Sample Question Paper

Basic (Code 241)

ANSWERS

Section A

1. (*c*)	2. (*a*)	3. (*c*)
4. (*b*)	5. (*b*)	6. (*b*)
7. (*c*)	8. (*d*)	9. (*b*)
10. (*b*)	11. (*d*)	12. (*d*)
13. (*a*)	14. (*c*)	15. (*a*)
16. (*b*)	17. (*a*)	18. (*c*)
19. (*d*)	20. (*c*)	

Section B

21. Since $PA = PB$

$\therefore$ $PA^2 = PB^2$

$\Rightarrow$ $(k - 1 - 3)^2 + (2 - k)^2 = (k - 1 - k)^2 + (2 - 5)^2$

$\Rightarrow$ $(k - 4)^2 + (2 - k)^2 = 1 + 9$

$\Rightarrow$ $k^2 - 8k + 16 + 4 + k^2 - 4k - 10 = 0$

$\Rightarrow$ $2k^2 - 12k + 10 = 0$

$\Rightarrow$ $k^2 - 6k + 5 = 0$

$\Rightarrow$ $k^2 - 5k - k + 5 = 0$

$\Rightarrow$ $k(k - 5) - 1(k - 5) = 0$

$\Rightarrow$ $(k - 5)(k - 1) = 0$

$\therefore$ Either $k - 5 = 0 \Rightarrow k = 5$

or $k - 1 = 0 \Rightarrow k = 1$

$\therefore$ The required values of k are **5** and **1**.

or

In right triangle ABC, we have

$AB^2 + BC^2 = AC^2$ [By Pythagoras' Theorem]

$\Rightarrow$ $(2 - a)^2 + (9 - 5)^2 + (a - 5)^2 + (5 - 5)^2 = (2 - 5)^2 + (9 - 5)^2$

$\Rightarrow$ $4 - 4a + a^2 + 16 + a^2 - 10a + 25 = 9 + 16$

$$\Rightarrow \quad 2a^2 - 14a + 20 = 0$$

$$\Rightarrow \quad a^2 - 7a + 10 = 0$$

$$\Rightarrow \quad a^2 - 5a - 2a + 10 = 0$$

$$\Rightarrow \quad a(a - 5) - 2(a - 5) = 0$$

$$\Rightarrow \quad (a - 5)(a - 2) = 0$$

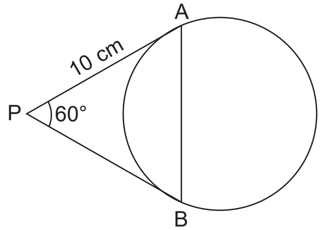
Either $(a - 5) = 0$ or $(a - 2) = 0$

$\Rightarrow \quad a = 5$ or $a = 2$

$a \neq 5$ as B and C cannot coincide.

$\therefore \quad a = 2$

22. Given that PA and PB are tangents to a circle from an outside point P such that PA = 10 cm and ∠APB = 60°. To find the length of the chord AB.



We know that $PB = PA = 10$ cm

$\therefore$ In ΔPAB,

$$\angle PAB = \angle PBA = \frac{180^\circ - 60^\circ}{2} = 60^\circ$$

$\therefore$ ΔPAB is an equilateral triangle.

$\therefore$ AB = PB = PA = **10 cm** which is the required length of AB.

23. Let the first term and common difference of the AP be 'a' and 'd' respectively.

$\therefore \quad a_9 = a + 8d \Rightarrow a + 8d = -2.6 \quad \ldots(1)$

$a_{23} = a + 22d \Rightarrow a + 22d = -5.4 \quad \ldots(2)$

Subtracting (1) from (2), we have

$$a + 22d = -5.4$$

$$a + 8d = -2.6$$

$$(-) \quad (-) \qquad (+)$$

$$\Rightarrow \quad (22 - 8)\, d = -5.4 + 2.6$$

$$\Rightarrow \quad 14d = -2.8$$

$$\Rightarrow \quad d = -\frac{2.8}{14} = -0.2$$

Substituting $d = -0.2$ in (1),

$$a + 8(-0.2) = -2.6$$

$$a + (-1.6) = -2.6$$

$$\Rightarrow \quad a = -2.6 + (1.6) = -1$$

Now, $a_2 = a + d = -1 + (-0.2) = -1.2$

Thus 2nd term is **–1.2**.

