5

Introduction to Euclid's Geometry

Checkpoint (Page 57)

1. How many lines can pass through
   (*a*) one point
   (*b*) two distinct points?

Sol. We know from Euclid's postulates that
(*a*) infinitely many lines can pass through one point.
(*b*) only one line can pass through two distinct points.

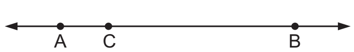
2. What is the least number of distinct points which determines a unique line?

Sol. From Euclid's postulates, we know that the least number of distinct points which determines a unique line is two.

3. Identify the portions of parallel and intersecting lines in the following:
   (*a*) Two opposite edges of a rectangular door
   (*b*) The adjacent edges of your geometry box

Sol. (*a*) We know that two opposite sides of a rectangle are parallel. Hence, two opposite edges of a rectangular door are parallel.
(*b*) We know that two adjacent sides of a rectangle represent portion of two intersecting lines meeting at a point. Hence, the adjacent edges of our geometry box which is rectangular in shape represent portion of two intersecting lines.

4. How many line(s) and line segments are there in the given figure? Name them.



Sol. From definitions of a line and line segment, we see that in the given figure, there is only one line AB and three line segments AC, CB and AB.

5. How many diameters can you draw in a circle?

Sol. We know that all diameters of a circle are line segments passing through the unique centre of a circle. Since, infinite number of line segments can be drawn through one point, hence there are infinitely many diameters which can be drawn in a circle.

6. Find the minimum and maximum number of points of intersection of three distinct lines in a plane?

Sol. We know that three lines can either be parallel to each other or they can form a triangle with three vertices. Hence, the minimum number of points of intersection of three distinct lines is 0 since parallel lines can never meet together, and the maximum number of points of intersection of three lines is 3, in this case they can form a triangle with three vertices.

7. Define equilateral triangle and right triangle.

Sol. An equilateral triangle is a triangle with three equal sides and a right-angled triangle is a triangle with one angle measuring 90°.

8. If the distance between two lines is the same everywhere, what kind of lines are they?

Sol. We know that the distance between two parallel lines is same everywhere. Hence, in this case, the two lines are parallel.

Check Your Progress

(Page 60)

Multiple-Choice Questions

1. Which of the following needs a proof?

(*a*) an axiom (*b*) a theorem

(*c*) a definition (*d*) a postulate

[CBSE SP 2010]

Sol. (*b*) a theorem

In order to verify the facts we prove a theorem using axioms, definitions and postulates.

2. If the area of a triangle is equal to that of a rectangle and the area of a rectangle is equal to that of a square, then the area of the triangle is equal to the area of the square. This follows from Euclid's

(*a*) fourth axiom (*b*) second axiom

(*c*) third axiom (*d*) first axiom

Sol. (*d*) first axiom

From Euclid's first axiom, we know that things which are equal to the same thing are equal to one another. Here, the area of a triangle and that of a rectangle are each equal to that of a square. Hence, the area of the triangle is equal to the area of the square.

3. Binod and Dinesh are of the same age. Sushil is also of the same age as Dinesh. State the Euclid's axiom which can be used to illustrate the relative ages of Binod and Sushil.

(*a*) First axiom (*b*) Second axiom

(*c*) Third axiom (*d*) Fourth axiom

Sol. (*a*) First axiom

4. Two lines l_1 and l_2 intersect at a point A. Choose the correct option which is true for the distance between the two lines l_1 and l_2, as they move away from the point A.

(*a*) The distance decreases continuously.

(*b*) The distance increases continuously.

(*c*) The distance remains constant.

(*d*) The distance increases and decreases depending upon the point of intersection.

Sol. (*b*) The distance increases continuously.

Very Short Answer Type Questions

5. What is the maximum number of point(s) of intersection of two distinct lines?

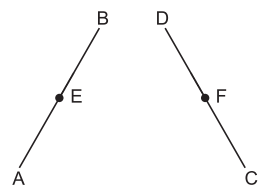
Sol. We know that two distinct lines can either intersect each other at only one point or else they may be parallel to each other. In the second case they cannot intersect each other at all. Hence, the required maximum number of point(s) of intersection will be only one.

6. How many maximum number of distinct lines can be drawn through three non-collinear points in a plane?

Sol. We know that a triangle can be drawn through three non-collinear points by three distinct lines. Hence, the maximum number of distinct lines that can be drawn through three non-collinear points is three.

