2

Polynomials

Checkpoint (Page 19)

1. Simplify $-3b(ab + b^2) + 300$ and find its values for $a = 3$ and $b = 4$.

Sol. We have

$-3b(ab + b^2) + 300 = -3ab^2 - 3b^3 + 300$

When $a = 3$ and $b = 4$, the given expression

$$= -3 \times 3 \times 4^2 - 3 \times 4^3 + 300$$
$$= -144 - 192 + 300$$
$$= -336 + 300$$
$$= -36$$

$\therefore$ The required value is -36.

2. Find the product: $x^3y^2(x^2 + y^2 + z^2)$.

Sol. $x^3y^2 (x^2 + y^2 + z^2) = x^3y^2 \times x^2 + x^3y^2 \times y^2 + x^3y^2 \times z^2$

$$= x^5y^2 + x^3y^4 + x^3y^2z^2$$

3. Using a suitable identity, determine the product: $\left(x+\frac{3}{4}\right)\left(x+\frac{3}{4}\right)$.

Sol. $$\left(x+\frac{3}{4}\right)\left(x+\frac{3}{4}\right) = \left(x+\frac{3}{4}\right)^2$$

$$= x^2 + 2x \times \frac{3}{4} + \left(\frac{3}{4}\right)^2$$

$$= x^2 + \frac{3}{2}x + \frac{9}{16}$$

4. By using suitable identity, simplify $(xy - yz)^2$.

Sol. $$(xy - yz)^2 = (xy)^2 + (yz)^2 - 2xy \times yz$$

$$= x^2y^2 + y^2z^2 - 2xy^2z$$

5. Using the identity

$$(x + a)(x + b) = x^2 + (a + b)x + ab,$$

find the product: $(2x + 3)(2x - 5)$.

Sol. $$(2x + 3)(2x - 5) = 2\left(x+\frac{3}{2}\right) \times 2\left(x-\frac{5}{2}\right)$$

$$= 2 \times 2\left(x+\frac{3}{2}\right)\left(x-\frac{5}{2}\right)$$

$$= 4\left[x^2 + \left(\frac{3}{2}-\frac{5}{2}\right)x + \left(\frac{3}{2}\right)\left(\frac{-5}{2}\right)\right]$$

$$= 4\left[x^2 - x - \frac{15}{4}\right]$$

$$= 4x^2 - 4x - 15$$

6. Evaluate $(91)^2$ by using identity.

Sol. We have $91^2 = (90 + 1)^2$

$$= 90^2 + 2 \times 90 \times 1 + 1^2$$
$$= 8100 + 180 + 1$$
$$= 8281$$

7. Simplify $(4x + 3)^2 - (4x - 3)^2$.

Sol. $(4x + 3)^2 - (4x - 3)^2$

$= (4x)^2 + 2 \times 4x \times 3 + 3^2 - \{(4x)^2 - 2 \times 4x \times 3 + 3^2\}$

$= 16x^2 + 24x + 9 - 16x^2 + 24x - 9$

$= 48x$

8. Find the continued product: $(x + 2)(x - 2)(x^2 + 4)$.

Sol. $(x + 2)(x - 2)(x^2 + 4)$

$$= (x^2 - 2^2)(x^2 + 4)$$
$$= (x^2 - 4)(x^2 + 4)$$
$$= (x^2)^2 - 4^2$$
$$= x^4 - 16$$

9. If $2x + 3y = 11$ and $xy = 8$, find the value of $4x^2 + 9y^2$.

Sol. $$4x^2 + 9y^2 = (2x)^2 + (3y)^2$$

$$= (2x + 3y)^2 - 2 \times 2x \times 3y$$

$$= 11^2 - 12 \times 8$$
$$= 121 - 96 = 25$$

10. If $x - \frac{1}{x} = 3$, find the value of $x^2 + \frac{1}{x^2}$.

Sol. We have

$$x^2 + \frac{1}{x^2} = \left(x - \frac{1}{x}\right)^2 + 2x \times \frac{1}{x}$$
$$= 3^2 + 2$$
$$= 11$$

Check Your Progress 1

(Page 21)

Multiple-Choice Questions

1. Consider the given expression $x^{p-3} + 7$. Find the least integer value of p, for which the given expression is a polynomial in degree 1.

(*a*) 1 (*b*) 2

(*c*) 3 (*d*) 4

Sol. (*d*) 4

For the expression to be a polynomial in degree 1,

$$x^{p-3} = x^1$$

Equating the coefficients,

$$p - 3 = 1$$
$$\Rightarrow \quad p = 1 + 3 = 4$$

2. Which of the following is a polynomial?

(*a*) $12x^{-2} + 5x^2 + 1$ (*b*) $\sqrt{3y} - 7$

(*c*) $p^{-1}z + p^{-2}z^2 + 1$ (*d*) $\frac{x^3}{2} - \frac{3}{x^2}$

Sol. (*c*) $p^{-1}z + p^{-2}z^2 + 1$

$p^{-1}z + p^{-2}z^2 + 1 = \frac{1}{p}z + \frac{1}{p^2}z^2 + 1$ has only non-negative integral powers of z. So, it is a polynomial.

3. $\sqrt{3}$ is a polynomial of degree

(*a*) 1 (*b*) 2

(*c*) 0 (*d*) $\frac{1}{2}$

Sol. (*c*) 0

We know that any constant is a polynomial of degree 0.

Since $\sqrt{3}$ is a constant, hence it is a polynomial of degree 0.

4. Degree of the zero polynomial is

(*a*) 0 (*b*) 1

(*c*) not defined (*d*) any natural number

Sol. (*c*) not defined

The degree of the zero polynomial is not defined.

5. Which of the following is a zero of the polynomial $x^3 + 2x^2 - 2x - 1$?

(*a*) –1 (*b*) 1

(*c*) –2 (*d*) 2

Sol. (*b*) 1

We see that when $x = 1$, then

$$x^3 + 2x^2 - 2x - 1 = 1^3 + 2 \times 1^2 - 2 \times 1 - 1$$
$$= 3 - 3 = 0$$

6. If $p(x) = x + 7$, then $p(x) + p(-x)$ is equal to

(*a*) $2x$ (*b*) 7

(*c*) 0 (*d*) 14

Sol. (*d*) 14

$$p(x) = x + 7$$
$$p(-x) = -x + 7$$
$$p(x) + p(-x) = x + 7 - x + 7 = 14$$

7. If $p(x) = x^2 - 3\sqrt{3}x + 3$, then $p(3\sqrt{3})$ is equal to

(*a*) 0 (*b*) $3\sqrt{3}$

(*c*) 27 (*d*) 3

Sol. (*d*) 3

$$p(x) = x^2 - 3\sqrt{3}\,x + 3$$
$$p(3\sqrt{3}) = (3\sqrt{3})^2 - 3\sqrt{3}\,(3\sqrt{3}) + 3$$
$$= 9 \times 3 - 9 \times 3 + 3$$
$$= 27 - 27 + 3$$
$$= 3$$

Very Short Answer Type Questions

8. What is the coefficient of x^3 in the polynomial $9x^3 + \frac{x}{8} + 6x^5 + \frac{3}{5}$?

Sol. The coefficient of x^3 in the given polynomial is 9.

9. Find the value of the polynomial $5x^2 - 4$ at $x = 2$.

Sol. The required value is $5 \times 2^2 - 4 = 20 - 4 = 16$.

10. Classify the following as constant, linear, quadratic and cubic polynomials:

(*a*) $x^2 + \sqrt{7}x + 9$ (*b*) –200

(*c*) $t - 5t^3$ (*d*) $4x - \sqrt{3}$

Sol. (*a*) Quadratic (degree 2)

(*b*) Constant (degree 0)

(*c*) Cubic (degree 3)

(*d*) Linear (degree 1)

11. Which of the following expressions are polynomials? Justify your answers.

(*a*) $\sqrt{3}x^3 - \frac{1}{5}x + \frac{\sqrt{3}}{\sqrt{7}}$

(*b*) $9\sqrt{x} - 7$

(*c*) $\frac{5}{x^2} - 8$

(*d*) $7x^{-2} - 3x^2 + 5$

(*e*) $\sqrt{8}x^3 - \frac{3}{\sqrt{5}}x + \frac{1}{\sqrt{7}}$

(*f*) $\sqrt{5}x^2 + 9x + 3$

Sol. We see that all exponents of x in each of the expressions in (*a*), (*e*) and (*f*) are non-negative integer, and those in the rest of the expression are not. Hence, only expressions in (*a*), (*e*) and (*f*) are polynomials.

Short Answer Type Questions

12. If $p(x) = ax^2 - 3x + 7b$ and 1, –2 are zeroes of $p(x)$, then find the value of a and b.

Sol. We see that $p(1) = 0$

$\therefore \quad a - 3 + 7b = 0 \quad \ldots(1)$

and $p(-2) = 0$

$\Rightarrow \quad a(-2)^2 + 3 \times 2 + 7b = 0$

$4a + 6 + 7b = 0 \quad \ldots(2)$

Subtracting (1) from (2), we get

$3a + 9 = 0$

$\therefore \quad a = -3$

$\therefore$ From (1), $7b = 3 - a$

$= 3 + 3$

$= 6$

$\Rightarrow \quad b = \frac{6}{7}$

$\therefore$ The required values of a and b are respectively -3 and $\frac{6}{7}$.

13. If $x = \frac{-1}{3}$ is a zero of a polynomial $p(x) = 27x^3 - ax^2 - x + 3$, then find the value of a.

Sol. We have $p\left(-\frac{1}{3}\right) = 0$

$\Rightarrow 27\left(-\frac{1}{3}\right)^3 - a\left(-\frac{1}{3}\right)^2 + \frac{1}{3} + 3 = 0$

$\Rightarrow \quad -1 - \frac{a}{9} + \frac{1}{3} + 3 = 0$

$\Rightarrow \quad 2 + \frac{1}{3} = \frac{a}{9}$

$\Rightarrow \quad a = 18 + \frac{1}{3} \times 9$

$= 18 + 3$

$= 21$

$\therefore$ The required value of a is 21.

Long Answer Type Questions

14. If $p(x) = x^3 + 3x - 2x^2 - 6$ and $q(x) = 3x^2 - 7x - 8$, then prove that $q(-1)$ is a zero of $p(x)$.

Sol. We have $q(-1) = 3(-1)^2 + 7 \times 1 - 8$

$= 3 + 7 - 8$

$= 2$

$\therefore \quad p\{q(-1)\} = p(2)$

$= 2^3 + 3 \times 2 - 2 \times 2^2 - 6$

$= 8 + 6 - 8 - 6$

$= 0$

$\therefore \quad p\{q(-1)\} = 0$

$\therefore$ $q(-1)$ is a zero of $p(x)$.

Hence, proved.

15. If 2 and –2 are the zeroes of the polynomial $p(x) = ax^4 + 2x^3 - 3x^2 + bx - 4$, find the values of a and b and hence $p(-3)$.

Sol. We have $p(2) = 0$

$\Rightarrow \quad a \times 2^4 + 2 \times 2^3 - 3 \times 2^2 + b \times 2 - 4 = 0$

$\Rightarrow \quad 16a + 16 - 12 + 2b - 4 = 0$

$\Rightarrow \quad 16a + 2b = 0$

$\Rightarrow \quad 8a + b = 0 \quad \ldots(1)$

Again, $p(-2) = 0$

$\therefore \; a \times (-2)^4 + 2 \times (-2)^3 - 3 \times (-2)^2 + b \times (-2) - 4 = 0$

$\Rightarrow \quad 16a - 16 - 12 - 2b - 4 = 0$

$\Rightarrow \quad 16a - 2b = 32$

$\Rightarrow \quad 8a - b = 16 \quad \ldots(2)$

Adding (1) and (2), we get

$16a = 16$

$\Rightarrow \quad a = 1$

$\therefore$ From (1), $8 + b = 0$

$\Rightarrow \quad b = -8$

$\therefore$ The required values of a and b are respectively 1 and –8.

Now, $p(-3) = a \times (-3)^4 + 2 \times (-3)^3 - 3(-3)^2 + b \times (-3) - 4$

$= 81a - 54 - 27 - 3b - 4$
$= 81a - 3b - 85$
$= 81 \times 1 - 3 \times (-8) - 85$
[Putting the values of a and b]
$= 81 + 24 - 85$
$= 20$

Hence, the required values of a, b and $p(-3)$ are respectively 1, −8 and 20.

