9

Circles

Checkpoint (Page 119)

1. The diameter of planet Venus is 12278 km. Find its circumference.

Sol. Diameter = 12278 km

$$\text{Radius } (R) = \frac{\text{Diameter}}{2}$$

$$= \frac{12278}{2} \text{ km} = 6139 \text{ km}$$

Radius (R) = 6139 km

Circumference = $2\pi R$

$$= 2 \times \frac{22}{7} \times 6139 \text{ km}$$

$$= 2 \times 22 \times 877 \text{ km}$$

$$= 38588 \text{ km}$$

Hence, circumference of planet venus is 38588 km.

2. Two circles having the same centre have radii 350 m and 490 m. What is the difference between their circumferences?

Sol. Radius of one circle (R_1) = 350 m

Radius of other circle (R_2) = 490 m

Circumference (C_1) = $2\pi R_1$

$$= 2 \times \frac{22}{7} \times 350 \text{ m}$$

$$= 2 \times 22 \times 50 \text{ m} = 2200 \text{ m}$$

Circumference (C_2) = $2\pi R_2$

$$= 2 \times \frac{22}{7} \times 490 \text{ m}$$

$$= 2 \times 22 \times 70 \text{ m} = 3080 \text{ m}$$

Difference between the circumference

$$= C_2 - C_1$$

$$= (3080 - 2200) \text{ m} = 880 \text{ m}$$

Hence, difference between the circumference of two circles is 880 m.

3. Find the perimeter of a semicircular plate of radius 3.85 cm.

Sol. Radius (R) = 3.85 cm

Perimeter of semi-circular plate = $\pi R + 2R$

$$= \left(\frac{22}{7} \times 3.85 + 2 \times 3.85\right) \text{ cm}$$

$$= (22 \times 0.55 + 2 \times 3.85) \text{ cm}$$

$$= (12.1 + 7.7) \text{ cm}$$

$$= 19.8 \text{ cm}$$

Hence, perimeter of semi-circular plate is 19.8 cm.

4. The area of a circle is 55.44 m^2. Find its radius.

Sol. Area of a circle (A) = 55.44 m^2

$$A = \pi R^2$$

$$55.44 \text{ m}^2 = \frac{22}{7} \times R^2$$

$$(55.44 \times 7) \text{ m}^2 = 22 \times R^2$$

$$\left(\frac{55.44 \times 7}{22}\right) \text{ m}^2 = R^2$$

$$R^2 = \left(\frac{55.44 \times 7}{22}\right) \text{m}^2$$

$$= (2.52 \times 7) \text{ m}^2$$

$$= (17.64) \text{ m}^2$$

$$R^2 = \left(\frac{1764}{100}\right) \text{ m}^2$$

$$R^2 = \left(\frac{42}{10}\right)^2 \text{ m}^2$$

$$R = 4.2 \text{ m}$$

Hence, the radius of a circle is 4.2 m.

5. The area of two circles are in the ratio 16 : 25. Find the ratio of their circumferences.

Sol. Let area of one circle be A_1

Let area of other circle be A_2

Ratio of area of two circles = 16 : 25

$$\frac{A_1}{A_2} = \frac{16}{25}$$

$$\frac{\pi R_1^2}{\pi R_2^2} = \frac{16}{25}$$

$$\frac{R_1^2}{R_2^2} = \frac{16}{25}$$

$$\left(\frac{R_1}{R_2}\right)^2 = \left(\frac{4}{5}\right)^2$$

$$\frac{R_1}{R_2} = \frac{4}{5}$$

Ratio of circumference of two circles are

$$\frac{C_1}{C_2} = \frac{2\pi R_1}{2\pi R_2} = \frac{R_1}{R_2} = \frac{4}{5}$$

Hence ratio of circumference of two circles is 4 : 5.

6. The minute hand of a circular clock is 11 cm long. How far does the tip of the minute hand move in 2 hours? [Take π = 3.14]

Sol. The minute hand of a circular clock (Radius) = 11 cm. We have to find circumference of a circular clock.

Circumference (C) = $2\pi R$

= 2 × 3.14 × 11 cm

= 69.08 cm

2 hours = 2 complete revolution

Distance covered by tip of minute hand in

2 hours = 2 × 69.08 cm

= 138.16 cm

Hence, the tip of the minute hand move 138.16 cm in 2 hours.

7. If the area of a circle is 24.64 cm^2, then find its circumference.

Sol. Area (A) = 24.64 cm^2

$A = \pi R^2$

$\Rightarrow \quad 24.64\ cm^2 = \frac{22}{7} \times R^2$

$\Rightarrow \quad (24.64 \times 7)\ cm^2 = 22 \times R^2$

$\Rightarrow \quad \left(\frac{24.64 \times 7}{22}\right) cm^2 = R^2$

$\Rightarrow \quad R^2 = \left(\frac{2.24 \times 7}{2}\right) cm^2$

$\Rightarrow \quad R^2 = (1.12 \times 7)\ cm^2$

$\Rightarrow \quad R^2 = 7.84\ cm^2$

$\Rightarrow \quad R = 2.8$ cm

Circumference (C) = $2\pi R$

$= 2 \times \frac{22}{7} \times 2.8$ cm

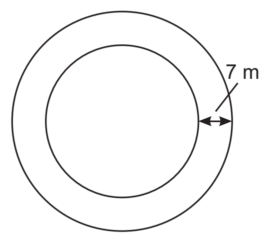
= (2 × 22 × 0.4) cm

= 17.6 cm

Hence, circumference of a circle is 17.6 cm.

8. The circumference of a circular park is 660 m. A 7 m wide path surrounds it. Find the cost of fencing the outer boundary at the rate of ₹ 60 per metre.

Sol. Circumference of a circular park (C) = 660 m



C = $2\pi R$

[R is the radius of a circle]

$\Rightarrow \quad 660\ m = 2 \times \frac{22}{7} \times R$

$\Rightarrow \quad (660 \times 7)\ m = 2 \times 22 \times R$

$\Rightarrow \quad \left(\frac{660 \times 7}{2 \times 22}\right) m = R$

$\Rightarrow \quad R = \left(\frac{30 \times 7}{2}\right) m$

= (15 × 7) m

$\Rightarrow \quad R = 105$ m

∴ Radius of circular park = 105 m

We need to find the circumference of the circular path around the park.

Circumference (C_1) = $2\pi R_1$

[$R_1 = R + 7 = (105 + 7)$ m = 112 m]

$= 2 \times \frac{22}{7} \times 112$ m

= (2 × 22 × 16) m

= 704 m

Cost of fencing 1 m outer boundary = ₹60

Cost of fencing 704 m outer boundary

= ₹60 × 704

= ₹42240

Hence, cost of fencing outer boundary is ₹42240.

9. Find the radius of a circle whose area is twice the area of a circle of radius 14 cm.

Sol. Radius of a circle (R) = 14 cm

Area of a circle (A) = πR^2

$$= \left(\frac{22}{7} \times 14 \times 14\right) \text{ cm}^2$$

$$= (22 \times 2 \times 14) \text{ cm}^2$$

$$= 616 \text{ cm}^2$$

Let area of the other circle be A_1 and radius be R_1.

Given that

Area of the other circle = 2 × area of a circle

$$A_1 = (2 \times 616) \text{ cm}^2$$

$$\pi R_1^2 = (2 \times 616) \text{ cm}^2$$

$$R_1^2 = \frac{2 \times 616 \times 7}{22} \text{ cm}^2$$

$$= \frac{8624}{22} \text{ cm}^2$$

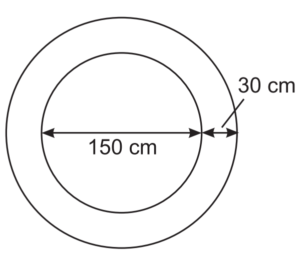
$$R_1^2 = 392 \text{ cm}^2$$

$$R_1 = 14\sqrt{2} \text{ cm}$$

Hence, radius of the other circle is $14\sqrt{2}$ cm.

10. A well of diameter 150 cm has a 30 cm wide parapet running around it. Find the area of the parapet.

Sol.



Radius of inner circle (R) = $\dfrac{\text{Diameter}}{2}$

$$= \frac{150}{2} \text{ cm} = 75 \text{ cm}$$

Area of inner circle = πR^2

$$= \left(\frac{22}{7} \times 75 \times 75\right) \text{cm}^2$$

Radius of outer circle (R_1) = R + 30

= (75 + 30) cm

= 105 cm

Area of outer circle = πR_1^2

$$= \left(\frac{22}{7} \times 105 \times 105\right) \text{cm}^2$$

Area of ring = Area of outer circle − Area of inner circle

$$= \pi R_1^2 - \pi R^2$$

$$= \pi\left(R_1^2 - R^2\right)$$

$$= \frac{22}{7}\left[105 \times 105 - 75 \times 75\right] \text{cm}^2$$

$$= \frac{22}{7}\left[(105)^2 - (75)^2\right] \text{cm}^2$$

$$= \frac{22}{7}(105 + 75)(105 - 75) \text{ cm}^2$$

[Using identity $a^2 - b^2 = (a + b)(a - b)$]

$$= \left(\frac{22}{7} \times 180 \times 30\right) \text{ cm}^2$$

$$= \frac{118800}{7} \text{ cm}^2$$

$$= 16971.42 \text{ cm}^2$$

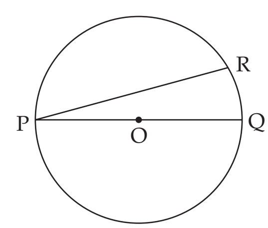
Hence, area of parapet is 16971 cm^2 (approx.)

Check Your Progress 1

(Page 125)

Multiple-Choice Questions

1. If PQ = 8.2 cm and PR = 8 cm, then find the distance of chord PR from the centre of the circle O.



(*a*) 0.009 cm (*b*) 0.09 cm

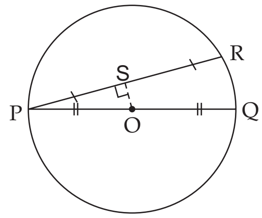
(*c*) 0.9 cm (*d*) 9 cm

Sol. (*b*) 0.9 cm

From O, draw perpendicular as on chord PR.

PS = SR

[Perpendicular from the centre bisects the chord]



In right triangle PSO,

$$PS = 4 \text{ cm}$$

$$PO = 4.1 \text{ cm}$$

$$PO^2 = OS^2 + PS^2$$

$\Rightarrow$ $OS^2 = PO^2 - PS^2$

$\Rightarrow$ $OS^2 = (4.1 \text{ cm})^2 - (4 \text{ cm})^2$

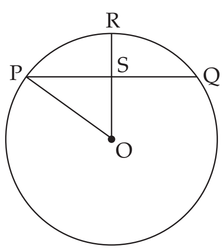
$\Rightarrow$ $OS^2 = (4.1 \text{ cm} + 4 \text{ cm})\ (4.1 \text{ cm} - 4 \text{ cm})$

$\Rightarrow$ $OS^2 = 8.1 \text{ cm} \times 0.1 \text{ cm}$

$\Rightarrow$ $OS^2 = 0.81 \text{ cm}^2$

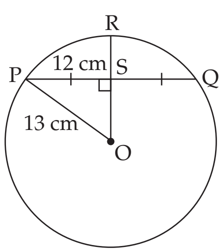
$\therefore$ $OS = 0.9 \text{ cm}$

2. In the given figure, the radius of the circle is 13 cm and length of the chord PQ is 24 cm. Then the measure of RS is



(*a*) 5 cm. (*b*) 8 cm.
(*c*) 12 cm. (*d*) 13 cm.

Sol. (*b*) 8 cm



In ΔOSP,

$OP = 13 \text{ cm}$ [Radius of the circle]

$SP = 12 \text{ cm}$

$OS^2 = OP^2 - SP^2$

$\Rightarrow$ $OS^2 = (13 \text{ cm})^2 - (12 \text{ cm})^2$

$\Rightarrow$ $OS^2 = 25 \text{ cm}^2$

$\Rightarrow$ $OS = 5 \text{ cm}$

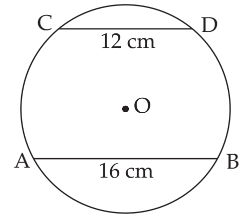
$OR = 13 \text{ cm}$ [Radius of the circle]

$RS = OR - OS$

$= 13 \text{ cm} - 5 \text{ cm}$

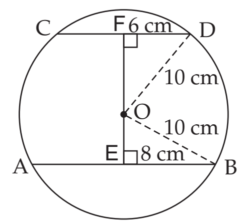
$\therefore$ $RS = 8 \text{ cm}$

3. In the given figure, the radius of the circle is 10 cm. Chords AB and CD are of lengths 16 cm and 12 cm respectively. Find the distance between the chords. What will be change in the distance if the chords are in the same direction?



(*a*) 2 cm. The distance will increase by 12 cm.
(*b*) 14 cm. The distance will decrease by 12 cm.
(*c*) 7 cm. The distance will remain the same.
(*d*) 14 cm. The distance will increase by 12 cm.

Sol. (*b*) 14 cm. The distance will decrease by 12 cm.



In ΔOFD,

$OF^2 = OD^2 - FD^2$

$OF^2 = 100 \text{ cm}^2 - 36 \text{ cm}^2$

$OF^2 = 64 \text{ cm}^2$

$\Rightarrow$ $OF = 8 \text{ cm}$ ...(1)

In ΔOEB

$OE^2 = OB^2 - BE^2$

$OE^2 = 100 \text{ cm}^2 - 64 \text{ cm}^2$

$OE^2 = 36 \text{ cm}^2$

$\Rightarrow$ $OE = 6 \text{ cm}$

Distance between chords A and B = 8 cm + 6 cm = 14 cm.

When both the chords are on the same side, distance between them = 8 cm – 6 cm = 2 cm.

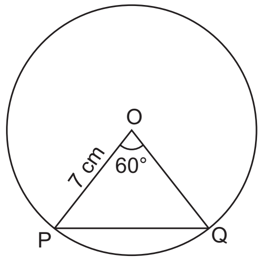
$\therefore$ The distance will decrease by 12 cm.

4. PQ is a chord of a circle with centre O and radius equal to 7 cm. If $\angle POQ = 60^\circ$, then the length of the chord PQ is

(*a*) $\frac{7}{2}$ cm (*b*) $\frac{7\sqrt{3}}{2}$ cm

(*c*) 7 cm (*d*) $\frac{7\sqrt{3}}{4}$ cm

Sol. (*c*) 7 cm



Given,

Radius = 7 cm, OP = OQ = 7 cm

Also, $\angle POQ = 60°$

In ΔPOQ, we have

$\angle POQ + \angle OPQ + \angle OQP = 180°$

[Angle sum property of a triangle]

$\Rightarrow$ $60° + \angle OPQ + \angle OPQ = 180°$

[$\angle OPQ = \angle OQP$,

$\because$ OP = OQ, ΔPOQ is an isosceles triangle]

$\Rightarrow$ $60° + 2\angle OPQ = 180°$

$\Rightarrow$ $2\angle OPQ = 180° - 60°$

$\Rightarrow$ $2\angle OPQ = 120°$

$\Rightarrow$ $\angle OPQ = 60°$

$\Rightarrow$ $\angle OQP = 60°$

Hence, ΔPOQ is an equilateral triangle as

$\angle POQ = \angle OPQ = \angle OQP = 60°$.

