrac{1}{x-2}+\dfrac{1}{x}=\dfrac{8}{2x+5};\quad x \neq 0, \dfrac{-5}{2}$

Sol. We have
$$\frac{1}{x-2}+\frac{1}{x} = \frac{8}{2x+5}$$
$$\Rightarrow \quad \frac{x+x-2}{x(x-2)} = \frac{8}{2x+5}$$
$$\Rightarrow \quad \frac{2x-2}{x^2-2x} = \frac{8}{2x+5}$$
$$\Rightarrow \quad 8x^2 - 16x = (2x+5)(2x-2)$$
$$= 4x^2 - 6x - 10$$
$$\Rightarrow \quad 4x^2 - 22x + 10 = 0$$
$$\Rightarrow \quad 2x^2 - 11x + 5 = 0$$
$$\Rightarrow \quad 2x^2 - 10x - x + 5 = 0$$
$$\Rightarrow \quad 2x(x-5) - 1(x-5) = 0$$
$$\Rightarrow \quad (x-5)(2x-1) = 0$$

$\therefore$ Either $\quad x - 5 = 0 \quad \ldots(1)$

or $\quad 2x - 1 = 0 \quad \ldots(2)$

$\therefore$ From (1) and (2), we have, $x = 5, \dfrac{1}{2}$ which are the required roots.

49. $\dfrac{2}{x+1}+\dfrac{3}{2(x-2)}=\dfrac{23}{5x};\quad x \neq 0, -1, 2$ **[CBSE 2015]**

Sol. We have
$$\frac{2}{x+1}+\frac{3}{2(x-2)} = \frac{23}{5x}$$
$$\Rightarrow \quad \frac{4(x-2)+3x+3}{2(x-2)(x+1)} = \frac{23}{5x}$$
$$\Rightarrow \quad \frac{4x-8+3x+3}{2\left(x^2-x-2\right)} = \frac{23}{5x}$$
$$\Rightarrow \quad \frac{7x-5}{2x^2-2x-4} = \frac{23}{5x}$$
$$\Rightarrow \quad 35x^2 - 25x = 46x^2 - 46x - 92$$
$$\Rightarrow \quad (35-46)x^2 - 25x + 46x + 92 = 0$$
$$\Rightarrow \quad -11x^2 + 21x + 92 = 0$$
$$\Rightarrow \quad 11x^2 - 21x - 92 = 0$$
$$\Rightarrow \quad 11x^2 + 23x - 44x - 92 = 0$$
$$\Rightarrow \quad x(11x+23) - 4(11x+23) = 0$$
$$\Rightarrow \quad (11x+23)(x-4) = 0$$

$\therefore$ Either $\quad 11x + 23 = 0 \quad \ldots(1)$

or $\quad x - 4 = 0 \quad \ldots(2)$

$\therefore$ From (1) and (2), we have $x = \dfrac{-23}{11}, 4$ which are the required roots.

50. $\dfrac{4}{z-1}-\dfrac{5}{z+2}=\dfrac{3}{z};\ z \neq 1, 0, -2$ **[CBSE 2014]**

Sol. We have
$$\frac{4}{z-1}-\frac{5}{z} = \frac{3}{z}$$
$$\Rightarrow \quad \frac{4z+8-5z+5}{(z-1)(z+2)} = \frac{3}{z}$$
$$\Rightarrow \quad \frac{13-z}{z^2+z-2} = \frac{3}{z}$$
$$\Rightarrow \quad 3z^2 + 3z - 6 = 13z - z^2$$
$$\Rightarrow \quad 4z^2 - 10z - 6 = 0$$
$$\Rightarrow \quad 2z^2 - 5z - 3 = 0$$
$$\Rightarrow \quad 2z^2 + z - 6z - 3 = 0$$
$$\Rightarrow \quad z(2z+1) - 3(2z+1) = 0$$
$$\Rightarrow \quad (2z+1)(z-3) = 0$$

$\therefore$ Either $\quad 2z + 1 = 0 \quad \ldots(1)$

or $\quad z - 3 = 0 \quad \ldots(2)$

$\therefore$ From (1) and (2), we have, $z = -\dfrac{1}{2}, 3$ which are the required roots.

51. $\left(\dfrac{2x}{x-3}\right)+\left(\dfrac{1}{2x+3}\right)+\dfrac{3x+9}{(x-3)(2x+3)} = 0,$

$x \neq 3, \dfrac{-3}{2}$ **[CBSE 2006, 2016, SP 2011]**

Sol. We have
$$\frac{2x}{x-3}+\frac{1}{2x+3}+\frac{3x+9}{(x-3)(2x+3)} = 0$$
$$\Rightarrow \quad \frac{2x(2x+3)+x-3+3x+9}{(2x+3)(x-3)} = 0$$
$$\Rightarrow \quad 4x^2 + 10x + 6 = 0$$
$$\Rightarrow \quad 2x^2 + 5x + 3 = 0$$
$$\Rightarrow \quad 2x^2 + 3x + 2x + 3 = 0$$
$$\Rightarrow \quad x(2x+3) + 1(2x+3) = 0$$
$$\Rightarrow \quad (2x+3)(x+1) = 0$$

$\therefore$ Either $\quad 2x + 3 = 0 \quad \ldots(1)$

or $\quad x + 1 = 0 \quad \ldots(2)$

$\therefore$ From (1) and (2), we have $x = \dfrac{-3}{2}, -1$ which are the required roots.

52. (*a*) $16 \times 4^{x+2} - 16 \times 2^{x+1} + 1 = 0$

(*b*) $5^{4x} - 3 \times 5^{2x+1} = 250$ **[CBSE 2000]**

Sol. (*a*) We have

$$16 \times 4^{x+2} - 16 \times 2^{x+1} + 1 = 0$$

$\Rightarrow$ $16 \times (2^2)^{x+2} - 16 \times 2^x \times 2 + 1 = 0$

$\Rightarrow$ $16 \times 2^{2x} \times 2^4 - 32 \times 2^x + 1 = 0$

$\Rightarrow$ $256a^2 - 32a + 1 = 0,$

where $a = 2^x$ ...(1)

$\Rightarrow$ $(16a)^2 - 2 \times 16a \times 1 + 1^2 = 0$

$\Rightarrow$ $(16a - 1)^2 = 0$

$\Rightarrow$ $16a - 1 = 0$

$\Rightarrow$ $a = \frac{1}{16}$

$= \frac{1}{2^4}$

$= 2^{-4}$

$\Rightarrow$ $2^x = 2^{-4}$

[From (1)]

$\Rightarrow$ $x = -4$ which is the required solution.

(*b*) We have $5^{4x} - 3 \times 5^{2x+1} = 250$

$\Rightarrow$ $(5^{2x})^2 - 3 \times 5^{2x} \times 5 = 250$

$\Rightarrow$ $a^2 - 15a - 250 = 0$ where $a = 5^{2x}$ ...(1)

$\Rightarrow$ $a^2 + 10a - 25a - 250 = 0$

$\Rightarrow$ $a(a + 10) - 25(a + 10) = 0$

$\Rightarrow$ $(a + 10)(a - 25) = 0$

$\therefore$ Either $a + 10 = 0$ ...(2)

or, $a - 25 = 0$ ...(3)

$\therefore$ From (2), we have $a = -10$

$\Rightarrow$ $5^{2x} = -10$

[From (1)]

which is absurd, since LHS is always positive for all values of x, but RHS < 0.

$\therefore$ From (3), we get $a = 25$

$\Rightarrow$ $5^{2x} = 5^2$ [From (1)]

$\Rightarrow$ $2x = 2$

$\Rightarrow$ $x = 1$

which is the required solution.

53. $\sqrt{2x+9} + x = 13$ **[CBSE 2016]**

Sol. We have $\sqrt{2x+9} + x = 13$

$\Rightarrow$ $\sqrt{2x+9} = 13 - x$ ...(1)

On squaring both sides, we get

$\Rightarrow$ $2x + 9 = (13 - x)^2$

$= 169 + x^2 - 26x$

$\Rightarrow$ $x^2 - 28x + 160 = 0$

$\Rightarrow$ $x^2 - 20x - 8x + 160 = 0$

$\Rightarrow$ $x(x - 20) - 8(x - 20) = 0$

$\Rightarrow$ $(x - 20)(x - 8) = 0$

$\therefore$ Either $x - 20 = 0$ ...(2)

or $x - 8 = 0$ ...(3)

From (2), we see that $x = 20$ which does not satisfy the given equation.

Hence, we reject this value of x.

$\therefore$ From (3), we get $x = 8$ which satisfies the given equation. Hence, $x = 8$ is the only solution.

54. Solve:

(*a*) $4x^2 - 16(p - q)x + (15p^2 - 34pq + 15q^2) = 0$

(*b*) $p^2x^2 + (p^2 - q^2)x - q^2 = 0$

(*c*) $12abx^2 - (9a^2 - 8b^2)x - 6ab = 0$

[CBSE 2006, SP 2011]

(*d*) $(a + b)^2x^2 - 8(a^2 - b^2)x - 20(a - b)^2 = 0$

[CBSE SP 2011]

Sol. (*a*) We have

$$4x^2 - 16(p - q)x + (15p^2 - 34pq + 15q^2) = 0$$

$\Rightarrow$ $4x^2 - 16(p - q)x + 15p^2 - 25pq - 9pq + 15q^2 = 0$

$\Rightarrow$ $4x^2 - 16(p - q)x + 5p(3p - 5q) - 3q(3p - 5q) = 0$

$\Rightarrow$ $4x^2 - 16(p - q)x + (3p - 5q)(5p - 3q) = 0$

$\Rightarrow$ $4x^2 - 2\{(3p - 5q) + (5p - 3q)\}x + (3p - 5q)$
$(5p - 3q) = 0$

$\Rightarrow$ $4x^2 - 2(a + b)x + ab = 0$

where $a = 3p - 5q$ ...(1)

and $b = 5p - 3q$ ...(2)

$\Rightarrow$ $4x^2 - 2ax - 2bx + ab = 0$

$\Rightarrow$ $2x(2x - a) - b(2x - a) = 0$

$\Rightarrow$ $(2x - a)(2x - b) = 0$

$\therefore$ Either $2x - a = 0$ ...(3)

or, $2x - b = 0$ ...(4)

From (1) and (3), we get

$$x = \frac{a}{2} = \frac{3p - 5q}{2}$$

and from (2) and (4), we get

$$x = \frac{b}{2} = \frac{5p - 3q}{2}$$

Hence, the required roots are $\frac{3p - 5q}{2}$ and $\frac{5p - 3q}{2}$.

(*b*) We have $p^2x^2 + (p^2 - q^2)x - q^2 = 0$

$\Rightarrow$ $p^2x^2 + p^2x - q^2x - q^2 = 0$

$$\Rightarrow \quad p^2x(x + 1) - q^2(x + 1) = 0$$

$$\Rightarrow \quad (x + 1)(p^2x - q^2) = 0$$

$\therefore$ Either $\quad x + 1 = 0 \quad$ ...(1)

or $\quad p^2x - q^2 = 0 \quad$ ...(2)

From (1) and (2), we have $x = -1, \dfrac{q^2}{p^2}$ which are the required roots.

(*c*) We have

$$12abx^2 - (9a^2 - 8b^2)x - 6ab = 0$$

$$\Rightarrow \quad 12abx^2 - 9a^2x + 8b^2x - 6ab = 0$$

$$\Rightarrow \quad 3ax(4bx - 3a) + 2b(4bx - 3a) = 0$$

$$\Rightarrow \quad (4bx - 3a)(3ax + 2b) = 0$$

$\therefore$ Either $\quad 4bx - 3a = 0 \quad$ ...(1)

or $\quad 3ax + 2b = 0 \quad$ ...(2)

From (1) and (2), we have $x = \dfrac{3a}{4b}, -\dfrac{2b}{3a}$

which are the required roots.

(*d*) We have

$$(a + b)^2x^2 - 8(a^2 - b^2)x - 20(a - b)^2 = 0$$

$$\Rightarrow \quad (a + b)^2x^2 - 8(a + b)(a - b)x - 20(a - b)^2 = 0$$

$$\Rightarrow \quad p^2x^2 - 8pqx - 20q^2 = 0$$

where $\quad p = a + b \quad$ ...(1)

and $\quad q = a - b \quad$ ...(2)

$$\Rightarrow \quad p^2x^2 - 10pqx + 2pqx - 20q^2 = 0$$

$$\Rightarrow \quad px(px - 10q) + 2q(px - 10q) = 0$$

$$\Rightarrow \quad (px - 10q)(px + 2q) = 0$$

$\therefore$ Either $\quad px - 10q = 0 \quad$ ...(3)

or, $\quad px + 2q = 0 \quad$ ...(4)

$\therefore$ From (1), (2) and (3), we get

$$x = \frac{10q}{p} = \frac{10(a - b)}{a + b}$$

and from (1), (2) and (4), we get

$$x = -\frac{2q}{p} = -\frac{2(a - b)}{a + b}$$

Hence, the required roots are $\dfrac{10(a - b)}{a + b}$ and $-\dfrac{2(a - b)}{a + b}$.

Check Your Progress 3

(Page 72)

Very Short Answer Type Questions

1. The sum of two numbers is 18 and their product is 56. Find the numbers.

Sol. Let the two numbers be x and $18 - x$.

$\therefore$ According to the problem,

$$x(18 - x) = 56$$

$$\Rightarrow \quad x^2 - 18x + 56 = 0$$

$$\Rightarrow \quad x^2 - 4x - 14x + 56 = 0$$

$$\Rightarrow \quad x(x - 4) - 14(x - 4) = 0$$

$$\Rightarrow \quad (x - 4)(x - 14) = 0$$

$\therefore$ Either $\quad x - 4 = 0 \quad$ ...(1)

or, $\quad x - 14 = 0 \quad$ ...(2)

From (1) and (2), we have $x = 4$ or 14.

Hence, the required numbers are 4 and 14.

2. (*a*) The sum of a number and its reciprocal is $-\dfrac{25}{12}$. Find the number.

(*b*) The sum of two numbers is 15 and the sum of their reciprocals is $\dfrac{3}{10}$. Find the numbers. **[CBSE 2023 Standard]**

(*c*) The sum of two numbers a and b is 15 and the sum of their reciprocals $\dfrac{1}{a}$ and $\dfrac{1}{b}$ is $\dfrac{3}{10}$.

Find the numbers a and b. **[CBSE 2000, 2005]**

Sol. (*a*) Let the number be x, where $x \neq 0$. Then according to the problem, we have $x + \dfrac{1}{x} = -\dfrac{25}{12}$

$$\Rightarrow \quad \frac{x^2 + 1}{x} = -\frac{25}{12}$$

$$\Rightarrow \quad 12x^2 + 12 + 25x = 0$$

$$\Rightarrow \quad 12x^2 + 25x + 12 = 0$$

$$\Rightarrow \quad 12x^2 + 16x + 9x + 12 = 0$$

$$\Rightarrow \quad 4x(3x + 4) + 3(3x + 4) = 0$$

$$\Rightarrow \quad (3x + 4)(4x + 3) = 0$$

$\therefore$ Either $\quad 3x + 4 = 0 \quad$ ...(1)

or, $\quad 4x + 3 = 0 \quad$ ...(2)

$\therefore$ From (1) and (2), we have $x = -\dfrac{4}{3}, -\dfrac{3}{4}$

$\therefore$ The required number is either $-\dfrac{4}{3}$ or $-\dfrac{3}{4}$.

(*b*) Let the two numbers be x and $15 - x$.

$\therefore$ According to the problem, we have

$$\frac{1}{x} + \frac{1}{15 - x} = \frac{3}{10}$$

$$\Rightarrow \quad \frac{15 - x + x}{x(15 - x)} = \frac{3}{10}$$

$$\Rightarrow \quad \frac{15}{15x - x^2} = \frac{3}{10}$$

$$\Rightarrow \quad 45x - 3x^2 = 150$$

$\Rightarrow \quad 3x^2 - 45x + 150 = 0$

$\Rightarrow \quad x^2 - 15x + 50 = 0$

$\Rightarrow \quad x^2 - 10x - 5x + 50 = 0$

$\Rightarrow \quad x(x - 10) - 5(x - 10) = 0$

$\Rightarrow \quad (x - 10)(x - 5) = 0$

$\therefore$ Either $\quad x - 10 = 0 \quad \ldots(1)$

or, $\quad x - 5 = 0 \quad \ldots(2)$

$\therefore$ From (1) and (2), we have $x = 10, 5$.

