1

Real Numbers

Checkpoint (Page 6)

1. Which of the following is a rational number?

(*a*) $\sqrt{2}$ (*b*) 3π

(*c*) $0.\overline{1256}$ (*d*) 0.12345678001…

Sol. (*c*) $0.\overline{1256}$

We know that any number which can be expressed either as a non-terminating recurring decimal or a terminating decimal is a rational number, otherwise it is an irrational number. Now, the number in (*c*) only can be written as a non-terminating recurring decimal and so only this number is rational.

2. The value of $0.\overline{35}+0.\overline{42}$ is

(*a*) $0.\overline{45}$ (*b*) $0.\overline{56}$

(*c*) $0.\overline{77}$ (*d*) 0.77

Sol. (*c*) $0.\overline{77}$

We have

$$0.\overline{35}+0.\overline{42} = 0.353535\ldots + 0.424242\ldots$$
$$= 0.777777\ldots$$
$$= 0.\overline{77}$$

3. The number $1.\overline{36}$ in the form of $\frac{p}{q}$, where p and q are non-negative integers and $q \neq 0$, is

(*a*) $\frac{13}{11}$ (*b*) $\frac{14}{11}$

(*c*) $\frac{15}{11}$ (*d*) $\frac{11}{15}$

Sol. (*c*) $\frac{15}{11}$

Let $x = 1.\overline{36} = 1.363636\ldots$ …(1)

$\therefore$ $100x = 136.363636\ldots$ …(2)

$\therefore$ Subtracting (1) from (2), we get

$$99x = 135$$

$$\therefore \quad x = \frac{135}{99} = \frac{15}{11}$$

4. If $3^4 \times 9^2 = 81^x$, then x is

(*a*) 2 (*b*) 3

(*c*) 4 (*d*) 5

Sol. (*a*) 2

We have $3^4 \times 9^2 = 81^x$

$$3^4 \times (3^2)^2 = (3^4)^x$$
$$3^{4+4} = 3^{4x}$$
$$\Rightarrow \quad 3^8 = 3^{4x}$$
$$\Rightarrow \quad 8 = 4x$$
$$\Rightarrow \quad x = 2$$

5. The simplified form of $(256)^{-\left(4^{\frac{-3}{2}}\right)}$ is

(*a*) 2 (*b*) $\frac{1}{2}$

(*c*) 8 (*d*) 4

Sol. (*b*) $\frac{1}{2}$

We have $4^{\frac{-3}{2}} = 2^{-2\times\frac{3}{2}}$

$$= 2^{-3} = \frac{1}{8}$$

$\therefore$ Given expression =

$$\left(4^4\right)^{-\frac{1}{8}} = 4^{-\frac{1}{8}\times 4} = 4^{\frac{-1}{2}} = \frac{1}{\sqrt{4}} = \frac{1}{2}$$

6. The value of $\sqrt{5+2\sqrt{6}}$ is

(*a*) $\sqrt{3} + \sqrt{2}$ (*b*) $\sqrt{3} - \sqrt{2}$

(*c*) $\sqrt{5} + \sqrt{6}$ (*d*) none of these

Sol. (*a*) $\sqrt{3} + \sqrt{2}$

We have

$$\begin{aligned} \sqrt{5+2\sqrt{6}} &= \sqrt{\left(\sqrt{3}\right)^2 + \left(\sqrt{2}\right)^2 + 2\sqrt{3}\times\sqrt{2}} \\ &= \sqrt{\left(\sqrt{3}+\sqrt{2}\right)^2} \\ &= \sqrt{3}+\sqrt{2} \end{aligned}$$

7. If $x+\frac{1}{x} = 3$, calculate $x^2+\frac{1}{x^2}$, $x^3+\frac{1}{x^3}$ and $x^4+\frac{1}{x^4}$.

Sol. We have

$$\begin{aligned} x^2+\frac{1}{x^2} &= \left(x+\frac{1}{x}\right)^2 - 2x\times\frac{1}{x} \\ &= 3^2-2 \\ &= 7 \end{aligned}$$

$$\begin{aligned} \therefore \quad x^4+\frac{1}{x^4} &= \left(x^2+\frac{1}{x}\right)^2 - 2x^2.\frac{1}{x^2} \\ &= 7^2-2 \\ &= 47 \end{aligned}$$

$$\begin{aligned} \text{and} \quad x^3+\frac{1}{x^3} &= \left(x+\frac{1}{x}\right)^3 - 3x.\frac{1}{x}\left(x+\frac{1}{x}\right) \\ &= 3^3-3\times 3 \\ &= 27-9 \\ &= 18 \end{aligned}$$

8. If $\frac{a}{b}+\frac{b}{a} = 1$, find a^3+b^3.

Sol. We have $\frac{a}{b}+\frac{b}{a} = 1$

$$\Rightarrow \quad \frac{a^2+b^2}{ab} = 1$$

$$\Rightarrow \quad a^2+b^2-ab = 0 \qquad \ldots(1)$$

$$\begin{aligned} \therefore \quad a^3+b^3 &= (a+b)(a^2+b^2-ab) \\ &= (a+b)\times 0 \qquad [\text{From } (1)] \\ &= 0 \end{aligned}$$

$\therefore$ The value of a^3+b^3 is 0.

9. Rationalise and simplify: $\frac{2\sqrt{6}-\sqrt{5}}{3\sqrt{5}-2\sqrt{6}}$

Sol. We have

$$\begin{aligned} \frac{2\sqrt{6}-\sqrt{5}}{3\sqrt{5}-2\sqrt{6}} &= \frac{\left(2\sqrt{6}-\sqrt{5}\right)\left(3\sqrt{5}+2\sqrt{6}\right)}{\left(3\sqrt{5}-2\sqrt{6}\right)\left(3\sqrt{5}+2\sqrt{6}\right)} \\ &= \frac{6\sqrt{30}+4\times 6-3\times 5-2\sqrt{30}}{\left(3\sqrt{5}\right)^2-\left(2\sqrt{6}\right)^2} \\ &= \frac{24-15+4\sqrt{30}}{45-24} \\ &= \frac{9+4\sqrt{30}}{21} \end{aligned}$$

which is the required value.

10. Find a and b if $\frac{5+3\sqrt{3}}{7+4\sqrt{3}} = a+b\sqrt{3}$.

Sol. We have $\frac{5+3\sqrt{3}}{7+4\sqrt{3}} = a+b\sqrt{3}$

$$\Rightarrow \quad \frac{\left(5+3\sqrt{3}\right)\left(7-4\sqrt{3}\right)}{\left(7+4\sqrt{3}\right)\left(7-4\sqrt{3}\right)} = a+b\sqrt{3}$$

$$\Rightarrow \quad \frac{35-12\times 3-20\sqrt{3}+21\sqrt{3}}{7^2-\left(4\sqrt{3}\right)^2} = a+b\sqrt{3}$$

$$\Rightarrow \quad \frac{35-36+\sqrt{3}}{49-48} = a+b\sqrt{3}$$

$$\Rightarrow \quad \sqrt{3}-1 = a+b\sqrt{3}$$

$\therefore$ $a = -1$, $b = 1$ which are the required values.

Check Your Progress 1

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Multiple-Choice Questions

1. If LCM (84, 24) = 168, then HCF (84, 24) is

(*a*) 11 (*b*) 12

(*c*) 13 (*d*) 14

Sol. (*b*) 12

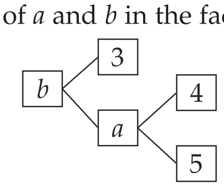
We know that

$$\text{LCM } (84, 24)\times\text{HCF } (84, 24) = 84\times 24$$

$$\Rightarrow \quad 168\times\text{HCF } (84, 24) = 84\times 24$$

$$\text{HCF } (84, 24) = \frac{84\times 24}{168} = 12$$

2. The values of a and b in the factor tree are



(*a*) 10, 20 (*b*) 20, 30

(*c*) 20, 60 (*d*) 10, 60

Sol. (*c*) 20, 60

From the given factor tree, we see that

$$a = 4 \times 5 = 20$$

and $\quad b = 3 \times a = 3 \times 20 = 60$

$\therefore$ The required values of a and b are 20 and 60 respectively.

3. LCM of $2^4 \times 3^3$ and $2^3 \times 3^4$ is

(*a*) 2^4 (*b*) 3^4

(*c*) $2^4 \times 3^4$ (*d*) $2^3 \times 3^3$

Sol. (*c*) $2^4 \times 3^4$

We have $\quad 2^4 \times 3^3 = 2^3 \times 3^3 \times 2$

and $\quad 2^3 \times 3^4 = 2^3 \times 3^3 \times 3$

$\therefore \quad \text{LCM} = 2^3 \times 3^3 \times 2 \times 3$

$\qquad\qquad = 2^4 \times 3^4$

4. The sum of the exponents of the prime factors in the prime factorisation of 324 is

(*a*) 3 (*b*) 4

(*c*) 5 (*d*) 6

Sol. (*d*) 6

We find the prime factors of 324 as follows:

$$324 = 2^2 \times 3^4$$

2	324
2	162
3	81
3	27
3	9
3	3
	1

$\therefore$ Sum of the exponents of 2 and 3 in the prime factorisation of 324 is 2 + 4 i.e. 6.

5. If n is any natural number, then $6^n - 5^n$ will always end with

(*a*) 0 (*b*) 1

(*c*) 2 (*d*) 3

Sol. (*b*) 1

We know that 6^n will end with 6 for all natural values of n and 5^n will end with 5 for all natural values n. Hence, $6^n - 5^n$ will always end with $6 - 5 = 1$ only.

6. The LCM and HCF of two numbers are equal, then the numbers will be

(*a*) prime. (*b*) coprime.

(*c*) composite. (*d*) equal.

Sol. (*d*) equal

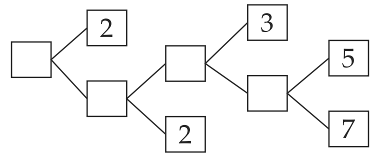
Let the two numbers be a and b and let K be their common LCM and HCF.