Check Your Progress 2

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Multiple-Choice Questions

1. The zero of the polynomial $p(x)$ where $p(x) = ax + 1$ where $a \neq 0$, is

(a) 1 (b) $-a$
(c) 0 (d) $-\frac{1}{a}$ **[CBSE SP 2010]**

Sol. (d) $\frac{-1}{a}$

Putting $p(x) = 0$, we get

$$ax + 1 = 0$$
$$\Rightarrow \quad x = \frac{-1}{a}$$

$\therefore$ The zero of $p(x)$ is $-\frac{1}{a}$

2. The value of k for which the polynomial $x^3 + 3x^2 - 3x + k$ has −3 as its zero, is

(a) −9 (b) −3
(c) 9 (d) 12 **[CBSE SP 2011]**

Sol. (a) − 9

Since −3 is a zero of the polynomial $p(x) = x^3 + 3x^2 - 3x + k$,

$$\therefore \quad p(-3) = 0$$
$$\Rightarrow \quad (-3)^3 + 3\,(-3)^2 + 3 \times 3 + k = 0$$
$$\Rightarrow \quad -27 + 27 + 9 + k = 0$$
$$\Rightarrow \quad k = -9$$

3. If $x + 1$ is a factor of the polynomial $2x^2 + kx$, then the value of k is

(a) −2 (b) 2
(c) −3 (d) 4

Sol. (b) 2

Since $x + 1$ is a factor of the polynomial $2x^2 + kx$, hence by factor theorem, $p(-1) = 0$

$$\therefore \quad 2\,(-1)^2 + k(-1) = 0$$
$$\Rightarrow \quad 2 - k = 0$$
$$\Rightarrow \quad k = 2$$

4. Which of these is a factor of the polynomial $p(x) = x^3 + 4x + 5$?

(a) $x - 1$ (b) $x - 2$
(c) $x + 1$ (d) $x + 2$

Sol. (c) $x + 1$

Putting $x = -1$ in the polynomial

$$p(x) = x^3 + 4x + 5$$
$$\Rightarrow \quad p(-1) = (-1)^3 + 4(-1) + 5$$
$$\Rightarrow \quad p(-1) = -1 - 4 + 5$$
$$\Rightarrow \quad p(-1) = -5 + 5$$
$$p(-1) = 0$$

$\therefore$ $x + 1$ is factor of the polynomial

$$p(x) = x^3 + 4x + 5$$

5. The polynomial $(x - p)$ is a factor of the polynomial $x^4 - 2x^2 + ax + a$, where a is a constant. Which of these is the correct relation between p and a?

(a) $a = \dfrac{p^2(2-p^2)}{1+p}$ (b) $a = \dfrac{p^2(2+p^2)}{1-p}$

(c) $a = \dfrac{p^2(2-p^2)}{1-p}$ (d) $a = \dfrac{p^2(2+p^2)}{1+p}$

Sol. (a) $a = \dfrac{p^2(2-p^2)}{1+p}$

Since $(x - p)$ is a factor of the polynomial $p(x)$, by factor theorem, $p(p) = 0$

$$\therefore \quad p^4 - 2p^2 + a\,p + a = 0$$
$$\Rightarrow \quad a(1 + p) = 2p^2 - p^4$$
$$\Rightarrow \quad a = \frac{p^2(2-p^2)}{1+p}$$

6. $x + 1$ is a factor of the polynomial

(a) $x^3 + x^2 - x + 1$
(b) $x^4 + x^3 + x^2 + 1$
(c) $x^3 + x^2 + x + 1$
(d) $x^4 + 3x^3 + 3x^2 + x + 1$

Sol. (c) $x^3 + x^2 + x + 1$

Since $(x + 1)$ is a factor of $p(x)$, by factor theorem $p(-1) = 0$

$$(-1)^3 + (-1)^2 + (-1) + 1 = 0$$
$$\Rightarrow \quad -1 + 1 - 1 + 1 = 0$$
$$0 = 0$$

7. The polynomial $p(x) = x^3 - 5x^2 - x + 5$ is such that $p(1) = 0$ and $p(-1) = 0$. Which of these is equivalent to $p(x)$?

(a) $(x + 1)\,(x - 5)$
(b) $(x - 1)\,(x + 5)$
(c) $(x - 1)\,(x + 1)\,(x - 5)$
(d) $(x - 1)\,(x + 1)\,(x + 5)$

Sol. (*c*) $(x-1)(x+1)(x-5)$

We have, $p(x) = x^3 - 5x^2 - x + 5$

$$= x^2(x-5) - 1(x-5)$$
$$= (x^2-1)(x-5)$$
$$= (x-1)(x+1)(x-5)$$

8. One of the factors of $(25x^2 - 1) + (1 + 5x)^2$ is

(*a*) $5 - x$ (*b*) $5 + x$

(*c*) $5x - 1$ (*d*) $10x$

Sol. (*d*) $10x$

$$(25x^2 - 1) + (1 + 5x)^2 = [(5x)^2 - (1)] + (1 + 5x)^2$$
$$= [(5x+1)(5x-1)] + (1+5x)^2$$
$$= (1+5x)[5x - 1 + 1 + 5x]$$
$$= 10x(1+5x)$$

Very Short Answer Type Questions

9. Show that $3x + 2$ is a factor of $3x^3 + x^2 - 20x - \frac{116}{9}$.

Sol. Putting $x = -\frac{2}{3}$ in $p(x) = 3x^3 + x^2 - 20x - \frac{116}{9}$, we get,

$$p\left(-\frac{2}{3}\right) = 3 \times \left(-\frac{2}{3}\right)^3 + \left(-\frac{2}{3}\right)^2 + 20 \times \frac{2}{3} - \frac{116}{9}$$
$$= \frac{-8}{9} + \frac{4}{9} + \frac{40}{3} - \frac{116}{9}$$
$$= \frac{-8 + 4 + 120 - 116}{9}$$
$$= \frac{0}{9} = 0$$

$\therefore$ By factor theorem, $x + \frac{2}{3}$ is a factor of $p(x)$

or, $\frac{3x+2}{3}$ is a factor of $p(x)$ or $3x + 2$ is a factor of $p(x)$.

Hence, proved.

10. Using the factor theorem, show that the polynomial $p(x) = x^3 - 2x^2 + 3x - 18$ is a multiple of $x - 3$.

Sol. We see that

$$p(3) = 3^3 - 2 \times 3^2 + 3 \times 3 - 18$$
$$= 27 - 18 + 9 - 18$$
$$= 36 - 36$$
$$= 0$$

$\therefore$ By factor theorem, $x - 3$ is a factor of $p(x)$, i.e. $p(x)$ is a multiple of $x - 3$.

11. Prove that $(x + 1)$ is a factor of each of the polynomial $x^{99} + 1$ and $x^{100} - 1$.

Sol. Since $(x + 1)$ is a factor of the polynomial $p(x)$, by factor theorem, $p(-1) = 0$

Putting $x = -1$ in the polynomial $x^{99} + 1$

$$x^{99} + 1 = (-1)^{99} + 1$$
$$= -1 + 1 = 0$$

Putting $x = -1$ in the polynomial $x^{100} - 1$

$$x^{100} - 1 = (-1)^{100} - 1$$
$$= +1 - 1 = 0$$

12. Use the factor theorem to determine whether $g(x)$ is a factor of $p(x)$, where $p(x) = x^3 + 7x^2 - 7x + 8$, $g(x) = x + 8$.

Sol. If $g(x) = x + 8$ is a factor of $p(x)$, then $p(-8) = 0$

Putting the value $x = -8$ in $p(x) = x^3 + 7x^2 - 7x + 8$

$$p(x) = x^3 + 7x^2 - 7x + 8$$
$$= (-8)^3 + 7(-8)^2 - 7(-8) + 8$$
$$= -512 + 448 + 56 + 8$$
$$= -512 + 512 = 0$$

$\therefore$ $g(x)$ is a factor of $p(x)$.

13. Find the value of a if $x - a$ is a factor of $x^3 - a^2x + x + 2$.

Sol. If $x - a$ is a factor of $p(x)$, by factor theorem

$$p(a) = 0$$
$$a^3 - a^2 \times a + a + 2 = 0$$
$$a^3 - a^3 + a = -2$$
$$a = -2$$

14. The polynomial $(4x - 3)$ is a factor of the polynomial $p(x) = 4x^3 + x^2 - 11x + 2r$. What is the value of r?

Sol. If $(4x - 3)$ is a factor of the polynomial $p(x)$, then by factor theorem

$$p\left(\frac{3}{4}\right) = 0$$
$$4\left(\frac{3}{4}\right)^3 + \left(\frac{3}{4}\right)^2 - 11 \times \frac{3}{4} + 2r = 0$$
$$4 \times \frac{27}{64} + \frac{9}{16} - \frac{33}{4} + 2r = 0$$
$$\frac{27}{16} + \frac{9}{16} - \frac{33}{4} + 2r = 0$$
$$\frac{18}{16} - \frac{33}{4} + 2r = 0$$
$$-\frac{114}{16} = -2r$$
$$r = \frac{114}{16 \times 2} = \frac{57}{16}$$

Short Answer Type Questions

15. If $x - 1$ and $x + 3$ are factors of $x^3 - ax^2 - 13x + b$, prove that a and b satisfy the relations $a - b + 12 = 0$ and $9a - b - 12 = 0$.

Sol. If $x - 1$ is a factor of $p(x)$, then by factor theorem

$$p(1) = 0$$
$$p(1) = (1)^3 - a(1)^2 - 13(1) + b$$
$$= 1 - a - 13 + b$$
$$p(1) = 0$$
$$\therefore \quad 1 - a - 13 + b = 0$$
$$\Rightarrow \quad -a - 12 + b = 0$$
$$\Rightarrow \quad a - b + 12 = 0$$

Again, if $x + 3$ is a factor of $p(x)$, then by factor theorem, $p(-3) = 0$

$$p(-3) = (-3)^3 - a(-3)^2 - 13(-3) + b$$
$$= -27 - 9a + 39 + b$$
$$p(-3) = -9a + 12 + b$$

Given $\quad p(-3) = 0$

$$\therefore \quad -9a + 12 + b = 0$$
$$\Rightarrow \quad 9a - b - 12 = 0$$

16. Show that both $x + 2$ and $3x - 2$ are factors of the polynomial $3x^4 + 4x^3 - x^2 + 4x - 4$.

Sol. If $x + 2$ is a factor of the polynomial, then by factor theorem $p(-2) = 0$.

$$p(-2) = 3(-2)^4 + 4(-2)^3 - (-2)^2 + 4(-2) - 4$$
$$= 3 \times 16 - 4 \times 8 - 2 \times 2 - 8 - 4$$
$$= 48 - 32 - 4 - 8 - 4$$
$$= 0$$

If $3x - 2$ is a factor of the polynomial, then by factor theorem $p\left(\frac{2}{3}\right) = 0$

$$p\left(\frac{2}{3}\right) = 3\times\left(\frac{2}{3}\right)^4 + 4\times\left(\frac{2}{3}\right)^3 - \left(\frac{2}{3}\right)^2 + 4\left(\frac{2}{3}\right) - 4$$
$$= 3\times\frac{16}{81} + 4\times\frac{8}{27} - \frac{4}{9} + \frac{8}{3} - 4$$
$$= \frac{16}{27} + \frac{32}{27} - \frac{4}{9} + \frac{8}{3} - 4$$
$$= \frac{16 + 32 - 12 + 72 - 108}{27}$$
$$= \frac{0}{27}$$
$$= 0$$

$\therefore \quad x + 2$ and $3x - 2$ are factors of the polynomial $3x^4 + 4x^3 - x^2 + 4x - 3$.

