TEACHER'S HANDBOOK

STELLAR LEARNING

Mathematics

10

On Board! BOOKS

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Real Numbers

Checkpoint (Page 6)

1. Which of the following is a rational number?

   (*a*) $\sqrt{2}$ (*b*) 3π

   (*c*) $0.\overline{1256}$ (*d*) 0.12345678001…

Sol. (*c*) $0.\overline{1256}$

We know that any number which can be expressed either as a non-terminating recurring decimal or a terminating decimal is a rational number, otherwise it is an irrational number. Now, the number in (*c*) only can be written as a non-terminating recurring decimal and so only this number is rational.

2. The value of $0.\overline{35}+0.\overline{42}$ is

   (*a*) $0.\overline{45}$ (*b*) $0.\overline{56}$

   (*c*) $0.\overline{77}$ (*d*) 0.77

Sol. (*c*) $0.\overline{77}$

We have

$$0.\overline{35} + 0.\overline{42} = 0.353535\ldots + 0.424242\ldots = 0.777777\ldots = 0.\overline{77}$$

3. The number $1.\overline{36}$ in the form of $\frac{p}{q}$, where p and q are non-negative integers and $q \neq 0$, is

   (*a*) $\frac{13}{11}$ (*b*) $\frac{14}{11}$

   (*c*) $\frac{15}{11}$ (*d*) $\frac{11}{15}$

Sol. (*c*) $\frac{15}{11}$

Let $x = 1.\overline{36} = 1.363636\ldots$ …(1)

$\therefore$ $100x = 136.363636\ldots$ …(2)

$\therefore$ Subtracting (1) from (2), we get

$$99x = 135$$

$$\therefore \quad x = \frac{135}{99} = \frac{15}{11}$$

4. If $3^4 \times 9^2 = 81^x$, then x is

   (*a*) 2 (*b*) 3

   (*c*) 4 (*d*) 5

Sol. (*a*) 2

We have $3^4 \times 9^2 = 81^x$

$$3^4 \times (3^2)^2 = (3^4)^x$$

$$3^{4+4} = 3^{4x}$$

$$\Rightarrow \quad 3^8 = 3^{4x}$$

$$\Rightarrow \quad 8 = 4x$$

$$\Rightarrow \quad x = 2$$

5. The simplified form of $(256)^{-\left(4^{\frac{-3}{2}}\right)}$ is

   (*a*) 2 (*b*) $\frac{1}{2}$

   (*c*) 8 (*d*) 4

Sol. (*b*) $\frac{1}{2}$

We have $4^{\frac{-3}{2}} = 2^{-2\times\frac{3}{2}} = 2^{-3} = \frac{1}{8}$

$\therefore$ Given expression =

$$\left(4^4\right)^{-\frac{1}{8}} = 4^{-\frac{1}{8}\times 4} = 4^{\frac{-1}{2}} = \frac{1}{\sqrt{4}} = \frac{1}{2}$$

6. The value of $\sqrt{5+2\sqrt{6}}$ is

(*a*) $\sqrt{3}+\sqrt{2}$ (*b*) $\sqrt{3}-\sqrt{2}$

(*c*) $\sqrt{5}+\sqrt{6}$ (*d*) none of these

Sol. (*a*) $\sqrt{3}+\sqrt{2}$

We have

$$\sqrt{5+2\sqrt{6}} = \sqrt{(\sqrt{3})^2+(\sqrt{2})^2+2\sqrt{3}\times\sqrt{2}}$$
$$= \sqrt{(\sqrt{3}+\sqrt{2})^2}$$
$$= \sqrt{3}+\sqrt{2}$$

7. If $x+\frac{1}{x}=3$, calculate $x^2+\frac{1}{x^2}$, $x^3+\frac{1}{x^3}$ and $x^4+\frac{1}{x^4}$.

Sol. We have

$$x^2+\frac{1}{x^2} = \left(x+\frac{1}{x}\right)^2 - 2x\times\frac{1}{x}$$
$$= 3^2-2$$
$$= 7$$

$$\therefore \quad x^4+\frac{1}{x^4} = \left(x^2+\frac{1}{x}\right)^2 - 2x^2.\frac{1}{x^2}$$
$$= 7^2-2$$
$$= 47$$

and $$x^3+\frac{1}{x^3} = \left(x+\frac{1}{x}\right)^3 - 3x.\frac{1}{x}\left(x+\frac{1}{x}\right)$$
$$= 3^3-3\times 3$$
$$= 27-9$$
$$= 18$$

8. If $\frac{a}{b}+\frac{b}{a}=1$, find a^3+b^3.

Sol. We have $\frac{a}{b}+\frac{b}{a}=1$

$$\Rightarrow \quad \frac{a^2+b^2}{ab} = 1$$

$$\Rightarrow \quad a^2+b^2-ab = 0 \qquad \ldots(1)$$

$$\therefore \quad a^3+b^3 = (a+b)(a^2+b^2-ab)$$
$$= (a+b)\times 0 \qquad \text{[From (1)]}$$
$$= 0$$

$\therefore$ The value of a^3+b^3 is 0.

9. Rationalise and simplify: $\frac{2\sqrt{6}-\sqrt{5}}{3\sqrt{5}-2\sqrt{6}}$

Sol. We have

$$\frac{2\sqrt{6}-\sqrt{5}}{3\sqrt{5}-2\sqrt{6}} = \frac{(2\sqrt{6}-\sqrt{5})(3\sqrt{5}+2\sqrt{6})}{(3\sqrt{5}-2\sqrt{6})(3\sqrt{5}+2\sqrt{6})}$$
$$= \frac{6\sqrt{30}+4\times 6-3\times 5-2\sqrt{30}}{(3\sqrt{5})^2-(2\sqrt{6})^2}$$
$$= \frac{24-15+4\sqrt{30}}{45-24}$$
$$= \frac{9+4\sqrt{30}}{21}$$

which is the required value.

10. Find a and b if $\frac{5+3\sqrt{3}}{7+4\sqrt{3}} = a+b\sqrt{3}$.

Sol. We have $$\frac{5+3\sqrt{3}}{7+4\sqrt{3}} = a+b\sqrt{3}$$

$$\Rightarrow \quad \frac{(5+3\sqrt{3})(7-4\sqrt{3})}{(7+4\sqrt{3})(7-4\sqrt{3})} = a+b\sqrt{3}$$

$$\Rightarrow \quad \frac{35-12\times 3-20\sqrt{3}+21\sqrt{3}}{7^2-(4\sqrt{3})^2} = a+b\sqrt{3}$$

$$\Rightarrow \quad \frac{35-36+\sqrt{3}}{49-48} = a+b\sqrt{3}$$

$$\Rightarrow \quad \sqrt{3}-1 = a+b\sqrt{3}$$

$\therefore$ $a=-1$, $b=1$ which are the required values.

Milestone 1

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Multiple-Choice Questions

1. For any positive integers a and 4, there exist unique integers q and r such that $a=4q+r$, where r must satisfy

(*a*) $0 \le r < 4$ (*b*) $1 < r < 4$

(*c*) $0 < r < 4$ (*d*) $0 < r \le 4$

Sol. (*a*) $0 \le r < 4$

From Euclid's division lemma, we know that given any two positive integers a and b where $a > b$, there exist unique integers q and r such that $a = bq + r$ where $0 \le r < b$. In the present problem, $b = 4$ and hence $0 \le r < 4$.

2. If the HCF of 55 and 99 is expressible in the form $55m - 99$, then the value of m is

(*a*) 1 (*b*) 2

(*c*) 3 (*d*) 4

Sol. (*b*) 2

We find the HCF of 55 and 99 is 11.

If this, HCF is $55m - 99$, then

$$55m - 99 = 11$$

$$\Rightarrow \quad 55m = 99 + 11 = 110$$

$$\therefore \quad m = \frac{110}{55} = 2$$

$$\begin{array}{r} 55\,)\,99\,(\,1 \\ 55 \\ \hline 44\,)\,55\,(\,1 \\ 44 \\ \hline 11\,)\,44\,(\,4 \\ 44 \\ \hline 0 \\ \hline \end{array}$$

which is the required value.

Very Short Answer Type Questions

3. Find the number which when divided by 35 gives 62 as quotient and 25 as remainder.

Sol. Let a be the required number, then if a is divided by b and the remainder is r, then by Euclid's division lemma, we have

$$a = bq + r$$

Here $b = 35$, $q = 62$ and $r = 25$

$$\therefore \quad a = 35 \times 62 + 25$$
$$= 2170 + 25$$
$$= 2195$$

$\therefore$ The required number is 2195.

4. A fruit seller has 210 pieces of mangoes and 245 pieces of apples. He wants to stack them in such a way that each stack has the same number of them and also they occupy minimum space. What is the maximum number of fruits that can be placed in each stack for this purpose?

Sol. Clearly, the required number of fruits will be the HCF of 210 and 245. So, we find the HCF by using Euclid's division lemma as follows:

$$245 = 210 \times 1 + 35$$
$$210 = 35 \times 6 + 0$$

Since, the final remainder is zero.

Hence, the HCF is 35.

$\therefore$ The required number of fruits is 35.

Short Answer Type-I Questions

5. Use Euclid's division algorithm to find HCF of

(*a*) 448, 1008 and 168 (*b*) 625 and 165

Sol. (*a*) We first find the HCF of 168 and 448 as follow by using Euclid's division lemma.

$$448 = 168 \times 2 + 112$$
$$168 = 112 \times 1 + 56$$
$$112 = 56 \times 2 + 0$$

Since, the final remainder is zero, hence the HCF of 168 and 448 is 56. Now, find the HCF of 56 and the third given number 1008 as follows:

$$1008 = 56 \times 18 + 0$$

Since, the final remainder is zero, hence, the HCF of 1008 and 56 is 56.

Hence, the required HCF of the three given numbers is 56.

(*b*) We find the HCF of 625 and 165 as follows by using Euclid's division lemma.

$$625 = 165 \times 3 + 130$$
$$165 = 130 \times 1 + 35$$
$$130 = 35 \times 3 + 25$$
$$35 = 25 \times 1 + 10$$
$$25 = 10 \times 2 + 5$$
$$10 = 5 \times 2 + 0$$

Since, the final remainder is zero, hence the HCF of 625 and 165 is 5.

6. Find the greatest number that will divide 617 and 966 leaving remainders 8 and 9 respectively.

Sol. According to the question, the greatest number that will divide 617 and 966 leaving remainders 8 and 9 respectively, will be the HCF [(617 – 8), (966 – 9)], i.e., HCF (609, 957).

$\therefore$ Applying Euclid's division lemma to 609 and 957, we get

$$957 = 609 \times 1 + 348$$
$$609 = 348 \times 1 + 261$$
$$348 = 261 \times 1 + 87$$
$$261 = 87 \times 3 + 0$$

Since, the final remainder is 0.

$\therefore$ 87 is the HCF of 609 and 957.

Hence, the required number is 87.

Short Answer Type-II Questions

7. Use Euclid's division algorithm to show that the cube of any positive integer is of the form $9m$, $9m + 1$ or $9m + 8$ for some integer m.

Sol. Let a be a positive integer and $b = 3$.

$\therefore$ By Euclid's division lemma, we have

$$a = bq + r$$

where $q \geq 0$ and $0 \leq r < 3$.

We now consider 3 cases:

Case (*i*) Let $\quad r = 0$

$$\therefore \quad a = 3q$$

$$\Rightarrow \quad a^3 = 27q^3 = 9(3q^3) = 9m$$

where $m = 3q^3$

Case (*ii*) Let $r = 1$

$\therefore \quad a = 3q + 1$

$\Rightarrow \quad a^3 = (3q + 1)^3$

$= 27q^3 + 3 \times 9q^2 + 3 \times 3q + 1$

$= 9(3q^3 + 3q^2 + q) + 1$

$= 9m + 1$

where $m = 3q^3 + 3q^2 + q$

Case (iii) Let $r = 2$

$\therefore \quad a = 3q + 2$

$\Rightarrow \quad a^3 = (3q + 2)^3$

$= 27q^3 + 3 \times 9q^2 \times 2 + 3 \times 3q \times 2 + 8$

$= 9(3q^3 + 6q^2 + 2q) + 8$

$= 9m + 8$

where $m = 3q^3 + 6q^2 + 2q$

Hence, the cube of any positive integer is of the form $9m$, $9m + 1$ or $9m + 8$ for some integer m.

8. Prove that the product of two consecutive positive integers is divisible by 2.

Sol. We first prove that any even integer is of the form $2q$ and any positive odd integer will be of the form $2q + 1$, where q is some positive integer.

Let a be any positive integer and $b = 2$. Then by Euclid's division lemma, we have $a = 2q + r$ for some integer $q \geq 0$ and $r = 0$ or 1, since $0 \leq r < 2$. When $r = 0$, $a = 2q$ and when $r = 1$, then $a = 2q + 1$.

Hence, an even integer is of the form $2q$ and an odd integer is of the form $2q + 1$ where q is some positive integer.

Now, we also know that two positive consecutive integers must be even and odd or odd and even. Hence, the product of two consecutive positive integers must be of the form $2q(2q + 1)$ where $q \geq 0$. Since $2q(2q + 1) = 2m$ where $m = q(2q + 1)$ is a positive integer, hence such a number is always divisible by 2.

Long Answer Type Questions

9. There are 156, 208 and 260 students in groups A, B and C respectively. Buses are hired for a field trip. Find the minimum number of buses to be hired if the same number of students should be accommodated in each bus and each bus can accomodate atmost 55 students.

Sol. We first find the HCF of 156 and 208 by using Euclid's division lemma.

$$208 = 156 \times 1 + 52$$

$$156 = 52 \times 3 + 0$$

Since, the final remainder is zero, hence the HCF of 156 and 208 is 52.

Now, find the HCF of 52 and 260 by using Euclid's division lemma.

$$260 = 52 \times 5 + 0$$

Since, the final remainder is zero, hence the HCF of 52 and 260 is 52.

Hence, 52 students will accommodate in each bus.

Total number of students = 156 + 208 + 260 = 624

$\therefore$ Minimum number of buses to be hired

$$= \frac{\text{Total number of students}}{\text{Number of students in each bus}}$$

$$= \frac{624}{52} = 12$$

Hence, the minimum number of buses to be hired is 12.

10. Show that only one of the numbers n, $n + 2$ and $n + 4$ is divisible by 3.

Sol. Let m be any positive integer and let r be the remainder when m is divided by 3. Then $0 \leq r < 3$. Hence, by Euclid's division lemma, we have

$$m = 3q + r \qquad \ldots(1)$$

where q is a positive integer and $0 \leq r < 3$.

Let $m = n$ be divisible by 3.

Then from (1), we get

$$n = 3q \text{ with } r = 0$$

$$\therefore \quad q = \frac{n}{3} \qquad \ldots(2)$$

If possible, let $m = n + 2$ be also divisible by 3. Then from (1), we get

$$n + 2 = 3q = 3 \times \frac{n}{3} = n \qquad \text{[From (2)]}$$

$\Rightarrow \quad n + 2 = n$ which is absurd.

$\therefore$ $n + 2$ is not divisible by 3.

Next, if possible, let $m = n + 4$ be divisible by 3. Then from (1) and (2), we get

$$n + 4 = 3q = 3 \times \frac{n}{3} = n$$

which is again impossible. Thus, $n + 4$ is also not divisible by 3.

Similarly, we can show that if $n + 2$ is divisible by 3, then n and $n + 4$ will not be divisible by 3 and if $n + 4$ is divisible by 3, then n and $n + 2$ will not be divisible by 3.

Thus we have proved that only one of the numbers n, $n + 2$ and $n + 4$ is divisible by 3.

11. Prove that the product of three consecutive positive integers is divisible by 6.

Sol. We first prove that any positive integer is of the form $6q$, $6q + 1$, $6q + 2$, $6q + 3$, $6q + 4$ or $6q + 5$ where $q \geq 0$.

Let a be any positive integer and $b = 6$. Applying Euclid's division lemma, we get

$$a = bq + r = 6q + r$$

where $0 \leq r < 6$ i.e., $r = 0, 1, 2, 3, 4$, or 5 and $q \geq 0$.

Thus, $a = 6q$ with $r = 0$ or $a = 6q + 1$ with $r = 1$, or $a = 6q + 2$ with $r = 2$, or $a = 6q + 3$ with $r = 3$, or $a = 6q + 4$ with $r = 4$ or $a = 6q + 5$ with $r = 5$.

Now, if $n = 6q$, then

$$n(n + 1)(n + 2) = 6q(6q + 1)(6q + 2)$$

which is divisible by 6.

If $n = 6q + 1$, then

$$\begin{aligned} n(n + 1)(n + 2) &= (6q + 1)(6q + 2)(6q + 3) \\ &= (6q + 1) \times 2(3q + 1) \times 3(2q + 1) \\ &= 6(6q + 1)(3q + 1)(2q + 1) \end{aligned}$$

which is divisible by 6.

If $n = 6q + 2$, then

$$\begin{aligned} n(n + 1)(n + 2) &= (6q + 2)(6q + 3)(6q + 4) \\ &= 2(3q + 1) \times 3(2q + 1) \times 2(3q + 2) \\ &= 12(3q + 1)(2q + 1)(3q + 2) \end{aligned}$$

which is divisible by 6.

Similarly, we can show that when $n = 6q + 3$, or $n = 6q + 4$ or $n = 6q + 5$, then $n(n + 1)(n + 2)$ is divisible by 6.

Hence, proved.

12. Find the HCF of 135 and 225 and express it as a linear combination of x and y.

Sol. We start with the greater number 225.

Applying Euclid's lemma, we get

$$225 = 135 \times 1 + 90 \quad \ldots(1)$$

$\because$ $r = 90 \neq 0$, take divisor 135 as the new dividend and remainder 90 as new divisor, we get

$$135 = 90 \times 1 + 45 \quad \ldots(2)$$

Continue this process till remainder $r = 0$

$\therefore$ $$90 = 45 \times 2 + 0 \quad \ldots(3)$$

$\because$ $r = 0$, the divisor at last step is the HCF.

$\therefore$ HCF of 135 and 225 is 45.

From (2), $135 = 90 \times 1 + 45$

$\Rightarrow$ $$45 = 135 - 90 \times 1 \quad \ldots(4)$$

From (1), $225 = 135 \times 1 + 90$

$\Rightarrow$ $$90 = 225 - 135 \times 1 \quad \ldots(5)$$

Using value of 90 from (5) in (4)

$$45 = 135 - (225 - 135 \times 1) \times 1$$

$\therefore$ $$45 = 135 \times 1 - 225 \times 1 + 135 \times 1$$

$$45 = 135 \times 2 - 225 \times 1$$

$\therefore$ $$45 = 135 \times 2 + 225 \times (-1)$$

$\Rightarrow$ $$45 = 135x + 225y$$

where $x = 2$, $y = -1$.

Milestone 2

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Multiple-Choice Questions

1. If LCM (84, 24) = 168, then HCF (84, 24) is

(*a*) 11 (*b*) 12

(*c*) 13 (*d*) 14

Sol. (*b*) 12

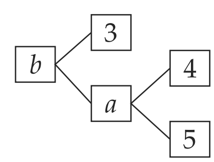
We know that

$$\text{LCM } (84, 24) \times \text{HCF } (84, 24) = 84 \times 24$$

$\Rightarrow$ $$168 \times \text{HCF } (84, 24) = 84 \times 24$$

$$\text{HCF } (84, 24) = \frac{84 \times 24}{168} = 12$$

2. The values of a and b in the factor tree are



(*a*) 10, 20 (*b*) 20, 30

(*c*) 20, 60 (*d*) 10, 60

Sol. (*c*) 20, 60

From the given factor tree, we see that

$$a = 4 \times 5 = 20$$

and $$b = 3 \times a = 3 \times 20 = 60$$

$\therefore$ The required values of a and b are 20 and 60 respectively.

3. LCM of $2^4 \times 3^3$ and $2^3 \times 3^4$ is

(*a*) 2^4 (*b*) 3^4

(*c*) $2^4 \times 3^4$ (*d*) $2^3 \times 3^3$

Sol. (*c*) $2^4 \times 3^4$

We have $2^4 \times 3^3 = 2^3 \times 3^3 \times 2$

and $2^3 \times 3^4 = 2^3 \times 3^3 \times 3$

$\therefore$ $$\text{LCM} = 2^3 \times 3^3 \times 2 \times 3 = 2^4 \times 3^4$$

4. The sum of the exponents of the prime factors in the prime factorisation of 324 is
(*a*) 3 (*b*) 4
(*c*) 5 (*d*) 6

Sol. (*d*) 6

We find the prime factors of 324 as follows:

2	324
2	162
3	81
3	27
3	9
3	3
	1

$$324 = 2^2 \times 3^4$$

$\therefore$ Sum of the exponents of 2 and 3 in the prime factorisation of 324 is 2 + 4 i.e. 6.

5. If n is any natural number, then $6^n - 5^n$ will always end with
(*a*) 0 (*b*) 1
(*c*) 2 (*d*) 3

Sol. (*b*) 1

We know that 6^n will end with 6 for all natural values of n and 5^n will end with 5 for all natural values n. Hence, $6^n - 5^n$ will always end with $6 - 5 = 1$ only.

6. The LCM and HCF of two numbers are equal, then the numbers will be
(*a*) prime. (*b*) coprime.
(*c*) composite. (*d*) equal.

Sol. (*d*) equal

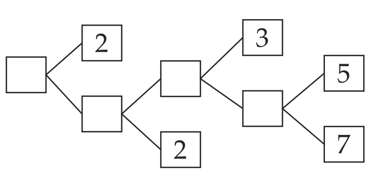
Let the two numbers be a and b and let K be their common LCM and HCF.

$\therefore$ $ab = K^2$

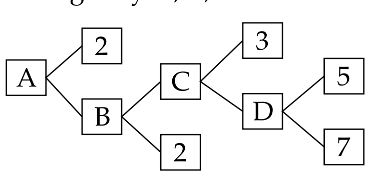
$\because$ a, b and K are natural numbers, hence, a and b be equal to each other. Two equal numbers may or may not always be prime, composite or coprime.

Very Short Answer Type Questions

7. Find the missing numbers in the following factor tree.



Sol. Let us denote the empty boxes starting from the left to the right by A, B, C and D.



Then the number in box D is 5 × 7 = 35.

$\therefore$ The number in box C is 35 × 3 = 105.

Also, the number in box B is 105 × 2 = 210

and the number in the box A is 210 × 2 = 420.

$\therefore$ The missing numbers are 420, 210, 105 and 35.

8. Find the prime factorisation of the following positive integers.
(*a*) 133100 (*b*) 8008

Sol. (*a*) We find the prime factors of 133100 as follows:

$\therefore$ $$133100 = 2^2 \times 5^2 \times 11^3$$

$\therefore$ The prime factorisation of 133100 is $2^2 \times 5^2 \times 11^3$.

2	133100
2	66550
5	33275
5	6655
11	1331
11	121
11	11
	1

(*b*) We find the prime factor of 8008 as follows:

$\therefore$ $$8008 = 2^3 \times 7 \times 11 \times 13$$

$\therefore$ The prime factorisation of 8008 is $2^3 \times 7 \times 11 \times 13$.

2	8008
2	4004
2	2002
7	1001
11	143
13	13
	1

9. Find HCF and LCM of the following numbers using Fundamental Theorem of Arithmetic.
(*a*) 600 and 750 (*b*) 240 and 720

Sol. (*a*) We first find the prime factors of 600 and 750 as follows:

$$600 = 2 \times 2 \times 2 \times 3 \times 5 \times 5$$
$$= 2^3 \times 3 \times 5^2$$
$$750 = 2 \times 3 \times 5 \times 5 \times 5$$
$$= 2 \times 3 \times 5^3$$

Hence, HCF (600, 750) = 150

We know that for any two positive integers a and b, we have

HCF (a, b) × LCM (a, b) = $a \times b$

$\Rightarrow$ HCF (600, 750) × LCM (600, 750)
= 600 × 750

$\Rightarrow$ 150 × LCM (600, 750) = 600 × 750

Hence, LCM (600, 750) = $\frac{600 \times 750}{150}$ = 3000

(*b*) We first find the prime factors of 240 and 720 as follows:

$$240 = 2 \times 2 \times 2 \times 2 \times 3 \times 5$$
$$= 2^4 \times 3 \times 5$$
$$720 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5$$
$$= 2^4 \times 3^2 \times 5$$

Hence, HCF(240, 720) = $2^4 \times 3 \times 5 = 240$

We know that for any two positive integers a and b, we have

HCF $(a, b) \times$ LCM $(a, b) = a \times b$

$\Rightarrow$ HCF (240, 720) × LCM (240, 720) = 240 × 720

$\Rightarrow$ 240 × LCM (240, 720) = 240 × 720

Hence, LCM (240, 720) = $\frac{240 \times 720}{240}$ = 720

10. Can two numbers have 16 as HCF and 644 as LCM?

Sol. No, two numbers cannot have 16 as HCF and 644 as LCM.

Since LCM of two numbers is always divisible by their HCF. Now, the given LCM is 644 which is not divisible by the HCF i.e. 16. Hence, it is not possible to get such two numbers whose HCF and LCM are 16 and 644 respectively.

11. Write the HCF of the smallest composite number and smallest prime number.

Sol. We know that the smallest composite number is 4 and the smallest prime number is 2 and the HCF of 2 and 4 is clearly 2 which is the required HCF.

Short Answer Type-I Questions

12. Find the HCF and LCM of following using prime factorisation.

(*a*) 50, 160 and 400 (*b*) 70, 90 and 250

Sol. (*a*) By prime factorisation, we get

$$50 = 2 \times 5^2$$
$$160 = 2^5 \times 5$$
$$400 = 2^4 \times 5^2$$

$$\begin{array}{r|r} 2 & 50 \\ \hline 5 & 25 \\ \hline 5 & 5 \\ \hline & 1 \end{array} \qquad \begin{array}{r|r} 2 & 160 \\ \hline 2 & 80 \\ \hline 2 & 40 \\ \hline 2 & 20 \\ \hline 2 & 10 \\ \hline 5 & 5 \\ \hline & 1 \end{array} \qquad \begin{array}{r|r} 2 & 400 \\ \hline 2 & 200 \\ \hline 2 & 100 \\ \hline 2 & 50 \\ \hline 5 & 25 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$

$\therefore$ HCF of 50,160 and 400 = $2 \times 5 = 10$

LCM of 50, 160 and 400 = $2^5 \times 5^2 = 800$

(*b*) By prime factorisation, we get

$$70 = 2 \times 5 \times 7$$
$$90 = 2 \times 5 \times 3^2$$
$$250 = 2 \times 5^3$$

$$\begin{array}{r|r} 2 & 70 \\ \hline 5 & 35 \\ \hline 7 & 7 \\ \hline & 1 \end{array} \qquad \begin{array}{r|r} 2 & 90 \\ \hline 3 & 45 \\ \hline 3 & 15 \\ \hline 5 & 5 \\ \hline & 1 \end{array} \qquad \begin{array}{r|r} 2 & 250 \\ \hline 5 & 125 \\ \hline 5 & 25 \\ \hline 5 & 5 \\ \hline & 1 \end{array}$$

$\therefore$ HCF of 70, 90 and 250 = $2 \times 5 = 10$

LCM of 70, 90 and 250 = $2 \times 3^2 \times 5^3 \times 7$

= 15750

13. The HCF and LCM of two numbers are 40 and 35960 respectively. If one of the number is 1160. Find the other.

Sol. We know that

HCF$(a, b) \times$ LCM$(a, b) = a \times$ b

$\Rightarrow$ $40 \times 35960 = 1160 \times b$

$\Rightarrow$ $\frac{40 \times 35960}{1160} = b$

$\Rightarrow$ $1240 = b$

$\therefore$ The other number is 1240.

14. Given that HCF (336, 486) = 54, find LCM (336, 486).

Sol. We know that

HCF$(a, b) \times$ LCM $(a, b) = a \times b$

HCF (336,486) × LCM (336, 486) = 336 × 486

$\Rightarrow$ 54 × LCM (336, 486) = 336 × 486

$\Rightarrow$ LCM(336, 486) = $\frac{336 \times 486}{54}$

= 56 × 54

= 3024

$\therefore$ LCM(336, 486) = 3024

15. Explain why $3 \times 5 \times 7 \times 11 + 5$ is a composite number.

Sol. $3 \times 5 \times 7 \times 11 + 5 = 1155 + 5$

$= 1160$

$= 2^3 \times 5 \times 29$

A number which has more than two factors (other than 1 and itself) is a composite number. Since, the above can be expressed as a product of more than two prime factors, therefore it is a composite number.

Short Answer Type-II Questions

16. The LCM of two numbers is 14 times their HCF. The sum of LCM and HCF is 600. If one of the numbers is 280, then find the other number. **[CBSE 2012]**

Sol. Let the other number be x and let the HCF $(280, x)$ be y.

Then according to the problem,

$$\text{LCM}(280, x) = 14y$$

and $\quad 14y + y = 600$

$\Rightarrow \quad 15y = 600$

$\Rightarrow \quad y = \frac{600}{15} = 40$

$\therefore \quad \text{HCF}(280, x) = 40$

and $\text{LCM}(280, x) = 14 \times 40 = 560$

$\because \quad \text{LCM}(280, x) \times \text{HCF}(280, x) = 280x$

$\therefore \quad 560 \times 40 = 280x$

$\Rightarrow \quad \frac{560 \times 40}{280} = x$

$\Rightarrow \quad x = 80$

$\therefore$ The other number is 80.

17. Find the smallest number which when increased by 17 is exactly divisible by 520 and 468.

Sol. We first find the smallest number which is divisible by 520 and 468 by finding their LCM as follows:

$\text{LCM} = 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 13$
$= 4680$

2	520, 468
2	260, 234
2	130, 117
3	65, 117
3	65, 39
5	65, 13
13	13, 13
	1, 1

Hence, the smallest number is 4680 – 17 i.e. 4663.

18. On a morning walk, three persons step off together and their steps measure 35 cm, 40 cm and 42 cm respectively. What is the minimum distance each should walk so that each can cover the same distance in complete steps?

Sol. The required minimum distance will be equal to the LCM of 35, 40 and 42 in cm. So, we find the LCM of 35, 40 and 42 as follows:

$\text{LCM} = 2 \times 2 \times 2 \times 3 \times 5 \times 7 = 840$

2	35, 40, 42
2	35, 20, 21
2	35, 10, 21
3	35, 5, 21
5	35, 5, 7
7	7, 1, 7
	1, 1, 1

Hence, the minimum distance walked is 840 cm.

19. Check whether 15^n can end with digit 0 for $n \in \text{N}$.

Sol. If 15^n end with digit zero, then the number should be divisible by 2 and 5.

This means the prime factorisation of 15^n should contain prime factors 2 and 5.

$\Rightarrow \quad 15^n = (3 \times 5)^n$

$\because$ 2 is not present in the prime factorisation, there is no natural number for which 15^n ends with the digit zero.

So, 15^n cannot end with digit zero.

Long Answer Type Questions

20. Using prime factorisation method, find HCF and LCM of 24, 30 and 70. Also, show that HCF × LCM ≠ Product of three numbers.

Sol. We find the prime factors of 24, 30 and 70 as follows:

$$24 = 2^3 \times 3$$

$$30 = 2 \times 3 \times 5$$

$$70 = 2 \times 5 \times 7$$

2	24
2	12
2	6
3	3
	1

2	30
3	15
5	5
	1

2	70
5	35
7	7
	1

$\therefore$ HCF of 24, 30 and 70 = 2

LCM of 24, 30 and 70 = $2^3 \times 3 \times 5 \times 7 = 840$

$\therefore \quad \text{HCF} \times \text{LCM} = 2 \times 840 = 1680$

Also, $24 \times 30 \times 70 = 50400$

$\therefore$ HCF × LCM ≠ Product of three given numbers.

21. Find the greatest number of five digits exactly divisible by 12, 15 and 36.

Sol. The greatest number of 5 digits exactly divisible by 12, 15 and 36 will be divisible by LCM (12, 15, 36).

$$12 = 2 \times 2 \times 3 = 2^2 \times 3$$

$$15 = 3 \times 5$$

$$36 = 2 \times 2 \times 3 \times 3 = 2^2 \times 3^2$$

$$\therefore \quad \text{LCM} = 2^2 \times 3^2 \times 5 = 180$$

The greatest number of 5 digits = 99999

$$99999 = 180 \times 555 + 99$$

$\therefore$ The greatest number that will be divisible by 180

$$= 99999 - 99 = 99900$$

Hence, the required number is 99900.

22. Red colour crayons are available in a pack of 18 and blue colour in a pack of 24. If I need to buy an equal number of crayons of both colours, what is the least number of packs of each kind I would need to buy?

Sol. We first find the LCM of 18 and 24 as follows:

$$\therefore \quad \text{LCM} = 2 \times 2 \times 2 \times 3 \times 3 = 72$$

2	18, 24
2	9, 12
2	9, 6
3	9, 3
3	3, 3
	1, 1

Hence, I should by 72 red color crayons and 72 blue color crayons. Now, 18 red color crayons are available in one pack and 24 blue color crayons are available in one pack. Hence, the required number of packs of red color crayons is 72 ÷ 18 i.e. 4 and the number of packs of blue color is 72 ÷ 24 i.e. 3.

23. Three bells toll at intervals of 12, 15 and 18 minutes respectively. If they start tolling together after what time will they toll together again?

Sol. The time required by the three bells to toll together again is the LCM of 12, 15 and 18 (in minutes).

$$12 = 2 \times 2 \times 3 = 2^2 \times 3$$

$$15 = 3 \times 5$$

$$18 = 2 \times 3 \times 3 = 2 \times 3^2$$

LCM of 12, 15 and 18 = $2^2 \times 3^2 \times 5 = 180$

Hence, the time at which the three bells toll together again is 180 minutes or 3 hours.

Milestone 3

(Page 15)

Multiple-Choice Questions

1. $\dfrac{1}{\pi} - \dfrac{7}{22}$ is

(*a*) a rational number. (*b*) an irrational number.
(*c*) a prime number. (*d*) an even number.

Sol. (*b*) an irrational number.

We know that π is an irrational number and $\dfrac{7}{22}$ is a rational number. Since a combination of a rational number and an irrational number is an irrational number, hence the given number is irrational.

2. Which of the following will have a terminating decimal expansion?

(*a*) $\dfrac{65}{484}$ (*b*) $\dfrac{66}{180}$

(*c*) $\dfrac{25}{1600}$ (*d*) $\dfrac{31}{2^2 \times 5^2 \times 7^2}$

Sol. (*c*) $\dfrac{25}{1600}$

For terminating decimal the prime factorisation of denominator should only contain the factor 2 or factor 5 or both factor 2 and 5.

$\therefore$ $\dfrac{25}{1600}$ is a terminating decimal. Since prime factorisation of 1600 is $2^6 \times 5^2$.

3. The decimal expansion of π is

(*a*) terminating.
(*b*) non-terminating non-repeating.
(*c*) non-terminating repeating.
(*d*) does not exist.

Sol. (*b*) non-terminating non-repeating.

Since π is an irrational number which can be expressed as a non-terminating non-repeating decimal expansion.

4. The decimal expansion of the rational number $\dfrac{26}{2^2 \times 5}$ will terminate after

(*a*) one decimal place.
(*b*) two decimal places.
(*c*) three decimal places.
(*d*) four decimal places.

Sol. (*a*) one decimal place.

We have $\dfrac{26}{2^2 \times 5} = \dfrac{26}{20} \times \dfrac{5}{5} = \dfrac{130}{100} = \dfrac{13}{10} = 1.3$

Hence, the decimal point terminates after one decimal place.

Very Short Answer Type Questions

5. After how many places of decimals will the decimal expansion of $\dfrac{31}{2^3 \times 5^4}$ terminate?

Sol. We have $\frac{31}{2^3 \times 5^4} = \frac{31}{8 \times 625} = \frac{31}{5000}$

$= \frac{31 \times 2}{5000 \times 2} = \frac{62}{10000}$

$= 0.0062$

Hence, the decimal expansion of $\frac{31}{2^3 \times 5^4}$ terminate after four decimal places.

6. Write the number of consecutive zeroes in the prime factorisation of a natural number N given by $2^4 \times 3^4 \times 5^3 \times 11$.

Sol. We have

$$2^4 \times 3^4 \times 5^3 \times 11 = 2^3 \times 2 \times 5^3 \times 3^4 \times 11$$
$$= (2 \times 5)^3 \times 2 \times 3^4 \times 11$$
$$= 22 \times 81 \times 10^3$$
$$= 22 \times 81 \times 1000$$

Hence, the required number of consecutive zeroes in the given number is 3.

Short Answer Type-I Questions

7. Without actually performing the long division, state whether the following rational numbers will have terminating or non-terminating decimal expansion.

(*a*) $\frac{173}{2^4 5^3}$ (*b*) $\frac{123}{2^5 \times 5^2 \times 7^3}$

(*c*) $\frac{65}{1200}$ (*d*) $\frac{64}{800}$

Sol. (*a*) $\frac{173}{2^4 \times 5^3}$, clearly it is in its simplest form. Also the denominator $2^4 \times 5^3$ is of the form $2^n 5^m$, where n and m are non-negative integers.

Hence, $\frac{173}{2^4 \times 5^3}$ has a terminating decimal expansion.

(*b*) $\frac{123}{2^5 \times 5^2 \times 7^3} = \frac{3 \times 41}{2^5 \times 5^2 \times 7^3}$, clearly fraction is in its simplest form. The denominator is not of the form $2^n 5^m$, where n and m are non-negative integers.

Hence, $\frac{173}{2^4 \times 5^3}$ has a non-terminating decimal expansion.

(*c*) $\frac{65}{1200} = \frac{13}{2^4 \times 3 \times 5}$, clearly fraction is in its simplest form. The denominator is not of the form $2^n 5^m$, where n and m are non-negative integers.

Hence, $\frac{65}{1200}$ has a non-terminating decimal expansion.

(*d*) $\frac{64}{800} = \frac{2}{5 \times 5} = \frac{2}{5^2}$, clearly fraction is in its simplest form. Also the denominator $5^2 = 2^0 \times 5^2$ is of the form $2^n 5^m$, where n and m are non-negative integers.

Hence, $\frac{64}{800}$ has a non-terminating decimal expansion.

8. What can you say about the prime factorisation of the denominators of the following rationals:

(*a*) 102.5325 (*b*) $26.\overline{1245}$

Sol. (*a*) Since the given decimal number is terminating, hence, the denominator of this rational number will be of the form $2^n 5^m$, where n and m are non-negative integers.

(*b*) Since the given decimal number is non-terminating repeating, hence, the denominator of this rational number is not of the form $2^n 5^m$, where n and m are non-negative integers.

9. If $\frac{241}{4000} = \frac{241}{2^m 5^n}$, find the values of m and n where m and n are non-negative integers. Hence, write its decimal expansion without its actual division.

[CBSE 2012]

Sol. We have $\frac{241}{4000} = \frac{241}{2^m 5^n}$

$\Rightarrow \quad 2^m 5^n = 4000 = 2^5 \times 5^3$

$\therefore$ $m = 5$ and $n = 3$

Now, $\frac{241}{2^5 5^3} = \frac{241}{4 \times 10^3} = \frac{60.25}{1000} = 0.06025$ which is the required expansion.

10. Write down the decimal expansion of the following rational numbers by writing their denominators in the form $2^n 5^m$, where n and m are non-negative integers.

(*a*) $\frac{16}{500}$ (*b*) $\frac{110}{125}$

(*c*) $\frac{3}{80}$ (*d*) $\frac{21}{250}$

Sol. (*a*) $\frac{16}{500} = \frac{16}{2^2 \times 5^3} = \frac{16 \times 2}{2^3 \times 5^3} = \frac{32}{10^3} = 0.032$

(*b*) $\frac{110}{125} = \frac{110}{5^3} = \frac{110 \times 2^3}{(5 \times 2)^3} = \frac{880}{10^3} = 0.88$

(c) $\frac{3}{80} = \frac{3}{2^4 \times 5} = \frac{3 \times 5^3}{2^4 \times 5^4} = \frac{375}{(2 \times 5)^4} = \frac{375}{10^4}$

$= 0.0375$

(d) $\frac{21}{250} = \frac{21}{2 \times 5^3} = \frac{21 \times 2^2}{(2 \times 5)^3} = \frac{84}{10^3} = 0.084$

Short Answer Type-II Questions

11. Prove that $\sqrt{11}$ is irrational.

Sol. Let us assume on the contrary that $\sqrt{11}$ is a rational number.

Then $\sqrt{11} = \frac{p}{q}$ where p and q are coprime and $q \neq 0$

$\therefore$ On squaring both sides

$\Rightarrow \quad 11 = \frac{p^2}{q^2}$

$\Rightarrow \quad p^2 = 11q^2 \quad \ldots(1)$

$\Rightarrow$ p^2 is divisible by 11

[$\because$ $11q^2$ is divisible by 11]

$\Rightarrow$ p is divisible by 11 $\quad \ldots(2)$

[$\because$ 11 is prime and divides p^2
$\Rightarrow$ 11 divides p]

Let $p = 11\,c$, for some integer c

Substituting $p = 11\,c$ in (1), we get

$11q^2 = (11c)^2$

$\Rightarrow \quad 11q^2 = 121c^2$

$\Rightarrow \quad q^2 = 11c^2$

$\Rightarrow$ q^2 is divisible by 11

[$\because$ $11\,c^2$ is divisible by 11]

$\Rightarrow$ q is divisible by 11 $\quad \ldots(3)$

[$\because$ 11 is prime and divides q^2
$\Rightarrow$ 11 divides q]

From (2) and (3)

$\Rightarrow$ Both p and q are divisible by 11.

$\therefore$ 11 is a common factor of p and q.

But this contradicts the fact that p and q are coprime.

This contradiction has arisen because of our incorrect assumption that $\sqrt{11}$ is rational.

Hence, $\sqrt{11}$ is irrational.

12. Prove each of the following is irrational.

(a) $\sqrt{3} - \sqrt{5}$ (b) $\frac{3\sqrt{2}}{5}$

(c) $5 + 3\sqrt{2}$ (d) $11 + 13\sqrt{2}$

Sol. (a) Let us assume on the contrary that $\sqrt{3} - \sqrt{5}$ is a rational number.

Then, there exists coprime p and q ($q \neq 0$) such that

$$\sqrt{3} - \sqrt{5} = \frac{p}{q}$$

$$\Rightarrow \quad \sqrt{5} = \sqrt{3} - \frac{p}{q}$$

On squaring both sides, we get

$$5 = \left(\sqrt{3} - \frac{p}{q}\right)^2$$

$$= \left(\sqrt{3}\right)^2 + \frac{p^2}{q^2} - \frac{2p}{q}\sqrt{3}$$

$$\Rightarrow \quad 5 - 3 - \frac{p^2}{q^2} = -\frac{2p}{q}\sqrt{3}$$

$$\Rightarrow \quad 2 - \frac{p^2}{q^2} = -\frac{2p}{q}\sqrt{3}$$

$$\Rightarrow \quad \frac{2q^2 - p^2}{q^2} = -\frac{2p}{q}\sqrt{3}$$

$$\Rightarrow \quad \sqrt{3} = \frac{p^2 - 2q^2}{2pq}$$

[$\therefore$ p, q are integer,
$\therefore$ $\frac{p^2 - 2q^2}{2pq}$ is a rational number]

$\Rightarrow$ $\sqrt{3}$ is a rational number.

This contradicts the fact that $\sqrt{3}$ is an irrational number. So, our assumption is wrong.

Hence, $\sqrt{3} - \sqrt{5}$ is an irrational number.

(b) Let us assume on contrary that $\frac{3\sqrt{2}}{5}$ is a rational.

Then, there exists coprime p and q ($q \neq 0$) such that

$$\frac{3\sqrt{2}}{5} = \frac{p}{q}$$

$$\Rightarrow \quad \sqrt{2} = \frac{p}{q} \times \frac{5}{3} = \frac{5p}{3q}$$

[$\because$ p, q are integer, $\therefore$ $\frac{5p}{3q}$ is a rational number]

$\Rightarrow$ $\sqrt{2}$ is a rational number.

This contradicts the fact that $\sqrt{2}$ is an irrational number.

So, our assumption is wrong.

Hence, $\frac{3\sqrt{2}}{5}$ is an irrational number.

(*c*) Let us assume on the contrary that $5 + 3\sqrt{2}$ is a rational number. Then, there exists coprime p and q ($q \neq 0$) such that

$$5 + 3\sqrt{2} = \frac{p}{q}$$

$$\Rightarrow \quad 3\sqrt{2} = \frac{p}{q} - 5$$

$$\Rightarrow \quad 3\sqrt{2} = \frac{p - 5q}{q}$$

$$\Rightarrow \quad \sqrt{2} = \frac{p - 5q}{3q}$$

[$\because$ p, q are integers,

$\therefore$ $\frac{p - 5q}{3q}$ is a rational number]

$\Rightarrow$ $\sqrt{2}$ is a rational number.

This contradicts the fact that $\sqrt{2}$ is an irrational number. So, our assumption is wrong.

Hence, $5 + 3\sqrt{2}$ is an irrational number.

(*d*) Let us assume on the contrary that $11 + 13\sqrt{2}$ is a rational number.

Then, there exists coprime p and q ($q \neq 0$) such that

$$11 + 13\sqrt{2} = \frac{p}{q}$$

$$\Rightarrow \quad 13\sqrt{2} = \frac{p}{q} - 11$$

$$\Rightarrow \quad 13\sqrt{2} = \frac{p - 11q}{q}$$

$$\Rightarrow \quad \sqrt{2} = \frac{p - 11q}{13q}$$

[$\because$ p, q are integers,

$\therefore$ $\frac{p - 11q}{13q}$ is a rational number]

$\Rightarrow$ $\sqrt{2}$ is a rational number.

This contradicts the fact that $\sqrt{2}$ is an irrational number.

So, our assumption is wrong.

Hence, $11 + 13\sqrt{2}$ is an irrational number.

13. Prove each of the following is irrational.

(*a*) $\frac{1}{\sqrt{2}}$ (*b*) $\frac{3}{\sqrt{5}}$

Sol. (*a*) $\frac{1}{\sqrt{2}} = \frac{1 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} = \frac{\sqrt{2}}{2}$

Let us assume on the contrary that $\frac{1}{\sqrt{2}}$ is a rational number but $\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\Rightarrow$ $\frac{\sqrt{2}}{2}$ is rational

Let $\frac{\sqrt{2}}{2} = \frac{p}{q}$, where p and q are coprime and non-zero integers.

$$\Rightarrow \quad \sqrt{2} = \frac{2p}{q},$$

$\Rightarrow$ $\sqrt{2}$ is rational.

[$\therefore$ p and q are integers,

$\therefore$ $\frac{2p}{q}$ is a rational number]

This contradicts the fact that $\sqrt{2}$ is an irrational number. So, our assumption is wrong.

Hence, $\frac{1}{\sqrt{2}}$ is an irrational number.

(*b*) $\frac{3}{\sqrt{5}} = \frac{3 \times \sqrt{5}}{\sqrt{5} \times \sqrt{5}} = \frac{3\sqrt{5}}{5}$

Let us assume on the contrary that $\frac{3}{\sqrt{5}}$ is rational but $\frac{3}{\sqrt{5}} = \frac{3\sqrt{5}}{5}$.

$\Rightarrow$ $\frac{3\sqrt{5}}{5}$ is rational.

Let $\frac{3\sqrt{5}}{5} = \frac{p}{q}$, where p and q are coprime and non-zero integers.

$$\sqrt{5} = \frac{5p}{3q}$$

$\Rightarrow$ $\sqrt{5}$ is rational

[$\because$ p and q are integers,

$\therefore$ $\frac{5p}{3q}$ is a rational number]

This contradicts the fact that $\sqrt{5}$ is an irrational number. So, our assumption is wrong.

Hence, $\frac{3}{\sqrt{5}}$ is an irrational number.

14. Write whether $\frac{2\sqrt{45} + 3\sqrt{20}}{2\sqrt{5}}$ on simplification is a rational or irrational. **[CBSE 2010]**

Sol. We have

$$\frac{2\sqrt{45}+3\sqrt{20}}{2\sqrt{5}} = \frac{2\sqrt{9\times 5}+3\sqrt{4\times 5}}{2\sqrt{5}}$$

$$= \frac{2\sqrt{9}\sqrt{5}+3\sqrt{4}\sqrt{5}}{2\sqrt{5}}$$

$$= \frac{2\times 3\sqrt{5}+3\times 2\sqrt{5}}{2\sqrt{5}}$$

$$= \frac{6\sqrt{5}+6\sqrt{5}}{2\sqrt{5}}$$

$$= \frac{12\sqrt{5}}{2\sqrt{5}} = 6$$

which is a rational number.

Long Answer Type Questions

15. Prove that $\sqrt{2}+\frac{3}{\sqrt{2}}$ is irrational.

Sol. Let us assume on the contrary that $\sqrt{2}+\frac{3}{\sqrt{2}}$ is rational number.

Then, there exist coprime p and q ($q \neq 0$) such that

$$\sqrt{2}+\frac{3}{\sqrt{2}} = \frac{p}{q}$$

$$\Rightarrow \quad \frac{2+3}{\sqrt{2}} = \frac{p}{q}$$

$$\Rightarrow \quad \frac{5}{\sqrt{2}} = \frac{p}{q}$$

$$\Rightarrow \quad \frac{1}{\sqrt{2}} = \frac{p}{5q}$$

$\Rightarrow \quad \frac{1}{\sqrt{2}}$ is rational.

[$\because$ p and q are integers,

$\therefore$ $\frac{p}{5q}$ is a rational number]

This contradicts the fact that $\frac{1}{\sqrt{2}}$ is irrational.

[Refer to solution of Q-13 (*a*)]

So, our assumption is wrong.

Hence, $\sqrt{2}+\frac{3}{\sqrt{2}}$ is irrational.

16. Has the rational number $\frac{441}{2^2\times 5^7\times 3}$ a terminating or non-terminating decimal representation?

Sol. We have

$$\frac{441}{2^2\times 5^7\times 3} = \frac{147}{2^2\times 5^7}$$

Since the denominator of the simplified number is of the form 2^m5^n where m and n are non-negative integer. Hence, $\frac{441}{2^2\times 5^7\times 3}$ has a terminating decimal representation.

Higher Order Thinking Skills (HOTS) Questions

(Page 16)

1. If p is any prime number and a^2 is divisible by p, then p will also divide a. Explain.

Sol. From the Fundamental Theorem of Arithmetic, the integer a can be factorised as the product of primes which are unique.

Let $a = p_1\, p_2\, p_3 \ldots p_n$, where $p_1, p_2, \ldots p_n$ are primes, not necessarily distinct.

$$\therefore \quad a^2 = (p_1p_2p_3\ldots p_n)(p_1p_2p_3\ldots p_n)$$

$$= p_1^2\; p_2^2\; p_3^2 \ldots p_n^2$$

Now, it is given that the prime p divides a^2. Hence, p is a prime factor of a^2.

$\therefore$ From the uniqueness part of the Fundamental Theorem of Arithmetic, it follows that the only prime factors of a^2 are $p_1, p_2 p_3 \ldots p_n$. So, p is one of $p_1, p_2, p_3 \ldots p_n$.

Now, since $a = p_1p_2p_3\ldots p_n$

$\therefore$ p divides a.

Hence, proved.

2. Find the greatest number of six digits exactly divisible by 12, 15 and 36.

Sol. The greatest number of 6 digit exactly divisible by 12, 15 and 36 will be divisible by LCM (12, 15, 36).

$$12 = 2\times 2\times 3 = 2^2\times 3$$

$$12 = 3\times 5$$

$$36 = 2\times 2\times 3\times 3 = 2^2\times 3^2$$

$$\therefore \quad \text{LCM} = 2^2\times 3^2\times 5 = 180$$

The greatest number of 6 digits = 999999

$$999999 = 180\times 5555 + 99$$

$\therefore$ The greatest number that will be divisible by 180

$$= 999999 - 99 = 999900$$

Hence, the required number is 999900.

3. Find the HCF of 145 and 155. Express it as a linear combination of x and y.

Sol. By using Euclid's division lemma, we have

$$155 = 145 \times 1 + 10 \quad \ldots(1)$$
$$145 = 10 \times 14 + 5 \quad \ldots(2)$$
$$10 = 5 \times 2 + 0 \quad \ldots(3)$$

$\therefore$ From (3), since the final remainder is zero, hence, the HCF is 5.

Now, from (2), we have

$$\text{HCF} = 5 = 145 - 10 \times 14 \quad \ldots(4)$$

From (1), $\quad 10 = 155 - 145 \quad \ldots(5)$

$\therefore$ From (4) and (5), we have

$$\begin{aligned} 5 &= 145 - (155 - 145) \times 14 \\ &= 145 - 155 \times 14 + 145 \times 14 \\ &= 145 \times (1 + 14) - 155 \times 14 \\ &= 145 \times 15 - 155 \times 14 \\ &= 145x + 155y \end{aligned}$$

where $x = 15$ and $y = -14$.

$\therefore$ HCF as a linear combination of x and y is $145x + 155y = 5$, where $x = 15$ and $y = -14$.

4. If p and q are odd positive integers, then prove that $p^2 + q^2$ is even but not divisible by 4.

Sol. Let $p = 2p_1 + 1$ and $q = 2q_1 + 1$, where p and q are odd positive integers and p_1, q_1 are positive integers.

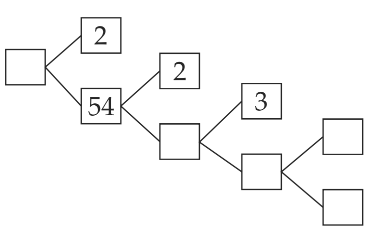
$$\begin{aligned} \therefore \quad p^2 + q^2 &= (2p_1 + 1)^2 + (2q_1 + 1)^2 \\ &= 4\left(p_1^2 + q_1^2\right) + 2 + 4\left(p_1 + q_1\right) \\ &= 2\left[2\left(p_1^2 + q_1^2\right) + 2\left(p_1 + q_1\right) + 1\right] \\ &= 2m \end{aligned}$$

where $m = 2\left(p_1^2 + q_1^2\right) + 2\left(p_1 + q_1\right) + 1$ is a positive integer.

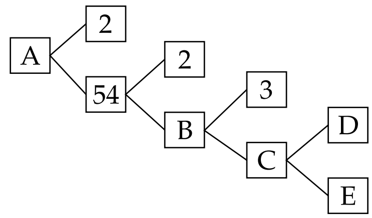
$\therefore$ 2 is a factor of $p^2 + q^2$, but not 4.

Hence, $p^2 + q^2$ is an even integer but not divisible by 4.

5. Find the missing numbers in the prime factor tree:



Sol. Let us denote the empty box on the extreme left by the letter A and the subsequent empty boxes on its right by B, C, D and E respectively. In the empty box A. The missing number is $54 \times 2 = 108$. The missing number in the empty box B is $54 \div 2 = 27$, and that is the empty box C is $27 \div 3 = 9$. Since $9 = 3 \times 3$, hence, the missing numbers in the empty boxes D and E are 3 and 3.



Hence, the required missing number in the empty boxes, starting from the left are 108, 27, 9, 3 and 3 respectively.

6. Four bells toll together at 10 am. They toll after 8, 12, 16, 24 seconds respectively. How many times will they toll together again in the next 2 hours?

Sol. The time required by the four bells to toll together is the LCM of 8, 12, 16 and 24 (in seconds).

$$8 = 2 \times 2 \times 2 = 2^3$$
$$12 = 2 \times 2 \times 3 = 2^2 \times 3$$
$$16 = 2 \times 2 \times 2 \times 2 = 2^4$$
$$24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3$$

$\therefore$ LCM of 8, 12, 16 and 24 $= 2^3 \times 3 = 24$

Hence, after 48 seconds, the four bells will toll together once again. Now, is the next 2 hours i.e. $2 \times 60 \times 60$ seconds.

The number of times the four bells will toll together $= \dfrac{2 \times 60 \times 60}{48} = 150$

$\therefore$ In the next 2 hours number of times the four bells toll together is 150.

7. For any positive integer n, prove that $n^3 - n$ is divisible by 6.

Sol. We have $n^3 - n = n(n^2 - 1) = (n - 1)\ n(n + 1)$ which is the product of three consecutive positive integer. We shall now prove that the product of three consecutive positive integers is divisible by 6.

We know that any positive integer is of the form $6q$, $6q + 1$, $6q + 2$, $6q + 4$ or $6q + 5$, for some integer q.

Case (i) $\quad n = 6q$

$$\begin{aligned} \Rightarrow \quad (n - 1)\ n\ (n + 1) &= (6q - 1)\ (6q)\ (6q + 1) \\ &= 6q\ (6q - 1)\ (6q + 1) \end{aligned}$$

$\Rightarrow$ $n^3 - n$ is divisible by 6.

Case (*ii*) $n = 6q + 1$

$$\Rightarrow \quad (n-1)\,n\,(n+1) = (6q)(6q+1)(6q+2)$$
$$= 6q\,(6q+1)(6q+2)$$

$\Rightarrow$ $n^3 - n$ is divisible by 6.

Case (*iii*) $n = 6q + 2$

$$\Rightarrow \quad (n-1)\,n\,(n+1) = (6q+1)(6q+2)(6q+3)$$
$$= 2.3(6q+1)(3q+1)(2q+1)$$
$$= 6(6q+1)(3q+1)(2q+1)$$

$\Rightarrow$ $n^3 - n$ is divisible by 6.

Case (*iv*) $n = 6q + 3$

$$\Rightarrow (n-1)\,n\,(n+1) = (6q+2)(6q+3)(6q+4)$$
$$= 2.3.2\,(3q+1)(2q+1)(3q+2)$$
$$= 12\,(3q+1)(2q+1)(3q+2)$$

$\Rightarrow$ $n^3 - n$ is divisible by 6.

Case (*v*) $n = 6q + 4$

$$\Rightarrow (n-1)\,n\,(n+1) = (6q+3)(6q+4)(6q+5)$$
$$= 3.2\,(2q+1)(3q+2)(6q+5)$$
$$= 6\,(2q+1)(3q+2)(6q+5)$$

$\Rightarrow$ $n^3 - n$ is divisible by 6.

Case (*vi*) $n = 6q + 5$

$$(n-1)\,n\,(n+1) = (6q+4)(6q+5)(6q+6)$$
$$= 2.6\,(3q+2)(6q+5)(q+1)$$
$$= 12\,(3q+2)(6q+5)(q+1)$$

$\Rightarrow$ $n^3 - n$ is divisible by 6.

Hence, $n^3 - n$ is divisible by 6 for every positive integer n.

8. If x and y are two odd positive integers such that $x > y$. Prove that one of the two numbers $\frac{x+y}{2}$ and $\frac{x-y}{2}$ is odd and the other is even.

Sol. Let $x = 2q + 3$ and $y = 2q + 1$ be two positive odd integers where $x > y$.

$$\text{Now,} \quad \frac{x+y}{2} = \frac{2q+3+2q+1}{2}$$
$$= \frac{2(2q+2)}{2}$$
$$= 2q + 2$$
$$= 2(q+1)$$

which is a positive even integer

$$\text{and} \quad \frac{x-y}{2} = \frac{2q+3-2q-1}{2}$$
$$= \frac{2}{2}$$
$$= 1$$

which is a positive odd integer.

Hence, proved.

Self-Assessment

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Multiple-Choice Questions

1. The least number that is divisible by all the numbers from 1 to 9 (both inclusive) is

(*a*) 10 (*b*) 180

(*c*) 540 (*d*) 2520

Sol. (*d*) 2520

We find the LCM of all numbers from 1 to 9 as follows:

$$\text{LCM} = 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 7$$
$$= 2^3 \times 3^2 \times 5 \times 7$$
$$= 2520$$

2	1, 2, 3, 4, 5, 6, 7, 8, 9
2	1, 1, 3, 2, 5, 3, 7, 4, 9
2	1, 1, 3, 1, 5, 3, 7, 2, 9
3	1, 1, 3, 1, 5, 3, 7, 1, 9
3	1, 1, 1, 1, 5, 1, 7, 1, 3
5	1, 1, 1, 1, 5, 1, 7, 1, 1
7	1, 1, 1, 1, 1, 1, 7, 1, 1
	1, 1, 1, 1, 1, 1, 1, 1, 1

$\therefore$ The least number that is divisible by all the numbers from 1 to 9 is 2520.

2. The decimal expansion of the rational number $\frac{67}{2^4 5^3}$ will terminate after

(*a*) one decimal place.

(*b*) three decimal places.

(*c*) two decimal places.

(*d*) four decimal places.

Sol. (*d*) four decimal places.

We have

$$\frac{67}{2^4 5^3} = \frac{67 \times 5}{2^4 \times 5^3 \times 5}$$
$$= \frac{335}{2^4 \times 5^4}$$
$$= \frac{335}{10^4}$$
$$= 0.0335$$

Hence, the given number will terminate after four decimal places.

3. $n^2 - 1$ is divisible by 8, if n is

(*a*) an integer.

(*b*) an odd integer.

(*c*) a natural number.

(*d*) an even integer.

Sol. (*b*) an odd integer.

We have $\frac{n^2-1}{8} = \frac{(n-1)(n+1)}{8}$ which is not always an integer when n is any integer.

Let n be any even integer, say $n = 2m$.

$$\therefore \qquad \frac{n^2-1}{8} = \frac{(2m-1)(2m+1)}{8} = \frac{4m^2-1}{8}$$

which is not an integer when m is any integer.

Let n be any odd integer, say $n = 2p + 1$.

$$\therefore \qquad \frac{n^2-1}{8} = \frac{(n-1)(n+1)}{8} = \frac{2p(2p+2)}{8} = \frac{p(p+1)}{2}$$

which is always an integer when p is any positive integer, since the product of two consecutive integers is always divisible by 2.

Hence, $n^2 - 1$ is divisible by 8 only when n is an odd integer.

Fill in the Blanks

4. If HCF $(a, 8) = 4$, LCM $(a, 8) = 24$, then a is **12**.

Sol. HCF $(a, b) \times$ LCM $(a, b) = a \times b$

$$4 \times 24 = a \times 8$$

$$\Rightarrow \qquad a = \frac{4 \times 24}{8} = 12$$

5. For some integer q every even integer is of the form **2*q***.

6. If a and b are coprime, then a^2 and b^2 are **coprime**.

Sol. Consider $a = 2$ and $b = 3$, then $a^2 = 4$, $b^2 = 9$

Note: 4 and 9 are coprime.

$\therefore$ a^2 and b^2 are coprime.

7. The maximum number of factors of a prime number is **2**.

Assertion-Reason Type Questions

Directions (Q. Nos. 8 to 10): Each of these questions contains an assertion followed by reason. Read them carefully, and answer the question on the basis of the following options, select the one that best describes the two statements.

(a) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.

(b) Both Assertion and Reason are true but Reason is not the correct explanation of Assertion.

(c) If Assertion is true but Reason is false.

(d) If Assertion is false but Reason is true.

8. **Assertion:** $6 - \sqrt{2} \times \sqrt{2}$ is a rational number.

Reason: $\sqrt{2} \times \sqrt{2}$ is a rational number.

Sol. The correct answer is (a) as both the statements are correct.

$\sqrt{2} \times \sqrt{2} = 2$ which is a rational number, 6 is a rational no. and the difference of two rational no. is always rational no.

Hence, the fact that $\sqrt{2} \times \sqrt{2}$ is rational is the correct explanation why $6 - \sqrt{2} \times \sqrt{2}$ is rational.

9. **Assertion:** The HCF of 844 and 244 is 4.

Reason: 6 is the highest number that can divide both 844 and 244.

Sol. The correct answer is (c) as the HCF of 844 and 244 is 4, hence, the assertion is correct. However, the highest no. which can divide both 844 and 244 is 4 and not 6.

Hence, the reason is incorrect.

10. **Assertion:** 1/3 is a non-terminating decimal.

Reason: The denominator cannot be expressed as 2^n5^m.

Sol. The correct answer is (a) as 1/3 when expressed in decimal gives 0.333 ... which is non-terminating. To be a terminating decimal, the denominator must be of the form 2^n5^n which can be expressed in terms of power of 10. Here the denominator 3 cannot be expressed as 2^n5^m.

Hence, both statements are correct and reason is the correct explanation of the assertion.

Case Study Based Questions

11. Starting out your day with a walk can offer your body a number of health benefits. So, three friends Nehal, Sukanya and Arpit plan to go for a morning walk. They step off together and their steps measure 90 cm, 80 cm and 85 cm respectively. Based on the above information, answer the following questions.

(*a*) What is the minimum distance each of them will cover before they meet again?

(*i*) 612000 cm

(*ii*) 306000 cm

(*iii*) 18000 cm

(*iv*) 12240 cm

Ans. (*iv*) 12240 cm

(*b*) What is the number of common steps covered by all of them?

(*i*) 2 (*ii*) 5

(*iii*) 10 (*iv*) 15

Ans. (*ii*) 5

(*c*) If a and b are two positive integers, then which of the following is correct?

(*i*) HCF $(a, b) \times$ LCM $(a, b) = a \times b$

(*ii*) $a \times$ LCM $(a, b) = b \times$ HCF (a, b)

(*iii*) HCF $(a, b) +$ LCM $(a, b) = a + b$

(*iv*) HCF $(a, b) -$ LCM $(a, b) = a - b$

Ans. (*i*) HCF $(a, b) \times$ LCM $(a, b) = a \times b$

(*d*) A largest positive integer that divides given two positive integers is called

(*i*) HCF.

(*ii*) LCM.

(*iii*) coprime.

(*iv*) none of these

Ans. (*i*) HCF.

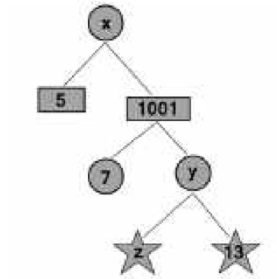
(*e*) The maximum number of factors of a prime number is

(*i*) 1 (*ii*) 2

(*iii*) 3 (*iv*) 4

Ans. (*ii*) 2

12. A mathematics teacher of class X asks one of your friends to make a model of a factor tree for Mathematics exhibition. She/he finds some difficulty and asks for your help in completing the task. Observe the following factor tree and answer the following questions.



(*a*) What is the value of x?

(*i*) 2058 (*ii*) 5005

(*iii*) 7429 (*iv*) 8232

Ans. (*ii*) 5005

(*b*) What is the value of y?

(*i*) 143 (*ii*) 145

(*iii*) 243 (*iv*) 343

Ans. (*i*) 143

(*c*) What is the value of z?

(*i*) 7 (*ii*) 11

(*iii*) 13 (*iv*) 17

Ans. (*ii*) 11

(*d*) Factor tree is used to determine the

(*i*) HCF.

(*ii*) LCM.

(*iii*) prime factors.

(*iv*) none of these

Ans. (*iii*) prime factors.

(*e*) The prime factorisation of the value of x is

(*i*) $5 \times 7 \times 11 \times 13$

(*ii*) $5^2 \times 7 \times 11 \times 13$

(*iii*) $5 \times 7^2 \times 11 \times 13$

(*iv*) $5 \times 7 \times 11^2 \times 13$

Ans. (*i*) $5 \times 7 \times 11 \times 13$

Very Short Answer Type Questions

13. If two positive integers a and b are written as $a = xy^2$, $b = x^3y$ where x and y are prime numbers, then find the LCM (a, b).

Sol. We have $a = x \times y^2$

$b = x^3 \times y$

$\therefore$ LCM of a and b is x^3y^2.

14. If the HCF of 65 and 117 is expressible in the form of $65a - 117$, then find the value of a.

Sol. We first find the HCF of 65 and 117 by using Euclid's division lemma as follows:

We have $11 = 65 \times 1 + 52$ ...(1)

$$65 = 52 \times 1 + 13 \qquad \ldots(2)$$
$$52 = 13 \times 4 + 0 \qquad \ldots(3)$$

$\therefore$ From (3), HCF = 13

Now, from (2),

$$13 = 65 - 52 \qquad \ldots(4)$$

From (1), $\quad 52 = 117 - 65 \qquad \ldots(5)$

$\therefore$ From (4) and (5), we have

$$13 = 65 - (117 - 65)$$
$$= 65 \times 2 - 117$$

Comparing $65a - 117$ with $65 \times 2 - 117$, we see that $a = 2$.

$\therefore$ The required value of a is 2.

15. Find the largest number which divides 615 and 963 leaving remainder 6 in each case.

Sol. The required number will be the HCF of $615 - 6 = 609$ and $963 - 6 = 957$. We now find the HCF of 609 and 957 by using Euclid's division lemma as follows:

We have
$$957 = 609 \times 1 + 348$$
$$609 = 348 \times 1 + 261$$
$$348 = 261 \times 1 + 87$$
$$261 = 87 \times 3 + 0$$

Since the final remainder is zero, hence, the required HCF is 87.

Short Answer Type-I Questions

16. Show that 12^n cannot end with digit 0 or 5 for any natural number n.

Sol. We have $\quad 12^n = (3 \times 2^2)^n = 3^n 2^{2n}$

Since, 3^n and 2^{2n} cannot end with 0 or 5 for any natural number n, hence, 12^n cannot end with 0 or 5, for any natural number n.

17. A rational number in its decimal form is 2.635. What can you say about the prime factors of q when this number is expressed in the form $\frac{p}{q}$?

Sol. We have
$$2.635 = 2\frac{635}{1000} = \frac{2635}{1000} = \frac{527}{200} = \frac{527}{2^3 \times 5^2}$$

Let
$$\frac{p}{q} = \frac{527}{2^3 \times 5^2}$$

Hence, the prime factors of q are 2 and 5.

18. Find the LCM and HCF of 198 and 144. Verify that HCF × LCM = product of two numbers.

Sol. We first find the prime factors of 198 and 144 as follows:

$$198 = 2 \times 3^2 \times 11$$
$$144 = 2^4 \times 3^2$$

$\therefore \quad$ HCF $= 2 \times 3^2 = 18$

and $\quad$ LCM $= 2^4 \times 3^2 \times 11 = 1584$

$$\begin{array}{r|r} 2 & 198 \\ \hline 3 & 99 \\ \hline 3 & 33 \\ \hline 11 & 11 \\ \hline & 1 \end{array} \qquad \begin{array}{r|r} 2 & 144 \\ \hline 2 & 72 \\ \hline 2 & 36 \\ \hline 2 & 18 \\ \hline 3 & 9 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

We see that

LCM (198, 144) = 1584

and $\quad$ HCF(198, 144) = 18

$\therefore \quad$ HCF × LCM $= 18 \times 1584$
$$= 18 \times 8 \times \frac{1584}{8}$$
$$= 144 \times 198$$
= Product of two numbers

Hence, proved.

19. Explain why $11 \times 13 \times 23 + 23 \times 5$ is a composite number.

Sol.
$$11 \times 13 \times 23 + 23 \times 5 = 23\,(11 \times 13 + 5)$$
$$= 23 \times 148$$
$$= 2^2 \times 23 \times 37$$

A number which has more than two factors (other than 1 and itself) is a composite number. Since, the above can be expressed as a product of more than two prime factors, therefore it is a composite number.

Short Answer Type-II Questions

20. Find the HCF using Euclid's division Algorithm.

(*a*) 255 and 867

(*b*) 377, 435 and 667

Sol. (*a*) By Euclid's division lemma, we have
$$867 = 255 \times 3 + 102$$
$$255 = 102 \times 2 + 51$$
$$102 = 51 \times 2 + 0$$

Since, the final remainder is zero.

$\therefore$ The HCF of 255 and 867 is 51.

(*b*) We first find the HCF of 377 and 435 as follows:

By Euclid's division lemma, we have

$$435 = 377 \times 1 + 58$$
$$377 = 58 \times 6 + 29$$
$$58 = 29 \times 2 + 0$$

$\therefore$ HCF (377, 435) = 29

We now find the HCF of 29 and 667 as follows:

$$667 = 29 \times 23 + 0$$

$\therefore$ HCF (29, 667) = 29

$\therefore$ The HCF of 377, 435 and 667 is 29.

21. What is the smallest number which when increased by 6 becomes divisible by 36, 63 and 108?

Sol. We first find the LCM of 36, 63 and 108 as follows:

2	36, 63, 108
2	18, 63, 54
3	9, 63, 27
3	3, 21, 9
3	1, 7, 3
7	1, 7, 1
	1, 1, 1

LCM $= 2 \times 2 \times 3 \times 3 \times 3 \times 7$
$= 756$

Hence, the smallest number which, when increased by 6 becomes divisible by 36, 63 and 108 is 756 – 6 i.e. 750.

22. Find the greatest number that will divide 445, 572, 699 leaving remainders 4, 5, 6 respectively.

Sol. Since 445, 572 and 699, divided by the required number leave remainders 4, 5 and 6 respectively.

Therefore 445 – 4 = 441, 572 – 5 = 567 and 699 – 6 = 693 are completely divisible by the required number.

Clearly, the required number is the HCF of 441, 567 and 693.

Applying Euclid's division lemma to 441 and 567, we get

$$567 = 441 \times 1 + 126$$
$$441 = 126 \times 3 + 63$$
$$126 = 63 \times 2 + 0$$

Since the remainder is zero.

$\therefore$ 63 is the HCF of 441 and 567.

Now applying the division lemma to 693 and 63, we get

$$693 = 63 \times 11 + 0$$

Since the remainder is zero.

$\therefore$ 63 is the HCF of 693 and 63.

$\therefore$ The HCF of 441, 567 and 693 is 63.

Hence, the required number is 63.

23. Prove that $\sqrt{3}$ is an irrational number.

Sol. Let us assume on the contrary that $\sqrt{3}$ is a rational number.

Then $\sqrt{3} = \dfrac{p}{q}$, where p and q are coprime, $q \neq 0$

$\therefore$ On squaring both sides

$$3 = \frac{p^2}{q^2}$$

$\Rightarrow \qquad 3q^2 = p^2 \qquad \ldots(1)$

$\Rightarrow$ p^2 is divisible by 3 $\quad$ [$\because$ $3q^2$ is divisible by 3]

$\Rightarrow$ p is divisible by 3 $\qquad \ldots(2)$

[$\because$ 3 is prime and divides $p^2 \Rightarrow$ 3 divides p]

Let $p = 3c$, for some integer c.

Substituting $p = 3c$ in (1), we get

$$3q^2 = (3c)^2 = 3q^2 = 9c^2$$

$\Rightarrow \qquad q^2 = 3c^2$

$\Rightarrow$ q^2 is divisible by 3 $\quad$ [$\because$ $3c^2$ is divisible by 3]

$\Rightarrow$ q is divisible by 3…(3)

[$\because$ 3 is prime and divides $q^2 \Rightarrow$ 3 divides q]

From (2) and (3)

$\Rightarrow$ both p and q are divisible by 3.

3 is a common factor of p and q.

But this contradicts the fact that p and q are coprime.

This contradiction has arisen because of our incorrect assumption that $\sqrt{3}$ is rational. Hence, $\sqrt{3}$ is irrational.

24. Show that reciprocal of an irrational number is irrational.

Sol. Let $\sqrt{p}$ be an irrational number.

$$\frac{1}{\sqrt{p}} = \frac{1 \times \sqrt{p}}{\sqrt{p} \times \sqrt{p}} = \frac{\sqrt{p}}{p}$$

Let us assume on the contrary that

$\dfrac{1}{\sqrt{p}}$ is rational but $\dfrac{1}{\sqrt{p}} = \dfrac{\sqrt{p}}{p}$.

$\Rightarrow \dfrac{\sqrt{p}}{p}$ is rational.

Let $\dfrac{\sqrt{p}}{p} = \dfrac{a}{b}$, where a and b are coprime and non-zero integers.

$$\Rightarrow \quad \sqrt{p} = \frac{ap}{b}$$

$\Rightarrow$ $\sqrt{p}$ is rational.

[$\because$ p, a and b are integers,

$\therefore$ $\frac{ap}{b}$ is a rational number]

This contradicts the fact that $\sqrt{p}$ is an irrational number. So, our assumption is wrong.

Hence, $\frac{1}{\sqrt{p}}$ is an irrational number.

25. In an examination, the number of students in Class VIII, IX, X are 60, 84, 108 respectively. Find the minimum number of rooms required if in each room, the same number of students are to be seated and all of them being in the same class.

Sol. We have to find the HCF of 60, 84 and 108.

Applying Euclid's division lemma to 60 and 84, we get

$$84 = 60 \times 1 + 24$$
$$60 = 24 \times 2 + 12$$
$$24 = 12 \times 2 + 0$$

Since the remainder is zero.

$\therefore$ HCF of 60 and 84 is 12.

Now, applying the division lemma to 12 and 108.

$$108 = 12 \times 9 + 0$$

Since the remainder is zero

$\therefore$ HCF of 12 and 108 os 12.

$\therefore$ The HCF of 60, 84 and 108 is 12.

$\therefore$ For 60 students of class VIII, we need a minimum of $60 \div 12 = 5$ rooms.

For 84 students of class IX, we need a minimum of $84 \div 12 = 7$ rooms.

Finally, for 108 students of class X, we need a minimum of $108 \div 12 = 9$ rooms.

Hence, the required total minimum number of rooms is $5 + 7 + 9$ i.e. 21.

Long Answer Type Questions

26. Show that if n is an odd integer, then $n^2 - 1$ is divisible by 8.

Sol. We know that any positive odd integer is of the form $2q + 1$ for some integer q.

$$\therefore \quad n = 2q + 1$$

Then, $$\frac{n^2-1}{8} = \frac{(n+1)(n-1)}{8}$$

$$= \frac{2q(2q+2)}{8}$$

$$= \frac{q(q+1)}{2}$$

Now, since the product of two consecutive integers is even i.e. divisible by 2

$\therefore$ $\frac{q(q+1)}{2}$ is an integer.

$\therefore$ $\frac{n^2-1}{8}$ is also an integer when n is an odd integer.

$\therefore$ $n^2 - 1$ is divisible by 8 when n is an odd integer.

27. Show that one and only one out of n, $n + 4$, $n + 8$, $n + 12$, $n + 16$ is divisible by 5, where n is any positive integer.

Sol. Let any positive integer a be divided by 5 and let q be the quotient and r be the remainder. Then, by Euclid's division lemma, we have

$$a = 5q + r \quad \ldots(1)$$

where q is a positive integer and $0 \le r < 5$.

Let $a = n$ be divisible by 5. Then from (1), we get

$$n = 5q + r \quad \ldots(2)$$

where $0 \le r < 5$

Putting $r = 0$ in (2), we get

$$n = 5q \quad \ldots(3)$$

$\Rightarrow$ n is divisible by 5.

Adding 4 both sides in (3), we get

$$n + 4 = 5q + 4$$

$\Rightarrow$ $n + 4$ is not divisible by 5.

Adding 8 both sides in (3), we get

$$n + 8 = 5q + 8$$

$\Rightarrow$ $n + 8$ is not divisible by 5.

Adding 12 both sides in (3), we get

$$n + 12 = 5q + 12$$

$\Rightarrow$ $n + 12$ is not divisible by 5.

Adding 16 both sides in (3), we get

$$n + 16 = 5q + 16$$

$\Rightarrow$ $n + 16$ is not divisible by 5.

Now, putting $r = 1$ in (2), we get

$$n = 5q + 1 \quad \ldots(4)$$

$\Rightarrow$ n is not divisible by 5.

Adding 4 both sides in (4), we get

$$n + 4 = 5q + 1 + 4$$
$$n + 4 = 5q + 5$$

$\Rightarrow \qquad n + 4 = 5(q + 1)$

$\Rightarrow$ $n + 4$ is divisible by 5.

Adding 8 both sides in (4), we get

$$n + 8 = 5q + 1 + 8$$

$\Rightarrow \qquad n + 8 = 5q + 9$

$\Rightarrow$ $n + 8$ is not divisible by 5.

Similarly, we can show that $n + 12$, $n + 16$ are also not divisible by 5.

In the same way, we can show that if any other number say, $n + 4$ is divisible by 5, then none of the remaining numbers n, $n + 8$, $n + 12$ and $n + 16$ is divisible by 5.

Thus, for each value of r such that $0 \le r < 5$ only one out of n, $n + 4$, $n + 8$, $n + 12$ and $n + 16$ is divisible by 5.

Hence, proved.

28. Find the HCF of 96 and 404. Express it as a linear combination of x and y.

Sol. We start with the greater number 404.

Applying Euclid's lemma, we get

$$404 = 96 \times 4 + 20 \qquad \ldots(1)$$

```
96 ) 404 ( 4
     384
     20 ) 96 ( 4
          80
          16 ) 20 ( 1
               16
                4 ) 16 ( 4
                    16
                     0
```

$\because$ $r = 20 \neq 0$, take divisor 96 as the new dividend and remainder 20 as new divisor, we get

$$96 = 20 \times 4 + 16 \qquad \ldots(2)$$

Continue the process till remainder, $r = 0$

$\therefore \qquad 20 = 16 \times 1 + 4 \qquad \ldots(3)$

$$16 = 4 \times 4 + 0 \qquad \ldots(4)$$

$\because$ $r = 0$, the divisor at last step is the HCF.

$\therefore$ HCF of 96 and 404 is 4.

From (3), $\qquad 20 = 16 \times 1 + 4$

$\Rightarrow \qquad 4 = 20 - 16 \times 1 \qquad \ldots(5)$

From (1),

$$404 = 96 \times 4 + 20$$

$\Rightarrow \qquad 20 = 404 - 96 \times 4 \qquad \ldots(6)$

From (2),

$$96 = 20 \times 4 + 16$$

$$16 = 96 - 20 \times 4$$

$\Rightarrow \qquad 16 = 96 - 4\,(404 - 96 \times 4)$ [From 6]

$\Rightarrow \qquad 16 = 96 - 4 \times 404 + 96 \times 16$

$\Rightarrow \qquad 16 = 96 \times 17 - 4 \times 404 \qquad \ldots(7)$

Using value of 20 and 16 from (6) and (7) in (5)

$$4 = 404 - 96 \times 4 - 96 \times 17 + 4 \times 404$$

$$4 = 404 \times 5 - 96 \times 21$$

$$4 = 404 \times (5) + 96 \times (-21)$$

$\Rightarrow \qquad 4 = 404x + 96y$

where $x = 5$, $y = -21$.

Let's Compete

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Multiple-Choice Questions

1. How many prime numbers are of the form $10n + 1$, where n is a natural number such that $1 \le n < 10$?

(*a*) 3 (*b*) 4

(*c*) 5 (*d*) 6

Sol. (*c*) 5

Putting n = 1, 2, 3, 4, 5, 6, 7, 8 and 9 in the expression $N = 10n + 1$, we get the following numbers respectively: 11, 21, 31, 41, 51, 61, 71, 81 and 91. Only 11, 31, 41, 61 and 71 are prime and the remaining numbers are composite. Hence, the total number of prime numbers is 5.

2. If 3 is the least prime factor of p and 5 is the least prime factor of q, then the least prime factor of $(p + q)$ is

(*a*) 2 (*b*) 3

(*c*) 5 (*d*) 11

Sol. (*a*) 2

The least prime factor of p is 3.

Let us assume that p is a product of 3 and x where x is a prime number greater than 3, say $x = 5$.

Now, $\qquad p = x \times 3 = 5 \times 3 = 15$

The least prime factor of q is 5. Let us assume that q is product of 5 and x, where x is a prime greater than 5, say $x = 7$.

Now, $(p + q)$ becomes $(15 + 35) = 50$

$\therefore$ The least prime factor of 50 will be 2 since $50 = 2 \times 5 \times 5$.

$\therefore$ The least prime factor of $(p + q)$ is 2.

3. $157^2 - 151^2$ is a

(*a*) prime number.

(*b*) composite number.

(*c*) an odd prime number.

(*d*) an odd composite number.

Sol. (*b*) composite number.

$$\begin{aligned} 157^2 - 151^2 &= (157 + 151)(157 - 151) \\ &= 308 \times 6 \\ &= 2^2 \times 7 \times 11 \times 3 \times 2 \\ &= 2^3 \times 7 \times 11 \times 3 \quad \ldots(1) \end{aligned}$$

Since, the given number has more than two prime factors. Hence, it must be a composite number and it is clear from (1) that this number will end with an even integer. So, this composite number cannot be odd.

4. The traffic lights at three different road crossings change after every 48 s, 72 s and 108 s respectively. If they change simultaneously at 9:00 am, at what time will they change together again?

(*a*) 10 : 00 am 10 min 5 s

(*b*) 9 : 00 am 7 min 12 s

(*c*) 9 : 00 am 12 min 7 s

(*d*) 10 : 00 am 5 min 12 s

Sol. (*b*) 9 : 00 am 7 min 12 s

We first find the LCM of 48, 72 and 108 as follows:

$$\text{LCM} = 2^4 \times 3^3 = 16 \times 27 = 432$$

2	48, 72, 108
2	24, 36, 54
2	12, 18, 27
2	6, 9, 27
3	3, 9, 27
3	1, 3, 9
3	1, 1, 3
	1, 1, 1

Hence, after 432 s i.e. after 7 min 12 s from 9 am, the traffic lights will change together again.

Hence, the required time is 9 : 00 am 7 min 12 s.

5. If $x = 2^2 \times 3^a$, $y = 2^3 \times 3 \times 5$, $z = 2^2 \times 3^2 \times 7$ and LCM $(x, y, z) = 7560$, then a is equal to

(*a*) 1 (*b*) 2

(*c*) 3 (*d*) 4

Sol. (*c*) 3

We have
$$x = 2^2 \times 3^a$$
$$y = 2^3 \times 3 \times 5$$
$$z = 2^2 \times 3^2 \times 7$$

LCM $(x, y, z) = 7560$

LCM $(x, y, z) = 2^3 \times 3^a \times 5 \times 7$

(Assuming that $a \geq 3$)

$$7560 = 2^3 \times 3^a \times 5 \times 7$$

$$\Rightarrow \quad \frac{7560}{2^3 \times 5 \times 7} = 3^a$$

$$\Rightarrow \quad 27 = 3^a$$

$$\Rightarrow \quad 3^3 = 3^a$$

$$\Rightarrow \quad a = 3$$

6. If n is any natural number, then $8^{2n} - 3^{2n}$ is always divisible by

(*a*) 5 (*b*) 11

(*c*) both 5 and 11 (*d*) none of these

Sol. (*c*) both 5 and 11

Given n is a natural number, we have $8^{2n} - 3^{2n}$

If $n = 1$ then $8^2 - 3^2 = 64 - 9 = 55$ is always divisible by both 5 and 11.

If $n = 2$ then $8^4 - 3^4 = 4096 - 81 = 4015$ is also divisible by both 5 and 11.

$\therefore$ For every n, $8^{2n} - 3^{2n}$ is always divisible by both 5 and 11.

7. The smallest irrational number by which $\sqrt{27}$ should be multiplied so as to get a rational number is

(*a*) $\sqrt{2}$ (*b*) $\sqrt{27}$

(*c*) $3\sqrt{3}$ (*d*) $\sqrt{3}$

Sol. (*d*) $\sqrt{3}$

Since $\sqrt{27} = \sqrt{9 \times 3} = 3\sqrt{3}$, we see that if we multiply this by the least irrational number $\sqrt{2}$, we get $3\sqrt{3} \times \sqrt{2} = 3\sqrt{6}$ which is again an irrational number. But if we multiply $3\sqrt{3}$ by $\sqrt{3}$, we get $3 \times 3 = 9$ which is a rational number.

Hence, the required smallest irrational number is $\sqrt{3}$.

8. A pair of irrational numbers whose product is a rational number is

(*a*) $\sqrt{16}\sqrt{4}$ (*b*) $\sqrt{5}\sqrt{2}$

(*c*) $\sqrt{3}\sqrt{27}$ (*d*) $\sqrt{36}\sqrt{2}$

Sol. (*c*) $\sqrt{3}\sqrt{27}$

Both $\sqrt{3}$ and $\sqrt{27} = \sqrt{9 \times 3} = 3\sqrt{3}$ are irrational numbers and their product $\sqrt{3} \times 3\sqrt{3} = 9$, which is a rational number.

