8

Quadrilaterals

Checkpoint (Page 103)

1. Check whether the following groups of four angles represent the angles of a quadrilateral or not.
   (*a*) 80°, 100°, 95°, 85°
   (*b*) 47°, 86°, 92°, 95°
   (*c*) 31°, 79°, 118°, 132°

Sol. We know that the sum of four angles of a quadrilateral is 360°.

(*a*) We see that

$$80° + 100° + 95° + 85° = 180° + 180° = 360°$$

Hence, the given angles are the angles of a quadrilateral.

(*b*) We see that

$$47° + 86° + 92° + 95° = 320° < 360°$$

Hence, the given angles are not the angles of any quadrilateral.

(*c*) We see that

$$31° + 79° + 118° + 132° = 110° + 250° = 360°$$

Hence, the given angles are the angles of a quadrilateral.

2. Three angles of quadrilateral are 57°, 120° and 73°. Find the measure of the fourth angle.

Sol. Let the fourth angle be $x°$.

$$\therefore \quad 57° + 120° + 73° + x° = 360°$$
$$\Rightarrow \quad 250° + x° = 360°$$
$$\Rightarrow \quad x° = 360° - 250° = 110°$$

Hence, the required angle is 110°.

3. The angles of a quadrilateral are in the ratio 3 : 4 : 5 : 6. Find the measures of the angles of the quadrilateral.

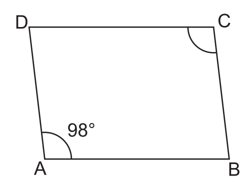
Sol. Let the angles of the quadrilateral be $(3x)°$, $(4x)°$, $(5x)°$ and $(6x)°$.

$$\therefore \quad 3x + 4x + 5x + 6x = 360$$
$$\Rightarrow \quad 18x = 360$$
$$\Rightarrow \quad x = \frac{360}{18} = 20$$

Hence, the required angles are $(3 \times 20)° = 60°$, $(4 \times 20)° = 80°$, $(5 \times 20)° = 100°$ and $(6 \times 20)° = 120°$.

4. In a parallelogram ABCD, ∠A = 98°. Find ∠B, ∠C and ∠D.

Sol. Let ABCD be a parallelogram and ∠A = 98°.



$$\because \quad \angle A = \angle C$$
$$\therefore \quad \angle C = 98°$$
$$\text{Also,} \quad \angle A + \angle B = 180°$$
$$\Rightarrow \quad \angle B = 180° - \angle A = 180° - 98° = 82°$$
$$\therefore \quad \angle D = \angle B = 82°$$

Hence, ∠B = 82°, ∠C = 98° and ∠D = 82°.

5. Find the measures of angles of a parallelogram if one angle is 40° less than thrice the smallest angle.

Sol. Let the smaller angle be $x°$. Then another angle is $(3x - 40)°$.

$\therefore$ $(2x)° + 2(3x - 40)°$ = Sum of four angles of the parallelogram

$= 360°$

$\Rightarrow \quad x + 3x - 40 = 180$

$\Rightarrow \quad 4x = 180 + 40$

$\Rightarrow \quad 4x = 220$

$\therefore \quad x = \dfrac{220}{4} = 55$

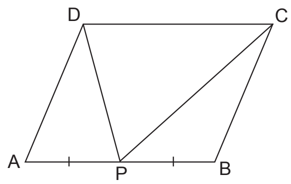
Also, $\quad (3x - 40)° = (3 \times 55 - 40)°$

$= (165 - 40)°$

$= 125°$

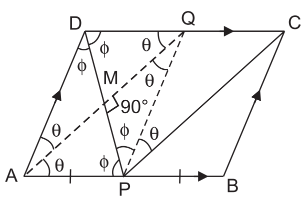
Hence, the required angles are 55° and 125°.

6. In the given figure, ABCD is a parallelogram. If AB = 2AD and P is the mid-point of AB, then find $\angle$CPD.



Sol. Given that ABCD is a parallelogram and P is a point on AB such that AB = 2AD and P is a mid-point of AB.

To find the measure of $\angle$CPD.



Construction: We draw PQ $\parallel$ AD to cut DC at Q.

We see that APQD is a rhombus and we know that the two diagonals AQ and PD of the rhombus bisect each other at right angles.

$\therefore \quad \angle PMQ = 90°$

Let PAD = 2θ and $\angle ADQ = 2\phi$.

$\therefore \quad 2\theta + 2\phi = 180°$

$\Rightarrow \quad \phi + \theta = 90° \quad \ldots(1)$

$\therefore \quad \angle PAQ = \angle QAD = \theta$

and $\quad \angle AQD = \angle AQP = \theta$

Also, $\quad \angle QPC = \angle AQP = \theta$

Also, $\quad \angle ADP = \angle QDP = \phi$

and $\quad \angle APD = \angle DPQ = \phi$

Now, in ΔPMQ, we have

$\angle MQP + \angle MPQ = \theta + \phi$

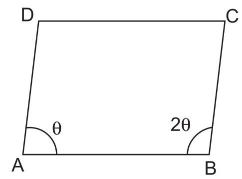
$\Rightarrow \quad \angle QPC + \angle MPQ = \theta + \phi$

$\Rightarrow \quad \angle CPD = \theta + \phi = 90° \quad$ [From (1)]

Hence, the required measure of $\angle$CPD is 90°.

7. If an angle of a parallelogram is half of its adjacent angle, find the angles of the parallelogram.

Sol. Let ABCD be a parallelogram with $\angle A = \theta$ and $\angle B = 2\theta$.



Now, $\quad \angle A + \angle B = 180°$

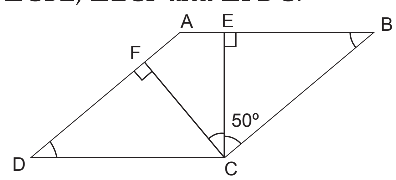
$\Rightarrow \quad 2\theta + \theta = 180°$

$\Rightarrow \quad 3\theta = 180°$

$\Rightarrow \quad \theta = \dfrac{180°}{3} = 60°$

Hence, the required angles are 60°, 120°, 60° and 120°.

8. In the given figure, ABCD is a parallelogram. If $\angle BCE = 50°$, $\angle BEC = 90°$ and $\angle CFD = 90°$, then find $\angle$CBE, $\angle$ECF and $\angle$FDC.



Sol. Given that ABCD is a parallelogram, CE $\perp$ AB, CF $\perp$ AD and $\angle BCE = 50°$.

To find the measures of $\angle$CBE, $\angle$ECF and $\angle$FDC.

In ΔECB, we have

$\angle CBE = 90° - 50° = 40°$

$[\because \angle CBE + \angle ECB = 90°]$

Also, $\quad \angle FDC = \angle CBE$

$[\because$ ABCD is a parallelogram$]$

$= 40°$

$\therefore$ In ΔFDC, $\angle FCD = 90° - \angle FDC$

$= 90° - 40° = 50°$

Now, $\quad \angle DAB + \angle ADC = 180°$

$\Rightarrow \quad \angle DAB + 40° = 180°$

$\therefore \quad \angle DAB = 180° - 40°$

$= 140°$

$\therefore \quad \angle DCB = \angle DAB$

$= 140°$

$\Rightarrow \quad \angle FCD + \angle ECF + \angle ECB = 140°$

$\Rightarrow \quad 50° + \angle ECF + 50° = 140°$

$\Rightarrow \quad \angle ECF = 140° - 100°$

$= 40°$

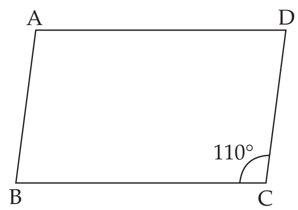
Hence, the required measures of $\angle CBE$, $\angle ECF$ and $\angle FDC$ are 40°, 40° and 40° respectively.

Check Your Progress 1

(Page 107)

Multiple-Choice Questions

1. In a parallelogram ABCD, $\angle C = 110°$. Find the measure of $\angle B$.



(*a*) 110° (*b*) 70°

(*c*) 60° (*d*) 50°

Sol. (*b*) 70°

In parallelogram ABCD,

$\angle B + \angle C = 180°$

$\Rightarrow \quad \angle B = 180° - 110°$

$\Rightarrow \quad \angle B = 70°$

2. The quadrilateral formed by joining the mid-points of the sides of a quadrilateral ABCD taken in order, is a rectangle, if

(*a*) ABCD is a rectangle.

(*b*) ABCD is a parallelogram.

(*c*) diagonals of ABCD are equal

(*d*) diagonals of ABCD are perpendicular.

Sol. (*d*) diagonals of ABCD are perpendicular.

3. The quadrilateral formed by joining the mid-points of the sides of a quadrilateral ABCD, taken in order is a rhombus, if

(*a*) ABCD is a parallelogram.

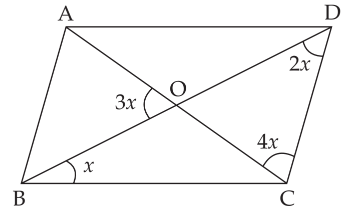
(*b*) ABCD is a rhombus.

(*c*) diagonals of ABCD are equal.

(*d*) diagonals of ABCD are perpendicular.

Sol. (*c*) diagonals of ABCD are equal.

4. The given figure ABCD is a parallelogram. The diagonals AC and BD intersect at O. Find all the angles of the parallelogram.



(*a*) 60°, 120°, 60°, 120°

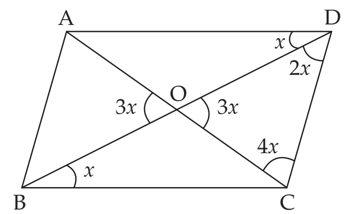
(*b*) 80°, 100°, 80°, 100°

(*c*) 90°, 90°, 90°, 90°

(*d*) 40°, 140°, 40°, 140°

Sol. (*a*) 60°, 120°, 60°, 120°

We have,



In ΔODC,

$\angle COD = \angle AOB = 3x$

[Vertically opposite angles]

$\angle COD + \angle OCD + \angle CDO = 180°$

$\Rightarrow \quad 3x + 4x + 2x = 180°$

$\Rightarrow \quad 9x = 180°$

$\Rightarrow \quad x = 20°$

$\angle DBC = \angle ADB$

[Alternate angles, AD || BC]

$\Rightarrow \quad \angle ADB = x$

$\angle D = \angle ADB + \angle BDC$

$= x + 2x$

$\angle D = 3x = 60° \quad [\because \ x = 20°]$

$\Rightarrow \quad \angle B = 60°$ [Opposite angles of a || gm are equal]

$\angle A + \angle D = 180°$

$\Rightarrow \quad \angle A = 180° - 60° = 120°$

$\therefore \quad \angle C = 120°$

The angles of the parallelogram are 60°, 120°, 60°, 120°.

5. If an angle of a parallelogram is one-third of its adjacent angle, find all the angles of the parallelogram.

(*a*) 45°, 135°, 45° and 135°

(*b*) 60°, 120°, 60° and 120°

(*c*) 70°, 110°, 70° and 110°

(*d*) 80°, 100°, 80° and 100°

Sol. (*a*) 45°, 135°, 45° and 135°

Let the adjacent angle = x

The other angle $= \frac{1}{3}x$

$$x + \frac{1}{3}x = 180°$$

$$\Rightarrow \quad \frac{4x}{3} = 180°$$

$$\Rightarrow \quad x = \frac{3}{4} \times 180° = 135°$$

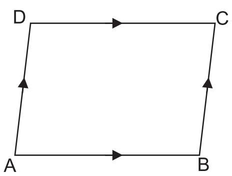
Therefore angles of the parallelogram are 45°, 135°, 45° and 135°.

Very Short Answer Type Questions

6. In a parallelogram ABCD, determine the sum of the angles of C and D.

Sol. Let ABCD is a parallelogram. Then AB ∥ DC and AD ∥ BC.

To find the measure of ∠C + ∠D.



∵ AD ∥ BC and DC is their transversal,

∴ sum of the interior angles, i.e.

$$\angle ADC + \angle BCD = 180°$$

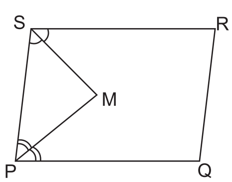
$$\Rightarrow \quad \angle C + \angle D = 180°$$

Hence, the sum of the angles of C and D is 180°.

7. In a parallelogram PQRS, the bisectors of ∠P and ∠S intersect at M. What is the measure of ∠PMS?

Sol. Given that PQRS is a parallelogram. PM and SM, the bisectors of ∠SPQ and ∠PSR respectively intersect each other at M.