Again, $a_k = a + (k - 1)\,d$

$= -1 + (k - 1) \times (-0.2)$

$= -1 + (-0.2k) + 0.2 = -0.8 - 0.2k$

Thus, kth term is **$(-0.8 - 0.2k)$**.

or

Here, first term = a = 6

Let common difference = d

$\because$ S_n = Sum of first n terms = $\frac{n}{2}\,[2a + (n - 1)d]$

$\therefore$ S_3 = Sum of first three terms

$= \frac{3}{2}\,[(2 \times 6) + (3 - 1)d]$

$= \frac{3}{2}\,[12 + 2d] = 18 + 3d$ ...(1)

S_6 = Sum of first six terms

$= \frac{6}{2}\,[(2 \times 6) + (6 - 1)d]$

$= 3[12 + 5d] = 36 + 15d$ ...(2)

Now, $S_3 = \frac{1}{2}\,(S_6 - S_3) \Rightarrow 2S_3 = S_6 - S_3$

$\Rightarrow$ $2S_3 + S_3 = S_6$ or $3S_3 = S_6$ ...(3)

From (1), (2) and (3), we get

$3[18 + 3d] = 36 + 15d$

$\Rightarrow$ $54 + 9d = 36 + 15d$ or $9d - 15d = 36 - 54$

$\Rightarrow$ $-6d = -18 \therefore d = \frac{-18}{-6} = 3$

24. $\tan\,(A + B) = \sqrt{3}$

$\Rightarrow$ $\tan\,(A + B) = \tan 60°$ $[\because \tan 60° = \sqrt{3}\,]$

$\Rightarrow$ $A + B = 60°$ ...(1)

Also $\tan\,(A - B) = \frac{1}{\sqrt{3}}$

$\Rightarrow$ $\tan\,(A - B) = \tan 30°$ $[\because \tan 30° = \frac{1}{\sqrt{3}}\,]$

$\Rightarrow$ $A - B = 30°$ ...(2)

Adding (1) and (2), we get

$2A = 90°$

$\Rightarrow$ $A = 45°$

Substituting A = 45° in (1), we get

$45° + B = 60°$

$\therefore$ $B = 60° - 45° = 15°$

25. Class size = Difference between any two consecutive mid values = 25 – 15 = 10.

Mid values 15 corresponds to class (15 – 5) – (15 + 5), *i.e.* 10 – 20, and so on.

Thus, cumulative frequency table for the given data is:

Class Interval	*Frequency* (f_i)	*Cumulative Frequency* (cf)
10 – 20	4	4
20 – 30	28	32
30 – 40	15	47
40 – 50	20	67
50 – 60	17	84
60 – 70	16	100
Total	$n = \Sigma f_i = 100$	

Here, $n = \Sigma f_i = 100 \Rightarrow \frac{n}{2} = \frac{100}{2} = 50.$

The cumulative frequency just greater than 50 is 67 and the corresponding class is 40 – 50. So, the median class is 40 – 50.

$\therefore$ $l = 40, cf = 47, f = 20$ and $h = 10$

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

$$= 40 + \left(\frac{50 - 47}{20}\right) \times 10 = 40 + \frac{3}{20} \times 10$$

$$= 40 + 1.5$$

$$= 41.5$$

Hence, the median is 41.5.

Section C

26. Let us assume on the contrary that $\sqrt{3}$ is a rational number and its simplest form is $\frac{a}{b}$, where a and b are integers having no common factor other than 1 and $b \neq 0$.

$\therefore$ $\sqrt{3} = \frac{a}{b}$

$\Rightarrow$ $3 = \frac{a^2}{b^2}$ [Squaring both sides]

$\Rightarrow$ $3b^2 = a^2$ ...(1)

$\therefore$ a^2 is divisible by 3 [$\because$ $3b^2$ is divisible by 3]

$\Rightarrow$ a is divisible by 3 [$\because$ 3 is prime and divides $a^2 \Rightarrow$ 3 divides a]

Let $a = 3c$ for some integer c.

Substituting $a = 3c$ in (1), we get

$\Rightarrow \quad 3b^2 = (3c)^2 \qquad \Rightarrow 3b^2 = 9c^2 \qquad \Rightarrow b^2 = 3c^2$

$\Rightarrow \quad b^2$ is divisible by 3 [$\because$ $3c^2$ is divisible by 3]

$\Rightarrow \quad b$ is divisible by 3 [$\because$ 3 is prime and divides $b^2 \Rightarrow$ 3 divides b]

Since, a and b are both divisible by 3, $\therefore$ 3 is a common factor of a and b.