$\therefore$ $\sqrt{3}\sqrt{27}$ is a rational number.

9. The LCM of two numbers is 1800. Which of the following cannot be their HCF?

(*a*) 500 (*b*) 200

(*c*) 300 (*d*) 600

Sol. (*a*) 500

We know that the LCM of any two positive numbers is divisible by their HCF. In the present problem, we see that 1800 is not exactly divisible by only 500 in (*a*), but it is exactly divisible by 200, 300 or 600 in (*b*), (*c*) and (*d*). Hence only 500 in (*a*) cannot be the HCF of the two number.

10. If d is the HCF of 45 and 63 and satisfying $d = 45x + 63y$, then the values of x and y are

(*a*) $x = 3, y = -2$

(*b*) $x = 2, y = -3$

(*c*) $x = -3, y = 2$

(*d*) $x = 3, y = 2$

Sol. (*a*) $x = 3, y = -2$

We first find the value of d, the HCF of 45 and 63 by using Euler's division lemma as follows:

$$63 = 45 \times 1 + 18 \quad \ldots(1)$$

$$45 = 18 \times 2 + 9 \quad \ldots(2)$$

$$18 = 9 \times 2 + 0 \quad \ldots(3)$$

$\therefore$ From (3), HCF of 45 and 63 is 9.

Now, from (2),

$$\begin{aligned} 9 &= 45 - 18 \times 2 \\ &= 45 - (63 - 45) \times 2 \quad \text{[From (1)]} \\ &= 45 - 63 \times 2 + 45 \times 2 \\ &= 45 \times 3 - 63 \times 2 \\ &= 45x + 63y \end{aligned}$$

where $x = 3$ and $y = -2$.

$\therefore$ The values of x and y are 3 and -2 respectively.

Value-based Question (Optional)

(Page 19)

1. Ritika has two ribbons which are 84 cm and 98 cm long. These ribbons are of same colour and material. She wants to cut them into equal pieces so that no ribbon is wasted.

(*a*) If she gifts one pair each to her two daughters then how many ribbon pieces are left with her?

(*b*) What values are shown by Ritika?

Sol. (*a*) We find the maximum length of each ribbon that can be cut from two ribbons. So, we first find the HCF of 84 and 98 by using Euclid's division lemma as follows:

$$98 = 84 \times 1 + 14$$

$$84 = 14 \times 6 + 0$$

Hence, HCF of 84 and 98 is 14.

Hence, the maximum length of each piece of each ribbon is 14 cm.

Now, the number of pieces of the first ribbon $= 84 \text{ cm} \div 14 \text{ cm} = 6$

and the number of piece of the second ribbon $= 98 \text{ cm} \div 14 \text{ cm} = 7$

Out of 6 pieces and 7 pieces of these two ribbons, she gifted 2 pieces to each of her two daughters. Hence, the total number of pieces of ribbons which are left with her is

$$(6 - 2) + (7 - 2) = 4 + 5 = 9$$

Hence, the required number of pieces of ribbons which are left with her is 9.

(*b*) Problem solving and empathy.

2

Polynomials

Checkpoint (Page 22)

1. Write any polynomial in x which has no real zero at all. Write down its value when $x = -1$.

Sol. Clearly, $x^2 + 1$ is a polynomial which has no real zero at all, since $x^2 + 1$ cannot be zero for any real value of x.

For if $x^2 + 1 = 0$, then $x = \pm\sqrt{-1}$ which are not real.

Now, when $x = -1$, then $x^2 + 1 = (-1)^2 + 1 = 2$ which is the required value of the polynomial $x^2 + 1$ at $x = -1$.

2. Find the remainder when $4\sqrt{2}x^2 + 3x + 5$ is divided by $x + \sqrt{2}$, by using remainder theorem.

Sol. Let $f(x) = 4\sqrt{2}x^2 + 3x + 5$.

$\therefore$ By remainder theorem, the required remainder is

$$\begin{aligned} f\left(-\sqrt{2}\right) &= 4\sqrt{2}\left(-\sqrt{2}\right)^2 + 3\left(-\sqrt{2}\right) + 5 \\ &= 8\sqrt{2} - 3\sqrt{2} + 5 \\ &= 5\sqrt{2} + 5 \\ &= 5\left(\sqrt{2} + 1\right) \end{aligned}$$

3. By using the factor theorem, show that $x + 3$ is a factor of the polynomial $2x^4 + 6x^3 - 3x^2 - 5x + 12$.

Sol. Let $f(x) = 2x^4 + 6x^3 - 3x^2 - 5x + 12$

We see that

$$\begin{aligned} f(-3) &= 2(-3)^4 + 6(-3)^3 - 3(-3)^2 - 5(-3) + 12 \\ &= 162 - 162 - 27 + 15 + 12 \\ &= 0 - 27 + 27 \\ &= 0 \end{aligned}$$

Hence, by factor theorem $x + 3$ is a factor of $f(x)$.

4. Find the value of a if $x - 1$ is a factor of $a^2x^3 - 4ax + 4a - 1$.

Sol. Let $f(x) = a^2x^3 - 4ax + 4a - 1$

$\because$ $x - 1$ is a factor of $f(x)$,

$\therefore$ By factor theorem, we have

$$f(1) = 0$$

$$\Rightarrow \quad a^2 - 4a + 4a - 1 = 0$$

$$\Rightarrow \quad a^2 = 1$$

$$\Rightarrow \quad a = \pm 1$$

which is the required value of a.

5. Using identities, find the product

$$\left(x - \frac{1}{x}\right)\left(x + \frac{1}{x}\right)\left(x^2 + \frac{1}{x^2}\right)\left(x^4 + \frac{1}{x^4}\right).$$

Sol. We have

$$\begin{aligned} &\left(x - \frac{1}{x}\right)\left(x + \frac{1}{x}\right)\left(x^2 + \frac{1}{x^2}\right)\left(x^4 + \frac{1}{x^4}\right) \\ &= \left(x^2 - \frac{1}{x^2}\right)\left(x^2 + \frac{1}{x^2}\right)\left(x^4 + \frac{1}{x^4}\right) \\ &= \left(x^4 - \frac{1}{x^4}\right)\left(x^4 + \frac{1}{x^4}\right) \\ &= \left(x^8 - \frac{1}{x^8}\right) \end{aligned}$$

which is the required product.

6. If $a + b = 4$ and $a - b = 2$, find the value of $a^2 + b^2$.

Sol. We have

$$\begin{aligned} a^2 + b^2 &= \frac{(a + b)^2 + (a - b)^2}{2} \\ &= \frac{4^2 + 2^2}{2} \end{aligned}$$

$$= \frac{16+4}{2}$$

$$= \frac{20}{2}$$

$$= 10$$

which is the required value.

7. Without actually calculating the cubes, evaluate $75^3 - 40^3 - 35^3$.

Sol. We have

$75^3 - 40^3 - 35^3$

$$= (75 - 40)(75^2 + 75 \times 40 + 40^2) - 35^3$$
$$= 35(75^2 + 3000 + 40^2) - 35^3$$
$$= 35[75^2 - 35^2 + 3000 + 1600]$$
$$= 35[(75 + 35)(75 - 35) + 4600]$$
$$= 35[110 \times 40 + 4600]$$
$$= 35 \times (4400 + 4600)$$
$$= 35 \times 9000$$
$$= 315000$$

which is the required value.

8. Factorise: $3 - 12(a - b)^2$

Sol. We have

$$3 - 12(a - b)^2 = 3[1 - 4(a - b)^2]$$
$$= 3[1^2 - \{2(a - b)\}^2]$$
$$= 3[1 + 2(a - b)][1 - 2(a - b)]$$
$$= 3(1 + 2a - 2b)(1 - 2a + 2b)$$

which are the required factors.

9. Factorise: $(a^2 - 2a)^2 - 23(a^2 - 2a) + 120$

Sol. We have

$(a^2 - 2a)^2 - 23(a^2 - 2a) + 120$

$$= x^2 - 23x + 120 \quad \text{where } x = a^2 - 2a \quad \ldots(1)$$
$$= x^2 - 8x - 15x + 120$$
$$= x(x - 8) - 15(x - 8)$$
$$= (x - 15)(x - 8)$$
$$= (a^2 - 2a - 15)(a^2 - 2a - 8)$$
$$= (a^2 + 3a - 5a - 15)(a^2 + 2a - 4a - 8)$$
$$= \{a(a + 3) - 5(a + 3)\}\{a(a + 2) - 4(a + 2)\}$$
$$= (a + 3)(a - 5)(a + 2)(a - 4)$$

which are the required factors.

10. Using factor theorem, factorise $2x^3 - x^2 - 13x - 6$.

Sol. We have $f(x) = 2x^3 - x^2 - 13x - 6$

We see that $f(-2) = 2 \times (-2)^3 - (-2)^2 - 13(-2) - 6$

$$= -16 - 4 + 26 - 6$$
$$= 0$$

$\therefore$ By factor theorem, $x + 2$ will be a factor of $f(x)$.

Now, we write

$$f(x) = 2x^3 - x^2 - 13x - 6$$
$$= 2x^3 + 4x^2 - 5x^2 - 10x - 3x - 6$$
$$= 2x^2(x + 2) - 5x(x + 2) - 3(x + 2)$$
$$= (x + 2)(2x^2 - 5x - 3)$$
$$= (x + 2)(2x^2 + x - 6x - 3)$$
$$= (x + 2)\{x(x + 1) - 3(2x + 1)\}$$
$$= (x + 2)(x - 3)(2x + 1)$$

which are the required factors.

Milestone 1

(Page 26)

Multiple-Choice Questions

1. A quadratic polynomial whose sum and product of zeroes are respectively 3 and −10 is

(*a*) $x^2 - 3x - 10$
(*b*) $x^2 + 3x + 10$
(*c*) $x^2 + 3x - 10$
(*d*) $x^2 - 3x + 10$

Sol. (*a*) $x^2 - 3x - 10$

We have,

Sum of the zeroes = 3 and

Product of the zeroes = −10

Hence, the required polynomial is

$x^2 -$ (Sum of the zeroes)x + Product of zeroes

$$= x^2 - 3x - 10$$

2. If the product of the zeroes of the quadratic polynomial $2x^2 - 3x - 5c$ is $\frac{1}{2}$, then the value of c is equal to

(*a*) −5 (*b*) 5
(*c*) $\frac{1}{5}$ (*d*) $-\frac{1}{5}$

Sol. (*d*) $-\frac{1}{5}$

We see that the product of zeroes of the given polynomial is $\frac{-5c}{2}$

$$\therefore \quad \frac{-5c}{2} = \frac{1}{2}$$

$$\Rightarrow \quad c = -\frac{1}{5}$$

Very Short Answer Type Questions

3. If $p(x) = 3x^2 - 9$, find the value of $p\left(a+\frac{1}{a}\right)$ where $a \neq 0$. Hence, find the value of $p(2)$.

Sol. We have

$$\begin{aligned} p(x) &= 3x^2 - 9 \\ &= 3(x^2 - 3) \\ &= 3\left(x+\sqrt{3}\right)\left(x-\sqrt{3}\right) \end{aligned}$$

$$\therefore \quad p\left(a+\frac{1}{a}\right) = 3\left(a+\frac{1}{a}+\sqrt{3}\right)\left(a+\frac{1}{a}-\sqrt{3}\right)$$

$\therefore$ Putting $a = 1$, we get

$$\begin{aligned} p(2) &= 3\left(2+\sqrt{3}\right)\left(2-\sqrt{3}\right) \\ &= 3(2^2 - 3) \\ &= 3(4-3) \\ &= 3 \end{aligned}$$

which is the required value.

4. One zero of a quadratic polynomial is $\sqrt{3}$ and the product of the two zeroes is $-5\sqrt{3}$. Find the quadratic polynomial.

Sol. Let α and β be two zeroes of the polynomial, where $\alpha = \sqrt{3}$,

Now it is given that

$$\alpha\beta = -5\sqrt{3}$$

$$\therefore \quad \beta = \frac{-5\sqrt{3}}{\sqrt{3}} = -5$$

$\therefore$ The required polynomial is

x^2 – (Sum of the zeroes)x + Product of the zeroes

$$\begin{aligned} &= x^2 - \left(-5+\sqrt{3}\right)x - 5\sqrt{3} \\ &= x^2 + \left(5-\sqrt{3}\right)x - 5\sqrt{3} \end{aligned}$$

5. Write down the quadratic polynomial whose zeroes are $3+\sqrt{5}$ and $3-\sqrt{5}$. **[CBSE SP 2011]**

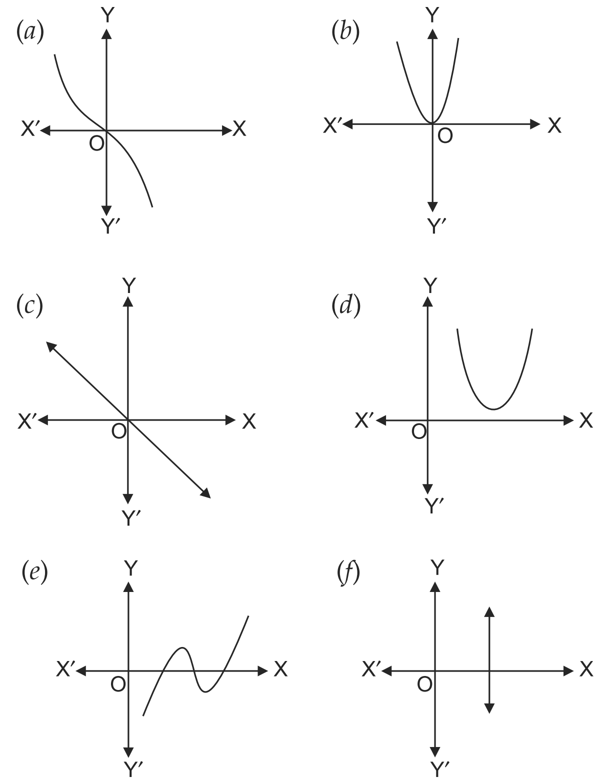
Sol. The required quadratic polynomial is

x^2 + (Sum of the zeroes)x + Product of the zeroes

$$\begin{aligned} &= x^2 - \left(3+\sqrt{5}+3-\sqrt{5}\right)x + \left(3+\sqrt{5}\right)\left(3-\sqrt{5}\right) \\ &= x^2 - 6x + (9-5) \\ &= x^2 - 6x + 4 \end{aligned}$$

Short Answer Type-I Questions

6. Which of the following correspond to the graph of a linear or quadratic polynomial equation and find the number of zeroes of each polynomial.



Sol. (*a*) The graph is neither a line nor a parabola. Hence, the graph is neither linear nor quadratic. Since the graph cuts the x-axis at only one point, hence this polynomial has only one zero.

(*b*) Since the graph is a parabola in shape, hence it is a graph of a quadratic polynomial equation. Since it cuts the x-axis at only one point, hence this polynomial has only one zero.

(*c*) This graph is a straight line and it cuts the x-axis at only one point. Hence, the polynomial is a linear one and this polynomial has only one zero.

(*d*) This graph is a parabola. Hence, it is the graph of a quadratic polynomial equation. Since this graph does not cut the x-axis at all. Hence, this polynomial has no zero.

(*e*) This graph is neither a parabola nor a line. Hence, the polynomial is neither linear nor quadratic. Again, this graph cuts the x-axis at three point. Hence, the polynomial has three zeroes.

(*f*) This graph is a line intersecting x-axis at only one point. Hence, the polynomial is linear and has only one zero.

7. If the sum of the zeroes of the quadratic polynomial $f(y) = ky^2 + 2y + 3k$ is equal to their product, find the value of k. **[CBSE SP 2013]**

Sol. We know that sum of the zeroes of the given polynomial $= -\frac{2}{k}$ and the product of the zeroes $= \frac{3k}{k} = 3$.

It is given that

$$-\frac{2}{k} = 3$$

$$\Rightarrow \qquad k = -\frac{2}{3}$$

which is the required value of k.

8. If 2 and −3 are the zeroes of the quadratic polynomial $x^2 + (a+1)x + b$, then find the values of a and b. **[CBSE SP 2016]**

Sol. We have

Sum of the zeroes $= -(a+1)$

and Product of the zeroes $= b$

$\therefore \qquad -(a+1) = 2 - 3 = -1$

$\therefore \qquad a = 0$

and $\qquad b = 2 \times (-3) = -6$

$\therefore$ The required values of a and b are 0 and −6 respectively.

Short Answer Type-II Questions

9. If α and β are the zeroes of the quadratic polynomial $3x^2 - 5x - 2$, find the value of $\alpha^2 + \beta^2$.

Sol. We have

$$\alpha + \beta = \text{Sum of zeroes} = \frac{5}{3} \qquad \ldots(1)$$

and $$\alpha\beta = \text{Product of zeroes} = -\frac{2}{3} \qquad \ldots(2)$$

$$\therefore \quad (\alpha^2 + \beta^2) = (\alpha + \beta)^2 - 2\alpha\beta$$

$$= \left(\frac{5}{3}\right)^2 + 2 \times \frac{2}{3} \quad \text{[From (1) and (2)]}$$

$$= \frac{25}{9} + \frac{4}{3}$$

$$= \frac{25 + 12}{9}$$

$$= \frac{37}{9}$$

which is the required value.

10. If α and β are zeroes of the polynomial $2x^2 - 5x + 7$, then find a quadratic polynomial whose zeroes are $(3\alpha + 4\beta)$ and $(4\alpha + 3\beta)$. **[CBSE SP 2013]**

Sol. We have

$$\alpha + \beta = \text{Sum of the zeroes} = \frac{5}{2} \qquad \ldots(1)$$

and $$\alpha\beta = \text{Product of zeroes} = \frac{7}{2} \qquad \ldots(2)$$

Now, the quadratic polynomial whose zeroes are $3\alpha + 4\beta$ and $4\alpha + 3\beta$ is

$x^2 - (\text{Sum of the zeroes})x + (\text{Product of the zeroes})$

$= x^2 - (3\alpha + 4\beta + 4\alpha + 3\beta)x + (3\alpha + 4\beta)(4\alpha + 3\beta)$

$= x^2 - 7(\alpha + \beta)x + 12\alpha^2 + 12\beta^2 + 25\,\alpha\beta$

$= x^2 - 7 \times \frac{5}{2}x + 12\{(\alpha + \beta)^2 - 2\,\alpha\beta\} + 25\alpha\beta$

[From (1)]

$= x^2 - \frac{35}{2}x + 12(\alpha + \beta)^2 + \alpha\beta$

$= x^2 - \frac{35}{2}x + 12 \times \frac{25}{4} + \frac{7}{2}$ [From (1) and (2)]

$= x^2 - \frac{35}{2}x + 75 + \frac{7}{2}$

$= x^2 - \frac{35}{2}x + \frac{157}{2}$

$= \frac{1}{2}\left(2x^2 - 35x + 157\right)$

Hence, the required polynomial is $2x^2 - 35x + 157$.

Long Answer Type Questions

11. If $\sqrt{5}$ and $-\sqrt{5}$ are two zeroes of the polynomial $x^3 + 3x^2 - 5x - 15$, then find its third zero. **[CBSE 2010]**

Sol. If α is the third zero of the polynomial $x^3 + 3x^2 - 5x - 15$, then

$$\text{product of the zeroes} = \alpha\sqrt{5}\left(-\sqrt{5}\right)$$

$\Rightarrow \qquad 15 = -5\alpha$

$\Rightarrow \qquad -5\alpha = 15$

$\Rightarrow \qquad \alpha = -3$

Hence, the required zero is −3.

12. The product of two zeroes of the polynomial $p(x) = x^3 - 6x^2 + 11x - 6$ is 6. Find all the zeroes of the polynomial.

Sol. It α, β and γ be the three zeroes of the polynomial

$$p(x) = x^3 - 6x^2 + 11x - 6$$

then $\qquad \alpha + \beta + \gamma = 6 \qquad \ldots(1)$

$\qquad \alpha\beta + \alpha\gamma + \beta\gamma = 11 \qquad \ldots(2)$

and $\qquad \alpha\beta\gamma = 6 \qquad \ldots(3)$

Let $\qquad \beta\gamma = 6 \qquad$ [Given] ...(4)

$\therefore \qquad \alpha = 1 \qquad$ [From (3)]

$\therefore$ Either $\beta = 3$ and $\gamma = 2$ or $\beta = 6$ and $\gamma = 1$.

But $\beta = 3$, $\gamma = 2$ and $\alpha = 1$ satisfy (1) and not $\beta = 6, \gamma = 1, \alpha = 1$.

Hence, the required zeroes of the given polynomial are 1, 2 and 3.

Milestone 2

(Page 28)

Multiple-Choice Questions

1. When a polynomial $p(x)$ is divided by the polynomial $x^2 + 2x + 3$, then the remainder is 4 and quotient is $x - 3$. Then $p(x)$ is equal to

(*a*) $x^3 - x^2 + 3x + 5$ (*b*) $x^3 - x^2 - 3x - 5$

(*c*) $x^3 + x^2 + 3x + 5$ (*d*) $x^3 + x^2 + 3x - 5$

Sol. (*b*) $x^3 - x^2 - 3x - 5$

We have by division algorithm,

$$\begin{aligned} p(x) &= (x^2 + 2x + 3)(x - 3) + 4 \\ &= x^3 + 2x^2 + 3x - 3x^2 - 6x - 9 + 4 \\ &= x^3 - x^2 - 3x - 5 \end{aligned}$$

2. The degree of the remainder when a cubic polynomial is divided by a linear polynomial is

(*a*) more than two. (*b*) two.

(*c*) one. (*d*) zero.

Sol. (*d*) zero.

We know that the degree of the remainder is less than that of the divisor which is linear. Since the degree of the polynomial in the divisor is 1, hence the required degree of the remainder will be zero.

Very Short Answer Type Questions

3. What must be added to the polynomial $3x^2 + 4x + 5$ so that the resulting polynomial is exactly divisible by $x - 2$?

Sol. We first of all divide $3x^2 + 4x + 5$ by $x - 2$ as follows:

$$\begin{array}{r|l|l} x-2 & 3x^2 + 4x + 5 & 3x + 10 \\ & \underline{\overset{-}{3x^2} \overset{+}{-} 6x} & \\ & 10x + 5 & \\ & \underline{\overset{-}{10x} \overset{+}{-} 20} & \\ & 25 & \end{array}$$

$\therefore$ We must subtract 25 from $3x^2 + 4x + 5$ so that the resulting polynomial is exactly divisible by $x - 2$. In other words, we must add -25 with the dividend so that the resulting polynomial is exactly divisible by $x - 2$.

Hence, the required number is -25.

4. When a polynomial $p(x)$ is divided by $x^2 + 2$, then the remainder is zero and the quotient is $(x^2 + 2)^2$. What is $p(x)$ and what is its degree?

Sol. By division algorithm, we have $p(x) = (x^2 + 2)^2 \times (x^2 + 2) = (x^2 + 2)^3$ which, when expanded, is a polynomial of degree 6.

$\therefore$ The required polynomial and degree are respectively $(x^2 + 2)^3$ and 6.

Short Answer Type-I Questions

5. Find the value of b for which $2x + 3$ is a factor of $2x^3 + 9x^2 - x - b$. **[CBSE SP 2012]**

Sol. If $f(x) = 2x^3 + 9x^2 - x - b$, then $2x + 3$ will be a factor of $f(x)$ if $f\left(-\frac{3}{2}\right) = 0$ by factor theorem.

$$\therefore \quad f\left(-\frac{3}{2}\right) = 2\left(-\frac{3}{2}\right)^3 + 9\left(-\frac{3}{2}\right)^2 - \left(-\frac{3}{2}\right) - b = 0$$

$$\Rightarrow \quad -\frac{27}{4} + \frac{9 \times 9}{4} + \frac{3}{2} - b = 0$$

$$\Rightarrow \quad b = -\frac{27}{4} + \frac{81}{4} + \frac{3}{2}$$

$$= \frac{-27 + 81 + 6}{4}$$

$$= \frac{60}{4}$$

$$= 15$$

Hence, the required value of b is 15.

6. Find all the zeroes of $p(x) = x^3 - 9x^2 - 12x + 20$ if $x + 2$ is a factor of $p(x)$. **[CBSE SP 2011]**

Sol. Since $x + 2$ is a factor of $p(x) = x^3 - 9x^2 - 12x + 20$, hence $p(x)$ will be divisible by $x + 2$ and the quotient will be the other factor. So, we first of all divide $p(x)$ by $x + 2$ as follows:

$$\begin{array}{r|l|l} x+2 & x^3 - 9x^2 - 12x + 20 & x^2 - 11x + 10 \\ & \underline{\overset{-}{x^3} \overset{-}{+} 2x^2} & \\ & -11x^2 - 12x + 20 & \\ & \underline{\overset{+}{-11x^2} \overset{+}{-} 22x} & \\ & 10x + 20 & \\ & \underline{\overset{-}{10x} \overset{-}{+} 20} & \\ & 0 & \end{array}$$

Hence, the other factor of $p(x)$ is $x^2 - 11x + 10$.

Now, $x^2 - 11x + 10 = x^2 - 10x - x + 10$

$= x(x - 10) - 1\,(x - 10)$

$= (x - 1)\,(x - 10)$

$\therefore$ The other two zeroes of $p(x)$ will be 1 and 10.

Hence, the required zeroes of $p(x)$ are 1, 10 and –2.

Short Answer Type-II Questions

7. Obtain all the zeroes of $x^4 + 5x^3 - 2x^2 - 40x - 48$ if two of its zeroes are $2\sqrt{2}$ and $-2\sqrt{2}$. **[CBSE SP 2011]**

Sol. Let $f(x) = x^4 + 5x^3 - 2x^2 - 40x - 48$

Since $2\sqrt{2}$ and $-2\sqrt{2}$ are two of its zeroes, hence $x - 2\sqrt{2}$ and $x + 2\sqrt{2}$ will both be factors of $f(x)$, i.e. $x^2 - \left(2\sqrt{2}\right)^2 = x^2 - 8$ will be a factor of $f(x)$. We now divide $f(x)$ by $x^2 - 8$ as follows to get another factor.

$$\begin{array}{r|l|l}
x^2 - 8 & x^4 + 5x^3 - 2x^2 - 40x - 48 & x^2 + 5x + 6 \\
 & \underline{\underset{-}{x^4} \qquad \underset{+}{-}\,8x^2} & \\
 & \quad 5x^3 + 6x^2 - 40x - 48 & \\
 & \quad \underline{\underset{-}{5x^3} \qquad \underset{+}{-}\,40x} & \\
 & \qquad 6x^2 - 48 & \\
 & \qquad \underline{\underset{-}{6x^2} \underset{+}{-}\,48} & \\
 & \qquad\quad 0 &
\end{array}$$

$\therefore$ The other factor is $x^2 + 5x + 6$

Now, $x^2 + 5x + 6 = x^2 + 3x + 2x + 6$

$= x(x + 3) + 2\,(x + 3)$

$= (x + 3)\,(x + 2)$

$\therefore$ The other two zeroes of $p(x)$ are -2 and -3.

Hence, the required four zeroes of $p(x)$ are –2, –3, $2\sqrt{2}$ and $-2\sqrt{2}$.

8. When a polynomial $f(x)$ is divided by $x^2 - 5$, the quotient is $x^2 - 2x - 3$ and remainder is 0. Find the polynomial and all its zeroes. **[CBSE SP 2011]**

Sol. By division algorithm, we have

$f(x) = (x^2 - 5)\,(x^2 - 2x - 3)$

$= x^4 - 2x^3 - 3x^2 - 5x^2 + 10x + 15$

$= x^4 - 2x^3 - 8x^2 + 10x + 15$

We factorise $f(x) = (x^2 - 5)\,(x^2 - 2x - 3)$ as follows:

$f(x) = \left(x + \sqrt{5}\right)\left(x - \sqrt{5}\right)(x^2 + x - 3x - 3)$

$= \left(x + \sqrt{5}\right)\left(x - \sqrt{5}\right)\{x(x + 1) - 3\,(x + 1)\}$

$= \left(x + \sqrt{5}\right)\left(x - \sqrt{5}\right)(x + 1)\,(x - 3)$

Hence, required zeroes of $f(x)$ are $-\sqrt{5}$, $\sqrt{5}$, –1 and 3.

Long Answer Type Questions

9. What must be subtracted from the polynomial $x^4 + 2x^3 - 7x^2 - 11x + 18$ so that the resulting polynomial is exactly divisible by $x^2 + x - 6$?

Sol. We divide $x^4 + 2x^3 - 7x^2 - 11x + 18$ by $x^2 + x - 6$ as follows:

$$\begin{array}{r|l|l}
x^2 + x - 6 & x^4 + 2x^3 - 7x^2 - 11x + 18 & x^2 + x - 2 \\
 & \underline{\underset{-}{x^4} \underset{-}{+}\, x^3 \underset{+}{-}\, 6x^2} & \\
 & \quad x^3 - x^2 - 11x + 18 & \\
 & \quad \underline{\underset{-}{x^3} \underset{-}{+}\, x^2 \underset{+}{-}\, 6x} & \\
 & \qquad -2x^2 - 5x + 18 & \\
 & \qquad \underline{\underset{+}{-2x^2} \underset{+}{-}\, 2x \underset{-}{+}\, 12} & \\
 & \qquad\qquad -3x + 6 &
\end{array}$$

Hence, the required expression which should be subtracted from the given polynomial is $-3x + 6$.

10. Find all the zeroes of the polynomial $3x^4 + 6x^3 - 2x^2 - 10x - 5$, if two of its zeroes are $\sqrt{\dfrac{5}{3}}$ and $-\sqrt{\dfrac{5}{3}}$. **[CBSE SP 2018]**

Sol. Let $f(x) = 3x^4 + 6x^3 - 2x^2 - 10x - 5$

Since $\sqrt{\dfrac{5}{3}}$ and $-\sqrt{\dfrac{5}{3}}$ are two of the zeroes of $f(x)$, hence, $x - \sqrt{\dfrac{5}{3}}$ and $x + \sqrt{\dfrac{5}{3}}$ are two of the factors of $f(x)$.

$\therefore \left(x - \sqrt{\dfrac{5}{3}}\right)\left(x + \sqrt{\dfrac{5}{3}}\right) = x^2 - \dfrac{5}{3}$ is a factor of $f(x)$,

we now find the other factor by dividing $f(x)$ by $x^2 - \dfrac{5}{3}$ as follows:

$$
\begin{array}{r|l|l}
x^2-\frac{5}{3} & 3x^4+6x^3-2x^2-10x-5 & 3x^2+6x+3 \\
 & \underline{\overset{-}{3x^4} \qquad \overset{+}{-5x^2}} & \\
 & 6x^3+3x^2-10x-5 & \\
 & \underline{\overset{-}{6x^3} \qquad \overset{+}{-10x}} & \\
 & 3x^2-5 & \\
 & \underline{\overset{-}{3x^2}\overset{+}{-5}} & \\
 & 0 &
\end{array}
$$

$\therefore$ The other factor is $3x^2 + 6x + 3$. This can be written as $3x^2 + 6x + 3 = 3(x^2 + 2x + 1) = 3(x + 1)^2$. Hence, the other two zeroes are -1 and -1. Hence, the required zeroes of $f(x)$ are $-\sqrt{\frac{5}{3}}, \sqrt{\frac{5}{3}}, -1, -1$.

Higher Order Thinking Skills (HOTS) Questions

(Page 29)

1. If α and β are the zeroes of the polynomial $f(x) = 4x^2 - 4x + 1$, find the value of $\frac{1}{\alpha^4} + \frac{1}{\beta^4}$.

Sol. We have $\quad \alpha + \beta = \frac{4}{4} = 1 \quad \ldots(1)$

and $\quad \alpha\beta = \frac{1}{4} \quad \ldots(2)$

Now,
$$\frac{1}{\alpha^4} + \frac{1}{\beta^4} = \frac{\alpha^4 + \beta^4}{(\alpha\beta)^4}$$
$$= \frac{(\alpha^2 + \beta^2)^2 - 2(\alpha\beta)^2}{(\alpha\beta)^4}$$
$$= \frac{\{(\alpha + \beta)^2 - 2\alpha\beta\}^2 - 2(\alpha\beta)^2}{(\alpha\beta)^4}$$
$$= \frac{\left(1 - 2 \times \frac{1}{4}\right)^2 - 2 \times \frac{1}{16}}{\left(\frac{1}{4}\right)^4}$$
[From (1) and (2)]
$$= \frac{\frac{1}{4} - \frac{1}{8}}{\frac{1}{256}}$$
$$= \frac{2-1}{8} \times 256$$
$$= \frac{1}{8} \times 256 = 32$$

which is the required value.

2. If the zeroes of the polynomial $x^2 + px + q$ are double in value to the zeroes of $2x^2 - 5x - 3$, find the values of p and q. **[CBSE SP 2012]**

Sol. Let α and β be the zeroes of the polynomial $2x^2 - 5x - 3$.

Then $\quad \alpha + \beta = \frac{5}{2} \quad \ldots(1)$

and $\quad \alpha\beta = -\frac{3}{2} \quad \ldots(2)$

$\therefore$ According to the problem, 2α and 2β are the zeroes of the polynomial $x^2 + px + q$.

$\therefore \quad 2(\alpha + \beta) = -p \quad \ldots(3)$

and $\quad 4\alpha\beta = q \quad \ldots(4)$

$\therefore$ From (1) and (3), we have
$$2\left(\frac{5}{2}\right) = -p$$
$\Rightarrow \quad 5 = -p$

$\Rightarrow \quad p = -5$

and from (2) and (4), we have
$$4 \times \left(-\frac{3}{2}\right) = q$$
$\Rightarrow \quad q = -6$

Hence, the required values of p and q are -5 and -6 respectively.

3. If the zeroes of the polynomial $x^3 - 15x^2 + 71x + k$ are in AP, find the value of k.

Sol. Let $\alpha - d$, α and $\alpha + d$ be the three zeroes of the polynomial $x^3 - 15x^2 + 71x + k$.

Then we have

$\alpha - d + \alpha + \alpha + d = 15$

$\Rightarrow \quad 3\alpha = 15$

$\Rightarrow \quad \alpha = 5 \quad \ldots(1)$

Also, $(\alpha - d)\,\alpha\,(\alpha + d) = -k$

$\Rightarrow \quad k = -\alpha(\alpha^2 - d^2)$

$= -5(25 - d^2)$

$= -125 + 5d^2 \quad \ldots(2)$

and $\quad (\alpha - d)\,\alpha + (\alpha - d)(\alpha + d) + \alpha(\alpha + d) = 71$

$\Rightarrow \quad (5 - d)\,5 + (5 - d)(5 + d) + 5(5 + d) = 71$ [From (1)]

$\Rightarrow \quad 25 - 5d + 25 - d^2 + 25 + 5d = 71$

$\Rightarrow \quad -d^2 + 75 = 71$

$\Rightarrow \quad d^2 = 4 \quad \ldots(3)$

$\therefore$ From (2) and (3), we get,
$$k = 5 \times 4 - 125 = -105$$
which is the required value of k.

4. Find the zeroes of the polynomial $f(x) = 2x^3 - x^2 - 4x + 2$ if two of its zeroes are equal in magnitude but opposite in sign.

Sol. Let the three zeroes of $f(x)$ be α, $-\alpha$ and β.

$$\alpha - \alpha + \beta = \frac{1}{2}$$

$$\Rightarrow \quad \beta = \frac{1}{2} \quad \ldots(1)$$

and $$\alpha(-\alpha)\beta = \frac{-2}{2}$$

$$\Rightarrow \quad \alpha^2\beta = 1$$

$$\Rightarrow \quad \alpha^2 . \frac{1}{2} = 1 \quad \text{[From (1)]}$$

$$\Rightarrow \quad \alpha^2 = 2$$

$$\Rightarrow \quad \alpha = \pm\sqrt{2}$$

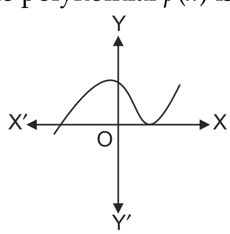
Hence, the required zeroes of $f(x)$ are $+\sqrt{2}$, $-\sqrt{2}$ and $\frac{1}{2}$.

Self-Assessment

(Page 29)

Multiple-Choice Questions

1. The graph of $y = p(x)$ is given. The number of zeroes of the polynomial $p(x)$ is



(*a*) 0 (*b*) 3

(*c*) 2 (*d*) 4 **[CBSE SP 2011]**

Sol. (*c*) 2

Since the graph cuts the x-axis at two points, hence the required number of zeroes of the given polynomial is 2.

2. A real number k is called a zero of the polynomial $p(x)$ when

(*a*) $p(k) = -2$ (*b*) $p(k) = 0$

(*c*) $p(k) = 1$ (*d*) $p(k) = -1$

Sol. (*b*) $p(k) = 0$

By definition, we know that k is a zero of the polynomial $p(x)$ if $p(k) = 0$

Fill in the Blanks

3. Graph of polynomial $f(x) = (x - 2)(x - 3)$ will intersect x-axis at exactly **two** points.

4. A polynomial of degree zero is called a **constant** polynomial.

5. Quadratic polynomial whose sum and product of zeroes are $\frac{-1}{2}$ and $\frac{1}{2}$ respectively is $\mathbf{k(2x^2 + x + 1)}$.

Sol. $k(x^2 - (\text{Sum of zeroes})\,x + (\text{Product of zeroes}))$

$$\Rightarrow k\left(x^2 - \left(-\frac{1}{2}\right)x + \frac{1}{2}\right)$$

$$\Rightarrow k\left(x^2 + \frac{1}{2}x + \frac{1}{2}\right)$$

$$\Rightarrow k\left(2x^2 + x + 1\right)$$

6. Polynomial $ax^2 - c$ has two zeroes which are **equal** but opposite in sign.

Assertion-Reason Type Questions

Directions (Q. Nos. 7 to 9): Each of these questions contains an assertion followed by reason. Read them carefully, and answer the question on the basis of the following options, select the one that best describes the two statements.

(a) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.

(b) Both Assertion and Reason are true but Reason is not the correct explanation of Assertion.

(c) If Assertion is true but Reason is false.

(d) If Assertion is false but Reason is true.

7. **Assertion:** $x + 4 = 0$ is a linear polynomial.

Reason: A polynomial of degree 1 is called a linear polynomial.

Sol. The correct answer is (a). Both Assertion and Reason are correct and the Reason is the correct explanation of Assertion.

8. **Assertion:** 4 is a zero of the polynomial $p(x) = x^2 - 3x - 4$

Reason: Putting $x = 4$, we get $p(x) = 1$

Sol. The correct answer is (c) as after putting $x = 4$, we get $p(x) = 0$.

Thus, the Assertion is correct but Reason is incorrect.

9. **Assertion:** The product of the zeroes of quadratic polynomial $x^2 + 7x + 10$ is 10.

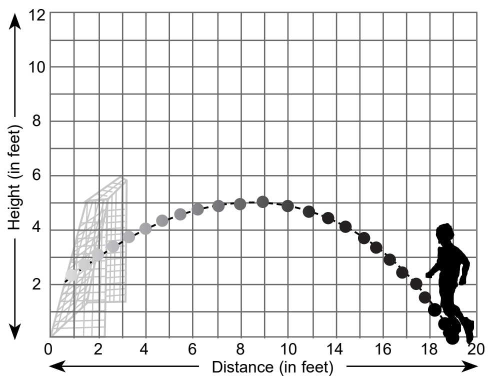
Reason: Sum of zeroes is given by the negative ratio of coefficient of x and that of x^2.

Sol. The correct answer is (b) as the products of zeroes is given by the ratio of constant term to coefficient of x^2.

Thus, Assertion is correct and Reason is also correct but it is not the correct explanation of Assertion, as it talks about sum instead of product.

Case Study Based Questions

10. A football tournament was going on between a Delhi school and a Haryana school. The score was levelled and only 5 minutes were left. A boy from Haryana school gave a goal in the last minute and won the tournament. The path of the football was placed on a graph. Here the variables x and y represents the horizontal distance (ft) and vertical height (ft) respectively. A quadratic function can be expressed as an expression in the form $ax^2 + bx + c$ where $a \neq 0$.



Based on above information, answer the following questions.

(*a*) Which curve is made by the football?

(*i*) Ellipse (*ii*) Hyperbola

(*iii*) Parabola (*iv*) Spherical

Ans. (*iii*) Parabola

(*b*) The standard form of quadratic polynomial is $ax^2 + bx + c$, where a, b and c are

(*i*) all real numbers.

(*ii*) all rational numbers.

(*iii*) all integers.

(*iv*) all real numbers with '*a*' a non-zero real number.

Ans. (*iv*) all real numbers with '*a*' a non-zero real number.

(*c*) What kind of equation $y = 0x^2 + 5x + 3$ is?

(*i*) Zero (*ii*) Linear

(*iii*) Quadratic (*iv*) Cubic

Ans. (*ii*) Linear

(*d*) From the graph, write the number of zeroes of the curve of the polynomial.

(*i*) 1 (*ii*) 2 (*iii*) 0 (*iv*) 3

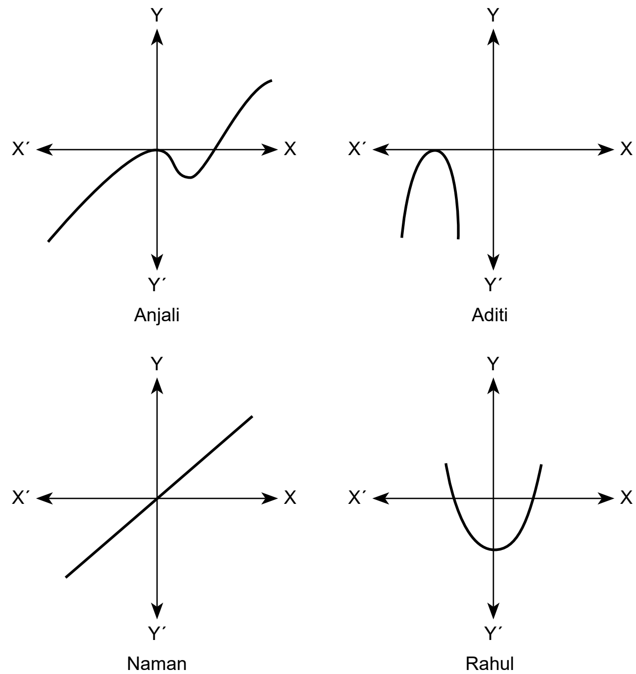
Ans. (*i*) 1

(*e*) At what height, did the ball reach the goal post?

(*i*) 2 ft (*ii*) 3 ft (*iii*) 0 ft (*iv*) 4 ft

Ans. (*i*) 2 ft

11. A mathematics teacher of a certain school asked four students of class 10 to draw graph of $f(x) = ax^2 + bx + c$. The graph drawn by four students Anjali, Aditi, Naman and Rahul are shown below:



(*a*) How many students have drawn the graph correctly?

(*i*) 1 (*ii*) 2 (*iii*) 3 (*iv*) 4

Ans. (*ii*) 2

(*b*) Which type of polynomial is represented by Naman's graph?

(*i*) Linear (*ii*) Parabola

(*iii*) Zig-Zag (*iv*) None of these

Ans. (*i*) Linear

(*c*) How many zeroes are there for the Anjali's graph?

(*i*) 0 (*ii*) 1 (*iii*) 2 (*iv*) 3

Ans. (*iii*) 2

(*d*) If $f(x) = ax^2 + bx + c$ and $a + b + c = 0$, then one zero is equal to

(*i*) $-\frac{b}{a}$ (*ii*) $-\frac{c}{a}$

(*iii*) $\frac{b}{c}$ (*iv*) $\frac{c}{a}$

Ans. (*iv*) $\frac{c}{a}$

(*e*) If $f(x) = ax^2 + bx + c$ and $a + c = b$, then one of the zeroes is equal to

(*i*) $-\frac{b}{a}$ (*ii*) $-\frac{c}{a}$

(*iii*) $\frac{b}{c}$ (*iv*) $\frac{c}{a}$

Ans. (*ii*) $-\frac{c}{a}$

Very Short Answer Type Questions

12. If $p(x) = 3x^2 - x + 5$, what is the value of $p(-2)$?

Sol. We have $p(-2) = 3(-2)^2 - (-2) + 5 = 12 + 2 + 5 = 19$ which is the required value.

13. A quadratic polynomial has two zeroes whose sum is 6. If one of the zeroes is $3-\sqrt{7}$, what is the polynomial?

Sol. The sum of two zeroes is 6 and one of the zeroes is $3 + \sqrt{7}$.

$\therefore$ The other zero of the polynomial

$$= 6 - 3 + \sqrt{7} = 3 + \sqrt{7}$$

$\therefore$ The required polynomial is

x^2 – (Sum of the zeroes)x + Product of the zeroes

$$= x^2 - 6x + \left(3 + \sqrt{7}\right)\left(3 - \sqrt{7}\right)$$
$$= x^2 - 6x + 9 - 7$$
$$= x^2 - 6x + 2$$

Short Answer Type-I Questions

14. Divide $3x^3 + 16x^2 + 21x + 20$ by $x + 4$ and find the quotient and the remainder. **[CBSE SP 2011]**

Sol. We divide $3x^3 + 16x^2 + 21x + 20$ by $x + 4$ as follows:

$$
\begin{array}{r|l|l}
x+4 & 3x^3 + 16x^2 + 21x + 20 & 3x^2 + 4x + 5 \\
 & \underline{\overset{-}{}3x^3 \overset{-}{+} 12x^2} & \\
 & \quad 4x^2 + 21x + 20 & \\
 & \quad \underline{4x^2 + 16x} & \\
 & \qquad 5x + 20 & \\
 & \qquad \underline{5x + 20} & \\
 & \qquad\quad 0 &
\end{array}
$$

Hence, the required quotient $= 3x^2 + 4x + 5$ and the remainder = 0.

15. Find the cubic polynomial with the sum of its zeroes, sum of the product of zeroes taken two at a time and product of its zeros are 0, –7 and –6 respectively. **[CBSE SP 2010]**

Sol. Let α, β and γ be the three zeroes of the polynomial $p(x)$.

Then $p(x) = (x - \alpha)(x - \beta)(x - \gamma)$

$= \{x^2 - (\alpha + \beta)x + \alpha\beta\}(x - \gamma)$

$= x^3 - (\alpha + \beta)x^2 + \alpha\beta x - \gamma x^2 + \gamma(\alpha + \beta)x - \alpha\beta\gamma$

$= x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \alpha\gamma + \beta\gamma)x - \alpha\beta\gamma$

$= x^3 - 0.x^2 - 7x + 6 = x^3 - 7x + 6$

which is the required polynomial.

16. Read the following passage and answer the questions that follows:

A teacher told 10 students to write a polynomial on the blackboard. Students wrote

(*i*) $x^2 + 2$ (*ii*) $2x + 3$

(*iii*) $x^3 + x^2 + 1$ (*iv*) $x^3 + 2x^2 + 1$

(*v*) $x^2 - 2x + 1$ (*vi*) $x - 3$

(*vii*) $x^4 + x^2 + 1$ (*viii*) $x^2 + 2x + 1$

(*ix*) $2x^3 - x^2$ (*x*) $x^4 - 1$

(*a*) How many students wrote cubic polynomial?

(*b*) Divide the polynomial $(x^2 + 2x + 1)$ by $(x + 1)$ **[CBSE SP(Basic) 2019]**

Sol. (*a*) 3 students

(*b*) $\frac{x^2 + 2x + 1}{x + 1} = \frac{(x + 1)^2}{x + 1} = x + 1$

Short Answer Type-II Questions

17. If two zeroes of the polynomial $f(x) = 2x^3 + 3x^2 - 9x - 10$ are 2 and –1, then find its third zero.

Sol. Let α be the third zero.

Then sum of the zeroes $= 2 - 1 + \alpha = -\frac{3}{2}$

$$\Rightarrow \quad \alpha = -\frac{3}{2} - 1 = -\frac{5}{2}$$

which is the required third zero.

18. If α and β are the zeroes of the polynomial $2t^2 + 7t + 5$, find the value of $\left(\sqrt{\frac{\alpha}{\beta}} + \sqrt{\frac{\beta}{\alpha}}\right) \div \left(\alpha^2 + \beta^2\right)$.

Sol. We have $\quad \alpha + \beta = -\frac{7}{2} \qquad \ldots(1)$

and $\qquad \alpha\beta = \dfrac{5}{2} \qquad \ldots(2)$

$$\left(\sqrt{\frac{\alpha}{\beta}} + \sqrt{\frac{\beta}{\alpha}}\right) \div (\alpha^2 + \beta^2)$$

$$= \frac{\alpha + \beta}{\sqrt{\alpha\beta}} \times \frac{1}{\alpha^2 + \beta^2}$$

$$= \frac{\alpha + \beta}{\sqrt{\alpha\beta}\left\{(\alpha + \beta)^2 - 2\alpha\beta\right\}}$$

$$= \frac{-\frac{7}{2}}{\sqrt{\frac{5}{2}}\left(\frac{49}{4} - 5\right)}$$

$$= \frac{-\frac{7}{2}}{\sqrt{\frac{5}{2}} \times \frac{29}{4}} = -\frac{7}{2} \times \frac{4\sqrt{2}}{29\sqrt{5}}$$

$$= \frac{-7}{2} \times \frac{4\sqrt{2}}{29\sqrt{5}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$= -\frac{7}{2} \times \frac{8}{29\sqrt{10}}$$

$$= \frac{-28}{29\sqrt{10}} = \frac{28\sqrt{10}}{29 \times 10}$$

$$= \frac{14\sqrt{10}}{29 \times 5}$$

$$= \frac{-14\sqrt{10}}{145}$$

Hence, the required value is $\dfrac{-14\sqrt{10}}{145}$.

Long Answer Type Questions

19. If the polynomial $x^4 - 6x^3 + 16x^2 - 25x + 10$ is divided by another polynomial $x^2 - 2x + k$, the remainder comes out to be $x + a$. Find k and a. **[CBSE 2011]**

Sol. We divide $x^4 - 6x^3 + 16x^2 - 25x + 10$ by $x^2 - 2x + k$ as follows:

$$\begin{array}{r|l|l}
x^2 - 2x + k & x^4 - 6x^3 + 16x^2 - 25x + 10 & x^2 - 4x + (8 - k) \\
 & \underset{-}{x^4} \underset{+}{-} 2x^3 \underset{-}{+} kx^2 & \\
\hline
 & -4x^3 + (16 - k)x^2 - 25x + 10 & \\
 & \underset{+}{-} 4x^3 \underset{-}{+} 8x^2 \underset{+}{-} 4kx & \\
\hline
 & (8 - k)x^2 + (4k - 25)x + 10 & \\
 & \underset{-}{(8 - k)x^2} \underset{-}{+} (2k - 16)x \underset{-}{+} k(8 - k) & \\
\hline
 & (2k - 9)x + 10 - k(8 - k) & \\
\hline
\end{array}$$

Since the remainder is given to be $x + a$, hence, we must have

$$2k - 9 = 1$$

$$\Rightarrow \qquad k = 5 \qquad \ldots(1)$$

and
$$\begin{aligned} a &= 10 - k(8 - k) \\ &= 10 - 5(8 - 5) \qquad \text{[From (1)]} \\ &= 10 - 5(3) \\ &= 10 - 15 \\ &= -5 \end{aligned}$$

$\therefore$ The required values of k and a are respectively 5 and -5.

20. If two zeroes of the polynomial $x^4 - 6x^3 - 26x^2 + 138x - 35$ are $2 \pm \sqrt{3}$, find the other zeroes. **[CBSE SP 2011]**

Sol. The two of the factors of the given polynomial are $x - 2 - \sqrt{3}$ and $x - 2 + \sqrt{3}$

Now, $\left(x - 2 - \sqrt{3}\right)\left(x - 2 + \sqrt{3}\right) = (x - 2)^2 - 3$

$$= x^2 - 4x + 1$$

We now divide the given polynomial by $x^2 - 4x + 1$ to get another factor, as follows:

$$\begin{array}{r|l|l}
x^2 - 4x + 1 & x^4 - 6x^3 - 26x^2 + 138x - 35 & x^2 - 2x - 35 \\
 & \underset{-}{x^4} \underset{+}{-} 4x^3 \underset{-}{+} x^2 & \\
\hline
 & -2x^3 - 27x^2 + 138x - 35 & \\
 & \underset{+}{-2x^3} \underset{-}{+} 8x^2 \underset{+}{-} 2x & \\
\hline
 & -35x^2 + 140x - 35 & \\
 & \underset{+}{-35x^2} \underset{-}{+} 140x \underset{+}{-} 35 & \\
\hline
 & 0 & \\
\hline
\end{array}$$

$\therefore$ The second factor of the given polynomial is $x^2 - 2x - 35$ which can be written as

$$\begin{aligned} x^2 - 2x - 35 &= x^2 + 5x - 7x - 35 \\ &= x(x + 5) - 7(x + 5) \\ &= (x + 5)(x - 7) \end{aligned}$$

Hence, the required two zeroes are 7 and -5.

Let's Compete

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Multiple-Choice Questions

1. The minimum number of zero or zeroes of the polynomial $x^3 - 6x^2 + 13x - 20$ is/are

(*a*) 1 $\qquad$ (*b*) 2

(*c*) 3 $\qquad$ (*d*) 0

Sol. (*a*) 1

Let $f(x) = x^3 - 6x^2 + 13x - 20$.

We see that $f(x) = 0$ for $x = 4$.

$\therefore$ By factor theorem, $x - 4$ is a factor of $f(x)$. We now divide $f(x)$ by $x - 4$ to get the factor of $f(x)$.

$$\begin{array}{r|l|l} x-4 & x^3 - 6x^2 + 13x - 20 & x^2 - 2x + 5 \\ & \underline{\overset{-}{x^3} \overset{+}{-} 4x^2} & \\ & -2x^2 + 13x - 20 & \\ & \underline{\overset{+}{-2x^2} \overset{-}{+} 8x} & \\ & 5x - 20 & \\ & \underline{\overset{-}{5x} \overset{+}{-} 20} & \\ & 0 & \end{array}$$

$\therefore$ The second factor is $x^2 - 2x + 5$. We can verify by direct substitution or otherwise that $x^2 - 2x + 5$ is not zero for any real value of x. Hence, it has no other zero. Hence, the given polynomial has only one real zero.

2. If $x - 2$ is a factor of $2x^3 - x^2 - bx - 2$, then the value of b is equal to

(*a*) 10 (*b*) −5

(*c*) 5 (*d*) −10

Sol. (*c*) 5

Since $x - 2$ is a factor of $2x^3 - x^2 - bx - 2$, hence $x = 2$ will be a zero of this polynomial.

$\therefore \quad 2 \times 2^3 - 2^2 - 2b - 2 = 0$

$\Rightarrow \quad 16 - 4 - 2b - 2 = 0$

$\Rightarrow \quad 2b = 10$

$\Rightarrow \quad b = 5$

3. The real number which should be subtracted from the polynomial $3x^3 + 10x^2 - 14x + 9$ so that the resulting polynomial is exactly divisible by $3x - 2$ is

(*a*) −7 (*b*) 5

(*c*) 7 (*d*) −5

Sol. (*b*) 5

We first divide $3x^3 + 10x^2 - 14x + 9$ by $3x - 2$ as follows:

$$\begin{array}{r|l|l} 3x-2 & 3x^3 + 10x^2 - 14x + 9 & x^2 + 4x - 2 \\ & \underline{\overset{-}{3x^3} \overset{+}{-} 2x^2} & \\ & 12x^2 - 14x + 9 & \\ & \underline{\overset{-}{12x^2} \overset{+}{-} 8x} & \\ & -6x + 9 & \\ & \underline{\overset{+}{-6x} \overset{-}{+} 4} & \\ & 5 & \end{array}$$

Hence, the required number is 5.

4. The value of k so that $x^2 + x + k$ is the only quadratic factor of the polynomial $3x^4 - 3x$ is

(*a*) −2 (*b*) 2

(*c*) −1 (*d*) 1

Sol. (*d*) 1

We have

$$3x^4 - 3x = 3x(x^3 - 1)$$
$$= 3x\,(x - 1)\,(x^2 + x + 1)$$

$\therefore$ The only quadratic factor of the given polynomial is $x^2 + x + 1$. Hence, the required value of k is 1.

5. If α and β are the zeroes of the polynomial $f(x) = px^2 - 2x + 3p$ and $\alpha + \beta = \alpha\beta$, then the value of p is

(*a*) $\frac{2}{3}$ (*b*) $-\frac{2}{3}$

(*c*) $\frac{3}{2}$ (*d*) $-\frac{3}{2}$ **[CBSE SP 2011]**

Sol. (*a*) $\frac{2}{3}$

We have $\quad \alpha + \beta = \frac{2}{p} \quad \ldots(1)$

and $\quad \alpha\beta = \frac{3p}{p} = 3 \quad \ldots(2)$

$\because \quad \alpha + \beta = \alpha\beta$

$\therefore \quad \frac{2}{p} = 3 \quad$ [From (1) and (2)]

$\Rightarrow \quad p = \frac{2}{3}$

6. If α and β are the zeroes of the quadratic polynomial $x^2 + 4kx + 4$ where k is a constant, and if $\alpha^2 + \beta^2 = 24$, then the values of k are given by

(*a*) 2, 3 (*b*) ± 2

(*c*) $\pm\sqrt{2}$ (*d*) 2, –3

Sol. (*c*) $\pm\sqrt{2}$

We have $\quad \alpha^2 + \beta^2 = 24$

$\Rightarrow \quad (\alpha + \beta)^2 - 2\alpha\beta = 24 \quad \ldots(1)$

Now, $\quad \alpha + \beta = -4k \quad \ldots(2)$

and $\quad \alpha\beta = 4 \quad \ldots(3)$

From (1), (2) and (3), we get

$$(-4k)^2 - 2(4) = 24$$

$\Rightarrow \quad 16k^2 - 8 = 24$

$\Rightarrow \quad k^2 = 2$

$\Rightarrow \qquad k = \pm\sqrt{2}$

7. If α, β and γ are the zeroes of the cubic polynomial $x^3 + px^2 + qx + 2$ such that $\beta = -\frac{1}{\alpha}$, then the value of $5 + 2p + q$ is

(*a*) –1 (*b*) 2

(*c*) 1 (*d*) 0

Sol. (*d*) 0

We have

$$\alpha + \beta + \gamma = -p \qquad \ldots(1)$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = q \qquad \ldots(2)$$

$$\alpha\beta\gamma = -2 \qquad \ldots(3)$$

and $\alpha\beta = -1$ [Given]...(4)

$\therefore$ (3) and (4), $\gamma = 2$.

Now, from (1) and (2),

$$\alpha + \beta = -p - 2$$

$$\Rightarrow \quad p = -2 - (\alpha + \beta) \qquad \ldots(5)$$

Also, from (2) and (4), we get

$$q = -1 + (\alpha + \beta)2 \qquad \ldots(6)$$

$\therefore$ From (5) and (6), we have

$$5 + 2p + q = 5 - 4 - 2(\alpha + \beta) - 1 + 2(\alpha + \beta)$$
$$= 0$$

8. If $a - b$, a and $a + b$ are the zeroes of the polynomial $x^3 + 3x^2 - 8$, then the value of $a + b$ is

(*a*) $-1 \pm \sqrt{7}$ (*b*) 2 or – 4

(*c*) 4 or – 2 (*d*) 4 or 3

Sol. (*b*) 2 or –4

We have

Sum of the zeroes $= a - b + a + a + b = -3$

$\Rightarrow \quad 3a = -3$

$\Rightarrow \quad a = -1$

Product of zeroes $= a(a - b)(a + b) = 8$

$\Rightarrow \quad a(a^2 - b^2) = 8$

$\Rightarrow \quad (-1)(1 - b^2) = 8$

$\Rightarrow \quad b^2 = 9$

$\Rightarrow \quad b = \pm 3$

$\therefore \quad a + b = -1 \pm 3 = 2$ or -4

9. If α, β and γ are the zeroes of the polynomial $-2x^3 + 5x - 4$, then the value of $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$ is

(*a*) $\frac{5}{4}$ (*b*) $-\frac{5}{4}$

(*c*) $\frac{4}{5}$ (*d*) $-\frac{4}{5}$

Sol. (*a*) $\frac{5}{4}$

We have

$$\alpha + \beta + \gamma = 0 \qquad \ldots(1)$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = -\frac{5}{2} \qquad \ldots(2)$$

$$\alpha\beta\gamma = -\frac{4}{2} = -2 \qquad \ldots(3)$$

$$\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = \frac{\beta\gamma + \alpha\gamma + \alpha\beta}{\alpha\beta\gamma}$$

$$= \frac{\frac{-5}{2}}{-2} \qquad \text{[From (2) and (3)]}$$

$$= \frac{5}{4}$$

10. If the graph of the polynomial equation $y = p(x)$ where $p(x)$ is a polynomial intersects the x-axis three times in three distinct points, then which of the following could be the expression for $p(x)$?

(*a*) $x^2 - 9$ (*b*) $3x + 3$

(*c*) $4 - 4x - x^2 + x^3$ (*d*) $3x^2 + 3x - 3$

[CBSE SP 2012]

Sol. (*c*) $4 - 4x - x^2 + x^3$

Since the graph of the given polynomial intersects x-axis at 3 distinct points, hence the polynomial must be cubic.

Value-based Question (Optional)

(Page 32)

1. A lady Principal of a well-known public school wanted to help a nearby secondary school for poor but meritorious students, which is run by an NGO. She asked each bright Senior Secondary student of her school to teach different topics of mathematics at the lower classes daily for at least one hour to the poor students as his/her project work. One day, one such student was selected from the Senior Secondary Public School and was asked to teach how to find the value of a polynomial at a particular point in some lower class. He prepared the following problem:

Find the values of each of the two polynomials $f(x)$ and $g(x)$ at the points $x = 1, -1, 2, -2$ where $f(x) = x^3 + 6x^2 + 5x - 12$ and $g(x) = 2x^3 + x^2 - 18x - 9$.

Some students got all the correct values as follows:

$f(1) = 0, f(-1) = -12, f(2) = 30, f(-2) = -6$

$g(1) = -24$, $g(-1) = 8$, $g(2) = -25$ and $g(-2) = 15$

(*a*) From the above values of $f(x)$ and $g(x)$, state with reasons, whether any of the values 1, –1, 2, –2 may be a zero or zeroes of $f(x)$ or $g(x)$.

(*b*) If –3 is a zero of $f(x)$, what is the third zero of $f(x)$, if any.

(*c*) If $-\frac{1}{2}$ and 3 are two zeroes of $g(x)$, what is its third zero?

(*d*) What values are exhibited by the Principal of the school?

Sol. (*a*) Since $f(1) = 0$ and $g(x) \neq 0$ for any value of x, hence it follows that 1 is a zero of $f(x)$, but none of the points +1, –1, +2 and –2 is a zero of $g(x)$.

(*b*) In this case $x + 3$ is a factor of $f(x)$. We write

$$\begin{aligned} f(x) &= x^3 + 6x^2 + 5x - 12 \\ &= x^3 + 3x^2 + 3x^2 + 9x - 4x - 12 \\ &= x^2(x + 3) + 3x\,(x + 3) - 4\,(x + 3) \\ &= (x + 3)\,(x^2 + 3x - 4) \\ &= (x + 3)\,(x^2 + 4x - x - 4) \\ &= (x + 3)\{x(x + 4) - 1\,(x + 4)\} \\ &= (x + 3)\,(x + 4)\,(x - 1) \end{aligned}$$

All the zeroes of $f(x)$ are 1, –3 and –4.

Hence, the required third zero is –4.

(*c*) We write $g(x)$ as

$$\begin{aligned} g(x) &= 2x^3 + x^2 - 18x - 9 \\ &= 2x^3 + 6x^2 - 5x^2 - 15x - 3x - 9 \\ &= 2x^2(x + 3) - 5x(x + 3) - 3\,(x + 3) \\ &= (x + 3)\,(2x^2 - 5x - 3) \\ &= (x + 3)\,(2x^2 - 6x + x - 3) \\ &= (x + 3)\,\{2x(x - 3) + 1\,(x - 3)\} \\ &= (x + 3)\,(x - 3)\,(2x + 1) \end{aligned}$$

Hence, all the zeroes of $g(x)$ are 3, $-\frac{1}{2}$ and –3.

$\therefore$ The required third zero is –3.

Alternatively, we can solve this problem in the following way also.

Clearly, $\left(x + \frac{1}{2}\right)(x - 3)$ is a factor of $g(x)$.

Now,

$$\begin{aligned} \left(x + \frac{1}{2}\right)(x - 3) &= x^2 + \left(\frac{1}{2} - 3\right)x - \frac{3}{2} \\ &= x^2 - \frac{5}{2}x - \frac{3}{2} \\ &= \frac{1}{2}\left(2x^2 - 5x - 3\right) \end{aligned}$$

We divide $g(x)$ by $2x^2 - 5x - 3$ as follows:

$$\begin{array}{r|l|l} 2x^2 - 5x - 3 & 2x^3 + x^2 - 18x - 9 & x + 3 \\ & \underline{\underset{-}{2x^3} \underset{+}{-} 5x^2 \underset{+}{-} 3x} & \\ & 6x^2 - 15x - 9 & \\ & \underline{\underset{-}{6x^2} \underset{+}{-} 15x \underset{+}{-} 9} & \\ & 0 & \end{array}$$

The remaining factor is $x + 3$. Hence the required third zero is –3.

(*d*) Kindness and sympathy towards poor and meritorious students and interest in mathematics.

3

Pair of Linear Equations in Two Variables

Checkpoint (Page 34)

1. Determine which values of x and y will satisfy the equation $2x - 3y - 5 = 0$?

(a) $x = 0, y = \frac{5}{2}$ (b) $x = -5, y = \frac{5}{3}$

(c) $x = -2, y = 3$ (d) $x = 4, y = 1$

Sol. (a) Putting $x = 0$ and $y = \frac{5}{2}$ on the LHS of the equation $2x - 3y - 5 = 0$, we get

$$\text{LHS} = 2 \times 0 - 3 \times \frac{5}{2} - 5 = \frac{-15}{2} - 5 = \frac{-15 - 10}{2} = -\frac{25}{2} \neq \text{RHS}$$

$\therefore$ The equation is not satisfied.

(b) Putting $x = -5$ and $y = \frac{5}{3}$ on the LHS of the equation $2x - 3y - 5 = 0$, we get

$$\text{LHS} = 2 \times (-5) - 3 \times \frac{5}{3} - 5 = -10 - 5 - 5 = -20 \neq \text{RHS}$$

$\therefore$ The equation is not satisfied.

(c) Putting $x = -2$ and $y = 3$ on the LHS of the equation $2x - 3y - 5 = 0$, we get

$$\text{LHS} = 2 \times (-2) - 3 \times 3 - 5 = -4 - 9 - 5 = -13 - 5 = -18 \neq \text{RHS}$$

$\therefore$ The equation is not satisfied.

(d) Putting $x = 4$ and $y = 1$ on the LHS of the equation $2x - 3y - 5 = 0$, we get

$\text{LHS} = 2 \times 4 - 3 \times 1 - 5 = 8 - 8 = 0 = \text{RHS}$

Thus, $x = 4$ and $y = 1$ satisfies the equation $2x - 3y - 5 = 0$.

2. Express the following statement in the form of a linear equation in two variables: $ax + by + c = 0$. The present ages of the father and his son are x years and y years respectively. Three years ago, the father's age was twice the age of the son.

Sol. According to the problem, we have

$$x - 3 = 2(y - 3)$$

$$\Rightarrow \quad x - 2y + 3 = 0$$

which is the required equation.

3. Find the solution of the equations $2x + 3 = 5$ and $3y - 8 = 7$. Is there any other solution of these two equations?

Sol. We have $2x + 3 = 5$

$$\Rightarrow \quad 2x = 5 - 3 = 2$$

$$\Rightarrow \quad x = \frac{2}{2} = 1$$

Also, we have

$$3y - 8 = 7$$

$$\Rightarrow \quad 3y = 7 + 8 = 15$$

$$\Rightarrow \quad y = \frac{15}{3} = 5$$

Hence, the required solution is $x = 1$ and $y = 5$.

No, these two equation do not have any other solution at all.

4. Show that the solution of the equations $5x - 7 = 3$ and $2 - y = 5$ will satisfy the equation $7x + 8y + 10 = 0$.

Sol. We have $5x - 7 = 3$

$$\Rightarrow \quad 5x = 7 + 3 = 10$$

$$\Rightarrow \quad x = \frac{10}{5} = 2$$

Also, we have

$$2 - y = 5$$

$$\Rightarrow \quad y = 2 - 5 = -3$$

Putting $x = 2$ and $y = -3$ on the LHS of the equation $7x + 8y + 10 = 0$, we get

$$\text{LHS} = 7 \times 2 + 8 \times (-3) + 10$$
$$= 14 - 24 + 10$$
$$= 24 - 24$$
$$= 0 = \text{RHS}$$

Hence, $x = 2$, $y = -3$ satisfies the equation $7x + 8y + 10 = 0$.

5. Determine the coordinates of the points where the line $3x - 2y = 6$ will intersect the

(*a*) x-axis and

(*b*) y-axis.

Sol. (*a*) The given line $3x - 2y = 6$ will intersect x-axis when $y = 0$.

Putting $y = 0$, we get

$$3x = 6$$

$$\Rightarrow \quad x = 2$$

Hence, the given line will intersect the x-axis at the point (2, 0).

(*b*) The given line $3x - 2y = 6$ will intersect y-axis when $x = 0$. Putting $x = 0$, we get

$$-2y = 6$$

$$\Rightarrow \quad y = -\frac{6}{2} = -3$$

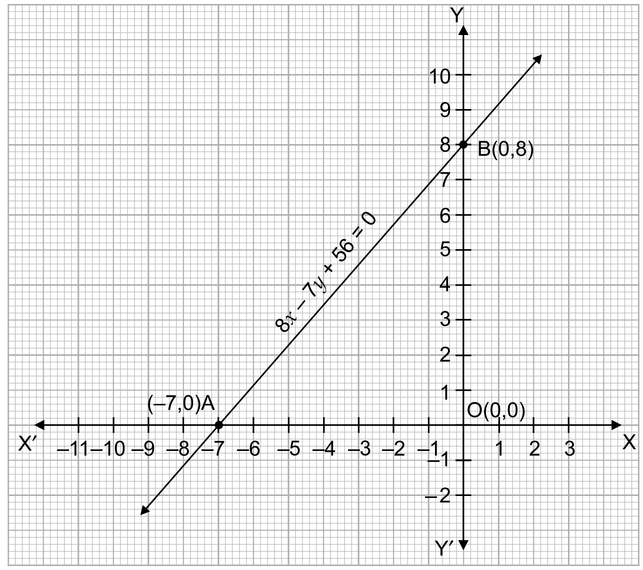
Hence, the given line will intersect the y-axis at the point (0, –3).

6. Find the coordinates of the vertices of the triangle formed by the line $8x - 7y + 56 = 0$, x-axis and the y-axis graphically.

Sol. Putting $y = 0$ in the given equation, we get

$$8x = -56$$

$$\Rightarrow \quad x = -\frac{56}{8} = -7$$



Hence, the line intersects the x-axis at the point A(–7, 0). Again, putting $x = 0$ in the given equation, we get

$$7y = 56$$

$$\Rightarrow \quad y = \frac{56}{7} = 8$$

$\therefore$ The line intersects the y-axis at the point B(0, 8).

Finally, x and y axes intersects each other at the point O(0, 0) which is the origin. We now join AB to get the required triangle OAB as shown in the graph.

7. Draw the graph of the lines $-3x + y = 0$ and $56x - 3y = 0$ and determine their point of intersection, if any, graphically.

Sol. From the equation $-3x + y = 0$ we get, $y = 3x$. A few values of x and y are shown in Table 1 below:

Table 1

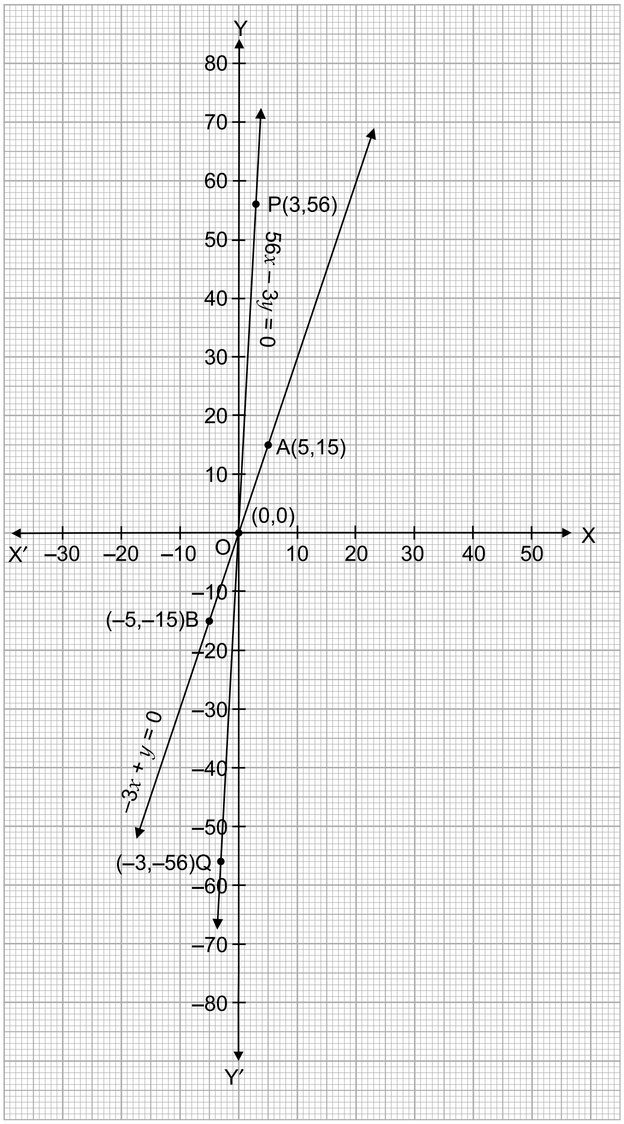
x	0	5	–5
y	0	15	–15

Similarly, from the equation $56x - 3y = 0$ we get, $y = \frac{56x}{3}$. A few values of x and y are shown in Table 2 below:

Table 2

x	0	3	–3
y	0	56	–56

We now plot the point O(0, 0), A(5, 15) and B(–5, –15) on a graph paper and join them by a straight line AOB.



We also plot the points O(0, 0), P(3, 56) and Q(–3, –56) on the same graph paper and join them by another straight line POQ. From the graph, we see that the two lines AB and PQ intersect each other at the origin O(0, 0) as shown in the graph.

8. A point (3, –4) lies on the line represented by $7x - 3y + 5k = 0$. Determine the value of k.

Sol. Putting $x = 3$ and $y = -4$ in the given equation, we get

$$7 \times 3 - 3 \times (-4) + 5k = 0$$

$$\Rightarrow \quad 21 + 12 + 5k = 0$$

$$\Rightarrow \quad k = -\frac{33}{5}$$

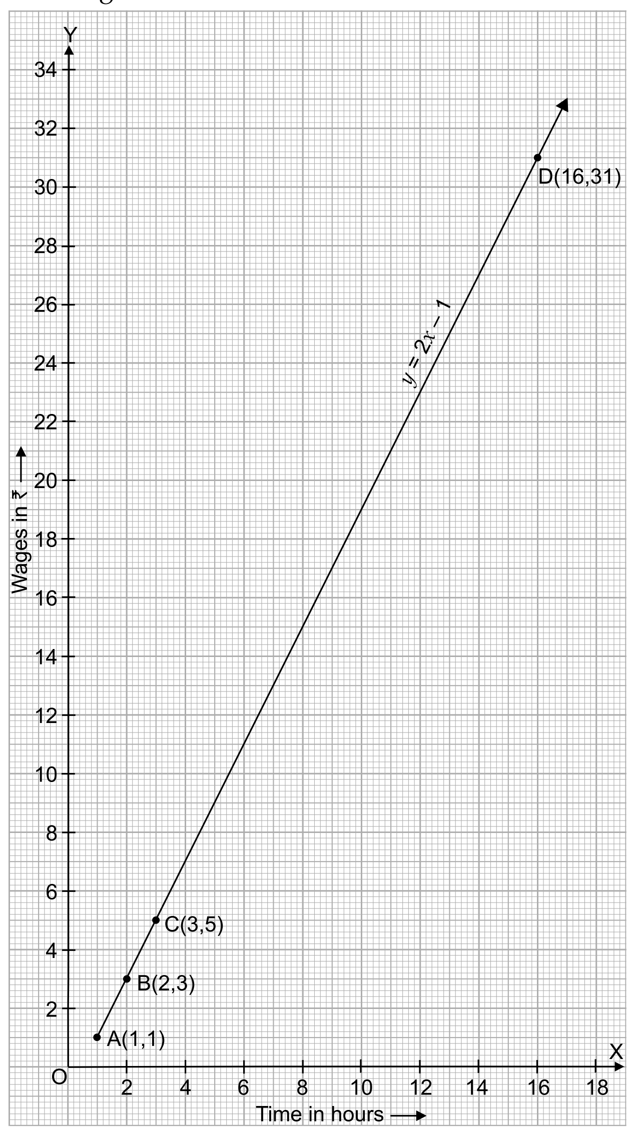
$\therefore$ The value of k is $-\dfrac{33}{5}$.

9. Working for x hours, a labour gets ₹y, where $y = 2x - 1$. Draw the graph of work-wage equation. From this graph, find the wages of the labour, if he works for 16 hours.

Sol. We first find some values of x and y from the given equation $y = 2x - 1$. These values of x and y are shown in the table below:

x	1	2	3	16
y	1	3	5	31

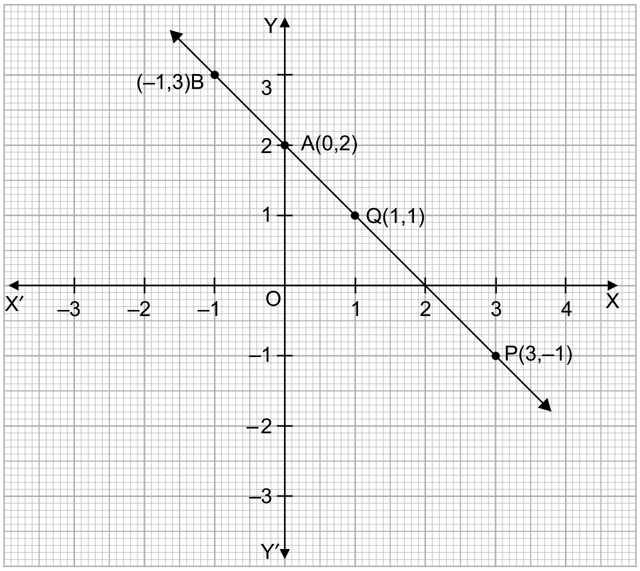
We now plot the points A(1, 1), B(2, 3), C(3, 5) and D(16, 31) on a graph paper and join them by a straight line AD to get the required graph of work-wage equation. In the graph, times in hours are shown along x-axis and wages in ₹ are shown along y-axis. Note that we have chosen different scales on x and y axes, since the unit of time and wages are different here.



From the graph, we see that when $x = 16$, $y = 31$. Hence, the required wages of the labour are ₹31, if he works for 16 hours.

10. Plot the points (0, 2) and (–1, 3) on a graph paper and draw a line through these two points. If the points (p, –1) and (1, q) lie on this line, then find the values of p and q.

Sol. We have plotted the points A(0, 2) and B(–1, 3) on the graph paper and join them by a straight line AB.



From the graph, we see that the points P(p, –1) and Q(1, q) will lie on the line only if $p = 3$ and $q = 1$.

Hence, the required values of p and q are 3 and 1 respectively.

Milestone 1

(Page 38)

Multiple-Choice Questions

1. The pair of equations $x = 4$ and $y = 3$ graphically represent lines which are

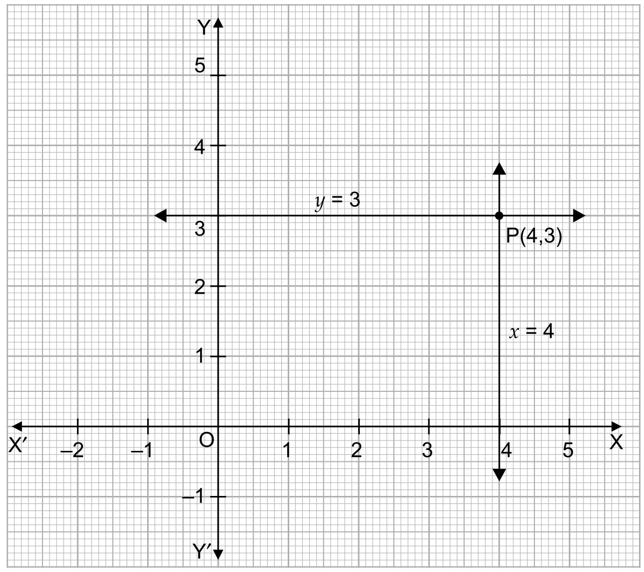
(a) coincident.

(b) parallel.

(c) intersecting at (3, 4).

(d) intersecting at (4, 3). **[CBSE 2012]**

Sol. (d) intersecting at (4, 3).



From the graph, we see that $x = 4$ is a line parallel to y-axis and $y = 3$ is a line parallel to x-axis. We see that these two lines intersect each other at the point P with coordinates (4, 3).

2. Let x years and y years represent the present ages of A and B respectively. Five years ago, A was thrice as old as B and ten years later A will be twice as old as B. This situation can be represented algebraically as

(a) $x + 3y - 10 = 0$ and $x + 2y + 10 = 0$

(b) $x - 3y + 10 = 0$ and $x - 2y - 10 = 0$

(c) $2x - 3y + 10 = 0$ and $2x + 3y - 10 = 0$

(d) $2x + 3y + 10 = 0$ and $2x - 3y + 10 = 0$

Sol. (b) $x - 3y + 10 = 0$ and $x - 2y - 10 = 0$

The present age of A = x years

The present age of B = y years

A's age, five years ago = $(x - 5)$ years

B's age, five years ago = $(y - 5)$ years

A's age, ten years later = $(x + 10)$ years

B's age, ten years later = $(y + 10)$ years

According to the problem, we have

$$x - 5 = 3(y - 5)$$

$$\Rightarrow \quad x - 3y - 5 + 15 = 0$$

$$\Rightarrow \quad x - 3y + 10 = 0$$

and $$x + 10 = 2(y + 10)$$

$$\Rightarrow \quad x - 2y + 10 - 20 = 0$$

$$\Rightarrow \quad x - 2y - 10 = 0$$

Very Short Answer Type Questions

3. Find the total number of point(s) of intersection of the line $2x + 3y + 5 = 0$ with the x-axis.

Sol. Putting $y = 0$ in the given equation, we get

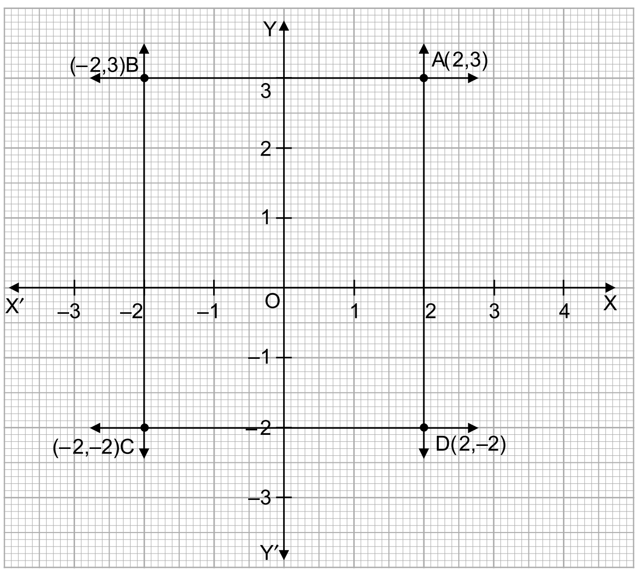
$$2x = -5$$

$$\Rightarrow \quad x = \frac{-5}{2}$$

$\therefore$ The given line intersects the x-axis at only one point *viz.* $\left(\frac{-5}{2}, 0\right)$.

4. Name the special quadrilateral formed by the lines $x = 2$, $x = -2$, $y = 3$ and $y = -2$ and hence find its area.

Sol. Lines $x = 2$ and $x = -2$ are parallel to y-axis and the lines $y = 3$ and $y = -2$ are parallel to x-axis as shown in the graph. These lines intersect pairwise at A, B, C and D as shown in the graph.



We see from the graph that the coordinates of the points A, B, C and D are respectively (2, 3), (–2, 3), (–2, –2) and (2, –2).

The length of the line segment AB = (2 + 2) units = 4 units = length of the line segment CD.

Also, the length of the line segment BC = (3 + 2) units = 5 units = length of the line segment AD.

Since, AB ∥ CD and BC ∥ AD

$\therefore$ ABCD is a parallelogram.

Also, each of the angles A, B, C and D is 90° and length and breadth are different. So, this parallelogram is a rectangle of area 4 × 5 sq units i.e. 20 sq units.

Short Answer Type-I Questions

5. Draw the graph of equation $3x + 2y - 6 = 0$ and name the triangle formed by this line with coordinates axes. What are the coordinates of its vertices? What kind of triangle is it with respect to angles of the triangle?

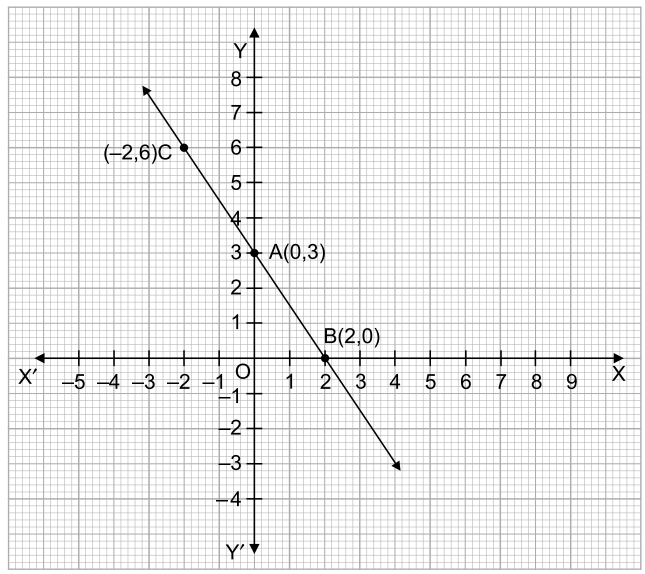
Sol. From the given equation, we have

$$2y = 6 - 3x$$

$$\Rightarrow \quad y = \frac{6 - 3x}{2}$$

We find some values of x and y as shown in the table below:

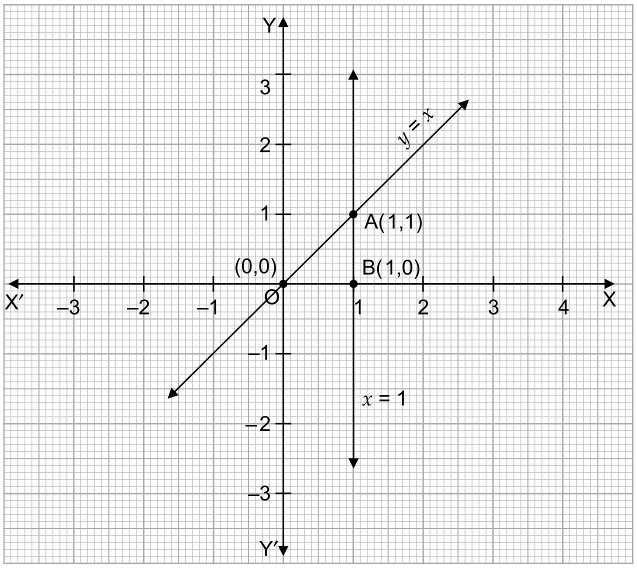
x	0	2	–2
y	3	0	6



We plot the points A(0, 3), B(2, 0) and C(–2, 6) on a graph paper and join them by a straight line BC. From the graph we see that this line forms a right angled triangle AOB where ∠AOB = 90° and O is the origin. The coordinates of the vertices A, O and B of the triangle AOB are respectively (0, 3), (0, 0) and (2, 0).

