10

Heron's Formula

Checkpoint (Page 142)

1. Find the area of an equilateral triangle with side 4 cm.

Sol. Here, side = 4 cm

Area of an equilateral triangle = $\frac{\sqrt{3}}{4}(\text{side})^2$

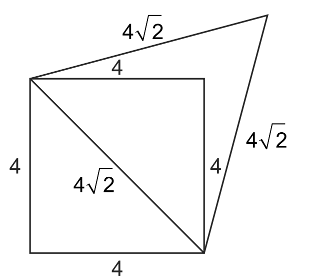
$= \frac{\sqrt{3}}{4}(4)^2 \text{ cm}^2$

$= 4\sqrt{3} \text{ cm}^2$

Hence, area of the equilateral triangle is $4\sqrt{3}$ cm^2.

2. Find the area of an equilateral triangle drawn from the diagonal of a square of side 4 cm.

Sol. Here, side of square = 4 cm



Diagonal of square = $\sqrt{2}$ (side)

$= \sqrt{2}(4)$ cm

$= 4\sqrt{2}$ cm

Area of an equilateral triangle drawn from the Diagonal of a square = $\frac{\sqrt{3}}{4}(\text{side})^2$

$= \frac{\sqrt{3}}{4}(4\sqrt{2})^2 \text{ cm}^2$

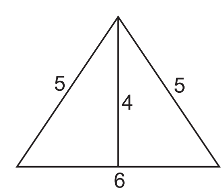
$= \frac{\sqrt{3}}{4}(4 \times 4 \times 2) \text{ cm}^2$

$= 8\sqrt{3} \text{ cm}^2$

Hence, the area of the equilateral triangle drawn from the diagonal of a square is $8\sqrt{3}$ cm^2.

3. Find the area of an isosceles triangle with base of length 6 cm and each equal side of length 5 cm, by using the formula: area = $\frac{1}{2}$ base × altitude.

Sol. Area of an isosceles triangle



$= \frac{1}{2} \times \text{base} \times \text{altitude}$

$= \frac{1}{2} \times 6 \times 4 \text{ cm}^2$

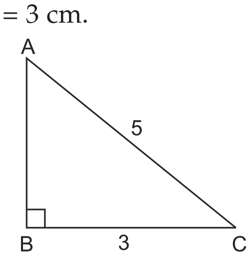
[By Pythagoras' Theorem, altitude = 4 cm]

$= 12 \text{ cm}^2$

Hence, the area of an isosceles triangle is 12 cm^2.

4. Find the length of the perimeter of a right-angled triangle whose hypotenuse and one side are of lengths 5 cm and 3 cm respectively.

Sol. Let ABC be right-angled triangle whose hypotenuse = AC = 5 cm and one of its side = BC = 3 cm.



By Pythagoras' theorem,

$$(AC)^2 = (AB)^2 + (BC)^2$$

$$\Rightarrow \quad (5\text{ cm})^2 = (AB)^2 + (3\text{ cm})^2$$

$$\Rightarrow \quad (25 - 9)\text{ cm}^2 = (AB)^2$$

$$\Rightarrow \quad AB = \sqrt{16}\text{ cm}$$

$$\Rightarrow \quad AB = 4\text{ cm}$$

$\therefore$ Perimeter of a right-angled triangle ABC

$$= AB + BC + AC$$

$$= (4 + 3 + 5)\text{ cm}$$

$$= 12\text{ cm}$$

Hence, the length of the perimeter of a right-angled triangle is 12 cm.

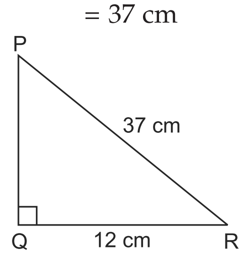
5. Taking the length of the perimeter of the right-angled triangle in problem no. 4 above as the length of one side of another right-angled triangle, the second right-angled triangle is constructed with hypotenuse of length 37 cm. Find the area of this second right-angled triangle.

Sol. Perimeter of right-angled triangle in problem number 5 = Length of one side of second right-angled triangle [Given]

$$= 12\text{ cm}$$

Hypotenuse of second right-angled triangle

$$= 37\text{ cm}$$



By Pythagoras' Theorem,

$$(PR)^2 = (PQ)^2 + (QR)^2$$

$$\Rightarrow \quad (37\text{ cm})^2 = (PQ)^2 + (12\text{ cm})^2$$

$$\Rightarrow \quad 1369\text{ cm}^2 = (PQ)^2 + 144\text{ cm}^2$$

$$\Rightarrow \quad 1225\text{ cm}^2 = (PQ)^2$$

$$\Rightarrow \quad PQ = 35\text{ cm}$$

Area of the second right-angled triangle

$$= \frac{1}{2} \times \text{base} \times \text{altitude}$$

$$= \frac{1}{2} \times QR \times PQ$$

$$= \left(\frac{1}{2} \times 12 \times 35\right)\text{ cm}^2$$

$$= 210\text{ cm}^2$$

Hence, the area of the second right-angled triangle is 210 cm^2.

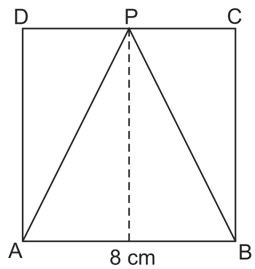
6. On the side AB of length 8 cm of a square ABCD, a triangle ABP is constructed where P is any point on another side CD of the square. Find the area of this triangle.

Sol. Here, altitude = 8 cm, AB = 8 cm.

$$\text{Area of triangle ABP} = \frac{1}{2} \times \text{base} \times \text{altitude}$$

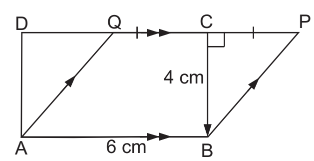
$$= \left(\frac{1}{2} \times 8 \times 8\right)\text{ cm}^2$$

$$= 32\text{ cm}^2$$



Hence, the area of the triangle is 32 cm^2.

7. ABCD is a rectangle of side AB = 6 cm and BC = 4 cm. If ABPQ is a parallelogram, where Q and P are points on DC and DC produced respectively as shown in the given figure and if C is the mid-point of PQ, then find the area of ΔCPB.



Sol. Given AB = 6 cm, BC = 4 cm

Since C is the mid-point of PQ.

$$\Rightarrow \quad CP = CQ$$

As ABCD is a square and ABPQ is a parallelogram

$\therefore$ Q is the mid-point of CD.

$$\Rightarrow \quad CQ = DQ = 3\text{ cm}$$

[$\because$ CD = AB = 6 cm]

$$\therefore \quad \text{Area of } \Delta CPB = \frac{1}{2} \times \text{base} \times \text{altitude}$$

$$= \frac{1}{2} \times 3 \times 4$$

$$= 6\text{ cm}^2$$

Hence, the area of ΔCPB is 6 cm^2.

Check Your Progress

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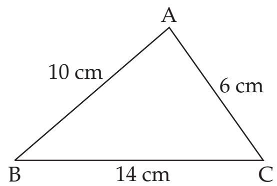
Multiple-Choice Questions

1. Which of the following is equal to the area of the triangle?



(*a*) $\sqrt{(9)(5)(1)}$ (*b*) $\sqrt{15(9)(5)(1)}$

(*c*) $\sqrt{(24)(20)(16)}$ (*d*) $\sqrt{30(24)(20)(16)}$

Sol. (*b*) $\sqrt{15(9)(5)(1)}$

By Heron's formula,

$$s = \frac{10+6+14}{2} = 15$$

Area of the ΔABC

$$= \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{15(15-6)(15-10)(15-14)}$$

$$= \sqrt{15(9)(5)(1)}$$

2. The sides of a triangle are 12 cm, 16 cm and 20 cm respectively. Its area is

(*a*) 48 cm^2 (*b*) 120 cm^2

(*c*) 96 cm^2 (*d*) 160 cm^2 **[CBSE SP 2012]**

Sol. (*c*) 96 cm^2

Here, a = 12 cm, b = 16 cm, c = 20 cm

$$\therefore \quad s = \frac{a+b+c}{2}$$

$$= \frac{12+16+20}{2} \text{ cm}$$

$$= \frac{48}{2} \text{ cm} = 24 \text{ cm}$$

Area of the triangle

$$= \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{24(24-12)(24-16)(24-20)} \text{ cm}^2$$

$$= \sqrt{24(12)(8)(4)} \text{ cm}^2$$

$$= \sqrt{12 \times 2 \times 12 \times 2 \times 4 \times 4} \text{ cm}^2$$

$$= 12 \times 2 \times 4 \text{ cm}^2$$

$$= 96 \text{ cm}^2$$

3. If the area of an equilateral triangle is $16\sqrt{3}$ cm^2, then the perimeter of the triangle is

(*a*) 12 cm (*b*) 24 cm

(*c*) 48 cm (*d*) 36 cm **[CBSE SP 2013]**

Sol. (*b*) 24 cm

Area of an equilateral triangle = $16\sqrt{3}$ cm^2 [Given]

$$\Rightarrow \quad \frac{\sqrt{3}}{4}a^2 = 16\sqrt{3}$$

$$\Rightarrow \quad a^2 = 64$$

$$\Rightarrow \quad a = \pm 8$$

$$\Rightarrow \quad a = 8 \quad [\because a = -8; \text{ rejected}]$$

$\therefore$ Side of an equilateral triangle = 8 cm

$\therefore$ Perimeter of an equilateral triangle

= (8 + 8 + 8) cm = 24 cm

4. The area of an isosceles triangle having base 2 cm and length of one of the equal sides 4 cm, is

(*a*) $\sqrt{15}$ cm^2 (*b*) $\sqrt{\frac{15}{2}}$ cm^2

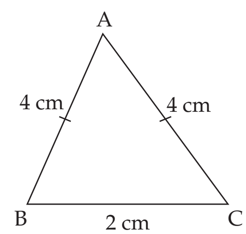
(*c*) $2\sqrt{15}$ cm^2 (*d*) $4\sqrt{15}$ cm^2

