Mathematics 10

Sample Question Paper

Standard (Code 041)

ANSWERS

Section A

1. (*b*)	2. (*d*)	3. (*a*)
4. (*d*)	5. (*b*)	6. (*a*)
7. (*a*)	8. (*d*)	9. (*a*)
10. (*c*)	11. (*d*)	12. (*c*)
13. (*c*)	14. (*b*)	15. (*b*)
16. (*b*)	17. (*a*)	18. (*b*)
19. (*a*)	20. (*a*)	

Section B

21. Let the two numbers be a and b where $a = 280$.

Let their LCM be x and HCF be y.

Then, $\text{LCM} = x = 14y$ … (1)

and $x + y = 600$ … (2)

$\therefore$ $14y + y = 600$ [From (1) and (2)]

$\Rightarrow$ $15y = 600$

$\Rightarrow$ $y = 40$

$\Rightarrow$ $\text{HCF} = 40$

$\therefore$ $\text{LCM} = x = 14y = 14 \times 40 = 560$

Now, product of two numbers = product of their LCM and HCF

$\therefore$ $280 \times b = 560 \times 40$

$\Rightarrow$ $b = \dfrac{560 \times 40}{280} = 80$

or

$45 = 27 \times 1 + 18$ … (1)

$27 = 18 \times 1 + 9$ … (2)

$$27\overline{)45}(1 \quad -27 \quad \overline{18} \qquad 18\overline{)27}(1 \quad -18 \quad \overline{9} \qquad 9\overline{)18}(2 \quad -18 \quad \overline{\times}$$

$18 = 9 \times 2 + 0$

∴ HCF of 45 and 27 is 9

$$9 = 27 - 18 \times 1 \quad \text{[From (2)]}$$
$$= 27 - (45 - 27 \times 1) \quad \text{[Using (1)]}$$
$$= 27 - 45 + 27$$
$$= 27 \times 2 + 45 \times (-1) = 27x + 45y$$

Hence, $x = 2$ and $y = -1$.

22. Distinct numbers in two dice are 1, 2, 3, 4, 5 and 6, and 1, 2, 3 respectively.

∴ Total number of possible distinct outcomes are 11, 12, 13, 21, 22, 23, 31, 32, 33, 41, 42, 43, 51, 52, 53, 61, 62, and 63.

Favourable distinct outcomes for sums 2, 3, 4, 5, 6, 7, 8 and 9 for two numbers of two dice are respectively 11; 12 and 21; 13, 31 and 22; 23, 32 and 41; 15, 51 and 42; 52, 61 and 43; 62 and 53; 63.

∴ $$P(2) = \frac{1}{18}; P(3) = \frac{2}{18} = \frac{1}{9}; P(4) = \frac{3}{18} = \frac{1}{6};$$
$$P(5) = \frac{3}{18} = \frac{1}{6}; P(6) = \frac{3}{18} = \frac{1}{6}; P(7) = \frac{3}{18} = \frac{1}{6},$$
$$P(8) = \frac{2}{18} = \frac{1}{9} \text{ and } P(9) = \frac{1}{18}.$$

or

$P(\text{toffees}) = \frac{3}{8}$.

Let the number of toffees = x.

Number of eclairs = $x + 6$, total outcomes = $2x + 6$.

So, $$\frac{3}{8} = \frac{x}{2x+6}$$
⇒ $$8x = 6x + 18$$
⇒ $$2x = 18$$
∴ $$x = 9 \text{ (Toffees)}$$
$$\text{Eclairs} = x + 6 = 9 + 6 = 15.$$