6. Draw the graphs of equations $y = x$ and $x = 1$. Hence, find the area of the triangle formed by these two lines and the x-axis.

Sol. The graph of $y = x$ and $x = 1$ are shown on the graph paper. These two lines intersect each other at the point A(1, 1). Also, the line $x = 1$ which is parallel to y-axis cuts the x-axis at the point B(1, 0). The given line form a right angled triangle OBA with the coordinate axes, where ∠OBA = 90°.



∴ The area of this triangle

$$= \frac{1}{2} \times OB \times AB$$

$$= \frac{1}{2} \times 1 \times 1 \text{ sq units}$$

$$= \frac{1}{2} \text{ sq units}$$

Short Answer Type-II Questions

7. Show graphically that the system of equations $2x + 3y = 10$ and $4x + 6y = 12$ has no solution. **[CBSE 2010]**

Sol. Graph of the equation

$$2x + 3y = 10$$

$$\Rightarrow \quad x = \frac{10 - 3y}{2} \quad \ldots(1)$$

and $\quad 4x + 6y = 12$

$$\Rightarrow \quad 2x + 3y = 6$$

$$\Rightarrow \quad x = \frac{6 - 3y}{2} \quad \ldots(2)$$

We find some value of x and y from (1) and those from (2). These values are shown in Table 1 and 2 below respectively.

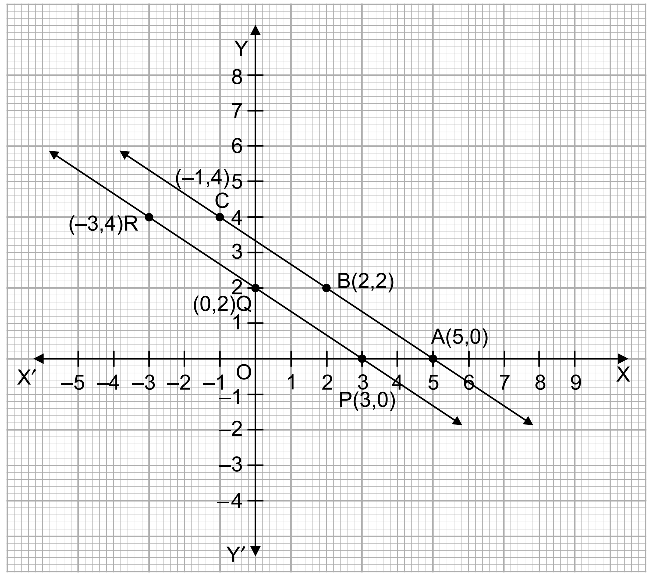
Table 1

x	5	2	–1
y	0	2	4

Table 2

x	3	0	–3
y	0	2	4

We plot the points A(5, 0), B(2, 2) and C(–1, 4). Also, we plot the points P(3, 0), Q(0, 2) and R(–3, 4) and join ABC and PQR by two distinct lines in the same graph paper.



We see from the graph that these two lines are parallel and so they cannot meet each other. Hence, the two given equations have no solution at all.

8. Show graphically that the lines represented by the equations $-7x + 5y = 17$, $3x - 4y = -11$ and $15x + 7y = -1$ are concurrent at the point (–1, 2).

Sol. From the given equation, we have

$$y = \frac{17 + 7x}{5} \quad \ldots(1)$$

$$y = \frac{3x + 11}{4} \quad \ldots(2)$$

and $$y = \frac{-15x - 1}{7} \quad \ldots(3)$$

We now calculate some values of x and y from (1), (2) and (3) and list these values in Table 1, 2 and 3 respectively.

Table 1

x	–1	–6
y	2	–5

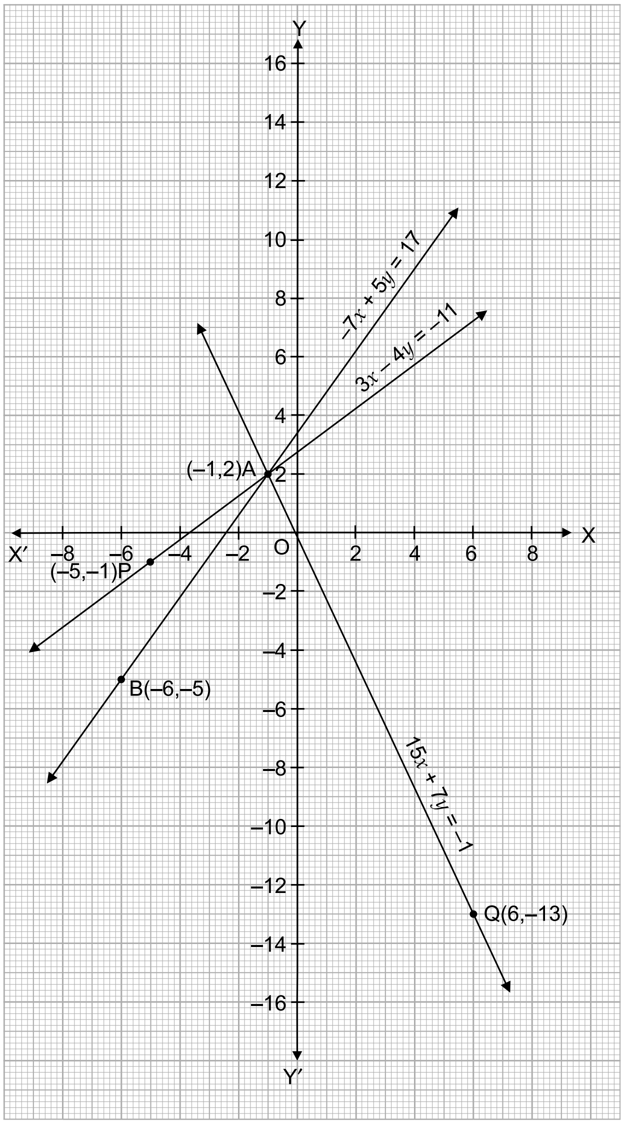
Table 2

x	–1	–5
y	2	–1

Table 3

x	–1	6
y	2	–13

We plot the points A(–1, 2), B(–6, –5), P(–5, –1) and Q(6, –13) and join the points in Table 1, Table 2 and table 3 respectively with three separate straight lines PA, BA and QA which are concurrent at the point A(–1, 2) as shown in the graph.



9. Determine graphically whether the system of equations $x - 2y = 2$, $4x - 2y = 5$ is consistent or inconsistent.

Sol. From the two given equations, we have

$$y = \frac{x-2}{2} \qquad \ldots(1)$$

and $$y = \frac{4x-5}{2} \qquad \ldots(2)$$

From (1) and (2), we list some values of x and y as shown below in Table 1 and Table 2 respectively.

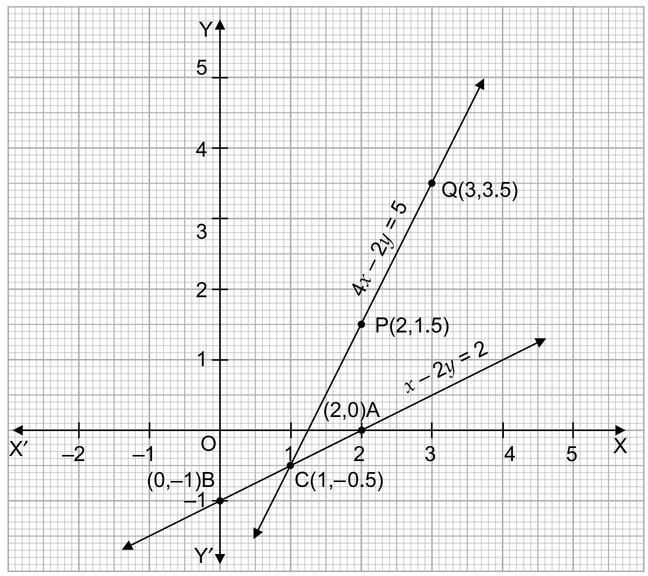
Table 1

x	2	0	1
y	0	–1	–0.5

Table 2

x	2	3	1
y	1.5	3.5	–0.5

We now plot the points A(2, 0), B(0, –1) and C(1, –0.5) from Table 1 and P(2, 1.5) and Q(3, 3.5) from Table 2 in the graph paper and join them by two separate line BCA and QPC respectively as shown in the graph. From the graph, we see that the two lines intersect each other at a point C(1, –0.5).



Hence, the two given equations have a unique solution. Hence, these two equations are consistent.

Long Answer Type Questions

10. Draw the graphs of the equations $y = x$, $y = 0$ and $2x + 2y = 20$. Hence, determine the area of the triangle formed by these three lines.

Sol. We find some values of x and y from the equation

$$y = x \qquad \ldots(1)$$

and $$2x + 2y = 20$$

$$\Rightarrow \quad y = 10 - x \qquad \ldots(2)$$

These values are listed in Table 1 and Table 2 respectively.

Table 1

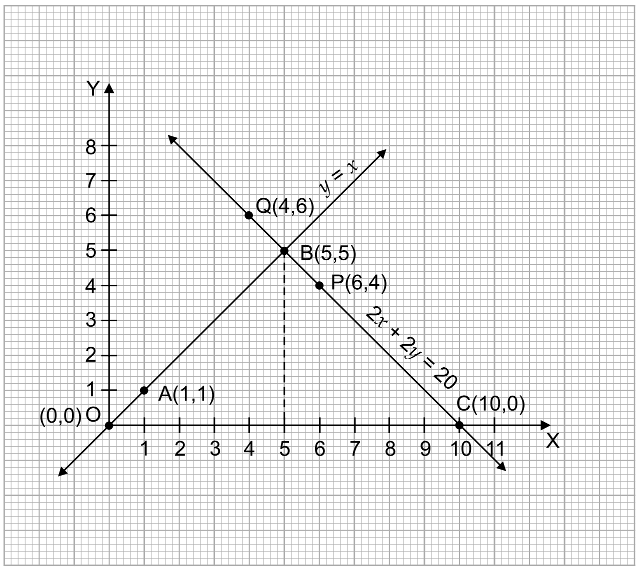
x	0	1	5
y	0	1	5

Table 2

x	6	4	5
y	4	6	5

We now plot the points O(0, 0), A(1, 1) and B(5, 5) from Table 1 and join them by a line OAB. We next plot the points P(6, 4) and Q(4, 6) from Table 2 and join them by another line PBQ in the same graph paper. These two lines intersect the line $y = 0$, i.e. the x-axis at the points O(0, 0) and C(10, 0) forming a triangle BOC.

We see that the base OC = 10 units and the height BM = 5 units. Hence, the area of ΔBOC is $\frac{1}{2} \times 10 \times 5$ sq units i.e. 25 sq units.



11. Draw the graphs of the equations $2y - x = 8$, $5y - x = 14$ and $y - 2x = 1$. Hence, find the coordinates of the vertices of the triangle formed by these three lines.

Sol. The given equation can be written as

$$x = 2y - 8 \quad \ldots(1)$$

$$x = 5y - 14 \quad \ldots(2)$$

and $$y = 1 + 2x \quad \ldots(3)$$

We list some values of x and y from (1), (2) and (3) respectively in Table 1, 2 and 3 below:

Table 1

x	2	–4	0
y	5	2	4

Table 2

x	–4	1	6
y	2	3	4

Table 3

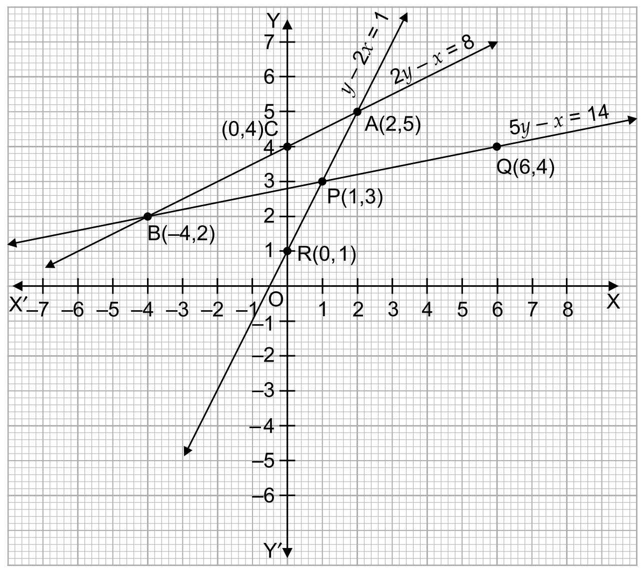
x	0	1	2
y	1	3	5

We now plot the points A(2, 5), B(–4, 2) and C(0, 4) from Table 1 and join them by a line ACB.

Similarly, we plot the points B(–4, 2), P(1, 3) and Q(6, 4) from Table 2 and join them by another line BPQ.

Finally, we plot the points R(0, 1), P(1, 3) and A(2, 5) from Table 3 and join them by a third line RPA.

The three lines AB, BP and PA drawn in the same graph are shown below in the graph paper. From the graph, we see that the vertices of the triangle ABP formed by these three lines are A(2, 5), B(–4, 2) and P(1, 3).



Milestone 2

(Page 44)

Multiple-Choice Questions

1. One of the equation of a pair of dependent linear equations is $-5x + 7y = 2$. Then the second equation can be

(*a*) $-10x - 14y + 4 = 0$ (*b*) $-10x + 14y + 4 = 0$

(*c*) $10x - 14y = -4$ (*d*) $10x + 14y + 4 = 0$

[CBSE SP 2011]

Sol. (*c*) $10x - 14y = -4$

We know that two equations $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are linearly dependent if $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$.

We see that the given equation $-5x + 7y = 2$ i.e. $5x - 7y + 2 = 0$ and $10x - 14y + 4 = 0$ in the choice (*c*) only are linearly dependent, for $\frac{5}{10} = \frac{-7}{-14} = \frac{2}{4}$ is true.

2. The point of intersection of the lines $y = 3x + 6$ and $y + 3x = 0$ is

(*a*) $(-1, 3)$ (*b*) $(1, 3)$

(*c*) $(1, -3)$ (*d*) $(-1, -3)$

Sol. (*a*) $(-1, 3)$

We have $y = 3x + 6$ ...(1)

and $y + 3x = 0$ ...(2)

From (1) and (2), we have

$$3x + 6 + 3x = 0$$

$$\Rightarrow \quad 6x = -6$$

$$\Rightarrow \quad x = -1$$

$\therefore$ From (1), $y = -3 + 6 = 3$

$\therefore$ The required point of intersection of the two given lines is $(-1, 3)$.

Very Short Answer Type Questions

3. The line $\dfrac{x}{a} + \dfrac{y}{b} = -2$ intersects the x-axis at A and y-axis at B. Find the coordinates of A and B.

Sol. Putting $y = 0$ in the given equation $\dfrac{x}{a} + \dfrac{y}{b} = -2$, we get $x = -2a$

Hence, A is the point $(-2a, 0)$. Again, putting $x = 0$ in the given equation, we get $y = -2b$. Hence, B is the point $(0, -2b)$.

4. If $x = a + b$ and $y = a - b$ is the solution of the equations $x - y = 2$ and $x + y = 4$, then find the values of a and b.

Sol. Putting $x = a + b$ and $y = a - b$ in the given equation

$x - y = 2$ ...(1)

and $x + y = 4$ ...(2)

we get $a + b - a + b = 2$

$$\Rightarrow \quad 2b = 2$$

$$\Rightarrow \quad b = 1$$

and $a + b + a - b = 4$

$$\Rightarrow \quad 2a = 4$$

$$\Rightarrow \quad a = 2$$

Hence, the required values of a and b are 2 and 1 respectively.

Short Answer Type-I Questions

5. If the lines represented by the equations $2x = -7py - 3$ and $-5y = -8 + 7px$ are parallel to each other, find the value(s) of p.

Sol. Since the lines

$2x + 7py + 3 = 0$ ...(1)

and $7px + 5y - 8 = 0$ ...(2)

are parallel, we have

$$\frac{2}{7p} = \frac{7p}{5} \neq -\frac{3}{8}$$

$$\Rightarrow \quad 49p^2 = 10$$

$$\Rightarrow \quad p^2 = \frac{10}{49}$$

$$\therefore \quad p = \pm\frac{\sqrt{10}}{\sqrt{49}} = \pm\frac{\sqrt{10}}{7}$$

Hence, the required value(s) of p are $\pm\dfrac{\sqrt{10}}{7}$.

6. Can the linear equations $5y = 3(q - 1)x - 10$ and $-3x = 7y + 14$ have infinite number of solution for any value of q? If so, find the value(s) of q.

Sol. We see that the equations

$3(q - 1)x - 5y - 10 = 0$ ...(1)

and $3x + 7y + 14 = 0$ ...(2)

will have infinite number of solutions if

$$\frac{3}{3(q-1)} = -\frac{7}{5} = -\frac{14}{10}$$

$$\Rightarrow \quad 7q - 7 = -5$$

$$\Rightarrow \quad q = \frac{2}{7}$$

Yes, the given equation will have infinite number of solutions if $q = \dfrac{2}{7}$.

7. Find the value of k for which the following pair of linear equations has a unique solution.

$4x + 7y - 30 = 0$

$5x - 9ky + 20 = 0$

Sol. The given equation will have a unique solution if

$$\frac{4}{5} = \frac{7}{-9k} \neq \frac{-30}{20}$$

i.e. $-36k = 35$

$$\Rightarrow \quad k = -\frac{35}{36}$$

Hence, the required value of k is $-\dfrac{35}{36}$.

Short Answer Type-II Questions

8. Solve by any suitable method:

$a(x + y) + b(x - y) = a^2 - ab + b^2$

$a(x + y) - b(x - y) = a^2 + ab + b^2$ **[CBSE SP 2010]**

Sol. We have

$$a(x + y) + b(x - y) = a^2 - ab + b^2 \quad \ldots(1)$$

and $$a(x + y) - b(x - y) = a^2 + ab + b^2 \quad \ldots(2)$$

Adding (1) and (2), we get

$$x + y = \frac{a^2 + b^2}{a} \quad \ldots(3)$$

Subtracting (2) from (1), we get

$$x - y = -a \quad \ldots(4)$$

Adding (3) and (4), we get

$$2x = \frac{a^2 + b^2 - a^2}{a} = \frac{b^2}{a}$$

$$\Rightarrow \quad x = \frac{b^2}{2a}$$

Subtracting (4) from (3), we get

$$2y = \frac{a^2 + b^2}{a} + a$$

$$= \frac{2a^2 + b^2}{a}$$

$$\Rightarrow \quad y = \frac{2a^2 + b^2}{2a}$$

$\therefore$ The required solution is $x = \dfrac{b^2}{2a}$ and $y = \dfrac{2a^2 + b^2}{2a}$.

9. Solve by any suitable method:

$$\frac{57}{x+y} + \frac{6}{x-y} = 5 \text{ and } \frac{38}{x+y} + \frac{21}{x-y} = 9$$

Sol. Let $$\frac{1}{x + y} = u \quad \ldots(1)$$

$$\frac{1}{x - y} = v \quad \ldots(2)$$

$\therefore$ The given equation $\dfrac{57}{x + y} + \dfrac{6}{x - y} = 5$, becomes

$$57u + 6v = 5$$

$$\Rightarrow \quad 57u + 6v - 5 = 0 \quad \ldots(3)$$

Similarly, the equation $\dfrac{38}{x + y} + \dfrac{21}{x - y} = 9$ becomes

$$38u + 21v = 9$$

$$\Rightarrow \quad 38u + 21v - 9 = 0 \quad \ldots(4)$$

$\therefore$ From (3) and (4) by method of cross-multiplication, we have

$$\frac{u}{6 \times (-9) - (-5) \times 21} = \frac{v}{-5 \times 38 - 57 \times (-9)} = \frac{1}{57 \times 21 - 6 \times 38}$$

$$\Rightarrow \quad \frac{u}{-54 + 105} = \frac{v}{-190 + 513} = \frac{1}{1197 - 228}$$

$$\Rightarrow \quad \frac{u}{51} = \frac{v}{323} = \frac{1}{969}$$

$$\therefore \quad u = \frac{51}{969} = \frac{1}{19}$$

and $$v = \frac{323}{969} = \frac{1}{3}$$

$\therefore$ From (1) and (2), we have

$$\frac{1}{x + y} = \frac{1}{19}$$

$$\Rightarrow \quad x + y = 19 \quad \ldots(5)$$

and $$\frac{1}{x - y} = \frac{1}{3}$$

$$\Rightarrow \quad x - y = 3 \quad \ldots(6)$$

Adding (5) and (6), we get

$$2x = 22$$

$$\Rightarrow \quad x = 11$$

Subtracting (6) from (5), we get

$$2y = 16$$

$$\Rightarrow \quad y = 8$$

Hence, the required solution is $x = 11$ and $y = 8$.

Long Answer Type Questions

10. Solve: $\dfrac{2}{x} + \dfrac{2}{3y} = \dfrac{1}{6}$ and $\dfrac{3}{x} + \dfrac{2}{y} = 0$ and hence find the value of a for which $y = ax - 4$.

Sol. Let $$\frac{1}{x} = u \quad \ldots(1)$$

$$\frac{1}{y} = v \quad \ldots(2)$$

$\therefore$ The given equation $\dfrac{2}{x} + \dfrac{2}{3y} = \dfrac{1}{6}$, becomes

$$2u + \frac{2}{3}v = \frac{1}{6}$$

$$\Rightarrow \quad 12u + 4v = 1 \quad \ldots(3)$$

Similarly, the equation $\dfrac{3}{x} + \dfrac{2}{y} = 0$, becomes

$$3u + 2v = 0 \quad \ldots(4)$$

Multiplying both sides by 2 in (4), we get

$$6u + 4v = 0 \quad \ldots(5)$$

$$\Rightarrow \quad 4v = -6u \quad \ldots(6)$$

$\therefore$ Substituting the value of $4v$ from (6) in (3), we get

$$12u - 6u = 1$$

$$\Rightarrow \quad 6u = 1$$

$\Rightarrow \quad u = \frac{1}{6}$

$\therefore$ From (6), $\quad 4v = -6 \times \frac{1}{6}$

$\Rightarrow \quad 4v = -1$

$\Rightarrow \quad v = -\frac{1}{4}$

Hence, from (1) and (2), we get

$$\frac{1}{x} = u = \frac{1}{6}$$

$\Rightarrow \quad x = 6$

and $\quad \frac{1}{y} = v = -\frac{1}{4}$

$\Rightarrow \quad y = -4$

Putting value of x and y in the given equation $y = ax - 4$, we get

$$-4 = a \times 6 - 4$$

$\Rightarrow \quad 0 = 6a$

$\Rightarrow \quad 6a = 0$

$\Rightarrow \quad a = 0$

Hence, the required value of a is 0.

11. Prove that the values of x and y obtained from the equations $\frac{1}{3x} + \frac{y}{9} = 5$ and $\frac{1}{5x} + \frac{y}{2} = 16$, $(x \neq 0)$ will satisfy the equations

$$\frac{1}{3u} + \frac{v}{9} = 5 \text{ and } \frac{1}{5u} + \frac{v}{2} = 16 \ (u \neq 0)$$

where $u = yx^2 - 1$ and $v = \frac{y^2x - 90}{3}$.

Sol. Let $\quad \frac{1}{x} = a$

$\therefore$ The given equation $\frac{1}{3x} + \frac{y}{9} = 5$, becomes

$$\frac{1}{3}a + \frac{y}{9} = 5$$

$\Rightarrow \quad 3a + y = 45 \quad \ldots(1)$

Similarly, the equation $\frac{1}{5x} + \frac{y}{2} = 16$, becomes

$$\frac{a}{5} + \frac{y}{2} = 16$$

$\Rightarrow \quad 2a + 5y = 160 \quad \ldots(2)$

From (1), we have

$$y = 45 - 3a \quad \ldots(3)$$

Putting the value of y from (3) in (2), we get

$$2a + 5(45 - 3a) = 160$$

$\Rightarrow \quad 2a + 225 - 15a = 160$

$\Rightarrow \quad 13a = 65$

$\Rightarrow \quad a = 5 \quad \ldots(4)$

$\Rightarrow \quad a = \frac{1}{x} = 5$

$\Rightarrow \quad x = \frac{1}{5} \quad \ldots(5)$

Putting value of a from (4) in (1), we get

$$3 \times 5 + y = 45$$

$\Rightarrow \quad 15 + y = 45$

$\Rightarrow \quad y = 30 \quad \ldots(6)$

$\therefore$ From the given equation $u = yx^2 - 1$, we have

$$u = 30 \times \frac{1}{25} - 1$$

[From (5) and (6)]

$\Rightarrow \quad u = \frac{6}{5} - 1 = \frac{1}{5}$

$\Rightarrow \quad u = \frac{1}{5} \quad \ldots(7)$

and from the given equation $v = \frac{y^2x - 90}{3}$, we have

$$v = \frac{900 \times \frac{1}{5} - 90}{3}$$

$$= \frac{90}{3} = 30$$

$\Rightarrow \quad v = 30 \quad \ldots(8)$

$\therefore$ From the remaining two equations, we have

$$\frac{1}{3u} + \frac{v}{9} = 5 \quad \ldots(9)$$

and $\quad \frac{1}{5u} + \frac{v}{2} = 16 \quad \ldots(10)$

Putting $\frac{1}{u} = 5$ and $v = 30$ from (7) and (8) in (9) and (10), we see that both the equations are satisfied.

Hence, proved.

12. Solve the pair of linear equations by any suitable method:

$$\frac{42}{3x+4y} + \frac{52}{4y-3x} = 5 \text{ and } \frac{7}{3x+4y} + \frac{13}{4y-3x} = 1$$

where $3x + 4y \neq 0$ and $4y - 3x \neq 0$.

Sol. Let $u = \dfrac{1}{3x + 4y}$

$v = \dfrac{1}{4y - 3x}$

$\therefore$ The given equation $\dfrac{42}{3x + 4y} + \dfrac{52}{4y - 3x} = 5,$

becomes

$$42u + 52v = 5 \quad \ldots(1)$$

Similarly, the equation $\dfrac{7}{3x + 4y} + \dfrac{13}{4y - 3x} = 1,$

becomes

$$7u + 13v = 1 \quad \ldots(2)$$

Multiplying (2) by 4, we get

$$28u + 52v = 4 \quad \ldots(3)$$

Subtracting (3) from (1), we get

$$14u = 1$$

$$\Rightarrow \quad u = \frac{1}{14} \quad \ldots(4)$$

$\therefore$ From (2), we get

$$7 \times \frac{1}{14} + 13v = 1$$

$$\frac{1}{2} + 13v = 1$$

$$\Rightarrow \quad 13v = \frac{1}{2}$$

$$\Rightarrow \quad v = \frac{1}{26} \quad \ldots(5)$$

From (4) and (5), we get

$$\frac{1}{3x + 4y} = \frac{1}{14}$$

$$\Rightarrow \quad 3x + 4y = 14 \quad \ldots(6)$$

$$\frac{1}{4y - 3x} = \frac{1}{26}$$

$$\Rightarrow \quad 4y - 3x = 26 \quad \ldots(7)$$

Adding (6) and (7), we get

$$8y = 40$$

$$\Rightarrow \quad y = 5$$

$\therefore$ From (6), we get

$$3x + 4 \times 5 = 14$$

$$\Rightarrow \quad 3x + 20 = 14$$

$$\Rightarrow \quad 3x = -6$$

$$\Rightarrow \quad x = -2$$

Hence, the required solution is $x = -2$ and $y = 5$.

Milestone 3

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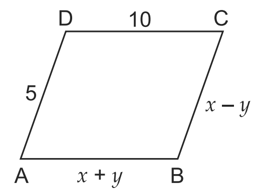
Multiple-Choice Questions

1. In the given figure, ABCD is a parallelogram. If AB = $x + y$, BC = $x - y$, CD = 10 units and AD = 5 units, then the values of x and y are respectively



(*a*) 8, 2 (*b*) 7.5, 2.5

(*c*) 6, 1 (*d*) 6, 4 **[CBSE SP 2011]**

Sol. (*b*) 7.5, 2.5

We know that in a parallelogram, the opposite sides are of equal length.

$$\therefore \quad x + y = 10 \quad \ldots(1)$$

$$\text{and} \quad x - y = 5 \quad \ldots(2)$$

Adding (1) and (2), we get

$$2x = 15$$

$$\Rightarrow \quad x = \frac{15}{2} = 7.5$$

Subtracting (2) from (1), we get

$$2y = 5$$

$$\Rightarrow \quad y = \frac{5}{2} = 2.5$$

$\therefore$ The required value of x and y are 7.5 and 2.5 respectively.

2. A two-digit number is 5 times the sum of its digits. If 9 is added to the number, then the digits interchange their places. Then the number is

(*a*) 46 (*b*) 64

(*c*) 54 (*d*) 45

Sol. (*d*) 45

Let the digits in the unit's place and the ten's place be x and y respectively. Then the number is $10y + x$.

The number obtained by interchanging the digits is $10x + y$.

According to the first condition of the problem, we have

$$10y + x = 5(x + y)$$

$$\Rightarrow \quad 10y + x = 5x + 5y$$

$$\Rightarrow \quad 4x - 5y = 0$$

$$\Rightarrow \quad x = \frac{5}{4}y \quad \ldots(1)$$

According to the second condition of the problem, we have

$$10y + x + 9 = 10x + y$$

$$\Rightarrow \quad 9x - 9y = 9$$

$$\Rightarrow \quad x - y = 1 \quad \ldots(2)$$

$\therefore$ From (1) and (2), we get

$$\frac{5}{4}y - y = 1$$

$$\Rightarrow \quad 5y - 4y = 4$$

$$\Rightarrow \quad y = 4$$

$\therefore$ From (1), $\quad x = \frac{5}{4} \times 4 = 5$

Hence, the required number is $10 \times 4 + 5$ i.e. 45.

Very Short Answer Type Questions

3. Find two numbers such that one-fourth of their sum is 11 and one-third of their difference is 2.

Sol. Let the two numbers be x and y and $x > y$.

$\therefore$ According to the problem, we have

$$\frac{x+y}{4} = 11$$

$$\Rightarrow \quad x + y = 44 \quad \ldots(1)$$

and $\quad \frac{x-y}{3} = 2$

$$\Rightarrow \quad x - y = 6 \quad \ldots(2)$$

Adding (1) and (2), we get

$$2x = 50$$

$$\therefore \quad x = \frac{50}{2} = 25$$

Subtracting (2) from (1), we get

$$2y = 38$$

$$\therefore \quad y = \frac{38}{2} = 19$$

Hence, the required numbers are 25 and 19.

4. A lady has 20 coins in her purse, consisting of ₹5 and ₹1 coins. If she has ₹40 in her purse, find the number of ₹1 and ₹5 coins separately.

Sol. Let the number of ₹1 coins be x and that of ₹5 coins be y.

$\therefore$ According to the problem, we have

$$x + y = 20 \quad \ldots(1)$$

and $\quad x + 5y = 40 \quad \ldots(2)$

Subtracting (1) from (2), we get

$$4y = 20$$

$$\Rightarrow \quad y = \frac{20}{4} = 5$$

$\therefore$ From (1), $x = 20 - 5 = 15$

$\therefore$ The required number of ₹1 coins and ₹5 coins are 15 and 5 respectively.

Short Answer Type-I Questions

5. The ratio of incomes of two persons is 9 : 7 and that of their expenditure is 4 : 3. If each of them saves ₹200 per month, find their monthly incomes.

Sol. Let the monthly incomes of two persons be ₹$9x$ and ₹$7x$, where x is a constant number and let their respective expenditures be ₹$4y$ and ₹$3y$, where y is a constant number.

Then the net income of the first person is ₹$(9x - 4y)$ and that of the second person is ₹$(7x - 3y)$.

$\therefore$ According to the problem, we have

$$9x - 4y - 200 = 0 \quad \ldots(1)$$

and $\quad 7x - 3y - 200 = 0 \quad \ldots(2)$

Solving (1) and (2) by the method of cross-multiplication, we get

$$\frac{x}{4 \times 200 - 3 \times 200} = \frac{y}{-7 \times 200 + 9 \times 200} = \frac{1}{-9 \times 3 + 4 \times 7}$$

$$\Rightarrow \quad \frac{x}{800 - 600} = \frac{y}{-1400 + 1800} = \frac{1}{-27 + 28}$$

$$\Rightarrow \quad \frac{x}{200} = \frac{y}{400} = 1$$

$\therefore$ $x = 200$ and $y = 400$

$\therefore$ The required monthly incomes of two persons are ₹$9x = 9 \times 200 =$ ₹1800 and ₹$7x =$ ₹$7 \times 200 =$ 1400.

6. 10 years ago, a father was twelve times as old as his son and 10 years hence, he will be twice as old as his son will be then. Find their present ages.

Sol. Let the present ages of the father and his son be x years and y years respectively. Then according to the problem, we have

$$x - 10 = 12(y - 10) \quad \ldots(1)$$

and $\quad x + 10 = 2(y + 10) \quad \ldots(2)$

From (1), we get

$$x - 10 = 12y - 120$$

$$\Rightarrow \quad x = 12y - 110 \quad \ldots(3)$$

$\therefore$ From (2) and (3), we get

$$x + 10 = 2y + 20$$

$$\Rightarrow \quad 12y - 110 + 10 = 2y + 20$$

$$\Rightarrow \quad 10y = 20 - 10 + 110$$

$$\Rightarrow \quad 10y = 120$$

$$\Rightarrow \quad y = \frac{120}{10} = 12$$

$\therefore$ From (3), $x = 12 \times 12 - 110 = 144 - 110 = 34$

Hence, the required present ages of the father and his son are 34 years and 12 years respectively.

Short Answer Type-II Questions

7. There is a number consisting of two digits. The number is equal to three times the sum of its digits and if it is multiplied by 3, then the result will be equal to the square of the sum of its digits. Find the number.

Sol. Let the digit in the unit's place of the number be x and that in the ten's place be y. Then the number is $10y + x$.

According to the first condition of the problem, we have

$$10y + x = 3(x + y) \quad \ldots(1)$$

According to the second condition of the problem, we have

$$3(10y + x) = (x + y)^2 \quad \ldots(2)$$

Dividing (2) by (1), we get

$$3 = \frac{(x + y)^2}{3(x + y)} = \frac{x + y}{3}$$

$$\Rightarrow \quad x + y = 9$$

$$\Rightarrow \quad y = 9 - x \quad \ldots(3)$$

$\therefore$ From (1) and (3), we get

$$10(9 - x) + x = 3 \times 9$$

$$\Rightarrow \quad 90 - 9x = 27$$

$$\Rightarrow \quad 9x = 63$$

$$\Rightarrow \quad x = \frac{63}{9} = 7$$

$\therefore$ From (3), $\quad y = 9 - 7 = 2$

Hence, the number is $10 \times 2 + 7$ i.e. 27.

8. The denominator of a fraction exceeds the numerator by 4 and if 5 is taken away from each, the sum of the reciprocal of the new fraction and 4 times the original fraction is 5. Find the original fraction.

Sol. Let the numerator and the denominator of the fraction be x and y respectively, where $y \neq 0$. Then the original fraction is $\frac{x}{y}$.

$\therefore$ According to the problem, we have

$$y = x + 4 \quad \ldots(1)$$

and $\quad \dfrac{y - 5}{x - 5} + \dfrac{4x}{y} = 5 \quad \ldots(2)$

$\therefore$ From (1) and (2), we have

$$\frac{x + 4 - 5}{x - 5} + \frac{4x}{x + 4} = 5$$

$$\Rightarrow \quad \frac{(x + 4 - 5)(x + 4) + 4x(x - 5)}{(x + 4)(x - 5)} = 5$$

$$\Rightarrow \quad (x - 1)(x + 4) + 4x(x - 5) = 5(x + 4)(x - 5)$$

$$\Rightarrow \quad x^2 + 4x - x - 4 + 4x^2 - 20x = 5(x^2 - x - 20)$$

$$\Rightarrow \quad 5x^2 - 17x - 4 = 5x^2 - 5x - 100$$

$$\Rightarrow \quad 12x = 96$$

$$\Rightarrow \quad x = \frac{96}{12} = 8$$

$\therefore$ From (1), $\quad y = 8 + 4 = 12$

$\therefore$ The required fraction is $\frac{8}{12}$.

Long Answer Type Questions

9. A train covered a certain distance at a uniform speed. If the train would have been 12 km/h faster, it would have taken 2 hours 40 minutes less time than the schedule time and if the train were slower by 12 km/h, it would have taken 4 hours more than the schedule time. Find the length of the journey and the speed of the train.

Sol. Let the distance of the journey be x km and the original speed of the train be y km/h.

Then the actual time taken by the train, say $t = \frac{x}{y}$ hours.

When the train moves 12 km/h faster, then let the time taken by the train be t_1 hours.

$$\therefore \quad t_1 = \frac{x}{y + 12} \text{ hours}$$

According to the problem, we have

$$t - t_1 = 2\frac{40}{60} \text{ hours}$$

$$= 2\frac{2}{3} \text{ hours}$$

$$= \frac{8}{3} \text{ hours}$$

$\therefore \quad \dfrac{x}{y} - \dfrac{x}{y+12} = \dfrac{8}{3}$ ...(1)

When the train moves 12 km/h slower, let the time taken by the train be t_2 hours.

$\therefore \quad t_2 = \dfrac{x}{y-12}$ hours

$\therefore \quad t_2 - t = 4$ hours

$\therefore \quad \dfrac{x}{y-12} - \dfrac{x}{y} = 4$ ...(2)

From (1), we have

$$\frac{x(y+12-y)}{y(y+12)} = \frac{8}{3}$$

$$\Rightarrow \quad \frac{12x}{y(y+12)} = \frac{8}{3}$$

$$\Rightarrow \quad \frac{3x}{y(y+12)} = \frac{2}{3}$$

$\Rightarrow \quad 2y(y+12) = 9x$ ...(3)

From (2), we have

$$\frac{x(y-y+12)}{y(y-12)} = 4$$

$$\Rightarrow \quad \frac{12x}{y(y-12)} = 4$$

$$\Rightarrow \quad \frac{3x}{y(y-12)} = 1$$

$\therefore \quad y(y-12) = 3x$ ...(4)

Dividing (3) by (4), we get

$$\frac{2(y+12)}{y-12} = 3$$

$$\Rightarrow \quad 2y + 24 = 3y - 36$$

$$\Rightarrow \quad y = 24 + 36 = 60$$

$\therefore$ From (4), $\quad 3x = 60(60-12) = 60 \times 48$

$$\Rightarrow \quad x = 20 \times 48 = 960$$

Hence, the length of the journey is 960 km and the speed of the train is 60 km/h.

10. A vessel contains a mixture of 24 L milk and 6 L water and the second vessel contains a mixture of 15 L milk and 10 L water. How much mixture of milk and water should be taken from the first and the second vessels separately and kept in a third vessel so that the third vessel may contain a mixture of 25 L milk and 10 L water?

Sol. Let x L of the mixture from the first vessel be mixed with y L of the mixture in the second vessel.

Now, in the first vessel, out of (24 + 6) L = 30 L of the mixture, the amount of milk is 24 L and that of water is 6 L.

Hence, x L of the mixture from this vessel contain $\dfrac{24x}{30}$ L = $\dfrac{4x}{5}$ L of milk and $\dfrac{6x}{30}$ L = $\dfrac{x}{5}$ L of water.

In the second vessel, out of (15 + 10)L = 25 L of the mixture, the amount of milk is 15 L and that of water is 10 L.

Hence, y L of the mixture from this vessel contains $\dfrac{15y}{25}$ L = $\dfrac{3y}{5}$ L of milk and $\dfrac{10y}{25}$ L = $\dfrac{2y}{5}$ L of water.

$\therefore$ The total amount of milk in the third vessel

$$= \left(\frac{4x}{5} + \frac{3y}{5}\right) \text{L} = \frac{4x+3y}{5} \text{ L}$$

$\therefore$ The total amount of water in this vessel is

$$\left(\frac{x}{5} + \frac{2y}{5}\right) \text{L} = \frac{x+2y}{5} \text{ L}$$

$\therefore$ According to the problem, we have

$$\frac{4x+3y}{5} = 25$$

$\Rightarrow \quad 4x + 3y - 125 = 0$ ...(1)

and $\quad \dfrac{x+2y}{5} = 10$

$\Rightarrow \quad x + 2y - 50 = 0$ ...(2)

Solving for x and y from (1) and (2) by the method of cross-multiplication, we get

$$\frac{x}{-3 \times 50 + 2 \times 125} = \frac{y}{-125 \times 1 + 4 \times 50} = \frac{1}{4 \times 2 - 3 \times 1}$$

$$\Rightarrow \quad \frac{x}{-150+250} = \frac{y}{-125+200} = \frac{1}{5}$$

$$\Rightarrow \quad \frac{x}{100} = \frac{y}{75} = \frac{1}{5}$$

$$\therefore \quad x = \frac{1}{5} \times 100 = 20$$

and $\quad y = \dfrac{1}{5} \times 75 = 15$

Hence, the required amount of mixture to be taken from the first vessel and the second vessel are 20 L and 15 L respectively.

Higher Order Thinking Skills (HOTS) Questions

(Page 50)

1. Determine graphically the vertices of the triangle, the equations of whose sides are given below:

$$y = x,\ y = 0 \text{ and } 3x + 3y = 10$$

Sol. We first find some values of x and y from the given equation

$$y = x \quad \ldots(1)$$

and $$y = \frac{10}{3} - x \quad \ldots(2)$$

and list them in Table 1 and 2 respectively.

Table 1

x	$\frac{5}{3}$	0	$\frac{1}{3}$
y	$\frac{5}{3}$	0	$\frac{1}{3}$

Table 2

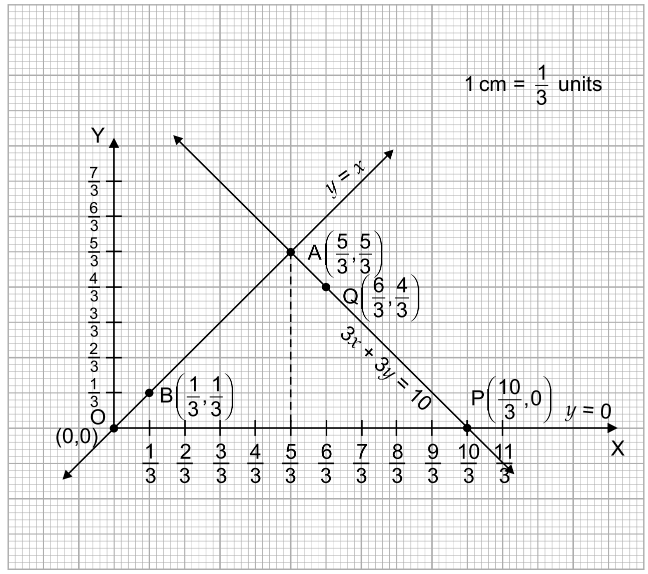
x	$\frac{5}{3}$	$\frac{6}{3}$	$\frac{10}{3}$
y	$\frac{5}{3}$	$\frac{4}{3}$	0

We now plot the points A$\left(\frac{5}{3}, \frac{5}{3}\right)$, B$\left(\frac{1}{3}, \frac{1}{3}\right)$ and O(0, 0) from Table 1 and join them by a straight line ABO.

We next plot the points P$\left(\frac{10}{3}, 0\right)$, A$\left(\frac{5}{3}, \frac{5}{3}\right)$ and Q$\left(\frac{6}{3}, \frac{4}{3}\right)$ from Table 2 and join them by a straight line PQA.

Now, $y = 0$ is the x-axis. The point of intersection of the line ABO with x-axis is O(0, 0) and that between the line PQA and the x-axis is $\left(\frac{10}{3}, 0\right)$.

From the graph, we see that the ordinates of the vertices A, O and B of ΔAOB are $\left(\frac{5}{3}, \frac{5}{3}\right)$, (0, 0) and $\left(\frac{10}{3}, 0\right)$ respectively.



2. Solve the following pair of linear equations by any suitable method:

$$\frac{2}{13}(2x+3y) = 3 + \frac{x-y}{4}$$

$$\frac{4y+5x}{3} = 2x + 7\frac{1}{6}$$

Sol. We have

$$\frac{2}{13}(2x + 3y) = 3 + \frac{x-y}{4}$$

$$\Rightarrow \quad \frac{4x+6y}{13} = \frac{12+x-y}{4}$$

$$\Rightarrow \quad 16x + 24y = 156 + 13x - 13y$$

$$\Rightarrow \quad 16x - 13x + 24y + 13y - 156 = 0$$

$$\Rightarrow \quad 3x + 37y - 156 = 0 \quad \ldots(1)$$

Also, $$\frac{4y+5x}{3} = 2x + 7\frac{1}{6}$$

$$\Rightarrow \quad \frac{4y+5x}{3} = 2x + \frac{43}{6}$$

$$\Rightarrow \quad \frac{4y+5x}{3} = \frac{12x+43}{6}$$

$$\Rightarrow \quad 24y + 30x = 36x + 129$$

$$\Rightarrow \quad 36x - 30x - 24y + 129 = 0$$

$$\Rightarrow \quad 6x - 24y + 129 = 0$$

$$\Rightarrow \quad 2x - 8y + 43 = 0 \quad \ldots(2)$$

From (1) and (2), by the method of cross-multiplications, we have

$$\frac{x}{37 \times 43 - 8 \times 156} = \frac{y}{-2 \times 156 - 3 \times 43}$$

$$= \frac{1}{-8 \times 3 - 2 \times 37}$$

$$\Rightarrow \quad \frac{x}{1591-1248} = \frac{y}{-312-129} = \frac{y}{-24-74}$$

$$\Rightarrow \quad \frac{x}{343} = \frac{y}{-441} = -\frac{1}{98}$$

$$\therefore \quad x = -\frac{343}{98} = -\frac{7}{2}$$

and $\quad y = \frac{441}{98} = \frac{9}{2}$

Hence, the required solution is $x = -\frac{7}{2}, y = \frac{9}{2}$.

3. Solve the following pair of linear equations by any suitable method:

$$119x - 381y = 643$$
$$381x - 119y = -143$$

Sol. We have

$$119x - 381y = 643 \quad \ldots(1)$$

and $\quad 381x - 119y = -143 \quad \ldots(2)$

Adding (1) and (2), we get

$$500x - 500y = 500$$

$$\Rightarrow \quad x - y = 1$$

$$\Rightarrow \quad y = x - 1 \quad \ldots(3)$$

$\therefore$ From (1) and (3), we get

$$119x - 381(x-1) = 643$$

$$\Rightarrow \quad 119x - 381x = 643 - 381$$

$$\Rightarrow \quad -262x = 262$$

$$\Rightarrow \quad x = -1$$

$\therefore$ From (3), $\quad y = -1 -1 = -2$

Hence, the required solution is $x = -1$, $y = -2$.

4. Prove that the pair of linear equations $2(k+2)x + ky + 5 = 0$ and $kx - 3(k+2)y - 8 = 0$ cannot have an infinite number of solutions for any real values of k. Also, prove that the equations will have a unique solution for all real values of k.

Sol. We know that the given equations may have an infinite number of solution if

$$\frac{2(k+2)}{k} = \frac{k}{-3(k+2)} = -\frac{5}{8} \quad \ldots(1)$$

From (1), we see that

$$\frac{2(k+2)}{k} = -\frac{5}{8}$$

$$\Rightarrow \quad 16k + 32 = -5k$$

$$\Rightarrow \quad 21k = -32$$

$$\Rightarrow \quad k = -\frac{32}{21} \quad \ldots(2)$$

Also, $\quad \frac{k}{-3(k+2)} = -\frac{5}{8}$

$$\Rightarrow \quad 8k = 15k + 30$$

$$\Rightarrow \quad 7k + 30 = 0$$

$$\Rightarrow \quad k = -\frac{30}{7} \quad \ldots(3)$$

From (2) and (3), we see that

$$k = -\frac{32}{21} = -\frac{30}{7}$$

which is absurd.

Hence, it follows that the two given equations cannot have an infinite number of solutions for any real value of k.

Also, the given equation will have a unique solution if

$$\frac{2(k+2)}{k} \neq \frac{k}{-3(k+2)}$$

$$\Rightarrow \quad -6(k+2)^2 \neq k^2$$

which is always true for all real values of k. Hence, the given equations can have a unique solution for all real values of k.

Self-Assessment

(Page 50)

Multiple-Choice Questions

1. If the pair of the linear equations is consistent, then the lines will be

(*a*) always intersecting.
(*b*) always coincident.
(*c*) intersecting or coincident.
(*d*) parallel.

Sol. (*c*) intersecting or coincident.

We know that two linear equations are consistent if they have either a finite number of solutions or infinite number of solutions. In the former case, the two lines will be intersecting at a single point and in the second case, the two lines will be coincident.

2. The point of intersection of the lines $y = 3x$ and $x = 3y$ is

(*a*) (3, 0) (*b*) (0, 3)
(*c*) (3, 3) (*d*) (0, 0) **[CBSE SP 2011]**

Sol. (*d*) (0, 0)

From the two given equations, we have

$$y = 3 \times 3y = 9y$$

$\Rightarrow \quad 8y = 0$

$\Rightarrow \quad y = 0$

$\therefore$ From the second equation, $x = 0$.

$\therefore$ The required point of intersection of the two given lines is (0, 0).

3. If $\frac{2}{x}+\frac{3}{y} = 13$ and $\frac{5}{x}-\frac{4}{y} = -2$, then $x + y$ equals

(*a*) $\frac{5}{6}$ (*b*) $-\frac{5}{6}$

(*c*) $-\frac{1}{6}$ (*d*) $\frac{1}{6}$ **[CBSE SP 2011]**

Sol. (*a*) $\frac{5}{6}$

Let $\frac{1}{x} = u$ and $\frac{1}{y} = v$

$\therefore$ The given equation $\frac{2}{x}+\frac{3}{y} = 13$, becomes

$$2u + 3v = 13 \qquad \ldots(1)$$

Similarly, the given equation $\frac{5}{x}-\frac{4}{y} = -2$, becomes

$$5u - 4v = -2 \qquad \ldots(2)$$

From (1) and (2), by the method of cross-multiplication, we have

$$\frac{u}{3 \times 2 - 13 \times 4} = \frac{v}{-13 \times 5 - 2 \times 2} = \frac{1}{-2 \times 4 - 3 \times 5}$$

$$\Rightarrow \frac{u}{6-52} = \frac{v}{-65-4} = \frac{1}{-8-15}$$

$$\frac{u}{-46} = \frac{v}{-69} = \frac{1}{-23}$$

$\Rightarrow \quad \frac{u}{2} = \frac{v}{3} = 1$

$\therefore \quad u = 2, v = 3$

$\therefore \quad \frac{1}{x} = u = 2$

$\Rightarrow \quad x = \frac{1}{2}$

and $\quad \frac{1}{y} = v = 3$

$\Rightarrow \quad y = \frac{1}{3}$

$\therefore$ The value of $x + y$ is $\left(\frac{1}{2}+\frac{1}{3}\right)$ i.e. $\frac{5}{6}$.

4. If the pair of linear equations $4x - 3y = 1$ and $kx + 6y = 3$ have no solution at all, then the value of k will be

(*a*) 8 (*b*) -8

(*c*) 7 (*d*) -7

Sol. (*b*) -8

If the two given equations do not have any solution, then these two equations will represent two parallel lines.

Hence, we have $\frac{4}{k} = \frac{-3}{6} \neq \frac{1}{3}$

$\therefore \quad k = -8 \qquad (\because -\frac{3}{6} \neq \frac{1}{3}$ is true)

Fill in the Blanks

5. If $(6, k)$ is a solution of the equation $3x + y - 22 = 0$, then the value of k is **4**.

Sol. Since $(6, k)$ is a solution of $3x + y - 22 = 0$

$\therefore \quad 3(6) + k - 22 = 0$

$\Rightarrow \quad 18 + k - 22 = 0$

$\Rightarrow \quad k = 22 - 18$

$\Rightarrow \quad k = 4$

6. The value of α for which the pair of equations $3x + \alpha y = 6$ and $6x + 8y = 7$ has no solution is **4**.

Sol. For no solution,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\therefore \quad \frac{3}{6} = \frac{\alpha}{8} \neq \frac{-6}{-7}$$

$$\alpha = 4$$

7. If the lines given by $3x - 4y + 7 = 0$ and $kx + 3y - 5 = 0$ are parallel, then the value of k is **–2.25**.

Sol. The graphs of given pair of linear equations are parallel lines when these equations have no solution.

For no solution,

$$\Rightarrow \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\Rightarrow \quad \frac{3}{k} = \frac{-4}{3} \neq \frac{7}{-5}$$

$$\Rightarrow \quad k = \frac{-9}{4} = -2.25$$

8. If the sum of two numbers is 6 and their difference is 4, then the numbers are **5** and **1**.

Sol. Let the two numbers be x and y where $x > y$.

Then, $\quad x + y = 6 \qquad$ ...(1)

and $\quad x - y = 4 \qquad$ ...(2)

Solving (1) and (2), we get $x = 5$ and $y = 1$

Hence, the numbers are 5 and 1.

Assertion-Reason Type Questions

Directions (Q. Nos. 9 to 11): Each of these questions contains an assertion followed by reason. Read them carefully, and answer the question on the basis of the following options, select the one that best describes the two statements.

(a) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.

(b) Both Assertion and Reason are true but Reason is not the correct explanation of Assertion.

(c) If Assertion is true but Reason is false.

(d) If Assertion is false but Reason is true.

9. **Assertion:** $2x + 3y = 8$ is a linear equation in two variables.

Reason: 2 and 3 are variables.

Sol. The correct answer is (*c*).

$2x + 3y = 8$ is a linear equation in two variables as it has two variables x and y with non-zero is efficient. 2 and 3 are not variables.

Thus Assertion is correct while Reason is wrong.

10. **Assertion:** (2, 3) is a solution of the equation $2x + 3y = 13$.

Reason: By substituting 2 and 3 for x and y, we get 13 in LHS.

Sol. The correct answer is (*a*).

Both the statements are correct. By substituting 2 and 3 for x and y, we get 13 in L.H.S. which is equal to R.H.S. That's why (2, 3) is a solution. Thus, the Reason is correct explanation of the Assertion.

11. **Assertion:** (1, 3) is a point of the line represented by $x + y = 8$.

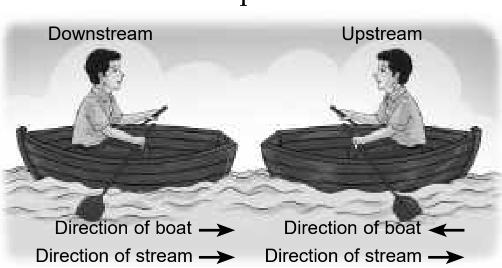
Reason: $1 + 3 = 4$

Sol. The correct answer is (*d*).

The Reason is correct. The Assertion is wrong as 1, 3 does not satisfy the equation $x + y = 8$.

Case Study Based Questions

12. A boatman rows his boat 64 km downstream in 4 hours and 48 km upstream in 4 hours.



Based on the above information, answer the following questions.

(*a*) If the speed of the boat in the still water be x km/h and the speed of the current be y km/h, then the pair of linear equations representing the situation is

(*i*) $x + y = 64, x - y = 48$

(*ii*) $x + y = 48, x - y = 64$

(*iii*) $x + y = 16, x - y = 12$

(*iv*) $x + y = 12, x - y = 16$

Ans. (*iii*) $x + y = 16, x - y = 12$

(*b*) What is the relation between speed, distance and time?

(*i*) Distance = Speed/Time

(*ii*) Speed = Distance/Time

(*iii*) Speed = Distance × Time

(*iv*) Time = Speed × Distance

Ans. (*ii*) Speed = Distance/Time

(*c*) What is the boatman's speed of rowing in still water?

(*i*) 10 km/h (*ii*) 12 km/h

(*iii*) 14 km/h (*iv*) 16 km/h

Ans. (*iii*) 14 km/h

(*d*) What is the speed of the current?

(*i*) 1 km/h (*ii*) 2 km/h

(*iii*) 3 km/h (*iv*) 4 km/h

Ans. (*ii*) 2 km/h

(*e*) If a pair of equations is consistent, then the lines will be

(*i*) always coincident.

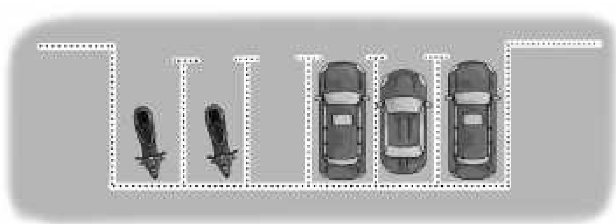
(*ii*) always intersecting.

(*iii*) parallel.

(*iv*) intersecting or coincident.

Ans. (*iv*) intersecting or coincident.

13. A parking lot is an area that is assigned for parking. Parking lots are common near shops, bars, restaurants and malls, etc. In a parking lot, parking charges for 20 cars and 15 scooters is ₹ 475 and parking charges for 65 cars and 12 scooters is ₹ 1360. Based on the above information, answer the following questions.



(*a*) If the parking charge for one car be ₹ x and the parking charge for one scooter be ₹ y, then the pair of linear equations representing the situation is

(*i*) $20x + 15y = 475$, $65x + 12y = 1360$

(*ii*) $20x + 15y = 1360$, $65x + 12y = 475$

(*iii*) $15x + 20y = 475$, $12x + 65y = 1360$

(*iv*) $15x + 20y = 1360$, $12x + 65y = 475$

Ans. (*i*) $20x + 15y = 475$, $65x + 12y = 1360$

(*b*) What is the parking charge for one car?

(*i*) ₹ 10 (*ii*) ₹ 20

(*iii*) ₹ 25 (*iv*) ₹ 30

Ans. (*ii*) ₹ 20

(*c*) What is the parking charge for one scooter?

(*i*) ₹ 5 (*ii*) ₹ 10

(*iii*) ₹ 15 (*iv*) ₹ 20

Ans. (*i*) ₹ 5

(*d*) What is the parking charge for 5 cars and 5 scooters?

(*i*) ₹ 95 (*ii*) ₹ 100

(*iii*) ₹ 125 (*iv*) ₹ 150

Ans. (*iii*) ₹ 125

(*e*) Graphically, if the pair of linear equations intersect at one point, then the pair of equations is

(*i*) consistent.

(*ii*) inconsistent.

(*iii*) both of these

(*iv*) none of these

Ans. (*i*) consistent.

Very Short Answer Type Questions

14. Find a common point on the lines $4x - 5 = y$ and $2x - y = 3$.

Sol. The required common point is the unique point of intersection of the two given lines.

Solving the two equation, we get

$$2x - (4x - 5) = 3$$

$$\Rightarrow \quad 2x - 4x + 5 = 3$$

$$\Rightarrow \quad -2x = -2$$

$$\therefore \quad x = 1$$

$\therefore$ From the first equation, we have

$$y = 4x - 5 = 4 - 5 = -1$$

$\therefore$ The required point of intersection is $(1, -1)$

15. Find the value of p so that the lines represented by $3x - 4y + 7 = 0$ and $px + 3y - 5 = 0$ are parallel.

Sol. If the two lines are parallel, then

$$\frac{3}{p} = -\frac{4}{3} \neq \frac{7}{-5}$$

$$\therefore \quad 4p = -9$$

$$\therefore \quad p = -\frac{9}{4}$$

Since $-\frac{4}{3} \neq \frac{-7}{5}$ is true.

Hence, the required value of p is $-\frac{9}{4}$.

Short Answer Type-I Questions

16. Find the coordinates of the point of intersection of the line $3x + 2y + 12 = 0$ with the coordinate axes.

Sol. Putting $y = 0$ in the equation $3x + 2y + 12 = 0$,

we get $\quad x = -\frac{12}{3} = -4$

Again, putting $x = 0$ within same equation, we get

$$y = -\frac{12}{2} = -6$$

Hence, the required point of intersection of the given line with x-axis is $(-4, 0)$ and that with y-axis is $(0, -6)$.

17. Solve the following pair of linear equations by the method of cross-multiplication:

$$2x - 3y = 1; \; 5 = x - 5y$$

Sol. We have

$$2x - 3y - 1 = 0 \quad \ldots(1)$$

and $\quad x - 5y - 5 = 0 \quad \ldots(2)$

Solving (1) and (2) by the method of cross-multiplication, we have

$$\frac{x}{3 \times 5 - 5 \times 1} = \frac{y}{-1 \times 1 + 5 \times 2} = \frac{1}{-2 \times 5 + 3 \times 1}$$

$$\Rightarrow \quad \frac{x}{10} = \frac{y}{9} = \frac{1}{-7}$$

$\therefore \; x = -\frac{10}{7}$, and $y = -\frac{9}{7}$

$\therefore$ The required solution is $x = -\frac{10}{7}$, $y = -\frac{9}{7}$.

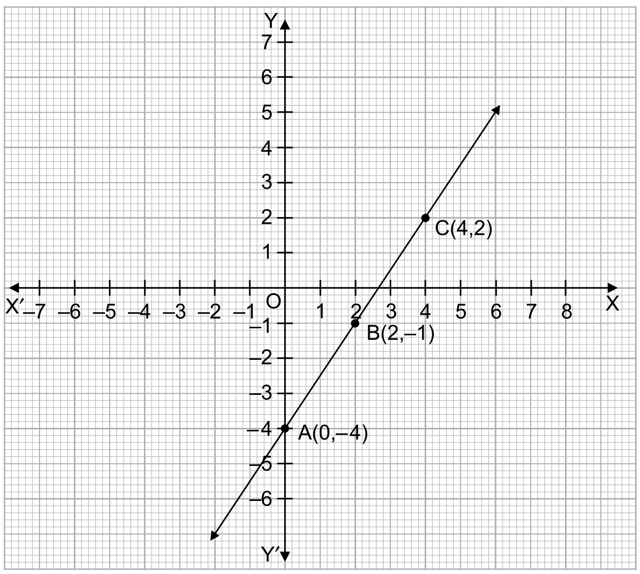
Short Answer Type-II Questions

18. Show graphically that the system of equations $3x - 2y = 8$ and $6x - 4y = 16$ is consistent, but they have infinitely many solutions.

Sol. We see that the two given equation $3x - 2y = 8$ and $6x - 4y = 16$ represent the same equation, since dividing both sides of the second equation by 2, we get the first equation. Hence, both the equations will represent the same line. In other words, both the lines will be coincident and so they will have infinitely many solutions. To draw the graph of the given equation, we write, it as $y = \dfrac{3x-8}{2}$. We tabulate some values of x and y as follows:

x	0	2	4
y	−4	−1	2

We plot the points A(0, −4), B(2, −1) and C(4, 2) on a graph paper and join them by a straight line ABC as shown below:



19. The sum of two numbers, as well as the difference between their squares is 25. Find the numbers.

Sol. Let the two numbers be x and y, where $x > y$. Then according to the problem, we have

$$x + y = 25 \qquad \text{...(1)}$$

and $$x^2 - y^2 = 25 \qquad \text{...(2)}$$

From (2), we have

$$(x + y)(x - y) = 25$$

$$\Rightarrow \quad 25(x - y) = 25 \qquad \text{[From (1)]}$$

$$\Rightarrow \quad x - y = 1 \qquad \text{...(3)}$$

Adding (1) and (3), we get

$$2x = 26$$

$$\Rightarrow \quad x = \frac{26}{2} = 13$$

Subtracting (3) from (1), we get

$$2y = 24$$

$$\Rightarrow \quad x = \frac{24}{2} = 12$$

∴ The required two numbers are 13 and 12.

Long Answer Type Questions

20. A man sold a chair and a table together for ₹760, thereby making a profit of 25% on chair and 10% on table. By selling them together for ₹767.50, he would have made a profit of 10% on the chair and 25% on the table. Find the cost price of each object.

Sol. Let the cost price of each chair be ₹x and that of each table be ₹y.

Then, the SP of the chair $= ₹\left(x + \dfrac{25x}{100}\right)$

$$= ₹\left(x + \frac{x}{4}\right)$$

$$= ₹\,\frac{5x}{4}$$

and the SP of the table $= ₹\left(y + \dfrac{10y}{100}\right)$

$$= ₹\left(y + \frac{y}{10}\right)$$

$$= ₹\,\frac{11y}{10}$$

∴ According to the problem, we have

$$\frac{5x}{4} + \frac{11y}{10} = 760$$

$$\Rightarrow \quad \frac{25x + 22y}{20} = 760$$

$$\Rightarrow \quad 25x + 22y = 15200 \qquad \text{...(1)}$$

Again, if the chair is sold at a profit of 10% and the table is sold at a profit of 25%, then

the SP of the chair $= ₹\left(x + \dfrac{10x}{100}\right)$

$$= ₹\left(x + \frac{x}{10}\right) = ₹\,\frac{11x}{10}$$

and the SP of the table $= ₹\left(y + \dfrac{25y}{100}\right)$

$$= ₹\left(y + \frac{y}{4}\right) = ₹\,\frac{5y}{4}$$

∴ According to the problem, we have

$$\frac{11x}{10} + \frac{5y}{4} = 767.50 = 767\frac{1}{2} = \frac{1535}{2}$$

$$\Rightarrow \quad \frac{22x + 25y}{20} = \frac{1535}{2}$$

$$\Rightarrow \quad 22x + 25y = 15350 \qquad \ldots(2)$$

Adding (1) and (2), we have

$$47(x + y) = 30550$$

$$\Rightarrow \quad x + y = \frac{30550}{47} = 650 \qquad \ldots(3)$$

Subtracting (1) from (2), we get

$$3(-x + y) = 150$$

$$\Rightarrow \quad -x + y = \frac{150}{3} = 50 \qquad \ldots(4)$$

Adding (3) and (4), we get

$$2y = 700$$

$$\Rightarrow \quad y = \frac{700}{2} = 350$$

Subtracting (4) from (3), we get

$$2x = 600$$

$$\Rightarrow \quad x = \frac{600}{2} = 300$$

Hence, the required cost price of each chair is ₹300 and that of each table is ₹350.

21. 8 men and 12 boys can finish a piece of work in 10 days, while 6 men and 8 boys can finish it in 14 days. Find the time taken by a single man and a single boy to do the same work separately.

[CBSE SP 2011]

Sol. Suppose one man alone takes x days to finish a piece of work and one boy alone can finish it in y days.

One man's one day work $= \frac{1}{x}$

One boy's one day work $= \frac{1}{y}$

Given, 8 men and 12 boys can finish the work in 10 days.

$$\Rightarrow \quad \frac{8}{x} + \frac{12}{y} = \frac{1}{10}$$

$$\Rightarrow \quad \frac{80}{x} + \frac{120}{y} = 1 \qquad \ldots(1)$$

Given, 6 men and 8 boys can finish the work in 14 days.

$$\Rightarrow \quad \frac{6}{x} + \frac{8}{y} = \frac{1}{14}$$

$$\Rightarrow \quad \frac{84}{x} + \frac{112}{y} = 1 \qquad \ldots(2)$$

Let $\frac{1}{x} = u$ and $\frac{1}{y} = v$

Then (1) and (2) becomes

$$80u + 120v - 1 = 0 \qquad \ldots(3)$$

$$84u + 112v - 1 = 0 \qquad \ldots(4)$$

From (3) and (4), by the method of cross-multiplication, we have

$$\frac{u}{120 \times (-1) - 112 \times (-1)} = \frac{v}{84 \times (-1) - 80 \times (-1)}$$

$$= \frac{1}{80 \times 112 - 120 \times 84}$$

$$\Rightarrow \quad \frac{u}{-120 + 112} = \frac{v}{-84 + 80} = \frac{1}{8960 - 10080}$$

$$\Rightarrow \quad \frac{u}{-8} = \frac{v}{-4} = \frac{1}{-1120}$$

$$\therefore \quad u = \frac{8}{1120} = \frac{1}{140}, \; v = \frac{4}{1120} = \frac{1}{280}$$

$$\therefore \quad x = 140, \; y = 280$$

Hence, 1 man can complete the work in 140 days and 1 boy can complete the same work in 280 days.

Let's Compete

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Multiple-Choice Questions

1. The value of m for which the pair of linear equations $1 = 9x + my$ and $2 - 4y = 3x$ has exactly one solution is

(*a*) all real values

(*b*) all real values except 12

(*c*) $p = 12$ only

(*d*) $p = \pm 12$

Sol. (*b*) all real values except 12

The condition that the two given equation $9x + my - 1 = 0$ and $3x + 4y - 2 = 0$ have only one solution is $\frac{9}{3} \neq \frac{m}{4}$

$$\Rightarrow \quad \frac{m}{4} \neq 3$$

$$\Rightarrow \quad m \neq 3 \times 4 = 12$$

Hence, two given equations will have only one solution for all real values of m except when $m = 12$.

2. The value of k for which the pair of linear equations $kx + 3y = k - 2$ and $12x + ky = k$ has no solution at all are

(*a*) 6 and 5

(*b*) −5 and 6

(*c*) −6 and 6

(*d*) −5 and 5

Sol. (*c*) −6 and 6

The condition that the equation $kx + 3y - k + 2 = 0$ and $12x + ky - k = 0$ have no solution at all is

$$\frac{k}{12} = \frac{3}{k} \neq \frac{2-k}{-k}$$

Now, from $\frac{k}{12} = \frac{3}{k}$, we get $k^2 = 36$

$\Rightarrow \quad k = \pm 6 \quad \ldots(1)$

and from $\frac{3}{k} \neq \frac{2-k}{-k}$, we have $-3k \neq 2k - k^2$

$\Rightarrow \quad k^2 - 5k \neq 0$

i.e. $\quad k(k-5) \neq 0$

$\Rightarrow \quad k \neq 0$ or $k \neq 5$

which is true from (1)

$\therefore$ The required values of k are −6 and 6.

3. The solution of the linear equations
$2(ax - by) + (a + 4b) = 0$
and $2(bx + ay) + (b - 4a) = 0$ is

(*a*) $x = -\frac{1}{2}, y = 2$

(*b*) $x = 2, y = -\frac{1}{2}$

(*c*) $x = -\frac{a}{2}, y = 2b$

(*d*) $x = 2b, y = -\frac{a}{2}$

Sol. (*a*) $x = -\frac{1}{2}, y = 2$

From the given equation, we have

$2ax - 2by + a + 4b = 0 \quad \ldots(1)$

and $\quad 2bx + 2ay + b - 4a = 0 \quad \ldots(2)$

Multiplying (1) by 'b', we get

$2abx - 2b^2y + ab + 4b^2 = 0 \quad \ldots(3)$

Multiplying (2) by 'a', we get

$2abx + 2a^2y + ab - 4a^2 = 0 \quad \ldots(4)$

Subtracting (4) from (3), we get

$$-2(a^2 + b^2)y + 4(a^2 + b^2) = 0$$

$\Rightarrow \quad y = \frac{4}{2} = 2$

$\therefore$ From (1), we get

$2ax - 4b + a + 4b = 0$

$\Rightarrow \quad 2x = -1$

$\Rightarrow \quad x = -\frac{1}{2}$

$\therefore$ The required solution is $x = -\frac{1}{2}, y = 2$.

4. The area of the region bounded by the lines $-5x + 3y = 15$, $x = 0$ and $y = 0$ is

(*a*) 16 sq units (*b*) 8 sq units

(*c*) 15 sq units (*d*) $\frac{15}{2}$ sq units

Sol. (*d*) $\frac{15}{2}$ sq units

Putting $x = 0$ and $y = 0$ successively in the given equation $-5x + 3y = 15$, we get $y = 5$ and $x = -3$.

$\therefore$ The coordinates of the vertices of ΔAOB are A(−3, 0), O(0, 0) and B(5, 0). ΔAOB is a right-angled triangle where ∠AOB = 90°.

$\therefore$ Area of ΔAOB $= \frac{1}{2} \times \text{OA} \times \text{OB}$

$= \frac{1}{2} \times 3 \times 5$ sq units

$= \frac{15}{2}$ sq units

5. The distance between two lines is the same non-zero quantity throughout. Then the two lines are

(*a*) consistent and have a unique solution.

(*b*) consistent and have infinite no. of solutions.

(*c*) inconsistent.

(*d*) dependent.

Sol. (*c*) inconsistent.

If the distance between two lines is a non-zero constant, then the two lines are clearly parallel to each other and so they do not intersect each other. Therefore, they do not have any solution at all. Hence, the two lines are inconsistent.

6. A and B are two brothers, B being younger than A. Two times A's age exceeds B's age by 5 years and $\frac{1}{4}$th of A's age is equal to $\frac{1}{3}$ of B's age. Then ages of A and B are respectively

(*a*) $4\frac{1}{2}$ years and $3\frac{1}{2}$ years

(*b*) 4 years and 3 years

(*c*) 5 years and 3 years

(*d*) 5 years and 4 years

Sol. (*b*) 4 years and 3 years

Let the ages of A and B be x years and y years respectively, where $x > y$. Then according to the problem, we have

$2x - y = 5 \quad \ldots(1)$

and $\quad \frac{x}{4} = \frac{y}{3}$

$$\Rightarrow \quad 3x = 4y \quad \ldots(2)$$

From (2), we have

$$y = \frac{3x}{4} \quad \ldots(3)$$

$\therefore$ From (1), we get

$$2x - \frac{3x}{4} = 5$$

$$\Rightarrow \quad \frac{5x}{4} = 5$$

$$\Rightarrow \quad x = 4$$

$\therefore$ From (3), $\quad y = \frac{3}{4} \times 4 = 3$

Hence, the required ages of A and B are 4 years and 3 years respectively.

7. A two-digit number is such that the product of its digits is 14. If 45 is added to the number, then the digits interchange their places. Then the number is

(*a*) 27 (*b*) 72
(*c*) 14 (*d*) 41

Sol. (*a*) 27

Let the digit in the unit's place of the number be x and that in the ten's place be y. Then the number is $10y + x$.

$\therefore$ According to the problem, we have

$$xy = 14 \quad \ldots(1)$$

and $\quad 10y + x + 45 = 10x + y$

$$\Rightarrow \quad 9y - 9x + 45 = 0$$

$$\Rightarrow \quad y - x + 5 = 0 \quad \ldots(2)$$

$\therefore$ From (1) and (2), we get

$$x(-5 + x) = 14$$

$$\Rightarrow \quad x^2 - 5x - 14 = 0$$

$$\Rightarrow \quad x^2 + 2x - 7x - 14 = 0$$

$$\Rightarrow \quad x(x + 2) -7(x + 2) = 0$$

$$\Rightarrow \quad (x + 2)(x - 7) = 0$$

$\therefore$ Either $x - 7 = 0 \quad \ldots(3)$

or $\quad x + 2 = 0 \quad \ldots(4)$

From (3), $x = 7$ and from (4), $x = -2$ which is not possible, since the digit of a number cannot be negative.

Hence, $x = 7$.

$\therefore$ From (1), $y = \frac{14}{7} = 2$

$\therefore$ The required number is $10 \times 2 + 7$ i.e. 27.

8. The value of c such that the pair of linear equations $cx + 3y = 3$, $12x + cy = 6$ represents a pair of parallel lines is

(*a*) 6, –6 (*b*) 6
(*c*) –6 (*d*) does not exist

Sol. (*c*) –6

If the two given lines are parallel, then

$$\frac{c}{12} = \frac{3}{6} \neq \frac{3}{6}$$

Now, from $\frac{c}{12} = \frac{3}{c}$, we have

$$c^2 = 3 \times 12 = 36$$

$$\therefore \quad c = \pm 6$$

When $c = 6$, then $\frac{3}{6} \neq \frac{3}{6}$ is not true.

When $c = -6$, then $\frac{3}{-6} \neq \frac{3}{6}$ is true.

Hence, the required value of c is –6.

9. The students of a class are made to stand in rows. If 1 student is extra in each row, there would be 2 rows less. On the other hand, if 1 student is less in each row, there would be 3 rows more. If the number of students in each row is x and the number of rows be y, then the total number of students in the class can be obtained by solving the following pair of linear equations

(*a*) $2x - y = 2$ and $3x + y = 3$
(*b*) $y - 2x = 2$ and $3x - y = 3$
(*c*) $x - 2y = 2$ and $y - 3x = 3$
(*d*) $y + 2x = 2$ and $3x - y = 3$

Sol. (*b*) $y - 2x = 2$ and $3x - y = 3$

The number of students in each row is x and the number of rows is y.

$\therefore$ The total number of students in the class is xy.

According to the problem, if 1 student is extra in each row i.e. $(x + 1)$, then the number of rows will be $(y - 2)$.

$\therefore$ The total number of students in the class is $(x + 1)(y - 2)$.

Hence, $\quad (x + 1)(y - 2) = xy$

$$\Rightarrow \quad xy - 2x + y - 2 = xy$$

$$\Rightarrow \quad y - 2x = 2 \quad \ldots(1)$$

Again, if 1 student is less in each row i.e. $(x - 1)$, then the number of rows will be $(y + 3)$.

$$\therefore \quad (x - 1)(y + 3) = xy$$

$$\Rightarrow \quad xy + 3x - y - 3 = xy$$

$$\Rightarrow \quad 3x - y = 3 \quad \ldots(2)$$

$\therefore$ The total number of students can be obtained by solving the equations $y - 2x = 2$ and $3x - y = 3$.

10. Two pipes take 12 hours to fill a water tank. If the pipe of larger diameter is used for 4 hours and the pipe of smaller diameter for 9 hours, only half of the tank can be filled. Then the pipe of smaller diameter can fill the water tank in

(*a*) 10 hours (*b*) 25 hours

(*c*) 30 hours (*d*) 20 hours

Sol. (*c*) 30 hours

Let the pipe of smaller diameter can fill the tank in x hours and the pipe of larger diameter can fill the tank in y hours. Clearly, $x > y$.

$\therefore$ In 1 hour, the pipe of smaller diameter can fill $\frac{1}{x}$th part of the tank and in one hour, the pipe of larger diameter can fill $\frac{1}{y}$th part of the tank.

$\therefore$ According to the problem, we have

$$\frac{1}{x} + \frac{1}{y} = \frac{1}{12} \qquad \ldots(1)$$

Also, it is given that when the pipe of larger diameter used for 4 hours and the pipe of smaller diameter for 9 hours, only half of the tank is filled.

$$\therefore \qquad \frac{9}{x} + \frac{4}{y} = \frac{1}{2} \qquad \ldots(2)$$

From (1) and (2), we have

$$u + v = \frac{1}{12} \qquad \ldots(3)$$

$$\text{and} \qquad 9u + 4v = \frac{1}{2} \qquad \ldots(4)$$

$$\text{where} \qquad u = \frac{1}{x} \qquad \ldots(5)$$

$$\text{and} \qquad v = \frac{1}{y} \qquad \ldots(6)$$

From (3), we have

$$v = \frac{1}{12} - u \qquad \ldots(7)$$

$\therefore$ From (4), we have

$$9u + 4\left(\frac{1}{12} - u\right) = \frac{1}{2}$$

$$\Rightarrow \qquad 5u = \frac{1}{2} - \frac{1}{3} = \frac{3-2}{6} = \frac{1}{6}$$

$$\therefore \qquad u = \frac{1}{30}$$

$$\Rightarrow \qquad x = 30 \qquad \text{[From (5)]}$$

$\therefore$ The pipe of smaller diameter can fill the tank in 30 hours.

Value-based Questions (Optional)

(Page 52)

1. While teaching about the Indian National Flag, the teacher asked students that how many spokes are there in the blue colour wheel? One student replies that it is 8 times the number of colours in the flag. While the other says that the sum of the number of colours in the flag and the number of lines in the wheel is 27.

(*a*) Convert the statement into linear equations in two variables and find the number of lines in the wheel.

(*b*) What does the wheel signify in the flag?

Sol. (*a*) Let the number of spokes in the wheel be x and the number of colours be y. Then according to the one student,

$$x = 8y$$

$$\Rightarrow \qquad x - 8y = 0 \qquad \ldots(1)$$

According to some other students,

$$x + y = 27 \qquad \ldots(2)$$

Since the number of lines in the wheel is same as the number of spokes in the wheel.

Hence, (1) and (2) are the required linear equations.

From (1) and (2), we have

$$8y + y = 27$$

$$\Rightarrow \qquad 9y = 27$$

$$\Rightarrow \qquad y = \frac{27}{9} = 3$$

$\therefore$ From (2), $x = 27 - 3 = 24$

Hence, the required number of spokes or lines in the wheel is 24.

(*b*) The wheel signifies the continuous progress of the country under the rule of law.

2. In a painting competition of a school, a child made Indian National Flag whose perimeter was 50 cm. Its area will be decreased by 6 sq cm, if length is decreased by 3 cm and breadth is increased by 2 cm.

(*a*) Find the dimensions of the flag.

(*b*) What does the saffron colour signify in the flag?

Sol. (*a*) Let x cm be the length and y cm be the breadth of the rectangular national flag.

Since the perimeter is 50 cm.

$$\therefore \qquad 2(x + y) = 50$$

$$\Rightarrow \qquad x + y = 25 \qquad \ldots(1)$$

Original area of the flag = xy cm^2.

$\therefore$ According to the problem, we have

$$(x - 3)(y + 2) = xy - 6$$

$$\Rightarrow \qquad xy + 2x - 3y - 6 = xy - 6$$

$$\Rightarrow \qquad 2x - 3y = 0 \qquad \ldots(2)$$

$\therefore$ From (1), we have

$$y = 25 - x \qquad \ldots(3)$$

$\therefore$ From (2), we have

$$2x - 3\,(25 - x) = 0$$

$$\Rightarrow \qquad 2x - 75 + 3x = 0$$

$$\Rightarrow \qquad 5x = 75$$

$$\Rightarrow \qquad x = \frac{75}{5} = 15$$

$\therefore$ From (3), $\quad y = 25 - 15 = 10$

Hence, the required dimensions of the flag are length = 15 cm and breadth = 10 cm.

(*b*) The saffron colour signifies the strength and courage of the people of the country.

4

Quadratic Equations

Checkpoint (Page 55)

1. Which of the following is a polynomial?

(*a*) $3x^2 + \frac{1}{x} - 5$

(*b*) $-2x^2 + 5\sqrt{x} + 8$

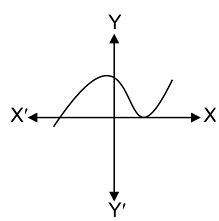
(*c*) $\sqrt{2}\,x^3 + \sqrt{3}\,x^2 + \sqrt{5}\,x - 3$

(*d*) $\frac{3}{x^3} + 4x^2 - 5x + \frac{1}{3}$

Sol. (*c*) $\sqrt{2}x^3 + \sqrt{3}x^2 + \sqrt{5}x - 3$

We know that a polynomial is an algebraic expression in which the exponents of x in each term is a non-negative integer. Only the expression given in option (*c*) of the choices is such an expression.

2. The graph of $y = p(x)$ is given. The number of zeroes of $p(x)$ are:



(*a*) 0

(*b*) 3

(*c*) 2

(*d*) 4

[CBSE SP 2011]

Sol. (*c*) 2

The given graph cuts and touches the x-axis only at two points. Hence, the required number of zeroes of $p(x)$ is 2.

3. If a pair of equations is consistent, then the lines will be

(*a*) always intersecting.

(*b*) always coincident.

(*c*) intersecting or coincident.

(*d*) parallel.

Sol. (*c*) intersecting or coincident.

We know that a pair of equations is consistent when one or more points of the graphs of these equations is or are common. In this case, the lines will be either intersecting or coincident.

4. Verify that 3 is a zero of polynomial $x^3 - 3x^2 - x + 3$.

Sol. Putting $x = 3$ in the polynomial $p(x) = x^3 - 3x^2 - x + 3$, we get

$$\begin{aligned} p(3) &= 3^3 - 3 \times 3^2 - 3 + 3 \\ &= 27 - 27 - 3 + 3 \\ &= 0 \end{aligned}$$

Hence, 3 is a zero of $p(x)$.

5. If α, β and γ are zeroes of cubic polynomial $x^3 + px^2 + qx + 2$ such that $\alpha\beta + 1 = 0$. Find the value of $2p + q + 5$.

Sol. We have,

$$\alpha + \beta + \gamma = -p \quad \ldots(1)$$

$$\alpha\beta + \alpha\gamma + \beta\gamma = q \quad \ldots(2)$$

and $$\alpha\beta\gamma = -2 \quad \ldots(3)$$

Also, $$\alpha\beta = -1 \quad \text{[Given]}\ldots(4)$$

$\therefore$ From (3) and (4), we have

$$\gamma = 2 \quad \ldots(5)$$

$\therefore$ From (1) and (5),

$$\alpha + \beta = -p - 2 \quad \ldots(6)$$

$\therefore$ From (2), (4) and (6), we have

$$-1 + \gamma(\alpha + \beta) = q$$

$$\Rightarrow \quad -1 + 2(-p - 2) = q$$

$$\Rightarrow \quad 2p + q + 5 = 0$$

6. If α, β and γ are the zeroes of the polynomial $x^3 + 3x^2 + 10x - 24$ then find the value of $\frac{1}{\gamma} + \frac{1}{\alpha} + \frac{1}{\beta}$.

Sol. We have

$$\alpha\beta + \alpha\gamma + \beta\gamma = 10 \quad \ldots(1)$$

and $$\alpha\beta\gamma = 24 \quad \ldots(2)$$

$\therefore$ We have

$$\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = \frac{\beta\gamma + \gamma\alpha + \alpha\beta}{\alpha\beta\gamma} = \frac{10}{24}$$

[From (1) and (2)]

$$= \frac{5}{12}$$

which is the required value.

7. If $\frac{x}{3} + \frac{y}{4} = 6$ and $\frac{x}{6} + \frac{y}{2} = 6$, find the value of $3y - 2x$ and $\frac{x}{y} + \frac{1}{2}$.

Sol. We have $$\frac{x}{3} + \frac{y}{4} = 6$$

$$\Rightarrow \quad \frac{4x + 3y}{12} = 6$$

$$\Rightarrow \quad 4x + 3y = 72 \quad \ldots(1)$$

and $$\frac{x}{6} + \frac{y}{2} = 6$$

$$\Rightarrow \quad \frac{x + 3y}{6} = 6$$

$$\Rightarrow \quad x + 3y = 36 \quad \ldots(2)$$

Multiplying (2) by 4, we get

$$4x + 12y = 144 \quad \ldots(3)$$

$\therefore$ Subtracting (1) from (3), we get

$$9y = 144 - 72 = 72$$

$$\Rightarrow \quad y = \frac{72}{9} = 8$$

$\therefore$ From (2), we get

$$x = 36 - 3 \times 8 = 12$$

Now, putting the values of x and y in the given expressions, we get

$$3y - 2x = 3 \times 8 - 2 \times 12$$
$$= 24 - 24 = 0$$

and $$\frac{x}{y} + \frac{1}{2} = \frac{12}{8} + \frac{1}{2}$$
$$= \frac{3}{2} + \frac{1}{2}$$
$$= \frac{3+1}{2}$$
$$= \frac{4}{2} = 2$$

Hence, the required values of $3y - 2x$ and $\frac{x}{y} + \frac{1}{2}$ are 0 and 2 respectively.

8. Solve for x and y: $149x - 330y = -511$ and $-330x + 149y = -32$

Sol. We have

$$149x - 330y = -511 \quad \ldots(1)$$

and $$-330x + 149y = -32 \quad \ldots(2)$$

Adding (1) and (2), we get

$$149(x + y) - 330(x + y) = -543$$

$$\Rightarrow \quad (149 - 330)(x + y) = -543$$

$$\Rightarrow \quad -181(x + y) = -543$$

$$\Rightarrow \quad x + y = \frac{543}{181} = 3 \quad \ldots(3)$$

Again, subtracting (2) from (1), we get

$$149(x - y) + 330(x - y) = -511 + 32$$

$$\Rightarrow \quad (149 + 330)(x - y) = -479$$

$$\Rightarrow \quad 479(x - y) = -479$$

$$\Rightarrow \quad x - y = -1 \quad \ldots(4)$$

Adding (3) and (4), we get

$$2x = 3 - 1 = 2$$

$$\Rightarrow \quad x = 1$$

$\therefore$ From (3), $$y = 3 - x$$
$$= 3 - 1 = 2$$

$\therefore$ The required solution is $x = 1$, $y = 2$.