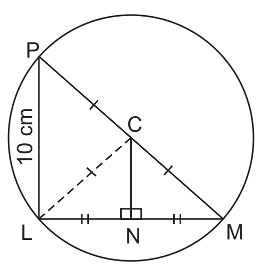
$\therefore$ Length of chord PQ is 7 cm.

5. LM is a chord of a circle with centre C. If CN $\perp$ LM, MC produced intersects the circle at P and if PL = 10 cm, then the length of CN will be

(*a*) 8 cm (*b*) 5 cm

(*c*) 6 cm (*d*) 9 cm

Sol. (*b*) 5 cm



Given,

C is the centre of a circle and let

PC = MC = LC = R

[Radius of a circle]

$\because$ ΔPCL and ΔCLM is an isosceles triangle

[PC = LC = MC]

Let $\angle CPL = \angle CLP = \angle CLM = \angle CML = x$

In ΔPLM, we have

$\angle LPM + \angle LMP + \angle PLM = 180°$

[Angle sum property of a triangle]

$\Rightarrow$ $\angle LPM + \angle LMP + \angle CLP + \angle CLM = 180°$

$\Rightarrow$ $x + x + x + x = 180°$

$\Rightarrow$ $4x = 180°$

$\Rightarrow$ $x = 45°$

$\therefore$ ΔPCL is a right-angled triangle.

By Pythagoras' Theorem, we have

$(PL)^2 = (LC)^2 + (PC)^2$

$\Rightarrow$ $(10\text{ cm})^2 = R^2 + R^2$

$\Rightarrow$ $100\text{ cm}^2 = 2R^2$

$\Rightarrow$ $R^2 = 50\text{ cm}^2$

$\Rightarrow$ $R = 5\sqrt{2}$ cm

PM = PC + MC

$= 5\sqrt{2} + 5\sqrt{2}$

$= 10\sqrt{2}$

$\because$ ΔPLM is a right-angled triangle,

$\therefore$ By Pythagoras' Theorem, we have

$(PM)^2 = (PL)^2 + (LM)^2$

$\Rightarrow$ $\left(10\sqrt{2}\text{ cm}\right)^2 = (10\text{ cm})^2 + (LM)^2$

$\Rightarrow$ $200\text{ cm}^2 = 100\text{ cm}^2 + (LM)^2$

$\Rightarrow$ $(LM)^2 = 100\text{ cm}^2$

$\Rightarrow$ LM = 10 cm

CN is perpendicular to LM, so it will bisect LM. [The $\perp$ from the centre of a circle to a chord bisects the chord]

LN = NM = 5 cm

$\because$ ΔCNL is a right-angled triangle,

$\therefore$ By Pythagoras' Theorem, we have

$(LC)^2 = (CN)^2 + (LN)^2$

$\Rightarrow$ $\left(5\sqrt{2}\text{ cm}\right)^2 = (CN)^2 + (5\text{ cm})^2$

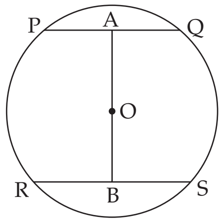
$\Rightarrow$ $50\text{ cm}^2 = (CN)^2 + 25\text{ cm}^2$

$\Rightarrow$ $(CN)^2 = 25\text{ cm}^2$

$\Rightarrow$ CN = 5 cm

Very Short Answer Type Questions

6. The radius of the circle is 5 cm. If the lengths of the chords PQ and RS are 6 cm and 8 cm, find the length of AB.

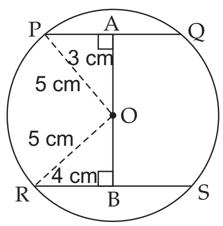


Sol. In ΔOAP

$AP = AQ$ [Perpendicular from the centre bisects the chord]

$OA^2 = OP^2 - PA^2$

[OP = radius of the circle]



$\Rightarrow \quad OA^2 = (5\text{ cm})^2 - (3\text{ cm})^2$

$\Rightarrow \quad OA^2 = 25\text{ cm}^2 - 9\text{ cm}^2 = 16\text{ cm}^2$

$\Rightarrow \quad OA = 4\text{ cm}$ ...(1)

In ΔOBR,

$OB^2 = OR^2 - RB^2$

[OR = radius of the circle]

$\Rightarrow \quad OB^2 = (5\text{ cm})^2 - (4\text{ cm})^2$

$\Rightarrow \quad OB^2 = 25\text{ cm}^2 - 16\text{ cm}^2 = 9\text{ cm}^2$

$\Rightarrow \quad OB = 3\text{ cm}$ ...(2)

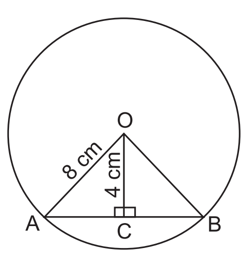
$AB = OA + OB$

$= 4\text{ cm} + 3\text{ cm}$ [From (1) and (2)]

$\therefore \quad AB = 7\text{ cm}$

7. Find the length of the chord of a circle of radius 8 cm, if its distance from the centre of the circle is 4 cm.

Sol. Let O is the centre of a circle and AB be the chord.



Given,

Radius(OA = OB) = 8 cm and OC = 4 cm is the distance of the chord from the centre of a circle.

$\because \quad \Delta ACO$ is a right-angled triangle,

$\therefore$ By Pythagoras' Theorem, we have

$(OA)^2 = (AC)^2 + (OC)^2$

$(8\text{ cm})^2 = (AC)^2 + (4)^2$

$\Rightarrow \quad 64\text{ cm}^2 = (AC)^2 + 16\text{ cm}^2$

$\Rightarrow \quad (AC)^2 = (64 - 16)\text{ cm}^2$

$\Rightarrow \quad (AC)^2 = 48\text{ cm}^2$

$\Rightarrow \quad AC = 4\sqrt{3}\text{ cm}$

$\because \quad AB = AC + CB$

[AC = CB, $\because$ The $\perp$ from the centre of a circle to a chord bisects the chord]

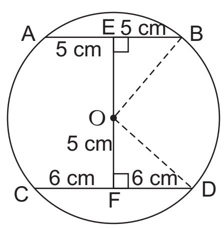
$= \left(4\sqrt{3} + 4\sqrt{3}\right)\text{cm}$

$= 8\sqrt{3}\text{ cm}$

Hence, length of the chord of a circle is $8\sqrt{3}$ cm.

8. The lengths of two parallel chords of a circle are 12 cm and 10 cm. If the distance of one of the chords from the centre of the circle is 5 cm, find the distance of the other chord from the centre of the circle.

Sol. Let the distance of chord of length 12 cm from the centre = 5 cm.



In ΔOFD

$OD^2 = OF^2 + FD^2$

$\Rightarrow \quad OD^2 = (5\text{ cm})^2 + (6\text{ cm})^2$

$\Rightarrow \quad OD^2 = 25\text{ cm}^2 + 36\text{ cm}^2$

$\Rightarrow \quad OD^2 = 61\text{ cm}^2$

$\therefore \quad OD = \sqrt{61}\text{ cm}$

But OD = radius of the circle = $\sqrt{61}$ cm

In ΔOEB,

$OB = OD$

= radius of the circle = $\sqrt{61}$ cm

$OE^2 = OB^2 - BE^2$

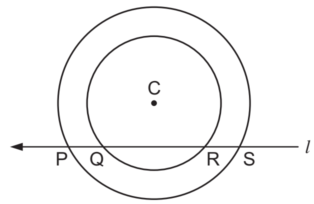
$\Rightarrow \quad OE^2 = 61\text{ cm}^2 - 25\text{ cm}^2$

$\Rightarrow \quad OE^2 = 36\text{ cm}^2$

$\Rightarrow \quad OE = 6\text{ cm}$

The distance of the other chord from the centre of the circle = 6 cm.

9. Two concentric circles with centre C are cut by a line l at P, Q, R and S as shown in the figure. Prove that PQ = RS.



Sol.

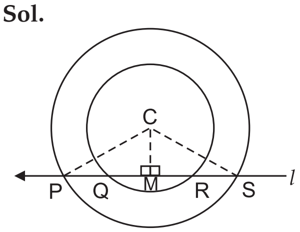


Fig (*i*)

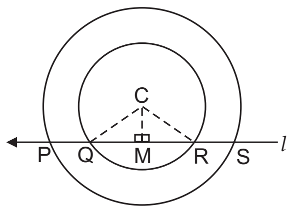


Fig (*ii*)

Construction: Draw a line CM ⊥ PS.

Also, join CP and CS in Fig (*i*)

∵ PS is a chord,

∴ PM = MS …(1)

[The ⊥ from the centre of a circle to a chord bisects the chord]

Construction: Draw CM ⊥ PS.

Also, join CQ and CR in Fig (*ii*)

∵ CM ⊥ QR

∴ QM = MR …(2)

[∵ The ⊥ from the centre of a circle to a chord bisects the chord]

Subtract (2) from (1), we get

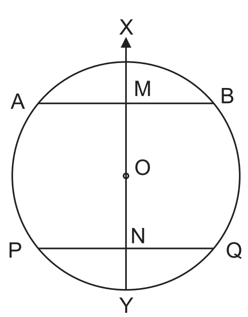
PM – QM = MS – MR

⇒ PQ = RS

Hence, proved.

10. If a diameter of a circle bisects each of the two chords of the circle, prove that the chords are parallel.

Sol.



Let XOY bisect the chord AB at M and chord PQ at N.

∵ M is mid-point of AB,

∴ OM ⊥ AB

[The line drawn joining the centre of a circle to the mid-point of a chord is perpendicular to the chord]

∴ ∠BMO is a right angle. …(1)

Similarly, ON ⊥ PQ

∴ ∠PNO is a right angle. …(2)

∴ ∠BMO = ∠PNO [From (1) and (2)]

⇒ ∠BMN = ∠PNM [Each 90°]

But ∠BMN and ∠PNM are alternate angles.

∴ AB ∥ PQ

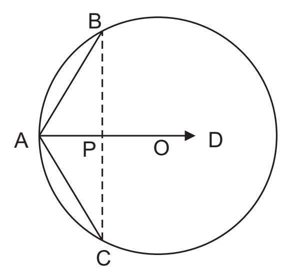
Hence, proved.

Short Answer Type Questions

11. Two chords AB and AC of a circle are equal. Prove that the centre of the circle lies on the bisector of ∠BAC.

Sol. Let AD be the bisector of ∠BAC.

Join BC and let intersect AD at P.



In ΔBAP and ΔCAP, we have

AB = AC [Given]

∠BAP = ∠CAP

[∵ AD is the bisector of ∠BAC]

AP = AP [Common]

∴ ΔBAP ≅ ΔCAP

[By SAS congruence]

⇒ PB = PC [By CPCT] …(1)

⇒ ∠APB = ∠APC [By CPCT] …(2)

Now, ∠APB + ∠APC = 180° [Linear pair] …(3)

∴ ∠APB = ∠APC = 90°

[From (2) and (3)] …(4)

∴ AP is perpendicular bisector of chord BC

[From (1) and (4)]

⇒ AD is the perpendicular bisector of chord BC. But the perpendicular bisector of a chord always passes through the centre of the circle.

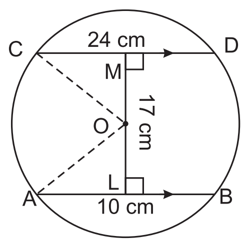
∴ AD passes through the center O of the circle.

⇒ O lies on AD.

Hence, the centre of the circle lies on the angle bisector of ∠BAC.

12. AB and CD are two parallel chords of a circle lying on the opposite sides of the centre such that AB = 10 cm and CD = 24 cm. If the distance between AB and CD is 17 cm, determine the radius of the circle.

Sol. Given, AB and CD are two chords of a circle such that AB ∥ CD, AB = 10 cm, CD = 24 cm and distance between AB and CD = 17 cm.



Draw OL ⊥ AB and OM ⊥ CD. Join OA and OC.

Then, $OA = OC = r$

[Radius of a circle]

∵ OL ⊥ AB, OM ⊥ CD and AB ∥ CD,

∴ The points L, O and M are collinear and LM = 17 cm

Let OL = x cm, OM = $(17 - x)$ cm

∵ Perpendicular from the centre of a circle to a chord bisects the chord.

∴ $$AL = \frac{1}{2}AB$$

$$= \frac{1}{2} \times 10 = 5 \text{ cm}$$

and $$CM = \frac{1}{2}CD$$

$$= \frac{1}{2} \times 24 = 12 \text{ cm}$$

∵ ΔOLA is a right-angled triangle,

∴ By Pythagoras' theorem, we have

$$(OA)^2 = (OL)^2 + (AL)^2$$

⇒ $$r^2 = x^2 + (5)^2 \quad \ldots(1)$$

∵ ΔOMC is a right-angled triangle,

∴ By Pythagoras' Theorem, we have

$$(OC)^2 = (OM)^2 + (CM)^2$$

⇒ $$r^2 = (17 - x)^2 + (12)^2 \quad \ldots(2)$$

From (1) and (2), we have

$$x^2 + (5)^2 = (17 - x)^2 + (12)^2$$

⇒ $$x^2 + 25 = 289 + x^2 - 34x + 144$$

⇒ $$34x = 408$$

⇒ $$x = 12$$

Now, substituting $x = 12$ in (1), we get

$$r^2 = (12)^2 + (5)^2$$

⇒ $$r^2 = 144 + 25$$

⇒ $$r^2 = 169$$

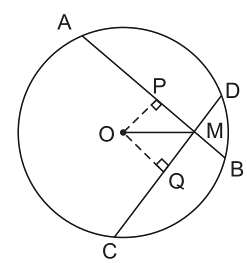
⇒ $$r = 13 \text{ cm}$$

Hence, the radius of a circle is 13 cm.

13. If two equal chords of a circle intersect within the circle, prove that the line joining the point of intersection to the centre makes equal angles with the chords.

Sol. Let AB and CD be two equal chords of a circle with centre O, such that they intersect at point M within the circle.

Draw OP ⊥ AB and OQ ⊥ CD. Also, join OM.



In ΔOPM and ΔOQM, we have

OP = OQ

[Equal chords are equidistant from the centre]

OM = OM [Common]

∴ ΔOPM ≅ ΔOQM

[By RHS congruence]

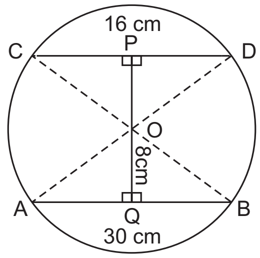
⇒ ∠OMP = ∠OMQ [By CPCT]

Thus, if two equal chords of a circle intersect within the circle, then the line joining the point of intersection to the centre makes equal angles with the chords.