17. If $ax^2 + bx - c$ is exactly divisible by both $2x + 1$ and $3x - \frac{1}{2}$, prove that $b = 4c$.

Sol. If $2x + 1$ exactly divides the polynomial $p(x)$, then by factor theorem, $p\left(-\frac{1}{2}\right) = 0$

$$p\left(-\frac{1}{2}\right) = a\left(-\frac{1}{2}\right)^2 + b\left(-\frac{1}{2}\right) - c$$
$$= a\times\frac{1}{4} - \frac{1}{2}b - c$$

But $\quad p\left(-\frac{1}{2}\right) = 0$

$$\therefore \quad \frac{a}{4} - \frac{b}{2} - c = 0$$
$$\Rightarrow \quad a - 2b = 4c \qquad ...(1)$$

Also $3x - \frac{1}{2}$ exactly divides the polynomial, then by factor theorem, $p\left(\frac{1}{6}\right) = 0$

$$p\left(\frac{1}{6}\right) = a\left(\frac{1}{6}\right)^2 + b\left(\frac{1}{6}\right) - c$$
$$= \frac{a}{36} + \frac{b}{6} - c$$

But $\quad p\left(\frac{1}{6}\right) = 0$

$$\therefore \quad \frac{a}{36} + \frac{b}{6} - c = 0$$
$$\Rightarrow \quad a + 6b = 36c \qquad ...(2)$$

From (2),

$$a = 36c - 6b$$

Putting the value of a from (2) in (1), we get

$$36c - 6b - 2b = 4c$$
$$-8b = -32c$$
$$\Rightarrow \quad b = 4c$$

18. If both $x - 2$ and $x - \frac{1}{2}$ are factors of $px^2 + 5x + r$, show that $p = r$.

Sol. If $x - 2$ is a factor of the $p(x)$, then $p(2) = 0$

$$p(2) = p(2)^2 + 5 \times 2 + r$$
$$= 4p + 10 + r$$

But $p(2) = 0$

$$\therefore \quad 4p + 10 + r = 0$$
$$\Rightarrow \quad 4p + r = -10 \qquad ...(1)$$

Also, $x - \frac{1}{2}$ is also factor of $px^2 + 5x + r$

$$p\left(\frac{1}{2}\right) = p\left(\frac{1}{2}\right)^2 + 5\left(\frac{1}{2}\right) + r$$
$$= \frac{p}{4} + \frac{5}{2} + r$$

But $p\left(\frac{1}{2}\right) = 0$

$$p + 10 + 4r = 0$$

$$p + 4r = -10 \qquad ...(2)$$

Subtracting (2) from (1), we get

$$3p - 3r = 0$$

$$\Rightarrow \qquad p = r$$

19. For what value of m is $x^3 - 2mx^2 + 16$ is divisible by $x + 2$?

Sol. If $p(x)$ is divisible by $x + 2$, then by factor theorem, $p(-2) = 0$

$$p(-2) = (-2)^3 - 2m(-2)^2 + 16$$
$$= -8 - 8m + 16$$
$$-8 - 8m + 16 = 0$$
$$\Rightarrow \qquad -8m + 8 = 0$$
$$\Rightarrow \qquad -8m = -8$$
$$\Rightarrow \qquad m = 1$$

Long Answer Type Questions

20. If $p(x) = x^3 + 3x^2 - 2x + 4$, then find the value of $p(2) + p(-2) - p(0)$.

Sol. $p(x) = x^3 + 3x^2 - 2x + 4$

$$p(2) = (2)^3 + 3(2)^2 - 2 \times 2 + 4$$
$$= 8 + 3 \times 4 - 4 + 4$$
$$= 8 + 12 - 4 + 4$$
$$p(2) = 20$$
$$p(-2) = (-2)^3 + 3(-2)^2 - 2 \times (-2) + 4$$
$$= -8 + 12 + 4 + 4$$
$$\Rightarrow \qquad p(-2) = 12$$
$$p(0) = 0 + 3 \times 0 - 2 \times 0 + 4$$
$$\Rightarrow \qquad p(0) = 4$$
$$\therefore \quad p(2) + p(-2) - p(0)$$
$$= 20 + 12 - 4$$
$$= 28$$

21. If $2x - 1$ and $3x + 2$ are factors of $ax^3 + bx^2 + 8x - 3$, show that a and b are given by the equations $a + 2b + 8 = 0$ and $8a - 12b + 225 = 0$.

Sol. If $2x - 1$ is a factor of $p(x)$, then by factor theorem,

$$p\left(\frac{1}{2}\right) = 0$$
$$p\left(\frac{1}{2}\right) = a\left(\frac{1}{2}\right)^3 + b\left(\frac{1}{2}\right)^2 + 8\left(\frac{1}{2}\right) - 3$$
$$= \frac{a}{8} + \frac{b}{4} + 4 - 3$$
$$= \frac{a}{8} + \frac{b}{4} + 1$$

But $p\left(\frac{1}{2}\right) = 0$

$$\therefore \qquad \frac{a}{8} + \frac{b}{4} + 1 = 0$$
$$\Rightarrow \qquad a + 2b + 8 = 0$$

Again, if $3x + 2$ is a factor of $p(x)$, then

$$p\left(-\frac{2}{3}\right) = 0$$
$$p\left(-\frac{2}{3}\right) = a\left(-\frac{2}{3}\right)^3 + b\left(-\frac{2}{3}\right)^2 + 8\left(-\frac{2}{3}\right) - 3$$
$$= -a \times \frac{8}{27} + b \times \frac{4}{9} - \frac{16}{3} - 3$$

But $p\left(-\frac{2}{3}\right) = 0$

$$\therefore \qquad \frac{-8a}{27} + \frac{4b}{9} - \frac{16}{3} - 3 = 0$$
$$\Rightarrow \qquad -8a + 12b - 144 - 81 = 0$$
$$\Rightarrow \qquad 8a - 12b + 225 = 0$$

Check Your Progress 3

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Multiple-Choice Questions

1. If $x + \frac{1}{x} = 3$, then $x^2 + \frac{1}{x^2}$ is equal to

(*a*) 11 (*b*) $9\frac{1}{9}$

(*c*) 7 (*d*) None of these

Sol. (*c*) 7

$$x + \frac{1}{x} = 3$$
$$\Rightarrow \qquad x^2 + \frac{1}{x^2} + 2(x)\left(\frac{1}{x}\right) = 9$$

[Squaring both sides]

$$\Rightarrow \qquad x^2 + \frac{1}{x^2} + 2 = 9$$
$$\Rightarrow \qquad x^2 + \frac{1}{x^2} = 9 - 2$$
$$\Rightarrow \qquad x^2 + \frac{1}{x^2} = 7$$

2. If $a + b + c = 4$ and $ab + bc + ca = 2$, then $a^2 + b^2 + c^2$ is equal to

(*a*) 12 (*b*) 28

(*c*) 14 (*d*) None of these

Sol. (*a*) 12

$(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

$\Rightarrow \quad 4^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

$\Rightarrow \quad 16 = a^2 + b^2 + c^2 + 2 \times 2$

$\Rightarrow \quad a^2 + b^2 + c^2 = 12$

3. Which of these identities can be used to find the value of the expression 98 × 102?

(*a*) $(x + y)^2 = x^2 + 2xy + y^2$

(*b*) $(x - y)^2 = x^2 - 2xy + y^2$

(c) $(x + y)(x - y) = x^2 - y^2$

(*d*) $(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

Sol. (*c*) $(x + y)(x - y) = x^2 - y^2$

98 × 102 can be expressed as (100 – 2) (100 + 2)

4. The value of $249^2 - 248^2$ is

(*a*) 1^2 (*b*) 477

(*c*) 487 (*d*) 497

Sol. (*d*) 497

$249^2 - 248^2 = (249 + 248)(249 - 248)$

$[a^2 - b^2 = (a + b)(a - b)]$

$= 497$

5. If $\frac{x}{y} + \frac{y}{x} = -1$ $(x, y \neq 0)$, the value of $x^3 - y^3$ is

(*a*) 1 (*b*) –1

(*c*) 0 (*d*) $\frac{1}{2}$

Sol. (*c*) 0

$$\frac{x}{y} + \frac{y}{x} = -1$$

$$\Rightarrow \quad \frac{x^2 + y^2}{xy} = -1$$

$$\Rightarrow \quad x^2 + y^2 = -xy$$

$$\Rightarrow \quad x^2 + y^2 + xy = 0 \qquad ...(1)$$

We have,

$$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$$

$$= (x - y) \times 0$$

[Putting value from (1)]

$$\Rightarrow \quad x^3 - y^3 = 0$$

6. The value of

1.75 × 1.75 – 2 × 0.75 × 1.75 + 0.75 × 0.75

is equal to

(*a*) 2.5 (*b*) 1.0

(*c*) 1.5 (*d*) 2.0

Sol. (*b*) 1.0

1.75 × 1.75 – 2 × 0.75 × 1.75 + 0.75 × 0.75

$= (1.75)^2 - 2 \times 1.75 \times 0.75 + (0.75)^2$

$= (1.75 - 0.75)^2 = 1^2 = 1.0$

7. The product

$$\left(\frac{5y}{2} + \frac{2x}{3}\right)\left(\frac{4x^2}{9} + \frac{25y^2}{4}\right)\left(\frac{2x}{3} - \frac{5y}{2}\right)$$

is equal to

(*a*) $\frac{25y^2}{4} - \frac{4x^2}{9}$ (*b*) $\frac{625y^4}{16} - \frac{16x^4}{81}$

(*c*) $\frac{16x^4}{81} - \frac{625y^4}{16}$ (*d*) $\frac{4x^2}{9} - \frac{25y^2}{4}$ **[CBSE SP 2011]**

Sol. (*c*) $\left(\frac{16x^4}{81} - \frac{625y^4}{16}\right)$

$$\left(\frac{5y}{2} + \frac{2x}{3}\right)\left(\frac{4x^2}{9} + \frac{25y^2}{4}\right)\left(\frac{2x}{3} - \frac{5y}{3}\right)$$

$$= \left(\frac{2x}{3} + \frac{5y}{2}\right)\left(\frac{2x}{3} - \frac{5y}{2}\right)\left(\frac{4x^2}{9} + \frac{25y^2}{4}\right)$$

$$= \left\{\left(\frac{2x}{3}\right)^2 - \left(\frac{5y}{2}\right)^2\right\}\left(\frac{4x^2}{9} + \frac{25y^2}{9}\right)$$

$$= \left(\frac{4x^2}{9} - \frac{25y^2}{4}\right)\left(\frac{4x^2}{9} + \frac{25y^2}{4}\right)$$

$$= \left(\frac{4x^2}{9}\right)^2 - \left(\frac{25y^2}{4}\right)^2$$

$$= \frac{16x^4}{81} - \frac{625y^4}{16}$$

Very Short Answer Type Questions

8. Using suitable identity evaluate $(995)^2$.

Sol. $(995)^2 = (1000 - 5)^2$

$= (1000)^2 - 2 \times 1000 \times 5 + (5)^2$

$[(a - b)^2 = a^2 - 2ab + b^2]$

$= 1000000 - 10000 + 25$

$= 990025$

$\therefore \quad (995)^2 = 990025$

9. Using suitable identity evaluate 101 × 103.

Sol. $101 \times 103 = (100 + 1) \times (100 + 3)$

$= (100)^2 + (1 + 3)(100) + 1 \times 3$

$[(x + a)(x + b) = x^2 + (a + b)x + ab]$

$= 10000 + 400 + 3$

$= 10403$

$\therefore \quad 101 \times 103 = 10403$

10. Using suitable identity evaluate $(33.2)^2 - (16.8)^2$.

Sol. $(33.2)^2 - (16.8)^2 = (33.2 + 16.8)(33.2 - 16.8)$

$[a^2 - b^2 = (a + b)(a - b)]$

$= 50 \times 16.4$

$= 820$

$\therefore \quad (33.2)^2 - (16.8)^2 = 820$

11. If $16x^2 + 8y^2 = 9$ and $xy = \frac{-\sqrt{2}}{9}$, find the value of $4x - 2\sqrt{2}y$.

Sol. We have

$$16x^2 + 8y^2 = 9$$

$$\Rightarrow \quad (4x)^2 + (2\sqrt{2}y)^2 = 9$$

$$\Rightarrow \quad (4x - 2\sqrt{2}y)^2 + 2 \times 4x \times 2\sqrt{2}y = 9$$

$$\Rightarrow \quad (4x - 2\sqrt{2}y)^2 + 16\sqrt{2} \times \left(-\frac{\sqrt{2}}{9}\right) = 9$$

$$[\because xy = -\frac{\sqrt{2}}{9}]$$

$$\Rightarrow \quad (4x - 2\sqrt{2}y)^2 - \frac{32}{9} = 9$$

$$\Rightarrow \quad (4x - 2\sqrt{2}y)^2 = \frac{32}{9} + 9 = \frac{113}{9}$$

$$\therefore \quad 4x - 2\sqrt{2}y = \pm\frac{\sqrt{113}}{3}$$

12. If $a + \frac{1}{a} = 5$, find the value of $a^3 + \frac{1}{a^3}$.

Sol. We have

$$a^3 + \frac{1}{a^3} = \left(a + \frac{1}{a}\right)^3 - 3a \times \frac{1}{a}\left(a + \frac{1}{a}\right)$$

$$= 5^3 - 3 \times 5$$

$$= 125 - 15$$

$$= 110$$

which is the required value.

13. Express $(x - a - b - c)\ (a - b + x + c)$ as the difference of two squares.

Sol. $(x - a - b - c)\ (a - b + x + c)$

$$= \{(x - b) - (a + c)\}\{(x - b) + (a + c)\}$$

$$= (x - b)^2 - (a + c)^2$$

which is the required expression.

14. If $a - b = 3$ and $a + b = 5$, find

(*a*) ab (*b*) $a^2 + b^2$

Sol. (*a*) We have

$$ab = \frac{(a + b)^2 - (a - b)^2}{4}$$

$$= \frac{5^2 - 3^2}{4}$$

$$= \frac{25 - 9}{4}$$

$$= \frac{16}{4} = 4$$

(*b*)

$$a^2 + b^2 = \frac{(a + b)^2 + (a - b)^2}{2}$$

$$= \frac{5^2 + 3^2}{2}$$

$$= \frac{25 + 9}{2}$$

$$= \frac{34}{2} = 17$$

Short Answer Type Questions

15. If $\left(a + \frac{1}{a}\right)^2 = 3$, prove that $a^3 + \frac{1}{a^3} = 0$.

Sol. Given $\left(a + \frac{1}{a}\right)^2 = 3$

$$\Rightarrow \quad a + \frac{1}{a} = \sqrt{3}$$

Using the formula,

$$\left(a + \frac{1}{a}\right)^3 = a^3 + \left(\frac{1}{a}\right)^3 + 3\left(a + \frac{1}{a}\right)$$

$$\Rightarrow \quad (\sqrt{3})^3 = a^3 + \frac{1}{a^3} + 3 \times \sqrt{3}$$

$$\Rightarrow \quad 3\sqrt{3} = a^3 + \frac{1}{a^3} + 3\sqrt{3}$$

$$\Rightarrow \quad a^3 + \frac{1}{a^3} = 0$$

16. If $2x - 3y = 5$, show that $8x^3 - 27y^3 - 90xy = 125$.

Sol. $2x - 3y = 5$

$$\Rightarrow \quad (2x - 3y)^3 = 5^3$$

$$\Rightarrow \quad (2x)^3 - (3y)^3 - 3 \times 2x \times 3y(2x - 3y) = 125$$

$$[(x - y)^3 = x^3 - y^3 - 3xy(x - y)]$$

$$\Rightarrow \quad 8x^3 - 27y^3 - 18xy \times 5 = 125$$

$$[\because \quad 2x - 3y = 5]$$

$$\Rightarrow \quad 8x^3 - 27y^3 - 90xy = 125$$

17. Find the value of $\frac{6.25 \times 6.25 - 1.75 \times 1.75}{4.5}$.

Sol.

$$\frac{6.25 \times 6.25 - 1.75 \times 1.75}{4.5} = \frac{(6.25)^2 - (1.75)^2}{4.5}$$

$$= \frac{(6.25 + 1.75)\,(6.25 - 1.75)}{4.5}$$

$$[a^2 - b^2 = (a + b)\ (a - b)]$$

$$= \frac{8 \times 4.5}{4.5} = 8$$

$$\therefore \quad \frac{6.25 \times 6.25 - 1.75 \times 1.75}{4.5} = 8$$

18. Express $-7.985^3 + 11.861^3 - 3.876^3$ as the product of any four distinct real numbers.

Sol. We see that

$$a + b + c = 11.861 - 7.985 - 3.876$$
$$= 11.861 - 11.861$$
$$= 0$$

where $a = 11.861$, $b = -7.985$ and $c = -3.876$

$\therefore \quad a^3 + b^3 + c^3 = 3abc$

i.e. $11.861^3 - 7.985^3 - 3.876^3$

$$= 3 \times 11.861 \times 7.985 \times 3.876$$

which is the required expression.

19. If $a + b + c = 5$, $ab + ac + bc = 3$ and $abc = -27$, find the value of $a^3 + b^3 + c^3$.

Sol. We have

$$(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + ac + bc)$$
$$\Rightarrow \quad 5^2 = a^2 + b^2 + c^2 + 2 \times 3$$
$$\Rightarrow \quad a^2 + b^2 + c^2 = 25 - 6$$
$$= 19 \quad \ldots(1)$$

Now, $a^3 + b^3 + c^3 - 3abc$

$$= (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$$
$$\Rightarrow a^3 + b^3 + c^3 + 3 \times 27 = 5 \times (19 - 3) \quad \text{[From (1)]}$$
$$= 5 \times 16$$
$$= 80$$
$$\therefore \quad a^3 + b^3 + c^3 = 80 - 81$$
$$= -1$$

which is the required value.

Long Answer Type Questions

20. If $x + y + z = 7$ and $x^2 + y^2 + z^2 = 33$, find the value of $x^3 + y^3 + z^3 - 3xyz$.

Sol. We have

$$(x + y + z)^2 = x^2 + y^2 + z^2 + 2(xy + xz + yz)$$
$$\Rightarrow \quad 7^2 = 33 + 2(xy + xz + yz)$$
$$\Rightarrow \quad xy + xz + yz = \frac{49 - 33}{2}$$
$$= \frac{16}{2} = 8 \quad \ldots(1)$$

$\therefore \quad x^3 + y^3 + z^3 - 3xyz$

$= (x + y + z)(x^2 + y^2 + z^2 - xy - xz - yz)$

$= 7 \times (33 - 8)$ [From (1)]

$= 7 \times 25$

$= 175$

which is the required value.

21. If $x^3 + y^3 + z^3 = 49$ and $x + y + z = 1$, find the value of $xy + yz + zx - xyz$.

Sol. We have $x^3 + y^3 + z^3 - 3xyz$

$$= (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$$
$$= [x^2 + y^2 + z^2 - (xy + yz + zx)]$$
$$= (x + y + z)^2 - 2(xy + yz + zx) - (xy + yz + zx)$$
$$= 1 - 3(xy + yz + zx)$$
$$\Rightarrow \quad 49 - 3xyz + 3(xy + yz + zx) = 1$$
$$\Rightarrow \quad 3[xy + yz + zx - xyz] = 1 - 49$$
$$= -48$$
$$\Rightarrow \quad xy + yz + zx - xyz = -\frac{48}{3} = -16$$

which is the required value of the given expression.

22. The volume of a cube is given by the expression $27x^3 + 8y^3 + 54x^2y + 36xy^2$. What is the expression for the length of the side of the cube?

Sol. Volume of the cube = $(\text{side})^3$

$27x^3 + 8y^3 + 54x^2y + 36xy^2$

$$= (3x)^3 + (2y)^3 + 3(3x)^2(2y) + 3(3x)(2y)$$
$$= (3x + 2y)^3$$

[Using the identity $(a + b)^3 = a^3 + b^3 + 3a^2b + 3ab^2$]

The expression for the side of the cube

$$= 3x + 2y$$

Check Your Progress 4

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Multiple-Choice Questions

1. The factorisation of $4x^2 + 2 - 2x^3 - x$ is

(*a*) $(2 - x)(2x^2 + 1)$ (*b*) $(2 + x)(2x^2 - 1)$

(*c*) $(2 - x)(2x^2 - 1)$ (*d*) None of these

Sol. (*a*) $(2 - x)(2x^2 + 1)$

$$4x^2 + 2 - 2x^3 - x = 4x^2 - 2x^3 + 2 - x$$
$$= 2x^2(2 - x) + 1(2 - x)$$
$$= (2 - x)(2x^2 + 1)$$

2. The factorisation of $-x^2 + (a + b)x - ab$ is

(*a*) $(a - x)(b - x)$ (*b*) $(-x + a)(b + x)$

(*c*) $(a - x)(-b + x)$ (*d*) $(-x - a)(-b + x)$

Sol. (*c*) $(a - x)(-b + x)$

$$-x^2 + (a + b)x - ab = -x^2 + ax + bx - ab$$
$$= x(-x + a) + b(x - a)$$
$$= x(-x + a) - b(-x + a)$$

$= (-x + a)\ (x - b)$

$= (a - x)\ (-b + x)$

3. The factorisation of $-x^4 + \left(ab + \frac{c}{a}\right) x^2 - bc$ is

(*a*) $\left(\frac{x^2}{a} + b\right)(c - x^2a)$ (*b*) $\left(\frac{x^2}{a} - b\right)(c - x^2a)$

(*c*) $\left(\frac{x^2}{a} - b\right)(c + x^2a)$ (*d*) None of these

Sol. (*b*) $\left(\frac{x^2}{a} - b\right)(c - x^2a)$

$$-x^4 + \left(ab + \frac{c}{a}\right)x^2 - bc$$

$$= -x^4 + abx^2 + \frac{c}{a}x^2 - bc$$

$$= -ax^2\left(\frac{x^2}{a} - b\right) + c\left(\frac{x^2}{a} - b\right)$$

$$= \left(\frac{x^2}{a} - b\right)(c - x^2a)$$

4. If the area of a triangle is $(6x^2 - x - 1)$ square units, then its possible dimensions in appropriate units are

(*a*) $2x + 1$ and $3x + 1$ (*b*) $3x + 1$ and $2x - 1$

(*c*) $3x - 1$ and $2x - 1$ (*d*) $3x - 1$ and $2x + 1$

Sol. (*b*) $3x + 1$ and $2x - 1$

We see that

$$6x^2 - x - 1 = 6x^2 + 2x - 3x - 1$$

$$= 2x(3x + 1) - 1(3x + 1)$$

$$= (2x - 1)\ (3x + 1)$$

Since the area of a triangle is $\frac{1}{2} \times$ base $\times$ altitude, hence the possible dimensions of the triangle are $3x + 1$ and $2x - 1$.

5. The factors of $\frac{1}{12} - \frac{x^2}{48}$ are

(*a*) $\frac{1}{48}(4 + x)(4 - x)$ (*b*) $\frac{1}{24}(4 + x)(4 - x)$

(*c*) $\frac{1}{12}\left(1 - \frac{x}{2}\right)\left(1 + \frac{x}{2}\right)$ (*d*) $\frac{1}{12}\left(1 + \frac{x}{4}\right)\left(1 - \frac{x}{4}\right)$

Sol. (*c*) $\frac{1}{12}\left(1 - \frac{x}{2}\right)\left(1 + \frac{x}{2}\right)$

We have

$$\frac{1}{12} - \frac{x^2}{48} = \frac{1}{12}\left(1 - \frac{x^2}{4}\right)$$

$$= \frac{1}{12}\left(1 + \frac{x}{2}\right)\left(1 - \frac{x}{2}\right)$$

Very Short Answer Type Questions

6. Find all the real factors of $1 - 16x^4$.

Sol. We have

$$1 - 16x^4 = 1^2 - (4x^2)^2$$

$$= (1 + 4x^2)\ (1 - 4x^2)$$

$$= (1 + 4x^2)\ \{1^2 - (2x)^2\}$$

$$= (1 + 4x^2)(1 + 2x)(1 - 2x)$$

which are the required factors.