9. Five years ago, A was thrice as old as B, and ten years later, A shall be twice as old as B. What are the present ages of A and B?

Sol. Let the present ages of A and B be x years and y years respectively.

$\therefore$ According to the problem, we have

$$x - 5 = 3(y - 5)$$

$$\Rightarrow \quad x - 3y + 10 = 0 \quad \ldots(1)$$

and $$x + 10 = 2(y + 10)$$

$$\Rightarrow \quad x - 2y - 10 = 0 \quad \ldots(2)$$

Subtracting (2) from (1), we get

$$-y + 20 = 0$$

$$\Rightarrow \quad y = 20$$

$\therefore$ From (1), $$x = 3y - 10$$
$$= 3 \times 20 - 10$$
$$= 50$$

$\therefore$ The required present ages of A and B are 50 years and 20 years respectively.

10. Divide 100 into two parts such that the sum of their reciprocals is $\frac{1}{24}$.

Sol. Let one of the two parts of the number be x. Then the other part is $100 - x$.

$\therefore$ According to the problem,

$$\frac{1}{x} + \frac{1}{100 - x} = \frac{1}{24}$$

$$\frac{100 - x + x}{x(100 - x)} = \frac{1}{24}$$

$\Rightarrow$ $x(100 - x) = 2400$

$\Rightarrow$ $x^2 - 100x + 2400 = 0$

$\Rightarrow$ $x^2 - 60x - 40x + 2400 = 0$

$\Rightarrow$ $x(x - 60) - 40(x - 60) = 0$

$\Rightarrow$ $(x - 60)(x - 40) = 0$

$\therefore$ Either $x - 60 = 0$ ...(1)

or $x - 40 = 0$ ...(2)

From (1), $x = 60$ and from (2), $x = 40$.

Hence, the required two parts are 60 and 40.

Milestone 1

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Multiple-Choice Questions

1. Which one of the following is not a quadratic equation?

(*a*) $3(x - 1)^2 = 4x^2 - 5x + 3$

(*b*) $5x - x^2 = 2x^2 + 3$

(*c*) $(\sqrt{3}x + \sqrt{2})^2 - x^2 = 2x^2 + 2x$

(*d*) $3(x - 1)^2 = 5x^2 - 2x + 1$

Sol. (*c*) $\left(\sqrt{3}x + \sqrt{2}\right)^2 - x^2 = 2x^2 + 2x$

When an algebraic expression can be simplified in the form $ax^2 + \mathrm{b}x + c$ where a, b and c are constants and $a \neq 0$ is called a quadratic expression and the corresponding equation $ax^2 + bx + c = 0$ is called a quadratic equation. We see that out of four expressions in (*a*), (*b*), (*c*) and (*d*), only the equation in (*c*) cannot be simplified in the form $ax^2 + bx + c = 0$ where $a \neq 0$, since the second degree terms from both sides of the equation cancel each other giving a linear equation.

2. If $\frac{1}{2}$ is a root of the equation $x^2 + kx - \frac{5}{4} = 0$, then the value of k is

(*a*) $\frac{1}{2}$ (*b*) -2

(*c*) $\frac{1}{4}$ (*d*) 2

Sol. (*d*) 2

Putting $x = \frac{1}{2}$ in the equation $x^2 + kx - \frac{5}{4} = 0$, we get

$$\left(\frac{1}{2}\right)^2 + \frac{k}{2} - \frac{5}{4} = 0$$

$\Rightarrow$ $\frac{1}{4} - \frac{5}{4} + \frac{k}{2} = 0$

$\Rightarrow$ $\frac{k}{2} = 1$

$\therefore$ $k = 2$

Very Short Answer Type Questions

3. Check whether the following equations are quadratic or not.

(*a*) $3x^2 = x + 4$

(*b*) $x^2 + x - 12 = 0$

(*c*) $x^3 - 4x^2 - 7x + 3 = 0$

(*d*) $(5x + 1)(2x + 3) = (10x + 1)(x + 2)$

Sol. (*a*) The given equation can be written as $3x^2 - x - 4 = 0$, which is of the form $ax^2 + bx + c = 0$ where $a \neq 0$.

$\therefore$ This equation is quadratic.

(*b*) The given equation being of the form $ax^2 + bx + c = 0$, $a \neq 0$, is quadratic.

(*c*) The highest exponent of x on the left side of the given equation is 3 and not 2. So, it is not quadratic.

(*d*) The given equation can be simplified as

$$10x^2 + 17x + 3 = 10x^2 + 21x + 2$$

$\Rightarrow$ $4x - 1 = 0$

which is a linear equation and so it is not quadratic.

4. Represent the following situations in the form of quadratic equations:

(*a*) The sum of two numbers is 18. The sum of their reciprocals is $\frac{1}{4}$. We need to find the numbers.

(*b*) The product of the ages (in years) of two sisters is 238. The difference in their ages (in years) is 3. We would like to find their present ages.

Sol. (*a*) Let one of the numbers be x. Then the other number is $18 - x$.

$\therefore$ According to the problem,

$$\frac{1}{x} + \frac{1}{18-x} = \frac{1}{4}$$

$$\Rightarrow \quad \frac{18-x+x}{x(18-x)} = \frac{1}{4}$$

$$\Rightarrow \quad x(18-x) = 72$$

$$\Rightarrow \quad x^2 - 18x + 72 = 0$$

which is the required quadratic equation.

(*b*) Let x years be the age of one of the sisters (say, elder sister)

Then the age of another sister is $\frac{238}{x}$ years.

According to the problem, $x - \frac{238}{x} = 3$

$$\Rightarrow \quad x^2 - 3x - 238 = 0$$

which is the required equation.

Short Answer Type-I Questions

5. For the quadratic equation $x^2 + 2x - 15 = 0$, determine which of the following are solution(s):

(*a*) $x = -5$ (*b*) $x = -3$ (*c*) $x = 3$

Sol. (*a*) Putting $x = -5$ in the given equation, we see that

$$\begin{aligned} \text{LHS} &= (-5)^2 + 2(-5) - 15 \\ &= 25 - 10 - 15 \\ &= 0 \\ &= \text{RHS} \end{aligned}$$

$\therefore$ This is a solution of the given equation.

(*b*) For $x = -3$, we have

$$\begin{aligned} \text{LHS} &= (-3)^2 + 2(-3) - 15 \\ &= 9 - 6 - 15 \\ &= -12 \\ &\neq \text{RHS} \end{aligned}$$

$\therefore$ This is not a solution of the given equation.

(*c*) For $x = 3$, we have

$$\begin{aligned} \text{LHS} &= 3^2 + 2 \times 3 - 15 \\ &= 9 + 6 - 15 \\ &= 15 - 15 \\ &= 0 \\ &= \text{RHS} \end{aligned}$$

$\therefore$ This is a solution of the given equation.

6. In each of the following, determine whether the given values of x are solutions of the given equation or not.

(*a*) $2x^2 + 5x - 25 = 0$; $x = -5$, $x = \frac{5}{2}$

(*b*) $24x^2 - 2x - 15 = 0$; $x = \frac{5}{6}$, $x = -\frac{3}{4}$

(*c*) $mnx^2 + (m^2 + n^2)x + mn = 0$; $x = -\frac{m}{n}$, $x = -\frac{n}{m}$

(*d*) $(2x + 3)(3x - 2) = 0$; $x = -\frac{3}{2}$, $x = -\frac{2}{3}$

Sol. (*a*) For $x = -5$,

$$\begin{aligned} \text{LHS} &= 2 \times (-5)^2 + 5 \times (-5) - 25 \\ &= 50 - 25 - 25 \\ &= 0 \\ &= \text{RHS} \end{aligned}$$

$\therefore$ $x = -5$ is a solution.

Again, for $x = \frac{5}{2}$,

$$\begin{aligned} \text{LHS} &= 2 \times \left(\frac{5}{2}\right)^2 - 25 \\ &= \frac{25}{2} + \frac{25}{2} - 25 \\ &= 25 - 25 \\ &= 0 \\ &= \text{RHS} \end{aligned}$$

$\therefore$ $x = \frac{5}{2}$ is also a solution.

(*b*) For $x = \frac{5}{6}$,

$$\begin{aligned} \text{LHS} &= 24 \times \left(\frac{5}{6}\right)^2 - 2 \times \frac{5}{6} - 15 \\ &= 24 \times \frac{25}{36} - \frac{5}{3} - 15 \\ &= \frac{2 \times 25}{3} - \frac{5}{3} - 15 \\ &= \frac{50-5}{3} - 15 \\ &= \frac{45}{3} - 15 \\ &= 15 - 15 \\ &= 0 \\ &= \text{RHS} \end{aligned}$$

$\therefore$ $x = \frac{5}{6}$ is a solution.

Again, for $x = -\frac{3}{4}$, we have

$$\begin{aligned} \text{LHS} &= 24 \times \left(-\frac{3}{4}\right)^2 - 2 \times \left(-\frac{3}{4}\right) - 15 \\ &= \frac{24 \times 9}{16} + \frac{3}{2} - 15 \\ &= \frac{27}{2} + \frac{3}{2} - 15 \end{aligned}$$

$$= \frac{27+3}{2} - 15$$
$$= 15 - 15$$
$$= 0$$
$$= \text{RHS}$$

$\therefore \quad x = -\frac{3}{4}$ is also a solution.

(*c*) For $x = -\frac{m}{n}$, we have

$$\text{LHS} = mn\left(-\frac{m}{n}\right)^2 + (m^2+n^2)\left(-\frac{m}{n}\right) + mn$$
$$= mn \times \frac{m^2}{n^2} - \frac{m^3}{n} - mn + mn$$
$$= \frac{m^3}{n} - \frac{m^3}{n} - mn + mn$$
$$= 0 + 0$$
$$= 0$$
$$= \text{RHS}$$

$\therefore \quad x = -\frac{m}{n}$ is a solution.

Again, for $x = -\frac{n}{m}$, we have

$$\text{LHS} = mn \times \left(-\frac{n}{m}\right)^2 + (m^2+n^2)\left(-\frac{n}{m}\right) + mn$$
$$= mn \times \frac{n^2}{m^2} - \frac{nm^2}{m} - \frac{n^3}{m} + mn$$
$$= \frac{n^3}{m} - nm - \frac{n^3}{m} + nm$$
$$= 0 + 0$$
$$= 0$$
$$= \text{RHS}$$

$\therefore \quad x = -\frac{n}{m}$ is also a solution.

(*d*) For $x = -\frac{3}{2}$,

$$\text{LHS} = \left(-2 \times \frac{3}{2} + 3\right)\left(-3 \times \frac{3}{2} - 2\right)$$
$$= 0 \times \left(-\frac{9}{2} - 2\right)$$
$$= 0$$
$$= \text{RHS}$$

$\therefore \quad x = -\frac{3}{2}$ is a solution.

Again, for $x = -\frac{2}{3}$, we have

$$\text{LHS} = \left(-2 \times \frac{2}{3} + 3\right)\left(-3 \times \frac{2}{3} - 2\right)$$
$$= \left(-\frac{4}{3} + 3\right)(-4)$$
$$= +\frac{5}{3} \times (-4)$$
$$\neq \text{RHS}$$

$\therefore \quad x = -\frac{2}{3}$ is not a solution.

Short Answer Type-II Questions

7. In each of the following, find the value of k for which the given value of x is a solution of the given equation.

(*a*) $kx^2 - x - 2 = 0$; $x = \frac{2}{3}$

(*b*) $\sqrt{5}x^2 + kx + 4\sqrt{5} = 0$; $x = -\sqrt{5}$

Sol. (*a*) For $x = \frac{2}{3}$, we have from the given equation

$$k\left(\frac{2}{3}\right)^2 - \frac{2}{3} - 2 = 0$$
$$\Rightarrow \quad \frac{4k}{9} - \frac{8}{3} = 0$$
$$\Rightarrow \quad \frac{4k}{9} = \frac{8}{3}$$
$$\Rightarrow \quad k = \frac{8}{3} \times \frac{9}{4} = 6$$

which is the required value of k.

(*b*) For $x = -\sqrt{5}$, we have from the given equation

$$\sqrt{5}\left(-\sqrt{5}\right)^2 - \sqrt{5}k + 4\sqrt{5} = 0$$
$$\Rightarrow \quad 5\sqrt{5} - \sqrt{5}k + 4\sqrt{5} = 0$$
$$\Rightarrow \quad 5 - k + 4 = 0$$

$\Rightarrow$ $k = 9$ which is the required value of k.

8. If one root of the quadratic equation $2x^2 + px + 4 = 0$ is 2, find the other root. Also find the value of p.

Sol. Let α be the other root.

Then, sum of the roots $= \alpha + 2 = -\frac{p}{2}$ ...(1)

and the product of the roots $= 2\alpha = \frac{4}{2} = 2$

$\therefore \quad \alpha = 1$ ...(2)

$\therefore$ From (1) and (2),

$$1 + 2 = -\frac{p}{2}$$
$$\Rightarrow \quad p = 3 \times -2$$
$$\Rightarrow \quad p = -6$$

$\therefore$ The other root is 1 and the required value of p is -6.

9. Find the value of λ for which $x = 2$ is a root of the equation $(2\lambda + 1)x^2 + 2x - 3 = 0$.

Sol. Putting $x = 2$ in the given equation, we get

$$(2\lambda + 1)2^2 + 2 \times 2 - 3 = 0$$

$$\Rightarrow \quad (2\lambda + 1)4 + 4 - 3 = 0$$

$$\Rightarrow \quad 8\lambda + 4 + 4 - 3 = 0$$

$$\Rightarrow \quad 8\lambda = -5$$

$$\therefore \quad \lambda = -\frac{5}{8}$$

which is the required value of λ.

Long Answer Type Questions

10. Find the values of p and q for which $x = \frac{1}{2}$ and $x = \frac{3}{4}$ are the roots of the equation $px^2 + qx - 3 = 0$.

Sol. We have

$$\text{Sum of the roots} = \frac{1}{2} + \frac{3}{4} = -\frac{q}{p}$$

$$\Rightarrow \quad \frac{q}{p} = -\left(\frac{2+3}{4}\right) = -\frac{5}{4}$$

$$\Rightarrow \quad q = -\frac{5p}{4} \quad \ldots(1)$$

$$\text{Product of the roots} = \frac{1}{2} \times \frac{3}{4} = -\frac{3}{p}$$

$$\Rightarrow \quad -\frac{3}{p} = \frac{3}{8}$$

$$\therefore \quad p = -8 \quad \ldots(2)$$

From (1) and (2),

$$q = -\frac{5}{4} \times (-8)$$

$$= 5 \times 2$$

$$= 10$$

$\therefore$ The required values of p and q are -8 and 10 respectively.

11. Find the values of m and n for which $x = -3$ and $x = \frac{2}{3}$ are the roots of the equation $mx^2 + 7x + n = 0$.

Sol.
$$\text{Sum of the roots} = -3 + \frac{2}{3} = -\frac{7}{m}$$

$$\Rightarrow \quad \frac{-7}{3} = -\frac{7}{m}$$

$$\Rightarrow \quad m = 3 \quad \ldots(1)$$

$$\text{Product of the roots} = -3 \times \frac{2}{3} = \frac{n}{m}$$

$$\Rightarrow \quad \frac{n}{3} = -2 \quad \text{[From (1)]}$$

$$\Rightarrow \quad n = -6.$$

$\therefore$ The required values of m and n are 3 and -6 respectively.

12. If $x = \frac{2}{3}$ and $x = -3$ are the roots of the equation $ax^2 + 7x + b = 0$, find the values of a and b.

Sol. Sum of the roots

$$= \frac{2}{3} - 3 = -\frac{7}{a}$$

$$\Rightarrow \quad \frac{7}{a} = \frac{7}{3}$$

$$\Rightarrow \quad a = 3 \quad \ldots(1)$$

$$\text{Product of the roots} = -3 \times \frac{2}{3} = \frac{b}{a}$$

$$\Rightarrow \quad \frac{b}{3} = -2 \quad \text{[From (1)]}$$

$$\Rightarrow \quad b = -6$$

$\therefore$ The required values of a and b are 3 and -6 respectively.

Milestone 2

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Multiple-Choice Questions

1. The roots of the quadratic equation $2x^2 - x - 6 = 0$ are

(*a*) $-2, \frac{3}{2}$ (*b*) $2, \frac{-3}{2}$

(*c*) $-2, \frac{-3}{2}$ (*d*) $2, \frac{3}{2}$ **[CBSE SP 2012]**

Sol. (*b*) $2, -\frac{3}{2}$

We have

$$2x^2 - x - 6 = 0$$

$$\Rightarrow \quad 2x^2 + 3x - 4x - 6 = 0$$

$$\Rightarrow \quad x(2x + 3) - 2(2x + 3) = 0$$

$$\Rightarrow \quad (x - 2)(2x + 3) = 0$$

$$\therefore \text{ Either} \quad x - 2 = 0 \quad \ldots(1)$$

$$\text{or} \quad 2x + 3 = 0 \quad \ldots(2)$$

From (1) and (2), we get $x = 2$ and $x = -\frac{3}{2}$ which are the required roots.

2. The roots of the quadratic equation $x^2 - 3x - m(m + 3) = 0$, where m is a constant, are

(*a*) $m, m + 3$ (*b*) $-m, m + 3$

(*c*) $m, -(m + 3)$ (*d*) $-m, -(m + 3)$

[CBSE 2011]

Sol. (*b*) $-m, m + 3$

We have,

$$x^2 - 3x - m(m + 3) = 0$$

$$\Rightarrow \quad x^2 - 3x - m^2 - 3m = 0$$

$$\Rightarrow \quad x^2 - m^2 - 3(x + m) = 0$$

$$\Rightarrow \quad (x + m)(x - m) - 3(x + m) = 0$$

$$\Rightarrow \quad (x + m)(x - m - 3) = 0$$

$\therefore$ Either $\quad x + m = 0 \quad$ ...(1)

or $\quad x - (m + 3) = 0 \quad$ ...(2)

From (1) & (2), we have $x = -m$ and $x = m + 3$ which are the required roots of the given equation.

3. If one root of the equation $2x^2 + kx - 6 = 0$ is 2, then the value of $k + 1$ is

(*a*) 1 (*b*) -1

(*c*) 0 (*d*) -2

Sol. (*c*) 0

Putting $x = 2$ in the given equation, we get

$$2 \times 2^2 + 2k - 6 = 0$$

$$\Rightarrow \quad 8 - 6 + 2k = 0$$

$$\Rightarrow \quad k + 1 = 0$$

which is the required value of $k + 1$.

4. The quadratic equation $2y^2 - \sqrt{3}y + 1 = 0$ has

(*a*) more than two real roots

(*b*) two equal real roots

(*c*) no real roots

(*d*) two distinct real roots

Sol. (*c*) no real roots

We have, the discriminant,

$$D = \left(-\sqrt{3}\right)^2 - 4 \times 2 \times 1$$

$$= 3 - 8$$

$$= -5 < 0$$

Since, $D < 0$, hence, the given equation has no real roots.

5. Which one of the following equations has two distinct roots?

(*a*) $x^2 + 2x - 7 = 0$ (*b*) $3y^2 - 3\sqrt{3}y + \frac{9}{4} = 0$

(*c*) $x^2 + 2x + 2\sqrt{3} = 0$ (*d*) $6x^2 - 3x + 1 = 0$

Sol. (*a*) $x^2 + 2x - 7 = 0$

We first calculate the discriminant D of each of the given quadratic equations.

For (*a*), $\quad D = 2^2 + 4 \times 7 > 0$

Hence, this equation has two real distinct roots.

For (*b*), $\quad D = \left(-3\sqrt{3}\right)^2 - 4 \times 3 \times \frac{9}{4}$

$= 27 - 27 = 0$

Hence, this equation has two equal real roots.

For (*c*), $\quad D = 2^2 - 4 \times 1 \times 2\sqrt{3}$

$= 4 - 8\sqrt{3} < 0$

Hence, this equation has no real roots at all.

For (*d*), $\quad D = (-3)^2 - 4 \times 6 \times 1$

$= 3 - 24$

$= -21 < 0$

Hence, this equation has no real roots at all.

$\therefore$ (*a*) is the only choice, since this equation has two distinct roots.

6. Which one of the following equations has no real roots?

(*a*) $x^2 - 2x - 2\sqrt{3} = 0$

(*b*) $x^2 - 4x + 4\sqrt{2} = 0$

(*c*) $3x^2 + 4\sqrt{3}x + 3 = 0$

(*d*) $x^2 + 4x - 2\sqrt{2} = 0$

Sol. (*b*) $x^2 - 4x + 4\sqrt{2} = 0$

For (*a*), $\quad D = (-2)^2 - 4 \times 1 \times \left(-2\sqrt{3}\right)$

$= 4 + 8\sqrt{3} > 0$

Hence, this equation has two distinct real roots.

For (*b*), $\quad D = (-4)^2 - 4 \times 1 \times 4\sqrt{2}$

$= 16 - 16\sqrt{2}$

$= 16\left(1 - \sqrt{2}\right) < 0$

$\therefore$ This equation has no real roots at all.

For (*c*), $\quad D = \left(4\sqrt{3}\right)^2 - 4 \times 3 \times 3$

$= 48 - 36$

$= 12 > 0$

$\therefore$ This equation has two distinct real roots.

For (*d*), $\quad D = 4^2 - 4 \times 1 \times \left(-2\sqrt{2}\right)$

$= 16 + 8\sqrt{2} > 0$

$\therefore$ This equation has two distinct real roots.

$\therefore$ (*b*) is the only choice, since this equation does not have any real roots.

7. $(x^2 + 2)^2 - x^2 = 0$ has

(*a*) four real roots (*b*) two real roots

(*c*) one real root (*d*) no real roots

Sol. (*d*) no real roots

From the given equation, we have

$$(x^2 + 2 + x)\,(x^2 + 2 - x) = 0$$

$\therefore$ Either $x^2 + x + 2 = 0$ ...(1)

or, $x^2 - x + 2 = 0$ ...(2)

We see that both (1) and (2) are quadratic equations and the discriminant D for both the quadratic equations is given by

$$D = (\pm 1)^2 - 4 \times 1 \times 2$$
$$= 1 - 8 = -7 < 0$$

$\therefore$ Both the roots of each of the two equations (1) and (2) are imaginary. In other words, the given equation does not have any real roots at all.

8. If the equation $x^2 + 4x + k = 0$ has real and distinct roots, then

(*a*) $k \le 4$ (*b*) $k < 4$

(*c*) $k > 4$ (*d*) $k \ge 4$

Sol. (*b*) $k < 4$

The discriminant D for the given quadratic equation is given by

$$D = 4^2 - 4 \times 1 \times k$$
$$= 16 - 4k$$

Now, the given equation will have two real and distinct roots only if

$$D > 0$$
$$\Rightarrow \quad 16 - 4k > 0$$
$$\Rightarrow \quad k < 4$$

9. If the equation $25x^2 - kx + 9 = 0$ has equal roots, then

(*a*) $k = \pm 30$ (*b*) $k = \pm 25$

(*c*) $k = \pm 9$ (*d*) $k = \pm 34$

Sol. (*a*) $k = \pm 30$

We have, the discriminant D for the given equation is given by

$$D = (-k)^2 - 4 \times 25 \times 9$$
$$= k^2 - 900$$

$\therefore$ The given equation will have two real equal roots if D = 0

i.e. $k^2 - 900 = 0$

$$\Rightarrow \quad k = \pm\sqrt{900} = \pm 30$$

10. If the roots of the equation $x^2 - 2x(1 + 3k) + 7(3 + 2k) = 0$ are real and equal, then

(*a*) $k = 2, \dfrac{-10}{9}$ (*b*) $k = -2, \dfrac{10}{9}$

(*c*) $k = 9, \dfrac{1}{10}$ (*d*) $k = -9, \dfrac{-1}{10}$

Sol. (*a*) $k = 2, -\dfrac{10}{9}$

The discriminant D for the given equation is given by

$$D = \{-2(1 + 3k)\}^2 - 4 \times 1 \times 7(3 + 2k)$$
$$= 4(1 + 3k)^2 - 28(3 + 2k)$$
$$= 4(1 + 9k^2 + 6k) - 84 - 56k$$
$$= 4 + 36k^2 + 24k - 84 - 56k$$
$$= 36k^2 - 32k - 80$$
$$= 4(9k^2 - 8k - 20)$$

$\therefore$ The given equation will have two real and equal roots if D = 0.

i.e. $9k^2 - 8k - 20 = 0$

$$\Rightarrow \quad 9k^2 + 10k - 18k - 20 = 0$$
$$\Rightarrow \quad k(9k + 10) - 2\,(9k + 10) = 0$$
$$\Rightarrow \quad (9k + 10)\,(k - 2) = 0$$

$\therefore$ Either $9k + 10 = 0$ ...(1)

or, $k - 2 = 0$ ...(2)

From (1), we have $k = -\dfrac{10}{9}$ and from (2), $k = 2$.

$\therefore$ The required values of k are 2 and $-\dfrac{10}{9}$.

Very Short Answer Type Questions

Solve:

11. $(x - 5)(x + 4) = 0$

Sol. From the given equation, we have

$$(x - 5)\,(x + 4) = 0$$

$\therefore$ Either $x - 5 = 0$ ...(1)

or, $x + 4 = 0$ ...(2)

From (1) & (2), we have, $x = 5$, $x = -4$

$\therefore$ Required roots are 5 and -4.

12. $(2x + 3)(3x + 2) = 0$

Sol. From the given equation, we have

$$(2x + 3)\,(3x + 2) = 0$$

$\therefore$ Either $2x + 3 = 0$ ...(1)

or, $3x + 2 = 0$ ...(2)

From (1), $x = -\dfrac{3}{2}$ and from (2), we have, $x = -\dfrac{2}{3}$

$\therefore$ Required roots are $-\frac{3}{2}$ and $-\frac{2}{3}$.

13. $\left(\frac{x}{2}-1\right)\left(\frac{x}{3}+5\right)=0$

Sol. From the given equation, we have

$$\left(\frac{x}{2}-1\right)\left(\frac{x}{3}+5\right)=0$$

$\therefore$ Either $\frac{x}{2}-1=0$ ...(1)

or $\frac{x}{3}+5=0$ ...(2)

From (1) and (2), we have, $x = 2$, $x = -15$.

$\therefore$ Required roots are 2 and –15.

14. $7x^2 + 3x = 0$

Sol. From the given equation, we have

$$7x^2 + 3x = 0$$

$\Rightarrow$ $x(7x + 3) = 0$

$\therefore$ Either $x = 0$ ...(1)

or $7x + 3 = 0$

$\Rightarrow$ $x = -\frac{3}{7}$ ...(2)

From (1) and (2), required roots are 0, $-\frac{3}{7}$.

Using factorization, solve each of the following quadratic equations (Q. 15 to Q. 18):

15. $16x^2 - 49 = 0$

Sol. We have $16x^2 - 49 = 0$

$\Rightarrow$ $(4x)^2 - 7^2 = 0$

$\Rightarrow$ $(4x + 7)(4x - 7) = 0$

$\therefore$ Either $4x + 7 = 0$ ...(1)

or $4x - 7 = 0$ ...(2)

$\therefore$ From (1) and (2), we have,

$$x = -\frac{7}{4}, \frac{7}{4}$$

which are the required roots.

16. $18x^2 - 50 = 0$

Sol. We have $18x^2 - 50 = 0$

$\Rightarrow$ $2(9x^2 - 25) = 0$

$\Rightarrow$ $(3x)^2 - 5^2 = 0$

$\Rightarrow$ $(3x + 5)(3x - 5) = 0$

$\therefore$ Either $3x + 5 = 0$ ...(1)

or $3x - 5 = 0$ ...(2)

From (1) and (2), we have $x = -\frac{5}{3}, \frac{5}{3}$ which are the required roots.

17. $(y - 2)^2 - 9 = 0$

Sol. We have $(y - 2)^2 - 9 = 0$

$\Rightarrow$ $(y - 2)^2 = 9$

$\Rightarrow$ $y - 2 = \pm 3$

$\therefore$ $y = 2 \pm 3 = 5, -1$

Hence, required roots are 5, –1.

18. $(x + 3)^2 - 36 = 0$

Sol. We have $(x + 3)^2 - 36 = 0$

$\Rightarrow$ $(x + 3)^2 = 36$

$\Rightarrow$ $x + 3 = \pm 6$

$\therefore$ $x = -3 \pm 6 = 3, -9$ which are the required roots.

19. Write the discriminant of each of the following quadratic equations.

(*a*) $x^2 - 6x + 8 = 0$

(*b*) $3\sqrt{3}x^2 + 10x + \sqrt{3} = 0$ **[CBSE 2009]**

Sol. (*a*) We have, discriminant,

$D = (-6)^2 - 4 \times 1 \times 8 = 36 - 32 = 4$

which is the required discriminant.

(*b*) We have, discriminant,

$$D = 10^2 - 4 \times 3\sqrt{3} \times \sqrt{3}$$
$$= 100 - 36 = 64$$

which is the required value of the discriminant.

20. Examine which of the following quadratic equations have real roots.

(*a*) $x^2 + 5x + 5 = 0$

(*b*) $2y - 1 - \frac{2}{y-2} = 3$

Sol. (*a*) Here discriminant,

$$D = 5^2 - 4 \times 1 \times 5$$
$$= 25 - 20 = 5 > 0$$

$\therefore$ This equation has two real and distinct roots.

(*b*) We have

$$\frac{(2y-1)(y-2)-2}{y-2} = 3$$

$\Rightarrow$ $2y^2 - 5y + 2 - 2 = 3y - 6$

$\Rightarrow$ $2y^2 - 8y + 6 = 0$

$\Rightarrow$ $y^2 - 4y + 3 = 0$

$\therefore$ Discriminant, $D = (-4)^2 - 4 \times 1 \times 3$

$= 16 - 12$

$= 4 > 0.$

$\therefore$ This equation has also two distinct real roots.

21. Which of the following equations have real roots?

(*a*) $4x^2 + 7x + 2 = 0$

(*b*) $x^2 + x + 1 = 0$

Sol. (*a*) We have discriminant,

$$D = 7^2 - 4 \times 4 \times 2$$
$$= 49 - 32$$
$$= 17 > 0$$

$\therefore$ This equation has two real distinct roots.

(*b*) Here, $D = 1^2 - 4 \times 1 \times 1 = 1 - 4 = -3 < 0$

$\therefore$ This equation has no real roots at all.

22. What is the nature of the roots of the following quadratic equations?

(*a*) $x^2 - 2\sqrt{3}\,x + 3 = 0$ **[CBSE 2008]**

(*b*) $4x^2 - 12x - 9 = 0$ **[CBSE SP 2012]**

Sol. (*a*) The discriminant,

$$D = \left(-2\sqrt{3}\right)^2 - 4 \times 1 \times 3$$
$$= 12 - 12 = 0$$

$\therefore$ This equation has two real and equal roots.

(*b*) Here, the discriminant,

$$D = (-12)^2 - 4 \times 4 \times (-9)$$
$$= 144 + 144 = 288 > 0$$

$\therefore$ This equation has two real and unequal roots.

23. Which of the following equations have both roots equal?

(*a*) $x^2 + 10x - 39 = 0$

(*b*) $4x^2 - 12\sqrt{3}\,x + 27 = 0$

(*c*) $4x^2 - 5x + \dfrac{25}{16} = 0$

Sol. (*a*) The discriminant,

$$D = 10^2 - 4 \times 1 \times (-39)$$
$$= 100 + 156 = 256 > 0.$$

$\therefore$ Both the roots of this equation are real but unequal.

(*b*) Here, the discriminant,

$$D = \left(-12\sqrt{3}\right)^2 - 4 \times 4 \times 27$$
$$= 432 - 432 = 0.$$

$\therefore$ Both the roots are real and equal.

(*c*) Here the discriminant,

$$D = (-5)^2 - 4 \times 4 \times \frac{25}{16}$$
$$= 25 - 25 = 0$$

$\therefore$ Both the roots are real and equal.

Short Answer Type-I Questions

Using factorization, solve each of the following quadratic equations (Q. 24 to Q. 28):

24. $x^2 + 11x - 152 = 0$

Sol. We have

$$x^2 + 11x - 152 = 0$$
$$\Rightarrow \quad x^2 + 19x - 8x - 152 = 0$$
$$\Rightarrow \quad x(x + 19) - 8(x + 19) = 0$$
$$\Rightarrow \quad (x + 19)(x - 8) = 0$$

$\therefore$ Either $\quad x + 19 = 0 \quad \ldots(1)$

or $\quad x - 8 = 0 \quad \ldots(2)$

$\therefore$ From (1) & (2), we have $x = -19, 8$ which are the required roots.

25. $x^2 - 7x - 44 = 0$

Sol. We have

$$x^2 - 7x - 44 = 0$$
$$\Rightarrow \quad x^2 + 4x - 11x - 44 = 0$$
$$\Rightarrow \quad x(x + 4) - 11(x + 4) = 0$$
$$\Rightarrow \quad (x + 4)(x - 11) = 0$$

$\therefore$ Either $\quad x + 4 = 0 \quad \ldots(1)$

or $\quad x - 11 = 0 \quad \ldots(2)$

$\therefore$ From (1) and (2), $x = -4, 11$ which are the required roots.

26. $12x^2 - x - 1 = 0$

Sol. We have

$$12x^2 - x - 1 = 0$$
$$\Rightarrow \quad 12x^2 + 3x - 4x - 1 = 0$$
$$\Rightarrow \quad 3x(4x + 1) - 1(4x + 1) = 0$$
$$\Rightarrow \quad (4x + 1)(3x - 1) = 0$$

$\therefore$ Either $\quad 4x + 1 = 0 \quad \ldots(1)$

or $\quad 3x - 1 = 0 \quad \ldots(2)$

From (1) and (2), we have $x = -\dfrac{1}{4}, \dfrac{1}{3}$ which are the required roots.

27. $\dfrac{2}{5}x^2 - x - \dfrac{3}{5} = 0$ **[CBSE SP 2011]**

Sol. We have

$$\frac{2}{5}x^2 - x - \frac{3}{5} = 0$$
$$\Rightarrow \quad 2x^2 - 5x - 3 = 0$$
$$\Rightarrow \quad 2x^2 + x - 6x - 3 = 0$$
$$\Rightarrow \quad x(2x + 1) - 3(2x + 1) = 0$$
$$\Rightarrow \quad (2x + 1)(x - 3) = 0$$

$\therefore$ Either $\quad 2x + 1 = 0 \quad \ldots(1)$

or $\quad x - 3 = 0 \quad \ldots(2)$

From (1) and (2), we have $x = -\frac{1}{2}, 3$ which are the required roots.

28. $x^2 - (\sqrt{3}+1)x + \sqrt{3} = 0$ **[CBSE 2015]**

Sol. We have

$$x^2 - \left(\sqrt{3}+1\right)x + \sqrt{3} = 0$$

$$\Rightarrow \quad x^2 - \sqrt{3}\,x - x + \sqrt{3} = 0$$

$$\Rightarrow \quad x\left(x-\sqrt{3}\right) - 1\left(x - \sqrt{3}\right) = 0$$

$$\Rightarrow \quad \left(x-\sqrt{3}\right)(x-1) = 0$$

$\therefore$ Either $\quad x - \sqrt{3} = 0 \quad \ldots(1)$

or $\quad x - 1 = 0 \quad \ldots(2)$

$\therefore$ From (1) and (2), we have $x = \sqrt{3}$, 1 which are the required roots.

Find the roots of the following quadratic equations, if they exist, by the method of completing the square (Q. 29 to Q. 31):

29. $x^2 - 5x + 6 = 0$

Sol. We have

$$x^2 - 5x + 6 = 0$$

$$\Rightarrow x^2 - 2 \times \frac{5}{2}x + \left(\frac{5}{2}\right)^2 + 6 - \left(\frac{5}{2}\right)^2 = 0$$

$$\Rightarrow \quad \left(x - \frac{5}{2}\right)^2 = \frac{25}{4} - 6 = \frac{25-24}{4} = \frac{1}{4}$$

$$\therefore \quad x - \frac{5}{2} = \pm\frac{1}{2}$$

$\Rightarrow x = \frac{5 \pm 1}{2} = 3, 2$, which are the required roots.

30. $2x + 8 = 3x^2$

Sol. We have

$$2x + 8 = 3x^2$$

$$\Rightarrow \quad 3x^2 - 2x - 8 = 0$$

$$\Rightarrow \quad x^2 - \frac{2}{3}x - \frac{8}{3} = 0$$

$$\Rightarrow \quad x - 2 \times \frac{1}{3}x + \left(\frac{1}{3}\right)^2 - \frac{8}{3} - \left(\frac{1}{3}\right)^2 = 0$$

$$\Rightarrow \quad \left(x - \frac{1}{3}\right)^2 = \frac{8}{3} + \frac{1}{9} = \frac{24+1}{9} = \frac{25}{9}$$

$$\therefore \quad x - \frac{1}{3} = \pm\frac{5}{3}$$

$$\Rightarrow \quad x = \frac{1}{3} \pm \frac{5}{3} = \frac{1 \pm 5}{3} = 2, -\frac{4}{3}$$

which are the required roots.

31. $4\sqrt{3}x^2 + 5x - 2\sqrt{3} = 0$

Sol. We have

$$4\sqrt{3}x^2 + 5x - 2\sqrt{3} = 0$$

$$\Rightarrow \quad x^2 + \frac{5}{4\sqrt{3}}x - \frac{2\sqrt{3}}{4\sqrt{3}} = 0$$

$$\Rightarrow x^2 + 2\,\frac{5}{8\sqrt{3}}x + \left(\frac{5}{8\sqrt{3}}\right)^2 - \left(\frac{5}{8\sqrt{3}}\right)^2 - \frac{1}{2} = 0$$

$$\Rightarrow \quad \left(x + \frac{5}{8\sqrt{3}}\right)^2 = \frac{1}{2} + \frac{25}{192} = \frac{96+25}{192} = \frac{121}{192} = \left(\frac{11}{8\sqrt{3}}\right)^2$$

$$\therefore \quad x + \frac{5}{8\sqrt{3}} = \pm\frac{11}{8\sqrt{3}}$$

$$\Rightarrow \quad x = -\frac{5}{8\sqrt{3}} \pm \frac{11}{8\sqrt{3}} = \frac{-5 \pm 11}{8\sqrt{3}} = \frac{6}{8\sqrt{3}}, \frac{-16}{8\sqrt{3}} = \frac{2 \times 3}{8\sqrt{3}}, \frac{-8 \times 2\sqrt{3}}{8 \times 3} = \frac{\sqrt{3}}{4}, -\frac{2\sqrt{3}}{4}$$

$\therefore$ Required roots are $\frac{\sqrt{3}}{4}$ and $-\frac{2\sqrt{3}}{3}$.

32. In the following, determine whether the given quadratic equations have real roots and if so, find the roots.

(*a*) $3x^2 - 6x + 5 = 0$

(*b*) $4\sqrt{3}\,x^2 + 5x - 2\sqrt{3} = 0$ **[CBSE 2013]**

Sol. (*a*) We have the discriminant,

$$D = (-6)^2 - 4 \times 3 \times 5$$
$$= 36 - 60$$
$$= -24 < 0$$

$\therefore$ This equation has no real roots.

(*b*) We have discriminant,

$$D = 5^2 - 4 \times 4\sqrt{3} \times \left(-2\sqrt{3}\right)$$
$$= 25 + 32 \times 3$$
$$= 25 + 96$$
$$= 121 > 0.$$

$\therefore$ This equation has two real and unequal roots.

Now, we have

$$4\sqrt{3}\,x^2 + 5x - 2\sqrt{3} = 0$$
$$\Rightarrow \quad 4\sqrt{3}\,x^2 + 8x - 3x - 2\sqrt{3} = 0$$
$$\Rightarrow 4x\left(\sqrt{3}x + 2\right) - \sqrt{3}\left(\sqrt{3}x + 2\right) = 0$$
$$\Rightarrow \quad \left(4x - \sqrt{3}\right)\left(\sqrt{3}x + 2\right) = 0$$

$\therefore$ Either $\quad 4x - \sqrt{3} = 0 \quad \ldots(1)$

or $\quad \sqrt{3}x + 2 = 0 \quad \ldots(2)$

$\therefore$ From (1) and (2), we have $x = \dfrac{\sqrt{3}}{4}, -\dfrac{2}{\sqrt{3}}$

which are the required roots.

33. Solve each of the following equations by using the quadratic formula.

(*a*) $x^2 + 2x - 4 = 0$

(*b*) $3x^2 - 32x + 12 = 0$

(*c*) $4x^2 + 4ax + (a^2 - b^2) = 0$

(*d*) $ab^2x\left(\dfrac{a}{d}x + 2\dfrac{c}{b}\right) + c^2d = 0$

Sol. (*a*) We have,

$$x^2 + 2x - 4 = 0$$

Comparing this equation with the standard quadratic equation $ax^2 + bx + c = 0$, we have $a = 1$, $b = 2$ and $c = -4$.

$$\therefore \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{-2 \pm \sqrt{4 + 4 \times 4}}{2}$$
$$= \frac{-2 \pm 2\sqrt{5}}{2} = -1 \pm \sqrt{5}$$

$\therefore$ Required roots are $-1 + \sqrt{5},\ -1 - \sqrt{5}$

(*b*) Comparing the given equation $3x^2 - 32x + 12 = 0$ with the equation $ax^2 + bx + c = 0$, we get $a = 3$, $b = -32$ and $c = 12$.

$$\therefore \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{32 \pm \sqrt{1024 - 4 \times 3 \times 12}}{2 \times 3}$$
$$= \frac{32 \pm \sqrt{1024 - 144}}{6}$$
$$= \frac{32 \pm \sqrt{880}}{6}$$
$$= \frac{32 \pm 4\sqrt{55}}{6}$$
$$= \frac{16 \pm 2\sqrt{55}}{3}$$

$\therefore$ Required roots are $\dfrac{16 + 2\sqrt{55}}{3}$ and $\dfrac{16 - 2\sqrt{55}}{3}$.

(*c*) Comparing the given equation $4x^2 + 4ax + (a^2 - b^2) = 0$ with the equation $Ax^2 + Bx + C = 0$, we get $A = 4$, $B = 4a$, $C = a^2 - b^2$.

$$\therefore \quad x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$
$$= \frac{-4a \pm \sqrt{16a^2 - 4 \times 4(a^2 - b^2)}}{2 \times 4}$$
$$= \frac{-4a \pm \sqrt{16a^2 - 16a^2 + 16b^2}}{8}$$
$$= \frac{-4a \pm 4b}{8}$$
$$= \frac{-(a \pm b)}{2}$$

$\therefore$ Required roots are $\dfrac{-(a + b)}{2}$ and $\dfrac{b - a}{2}$.

(*d*) We have

$$ab^2x \times \frac{a}{d}x + \frac{2c}{b} \times ab^2x + c^2d = 0$$
$$\Rightarrow \quad \frac{a^2b^2x^2}{d} + 2abcx + c^2d = 0$$
$$\Rightarrow \quad a^2b^2x^2 + 2abc\,dx + c^2d^2 = 0$$

Comparing this equation with the equation $Ax^2 + Bx + C = 0$, we get $A = a^2b^2$, $B = 2abcd$ and $C = c^2d^2$, we get

$$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$= \frac{-2abcd \pm \sqrt{4a^2b^2c^2d^2 - 4a^2b^2c^2d^2}}{2a^2b^2}$$

$$= \frac{-2abcd}{2a^2b^2}$$

$$= \frac{-cd}{ab}$$

$\therefore$ The two required roots are $\frac{-cd}{ab}$, $-\frac{cd}{ab}$.

34. In each of the following, determine value(s) of p for which the given quadratic equation has real roots:

(*a*) $px(x - 2) + 6 = 0$ **[CBSE SP 2012]**

(*b*) $x^2 + 6x + 2p + 1 = 0$

Sol. (*a*) We have

$$px(x - 2) + 6 = 0$$

$\Rightarrow \quad px^2 - 2px + 6 = 0$

$\therefore$ Discriminant, $D = (-2p)^2 - 4p \times 6$

$= 4p^2 - 24p$

For real roots, $D \geq 0$

$\Rightarrow \quad 4p^2 - 24p \geq 0$

$\Rightarrow \quad p^2 - 6p \geq 0$

$\Rightarrow \quad p - 6 \geq 0 \quad [\because p \neq 0]$

$\Rightarrow \quad p \geq 6$

which are the required value(s) of p.

(*b*) We have

$$x^2 + 6x + 2p + 1 = 0$$

$\therefore$ Discriminant, $D = 6^2 - 4 \times 1\,(2p + 1)$

$= 36 - 8p - 4$

$= 32 - 8p$

$= 8(4 - p)$

For real roots, $D \geq 0$

$\Rightarrow \quad 8(4 - p) \geq 0$

$\Rightarrow \quad 4 - p \geq 0$

$\Rightarrow \quad p \leq 4$

which are the required value(*s*) of p.

35. Find the values of k for which the roots are real and equal in each of the following equations:

(*a*) $kx^2 - kx + 1 = 0$ **[CBSE SP 2016]**

(*b*) $(k + 4)x^2 + (k + 1)x + 1 = 0$ **[CBSE 2002 C, SP 2011]**

(*c*) $x^2 - 2(1 + 3k)x + 7(3 + 2k) = 0$

(*d*) $(k + 1)x^2 - 2(k - 1)x + 1 = 0$ **[CBSE 2002]**

Sol. (*a*) We have $kx^2 - kx + 1 = 0$

$\therefore$ Discriminant, $D = (-k)^2 - 4k \times 1$

$= k^2 - 4k$

Now, for real and equal roots, D = 0.

$\therefore \quad k^2 - 4k = 0$

$\Rightarrow \quad k - 4 = 0$

[$\because k \neq 0$; for if $k = 0$, then the given equation is absurd]

$\therefore$ $k = 4$ which is the required value of k.

(*b*) We have $(k + 4)x^2 + (k + 1)x + 1 = 0$

$\therefore$ Discriminant, $D = (k + 1)^2 - 4(k + 4) \times 1$

$= k^2 + 2k + 1 - 4k - 16$

$= k^2 - 2k - 15$

For real and equal roots, we have D = 0.

$\Rightarrow \quad k^2 - 2k - 15 = 0$

$\Rightarrow \quad k^2 + 3k - 5k - 15 = 0$

$\Rightarrow k(k + 3) - 5(k + 3) = 0$

$\Rightarrow \quad (k + 3)(k - 5) = 0$

$\therefore$ Either $\quad k - 5 = 0 \quad \ldots(1)$

or $\quad k + 3 = 0 \quad \ldots(2)$

From (1) and (2), we have $k = 5, -3$ which are the required values of k.

(*c*) We have $x^2 - 2(1 + 3k)x + 7(3 + 2k) = 0$

$\therefore$ Discriminant, $D = \{-2(1 + 3k)\}^2 - 4 \times 1 \times 7\,(3 + 2k)$

$= 4(1 + 3k)^2 - 28\,(3 + 2k)$

$= 4(1 + 9k^2 + 6k) - 84 - 56k$

$= 36k^2 + 24k - 56k + 4 - 84$

$= 36k^2 - 32k - 80$

$= 4(9k^2 - 8k - 20)$

Now, for real and equal roots, D = 0.

$\therefore \quad 9k^2 - 8k - 20 = 0$

$\Rightarrow \quad 9k^2 + 10k - 18k - 20 = 0$

$\Rightarrow \quad k(9k + 10) -2(9k + 10) = 0$

$\Rightarrow \quad (9k + 10)(k - 2) = 0$

$\therefore$ Either $\quad 9k + 10 = 0 \quad \ldots(1)$

or $\quad k - 2 = 0 \quad \ldots(2)$

From (1) and (2), we have $k = \frac{-10}{9}$, 2 which are the required values of k.

(*d*) We have $(k + 1)x^2 - 2\,(k - 1)x + 1 = 0$

$\therefore$ Discriminant, $D = \{-2(k - 1)\}^2 - 4(k + 1) \times 1$

$= 4(k - 1)^2 - 4(k + 1)$

$= 4k^2 - 8k + 4 - 4k - 4$

$= 4k^2 - 12k$

For real and equal roots, we have D = 0.

$\therefore \quad 4k^2 - 12k = 0$

$\Rightarrow \quad 4k(k - 3) = 0$

$\Rightarrow \quad k = 0, 3$

which are the required values of k.

Short Answer Type-II Questions

Using factorization, solve each of the following quadratic equations (Q. 36 to Q. 40):

36. $3\sqrt{5}\,x^2 + 2x - \sqrt{5} = 0$

Sol. We have

$$3\sqrt{5}x^2 + 2x - \sqrt{5} = 0$$

$$\Rightarrow \quad 3\sqrt{5}x^2 + 5x - 3x - \sqrt{5} = 0$$

$$\Rightarrow \quad \sqrt{5}x\left(3x + \sqrt{5}\right) - 1\left(3x + \sqrt{5}\right) = 0$$

$$\Rightarrow \quad \left(3x + \sqrt{5}\right)\left(\sqrt{5}x - 1\right) = 0$$

$\therefore$ Either $\quad 3x + \sqrt{5} = 0 \quad \ldots(1)$

or $\quad \sqrt{5}x - 1 = 0 \quad \ldots(2)$

$\therefore$ From (1) and (2), we have $x = \dfrac{-\sqrt{5}}{3}, \dfrac{1}{\sqrt{5}}$ which are the required roots.

37. $9x + \dfrac{1}{x} = 6$

Sol. We have $\quad 9x + \dfrac{1}{x} = 6$

$$\Rightarrow \quad 9x^2 + 1 = 6x$$

$$\Rightarrow \quad 9x^2 - 6x + 1 = 0$$

$$\Rightarrow \quad (3x)^2 - 2 \times 3x + 1^2 = 0$$

$$\Rightarrow \quad (3x - 1)^2 = 0$$

$\therefore$ $x = \dfrac{1}{3}, \dfrac{1}{3}$ which are the required roots.

38. $\dfrac{3}{x^2} + \dfrac{14}{x} + 8 = 0$

Sol. We have $\quad \dfrac{3}{x^2} + \dfrac{14}{x} + 8 = 0$

$$\Rightarrow \quad 3 + 14x + 8x^2 = 0$$

$$\Rightarrow \quad 8x^2 + 14x + 3 = 0$$

$$\Rightarrow \quad 8x^2 + 2x + 12x + 3 = 0$$

$$\Rightarrow \quad 2x(4x + 1) + 3(4x + 1) = 0$$

$$\Rightarrow \quad (4x + 1)(2x + 3) = 0$$

$\therefore$ Either $\quad 4x + 1 = 0 \quad \ldots(1)$

or $\quad 2x + 3 = 0 \quad \ldots(2)$

$\therefore$ From (1) and (2), we have $x = -\dfrac{1}{4}, -\dfrac{3}{2}$ which are the required roots.

39. $\dfrac{4}{9}x^2 - \dfrac{4}{3}x + 1 = 0$

Sol. We have $\quad \dfrac{4}{9}x^2 - \dfrac{4}{3} + 1 = 0$

$$\Rightarrow \quad 4x^2 - \frac{4}{3} \times 9x + 9 = 0$$

$$\Rightarrow \quad 4x^2 - 12x + 9 = 0$$

$$\Rightarrow \quad (2x)^2 - 2 \times 2x \times 3 + 3^2 = 0$$

$$\Rightarrow \quad (2x - 3)^2 = 0$$

$\Rightarrow$ $x = \dfrac{3}{2}, \dfrac{3}{2}$ which are the required roots.

40. $a(x^2 + 1) + (a^2 + 1)x = 0$

Sol. We have $\quad a(x^2 + 1) + (a^2 + 1)x = 0$

$$\Rightarrow \quad ax^2 + (a^2 + 1)x + a = 0$$

$$\Rightarrow \quad ax^2 + a^2x + x + a = 0$$

$$\Rightarrow \quad ax(x + a) + 1(x + a) = 0$$

$$\Rightarrow \quad (x + a)(ax + 1) = 0$$

$\therefore$ Either $\quad x + a = 0 \quad \ldots(1)$

or $\quad ax + 1 = 0 \quad \ldots(2)$

From (1) and (2), we have $x = -a, -\dfrac{1}{a}$ which are the required roots.

Find the roots of the following quadratic equations, if they exist, by the method of completing the square (Q. 41 to Q. 44):

41. $4x^2 - 2x + \dfrac{1}{4} = 0$

Sol. We have

$$4x^2 - 2x + \frac{1}{4} = 0$$

$$\Rightarrow \quad x^2 - \frac{x}{2} + \frac{1}{16} = 0$$

$$\Rightarrow \quad x^2 - 2 \times \frac{1}{4}x + \left(\frac{1}{4}\right)^2 = 0$$

$$\Rightarrow \quad \left(x - \frac{1}{4}\right)^2 = 0$$

$\therefore$ $x = \dfrac{1}{4}, \dfrac{1}{4}$ which are the required roots.

42. $2x^2 + 5x + 7 = 0$

Sol. We have $\quad 2x^2 + 5x + 7 = 0$

$$\Rightarrow \quad x^2 + \frac{5}{2}x + \frac{7}{2} = 0$$

$$\Rightarrow \quad x^2 + 2 \times \frac{5}{4}x + \left(\frac{5}{4}\right)^2 - \left(\frac{5}{4}\right)^2 + \frac{7}{2} = 0$$

$$\Rightarrow \quad \left(x + \frac{5}{4}\right)^2 - \frac{25}{16} + \frac{7}{2} = 0$$

$$\Rightarrow \quad \left(x + \frac{5}{4}\right)^2 + \frac{56 - 25}{16} = 0$$

$$\Rightarrow \quad \left(x + \frac{5}{4}\right)^2 = -\frac{31}{16}$$

$\therefore \quad x + \dfrac{5}{4} = \pm\sqrt{-\dfrac{31}{16}}$ which are not real.

Hence, this equation has no real roots at all.

43. $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

Sol. We have $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

$$\Rightarrow \quad x^2 + \frac{7}{\sqrt{2}}x + 5 = 0$$

$$\Rightarrow \quad x^2 + 2 \times \frac{7}{2\sqrt{2}}x + \left(\frac{7}{2\sqrt{2}}\right)^2 - \left(\frac{7}{2\sqrt{2}}\right)^2 + 5 = 0$$

$$\Rightarrow \quad \left(x + \frac{7}{2\sqrt{2}}\right)^2 + 5 - \frac{49}{8} = 0$$

$$\Rightarrow \quad \left(x + \frac{7}{2\sqrt{2}}\right)^2 - \frac{9}{8} = 0$$

$$\Rightarrow \quad \left(x + \frac{7}{2\sqrt{2}}\right)^2 = \frac{9}{8}$$

$$\Rightarrow \quad x + \frac{7}{2\sqrt{2}} = \pm\frac{3}{2\sqrt{2}}$$

$$\therefore \quad x = -\frac{7}{2\sqrt{2}} \pm \frac{3}{2\sqrt{2}} = \frac{-7 \pm 3}{2\sqrt{2}} = \frac{-4}{2\sqrt{2}}, \frac{-10}{2\sqrt{2}}$$

$= -\sqrt{2}, -\dfrac{5}{\sqrt{2}}$ which are the required roots.

44. $x^2 - 3x - 10 = 0$

Sol. We have $x^2 - 3x - 10 = 0$

$$\Rightarrow \quad x^2 - 2 \times \frac{3}{2}x + \left(\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^2 - 10 = 0$$

$$\Rightarrow \quad \left(x - \frac{3}{2}\right)^2 = 10 + \frac{9}{4}$$

$$= \frac{49}{4}$$

$$\Rightarrow \quad x - \frac{3}{2} = \pm\frac{7}{2}$$

$\Rightarrow \quad x = \dfrac{3}{2} \pm \dfrac{7}{2} = \dfrac{3 \pm 7}{2} = 5, -2$ which are the required roots.

45. Solve:

(*a*) $\dfrac{6}{x} - \dfrac{2}{x-1} = \dfrac{1}{x+2}; x \neq 0, 1, -2$ **[CBSE SP 2006]**

(*b*) $\dfrac{a}{x-b} + \dfrac{b}{x-a} = 2; (x \neq a, b)$ **[CBSE 2016]**

Sol. (*a*) We have

$$\frac{6}{x} - \frac{2}{x-1} = \frac{1}{x+2}$$

$$\Rightarrow \quad \frac{6(x-1) - 2x}{x(x-1)} = \frac{1}{x+2}$$

$$\Rightarrow \quad \frac{6x - 6 - 2x}{x^2 - x} = \frac{1}{x+2}$$

$$\Rightarrow \quad (4x - 6)(x + 2) = x^2 - x$$

$$\Rightarrow \quad 4x^2 + 2x - 12 - x^2 + x = 0$$

$$\Rightarrow \quad 3x^2 + 3x - 12 = 0$$

$$\Rightarrow \quad x^2 + x - 4 = 0$$

$$\therefore \quad x = \frac{-1 \pm \sqrt{1 - 4 \times (-4)}}{2}$$

$$= \frac{-1 \pm \sqrt{17}}{2}$$

$\therefore$ The required roots are $\dfrac{-1 \pm \sqrt{17}}{2}$ and $\dfrac{-1 - \sqrt{17}}{2}$.

(*b*) We have

$$\frac{a}{x-b} + \frac{b}{x-a} = 2$$

$$\Rightarrow \quad \frac{a(x-a) + b(x-b)}{(x-b)(x-a)} = 2$$

$$\Rightarrow \quad \frac{(a+b)x - a^2 - b^2}{x^2 - (a+b)x + ab} = 2$$

$$\Rightarrow \quad 2x^2 - 2(a+b)x + 2ab = (a+b)x - (a^2 + b^2)$$

$$\Rightarrow \quad 2x^2 - 3(a+b)x + a^2 + b^2 + 2ab = 0$$

$$\Rightarrow \quad 2x^2 - 3(a+b)x + (a+b)^2 = 0$$

$$x = \frac{+3(a+b) \pm \sqrt{9(a+b)^2 - 8(a+b)^2}}{4}$$

$$= \frac{+3(a+b) \pm (a+b)}{4} = (a+b), \frac{a+b}{2}$$

which are the required roots.

46. Determine k so that the equation $x^2 - 4x + k = 0$ has

(*a*) two distinct real roots

(*b*) coincident roots

Sol. We have discriminant, $D = (-4)^2 - 4k$

$= 16 - 4k$

$= 4(4 - k)$

(*a*) For distinct real roots, we have $D > 0$

$\Rightarrow \quad 4 - k > 0$

$\Rightarrow \quad k < 4$

which are the required values of k.

(*b*) For coincident roots, $D = 0$

$\Rightarrow \quad 4 - k = 0$

$\Rightarrow \quad k = 4$

which is the required value of k.

47. For what values of m will the equation $2mx^2 - 2(1 + 2m)x + (3 + 2m) = 0$ have real but distinct roots? When will the roots be equal?

Sol. We have discriminant,

$$D = \{-2(1 + 2m)\}^2 - 4 \times 2m\,(3 + 2m)$$
$$= 4(1 + 2m)^2 - 8m\,(3 + 2m)$$
$$= 4 + 16m^2 + 16m - 24m - 16m^2$$
$$= 4 - 8m$$
$$= 4(1 - 2m)$$

$\therefore$ For real distinct roots, $D > 0$

$\Rightarrow \quad 1 - 2m > 0$

$\Rightarrow \quad m < \dfrac{1}{2}$

For real and equal roots, $D = 0$

$\Rightarrow \quad 1 - 2m = 0$

$\Rightarrow \quad m = \dfrac{1}{2}$

which are the required values of m.

Long Answer Type Questions

Using factorization, solve each of the following quadratic equations (Q. 48 to Q. 54):

48. $\dfrac{1}{x-5} - \dfrac{1}{x+1} = \dfrac{6}{7}$

Sol. We have

$$\frac{1}{x-5} - \frac{1}{x+1} = \frac{6}{7}$$

$\Rightarrow \quad \dfrac{x+1-x+5}{(x-5)(x+1)} = \dfrac{6}{7}$

$\Rightarrow \quad \dfrac{6}{x^2-4x-5} = \dfrac{6}{7}$

$\Rightarrow \quad x^2 - 4x - 5 = 7$

$\Rightarrow \quad x^2 - 4x - 12 = 0$

$\Rightarrow \quad x^2 + 2x - 6x - 12 = 0$

$\Rightarrow \quad x(x + 2) - 6(x + 2) = 0$

$\Rightarrow \quad (x + 2)\,(x - 6) = 0$

$\therefore$ Either $\quad x + 2 = 0 \quad$ …(1)

or, $\quad x - 6 = 0 \quad$ …(2)

$\therefore$ From (1) and (2), we have $x = -2, 6$ which are the required roots.

49. $\dfrac{1}{x-2} + \dfrac{1}{x} = \dfrac{8}{2x+5};\ x \neq 0, \dfrac{-5}{2}$

Sol. We have $\quad \dfrac{1}{x-2} + \dfrac{1}{x} = \dfrac{8}{2x+5}$

$\Rightarrow \quad \dfrac{x+x-2}{x(x-2)} = \dfrac{8}{2x+5}$

$\Rightarrow \quad \dfrac{2x-2}{x^2-2x} = \dfrac{8}{2x+5}$

$\Rightarrow \quad 8x^2 - 16x = (2x+5)(2x-2)$

$\quad = 4x^2 - 6x - 10$

$\Rightarrow \quad 4x^2 - 22x + 10 = 0$

$\Rightarrow \quad 2x^2 - 11x + 5 = 0$

$\Rightarrow \quad 2x^2 - 10x - x + 5 = 0$

$\Rightarrow \quad 2x(x - 5) - 1(x - 5) = 0$

$\Rightarrow \quad (x - 5)\,(2x - 1) = 0$

$\therefore$ Either $\quad x - 5 = 0 \quad$ …(1)

or $\quad 2x - 1 = 0 \quad$ …(2)

$\therefore$ From (1) and (2), we have, $x = 5, \dfrac{1}{2}$ which are the required roots.

50. $\dfrac{2}{x+1} + \dfrac{3}{2(x-2)} = \dfrac{23}{5x};\ x \neq 0, -1, 2$ **[CBSE 2015]**

Sol. We have $\quad \dfrac{2}{x+1} + \dfrac{3}{2(x-2)} = \dfrac{23}{5x}$

$\Rightarrow \quad \dfrac{4(x-2)+3x+3}{2(x-2)(x+1)} = \dfrac{23}{5x}$

$\Rightarrow \quad \dfrac{4x-8+3x+3}{2\left(x^2-x-2\right)} = \dfrac{23}{5x}$

$\Rightarrow \quad \dfrac{7x-5}{2x^2-2x-4} = \dfrac{23}{5x}$

$\Rightarrow \quad 35x^2 - 25x = 46x^2 - 46x - 92$

$\Rightarrow \quad (35 - 46)x^2 - 25x + 46x + 92 = 0$

$\Rightarrow \quad -11x^2 + 21x + 92 = 0$

$\Rightarrow \quad 11x^2 - 21x - 92 = 0$

$\Rightarrow \quad 11x^2 + 23x - 44x - 92 = 0$

$\Rightarrow \quad x(11x + 23) - 4(11x + 23) = 0$

$\Rightarrow \quad (11x + 23)\,(x - 4) = 0$

$\therefore$ Either $\quad 11x + 23 = 0 \quad$ …(1)

or $\quad x - 4 = 0 \quad$ …(2)

$\therefore$ From (1) and (2), we have $x = \dfrac{-23}{11}, 4$ which are the required roots.

51. $\dfrac{4}{z-1} - \dfrac{5}{z+2} = \dfrac{3}{z};\ z \neq 1, 0, -2$ **[CBSE 2014]**

Sol. We have

$$\frac{4}{z-1} - \frac{5}{z} = \frac{3}{z}$$

$\Rightarrow \quad \dfrac{4z+8-5z+5}{(z-1)(z+2)} = \dfrac{3}{z}$

$\Rightarrow \quad \dfrac{13-z}{z^2+z-2} = \dfrac{3}{z}$

$$\Rightarrow \quad 3z^2 + 3z - 6 = 13z - z^2$$
$$\Rightarrow \quad 4z^2 - 10z - 6 = 0$$
$$\Rightarrow \quad 2z^2 - 5z - 3 = 0$$
$$\Rightarrow \quad 2z^2 + z - 6z - 3 = 0$$
$$\Rightarrow \quad z(2z + 1) - 3(2z + 1) = 0$$
$$\Rightarrow \quad (2z + 1)(z - 3) = 0$$
$\therefore$ Either $\quad 2z + 1 = 0 \quad$ ...(1)

or $\quad z - 3 = 0 \quad$ ...(2)

$\therefore$ From (1) and (2), we have, $z = -\frac{1}{2}$, 3 which are the required roots.

52. $\left(\frac{2x}{x-3}\right) + \left(\frac{1}{2x+3}\right) + \frac{3x+9}{(x-3)(2x+3)} = 0$, $x \neq 3, \frac{-3}{2}$ **[CBSE 2006, 2016, SP 2011]**

Sol. We have
$$\frac{2x}{x-3} + \frac{1}{2x+3} + \frac{3x+9}{(x-3)(2x+3)} = 0$$
$$\Rightarrow \quad \frac{2x(2x+3) + x - 3 + 3x + 9}{(2x+3)(x-3)} = 0$$
$$\Rightarrow \quad 4x^2 + 10x + 6 = 0$$
$$\Rightarrow \quad 2x^2 + 5x + 3 = 0$$
$$\Rightarrow \quad 2x^2 + 3x + 2x + 3 = 0$$
$$\Rightarrow \quad x(2x + 3) + 1(2x + 3) = 0$$
$$\Rightarrow \quad (2x + 3)(x + 1) = 0$$
$\therefore$ Either $\quad 2x + 3 = 0 \quad$ ...(1)

or $\quad x + 1 = 0 \quad$ ...(2)

$\therefore$ From (1) and (2), we have $x = \frac{-3}{2}$, -1 which are the required roots.

53. (*a*) $16 \times 4^{x+2} - 16 \times 2^{x+1} + 1 = 0$

(*b*) $5^{4x} - 3 \times 5^{2x+1} = 250$ **[CBSE 2000]**

Sol. (*a*) We have
$$16 \times 4^{x+2} - 16 \times 2^{x+1} + 1 = 0$$
$$\Rightarrow \quad 16 \times (2^2)^{x+2} - 16 \times 2^x \times 2 + 1 = 0$$
$$\Rightarrow \quad 16 \times 2^{2x} \times 2^4 - 32 \times 2^x + 1 = 0$$
$$\Rightarrow \quad 256a^2 - 32a + 1 = 0,$$
where $a = 2^x$ ...(1)
$$\Rightarrow \quad (16a)^2 - 2 \times 16a \times 1 + 1^2 = 0$$
$$\Rightarrow \quad (16a - 1)^2 = 0$$
$$\Rightarrow \quad 16a - 1 = 0$$
$$\Rightarrow \quad a = \frac{1}{16}$$
$$= \frac{1}{2^4}$$
$$= 2^{-4}$$
$$\Rightarrow \quad 2^x = 2^{-4}$$
[From (1)]

$\Rightarrow$ $x = -4$ which is the required solution.

(*b*) We have $\quad 5^{4x} - 3 \times 5^{2x+1} = 250$
$$\Rightarrow \quad (5^{2x})^2 - 3 \times 5^{2x} \times 5 = 250$$
$\Rightarrow \quad a^2 - 15a - 250 = 0$ where $a = 5^{2x}$ ...(1)
$$\Rightarrow \quad a^2 + 10a - 25a - 250 = 0$$
$$\Rightarrow \quad a(a + 10) - 25(a + 10) = 0$$
$$\Rightarrow \quad (a + 10)(a - 25) = 0$$
$\therefore$ Either $\quad a + 10 = 0 \quad$ ...(2)

or, $\quad a - 25 = 0 \quad$ ...(3)

$\therefore$ From (2), we have $\quad a = -10$
$$\Rightarrow \quad 5^{2x} = -10$$
[From (1)]

which is absurd, since LHS is always positive for all values of x, but RHS < 0.

$\therefore$ From (3), we get $\quad a = 25$

$\Rightarrow \quad 5^{2x} = 5^2 \quad$ [From (1)]
$$\Rightarrow \quad 2x = 2$$
$$\Rightarrow \quad x = 1$$
which is the required solution.

54. $\sqrt{2x+9} + x = 13$ **[CBSE 2016]**

Sol. We have $\quad \sqrt{2x+9} + x = 13$

$\Rightarrow \quad \sqrt{2x+9} = 13 - x \quad$ ...(1)

On squaring both sides, we get
$$\Rightarrow \quad 2x + 9 = (13 - x)^2$$
$$= 169 + x^2 - 26x$$
$$\Rightarrow \quad x^2 - 28x + 160 = 0$$
$$\Rightarrow \quad x^2 - 20x - 8x + 160 = 0$$
$$\Rightarrow \quad x(x - 20) - 8(x - 20) = 0$$
$$\Rightarrow \quad (x - 20)(x - 8) = 0$$
$\therefore$ Either $\quad x - 20 = 0 \quad$ ...(2)

or $\quad x - 8 = 0 \quad$ ...(3)

From (2), we see that $x = 20$ which does not satisfy the given equation.

Hence, we reject this value of x.

$\therefore$ From (3), we get $x = 8$ which satisfies the given equation. Hence, $x = 8$ is the only solution.

Find the roots of the following quadratic equations, if they exist, by the method of completing the square (Q. 55 to Q. 56):

55. $(a + b)x^2 - 2ax + a - b = 0$

Sol. We have

$$(a + b)x^2 - 2ax + (a - b) = 0$$

$$\Rightarrow \quad x^2 - \frac{2ax}{a+b} + \frac{a-b}{a+b} = 0$$

$$\Rightarrow x^2 - 2\times\frac{a}{a+b}\times x + \left(\frac{a}{a+b}\right)^2 - \left(\frac{a}{a+b}\right)^2 + \frac{a-b}{a+b} = 0$$

$$\Rightarrow \quad \left(x - \frac{a}{a+b}\right)^2 = \frac{a^2}{(a+b)^2} - \frac{a-b}{a+b}$$

$$= \frac{a^2 - a^2 + b^2}{(a+b)^2}$$

$$= \frac{b^2}{(a+b)^2}$$

$$\therefore \quad x - \frac{a}{a+b} = \pm\frac{b}{a+b}$$

$$\Rightarrow \quad x = \frac{a}{a+b} \pm \frac{b}{a+b}$$

$$= \frac{a \pm b}{a+b} = 1 \text{ or } \frac{a-b}{a+b}$$

$\therefore$ The required roots are 1, $\frac{a-b}{a+b}$.

56. $x^2 - (\sqrt{7} + 1)x + \sqrt{7} = 0$

Sol. We have $x^2 - \left(\sqrt{7} + 1\right)x + \sqrt{7} = 0$

$$\Rightarrow x^2 - 2 \times \left(\frac{\sqrt{7}+1}{2}\right) \times x + \left(\frac{\sqrt{7}+1}{2}\right)^2 - \frac{\left(\sqrt{7}+1\right)^2}{4} + \sqrt{7} = 0$$

$$\Rightarrow \quad \left(x - \frac{\sqrt{7}+1}{2}\right)^2 = \frac{\left(\sqrt{7}+1\right)^2}{4} - \sqrt{7}$$

$$= \frac{7 + 1 + 2\sqrt{7} - 4\sqrt{7}}{4}$$

$$= \frac{7 + 1 - 2\sqrt{7}}{4}$$

$$= \frac{\left(\sqrt{7}\right)^2 + 1^2 - 2\sqrt{7}}{4}$$

$$= \left(\frac{\sqrt{7}-1}{2}\right)^2$$

$$\Rightarrow \quad \left(x - \frac{\sqrt{7}+1}{2}\right)^2 = \left(\frac{\sqrt{7}-1}{2}\right)^2$$

$$\Rightarrow \quad x - \frac{\sqrt{7}+1}{2} = \pm\frac{\sqrt{7}-1}{2}$$

$$\Rightarrow \quad x = \frac{\sqrt{7}+1}{2} \pm \frac{\sqrt{7}-1}{2}$$

$$= \frac{\left(\sqrt{7}+1\right) \pm \left(\sqrt{7}-1\right)}{2}$$

$$= \sqrt{7}, 1$$

which are the required roots.

57. Solve:

(*a*) $4x^2 - 16(p - q)x + (15p^2 - 34pq + 15q^2) = 0$

(*b*) $p^2x^2 + (p^2 - q^2)x - q^2 = 0$

(*c*) $12abx^2 - (9a^2 - 8b^2)x - 6ab = 0$

[CBSE 2006, SP 2011]

(*d*) $(a + b)^2x^2 - 8(a^2 - b^2)x - 20(a - b)^2 = 0$

[CBSE SP 2011]

Sol. (*a*) We have

$$4x^2 - 16\,(p - q)x + (15p^2 - 34pq + 15q^2) = 0$$

$$\Rightarrow 4x^2 - 16\,(p - q)x + 15p^2 - 25pq - 9pq + 15q^2 = 0$$

$$\Rightarrow 4x^2 - 16\,(p - q)x + 5p\,(3p - 5q) - 3q\,(3p - 5q) = 0$$

$$\Rightarrow \quad 4x^2 - 16\,(p - q)x + (3p - 5q)\,(5p - 3q) = 0$$

$$\Rightarrow 4x^2 - 2\{(3p - 5q) + (5p - 3q)\}x + (3p - 5q)(5p - 3q) = 0$$

$$\Rightarrow \quad 4x^2 - 2(a + b)x + ab = 0$$

where $\quad a = 3p - 5q \quad$ ...(1)

and $\quad b = 5p - 3q \quad$ ...(2)

$$\Rightarrow \quad 4x^2 - 2ax - 2bx + ab = 0$$

$$\Rightarrow \quad 2x(2x - a) - b\,(2x - a) = 0$$

$$\Rightarrow \quad (2x - a)\,(2x - b) = 0$$

$\therefore$ Either $\quad 2x - a = 0 \quad$ ...(3)

or, $\quad 2x - b = 0 \quad$ ...(4)

From (1) and (3), we get

$$x = \frac{a}{2} = \frac{3p-5q}{2}$$

and from (2) and (4), we get

$$x = \frac{b}{2} = \frac{5p-3q}{2}$$

Hence, the required roots are $\frac{3p-5q}{2}$ and $\frac{5p-3q}{2}$.

(*b*) We have $\quad p^2x^2 + (p^2 - q^2)x - q^2 = 0$

$$\Rightarrow \quad p^2x^2 + p^2x - q^2x - q^2 = 0$$

$$\Rightarrow \quad p^2x(x + 1) - q^2(x + 1) = 0$$

$$\Rightarrow \quad (x + 1)\,(p^2x - q^2) = 0$$

$\therefore$ Either $\quad x + 1 = 0 \quad \ldots(1)$

or $\quad p^2x - q^2 = 0 \quad \ldots(2)$

From (1) and (2), we have $x = -1, \dfrac{q^2}{p^2}$ which are the required roots.

(*c*) We have

$$12abx^2 - (9a^2 - 8b^2)x - 6ab = 0$$

$$\Rightarrow \quad 12abx^2 - 9a^2x + 8b^2x - 6ab = 0$$

$$\Rightarrow \quad 3ax\,(4bx - 3a) + 2b(4bx - 3a) = 0$$

$$\Rightarrow \quad (4bx - 3a)\,(3ax + 2b) = 0$$

$\therefore$ Either $\quad 4bx - 3a = 0 \quad \ldots(1)$

or $\quad 3ax + 2b = 0 \quad \ldots(2)$

From (1) and (2), we have $x = \dfrac{3a}{4b}, -\dfrac{2b}{3a}$

which are the required roots.

(*d*) We have

$$(a + b)^2x^2 - 8(a^2 - b^2)x - 20\,(a - b)^2 = 0$$

$$\Rightarrow \quad (a + b)^2x^2 - 8(a + b)\,(a - b)x - 20\,(a - b)^2 = 0$$

$$\Rightarrow \quad p^2x^2 - 8pqx - 20q^2 = 0$$

where $\quad p = a + b \quad \ldots(1)$

and $\quad q = a - b \quad \ldots(2)$

$$\Rightarrow \quad p^2x^2 - 10pqx + 2pqx - 20q^2 = 0$$

$$\Rightarrow \quad px(px - 10q) + 2q(px - 10q) = 0$$

$$\Rightarrow \quad (px - 10q)\,(px + 2q) = 0$$

$\therefore$ Either $\quad px - 10q = 0 \quad \ldots(3)$

or, $\quad px + 2q = 0 \quad \ldots(4)$

$\therefore$ From (1), (2) and (3), we get

$$x = \frac{10q}{p} = \frac{10(a - b)}{a + b}$$

and from (1), (2) and (4), we get

$$x = -\frac{2q}{p} = -\frac{2(a - b)}{a + b}$$

Hence, the required roots are $\dfrac{10(a - b)}{a + b}$ and $-\dfrac{2(a - b)}{a + b}$.

Milestone 3

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Short Answer Type-I Questions

1. The sum of two numbers is 18 and their product is 56. Find the numbers.

Sol. Let the two numbers be x and $18 - x$.

$\therefore$ According to the problem,

$$x(18 - x) = 56$$

$$\Rightarrow \quad x^2 - 18x + 56 = 0$$

$$\Rightarrow \quad x^2 - 4x - 14x + 56 = 0$$

$$\Rightarrow \quad x(x - 4) - 14(x - 4) = 0$$

$$\Rightarrow \quad (x - 4)\,(x - 14) = 0$$

$\therefore$ Either $\quad x - 4 = 0 \quad \ldots(1)$

or, $\quad x - 14 = 0 \quad \ldots(2)$

From (1) and (2), we have $x = 4$ or 14.

Hence, the required numbers are 4 and 14.

2. (*a*) The sum of a number and its reciprocal is $-\dfrac{25}{12}$. Find the number.

(*b*) The sum of two numbers is 9 and the sum of their reciprocals is $\dfrac{1}{2}$. Find the numbers. **[CBSE 2012]**

(*c*) The sum of two numbers a and b is 15 and the sum of their reciprocals $\dfrac{1}{a}$ and $\dfrac{1}{b}$ is $\dfrac{3}{10}$. Find the numbers a and b. **[CBSE 2000, 2005]**

Sol. (*a*) Let the number be x, where $x \neq 0$. Then according to the problem, we have $x + \dfrac{1}{x} = -\dfrac{25}{12}$

$$\Rightarrow \quad \frac{x^2 + 1}{x} = -\frac{25}{12}$$

$$\Rightarrow \quad 12x^2 + 12 + 25x = 0$$

$$\Rightarrow \quad 12x^2 + 25x + 12 = 0$$

$$\Rightarrow \quad 12x^2 + 16x + 9x + 12 = 0$$

$$\Rightarrow \quad 4x(3x + 4) + 3(3x + 4) = 0$$

$$\Rightarrow \quad (3x + 4)\,(4x + 3) = 0$$

$\therefore$ Either $\quad 3x + 4 = 0 \quad \ldots(1)$

or, $\quad 4x + 3 = 0 \quad \ldots(2)$

$\therefore$ From (1) and (2), we have $x = -\dfrac{4}{3}, -\dfrac{3}{4}$

$\therefore$ The required number is either $-\dfrac{4}{3}$ or $-\dfrac{3}{4}$.

(*b*) Let the two numbers be x and $9 - x$.

$\therefore$ According to the problem, we have

$$\frac{1}{x} + \frac{1}{9 - x} = \frac{1}{2}$$

$$\Rightarrow \quad \frac{9 - x + x}{x(9 - x)} = \frac{1}{2}$$

$$\Rightarrow \quad \frac{9}{9x - x^2} = \frac{1}{2}$$

$$\Rightarrow \quad 9x - x^2 = 18$$

$\Rightarrow \quad x^2 - 9x + 18 = 0$

$\Rightarrow \quad x^2 - 6x - 3x + 18 = 0$

$\Rightarrow \quad x(x - 6) - 3(x - 6) = 0$

$\Rightarrow \quad (x - 6)(x - 3) = 0$

$\therefore$ Either $\quad x - 6 = 0 \quad \ldots(1)$

or, $\quad x - 3 = 0 \quad \ldots(2)$

$\therefore$ From (1) and (2), we have $x = 6, 3$.

$\therefore$ When one number is 3, the other number is 6.

Hence, the required two numbers are 3 and 6.

(*c*) According to the problem, we have

$$a + b = 15 \quad \ldots(1)$$

and $$\frac{1}{a} + \frac{1}{b} = \frac{3}{10} \quad \ldots(2)$$

From (1), $\quad b = 15 - a \quad \ldots(3)$

$\therefore$ From (2),

$$\frac{1}{a} + \frac{1}{15 - a} = \frac{3}{10}$$

$$\Rightarrow \quad \frac{15 - a + a}{a(15 - a)} = \frac{3}{10}$$

$$\Rightarrow \quad \frac{15}{a(15 - a)} = \frac{3}{10}$$

$$\Rightarrow \quad \frac{5}{15a - a^2} = \frac{1}{10}$$

$\Rightarrow \quad 15a - a^2 = 50$

$\Rightarrow \quad a^2 - 15a + 50 = 0$

$\Rightarrow \quad a^2 - 10a - 5a + 50 = 0$

$\Rightarrow \quad a(a - 10) - 5(a - 10) = 0$

$\Rightarrow \quad (a - 10)(a - 5) = 0$

$\therefore$ Either $\quad a - 10 = 0 \quad \ldots(1)$

or, $\quad a - 5 = 0 \quad \ldots(2)$

$\therefore$ From (1) and (2), we have $a = 10, 5$.

We see from (3) that when $a = 10$, then $b = 5$ and when $a = 5$, then $b = 10$.

Thus, the required two numbers are 5 and 10.

3. (*a*) A natural number when increased by 12 becomes equal to 160 times its reciprocal. Find the number. **[CBSE 2012]**

(*b*) Find possible integer(s) which, when decreased by 20, is equal to 69 times the reciprocal of the number.

Sol. (*a*) Let the number be x.

$\therefore$ According to the problem, we have

$$x + 12 = \frac{160}{x}$$

$\Rightarrow \quad x^2 + 12x - 160 = 0$

$\Rightarrow \quad x^2 + 20x - 8x - 160 = 0$

$\Rightarrow \quad x(x + 20) - 8(x + 20) = 0$

$\Rightarrow \quad (x + 20)(x - 8) = 0$

$\therefore$ Either $\quad x + 20 = 0 \quad \ldots(1)$

or, $\quad x - 8 = 0 \quad \ldots(2)$

From (1) and (2), we have

$$x = -20, 8$$

Now, since –20 is not a natural number, hence, we reject it.

$\therefore \quad x = 8$

which is the required number.

(*b*) Let one of the integers be x.

$\therefore$ According to the Problem,

$$x - 20 = \frac{69}{x}$$

$\Rightarrow \quad x^2 - 20x - 69 = 0$

$\Rightarrow \quad x^2 + 3x - 23x - 69 = 0$

$\Rightarrow \quad x(x + 3) - 23(x + 3) = 0$

$\Rightarrow \quad (x + 3)(x - 23) = 0$

$\therefore$ Either $\quad x + 3 = 0 \quad \ldots(1)$

or, $\quad x - 23 = 0 \quad \ldots(2)$

From (1) and (2), we have $x = -3, 23$ which are the required integers.

4. The sum S of first n natural numbers is given by the relation $S = n\frac{(n+1)}{2}$. Find n if the sum is 465.

Sol. We have $$S = \frac{n(n+1)}{2}$$

$$\Rightarrow \quad 465 = \frac{n^2 + n}{2}$$

$\Rightarrow \quad n^2 + n - 930 = 0$

$\Rightarrow \quad n^2 + 31n - 30n - 930 = 0$

$\Rightarrow \quad n(n + 31) - 30(n + 31) = 0$

$\Rightarrow \quad (n - 30)(n + 31) = 0$

$\therefore$ Either $\quad n - 30 = 0 \quad \ldots(1)$

or, $\quad n + 31 = 0 \quad \ldots(2)$

From (1) and (2), we have $n = 30, -31$.

Since, –31 is not a natural number, we reject it.

So, $n = 30$ which is the required value of n.

5. There are three consecutive integers such that the square of the first increased by the product of the other two gives 154. What are the integers? **[CBSE SP 2011]**

Sol. Let the three consecutive integers be x, $x + 1$, $x + 2$.

$\therefore$ According to the problem, we have

$$x^2 + (x + 1)(x + 2) = 154$$

$\Rightarrow$ $x^2 + x^2 + 3x + 2 - 154 = 0$

$\Rightarrow$ $2x^2 + 3x - 152 = 0$

$\Rightarrow$ $2x^2 + 19x - 16x - 152 = 0$

$\Rightarrow$ $x(2x + 19) - 8(2x + 19) = 0$

$\Rightarrow$ $(x - 8)(2x + 19) = 0$

$\therefore$ Either $x - 8 = 0$ ...(1)

or, $2x + 19 = 0$ ...(2)

$\therefore$ From (1) and (2), $x = 8, -\frac{19}{2}$

But $-\frac{19}{2}$ is not an integer and so we reject this value.

$\therefore$ $x = 8$

$\therefore$ The required consecutive integers are 8, 9 and 10.

6. The product of two successive multiples of 5 is 300. Find the multiples.

Sol. Let two multiples of 5 be $5x$ and $5(x + 1)$.

$\therefore$ According to the problem,

$$5x \times 5(x + 1) = 300$$

$\Rightarrow$ $x^2 + x = \frac{300}{25} = 12$

$\Rightarrow$ $x^2 + x - 12 = 0$

$\Rightarrow$ $x^2 + 4x - 3x - 12 = 0$

$\Rightarrow$ $x(x + 4) - 3(x + 4) = 0$

$\Rightarrow$ $(x + 4)(x - 3) = 0$

$\therefore$ Either $x + 4 = 0$ ...(1)

or, $x - 3 = 0$ ...(2)

From (1), $x = -4$ and from (2) $x = 3$.

Since the multiples of a natural number cannot be negative, we reject $x = -4$.

$\therefore$ We have $x = 3$.

$\therefore$ The required multiples of 5 are 5×3 and $5 \times (3 + 1)$, i.e. 15 and 20.

7. The age of father is equal to the square of the age of his son. The sum of the age of the father and five times the age of the son is 84 years. Find their ages.

Sol. Let the ages of the father and his son be x years and y years respectively.

$\therefore$ According to the problem, we have

$x = y^2$ ...(1)

and $x + 5y = 84$ ...(2)

$\therefore$ From (1) and (2), we have

$$y^2 + 5y - 84 = 0$$

$\Rightarrow$ $y^2 + 12y - 7y - 84 = 0$

$\Rightarrow$ $y(y + 12) - 7(y + 12) = 0$

$\Rightarrow$ $(y + 12)(y - 7) = 0$

$\therefore$ Either $y + 12 = 0$ ...(3)

or, $y - 7 = 0$ ...(4)

From (3) and (4), we have $y = -12$ or 7.

Since the age cannot be negative, we reject $y = -12$.

$\therefore$ We have $y = 7$.

$\therefore$ From (1), we have $x = 7^2 = 49$.

$\therefore$ The required ages of the son and his father are 7 years and 49 years respectively.

8. When Deepica was asked her age, she replied "If you subtract eleven times my age from the square of my age the result is 210". Find her age.

Sol. Let the age of Deepica be x years. Then according to the problem, we have

$$x^2 - 11x = 210$$

$\Rightarrow$ $x^2 - 11x - 210 = 0$

$\Rightarrow$ $x^2 + 10x - 21x - 210 = 0$

$\Rightarrow$ $x(x + 10) - 21(x + 10) = 0$

$\Rightarrow$ $(x + 10)(x - 21) = 0$

$\therefore$ Either $x + 10 = 0$ ...(1)

or, $x - 21 = 0$ ...(2)

$\therefore$ From (1) and (2), we have $x = -10, 21$.