$\therefore$ When one number is 5, the other number is 10. Hence, the required two numbers are 5 and 10.

(*c*) According to the problem, we have

$$a + b = 15 \quad \ldots(1)$$

and $$\frac{1}{a} + \frac{1}{b} = \frac{3}{10} \quad \ldots(2)$$

From (1), $\quad b = 15 - a \quad \ldots(3)$

$\therefore$ From (2),

$$\frac{1}{a} + \frac{1}{15 - a} = \frac{3}{10}$$

$$\Rightarrow \quad \frac{15 - a + a}{a(15 - a)} = \frac{3}{10}$$

$$\Rightarrow \quad \frac{15}{a(15 - a)} = \frac{3}{10}$$

$$\Rightarrow \quad \frac{5}{15a - a^2} = \frac{1}{10}$$

$\Rightarrow \quad 15a - a^2 = 50$

$\Rightarrow \quad a^2 - 15a + 50 = 0$

$\Rightarrow \quad a^2 - 10a - 5a + 50 = 0$

$\Rightarrow \quad a(a - 10) - 5(a - 10) = 0$

$\Rightarrow \quad (a - 10)(a - 5) = 0$

$\therefore$ Either $\quad a - 10 = 0 \quad \ldots(1)$

or, $\quad a - 5 = 0 \quad \ldots(2)$

$\therefore$ From (1) and (2), we have $a = 10, 5$.

We see from (3) that when $a = 10$, then $b = 5$ and when $a = 5$, then $b = 10$.

Thus, the required two numbers are 5 and 10.

3. (*a*) A natural number when increased by 12 becomes equal to 160 times its reciprocal. Find the number. **[CBSE 2012]**

(*b*) Find possible integer(s) which, when decreased by 20, is equal to 69 times the reciprocal of the number.

Sol. (*a*) Let the number be x.

$\therefore$ According to the problem, we have

$$x + 12 = \frac{160}{x}$$

$\Rightarrow \quad x^2 + 12x - 160 = 0$

$\Rightarrow \quad x^2 + 20x - 8x - 160 = 0$

$\Rightarrow \quad x(x + 20) - 8(x + 20) = 0$

$\Rightarrow \quad (x + 20)(x - 8) = 0$

$\therefore$ Either $\quad x + 20 = 0 \quad \ldots(1)$

or, $\quad x - 8 = 0 \quad \ldots(2)$

From (1) and (2), we have

$$x = -20, 8$$

Now, since –20 is not a natural number, hence, we reject it.

$\therefore \quad x = 8$

which is the required number.

(*b*) Let one of the integers be x.

$\therefore$ According to the Problem,

$$x - 20 = \frac{69}{x}$$

$\Rightarrow \quad x^2 - 20x - 69 = 0$

$\Rightarrow \quad x^2 + 3x - 23x - 69 = 0$

$\Rightarrow \quad x(x + 3) - 23(x + 3) = 0$

$\Rightarrow \quad (x + 3)(x - 23) = 0$

$\therefore$ Either $\quad x + 3 = 0 \quad \ldots(1)$

or, $\quad x - 23 = 0 \quad \ldots(2)$

From (1) and (2), we have $x = -3, 23$ which are the required integers.

4. The sum S of first n natural numbers is given by the relation $S = n\frac{(n+1)}{2}$. Find n if the sum is 465.

Sol. We have $$S = \frac{n(n+1)}{2}$$

$$\Rightarrow \quad 465 = \frac{n^2 + n}{2}$$

$\Rightarrow \quad n^2 + n - 930 = 0$

$\Rightarrow \quad n^2 + 31n - 30n - 930 = 0$

$\Rightarrow \quad n(n + 31) - 30(n + 31) = 0$

$\Rightarrow \quad (n - 30)(n + 31) = 0$

$\therefore$ Either $\quad n - 30 = 0 \quad \ldots(1)$

or, $\quad n + 31 = 0 \quad \ldots(2)$

From (1) and (2), we have $n = 30, -31$.

Since, –31 is not a natural number, we reject it.

So, $n = 30$ which is the required value of n.

5. There are three consecutive integers such that the square of the first increased by the product of the other two gives 154. What are the integers? **[CBSE SP 2011]**

Sol. Let the three consecutive integers be x, $x + 1$, $x + 2$.

$\therefore$ According to the problem, we have

$$x^2 + (x + 1)(x + 2) = 154$$

$\Rightarrow \quad x^2 + x^2 + 3x + 2 - 154 = 0$

$\Rightarrow \quad 2x^2 + 3x - 152 = 0$

$\Rightarrow \quad 2x^2 + 19x - 16x - 152 = 0$

$\Rightarrow \quad x(2x + 19) - 8(2x + 19) = 0$

$\Rightarrow \quad (x - 8)(2x + 19) = 0$

$\therefore$ Either $\quad x - 8 = 0 \quad$ ...(1)

or, $\quad 2x + 19 = 0 \quad$ ...(2)

$\therefore$ From (1) and (2), $\quad x = 8, -\frac{19}{2}$

But $-\frac{19}{2}$ is not an integer and so we reject this value.

$\therefore \quad x = 8$

$\therefore$ The required consecutive integers are 8, 9 and 10.

6. The product of two successive multiples of 5 is 300. Find the multiples.

Sol. Let two multiples of 5 be $5x$ and $5(x + 1)$.

$\therefore$ According to the problem,

$$5x \times 5(x + 1) = 300$$

$\Rightarrow \quad x^2 + x = \frac{300}{25} = 12$

$\Rightarrow \quad x^2 + x - 12 = 0$

$\Rightarrow \quad x^2 + 4x - 3x - 12 = 0$

$\Rightarrow \quad x(x + 4) - 3(x + 4) = 0$

$\Rightarrow \quad (x + 4)(x - 3) = 0$

$\therefore$ Either $\quad x + 4 = 0 \quad$ ...(1)

or, $\quad x - 3 = 0 \quad$ ...(2)

From (1), $x = -4$ and from (2) $x = 3$.

Since the multiples of a natural number cannot be negative, we reject $x = -4$.

$\therefore$ We have $x = 3$.

$\therefore$ The required multiples of 5 are 5×3 and $5 \times (3 + 1)$, i.e. 15 and 20.

7. The age of father is equal to the square of the age of his son. The sum of the age of the father and five times the age of the son is 84 years. Find their ages.

Sol. Let the ages of the father and his son be x years and y years respectively.

$\therefore$ According to the problem, we have

$$x = y^2 \quad \text{...(1)}$$

and $\quad x + 5y = 84 \quad$ ...(2)

$\therefore$ From (1) and (2), we have

$$y^2 + 5y - 84 = 0$$

$\Rightarrow \quad y^2 + 12y - 7y - 84 = 0$

$\Rightarrow \quad y(y + 12) - 7(y + 12) = 0$

$\Rightarrow \quad (y + 12)(y - 7) = 0$

$\therefore$ Either $\quad y + 12 = 0 \quad$ ...(3)

or, $\quad y - 7 = 0 \quad$ ...(4)

From (3) and (4), we have $y = -12$ or 7.

Since the age cannot be negative, we reject $y = -12$.

$\therefore$ We have $y = 7$.

$\therefore$ From (1), we have $x = 7^2 = 49$.

$\therefore$ The required ages of the son and his father are 7 years and 49 years respectively.

8. When Deepica was asked her age, she replied "If you subtract eleven times my age from the square of my age the result is 210". Find her age.

Sol. Let the age of Deepica be x years. Then according to the problem, we have

$$x^2 - 11x = 210$$

$\Rightarrow \quad x^2 - 11x - 210 = 0$

$\Rightarrow \quad x^2 + 10x - 21x - 210 = 0$

$\Rightarrow \quad x(x + 10) - 21(x + 10) = 0$

$\Rightarrow \quad (x + 10)(x - 21) = 0$

$\therefore$ Either $\quad x + 10 = 0 \quad$ ...(1)

or, $\quad x - 21 = 0 \quad$ ...(2)

$\therefore$ From (1) and (2), we have $x = -10, 21$.

Since age cannot be negative, we reject $x = -10$.

$\therefore$ The required age of Deepica is 21 years.

9. Twenty-seven years hence Ashish's age will be square of what it was 29 years ago. Find his present age.

Sol. Let the present age of Ashish be x years. Hence, according to the problem, we have

$$x + 27 = (x - 29)^2$$

$\Rightarrow \quad x^2 - 58x + 29^2 - x - 27 = 0$

$\Rightarrow \quad x^2 - 59x + 841 - 27 = 0$

$\Rightarrow \quad x^2 - 59x + 814 = 0$

$\Rightarrow \quad x^2 - 37x - 22x + 814 = 0$

$\Rightarrow \quad x(x - 37) - 22(x - 37) = 0$

$\Rightarrow \quad (x - 37)(x - 22) = 0$

$\therefore$ Either $\quad x - 37 = 0 \quad$ ...(1)

or, $\quad x - 22 = 0 \quad$ ...(2)

From (1), $x = 37$ and from (2) $x = 22$.

Now, $x = 22$ is impossible, since $22 < 29$ and hence, 29 years ago Ashish's age was less than 0 which is absurd.

Hence, we reject $x = 22$ and accept $x = 37$.

Hence, the required present age of Ashish is 37 years.

10. In a class test, the sum of marks obtained by P in Mathematics and Science is 28. Had he got 3 more marks in Mathematics and 4 marks less in Science, the product of marks obtained in the two subjects would have been 180. Find the marks obtained in the two subjects separately.

[CBSE 2008]

Sol. Let the marks obtained by P in mathematics and science be x and y respectively.

Then according to the problem, we have

$$x + y = 28 \quad \ldots(1)$$

and $$(x + 3)(y - 4) = 180 \quad \ldots(2)$$

From (1), we have $$y = 28 - x \quad \ldots(3)$$

$\therefore$ From (2), we have

$$(x + 3)(28 - x - 4) = 180$$

$$\Rightarrow \quad (x + 3)(24 - x) - 180 = 0$$

$$\Rightarrow \quad -x^2 + 21x + 72 - 180 = 0$$

$$\Rightarrow \quad x^2 - 21x + 108 = 0$$

$$\Rightarrow \quad x^2 - 12x - 9x + 108 = 0$$

$$\Rightarrow \quad x(x - 12) - 9(x - 12) = 0$$

$$\Rightarrow \quad (x - 12)(x - 9) = 0$$

$\therefore$ Either $$x - 12 = 0 \quad \ldots(4)$$

or, $$x - 9 = 0 \quad \ldots(5)$$

From (4), $$x = 12 \quad \ldots(6)$$

and from (5), $$x = 9 \quad \ldots(7)$$

When $x = 12$, then from (3), we see that

$$y = 28 - 12 = 16$$

From (7), we see that when $x = 9$, then from (3), $y = 28 - 9 = 19$.

Hence, P obtained either 12 marks in mathematics and 16 marks in science or 9 marks in mathematics and 19 marks in science.

11. ₹1200 was distributed equally among certain number of students. Had there been 8 more students, each would have received ₹ 5 less. Find the number of students. **[CBSE SP 2006]**

Sol. Let the number of students be x. Then each student received ₹ $\frac{1200}{x}$.

$\therefore$ According to the condition of the problem, we have

$$(x + 8)\left(\frac{1200}{x} - 5\right) = 1200$$

$$\Rightarrow \quad 1200 - 5x + \frac{9600}{x} - 40 - 1200 = 0$$

$$\Rightarrow \quad -5x^2 + 9600 - 40x = 0$$

$$\Rightarrow \quad x^2 + 8x - 1920 = 0$$

$$\Rightarrow \quad x^2 + 48x - 40x - 1920 = 0$$

$$\Rightarrow \quad x(x + 48) - 40(x + 48) = 0$$

$$\Rightarrow \quad (x - 40)(x + 48) = 0$$

$\therefore$ Either $$x - 40 = 0 \quad \ldots(1)$$

or, $$x + 48 = 0 \quad \ldots(2)$$

From (2) $x = -48$ which is absurd, since the number of students cannot be negative. From (1), $x = 40$ which is the required number of students.

12. The sum of the areas of two squares is 325 cm^2. The side of the larger square is 5 cm longer than the side of the smaller square. Find the side of each square.

Sol. Let the side of the smaller square be x cm. Then the side of the larger square is $(x + 5)$ cm. Hence, according to the problem, we have

$$x^2 + (x + 5)^2 = 325$$

$$\Rightarrow \quad x^2 + x^2 + 10x + 25 - 325 = 0$$

$$\Rightarrow \quad 2x^2 + 10x - 300 = 0$$

$$\Rightarrow \quad x^2 + 5x - 150 = 0$$

$$\Rightarrow \quad x^2 + 15x - 10x - 150 = 0$$

$$\Rightarrow \quad x(x + 15) - 10(x + 15) = 0$$

$$\Rightarrow \quad (x + 15)(x - 10) = 0$$

$\therefore$ Either $$x + 15 = 0 \quad \ldots(1)$$

or, $$x - 10 = 0 \quad \ldots(2)$$

From (1), $x = -15$ which is absurd, since the length of a side of a square cannot be negative.

$\therefore$ We reject this value.

$\therefore$ From (2), we get $x = 10$

Hence, the required lengths of the sides of the two squares are 10 cm and 15 cm.

13. The length of the hypotenuse of a right triangle exceeds the length of the base by 2 cm and exceeds twice the length of the altitude by 1 cm. Find the length of each side of the triangle.

[CBSE 2002]

Sol. Let the lengths of the base and altitude of the right angled triangle be x cm and y cm respectively.

Hence, the length of the hypotenuse of the triangle is either $(x + 2)$ cm or $2y + 1$ cm.

Hence, $x + 2 = 2y + 1$

$\Rightarrow \quad x - 2y + 1 = 0$

$\Rightarrow \quad y = \dfrac{x+1}{2} \quad \ldots(1)$

When the length of the hypotenuse is $(x + 2)$ cm, then

$$h^2 = x^2 + y^2$$

$$= x^2 + \left(\frac{x+1}{2}\right)^2$$

[By Pythagoras theorem]

$\Rightarrow \quad (x+2)^2 = x^2 + \dfrac{x^2 + 2x + 1}{4}$

$\Rightarrow \quad x^2 + 4x + 4 = x^2 + \dfrac{x^2 + 2x + 1}{4}$

$\Rightarrow \quad 16x + 16 = x^2 + 2x + 1$

$\Rightarrow \quad x^2 - 14x - 15 = 0$

$\Rightarrow \quad x^2 + x - 15x - 15 = 0$

$\Rightarrow \quad x(x + 1) - 15(x + 1) = 0$

$\Rightarrow \quad (x + 1)(x - 15) = 0$

$\therefore$ Either $\quad x + 1 = 0 \quad \ldots(2)$

or, $\quad x - 15 = 0 \quad \ldots(3)$

From (2) $x = -1$ which is absurd, since the length cannot be negative

$\therefore$ We reject $x = -1$.

$\therefore$ From (3), $x = 15$ and from (1), $y = 8$.

Hence, the required lengths of the base, altitude and hypotenuse are 15 cm, 8 cm and $(15 + 2)$ cm or $(2 \times 8 + 1)$ cm, i.e. 17 cm respectively.

14. The diagonal of a rectangular field is 16 metres more than the shorter side. If the longer side is 14 metres more than the shorter side, then find the lengths of the sides of the field. **[CBSE 2015]**

Sol. Let the length of the shorter side be x cm. Then the length of the longer side and the length of the diagonal of the rectangle are $(x + 14)$cm and $(x + 16)$ cm respectively.

$\therefore$ By Pythagoras theorem, we have

$$(x + 14)^2 + x^2 = (x + 16)^2$$

$\Rightarrow \quad x^2 + 28x + 196 + x^2 = x^2 + 32x + 256$

$\Rightarrow \quad x^2 - 4x - 60 = 0$

$\Rightarrow \quad x^2 + 6x - 10x - 60 = 0$

$\Rightarrow \quad x(x + 6) - 10(x + 6) = 0$

$\Rightarrow \quad (x + 6)(x - 10) = 0$

$\therefore$ Either $\quad x + 6 = 0 \quad \ldots(1)$

or, $\quad x - 10 = 0 \quad \ldots(2)$

From (1), $x = -6$ which is rejected, since the length cannot be negative.

