6

Lines and Angles

Checkpoint (Page 67)

1. State why it is not possible to divide a line into two equal parts.

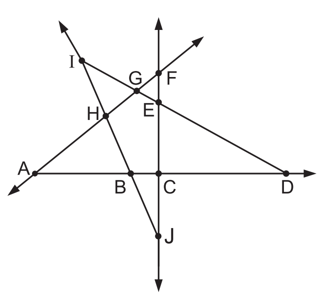
Sol. It is not possible to divide a line into two equal parts, because a line does not have a finite length.

2. In the figure given along side, find

   (*a*) the maximum number of distinct closed regions bounded by line segments such that no part of a region may lie within the other region and the minimum number of such closed regions.

   (*b*) any three rays and two lines.

   (*c*) the total number of distinct points common to each pair of line segments among all the pairs. Name any ten pairs of line segments passing through these points.



Sol. (*a*) The maximum number of distinct closed regions is six, *viz*. IGH, ABH, GEF, CDE, BCJ and BCEGH and the minimum number of such regions is only one, *viz*. ABJCDEFGIH.

(*b*) Ray CD, ray HI and ray GF are any three rays, and AF, FJ are any two lines.

(*c*) The total number of distinct points common to each pair of line segments among all the pairs are A, B, C, D, E, F, G, H, I, J.

Any ten pairs of line segments passing through the above points are:

AH, AB; IH, IG; GI, GH; FE, FG; EF, EG; HG, HI; BC, BJ; JC, JB; CD, CE and DE, DC.

3. Three distinct points are given in a plane. How many lines can be drawn through them?

Sol. If the three distinct points are collinear as shown in Fig. (*i*), then only one line can be drawn through those points. If the three points are not collinear, then three distinct lines can be drawn through them as shown in Fig. (*ii*).

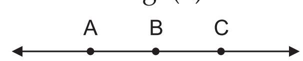


Fig. (*i*)

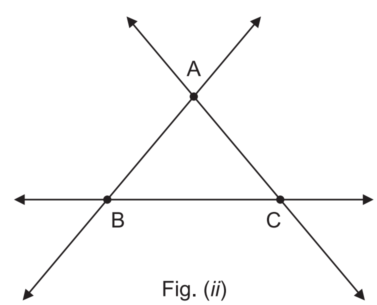


Fig. (*ii*)

4. A line and a point, not on the line, are given. How many planes can be made to pass through the line and the point?

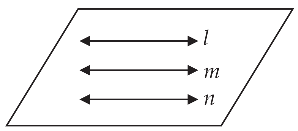
Sol. We know that only one plane can be drawn through a line and a point not lying on the line.

5. What is the least number of distinct points which determines a unique line?

Sol. The least number of distinct points which determines a unique line is 2.

6. Draw three lines in a plane so that you may get the minimum number of points of intersection. What is the number of such points of intersection?

Sol. Three parallel lines l, m and n are drawn on a plane shown in the figure. The lines do not intersect each other at all. Hence, the required minimum number of points of intersection of these three lines on a plane is zero.



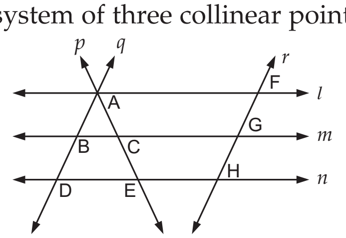
7. In the given figure, write the name(s) of

(*a*) all pairs of parallel lines.

(*b*) all pairs of intersecting lines.

(*c*) the point through which three lines pass and also name these lines.

(*d*) all system of three collinear points.



Sol. (*a*) Pairs of parallel lines are: l, m; l, n; m, n; q, r.

(*b*) Pairs of intersecting lines are: p, q; p, l; p, m; p, n; q, l; q, m; q, n; r, l; r, m; r, n and p, r when extended.

(*c*) The three lines p, q and l passes through a single point A.

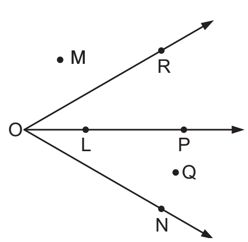
(*d*) All systems of three collinear points are A, B and D; A, C and E; F, G and H; B, C and G; D, E and H.

8. In the given diagram, name the point(s)

(*a*) in the exterior of ∠POR.

(*b*) in the interior of ∠NOP.

(*c*) on ∠POR.



Sol. (*a*) The points M, Q and N lie in the exterior of ∠POR.

(*b*) The only point Q lies in the interior of ∠NOP.

(*c*) The points P, L, O and R lie on ∠POR.

Check Your Progress 1

(Page 72)

Multiple-Choice Questions

1. The supplement of an angle is less than three times its complement by 20°. Then the angle is

(*a*) 50° (*b*) 30°

(*c*) 35° (*d*) 45°

Sol. (*c*) 35°

Let the required angle be of measure x in degrees. Then the supplement and complement of x are $180° - x$ and $90° - x$ respectively.

∴ According to the problem, we have

$$180° - x = 3(90° - x) - 20°$$

$$\Rightarrow \quad 3(90° - x) - (180° - x) = 20°$$

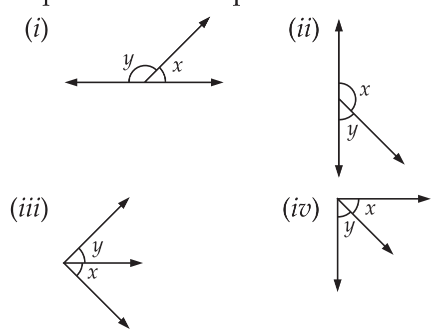
$$\Rightarrow \quad 270° - 3x - 180° + x = 20°$$

$$\Rightarrow \quad 2x = 270° - 180° - 20° = 70°$$

$$\therefore \quad x = \frac{70°}{2} = 35°$$

∴ The angle is 35°.

2. In the given figures, which pairs of angles represent a linear pair?



(*a*) (*i*) and (*iv*) (*b*) (*iii*) and (*iv*)

(*c*) (*i*) and (*iii*) (*d*) (*i*) and (*ii*)

Sol. (*d*) (*i*) and (*ii*)

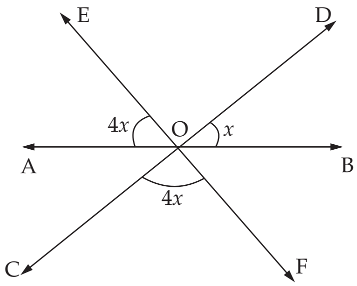
In Fig. (*i*) and (*ii*), we see that $x + y = 180°$, but in Fig. (*iii*) and (*iv*) $x + y < 180°$. Hence, the pair of angles only in Fig. (*i*) and (*ii*) represents a linear pair.

3. Two angles are supplementary. Given that one of the angles is an acute angle. Which of the following angles could be the measure of the other angle.

(*a*) 60° (*b*) 90°
(*c*) 120° (*d*) 180°

Sol. (*c*) 120°

4. In the given figure, determine the value of x.



(*a*) 15° (*b*) 20°
(*c*) 30° (*d*) 45°

Sol. (*b*) 20°

$$\angle COF = \angle DOE$$

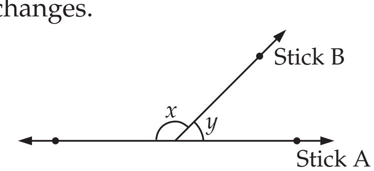
[Vertically opposite angles]

$\therefore$ $4x + 4x + x = 180°$

$\Rightarrow$ $9x = 180°$

$\Rightarrow$ $x = 20°$

5. Savita placed two sticks forming angles x and y as shown in the figure. Savita moves stick B such that the value of y doubles. How does the value of x changes.



(*a*) The value of x doubles
(*b*) The value of x reduces by y
(*c*) The value of x increases by y
(*d*) The value of x becomes $\frac{1}{2}$ times

Sol. (*b*) The value of x reduces by y

Very Short Answer Type Questions

6. Find the measure of an angle which is three times its supplement.

Sol. Let the required measure of the angle be x in degrees.

$\therefore$ According to the problem, we have

$$x = 3(180° - x)$$

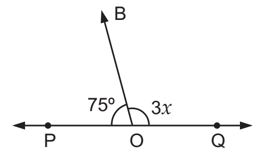
$\Rightarrow$ $3x + x = 540°$

$\Rightarrow$ $4x = 540°$

$\Rightarrow$ $x = \frac{540°}{4} = 135°$

Hence, the required measure of the angle is 135°.