Long Answer Type Questions

14. If the length of a chord of a circle 8 cm away from the centre of the circle, is 30 cm, find the distance of a chord of length 16 cm, from the centre of the circle.

Sol. O is the centre of a circle.



Chord AB = 30 cm

Distance of a chord AB from centre, OQ = 8 cm

$\because$ $OQ \perp AB$

[The $\perp$ from the centre of a circle to a chord bisects the chord]

$\therefore$ AQ = 15 cm

$\because$ ΔAQO is a right-angled triangle,

$\because$ By Pythagoras' Theorem, we have

$$(AO)^2 = (AQ)^2 + (OQ)^2$$
$$= (15 \text{ cm})^2 + (8 \text{ cm})^2$$
$$= (225 + 64) \text{ cm}^2$$

$\Rightarrow$ $(AO)^2 = 289 \text{ cm}^2$

$\Rightarrow$ AO = 17 cm

$\Rightarrow$ AO = BO = CO = DO = 17 cm

[Radius of a circle]

$\because$ $OP \perp CD$

[The $\perp$ from the centre of a circle to a chord bisects the chord]

$\therefore$ CP = 8 cm

$\because$ ΔCPO is a right-angled triangle,

$\therefore$ By Pythagoras' Theorem, we have

$$(CO)^2 = (CP)^2 + (OP)^2$$

$\Rightarrow$ $(17 \text{ cm})^2 = (8 \text{ cm})^2 + (OP)^2$

$\Rightarrow$ $289 \text{ cm}^2 = 64 \text{ cm}^2 + (OP)^2$

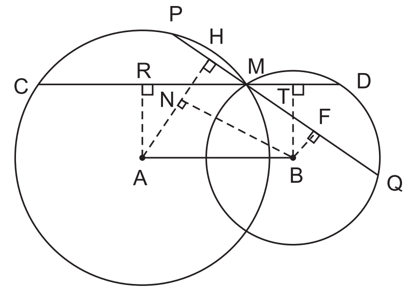
$\Rightarrow$ $(OP)^2 = 225 \text{ cm}^2$

$\Rightarrow$ OP = 15 cm

Hence, the distance of a chord of length 16 cm from the centre is 15 cm.

15. Prove that of all the line segments drawn through a point of intersection of two circles and terminated by them, the one which is parallel to the line of centres is the greatest.

Sol. Given that two circles with centres A and B intersect and M is one of the points of intersection. CD and PQ are two line-segments through M, terminated by the two circles such that CD $\parallel$ AB.



To prove that CD > PQ.

Construction: We draw $AR \perp CD$, $BT \perp CD$, $AH \perp PQ$, $BF \perp PQ$ and $BN \perp AH$.

$\because$ AH is perpendicular to the chord PM.

$\therefore$ H is the mid-point of PM

$\therefore$ PM = 2HM …(1)

Again, since BF is perpendicular to the chord MQ,

$\therefore$ F is the mid-point of MQ

$\therefore$ MQ = 2MF …(2)

Adding (1) and (2), we have

PM + MQ = 2(HM + MF)

$\Rightarrow$ PQ = 2HF …(3)

Now, since $BF \perp HF$, $NH \perp HF$, $BN \perp HN$ and $FH \perp NH$, the figure NBFH is a rectangle.

$\therefore$ HF = NB

$\therefore$ From (3), PQ = 2NB …(4)

Now, in right-angled triangle ANB,

AB > NB

$\therefore$ 2AB > 2NB = PQ [From (4)] …(5)

Now, AR is perpendicular to the chord CM. Hence, R is the mid-point of CM.

$\therefore$ CM = 2RM …(6)

Again, BT is perpendicular to the chord MD. Hence, T is the mid-point of MD.

$\therefore$ MD = 2MT …(7)

Adding (6) and (7), we get

CM + MD = 2(RM + MT)

$\Rightarrow$ CD = 2RT = 2AB …(6)

From (5) and (6), we have

CD > PQ

Hence, proved.

Check Your Progress 2

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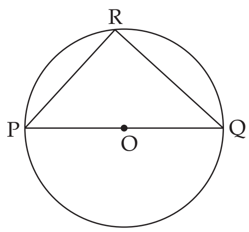
Multiple-Choice Questions

1. In the given figure, PQ is the diameter of the circle with centre O. If PR = QR, then ∠PRQ is equal to

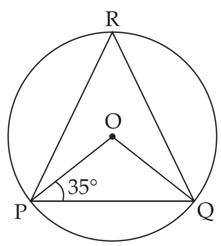


(*a*) 30° (*b*) 60°
(*c*) 45° (*d*) 90°

Sol. (*d*) 90°

The angle in a semicircle is a right angle.

2. In the given figure, if ∠OPR = 35°, then ∠PRQ is equal to



(*a*) 35° (*b*) 55°
(*c*) 70° (*d*) 110°

Sol. (*b*) 55°

In ΔPOQ,

∠OQP = 35° [OP = OQ]

∠POQ = 180° – 35° – 35°

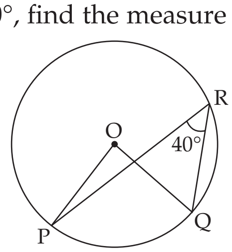
⇒ ∠POQ = 110°

$\angle PRQ = \frac{1}{2}\angle POQ = \frac{1}{2} \times 110°$

[By Theorem 9.7]

∴ ∠PRQ = 55°

3. In the given figure, O is the centre of the circle. If ∠PRQ is 40°, find the measure of ∠POQ.



(*a*) 40° (*b*) 80°
(*c*) 140° (*d*) 90°

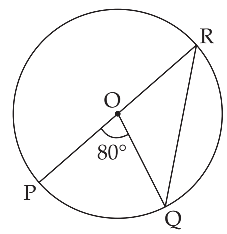
Sol. (*b*) 80°

∠POQ = 2∠PRQ

= 2 × 40°

∴ ∠POQ = 80°

4. In the given figure, O is the centre of the circle. PR is a diameter of the circle and Q is any point on its circumference. If ∠POQ = 80°, find the measure of ∠PRQ.



(*a*) 40° (*b*) 80°
(*c*) 100° (*d*) 120°

Sol. (*a*) 40°

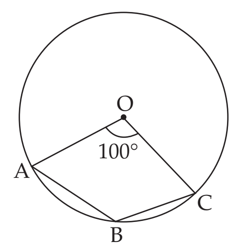
∠POQ = 2∠PRQ

⇒ $\angle PRQ = \frac{1}{2} \times \angle POQ$

$= \frac{1}{2} \times 80°$

∴ ∠PRQ = 40°

5. O is the centre of the circle. If ∠AOC = 100°, find the measure of ∠ABC.



(*a*) 80° (*b*) 100°
(*c*) 130° (*d*) 150°

Sol. (*c*) 130°

∠AOC = 100°

reflex ∠AOC = 360° – 100° = 260°

[Angles about a point]

$\angle ABC = \frac{1}{2}$ reflex ∠AOC

$= \frac{1}{2} \times 260°$

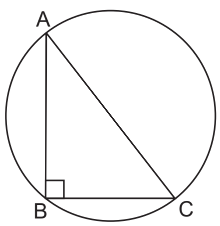
∴ ∠ABC = 130°

Very Short Answer Type Questions

6. ΔABC is a right-angled triangle with $\angle B = 90°$. A circle is drawn circumscribing the ΔABC. State with reason, what will be the position of the centre of this circle.

Sol. Given, ΔABC is a right-angled triangle.

We know, the arc of a circle subtending a right-angle at any point on the remaining part of the circle is a semi-circle.



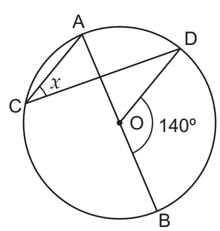
$\therefore$ $\overset{\frown}{AC}$ is a semi-circle.

$\Rightarrow$ AC is the diameter of a circle.

$\Rightarrow$ Centre of a circle will lie on AC, i.e hypotenuse of a right-angled ΔABC.

$\therefore$ Position of the centre of this circle is mid-point of the hypotenuse AC, since angle in a semicircle is 90°.

7. In the given figure, if O is the centre of a circle, AB is a diameter, CD is a chord, and $\angle DOB = 140°$, what is the value of x where $\angle ACD = x$?



Sol. $\angle DOB + \angle AOD = 180°$ [Linear pair]

$\Rightarrow$ $140° + \angle AOD = 180°$

$\Rightarrow$ $\angle AOD = 40°$

$\angle AOD = 2\angle ACD$

[The angle subtended by an arc of a circle at the centre is double the angle subtended by it at any point on the remaining part of the circle]

$\Rightarrow$ $40° = 2\angle ACD$

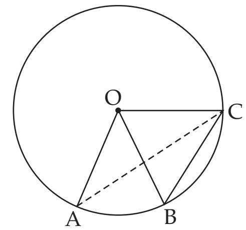
$\Rightarrow$ $\angle ACD = 20°$

$\angle ACD = x = 20°$

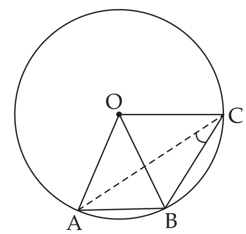
Hence, value of x, where $\angle ACD = x$ is 20°.

8. In the given figure, OABC is a quadrilateral in which OA = OB = OC. Prove that,

$\angle AOC = 2 (\angle BAC + \angle ACB)$



Sol. Join AB. Since OA = OB = OC, hence points A, B and C lie on the circle with centre O and radius OA, OB or OC.



Now, $\angle AOB$ is the angle made by the arc AB at the centre O of the circle. $\angle ACB$ is the angle made by the arc AB at a point C on the remaining part of the circumference.

$\therefore$ $\angle AOB = 2\angle ACB$ ...(1)

$\angle BOC$ is the angle made by the arc BC at the centre O of the circle. $\angle BAC$ is the angle made by the arc BC at a point A on the remaining part of the circumference.

$\therefore$ $\angle BOC = 2\angle BAC$ ...(2)

Adding (1) and (2), we get

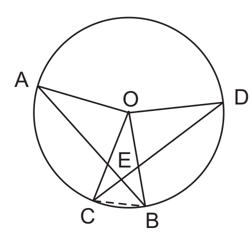
$\angle AOB + \angle BOC = 2\angle ACB + 2\angle BAC$

$\therefore$ $\angle AOC = 2(\angle BAC + \angle ACB)$

9. In a circle with centre O, chords AB and CD intersect inside the circumference at E. Prove that $\angle AOC + \angle BOD = 2\angle AEC$ **[CBSE SP 2011]**

Sol. *Construction*: Join CB.

Since the angle made by an arc of a circle at the centre is double the angle made by it at a point on the remaining part of the circle.



$\therefore$ $\angle AOC = 2\angle ABC$ …(1)

and $\angle BOD = 2\angle DCB$ …(2)

Adding (1) and (2), we have

$\angle AOC + \angle BOD = 2\angle ABC + 2\angle DCB$

$= 2 (\angle ABC + \angle DCB)$

$= 2 (\angle EBC + \angle BCE)$

$[\because \angle ABC = \angle EBC$ and $\angle DCB = \angle BCE]$

$= 2\angle AEC$

[Exterior angle of ΔBEC = Sum of interior opposite angles]

$\therefore \quad \angle AOC + \angle BOD = 2 \angle AEC$

Hence, proved.

10. A right triangle PQR, right-angled at R, in which a circle is drawn on the hypotenuse PQ as a diameter. Prove that the circle passes through the point R.

Sol. Given, a triangle PQR, right-angled at R. A circle is drawn on PQ as diameter.

Suppose the circle does not pass through R and cuts PR produced in S (Fig (*i*)) or PR in S (Fig (*ii*)).

$\angle PRQ = 90°$ [Given]

$\angle PSQ = 90°$ [Angle in a semi-circle]

$\Rightarrow \quad \angle PRQ = \angle PSQ$

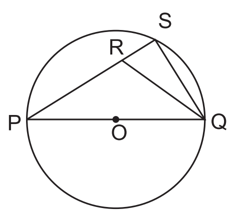


Fig. (*i*)

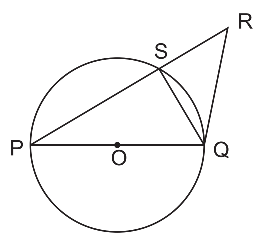


Fig. (*ii*)

Exterior angle of a triangle cannot be equal to its interior opposite angle.

$\therefore$ We reach a contradiction. So, our supposition is wrong.

Hence, the required result is proved.

11. PQ is a diameter of a circle with centre at O. R and S are two points on the opposite sides of the diameter. If $\angle QPR = 30°$ and $\angle PQS = 50°$, find the measures of $\angle RPS$ and $\angle RQS$.

Sol. In ΔPQR,

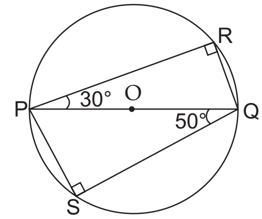
$\angle PRQ = 90°$ [Angle in a semicircle]

$\angle QPR = 30°$ [Given]

$\angle RPQ + \angle PQR + \angle PRQ = 180°$

$\Rightarrow \quad \angle PQR = 180° - 90° - 30°$

$\Rightarrow \quad \angle PQR = 60°$ ...(1)



In ΔPQS,

$\angle PSQ = 90°$ [Angle in a semicircle]

$\angle PQS = 50°$ [Given]

$\angle PQS + \angle PSQ + \angle QPS = 180°$

$\Rightarrow \quad \angle QPS = 180° - 90° - 50°$

$\Rightarrow \quad \angle QPS = 40°$ ...(2)

$\angle RPS = \angle RPQ + \angle QPS$

$= 30° + 40°$ [From (2)]

$\Rightarrow \quad \angle RPS = 70°$

$\angle RQS = \angle PQR + \angle PQS$

$= 60° + 50°$ [From (1)]

$= 110°$

$\Rightarrow \quad \angle RQS = 110°$

Short Answer Type Questions

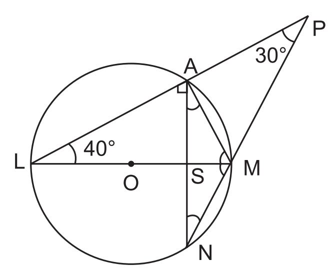
12. LAM is a right-angled triangle with $\angle LAM = 90°$. A circle drawn with the centre at O, the mid-point of LM. AN is a chord of the circle cutting LM at S and the circle at N. NM produced cuts LA produced at P. If $\angle LPM = 30°$ and $\angle ALM = 40°$, find $\angle ANM$, $\angle AMN$ and $\angle NAM$.