7. In the given figure, AE = DF. E is the mid-point of AB and F is the mid-point of DC. Using Euclid's axiom, show that

AB = DC **[CBSE SP 2012]**



Sol. We have AE = DF [Given]

Also, AE × 2 = DE × 2

[By Euclid's axiom, i.e. if equals are multiplied by equals then their products are equal]

⇒ AB = DC

[E and F are mid-point of AB and CD respectively]

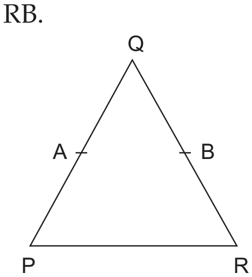
Hence, proved.

8. The area of an equilateral triangle is equal to the area of a square. The area of the square is equal to the area of a rhombus. What is the relation between the area of an equilateral triangle and the area of a rhombus?

Sol. The area of the equilateral triangle is equal to the area of a rhombus.

Short Answer Type Questions

9. In the given figure, PQ = QR, AQ = BQ. Show that PA = RB.



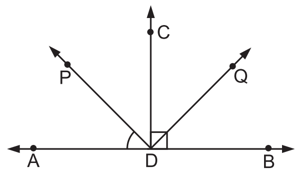
Sol. PQ = QR [Given]

$\Rightarrow$ PA + AQ = RB + BQ

[A and B lies between PQ and QR respectively]

$\Rightarrow$ PA = RB [AQ = BQ and by axiom 3]

10. In the given figure, AB $\perp$ CD. If $\angle$PDA = $\angle$QDB, prove that $\angle$PDC = $\angle$QDC.



State the axiom(s) used to prove this.

Sol. Given that line AB is perpendicular to the ray DC. Also, DP and DQ are two rays such that

$$\angle PDA = \angle QDB$$

To prove,

$$\angle PDC = \angle QDC$$

We have $\angle CDB = \angle CDA = 90°$ [Given]

Now, subtract $\angle$QDB from both sides, we get

$$\angle CDB - \angle QDB = \angle CDA - \angle QDB$$

$$= \angle CDA - \angle PDA$$

[$\because$ $\angle QDB = \angle PDA$]

$\Rightarrow$ $\angle QDC = \angle PDC$

Hence, proved.

Euclid's axiom used here is:

"If equals are subtracted from equals, the remainders are equal".

We have also used Euclid's postulate: "All right angles are equal to one another".

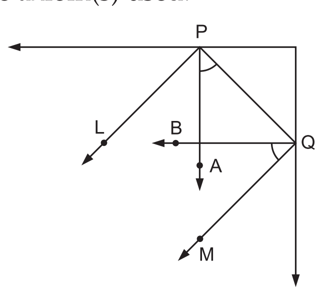
Long Answer Type Questions

11. In the given figure, $\angle$APQ = $\angle$BQM. PA bisects $\angle$LPQ and QB bisects $\angle$PQM.

Show that

$$\angle LPQ = \angle PQM.$$

State the axiom(s) used.



Sol. Given that $\angle APQ = \angle BQM$

$\angle LPQ = 2\angle APQ$

[$\because$ PA bisects $\angle$LPQ] …(1)

and $\angle PQM = 2\angle BQM$

[$\because$ QB bisects $\angle$PQM] …(2)

To prove,

$$\angle LPQ = \angle PQM$$

We have

$\angle APQ = \angle BQM$ [Given]

$\Rightarrow$ $2\angle APQ = 2\angle BQM$ ...[3]

$\Rightarrow$ $\angle LPQ = \angle PQM$

[From (1), (2) and (3)]

Hence, proved.

Euclid's axiom used here is:

"It equals are multiplied by equals, then their products are equal".

or

"Things which are double of equal things are equal to one another".

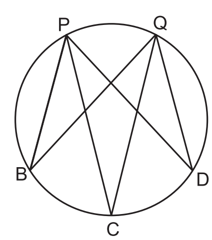
12. In the given figure,

$\angle BPD = \angle BQD$

and $\angle BPC = \angle BQC$

Prove that

$$\angle CPD = \angle CQD$$

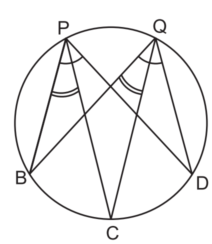


State the axiom(s) used.