7. In the given figure, POQ is a straight line. Find x.



Sol. From the figure, we see that

$3x + 75° = 180°$ [Linear pair]

$\Rightarrow 3x = 180° - 75° = 105°$

$\Rightarrow$ $x = \frac{105°}{3} = 35°$

Hence, the required value of x is 35°.

8. Two supplementary angles are in the ratio 5 : 4. Find the angles.

Sol. Let the two supplementary angles measure x and $180 - x$ in degrees.

$\therefore$ According to the problem, we have

$$\frac{x}{180 - x} = \frac{5}{4}$$

$\Rightarrow$ $900 - 5x = 4x$

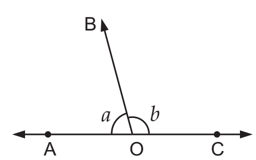
$\Rightarrow$ $5x + 4x = 900$

$\Rightarrow$ $9x = 900$

$\therefore$ $x = \frac{900}{9} = 100$

$\therefore$ The required angles measure 100° and 180° – 100° = 80°.

9. In the given figure, ∠AOB and ∠COB form a linear pair. Find the values of a and b if $2a = b - 30°$.



Sol. According to the problem, we have

$a + b = 180°$ …(1)

and $2a = b - 30°$

$b = 2a + 30°$ …(2)

$\therefore$ From (1) and (2), we have

$a + 2a + 30° = 180°$ [Linear pair]

$\Rightarrow$ $3a = 180° - 30° = 150°$

$\Rightarrow$ $a = 50°$

$\therefore$ From (2), $b = 2 \times 50° + 30° = 130°$

$\therefore$ The required values of a and b are 50° and 130° respectively.

Short Answer Type Questions

10. If the complement of an angle is one-sixth of its supplement, find the angle.

Sol. Let the required angle measures x in degrees.

Then according to the problem, we have

$$90° - x = \frac{1}{6}(180° - x)$$

$$\Rightarrow \quad 540° - 6x = 180° - x$$

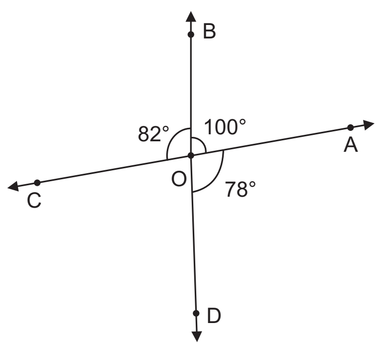
$$\Rightarrow \quad 5x = 540° - 180° = 360°$$

$$\Rightarrow \quad x = \frac{360°}{5} = 72°$$

Hence, the required angle is 72°.

11. Let OA, OB, OC and OD be rays in anticlockwise direction starting from OA such that ∠AOB = ∠COD = 100°, ∠BOC = 82° and ∠AOD = 78°. Is it true that AOC and BOD are straight lines? Justify your answer. **[CBSE SP 2012]**

Sol. From the figure,



We have

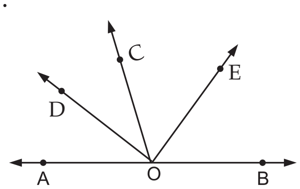
$$\angle AOC = \angle AOB + \angle BOC$$
$$= 100° + 82°$$
$$= 182° \neq 180°$$

and

$$\angle BOD = \angle BOA + \angle AOD$$
$$= 100° + 78°$$
$$= 178° \neq 180°$$

Since, neither ∠AOC nor ∠BOD is 180°, hence, AOC and BOD are not straight lines.

12. In the given figure, OD is the bisector of ∠AOC, OE is the bisector of ∠BOC and OD ⊥ OE. Show that the points A, O and B are collinear.



Sol. $\angle AOD = \angle DOC$ ...(1)

[OD is the bisector of ∠AOC]

$\angle COE = \angle BOE$ ...(2)

[OE is the bisector of ∠BOC]

Adding (1) and (2), we get

$$\angle AOD + \angle BOE = \angle DOC + \angle COE$$

Given, OD ⊥ OE

$\Rightarrow \quad \angle DOC + \angle COE = 90°$ ...(3)

$\Rightarrow \quad \angle AOD + \angle BOE = 90°$ ...(4)

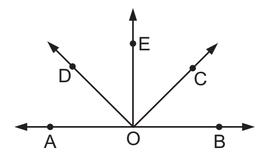
$\Rightarrow \quad \angle AOD + \angle DOC + \angle COE + \angle BOE = 180°$

[From (1), (2), (3) and (4)]

⇒ AB is a straight line and points A, O and B are collinear.

Long Answer Type Questions

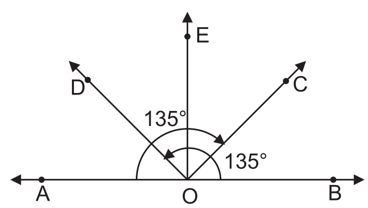
13. In the given figure, ∠AOC = ∠BOD = 135° and ∠DOE = ∠COE. Find the measures of ∠AOD, ∠DOC and ∠EOC.



Sol. Given that

$\angle AOC = \angle BOD = 135°$ …(1)

Let $\angle DOE = \angle COE = x$ …(2)



Now, $\angle AOD = \angle AOC - (\angle DOE + \angle COE)$

$= 135° - (x + x)$

[From (1) and (2)]

$\Rightarrow \quad \angle AOD = 135° - 2x$ …(3)

Similarly, $\angle BOC = 135° - 2x$ …(4)

$\angle AOD + \angle DOE + \angle COE + \angle BOC = 180°$

[∵ AOB is a line]

$\Rightarrow 135° - 2x + x + x + 135° - 2x = 180°$

[From (1), (2), (3) and (4)]

$$\Rightarrow \quad 270° - 2x = 180°$$

$$\Rightarrow \quad 270° - 180° = 2x$$

$$\Rightarrow \quad 90° = 2x$$

$$\Rightarrow \quad x = \frac{90°}{2}$$

$$\Rightarrow \quad x = 45°$$

$\Rightarrow \quad \angle AOD = 135° - 2(45°)$

$= 135° - 90°$

$= 45°$

$\Rightarrow \quad \angle EOC = 45°$

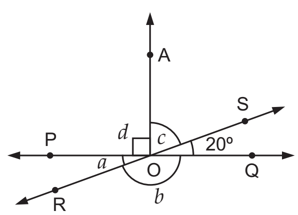
$\Rightarrow \quad \angle DOC = \angle DOE + \angle COE$

$= 45° + 45°$

$= 90°$

$\therefore$ The measure of $\angle AOD = 45°$, $\angle DOC = 90°$ and $\angle EOC = 45°$.

14. In the given figure, lines PQ and RS intersect at O. AO $\perp$ PQ. If $\angle SOQ = 20°$, find the angles a, b, c and d.



Sol. Given that POQ and ROS are two straight lines which intersect each other at O. The ray OA is perpendicular to line PQ.

Also, $\angle AOP = d$, $\angle POR = a$, $\angle QOR = b$, $\angle AOS = c$ and $\angle SOQ = 20°$.

To find the angles a, b, c and d, we have

$a = \angle POR = \angle SOQ$ [Vertically opposite angles]

$\Rightarrow \quad a = 20° \quad \ldots(1)$

$c = \angle AOS$

$= \angle AOQ - \angle SOQ$

$= 90° - 20°$

$\Rightarrow \quad c = 70°$

$b = \angle QOR$

$= \angle POQ - a$

$= 180° - 20°$ [From (1)]

$\Rightarrow \quad b = 160°$

$d = \angle AOP = 90°$ [$\because$ OA $\perp$ PQ]

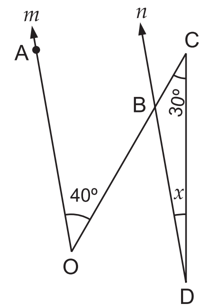
Hence, the required values of a, b, c and d are 20°, 160°, 70° and 90° respectively.

Check Your Progress 2

(Page 76)

Multiple-Choice Questions

1. In the given figure, if $m \parallel n$, then the measure of x is



(*a*) 10° (*b*) 40°

(*c*) 70° (*d*) 30°

Sol. (*a*) 10°

Given that rays m = OA and n = DB are parallel. Also, $\angle AOB = 40°$, $\angle BCD = 30°$ and $\angle BDC = x$

$\because$ OA $\parallel$ DB and OBC is a transversal,

$\therefore \quad \angle OBD = \angle AOB = 40°$ [Alternate angles]

$\because$ $\angle OBD$ is an exterior angle of ΔBCD.