Since age cannot be negative, we reject $x = -10$.

$\therefore$ The required age of Deepica is 21 years.

9. Twenty-seven years hence Ashish's age will be square of what it was 29 years ago. Find his present age.

Sol. Let the present age of Ashish be x years. Hence, according to the problem, we have

$$x + 27 = (x - 29)^2$$

$\Rightarrow$ $x^2 - 58x + 29^2 - x - 27 = 0$

$\Rightarrow$ $x^2 - 59x + 841 - 27 = 0$

$\Rightarrow$ $x^2 - 59x + 814 = 0$

$\Rightarrow$ $x^2 - 37x - 22x + 814 = 0$

$\Rightarrow$ $x(x - 37) - 22(x - 37) = 0$

$\Rightarrow$ $(x - 37)(x - 22) = 0$

$\therefore$ Either $x - 37 = 0$ ...(1)

or, $x - 22 = 0$ ...(2)

From (1), $x = 37$ and from (2) $x = 22$.

Now, $x = 22$ is impossible, since $22 < 29$ and hence, 29 years ago Ashish's age was less than 0 which is absurd.

Hence, we reject $x = 22$ and accept $x = 37$.

Hence, the required present age of Ashish is 37 years.

10. In a class test, the sum of marks obtained by P in Mathematics and Science is 28. Had he got 3 more marks in Mathematics and 4 marks less in Science, the product of marks obtained in the two subjects would have been 180. Find the marks obtained in the two subjects separately.

[CBSE 2008]

Sol. Let the marks obtained by P in mathematics and science be x and y respectively.

Then according to the problem, we have

$$x + y = 28 \qquad \ldots(1)$$

and $$(x + 3)(y - 4) = 180 \qquad \ldots(2)$$

From (1), we have $$y = 28 - x \qquad \ldots(3)$$

$\therefore$ From (2), we have

$$(x + 3)(28 - x - 4) = 180$$

$$\Rightarrow \quad (x + 3)(24 - x) - 180 = 0$$

$$\Rightarrow \quad -x^2 + 21x + 72 - 180 = 0$$

$$\Rightarrow \quad x^2 - 21x + 108 = 0$$

$$\Rightarrow \quad x^2 - 12x - 9x + 108 = 0$$

$$\Rightarrow \quad x(x - 12) - 9(x - 12) = 0$$

$$\Rightarrow \quad (x - 12)(x - 9) = 0$$

$\therefore$ Either $$x - 12 = 0 \qquad \ldots(4)$$

or, $$x - 9 = 0 \qquad \ldots(5)$$

From (4), $$x = 12 \qquad \ldots(6)$$

and from (5), $$x = 9 \qquad \ldots(7)$$

When $x = 12$, then from (3), we see that

$$y = 28 - 12 = 16$$

From (7), we see that when $x = 9$, then from (3), $y = 28 - 9 = 19$.

Hence, P obtained either 12 marks in mathematics and 16 marks in science or 9 marks in mathematics and 19 marks in science.

11. ₹1200 was distributed equally among certain number of students. Had there been 8 more students, each would have received ₹ 5 less. Find the number of students. **[CBSE SP 2006]**

Sol. Let the number of students be x. Then each student received ₹ $\dfrac{1200}{x}$.

$\therefore$ According to the condition of the problem, we have

$$(x + 8)\left(\frac{1200}{x} - 5\right) = 1200$$

$$\Rightarrow \quad 1200 - 5x + \frac{9600}{x} - 40 - 1200 = 0$$

$$\Rightarrow \quad -5x^2 + 9600 - 40x = 0$$

$$\Rightarrow \quad x^2 + 8x - 1920 = 0$$

$$\Rightarrow \quad x^2 + 48x - 40x - 1920 = 0$$

$$\Rightarrow \quad x(x + 48) - 40(x + 48) = 0$$

$$\Rightarrow \quad (x - 40)(x + 48) = 0$$

$\therefore$ Either $$x - 40 = 0 \qquad \ldots(1)$$

or, $$x + 48 = 0 \qquad \ldots(2)$$

From (2) $x = -48$ which is absurd, since the number of students cannot be negative. From (1), $x = 40$ which is the required number of students.

12. The sum of the areas of two squares is 325 cm^2. The side of the larger square is 5 cm longer than the side of the smaller square. Find the side of each square.

Sol. Let the side of the smaller square be x cm. Then the side of the larger square is $(x + 5)$ cm. Hence, according to the problem, we have

$$x^2 + (x + 5)^2 = 325$$

$$\Rightarrow \quad x^2 + x^2 + 10x + 25 - 325 = 0$$

$$\Rightarrow \quad 2x^2 + 10x - 300 = 0$$

$$\Rightarrow \quad x^2 + 5x - 150 = 0$$

$$\Rightarrow \quad x^2 + 15x - 10x - 150 = 0$$

$$\Rightarrow \quad x(x + 15) - 10(x + 15) = 0$$

$$\Rightarrow \quad (x + 15)(x - 10) = 0$$

$\therefore$ Either $$x + 15 = 0 \qquad \ldots(1)$$

or, $$x - 10 = 0 \qquad \ldots(2)$$

From (1), $x = -15$ which is absurd, since the length of a side of a square cannot be negative.

$\therefore$ We reject this value.

$\therefore$ From (2), we get $x = 10$

Hence, the required lengths of the sides of the two squares are 10 cm and 15 cm.

13. The length of the hypotenuse of a right triangle exceeds the length of the base by 2 cm and exceeds twice the length of the altitude by 1 cm. Find the length of each side of the triangle.

[CBSE 2002]

Sol. Let the lengths of the base and altitude of the right angled triangle be x cm and y cm respectively.

Hence, the length of the hypotenuse of the triangle is either $(x + 2)$ cm or $2y + 1$ cm.

Hence, $x + 2 = 2y + 1$

$$\Rightarrow \quad x - 2y + 1 = 0$$

$$\Rightarrow \quad y = \frac{x+1}{2} \quad \ldots(1)$$

When the length of the hypotenuse is $(x + 2)$cm, then

$$h^2 = x^2 + y^2$$

$$= x^2 + \left(\frac{x+1}{2}\right)^2$$

[By Pythagoras theorem]

$$\Rightarrow \quad (x+2)^2 = x^2 + \frac{x^2+2x+1}{4}$$

$$\Rightarrow \quad x^2 + 4x + 4 = x^2 + \frac{x^2+2x+1}{4}$$

$$\Rightarrow \quad 16x + 16 = x^2 + 2x + 1$$

$$\Rightarrow \quad x^2 - 14x - 15 = 0$$

$$\Rightarrow \quad x^2 + x - 15x - 15 = 0$$

$$\Rightarrow \quad x(x+1) - 15(x+1) = 0$$

$$\Rightarrow \quad (x+1)(x-15) = 0$$

$\therefore$ Either $\quad x + 1 = 0 \quad \ldots(2)$

or, $\quad x - 15 = 0 \quad \ldots(3)$

From (2) $x = –1$ which is absurd, since the length cannot be negative

$\therefore$ We reject $x = –1$.

$\therefore$ From (3), $x = 15$ and from (1), $y = 8$.

Hence, the required lengths of the base, altitude and hypotenuse are 15 cm, 8 cm and (15 + 2) cm or (2 × 8 + 1) cm, i.e. 17 cm respectively.

14. The diagonal of a rectangular field is 16 metres more than the shorter side. If the longer side is 14 metres more than the shorter side, then find the lengths of the sides of the field. **[CBSE 2015]**

Sol. Let the length of the shorter side be x cm. Then the length of the longer side and the length of the diagonal of the rectangle are $(x + 14)$cm and $(x + 16)$ cm respectively.

$\therefore$ By Pythagoras theorem, we have

$$(x+14)^2 + x^2 = (x+16)^2$$

$$\Rightarrow \quad x^2 + 28x + 196 + x^2 = x^2 + 32x + 256$$

$$\Rightarrow \quad x^2 - 4x - 60 = 0$$

$$\Rightarrow \quad x^2 + 6x - 10x - 60 = 0$$

$$\Rightarrow \quad x(x+6) - 10(x+6) = 0$$

$$\Rightarrow \quad (x+6)(x-10) = 0$$

$\therefore$ Either $\quad x + 6 = 0 \quad \ldots(1)$

or, $\quad x - 10 = 0 \quad \ldots(2)$

From (1), $x = -6$ which is rejected, since the length cannot be negative.

$\therefore$ From (2), we have $x = 10$.

Hence, the required lengths of the sides are 10 cm, (10 + 14) cm, i.e. 24 cm and (10 + 16) cm, i.e. 26 cm.

15. A passenger train takes one hour less when its speed is increased by 15 km/hour than its usual speed for a journey of 300 km. Find the usual speed of the train. **[CBSE SP 2006]**

Sol. Let the usual speed of the train be x km/h.

$\therefore$ The usual time taken by the train to cover a distance of 300 km is $\frac{300}{x}$. Now, if the speed is $(x + 15)$ km/h then the time taken by the train to cover the same distance is $\frac{300}{x+15}$

$\therefore$ According to the problem, we have

$$\frac{300}{x} - \frac{300}{x+15} = 1$$

$$\Rightarrow \quad \frac{300(x+15-x)}{x(x+15)} = 1$$

$$\Rightarrow \quad \frac{4500}{x^2+15x} = 1$$

$$\Rightarrow \quad x^2 + 15x - 4500 = 0$$

$$\Rightarrow \quad x^2 + 75x - 60x - 4500 = 0$$

$$\Rightarrow \quad x(x+75) - 60(x+75) = 0$$

$$\Rightarrow \quad (x+75)(x-60) = 0$$

$\therefore$ Either $\quad x + 75 = 0 \quad \ldots(1)$

or, $\quad x - 60 = 0 \quad \ldots(2)$

From (1), $x = -75$ which is rejected, since speed cannot be negative here.

$\therefore$ From (2), we get $x = 60$.

$\therefore$ The required usual speed of the train is 60 km/h.

Short Answer Type-II Questions

16. The difference of two numbers is 4. If the difference of their reciprocals is $\frac{4}{21}$, find the two numbers. **[CBSE 2008]**

Sol. Let the two numbers be x and y, where $x > y$.

Then according to the problem, we have

$$x - y = 4 \quad \ldots(1)$$

and $$\frac{1}{y} - \frac{1}{x} = \frac{4}{21} \quad \ldots(2)$$

since $\frac{1}{x} < \frac{1}{y}$

From (1), $$y = x - 4 \quad \ldots(3)$$

$\therefore$ From (2), $$\frac{1}{x-4} - \frac{1}{x} = \frac{4}{21}$$

$$\Rightarrow \frac{x - x + 4}{x(x-4)} = \frac{4}{21}$$

$$\Rightarrow \frac{1}{x^2 - 4x} = \frac{1}{21}$$

$$\Rightarrow x^2 - 4x - 21 = 0$$

$$\Rightarrow x^2 + 3x - 7x - 21 = 0$$

$$\Rightarrow x(x + 3) - 7(x + 3) = 0$$

$$\Rightarrow (x + 3)(x - 7) = 0$$

$\therefore$ Either $$x + 3 = 0 \quad \ldots(4)$$

or, $$x - 7 = 0 \quad \ldots(5)$$

From (3) and (4), we have $x = -3$, $y = -7$

and from (3) and (5), we have $x = 7$, $y = 3$.

Hence, the required two numbers are either 3 and 7 or, –3 and –7.

17. (*a*) The denominator of a fraction is 3 more than its numerator. The sum of the fraction and its reciprocal is $2\frac{9}{10}$. Find the fraction. **[CBSE 2015]**

(*b*) The denominator of a fraction is one more than twice its numerator. If the sum of the fraction and its reciprocal is $2\frac{16}{21}$, find the fraction. **[CBSE 2016]**

Sol. (*a*) Let the numerator of the fraction be x. Then its denominator is $x + 3$. Then the fraction is $\frac{x}{x+3}$.

$\therefore$ According to the problem, we have

$$\frac{x}{x+3} + \frac{x+3}{x} = 2\frac{9}{10} = \frac{29}{10}$$

$$\Rightarrow \frac{x^2 + (x+3)^2}{x(x+3)} = \frac{29}{10}$$

$$\Rightarrow \frac{x^2 + x^2 + 6x + 9}{x^2 + 3x} = \frac{29}{10}$$

$$\Rightarrow 20x^2 + 60x + 90 = 29x^2 + 87x$$

$$\Rightarrow 9x^2 + 27x - 90 = 0$$

$$\Rightarrow x^2 + 3x - 10 = 0$$

$$\Rightarrow x^2 + 5x - 2x - 10 = 0$$

$$\Rightarrow x(x + 5) - 2(x + 5) = 0$$

$$\Rightarrow (x + 5)(x - 2) = 0$$

$\therefore$ Either $$x + 5 = 0 \quad \ldots(1)$$

or, $$x - 2 = 0 \quad \ldots(2)$$

From (1), $x = -5$ and from (2), $x = 2$

If it is assumed that both the numerator and the denominator of the fraction may be negative, then $x = -5$ is accepted. In this case, the numerator and the denominator are respectively –5 and –5 + 3, i.e. –2 and so the fraction is $\frac{-5}{-2}$ i.e. $\frac{5}{2}$.

In case, if it is assumed that both the numerator and the denominator should be positive, then we accept only $x = 2$.

In this case, the required fraction will be $\frac{2}{2+3}$ i.e. $\frac{2}{5}$.

(*b*) Let the numerator be x. Then the denominator is $2x + 1$.

$\therefore$ The original fraction is $\frac{x}{2x+1}$.

$\therefore$ According to the problem, we have

$$\frac{x}{2x+1} + \frac{2x+1}{x} = 2\frac{16}{21} = \frac{58}{21}$$

$$\Rightarrow \frac{x^2 + (2x+1)^2}{x(2x+1)} = \frac{58}{21}$$

$$\Rightarrow \frac{x^2 + 4x^2 + 4x + 1}{2x^2 + x} = \frac{58}{21}$$

$$\Rightarrow \frac{5x^2 + 4x + 1}{2x^2 + x} = \frac{58}{21}$$

$$\Rightarrow 150x^2 + 84x + 21 = 116x^2 + 58x$$

$$\Rightarrow 11x^2 - 26x - 21 = 0$$

$$\Rightarrow 11x^2 - 33x + 7x - 21 = 0$$

$$\Rightarrow 11x(x - 3) + 7(x - 3) = 0$$

$$\Rightarrow (x - 3)(11x + 7) = 0$$

$\therefore$ Either $$x - 3 = 0 \quad \ldots(1)$$

or, $$11x + 7 = 0 \quad \ldots(2)$$

From (1), $x = 3$ and from (2), $x = -\frac{7}{11}$ which is rejected, since numerator and denominator should be integers.

$\therefore$ $x = 3$ and so the original fraction is $\dfrac{3}{2 \times 3 + 1}$

$= \dfrac{3}{7}$.

$\therefore$ The required fraction is $\dfrac{3}{7}$.

18. A two-digit number is four times the sum of the digits. It is also equal to three times the product of the digits. Find the number. **[CBSE 2016]**

Sol. Let the digit in the unit's place of the number be x and that in the ten's place be y. Then the number is $10y + x$.

$\therefore$ According to the problem,

$$10y + x = 4(x + y)$$
$$= 4x + 4y$$
$$\Rightarrow \quad 3x - 6y = 0$$
$$\Rightarrow \quad y = \frac{3x}{6} = \frac{x}{2} \quad \ldots(1)$$
$$\text{and} \quad 10y + x = 3xy \quad \ldots(2)$$

From (1) and (2), we get

$$10 \times \frac{x}{2} + x = 3x \times \frac{x}{2}$$
$$\Rightarrow \quad 5x + x = \frac{3x^2}{2}$$
$$\Rightarrow \quad 12x = 3x^2$$
$$\Rightarrow \quad x^2 - 4x = 0$$
$$\Rightarrow \quad x(x - 4) = 0$$

$\therefore$ Either $\quad x = 0 \quad \ldots(1)$

or, $\quad x - 4 = 0 \quad \ldots(2)$

From (2), we get $x = 4$

When $x = 0$, we see from (1) that $y = 0$

and when $x = 4$, $y = \dfrac{4}{2} = 2$

For $x = 0$ $y = 0$, we do not get any non-zero number.

Hence, for $x = 4$ and $y = 2$, the required number is $10 \times 2 + 4 = 24$.

Hence, the required number is 24.

19. Vijay is x years old and his father is nine times the square of Vijay's age. Thirty-two years hence, his father's age will be double of his age then. Find their present ages.

Sol. The age of Vijay is x years. The age of Vijay's father is $9x^2$ years.

According to the problem,

$$9x^2 + 32 = 2(x + 32)$$
$$\Rightarrow \quad 9x^2 - 2x + 32 - 64 = 0$$
$$\Rightarrow \quad 9x^2 - 2x - 32 = 0$$
$$\Rightarrow \quad 9x^2 + 16x - 18x - 32 = 0$$
$$\Rightarrow \quad x(9x + 16) - 2(9x + 16) = 0$$
$$\Rightarrow \quad (x - 2)(9x + 16) = 0$$

$\therefore$ Either $\quad x - 2 = 0 \quad \ldots(1)$

or, $\quad 9x + 16 = 0 \quad \ldots(2)$

From (1), $x = 2$ and from (2), $x = -\dfrac{16}{9}$ which is rejected, since the age cannot be negative.

The required present age of Vijay is 2 years and his father is $9 \times (2)^2 = 9 \times 4$, i.e. 36 years.

20. A shopkeeper buys a number of books for ₹ 80. If he had bought 4 more books for the same amount, each book would have cost ₹ 1 less. Find the number of books he bought. **[CBSE 2012]**

Sol. Let the number of books bought by the shopkeeper be x.

$\therefore$ The original cost of each book = ₹ $\dfrac{80}{x}$

$\therefore$ According to the problem, we have

$$(x + 4)\left(\frac{80}{x} - 1\right) = 80$$
$$\Rightarrow \quad 80 - x + \frac{320}{x} - 4 - 80 = 0$$
$$\Rightarrow \quad -x^2 + 320 - 4x = 0$$
$$\Rightarrow \quad x^2 + 4x - 320 = 0$$
$$\Rightarrow \quad x^2 + 20x - 16x - 320 = 0$$
$$\Rightarrow \quad x(x + 20) - 16(x + 20) = 0$$
$$\Rightarrow \quad (x + 20)(x - 16) = 0$$

$\therefore$ Either $\quad x + 20 = 0 \quad \ldots(1)$

or, $\quad x - 16 = 0 \quad \ldots(2)$

From (1), $x = -20$ which is rejected, since the number of books cannot be negative.

$\therefore$ From (2), we get $x = 16$ which is the required number of books.

21. A person on tour has ₹ 360 for his daily expenses. If he exceeds his tour programme by 4 days, he must cut down his daily expenses by ₹ 3 per day. Find the number of days of his tour programme? **[CBSE SP 2011]**

Sol. Let the number of days of the person's tour programme be x.

Then his usual daily expense $= \dfrac{360}{x}$.

$\therefore$ According to the problem, we have

$$(x + 4)\left(\frac{360}{x} - 3\right) = 360$$

$\Rightarrow \quad 360 - 3x + \dfrac{1440}{x} = 372$

$\Rightarrow \quad \dfrac{-3x^2 + 1440}{x} = 12$

$\Rightarrow \quad 3x^2 + 12x - 1440 = 0$

$\Rightarrow \quad x^2 + 4x - 480 = 0$

$\Rightarrow \quad x^2 + 24x - 20x - 480 = 0$

$\Rightarrow \quad x(x + 24) - 20(x + 24) = 0$

$\Rightarrow \quad (x + 24)(x - 20) = 0$

$\therefore$ Either $\quad x + 24 = 0 \quad \ldots(1)$

or, $\quad x - 20 = 0 \quad \ldots(2)$

$\therefore$ From (1), $x = -24$, which is rejected, the number of days cannot be negative. From (2), $x = 20$ which is the required number of days.

22. The perimeter of a rectangle is 94 cm and its area is 246 cm^2. Find the dimensions of the rectangle.

Sol. Let the length and breadth of the rectangle be x cm and y cm respectively, where $x > y$.

Then, according to the problem, we have

$$2(x + y) = 94$$

$\Rightarrow \quad x + y = 47 \quad \ldots(1)$

and $\quad xy = 246 \quad \ldots(2)$

From (1), $\quad y = 47 - x \quad \ldots(3)$

$\therefore$ From (2), we have

$$x(47 - x) = 246$$

$\Rightarrow \quad x^2 - 47x + 246 = 0$

$\Rightarrow \quad x^2 - 41x - 6x + 246 = 0$

$\Rightarrow \quad x(x - 41) - 6(x - 41) = 0$

$\Rightarrow \quad (x - 41)(x - 6) = 0$

$\therefore$ Either $\quad x - 41 = 0 \quad \ldots(1)$

or, $\quad x - 6 = 0 \quad \ldots(2)$

From (1), $x = 41$ and from (2), $x = 6$.

$\therefore$ From (3), $y = 47 - 41 = 6$ when $x = 41$ and $y = 47 - 6 = 41$ when $x = 6$.

$\because x > y$, $\therefore x = 41$ and $y = 6$.

Hence, the required length and breadth are 41 cm and 6 cm respectively.

23. A farmer wishes to start a 100 m^2 rectangular vegetable garden. Since, he has only 30 m of barbed wire, he fences three sides of the rectangular garden letting his house compound wall act as the fourth side fence. Find the dimensions of the garden. **[CBSE SP 2011]**

Sol. Let the length and breadth of the rectangle be y m and x m respectively so that $x > y$.

Then according to the problem, we have

$2x + y = 30 \quad \ldots(1)$

and $\quad xy = 100 \quad \ldots(2)$

From (1), $\quad y = 30 - 2x \quad \ldots(3)$

$\therefore$ From (2) and (3), we have

$$x(30 - 2x) = 100$$

$\Rightarrow \quad 2x^2 - 30x + 100 = 0$

$\Rightarrow \quad x^2 - 15x + 50 = 0$

$\Rightarrow \quad x^2 - 10x - 5x + 50 = 0$

$\Rightarrow \quad x(x - 10) - 5(x - 10) = 0$

$\Rightarrow \quad (x - 10)(x - 5) = 0$

$\therefore$ Either $x - 10 = 0$ or $x - 5 = 0$

$\Rightarrow \quad x = 10$ or $x = 5$

$\therefore$ From (3), $y = 30 - 2 \times 10 = 10$ when $x = 10$ and $y = 30 - 2 \times 5 = 20$ when $x = 5$.

$\therefore$ We have either $x = 10, y = 10$

or $\quad x = 5, y = 20$

[satisfies the given condition as the garden is rectangular]

Hence, the required length and breadth of the rectangle are 20 m and 5 m respectively.

24. The hypotenuse of a right triangle is $3\sqrt{5}$ cm. If the smaller side is tripled and the larger side is doubled, the new hypotenuse will be 15 cm. Find the length of each side.

Sol. Let the lengths of two sides of the right angled triangle be x cm and y cm respectively, where $x > y > 0$.

Then by using Pythagoras theorem, we have

$$\left(3\sqrt{5}\right)^2 = x^2 + y^2$$

$\Rightarrow \quad x^2 + y^2 = 45 \quad \ldots(1)$

and $\quad (3y)^2 + (2x)^2 = 15^2$

$\Rightarrow \quad 9y^2 + 4x^2 = 225 \quad \ldots(2)$

From (1) and (2), we have

$$p + q = 45$$

$\Rightarrow \quad q = 45 - p \quad \ldots(3)$

and $\quad 4p + 9q = 225 \quad \ldots(4)$

where $\quad p = x^2 \quad \ldots(5)$

and $\quad q = y^2 \quad \ldots(6)$

$\therefore$ From (3) and (4), we get

$$4p + 9(45 - p) = 225$$

$$\Rightarrow \quad 4p - 9p + 405 = 225$$
$$\Rightarrow \quad 5p = 180$$
$$\Rightarrow \quad p = \frac{180}{5} = 36$$
$$\Rightarrow \quad x^2 = 36 \qquad \text{[From (5)]}$$
$$\Rightarrow \quad x = \pm 6$$

Since $x > 0$, we take $x = 6$.

when $p = 36$, we have

$$q = 45 - 36 = 9 \qquad \text{[From (3)]}$$
$$\Rightarrow \quad y^2 = 9 \qquad \text{[From (6)]}$$
$$\Rightarrow \quad y = \pm 3$$

$\because$ $y > 0$, we take $y = 3$.

Hence, the required lengths of the two sides of the triangle are 3 cm and 6 cm.

25. A rectangle has a perimeter 46 cm and either of the diagonals measures 17 cm. Find the dimensions of the rectangle.

Sol. Let the length and breadth of the rectangle be x cm and y cm respectively, where $x > y > 0$.

$\therefore$ According to the problem, we have

$$2(x + y) = 46$$
$$\Rightarrow \quad x + y = 23$$
$$\Rightarrow \quad y = 23 - x \qquad \ldots(1)$$

and by Pythagoras theorem, we have

$$x^2 + y^2 = 17^2 = 289 \qquad \ldots(2)$$

$\therefore$ From (1) and (2), we get

$$x^2 + (23 - x)^2 - 289 = 0$$
$$\Rightarrow \quad 2x^2 - 46x + 529 - 289 = 0$$
$$\Rightarrow \quad 2x^2 - 46x + 240 = 0$$
$$\Rightarrow \quad x^2 - 23x + 120 = 0$$
$$\Rightarrow \quad x^2 - 15x - 8x + 120 = 0$$
$$\Rightarrow \quad x(x - 15) - 8(x - 15) = 0$$
$$\Rightarrow \quad (x - 8)(x - 15) = 0$$

$\therefore$ Either $\quad x - 8 = 0 \qquad \ldots(3)$

or, $\quad x - 15 = 0 \qquad \ldots(4)$

$\therefore$ From (3) and (4), $x = 8, 15$

$\therefore$ From (1), $y = 23 - 8 = 15$ when $x = 8$

which is rejected, since $x < y$ in this case.

Again, when $x = 15$, from (1), we get

$$y = 23 - 15 = 8$$

which is accepted, since $x > y$.

Hence, the required length and breadth of the rectangle are 15 cm and 8 cm respectively.

26. A bus moving at its usual speed covers distance between towns A and B which are 550 km apart in 1 hour less than it takes to cover the same distance, when it is raining and the bus has to reduce the speed by 5 km/h. Calculate the time taken by the bus to cover the distance between A and B when it is raining.

Sol. Let the usual speed of the bus be x km/h. Then the usual time taken by the bus to cover a distance of 550 km is $\frac{550}{x}$ h.

Now, according to the problem, we have

$$\frac{550}{x - 5} - \frac{550}{x} = 1$$
$$\Rightarrow \quad 550\left(\frac{x - x + 5}{x(x - 5)}\right) = 1$$
$$\Rightarrow \quad \frac{2750}{x^2 - 5x} = 1$$
$$\Rightarrow \quad x^2 - 5x - 2750 = 0$$
$$\Rightarrow \quad x^2 - 55x + 50x - 2750 = 0$$
$$\Rightarrow \quad x(x - 55) + 50(x - 55) = 0$$
$$\Rightarrow \quad (x - 55)(x + 50) = 0$$

$\therefore$ Either $\quad x - 55 = 0 \qquad \ldots(1)$

or, $\quad x + 50 = 0 \qquad \ldots(2)$

From (1), $x = 55$ and from (2), $x = -50$ which is rejected, since x cannot be negative.

Hence, the usual speed of the bus is 55 km/h.

$\therefore$ The required usual time taken by the bus to travel a distance of 550 km is $\frac{550}{50}$ hours i.e. 11 hours.

27. A train travels at a uniform speed for a distance of 54 km and then travels a distance of 63 km at an average speed of 6 km/h more than the first speed. If it takes 3 hours to complete the total journey what is the first speed of the train? **[CBSE 2015]**

Sol. Let the first speed of the train be x km/h. Then according to the problem, we have

$$\frac{54}{x} + \frac{63}{x + 6} = 3$$
$$\Rightarrow \quad \frac{54(x + 6) + 63x}{x(x + 6)} = 3$$
$$\Rightarrow \quad 54x + 324 + 63x = 3x(x + 6)$$
$$= 3x^2 + 18x$$
$$\Rightarrow \quad 3x^2 + 18x - 117x - 324 = 0$$

$\Rightarrow \quad 3x^2 - 99x - 324 = 0$

$\Rightarrow \quad x^2 - 33x - 108 = 0$

$\Rightarrow \quad x^2 + 3x - 36x - 108 = 0$

$\Rightarrow \quad x(x + 3) - 36(x + 3) = 0$

$\Rightarrow \quad (x + 3)(x - 36) = 0$

$\therefore$ Either $\quad x + 3 = 0 \quad \ldots(1)$

or, $\quad x - 36 = 0 \quad \ldots(2)$

From (1), $x = -3$ which is rejected, since x cannot be negative.

$\therefore$ From (2), we get $x = 36$ which is accepted.

Hence, the required speed is 36 km/h.

28. The time taken by a person to cover 150 km was $2\frac{1}{2}$ hours more than the time taken in the return journey. If he returned at a speed of 10 km/hour more than the speed while going, find the speed per hour in each direction. **[CBSE 2016]**

Sol. Let the speed of the man for onward journey be x km/h. Then, according to the problem, we have

$$\frac{150}{x} - \frac{150}{x+10} = 2\frac{1}{2} = \frac{5}{2}$$

$$\Rightarrow \quad \frac{30}{x} - \frac{30}{x+10} = \frac{1}{2}$$

$$\Rightarrow \quad \frac{30x + 300 - 30x}{x(x+10)} = \frac{1}{2}$$

$\Rightarrow \quad x^2 + 10x - 600 = 0$

$\Rightarrow \quad x^2 + 30x - 20x - 600 = 0$

$\Rightarrow \quad x(x + 30) - 20(x + 30) = 0$

$\Rightarrow \quad (x + 30)(x - 20) = 0$

$\therefore$ Either $\quad x + 30 = 0 \quad \ldots(1)$

or, $\quad x - 20 = 0 \quad \ldots(2)$

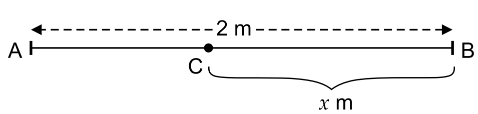
From (1), $x = -30$ which is rejected, since x cannot be negative.

$\therefore$ From (2), $x = 20$ which is accepted.

$\therefore$ The required speed for onward journey is 20 km/h and that for return journey is (20 + 10) km/h i.e. 30 km/h.

29. A segment AB of 2 m length is divided at C, into two parts such that $AC^2 = AB \times CB$. Find the length of the part CB.

Sol.



Let CB = x m.

$\therefore \quad AC = (2 - x)$ m

Also, $\quad AB = 2$ m

$\therefore \quad x < 2$

$\therefore$ From $AC^2 = AB \times CB$, we get

$\Rightarrow \quad (2 - x)^2 = 2x$

$\Rightarrow \quad 4 + x^2 - 4x - 2x = 0$

$\Rightarrow \quad x^2 - 6x + 4 = 0$

$$\therefore \quad x = \frac{6 \pm \sqrt{6^2 - 4 \times 4}}{2 \times 1}$$

$$= \frac{6 \pm \sqrt{20}}{2}$$

$$= 3 \pm \sqrt{5}$$

$\therefore \quad x = 3 + \sqrt{5}$ or $x = 3 - \sqrt{5}$

But since $x < 2$, we reject $x = 3 + \sqrt{5}$.

Hence, $x = 3 - \sqrt{5}$ is accepted.

$\therefore$ The required length of the part CB is $(3 - \sqrt{5})$ m.

Long Answer Type Questions

30. A two-digit number is such that the product of the digits is 20. If 9 is subtracted from the number, the digits interchange their places. Find the number. **[CBSE SP 2006, 2011]**

Sol. Let the digit in the unit place of the number be x and that in the ten's place be y. Then the number is $10y + x$.

$\therefore$ According to the problem, we have

$xy = 20 \quad \ldots(1)$

and $\quad 10y + x - 9 = 10x + y$

$\Rightarrow \quad 9x - 9y + 9 = 0$

$\Rightarrow \quad x - y + 1 = 0 \quad \ldots(2)$

$\therefore$ From (2), wet get $\quad y = x + 1 \quad \ldots(3)$

$\therefore$ From (1), we have

$x(x + 1) = 20$

$\Rightarrow \quad x^2 + x - 20 = 0$

$\Rightarrow \quad x^2 + 5x - 4x - 20 = 0$

$\Rightarrow \quad x(x + 5) - 4(x + 5) = 0$

$\Rightarrow \quad (x + 5)(x - 4) = 0$

$\therefore$ Either $\quad x + 5 = 0 \quad \ldots(4)$

or, $\quad x - 4 = 0 \quad \ldots(5)$

From (4), $x = -5$ which is rejected, since x cannot be negative.

$\therefore$ From (5), we have $x = 4$ which is accepted.

Now, when $x = 4$, from (3), $y = 4 + 1 = 5$

$\therefore$ The required number is $10 \times 5 + 4$ i.e 54.

31. A worker earns ₹ 400 in a certain number of days. If his daily wage had been ₹ 10 more, he would have taken 2 days less to earn the same amount. Find how many days he worked at a higher wage.

Sol. Let the worker worked for x days at the usual wage.

Then he worked for $(x - 2)$ days at higher wage.

$\therefore$ His daily wage at higher rate is ₹ $\dfrac{400}{x-2}$

and his daily wage at the usual rate is ₹ $\dfrac{400}{x}$.

$\therefore$ According to the problem, we have

$$\frac{400}{x-2} - \frac{400}{x} = 10$$

$$\Rightarrow \quad \frac{40}{x-2} - \frac{40}{x} = 1$$

$$\Rightarrow \quad \frac{40x - 40x + 80}{x(x-2)} = 1$$

$$\Rightarrow \quad \frac{80}{x^2 - 2x} = 1$$

$$\Rightarrow \quad x^2 - 2x - 80 = 0$$

$$\Rightarrow \quad x^2 + 8x - 10x - 80 = 0$$

$$\Rightarrow \quad x(x + 8) - 10(x + 8) = 0$$

$$\Rightarrow \quad (x + 8)(x - 10) = 0$$

$\therefore$ Either $\quad x + 8 = 0 \quad \ldots(1)$

or, $\quad x - 10 = 0 \quad \ldots(2)$

From (1), $x = -8$ which is rejected, since x cannot be negative.

$\therefore$ From (2), we have $x = 10$ which is accepted.

Hence, he worked for 10 days at the usual wage and so he worked for 10 – 2 = 8 days at higher wage.

$\therefore$ The required number of days a worker worked at a higher wage is 8 days.

32. The ratio of the areas of a rectangle and a square is 24 : 25. If the perimeter of each is 80 units, find the dimensions of the rectangle.

Sol. Let the length of the rectangle be x cm and the breadth be y cm so that $x > y > 0$.

$\therefore$ The area of the rectangle $= xy$ cm^2 and perimeter of the rectangle $= 2(x + y) = 80$ [Given]

$$\Rightarrow \quad x + y = 40$$

$$\Rightarrow \quad y = 40 - x \quad \ldots(1)$$

If a cm be the side of the square, then its perimeter $= 4a$ cm.

$$\therefore \quad 4a = 80 \quad \text{[Given]}$$

$$\Rightarrow \quad a = \frac{80}{4} = 20$$

Hence, the side of the square is 20 cm and so the area of the square $= a^2$ cm^2 $= 20^2$ cm^2 $= 400$ cm^2

$$\therefore \quad \frac{\text{Area of the rectangle}}{\text{Area of the square}} = \frac{24}{25}$$

$$\Rightarrow \quad \frac{xy}{400} = \frac{24}{25} \quad \text{[Given]}$$

$$\Rightarrow \quad 400 \times 24 = 25xy$$

$$\Rightarrow \quad xy = \frac{450 \times 24}{25}$$

$$= 16 \times 24$$

$$= 384 \quad \ldots(2)$$

$$\Rightarrow \quad x(40 - x) = 384 \quad \text{[From (1)]}$$

$$\Rightarrow \quad x^2 - 40x + 384 = 0$$

$$\Rightarrow \quad x^2 - 16x - 24x + 384 = 0$$

$$\Rightarrow \quad x(x - 16) - 24(x - 16) = 0$$

$$\Rightarrow \quad (x - 16)(x - 24) = 0$$

$\therefore$ Either $\quad x - 16 = 0 \quad \ldots(1)$

or, $\quad x - 24 = 0 \quad \ldots(2)$

From (1) and (2), we have $x = 24, 16$.

$\therefore$ From (1), $y = 16, 24$ when $x = 24, 16$ respectively.

$\because x > y, \therefore \; x = 24, y = 16$

Hence, the required length and breadth of the rectangle are 24 cm and 16 cm respectively.

33. The hypotenuse of a right-angled triangle is 50 cm and the longer of the other two sides, exceeds the shorter by 10 cm. Calculate

(*a*) the lengths of the sides

(*b*) area of the triangle

Sol. (*a*) Let the shorter side of the right angled triangle be x cm. Then the longer side of the triangle is $(x + 10)$ cm. Since, the hypotenuse of the triangle is 50 cm, hence by using Pythagoras theorem in this right angled triangle, we have

$$x^2 + (x + 10)^2 = 50^2$$

$$\Rightarrow \quad x^2 + x^2 + 20x + 100 = 2500$$

$$\Rightarrow \quad 2x^2 + 20x - 2400 = 0$$

$$\Rightarrow \quad x^2 + 10x - 1200 = 0$$

$$\Rightarrow \quad x^2 + 40x - 30x - 1200 = 0$$

$$\Rightarrow \quad x(x + 40) - 30(x + 40) = 0$$

$$\Rightarrow \quad (x + 40)(x - 30) = 0$$

$\therefore$ Either $\quad x + 40 = 0 \quad \ldots(1)$

or, $\quad x - 30 = 0 \quad \ldots(2)$

From (1), $x = -40$ which is rejected, since x is negative.

$\therefore$ From (2), $x = 30$ which is accepted.

$\therefore$ The required shorter side and the longer side of the triangle are 30 cm and (30 + 10) cm i.e. 40 cm respectively.

(*b*) Hence, the required area of the triangle is $\frac{1}{2} \times 30 \times 40 \text{ cm}^2$ i.e. 600 cm^2.

34. A rectangular park is to be designed whose breadth is 3 m less than its length. Its area is to be 4 square metres more than the area of a park that has already been made in the shape of an isosceles triangle with its base as the breadth of the rectangular park and of altitude 12 m. Find the length and breadth of the rectangular park.

[CBSE 2016]

Sol. Let the length of the rectangle be x m. Then its breadth = $(x - 3)$ m.

$\therefore$ Its area = $x(x - 3)$ m^2.

The area of the isosceles triangle with base $(x - 3)$ m and altitude 12 m = $\frac{1}{2} \times 12 \times (x-3)$ m^2

= $(6x - 18)$ m^2.

$\therefore$ According to the problem, we have

$$x(x - 3) = 6x - 18 + 4 = 6x - 14$$

$\Rightarrow$ $x^2 - 3x - 6x + 14 = 0$

$\Rightarrow$ $x^2 - 9x + 14 = 0$

$\Rightarrow$ $x^2 - 7x - 2x + 14 = 0$

$\Rightarrow$ $x(x - 7) - 2(x - 7) = 0$

$\Rightarrow$ $(x - 2)(x - 7) = 0$

$\therefore$ Either $x - 2 = 0$ ...(1)

or, $x - 7 = 0$ ...(2)

From (1) and (2), we have $x = 2, 7$

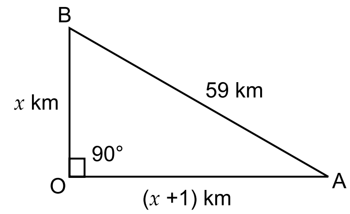
Since the length x m > 3 m, therefore, we reject $x = 2$.

So, we accept $x = 7$.

$\therefore$ The required length of the rectangle is 7 m and the breadth is (7 – 3) m i.e. 4 m.

35. Two ships leave simultaneously in directions at right angles to each other. The speed of one of them exceeds the other by 1 km per hour. The distance between the ships after 1 hour is 29 km. Find their speeds.

Sol. Let the slower speed be x km/h and the faster speed is $(x + 1)$ km/h. Let after 1 hour two ships S_1 and S_2 moving along OA and OB, where OA is perpendicular to OB, come to B and A respectively.



$\therefore$ OA = $(x + 1)$ km, OB = x km. It is given that AB = 59 km.

$\because$ $\angle AOB = 90°$,

$\therefore$ By using Pythagoras theorem, we get

$$x^2 + (x + 1)^2 = 59^2$$

$\Rightarrow$ $2x^2 + 2x + 1 - 841 = 0$

$\Rightarrow$ $2x^2 + 2x - 840 = 0$

$\Rightarrow$ $x^2 + x - 420 = 0$

$\Rightarrow$ $x^2 + 21x - 20x - 420 = 0$

$\Rightarrow$ $x(x + 21) - 20(x + 21) = 0$

$\Rightarrow$ $(x + 21)(x - 20) = 0$

$\therefore$ Either $x + 21 = 0$ ...(1)

or, $x - 20 = 0$ ...(2)

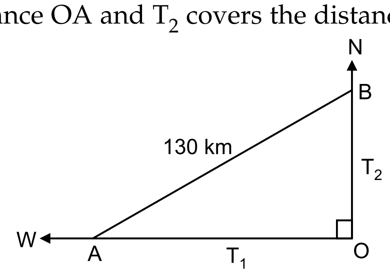
From (1), $x = -21$ which is rejected, since x cannot be negative.

$\therefore$ From (2), we get $x = 20$ which is accepted.

$\therefore$ The required slower speed is 20 km/h and the faster speed is (20 + 1) km/h i.e. 21 km/h.

36. Two trains leave a railway station at the same time. The first train travels due west and the second train due north. The first train travels 35 km/h faster than the second train. If after two hours, they are 130 km apart, find the average speed of each train. **[CBSE SP 2011]**

Sol. Let the two trains T_2 and T_1 move with speeds x km/h and $(x + 35)$ km/h towards north and west respectively. The speed of T_1 being more than that of T_2 by 35 km/h, starting from the same station O. After 2 hours let T_1 covers the distance OA and T_2 covers the distance OB.



Then $\quad OA = 2(x + 35)$ km

$\quad = (2x + 70)$ km

and $\quad OB = 2x$ km

Given that AB = 130 km

$\because$ $\angle AOB = 90°$, hence by using Pythagoras theorem in ΔOAB, we get

$$(2x)^2 + (2x + 70)^2 = 130^2$$

$$\Rightarrow \quad 4x^2 + 4x^2 + 280x + 4900 - 16900 = 0$$

$$\Rightarrow \quad 8x^2 + 280x - 12000 = 0$$

$$\Rightarrow \quad x^2 + 35x - 1500 = 0$$

$$\Rightarrow \quad x^2 + 60x - 25x - 1500 = 0$$

$$\Rightarrow \quad x(x + 60) - 25(x + 60) = 0$$

$$\Rightarrow \quad (x + 60)(x - 25) = 0$$

$\therefore$ Either $\quad x + 60 = 0 \quad \ldots(1)$

or, $\quad x - 25 = 0 \quad \ldots(2)$

From (1), $x = -60$ which is rejected, since x is negative.

$\therefore$ From (2), $x = 25$ which is accepted.

Hence, the required speeds are 25 km/h and (25 + 35) km/h i.e. 60 km/h.

37. A boat takes 2 hour longer to go 30 km up a river than to go the same distance down the river. Calculate the rate at which the boat travels in still water, given that the river is flowing at 2 km/hour.

Sol. Let the speed of the boat in still water be x km/h. Then, the speed of the boat in favour of the current = Speed of the boat in still water + Speed of the current = $(x + 2)$ km/h.

Also, the speed of the boat against the current = Speed of the boat in still water – Speed of the current = $(x - 2)$ km/h.

$\therefore$ Time taken by the boat to travel a distance of 30 km in favour of the current is $\frac{30}{x+2}$ h and that against the current is $\frac{30}{x-2}$ h.

$\therefore$ According to the problem, we have

$$\frac{30}{x-2} - \frac{30}{x+2} = 2$$

$$\Rightarrow \quad \frac{15}{x-2} - \frac{15}{x+2} = 1$$

$$\Rightarrow \quad \frac{15x + 30 - 15x + 30}{(x+2)(x-2)} = 1$$

$$\Rightarrow \quad \frac{60}{x^2 - 4} = 1$$

$$\Rightarrow \quad x^2 - 4 = 60$$

$$\Rightarrow \quad x^2 = 60 + 4 = 64$$

$$\therefore \quad x = \pm\sqrt{64} = \pm 8$$

$\because$ x cannot be negative, we take $x = 8$.

Hence, the required speed of the boat in still water is 8 km/h.

38. Two pipes running together can fill a tank in $11\frac{1}{9}$ minutes. If one pipe takes 5 minutes more than the other to fill the tank separately, find the time in which each pipe would fill the tank separately. **[CBSE 2016]**

Sol. Let the pipe of smaller diameter can fill the tank in x minutes and that of bigger diameter can fill the tank in y minutes. Then clearly, $x > y > 0$.

Then according to the problem, we have

$$x = y + 5$$

$$\Rightarrow \quad y = x - 5 \quad \ldots(1)$$

and $$\frac{1}{x} + \frac{1}{y} = \frac{1}{11\frac{1}{9}} = \frac{9}{100} \quad \ldots(2)$$

$\therefore$ From (1) and (2), we have

$$\frac{1}{x} + \frac{1}{x-5} = \frac{9}{100}$$

$$\Rightarrow \quad \frac{x - 5 + x}{x(x-5)} = \frac{9}{100}$$

$$\Rightarrow \quad \frac{2x - 5}{x^2 - 5x} = \frac{9}{100}$$

$$\Rightarrow \quad 9x^2 - 45x = 200x - 500$$

$$\Rightarrow \quad 9x^2 - 245x + 500 = 0$$

$$\therefore \quad x = \frac{245 \pm \sqrt{245^2 - 4 \times 9 \times 500}}{2 \times 9}$$

$$= \frac{245 \pm \sqrt{60025 - 18000}}{18}$$

$$= \frac{245 \pm \sqrt{42025}}{18}$$

$$= \frac{245 \pm 205}{18}$$

$$= \frac{245 + 205}{18} \text{ or } \frac{245 - 205}{18}$$

$$= \frac{450}{18} \text{ or } \frac{40}{18}$$

$$= 25 \text{ or } \frac{20}{9}$$

When $x = 25$, $y = 25 - 5 = 20$ [From (1)]

When $x = \frac{20}{9}$, $y = \frac{20}{9} - 5 = -\frac{25}{9}$ which is rejected, since y cannot be negative.

Hence, the required time in which each pipe would fill the tank separately is 20 minutes and 25 minutes.

39. To fill a swimming pool two pipes are to be used. If the pipe of larger diameter is used for 4 hours and the pipe of smaller diameter for 9 hours, only half the pool can be filled. Find how long it would take for each pipe to fill the pool separately, if the pipe of smaller diameter takes 10 hours more than the pipe of larger diameter to fill the pool. **[CBSE 2015]**

Sol. Let the pipe of larger diameter take x hours to fill the pool. Then the pipe of smaller diameter will take $(x + 10)$ hours to fill the same pool.

$\therefore$ According to the problem, we have

$$\frac{4}{x} + \frac{9}{x+10} = \frac{1}{2}$$

$$\Rightarrow \quad \frac{4x + 40 + 9x}{x(x+10)} = \frac{1}{2}$$

$$\Rightarrow \quad \frac{13x + 40}{x^2 + 10x} = \frac{1}{2}$$

$$\Rightarrow \quad x^2 + 10x = 26x + 80$$

$$\Rightarrow \quad x^2 - 16x - 80 = 0$$

$$\Rightarrow \quad x^2 + 4x - 20x - 80 = 0$$

$$\Rightarrow \quad x(x + 4) - 20(x + 4) = 0$$

$$\Rightarrow \quad (x + 4)(x - 20) = 0$$

$\therefore$ Either $\quad x + 4 = 0 \quad \ldots(1)$

or, $\quad x - 20 = 0 \quad \ldots(2)$

From (1), $x = -4$ which is rejected, since x cannot be negative.

From (2), $x = 20$ which is accepted.

Hence, the pipe of larger diameter takes 20 hours to fill the pool and that of smaller diameter takes (20 + 10) hours = 30 hours to fill the same pool.

Hence, the required time taken by each pipe to fill the pool is 20 hours and 30 hours.

40. A pool is filled by three pipes with uniform flow. The first two pipes operating simultaneously, fill the pool in the same time during which the pool is filled by the third pipe alone. The second pipe fills the pool five hours faster than the first pipe and four hours slower than the third pipe. Find the time required by each pipe to fill the pool separately. **[CBSE SP 2011]**

Sol. Let the first pipe fill the pool in x hours. Then according to the problem, the second pipe takes 5 hours less than the first to fill the pool. Hence, the second pipe fills the pool in $(x - 5)$ hours. Also, the second pipe takes 4 hours more than the third pipe. In other words, the third pipe takes 4 hours less than the second pipe. Hence, the third pipe fills the pool in $(x - 5 - 4)$ hours i.e. in $(x - 9)$ hours.

Now, according to the problem, we have

$$\frac{1}{x} + \frac{1}{x-5} = \frac{1}{x-9}$$

$$\Rightarrow \quad \frac{x - 5 + x}{x(x-5)} = \frac{1}{x-9}$$

$$\Rightarrow \quad \frac{2x - 5}{x^2 - 5x} = \frac{1}{x-9}$$

$$\Rightarrow \quad x^2 - 5x = (x - 9)(2x - 5)$$

$$\Rightarrow \quad x^2 - 5x = 2x^2 - 23x + 45$$

$$\Rightarrow \quad x^2 - 18x + 45 = 0$$

$$\Rightarrow \quad x^2 - 3x - 15x + 45 = 0$$

$$\Rightarrow \quad x(x - 3) - 15(x - 3) = 0$$

$$\Rightarrow \quad (x - 3)(x - 15) = 0$$

$\therefore$ Either $\quad x - 3 = 0 \quad \ldots(1)$

or, $\quad x - 15 = 0 \quad \ldots(2)$

From (1), $x = 3$ which is rejected, since in this case $x - 5$ becomes negative. Hence, from (2), $x = 15$ which is accepted.

Hence, the required time are 15 hours, (15 – 5) hours i.e. 10 hours and (15 – 9) hours i.e. 6 hours.

41. One-fourth of a herd of camels was seen in the forest. Twice the square root of the herd had gone to the mountains and 15 camels were on the bank of the river. Find the number of camels.

Sol. Let the total number of camels be x.

Then according to the problem, we have

$$\frac{x}{4} + 2\sqrt{x} + 15 = x$$

$$\Rightarrow \quad x + 8\sqrt{x} + 60 = 4x$$

$$\Rightarrow \quad 8\sqrt{x} = 3x - 60$$

On squaring both sides

$$\Rightarrow \quad 64x = (3x - 60)^2$$

$$\Rightarrow \quad 64x = 9x^2 + 3600 - 360x$$

$$\Rightarrow \quad 9x^2 - 424x + 3600 = 0$$

$$\therefore \quad x = \frac{424 \pm \sqrt{(424)^2 - 4 \times 9 \times 3600}}{18}$$

Now, $(424)^2 - 4 \times 9 \times 3600$

$= (424)^2 - 36 \times 3600$

$= (424)^2 - (6 \times 60)^2$

$= (424)^2 - (360)^2$

$= (424 + 360)(424 - 360)$

$= 784 \times 64$

$\therefore \sqrt{424^2 - 4 \times 9 \times 3600}$

$= \sqrt{784}\,\sqrt{64}$

$= 28 \times 8$

$= 224$

$\therefore x = \dfrac{424 \pm 224}{18} = \dfrac{648}{18}, \dfrac{200}{18} = 36, \dfrac{100}{9}$

We reject $x = \dfrac{100}{9}$, since x cannot be a fraction.

$\therefore x = 36$

Hence, the required number of camels are 36.

Higher Order Thinking Skills (HOTS) Questions

(Page 76)

1. Solve: $4x^2 - 4a^2x + (a^4 - b^4) = 0$ **[CBSE 2004, SP 2011]**

Sol. We have $4x^2 - 4a^2x + a^4 - b^4 = 0$

$$4x^2 - 4a^2x + (a^2)^2 - (b^2)^2 = 0$$

$$\Rightarrow (2x)^2 - 2 \times 2x \times a^2 + (a^2)^2 - (b^2)^2 = 0$$

$$\Rightarrow (2x - a^2)^2 - b^2 = 0$$

$$\Rightarrow (2x - a^2 + b^2)(2x - a^2 - b^2) = 0$$

$\therefore$ Either $2x - a^2 + b^2 = 0$ ...(1)

or, $2x - a^2 - b^2 = 0$ ...(2)

From (1), $x = \dfrac{a^2 - b^2}{2}$ and from (2), $x = \dfrac{a^2 + b^2}{2}$

$\therefore$ The required solutions are $\dfrac{a^2 - b^2}{2}$ and $\dfrac{a^2 + b^2}{2}$.

2. Solve: $\dfrac{1}{a+b+x} = \dfrac{1}{a} + \dfrac{1}{b} + \dfrac{1}{x}$; $a \neq 0, b \neq 0, x \neq 0$ **[CBSE 2005, 2017, SP 2011]**

Sol. We have $\dfrac{1}{p+x} = \dfrac{(a+b)x + ab}{abx} = \dfrac{px + q}{qx}$, where

$p = a + b$ ...(1)

and $q = ab$ ...(2)

$$\Rightarrow \frac{1}{p+x} = \frac{px+q}{qx}$$

$$\Rightarrow p^2x + pq + px^2 + qx = qx$$

$$\Rightarrow px^2 + p^2x + pq = 0$$

$$\Rightarrow x^2 + px + q = 0$$

$$\Rightarrow x^2 + (a+b)x + ab = 0 \text{ [From (1) and (2)]}$$

$$\Rightarrow x^2 + ax + bx + ab = 0$$

$$\Rightarrow x(x+a) + b(x+a) = 0$$

$$\Rightarrow (x+a)(x+b) = 0$$

$\therefore$ Either $x + a = 0$ ...(3)

or, $x + b = 0$ ...(4)

$\therefore$ From (3) and (4), we have $x = -a, -b$ which are the required solutions.

3. By using the method of completing the square, show that the equation $2x^2 + x + 5 = 0$ has no real roots.

Sol. We have $2x^2 + x + 5 = 0$

$$\Rightarrow x^2 + \frac{1}{2}x + \frac{5}{2} = 0$$

$$\Rightarrow x^2 + 2.\frac{1}{4}x + \left(\frac{1}{4}\right)^2 - \left(\frac{1}{4}\right)^2 + \frac{5}{2} = 0$$

$$\Rightarrow \left(x + \frac{1}{4}\right)^2 + \frac{5}{2} - \frac{1}{16} = 0$$

$$\Rightarrow \left(x + \frac{1}{4}\right)^2 = -\frac{39}{16}$$

$$\therefore x + \frac{1}{4} = \pm\sqrt{-\frac{39}{16}}$$

which is not real.

Hence, the given equation does not have any real roots.

4. In the following, determine value(s) of k for which the given quadratic equation has real roots:

$4x^2 - 3kx + 9 = 0$ **[CBSE SP 2013]**

Sol. We know that for real roots,

discriminant, $D = (-3k)^2 - 4 \times 4 \times 9$

$= 9k^2 - 144$

For real roots, we have $D \geq 0$

$$\therefore 9k^2 - 144 \geq 0$$

$$\Rightarrow (3k)^2 - 12^2 \geq 0$$

$$\Rightarrow (3k + 12)(3k - 12) \geq 0$$

$\therefore$ Either $3k + 12 \geq 0$ ...(1)

and $3k - 12 \geq 0$ ...(2)

or $3k + 12 \leq 0$ ...(3)

and $3k - 12 \leq 0$ ...(4)

From (1) and (2), we have

$$k \geq -\frac{12}{3} = -4$$

and $k \geq \dfrac{12}{3} = 4$

$$\Rightarrow \quad k \geq 4 \quad \dots(5)$$

From (3) and (4), we have

$$k \leq \frac{-12}{3} = -4$$

and $\quad k \leq \frac{12}{3} = 4$

$$\Rightarrow \quad k \leq -4 \quad \dots(6)$$

Hence, from (5) and (6), the required values of k are either $k \geq 4$ or $k \leq -4$.

5. A dealer sells an article for ₹ 75 and gains as much per cent as the cost price of the article. What is the cost price of the article?

Sol. Let the cost price of the article be ₹ x.

The selling price of the article = ₹ 75

$\therefore \quad$ Gain = ₹ $(75 - x)$

$\therefore$ According to the problem, we have

$$\frac{\text{Gain}}{\text{CP}} \times 100 = \text{SP}$$

$$\Rightarrow \quad \frac{(75 - x)100}{x} = x$$

$$\Rightarrow \quad x^2 = 7500 - 100x$$

$$\Rightarrow \quad x^2 + 100x - 7500 = 0$$

$$\Rightarrow \quad x^2 + 150x - 50x - 7500 = 0$$

$$\Rightarrow \quad x(x + 150) - 50(x + 150) = 0$$

$$\Rightarrow \quad (x + 150)(x - 50) = 0$$

$\therefore$ Either $\quad x + 150 = 0 \quad \dots(1)$

or, $\quad x - 50 = 0 \quad \dots(2)$

From (1), $x = -150$ which is rejected, since x cannot be negative.

$\therefore$ From (2), $x = 50$ which is accepted.

$\therefore$ The required cost price is ₹50.

6. A factory kept increasing its output by the same percentage every year. Find the percentage if it is known that the output is doubled in the last two years. **[CBSE SP 2011]**

Sol. Let the initial output be ₹y and the required percentage increase be x %. Then the output for 2 years = ₹$y\left(1 + \frac{x}{100}\right)^2$

$\therefore$ According to the problem, we have

$$y\left(1 + \frac{x}{100}\right)^2 = 2y$$

$$\Rightarrow \quad 1 + \frac{x}{100} = \pm\sqrt{2}$$

$$\Rightarrow \quad x = 100(\pm\sqrt{2} - 1)$$

But x cannot be negative. So, we reject $100(-1 - \sqrt{2})$ and accept $x = 100(\sqrt{2} - 1)$.

$\therefore$ The required percentage is $100(\sqrt{2} - 1)\%$.

7. Natasha is x years old and her mother is x^2 years old. When her mother becomes $11x$ years old, Natasha becomes x^2 years. Find their present ages.

Sol. Natasha's has mother will be $11x$ years after $(11x - x^2)$ years.

After $(11x - x^2)$ years, Natasha's age will be $(x + 11x - x^2)$ years = $(12x - x^2)$ years = $x(12 - x)$ years.

$\therefore$ According to the problem, we have

$$x(12 - x) = x^2$$

$$\Rightarrow \quad 12 - x = x$$

$$\Rightarrow \quad 2x = 12$$

$$\Rightarrow \quad x = 6$$

$\therefore$ The required present ages of Natasha and her mother are 6 years and 6^2 years i.e. 36 years respectively.

8. The sum of the areas of two squares is 400 cm^2. If the difference of their perimeters is 16 cm, find the sides of the two squares. **[CBSE 2013]**

Sol. Let the sides of the two squares be x cm and y cm, where $x > y$.

$\therefore$ According to the problem, we have

$$x^2 + y^2 = 400 \quad \dots(1)$$

and $\quad 4x - 4y = 16$

$$\Rightarrow \quad x - y = 4$$

$$y = x - 4 \quad \dots(2)$$

Now, $\quad x^2 + (x - 4)^2 = 400$

[From (1) and (2)]

$$\Rightarrow \quad x^2 + x^2 - 8x + 16 - 400 = 0$$

$$\Rightarrow \quad 2x^2 - 8x - 384 = 0$$

$$\Rightarrow \quad x^2 - 4x - 192 = 0$$

$$\Rightarrow \quad x^2 + 12x - 16x - 192 = 0$$

$$\Rightarrow \quad x(x + 12) - 16(x + 12) = 0$$

$$\Rightarrow \quad (x + 12)(x - 16) = 0$$

$\therefore$ Either $\quad x + 12 = 0 \quad \dots(3)$

or, $\quad x - 16 = 0 \quad \dots(4)$

From (3), $x = -12$ which is rejected, since x cannot be negative.

From (4), $x = 16$ which is accepted.

$\therefore$ From (2), $\quad y = 16 - 4 = 12$

Hence, the required sides of the two squares are 16 cm and 12 cm.

9. The perimeter of a right-angled triangle is four times the length of the shortest side. The numerical value of the area of the triangle is eight times the numerical value of the length of the shortest side. Find the lengths of the three sides of the triangle.

Sol. Let the length of the shortest side be x cm and the length of the bigger side of the right angled triangle be y cm.

Then the length of its hypotenuse is $\sqrt{x^2 + y^2}$ cm

[By Pythagoras theorem]

Now, according to the problem, the perimeter of the triangle = $4x$

$$\therefore \quad 4x = x + y + \sqrt{x^2 + y^2}$$

$$\Rightarrow \quad 3x = y + \sqrt{x^2 + y^2}$$

$$\Rightarrow \quad 3x - y = \sqrt{x^2 + y^2} \quad \ldots(1)$$

Also, area of the triangle = $8x$

According to the problem, we have

$$8x = \frac{1}{2} xy$$

$$\Rightarrow \quad y = 16 \quad \ldots(2)$$

$\therefore$ From (1) and (2), we get

$$(3x - 16)^2 = x^2 + 256$$

$$\Rightarrow \quad 9x^2 + 256 - 96x = x^2 + 256$$

$$\Rightarrow \quad 8x^2 = 96x$$

$$\Rightarrow \quad x = \frac{96}{8} \quad [\because x \neq 0]$$

$$\Rightarrow \quad x = 12 \quad \ldots(3)$$

$$\therefore \quad \sqrt{x^2 + y^2} = \sqrt{256 + 144}$$

$$= \sqrt{400} = 20$$

Hence, the required lengths of the sides of the triangle are 12 units, 16 units and 20 units.

10. A dealer sells a pen for ₹ 24 and gains as much per cent as the cost price of the pen. Find the cost price of the pen.

Sol. Let the cost price of the pen be ₹ x.

The selling price of the pen = ₹ 24

$\therefore$ Gain = ₹ $(24 - x)$

$\therefore$ According to the problem, we have

$$24 - x = x \text{ \% of } x = \frac{x^2}{100}$$

$$\Rightarrow \quad x^2 + 100x - 2400 = 0$$

$$\Rightarrow \quad x^2 + 120x - 20x - 2400 = 0$$

$$\Rightarrow \quad x(x + 120) - 20(x + 120) = 0$$

$$\Rightarrow \quad (x + 120)(x - 20) = 0$$

$\therefore$ Either $x + 120 = 0$ ...(1)

or, $x - 20 = 0$ ...(2)

From (1), $x = -120$, which is rejected, since x cannot be negative.

$\therefore$ From (2), we have, $x = 20$ which is accepted.

$\therefore$ The required cost price is ₹ 20.

Self-Assessment

(Page 76)

Multiple-Choice Questions

1. The quadratic equation $x^2 - x - 2 = 0$ has roots which are

(*a*) real and equal

(*b*) real, unequal and rational

(*c*) real, unequal and irrational

(*d*) not real

Sol. (*b*) real, unequal and rational

We see that the discriminant,

$$D = (-1)^2 - 4 \times 1(-2)$$
$$= 1^2 + 8$$
$$= 9 > 0$$

$$\because \quad D > 0$$

$\therefore$ The roots are real and unequal.

Also, since $\sqrt{D} = \sqrt{9} = 3$ which is a rational number, hence the roots are also rational.

2. The value of k for which $3x^2 + 2x + k = 0$ has real roots, is

(*a*) $k > \frac{1}{3}$ (*b*) $k \leq \frac{1}{3}$

(*c*) $k \geq \frac{1}{3}$ (*d*) $k < \frac{1}{3}$

Sol. (*b*) $k \leq \frac{1}{3}$

For real roots, discriminant, $D \geq 0$

$$\Rightarrow \quad 2^2 - 4 \times 3k \geq 0$$

$$\Rightarrow \quad 4 - 12k \geq 0$$

$$\Rightarrow \quad 12k \leq 4$$

$$\Rightarrow \quad k \leq \frac{1}{3}$$

3. If the equation $px^2 + 2x + p = 0$ has two equal roots, then

(a) $p = 0$ (b) $p = 1, 0$

(c) $p = \pm 1$ (d) $p = -1, 0$

Sol. (c) $p = \pm 1$

We have the discriminant,

$$D = 2^2 - 4p \times p$$
$$= 4 - 4p^2$$

Now, for real and equal roots, D = 0

$\therefore \quad 4 - 4p^2 = 0$

$\Rightarrow \quad p^2 = 1$

$\Rightarrow \quad p = \pm 1$

4. If one root of the quadratic equation $2x^2 + px + 4 = 0$, is 2, then the other root and p are respectively

(a) 6, – 1 (b) – 1, – 6

(c) – 1, 6 (d) 1, – 6

Sol. (d) 1, –6

Since $x = 2$ is a root, hence

$2 \times 2^2 + p \times 2 + 4 = 0$

$\Rightarrow \quad 12 + 2p = 0$

$\Rightarrow \quad p = -6$

If the other root is α, then

$$\text{Sum of the roots} = -\frac{p}{2} = +\frac{6}{2} = 3$$

$\therefore \quad \alpha + 2 = 3$

$\Rightarrow \quad \alpha = 3 - 2 = 1$

$\therefore$ The other root α is 1.

5. If m and n are roots of the equation $x^2 + mx + n = 0$, then

(a) $m = 2, n = 1$

(b) $m = -2, n = -1$

(c) $m = 1, n = -2$

(d) $m = -1, n = 2$

Sol. (c) $m = 1, n = -2$

$\because$ m and n are the two roots of the equation

$$x^2 + mx + n = 0$$

$\therefore \quad m + n = -m$

$\Rightarrow \quad 2m + n = 0 \quad \ldots(1)$

Also, $\quad mn = n$

$\Rightarrow \quad m = 1$, assuming $n \neq 0$.

$\therefore$ From (1), $\quad 2 + n = 0$

$\Rightarrow \quad n = -2$

$\therefore$ The required values of m and n are 1 and –2 respectively.

6. If the roots of a quadratic equation are 4 and – 3, then the equation is

(a) $x^2 + x - 12 = 0$ (b) $x^2 - x - 12 = 0$

(c) $x^2 + x + 12 = 0$ (d) $x^2 - x + 12 = 0$

Sol. (b) $\quad x^2 - x - 12 = 0$

The required equation is

x^2 – (sum of the roots)x + product of the roots = 0

$\Rightarrow \quad x^2 - (4 - 3)\, x - 4 \times 3 = 0$

$\Rightarrow \quad x^2 - x - 12 = 0$

7. If the sum of the roots of the equation $x^2 - x = p(2x - 1)$ is zero, then

(a) $p = -2$ (b) $p = 2$

(c) $p = -\frac{1}{2}$ (d) $p = \frac{1}{2}$

Sol. (c) $p = -\frac{1}{2}$

The given equation is

$$x^2 - x - 2px + p = 0$$

$\Rightarrow \quad x^2 - (1 + 2p)x + p = 0$

$\therefore \quad$ Sum of the roots = $1 + 2p$

It is given that this sum is zero.

$\therefore \quad 1 + 2p = 0$

$\Rightarrow \quad p = -\frac{1}{2}$

8. A quadratic equation whose one root is 3 and the sum of the roots is 0, is

(a) $x^2 + 3 = 0$ (b) $x^2 + 9 = 0$

(c) $x^2 - 9 = 0$ (d) $x^2 - 3 = 0$

Sol. (c) $x^2 - 9 = 0$

Sum of the roots = 0 and one root is 3. Hence, the other root must be –3.

$\therefore$ The required equation is x^2 + (sum of the roots)x + product of the roots = 0

$\Rightarrow \quad x^2 - 3 \times 3 = 0$

$\Rightarrow \quad x^2 - 9 = 0$

Fill in the Blanks

9. If the discriminant of a quadratic equation is **positive** then it has real and unequal roots.

10. Quadratic equation whose roots are 1 and – 2 is $\boldsymbol{x^2 + x - 2 = 0}$.

11. If the roots of the equation $x^2 - kx + p = 0$ differ by one, then $k^2 - 4p$ is equal to **1**.

12. Product of the roots of quadratic equation $x^2 - 9x + 18 = 0$ is **double** the sum of its roots.

Sol. $x^2 - 9x + 18 = 0$

Sum of the roots = $\frac{-b}{a} = \frac{-(-9)}{1} = 9$

Product of the roots = $\frac{c}{a} = \frac{18}{1} = 18$

$\therefore$ Product of the roots = 2 (Sum of roots)

Assertion-Reason Type Questions

Directions (Q. Nos. 13 to 15): Each of these questions contains an assertion followed by reason. Read them carefully, and answer the question on the basis of the following options, select the one that best describes the two statements.

(a) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.

(b) Both Assertion and Reason are true but Reason is not the correct explanation of Assertion.

(c) If Assertion is true but Reason is false.

(d) If Assertion is false but Reason is true.

13. Assertion: $ax^2 + bx + c = 0$ is not a quadratic equation if $a = 0$.

Reason: If $a = 0$, then there will be no term containing the 2nd power of x.

Sol. The correct answer is (*a*) both the statements are corrrect. A quadratic polynomial must have a non-zero coeffcient.

Thus, the reason is a proper explanation of assertion.

14. Assertion: The roots of the equation $2x^2 - 5x + 3 = 0$ and the zeroes of the polynomial $2x^2 - 5x + 3$ are the same.

Reason: Zeroes of a polynomial satisfy the corresponding equation.

Sol. The correct answer is (*a*) Both the statements are correct since the zeroes of a polynomial can satisfy the corresponding equation, hence they are the solutions also.

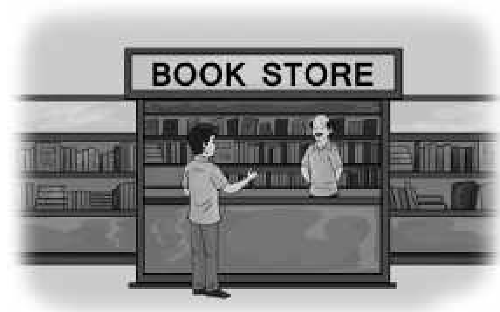
15. Assertion: $2x^2 + x - 528 = 0$ has one real and one imaginary roots.

Reason: The constant term is negative and the coefficient of x^2 is positive.

Sol. The correct answer is (*d*). The assertion is wrong because a quadratic equation cannot have one real and one imaginary root. The reason is correct.

Case Study Based Questions

16. Ankit had bought a number of books from the bookstore for ₹ 640. Also, he had bought same number of books with 8 more books for the same amount, each book would have cost ₹ 4 less. Based on the above information, answer the following questions.



(*a*) Let the number of books bought be x, then the number of books bought later will be

(*i*) x (*ii*) $x + 8$

(*iii*) $x - 8$ (*iv*) $8x$

Ans. (*ii*) $x + 8$

(*b*) What is the cost of each book if the number of books be 16 and the amount paid is ₹ 80?

(*i*) ₹ 5 (*ii*) ₹ 10

(*iii*) ₹ 15 (*iv*) ₹ 20

Ans. (*i*) ₹ 5

(*c*) If same number of books with 8 more books are bought for ₹ 640, then cost of each book reduces by

(*i*) ₹ 2 (*ii*) ₹ 4

(*iii*) ₹ 6 (*iv*) ₹ 8

Ans. (*ii*) ₹ 4

(*d*) Which of the following is the quadratic equation for the number of books bought?

(*i*) $x^2 + 4x - 320 = 0$

(*ii*) $x^2 - 4x + 320 = 0$

(*iii*) $x^2 + 8x - 1280 = 0$

(*iv*) $x^2 + 8x - 640 = 0$

Ans. (*iii*) $x^2 + 8x - 1280 = 0$

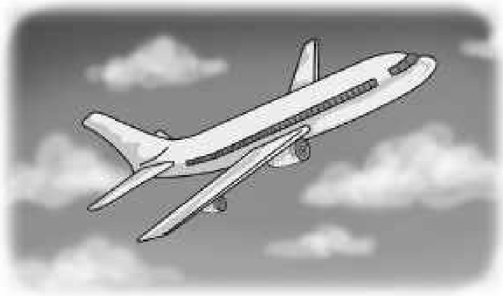
(*e*) The number of books bought initially are

(*i*) 16 (*ii*) 20

(*iii*) 24 (*iv*) 32

Ans. (*iv*) 32

17. An aircraft is a vehicle such as a plane or a helicopter that can fly and carry goods or passengers. An aircraft was slowed down due to bad weather. In a flight of 600 km, its average speed for the trip was reduced by 200 km/hour and the time of the flight increased by 30 minutes. Based on the above situation, answer the following questions.



(*a*) Let the original speed of the aircraft be x km/h, then its reduced speed will be

(*i*) 200 km/h

(*ii*) $(x - 200)$ km/h

(*iii*) $(x + 200)$ km/h

(*iv*) 20 km/h

Ans. (*ii*) $(x - 200)$ km/h

(*b*) What is the relationship between speed, distance and time?

(*i*) Speed = Distance × Time

(*ii*) Time = Speed × Distance

(*iii*) Speed = Distance/Time

(*iv*) Time = Speed/Distance

Ans. (*iii*) Speed = Distance/Time

(*c*) Which of the following is the correct quadratic equation for the average speed of the aircraft?

(*i*) $x^2 - 200x - 240000 = 0$

(*ii*) $x^2 - 200x + 120000 = 0$

(*iii*) $x^2 - 400x + 240000 = 0$

(*iv*) $x^2 + 400x - 120000 = 0$

Ans. (*i*) $x^2 - 200x - 240000 = 0$

(*d*) What is the average speed of the aircraft?

(*i*) 300 km/h (*ii*) 400 km/h

(*iii*) 500 km/h (*iv*) 600 km/h

Ans. (*iv*) 600 km/h

(*e*) What is the duration of the flight?

(*i*) $\frac{1}{2}$ hour (*ii*) 1 hour

(*iii*) 2 hours (*iv*) 3 hours

Ans. (*ii*) 1 hour

Short Answer Type-I Questions

Represent the following situations in the form of quadratic equations (Q. 18 – Q. 20).

18. A two-digit number is such that its ten's digit is the square of the unit's digit and the sum of digits is 12. We need to find the number.

Sol. Let the digit in the unit's place be x.

Then, the digit in the ten's place is x^2.

$\therefore$ According to the problem, we have

$$x^2 + x = 12$$

$$\Rightarrow \quad x^2 + x - 12 = 0$$

$$\Rightarrow \quad x^2 + 4x - 3x - 12 = 0$$

$$\Rightarrow \quad x(x + 4) - 3(x + 4) = 0$$

$$\Rightarrow \quad (x - 3)(x + 4) = 0$$

$\therefore$ Either $\quad x - 3 = 0 \quad$ ...(1)

or, $\quad x + 4 = 0 \quad$ ...(2)

From (1), we have, $x = 3$ which is accepted.

From (2), $x = -4$, which is rejected.

$\therefore$ The digit in the unit's place is 3 and the digit in the ten's place is 3^2 i.e. 9.

$\therefore$ The required number is 39.

19. The cost of $2x$ articles is ₹ $(5x + 54)$ while the cost of $(x + 2)$ articles is ₹ $(10x - 4)$. We need to find x.

Sol. The cost of $2x$ articles is ₹ $(5x + 54)$

Then, the cost of 1 article = ₹ $\frac{5x + 54}{2x}$

The cost of $(x + 2)$ articles is ₹ $(10x - 4)$

Then, the cost of 1 article = ₹ $\frac{10x - 4}{x + 2}$

$\therefore$ According to the problem, we have

$$₹\ \frac{5x + 54}{2x} = ₹\ \frac{10x - 4}{x + 2}$$

$$\Rightarrow \quad (5x + 54)(x + 2) = 2x(10x - 4)$$

$$\Rightarrow \quad 5x^2 + 64x + 108 = 20x^2 - 8x$$

$$\Rightarrow \quad 15x^2 - 72x - 108 = 0$$

$$\Rightarrow \quad 5x^2 - 24x - 36 = 0$$

$$\Rightarrow \quad 5x^2 - 30x + 6x - 36 = 0$$

$$\Rightarrow \quad 5x(x - 6) + 6(x - 6) = 0$$

$$\Rightarrow \quad (5x + 6)(x - 6) = 0$$

$\therefore$ Either $\quad 5x + 6 = 0 \quad$ ...(1)

or, $\quad x - 6 = 0 \quad$ ...(2)

From (1), we have, $x = \frac{-6}{5}$, which is rejected.

From (2), we have, $x = 6$, which is accepted.

$\therefore$ The value of x is 6.

20. Had Neelu scored 10 more marks in her English test out of 30 marks, 9 times these marks would have been the square of her actual marks. We need to find how many marks did she get in the test.

Sol. Let Neelu secured x marks in English.

$\therefore$ According to the problem, we have

$$9(x + 10) = x^2$$

$$\Rightarrow \quad 9x + 90 = x^2$$

$$\Rightarrow \quad x^2 - 9x - 90 = 0$$

$$\Rightarrow \quad x^2 - 10x + 9x - 90 = 0$$

$$\Rightarrow \quad x(x - 10) + 9(x - 10) = 0$$

$$\Rightarrow \quad (x - 10)(x + 9) = 0$$

$\therefore$ Either $\quad x - 10 = 0 \quad \ldots(1)$

or, $\quad x + 9 = 0 \quad \ldots(2)$

From (1), we have, $x = 10$, which is accepted.

From (2), we have, $x = -9$ which is rejected, since x cannot be negative.

$\therefore$ Neelu get 10 marks in the test.

Short Answer Type-II Questions

21. If $ax^2 - 7x + c = 0$ has 14 as the sum of roots and also as the product of roots, find the values of a and c.

Sol. Let α and β be the roots of the given equation. Then

$$\alpha + \beta = \frac{7}{a} = 14$$

$$\Rightarrow \quad a = \frac{7}{14} = \frac{1}{2}$$

and $$\alpha\beta = \frac{c}{a} = 14$$

$$\Rightarrow \quad \frac{c}{\frac{1}{2}} = 14$$

$$\Rightarrow \quad c = \frac{14}{2} = 7$$

Hence, the required values of a and c are $\frac{1}{2}$ i.e. 0.5 and 7 respectively.

22. Find the value of p for which the root of the quadratic equation $px^2 - 14x + 8 = 0$ is six times the other. **[CBSE 2017]**

Sol. If α be one root of the given equation, then the other root is 6α.

$$\therefore \quad \alpha + 6\alpha = \frac{14}{p}$$

$$\Rightarrow \quad 7\alpha = \frac{14}{p} \quad \ldots(1)$$

and $$\alpha \cdot 6\alpha = \frac{8}{p}$$

$$\Rightarrow \quad 3\alpha^2 = \frac{4}{p} \quad \ldots(2)$$

From (1), $$\alpha = \frac{2}{p}$$

$$\Rightarrow \quad \alpha^2 = \frac{4}{p^2}$$

$\therefore$ From (2), $$3 \times \frac{4}{p^2} = \frac{4}{p}$$

$$\Rightarrow \quad p^2 = 3p$$

$$\Rightarrow \quad p = 3 \quad [\because p \neq 0]$$

$\therefore$ The required value of p is 3.

23. If the roots of the equation $(b - c)x^2 + (c - a)x + (a - b) = 0$ are equal, then prove that $2b = a + c$. **[CBSE 2002 C]**

Sol. The discriminant, D of the given equation is given by

$$D = (c - a)^2 - 4(b - c)(a - b)$$

Now, for real and equal roots, D = 0

$$\Rightarrow \quad (c - a)^2 - 4(b - c)(a - b) = 0$$

$$\Rightarrow \quad c^2 + a^2 - 2ac - 4(ab - b^2 - ac + bc) = 0$$

$$\Rightarrow \quad c^2 + a^2 + 4b^2 + 2ac - 4ab - 4bc = 0$$

$$\Rightarrow \quad (2b - a - c)^2 = 0$$

$$\Rightarrow \quad 2b - a - c = 0$$

$$\Rightarrow \quad 2b = a + c$$

Hence, proved.

24. Form an equation whose roots are cubes of the roots of the equation $x^2 + bx + c = 0$.

Sol. Let α and β be the roots of the equation

$$x^2 + bx + c = 0$$

Then $\quad \alpha + \beta = -b \quad \ldots(1)$

and $\quad \alpha\beta = c \quad \ldots(2)$

Now, the equation whose roots are α^3 and β^3 is

x^2 – (sum of the roots)x + product of two roots = 0

$\Rightarrow \; x^2 - (\alpha^3 + \beta^3)x + \alpha^3\beta^3 = 0$

$\Rightarrow \; x^2 - \{(\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)\}x + (\alpha\beta)^3 = 0$

$\Rightarrow \; x^2 - (-b^3 + 3bc)x + c^3 = 0 \quad$ [From (1) and (2)]

$\Rightarrow \; x^2 + b(b^2 - 3c)x + c^3 = 0$

which is the required equation.

25. Form a quadratic equation whose roots are $\frac{1}{a + b}$ and $\frac{1}{a - b}$.

Sol. The equation whose roots are $\frac{1}{a + b}$ and $\frac{1}{a - b}$ is

x^2 – (sum of the roots)x + product of the roots = 0

$$\Rightarrow \quad x^2 - \left(\frac{1}{a+b} + \frac{1}{a-b}\right)x + \frac{1}{a^2-b^2} = 0$$

$$\Rightarrow \quad x^2 - \frac{2ax}{(a+b)(a-b)} + \frac{1}{a^2-b^2} = 0$$

$$\Rightarrow \quad x^2 - \frac{2ax}{a^2-b^2} + \frac{1}{a^2-b^2} = 0$$

$$\Rightarrow \quad (a^2-b^2)x^2 - 2ax + 1 = 0$$

which is the required equation.

26. If α and β are the roots of the equation $3x^2 - 4x + 1 = 0$, form an equation whose roots are $\frac{\alpha^2}{\beta}$ and $\frac{\beta^2}{\alpha}$.

Sol. We have $\quad \alpha + \beta = \frac{4}{3} \quad \ldots(1)$

and $\quad \alpha\beta = \frac{1}{3} \quad \ldots(2)$

$\therefore$ The equation whose roots are $\frac{\alpha^2}{\beta}$ and $\frac{\beta^2}{\alpha}$ is

$$x^2 - \left(\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}\right)x + \frac{\alpha^2}{\beta} \times \frac{\beta^2}{\alpha} = 0$$

$$\Rightarrow \quad x^2 - \frac{\alpha^3+\beta^3}{\alpha\beta}x + \alpha\beta = 0$$

$$\Rightarrow \quad x^2 - \frac{\left\{(\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)\right\}x}{\alpha\beta} + \alpha\beta = 0$$

$$\Rightarrow \quad x^2 - \frac{\left\{\left(\frac{4}{3}\right)^3 - 3 \times \frac{1}{3} \times \frac{4}{3}\right\}x}{\frac{1}{3}} + \frac{1}{3} = 0$$

$$\Rightarrow \quad x^2 - 3\left(\frac{64}{27} - \frac{4}{3}\right)x + \frac{1}{3} = 0$$

$$\Rightarrow \quad x^2 - 3 \times \frac{64-36}{27}x + \frac{1}{3} = 0$$

$$\Rightarrow \quad x^2 - \frac{28}{9}x + \frac{1}{3} = 0$$

$$\Rightarrow \quad 9x^2 - 28x + 3 = 0$$

which is the required equation.

27. If roots of the quadratic equation $x^2 + 2px + mn = 0$ are real and equal, show that the roots of the quadratic equation $x^2 - 2(m+n)x + (m^2 + n^2 + 2p^2) = 0$ are also equal. **[CBSE 2016]**

Sol. The discriminant, D of the equation $x^2 + 2px + mn = 0$ is given by $D = 4p^2 - 4mn$.

Now, for real and equal roots, D = 0.

$$4p^2 - 4mn = 0$$

$$\Rightarrow \quad p^2 = mn \quad \ldots(1)$$

The discriminant, D′ of the second quadratic equation $x^2 - 2(m+n)x + (m^2 + n^2 + 2p^2) = 0$ is given by

$$\begin{aligned} D' &= \{2(m+n)\}^2 - 4(m^2+n^2+2p^2) \\ &= 4(m+n)^2 - 4(m^2+n^2+2p^2) \\ &= 4[m^2+n^2+2mn-m^2-n^2-2p^2] \\ &= 4[2mn-2p^2] \\ &= 4[2p^2-2p^2] \qquad \text{[From (1)]} \\ &= 0 \end{aligned}$$

Hence, the two roots of the second equation are also real and equal.

Long Answer Type Questions

28. Solve the following for x:

$$\frac{1}{2a+b+2x} = \frac{1}{2a} + \frac{1}{b} + \frac{1}{2x}$$

Sol. We have $\quad \frac{1}{2a+b+2x} = \frac{2a+b}{2ab} + \frac{1}{2x}$

$$\Rightarrow \quad \frac{1}{p+2x} = \frac{p}{2ab} + \frac{1}{2x} = \frac{px+ab}{2abx},$$

where $\quad p = 2a + b \quad \ldots(1)$

$\Rightarrow \quad (p+2x)(px+ab) = 2abx$

$\Rightarrow \quad p^2x + pab + 2px^2 + 2abx = 2abx$

$\Rightarrow \quad p^2x + pab + 2px^2 = 0$

$\Rightarrow \quad 2x^2 + px + ab = 0$

$\Rightarrow \quad 2x^2 + (2a+b)x + ab = 0$

[Putting the value of p from (1)]

$\Rightarrow \quad 2x^2 + 2ax + bx + ab = 0$

$\Rightarrow \quad 2x(x+a) + b(x+a) = 0$

$\Rightarrow \quad (x+a)(2x+b) = 0$

$\therefore$ Either $\quad x + a = 0 \quad \ldots(1)$

or, $\quad 2x + b = 0 \quad \ldots(2)$

From (1) and (2), we have $x = -a, -\frac{b}{2}$ which are the required solution.

29. Some students planned a picnic. The total budget for food was ₹ 2000. But 5 students failed to attend the picnic and thus the cost of food for each member increased by ₹ 20. How many students attended the picnic and how much did each student pay for the food?

Sol. Let the number of students who attended the picnic be x. Then the original number of students was $x+5$. Hence, original contribution per student = ₹ $\frac{2000}{x+5}$.

According to the problem, we have

$$\left(\frac{2000}{x+5}+20\right)x = 2000$$

$$\Rightarrow \quad \frac{2000}{x+5} + 20x = 2000$$

$$\Rightarrow \quad \frac{100x}{x+5} + x = 100$$

$$\Rightarrow \quad 100x + x^2 + 5x = 100x + 500$$

$$\Rightarrow \quad x^2 + 5x - 500 = 0$$

$$\Rightarrow \quad x^2 + 25x - 20x - 500 = 0$$

$$\Rightarrow \quad x(x+25) - 20(x+25) = 0$$

$$\Rightarrow \quad (x+25)(x-20) = 0$$

$\therefore$ Either $\quad x + 25 = 0 \quad \ldots(1)$

or, $\quad x - 20 = 0 \quad \ldots(2)$

From (1), $x = -25$ which is rejected, since x cannot be negative.

$\therefore$ From (2), $x = 20$ which is accepted.

Hence, the required number of students who attended the picnic is 20 and the contribution per student is ₹ $\frac{2000}{20}$, i.e. ₹100.