7. Resolve $27a^3 + 8b^3$ into two real factors.

Sol. We have

$$27a^3 + 8b^3 = (3a)^3 + (2b)^3$$

$$= (3a + 2b)\ \{(3a)^2 - 3a \times 2b + (2b)^2\}$$

$$= (3a + 2b)\ (9a^2 - 6ab + 4b^2).$$

Factorise:

8. $x^2 - ab + (a - b)x$

Sol. $x^2 - ab + (a - b)x = x^2 - ab + ax - bx$

$$= x^2 + ax - bx - ab$$

$$= x(x + a) - b\ (x + a)$$

$$= (x + a)\ (x - b)$$

$\therefore\ x^2 - ab + (a - b)x = (x + a)\ (x - b)$

9. $(x + 3y)^2 - 25z^2$

Sol. $(x + 3y)^2 - 25z^2 = (x + 3y)^2 - (5z)^2$

$$= (x + 3y + 5z)\ (x + 3y - 5z)$$

$\therefore\ (x + 3y)^2 - 25z^2 = (x + 3y + 5z)\ (x + 3y - 5z)$

10. $4 + 4xy - y^2 - 4x^2$

Sol. $4 + 4xy - y^2 - 4x^2 = 4 - [4x^2 - 4xy + y^2]$

$$= 4 - [(2x)^2 - 2 \times 2x \times y + y^2]$$

$$= 4 - [(2x - y)^2]$$

$$= (2)^2 - (2x - y)^2$$

$$= (2 + 2x - y)\ (2 - 2x + y)$$

$\therefore\ 4 + 4xy - y^2 - 4x^2 = (2 + 2x - y)\ (2 - 2x + y)$

11. $3 - 12(a - b)^2$ **[CBSE SP 2011]**

Sol. We have

$$3 - 12\ (a - b)^2 = 3 - 12a^2 - 12b^2 + 24ab$$

$$= 3\ (1 - 4a^2 - 4b^2 + 8ab)$$

$$= 3[1^2 - \{(2a)^2 + (2b)^2 - 2 \times 2a \times 2b\}]$$

$$= 3[1^2 - (2a - 2b)^2]$$

$$= 3(1 + 2a - 2b)\ (1 - 2a + 2b)$$

which are the required factors.

12. $x^{12} - y^{12}$ **[CBSE 2010]**

Sol. We have

$x^{12} - y^{12}$

$= (x^4)^3 - (y^4)^3$
$= (x^4 - y^4)(x^8 + y^8 + x^4y^4)$
$= (x^2 + y^2)(x^2 - y^2)\{(x^4 + y^4)^2 - (x^2y^2)^2\}$
$= (x^2 + y^2)(x + y)(x - y)(x^4 + y^4 + x^2y^2)(x^4 + y^4 - x^2y^2)$
$= (x^2 + y^2)(x + y)(x - y)(x^4 + y^4 - x^2y^2)\{(x^2)^2 + (y^2)^2 + 2x^2y^2 - (xy)^2\}$
$= (x^2 + y^2)(x + y)(x - y)(x^4 + y^4 - x^2y^2)\{(x^2 + y^2)^2 - (xy)^2\}$
$= (x^2 + y^2)(x + y)(x - y)(x^4 + y^4 - x^2y^2)(x^2 + y^2 + xy)(x^2 + y^2 - xy)$

which are the required factors.

Short Answer Type Questions

Factorise into linear factors:

13. $18x^2 + 41x - 10$

Sol. We have

$$18x^2 + 41x - 10 = 18x^2 + 45x - 4x - 10 = 9x(2x + 5) - 2(2x + 5) = (2x + 5)(9x - 2)$$

which are the required factors.

14. $(a^2 - 2a)^2 - 23(a^2 - 2a) + 120$ **[CBSE SP 2013]**

Sol. We have $(a^2 - 2a)^2 - 23(a^2 - 2a) + 120$

Let $a^2 - 2a = b$ … (1)

Then $(a^2 - 2a)^2 - 23(a^2 - 2a) + 120$
$= b^2 - 23b + 120$
$= b^2 - 8b - 15b + 120$
$= b(b - 8) - 15(b - 8)$
$= (b - 8)(b - 15)$
$= (a^2 - 2a - 8)(a^2 - 2a - 15)$ [From (1)]
$= (a^2 + 2a - 4a - 8)(a^2 + 3a - 5a - 15)$
$= \{a(a + 2) - 4(a + 2)\}\{a(a + 3) - 5(a + 3)\}$
$= (a + 2)(a - 4)(a + 3)(a - 5)$

which are the required factors.

15. Without actually calculating the cubes find the value of

$$\left(\frac{1}{2}\right)^3 + \left(\frac{1}{3}\right)^3 - \left(\frac{5}{6}\right)^3$$

Sol. $$\left(\frac{1}{2}\right)^3 + \left(\frac{1}{3}\right)^3 - \left(\frac{5}{6}\right)^3 = \left[\left(\frac{1}{2}\right)^3 + \left(\frac{1}{3}\right)^3\right] - \left(\frac{5}{6}\right)^3$$

$$= \left[\left(\frac{1}{2} + \frac{1}{3}\right)^3 - 3 \times \frac{1}{2} \times \frac{1}{3}\left(\frac{1}{2} + \frac{1}{3}\right)\right] - \left(\frac{5}{6}\right)^3$$

$[\because \ x^3 + y^3 = (x + y)^3 - 3xy(x + y)]$

$$= \left(\frac{5}{6}\right)^3 - 3 \times \frac{1}{6} \times \frac{5}{6} - \left(\frac{5}{6}\right)^3$$

$$= -\frac{5}{12}$$

$$\therefore \ \left(\frac{1}{2}\right)^3 + \left(\frac{1}{3}\right)^3 - \left(\frac{5}{6}\right)^3 = \frac{-5}{12}$$

16. Lipika donated a certain amount of money to a charitable dispensary. Her friend wanted to know the amount she donated. Instead of disclosing the amount by Lipika gave the hint that if $\left(x + \frac{1}{x}\right) =$ ₹ 7, then the amount donated by her is ₹$\left(x^3 + \frac{1}{x^3}\right)$. Find the amount donated by Lipika.

Sol. Given, $\left(x + \frac{1}{x}\right) =$ ₹ 7

Amount donated = ₹ $\left(x^3 + \frac{1}{x^3}\right)$

$$\left(x + \frac{1}{x}\right)^3 = x^3 + \frac{1}{x^3} + 3 \times x \times \frac{1}{x}\left(x + \frac{1}{x}\right)$$

$$(7)^3 = x^3 + \frac{1}{x^3} + 3 \times 7$$

$$343 = x^3 + \frac{1}{x^3} + 21$$

$$x^3 + \frac{1}{x^3} = 343 - 21 = 322$$

Amount donated by Lipika = $x^3 + \frac{1}{x^3}$ = ₹ 322

Long Answer Type Questions

17. Simplify:

$$\frac{(25y^2 - 16z^2)^3 + (16z^2 - 9x^2)^3 + (9x^2 - 25y^2)^3}{(5y - 4z)^3 + (4z - 3x)^3 + (3x - 5y)^3}$$

Sol. Since,

$(25y^2 - 16z^2) + (16z^2 - 9x^2) + (9x^2 - 25y^2) = 0$

$\therefore \ (25y^2 - 16z^2)^3 + (16z^2 - 9x^2)^3 + (9x^2 - 25y^2)^3$
$= 3(25y^2 - 16z^2)(16z^2 - 9x^2)(9x^2 - 25y^2)$
$= 3\{(5y)^2 - (4z)^2\}\{(4z)^2 - (3x)^2\}\{(3x)^2 - (5y)^2\}$
$= 3(5y + 4z)(5y - 4z)(4z + 3x)(4z - 3x)(3x + 5y)(3x - 5y)$

Again, since

$(5y - 4z) + (4z - 3x) + (3x - 5y) = 0$

$\therefore \ (5y - 4z)^3 + (4z - 3x)^3 + (3x - 5y)^3 = 3(5y - 4z)(4z - 3x)(3x - 5y)$

$$\therefore \ \frac{(25y^2 - 16z^2)^3 + (16z^2 - 9x^2)^3 + (9x^2 - 25y^2)^3}{(5y - 4z)^3 + (4z - 3x)^3 + (3x - 5y)^3}$$

$$= \frac{3(5y+4z)(4z+3x)(3x+5y)(5y-4z)(4z-3x)(3x-5y)}{3(5y-4z)(4z-3x)(3x-55y)}$$

$= (5y + 4z)\ (4z + 3x)\ (3x + 5y)$

which is the required simplified expression.

18. Prove that

$(x + y)^3 + (y + z)^3 + (z + x)^3 - 3(x + y)(y + z)(z + x)$
$= 2(x^3 + y^3 + z^3 - 3xyz)$ **[CBSE SP 2010]**

Sol. We have,

LHS

$= (x + y)^3 + (y + z)^3 + (z + x)^3 - 3(x + y)(y + z)(x + x)$

$= (x + y + y + z + z + x)\ \{(x + y)^2 + (y + z)^2 + (z + x)^2 - (x + y)\ (y + z) - (x + y)\ (z + x) - (y + z)\ (z + y)\}$

$= 2(x + y + z)\ (x^2 + y^2 + y^2 + z^2 + z^2 + x^2 + 2xy + 2yz + 2zx - x^2 - y^2 - z^2 - xy - yz - zx - xz - yz - xy - yz - xy - zx)$

$= 2(x + y + z)\ (x^2 + y^2 + z^2 - xy - yz - zx)$

$= 2(x^3 + y^3 + z^3 - 3xyz)$

Hence, proved.

19. The volume of a cuboid is given by the algebraic expression $2x^3 - x^2 - 13x - 6$ in appropriate unit. Find the possible expressions for the dimensions of the cuboid in appropriate units.

Sol. Since the volume of a cuboid = Length × Breadth × Height

$\therefore$ The possible dimensions of the cuboid will be three factors of the given expression. So, we factorize $2x^3 - x^2 - 13x - 6$.

We see that this expression is zero when $x = -2$. Hence, $x + 2$ will be a factor of this expression.

$\therefore$ We rewrite this expression as follows:

$2x^3 - x^2 - 13x - 6$

$= 2x^3 + 4x^2 - 5x^2 - 10x - 3x - 6$

$= 2x^2\ (x + 2) - 5x\ (x + 2) - 3\ (x + 2)$

$= (x + 2)\ (2x^2 - 5x - 3)$

$= (x + 2)\ (2x^2 + x - 6x - 3)$

$= (x + 2)\ \{x(2x + 1) - 3(2x + 1)\}$

$= (x + 2)\ (2x + 1)\ (x - 3)$

Hence, the required dimensions of the cuboid in appropriate units are $x + 2$, $2x + 1$ and $x - 3$

20. If $3x + y + z = 0$, show that $27x^3 + y^3 + z^3 = 9xyz$.

[CBSE SP 2011]

Sol. We have

$27x^3 + y^3 + z^3 - 9xyz$

$= (3x)^3 + y^3 + z^3 - 3 \times 3xyz$

$= (3x + y + z)\ (9x^2 + y^2 + z^2 - 3xy - 3xz - yz)$

$= 0 \times (9x^2 + y^2 + z^2 - 3xy - 3xz - yz)$

$= 0$

$\therefore\ 27x^3 + y^3 + z^3 = 9xyz$

Higher Order Thinking Skills (HOTS) Questions

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Factorise the following:

1. $x^2 + 3y^2 - z^2 + 2yz - 4xy$

Sol. We have

$x^2 + 3y^2 - z^2 + 2yz - 4xy$

$= x^2 - 4xy + 4y^2 - y^2 - z^2 + 2yz$

$= \{x^2 - 2x \times 2y + (2y)^2\} - (y^2 + z^2 - 2yz)$

$= (x - 2y)^2 - (y - z)^2$

$= (x - 2y + y - z)\ (x - 2y - y + z)$

$= (x - y - z)\ (x - 3y + z)$

which are the required factors.

2. $a^2 + 2ab - ac - 3b^2 + 5bc - 2c^2$

Sol. We have

$a^2 + 2ab - ac - 3b^2 + 5bc - 2c^2$

$$= a^2 + a(2b - c) + \left(\frac{2b-c}{2}\right)^2 - \left(\frac{2b-c}{2}\right)^2 - 3b^2 + 5bc - 2c^2$$

$$= \left(a + \frac{2b-c}{2}\right)^2 - \frac{(2b-c)^2}{4} - 3b^2 + 5bc - 2c^2$$

$$= \frac{(2a+2b-c)^2}{4} - \frac{(2b-c)^2 + 12b^2 - 20bc + 8c^2}{4}$$

$$= \frac{(2a+2b-c)^2}{4} - \frac{4b^2 + c^2 - 4bc + 12b^2 - 20bc + 8c^2}{4}$$

$$= \frac{(2a+2b-c)^2}{4} - \frac{(4b)^2 + (3c)^2 - 2 \times 4b \times 3c}{4}$$

$$= \frac{(2a+2b-c)^2 - (4b-3c)^2}{4}$$

$$= \frac{(2a+2b-c+4b-3c)(2a+2b-c-4b+3c)}{4}$$

$$= \frac{(2a+6b-4c)(2a-2b+2c)}{4}$$

$= (a + 3b - 2c)\ (a - b + c)$

which are the required factors.

3. $y^4 + y^2 - 2ay + 1 - a^2$

Sol. $y^4 + y^2 - 2ay + 1 - a^2$

$$= y^4 + 1^2 + 2y^2 - y^2 - 2ay - a^2$$
$$= (y^2 + 1)^2 - (y + a)^2$$
$$= (y^2 + 1 + y + a)(y^2 + 1 - y - a)$$

which are the required factors.

4. $a^4 + b^4 + c^4 - 2b^2c^2 - 2a^2c^2 - 2a^2b^2$

Sol. $a^4 + b^4 + c^4 - 2b^2c^2 - 2a^2c^2 - 2a^2b^2$

$$= a^4 + b^4 + c^4 + 2b^2c^2 - 2a^2c^2 - 2a^2b^2 - 4b^2c^2$$
$$= (a^2 - b^2 - c^2)^2 - (2bc)^2$$
$$= (a^2 - b^2 - c^2 + 2bc)(a^2 - b^2 - c^2 - 2bc)$$
$$= \{a^2 - (b - c)^2\}\{a^2 - (b + c)^2\}$$
$$= (a + b - c)(a - b + c)(a + b + c)(a - b - c)$$

which are the required factors.

5. $(x + 1)(x + 3)(x + 5)(x + 7) + 15$

Sol. $(x + 1)(x + 3)(x + 5)(x + 7) + 15$

$= \{(x + 1)(x + 7)\}\{(x + 3)(x + 5)\} + 15$

$= (x^2 + 8x + 7)(x^2 + 8x + 15) + 15$

$= (a + 7)(a + 15) + 15,$ where $x^2 + 8x = a$ ... (1)

$= a^2 + 22a + 105 + 15$

$= a^2 + 22a + 120$

$= a^2 + 12a + 10a + 120$

$= a(a + 12) + 10(a + 12)$

$= (a + 12)(a + 10)$

$= (x^2 + 8x + 12)(x^2 + 8x + 10)$ [From (1)]

$= (x^2 + 6x + 2x + 12)(x^2 + 8x + 10)$

$= \{x(x + 6) + 2(x + 6)\}(x^2 + 8x + 10)$

$= (x + 6)(x + 2)(x^2 + 8x + 10)$

which are the required factors.

6. Prove that

$(y + z - x)^3 + (z + x - y)^3 + (x + y - z)^3$

$= 3(y + z - x)(z + x - y)(x + y - z)$

$= -24xyz$, if $x + y + z = 0$

Sol. We see that

$(y + z - x) + (z + x - y) + (x + y - z)$

$= x + y + z = 0$ [Given]

$\because$ $(y + z - x)^3 + (z + x - y)^3 + (x + y - z)^3$

$= 3(y + z - x)(z + x - y)(x + y - z)$

$= 3(-x - x)(-y - y)(-z - z)$ [$\because x + y + z = 0$]

$= 3(-2x)(-2y)(-2z)$

$= -24\,xyz$

Hence, proved.

7. If $x + y = a$, $x^2 + y^2 = b^2$ and $x^3 + y^3 = c^3$, prove that $a^3 + 2c^3 = 3ab^2$.

Sol. We have

$$x + y = a \quad \text{...(1)}$$
$$x^2 + y^2 = b^2 \quad \text{...(2)}$$
$$\Rightarrow \quad (x + y)^2 - 2xy = b^2$$
$$\Rightarrow \quad a^2 - 2xy = b^2 \quad \text{[From (1)]}$$
$$\Rightarrow \quad xy = \frac{a^2 - b^2}{2} \quad \text{...(3)}$$

Also, $x^3 + y^3 = c^3$

$$\Rightarrow \quad (x + y)(x^2 + y^2 - xy) = c^3$$
$$\Rightarrow \quad a\left(b^2 - \frac{a^2 - b^2}{2}\right) = c^3 \text{ [From (2) and (3)]}$$
$$\Rightarrow \quad a(3b^2 - a^2) = 2c^3$$
$$\Rightarrow \quad 3ab^2 - a^3 = 2c^3$$
$$\Rightarrow \quad a^3 + 2c^3 = 3ab^2$$

Hence, proved.

8. If $3s = a + b + c$, show that

$(s - a)^3 + (s - b)^3 + (s - c)^3 = 3(s - a)(s - b)(s - c)$

Sol. We have $s + s + s = a + b + c$

$\Rightarrow$ $(s - a) + (s - b) + (s - c) = 0$

$\therefore$ $(s - a)^3 + (s - b)^3 + (s - c)^3$

$= 3(s - a)(s - b)(s - c)$

Hence, proved.

9. If $2s = a + b + c$, show that

$(s - a)^3 + (s - b)^3 + (s - c)^3 - 3(s - a)(s - b)(s - c)$

$= \frac{1}{2}(a^3 + b^3 + c^3 - 3abc)$

Sol. We have

$(s - a)^3 + (s - b)^3 + (s - c)^3 - 3(s - a)(s - b)(s - c)$

$= (s - a + s - b + s - c)[(s - a)^2 + (s - b)^2 + (s - c)^2$
$- (s - a)(s - b) - (s - b)(s - c) - (s - c)(s - a)]$

$= [3s - (a + b + c)][3s^2 + a^2 + b^2 + c^2 - 2s(a + b + c)$
$- \{s^2 - (a + b)s + ab\} - \{s^2 - (b + c)s + bc\}$
$- \{s^2 - (c + a)s + ca\}]$

$= (3s - 2s)[a^2 + b^2 + c^2 - 2s \times 2s + 2s(a + b + c)$
$- ab - bc - ca]$

$= s[a^2 + b^2 + c^2 - 4s^2 + 4s^2 - ab - bc - ca]$

$$= \frac{a + b + c}{2}(a^2 + b^2 + c^2 - ab - bc - ca)$$

$$= \frac{1}{2}\left(a^3 + b^3 + c^3 - 3abc\right)$$

Hence, proved.

10. Factorise:

$-7x^2 - 3y^2 + 22xy - 44x + 12y - 12$

Sol. We have

$-7x^2 - 3y^2 + 22xy - 44x + 12y - 12$

$= -7x^2 + 22x(y-2) - 3y^2 + 12y - 12$

$= -7x^2 + 22x(y-2) - 3(y-2)^2$

$= -7x^2 + 21x(y-2) + (y-2)x - 3(y-2)^2$

$= -7x\{x - 3(y-2)\} + (y-2)\{x - 3(y-2)\}$

$= (x - 3y + 6)(y - 2 - 7x)$

which are the required factors.

Self-Assessment

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Multiple-Choice Questions

1. $\sqrt{2}$ is a polynomial of degree

(*a*) 2 (*b*) 0

(*c*) 1 (*d*) $\frac{1}{2}$ **[CBSE SP 2012]**

Sol. (*b*) 0

Since any constant is a polynomial of degree 0 and since $\sqrt{2}$ is a constant, hence, it is a polynomial of degree 0.

2. Degree of polynomial $(x^3 - 2)(x^2 + 11)$ is

(*a*) 0 (*b*) 3

(*c*) 5 (*d*) 2 **[CBSE SP 2012]**

Sol. (*c*) 5

We have $(x^3 - 2)(x^2 + 11) = x^5 + 11x^3 - 2x^2 - 22$ which is a polynomial of degree 5.

3. The value of the polynomial $5x^3 - 4x^2 + 3$ when $x = -1$ is

(*a*) –2 (*b*) 2

(*c*) –6 (*d*) 6

Sol. (*c*) –6

$$p(x) = 5x^3 - 4x^2 + 3$$
$$p(-1) = 5(-1)^3 - 4(-1)^2 + 3$$
$$= -5 - 4 + 3$$
$$= -6$$
$$\therefore \quad p(-1) = -6$$

4. The polynomial $(x - a)$, where $a > 0$, is the factor of the polynomial $p(x) = 4\sqrt{2}x^2 - \sqrt{2}$. Which of these is a polynomial whose factor is $\left(x - \frac{1}{a}\right)$?

(*a*) $x^2 + x - 6$ (*b*) $x^2 + x + 6$

(*c*) $x^2 + 4x - 3$ (*d*) $x^2 - 5x + 4$

Sol. (*a*) $x^2 + x - 6$

The polynomial $(x - a)$ is a factor of the polynomial $p(x) = 4\sqrt{2}x^2 - \sqrt{2}$.

By factor theorem,

$$p(a) = 0$$
$$p(a) = 4\sqrt{2}a^2 - \sqrt{2}$$
$$\Rightarrow \quad 4\sqrt{2}a^2 - \sqrt{2} = 0$$
$$\Rightarrow \quad a^2 = \frac{\sqrt{2}}{4\sqrt{2}}$$
$$\Rightarrow \quad a^2 = \frac{1}{4}$$
$$\Rightarrow \quad a = \frac{1}{2}$$
$$\therefore \quad (x - a) = (x - 2)$$

$(x - 2)$ is a factor of the polynomial $x^2 + x - 6$.

$$p(2) = 2^2 + 2 - 6$$
$$\Rightarrow \quad = 4 + 2 - 6$$
$$\Rightarrow \quad p(2) = 0$$

5. Which of the following is a factor of $(x + y)^3 - (x^3 + y^3)$?

(*a*) $x^2 + y^2 - xy$ (*b*) $x^2 + y^2 + 2xy$

(*c*) xy^2 (*d*) $3xy$

Sol. (*d*) $3xy$

$$(x + y)^3 - (x^3 + y^3) = x^3 + y^3 + 3xy(x + y) - x^3 - y^3$$
$$= 3xy(x + y)$$

$\therefore$ $3xy$ is a factor of $(x + y)^3 - (x^3 + y^3)$.

6. Given that $a^2 = 100^2$. Which of the following expressions give the value of the expression 103×108?

(*a*) $a^2 + 11a + 11$ (*b*) $a^2 + 11a + 24$

(*c*) $a^2 + 24a + 11$ (*d*) $a^2 + 24a + 24$

Sol. (*b*) $a^2 + 11a + 24$

Given $a^2 = 100^2$

$103 \times 108 = (100 + 3) \times (100 + 8)$

$\Rightarrow \quad = 100 \times 100 + 3 \times 100 + 8 \times 100 + 3 \times 8$

$\Rightarrow \quad = 100^2 + 100(3 + 8) + 24$

$\Rightarrow \quad = a^2 + 11a + 24$

$\therefore \quad 103 \times 108 = a^2 + 11a + 24$

7. The area of a rectangle is $6x^2 + 5x - 6$. The possible dimensions of its length and breadth are as follows.

(*a*) $(2x + 3), (3x + 2)$ (*b*) $(2x + 3), (3x - 2)$

(*c*) $(2x - 3), (3x - 2)$ (*d*) $(2x - 3), (3x + 2)$

Sol. (*b*) $(2x + 3), (3x - 2)$

Factorising $6x^2 + 5x - 6$

$6x^2 + 5x - 6 = 6x^2 + 9x - 4x - 6$

$\Rightarrow \; = 3x(2x + 3) - 2(2x + 3)$

$= (2x + 3)\,(3x - 2)$

The possible dimensions of its length and breadth are $(2x + 3)$, $(3x - 2)$.

8. If $64x^2 - a = \left(8x + \frac{3}{4}\right)\left(8x - \frac{3}{4}\right)$, then the value of a is

(*a*) $\frac{3}{4}$ (*b*) 0

(*c*) $\frac{9}{16}$ (*d*) $\frac{3}{2}$

Sol. (*c*) $\frac{9}{16}$

$$64x^2 - a = \left(8x + \frac{3}{4}\right)\left(8x - \frac{3}{4}\right)$$

$$(8x)^2 - (\sqrt{a})^2 = \left(8x + \frac{3}{4}\right)\left(8x - \frac{3}{4}\right)$$

$$\Rightarrow \quad \sqrt{a} = \frac{3}{4}$$

$$\therefore \quad a^2 = \frac{9}{16}$$

9. If $a + b + c = 0$, then $a^3 + b^3 + c^3$ is equal to

(*a*) *abc* (*b*) 3*abc*

(*c*) 2*abc* (*d*) 0

Sol. (*b*) 3*abc*

We have $a^3 + b^3 + c^3 - 3abc$

$= (a + b + c)\,(a^2 + b^2 + c^2 - ab - bc - c)$

If $a + b + c = 0$, then

$$a^3 + b^3 + c^3 - 3abc = 0$$

$$\Rightarrow \quad a^3 + b^3 + c^3 = 3abc$$

Fill in the Blanks

10. Degree of zero polynomial is **not defined**.

11. The coefficient of x^2 in $(2x^2 - 5)\,(4 + 3x^2)$ is **–7**.

12. Zeroes of the polynomial $p(x) = (x + 2)\,(x + 5)$ are **–2** and **–5**.

13. A polynomial of degree 5 in x has at most **6** terms.

Assertion-Reason Type Questions

Directions (Q. Nos. 14 to 17): Each of these questions contains an assertion followed by reason. Read them carefully, and answer the question on the basis of the following options, select the one that best describes the two statements.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

14. **Assertion (A):** $x + 8 = 0$ is a linear polynomial.