23. $4(\sin^4 30° + \cos^4 60°) - \frac{2}{3}(\sin^2 60° - \cos^2 45°) + \frac{1}{2}\tan^2 60°$

$$= 4\left[\left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^4\right] - \frac{2}{3}\left[\left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{\sqrt{2}}\right)^2\right] + \frac{1}{2}(\sqrt{3})^2$$
$$= 4\left(\frac{1}{16} + \frac{1}{16}\right) - \frac{2}{3}\left(\frac{3}{4} - \frac{1}{2}\right) + \frac{1}{2}(3)$$
$$= 4\left(\frac{2}{16}\right) - \frac{2}{3}\left(\frac{3-2}{4}\right) + \frac{3}{2}$$
$$= \frac{1}{2} - \frac{2}{3}\left(\frac{1}{4}\right) + \frac{3}{2}$$
$$= \frac{1}{2} - \frac{1}{6} + \frac{3}{2}$$
$$= \frac{3-1+9}{6}$$

$$= \frac{12-1}{6}$$

$$= \frac{11}{6}$$

24. Let the point on the y-axis be P(0, y).

Then, $\quad AP = \sqrt{(0-5)^2+(y-2)^2}$

and $\quad BP = \sqrt{(0-8)^2+(y-8)^2}$

According to the question,

$$4\sqrt{(0-5)^2+(y-2)^2} = 2\sqrt{(0-8)^2+(y-8)^2}$$

$$\Rightarrow \quad 4\sqrt{25+y^2-4y+4} = 2\sqrt{64+y^2-16y+64}$$

$$\Rightarrow \quad 2\sqrt{y^2-4y+29} = \sqrt{y^2-16y+128}$$

Squaring both the sides, we get

$$4(y^2 - 4y + 29) = y^2 - 16y + 128$$

$$\Rightarrow \quad 4y^2 - 16y + 116 = y^2 -- 16y + 128$$

$$\Rightarrow \quad 3y^2 = 12$$

$$\Rightarrow \quad y^2 = 4$$

$$\Rightarrow \quad y = \pm 2$$

Hence, the coordinates of the required point on y-axis is (0, 2) or (0, –2).

25. A (4, 7)

B (p, 3)

C (7, 3)

$$AB = \sqrt{(p-4)^2+16}$$

$$BC = \sqrt{(p-7)^2+(0)^2}$$

$$AC = \sqrt{(3)^2+(4)^2}$$

Since ABC is a right-angled triangle

$$\therefore \quad AB^2 + BC^2 = AC^2$$

$$(p-4)^2 + 16 + (p-7)^2 = 25$$

$$p^2 + 16 - 8p + 16 + p^2 + 49 - 14p = 25$$

$$2p^2 - 22p + 81 = 25$$

$$2p^2 - 22p + 56 = 0$$

$$p^2 - 11p + 28 = 0$$

$$p^2 - 4p - 7p + 28 = 0$$

$$p(p-4) - 7(p-4) = 0$$

$$(p-4)(p-7) = 0$$

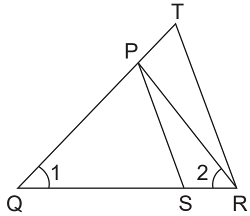
$$p = 4, 7$$

p cannot be 7 hence we will reject it as it does not satisfy the condition.

$$\therefore \quad p = 4$$

Section C

26.



$$\frac{QR}{QS} = \frac{QT}{PR}$$ [Given]

$\Rightarrow$ $$\frac{QS}{QR} = \frac{PR}{QT}$$ [Taking reciprocals]

$\Rightarrow$ $$\frac{PQ}{QT} = \frac{QS}{QR}$$

[$\therefore$ PR = PQ, sides opposite equal $\angle$s of ΔPQR].

In ΔPQS and ΔTQR, we have

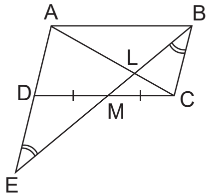
$$\frac{PQ}{QT} = \frac{QS}{QR}$$

and $\angle PQS = \angle TQR = \angle\theta$

$\therefore$ $\Delta PQS \sim \Delta TQR$

or

In ΔBMC and ΔEMD,



(i) CM = DM [$\therefore$ M is the mid-point of DC]

(ii) $\angle BMC = \angle EMD$ [Vertically opposite angles]

(iii) $\angle CBM = \angle DEM$ [Alt. $\angle$s, BC $\parallel$ ADE]

$\therefore$ $\Delta BMC \cong \Delta EMD$ [By AAS criterion of congruence]

$\therefore$ BC = ED [By CPCT] … (1)

