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Number Systems

Checkpoint (Page 6)

1. Find a rational number between $-\frac{2}{5}$ and $\frac{1}{4}$ with the denominator as 40.

Sol. We have

$$-\frac{2}{5} = \frac{-2 \times 8}{5 \times 8} = -\frac{16}{40}$$

and $$\frac{1}{4} = \frac{1 \times 10}{4 \times 10} = \frac{10}{40}$$

Any rational number between $-\frac{2}{5}$ and $\frac{1}{4}$ is $\frac{-3}{40}$.

Note: Here the answer is not unique. So, there may be many other rational numbers between $-\frac{2}{5}$ and $\frac{1}{4}$.

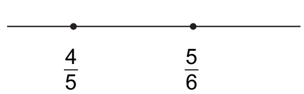
2. Will the rational number $\frac{4}{5}$ lie on the left or the right of the rational number $\frac{5}{6}$ on the number line? Explain why?

Sol. We have

$$\frac{4}{5} = \frac{4 \times 6}{5 \times 6} = \frac{24}{30}$$

and $$\frac{5}{6} = \frac{5 \times 5}{6 \times 5} = \frac{25}{30}$$

$\because \frac{24}{30} < \frac{25}{30}$



$\therefore \frac{4}{5} < \frac{5}{6}$

$\therefore \frac{4}{5}$ will lie on the left of the rational number $\frac{5}{6}$ on the number line.

3. Write the multiplicative inverse of $-\frac{13}{17}$.

Sol. The required multiplicative inverse of $-\frac{13}{17}$ is

$$\frac{1}{-\frac{13}{17}} = -\frac{17}{13}.$$

4. Evaluate $5\sqrt[5]{32} - 3\sqrt[4]{81}$.

Sol. We have

$$5\sqrt[5]{32} - 3\sqrt[4]{81} = 5 \times \sqrt[5]{2^5} - 3 \times \sqrt[4]{3^4}$$
$$= 5 \times 2^{\frac{5}{5}} - 3 \times 3^{\frac{4}{4}}$$
$$= 5 \times 2 - 3 \times 3$$
$$= 10 - 9$$
$$= 1$$

5. If $\frac{n5^{n+1} - 7 \times 5^{n+2}}{5^{n-1} + 7 \times 5^n} = 5$, then find the value of n, where n is a rational number.

Sol. We have

$$\frac{n5^{n+1} - 7 \times 5^{n+2}}{5^{n-1} + 7 \times 5^n} = 5$$

$$\Rightarrow \quad n5^{n+1} - 7 \times 5^{n+2} = 5(5^{n-1} + 7 \times 5^n)$$
$$= 5^n + 7 \times 5^{n+1}$$
$$\Rightarrow \quad n5^n \times 5 - 7 \times 5^n \times 25 = 5^n (1 + 7 \times 5)$$
$$5^n (5n - 175) = 5^n \times 36$$
$$\Rightarrow \quad 5n = 36 + 175$$
$$\therefore \quad n = \frac{211}{5}$$
$$= 42.2$$

$\therefore$ The required value of n is 42.2.

6. Evaluate $\left(\frac{1024}{3125}\right)^{-\frac{3}{5}}$.

Sol. We have

$$\left(\frac{1024}{3125}\right)^{-\frac{3}{5}} = \left(\frac{3125}{1024}\right)^{\frac{3}{5}}$$

$$= \frac{\left(5^5\right)^{\frac{3}{5}}}{\left(2^{10}\right)^{\frac{3}{5}}} = \frac{5^{5\times\frac{3}{5}}}{2^{10\times\frac{3}{5}}}$$

$$= \frac{5^3}{2^6} = \frac{125}{64}$$

7. A drum full of rice weighs $31\frac{1}{6}$ kg. If the weight of the empty drum is $11\frac{3}{4}$ kg, then find the weight of rice in the drum.

Sol. The required weight of the rice in the drum = Weight of drum full of rice – Weight of empty drum

$$= \left(31\frac{1}{6} - 11\frac{3}{4}\right) \text{ kg}$$

$$= \left(\frac{187}{6} - \frac{47}{4}\right) \text{ kg}$$

$$= \frac{374 - 141}{12} \text{ kg}$$

$$= \frac{233}{12} \text{ kg}$$

$$= 19\frac{5}{12} \text{ kg}$$

Hence, the weight of rice in the drum is $19\frac{5}{12}$ kg.

8. Solve for x:

$$25^{2x+1} \times 125^{-x+2} = 5$$

Sol. We have

$$25^{2x+1} \times 125^{-x+2} = 5$$

$$\Rightarrow (5^2)^{2x+1} \times (5^3)^{-x+2} = 5$$

$$\Rightarrow 5^{4x+2} \times 5^{-3x+6} = 5$$

$$\Rightarrow 5^{4x+2-3x+6} = 5$$

$$\Rightarrow 5^{x+8} = 5^1$$

$$\Rightarrow x + 8 = 1$$

$$\Rightarrow x = 1 - 8$$

$$= -7$$

$\therefore$ The required value of x is –7.

9. Find the value of x, if $\frac{8}{13} + x = \frac{16}{3}$.

Sol. We have

$$\frac{8}{13} + x = \frac{16}{3}$$

$$x = \frac{16}{3} - \frac{8}{13}$$

$$= \frac{208 - 24}{39} = \frac{184}{39}$$

$\therefore$ The required value of x is $\frac{184}{39}$.

10. What is the value of $\sqrt[3]{\sqrt[4]{x}}$ when $x = 4096$?

Sol. We have

$$\sqrt[3]{\sqrt[4]{x}} = x^{\frac{1}{3}\times\frac{1}{4}}$$

$$= x^{\frac{1}{12}} = 4096^{\frac{1}{12}}$$

$$= \left(2^{12}\right)^{\frac{1}{12}} = 2$$

$\therefore$ The required value of the expression is 2.

Check Your Progress

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Multiple-Choice Questions

1. A rational number between 2 and 3 is

(*a*) 2.010010001 … (*b*) $\sqrt{6}$

(*c*) $\frac{5}{2}$ (*d*) $4 - \sqrt{2}$ **[CBSE SP 2013]**

Sol. (*c*) $\frac{5}{2}$

A rational number between 2 and 3 is

$\frac{2+3}{2} = \frac{5}{2}$.

2. The decimal expansion of an irrational number is

(*a*) recurring decimal.

(*b*) terminating decimal.

(*c*) either terminating or non-terminating.

(*d*) non-terminating and non-recurring.

Sol. (*d*) non-terminating and non-recurring.

3. Which of the following is an irrational number?

(*a*) $\frac{\sqrt{18}}{\sqrt{2}}$ (*b*) $\frac{\sqrt{12}}{\sqrt{3}}$

(*c*) $\frac{\sqrt{45}}{\sqrt{5}}$ (*d*) $\frac{\sqrt{42}}{\sqrt{7}}$

Sol. (*d*) $\frac{\sqrt{42}}{\sqrt{7}}$

Solving $\frac{\sqrt{42}}{\sqrt{7}}$

$$\frac{\sqrt{42}}{\sqrt{7}} = \frac{\sqrt{7\times 6}}{\sqrt{7}} = \sqrt{6}$$

$\sqrt{6}$ is irrational number.