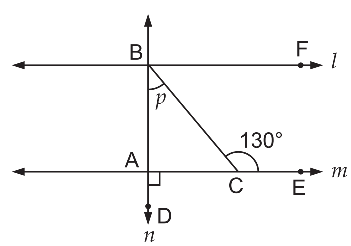
$\therefore \quad \angle OBD = 40° = \angle BCD + \angle BDC$ [Property of exterior angle of a triangle]

$\Rightarrow \quad 40° = 30° + x$

$\Rightarrow \quad x = 40° - 30° = 10°$

$\therefore$ The measure of x is 10°.

2. In the given figure, if $l \parallel m$ and $n \perp m$, then the measure of angle p is



(*a*) 30°

(*b*) 40°

(*c*) 50°

(*d*) 60°

Sol. (*b*) 40°

Given that lines l and m are parallel to each other and n is perpendicular to m. Also, $\angle BCE = 130°$, $\angle ABC = p$ and $\angle CAD = 90°$.

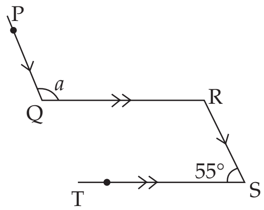
$\therefore$ $\angle BCE$ is an exterior angle of a ΔBAC

$\therefore \quad \angle BCE = \angle ABC + \angle BAC$ [Property of exterior angle of a triangle]

$\Rightarrow \quad 130° = p + 90°$

$\Rightarrow \quad p = 130° - 90° = 40°$

3. In the given figure PQ || RS and QR || TS. Find the value of a.



(a) 125° (b) 55°

(c) 35° (d) 90°

Sol. (a) 125°

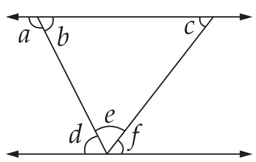
$\angle PQR = \angle QRS = a$ [Alt. ∠s PQ || RS]

$\angle QRS + \angle RST = 180°$ [Coint. ∠s QR || TS]

$a + 55° = 180°$

$a = 180° - 55° = 125°$

4. Consider the given figure. If $\angle a = \angle e + \angle c$, then which of the following options explains it.



(a) $\angle a = \angle e + \angle f$ and $\angle f = \angle c$

(b) $\angle a = \angle e$ and $\angle f = \angle c$

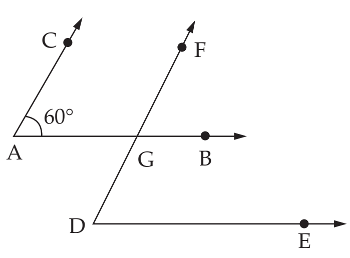
(c) $\angle a = \angle e$ and $\angle c = \angle e + \angle f$

(d) The data is not sufficient to explain it.

Sol. (a) $\angle a = \angle e + \angle f$ and $\angle f = \angle c$

Very Short Answer Type Questions

5. In the given figure, AC || DF and AB || DE. If ∠BAC = 60°, find ∠EDF.

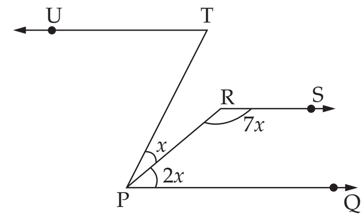


Sol. ∠CAB = ∠FGB = 60° [Corr. ∠s, AC || DE]

∠FGB = ∠FDE = 60° [Corr. ∠s, AB || DE]

⇒ ∠EDF = 60°

6. In the given figure, PQ || TU and TU || RS. Find the value of ∠PTU.



Sol. ∠RPQ + ∠PRS = 180° [Coint. ∠s, PQ || RS]

⇒ $2x + 7x = 180°$

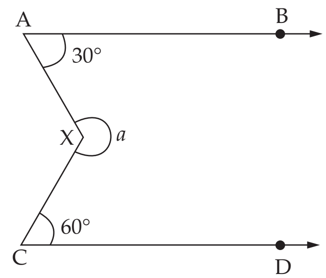
⇒ $9x = 180°$

⇒ $x = 20°$

∴ $\angle TPQ = x + 2x = 3x = 60°$

⇒ ∠PTU = 60° [Alt. ∠s, PQ || TU]

7. In the given figure, AB || CD. Find the value of a.



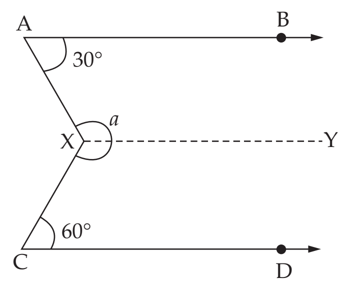
Sol. Draw XY || AB.

∠BAX + ∠AXY = 180°

[Coint. ∠s, AB || XY]

⇒ ∠AXY = 180° – 30°

⇒ ∠AXY = 150° ...(1)



XY || CD as AB || CD.

⇒ ∠XCD + ∠CXY = 180°

[Coint. ∠s, XY || CD]

⇒ 60° + ∠CXY = 180°

⇒ ∠CXY = 180° – 60°

⇒ ∠CXY = 120° ...(2)

Again

$\angle a = \angle AXY + \angle CXY$

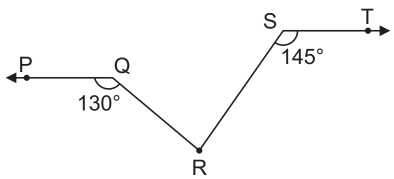
$= 150° + 120°$ [From (1) and (2)]

⇒ $\angle a = 270°$

8. In the given figure, PQ ∥ ST.

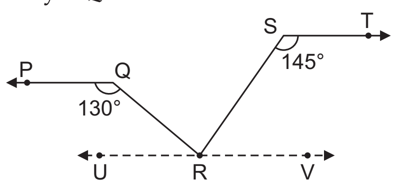
∠PQR = 130° and ∠RST = 145°.

Find the measure of ∠QRS.



Sol. Given that PQ ∥ ST, ∠PQR = 130° and ∠RST = 145°.

Construction: Draw a line UV through R parallel to the ray PQ or ST.



∵ PQ ∥ UR and QR is a transversal,

∴ ∠PQR + ∠QRU = 180° [Coint. ∠s, PQ ∥ UV]

⇒ 130° + ∠QRU = 180°

⇒ ∠QRU = 180° – 130° = 50° …(1)

Similarly, since ST ∥ RV,

∴ ∠TSR + ∠SRV = 180° [Coint. ∠s, ST ∥ UV]

⇒ 145° + ∠SRV = 180°

∠SRV = 180° – 145 = 35° …(2)

Hence, ∠QRS = ∠URV – ∠QRU – ∠SRV

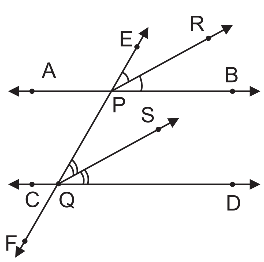
= 180° – 50° – 35° [From (1) and (2)]

= 180° – 85° = 95°

∴ The measure of ∠QRS is 95°.

9. Prove that if two parallel lines are intersected by a transversal, then the bisectors of any two corresponding angles are parallel.

Sol. Given that AB and CD are two parallel lines and EF is their transversal. PR and QS are the bisectors of the two corresponding angles, ∠EPB and ∠PQD respectively.



To prove that PR ∥ QS.

We have

∠EPB = ∠PQD [Corr. ∠s, AB ∥ CD]

⇒ $\frac{1}{2}$∠EPB = $\frac{1}{2}$∠PQD

⇒ ∠EPR = ∠PQS [∵ PR and QS are the bisectors of ∠EPB and ∠PQD respectively]

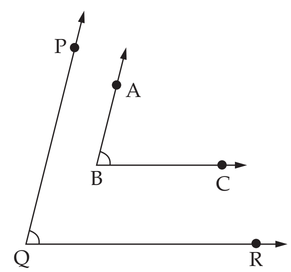
But these two are corresponding angles with respect to rays PR and QS and their transversal EF.