Also, BC = AD [Opp. sides of a $\parallel$gm] … (2)

$\Rightarrow$ 2BC = ED + AD [Adding (1) and (2)]

$\Rightarrow$ 2BC = AE … (3)

Now, in ΔAEL and ΔCBL, we have

$\angle ALE = \angle CLB$ [Vertically opposite angles]

$\angle EAL = \angle BCL$ [Alt $\angle$s, BC $\parallel$ ADE]

$\therefore$ $\Delta AEL \sim \Delta CBL$ [By AA similarity]

$$\Rightarrow \quad \frac{EL}{BL} = \frac{AE}{CB}$$

[Corresponding sides of similar triangles are proportional]

$$\Rightarrow \quad \frac{EL}{BL} = \frac{2BC}{BC} \qquad \text{[Using (3)]}$$

$$\Rightarrow \quad EL = 2BL$$

27. Let the two-digit number be $10x + y$.

According to the given conditions

$$4(x + y) = 10x + y \qquad ... (1)$$

and $$10x + y = 3xy \qquad ... (2)$$

Simplifying eq. (1), we get

$$4x + 4y = 10x + y$$

$$3y = 6x$$

$$y = 2x \qquad ... (3)$$

Putting the value of y from eq. (3) in eq. (2), we get

$$\Rightarrow \quad 10x + 2x = 3x \times 2x$$

$$\Rightarrow \quad 6x^2 - 12x = 0$$

$$\Rightarrow \quad 6x(x - 2) = 0$$

$$\Rightarrow \quad x = 0 \text{ or } x = 2$$

Since $x = 0$ does not satisfy the given condition, hence we will reject it.

$$\Rightarrow \quad x = 2$$

$$\therefore \quad y = 2x = 4$$

Hence, the number is

$$10x + y = 10(2) + 4 = 24$$

28. α, β are the zeroes of the polynomial $x^2 + 4x + 3$

$$\therefore \quad \alpha + \beta = -4 \text{ and } \alpha\beta = 3 \qquad ... (1)$$

Zeroes of required polynomial are $1 + \frac{\beta}{\alpha}$ and $1 + \frac{\alpha}{\beta}$.

$$\text{Sum of zeroes} = 1 + \frac{\beta}{\alpha} + 1 + \frac{\alpha}{\beta}$$

$$= \frac{\alpha\beta + \beta^2 + \alpha\beta + \beta^2}{\alpha\beta}$$

$$= \frac{\alpha^2 + 2\alpha\beta + \beta^2}{\alpha\beta}$$

$$= \frac{(\alpha + \beta)^2}{\alpha\beta}$$

$$= \frac{(-4)^2}{3} = \frac{16}{3} \qquad \text{[Using (1)]}$$

$$\text{Product of zeroes} = \left(1+\frac{\beta}{\alpha}\right)\left(1+\frac{\alpha}{\beta}\right)$$

$$= 1 + \frac{\alpha}{\beta} + \frac{\beta}{\alpha} + \frac{\alpha\beta}{\alpha\beta}$$

$$= 1 + \frac{\alpha}{\beta} + \frac{\beta}{\alpha} + 1$$

$$= \frac{\alpha\beta + \alpha^2 + \beta^2 + \alpha\beta}{\alpha\beta}$$

$$= \frac{(\alpha+\beta)^2}{\alpha\beta}$$

$$= \frac{(-4)^2}{3} = \frac{16}{3}$$

So, the required polynomial is $x^2 - \frac{16}{3}x + \frac{16}{3}$ or $k\left(x^2 - \frac{16x}{3} + \frac{16}{3}\right)$ where k is a non-zero constant. If $k = 3$, then the polynomial is $3x^2 - 16x + 16$.