Sol. In the given figure,

$\angle BPD = \angle BQD$ [Given]

and $\angle BPC = \angle BQC$ [Given]



To prove,

$$\angle CPD = \angle CQD$$

We have $\angle BPD = \angle BQD$ [Given]

$\Rightarrow \angle BPD - \angle BPC = \angle BQD - \angle BQC$

[$\because$ $\angle BPC = \angle BQC$]

$\Rightarrow$ $\angle CPD = \angle CQD$

Hence, proved.

Euclid's axiom used here is:

"If equals are subtracted from equals, then the remainders are equals".

Higher Order Thinking Skills (HOTS) Question

(Page 61)

1. In finding each angle of a regular polygon of n sides by using the formula:

   Sum of all interior angles of a polygon of n sides $= (n - 2) \times 180°$, it was found that each angle of a regular polygon was 168°. Find the number of sides of this polygon mentioning the Euclid's axiom(s) used.

Sol. A regular polygon of n sides has n equal angles. Since each angle measures 168°, hence, n such angles will measure $(168n)°$.

$\therefore$ According to the problem,

$$(n - 2) \times 180 = 168n$$

$$180n - 168n = 360$$

$$\Rightarrow \quad 12n = 360$$

$$\Rightarrow \quad n = \frac{360}{12} = 30$$

Hence, the required number of sides of the regular polygon is 30.

Euclid's axiom used here is:

"If equals are added to equals, then the whole are equal".

Note: In the above solution, we have added n equal angles, each of measures 168°.

$\therefore$ The sum of angles of the polygon measure $(168n)°$.

Self-Assessment

(Page 62)

Multiple-Choice Questions

1. Which of the following are known as the boundaries of solids?

   (*a*) Curves (*b*) Lines
   (*c*) Points (*d*) Surfaces

Sol. (*d*) Surfaces

We know that any solid is bounded by a surface and not by a curve, a line or a point.

2. The number of dimensions, a cuboid has

   (*a*) 1 (*b*) 2
   (*c*) 3 (*d*) 4

Sol. (*c*) 3

We know that a cuboid has length, breadth and height. Hence, it has 3 dimensions.

3. Which of the following statements is not correct?

   (*a*) A line has one dimension.
   (*b*) A plane has two dimensions.
   (*c*) A circle can be drawn with any radius and at any point.
   (*d*) Two distinct lines can pass through a point in the same direction.

Sol. (*d*) Two distinct lines can never pass through a point in the same direction.

4. The three steps from solids to points are

   (*a*) solids – surfaces – lines – points
   (*b*) solids – lines – surfaces – points
   (*c*) lines – surfaces – points – solids
   (*d*) lines – points – surfaces – solids

Sol. (*a*) solids – surfaces – lines – points

Assertion-Reason Type Questions

Directions (Q. Nos. 5 to 8): Each of these questions contains an assertion followed by reason. Read them carefully, and answer the question on the basis of the following options, select the one that best describes the two statements.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

5. **Assertion (A):** A straight line may be drawn from any one point to any other point.

   Reason (R): A straight line joins all the points on a plane.

Sol. (*c*) Assertion (A) is true but Reason (R) is false.

A straight line may be drawn from any one point to any other point but it is not true that straight line joins all the points on a plane.

6. **Assertion (A):** Only one circle can be drawn from a given center.

Reason (R): From a point many circles can be drawn with different radii.

Sol. (*d*) Assertion (A) is false but Reason (R) is true.

From a point many circles with different radii can be drawn (concentric circles).

7. **Assertion (A):** Two distinct lines cannot have more than one point in common.

Reason (R): Only parallel lines can have more than one common point.

Sol. (*c*) Assertion (A) is true but Reason (R) is false.

Parallel lines do not intersect, so they do not have any point in common.

8. **Assertion (A):** The two lines which are parallel to the same line, are perpendicular to each other.

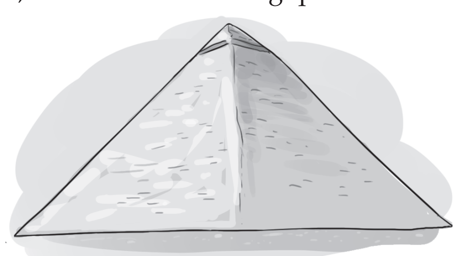
Reason (R): The two lines which are parallel to the same line, are parallel to each other.

Sol. (*d*) Assertion (A) is false but Reason (R) is true.