Sol. (*a*) $\sqrt{15}$ cm^2

$$s = \frac{4 \text{ cm} + 4 \text{ cm} + 2 \text{ cm}}{2}$$

$$= \frac{10}{2} \text{ cm}$$

$$s = 5 \text{ cm}$$



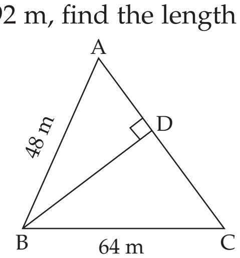
Area of ΔABC $= \sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{5(5-4)(5-4)(5-2)} \text{ cm}^2$$

$$= \sqrt{5 \times 1 \times 1 \times 3} \text{ cm}^2$$

$$= \sqrt{15} \text{ cm}^2$$

5. If the perimeter of the triangle as shown in the figure is 192 m, find the length of BD.



(*a*) 38.4 m (*b*) 40 m
(*c*) 76.8 m (*d*) 80 m

Sol. (*a*) 38.4 m

Perimeter of the triangle = 192 m

$$AB + BC + AC = 192 \text{ m}$$
$$48 \text{ m} + 64 \text{ m} + AC = 192 \text{ m}$$
$$\Rightarrow \quad AC = 192 \text{ m} - 112 \text{ m}$$
$$\Rightarrow \quad AC = 80 \text{ m}$$
$$s = \frac{192}{2} \text{ m} = 96 \text{ m}$$

Area of ΔABC

$$= \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{96 \times (96-48)(96-64)(96-80)}$$
$$= \sqrt{96 \times 48 \times 32 \times 16} \text{ m}^2$$
$$= \sqrt{16 \times 6 \times 16 \times 3 \times 16 \times 2 \times 16} \text{ m}^2$$
$$= 16 \times 16 \times 6 \text{ m}^2$$

Also, area of $\Delta ABC = \frac{1}{2} \times AC \times BD$

$$= \frac{1}{2} \times 80 \text{ m} \times BD$$
$$= 16 \times 16 \times 6 \text{ m}^2$$
$$\Rightarrow \frac{1}{2} \times 80 \text{ m} \times BD = 16 \times 16 \times 6 \text{ m}^2$$
$$\Rightarrow \quad BD = \frac{16 \times 16 \times 6 \times 2}{80} \text{ m}$$
$$\Rightarrow \quad BD = 38.4 \text{ m}$$

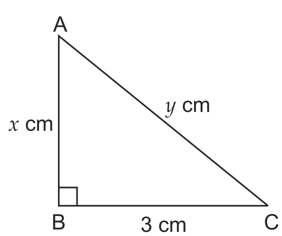
Very Short Answer Type Questions

6. Find the perimeter of a right-angled triangle of area 6 cm^2 and one side 3 cm.

Sol. Given, area = 6 cm^2

One Side = 3 cm

Let other side = x cm



∵ Area of right-angled triangle

$$= \frac{1}{2} \times \text{base} \times \text{altitude}$$
$$\Rightarrow \quad 6 \text{ cm}^2 = \frac{1}{2} \times 3 \text{ cm} \times x$$
$$\Rightarrow \quad 4 \text{ cm} = x$$
$$\Rightarrow \quad AB = 4 \text{ cm} \quad \ldots(1)$$

In right triangle ΔABC, we have

$$AC^2 = AB^2 + BC^2$$
$$= (4 \text{ cm})^2 + (3 \text{ cm})^2$$
$$= (16 + 9) \text{ cm}^2$$
$$= 25 \text{ cm}^2 \quad \text{[From (1)]}$$
$$\Rightarrow \quad AC = 5 \text{ cm}$$

∴ Perimeter of a right-angled triangle

$$= (3 + 4 + 5) \text{ cm}$$
$$= 12 \text{ cm}$$

Hence, the perimeter of a right-angled triangle is 12 cm.

7. The perimeter of a triangle whose sides are in the ratio 3 : 2 : 4 is 72 cm. Find the sum of the lengths of its longest and shortest sides.

Sol. Let the sides of the triangle be $a = 3x$ cm, $b = 2x$ cm and $c = 4x$ cm.

$$\text{Perimeter} = 72 \text{ cm}$$
$$\Rightarrow \quad 3x + 2x + 4x = 72 \text{ cm}$$
$$\Rightarrow \quad 9x = 72 \text{ cm}$$
$$\Rightarrow \quad x = 8 \text{ cm}$$
$$\therefore \quad a = 3x = (3 \times 8) \text{ cm} = 24 \text{ cm}$$
$$b = 2x = (2 \times 8) \text{ cm} = 16 \text{ cm}$$
$$c = 4x = (4 \times 8) \text{ cm} = 32 \text{ cm}$$

∴ Sum of the lengths of its longest and shortest sides = (32 + 16) cm = 48 cm.

8. Find the area of a triangle whose perimeter is 68 cm and two of its sides are 25 cm and 26 cm.

Sol. Let a = 25 cm, b = 26 cm, c = x cm

$$\text{Perimeter} = 68 \text{ cm}$$
$$\Rightarrow \quad 25 + 26 + x = 68 \text{ cm}$$
$$\Rightarrow \quad 51 + x = 68 \text{ cm}$$
$$\Rightarrow \quad x = 17 \text{ cm}$$
$$\therefore \quad c = 17 \text{ cm}$$
$$s = \frac{\text{Perimeter}}{2}$$
$$= \frac{68}{2} \text{ cm} = 34 \text{ cm}$$

Area of triangle

$$= \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{34(34-25)(34-26)(34-17)} \text{ cm}^2$$
$$= \sqrt{34(9)(8)(17)} \text{ cm}^2$$

$= \sqrt{2 \times 17 \times 3 \times 3 \times 2 \times 2 \times 2 \times 17}$ cm^2

$= 2 \times 17 \times 3 \times 2$ cm^2

$= 204$ cm^2

Hence, the area of a triangle is 204 cm^2.

9. One side of an equilateral triangle measures 8 cm. Find its area and altitude.

Sol. Here, $a = 8$ cm, $b = 8$ cm and $c = 8$ cm

$$s = \frac{a+b+c}{2} = \frac{8+8+8}{2} \text{ cm} = \frac{24}{2} \text{ cm} = 12 \text{ cm}$$

Area of the given equilateral triangle

$$= \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{12(12-8)(12-8)(12-8)} \text{ cm}^2$$
$$= \sqrt{12 \times 4 \times 4 \times 4} \text{ cm}^2$$
$$= \sqrt{3 \times 4 \times 4 \times 4 \times 4} \text{ cm}^2$$
$$= 4 \times 4\sqrt{3} \text{ cm}^2$$
$$= 16\sqrt{3} \text{ cm}^2$$

Also, $\frac{1}{2} \times$ base $\times$ altitude = Area of the triangle

$\Rightarrow \quad \frac{1}{2} \times 8$ cm $\times$ altitude $= 16\sqrt{3}$ cm^2

$\Rightarrow \quad$ Altitude $= \frac{16\sqrt{3} \times 2}{8}$ cm

$= 4\sqrt{3}$ cm

Hence, the area of the given equilateral triangle is $16\sqrt{3}$ cm^2 and its altitude is $4\sqrt{3}$ cm.

Short Answer Type Questions

10. The park in a housing society is in the shape of a triangle of sides 65 m, 70 m and 75 m. Find the cost of laying grass in the park at the rate of ₹ 50 per m^2.

Sol. $s = \frac{65 \text{ m} + 70 \text{ m} + 75 \text{ m}}{2}$

$\Rightarrow s = 105$ m

Area of the park

$$= \sqrt{105(105-65)(105-70)(105-75)} \text{ m}^2$$
$$= \sqrt{105 \times 40 \times 35 \times 30} \text{ m}^2$$
$$= \sqrt{3 \times 5 \times 7 \times 2 \times 2 \times 2 \times 5 \times 5 \times 7 \times 2 \times 3 \times 5}$$
$$= 3 \times 5 \times 5 \times 7 \times 2 \times 2 \text{ m}^2$$
$$= 2100 \text{ m}^2$$

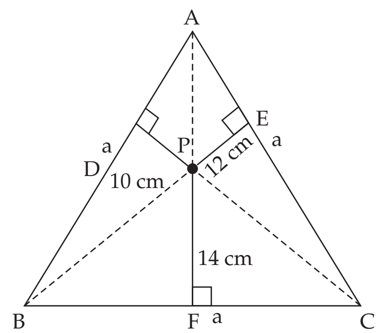
The cost of laying the grass

= ₹50 × 2100 = ₹1,05,000

11. From a point in the interior of an equilateral triangle, perpendiculars are drawn on the three sides. The lengths of the perpendiculars are 10 cm, 12 cm and 14 cm. Find the area of the triangle.

Sol. Let the side of the equilateral triangle ABC = a

$$s = \frac{a+a+a}{2} = \frac{3a}{2}$$



By Heron's formula

Area of ΔABC $= \sqrt{s(s-a)(s-a)(s-a)}$

$$= \sqrt{\frac{3}{2}a\left(\frac{3}{2}a-a\right)\left(\frac{3}{2}a-a\right)\left(\frac{3}{2}a-a\right)}$$
$$= \sqrt{\frac{3}{2}a \times \frac{a}{2} \times \frac{a}{2} \times \frac{a}{2}}$$
$$= \sqrt{3}\frac{a^2}{4} \quad \text{...(1)}$$

Area of ΔAPC $= \frac{1}{2} \times$ AC $\times$ PE

$= \frac{1}{2} \times a \times 12$ cm

$= 6a$ cm ...(2)

Area of ΔABP $= \frac{1}{2} \times$ AB $\times$ DP $= \frac{1}{2} \times a \times 10$ cm

$= 5a$ cm ...(3)

Area of ΔBPC $= \frac{1}{2} \times$ BC $\times$ PF $= \frac{1}{2} \times a \times 14$ cm

$= 7a$ cm ...(4)

Area of ΔABC = Area of ΔAPC + Area of ΔABP + Area of ΔBPC

$= 6a$ cm $+ 5a$ cm $+ 7a$ cm

$= 18a$ cm ...(5)

Equating (1) and (5)

$$\sqrt{3}\frac{a^2}{4} = 18a$$

$\Rightarrow \quad a = \frac{18 \times 4}{\sqrt{3}}$

$\Rightarrow \qquad a = \sqrt{3} \times 6 \times 4$

$\Rightarrow \qquad a = 24\sqrt{3}$ cm

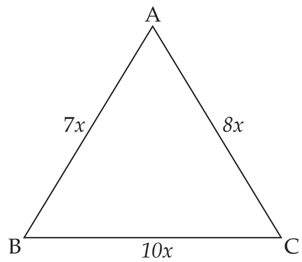
$\therefore$ Area of ΔABC $= \dfrac{\sqrt{3}}{4} \times \left(24\sqrt{3}\right)^2$ cm^2

$= \dfrac{\sqrt{3}}{4} \times 24 \times 24 \times 3$ cm^2

$= 432\sqrt{3}$ cm^2

12. The perimeter of a triangular field is 250 m and its sides are in the ratio 7 : 8 : 10. Find the area of the triangular field.