To find the measure of ∠PMS.



We know that

$$\angle SPQ + \angle PSR = 180°$$

$$\Rightarrow \quad \frac{1}{2}\angle SPQ + \frac{1}{2}\angle PSR = 90°$$

$$\Rightarrow \quad \angle SPM + \angle PSM = 90°$$

In ΔPMS, we have

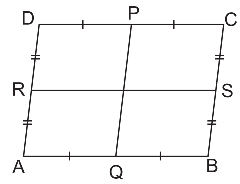
$$\angle PMS = 180° - (\angle SPM + \angle PSM)$$
$$= 180° - 90°$$
$$= 90°$$

∴ The required measure of ∠PMS is 90°.

8. Prove that the line segment joining the mid-points of the opposite sides of a parallelogram is parallel to the other pair of parallel sides.

Sol. Given that ABCD is a parallelogram and Q, S, P, R are respectively the mid-points of AB, BC, CD and DA. PQ and RS are joined.

To prove that PQ ∥ AD and RS ∥ AB.



∵ ABCD is a parallelogram,

∴ AB ∥ DC

and AB = DC

∴ AQ ∥ DP

and $\frac{1}{2}$ AB = $\frac{1}{2}$ DC

⇒ AQ = DP

∴ AQPD must be a parallelogram.

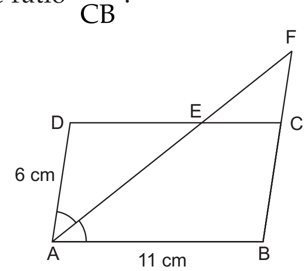
∴ PQ ∥ DA

Similarly, we can show that RS ∥ AB.

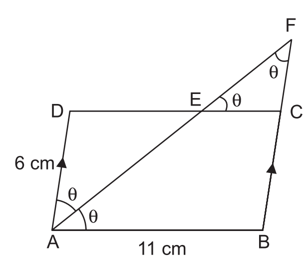
Hence, proved.

9. In a parallelogram ABCD, AB = 11 cm and AD = 6 cm. The bisector of ∠A meets DC at E. AE and BC produced meet at the point F.

Find the ratio $\frac{CF}{CB}$.



Sol. Given that ABCD is a parallelogram, AB = 11 cm, AD = 6 cm and E is a point on DC such that AE is the bisector of ∠DAB. AE and BC produced meet each other at F.



To find the ratio $\frac{CF}{CB}$.

Let $\angle DAE = \angle EAB = \theta$

Then $\angle FEC = \angle BAE$ [Corresponding angles]

$= \theta$

Also, $\angle EFC = \angle DAE$ [Alternate angles]

$= \theta$

$\therefore$ In ΔABF, $BF = AB = 11$ cm

$\therefore$ $CF = BF - BC$

$= BF - AD$ [$\because$ $BC = AD$]

$= (11 - 6)$ cm

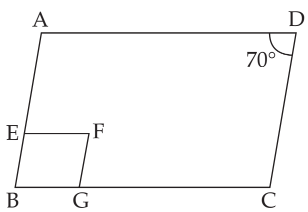
$= 5$ cm

and $CB = DA$

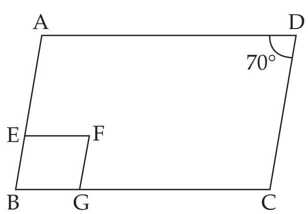
$= 6$ cm

$\therefore$ Required ratio of $\frac{CF}{CB} = \frac{5}{6}$.

10. In the given figure, ABCD and EFGB are both parallelograms. If $\angle D = 70°$, find all the angles of the parallelogram EBGF.



Sol. $\angle B = \angle D$ [Opposite angles of a $\parallel$gm are equal]



$\Rightarrow$ $\angle B = 70°$

$\angle B + \angle G = 180°$

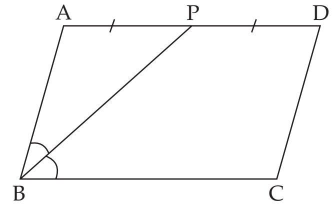
$\Rightarrow$ $\angle G = 180° - 70° = 110°$

$\angle F = \angle B = 70°$

$\angle E = \angle G = 110°$

The angles of the parallelogram BGFE are 70°, 110°, 70° and 110°.

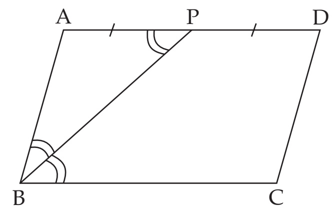
11. In the given figure, ABCD is a parallelogram. The bisector of $\angle B$ also bisects AD at P. Prove that BC = 2AB.



Sol. $\angle ABP = \angle PBC$ [BP bisects $\angle B$]

$\angle PBC = \angle APB$ [Alternate angles, AD $\parallel$ BC]

$\Rightarrow$ $\angle APB = \angle ABP$



$\therefore$ $AP = AB$

[Sides opposite to equal angles]

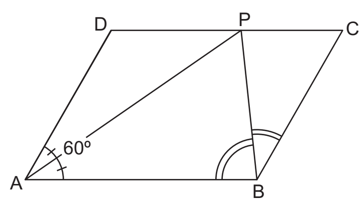
$2AP = 2AB$

$AD = 2AB$

$\Rightarrow$ $BC = 2AB$ [AD = BC]

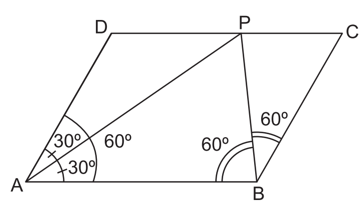
Short Answer Type Questions

12. In the given figure, ABCD is a parallelogram and $\angle A = 60°$. If the bisectors of $\angle A$ and $\angle B$ meet at P, prove that AD = DP, PC = BC and DC = 2AD.



Sol. Given that ABCD is a parallelogram and $\angle A = 60°$. P is a point on DC such that PA and PB are respectively the bisectors of $\angle DAB$ and $\angle ABC$.

To prove that AD = DP, PC = BC and DC = 2AD.



We have $\angle DAP = \angle PAB$

$= \frac{60°}{2} = 30°$ ...(1)

Also, $\angle APD = \angle PAB$

[Alternate angles, $\because$ AB $\parallel$ DC]

$\therefore$ $\angle APD = 30°$

$\therefore \quad \angle APD = \angle DAP$ [From (1)]

$\therefore \quad AD = DP$

Now, $\angle A + \angle B = 180°$

$\therefore \quad \angle B = 180° - \angle A$

$= 180° - 60° = 120°$

$\therefore \quad \angle PBC = \angle PBA$

$= \text{alternate } \angle CPB$

[$\because$ AB $\parallel$ DC]

$\therefore \quad PC = BC$

Also, $DC = DP + PC$

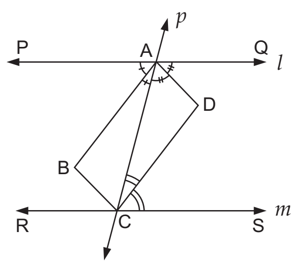
$= AD + BC$

$= AD + AD = 2AD$

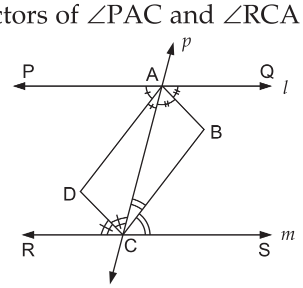
$\therefore \quad DC = 2AD$

Hence, proved.

13. Two parallel lines l and m are intersected by a transversal p as shown in the figure. Show that the quadrilateral formed by the bisectors of interior angles is a rectangle. **[CBSE SP 2013]**



Sol. Given that PQ and RS denoted by l and m respectively are two parallel lines and AC denoted by p is a transversal. AD and CD are the bisectors of $\angle PAC$ and $\angle RCA$ respectively.



Also, AB and CB are the bisectors of $\angle QAC$ and $\angle SCA$ respectively forming a quadrilateral ABCD.

To prove that ABCD is a rectangle.

We have,

$\angle PAC = \angle SCA$

[Alternate angles, $\because$ PQ $\parallel$ RS]

$\Rightarrow \quad \frac{1}{2}\angle PAC = \frac{1}{2}\angle SCA$

$\Rightarrow \quad \angle DAC = \angle BCA$

$\therefore \quad AD \parallel BC$ ...(1)

Also, $\angle QAC = \angle RCA$

[Alternate angles, $\because$ PQ $\parallel$ RS]

$\Rightarrow \quad \frac{1}{2}\angle QAC = \frac{1}{2}\angle RCA$

$\Rightarrow \quad \angle BAC = \angle DCA$

$\therefore \quad AB \parallel DC$ ...(2)

$\therefore$ From (1) and (2), we see that ABCD is a parallelogram.

Again, $\angle PAC + \angle RCA = 180°$

[Sum of interior angles between two parallel lines]

$\Rightarrow \quad \frac{1}{2}\angle PAC + \frac{1}{2}\angle RCA = \frac{180°}{2} = 90°$

$\Rightarrow \quad \angle DAC + \angle DCA = 90°$

$\therefore$ In ΔADC,

$\angle ADC = 180° - (\angle DAC + \angle DCA)$

$= 180° - 90°$

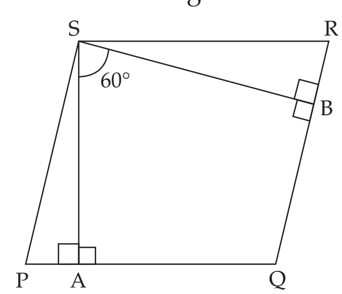
$= 90°$

$\therefore$ The parallelogram ABCD is a rectangle.

Hence, proved.

14. The angle between two altitudes of a parallelogram through the vertex of an obtuse angle of the parallelogram is 60°. Find the angles of the parallelogram.

Sol. $\angle S$ and $\angle Q$ are two obtuse angles. SA and SB are two altitudes of the parallelogram PQRS from the vertex of an obtuse angle.



$\angle ASB = 60°$ [Given]

ASBQ is a quadrilateral.

$\therefore \quad \angle SAQ + \angle ASB + \angle SBQ + \angle Q = 360°$

$\Rightarrow \quad 90° + 60° + 90° + \angle Q = 360°$

[SA and SB are altitudes]

$\Rightarrow \quad \angle Q = 360° - 240°$

$\Rightarrow \quad \angle Q = 120°$ ...(1)

$\angle S = \angle Q$ [Opposite angles are equal in a $\parallel$gm]

$\Rightarrow \qquad \angle S = 120°$ ...(2)

Again, $\angle P + \angle Q = 180°$

[Sum of coint. angles is 180°]

$\Rightarrow \qquad \angle P = 180° - 120°$

$\Rightarrow \qquad \angle P = 60°$ ...(3)

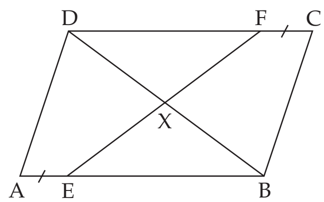
$\angle P = \angle R$

[Opp. angles are equal in ∥ gm]

$\Rightarrow \qquad \angle R = 60°$

Therefore, the angles of the parallelogram are $\angle P = 60°$, $\angle Q = 120°$, $\angle R = 60°$, $\angle S = 120°$.

15. In the given figure, ABCD is a parallelogram. E and F are two points on sides AB and CD such that AE = CF. Prove that BD and EF bisect each other.



Sol. AB = CD

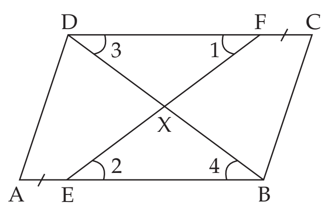
[Opp. sides of a ∥gm are equal]

AB = AE + BE

CD = DF + FC

AE = CF [Given]

$\Rightarrow$ BE = DF ...(1)



In ΔEXB and ΔFXD

$\angle 1 = \angle 2$ [Alt. angles, DF ∥ EB]

$\angle 3 = \angle 4$ [Alt. angles, DF ∥ EB]

DF = BE [From (1)]

$\therefore$ By ASA congruence criterion,

ΔEXB ≅ ΔFXD

$\therefore$ EX = XF [By CPCT]

DX = XB [By CPCT]

Hence, BD and EF bisect each other.