The two lines parallel to the same line are parallel to each other.

Case Study Based Questions

9. Roshni and her family had their last summer vacation in Egypt. They visited the iconic pyramids of Giza, as well as other tourist attractions in and around. The Great Pyramid of Giza is the oldest and the largest of the pyramids in the Giza pyramid complex. Roshni was astounded to discover that the ancient Egyptians were adept at geometrical calculations. They built pyramids using their understanding of geometry. Now, answer the following questions.



(*a*) What is the shape of the base of a pyramid which is a solid figure?

Sol. Any polygon.

Base of a pyramid is a triangle or square or some other polygon.

(*b*) What is the shape of the side faces of a pyramid?

Sol. Triangles.

(*c*) (*i*) How many dimensions, a surface has?

Sol. 2

Surface has only two dimensions, length and breadth.

or

(*ii*) How many dimensions, a solid has?

Sol. 3

Every solid has 3 dimensions, length, breadth and height.

10. Kavya is passionate to know about India's ancient heritage and culture. She had recently studied about the magnificent Indus Valley civilization in her history classes. She had also studied two of the Indus Valley civilization's major settlements, Harappa and Mohenjodaro. Geometry was used extensively in the Indus Valley culture, as evident by the excavations at these locations. Now, answer the following questions.

(*a*) What was the ratio length : breadth : thickness of the bricks used in the construction work in the Indus Valley civilization?

Sol. 4 : 2 : 1

(*b*) What was the shapes of altars used for household rituals in ancient India?

Sol. Squares and circles.

(*c*) (*i*) How many dimensions, a line has?

Sol. 1

or

(*ii*) What is the number of interwoven isosceles triangles in Sriyantras (in the Atharva Veda)?

Sol. 9

Very Short Answer Type Questions

11. Two distinct lines in a plane have x common point(s). What is the value of x?

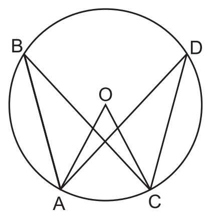
Sol. We know that two distinct lines in a plane can intersect each other at only one point. In other words, only one common point is possible for two distinct lines in a plane. Hence, the required value of x is 1.

12. Can you draw an equilateral triangle with one side equal to the length of the line? If not, state why?

Sol. We know that the three sides of an equilateral triangle are line segments and not lines, since a line does not have a definite length. Hence, the required answer to this problem is "no".

13. In the given figure,

$$\angle ABC = \frac{1}{2}\angle AOC \text{ and } \angle AOC = 2\angle ADC$$



Show that $\angle ABC = \angle ADC$. State the axiom(s) used.

Sol. In the given figure, it is given that

$$\angle ABC = \frac{1}{2}\angle AOC$$

and $\quad \angle AOC = 2\angle ADC$

To prove,

$$\angle ABC = \angle ADC$$

We have

$$2\angle ADC = \angle AOC \qquad \text{[Given]}$$

$$\Rightarrow \quad \angle ADC = \frac{1}{2}\angle AOC \qquad \ldots(1)$$

Given, $\quad \angle ABC = \frac{1}{2}\angle AOC \qquad \ldots(2)$

$\therefore$ From (1) and (2),

$$\angle ABC = \angle ADC$$

Hence, proved.

Euclid's axioms used here are:

(*i*) "If equals are divided by equals, then their quotients are equal" or "Things which are halves of equal things are equal".

(*ii*) "Things which are equal to the same thing are equal to one another".

14. Show that an obtuse angle is greater than an acute angle. State the axiom(s) used.

Sol. We know that an obtuse angle is greater than 90° and an acute angle is less than 90°. Hence, an acute angle is a part of an obtuse angle.

Euclid's axiom used here is:

"The whole is greater than the part".

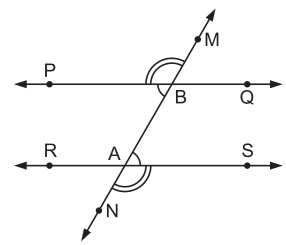
Short Answer Type Questions

15. In the given figure,

$$\angle BAS = \angle PBA \text{ and } \angle PBM = \angle SAN$$

Prove that $\angle MBA = \angle BAN$

State the axiom(s) used.