$\therefore \quad ab = K^2$

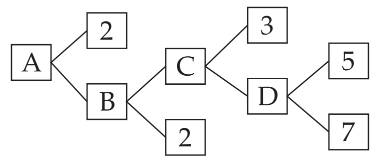
$\because$ a, b and K are natural numbers, hence, a and b be equal to each other. Two equal numbers may or may not always be prime, composite or coprime.

Very Short Answer Type Questions

7. Find the missing numbers in the following factor tree.



Sol. Let us denote the empty boxes starting from the left to the right by A, B, C and D.



Then the number in box D is $5 \times 7 = 35$.

$\therefore$ The number in box C is $35 \times 3 = 105$.

Also, the number in box B is $105 \times 2 = 210$

and the number in the box A is $210 \times 2 = 420$.

$\therefore$ The missing numbers are 420, 210, 105 and 35.

8. Find the prime factorisation of the following positive integers.

(*a*) 133100 (*b*) 8008

Sol. (*a*) We find the prime factors of 133100 as follows:

$\therefore \quad 133100 = 2^2 \times 5^2 \times 11^3$

$\therefore$ The prime factorisation of 133100 is $2^2 \times 5^2 \times 11^3$.

2	133100
2	66550
5	33275
5	6655
11	1331
11	121
11	11
	1

(*b*) We find the prime factor of 8008 as follows:

$\therefore \quad 8008 = 2^3 \times 7 \times 11 \times 13$

$\therefore$ The prime factorisation of 8008 is $2^3 \times 7 \times 11 \times 13$.

2	8008
2	4004
2	2002
7	1001
11	143
13	13
	1

9. Find HCF and LCM of the following numbers using Fundamental Theorem of Arithmetic.

(*a*) 600 and 750 (*b*) 240 and 720

Sol. (*a*) We first find the prime factors of 600 and 750 as follows:

$$600 = 2 \times 2 \times 2 \times 3 \times 5 \times 5$$
$$= 2^3 \times 3 \times 5^2$$
$$750 = 2 \times 3 \times 5 \times 5 \times 5$$
$$= 2 \times 3 \times 5^3$$

Hence, HCF (600, 750) = 150

We know that for any two positive integers a and b, we have

HCF (a, b) × LCM $(a, b) = a \times b$

$\Rightarrow$ HCF (600, 750) × LCM (600, 750) $= 600 \times 750$

$\Rightarrow$ 150 × LCM (600, 750) $= 600 \times 750$

Hence, LCM (600, 750) $= \dfrac{600 \times 750}{150} = 3000$

(*b*) We first find the prime factors of 240 and 720 as follows:

$$240 = 2 \times 2 \times 2 \times 2 \times 3 \times 5$$
$$= 2^4 \times 3 \times 5$$
$$720 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5$$
$$= 2^4 \times 3^2 \times 5$$

Hence, HCF(240, 720) $= 2^4 \times 3 \times 5 = 240$

We know that for any two positive integers a and b, we have

HCF (a, b) × LCM $(a, b) = a \times b$

$\Rightarrow$ HCF (240, 720) × LCM (240, 720) $= 240 \times 720$

$\Rightarrow$ 240 × LCM (240, 720) $= 240 \times 720$

Hence, LCM (240, 720) $= \dfrac{240 \times 720}{240} = 720$

10. Can two numbers have 16 as HCF and 644 as LCM?

Sol. No, two numbers cannot have 16 as HCF and 644 as LCM.

Since LCM of two numbers is always divisible by their HCF. Now, the given LCM is 644 which is not divisible by the HCF i.e. 16. Hence, it is not possible to get such two numbers whose HCF and LCM are 16 and 644 respectively.

11. Write the HCF of the smallest composite number and smallest prime number.

Sol. We know that the smallest composite number is 4 and the smallest prime number is 2 and the HCF of 2 and 4 is clearly 2 which is the required HCF.

Short Answer Type Questions

12. Find the HCF and LCM of following using prime factorisation.

(*a*) 50, 160 and 400 (*b*) 70, 90 and 250

Sol. (*a*) By prime factorisation, we get

$$50 = 2 \times 5^2$$
$$160 = 2^5 \times 5$$
$$400 = 2^4 \times 5^2$$

$$\begin{array}{r|r} 2 & 50 \\ \hline 5 & 25 \\ \hline 5 & 5 \\ \hline & 1 \end{array} \qquad \begin{array}{r|r} 2 & 160 \\ \hline 2 & 80 \\ \hline 2 & 40 \\ \hline 2 & 20 \\ \hline 2 & 10 \\ \hline 5 & 5 \\ \hline & 1 \end{array} \qquad \begin{array}{r|r} 2 & 400 \\ \hline 2 & 200 \\ \hline 2 & 100 \\ \hline 2 & 50 \\ \hline 5 & 25 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$

$\therefore$ HCF of 50,160 and 400 $= 2 \times 5 = 10$

LCM of 50, 160 and 400 $= 2^5 \times 5^2 = 800$

(*b*) By prime factorisation, we get

$$70 = 2 \times 5 \times 7$$
$$90 = 2 \times 5 \times 3^2$$
$$250 = 2 \times 5^3$$

$$\begin{array}{r|r} 2 & 70 \\ \hline 5 & 35 \\ \hline 7 & 7 \\ \hline & 1 \end{array} \qquad \begin{array}{r|r} 2 & 90 \\ \hline 3 & 45 \\ \hline 3 & 15 \\ \hline 5 & 5 \\ \hline & 1 \end{array} \qquad \begin{array}{r|r} 2 & 250 \\ \hline 5 & 125 \\ \hline 5 & 25 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$

$\therefore$ HCF of 70, 90 and 250 $= 2 \times 5 = 10$

LCM of 70, 90 and 250 $= 2 \times 3^2 \times 5^3 \times 7$
$= 15750$

13. The HCF and LCM of two numbers are 40 and 35960 respectively. If one of the number is 1160. Find the other.

Sol. We know that

HCF(a, b) × LCM$(a, b) = a \times$ b

$\Rightarrow$ $40 \times 35960 = 1160 \times b$

$\Rightarrow$ $\dfrac{40 \times 35960}{1160} = b$

$\Rightarrow$ $1240 = b$

$\therefore$ The other number is 1240.

14. Given that HCF (336, 486) = 54, find LCM (336, 486).

Sol. We know that

HCF(a, b) × LCM $(a, b) = a \times b$

HCF (336,486) × LCM (336, 486) $= 336 \times 486$

$\Rightarrow$ 54 × LCM (336, 486) $= 336 \times 486$

$\Rightarrow$ LCM(336, 486) $= \dfrac{336 \times 486}{54}$

$= 56 \times 54$

$= 3024$

$\therefore$ LCM(336, 486) = 3024

15. Explain why $3 \times 5 \times 7 \times 11 + 5$ is a composite number.

Sol. $3 \times 5 \times 7 \times 11 + 5 = 1155 + 5$

$$= 1160$$

$$= 2^3 \times 5 \times 29$$

A number which has more than two factors (other than 1 and itself) is a composite number. Since, the above can be expressed as a product of more than two prime factors, therefore it is a composite number.

16. The LCM of two numbers is 14 times their HCF. The sum of LCM and HCF is 600. If one of the numbers is 280, then find the other number.

[CBSE 2012]

Sol. Let the other number be x and let the HCF $(280, x)$ be y.

Then according to the problem,

$$\text{LCM}(280, x) = 14y$$

and $14y + y = 600$

$\Rightarrow \quad 15y = 600$

$\Rightarrow \quad y = \frac{600}{15} = 40$

$\therefore \quad \text{HCF}(280, x) = 40$

and $\text{LCM}(280, x) = 14 \times 40 = 560$

$\because \quad \text{LCM}(280, x) \times \text{HCF}(280, x) = 280x$

$\therefore \quad 560 \times 40 = 280x$

$\Rightarrow \quad \frac{560 \times 40}{280} = x$

$\Rightarrow \quad x = 80$

$\therefore$ The other number is 80.

17. Find the smallest number which when increased by 17 is exactly divisible by 520 and 468.

Sol. We first find the smallest number which is divisible by 520 and 468 by finding their LCM as follows:

$\text{LCM} = 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 13$
$= 4680$

2	520, 468
2	260, 234
2	130, 117
3	65, 117
3	65, 39
5	65, 13
13	13, 13
	1, 1

Hence, the smallest number is 4680 – 17 i.e. 4663.

18. On a morning walk, three persons step off together and their steps measure 35 cm, 40 cm and 42 cm respectively. What is the minimum distance each should walk so that each can cover the same distance in complete steps?

Sol. The required minimum distance will be equal to the LCM of 35, 40 and 42 in cm. So, we find the LCM of 35, 40 and 42 as follows:

$\text{LCM} = 2 \times 2 \times 2 \times 3 \times 5 \times 7 = 840$

2	35, 40, 42
2	35, 20, 21
2	35, 10, 21
3	35, 5, 21
5	35, 5, 7
7	7, 1, 7
	1, 1, 1

Hence, the minimum distance walked is 840 cm.

19. Check whether 15^n can end with digit 0 for $n \in \text{N}$.

Sol. If 15^n end with digit zero, then the number should be divisible by 2 and 5.

This means the prime factorisation of 15^n should contain prime factors 2 and 5.

$\Rightarrow \quad 15^n = (3 \times 5)^n$

$\because$ 2 is not present in the prime factorisation, there is no natural number for which 15^n ends with the digit zero.

So, 15^n cannot end with digit zero.

Long Answer Type Questions

20. Using prime factorisation method, find HCF and LCM of 24, 30 and 70. Also, show that HCF × LCM ≠ Product of three numbers.