Sol. Given, $\angle LPM = 30°$,

$\angle ALM = 40°$,

$\angle LAM = 90°$

and $\angle PAM = \angle LAM = 90°$ [Linear pair]



$\because \angle AMN$ is exterior angle of ΔPAM,

$\therefore \angle PAM + \angle APM = \angle AMN$

[Exterior angle is equal to sum of its two opposite interior angles]

$\Rightarrow \quad 90° + 30° = \angle AMN$

$\Rightarrow \quad \angle AMN = 120°$

$\because \quad \angle ALM = \angle ANM$

[Angles in the same segment]

$\therefore \quad \angle ANM = 40°$

In ΔAMN, we have

$$\angle ANM + \angle AMN + \angle NAM = 180°$$

$$40° + 120° + \angle NAM = 180°$$

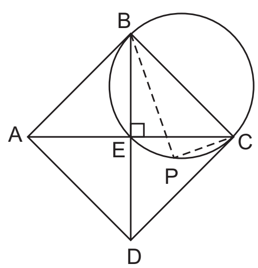
$$160° + \angle NAM = 180°$$

$$\angle NAM = 20°$$

Hence, $\angle ANM = 40°$, $\angle AMN = 120°$ and $\angle NAM = 20°$.

13. Prove that the circle drawn with any side of a rhombus as a diameter, passes through the point of intersection of its diagonals.

Sol. Let ABCD be the rhombus with its diagonal AC and BD intersecting at E.



Draw a circle with BC as diameter. Take a point P on the circle and join PB and PC.

$\angle BPC = 90°$ [Angle in a semi-circle] …(1)

But $\angle BEC = 90°$ [Diagonals of a rhombus are $\perp$ to each other] …(2)

$\therefore \quad \angle BEC = \angle BPC$ [From (1) and (2)]

$\Rightarrow$ BC subtends equal angles at points E and P which are on the same side of it.

We know that if a line segment joining two points subtends equal angles on the same side of the line containing the line segment, the four points lie on a circle.

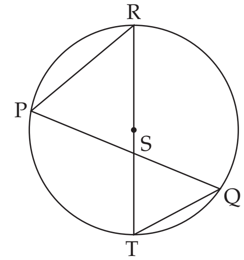
$\therefore$ Points B, E, P and C are concyclic.

$\therefore$ E lies on the circle with BC as a diameter.

Similarly, it can be proved that E lies on circle with AB, AD and CD as diameters.

Hence, the circle drawn with any side of rhombus as diameter passes through the point of intersection of its diagonals.

14. In the given figure, S is any point on the chord PQ. R is a point on the circle such that PR = PS. Prove that QS = QT.



Sol. $PR = RS$ [Given]

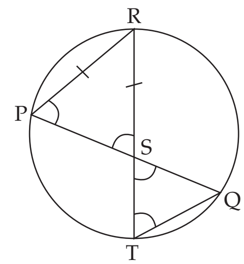
$\Rightarrow \quad \angle RPS = \angle PSR$

$= \angle QST$ [Vert. opp. $\angle$s] ...(1)

$\angle RPQ = \angle RTQ$

[Angles in the same segment]

$\Rightarrow \quad \angle RPS = \angle STQ$ ...(2)



From (1) and (2)

$\angle QST = \angle STQ$

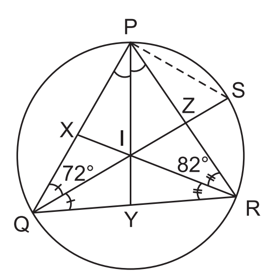
$\therefore \quad QS = QT$

[Sides opposite to equal angles]

Long Answer Type Questions

15. In $\angle PQR$, $\angle PQR = 72°$ and $\angle QRP = 82°$ and I is the incentre of ΔPQR. When QI is produced, it meets the circumcircle of ΔPQR at S. SQ is joined. Calculate $\angle SPR$, $\angle IPR$ and $\angle PIS$.

Sol. The incentre of a triangle is the intersection of the angle bisectors of the triangle.



Given, $\angle PQR = 72°$, $\angle QRP = 82°$

In ΔPQR, we have

$$\angle PQR + \angle QRP + \angle QPR = 180°$$

$$\Rightarrow \quad 72° + 82° + \angle QPR = 180°$$

$\Rightarrow \quad 154° + \angle QPR = 180°$

$\Rightarrow \quad \angle QPR = 26°$

$$\angle IPR = \frac{1}{2}\angle QPR$$

[PY is the angle bisector of $\angle QPR$]

$\Rightarrow \quad \angle IPR = \frac{1}{2} \times 26° = 13$

$\Rightarrow \quad \angle IPR = 13°$

$$\angle SQR = \frac{1}{2}\angle PQR$$

[QZ is the angle bisector of $\angle PQR$]

$\Rightarrow \quad \angle SQR = \frac{1}{2} \times 72°$

$\Rightarrow \quad \angle SQR = 36°$

$$\angle SQR = \angle SPR$$

[Angles in the same segment of a circle are equal]

$\Rightarrow \quad \angle SPR = 36°$

In ΔQZR, we have

$$\angle ZQR + \angle ZRQ = \angle QZP$$

[Exterior angle of a triangle is equal to sum of its two opposite interior angles]

$\Rightarrow \quad 36° + 82° = \angle QZP$

$\Rightarrow \quad \angle QZP = 118°$

In ΔPIZ, we have

$\angle IZP + \angle PIZ + \angle IPZ = 180°$

[Angle sum property of a triangle]

$118° + \angle PIZ + 13° = 180°$

[$\because \angle QZP = \angle IZP$ and $\angle IPR = \angle IPZ$]

$131° + \angle PIZ = 180°$

$\angle PIZ = 49°$

$\angle PIS = 49° \quad$ [$\because \angle PIS = \angle PIZ$]

Hence, measure of $\angle PIS = 49°$, $\angle SPR = 36°$ and $\angle IPR = 13°$.

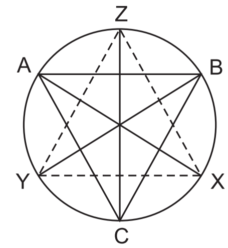
16. ΔABC is inscribed in a circle and the bisectors of $\angle A$, $\angle B$ and $\angle C$ meet the circumference of the circle at X, Y and Z respectively. Prove that

$$\angle X = 90° - \frac{\angle A}{2},$$

$$\angle Y = 90° - \frac{\angle B}{2}$$

and $$\angle Z = 90° - \frac{\angle C}{2}.$$

Sol. Let bisectors of $\angle A$, $\angle B$ and $\angle C$ of a triangle ABC intersect the circumference of the circle at X, Y and Z respectively.



Now, from figure

$$\angle X = \angle YXZ$$

$$\angle X = \angle YXA + \angle AXZ$$

$\because \angle YXA$ and $\angle YBA$ are the angles in the same segment of the circle.

$\therefore \quad \angle YXA = \angle YBA$

Hence, $\quad \angle X = \angle YBA + \angle AXZ$

Again, $\angle AXZ$ and $\angle YCA$ are the angles in the same segment of the circle.

Hence, $\quad \angle AXZ = \angle YCA$

Again, $\because$ BY is the bisector of $\angle B$ and CZ is the bisector of $\angle C$

So, $\quad \angle X = \frac{1}{2}\angle B + \frac{1}{2}\angle C$

Similarly, $\quad \angle Y = \frac{1}{2}\angle C + \frac{1}{2}\angle A$

and $\quad \angle Z = \frac{1}{2}\angle A + \frac{1}{2}\angle B$

Now, $\quad \angle X = \frac{1}{2}\angle B + \frac{1}{2}\angle C$

$$= \frac{1}{2}(\angle B + \angle C)$$

$\Rightarrow \quad \angle X = \frac{1}{2}(180° - \angle A)$

[$\angle A + \angle B + \angle C = 180°$]

$\Rightarrow \quad \angle X = 90° - \frac{1}{2}\angle A$

Similarly, $\quad \angle Y = \frac{1}{2}(180° - \angle B)$

$$= 90° - \frac{1}{2}\angle B$$

$$\angle Y = 90° - \frac{1}{2}\angle B$$

and

$$\angle Z = \frac{1}{2}(180° - \angle C)$$

$$= 90° - \frac{1}{2}\angle C$$

$$\angle Z = 90° - \frac{1}{2}\angle C$$

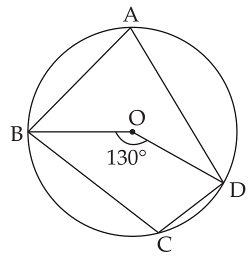
Hence, proved.

Check Your Progress 3

(Page 133)

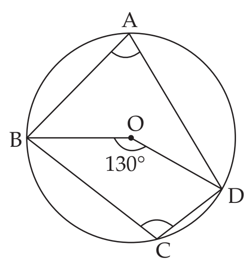
Multiple-Choice Questions

1. In the given figure, find the measure of ∠BCD.



(*a*) 50° (*b*) 130°
(*c*) 115° (*d*) 60°

Sol. (*c*) 115°



$$\angle BAD = \frac{1}{2}\angle BOD$$

$$= \frac{1}{2} \times 130° = 65°$$

$\angle BAD + \angle BCD = 180°$

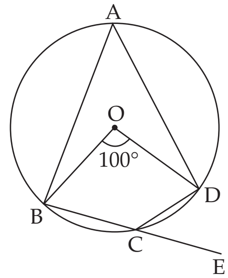
[Opp. ∠s of a cyclic quadrilateral are supplementary]

$\Rightarrow \quad \angle BCD = 180° - \angle BAD$

$= 180° - 65°$

$\therefore \quad \angle BCD = 115°$

2. In the given figure, O is the centre of the circle. If ∠BOD = 100°, find the measure of ∠DCE.



(*a*) 50° (*b*) 80°
(*c*) 100° (*d*) 180°

Sol. (*a*) 50°

$$\angle BAD = \frac{1}{2} \times \angle BOD$$

$$= \frac{1}{2} \times 100° = 50°$$

$\Rightarrow \quad \angle BAD = 50°$

$\angle BAD + \angle BCD = 180°$

$\Rightarrow \quad \angle BCD = 180° - 50°$

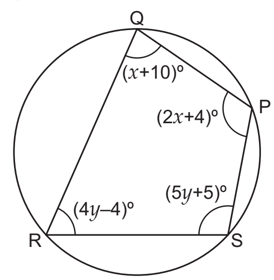
$\Rightarrow \quad \angle BCD = 130°$

$\angle BCD + \angle DCE = 180°$ [BE is a straight line]

$\Rightarrow \quad \angle DCE = 180° - 130°$

$\therefore \quad \angle DCE = 50°$

3. In the given figure, PQRS is a cyclic quadrilateral. If $\angle P = (2x + 4)°$, $\angle Q = (x + 10)°$, $\angle R = (4y - 4)°$ and $\angle S = (5y + 5)°$, then the values of x and y are respectively



(*a*) 25 and 40 (*b*) 40 and 25
(*c*) 27 and 38 (*d*) 38 and 27

Sol. (*b*) 40 and 25

Given, PQRS is a cyclic quadrilateral.

So, opposite angles of a cyclic quadrilateral are supplementary.

$\angle P + \angle R = 180°$

$2x + 4 + 4y - 4 = 180°$

$2x + 4y = 180°$

$x + 2y = 90°$ …(1)

$\angle Q + \angle S = 180°$

$x + 10 + 5y + 5 = 180°$

$x + 5y + 15 = 180°$

$x + 5y = 165°$ …(2)

Solving (1) and (2), we get

$x + 2y = 90°$

$\underset{-}{x} + \underset{-}{5y} = \underset{-}{165°}$

$-3y = -75°$

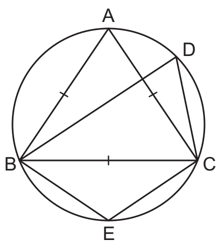
$\Rightarrow \quad y = 25°$

Putting value of 'y' in (1), we get

$$x + 2 \times 25° = 90°$$
$$x + 50° = 90°$$
$$x = 40°$$

4. If ABC is an equilateral triangle, then the measure of ∠BDC and ∠BEC are respectively

[CBSE SP 2011]



(*a*) 120° and 60° (*b*) 60° and 100°

(*c*) 120° and 50° (*d*) 60° and 120°

Sol. (*d*) 60° and 120°

Given, ΔABC is an equilateral triangle. So, all the angles of ΔABC will be equal to 60°.

$$\angle ABC = \angle BAC = \angle ACB = 60°$$
$$\angle BAC = \angle BDC = 60°$$

[Angles in the same segment of a circle are equal]

∵ BDCE is a cyclic quadrilateral.

[B, D, C and E lie on the circle]

∴ $\angle BDC + \angle BEC = 180°$

[Opposite angles of a cyclic quadrilateral are supplementary]

⇒ $60° + \angle BEC = 180°$

⇒ $\angle BEC = 180° - 60°$

∴ $\angle BEC = 120°$

5. ABCD is a quadrilateral with A as the centre of a circle passing through the points B, C and D such that AB = AC = AD, ∠CBD = 20° and ∠CDB = 30°. Then ∠BAD is equal to

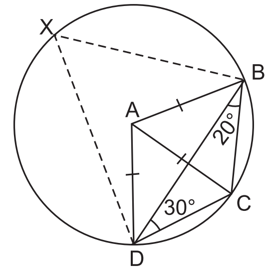
(*a*) 80° (*b*) 70°

(*c*) 140° (*d*) 100°

Sol. (*d*) 100°

In ΔDCB,

$$\angle BDC + \angle DBC + \angle DCB = 180°$$
$$30° + 20° + \angle DCB = 180°$$
$$50° + \angle DCB = 180°$$
$$\angle DCB = 130°$$



Mark a point X on the circle and join XD and XB.

$$\angle BAD = 2\angle DXB \quad \text{...(1)}$$

∵ XBCD is a cyclic quadrilateral [X, B, C, D lie on the circle]

∴ $\angle DCB + \angle DXB = 180°$

[Opposite angles of a cyclic quadrilateral is supplementary]

$$130° + \angle DXB = 180°$$
$$\angle DXB = 50° \quad \text{...(2)}$$

Now, substitute (2) in (1),

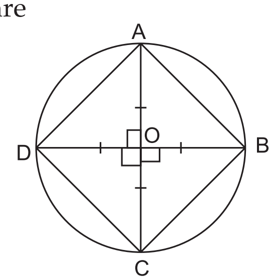
$$\angle BAD = 2 \times 50° = 100°$$

6. Two diameters of a circle intersect each other at right angles. Then the quadrilateral formed by joining their end points is

(*a*) a rhombus (*b*) a square

(*c*) a rectangle (*d*) a kite

Sol. (*b*) a square



In ΔAOD and ΔAOB, we have

AO = AO [Common]

OD = OB [Radius of a circle]

∠AOD = ∠AOB [Each 90°]

∴ ΔAOD ≅ ΔAOB

[By SAS congruence]

⇒ AD = AB [By CPCT]

Similarly, DC = BC

AD = DC

AB = BC

∴ All sides of a quadrilateral are equal. ...(1)

∵ ΔAOD is a right-angled isosceles triangle.

∴ $\angle ODA = \angle OAD = 45°$

$$\angle OAB = 45° = \angle OAD$$

[ΔAOD ≅ ΔAOB]

∴ $\angle DAB = \angle OAD + \angle OAB$
$= 45° + 45°$
$= 90°$

Similarly, $\angle ADC = 90°$

∵ ABCD is a cyclic quadrilateral. So, opposite angles of a cyclic quadrilateral are supplementary.