Sol. In the given figure, it is given that

$$\angle BAS = \angle PBA$$

and $\quad \angle PBM = \angle SAN$

To prove,

$$\angle MBA = \angle BAN$$

We have

$$\angle PBA = \angle BAS \qquad \ldots(1)$$

and $\quad \angle PBM = \angle SAN \qquad \ldots(2)$

Adding (1) and (2), we get

$$\angle PBA + \angle PBM = \angle BAS + \angle SAN$$

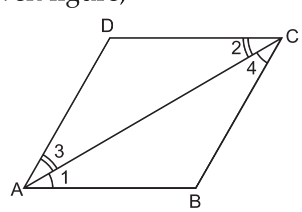
$$\Rightarrow \quad \angle MBA = \angle BAN$$

Hence, proved.

Euclid's axiom used here is the following:

"If equals are added to equals, then the wholes are equal".

16. In the given figure,



$$\angle 1 = \angle 4, \angle 3 = \angle 2$$

and $\quad \angle 2 = \angle 4$

Prove that $\quad \angle 1 = \angle 3$.

State the axiom(s) used. **[CBSE SP 2010]**

Sol. In the given figure, it is given that $\angle 1 = \angle 4$, $\angle 3 = \angle 2$ and $\angle 2 = \angle 4$.

To prove,

$$\angle 1 = \angle 3$$

We have

$$\angle 1 = \angle 4 \quad \ldots(1)$$

and $\angle 2 = \angle 4$ ...(2)

$\therefore$ From (1) and (2), we have

$$\angle 1 = \angle 2 \quad \ldots(3)$$

Also, $\angle 2 = \angle 3$ ...(4)

$\therefore$ From (3) and (4), we have,

$$\angle 1 = \angle 3$$

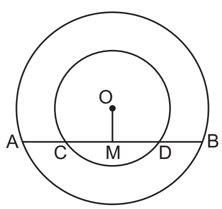
Hence, proved.

Euclid's axiom used here is:

"Things which are equal to the same thing are equal to one another".

Long Answer Type Questions

17. In the given figure, O is the centre of two concentric circles. It is given that AM = BM and CM = DM. Show that AC = BD.



State the axiom(s) used.

Sol. In the given figure, O is the centre of two concentric circles. It is given that

$$AM = BM \quad \ldots(1)$$

and CM = DM ...(2)

To prove, AC = BD.

We have

$$AB = BM$$

Subtract CM from both sides, we get

$$\Rightarrow \quad AM - CM = BM - CM$$

$$= BM - DM \quad \text{[From (2)]}$$

$$\Rightarrow \quad AC = BD$$

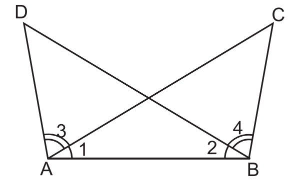
Hence, proved.

Euclid's axiom used here is:

"If equals are subtracted from equals, then the remainders are equal".

18. In the given figure,

(*a*) If $\angle 1 = \angle 2$
and $\angle 3 = \angle 4$.



show that

$$\angle DAB = \angle ABC$$

State the axiom(s) used.

(*b*) If $\angle DAB = \angle ABX$ and $\angle 3 = \angle 4$
Show that $\angle 1 = \angle 2$
State the axiom(s) used.

Sol. (*a*) In the given figure, it is given that

$$\angle 1 = \angle 2$$

and $\angle 3 = \angle 4$

To show,

$$\angle DAB = \angle ABC$$

We have

$$\angle 1 = \angle 2 \quad \ldots(1)$$

$$\angle 3 = \angle 4 \quad \ldots(2)$$

Adding $\angle 3$ both sides in equation (1), we get

$$\angle 1 + \angle 3 = \angle 2 + \angle 3$$

$$= \angle 2 + \angle 4 \quad \text{[From (2)]}$$

$$\angle DAB = \angle ABC$$

Hence, proved.

Euclid's axiom used here is:

"If equals are added to equals, then the wholes are equal".

(*b*) In this case, it is given that

$$\angle DAB = \angle ABC \quad \ldots(3)$$

and $\angle 3 = \angle 4$ ...(4)

To show,

$$\angle 1 = \angle 2$$

From (3), we have

$$\angle DAB = \angle ABC$$

Subtracting $\angle 3$ from both sides, we get

$$\angle DAB - \angle 3 = \angle ABC - \angle 3$$

$$= \angle ABC - \angle 4 \quad \text{[From (4)]}$$

$$\angle 1 = \angle 2$$

Hence, proved.