But this contradicts the fact a and b have no common factor other than 1.

This contradiction has arisen because of our incorrect assumption that $\sqrt{3}$ is rational.

Hence, $\sqrt{3}$ is an irrational number.

27. Coordinates of C which is the mid-point of A(0, 4) and B(6, 0) are $\left(\frac{0+6}{2}, \frac{4+0}{2}\right)$, i.e. **C(3, 2)**.

Let the coordinates of P be (x, y).

Also, coordinates of the origin O are (0, 0).

So, $\qquad 3 = \frac{x+0}{4} \qquad \Rightarrow x = 12$

and $\qquad 2 = \frac{y+0}{4} \qquad \Rightarrow y = 8$

So, coordinates of P are **(12, 8)**.

Now, $\qquad \text{BP} = \sqrt{(12-6)^2 + (8-0)^2}$

and $\qquad = \sqrt{36+64}$

and $\qquad = \sqrt{100}$ = **10 units**.

or

Let the given line divides the line segment joining the points (2, –2) and (3, 7) in the ratio $k : 1$.

Then, the coordinates of the point to divide the line segment are $\frac{3k+2}{k+1}$ and $\frac{7k-2}{k+1}$.

Since, this point lies on the given line, so

$$2\left(\frac{3k+2}{k+1}\right) + \frac{7k-2}{k+1} - 4 = 0$$

$$\Rightarrow \quad 6k + 4 + 7k - 2 - 4k - 4 = 0$$

$$\Rightarrow \quad 9k - 2 = 0$$

$$\Rightarrow \quad k = \frac{2}{9}$$

Thus, the given line divides the line segment joining the given point in the ratio **2 : 9**.

28.
$$\text{LHS} = \frac{(\sin\theta+\cos\theta)^2-1}{\tan\theta-\sin\theta\cos\theta}$$
$$= \frac{\sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta-1}{\frac{\sin\theta}{\cos\theta}-\sin\theta\cos\theta}$$
$$= \frac{1+2\sin\theta\cos\theta-1}{\sin\theta\left(\frac{1}{\cos\theta}-\cos\theta\right)} \qquad [\because \sin^2\theta+\cos^2\theta=1]$$
$$= \frac{2\sin\theta\cos\theta}{\sin\theta\left(\frac{1-\cos^2\theta}{\cos\theta}\right)}$$
$$= \left(\frac{2\cos\theta}{1-\cos^2\theta}\right)(\cos\theta)$$
$$= \frac{2\cos^2\theta}{\sin^2\theta}$$
$$[\because \sin^2\theta+\cos^2\theta=1 \Rightarrow 1-\cos^2\theta=\sin^2\theta]$$
$$= \mathbf{2\cot^2}\,\theta$$
$$= \text{RHS}$$

29. Here, we apply the step derivation method to calculate the mean.

Here, the assumed mean, $a = 70$ (in %) and the length of each class interval, $h = 10$.

Literacy rate (in %)	*Class marks x_i (in %)*	*Frequency f_i*	$u_i = \frac{x_i - 70}{10}$	$u_i f_i$
45–55	50	4	–2	–8
55–65	60	11	–1	–11
65–75	70	12	0	0
75–85	80	9	1	9
85–95	90	4	2	8
		$\Sigma f_i = 40$		$\Sigma u_i f_i = -2$

$\therefore$ Required mean $= \bar{x} = a + h\dfrac{\Sigma u_i f_i}{\Sigma f_i}$
$$= \left(70 - 10\times\frac{2}{40}\right)\%$$
$$= (70 - 0.5)\%$$
$$= \mathbf{69.5\%}.$$

30. (*a*) Since the perpendicular drawn from the centre of a circle to a chord bisects the chord.

$\therefore$ ON bisects AB $\Rightarrow BN = AN$ ...(1)

Now $PA \cdot PB = (PN - AN)(PN + BN)$

$= (PN - AN)(PN + AN)$ [Using (1)]

$= PN^2 - AN^2$

Hence, $PA \cdot PB = PN^2 - AN^2$

(*b*) In right triangle ONP, we have

$ON^2 + PN^2 = OP^2$ [By Pythagoras' Theorem] ...(2)

$\Rightarrow PN^2 = OP^2 - ON^2$

$\Rightarrow PN^2 - AN^2 = OP^2 - ON^2 - AN^2$

[Subtracting AN^2 from both sides]

$= OP^2 - (ON^2 + AN^2)$

$= OP^2 - OA^2$

[Using Pythagoras' Theorem in right ΔONA]

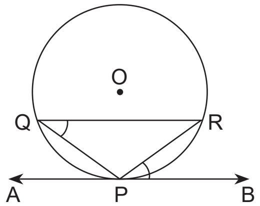
$= OP^2 - OT^2$ [$\because$ OA = OT, radii of a circle]

Hence, $PN^2 - AN^2 = OP^2 - OT^2$

or

Given that P is the mid-point of arc QR of a circle with centre at O and AB is a tangent to the circle at P.