4. On a number line, $\frac{3}{\sqrt{18}}$ is located halfway between 0 and $\sqrt{x}$. What is the value of x?

(*a*) 1 (*b*) 2
(*c*) 3 (*d*) 5

Sol. (*b*) 2

Given $\frac{3}{\sqrt{18}}$ is located halfway between 0 and $\sqrt{x}$.

$$\frac{0+\sqrt{x}}{2} = \frac{3}{\sqrt{18}}$$

$$\Rightarrow \quad \sqrt{x} = \frac{3\times 2}{\sqrt{18}} = \frac{6}{\sqrt{3\times 6}}$$

$$\Rightarrow \quad = \frac{\sqrt{6}}{\sqrt{3}} = \sqrt{2}$$

$$\Rightarrow \quad \sqrt{x} = \sqrt{2}$$

$$\Rightarrow \quad x = 2$$

5. The value of $0.\overline{3}$ in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$ is

(*a*) $\frac{33}{100}$ (*b*) $\frac{3}{10}$
(*c*) $\frac{1}{3}$ (*d*) $\frac{3}{100}$

Sol. (*c*) $\frac{1}{3}$

We have $x = 0.\overline{3}$

$$x = 0.3333... \quad ...(1)$$
$$10x = 3.3333... \quad ...(2)$$

On subtracting (1) from (2), we get

$$9x = 3$$

$$\Rightarrow \quad x = \frac{3}{9} = \frac{1}{3}$$

6. If $x(5+\sqrt{7})$ is a rational number, then x must be equal to

(*a*) $5+\sqrt{7}$ (*b*) $\sqrt{7}-5$
(*c*) $\sqrt{7}+\sqrt{5}$ (*d*) $7+\sqrt{5}$

Sol. (*b*) $\sqrt{7}-5$

We know that

$$\left(\sqrt{7}+5\right)\left(\sqrt{7}-5\right) = 7-25 = -18$$

which is a rational number.

$\therefore$ The required value of x is $\sqrt{7}-5$.

7. Which of these is equivalent to $9^{3/4} \times 27^{1/5}$?

(*a*) $3^{\left(\frac{3}{4}+\frac{3}{5}\right)}$ (*b*) $3^{\left(\frac{3}{2}+\frac{3}{5}\right)}$
(*c*) $3^{\left(\frac{3}{4}-\frac{3}{5}\right)}$ (*d*) $3^{\left(\frac{3}{2}-\frac{3}{5}\right)}$

Sol. (*b*) $3^{\left(\frac{3}{2}+\frac{3}{5}\right)}$

Solving $9^{3/4} \times 27^{1/5}$

$$9^{3/4} \times 27^{1/5} = 3^{2\left(\frac{3}{4}\right)} \times 3^{3\left(\frac{1}{5}\right)}$$

$$= 3^{\frac{3}{2}} \times 3^{\frac{3}{5}}$$

$$= 3^{\left(\frac{3}{2}+\frac{3}{5}\right)}$$

8. If $\left(x^{\frac{1}{3}} \times 5^{\frac{1}{4}}\right)^{\frac{1}{2}} \times \left(\frac{3^{\frac{1}{4}}}{y^{\frac{1}{5}}}\right)^{2} = \frac{3^{\frac{5}{6}}}{5^{\frac{3}{40}}}$, then find the values of x and y.

(*a*) $x = 3$ and $y = 5$ (*b*) $x = 3$ and $y = 25$
(*c*) $x = 9$ and $y = 5$ (*d*) $x = 9$ and $y = \sqrt{5}$

Sol. (*d*) $x = 9$ and $y = \sqrt{5}$

Solving $\left(x^{\frac{1}{3}} \times 5^{\frac{1}{4}}\right)^{\frac{1}{2}} \times \left(\frac{3^{\frac{1}{4}}}{y^{\frac{1}{5}}}\right)^{2} = \frac{3^{\frac{5}{6}}}{5^{\frac{3}{40}}}$

$$\Rightarrow \quad x^{\frac{1}{6}} \times 5^{\frac{1}{8}} \times \frac{3^{\frac{1}{2}}}{y^{\frac{2}{5}}} = \frac{3^{\frac{5}{6}}}{5^{\frac{3}{40}}}$$

$$\Rightarrow \quad \frac{x^{\frac{1}{6}}}{y^{\frac{2}{5}}} = \frac{3^{\frac{5}{6}-\frac{1}{2}}}{5^{\frac{3}{40}}+5^{\frac{1}{8}}}$$

$$= \frac{3^{\frac{5-3}{6}}}{5^{\frac{3+5}{40}}}$$

$$= \frac{3^{\frac{2}{6}}}{5^{\frac{8}{40}}} = \frac{3^{\frac{1}{3}}}{5^{\frac{1}{5}}}$$

$$\Rightarrow \quad \frac{x^{\frac{1}{6}}}{y^{\frac{2}{5}}} = \frac{3^{\frac{2}{6}}}{5^{\frac{1}{5}}}$$

Equating the variables,

$$x = 3^2 = 9$$

$$y^2 = 5$$

$$\Rightarrow \quad y = \sqrt{5}$$

9. If $\sqrt{2} = 1.4142$, then $\sqrt{\dfrac{\sqrt{2}-1}{\sqrt{2}+1}}$ is equal to

(*a*) 2.4142 (*b*) 5.8282

(*c*) 0.4142 (*d*) 0.1718

Sol. (*c*) 0.41542

Solving $\sqrt{\dfrac{\sqrt{2}-1}{\sqrt{2}+1}}$

$$= \sqrt{\frac{(\sqrt{2}-1)(\sqrt{2}-1)}{(\sqrt{2}+1)(\sqrt{2}-1)}}$$

[Rationalising denominator by multiplying denominator and numerator by $((\sqrt{2}-1).)$]

$$= \sqrt{\frac{(\sqrt{2}-1)^2}{2-1}}$$

$$= (\sqrt{2}-1)$$

$$= 1.4142 - 1$$

$$= 0.4142$$

Very Short Answer Type Questions

10. What would be the denominator after rationalising $\dfrac{11}{7\sqrt{5}-7\sqrt{3}}$?

Sol. 98

Rationalising the fraction by multiplying the numerator and denominator by $(7\sqrt{5}+7\sqrt{3})$. We get,

$$\frac{11}{7\sqrt{5}-7\sqrt{3}} \times \frac{7\sqrt{5}+7\sqrt{3}}{7\sqrt{5}+7\sqrt{3}}$$

$$= \frac{11(7\sqrt{5}+7\sqrt{3})}{49\times 5 - 49 \times 3}$$

$$= \frac{11(7\sqrt{5}+7\sqrt{3})}{98}$$

11. Show that 1.9999 ... is equal to 2?

Sol. Let $x = 1.9999 \ldots$ ...(1)

$10x = 19.9999 \ldots$ ...(2)

Subtracting (1) from (2), we get

$$9x = 18$$

$$x = 2$$

12. If $x = \sqrt{3}-1$, then find the value of $x + \dfrac{1}{x}$ with a rational denominator.

Sol. $x = \sqrt{3}-1$

$$\frac{1}{x} = \frac{1}{\sqrt{3}-1}$$

$$= \frac{1}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$= \frac{\sqrt{3}+1}{3-1} = \frac{\sqrt{3}+1}{2}$$

$$x + \frac{1}{x} = \sqrt{3}-1+\frac{\sqrt{3}+1}{2}$$

$$= \frac{2\sqrt{3}-2+\sqrt{3}+1}{2}$$

$$= \frac{3\sqrt{3}-1}{2}$$

13. If $a = b^x$, $b = c^y$ and $a = c^z$, then show that $z = xy$.

Sol. We have

$$a = b^x = (c^y)^x = c^{xy} \quad \ldots(1)$$

Also, $a = c^z$ ...(2)

$\therefore \quad c^z = c^{xy}$ [From (1) and (2)]

$\Rightarrow \quad z = xy$

Hence, proved.