∴ $\angle DCB = 90°$

and $\angle ABC = 90°$

∵ All angles of a quadrilateral are of 90°. …(2)

∴ From (1) and (2),

Quadrilateral is a square.

Hence, proved.

Very Short Answer Type Questions

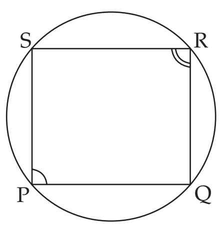
7. PQRS is a cyclic quadrilateral such that $\angle QPS : \angle QRS = 5 : 4$. Find the measures of $\angle QPS$ and $\angle QRS$.

Sol. PQRS is a cyclic quadrilateral.

$\angle QPS : \angle QRS = 5 : 4$

Let $\angle QPS = 5x$

then $\angle QRS = 4x$



Since opposite angles of a cyclic quadrilateral are supplementary, hence

$\angle QPS + \angle QRS = 180°$

⇒ $5x + 4x = 180°$

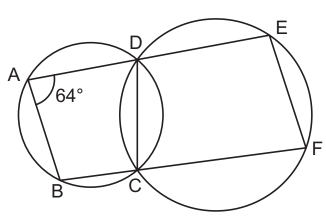
⇒ $9x = 180°$

⇒ $x = 20°$

∴ $\angle QPS = 5x = 5 \times 20° = 100°$

$\angle QRS = 4x = 4 \times 20° = 80°$

8. In the given figure, if $\angle BAD = 64°$, find the measures of $\angle DCF$ and $\angle DEF$.



Sol. Given, $\angle BAD = 64°$

ABCD is a cyclic quadrilateral [A, B, C, D lie on the circle]

$\angle BAD + \angle BCD = 180°$

[Opposite angles of a cyclic quadrilateral are supplementary]

⇒ $64° + \angle BCD = 180°$

⇒ $\angle BCD = 180° - 64°$

$= 116°$

$\angle BCD + \angle DCF = 180°$ [Linear pair]

⇒ $116° + \angle DCF = 180°$

⇒ $\angle DCF = 64°$

DEFC is a cyclic quadrilateral (D, E, F, C lie on the circle)

$\angle DCF + \angle DEF = 180°$

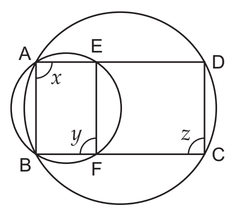
[Opposite angles of a cyclic quadrilateral are supplementary]

⇒ $64° + \angle DEF = 180°$

⇒ $\angle DEF = 116°$

Hence, measure of $\angle DCF = 64°$ and $\angle DEF = 116°$.

9. In the given figure below, ABCD is a cyclic quadrilateral. A second circle passing through A and B meets AD and BC at E and F respectively. If $\angle BAD = x$, $\angle EFB = y$ and $\angle BCD = z$, what is the relation between y and z?



Sol. Given, $\angle BAD = x$

$\angle EFB = y$

$\angle BCD = z$

ABCD is a cyclic quadrilateral

$x + z = 180°$

$x = 180° - z$ …(1)

AEFB is a cyclic quadrilateral

$x + y = 180°$ …(2)

Put the value of x from (1) in (2), we get

$180° - z + y = 180°$

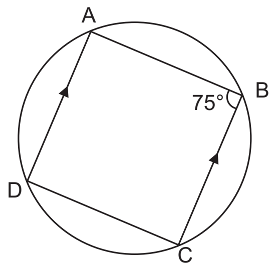
$-z + y = 0$

$y = z$

Hence, relation between y and z is $y = z$.

10. ABCD is a cyclic quadrilateral with AD ∥ BC if ∠B = 75º, determine the remaining angles.

Sol. Given, ABCD is a cyclic quadrilateral with AD ∥ BC and ∠B = 75°



$\angle A + \angle B = 180°$ [Cointerior ∠s, AD ∥ BC]

$\Rightarrow \quad \angle A + 75° = 180°$

$\Rightarrow \quad \angle A = 105°$

$\angle B + \angle D = 180°$ [Opposite angles of a cyclic quadrilateral are supplementary]

$75° + \angle D = 180°$

$\angle D = 105°$

Similarly, $\angle A + \angle C = 180°$

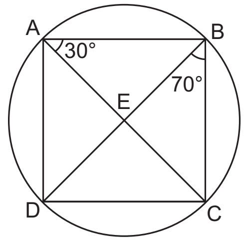
$\Rightarrow \quad 105° + \angle C = 180°$

$\Rightarrow \quad \angle C = 75°$

Hence, the remaining angles are ∠A = 105°, ∠C = 75° and ∠D = 105°.

11. ABCD is a cyclic quadrilateral whose diagonals intersect each other at a point E. If ∠DBC = 70º, ∠BAC = 30º, find ∠BCD. Further, if AB = BC, find ∠ECD. **[CBSE SP 2011]**

Sol. Given, ∠DBC = 70° and ∠BAC = 30°



$\angle DBC = \angle DAC$

[Angles in the same segment of a circle are equal]

$\angle DAC = 70°$

$\angle DAB = \angle DAC + \angle BAC$

$= 70° + 30°$

$= 100°$

$\Rightarrow \quad \angle DAB = 100°$

$\angle DAB + \angle BCD = 180°$ [Opposite angles of a cyclic quadrilateral are supplementary]

$100° + \angle BCD = 180°$

$\Rightarrow \quad \angle BCD = 80°$

If AB = BC, then ΔABC is an isosceles triangle

$\angle BAC = \angle BCA = 30°$

$\angle ECD = \angle BCD - \angle BCA$

$= 80° - 30°$

$= 50°$

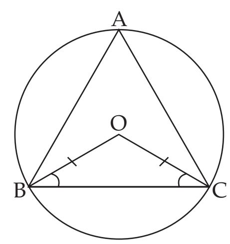
$\Rightarrow \quad \angle ECD = 50°$

Hence, measure of ∠BCD = 80° and ∠ECD = 50°.

Short Answer Type Questions

12. The circumcentre of the triangle ABC is O. Prove that ∠OBC + ∠BAC = 90°.

Sol.



In ΔOBC,

$\angle OBC = \angle OCB$ [Angles opposite to equal side]

$\angle OBC + \angle OCB + \angle BOC = 180°$ [Angle sum property]

$2\angle OBC + \angle BOC = 180°$ ...(1)

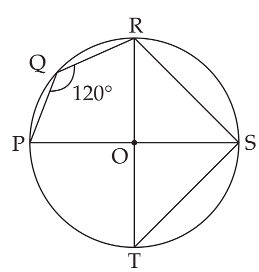
Now, $\angle BOC = 2\angle BAC$ ...(2)

From (1) and (2)

$2\angle OBC + 2\angle BAC = 180°$

$\therefore \quad \angle OBC + \angle BAC = 90°$

13. In the given figure, ∠PQR = 120° and chord RS = ST. O is the centre of the circle. Find the measure of ∠PST.



Sol. In cyclic quadrilateral PQRS,

$\angle PQR + \angle RSP = 180°$

$\Rightarrow \quad \angle RSP = 180° - 120°$

$\Rightarrow \quad \angle RSP = 60°$ ...(1)

$\angle RST = 90°$ [Angle in a semicircle]

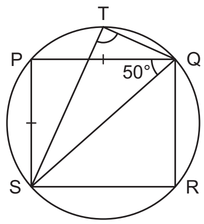
$\Rightarrow \quad \angle PST = \angle RST - \angle RSP$

$= 90° - 60°$

$= 30°$

$\therefore \quad \angle PST = 30°$

14. PQRS is a cyclic quadrilateral such that PQ = PS and $\angle PQS = 50°$. T is a point on the circle such that P and T lie on the same side of QS. TQ and TS are joined. Find $m\angle QTS$ and $m\angle QRS$.



Sol. Given, $\angle PQS = 50°$ and ΔPSQ is an isosceles triangle such that PS = PQ

$$\angle PQS = \angle PSQ = 50°$$

In ΔPSQ, we have

$$\angle PQS + \angle PSQ + \angle SPQ = 180°$$

$\Rightarrow \quad 50° + 50° + \angle SPQ = 180°$

$\Rightarrow \quad 100° + \angle SPQ = 180°$

$\Rightarrow \quad \angle SPQ = 80°$

$\angle SPQ = \angle QTS$

[Angles in the same segment of a circle are equal]

$\Rightarrow \quad \angle QTS = 80°$

PQRS is a cyclic quadrilateral

[P, Q, R, S lie on a circle]

$\angle SPQ + \angle QRS = 180°$

[Opposite $\angle$s of a cyclic quadrilateral are supplementary]

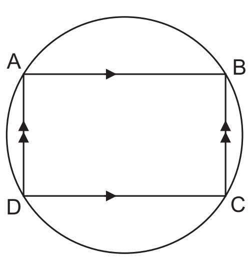
$80° + \angle QRS = 180°$

$\angle QRS = 100°$

Hence, measure of $\angle QTS = 80°$ and $\angle QRS = 100°$, i.e. $m\angle QTS = 80°$ and $m\angle QRS = 100°$.

15. Prove that a cyclic parallelogram is a rectangle.

Sol. Given, ABCD is a cyclic parallelogram.



$\because$ Opposite angles of a cyclic parallelogram are supplementary,

$\therefore \quad \angle ABC + \angle ADC = 180° \quad \ldots(1)$

$\angle ABC = \angle ADC$

[Opposite angles of a parallelogram are equal) …(2)

$\therefore \quad 2\angle ABC = 180°$ [From (1) and (2)]

$\Rightarrow \quad \angle ABC = 90°$

Also, $\angle ADC = 90°$ [From (2)]

$\therefore$ Each angle of a parallelogram ABCD is a right angle.

Hence, ABCD is a rectangle.

Long Answer Type Questions

16. Angles in a quadrilateral are in the ratio 1 : 5 : 7 : 3. Prove that it is a cyclic quadrilateral.

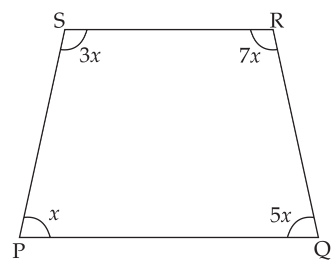
Sol. Let PQRS be a quadrilateral.

$\angle P = x$

$\angle Q = 5x$

$\angle R = 7x$

$\angle S = 3x$



Since, sum of angles of a quadrilateral is 360°,

$\Rightarrow \angle P + \angle Q + \angle R + \angle S = 360°$

$x + 5x + 7x + 3x = 360°$

$16x = 360°$

$\Rightarrow \quad x = 22.5°$

Hence,

$\angle P = 22.5°$

$\angle Q = 5x = 5 \times 22.5° = 112.5°$

$\angle R = 7x = 7 \times 22.5° = 157.5°$

$\angle S = 3x = 3 \times 22.5° = 67.5°$

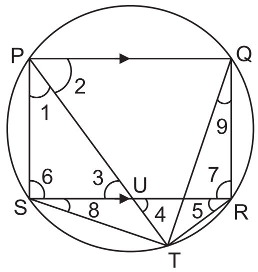
$\angle P + \angle R = 22.5° + 157.5° = 180°$

$\angle Q + \angle S = 112.5° + 67.5° = 180°$

$\therefore$ Quad. PQRS is a cyclic quadrilateral as opposite angles are supplementary.

17. In a cyclic quadrilateral PQRS, PQ || SR. The internal bisector of $\angle P$ meets RS at U and the circle at T. TR, TS and TQ are joined. Prove that (*a*) UT = TR and (*b*) $\Delta QRT \cong \Delta SUT$.

Sol.



(*a*) $\angle 1 = \angle 2$ [PT is the bisector of $\angle$P] …(1)

$\angle 2 = \angle 3$ [Alternate $\angle$s, PQ $\parallel$ SR] …(2)

$\angle 3 = \angle 4$ [Vertically opposite $\angle$s] …(3)

$\angle 1 = \angle 5$

[Angles in the same segment] …(4)

From equation (1), (2), (3) and (4), we have

$\angle 5 = \angle 4$

$\Rightarrow$ UT = TR

[Sides opposite to equal angles]

Hence, proved.

(*b*) $\angle QPS + \angle PSR = 180°$

[Cointerior $\angle$s, PQ $\parallel$ SR]

$\angle 1 + \angle 2 + \angle 6 = 180°$ …(5)

$\because$ PQRS is a cyclic quadrilateral. So, opposite $\angle$s of a cyclic quadrilateral are supplementary,

$\therefore$ $\angle QPS + \angle QRS = 180°$

$\Rightarrow$ $\angle 1 + \angle 2 + \angle 7 = 180°$ …(6)

$\angle 6 = \angle 7$

[From (5) and (6)] …(7)

Adding equation (4) and (7), we get

$\angle 1 + \angle 6 = \angle 5 + \angle 7$

$\Rightarrow$ $\angle SUT = \angle QRT$

[Ext. $\angle$SUT of ΔPSU = sum of int. opp. $\angle$s] …(8)

In ΔQRT and ΔSUT, we have

$\angle 9 = \angle 8$

[Angles in the same segment]

$\angle QRT = \angle SUT$ [From (8)]

UT = TR [Proved in part (*a*)]

$\therefore$ $\Delta QRT \cong \Delta SUT$

[By AAS congruence]

Hence, proved.

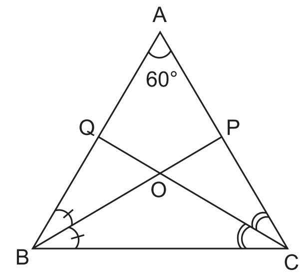
18. In a ΔABC, if $\angle A = 60°$ and the bisectors of $\angle$B and $\angle$C meet AC and AB at P and Q respectively and intersect each other at O. Prove that APOQ is a cyclic quadrilateral.

Sol. Let $\angle ABP = \angle PBC = x$

[$\because$ BP is the bisector of $\angle$B] …(1)

Let $\angle ACQ = \angle QCB = y$

[$\because$ QC is the bisector of $\angle$C] …(2)



In ΔOBC, we have

$\angle OBC + \angle OCB + \angle BOC = 180°$

[Angle sum property of a triangle]

$\Rightarrow$ $\angle BOC + \angle PBC + \angle QCB = 180°$

[$\angle OBC = \angle PBC$ and $\angle OCB = \angle QCB$]

$\Rightarrow$ $\angle BOC + x + y = 180°$

$\Rightarrow$ $\angle QOP + x + y = 180°$

[$\angle BOC = \angle QOP$, vertically opp. $\angle$s]

$\angle QOP = 180° - (x + y)$ …(3)

In ΔABC, we have

$\angle ABC + \angle ACB + \angle BAC = 180°$

$2x + 2y + 60° = 180°$

$2x + 2y = 120°$

$x + y = 60°$ …(4)

From (3) and (4), we have

$\angle QOP = 180° - 60° = 120°$

In quadrilateral APOQ, we have

$\angle QAP + \angle QOP = 60° + 120°$

$\Rightarrow$ $\angle QAP + \angle QOP = 180°$

But $\angle$QAP and $\angle$QOP are opposite angles of a quadrilateral APOQ.