Sol. We find the prime factors of 24, 30 and 70 as follows:

$$24 = 2^3 \times 3$$
$$30 = 2 \times 3 \times 5$$
$$70 = 2 \times 5 \times 7$$

2	24
2	12
2	6
3	3
	1

2	30
3	15
5	5
	1

2	70
5	35
7	7
	1

$\therefore$ HCF of 24, 30 and 70 = 2

LCM of 24, 30 and 70 $= 2^3 \times 3 \times 5 \times 7 = 840$

$\therefore \quad \text{HCF} \times \text{LCM} = 2 \times 840 = 1680$

Also, $24 \times 30 \times 70 = 50400$

$\therefore$ HCF × LCM ≠ Product of three given numbers.

21. Find the greatest number of five digits exactly divisible by 12, 15 and 36.

Sol. The greatest number of 5 digits exactly divisible by 12, 15 and 36 will be divisible by LCM (12, 15, 36).

$$12 = 2 \times 2 \times 3 = 2^2 \times 3$$
$$15 = 3 \times 5$$
$$36 = 2 \times 2 \times 3 \times 3 = 2^2 \times 3^2$$
$$\therefore \quad \text{LCM} = 2^2 \times 3^2 \times 5 = 180$$

The greatest number of 5 digits = 99999

$$99999 = 180 \times 555 + 99$$

$\therefore$ The greatest number that will be divisible by 180

$$= 99999 - 99 = 99900$$

Hence, the required number is 99900.

22. Red colour crayons are available in a pack of 18 and blue colour in a pack of 24. If I need to buy an equal number of crayons of both colours, what is the least number of packs of each kind I would need to buy?

Sol. We first find the LCM of 18 and 24 as follows:

$$\therefore \quad \text{LCM} = 2 \times 2 \times 2 \times 3 \times 3 = 72$$

2	18, 24
2	9, 12
2	9, 6
3	9, 3
3	3, 3
	1, 1

Hence, I should by 72 red color crayons and 72 blue color crayons. Now, 18 red color crayons are available in one pack and 24 blue color crayons are available in one pack. Hence, the required number of packs of red colour crayons is 72 ÷ 18 i.e. 4 and the number of packs of blue colour is 72 ÷ 24 i.e. 3.

23. Three bells toll at intervals of 12, 15 and 18 minutes respectively. If they start tolling together after what time will they toll together again?

Sol. The time required by the three bells to toll together again is the LCM of 12, 15 and 18 (in minutes).

$$12 = 2 \times 2 \times 3 = 2^2 \times 3$$
$$15 = 3 \times 5$$
$$18 = 2 \times 3 \times 3 = 2 \times 3^2$$

LCM of 12, 15 and 18 = $2^2 \times 3^2 \times 5 = 180$

Hence, the time at which the three bells toll together again is 180 minutes or 3 hours.

Check Your Progress 2

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Multiple-Choice Questions

1. $\dfrac{1}{\pi} - \dfrac{7}{22}$ is

(*a*) a rational number. (*b*) an irrational number.
(*c*) a prime number. (*d*) an even number.

Sol. (*b*) an irrational number.

We know that π is an irrational number and $\dfrac{7}{22}$ is a rational number. Since a combination of a rational number and an irrational number is an irrational number, hence the given number is irrational.

2. The number $(5 - 3\sqrt{5} + \sqrt{5})$ is

(*a*) an integer.
(*b*) a rational number.
(*c*) an irrational number.
(*d*) a whole number. **[CBSE 2023]**

Sol. (*c*) an irrational number

$5 - 3\sqrt{5} + \sqrt{5} = 5 - 2\sqrt{5}$ is an irrational number.

3. Which of the following is an irrational number?

(*a*) $\left(2\sqrt{3} - \dfrac{1}{\sqrt{3}}\right)^2$ (*b*) $(\sqrt{2} - 1)^2$

(*c*) $\sqrt{2} - (2 + \sqrt{2})$ (*d*) $\dfrac{(\sqrt{2} + 5\sqrt{2})}{\sqrt{2}}$ **[CBSE 2023]**

Sol. (*b*) $\left(\sqrt{2} - 1\right)^2$

Expanding $\left(\sqrt{2} - 1\right)^2 = \left(\sqrt{2}\right)^2 + (1)^2 - 2\left(\sqrt{2}\right)(1)$

$$= 2 + 1 - 2\sqrt{2}$$
$$= 3 - 2\sqrt{2}$$

4. If $p^2 = \dfrac{32}{50}$, then p is a/an

(*a*) whole number.
(*b*) integer.
(*c*) rational number.
(*d*) irrational number. **[CBSE 2023]**

Sol. (*c*) rational number

$$p^2 = \frac{32}{50}$$
$$\Rightarrow \quad p = \sqrt{\frac{32}{50}}$$
$$\Rightarrow \quad p = \sqrt{\frac{16}{25}}$$

$\Rightarrow \qquad p = \dfrac{4}{5}$

Therefore p is a rational number.

Very Short Answer Type Questions

5. Give example of two irrational number whose
(*a*) sum is a rational number.
(*b*) product is an irrational number.

Sol. Consider the two irrational numbers $3+\sqrt{2}$ and $6-\sqrt{2}$

(*a*) Sum of $3+\sqrt{2}$ and $6-\sqrt{2}$

$3+\sqrt{2}+6-\sqrt{2}$
= 9, which is a rational number.

(*b*) Product of $3+\sqrt{2}$ and $6-\sqrt{2}$

$$\left(3+\sqrt{2}\right)\left(6-\sqrt{2}\right) = 18-3\sqrt{2}+6\sqrt{2}-2$$
$$= 16+3\sqrt{2}$$

$16+3\sqrt{2}$ is an irrational number.

6. Write the number of consecutive zeroes in the prime factorisation of a natural number N given by $2^4 \times 3^4 \times 5^3 \times 11$.

Sol. We have

$$2^4 \times 3^4 \times 5^3 \times 11 = 2^3 \times 2 \times 5^3 \times 3^4 \times 11$$
$$= (2\times 5)^3 \times 2 \times 3^4 \times 11$$
$$= 22 \times 81 \times 10^3$$
$$= 22 \times 81 \times 1000$$

Hence, the required number of consecutive zeroes in the given number is 3.

Short Answer Type Questions

7. Show that 8^n can never end with the digit 0 for any natural number n. **[CBSE 2023]**

Sol. $8^n = (2^3)^n = 2^{3n}$

The only prime in the factorisation of 8^n is 2. By the uniqueness of fundamental theorem of arithmetic, there is no other prime in the factorisation of $8^n = 2^{3n}$.

$\therefore$ 5 does not occur in the prime factorisation of 8^n for any $n \in$ N.

For 8^n to end with digit 0 for any natural number n, it must have 5 also a factor.

$\therefore$ 8^n cannot end with digit 0 for any $n \in$ N.

8. Prove that $6-4\sqrt{5}$ is an irrational number, given that $\sqrt{5}$ is an irrational number. **[CBSE 2024]**

Sol. Let us assume on the contrary that $6-4\sqrt{5}$ is a rational number.

Then, there exist coprime a and b ($b \neq 0$), such that

$$6-4\sqrt{5} = \frac{a}{b}$$

$$\Rightarrow \quad 6-\frac{a}{b} = 4\sqrt{5}$$

$$\Rightarrow \quad \frac{6b-a}{b} = 4\sqrt{5}$$

$$\Rightarrow \quad \frac{6b-a}{4b} = \sqrt{5}$$

$\Rightarrow$ $\sqrt{5}$ is rational [$\because$ 4, 6, a and b are integers

$\therefore$ $\dfrac{6b-a}{4b}$ is a rational number]

This contradicts the fact that $\sqrt{5}$ is irrational. So our assumption is incorrect.

$\therefore$ $6-4\sqrt{5}$ is irrational.

9. Show that $11 \times 19 \times 23 + 3 \times 11$ is not a prime number. **[CBSE 2024]**

Sol. $11 \times 19 \times 23 + 3 \times 11 = 11[19 \times 23 + 3]$
$= 11\,[437 + 3]$
$= 11\,[440]$
$= 11\,[2 \times 2 \times 10 \times 11]$
$= 2 \times 2 \times 10 \times 11 \times 11$

Therefore, the given number is not a prime number as it has more than two factors. A prime number has only two factors namely 1 and the number itself.

10. Two positive integers p and q are expressed as $p = 18a^2b^4$ and $q = a^3b^2$, where a and b are prime numbers. Find LCM (p, q).

Sol.
$$p = 18a^2b^4 = 2 \times 3^2 \times a^2 \times b^4$$
$$q = a^3b^2$$

Prime factors of 2, 3, a and b with greatest exponents are 1, 2, 3 and 4 respectively.

Hence, required LCM is $2 \times 3^2 \times a^3 \times b^4$

$$\text{LCM} = 18a^3b^4$$

11. Prove that $\sqrt{11}$ is irrational.

Sol. Let us assume on the contrary that $\sqrt{11}$ is a rational number.

Then $\sqrt{11} = \dfrac{p}{q}$ where p and q are coprime and $q \neq 0$

$\therefore$ On squaring both sides

$$\Rightarrow \quad 11 = \frac{p^2}{q^2}$$

$$\Rightarrow \quad p^2 = 11q^2 \qquad \ldots(1)$$

$\Rightarrow$ p^2 is divisible by 11

[$\because$ $11q^2$ is divisible by 11]

$\Rightarrow$ p is divisible by 11 ...(2)

[$\because$ 11 is prime and divides p^2 $\Rightarrow$ 11 divides p]

Let $p = 11\,c$, for some integer c

Substituting $p = 11\,c$ in (1), we get

$$11q^2 = (11c)^2$$

$$\Rightarrow \quad 11q^2 = 121c^2$$

$$\Rightarrow \quad q^2 = 11c^2$$

$\Rightarrow$ q^2 is divisible by 11

[$\because$ $11\,c^2$ is divisible by 11]

$\Rightarrow$ q is divisible by 11 ...(3)

[$\because$ 11 is prime and divides q^2 $\Rightarrow$ 11 divides q]

From (2) and (3)

$\Rightarrow$ Both p and q are divisible by 11.

$\therefore$ 11 is a common factor of p and q.

But this contradicts the fact that p and q are coprime.

This contradiction has arisen because of our incorrect assumption that $\sqrt{11}$ is rational.