14. If $(\sqrt{a^3})^{\frac{2}{3}} \div \sqrt[12]{(a^4)^{\frac{1}{3}}} = x$, then prove that $\sqrt[8]{x^9} = a$.

Sol. We have

$$\Rightarrow \quad \left(\sqrt{a^3}\right)^{\frac{2}{3}} \div \sqrt[12]{\left(a^4\right)^{\frac{1}{3}}} = x$$

$$\Rightarrow \quad a^{\frac{3}{2}\times\frac{2}{3}} \div a^{\frac{4}{3}\times\frac{1}{12}} = x$$

$$\Rightarrow \quad a \div a^{\frac{1}{9}} = x$$

$$\Rightarrow \quad a^{\frac{8}{9}} = x$$

$$\therefore \quad a = x^{\frac{9}{8}} = \sqrt[8]{x^9}$$

Hence, proved.

15. If $\sqrt{3} = 1.732$, find the approximate value of $\dfrac{2+\sqrt{3}}{\sqrt{3}}$.

Sol. We have

$$\frac{2+\sqrt{3}}{\sqrt{3}} = \frac{\left(2+\sqrt{3}\right)\sqrt{3}}{\sqrt{3}\times\sqrt{3}}$$

$$= \frac{3+2\sqrt{3}}{3}$$

$$= 1 + \frac{2}{3}\times\sqrt{3}$$

$$= 1 + \frac{2 \times 1.732}{3}$$

$$= 1 + \frac{3.464}{3}$$

$$= 1 + 1.155$$

$$= 2.155 \text{ (approx.)}$$

16. If $\frac{5+2\sqrt{3}}{7+4\sqrt{3}} = x + y\sqrt{3}$, where x and y are rational numbers, prove that $\frac{x}{y} = -\frac{11}{6}$.

Sol. We have

$$\frac{5 + 2\sqrt{3}}{7 + 4\sqrt{3}} = x + y\sqrt{3}$$

$$\Rightarrow \quad \frac{\left(5 + 2\sqrt{3}\right)\left(7 - 4\sqrt{3}\right)}{7^2 - \left(4\sqrt{3}\right)^2} = x + y\sqrt{3}$$

$$\Rightarrow \quad \frac{35 - 24 + 14\sqrt{3} - 20\sqrt{3}}{49 - 48} = x + y\sqrt{3}$$

$$\Rightarrow \quad 11 - 6\sqrt{3} = x + y\sqrt{3}$$

$$\therefore \quad x = 11 \text{ and } y = -6$$

$$\therefore \quad \frac{x}{y} = -\frac{11}{6}$$

Hence, proved.

Short Answer Type Questions

17. Express $\frac{\sqrt{7}+\sqrt{2}}{9+2\sqrt{14}}$ with a rational denominator.

Sol. We have

$$\frac{\sqrt{7} + \sqrt{2}}{9 + 2\sqrt{14}} = \frac{\left(\sqrt{7} + \sqrt{2}\right)\left(9 - 2\sqrt{14}\right)}{9^2 - \left(2\sqrt{14}\right)^2}$$

$$= \frac{9\sqrt{7} + 9\sqrt{2} - 2\sqrt{98} - 2\sqrt{28}}{81 - 56}$$

$$= \frac{9\sqrt{7} + 9\sqrt{2} - 14\sqrt{2} - 4\sqrt{7}}{25}$$

$$= \frac{5\sqrt{7} - 5\sqrt{2}}{25}$$

$$= \frac{5\left(\sqrt{7} - \sqrt{2}\right)}{25}$$

$$= \frac{\sqrt{7} - \sqrt{2}}{5}$$

18. Simplify:

$$\frac{1}{2+\sqrt{5}} + \frac{1}{\sqrt{5}+\sqrt{6}} + \frac{1}{\sqrt{6}+\sqrt{7}} + \frac{1}{\sqrt{7}+\sqrt{8}}$$

[CBSE SP 2011]

Sol. We have

$$\frac{1}{2 + \sqrt{5}} = \frac{2 - \sqrt{5}}{2^2 - \left(\sqrt{5}\right)^2}$$

$$= \frac{2 - \sqrt{5}}{4 - 5} = \sqrt{5} - 2 \qquad \ldots(1)$$

$$\frac{1}{\sqrt{5} + \sqrt{6}} = \frac{\sqrt{5} - \sqrt{6}}{\left(\sqrt{5}\right)^2 - \left(\sqrt{6}\right)^2}$$

$$= \frac{\sqrt{5} - \sqrt{6}}{5 - 6} = \sqrt{6} - \sqrt{5} \qquad \ldots(2)$$

$$\frac{1}{\sqrt{6} + \sqrt{7}} = \frac{\sqrt{6} - \sqrt{7}}{\left(\sqrt{6}\right)^2 - \left(\sqrt{7}\right)^2}$$

$$= \frac{\sqrt{6} - \sqrt{7}}{6 - 7} = \sqrt{7} - \sqrt{6} \qquad \ldots(3)$$

$$\frac{1}{\sqrt{7} + \sqrt{8}} = \frac{\sqrt{7} - \sqrt{8}}{\left(\sqrt{7}\right)^2 - \left(\sqrt{8}\right)^2}$$

$$= \frac{\sqrt{7} - \sqrt{8}}{7 - 8} = \sqrt{8} - \sqrt{7} \qquad \ldots(4)$$

$$\frac{1}{2 + \sqrt{5}} + \frac{1}{\sqrt{5} + \sqrt{6}} + \frac{1}{\sqrt{6} + \sqrt{7}} + \frac{1}{\sqrt{7} + \sqrt{8}}$$

$$= \sqrt{5} - 2 + \sqrt{6} - \sqrt{5} + \sqrt{7} - \sqrt{6} + \sqrt{8} - \sqrt{7}$$

[From (1), (2), (3) and (4)]

$$= \sqrt{8} - 2 = 2\sqrt{2} - 2$$

$$= 2\left(\sqrt{2} - 1\right)$$

Long Answer Type Questions

19. If $a = \frac{\sqrt{5}+\sqrt{10}}{\sqrt{10}-\sqrt{5}}$ and $b = \frac{\sqrt{10}-\sqrt{5}}{\sqrt{10}+\sqrt{5}}$, show that $\sqrt{a} - \sqrt{b} - 2\sqrt{ab} = 0$. **[CBSE SP 2013]**

Sol. We have

$$a = \frac{\left(\sqrt{10} + \sqrt{5}\right)^2}{\left(\sqrt{10}\right)^2 - \left(\sqrt{5}\right)^2}$$

$$= \frac{\left(\sqrt{10} + \sqrt{5}\right)^2}{10 - 5}$$

$$= \frac{\left(\sqrt{10} + \sqrt{5}\right)^2}{5} \qquad \ldots(1)$$

$$b = \frac{\left(\sqrt{10} - \sqrt{5}\right)^2}{\left(\sqrt{10}\right)^2 - \left(\sqrt{5}\right)^2}$$

$$= \frac{\left(\sqrt{10} - \sqrt{5}\right)^2}{10 - 5}$$

$$= \frac{\left(\sqrt{10} - \sqrt{5}\right)^2}{5} \qquad \ldots(2)$$

$$\therefore \quad \sqrt{a} = \frac{\sqrt{10} + \sqrt{5}}{\sqrt{5}}$$

and $\sqrt{b} = \dfrac{\sqrt{10}-\sqrt{5}}{\sqrt{5}}$

$\therefore \quad \sqrt{ab} = \dfrac{\left(\sqrt{10}\right)^2-\left(\sqrt{5}\right)^2}{5}$

$= \dfrac{10-5}{5} = 1 \qquad \ldots(3)$

$\therefore \quad \sqrt{a}-\sqrt{b}-2\sqrt{ab}$

$= \dfrac{\sqrt{10}+\sqrt{5}-\sqrt{10}+\sqrt{5}}{\sqrt{5}} - 2\times 1$

[From (1), (2) and (3)]

$= \dfrac{2\sqrt{5}}{\sqrt{5}} - 2\times 1$

$= 2-2 = 0$

Hence, proved.