Euclid's axiom used here is:

"If equals are subtracted from equals, then the remainders are equals".

Let's Compete

(Page 64)

Multiple-Choice Questions

1. "Lines are parallel if they do not intersect" is stated in the form of

(*a*) an axiom. (*b*) a postulate.
(*c*) a definition. (*d*) a proof.

Sol. (*c*) a definition.

We know from the definition of two parallel lines that they do not intersect each other.

2. If $x = y + z$ where $y, z > 0$, then $x > y$ and $x > z$. In this statement, which of the following axioms of Euclid is used?

(*a*) Things which are equal to the same thing are equal to one another.
(*b*) The whole is greater than the part.
(*c*) If equals are subtracted from equals, the remainders are equal.
(*d*) If equals are added to equals, the whole are equal.

Sol. (*b*) The whole is greater than the part.

We see from the given equation $x = y + z$ where $y, z > 0$ that $y + z$ is greater than both y and z.

In other words,

Each of y and z is a part of the whole $y + z$ i.e. x.

$\therefore$ $x > y$ and $x > z$ follows from Euclid's axiom: "The whole is greater than the part".

3. One of Euclid's axioms is "Things which are double of the same things are equal to one another".
An example of this axiom is

(*a*) If $2x = 4y$, then $2x - 4y = 0$.
(*b*) If AB = CD, then AB + x = CD + x.
(*c*) An angle subtended by an arc of a circle at its centre is double the angle subtended by the arc at a point on its circumference.
(*d*) All diameters of a circle are equal.

Sol. (*d*) All diameters of a circle are equal.

We know that a diameter of a circle is twice the length of its radius. Also, all radii of a circle are equal in length. Hence, all diameters are also equal.

4. Trichotomy law or axiom is followed in which of the following example.

(*a*) If x and y are two numbers, then only one of the following statements is true: $x = y$, $x > y$ and $x < y$.
(*b*) If $x = y$, then $\frac{x}{a} = \frac{x}{b}$ where $a, b \neq 0$.
(*c*) The lengths of all sides of an equilateral triangle are equal.
(*d*) The sum of three angles of any triangle is 180°.

Sol. (*a*) If x and y are two numbers, then only one of the following statements is true: $x = y$, $x > y$ and $x < y$.

Euclid's trichotomy law states that of two quantities of the same kind, the first is greater than, equal to or less than the second.

5. Assuming that the sum of three angles of a triangle is 180°, we can prove, by using some axiom(s) of Euclid that the sum of four angles of a quadrilateral is

(*a*) 190° (*b*) 200°
(*c*) 360° (*d*) 300°

Sol. (*c*) 360°

A quadrilateral can be divided into two triangles by drawing a diagonal. We know that the sum of three angles of a triangle is 180°. Hence, the sum of three angles of each of two triangles within the quadrilateral is 180°. We can prove that the sum of four angles of the quadrilateral is 360°, by using suitable axioms of Euclid.

6. The sum of interior angles of any regular polygon of n sides is given by $(n - 2) \times 180°$. By using same axiom(s) of Euclid we can show that each interior angle of a regular polygon of 60 sides is

(*a*) 164° (*b*) 174°
(*c*) 175° (*d*) 100°

Sol. (*b*) 174°

In this case, the sum of all angles of a regular polygon of 60 sides = $(60 - 2) \times 180° = 58 \times 180°$.
Since all angles of 60 sided regular polygon are equal, hence each angle will measure

$$\frac{58 \times 180°}{60} = 174°$$

7. The ratio of the radii of two circles is equal to the ratio of their circumferences. This result can be proved by using the result $C = 2\pi r$ where r is the radius of a circle and C is its circumference, by using the Euclid's axiom.

(*a*) If equals are added to equals, the whole are equal.
(*b*) If equals are subtracted from equals, the remainders are equal.
(*c*) The whole is the greater than the part.
(*d*) Things which are equal to the same thing are equal to one another.

Sol. (*d*) Things which are equal to the same things are equal to one another.