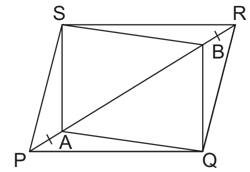
Long Answer Type Questions

16. In parallelogram PQRS, two points A and B are taken on the diagonal PR such that RB = PA as shown in the given figure. Show that

(*a*) ΔSBR ≅ ΔQAP (*b*) SB = QA

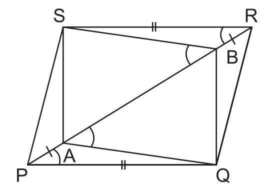
(*c*) ΔSAP ≅ ΔQBR (*d*) SA = QB

(*e*) SBQA is a parallelogram



Sol. Given that PQRS is a parallelogram and PR is a diagonal. A and B are two points on PR such that PA = RB.

To prove that (*a*) ΔSBR ≅ ΔQAP (*b*) SB = QA (*c*) ΔSAP ≅ ΔQBR (*d*) SA = QB (*e*) SBQA is a parallelogram.



(*a*) In ΔSBR and ΔQAP, we have

RB = PA [Given]

SR = QP

[∵ PQRS is a parallelogram]

$\angle SRB$ = alternate $\angle QPA$.

$\therefore$ By SAS congruence criterion, we have

ΔSBR ≅ ΔQAP

(*b*) $\therefore$ SB = QA [By CPCT]

(*c*) In ΔSAP and ΔQBR, we have

PA = RB [Given]

SP = QR

[∵ PQRS is a parallelogram]

$\angle SPA$ = alternate $\angle QRB$

$\therefore$ By SAS congruence criterion,

ΔSAP ≅ ΔQBR

(*d*) $\therefore$ SA = QB [By CPCT]

(*e*) ∵ ΔSBR ≅ ΔQAP,

$\therefore \qquad \angle SBR = \angle QAP$

$\Rightarrow \quad 180° - \angle SBA = 180° - \angle QAB$

$\Rightarrow \qquad \angle SBA = \angle QAB$

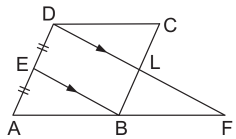
But these are alternate angles.

$\therefore$ SB ∥ AQ

Similarly, from the congruency of ΔSAP and ΔQBR we can show that AS ∥ QB

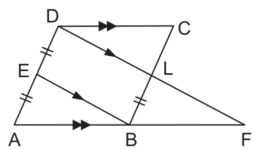
$\therefore$ The figure SBQA is a parallelogram.

17. In the given figure, ABCD is a parallelogram and E is mid-point of AD. DL || BE meets AB produced at F. Prove that B is the mid-point of AF and EB = FL. **[CBSE SP 2012]**



Sol. Given that ABCD is a parallelogram and E is the mid-point of AD. L is a point on BC such that DL ∥ BE. DL and AB produced meet each other at the point F.

To prove that B is the mid-point of AF and EB = FL.



$\because$ EBLD is a parallelogram,

$\therefore$ ED = BL

Also, since E is the mid-point of AD,

$\therefore$ AE = ED

$\therefore$ AE = BL

Now, in ΔABE and ΔBFL, we have

AE = BL

∠EAB = corresponding ∠LBF

[$\because$ AD ∥ BC]

∠EBA = ∠LFB [$\because$ EB ∥ DF]

$\therefore$ By AAS congruence criterion,

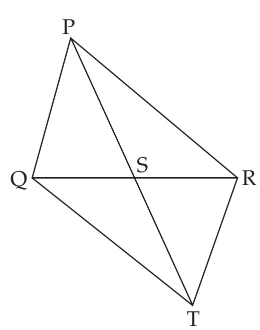
ΔABE ≅ ΔBFL

$\therefore$ EB = LF = FL

and AB = BF [By CPCT]

$\therefore$ B is the mid-point of AF.

18. The median PS of a ΔPQR is produced to T so that PS = ST. Prove that the quadrilateral PQTR is a parallelogram.

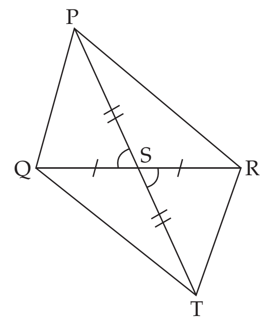


Sol. In ΔPSQ and ΔTSR,

PS = TS [Given]

∠PSQ = ∠TSR [Vert. opp. ∠s]

QS = RS [$\because$ PS is the median]



$\therefore$ By SAS congruence criterion,

ΔPSQ ≅ ΔTSR

$\therefore$ PQ = RT [By CPCT] ...(1)

and ∠PQS = ∠TRS [By CPCT]

$\Rightarrow$ ∠PQR = ∠TRQ [Same angles]

But ∠PQR and ∠TRQ are alternate interior angles formed when transversal QR intersects PQ and RT at Q and R respectively.

$\therefore$ PQ ∥ RT ...(2)

Thus, in quadrilateral PQTR,

PQ = RT and PQ ∥ RT [From (1) and (2)]

$\Rightarrow$ PQTR is a parallelogram.

Check Your Progress 2

(Page 111)

Multiple-Choice Questions

1. In ΔPQR, A and B are mid-points of sides PQ and PR respectively. X is any point on side QR. X is joined to P. If C and D are mid-points of QX and XR respectively, then ABDC is a

(*a*) square. (*b*) rectangle.

(*c*) rhombus. (*d*) parallelogram.

Sol. (*d*) parallelogram

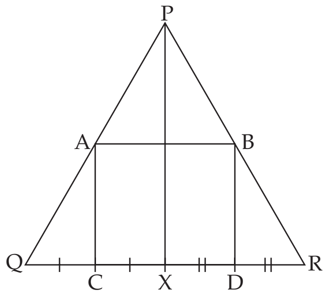
We have, QR = QX + XR

= 2CX + 2XD

[XC = CQ and XD = DR]

= 2[CX + XD]

QR = 2CD ...(1)



In ΔPQR, by Mid-point Theorem,

$$AB \parallel CD$$

and $AB = \frac{1}{2} QR$

or, $AB = CD$ [From (1)] ...(2)

In ΔPQX,

A is the mid-point of PQ and C is the mid-point of QX

$\Rightarrow$ $AC \parallel PX$ and $AC = \frac{1}{2} PX$

[By Mid-point Theorem] ...(3)

In ΔPXR,

B is the mid-point of PR and D is the mid-point of XR.

$\Rightarrow$ $BD \parallel PX$ and $BD = \frac{1}{2} PX$ ...(4)

From (3) and (4),

$AC \parallel BD$ and $AC = BD$ ...(5)

$\therefore$ From (2) and (5), ABDC is a parallelogram.

2. D and E are the mid-points of the sides AB and AC respectively of ΔABC. DE is produce to F. To prove that CF is equal and parallel to DA, we need an additional information which

(*a*) $\angle ADE = \angle ECF$ (*b*) $\angle EFC = \angle DAE$

(*c*) $AE = EF$ (*d*) $DE = EF$

Sol. (*d*) DE = EF

3. D, E and F are the mid-points of the sides BC, CA and AB of ΔABC. If $\angle A = 40°$, $\angle B = 80°$ and $\angle C = 60°$, find $\angle D$, $\angle E$ and $\angle F$ of ΔDEF.

(*a*) 60°, 80°, 40° (*b*) 40°, 80°, 60°

(*c*) 80°, 60°, 40° (*d*) 40°, 60°, 80°

Sol. (*b*) 40°, 80°, 60°

In ΔABC and ΔDCE,

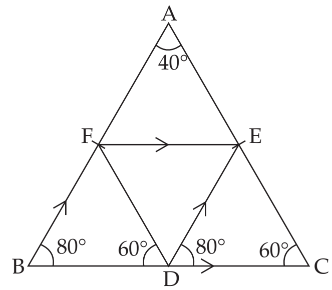
E is the mid-point of AC and D is the mid-point of BC.

By mid-point theorem,

$DE \parallel AB$ and $DE = \frac{1}{2} AB$

$\angle B = \angle EDC$ [Corresponding angles]

$\therefore$ $\angle EDC = 80°$ ...(1)



Similarly, F is the mid-point of AB and D is the mid-point of BC.

By mid-point theorem,

$FD \parallel AC$ and $FD = \frac{1}{2} AC$

$\therefore$ $\angle C = \angle FDB = 60°$ [Corr. $\angle$s] ...(2)

$\angle FDB + \angle FDE + \angle EDC = 180°$

$\Rightarrow$ $\angle FDE = 180° - 60° - 80°$

$\Rightarrow$ $\angle FDE = 40°$

or, $\angle D = 40°$ ...(3)

$\angle FED = \angle EDC = 80°$ [Alt. angles] ...(4)

$\angle EFD = \angle FDB = 60°$ [Alt. angles] ...(5)

From (3), (4) and (5)

$\angle D = 40°$, $\angle E = 80°$, $\angle F = 60°$.

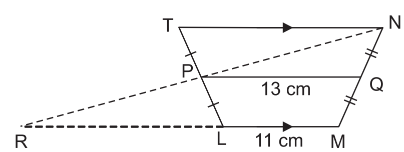
4. LMNT is a trapezium in which LM $\parallel$ TN. P and Q are the mid-points of TL and NM respectively. If LM = 11 cm, PQ = 13 cm, then the length of TN is

(*a*) 18 cm (*b*) 17 cm

(*c*) 16 cm (*d*) 15 cm

Sol. (*d*) 15 cm

Given that P and Q are respectively the mid-points of non parallel sides TL and MN of a trapezium LMNT, where LM $\parallel$ TN, LM = 11 cm and PQ = 13 cm.



To prove that PQ $\parallel$ TN or LM and $PQ = \frac{1}{2}(LM + TN)$

and hence to find the length of TN.

Construction: We join NP and produce it to cut ML produced at the point R.

In ΔTPN and ΔLPR, we have

$$TP = LP$$

$$\angle TNP = \text{alternate } \angle LRP \quad [\because RM \parallel TN]$$

$$\angle TPN = \angle LPR \quad \text{[Vertically opposite angles]}$$

$\therefore$ By AAS congruence criterion, we have

$$\Delta TPN \cong \Delta LPR$$

$\Rightarrow$ $NP = RP$ [By CPCT]

and $TN = LR$ ...(1)

i.e., P is the mid-point of RN.

Now, in ΔRNM, P and Q are the mid-points of RN and MN respectively.

$\therefore$ By the mid-point theorem of a triangle,

$PQ \parallel RM$ or TN

and
$$PQ = \frac{1}{2} \times RM$$
$$= \frac{1}{2}(RL + LM)$$
$$= \frac{1}{2}(TN + LM) \quad \text{[From (1)]}$$
$$\Rightarrow \quad 13 = \frac{1}{2}(TN + 11)$$
$$\Rightarrow \quad 26 = TN + 11$$
$$\Rightarrow \quad TN = 26 - 11$$
$$= 15$$

Hence, the required length of TN is 15 cm.

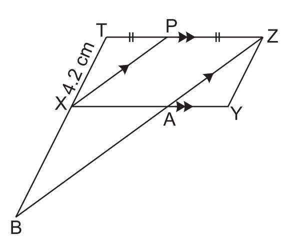
5. P is the mid-point of the side TZ of a parallelogram XYZT. A line through Z parallel to PX intersects XY at A and TX produced at B. If TX = 4.2 cm, then the length of TB is

(*a*) 4.2 cm (*b*) 8.4 cm

(*c*) 12.6 cm (*d*) 6.3 cm

Sol. (*b*) 8.4 cm

Given that P is the mid-point of the side TZ of the parallelogram XYZT. A is a point on XY such that $ZA \parallel PX$. ZA produced intersects TX produced at the point B.



To find the measure of TB if TX = 4.2 cm.

In ΔTBZ, P is the mid-point of TZ and $PX \parallel ZB$.

$\therefore$ By the converse of mid-point theorem for a triangle, X is the mid-point of TB.

$$\therefore \quad TB = TX + XB$$
$$= TX + TX$$
$$= 2TX$$
$$= 2 \times 4.2$$
$$= 8.4$$

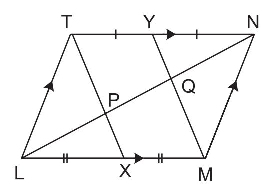
$\therefore$ The required length of TB is 8.4 cm.

Very Short Answer Type Questions

6. LMNT is a parallelogram. X and Y are respectively the mid-points of the sides LM and TN. TX and MY meet the diagonal LN at P and Q respectively. If the length of the diagonal LN of the parallelogram is 15 cm, what is the length of PQ?

Sol. Given that LMNT is a parallelogram, X and Y are the mid-points of the sides LM and TN respectively. LN is a diagonal of the parallelogram. TX and YM cut LN at the points P and Q respectively. It is also given that LN = 15 cm.