∴ PR ∥ QS

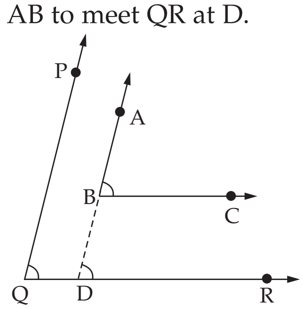
Hence, proved.

Short Answer Type Questions

10. In the given figure, PQ ∥ AB and BC ∥ QR. Show that ∠PQR = ∠ABC.



Sol. Produce AB to meet QR at D.

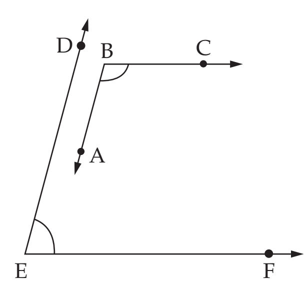


∠PQR = ∠ADR [Corr. ∠s, PQ ∥ AD]

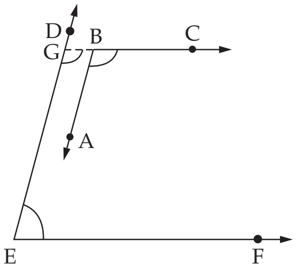
∠ADR = ∠ABC [Corr. ∠s, QR ∥ BC]

⇒ ∠PQR = ∠ABC

11. In the given figure, AB ∥ DE and BC ∥ EF. Show that ∠DEF + ∠ABC = 180°.



Sol. Extend BC to meet DE at G.



AB || DE

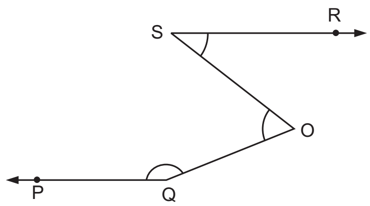
$\Rightarrow \quad \angle EGC = \angle ABC$ [Corr. angles] ...(1)

$\angle EGC + \angle DEF = 180°$ [Coint. angles] ...(2)

From (1) and (2)

$\angle ABC + \angle DEF = 180°$

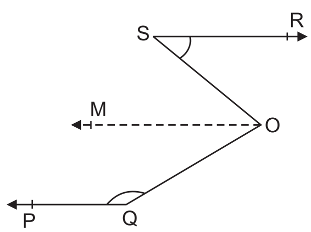
12. In the given figure, PQ || RS. Prove that $\angle PQO + \angle QOS = 180° + \angle OSR$.



Sol. Given that PQ || RS

To prove that $\angle PQO + \angle QOS = 180° + \angle OSR$.

Construction: We draw a ray OM through O parallel to SR or PQ.



$\because$ PQ || OM and OQ is a transversal,

$\therefore \quad \angle PQO + \angle QOM = 180°$ [Coint. $\angle$s]

$\Rightarrow \quad \angle PQO + \angle QOS - \angle MOS = 180°$

$\Rightarrow \quad \angle PQO + \angle QOS = 180° + \angle MOS$

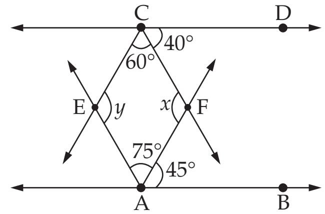
$= 180° + \angle OSR$ [Alt. $\angle$s, OM || RS]

$\Rightarrow \quad \angle PQO + \angle QOS = 180° + \angle OSR$

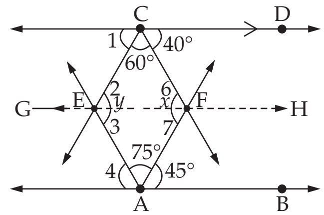
Hence, proved.

Long Answer Type Questions

13. In the given figure, AB || CD. Find the values of angles x and y.



Sol. Through E and F draw EG || CD and FH || AB.



$\angle 1 = 180° - 60° - 40°$ [Linear pair]

$\angle 1 = 80°$

$\angle 1 = \angle 2 = 80°$ [Alt. angles]

$\angle 4 = 180° - 75° - 45° = 180° - 120°$

$\angle 4 = 60°$

$\angle 3 = \angle 4$ [Alt. angles]

$\therefore \quad \angle 3 = 60°$

$\Rightarrow \quad y = \angle 2 + \angle 3$

$= 80° + 60°$

$y = 140°$

Since FH || AB.

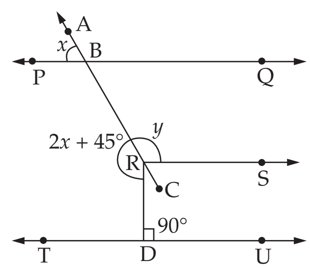
$\angle 6 = 40°$ [Alt. angles]

$\angle 7 = 45°$ [Alt. angles]

$x = \angle 6 + \angle 7 = 40° + 45°$

$\Rightarrow \quad x = 85°$

14. In the given figure, PQ || RS, RS || TU and RD is perpendicular on TU. Find the values of x and y.

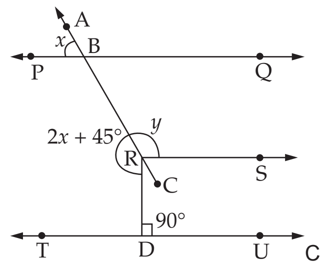


Sol. $\angle ABQ = \angle BRS = y$ [Corresponding angles]

$\angle ABP + \angle ABQ = 180°$ [PQ is a straight line]

$\Rightarrow \quad x + y = 180°$ ...(1)

$\angle SRD = 90°$



$$\angle SRD = 90°$$

$\therefore \quad \angle BRS + \angle BRD + \angle SRD = 360°$

$\Rightarrow \quad y + 2x + 45° + 90° = 360°$

$\Rightarrow \quad y + 2x + 135° = 360°$

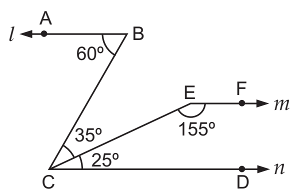
$\Rightarrow \quad 2x + y = 225° \qquad ...(2)$

Subtracting (1) from (2) we get

$$x = 45°$$

$\Rightarrow \quad y = 180° - 45° = 135°$

15. In the given figure, show that $l \parallel m$.



Sol. We have

$$\angle BCD = \angle BCE + \angle ECD$$
$$= 35° + 25°$$
$$= 60°$$
$$= \angle ABC$$

But these two are alternate angles with respect to the pair of lines AB and CD and their transversal BC.

$\therefore \quad AB \parallel CD, \quad \text{i.e. } l \parallel n \qquad ...(1)$

Again,

$\angle ECD + \angle CEF = 25° + 155° = 180°$

$\because$ The sum of two interior angles is 180°,

$\therefore \quad EF \parallel CD \quad \Rightarrow \quad m \parallel n \qquad ...(2)$

$\therefore$ From (1) and (2), we see that $l \parallel m$.

Higher Order Thinking Skills (HOTS) Questions

(Page 78)

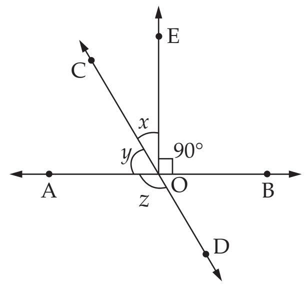
1. If A, B, C and D are four collinear points, name all the line segments that can be obtained.

Sol.

The line segments are

AB, BC, CD, AC, AD, BD

2. In the figure given, lines AB and CD intersect at O. OE ⊥ AB.

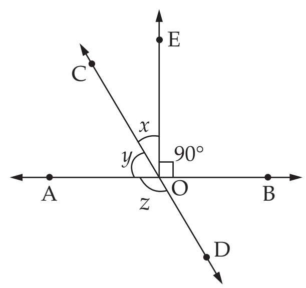


Find the value(s) of

(*a*) z, if $x = 45°$ (*b*) z, if $x : y = 3 : 2$

(*c*) x, y and z, if $z : x = 5 : 2$

Sol.