Sol. The sides of the triangular field are in the ratio 7 : 8 : 10.



Let the sides of the triangular field be $7x$, $8x$ and $10x$.

Then, $7x + 8x + 10x = 250$ m [Given]

$\Rightarrow \qquad 25x = 250$ m

$\Rightarrow \qquad x = 10$ m

Therefore, sides of the triangular field are 70 m, 80 m and 100 m.

$$s = \frac{70 + 80 + 100}{2} \text{ m} = 125 \text{ m}$$

By Heron's formula,

Area of the triangular field

$= \sqrt{125\,(125-70)\,(125-80)\,(125-100)}$

$= \sqrt{125 \times 55 \times 45 \times 25}$ m^2

$= \sqrt{5 \times 25 \times 5 \times 11 \times 5 \times 9 \times 25}$ m^2

$= 5 \times 25 \times 3\sqrt{55}$ m^2

$= 375\sqrt{55}$ m^2

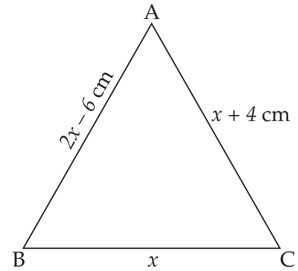
Long Answer Type Questions

13. The perimeter of a triangle is 50 cm. One side of a triangle is 4 cm longer than the smaller side and the third side is 6 cm less than twice the smaller side. Find the area of the triangle.

Sol. Let the smaller side of the triangle $= x$ cm

AC $= x + 4$ cm

AB $= 2x - 6$ cm



Perimeter of ΔABC

$= x + x + 4 + 2x - 6$

$= 4x - 2$

But perimeter of ΔABC = 50 cm

$\therefore \quad 4x - 2 = 50$

$\Rightarrow \quad 4x = 52$

$\Rightarrow \quad x = \dfrac{52}{4} = 13$ cm

$\therefore$ BC = 13 cm

AC = (13 + 4) cm = 17 cm

AB = (26 – 6) cm = 20 cm

$$s = \frac{13 + 17 + 20}{2} \text{ cm} = 25 \text{ cm}$$

By Heron's formula,

Area of ΔABC

$= \sqrt{25\,(25-13)\,(25-17)\,(25-20)}$ cm^2

$= \sqrt{25 \times 12 \times 8 \times 5}$ cm^2

$= \sqrt{5 \times 5 \times 3 \times 4 \times 2 \times 4 \times 5}$ cm^2

$= 5 \times 4\sqrt{30}$ cm^2

$= 20\sqrt{30}$ cm^2

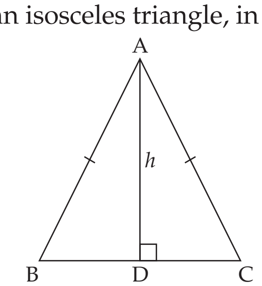
14. The perimeter of an isosceles triangle is 49 cm and the base is $1\frac{1}{2}$ times each of the equal sides. Find the

(*a*) length of each side of the triangle.

(*b*) area of the triangle.

(*c*) height of the triangle.

Sol. ΔABC is an isosceles triangle, in which AB = AC.



Let $\qquad$ AB $= x$

$\therefore \qquad$ AC $= x$

$$\text{Base, BC} = \frac{3}{2}x$$

(*a*) Given, perimeter = 49 cm

$$\therefore \quad x + x + \frac{3}{2}x = 49 \text{ cm}$$

$$\Rightarrow \quad \frac{2x + 2x + 3x}{2} = 49 \text{ cm}$$

$$\Rightarrow \quad \frac{7}{2}x = 49 \text{ cm}$$

$$\Rightarrow \quad x = 49 \times \frac{2}{7} \text{ cm} = 14 \text{ cm}$$

$$\therefore \quad AB = 14 \text{ cm}$$

$$AC = 14 \text{ cm}$$

$$BC = \frac{3}{2}x = \frac{3}{2} \times 14 \text{ cm}$$

$$= 21 \text{ cm}$$

(*b*) $s = \frac{49}{2}$ cm

By Heron's formula

Area of ΔABC

$$= \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{\frac{49}{2}\left(\frac{49}{2} - 14\right)\left(\frac{49}{2} - 14\right)\left(\frac{49}{2} - 21\right)} \text{ cm}^2$$

$$= \sqrt{\frac{49}{2}\left(\frac{49-28}{2}\right)\left(\frac{49-28}{2}\right)\left(\frac{49-42}{2}\right)} \text{ cm}^2$$

$$= \sqrt{\frac{49}{2} \times \frac{21}{2} \times \frac{21}{2} \times \frac{7}{2}} \text{ cm}^2$$

$$= \frac{7}{2} \times \frac{21}{2}\sqrt{7} \text{ cm}^2$$

$$= \frac{147}{4}\sqrt{7} \text{ cm}^2$$

(*c*) Let AD = height of the triangle = *h*

$$\frac{1}{2} \times BC \times AD = \frac{147}{4}\sqrt{7} \text{ cm}^2$$

$$\frac{1}{2} \times 21 \times AD = \frac{147}{4}\sqrt{7} \text{ cm}$$

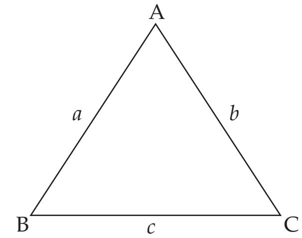
$$\Rightarrow \quad AD = \frac{2}{21} \times \frac{147}{4}\sqrt{7} \text{ cm}$$

$$= \frac{7}{2}\sqrt{7} \text{ cm}$$

15. Find the percentage decrease in the area of a triangle if its each side is halved.

Sol. Let the lengths of the sides of the triangle be *a*, *b* and *c*.

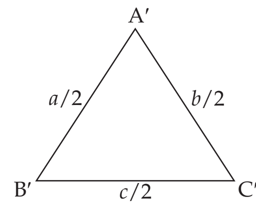
Then $$s = \frac{a+b+c}{2}$$



From Heron's formula,

Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$

Now, sides of the triangle are halved.



So, $$s' = \frac{\frac{a}{2} + \frac{b}{2} + \frac{c}{2}}{2} = \frac{s}{2}$$

$$\text{Area of } \Delta A'B'C' = \sqrt{s'\left(s' - \frac{a}{2}\right)\left(s' - \frac{b}{2}\right)\left(s' - \frac{c}{2}\right)}$$

$$= \sqrt{\frac{s}{2}\left(\frac{s}{2} - \frac{a}{2}\right)\left(\frac{s}{2} - \frac{b}{2}\right)\left(\frac{s}{2} - \frac{c}{2}\right)}$$

$$= \sqrt{\frac{s(s-a)(s-b)(s-c)}{16}}$$

$$= \frac{1}{4} \times \text{Area of } \Delta ABC$$

So,

Percentage decrease in area

$$= \frac{\text{Area of } \Delta ABC - \text{Area of } \Delta A'B'C'}{\text{Area of } \Delta ABC} \times 100\,\%$$

$$= \frac{\text{Area of } \Delta ABC - \frac{1}{4}\text{Area of } \Delta ABC}{\text{Area of } \Delta ABC} \times 100\%$$

$$= \frac{3}{4} \times 100\%$$

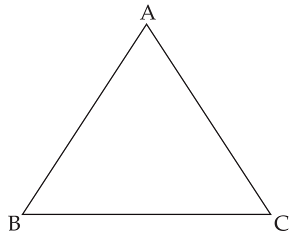
$$= 75\%.$$

Higher Order Thinking Skills (HOTS) Questions

(Page 147)

1. The perimeter of an isosceles triangle is equal to 28 cm. If the ratio of one of the equal side to the base is 5 : 4, find the area of the triangle.

Sol. Let AB and AC be the equal sides of an isosceles triangle ABC.



$$\frac{AB}{BC} = \frac{5}{4} \quad \text{[Given]}$$

Let $AB = 5x$

$BC = 4x$

$\Rightarrow AC = 5x$

Given perimeter of the isosceles triangle = 28 cm

$\Rightarrow AB + BC + AC = 28$ cm

$\Rightarrow 5x + 4x + 5x = 28$ cm

$\Rightarrow 14x = 28$ cm

$\Rightarrow x = 2$ cm

$\therefore AB = 10$ cm

$BC = 8$ cm

$AC = 10$ cm

$$s = \frac{AB + BC + AC}{2}$$

$$= \frac{10 \text{ cm} + 8 \text{ cm } 10 \text{ cm}}{2}$$

$s = 14$ cm

Area of the isosceles triangle ABC,

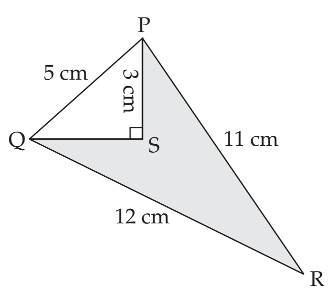
$$= \sqrt{14(14-10)(14-8)(14-10)} \text{ cm}^2$$

$$= \sqrt{14 \times 4 \times 6 \times 4} \text{ cm}^2$$

$$= \sqrt{2 \times 7 \times 4 \times 2 \times 3 \times 4} \text{ cm}^2$$

$$= 8\sqrt{21} \text{ cm}^2$$

2. Find the area of the shaded region.



Sol. In ΔPQR

$$s = \frac{PQ + PR + QR}{2} = \frac{5 \text{ cm} + 11 \text{ cm} + 12 \text{ cm}}{2}$$

$$= \frac{28}{2} \text{ cm} = 14 \text{ cm}$$

Area of ΔPQR

$$= \sqrt{14(14-5)(14-11)(14-12)} \text{ cm}^2$$

$$= \sqrt{14 \times 9 \times 3 \times 2} \text{ cm}^2$$

$$= \sqrt{2 \times 7 \times 9 \times 2 \times 3} \text{ cm}^2$$

$$= 6\sqrt{21} \text{ cm}^2$$

In right-triangle PQS,

$PS^2 + QS^2 = PQ^2$

$\Rightarrow QS^2 = PQ^2 - PS^2$

$\Rightarrow \quad = (5 \text{ cm})^2 - (3 \text{ cm})^2$

$\Rightarrow QS^2 = 16 \text{ cm}^2$

$\Rightarrow QS = 4$ cm

$$\text{Area of } \Delta PQS = \frac{1}{2} \times \text{base} \times \text{height}$$

$$= \frac{1}{2} \times 4 \text{ cm} \times 3 \text{ cm}$$

$$= 6 \text{ cm}^2$$

Area of the shaded region

$= \text{Area of } \Delta PQR - \text{Area of } \Delta PQS$

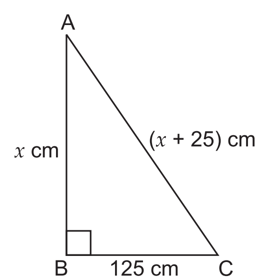
$= 6\sqrt{21} \text{ cm}^2 - 6 \text{ cm}^2$

$= 6\left(\sqrt{21} - 1\right) \text{ cm}^2$

3. One side of a right-angled triangle measures 125 cm and the difference in length of its hypotenuse and the other side is 25 cm. Find the lengths of its two unknown sides and calculate its area. Verify the result by Heron's formula.