To find the length of PQ.



We see that $XM = \frac{1}{2} LM = \frac{1}{2} TN$

Also, $XM \parallel TY$.

$\therefore$ The figure TXMY is a parallelogram.

$$\therefore \quad TX \parallel MY$$
$$\Rightarrow \quad TP \parallel QY$$

Now, in ΔTNP, Y is the mid-point of TN and $TP \parallel QY$.

$\therefore$ Q is the mid-point of PN

[By mid-point theorem for a triangle]

$$\therefore \quad PQ = \frac{1}{2} PN \quad \text{...(1)}$$

Similarly, in ΔLMQ, $\because$ X is the mid-point of LM and $PX \parallel QM$,

$$\therefore \quad PQ = \frac{1}{2} LQ \quad \text{...(2)}$$

Adding (1) and (2), we get

$$2PQ = \frac{1}{2}(PN + LQ)$$

$$= \frac{1}{2}(PQ + QN + PQ + LP)$$

$$= \frac{1}{2}(PQ + QN + LP) + \frac{1}{2}PQ$$

$$= \frac{1}{2}LN + \frac{1}{2}PQ$$

$$\Rightarrow \quad \left(2 - \frac{1}{2}\right)PQ = \frac{1}{2}LN$$

$$\Rightarrow \quad 3PQ = LN$$

$$\Rightarrow \quad PQ = \frac{LN}{3}$$

$$= \frac{15}{3}$$

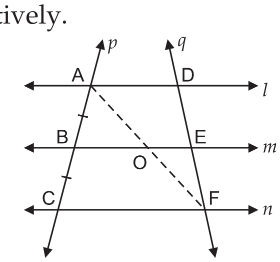
$$= 5$$

Hence, the required length of PQ is 5 cm.

7. If there are three or more parallel lines and the intercepts made by them on one transversal are equal, then prove that the intercepts made on any other transversal are also equal.

[CBSE SP 2011, 2012]

Sol. Given that l, m, n are three parallel lines cutting the two transversals p and q at A, B, C and D, E, F respectively.



It is given that intercept AB = intercept BC.

To prove that intercept DE = intercept EF.

Construction: We join AF to cut the line m or BE at O.

In ΔACF, B is the mid-point of AC and BO ∥ CF.

∴ By converse of mid-point theorem for a triangle, O is the mid-point of AF.

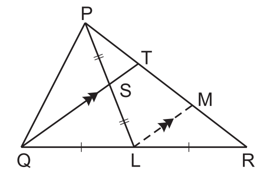
Now, in ΔAFD, O is the mid point of AF and OE ∥ AD.

∴ E is the mid-point of DF, i.e. DE = EF.

Hence, proved.

8. In ΔPQR, PL is the median through P and S is the mid-point of PL. QS produced meets PR at T. Prove that $PT = \frac{1}{3}PR$.

Sol. Given that PL is a median of a triangle PQR and S is the mid-point of PL. QS produced meets PR at T.



To prove that $PT = \frac{1}{3}PR$

Construction: We draw LM ∥ QT where M is a point on PR.

In ΔTQR, L is the mid-point of QR and LM ∥ QT.

∴ By converse of mid-point theorem for a triangle, M is the mid-point of TR.

i.e. $\quad TM = MR \quad$ ...(1)

Again, in ΔPLM, S is the mid-point of PL and ST ∥ LM.

∴ By converse of mid-point theorem for a triangle, T is the mid-point of PM.

i.e. $\quad TM = PT \quad$ ...(2)

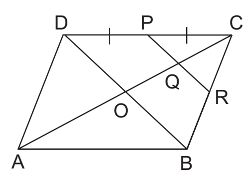
∴ From (1) and (2), we have

$$PT = TM = MR$$

i.e. $\quad PT = \frac{1}{3}PR$

Hence, proved.

9. In the given figure, ABCD is a parallelogram. P is the mid-point of the side DC and Q is a point on AC such that $CQ = \frac{1}{4}AC$. If PQ produced meets BC at R, prove that R is the mid-point of BC.



Sol. Given that ABCD is a parallelogram, P is the mid-point of DC and Q is a point on the diagonal AC such that $CQ = \frac{1}{4}AC$. BD is another diagonal of the parallelogram and let the two diagonals meet each other at the point O.

Let PQ produced meet BC at R.

To prove that R is the mid-point of BC.

Since diagonals of a parallelogram bisect each other,

∴ $\quad AC = 2OC$

$\therefore \quad CQ = \frac{1}{4} AC$

$$= \frac{1}{4} \times 2\, OC$$

$$= \frac{1}{2} OC,$$

i.e. Q is the mid-point of CO.

$\therefore$ By mid-point theorem for a triangle, PQ $\parallel$ DO

$\Rightarrow$ PR $\parallel$ DB

Now, in ΔCOB, QR $\parallel$ OB and Q is the mid-point of CO.

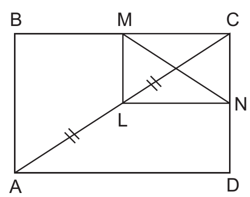
$\therefore$ By converse of mid-point theorem of a triangle, R is the mid-point of BC.

Hence, proved.

Short Answer Type Questions

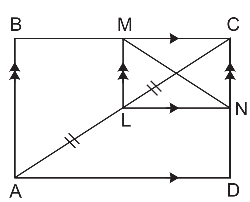
10. In the given figure, ABCD is a rectangle and L is the mid-point of the diagonal AC. LM and LN are respectively parallel to AB and AD. Prove that N is the mid-point of DC. Also prove that

$$MN = \frac{1}{2} AC.$$



Sol. Given that ABCD is a rectangle and L is the mid-point of the diagonal AC. LM and LN are respectively parallel to AB and AD, where M and N are points on BC and CD respectively.

To prove that N is the mid-point of DC and $MN = \frac{1}{2} AC$.



In ΔADC, since L is the mid-point of AC and LN $\parallel$ AD, hence, by converse of mid-point theorem, N is also the mid-point of DC.

Again, in ΔABC,

$\because$ L is the mid-point of AC and LM $\parallel$ AB, hence, by converse of mid-point theorem for a triangle, M is also the mid-point of BC.

Now, since ML $\parallel$ CN and LN $\parallel$ MC,

$\therefore$ MLNC is a rectangle with diagonals LC = MN.

Since L is the mid-point of AC,

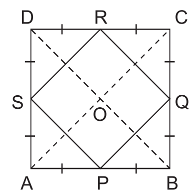
$\therefore \quad MN = LC = \frac{1}{2} AC$

Hence, proved.

11. Quadrilateral ABCD is a rhombus and P, Q, R and S are the mid-points of AB, BC, CD and DA respectively. Prove that PQRS is a rectangle.

Sol. Given that ABCD is a rhombus and P, Q, R, S are the mid-points of AB, BC, CD and DA respectively.

To prove that the quadrilateral PQRS is a rectangle.



Construction: We join the diagonals AC and BD of the rhombus ABCD. Let them intersect each other at the point O.

By mid-point theorem of a triangle, we have

In ΔADC or ΔABC, RS or QP $\parallel$ AC and $RS = \frac{1}{2} AC = QP$

Similarly, in ΔADB or ΔDCB, SP or RQ $\parallel$ DB and $SP = \frac{1}{2} DB = RQ$

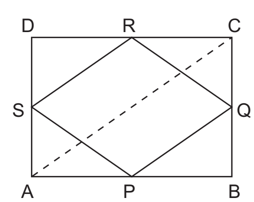
Now, we know that in a rhombus two diagonals are at right angles to each other and unequal in length. Hence, in rhombus ABCD, AC $\perp$ DB and AC $\neq$ DB.

$\therefore$ In the quadrilateral PQRS, RS $\perp$ RQ, RS $\neq$ RQ, but RQ $\parallel$ SP and RQ = SP.

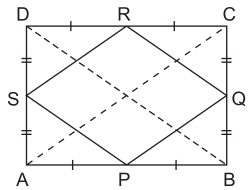
$\therefore$ PQRS is a parallelogram and this parallelogram is a rectangle, since RS $\perp$ RQ and RS $\neq$ RQ.

Long Answer Type Questions

12. In the given figure, ABCD is a rectangle and P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively. Prove that quadrilateral PQRS is a rhombus.



Sol. Given that ABCD is a rectangle with AC and DB as its two diagonals. P, Q, R, S are the mid-points of the sides AB, BC, CD and DA respectively. To prove that the quadrilateral PQRS is a rhombus.



In ΔADC, since S and R are the mid-points of AD and DC respectively, hence, by mid-point theorem for a triangle, SR $\parallel$ AC

and $\quad SR = \frac{1}{2} AC \quad \ldots(1)$

Similarly, from ΔADB,

$$SP = \frac{1}{2} DB = \frac{1}{2} AC \quad \ldots(2)$$

[$\because$ Two diagonals AC and DB of a rectangle are equal]

$\therefore$ From (1) and (2),

$$SP = SR$$

Also, $\quad SP \parallel RQ \quad [\because$ SP or RQ $\parallel$ DB]

and $\quad SR \parallel PQ \quad [\because$ SR or PQ $\parallel$ AC]

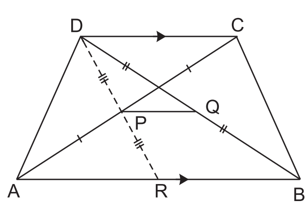
$\therefore$ The quadrilateral PQRS is a parallelogram with adjacent sides SP and RS equal. Hence, PQRS is either a rhombus or a square.

But since angle between two diagonals AC and BD of a rectangle is not 90°, hence, angle between SR ($\parallel$ AC) and SP ($\parallel$ DB) is not 90°. Hence, the parallelogram PQRS must be a rhombus.

13. Prove that the line segment joining the mid-points of the diagonals of a trapezium is parallel to each of the parallel sides and is equal to half the difference of these sides.

Sol. Given that AD and BC are two non parallel sides of a trapezium ABCD where AB $\parallel$ DC. AC and BD are its two diagonals. P and Q are the mid-points of AC and BD respectively.

To prove that PQ $\parallel$ AB or DC and $PQ = \frac{1}{2}(AB - DC)$.



Construction: We join DP and produce it to cut AB at R.

In ΔAPR and ΔCPD, we have

$$AP = CP$$

[$\because$ P is the mid-point of AC]

$$\angle PAR = \text{alternate } \angle PCD$$

[$\because$ AB $\parallel$ DC and AC is a transversal]

$$\angle APR = \angle CPD$$

[Vertically opposite angles]

$\therefore$ By ASA congruence criterion, we have

$$\Delta APR \cong \Delta CPD \quad \ldots(1)$$

$\therefore \quad AR = CD \quad$ [By CPCT]

and $\quad RP = DP \quad$ [By CPCT]

Now, in ΔDRB, P and Q are the mid-points of DR and DB respectively. Hence, by mid-point theorem for a triangle, PQ $\parallel$ AB or DC.

Also, $\quad PQ = \frac{1}{2} RB$

$$= \frac{1}{2}(AB - AR)$$

$$= \frac{1}{2}(AB - CD) \quad \text{[From (1)]}$$

Hence, proved.

Higher Order Thinking Skills (HOTS) Questions

(Page 113)

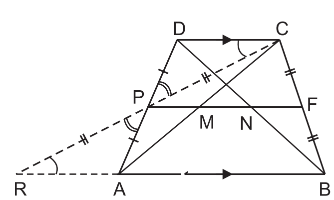
1. Prove that the line segment joining the mid-points of two non parallel sides of a trapezium

(*a*) is parallel to each of the parallel sides of the trapezium,

(*b*) is equal to half of their sum and

(*c*) bisects the two diagonals of the trapezium.

Sol. Given that ABCD is a trapezium, AD and BC are its non parallel sides and AB $\parallel$ DC. P and Q are the mid-points of the sides AD and BC respectively. The line segment PQ intersects the two diagonals AC and BD of the trapezium at the points M and N respectively.



To prove that

(a) PQ $\parallel$ AB and DC

(b) $PQ = \frac{1}{2}(AB + DC)$

(c) M and N are the mid-points of AC and BD respectively.

Construction: We join CP and produce it to cut BA produced at R.