20. If $\dfrac{3+\sqrt{7}}{3-4\sqrt{7}} = a+b\sqrt{7}$, where a and b are rational numbers, find the values of a and b.

[CBSE SP 2013]

Sol. We have

$$\frac{3+\sqrt{7}}{3-4\sqrt{7}} = a+b\sqrt{7}$$

$$\Rightarrow \quad \frac{\left(3+\sqrt{7}\right)\left(3+4\sqrt{7}\right)}{3^2-\left(4\sqrt{7}\right)^2} = a+b\sqrt{7}$$

$$\Rightarrow \quad \frac{9+28+12\sqrt{7}+3\sqrt{7}}{9-112} = a+b\sqrt{7}$$

$$\Rightarrow \quad \frac{37+15\sqrt{7}}{-103} = a+b\sqrt{7}$$

$$\therefore \quad a = -\frac{37}{103}$$

and $$b = -\frac{15}{103}$$

$\therefore$ The required value of a is $-\dfrac{37}{103}$ and b is $-\dfrac{15}{103}$.

21. Find the value of

$\dfrac{\sqrt{\sqrt{5}+2}+\sqrt{\sqrt{5}-2}}{\sqrt{\sqrt{5}+1}}$. **[CBSE SP 2011]**

Sol. Let $x = \dfrac{\sqrt{\sqrt{5}+2}+\sqrt{\sqrt{5}-2}}{\sqrt{\sqrt{5}+1}}$

Then,

$$x^2 = \frac{\sqrt{5}+2+\sqrt{5}-2+2\sqrt{\left(\sqrt{5}+2\right)\left(\sqrt{5}-2\right)}}{\sqrt{5}+1}$$

$$= \frac{2\sqrt{5}+2\sqrt{5-4}}{\sqrt{5}+1}$$

$$= \frac{2\left(\sqrt{5}+1\right)}{\sqrt{5}+1} = 2$$

$\therefore \quad x = \sqrt{2}$ (Taking the positive value only)

$\therefore$ The required value of the given expression is $\sqrt{2}$.

22. If $x = 7+\sqrt{40}$, find the value of $\sqrt{x}+\dfrac{1}{\sqrt{x}}$.

[CBSE SP 2011]

Sol. We have

$$x = 7+\sqrt{40}$$

$$= 5+2+2\sqrt{10}$$

$$= \left(\sqrt{5}\right)^2+\left(\sqrt{2}\right)^2+2\sqrt{5}\times\sqrt{2}$$

$$= \left(\sqrt{5}+\sqrt{2}\right)^2$$

$$\therefore \quad \sqrt{x} = \sqrt{5}+\sqrt{2}$$

and $$\frac{1}{\sqrt{x}} = \frac{1}{\sqrt{5}+\sqrt{2}}$$

$$= \frac{\sqrt{5}-\sqrt{2}}{\left(\sqrt{5}\right)^2-\left(\sqrt{2}\right)^2}$$

$$= \frac{\sqrt{5}-\sqrt{2}}{5-2}$$

$$\therefore \quad \sqrt{x}+\frac{1}{\sqrt{x}} = \sqrt{5}+\sqrt{2}+\frac{\sqrt{5}-\sqrt{2}}{3}$$

$$= \frac{4\sqrt{5}+2\sqrt{2}}{3}$$

$$= \frac{2\left(2\sqrt{5}+\sqrt{2}\right)}{3}$$

$$= \frac{2}{3}\left(2\sqrt{5}+\sqrt{2}\right)$$

$\therefore$ The required value of $\sqrt{x}+\dfrac{1}{\sqrt{x}}$ is $\dfrac{2}{3}\left(2\sqrt{5}+\sqrt{2}\right)$.

Higher Order Thinking Skills (HOTS) Questions

(Page 13)

1. If $x = \dfrac{2\sqrt{5}+3\sqrt{2}}{2\sqrt{5}-3\sqrt{2}}$, find the value of $x^3-\dfrac{1}{x^3}$.

Sol. We have

$$x = \frac{2\sqrt{5}+3\sqrt{2}}{2\sqrt{5}-3\sqrt{2}}$$

$$= \frac{(2\sqrt{5}+3\sqrt{2})(2\sqrt{5}+3\sqrt{2})}{(2\sqrt{5}-3\sqrt{2})(2\sqrt{5}+3\sqrt{2})}$$

$$= \frac{(2\sqrt{5}+3\sqrt{2})^2}{(2\sqrt{5})^2-(3\sqrt{2})^2}$$

$$= \frac{20+18+2\times 2\sqrt{5}\times 3\sqrt{2}}{20-18}$$

$$= \frac{38+12\sqrt{10}}{2}$$

$$\therefore \quad x-\frac{1}{x} = 19+6\sqrt{10}-\frac{1}{19+6\sqrt{10}}$$

$$= 19+6\sqrt{10}-\frac{19-6\sqrt{10}}{19^2-(6\sqrt{10})^2}$$

$$= 19+6\sqrt{10}-\frac{19-6\sqrt{10}}{361-360}$$

$$= 19+6\sqrt{10}-19+6\sqrt{10}$$

$$= 12\sqrt{10} \quad \ldots(1)$$

$$\therefore \quad x^3-\frac{1}{x^3} = \left(x-\frac{1}{x}\right)^3+3\left(x-\frac{1}{x}\right)$$

$$= (12\sqrt{10})^3+3\times 12\sqrt{10}$$

[From (1)]

$$= 17280\sqrt{10}+36\sqrt{10}$$

$$= 17316\sqrt{10}$$

$\therefore$ The required value of $x^3-\frac{1}{x^3}$ is $17316\sqrt{10}$.

2. If $x = \frac{1}{4+\sqrt{15}}$, find the value of

$x^4-16x^3+66x^2-16x+72$.

Sol. We have

$$x = \frac{1}{4+\sqrt{15}}$$

$$= \frac{4-\sqrt{15}}{4^2-(\sqrt{15})^2}$$

$$= \frac{4-\sqrt{15}}{16-15}$$

$$= 4-\sqrt{15}$$

$$\therefore \quad (x-4)^2 = (-\sqrt{15})^2 = 15$$

$$\Rightarrow \quad x^2-8x+16-15 = 0$$

$$\Rightarrow \quad x^2 = 8x-1 \quad \ldots(1)$$

$\therefore$ $x^4-16x^3+66x^2-16x+72$

$= (x^2)^2-16x^2.\,x+66x^2-16x+72$

$= (8x-1)^2-16(8x-1)x+66\,(8x-1)-16x+72$ [From (1)]

$= 64x^2+1-16x-128x^2+16x+528x-66-16x+72$

$= -64x^2+512x+7$

$= -64\,(8x-1)+512x+7$ [From (1)]

$= -512x+64+512x+7 = 71$

$\therefore$ The required value of the given expression is 71.

3. If $x = \frac{1}{5+3\sqrt{3}}$ and $y = \frac{1}{5-3\sqrt{3}}$, show that $xy^2+x^2y = \frac{5}{2}$.

Sol. We have

$$x = \frac{1}{5+3\sqrt{3}}$$

$$= \frac{5-3\sqrt{3}}{5^2-(3\sqrt{3})^2}$$

$$= \frac{5-3\sqrt{3}}{25-27}$$

$$= \frac{3\sqrt{3}-5}{2}$$

and

$$y = \frac{1}{5-3\sqrt{3}}$$

$$= \frac{5+3\sqrt{3}}{5^2-(3\sqrt{3})^2}$$

$$= \frac{5+3\sqrt{3}}{25-27}$$

$$= \frac{-5-3\sqrt{3}}{2}$$

$$\therefore \quad x+y = \frac{3\sqrt{3}-5-5-3\sqrt{3}}{2}$$

$$= -5 \quad \ldots(1)$$

Also,

$$xy = \frac{5-3\sqrt{3}}{2}\times\frac{5+3\sqrt{3}}{2}$$

$$= \frac{5^2-(3\sqrt{3})^2}{4}$$

$$= \frac{25-27}{4} = -\frac{1}{2} \quad \ldots(2)$$

$\therefore \quad xy^2 + x^2y = xy(x+y)$

$= 5 \times \frac{1}{2}$ [From (1) and (2)]

$= \frac{5}{2}$

Hence, proved.