Sol. Let ΔABC be the right-angled triangle at B.

Let BC = 125 cm, AB = x cm, AC = $(x + 25)$ cm



In ΔABC,

$$(AC)^2 = (AB)^2 + (BC)^2$$

[By Pythagoras' Theorem]

$\Rightarrow (x + 25)^2 = (x)^2 + (125)^2$

$\Rightarrow x^2 + 625 + 50x = x^2 + 15625$

$\Rightarrow 50x = 15000$

$\Rightarrow x = 300$

$\Rightarrow AB = 300$ cm

Now, $AC = (x + 25)$ cm

$= (300 + 25)$ cm

$AC = 325$ cm

Area of right $\Delta ABC = \frac{1}{2} \times BC \times AB$

$= \left(\frac{1}{2} \times 125 \times 300\right)$ cm^2

$= 18750$ cm^2

Verification by using Heron's formula:

In ΔABC, we have

$a = AB = 300$ cm, $b = BC = 125$ cm, $c = AC = 325$ cm

$\therefore \quad s = \frac{a+b+c}{2}$

$= \frac{300 + 125 + 325}{2}$ cm

$= 375$ cm

$ar(\Delta ABC) = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{375(375-300)(375-125)(375-325)}$ cm^2

$= \sqrt{375(75)(250)(50)}$ cm^2

$= \sqrt{5 \times 75 \times 75 \times 50 \times 5 \times 50}$ cm^2

$= 5 \times 75 \times 50$ cm^2 $= 18750$ cm^2

Hence, the length of two unknown sides of the ΔABC are 300 cm and 325 cm and its area is 18750 cm^2.

Self-Assessment

(Page 148)

Multiple-Choice Questions

1. A triangle ABC in which AB = AC = 4 cm and $\angle A = 90°$, has an area

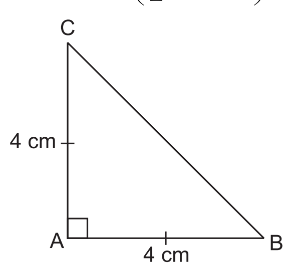
(*a*) 4 cm^2 (*b*) 16 cm^2

(*c*) 8 cm^2 (*d*) 12 cm^2

Sol. (*c*) 8 cm^2

Area of $\Delta ABC = \frac{1}{2} \times$ base $\times$ altitude

$= \left(\frac{1}{2} \times 4 \times 4\right)$ cm^2 $= 8$ cm^2



2. The area of a triangle whose sides are 7 cm, 9 cm and 8 cm is

(*a*) $12\sqrt{5}$ cm^2 (*b*) $15\sqrt{5}$ cm^2

(*c*) $15\sqrt{3}$ cm^2 (*d*) $12\sqrt{15}$ cm^2

Sol. (*a*) $12\sqrt{5}$ cm^2

Let $a = 7$ cm, $b = 9$ cm, $c = 8$ cm

Semi-perimeter of triangle $= \frac{a+b+c}{2}$

$= \left(\frac{7+9+8}{2}\right)$ cm

$= \left(\frac{24}{2}\right)$ cm

$= 12$ cm

Area of the triangle

$= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{12(12-7)(12-9)(12-8)}$ cm^2

$= \sqrt{12 \times 5 \times 3 \times 4}$ cm^2

$= \sqrt{3 \times 4 \times 5 \times 3 \times 4}$ cm^2

$= 3 \times 4\sqrt{5}$ cm^2

$= 12\sqrt{5}$ cm^2

3. The edges of a triangular metal sheet are 12 cm, 16 cm and 20 cm. Find the cost of painting it at the ₹ 10 per cm^2 is

(*a*) ₹ 9.60 (*b*) ₹ 96.00

(*c*) ₹ 960.00 (*d*) ₹ 9600.00

Sol. (*c*) ₹ 960.00

$s = \frac{12 \text{ cm} + 16 \text{ cm} + 20 \text{ cm}}{2}$

$= \frac{48}{2}$ cm

$\Rightarrow \quad s = 24$ cm

Area of triangular metal sheet

$= \sqrt{24(24-12)(24-16)(24-20)}$ cm^2

$= \sqrt{24 \times 12 \times 8 \times 4}$ cm^2

$= \sqrt{12 \times 2 \times 12 \times 8 \times 4}$ cm^2

$= 96$ cm^2

The cost of painting it

$=$ ₹ $10 \times 96 =$ ₹ 960

4. In a triangle, if each side is reduced to half its original side, then what is the percentage change in its area?

(*a*) 25% decrease (*b*) 25% increase

(*c*) 50% decrease (*d*) 75% decrease

Sol. (*d*) 75% decrease

Let each side of the triangle be a, b and c.

$$s = \frac{a+b+c}{2} \quad ...(1)$$

Area of the triangle = A

$$A = \sqrt{s(s-a)(s-b)(s-c)} \quad ...(2)$$

Now each side is reduced to half its original side.

$$s' = \frac{\frac{a}{2}+\frac{b}{2}+\frac{c}{2}}{2}$$

$$\Rightarrow \quad s' = \frac{a+b+c}{2\times 2}$$

$$\Rightarrow \quad s' = \frac{s}{2}$$

$$\Rightarrow \quad s' = 2s' \quad ...(3)$$

Now, area of the new triangle = A′

$$A' = \sqrt{s'\left(s'-\frac{a}{2}\right)\left(s'-\frac{b}{2}\right)\left(s'-\frac{c}{2}\right)}$$

$$= \sqrt{s'\left(\frac{2s'-a}{2}\right)\left(\frac{2s'-b}{2}\right)\left(\frac{2s'-c}{2}\right)}$$

$$= \sqrt{s'\left(\frac{s-a}{2}\right)\left(\frac{s-b}{2}\right)\left(\frac{s-c}{2}\right)}$$

$$= \sqrt{\frac{s}{2}\left(\frac{s-a}{2}\right)\left(\frac{s-b}{2}\right)\left(\frac{s-c}{2}\right)}$$

$$= \frac{1}{4}\text{ A}$$

$$A' = \frac{1}{4}\text{ A} \quad ...(4)$$

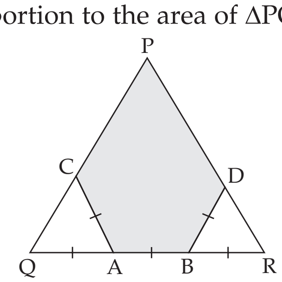
Percentage decrease in area

$$= \frac{A-A'}{A} \times 100$$

$$= \frac{A-\frac{1}{4}A}{A} \times 100$$

$$= \frac{3}{4} \times 100 = 75\% \text{ decrease}$$

5. In the given figure, PQR is an equilateral triangle. AQ = AC = AB = BD = BR. The ratio of the shaded portion to the area of ΔPQR is

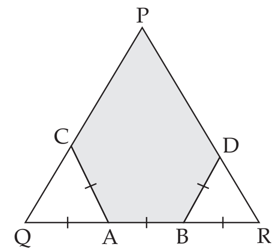


(*a*) $\frac{3}{9}$ (*b*) $\frac{5}{9}$ (*c*) $\frac{7}{9}$ (*d*) $\frac{9}{11}$

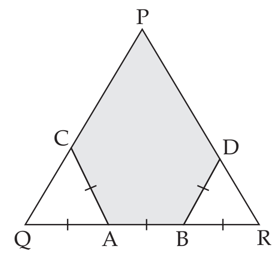
Sol. (*c*) $\frac{7}{9}$

ΔPQR is an equilateral triangle.