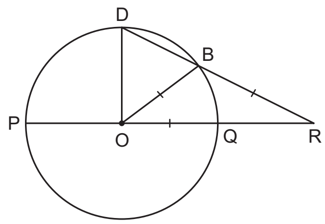
Hence, APOQ is a cyclic quadrilateral.

Hence, proved.

Higher Order Thinking Skills (HOTS) Questions

(Page 135)

1. From a point R outside a circle with centre O, a line segment RQP is drawn through O cutting the circle at Q and P as shown in the figure. Another line segment RBD is drawn cutting the circle at B and D. If RB = OQ, show that $\overset{\frown}{PD} = 3\overset{\frown}{QB}$.



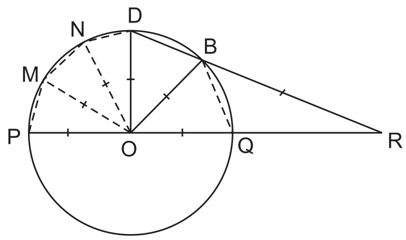
Sol. Join BQ, ND, MN and PM.

In ΔQOB and ΔBOD, we have

$OQ = OD$ [Radius of a circle]

$\angle OBQ = \angle ODB$

[$OQ = OB = OD$; angle opposite to same sides are equal]



$\angle OQB = \angle OBD$ [$OQ = OB = OD$]

$\therefore \quad \Delta QOB \cong \Delta BOD$

[By AAS congruence]

$\angle QOB = \angle BOD$ [By CPCT]

$\Rightarrow \quad \overset{\frown}{QB} = \overset{\frown}{DB}$ ...(1)

Similarly, $\Delta ODB \cong \Delta ODN$

$\Rightarrow \quad \overset{\frown}{DB} = \overset{\frown}{ND}$ ...(2)

$\Delta ODN \cong \Delta ONM$

$\Rightarrow \quad \overset{\frown}{ND} = \overset{\frown}{MN}$ ...(3)

$\Delta ONM \cong \Delta OMP$

$\Rightarrow \quad \overset{\frown}{MN} = \overset{\frown}{PM}$ ...(4)

Adding (2), (3) and (4), we get

$$\overset{\frown}{DB} + \overset{\frown}{ND} + \overset{\frown}{MN} = \overset{\frown}{ND} + \overset{\frown}{MN} + \overset{\frown}{PM}$$

$$\Rightarrow \quad \overset{\frown}{DB} + \overset{\frown}{ND} + \overset{\frown}{MN} = \overset{\frown}{PD}$$

$$\Rightarrow \quad \overset{\frown}{QB} + \overset{\frown}{QB} + \overset{\frown}{QB} = \overset{\frown}{PD}$$

$$[\overset{\frown}{QB} = \overset{\frown}{DB} = \overset{\frown}{ND} = \overset{\frown}{MN} = \overset{\frown}{PM}]$$

$$\therefore \quad \overset{\frown}{PD} = 3\overset{\frown}{QB}$$

Hence, proved.

2. A is the centre of a circle and B, C, D are points on the circle forming a quadrilateral ABCD in which AB = AC = AD. Prove that

$$\angle BAD = 2(\angle CBD + \angle CDB)$$

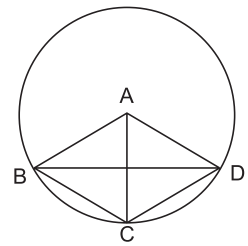
Sol. We know that the angle subtended by an arc of a circle at the centre is double the angle subtended by it at any point on the remaining part of the circle.

$\therefore \quad \angle CAD = 2\angle CBD$

[Angles subtended by $\overset{\frown}{CD}$] ...(1)

Also, $\angle BAC = 2\angle CDB$

[Angles subtended by $\overset{\frown}{BC}$] ...(2)



Adding (1) and (2), we get

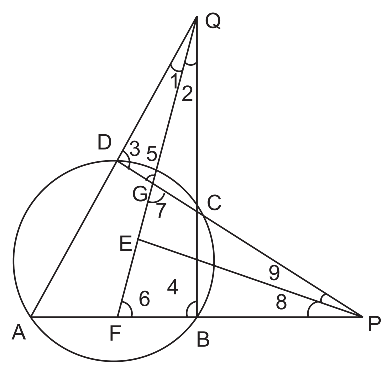
$$\angle CAD + \angle BAC = 2\angle CBD + 2\angle CDB$$

$$\angle BAD = 2(\angle CBD + \angle CDB)$$

Hence, proved.

3. Prove that the bisectors of the angles formed by producing the opposite sides of a cyclic quadrilateral intersect at right angles.

Sol. ABCD is a cyclic quadrilateral whose opposite sides when produced intersect at the points P and Q respectively. The bisectors PE and QF of $\angle P$ and $\angle Q$ meet at E and F respectively.



In ΔQDG and ΔQBF, we have

$\angle 1 = \angle 2$

[QE is bisector of $\angle DQC$]

$\angle 3 = \angle 4$

[Exterior $\angle$s of cyclic quadrilateral is equal to its interior opp. $\angle$s]

$\angle 5 = \angle 6$ ...(1)

But, $\angle 5 = \angle 7$

[Vertically opposite $\angle$s] ...(2)

From (1) and (2), we have

$\angle 6 = \angle 7$

Now, in ΔPGE and ΔPFE,

$$\angle 9 = \angle 8$$

[PE is bisector of $\angle$CPB]

$$\angle 6 = \angle 7 \quad \text{[Already proved]}$$

$\therefore$ $\angle PEG = \angle PEF$

But $\angle PEG + PEF = 180°$

$\therefore$ $\angle PEG = \angle PEF = 90°$

So, $\angle PEG = 90°$

Hence, proved.

4. If the bisectors of the opposite angles of a cyclic quadrilateral intersect the corresponding circle at two points, then prove that the line segment joining these two points is a diameter of the circle.

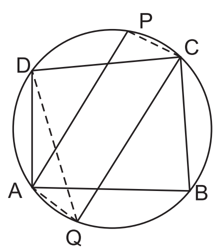
Sol. $\because$ ABCD is a cyclic quadrilateral,

$\therefore$ $\angle A + \angle C = 180°$

[Opposite $\angle$s of a cyclic quadrilateral are supplementary]

$\Rightarrow$ $\frac{1}{2}\angle A + \frac{1}{2}\angle C = 90°$

$\Rightarrow$ $\angle PAB + \angle BCQ = 90°$



But $\angle BCQ = \angle BAQ$

[Angles in same segment of a circle are equal]

$\therefore$ $\angle PAB + \angle BAQ = 90°$

$\Rightarrow$ $\angle PAQ = 90°$

$\Rightarrow$ $\angle$PAQ is in a semi-circle.

$\Rightarrow$ PQ is a diameter of a circle.

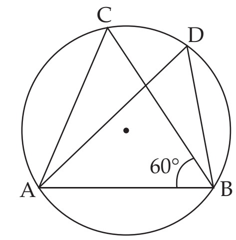
Hence, proved.

Self-Assessment

(Page 135)

Multiple-Choice Questions

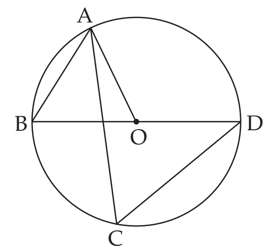
1. In the given figure, $\angle ACB = 70°$ and $\angle ABC = 60°$. The measure of $\angle$ADB is equal to



(*a*) 50° (*b*) 60°
(*c*) 70° (*d*) 80°

Sol. (*c*) 70°

2. BD is the diameter of the circle with centre O. If $\angle BAO = 60°$, then measure of $\angle$ACD is



(*a*) 30° (*b*) 45°
(*c*) 60° (*d*) 90°

Sol. (*c*) 60°

In ΔAOB,

AO = BO [Radius of the circle]

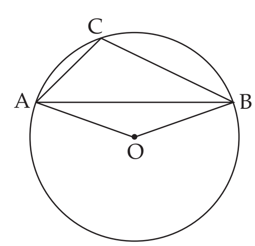
$\angle BAO = 60°$ [Given]

$\Rightarrow$ $\angle ABO = 60°$ [Angles opposite to equal sides are equal]

$\angle ABO = \angle ACD = 60°$

[Angles in the segment of the circle are equal]

3. In the given figure $\angle ACB = 100°$. Find the measure of $\angle$AOB.



(*a*) 80° (*b*) 100°
(*c*) 200° (*d*) 160°

Sol. (*d*) 160°

$\angle ACB = 100°$ [Given]

$\Rightarrow$ Reflex $\angle AOB = 2 \times 100° = 200°$

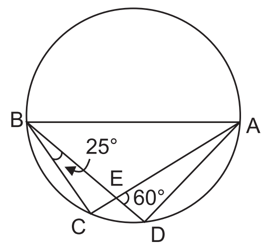
$\therefore$ $\angle AOB = 360° - 200° = 160°$

4. Two chords BD and AC of a circle intersect each other at E. If A and B are the ends of a diameter of the circle and if ∠CBE = 25° and ∠DEA = 60°, then the measure of ∠ADB is

(*a*) 90° (*b*) 85°

(*c*) 95° (*d*) 120°

Sol. (*c*) 95°



Given, ∠CBE =25° and ∠DEA = 60°

Also, AB is a diameter of a circle.

∠DEA = ∠CEB = 60° [Vertically opposite ∠s]

In ΔBCE, we have

∠CEB + ∠CBE + ∠BCE = 180°

[Angle sum property of a triangle]

⇒ 60° + 25° + ∠BCE = 180°

⇒ 85° + ∠BCE = 180°

⇒ ∠BCE = 95°

∠ADB = ∠BCE

[Angle in a same segment of a circle are equal]

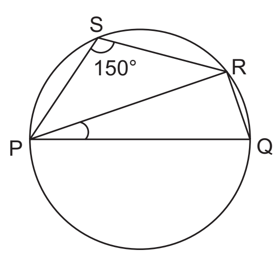
∴ ∠ADB = 95°

5. PQRS is a cyclic quadrilateral such that PQ is a diameter of the circle circumscribing the quadrilateral and ∠PSR = 150°, then the measure of ∠QPR is

(*a*) 60° (*b*) 50°

(*c*) 40° (*d*) 30°

Sol. (*a*) 60°



∵ PQRS is a cyclic quadrilateral,

∴ ∠PSR + ∠RQP = 180°

[Opposite ∠s of a cyclic quadrilateral are supplementary]

⇒ 150° + ∠RQP = 180°

∴ ∠RQP = 30°

∵ PQ is a diameter,

∴ ∠PRQ = 90°

[Angle in a semi-circle is a right angle]

In ΔPRQ, we have

∠RQP + ∠PRQ + ∠QPR = 180°

[Angle sum property of a triangle]

⇒ 30° + 90° + ∠QPR = 180°

⇒ 120° + ∠QPR = 180°

∴ ∠QPR = 60°

Hence, measure of ∠QPR is 60°.

Fill in the Blanks

6. Angle formed in minor segment of a circle is **an obtuse angle**.

7. Number of circles that can be drawn through three non-collinear points is **1**.

8. Greatest chord of a circle is called its **diameter**.

9. The region between a chord and either of the arc is called **a segment**.

Assertion-Reason Type Questions

Directions (Q. Nos. 10 to 13): Each of these questions contains an assertion followed by reason. Read them carefully, and answer the question on the basis of the following options, select the one that best describes the two statements.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

10. **Assertion (A):** Two conjugate arcs of a circle will have same end points.

Reason (R): Two conjugate arcs of a circle complete the circle.

Sol. (*a*)

The assertion and reason both are correct and reason is correct explanation of assertion.

11. **Assertion (A):** The degree measure of a semicircle is 90.

Reason (R): Semicircle is half of a circle.

Sol. (*d*)

Semicircle is half of a circle and its degree measure is 180°.

∴ Reason is correct but assertion is incorrect.

12. **Assertion (A):** Diagonal of a circle is its greatest chord.

Reason (R): A chord divides a circle into segments.

Sol. (*b*)

Both assertion and reason are correct but reason is not correct explanation of assertion.

13. **Assertion (A):** There is one and only one circle passing through three given non-collinear points.

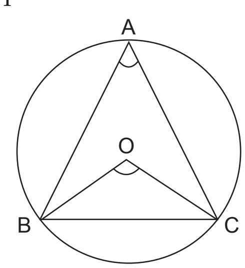
Reason (R): Circles with same area have equal radii.

Sol. (*b*)

Both assertion and reason are correct but reason is not the correct explanation of assertion.

Case Study Based Questions

14. Coronavirus disease (COVID-19) is an infectious disease caused by a newly discovered coronavirus. The COVID-19 pandemic has led to a dramatic loss of human life and greatest challenge we have faced since World War II. In order to protect people against severe COVID-19 disease, Government of India is working regularly in speeding up Covid-19 vaccination. For this, there are three vaccination centres in a village situated at A, B and C as shown in the figure. These three vaccination centres are equidistant from each other as shown in the figure. Based on the above situation, answer the following questions.



(*a*) What is the measure of ∠ABC?

Ans. 60°

(*b*) If the length of AB is 6 km, then find the value of BC + CA.

Ans. 12 km

(*c*) (*i*) What is the measure of ∠BOC?

or

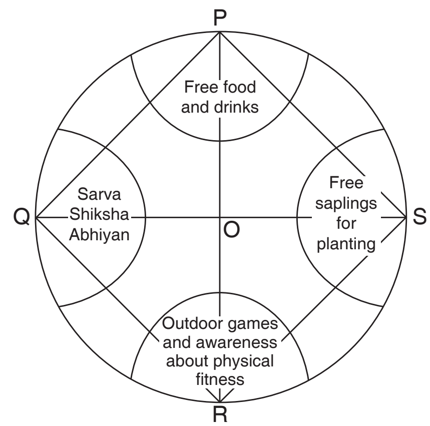
(*ii*) What is the value of (∠OBC + ∠OCB)?

Ans. (*i*) 120°

or

(*ii*) 60°

15. A group of social workers organised a mela for slum children in a circular park. They set-up four equidistant stalls P, Q, R and S along the boundary of the park. Stall P provided free food and drinks. Stall Q provided the awareness about the importance of education, while stalls R and S dealt with physical fitness and environment protection respectively.



If PR and QS intersect at right angle, then answer the following questions.

(*a*) What is the measure of ∠PSO?

Ans. 45°

(*b*) What is the measure of ∠RSQ?

Ans. 45°

(*c*) (*i*) What is the measure of ∠PSR?

or

(*ii*) What is the property of cyclic quadrilateral?

Ans. (*i*) 90°

or

(*ii*) Opposite angles are supplementary.

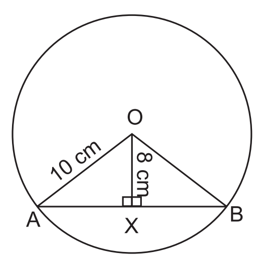
Very Short Answer Type Questions

16. The distance of a chord from the centre of a circle of radius 10 cm is 8 cm. What is the length of length of this chord?