(a) In ΔDPC and ΔAPR, we have

$$PD = PA \quad \text{[Given]}$$

$$\angle DCP = \text{alternate } \angle ARP$$

[$\because$ AB $\parallel$ DC and RC is a transversal]

$$\angle DPC = \angle APR$$

[Vertically opposite angles]

$\therefore$ By AAS congruence criterion,

$$\Delta DPC \cong \Delta APR$$

$\therefore$ CP = RP [By CPCT] …(1)

and DC = AR [By CPCT] …(2)

Now, in ΔCRB, PQ $\parallel$ AB or DC by mid-point theorem for a triangle, since P and Q are the mid-points of CR and CB.

(b) Since, P and Q are the mid-points of the sides CR and CB of ΔCRB, hence by mid-point theorem for a triangle,

$$PQ = \frac{1}{2} RB$$

$$= \frac{1}{2}(AR + AB)$$

$$= \frac{1}{2}(DC + AB)$$

[From (2)]

(c) In ΔADC, P is the mid-point of AD and PM $\parallel$ DC.

$\therefore$ By converse of mid-point theorem for a triangle, M is the mid-point of AC.

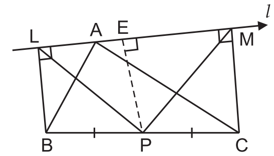
Again, in ΔBDC, Q is the mid-point of BC and QN $\parallel$ DC.

$\therefore$ By converse of mid-point theorem for a triangle, N is the mid-point of BD.

Hence, proved.

2. In a ΔABC, BL and CM are drawn perpendiculars to a line segment through A, from B and C respectively. If P is the mid-point of BC, prove that PL = PM.

Sol. Given that ABC is a triangle and LM is a line segment through the point A. BL and CM are drawn perpendiculars to a line segment through A, from B and C respectively. P is the mid-point of BC.



To prove that PL = PM

Construction: Draw PE $\perp$ LM

Since BL, PE and CM are perpendiculars to the same line segment LM, they are parallel to one another. Now, BC and LM are transversals between these three parallel line segments. Since intercepts BP and PC between these three parallel line segments are equal, hence, intercepts LE and ME on the other transversal LM will also be equal.

Hence, LE = ME …(1)

Now, in ΔLEP and ΔMEP, we have

$$LE = ME \quad \text{[From (1)]}$$

$$\angle LEP = \angle MEP = 90^\circ$$

[By construction]

$$PE = PE$$

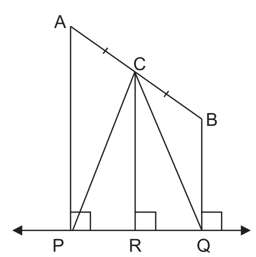
$\therefore$ By SAS congruence criterion,

$$\Delta LEP \cong \Delta MEP$$

$\therefore$ PL = PM [By CPCT]

Hence, proved.

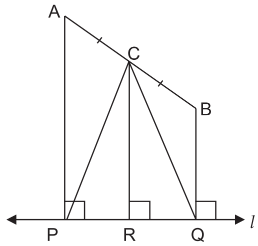
3. In the given figure, AP and BQ are both perpendiculars to the line l. C is the mid-point of line segment AB. Prove that CP = CQ.



Sol. Given that AB is a line segment and C is its mid-point. P, R, Q are three points on a line l such that AP, CR and BQ are perpendicular to the line l. CP and CQ are joined.

To prove that CP = CQ

We see that AP $\parallel$ CR $\parallel$ BQ and PR, RQ are the intercepts on l and AC and CB are intercepts on AB.



$\because$ AC = CB,

$\therefore$ PR = RQ ...(1)

Now, in ΔCRP and ΔCRQ, we have

PR = RQ

CR = CR

$\angle$CRP = $\angle$CRQ

= 90°

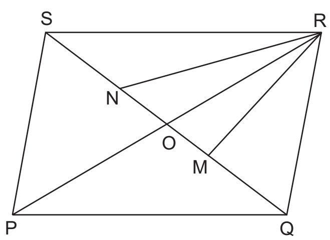
$\therefore$ By SAS congruence criterion, we have

$\Delta CRP \cong \Delta CRQ$

$\therefore$ CP = CQ [By CPCT]

Hence, proved.

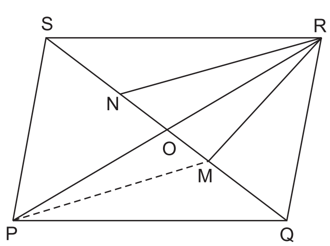
4. In the given figure, PQRS is a parallelogram. The diagonal QS is trisected at M and N. Prove that PM = RN, PM || RN and the line segment MN is bisected by PR.



Sol. Given that PQRS is a parallelogram. The diagonals PR and QS bisect each other at O. N and M are the points of trisection on SQ, so that

$$SN = NM = MQ = \frac{1}{3}SQ \quad ...(1)$$

To prove that PM = RN, PM $\parallel$ RN and the line segment MN is bisected by PR.



Construction: We join PM.

We have $SO = OQ = \frac{1}{2}SQ$ ...(1)

and PO = RO ...(2)

since the diagonals of a parallelogram bisect each other.

$SN = NM = MQ = \frac{1}{3}SQ$

[Given] ...(3)

$\therefore$ $SO = \frac{1}{2}SQ = OQ$

$\therefore$ $ON = OS - SN = \frac{1}{2}SQ - \frac{1}{3}SQ$

[From (1) and (3)]

$= \frac{SQ}{6}$

$OM = OQ - MQ$

$= \frac{1}{2}SQ - \frac{1}{3}SQ$

$= \frac{SQ}{6}$ [From (1) and (3)]

$\therefore$ ON = OM ...(4)

Hence, MN is bisected by PR at O.

Now, in ΔRON and ΔPOM, we have

RO = PO [From (2)]

ON = OM [From (4)]

$\angle$RON = $\angle$POM

[Vertically opposite angles]

$\therefore$ By SAS congruence criterion, we have

$\Delta RON \cong \Delta POM$

$\Rightarrow$ RN = PM [By CPCT]

and $\angle$MPO = alternate $\angle$NRO

$\therefore$ PM $\parallel$ RN

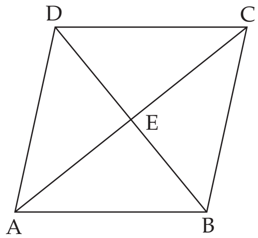
Hence, proved.

Self-Assessment

(Page 113)

Multiple-Choice Questions

1. In the given figure, ABCD is a rhombus whose diagonals intersect at E. If $\angle EAB : \angle EBA = 2 : 3$, find the angles $\angle$EAB, $\angle$EBA and $\angle$AEB of ΔAEB.

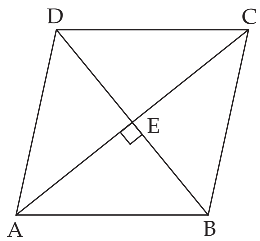


(*a*) 36°, 54° and 90° (*b*) 54°, 36° and 90°
(*c*) 60°, 60° and 60° (*d*) 90°, 36° and 54°

Sol. (*a*) 36°, 54° and 90°

The diagonals of a rhombus are perpendicular to each other.

$\therefore \quad \angle AEB = 90°$



In ΔAEB,

$$\angle EAB + \angle AEB + \angle EBA = 180°$$

$$\Rightarrow \quad \angle EAB + \angle EBA = 90°$$

But $\quad \angle EAB : \angle EBA = 2 : 3$

$$\therefore \quad \angle EAB = \frac{2}{5} \times 90° = 36°$$

$$\angle EBA = \frac{3}{5} \times 90° = 54°$$

2. Find all the angles of a parallelogram, if one angle is 18° more than twice the smaller angle.

(*a*) 54°, 126°, 54°, 126°
(*b*) 60°, 120°, 60°, 120°
(*c*) 40°, 140°, 40°, 140°
(*d*) 80°, 100°, 80°, 100°

Sol. (*a*) 54°, 126°, 54°, 126°

Let the smaller angle be x.

Then the obtuse angle = $18° + 2x$

The adjacent angles of a ∥gm are supplementary,

$$\therefore \quad x + 18° + 2x = 180°$$

$$\Rightarrow \quad 3x = 162°$$

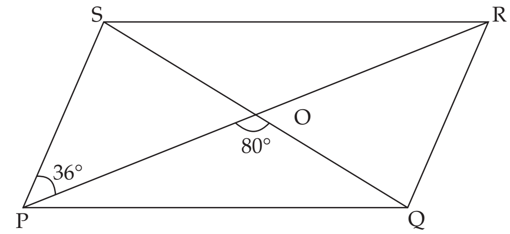
$$\Rightarrow \quad x = 54°$$

Therefore, the angles of the parallelogram are, 54°, 126°, 54°, 126°.

3. The diagonals PR and QS of a parallelogram PQRS intersect at O. If ∠POQ = 80° and ∠RPS = 36°, then ∠RQS is equal to

(*a*) 36° (*b*) 80°
(*c*) 44° (*d*) 100°

Sol. (*c*) 44°



$$\angle POS = 180° - 80° = 100°$$

[Linear pair]

In ΔPOS,

$$\angle SPO + \angle POS + \angle PSO = 180°$$

[Angle sum property]

$$\angle PSO = 180° - 36° - 100° = 44°$$

$\angle PSO = \angle RQS$ [Alt. ∠s, PS ∥ QR]

$$\therefore \quad \angle RQS = 44°$$

4. Which of the following may not be true for any parallelogram?

(*a*) Opposite sides are equal.
(*b*) Opposite angles are equal.
(*c*) Opposite angles are always bisected by the diagonals.
(*d*) Diagonals bisect each other.

Sol. (*c*) Opposite angles are always bisected by the diagonals.

We know that for any parallelogram, opposite sides are always equal, opposite angles are also always equal and the two diagonals always bisect each other. The only property which does not hold good for all parallelograms is that the diagonals bisect opposite angles.

Only in the case of a square or a rhombus, the diagonals bisect the opposite angles, but not in the case of a rectangle.

Fill in the Blanks

5. Three angles of a quadrilateral are 60°, 86°, 110°, then its fourth angle is **104°**.

Sol. $60° + 86° + 110° + x = 360°$ [∠s of a quadrilateral]

$$x = 360° - 256° \quad 104°$$

6. In a parallelogram ABCD, if ∠A = 60°, then ∠D is equal to **120°**.

Sol. $\angle A + \angle D = 180°$ [Coint. ∠s, AB ∥ CD]

$$\Rightarrow \quad 60° + \angle D = 180°$$

$$\Rightarrow \quad \angle D = 180° - 60° = 120°$$

7. If four angles of a quadrilateral are $(2x + 20)°$, $(3x - 30)°$, $(x + 10)°$ and $(2x)°$, then value of x is **45°**.

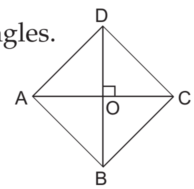
Sol. $(2x + 20)° + (3x - 30)° + (x + 10)° + (2x)° = 360°$

$\Rightarrow \quad 8x = 360$

$\Rightarrow \quad x = 45$

8. If the diagonals of a rhombus are 18 cm and 24 cm respectively, then its side is equal to **15 cm**.

Sol. ABCD is a rhombus.



$\therefore$ Its diagonal bisects at right angles.

$\therefore$ In ΔCOD, OC = 12 cm and OD = 9 cm

$\therefore$ Using Pythagoras' Theorem

$$DC = \sqrt{12^2 + 9^2} = \sqrt{144 + 81} = \sqrt{225} = 15 \text{ cm}$$

Assertion-Reason Type Questions

Directions (Q. Nos. 9 to 14): Each of these questions contains an assertion followed by reason. Read them carefully, and answer the question on the basis of the following options, select the one that best describes the two statements.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

9. **Assertion (A):** A parallelogram has only one pair of parallel sides.

Reason (R): A trapezium has only one pair of parallel sides.

Sol. (*d*)

A parallelogram has two pair of parallel sides whereas a trapezium has only one pair of parallel sides.

$\therefore$ Assertion is false but reason is true.

10. **Assertion (A):** All rectangles are squares.

Reason (R): Squares are rectangles with equal sides.

Sol. (*d*)

All rectangles are not squares whereas squares are rectangles with equal sides.

$\therefore$ Assertion is false but reason is true.

11. **Assertion (A):** Two opposite angles of a parallelogram are $(2x + 8)°$ and $(5x - 82)°$. The measure of one of the angle is 68°.

Reason (R): The adjacent angles of a parallelogram are supplementary.

Sol. (*d*)

$5x - 82 = 68$

$\Rightarrow \quad 5x = 150$

$\Rightarrow \quad x = 30$

Therefore, the other opposite angle

$= 2x + 8$

$= 68°$

Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

12. **Assertion (A):** The diagonals AC and BD of a rhombus ABCD intersect at a point O and are perpendicular to each other.