∴ PQ = QR = PR = a (say)



$$\text{Area of } \Delta PQR = \frac{\sqrt{3}}{4}a^2 \quad ...(1)$$



In ΔAQC

$$AC = QA$$

$$\therefore \quad \angle Q = \angle C$$

$$\text{But} \quad \angle Q = 60°$$

$$\therefore \quad \angle C = 60°$$

$$\Rightarrow \quad \angle A = 60°$$

⇒ ΔAQC is an equilateral triangle.

$$QA = AB = BR$$

$$\Rightarrow \quad QA = \frac{a}{3}$$

$$\text{Area of } \Delta AQC = \frac{\sqrt{3}}{4}\times\left(\frac{a}{3}\right)^2 = \frac{\sqrt{3}}{4}\times\frac{a^2}{9}$$

$$\text{Similarly area of } \Delta DBR = \frac{\sqrt{3}}{4}\times\frac{a^2}{9}$$

Area of the shaded portion

= Area of ΔPQR – Area of ΔAQC – Area of ΔDBR

$$= \frac{\sqrt{3}}{4}a^2 - \frac{\sqrt{3}}{4}\frac{a^2}{9} - \frac{\sqrt{3}}{4}\frac{a^2}{9}$$

$$= \frac{\sqrt{3}}{4}a^2\left[1-\frac{1}{9}-\frac{1}{9}\right]$$

$$= \frac{\sqrt{3}}{4}a^2\left[\frac{9-1-1}{9}\right]$$

$$= \frac{\sqrt{3}}{4} a^2 \times \frac{7}{9}$$

Ratio of the shaded portion to the area of ΔPQR

$$= \frac{\frac{\sqrt{3}}{4} a^2 \times \frac{7}{9}}{\frac{\sqrt{3}}{4} a^2} = \frac{7}{9}$$

Fill in the Blanks

6. The area of a triangle with base 8 cm and height 10 cm is **40 cm²**.

Sol.
$$\text{Area} = \frac{1}{2} \times \text{Base} \times \text{Height}$$
$$\therefore \quad = \frac{1}{2} \times 8 \times 10 \text{ cm}^2$$
$$= 40 \text{ cm}^2$$

7. The area of a triangle whose sides are 3 cm, 4 cm and 5 cm is **6 cm²**.

Sol. $\therefore$
$$s = \frac{3+4+5}{2} = \frac{12}{2} = 6$$
$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{6(6-3)(6-4)(6-5)} \text{ cm}^2$$
$$= \sqrt{6(3)(2)(1)} \text{ cm}^2$$
$$= 6 \text{ cm}^2$$

8. If the perimeter and base of an isosceles triangle are 11 cm and 5 cm respectively, then its area is $\mathbf{\frac{5}{4}\sqrt{11}}$ **cm²**.

Sol. Let each equal side be x. Perimeter = 11 cm
$$x + x + 5 \text{ cm} = 11 \text{ cm}$$
$$\Rightarrow \quad 2x = 6 \text{ cm}$$
$$\Rightarrow \quad x = 3 \text{ cm}$$

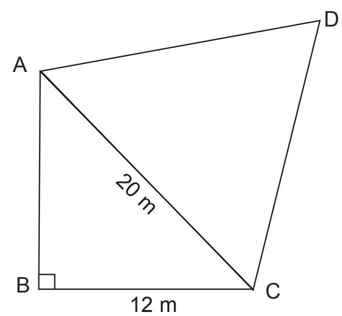
Here, a = 3 cm, b = 5 cm and c = 3 cm
$$s = \frac{a+b+c}{2} = \frac{3+5+3}{2} \text{ cm} = \frac{11}{2} \text{ cm}$$
$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{\frac{11}{2}\left(\frac{11}{2}-3\right)\left(\frac{11}{2}-5\right)\left(\frac{11}{2}-3\right)} \text{ cm}^2$$
$$= \sqrt{\frac{11}{2}\left(\frac{5}{2}\right)\left(\frac{1}{2}\right)\left(\frac{5}{2}\right)} \text{ cm}^2$$
$$= \frac{5}{4}\sqrt{11} \text{ cm}^2$$

9. The area of an isosceles triangle having base 24 cm and length of one of the equal sides 20 cm is **192 cm²**.

Sol. Here a = 20 cm, b = 20 cm and c = 24 cm
$$s = \frac{a+b+c}{2} = \frac{20+20+24}{2} \text{ cm} = \frac{64}{2} \text{ cm} = 32 \text{ cm}$$
$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{32(32-20)(32-20)(32-24)} \text{ cm}^2$$
$$= \sqrt{32(12)(12)(8)} \text{ cm}^2$$
$$= 8 \times 2 \times 12 \text{ cm}^2$$
$$= 192 \text{ cm}^2$$

Case Study Based Question

10. Some students of a school staged a rally for cleanliness drive in the park as given in the figure. They walked through the lanes in two groups. First group walked through the lanes AB, BC and CA, while the second group through AC, CD and DA. Then they cleaned the area enclosed within their lanes. If BC = 12 m, AC = 20 m, $\angle ABC = 90°$ and ΔACD is an equilateral triangle, then answer the following questions.



(*a*) What is the area cleaned by first group (area of ΔABC)?

Ans. 96 m²

(*b*) What is the area cleaned by second group (area of ΔACD)?

Ans. 173 m² approx.

(*c*) (*i*) Which group cleaned more area and by how much?

Ans. Second group, 77 m²

or

(*ii*) What is the total area cleaned by the students (neglecting the width of the lanes)?

Ans. 269 m² approx.

Very Short Answer Type Questions

11. Find the area of an equilateral triangle whose perimeter is 18 cm. (Use $\sqrt{3}$ = 1.732)

Sol. Perimeter of the given equilateral triangle = 18 cm

$\therefore \quad s = \dfrac{\text{Perimeter}}{2}$

$= \dfrac{18}{2}$ cm = 9 cm

and each side $= \dfrac{18}{3}$ cm = 6 cm

So, a = 6 cm, b = 6 cm, c = 6 cm

Area of the given equilateral triangle

$= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{9(9-6)(9-6)(9-6)}\ \text{cm}^2$

$= \sqrt{9(3)(3)(3)}\ \text{cm}^2$

$= \sqrt{3 \times 3 \times 3 \times 3 \times 3}\ \text{cm}^2$

$= 3 \times 3\sqrt{3}\ \text{cm}^2$

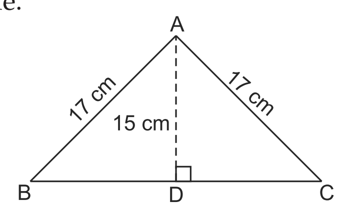
$= 9 \times 1.732\ \text{cm}^2$

$= 15.588\ \text{cm}^2$

Hence, the area of an equilateral triangle is $15.588\ \text{cm}^2$.

12. Find the area of an isosceles triangle whose altitude is 15 cm and one of the equal sides is 17 cm.

Sol. Let ABC be an isosceles triangle with AD as altitude.



Here, AB = AC = 17 cm, AD = 15 cm

In right ΔADC, we have

$(AC)^2 = (AD)^2 + (DC)^2$

[By Pythagoras' Theorem]

$\Rightarrow \quad (17\ \text{cm})^2 = (15\ \text{cm})^2 + (DC)^2$

$\Rightarrow \quad 289\ \text{cm}^2 = 225\ \text{cm}^2 + (DC)^2$

$\Rightarrow \quad (289 - 225)\ \text{cm}^2 = (DC)^2$

$\Rightarrow \quad (DC)^2 = 64\ \text{cm}^2$

$\Rightarrow \quad DC = 8\ \text{cm}$

Now, $\quad BC = 2(DC)$

$= 2(8)$ cm

$= 16$ cm

In ΔABC, we have

a = AB = 17 cm, b = AC = 17 cm, c = BC = 16 cm

$\therefore \quad s = \dfrac{a+b+c}{2}$

$= \dfrac{17+17+16}{2}$ cm

$= \dfrac{50}{2}$ cm = 25 cm

Area of ΔABC $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{25(25-17)(25-17)(25-16)}\ \text{cm}^2$

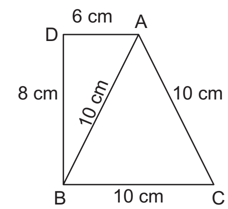
$= \sqrt{25 \times 8 \times 8 \times 9}\ \text{cm}^2$

$= \sqrt{5 \times 5 \times 8 \times 8 \times 3 \times 3}\ \text{cm}^2$

$= (5 \times 8 \times 3)\ \text{cm}^2 = 120\ \text{cm}^2$

Hence, the area of an isosceles triangle is $120\ \text{cm}^2$.

13. Find the area that is to be added to the area of ΔADB with dimensions shown in the given figure, so that it becomes equal to the area of ΔABC. (Use $\sqrt{3}$ = 1.732)



Sol. In ΔADB, we have

a = AB = 10 cm, b = BD = 8 cm, c = AD = 6 cm

$\therefore \quad s = \dfrac{a+b+c}{2}$

$= \dfrac{10+8+6}{2}$ cm

$= \dfrac{24}{2}$ cm = 12 cm

Area of ΔADB $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{12(12-10)(12-8)(12-6)}\ \text{cm}^2$

$= \sqrt{12(2)(4)(6)}\ \text{cm}^2$

$= \sqrt{4 \times 3 \times 2 \times 4 \times 3 \times 2}\ \text{cm}^2$

$= (4 \times 3 \times 2)\ \text{cm}^2$

$= 24\ \text{cm}^2$

In ΔABC,

Area of ΔABC $= \dfrac{\sqrt{3}}{4}(\text{side})^2$

$= \dfrac{\sqrt{3}}{4}(10)^2\ \text{cm}^2$

$= \dfrac{\sqrt{3}}{4} \times 100\ \text{cm}^2$

$= 25\sqrt{3}\ \text{cm}^2$

$= 25\ (1.732)\ \text{cm}^2$

$= 43.3\ \text{cm}^2$

$\therefore$ Area that is to be added to the area of ΔADB so that it becomes equal to the area of ΔABC

$$= \text{Area of } \Delta ABC - \text{Area of } \Delta ADB$$
$$= (43.3 - 24)\text{ cm}^2 = 19.3\text{ cm}^2$$

14. Find the area of a triangle whose sides are 13 cm, 14 cm and 15 cm. **[CBSE SP 2011]**

Sol. Here, $a = 13$ cm, $b = 14$ cm, $c = 15$ cm

$$\therefore \quad s = \frac{a+b+c}{2}$$
$$= \left(\frac{13+14+15}{2}\right)\text{ cm}$$
$$= \frac{42}{2}\text{ cm} = 21\text{ cm}$$

Area of a triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{21(21-13)(21-14)(21-15)}\text{ cm}^2$$
$$= \sqrt{21 \times 8 \times 7 \times 6}\text{ cm}^2$$
$$= \sqrt{7 \times 3 \times 2 \times 2 \times 2 \times 7 \times 2 \times 3}\text{ cm}^2$$
$$= (7 \times 3 \times 2 \times 2)\text{ cm}^2$$
$$= 84\text{ cm}^2$$

15. Find the cost of levelling the lawn in the form of a triangle having the sides 51 m, 37 m and 20 m at the rate of ₹5 per m^2.