Sol. Given, OA = OB = 10 cm [Radius of a circle]

OX = 8 cm

[Distance of a chord from the centre of a circle]



$\because$ OX $\perp$ AX

$\therefore$ ΔOXA is a right-angled triangle.

$\therefore$ By Pythagoras' Theorem, we have

$$(OA)^2 = (AX)^2 + (OX)^2$$

$\Rightarrow$ $(10)^2 = (AX)^2 + (8)^2$

$\Rightarrow$ $100 = (AX)^2 + 64$

$\Rightarrow$ $(AX)^2 = 36$

$\Rightarrow$ AX = 6 cm

$\therefore$ AB = 2 AX

[OX $\perp$ AB, The $\perp$ from the centre of a circle to a chord bisects the chord]

$= 2 \times 6$

$= 12$

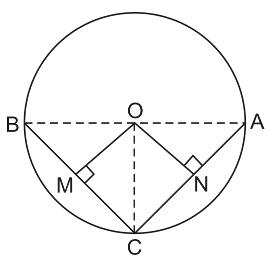
AB = 12 cm

Hence, length of the chord AB is 12 cm.

17. The two perpendicular bisectors of two chords BC and CA of a circle meet at a point O. Then O will be the centre of the circle and the radius of the circle will be OA, OB or OC. Is this statement true or false?

Sol. The statement is true.

To prove that O is the centre and OA, OB and OC are radius.



In ΔBOM and ΔCOM, we have

BM = MC

[OM is perpendicular bisector of BC]

OM = OM [Common]

$\angle$OMB = $\angle$OMC [Each 90°]

$\Rightarrow$ ΔBOM $\cong$ ΔCOM

[By SAS congruence]

$\Rightarrow$ OB = OC [By CPCT] …(1)

In ΔCON and ΔNOA

CN = NA

[ON is perpendicular bisector of CA]

ON = ON [Common]

$\angle$CNO = $\angle$ANO [Each 90°]

$\Rightarrow$ ΔCON $\cong$ ΔNOA

[By SAS congruence]

$\Rightarrow$ OC = OA [By CPCT] …(2)

From (1) and (2), we have

OA = OB = OC

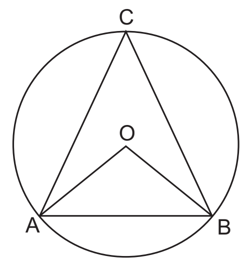
$\Rightarrow$ O is the centre of a circle

The centre of the circle is the only point within the circle that has points on the circumference which are equal in distance from it.

Hence, O is the centre of a circle and OA, OB and OC are the radius.

18. Prove that an angle in a major segment of a circle is acute.

Sol. Let $\angle$ACB be an angle formed in the major segment of a circle with centre O.



$$\angle ACB = \frac{1}{2}\angle AOB \quad \text{…(1)}$$

[Angle subtended by an arc of a circle at the centre is twice the angle subtented by it on the remaining part of the circle]

But $\angle$AOB < 180°

$\Rightarrow$ $\frac{1}{2}\angle AOB < 90°$

$\Rightarrow$ $\angle$ACB < 90° [From (1)]

$\therefore$ $\angle$ACB is an acute angle in the major segment.

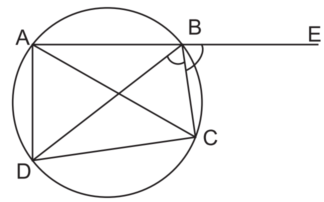
Hence, an angle in a measure segment of a circle is acute.

19. ABCD is a cyclic quadrilateral. The side AB is produced to E outside the circle such that BC is the internal bisector of $\angle$DBE. Prove that AC = CD.

Sol. We have $\angle$EBC = $\angle$ADC

[Exterior $\angle$s of cyclic quadrilateral is equal to interior opposite $\angle$s] …(1)

$\angle$EBC = $\angle$DBC [Given] …(2)



From (1) and (2), we have

$\angle$ADC = $\angle$DBC …(3)

But $\angle DAC = \angle DBC$

[Angles in the same segment] …(4)

From (3) and (4), we have

$\angle ADC = \angle DAC$

$\Rightarrow$ AC = CD

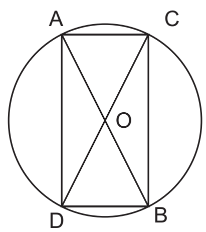
[Sides opposite to equal angles]

Hence, proved.

Short Answer Type Questions

20. Prove that if two chords of a circle bisect each other at a point within the circle, then these chords are the diameters of the circle.

Sol. Let AB and CD be two chords.



In ΔAOD and ΔCOB

AO = OB

DO = OC

[Given, chords are bisecting each other]

$\angle AOD = \angle COB$ [Vertically opp. $\angle$s]

$\therefore$ $\Delta AOD \cong \Delta COB$

[By SAS congruence]

$\Rightarrow$ AD = CB [By CPCT]

Also, $\overset{\frown}{AD} \cong \overset{\frown}{CB}$

[If two chords are equal then their corresponding arcs are congruent] …(1)

Similarly, $\Delta AOC \cong \Delta BOD$

[By SAS congruence]

AC = BD [By CPCT]

$\overset{\frown}{AC} \cong \overset{\frown}{BD}$ …(2)

Adding (1) and (2), we get

$\overset{\frown}{AC} + \overset{\frown}{AD} \cong \overset{\frown}{BD} + \overset{\frown}{CB}$

$\Rightarrow$ $\overset{\frown}{CAD} \cong \overset{\frown}{CBD}$

CD divides the circle into two equal parts. So, CD and AB are diameters of a circle.

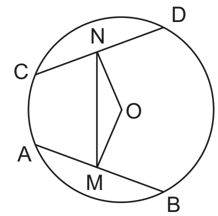
Hence, proved.

21. Prove that the line joining the mid-points of two equal chords of a circle subtends equal angles with the chords.

Sol. To Prove that $\angle AMN = \angle CNM$ and $\angle BMN = \angle DNM$

Construction: Join OM and ON

$\because$ The line segment joining the centre of a circle to the mid-point of a chord is perpendicular to the chord.



Given, AB and CD are equal chords.

$\therefore$ They are equidistant from the centre,

i.e. OM = ON

In ΔOMN, we have

OM = ON

$\angle OMN = \angle ONM$

[Angles opposite to equal sides] …(1)

$\angle OMA = \angle ONC$ [Each 90°] …(2)

$\angle OMB = \angle OND$ [Each 90°] …(3)

Subtracting (2) from (1), we have

$\angle OMA - \angle OMN = \angle ONC - \angle ONM$

$\Rightarrow$ $\angle AMN = \angle CNM$

Adding (1) and (3), we have

$\angle OMB + \angle OMN = \angle OND + \angle ONM$

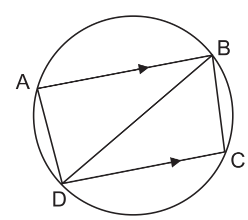
$\Rightarrow$ $\angle BMN = \angle DNM$

Hence, proved.

22. If two sides of a cyclic quadrilateral are parallel, prove that the remaining two sides are equal.

[CBSE SP 2011]

Sol. Let ABCD be the cyclic quadrilateral with AB $\parallel$ DC. Join BD.



Now, since AB $\parallel$ DC and BD is the transversal.

$\angle ABD = \angle CDB$ [Alternate $\angle$s]

$\angle ABD$ is subtented by chord AD on the circumference and $\angle CBD$ is subtented by chord BC.

We know, if the angles subtended by two chords on the circumference of the circle are equal, then the length of the chords is also equal.

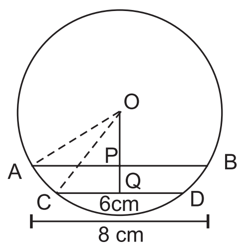
Hence, AD = BC, i.e the remaining two sides of the cyclic quadrilateral are equal.

Long Answer Type Questions

23. The lengths of two parallel chords on the same side of the centre are 6 cm and 8 cm. If the smaller chord is at a distance of 4 cm from the centre, what is the distance of the other chord from the centre? **[CBSE SP 2011]**

Sol. Let AB and CD be two parallel chords of a circle with centre O such that AB = 8 cm and CD = 6 cm.

Draw $OQ \perp CD$ and $OP \perp AB$.



$\because$ $AB \parallel CD$ and $OQ \perp CD$, $OP \perp AB$,

$\therefore$ Points O, P, and Q are collinear.

$$CQ = \frac{1}{2}CD = \frac{1}{2} \times 6 = 3 \text{ cm}$$

[As the $\perp$ from the centre of a circle to the chord bisects the chord)

$$AP = \frac{1}{2}AB = \frac{1}{2} \times 8 = 4 \text{ cm}$$

$\because$ ΔOQC is a right-angled triangle,

$\therefore$ By Pythagoras' Theorem, we have

$$(OC)^2 = (CQ)^2 + (OQ)^2$$

$\Rightarrow$ $(OC)^2 = (3)^2 + (4)^2$

$\Rightarrow$ $(OC)^2 = 9 + 16$

$\Rightarrow$ $(OC)^2 = 25$

$\Rightarrow$ $OC = 5$ cm

$\Rightarrow$ $OA = OC$ [Radii of the circle]

$\Rightarrow$ $OA = 5$ cm

$\because$ ΔOPA is a right-angled triangle,

$\therefore$ By Pythagoras' Theorem, we have

$$(OA)^2 = (AP)^2 + (OP)^2$$

$\Rightarrow$ $(5)^2 = (4)^2 + (OP)^2$

$\Rightarrow$ $25 = 16 + (OP)^2$

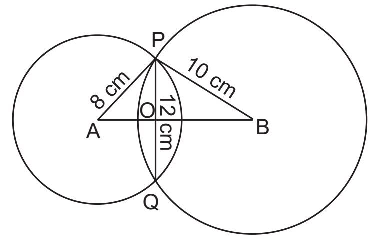
$\Rightarrow$ $(OP)^2 = 9$

$\Rightarrow$ $OP = 3$ cm

Hence, distance of the bigger chord from the centre is 3 cm.

24. Two circles of radii 10 cm and 8 cm intersect each other at P and Q and the length of their common chord is 12 cm. Find the distance between their centres. **[CBSE SP 2011]**

Sol. Let A and B are the centres of two circles. PQ is the common chord of length 12 cm.



Given, $AP = 8$ cm

$BP = 10$ cm

$$PO = \frac{1}{2}PQ$$

[Perpendicular from the centre of the circle to a chord bisects the chord]

$$= \frac{1}{2} \times 12 = 6$$

$PO = 6$ cm

$\because$ ΔPOA is a right-angled triangle,

$\therefore$ By Pythagoras' Therorem, we have

$$(AP)^2 = (AO)^2 + (PO)^2$$

$\Rightarrow$ $(8 \text{ cm})^2 = (AO)^2 + (6 \text{ cm})^2$

$\Rightarrow$ $64 \text{ cm}^2 = (AO)^2 + 36 \text{ cm}^2$

$\Rightarrow$ $(AO)^2 = 28 \text{ cm}^2$

$\Rightarrow$ $AO = 2\sqrt{7}$ cm

$= 2 \times 2.64$ cm

$= 5.29$ cm

$\Rightarrow$ $AO = 5.29$ cm

$\because$ ΔPOB is a right-angled triangle,

$\therefore$ By Pythagoras' Theorem, we have

$$(BP)^2 = (BO)^2 + (PO)^2$$

$\Rightarrow$ $(10 \text{ cm})^2 = (BO)^2 + (6 \text{ cm})^2$

$\Rightarrow$ $100 \text{ cm}^2 = (BO)^2 + 36 \text{ cm}^2$

$\Rightarrow$ $(BO)^2 = 64 \text{ cm}^2$

$\Rightarrow$ $BO = 8$ cm

$\because$ $AB = AO + BO$

$= (5.29 + 8)$ cm

$= 13.29$ cm

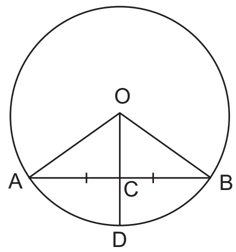
$\therefore$ $AB = 13.29$ cm

Hence, distance between the centres of two circles is 13.29 cm.

25. Prove that the line joining the mid-point of a chord to the centre of a circle passes through the mid-point of the corresponding minor arc.

Sol. Given, C is the mid-point of chord AB.

To prove that D is the mid-point of arc AB.



In ΔOAC and ΔOBC, we have

$OA = OB$ [Radius of a circle]

$OC = OC$ [Common]

$AC = BC$ [C is the mid-point of AB]

$\therefore \quad \Delta OAC \cong \Delta OBC$ [By SSS congruence]

$\Rightarrow \quad \angle AOC = \angle BOC$ [By CPCT]

$\Rightarrow \quad \widehat{AD} \cong \widehat{BD}$

Hence, D is the mid-point of arc AB.

Let's Compete

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Multiple-Choice Questions

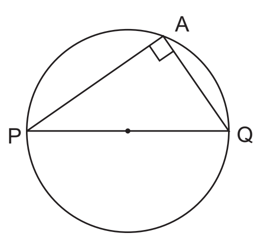
1. PQ is a chord of a circle with radius r. If A is any point on the circle such that $\angle PAQ = 90^\circ$, then PQ is equal to

(*a*) r (*b*) $2r$

(*c*) $3r$ (*d*) $4r$

Sol. (*b*) $2r$

We know, angle in a semi-circle is a right angle.



Given, radius = r

Diameter = $2r$

PQ = Diameter = $2r$

Hence, PQ is $2r$.

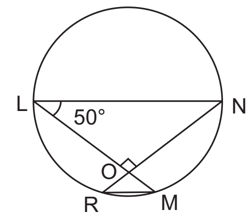
2. Chords LM and NR intersect each other within a circle at right angles. If $\angle NLM = 50^\circ$, then $\angle LMR$ is equal to

(*a*) 60° (*b*) 50°

(*c*) 40° (*d*) 70°

Sol. (*c*) 40°

Given, $\angle NLM = 50^\circ$



In ΔLON, we have

$\angle LNO + \angle NLO + \angle LON = 180^\circ$

$\Rightarrow \quad \angle LNO + 50^\circ + 90^\circ = 180^\circ$

$\Rightarrow \quad \angle LNO + 140^\circ = 180^\circ$

$\angle LNO = 40^\circ$

$\angle LMR = \angle LNO$

[Angle in a same segment of a circle are equal]

$\therefore \quad \angle LMR = 40^\circ$

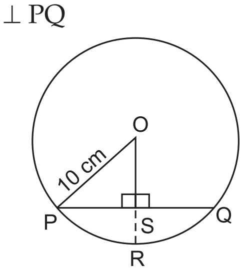
3. PQ is a chord of a circle with centre at O. If OP = 10 cm, PQ = 16 cm and $OS \perp PQ$, then the length of SR is equal to

(*a*) 4 cm (*b*) 6 cm

(*c*) 3 cm (*d*) 5 cm

Sol. (*a*) 4 cm

Given $OS \perp PQ$



$\because$ The perpendicular from the centre of a circle to a chord bisects the chord.