(*a*) $x + y = 90°$ [OE ⊥ AB]

$\Rightarrow \quad y = 90° - 45°$ [Given $x = 45°$]

$\Rightarrow \quad y = 45°$

$\angle BOD = 45°$ [Vertically opposite angles]

$\angle BOE + x + y + z + \angle BOD = 360°$

[Angle around a point is 360°]

$\Rightarrow \quad 90° + 45° + 45° + z + 45° = 360°$

$\Rightarrow \quad z = 360° - 225°$

$\Rightarrow \quad z = 135°$

(*b*) Given $x : y = 3 : 2$

$$\frac{x}{y} = \frac{3}{2}$$

$$x = \frac{3}{2}y \qquad ...(1)$$

Also, $\quad x + y = 90°$

$$\frac{3}{2}y + y = 90°$$

$\Rightarrow \quad \dfrac{3y + 2y}{2} = 90°$

$\Rightarrow \quad 5y = 90° \times 2$

$\Rightarrow \quad y = \dfrac{90° \times 2}{5}$

$\Rightarrow \quad y = 36°$

$\therefore \quad x = 90° - 36°$

$\Rightarrow \quad x = 54°$

$\angle BOD = 36°$

$\angle BOE + x + y + z + \angle BOD = 360°$

$90° + 54° + 36° + z + 36° = 360°$

$\Rightarrow \quad z = 360° - 216°$

$\Rightarrow \quad z = 144°$

(*c*) $z : x = 5 : 2$

$\Rightarrow \quad \frac{z}{x} = \frac{5}{2}$

$\Rightarrow \quad z = \frac{5}{2}x$ ...(1)

Again $x + y = 90°$

$\Rightarrow \quad y = 90° - x$ ...(2)

$\therefore \quad \angle BOD = 90° - x$

$\angle BOE + x + y + z + \angle BOD = 360°$

$\Rightarrow \quad 90° + x + 90° - x + \frac{5}{2}x + 90° - x = 360°$

[From (1) and (2)]

$\Rightarrow \quad 270° + \frac{5}{2}x - x = 360°$

$\Rightarrow \quad \frac{5x - 2x}{2} = 360° - 270°$

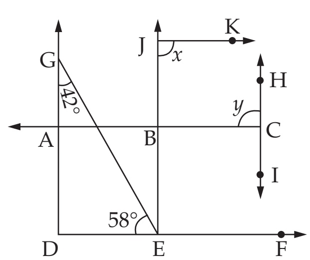
$\Rightarrow \quad \frac{3}{2}x = 90°$

$\therefore \quad x = 90° \times \frac{2}{3} = 60°$

$\therefore \quad y = 90° - x = 90° - 60° = 30°$

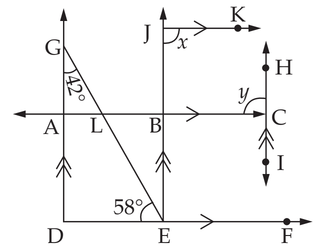
$\therefore \quad z = \frac{5}{2}x = \frac{5}{2} \times 60° = 150°$

3. In the given figure, AC ∥ DF, JK ∥ AC, DG ∥ EJ and EJ ∥ HI. Find the values of x and y.



Sol. $\angle BLE = 58°$ [Alternate angle]

$\angle BEL = 42°$ [Alternate angle]



In ΔBEL

$\angle BLE + \angle BEL + \angle EBL = 180°$

$58° + 42° + \angle EBL = 180°$

$\Rightarrow \quad \angle EBL = 180° - 100°$

$\Rightarrow \quad \angle EBL = 80°$

$\therefore \quad \angle JBC = 80°$ [Vertically opposite angles]

$\angle JBC + y = 180°$

[Cointerior angles, EJ ∥ HI]

$\Rightarrow \quad y = 180° - 80°$

$y = 100°$

$\angle JBC + x = 180°$

[Cointerior angles, JK ∥ AC]

$\Rightarrow \quad x + 180° - 80° = 100°$

Self-Assessment

(Page 79)

Multiple-Choice Questions

1. The complement of an angle is twice the angle itself. The measure of the angle is

(*a*) 30° (*b*) 15°

(*c*) 45° (*d*) 10°

Sol. (*a*) 30°

Let the angle be x.

Then complement = $90° - x$

According to problem, $90° - x = 2x$

$\Rightarrow \quad x + 2x = 90°$

$\Rightarrow \quad 3x = 90°$

$\therefore \quad x = 30°$

2. The supplement of an angle is equal to seven-fifth of the number itself. The measure of the angle is

(*a*) 45° (*b*) 75°

(*c*) 90° (*d*) 105°

Sol. (*b*) 75°

Let the angle be x.

Supplement = $180° - x$

According to problem, $180° - x = \frac{7}{5}x$

$\Rightarrow \quad x + \frac{7}{5}x = 180°$

$\Rightarrow \quad \frac{12x}{5} = 180°$

$\therefore \quad x = 75°$

3. The supplement of an angle is equal to three times its complement. Then the measure of the angle is

(*a*) 60° (*b*) 50°
(*c*) 45° (*d*) 30°

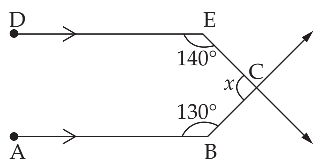
Sol. (*c*) 45°

Let the measure of the angle be x in degrees.

$\therefore$ According to the problem, we have

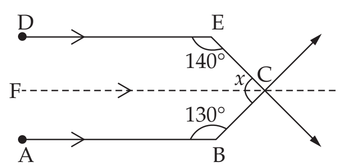
$$180° - x = 3(90° - x)$$
$$= 270° - 3x$$
$$\Rightarrow \quad 3x - x = 270° - 180°$$
$$\Rightarrow \quad 2x = 90°$$
$$\Rightarrow \quad x = 45°$$

4. In the given figure DE || AB, ∠DEC = 140° and ∠ABC = 130°, then x is



(*a*) 60° (*b*) 90°
(*c*) 120° (*d*) 130°

Sol. (*b*) 90°



Draw CF || DE

$\therefore$ CF || AB

∠DEC + ∠ECF = 180° [Cointerior angles, DE || CF]

$\Rightarrow$ ∠ECF = 180° – 140°

$\Rightarrow$ ∠ECF = 40°

∠ABC + ∠BCF = 180° [Cointerior angles, CF || AB]

$\Rightarrow$ ∠BCF = 180° – 130°

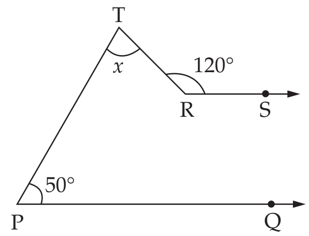
$\Rightarrow$ ∠BCF = 50°

x = ∠ECF + BCF

$\therefore$ $x = 40° + 50°$

$\Rightarrow$ $x = 90°$

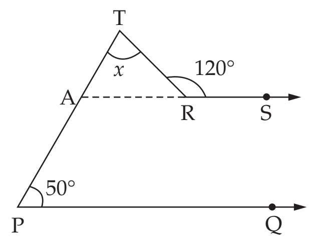
5. In the given figure, PQ || RS. The measure of angle x is



(*a*) 50° (*b*) 60°
(*c*) 70° (*d*) 120°

Sol. (*c*) 70°

Extend RS to meet PT at A.



AS || PQ

∠TRA = 180° – 120°

$\Rightarrow$ ∠TRA = 60° ...(1)

∠TAR = ∠APQ = 50° ...(2)
[Corresponding angles]

In ΔATR,

x + ∠TAR + ∠TRA = 180° [From (1) and (2)]

$\Rightarrow \quad x + 50° + 60° = 180°$

$\Rightarrow \quad x + 110° = 180°$

$\Rightarrow \quad x = 180° - 110°$

$\Rightarrow \quad x = 70°$

Fill in the Blanks

6. If one angle of a linear pair is acute, then the other angle be **obtuse.**

7. The measure of an angle which is four times its complement is **72°**.

Sol. Angle = 4 (Its complement)

$\Rightarrow \quad x = 4(90° - x)$

$\Rightarrow \quad 5x = 360°$

$\Rightarrow \quad x = 72°$

8. If two complementary angles are in the ratio 2 : 3, then the angles are **36°** and **54°**.

Sol. $2x + 3x = 90°$

$\Rightarrow \quad 5x = 90°$

$\Rightarrow \quad x = 18°$

Then, $2x = 2 \times 18° = 36°$

and $3x = 3 \times 18° = 54°$

9. The measure of an angle which is 24° more than its complement is **57°**.

Sol. Angle = Its complement + 24°

$x = (90° - x) + 24°$

$\Rightarrow 2x = 114°$

$\Rightarrow x = 57°$

Assertion-Reason Type Questions

Directions (Q. Nos. 10 to 13): Each of these questions contains an assertion followed by reason. Read them carefully, and answer the question on the basis of the following options, select the one that best describes the two statements.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

10. **Assertion (A):** Angles of 160° and 20° are supplementary.