Sol.
$$s = \frac{51+37+20}{2}\text{ m}$$
$$= \frac{108}{2}\text{ m} = 54\text{ m}$$

Area of the lawn

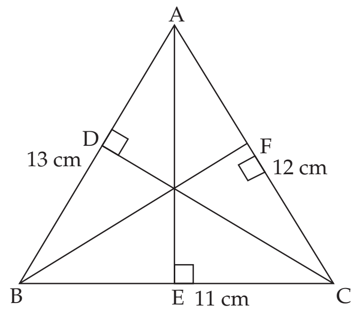
$$= \sqrt{54(54-51)(54-37)(54-20)}\text{ m}^2$$
$$= \sqrt{54 \times 3 \times 17 \times 34}\text{ m}^2$$
$$= \sqrt{9 \times 6 \times 3 \times 17 \times 17 \times 2}\text{ m}^2$$
$$= 3 \times 6 \times 17\text{ m}^2$$
$$= 306\text{ m}^2$$

Cost of levelling the lawn

= ₹5 × 306 = ₹1530

16. The sides of a triangle are 11 cm, 12 cm and 13 cm. Find the length of the altitude corresponding to the sides.

Sol.
$$s = \frac{11+12+13}{2}\text{ cm}$$
$$= \frac{36}{2}\text{ cm}$$
$$\Rightarrow \quad s = 18\text{ cm}$$



Area of the triangle

$$= \sqrt{18(18-11)(18-12)(18-13)}\text{ cm}^2$$
$$= \sqrt{18 \times 7 \times 6 \times 5}\text{ cm}^2$$
$$= 6\sqrt{3 \times 5 \times 7}\text{ cm}^2$$
$$= 6\sqrt{105}\text{ cm}^2$$

Considering AC as base,

$$\text{Area of } \Delta ABC = \frac{1}{2} \times AC \times BF$$
$$\frac{1}{2} \times AC \times BF = 6\sqrt{105}\text{ cm}^2$$
$$BF = \frac{2 \times 6}{12}\sqrt{105}\text{ cm}$$
$$= \sqrt{105}\text{ cm}$$

Considering BC as base,

$$\text{Area of } \Delta ABC = \frac{1}{2} \times BC \times AE$$
$$\frac{1}{2} \times BC \times AE = 6\sqrt{105}\text{ cm}^2$$
$$\Rightarrow \quad AE = \frac{6 \times 2}{11}\sqrt{105}\text{ cm}$$
$$\Rightarrow \quad AE = \frac{12}{11}\sqrt{105}\text{ cm}$$

Considering AB as base,

$$\text{Area of } \Delta ABC = \frac{1}{2} \times AB \times CD$$
$$\frac{1}{2} \times AB \times CD = 6\sqrt{105}\text{ cm}^2$$
$$\Rightarrow \quad CD = \frac{6 \times 2}{13}\sqrt{105}\text{ cm}$$
$$\Rightarrow \quad CD = \frac{12}{13}\sqrt{105}\text{ cm}$$

Therefore, lengths of the altitudes corresponding to the sides AB, BC and AC are $\frac{12}{13}\sqrt{105}$ cm, $\frac{12}{11}\sqrt{105}$ cm and $\sqrt{105}$ cm respectively.

Short Answer Type Questions

17. Using Heron's formula, find the area of an equilateral triangle whose perimeter is 24 cm. **[CBSE SP 2013]**

Sol. Perimeter of the given equilateral triangle = 24 cm

$$\therefore \quad s = \frac{\text{Perimeter}}{2} = \frac{24}{2} \text{ cm} = 12 \text{ cm}$$

and each side $= \frac{24}{3}$ cm = 8 cm

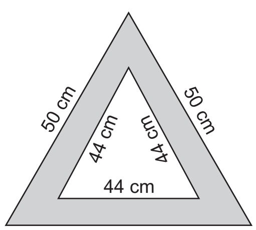
So, a = 8 cm, b = 8 cm, c = 8 cm

Area of the given equilateral triangle

$$= \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{12(12-8)(12-8)(12-8)} \text{ cm}^2$$
$$= \sqrt{12(4)(4)(4)} \text{ cm}^2$$
$$= \sqrt{3 \times 4 \times 4 \times 4 \times 4} \text{ cm}^2$$
$$= 4 \times 4\sqrt{3} \text{ cm}^2$$
$$= 16\sqrt{3} \text{ cm}^2$$

Hence, the area of an equilateral triangle is $16\sqrt{3}$ cm^2.

18. Find the cost of painting the shaded area shown in the given figure at the rate of ₹ 2 per cm^2. (Use $\sqrt{3}$ = 1.73)



Sol. Let a, b, c be sides of the given larger triangle.

Then, $a = b = c = 50$ cm

$$\therefore \quad s = \frac{a+b+c}{2} = \frac{50+50+50}{2} \text{ cm} = \frac{150}{2} \text{ cm} = 75 \text{ cm}$$

Area of the given larger triangle

$$= \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{75(75-50)(75-50)(75-50)} \text{ cm}^2$$
$$= \sqrt{75(25)(25)(25)} \text{ cm}^2$$
$$= \sqrt{3 \times 25 \times 25 \times 25 \times 25} \text{ cm}^2$$
$$= 25 \times 25\sqrt{3} \text{ cm}^2$$
$$= 625\sqrt{3} \text{ cm}^2$$
$$= A_1 \text{ (say)}$$

Let a_1, b_1, c_1, be the side of the given smaller triangle.

Then, $a_1 = b_1 = c_1 = 44$ cm

$$\therefore \quad s_1 = \frac{a_1+b_1+c_1}{2} = \frac{44+44+44}{2} \text{ cm} = \frac{132}{2} \text{ cm} = 66 \text{ cm}$$

Area of the given smaller

$$= \sqrt{s_1(s_1-a_1)(s_1-b_1)(s_1-c_1)}$$
$$= \sqrt{66(66-44)(66-44)(66-44)} \text{ cm}^2$$
$$= \sqrt{66(22)(22)(22)} \text{ cm}^2$$
$$= \sqrt{3 \times 22 \times 22 \times 22 \times 22} \text{ cm}^2$$
$$= 22 \times 22\sqrt{3} \text{ cm}^2$$
$$= 484\sqrt{3} \text{ cm}^2$$
$$= A_2 \text{ (say)}$$

Area to be painted $= A_1 - A_2$

$$= \left(625\sqrt{3} - 484\sqrt{3}\right) \text{ cm}^2$$
$$= 141\sqrt{3} \text{ cm}^2$$
$$= 141 \times 1.73 \text{ cm}^2$$
$$= 243.93 \text{ cm}^2$$

$\therefore$ Cost of painting = 243.93 × ₹2 = ₹487.86

Hence, the cost of painting the shaded area is ₹487.86.

19. Flyovers are built in the city to ease the flow of traffic. The side walls of these flyovers are used for advertisement and generate revenue for municipal corporations. One such wall of a flyover is triangular in shape having dimensions 26 m, 28 m and 30 m. The advertisement cost is ₹ 5000 per m^2 for a single year. Find the rent paid by a company for using the side wall for advertisement for two years.

Sol.
$$s = \frac{26+28+30}{2} \text{ m}$$
$$\Rightarrow \quad s = \frac{84}{2} \text{ m} = 42 \text{ m}$$

Area of the side wall of the flyover

$$= \sqrt{42(42-26)(42-28)(42-30)}\ m^2$$

$$= \sqrt{42 \times 16 \times 14 \times 12}\ m^2$$

$$= \sqrt{14 \times 3 \times 16 \times 14 \times 12}\ m^2$$

$$= 14 \times 4 \times 6\ m^2$$

$$= 336\ m^2$$

Rent paid for using the side wall for advertisement for two years

$$= ₹5000 \times 2 \times 336$$

$$= ₹33,60,000$$

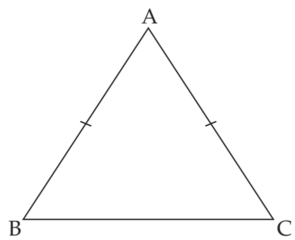
20. The perimeter of an isosceles triangle is 32 cm. The ratio of the equal side to its base is 3 : 2. Find the area of the triangle.

Sol. Let ΔABC be an isosceles triangle in which AB = AC.

Let $AC = 3x$

$BC = 2x$

$\therefore$ $AB = 3x$



Given, perimeter = 32 cm

$3x + 2x + 3x = 32$ cm

$\Rightarrow$ $8x = 32$ cm

$\Rightarrow$ $x = 4$ cm

$\therefore$ AC = 12 cm

BC = 8 cm

AB = 12 cm

$$s = \frac{12+8+12}{2}\text{ cm} = 16\text{ cm}$$

Area of ΔABC

$$= \sqrt{16(16-12)(16-8)(16-12)}\ cm^2$$

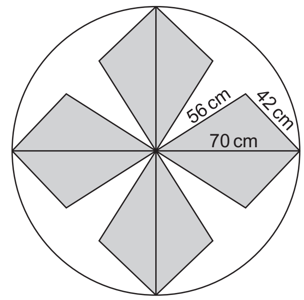
$$= \sqrt{16 \times 4 \times 8 \times 4}\ cm^2$$

$$= 8 \times 4 \times \sqrt{2}\ cm^2$$

$$= 32\sqrt{2}\ cm^2$$

Long Answer Type Questions

21. A design made up of 8 congruent triangles each having sides as 70 cm, 56 cm and 42 cm is painted on a circular glass window of radius 70 cm as shown in the given figure. Find the painted area of the glass window and remaining area.



Sol. Let the sides of each painted triangle be a, b and c, such that

$a = 70$ cm, $b = 56$ cm and $c = 42$ cm

Then, $$s = \frac{a+b+c}{2}$$

$$= \frac{70+56+42}{2}\text{ cm}$$

$$= \frac{168}{2}\text{ cm} = 84\text{ cm}$$

Painted area of the glass window

$$= 8\sqrt{s(s-a)(s-b)(s-c)}$$

$$= 8\sqrt{84(84-70)(84-56)(84-42)}\ cm^2$$

$$= 8\sqrt{84(14 \times 28 \times 42)}\ cm^2$$

$$= 8\sqrt{2 \times 42 \times 14 \times 14 \times 2 \times 42}\ cm^2$$

$$= 8(2 \times 42 \times 14)\ cm^2$$

$$= 9408\ cm^2$$

Remaining Area = Area of the circular window − Area to be painted

$$= \pi r^2 - 9408\ cm^2$$

$$= \left(\frac{22}{7} \times 70 \times 70 - 9408\right)\ cm^2$$

$$= (15400 - 9408)\ cm^2$$

$$= 5992\ cm^2$$

Hence, the painted area of the glass window is 9408 cm^2 and remaining area is 5992 cm^2.