29. $x = a \sec\theta + b \tan\theta$ [Given]

$\Rightarrow$ $x^2 = a^2 \sec^2\theta + b^2 \tan^2\theta + 2ab \sec\theta \tan\theta$

[Squaring the given equation] … (1)

$y = a \tan\theta + b \sec\theta$ [Given]

$\Rightarrow$ $y^2 = a^2 \tan^2\theta + b^2 \sec^2\theta + 2ab \tan\theta \sec\theta$

[Squaring the given equation] … (2)

Subtracting (2) from (1) we get

$\Rightarrow$ $x^2 - y^2 = a^2(\sec^2\theta - \tan^2\theta) + b^2(\tan^2\theta - \sec^2\theta)$

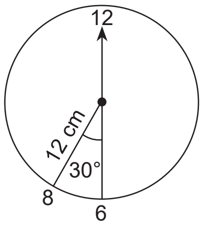
$\Rightarrow$ $x^2 - y^2 = a^2(1) + b^2(-1)$ [$\because 1 + \tan^2\theta = \sec^2\theta$]

$\Rightarrow$ $x^2 - y^2 = a^2 - b^2$

Hence, proved.

30. Minute hand (radius) = 12 cm

area covered from 8:00 am to 8:35 am



$$= \frac{1}{2}\pi r^2 + \frac{\theta}{360}\pi r^2$$

1 min = 6°

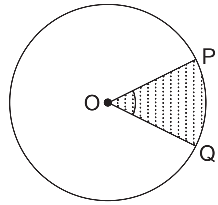
$\therefore$ 5 min = 30°

$$\text{Area} = \frac{1}{2}\pi r^2 + \frac{\theta}{360}\pi r^2$$

$$= \pi r^2\left(\frac{1}{2} + \frac{30}{360}\right)$$

$$= \frac{22}{7} \times 12 \times 12 \times \frac{7}{12} = 264 \text{ cm}^2$$

or



$$\text{Area of the circle} = \pi r^2$$

$$\text{Area of the sector} = \frac{\theta}{360} \times \pi r^2$$

According to given condition,

$$\text{Area of the sector} = \frac{3}{20} \text{ of } \pi r^2$$

$$\therefore \quad \frac{\theta}{360} \times \pi r^2 = \frac{3}{20} \times \pi r^2$$

$$\theta = \frac{3 \times 360}{20}$$

$$\theta = 54°.$$

31. Let us assume on the contrary that $3+5\sqrt{2}$ is rational.

Then, there exist coprime a and b ($b \neq 0$) such that

$$3 + 5\sqrt{2} = \frac{a}{b}$$

$$\Rightarrow \quad 5\sqrt{2} = \frac{a}{b} - 3 = \frac{a-3b}{b}$$

$$\Rightarrow \quad \sqrt{2} = \frac{a-3b}{5b}$$

$\Rightarrow$ $\sqrt{2}$ is rational [$\because$ 3, 5, a and b are integers $\therefore$ $\frac{a-3b}{5b}$ is a rational number]

This contradicts the fact that $\sqrt{2}$ is irrational.

The contradiction has arisen because of our incorrect assumption that $3 + 5\sqrt{2}$ is rational.

Hence, $3 + 5\sqrt{2}$ is irrational.

Section D

32. $$x + y = 7$$

$$\Rightarrow \quad y = 7 - x$$

Table for $x + y = 7$

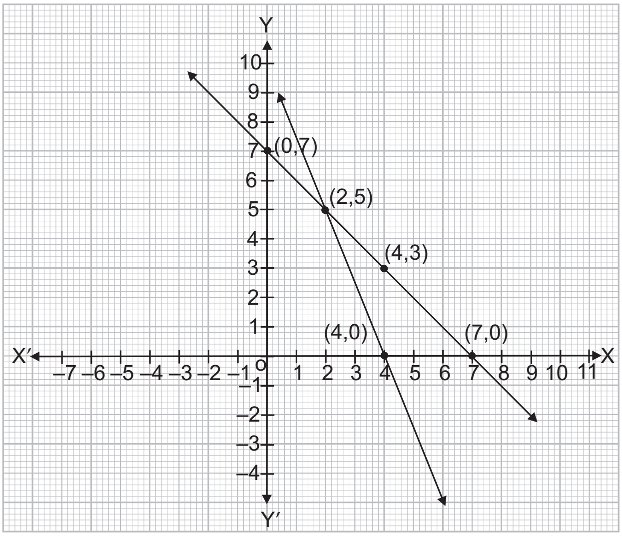
x	1	4
y	6	3

$$5x + 2y = 20$$

$$\Rightarrow \quad y = \frac{20-5x}{2}$$

Table for $5x + 2y = 20$

x	2	4
y	5	0



The two graph lines intersect at (2, 5).