To prove that QR $\parallel$ PB.



Since arc PQ = arc PR

$\therefore$ chord PQ = chord PR

$\therefore$ In ΔPQR, PQ = PR

$\therefore \angle PQR = \angle PRQ$

But $\angle PQR = \angle RPB$

$\therefore \angle RPB = \angle PRQ$

But these two angles are alternate angles between the line AB and chord QR. Hence, QR $\parallel$ PB.

31. Let the required two digit number be $10x + y$.

Then, $x \times y = 20$

$\Rightarrow \quad y = \frac{20}{x} \quad \ldots(1)$

Given, number – 9 = number with interchanged digits

$\Rightarrow \quad 10x + y - 9 = 10y + x$

$\Rightarrow \quad 10x - x + y - 10y - 9 = 0$

$\Rightarrow \quad 9x - 9y - 9 = 0$

$\Rightarrow \quad x - y - 1 = 0$

$\Rightarrow \quad x - \frac{20}{x} - 1 = 0 \quad$ [Using (1)]

$\Rightarrow \quad x^2 - x - 20 = 0$

$\Rightarrow \quad x^2 - 5x + 4x - 20 = 0$

$\Rightarrow \quad x(x - 5) + 4(x - 5) = 0$

$\Rightarrow \quad$ Either $(x - 5) = 0$ or $(x + 4) = 0$

$\Rightarrow \quad x = 5$ or $x = -4$ (rejected)

Substituting $x = 5$ in equation (1), we get

$y = \frac{20}{5} = 4$ we get

Required number $= 10x + y = 10(5) + 4 = 54$

Hence, the required number is **54**.

Section D

32. Let the usual speed of the bus be x km/h

Reduced speed of the bus = $(x - 5)$ km/h

Measurement	*Bus moving at the usual speed*	*Bus moving at the reduced speed*
Distance between A and B	550 km	550 km
Speed	x km/h	$(x - 5)$ km/h
Time	$\frac{550}{x}$ h	$\frac{550}{(x-5)}$ h

Given, $\begin{bmatrix}\text{Time taken by the} \\ \text{bus moving at the} \\ \text{reduced speed}\end{bmatrix} - \begin{bmatrix}\text{Time taken by the} \\ \text{bus moving at the} \\ \text{usual speed}\end{bmatrix} = 1$ hour

$\Rightarrow \quad \frac{550}{x-5} - \frac{550}{x} = 1$

$\Rightarrow \quad \frac{550(x-x+5)}{x(x-5)} = 1$

$$\Rightarrow \quad 2750 = x^2 - 5x$$

$$\Rightarrow \quad x^2 - 5x - 2750 = 0$$

$$\Rightarrow \quad x^2 - 55x + 50x - 2750 = 0$$

$$\Rightarrow \quad x(x - 55) + 50(x - 55) = 0$$

$$\Rightarrow \quad (x - 55)(x + 50) = 0$$

$$\Rightarrow \quad \text{Either } (x - 55) = 0 \text{ or } (x + 50) = 0$$

$$\Rightarrow \quad x = 55 \text{ or } x = -50 \text{ (rejected)}$$

Time taken by the bus to cover the distance between A and B when its raining

$$= \frac{550}{x-5} \text{ hours} = \frac{550}{55-5} \text{ hours}$$

$$= \frac{550}{50} \text{ hours} = 11 \text{ hours}$$

Hence, the time taken is **11 hours**.

or

Let the number of books bought = x

Amount	₹ 80	₹ 80
No. of books	x	$x + 4$
Cost of each book	₹ $\frac{80}{x}$	₹ $\frac{80}{x+4}$

If 4 more books are bought for ₹ 80, the cost of each book reduces by ₹ 1.