Let's Compete

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Multiple-Choice Questions

1. Quadratic equation whose roots are the reciprocal of the roots of the equation $ax^2 + bx + c = 0$ is

(*a*) $ax^2 + cx + b = 0$

(*b*) $cx^2 + bx + a = 0$

(*c*) $cx^2 - bx + a = 0$

(*d*) $cx^2 + bx - a = 0$

Sol. (*b*) $cx^2 + bx + a = 0$

Let the roots of the equation $ax^2 + bx + c = 0$ be α and β.

Then $\quad \alpha + \beta = -\frac{b}{a} \quad \ldots(1)$

and $\quad \alpha\beta = \frac{c}{a} \quad \ldots(2)$

$\therefore$ The equation whose roots are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ is

$$x^2 - \left(\frac{1}{\alpha}+\frac{1}{\beta}\right)x + \frac{1}{\alpha}\cdot\frac{1}{\beta} = 0$$

$$\Rightarrow \quad x^2 - \frac{\alpha+\beta}{\alpha\beta}x + \frac{1}{\alpha\beta} = 0$$

$$\Rightarrow \quad \alpha\beta x^2 - (\alpha+\beta)x + 1 = 0$$

$$\Rightarrow \quad \frac{c}{a}x^2 + \frac{b}{a}x + 1 = 0$$

$$\Rightarrow \quad cx^2 + bx + a = 0$$

2. Which constant must be added and subtracted to solve the quadratic equation $5x^2 - 6x - 2 = 0$ by the method of completing the square?

(*a*) $\frac{3}{5}$ (*b*) $\frac{36}{25}$

(*c*) $\frac{25}{36}$ (*d*) $\frac{9}{25}$

Sol. (*d*) $\frac{9}{25}$

We have $5x^2 - 6x - 2 = 0$

$$\Rightarrow \quad x^2 - \frac{6}{5}x - \frac{2}{5} = 0$$

$$x^2 - 2\cdot\frac{3}{5}x + \left(\frac{3}{5}\right)^2 - \left(\frac{3}{5}\right)^2 = \frac{2}{5}$$

So, we see that we shall have to add and subtract $\left(\frac{3}{5}\right)^2 = \frac{9}{25}$ in order to make LHS a perfect square.

3. If $x = -2$ and $x = \frac{3}{4}$ are solutions of the equation $px^2 + qx - 6 = 0$, then the values of p and q are respectively

(*a*) 1, 6 (*b*) 5, 4

(*c*) 4, 5 (*d*) 6, 1

Sol. (*c*) 4, 5

We have,

$$\text{Sum of the roots} = -2 + \frac{3}{4} = -\frac{5}{4} = -\frac{q}{p}$$

$$\Rightarrow \quad 5p = 4q \quad \ldots(1)$$

Also, product of the roots

$$= \quad -2 \times \frac{3}{4} = -\frac{6}{p}$$

$$\Rightarrow \quad 3p = 12$$

$$\Rightarrow \quad p = \frac{12}{3} = 4$$

$$\therefore \quad q = \frac{5}{4} \times 4 \quad \text{[From (1)]}$$

$$= 5$$

$\therefore$ The values of p and q are 4 and 5 respectively.

4. If two numbers m and n are such that the quadratic equation $mx^2 + 3x + 2n = 0$ has -6 as the sum of the roots and also as the product of roots then

(*a*) $m = \frac{1}{2}, n = \frac{-3}{2}$

(*b*) $m = \frac{-3}{2}, n = \frac{1}{2}$

(*c*) $m = \frac{2}{3}, n = \frac{-1}{2}$

(*d*) $m = \frac{-2}{3}, n = \frac{3}{2}$

Sol. (*a*) $m = \frac{1}{2}, n = \frac{-3}{2}$

According to the problem, we have

Sum of the roots = -6

$$\Rightarrow \quad \frac{-3}{m} = -6$$

$$\Rightarrow \quad m = \frac{3}{6} = \frac{1}{2} \quad \ldots(1)$$

and product of the roots = -6

$$\Rightarrow \quad \frac{2n}{m} = -6$$

$$\Rightarrow \quad \frac{2n}{\frac{1}{2}} = -6 \quad \text{[From (1)]}$$

$$\Rightarrow \quad n = -\frac{6}{4} = -\frac{3}{2}$$

$$\therefore \quad m = \frac{1}{2} \text{ and } n = -\frac{3}{2}$$

5. The value of y which satisfies the equation $1 + \frac{y^2}{13} = \sqrt{\frac{27}{169} + 1}$ is

(*a*) ± 2 (*b*) ± 1

(*c*) ± 3 (*d*) ± 4

Sol. (*b*) ± 1

We have $$1 + \frac{y^2}{13} = \sqrt{\frac{27}{169} + 1} = \sqrt{\frac{196}{169}} = \frac{14}{13}$$

$$\Rightarrow \quad \frac{y^2}{13} = \frac{14}{13} - 1 = \frac{1}{13}$$

$$\therefore \quad y^2 = 1$$

$$\Rightarrow \quad y = \pm 1$$

6. If $x = \sqrt{6 + \sqrt{6 + \sqrt{6 \ldots}}}$, then the value of x is

(*a*) 1 (*b*) 2

(*c*) 3 (*d*) 4

Sol. (*c*) 3

We have $$x = \sqrt{6 + x}$$

$$\Rightarrow \quad x^2 = 6 + x$$

[Squaring both sides]

$$\Rightarrow \quad x^2 - x - 6 = 0$$

$$\Rightarrow \quad x^2 - 3x + 2x - 6 = 0$$

$$\Rightarrow \quad x(x - 3) + 2(x - 3) = 0$$

$$\Rightarrow \quad (x - 3)(x + 2) = 0$$

$\therefore$ Either $x + 2 = 0$ …(1)

or, $x - 3 = 0$ …(2)

From (1), $x = -2$ which is rejected, since x is not negative.

From (2), $x = 3$ which is accepted.

7. If $x = 1$ is a common root of $ax^2 + ax + 2 = 0$ and $x^2 + x + b = 0$, then $a : b$ is equal to

(*a*) 1 : 2 (*b*) 2 : 1

(*c*) 1 : 4 (*d*) 4 : 1

Sol. (*a*) 1 : 2

Since $x = 1$ is a common root of both the given equation, we have

$$a + a + 2 = 0$$

$$\Rightarrow \quad a = -1 \text{ and } 1 + 1 + b = 0$$

$$\Rightarrow \quad b = -2$$

$$\therefore \quad a : b = 1 : 2$$

8. The ratio of sum and products of the roots of the equation $3x^2 + 12 - 13x = 0$ is

(*a*) 12 : 13 (*b*) 13 : 12

(*c*) 6 : 7 (*d*) 7 : 6

Sol. (*b*) 13 : 12

Let α and β be the roots of the given equation $3x^2 + 12 - 13x$.

We have $x^2 + 4 - \frac{13}{3}x$

$$\Rightarrow \quad x^2 - \frac{13}{3}x + 4$$

Then, $\alpha + \beta = \frac{13}{3}$ and $\alpha\beta = 4$

$\therefore$ The required ratio $\frac{\alpha + \beta}{\alpha\beta}$ is $\frac{\frac{13}{3}}{4}$ i.e. $\frac{13}{12}$

9. If 1 is a root of the equations $ay^2 + ay + 3 = 0$ and $y^2 + y + b = 0$, then ab equals

(*a*) 3 (*b*) $\frac{-7}{2}$

(*c*) 6 (*d*) -3 **[CBSE SP 2012]**

Sol. (*a*) 3

We see that $y = 1$ will satisfy both the given equations

$\therefore \quad a + a + 3 = 0$

$\Rightarrow \quad a = -\frac{3}{2} \quad \ldots(1)$

and $\quad 1 + 1 + b = 0$

$\Rightarrow \quad b = -2 \quad \ldots(2)$

$\therefore \quad ab = -\frac{3}{2} \times (-2)$ [From (1) and (2)]

$= 3$

10. If one root of $3x^2 = 8x + (2k + 1)$ is seven times the other, then the roots are

(*a*) $-3, -\frac{3}{7}$ (*b*) $\frac{1}{3}, \frac{7}{3}$

(*c*) $-\frac{1}{3}, -\frac{7}{3}$ (*d*) $3, \frac{3}{7}$

Sol. (*b*) $\frac{1}{3}, \frac{7}{3}$

Let α and 7α be the roots of the given equation

$$3x^2 = 8x + (2k + 1)$$

Then,

Sum of the roots $= \alpha + 7\alpha = \frac{8}{3}$

$\Rightarrow \quad 8\alpha = \frac{8}{3}$

$\Rightarrow \quad \alpha = \frac{1}{3}$

$\therefore$ The required roots are $\frac{1}{3}$ and $\frac{7}{3}$.

Value-based Questions (Optional)

(Page 79)

1. A passenger, while boarding the plane, slipped from the stairs and got hurt. The pilot took the passenger in the emergency clinic at the airport for treatment. Due to this, the plane got delayed by half an hour. To reach the destination 1500 km away in time, so that the passengers could catch the connecting flight, the speed of the plane was increased by 250 km/hour than the usual speed. Find the usual speed of the plane. What value is depicted in this question? **[CBSE 2016]**

Sol. Let the usual speed of the aeroplane be x km/h. Then the increased speed is $(x + 250)$ km/h.

$\therefore$ According to the problem, we have

$$\frac{1500}{x} = \frac{1500}{x + 250} + \frac{1}{2}$$

$\Rightarrow \quad 1500\left(\frac{1}{x} - \frac{1}{x + 250}\right) = \frac{1}{2}$

$\Rightarrow \quad \frac{1500(x + 250 - x)}{x(x + 250)} = \frac{1}{2}$

$\Rightarrow \quad x^2 + 250x - 750000 = 0$

$\Rightarrow \quad x^2 - 750x + 1000x - 750000 = 0$

$\Rightarrow \quad x(x - 750) + 1000(x - 750) = 0$

$\Rightarrow \quad (x - 750)(x + 1000) = 0$

$\therefore$ Either $\quad x - 750 = 0 \quad \ldots(1)$

or, $\quad x + 1000 = 0 \quad \ldots(2)$

From (1), $x = 750$ and from (2), $x = -1000$ which is rejected, since x is negative.

Hence, the required speed of the aeroplane is 750 km/h.

Values: Critical thinking and decision making.

2. On Van Mahotsav day some students planted trees in horizontal and vertical rows. They planted 480 trees in all such that there were four trees more in each horizontal row than in vertical row.

(*a*) Find the number of trees in each horizontal row.

(*b*) What value was exhibited by the students who planted trees?

Sol. (*a*) Let the number of trees planted in each horizontal row $= x$.

Then, the number of trees planted in each vertical row $= (x - 4)$

Given, total number of trees = 480

$\therefore \quad x(x - 4) = 480$

$\Rightarrow \quad x^2 - 4x - 480 = 0$

$\Rightarrow \quad x^2 - 24x + 20x - 480 = 0$

$\Rightarrow \quad x(x - 24) + 20(x - 24) = 0$

$\Rightarrow \quad (x - 24)(x + 20) = 0$

$\therefore$ Either $\quad x - 24 = 0 \quad \ldots(1)$

or $\quad x + 20 = 0 \quad \ldots(2)$

From (1), we have, $x = 24$, which is accepted.

From (2), we have $x = -20$, which is rejected, since x cannot be negative.

$\therefore$ The required number of trees in each horizontal row is 24.

(*b*) Environmental awareness.

3. Three-eighth of the students of a class opted for visiting an old age home. Sixteen students opted for having a nature walk. Square root of total number of students in the class opted for tree plantation in the school. The number of students who visited an old age home is same as the number of students who went for a nature walk and did tree plantation. Find the total number of students. What values are inculcated in students through such activities? **[CBSE SP 2015]**

Sol. Let the total number of students be x.

Then, according to the problem, we have

$$\frac{3x}{8} = 16 + \sqrt{x}$$

$$\Rightarrow \quad 3x - 8\sqrt{x} - 128 = 0$$

$$\Rightarrow \quad 3a^2 - 8a - 128 = 0$$

where $\quad a = \sqrt{x} \quad \ldots(1)$

$$\Rightarrow \quad 3a^2 - 24a + 16a - 128 = 0$$

$$\Rightarrow \quad 3a(a - 8) + 16(a - 8) = 0$$

$$\Rightarrow \quad (a - 8)(3a + 16) = 0$$

$\therefore$ Either $\quad 3a + 16 = 0 \quad \ldots(2)$

or $\quad a - 8 = 0 \quad \ldots(3)$

From (2), $a = -\dfrac{16}{3}$, which is rejected since a is not negative.

$\therefore$ From (3), $a = 8$ which is accepted

$\therefore \quad a = 8$

$\Rightarrow \quad \sqrt{x} = 8 \quad$ [From (1)]

$\Rightarrow \quad x = 64 \quad$ [Squaring both sides]

Hence, the required number of students are 64.

Values: Environmental awareness, caring and concern.

5

Arithmetic Progressions

Checkpoint (Page 82)

1. Which of the following is (or are) a linear equation (or linear equations) in one variable?

(*a*) $ax^2 + bx + c = 0$ (*b*) $\sqrt{2}x - 3 = 6$

(*c*) $2x - y = 3$ (*d*) $x^2 + y^2 = a^2$

(*e*) $-\frac{2}{x} + 5x + 7 = 0$ (*f*) $n(n+1) - \frac{2n+1}{6} = 3$

Sol. (*b*) $\sqrt{2}x - 3 = 6$

Only in (*b*), we see that there is only one variable x with exponent 1.

2. If $2y - 3 = -5y + 11$, then the solution of this equation is

(*a*) $y = -2$ (*b*) $y = 3$

(*c*) $y = -3$ (*d*) $y = 2$

Sol. (*d*) $y = 2$

From the given equation, we see that

$$2y + 5y = 11 + 3$$

$$\Rightarrow \quad 7y = 14$$

$$\Rightarrow \quad y = \frac{14}{7} = 2$$

3. Find the correct answer from the following. The solution of the inequation $-3x - 2 > -6 + x$ is

(*a*) $x > 1$ (*b*) $x < -1$

(*c*) $x < 1$ (*d*) $-1 < x < 1$

Sol. (*c*) $x < 1$

We have $\quad -3x - x > -6 + 2$

$$\Rightarrow \quad -4x > -4$$

$$\Rightarrow \quad x < 1$$

4. If $t_n = (-1)^n n + 5$, find the values of

(*a*) t_4 (*b*) $t_7 t_9$ (*c*) $\frac{t_3}{t_2}$

Sol. (*a*) For $n = 4$,

$$t_4 = (-1)^4 \times 4 + 5 = 4 + 5 = 9$$

(*b*) For $n = 7$,

$$t_7 = (-1)^7 \times 7 + 5 = -7 + 5 = -2$$

and for $n = 9$,

$$t_9 = (-1)^9 \times 9 + 5 = -9 + 5 = -4$$

$$\therefore \quad t_7 t_9 = (-2) \times (-4) = 8$$

which is the required value.

(*c*) For $n = 3$, $t_3 = (-1)^3 \times 3 + 5 = -3 + 5 = 2$

and for $n = 2$,

$$t_2 = (-1)^2 \times 2 + 5 = 2 + 5 = 7$$

$$\therefore \quad \frac{t_3}{t_2} = \frac{2}{7}$$

which is the required value.

5. If $t_n = \frac{2n+1}{n+1}$, find the value of t_{n-1} in terms of n.

Sol. Replacing n by $n - 1$, we get

$$t_{n-1} = \frac{2(n-1)+1}{n-1+1} = \frac{2n-2+1}{n} = \frac{2n-1}{n} = 2 - \frac{1}{n}$$

6. If $(x + y)^2 = 4xy$, prove that $x = y$.

Sol. We have $(x + y)^2 - 4xy = 0$

$\Rightarrow \quad x^2 + y^2 + 2xy - 4xy = 0$

$\Rightarrow \quad x^2 + y^2 - 2xy = 0$

$\Rightarrow \quad (x - y)^2 = 0$

$\Rightarrow \quad x = y$

Hence, proved.

7. Find three consecutive positive integers whose sum is 219.

Sol. Let three consecutive positive integers be x, $x+1$ and $x + 2$. According to the problem, we have

$x + x + 1 + x + 2 = 219$

$\Rightarrow \quad 3x + 3 = 219$

$\Rightarrow \quad 3x = 219 - 3 = 216$

$\Rightarrow \quad x = \dfrac{216}{3} = 72$

$\therefore$ The required numbers are 72, 73 and 74.

8. The angles of a triangle are $3x°$, $(2x + 20)°$ and $(5x - 40)°$. Find the angles.

Sol. Using angle sum property of a triangle, we have

$3x° + (2x + 20)° + (5x - 40)° = 180°$

$\Rightarrow \quad 10x° = 180° + 20°$

$= 200°$

$\therefore \quad x = \dfrac{200°}{10}$

$= 20°$

$\therefore$ The required angles are $3 \times 20°$, $(2 \times 20 + 20)°$ and $(5 \times 20 - 40)°$, i.e. 60°, 60° and 60°, i.e. 60° each.

Milestone 1

(Page 88)

Multiple-Choice Questions

1. The 15th term of the AP $-5, \dfrac{-5}{2}, 0, \dfrac{5}{2}, \ldots$ is

(*a*) –30 (*b*) 20

(*c*) 30 (*d*) –20

Sol. (*c*) 30

For the given AP, first term of $a = -5$, common difference, $d = -\dfrac{5}{2} - (-5) = \dfrac{-5}{2} + 5 = \dfrac{5}{2}$.

Let us denote the nth term of the AP by a_n.

$\therefore \quad a_n = a + (n - 1)d$

$= -5 + (n - 1) \times \dfrac{5}{2}$

$= -5 + \dfrac{5n}{2} - \dfrac{5}{2}$

$\Rightarrow \quad a_n = -\dfrac{15}{2} + \dfrac{5n}{2}$

$\therefore \quad a_{15} = -\dfrac{15}{2} + \dfrac{5}{2} \times 15$

$= \dfrac{75 - 15}{2}$

$= \dfrac{60}{2} = 30$

2. The 10th term from the end of the AP 7, 11, 15, 19, …, 103 is

(*a*) 64 (*b*) 67

(*c*) 70 (*d*) 54

Sol. (*b*) 67

Here, first term, $a = 7$, common difference, $d = 11 - 7 = 4$. Let n be the total number of terms of the AP and a_n be the nth term of the AP.

$\therefore \quad a_n = 103$

$\Rightarrow \quad a + (n - 1)d = 103$

$\Rightarrow \quad 7 + (n - 1)4 = 103$

$\Rightarrow \quad n - 1 = \dfrac{103 - 7}{4}$

$= \dfrac{96}{4} = 24$

$\therefore \quad n = 24 + 1$

$= 25$

Now, 10th term from the end = (25 – 10 + 1)th = 16th term from the beginning.

$\therefore \quad a_{16} = 7 + (16 - 1) \times 4$

$= 7 + 15 \times 4 = 67$

3. The 8th term of an AP whose first two terms are 5 and 11 is

(*a*) 40 (*b*) 46

(*c*) 35 (*d*) 47

Sol. (*d*) 47

Here, the first term, $a = 5$, common difference, $d = 11 - 5 = 6$.

$\therefore \quad a_8 = a + (8 - 1)d$

$= 5 + 7 \times 6$

$= 5 + 42$

$= 47$

Very Short Answer Type Questions

4. If d be the common difference of an AP, and if each term of the AP is decreased by 4, what is the common difference of this new AP?

Sol. Let the original AP be $a, a+d, a+2d, \dots a+(n-1)d$.

Then the new AP is $a-4, a+d-4, a+2d-4, \dots a+(n-1)d-4$.

If D be the new common difference for the new AP, then

$$D = (a+d-4)-(a-4) = d$$

Hence, the required common difference of the new AP is also d.

5. If each term of an AP is multiplied by 3, will the resulting sequence be an AP? If so, what will be its common difference?

Sol. Let the original AP be $a,\ a+d,\ a+2d,\ \dots\dots,\ a+(n-1)d$.

Then the new sequence will be $3a$, $3(a+d)$, $3(a+2d)$, ... , $3\{a+(n-1)d\}$

Here $\quad a_n = 3\{a+(n-1)d\}$

$\therefore \quad a_{n-1} = 3\{a+(n-2)d\}$

$\therefore$ If D be the common difference of this new sequence, then

$$\begin{aligned} D &= a_n - a_{n-1} \\ &= 3a + 3(n-1)d - 3a - 3(n-2)d \\ &= 3nd - 3d - 3nd + 6d \\ &= 3d \end{aligned}$$

which is independent of n.

$\because$ D is a constant

$\therefore$ Yes, the new sequence will also be an AP with common difference equal to 3 times the common difference of the original AP.

6. There are 1000 terms in an AP. If $a_{107} = a_{106} + 3a + b$ where $a > 0$ and $b > 0$ and the successive terms of the AP are in increasing order, what is its common difference?

Sol. Successive terms of the given AP are a_{106} and a_{107}. We have

$$\begin{aligned} a_{107} - a_{106} &= 3a + b \\ a + 106d - a - 105d &= 3a + b \\ d &= 3a + b \end{aligned}$$

Hence, the required common difference is $3a+b$.

7. If there are 60 terms in an AP, what will be the position of a certain term of the AP from the beginning, if its position from the end is 19th?

Sol. 19th term from the end of the given AP is $(60 - 19 + 1)$th term, i.e. 42nd term from the beginning. Hence, the required position is 42nd term from the beginning.

8. If the simple interest rate on ₹200 is 5% every year, then what will be its total interest at the end of 4th year? Are the total interests at the end of 1st year, 2nd year, 3rd year, etc. in AP? If so, what is the common difference of this AP?

Sol. We see that simple interest for 1 year on ₹200 is ₹ $\dfrac{200 \times 5 \times 1}{100}$, i.e. ₹ $\dfrac{2 \times 5}{1}$, i.e. 10

$\therefore$ Interests at the end of 1st year, 2nd year, 3rd year, 4th year, etc. will be ₹10, ₹20, ₹30, ₹40, ... etc respectively, which is clearly an AP with common difference of ₹10. Also, the required total interest at the end of 4 years is ₹40.

9. Insert a number between 2 and 3 such that these three numbers are in AP.

Sol. Let the three numbers in AP be $2, 2+d, 3$, where d is the common difference.

$$\begin{aligned} \therefore \quad 2 + d - 2 &= 3 - 2 - d \\ &= 1 - d \\ \Rightarrow \quad 2d &= 1 \\ \Rightarrow \quad d &= \frac{1}{2} \end{aligned}$$

$\therefore$ The required middle term between 2 and 3 is $2 + d$, i.e. $2 + \dfrac{1}{2}$, i.e. $\dfrac{5}{2}$.

Short Answer Type-I Questions

10. Find the sequence whose nth term is $\dfrac{n^2}{n+3}$.

Sol. If a_n be the nth term of the sequence, then

$$a_n = \frac{n^2}{n+3}$$

$$\begin{aligned} \therefore \quad a_1 &= \frac{1}{1+3} = \frac{1}{4}, \\ a_2 &= \frac{2^2}{2+3} = \frac{4}{5}, \\ a_3 &= \frac{3^2}{3+3} = \frac{9}{6} = \frac{3}{2}, \\ a_4 &= \frac{4^2}{4+3} = \frac{16}{7}, \text{ etc.} \end{aligned}$$

Hence, the required sequence is $\dfrac{1}{4}, \dfrac{4}{5}, \dfrac{3}{2}, \dfrac{16}{7}, \dots$

11. Find the 7th term of the AP 1, 6, 11, 16,...

Sol. Here, first term, $a = 1$, common difference, $d = 6 - 1 = 5$.

$\therefore$ 7th term, $a_7 = a + (7-1)d = 1 + 6 \times 5 = 31$

$\therefore$ The required 7th term is 31.

12. Find the 10th term of the AP $\sqrt{2}, \sqrt{8}, \sqrt{18}, \ldots$

[CBSE 2015]

Sol. Here, first term, $a = \sqrt{2}$, common difference, $d = \sqrt{8} - \sqrt{2} = 2\sqrt{2} - \sqrt{2} = \sqrt{2}$

$$\therefore \quad a_{10} = a + (10-1)d = \sqrt{2} + 9 \times \sqrt{2} = 10\sqrt{2} = \sqrt{200}$$

$\therefore$ The required 10th term is $\sqrt{200}$.

13. Find the 25th term of the AP $-5, -\frac{5}{2}, 0, \frac{5}{2}, \ldots$

[CBSE 2015]

Sol. Here, first term, $a = -5$, common difference, $d = -\frac{5}{2} + 5 = \frac{5}{2}$.

$$\therefore \quad a_{25} = a + (25-1)d = -5 + 24 \times \frac{5}{2} = 60 - 5 = 55$$

$\therefore$ The required 25th term is 55.

14. The nth term of an AP is $7 - 4n$. Find its common difference. **[CBSE 2008]**

Sol. We have $a_n = 7 - 4n$

$$\therefore \quad a_{n-1} = 7 - 4(n-1) = 7 - 4n + 4 = 11 - 4n$$

$$\therefore \quad a_n - a_{n-1} = 7 - 4n - (11 - 4n) = -4$$

$\therefore$ The required common difference is -4.

Short Answer Type-II Questions

15. Which term of the AP 21, 18, 15, … is 0?

[CBSE SP 2012]

Sol. Here, first term $a = 21$, common difference, $d = 18 - 21 = -3$. If a_n be 0, then

$$a_n = a + (n-1)d = 0$$

$$\Rightarrow \quad 21 - 3(n-1) = 0$$

$$\Rightarrow \quad n - 1 = \frac{21}{3} = 7$$

$$\Rightarrow \quad n = 7 + 1 = 8$$

Hence, the required term is 8th term.

16. Which term of the AP 3, 10, 17, 24,… will be 84 more than its 13th term? **[CBSE 2004]**

Sol. Here, first term, $a = 3$, common difference, $d = 10 - 3 = 7$.

$$\therefore \quad a_{13} = a + (13-1)d = 3 + 12 \times 7 = 3 + 84 = 87$$

Let nth term, a_n be $84 + 87 = 171$

$$\therefore \quad a_n = 171$$

$$\Rightarrow \quad a + (n-1)d = 171$$

$$\Rightarrow \quad 3 + (n-1) \times 7 = 171$$

$$\Rightarrow \quad (n-1) \times 7 = 171 - 3 = 168$$

$$\Rightarrow \quad n - 1 = \frac{168}{7} = 24$$

$$\Rightarrow \quad n = 24 + 1 = 25$$

$\therefore$ The required term is 25th term.

17. The 7th term of an AP is -4 and its 13th term is -16. Find the AP. **[CBSE 2004, SP 2011]**

Sol. Let a be the 1st term and d be the common difference of the AP.

Then according to the problem, we have

$$a_7 = -4$$

$$\Rightarrow \quad a + (7-1)d = -4$$

$$\Rightarrow \quad a + 6d + 4 = 0 \qquad \ldots(1)$$

and $a_{13} = -16$

$$\Rightarrow \quad a + (13-1)d = -16$$

$$\Rightarrow \quad a + 12d + 16 = 0 \qquad \ldots(2)$$

Subtracting (1) from (2), we get

$$6d = -12$$

$$\Rightarrow \quad d = -\frac{12}{6} = -2$$

$\therefore$ From (1),

$$a = -4 - 6d = -4 - 6 \times (-2) = -4 + 12 = 8$$

Hence, the required AP is 8, 6, 4, 2, 0, –2, …

18. The 9th term of an AP is -32 and the sum of its 11th and 13th terms is -94. Find the common difference of the AP. **[CBSE 2013]**

Sol. Let a be the 1st term, d be the common difference and let a_n be the nth term. Now, according to the problem, we have

$$a_9 = -32$$

$$\Rightarrow \quad a + (9-1)d = -32$$

$$\Rightarrow \quad a + 8d = -32 \qquad \ldots(1)$$

$$a_{11} + a_{13} = -94$$

$$\Rightarrow \quad a + (11 - 1)d + a + (13 - 1)d = -94$$

$$\Rightarrow \quad 2a + 22d = -94$$

$$\Rightarrow \quad a + 11d = -47 \qquad \ldots(2)$$

$\therefore$ Subtracting (2) from (1), we get

$$-3d = 47 - 32 = 15$$

$$\Rightarrow \quad d = \frac{15}{-3} = -5$$

Hence, the required common difference is –5.

19. The angles of a quadrilateral are in AP. If the least angle is 60°, find the other angles.

Sol. The least angle of a quadrilateral = 60° [Given]

Let the other angles of the quadrilateral be $60° + d°$, $60° + 2d°$ and $60° + 3d°$.

$$\therefore\ 60° + 60° + d° + 60° + 2d° + 60° + 3d° = 360°$$

[By angle sum property of a quadrilateral]

$$\Rightarrow \quad 6d° = 360° - 240° = 120°$$

$$\therefore \quad d° = \frac{120°}{6} = 20°$$

$\therefore$ The required angles of the quadrilateral are $60° + 20°$, $60° + 40°$, $60° + 60°$, i.e. 80°, 100° and 120°.

Long Answer Type Questions

20. Each of the AP's 2, 4, 6, 8,… and 3, 6, 9, 12, … is continued to 200 terms. How many terms of these two AP's are identical?

Sol. We see that for the 1st AP, the first term, $a = 2$, common difference, $d = 4 - 2 = 2$. If a_n denote the nth term of this AP, then

$$\begin{aligned} a_{200} &= a + (n - 1)d \\ &= 2 + 199 \times 2 \\ &= 2 \times 200 \\ &= 400 \end{aligned}$$

Similarly, for the 2nd AP, $a' = 3$, $d' = 6 - 3 = 3$

$$\begin{aligned} \therefore \quad a'_{200} &= a' + (200 - 1)d' \\ &= 3 + 199 \times 3 \\ &= 3 \times 200 \\ &= 600 \end{aligned}$$

$$\because \quad 400 < 600$$

$\therefore$ The identical terms in the two AP's will be upto 400 only.

Now, the terms in the 1st AP are multiples of 2 and those in the 2nd AP are multiples of 3. Hence, the identical terms of the two AP's will be multiples of $3 \times 2 = 6$. Hence, the terms which are identical to the two AP's are 6, 12, 18, 24, … which is again another AP with $a = 6$ and $d = 12 - 6 = 6$. Let the mth term of this AP be 400.

$$\therefore \quad a_m = 6 + (m - 1)6 \le 400$$

$$\Rightarrow \quad 6m \le 400$$

$$\Rightarrow \quad m \le 66\frac{2}{3}$$

Since m is a positive integer, hence $m = 66$ which is the required number of terms.

21. If $\frac{b+c-a}{a}$, $\frac{c+a-b}{b}$ and $\frac{a+b-c}{c}$ are in AP and $a + b + c \neq 0$, then show that $\frac{1}{a}$, $\frac{1}{b}$ and $\frac{1}{c}$ are in AP.

Sol. Since, $\frac{b+c-a}{a}$, $\frac{c+a-b}{b}$ and $\frac{a+b-c}{c}$ are in AP

$$\therefore \quad \frac{2(c+a-b)}{b} + 2 = \frac{b+c-a}{a} + 1 + \frac{a+b-c}{c} + 1$$

$$\frac{2(a+c)}{b} - \frac{2b}{b} + 2 = \frac{b+c}{a} - \frac{a}{a} + 1 + \frac{a+b}{c} - \frac{c}{c} + 1$$

$$\frac{2(a+c)}{b} - 2 + 2 = \frac{b+c}{a} - 1 + 1 + \frac{a+b}{c} - 1 + 1$$

$$\frac{2(a+c)}{b} = \frac{b+c}{a} + \frac{a+b}{c}$$

$$\Rightarrow \quad \frac{2(a+c)}{b} + 2 = \left(\frac{b+c}{a} + 1\right) + \left(\frac{a+b}{c} + 1\right)$$

$$\Rightarrow \quad \frac{2(a+b+c)}{b} = \frac{a+b+c}{b} + \frac{a+b+c}{c}$$

$$\Rightarrow \quad \frac{2}{b} = \frac{1}{a} + \frac{1}{c}$$

[Dividing both sides by $a + b + c$, where $a + b + c \neq 0$]

Hence, $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in AP.

22. If $l^2(m + n)$, $m^2(n + l)$ and $n^2(l + m)$ are in AP, prove that either l, m, n are in AP or $lm + mn + nl = 0$.

Sol. Given that $l^2(m + n)$, $m^2(n + l)$ and $n^2(l + m)$ are in AP

$$\therefore \quad 2m^2(n + l) = l^2(m + n) + n^2(l + m)$$

$$\Rightarrow \quad 2m^2n + 2m^2l - l^2m - l^2n - n^2l - n^2m = 0 \quad \ldots(1)$$

Now, l, m, n will be in AP if $2m = l + n$ …(2)

$\therefore$ l, m, n will be in AP or, $lm + mn + nl = 0$ …(3)

If either (2) is true or (3) is true.

i.e. if $(2m - l - n)(lm + mn + nl) = 0$

i.e. if $2m^2l + 2m^2n + 2mnl - l^2m - lmn - nl^2$

$$- nlm - mn^2 - n^2l = 0$$

i.e. if $2m^2n + 2m^2l - l^2m - l^2n - n^2l - n^2m = 0$

which is true. [From (1)]

Hence, proved.

23. Divide 30 into four parts which are in AP such that the ratio of the product of the first and the fourth terms and the product of the second and third terms is 2 : 3.

Sol. Let a, $a + d$, $a + 2d$, $a + 3d$ be four terms of an AP

Given that

$$a + a + d + a + 2d + a + 3d = 4a + 6d = 30$$

$$\Rightarrow \quad 2a + 3d = 15$$

$$\Rightarrow \quad a = \frac{15 - 3d}{2} \quad \ldots(1)$$

Also, $$\frac{a(a + 3d)}{(a + d)(a + 2d)} = \frac{2}{3}$$

$$\Rightarrow \quad 3a^2 + 9ad = 2a^2 + 6ad + 4d^2$$

$$\Rightarrow \quad a^2 + 3ad - 4d^2 = 0$$

$$\Rightarrow \quad \frac{(15 - 3d)^2}{4} + 3d \times \frac{15 - 3d}{2} - 4d^2 = 0$$

[From (1)]

$$\Rightarrow \quad \frac{225 + 9d^2 - 90d}{4} + \frac{45d - 9d^2}{2} - 4d^2 = 0$$

$$\Rightarrow \quad 225 + 9d^2 - 90d + 90d - 18d^2 - 16d^2 = 0$$

$$\Rightarrow \quad 25d^2 = 225$$

$$\Rightarrow \quad d^2 = 9$$

$$\Rightarrow \quad d = \pm 3 \quad \ldots(2)$$

When $d = 3$, then from (1), $a = \dfrac{15 - 3 \times 3}{2} = 3$

When $d = -3$, then from (1), $a = \dfrac{15 + 3 \times 3}{2} = 12$

Hence, the required terms are either 3, 6, 9, 12 or 12, 9, 6, 3.

24. If the pth terms of the progressions 2, $3\dfrac{11}{18}$, $5\dfrac{2}{9}$, $6\dfrac{5}{6}$, … and 94, $91\dfrac{7}{9}$, $89\dfrac{5}{9}$, $87\dfrac{1}{3}$, … be the same, find the value of p and the value of the term.

Sol. For the 1st AP, the first term, $a = 2$ and the common difference, $d = 3\dfrac{11}{18} - 2 = \dfrac{65}{18} - 2$

$= \dfrac{65 - 36}{18} = \dfrac{29}{18}$, and for the 2nd AP, $a' = 94$ and

$$d' = 91\frac{7}{9} - 94 = \frac{826 - 846}{9} = -\frac{20}{9}$$

Given, the pth terms for both the AP's are same.

Now, for the 1st AP, $a_p = 2 + (p - 1)\dfrac{29}{18}$

and for the 2nd AP, $a'_p = 94 - (p - 1)\dfrac{20}{9}$

$$\therefore \quad 2 + (p - 1)\frac{29}{18} = 94 - (p - 1)\frac{20}{9}$$

$$\Rightarrow \quad (p - 1)\left(\frac{29}{18} + \frac{20}{9}\right) = 94 - 2$$

$$\Rightarrow \quad (p - 1)\left(\frac{29 + 40}{18}\right) = 92$$

$$\Rightarrow \quad p - 1 = 92 \times \frac{18}{69}$$

$$\Rightarrow \quad p - 1 = 4 \times 6$$

$$\Rightarrow \quad p - 1 = 24$$

$$\therefore \quad p = 25$$

Now, $$a_{25} = 2 + 24 \times \frac{29}{18}$$

$$= 2 + 4 \times \frac{29}{3}$$

$$= 2 + \frac{116}{3}$$

$$= \frac{6 + 116}{3}$$

$$= \frac{122}{3}$$

$$= 40\frac{2}{3}$$

Hence, the required value of p is 25 and the required 25th term is $40\dfrac{2}{3}$.

Milestone 2

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Multiple-Choice Questions

1. The nth term of an AP whose sum of n terms is S_n, is

(*a*) $S_n - S_{n+1}$ (*b*) $S_n + S_{n-1}$

(*c*) $S_n + S_{n+1}$ (*d*) $S_n - S_{n-1}$

Sol. (*d*) $S_n - S_{n-1}$

We have $$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$= na + \frac{n(n-1)d}{2}$$

and $$S_{n-1} = \frac{n-1}{2}[2a + (n-2)d]$$

$$= (n-1)a + \frac{(n-1)(n-2)d}{2}$$

$\therefore$ $S_n - S_{n-1}$

$$= na + \frac{n(n-1)d}{2} - (n-1)a - \frac{(n-1)(n-2)d}{2}$$

$$= na - na + a + \frac{d\left(n^2 - n - n^2 + 3n - 2\right)}{2}$$

$$= a + \frac{d(2n-2)}{2}$$

$$= a + (n-1)d$$

$$= a_n$$

$\therefore$ The nth term of an AP whose sum of n terms is S_n, is $S_n - S_{n-1}$.

2. The nth term of an AP whose sum is given by $S_n = \frac{5n^2}{2} + \frac{3n}{2}$, will be

(a) $7n - 1$ (b) $5n + 1$
(c) $6n - 1$ (d) $5n - 1$

Sol. (d) $5n - 1$

We have

$$a_n = S_n - S_{n-1}$$

$$= \frac{5n^2}{2} + \frac{3n}{2} - \frac{5(n-1)^2}{2} - \frac{3(n-1)}{2}$$

$$= \frac{5n^2 - 5(n-1)^2}{2} + \frac{3n}{2} - \frac{3n}{2} + \frac{3}{2}$$

$$= \frac{5(n+n-1)(n-n+1)}{2} + \frac{3}{2}$$

$$= \frac{5(2n-1)+3}{2}$$

$$= \frac{10n-2}{2}$$

$$= 5n - 1$$

$\therefore$ The required nth term of an AP is $5n - 1$.

3. If the nth term of an AP is $(2n + 1)$, then the sum of the first n terms of the AP is

(a) $n(n + 2)$ (b) $n(n - 2)$
(c) $n(n + 1)$ (d) $n(n - 1)$

Sol. (a) $n(n + 2)$

We have

$$a_p = 2p + 1$$

$$\therefore \quad S_n = \sum_{p=1}^{n} a_p$$

$$= 2\sum_{p=1}^{n} p + \sum_{p=1}^{n} 1$$

$$= 2(1 + 2 + 3 + \ldots + n) + n$$

$$= 2 \times \frac{n(n+1)}{2} + n$$

$$= n(n + 1 + 1)$$

$$= n(n + 2)$$

which is the required sum.

4. The sum of first seven terms of an AP is 49 and that of 17 terms is 289. The sum of first n terms is

(a) $2n$ (b) $\frac{n(n+1)}{2}$
(c) n^2 (d) $\frac{n^2+1}{2}$

Sol. (c) n^2

Let a be the 1st term, d be the common difference and S_n be the sum of n terms of an AP

Then $$S_7 = \frac{7}{2}\{2a + (7-1)d\} = 49$$

$$\Rightarrow \quad a + 3d = 7 \quad \ldots(1)$$

and $$S_{17} = \frac{17}{2}\{2a + (17-1)d\}$$

$$= 289$$

$$\Rightarrow \quad a + 8d = 17 \quad \ldots(2)$$

Subtracting (1) from (2), we have

$$5d = 10$$

$$\Rightarrow \quad d = \frac{10}{5} = 2$$

$\therefore$ From (1), $a = 7 - 3 \times 2 = 1$

Now, $$S_n = \frac{n}{2}\{2a + (n-1)d\}$$

$$= \frac{n}{2}\{2 + (n-1)2\}$$

$$= n^2$$

$\therefore$ The sum of first n terms is n^2.

Very Short Answer Type Questions

5. If all the terms of a sequence are equal, can it be an AP? If so, what is its common difference?

Sol. Let the sequence be $a, a, a, a, \ldots$

Here, we see that the common difference d is $a - a = 0$ which is a constant. Hence, it can be called an AP with common difference 0.

6. If the first terms and the common differences of two AP's are the same. Will they have the same number of terms? Give an example.

Sol. Not necessarily, because the total number of terms of the two AP's may be different. For example:

Two AP's are

(*i*) $a, a + d, a + 2d, \dots a + 100d$ and

(*ii*) $a, a + d, a + 2d, \dots a + 100d, a + 101d$

They have 101 and 102 terms respectively, and so they are different AP's.

7. If the sum of the first four terms of an AP is 19 and that of first three terms is 16, what is its fourth term?

Sol. Let a be the 1st term and d be the common difference.

Then, we have

$$a + (a + d) + (a + 2d) + (a + 3d) = 19$$

$$\Rightarrow \quad 4a + 6d = 19 \qquad \dots(1)$$

Also, $$a + (a + d) + (a + 2d) = 16$$

$$\Rightarrow \quad 3a + 3d = 16$$

$$\Rightarrow \quad 6a + 6d = 32 \qquad \dots(2)$$

Subtracting (1) from (2), we get

$$2a = 32 - 19 = 13$$

$$\Rightarrow \quad a = \frac{13}{2}$$

$\therefore$ From (1), $$6d = 19 - 4a = 19 - 4 \times \frac{13}{2} = 19 - 13 \times 2 = 19 - 26 = -7$$

$$\therefore \quad d = \frac{-7}{6}$$

$\therefore$ The fourth term of the AP is

$$a + 3d = \frac{13}{2} - 3 \times \frac{7}{6} = \frac{13}{2} - \frac{7}{2} = \frac{6}{2} = 3$$

Short Answer Type-I Questions

8. Find the sum of the AP $x + y, x - y, x - 3y, \dots$ to 20 terms.

Sol. The first term of an AP, $a = x + y$ and the common difference, $d = x - y - x - y = -2y$.

We have $$S_{20} = \frac{20}{2}(2a + (20 - 1)d)$$

$$S_{20} = \frac{20}{2}\{2(x + y) + 19 \times (-2y)\}$$
$$= 10\{2(x + y) + 19 \times (-2y)\}$$
$$= 20(x + y) + 190 \times (-2y)$$
$$= 20x + 20y - 380y$$
$$= 20x - 360y$$

$\therefore$ The required sum is $20x - 360y$.

9. Find the sum of the first 24 terms of the AP 99, 96, 93, 90, …

Sol. The first term, $a = 99$ and the common difference, $d = 96 - 99 = -3$. Let S_n denote the sum to n terms of the AP.

$$\therefore \quad S_{24} = \frac{24}{2}\{2a + (24 - 1)d\}$$
$$= \frac{24}{2}\{2 \times 99 + 23 \times (-3)\}$$
$$= 24 \times 99 - 12 \times 23 \times 3$$
$$= 2376 - 828 = 1548$$

$\therefore$ The required sum is 1548.

10. If the sum of n terms of an AP is given by $S_n = \dfrac{n(5n+7)}{12}$, find its a_n.

Sol. We know that the nth term, a_n is given by

$$a_n = S_n - S_{n-1}$$
$$= \frac{n(5n+7)}{12} - \frac{(n-1)[5(n-1)+7]}{12}$$
$$= \frac{n(5n+7)}{12} - \frac{(n-1)(5n+2)}{12}$$
$$= \frac{5n^2 + 7n - 5n^2 - 2n + 5n + 2}{12}$$
$$= \frac{10n + 2}{12}$$
$$= \frac{5n + 1}{6}$$

$\therefore$ The value of a_n is $\dfrac{5n+1}{6}$.

11. Find the sum of all even numbers between 9 and 99.

Sol. All even numbers between 9 and 99 are 10, 12, 14,… 98 which is in AP with 1st term, $a = 10$ and common difference, $d = 12 - 10 = 2$.

Let S_n denote the sum of all the terms of this AP. If a_n denote the nth term, where n is the number of terms of this AP, then

$$a_n = 98$$
$$\Rightarrow \quad 98 = a + (n-1)d$$
$$\Rightarrow \quad 98 = 10 + (n-1) \times 2$$
$$\Rightarrow \quad 98 = 10 + 2n - 2$$
$$\Rightarrow \quad 90 = 2n$$
$$\Rightarrow \quad n = \frac{90}{2} = 45$$
$$\therefore \quad S_{45} = \frac{45}{2}\{2a + (45-1)d\}$$
$$= \frac{45}{2}(2 \times 10 + 44 \times 2)$$
$$= 450 + 45 \times 44$$
$$= 450 + 1980$$
$$= 2430$$

$\therefore$ The required sum is 2430.

12. Find the sum of all numbers between 100 and 200 which are multiples of 5.

Sol. Numbers between 100 and 200 which are multiples of 5 are 105, 110, 115, …195, which is in AP.

If a is the 1st term, d is the common difference and n is the number of terms of this AP, then $a = 105$, $d = 110 - 105 = 5$

$$a_n = 195$$
$$\Rightarrow \quad a + (n-1)d = 195$$
$$\Rightarrow \quad 105 + (n-1)5 = 195$$
$$\Rightarrow \quad (n-1)5 = 90$$
$$\Rightarrow \quad n - 1 = 18$$
$$\Rightarrow \quad n = 19$$
$$\therefore \quad S_{19} = \frac{19}{2}\{2a + (19-1)d\}$$
$$= \frac{19}{2}(2 \times 105 + 18 \times 5)$$
$$= 105 \times 19 + 19 \times 45$$
$$= 19 \times (105 + 45)$$
$$= 19 \times 150$$
$$= 2850$$

$\therefore$ The required sum is 2850.

Short Answer Type-II Questions

13. Find the sum of $\left(1-\frac{1}{n}\right)+\left(1-\frac{2}{n}\right)+\left(1-\frac{3}{n}\right)+\ldots$ upto n terms.

Sol. We have

$$\left(1-\frac{1}{n}\right)+\left(1-\frac{2}{n}\right)+\left(1-\frac{3}{n}\right)+\ldots \text{ upto } n \text{ terms}$$
$$= (1 + 1 + \ldots \text{ upto } n \text{ terms}) - \frac{1}{n}(1 + 2 + 3 + \ldots + n)$$
$$= n - \frac{1}{n} \times \frac{n(n+1)}{2}$$
$$= \frac{2n - n - 1}{2}$$
$$= \frac{n-1}{2}$$

$\therefore$ The required sum is $\frac{n-1}{2}$.

14. Find the sum of the first 54 terms of the AP whose third term is -103 and the 7th term is -63.

Sol. Let a be the 1st term and d be the common difference of the AP. We denote nth term by a_n and sum to n terms of the AP by S_n.

Then, we have $\quad a_3 = -103$
$$\Rightarrow \quad a + (3-1)d = -103$$
$$\Rightarrow \quad a + 2d = -103 \qquad \ldots(1)$$
and $\quad a_7 = -63$
$$\Rightarrow \quad a + (7-1)d = -63$$
$$\Rightarrow \quad a + 6d = -63 \qquad \ldots(2)$$

Subtracting (1) from (2), we get
$$4d = 40$$
$$\Rightarrow \quad d = \frac{40}{4} = 10$$
$\therefore$ From (1), $\quad a = -103 - 2d$
$$= -103 - 2 \times 10$$
$$= -103 - 20$$
$$= -123$$
$$\therefore \quad S_{54} = \frac{54}{2}\{2a + (54-1)d\}$$
$$= 27(2 \times (-123) + 53 \times 10)$$
$$= 27 \times (-246 + 530)$$
$$= 27 \times 284 = 7668$$

$\therefore$ The required sum is 7668.

15. In an AP, if the 12th term is -13 and the sum of its first 4 terms is 24, find the sum of its first 10 terms. **[CBSE 2015]**

Sol. Let a be the 1st term and d be the common difference of the AP. Let a_n denote the nth term and S_n denote the sum of the n terms of the AP. Then,

$$a_{12} = -13$$
$$\Rightarrow \quad a + (12-1)d = -13$$
$$\Rightarrow \quad a + 11d = -13 \qquad \ldots(1)$$

Also, $S_4 = \frac{4}{2}\{2a + (4-1)d\}$

$24 = 2(2a + 3d)$

$\Rightarrow \quad 2a + 3d = 12 \quad \ldots(2)$

Multiplying (1) by 2 and subtracting it from (2), we get

$-19d = 38$

$\Rightarrow \quad d = -2$

$\therefore$ From (1), $a = -13 - 11 \times (-2)$

$= -13 + 22$

$= 9$

$\therefore \quad S_{10} = \frac{10}{2}\{2a + (10-1)d\}$

$= 5\{2 \times 9 + 9 \times (-2)\}$

$= 5(18 - 18)$

$= 5 \times 0 = 0$

$\therefore$ The required sum is 0.

16. Find the sum of n terms of the sequence 1, –3, 5, –7, 9, –11, … when (*a*) n is even and (*b*) n is odd.

Sol. (*a*) Let $n = 2m$, where m is any positive integer.

Now, taking only positive terms in the odd positions only of the given sequence, we have the sequence 1, 5, 9, … which is an AP with 1st term, $a = 1$ and the common difference, $d = 5 - 1 = 4$. If a_m denote the mth term of this AP, then $a_m = a + (m-1)d = 1 + (m-1)4 = 4m - 3$ and if S_m denote the sum of m terms of this AP, then

$$S_m = \frac{m}{2}\{2a + (m-1)d\}$$

$$= ma + \frac{m(m-1)d}{2}$$

$$= m + \frac{m(m-1) \times 4}{2}$$

$$= m + 2m(m-1)$$

$$= m + 2m^2 - 2m$$

$$= 2m^2 - m$$

Again, taking negative terms in the even positions of the given sequence, we have the sequence –3, –7, –11, … which is another AP with $a = -3$, $d = -7 + 3 = -4$.

$\therefore$ Sum S'_m of m terms of this AP is given by

$$S'_m = \frac{m}{2}\{2 \times (-3) + (m-1)(-4)\}$$

$$= -3m - 2m(m-1)$$

$$= -3m - 2m^2 + 2m$$

$$= -2m^2 - m$$

$\therefore$ Sum of $2m$ terms of the given sequence

$$= S_m + S'_m$$

$$= 2m^2 - m - 2m^2 - m$$

$$= -2m$$

$$= -n$$

Hence, the required sum is $-n$, when n is even.

(*b*) Let $n = 2m + 1$, where m is any positive integer.

We see that each number in the odd position of the given sequence is a positive number of the AP 1, 5, 9, 13, …

$\therefore$ Sum of $2m + 1$ terms of the given sequence

$$= S_m + S'_m + a_{m+1}$$

$$= -2m + 4(m+1) - 3 \qquad \text{[From part } (a)\text{]}$$

$$= -2m + 4m + 4 - 3$$

$$= 2m + 1 = n$$

$\therefore$ The required sum is n, when n is odd.

17. Find the sum of the integers between 1 and 499 which are multiples of 3 and 5.

Sol. Numbers between 1 to 499 which are multiples of 3 and 5, i.e. $3 \times 5 = 15$ are 15, 30, 45, … which is in AP with 1st term, $a = 15$ and common difference, $d = 30 - 15 = 15$.

If n be the number of terms of this AP, then $a_n < 499$

i.e. $a + (n-1)d < 499$

$\Rightarrow \quad 15 + (n-1)15 < 499$

$\Rightarrow \quad 15n < 499$

$\Rightarrow \quad n < \frac{499}{15} = 33\frac{4}{15}$

$\therefore$ $n = 33$ which is a positive integer.

$\therefore \quad S_{33} = \frac{33}{2}[2 \times 15 + (33-1) \times 15]$

$= \frac{33}{2} \times 15 \times (2 + 32)$

$= 33 \times 15 \times 17$

$= 8415$

$\therefore$ The required sum is 8415.

18. A manager was appointed in an office at a salary of ₹15000 per month. It was decided that his annual increment will be ₹500. Find his salary in the 15th year and also his total salary after 15 years of service.

Sol. Here, we shall find the 15th term of an AP with the 1st term $a = 15000$ and the common difference, $d = 500$.

If a_n denote the nth term of this AP, then

$$a_{15} = 15000 + (15 - 1) \times 500$$
$$= 15000 + 14 \times 500$$
$$= 15000 + 7000$$
$$= 22000$$

Hence, his required salary in the 15th year will be ₹22000.

His yearly salary = ₹15000 × 12 = ₹180000. Yearly increment is ₹500. Hence, his total salary after 15 years = Sum of 15 terms of the AP 180000, 180500, 181000, … to 15th term

$$\therefore \quad S_{15} = \frac{15}{2}\{2 \times 180000 + (15 - 1) \times 500\}$$
$$= \frac{15}{2}\{360000 + 14 \times 500\}$$
$$= \frac{15}{2}\{360000 + 7000\}$$
$$= \frac{15}{2}\{367000\} = 15 \times 183500$$
$$= 2752500$$

Hence, his total salary after 15 years will be ₹2752500.

Long Answer Type Questions

19. Find the maximum sum of the terms of the AP 148, 138, 128, 118, …

Sol. We see that the 1st term of the AP, $a = 148$ and the common difference, $d = 138 - 148 = -10$. The terms of the AP gradually decreases, so, the sum of the terms of the AP will be maximum only when all the terms are positive. Let n be the number of terms of the AP till the last term is near 0.

Now,

$$a_n = a + (n - 1)d$$
$$= 148 + (n - 1) \times (-10)$$
$$= 148 - (n - 1) \times 10$$

We see that $\quad a_n \geq 0$

$\Rightarrow \quad 148 - (n - 1) \times 10 \geq 0$

$\Rightarrow \quad (n - 1) \leq 14.8$

$\Rightarrow \quad n \leq 15.8$

$\because$ n is an integer, we have

$$n = 15$$

$\therefore$ The required maximum sum is given by

$$S_{15} = \frac{15}{2}[2 \times 148 - 14 \times 10]$$
$$= 15 \times 148 - 15 \times 70$$
$$= 15 \times (148 - 70)$$
$$= 15 \times 78$$
$$= 1170$$

20. If the ratio of the sums of the first m and n terms of an AP is $m^2 : n^2$, show that the ratio of its mth and nth terms is $(2m - 1) : (2n - 1)$.

[CBSE 2016, 2017]

Sol. We have

$$\frac{S_m}{S_n} = \frac{m^2}{n^2}$$

Let $S_m = km^2$ and $S_n = kn^2$, where k is a non-zero constant.

Now, $\quad a_m = S_m - S_{m-1}$
$$= k\{m^2 - (m - 1)^2\}$$
$$= k(m + m - 1)(m - m + 1)$$
$$= k(2m - 1)$$

and $\quad a_n = S_n - S_{n-1}$
$$= k\{n^2 - (n - 1)^2\}$$
$$= k(n + n - 1)(n - n + 1)$$
$$= k(2n - 1)$$

$$\therefore \quad \frac{a_m}{a_n} = \frac{(2m - 1)k}{(2n - 1)k}$$
$$= \frac{2m - 1}{2n - 1}$$

Hence, proved.

21. The sum of three numbers in AP is 12 and the sum of their cubes is 288. Find the numbers.

[CBSE 2016]

Sol. Let the three numbers in AP be $\alpha - d$, α and $\alpha + d$.

Then, according to the problem, we have

$$\alpha - d + \alpha + \alpha + d = 12$$

$\Rightarrow \quad 3\alpha = 12$

$\Rightarrow \quad \alpha = 4$

Also, $\quad (\alpha - d)^3 + \alpha^3 + (\alpha + d)^3 = 288$

$\Rightarrow \quad (4 - d)^3 + 4^3 + (4 + d)^3 = 288$

$\Rightarrow \quad (4 - d)^3 + (4 + d)^3 = 288 - 64 = 224$

$\Rightarrow \quad (4 - d + 4 + d)\{(4 - d)^2 - (4 - d)(4 + d) + (4 + d)^2\} = 224$

$\Rightarrow \quad 8\{2(4^2 + d^2) - (16 - d^2)\} = 224$

$\Rightarrow \quad 8\{32 + 2d^2 - 16 + d^2\} = 224$

$\Rightarrow \quad 32 + 2d^2 - 16 + d^2 = \frac{224}{8} = 28$

$\Rightarrow \quad 3d^2 = 28 - 16 = 12$

$\Rightarrow \quad d^2 = 4$

$\Rightarrow$ $d = \pm 2$

$\therefore$ The three numbers are $4 - 2, 4, 4 + 2$, when $d = 2$, i.e. 2, 4, 6 or, $4 + 2, 4, 4 - 2$, when $d = -2$, i.e. 6, 4, 2.

Hence, the required numbers are 2, 4 and 6.

22. If the sums of n terms of two AP's are in the ratio $(3n - 13) : (5n + 21)$, find the ratio of their 24th terms.

Sol. Let a and a' be the 1st term and d and d' be the common difference of the two AP's. Let S and S' be the respective sums of the two AP's.

Then $$S = \frac{n}{2}[2a + (n-1)d] \quad \ldots(1)$$

and $$S' = \frac{n}{2}[2a' + (n-1)d'] \quad \ldots(2)$$

From (1) and (2), we have

$$\frac{S}{S'} = \frac{2a + (n-1)d}{2a' + (n-1)d'}$$

$$= \frac{3n - 13}{5n + 21} \quad \text{[Given]} \ldots(3)$$

Now, the ratio of the 24th terms of the two AP's is

$$\frac{a + (24-1)d}{a' + (24-1)d'} = \frac{a + 23d}{a' + 23d'}$$

$$= \frac{2a + 46d}{2a' + 46d'} \quad \ldots(4)$$

Comparing (3) and (4), i.e. $\frac{2a + (n-1)d}{2a' + (n-1)d'}$ and $\frac{2a + 46d}{2a' + 46d'}$

We see that $n - 1 = 46$

$\Rightarrow$ $n = 46 + 1 = 47$

Putting $n = 47$ in (3), we get

$$\frac{2a + 46d}{2a' + 46d'} = \frac{3 \times 47 - 13}{5 \times 47 + 21}$$

$$= \frac{141 - 13}{235 + 21}$$

$$= \frac{128}{256}$$

$$= \frac{1}{2}$$

$\therefore$ The required ratio is 1 : 2.

23. The interior angles of a polygon are in AP. The smallest angle is 120° and the common difference is 5°. Find the number of sides of the polygon.

Sol. The first term of the given AP will be $a = 120°$ and the common difference, $d = 5°$. Let n be the number of sides of the polygon. Then the sum of all interior angles of the polygon is $(n - 2) \times 180°$.

$\therefore$ $(n - 2) \times 180° = 120° + 125° + 130° + \ldots$ to n terms

$\Rightarrow$ $(n-2) \times 180° = \frac{n}{2}[2 \times 120° + (n-1) \times 5°]$

$\Rightarrow$ $(n-2) \times 36 = \frac{n}{2}[2 \times 24 + n - 1]$

$\Rightarrow$ $(n-2) \times 36 = \frac{n}{2}(48 + n - 1)$

$\Rightarrow$ $(n-2) \times 36 = \frac{n}{2}(47 + n)$

$\Rightarrow$ $72n - 144 - 47n - n^2 = 0$

$\Rightarrow$ $n^2 - 25n + 144 = 0$

$\Rightarrow$ $n^2 - 16n - 9n + 144 = 0$

$\Rightarrow$ $n(n - 16) - 9(n - 16) = 0$

$\Rightarrow$ $(n - 16)(n - 9) = 0$

$\therefore$ Either $n = 16$ or $n = 9$.

But $n = 16$ is not possible, since in this case the 16th angle of the polygon, i.e. $a_{16} = 120° + 15 \times 5° = 120° + 75° = 195° > 180°$ which is absurd.

Hence, the required number of sides is 9.

Milestone 3

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Very Short Answer Type Questions

1. A sum of ₹500 is invested at 6% simple interest per annum. Will the interests at the end of first year, second year, third year, fourth year, … form an AP? If so, what is the common difference of the AP ?

Sol. Interest at the end of 1st year

$$= ₹\,\frac{500 \times 6}{100} = ₹\,30$$

We see that this interest on each year will be the same, i.e. ₹30

$\therefore$ Total interest at the end of 2nd year

= ₹30 + ₹30

= ₹60

Total interest at the end of 3rd year

= (60 + 30)

= 90 and so on.

$\therefore$ Total interests at the end of 1st year, 2nd year, 3rd year, 4th year, and so on are respectively ₹30, ₹60, ₹90, ₹120, and so on.

The sequence of numbers 30, 60, 90, 120, … forms an AP with common difference, $d = 60 - 30 = 30$.

2. In the month of July 2017, the total number of visitors in a zoo was 1550. If the number of visitors to the zoo decreased daily by 6 from 1st July to 31st July, 2017, then find the number of visitors to the zoo on 1st July, 2017.

Sol. The sum of the number of all daily visitors in the zoo for 31 days is given to be 1550. The number of daily visitors to the zoo form an AP with first term, a and the common difference, $d = -6$.

If S_{31} is the sum of the numbers of all daily visitors, then

$$S_{31} = \frac{31}{2}[2a - 30 \times 6]$$

$$\Rightarrow \quad 1550 = \frac{31}{2}[2a - 180]$$

$$\Rightarrow \quad 1550 = 31a - 2790$$

$$\Rightarrow \quad 4340 = 31a$$

$$\Rightarrow \quad a = \frac{4340}{31} = 140$$

$\therefore$ The number of visitors to the zoo on 1st July, 2017 was 140.

Short Answer Type-I Questions

3. A manufacturer of TV sets produced 600 units in the third year and 700 units in the seventh year. Assuming that the production increases uniformly by a fixed number of units every year, find

(*a*) the production in the first year.

(*b*) the production in the 10th year.

(*c*) the total production in 7 years.

[CBSE SP 2016]

Sol. (*a*) Let the initial production in the 1st year be a units, and let d be the increase in production every year.

We see that the numbers of yearly production form an AP with 1st term, a and common difference, d.

According to the problem,

a_3 = production in the third year = 600

$$\Rightarrow \quad a + 2d = 600 \quad \ldots(1)$$

and a_7 = production in the seventh year = 700

$$\Rightarrow \quad a + 6d = 700 \quad \ldots(2)$$

Subtracting (1) from (2), we get

$$4d = 700 - 600 = 100$$

$$\Rightarrow \quad d = \frac{100}{4} = 25$$

$\therefore$ From (1), $a = 600 - 2 \times 25$
$= 600 – 50 = 550$

Hence, the required production in the first year is 550 units.

(*b*) Now, production in the 10th year $= a_{10}$

$$= a + 9d$$
$$= 550 + 9 \times 25$$
$$= 550 + 225$$
$$= 775$$

Hence, the required production in the 10th year is 775 units.

(*c*) Finally, if S_7 denote the total production in 7 years, then

$$S_7 = \frac{7}{2}\{2a + (7-1)d\}$$
$$= \frac{7}{2}\{2 \times 550 + 6 \times 25\}$$
$$= 7 \times 550 + 75 \times 7$$
$$= 7 \times (550 + 75)$$
$$= 7 \times 625$$
$$= 4375$$

Hence, the required total production in 7 years is 4375 units.

4. You are saving ₹1 today, ₹2 the next day and ₹3 the third day and so on and your friend is saving ₹2, ₹6, ₹10, ₹14 … successively in every alternate day in the month of February, 2018, then who will save more and by how much?

Sol. We see that the month of February, 2018 contains 28 days.

You are saving ₹(1 + 2 + 3 + … + 28)

$$= ₹\ \frac{28 \times 29}{2} = 406$$

and your friend are saving ₹(2 + 6 + 10 + 14 + …) for 14 alternate days in February, 2018. Now the sequence of numbers 2, 6, 10, 14, … form an AP with first term, $a = 2$ and the common difference, $d = 6 - 2 = 4$.

$\therefore$ Total amount of your friend's saving in February 2018 is

$$S_{14} = ₹\ \frac{14}{2}\{2a + (14-1) \times d\}$$
$$= ₹7 \times (2 \times 2 + 13 \times 4)$$
$$= ₹7 \times (4 + 52)$$
$$= ₹7 \times 56$$
$$= ₹392$$

$\therefore$ You are saving ₹(406 – 392) = ₹14 more than your friend in February, 2018.

5. A man buys infrastructure bonds every year. The value of these bonds exceeds the previous year purchase value by ₹200. After 15 years, the total value of the infrastructure bonds purchased by him is ₹96000. Find the face value of the bonds bought by him in the first year.

Sol. Let the face value of the bonds in the 1st year be ₹a. Then the value of the bonds increases by ₹200 every year for 15 years and the total value of the bonds in 15 years is ₹96000. We see that the values of the bonds every year for 15 years form an AP with 1st term, a and the common difference, $d = 200$.

If S_{15} is the sum of 15 terms of this AP,

then $$S_{15} = \frac{15}{2}\{2a + (15-1) \times 200\}$$

$$\Rightarrow \quad 96000 = \frac{15}{2}\{2a + 14 \times 200\}$$

$$\Rightarrow \quad 96000 = 15a + 15 \times 14 \times 100$$

$$\Rightarrow \quad 96000 = 15a + 21000$$

$$\Rightarrow \quad 75000 = 15a$$

$$\Rightarrow \quad a = \frac{75000}{15}$$

$$= 5000$$

Hence, the required face value of the bonds in the 1st year is ₹5000.

6. A man takes up a job of ₹60000 per month with annual increment of ₹2000. How much will he earn in 10 years?

Sol. We see that the earnings of the man each year for 10 years form an AP with 1st term, a = ₹60000 × 12 = ₹720000 and common difference, d = ₹2000. Let S_{10} denote his total earning in 10 years. Then S_{10} is the sum of 10 terms of this AP.

$$\therefore \quad S_{10} = ₹\,\frac{10}{2}\{2a + (10-1)d\}$$

$$= ₹5 \times (2 \times 720000 + 9 \times 2000)$$

$$= ₹5 \times (1440000 + 18000)$$

$$= ₹5 \times 1458000$$

$$= ₹7290000$$

∴ A man will earn ₹7290000 in 10 years.

Short Answer Type-II Questions

7. The minimum age of children to participate in a painting competition is 8 years. It is observed that the age of the youngest boy was 8 years and the ages of the rest of the participants are having a common difference of 4 months. If the sum of the ages of all participants is 168 years, find the age of the eldest participant in the painting competition. **[CBSE SP 2015]**

Sol. We see that the ages of different boys form an AP with 1st term, a = 8 years and the common difference, d = 4 months = $\frac{1}{3}$ year.

The sum S_n of the ages of all participants is given by

$$S_n = 168 \qquad \text{[Given]}$$

$$\Rightarrow \quad \frac{n}{2}\left[2a + \frac{n-1}{3}\right] = 168$$

$$\Rightarrow \quad \frac{n}{2}\left[2 \times 8 + \frac{n-1}{3}\right] = 168$$

$$\Rightarrow \quad 8n + \frac{n(n-1)}{6} = 168$$

$$\Rightarrow \quad \frac{n^2 - n + 48n}{6} = 168$$

$$\Rightarrow \quad n^2 + 47n - 1008 = 0$$

$$\Rightarrow \quad n^2 + 63n - 16n - 1008 = 0$$

$$\Rightarrow \quad n(n + 63) - 16(n + 63) = 0$$

$$\Rightarrow \quad (n + 63)(n - 16) = 0$$

∴ Either $n + 63 = 0$

$$\Rightarrow \quad n = -63$$

which is rejected, since n cannot be negative.

or, $n - 16 = 0$

$$\Rightarrow \quad n = 16$$

which is accepted.

Hence, the total number of participants is 16 and the age of the eldest participant will be the 16th term, a_{16}, of the AP.

Now, $$a_{16} = 8 + (16-1) \times \frac{1}{3} = 13$$

Hence, the required age of the eldest participant is 13 years.

8. Jaipal Singh repays the total loan of ₹118000 by paying every month starting with the first instalment of ₹1000. If he increases the instalment by ₹100 every month, what amount will be paid by him in 30th instalment? What amount of loan does he still have to pay after 30th instalment? **[CBSE SP 2012]**

Sol. We see that the amounts of different monthly instalments form an AP with 1st term, a = ₹1000 and the common difference, d = ₹100. If a_{30}

represents the amount in the 30th instalment and S_{30} represents the total amount of loan paid by Jaipal singh in 30 instalments, then

$$a_{30} = a + (30-1)d$$
$$= (1000 + 29 \times 100)$$
$$= ₹3900$$

and $$S_{30} = ₹\ \frac{30}{2}(2 \times 1000 + 29 \times 100)$$
$$= ₹15 \times (2000 + 2900)$$
$$= ₹15 \times 4900$$
$$= ₹73500$$

$\because$ Total amount of loan is ₹118000.

$\therefore$ Balance amount of loan, he will still have to pay after 30th instalment is (₹118000 – ₹73500), i.e ₹44500.

9. Two cyclists C_1 and C_2 start together in the same direction from the same place. The cyclist C_1 cycles at a uniform speed of $11\frac{1}{4}$ km/h and the cyclist C_2 cycles at a uniform speed of $9\frac{3}{4}$ km/h in the first hour and then increases his speed by $\frac{1}{4}$ km/h in each succeeding hour. After how many hour(s) will the cyclist C_2 overtake C_1, if both the cyclists cycle non-stop?

Sol. Let the two cyclists meet together after t hours.

Now, in t hours, the cyclist C_1 travels a distance of

$$11\frac{1}{4} \times t \text{ km} = \frac{45t}{4} \text{ km}.$$

We see that the distances travelled by the cyclist C_2 in different hours form an AP with the 1st term, $a = 9\frac{3}{4}$ km $= \frac{39}{4}$ km and the common difference, $d = \frac{1}{4}$ km. Hence, the total distance travelled by the cyclist C_2 in t hours will be the sum of t terms, S_t of this AP.

$\therefore$ $$S_t = \frac{t}{2}[2a + (t-1)d]$$
$$= \frac{t}{2}\left[2 \times \frac{39}{4} + (t-1) \times \frac{1}{4}\right] \text{ km}$$
$$= \left[\frac{39}{4}t + \frac{t(t-1)}{8}\right] \text{ km}$$
$$= \frac{78t + t^2 - t}{8} \text{ km}$$
$$= \frac{77t + t^2}{8} \text{ km}$$

We have

$$\frac{77t + t^2}{8} = \frac{45t}{4}$$

$\Rightarrow$ $$\frac{77 + t}{8} = \frac{45}{4}$$

$\Rightarrow$ $$77 + t = 90$$

$\Rightarrow$ $$t = 90 - 77 = 13$$

Hence, the required time is 13 hours.

Long Answer Type Questions

10. A polygon has 41 sides, the length of which, starting from the smallest, are in AP. If the perimeter of the polygon is 902 cm and the length of the largest side is 21 times the smallest, find the length of the smallest side and the common difference of the AP.

Sol. Let the smallest side of the polygon be of length a cm. Then the largest side is of length $21a$ cm.

Clearly, the different lengths of the sides of the polygon form an AP with 1st term be a cm and the last or 41st term be $21a$ cm.

Let d cm be the common difference of this AP.

Then $$a_{41} = a + 40d = 21a$$

$\Rightarrow$ $$40d = 20a$$

$\Rightarrow$ $$d = \frac{a}{2} \quad \ldots(1)$$

The perimeter of the polygon will be the sum of all 41 terms of the AP, i.e. S_{41}.

Now, $$S_{41} = \frac{41}{2}(2a + 40d)$$
$$= \frac{41}{2}\left(2a + 40 \times \frac{a}{2}\right) \text{ cm}$$

[From (1)]

$$= \frac{41}{2}(2a + 20a) \text{ cm}$$
$$= 41 \times 11a \text{ cm}$$
$$= 451a \text{ cm}$$

$\therefore$ According to the problem, we have

$$451a = 902$$

$\Rightarrow$ $$a = \frac{902}{451}$$

$\Rightarrow$ $$a = 2 \quad \ldots(2)$$

Now, $$d = \frac{2}{2} \quad \text{[From (2)]}$$

$\Rightarrow$ $$d = 1$$

Hence, the required smallest side is 2 cm and common difference, d is 1 cm.

11. A thief, after committing a theft, runs at a uniform speed of 50 m/min. After 2 minutes, a policeman runs to catch him. He goes 60 m in the first minute and increases his speed by 5 m/min every succeeding minute. After how many minutes will the policeman catch the thief? **[CBSE 2016]**

Sol. When the police starts running, the thief is 100 m apart.

Speed for 1st minute is 60 m/minute and increases by 5 m/minute.

AP: 10, 15 …

a = 10 m/minute (distance reduced in 1st minute)

$$d = 5$$

$$S_n = 100$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\Rightarrow \quad 100 = \frac{n}{2}[20 + (n-1)5]$$

$$\Rightarrow \quad 200 = n[20 + 5n - 5]$$

$$\Rightarrow \quad 200 = n[15 + 5n]$$

$$\Rightarrow \quad 40 = n[3 + n]$$

$$\Rightarrow \quad 40 = 3n + n^2$$

$$\Rightarrow \quad n^2 + 3n - 40 = 0$$

$$\Rightarrow \quad n^2 + 8n - 5n - 40 = 0$$

$$\Rightarrow \quad n(n+8) - 5(n+8) = 0$$

$$\Rightarrow \quad (n+8)(n-5) = 0$$

$\therefore$ Either $n = -8$

or $n = 5$

Since, time cannot be negative, hence we will reject –8.

$\therefore$ Policeman will catch the thief in 5 minutes.

12. The houses of a row are numbered consecutively from 1 to 49. Show that there is a value of x such that the sum of the numbers of the houses preceding the house numbered x is equal to the sum of the numbers of the houses following it. Find the value of x. **[CBSE 2016]**

Sol. Let us first calculate the sum of houses before x.

$a = 1, d = 1, l = x - 1, n = x - 1$

$$S_n = \frac{n}{2}[a + l]$$

$$S_x = \frac{x-1}{2}[1 + x - 1]$$

$$S_x = \frac{x(x-1)}{2} \qquad \ldots(1)$$

Now, we will calculate the sum of houses after x.

$a = x + 1, d = 1, l = 49, n = 49 - x$

$$S_x = \frac{49-x}{2}[x + 1 + 49]$$

$$= \frac{(49-x)}{2}(x + 50) \qquad \ldots(2)$$

We will now equate (1) and (2), we get

$$\frac{x(x-1)}{2} = \frac{(49-x)(x+50)}{2}$$

$$\Rightarrow \quad x^2 - x = 49x + 2450 - x^2 - 50x$$

$$\Rightarrow \quad 2x^2 - x = -x + 2450$$

$$\Rightarrow \quad 2x^2 = 2450$$

$$\Rightarrow \quad x^2 = 1225$$

$$\Rightarrow \quad x = \pm 35$$

Since, house number cannot be negative, hence we will reject –35.

$\therefore$ The house number is 35.

Higher Order Thinking Skills (HOTS) Questions

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1. If p^2, q^2 and r^2 are in AP, prove that $\frac{1}{q+r}$, $\frac{1}{r+p}$ and $\frac{1}{p+q}$ are also in AP.

Sol. Since p^2, q^2 and r^2 are in AP,

$$\therefore \quad 2q^2 = p^2 + r^2 \qquad \ldots(1)$$

Now, $\frac{1}{q+r}$, $\frac{1}{r+p}$ and $\frac{1}{p+q}$ will be in AP if

$$\frac{2}{r+p} = \frac{1}{q+r} + \frac{1}{p+q}$$

$$= \frac{p+q+q+r}{(q+r)(p+q)}$$

$$= \frac{p+2q+r}{(q+r)(p+q)}$$

i.e. if $(r + p)(p + 2q + r) = 2(q + r)(p + q)$

i.e. if $rp + 2rq + r^2 + p^2 + 2pq + pr$
$= 2pq + 2q^2 + 2rp + 2rq$

i.e. if $r^2 + p^2 = 2q^2$ which is true by (1).

Hence, proved

2. Find the first negative term of the AP 2000, 1990, 1980, 1970, …

Sol. Here, the 1st term, $a = 2000$, common difference, $d = 1990 - 2000 = -10$.

$\therefore$ If a_n be the nth term, then

$$a_n = a + (n-1)d$$
$$= 2000 - (n-1) \times 10 \quad \ldots(1)$$

We shall find the maximum value of n for which $a_n \geq 0$.

i.e. $\quad 2000 - (n-1) \times 10 \geq 0 \qquad$ [From (1)]

$\Rightarrow \qquad n - 1 \leq 200$

$\Rightarrow \qquad n \leq 201$

$\therefore$ Maximum value of n for which n is positive is $n = 201$.

$\therefore$ The 1st negative term will be negative when $n = 202$.

Hence, the required term is 202nd term.

Also, $\qquad a_{202} = 2000 - (202 - 1) \times 10 \qquad$ [From (1)]

$$= 2000 - 2010$$
$$= -10$$

Hence, the required 1st negative term is -10.

3. How many terms are identical in the two AP's 3, 6, 9, 12, … upto 300 terms and 5, 10, 15, 20, … upto 200 terms?

Sol. In the two AP's, the successive terms are multiples of 3 and 5 respectively. Hence, the identical terms of the resulting AP will be those which are multiples of 3 as well as 5, i.e. multiples of $3 \times 5 = 15$. Now, in the 1st AP, the last term

$$a_{300} = 3 + 299 \times 3$$
$$= 3 + 897$$
$$= 900$$

Also, in the second AP, the last term

$$a_{200} = 5 + 199 \times 5$$
$$= 5 \times (199 + 1)$$
$$= 5 \times 200$$
$$= 1000$$

Hence, the terms of the resulting AP which are multiples of 15 will be upto 900.

$\therefore$ If a_n be the nth term, then

$$a_n \leq 900$$

$\Rightarrow \quad 15 + (n-1) \times 15 \leq 900$

$\Rightarrow \qquad 15n \leq 900$

$\Rightarrow \qquad n \leq \dfrac{900}{15} = 60$

$\therefore$ The maximum value of n is 60.

Hence, there are 60 identical terms in the two given AP's.

4. The 9th term of an AP is 11 times the first term. Prove that the 17th term is 6 times as great as the third term.

Sol. Let a be the 1st term and d be the common difference of the AP. If a_n denotes its nth term, then

$$a_n = a + (n-1)d$$

Given that $\qquad a_9 = 11a$

$\Rightarrow \qquad a + (9-1)d = 11a$

$\Rightarrow \qquad 8d = 10a$

$\Rightarrow \qquad d = \dfrac{5a}{4} \qquad \ldots(1)$

We shall now prove that

$$a_{17} = 6a_3$$

i.e. $\qquad \dfrac{a_{17}}{a_3} = 6$

Now, $\qquad \dfrac{a_{17}}{a_3} = \dfrac{a + 16d}{a + 2d}$

$$= \frac{a + 16 \times \frac{5a}{4}}{a + 2 \times \frac{5a}{4}} \qquad \text{[From (1)]}$$

$$= \frac{4a + 16 \times 5a}{4a + 2 \times 5a}$$

$$= \frac{4a + 80a}{4a + 10a} = \frac{84a}{14a}$$

$\therefore \qquad \dfrac{a_{17}}{a_3} = 6$

$\Rightarrow \qquad a_{17} = 6a_3$

Hence, proved.

5. Find the sum of integers between 1 and 100 which are divisible by 2 or 5.

Sol. Numbers between 1 and 100, which are divisible by 2 are 2, 4, 6, 8, …, 98. If S be the sum of these numbers, then

$$S = 2 + 4 + 6 + 8 + \ldots + 98$$
$$= 2(1 + 2 + 3 + 4 + \ldots + 49)$$
$$= 2 \times \frac{49 \times 50}{2}$$

$\therefore \qquad S = 2450 \qquad \ldots(1)$

Also, numbers between 1 and 100, which are divisible by 5 are 5, 10, 15, 25, … , 95. If S' be the sum of these numbers, then

$$S' = 5 + 10 + 15 + 20 + 25 + \ldots + 95$$
$$= 5\,(1 + 2 + 3 + 4 + 5 + \ldots + 19)$$
$$= 5 \times \frac{19 \times 20}{2}$$

$\therefore \quad S' = 950 \quad \ldots(2)$

Among all these numbers which are divisible by 2 or 5, there are some common numbers, which are divisible by both 2 and 5, i.e. by $2 \times 5 = 10$.

These numbers are 10, 20, 30, 40, … 90.

If S'' be the sum of all these numbers, then

$$S'' = 10 + 20 + 30 + \ldots + 90$$
$$= 10(1 + 2 + 3 + \ldots + 9)$$
$$= 10 \times \frac{9 \times 10}{2}$$

$\therefore \quad S'' = 450 \quad \ldots(3)$

Hence, the required sum

$$= S + S' - S''$$
$$= 2450 + 950 - 450$$

[From (1), (2) and (3)]

$$= 2450 + 500$$
$$= 2950$$

$\therefore$ The required sum is 2950.

6. Find the sum of 11 terms of an AP whose middle term is 11.

Sol. Let a be the 1st term and d be the common difference of the AP. If n be the nth term of the AP, then

$$a_n = a + (n - 1)d \quad \ldots(1)$$

We know that the middle term is $a_{\frac{n+1}{2}}$ if n is odd.

Here, $n = 11$ which is odd.

$\therefore$ Middle term $= a_{\frac{11+1}{2}} = a_6$

$= a + 5d$ [From (1)]

It is given that $a_6 = 11$

$\therefore \quad a + 5d = 11 \quad \ldots(2)$

Also, $S_{11} = \frac{11}{2}[2a + (11 - 1)d]$

$= 11(a + 5d)$

$= 11 \times 11$ [From (2)]

$= 121$

$\therefore$ The required sum is 121.

Self-Assessment

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Multiple-Choice Questions

1. If $2 + 4 + 6 + 8 + \ldots$ to nth term = 110, then the value of n is equal to

(*a*) 11 (*b*) 12

(*c*) 10 (*d*) 9

Sol. (*c*) 10

We have

$2 + 4 + 6 + \ldots$ to nth term

$$= 2(1 + 2 + 3 + \ldots \text{ to } n \text{ terms})$$
$$= 2 \times \frac{n(n+1)}{2}$$
$$= n(n + 1)$$

$\therefore \quad n(n + 1) = 110$ [Given]

$\Rightarrow \quad n^2 + n - 110 = 0$

$\Rightarrow \quad n^2 + 11n - 10n - 110 = 0$

$\Rightarrow \quad n(n + 11) - 10(n + 11) = 0$

$\Rightarrow \quad (n + 11)(n - 10) = 0$

$\therefore$ Either $n + 11 = 0$

$\Rightarrow \quad n = -11$

which is rejected, since n cannot be negative.

or $n - 10 = 0$

$\Rightarrow \quad n = 10$

which is accepted.

$\therefore$ The value of n is 10.

2. If the sum of three numbers in AP is 18 and their product is 120, then the numbers are

(*a*) 9, 6, 3 (*b*) 5, 6, 7

(*c*) 1, 6, 11 (*d*) 2, 6, 10

Sol. (*d*) 2, 6, 10

Let $\alpha - d$, α and $\alpha + d$ be any three terms of an AP

$\therefore$ According to the problem,

$$\alpha - d + \alpha + \alpha + d = 18$$

$\Rightarrow \quad 3\alpha = 18$

$\Rightarrow \quad \alpha = 6 \quad \ldots(1)$

Also, $(\alpha - d)\,\alpha\,(\alpha + d) = 120$

$6(\alpha^2 - d^2) = 120$

$\Rightarrow \quad \alpha^2 - d^2 = 20$

$\Rightarrow \quad 36 - d^2 = 20$ [From (1)]

$\Rightarrow \quad d^2 = 16$

$\Rightarrow \quad d = \pm 4$

$\therefore$ The required AP is 6 + 4, 6 and 6 – 4, i.e. 10, 6 and 2 or 6 – 4, 6, 6 + 4, i.e. 2, 6, 10.

$\therefore$ The required numbers are 2, 6, 10.

Fill in the Blanks

3. The tenth term of the AP: 2.5, 4.5, 6.5, ... is **20.5**.

Sol. $a_n = a + (n - 1)d$

$$a_{10} = 2.5 + (10 - 1)(2)$$
$$= 2.5 + 9(2)$$
$$= 2.5 + 18$$
$$= 20.5$$

4. If $a_{20} - a_{12} = -32$, then the common difference of the AP is **–4**.

Sol. $a_{20} - a_{12} = -32$

$\Rightarrow \quad a + 19d - a - 11d = -32$

$\Rightarrow \quad 8d = -32$

$\Rightarrow \quad d = -4$

5. If k, $2k - 1$ and $2k + 1$ are three consecutive terms of an AP, the value of k is **3**.

Sol. $d = 2k - 1 - k$
$= k - 1$
$d = 2k + 1 - 2k + 1$
$= 2$

On comparing, we have
$$k - 1 = 2$$
$\Rightarrow \quad k = 3$

6. The sum of first five terms of the AP : 3, 7, 11, 15, ... is **55**.

Sol. Here, $d = 7 - 3 = 4$

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$
$$S_5 = \frac{5}{2}[2(3) + (5 - 1)(4)]$$
$$= \frac{5}{2}[6 + 4(4)]$$
$$= \frac{5}{2}(6 + 16) = \frac{5}{2} \times 22$$
$$= 5 \times 11 = 55$$

Assertion-Reason Type Questions

Directions (Q. Nos. 7 to 9): Each of these questions contains an assertion followed by a reason. Read them carefully, and answer the question on the basis of the following options, select the one that best describes the two statements.

(a) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.

(b) Both Assertion and Reason are true but Reason is not the correct explanation of Assertion.

(c) If Assertion is true but Reason is false.

(d) If Assertion is false but Reason is true.

7. **Assertion:** The 10th term in the series 2, 4, 6, 8,… is 20.

Reason: The series is an arithmetic progression with common difference of 3.

Sol. The correct answer is (*c*).

The Assertion is correct but the Reason is wrong as common difference is 2 and not 3.

8. **Assertion:** The sum of first 8 terms in the series 45, 43, 41, 39,… is 304.

Reason: The first term is 45 and the common difference is – 2.

Sol. The correct answer is (*a*).

The first term is 45 and the common difference is –2.

So, sum of 8 terms is 304.

Thus, both Assertion and Reason are correct and Reason is the correct explanation of Assertion.

9. **Assertion:** 89 is not a term of the list of numbers 5, 11, 17, 23,…

Reason: The first term is 5 and common difference is 6.

Sol. The correct answer is (*d*).

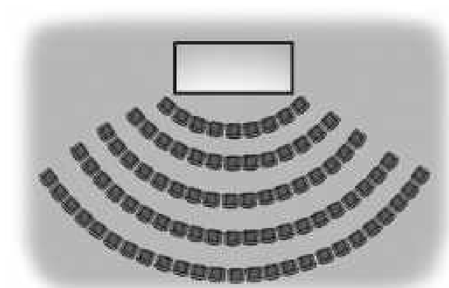
In the series, the common difference is 6.

So, 15th term will be 89.

Thus, the Assertion is wrong but Reason is correct.

Case Study Based Questions

10. In the school auditorium, annual day function is being organised. Organisers observe that, 9 guests occupied the 1st row, 13 guests occupied the 2nd row, 17 guests occupied the 3rd row and so on. There were 25 guests in the last row.



Read the above situation and answer the following questions.

(*a*) Write a series of number of guests based on their seating arrangement.

(*i*) 9, 13, 17 (*ii*) 9, 13, 17,…,25

(*iii*) 17, 13, 9 (*iv*) 25…9

Ans. (*ii*) 9, 13, 17,…,25

(*b*) Write the formula used in finding the number of rows in the auditorium.