Let r_1 and r_2 be the radii of the two circles and C_1 and C_2 be their respective circumferences. Then we have

$$C_1 = 2\pi r_1 \qquad \ldots(1)$$

and $$C_2 = 2\pi r_2 \qquad \ldots(2)$$

From (1), $$\frac{C_1}{r_1} = 2\pi$$

[Dividing both sides by r_1] …(3)

and From (2),

$$\frac{C_2}{r_2} = 2\pi$$

[Dividing both sides by r_2] …(4)

From (3) and (4), we have

$$\frac{C_1}{r_1} = \frac{C_2}{r_2} \qquad \ldots(5)$$

Multiplying both sides of equation (5) by $\frac{r_2}{C_1}$, we get

$$\frac{C_1}{r_1} \times \frac{r_2}{C_1} = \frac{C_2}{r_2} \times \frac{r_2}{C_1}$$

$$\Rightarrow \qquad \frac{r_2}{r_1} = \frac{C_2}{C_1}$$

Hence, proved.

Euclid's axiom used here is:

"Things which are equal to the same thing are equal to one another".

8. Euclid stated that all right angles are equal to one another in the form of

(*a*) a postulate.

(*b*) a definition.

(*c*) a proof.

(*d*) an axiom.

Sol. (*a*) a postulate.

The given statement is a universal truth which is used in geometry only. Hence, it is a postulate.

9. "If equals are multiplied by equals then their products are equal". An example of this axiom is

(*a*) The area and the circumference of a circle of radius r are respectively πr^2 and $2\pi r$.

(*b*) If $2x = 3y$, then $2x \pm k = 3y \pm k$.

(*c*) If A > B, then $-\frac{A}{2} < \frac{B}{2}$.

(*d*) If AC is an arc of a circle with centre at O and if D and B are two points on the remaining part of the circumference, and if $\angle AOB = 2\angle ADC$ and $\angle ABC = \frac{1}{2}\angle AOB$, then $\angle ABC = \angle ADC$.

Sol. (*c*) If A > B, then $\frac{-A}{2} < \frac{B}{2}$

Let A, B > 0 and A = B + x where $x > 0$. Clearly A > B, by axiom: "The whole is greater than the part".

$$\therefore \qquad -\frac{A}{2} = -\frac{B}{2} - \frac{x}{2}$$

By using axiom: "If equals are multiplied by equals, then their products are equal". But by definition, $-\frac{B}{2} - \frac{x}{2} < \frac{B}{2}$, since a negative number is always less than a positive number, by definition. Hence, $-\frac{A}{2} < \frac{B}{2}$.

10. If $\Delta ABC \cong \Delta DEF$, then AB = DE, AC = DF, BC = EF, $\angle A = \angle D$, $\angle B = \angle E$ and $\angle C = \angle F$. This important result follows Euclid's axiom stated as

(*a*) Things which are equal to the same thing are equal to one another.

(*b*) Things which coincide with one another are equal in all respects.

(*c*) If equals are subtracted from equals, the remainders are equal.

(*d*) Things which are halves to the same things are equal to one another.

Sol. (*b*) Things which coincide with one another are equal in all respects.

We know that three angles and three sides are six components of a triangle. If these six components of each triangle coincide respectively with those of another triangle, then the two triangles will be congruent to each other. This clearly follows from Euclid's axiom: "Things which coincide with one another are equal in all respects".

Life Skills

(Page 65)

1. Two businessmen, Mahesh and Badal, were rich people in a village. Mahesh's income was more than that of Badal. They decided to donate 25% of their income for the education of poor students in a school in the village. After donation, it was found that the balance of Mahesh's income was double that of Badal.

(*a*) Prove that Mahesh's income was double the income of Badal.

(*b*) In proving this result, which axiom(s) of Euclid will you use?

Sol. (*a*) Let the incomes of Mahesh and Badal be ₹ x and ₹ y respectively, where $x > y > 0$.

According to the problem,

$$x - \frac{x}{4} = 2\left(y - \frac{y}{4}\right)$$

$$\Rightarrow \qquad \frac{3x}{4} = \frac{3y}{2}$$

$$\Rightarrow \qquad \frac{3x}{4} \div \frac{3}{4} = \frac{3y}{2} \div \frac{3}{4} \qquad \ldots(1)$$

$$\Rightarrow \qquad \frac{3x}{4} \times \frac{4}{3} = \frac{3y}{2} \times \frac{4}{3}$$

$$\Rightarrow \qquad x = 2y \qquad \ldots(2)$$

From (2), we see that Mahesh's income was double the income of Badal.

Hence, proved.

(*b*) We have obtained equation (2) from (1) by dividing both sides by the same number $\frac{3}{4}$.

Hence, we have used the following axiom of Euclid's:

"If equals are divided by equals, then the quotients are equal".