$$\therefore \quad \frac{80}{x} - \frac{80}{x+4} = 1$$

$$\Rightarrow \quad 80\left[\frac{x+4-x}{x(x+4)}\right] = 1$$

$$\Rightarrow \quad 80 \times 4 = x^2 + 4x$$

$$\Rightarrow \quad x^2 + 4x - 320 = 0$$

$$\Rightarrow \quad x^2 + 20x - 16x - 320 = 0$$

$$\Rightarrow \quad x(x + 20) - 16(x + 20) = 0$$

$$\Rightarrow \quad (x + 20)(x - 16) = 0$$

$$\Rightarrow \quad \text{Either } (x + 20) = 0 \quad \text{or} \quad (x - 16) = 0$$

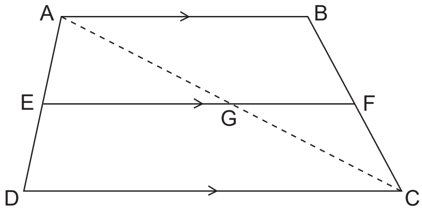
$$\Rightarrow \quad x = -20 \text{ (rejected)} \quad \text{or} \quad x = 16$$

Hence, the number of books bought = **16**.

The initial price of the book = ₹ $\frac{80}{16}$

= ₹ 5

33. Join AC and let it intersect EF at G.



In ΔADC, EG || DC [$\because$ EF || DC]

$$\therefore \quad \frac{AE}{ED} = \frac{AG}{GC} \quad \text{[By BPT] ...(1)}$$

In ΔCBA, EG || BA [$\because$ EF || AB]

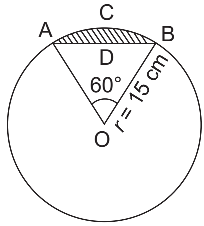
$$\therefore \quad \frac{CF}{FB} = \frac{CG}{GA}$$

$$\Rightarrow \quad \frac{BF}{FC} = \frac{AG}{GC} \quad \text{[Taking reciprocals] ...(2)}$$

From (1) and (2), we have

$$\frac{AE}{ED} = \frac{BF}{FC} \quad \left[\text{Each is equal to } \frac{AG}{GC}\right]$$

34. Let r cm be the radius of the circle with centre at O, so that $r = 15$. The chord AB subtends an angle, $\theta = 60°$ at the centre O.



$\therefore$ Area of the sector OACB of the circle

$$= \pi r^2 \times \frac{\theta}{360}$$

$$= 3.14 \times 15^2 \times \frac{60°}{360°} \text{ cm}^2$$

$$= 3.14 \times 225 \times \frac{1}{6} \text{ cm}^2$$

$$= 117.75 \text{ cm}^2$$

Now, area of the equilateral ΔAOB

$$= \frac{\sqrt{3}}{4} \times 15 \times 15 \text{ cm}^2$$

$$= \frac{225 \times 1.73}{4} \text{ cm}^2$$

$$= 97.32 \text{ cm}^2$$

∴ Required area of the minor segment of the circle, ACBD (the shaded region)

$= (117.75 - 97.32) \text{ cm}^2$

$= 20.43 \text{ cm}^2$ ...(1)

Now, area of the whole circle

$= \pi r^2$

$= 3.14 \times 225 \text{ cm}^2$

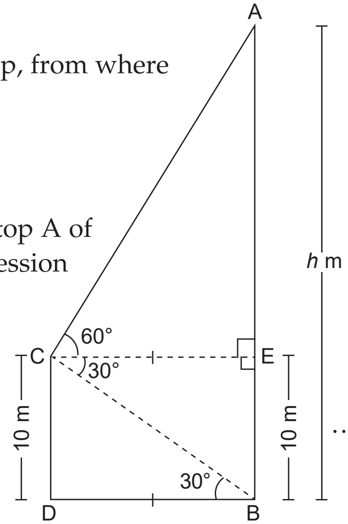
$= 706.50 \text{ cm}^2$ ...(2)

∴ Required area of the major circle = Area of the whole circle – Area of the minor circle

$= (706.50 - 20.43) \text{ cm}^2$ [From (1) and (2)]

$= \mathbf{686.07\ cm^2}$.

35. Let AB (= h metres) be the hill.



Let C be the point on the deck CD of the ship, from where the man is observing the hill.