$\therefore$ From (2), we have $x = 10$.

Hence, the required lengths of the sides are 10 cm, $(10 + 14)$ cm, i.e. 24 cm and $(10 + 16)$ cm, i.e. 26 cm.

15. A passenger train takes one hour less when its speed is increased by 15 km/hour than its usual speed for a journey of 300 km. Find the usual speed of the train. **[CBSE SP 2006]**

Sol. Let the usual speed of the train be x km/h.

$\therefore$ The usual time taken by the train to cover a distance of 300 km is $\dfrac{300}{x}$. Now, if the speed is $(x + 15)$ km/h then the time taken by the train to cover the same distance is $\dfrac{300}{x+15}$

$\therefore$ According to the problem, we have

$$\frac{300}{x} - \frac{300}{x+15} = 1$$

$\Rightarrow \quad \dfrac{300(x + 15 - x)}{x(x+15)} = 1$

$\Rightarrow \quad \dfrac{4500}{x^2 + 15x} = 1$

$\Rightarrow \quad x^2 + 15x - 4500 = 0$

$\Rightarrow \quad x^2 + 75x - 60x - 4500 = 0$

$\Rightarrow \quad x(x + 75) - 60(x + 75) = 0$

$\Rightarrow \quad (x + 75)(x - 60) = 0$

$\therefore$ Either $\quad x + 75 = 0 \quad \ldots(1)$

or, $\quad x - 60 = 0 \quad \ldots(2)$

From (1), $x = -75$ which is rejected, since speed cannot be negative here.

$\therefore$ From (2), we get $x = 60$.

$\therefore$ The required usual speed of the train is 60 km/h.

Short Answer Type Questions

16. The difference of two numbers is 4. If the difference of their reciprocals is $\dfrac{4}{21}$, find the two numbers. **[CBSE 2008]**

Sol. Let the two numbers be x and y, where $x > y$.

Then according to the problem, we have

$$x - y = 4 \quad \ldots(1)$$

and $$\frac{1}{y} - \frac{1}{x} = \frac{4}{21} \quad \ldots(2)$$

since $\frac{1}{x} < \frac{1}{y}$

From (1), $$y = x - 4 \quad \ldots(3)$$

$\therefore$ From (2), $$\frac{1}{x-4} - \frac{1}{x} = \frac{4}{21}$$

$$\Rightarrow \quad \frac{x - x + 4}{x(x-4)} = \frac{4}{21}$$

$$\Rightarrow \quad \frac{1}{x^2 - 4x} = \frac{1}{21}$$

$$\Rightarrow \quad x^2 - 4x - 21 = 0$$

$$\Rightarrow \quad x^2 + 3x - 7x - 21 = 0$$

$$\Rightarrow \quad x(x + 3) - 7(x + 3) = 0$$

$$\Rightarrow \quad (x + 3)(x - 7) = 0$$

$\therefore$ Either $$x + 3 = 0 \quad \ldots(4)$$

or, $$x - 7 = 0 \quad \ldots(5)$$

From (3) and (4), we have $x = -3$, $y = -7$

and from (3) and (5), we have $x = 7$, $y = 3$.

Hence, the required two numbers are either 3 and 7 or, –3 and –7.

17. (*a*) The denominator of a fraction is 3 more than its numerator. The sum of the fraction and its reciprocal is $2\frac{9}{10}$. Find the fraction. **[CBSE 2015]**

(*b*) The denominator of a fraction is one more than twice its numerator. If the sum of the fraction and its reciprocal is $2\frac{16}{21}$, find the fraction. **[CBSE 2016]**

Sol. (*a*) Let the numerator of the fraction be x. Then its denominator is $x + 3$. Then the fraction is $\frac{x}{x+3}$.

$\therefore$ According to the problem, we have

$$\frac{x}{x+3} + \frac{x+3}{x} = 2\frac{9}{10} = \frac{29}{10}$$

$$\Rightarrow \quad \frac{x^2 + (x+3)^2}{x(x+3)} = \frac{29}{10}$$

$$\Rightarrow \quad \frac{x^2 + x^2 + 6x + 9}{x^2 + 3x} = \frac{29}{10}$$

$$\Rightarrow \quad 20x^2 + 60x + 90 = 29x^2 + 87x$$

$$\Rightarrow \quad 9x^2 + 27x - 90 = 0$$

$$\Rightarrow \quad x^2 + 3x - 10 = 0$$

$$\Rightarrow \quad x^2 + 5x - 2x - 10 = 0$$

$$\Rightarrow \quad x(x + 5) - 2(x + 5) = 0$$

$$\Rightarrow \quad (x + 5)(x - 2) = 0$$

$\therefore$ Either $$x + 5 = 0 \quad \ldots(1)$$

or, $$x - 2 = 0 \quad \ldots(2)$$

From (1), $x = -5$ and from (2), $x = 2$

If it is assumed that both the numerator and the denominator of the fraction may be negative, then $x = -5$ is accepted. In this case, the numerator and the denominator are respectively –5 and –5 + 3, i.e. –2 and so the fraction is $\frac{-5}{-2}$ i.e. $\frac{5}{2}$.

In case, if it is assumed that both the numerator and the denominator should be positive, then we accept only $x = 2$.

In this case, the required fraction will be $\frac{2}{2+3}$ i.e. $\frac{2}{5}$.

(*b*) Let the numerator be x. Then the denominator is $2x + 1$.

$\therefore$ The original fraction is $\frac{x}{2x+1}$.

$\therefore$ According to the problem, we have

$$\frac{x}{2x+1} + \frac{2x+1}{x} = 2\frac{16}{21} = \frac{58}{21}$$

$$\Rightarrow \quad \frac{x^2 + (2x+1)^2}{x(2x+1)} = \frac{58}{21}$$

$$\Rightarrow \quad \frac{x^2 + 4x^2 + 4x + 1}{2x^2 + x} = \frac{58}{21}$$

$$\Rightarrow \quad \frac{5x^2 + 4x + 1}{2x^2 + x} = \frac{58}{21}$$

$$\Rightarrow \quad 150x^2 + 84x + 21 = 116x^2 + 58x$$

$$\Rightarrow \quad 11x^2 - 26x - 21 = 0$$

$$\Rightarrow \quad 11x^2 - 33x + 7x - 21 = 0$$

$$\Rightarrow \quad 11x(x - 3) + 7(x - 3) = 0$$

$$\Rightarrow \quad (x - 3)(11x + 7) = 0$$

$\therefore$ Either $$x - 3 = 0 \quad \ldots(1)$$

or, $$11x + 7 = 0 \quad \ldots(2)$$

From (1), $x = 3$ and from (2), $x = -\frac{7}{11}$ which is rejected, since numerator and denominator should be integers.

$\therefore$ $x = 3$ and so the original fraction is $\frac{3}{2 \times 3 + 1}$

$= \frac{3}{7}$.

$\therefore$ The required fraction is $\frac{3}{7}$.

18. A two-digit number is four times the sum of the digits. It is also equal to three times the product of the digits. Find the number. **[CBSE 2016]**

Sol. Let the digit in the unit's place of the number be x and that in the ten's place be y. Then the number is $10y + x$.

$\therefore$ According to the problem,

$$10y + x = 4(x + y)$$
$$= 4x + 4y$$
$$\Rightarrow \quad 3x - 6y = 0$$
$$\Rightarrow \quad y = \frac{3x}{6} = \frac{x}{2} \quad \ldots(1)$$
$$\text{and} \quad 10y + x = 3xy \quad \ldots(2)$$

From (1) and (2), we get

$$10 \times \frac{x}{2} + x = 3x \times \frac{x}{2}$$
$$\Rightarrow \quad 5x + x = \frac{3x^2}{2}$$
$$\Rightarrow \quad 12x = 3x^2$$
$$\Rightarrow \quad x^2 - 4x = 0$$
$$\Rightarrow \quad x(x - 4) = 0$$

$\therefore$ Either $\quad x = 0 \quad \ldots(1)$

or, $\quad x - 4 = 0 \quad \ldots(2)$

From (2), we get $x = 4$

When $x = 0$, we see from (1) that $y = 0$

and when $x = 4$, $y = \frac{4}{2} = 2$

For $x = 0$ $y = 0$, we do not get any non-zero number.

Hence, for $x = 4$ and $y = 2$, the required number is $10 \times 2 + 4 = 24$.

Hence, the required number is 24.

19. Vijay is x years old and his father is nine times the square of Vijay's age. Thirty-two years hence, his father's age will be double of his age then. Find their present ages.

Sol. The age of Vijay is x years. The age of Vijay's father is $9x^2$ years.

According to the problem,

$$9x^2 + 32 = 2(x + 32)$$
$$\Rightarrow \quad 9x^2 - 2x + 32 - 64 = 0$$
$$\Rightarrow \quad 9x^2 - 2x - 32 = 0$$
$$\Rightarrow \quad 9x^2 + 16x - 18x - 32 = 0$$
$$\Rightarrow \quad x(9x + 16) - 2(9x + 16) = 0$$
$$\Rightarrow \quad (x - 2)(9x + 16) = 0$$

$\therefore$ Either $\quad x - 2 = 0 \quad \ldots(1)$

or, $\quad 9x + 16 = 0 \quad \ldots(2)$

From (1), $x = 2$ and from (2), $x = -\frac{16}{9}$ which is rejected, since the age cannot be negative.

The required present age of Vijay is 2 years and his father is $9 \times (2)^2 = 9 \times 4$, i.e. 36 years.

20. A shopkeeper buys a number of books for ₹ 80. If he had bought 4 more books for the same amount, each book would have cost ₹ 1 less. Find the number of books he bought. **[CBSE 2012]**

Sol. Let the number of books bought by the shopkeeper be x.

$\therefore$ The original cost of each book = ₹ $\frac{80}{x}$

$\therefore$ According to the problem, we have

$$(x + 4)\left(\frac{80}{x} - 1\right) = 80$$
$$\Rightarrow \quad 80 - x + \frac{320}{x} - 4 - 80 = 0$$
$$\Rightarrow \quad -x^2 + 320 - 4x = 0$$
$$\Rightarrow \quad x^2 + 4x - 320 = 0$$
$$\Rightarrow \quad x^2 + 20x - 16x - 320 = 0$$
$$\Rightarrow \quad x(x + 20) - 16(x + 20) = 0$$
$$\Rightarrow \quad (x + 20)(x - 16) = 0$$

$\therefore$ Either $\quad x + 20 = 0 \quad \ldots(1)$

or, $\quad x - 16 = 0 \quad \ldots(2)$

From (1), $x = -20$ which is rejected, since the number of books cannot be negative.

$\therefore$ From (2), we get $x = 16$ which is the required number of books.

21. A person on tour has ₹ 360 for his daily expenses. If he exceeds his tour programme by 4 days, he must cut down his daily expenses by ₹ 3 per day. Find the number of days of his tour programme? **[CBSE SP 2011]**

Sol. Let the number of days of the person's tour programme be x.

Then his usual daily expense = $\frac{360}{x}$.

$\therefore$ According to the problem, we have

$$(x + 4)\left(\frac{360}{x} - 3\right) = 360$$

$\Rightarrow \quad 360 - 3x + \dfrac{1440}{x} = 372$

$\Rightarrow \quad \dfrac{-3x^2 + 1440}{x} = 12$

$\Rightarrow \quad 3x^2 + 12x - 1440 = 0$

$\Rightarrow \quad x^2 + 4x - 480 = 0$

$\Rightarrow \quad x^2 + 24x - 20x - 480 = 0$

$\Rightarrow \quad x(x + 24) - 20(x + 24) = 0$

$\Rightarrow \quad (x + 24)(x - 20) = 0$

$\therefore$ Either $\quad x + 24 = 0 \quad \ldots(1)$

or, $\quad x - 20 = 0 \quad \ldots(2)$

$\therefore$ From (1), $x = -24$, which is rejected, the number of days cannot be negative. From (2), $x = 20$ which is the required number of days.

22. The perimeter of a rectangle is 94 cm and its area is 246 cm^2. Find the dimensions of the rectangle.

Sol. Let the length and breadth of the rectangle be x cm and y cm respectively, where $x > y$.

Then, according to the problem, we have

$$2(x + y) = 94$$

$\Rightarrow \quad x + y = 47 \quad \ldots(1)$

and $\quad xy = 246 \quad \ldots(2)$

From (1), $\quad y = 47 - x \quad \ldots(3)$

$\therefore$ From (2), we have

$$x(47 - x) = 246$$

$\Rightarrow \quad x^2 - 47x + 246 = 0$

$\Rightarrow \quad x^2 - 41x - 6x + 246 = 0$

$\Rightarrow \quad x(x - 41) - 6(x - 41) = 0$

$\Rightarrow \quad (x - 41)(x - 6) = 0$

$\therefore$ Either $\quad x - 41 = 0 \quad \ldots(1)$

or, $\quad x - 6 = 0 \quad \ldots(2)$

From (1), $x = 41$ and from (2), $x = 6$.

$\therefore$ From (3), $y = 47 - 41 = 6$ when $x = 41$ and $y = 47 - 6 = 41$ when $x = 6$.

$\because x > y$, $\therefore x = 41$ and $y = 6$.

Hence, the required length and breadth are 41 cm and 6 cm respectively.

23. A farmer wishes to start a 100 m^2 rectangular vegetable garden. Since, he has only 30 m of barbed wire, he fences three sides of the rectangular garden letting his house compound wall act as the fourth side fence. Find the dimensions of the garden. **[CBSE SP 2011]**

Sol. Let the length and breadth of the rectangle be y m and x m respectively so that $x > y$.

Then according to the problem, we have

$2x + y = 30 \quad \ldots(1)$

and $\quad xy = 100 \quad \ldots(2)$

From (1), $\quad y = 30 - 2x \quad \ldots(3)$

$\therefore$ From (2) and (3), we have

$$x(30 - 2x) = 100$$

$\Rightarrow \quad 2x^2 - 30x + 100 = 0$

$\Rightarrow \quad x^2 - 15x + 50 = 0$

$\Rightarrow \quad x^2 - 10x - 5x + 50 = 0$

$\Rightarrow \quad x(x - 10) - 5(x - 10) = 0$

$\Rightarrow \quad (x - 10)(x - 5) = 0$

$\therefore$ Either $x - 10 = 0$ or $x - 5 = 0$

$\Rightarrow \quad x = 10$ or $x = 5$

$\therefore$ From (3), $y = 30 - 2 \times 10 = 10$ when $x = 10$ and $y = 30 - 2 \times 5 = 20$ when $x = 5$.

$\therefore$ We have either $x = 10, y = 10$

or $\quad x = 5, y = 20$

[satisfies the given condition as the garden is rectangular]

Hence, the required length and breadth of the rectangle are 20 m and 5 m respectively.

24. The hypotenuse of a right triangle is $3\sqrt{5}$ cm. If the smaller side is tripled and the larger side is doubled, the new hypotenuse will be 15 cm. Find the length of each side.

Sol. Let the lengths of two sides of the right angled triangle be x cm and y cm respectively, where $x > y > 0$.

Then by using Pythagoras theorem, we have

$$\left(3\sqrt{5}\right)^2 = x^2 + y^2$$

$\Rightarrow \quad x^2 + y^2 = 45 \quad \ldots(1)$

and $\quad (3y)^2 + (2x)^2 = 15^2$

$\Rightarrow \quad 9y^2 + 4x^2 = 225 \quad \ldots(2)$

From (1) and (2), we have

$$p + q = 45$$

$\Rightarrow \quad q = 45 - p \quad \ldots(3)$

and $\quad 4p + 9q = 225 \quad \ldots(4)$

where $\quad p = x^2 \quad \ldots(5)$

and $\quad q = y^2 \quad \ldots(6)$

$\therefore$ From (3) and (4), we get

$$4p + 9(45 - p) = 225$$

$$\Rightarrow \quad 4p - 9p + 405 = 225$$

$$\Rightarrow \quad 5p = 180$$

$$\Rightarrow \quad p = \frac{180}{5} = 36$$

$$\Rightarrow \quad x^2 = 36 \qquad \text{[From (5)]}$$

$$\Rightarrow \quad x = \pm 6$$

Since $x > 0$, we take $x = 6$.

when $p = 36$, we have

$$q = 45 - 36 = 9 \qquad \text{[From (3)]}$$

$$\Rightarrow \quad y^2 = 9 \qquad \text{[From (6)]}$$

$$\Rightarrow \quad y = \pm 3$$

$\because$ $y > 0$, we take $y = 3$.