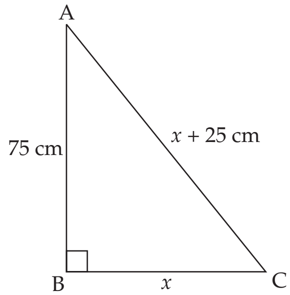
22. The length of one side of a right angled triangle is 75 cm and the difference in length of its hypotenuse and the other side is 25 cm. Find the lengths of its two unknown side and calculate its area. Verify the result by Heron's formula.

Sol. ΔABC is a right-angled triangle.

Let AB = 75 cm

BC = x

$\therefore$ AC = x + 25 cm



$$AC^2 = AB^2 + BC^2$$

$$\Rightarrow \quad (x + 25)^2 = (75)^2 + x^2$$

$$\Rightarrow \quad x^2 + 50x + 625 = 5625 + x^2$$

$$\Rightarrow \quad 50x = 5625 - 625$$

$$\Rightarrow \quad 50x = 5000$$

$$x = 100 \text{ cm}$$

$$\therefore \quad BC = 100 \text{ cm}$$

$$AC = 125 \text{ cm}$$

$$\text{Area of } \Delta ABC = \frac{1}{2} \times 100 \times 75 \text{ cm}^2$$

$$= 3750 \text{ cm}^2$$

$$s = \frac{75 + 100 + 125}{2} \text{ cm}$$

$$= \frac{300}{2} \text{ cm} = 150 \text{ cm}$$

Area of ΔABC by Heron's formula

$$= \sqrt{150 \times (150 - 75)(150 - 100)(150 - 125)} \text{ cm}^2$$

$$= \sqrt{150 \times 75 \times 50 \times 25} \text{ cm}^2$$

$$= \sqrt{50 \times 3 \times 3 \times 25 \times 50 \times 25} \text{ cm}^2$$

$$= 50 \times 3 \times 25 \text{ cm}^2$$

$$= 3750 \text{ cm}^2$$

Let's Compete

(Page 149)

Multiple-Choice Questions

1. The edges of a triangular board are 6 cm, 8 cm and 10 cm. The cost of painting it at the rate of 90 paise per cm^2 is

(*a*) ₹ 19.75 (*b*) ₹ 20.50

(*c*) ₹ 21.60 (*d*) ₹ 23.70

Sol. (*c*) ₹ 21.60

Let the edges of a triangular board be a, b, and c such that

$a = 6$ cm, $b = 8$ cm, $c = 10$ cm

Then, $$s = \frac{a + b + c}{2}$$

$$= \frac{6 + 8 + 10}{2} \text{ cm}$$

$$= \frac{24}{2} \text{ cm} = 12 \text{ cm}$$

Area of the triangular board

$$= \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{12(12-6)(12-8)(12-10)} \text{ cm}^2$$

$$= \sqrt{12(6)(4)(2)} \text{ cm}^2$$

$$= \sqrt{6 \times 2 \times 6 \times 2 \times 2 \times 2} \text{ cm}^2$$

$$= (6 \times 2 \times 2) \text{ cm}^2$$

$$= 24 \text{ cm}^2$$

Cost of painting the triangular board

$$= 24 \times ₹ \frac{90}{100}$$

$$= ₹ 21.60$$

Hence, the cost of painting the traingular board is ₹ 21.60.

2. The perimeter of a rhombus is 40 cm. If one of its diagonals is 12 cm, then its area is

(*a*) 30 cm^2 (*b*) 24 cm^2

(*c*) 48 cm^2 (*d*) 96 cm^2

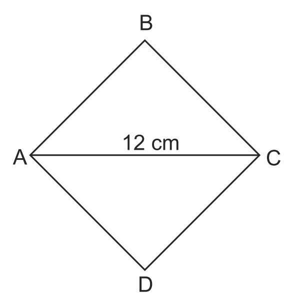
Sol. (*d*) 96 cm^2

Let ABCD be the given rhombus in which

$$AB = BC = CD = DA = \frac{\text{Perimeter}}{4}$$

$$= \frac{40}{4} \text{ cm} = 10 \text{ cm}$$

and diagonal AC = 12 cm



In ΔABC,

Let $a = 10$ cm, $b = 12$ cm and $c = 10$ cm

Then, $$s = \frac{a + b + c}{2}$$

$$= \frac{10 + 12 + 10}{2} \text{ cm}$$

$$= \frac{32}{2} \text{ cm} = 16 \text{ cm}$$

$$\text{Area of } \Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{16(16-10)(16-12)(16-10)} \text{ cm}^2$$
$$= \sqrt{16(6)(4)(6)} \text{ cm}^2$$
$$= \sqrt{4 \times 4 \times 6 \times 2 \times 2 \times 6} \text{ cm}^2$$
$$= 4 \times 6 \times 2 \text{ cm}^2$$
$$= 48 \text{ cm}^2$$

Diagonal of a rhombus divides it in two congruent triangles.

$\Rightarrow$ $\Delta ABC \cong \Delta ADC$

$\Rightarrow$ Area of ΔABC = Area of ΔADC

Area of rhombus ABCD = Area of ΔABC + Area of ΔADC

= 2 Area of ΔABC

= 2 (48) cm^2

= 96 cm^2

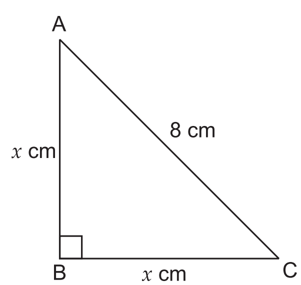
Hence, the area of the rhombus is 96 cm^2.

3. If the length of hypotenuse of a right-angled isosceles triangle is 8 cm, then its area is

(*a*) 16 cm^2 (*b*) 32 cm^2

(*c*) 64 cm^2 (*d*) 40 cm^2

Sol. (*a*) 16 cm^2



Let ABC be a right triangle such that AB = AC = x cm and hypotenuse = AC = 8 cm.

In right ΔABC, we have

$$AC^2 = AB^2 + BC^2$$
$$\Rightarrow \quad (8)^2 = (x)^2 + (x)^2$$
$$\Rightarrow \quad 64 = 2x^2$$
$$\Rightarrow \quad 32 = x^2$$
$$\Rightarrow \quad x = \pm\sqrt{32}$$
$$\Rightarrow \quad x = 4\sqrt{2}$$

So, AB = AC = $4\sqrt{2}$ cm

$$\text{Area of } \Delta ABC = \frac{1}{2} \times BC \times AB$$
$$= \frac{1}{2} \times 4\sqrt{2} \times 4\sqrt{2} \text{ cm}^2$$
$$= \frac{1}{2} \times 16 \times 2 \text{ cm}^2$$
$$= 16 \text{ cm}^2$$

Hence, the area of a right-angled isosceles triangle is 16 cm^2.

4. The ratio of the equal side to its base of an isosceles triangle is 5 : 3. If the perimeter of the triangle is 26 cm, then the area of the triangle is

(*a*) 91 cm^2 (*b*) $91\sqrt{3}$ cm^2

(*c*) $3\sqrt{91}$ cm^2 (*d*) 273 cm^2

Sol. (*c*) $3\sqrt{91}$ cm^2

Given ratio of equal sides to its base of an isosceles triangle is 5 : 3.

Let the sides of the triangle be $5x$ cm, $3x$ cm and $5x$ cm.

Then, $5x + 3x + 5x = 26$ cm

$\Rightarrow$ $13x = 26$ cm

$\Rightarrow$ $x = 2$ cm

So, the sides of the triangle are (5 × 2) cm, (3 × 2) cm and (5 × 2) cm, i.e. 10 cm, 6 cm and 10 cm.

Here, a = 10 cm, b = 6 cm, c = 10 cm

$$\therefore \quad s = \frac{a+b+c}{2}$$
$$= \frac{10+6+10}{2} \text{ cm}$$
$$= \frac{26}{2} \text{ cm}$$
$$= 13 \text{ cm}$$

$$\text{Area of the triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{13(13-10)(13-6)(13-10)} \text{ cm}^2$$
$$= \sqrt{13 \times 3 \times 7 \times 3} \text{ cm}^2$$
$$= 3\sqrt{91} \text{ cm}^2$$

Hence, the area of the triangle is $3\sqrt{91}$ cm^2.

5. If the differences between the semi-perimeter and the sides of a triangle are 12 cm, 8 cm and 4 cm, then the area of the triangle is

(*a*) 48 cm^2 (*b*) 96 cm^2

(*c*) 80 cm^2 (*d*) 90 cm^2

Sol. (*b*) 96 cm^2

Given, $(s-a) = 12$ cm

$(s-b) = 8$ cm

$(s-c) = 4$ cm

$\Rightarrow$ $(s-a+s-b+s-c) = (12+8+4)$ cm

$\Rightarrow$ $3s-(a+b+c) = 24$ cm

$\Rightarrow$ $3s-2s = 24$ cm

$[\because \frac{a+b+c}{2} = s]$

$\Rightarrow \quad s = 24$ cm

Area of the triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{24(12)(8)(4)}$ cm^2

$= \sqrt{12 \times 2 \times 12 \times 2 \times 4 \times 4}$ cm^2

$= (12 \times 2 \times 4)$ cm^2

$= 96$ cm^2

Hence, the area of the triangle is 96 cm^2.

6. If the area of a triangle is 384 cm^2 and its perimeter is 96 cm, then the numerical value of the product of the differences of its semi-perimeter and the lengths of its sides is

(*a*) 3072 (*b*) 1536

(*c*) 8 (*d*) 4

Sol. (*a*) 3072

Area of a triangle = 384 cm^2

Perimeter of a triangle = 96 cm

$\Rightarrow \quad a + b + c = 96$ cm

Then, $\quad s = \frac{a+b+c}{2}$

$= \frac{96}{2}$ cm

$= 48$ cm

Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$\Rightarrow \quad 384 = \sqrt{48(s-a)(s-b)(s-c)}$

Squaring both sides

$\Rightarrow \quad \frac{384 \times 384}{48} = (s-a)(s-b)(s-c)$

$\Rightarrow \quad 3072 = (s-a)(s-b)(s-c)$

Hence, the numerical value of the product of the differences of its semi-perimeter and the length of its sides is 3072.