Hence, $x = 2$ and $y = 5$.

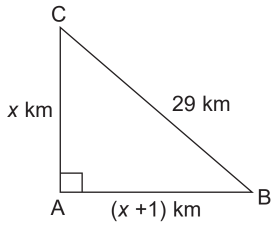
Graph of $x + y = 7$ intersects the x-axis at (7, 0) and y-axis at (0,7).

Graph of $5x + 2y = 20$ intersects the x-axis at (4, 0) and y-axis at (0, 10).

or

Let A be the point from where the two ships leave simultaneously in directions at right angles to each other.

Let the ship moving at the speed of x km/h reach point C. Then AC = x km.



Let the ship moving at the speed of $(x + 1)$ km/h reach the point B.

Then, $\quad AB = (x + 1)$ km

and $\quad BC = 29$ km

In right ΔCAB, we have

$$AC^2 + AB^2 = BC^2 \qquad \text{[By Pythagoras' Theorem]}$$

$$\Rightarrow \quad x^2 + (x + 1)^2 = 29^2$$

$$\Rightarrow \quad x^2 + x^2 + 2x + 1 = 841$$

$$\Rightarrow \quad 2x^2 + 2x - 840 = 0$$

$$\Rightarrow \quad x^2 + x - 420 = 0$$

$$\Rightarrow \quad x - 20x + 21x - 420 = 0$$

$\Rightarrow \quad x(x-20)+21(x-20)=0$

$\Rightarrow \quad (x-20)(x+21)=0$

$\Rightarrow \quad$ Either $(x-20)=0$ or $(x+21)=0$

$\Rightarrow \quad x=20$ or $x=-21$ (rejected)

The speeds of the ships are

$$x \text{ km/h} = 20 \text{ km/h}$$

and $(x+1)$ km/h $= (20+1)$ km/h $= 21$ km/h

Hence, the speeds of the ships are 20 km/h and 21 km/h.

33. Since the tangent at any point of a circle is perpendicular to the radius through the point of contact

$\therefore \quad OD \perp BC$ and $OE \perp AC$

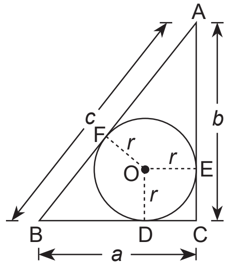
$\Rightarrow \quad \angle ODC = 90°$ and $\angle OEC = 90°$

Also $\quad \angle ECD = 90°$ [Given]

$\therefore$ In quadrilateral OECD, each angle is a right angle and adjacent sides OD and OE are equal (OD and OE are radii of the same circle).

So, quadrilateral OECD is a square.

Thus $\quad CD = CE = OE$ or $OD = r$ ...(1)



Since, the lengths of tangents drawn from an external point to a circle are equal.

$\therefore \quad AE = AF$ [Tangents from A] ...(2)

$BD = BF$ [Tangents from B] ...(3)

$AE = AC - CE = b - r$ [Using (1)]

$AF = b - r$ [Using (2)]

$BF = c - AF = c - b + r$

$BD = c - b + r$ [Using (3)] ...(4)

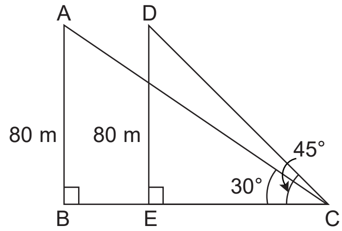
Now, $\quad BC = CD + BD$

$\Rightarrow \quad a = r + c - b + r$ [Using (1) and (4)]

$\Rightarrow \quad a + b = c + 2r$

Hence, $2r + c = a + b$.

34. Let A and D be the positions of the bird from the ground. The angle of elevation of A and D at C are 30° and 45° respectively.



In ΔABC, we have

$$\tan 30^\circ = \frac{AB}{BC}$$

$$\frac{1}{\sqrt{3}} = \frac{80}{BC}$$

$$BC = 80\sqrt{3} \text{ m}$$

In ΔDEC, we have

$$\tan 45^\circ = \frac{DE}{EC}$$

$$1 = \frac{80}{EC}$$

$$EC = 80 \text{ m}$$

Distance travelled by bird = BC – EC

$$= 80\sqrt{3} - 80$$

$$= 80\left(\sqrt{3}-1\right)$$

$$= 80 \times 0.732 = 58.56 \text{ m}$$

We know

$$\text{Speed} = \frac{\text{distance}}{\text{time}}$$

$$= \frac{58.56}{2}$$

$$= 29.28 \text{ m/s}$$

35.

Class interval	*Mid-value* x_i	*Frequency* f_i	*Cumulative frequency cf*	$f_i x_i$
0–10	5	8	8	40
10–20	15	7	15	105
20–30	25	15	30	375
30–40	35	20	50	700
40–50	45	12	62	540
50–60	55	8	70	440
60–70	65	10	80	650
		$n = \Sigma f_i = 80$		$\Sigma f_i x_i = 2850$

$$\text{Mean} = \bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{2850}{80} = 35.625$$

Median: Here, $n = \Sigma f_i = 80$. So, $\frac{n}{2} = \frac{80}{2} = 40.$

Cumulative frequency just greater than 40 is 50 and the corresponding class is 30–40.

So, the median class is 30–40.

$\therefore$ $l = 30$, $cf = 30$, $f = 20$ and $h = 10$

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

$$= 30 + \left(\frac{40-30}{20}\right) \times 10$$

$$= 30 + \left(\frac{10}{20}\right) \times 10$$

$$= 30 + 5$$

$$= 35$$

Hence, Mean = 35.625, and Median = 35.

or

From the given data, we construct the cumulative frequency table as follows:

Marks	*Frequency* (f_i)	*Cumulative frequency* (cf)
0–10	10	10
10–20	f_1	$10 + f_1$
20–30	25	$35 + f_1$
30–40	30	$65 + f_1$
40–50	f_2	$65 + f_1 + f_2$
50–60	10	$75 + f_1 + f_2$
	$n = \Sigma f_i = 75 + f_1 + f_2$	

Since total frequencies are given to be 100.

$\therefore$ $75 + f_1 + f_2 = 100$

$\Rightarrow$ $f_1 + f_2 = 100 - 75 = 25$ …(1)

Since the median is given to be 32.

$\therefore$ Median class is 30–40 in which 32 lies.

Hence, l = the lower limit of the median class = 30

f_m = the frequency of the median class = 30

cf_{-1} = cumulative frequency of the class preceding the median class = $35 + f_1$

$$\frac{n}{2} = \frac{75 + f_1 + f_2}{2} = \frac{75 + 25}{2} \quad \text{[From (1)]}$$

$$= 50$$

$$h = \text{class size} = 10$$

$$\therefore \quad \text{Median} = l + \frac{\frac{n}{2} - cf_{-1}}{f_m} \times h$$

$$= 30 + \frac{50 - 35 - f_1}{30} \times 10$$

$$\Rightarrow \quad 30 + \frac{15 - f_1}{3} = 32 \quad \text{(Given)}$$

$$\Rightarrow \quad \frac{15 - f_1}{3} = 32 - 30 = 2$$

$$\Rightarrow \quad 15 - f_1 = 6$$

$$\Rightarrow \quad f_1 = 9$$

$\therefore$ From (1), $f_2 = 25 - 9 = 16$

Hence, the required values of f_1 and f_2 are respectively 9 and 16.

Section E

36. (*a*) 1000
    (*b*) 1000
    (*c*) (*i*) 4000

    or

    (*ii*) 21000

37. (*a*) 63.64 *ft*
    (*b*) 95.45 *ft*
    (*c*) (*i*) 800 cm

    or

    (*ii*) 14 m

38. (*a*) 21
    (*b*) 134 cm^3 (approx.)
    (*c*) (*i*) 2110 cm^2

    or

    (*ii*) 24186 cm^3 (approx.)