Hence, the required lengths of the two sides of the triangle are 3 cm and 6 cm.

25. A rectangle has a perimeter 46 cm and either of the diagonals measures 17 cm. Find the dimensions of the rectangle.

Sol. Let the length and breadth of the rectangle be x cm and y cm respectively, where $x > y > 0$.

$\therefore$ According to the problem, we have

$$2(x + y) = 46$$

$$\Rightarrow \quad x + y = 23$$

$$\Rightarrow \quad y = 23 - x \qquad \ldots(1)$$

and by Pythagoras theorem, we have

$$x^2 + y^2 = 17^2 = 289 \qquad \ldots(2)$$

$\therefore$ From (1) and (2), we get

$$x^2 + (23 - x)^2 - 289 = 0$$

$$\Rightarrow \quad 2x^2 - 46x + 529 - 289 = 0$$

$$\Rightarrow \quad 2x^2 - 46x + 240 = 0$$

$$\Rightarrow \quad x^2 - 23x + 120 = 0$$

$$\Rightarrow \quad x^2 - 15x - 8x + 120 = 0$$

$$\Rightarrow \quad x(x - 15) - 8(x - 15) = 0$$

$$\Rightarrow \quad (x - 8)(x - 15) = 0$$

$\therefore$ Either $\quad x - 8 = 0 \qquad \ldots(3)$

or, $\quad x - 15 = 0 \qquad \ldots(4)$

$\therefore$ From (3) and (4), $x = 8, 15$

$\therefore$ From (1), $y = 23 - 8 = 15$ when $x = 8$

which is rejected, since $x < y$ in this case.

Again, when $x = 15$, from (1), we get

$$y = 23 - 15 = 8$$

which is accepted, since $x > y$.

Hence, the required length and breadth of the rectangle are 15 cm and 8 cm respectively.

26. A bus moving at its usual speed covers distance between towns A and B which are 550 km apart in 1 hour less than it takes to cover the same distance, when it is raining and the bus has to reduce the speed by 5 km/h. Calculate the time taken by the bus to cover the distance between A and B when it is raining.

Sol. Let the usual speed of the bus be x km/h. Then the usual time taken by the bus to cover a distance of 550 km is $\frac{550}{x}$ h.

Now, according to the problem, we have

$$\frac{550}{x-5} - \frac{550}{x} = 1$$

$$\Rightarrow \quad 550\left(\frac{x - x + 5}{x(x-5)}\right) = 1$$

$$\Rightarrow \quad \frac{2750}{x^2 - 5x} = 1$$

$$\Rightarrow \quad x^2 - 5x - 2750 = 0$$

$$\Rightarrow \quad x^2 - 55x + 50x - 2750 = 0$$

$$\Rightarrow \quad x(x - 55) + 50(x - 55) = 0$$

$$\Rightarrow \quad (x - 55)(x + 50) = 0$$

$\therefore$ Either $\quad x - 55 = 0 \qquad \ldots(1)$

or, $\quad x + 50 = 0 \qquad \ldots(2)$

From (1), $x = 55$ and from (2), $x = -50$ which is rejected, since x cannot be negative.

Hence, the usual speed of the bus is 55 km/h.

$\therefore$ The required usual time taken by the bus to travel a distance of 550 km is $\frac{550}{50}$ hours i.e. 11 hours.

27. A train travels at a uniform speed for a distance of 54 km and then travels a distance of 63 km at an average speed of 6 km/h more than the first speed. If it takes 3 hours to complete the total journey what is the first speed of the train?

[CBSE 2015]

Sol. Let the first speed of the train be x km/h. Then according to the problem, we have

$$\frac{54}{x} + \frac{63}{x+6} = 3$$

$$\Rightarrow \quad \frac{54(x+6) + 63x}{x(x+6)} = 3$$

$$\Rightarrow \quad 54x + 324 + 63x = 3x(x + 6)$$

$$= 3x^2 + 18x$$

$$\Rightarrow \quad 3x^2 + 18x - 117x - 324 = 0$$

$$\Rightarrow \quad 3x^2 - 99x - 324 = 0$$

$\Rightarrow \quad x^2 - 33x - 108 = 0$

$\Rightarrow \quad x^2 + 3x - 36x - 108 = 0$

$\Rightarrow \quad x(x + 3) - 36(x + 3) = 0$

$\Rightarrow \quad (x + 3)(x - 36) = 0$

$\therefore$ Either $\quad x + 3 = 0 \quad \ldots(1)$

or, $\quad x - 36 = 0 \quad \ldots(2)$

From (1), $x = -3$ which is rejected, since x cannot be negative.

$\therefore$ From (2), we get $x = 36$ which is accepted.

Hence, the required speed is 36 km/h.

28. The time taken by a person to cover 150 km was $2\frac{1}{2}$ hours more than the time taken in the return journey. If he returned at a speed of 10 km/hour more than the speed while going, find the speed per hour in each direction. **[CBSE 2016]**

Sol. Let the speed of the man for onward journey be x km/h. Then, according to the problem, we have

$$\frac{150}{x} - \frac{150}{x+10} = 2\frac{1}{2} = \frac{5}{2}$$

$\Rightarrow \quad \dfrac{30}{x} - \dfrac{30}{x+10} = \dfrac{1}{2}$

$\Rightarrow \quad \dfrac{30x + 300 - 30x}{x(x+10)} = \dfrac{1}{2}$

$\Rightarrow \quad x^2 + 10x - 600 = 0$

$\Rightarrow \quad x^2 + 30x - 20x - 600 = 0$

$\Rightarrow \quad x(x + 30) - 20(x + 30) = 0$

$\Rightarrow \quad (x + 30)(x - 20) = 0$

$\therefore$ Either $\quad x + 30 = 0 \quad \ldots(1)$

or, $\quad x - 20 = 0 \quad \ldots(2)$

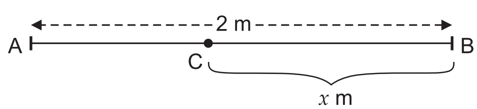
From (1), $x = -30$ which is rejected, since x cannot be negative.

$\therefore$ From (2), $x = 20$ which is accepted.

$\therefore$ The required speed for onward journey is 20 km/h and that for return journey is (20 + 10) km/h i.e. 30 km/h.

29. A segment AB of 2 m length is divided at C, into two parts such that $AC^2 = AB \times CB$. Find the length of the part CB.

Sol.



Let CB = x m.

$\therefore \quad AC = (2 - x)$ m

Also, $\quad AB = 2$ m

$\therefore \quad x < 2$

$\therefore$ From $AC^2 = AB \times CB$, we get

$\Rightarrow \quad (2 - x)^2 = 2x$

$\Rightarrow \quad 4 + x^2 - 4x - 2x = 0$

$\Rightarrow \quad x^2 - 6x + 4 = 0$

$$\therefore \quad x = \frac{6 \pm \sqrt{6^2 - 4 \times 4}}{2 \times 1} = \frac{6 \pm \sqrt{20}}{2} = 3 \pm \sqrt{5}$$

$\therefore \quad x = 3 + \sqrt{5}$ or $x = 3 - \sqrt{5}$

But since $x < 2$, we reject $x = 3 + \sqrt{5}$.

Hence, $x = 3 - \sqrt{5}$ is accepted.

$\therefore$ The required length of the part CB is $(3 - \sqrt{5})$ m.

Long Answer Type Questions

30. A two-digit number is such that the product of the digits is 20. If 9 is subtracted from the number, the digits interchange their places. Find the number. **[CBSE SP 2006, 2011]**

Sol. Let the digit in the unit place of the number be x and that in the ten's place be y. Then the number is $10y + x$.

$\therefore$ According to the problem, we have

$xy = 20 \quad \ldots(1)$

and $\quad 10y + x - 9 = 10x + y$

$\Rightarrow \quad 9x - 9y + 9 = 0$

$\Rightarrow \quad x - y + 1 = 0 \quad \ldots(2)$

$\therefore$ From (2), wet get $\quad y = x + 1 \quad \ldots(3)$

$\therefore$ From (1), we have

$x(x + 1) = 20$

$\Rightarrow \quad x^2 + x - 20 = 0$

$\Rightarrow \quad x^2 + 5x - 4x - 20 = 0$

$\Rightarrow \quad x(x + 5) - 4(x + 5) = 0$

$\Rightarrow \quad (x + 5)(x - 4) = 0$

$\therefore$ Either $\quad x + 5 = 0 \quad \ldots(4)$

or, $\quad x - 4 = 0 \quad \ldots(5)$

From (4), $x = -5$ which is rejected, since x cannot be negative.

$\therefore$ From (5), we have $x = 4$ which is accepted.

Now, when $x = 4$, from (3), $y = 4 + 1 = 5$

$\therefore$ The required number is $10 \times 5 + 4$ i.e 54.

31. A worker earns ₹ 400 in a certain number of days. If his daily wage had been ₹ 10 more, he would have taken 2 days less to earn the same amount. Find how many days he worked at a higher wage.

Sol. Let the worker worked for x days at the usual wage.

Then he worked for $(x - 2)$ days at higher wage.

$\therefore$ His daily wage at higher rate is ₹ $\frac{400}{x-2}$

and his daily wage at the usual rate is ₹ $\frac{400}{x}$.

$\therefore$ According to the problem, we have

$$\frac{400}{x-2} - \frac{400}{x} = 10$$

$$\Rightarrow \quad \frac{40}{x-2} - \frac{40}{x} = 1$$

$$\Rightarrow \quad \frac{40x - 40x + 80}{x(x-2)} = 1$$

$$\Rightarrow \quad \frac{80}{x^2 - 2x} = 1$$

$$\Rightarrow \quad x^2 - 2x - 80 = 0$$

$$\Rightarrow \quad x^2 + 8x - 10x - 80 = 0$$

$$\Rightarrow \quad x(x + 8) - 10(x + 8) = 0$$

$$\Rightarrow \quad (x + 8)(x - 10) = 0$$

$\therefore$ Either $x + 8 = 0$ ...(1)

or, $x - 10 = 0$ ...(2)

From (1), $x = -8$ which is rejected, since x cannot be negative.

$\therefore$ From (2), we have $x = 10$ which is accepted.

Hence, he worked for 10 days at the usual wage and so he worked for $10 - 2 = 8$ days at higher wage.

$\therefore$ The required number of days a worker worked at a higher wage is 8 days.

32. The ratio of the areas of a rectangle and a square is 24 : 25. If the perimeter of each is 80 units, find the dimensions of the rectangle.

Sol. Let the length of the rectangle be x cm and the breadth be y cm so that $x > y > 0$.

$\therefore$ The area of the rectangle $= xy$ cm^2 and perimeter of the rectangle $= 2(x + y) = 80$ [Given]

$$\Rightarrow \quad x + y = 40$$

$$\Rightarrow \quad y = 40 - x \quad \text{...(1)}$$

If a cm be the side of the square, then its perimeter $= 4a$ cm.

$$\therefore \quad 4a = 80 \quad \text{[Given]}$$

$$\Rightarrow \quad a = \frac{80}{4} = 20$$

Hence, the side of the square is 20 cm and so the area of the square $= a^2$ cm^2 $= 20^2$ cm^2 $= 400$ cm^2

$$\therefore \quad \frac{\text{Area of the rectangle}}{\text{Area of the square}} = \frac{24}{25}$$

$$\Rightarrow \quad \frac{xy}{400} = \frac{24}{25} \quad \text{[Given]}$$

$$\Rightarrow \quad 400 \times 24 = 25xy$$

$$\Rightarrow \quad xy = \frac{450 \times 24}{25}$$

$$= 16 \times 24$$

$$= 384 \quad \text{...(2)}$$

$$\Rightarrow \quad x(40 - x) = 384 \quad \text{[From (1)]}$$

$$\Rightarrow \quad x^2 - 40x + 384 = 0$$

$$\Rightarrow \quad x^2 - 16x - 24x + 384 = 0$$

$$\Rightarrow \quad x(x - 16) - 24(x - 16) = 0$$

$$\Rightarrow \quad (x - 16)(x - 24) = 0$$

$\therefore$ Either $x - 16 = 0$ ...(1)

or, $x - 24 = 0$ ...(2)

From (1) and (2), we have $x = 24, 16$.

$\therefore$ From (1), $y = 16, 24$ when $x = 24, 16$ respectively.

$\because$ $x > y$, $\therefore$ $x = 24$, $y = 16$

Hence, the required length and breadth of the rectangle are 24 cm and 16 cm respectively.

33. The hypotenuse of a right-angled triangle is 50 cm and the longer of the other two sides, exceeds the shorter by 10 cm. Calculate

(*a*) the lengths of the sides

(*b*) area of the triangle

Sol. (*a*) Let the shorter side of the right angled triangle be x cm. Then the longer side of the triangle is $(x + 10)$ cm. Since, the hypotenuse of the triangle is 50 cm, hence by using Pythagoras theorem in this right angled triangle, we have

$$x^2 + (x + 10)^2 = 50^2$$

$$\Rightarrow \quad x^2 + x^2 + 20x + 100 = 2500$$

$$\Rightarrow \quad 2x^2 + 20x - 2400 = 0$$

$$\Rightarrow \quad x^2 + 10x - 1200 = 0$$

$$\Rightarrow \quad x^2 + 40x - 30x - 1200 = 0$$

$$\Rightarrow \quad x(x + 40) - 30(x + 40) = 0$$

$$\Rightarrow \quad (x + 40)(x - 30) = 0$$

$\therefore$ Either $x + 40 = 0$ ...(1)

or, $x - 30 = 0$ ...(2)

From (1), $x = -40$ which is rejected, since x is negative.

$\therefore$ From (2), $x = 30$ which is accepted.

$\therefore$ The required shorter side and the longer side of the triangle are 30 cm and (30 + 10) cm i.e. 40 cm respectively.

(*b*) Hence, the required area of the triangle is $\frac{1}{2} \times 30 \times 40$ cm^2 i.e. 600 cm^2.

34. A rectangular park is to be designed whose breadth is 3 m less than its length. Its area is to be 4 square metres more than the area of a park that has already been made in the shape of an isosceles triangle with its base as the breadth of the rectangular park and of altitude 12 m. Find the length and breadth of the rectangular park.

[CBSE 2016]

Sol. Let the length of the rectangle be x m. Then its breadth = $(x - 3)$ m.

$\therefore$ Its area = $x(x - 3)$ m^2.

The area of the isosceles triangle with base $(x - 3)$ m and altitude 12 m = $\frac{1}{2} \times 12 \times (x - 3)$ m^2

= $(6x - 18)$ m^2.

$\therefore$ According to the problem, we have

$$x(x - 3) = 6x - 18 + 4 = 6x - 14$$

$$\Rightarrow \quad x^2 - 3x - 6x + 14 = 0$$

$$\Rightarrow \quad x^2 - 9x + 14 = 0$$

$$\Rightarrow \quad x^2 - 7x - 2x + 14 = 0$$

$$\Rightarrow \quad x(x - 7) - 2(x - 7) = 0$$

$$\Rightarrow \quad (x - 2)(x - 7) = 0$$

$\therefore$ Either $\quad x - 2 = 0 \quad$ ...(1)

or, $\quad x - 7 = 0 \quad$ ...(2)

From (1) and (2), we have $x = 2, 7$

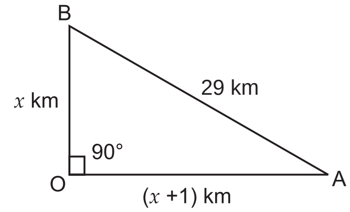
Since the length x m > 3 m, therefore, we reject $x = 2$.

So, we accept $x = 7$.

$\therefore$ The required length of the rectangle is 7 m and the breadth is (7 – 3) m i.e. 4 m.

35. Two ships leave simultaneously in directions at right angles to each other. The speed of one of them exceeds the other by 1 km per hour. The distance between the ships after 1 hour is 29 km. Find their speeds.

Sol. Let the slower speed be x km/h and the faster speed is $(x + 1)$ km/h. Let after 1 hour two ships S_1 and S_2 moving along OA and OB, where OA is perpendicular to OB, come to B and A respectively.



$\therefore$ OA = $(x + 1)$ km, OB = x km. It is given that AB = 29 km.