Hence, $\sqrt{11}$ is irrational.

12. Prove each of the following is irrational.

(*a*) $\sqrt{3}-\sqrt{5}$ (*b*) $\dfrac{3\sqrt{2}}{5}$

(*c*) $5 + 3\sqrt{2}$ (*d*) $11 + 13\sqrt{2}$

Sol. (*a*) Let us assume on the contrary that $\sqrt{3}-\sqrt{5}$ is a rational number.

Then, there exists coprime p and q ($q \neq 0$) such that

$$\sqrt{3}-\sqrt{5} = \frac{p}{q}$$

$$\Rightarrow \quad \sqrt{5} = \sqrt{3}-\frac{p}{q}$$

On squaring both sides, we get

$$5 = \left(\sqrt{3}-\frac{p}{q}\right)^2$$

$$= \left(\sqrt{3}\right)^2 + \frac{p^2}{q^2} - \frac{2p}{q}\sqrt{3}$$

$$\Rightarrow \quad 5-3-\frac{p^2}{q^2} = -\frac{2p}{q}\sqrt{3}$$

$$\Rightarrow \quad 2-\frac{p^2}{q^2} = -\frac{2p}{q}\sqrt{3}$$

$$\Rightarrow \quad \frac{2q^2-p^2}{q^2} = -\frac{2p}{q}\sqrt{3}$$

$$\Rightarrow \quad \sqrt{3} = \frac{p^2-2q^2}{2pq}$$

[$\therefore$ p, q are integer, $\therefore$ $\dfrac{p^2-2q^2}{2pq}$ is a rational number]

$\Rightarrow$ $\sqrt{3}$ is a rational number.

This contradicts the fact that $\sqrt{3}$ is an irrational number. So, our assumption is wrong.

Hence, $\sqrt{3}-\sqrt{5}$ is an irrational number.

(*b*) Let us assume on contrary that $\dfrac{3\sqrt{2}}{5}$ is a rational.

Then, there exists coprime p and q ($q \neq 0$) such that

$$\frac{3\sqrt{2}}{5} = \frac{p}{q}$$

$$\Rightarrow \quad \sqrt{2} = \frac{p}{q}\times\frac{5}{3} = \frac{5p}{3q}$$

[$\because$ p, q are integer, $\therefore$ $\dfrac{5p}{3q}$ is a rational number]

$\Rightarrow$ $\sqrt{2}$ is a rational number.

This contradicts the fact that $\sqrt{2}$ is an irrational number.

So, our assumption is wrong.

Hence, $\dfrac{3\sqrt{2}}{5}$ is an irrational number.

(*c*) Let us assume on the contrary that $5 + 3\sqrt{2}$ is a rational number. Then, there exists coprime p and q ($q \neq 0$) such that

$$5+3\sqrt{2} = \frac{p}{q}$$

$$\Rightarrow \quad 3\sqrt{2} = \frac{p}{q}-5$$

$$\Rightarrow \quad 3\sqrt{2} = \frac{p-5q}{q}$$

$$\Rightarrow \quad \sqrt{2} = \frac{p-5q}{3q}$$

[$\because$ p, q are integers, $\therefore$ $\dfrac{p-5q}{3q}$ is a rational number]

$\Rightarrow$ $\sqrt{2}$ is a rational number.

This contradicts the fact that $\sqrt{2}$ is an irrational number. So, our assumption is wrong.

Hence, $5+3\sqrt{2}$ is an irrational number.

(*d*) Let us assume on the contrary that $11+13\sqrt{2}$ is a rational number.

Then, there exists coprime p and q $(q \neq 0)$ such that

$$11+13\sqrt{2} = \frac{p}{q}$$

$$\Rightarrow \quad 13\sqrt{2} = \frac{p}{q}-11$$

$$\Rightarrow \quad 13\sqrt{2} = \frac{p-11q}{q}$$

$$\Rightarrow \quad \sqrt{2} = \frac{p-11q}{13q}$$

[$\because$ p, q are integers,

$\therefore$ $\frac{p-11q}{13q}$ is a rational number]

$\Rightarrow$ $\sqrt{2}$ is a rational number.

This contradicts the fact that $\sqrt{2}$ is an irrational number.

So, our assumption is wrong.

Hence, $11+13\sqrt{2}$ is an irrational number.

13. Prove each of the following is irrational.

(*a*) $\frac{1}{\sqrt{2}}$ (*b*) $\frac{3}{\sqrt{5}}$

Sol. (*a*) $\frac{1}{\sqrt{2}} = \frac{1\times\sqrt{2}}{\sqrt{2}\times\sqrt{2}} = \frac{\sqrt{2}}{2}$

Let us assume on the contrary that $\frac{1}{\sqrt{2}}$ is a rational number but $\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\Rightarrow$ $\frac{\sqrt{2}}{2}$ is rational

Let $\frac{\sqrt{2}}{2} = \frac{p}{q}$, where p and q are coprime and non-zero integers.

$\Rightarrow$ $\sqrt{2} = \frac{2p}{q}$,

$\Rightarrow$ $\sqrt{2}$ is rational.

[$\therefore$ p and q are integers,

$\therefore$ $\frac{2p}{q}$ is a rational number]

This contradicts the fact that $\sqrt{2}$ is an irrational number. So, our assumption is wrong.

Hence, $\frac{1}{\sqrt{2}}$ is an irrational number.

(*b*) $\frac{3}{\sqrt{5}} = \frac{3\times\sqrt{5}}{\sqrt{5}\times\sqrt{5}} = \frac{3\sqrt{5}}{5}$

Let us assume on the contrary that $\frac{3}{\sqrt{5}}$ is rational but $\frac{3}{\sqrt{5}} = \frac{3\sqrt{5}}{5}$.

$\Rightarrow$ $\frac{3\sqrt{5}}{5}$ is rational.

Let $\frac{3\sqrt{5}}{5} = \frac{p}{q}$, where p and q are coprime and non-zero integers.

$$\sqrt{5} = \frac{5p}{3q}$$

$\Rightarrow$ $\sqrt{5}$ is rational

[$\because$ p and q are integers,

$\therefore$ $\frac{5p}{3q}$ is a rational number]

This contradicts the fact that $\sqrt{5}$ is an irrational number. So, our assumption is wrong.

Hence, $\frac{3}{\sqrt{5}}$ is an irrational number.

14. Write whether $\frac{2\sqrt{45}+3\sqrt{20}}{2\sqrt{5}}$ on simplification is a rational or irrational. **[CBSE 2010]**

Sol. We have

$$\frac{2\sqrt{45}+3\sqrt{20}}{2\sqrt{5}} = \frac{2\sqrt{9\times5}+3\sqrt{4\times5}}{2\sqrt{5}}$$

$$= \frac{2\sqrt{9}\sqrt{5}+3\sqrt{4}\sqrt{5}}{2\sqrt{5}}$$

$$= \frac{2\times3\sqrt{5}+3\times2\sqrt{5}}{2\sqrt{5}}$$

$$= \frac{6\sqrt{5}+6\sqrt{5}}{2\sqrt{5}}$$

$$= \frac{12\sqrt{5}}{2\sqrt{5}} = 6$$

which is a rational number.

Long Answer Type Questions

15. Prove that $\sqrt{2}+\frac{3}{\sqrt{2}}$ is irrational.

Sol. Let us assume on the contrary that $\sqrt{2}+\frac{3}{\sqrt{2}}$ is rational number.

Then, there exist coprime p and q ($q \neq 0$) such that

$$\sqrt{2}+\frac{3}{\sqrt{2}} = \frac{p}{q}$$

$$\Rightarrow \quad \frac{2+3}{\sqrt{2}} = \frac{p}{q}$$

$$\Rightarrow \quad \frac{5}{\sqrt{2}} = \frac{p}{q}$$

$$\Rightarrow \quad \frac{1}{\sqrt{2}} = \frac{p}{5q}$$

$\Rightarrow$ $\frac{1}{\sqrt{2}}$ is rational.

[$\because$ p and q are integers,

$\therefore$ $\frac{p}{5q}$ is a rational number]

This contradicts the fact that $\frac{1}{\sqrt{2}}$ is irrational.

[Refer to solution of Q-13 (*a*)]

So, our assumption is wrong.

Hence, $\sqrt{2}+\frac{3}{\sqrt{2}}$ is irrational.

Higher Order Thinking Skills (HOTS) Questions

(Page 12)

1. If p is any prime number and a^2 is divisible by p, then p will also divide a. Explain.

Sol. From the Fundamental Theorem of Arithmetic, the integer a can be factorised as the product of primes which are unique.

Let $a = p_1 p_2 p_3 \ldots p_n$, where $p_1, p_2, \ldots p_n$ are primes, not necessarily distinct.

$$\therefore \quad a^2 = (p_1p_2p_3\ldots p_n)(p_1p_2p_3\ldots p_n)$$

$$= p_1^2\, p_2^2\, p_3^2 \ldots p_n^2$$

Now, it is given that the prime p divides a^2. Hence, p is a prime factor of a^2.

$\therefore$ From the uniqueness part of the Fundamental Theorem of Arithmetic, it follows that the only prime factors of a^2 are $p_1, p_2 p_3 \ldots p_n$. So, p is one of $p_1, p_2, p_3 \ldots p_n$.

Now, since $a = p_1p_2p_3\ldots p_n$

$\therefore$ p divides a.

Hence, proved.

2. Find the greatest number of six digits exactly divisible by 12, 15 and 36.

Sol. The greatest number of 6 digit exactly divisible by 12, 15 and 36 will be divisible by LCM (12, 15, 36).

$$12 = 2 \times 2 \times 3 = 2^2 \times 3$$

$$12 = 3 \times 5$$

$$36 = 2 \times 2 \times 3 \times 3 = 2^2 \times 3^2$$

$$\therefore \quad \text{LCM} = 2^2 \times 3^2 \times 5 = 180$$

The greatest number of 6 digits = 999999

$$999999 = 180 \times 5555 + 99$$

$\therefore$ The greatest number that will be divisible by 180

$$= 999999 - 99 = 999900$$

Hence, the required number is 999900.