Then, CD, the height of the deck = 10 m

Draw CE ⊥ AB. Then, ∠AEC = ∠BEC = 90°

It is given that the angle of elevation of the top A of the hill AB at C is 60° and the angle of depression of the base B of the hill AB at C is 30°,

i.e. ∠ACE = 60° and

∠BCE = 30°

EB = CD = 10 m ... (1)

In right ΔAEC, we have

$$\tan 60° = \frac{AE}{CE}$$

$$\Rightarrow \sqrt{3} = \frac{AB - EB}{CE}$$

$$\Rightarrow CE = \frac{h - 10}{\sqrt{3}} \text{ m}$$ [Using (1)] ... (2)

In right ΔBEC, we have

$$\tan 30° = \frac{BE}{CE}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{10 \text{ m}}{CE}$$

$$\Rightarrow CE = 10\sqrt{3} \text{ m}$$ ... (3)

$$\Rightarrow \frac{h - 10}{\sqrt{3}} = 10\sqrt{3}$$ [From (2) and (3)]

$$\Rightarrow h - 10 = 10 \times 3$$

$$\Rightarrow h = 40$$

Distance of the hill from the ship

$$\Rightarrow \quad = DB = CE = 10\sqrt{3} \text{ m} \qquad \text{[Using (3)]}$$

$$= 10\,(1.\,73) \text{ m} = 17.3 \text{ m}$$

Hence, the distance of the hill from the ship is 17.3 m
and the height of the hill is 40 m.

or

Let AB be the height of the lighthouse. The angle of depression of a ship as observed from the top of the lighthouse at the point D and C are respectively 30° and 60°. Then, CD is the distance travelled by the ship during the period of observation.

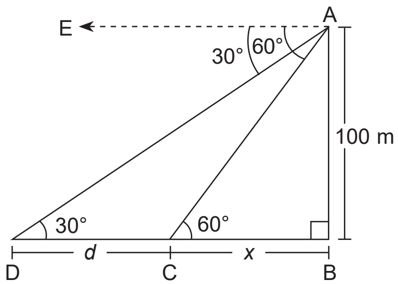
Then, $\angle EAD = 30^\circ$ and $\angle EAC = 60^\circ$.

Now, $AE \parallel BC$.

Thus, $\angle EAD = \angle ADB$

$\Rightarrow$ $\angle ADB = 30^\circ$

and $\angle EAC = \angle ACB$



$\Rightarrow$ $\angle ACB = 60^\circ$

Then, $AB = 100$ m, $CD = d$, $CB = x$.

$\angle ABC = \angle ABD = 90^\circ$

$DB = d + x$

In right ΔABC, we have

$$\tan 60^\circ = \frac{AB}{CB}$$

$$\Rightarrow \quad \sqrt{3} = \frac{100 \text{ m}}{x}$$

$$\Rightarrow \quad x = \frac{100 \text{ m}}{\sqrt{3}} \qquad \ldots (1)$$

In right ΔABD, we have

$$\tan 30^\circ = \frac{AB}{DB}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{100 \text{ m}}{d - x}$$

$$\Rightarrow \quad d + x = 100\sqrt{3}$$

$$\Rightarrow \quad d = 100\sqrt{3} - x \qquad \text{[Using equation (1)]}$$

$$= 100\sqrt{3} - \frac{100}{\sqrt{3}}$$

$$= \frac{100 \times 2}{\sqrt{3}} = \frac{200}{\sqrt{3}}$$

$$= \frac{200}{1.732}$$

$$= 115.47 \text{ (approx.)}$$

Hence, the distance travelled by the ship is 115.47 m (approx.).

Section E

36. (*a*) Parabola

(*b*) Linear

(*c*) (*i*) 1

or

(*ii*) $2ft$

37. (*a*) Percentage of people with white hair

$$= (100 - 65 - 25) = 10\%$$

$$P(\text{white}) = \frac{10}{100} = \frac{1}{10}$$

(*b*) Percentage of people with brown or black hair

$$= (65 + 25) = 90\%$$

$$P(\text{brown or black}) = \frac{90}{100} = \frac{9}{10}$$

(*c*) (*i*) Percentage of people with white or black hair

$$= (10 + 65) = 75\%$$

$$P(\text{white or black}) = \frac{75}{100} = \frac{3}{4}$$

or

(*ii*) Percentage of people with neither brown nor white hair = 65%

$$P(\text{neither brown nor white}) = \frac{65}{100} = \frac{13}{20}$$

38. (*a*) 25 m

(*b*) 1571.43 m^2

(*c*) (*i*) 4400 m^2

or

(*ii*) ₹ 83600 (approx.)