$\Rightarrow \quad PS = 8$ cm

$\because \Delta PSO$ is a right-angled triangle,

$\therefore$ By Pythagoras' Theorem, we have

$(PO)^2 = (PS)^2 + (SO)^2$

$\Rightarrow \quad (10)^2 = (8)^2 + (SO)^2$

$\Rightarrow \quad 100 = 64 + (SO)^2$

$\Rightarrow \quad (SO)^2 = 36$

$\Rightarrow \quad SO = 6$ cm

Since, $OP = OR = 10$ cm [Radius]

$\therefore \quad SR = OR - SO$

$= 10 \text{ cm} - 6 \text{ cm}$

$= 4$ cm

Hence, the length of SR is 4 cm.

4. PQRS is a cyclic trapezium in which PS || QR and $\angle Q = 70^o$. Then $\angle QRS$ is equal to

(*a*) 50° (*b*) 60°

(*c*) 80° (*d*) 70°

Sol. (*d*) 70°

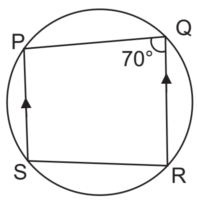
Given, $\angle Q = 70°$

$\angle P + \angle Q = 180°$

[Cointerior $\angle$s, PS || QR]

$\Rightarrow \angle P + 70° = 180°$

$\angle P = 110°$



$\because$ PQRS is a cyclic trapezium,

$\therefore \angle P + \angle R = 180°$

[Opposite $\angle$s of cyclic quadrilateral are supplementary]

$\Rightarrow 110° + \angle R = 180°$

$\angle R = 70°$

$\angle QRS = 70°$

5. If PQ = 15 cm and QR = 8 cm are two line segments intersecting each other at Q at right angles. Then the radius of the circle passing through the points P, Q and R is

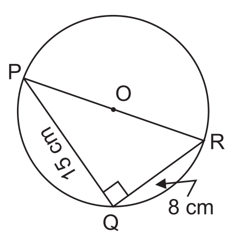
(*a*) 8 cm (*b*) 8.5 cm

(*c*) 9 cm (*d*) 9.5 cm

Sol. (*b*) 8.5 cm

Given, PQ = 15 cm

QR = 8 cm



$\because$ The arc of a circle subtending a right angle at any point on the remaining part of the circle is a semi-circle.

$\therefore$ PR is the diameter of a circle.

$\because$ ΔPQR is a right-angled triangle,

$\therefore$ By Pythagoras' Theorem, we have

$(PR)^2 = (QP)^2 + (QR)^2$

$= (15 \text{ cm})^2 + (8 \text{ cm})^2$

$= 225 \text{ cm}^2 + 64 \text{ cm}^2$

$= 289 \text{ cm}^2$

PR = 17 cm

$\text{Radius} = \frac{PR}{2}$

$= \frac{17}{2} \text{ cm} = 8.5 \text{ cm}$

Radius = 8.5 cm

Hence, radius of a circle is 8.5 cm.

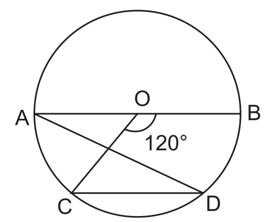
6. AOB is a diameter of a circle with centre at O. CD is a chord of the circle such that $\angle COB = 120^o$. Then the measure of $\angle ADC$ is equal to

(*a*) 50° (*b*) 40°

(*c*) 30° (*d*) 60°

Sol. (*c*) 30°

Given, $\angle COB = 120°$



$\because$ AOB is a line,

$\therefore \angle AOC + \angle COB = 180°$ [Linear pair]

$\Rightarrow \angle AOC + 120° = 180°$

$\angle AOC = 60°$

$\angle AOC = 2 \angle ADC$

[Angle subtended by an arc of a circle at the centre is double the angle subtented by it at any point on the remaining part of the circle]

$\Rightarrow 60° = 2\angle ADC$

$\Rightarrow \angle ADC = 30°$

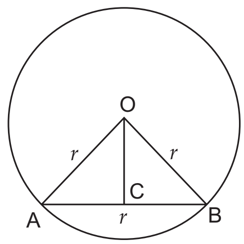
Hence, measure of $\angle ADC$ is 30°

7. If a chord of a circle is equal to its radius r, then the distance of this chord from the centre of the circle is

(*a*) $\frac{\sqrt{2}}{3} r$ (*b*) $\frac{2}{3} r$

(*c*) $\frac{\sqrt{3}}{2} r$ (*d*) $\frac{2}{\sqrt{3}} r$

Sol. (*c*) $\frac{\sqrt{3}}{2} r$



Given, radius (r) = chord AB

$\therefore$ $OA = OB = AB = r$

$OC \perp AB$

[$\perp$ from the centre to the chord bisects the chord]

$$AC = CB = \frac{r}{2}$$

$\because$ ΔACO is a right-angled triangle.

$\therefore$ By Pythagoras' theorem, we have

$$(AO)^2 = (AC)^2 + (CO)^2$$

$$\Rightarrow \quad r^2 = \left(\frac{r}{2}\right)^2 + (CO)^2$$

$$\Rightarrow \quad (CO)^2 = r^2 - \frac{r^2}{4} = \frac{3r^2}{4}$$

$$\Rightarrow \quad CO = \frac{r\sqrt{3}}{2}$$

Hence, distance of the chord from the centre of a circle is $\frac{\sqrt{3}}{2}r$.

8. PQ is the common chord of two circles intersecting each other at P and Q. If AD and BC are two chords of the smaller and bigger circles respectively such that A, P, B lie on a line and D, Q, C lie on the second line and if $\angle ABC = 75^\circ$, then the measure of $\angle PAD$ is equal to

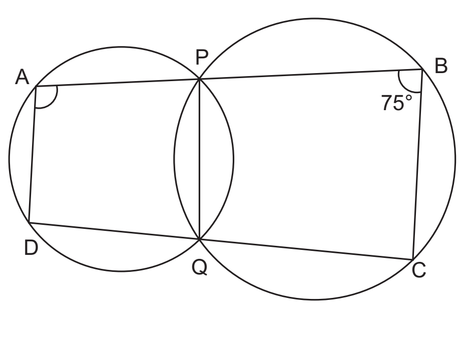
(*a*) 125° (*b*) 150°

(*c*) 75° (*d*) 105°

Sol. (*d*) 105°

PBCQ is a cyclic quadrilateral

[P, B, C, Q lie on circle]



$$\angle PBC + \angle PQC = 180^\circ$$

[Opposite $\angle$s of a cyclic quadrilateral are supplementary]

$$\Rightarrow \quad 75^\circ + \angle PQC = 180^\circ$$

$$\angle PQC = 105^\circ$$

$\because$ DQC is a line,

$\therefore$ $\angle DQP + \angle PQC = 180^\circ$ [Linear pair]

$$\Rightarrow \quad \angle DQP + 105^\circ = 180^\circ$$

$$\angle DQP = 75^\circ$$

$\because$ APQD is a cyclic quadrilateral,

$\therefore$ $\angle PAD + \angle DQP = 180^\circ$

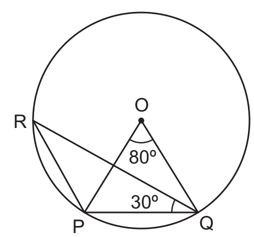
[Opposite $\angle$s of a cyclic quadrilateral are supplementary]

$$\Rightarrow \quad \angle PAD + 75^\circ = 180^\circ$$

$$\angle PAD = 105^\circ$$

Hence, measure of $\angle PAD$ is 105°.

9. In the given figure, if $\angle POQ = 80^\circ$ and $\angle PQR = 30^\circ$, then $\angle RPO$ is equal to



(*a*) 30° (*b*) 60°

(*c*) 80° (*d*) 40°

Sol. (*b*) 60°

$$\angle POQ = 2\angle PRQ$$

[Angle subtended by an arc of a circle at the centre is double the angle subtended by it at any point on the remaining part of the circle]

$$\Rightarrow \quad 80^\circ = 2\angle PRQ$$

$$\angle PRQ = 40^\circ$$

In ΔOPQ, we have

$$\angle OPQ + \angle OQP + \angle POQ = 180^\circ$$

[Angle sum property of a triangle]

$$\Rightarrow \quad 2\angle OPQ + 80^\circ = 180^\circ$$

[$\because$ $\angle OPQ = \angle OQP$, angles opposite to equal sides (radii) OP and OQ of ΔOPQ]

$$\Rightarrow \quad 2\angle OPQ = 100^\circ$$

$$\angle OPQ = 50^\circ$$

In ΔQRP, we have

$$\angle RQP + \angle RPQ + \angle PRQ = 180^\circ$$

$$30^\circ + \angle RPO + \angle OPQ + 40^\circ = 180^\circ$$

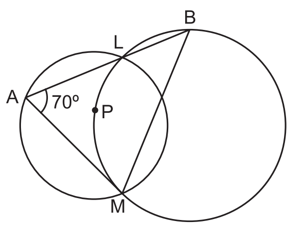
$\Rightarrow \quad 70° + \angle RPO + 50° = 180°$

$\Rightarrow \quad 120° + \angle RPO = 180°$

$\angle RPO = 60°$

Hence, measure of $\angle RPO$ is 60°.

10. In the figure, two circles intersect each other at L and M. The centre P of the smaller circle lies on the circumference of the larger circle. If $\angle LAM = 70°$, then the measure of $\angle LBM$ is



(*a*) 40° (*b*) 70°
(*c*) 60° (*d*) 50°

Sol. (*a*) 40°

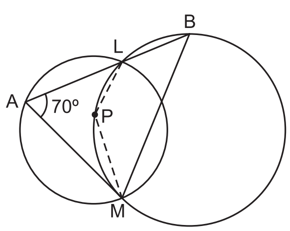
$\angle LPM = 2\angle LAM$

[Angle subtended by the arc of a circle at the centre is double the angle subtended by it at any point on the remaining part of the circle]

$= 2 \times 70°$

$= 140°$

$\angle LPM = 140° \quad \ldots(1)$



$\because$ Since the opposite $\angle$s of a cyclic quadrilateral are supplementary and BMPL is a cyclic quadrilateral,

$\therefore \quad \angle LPM + \angle LBM = 180°$

$\Rightarrow \quad 140° + \angle LBM = 180° \quad$ [From (1)]

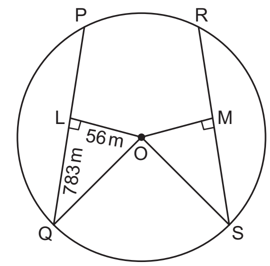
$\angle LBM = 40°$

Hence, measure of $\angle LBM$ is 40°.

Life Skills

(Page 138)

1. Two roads PQ and RS each of length 1566 m, run through a circular park as shown in the figure.



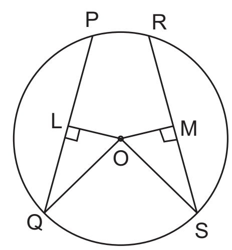
(*a*) If the road PQ is at a distance of 56 m from the centre O of the circular park, then find the distance of the road RS from the centre.

(*b*) Find the radius of the circular park.

(*c*) On Van Mahotsav Day, some students planted a few trees at equal distance of nearly 6.3 m from each other. Find the approximate number of trees planted around the park.

Sol. Given, length of road PQ = 1566 m.

Length of road RS = 1566 m.



(*a*) Distance of PQ from centre O = 56 m [Given]

We know equal chords of a circle are equidistant from the centre of the circle.

$\because$ PQ and RS are of equal length,

$\therefore$ They are equidistant from the centre.

$\therefore$ Distance of RS from centre = 56 m

(*b*) Let $OL \perp PQ$ and $OM \perp RS$

$PQ = 2\angle Q$

[Perpendicular drawn from the centre of the circle to the chord bisects the chord]

$\angle Q = \frac{1}{2} PQ$

$= \frac{1}{2} \times 1566 \text{ m}$

$= 783 \text{ m}$

$\angle Q = 783 \text{ m}$

$\because$ ΔOLQ is a right-angled triangle,

$\therefore$ By Pythagoras' Theorem, we have

$(OQ)^2 = (OL)^2 + (LQ)^2$

$= (56 \text{ m})^2 + (783 \text{ m})^2$

$= (3136 + 613089) \text{ m}^2$

$(OQ)^2 = 616225 \text{ m}^2$

$OQ = 785 \text{ m}$

Hence, radius of the circular park is 785 m.

(*c*) Circumference of circular park = $2\pi r$

$$= 2 \times \frac{22}{7} \times 785 \text{ m}$$

$$= \frac{34540}{7} \text{ m}$$

Given, students planted few trees at equal distance of nearly 6.3 m from each other along the circumference of a circular park.

Number of trees planted

$$= \frac{\text{Circumference of park}}{\text{Distance between one tree from other}}$$

$$= \frac{34540}{7 \times 6.3}$$

$$= \frac{34540 \times 10}{7 \times 63}$$

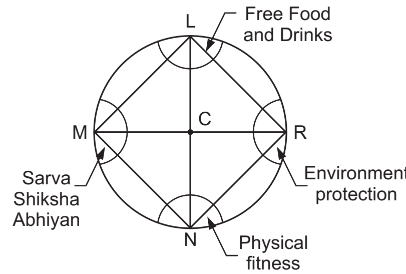
$$= \frac{345400}{441}$$

$$= 783 \text{ (approx.)}$$

Hence, approximate number of trees planted around the park is 783.

2. A group of social workers organised a mela for slum children in a circular park. They set-up four equidistant stalls at L, M, N and R along the boundary of the park. Stall L provided free food and drinks. Stall M provided awareness about the importance of education, while the stalls N and R dealt with physical fitness and environment protection respectively.

If LN and MR intersect each other at C at right angles to each other, prove that the quadrilateral LMNR is a square.



Sol. $\angle LCM = 2\angle LRM$

$\Rightarrow \quad 90° = 2\angle LRM$

$\Rightarrow \quad \angle LRM = 45°$ …(1)

$\angle MCN = 2\angle MRN$

$\Rightarrow \quad 90° = 2\angle MRN$

$\Rightarrow \quad \angle MRN = 45°$ …(2)

Adding (1) and (2), we get

$\angle LRM + \angle MRN = 45° + 45°$

$\Rightarrow \quad LRN = 90°$

Similarly,

$\angle LMN = \angle MNR = \angle RLM = 90°$ …(3)

In ΔLCR and ΔNCR, we have

$\angle LCR = \angle NCR$ [Each is 90°]

$CR = CR$ [Common]

$\angle LRC = \angle NRC$ [Each is 45°]

$\therefore \quad \Delta LCR \cong \Delta NCR$

[By ASA congruence]

$\Rightarrow \quad LR = RN$ [By CPCT]

Similarly, $LM = MN = LR = RN$ …(4)

$\Rightarrow$ LMNR is a quadrilateral in which all sides are equal and each angle is 90°.

[From (3) and (4)]

Hence, LMNR is a square.