Reason (R): $\Delta AOD \cong \Delta COD$.

Sol. (*a*) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

13. **Assertion (A):** If an angle of a parallelogram is two-thirds of its adjacent angle, then one of the angle is 108°.

Reason (R): The adjacent angles of a parallelogram are supplementary.

Sol. (*a*)

Let one angle of the ||gm = x

Adjacent angle $= \dfrac{2}{3}x$

Since adjacent angles of a ||gm are supplementary,

$\therefore \quad x + \dfrac{2}{3}x = 180°$

$\Rightarrow \quad \dfrac{5}{3}x = 180°$

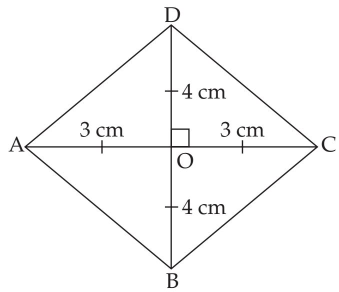
$\Rightarrow \quad x = \dfrac{180° \times 3}{5} = 108°$

Therefore, both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

14. **Assertion (A):** If the diagonals of a rhombus are 6 cm and 8 cm respectively, then its side is equal to 5 cm.

Reason (R): The diagonals of a rhombus bisect each other at right angle.

Sol. (*a*)



In ΔCOD,

$$CD^2 = OC^2 + OD^2$$

$$CD^2 = 9 \text{ cm}^2 + 16 \text{ cm}^2$$

$$CD^2 = 25 \text{ cm}^2$$

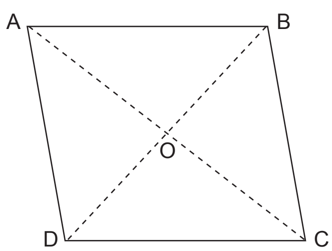
$\therefore$ $CD = 5$ cm

[Diagonals of a rhombus bisect each other at right angles]

$\therefore$ Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

Case Study Based Questions

15. Ankur and Shubham are two friends and live in the same city. To help out the senior citizens, they go to the old age home every Saturday which is located at point O. Ankur's home is located at point A and Shubham's house is located at point B. ABCD is a rhombus in which $\angle ABC = 110°$.



Based on the above situation, answer the following questions.

(*a*) Find the measure of $\angle AOB$.

Ans. 90°

(*b*) Find the measure of $\angle AOD$.

Ans. 90°

(*c*) (*i*) Find the measure of $\angle OAB$.

or

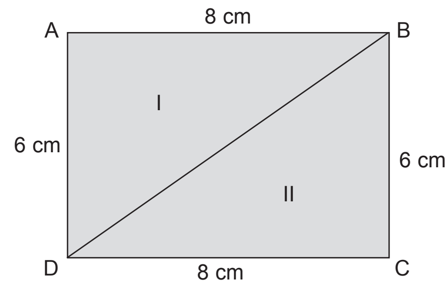
(*ii*) If Ankur and Shubham go along AO and BO respectively to help out the senior citizens living in that old age home, then which of them has to cover shorter distance to reach there?

Ans. (*i*) 35°

or

(*ii*) Shubham

16. Biscuit that is in the form of quadrilateral with sides 8 cm, 6 cm, 8 cm and 6 cm as shown in the given figure. A mother divides it into two parts on one of it's diagonal. She gives part I to her daughter and part II to her son. It is given that one of the angles of this quadrilateral is a right angle. She asks following questions from her children.



(*a*) Find the sum of all the angles of a quadrilateral.

Ans. 360°

(*b*) Which type of quadrilateral is formed in the given figure?

Ans. Rectangle

(*c*) (*i*) What is the length of the diagonal BD?

or

(*ii*) Find the measure of each angle of the rectangle.

Ans. (*i*) 10 cm

or

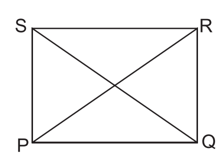
(*ii*) equal to 90°

Very Short Answer Type Questions

17. PQRS is a parallelogram. If the two diagonals PR and QS are of equal length, what will be the measure of each angle of the parallelogram?

Sol. Given that PQRS is a parallelogram and PR, QS are its diagonals such that PR = QS.

To find each angle of the parallelogram.



In ΔPQR and ΔQPS, we have

PR = QS [Given]

QR = PS

[∵ Opposite sides of a parallelogram]

PQ = QP [Common]

∴ By SSS congruence criterion, $\Delta PQR \cong \Delta QPS$.

⇒ $\angle RQP = \angle SPQ$ [By CPCT] …(1)

Also, since PS ∥ QR,

∴ $\angle RQP + \angle SPQ = 180°$

∴ From (1), $\angle RQP = \angle SPQ = 90°$

∴ $\angle PSR = \angle RQP = 90°$

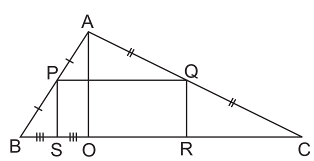
and $\angle SRQ = \angle SPQ = 90°$

∴ Each angle of the parallelogram is 90°.

18. Let P and Q be the mid-point of the sides AB and AC of ΔABC and O be any point on the side BC. O is joined to A. If S and R are the mid-points of OB and OC respectively, then what special name of the quadrilateral PQRS will you give so that the statement is always true?

Sol. Given that P, Q are the mid-points of the sides AB and AC respectively of ΔABC. O is any point on BC. BO and CO are bisected by the points S and R respectively on BC. PQ, QR and PS are joined.

To find the special name of the quadrilateral PQRS.



Since, P and Q are the mid-points of the sides AB and AC of ΔABC, respectively, hence by mid-point theorem for a triangle, PQ ∥ BC

and $PQ = \frac{1}{2} BC$ …(1)

Now, $SR = SO + OR$

$= \frac{1}{2} OB + \frac{1}{2} OC$

[∵ S and R are the mid-points of OB and OC respectively]

$= \frac{1}{2}(OB + OC)$

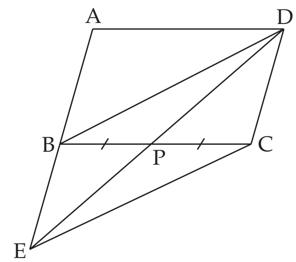
⇒ $SR = \frac{1}{2} BC$ …(2)

∴ From (1) and (2),

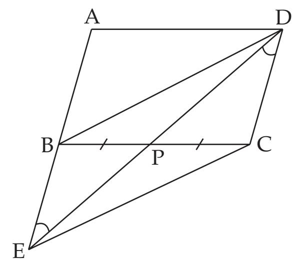
PQ = SR and PQ ∥ SR

Hence, the quadrilateral PQRS is a parallelogram.

19. In the given figure, ABCD is a parallelogram and P is the mid-point of BC. DP and AB meet in E when produced. Prove that BECD is a parallelogram.



Sol.



In ΔBPE and ΔCPD

BP = CP [Given]

$\angle BPE = \angle CPD$ [Vert. opp. ∠s]

$\angle BEP = \angle CDP$ [Alt. ∠s, BE ∥ DC]

∴ By AAS congruence criterion,

$\Delta BPE \cong \Delta CPD$

∴ BE = CD [By CPCT]

and BE ∥ CD [Given]

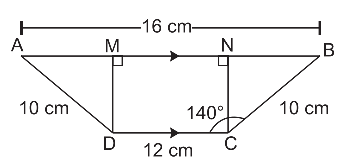
Therefore, BECD is a parallelogram as one pair of opposite sides is equal and parallel.

20. ABCD is a trapezium such that AB = 16 cm, CD = 12 cm, BC = AD = 10 cm, $\angle DCB = 140°$ and AB ∥ DC. Find the measure of ∠BAD.

Sol. Given that ABCD is a trapezium in which AB ∥ DC, $\angle DCB = 140°$, AB = 16 cm, CD = 12 cm, BC = AD = 10 cm.

To find the measure of ∠BAD.

Construction: We draw DM ⊥ AB and CN ⊥ AB where M and N are points on AB.



In ΔAMD and ΔBNC, we have

$\angle AMD = \angle BNC = 90°$

AD = BC = 10 cm

$$DM = CN$$

[$\because$ AB $\parallel$ CD, DM $\perp$ AB and CN $\perp$ AB]

$\therefore$ By RHS congruence criterion, we have

$$\Delta AMD \cong \Delta BNC$$

$\Rightarrow$ $\angle ADM = \angle BCN$ [By CPCT] ...(1)

and $\angle DAM = \angle CBN$ [By CPCT]

Now, $\angle BCN = \angle BCD - \angle NCD$

$= 140° - 90°$

$= 50°$

$\therefore$ From (1), $\angle ADM = \angle BCN = 50°$

$\therefore$ $\angle DAM = 90° - 50°$

$= 40°$

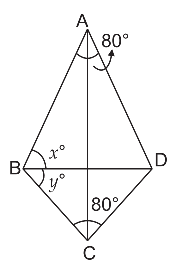
$\Rightarrow$ $\angle BAD = 40°$

which is the required measure of the angle.

21. A diagonal BD of a rhombus ABCD makes angle of measure $x°$ and $y°$ respectively with the sides BA and BC. If $\angle BCD = 80°$, then what is the value of $x° + 2y°$?

Sol. Given that ABCD is a rhombus, BD and AC are its two diagonals and $\angle BCD = 80°$, $\angle ABD = x°$ and $\angle CBD = y°$.

To find the value of $x° + 2y°$.



In ΔABD,

$\because$ $AB = AD$

$\therefore$ $\angle ABD = \angle ADB$

$\therefore$ $\angle ADB = x°$

$\therefore$ From ΔABD,

$\angle ABD + \angle ADB + \angle BAD = 180°$

[Angle sum property of a triangle]

$\Rightarrow$ $x° + x° + 80° = 180°$

$\Rightarrow$ $2x° = 100°$

$\therefore$ $x° = 50°$

Similarly, from ΔBCD, we can show that

$$2y° + 80° = 180°$$

$\therefore$ $y° = 50°$

$\therefore$ $x° + 2y° = 50° + 2 \times 50°$

$= 150°$

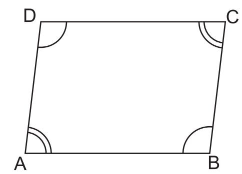
which is the required value.

Short Answer Type Questions

22. Prove that if each pair of opposite angles of a quadrilateral is equal, then it is a parallelogram.

Sol. Given that ABCD is a quadrilateral in which $\angle A = \angle C$ and $\angle B = \angle D$.

To prove that ABCD is a parallelogram.



We have $\angle A = \angle C$ and $\angle B = \angle D$.

$\therefore$ Adding, we get

$\angle A + \angle B = \angle C + \angle D$ ...(1)

Also, $\angle A + \angle B + \angle C + \angle D = 360°$

[Angle sum property of a quadrilateral]

$\Rightarrow$ $2(\angle A + \angle B) = 360°$ [From (1)]

$\Rightarrow$ $\angle A + \angle B = \frac{360Υ}{2} = 180°$ ...(2)

Now, AB intersects AD and BC at A and B respectively such that the sum of the consecutive interior angles $\angle A$ and $\angle B$ is 180°.

$\therefore$ AD $\parallel$ BC ...(3)

Again, from (2), we have

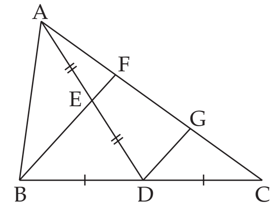
$\angle C + \angle B = 180°$ [$\because \angle A = \angle C$]

Now, BC intersects DC and AB at C and B respectively such that the sum of the consecutive interior angles $\angle B$ and $\angle C$ is 180°.

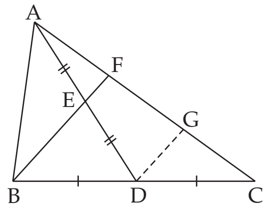
$\therefore$ AB $\parallel$ DC ...(4)

From (3) and (4), we conclude that the opposite sides of a quadrilateral ABCD are parallel. Hence, ABCD is a parallelogram.

23. In the given figure, ABC is a triangle in which D is the mid-point of BC. E is the mid-point of AD. BE when produced meets AC in F. Prove that $AF = \frac{1}{3}AC$.



Sol. Draw DG ∥ BF to cut AC at G.



In ΔBCF,

D is the mid-point of BC. [Given]

DG ∥ BF [By construction]

$\therefore$ G is the mid-point of FC.

$\therefore$ CG = GF ...(1)

In ΔADG,

E is the mid-point of AD. [Given]

EF ∥ DG [By construction]

$\therefore$ F is the mid-point of AG.