4. $\frac{\sqrt{a+2b}+\sqrt{a-2b}}{\sqrt{a+2b}-\sqrt{a-2b}} = x$, prove that $bx^2 - ax + b = 0$.

Sol. We have

$$\frac{x}{1} = \frac{\sqrt{a+2b}+\sqrt{a-2b}}{\sqrt{a+2b}-\sqrt{a-2b}}$$

$$\Rightarrow \frac{x+1}{x-1} = \frac{\sqrt{a+2b}+\sqrt{a-2b}+\sqrt{a+2b}-\sqrt{a-2b}}{\sqrt{a+2b}+\sqrt{a-2b}-\sqrt{a+2b}+\sqrt{a-2b}}$$

[By applying componendo and dividendo on both sides]

$$\Rightarrow \frac{x+1}{x-1} = \frac{2\sqrt{a+2b}}{2\sqrt{a-2b}} = \frac{\sqrt{a+2b}}{\sqrt{a-2b}}$$

$$\Rightarrow \frac{(x+1)^2}{(x-1)^2} = \frac{\left(\sqrt{a+2b}\right)^2}{\left(\sqrt{a-2b}\right)^2} = \frac{a+2b}{a-2b}$$

$$\Rightarrow \frac{(x+1)^2+(x-1)^2}{(x+1)^2-(x-1)^2} = \frac{a+2b+a-2b}{a+2b-a+2b}$$

[By applying componendo and dividendo on both sides]

$$\Rightarrow \frac{2\left(x^2+1\right)}{4x} = \frac{2a}{4b}$$

$$\Rightarrow \frac{x^2+1}{x} = \frac{a}{b}$$

$$\Rightarrow bx^2 + b = ax$$

$$\Rightarrow bx^2 - ax + b = 0$$

Hence, proved.

5. Find the value of

$$\frac{1+x}{1+\sqrt{1+x}} + \frac{1-x}{1-\sqrt{1-x}} \text{ when } x = \frac{\sqrt{3}}{2}.$$

Sol. We have

$$1 + x = 1 + \frac{\sqrt{3}}{2}$$

$$= \frac{2+\sqrt{3}}{2} = \frac{4+2\sqrt{3}}{4}$$

$$= \frac{3+1+2\sqrt{3}}{4}$$

$$= \frac{\left(\sqrt{3}\right)^2 + 1^2 + 2\sqrt{3}\times 1}{2^2}$$

$$= \left(\frac{\sqrt{3}+1}{2}\right)^2 \quad \ldots(1)$$

Similarly, $1 - x = \left(\frac{\sqrt{3}-1}{2}\right)^2 \quad \ldots(2)$

$$\therefore \frac{1+x}{1+\sqrt{1+x}} + \frac{1-x}{1-\sqrt{1-x}}$$

$$= \frac{\frac{\left(\sqrt{3}+1\right)^2}{4}}{1+\frac{\sqrt{3}+1}{2}} + \frac{\frac{\left(\sqrt{3}-1\right)^2}{4}}{1-\frac{\sqrt{3}-1}{2}} \quad \text{[From (1) and (2)]}$$

$$= \frac{\left(\sqrt{3}+1\right)^2}{4+2\left(\sqrt{3}+1\right)} + \frac{\left(\sqrt{3}-1\right)^2}{4-2\left(\sqrt{3}-1\right)}$$

$$= \frac{\left(\sqrt{3}\right)^2+1+2\sqrt{3}}{6+2\sqrt{3}} + \frac{\left(\sqrt{3}\right)^2+1-2\sqrt{3}}{6-2\sqrt{3}}$$

$$= \frac{4+2\sqrt{3}}{6+2\sqrt{3}} + \frac{4-2\sqrt{3}}{6-2\sqrt{3}}$$

$$= \frac{2+\sqrt{3}}{3+\sqrt{3}} + \frac{2-\sqrt{3}}{3-\sqrt{3}}$$

$$= \frac{\left(2+\sqrt{3}\right)\left(3-\sqrt{3}\right)+\left(2-\sqrt{3}\right)\left(3+\sqrt{3}\right)}{\left(3+\sqrt{3}\right)\left(3-\sqrt{3}\right)}$$

$$= \frac{6-2\sqrt{3}+3\sqrt{3}-3+6+2\sqrt{3}-3\sqrt{3}-3}{3^2-\left(\sqrt{3}\right)^2}$$

$$= \frac{6}{9-3} = \frac{6}{6} = 1$$

Hence, the required value of the given expression is 1.

Self-Assessment

(Page 13)

Multiple-Choice Questions

1. Every rational number is

(*a*) a natural number.

(*b*) an integer.

(*c*) a real number.

(*d*) a whole number.

Sol. (*c*) a real number

2. Decimal representation of a rational number cannot be

(*a*) terminating.

(*b*) non-terminating.

(*c*) non-terminating repeating.
(*d*) non-terminating non-repeating.

Sol. (*d*) non-terminating non-repeating.

3. The product of any two irrational numbers is
(*a*) always an irrational number.
(*b*) always a rational number.
(*c*) always an integer.
(*d*) sometimes rational, sometimes irrational.

Sol. (*d*) sometimes rational, sometimes irrational.

4. An expression $2(\sqrt{x}-1) + \sqrt{8}$ is given. When $-8\sqrt{2}$ is added to the expression it becomes a rational number. What is the value of x?
(*a*) 6 (*b*) 12
(*c*) 18 (*d*) 36

Sol. (*c*) 18

Given $2(\sqrt{x}-1)+\sqrt{8}$

On adding $-8\sqrt{2}$, the expression becomes a rational number.

$$2(\sqrt{x}-1)+\sqrt{8}-8\sqrt{2}$$
$$\Rightarrow \quad 2\sqrt{x}-2+2\sqrt{2}-\sqrt{2}$$
$$\Rightarrow \quad 2\sqrt{x}-2-6\sqrt{2}$$

The expression to be rational,

$$2\sqrt{x} = 6\sqrt{2}$$
$$\sqrt{x} = 3\sqrt{2}$$
$$\therefore \quad x = 18$$

5. A rational number which is equivalent to $\frac{2}{13}$ is
(*a*) $\frac{4}{13}$ (*b*) $\frac{4}{26}$
(*c*) $\frac{13}{2}$ (*d*) $\frac{2}{26}$

Sol. (*b*) $\frac{4}{26}$

We have $\frac{2}{13} = \frac{2\times 2}{13\times 2} = \frac{4}{26}$

6. A number which is irrational is
(*a*) $0.\overline{142857}$ (*b*) 2.3785
(*c*) $0.0\overline{53}$ (*d*) 3.050050005 …

Sol. (*d*) 3.050050005….

We know that any non-recurring and non-terminating number is an irrational number.

7. Find the value of $729^{-\frac{1}{3}}$.
(*a*) -9 (*b*) $-\frac{1}{9}$
(*c*) $\frac{1}{9}$ (*d*) 9

Sol. (*c*) $\frac{1}{9}$

Solving $729^{-1/3}$

$$= \frac{1}{729^{1/3}}$$
$$= \frac{1}{(9\times 9\times 9)^{1/3}}$$
$$= \frac{1}{(9^3)^{1/3}}$$
$$= \frac{1}{9}$$

8. What is the value of $(1^3+2^3+3^3)^{\frac{1}{2}}$?
(*a*) 4 (*b*) 6
(*c*) 8 (*d*) 10

Sol. (*b*) 6

Solving $(1^3+2^3+3^3)^{\frac{1}{2}}$

$$= (1+2\times 2\times 2+3\times 3\times 3)^{\frac{1}{2}}$$
$$= (1+8+27)^{\frac{1}{2}}$$
$$= (36)^{\frac{1}{2}}$$
$$= (6^2)^{\frac{1}{2}}$$
$$= 6^{2\times\frac{1}{2}}$$
$$= 6^1$$
$$= 6$$

9. $\sqrt[4]{\sqrt[3]{2^2}}$ equals
(*a*) $2^{-\frac{1}{6}}$ (*b*) 2^{-6}
(*c*) $2^{\frac{1}{6}}$ (*d*) 2^6

Sol. (*c*) $2^{\frac{1}{6}}$

Solving $\sqrt[4]{\sqrt[3]{2^2}}$

$$= \sqrt[4]{2^{2\times\frac{1}{3}}}$$
$$= \sqrt[4]{2^{\frac{2}{3}}}$$
$$= 2^{\frac{2}{3}\times\frac{1}{4}}$$
$$= 2^{\frac{2}{12}}$$
$$= 2^{\frac{1}{6}}$$

10. Which of the following is equal to *p*?

(*a*) $p^{\frac{12}{7}} - p^{\frac{5}{7}}$ (*b*) $\sqrt[12]{(p^4)^{\frac{1}{3}}}$

(*c*) $(\sqrt{p^3})^{\frac{2}{3}}$ (*d*) $p^{\frac{12}{7}} \times p^{\frac{7}{12}}$

Sol. (*d*) $p^{\frac{12}{7}} \times p^{\frac{7}{12}}$

Solving $p^{\frac{12}{7}} \times p^{\frac{7}{12}}$

$= p^{\frac{12}{7} \times \frac{7}{12}}$

$= p^1$

$= p$

Fill in the Blanks

11. π is an irrational number because its decimal expansion is **non-terminating non-repeating**.

12. Every point on a number line represents **a unique real** number.

13. The value of $\sqrt{(3^{-2})}$ is $\mathbf{\frac{1}{3}}$.

Sol. $\sqrt{(3^{-2})} = \sqrt{\frac{1}{3^2}} = \frac{1}{3}$

14. The value of $\frac{2^0 + 7^0}{5^0}$ is **2**.

Sol. $\frac{2^0 + 7^0}{5^0} = \frac{1+1}{1} = 2$

Assertion-Reason Type Questions

Directions (Q. Nos. 15 to 18): Each of these questions contains an assertion followed by reason. Read them carefully, and answer the question on the basis of the following options, select the one that best describes the two statements.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

15. **Assertion (A):** 2/6 and 3/9 are equivalent rational numbers.

Reason (R): Both 2/6 and 3/9 are equal to 1/3 in their lowest form.

Sol. The correct answer is (a). Both the Assertion and Reason are correct and Reason is correct explanation of Assertion, since all the fractions are equivalent.

16. **Assertion (A):** 7 is a rational number.

Reason (R): The square roots of all positive integers are irrationals.

Sol. The correct answer is (c). 7 is a rational number, as it can be written as $\frac{7}{1}$, which is a rational number.

The square roots of all positive integers are not irrational. For example , $\sqrt{4} = \pm 2$, where 2 and –2 are both rational numbers.

Thus, Assertion is correct but Reason is incorrect.

17. **Assertion (A):** $\frac{13^5}{13^2} = 13^7$

Reason (R): If $a > 0$ be real number and *m* and *n* be rational numbers, then $\frac{a^m}{a^n} = a^{m-n}$

Sol. The correct answer is (d). Assertion is incorrect, as $\frac{13^5}{13^2} \neq 13^7$, it is $\frac{13^5}{13^2} = 13^{5-2} = 13^3$.

Reason is correct, as it explains the concept.

Thus, Assertion is incorrect but Reason is correct.

18. **Assertion (A):** $\sqrt{3}$ is an irrational number.

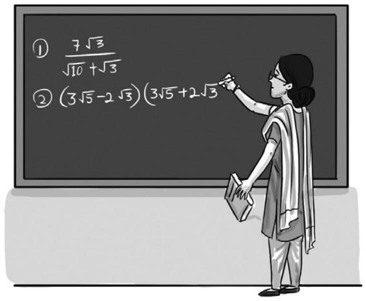
Reason (R): A number which is not in the form $\frac{p}{q}$, where *p* and *q* are integers and $q \neq 0$ is called an irrational number.

Sol. The correct answer is (a).

$\sqrt{3}$ is an irrational number because it cannot be written in the form $\frac{p}{q}$. Thus, both Assertion and Reason are correct and Reason is the correct explanation of Assertion.

Case Study Based Questions

19. The Mathematics teacher of class IX of a certain school wrote two different expressions on the blackboard. She selects two students Anu and Mayank to solve two different expressions on the blackboard and asks them to explain their simplifications. Anu explains simplification of $\frac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}}$ by rationalising the denominator and Mayank explains simplification of $(3\sqrt{5} - 2\sqrt{3})\ (3\sqrt{5} + 2\sqrt{3})$ by using the identity $(a - b)\ (a + b)$. Now, answer the following questions.



(*a*) What is the conjugate of $\sqrt{10}+\sqrt{3}$?

Ans. $\sqrt{10}-\sqrt{3}$

(*b*) What is the answer got by Anu by rationalising the denominator of $\dfrac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}}$?

Ans. $\sqrt{30}-3$

(*c*) (*i*) What answer Mayank got by simplifying $(3\sqrt{5}-2\sqrt{3})\,(3\sqrt{5}+2\sqrt{3})$?

Ans. 33

or

(*ii*) Given an example of any two rational numbers whose sum and product both are rationals?

Ans. The sum and product of any two rational numbers will always be rational.

20. In today's fast-paced world, everyone wants to raise their living standards. They are always working hard to give themselves and their loved ones a better life. But in this race of life, they do not get the time for social service. Hence they donate money to NGOs that are dedicated to helping the people in need. A survey was conducted, it was found that 7 out of every 11 households are donating some amount of their income to NGOs. Based on the above information, answer the following questions.

(*a*) What is the fraction of households which are donating?

Ans. $\dfrac{7}{11}$

(*b*) What is the fraction of households which are not donating?

Ans. $\dfrac{4}{11}$

(*c*) (*i*) What is the decimal form of the fraction of households, which are donating?

Ans. $0.\overline{63}$

or

(*ii*) What is the decimal form of the fraction of households, which are not donating?

Ans. $0.\overline{36}$

Very Short Answer Type Questions

21. What is the simplest rationalisation factor of $\sqrt[3]{40}$ among the numbers $\sqrt[3]{5}, \sqrt{40}, \sqrt[3]{40}, \sqrt{5}, \sqrt[3]{25}$?

Sol. We know that

$$\sqrt[3]{40} = \sqrt[3]{8\times 5} = \sqrt[3]{2^3\times 5} = 2\times\sqrt[3]{5} = 2\sqrt[3]{5}$$

Also, the simplest rationalisation factor of $\sqrt[3]{5}$ is $\sqrt[3]{5^2} = \sqrt[3]{25}$.

Hence, $\sqrt[3]{25}$ is the only simplest rationalisation factor of $\sqrt[3]{40}$.

22. Write any two irrational numbers between 0.21 and 0.22.

Sol. We know that $\sqrt{5} \simeq 2.236$

$\therefore \quad \dfrac{\sqrt{5}}{10} \simeq 0.224$

Also, any two rational numbers between 0.210 and 0.220 are 0.214 and 0.215 which are less than 0.224 by 0.010 and 0.009 respectively. Hence, any two irrational numbers between 0.21 and 0.22 can be taken as $\dfrac{\sqrt{5}}{10}-0.010$ and $\dfrac{\sqrt{5}}{10}-0.009$.

Alternatively, any two non-terminating and non-recurring numbers between 0.21 and 0.22 can be taken as 0.2101001000100001… and 0.212122122212222…

Note: Here the answer is not unique and so there may be many such irrational numbers between 0.21 and 0.22.

23. Express $0.\overline{31}$ in the form $\dfrac{p}{q}$, where p and q are integers and $q \neq 0$.

Sol. Let $x = 0.\overline{31} = 0.313131\ldots$

$\therefore \quad 100x = 31.313131\ldots$

$\therefore \quad 100x - x = 31$

$\Rightarrow \quad 99x = 31$

$\Rightarrow \quad x = \dfrac{31}{99}$

which is the required expression.

24. Express $\sqrt{5}$ correct upto three places of decimals.

Sol. We find $\sqrt{5}$ as follows:

	2.236…
2	$5.\overline{00}\,\overline{00}\,\overline{00}$
	4
42	100
	84
443	1600
	1329
4466	27100
	26796
	304

$\therefore \sqrt{5} = 2.236$

Short Answer Type Questions

25. Simplify: $\sqrt{432} - \dfrac{5}{2}\sqrt{\dfrac{1}{3}} + 4\sqrt{3}$

Sol. We have

$$\sqrt{432} - \frac{5}{2}\sqrt{\frac{1}{3}} + 4\sqrt{3}$$

$$= \sqrt{2^2 \times 2^2 \times 3^2 \times 3} - \frac{5}{2} \times \frac{\sqrt{3}}{3} + 4\sqrt{3}$$

$$= 12\sqrt{3} - \frac{5\sqrt{3}}{6} + 4\sqrt{3}$$

$$= \frac{72\sqrt{3} - 5\sqrt{3} + 24\sqrt{3}}{6} = \frac{91\sqrt{3}}{6}$$

26. Write $\dfrac{2-\sqrt{3}}{3-7\sqrt{2}}$ in the form of an expression with a rational denominator.

Sol. We have $\dfrac{2-\sqrt{3}}{3-7\sqrt{2}} = \dfrac{(2-\sqrt{3})(3+7\sqrt{2})}{(3-7\sqrt{2})(3+7\sqrt{2})}$

$$= \frac{6 + 14\sqrt{2} - 3\sqrt{3} - 7\sqrt{6}}{3^2 - (7\sqrt{2})^2}$$

$$= \frac{6 + 14\sqrt{2} - 3\sqrt{3} - 7\sqrt{6}}{9 - 98}$$

$$= \frac{3\sqrt{3} + 7\sqrt{6} - 6 - 14\sqrt{2}}{89}$$

which is the required expression.

Long Answer Type Questions

27. Simplify:

$$\frac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}} - \frac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}} - \frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}}$$

Sol. We have

$$\frac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}} = \frac{7\sqrt{3}(\sqrt{10}-\sqrt{3})}{(\sqrt{10})^2 - (\sqrt{3})^2}$$

$$= \frac{7\sqrt{30} - 21}{10 - 3}$$

$$= \frac{7\sqrt{30} - 21}{7}$$

$$= \sqrt{30} - 3 \qquad \ldots(1)$$

$$\frac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}} = \frac{2\sqrt{5}(\sqrt{6}-\sqrt{5})}{(\sqrt{6})^2 - (\sqrt{5})^2}$$

$$= \frac{2\sqrt{30} - 10}{6 - 5}$$

$$= 2\sqrt{30} - 10 \qquad \ldots(2)$$

$$\frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}} = \frac{3\sqrt{2}(\sqrt{15}-3\sqrt{2})}{(\sqrt{15})^2 - (3\sqrt{2})^2}$$

$$= \frac{3\sqrt{30} - 18}{15 - 18}$$

$$= \frac{18 - 3\sqrt{30}}{3}$$

$$= 6 - \sqrt{30} \qquad \ldots(3)$$

$$\therefore \frac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}} - \frac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}} - \frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}}$$

$$= \sqrt{30} - 3 - 2\sqrt{30} + 10 - 6 + \sqrt{30}$$

$$= 10 - 9 = 1 \qquad \text{[From (1), (2) and (3)]}$$

28. If $x = \dfrac{1}{2-\sqrt{3}}$, find the value of $x^3 - 2x^2 - 7x + 5$.

[CBSE SP 2011]

Sol. We have $x = \dfrac{1}{2-\sqrt{3}}$

$$= \frac{2+\sqrt{3}}{2^2 - (\sqrt{3})^2}$$

$$= \frac{2+\sqrt{3}}{1}$$

$= 2 + \sqrt{3}$

$\therefore \quad (x-2)^2 = 3$

$\Rightarrow \quad x^2 - 4x + 4 = 3$

$\Rightarrow \quad x^2 = 4x - 1 \quad \ldots(1)$

$\therefore \quad x^3 - 2x^2 - 7x + 5$

$= x \times (4x-1) - 2(4x-1) - 7x + 5 \quad$ [From (1)]

$= 4x^2 - x - 8x + 2 - 7x + 5$

$= 4(4x-1) - 16x + 7 \quad$ [From (1)]

$= 16x - 4 - 16x + 7 = 3$

Hence, the value of $x^3 - 2x^2 - 7x + 5$ is 3.

Let's Compete

(Page 15)

Multiple-Choice Questions

1. A rational number between $\sqrt{2}$ and $\sqrt{3}$ is

(*a*) 1.31 (*b*) 1.42

(*c*) 1.41 (*d*) 1.75

Sol. (*b*) 1.42

We know that $\sqrt{2} \simeq 1.414$ and $\sqrt{3} \simeq 1.732$

We see that out of four choices, only 1.42 lies between 1.414 and 1.732.

2. An irrational number between two rational numbers $2.\overline{19}$ and $2.\overline{33}$ is

(*a*) $\sqrt{\frac{11}{2}}$ (*b*) $\frac{\sqrt{17}}{3}$

(*c*) $\frac{\sqrt{15}}{2}$ (*d*) $\sqrt{5}$

Sol. (*d*) $\sqrt{5}$

We see by actual calculation that

$\sqrt{\frac{11}{2}} = \frac{\sqrt{22}}{2} \simeq 2.345,$

$\frac{\sqrt{17}}{3} \simeq 1.37,\ \frac{\sqrt{15}}{2} \simeq 1.9$ and $\sqrt{5} \simeq 2.236$

Out of these numbers only 2.236 i.e. $\sqrt{5}$ lies between two given numbers 2.191919… and 2.33333…

3. The real number $2.1\overline{06}$ when expressed in the form $\frac{p}{q}$ where p and q are integers and $q \neq 0$ is

(*a*) $\frac{15}{7}$ (*b*) $\frac{13}{6}$

(*c*) $\frac{139}{66}$ (*d*) $\frac{141}{67}$

Sol. (*c*) $\frac{139}{66}$

Let $x = 2.1\overline{06}$

$= 2.1060606\ldots$

$\therefore \quad 100x = 210.606060 \quad \ldots(1)$

$\therefore \quad 10000x = 21060.606060 \quad \ldots(2)$

Subtracting (1) from (2), we get

$9900\,x = 20850$

$\Rightarrow \quad x = \frac{20850}{9900} = \frac{139}{66}$

4. If $a + b\sqrt{7} = \frac{3+\sqrt{7}}{3-\sqrt{7}}$, where a and b are rational numbers, then the values of a and b are respectively

(*a*) 8 and 3 (*b*) 3 and 8

(*c*) 3 and 7 (*d*) 7 and 3

Sol. (*a*) 8 and 3

We have

$$a + b\sqrt{7} = \frac{3+\sqrt{7}}{3-\sqrt{7}}$$

$$= \frac{\left(3+\sqrt{7}\right)^2}{\left(3-\sqrt{7}\right)\left(3+\sqrt{7}\right)}$$

$$= \frac{9 + 7 + 6\sqrt{7}}{3^2 - \left(\sqrt{7}\right)^2}$$

$$= \frac{16 + 6\sqrt{7}}{9 - 7}$$

$$= 8 + 3\sqrt{7}$$

$\therefore \quad a = 8$ and $b = 3$

5. The value of $\frac{5^{\frac{1}{3}} \times 25^{\frac{-2}{3}} \times 625^{\frac{1}{3}}}{\sqrt[3]{125}}$ is equal to

(*a*) $\sqrt[3]{25}$ (*b*) $5^{\frac{4}{3}}$

(*c*) $\frac{1}{\sqrt[3]{25}}$ (*d*) $5^{-\frac{4}{3}}$

Sol. (*c*) $\frac{1}{\sqrt[3]{25}}$

$$\frac{5^{\frac{1}{3}} \times 25^{-\frac{2}{3}} \times 625^{\frac{1}{3}}}{\sqrt[3]{125}} = \frac{5^{\frac{1}{3}} \times \left(5^2\right)^{-\frac{2}{3}} \times \left(5^4\right)^{\frac{1}{3}}}{\left(5^3\right)^{\frac{1}{3}}}$$

$$= \frac{5^{\frac{1}{3}} \times 5^{-\frac{4}{3}} \times 5^{\frac{4}{3}}}{5}$$

$= 5^{\frac{1}{3}-\frac{4}{3}+\frac{4}{3}-1}$

$= 5^{-\frac{2}{3}}$

$= \frac{1}{5^{\frac{2}{3}}} = \frac{1}{\sqrt[3]{25}}$

6. The value of $\frac{4}{(216)^{-\frac{2}{3}}} - \frac{1}{(256)^{-\frac{3}{4}}}$ is equal to

(*a*) −60 (*b*) 60

(*c*) −80 (*d*) 80

Sol. (*d*) 80

$$\frac{4}{(216)^{-\frac{2}{3}}} - \frac{1}{(256)^{-\frac{3}{4}}} = \frac{4}{\left(2^3 \times 3^3\right)^{-\frac{2}{3}}} - \frac{1}{\left(2^8\right)^{-\frac{3}{4}}}$$

$$= \frac{4}{2^{-3\times\frac{2}{3}} \times 3^{-3\times\frac{2}{3}}} - \frac{1}{2^{-8\times\frac{3}{4}}}$$

$= 4 \times 2^2 \times 3^2 - 2^6$

$= 144 - 64$

$= 80$

7. If $a = 2$ and $b = 3$, then the value of $(a^a + b^b)^{-1}$ is equal to

(*a*) 31 (*b*) $\frac{1}{31}$

(*c*) $\frac{1}{13}$ (*d*) 13

Sol. (*b*) $\frac{1}{31}$

We have

$(a^a + b^b)^{-1} = (2^2 + 3^3)^{-1}$

$= (4 + 27)^{-1} = \frac{1}{31}$

8. If $\left(\frac{2}{7}\right) \times \left(\frac{7}{5}\right)^{3x} = \frac{98}{125}$, then the value of x is equal to

(*a*) 1 (*b*) 2

(*c*) −1 (*d*) −2

Sol. (*a*) 1

We have

$$\left(\frac{2}{7}\right) \times \left(\frac{7}{5}\right)^{3x} = \frac{98}{125}$$

$$\left(\frac{7}{5}\right)^{3x} = \frac{98}{125} \times \frac{7}{2}$$

$$= \frac{49 \times 7}{125}$$

$$= \frac{343}{125}$$

$$\Rightarrow \quad \left(\frac{7}{5}\right)^{3x} = \left(\frac{7}{5}\right)^{3}$$

On comparing, we get

$3x = 3$

$\Rightarrow \quad x = 1$

∴ The required value of x is 1.

9. The value of $\sqrt{5 - \sqrt{24}}$ is equal to

(*a*) $\sqrt{5} - 1$ (*b*) $\sqrt{6} - \sqrt{5}$

(*c*) $\sqrt{6} - 1$ (*d*) $\sqrt{3} - \sqrt{2}$

Sol. (*d*) $\sqrt{3} - \sqrt{2}$

We have

$$5 - \sqrt{24} = 3 + 2 - 2\sqrt{6}$$

$$= \left(\sqrt{3}\right)^2 + \left(\sqrt{2}\right)^2 - 2 \times \sqrt{2} \times \sqrt{3}$$

$$= \left(\sqrt{3} - \sqrt{2}\right)^2$$

$$\therefore \quad \sqrt{5 - \sqrt{24}} = \sqrt{3} - \sqrt{2}$$

10. The simplified value of $\frac{5^{89} + 5^{90}}{5^{92} - 5^{91}}$ is equal to

(*a*) $\frac{3}{25}$ (*b*) $\frac{2}{25}$

(*c*) 0.06 (*d*) 0.05

Sol. (*c*) 0.06

Let $5^{89} = a$

∴ $5^{90} = 5a$, $5^{92} = 125a$ and $5^{91} = 25a$

$$\frac{5^{89} + 5^{90}}{5^{92} - 5^{91}} = \frac{a + 5a}{125a - 25a}$$

$$= \frac{6}{100}$$

$= 0.06$

Life Skills

(Page 16)

1. Two classmates Salma and Anil simplified two different expressions during the revision hour and explained to each other their simplifications.

Salma explains simplification of $\frac{\sqrt{2}}{\sqrt{5}+\sqrt{3}}$ and Anil explains simplification of $\sqrt{28} + \sqrt{98} + \sqrt{147}$.

Write both the simplifications.

Sol. We have

$$\frac{\sqrt{2}}{\sqrt{5}+\sqrt{3}} = \frac{\sqrt{2}\left(\sqrt{5}-\sqrt{3}\right)}{\left(\sqrt{5}\right)^2-\left(\sqrt{3}\right)^2}$$

$$= \frac{\sqrt{10}-\sqrt{6}}{5-3}$$

$$= \frac{\sqrt{10}-\sqrt{6}}{2}$$

and $\sqrt{28}+\sqrt{98}+\sqrt{147}$

$$= \sqrt{4\times 7}+\sqrt{49\times 2}+\sqrt{49\times 3}$$

$$= 2\sqrt{7}+7\sqrt{2}+7\sqrt{3}$$

2. Varun was facing some difficulty in simplifying $\frac{1}{\sqrt{7}-\sqrt{3}}$. His classmate Priya gave him a clue to rationalise the denominator for simplification. Varun simplified the expression and thanked Priya for this goodwill. How did Varun simplify $\frac{1}{\sqrt{7}-\sqrt{3}}$?

Sol. Varun's simplification

$$\frac{1}{\sqrt{7}-\sqrt{3}} = \frac{\sqrt{7}+\sqrt{3}}{\left(\sqrt{7}-\sqrt{3}\right)\left(\sqrt{7}+\sqrt{3}\right)}$$

$$= \frac{\sqrt{7}+\sqrt{3}}{\left(\sqrt{7}\right)^2-\left(\sqrt{3}\right)^2}$$

$$= \frac{\sqrt{7}+\sqrt{3}}{7-3}$$

$$= \frac{\sqrt{7}+\sqrt{3}}{4}$$