Reason (R): A polynomial of degree 1 is called a linear polynomial.

Sol. The correct answer is (a).

Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.

15. **Assertion (A):** 2 is a zero of the polynomial

$p(x) = x^2 - 3x - 4$

Reason (R): Putting $x = 2$, we get $p(x) = -6$

Sol. The correct answer is (d).

Putting $x = 2$, we get $p(x) = -6$.

Thus, Assertion is incorrect and Reason is correct.

16. **Assertion (A):** $99^3 = 100^3 - 3 \times 100 \times 99 + 99^3$

Reason (R): $(x - y)^3 = x^3 - 3xy(x - y) - y^3$

Sol. The correct answer is (d).

Assertion is incorrect and Reason is correct as it is a direct formula.

17. **Assertion (A):** $(a + b)^2 = a^2 + 2ab + b^2$ is an algebraic identity.

Reason (R): $(a + b)^2 = a^2 + 2ab + b^2$ holds true only for a particular pair of a and b.

Sol. The correct answer is (c).

$(a + b)^2 = a^2 + 2ab + b^2$ is an algebraic identity and it holds true for any pair of a and b. Thus, Assertion is correct but Reason is incorrect.

Case Study Based Questions

18. CBSE Udaan is a project started by Central Board of Secondary Education under the guidance of the Ministry of Human Resource Development, Government of India. The main objective of this program is to increase the enrolment rate of girls in the engineering colleges of the country.

UDAAN

A Programme To Give Wings To Girl Students

In a college, a group of $(x + y)$ lecturers, $(x^2 + y^2)$ girls and $(x^3 + y^3)$ boys organised a campaign on CBSE Udaan. Based on the above information, answer the following questions.

(*a*) Name the mathematical concepts used here?

Ans. Polynomial

(*b*) What is the expansion of $(x + y)^3$?

Ans. $x^3 + y^3 + 3x^2y + 3xy^2$

(*c*) (*i*) If in the group, there are 12 lecturers and 80 girls, then what is the number of boys?

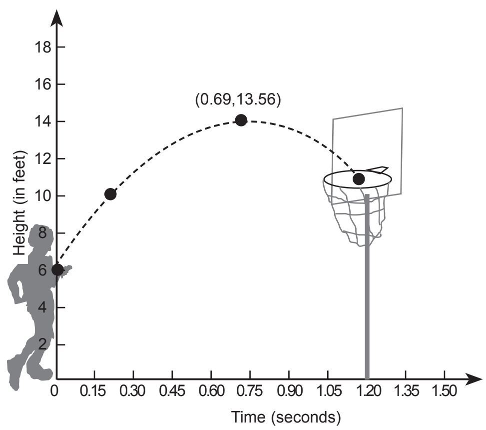
Ans. 576

or

(*ii*) If in the group, there are 8 lecturers and the value of xy is equal to 11, then what is the number of boys?

Ans. 248

19. Two teams from a certain school were playing basketball match in the school playground. One of the team members, Varun observes that the path followed by the basketball is parabolic in shape. Then the path of the basketball was traced on the graph paper.



If the equation of the height of a ball (in feet) at a given time (t) is

$$h(t) = -16t^2 + 22t + 6$$

then answer the following questions.

(*a*) What is the degree of the polynomial?

Ans. 2

(*b*) What is the name of the given polynomial on the basis of degree?

Ans. Quadratic polynomial

(*c*) (*i*) At what times (in sec) does the ball reach 10 feet?

Ans. 0.22 sec, 1.16 sec

or

(*ii*) From the graph, at what time does the ball reach its maximum height?

Ans. 0.69 sec

Very Short Answer Type Questions

20. Write the algebraic expression $\frac{2}{x^{-5}} + \frac{3}{x^{-1}} - \frac{\sqrt{3}}{4}x^6 + \sqrt{8} - 7x^3$ in the standard form of a polynomial.

Sol. $\frac{2}{x^{-5}} + \frac{3}{x^{-1}} - \frac{\sqrt{3}}{4}x^6 + \sqrt{8} - 7x^3$

$$= 2x^5 + 3x - \frac{\sqrt{3}}{4}x^6 + \sqrt{8} - 7x^3$$

$\therefore$ The required polynomial in standard form is

$$-\frac{\sqrt{3}}{4}x^6 + 2x^5 - 7x^3 + 3x + \sqrt{8}$$

21. If $f(x) = 3x^2 + 5x - 7$, show that $\frac{f(1) - f(0)}{f(2)} = \frac{8}{15}$.

Sol. We have $f(1) = 3 + 5 - 7 = 1$

$f(0) = -7, f(2) = 3 \times 4 + 5 \times 2 - 7 = 15$

$$\therefore \frac{f(1) - f(0)}{f(2)} = \frac{1 + 7}{15} = \frac{8}{15}$$

Hence, proved.

22. Consider the polynomial $x^2 + kx + 5$, where k is a constant. At $x = 2$, the value of the polynomial is 15. What is the value of the polynomial at $x = 5$?

Sol. $p(x) = x^2 + kx + 5$

$$p(2) = 2^2 + k.2 + 5 = 4 + 2k + 5 = 9 + 2k$$

Given that $p(2) = 15$

$\therefore \quad 9 + 2k = 15$

$\Rightarrow \quad 2k = 15 - 9 = 6$

$\Rightarrow \quad k = 3$

Therefore, the polynomial $p(x) = x^2 + 3x + 5$

$$p(5) = 5^2 + 3 \times 5 + 5 = 25 + 15 + 5$$

$\Rightarrow \quad p(5) = 45$

23. Consider the expression $x^{(a^2-1)} + 3x^{\frac{a}{2}}$, where a is a constant. For what value of a, will the expression be a cubic polynomial?

Sol. For the expression to be a cubic polynomial,

$$x^{(a^2-1)} = x^3$$

$\Rightarrow \quad a^2 - 1 = 3$

$\Rightarrow \quad a^2 = 4$

$a = 2$

24. Simplify by using an algebraic identity:

$$\frac{18.31 \times 18.31 - 8.31 \times 8.31}{26.62}$$

Sol. $\dfrac{18.31 \times 18.31 - 8.31 \times 8.31}{26.62}$

$$= \frac{18.31^2 - 8.31^2}{26.62}$$

$$= \frac{(18.31 + 8.31)(18.31 - 8.31)}{26.62}$$

$$= \frac{26.62 \times 10}{26.62}$$

$$= 10$$

25. If $x + \dfrac{1}{x} = 2$, find the value of $x^3 + \dfrac{1}{x^3}$.

Sol. We have

$$x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x}\right)^3 - 3x \times \frac{1}{x}\left(x + \frac{1}{x}\right)$$

$$= 2^3 - 3 \times 2$$

$$= 8 - 6$$

$$= 2$$

which is the required value.

Short Answer Type Questions

26. Express $3a^2 + 2b^2 + c^2 - 2\sqrt{6}\,ab + 2\sqrt{3}\,ac - 2\sqrt{2}bc$ as the square of a trinomial.

Sol. $3a^2 + 2b^2 + c^2 - 2\sqrt{6}\,ab + 2\sqrt{3}\,ac - 2\sqrt{2}\,bc$

$$= \left(\sqrt{3}a\right)^2 + \left(\sqrt{2}b\right)^2 + c^2 - 2\sqrt{2} \times \sqrt{3}\,ab + 2 \times \sqrt{3}\,ac - 2 \times \sqrt{2}\,bc$$

$$= \left(\sqrt{3}a - \sqrt{2}b + c\right)^2$$

which is the required expression.

27. If $x^2 + \dfrac{9}{x^2} = 12$, find the value of $x^3 + \dfrac{27}{x^3}$.

Sol. We have

$$x^2 + \frac{9}{x^2} = 12$$

$$\Rightarrow \quad x^2 + \left(\frac{3}{x}\right)^2 = 12$$

$$\Rightarrow \quad \left(x + \frac{3}{x}\right)^2 - 2 \times x \times \frac{3}{x} = 12$$

$$\Rightarrow \quad \left(x + \frac{3}{x}\right)^2 = 18$$

$$\Rightarrow \quad x + \frac{3}{x} = \pm 3\sqrt{2} \qquad \ldots(1)$$

When $x + \dfrac{3}{x} = 3\sqrt{2}$, we have

$$x^3 + \frac{27}{x^3} = x^3 + \left(\frac{3}{x}\right)^3$$

$$= \left(x + \frac{3}{x}\right)^3 - 3 \times x \times \frac{3}{x}\left(x + \frac{3}{x}\right)$$

$$= \left(3\sqrt{2}\right)^3 - 9 \times 3\sqrt{2}$$

$$= 54\sqrt{2} - 27\sqrt{2}$$

$$= 27\sqrt{2}$$

When $x + \dfrac{3}{x} = -3\sqrt{2}$, we have

$$x^3 + \frac{27}{23} = \left(-3\sqrt{2}\right)^3 - 9 \times \left(-3\sqrt{2}\right)$$

$$= -54\sqrt{2} + 27\sqrt{2}$$

$$= -27\sqrt{2}$$

$\therefore$ The required value of the given expression is $\pm 27\sqrt{2}$.

28. Without actually calculating the cubes, find the value of $(0.2)^3 - (0.3)^3 + (0.1)^3$.

Sol. $(0.2)^3 - (0.3)^3 + (0.1)^3 = -1[(0.3)^3 - (0.2)^3] + (0.1)^3$

$= -1[(0.3 - 0.2)^3 + 3 \times 0.3 \times 0.2\,(0.3 - 0.2)] + (0.1)^3$

[Using the identify $(x - y)^3 = x^3 - y^3 - 3xy\,(x - y)$]

$$= -1\,[(0.1)^3 + 3 \times 0.3 \times 0.2 \times 0.1] + (0.1)^3$$

$$= -(0.1)^3 - 3 \times 0.3 \times 0.2 \times 0.1 + (0.1)^3$$

$$= -3 \times 0.3 \times 0.2 \times 0.1$$

$$= 0.018$$

$\Rightarrow \quad (0.2)^3 - (0.3)^3 + (0.1)^3 = 0.018$

29. Factorise:

$2\sqrt{2}a^3 + 8b^3 - 27c^3 + 18\sqrt{2}\,abc$

Sol. $2\sqrt{2}a^3 + 8b^3 - 27c^3 + 18\sqrt{2}abc$

$$= (\sqrt{2}a)^3 + (2b)^3 + (-3c)^3 - 3(\sqrt{2}a)\,(2b)\,(-3c)$$

$$= (\sqrt{2}a + 2b - 3c)\,[(\sqrt{2}a)^2 + (2b)^2 + (-3c)^2 - (\sqrt{2}a)(2b) - (2b)(-3c) - (-3c)(\sqrt{2}a)]$$

$$= (\sqrt{2}a + 2b - 3c)\,[2a^2 + 4b^2 + 9c^2 - 2\sqrt{2}\,ab + 6bc + 3\sqrt{2}\,ac]$$

$\therefore \quad 2\sqrt{2}a^3 + 8b^3 - 27c^3 + 18\sqrt{2}abc$

$$= (\sqrt{2}a + 2b - 3c)\,(2a^2 + 4b^2 + 9c^2 - 2\sqrt{2}\,ab + 6bc + 3\sqrt{2}\,ac)$$

Long Answer Type Questions

30. Show that

$x^{18} - y^{18} = (x + y)(x^2 - xy + y^2)(x - y)(x^2 + xy + y^2)$
$\times(x^6 - x^3y^3 + y^6)(x^6 + x^3y^3 + y^6).$

Sol. We have

$$\begin{aligned} x^{18} - y^{18} &= (x^9)^2 - (y^9)^2 \\ &= (x^9 + y^9)(x^9 - y^9) \\ &= \{(x^3)^3 + (y^3)^3\}\{(x^3)^3 - (y^3)^3\} \end{aligned}$$

$$= (x^3 + y^3)(x^6 - x^3y^3 + y^6)(x^3 - y^3)(x^6 + x^3y^3 + y^6)$$
$$= (x + y)(x^2 - xy + y^2)(x - y)(x^2 + xy + y^2)$$
$$(x^6 - x^3y^3 + y^6)(x^6 + x^3y^3 + y^6)$$

Hence, proved.

31. Find all the real factors of

$2x^5 - 6x^4 + 13x^3 - 39x^2 + 20x - 60.$

Sol. We have

$2x^5 - 6x^4 + 13x^3 - 39x^2 + 20x - 60$

$= 2x^4(x - 3) + 13x^2(x - 3) + 20(x - 3)$

$= (x - 3)(2x^4 + 13x^2 + 20)$ ...(1)

Now, $2x^4 + 13x^2 + 20$

$$= 2\left(x^4 + \frac{13}{2}x^2 + 10\right)$$

$$= 2\left[\left(x^2\right)^2 + 2x^2 \times \frac{13}{4} + \left(\frac{13}{4}\right)^2 - \left(\frac{13}{4}\right)^2 + 10\right]$$

$$= 2\left[\left(x^2 + \frac{13}{4}\right)^2 + 10 - \frac{169}{16}\right]$$

$$= 2\left[\left(x^2 + \frac{13}{4}\right)^2 - \left(\frac{3}{4}\right)^2\right]$$

$$= 2\left[\left(x^2 + \frac{13}{4} + \frac{3}{4}\right)\left(x^2 + \frac{13}{4} - \frac{3}{4}\right)\right]$$

$$= 2\left[\left(x^2 + 4\right)\left(x^2 + \frac{5}{2}\right)\right]$$

$= (x^2 + 4)(2x^2 + 5)$

$\therefore\ 2x^5 - 6x^4 + 13x^3 - 39x^2 + 20x - 60$

$= (x - 3)(x^2 + 4)(2x^2 + 5)$ [From (1)]

which are the required factors.

32. If $a + b + c = 5$ and $ab + bc + ca = 10$, then prove that $a^3 + b^3 + c^3 - 3abc = -25$.

Sol. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

$\Rightarrow\ a^2 + b^2 + c^2 = (a + b + c)^2 - 2(ab + bc + ca)$

$= 5^2 - 2 \times 10$

$= 25 - 20$

$\Rightarrow\ a^2 + b^2 + c^2 = 5$ ...(1)

Now, $a^3 + b^3 + c^3 - 3abc$

$= (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

$= (a + b + c)[(a^2 + b^2 + c^2) - (ab + bc + ca)]$

$= 5[5 - 10]$ [Using (1)]

$= -25$

$\Rightarrow\ a^3 + b^3 + c^3 - 3abc = -25$

Let's Compete

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Multiple-Choice Questions

1. A polynomial $f(x)$ has degree 10. Then the maximum and minimum numbers of terms that $f(x)$ may have are

(*a*) 11 and 1	(*b*) 10 and 1
(*c*) 10 and 2	(*d*) 11 and 10

Sol. (*a*) 11 and 1

A polynomial $f(x)$ of degree 10 with maximum number of term is as follows:

$$f(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5 + a_6x^6 + a_7x^7 + a_8x^8 + a_9x^9 + a_{10}x^{10}$$

$\therefore$ It has at most 11 terms and at least 1 term *viz.* $f(x) = a_0$.

2. If each of the last two terms of the polynomial $2x^3 + 5x^2 - 9x + 10$ is increased by d so that the resulting polynomial has 1 as its zero, then the value of d is equal to

(*a*) 1	(*b*) –1
(*c*) 2	(*d*) –4

Sol. (*d*) –4

Let the resulting polynomial be

$$\begin{aligned} f(x) &= 2x^3 + 5x^2 - 9x + d + 10 + d \\ &= 2x^3 + 5x^2 - 9x + 10 + 2d \end{aligned}$$

Now, $f(1) = 0$

$\Rightarrow\ 2 + 5 - 9 + 10 + 2d = 0$

$\Rightarrow\ 8 + 2d = 0$

$\Rightarrow\ d = -4$

3. The number to be subtracted from the polynomial $x^4 + 2x^3 - 3x^2 + 5$ so that –3 becomes its zero, is

(*a*) –5	(*b*) 5
(*c*) 1	(*d*) 4

Sol. (*b*) 5

Let the required number to be subtracted from the given polynomial be n. Then the new polynomial $p(x)$ is given by

$$p(x) = x^4 + 2x^3 - 3x^2 + 5 - n$$

Since –3 is a zero of this polynomial,

$\therefore \quad p(-3) = 0$

$\Rightarrow \quad (-3)^4 + 2(-3)^3 - 3(-3)^2 + 5 - n = 0$

$\Rightarrow \quad 81 - 54 - 27 + 5 - n = 0$

$\Rightarrow \quad 86 - 81 - n = 0$

$\Rightarrow \quad n = 5$

4. When $x^3 + 4x^2 - 3x + b$ is divided by $x - 2$, then the remainder is the zero of another polynomial $x^3 - 19x^2 + x - 19$. Then the value of b is equal to

(*a*) –2 (*b*) –1

(*c*) 3 (*d*) 1

Sol. (*d*) 1

Let $f(x) = x^3 + 4x^2 - 3x + b$

and $g(x) = x^3 - 19x^2 + x - 19$

The zero of the polynomial $g(x)$ is given by the equation $g(x) = 0$

$\Rightarrow \quad x^2(x - 19) + 1(x - 19) = 0$

$\Rightarrow \quad (x - 19)(x^2 + 1) = 0$

$\Rightarrow \quad x = 19 \quad (\because x^2 + 1 \neq 0)$

$\therefore$ Zero of $g(x)$ is 19.

Hence, 19 is the remainder when $f(x)$ is divided by $x - 2$.

$\therefore \quad f(2) = 2^3 + 4 \times 2^2 - 3 \times 2 + b$

$= 19$

$\Rightarrow \quad 8 + 16 - 6 + b = 19$

$\Rightarrow \quad 18 + b = 19$

$\Rightarrow \quad b = 1$

5. $x + 2$ is a factor of the polynomial

(*a*) $x^3 - 2x^2 + 3x - 6$ (*b*) $x^3 + 2x^2 - 3x - 6$

(*c*) $x^3 + 2x^2 + 3x - 6$ (*d*) $x^3 + 2x^2 + 3x + 6$

Sol. (*b*) $x^2 + 2x^2 - 3x - 6$

We see that the polynomial in only (*b*) is 0 when $x = -2$, since,

$(-2)^3 + 2(-2)^2 - 3(-2) - 6 = -8 + 8 + 6 - 6 = 0$

6. The product of two expressions is $9x^2 - 16y^2 - 12x + 16y$. If factor is $3x + 4(y - 1)$, then the other factor is

(*a*) $3x - 4y$ (*b*) $4y - 3x$

(*c*) $3x + 4y$ (*d*) $-3x - 4y$

Sol. (*a*) $3x - 4y$

We have

$9x^2 - 16y^2 - 12x + 16y$

$= (3x)^2 - (4y)^2 - 12x + 16y$

$= (3x + 4y)(3x - 4y) - 4(3x - 4y)$

$= (3x - 4y)(3x + 4y - 4)$

$= (3x - 4y)\{3x + 4(y - 1)\}$

$\therefore$ If one factor is $3x + 4(y - 1)$, then the other factor is $3x - 4y$.

7. The linear polynomial in '*a*' which must be added with the polynomial $a^4 + 2a^3 - 2a^2 + a - 1$ so that the resulting polynomial is exactly divisible by $a^2 + 2a + 3$, is

(*a*) $11a - 14$ (*b*) $14 - 11a$

(*c*) $11a + 14$ (*d*) $-11a - 14$

Sol. (*d*) $-11a - 14$

Let the required linear polynomial in '*a*' be $k_1a + k_2$ where k_1 and k_2 are some constants.

$\therefore$ The new polynomial is $a^4 + 2a^3 - 2a^2 + (k_1 + 1)a + k_2 - 1$. If we divide this polynomial by $a^2 + 2a + 3$ by long division method, then the remainder becomes $(k_1 + 11)a + k_2 + 14$. If the remainder is zero, then

$k_1 = -11$

and $k_2 = -14$.

$\therefore$ The required linear polynomial is $-11a - 14$.

8. The zeroes of the polynomial $-6x^3 - 23x^2 + 5x + 4$ are $-\frac{1}{3}, \frac{1}{2}$ and -4. Then the factors of the polynomial are

(*a*) $3x - 1, 1 + 2x, 4 + x$ and 6

(*b*) $3x + 1, 1 - 2x, 4 + x$ and – 6

(*c*) $3x - 2, 2 + x, 4x + 1$ and 24

(*d*) $1 + 3x, 1 + 2x, 4 - x$ and 24

Sol. (*b*) $3x + 1, 1 - 2x, 4 + x$ and -6

We have $\left(x + \frac{1}{3}\right)\left(x - \frac{1}{2}\right)(x + 4)$

$$= \frac{(3x + 1)(2x - 1)(x + 4)}{6}$$

$$= \frac{(6x^2 - x - 1)(x + 4)}{6}$$

$$= \frac{6x^3 - x^2 - x + 24x^2 - 4x - 4}{6}$$

$$= \frac{6x^3 + 23x^2 - 5x - 4}{6}$$

$$= \frac{-6x^3 - 23x^2 + 5x + 4}{-6}$$

Hence, the required factors of the given polynomial are $3x + 1, 1 - 2x, 4 + x$ and -6.

9. If $x^2 + y^2 + z^2 = 16$ and $x + y + z = 12$, then the value of $xy + yz + zx$ is

(*a*) 128 (*b*) ±8

(*c*) 64 (*d*) ±16

Sol. (*c*) 64

We have $x^2 + y^2 + z^2 = 16$

$\Rightarrow \quad (x + y + z)^2 - (xy + yz + zx) = 16$

$\Rightarrow \quad 12^2 - 2(xy + yz + zx) = 16$

$\Rightarrow \quad xy + yz + zx = \dfrac{144 - 16}{2}$

$= \dfrac{128}{2} = 64$

10. If $x^3 + y^3 + z^3 = 42$, $x^2 + y^2 + z^2 = 16$, and $x + y + z = 6$, then the value of xyz is

(*a*) –2 (*b*) 2

(*c*) 6 (*d*) –6

Sol. (*b*) 2

We have

$$x^2 + y^2 + z^2 = 16$$

$\Rightarrow \quad (x + y + z)^2 - 2(xy + yz + zx) = 16$

$\Rightarrow \quad 6^2 - 2(xy + yz + zx) = 16$

$\Rightarrow \quad xy + yz + zx = \dfrac{36 - 16}{2}$

$= 10 \quad \ldots(1)$

Now, $x^3 + y^3 + z^3 - 3xyz$

$= (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

$\Rightarrow \quad 42 - 3xyz = 6 \times (16 - 10) = 36$ [From (1)]

$\Rightarrow \quad xyz = \dfrac{42 - 36}{3} = \dfrac{6}{3} = 2$

Life Skills

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1. Two brothers were studying in a school in classes X and XII. The younger brother, a student of class X could not find the factors of the polynomial $x^3 + 4x^2 + x - 6$. He asked his elder brother, a student of class XII to help him to solve the problem. The elder brother did not remember theorems to solve the problem. Hence, out of affection to his younger brother, the elder brother promised to help his younger brother to solve the problem. The elder brother had to study the remainder and factor theorems in order to explain the solution of the problem to his younger brother. Ultimately, he solved the problem and made the younger brother happy.

Find all the factors of the polynomial.

Sol. We have

$$f(x) = x^3 + 4x^2 + x - 6$$

$f(x) = 0$ when $x = 1$

$\therefore$ $x - 1$ is a factor of $f(x)$. So, we write

$f(x) = x^3 - x^2 + 5x^2 - 5x + 6x - 6$

$= x^2(x - 1) + 5x(x - 1) + 6(x - 1)$

$= (x^2 + 5x + 6)(x - 1)$

$= (x^2 + 3x + 2x + 6) 6(x - 1)$

$= \{x(x + 3) + 2(x + 3)\}(x - 1)$

$= (x + 3)(x + 2)(x - 1)$

which are the required factors of the given polynomial.