$\because \quad \angle AOB = 90°$,

$\therefore$ By using Pythagoras theorem, we get

$$x^2 + (x + 1)^2 = 29^2$$

$$\Rightarrow \quad 2x^2 + 2x + 1 - 841 = 0$$

$$\Rightarrow \quad 2x^2 + 2x - 840 = 0$$

$$\Rightarrow \quad x^2 + x - 420 = 0$$

$$\Rightarrow \quad x^2 + 21x - 20x - 420 = 0$$

$$\Rightarrow \quad x(x + 21) - 20(x + 21) = 0$$

$$\Rightarrow \quad (x + 21)(x - 20) = 0$$

$\therefore$ Either $\quad x + 21 = 0 \quad$ ...(1)

or, $\quad x - 20 = 0 \quad$ ...(2)

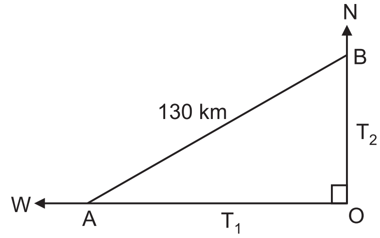
From (1), $x = -21$ which is rejected, since x cannot be negative.

$\therefore$ From (2), we get $x = 20$ which is accepted.

$\therefore$ The required slower speed is 20 km/h and the faster speed is (20 + 1) km/h i.e. 21 km/h.

36. Two trains leave a railway station at the same time. The first train travels due west and the second train due north. The first train travels 35 km/h faster than the second train. If after two hours, they are 130 km apart, find the average speed of each train. **[CBSE SP 2011]**

Sol. Let the two trains T_2 and T_1 move with speeds x km/h and $(x + 35)$ km/h towards north and west respectively. The speed of T_1 being more than that of T_2 by 35 km/h, starting from the same station O. After 2 hours let T_1 covers the distance OA and T_2 covers the distance OB.



Then $\quad OA = 2(x + 35)$ km

$= (2x + 70)$ km

and $\quad OB = 2x$ km

Given that AB = 130 km

$\because$ $\angle AOB = 90°$, hence by using Pythagoras theorem in ΔOAB, we get

$$(2x)^2 + (2x + 70)^2 = 130^2$$

$$\Rightarrow \quad 4x^2 + 4x^2 + 280x + 4900 - 16900 = 0$$

$$\Rightarrow \quad 8x^2 + 280x - 12000 = 0$$

$$\Rightarrow \quad x^2 + 35x - 1500 = 0$$

$$\Rightarrow \quad x^2 + 60x - 25x - 1500 = 0$$

$$\Rightarrow \quad x(x + 60) - 25(x + 60) = 0$$

$$\Rightarrow \quad (x + 60)(x - 25) = 0$$

$\therefore$ Either $\quad x + 60 = 0 \quad \ldots(1)$

or, $\quad x - 25 = 0 \quad \ldots(2)$

From (1), $x = -60$ which is rejected, since x is negative.

$\therefore$ From (2), $x = 25$ which is accepted.

Hence, the required speeds are 25 km/h and (25 + 35) km/h i.e. 60 km/h.

37. A boat takes 2 hour longer to go 30 km up a river than to go the same distance down the river. Calculate the rate at which the boat travels in still water, given that the river is flowing at 2 km/hour.

Sol. Let the speed of the boat in still water be x km/h. Then, the speed of the boat in favour of the current = Speed of the boat in still water + Speed of the current = $(x + 2)$ km/h.

Also, the speed of the boat against the current = Speed of the boat in still water – Speed of the current = $(x - 2)$ km/h.

$\therefore$ Time taken by the boat to travel a distance of 30 km in favour of the current is $\frac{30}{x+2}$ h and that against the current is $\frac{30}{x-2}$ h.

$\therefore$ According to the problem, we have

$$\frac{30}{x-2} - \frac{30}{x+2} = 2$$

$$\Rightarrow \quad \frac{15}{x-2} - \frac{15}{x+2} = 1$$

$$\Rightarrow \quad \frac{15x + 30 - 15x + 30}{(x+2)(x-2)} = 1$$

$$\Rightarrow \quad \frac{60}{x^2 - 4} = 1$$

$$\Rightarrow \quad x^2 - 4 = 60$$

$$\Rightarrow \quad x^2 = 60 + 4 = 64$$

$$\therefore \quad x = \pm\sqrt{64} = \pm 8$$

$\because$ x cannot be negative, we take $x = 8$.

Hence, the required speed of the boat in still water is 8 km/h.

38. Two pipes running together can fill a tank in $11\frac{1}{9}$ minutes. If one pipe takes 5 minutes more than the other to fill the tank separately, find the time in which each pipe would fill the tank separately. **[CBSE 2016]**

Sol. Let the pipe of smaller diameter can fill the tank in x minutes and that of bigger diameter can fill the tank in y minutes. Then clearly, $x > y > 0$.

Then according to the problem, we have

$$x = y + 5$$

$$\Rightarrow \quad y = x - 5 \quad \ldots(1)$$

and $$\frac{1}{x} + \frac{1}{y} = \frac{1}{11\frac{1}{9}} = \frac{9}{100} \quad \ldots(2)$$

$\therefore$ From (1) and (2), we have

$$\frac{1}{x} + \frac{1}{x-5} = \frac{9}{100}$$

$$\Rightarrow \quad \frac{x - 5 + x}{x(x-5)} = \frac{9}{100}$$

$$\Rightarrow \quad \frac{2x - 5}{x^2 - 5x} = \frac{9}{100}$$

$$\Rightarrow \quad 9x^2 - 45x = 200x - 500$$

$$\Rightarrow \quad 9x^2 - 245x + 500 = 0$$

$$\therefore \quad x = \frac{245 \pm \sqrt{245^2 - 4 \times 9 \times 500}}{2 \times 9}$$

$$= \frac{245 \pm \sqrt{60025 - 18000}}{18}$$

$$= \frac{245 \pm \sqrt{42025}}{18}$$

$$= \frac{245 \pm 205}{18}$$

$$= \frac{245 + 205}{18} \text{ or } \frac{245 - 205}{18}$$

$$= \frac{450}{18} \text{ or } \frac{40}{18}$$

$$= 25 \text{ or } \frac{20}{9}$$

When $x = 25$, $y = 25 - 5 = 20$ [From (1)]

When $x = \frac{20}{9}$, $y = \frac{20}{9} - 5 = -\frac{25}{9}$ which is rejected, since y cannot be negative.

Hence, the required time in which each pipe would fill the tank separately is 20 minutes and 25 minutes.

39. To fill a swimming pool two pipes are to be used. If the pipe of larger diameter is used for 4 hours and the pipe of smaller diameter for 9 hours, only half the pool can be filled. Find how long it would take for each pipe to fill the pool separately, if the pipe of smaller diameter takes 10 hours more than the pipe of larger diameter to fill the pool. **[CBSE 2015]**

Sol. Let the pipe of larger diameter take x hours to fill the pool. Then the pipe of smaller diameter will take $(x + 10)$ hours to fill the same pool.

$\therefore$ According to the problem, we have

$$\frac{4}{x} + \frac{9}{x+10} = \frac{1}{2}$$

$$\Rightarrow \quad \frac{4x + 40 + 9x}{x(x+10)} = \frac{1}{2}$$

$$\Rightarrow \quad \frac{13x + 40}{x^2 + 10x} = \frac{1}{2}$$

$$\Rightarrow \quad x^2 + 10x = 26x + 80$$

$$\Rightarrow \quad x^2 - 16x - 80 = 0$$

$$\Rightarrow \quad x^2 + 4x - 20x - 80 = 0$$

$$\Rightarrow \quad x(x + 4) - 20(x + 4) = 0$$

$$\Rightarrow \quad (x + 4)(x - 20) = 0$$

$\therefore$ Either $\quad x + 4 = 0 \quad$ ...(1)

or, $\quad x - 20 = 0 \quad$ ...(2)

From (1), $x = -4$ which is rejected, since x cannot be negative.

From (2), $x = 20$ which is accepted.

Hence, the pipe of larger diameter takes 20 hours to fill the pool and that of smaller diameter takes (20 + 10) hours = 30 hours to fill the same pool.

Hence, the required time taken by each pipe to fill the pool is 20 hours and 30 hours.

40. A pool is filled by three pipes with uniform flow. The first two pipes operating simultaneously, fill the pool in the same time during which the pool is filled by the third pipe alone. The second pipe fills the pool five hours faster than the first pipe and four hours slower than the third pipe. Find the time required by each pipe to fill the pool separately. **[CBSE SP 2011]**

Sol. Let the first pipe fill the pool in x hours. Then according to the problem, the second pipe takes 5 hours less than the first to fill the pool. Hence, the second pipe fills the pool in $(x - 5)$ hours. Also, the second pipe takes 4 hours more than the third pipe. In other words, the third pipe takes 4 hours less than the second pipe. Hence, the third pipe fills the pool in $(x - 5 - 4)$ hours i.e. in $(x - 9)$ hours.

Now, according to the problem, we have

$$\frac{1}{x} + \frac{1}{x-5} = \frac{1}{x-9}$$

$$\Rightarrow \quad \frac{x - 5 + x}{x(x-5)} = \frac{1}{x-9}$$

$$\Rightarrow \quad \frac{2x - 5}{x^2 - 5x} = \frac{1}{x-9}$$

$$\Rightarrow \quad x^2 - 5x = (x - 9)(2x - 5)$$

$$\Rightarrow \quad x^2 - 5x = 2x^2 - 23x + 45$$

$$\Rightarrow \quad x^2 - 18x + 45 = 0$$

$$\Rightarrow \quad x^2 - 3x - 15x + 45 = 0$$

$$\Rightarrow \quad x(x - 3) - 15(x - 3) = 0$$

$$\Rightarrow \quad (x - 3)(x - 15) = 0$$

$\therefore$ Either $\quad x - 3 = 0 \quad$ ...(1)

or, $\quad x - 15 = 0 \quad$ ...(2)

From (1), $x = 3$ which is rejected, since in this case $x - 5$ becomes negative. Hence, from (2), $x = 15$ which is accepted.

Hence, the required times are 15 hours, (15 – 5) hours i.e. 10 hours and (15 – 9) hours i.e. 6 hours.

41. One-fourth of a herd of camels was seen in the forest. Twice the square root of the herd had gone to the mountains and 15 camels were on the bank of the river. Find the number of camels.

Sol. Let the total number of camels be x.

Then according to the problem, we have

$$\frac{x}{4} + 2\sqrt{x} + 15 = x$$

$$\Rightarrow \quad x + 8\sqrt{x} + 60 = 4x$$

$$\Rightarrow \quad 8\sqrt{x} = 3x - 60$$

On squaring both sides

$$\Rightarrow \quad 64x = (3x - 60)^2$$

$$\Rightarrow \quad 64x = 9x^2 + 3600 - 360x$$

$$\Rightarrow \quad 9x^2 - 424x + 3600 = 0$$

$$\therefore \quad x = \frac{424 \pm \sqrt{(424)^2 - 4 \times 9 \times 3600}}{18}$$

Now, $(424)^2 - 4 \times 9 \times 3600$

$$= (424)^2 - 36 \times 3600$$

$$= (424)^2 - (6 \times 60)^2$$

$$= (424)^2 - (360)^2$$
$$= (424 + 360)(424 - 360)$$
$$= 784 \times 64$$

$\therefore \sqrt{424^2 - 4 \times 9 \times 3600}$
$$= \sqrt{784}\ \sqrt{64}$$
$$= 28 \times 8$$
$$= 224$$

$\therefore x = \dfrac{424 \pm 224}{18} = \dfrac{648}{18}, \dfrac{200}{18} = 36, \dfrac{100}{9}$

We reject $x = \dfrac{100}{9}$, since x cannot be a fraction.

$\therefore x = 36$

Hence, the required number of camels are 36.

Higher Order Thinking Skills (HOTS) Questions

(Page 75)

1. Solve: $4x^2 - 4a^2x + (a^4 - b^4) = 0$ **[CBSE 2004, SP 2011]**

Sol. We have $\quad 4x^2 - 4a^2x + a^4 - b^4 = 0$

$$4x^2 - 4a^2x + (a^2)^2 - (b^2)^2 = 0$$

$\Rightarrow \quad (2x)^2 - 2 \times 2x \times a^2 + (a^2)^2 - (b^2)^2 = 0$

$\Rightarrow \quad (2x - a^2)^2 - b^2 = 0$

$\Rightarrow \quad (2x - a^2 + b^2)(2x - a^2 - b^2) = 0$

$\therefore$ Either $\quad 2x - a^2 + b^2 = 0 \quad \ldots(1)$

or, $\quad 2x - a^2 - b^2 = 0 \quad \ldots(2)$

From (1), $x = \dfrac{a^2 - b^2}{2}$ and from (2), $x = \dfrac{a^2 + b^2}{2}$

$\therefore$ The required solutions are $\dfrac{a^2 - b^2}{2}$ and $\dfrac{a^2 + b^2}{2}$.

2. Solve: $\dfrac{1}{a+b+x} = \dfrac{1}{a} + \dfrac{1}{b} + \dfrac{1}{x}$; $a \neq 0, b \neq 0, x \neq 0$

[CBSE 2005, 2017, SP 2011]

Sol. We have $\dfrac{1}{p+x} = \dfrac{(a+b)x + ab}{abx} = \dfrac{px + q}{qx}$, where

$$p = a + b \quad \ldots(1)$$

and $\quad q = ab \quad \ldots(2)$

$\Rightarrow \quad \dfrac{1}{p+x} = \dfrac{px+q}{qx}$

$\Rightarrow \quad p^2x + pq + px^2 + qx = qx$

$\Rightarrow \quad px^2 + p^2x + pq = 0$

$\Rightarrow \quad x^2 + px + q = 0$

$\Rightarrow \quad x^2 + (a+b)x + ab = 0$ [From (1) and (2)]

$\Rightarrow \quad x^2 + ax + bx + ab = 0$

$\Rightarrow \quad x(x + a) + b(x + a) = 0$

$\Rightarrow \quad (x + a)(x + b) = 0$

$\therefore$ Either $\quad x + a = 0 \quad \ldots(3)$

or, $\quad x + b = 0 \quad \ldots(4)$

$\therefore$ From (3) and (4), we have $x = -a, -b$ which are the required solutions.

3. By using the method of completing the square, show that the equation $2x^2 + x + 5 = 0$ has no real roots.

Sol. We have $\quad 2x^2 + x + 5 = 0$

$\Rightarrow \quad x^2 + \dfrac{1}{2}x + \dfrac{5}{2} = 0$

$\Rightarrow \quad x^2 + 2 \cdot \dfrac{1}{4}x + \left(\dfrac{1}{4}\right)^2 - \left(\dfrac{1}{4}\right)^2 + \dfrac{5}{2} = 0$

$\Rightarrow \quad \left(x + \dfrac{1}{4}\right)^2 + \dfrac{5}{2} - \dfrac{1}{16} = 0$

$\Rightarrow \quad \left(x + \dfrac{1}{4}\right)^2 = -\dfrac{39}{16}$

$\therefore \quad x + \dfrac{1}{4} = \pm\sqrt{-\dfrac{39}{16}}$

which is not real.

Hence, the given equation does not have any real roots.

4. In the following, determine value(s) of k for which the given quadratic equation has real roots:

$4x^2 - 3kx + 9 = 0$ **[CBSE SP 2013]**

Sol. We know that for real roots,

$$\text{discriminant, D} = (-3k)^2 - 4 \times 4 \times 9$$
$$= 9k^2 - 144$$

For real roots, we have $D \geq 0$

$\therefore \quad 9k^2 - 144 \geq 0$

$\Rightarrow \quad (3k)^2 - 12^2 \geq 0$

$\Rightarrow \quad (3k + 12)(3k - 12) \geq 0$

$\therefore$ Either $\quad 3k + 12 \geq 0 \quad \ldots(1)$

and $\quad 3k - 12 \geq 0 \quad \ldots(2)$

or $\quad 3k + 12 \leq 0 \quad \ldots(3)$

and $\quad 3k - 12 \leq 0 \quad \ldots(4)$

From (1) and (2), we have

$$k \geq -\frac{12}{3} = -4$$

and $\quad k \geq \dfrac{12}{3} = 4$

$$\Rightarrow \qquad k \geq 4 \qquad \ldots(5)$$

From (3) and (4), we have

$$k \leq \frac{-12}{3} = -4$$

and $\qquad k \leq \frac{12}{3} = 4$

$$\Rightarrow \qquad k \leq -4 \qquad \ldots(6)$$

Hence, from (5) and (6), the required values of k are either $k \geq 4$ or $k \leq -4$.