$\therefore$ AF = FG ...(2)

From (1) and (2)

AF = FG = GC

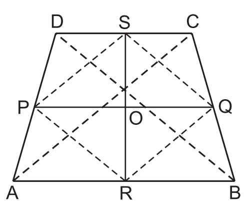
$\therefore$ $AF = \frac{1}{3}(AF + FG + GC) = \frac{1}{3} AC$

Long Answer Type Questions

24. Show that the line segments joining the mid-points of the opposite sides of a quadrilateral bisect each other. **[CBSE SP 2012]**

Sol. Given that ABCD is a quadrilateral and P, Q, R, S are the mid-point of the sides AD, BC, AB and DC respectively. PQ and RS are joined.

To prove that PQ and RS bisect each other at a point O.



Construction: We join PS, SQ, RQ and PR to form a quadrilateral PSQR with diagonals PQ and RS. We also join AC and DB.

We first prove that the quadrilateral PSQR is a parallelogram.

In ΔABD, P and R are the mid-points of AD and AB respectively.

$\therefore$ By mid-point theorem for a triangle, PR ∥ DB

and $PR = \frac{1}{2} DB$ …(1)

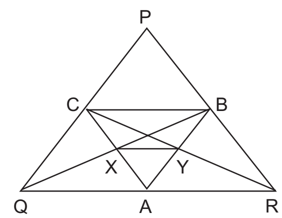
Similarly, in ΔDCB, S and Q are the mid-points of DC and CB respectively.

$\therefore$ By mid-point theorem for a triangle, SQ ∥ DB

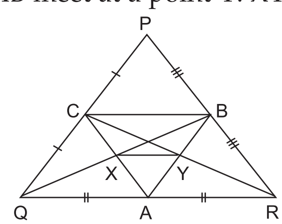
and $SQ = \frac{1}{2} DB$ …(2)

From (1) and (2), we see that PR ∥ SQ and PR = SQ, i.e. one pair of opposite sides of PRQS is equal and parallel and so PRQS is a parallelogram with PQ and SR as two diagonals. Since two diagonals of a parallelogram bisect each other, hence, it follows that PQ and RS bisect each other at a point O.

25. In the given figure, A, B and C are respectively the mid-points of sides QR, RP and PQ of ΔPQR. AC and QB meet at X. RC and AB meet at Y. Prove that $XY = \frac{1}{4} QR$.



Sol. Given that A, B and C are the mid-points of the sides QR, RP and PQ of a triangle PQR, respectively. QB and AC meet at a point X and RC and AB meet at a point Y. XY is joined.



To prove that $XY = \frac{1}{4} QR$

In ΔPQR, C is the mid-point of PQ and B is the mid-point of PR.

$\therefore$ CB ∥ QR and $CB = \frac{1}{2} QR = QA$ …(1)

[By mid-point theorem for a triangle]

From (1), we see that CB ∥ QA and CB = QA.

$\therefore$ The quadrilateral QABC is a parallelogram. Since the diagonals of a parallelogram bisect each other, hence X is the mid-point of the diagonal AC of the parallelogram QABC.

Similarly, Y is the mid-point of the diagonal AB of the parallelogram ARBC.

$\therefore$ In ΔABC, X is the mid-point of AC and Y is the mid-point of AB.

$\therefore \quad XY = \frac{1}{2} BC$

[By mid-point theorem for a triangle]

$= \frac{1}{2} \cdot \frac{1}{2} QR \quad$ [From (1)]

$= \frac{1}{4} QR$

Hence, proved.

Let's Compete

(Page 115)

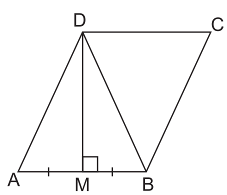
Multiple-Choice Questions

1. ABCD is a rhombus in which the altitude from D to side AB bisects AB. Then the angles of the rhombus are

(*a*) 100°, 80°, 100°, 80°
(*b*) 110°, 70°, 110°, 70°
(*c*) 120°, 60°, 120°, 60°
(*d*) 130°, 50°, 130°, 50°

Sol. (*c*) 120°, 60°, 120°, 60°

Given that ABCD is a rhombus, DM ⊥ AB where M is a point on AB and AM = MB.



To find the angles of the rhombus.

Construction: We join DM.

In ΔMAD and ΔMBD, we have

$MA = MB \quad$ [Given]

$MD = MD \quad$ [Common]

$\angle DMA = \angle DMB = 90° \quad$ [Given]

$\therefore$ By SAS congruence criterion,

$\Delta MAD \cong \Delta MBD$

$\Rightarrow \quad BD = AD \quad$ [By CPCT]

$= AB$

[$\because$ ABCD is a rhombus]

$\therefore$ ΔABD is an equilateral triangle.

$\therefore \quad \angle DAB = 60°$

$\therefore \quad \angle BCD = \angle DAB = 60°$

Also, $\quad \angle ABC = 180° - \angle DAB$

$= 180° - 60°$

$= 120°$

$\therefore \quad \angle ADC = \angle ABC$

$= 120°$

Hence, the required angles of the rhombus are 120°, 60°, 120° and 60°.

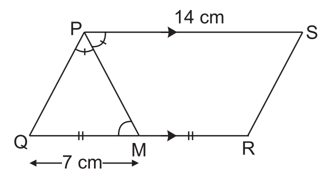
2. PQRS is a parallelogram and M is the mid-point of the side QR and ∠SPM = ∠QPM. If PS = 14 cm, then the length of SR is

(*a*) 7 cm (*b*) 14 cm
(*c*) 6 cm (*d*) 15 cm

Sol. (*a*) 7 cm

Given that PQRS is a parallelogram and M is the mid-point of the side QR. Also, ∠SPM = ∠QPM and PS = 14 cm.

To find the length of SR.



We have

$\angle PMQ =$ alternate $\angle SPM$

[$\because$ PS ∥ QR and PM is a transversal]

$= \angle QPM \quad$ [Given]

$\therefore$ In ΔPQM, $\angle PMQ = \angle QPM$

$\therefore \quad PQ = QM$

$= \frac{1}{2} QR$

$= \frac{1}{2} PS$

$= \frac{1}{2} \times 14$ cm

$= 7$ cm

But $\quad PQ = SR$

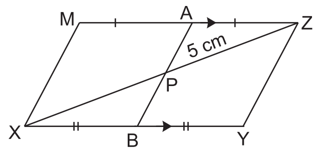
$\therefore \quad SR = 7$ cm

3. XYZM is a parallelogram. A and B are respectively the mid-points of MZ and XY respectively. If the diagonal XZ intersect AB at P and if PZ = 5 cm then the length of XZ is

(*a*) 5 cm (*b*) 10 cm
(*c*) 6 cm (*d*) 12 cm

Sol. (*b*) 10 cm

Given that XYZM is a parallelogram, A and B are the mid-points of MZ and XY respectively. XZ is a diagonal of the parallelogram. The line segment AB intersects XZ at P. Also, PZ = 5 cm.



To find the length of XZ.

In ΔMZX, A and B are the mid-points of MZ and XY respectively.

$\therefore \quad MA = \frac{1}{2} MZ = \frac{1}{2} XY = BX$

$\therefore$ MABX is a parallelogram.

$\therefore \quad AB \parallel MX$

$\Rightarrow \quad AP \parallel MX$

$\therefore$ In ΔMZX, AP $\parallel$ MX and A is the mid-point of the side MZ.

$\therefore$ By the converse of mid-point theorem for a triangle, P is the mid-point of XZ.

$$\therefore \quad XZ = 2PZ$$
$$= 2 \times 5 \text{ cm}$$
$$= 10 \text{ cm}$$

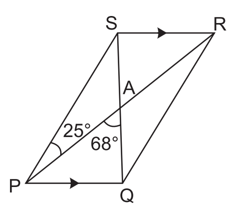
4. Two diagonals PR and QS of a parallelogram PQRS intersect each other at A. If $\angle SPA = 25°$ and $\angle PAQ = 68°$, then $\angle SQR$ is equal to

(*a*) 40° (*b*) 68°
(*c*) 25° (*d*) 43°

Sol. (*d*) 43°

Given that PQRS is a parallelogram and the two diagonals PR and QS intersect each other at a point A. Also, $\angle SPA = 25°$ and $\angle PAQ = 68°$.

To find the measure of $\angle SQR$.



We have

$$\angle SQR = \text{alternate } \angle PSQ \quad \ldots(1)$$
[$\because$ PS $\parallel$ QR and SQ is a transversal]

Now, $\angle PAQ + \angle PAS = 180°$

$\Rightarrow \quad 68° + \angle PAS = 180°$

$\Rightarrow \quad \angle PAS = 180° - 68° = 112° \quad \ldots(2)$

$\therefore$ From ΔPAS,

$\angle PAS + \angle SPA + \angle PSA = 180°$

[Angle sum property of a triangle]

$\Rightarrow \quad 112° + 25° + \angle PSA = 180°$

$\Rightarrow \quad \angle PSA = 180° - 112° - 25°$
$$= 180° - 137°$$
$$= 43°$$

$\Rightarrow \quad \angle PSQ = 43° \quad \ldots(3)$

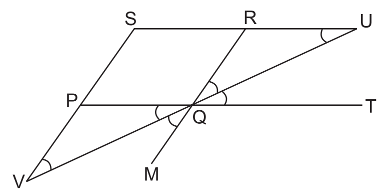
$\therefore$ From (1), $\quad \angle SQR = \angle PSQ = 43°$ [From (3)]

5. In parallelogram PQRS, the side PQ is produced to the point T. If the bisector of $\angle RQT$ meets SR produced and SP produced at U and V respectively and if SV = 30 cm, then the length of SU is

(*a*) 30 cm (*b*) 15 cm
(*c*) 7.5 cm (*d*) 25 cm

Sol. (*a*) 30 cm

Given that PQRS is a parallelogram. The side PQ is produced to a point T. The bisector of $\angle RQT$ meets SR produced and SP produced at the points U and V respectively.



It is given that SV = 30 cm.

To find the length of SU.

Construction: We produce RQ to M.

We have

$$\angle RQU = \angle UQT \quad \text{[Given]}$$
$$= \text{alternate } \angle RUQ$$
[$\because$ PT $\parallel$ SR and UV is a transversal]

$\therefore$ In ΔRQU, we have

$$\angle RQU = \angle RUQ$$

$\therefore \quad RU = RQ = PS \quad \ldots(1)$

$\therefore \quad SU = SR + RU$
$$= SR + RQ$$
$$= SR + SP \quad \ldots(2)$$

Now, $\quad \angle PVQ = \text{alternate } \angle VQM$

[$\because$ SV $\parallel$ RM]

$$= \angle PQV$$

$\therefore$ In ΔPQV,

$$PV = PQ = SR$$

$\therefore \quad SV = SP + PV$
$$= RQ + SR = SU \quad \text{[From (2)]}$$

$\therefore \quad 30 \text{ cm} = SU$

$\therefore$ Required length of SU is 30 cm.

6. The angles of a triangle ABC are 50°, 60° and 70°. Let the triangle formed by joining the mid-points of the sides of ΔABC be called $\Delta A_1B_1C_1$. Then the angles of the triangle formed by joining the mid-points of the sides of $\Delta A_1B_1C_1$ are

(*a*) 70°, 70° and 40°

(*b*) 60°, 40° and 80°

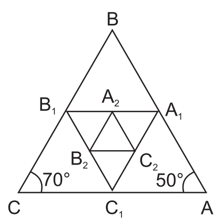
(*c*) 50°, 60° and 70°

(*d*) 40°, 90° and 50°

Sol. (*c*) 50°, 60° and 70°

Given that in ΔABC, A_1, B_1, and C_1, are the mid-points of the sides AB, BC and CA respectively. Also, A_2, B_2 and C_2 are the mid-points of the sides A_1B_1, B_1C_1 and C_1A_1 respectively.

To find the angles of $\Delta\ A_2B_2C_2$ if $\angle A = 50°$, $\angle B = 60°$ and $\angle C = 70°$.