Reason (R): Two angles are said to be supplementary if their sum is 180°.

Sol. (*a*) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

11. **Assertion (A):** Two intersecting lines have only one point in common.

Reason (R): Intersecting lines have two common points.

Sol. (*c*) Assertion (A) is correct but Reason (R) is incorrect as two intersecting lines have only one point in common.

12. **Assertion (A):** If angles forming a linear pair are equal, then each of these angles is 90°.

Reason (R): Angles forming a linear pair can both be either obtuse angles or acute angles.

Sol. (*c*) Assertion (A) is true but Reason (R) is false.

The sum of angles of a linear pair is 180°.

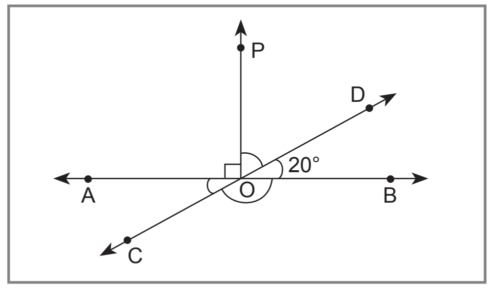
13. **Assertion (A):** Two adjacent angles have a common vertex.

Reason (R): Adjacent angles originate from the same vertex and have one common arm.

Sol. (*a*) Assertion (A) and Reason (R) are correct and reason is correct explanation of assertion as adjacent angles have a common vertex and one common arm.

Case Study Based Questions

14. To judge the preparation of students of class IX on the topic 'Lines and angles', Mathematics teacher draws the figure on the whiteboard such that lines AB and CD intersect at O. If PO ⊥ AB and ∠DOB = 20°, then answer the following questions.



(*a*) What is the measure of ∠AOC?

Sol. 20°

(*b*) What is the measure of ∠AOP?

Sol. 90°

(*c*) (*i*) What is the measure of ∠POD?

Sol. 70°

or

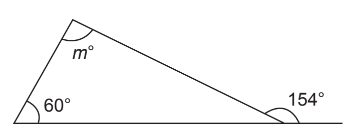
(*ii*) What is the measure of ∠COB?

Sol. 160°

15. Our Maths teacher knows yoga. She asked Nitya to show yoga postures. She made marks on angles. She showed acute, obtuse and right angles through yoga postures.

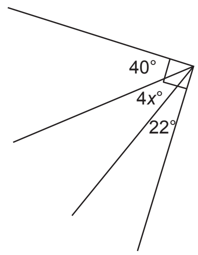
Based on the above information, answer the following questions.

(*a*) Find the value of *m* in the following figure.



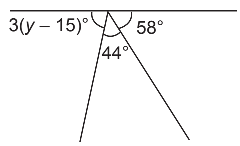
Sol. 94°

(*b*) What is the value of x in the following figure?



Sol. 7°

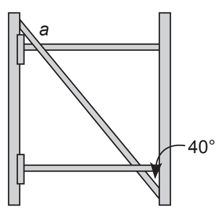
(*c*) (*i*) Find the value of y in the following figure.



Sol. 41°

or

(*ii*) Two gates consist of vertical posts, horizontal struts and diagonal beams. Find the angle a from the following figure.



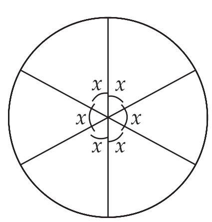
Sol. 140°

Very Short Answer Type Questions

16. A wheel has 6 spokes equally spaced. Find the angle between any two adjacent spokes.

Sol. Let the distance between two adjacent spokes

$= x°$



From the figure,

$$6x = 360° \quad \text{[Angle about a point]}$$

$$\therefore \quad x = \frac{360°}{6}$$

$$\Rightarrow \quad x = 60°$$

$\therefore$ Angle between any two adjacent spokes

$= 60°$

17. Two adjacent angles are such that one angle is two-thirds of the other angle. Find the angles, if they form a linear pair.

Sol. Let, one angle $= x$

Adjacent angle $= \dfrac{2x}{3}$

$$x + \frac{2x}{3} = 180°$$

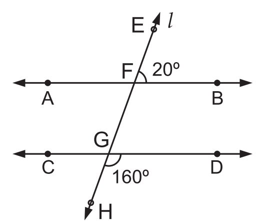
[Angles form a linear pair]

$$\Rightarrow \quad \frac{5x}{3} = 180°$$

$$\Rightarrow \quad x = 180° \times \frac{3}{5} = 108°$$

Therefore, the angles are 108° and 72°.

18. In the given figure, show that AB ∥ CD.



Sol. In the given figure, it is given that ∠EFB = 20° and ∠HGD = 160°.

To prove that AB ∥ CD.

We have ∠FGD = 180° – ∠HGD

= 180° – 160°

= 20° = ∠EFB

∴ ∠FGD = ∠EFB

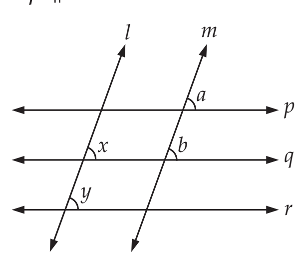
But these two angles are corresponding angles with respect to the pair of lines AB and CD and the transversal l.

Since, these corresponding angles are equal.

∴ AB ∥ CD

Hence, proved.

19. In the given figure, if $\angle a = \angle b$ and $\angle x = \angle y$, prove that $p \parallel r$.



Sol. Given that x and y, the corresponding angles with respect to the pair of lines q and r are equal. Also, a and b, the corresponding angles with respect to the pair of lines p and q are equal.

To prove that $p \parallel r$.

$\because$ The corresponding angles with respect to q and r are equal.

$\therefore \quad q \parallel r \quad \ldots(1)$

Again, the corresponding angles with respect to p and q are equal.

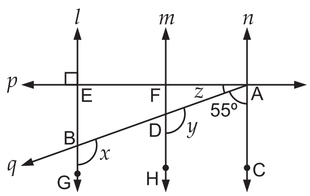
$\therefore \quad p \parallel q \quad \ldots(2)$

From (1) and (2), we have

$$p \parallel r$$

Hence, proved.

20. In the given figure, $l \parallel m \parallel n$. If $p \perp l$, $\angle BAC = 55°$, find the values of x, y and z. **[CBSE SP 2012]**



Sol. Given that $l \parallel m \parallel n$, $p \perp l$ and $\angle BAC = 55°$

Also, $\angle FAD = z$, $\angle ADH = y$ and $\angle DBG = x$

To find the values of x, y and z.

$\because p \perp l$

$\therefore p \perp m$ and $p \perp n$

$\therefore \quad \angle FAC = 90°$

$\therefore \quad z = \angle FAD$

$= \angle FAC - \angle DAC$

$= 90° - 55°$

$= 35°$

$\angle ADH + \angle DAC = 180° \quad [\because m \parallel n]$

$\Rightarrow \quad y + 55° = 180°$

$\Rightarrow \quad y = 180° - 55°$

$= 125°$

Also, $\quad \angle DBG = \angle ADH$

[Corresponding $\angle$s, $l \parallel m$]

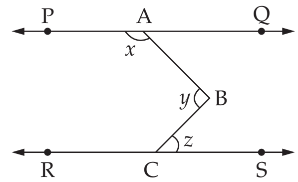
$\Rightarrow \quad x = y = 125°$

Hence, the required values of x, y, and z are 125°, 125° and 35° respectively.

Short Answer Type Questions

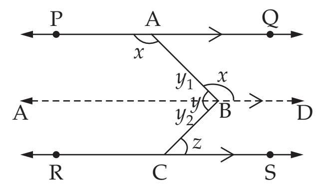
21. In the given figure, $PQ \parallel RS$, $\angle PAB = x$, $\angle ABC = y$ and $\angle BCS = z$. Prove that

$$x + y - z = 180°$$



Sol. Through B, draw AD parallel to PQ.