(*i*) $a + (n - 1)d$ (*ii*) $a + l$

(*iii*) $a + (n - 1)(-d)$ (*iv*) $\frac{n}{2}(a + l)$

Ans. (*i*) $a + (n - 1)d$

(*c*) How many rows are in the auditorium?

(*i*) 7 (*ii*) 12

(*iii*) 5 (*iv*) 8

Ans. (*iii*) 5

(*d*) Write the number of guests seated in 4th row.

(*i*) 18 (*ii*) 19

(*iii*) 20 (*iv*) 21

Ans. (*iv*) 21

(*e*) Write the total number of guests seated in the auditorium.

(*i*) 80 (*ii*) 82

(*iii*) 84 (*iv*) 85

Ans. (*iv*) 85

11. In an energy drink factory, manager observes that, in the 3rd year, the factory manufactured 3000 drinks and in the 6th year, the factory manufactured 6000 drinks (assuming that the production increases uniformly by a fixed number every year). Read the above information and answer the following questions:

(*a*) Does the production of drinks form arithmetic progression series?

(*i*) Yes

(*ii*) No

(*iii*) May be sometimes

(*iv*) Not always

Ans. (*i*) Yes

(*b*) What is the production of drinks in the 1st year?

(*i*) 500 (*ii*) 1000

(*iii*) 2000 (*iv*) 3000

Ans. (*ii*) 1000

(*c*) What is the production increase uniformly by a fixed number every year?

(*i*) 1000 (*ii*) 2000

(*iii*) 500 (*iv*) 3000

Ans. (*i*) 1000

(*d*) What is the production of drinks in the 4th year?

(*i*) 3000 (*ii*) 4000

(*iii*) 5000 (*iv*) 6000

Ans. (*ii*) 4000

(*e*) What is the total production of energy drinks in first 6 years?

(*i*) 18000 (*ii*) 21000

(*iii*) 25000 (*iv*) 16000

Ans. (*ii*) 21000

Very Short Answer Type Questions

12. Are the numbers $\sqrt{18}$, $\sqrt{50}$, $\sqrt{98}$, ... in AP? If so, what is its fourth term?

Sol. We have
$$\sqrt{18} = \sqrt{9 \times 2} = 3\sqrt{2},$$
$$\sqrt{50} = \sqrt{25 \times 2} = 5\sqrt{2},$$
$$\sqrt{98} = \sqrt{49 \times 2} = 7\sqrt{2}, \text{ etc.}$$

Hence, the numbers $\sqrt{18}$, $\sqrt{50}$, $\sqrt{98}$, ..., i.e. $3\sqrt{2}$, $5\sqrt{2}$, $7\sqrt{2}$, ... form an AP with common difference, $d = 5\sqrt{2} - 3\sqrt{2} = 2\sqrt{2}$

Here, the 1st term, $a = 3\sqrt{2}$. If a_n denotes the nth term of this AP, then $a_n = a + (n - 1)d$

$$\therefore \quad a_4 = a + 3d$$
$$= 3\sqrt{2} + 3 \times 2\sqrt{2}$$
$$= 9\sqrt{2}$$
$$= \sqrt{9^2 \times 2}$$
$$= \sqrt{162}$$

$\therefore$ Its fourth term is $\sqrt{162}$.

13. If the 10th term of an AP with common difference –3 is –20, what is its first term?

Sol. Let a be the 1st term and the common difference, $d = -3$. If a_n be the nth term of this AP, then

$$a_n = a + (n - 1)d$$
$$\therefore \quad a_{10} = a + 9d = -20 \qquad \text{[Given]}$$
$$\Rightarrow \quad a - 9 \times 3 = -20$$
$$\Rightarrow \quad a = 27 - 20 = 7$$

$\therefore$ Its first term is 7.

Short Answer Type-I Questions

14. Find the sum of two middle-most terms of the AP $-\frac{4}{3}, -1, -\frac{2}{3}, \ldots, 4\frac{1}{3}$. **[CBSE 2014]**

Sol. For the given AP, 1st term, $a = -\frac{4}{3}$ and the common difference, $d = -1 + \frac{4}{3} = \frac{1}{3}$

If a_n denote the nth term of the AP and n be the total number of terms of the given AP, then

$$a_n = a + (n-1)d$$

$$\Rightarrow \quad a_n = -\frac{4}{3} + \frac{n-1}{3}$$

$$\Rightarrow \quad \frac{13}{3} = \frac{-4}{3} + \frac{n-1}{3}$$

$$\Rightarrow \quad \frac{n-1}{3} = \frac{13+4}{3} = \frac{17}{3}$$

$$\therefore \quad n = 17 + 1 = 18$$

$\therefore$ The total number of terms of the given AP is 18, which is even.

$\therefore$ There are two middle terms of this AP which are $a_{\frac{18}{2}}$ and $a_{\frac{18}{2}+1}$, i.e. a_9 and a_{10}.

Now,
$$a_9 = -\frac{4}{3} + \frac{9-1}{3} = \frac{-4}{3} + \frac{8}{3} = \frac{-4+8}{3} = \frac{4}{3}$$

$$a_{10} = a_9 + d = \frac{4}{3} + \frac{1}{3} = \frac{5}{3}$$

$$\therefore \quad a_9 + a_{10} = \frac{4}{3} + \frac{5}{3} = \frac{9}{3} = 3$$

$\therefore$ The sum of two middle most terms of the AP is 3.

15. In an AP, the sum of n terms is $\frac{3n^2}{2} + \frac{13n}{2}$. Find its 25th term. **[CBSE 2015, 2006C]**

Sol. We have

$$S_n = \frac{3n^2}{2} + \frac{13n}{2}$$

where S_n denotes the sum of n terms of the AP. Let a_n denote the nth terms of the AP.

Then
$$a_n = S_n - S_{n-1}$$
$$= \frac{3n^2}{2} + \frac{13n}{2} - \frac{3(n-1)^2}{2} - \frac{13(n-1)}{2}$$
$$= \frac{3n^2 + 13n - 3n^2 - 3 + 6n - 13n + 13}{2}$$
$$= \frac{6n - 3 + 13}{2}$$
$$= \frac{6n + 10}{2}$$
$$= 3n + 5$$

$$\therefore \quad a_n = 3n + 5$$

$$\Rightarrow \quad a_{25} = 3 \times 25 + 5 = 75 + 5 = 80$$

$\therefore$ Its 25th term is 80.

Short Answer Type-II Questions

16. The fourth term of an AP is zero. Prove that the 25th term of the AP is three times its 11th term. **[CBSE 2016]**

Sol. Let a be the 1st term and d be its common difference. If a_n denote the nth term of the AP, then

$$a_n = a + (n-1)d$$

It is given that $a_4 = 0$

$$\Rightarrow \quad a + 3d = 0$$

$$\therefore \quad a = -3d \qquad \ldots(1)$$

Now,
$$a_{25} = a + 24d = -3d + 24d \quad \text{[From (1)]} = 21d$$

and
$$a_{11} = a + 10d = -3d + 10d = 7d$$

$$\therefore \quad \frac{a_{25}}{a_{11}} = \frac{21d}{7d} = 3$$

$$\therefore \quad a_{25} = 3a_{11}$$

Hence, proved.

17. Find the sum of all possible integral multiples of 3, which are less than 200.

Sol. Numbers which are less than 200 but multiples of 3 are 3, 6, 9, 12, … 198

If S be their sum, then

$$S = 3 + 6 + 9 + \ldots + 198$$
$$= 3(1 + 2 + 3 + \ldots + 66)$$
$$= 3 \times \frac{66 \times 67}{2}$$

$$= 99 \times 67$$
$$= 6633$$

$\therefore$ The required sum is 6633.

18. How many terms of the AP 1, 4, 7, … are needed to give the sum 715?

Sol. Let a be the 1st term and d be the common difference of the given AP. Then $a = 1$ and $d = 4 - 1 = 3$. If S_n denote the sum of n terms of the AP, then

$$S_n = \frac{n}{2}[2a + (n-1)d]$$
$$= na + \frac{n(n-1)d}{2}$$

When $S_n = 715$, then

$$S_n = 715 = n + \frac{n(3n-3)}{2}$$
$$\Rightarrow \quad 715 = \frac{2n + 3n^2 - 3n}{2}$$
$$\Rightarrow \quad 3n^2 - n - 1430 = 0$$
$$\therefore \quad n = \frac{1 \pm \sqrt{(-1)^2 + 4 \times 3 \times 1430}}{6}$$
$$= \frac{1 \pm \sqrt{17161}}{6}$$
$$= \frac{1 \pm 131}{6}$$
$$= \frac{132}{6} \text{ or } \frac{-130}{6}$$
$$= 22 \quad \text{or } \frac{-65}{3}$$

We reject the negative value since number of terms cannot be negative.

Hence, the required term is 22.

Long Answer Type Question

19. A gentleman buys every year bank's certificates of value exceeding the last year's purchase by ₹250. After 20 years, he finds that the total value of the bank certificates purchased by him is ₹72500. Find the value of the certificates purchased by him in

(*a*) the first year and (*b*) the 13th year.

Sol. (*a*) We see that the values of the certificates in successive years form an AP with the 1st term, a and the common difference, d = ₹250. We denote the sum of n terms of this AP by S_n.

Then $$S_n = \frac{n}{2}[2a + (n-1)d]$$

Given that S_{20} = ₹72500

$$\therefore \quad \frac{20}{2}[2a + (20-1) \times 250] = 72500$$
$$\Rightarrow \quad 20a + 190 \times 250 = 72500$$
$$\Rightarrow \quad 20a + 47500 = 72500$$
$$\Rightarrow \quad 20a = 25000$$
$$\Rightarrow \quad a = \frac{25000}{20}$$
$$= 1250$$

$\therefore$ The required value of the certificate in the 1st year is ₹1250.

(*b*) Here, we shall have to find a_{13}.

We have $$a_{13} = a + (13-1)d$$
$$= 1250 + 12 \times 250$$
$$= 1250 + 3000$$
$$= 4250$$

$\therefore$ The required value of the certificate in the 13th year is ₹4250.

Let's Compete

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Multiple-Choice Questions

1. The 10th term of the AP –1.0, –1.5, –2.0, … is

(*a*) 3.5 (*b*) –5.5
(*c*) 5.5 (*d*) –6.5

[CBSE SP 2012]

Sol. (*b*) –5.5

Here, the 1st term, $a = -1$, common difference, $d = -1.5 + 1.0 = -0.5$

Let a_n denote the nth term.

Then $$a_n = a + (n-1)d$$
$$\therefore \quad a_{10} = a + 9d$$
$$= -1 - 9 \times 0.5$$
$$= -1 - 4.5$$
$$= -5.5$$

2. In an AP, if the common difference, $d = -\frac{1}{2}$, the number of terms, $n = 37$ and the sum to 37 terms is $425\frac{1}{2}$, then the first term is equal to

(*a*) $\frac{41}{2}$ (*b*) $\frac{1}{2}$
(*c*) $-\frac{1}{2}$ (*d*) $-\frac{41}{2}$

Sol. (*a*) $\frac{41}{2}$

Let a be the 1st term.

If S_n denote the sum of n terms, then

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$= an + \frac{n(n-1)d}{2} \qquad \ldots(1)$$

It is given that $S_{37} = 425\frac{1}{2} = \frac{851}{2}$

$$\therefore \quad \frac{851}{2} = 37a + \frac{37 \times 36}{2} \times \frac{-1}{2}$$

[From (1)]

$$= 37a - \frac{37 \times 36}{4}$$

$$= 37a - \frac{1332}{4}$$

$$= 37a - 333$$

$$\Rightarrow \quad 37a = \frac{851}{2} + 333$$

$$= \frac{851 + 666}{2}$$

$$= \frac{1517}{2}$$

$$\therefore \quad a = \frac{1517}{2 \times 37}$$

$$= \frac{1517}{74} = \frac{41}{2}$$

3. If $3k$, $8k - 1$ and $5k + 1$ are three consecutive terms of an AP, then the value of k is

(*a*) $\frac{1}{4}$ (*b*) $\frac{2}{5}$

(*c*) $\frac{3}{8}$ (*d*) $\frac{3}{4}$

Sol. (*c*) $\frac{3}{8}$

Clearly, the common difference of any two successive terms will be constant.

$$\therefore \quad 8k - 1 - 3k = 5k + 1 - (8k - 1)$$

$$\Rightarrow \quad 8k - 1 - 3k = 5k + 1 - 8k + 1$$

$$\Rightarrow \quad 5k - 1 = -3k + 2$$

$$\Rightarrow \quad 8k = 3$$

$$\Rightarrow \quad k = \frac{3}{8}$$

4. If 3 times the 3rd term of an AP is equal to 4 times its 4th term, then its 7th term is equal to

(*a*) 18 (*b*) 11

(*c*) 7 (*d*) 0

Sol. (*d*) 0

Let a be the 1st term and d be the common difference of the AP. Let a_n be the nth term of the AP.

Given that $3a_3 = a_4$

$$\therefore \quad 3(a + 2d) = 4(a + 3d)$$

$$\Rightarrow \quad a + 6d = 0 \qquad \ldots(1)$$

$$\therefore \quad a_7 = a + 6d = 0 \qquad \text{[From (1)]}$$

5. The sum of $1^2 - 2^2 + 3^2 - 4^2 + 5^2 - 6^2 + \ldots$ to 50 terms is equal to

(*a*) -1175 (*b*) -1275

(*c*) -1375 (*d*) -125

Sol. (*b*) -1275

We have, $S = 1^2 - 2^2 + 3^2 - 4^2 + \ldots$ to 50 terms.

Now, writing two consecutive terms in a bracket to form a group, we have

$S = (1^2 - 2^2) + (3^2 - 4^2) + (5^2 - 6^2) + \ldots$ to 25 groups.

$= \{(1 - 2)(1 + 2)\} + \{(3 - 4)(3 + 4)\} + \{(5 - 6)(5 + 6)\} + \ldots$ to 25 groups

$= -1 \times (3 + 7 + 11 + \ldots$ to 25 terms$)$

$= -1 \times \frac{25}{2}(2 \times 3 + 24 \times 4)$

$= -1 \times (75 + 1200)$

$= -1275$

6. If the sum of the first p even numbers is equal to q times the sum of the first p odd numbers, then q is equal to

(*a*) $1 - \frac{1}{1+p}$ (*b*) $1 + \frac{1}{1+p}$

(*c*) $1 - \frac{1}{p}$ (*d*) $1 + \frac{1}{p}$

Sol. (*d*) $1 + \frac{1}{p}$

It is given that

$2 + 4 + 6 + 8 + \ldots$ to p terms

$= q(1 + 3 + 5 + 7 + \ldots$ to p terms$) \qquad \ldots(1)$

Now, for the 1st AP, the 1st term = 2 and the common difference = 4 – 2 = 2 and for the 2nd AP, the 1st term = 1 and the common difference = 3 – 1 = 2.

Also, for each AP, the total number of terms = p.

$\therefore$ From (1), we have

$$\frac{p}{2}\{2 \times 2 + (p - 1)2\} = q\{2 \times 1 + (p - 1)2\} \times \frac{p}{2}$$

$$\Rightarrow \quad 4 + 2p - 2 = 2q + 2pq - 2q$$

$$\Rightarrow \quad 2 + 2p = 2pq$$

$$\Rightarrow \quad 1 + p = pq$$

$$\Rightarrow \quad q = \frac{1+p}{p}$$

$$\Rightarrow \quad q = \frac{1}{p} + 1$$

$$\therefore \quad q = 1 + \frac{1}{p}$$

7. If S_2 and S_3 be the sums of two and three terms respectively of an AP with the first term a, then

(*a*) $3S_2 - S_3 = 3a$ (*b*) $S_3 - 3S_2 = 3a$

(*c*) $3S_2 + S_3 = 3a$ (*d*) $S_3 + 3S_2 = 5a$

Sol. (*a*) $3S_2 - S_3 = 3a$

Let a be the 1st term, d be the common difference and S_n be the sum of n terms of the AP.

Then $\quad S_n = \frac{n}{2}\{2a + (n-1)d\}$

$$\therefore \quad S_2 = \frac{2}{2}(2a + d)$$

$$\Rightarrow \quad S_2 = 2a + d \qquad \ldots(1)$$

and $\quad S_3 = \frac{3}{2}(2a + 2d)$

$$\Rightarrow \quad S_3 = 3a + 3d \qquad \ldots(2)$$

Multiplying (1) by 3, we get

$$3S_2 = 6a + 3d \qquad \ldots(3)$$

Subtracting (2) from (3) to eliminate d, we get

$$3S_2 - S_3 = 3a$$

8. The sum of three consecutive terms of an increasing AP is 36 and the product of its first and third terms is 23. Then its third term is

(*a*) 1 (*b*) 12

(*c*) 11 (*d*) 23

Sol. (*d*) 23

Let $\alpha - d$, α and $\alpha + d$, where $d > 0$, be three consecutive terms of an increasing AP. Then, according to the problem, we have

$$(\alpha - d) + \alpha + (\alpha + d) = 36$$

$$\Rightarrow \quad 3\alpha = 36$$

$$\Rightarrow \quad \alpha = 12$$

Also, $(\alpha - d)(\alpha + d) = 23$ [Given]

$$\Rightarrow \quad \alpha^2 - d^2 = 23$$

$$\Rightarrow \quad 12^2 - d^2 = 23$$

$$\Rightarrow \quad d^2 = 144 - 23$$

$$\Rightarrow \quad d^2 = 121$$

$$\Rightarrow \quad d = \pm 11$$

Hence, the required 3rd term is $\alpha + d = 12 + 11 = 23$

9. The sum of all two-digit odd positive numbers is

(*a*) 2250 (*b*) 2475

(*c*) 2275 (*d*) 2450

Sol. (*b*) 2475

We see that the two-digit odd numbers are 11, 13, 15, 17, … 99 which form an AP with the 1st term, $a = 11$ and the common difference, $d = 13 - 11 = 2$.

Let n denote the nth term of the AP. If n be the total number of terms of this AP, then

$$a_n = 99$$

$$\Rightarrow \quad a + (n-1)d = 99$$

$$\Rightarrow \quad 11 + (n-1)2 = 99$$

$$\Rightarrow \quad n - 1 = \frac{99 - 11}{2} = 44$$

$$\therefore \quad n = 44 + 1 = 45$$

Hence, the total number of terms of this AP is 45.

Now, 11 + 13 + 15 + 17 +… to 45 terms

$$S_n = \frac{45}{2} \times \{2a + (45-1)d\}$$

$$= \frac{45}{2}\{2 \times 11 + 44 \times 2\}$$

$$= 45 \times 11 + 45 \times 22 \times 2$$

$$= 45\,(11 + 44)$$

$$= 45 \times 55$$

$$= 2475$$

10. If the ratio of the 16th term to the 4th term of an AP is 1 : 4, then the ratio of the 72nd term to its 84th term is

(*a*) 16 : 13 (*b*) 4 : 5

(*c*) 13 : 16 (*d*) 5 : 4

Sol. (*c*) 13 : 16

Let a be the 1st term, d be the common difference of the AP, and a_n denote the nth term of the AP.

Then $\quad a_n = a + (n-1)d$

Given that $\quad \frac{a_{16}}{a_4} = \frac{1}{4}$

$$\therefore \quad \frac{a + 15d}{a + 3d} = \frac{1}{4}$$

$$\Rightarrow \quad 4a + 60d = a + 3d$$

$$\Rightarrow \quad 3a = -57d$$

$$\therefore \quad a = -\frac{57}{3}d = -19d \qquad \ldots(1)$$

$$\therefore \quad \frac{a_{72}}{a_{84}} = \frac{a + 71d}{a + 83d}$$

$$= \frac{-19d + 71d}{-19d + 83d} \qquad \text{[From (1)]}$$

$$= \frac{52}{64} = \frac{13}{16}$$

$\therefore \quad a_{72} : a_{84} = 13 : 16$

Value-based Questions (Optional)

(Page 101)

1. Yasmeen saves ₹32 during the first month, ₹36 in the second month and ₹40 in the third month. If she continues to save in this manner, in how many months she will save ₹2000, which she has intended to give for the college fee of her maid's daughter. What value is reflected here? **[CBSE 2014]**

Sol. Let n be the required number of months.

It is given that

₹32 + ₹36 + ₹40 + … to n term = ₹2000

Now, numbers 32, 36, 40, … form an AP with the 1st term, $a = 32$ and the common difference, $d = 36 - 32 = 4$

$\therefore$ The sum $S_n = 32 + 36 + 40 + \dots$ to n terms

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\therefore \quad 2000 = na + \frac{n(n-1)d}{2}$$

$$\Rightarrow \quad 2000 = 32n + 2n(n-1)$$

$$\Rightarrow \quad 2000 = 2n^2 + 30n$$

$$\Rightarrow \quad 2n^2 + 30n - 2000 = 0$$

$$\Rightarrow \quad n^2 + 15n - 1000 = 0$$

$$\Rightarrow \quad n^2 + 40n - 25n - 1000 = 0$$

$$\Rightarrow \quad n(n+40) - 25(n+40) = 0$$

$$\Rightarrow \quad (n-25)(n+40) = 0$$

$$\therefore \quad n = 25 \qquad [\because n > 0]$$

Hence, the required number of months is 25.

Values: Helpfulness and sympathy over the needy people.

2. Reshma wanted to save at least ₹6500 for sending her daughter to school next year (after 12 months). She saved ₹450 in the 1st month and raised her saving by ₹20 every next month. How much will she be able to save in the next 12 months? Will she be able to send her daughter to the school next year? What value is reflected in this question?

Sol. Savings in the 1st month, 2nd month, 3rd month, 4th month, … are ₹450, ₹470, ₹490, ₹510, … respectively which are in AP with 1st term, a = ₹450 and the common difference, d = ₹20, If S_n denote the sum of n terms of this AP, then

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

We now find S_{12}.

Now, $$S_{12} = \frac{12}{2}[2 \times 450 + 11 \times 20]$$

$$= 12 \times 450 + 66 \times 20$$

$$= 5400 + 1320 = 6720$$

$\therefore \quad S_{12}$ = ₹6720

which is greater than ₹6500.

Hence, she will be able to send her daughter to a school.

Value: Importance of education for all.

3. The sports teacher of a school wants the students to make a pyramid formation for one of the items for the sports day. She wants all 184 students of the school to participate in this item. She makes the first row of 16 students, the row above it of 14 students and the next higher row of 12 students and so on.

(*a*) Will she be able to involve all the 184 students? If yes, then how many rows will the pyramid consist of?

(*b*) What are the values exhibited by the sports teacher?

Sol. (*a*) We find the sum 16 + 14 + 12 + 10 + … + 2

Now, 16, 14, 12, 10, … , 2 form an AP with 1st term, $a = 16$, common difference, $d = 14 - 16 = -2$.

Let a_n be its nth term and S_n be the sum to n terms.

Then, $$a_n = a + (n-1)d$$

$$= 16 - (n-1)2 = 18 - 2n$$

When $a_n = 2$, then

$$18 - 2n = 2$$

$$\Rightarrow \quad n = 8$$

$$\therefore \quad S_8 = \frac{8}{2}[2 \times 16 - 7 \times 2]$$

$$= 8 \times 16 - 8 \times 7$$

$$= 128 - 56$$

$$= 72$$

$\therefore$ 72 students are needed to form the pyramid. So, with 184 students, it is possible to form a pyramid which will consist of 8 rows.

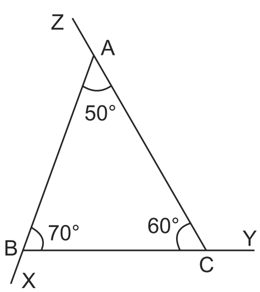
(*b*) Empathy and decision making.

6

Triangles

Checkpoint (Page 104)

1. Name the interior angles of the triangle ABC given below. Also, write the measurements of the interior angles.



Sol. The interior angles of the triangle are

$\angle ABC = 70°$, $\angle BCA = 60°$ and $\angle CAB = 50°$.

2. What is the sum of

(*a*) the three interior angles of a triangle?

(*b*) the three exterior angles of a triangle?

Sol. (*a*) The sum of the three interior angles of a triangle is 180° by angle sum property of the triangle.

(*b*) The sum of the three exterior angles of a triangle is 360°.

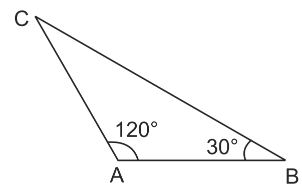
3. Can you draw a triangle ABC with sides AB = 3 cm, BC = 4 cm and CA = 8 cm? If not, explain why.

Sol. No, since in this case AB + BC = (3 + 4) cm = 7 cm < CA. This is because the sum of any two sides of a triangle is greater than its third side.

4. What is the relation between the three sides AB, BC and CA of an equilateral triangle ABC?

Sol. For an equilateral triangle, since all the three sides are of equal length, hence, AB = BC = CA.

5. In ΔABC, if $\angle CAB = 120°$ and $\angle ABC = 30°$,



(*a*) what is the measurement of $\angle ACB$?

(*b*) what is the relation between the sides AB and AC?

(*c*) what kind of triangle is it – (*i*) an equilateral triangle, (*ii*) an isosceles triangle, (*iii*) a scalene triangle, (*iv*) an obtuse-angled triangle, (*v*) an acute-angled triangle or (*vi*) a right-angled triangle?

Sol. (*a*) By angle sum property of a triangle, we know that

$$\angle ABC + \angle CAB + \angle ACB = 180°$$

$$\Rightarrow \quad 30° + 120° + \angle ACB = 180°$$

$$\Rightarrow \quad \angle ACB = 180° - 150° = 30°$$

which is the required measurement of $\angle ACB$.

(*b*) Since $\angle ABC = \angle ACB = 30°$

$$\therefore \quad AB = AC$$

which is the required relation.

(*c*) (*i*) No, since for an equilateral triangle, all the three sides are of equal length.

(*ii*) Yes, since the two base angles are equal, hence it is an isosceles triangle.

(*iii*) No, since the length of three sides are not unequal.

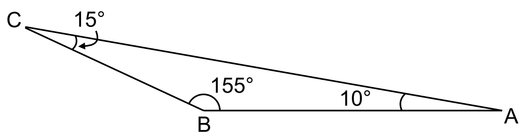
(*iv*) Yes, since one angle is 120° which is obtuse, hence it is an obtuse-angled triangle.

(*v*) No, since all the angles of this triangle are not acute.

(*vi*) No, since no angle of this triangle is right-angled.

6. If in ΔABC, $\angle BAC = 10°$ and $\angle ACB = 15°$, what is the relation between the sides AB, BC and AC of ΔABC?

Sol.



In ΔABC,

$$\angle BAC + \angle ACB + \angle ABC = 180°$$

[By angle sum property of a triangle]

$$\Rightarrow \quad 10° + 15° + \angle ABC = 180°$$

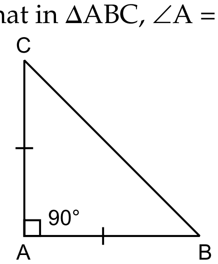
$$\Rightarrow \quad \angle ABC = 180° - 25° = 155°$$

Since $\angle BAC < \angle ACB < \angle ABC$,

$\therefore$ BC < AB < AC [since the side opposite to the smaller angle of a triangle is less than the side opposite to the greater angle]

7. In a right triangle ABC, $\angle A = 90°$ and ΔABC is also an isosceles triangle with AB = AC, what are the measures of $\angle ABC$ and $\angle ACB$?

Sol. It is given that in ΔABC, $\angle A = 90°$ and AB = AC.



$$\therefore \quad \angle ABC = \angle ACB = \theta \text{ (say)}$$

Since $\quad \angle A + \angle B + \angle C = 180°$

[By angle sum property of a triangle]

$$\Rightarrow \quad 90° + \theta + \theta = 180°$$

$$\Rightarrow \quad 2\theta = 180° - 90° = 90°$$

$$\Rightarrow \quad \theta = \frac{90°}{2} = 45°$$

$$\therefore \quad \angle ABC = \angle ACB = 45°$$

are the required measures of the angles.

8. Can you construct a unique triangle ABC with

(*a*) $\angle ABC = 95°$, $\angle ACB = 69°$ and $\angle BAC = 16°$?

(*b*) $\angle ABC = 70°$, $\angle ACB = 80°$ and $\angle BAC = 40°$?

Explain why.

Sol. (*a*) No, since when three angles are given, we can draw infinite number of similar triangles, although the sum of three angles in each case is 180°.

(*b*) No, since $\angle ABC + \angle ACB + \angle BAC = 70° + 80° + 40° = 190° > 180°$.

Hence, in this case, not a single triangle can be constructed.

9. In each of the following problems, the measures of the three angles of a triangle are given. In each case, identify the type of triangle on the basis of three angles.

(*a*) 60°, 60°, 60° (*b*) 75°, 15°, 90°

(*c*) 130°, 20°, 30° (*d*) 45°, 95°, 40°

Sol. (*a*) Since each angle is acute being less than 90°, hence, the triangle is an acute-angled triangle.

(*b*) In this case, one of the angles is 90° which is a right angle. Hence, the triangle is a right-angled triangle.

(*c*) In this case, one of the angles is 130° which is obtuse, since it greater than 90°. Hence, the triangle is obtuse-angled triangle.

(*d*) In this case, one of the angle is 95° which is also obtuse, since it is greater than 90°. Hence, the triangle is obtuse-angled triangle.

10. Find the angles of a triangle, which are in the ratio 3 : 5 : 10.

Sol. Let the three angle be $(3k)°$, $(5k)°$ and $(10k)°$ where k is a non-zero positive constant.

$\therefore$ By angle sum property of a triangle, we have

$$3k + 5k + 10k = 180$$

$$\Rightarrow \quad 18k = 180$$

$$\Rightarrow \quad k = \frac{180}{18} = 10$$

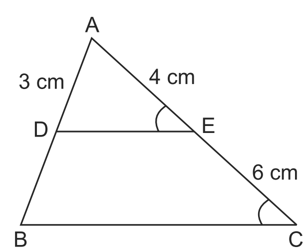
Hence, the required angles are $(3 \times 10)°$, $(5 \times 10)°$ and $(10 \times 10)°$, i.e. 30°, 50° and 100°.

Milestone 1

(Page 109)

Multiple-Choice Questions

1. In the given figure, if $\angle AED = \angle ACB$, then DB is equal to



(*a*) 4.5 cm (*b*) 4 cm

(*c*) 3 cm (*d*) 3.5 cm

Sol. (*a*) 4.5 cm

Given that D and E are two points on AB and AC of ΔABC, respectively such that $\angle$AED = $\angle$ACB.

Also, AD = 3 cm, AE = 4 cm, EC = 6 cm.

To find DB.

Given that $\angle$AED = $\angle$ACB. But these two are corresponding angles with respect to line segments DE and BC and their transversal AC. Hence, DE $\parallel$ BC.

By the basic proportionality theorem, we have

$$\frac{AD}{DB} = \frac{AE}{EC}$$

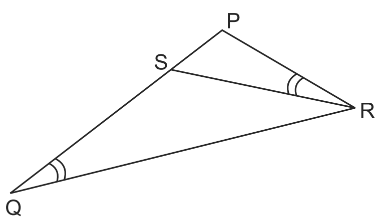
$$\Rightarrow \quad \frac{3}{DB} = \frac{4}{6} = \frac{2}{3}$$

$$\Rightarrow \quad 2\,DB = 9$$

$$\Rightarrow \quad DB = \frac{9}{2} = 4.5$$

Hence, the required length of DB is 4.5 cm.

2. In the given figure, $\angle$PQR = $\angle$PRS. If PR = 6 cm and PS = 2 cm, then PQ is equal to

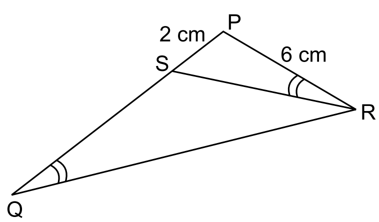


(*a*) 12 cm (*b*) 16 cm

(*c*) 18 cm (*d*) 20 cm

Sol. (*c*) 18 cm

Given that S is a point on the side PQ of a triangle PQR such that $\angle$PRS = $\angle$PQR.



Also, PR = 6 cm and PS = 2 cm.

To find the length of PQ.

In ΔPQR and ΔPRS, we have

$\angle$PQR = $\angle$PRS [Given]

$\angle$QPR = $\angle$RPS [Common]

$\therefore$ $\angle$PRQ = $\angle$PSR [Remaining angles]

By AAA similarity criterion, we have

$$\therefore \quad \Delta PQR \sim \Delta PRS$$

$$\Rightarrow \quad \frac{PR}{PS} = \frac{QR}{RS} = \frac{PQ}{PR} \quad \text{[By BPT]}\ldots(1)$$

Now, from (1), we get

$$\frac{PR}{PS} = \frac{PQ}{PR}$$

$$\Rightarrow \quad \frac{6}{2} = \frac{PQ}{6}$$

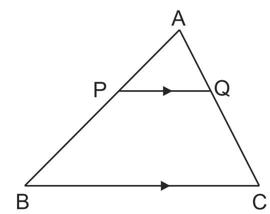
$$\Rightarrow \quad 2PQ = 36$$

$$\Rightarrow \quad PQ = \frac{36}{2} = 18$$

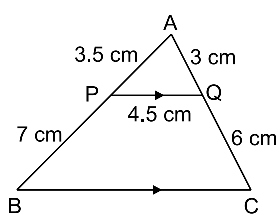
$\therefore$ Required length of PQ is 18 cm.

Very Short Answer Type Questions

3. In the given figure, P and Q are points on the sides AB and AC respectively of ΔABC such that AP = 3.5 cm, PB = 7 cm, AQ = 3 cm, QC = 6 cm and PQ = 4.5 cm, find BC. **[CBSE 2008]**



Sol. Given that P and Q are two points on the sides AB and AC respectively of ΔABC. Also, AP = 3.5 cm, PB = 7 cm, AQ = 3 cm, QC = 6 cm and PQ = 4.5 cm.



To find the length of BC.

We see that $\frac{AP}{PB} = \frac{3.5}{7} = \frac{1}{2}$

and $\frac{AB}{QC} = \frac{3}{6} = \frac{1}{2}$

$$\therefore \quad \frac{AP}{PB} = \frac{AQ}{QC}$$

$\therefore$ By the converse of basic proportionality theorem, we have PQ $\parallel$ BC.

$\therefore$ $\angle$AQP = corresponding $\angle$ACB

and $\angle$APQ = corresponding $\angle$ABC

Also, $\angle PAQ = \angle BAC$ [Common]

$\therefore$ $\Delta APQ \sim \Delta ABC$

$\Rightarrow$ $\dfrac{AP}{AB} = \dfrac{AQ}{AC} = \dfrac{PQ}{BC}$ [By BPT]

$\Rightarrow$ $\dfrac{3.5}{3.5+7} = \dfrac{3}{3+6} = \dfrac{4.5}{BC}$

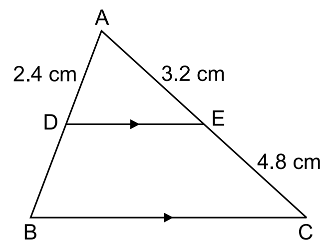
$\Rightarrow$ $\dfrac{1}{3} = \dfrac{4.5}{BC}$

$\Rightarrow$ $BC = 4.5 \times 3 = 13.5$

Hence, the required length of BC is 13.5 cm.

4. In ΔABC, D and E are the points on the sides AB and AC respectively such that DE $\parallel$ BC. If AD = 2.4 cm, AE = 3.2 cm and CE = 4.8 cm, find BD. **[CBSE SP 2011]**

Sol. Given that D and E are two points on the sides AB and AC respectively of ΔABC such that DE $\parallel$ BC.



$\therefore$ By basic proportionality theorem, we have

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$\Rightarrow$ $\dfrac{2.4}{DB} = \dfrac{3.2}{4.8}$

$\Rightarrow$ $\dfrac{2.4}{DB} = \dfrac{2}{3}$

$\Rightarrow$ $2DB = 2.4 \times 3 = 7.2$

$\Rightarrow$ $DB = \dfrac{7.2}{2} = 3.6$

Hence, the required length of DB is 3.6 cm.

Short Answer Type-I Questions

5. In ΔABC, D and E are the points on the sides AB and AC respectively such that AD × EC = AE × DB. Prove that DE $\parallel$ BC.

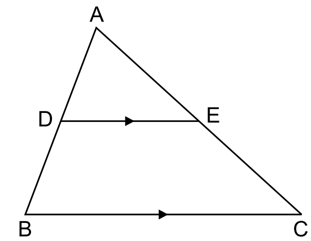
[CBSE SP 2010, 2011]

Sol. Given that D and E are the points on the sides AB and AC respectively such that AD × EC = AE × DB.

To prove that DE $\parallel$ BC.

We have

$$AD \times EC = AE \times DB$$



$\Rightarrow$ $\dfrac{AD}{DB} = \dfrac{AE}{EC}$

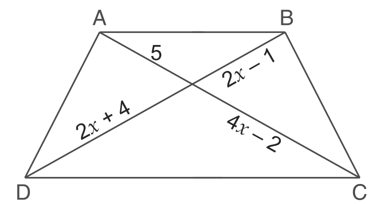
$\therefore$ By the converse of basic proportionality theorem, we get

$$DE \parallel BC$$

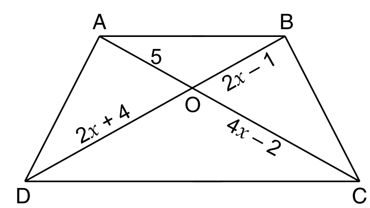
Hence, proved.

6. In the given figure, AB $\parallel$ DC. Find the value of x.

[CBSE SP 2010]



Sol. Given that ABCD is a trapezium in which AB $\parallel$ DC. Let the two diagonals AC and DB intersect each other at O such that AO = 5, CO = $4x - 2$, BO = $2x - 1$ and DO = $2x + 4$ units. To find the value of x.



In ΔAOB and ΔCOD, we have

$\angle BAO = \angle DCO$

[Alternate angles, $\because$ AB $\parallel$ DC]

$\angle ABO = \angle CDO$ [Alternate $\angle$s]

Hence, $\angle AOB = \angle COD$

$\therefore$ $\Delta AOB \sim \Delta COD$

$\Rightarrow$ $\dfrac{AO}{CO} = \dfrac{BO}{DO}$ [By BPT]

$\Rightarrow$ $\dfrac{5}{4x-2} = \dfrac{2x-1}{2x+4}$

$\Rightarrow$ $\dfrac{5}{2(2x-1)} = \dfrac{2x-1}{2(x+2)}$

$\Rightarrow$ $\dfrac{5}{2x-1} = \dfrac{2x-1}{x+2}$

$\Rightarrow$ $5x + 10 = (2x-1)^2 = 4x^2 - 4x + 1$

$$\Rightarrow \quad 4x^2 - 9x - 9 = 0$$
$$\Rightarrow \quad 4x^2 + 3x - 12x - 9 = 0$$
$$\Rightarrow \quad x(4x + 3) - 3(4x + 3) = 0$$
$$\Rightarrow \quad (x - 3)(4x + 3) = 0$$
$\therefore$ Either $\quad x - 3 = 0$

$$\Rightarrow \quad x = 3$$
or $\quad 4x + 3 = 0$

$$\Rightarrow \quad x = -\frac{3}{4}$$

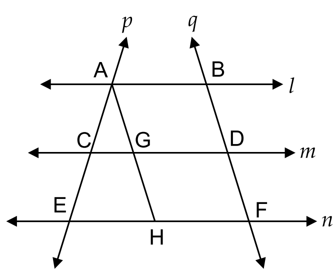
Now, when $x = -\frac{3}{4}$, OB $= 2x - 1$ becomes negative which is not possible.

$\therefore$ $x = 3$ which is the required value.

Short Answer Type-II Questions

7. If three or more parallel lines are intersected by two transversals, prove that the intercepts made by them on the transversals are proportional.

Sol. Given that l, m, n are three lines parallel to each other and these lines are intersects by two transversals p and q at A, C, E and B, D, F respectively forming the intercepts AC, CE, BD and DF.



To prove that
$$\frac{AC}{CE} = \frac{BD}{DF}$$

Construction: Draw the line segment AH parallel to the line q meeting the lines m and n at G and H respectively.

$\because \quad AB \parallel GD \quad$ [Given]

and $\quad AG \parallel BD \quad$ [By construction]

Hence, the figure ABDG is a parallelogram.

$\therefore \quad AG = BD \quad$ ...(1)

Again, $\quad GD \parallel HF \quad$ [Given]

and $\quad GH \parallel DF \quad$ [By construction]

$\therefore$ GDFH is a parallelogram.

$\therefore \quad GH = DF \quad$ ...(2)

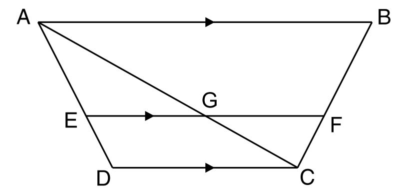
Now, in ΔAEH, $CG \parallel EH$

$\therefore$ By basic proportionality theorem, we have
$$\frac{AC}{CE} = \frac{AG}{GH} = \frac{BD}{DF}$$
[From (1) and (2)]
$$\Rightarrow \quad \frac{AC}{CE} = \frac{BD}{DF}$$

Hence, proved.

8. Prove that any line parallel to parallel sides of a trapezium divides the non-parallel sides proportionally. **[CBSE SP 2003, 2006, 2010]**

Sol. Given that ABCD is a trapezium with AB $\parallel$ DC. EF is a line segment parallel to AB and DC. To prove that $\frac{AE}{ED} = \frac{BF}{FC}$.

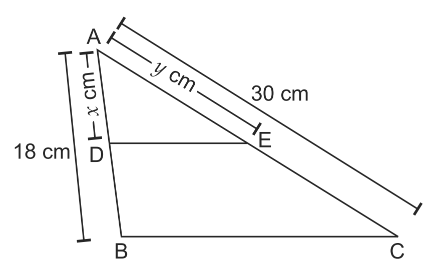


Construction: We join AC intersecting EF at G.

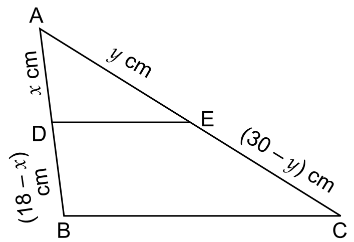
In ΔADC, we have
$$EG \parallel DC$$
$\therefore$ By basic proportionality theorem, we have
$$\frac{AE}{ED} = \frac{AG}{GC} \qquad \text{...(1)}$$
Again, in ΔACB, we have
$$GF \parallel AB$$
$\therefore$ By basic proportionality theorem, we have
$$\frac{AG}{GC} = \frac{BF}{FC} \qquad \text{...(2)}$$
$\therefore$ From (1) and (2), we have
$$\frac{AE}{ED} = \frac{BF}{FC}$$
Hence, proved.

Long Answer Type Questions

9. In the given figure, D is a point on AB and E is a point on AC of a triangle ABC such that DE $\parallel$ BC. If AD : DB = 1 : 2, AB = 18 cm and AC = 30 cm, find AE, EC, AD and DB.



Sol. Given that D and E are points on the sides AB and AC respectively of a triangle ABC such that DE $\parallel$ BC.



Also, $\frac{AD}{DB} = \frac{1}{2}$, AB = 18 cm and AC = 30 cm.

Let AD = x cm and AE = y cm

Then from the figure,

$$DB = (18 - x) \text{ cm}$$

and $$EC = (30 - y) \text{ cm}$$

To find AE, EC, AD and DB.

In ΔABC, DE $\parallel$ BC [Given]

$\therefore$ By basic proportionality theorem, we have

$$\frac{AE}{EC} = \frac{AD}{DB} = \frac{1}{2} \qquad \text{[Given]}$$

$$\Rightarrow \quad \frac{y}{30-y} = \frac{x}{18-x} = \frac{1}{2} \qquad \ldots(1)$$

$\therefore$ From (1), we have

$$30 - y = 2y$$

$$\Rightarrow \quad 3y = 30$$

$$\Rightarrow \quad y = \frac{30}{3} = 10$$

and $$18 - x = 2x$$

$$\Rightarrow \quad 3x = 18$$

$$\Rightarrow \quad x = \frac{18}{3} = 6$$

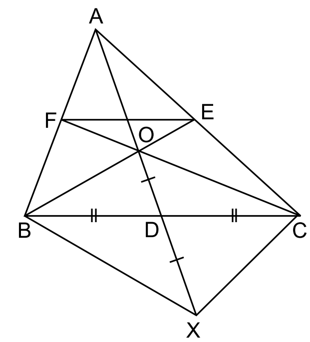
Hence, AE = y cm = 10 cm, EC = $(30 - y)$ cm = $(30 - 10)$ cm = 20 cm, AD = x cm = 6 cm and DB = $(18 - x)$ cm = $(18 - 6)$ = 12 cm.

$\therefore$ Required length of AE, EC, AD and DB are 10 cm, 20 cm, 6 cm and 12 cm respectively.

10. The side BC of ΔABC is bisected at D. O is any point on AD. BO and CO are produced to meet AC and AB at E and F respectively and AD is produced to X so that D is the mid-point of OX. Prove that AO : AX = AF : AB = AE : AC and show that EF $\parallel$ BC.

Sol. Given that ΔABC is a triangle, D is a point on BC such that BD = DC. O is any point on the median AD.

BO produced meets AC at E and CO produced meets AB at F as shown in the figure. AD is produced to a point X such that OD = DX.



To prove that

(*i*) $\frac{AO}{AX} = \frac{AF}{AB} = \frac{AE}{AC}$ and (*ii*) EF $\parallel$ BC.

Construction: We join FE, BX and CX.

(*i*) We see that in the quadrilateral OBXC, OD = DX and BD = DC, i.e. the two diagonals OX and BC bisect each other at D. Hence, the quadrilateral OBXC is a parallelogram.

$\therefore$ OC $\parallel$ BX

i.e., OF $\parallel$ BX

and OB $\parallel$ CX

i.e., OE $\parallel$ CX

Now, in ΔABX,

$\because$ BX $\parallel$ OF

$\therefore$ By basic proportionality theorem, we have

$$\frac{AF}{FB} = \frac{AO}{OX}$$

$$\Rightarrow \quad \frac{AF}{AF + FB} = \frac{AO}{AO + OX}$$

$$\Rightarrow \quad \frac{AF}{AB} = \frac{AO}{AX} \qquad \ldots(1)$$

Also, in ΔACX,

$\therefore$ By basic proportionality theorem, we have

$$\frac{AE}{EC} = \frac{AO}{OX}$$

$$\Rightarrow \quad \frac{AE}{AE + EC} = \frac{AO}{AO + OX}$$

$$\Rightarrow \quad \frac{AE}{AC} = \frac{AO}{AX} \qquad \ldots(2)$$

$\therefore$ From (1) and (2), we have

$$\frac{AO}{AX} = \frac{AF}{AB} = \frac{AE}{AC} \qquad \ldots(3)$$

(*ii*) Now, in ΔABC, since

$$\frac{AF}{FB} = \frac{AE}{EC} \qquad \text{[From (3)]}$$

$\therefore$ By the converse of basic proportionality theorem, we get

$$FE \parallel BC$$

Milestone 2

(Page 112)

Multiple-Choice Questions

1. In ΔABC and ΔPQR, $\angle A = \angle P$ and $\frac{AB}{PQ} = \frac{AC}{PR} = \frac{2}{3}$, then BC : QR is equal to

(*a*) 3 : 2 (*b*) 1 : 2

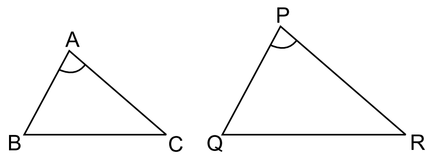
(*c*) 2 : 1 (*d*) 2 : 3

Sol. (*d*) 2 : 3

Given that in ΔABC and ΔPQR,

$$\angle A = \angle P$$

and $$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{2}{3}$$



$\therefore$ By SAS similarity criterion, we have

$$\Delta ABC \sim \Delta PQR$$

$$\Rightarrow \quad \frac{BC}{QR} = \frac{AB}{PQ} = \frac{AC}{PR} = \frac{2}{3}$$

$$\therefore \quad BC : QR = 2 : 3$$

2. In ΔABD, $\angle DAB = 90°$. C is a point on the hypotenuse BD such that $AC \perp BD$. Then AB^2 is equal to

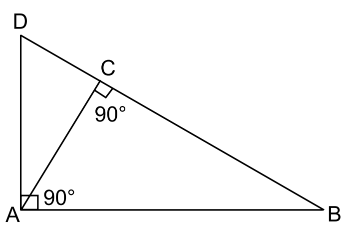
(*a*) $BC \times DC$ (*b*) $BD \times CD$

(*c*) $BC \times BD$ (*d*) $AD \times AC$

Sol. (*c*) $AC \times BD$

Given that ΔDAB is a right-angled triangle, with $\angle DAB = 90°$. C is a point on the hypotenuse BD such that $AC \perp DB$.

To find AB^2 in terms of other two sides of the triangles.



In ΔDAB and ΔACB, we have

$$\angle DAB = \angle ACB = 90°$$

$$\angle ABD = \angle CBA \qquad \text{[Common]}$$

$\therefore$ By AA similarity criterion, we have

$$\Delta DAB \sim \Delta ACB$$

$$\Rightarrow \quad \frac{DA}{AC} = \frac{DB}{AB} = \frac{AB}{CB} \qquad \ldots(1)$$

$\therefore$ From (1), we have

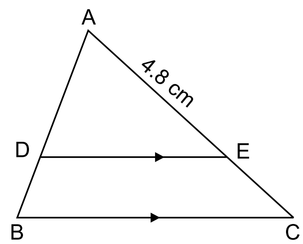
$$AB^2 = DB \times CB$$

i.e., $$AB^2 = BC \times BD$$

Very Short Answer Type Questions

3. In ΔABC, D and E are points on the sides AB and AC respectively such that $DE \parallel BC$. If $\frac{AD}{DB} = \frac{3}{2}$ and AE = 4.8 cm, find EC. **[CBSE SP 2011]**

Sol. Given that D and E are two points on the sides AB and AC respectively of ΔABC such that $DE \parallel BC$, $\frac{AD}{DB} = \frac{3}{2}$ and AE = 4.8 cm.



To find the length of EC.

Since $DE \parallel BC$, hence by the basic proportionality theorem, we have

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\Rightarrow \quad \frac{3}{2} = \frac{4.8}{EC}$$

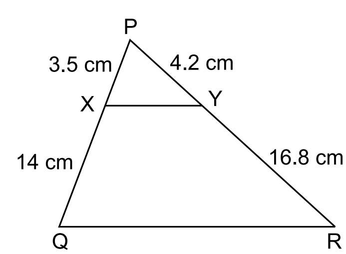
$$\Rightarrow \quad 3\,EC = 4.8 \times 2$$

$$\Rightarrow \quad EC = \frac{9.6}{3} = 3.2$$

Hence, the required length of EC is 3.2 cm.

4. X and Y are points on the sides PQ and PR respectively of a ΔPQR such that PX = 3.5 cm, XQ = 14 cm, PY = 4.2 cm and YR = 16.8 cm, then state if $XY \parallel QR$ or not.

Sol. Given that X and Y are two points on the sides PQ and PR respectively of a triangle PQR such that PX = 3.5 cm, XQ = 14 cm, PY = 4.2 cm and YR = 16.8 cm.



To test whether $XY \parallel QR$, or not.

We have $$\frac{PX}{XQ} = \frac{3.5}{14} = \frac{35}{140} = \frac{1}{4}$$

and $\dfrac{PY}{YR} = \dfrac{4.2}{16.8} = \dfrac{42}{168} = \dfrac{1}{4}$

$\therefore \quad \dfrac{PX}{XQ} = \dfrac{PY}{YR}$

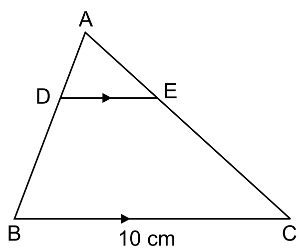
$\therefore$ By the converse of the basic proportionality theorem, we get

$$XY \parallel QR$$

Short Answer Type-I Questions

5. ABC is an equilateral triangle with points D and E on sides AB and AC respectively such that DE ∥ BC. If BC = 10 cm and $\dfrac{AE}{EC} = \dfrac{1}{3} = \dfrac{AD}{DB}$, then find the length of AE.

Sol. Given that ΔABC is an equilateral triangle with the side BC = 10 cm. Also, D and E are two points on the sides AB and AC respectively of ΔABC such the DE ∥ BC.



Also, $\dfrac{AE}{EC} = 3 = \dfrac{AD}{DB}$.

To find the length of AE.

Since, $\dfrac{AE}{EC} = \dfrac{1}{3}$ [Given]

$\Rightarrow$ EC = 3AE

$\Rightarrow$ AC − AE = 3AE

$\Rightarrow$ AC = 3AE + AE

$\Rightarrow$ 10 cm = 4 AE [$\because$ AC = BC = 10 cm]

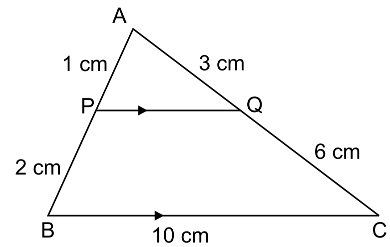
$\Rightarrow$ AE = $\dfrac{10}{4}$ cm

$\Rightarrow$ AE = 2.5 cm

Hence, the required length of AE is 2.5 cm.

6. P and Q are points on sides AB and AC respectively of ΔABC. If AP = 1 cm, PB = 2 cm, AQ = 3 cm and QC = 6 cm, show that BC = 3 PQ. **[CBSE SP 2011]**

Sol. Given that P and Q are two points on the sides AB and AC respectively of ΔABC such that AP = 1 cm, PB = 2 cm, AQ = 3 cm and QC = 6 cm. To show that BC = 3PQ.



Since, $\dfrac{AP}{PB} = \dfrac{1}{2}$ and $\dfrac{AQ}{QC} = \dfrac{3}{6} = \dfrac{1}{2}$

$\therefore \quad \dfrac{AP}{PB} = \dfrac{AQ}{QC}$

$\therefore$ PQ ∥ BC, by the converse of the basic proportionality theorem.

In ΔAPQ and ΔABC, we have

∠APQ = ∠ABC [Corresponding ∠s]

∠AQP = ∠ACB [Corresponding ∠s]

∠PAQ = ∠BAC [Common]

$\therefore$ By AAA similarity criterion, we have

ΔAPQ ~ ΔABC

$\Rightarrow \quad \dfrac{PQ}{BC} = \dfrac{AP}{AB} = \dfrac{1}{1+2} = \dfrac{1}{3}$

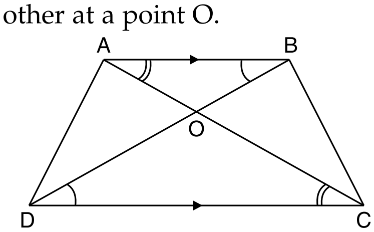
$\therefore$ BC = 3PQ

Hence, proved.

Short Answer Type-II Questions

7. Prove that the diagonals of a trapezium divide each other proportionally.

Sol. Given that ABCD is a trapezium such that AB ∥ DC and the two diagonals AC and BD intersect each other at a point O.



To prove that

$$\frac{AO}{CO} = \frac{BO}{DO}$$

In ΔAOB and ΔCOD, we have

∠AOB = ∠CDO [Vertically opposite angles]

∠ABO = ∠CDO [Alternate angles, $\because$ AB ∥ DC]

∠BAO = ∠DCO [Alternate angles]

$\therefore$ By AAA similarity criterion, we have

ΔAOB ~ ΔCOD

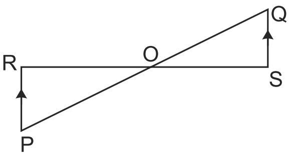
$$\Rightarrow \quad \frac{AO}{CO} = \frac{BO}{DO}$$

Hence, proved.

8. In the given figure, prove that

(*a*) $\Delta PRO \sim \Delta QSO$

(*b*) $\frac{PO}{RO} = \frac{QO}{SO}$

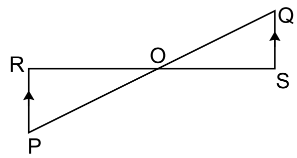


Sol. Given that two line segments RS and PQ intersect each other at a point O. Also, RP ∥ QS.

To prove that

(*a*) $\Delta PRO \sim \Delta QSO$

(*b*) $\frac{PO}{RO} = \frac{QO}{SO}$



(*a*) In ΔPRO and ΔQSO, we have

$$\angle PRO = \angle QSO$$

[Alternate angles, ∵ RP ∥ QS]

$$\angle ROP = \angle SOQ$$

[Vertically opposite angles]

∴ By AA similarity criterion, we have

$$\Delta PRO \sim \Delta QSO$$

(*b*) From (*a*), we have

$$\frac{PO}{QO} = \frac{RO}{SO}$$

$$\Rightarrow \quad \frac{PO}{RO} = \frac{QO}{SO}$$

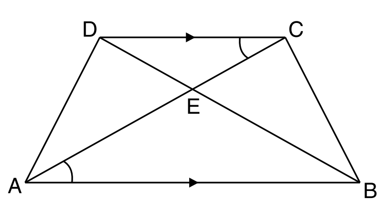
Hence, proved.

Long Answer Type Questions

9. ABCD is a trapezium in which AB ∥ CD. If diagonals AC and BD intersect each other at E, and if $\Delta AED \sim \Delta BEC$, prove that BD = AC and AD = BC.

Sol. Given that ABCD is a trapezium in which AB ∥ CD. The two diagonals AC and BD intersect each other at a point E. It is given that $\Delta AED \sim \Delta BEC$.

To prove that BD = AC and AD = BC.



We have

$$\Delta AED \sim \Delta BEC \quad \text{[Given]}$$

$$\frac{AE}{BE} = \frac{AD}{BC} = \frac{ED}{EC} \qquad \ldots(1)$$

Also, since $\angle AEB = \angle CED$

[Vertically opposite angles]

$$\angle BAE = \angle DCE$$

[Alternate angles, since AB ∥ CD]

∴ By AAA similarity criterion, we have

$$\Delta AEB \sim \Delta CED$$

$$\Rightarrow \quad \frac{AE}{CE} = \frac{AB}{CD} = \frac{EB}{ED} \qquad \ldots(2)$$

From (1), we have

$$\frac{AE}{BE} = \frac{ED}{EC} \qquad \ldots(3)$$

From (2), we have

$$\frac{AE}{CE} = \frac{BE}{DE}$$

$$\frac{AE}{BE} = \frac{EC}{ED} \qquad \ldots(4)$$

∴ From (3) and (4), we have

$$\frac{ED}{EC} = \frac{EC}{ED}$$

$$\Rightarrow \quad EC^2 = ED^2$$

$$\Rightarrow \quad EC = ED \qquad \ldots(5)$$

∴ From (3), we have

$$\frac{AE}{BE} = 1 \quad \left[\because \frac{EC}{ED} = 1, \text{ from (5)}\right]$$

$$\Rightarrow \quad AE = BE \qquad \ldots(6)$$

Now,
$$AC = AE + EC$$
$$= BE + ED$$
[From (5) and (6)]
$$= BD$$

Hence, AC = BD.

$$\therefore \text{ From (1)}, \quad \frac{AD}{BC} = \frac{AE}{BE} = 1$$

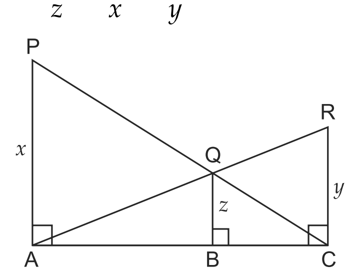
[∵ From (6), AE = BE]

$$\therefore \quad AD = BC$$

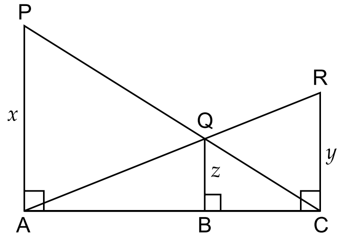
Hence, proved.

10. In the given figure, PA, QB and RC are perpendicular to AC and if PA = x, RC = y and QB = z, prove that

$$\frac{1}{z} = \frac{1}{x} + \frac{1}{y}$$



Sol. Given that PA, QB and RC are perpendiculars to the line segment ABC at the points A, B and C respectively. Also, PA = x, QB = z and RC = y.



In ΔBCQ and ΔACP,

$$QB \parallel PA$$

$$\therefore \quad \Delta BCQ \sim \Delta ACP$$

$$\Rightarrow \quad \frac{QB}{PA} = \frac{BC}{AC}$$

$$\Rightarrow \quad \frac{z}{x} = \frac{BC}{AC} \quad \ldots(1)$$

Also, in ΔBAQ and ΔCAR,

$$QB \parallel RC$$

$$\therefore \quad \Delta BAQ \sim \Delta CAR$$

$$\Rightarrow \quad \frac{QB}{RC} = \frac{AB}{AC}$$

$$\Rightarrow \quad \frac{z}{y} = \frac{AB}{AC}$$

$$= \frac{AC - BC}{AC} = 1 - \frac{BC}{AC}$$

$$\Rightarrow \quad 1 - \frac{z}{y} = \frac{BC}{AC} \quad \ldots(2)$$

$\therefore$ From (1) and (2), we have

$$\frac{z}{x} = 1 - \frac{z}{y}$$

$$\Rightarrow \quad \frac{1}{x} = \frac{1}{z} - \frac{1}{y}$$

$$\Rightarrow \quad \frac{1}{z} = \frac{1}{x} + \frac{1}{y}$$

Hence, proved.

Milestone 3

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Multiple-Choice Questions

1. The areas of two similar triangles LMN and UVW are respectively 121 cm^2 and 144 cm^2. If MN = 22 cm, then the length of VW is

(*a*) 12 cm (*b*) 24 cm

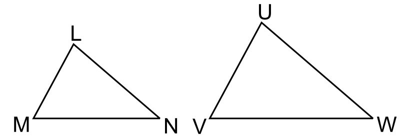
(*c*) 11 cm (*d*) 20 cm

Sol. (*b*) 24 cm

Given that $\Delta LMN \sim \Delta UVW$

and $$\frac{ar(\Delta LMN)}{ar(\Delta UVM)} = \frac{121}{144}$$

Also, MN = 22 cm.



To find the length of VW.

We have

$$\frac{ar(\Delta LMN)}{ar(\Delta UVM)} = \frac{MN^2}{UV^2}$$

$$[\because \Delta LMN \sim \Delta UVM]$$

$$\Rightarrow \quad \frac{121}{144} = \frac{22^2}{UV^2}$$

$$\Rightarrow \quad \frac{22}{UV} = \sqrt{\frac{121}{144}} = \frac{11}{12}$$

$$\therefore \quad UV \times 11 = 12 \times 22$$

$$\Rightarrow \quad UV = \frac{22 \times 12}{11} = 24$$

$\therefore$ Required length of UV is 24 cm.

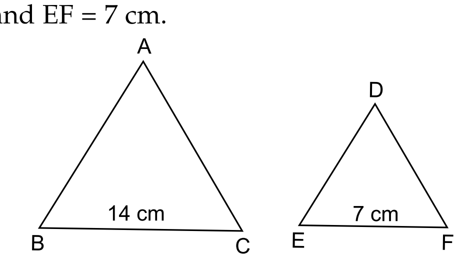
2. Two similar triangles ABC and DEF are such that BC = 14 cm and EF = 7 cm. Then the ratio of the areas of two triangles will be

(*a*) 1 : 4 (*b*) 2 : 1

(*c*) 1 : 2 (*d*) 4 : 1

Sol. (*d*) 4 : 1

Given that $\Delta ABC \sim \Delta DEF$, BC = 14 cm and EF = 7 cm.



To find ar(ΔABC) : ar (ΔDEF).

$$\because \quad \Delta ABC \sim \Delta DEF$$

$$\therefore \quad \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{BC^2}{EF^2} = \frac{14^2}{7^2}$$

$$= \frac{14 \times 14}{7 \times 7}$$

$$= \frac{2 \times 2}{1}$$

$$= \frac{4}{1}$$

$\therefore$ Required ratio = 4 : 1

3. The length of the hypotenuse of an isosceles right triangle whose one side is $5\sqrt{2}$ cm is

(*a*) 15 cm

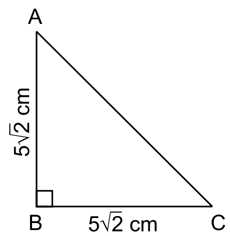
(*b*) 10 cm

(*c*) $10\sqrt{2}$ cm

(*d*) $15\sqrt{2}$ cm

Sol. (*b*) 10 cm

Given that ΔABC is a right-angled isosceles triangle so that $\angle ABC = 90°$ and $AB = BC = 5\sqrt{2}$ cm.



To find the length of the hypotenuse AC.

By Pythagoras' Theorem, we have

$$AC^2 = AB^2 + BC^2$$

$$= \left(5\sqrt{2}\right)^2 + \left(5\sqrt{2}\right)^2 = 100$$

$$\Rightarrow \quad AC = \sqrt{100} = 10$$

$\therefore$ Required length of AC is 10 cm.

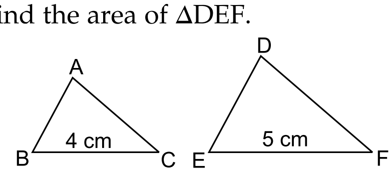
Very Short Answer Type Questions

4. ΔABC and ΔDEF are two similar triangles. If $BC = 4$ cm, $EF = 5$ cm and ar(ΔABC) = 64 cm^2, then what is the area of ΔDEF?

[CBSE 2002C, SP 2010]

Sol. Given that $\Delta ABC \sim \Delta DEF$, $BC = 4$ cm, $EF = 5$ cm and ar(ΔABC) = 64 cm^2.

To find the area of ΔDEF.



We have

$$\frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{BC^2}{EF^2} \quad [\because \Delta ABC \sim \Delta DEF]$$

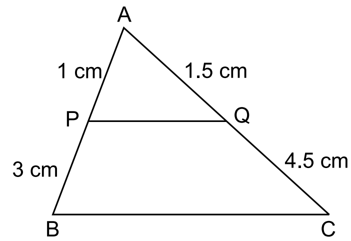
$$\Rightarrow \quad \frac{64}{ar(\Delta DEF)} = \frac{4^2}{5^2} = \frac{16}{25}$$

$$\therefore \quad ar(\Delta DEF) = \frac{64 \times 25}{16} = 100$$

Hence, the required area of ΔDEF is 100 cm^2.

5. ABC is a triangle and PQ is a straight line meeting AB in P and AC in Q. If $AP = 1$ cm, $PB = 3$ cm, $AQ = 1.5$ cm and $QC = 4.5$ cm, then what is the ratio of the areas of ΔAPQ and ΔABC?

Sol. Given that P and Q are two points on the sides AB and AC respectively of ΔABC such that $AP = 1$ cm, $PB = 3$ cm, $AQ = 1.5$ cm and $QC = 4.5$ cm. To find ar(ΔAPQ) : ar(ΔABC).



We have $\frac{AP}{PB} = \frac{1}{3}$ and $\frac{AQ}{QC} = \frac{1.5}{4.5} = \frac{1}{3}$.

$$\therefore \quad \frac{AP}{PB} = \frac{AQ}{QC}$$

$\therefore$ By the converse of the basic proportionality theorem, we have

$$PQ \parallel BC$$

$$\therefore \quad \Delta APQ \sim \Delta ABC$$

$$\therefore \quad \frac{ar(\Delta APQ)}{ar(\Delta ABC)} = \frac{AP^2}{AB^2}$$

$$= \frac{1^2}{(AP + PB)^2}$$

$$= \frac{1}{(1+3)^2}$$

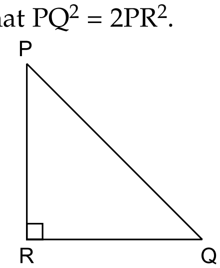
$$= \frac{1}{4^2} = \frac{1}{16}$$

$\therefore$ Required ratio = 1 : 16

6. Triangle PQR is an isosceles triangle right-angled at R. Prove that $PQ = 2PR^2$. **[CBSE SP 2011]**

Sol. Given that in $\angle PQR$, $\angle PRQ = 90°$, $PR = RQ$.

To prove that $PQ^2 = 2PR^2$.



$$\because \quad \angle PRQ = 90°$$

$\because$ By Pythagoras' Theorem, we have

$$PQ^2 = PR^2 + RQ^2$$

$$= PR^2 + PR^2 = 2PR^2$$

Hence, proved.

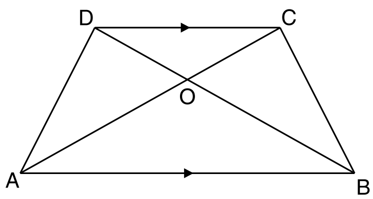
Short Answer Type-I Questions

7. ABCD is a trapezium in which AB $\parallel$ DC and AB = 2CD. If two diagonals AC and BD intersect each other at O, find the ratio of the areas of the triangles AOB and COD.

[CBSE 2000, 2005, 2009, SP 2011]

Sol. Given that ABCD is a trapezium in which AB $\parallel$ DC and AB = 2 CD. Two diagonals AC and BD intersect each other at a point O.

To find the ratio of ar(ΔAOB) and ar(ΔCOD).



We have in ΔAOB and ΔCOD,

$$\angle AOB = \angle COD$$

[Vertically opposite angels]

$$\angle OAB = \angle OCD$$

[Alternate angles, $\because$ AB $\parallel$ DC]

$$\angle OBA = \angle ODC \text{ [Alternate angles]}$$

$\therefore$ By AAA similarity criterion, we have

$$\Delta AOB \sim \Delta COD$$

$$\therefore \quad \frac{ar(\Delta AOB)}{ar(\Delta COD)} = \frac{AB^2}{CD^2}$$

$$= \frac{(2CD)^2}{CD^2} \quad \text{[Given]}$$

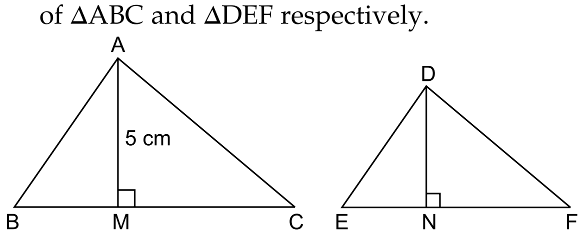
$$= \frac{4CD^2}{CD^2} = \frac{4}{1}$$

$\therefore$ Required ratio = 4 : 1

8. The areas of two similar triangles are 100 cm^2 and 49 cm^2. If the altitude of the larger triangle is 5 cm, find the corresponding altitude of the other.

[CBSE SP 2011]

Sol. Given that $\Delta ABC \sim \Delta DEF$, ar($\Delta$ABC) = 100 cm^2 and ar(ΔDEF) = 49 cm^2. AM $\perp$ BC and DN $\perp$ EF, where M and N are points on BC and EF respectively, clearly, AM and DN are the altitude of ΔABC and ΔDEF respectively.



Also, given that AM = 5 cm. To find the length of DN.

We have

$$\frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{100}{49} \qquad \ldots(1)$$

$$\Rightarrow \quad \frac{BC^2}{EF^2} = \frac{100}{49}$$

$$\Rightarrow \quad \frac{BC}{EF} = \sqrt{\frac{100}{49}} = \frac{10}{7} \qquad \ldots(2)$$

Now, $\quad ar(\Delta ABC) = \frac{1}{2} \times BC \times AM$

$$= \frac{1}{2} \times BC \times 5$$

and $\quad ar(\Delta DEF) = \frac{1}{2} \times EF \times DN$

$$= \frac{1}{2} \times EF \times DN$$

$\therefore$ From (1),

$$\frac{\frac{1}{2} \times BC \times 5}{\frac{1}{2} \times EF \times DN} = \frac{100}{49}$$

$$\Rightarrow \quad \frac{BC}{EF} \times \frac{5}{DN} = \frac{100}{49}$$

$$\Rightarrow \quad \frac{10}{7} \times \frac{5}{DN} = \frac{100}{49} \qquad \text{[From (2)]}$$

$$\Rightarrow \quad \frac{1}{DN} = \frac{100}{49} \times \frac{7}{50} = \frac{2}{7}$$

$$\therefore \quad DN = \frac{7}{2} = 3.5$$

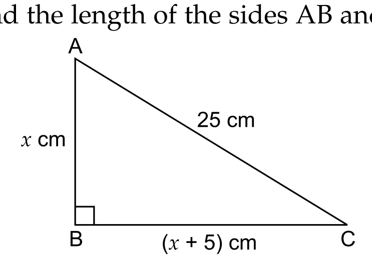
Hence, the required length of DN is 3.5 cm.

9. The hypotenuse of a right-triangle is 25 cm and out of the remaining two sides, one is longer than the other by 5 cm. Find the length of the other sides.

[CBSE SP 2013]

Sol. Given that ΔABC is a right-angled triangle with $\angle ABC = 90°$ and the hypotenuse AC = 25 cm. BC is greater than AB by 5 cm.

To find the length of the sides AB and BC.



Let the length of the shorter side AB be x cm. Then the length of the bigger side BC is $(x + 5)$ cm.

$\therefore$ By Pythagoras' Theorem, we have

$$AC^2 = AB^2 + AC^2 \quad [\because \angle ABC = 90°]$$

$$\Rightarrow \quad 25^2 = x^2 + (x + 5)^2$$

$$\Rightarrow \quad 625 = x^2 + x^2 + 10x + 25$$

$$\Rightarrow \quad 2x^2 + 10x - 600 = 0$$

$$\Rightarrow \quad x^2 + 5x - 300 = 0$$

$$\Rightarrow \quad x^2 + 20x - 15x - 300 = 0$$

$$\Rightarrow \quad x(x + 20) - 15(x + 20) = 0$$

$$\Rightarrow \quad (x + 20)(x - 15) = 0$$

$\therefore$ Either $x + 20 = 0$

$$\Rightarrow \quad x = -20$$

which is rejected, since length cannot be negative.

or $x - 15 = 0$

$$\Rightarrow \quad x = 15$$

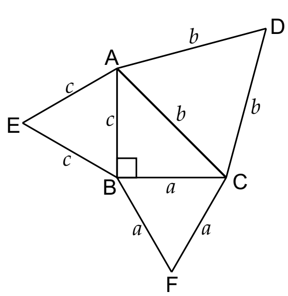
which is accepted.

$\therefore$ The required length of two sides are 15 cm and (15 + 5)cm, i.e. 20 cm.

Short Answer Type-II Questions

10. Equilateral triangles are drawn on the three sides of a right-angled triangle. Show that the area of the triangle on the hypotenuse is equal to the sum of the areas of the two triangles on the other two sides.

Sol. Given that ΔABC is a right-angled triangle with $\angle ABC = 90°$, BC = a units, CA = b units and AB = c units. 3 equilateral triangle ADC, AEB and BFC are drawn on the three sides AC, AB and BC respectively of ΔABC.



To prove that

$$ar(\Delta ADC) = ar(\Delta AEB) + ar(\Delta BFC)$$

$\because \quad \angle ABC = 90°,$

$\therefore$ By Pythagoras' Theorem, we have

$$AC^2 = AB^2 + BC^2 = c^2 + a^2$$

$$\Rightarrow \quad b^2 = c^2 + a^2 \quad \ldots(1)$$

We know that area of an equilateral triangle of side p units is $\frac{\sqrt{3}}{4}p^2$ sq units.

Now, $ar(\Delta ADC) = \frac{\sqrt{3}}{4}b^2$

$$ar(\Delta AEB) = \frac{\sqrt{3}}{4}c^2$$

$$ar(\Delta BFC) = \frac{\sqrt{3}}{4}a^2$$

$\therefore$ From (1), we see that

$$\frac{\sqrt{3}}{4}b^2 = \frac{\sqrt{3}}{4}c^2 + \frac{\sqrt{3}}{4}a^2$$

$$\Rightarrow \quad ar(\Delta ADC) = ar(\Delta AEB) + ar(\Delta BFC)$$

Hence, proved.

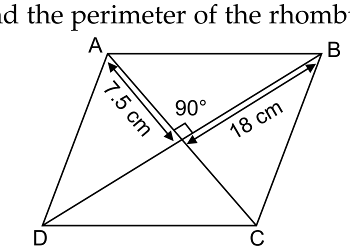
11. The diagonals of a rhombus are 15 cm and 36 cm long. Find its perimeter. **[CBSE SP 2011]**

Sol. Given that ABCD is a rhombus with diagonals AC and BD meeting each other at a point O such that

$$OB = \frac{1}{2}DB = \frac{1}{2} \times 36 \text{ cm} = 18 \text{ cm}$$

and $OA = \frac{1}{2}AC = \frac{1}{2} \times 15 \text{ cm} = 7.5 \text{ cm}$

To find the perimeter of the rhombus.



We know that the two diagonals of a rhombus bisect each other at right angles.

$$\therefore \quad \angle AOB = 90°$$

In ΔAOB,

$\therefore$ By Pythagoras' Theorem, we have

$$AB^2 = AO^2 + OB^2 = (7.5)^2 + 18^2 = 56.25 + 324 = 380.25$$

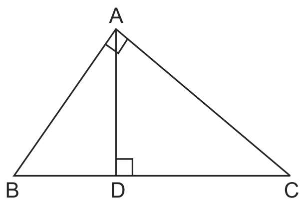
$$\therefore \quad AB = \sqrt{380.25} = 19.50$$

Hence, the required perimeter of the rhombus

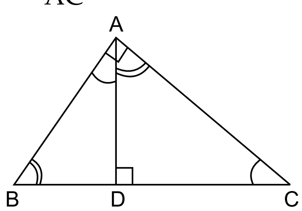
$$= 4\ AB$$
$$= 4 \times 19.50 \text{ cm}$$
$$= 78 \text{ cm}$$

Long Answer Type Questions

12. In the given figure, ABC is a triangle such that $\angle BAC = 90°$ and $AD \perp BC$, where D is a point on BC. Express the ratio $\dfrac{ar(\Delta ABD)}{ar(\Delta CAD)}$ in four different ways. Also, prove that $\dfrac{BD}{DC} = \dfrac{AB^2}{AC^2}$.



Sol. Given that in ΔABC, $\angle BAC = 90°$ and D is a point on the side BC such that $AD \perp BC$. To express $\dfrac{ar(\Delta ABD)}{ar(\Delta CAD)}$ in four different ways. Also, to prove that $\dfrac{BD}{DC} = \dfrac{AB^2}{AC^2}$.



In ΔABD and ΔCBA, we have

$$\angle ABD = \angle CBA \quad \text{[Common]}$$
$$\angle ADB = \angle CAB = 90°$$
$$\angle BAD = \angle BCA \quad \ldots(1)$$
$$\therefore \quad \Delta ABD \sim \Delta CBA$$

[By AAA similarity criterion]

$$\Rightarrow \quad \frac{AB}{CB} = \frac{AD}{CA} = \frac{BD}{AB}$$

$\therefore$ From above, we have

$$AB^2 = BD \times CB \quad \ldots(2)$$

Again, in ΔACD and ΔBCA,

$$\angle ACD = \angle BCA \quad \text{[Common]}$$
$$\angle ADC = \angle BAC = 90°$$
$$\therefore \quad \angle CAD = \angle CBA \quad \ldots(3)$$

$\therefore$ By AAA similarity criterion, we have

$$\Delta ACD \sim \Delta BCA$$

$$\therefore \quad \frac{AC}{BC} = \frac{AD}{BA} = \frac{DC}{CA}$$
$$\Rightarrow \quad \frac{AC}{BC} = \frac{AD}{AB} = \frac{DC}{AC}$$

$\therefore$ From above, we have

$$AC^2 = CB \times DC \quad \ldots(4)$$

$\therefore$ From (2) and (4), we have

$$\frac{AB^2}{AC^2} = \frac{BD \times CB}{CB \times DC} = \frac{BD}{DC} \quad \ldots(5)$$

Finally, in ΔABD and ΔCAD, we have

$$\angle ADB = \angle CDA = 90°$$
$$\angle ABD = \angle CAD \quad \text{[From (3)]}$$
$$\angle BAD = \angle ACD \quad \text{[From (1)]}$$

$\therefore$ By AAA similarity criterion, we have

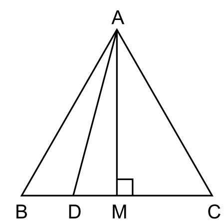
$$\Delta ABD \sim \Delta CAD$$

$$\therefore \quad \frac{ar(\Delta ABD)}{ar(\Delta CAD)} = \frac{AB^2}{AC^2} = \frac{AD^2}{DC^2} = \frac{BD^2}{AD^2} = \frac{BD}{DC}$$

[From (5)]

13. In an equilateral triangle ABC, D is a point on the side BC such that 4BD = BC. Prove that $16AD^2 = 13BC^2$. **[CBSE SP 2011]**

Sol. Given that ΔABC is an equilateral triangle. D is a point on the side BC such that 4 BD = BC. To prove that $16\ AD^2 = 13\ BC^2$.



Construction: Draw $AM \perp BC$, where M is a point on BC.

Clearly, M is the mid-point of BC,

$$\therefore \quad \Delta ABM \sim \Delta ACM$$
$$\therefore \quad BM = \frac{1}{2} BC$$

Now, in ΔADM,

$$\because \quad \angle AMD = 90°$$

$\therefore$ By Pythagoras' Theorem, we have

$$AD^2 = AM^2 + DM^2$$
$$= AM^2 + (BM - BD)^2$$
$$= AM^2 + BM^2 + BD^2 - 2BM \times BD$$
$$= AB^2 + BD^2 - 2BD \times \frac{1}{2} BC$$

[$\because$ From ΔABM, by Pythagoras' Theorem, $AB^2 = AM^2 + BM^2$]

$$= BC^2 + \frac{BC^2}{16} - \frac{BC}{4} \times BC$$

$$[\because BD = \frac{BC}{4}, \text{ given}]$$

$$= BC^2 + \frac{BC^2}{16} - \frac{BC^2}{4}$$

$$= \frac{13BC^2}{16}$$

$$\Rightarrow \quad 16AD^2 = 13BC^2$$

Hence, proved.

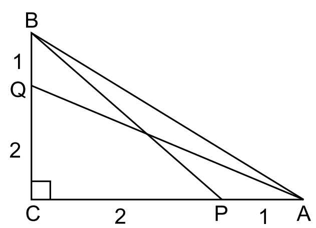
14. In a right triangle ABC right angled at C, P and Q are points on the sides CA and CB respectively which divide these sides in the ratio 2 : 1. Prove that

(*a*) $9AQ^2 = 9AC^2 + 4BC^2$

(*b*) $9BP^2 = 9BC^2 + 4AC^2$

(*c*) $9(AQ^2 + BP^2) = 13AB^2$ **[CBSE SP 2011]**

Sol. Given that BCA is a right-angled triangle where $\angle BCA = 90°$. P and Q are points on the sides CA and CB respectively such that CP : PA = 2 : 1 and CQ : QB = 2 : 1.



To prove that

(*a*) $9AQ^2 = 9\ AC^2 + 4\ BC^2$

(*b*) $9\ BP^2 = 9\ BC^2 + 4\ AC^2$ and

(*c*) $9\ (AQ^2 + BP^2) = 13\ AB^2$

Construction: Join AQ and BP.

We have $\quad CP = \frac{2}{3}CA \quad$ …(1)

and $\quad CQ = \frac{2}{3}CB \quad$ …(2)

(*a*) Now, in ΔAQC,

$\because \quad \angle ACQ = 90°$

$\therefore$ By Pythagoras' Theorem, we have

$$AQ^2 = AC^2 + CQ^2$$

$$= AC^2 + \left(\frac{2}{3}CB\right)^2 \quad \text{[From (2)]}$$

$$= AC^2 + \frac{4}{9}CB^2$$

$$\Rightarrow \quad 9AQ^2 = 9AC^2 + 4BC^2 \quad \text{…(3)}$$

(*b*) Again, in ΔBPC,

$\because \quad \angle BCP = 90°$

$\therefore$ By Pythagoras' Theorem, we have

$$BP^2 = BC^2 + CP^2$$

$$= BC^2 + \left(\frac{2}{3}CA\right)^2 \quad \text{[From (1)]}$$

$$= BC^2 + \frac{4}{9}CA^2$$

$$\Rightarrow \quad 9BP^2 = 9\ BC^2 + 4AC^2 \quad \text{…(4)}$$

(*c*) Adding (3) and (4), we get

$$9(AQ^2 + BP^2) = (4 + 9)\ AC^2 + (4 + 9)\ BC^2$$

$$= 13\ (AC^2 + BC^2)$$

$$= 13\ AB^2$$

$[\because \angle ACB = 90°, \therefore$ By Pythagoras' Theorem, $AC^2 + BC^2 = AB^2]$

Hence, proved.

Milestone 4

(Page 118)

Multiple-Choice Questions

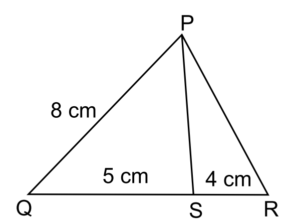
1. In ΔPQR, PS is the bisector of $\angle P$. If PQ = 8 cm, QS = 5 cm, SR = 4 cm, then the value of PR is

(*a*) 2 cm (*b*) 6.4 cm

(*c*) 4.9 cm (*d*) 4 cm

Sol. (*b*) 6.4 cm

Given that PS is the internal bisector of $\angle QPR$ and PQ = 8 cm, QS = 5 cm and SR = 4 cm. To find the value of PR.



$\because$ PS is the internal bisector of $\angle QPR$.

$\therefore$ By the internal bisector theorem, we have

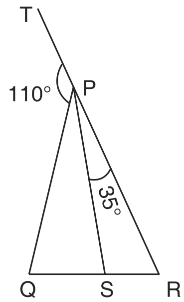
$$\frac{PQ}{PR} = \frac{QS}{SR}$$

$$\Rightarrow \quad \frac{8}{PR} = \frac{5}{4}$$

$$\Rightarrow \quad PR = \frac{4 \times 8}{5} = \frac{32}{5} = 6.4$$

$\therefore$ Required length of PR is 6.4 cm.

2. In the given figure, if exterior $\angle TPQ = 110°$, $\angle RPS = 35°$, PQ = 5 cm, PR = 7 cm and QR = 3 cm, then RS is equal to

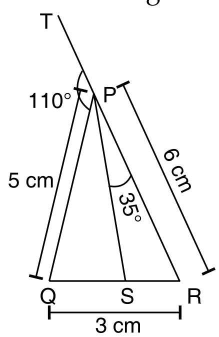


(*a*) 2.25 cm (*b*) 1.9 cm

(*c*) 2 cm (*d*) 1.75 cm

Sol. (*d*) 1.75 cm

Given that PQR is a triangle, RP is produced to T such that $\angle TPQ = 110°$, $\angle SPR = 35°$, PQ = 5 cm, QR = 3 cm, PR = 7 cm and S is a point on QR. PS is joined. To find the length of RS.



We have

$$\angle QPR = \angle TPR - \angle TPQ$$
$$= 180° - 110°$$
$$= 70°$$

Also,
$$\angle QPS = \angle QPR - \angle SPR$$
$$= 70° - 35°$$
$$= 35°$$
$$= \angle SPR$$

$\therefore$ PS is the internal bisector of $\angle QPR$.

Hence, by the theorem of internal bisector of an angle, we have

$$\frac{PQ}{PR} = \frac{QS}{SR}$$

$$\Rightarrow \quad \frac{5}{7} = \frac{QR - SR}{SR} = \frac{3 - SR}{SR}$$

$$\Rightarrow \quad 5SR = 21 - 7SR$$

$$\Rightarrow \quad (7 + 5)SR = 21$$

$$\Rightarrow \quad SR = \frac{21}{12} = \frac{7}{4} = 1.75$$

Hence, the required length SR is 1.75 cm.