3. Find the HCF of 145 and 155. Express it as a linear combination of x and y.

Sol. By using Euclid's division lemma, we have

$$155 = 145 \times 1 + 10 \quad \ldots(1)$$

$$145 = 10 \times 14 + 5 \quad \ldots(2)$$

$$10 = 5 \times 2 + 0 \quad \ldots(3)$$

$\therefore$ From (3), since the final remainder is zero, hence, the HCF is 5.

Now, from (2), we have

$$\text{HCF} = 5 = 145 - 10 \times 14 \quad \ldots(4)$$

From (1), $\quad 10 = 155 - 145 \quad \ldots(5)$

$\therefore$ From (4) and (5), we have

$$5 = 145 - (155 - 145) \times 14$$

$$= 145 - 155 \times 14 + 145 \times 14$$

$$= 145 \times (1 + 14) - 155 \times 14$$

$$= 145 \times 15 - 155 \times 14$$

$$= 145x + 155y$$

where $x = 15$ and $y = -14$.

$\therefore$ HCF as a linear combination of x and y is $145x + 155y = 5$, where $x = 15$ and $y = -14$.

4. If p and q are odd positive integers, then prove that $p^2 + q^2$ is even but not divisible by 4.

Sol. Let $p = 2p_1 + 1$ and $q = 2q_1 + 1$, where p and q are odd positive integers and p_1, q_1 are positive integers.

$$\therefore \quad p^2 + q^2 = (2p_1 + 1)^2 + (2q_1 + 1)^2$$

$$= 4\left(p_1^2 + q_1^2\right) + 2 + 4\left(p_1 + q_1\right)$$

$$= 2\left[2\left(p_1^2 + q_1^2\right) + 2\left(p_1 + q_1\right) + 1\right]$$

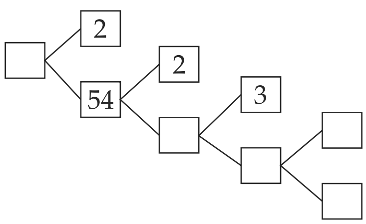
$$= 2m$$

where $m = 2\left(p_1^2 + q_1^2\right) + 2\left(p_1 + q_1\right) + 1$ is a positive integer.

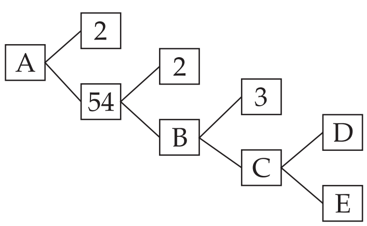
$\therefore$ 2 is a factor of $p^2 + q^2$, but not 4.

Hence, $p^2 + q^2$ is an even integer but not divisible by 4.

5. Find the missing numbers in the prime factor tree:



Sol. Let us denote the empty box on the extreme left by the letter A and the subsequent empty boxes on its right by B, C, D and E respectively. In the empty box A, the missing number is 54 × 2 = 108. The missing number in the empty box B is 54 ÷ 2 = 27, and that in the empty box C is 27 ÷ 3 = 9. Since 9 = 3 × 3, hence, the missing numbers in the empty boxes D and E are 3 and 3.



Hence, the required missing number in the empty boxes, starting from the left are 108, 27, 9, 3 and 3 respectively.

6. Four bells toll together at 10 am. They toll after 8, 12, 16, 24 seconds respectively. How many times will they toll together again in the next 2 hours?

Sol. The time required by the four bells to toll together is the LCM of 8, 12, 16 and 24 (in seconds).

$$8 = 2 \times 2 \times 2 = 2^3$$

$$12 = 2 \times 2 \times 3 = 2^2 \times 3$$

$$16 = 2 \times 2 \times 2 \times 2 = 2^4$$

$$24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3$$

$\therefore$ LCM of 8, 12, 16 and 24 = $2^3 \times 3 = 24$

Hence, after 48 seconds, the four bells will toll together once again. Now, in the next 2 hours i.e. 2 × 60 × 60 seconds.

The number of times the four bells will toll together = $\dfrac{2 \times 60 \times 60}{48} = 150$

$\therefore$ In the next 2 hours number of times the four bells toll together is 150.

7. For any positive integer n, prove that $n^3 - n$ is divisible by 6.

Sol. We have $n^3 - n = n(n^2 - 1) = (n - 1)\, n(n + 1)$ which is the product of three consecutive positive integer. We shall now prove that the product of three consecutive positive integers is divisible by 6.

We know that any positive integer is of the form $6q$, $6q + 1$, $6q + 2$, $6q + 4$ or $6q + 5$, for some integer q.

Case (*i*) $\quad n = 6q$

$\Rightarrow \quad (n - 1)\, n\, (n + 1) = (6q - 1)\, (6q)\, (6q + 1)$

$\qquad = 6q\, (6q - 1)\, (6q + 1)$

$\Rightarrow$ $n^3 - n$ is divisible by 6.

Case (*ii*) $\quad n = 6q + 1$

$\Rightarrow \quad (n - 1)\, n\, (n + 1) = (6q)\, (6q + 1)\, (6q + 2)$

$\qquad = 6q\, (6q + 1)\, (6q + 2)$

$\Rightarrow$ $n^3 - n$ is divisible by 6.

Case (*iii*) $\quad n = 6q + 2$

$\Rightarrow \quad (n - 1)\, n\, (n + 1) = (6q + 1)\, (6q + 2)\, (6q + 3)$

$\qquad = 2.3(6q + 1)\, (3q + 1)\, (2q + 1)$

$\qquad = 6(6q + 1)\, (3q + 1)\, (2q + 1)$

$\Rightarrow$ $n^3 - n$ is divisible by 6.

Case (*iv*) $\quad n = 6q + 3$

$\Rightarrow (n - 1)\, n\, (n + 1) = (6q + 2)\, (6q + 3)\, (6q + 4)$

$\qquad = 2.3.2\, (3q + 1)\, (2q + 1)\, (3q + 2)$

$\qquad = 12\, (3q + 1)\, (2q + 1)\, (3q + 2)$

$\Rightarrow$ $n^3 - n$ is divisible by 6.

Case (*v*) $\quad n = 6q + 4$

$\Rightarrow \; (n - 1)\, n\, (n + 1) = (6q + 3)\, (6q + 4)\, (6q + 5)$

$\qquad = 3.2\, (2q + 1)\, (3q + 2)\, (6q + 5)$

$\qquad = 6\, (2q + 1)\, (3q + 2)\, (6q + 5)$

$\Rightarrow$ $n^3 - n$ is divisible by 6.

Case (*vi*) $\quad n = 6q + 5$

$(n - 1)\, n\, (n + 1) = (6q + 4)\, (6q + 5)\, (6q + 6)$

$\qquad = 2.6\, (3q + 2)\, (6q + 5)\, (q + 1)$

$\qquad = 12\, (3q + 2)\, (6q + 5)\, (q + 1)$

$\Rightarrow \quad n^3 - n$ is divisible by 6.

Hence, $n^3 - n$ is divisible by 6 for every positive integer n.

8. If x and y are two odd positive integers such that $x > y$. Prove that one of the two numbers $\frac{x+y}{2}$ and $\frac{x-y}{2}$ is odd and the other is even.

Sol. Let $x = 2q + 3$ and $y = 2q + 1$ be two positive odd integers where $x > y$.

Now,
$$\frac{x+y}{2} = \frac{2q+3+2q+1}{2} = \frac{2(2q+2)}{2} = 2q+2 = 2(q+1)$$

which is a positive even integer

and
$$\frac{x-y}{2} = \frac{2q+3-2q-1}{2} = \frac{2}{2} = 1$$

which is a positive odd integer.

Hence, proved.

Self-Assessment

(Page 13)

Multiple-Choice Questions

1. The least number that is divisible by all the numbers from 1 to 9 (both inclusive) is

(*a*) 10 (*b*) 180

(*c*) 540 (*d*) 2520

Sol. (*d*) 2520

We find the LCM of all numbers from 1 to 9 as follows:

$$\text{LCM} = 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 7 = 2^3 \times 3^2 \times 5 \times 7 = 2520$$

2	1, 2, 3, 4, 5, 6, 7, 8, 9
2	1, 1, 3, 2, 5, 3, 7, 4, 9
2	1, 1, 3, 1, 5, 3, 7, 2, 9
3	1, 1, 3, 1, 5, 3, 7, 1, 9
3	1, 1, 1, 1, 5, 1, 7, 1, 3
5	1, 1, 1, 1, 5, 1, 7, 1, 1
7	1, 1, 1, 1, 1, 1, 7, 1, 1
	1, 1, 1, 1, 1, 1, 1, 1, 1

$\therefore$ The least number that is divisible by all the numbers from 1 to 9 is 2520.

2. If 'n' is a natural number, then which of the following numbers end with zero?

(*a*) $(3 \times 2)^n$ (*b*) $(2 \times 5)^n$

(*c*) $(6 \times 2)^n$ (*d*) $(5 \times 3)^n$ **[CBSE 2023]**

Sol. (*b*) $(2 \times 5)^n$

For $n = 1$,

$$(3 \times 2)^n = (3 \times 2)^1 = 6$$
$$(2 \times 5)^n = (2 \times 5)^1 = 10$$
$$(6 \times 2)^n = (6 \times 2)^1 = 12$$
$$(5 \times 3)^n = (5 \times 3)^1 = 15$$

From above,

$(2 \times 5)^n$ will end with zero.

3. $n^2 - 1$ is divisible by 8, if n is

(*a*) an integer. (*b*) an odd integer.

(*c*) a natural number. (*d*) an even integer.

Sol. (*b*) an odd integer.

We have $\frac{n^2-1}{8} = \frac{(n-1)(n+1)}{8}$ which is not always an integer when n is any integer.

Let n be any even integer, say $n = 2m$.

$$\therefore \quad \frac{n^2-1}{8} = \frac{(2m-1)(2m+1)}{8} = \frac{4m^2-1}{8}$$

which is not an integer when m is any integer.

Let n be any odd integer, say $n = 2p + 1$.

$$\therefore \quad \frac{n^2-1}{8} = \frac{(n-1)(n+1)}{8} = \frac{2p(2p+2)}{8} = \frac{p(p+1)}{2}$$

which is always an integer when p is any positive integer, since the product of two consecutive integers is always divisible by 2.

Hence, $n^2 - 1$ is divisible by 8 only when n is an odd integer.

Fill in the Blanks

4. If HCF $(a, 8) = 4$, LCM $(a, 8) = 24$, then a is **12**.

Sol. HCF $(a, b) \times$ LCM $(a, b) = a \times b$

$$4 \times 24 = a \times 8$$

$$\Rightarrow \quad a = \frac{4 \times 24}{8} = 12$$

5. For some integer q every even integer is of the form $\mathbf{2q}$.

6. If a and b are coprime, then a^2 and b^2 are **coprime**.

Sol. Consider $a = 2$ and $b = 3$, then $a^2 = 4$, $b^2 = 9$

Note: 4 and 9 are coprime.

$\therefore$ a^2 and b^2 are coprime.

7. The maximum number of factors of a prime number is **2**.

Assertion-Reason Type Questions

Directions (Q. Nos. 8 to 10): Each of these questions contains an assertion followed by reason. Read them carefully, and answer the question on the basis of the following options, select the one that best describes the two statements.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

8. **Assertion (A):** $6 - \sqrt{2} \times \sqrt{2}$ is a rational number.

Reason (R): $\sqrt{2} \times \sqrt{2}$ is a rational number.

Sol. The correct answer is (a) as both the statements are correct.

$\sqrt{2} \times \sqrt{2} = 2$ which is a rational number, 6 is a rational no. and the difference of two rational no. is always rational no.

Hence, the fact that $\sqrt{2} \times \sqrt{2}$ is rational is the correct explanation why $6 - \sqrt{2} \times \sqrt{2}$ is rational.

9. **Assertion (A):** The HCF of 844 and 244 is 4.

Reason (R): 6 is the highest number that can divide both 844 and 244.

Sol. The correct answer is (c) as the HCF of 844 and 244 is 4, hence, the assertion is correct. However, the highest no. which can divide both 844 and 244 is 4 and not 6.

Hence, the reason is incorrect.

10. **Assertion (A):** The number 5^n cannot end with digit 0, where n is a natural number.

Reason (R): Prime factorisation of 5 has only two factors, 1 and 5. **[CBSE 2023]**

Sol. (*b*) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

Any number that ends with digit 0 has at least one factor of 2 and one factor of 5 in its factorisation. The number 5^n has only prime factor of 5 and therefore cannot end with digit 0. Thus, Assertion (A) is true.

The prime factorisation of 5 has only two factors 1 and 5. Reason (R) is true but it does not explain Assertion (A).

Case Study Based Questions

11. Starting out your day with a walk can offer your body a number of health benefits. So, three friends Nehal, Sukanya and Arpit plan to go for a morning walk. They step off together and their steps measure 90 cm, 80 cm and 85 cm respectively. Based on the above information, answer the following questions.

(*a*) What is the minimum distance Nehal and Sukanya will cover before they meet again?

Ans. Nehal steps will cover 90 cm, 180 cm, 270 cm, ... and so on.

Sukanya steps will cover 80 cm, 160 cm, 240 cm, ... and so on.

Minimum distance Nehal and Sukanya will cover before they meet again = LCM of 90 and 80.

$$90 = 2 \times 3^2 \times 5$$
$$80 = 2^4 \times 5$$

LCM of 90 and 80 = $2^4 \times 3^2 \times 5 = 720$

Hence, the minimum distance Nehal and Sukanya will cover before they meet again = 720 cm.

(*b*) What is the minimum distance Sukanya and Arpit will cover before they meet again?

Ans. Sukanya steps will cover 80 cm, 160 cm, 240 cm, ... and so on.

Arpit steps will cover 85 cm, 170 cm, 255 cm, ... and so on.

Minimum distance Sukanya and Arpit will cover before they meet again = LCM of 80 and 85.

$$80 = 2^4 \times 5$$
$$85 = 5 \times 17$$
$$\text{LCM of 80 and 85} = 2^4 \times 5 \times 17$$
$$= 16 \times 5 \times 17$$
$$= 1360$$

Hence, the minimum distance Sukanya and Arpit will cover before they meet again

$$= 1360 \text{ cm.}$$

(*c*) (*i*) What is the minimum distance each of them will cover before they meet again?

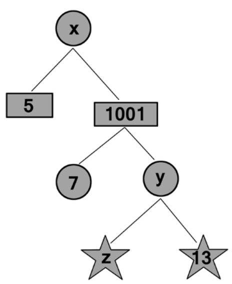
Ans. 12240 cm

or

(*ii*) What is the number of common steps covered by all of them?

Ans. 5

12. A mathematics teacher of class X asks one of your friends to make a model of a factor tree for Mathematics exhibition. She/he finds some difficulty and asks for your help in completing the task. Observe the following factor tree and answer the following questions.



(*a*) What is the value of x?

Ans. 5005

(*b*) What is the value of y?

Ans. 143

(*c*) (*i*) What is the value of z?

Ans. 11

or

(*ii*) What is the prime factorisation of the value of x?

Ans. $5 \times 7 \times 11 \times 13$

Very Short Answer Type Questions

13. If two positive integers a and b are written as $a = xy^2$, $b = x^3y$ where x and y are prime numbers, then find the LCM (a, b).

Sol. We have $a = x \times y^2$

$$b = x^3 \times y$$

$\therefore$ LCM of a and b is x^3y^2.

14. If the HCF of 65 and 117 is expressible in the form of $65a - 117$, then find the value of a.

Sol. We first find the HCF of 65 and 117 by using Euclid's division lemma as follows:

We have $11 = 65 \times 1 + 52$ ...(1)

$65 = 52 \times 1 + 13$ ...(2)

$52 = 13 \times 4 + 0$ ...(3)

$\therefore$ From (3), HCF = 13

Now, from (2),

$13 = 65 - 52$ ...(4)

From (1), $52 = 117 - 65$ ...(5)

$\therefore$ From (4) and (5), we have

$$13 = 65 - (117 - 65)$$
$$= 65 \times 2 - 117$$

Comparing $65a - 117$ with $65 \times 2 - 117$, we see that $a = 2$.

$\therefore$ The required value of a is 2.

15. Find the largest number which divides 615 and 963 leaving remainder 6 in each case.

Sol. The required number will be the HCF of $615 - 6 = 609$ and $963 - 6 = 957$. We now find the HCF of 609 and 957 by using Euclid's division lemma as follows:

We have $957 = 609 \times 1 + 348$

$609 = 348 \times 1 + 261$

$348 = 261 \times 1 + 87$

$261 = 87 \times 3 + 0$

Since the final remainder is zero, hence, the required HCF is 87.

16. Show that 12^n cannot end with digit 0 or 5 for any natural number n.

Sol. We have $12^n = (3 \times 2^2)^n = 3^n 2^{2n}$

Since, 3^n and 2^{2n} cannot end with 0 or 5 for any natural number n, hence, 12^n cannot end with 0 or 5, for any natural number n.

17. Find the least number which when divided by 12, 13 and 24 leaves remainder 7 in each case.

[CBSE 2023]

Sol. $12 = 2^2 \times 3$

$13 = 1 \times 13$

$24 = 2^2 \times 3$

$$\text{LCM of 12, 13 and 24} = 2^3 \times 3 \times 13$$
$$= 8 \times 3 \times 13$$
$$= 312$$

$\therefore$ Smallest number which leaves no remainder when divided by 12, 13 and 24 = 312.

$\therefore$ Smallest number which when divided by 12, 13 and 24 leaves a remainder 7 is each case = 312 + 7 = 319.

18. Find the LCM and HCF of 198 and 144. Verify that HCF × LCM = product of two numbers.

Sol. We first find the prime factors of 198 and 144 as follows:

$$198 = 2 \times 3^2 \times 11$$

$$144 = 2^4 \times 3^2$$

$$\therefore \quad HCF = 2 \times 3^2 = 18$$

and $\quad LCM = 2^4 \times 3^2 \times 11 = 1584$

$$\begin{array}{r|r} 2 & 198 \\ \hline 3 & 99 \\ \hline 3 & 33 \\ \hline 11 & 11 \\ \hline & 1 \end{array} \qquad \begin{array}{r|r} 2 & 144 \\ \hline 2 & 72 \\ \hline 2 & 36 \\ \hline 2 & 18 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

We see that

$$LCM\,(198, 144) = 1584$$

and $\quad HCF(198, 144) = 18$

$$\therefore \quad HCF \times LCM = 18 \times 1584$$

$$= 18 \times 8 \times \frac{1584}{8}$$

$$= 144 \times 198$$

$$= \text{Product of two numbers}$$

Hence, proved.

19. Explain why $11 \times 13 \times 23 + 23 \times 5$ is a composite number.

Sol. $11 \times 13 \times 23 + 23 \times 5 = 23\,(11 \times 13 + 5)$

$$= 23 \times 148$$

$$= 2^2 \times 23 \times 37$$

A number which has more than two factors (other than 1 and itself) is a composite number. Since, the above can be expressed as a product of more than two prime factors, therefore it is a composite number.

Short Answer Type Questions

20. Find the HCF using Euclid's division Algorithm.

(*a*) 255 and 867

(*b*) 377, 435 and 667

Sol. (*a*) By Euclid's division lemma, we have

$$867 = 255 \times 3 + 102$$

$$255 = 102 \times 2 + 51$$

$$102 = 51 \times 2 + 0$$

Since, the final remainder is zero.

$\therefore$ The HCF of 255 and 867 is 51.

(*b*) We first find the HCF of 377 and 435 as follows:

By Euclid's division lemma, we have

$$435 = 377 \times 1 + 58$$

$$377 = 58 \times 6 + 29$$

$$58 = 29 \times 2 + 0$$

$$\therefore \quad HCF\,(377, 435) = 29$$

We now find the HCF of 29 and 667 as follows:

$$667 = 29 \times 23 + 0$$

$$\therefore \quad HCF\,(29, 667) = 29$$

$\therefore$ The HCF of 377, 435 and 667 is 29.

21. What is the smallest number which when increased by 6 becomes divisible by 36, 63 and 108?

Sol. We first find the LCM of 36, 63 and 108 as follows:

$$\begin{array}{r|l} 2 & 36, 63, 108 \\ \hline 2 & 18, 63, 54 \\ \hline 3 & 9, 63, 27 \\ \hline 3 & 3, 21, 9 \\ \hline 3 & 1, 7, 3 \\ \hline 7 & 1, 7, 1 \\ \hline & 1, 1, 1 \end{array}$$

$$LCM = 2 \times 2 \times 3 \times 3 \times 3 \times 7$$

$$= 756$$

Hence, the smallest number which, when increased by 6 becomes divisible by 36, 63 and 108 is 756 – 6 i.e. 750.

22. Find the greatest number that will divide 445, 572, 699 leaving remainders 4, 5, 6 respectively.

Sol. Since 445, 572 and 699, divided by the required number leave remainders 4, 5 and 6 respectively.

Therefore 445 – 4 = 441, 572 – 5 = 567 and 699 – 6 = 693 are completely divisible by the required number.

Clearly, the required number is the HCF of 441, 567 and 693.

Applying Euclid's division lemma to 441 and 567, we get

$$567 = 441 \times 1 + 126$$

$$441 = 126 \times 3 + 63$$

$$126 = 63 \times 2 + 0$$

Since the remainder is zero.

$\therefore$ 63 is the HCF of 441 and 567.

Now applying the division lemma to 693 and 63, we get

$$693 = 63 \times 11 + 0$$

Since the remainder is zero.

$\therefore$ 63 is the HCF of 693 and 63.

$\therefore$ The HCF of 441, 567 and 693 is 63.

Hence, the required number is 63.

23. Prove that $\sqrt{3}$ is an irrational number.

Sol. Let us assume on the contrary that $\sqrt{3}$ is a rational number.

Then $\sqrt{3} = \frac{p}{q}$, where p and q are coprime, $q \neq 0$

$\therefore$ On squaring both sides

$$3 = \frac{p^2}{q^2}$$

$\Rightarrow \quad 3q^2 = p^2 \quad \ldots(1)$

$\Rightarrow$ p^2 is divisible by 3 $\quad [\because 3q^2$ is divisible by 3]

$\Rightarrow$ p is divisible by 3 $\quad \ldots(2)$

$[\because 3$ is prime and divides $p^2 \Rightarrow 3$ divides $p]$

Let $p = 3c$, for some integer c.

Substituting $p = 3c$ in (1), we get

$$3q^2 = (3c)^2 = 3q^2 = 9c^2$$

$\Rightarrow \quad q^2 = 3c^2$

$\Rightarrow$ q^2 is divisible by 3 $\quad [\because 3c^2$ is divisible by 3]

$\Rightarrow$ q is divisible by 3 $\quad \ldots(3)$

$[\because 3$ is prime and divides $q^2 \Rightarrow 3$ divides $q]$

From (2) and (3)

$\Rightarrow$ both p and q are divisible by 3.

3 is a common factor of p and q.

But this contradicts the fact that p and q are coprime.

This contradiction has arisen because of our incorrect assumption that $\sqrt{3}$ is rational. Hence, $\sqrt{3}$ is irrational.

24. Three bells ring at intervals of 8, 12 and 18 minutes. If all the three bells rang at 6 am, when will they ring together again. **[CBSE 2023]**

Sol. First bell rings after 8, 16, 24, 32 (minutes), . . . and so on.

Second bell rings after 12, 24, 36 (minutes), . . . and so on.

Third bell rings after 18, 36, 54 (minutes), . . . and so on.

Minutes after which the three bells will ring together again = LCM of 8, 12 and 18 (in minutes)

$$8 = 2 \times 2 \times 2 = 2^3$$

$$12 = 2 \times 2 \times 3 = 2^2 \times 3$$

$$18 = 2 \times 3 \times 3 = 2 \times 3^2$$

LCM of 8, 12 and 18 $= 2^3 \times 3^2 = 8 \times 9 = 72$

$= 72$ minutes

$= 1$ hour 12 minutes

The three bells will ring after 72 minutes, i.e. at 7.12 am.

25. In an examination, the number of students in Class VIII, IX, X are 60, 84, 108 respectively. Find the minimum number of rooms required if in each room, the same number of students are to be seated and all of them being in the same class.

Sol. We have to find the HCF of 60, 84 and 108.

Applying Euclid's division lemma to 60 and 84, we get

$$84 = 60 \times 1 + 24$$

$$60 = 24 \times 2 + 12$$

$$24 = 12 \times 2 + 0$$

Since the remainder is zero.

$\therefore$ HCF of 60 and 84 is 12.

Now, applying the division lemma to 12 and 108.

$$108 = 12 \times 9 + 0$$

Since the remainder is zero

$\therefore$ HCF of 12 and 108 is 12.

$\therefore$ The HCF of 60, 84 and 108 is 12.

$\therefore$ For 60 students of class VIII, we need a minimum of $60 \div 12 = 5$ rooms.

For 84 students of class IX, we need a minimum of $84 \div 12 = 7$ rooms.

Finally, for 108 students of class X, we need a minimum of $108 \div 12 = 9$ rooms.

Hence, the required total minimum number of rooms is $5 + 7 + 9$ i.e. 21.

Long Answer Type Questions

26. Show that if n is an odd integer, then $n^2 - 1$ is divisible by 8.

Sol. We know that any positive odd integer is of the form $2q + 1$ for some integer q.

$\therefore \quad n = 2q + 1$

Then, $\quad \frac{n^2 - 1}{8} = \frac{(n+1)(n-1)}{8}$

$$= \frac{2q(2q+2)}{8}$$

$$= \frac{q(q+1)}{2}$$

Now, since the product of two consecutive integers is even i.e. divisible by 2

$\therefore \quad \frac{q(q+1)}{2}$ is an integer.

$\therefore \quad \frac{n^2-1}{8}$ is also an integer when n is an odd integer.

$\therefore$ $n^2 - 1$ is divisible by 8 when n is an odd integer.

27. Show that one and only one out of n, $n + 4$, $n + 8$, $n + 12$, $n + 16$ is divisible by 5, where n is any positive integer.

Sol. Let any positive integer a be divided by 5 and let q be the quotient and r be the remainder. Then, by Euclid's division lemma, we have

$$a = 5q + r \qquad \ldots(1)$$

where q is a positive integer and $0 \le r < 5$.

Let $a = n$ be divisible by 5. Then from (1), we get

$$n = 5q + r \qquad \ldots(2)$$

where $0 \le r < 5$

Putting $r = 0$ in (2), we get

$$n = 5q \qquad \ldots(3)$$

$\Rightarrow$ n is divisible by 5.

Adding 4 both sides in (3), we get

$$n + 4 = 5q + 4$$

$\Rightarrow$ $n + 4$ is not divisible by 5.

Adding 8 both sides in (3), we get

$$n + 8 = 5q + 8$$

$\Rightarrow$ $n + 8$ is not divisible by 5.

Adding 12 both sides in (3), we get

$$n + 12 = 5q + 12$$

$\Rightarrow$ $n + 12$ is not divisible by 5.

Adding 16 both sides in (3), we get

$$n + 16 = 5q + 16$$

$\Rightarrow$ $n + 16$ is not divisible by 5.

Now, putting $r = 1$ in (2), we get

$$n = 5q + 1 \qquad \ldots(4)$$

$\Rightarrow$ n is not divisible by 5.

Adding 4 both sides in (4), we get

$$n + 4 = 5q + 1 + 4$$

$$n + 4 = 5q + 5$$

$$\Rightarrow \qquad n + 4 = 5(q + 1)$$

$\Rightarrow$ $n + 4$ is divisible by 5.

Adding 8 both sides in (4), we get

$$n + 8 = 5q + 1 + 8$$

$$\Rightarrow \qquad n + 8 = 5q + 9$$

$\Rightarrow$ $n + 8$ is not divisible by 5.

Similarly, we can show that $n + 12$, $n + 16$ are also not divisible by 5.

In the same way, we can show that if any other number say, $n + 4$ is divisible by 5, then none of the remaining numbers n, $n + 8$, $n + 12$ and $n + 16$ is divisible by 5.

Thus, for each value of r such that $0 \le r < 5$ only one out of n, $n + 4$, $n + 8$, $n + 12$ and $n + 16$ is divisible by 5.

Hence, proved.

28. Find the HCF of 96 and 404. Express it as a linear combination of x and y.

Sol. We start with the greater number 404.

Applying Euclid's lemma, we get

$$404 = 96 \times 4 + 20 \qquad \ldots(1)$$

$$\begin{array}{l}
96\,)\,404\,(\,4 \\
\quad\ \ \underline{384} \\
\quad\ \ 20\,)\,96\,(\,4 \\
\qquad\quad \underline{80} \\
\qquad\quad 16\,)\,20\,(\,1 \\
\qquad\qquad\ \underline{16} \\
\qquad\qquad\ 4\,)\,16\,(\,4 \\
\qquad\qquad\quad\ \underline{16} \\
\qquad\qquad\quad\ \ \underline{0}
\end{array}$$

$\because$ $r = 20 \ne 0$, take divisor 96 as the new dividend and remainder 20 as new divisor, we get

$$96 = 20 \times 4 + 16 \qquad \ldots(2)$$

Continue the process till remainder, $r = 0$

$$\therefore \qquad 20 = 16 \times 1 + 4 \qquad \ldots(3)$$

$$16 = 4 \times 4 + 0 \qquad \ldots(4)$$

$\because$ $r = 0$, the divisor at last step is the HCF.

$\therefore$ HCF of 96 and 404 is 4.

From (3), $\quad 20 = 16 \times 1 + 4$

$$\Rightarrow \qquad 4 = 20 - 16 \times 1 \qquad \ldots(5)$$

From (1),

$$404 = 96 \times 4 + 20$$

$$\Rightarrow \qquad 20 = 404 - 96 \times 4 \qquad \ldots(6)$$

From (2),

$$96 = 20 \times 4 + 16$$

$$16 = 96 - 20 \times 4$$

$$\Rightarrow \qquad 16 = 96 - 4\,(404 - 96 \times 4) \text{ [From 6]}$$

$$\Rightarrow \qquad 16 = 96 - 4 \times 404 + 96 \times 16$$

$$\Rightarrow \qquad 16 = 96 \times 17 - 4 \times 404 \qquad \ldots(7)$$

Using value of 20 and 16 from (6) and (7) in (5)

$$4 = 404 - 96 \times 4 - 96 \times 17 + 4 \times 404$$

$$4 = 404 \times 5 - 96 \times 21$$

$$4 = 404 \times (5) + 96 \times (-21)$$

$$\Rightarrow \qquad 4 = 404x + 96y$$

where $x = 5$, $y = -21$.

Let's Compete

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Multiple-Choice Questions

1. How many prime numbers are of the form $10n + 1$, where n is a natural number such that $1 \le n < 10$?

(*a*) 3 (*b*) 4
(*c*) 5 (*d*) 6

Sol. (*c*) 5

Putting $n = 1, 2, 3, 4, 5, 6, 7, 8$ and 9 in the expression $N = 10n + 1$, we get the following numbers respectively: 11, 21, 31, 41, 51, 61, 71, 81 and 91. Only 11, 31, 41, 61 and 71 are prime and the remaining numbers are composite. Hence, the total number of prime numbers is 5.

2. If 3 is the least prime factor of p and 5 is the least prime factor of q, then the least prime factor of $(p + q)$ is

(*a*) 2 (*b*) 3
(*c*) 5 (*d*) 11

Sol. (*a*) 2

The least prime factor of p is 3.

Let us assume that p is a product of 3 and x where x is a prime number greater than 3, say $x = 5$.

Now, $\qquad p = x \times 3 = 5 \times 3 = 15$

The least prime factor of q is 5. Let us assume that q is product of 5 and x, where x is a prime greater than 5, say $x = 7$.

Now, $(p + q)$ becomes $(15 + 35) = 50$

$\therefore$ The least prime factor of 50 will be 2 since $50 = 2 \times 5 \times 5$.

$\therefore$ The least prime factor of $(p + q)$ is 2.

3. $157^2 - 151^2$ is a

(*a*) prime number.
(*b*) composite number.
(*c*) an odd prime number.
(*d*) an odd composite number.

Sol. (*b*) composite number.

$$\begin{aligned} 157^2 - 151^2 &= (157 + 151)(157 - 151) \\ &= 308 \times 6 \\ &= 2^2 \times 7 \times 11 \times 3 \times 2 \\ &= 2^3 \times 7 \times 11 \times 3 \qquad \ldots(1) \end{aligned}$$

Since, the given number has more than two prime factors. Hence, it must be a composite number and it is clear from (1) that this number will end with an even integer. So, this composite number cannot be odd.

4. The traffic lights at three different road crossings change after every 48 s, 72 s and 108 s respectively. If they change simultaneously at 9:00 am, at what time will they change together again?

(*a*) 10:00 am 10 min 5 s
(*b*) 9:00 am 7 min 12 s
(*c*) 9:00 am 12 min 7 s
(*d*) 10:00 am 5 min 12 s

Sol. (*b*) 9 : 00 am 7 min 12 s

We first find the LCM of 48, 72 and 108 as follows:

$$\text{LCM} = 2^4 \times 3^3 = 16 \times 27 = 432$$

2	48, 72, 108
2	24, 36, 54
2	12, 18, 27
2	6, 9, 27
3	3, 9, 27
3	1, 3, 9
3	1, 1, 3
	1, 1, 1

Hence, after 432 s i.e. after 7 min 12 s from 9 am, the traffic lights will change together again.

Hence, the required time is 9 : 00 am 7 min 12 s.

5. If $x = 2^2 \times 3^a$, $y = 2^3 \times 3 \times 5$, $z = 2^2 \times 3^2 \times 7$ and LCM $(x, y, z) = 7560$, then a is equal to

(*a*) 1 (*b*) 2
(*c*) 3 (*d*) 4

Sol. (*c*) 3

We have $\qquad x = 2^2 \times 3^a$

$$y = 2^3 \times 3 \times 5$$

$$z = 2^2 \times 3^2 \times 7$$

$$\text{LCM}(x, y, z) = 7560$$

$$\text{LCM}(x, y, z) = 2^3 \times 3^a \times 5 \times 7$$

(Assuming that $a \ge 3$)

$$7560 = 2^3 \times 3^a \times 5 \times 7$$

$$\Rightarrow \quad \frac{7560}{2^3 \times 5 \times 7} = 3^a$$

$$\Rightarrow \quad 27 = 3^a$$

$$\Rightarrow \quad 3^3 = 3^a$$

$$\Rightarrow \quad a = 3$$

6. If n is any natural number, then $8^{2n} - 3^{2n}$ is always divisible by

(*a*) 5 (*b*) 11

(*c*) both 5 and 11 (*d*) none of these

Sol. (*c*) both 5 and 11

Given n is a natural number, we have $8^{2n} - 3^{2n}$

If $n = 1$ then $8^2 - 3^2 = 64 - 9 = 55$ is always divisible by both 5 and 11.

If $n = 2$ then $8^4 - 3^4 = 4096 - 81 = 4015$ is also divisible by both 5 and 11.

$\therefore$ For every n, $8^{2n} - 3^{2n}$ is always divisible by both 5 and 11.

7. The smallest irrational number by which $\sqrt{27}$ should be multiplied so as to get a rational number is

(*a*) $\sqrt{2}$ (*b*) $\sqrt{27}$

(*c*) $3\sqrt{3}$ (*d*) $\sqrt{3}$

Sol. (*d*) $\sqrt{3}$

Since $\sqrt{27} = \sqrt{9 \times 3} = 3\sqrt{3}$, we see that if we multiply this by the least irrational number $\sqrt{2}$, we get $3\sqrt{3} \times \sqrt{2} = 3\sqrt{6}$ which is again an irrational number. But if we multiply $3\sqrt{3}$ by $\sqrt{3}$, we get $3 \times 3 = 9$ which is a rational number. Hence, the required smallest irrational number is $\sqrt{3}$.

8. A pair of irrational numbers whose product is a rational number is

(*a*) $\sqrt{16}\sqrt{4}$ (*b*) $\sqrt{5}\sqrt{2}$

(*c*) $\sqrt{3}\sqrt{27}$ (*d*) $\sqrt{36}\sqrt{2}$

Sol. (*c*) $\sqrt{3}\sqrt{27}$

Both $\sqrt{3}$ and $\sqrt{27} = \sqrt{9 \times 3} = 3\sqrt{3}$ are irrational numbers and their product $\sqrt{3} \times 3\sqrt{3} = 9$, which is a rational number.

$\therefore$ $\sqrt{3}\sqrt{27}$ is a rational number.

9. The LCM of two numbers is 1800. Which of the following cannot be their HCF?

(*a*) 500 (*b*) 200

(*c*) 300 (*d*) 600

Sol. (*a*) 500

We know that the LCM of any two positive numbers is divisible by their HCF. In the present problem, we see that 1800 is not exactly divisible by only 500 in (*a*), but it is exactly divisible by 200, 300 or 600 in (*b*), (*c*) and (*d*). Hence only 500 in (*a*) cannot be the HCF of the two number.

10. If d is the HCF of 45 and 63 and satisfying $d = 45x + 63y$, then the values of x and y are

(*a*) $x = 3, y = -2$

(*b*) $x = 2, y = -3$

(*c*) $x = -3, y = 2$

(*d*) $x = 3, y = 2$

Sol. (*a*) $x = 3, y = -2$

We first find the value of d, the HCF of 45 and 63 by using Euler's division lemma as follows:

$$63 = 45 \times 1 + 18 \quad \ldots(1)$$

$$45 = 18 \times 2 + 9 \quad \ldots(2)$$

$$18 = 9 \times 2 + 0 \quad \ldots(3)$$

$\therefore$ From (3), HCF of 45 and 63 is 9.

Now, from (2),

$$\begin{aligned} 9 &= 45 - 18 \times 2 \\ &= 45 - (63 - 45) \times 2 \quad \text{[From (1)]} \\ &= 45 - 63 \times 2 + 45 \times 2 \\ &= 45 \times 3 - 63 \times 2 \\ &= 45x + 63y \end{aligned}$$

where $x = 3$ and $y = -2$.

$\therefore$ The values of x and y are 3 and -2 respectively.

Life Skills

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1. Ritika has two ribbons which are 84 cm and 98 cm long. These ribbons are of same colour and material. She wants to cut them into equal pieces so that no ribbon is wasted. If she gifts one pair each to her two daughters then how many ribbon pieces are left with her?

Sol. We find the maximum length of each ribbon that can be cut from two ribbons. So, we first find the HCF of 84 and 98 by using Euclid's division lemma as follows:

$$98 = 84 \times 1 + 14$$

$$84 = 14 \times 6 + 0$$

Hence, HCF of 84 and 98 is 14.

Hence, the maximum length of each piece of each ribbon is 14 cm.

Now, the number of pieces of the first ribbon = 84 cm ÷ 14 cm = 6

and the number of piece of the second ribbon = 98 cm ÷ 14 cm = 7

Out of 6 pieces and 7 pieces of these two ribbons, she gifted 2 pieces to each of her two daughters. Hence, the total number of pieces of ribbons which are left with her is

$$(6 - 2) + (7 - 2) = 4 + 5 = 9$$

Hence, the required number of pieces of ribbons which are left with her is 9.