Let $y = y_1 + y_2$



$\angle ABD = x$

[Alternate angles, $PQ \parallel AD$]

$\angle DBC = 180° - z$

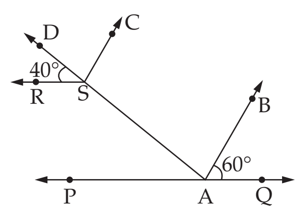
[Cointerior angles, $RS \parallel AC$]

$\therefore \quad x + y_1 + y_2 + 180 - z = 360°$

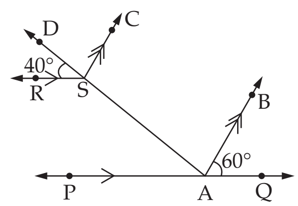
$\therefore \quad x + y - z = 180°$

Hence proved.

22. In the given figure, $PQ \parallel RS$ and $AB \parallel CS$. Find $\angle DSC$, $\angle PAS$ and $\angle BAS$.



Sol. We have



$\angle RSD = 40°$

$\angle RSD = \angle PAS$

$= 40° \quad$ [Corr. $\angle$s, $PQ \parallel RS$] ...(1)

$\angle PAS + \angle BAS + \angle BAQ = 180°$

[PQ is a straight line]

$\Rightarrow \quad 40° + \angle BAS + 60° = 180°$

$\Rightarrow \quad \angle BAS = 180° - 100°$

$\Rightarrow \quad \angle BAS = 80° \quad$ ...(2)

$\angle BAS + \angle ASC = 180° \quad$ [Cointerior angles]

$\Rightarrow \quad \angle ASC = 180° - 80°$

$\Rightarrow \quad \angle ASC = 100°$

$\angle DSC = 180° - 100° = 80° \quad$ ...(3)

From (1), (2) and (3)

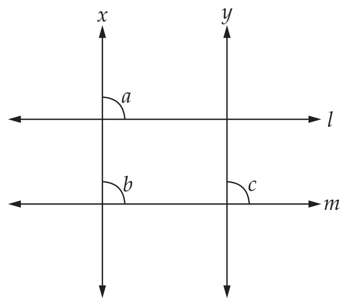
$\angle PAS = 40°$

$\angle BAS = 80°$

$\angle DSC = 80°$

23. Two lines are respectively perpendicular to two parallel lines. Show that they are parallel to each other.

Sol. Given, lines l and m are parallel lines. Line x is perpendicular on line l. Line y is perpendicular on line m.



$\angle a = 90°$ $[x \perp l]$

$\angle c = 90°$ $[y \perp m]$

$\Rightarrow$ $\angle a = \angle c = 90°$

$\angle a = \angle b$ [Corresponding angles]

$\Rightarrow$ $\angle a = \angle b = 90°$

$\Rightarrow$ $\angle a = \angle b = \angle c = 90°$

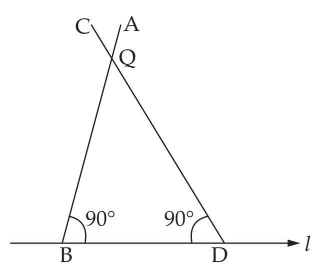
$\Rightarrow$ $\angle b = \angle c$

line $x \parallel$ line y [Corresponding angles are equal]

Long Answer Type Questions

24. Prove that through a given point, we can draw only one perpendicular to a given line.

Sol. Assume that from point Q, two perpendiculars AB and CD are drawn on the line l.



$\angle ABD = 90°$ $[AB \perp l]$

$\angle CDB = 90°$ $[CD \perp l]$

In ΔBQD,

$\angle BDQ + \angle QBD + \angle BQD = 180°$

[Angle sum property]

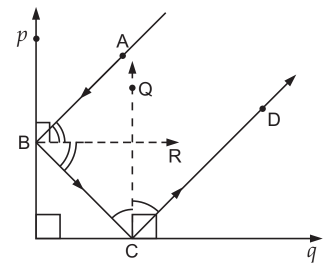
$\Rightarrow$ $90° + 90° + \angle BQD = 180°$

$\Rightarrow$ $\angle BQD = 0°$

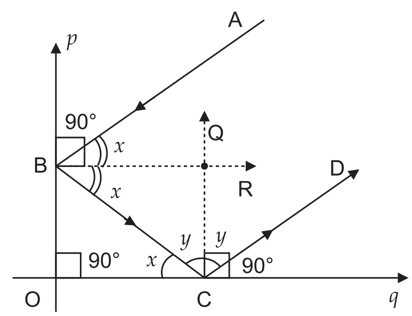
$\Rightarrow$ Lines AB and CD coincide.

Therefore, through a given point, we can draw only one perpendicular to a given line.

25. In the given figure, p and q are two plane mirrors perpendicular to each other. An incident ray AB strikes the mirror p at B, gets reflected along BC and then strikes the mirror q at C and finally gets reflected along CD. Prove that AB $\parallel$ CD.



Sol. Given that two mirrors p and q along OB and OC respectively are perpendicular to each other and intersect each other at O.



An incident ray AB strikes the mirror p at B and gets reflected along BC and then strikes the mirror q at C.

Finally, this ray gets reflected along CD.

Also, BR is drawn perpendicular to OB and CQ is drawn perpendicular to OC.

To prove that AB $\parallel$ CD.

We know that angle of incidence $\angle ABR$ = angle of reflection $\angle CBR = x$, say

and similarly, $\angle BCQ = \angle DCQ = y$, say.

$\because$ $\angle RBO + \angle QCO = 90° + 90° = 180°$

$\therefore$ BR $\parallel$ OC

Similarly, CQ $\parallel$ OB

$\therefore$ $\angle CBR = \angle OCB$

[Alternate $\angle$s, BR $\parallel$ OC]

$\Rightarrow$ $x = \angle OCB$

But $x + y = 90°$ ...(1)

$\therefore\ \angle ABC + \angle BCD = 2x + 2y$

$= 2(x + y)$

$= 2 \times 90° = 180°$ [From (1)]

$\therefore$ Sum of the interior angles with respect to the pair of lines BA and CD is 180°.

$\therefore$ AB ∥ CD

Hence, proved.

Let's Compete

(Page 81)

Multiple-Choice Questions

1. Find two supplementary angles whose ratio is 13 : 5.

(*a*) 140°, 40° (*b*) 120°, 60°
(*c*) 130°, 50° (*d*) 110°, 70°

Sol. (*c*) 130°, 50°

Two supplementary angles are

$\frac{13}{18} \times 180° = 130°$ and $\frac{5}{18} \times 180° = 50°$

2. The supplement of an angle is 60° more than twice the complement of the angle. Find the angle.

(*a*) 30° (*b*) 45°
(*c*) 60° (*d*) 90°

Sol. (*c*) 60°

Let, the angle = x

Complement of the angle = $90° - x$

Supplement = $2(90° - x) + 60°$

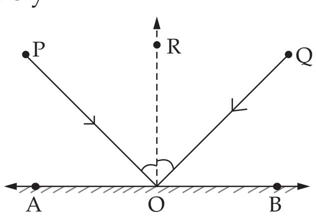
$\Rightarrow\ 2(90° - x) + 60° = 180° - x$

$\Rightarrow\ x + 180° - 2x + 60° = 180°$

$\Rightarrow\ -x = -60°$

$\Rightarrow\ x = 60°$

3. In the given figure, AB is a plane mirror. PO and OQ are the incident and reflected rays respectively.



If ∠POQ = 120°, then ∠AOP is

(*a*) 60° (*b*) 120°
(*c*) 40° (*d*) 30°

Sol. (*d*) 30°

∠POR = ∠QOR

[incident ray = reflected ray]

$\Rightarrow\ \angle POR = \frac{120°}{2} = 60°$

∠AOR = 90°

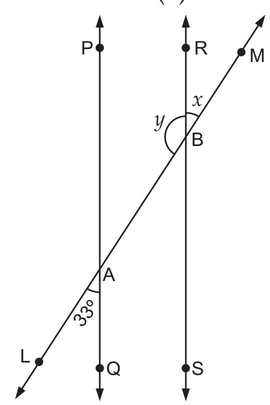
[The normal is at right angles]

$\therefore\ \angle AOP = 90° - 60°$

$\Rightarrow\ \angle AOP = 30°$

4. In the given figure, PQ ∥ RS. A transversal l intersects PQ and RS at the points A and B respectively such that ∠RBM = x and ∠RBA = y. Then the values of x and y are respectively

(*a*) 23° and 157° (*b*) 33° and 147°
(*c*) 33° and 157° (*d*) 23° and 147°



Sol. (*b*) 33° and 147°

Given that PQ and RS are two parallel lines and the transversal l intersect these two lines at the points A and B respectively such that ∠RBM = x and ∠RBA = y.