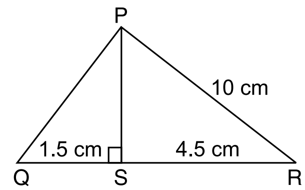
Very Short Answer Type Questions

3. Check whether in the following, PS is the bisector of $\angle P$ of ΔPQR.

(*a*) PQ = 5 cm , PR = 10 cm , QS = 1.5 cm and RS = 4.5 cm.

(*b*) PQ = 4 cm, PR = 8 cm, QS = 1.4 cm and RS = 2.8 cm.

Sol. (*a*)



We see that

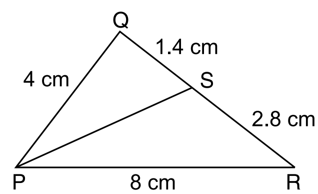
$$\frac{PQ}{PR} = \frac{5}{10} = \frac{1}{2}$$

and
$$\frac{QS}{RS} = \frac{1.5}{4.5} = \frac{1}{3}$$

$$\therefore \quad \frac{PQ}{PR} \neq \frac{QS}{RS}$$

$\therefore$ PS is not the internal bisector of $\angle QPR$.

(*b*)



In this case, we see that

$$\frac{PQ}{PR} = \frac{4}{8} = \frac{1}{2}$$

and
$$\frac{QS}{QR} = \frac{1.4}{2.8} = \frac{1}{2}$$

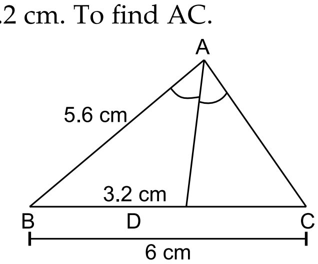
$$\therefore \quad \frac{PQ}{PR} = \frac{QS}{RS}$$

$\therefore$ By the converse of the theorem of internal bisector of an angle, PS is the internal bisector of $\angle PQR$.

4. In ΔABC, AD is the bisector of $\angle A$. If AB = 5.6 cm, BC = 6 cm and BD = 3.2 cm, find AC.

[CBSE 2001C]

Sol. Given that AD is the bisector of $\angle BAC$ of ΔABC such that AB = 5.6 cm, BC = 6 cm and BD = 3.2 cm. To find AC.



By the theorem of internal bisector, we have

$$\frac{AB}{AC} = \frac{BD}{DC}$$

$$\Rightarrow \quad \frac{5.6}{AC} = \frac{3.2}{BC - BD}$$

$$= \frac{3.2}{BC - 3.2}$$

$$= \frac{3.2}{6 - 3.2}$$

$$= \frac{3.2}{2.8}$$

$$= \frac{32}{28} = \frac{8}{7}$$

$$\Rightarrow \quad 8\ AC = 7 \times 5.6$$

$$\Rightarrow \quad AC = \frac{7 \times 5.6}{8} = 7 \times 0.7 = 4.9$$

Hence, required length of AC is 4.9 cm.

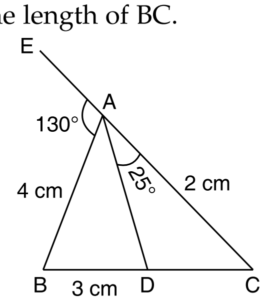
Short Answer Type-I Questions

5. In ΔABC, the exterior angle EAB is 130°. If D is a point on BC such that $\angle CAD = 25°$, AB = 4 cm, AC = 2 cm and BD = 3 cm, then what is the length of the side BC of ΔABC?

Sol. Given that $\angle EAB$ is an exterior angle of ΔABC such that $\angle EAB = \angle B = 130°$.

D is a point on BC such that $\angle CAD = 25°$, AB = 4 cm, AC = 2 cm and BD = 3 cm.

To find the length of BC.



We have $\angle BAD = \angle BAC - \angle DAC$

$$= (180° - 130°) - 25°$$

$$= 50° - 25°$$

$$= 25°$$

$$= \angle DAC$$

$\therefore$ AD is the internal bisector of $\angle BAC$.

$\therefore$ By the theorem of internal bisector of an angle, we have

$$\frac{AB}{AC} = \frac{BD}{DC}$$

$$\Rightarrow \quad \frac{4}{2} = \frac{3}{BC - BD}$$

$$\Rightarrow \quad 2 = \frac{3}{BC - 3}$$

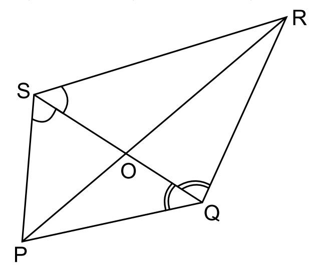
$$\Rightarrow \quad 2BC - 6 = 3$$

$$\Rightarrow \quad BC = \frac{9}{2} = 4.5$$

Hence, the required length of BC is 4.5 cm.

6. If the diagonal QS of a quadrilateral PQRS bisects both $\angle Q$ and $\angle S$, show that $\frac{PQ}{QR} = \frac{PS}{RS}$.

Sol. Given that PQRS is a quadrilateral and the diagonal QS bisects both $\angle PQR$ and $\angle PSR$ so that $\angle PSQ = \angle RSQ$ and $\angle PQS = \angle RQS$.



To prove that $\frac{PQ}{QR} = \frac{PS}{RS}$

Construction: Join the second diagonal PR. Let SQ and PR intersect each other at a point O.

In ΔPSR, SO is the internal bisector of $\angle PSR$.

$\therefore$ By the theorem of internal bisector, we have

$$\frac{PS}{RS} = \frac{PO}{RO} \qquad \ldots(1)$$

Again, in ΔPQR, QO is the internal bisector of $\angle PQR$.

$\therefore$ By the theorem of internal bisector, we have

$$\frac{PQ}{QR} = \frac{PO}{RO} \qquad \ldots(2)$$

$\therefore$ From (1) and (2), we have

$$\frac{PQ}{QR} = \frac{PS}{QR}$$

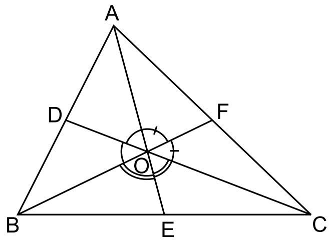
Hence, proved.

Short Answer Type-II Questions

7. O is a point in the interior of ΔABC. The bisectors of $\angle AOB$, $\angle BOC$ and $\angle COA$ meet the sides AB, BC and CA in points D, E and F respectively. Prove that AD × BE × CF = DB × EC × FA.

Sol. Given that O is any point within ΔABC.

D, E, F are points on AB, BC and CA respectively such that OD, OE and OF are the internal bisectors of $\angle AOB$, $\angle BOC$ and $\angle COA$ respectively.



To prove that AD × BE × CF = DB × EC × FA.

In ΔAOB, OD is the internal bisector of $\angle$AOB, in ΔBOC, OE is the internal bisector of $\angle$BOC and in ΔAOC, OF is the internal bisector of $\angle$COA.

$\therefore$ By the theorem of internal bisector, we have

$$\frac{DB}{AD} = \frac{OB}{OA} \quad \ldots(1)$$

$$\frac{EC}{BE} = \frac{OC}{OB} \quad \ldots(2)$$

and $$\frac{FA}{CF} = \frac{OA}{OC} \quad \ldots(3)$$

$\therefore$ From (1), (2) and (3), we have

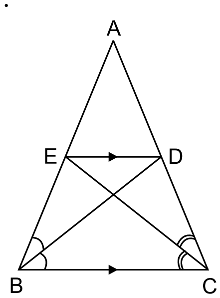
$$\frac{DB}{AD} \times \frac{EC}{BE} \times \frac{FA}{CF} = \frac{OB}{OA} \times \frac{OC}{OB} \times \frac{OA}{OC}$$

$\therefore$ AD × BE × CF = DB × EC × FA = 1

Hence, proved.

8. The bisectors of $\angle$B and $\angle$C of a triangle ABC meet the opposite sides at D and E respectively. If DE $\parallel$ CB, prove that the triangle is isosceles.

Sol. Given that D and E are points on the sides AC and AB respectively such that BD and CE are the internal bisectors of $\angle$ABC and $\angle$ACB respectively.



Constructions: Join E and D.

Given that ED $\parallel$ BC.

To prove that ΔABC is an isosceles triangle.

Since, BD and CE are the internal bisectors of $\angle$ABC and $\angle$ACB respectively, we have

By the theorem of internal bisector of an angle,

$$\frac{AE}{EB} = \frac{AC}{BC} \quad \ldots(1)$$

and $$\frac{AD}{DC} = \frac{AB}{BC} \quad \ldots(2)$$

Now, since ED $\parallel$ BC, hence by the converse of the basic proportionality theorem, we have

$$\frac{AE}{EB} = \frac{AD}{DC} \quad \ldots(3)$$

$\therefore$ From (1), (2) and (3), we have

$$\frac{AC}{BC} = \frac{AB}{BC}$$

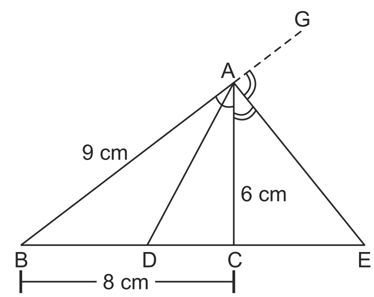
$\Rightarrow$ AC = AB

$\therefore$ ΔABC is an isosceles triangle.

Hence, proved.

Long Answer Type Questions

9. In the given figure, the internal and external bisectors of $\angle$A of ΔABC meet BC and BC produced at D and E respectively. If AB = 9 cm, AC = 6 cm, BC = 8 cm, find the length of BD and BE.



Sol. Given that ABC is a triangle.

BA is extended to the point G and BC is extended to the point E. AD is the internal bisector of $\angle$BAC and AE is the external bisector of $\angle$BAC.

To find the length of BD and BE.

Also, AB = 9 cm, AC = 6 cm and BC = 8 cm.

Since AD is the internal bisector of $\angle$BAC,

$\therefore$ By the theorem of internal bisector of an angle, we have

$$\frac{AB}{AC} = \frac{BD}{DC}$$

$$\Rightarrow \quad \frac{9}{6} = \frac{BD}{BC - BD} = \frac{BD}{8 - BD}$$

$$\Rightarrow \quad \frac{3}{2} = \frac{BD}{8 - BD}$$

$\Rightarrow$ 2BD = 24 – 3BD

$\Rightarrow$ 5BD = 24

$$\Rightarrow \quad BD = \frac{24}{5} = 4.8$$

Hence, the required length of BD is 4.8 cm.

Again, since AE is the external bisector of $\angle BAC$,

$\therefore$ By the theorem of external bisector of an angle, we have

$$\frac{AB}{AC} = \frac{BE}{EC}$$

$$\Rightarrow \quad \frac{9}{6} = \frac{BE}{BE - BC}$$

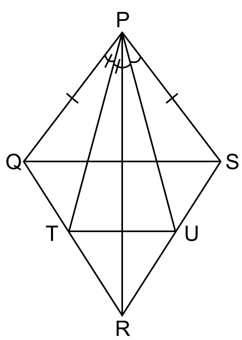
$$\Rightarrow \quad \frac{3}{2} = \frac{BE}{BE - 8}$$

$$\Rightarrow \quad 2\,BE = 3\,BE - 24$$

$$\Rightarrow \quad BE = 24$$

Hence, the required length of BE is 24 cm.

10. In the given figure, PQRS is a quadrilateral with PQ = PS, PT and PU are respectively bisectors of $\angle QPR$ and $\angle SPR$. Prove that TU $\parallel$ QS.



Sol. Given that PQRS is quadrilateral with PQ = PS. T and U are two points on the sides QR and SR respectively of the quadrilateral such that PT and PU are respectively, the internal bisectors of $\angle QPR$ and $\angle RPS$. To prove that TU $\parallel$ QS.

We have

By the theorem of internal bisector of an angle, In ΔPQR,

$$\frac{PQ}{PR} = \frac{QT}{TR} \qquad \ldots(1)$$

and in ΔPRS,

$$\frac{PS}{PR} = \frac{SU}{RU} \qquad \ldots(2)$$

$\therefore$ From (1) and (2), we have

$$\frac{QT}{TR} = \frac{SU}{RU} \qquad [\because PQ = PS]$$

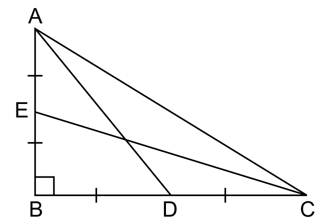
Hence, by the converse of basic proportionality theorem, we have TU $\parallel$ QS.

Higher Order Thinking Skills (HOTS) Questions

(Page 120)

1. ABC is a right-angled triangle, right-angled at B. D and E are points on BC and AB respectively such that AD and CE are two medians drawn from A and C respectively. If AC = 5 cm and AD = $\frac{3\sqrt{5}}{2}$ cm, find CE.

Sol. Given that ΔABC is a right-angled triangle with $\angle ABC = 90°$. D and E are the mid-points of BC and AB respectively so that AD and CE are two medians of ΔABC.



Given that AC = 5 cm and AD = $\frac{3\sqrt{5}}{2}$.

To find the length of CE.

Since $\angle ABC$ is 90°, hence, by using Pythagoras' Theorem, we have

In ΔABC,

$$AC^2 = AB^2 + BC^2$$

$$\Rightarrow \quad 25 = AB^2 + BC^2 \qquad \ldots(1)$$

In ΔABD,

$$AD^2 = AB^2 + BD^2$$

$$= AB^2 + \left(\frac{1}{2}BC\right)^2$$

$$= AB^2 + \frac{BC^2}{4}$$

$$\Rightarrow \quad \left(\frac{3\sqrt{5}}{2}\right)^2 = AB^2 + \frac{BC^2}{4}$$

$$\Rightarrow \quad \frac{45}{4} = AB^2 + \frac{BC^2}{4}$$

$$\Rightarrow \quad 4AB^2 + BC^2 = 45 \qquad \ldots(2)$$

From (1), $\quad AB^2 = 25 - BC^2 \qquad \ldots(3)$

$\therefore$ From (2),

$$4 \times (25 - BC^2) + BC^2 = 45$$

$$\Rightarrow \quad 100 - 4\,BC^2 + BC^2 = 45$$

$$\Rightarrow \quad 100 - 45 = 3\,BC^2$$
$$\Rightarrow \quad 55 = 3\,BC^2$$
$$\Rightarrow \quad BC^2 = \frac{55}{3} \qquad \ldots(4)$$
$$\therefore \text{ From (3),} \quad AB^2 = 25 - \frac{55}{3}$$
$$= \frac{75 - 55}{3} = \frac{20}{3}$$
$$\therefore \quad AB^2 = \frac{20}{3} \qquad \ldots(5)$$

Finally, in ΔEBC,

$$CE^2 = EB^2 + BC^2$$
$$= \left(\frac{1}{2}AB\right)^2 + BC^2$$
$$= \frac{AB^2}{4} + BC^2$$
$$= \frac{1}{4} \times \frac{20}{3} + \frac{55}{3}$$

[From (4) and (5)]

$$= \frac{5}{3} + \frac{55}{3}$$
$$= \frac{60}{3} = 20$$
$$\therefore \quad CE = \sqrt{20} = 2\sqrt{5}$$

Hence, the required length of CE is $2\sqrt{5}$ cm.

2. If AD is a median of a triangle ABC, then prove that $AB^2 + AC^2 = 2BD^2 + 2AD^2$.

or

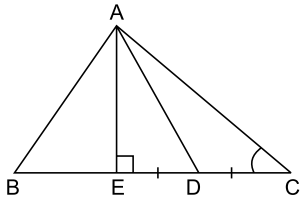
Prove that in any triangle, the sum of the squares of any two sides is equal to twice the square of half the third side together with twice the square of the median which bisects the third side.

Sol. Given that AD is a median of any triangle ABC so that BD = DC.

To prove that

$$AB^2 + AC^2 = 2BD^2 + 2AD^2$$

Construction: Draw $AE \perp BC$, where E is a point on BC.



In ΔABE and ΔAEC,

$$\angle AEB = \angle AEC = 90°$$

By Pythagoras' Theorem, we have

$$AB^2 + AC^2 = (AE^2 + BE^2) + (AE^2 + EC^2)$$
$$= 2\,AE^2 + BE^2 + CE^2$$
$$= 2AE^2 + (BD - ED)^2 + (CD + ED)^2$$
$$= 2AE^2 + BD^2 + ED^2 - 2\,BD \times ED + CD^2 + ED^2 + 2\,CD \times ED$$
$$= 2\,AE^2 + \left(\frac{1}{2}BC\right)^2 + 2ED^2 + \left(\frac{1}{2}BC\right)^2 + 2ED(CD - BD)$$
$$= 2(AE^2 + ED^2) + \frac{1}{2}BC^2 \quad \left[\because CD = BD = \frac{1}{2}BC\right]$$
$$= 2\,AD^2 + \frac{1}{2} \times (2\,BD)^2$$

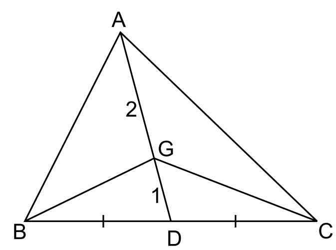
[$\because$ From ΔAED, $\angle AED = 90°$,
$\therefore AE^2 + ED^2 = AD^2$]

$$= 2\,AD^2 + 2\,BD^2$$

Hence, proved.

3. If G be the centroid of ΔABC, prove that $AB^2 + BC^2 + CA^2 = 3(GA^2 + GB^2 + GC^2)$.

Sol. Given that G is the centroid of ΔABC, AD is one of the medians of ΔABC so that AG : GD = 2 : 1.



To prove that

$$AB^2 + BC^2 + CA^2 = 3(GA^2 + GB^2 + GC^2)$$

Construction: Join GB and GC.

In ΔABC, since AD is a median, hence, we have

$$AB^2 + AC^2 = 2BD^2 + 2AD^2 \qquad \ldots(1)$$

and in ΔGBC, since GD is a median, hence, we have

$$GB^2 + GC^2 = 2BD^2 + 2DG^2 \qquad \ldots(2)$$

[By problem number 2]

Now, since G is the centroid of ΔABC,

$$\therefore GA = \frac{2}{3}AD \text{ and } GD = \frac{1}{3}AD \qquad \ldots(3)$$

Now,

$$AB^2 + AC^2 + BC^2 = 2BD^2 + 2AD^2 + 4BD^2$$

[$\because$ BC = 2BD and from (1)]

$$= 6BD^2 + 2AD^2 \qquad \ldots(4)$$

Also, $GB^2 + GC^2 + GA^2$

$$= 2BD^2 + 2DG^2 + \left(\frac{2}{3}AD^2\right)$$

[From (2) and (3)]

$$= 2BD^2 + 2 \times \frac{1}{9}AD^2 + \frac{4}{9}AD^2 \quad \text{[From (3)]}$$

$$= 2BD^2 + \frac{2}{3}AD^2$$

$\therefore$ $3(GA^2 + GB^2 + GC^2) = 6BD^2 + 2AD^2$ …(5)

$\therefore$ From (4) and (5), we have

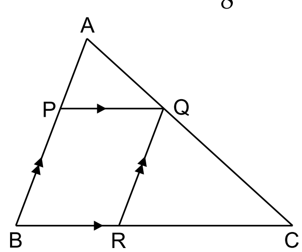
$$AB^2 + AC^2 + BC^2 = 3(GA^2 + GB^2 + GC^2)$$

Hence, proved.

4. P is a point on the side AB of ΔABC such that $\frac{AP}{PB} = \frac{1}{3}$. Q is a point on AC such that PQ ∥ BC and R is a point on BC such that QR ∥ AB. Prove that $ar(BPQR) = \frac{3}{8} ar(\Delta ABC)$.

Sol. Given that P and Q are points on AB and AC respectively of ΔABC such that $\frac{AP}{PB} = \frac{1}{3}$ and PQ ∥ BC. Also, R is a point on BC such that QR ∥ AB.

To prove that $ar(BPQR) = \frac{3}{8} ar(\Delta ABC)$.



By the basic proportionality theorem, we have

$$\frac{AP}{PB} = \frac{AQ}{QC} = \frac{1}{3} \quad \text{…(1)}$$

$\therefore$ $\Delta APQ \sim \Delta ABC$

$$\therefore \quad \frac{ar(\Delta ABC)}{ar(\Delta APQ)} = \frac{AB^2}{AP^2}$$

$$= \frac{(AP + PB)^2}{AP^2}$$

$$= \left(\frac{AP + PB}{AP}\right)^2$$

$$= \left(1 + \frac{PB}{AP}\right)^2$$

$$= (1 + 3)^2 \quad \text{[From (1)]}$$

$$= 16$$

$$\therefore \quad ar(\Delta APQ) = \frac{ar(\Delta ABC)}{16} \quad \text{…(2)}$$

Again, since QR ∥ AB,

$\therefore$ By the basic proportionality theorem,

$$\frac{CR}{RB} = \frac{CQ}{QA} = \frac{3}{1} \quad \text{[From (1)] …(3)}$$

Now, $\Delta CQR \sim \Delta CAB$

$$\frac{ar(\Delta CQR)}{ar(\Delta CAB)} = \frac{CQ^2}{CA^2}$$

$$\frac{ar(\Delta ABC)}{ar(\Delta CQR)} = \frac{CA^2}{CQ^2}$$

$$= \left(\frac{CQ + QA}{CQ}\right)^2$$

$$= \left(1 + \frac{QA}{CQ}\right)^2$$

$$= \left(1 + \frac{1}{3}\right)^2 \quad \text{[From (3)]}$$

$$= \frac{16}{9}$$

$$\Rightarrow \quad \frac{ar(\Delta ABC)}{ar(\Delta CQR)} = \frac{16}{9}$$

$$\Rightarrow \quad ar(\Delta CQR) = \frac{9}{16} ar(\Delta ABC) \quad \text{…(4)}$$

From (2) and (4), we have

$$ar(\Delta APQ) + ar(\Delta CQR) = \frac{ar(\Delta ABC)}{16} + \frac{9}{16} ar(\Delta ABC)$$

$$= \frac{5}{8} ar(\Delta ABC)$$

$$\therefore \quad ar(BPQR) = ar(\Delta ABC) - \{ar(\Delta APQ) + ar(\Delta CQR)\}$$

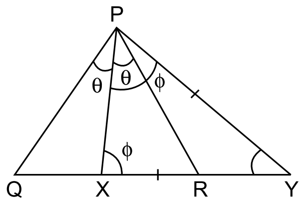
$$= ar(\Delta ABC) - \frac{5}{8} ar(\Delta ABC)$$

$$= \frac{3}{8} ar(\Delta ABC)$$

Hence, proved.

5. In ΔPQR, the angle P is bisected by PX which meets QR in X. Y is a point in QR produced such that PY = XY. Prove that $\Delta PRY \sim \Delta QPY$ and hence show that XY is a mean proportional between QY and RY, i.e., show that $XY^2 = QY \times RY$.

Sol. Given that X is a point on the side QR of ΔPQR such that PX is the bisector of $\angle QPR$. QR is produced to Y such that PY = XY. To prove that $\Delta PRY \sim \Delta QPY$ and $XY^2 = QY \times RY$



Let $\angle QPX = \angle RPX = \theta$ and $\angle XPY = \phi$

Since $\quad PY = XY$

$\therefore \quad \angle PXY = \angle XPY = \phi$

$\therefore$ In Δ PXR,

exterior $\angle PRY = \theta + \phi = \angle QPY \quad \ldots(1)$

Now, in ΔPRY and ΔQPY, we have

$\angle PRY = \angle QPY \quad$ [From (1)]

$\angle PYR = \angle QYP \quad$ [Common]

$\therefore$ By AA similarity criterion, we have

$\Delta PRY \sim \Delta QPY$

$$\therefore \quad \frac{PR}{QP} = \frac{PY}{QY} = \frac{RY}{PY} \quad \ldots(2)$$

$\therefore$ From (2), we have

$$\frac{PY}{QY} = \frac{RY}{PY}$$

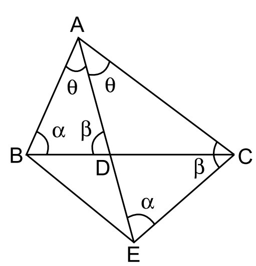
$\Rightarrow \quad PY^2 = QY \times RY$

$\Rightarrow \quad XY^2 = QY \times RY \quad [\because PY = XY]$

Hence, proved.

6. D is a point on the side BC of ΔABC such that $\frac{BD}{DC} = \frac{AB}{AC}$. AD is produced to E so that $\frac{AB}{AE} = \frac{AD}{AC}$. Prove that ΔBEC is isosceles.

Sol. Given that D is a point on the side BC of ΔABC such that $\frac{BD}{DC} = \frac{AB}{AC}$. AD is produced to E so that $\frac{AB}{AE} = \frac{AD}{AC}$.



To prove that ΔBEC is an isosceles triangle.

Construction: Join EB and EC.

Since $\frac{BD}{DC} = \frac{AB}{AC}$, hence, by the converse of the theorem of internal bisector of an angle, we have AD is the internal bisector of $\angle BAC$.

Let $\angle BAD = \angle EAC = \theta$

Now, in ΔBAD and ΔEAC, we have

$\angle BAD = \angle EAC = \theta$ and $\frac{AB}{AE} = \frac{AD}{AC}$

Hence, by SAS similarity criterion, we have

$\Delta BAD \sim \Delta EAC$

$\therefore \quad \angle BDA = \angle ECA = \beta$, say $\quad \ldots(1)$

and $\quad \angle ABD = \angle AEC = \alpha$, say $\quad \ldots(2)$

Again, in ΔCAD and ΔEAB, we have

$\angle CAD = \angle EAB = \theta$

and $\quad \frac{AC}{AE} = \frac{AD}{AB}$

Hence, by SAS similarity criterion, we have

$\Delta CAD \sim \Delta EAB$

$\therefore \quad \angle CDA = \angle EBA$

But $\quad \angle CDA = \theta + \alpha$

$[\because$ In ΔABD, exterior angle $CDA = \theta + \alpha]$

$\therefore \quad \angle CDA = \angle EBA = \theta + \alpha \quad \ldots(3)$

But since $\quad \angle CDA = 180° - \beta$

$\therefore \quad \angle CDA = \angle EBA = 180° - \beta \quad \ldots(4)$

Also, $\quad \angle ADC = \angle ABE$

$= \theta + \alpha = 180° - \beta \quad \ldots(5)$

Now, in ΔEBC

$\angle EBC = \angle ABE - \angle ABD$

$= \theta + \alpha - \alpha$

[From (5) and (2)]

$= \theta \quad \ldots(6)$

Also, $\quad \angle ECB = \angle ECA - \angle ACB$

Now, $\quad \angle ACB = 180° - 2\theta - \alpha$

and $\quad \angle ECA = \beta \quad$ [From (1)]

$\therefore \quad \angle ECB = \beta - 180° + 2\theta + \alpha$

$= -\theta - \alpha + 2\theta + \alpha$

[From (5)]

$\therefore \quad \angle EBC = \angle ECB = \theta$

[From (5) and (7)]

$\therefore \quad \Delta BEC$ is an isosceles.

Self-Assessment

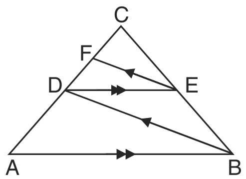
(Page 120)

Multiple-Choice Questions

1. In the given figure, AB $\parallel$ DE and BD $\parallel$ EF. Then

(*a*) $BC^2 = AB \times CE$ $\quad$ (*b*) $AB^2 = AC \times DE$

(*c*) $AC^2 = BC \times DC$ $\quad$ (*d*) $DC^2 = CF \times AC$



Sol. (*d*) $DC^2 = CF \times AC$

In ΔABC, we have

$$DE \parallel AB$$

$\therefore$ By the basic proportionality theorem, we have

$$\frac{AD}{CD} = \frac{BE}{EC} \quad \ldots(1)$$

In ΔBDC, $BD \parallel CF$

$\therefore$ By the basic proportionality theorem, we have

$$\frac{DF}{FC} = \frac{BE}{EC} \quad \ldots(2)$$

From (1) and (2), we have

$$\frac{AD}{CD} = \frac{DF}{FC}$$

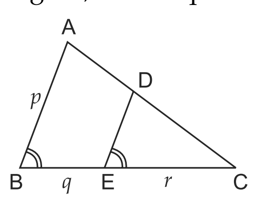
$$\Rightarrow \quad \frac{AC - CD}{CD} = \frac{CD - FC}{FC}$$

$$\Rightarrow \quad \frac{AC}{CD} - 1 = \frac{CD}{FC} - 1$$

$$\Rightarrow \quad \frac{AC}{CD} = \frac{CD}{FC}$$

$$\Rightarrow \quad CD^2 = AC \times CF$$

2. In the given figure, DE is equal to

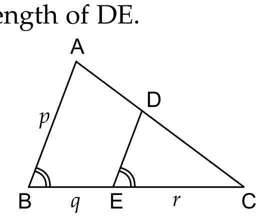


(*a*) $\dfrac{pq}{p+q}$ (*b*) $\dfrac{pr}{q+r}$

(*c*) $\dfrac{qr}{p+r}$ (*d*) $\dfrac{qr}{q+r}$

Sol. (*b*) $\dfrac{pr}{q+r}$

Given that in ΔABC, D and E are points on AC and BC respectively such that $\angle ABC = \angle DEC$, $EC = r$ units, $BE = q$ units and $AB = p$ units. To find the length of DE.



$\because \quad \angle DEC = \angle ABC$ [Corresponding $\angle$s]

$\therefore$ $AB \parallel DE$

$\therefore$ $\Delta DEC \sim \Delta ABC$

$$\therefore \quad \frac{BC}{EC} = \frac{AB}{DE}$$

$$\Rightarrow \quad \frac{q+r}{r} = \frac{p}{DE}$$

$\therefore$ Required length of DE is $\dfrac{pr}{q+r}$ units.

Fill in the Blanks

3. If $\Delta ABC \sim \Delta DEF$ and $EF = \dfrac{1}{3}BC$, then

ar (ΔABC): ar(ΔDEF) is **9 : 1**.

Sol. $\Delta ABC \sim \Delta DEF$ [Given]

$\because$ The ratio of the areas of two similar triangles is equal to the ratio of the squares of any two corresponding sides,

$$\therefore \quad \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{BC^2}{EF^2} = \frac{BC^2}{\left(\frac{1}{3}BC\right)^2} = \frac{9}{1}$$

Hence, ar(ΔABC) : ar(ΔDEF) = 9 : 1

4. The perimeter of an isosceles right triangle, the length of whose hypotenuse is 10 cm is $10\left(\sqrt{2}+1\right)$ cm.

Sol. Let ABC be an isosceles right triangle in which $\angle B = 90°$ and $AB = BC$

In right ΔABC, we have

$$AC^2 = AB^2 + BC^2$$

$$\Rightarrow \quad (10\text{ cm})^2 = 2AB^2$$

$$\Rightarrow \quad 100\text{ cm}^2 = 2AB^2$$

$$\Rightarrow \quad AB^2 = 50\text{ cm}^2$$

$$\Rightarrow \quad AB = 5\sqrt{2}\text{ cm}$$

Perimeter of $\Delta ABC = AB + BC + AC$

$= 2AB + AC$

$= (2 \times 5\sqrt{2} + 10)$ cm

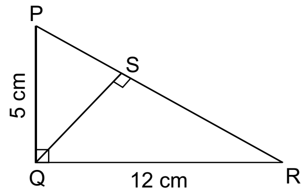
$= (10\sqrt{2} + 10)$ cm

$= 10(\sqrt{2} + 1)$ cm

5. In ΔPQR, $\angle Q = 90°$, PQ = 5 cm, QR = 12 cm. If $QS \perp PR$, then QS is equal to $\mathbf{\dfrac{60}{13}}$ **cm.**

Sol. In ΔPQR, we have

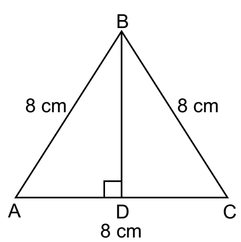
$$PR^2 = PQ^2 + QR^2$$



$$= (5\text{ cm})^2 + (12\text{ cm})^2$$
$$= 25\text{ cm}^2 + 144\text{ cm}^2$$
$$= 169\text{ cm}^2$$
$$\Rightarrow \quad PR = 13\text{ cm} \quad ...(1)$$
$$ar(\Delta PQR) = \frac{1}{2} \times QR \times PQ$$
$$= \frac{1}{2} \times 12 \times 5\text{ cm}^2$$
$$= 30\text{ cm}^2$$
[Taking QR as base] ...(2)
$$ar(\Delta PQR) = \frac{1}{2} \times PR \times QS$$
$$= \frac{1}{2} \times 13\text{ cm} \times QS$$
[Using (1)] ...(3)

From (2) and (3), we get
$$\frac{1}{2} \times 13\text{ cm} \times QS = 30\text{ cm}^2$$
$$\Rightarrow \quad QS = \frac{30 \times 2}{13}\text{ cm} = \frac{60}{13}\text{ cm}$$

6. The length of an altitude of an equilateral triangle of side 8 cm is $\mathbf{4\sqrt{3}}$ **cm.**



Sol. In right ΔADB, we have
$$AB^2 = AD^2 + BD^2$$
$$\Rightarrow \quad (8\text{ cm})^2 = AD^2 + (4\text{ cm})^2$$
$$AD^2 = (64 - 16)\text{ cm}^2 = 48\text{ cm}^2$$
$$AD = 4\sqrt{3}\text{ cm}$$

Assertion-Reason Type Questions

Directions (Q. Nos. 7 to 9): Each of these questions contains an assertion followed by reason. Read them carefully, and answer the question on the basis of the following options, select the one that best describes the two statements.

(a) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.

(b) Both Assertion and Reason are true but Reason is not the correct explanation of Assertion.

(c) If Assertion is true but Reason is false.

(d) If Assertion is false but Reason is true.

7. Assertion: Two similar triangles will have same size.

Reason: Similar triangles have the same measurement of angles.

Sol. The correct answer is (*d*) as similar triangles have same measurement of angles but not sides. The Assertion is wrong but Reason is correct.

8. Assertion: A square and rectangle are not similar.

Reason: Ratio of sides of a square and rectangle are not same.

Sol. The correct answer is (*a*).

Both the statements are correct. Rectangles have different size of length and breadth. Hence their ratios are not same, so they are not similar. Thus both statements are correct and Reason is correct explanation of the Assertion.

9. Assertion: All right-angled isosceles triangles are similar.

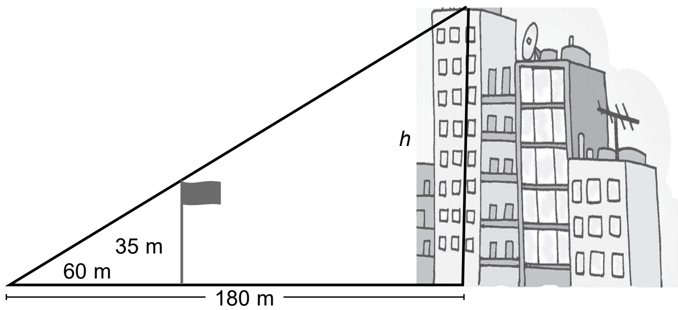
Reason: All isosceles triangles have all angles equal.

Sol. The correct answer is (*c*).

Assertion is correct as all right-angled isosceles triangles have the same angles of 90, 45 and 45 degrees. Thus Assertion is correct but Reason is wrong.

Case Study Based Questions

10. On the occasion of 15th August, lots of programmes are scheduled to be held. A flagpole stand is made outside the apartment. Nidhi looks up from the ground which is 60 m away from the flagpole. She looks up to the top of the building and the top of the flagpole in the same line up. If the flagpole is 35 m tall, and Nidhi is 180 m away from the building. Based on this given situation, answer the following questions.



(*a*) Are the triangles in the diagram similar?

(*i*) Yes (*ii*) No
(*iii*) May be (*iv*) Can't say

Ans. (*i*) Yes

(*b*) Find the height of the building.

(*i*) 90 m (*ii*) 95 m
(*iii*) 100 m (*iv*) 105 m

Ans. (*iv*) 105 m

(*c*) You are standing at the same distance 180 m away from the building and place the flagpole 85 m away from you, what will be the distance of the flagpole from the building?

(*i*) 86 m (*ii*) 90 m
(*iii*) 92 m (*iv*) 95 m

Ans. (*iv*) 95 m

(*d*) If the height of the flagpole is reduced by 3 m, then what is the height of the building in line?

(*i*) 92 m (*ii*) 96 m
(*iii*) 98 m (*iv*) 100 m

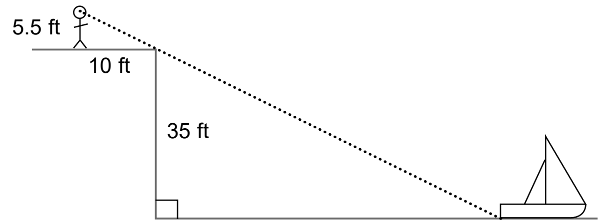
Ans. (*ii*) 96 m

(*e*) If the height of the flagpole is 30 m, then what is the height of the building in line?

(*i*) 80 m (*ii*) 85 m
(*iii*) 90 m (*iv*) 95 m

Ans. (*iii*) 90 m

11. A teacher of secondary school took 10 students of class 10 to Nongpoh, Meghalaya in summer vacation. One of the students, Amit was standing on a cliff. The cliff was 35 ft above the lake. Amit's height was 5.5 ft. Amit was standing 10 ft away from the edge of the cliff. Amit could visually align the top of the cliff with the water at the back of the boat. The situation was drawn and levelled. Based on this situation, answer the following questions.



(*a*) Find the distance of the boat from the cliff.

(*i*) 56 ft
(*ii*) 60.21 ft
(*iii*) 62.43 ft
(*iv*) 63.64 ft

Ans. (*iv*) 63.64 ft

(*b*) If Amit stands 15 ft away from the cliff, what will be the distance of the boat from the cliff?

(*i*) 94.95 ft (*ii*) 95.05 ft
(*iii*) 95.45 ft (*iv*) 96 ft

Ans. (*iii*) 95.45 ft

(*c*) A girl 160 cm tall, stands 360 cm from a lamp post at night. Her shadow from the light is 90 cm long. What is the height of the lamp post?

(*i*) 780 cm (*ii*) 798 cm
(*iii*) 800 cm (*iv*) 818 cm

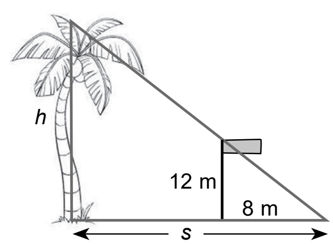
Ans. (*iii*) 800 cm

(*d*) A tower casts a shadow 7 m long. A vertical stick casts a shadow 0.6 m long. If the stick is 1.2 m high, what is the height of the tower?

(*i*) 13 m (*ii*) 14 m
(*iii*) 15 m (*iv*) 16 m

Ans. (*ii*) 14 m

(*e*) From the following figure, find *h*.



(*i*) $\frac{s}{2}$ (*ii*) $\frac{3s}{2}$
(*iii*) $\frac{4s}{3}$ (*iv*) $\frac{5s}{3}$

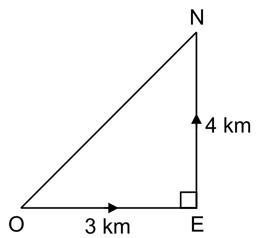
Ans. (*ii*) $\frac{3s}{2}$

Very Short Answer Type Questions

12. A man goes 3 km due east and then 4 km due north. How far is he now from the starting point?

Sol. Let O be the starting point, OE = 3 km towards east and EN = 4 km towards north. Since, OE $\perp$ NE, hence, by Pythagoras' Theorem, from ΔOEN, we have

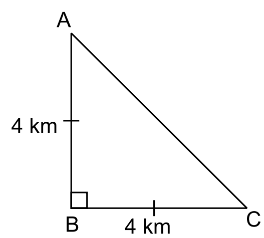
$$ON^2 = OE^2 + EN^2$$
$$= 3^2 + 4^2 = 9 + 16 = 25$$
$$\Rightarrow \quad ON = \sqrt{25} = 5$$



Hence, the man is now 5 km away from the starting point.

13. What is the length of the hypotenuse of a right angled isosceles triangle whose length of each equal side is 4 cm?

Sol. Let ΔABC be a right-angled isosceles triangle with $\angle$ABC = 90°, AB = BC = 4 cm.



To find the length of the hypotenuse AC.

$\because \quad \angle ABC = 90°$

$\therefore$ By Pythagoras' Theorem, we have

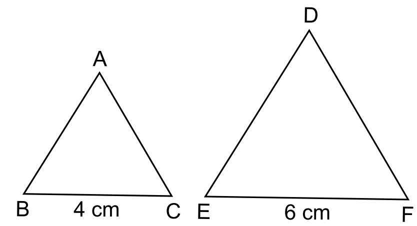
$$AC^2 = AB^2 + BC^2$$
$$= 4^2 + 4^2 = 32$$
$$\therefore \quad AC = \sqrt{32} = 4\sqrt{2}$$

Hence, the required length of the hypotenuse is $4\sqrt{2}$ cm.

Short Answer Type-I Questions

14. What is the ratio of the areas of two similar equilateral triangles of sides 4 cm and 6 cm?

Sol. Let ΔABC and ΔDEF be two equilateral triangles of sides 4 cm and 6 cm respectively.



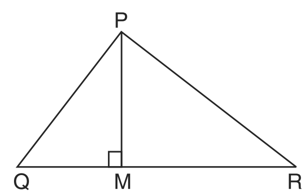
Now, all equilateral triangles are similar, since each angle of each equilateral triangle is 60°.

$$\therefore \quad \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{BC^2}{EF^2} = \frac{4^2}{6^2} = \frac{16}{36} = \frac{4}{9}$$

Hence, the required ratio is 4 : 9.

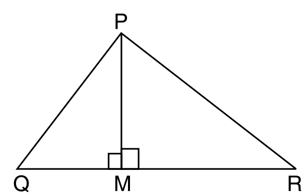
15. Use Pythagoras' theorem in the given figure to prove that $PR^2 = PQ^2 + QR^2 - 2QM \times QR$.

[CBSE SP 2006]



Sol. In ΔPQR, M is a point on QR such that PM $\perp$ QR. To prove that

$$PR^2 = PQ^2 + QR^2 - 2\,QM \times QR$$



In ΔPMR and ΔPMQ, we have

$$\angle PMR = \angle PMQ = 90°$$

Hence, by Pythagoras' Theorem, we have

$$PR^2 = PM^2 + MR^2 \qquad \text{[From } \Delta PMR] \ldots(1)$$

and
$$PQ^2 = PM^2 + MQ^2 \qquad \text{[From } \Delta PMQ] \ldots(2)$$

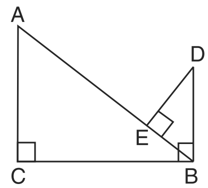
Subtracting (2) from (1), we get

$$PR^2 - PQ^2 = MR^2 - MQ^2$$
$$= (QR - QM)^2 - QM^2$$
$$= QR^2 + QM^2 - 2\,QR \times QM - QM^2$$
$$= QR^2 - 2QM \times QR$$
$$\Rightarrow \quad PR^2 = PQ^2 + QR^2 - 2QM \times QR$$

Hence, proved.

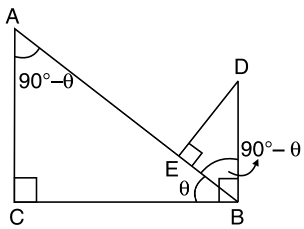
Short Answer Type-II Questions

16. In the given figure, $DB \perp BC$, $AC \perp BC$ and $DE \perp AB$. Prove that $\frac{BE}{DE} = \frac{AC}{BC}$. **[CBSE 2008]**



Sol. Given that ΔABC and ΔDEB, where E is a point on AB, are two triangle such that

$$\angle ACB = \angle DEB = \angle DBC = 90°$$



To prove that $\frac{BE}{DE} = \frac{AC}{BC}$

Let $\angle ABC = \theta$

$\therefore \quad \angle DBE = \angle DBC - \angle ABC$

$= 90° - \theta \quad \ldots(1)$

and $\quad \angle CAB = 90° - \theta \quad \ldots(2)$

Now, in ΔACB and ΔBED, we have

$\angle ACB = \angle BED = 90° \quad$ [Given]

$\angle CAB = \angle EBD = 90° - \theta$

[From (1) and (2)]

$\therefore$ By AA similarity criterion, we have

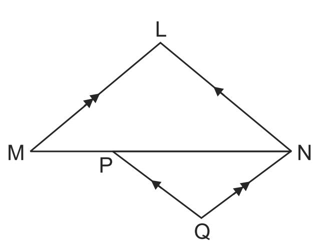
$$\Delta ACB \sim \Delta BED$$

$$\therefore \quad \frac{AC}{BE} = \frac{BC}{DE}$$

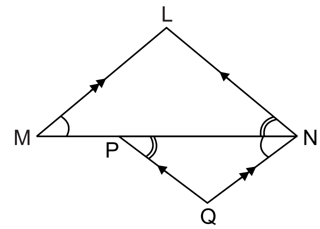
$$\Rightarrow \quad \frac{BE}{DE} = \frac{AC}{BC}$$

Hence, proved.

17. In the given figure, if $LM \parallel NQ$ and $LN \parallel PQ$. If $MP = \frac{1}{3} MN$, find the ratio of the areas of ΔLMN and ΔQNP. **[CBSE 2008]**



Sol. Given that LMN is a triangle and P is a point on MN such that PQN is another triangle, where $QN \parallel ML$ and $QP \parallel NL$.



Also, given that $MP = \frac{1}{3} MN$.

To find the ratio of the areas of ΔLMN and ΔQNP.

In ΔMLN and ΔNQP, we have

$\angle LMN = \angle QNP \quad$ [Alternate $\angle$s, $LM \parallel NQ$]

$\angle LNM = \angle QPN \quad$ [Alternate $\angle$s, $QP \parallel NL$]

$\therefore$ By AA similarity criterion, we have

$$\Delta MLN \sim \Delta NQP$$

$$\therefore \quad \frac{ar(\Delta LMN)}{ar(\Delta QNP)} = \frac{MN^2}{PN^2}$$

$$= \left(\frac{MN}{PN}\right)^2$$

$$= \left(\frac{MN}{MN - MP}\right)^2$$

$$= \left(\frac{1}{1 - \frac{MP}{MN}}\right)^2$$

$$= \frac{1}{\left(1 - \frac{1}{3}\right)^2}$$

$$= \frac{1}{\left(\frac{2}{3}\right)^2}$$

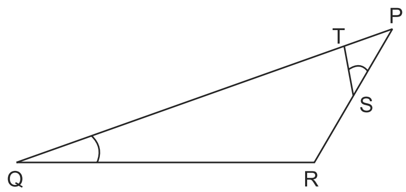
$$= \frac{1}{\frac{4}{9}}$$

$$= \frac{9}{4}$$

$\therefore$ Required ratio is 9 : 4.

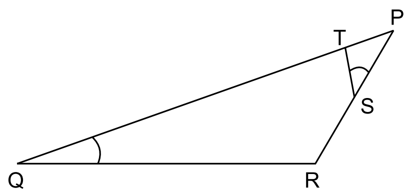
Long Answer Type Questions

18. In the given figure, $\angle PST = \angle PQR$. Prove that $\Delta PQR \sim \Delta PST$.



Hence, show that PQ × ST = PS × QR and PT × QR = ST × PR.

Sol. Given that PQR is a triangle and S, T are points on RP and QP respectively such that TSP is a triangle with ∠TSP = ∠PQR.



To prove that ΔPQR ~ ΔPST and PQ × ST = PS × QR, PT × QR = ST × PR.

In ΔPQR and ΔPST, we have

$$\angle PQR = \angle PST \quad \text{[Given]}$$
$$\angle QPR = \angle SPT \quad \text{[Common]}$$

∴ By AA similarity criterion, we have

$$\Delta PQR \sim \Delta PST$$

$$\Rightarrow \quad \frac{PQ}{PS} = \frac{PR}{PT} = \frac{QR}{ST} \quad \ldots(1)$$

From (1), we have

$$\frac{PQ}{PS} = \frac{QR}{ST}$$
$$PQ \times ST = PS \times QR$$

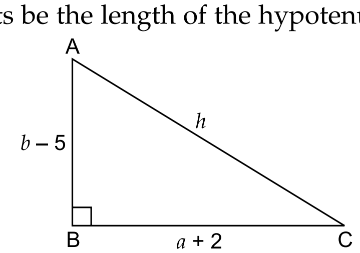
Again, from (1), we have

$$PT \times QR = ST \times PR$$

Hence, proved.

19. Find the length of the hypotenuse of a right angled triangle, if its sides are $(a + 2)$ cm, $(b - 5)$ cm where $a - b = 2$ and $a^2 + b^2 = 7$.

Sol. Let ABC be a right-angled triangle, with ∠ABC = 90°, BC = $(a + 2)$ units, AB = $(b - 5)$ units and h units be the length of the hypotenuse AC.



Also, given that $a - b = 2$ and $a^2 + b^2 = 7$.

To find the value of h.

We have $\quad a^2 + b^2 = 7 \quad \ldots(1)$

and $\quad a - b = 2 \quad \ldots(2)$

∵ $\quad \angle ABC = 90°$

Hence, by Pythagoras' Theorem, we have

$$\begin{aligned}
h^2 &= BC^2 + AB^2 \\
&= (a + 2)^2 + (b - 5)^2 \\
&= a^2 + 4 + 4a + b^2 + 25 - 10b \\
&= a^2 + b^2 + 4a - 10b + 29 \\
&= 7 + 4(2 + b) - 10b + 29 \\
&\qquad\qquad \text{[From (1) and (2)]} \\
&= 7 + 8 + 29 + 4b - 10b \\
&= 44 - 6b \qquad \ldots(3)
\end{aligned}$$

Now, from (1) and (2),

$$(2 + b)^2 + b^2 = 7$$
$$\Rightarrow \quad 4 + b^2 + 4b + b^2 = 7$$
$$\Rightarrow \quad 2b^2 + 4b - 3 = 0$$
$$\Rightarrow \quad b = \frac{-4 \pm \sqrt{16 + 4 \times 3 \times 2}}{2(2)}$$
$$= \frac{-4 \pm \sqrt{40}}{4}$$
$$= \frac{-4 \pm 2\sqrt{10}}{4}$$
$$= \frac{-2 \pm \sqrt{10}}{2}$$
$$= -1 \pm \frac{\sqrt{10}}{2} \quad \ldots(4)$$

∴ From (3) and (4), we have

$$h^2 = 44 - 6 \times \left(-1 \pm \frac{\sqrt{10}}{2}\right)$$
$$= 44 + 6 \mp 3\sqrt{10}$$
$$= 50 \mp 3\sqrt{10}$$

∴ The required length of the hypotenuse is $\sqrt{50 - 3\sqrt{10}}$ units or $\sqrt{50 + 3\sqrt{10}}$ units.

Let's Compete

(Page 122)

Multiple-Choice Questions

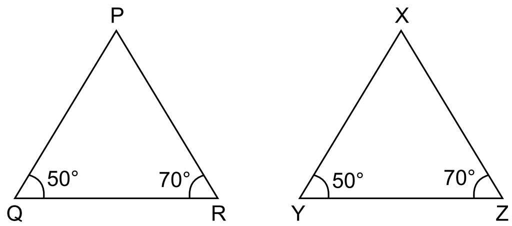
1. If ΔPQR ~ ΔXYZ, ∠Q = 50° and ∠R = 70°, then ∠X + ∠Y is equal to

(*a*) 70° (*b*) 110°

(*c*) 120° (*d*) 50°

Sol. (*b*) 110°

Given that $\Delta PQR \sim \Delta XYZ$, where $\angle Q = 50°$, $\angle R = 70°$.



To find $\angle X + \angle Y$.

In ΔPQR, we have

$\angle Q = 50°, \angle R = 70°$

$\therefore \quad \angle P = 180° - (50° + 70°)$

[By angle sum property of a triangle]

$\because \quad \Delta PQR \sim \Delta XYZ$

$\therefore \quad \angle X = \angle P = 60°, \angle Y = \angle Q = 50°$

$\therefore \quad \angle X + \angle Y = 60° + 50° = 110°$

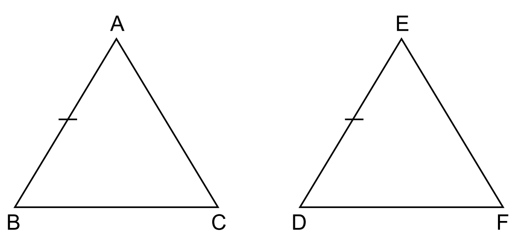
2. If in ΔABC and ΔDEF, $\dfrac{AB}{DE} = \dfrac{BC}{FD}$, then they will be similar when

(*a*) $\angle A = \angle F$ (*b*) $\angle A = \angle D$

(*c*) $\angle B = \angle D$ (*d*) $\angle B = \angle E$

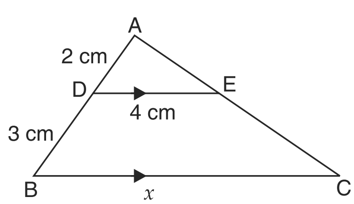
Sol. (*c*) $\angle B = \angle D$

Given that $\dfrac{AB}{DE} = \dfrac{BC}{FD}$ in ΔABC and ΔDEF.



Now, $\Delta ABC \sim \Delta EDF$ only if $\angle B = \angle D$ by SAS similarity criterion, since only $\angle B$ corresponds to $\angle D$ only.

3. In the given figure, DE ∥ BC. Then x equals to



(*a*) 6 cm (*b*) 8 cm

(*c*) 12 cm (*d*) 10 cm

Sol. (*d*) 10 cm

Given that D and E are two points on the sides AB and AC respectively of ΔABC such that DE ∥ BC. Also, given that AD = 2 cm, DB = 3 cm, DE = 4 cm and BC = x.

To find the value of x.

$\because \quad DE \parallel BC$

$\therefore \quad \Delta ADE \sim \Delta ABC$

$\Rightarrow \quad \dfrac{AB}{AD} = \dfrac{BC}{DE}$

$\Rightarrow \quad \dfrac{2+3}{2} = \dfrac{x}{4}$

$\Rightarrow \quad \dfrac{5}{2} \times 4 = x$

$\Rightarrow \quad x = 10$

Hence, the value of x is 10 cm.

4. P and Q are two points on the sides AB and AC respectively of a triangle ABC such that PQ ∥ BC, AP = 3.5 cm, PB = 7 cm, AQ = 3 cm, QC = 6 cm and PQ = 4.5 cm. Then the measure of BC is equal to

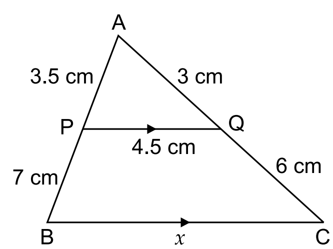
(*a*) 13.5 cm (*b*) 9 cm

(*c*) 12.5 cm (*d*) 15 cm

Sol. (*a*) 13.5 cm

Given that P and Q are two points on the sides AB and AC respectively of ΔABC, such that PQ ∥ BC. Also, given that AP = 3.5 cm, PB = 7 cm, AQ = 3 cm, QC = 6 cm and PQ = 4.5 cm.

To find the length of BC.



Let BC = x

Then, since PQ ∥ BC

$\therefore \quad \Delta APQ \sim \Delta ABC$

$\Rightarrow \quad \dfrac{PQ}{BC} = \dfrac{AQ}{AC}$

$\Rightarrow \quad \dfrac{4.5}{x} = \dfrac{3}{3+6} = \dfrac{3}{9} = \dfrac{1}{3}$

$\Rightarrow \quad x = 4.5 \times 3 = 13.5$

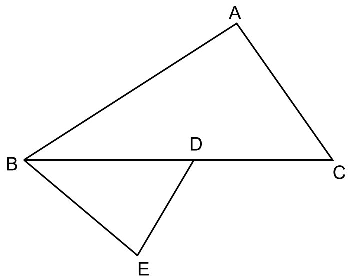
Hence, the required length of BC is 13.5 cm.

5. ABC and BDE are two equilateral triangles such that D is the mid-point of BC. Then the ratio of the areas of ΔABC and ΔBDE is (*a*) 2 : 1 (*b*) 1 : 2

(*c*) 4 : 1 (*d*) 1 : 4

Sol. (*c*) 4 : 1

Given that ΔABC is an equilateral triangle and D is the mid-point of BC.



Also, ΔBDE is another equilateral triangle. To find the ratio of the areas of ΔABC and ΔBDE.

Since all equilateral triangles are similar.

$\therefore \quad \Delta ABC \sim \Delta BDE$

$$\Rightarrow \quad \frac{\text{Area of}(\Delta ABC)}{\text{Area of}(\Delta BDE)} = \frac{BC^2}{BD^2} = \frac{(2BD)^2}{BD^2} = \frac{4}{1}$$

$\therefore$ Required ratio = 4 : 1

6. A vertical stick 30 m long casts a shadow 15 m long on the ground. At the same time, a tower casts a shadow 75 m long on the ground. Then the height of the tower is

(*a*) 150 m (*b*) 100 m

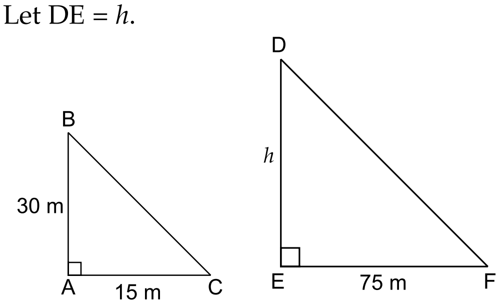
(*c*) 25 m (*d*) 200 m

Sol. (*a*) 150 m

Let AB be the vertical stick and AC be its shadow on the horizontal ground. Given that AB = 30 m and AC = 15 m. Let DE be the vertical tower and EF be its shadow on the horizontal ground.

To find the height of the tower ED.

Let DE = h.



Since the shadows are formed by the same sun, hence, $\angle ACB = \angle EFD$.

Also, $\angle BAC = \angle DEF = 90°$

$\therefore$ By AA similarity criterion, we have

$\Delta ABC \sim \Delta EDF$

$$\Rightarrow \quad \frac{AB}{ED} = \frac{AC}{EF}$$

$$\Rightarrow \quad \frac{30}{h} = \frac{15}{75} = \frac{1}{5}$$

$$\Rightarrow \quad h = 30 \times 5 = 150$$

Hence, the required height of the tower is 150 m.

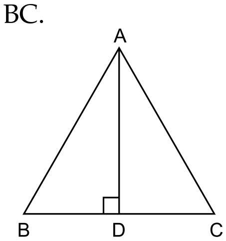
7. The length of an altitude of an equilateral triangle of side a is

(*a*) $\frac{2a}{\sqrt{3}}$ (*b*) $\frac{\sqrt{3}}{2a}$

(*c*) $\frac{a\sqrt{3}}{2}$ (*d*) $\frac{a}{2\sqrt{3}}$

Sol. (*c*) $\frac{a\sqrt{3}}{2}$

Given that ΔABC is an equilateral triangle of side a. D is a point on BC, such that AD is its altitude on BC.



$\therefore \quad \angle ADB = 90°$

To find the length of AD in terms of a.

We see that $\quad \Delta ABD \cong \Delta ACD$

Since $\quad AB = AC$

$AD = AD$ [Common]

$\angle ADB = \angle ADC = 90°$

$\therefore$ By RHS congruence criterion, we have

$\Delta ABD \cong \Delta ACD$

$\Rightarrow \quad BD = DC$

Now, in ΔABD, since $\angle ADB = 90°$,

hence, by Pythagoras' Theorem, we have

$$AB^2 = AD^2 + BD^2$$

$$\Rightarrow \quad a^2 = AD^2 + \left(\frac{1}{2}a\right)^2$$

$$\Rightarrow \quad AD^2 = a^2 - \frac{a^2}{4}$$

$$\Rightarrow \quad AD^2 = \frac{3a^2}{4}$$

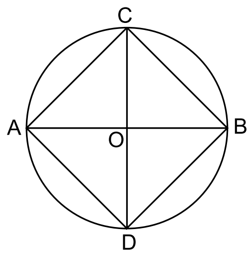
$$\therefore \quad AD = \frac{\sqrt{3}}{2}a$$

8. The area of a square inscribed in a circle of radius 8 cm is

(*a*) 64 cm^2 (*b*) 100 cm^2
(*c*) 120 cm^2 (*d*) 128 cm^2

Sol. (*d*) 128 cm^2

Let ACBD be a square inscribed in a circle with centre at O and diameter AOB. OB = OA = radius of the circle = 8 cm (given). The two diameters AOB and COD of the circle are the diagonals of the square so that $\angle AOC = 90°$.



In ΔAOC,

$\therefore$ By Pythagoras' Theorem, we have

$$AC^2 = AO^2 + OC^2$$
$$= 8^2 + 8^2 \quad [\because OC = OA = 8 \text{ cm}]$$
$$= 8^2 \times 2$$

$\therefore$ $$AC = \pm\sqrt{8^2 \times 2} = \pm 8\sqrt{2}$$

$\therefore$ Required area of the square

$$= AC^2 \text{ cm}^2$$
$$= \left(8\sqrt{2}\right)^2 \text{ cm}^2$$
$$= 128 \text{ cm}^2$$

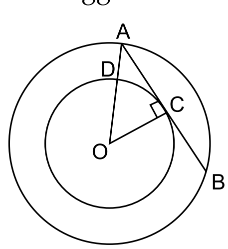
Hence, the area of a square inscribed in a circle of radius 8 cm is 128 cm^2.

9. The radii of two concentric circles are 15 cm and 17 cm. Then the length of chord of one circle which is tangent to the other is

(*a*) 8 cm (*b*) 16 cm
(*c*) 30 cm (*d*) 17 cm

Sol. (*b*) 16 cm

Let AB be a chord of the circle with radius OA = 17 cm and let AB touch a smaller concentric circle of radius OD = 15 cm at a point C. Let O be the common centre of the two circles. Now, since, OC is the radius of the smaller circle meets the chord AB of the bigger circle at C.



$\therefore$ $$\angle OCA = 90°$$

Now, from ΔOCA, using Pythagoras' Theorem, we have

$$AO^2 = OC^2 + CA^2$$

$\Rightarrow$ $$17^2 = 15^2 + \left(\frac{1}{2}AB\right)^2$$

[$\because$ C is the mid-point of the chord AB]

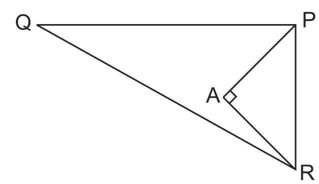
$\Rightarrow$ $$17^2 - 15^2 = \frac{1}{4}AB^2$$

$\Rightarrow$ $$AB^2 = 4(17 + 15)(17 - 15)$$
$$= 4 \times 32 \times 2$$
$$= 4 \times 64$$

$\Rightarrow$ $$AB = \sqrt{4 \times 64} = 2 \times 8 = 16$$

$\therefore$ Required length of the chord AB = 16 cm.

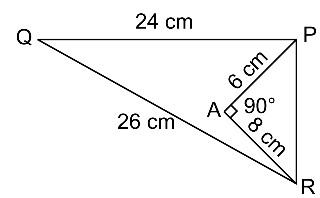
10. In the given figure, if PQ = 24 cm, QR = 26 cm, $\angle PAR = 90°$, PA = 6 cm and AR = 8 cm, then $\angle QPR$ is



(*a*) 30° (*b*) 90°
(*c*) 60° (*d*) 45°

Sol. (*b*) 90°

Given that PQR is a triangle with PQ = 24 cm, and QR = 26 cm.



A is a point within ΔPQR such that PAR is a right-angled triangle with $\angle PAR = 90°$, PA = 6 cm and AR = 8 cm.

To find $\angle QPR$.

In ΔPAR,

$\because$ $$\angle PAR = 90°$$

$\therefore$ By Pythagoras' Theorem, we have

$$PR^2 = PA^2 + AR^2$$
$$= 6^2 + 8^2$$
$$= 36 + 64$$
$$= 100$$

$\Rightarrow$ $$PR = \sqrt{100} = 10$$

Now, in ΔPQR, we see that

$$PQ^2 + PR^2 = 24^2 + 10^2$$
$$= 576 + 100 = 676$$

Also, $$QR^2 = 26^2$$
$$= 676$$

$\therefore$ $$PQ^2 + PR^2 = QR^2$$

$\therefore$ By converse of Pythagoras' Theorem, $\angle QPR = 90°$.

Value-based Questions (Optional)

(Page 123)

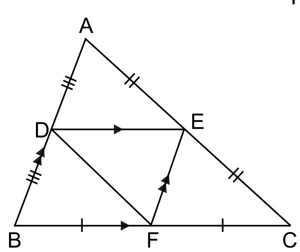
1. A man had a triangular plot of land ABC. He wanted to donate one-fourth of this plot for building a hospital and divide the remaining portion of land equally between a primary school, a secondary school and an orphanage.

(*a*) How can he divide the land? Justify your answer.

(*b*) What are the values exhibited by the man?

Sol. (*a*) Let the total area of the triangle ABC be Δ sq units. He wants to divide this land into 4 equal parts. Then the area of each part will be $\frac{\Delta}{4}$ sq units.

If D and E are two points on AB and AC such that DE $\parallel$ BC, then ar(ΔADE) = $\frac{\Delta}{4}$ sq units.



$$\therefore \quad \frac{ar(\Delta ADE)}{ar(\Delta ABC)} = \frac{1}{4} = \frac{AD^2}{AB^2} = \frac{AD^2}{4AD^2}$$

If $AB^2 = 4AD^2$

i.e. $AB = 2AD$

i.e. D is the mid-point of AB.

Hence, E is also the mid-point of AC.

$\therefore$ He denote a triangular part ADE for building a hospital.

Then we can divide the entire triangle into two equal parts, i.e. ΔADE of area $\frac{\Delta}{4}$ and trapezium DECB of the remaining area $\Delta - \frac{\Delta}{4} = \frac{3\Delta}{4}$. We shall now divide this trapezium into three equal parts, each of area $\frac{\Delta}{4}$ sq units. If F is the mid-point of BC and if we join EF, then EF $\parallel$ AB.

$$\therefore \quad \frac{ar(\Delta EFC)}{ar(\Delta ABC)} = \frac{1}{4} = \frac{CF^2}{BC^2} = \frac{CF^2}{4CF^2}$$

[$\because$ F is the mid-point of BC]

Thus, the ar(ΔEFC) = $\frac{\Delta}{4}$.

Hence, the area of the remaining portion DEFB is

$$\Delta - \left(\frac{\Delta}{4} + \frac{\Delta}{4}\right) = \frac{\Delta}{4}$$

We shall now divide this quadrilateral DEFB into two equal areas $\frac{\Delta}{4}$ and $\frac{\Delta}{4}$ sq units. We see that since BF $\parallel$ DE and EF $\parallel$ DB, therefore, the quadrilateral DEFB is a parallelogram. If we join one diagonal DF of this parallelogram, then ar(ΔDBF) = ar(ΔDEF) = $\frac{\Delta}{4}$, since each diagonal divides a parallelogram into two equal areas.

Thus, out of these three equal areas of triangles EFC, DBF and DEF, there will be a primary school, a secondary school and an orphanage.

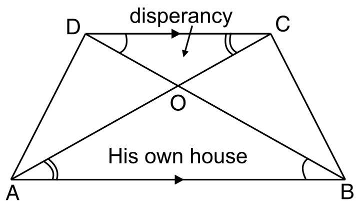
(*b*) Empathy, problem solving ability and humanity.

2. A man had a plot of land in the shape of a trapezium ABCD with AB $\parallel$ CD and AB = 3CD. The man divides his land into four triangles by joining the diagonals AC and BD meeting each other at O. He donated the triangular part CDO for building a dispensary for the villagers and uses the triangular piece AOB for building his own house. He planted trees in the remaining triangular regions AOD and COB.

(*a*) If ar(ΔAOB) = 198 m^2, what is the area of the land donated by the man for building a dispensary?

(*b*) What values were exhibited by the man?

Sol. Let ABCD be a trapezium with AB $\parallel$ CD and AB = 3CD. Diagonals AC and BD intersect each other at O forming triangles : ΔODC, ΔOAB, ΔOAD and ΔOBC. He donated ΔCOD for building a dispensary for the villagers and he uses ΔAOB for building his own house. To find the area of ΔODC.



(*a*) We see that in ΔAOB and ΔCOD

We have $\angle AOB = \angle COD$

[Vertically opposite angles]

$\angle OAB = \angle OCD$

[Alternate angles, $\because$ AB $\parallel$ CD]

$\angle OBA = \angle ODC$

[Alternate angles, $\because$ AB $\parallel$ CD]

$\therefore$ By AAA similarity criterion, we have

$$\Delta AOB \sim \Delta COD$$

$$\therefore \quad \frac{ar(\Delta AOB)}{ar(\Delta COD)} = \frac{AB^2}{CD^2}$$

$$= \frac{(3CD)^2}{CD^2}$$

$$= \frac{9CD^2}{CD^2} = 9$$

$$\Rightarrow \quad \frac{198m^2}{ar(\Delta COD)} = 9$$

$$\Rightarrow \quad ar(\Delta COD) = \frac{198m^2}{9} = 22\ m^2$$

Hence, the required area is 22 m^2.

(*b*) Empathy and awareness about environment and pollution.

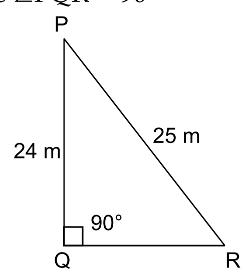
3. A boy observes an injured pigeon on a ventilator of his building at a height of 24 m from the ground. The ladder available for him just to reach the pigeon is 25 m long.

(*a*) At what distance from the wall of his building will he have to place the foot of the ladder so that he just reaches the pigeon to provide it the required help?

(*b*) What values does the boy shows?

Sol. (*a*) Let PQ be the height of the vertical building, P being the position of the pigeon. Let QR be the horizontal ground so that $\angle PQR = 90°$. PR is the ladder of length PR = 25 m. Also, PQ = 24 m. To find the distance QR.

Now, since $\angle PQR = 90°$



$\therefore$ By Pythagoras' Theorem, we have

$$PR^2 = PQ^2 + QR^2$$

$$\Rightarrow \quad 25^2 = 24^2 + QR^2$$

$$\Rightarrow \quad QR = \sqrt{25^2 - 24^2}$$

$$= \sqrt{(25+24)(25-24)}$$

$$= \sqrt{49} = 7$$

Hence, the required distance of QR is 7 m.

(*b*) Empathy, kindness and helpfulness.

7

Coordinate Geometry

Checkpoint (Page 126)

1. In which quadrant does point (–3, 5) lie?
   (*a*) first quadrant (*b*) second quadrant
   (*c*) third quadrant (*d*) fourth quadrant

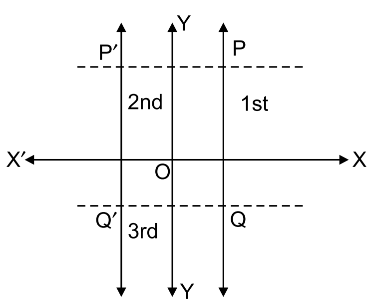
Sol. (*b*) second quadrant

Since x-coordinate is negative and y-coordinate is positive, hence, the given point lies in the second quadrant.

2. Two points having same abscissae but different ordinates lie on
   (*a*) x-axis
   (*b*) y-axis
   (*c*) a line parallel to y-axis
   (*d*) a line parallel to x-axis

Sol. (*c*) a line parallel to y-axis

Two such points, P and Q, must lie either in the first and fourth quadrants or, in the second and third quadrants as shown in the figure. Hence, these two points, P and Q must lie on a line parallel to y-axis as shown in the figure.



3. Which of the following is a solution set of the equation $3x + 2y = 10$?
   (*a*) (1, 3) (*b*) (2, 2)
   (*c*) (2, 3) (*d*) (3, 2)

Sol. (*b*) (2, 2)

We see that $x = 2$ and $y = 2$ satisfies the given equation $3x + 2y = 10$.

$$\begin{aligned} \text{LHS} &= 3 \times 2 + 2 \times 2 \\ &= 6 + 4 \\ &= 10 \\ &= \text{RHS} \end{aligned}$$

4. Find the area of the triangle ABC, formed joining the A(0, 1), B(0, 5) and C(3, 4).
   (*a*) 4 sq units (*b*) 5 sq units
   (*c*) 6 sq units (*d*) 8 sq units

Sol. (*c*) 6 sq units

We know that the area of ΔABC with vertices A(x_1, y_1), B(x_2, y_2), and C(x_3, y_3) is

$$\frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right]$$

$$= \left|\frac{1}{2}\left[0 \times (5-4) + 0 \times (4-1) + 3 \times (1-5)\right]\right| \text{ sq units}$$

$$= \left|\frac{1}{2} \times 3 \times -4\right| \text{ sq units}$$

$$= |3 \times -2| \text{ sq units}$$

$$= |-6|$$

$$= 6$$

Hence, the required area of the triangle ABC is 6 sq units.

5. The distance of point P(4, 3) from origin is
   (*a*) 7 units (*b*) 5 units
   (*c*) 8 units (*d*) 10 units

Sol. (*b*) 5 units

The required distance is $\sqrt{3^2 + 4^2}$ units, i.e. 5 units.

6. On which axes do the following points lie?

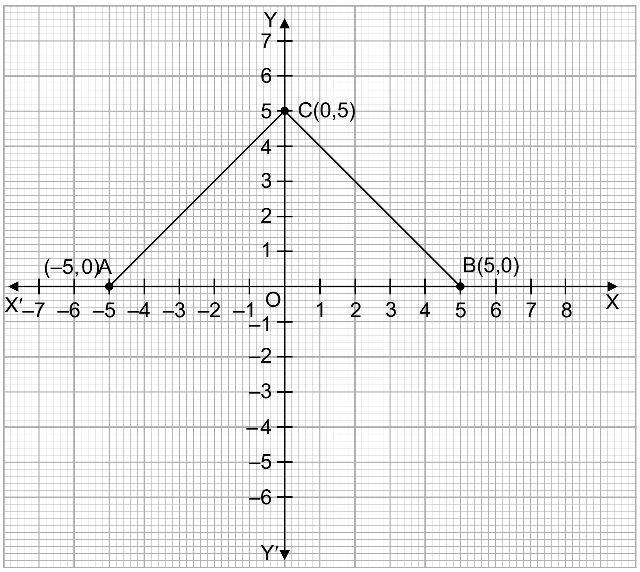
(a) (0, 1) (b) (–3, 0)

Sol. (a) Since x-coordinate is 0, hence, the point (0, 1) lies on the y-axis.

(b) Since y-coordinate is 0, hence, the point (–3, 0) lies on the x-axis.

7. Plot the points (–5, 0), (0, 5) and (5, 0) in rectangular coordinate system. Join them and name the type of the triangle formed.

Sol. We plot the points A(–5, 0), B(5, 0) and C(0, 5) on the graph paper and join AC and BC.



Let O be the origin.

We see that OA = OB = OC = 5 units.

$\therefore$ In ΔAOC,

$$\angle OAC = \angle OCA = 45°$$

and in ΔOBC,

$$OB = OC = 5 \text{ units}$$

$$\therefore \quad \angle OBC = \angle OCB = 45°$$

$$\therefore \quad \angle ACB = \angle ACO + \angle BCO = 45° + 45° = 90°$$

$\therefore$ In ΔABC,

$$AC = \sqrt{OA^2 + OC^2} = \sqrt{5^2 + 5^2} = \sqrt{50} = 5\sqrt{2}$$

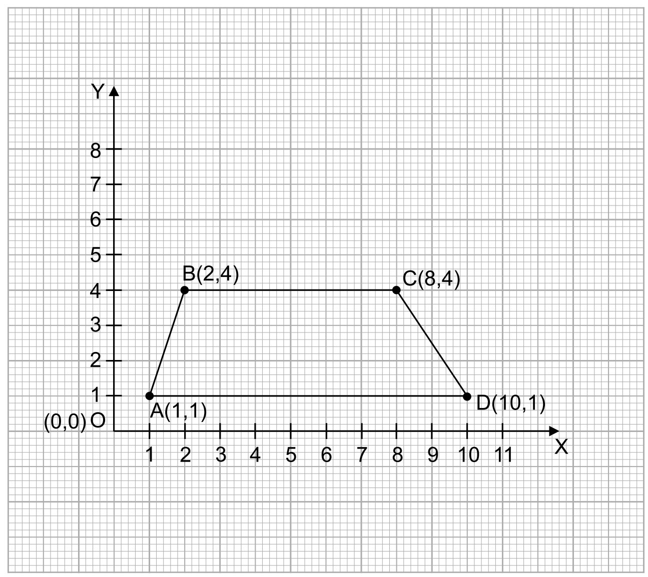
$$BC = \sqrt{OB^2 + OC^2} = \sqrt{5^2 + 5^2} = \sqrt{50} = 5\sqrt{2}$$

$\therefore$ In ΔABC, AC = BC and $\angle$ACB = 90°.

$\therefore$ ΔABC is a right-angled isosceles triangle.

8. Draw the quadrilateral whose vertices are A(1, 1), B(2, 4), C(8, 4) and D(10, 1). Name the type of quadrilateral formed.

Sol. We plot the points A(1, 1), B(2, 4), C(8, 4) and D (10, 1) on a graph paper with a suitable scale. We join the points A, B, C, and D by 4 line segments AB, BC, CD and DA to form a quadrilateral ABCD. Since, B and C are at a distance of 4 units and A and C are at a distance of 1 unit from the x-axis, hence, BC $\parallel$ AD. But AB is not parallel to CD. Hence, the quadrilateral ABCD is a trapezium.



9. Write the equation of

(a) x-axis

(b) y-axis

(c) a line parallel to x-axis

Sol. (a) On x-axis, the y-coordinate of every point is zero.

$\therefore$ The required equation of x-axis is $y = 0$.

(b) On the y-axis, the x-coordinate of every point is zero.

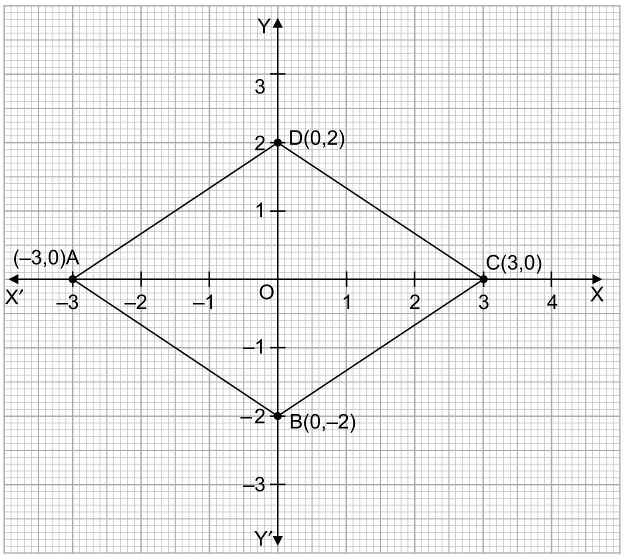
Hence, the required equation of y-axis is $x = 0$.

(c) Every point on the line parallel to x-axis is at a constant distance, say c, from the x-axis. Hence, the required equation of such a line is $y = c$, where c is a constant.

10. If coordinates of three vertices of a rhombus are (–3, 0), (0, –2) and (3, 0), then find the coordinates of the fourth vertex.

Sol. Let A (–3, 0), B(0, –2), C(3, 0) be the three coordinate of the three of the three vertices of a rhombus ABCD. We know that the two diagonals AC and BD of a rhombus bisect each other at right-angle at the origin 0, so that OA = OC – 3 units and OB = OD. But OB = 2 units.

$\therefore$ OD = 2 units



$\therefore$ The required coordinates of the fourth vertex are (0, 2)

Milestone 1

(Page 131)

Multiple-Choice Questions

1. The distance between two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is given by

(*a*) $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

(*b*) $\sqrt{(x_1+x_2)^2+(y_1+y_2)^2}$

(*c*) $\sqrt{(x_1+y_1)^2+(x_2+y_2)^2}$

(*d*) $\sqrt{(x_1-y_1)^2+(x_2-y_2)^2}$

Sol. (*a*) $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

We know the well-known formula for distance between two points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$.

2. The distance between the points (–1, 6) and (2, 2) is

(*a*) 5 units (*b*) 8 units

(*c*) 4 units (*d*) – 5 units

Sol. (*a*) 5 units

$$\text{The required distance} = \sqrt{(2+1)^2+(2-6)^2} \text{ units}$$
$$= \sqrt{9+16} \text{ units}$$
$$= \sqrt{25} \text{ units}$$
$$= 5 \text{ units}$$

Very Short Answer Type Questions

3. Find the distance of the point (3, –4) from origin.

Sol. $$\text{The required distance} = \sqrt{(3-0)^2+(-4-0)^2} \text{ units}$$
$$= \sqrt{9+16} \text{ units}$$
$$= \sqrt{25} \text{ units}$$
$$= 5 \text{ units}$$

4. On which axis do the point A(0, –5) lie?

Sol. Since the abscissa of the point A(0, –5) is 0, hence, A lies on the *y*-axis.

Short Answer Type-I Questions

5. For what value of x is the distance between the points A(–3, 2) and B(x, 8) is 10 units?

Sol. Here, A(–3, 2) and B(x, 8) be the given points $x_1 = -3, y_1 = 2$ and $x_2 = x, y_2 = 8$

Given AB = 10 units

By distance formula,

$$AB = \sqrt{\left[x-(-3)^2\right]+(8-2)^2}$$
$$\Rightarrow \quad 10 = \sqrt{(x+3)^2+(6)^2}$$
$$\Rightarrow \quad \sqrt{x^2+9+6x+36} = 10$$
$$\Rightarrow \quad \sqrt{x^2+6x+45} = 10$$

On squaring both sides,

$$x^2+6x+45 = 100$$
$$\Rightarrow \quad x^2+6x-55 = 0$$
$$\Rightarrow \quad x^2+11x-5x-55 = 0$$
$$\Rightarrow \quad x(x+11)-5(x+11) = 0$$
$$\Rightarrow \quad (x+11)(x-5) = 0$$
$$\Rightarrow \quad x = -11 \text{ or } x = 5$$

Hence, the required value of x is –11 or 5.

6. Find a point on the x-axis which is equidistant from the points A(5, 2) and B(1, –2).

Sol. Let any point on the x-axis be $P(x_1, 0)$.

According to the problem, we have PA = PB, where A(5, 2) and B(1, –2).

∴ By distance formula

$$\sqrt{(x_1-5)^2+(0-2)^2} = \sqrt{(x_1-1)^2+(0+2)^2}$$

$$\Rightarrow \quad x_1^2 - 10x_1 + 25 + 4 = x_1^2 - 2x_1 + 1 + 4$$

$$\Rightarrow \quad -10x_1 + 29 = -2x_1 + 5$$

$$\Rightarrow \quad 8x_1 + 5 - 29 = 0$$

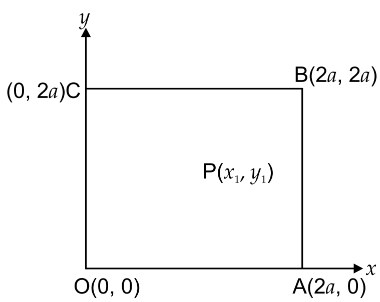
$$\Rightarrow \quad 8x_1 = 24$$

$$\Rightarrow \quad x_1 = \frac{24}{8} = 3$$

∴ The required point is (3, 0).

7. Find the vertices of the square of side '2a' with one vertex as origin such that other two vertices lie on the positive x and y axes.

Sol. If one vertex O(0, 0) lie at the origin and the other two vertices A and C lie on the positive x and y-axes respectively, then the coordinates of A are (2a, 0) and those of C are (0, 2a). Hence, the fourth vertex B is the point (2a, 2a).



Short Answer Type-II Questions

8. If the point P(x, y) is equidistant from A(3, 6) and B(–3, 4), prove that $3x + y - 5 = 0$. **[CBSE 2008]**

Sol. We have

$$PA = PB$$

∴ By distance formula,

$$\sqrt{(x-3)^2+(y-6)^2} = \sqrt{(x+3)^2+(y-4)^2}$$

On squaring both sides,

$$(x-3)^2 + (y-6)^2 = (x+3)^2 + (y-4)^2$$

$$\Rightarrow \quad x^2 + y^2 - 6x - 12y + 9 + 36 = x^2 + y^2 + 6x - 8y + 9 + 16$$

$$\Rightarrow \quad 12x + 4y + 25 - 45 = 0$$

$$\Rightarrow \quad 12x + 4y - 20 = 0$$

$$\Rightarrow \quad 3x + y - 5 = 0$$

Hence, proved.

9. Find a point which is equidistant from points (7, –6), (–1, 0), and (–2, –3).

Sol. Let the point P(x, y) be equidistant from the points A(7, –6), B(–1, 0) and C(–2, –3).

$$\therefore \quad PA^2 = PB^2 = PC^2$$

$$\Rightarrow \quad (7-x)^2 + (-6-y)^2 = (-1-x)^2 + y^2 = (-2-x)^2 + (-3-y)^2$$

$$\Rightarrow \quad x^2 + y^2 - 14x + 12y + 49 + 36 = x^2 + y^2 + 2x + 1 = x^2 + y^2 + 4x + 6y + 4 + 9$$

$$\Rightarrow \quad -14x + 12y + 85 = 2x + 1 = 4x + 6y + 13$$

∴ From the first and the middle expressions, we have

$$14x + 2x - 12y + 1 - 85 = 0$$

$$\Rightarrow \quad 16x - 12y - 84 = 0$$

$$\Rightarrow \quad 4x - 3y - 21 = 0 \qquad \ldots(1)$$

Again, from the first and the last expressions, we have

$$-14x + 12y + 85 = 4x + 6y + 13$$

$$\Rightarrow \quad 18x - 6y - 72 = 0$$

$$\Rightarrow \quad 3x - y - 12 = 0 \qquad \ldots(2)$$

From (2), $\quad y = 3x - 12 \qquad \ldots(3)$

∴ From (1),

$$4x - 3(3x - 12) - 21 = 0$$

$$\Rightarrow \quad 4x - 9x + 36 - 21 = 0$$

$$\Rightarrow \quad 5x = 15$$

$$\Rightarrow \quad x = \frac{15}{5} = 3$$

∴ From (3), $\quad y = 3 \times 3 - 12 = -3$

∴ The required point is (3, –3).

10. Using distance formula, show that the points (–4, –1), (0, 3) and (6, 9) are collinear.

Sol. Let A, B and C be the points (–4, –1), (0, 3) and (6, 9) respectively.

By distance formula,

$$AB = \sqrt{(0+4)^2+(3+1)^2}$$

$$= \sqrt{4^2+4^2}$$

$$= \sqrt{16+16}$$

$$= \sqrt{32}$$

$$= 4\sqrt{2}$$

$$BC = \sqrt{(6-0)^2(9-3)^2}$$

$$= \sqrt{6^2 + 6^2}$$

$$= \sqrt{36 + 36}$$

$$= \sqrt{72}$$

$$= 6\sqrt{2}$$

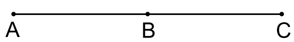
$$AC = \sqrt{(6+4)^2 + (9+1)^2}$$

$$= \sqrt{10^2 + 10^2}$$

$$= \sqrt{100 + 100}$$

$$= \sqrt{200}$$

$$= 10\sqrt{2}$$



$\therefore$ We see that

$$AB + BC = 4\sqrt{2} + 6\sqrt{2}$$

$$= 10\sqrt{2} = AC$$

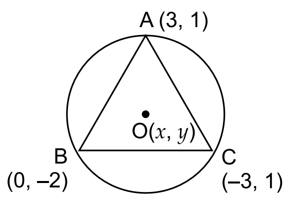
Hence, A, B, C are collinear.

11. