2

Polynomials

Checkpoint (Page 18)

1. Write any polynomial in x which has no real zero at all. Write down its value when $x = -1$.

Sol. Clearly, $x^2 + 1$ is a polynomial which has no real zero at all, since $x^2 + 1$ cannot be zero for any real value of x.

For if $x^2 + 1 = 0$, then $x = \pm\sqrt{-1}$ which are not real.

Now, when $x = -1$, then $x^2 + 1 = (-1)^2 + 1 = 2$ which is the required value of the polynomial $x^2 + 1$ at $x = -1$.

2. Find the remainder when $4\sqrt{2}x^2 + 3x + 5$ is divided by $x + \sqrt{2}$, by using remainder theorem.

Sol. Let $f(x) = 4\sqrt{2}x^2 + 3x + 5$.

$\therefore$ By remainder theorem, the required remainder is

$$f(-\sqrt{2}) = 4\sqrt{2}(-\sqrt{2})^2 + 3(-\sqrt{2}) + 5$$
$$= 8\sqrt{2} - 3\sqrt{2} + 5$$
$$= 5\sqrt{2} + 5$$
$$= 5(\sqrt{2} + 1)$$

3. By using the factor theorem, show that $x + 3$ is a factor of the polynomial $2x^4 + 6x^3 - 3x^2 - 5x + 12$.

Sol. Let $f(x) = 2x^4 + 6x^3 - 3x^2 - 5x + 12$

We see that

$$f(-3) = 2(-3)^4 + 6(-3)^3 - 3(-3)^2 - 5(-3) + 12$$
$$= 162 - 162 - 27 + 15 + 12$$
$$= 0 - 27 + 27$$
$$= 0$$

Hence, by factor theorem $x + 3$ is a factor of $f(x)$.

4. Find the value of a if $x - 1$ is a factor of $a^2x^3 - 4ax + 4a - 1$.

Sol. Let $f(x) = a^2x^3 - 4ax + 4a - 1$

$\because$ $x - 1$ is a factor of $f(x)$,

$\therefore$ By factor theorem, we have

$$f(1) = 0$$
$$\Rightarrow a^2 - 4a + 4a - 1 = 0$$
$$\Rightarrow a^2 = 1$$
$$\Rightarrow a = \pm 1$$

which is the required value of a.

5. Using identities, find the product

$$\left(x - \frac{1}{x}\right)\left(x + \frac{1}{x}\right)\left(x^2 + \frac{1}{x^2}\right)\left(x^4 + \frac{1}{x^4}\right).$$

Sol. We have

$$\left(x - \frac{1}{x}\right)\left(x + \frac{1}{x}\right)\left(x^2 + \frac{1}{x^2}\right)\left(x^4 + \frac{1}{x^4}\right)$$
$$= \left(x^2 - \frac{1}{x^2}\right)\left(x^2 + \frac{1}{x^2}\right)\left(x^4 + \frac{1}{x^4}\right)$$
$$= \left(x^4 - \frac{1}{x^4}\right)\left(x^4 + \frac{1}{x^4}\right)$$
$$= \left(x^8 - \frac{1}{x^8}\right)$$

which is the required product.

6. If $a + b = 4$ and $a - b = 2$, find the value of $a^2 + b^2$.

Sol. We have

$$a^2 + b^2 = \frac{(a + b)^2 + (a - b)^2}{2}$$
$$= \frac{4^2 + 2^2}{2}$$

$$= \frac{16 + 4}{2}$$

$$= \frac{20}{2}$$

$$= 10$$

which is the required value.

7. Without actually calculating the cubes, evaluate $75^3 - 40^3 - 35^3$.

Sol. We have

$75^3 - 40^3 - 35^3$

$$= (75 - 40)(75^2 + 75 \times 40 + 40^2) - 35^3$$
$$= 35(75^2 + 3000 + 40^2) - 35^3$$
$$= 35[75^2 - 35^2 + 3000 + 1600]$$
$$= 35[(75 + 35)(75 - 35) + 4600]$$
$$= 35[110 \times 40 + 4600]$$
$$= 35 \times (4400 + 4600)$$
$$= 35 \times 9000$$
$$= 315000$$

which is the required value.

8. Factorise: $3 - 12(a - b)^2$

Sol. We have

$$3 - 12(a - b)^2 = 3[1 - 4(a - b)^2]$$
$$= 3[1^2 - \{2(a - b)\}^2]$$
$$= 3[1 + 2(a - b)][1 - 2(a - b)]$$
$$= 3(1 + 2a - 2b)(1 - 2a + 2b)$$

which are the required factors.

9. Factorise: $(a^2 - 2a)^2 - 23(a^2 - 2a) + 120$

Sol. We have

$(a^2 - 2a)^2 - 23(a^2 - 2a) + 120$

$$= x^2 - 23x + 120 \quad \text{where } x = a^2 - 2a \quad \ldots(1)$$
$$= x^2 - 8x - 15x + 120$$
$$= x(x - 8) - 15(x - 8)$$
$$= (x - 15)(x - 8)$$
$$= (a^2 - 2a - 15)(a^2 - 2a - 8)$$
$$= (a^2 + 3a - 5a - 15)(a^2 + 2a - 4a - 8)$$
$$= \{a(a + 3) - 5(a + 3)\}\{a(a + 2) - 4(a + 2)\}$$
$$= (a + 3)(a - 5)(a + 2)(a - 4)$$

which are the required factors.

10. Using factor theorem, factorise $2x^3 - x^2 - 13x - 6$.

Sol. We have $f(x) = 2x^3 - x^2 - 13x - 6$

We see that $f(-2) = 2 \times (-2)^3 - (-2)^2 - 13(-2) - 6$

$$= -16 - 4 + 26 - 6$$
$$= 0$$

$\therefore$ By factor theorem, $x + 2$ will be a factor of $f(x)$.

Now, we write

$$f(x) = 2x^3 - x^2 - 13x - 6$$
$$= 2x^3 + 4x^2 - 5x^2 - 10x - 3x - 6$$
$$= 2x^2(x + 2) - 5x(x + 2) - 3(x + 2)$$
$$= (x + 2)(2x^2 - 5x - 3)$$
$$= (x + 2)(2x^2 + x - 6x - 3)$$
$$= (x + 2)\{x(x + 1) - 3(2x + 1)\}$$
$$= (x + 2)(x - 3)(2x + 1)$$

which are the required factors.

Check Your Progress

(Page 22)

Multiple-Choice Questions

1. A quadratic polynomial whose sum and product of zeroes are respectively 3 and −10 is

(*a*) $x^2 - 3x - 10$

(*b*) $x^2 + 3x + 10$

(*c*) $x^2 + 3x - 10$

(*d*) $x^2 - 3x + 10$

Sol. (*a*) $x^2 - 3x - 10$

We have,

Sum of the zeroes = 3 and

Product of the zeroes = −10

Hence, the required polynomial is

x^2 − (Sum of the zeroes)x + Product of zeroes

$$= x^2 - 3x - 10$$

2. If the product of the zeroes of the quadratic polynomial $2x^2 - 3x - 5c$ is $\frac{1}{2}$, then the value of c is equal to

(*a*) −5 (*b*) 5

(*c*) $\frac{1}{5}$ (*d*) $-\frac{1}{5}$

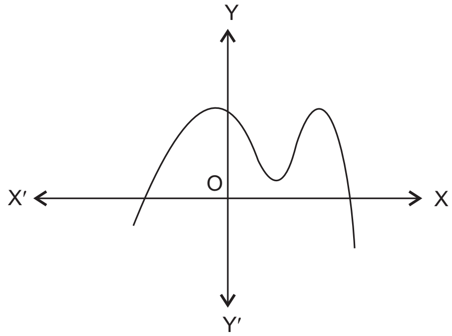
Sol. (*d*) $-\frac{1}{5}$

We see that the product of zeroes of the given polynomial is $\frac{-5c}{2}$

$$\therefore \quad \frac{-5c}{2} = \frac{1}{2}$$

$$\Rightarrow \quad c = -\frac{1}{5}$$

3. Graph of a polynomial $p(x)$ is given in the figure. The number of zeroes of $p(x)$ is:



(*a*) 2 (*b*) 3

(*c*) 4 (*d*) 5 **[CBSE 2023 Basic]**

Sol. (*a*) 2

The graph of $y = p(x)$ cuts the x-axis at two points.

$\therefore$ $p(x)$ has two zeroes.

4. If $p(x) = x^2 + 5x + 6$, then $p(-2)$ is

(*a*) 20 (*b*) 0

(*c*) –8 (*d*) 8 **[CBSE 2023 Basic]**

Sol. (*b*) 0

$$p(x) = x^2 + 5x + 6$$
$$p(-2) = (-2)^2 + 5(-2) + 6$$
$$= 4 - 10 + 6$$
$$\Rightarrow \quad p(-2) = 0$$

5. The number of polynomials having zeroes –1 and 2 is

(*a*) exactly 2. (*b*) only 1.

(*c*) at most 2. (*d*) infinite.

[CBSE 2023 Standard]

Sol. (*d*) infinite

A polynomial with given zeroes can be written as $k(x - \alpha)(x - \beta)$ where k is a non-zero constant and α and β are the zeroes. Since k can be any non-zero constant, hence, there will be infinite such polynomials.

6. The zeroes of a polynomial $x^2 + px + q$ are twice the zeroes of the polynomial $4x^2 - 5x - 6$. The value of p is

(*a*) $-\frac{5}{2}$ (*b*) $\frac{5}{2}$

(*c*) –5 (*d*) 10

[CBSE 2024 Standard]

Sol. (*a*) $-\frac{5}{2}$

Let α and β be roots of the polynomial $4x^2 - 5x - 6$.

$$\alpha + \beta = \frac{5}{4} \quad \text{...(1)}$$

According to the question 2α and 2β are the roots of the equation $x^2 + px + q$

$$2\alpha + 2\beta = -p$$
$$2(\alpha + \beta) = -p$$
$$\Rightarrow \quad p = -2 \times \frac{5}{4} = -\frac{5}{2} \quad \text{[from (1)]}$$

Very Short Answer Type Questions

7. If $p(x) = 3x^2 - 9$, find the value of $p\left(a + \frac{1}{a}\right)$ where $a \neq 0$. Hence, find the value of $p(2)$.

Sol. We have $p(x) = 3x^2 - 9$

$$= 3(x^2 - 3)$$
$$= 3\left(x + \sqrt{3}\right)\left(x - \sqrt{3}\right)$$
$$\therefore \quad p\left(a + \frac{1}{a}\right) = 3\left(a + \frac{1}{a} + \sqrt{3}\right)\left(a + \frac{1}{a} - \sqrt{3}\right)$$

$\therefore$ Putting $a = 1$, we get

$$p(2) = 3\left(2 + \sqrt{3}\right)\left(2 - \sqrt{3}\right)$$
$$= 3(2^2 - 3)$$
$$= 3(4 - 3)$$
$$= 3$$

which is the required value.

8. One zero of a quadratic polynomial is $\sqrt{3}$ and the product of the two zeroes is $-5\sqrt{3}$. Find the quadratic polynomial.

Sol. Let α and β be two zeroes of the polynomial, where $\alpha = \sqrt{3}$,

Now it is given that

$$\alpha\beta = -5\sqrt{3}$$
$$\therefore \quad \beta = \frac{-5\sqrt{3}}{\sqrt{3}} = -5$$

$\therefore$ The required polynomial is

x^2 – (Sum of the zeroes)x + Product of the zeroes

$$= x^2 - \left(-5 + \sqrt{3}\right)x - 5\sqrt{3}$$
$$= x^2 + \left(5 - \sqrt{3}\right)x - 5\sqrt{3}$$

9. Write down the quadratic polynomial whose zeroes are $3 + \sqrt{5}$ and $3 - \sqrt{5}$. **[CBSE SP 2011]**

Sol. The required quadratic polynomial is

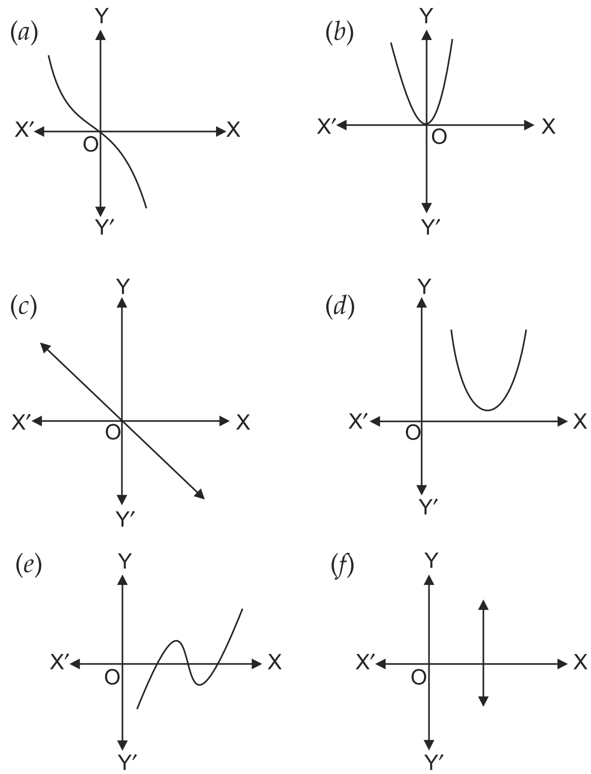
x^2 + (Sum of the zeroes)x + Product of the zeroes

$$= x^2 - \left(3 + \sqrt{5} + 3 - \sqrt{5}\right)x + \left(3 + \sqrt{5}\right)\left(3 - \sqrt{5}\right)$$

$$= x^2 - 6x + (9 - 5)$$

$$= x^2 - 6x + 4$$

10. Which of the following correspond to the graph of a linear or quadratic polynomial equation and find the number of zeroes of each polynomial.



Sol. (*a*) The graph is neither a line nor a parabola. Hence, the graph is neither linear nor quadratic. Since the graph cuts the x-axis at only one point, hence this polynomial has only one zero.

(*b*) Since the graph is a parabola in shape, hence it is a graph of a quadratic polynomial equation. Since it cuts the x-axis at only one point, hence this polynomial has only one zero.

(*c*) This graph is a straight line and it cuts the x-axis at only one point. Hence, the polynomial is a linear one and this polynomial has only one zero.

(*d*) This graph is a parabola. Hence, it is the graph of a quadratic polynomial equation. Since this graph does not cut the x-axis at all. Hence, this polynomial has no zero.

(*e*) This graph is neither a parabola nor a line. Hence, the polynomial is neither linear nor quadratic. Again, this graph cuts the x-axis at three point. Hence, the polynomial has three zeroes.

(*f*) This graph is a line intersecting x-axis at only one point. Hence, the polynomial is linear and has only one zero.

11. If the sum of the zeroes of the quadratic polynomial $f(y) = ky^2 + 2y + 3k$ is equal to their product, find the value of k. **[CBSE SP 2013]**

Sol. We know that sum of the zeroes of the given polynomial $= -\frac{2}{k}$ and the product of the zeroes $= \frac{3k}{k} = 3$.

It is given that

$$-\frac{2}{k} = 3$$

$$\Rightarrow \qquad k = -\frac{2}{3}$$

which is the required value of k.

12. If 2 and −3 are the zeroes of the quadratic polynomial $x^2 + (a + 1)x + b$, then find the values of a and b. **[CBSE SP 2016]**

Sol. We have

Sum of the zeroes $= -(a + 1)$

and Product of the zeroes $= b$

$$\therefore \qquad -(a + 1) = 2 - 3 = -1$$

$$\therefore \qquad a = 0$$

$$\text{and} \qquad b = 2 \times (-3) = -6$$

$\therefore$ The required values of a and b are 0 and −6 respectively.

Short Answer Type Questions

13. If α and β are the zeroes of the quadratic polynomial $3x^2 - 5x - 2$, find the value of $\alpha^2 + \beta^2$.

Sol. We have

$$\alpha + \beta = \text{Sum of zeroes} = \frac{5}{3} \qquad \ldots(1)$$

$$\text{and} \qquad \alpha\beta = \text{Product of zeroes} = -\frac{2}{3} \qquad \ldots(2)$$

$$\therefore \quad (\alpha^2 + \beta^2) = (\alpha + \beta)^2 - 2\alpha\beta$$

$$= \left(\frac{5}{3}\right)^2 + 2 \times \frac{2}{3} \qquad \text{[From (1) and (2)]}$$

$$= \frac{25}{9} + \frac{4}{3}$$

$$= \frac{25 + 12}{9}$$

$$= \frac{37}{9}$$

which is the required value.

14. If α and β are zeroes of the polynomial $2x^2 - 5x + 7$, then find a quadratic polynomial whose zeroes are (3α + 4β) and (4α + 3β). **[CBSE SP 2013]**

Sol. We have

$$\alpha + \beta = \text{Sum of the zeroes} = \frac{5}{2} \quad \text{...(1)}$$

and $$\alpha\beta = \text{Product of zeroes} = \frac{7}{2} \quad \text{...(2)}$$

Now, the quadratic polynomial whose zeroes are 3α + 4β and 4α + 3β is

x^2 – (Sum of the zeroes)x + (Product of the zeroes)

$= x^2 - (3\alpha + 4\beta + 4\alpha + 3\beta)x + (3\alpha + 4\beta)(4\alpha + 3\beta)$

$= x^2 - 7(\alpha + \beta)x + 12\alpha^2 + 12\beta^2 + 25\,\alpha\beta$

$= x^2 - 7 \times \frac{5}{2}x + 12\{(\alpha + \beta)^2 - 2\,\alpha\beta\} + 25\alpha\beta$

[From (1)]

$= x^2 - \frac{35}{2}x + 12(\alpha + \beta)^2 + \alpha\beta$

$= x^2 - \frac{35}{2}x + 12 \times \frac{25}{4} + \frac{7}{2}$ [From (1) and (2)]

$= x^2 - \frac{35}{2}x + 75 + \frac{7}{2}$

$= x^2 - \frac{35}{2}x + \frac{157}{2}$

$= \frac{1}{2}\left(2x^2 - 35x + 157\right)$

Hence, the required polynomial is $2x^2 - 35x + 157$.

15. If α, β are zeroes of the quadratic polynomial $x^2 - 5x + 6$, form another quadratic polynomial whose zeroes are $\frac{1}{\alpha}, \frac{1}{\beta}$. **[CBSE 2023 Basic]**

Sol. $x^2 - 5x + 6$

$$\alpha + \beta = \frac{-b}{a} = \frac{-(-5)}{1}$$

$\Rightarrow$ $$\alpha + \beta = 5 \quad \text{...(1)}$$

$$\alpha\beta = 6 \quad \text{...(2)}$$

Sum of the zeroes $\frac{1}{\alpha}$ and $\frac{1}{\beta}$

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta + \alpha}{\alpha\beta}$$

$\Rightarrow$ $$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = \frac{5}{6}$$

Product of the zeroes = $\frac{1}{\alpha} \cdot \frac{1}{\beta}$

$$\frac{1}{\alpha} \cdot \frac{1}{\beta} = \frac{1}{\alpha\beta}$$

$$\frac{1}{\alpha\beta} = \frac{1}{6}$$

Therefore, the required polynomial $p(x)$ is given by

$p(x) = k\,[x^2$ – (sum of zeroes) x + product of zeroes]

where k is any non-zero real number

$\Rightarrow$ $$p(x) = k\left[x^2 - \frac{5}{6}x + \frac{1}{6}\right]$$

$\Rightarrow$ $$p(x) = k\left[x^2 - \frac{5}{6}x + \frac{1}{6}\right]$$

$\Rightarrow$ $$p(x) = 6x^2 - 5x + 1, \text{ for } k = 6.$$

Long Answer Type Questions

16. If $\sqrt{5}$ and $-\sqrt{5}$ are two zeroes of the polynomial $x^3 + 3x^2 - 5x - 15$, then find its third zero. **[CBSE 2010]**

Sol. If α is the third zero of the polynomial $x^3 + 3x^2 - 5x - 15$, then

product of the zeroes = $\alpha\sqrt{5}\left(-\sqrt{5}\right)$

$\Rightarrow$ $$15 = -5\alpha$$

$\Rightarrow$ $$-5\alpha = 15$$

$\Rightarrow$ $$\alpha = -3$$

Hence, the required zero is –3.

17. The product of two zeroes of the polynomial $p(x) = x^3 - 6x^2 + 11x - 6$ is 6. Find all the zeroes of the polynomial.

Sol. It α, β and γ be the three zeroes of the polynomial

$$p(x) = x^3 - 6x^2 + 11x - 6$$

then $\alpha + \beta + \gamma = 6$ ...(1)

$\alpha\beta + \alpha\gamma + \beta\gamma = 11$ ...(2)

and $\alpha\beta\gamma = 6$ ...(3)

Let $\beta\gamma = 6$ [Given] ...(4)

∴ $\alpha = 1$ [From (3)]

∴ Either β = 3 and γ = 2 or β = 6 and γ = 1.

But β = 3, γ = 2 and α = 1 satisfy (1) and not β = 6, γ = 1, α = 1.

Hence, the required zeroes of the given polynomial are 1, 2 and 3.

Higher Order Thinking Skills (HOTS) Questions

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1. If α and β are the zeroes of the polynomial $f(x) = 4x^2 - 4x + 1$, find the value of $\frac{1}{\alpha^4} + \frac{1}{\beta^4}$.

Sol. We have $\alpha + \beta = \frac{4}{4} = 1$ ...(1)

and $\alpha\beta = \frac{1}{4}$ ...(2)

Now, $\frac{1}{\alpha^4} + \frac{1}{\beta^4} = \frac{\alpha^4 + \beta^4}{(\alpha\beta)^4}$

$= \frac{(\alpha^2 + \beta^2)^2 - 2(\alpha\beta)^2}{(\alpha\beta)^4}$

$= \frac{\{(\alpha + \beta)^2 - 2\alpha\beta\}^2 - 2(\alpha\beta)^2}{(\alpha\beta)^4}$

$= \frac{\left(1 - 2 \times \frac{1}{4}\right)^2 - 2 \times \frac{1}{16}}{\left(\frac{1}{4}\right)^4}$

[From (1) and (2)]

$= \frac{\frac{1}{4} - \frac{1}{8}}{\frac{1}{256}}$

$= \frac{2-1}{8} \times 256$

$= \frac{1}{8} \times 256 = 32$

which is the required value.

2. If the zeroes of the polynomial $x^2 + px + q$ are double in value to the zeroes of $2x^2 - 5x - 3$, find the values of p and q. **[CBSE SP 2012]**

Sol. Let α and β be the zeroes of the polynomial $2x^2 - 5x - 3$.

Then $\alpha + \beta = \frac{5}{2}$ ...(1)

and $\alpha\beta = -\frac{3}{2}$ ...(2)

$\therefore$ According to the problem, 2α and 2β are the zeroes of the polynomial $x^2 + px + q$.

$\therefore$ $2(\alpha + \beta) = -p$ ...(3)

and $4\alpha\beta = q$ ...(4)

$\therefore$ From (1) and (3), we have

$2\left(\frac{5}{2}\right) = -p$

$\Rightarrow$ $5 = -p$

$\Rightarrow$ $p = -5$

and from (2) and (4), we have

$4 \times \left(-\frac{3}{2}\right) = q$

$\Rightarrow$ $q = -6$

Hence, the required values of p and q are -5 and -6 respectively.

3. If the zeroes of the polynomial $x^3 - 15x^2 + 71x + k$ are in AP, find the value of k.

Sol. Let $\alpha - d$, α and $\alpha + d$ be the three zeroes of the polynomial $x^3 - 15x^2 + 71x + k$.

Then we have

$\alpha - d + \alpha + \alpha + d = 15$

$\Rightarrow$ $3\alpha = 15$

$\Rightarrow$ $\alpha = 5$ ...(1)

Also, $(\alpha - d)\,\alpha\,(\alpha + d) = -k$

$\Rightarrow$ $k = -\alpha\,(\alpha^2 - d^2)$

$= -5\,(25 - d^2)$

$= -125 + 5d^2$ ...(2)

and $(\alpha - d)\,\alpha + (\alpha - d)(\alpha + d) + \alpha\,(\alpha + d) = 71$

$\Rightarrow$ $(5 - d)\,5 + (5 - d)(5 + d) + 5(5 + d) = 71$

[From (1)]

$\Rightarrow$ $25 - 5d + 25 - d^2 + 25 + 5d = 71$

$\Rightarrow$ $-d^2 + 75 = 71$

$\Rightarrow$ $d^2 = 4$ ...(3)

$\therefore$ From (2) and (3), we get,

$k = 5 \times 4 - 125 = -105$

which is the required value of k.

4. Find the zeroes of the polynomial $f(x) = 2x^3 - x^2 - 4x + 2$ if two of its zeroes are equal in magnitude but opposite in sign.

Sol. Let the three zeroes of $f(x)$ be α, $-\alpha$ and β.

$\alpha - \alpha + \beta = \frac{1}{2}$

$\Rightarrow$ $\beta = \frac{1}{2}$ ...(1)

and $\alpha(-\alpha)\beta = \frac{-2}{2}$

$\Rightarrow$ $\alpha^2\beta = 1$

$\Rightarrow$ $\alpha^2 . \frac{1}{2} = 1$ [From (1)]

$\Rightarrow$ $\alpha^2 = 2$

$\Rightarrow$ $\alpha = \pm\sqrt{2}$

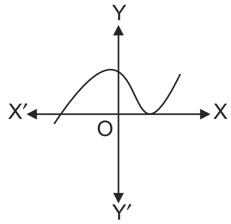
Hence, the required zeroes of $f(x)$ are $+\sqrt{2}$, $-\sqrt{2}$ and $\frac{1}{2}$.

Self-Assessment

(Page 24)

Multiple-Choice Questions

1. The graph of $y = p(x)$ is given. The number of zeroes of the polynomial $p(x)$ is



(*a*) 0 (*b*) 3

(*c*) 2 (*d*) 4 **[CBSE SP 2011]**

Sol. (*c*) 2

Since the graph cuts the x-axis at two points, hence the required number of zeroes of the given polynomial is 2.

2. A real number k is called a zero of the polynomial $p(x)$ when

(*a*) $p(k) = -2$ (*b*) $p(k) = 0$

(*c*) $p(k) = 1$ (*d*) $p(k) = -1$

Sol. (*b*) $p(k) = 0$

By definition, we know that k is a zero of the polynomial $p(x)$ if $p(k) = 0$

3. If one zero of the quadratic polynomial $kx^2 + 3x + k$ is 2, then the value of k is

(*a*) $-\frac{6}{5}$ (*b*) $\frac{6}{5}$

(*c*) $\frac{5}{6}$ (*d*) $-\frac{5}{6}$ **[CBSE 2023 Basic]**

Sol. (*a*) $-\frac{6}{5}$

One zero of the quadratic polynomial $kx^2 + 3x + k$ is 2

$$\Rightarrow \quad p(2) = 0$$
$$\Rightarrow \quad k(2)^2 + 3(2) + k = 0$$
$$\Rightarrow \quad 4k + 6 + k = 0$$
$$\Rightarrow \quad 5k = -6$$
$$\Rightarrow \quad k = -\frac{6}{5}$$

4. If one of the zeroes of the quadratic polynomial $(\alpha - 1)x^2 + \alpha x + 1$ is -3, then the value of α is

(*a*) $-\frac{2}{3}$ (*b*) $\frac{2}{3}$

(*c*) $\frac{4}{3}$ (*d*) $\frac{3}{4}$ **[CBSE 2024 Basic]**

Sol. (*c*) $\frac{4}{3}$

Since -3 is one of the zeroes of the quadratic polynomial, then $p(-3) = 0$

$$(\alpha - 1)(-3)^2 + \alpha(-3) + 1 = 0$$
$$\Rightarrow \quad 9(\alpha - 1) - 3\alpha + 1 = 0$$
$$\Rightarrow \quad 9\alpha - 9 - 3\alpha + 1 = 0$$
$$\Rightarrow \quad 6\alpha - 8 = 0$$
$$\Rightarrow \quad x = \frac{8}{6}$$
$$\Rightarrow \quad x = \frac{4}{3}$$

5. The sum of zeroes of the polynomial $\sqrt{2}x^2 - 17$ is given as

(*a*) $\frac{17\sqrt{2}}{2}$ (*b*) $\frac{-17\sqrt{2}}{2}$

(*c*) 0 (*d*) 1

[CBSE 2023 Standard]

Sol. (*c*) 0

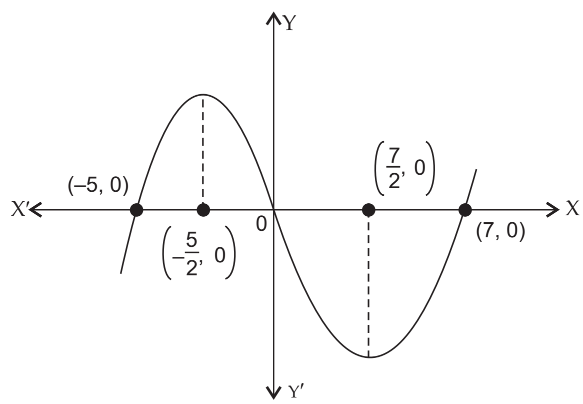
$$p(x) = \sqrt{2}\,x^2 - 17$$

Writing the above polynomial in the standard form,

$$p(x) = \sqrt{2}\,x^2 + 0x - 17$$

Sum of roots $= \frac{-b}{a} = \frac{0}{\sqrt{2}} = 0$

6. The graph of $y = p(x)$ is given in the figure below. Zeroes of the polynomial $p(x)$ are



(*a*) $-5, 7$ (*b*) $\frac{-5}{2}, \frac{-7}{2}$

(*c*) $-5, 0, 7$ (*d*) $-5, \frac{-5}{2}, \frac{7}{2}, 7$

Sol. (*c*) $-5, 0, 7$

The graph of the polynomial $p(x)$ cuts the x-axis at three points namely -5, 0 and 7. Hence zeroes of the polynomial are $-5, 0, 7$.

Fill in the Blanks

7. Graph of polynomial $f(x) = (x - 2)(x - 3)$ will intersect x-axis at exactly **two** points.

8. A polynomial of degree zero is called a **constant** polynomial.

9. Quadratic polynomial whose sum and product of zeroes are $\frac{-1}{2}$ and $\frac{1}{2}$ respectively is $\boldsymbol{k(2x^2 + x + 1)}$.

Sol. $k(x^2$ – (Sum of zeroes) x + (Product of zeroes))

$$\Rightarrow k\left(x^2 - \left(-\frac{1}{2}\right)x + \frac{1}{2}\right)$$

$$\Rightarrow k\left(x^2 + \frac{1}{2}x + \frac{1}{2}\right)$$

$$\Rightarrow k\left(2x^2 + x + 1\right)$$

10. Polynomial $ax^2 - c$ has two zeroes which are **equal** but opposite in sign.

Assertion-Reason Type Questions

Directions (Q. Nos. 11 to 15): Each of these questions contains an assertion followed by reason. Read them carefully, and answer the question on the basis of the following options, select the one that best describes the two statements.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

11. **Assertion (A):** $x + 4 = 0$ is a linear polynomial.

Reason (R): A polynomial of degree 1 is called a linear polynomial.

Sol. The correct answer is (a). Both Assertion and Reason are correct and the Reason is the correct explanation of Assertion.

12. **Assertion (A):** 4 is a zero of the polynomial $p(x) = x^2 - 3x - 4$

Reason (R): Putting $x = 4$, we get $p(x) = 1$

Sol. The correct answer is (c) as after putting $x = 4$, we get $p(x) = 0$.

Thus, the Assertion is correct but Reason is incorrect.

13. **Assertion (A):** The product of the zeroes of quadratic polynomial $x^2 + 7x + 10$ is 10.

Reason (R): Sum of zeroes is given by the negative ratio of coefficient of x and that of x^2.

Sol. The correct answer is (b) as the products of zeroes is given by the ratio of constant term to coefficient of x^2.

Thus, Assertion is correct and Reason is also correct but it is not the correct explanation of Assertion, as it talks about sum instead of product.

14. **Assertion (A):** Zeroes of a polynomial $p(x) = x^2 - 2x - 3$ are –1 and 3.

Reason (R): The graph of polynomial $p(x) = x^2 - 2x - 3$ intersects x-axis at (–1, 0) and (3, 0). **[CBSE 2024 Basic]**

Sol. (*a*) Both assertion and reason are true and reason is the correct explanation of assertion.

$$p(x) = x^2 - 2x - 3$$

$$x^2 - 2x - 3 = x^2 - 3x + x - 3$$

$$= x(x - 3) + 1(x - 3)$$

$$= (x - 3)(x + 1)$$

$\therefore$ Roots are $x = 3$ and $x = -1$.

Therefore, the graph of the polynomial intersects the x-axis at (–1, 0) and (3, 0).

15. **Assertion (A):** If the graph of a polynomial touches x-axis at only one point, then the polynomial cannot be a quadratic polynomial.

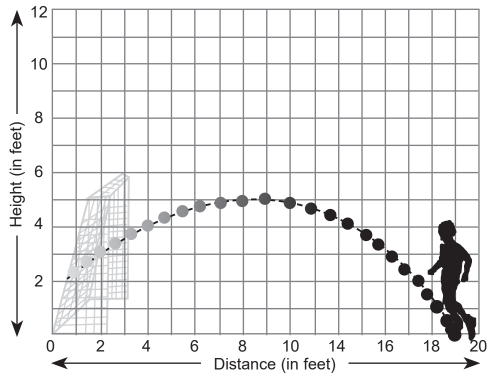
Reason (R): A polynomial of degree $n(n > 1)$ can have at most n zeroes. **[CBSE 2024 Standard]**

Sol. (*d*) Assertion is false but reason is true.

The graph of a quadratic polynomial is parabola. If the roots are equal, then the graph will touch the x-axis at only one point.

Case Study Based Questions

16. A football tournament was going on between a Delhi school and a Haryana school. The score was levelled and only 5 minutes were left. A boy from Haryana school gave a goal in the last minute and won the tournament. The path of the football was traced on a graph paper as shown. Here the variables x and y represents the horizontal distance (ft) and vertical height (ft) respectively. A quadratic function can be expressed as an expression in the form $ax^2 + bx + c$ where $a \neq 0$.



Based on above information, answer the following questions.

(*a*) What is the shape of the path followed by the football?

Ans. Parabola

(*b*) What kind of polynomial is $f(x) = 0x^2 + 5x + 3$?

Ans. Linear

(*c*) (*i*) From the graph, write the number of zeroes of the curve of the polynomial.

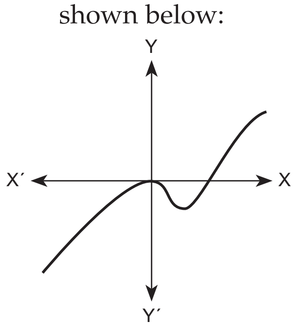
Ans. 1

or

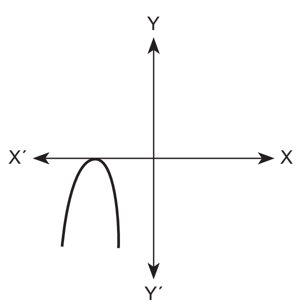
(*ii*) At what height, did the ball reach the goal post?

Ans. 2 ft

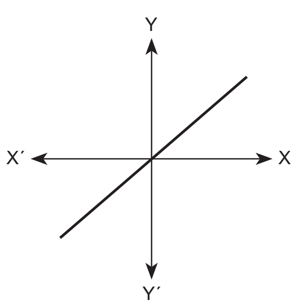
17. A mathematics teacher of a certain school asked four students of class 10 to draw graph of $f(x) = ax^2 + bx + c$. The graph drawn by four students Anjali, Aditi, Naman and Rahul are shown below:



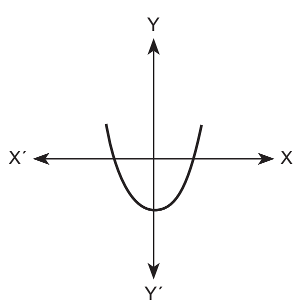
Anjali



Aditi



Naman



Rahul

(*a*) How many students have drawn the graph correctly?

Ans. 2

(*b*) Which type of polynomial is represented by Naman's graph?

Ans. Linear

(*c*) (*i*) How many zeroes are there for the Anjali's graph?

Ans. 2

or

(*ii*) How many zeroes are there for Rahul's graph?

Ans. 2

18. In a pool at an aquarium, a dolphin jumps out of the water travelling at 20 cm per second. Its height above water level after t seconds is given by $h = 20t - 16t^2$.

Based on the above, answer the following questions:

(*a*) Find zeroes of polynomial $p(t) = 20t - 16t^2$.

Ans. $p(t) = 20t - 16t^2$

For zeroes $p(t) = 0$

$\Rightarrow \quad 20t - 16t^2 = 0$

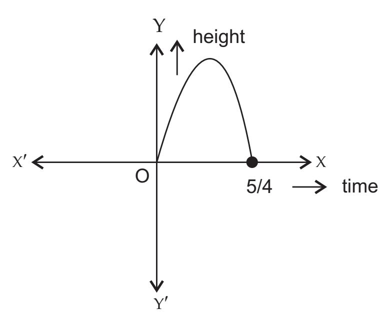
$\Rightarrow \quad 4t(5 - 4t) = 0$

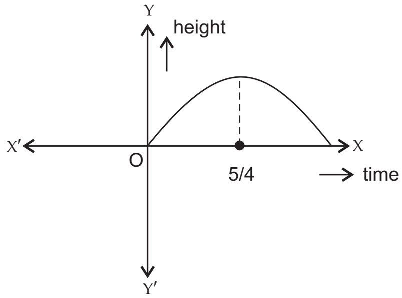
$\Rightarrow \quad t = 0 \quad \text{or} \quad t = \frac{5}{4}$

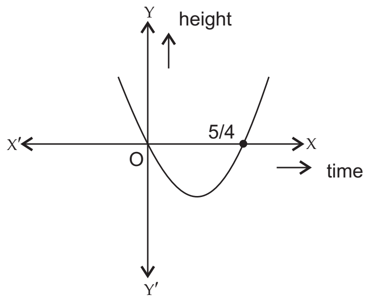
The zeroes of the polynomial $p(t) = 20t - 16t^2$ are 0 and $\frac{5}{4}$.

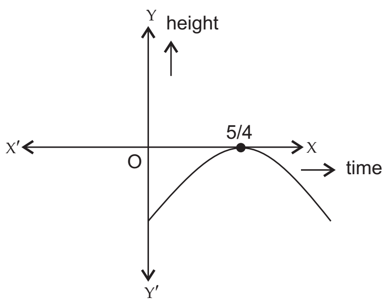
(*b*) Which of the following types of graph represents $p(t)$?

(*i*)



(*ii*) 

(*iii*) 

(*iv*) 

Ans. (*i*)

(*c*) (*i*) What would be the value of h at $t = \frac{3}{2}$?

Interpret the result.

Ans. We have $p(t) = 20t - 16t^2$

at $t = \frac{3}{2}$

$$p\left(\frac{3}{2}\right) = 20 \times \frac{3}{2} - 16\left(\frac{3}{2}\right)^2$$

$$= 30 - 36$$

$$p\left(\frac{3}{2}\right) = -6 \text{ cm}$$

The negative sign implies that it is below the XX' axis, that is under water.

$\therefore$ $|h| = 6$ cm

At $t = \frac{3}{2}$, dolphin is 6 cm under the water.

or

(*ii*) How much distance has the dolphin covered before hitting the water level again? **[CBSE 2023 Standard]**

Ans. Time taken to hit the water level again $= \frac{5}{4}$ sec

Speed of the dolphin = 20 cm/sec [Given]

Distance covered = speed × time

$$= 20 \times \frac{5}{4} \text{ cm} = 25 \text{ cm}$$

$\Rightarrow$ Distance covered by the dolphin before hitting the water level again = 25 cm.

Very Short Answer Type Questions

19. If $p(x) = 3x^2 - x + 5$, what is the value of $p(-2)$?

Sol. We have $p(-2) = 3(-2)^2 - (-2) + 5 = 12 + 2 + 5 = 19$ which is the required value.

20. A quadratic polynomial has two zeroes whose sum is 6. If one of the zeroes is $3-\sqrt{7}$, what is the polynomial?

Sol. The sum of two zeroes is 6 and one of the zeroes is $3 + \sqrt{7}$.

$\therefore$ The other zero of the polynomial

$$= 6 - 3 + \sqrt{7} = 3 + \sqrt{7}$$

$\therefore$ The required polynomial is

x^2 – (Sum of the zeroes)x + Product of the zeroes

$$= x^2 - 6x + \left(3 + \sqrt{7}\right)\left(3 - \sqrt{7}\right)$$

$$= x^2 - 6x + 9 - 7$$

$$= x^2 - 6x + 2$$

21. If one zero of the polynomial $p(x) = 6x^2 + 37x - (k - 2)$ is reciprocal of the other, then find the value of k.

[CBSE 2023 Standard]

Sol. The given polynomial is $6x^2 + 37x - (k - 2)$

Let α and $\frac{1}{\alpha}$ be the zeroes of the above polynomial.

$$\text{Product of the zeroes} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

$$\Rightarrow \quad \alpha \times \frac{1}{\alpha} = \frac{-(k-2)}{6}$$

$$\Rightarrow \quad -(k - 2) = 6$$

$$\Rightarrow \quad k - 2 = -6$$

$$\Rightarrow \quad k = 4$$

22. Find the cubic polynomial with the sum of its zeroes, sum of the product of zeroes taken two at a time and product of its zeros are 0, –7 and –6 respectively. **[CBSE SP 2010]**

Sol. Let α, β and γ be the three zeroes of the polynomial $p(x)$.

Then $p(x) = (x - \alpha)(x - \beta)(x - \gamma)$

$= \{x^2 - (\alpha + \beta)x + \alpha\beta\}(x - \gamma)$

$= x^3 - (\alpha + \beta)x^2 + \alpha\beta x - \gamma x^2 + \gamma(\alpha + \beta)x - \alpha\beta\gamma$

$= x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \alpha\gamma + \beta\gamma)x - \alpha\beta\gamma$

$= x^3 - 0.x^2 - 7x + 6 = x^3 - 7x + 6$

which is the required polynomial.

23. Read the following passage and answer the questions that follows:

A teacher told 10 students to write a polynomial on the blackboard. Students wrote

(*i*) $x^2 + 2$	(*ii*) $2x + 3$
(*iii*) $x^3 + x^2 + 1$	(*iv*) $x^3 + 2x^2 + 1$
(*v*) $x^2 - 2x + 1$	(*vi*) $x - 3$
(*vii*) $x^4 + x^2 + 1$	(*viii*) $x^2 + 2x + 1$
(*ix*) $2x^3 - x^2$	(*x*) $x^4 - 1$

(*a*) How many students wrote cubic polynomial?

(*b*) Divide the polynomial $(x^2 + 2x + 1)$ by $(x + 1)$

[CBSE SP(Basic) 2019]

Sol. (*a*) 3 students

(*b*) $\dfrac{x^2 + 2x + 1}{x + 1} = \dfrac{(x+1)^2}{x+1} = x + 1$

Short Answer Type Questions

24. If two zeroes of the polynomial $f(x) = 2x^3 + 3x^2 - 9x - 10$ are 2 and –1, then find its third zero.

Sol. Let α be the third zero.

Then sum of the zeroes $= 2 - 1 + \alpha = -\dfrac{3}{2}$

$$\Rightarrow \quad \alpha = -\frac{3}{2} - 1 = -\frac{5}{2}$$

which is the required third zero.

25. If α and β are the zeroes of the polynomial $2t^2 + 7t + 5$, find the value of $\left(\sqrt{\dfrac{\alpha}{\beta}} + \sqrt{\dfrac{\beta}{\alpha}}\right) \div \left(\alpha^2 + \beta^2\right)$.

Sol. We have $\quad \alpha + \beta = -\dfrac{7}{2} \quad \ldots(1)$

and $\quad \alpha\beta = \dfrac{5}{2} \quad \ldots(2)$

$$\left(\sqrt{\frac{\alpha}{\beta}} + \sqrt{\frac{\beta}{\alpha}}\right) \div (\alpha^2 + \beta^2)$$

$$= \frac{\alpha + \beta}{\sqrt{\alpha\beta}} \times \frac{1}{\alpha^2 + \beta^2}$$

$$= \frac{\alpha + \beta}{\sqrt{\alpha\beta}\left\{(\alpha + \beta)^2 - 2\alpha\beta\right\}}$$

$$= \frac{-\frac{7}{2}}{\sqrt{\frac{5}{2}}\left(\frac{49}{4} - 5\right)}$$

$$= \frac{-\frac{7}{2}}{\sqrt{\frac{5}{2}} \times \frac{29}{4}} = -\frac{7}{2} \times \frac{4\sqrt{2}}{29\sqrt{5}}$$

$$= \frac{-7}{2} \times \frac{4\sqrt{2}}{29\sqrt{5}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$= -\frac{7}{2} \times \frac{8}{29\sqrt{10}}$$

$$= \frac{-28}{29\sqrt{10}} = \frac{28\sqrt{10}}{29 \times 10}$$

$$= \frac{14\sqrt{10}}{29 \times 5}$$

$$= \frac{-14\sqrt{10}}{145}$$

Hence, the required value is $\dfrac{-14\sqrt{10}}{145}$.

Long Answer Type Questions

26. If the polynomial $x^4 - 6x^3 + 16x^2 - 25x + 10$ is divided by another polynomial $x^2 - 2x + k$, the remainder comes out to be $x + a$. Find k and a.

[CBSE 2011]

Sol. We divide $x^4 - 6x^3 + 16x^2 - 25x + 10$ by $x^2 - 2x + k$ as follows:

$$\begin{array}{l}
x^2 - 2x + k \,\big)\, x^4 - 6x^3 + 16x^2 - 25x + 10 \,\big(\, x^2 - 4x + (8 - k) \\
\qquad\qquad\quad \underset{-}{x^4} \underset{+}{-} 2x^3 \underset{-}{+} kx^2 \\
\hline
\qquad\qquad\quad -4x^3 + (16 - k)x^2 - 25x + 10 \\
\qquad\qquad\quad \underset{+}{-} 4x^3 \underset{-}{+} 8x^2 \underset{+}{-} 4kx \\
\hline
\qquad\qquad\qquad\quad (8 - k)x^2 + (4k - 25)x + 10 \\
\qquad\qquad\qquad\quad \underset{-}{(8 - k)x^2} \underset{-}{+} (2k - 16)x \underset{-}{+} k(8 - k) \\
\hline
\qquad\qquad\qquad\qquad\qquad (2k - 9)x + 10 - k(8 - k) \\
\hline
\end{array}$$

Since the remainder is given to be $x + a$, hence, we must have

$$2k - 9 = 1$$

$\Rightarrow \quad k = 5 \quad \ldots(1)$

and $\quad a = 10 - k(8 - k)$

$= 10 - 5(8 - 5) \quad$ [From (1)]

$= 10 - 5(3)$

$= 10 - 15$

$= -5$

$\therefore$ The required values of k and a are respectively 5 and -5.

27. If two zeroes of the polynomial $x^4 - 6x^3 - 26x^2 + 138x - 35$ are $2 \pm \sqrt{3}$, find the other zeroes. **[CBSE SP 2011]**

Sol. The two of the factors of the given polynomial are $x - 2 - \sqrt{3}$ and $x - 2 + \sqrt{3}$

Now, $\left(x - 2 - \sqrt{3}\right)\left(x - 2 + \sqrt{3}\right) = (x - 2)^2 - 3$

$= x^2 - 4x + 1$

We now divide the given polynomial by $x^2 - 4x + 1$ to get another factor, as follows:

$$
\begin{array}{r|l|l}
x^2 - 4x + 1 & x^4 - 6x^3 - 26x^2 + 138x - 35 & x^2 - 2x - 35 \\
 & x^4 - 4x^3 + x^2 & \\
 & \quad - \quad + \quad - & \\
\hline
 & -2x^3 - 27x^2 + 138x - 35 & \\
 & -2x^3 + 8x^2 - 2x & \\
 & \quad + \quad - \quad + & \\
\hline
 & -35x^2 + 140x - 35 & \\
 & -35x^2 + 140x - 35 & \\
 & \quad + \quad - \quad + & \\
\hline
 & 0 &
\end{array}
$$

$\therefore$ The second factor of the given polynomial is $x^2 - 2x - 35$ which can be written as

$x^2 - 2x - 35 = x^2 + 5x - 7x - 35$

$= x(x + 5) - 7(x + 5)$

$= (x + 5)(x - 7)$

Hence, the required two zeroes are 7 and -5.

Let's Compete

(Page 27)

Multiple-Choice Questions

1. The minimum number of zero or zeroes of the polynomial $x^3 - 6x^2 + 13x - 20$ is/are

(*a*) 1 (*b*) 2

(*c*) 3 (*d*) 0

Sol. (*a*) 1

Let $f(x) = x^3 - 6x^2 + 13x - 20$.

We see that $f(x) = 0$ for $x = 4$.

$\therefore$ By factor theorem, $x - 4$ is a factor of $f(x)$. We now divide $f(x)$ by $x - 4$ to get the factor of $f(x)$.

$$
\begin{array}{r|l|l}
x - 4 & x^3 - 6x^2 + 13x - 20 & x^2 - 2x + 5 \\
 & x^3 - 4x^2 & \\
 & - \quad + & \\
\hline
 & -2x^2 + 13x - 20 & \\
 & -2x^2 + 8x & \\
 & + \quad - & \\
\hline
 & 5x - 20 & \\
 & 5x - 20 & \\
 & - \quad + & \\
\hline
 & 0 &
\end{array}
$$

$\therefore$ The second factor is $x^2 - 2x + 5$. We can verify by direct substitution or otherwise that $x^2 - 2x + 5$ is not zero for any real value of x. Hence, it has no other zero. Hence, the given polynomial has only one real zero.

2. If α and β are the zeroes of the polynomial $ax^2 - 5x + c$ and $\alpha + \beta = \alpha\beta = 10$, then

(*a*) $a = 5, c = \frac{1}{2}$ (*b*) $a = 1, c = \frac{5}{2}$

(*c*) $a = \frac{5}{2}, c = 1$ (*d*) $a = \frac{1}{2}, c = 5$

[CBSE 2023 Standard]

Sol. (*d*) $a = \frac{1}{2}, c = 5$

The given polynomial is $ax^2 - 5x + c$.

α and β are the zeroes of the polynomial.

$\alpha + \beta = \frac{-b}{a} = \frac{5}{a}$

$\alpha\beta = \frac{c}{a} = \frac{c}{a}$

Given $\alpha + \beta = 10$

$\alpha\beta = 10$

$\frac{5}{a} = 10$

$\Rightarrow$ $a = \frac{5}{10}$

$\Rightarrow$ $a = \frac{1}{2}$

$\alpha\beta = 10$

$\frac{c}{a} = 10$

$\Rightarrow$ $c = 10 \times \frac{1}{2} = 5$

$\Rightarrow$ $c = 5$

$\therefore$ $a = \frac{1}{2}, c = 5$

3. If α, β are zeroes of a polynomial $p(x) = 2x^2 - x - 1$, then $\alpha^2 + \beta^2$ is equal to

(a) $-\frac{3}{4}$ (b) $\frac{5}{4}$

(c) $\frac{1}{4}$ (d) $\frac{3}{4}$ **[CBSE 2023 Standard]**

Sol. (b) $\frac{5}{4}$

The polynomial is $2x^2 - x - 1$.

$$a = 2$$
$$b = -1$$
$$c = -1$$
$$\alpha + \beta = \frac{-b}{a} = \frac{1}{2} \quad ...(1)$$
$$\alpha\beta = \frac{c}{a} = \frac{-1}{2} \quad ...(2)$$
$$(\alpha + \beta)^2 = \alpha^2 + \beta^2 + 2\alpha\beta$$
$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$$
$$= \left(\frac{1}{2}\right)^2 - \left[-2 \times \frac{-1}{2}\right]$$
$$= \frac{1}{4} + 1 = \frac{5}{4}$$

4. The value of k so that $x^2 + x + k$ is the only quadratic factor of the polynomial $3x^4 - 3x$ is

(a) -2 (b) 2

(c) -1 (d) 1

Sol. (d) 1

We have

$$3x^4 - 3x = 3x(x^3 - 1)$$
$$= 3x\,(x - 1)\,(x^2 + x + 1)$$

∴ The only quadratic factor of the given polynomial is $x^2 + x + 1$. Hence, the required value of k is 1.

5. If α and β are the zeroes of the polynomial $f(x) = px^2 - 2x + 3p$ and $\alpha + \beta = \alpha\beta$, then the value of p is

(a) $\frac{2}{3}$ (b) $-\frac{2}{3}$

(c) $\frac{3}{2}$ (d) $-\frac{3}{2}$ **[CBSE SP 2011]**

Sol. (a) $\frac{2}{3}$

We have $\alpha + \beta = \frac{2}{p}$ …(1)

and $\alpha\beta = \frac{3p}{p} = 3$ …(2)

$\because \quad \alpha + \beta = \alpha\beta$

$\therefore \quad \frac{2}{p} = 3$ [From (1) and (2)]

$\Rightarrow \quad p = \frac{2}{3}$

6. If α and β are the zeroes of the quadratic polynomial $x^2 + 4kx + 4$ where k is a constant, and if $\alpha^2 + \beta^2 = 24$, then the values of k are given by

(a) 2, 3 (b) ± 2

(c) $\pm\sqrt{2}$ (d) 2, –3

Sol. (c) $\pm\sqrt{2}$

We have $\alpha^2 + \beta^2 = 24$

$\Rightarrow \quad (\alpha + \beta)^2 - 2\alpha\beta = 24$ …(1)

Now, $\alpha + \beta = -4k$ …(2)

and $\alpha\beta = 4$ …(3)

From (1), (2) and (3), we get

$$(-4k)^2 - 2(4) = 24$$
$\Rightarrow \quad 16k^2 - 8 = 24$

$\Rightarrow \quad k^2 = 2$

$\Rightarrow \quad k = \pm\sqrt{2}$

7. If α, β and γ are the zeroes of the cubic polynomial $x^3 + px^2 + qx + 2$ such that $\beta = -\frac{1}{\alpha}$, then the value of $5 + 2p + q$ is

(a) –1 (b) 2

(c) 1 (d) 0

Sol. (d) 0

We have

$$\alpha + \beta + \gamma = -p \quad …(1)$$
$$\alpha\beta + \alpha\gamma + \beta\gamma = q \quad …(2)$$
$$\alpha\beta\gamma = -2 \quad …(3)$$

and $\alpha\beta = -1$ [Given]…(4)

∴ (3) and (4), $\gamma = 2$.