To find the values of x and y, we have

∠ABS = ∠LAQ

[Corresponding ∠s, PQ ∥ RS]

$\Rightarrow\ \angle ABS = 33°$

∠ABS = ∠RBM = 33°

[Vertically opposite ∠s]

$\Rightarrow\ x = 33°$

$\therefore\ y = 180° - x$ [Linear pair]

$= 180° - 33° = 147°$

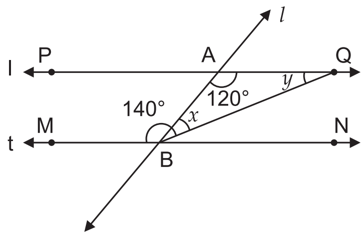
$\therefore$ The values of x and y are 33° and 147° respectively.

5. Lines PQ and MN are parallel to each other. A transversal l intersects PQ and MN at the points A and B respectively such that ∠BAQ = 120°. If ∠MBQ = 140°, ∠ABQ = x and ∠AQB = y, then the values of x and y are respectively

(*a*) 30° and 50° (*b*) 20° and 40°
(*c*) 40° and 20° (*d*) 50° and 30°

Sol. (*b*) 20° and 40°

Given that PQ and MN are two parallel lines and *l* is a transversal which intersects PQ and MN at the points A and B respectively such that ∠BAQ = 120°.



Also, ∠MBQ = 140°

∠ABQ = x

and ∠AQB = y

To find the values of x and y, we have

∠MBA = ∠BAQ

[Alternate ∠s, PQ ∥ MN]

⇒ ∠MBQ – ∠ABQ = 120°

⇒ 140° – x = 120°

⇒ x = 140° – 120° = 20° …(1)

Now, in ΔABQ, we have

∠ABQ + ∠AQB + ∠BAQ = 180°

[Angle sum property of a triangle]

⇒ $x + y$ + 120° = 180°

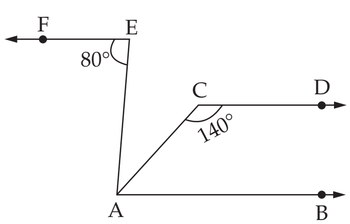
y = 180° – 120° – x

= 60° – x

= 60° – 20° = 40° [From (1)]

Hence, the required values of x and y are 20° and 40° respectively.

6. In the given figure, AB ∥ CD and CD ∥ EF. Find ∠EAC.



(*a*) 30° (*b*) 40°
(*c*) 80° (*d*) 100°

Sol. (*b*) 40°

∠BAC + ∠ACD = 180° [Cointerior angles]

⇒ ∠BAC + 140° = 180°

⇒ ∠BAC = 180° – 140°

⇒ ∠BAC = 40°

∠AEF = ∠BAE [Alternate angles]

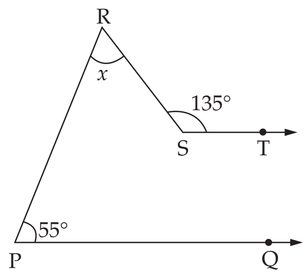
⇒ ∠BAE = 80°

⇒ ∠EAC = 80° – ∠BAC

⇒ ∠EAC = 80° – 40°

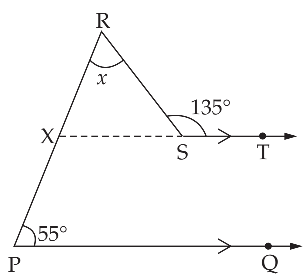
⇒ ∠EAC = 40°

7. In the given figure, PQ ∥ ST. Find the value of x.



(*a*) 55° (*b*) 90°
(*c*) 80° (*d*) 135°

Sol. (*c*) 80°



Extend ST to meet PR at X.

∠RSX = 180° – 135°

⇒ ∠RSX = 45°

∠RXS = ∠RPQ [Alternate angles]

⇒ ∠RXS = 55°

In ΔRXS,

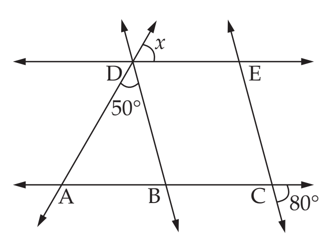
x + ∠RXS + ∠RSX = 180°

[Sum of angles in a triangle is 180°]

⇒ x + 55° + 45° = 180°

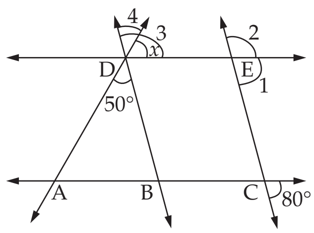
⇒ x = 80°

8. In the given figure, AC ∥ DE and BD ∥ CE. Find the value of x.



(*a*) 40° (*b*) 50°
(*c*) 70° (*d*) 80°

Sol. (*b*) 50°



$\angle 1 = 80°$ [Corresponding angles]

$\angle 2 = 180° - 80°$

$\Rightarrow \quad \angle 2 = 100°$

$\angle 2 = \angle 3$ [Corresponding angles]

$\Rightarrow \quad \angle 3 = 100°$

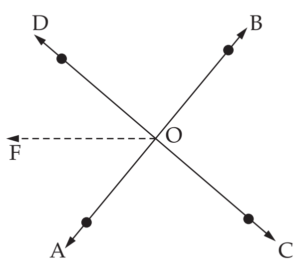
$\angle 4 = 50°$
[Vertically opposite angles]

$\therefore \quad x = \angle 3 - \angle 4$

$\Rightarrow \quad x = 100° - 50°$

$\Rightarrow \quad x = 50°$

9. In the given figure, lines AB and CD intersect at the point O. ∠BOD = 80°. OF is the bisector of ∠AOD. Find the measure of reflex ∠FOC.

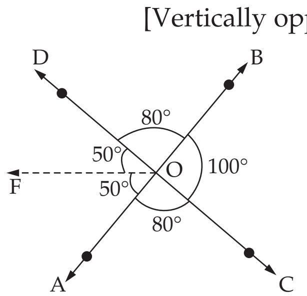


(*a*) 130° (*b*) 180°
(*c*) 220° (*d*) 230°

Sol. (*d*) 230°

$\angle BOD = 80°$ [Given]

$\angle AOC = \angle BOD$
[Vertically opposite angles]



$\Rightarrow \quad \angle AOC = 80°$

$\angle AOD = 180° - 80°$
[AB is a straight line]

$\Rightarrow \quad \angle AOD = 100°$

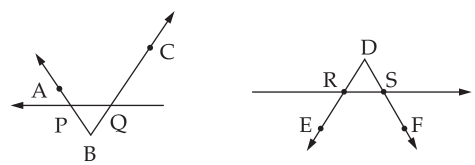
$\angle BOC = \angle AOD = 100°$
[Vertically opposite angles]

$\angle DOF = \angle AOF = 50°$ [OF bisects ∠AOD]

reflex $\angle FOC = 100° + 80° + 50°$

$= 230°$

10. In the given figure AB || DF and BC || DE. If ∠DSR = 50° and ∠ERS = 120°, then find the measure of ∠QBP.

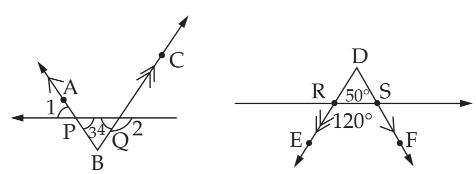


(*a*) 60° (*b*) 70°
(*c*) 80° (*d*) 120°

Sol. (*b*) 70°

$\angle 1 = 50°$ [Alternate angles]

$\angle 2 = 120°$ [Alternate angles]



$\angle 1 = \angle 3 = 50°$
[Vertically opposite angles]

$\angle 4 = 180° - \angle 2$

$\Rightarrow \quad \angle 4 = 180° - 120°$

$\Rightarrow \quad \angle 4 = 60°$

In ΔPBQ

$\angle 3 + \angle 4 + \angle QBP = 180°$

$\Rightarrow \quad \angle QBP = 180° - 50° - 60°$

$\Rightarrow \quad \angle QBP = 70°$