7. If the differences of the semi-perimeter and the sides of a triangle are 60 m, 30 m and 10 m and the area of the triangle is 1344 m^2, then the height of the triangle corresponding to the smallest side as the base is

(*a*) 67.2 m (*b*) 38.4 m

(*c*) 29.9 m (*d*) 50.3 m

Sol. (*a*) 67.2 m

Given $\quad (s-a) = 60$ m $\quad$ ...(1)

$(s-b) = 30$ m $\quad$ ...(2)

$(s-c) = 10$ m $\quad$ ...(3)

Area of the triangle = 1344 m^2

Adding (1), (2) and (3), we get

$s - a + s - b + s - c = (60 + 30 + 10)$ m

$\Rightarrow \quad 3s - (a + b + c) = 100$ m

$\Rightarrow \quad 3s - 2s = 100$ m $\quad [\because \frac{a+b+c}{2} = s]$

$\Rightarrow \quad s = 100$ m

So, the sides of the triangle are $s - a = 60$

$\Rightarrow \quad 100 - a = 60$

$\Rightarrow \quad a = 40$

and $\quad s - b = 30$

$\Rightarrow \quad 100 - b = 30$

$\Rightarrow \quad b = 70$

and $\quad s - c = 10$

$\Rightarrow \quad 100 - c = 10$

$\Rightarrow \quad c = 90$

i.e. $a = 40$ m, $b = 70$ m, $c = 90$ m

$\therefore$ Smallest side as base = 40 m

We know that

Area of the triangle $= \frac{1}{2} \times$ base $\times$ height

$\Rightarrow \quad 1344$ m$^2 = \frac{1}{2} \times 40$ m $\times$ height

$\Rightarrow \quad \left(\frac{1344 \times 2}{40}\right)$ m = height

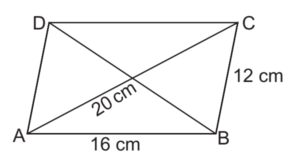
$\Rightarrow \quad$ height = 67.2 m

$\therefore$ The height of the triangle corresponding to the smallest side as the base is 67.2 m.

8. The two adjacent sides of a parallelogram ABCD are of lengths AB = 16 cm and BC = 12 cm and one diagonal AC of the parallelogram is of length 20 cm. Then the area of ΔDBC is

(*a*) 240 cm^2 (*b*) 96 cm^2

(*c*) 336 cm^2 (*d*) 192 cm^2



Sol. (*b*) 96 cm^2

Given ABCD is a parallelogram such that AB = 16 cm, BC = 12 cm and AC = 20 cm

$\therefore$ AB = CD = 16 cm, BC = AD = 12 cm

In ΔDBC, we have

a = CD = 16 cm, b = BC = 12 cm, c = AC = 20 cm

Then, $\quad s = \frac{a+b+c}{2}$

$$= \left(\frac{16 + 12 + 20}{2}\right) \text{cm}$$

$$= \left(\frac{48}{2}\right) \text{cm} = 24 \text{ cm}$$

Area of ΔDBC = $\sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{24(24-16)(24-12)(24-20)} \text{ cm}^2$$

$$= \sqrt{24(8)(12)(4)} \text{ cm}^2$$

$$= \sqrt{12 \times 2 \times 2 \times 4 \times 12 \times 4} \text{ cm}^2$$

$$= (12 \times 2 \times 4) \text{ cm}^2$$

$$= 96 \text{ cm}^2$$

$\therefore$ Area of ΔDBC is 96 cm^2.

9. If the differences of the perimeter and the sides of a triangle are 57 cm, 71 cm and 88 cm, then the area of this triangle is

(*a*) 326 cm^2 (*b*) 300 cm^2

(*c*) 320 cm^2 (*d*) 306 cm^2

Sol. (*d*) 306 cm^2

Let perimeter = P and semi-perimeter = s

Given $(P - a) = 57$ cm

$(P - b) = 71$ cm

$(P - c) = 88$ cm

Adding these, we get

$P - a + P - b + P - c = (57 + 71 + 88)$ cm

$\Rightarrow$ $3P - (a + b + c) = 216$ cm

$\Rightarrow$ $3(2s) - (2s) = 216$ cm

[$\because$ Perimeter = 2 semi-perimeter and $\frac{a+b+c}{2} = s$]

$\Rightarrow$ $6s - 2s = 216$ cm

$\Rightarrow$ $4s = 216$ cm

$\Rightarrow$ $s = 54$ cm

$\therefore$ Perimeter, $P = 2s$

$= 2(54 \text{ cm})$

$\Rightarrow$ $P = 108$ cm

So, the sides of the triangle and $P - a = 57$

$\Rightarrow$ $108 - a = 57$

$\Rightarrow$ $a = 51$

and $P - b = 71$

$\Rightarrow$ $108 - b = 71$

$\Rightarrow$ $b = 37$

and $P - c = 88$

$\Rightarrow$ $108 - c = 88$

$\Rightarrow$ $c = 20$

i.e. $a = 51$ cm, $b = 37$ cm, $c = 20$ cm

Area of the triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{54(54-51)(54-37)(54-20)} \text{ cm}^2$$

$$= \sqrt{54(3)(17)(34)} \text{ cm}^2$$

$$= \sqrt{3 \times 3 \times 3 \times 2 \times 3 \times 17 \times 17 \times 2} \text{ cm}^2$$

$$= (3 \times 3 \times 2 \times 17) \text{ cm}^2$$

$$= 306 \text{ cm}^2$$

Hence, the area of the triangle is 306 cm^2.

10. A tent is made by stitching 4 triangular pieces of canvas, each piece 13 m, 13 m and 24 m. Then the total area of the canvas required to make the tent is

(*a*) 120 m^2 (*b*) 200 m^2

(*c*) 240 m^2 (*d*) 60 m^2

Sol. (*c*) 240 m^2

Here, $a = 13$ m, $b = 13$ m, $c = 24$ m

Then, $s = \frac{a+b+c}{2}$

$$= \left(\frac{13 + 13 + 24}{2}\right) \text{m}$$

$$= \left(\frac{50}{2}\right) \text{m}$$

$$= 25 \text{ m}$$

Area of a triangular piece of canvas

$$= \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{25(25-13)(25-13)(25-24)} \text{ m}^2$$

$$= \sqrt{25(12)(12)(1)} \text{ m}^2$$

$$= \sqrt{5 \times 5 \times 12 \times 12} \text{ m}^2$$

$$= 5 \times 12 \text{ m}^2$$

$$= 60 \text{ m}^2$$

Total area of the canvas required to make the tent

= 4 × Area of a triangular piece of canvas

$$= (4 \times 60) \text{ m}^2$$

$$= 240 \text{ m}^2$$

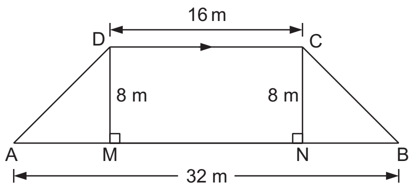
Hence, the total area of the canvas required to make the tent is 240 m^2.

Life Skills

(Page 150)

1. ABCD in the shape of an isosceles trapezium with

AB ∥ CD represents a plot of land owned by an old man, where AB = 32 m and CD = 16 m. He divides the whole land into three parts by drawing DM ⊥ AB and CN ⊥ AB, where M and N are points on AB and DM = CN = 8 m. He donates both the triangular parts to an orphanage and keeps the rectangular part MNCD for himself.



Find the total area of land donated by him to the orphanage.

Sol. Given ABCD is an isosceles trapezium with AB ∥ CD where AB = 32 m and CD = 16 m, DM = CN = 8 m.

∴ MN = CD = 16 m

AM = NB …(1)

[∵ ABCD is an isosceles trapezium]

Now, AB = AM + MN + NB

⇒ 32 m = AM + 16 m + NB

⇒ 16 m = AM + NB

⇒ 16 m = 2 AM [∵ NB = AM]

⇒ AM = 8 m [From (1)]

In ΔAMD,

$$\text{Area of } \Delta AMD = \frac{1}{2} \times \text{base} \times \text{height}$$

$$= \frac{1}{2} \times AM \times DM$$

$$= \frac{1}{2} \times 8 \text{ m} \times 8 \text{ m}$$

$$= 32 \text{ m}^2 \quad \ldots(2)$$

Since ABCD is an isosceles trapezium

∴ ΔAMD ≅ ΔBNC

⇒ Area of ΔAMD = Area of ΔBNC

∴ Area of ΔBNC = 32 m^2 …(3)

Total area of land donated by him to orphanage

= Area of ΔAMD + Area of ΔBNC

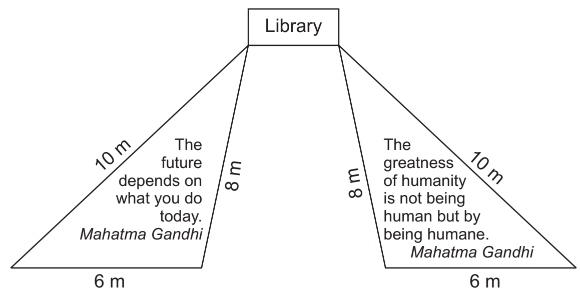
= 32 m^2 + 32 m^2 [From (2) and (3)]

= 64 m^2

Hence, he donated 64 m^2 of land to an orphanage.

2. The entrance to a library consists of two congruent triangular side walls as shown below with quotes of Mahatma Gandhi about the greatness of humanity, hardworking, helpfulness etc.

Find the area of each triangle if the sides of each triangle are 10 m, 8 m and 6 m.



Sol. For each triangular wall.

$a = 10$ m, $b = 8$ m, $c = 6$ m

Then, $$s = \frac{a+b+c}{2}$$

$$= \frac{10+8+6}{2} \text{ m}$$

$$= \frac{24}{2} \text{ m} = 12 \text{ m}$$

Area of each triangular wall

$$= \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{12(12-10)(12-8)(12-6)} \text{ m}^2$$

$$= \sqrt{12(2)(4)(6)} \text{ m}^2$$

$$= \sqrt{2 \times 6 \times 2 \times 2 \times 2 \times 6} \text{ m}^2$$

$$= 2 \times 6 \times 2 \text{ m}^2$$

$$= 24 \text{ m}^2$$

Hence, the area of each triangle is 24 m^2.