Now, from (1) and (2),

$$\alpha + \beta = -p - 2$$
$\Rightarrow \quad p = -2 - (\alpha + \beta)$ …(5)

Also, from (2) and (4), we get

$$q = -1 + (\alpha + \beta)2 \quad …(6)$$

∴ From (5) and (6), we have

$$5 + 2p + q = 5 - 4 - 2(\alpha + \beta) - 1 + 2\,(\alpha + \beta)$$
$$= 0$$

8. If $a - b$, a and $a + b$ are the zeroes of the polynomial $x^3 + 3x^2 - 8$, then the value of $a + b$ is

(a) $-1 \pm \sqrt{7}$ (b) 2 or – 4

(c) 4 or – 2 (d) 4 or 3

Sol. (*b*) 2 or –4

We have

Sum of the zeroes = $a - b + a + a + b = -3$

$\Rightarrow \quad 3a = -3$

$\Rightarrow \quad a = -1$

Product of zeroes = $a(a - b)(a + b) = 8$

$\Rightarrow \quad a(a^2 - b^2) = 8$

$\Rightarrow \quad (-1)(1 - b^2) = 8$

$\Rightarrow \quad b^2 = 9$

$\Rightarrow \quad b = \pm 3$

$\therefore \quad a + b = -1 \pm 3 = 2 \text{ or } -4$

9. If α, β and γ are the zeroes of the polynomial $-2x^3 + 5x - 4$, then the value of $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$ is

(*a*) $\frac{5}{4}$ (*b*) $-\frac{5}{4}$

(*c*) $\frac{4}{5}$ (*d*) $-\frac{4}{5}$

Sol. (*a*) $\frac{5}{4}$

We have

$$\alpha + \beta + \gamma = 0 \qquad \ldots(1)$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = -\frac{5}{2} \qquad \ldots(2)$$

$$\alpha\beta\gamma = -\frac{4}{2} = -2 \qquad \ldots(3)$$

$$\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = \frac{\beta\gamma + \alpha\gamma + \alpha\beta}{\alpha\beta\gamma}$$

$$= \frac{\frac{-5}{2}}{-2} \qquad \text{[From (2) and (3)]}$$

$$= \frac{5}{4}$$

10. If the graph of the polynomial equation $y = p(x)$ where $p(x)$ is a polynomial intersects the x-axis three times in three distinct points, then which of the following could be the expression for $p(x)$?

(*a*) $x^2 - 9$ (*b*) $3x + 3$

(*c*) $4 - 4x - x^2 + x^3$ (*d*) $3x^2 + 3x - 3$

[CBSE SP 2012]

Sol. (*c*) $4 - 4x - x^2 + x^3$

Since the graph of the given polynomial intersects x-axis at 3 distinct points, hence the polynomial must be cubic.

Life Skills

(Page 28)

1. A lady Principal of a well-known public school wanted to help a nearby secondary school for poor but meritorious students, which is run by an NGO. She asked each bright Senior Secondary student of her school to teach different topics of mathematics at the lower classes daily for at least one hour to the poor students as her/his project work. One day, one such student was selected from the Senior Secondary Public School and was asked to teach how to find the value of a polynomial at a particular point in some lower class. He prepared the following problem:

Find the values of each of the two polynomials $f(x)$ and $g(x)$ at the points $x = 1, -1, 2, -2$ where $f(x) = x^3 + 6x^2 + 5x - 12$ and $g(x) = 2x^3 + x^2 - 18x - 9$.

Some students got all the correct values as follows:

$f(1) = 0, f(-1) = -12, f(2) = 30, f(-2) = -6$

$g(1) = -24, g(-1) = 8, g(2) = -25$ and $g(-2) = 15$

(*a*) From the above values of $f(x)$ and $g(x)$, state with reasons, whether any of the values 1, –1, 2, –2 may be a zero or zeroes of $f(x)$ or $g(x)$.

(*b*) If –3 is a zero of $f(x)$, what is the third zero of $f(x)$, if any.

(*c*) If $-\frac{1}{2}$ and 3 are two zeroes of $g(x)$, what is its third zero?

Sol. (*a*) Since $f(1) = 0$ and $g(x) \neq 0$ for any value of x, hence it follows that 1 is a zero of $f(x)$, but none of the points +1, –1, +2 and –2 is a zero of $g(x)$.

(*b*) In this case $x + 3$ is a factor of $f(x)$. We write

$$f(x) = x^3 + 6x^2 + 5x - 12$$
$$= x^3 + 3x^2 + 3x^2 + 9x - 4x - 12$$
$$= x^2(x + 3) + 3x(x + 3) - 4(x + 3)$$
$$= (x + 3)(x^2 + 3x - 4)$$
$$= (x + 3)(x^2 + 4x - x - 4)$$
$$= (x + 3)\{x(x + 4) - 1(x + 4)\}$$
$$= (x + 3)(x + 4)(x - 1)$$

All the zeroes of $f(x)$ are 1, –3 and –4.

Hence, the required third zero is –4.

(*c*) We write $g(x)$ as

$$\begin{aligned} g(x) &= 2x^3 + x^2 - 18x - 9 \\ &= 2x^3 + 6x^2 - 5x^2 - 15x - 3x - 9 \\ &= 2x^2(x + 3) - 5x(x + 3) - 3\,(x + 3) \\ &= (x + 3)\,(2x^2 - 5x - 3) \\ &= (x + 3)\,(2x^2 - 6x + x - 3) \\ &= (x + 3)\,\{2x(x - 3) + 1\,(x - 3)\} \\ &= (x + 3)\,(x - 3)\,(2x + 1) \end{aligned}$$

Hence, all the zeroes of $g(x)$ are 3, $-\frac{1}{2}$ and –3.

$\therefore$ The required third zero is –3.

Alternatively, we can solve this problem in the following way also.

Clearly, $\left(x + \frac{1}{2}\right)(x - 3)$ is a factor of $g(x)$.

Now,

$$\begin{aligned} \left(x + \frac{1}{2}\right)(x - 3) &= x^2 + \left(\frac{1}{2} - 3\right)x - \frac{3}{2} \\ &= x^2 - \frac{5}{2}x - \frac{3}{2} \\ &= \frac{1}{2}\left(2x^2 - 5x - 3\right) \end{aligned}$$

We divide $g(x)$ by $2x^2 - 5x - 3$ as follows:

$$\begin{array}{r|l|l} 2x^2 - 5x - 3 & 2x^3 + x^2 - 18x - 9 & x + 3 \\ & \underline{\overset{-}{2x^3} \overset{+}{-} 5x^2 \overset{+}{-} 3x} & \\ & \quad 6x^2 - 15x - 9 & \\ & \quad \underline{\overset{-}{6x^2} \overset{+}{-} 15x \overset{+}{-} 9} & \\ & \qquad 0 & \end{array}$$

The remaining factor is $x + 3$. Hence the required third zero is −3.