Since A_1, B_1, C_1 are the mid-points of the sides AB, BC and CA of ΔABC,

$\therefore$ By mid-point theorem, we have

$A_1B_1 \parallel AC$, $B_1C_1 \parallel AB$ and $C_1A_1 \parallel BC$ ...(1)

Again, since A_2, B_2, C_2 are the mid-points of the sides A_1B_1, B_1C_1, and C_1A_1 of $\Delta A_1B_1C_1$

$\therefore$ By mid-point theorem, we have

$A_2B_2 \parallel A_1C_1$, $B_2C_2 \parallel A_1B_1$ and $C_2A_2 \parallel B_1C_1$ ...(2)

$\therefore$ From (1) and (2), we have

$A_2B_2 \parallel BC$, $B_2C_2 \parallel AC$ and $C_2A_2 \parallel AB$

$\therefore$ Angle between A_2B_2 and A_2C_2 will be the same as that between BC and AB, i.e. 60°, the angle between A_2B_2 and B_2C_2 will be the same as that between BC and AC, i.e. 70° and the angle between B_2C_2 and C_2A_2 will be the same as that between AC and AB, i.e. 50°.

Hence, the angles of $\Delta A_2B_2C_2$ will be 50°, 60° and 70°.

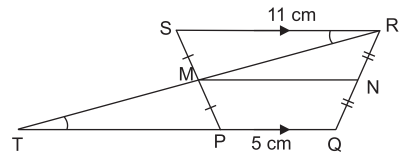
7. PQRS is a trapezium with PQ || SR and M, N are the mid-points of the sides PS and QR respectively. If PQ = 5 cm and SR = 11 cm, then the length of MN is

(*a*) 9 cm (*b*) 10 cm

(*c*) 7 cm (*d*) 8 cm

Sol. (*d*) 8 cm

Given that PQRS is a trapezium with $PQ \parallel SR$; M and N are the mid-points of SP and RQ respectively. PQ = 5 cm and SR = 11 cm. To find the length of MN.



Construction: We join RM and produce it to cut QP produced at T.

In ΔSMR and ΔPMT, we have

$$SM = PM \quad \text{[Given]}$$

$$\angle SRM = \text{alternate } \angle PTM$$

[$\because$ SR $\parallel$ PQ and TR is a transversal]

$$\angle SMR = \angle PMT$$

[Vertically opposite angles]

$\therefore$ By AAS congruence criterion,

$$\Delta SMR \cong \Delta PMT$$

$\Rightarrow$ TM = RM [By CPCT] ...(1)

and SR = PT [By CPCT] ...(2)

Now, in ΔRTQ, M and N are the mid-points of the sides RT and RQ respectively.

$\therefore$ By mid-point theorem for a triangle, MN $\parallel$ TQ

and $MN = \frac{1}{2} TQ$ ...(3)

Now, $TQ = PQ + PT$

$= PQ + SR$ [From (2)]

$= (5 + 11)$ cm

$= 16$ cm

$\therefore$ From (3), $MN = \frac{1}{2} TQ$

$= \frac{1}{2} \times 16$ cm

$= 8$ cm

8. The mid-points A, B, C and D of the sides MQ, QP, PN and NM respectively of a rhombus MNPQ whose diagonals MP and NQ are of lengths 6 cm and 8 cm respectively are the vertices of a parallelogram ABCD whose two diagonals are of length

(*a*) 5 cm and 5 cm

(*b*) 6 cm and 5 cm

(*c*) 6 cm and 6 cm

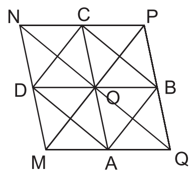
(*d*) 8 cm and 6 cm

Sol. (*a*) 5 cm and 5 cm

Given that MNPQ is a rhombus and A, B, C, D are the mid-points of the sides MQ, QP, PN and NM respectively. MP and NQ are the diagonals of the rhombus, intersecting each other at a point O. AB, BC, CD and DA are joined to form a parallelogram ABCD. The diagonals AC and DB of this parallelogram pass through O.

To find the length of AC and DB.

We shall first prove that the parallelogram ABCD is a rectangle.



Since, C and D are the mid-points of the sides NP and NM respectively of ΔNMP, hence by mid-point theorem for a triangle,

$$DC \parallel MP \text{ and } DC = \frac{1}{2}MP \quad \ldots(1)$$

Similarly, from ΔQMP,

$$AB \parallel MP \text{ and } AB = \frac{1}{2}MP \quad \ldots(2)$$

From (1) and (2), we see that

$$DC = AB \text{ and } DC \parallel AB.$$

$\therefore$ ABCD is a parallelogram.

Now, $AB \parallel MP$ and $BC \parallel NQ$.

$\therefore$ Angle between AB and BC will be the same as that between MP and NQ.

But we know that angle between two diagonals MP and NQ of a rhombus is 90°.

$\therefore \quad AB \perp BC$

$\therefore$ The parallelogram ABCD is a rectangle.

$\therefore$ From (1) and (2),

$$DC = AB$$
$$= \frac{1}{2} \times MP \quad \text{[From 2]}$$
$$= \frac{1}{2} \times 6 \text{ cm}$$
$$= 3 \text{ cm}$$

and $\quad AD = BC$

$$= \frac{1}{2} \times NQ$$
$$= \frac{1}{2} \times 8 \text{ cm}$$
$$= 4 \text{ cm}$$

$\therefore$ By Pythagoras' theorem, we get

$$AC^2 = AD^2 + DC^2$$

[From right-angled ΔADC]

$$= 4^2 + 3^2$$
$$= 25$$

$\therefore \quad AC = 5$

Also, $\quad AC = BD$

[$\because$ For a rectangle, two diagonals are of equal length]

$\therefore \quad BD = 5$

Hence, the required lengths of two diagonals of the rectangle ABCD are 5 cm and 5 cm.

9. The two diagonals of a quadrilateral ABCD are equal and perpendicular to each other. Another quadrilateral PQRS is formed by joining the mid-points of the sides of the former quadrilateral ABCD. Then the diagonals of the quadrilateral PQRS are

(*a*) equal but not perpendicular to each other.

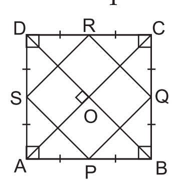
(*b*) equal and perpendicular to each other.

(*c*) neither equal nor perpendicular to each other.

(*d*) not equal but perpendicular to each other.

Sol. (*b*) equal and perpendicular to each other.

Given that ABCD is a quadrilateral such that the two diagonals AC and BD are of equal length and perpendicular to each other. Hence, the quadrilateral ABCD is a square. Let the two diagonals meet together at O. P, Q, R, S are the mid-points of the sides AB, BC, CD and DA of the square ABCD. We join PQ, QR, RS and SP to form a quadrilateral PQRS. We shall first prove that PQRS is another square.



$\because$ P, Q, R, S are the mid-points of the sides AB, BC, CD and DA of the square ABCD, hence by mid-point theorem for a triangle, we get

$SR = \frac{1}{2}AC$ and $SR \parallel AC$ from ΔADC,

$SP \parallel BD$ and $SP = \frac{1}{2}BD$ from ΔADB,

$PQ = \frac{1}{2}AC$ and $PQ \parallel AC$ from ΔABC

and $QR = \frac{1}{2}BD$ and $QR \parallel BD$ from ΔBCD.

Also, angle between SR and SP is equal to that between AC and BD, i.e. 90°.

Hence, the quadrilateral PQRS is such that $PQ = SR = \frac{1}{2} AC$, $SP = QR = \frac{1}{2} BD$.

$\therefore$ $PQ = SP$ [$\because AC = BD$]

and $\angle RSP = \angle DOA = 90°$

$\therefore$ The figure PQRS is also a square. Now, we know that the two diagonals of a square are perpendicular to each other. Also, they are equal to each other.

10. The quadrilateral PQRS formed by joining the mid-points P, Q, R and S of sides AB, BC, CD and DA respectively of another quadrilateral ABCD such that $PR \perp QS$, PR = 8 cm and QS = 6 cm. Then the two diagonals of the original quadrilateral ABCD are always

(*a*) equal and perpendicular to each other.

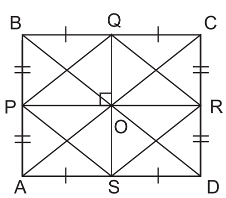
(*b*) equal but not perpendicular to each other.

(*c*) not equal but perpendicular to each other.

(*d*) neither equal nor perpendicular to each other.

Sol. (*b*) equal but not perpendicular to each other.

Given that P, Q, R, S are the mid-points of the sides AB, BC, CD and DA respectively of the original quadrilateral ABCD. We join PQ, QR, RS, SP, PR and QS to form a quadrilateral PQRS with diagonals PR and QS, where PR = 8 cm, QS = 6 cm and $PR \perp QS$. Since $PR \neq QS$ and $PR \perp QS$, hence, the second quadrilateral PQRS must be a rhombus.



We join AC and BD. We shall now determine the special name of the quadrilateral ABCD.

Since P, Q, R, S are the mid-points of AB, BC, CD and DA respectively of the quadrilateral ABCD, hence by mid-point theorem for a triangle, we have

$PQ = \frac{1}{2} AC$ and $PQ \parallel AC$ from ΔABC,

$QR = \frac{1}{2} BD$ and $QR \parallel BD$ from ΔBCD,

$RS = \frac{1}{2} AC$ and $RS \parallel AC$ from ΔADC,

and $PS = \frac{1}{2} BD$ and $PS \parallel BD$ from ΔBAD.

$\therefore$ $AC = 2PQ = 2RS$ and $BD = 2QR = 2PS$, $PQ \parallel RS$ and $QR \parallel PS$.

Now, since PQRS is a rhombus, hence RS = PS and so AC = BD.

$\therefore$ The quadrilateral ABCD is a parallelogram with diagonals AC and BD such that AC = BD.

$\therefore$ ABCD may be either a rectangle or a square.

Now, we see in ΔBCD where $\angle BCD = 90°$, that

$BC^2 = BD^2 - CD^2$

[By Pythagoras' theorem]

$= BD^2 - QS^2$ ...(1)

and in ΔABD, $AB^2 = BD^2 - AD^2$

$= BD^2 - PR^2$ ...(2)

$\because$ PR = 8 cm

and QS = 6 cm

$\therefore$ $PR \neq QS$

$\therefore$ From (1) and (2), we see that $AB \neq BC$.

$\therefore$ ABCD is not a square. So, finally we see that ABCD is a rectangle. Since two diagonals of a rectangle are equal but not perpendicular to each other, hence choice (*b*) is correct.

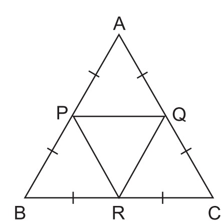
Life Skills

(Page 116)

1. The sports teacher of a school drew an equilateral triangle ABC by putting white powder on the school grassy field. A few students were asked to stand along the sides of this triangle. He suggested all students to draw another equilateral triangle within ΔABC such that the vertices P, Q and R of this triangle lie in the mid-points of sides AB, AC and BC respectively and then join PQ, QR and PR. Was the suggestion of the sports teacher right? Justify your answer.

Sol. Yes, the suggestion of the sports teacher was correct. This is due to the reasons as follows:

Since P, Q, R are the mid-points of AB, AC and BC.



$\therefore$ By mid-point theorem for a triangle,

$PQ = \frac{1}{2} BC$, $QR = \frac{1}{2} AB$ and $PR = \frac{1}{2} AC$.

$\because \quad AB = BC = AC,$

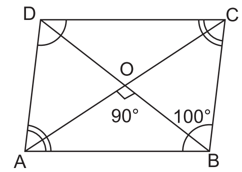
$\therefore \quad PQ = QR = PR$

$\therefore \Delta PQR$ is also an equilateral triangle.

2. ABCD is a rhombus in which $\angle ABC = 100°$. Arvinda's house is at A and Parimal's house is at B. There is an old age home at O, the point of intersection of two diagonals of the rhombus ABCD. Both the men have to come to the old age home at O from their respective houses everyday to help the old people there. They have to walk along AO and BO respectively. Who used to cover the shorter distance to reach to old age home?

Sol. We shall determine whether AO > BO or AO < BO.

It is given that ABCD is a rhombus and its two diagonals AC and BD bisect each other at a point O and $\angle AOB = 90°$, $\angle ABC = 100°$.



$\because \quad \angle DAB + \angle ABC = 180°$

$\therefore \quad \angle DAB = 180° - \angle ABC$

$= 180° - 100° = 80°$

Also, since $\quad \angle OAB = \frac{1}{2} \angle DAB$

$= \frac{1}{2} \times 80° = 40°$

and $\quad \angle OBA = \frac{1}{2} \times \angle ABC$

$= \frac{1}{2} \times 100°$

$= 50°$

$\therefore \quad \angle OBA > \angle OAB$

$\therefore \quad AO > BO$

[By triangular inequality]

Hence, Parimal whose house is at B used to cover the shorter distance to reach the old age home at O.