3

Pair of Linear Equations in Two Variables

Checkpoint (Page 31)

1. Determine which values of x and y will satisfy the equation $2x - 3y - 5 = 0$?

(*a*) $x = 0, y = \frac{5}{2}$ (*b*) $x = -5, y = \frac{5}{3}$

(*c*) $x = -2, y = 3$ (*d*) $x = 4, y = 1$

Sol. (*a*) Putting $x = 0$ and $y = \frac{5}{2}$ on the LHS of the equation $2x - 3y - 5 = 0$, we get

$$\text{LHS} = 2 \times 0 - 3 \times \frac{5}{2} - 5$$
$$= \frac{-15}{2} - 5$$
$$= \frac{-15 - 10}{2}$$
$$= -\frac{25}{2} \neq \text{RHS}$$

$\therefore$ The equation is not satisfied.

(*b*) Putting $x = -5$ and $y = \frac{5}{3}$ on the LHS of the equation $2x - 3y - 5 = 0$, we get

$$\text{LHS} = 2 \times (-5) - 3 \times \frac{5}{3} - 5$$
$$= -10 - 5 - 5$$
$$= -20$$
$$\neq \text{RHS}$$

$\therefore$ The equation is not satisfied.

(*c*) Putting $x = -2$ and $y = 3$ on the LHS of the equation $2x - 3y - 5 = 0$, we get

$$\text{LHS} = 2 \times (-2) - 3 \times 3 - 5$$
$$= -4 - 9 - 5$$
$$= -13 - 5$$
$$= -18 \neq \text{RHS}$$

$\therefore$ The equation is not satisfied.

(*d*) Putting $x = 4$ and $y = 1$ on the LHS of the equation $2x - 3y - 5 = 0$, we get

$$\text{LHS} = 2 \times 4 - 3 \times 1 - 5 = 8 - 8 = 0 = \text{RHS}$$

Thus, $x = 4$ and $y = 1$ satisfies the equation $2x - 3y - 5 = 0$.

2. Express the following statement in the form of a linear equation in two variables: $ax + by + c = 0$. The present ages of the father and his son are x years and y years respectively. Three years ago, the father's age was twice the age of the son.

Sol. According to the problem, we have

$$x - 3 = 2(y - 3)$$
$$\Rightarrow \quad x - 2y + 3 = 0$$

which is the required equation.

3. Find the solution of the equations $2x + 3 = 5$ and $3y - 8 = 7$. Is there any other solution of these two equations?

Sol. We have $2x + 3 = 5$

$$\Rightarrow \quad 2x = 5 - 3 = 2$$
$$\Rightarrow \quad x = \frac{2}{2} = 1$$

Also, we have

$$3y - 8 = 7$$
$$\Rightarrow \quad 3y = 7 + 8 = 15$$
$$\Rightarrow \quad y = \frac{15}{3} = 5$$

Hence, the required solution is $x = 1$ and $y = 5$.

No, these two equations do not have any other solution at all.

4. Show that the solution of the equations $5x - 7 = 3$ and $2 - y = 5$ will satisfy the equation $7x + 8y + 10 = 0$.

Sol. We have $5x - 7 = 3$

$$\Rightarrow \quad 5x = 7 + 3 = 10$$

$$\Rightarrow \quad x = \frac{10}{5} = 2$$

Also, we have

$$2 - y = 5$$

$$\Rightarrow \quad y = 2 - 5 = -3$$

Putting $x = 2$ and $y = -3$ on the LHS of the equation $7x + 8y + 10 = 0$, we get

$$\begin{aligned} \text{LHS} &= 7 \times 2 + 8 \times (-3) + 10 \\ &= 14 - 24 + 10 \\ &= 24 - 24 \\ &= 0 = \text{RHS} \end{aligned}$$

Hence, $x = 2$, $y = -3$ satisfies the equation $7x + 8y + 10 = 0$.

5. Determine the coordinates of the points where the line $3x - 2y = 6$ will intersect the

(*a*) x-axis and

(*b*) y-axis.

Sol. (*a*) The given line $3x - 2y = 6$ will intersect x-axis when $y = 0$.

Putting $y = 0$, we get

$$3x = 6$$

$$\Rightarrow \quad x = 2$$

Hence, the given line will intersect the x-axis at the point (2, 0).

(*b*) The given line $3x - 2y = 6$ will intersect y-axis when $x = 0$. Putting $x = 0$, we get

$$-2y = 6$$

$$\Rightarrow \quad y = -\frac{6}{2} = -3$$

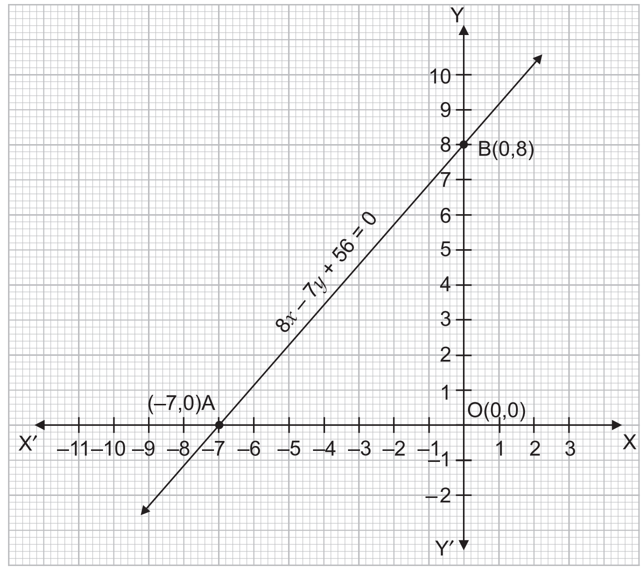
Hence, the given line will intersect the y-axis at the point (0, –3).

6. Find the coordinates of the vertices of the triangle formed by the line $8x - 7y + 56 = 0$, x-axis and the y-axis graphically.

Sol. Putting $y = 0$ in the given equation, we get

$$8x = -56$$

$$\Rightarrow \quad x = -\frac{56}{8} = -7$$



Hence, the line intersects the x-axis at the point A(–7, 0). Again, putting $x = 0$ in the given equation, we get

$$7y = 56$$

$$\Rightarrow \quad y = \frac{56}{7} = 8$$

$\therefore$ The line intersects the y-axis at the point B(0, 8).

Finally, x and y axes intersects each other at the point O(0, 0) which is the origin. We now join AB to get the required triangle OAB as shown in the graph.

7. Draw the graph of the lines $-3x + y = 0$ and $56x - 3y = 0$ and determine their point of intersection, if any, graphically.

Sol. From the equation $-3x + y = 0$ we get, $y = 3x$. A few values of x and y are shown in Table 1 below:

Table 1

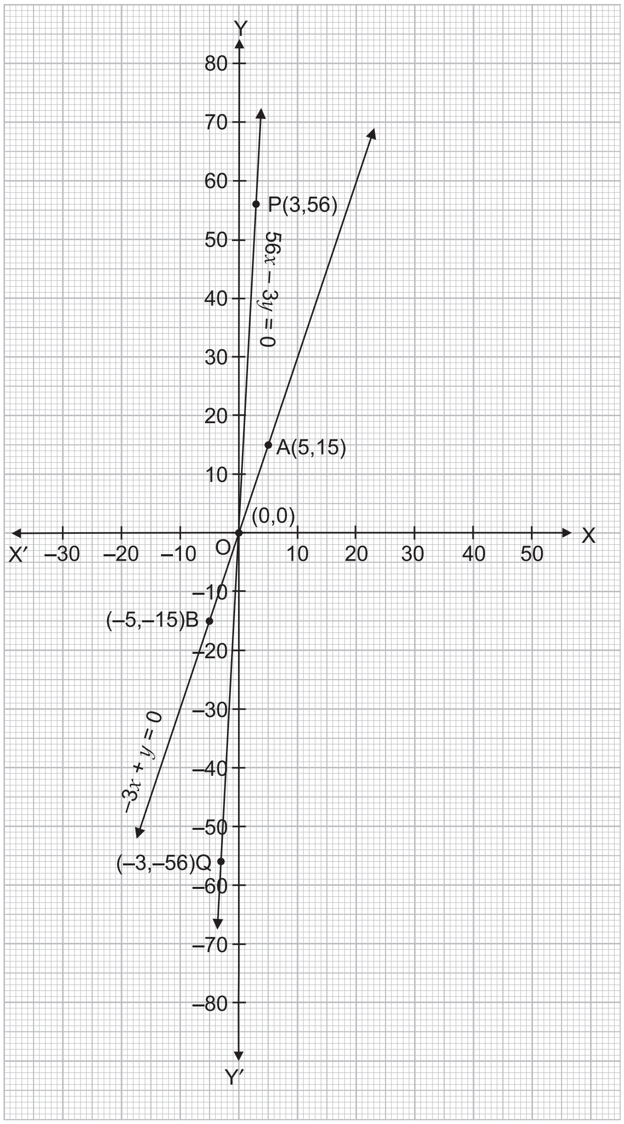
x	0	5	–5
y	0	15	–15

Similarly, from the equation $56x - 3y = 0$ we get, $y = \frac{56x}{3}$. A few values of x and y are shown in Table 2 below:

Table 2

x	0	3	–3
y	0	56	–56

We now plot the point O(0, 0), A(5, 15) and B(–5, –15) on a graph paper and join them by a straight line AOB.



We also plot the points O(0, 0), P(3, 56) and Q(–3, –56) on the same graph paper and join them by another straight line POQ. From the graph, we see that the two lines AB and PQ intersect each other at the origin O(0, 0) as shown in the graph.

8. A point (3, –4) lies on the line represented by $7x - 3y + 5k = 0$. Determine the value of k.

Sol. Putting $x = 3$ and $y = -4$ in the given equation, we get

$$7 \times 3 - 3 \times (-4) + 5k = 0$$

$$\Rightarrow \quad 21 + 12 + 5k = 0$$

$$\Rightarrow \quad k = -\frac{33}{5}$$

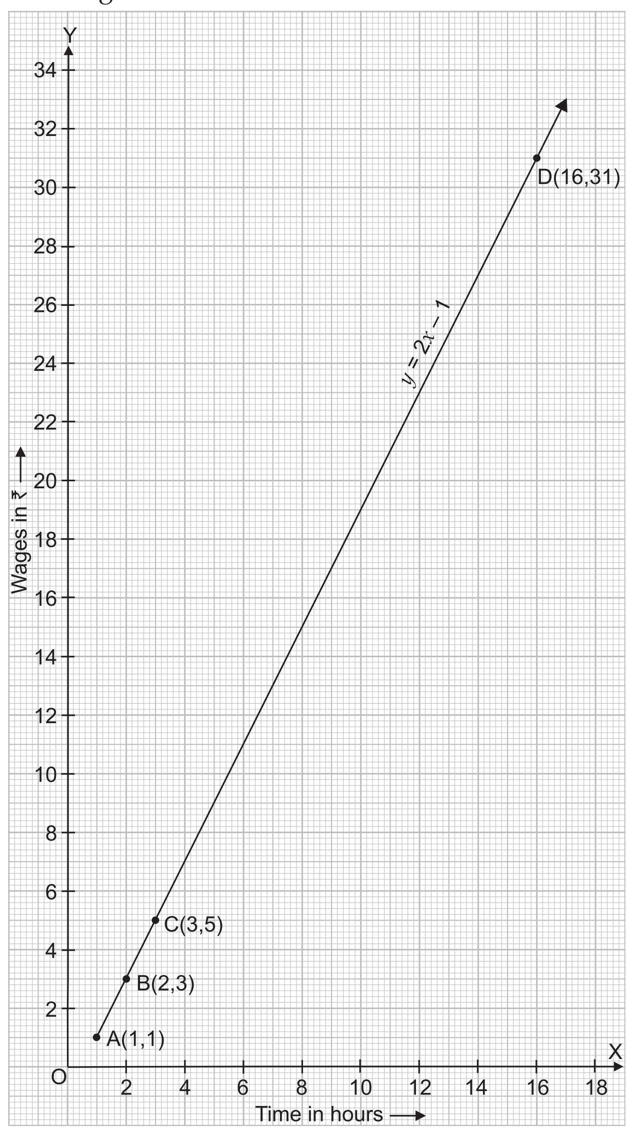
$\therefore$ The value of k is $-\frac{33}{5}$.

9. Working for x hours, a labour gets ₹y, where $y = 2x - 1$. Draw the graph of work-wage equation. From this graph, find the wages of the labour, if he works for 16 hours.

Sol. We first find some values of x and y from the given equation $y = 2x - 1$. These values of x and y are shown in the table below:

x	1	2	3	16
y	1	3	5	31

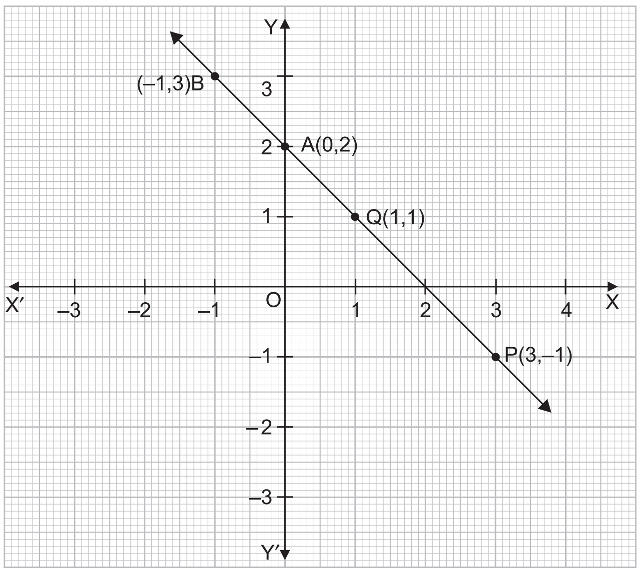
We now plot the points A(1, 1), B(2, 3), C(3, 5) and D(16, 31) on a graph paper and join them by a straight line AD to get the required graph of work-wage equation. In the graph, times in hours are shown along x-axis and wages in ₹ are shown along y-axis. Note that we have chosen different scales on x and y axes, since the unit of time and wages are different here.



From the graph, we see that when $x = 16$, $y = 31$. Hence, the required wages of the labour are ₹31, if he works for 16 hours.