5. A dealer sells an article for ₹ 75 and gains as much per cent as the cost price of the article. What is the cost price of the article?

Sol. Let the cost price of the article be ₹ x.

The selling price of the article = ₹ 75

$\therefore$ Gain = ₹ $(75 - x)$ and x% on CP

$\therefore$ According to the problem, we have

$$\Rightarrow \qquad 75 - x = \frac{x^2}{100}$$

$$\Rightarrow \qquad x^2 = 7500 - 100x$$

$$\Rightarrow \qquad x^2 + 100x - 7500 = 0$$

$$\Rightarrow \qquad x^2 + 150x - 50x - 7500 = 0$$

$$\Rightarrow \qquad x(x + 150) - 50(x + 150) = 0$$

$$\Rightarrow \qquad (x + 150)(x - 50) = 0$$

$\therefore$ Either $\qquad x + 150 = 0 \qquad \ldots(1)$

or, $\qquad x - 50 = 0 \qquad \ldots(2)$

From (1), $x = -150$ which is rejected, since x cannot be negative.

$\therefore$ From (2), $x = 50$ which is accepted.

$\therefore$ The required cost price is ₹50.

6. A factory kept increasing its output by the same percentage every year. Find the percentage if it is known that the output is doubled in the last two years. **[CBSE SP 2011]**

Sol. Let the initial output be ₹y and the required percentage increase be x %. Then the output for 2 years = ₹$y\left(1 + \frac{x}{100}\right)^2$

$\therefore$ According to the problem, we have

$$y\left(1 + \frac{x}{100}\right)^2 = 2y$$

$$\Rightarrow \qquad 1 + \frac{x}{100} = \pm\sqrt{2}$$

$$\Rightarrow \qquad x = 100(\pm\sqrt{2} - 1)$$

But x cannot be negative. So, we reject $100(-1 - \sqrt{2})$ and accept $x = 100(\sqrt{2} - 1)$.

$\therefore$ The required percentage is $100(\sqrt{2} - 1)$%.

7. Natasha is x years old and her mother is x^2 years old. When her mother becomes $11x$ years old, Natasha becomes x^2 years. Find their present ages.

Sol. Natasha's mother will be $11x$ years after $(11x - x^2)$ years.

After $(11x - x^2)$ years, Natasha's age will be $(x + 11x - x^2)$ years = $(12x - x^2)$ years = $x(12 - x)$ years.

$\therefore$ According to the problem, we have

$$x(12 - x) = x^2$$

$$\Rightarrow \qquad 12 - x = x$$

$$\Rightarrow \qquad 2x = 12$$

$$\Rightarrow \qquad x = 6$$

$\therefore$ The required present ages of Natasha and her mother are 6 years and 6^2 years i.e. 36 years respectively.

8. The sum of the areas of two squares is 400 cm^2. If the difference of their perimeters is 16 cm, find the sides of the two squares. **[CBSE 2013]**

Sol. Let the sides of the two squares be x cm and y cm, where $x > y$.

$\therefore$ According to the problem, we have

$$x^2 + y^2 = 400 \qquad \ldots(1)$$

and $\qquad 4x - 4y = 16$

$$\Rightarrow \qquad x - y = 4$$

$$y = x - 4 \qquad \ldots(2)$$

Now, $\qquad x^2 + (x - 4)^2 = 400$

[From (1) and (2)]

$$\Rightarrow \qquad x^2 + x^2 - 8x + 16 - 400 = 0$$

$$\Rightarrow \qquad 2x^2 - 8x - 384 = 0$$

$$\Rightarrow \qquad x^2 - 4x - 192 = 0$$

$$\Rightarrow \qquad x^2 + 12x - 16x - 192 = 0$$

$$\Rightarrow \qquad x(x + 12) - 16(x + 12) = 0$$

$$\Rightarrow \qquad (x + 12)(x - 16) = 0$$

$\therefore$ Either $\qquad x + 12 = 0 \qquad \ldots(3)$

or, $\qquad x - 16 = 0 \qquad \ldots(4)$

From (3), $x = -12$ which is rejected, since x cannot be negative.

From (4), $x = 16$ which is accepted.

$\therefore$ From (2), $\qquad y = 16 - 4 = 12$

Hence, the required sides of the two squares are 16 cm and 12 cm.

9. The perimeter of a right-angled triangle is four times the length of the shortest side. The numerical value of the area of the triangle is eight times the numerical value of the length of the shortest side. Find the lengths of the three sides of the triangle.

Sol. Let the length of the shortest side be x cm and the length of the bigger side of the right angled triangle be y cm.

Then the length of its hypotenuse is $\sqrt{x^2 + y^2}$ cm

[By Pythagoras theorem]

Now, according to the problem, the perimeter of the triangle = $4x$

$$\therefore \quad 4x = x + y + \sqrt{x^2 + y^2}$$

$$\Rightarrow \quad 3x = y + \sqrt{x^2 + y^2}$$

$$\Rightarrow \quad 3x - y = \sqrt{x^2 + y^2} \quad \ldots(1)$$

Also, area of the triangle = $8x$

According to the problem, we have

$$8x = \frac{1}{2}xy$$

$$\Rightarrow \quad y = 16 \quad \ldots(2)$$

$\therefore$ From (1) and (2), we get

$$(3x - 16)^2 = x^2 + 256$$

$$\Rightarrow \quad 9x^2 + 256 - 96x = x^2 + 256$$

$$\Rightarrow \quad 8x^2 = 96x$$

$$\Rightarrow \quad x = \frac{96}{8} \quad [\because x \neq 0]$$

$$\Rightarrow \quad x = 12 \quad \ldots(3)$$

$$\therefore \quad \sqrt{x^2 + y^2} = \sqrt{256 + 144}$$

$$= \sqrt{400} = 20$$

Hence, the required lengths of the sides of the triangle are 12 units, 16 units and 20 units.

10. A dealer sells a pen for ₹ 24 and gains as much per cent as the cost price of the pen. Find the cost price of the pen.

Sol. Let the cost price of the pen be ₹ x.

The selling price of the pen = ₹ 24

$$\therefore \quad \text{Gain} = ₹\ (24 - x)$$

$\therefore$ According to the problem, we have

$$24 - x = x\ \%\ \text{of}\ x = \frac{x^2}{100}$$

$$\Rightarrow \quad x^2 + 100x - 2400 = 0$$

$$\Rightarrow \quad x^2 + 120x - 20x - 2400 = 0$$

$$\Rightarrow \quad x(x + 120) - 20(x + 120) = 0$$

$$\Rightarrow \quad (x + 120)(x - 20) = 0$$

$\therefore$ Either $\quad x + 120 = 0 \quad \ldots(1)$

or, $\quad x - 20 = 0 \quad \ldots(2)$

From (1), $x = -120$, which is rejected, since x cannot be negative.

$\therefore$ From (2), we have, $x = 20$ which is accepted.

$\therefore$ The required cost price is ₹ 20.

Self-Assessment

(Page 76)

Multiple-Choice Questions

1. The discriminant of the quadratic equation $2x^2 - 5x - 3 = 0$ is

(*a*) 1 (*b*) 49

(*c*) 7 (*d*) 19 **[CBSE 2023 Basic]**

Sol. (*b*) 49

The quadratic equation is $2x^2 - 5x - 3 = 0$.

$$a = 2,\ b = -5,\ c = -3$$

$$\text{Discriminant, D} = b^2 - 4ac$$

$$= (-5)^2 - 4(2)(-3)$$

$$= 25 + 24$$

$$\text{D} = 49$$

2. The quadratic equation $x^2 - x - 2 = 0$ has roots which are

(*a*) real and equal

(*b*) real, unequal and rational

(*c*) real, unequal and irrational

(*d*) not real

Sol. (*b*) real, unequal and rational

We see that the discriminant,

$$\text{D} = (-1)^2 - 4 \times 1 \times (-2)$$

$$= 1 + 8$$

$$= 9 > 0$$

$$\because \quad \text{D} > 0$$

$\therefore$ The roots are real and unequal.

Also, since $\sqrt{\text{D}} = \sqrt{9} = 3$ which is a rational number, hence the roots are also rational.

3. The value of k for which $x^2 + 4x + k = 0$ has real roots, is

(*a*) $k \geq 4$ (*b*) $k \leq 4$

(*c*) $k \geq -4$ (*d*) $k \leq -4$

[CBSE 2023 Basic]

Sol. (*b*) $k \leq 4$

For real roots, discriminant, $D \geq 0$

$\Rightarrow \quad 4^2 - 4 \times 1 \times k \geq 0$

$\Rightarrow \quad 16 - 4k \geq 0$

$\Rightarrow \quad 4k \leq 16$

$\Rightarrow \quad k \leq 4$

4. If the equation $9x^2 + bx + \frac{1}{4} = 0$ has two equal roots, then the value of b is

(*a*) 0 (*b*) –3 only

(*c*) 3 only (*d*) ±3 **[CBSE 2023 Basic]**

Sol. (*d*) ±3

We have the discriminant,

$$D = b^2 - 4 \times 9 \times \frac{1}{4}$$
$$= b^2 - 9$$

Now, for real and equal roots, $D = 0$

$\therefore \quad b^2 - 9 = 0$

$\Rightarrow \quad b^2 = 9$

$\Rightarrow \quad b = \pm 3$

5. If one root of the quadratic equation $2x^2 + px + 4 = 0$, is 2, then the other root and p are respectively

(*a*) 6, – 1 (*b*) – 1, – 6

(*c*) – 1, 6 (*d*) 1, – 6

Sol. (*d*) 1, –6

Since $x = 2$ is a root, hence

$2 \times 2^2 + p \times 2 + 4 = 0$

$\Rightarrow \quad 12 + 2p = 0$

$\Rightarrow \quad p = -6$

If the other root is α, then

$$\text{Sum of the roots} = -\frac{p}{2} = +\frac{6}{2} = 3$$

$\therefore \quad \alpha + 2 = 3$

$\Rightarrow \quad \alpha = 3 - 2 = 1$

$\therefore$ The other root α is 1.

6. If m and n are roots of the equation $x^2 + mx + n = 0$, then

(*a*) $m = 2, n = 1$ (*b*) $m = -2, n = -1$

(*c*) $m = 1, n = -2$ (*d*) $m = -1, n = 2$

Sol. (*c*) $m = 1, n = -2$

$\because$ m and n are the two roots of the equation

$x^2 + mx + n = 0$

$\therefore \quad m + n = -m$

$\Rightarrow \quad 2m + n = 0 \quad \ldots(1)$

Also, $\quad mn = n$

$\Rightarrow \quad m = 1$, assuming $n \neq 0$.

$\therefore$ From (1), $\quad 2 + n = 0$

$\Rightarrow \quad n = -2$

$\therefore$ The required values of m and n are 1 and –2 respectively.

7. If the sum of the roots of the equation $x^2 - x = p(2x - 1)$ is zero, then

(*a*) $p = -2$ (*b*) $p = 2$

(*c*) $p = -\frac{1}{2}$ (*d*) $p = \frac{1}{2}$

Sol. (*c*) $p = -\frac{1}{2}$

The given equation is

$x^2 - x - 2px + p = 0$

$\Rightarrow \quad x^2 - (1 + 2p)x + p = 0$

$\therefore \quad$ Sum of the roots $= 1 + 2p$

It is given that this sum is zero.

$\therefore \quad 1 + 2p = 0$

$\Rightarrow \quad p = -\frac{1}{2}$

8. A quadratic equation whose one root is 2 and the sum of the roots is 0, is

(*a*) $x^2 + 4 = 0$ (*b*) $x^2 - 2 = 0$

(*c*) $4x^2 - 1 = 0$ (*d*) $x^2 - 4 = 0$

[CBSE 2023 Basic]

Sol. (*d*) $x^2 - 4 = 0$

Sum of the roots = 0 and one root is 2. Hence, the other root must be –2.

$\therefore$ The required equation is x^2 – (sum of the roots)x + product of the roots = 0

$\Rightarrow \quad x^2 - 2 \times 2 = 0$

$\Rightarrow \quad x^2 - 4 = 0$

9. If $x = 0.3$ is a root of the equation, $x^2 - 0.9k = 0$, then k is equal to

(*a*) 1 (*b*) 10

(*c*) 0.1 (*d*) 100

[CBSE 2023 Standard]

Sol. (*c*) 0.1

$x = 0.3$ is a root of the equation $x^2 - 0.9k = 0$.

$\Rightarrow \quad (0.3)^2 - 0.9k = 0$

$\Rightarrow \quad 0.09 - 0.9k = 0$

$\Rightarrow \quad -0.9k = -0.09$

$\Rightarrow \quad k = \dfrac{0.09}{0.9} = 0.1$

10. If the roots of equation $ax^2 + bx + c = 0$, $a \neq 0$ are real and equal, then which of the following relation is true?

(*a*) $a = \dfrac{b^2}{c}$ (*b*) $b^2 = ac$

(*c*) $ac = \dfrac{b^2}{4}$ (*d*) $c = \dfrac{b^2}{a}$

[CBSE 2024 Standard]

Sol. (*c*) $ac = \dfrac{b^2}{4}$

For real and equal roots,

$$b^2 - 4ac = 0$$

$$\Rightarrow \quad ac = \frac{b^2}{4}$$

Fill in the Blanks

11. If the discriminant of a quadratic equation is **positive** then it has real and unequal roots.

12. Quadratic equation whose roots are 1 and – 2 is $\boldsymbol{x^2 + x - 2 = 0}$.

13. If the roots of the equation $x^2 - kx + p = 0$ differ by one, then $k^2 - 4p$ is equal to **1**.

14. Product of the roots of quadratic equation $x^2 - 9x + 18 = 0$ is **double** the sum of its roots.

Sol. $x^2 - 9x + 18 = 0$

Sum of the roots $= \dfrac{-b}{a} = \dfrac{-(-9)}{1} = 9$

Product of the roots $= \dfrac{c}{a} = \dfrac{18}{1} = 18$

$\therefore$ Product of the roots = 2 (Sum of roots)

Assertion-Reason Type Questions

Directions (Q. Nos. 15 to 19): Each of these questions contains an assertion followed by reason. Read them carefully, and answer the question on the basis of the following options, select the one that best describes the two statements.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

15. Assertion (A): $ax^2 + bx + c = 0$ is not a quadratic equation if $a = 0$.

Reason (R): If $a = 0$, then there will be no term containing the 2nd power of x.

Sol. The correct answer is (*a*) both the statements are corrrect. A quadratic polynomial must have a non-zero coeffcient of x^2.

Thus, the reason is a proper explanation of assertion.

16. Assertion (A): The roots of the equation $2x^2 - 5x + 3 = 0$ and the zeroes of the polynomial $2x^2 - 5x + 3$ are the same.

Reason (R): Zeroes of a polynomial satisfy the corresponding equation.

Sol. The correct answer is (*a*) Both the statements are correct since the zeroes of a polynomial can satisfy the corresponding equation, hence they are the solutions also.

17. Assertion (A): $2x^2 + x - 528 = 0$ has one real and one imaginary roots.

Reason (R): The constant term is negative and the coefficient of x^2 is positive.

Sol. The correct answer is (*d*). The assertion is wrong because a quadratic equation cannot have one real and one imaginary root. The reason is correct.

18. Assertion (A): If one root of the quadratic equation $4x^2 - 10x + (k - 4) = 0$ is reciprocal of the other, then the value of k is 8.

Reason (R): Roots of the quadratic equation $x^2 - x + 1 = 0$ are real. **[CBSE 2023 Basic]**

Sol. (*c*) Assertion (A) is true but Reason (R) is false.

$$4x^2 - 10x + (k - 4) = 0$$

Let the roots be α and $\dfrac{1}{\alpha}$

Product of the roots $= \alpha \times \dfrac{1}{\alpha} = 1$

But Product of the roots $= \dfrac{c}{a}$

$$= \frac{k-4}{4}$$

$$\Rightarrow \quad \frac{k-4}{4} = 1$$

$$\Rightarrow \quad k = 8$$

Therefore, Assertion (A) is correct.

The quadratic equation is $x^2 - x + 1 = 0$

Discriminant, $D = (-1)^2 - 4(1)(1)$

$\Rightarrow$ $D = 1 - 4$

$\Rightarrow$ $D = -3$

$\therefore$ $D < 0$, no real roots are possible Reason (R) is false.

19. **Assertion (A):** If $5 + \sqrt{7}$ is a root of a quadratic equation with rational coefficients, then its other root is $5 - \sqrt{7}$.

Reason (R): Surd roots of a quadratic equation with rational coefficient occur in conjugate pairs.
[CBSE 2023 Standard]

Sol. (*a*) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

Case Study Based Questions

20. Ankit had bought a number of books from the bookstore for ₹ 640. Also, he had bought same number of books with 8 more books for the same amount, each book would have cost ₹ 4 less. Based on the above information, answer the following questions.

(*a*) If the number of books bought be x, then what is the number of books bought later?

Ans. $x + 8$

(*b*) What is the cost of each book if the number of books be 16 and the amount paid is ₹ 80?

Ans. ₹ 5

(*c*) (*i*) If same number of books with 8 more books are bought for ₹ 640, then what is the reduction in cost of each book?

Ans. ₹ 4

or

(*ii*) What is the number of books bought initially?

Ans. 32

21. An aircraft is a vehicle such as a plane or a helicopter that can fly and carry goods or passengers. An aircraft was slowed down due to bad weather. In a flight of 600 km, its average speed for the trip was reduced by 200 km/h and the time of the flight increased by 30 minutes. Based on the above situation, answer the following questions.

(*a*) If the original speed of the aircraft be x km/h, then what will be its reduced speed?

Ans. $(x - 200)$ km/h

(*b*) Form the quadratic equation for the average speed of the aircraft.

Ans. $x^2 - 200x - 240000 = 0$

(*c*) (*i*) What is the average speed of the aircraft?

Ans. 600 km/h

or

(*ii*) What is the duration of the flight?

Ans. 1 hour

22. A rectangular floor area can be completely tiled with 200 square tiles. If the side length of each tile is increased by 1 unit, it would take only 128 tiles to cover the floor.

(*a*) Assuming the original length of each side of a tile be x units, make a quadratic equation from the above information.

Ans. Let the original length of each tile = x units.

Area of the floor = $200x^2$ ...(1)

Also, Area of the floor = $128(x + 1)^2$ ...(2)

From (1) and (2)

$$200x^2 = 128(x + 1)^2$$

(*b*) Write the corresponding quadratic equation in standard form.

Ans. $$200x^2 = 128(x + 1)^2$$

$$\Rightarrow \quad 25x^2 = 16(x^2 + 2x + 1)$$

$$\Rightarrow \quad 25x^2 = 16x^2 + 32x + 16$$

$$\Rightarrow \quad 9x^2 - 32x - 16 = 0,$$

which is the standard form of the equation.

(*c*) (*i*) Find the value of x, the length of side of a tile by factorisation.

Ans. We have,

$$\Rightarrow \quad 9x^2 - 32x - 16 = 0$$

$$\Rightarrow \quad 9x^2 - 36x + 4x - 16 = 0$$

$$\Rightarrow \quad 9x(x - 4) + 4(x - 4) = 0$$

$$\Rightarrow \quad (9x + 4)(x - 4) = 0$$

$$\Rightarrow \quad 9x + 4 = 0 \text{ or } x - 4 = 0$$

$$\Rightarrow \quad x = \frac{-4}{9} \text{ or } x = 4$$

Since length cannot be negative

$\therefore$ $x = 4$ units

or

(*ii*) Solve the quadratic equation for x, using quadratic formula. **[CBSE 2024 Standard]**

Ans. We have, $9x^2 - 32x - 16 = 0$

Here, $a = 9, b = -32, c = -16$

Using quadratic formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow \quad x = \frac{32 \pm \sqrt{(-32)^2 - 4(9)(-16)}}{2 \times 9}$$

$$\Rightarrow \quad x = \frac{32 \pm \sqrt{1024 + 576}}{18}$$

$$= \frac{32 \pm \sqrt{1600}}{18}$$

$$\Rightarrow \quad x = \frac{32 \pm 40}{18}$$

$$\Rightarrow \quad x = \frac{32 + 40}{18} \quad \text{or} \quad x = \frac{32 - 40}{18}$$

$$\Rightarrow \quad x = \frac{72}{18} \quad \text{or} \quad x = \frac{-8}{18}$$

$$\Rightarrow \quad x = 4 \quad \text{or} \quad x = \frac{-4}{9}$$

Very Short Answer Type Questions

Represent the following situations in the form of quadratic equations (Q. 23 – Q. 25).

23. A two-digit number is such that its ten's digit is the square of the unit's digit and the sum of digits is 12. We need to find the number.

Sol. Let the digit in the unit's place be x.

Then, the digit in the ten's place is x^2.

$\therefore$ According to the problem, we have

$$x^2 + x = 12$$

$$\Rightarrow \quad x^2 + x - 12 = 0$$

$$\Rightarrow \quad x^2 + 4x - 3x - 12 = 0$$

$$\Rightarrow \quad x(x + 4) - 3(x + 4) = 0$$

$$\Rightarrow \quad (x - 3)(x + 4) = 0$$

$\therefore$ Either $x - 3 = 0$ ...(1)

or, $x + 4 = 0$ ...(2)

From (1), we have, $x = 3$ which is accepted.

From (2), $x = -4$, which is rejected.

$\therefore$ The digit in the unit's place is 3 and the digit in the ten's place is 3^2 i.e. 9.

$\therefore$ The required number is 39.

24. The cost of $2x$ articles is ₹ $(5x + 54)$ while the cost of $(x + 2)$ articles is ₹ $(10x - 4)$. We need to find x.

Sol. The cost of $2x$ articles is ₹ $(5x + 54)$

Then, the cost of 1 article = ₹ $\frac{5x + 54}{2x}$

The cost of $(x + 2)$ articles is ₹ $(10x - 4)$

Then, the cost of 1 article = ₹ $\frac{10x - 4}{x + 2}$

$\therefore$ According to the problem, we have

$$₹\ \frac{5x + 54}{2x} = ₹\ \frac{10x - 4}{x + 2}$$

$$\Rightarrow \quad (5x + 54)(x + 2) = 2x(10x - 4)$$

$$\Rightarrow \quad 5x^2 + 64x + 108 = 20x^2 - 8x$$

$$\Rightarrow \quad 15x^2 - 72x - 108 = 0$$

$$\Rightarrow \quad 5x^2 - 24x - 36 = 0$$

$$\Rightarrow \quad 5x^2 - 30x + 6x - 36 = 0$$

$$\Rightarrow \quad 5x(x - 6) + 6(x - 6) = 0$$

$$\Rightarrow \quad (5x + 6)(x - 6) = 0$$

$\therefore$ Either $5x + 6 = 0$ ...(1)

or, $x - 6 = 0$ ...(2)

From (1), we have, $x = \frac{-6}{5}$, which is rejected.

From (2), we have, $x = 6$, which is accepted.

$\therefore$ The value of x is 6.

25. Had Neelu scored 10 more marks in her English test out of 30 marks, 9 times these marks would have been the square of her actual marks. We need to find how many marks did she get in the test.

Sol. Let Neelu secured x marks in English.

$\therefore$ According to the problem, we have

$$9(x + 10) = x^2$$
$$\Rightarrow \quad 9x + 90 = x^2$$
$$\Rightarrow \quad x^2 - 9x - 90 = 0$$
$$\Rightarrow \quad x^2 - 15x + 6x - 90 = 0$$
$$\Rightarrow \quad x(x - 15) + 6(x - 15) = 0$$
$$\Rightarrow \quad (x - 15)(x + 6) = 0$$

$\therefore$ Either $\quad x - 15 = 0 \quad \ldots(1)$

or, $\quad x + 6 = 0 \quad \ldots(2)$

From (1), we have, $x = 15$, which is accepted.

From (2), we have, $x = -6$ which is rejected, since x cannot be negative.

$\therefore$ Neelu get 15 marks in the test.

26. Find the discriminant of the quadratic equation $3x^2 - 2x + \frac{1}{3} = 0$ and hence find the nature of its roots. **[CBSE 2023 Basic]**

Sol. The quadratic equation is $3x^2 - 2x + \frac{1}{3} = 0$

Comparing it with standard form of the quadratic equation, $ax^2 + bx + c = 0$, we have

$$a = 3,\ b = -2,\ c = \frac{1}{3}$$

Discriminant, $\quad D = b^2 - 4ac$

$$\Rightarrow \quad D = (-2)^2 - 4(3)\left(\frac{1}{3}\right)$$
$$= 4 - 4 = 0$$

Hence, the roots are real and equal.

27. If α and β are roots of the quadratic equation $x^2 - 7x + 10 = 0$, find the quadratic equation whose roots are α^2 and β^2. **[CBSE 2023 Standard]**

Sol. α and β are roots of the quadratic equation

$$x^2 - 7x + 10 = 0$$
$$a = 1,\ b = -7,\ c = 10$$

Sum of the roots $= \dfrac{-b}{a}$

$$\alpha + \beta = -\frac{(-7)}{1}$$
$$\Rightarrow \quad \alpha + \beta = 7 \quad \ldots(1)$$

Product of the roots $= \dfrac{c}{a}$

$$\alpha\beta = \frac{10}{1}$$
$$\Rightarrow \quad \alpha\beta = 10 \quad \ldots(2)$$

We have,

$$(\alpha + \beta)^2 = \alpha^2 + \beta^2 + 2\alpha\beta$$
$$\Rightarrow \quad (7)^2 = \alpha^2 + \beta^2 + 2 \times 10$$

[From (1) and (2)]

$$\Rightarrow \quad 49 = \alpha^2 + \beta^2 + 20$$
$$\Rightarrow \quad \alpha^2 + \beta^2 = 49 - 20$$
$$\Rightarrow \quad \alpha^2 + \beta^2 = 29 \quad \ldots(3)$$
$$\Rightarrow \quad (\alpha\beta)^2 = (10)^2 \quad \text{[From (2)]}$$
$$\Rightarrow \quad \alpha^2\beta^2 = 100 \quad \ldots(4)$$

The quadratic equation will be

x^2 – (Sum of the root)x + Product of the roots = 0

$$\Rightarrow \quad x^2 - 29x + 100 = 0 \quad \text{[From (3) and (4)]}$$

Short Answer Type Questions

28. If $ax^2 - 7x + c = 0$ has 14 as the sum of roots and also as the product of roots, find the values of a and c.

Sol. Let α and β be the roots of the given equation. Then

$$\alpha + \beta = \frac{7}{a} = 14$$
$$\Rightarrow \quad a = \frac{7}{14} = \frac{1}{2}$$

and $\quad \alpha\beta = \dfrac{c}{a} = 14$

$$\Rightarrow \quad \frac{c}{\frac{1}{2}} = 14$$
$$\Rightarrow \quad c = \frac{14}{2} = 7$$

Hence, the required values of a and c are $\frac{1}{2}$ i.e. 0.5 and 7 respectively.

29. Find the value of p for which the root of the quadratic equation $px^2 - 14x + 8 = 0$ is six times the other. **[CBSE 2017]**

Sol. If α be one root of the given equation, then the other root is 6α.

$$\therefore \quad \alpha + 6\alpha = \frac{14}{p}$$
$$\Rightarrow \quad 7\alpha = \frac{14}{p} \quad \ldots(1)$$

and $\quad \alpha \cdot 6\alpha = \dfrac{8}{p}$

$$\Rightarrow \quad 3\alpha^2 = \frac{4}{p} \quad \ldots(2)$$

From (1), $\alpha = \frac{2}{p}$

$$\Rightarrow \quad \alpha^2 = \frac{4}{p^2}$$

$\therefore$ From (2), $3 \times \frac{4}{p^2} = \frac{4}{p}$

$$\Rightarrow \quad p^2 = 3p$$

$$\Rightarrow \quad p = 3 \quad [\because p \neq 0]$$

$\therefore$ The required value of p is 3.

30. If the roots of the equation $(b - c)x^2 + (c - a)x + (a - b) = 0$ are equal, then prove that $2b = a + c$. **[CBSE 2002 C]**

Sol. The discriminant, D of the given equation is given by

$$D = (c - a)^2 - 4(b - c)(a - b)$$

Now, for real and equal roots, D = 0

$$\Rightarrow \quad (c - a)^2 - 4(b - c)(a - b) = 0$$

$$\Rightarrow \quad c^2 + a^2 - 2ac - 4(ab - b^2 - ac + bc) = 0$$

$$\Rightarrow \quad c^2 + a^2 + 4b^2 + 2ac - 4ab - 4bc = 0$$

$$\Rightarrow \quad (2b - a - c)^2 = 0$$

$$\Rightarrow \quad 2b - a - c = 0$$

$$\Rightarrow \quad 2b = a + c$$

Hence, proved.

31. Form an equation whose roots are cubes of the roots of the equation $x^2 + bx + c = 0$.

Sol. Let α and β be the roots of the equation

$$x^2 + bx + c = 0$$

Then $\alpha + \beta = -b \quad \ldots(1)$

and $\alpha\beta = c \quad \ldots(2)$

Now, the equation whose roots are α^3 and β^3 is

x^2 – (sum of the roots)x + product of two roots = 0

$\Rightarrow x^2 - (\alpha^3 + \beta^3)x + \alpha^3\beta^3 = 0$

$\Rightarrow x^2 - \{(\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)\}x + (\alpha\beta)^3 = 0$

$\Rightarrow x^2 - (-b^3 + 3bc)x + c^3 = 0$ [From (1) and (2)]

$\Rightarrow x^2 + b(b^2 - 3c)x + c^3 = 0$

which is the required equation.

32. Form a quadratic equation whose roots are $\frac{1}{a+b}$ and $\frac{1}{a-b}$.

Sol. The equation whose roots are $\frac{1}{a+b}$ and $\frac{1}{a-b}$ is

x^2 – (sum of the roots)x + product of the roots = 0

$$\Rightarrow \quad x^2 - \left(\frac{1}{a+b} + \frac{1}{a-b}\right)x + \frac{1}{a^2 - b^2} = 0$$

$$\Rightarrow \quad x^2 - \frac{2ax}{(a+b)(a-b)} + \frac{1}{a^2 - b^2} = 0$$

$$\Rightarrow \quad x^2 - \frac{2ax}{a^2 - b^2} + \frac{1}{a^2 - b^2} = 0$$

$$\Rightarrow \quad (a^2 - b^2)x^2 - 2ax + 1 = 0$$

which is the required equation.

33. If α and β are the roots of the equation $3x^2 - 4x + 1 = 0$, form an equation whose roots are $\frac{\alpha^2}{\beta}$ and $\frac{\beta^2}{\alpha}$.

Sol. We have $\alpha + \beta = \frac{4}{3} \quad \ldots(1)$

and $\alpha\beta = \frac{1}{3} \quad \ldots(2)$

$\therefore$ The equation whose roots are $\frac{\alpha^2}{\beta}$ and $\frac{\beta^2}{\alpha}$ is

$$x^2 - \left(\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}\right)x + \frac{\alpha^2}{\beta} \times \frac{\beta^2}{\alpha} = 0$$

$$\Rightarrow \quad x^2 - \frac{\alpha^3 + \beta^3}{\alpha\beta}x + \alpha\beta = 0$$

$$\Rightarrow \quad x^2 - \frac{\left\{(\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)\right\}x}{\alpha\beta} + \alpha\beta = 0$$

$$\Rightarrow \quad x^2 - \frac{\left\{\left(\frac{4}{3}\right)^3 - 3 \times \frac{1}{3} \times \frac{4}{3}\right\}x}{\frac{1}{3}} + \frac{1}{3} = 0$$

$$\Rightarrow \quad x^2 - 3\left(\frac{64}{27} - \frac{4}{3}\right)x + \frac{1}{3} = 0$$

$$\Rightarrow \quad x^2 - 3 \times \frac{64 - 36}{27}x + \frac{1}{3} = 0$$

$$\Rightarrow \quad x^2 - \frac{28}{9}x + \frac{1}{3} = 0$$

$$\Rightarrow \quad 9x^2 - 28x + 3 = 0$$

which is the required equation.

34. If roots of the quadratic equation $x^2 + 2px + mn = 0$ are real and equal, show that the roots of the quadratic equation $x^2 - 2(m + n)x + (m^2 + n^2 + 2p^2) = 0$ are also equal. **[CBSE 2016]**

Sol. The discriminant, D of the equation $x^2 + 2px + mn = 0$ is given by $D = 4p^2 - 4mn$.

Now, for real and equal roots, D = 0.

$$4p^2 - 4mn = 0$$

$$\Rightarrow \qquad p^2 = mn \qquad \ldots(1)$$

The discriminant, D′ of the second quadratic equation $x^2 - 2(m + n)x + (m^2 + n^2 + 2p^2) = 0$ is given by

$$\begin{aligned} D' &= \{2(m + n)\}^2 - 4\,(m^2 + n^2 + 2p^2) \\ &= 4\,(m + n)^2 - 4\,(m^2 + n^2 + 2p^2) \\ &= 4\,[m^2 + n^2 + 2mn - m^2 - n^2 - 2p^2] \\ &= 4\,[2mn - 2p^2] \\ &= 4\,[2p^2 - 2p^2] \qquad \text{[From (1)]} \\ &= 0 \end{aligned}$$

Hence, the two roots of the second equation are also real and equal.

Long Answer Type Questions

35. Solve the following for x:

$$\frac{1}{2a+b+2x} = \frac{1}{2a}+\frac{1}{b}+\frac{1}{2x}$$

Sol. We have $\qquad \dfrac{1}{2a + b + 2x} = \dfrac{2a + b}{2ab} + \dfrac{1}{2x}$

$$\Rightarrow \qquad \frac{1}{p + 2x} = \frac{p}{2ab} + \frac{1}{2x} = \frac{px + ab}{2abx},$$

where $\qquad p = 2a + b \qquad \ldots(1)$

$$\Rightarrow \qquad (p + 2x)\,(px + ab) = 2abx$$

$$\Rightarrow \qquad p^2x + pab + 2px^2 + 2abx = 2abx$$

$$\Rightarrow \qquad p^2x + pab + 2px^2 = 0$$

$$\Rightarrow \qquad 2x^2 + px + ab = 0$$

$$\Rightarrow \qquad 2x^2 + (2a + b)x + ab = 0$$

[Putting the value of p from (1)]

$$\Rightarrow \qquad 2x^2 + 2ax + bx + ab = 0$$

$$\Rightarrow \qquad 2x(x + a) + b\,(x + a) = 0$$

$$\Rightarrow \qquad (x + a)\,(2x + b) = 0$$

$\therefore$ Either $\qquad x + a = 0 \qquad \ldots(1)$

or, $\qquad 2x + b = 0 \qquad \ldots(2)$

From (1) and (2), we have $x = -a,\ -\dfrac{b}{2}$ which are the required solution.

36. Some students planned a picnic. The total budget for food was ₹ 2000. But 5 students failed to attend the picnic and thus the cost of food for each member increased by ₹ 20. How many students attended the picnic and how much did each student pay for the food?

Sol. Let the number of students who attended the picnic be x. Then the original number of students was $x+5$. Hence, original contribution per student = ₹ $\dfrac{2000}{x + 5}$.

According to the problem, we have

$$\left(\frac{2000}{x + 5} + 20\right) x = 2000$$

$$\Rightarrow \qquad \frac{2000}{x + 5} + 20x = 2000$$

$$\Rightarrow \qquad \frac{100x}{x + 5} + x = 100$$

$$\Rightarrow \qquad 100x + x^2 + 5x = 100x + 500$$

$$\Rightarrow \qquad x^2 + 5x - 500 = 0$$

$$\Rightarrow \qquad x^2 + 25x - 20x - 500 = 0$$

$$\Rightarrow \qquad x(x + 25) - 20\,(x + 25) = 0$$

$$\Rightarrow \qquad (x + 25)\,(x - 20) = 0$$

$\therefore$ Either $\qquad x + 25 = 0 \qquad \ldots(1)$

or, $\qquad x - 20 = 0 \qquad \ldots(2)$

From (1), $x = -25$ which is rejected, since x cannot be negative.

$\therefore$ From (2), $x = 20$ which is accepted.

Hence, the required number of students who attended the picnic is 20 and the contribution per student is ₹ $\dfrac{2000}{20}$, i.e. ₹100.

Let's Compete

(Page 78)

Multiple-Choice Questions

1. Quadratic equation whose roots are the reciprocal of the roots of the equation $ax^2 + bx + c = 0$ is

(*a*) $ax^2 + cx + b = 0$

(*b*) $cx^2 + bx + a = 0$

(*c*) $cx^2 - bx + a = 0$

(*d*) $cx^2 + bx - a = 0$

Sol. (*b*) $cx^2 + bx + a = 0$

Let the roots of the equation $ax^2 + bx + c = 0$ be α and β.

Then $\qquad \alpha + \beta = -\dfrac{b}{a} \qquad \ldots(1)$

and $\qquad \alpha\beta = \dfrac{c}{a} \qquad \ldots(2)$

$\therefore$ The equation whose roots are $\dfrac{1}{\alpha}$ and $\dfrac{1}{\beta}$ is

$$x^2 - \left(\frac{1}{\alpha} + \frac{1}{\beta}\right)x + \frac{1}{\alpha}\cdot\frac{1}{\beta} = 0$$

$$\Rightarrow \quad x^2 - \frac{\alpha+\beta}{\alpha\beta}x + \frac{1}{\alpha\beta} = 0$$

$$\Rightarrow \quad \alpha\beta x^2 - (\alpha+\beta)x + 1 = 0$$

$$\Rightarrow \quad \frac{c}{a}x^2 + \frac{b}{a}x + 1 = 0$$

$$\Rightarrow \quad cx^2 + bx + a = 0$$

2. The sum of a number x and its reciprocal is 4. Which of these correctly represents the above statement?

(*a*) $x^2 + 1 = 4$ (*b*) $x^2 + 1 = 4x$

(*c*) $x^2 + x = 4x$ (*d*) $x^2 + 1 = -4x$

Sol. (*b*) $x^2 + 1 = 4x$

3. If $x = -2$ and $x = \frac{3}{4}$ are solutions of the equation $px^2 + qx - 6 = 0$, then the values of p and q are respectively

(*a*) 1, 6 (*b*) 5, 4

(*c*) 4, 5 (*d*) 6, 1

Sol. (*c*) 4, 5

We have,

$$\text{Sum of the roots} = -2 + \frac{3}{4} = -\frac{5}{4} = -\frac{q}{p}$$

$$\Rightarrow \quad 5p = 4q \quad \ldots(1)$$

Also, product of the roots

$$= \quad -2 \times \frac{3}{4} = -\frac{6}{p}$$

$$\Rightarrow \quad 3p = 12$$

$$\Rightarrow \quad p = \frac{12}{3} = 4$$

$$\therefore \quad q = \frac{5}{4} \times 4 \quad \text{[From (1)]}$$

$$= 5$$

$\therefore$ The values of p and q are 4 and 5 respectively.

4. If two numbers m and n are such that the quadratic equation $mx^2 + 3x + 2n = 0$ has –6 as the sum of the roots and also as the product of roots then

(*a*) $m = \frac{1}{2}, n = \frac{-3}{2}$ (*b*) $m = \frac{-3}{2}, n = \frac{1}{2}$

(*c*) $m = \frac{2}{3}, n = \frac{-1}{2}$ (*d*) $m = \frac{-2}{3}, n = \frac{3}{2}$

Sol. (*a*) $m = \frac{1}{2}, n = \frac{-3}{2}$

According to the problem, we have

Sum of the roots = –6

$$\Rightarrow \quad \frac{-3}{m} = -6$$

$$\Rightarrow \quad m = \frac{3}{6} = \frac{1}{2} \quad \ldots(1)$$

and product of the roots = –6

$$\Rightarrow \quad \frac{2n}{m} = -6$$

$$\Rightarrow \quad \frac{2n}{\frac{1}{2}} = -6 \quad \text{[From (1)]}$$

$$\Rightarrow \quad n = -\frac{6}{4} = -\frac{3}{2}$$

$$\therefore \quad m = \frac{1}{2} \text{ and } n = -\frac{3}{2}$$

5. The value of y which satisfies the equation $1 + \frac{y^2}{13} = \sqrt{\frac{27}{169} + 1}$ is

(*a*) ± 2 (*b*) ± 1

(*c*) ± 3 (*d*) ± 4

Sol. (*b*) ± 1

$$\text{We have} \quad 1 + \frac{y^2}{13} = \sqrt{\frac{27}{169} + 1} = \sqrt{\frac{196}{169}} = \frac{14}{13}$$

$$\Rightarrow \quad \frac{y^2}{13} = \frac{14}{13} - 1 = \frac{1}{13}$$

$$\therefore \quad y^2 = 1$$

$$\Rightarrow \quad y = \pm 1$$

6. If $x = \sqrt{6 + \sqrt{6 + \sqrt{6 \ldots}}}$, then the value of x is

(*a*) 1 (*b*) 2

(*c*) 3 (*d*) 4

Sol. (*c*) 3

$$\text{We have} \quad x = \sqrt{6 + x}$$

$$\Rightarrow \quad x^2 = 6 + x$$

[Squaring both sides]

$$\Rightarrow \quad x^2 - x - 6 = 0$$

$$\Rightarrow \quad x^2 - 3x + 2x - 6 = 0$$

$$\Rightarrow \quad x(x-3) + 2(x-3) = 0$$

$$\Rightarrow \quad (x-3)(x+2) = 0$$

$\therefore$ Either $x + 2 = 0$ …(1)

or, $x - 3 = 0$ …(2)

From (1), $x = -2$ which is rejected, since x is not negative.

From (2), $x = 3$ which is accepted.

7. If $x = 1$ is a common root of $ax^2 + ax + 2 = 0$ and $x^2 + x + b = 0$, then $a : b$ is equal to

(*a*) 1 : 2 (*b*) 2 : 1
(*c*) 1 : 4 (*d*) 4 : 1

Sol. (*a*) 1 : 2

Since $x = 1$ is a common root of both the given equation, we have

$$a + a + 2 = 0$$

$$\Rightarrow \quad a = -1 \text{ and } 1 + 1 + b = 0$$

$$\Rightarrow \quad b = -2$$

$$\therefore \quad a : b = 1 : 2$$

8. The ratio of sum and products of the roots of the equation $3x^2 + 12 - 13x = 0$ is

(*a*) 12 : 13 (*b*) 13 : 12
(*c*) 6 : 7 (*d*) 7 : 6

Sol. (*b*) 13 : 12

Let α and β be the roots of the given equation $3x^2 + 12 - 13x$.

We have $x^2 + 4 - \frac{13}{3}x = 0$

$$\Rightarrow \quad x^2 - \frac{13}{3}x + 4 = 0$$

Then, $\alpha + \beta = \frac{13}{3}$ and $\alpha\beta = 4$

$\therefore$ The required ratio $\frac{\alpha + \beta}{\alpha\beta}$ is $\frac{\frac{13}{3}}{4}$ i.e. $\frac{13}{12}$.

9. If 1 is a root of the equations $ay^2 + ay + 3 = 0$ and $y^2 + y + b = 0$, then ab equals

(*a*) 3 (*b*) $\frac{-7}{2}$
(*c*) 6 (*d*) -3 **[CBSE SP 2012]**

Sol. (*a*) 3

We see that $y = 1$ will satisfy both the given equations

$$\therefore \quad a + a + 3 = 0$$

$$\Rightarrow \quad a = -\frac{3}{2} \quad \ldots(1)$$

$$\text{and} \quad 1 + 1 + b = 0$$

$$\Rightarrow \quad b = -2 \quad \ldots(2)$$

$$\therefore \quad ab = -\frac{3}{2} \times (-2) \quad \text{[From (1) and (2)]}$$

$$= 3$$

10. If one root of $3x^2 = 8x + (2k + 1)$ is seven times the other, then the roots are

(*a*) $-3, -\frac{3}{7}$ (*b*) $\frac{1}{3}, \frac{7}{3}$
(*c*) $-\frac{1}{3}, -\frac{7}{3}$ (*d*) $3, \frac{3}{7}$

Sol. (*b*) $\frac{1}{3}, \frac{7}{3}$

Let α and 7α be the roots of the given equation

$$3x^2 = 8x + (2k + 1)$$

Then,

$$\text{Sum of the roots} = \alpha + 7\alpha = \frac{8}{3}$$

$$\Rightarrow \quad 8\alpha = \frac{8}{3}$$

$$\Rightarrow \quad \alpha = \frac{1}{3}$$

$\therefore$ The required roots are $\frac{1}{3}$ and $\frac{7}{3}$.

Life Skills

(Page 79)

1. A passenger, while boarding the plane, slipped from the stairs and got hurt. The pilot took the passenger in the emergency clinic at the airport for treatment. Due to this, the plane got delayed by half an hour. To reach the destination 1500 km away in time, so that the passengers could catch the connecting flight, the speed of the plane was increased by 250 km/h than the usual speed. Find the usual speed of the plane. **[CBSE 2016]**

Sol. Let the usual speed of the aeroplane be x km/h. Then the increased speed is $(x + 250)$ km/h.

$\therefore$ According to the problem, we have

$$\frac{1500}{x} = \frac{1500}{x + 250} + \frac{1}{2}$$

$$\Rightarrow \quad 1500\left(\frac{1}{x} - \frac{1}{x + 250}\right) = \frac{1}{2}$$

$$\Rightarrow \quad \frac{1500(x + 250 - x)}{x(x + 250)} = \frac{1}{2}$$

$$\Rightarrow \quad x^2 + 250x - 750000 = 0$$

$$\Rightarrow \quad x^2 - 750x + 1000x - 750000 = 0$$

$$\Rightarrow \quad x(x - 750) + 1000(x - 750) = 0$$

$$\Rightarrow \quad (x - 750)(x + 1000) = 0$$

$$\therefore \text{ Either} \quad x - 750 = 0 \quad \ldots(1)$$

$$\text{or,} \quad x + 1000 = 0 \quad \ldots(2)$$

From (1), $x = 750$ and from (2), $x = -1000$ which is rejected, since x is negative.

Hence, the required speed of the aeroplane is 750 km/h.

2. On Van Mahotsav day some students planted trees in horizontal and vertical rows. They planted 480 trees in all such that there were four trees more in each horizontal row than in vertical row.

Find the number of trees in each horizontal row.

Sol. Let the number of trees planted in each horizontal row = x.

Then, the number of trees planted in each vertical row = $(x - 4)$

Given, total number of trees = 480

$\therefore \quad x(x - 4) = 480$

$\Rightarrow \quad x^2 - 4x - 480 = 0$

$\Rightarrow \quad x^2 - 24x + 20x - 480 = 0$

$\Rightarrow \quad x(x - 24) + 20(x - 24) = 0$

$\Rightarrow \quad (x - 24)(x + 20) = 0$

$\therefore$ Either $\quad x - 24 = 0 \quad \ldots(1)$

or $\quad x + 20 = 0 \quad \ldots(2)$

From (1), we have, $x = 24$, which is accepted.

From (2), we have $x = -20$, which is rejected, since x cannot be negative.

$\therefore$ The required number of trees in each horizontal row is 24.

3. Three-eighth of the students of a class opted for visiting an old age home. Sixteen students opted for having a nature walk. Square root of total number of students in the class opted for tree plantation in the school. The number of students who visited an old age home is same as the number of students who went for a nature walk and did tree plantation. Find the total number of students. **[CBSE SP 2015]**

Sol. Let the total number of students be x.

Then, according to the problem, we have

$$\frac{3x}{8} = 16 + \sqrt{x}$$

$\Rightarrow \quad 3x - 8\sqrt{x} - 128 = 0$

$\Rightarrow \quad 3a^2 - 8a - 128 = 0$

where $\quad a = \sqrt{x} \quad \ldots(1)$

$\Rightarrow \quad 3a^2 - 24a + 16a - 128 = 0$

$\Rightarrow \quad 3a(a - 8) + 16(a - 8) = 0$

$\Rightarrow \quad (a - 8)(3a + 16) = 0$

$\therefore$ Either $\quad 3a + 16 = 0 \quad \ldots(2)$

or $\quad a - 8 = 0 \quad \ldots(3)$

From (2), $a = -\dfrac{16}{3}$, which is rejected since a is not negative.

$\therefore$ From (3), $a = 8$ which is accepted

$\therefore \quad a = 8$

$\Rightarrow \quad \sqrt{x} = 8 \quad$ [From (1)]

$\Rightarrow \quad x = 64 \quad$ [Squaring both sides]

Hence, the required number of students are 64.