10. Plot the points (0, 2) and (–1, 3) on a graph paper and draw a line through these two points. If the points $(p, -1)$ and $(1, q)$ lie on this line, then find the values of p and q.

Sol. We have plotted the points A(0, 2) and B(–1, 3) on the graph paper and join them by a straight line AB.



From the graph, we see that the points $P(p, -1)$ and $Q(1, q)$ will lie on the line only if $p = 3$ and $q = 1$.

Hence, the required values of p and q are 3 and 1 respectively.

Check Your Progress 1

(Page 36)

Multiple-Choice Questions

1. The pair of equations $x = a$ and $y = b$ graphically represent lines which are

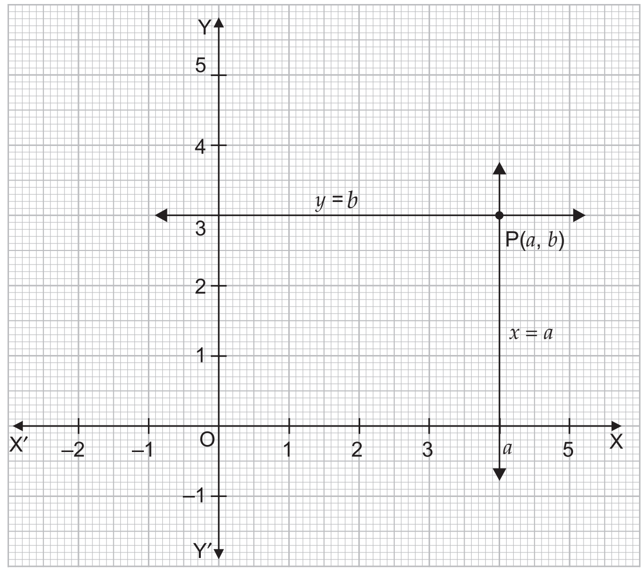
(*a*) coincident.

(*b*) parallel.

(*c*) intersecting at (b, a).

(*d*) intersecting at (a, b). **[CBSE 2023 Standard]**

Sol. (*d*) intersecting at (a, b).



From the graph, we see that $x = a$ is a line parallel to y-axis and $y = b$ is a line parallel to x-axis. We see that these two lines intersect each other at the point P with coordinates (a, b).

2. Let x years and y years represent the present ages of A and B respectively. Five years ago, A was thrice as old as B and ten years later A will be twice as old as B. This situation can be represented algebraically as

(*a*) $x + 3y - 10 = 0$ and $x + 2y + 10 = 0$

(*b*) $x - 3y + 10 = 0$ and $x - 2y - 10 = 0$

(*c*) $2x - 3y + 10 = 0$ and $2x + 3y - 10 = 0$

(*d*) $2x + 3y + 10 = 0$ and $2x - 3y + 10 = 0$

Sol. (*b*) $x - 3y + 10 = 0$ and $x - 2y - 10 = 0$

The present age of A = x years

The present age of B = y years

A's age, five years ago = $(x - 5)$ years

B's age, five years ago = $(y - 5)$ years

A's age, ten years later = $(x + 10)$ years

B's age, ten years later = $(y + 10)$ years

According to the problem, we have

$$x - 5 = 3(y - 5)$$

$$\Rightarrow \quad x - 3y - 5 + 15 = 0$$

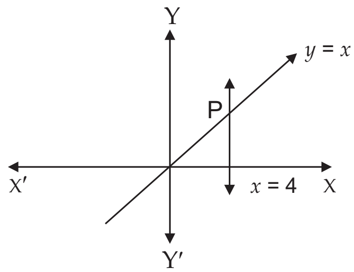
$$\Rightarrow \quad x - 3y + 10 = 0$$

and $$x + 10 = 2(y + 10)$$

$$\Rightarrow \quad x - 2y + 10 - 20 = 0$$

$$\Rightarrow \quad x - 2y - 10 = 0$$

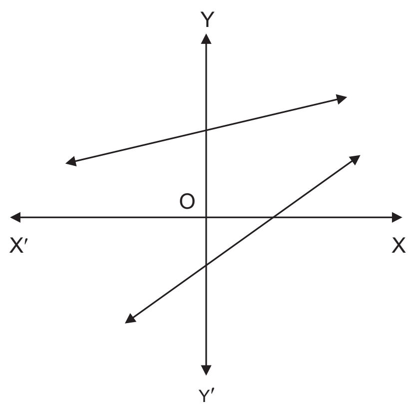
3. The lines represented by linear equations $y = x$ and $x = 4$ intersect at P. The coordinate of P are



(*a*) (4, 0)
(*b*) (4, 4)
(*c*) (0, 4)
(*d*) (–4, 4) **[CBSE 2023 Basic]**

Sol. (*a*) (4, 4)

4. In the given figure, graph of two linear equations are shown. The pair of these linear equations is



(*a*) consistent with unique solution.
(*b*) consistent with infinitely many solutions.
(*c*) inconsistent.
(*d*) inconsistent but can be made consistent by extending these lines. **[CBSE 2024 Standard]**

Sol. (*d*) inconsistent but can be made consistent by extending these lines.

Very Short Answer Type Questions

5. Find the total number of point(s) of intersection of the line $2x + 3y + 5 = 0$ with the x-axis.

Sol. Putting $y = 0$ in the given equation, we get

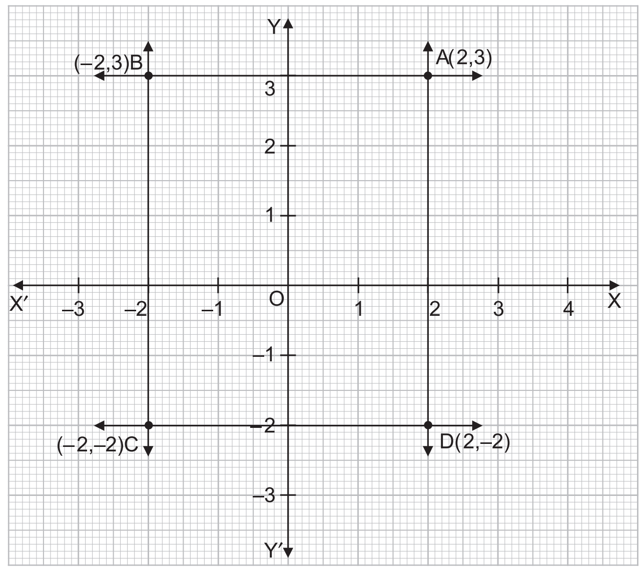
$$2x = -5$$

$$\Rightarrow \quad x = \frac{-5}{2}$$

$\therefore$ The given line intersects the x-axis at only one point *viz.* $\left(\frac{-5}{2}, 0\right)$.

6. Name the special quadrilateral formed by the lines $x = 2$, $x = -2$, $y = 3$ and $y = -2$ and hence find its area.

Sol. Lines $x = 2$ and $x = -2$ are parallel to y-axis and the lines $y = 3$ and $y = -2$ are parallel to x-axis as shown in the graph. These lines intersect pairwise at A, B, C and D as shown in the graph.



We see from the graph that the coordinates of the points A, B, C and D are respectively (2, 3), (–2, 3), (–2, –2) and (2, –2).

The length of the line segment AB = (2 + 2) units = 4 units = length of the line segment CD.

Also, the length of the line segment BC = (3 + 2) units = 5 units = length of the line segment AD.

Since, AB ∥ CD and BC ∥ AD

$\therefore$ ABCD is a parallelogram.

Also, each of the angles A, B, C and D is 90° and length and breadth are different. So, this parallelogram is a rectangle of area 4 × 5 sq units i.e. 20 sq units.

7. Draw the graph of equation $3x + 2y - 6 = 0$ and name the triangle formed by this line with coordinates axes. What are the coordinates of its vertices? What kind of triangle is it with respect to angles of the triangle?

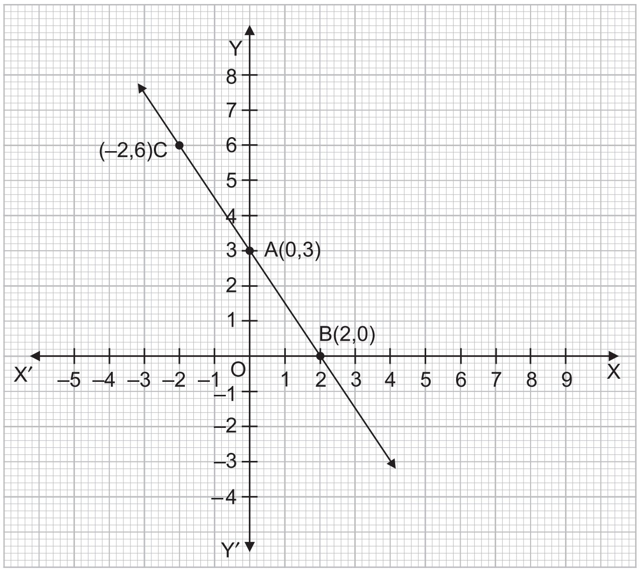
Sol. From the given equation, we have

$$2y = 6 - 3x$$

$$\Rightarrow \quad y = \frac{6 - 3x}{2}$$

We find some values of x and y as shown in the table below:

x	0	2	–2
y	3	0	6



We plot the points A(0, 3), B(2, 0) and C(–2, 6) on a graph paper and join them by a straight line BC. From the graph we see that this line forms a right angled triangle AOB where $\angle$AOB = 90° and O is the origin. The coordinates of the vertices A, O and B of the triangle AOB are respectively (0, 3), (0, 0) and (2, 0).

8. Draw the graphs of equations $y = x$ and $x = 1$. Hence, find the area of the triangle formed by these two lines and the x-axis.

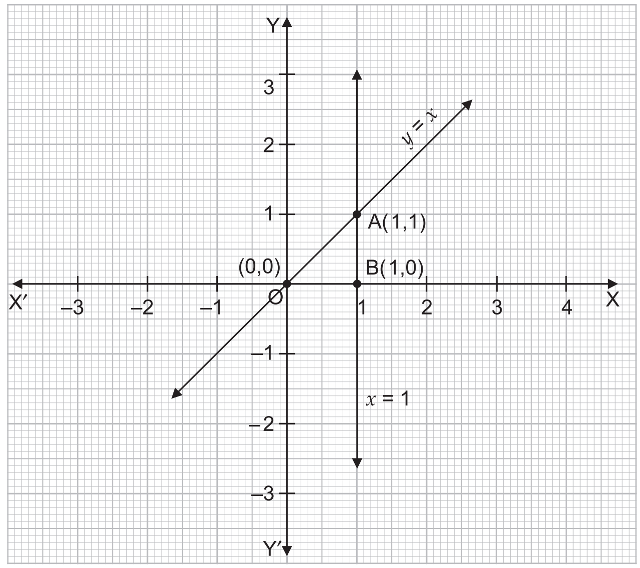
Sol. The graph of $y = x$ and $x = 1$ are shown on the graph paper. These two lines intersect each other at the point A(1, 1). Also, the line $x = 1$ which is parallel to y-axis cuts the x-axis at the point B(1, 0). The given line form a right angled triangle OBA with the coordinate axes, where $\angle$OBA = 90°.

$\therefore$ The area of this triangle

$$= \frac{1}{2} \times \text{OB} \times \text{AB}$$

$$= \frac{1}{2} \times 1 \times 1 \text{ sq units}$$

$$= \frac{1}{2} \text{ sq units}$$



Short Answer Type Questions

9. Show graphically that the system of equations $2x + 3y = 10$ and $4x + 6y = 12$ has no solution.

[CBSE 2010]

Sol. Graph of the equation

$$2x + 3y = 10$$

$$\Rightarrow \quad x = \frac{10 - 3y}{2} \quad \text{...(1)}$$

and $\quad 4x + 6y = 12$

$$\Rightarrow \quad 2x + 3y = 6$$

$$\Rightarrow \quad x = \frac{6 - 3y}{2} \quad \text{...(2)}$$

We find some value of x and y from (1) and those from (2). These values are shown in Table 1 and 2 below respectively.

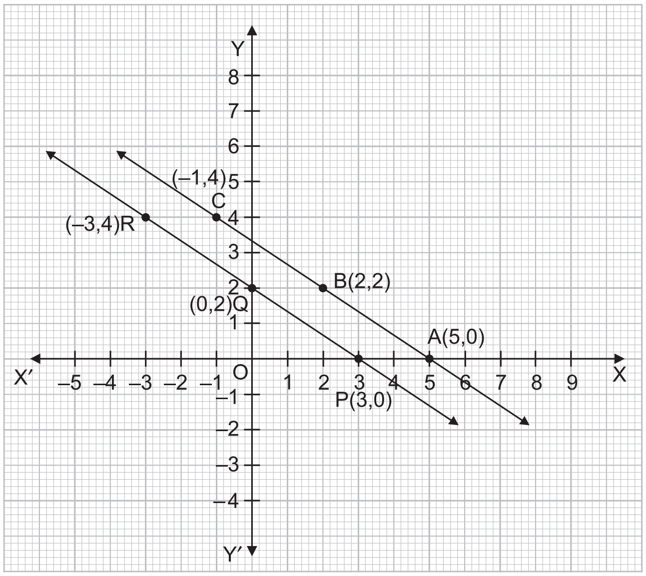
Table 1

x	5	2	–1
y	0	2	4

Table 2

x	3	0	–3
y	0	2	4

We plot the points A(5, 0), B(2, 2) and C(–1, 4). Also, we plot the points P(3, 0), Q(0, 2) and R(–3, 4) and join ABC and PQR by two distinct lines in the same graph paper.



We see from the graph that these two lines are parallel and so they cannot meet each other. Hence, the two given equations have no solution at all.

10. Show graphically that the lines represented by the equations $-7x + 5y = 17$, $3x - 4y = -11$ and $15x + 7y = -1$ are concurrent at the point $(-1, 2)$.

Sol. From the given equation, we have

$$y = \frac{17 + 7x}{5} \qquad \ldots(1)$$

$$y = \frac{3x + 11}{4} \qquad \ldots(2)$$

and
$$y = \frac{-15x - 1}{7} \qquad \ldots(3)$$

We now calculate some values of x and y from (1), (2) and (3) and list these values in Table 1, 2 and 3 respectively.

Table 1

x	–1	–6
y	2	–5

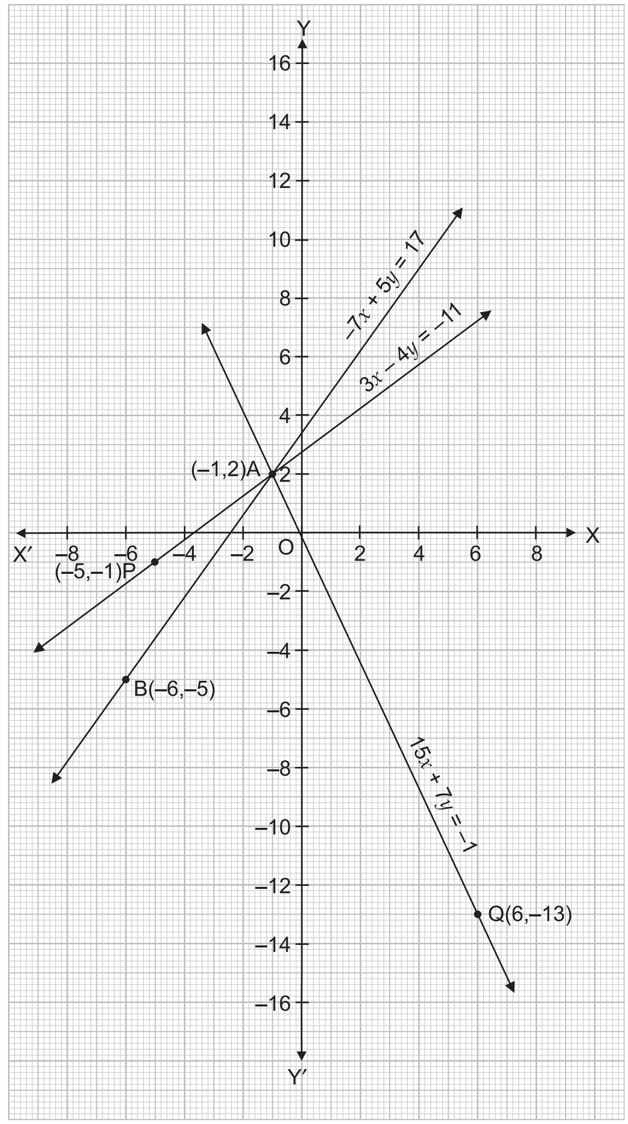
Table 2

x	–1	–5
y	2	–1

Table 3

x	–1	6
y	2	–13

We plot the points A(–1, 2), B(–6, –5), P(–5, –1) and Q(6, –13) and join the points in Table 1, Table 2 and Table 3 respectively with three separate straight lines PA, BA and QA which are concurrent at the point A(–1, 2) as shown in the graph.



11. Determine graphically whether the system of equations $x - 2y = 2$, $4x - 2y = 5$ is consistent or inconsistent.

Sol. From the two given equations, we have

$$y = \frac{x - 2}{2} \qquad \ldots(1)$$

and
$$y = \frac{4x - 5}{2} \qquad \ldots(2)$$

From (1) and (2), we list some values of x and y as shown below in Table 1 and Table 2 respectively.

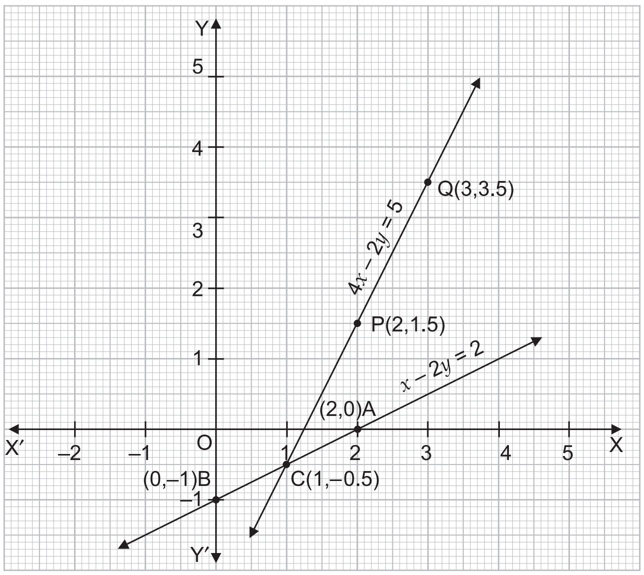
Table 1

x	2	0	1
y	0	–1	–0.5

Table 2

x	2	3	1
y	1.5	3.5	−0.5

We now plot the points A(2, 0), B(0, −1) and C(1, −0.5) from Table 1 and P(2, 1.5) and Q(3, 3.5) from Table 2 in the graph paper and join them by two separate line BCA and QPC respectively as shown in the graph. From the graph, we see that the two lines intersect each other at a point C(1, −0.5).



Hence, the two given equations have a unique solution. Hence, these two equations are consistent.

Long Answer Type Questions

12. Draw the graphs of the equations $y = x$, $y = 0$ and $2x + 2y = 20$. Hence, determine the area of the triangle formed by these three lines.

Sol. We find some values of x and y from the equation

$$y = x \quad \ldots(1)$$

and $$2x + 2y = 20$$

$$\Rightarrow \quad y = 10 - x \quad \ldots(2)$$

These values are listed in Table 1 and Table 2 respectively.

Table 1

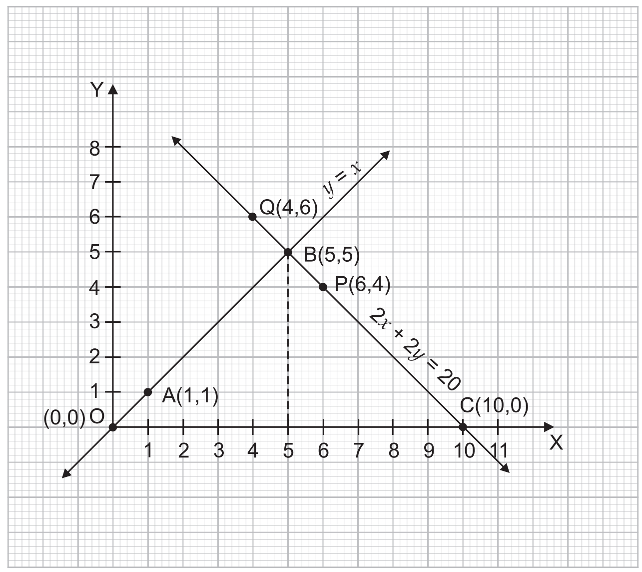
x	0	1	5
y	0	1	5

Table 2

x	6	4	5
y	4	6	5

We now plot the points O(0, 0), A(1, 1) and B(5, 5) from Table 1 and join them by a line OAB. We next plot the points P(6, 4) and Q(4, 6) from Table 2 and join them by another line PBQ in the same graph paper. These two lines intersect the line $y = 0$, i.e. the x-axis at the points O(0, 0) and C(10, 0) forming a triangle BOC.

We see that the base OC = 10 units and the height BM = 5 units. Hence, the area of ΔBOC is $\frac{1}{2} \times 10 \times 5$ sq units i.e. 25 sq units.



13. Draw the graphs of the equations $2y - x = 8$, $5y - x = 14$ and $y - 2x = 1$. Hence, find the coordinates of the vertices of the triangle formed by these three lines.

Sol. The given equation can be written as

$$x = 2y - 8 \quad \ldots(1)$$

$$x = 5y - 14 \quad \ldots(2)$$

and $$y = 1 + 2x \quad \ldots(3)$$

We list some values of x and y from (1), (2) and (3) respectively in Table 1, 2 and 3 below:

Table 1

x	2	−4	0
y	5	2	4

Table 2

x	−4	1	6
y	2	3	4

Table 3

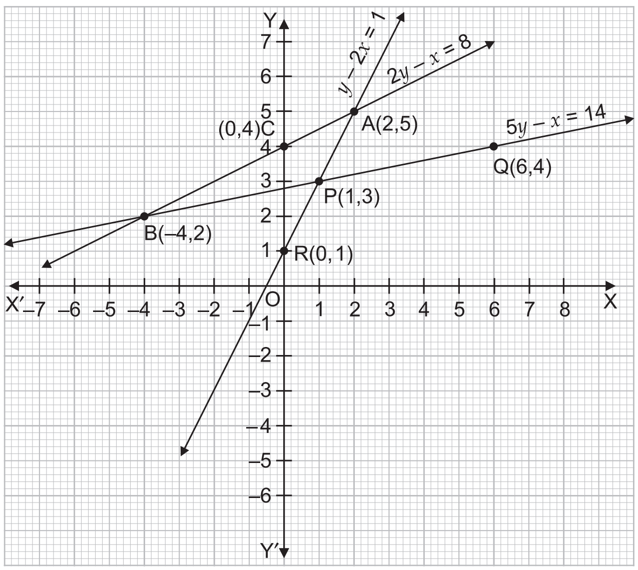
x	0	1	2
y	1	3	5

We now plot the points A(2, 5), B(–4, 2) and C(0, 4) from Table 1 and join them by a line ACB.

Similarly, we plot the points B(–4, 2), P(1, 3) and Q(6, 4) from Table 2 and join them by another line BPQ.

Finally, we plot the points R(0, 1), P(1, 3) and A(2, 5) from Table 3 and join them by a third line RPA.

The three lines AB, BP and PA drawn in the same graph are shown below in the graph paper. From the graph, we see that the vertices of the triangle ABP formed by these three lines are A(2, 5), B(–4, 2) and P(1, 3).



Check Your Progress 2

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Multiple-Choice Questions

1. One of the equation of a pair of dependent linear equations is $-5x + 7y = 2$. Then the second equation can be

(*a*) $-10x - 14y + 4 = 0$ (*b*) $-10x + 14y + 4 = 0$

(*c*) $10x - 14y = -4$ (*d*) $10x + 14y + 4 = 0$

[CBSE SP 2011]

Sol. (*c*) $10x - 14y = -4$

We know that two equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are linearly dependent if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$.

We see that the given equation $-5x + 7y = 2$ i.e. $5x - 7y + 2 = 0$ and $10x - 14y + 4 = 0$ in the choice (*c*) only are linearly dependent, for $\frac{5}{10} = \frac{-7}{-14} = \frac{2}{4}$ is true.

2. The point of intersection of the lines $y = 3x + 6$ and $y + 3x = 0$ is

(*a*) (–1, 3) (*b*) (1, 3)

(*c*) (1, –3) (*d*) (–1, –3)

Sol. (*a*) (–1, 3)

We have $y = 3x + 6$ ...(1)

and $y + 3x = 0$ ...(2)

From (1) and (2), we have

$$3x + 6 + 3x = 0$$

$$\Rightarrow \quad 6x = -6$$

$$\Rightarrow \quad x = -1$$

$\therefore$ From (1), $y = -3 + 6 = 3$

$\therefore$ The required point of intersection of the two given lines is (–1, 3).

3. The pair of linear equation $x + 2y - 5 = 0$ and $2x - 4y + 6 = 0$ is

(*a*) consistent.

(*b*) consistent with many solutions.

(*c*) consistent with two solutions.

(*d*) consistent with a unique solution.

[CBSE 2023 Basic]

Sol. (*d*) consistent with a unique solution.

The pair of linear equations is

$$x + 2y - 5 = 0$$

$$2x - 4y + 6 = 0$$

$a_1 = 1 \quad b_2 = 2 \quad c_1 = -5$

$a_2 = 2 \quad b_2 = -4 \quad c_1 = 6$

Since $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, therefore the two equations are consistent with a unique solution.

4. The point of intersection of the line represented by $3x - y = 3$ and y-axis is given by

(*a*) (0, –3) (*b*) (0, 3)

(*c*) (2, 0) (*d*) (–2, 0)

[CBSE 2023 Standard]

Sol. (*a*) (0, –3)

At y-axis, $x = 0$

$$3x - y = 3$$

Putting $x = 0$, in the above equation

$$3 \times 0 - y = 3$$

$$\Rightarrow \quad y = -3$$

$\therefore$ The point of intersection is (0, –3).

5. The value of k for which the pair of linear equations $5x + 2y - 7 = 0$ and $2x + ky + 1 = 0$ don't have a solution is

(a) 5 (b) $\frac{4}{5}$

(c) $\frac{5}{4}$ (d) $\frac{5}{2}$ **[CBSE 2024 Basic]**

Sol. (b) $\frac{4}{5}$

The given pair of linear equations is

$$5x + 2y - 7 = 0$$
$$2x + ky + 1 = 0$$
$$a_1 = 5 \quad b_1 = 2 \quad c_1 = -7$$
$$a_2 = 2 \quad b_2 = k \quad c_2 = 1$$
$$\frac{a_1}{a_2} = \frac{5}{2} \quad \frac{b_1}{b_2} = \frac{2}{k} \quad \frac{c_1}{c_2} = \frac{-7}{1}$$

For the two equations having no solution,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$
$$\Rightarrow \quad \frac{5}{2} = \frac{2}{k}$$
$$\Rightarrow \quad k = \frac{4}{5}$$

6. The value of k for which the pair of equations $kx = y + 2$ and $6x = 2y + 3$ has infinitely many solutions

(a) is $k = 3$. (b) does not exist.
(c) is $k = -3$. (d) is $k = 4$.

Sol. (b) does not exist.

The given pair of linear equations is

$$kx = y + 2 \text{ and } 6x = 2y + 3$$
$$\Rightarrow \quad kx - y - 2 = 0 \text{ and } 6x - 2y - 3 = 0$$
$$a_1 = k \quad b_1 = -1 \quad c_1 = -2$$
$$a_2 = 6 \quad b_2 = -2 \quad c_2 = -3$$

For the equations to have infinitely many solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

But $$\frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$
$$\frac{-1}{-2} \neq \frac{-2}{-3}$$
$$\Rightarrow \quad \frac{1}{2} \neq \frac{2}{3}$$

Hence, for any value of k, the given pair of linear equations will not have infinitely many solutions. Hence, the value of k does not exist.

Very Short Answer Type Questions

7. The line $\frac{x}{a} + \frac{y}{b} = -2$ intersects the x-axis at A and y-axis at B. Find the coordinates of A and B.

Sol. Putting $y = 0$ in the given equation $\frac{x}{a} + \frac{y}{b} = -2$, we get $x = -2a$

Hence, A is the point $(-2a, 0)$. Again, putting $x = 0$ in the given equation, we get $y = -2b$. Hence, B is the point $(0, -2b)$.

8. If $x = a + b$ and $y = a - b$ is the solution of the equations $x - y = 2$ and $x + y = 4$, then find the values of a and b.

Sol. Putting $x = a + b$ and $y = a - b$ in the given equation

$$x - y = 2 \quad \ldots(1)$$
and $$x + y = 4 \quad \ldots(2)$$
we get $$a + b - a + b = 2$$
$$\Rightarrow \quad 2b = 2$$
$$\Rightarrow \quad b = 1$$
and $$a + b + a - b = 4$$
$$\Rightarrow \quad 2a = 4$$
$$\Rightarrow \quad a = 2$$

Hence, the required values of a and b are 2 and 1 respectively.

9. If the lines represented by the equations $2x = -7py - 3$ and $-5y = -8 + 7px$ are parallel to each other, find the value(s) of p.

Sol. Since the lines

$$2x + 7py + 3 = 0 \quad \ldots(1)$$
and $$7px + 5y - 8 = 0 \quad \ldots(2)$$
are parallel, we have

$$\frac{2}{7p} = \frac{7p}{5} \neq -\frac{3}{8}$$
$$\Rightarrow \quad 49p^2 = 10$$
$$\Rightarrow \quad p^2 = \frac{10}{49}$$
$$\therefore \quad p = \pm\frac{\sqrt{10}}{\sqrt{49}} = \pm\frac{\sqrt{10}}{7}$$

Hence, the required value(s) of p are $\pm\frac{\sqrt{10}}{7}$.

10. Can the linear equations $5y = 3(q - 1)x - 10$ and $-3x = 7y + 14$ have infinite number of solution for any value of q? If so, find the value(s) of q.

Sol. We see that the equations

$$3(q-1)x - 5y - 10 = 0 \quad \ldots(1)$$

and $$3x + 7y + 14 = 0 \quad \ldots(2)$$

will have infinite number of solutions if

$$\frac{3}{3(q-1)} = -\frac{7}{5} = -\frac{14}{10}$$

$$\Rightarrow \quad 7q - 7 = -5$$

$$\Rightarrow \quad q = \frac{2}{7}$$

Yes, the given equation will have infinite number of solutions if $q = \frac{2}{7}$.

11. Find the value of k for which the following pair of linear equations has a unique solution.

$$4x + 7y - 30 = 0$$

$$5x - 9ky + 20 = 0$$

Sol. The given equation will have a unique solution if

$$\frac{4}{5} = \frac{7}{-9k} \neq \frac{-30}{20}$$

i.e. $$-36k = 35$$

$$\Rightarrow \quad k = -\frac{35}{36}$$

Hence, the required value of k is $-\frac{35}{36}$.

Short Answer Type Questions

12. Solve by any suitable method:

$a(x+y) + b(x-y) = a^2 - ab + b^2$

$a(x+y) - b(x-y) = a^2 + ab + b^2$ **[CBSE SP 2010]**

Sol. We have

$$a(x+y) + b(x-y) = a^2 - ab + b^2 \quad \ldots(1)$$

and $$a(x+y) - b(x-y) = a^2 + ab + b^2 \quad \ldots(2)$$

Adding (1) and (2), we get

$$x + y = \frac{a^2 + b^2}{a} \quad \ldots(3)$$

Subtracting (2) from (1), we get

$$x - y = -a \quad \ldots(4)$$

Adding (3) and (4), we get

$$2x = \frac{a^2 + b^2 - a^2}{a} = \frac{b^2}{a}$$

$$\Rightarrow \quad x = \frac{b^2}{2a}$$

Subtracting (4) from (3), we get

$$2y = \frac{a^2 + b^2}{a} + a$$

$$= \frac{2a^2 + b^2}{a}$$

$$\Rightarrow \quad y = \frac{2a^2 + b^2}{2a}$$

$\therefore$ The required solution is $x = \frac{b^2}{2a}$ and

$y = \frac{2a^2 + b^2}{2a}$.

13. If $217x + 131y = 913$ and $131x + 217y = 827$, then solve the equations for the values of x and y. **[CBSE 2023 Standard]**

Sol.

$$217x + 131y = 913 \quad \ldots(1)$$

$$131x + 217y = 827 \quad \ldots(2)$$

Adding equations (1) and (2), we get

$$348x + 348y = 1740$$

$$\Rightarrow \quad x + y = 5 \quad \ldots(3)$$

Subtracting equation (2) from (1)

$$86x - 86y = 86$$

$$\Rightarrow \quad x - y = 1 \quad \ldots(4)$$

Adding equations (3) and (4)

$$x + y = 5 \quad \ldots(3)$$

$$x - y = 1 \quad \ldots(4)$$

$$2x \quad = 6$$

$$\Rightarrow \quad x = 3$$

Putting value of $x = 3$ in equation (3), we get

$$3 + y = 5$$

$$\Rightarrow \quad y = 2$$

Long Answer Type Questions

14. Solve the following linear equations by using the method of elimination.

$$\frac{a}{10} + \frac{b}{5} + 2 = 16$$

and $$\frac{a}{8} + \frac{b}{6} = 15$$

Sol.

$$\frac{a}{10} + \frac{b}{5} + 2 = 16$$

$$\Rightarrow \quad \frac{a + 2b}{10} = 14$$

$$\Rightarrow \quad a + 2b = 140 \quad \ldots(1)$$

$$\Rightarrow \quad a = 140 - 2b \quad \ldots(2)$$

$$\frac{a}{8} + \frac{b}{6} = 15$$

$$\Rightarrow \quad \frac{3a + 4b}{24} = 15$$

$$\Rightarrow \quad 3a + 4b = 360 \quad \ldots(3)$$

Putting value of a from (2) in equation (3)

$\Rightarrow\ 3(140 - 2b) + 4b = 360$

$\Rightarrow\ 420 - 6b + 4b = 360$

$\Rightarrow\ -2b = -60$

$\Rightarrow\ b = 30$

From (2)

$a = 140 - 2b$

$\Rightarrow\ a = 140 - 60$

$\Rightarrow\ a = 80$

15. Solve the following pair of linear equations.

$217x + 131y = 913$

$131x + 217y = 827$

Sol. $217x + 131y = 913$ ...(1)

$131x + 217y = 827$ ...(2)

Adding equations (1) and (2), we get

$348x + 348y = 1740$

$\Rightarrow\ x + y = 5$ ...(3)

Subtracting equation (2) from (1)

$86x - 86y = 86$

$\Rightarrow\ x - y = 1$ ...(4)

Adding equations (3) and (4)

$$\begin{aligned} x + y &= 5 \\ x - y &= 1 \\ \hline 2x \quad &= 6 \end{aligned}$$

$\Rightarrow\ x = 3$

Putting value of $x = 3$ in equation (3), we get

$3 + y = 5$

$\Rightarrow\ y = 2$

16. Solve the following pair of linear equations.

$37x + 41y = 70$

$41x + 37y = 86$

Sol. $37x + 41y = 70$ ...(1)

$41x + 37y = 86$ ...(2)

Adding equations (1) and (2), we get

$78x + 78y = 156$

$x + y = 2$ ...(3)

Subtracting equation (2) from equation (1), we get

$-4x + 4y = -16$

$x - y = 4$ ...(4)

Adding equations (3) and (4), we get

$2x = 6$

$\Rightarrow\ x = 3$

Putting value of $x = 3$ in equation (3), we get

$3 + y = 2$

$\Rightarrow\ y = 2 - 3$

$\Rightarrow\ y = -1$

Check Your Progress 3

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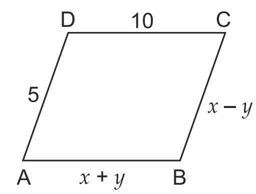
Multiple-Choice Questions

1. In the given figure, ABCD is a parallelogram. If AB = $x + y$, BC = $x - y$, CD = 10 units and AD = 5 units, then the values of x and y are respectively



(*a*) 8, 2 (*b*) 7.5, 2.5

(*c*) 6, 1 (*d*) 6, 4 **[CBSE SP 2011]**

Sol. (*b*) 7.5, 2.5

We know that in a parallelogram, the opposite sides are of equal length.

$\therefore\ x + y = 10$ …(1)

and $x - y = 5$ …(2)

Adding (1) and (2), we get

$2x = 15$

$\Rightarrow\ x = \frac{15}{2} = 7.5$

Subtracting (2) from (1), we get

$2y = 5$

$\Rightarrow\ y = \frac{5}{2} = 2.5$

$\therefore$ The required value of x and y are 7.5 and 2.5 respectively.

2. A two-digit number is 5 times the sum of its digits. If 9 is added to the number, then the digits interchange their places. Then the number is

(*a*) 46 (*b*) 64

(*c*) 54 (*d*) 45

Sol. (*d*) 45

Let the digits in the unit's place and the ten's place be x and y respectively. Then the number is $10y + x$.

The number obtained by interchanging the digits is $10x + y$.

According to the first condition of the problem, we have

$$10y + x = 5(x + y)$$

$$\Rightarrow \quad 10y + x = 5x + 5y$$

$$\Rightarrow \quad 4x - 5y = 0$$

$$\Rightarrow \quad x = \frac{5}{4}y \qquad \ldots(1)$$

According to the second condition of the problem, we have

$$10y + x + 9 = 10x + y$$

$$\Rightarrow \quad 9x - 9y = 9$$

$$\Rightarrow \quad x - y = 1 \qquad \ldots(2)$$

$\therefore$ From (1) and (2), we get

$$\frac{5}{4}y - y = 1$$

$$\Rightarrow \quad 5y - 4y = 4$$

$$\Rightarrow \quad y = 4$$

$\therefore$ From (1), $\quad x = \frac{5}{4} \times 4 = 5$

Hence, the required number is $10 \times 4 + 5$ i.e. 45.

Very Short Answer Type Questions

3. Find two numbers such that one-fourth of their sum is 11 and one-third of their difference is 2.

Sol. Let the two numbers be x and y and $x > y$.

$\therefore$ According to the problem, we have

$$\frac{x + y}{4} = 11$$

$$\Rightarrow \quad x + y = 44 \qquad \ldots(1)$$

and $\quad \frac{x - y}{3} = 2$

$$\Rightarrow \quad x - y = 6 \qquad \ldots(2)$$

Adding (1) and (2), we get

$$2x = 50$$

$$\therefore \quad x = \frac{50}{2} = 25$$

Subtracting (2) from (1), we get

$$2y = 38$$

$$\therefore \quad y = \frac{38}{2} = 19$$

Hence, the required numbers are 25 and 19.

4. A lady has 20 coins in her purse, consisting of ₹5 and ₹1 coins. If she has ₹40 in her purse, find the number of ₹1 and ₹5 coins separately.

Sol. Let the number of ₹1 coins be x and that of ₹5 coins be y.

$\therefore$ According to the problem, we have

$$x + y = 20 \qquad \ldots(1)$$

and $\quad x + 5y = 40 \qquad \ldots(2)$

Subtracting (1) from (2), we get

$$4y = 20$$

$$\Rightarrow \quad y = \frac{20}{4} = 5$$

$\therefore$ From (1), $x = 20 - 5 = 15$

$\therefore$ The required number of ₹1 coins and ₹5 coins are 15 and 5 respectively.

5. The ratio of incomes of two persons is 9 : 7 and that of their expenditure is 4 : 3. If each of them saves ₹200 per month, find their monthly incomes.

Sol. Let the monthly incomes of two persons be ₹$9x$ and ₹$7x$, where x is a constant number and let their respective expenditures be ₹$4y$ and ₹$3y$, where y is a constant number.

Then the net income of the first person is ₹$(9x - 4y)$ and that of the second person is ₹$(7x - 3y)$.

$\therefore$ According to the problem, we have

$$9x - 4y - 200 = 0 \qquad \ldots(1)$$

and $\quad 7x - 3y - 200 = 0 \qquad \ldots(2)$

Multiplying equation (1) by 3 and equation (2) by 4 and subtracting equation (2) from (1), we get

$$\begin{array}{rcrcrl} 27x & - & 12y & - & 600 & = 0 \\ 28x & - & 12y & - & 800 & = 0 \\ - & & + & & + & \\ \hline -x & & & + & 200 & = 0 \end{array}$$

$$\Rightarrow \quad x = 200$$

Putting $x = 200$ in equation (1), we get

$$1800 - 4y - 200 = 0$$

$$\Rightarrow \quad 1600 - 4y = 0$$

$$\Rightarrow \quad y = 400$$

$\therefore \quad x = 200$ and $y = 400$

$\therefore$ The required monthly incomes of two persons are ₹$9x = 9 \times 200 =$ ₹1800 and ₹$7x =$ ₹$7 \times 200 =$ 1400.

6. 10 years ago, a father was twelve times as old as his son and 10 years hence, he will be twice as old as his son will be then. Find their present ages.

Sol. Let the present ages of the father and his son be x years and y years respectively. Then according to the problem, we have

$$x - 10 = 12(y - 10) \qquad \ldots(1)$$

and $\quad x + 10 = 2(y + 10) \qquad \ldots(2)$

From (1), we get

$$x - 10 = 12y - 120$$

$\Rightarrow \quad x = 12y - 110 \quad \ldots(3)$

$\therefore$ From (2) and (3), we get

$$x + 10 = 2y + 20$$

$\Rightarrow \quad 12y - 110 + 10 = 2y + 20$

$\Rightarrow \quad 10y = 20 - 10 + 110$

$\Rightarrow \quad 10y = 120$

$\Rightarrow \quad y = \frac{120}{10} = 12$

$\therefore$ From (3), $x = 12 \times 12 - 110 = 144 - 110 = 34$

Hence, the required present ages of the father and his son are 34 years and 12 years respectively.

Short Answer Type Questions

7. The sum of the reciprocals of Varun's age (in years) 3 years ago and 5 years from now is $\frac{1}{3}$. Find his present age. **[CBSE 2023 Basic]**

Sol. Let the present age of Varun be x years.

Varun's age 3 years ago $= x - 3$

Varun's age 5 years hence $= x + 5$

According to the question,

$$\frac{1}{x-3} + \frac{1}{x+5} = \frac{1}{3}$$

$\Rightarrow \quad \frac{x+5+x-3}{(x-3)(x+5)} = \frac{1}{3}$

$\Rightarrow \quad \frac{2x+2}{x^2+2x-15} = \frac{1}{3}$

$\Rightarrow \quad 6x + 6 = x^2 + 2x - 15$

$\Rightarrow \quad x^2 - 4x - 21 = 0$

$\Rightarrow \quad x^2 - 7x + 3x - 21 = 0$

$\Rightarrow \quad x(x - 7) + 3(x - 7) = 0$

$\Rightarrow \quad (x - 7)(x + 3) = 0$

$\Rightarrow \quad x = 7$ or $x = -3$

Since age cannot be negative, $\therefore$ $x = 7$ years.

8. Half of the difference between two numbers is 2. The sum of the greater number and twice the smaller number is 13. Find the numbers. **[CBSE 2023 Standard]**

Sol. Let the two numbers be x and y. Let x be the great number.

Given that half the difference between two number is 2.

$$\frac{1}{2}(x - y) = 2$$

$\Rightarrow \quad x - y = 4 \quad \ldots(1)$

Again,

$$x + 2y = 13 \quad \ldots(2)$$

Subtracting equation (1) from equation (2), we get

$$\begin{array}{r} x + 2y = 13 \\ x - \;\; y = 4 \\ \underline{- \quad + \quad -} \\ 3y = 9 \end{array}$$

$\Rightarrow \quad y = 3$

Putting $y = 3$ in equation (1),

$$x - 3 = 4$$

$\Rightarrow \quad x = 7$

$\therefore$ The greater number is 7 and the smaller number is 3.

9. Two people are 16 km apart on a straight road. They start walking at the same time. If they walk towards each other with different speeds, they will meet in 2 hours. Had they walked in the same direction with same speeds as before, they would have met in 8 hours. Find their walking speeds. **[CBSE 2023 Standard]**

Sol. Let the speed of the two people be x km/h and y km/h.

Relative speed when both the people walk towards each other $= x + y$

$$\frac{\text{Distance}}{\text{Speed}} = \text{Time}$$

$$\frac{16}{x+y} = 2$$

$\Rightarrow \quad x + y = 8 \quad \ldots(1)$

Relative speed when both the people walk in the same direction $= x - y$

$$\frac{16}{x-y} = 8$$

$\Rightarrow \quad x - y = 2 \quad \ldots(2)$

Adding equations (1) and (2)

$$2x = 10$$

$\Rightarrow \quad x = 5$ km/h

Putting value of x in equation (1)

$$y = 8 - 5$$

$\Rightarrow \quad y = 3$ km/h

10. There is a number consisting of two digits. The number is equal to three times the sum of its digits and if it is multiplied by 3, then the result will be equal to the square of the sum of its digits. Find the number.

Sol. Let the digit in the unit's place of the number be x and that in the ten's place be y. Then the number is $10y + x$.

According to the first condition of the problem, we have

$$10y + x = 3(x + y) \quad \ldots(1)$$

According to the second condition of the problem, we have

$$3(10y + x) = (x + y)^2 \quad \ldots(2)$$

Dividing (2) by (1), we get

$$3 = \frac{(x+y)^2}{3(x+y)} = \frac{x+y}{3}$$

$\Rightarrow \quad x + y = 9$

$\Rightarrow \quad y = 9 - x \quad \ldots(3)$

$\therefore$ From (1) and (3), we get

$10(9 - x) + x = 3 \times 9$

$\Rightarrow \quad 90 - 9x = 27$

$\Rightarrow \quad 9x = 63$

$\Rightarrow \quad x = \frac{63}{9} = 7$

$\therefore$ From (3), $\quad y = 9 - 7 = 2$

Hence, the number is $10 \times 2 + 7$ i.e. 27.

11. The denominator of a fraction exceeds the numerator by 4 and if 5 is taken away from each, the sum of the reciprocal of the new fraction and 4 times the original fraction is 5. Find the original fraction.

Sol. Let the numerator and the denominator of the fraction be x and y respectively, where $y \neq 0$. Then the original fraction is $\frac{x}{y}$.

$\therefore$ According to the problem, we have

$$y = x + 4 \quad \ldots(1)$$

and $$\frac{y-5}{x-5} + \frac{4x}{y} = 5 \quad \ldots(2)$$

$\therefore$ From (1) and (2), we have

$$\frac{x+4-5}{x-5} + \frac{4x}{x+4} = 5$$

$\Rightarrow \quad \frac{(x+4-5)(x+4) + 4x(x-5)}{(x+4)(x-5)} = 5$

$\Rightarrow \quad (x - 1)(x + 4) + 4x(x - 5) = 5(x + 4)(x - 5)$

$\Rightarrow \quad x^2 + 4x - x - 4 + 4x^2 - 20x = 5(x^2 - x - 20)$

$\Rightarrow \quad 5x^2 - 17x - 4 = 5x^2 - 5x - 100$

$\Rightarrow \quad 12x = 96$

$\Rightarrow \quad x = \frac{96}{12} = 8$

$\therefore$ From (1), $\quad y = 8 + 4 = 12$

$\therefore$ The required fraction is $\frac{8}{12}$.

Long Answer Type Questions

12. A 2-digit number is four times the sum of its digits and twice the product of its digits. Find the number. **[CBSE 2023 Basic]**

Sol. Let the digits in the unit's place and the ten's place be x and y respectively. Then the number is $10y + x$.

According to the first condition of the problem, we have

$10y + x = 4(x + y)$

$\Rightarrow \quad 10y + x = 4x + 4y$

$\Rightarrow \quad 6y - 3x = 0$

$\Rightarrow \quad x = 2y \quad \ldots(1)$

According to the second condition of the problem, we have

$$10y + x = 2xy$$

Dividing both sides by xy, we get

$$\frac{10}{x} + \frac{1}{y} = 2$$

Putting $x = 2y$ from (1)

$$\frac{10}{2y} + \frac{1}{y} = 2$$

$\Rightarrow \quad \frac{5}{y} + \frac{1}{y} = 2$

$\Rightarrow \quad \frac{5+1}{y} = 2$

$\Rightarrow \quad y = \frac{6}{2} = 3$

$\Rightarrow \quad x = 2y = 6$

The two digit number $= 10y + x$

$= 10(3) + 6 = 36$

13. Five years ago, Amit was thrice as old as Baljeet. Ten years hence, Amit shall be twice as old as Baljeet. What are their present ages? **[CBSE 2023 Basic]**

Sol. Let Amit's and Baljeet's present ages be x years and y years respectively. Then according to the first condition of the problem,

$(x - 5) = 3(y - 5)$

$\Rightarrow \quad x - 5 = 3y - 15$

$\Rightarrow \quad x - 3y = -10$

$\Rightarrow \quad x = 3y - 10 \quad ...(1)$

According to the second condition of the problem,

$(x + 10) = 2(y + 10)$

$\Rightarrow \quad x + 10 = 2y + 20 \quad ...(2)$

Putting the value of x from (1) in equation (2)

$3y - 10 + 10 = 2y + 20$

$\Rightarrow \quad y = 20$

$\Rightarrow \quad x = 3y - 10$

$\Rightarrow \quad x = 60 - 10$

$\Rightarrow \quad x = 50$

Therefore, present ages of Amit and Baljeet are 50 years and 20 years respectively.

14. A train covered a certain distance at a uniform speed. If the train would have been 12 km/h faster, it would have taken 2 hours 40 minutes less time than the schedule time and if the train were slower by 12 km/h, it would have taken 4 hours more than the schedule time. Find the length of the journey and the speed of the train.

Sol. Let the distance of the journey be x km and the original speed of the train be y km/h.

Then the actual time taken by the train, say $t = \frac{x}{y}$ hours.

When the train moves 12 km/h faster, then let the time taken by the train be t_1 hours.

$\therefore \quad t_1 = \frac{x}{y + 12}$ hours

According to the problem, we have

$$t - t_1 = 2\frac{40}{60} \text{ hours}$$

$$= 2\frac{2}{3} \text{ hours}$$

$$= \frac{8}{3} \text{ hours}$$

$\therefore \quad \frac{x}{y} - \frac{x}{y + 12} = \frac{8}{3} \quad ...(1)$

When the train moves 12 km/h slower, let the time taken by the train be t_2 hours.

$\therefore \quad t_2 = \frac{x}{y - 12}$ hours

$\therefore \quad t_2 - t = 4$ hours

$\therefore \quad \frac{x}{y - 12} - \frac{x}{y} = 4 \quad ...(2)$

From (1), we have

$$\frac{x(y + 12 - y)}{y(y + 12)} = \frac{8}{3}$$

$\Rightarrow \quad \frac{12x}{y(y + 12)} = \frac{8}{3}$

$\Rightarrow \quad \frac{3x}{y(y + 12)} = \frac{2}{3}$

$\Rightarrow \quad 2y(y + 12) = 9x \quad ...(3)$

From (2), we have

$$\frac{x(y - y + 12)}{y(y - 12)} = 4$$

$\Rightarrow \quad \frac{12x}{y(y - 12)} = 4$

$\Rightarrow \quad \frac{3x}{y(y - 12)} = 1$

$\therefore \quad y(y - 12) = 3x \quad ...(4)$

Dividing (3) by (4), we get

$$\frac{2(y + 12)}{y - 12} = 3$$

$\Rightarrow \quad 2y + 24 = 3y - 36$

$\Rightarrow \quad y = 24 + 36 = 60$

$\therefore$ From (4), $\quad 3x = 60(60 - 12) = 60 \times 48$

$\Rightarrow \quad x = 20 \times 48 = 960$

Hence, the length of the journey is 960 km and the speed of the train is 60 km/h.

15. A vessel contains a mixture of 24 L milk and 6 L water and the second vessel contains a mixture of 15 L milk and 10 L water. How much mixture of milk and water should be taken from the first and the second vessels separately and kept in a third vessel so that the third vessel may contain a mixture of 25 L milk and 10 L water?

Sol. Let x L of the mixture from the first vessel be mixed with y L of the mixture in the second vessel.

Now, in the first vessel, out of $(24 + 6)$ L = 30 L of the mixture, the amount of milk is 24 L and that of water is 6 L.

Hence, x L of the mixture from this vessel contain $\frac{24x}{30}$ L = $\frac{4x}{5}$ L of milk and $\frac{6x}{30}$ L = $\frac{x}{5}$ L of water.

In the second vessel, out of $(15 + 10)$L = 25 L of the mixture, the amount of milk is 15 L and that of water is 10 L.

Hence, y L of the mixture from this vessel contains $\frac{15y}{25}$ L = $\frac{3y}{5}$ L of milk and $\frac{10y}{25}$ L = $\frac{2y}{5}$ L of water.

$\therefore$ The total amount of milk in the third vessel

$$= \left(\frac{4x}{5} + \frac{3y}{5}\right) \text{ L}$$

$$= \frac{4x + 3y}{5} \text{ L}$$

$\therefore$ The total amount of water in this vessel is

$$\left(\frac{x}{5} + \frac{2y}{5}\right) \text{ L} = \frac{x + 2y}{5} \text{ L}$$

$\therefore$ According to the problem, we have

$$\frac{4x + 3y}{5} = 25$$

$\Rightarrow \quad 4x + 3y - 125 = 0 \qquad \ldots(1)$

and $\quad \frac{x + 2y}{5} = 10$

$\Rightarrow \quad x + 2y - 50 = 0 \qquad \ldots(2)$

Multiplying equation (2) by 4 and subtracting from equation (1), we get

$$\begin{array}{r} 4x + 3y - 125 = 0 \\ 4x + 8y - 200 = 0 \\ - \quad - \quad + \quad \\ \hline -5y + 75 = 0 \end{array}$$

$\Rightarrow \quad y = \frac{75}{5} = 15$

Putting $y = 15$ in equation (2), we get

$$x + 30 - 50 = 0$$

$\Rightarrow \quad x = 20$

$\therefore \quad x = 20, y = 15$

Hence, the required amount of mixture to be taken from the first vessel and the second vessel are 20 L and 15 L respectively.

Higher Order Thinking Skills (HOTS) Questions

(Page 47)

1. Determine graphically the vertices of the triangle, the equations of whose sides are given below:

$$y = x, y = 0 \text{ and } 3x + 3y = 10$$

Sol. We first find some values of x and y from the given equation

$y = x \qquad \ldots(1)$

and $\quad y = \frac{10}{3} - x \qquad \ldots(2)$

and list them in Table 1 and 2 respectively.

Table 1

x	$\frac{5}{3}$	0	$\frac{1}{3}$
y	$\frac{5}{3}$	0	$\frac{1}{3}$

Table 2

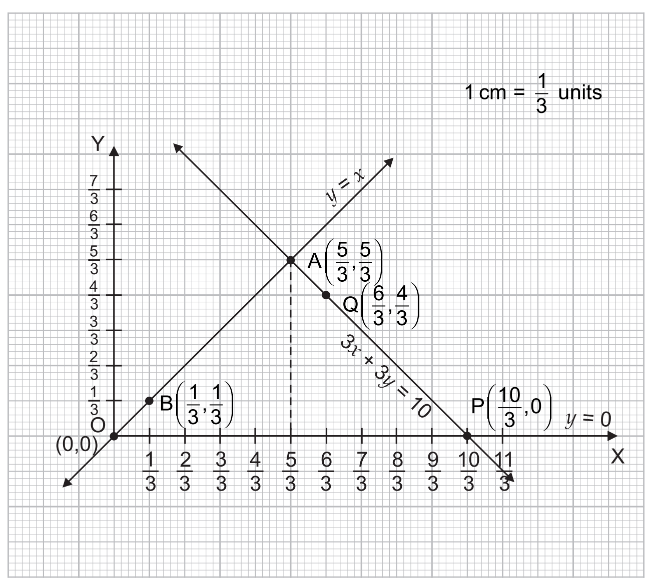
x	$\frac{5}{3}$	$\frac{6}{3}$	$\frac{10}{3}$
y	$\frac{5}{3}$	$\frac{4}{3}$	0

We now plot the points $A\left(\frac{5}{3}, \frac{5}{3}\right)$, $B\left(\frac{1}{3}, \frac{1}{3}\right)$ and O(0, 0) from Table 1 and join them by a straight line ABO.

We next plot the points $P\left(\frac{10}{3}, 0\right)$, $A\left(\frac{5}{3}, \frac{5}{3}\right)$ and $Q\left(\frac{6}{3}, \frac{4}{3}\right)$ from Table 2 and join them by a straight line PQA.

Now, $y = 0$ is the x-axis. The point of intersection of the line ABO with x-axis is O(0, 0) and that between the line PQA and the x-axis is $\left(\frac{10}{3}, 0\right)$.

From the graph, we see that the coordinates of the vertices A, O and B of ΔAOB are $\left(\frac{5}{3}, \frac{5}{3}\right)$, (0, 0) and $\left(\frac{10}{3}, 0\right)$ respectively.



2. Solve the following pair of linear equations by any suitable method:

$$\frac{2}{13}(2x+3y) = 3+\frac{x-y}{4}$$

$$\frac{4y+5x}{3} = 2x+7\frac{1}{6}$$

Sol. We have

$$\frac{2}{13}(2x+3y) = 3+\frac{x-y}{4}$$

$\Rightarrow$ $$\frac{4x+6y}{13} = \frac{12+x-y}{4}$$

$\Rightarrow$ $$16x + 24y = 156 + 13x - 13y$$

$\Rightarrow$ $$16x - 13x + 24y + 13y - 156 = 0$$

$\Rightarrow$ $$3x + 37y - 156 = 0 \quad \ldots(1)$$

Also, $$\frac{4y+5x}{3} = 2x+7\frac{1}{6}$$

$\Rightarrow$ $$\frac{4y+5x}{3} = 2x+\frac{43}{6}$$

$\Rightarrow$ $$\frac{4y+5x}{3} = \frac{12x+43}{6}$$

$\Rightarrow$ $$24y + 30x = 36x + 129$$

$\Rightarrow$ $$36x - 30x - 24y + 129 = 0$$

$\Rightarrow$ $$6x - 24y + 129 = 0$$

$\Rightarrow$ $$2x - 8y + 43 = 0 \quad \ldots(2)$$

Multiplying equation (1) by 2 and equation (2) by 3 and subtracting equation (2) from equation (1), we get

$$\begin{array}{r} 6x + 74y - 312 = 0 \\ 6x - 24y + 129 = 0 \\ - \quad + \quad - \quad\quad \\ \hline 98y - 441 = 0 \end{array}$$

$\Rightarrow$ $$y = \frac{441}{98} = \frac{9}{2}$$

Putting $y = \frac{9}{2}$ in equation (1), we get

$$3x + \frac{333}{2} - 156 = 0$$

$$3x + \frac{21}{2} = 0$$

$$x = -\frac{7}{2}$$

Hence, the required solution is $x = -\frac{7}{2}, y = \frac{9}{2}$.

3. Solve the following pair of linear equations by any suitable method:

$$119x - 381y = 643$$

$$381x - 119y = -143$$

Sol. We have

$$119x - 381y = 643 \quad \ldots(1)$$

and $$381x - 119y = -143 \quad \ldots(2)$$

Adding (1) and (2), we get

$$500x - 500y = 500$$

$\Rightarrow$ $$x - y = 1$$

$\Rightarrow$ $$y = x - 1 \quad \ldots(3)$$

$\therefore$ From (1) and (3), we get

$$119x - 381(x - 1) = 643$$

$\Rightarrow$ $$119x - 381x = 643 - 381$$

$\Rightarrow$ $$-262x = 262$$

$\Rightarrow$ $$x = -1$$

$\therefore$ From (3), $y = -1 -1 = -2$

Hence, the required solution is $x = -1, y = -2$.

4. Prove that the pair of linear equations $2(k + 2)x + ky + 5 = 0$ and $kx - 3(k + 2)y - 8 = 0$ cannot have an infinite number of solutions for any real values of k. Also, prove that the equations will have a unique solution for all real values of k.

Sol. We know that the given equations may have an infinite number of solutions if

$$\frac{2(k+2)}{k} = \frac{k}{-3(k+2)} = -\frac{5}{8} \quad \ldots(1)$$

From (1), we see that

$$\frac{2(k+2)}{k} = -\frac{5}{8}$$

$\Rightarrow$ $$16k + 32 = -5k$$

$\Rightarrow$ $$21k = -32$$

$\Rightarrow$ $$k = -\frac{32}{21} \quad \ldots(2)$$

Also, $$\frac{k}{-3(k+2)} = -\frac{5}{8}$$

$\Rightarrow$ $$8k = 15k + 30$$

$\Rightarrow$ $$7k + 30 = 0$$

$\Rightarrow$ $$k = -\frac{30}{7} \quad \ldots(3)$$

From (2) and (3), we see that

$$k = -\frac{32}{21} = -\frac{30}{7}$$

which is absurd.

Hence, it follows that the two given equations cannot have an infinite number of solutions for any real value of k.

Also, the given equation will have a unique solution if

$$\frac{2(k+2)}{k} \neq \frac{k}{-3(k+2)}$$

$\Rightarrow \quad -6(k+2)^2 \neq k^2$

which is always true for all real values of k. Hence, the given equations can have a unique solution for all real values of k.

Self-Assessment

(Page 47)

Multiple-Choice Questions

1. If the pair of the linear equations is consistent, then the lines will be
   (*a*) always intersecting.
   (*b*) always coincident.
   (*c*) intersecting or coincident.
   (*d*) parallel.

Sol. (*c*) intersecting or coincident.

We know that two linear equations are consistent if they have either a finite number of solutions or infinite number of solutions. In the former case, the two lines will be intersecting at a single point and in the second case, the two lines will be coincident.

2. The point of intersection of the lines $y = 3x$ and $x = 3y$ is
   (*a*) (3, 0) (*b*) (0, 3)
   (*c*) (3, 3) (*d*) (0, 0) **[CBSE SP 2011]**

Sol. (*d*) (0, 0)

From the two given equations, we have

$$y = 3 \times 3y = 9y$$

$\Rightarrow \quad 8y = 0$

$\Rightarrow \quad y = 0$

$\therefore$ From the second equation, $x = 0$.

$\therefore$ The required point of intersection of the two given lines is (0, 0).

3. If $\frac{2}{x}+\frac{3}{y} = 13$ and $\frac{5}{x}-\frac{4}{y} = -2$, then $x + y$ equals
   (*a*) $\frac{5}{6}$ (*b*) $-\frac{5}{6}$
   (*c*) $-\frac{1}{6}$ (*d*) $\frac{1}{6}$ **[CBSE SP 2011]**

Sol. (*a*) $\frac{5}{6}$

Let $\frac{1}{x} = u$ and $\frac{1}{y} = v$

$\therefore$ The given equation $\frac{2}{x} + \frac{3}{y} = 13$, becomes

$$2u + 3v = 13 \qquad \ldots(1)$$

Similarly, the given equation $\frac{5}{x} - \frac{4}{y} = -2$, becomes

$$5u - 4v = -2 \qquad \ldots(2)$$

Multiplying equation (1) by 4 and equation (2) by 3. Adding the two equations, we get

$$\begin{array}{rl} 8u + 12v &= 52 \\ 15u - 12v &= -6 \\ \hline 23u \qquad &= 46 \end{array}$$

$\Rightarrow \quad u = \frac{46}{23} = 2$

Putting value of $u = 2$ in equation (1), we get

$$4 + 3v = 13$$

$\Rightarrow \quad 3v = 9$

$\Rightarrow \quad v = 3$

$\therefore \quad u = 2, v = 3$

$\therefore \quad \frac{1}{x} = u = 2$

$\Rightarrow \quad x = \frac{1}{2}$

and $\quad \frac{1}{y} = v = 3$

$\Rightarrow \quad y = \frac{1}{3}$

$\therefore$ The value of $x + y$ is $\left(\frac{1}{2}+\frac{1}{3}\right)$ i.e. $\frac{5}{6}$.

4. If the pair of linear equations $4x - 3y = 1$ and $kx + 6y = 3$ have no solution at all, then the value of k will be
   (*a*) 8 (*b*) −8
   (*c*) 7 (*d*) −7

Sol. (*b*) −8

If the two given equations do not have any solution, then these two equations will represent two parallel lines.

Hence, we have $\frac{4}{k} = \frac{-3}{6} \neq \frac{1}{3}$

$\therefore \quad k = -8 \qquad \left(\because -\frac{3}{6} \neq \frac{1}{3} \text{ is true}\right)$

5. The pair of linear equations $x + 2y + 5 = 0$ and $-3x - 6y + 1 = 0$ has
   (*a*) a unique solution.
   (*b*) exactly two solutions.
   (*c*) infinitely many solutions.
   (*d*) no solution. **[CBSE 2023 Basic]**

Sol. (*d*) no solution.

The pair of linear equations is

$$x + 2y + 5 = 0$$
$$-3x - 6y + 1 = 0$$
$$a_1 = 1 \quad b_1 = 2 \quad c_1 = 5$$
$$a_2 = -3 \quad b_2 = -6 \quad c_2 = 1$$
$$\frac{a_1}{a_2} = \frac{-1}{3} \quad \frac{b_1}{b_2} = \frac{2}{-6} = \frac{-1}{3} \quad \frac{c_1}{c_2} = \frac{5}{1}$$

Thus, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Therefore, the pair of linear equations do not have any solution.

6. The value of k, if $(6, k)$ lies on the line represented by $x - 3y + 6 = 0$ is

(*a*) –4 (*b*) 12
(*c*) –12 (*d*) 4

Sol. (*d*) 4

The point $(6, k)$ lies on the line $x - 3y + 6 = 0$

$\Rightarrow \quad 6 - 3(k) + 6 = 0$

$\Rightarrow \quad 12 - 3k = 0$

$\Rightarrow \quad k = 4$

7. The pair of equations $ax + 2y = 9$ and $3x + by = 18$ represent parallel lines, where a, b are integers, if

(*a*) $a = b$
(*b*) $3a = 2b$
(*c*) $2a = 3b$
(*d*) $ab = 6$ **[CBSE 2023 Standard]**

Sol. (*d*) $ab = 6$

$$a_1 = a \quad b_1 = 2 \quad c_1 = -9$$
$$a_2 = 3 \quad b_2 = b \quad c_2 = -18$$

For lines to be parallel,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$\Rightarrow \quad \frac{a}{3} = \frac{2}{b}$

$\Rightarrow \quad ab = 6$

Fill in the Blanks

8. If $(6, k)$ is a solution of the equation $3x + y - 22 = 0$, then the value of k is **4**.

Sol. Since $(6, k)$ is a solution of $3x + y - 22 = 0$

$\therefore \quad 3(6) + k - 22 = 0$

$\Rightarrow \quad 18 + k - 22 = 0$

$\Rightarrow \quad k = 22 - 18$

$\Rightarrow \quad k = 4$

9. The value of α for which the pair of equations $3x + \alpha y = 6$ and $6x + 8y = 7$ has no solution is **4**.

Sol. For no solution,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$\therefore \quad \frac{3}{6} = \frac{\alpha}{8} \neq \frac{-6}{-7}$

$$\alpha = 4$$

10. If the lines given by $3x - 4y + 7 = 0$ and $kx + 3y - 5 = 0$ are parallel, then the value of k is **–2.25**.

Sol. The graphs of given pair of linear equations are parallel lines when these equations have no solution.

For no solution,

$\Rightarrow \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\Rightarrow \quad \frac{3}{k} = \frac{-4}{3} \neq \frac{7}{-5}$

$\Rightarrow \quad k = \frac{-9}{4} = -2.25$

11. If the sum of two numbers is 6 and their difference is 4, then the numbers are **5** and **1**.

Sol. Let the two numbers be x and y where $x > y$.

Then, $x + y = 6$ ...(1)

and $x - y = 4$ ...(2)

Solving (1) and (2), we get $x = 5$ and $y = 1$

Hence, the numbers are 5 and 1.

Assertion-Reason Type Questions

Directions (Q. Nos. 12 to 14): Each of these questions contains an assertion followed by reason. Read them carefully, and answer the question on the basis of the following options, select the one that best describes the two statements.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

12. **Assertion (A):** $2x + 3y = 8$ is a linear equation in two variables.

Reason (R): 2 and 3 are variables.

Sol. The correct answer is (*c*).

$2x + 3y = 8$ is a linear equation in two variables as it has two variables x and y with non-zero is efficient. 2 and 3 are not variables.

Thus, Assertion is correct while Reason is wrong.

13. **Assertion (A):** (2, 3) is a solution of the equation $2x + 3y = 13$.

Reason (R): By substituting 2 and 3 for x and y, we get 13 in LHS.

Sol. The correct answer is (*a*).

Both the statements are correct. By substituting 2 and 3 for x and y, we get 13 in LHS. which is equal to RHS. That's why (2, 3) is a solution. Thus, the Reason is correct explanation of the Assertion.

14. **Assertion (A):** (1, 3) is a point of the line represented by $x + y = 8$.

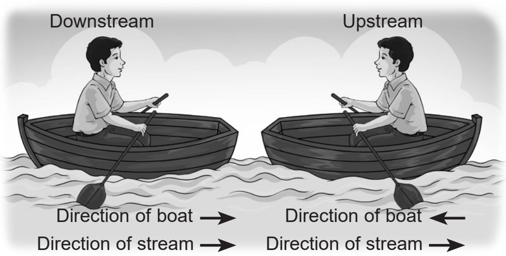
Reason (R): $1 + 3 = 4$

Sol. The correct answer is (*d*).

The Reason is correct. The Assertion is wrong as 1, 3 does not satisfy the equation $x + y = 8$.

Case Study Based Questions

15. A boatman rows his boat 64 km downstream in 4 hours and 48 km upstream in 4 hours.



Based on the above information, answer the following questions.

(*a*) If the speed of the boat in the still water be x km/h and the speed of the current be y km/h, then form the pair of linear equations representing the situation.

Ans. $x + y = 16$, $x - y = 12$

(*b*) What is the relation between speed, distance and time?

Ans. Speed = Distance/Time

(*c*) (*i*) What is the boatman's speed of rowing in still water?

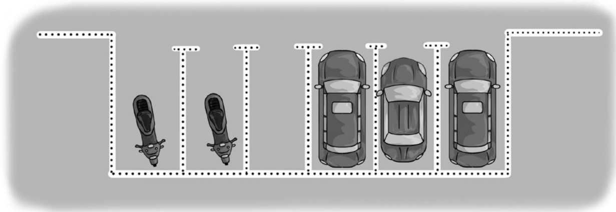
Ans. 14 km/h

or

(*ii*) What is the speed of the current?

Ans. 2 km/h

16. A parking lot is an area that is assigned for parking. Parking lots are common near shops, bars, restaurants and malls, etc. In a parking lot, parking charges for 20 cars and 15 scooters is ₹ 475 and parking charges for 65 cars and 12 scooters is ₹ 1360. Based on the above information, answer the following questions.



(*a*) If the parking charge for one car be ₹ x and the parking charge for one scooter be ₹ y, form the pair of linear equations representing the situation.

Ans. $20x + 15y = 475$, $65x + 12y = 1360$

(*b*) What is the parking charge for one car?

Ans. ₹ 20

(*c*) (*i*) What is the parking charge for one scooter?

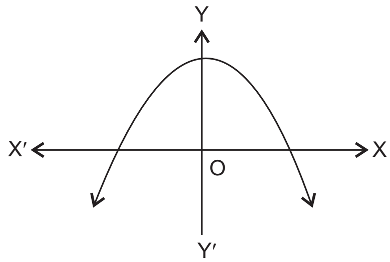
Ans. ₹ 5

or

(*ii*) What is the parking charge for 5 cars and 5 scooters?

Ans. ₹ 125

17. Rainbow is an arch of colours that is visible in the sky after rain or when water droplets are present in the atmosphere. The colours of the rainbow are generally, red, orange, yellow, green, blue, indigo and violet. Each colour of the rainbow makes a parabola. We know that any quadratic polynomial $p(x) = ax^2 + bx + c$ ($a \neq 0$) represents a parabola on the graph paper.



Based on the above, answer the following questions:

(*a*) The graph of a rainbow $y = f(x)$ is shown in the figure. Write the number of zeroes of the curve.

Ans. The graph cuts the x-axis at two points. Therefore, the curve has two zeroes.

(*b*) If the graph of a rainbow does not intersect the x-axis but intersects y-axis at one point, then how many zeroes will it have?

Ans. If the graph does not intersect, the x-axis, then it will have no zeroes.

(*c*) (*i*) If a rainbow is represented by the quadratic polynomial $p(x) = x^2 + (a + 1)\,x + b$, whose zeroes are 2 and –3, find the values of a and b.

Ans. $p(x) = x^2 + (a + 1)\,x + b$

$a_1 = 1$

$b_1 = (a + 1)$

$c_1 = b$

Roots are 2 and –3

Sum of the roots = 2 + (–3) = –1

$$\frac{-b}{a} = -1$$

$$\Rightarrow \quad \frac{-(a+1)}{1} = -1$$

$$\Rightarrow \quad -(a + 1) = -1$$

$$\Rightarrow \quad -a - 1 = -1$$

$$\Rightarrow \quad a = 0$$

Product of the roots = 2 × –3 = –6

$$\frac{c}{a} = -6$$

$$\Rightarrow \quad b = -6$$

Hence, $a = 0$ and $b = -6$

or

(*ii*) The polynomial $x^2 - 2x - (7p + 3)$ represents a rainbow. If –4 is a zero of it, find the value of p. **[CBSE 2023 Basic]**

Ans. $p(x) = x^2 - 2x - (7p + 3)$

$$p(-4) = 0$$

$$(-4)^2 - 2 \times (-4) - [7p + 3] = 0$$

$$\Rightarrow \quad 16 + 8 - 7p - 3 = 0$$

$$\Rightarrow \quad 21 - 7p = 0$$

$$\Rightarrow \quad -7p = -21$$

$$\Rightarrow \quad p = 3$$

Very Short Answer Type Questions

18. Find a common point on the lines $4x - 5 = y$ and $2x - y = 3$.

Sol. The required common point is the unique point of intersection of the two given lines.

Solving the two equation, we get

$$2x - (4x - 5) = 3$$

$$\Rightarrow \quad 2x - 4x + 5 = 3$$

$$\Rightarrow \quad -2x = -2$$

$$\therefore \quad x = 1$$

∴ From the first equation, we have

$$y = 4x - 5$$

$$= 4 - 5$$

$$= -1$$

∴ The required point of intersection is (1, –1).

19. Find the value of p so that the lines represented by $3x - 4y + 7 = 0$ and $px + 3y - 5 = 0$ are parallel.

Sol. If the two lines are parallel, then

$$\frac{3}{p} = -\frac{4}{3} \neq \frac{7}{-5}$$

$$\therefore \quad 4p = -9$$

$$\therefore \quad p = -\frac{9}{4}$$

Since $-\frac{4}{3} \neq \frac{-7}{5}$ is true.

Hence, the required value of p is $-\frac{9}{4}$.

20. Find the coordinates of the point of intersection of the line $3x + 2y + 12 = 0$ with the coordinate axes.

Sol. Putting $y = 0$ in the equation $3x + 2y + 12 = 0$,

we get $\quad x = -\frac{12}{3} = -4$

Again, putting $x = 0$ within same equation, we get

$$y = -\frac{12}{2} = -6$$

Hence, the required point of intersection of the given line with x-axis is $(-4, 0)$ and that with y-axis is $(0, -6)$.

21. Solve the following pair of linear equations by the method of elimination.

$$2x - 3y = 1;\ 5 = x - 5y$$

Sol. We have

$$2x - 3y - 1 = 0 \quad \ldots(1)$$

and $$x - 5y - 5 = 0 \quad \ldots(2)$$

Multiplying equation (2) by 2 and subtracting from equation (1).

$$\begin{array}{r} 2x - \ \ 3y - \ \ 1 = 0 \\ 2x - 10y - 10 = 0 \\ - \quad + \quad\ \ + \quad\quad \\ \hline 7y + \ \ 9 = 0 \end{array}$$

$$\Rightarrow \quad y = \frac{-9}{7}$$

Putting $y = \frac{-9}{7}$ in equation (2), we get

$$x + \frac{45}{7} - 5 = 0 \Rightarrow x = \frac{-10}{7}$$

The required solution is

$$x = \frac{-10}{7},\ y = \frac{-9}{7}.$$

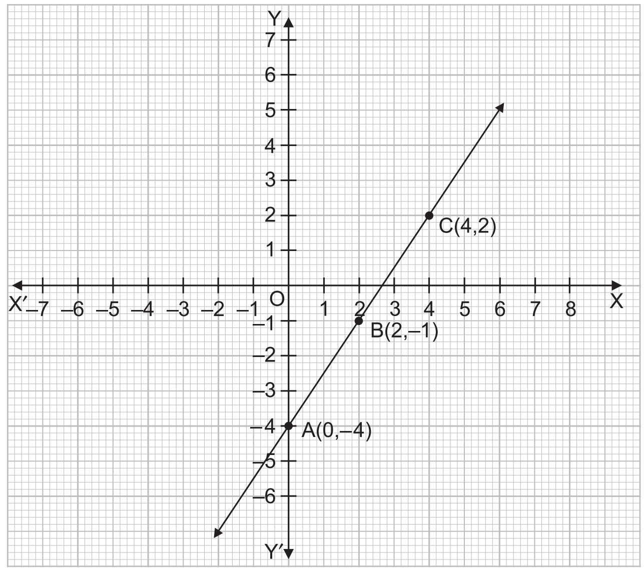
Short Answer Type Questions

22. Show graphically that the system of equations $3x - 2y = 8$ and $6x - 4y = 16$ is consistent, but they have infinitely many solutions.

Sol. We see that the two given equation $3x - 2y = 8$ and $6x - 4y = 16$ represent the same equation, since dividing both sides of the second equation by 2, we get the first equation. Hence, both the equations will represent the same line. In other words, both the lines will be coincident and so they will have infinitely many solutions. To draw the graph of the given equation, we write, it as $y = \frac{3x - 8}{2}$. We tabulate some values of x and y as follows:

x	0	2	4
y	-4	-1	2

First we plot the points A(0, −4), B(2, −1) and C(4, 2) on a graph paper, then join them by a straight line ABC as shown in the given figure.



23. The sum of two numbers, as well as the difference between their squares is 25. Find the numbers.

Sol. Let the two numbers be x and y, where $x > y$. Then according to the problem, we have

$$x + y = 25 \quad \ldots(1)$$

and $$x^2 - y^2 = 25 \quad \ldots(2)$$

From (2), we have

$$(x + y)(x - y) = 25$$

$$\Rightarrow \quad 25(x - y) = 25 \quad \text{[From (1)]}$$

$$\Rightarrow \quad x - y = 1 \quad \ldots(3)$$

Adding (1) and (3), we get

$$2x = 26$$

$$\Rightarrow \quad x = \frac{26}{2} = 13$$

Subtracting (3) from (1), we get

$$2y = 24$$

$$\Rightarrow \quad x = \frac{24}{2} = 12$$

$\therefore$ The required two numbers are 13 and 12.

24. If the system of linear equations $2x + 3y = 7$ and $2ax + (a + b)y = 28$ have infinite number of solutions, then find the value of 'a' and 'b'.

[CBSE 2023 Standard]

Sol. The given system of linear equations can be written as,

$$2x + 3y - 7 = 0 \quad \ldots(1)$$

$$2ax + (a + b)\,y - 28 = 0 \quad \ldots(2)$$

$$a_1 = 2 \quad b_1 = 3 \quad c_1 = -7$$

$$a_2 = 2a \quad b_2 = (a + b) \quad c_2 = -28$$

The given system of linear equations will have an infinite number of solutions, when

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\Rightarrow \quad \frac{2}{2a} = \frac{3}{(a+b)} = \frac{-7}{-28}$$

$$\frac{2}{2a} = \frac{-7}{-28}$$

$$\Rightarrow \quad \frac{1}{a} = \frac{1}{4}$$

$$\Rightarrow \quad a = 4$$

$$\left(\frac{3}{a+b}\right) = \frac{-7}{-28}$$

$$\Rightarrow \quad \frac{3}{(a+b)} = \frac{1}{4}$$

$$\Rightarrow \quad 12 = a + b$$

$$\Rightarrow \quad b = 12 - 4 = 8$$

Hence, $a = 4$ and $b = 8$.

25. In a chemistry laboratory, there is some quantity of 50% acid solution and some quantity of 25% acid solution. How much of each should be mixed to make 10 litres of 40% solution.

[CBSE 2024 Standard]

Sol. Let x litres of 50% acid solution and y litres of 25% acid solution are mixed.

$$x + y = 10 \quad \text{...(1)}$$

$$\frac{50}{100}x + \frac{25}{100}y = \frac{40}{100} \times 10$$

$$\Rightarrow \quad \frac{1}{2}x + \frac{1}{4}y = 4$$

$$\Rightarrow \quad 2x + y = 16 \quad \text{...(2)}$$

Subtracting equation (1) from equation (2), we have

$$x = 6$$

Putting $x = 6$ in eqn. (1)

$$6 + y = 10$$

$$\Rightarrow \quad y = 4$$

Hence, 6 litres of 50% acid solution and 4 litres of 25% acid solution are mixed.

Long Answer Type Questions

26. A man sold a chair and a table together for ₹760, thereby making a profit of 25% on chair and 10% on table. By selling them together for ₹767.50, he would have made a profit of 10% on the chair and 25% on the table. Find the cost price of each object.

Sol. Let the cost price of each chair be ₹x and that of each table be ₹y.

Then, the SP of the chair $= ₹\left(x + \frac{25x}{100}\right)$

$$= ₹\left(x + \frac{x}{4}\right)$$

$$= ₹\,\frac{5x}{4}$$

and the SP of the table $= ₹\left(y + \frac{10y}{100}\right)$

$$= ₹\left(y + \frac{y}{10}\right)$$

$$= ₹\,\frac{11y}{10}$$

∴ According to the problem, we have

$$\frac{5x}{4} + \frac{11y}{10} = 760$$

$$\Rightarrow \quad \frac{25x + 22y}{20} = 760$$

$$\Rightarrow \quad 25x + 22y = 15200 \quad \text{...(1)}$$

Again, if the chair is sold at a profit of 10% and the table is sold at a profit of 25%, then

the SP of the chair $= ₹\left(x + \frac{10x}{100}\right)$

$$= ₹\left(x + \frac{x}{10}\right) = ₹\,\frac{11x}{10}$$

and the SP of the table $= ₹\left(y + \frac{25y}{100}\right)$

$$= ₹\left(y + \frac{y}{4}\right) = ₹\,\frac{5y}{4}$$

∴ According to the problem, we have

$$\frac{11x}{10} + \frac{5y}{4} = 767.50 = 767\frac{1}{2} = \frac{1535}{2}$$

$$\Rightarrow \quad \frac{22x + 25y}{20} = \frac{1535}{2}$$

$$\Rightarrow \quad 22x + 25y = 15350 \quad \text{...(2)}$$

Adding (1) and (2), we have

$$47(x + y) = 30550$$

$$\Rightarrow \quad x + y = \frac{30550}{47} = 650 \quad \text{...(3)}$$

Subtracting (1) from (2), we get

$$3(-x + y) = 150$$

$$\Rightarrow \quad -x + y = \frac{150}{3} = 50 \quad \text{...(4)}$$

Adding (3) and (4), we get

$$2y = 700$$

$$\Rightarrow \qquad y = \frac{700}{2} = 350$$

Subtracting (4) from (3), we get

$$2x = 600$$

$$\Rightarrow \qquad x = \frac{600}{2} = 300$$

Hence, the required cost price of each chair is ₹300 and that of each table is ₹350.

27. A motor boat who speed is 18 km/h in still water takes 1 hour more to go 24 km upstream than to return to the same point. Find the speed of the stream and total time of the journey.

[CBSE 2023 Basic]

Sol. Let the speed of the stream be x km/h.

Speed of the motor boat in still water

$$= 18 \text{ km/h}$$

Speed of the motor boat upstream

$$= (18 - x) \text{ km/h}$$

Speed of the motor boat downstream

$$= (18 + x) \text{ km/h}$$

Time taken to go upstream $= \dfrac{24}{18 - x}$ h $= T_1$

Time taken to go downstream $= \dfrac{24}{18 + x}$ h $= T_2$

Given that $T_1 - T_2 = 1$ h

$$\frac{24}{18 - x} - \frac{24}{18 + x} = 1$$

$$\Rightarrow \qquad 24[18 + x - 18 + x] = (18)^2 - x^2$$

$$\Rightarrow \qquad 48x = 324 - x^2$$

$$\Rightarrow \qquad x^2 - 48x - 324 = 0$$

$$\Rightarrow \qquad x^2 + 54x - 6x - 324 = 0$$

$$\Rightarrow \qquad x(x + 54) - 6(x + 54) = 0$$

$$\Rightarrow \qquad (x - 6)(x + 54) = 0$$

$$\Rightarrow \qquad x = 6 \text{ or } x = -54$$

Since speed cannot be negative, therefore speed of the stream is 6 km/h.

Let's Compete

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Multiple-Choice Questions

1. The value of m for which the pair of linear equations $1 = 9x + my$ and $2 - 4y = 3x$ has exactly one solution is

(*a*) all real values

(*b*) all real values except 12

(*c*) $p = 12$ only

(*d*) $p = \pm 12$

Sol. (*b*) all real values except 12

The condition that the two given equation $9x + my - 1 = 0$ and $3x + 4y - 2 = 0$ have only one solution is $\dfrac{9}{3} \neq \dfrac{m}{4}$

$$\Rightarrow \qquad \frac{m}{4} \neq 3$$

$$\Rightarrow \qquad m \neq 3 \times 4 = 12$$

Hence, two given equations will have only one solution for all real values of m except when $m = 12$.

2. The value of k for which the pair of linear equations $kx + 3y = k - 2$ and $12x + ky = k$ has no solution at all are

(*a*) 6 and 5

(*b*) −5 and 6

(*c*) −6 and 6

(*d*) −5 and 5

Sol. (*c*) −6 and 6

The condition that the equation $kx + 3y - k + 2 = 0$ and $12x + ky - k = 0$ have no solution at all is

$$\frac{k}{12} = \frac{3}{k} \neq \frac{2 - k}{-k}$$

Now, from $\dfrac{k}{12} = \dfrac{3}{k}$, we get $k^2 = 36$

$$\Rightarrow \qquad k = \pm 6 \qquad \ldots(1)$$

and from $\dfrac{3}{k} \neq \dfrac{2 - k}{-k}$, we have $-3k \neq 2k - k^2$

$$\Rightarrow \qquad k^2 - 5k \neq 0$$

i.e. $\qquad k(k - 5) \neq 0$

$$\Rightarrow \qquad k \neq 0 \text{ or } k \neq 5$$

which is true from (1)

$\therefore$ The required values of k are −6 and 6.

3. The solution of the linear equations
$2(ax - by) + (a + 4b) = 0$
and $2(bx + ay) + (b - 4a) = 0$ is

(*a*) $x = -\dfrac{1}{2}, y = 2$

(*b*) $x = 2, y = -\dfrac{1}{2}$

(*c*) $x = -\dfrac{a}{2}, y = 2b$

(*d*) $x = 2b, y = -\dfrac{a}{2}$

Sol. (*a*) $x = -\frac{1}{2}, y = 2$

From the given equation, we have

$$2ax - 2by + a + 4b = 0 \quad \ldots(1)$$

and $$2bx + 2ay + b - 4a = 0 \quad \ldots(2)$$

Multiplying (1) by '*b*', we get

$$2abx - 2b^2y + ab + 4b^2 = 0 \quad \ldots(3)$$

Multiplying (2) by '*a*', we get

$$2abx + 2a^2y + ab - 4a^2 = 0 \quad \ldots(4)$$

Subtracting (4) from (3), we get

$$-2(a^2 + b^2)y + 4(a^2 + b^2) = 0$$

$$\Rightarrow \quad y = \frac{4}{2} = 2$$

$\therefore$ From (1), we get

$$2ax - 4b + a + 4b = 0$$

$$\Rightarrow \quad 2x = -1$$

$$\Rightarrow \quad x = -\frac{1}{2}$$

$\therefore$ The required solution is $x = -\frac{1}{2}, y = 2$.

4. The area of the region bounded by the lines $-5x + 3y = 15$, $x = 0$ and $y = 0$ is

(*a*) 16 sq units (*b*) 8 sq units

(*c*) 15 sq units (*d*) $\frac{15}{2}$ sq units

Sol. (*d*) $\frac{15}{2}$ sq units

Putting $x = 0$ and $y = 0$ successively in the given equation $-5x + 3y = 15$, we get $y = 5$ and $x = -3$.

$\therefore$ The coordinates of the vertices of ΔAOB are A(–3, 0), O(0, 0) and B(5, 0). ΔAOB is a right-angled triangle where $\angle$AOB = 90°.

$$\therefore \quad \text{Area of } \Delta AOB = \frac{1}{2} \times OA \times OB$$

$$= \frac{1}{2} \times 3 \times 5 \text{ sq units}$$

$$= \frac{15}{2} \text{ sq units}$$

5. The distance between two lines is the same non-zero quantity throughout. Then the two lines are

(*a*) consistent and have a unique solution.

(*b*) consistent and have infinite no. of solutions.

(*c*) inconsistent.

(*d*) dependent.

Sol. (*c*) inconsistent.

If the distance between two lines is a non-zero constant, then the two lines are clearly parallel to each other and so they do not intersect each other. Therefore, they do not have any solution at all. Hence, the two lines are inconsistent.

6. A and B are two brothers, B being younger than A. Two times A's age exceeds B's age by 5 years and $\frac{1}{4}$th of A's age is equal to $\frac{1}{3}$ of B's age. Then ages of A and B are respectively

(*a*) $4\frac{1}{2}$ years and $3\frac{1}{2}$ years

(*b*) 4 years and 3 years

(*c*) 5 years and 3 years

(*d*) 5 years and 4 years

Sol. (*b*) 4 years and 3 years

Let the ages of A and B be x years and y years respectively, where $x > y$. Then according to the problem, we have

$$2x - y = 5 \quad \ldots(1)$$

and $$\frac{x}{4} = \frac{y}{3}$$

$$\Rightarrow \quad 3x = 4y \quad \ldots(2)$$

From (2), we have

$$y = \frac{3x}{4} \quad \ldots(3)$$

$\therefore$ From (1), we get

$$2x - \frac{3x}{4} = 5$$

$$\Rightarrow \quad \frac{5x}{4} = 5$$

$$\Rightarrow \quad x = 4$$

$\therefore$ From (3), $y = \frac{3}{4} \times 4 = 3$

Hence, the required ages of A and B are 4 years and 3 years respectively.

7. A two-digit number is such that the product of its digits is 14. If 45 is added to the number, then the digits interchange their places. Then the number is

(*a*) 27 (*b*) 72

(*c*) 14 (*d*) 41

Sol. (*a*) 27

Let the digit in the unit's place of the number be x and that in the ten's place be y. Then the number is $10y + x$.

$\therefore$ According to the problem, we have

$$xy = 14 \quad \ldots(1)$$

and $$10y + x + 45 = 10x + y$$

$\Rightarrow \quad 9y - 9x + 45 = 0$

$\Rightarrow \quad y - x + 5 = 0 \quad \ldots(2)$

$\therefore$ From (1) and (2), we get

$$x(-5 + x) = 14$$

$\Rightarrow \quad x^2 - 5x - 14 = 0$

$\Rightarrow \quad x^2 + 2x - 7x - 14 = 0$

$\Rightarrow \quad x(x + 2) -7 (x + 2) = 0$

$\Rightarrow \quad (x + 2) (x - 7) = 0$

$\therefore \quad$ Either $x - 7 = 0 \quad \ldots(3)$

or $\quad x + 2 = 0 \quad \ldots(4)$

From (3), $x = 7$ and from (4), $x = -2$ which is not possible, since the digit of a number cannot be negative.

Hence, $x = 7$.

$\therefore$ From (1), $y = \dfrac{14}{7} = 2$

$\therefore$ The required number is $10 \times 2 + 7$ i.e. 27.

8. The value of c such that the pair of linear equations $cx + 3y = 3$, $12x + cy = 6$ represents a pair of parallel lines is

(*a*) 6, –6 (*b*) 6

(*c*) –6 (*d*) does not exist

Sol. (*c*) –6

If the two given lines are parallel, then

$$\frac{c}{12} = \frac{3}{6} \neq \frac{3}{6}$$

Now, from $\dfrac{c}{12} = \dfrac{3}{c}$, we have

$$c^2 = 3 \times 12 = 36$$

$\therefore \quad c = \pm 6$

When $c = 6$, then $\dfrac{3}{6} \neq \dfrac{3}{6}$ is not true.

When $c = -6$, then $\dfrac{3}{-6} \neq \dfrac{3}{6}$ is true.

Hence, the required value of c is –6.

9. The students of a class are made to stand in rows. If 1 student is extra in each row, there would be 2 rows less. On the other hand, if 1 student is less in each row, there would be 3 rows more. If the number of students in each row is x and the number of rows be y, then the total number of students in the class can be obtained by solving the following pair of linear equations

(*a*) $2x - y = 2$ and $3x + y = 3$

(*b*) $y - 2x = 2$ and $3x - y = 3$

(*c*) $x - 2y = 2$ and $y - 3x = 3$

(*d*) $y + 2x = 2$ and $3x - y = 3$

Sol. (*b*) $y - 2x = 2$ and $3x - y = 3$

The number of students in each row is x and the number of rows is y.

$\therefore$ The total number of students in the class is xy.

According to the problem, if 1 student is extra in each row i.e. $(x + 1)$, then the number of rows will be $(y - 2)$.

$\therefore$ The total number of students in the class is $(x + 1) (y - 2)$.

Hence, $\quad (x + 1) (y - 2) = xy$

$\Rightarrow \quad xy - 2x + y - 2 = xy$

$\Rightarrow \quad y - 2x = 2 \quad \ldots(1)$

Again, if 1 student is less in each row i.e. $(x - 1)$, then the number of rows will be $(y + 3)$.

$\therefore \quad (x - 1) (y + 3) = xy$

$\Rightarrow \quad xy + 3x - y - 3 = xy$

$\Rightarrow \quad 3x - y = 3 \quad \ldots(2)$

$\therefore$ The total number of students can be obtained by solving the equations $y - 2x = 2$ and $3x - y = 3$.

10. Two pipes take 12 hours to fill a water tank. If the pipe of larger diameter is used for 4 hours and the pipe of smaller diameter for 9 hours, only half of the tank can be filled. Then the pipe of smaller diameter can fill the water tank in

(*a*) 10 hours (*b*) 25 hours

(*c*) 30 hours (*d*) 20 hours

Sol. (*c*) 30 hours

Let the pipe of smaller diameter can fill the tank in x hours and the pipe of larger diameter can fill the tank in y hours. Clearly, $x > y$.

$\therefore$ In 1 hour, the pipe of smaller diameter can fill $\dfrac{1}{x}$ th part of the tank and in one hour, the pipe of larger diameter can fill $\dfrac{1}{y}$ th part of the tank.

$\therefore$ According to the problem, we have

$$\frac{1}{x} + \frac{1}{y} = \frac{1}{12} \quad \ldots(1)$$

Also, it is given that when the pipe of larger diameter used for 4 hours and the pipe of smaller diameter for 9 hours, only half of the tank is filled.

$$\therefore \quad \frac{9}{x} + \frac{4}{y} = \frac{1}{2} \quad \ldots(2)$$

From (1) and (2), we have

$$u + v = \frac{1}{12} \quad \ldots(3)$$

and $\quad 9u + 4v = \frac{1}{2}$ ...(4)

where $\quad u = \frac{1}{x}$ ...(5)

and $\quad v = \frac{1}{y}$ ...(6)

From (3), we have

$$v = \frac{1}{12} - u \quad \text{...(7)}$$

$\therefore$ From (4), we have

$$9u + 4\left(\frac{1}{12} - u\right) = \frac{1}{2}$$

$$\Rightarrow \quad 5u = \frac{1}{2} - \frac{1}{3} = \frac{3-2}{6} = \frac{1}{6}$$

$$\therefore \quad u = \frac{1}{30}$$

$$\Rightarrow \quad x = 30 \quad \text{[From (5)]}$$

$\therefore$ The pipe of smaller diameter can fill the tank in 30 hours.

Life Skills

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1. While teaching about the Indian National Flag, the teacher asked students that how many spokes are there in the blue colour wheel? One student replies that it is 8 times the number of colours in the flag. While the other says that the sum of the number of colours in the flag and the number of lines in the wheel is 27.

(*a*) Convert the statement into linear equations in two variables and find the number of lines in the wheel.

(*b*) What does the wheel signify in the flag?

Sol. (*a*) Let the number of spokes in the wheel be x and the number of colours be y. Then according to the one student,

$$x = 8y$$

$$\Rightarrow \quad x - 8y = 0 \quad \text{...(1)}$$

According to some other students,

$$x + y = 27 \quad \text{...(2)}$$

Since the number of lines in the wheel is same as the number of spokes in the wheel.

Hence, (1) and (2) are the required linear equations.

From (1) and (2), we have

$$8y + y = 27$$

$$\Rightarrow \quad 9y = 27$$

$$\Rightarrow \quad y = \frac{27}{9} = 3$$

$\therefore$ From (2), $\quad x = 27 - 3 = 24$

Hence, the required number of spokes or lines in the wheel is 24.

(*b*) The wheel signifies the continuous progress of the country under the rule of law.

2. In a painting competition of a school, a child made Indian National Flag whose perimeter was 50 cm. Its area will be decreased by 6 sq cm, if length is decreased by 3 cm and breadth is increased by 2 cm.

Find the dimensions of the flag.

Sol. Let x cm be the length and y cm be the breadth of the rectangular national flag.

Since the perimeter is 50 cm.

$$\therefore \quad 2(x + y) = 50$$

$$\Rightarrow \quad x + y = 25 \quad \text{...(1)}$$

Original area of the flag = xy cm^2.

$\therefore$ According to the problem, we have

$$(x - 3)(y + 2) = xy - 6$$

$$\Rightarrow \quad xy + 2x - 3y - 6 = xy - 6$$

$$\Rightarrow \quad 2x - 3y = 0 \quad \text{...(2)}$$

$\therefore$ From (1), we have

$$y = 25 - x \quad \text{...(3)}$$

$\therefore$ From (2), we have

$$2x - 3(25 - x) = 0$$

$$\Rightarrow \quad 2x - 75 + 3x = 0$$

$$\Rightarrow \quad 5x = 75$$

$$\Rightarrow \quad x = \frac{75}{5} = 15$$

$\therefore$ From (3), $\quad y = 25 - 15 = 10$

Hence, the required dimensions of the flag are length = 15 cm and breadth = 10 cm.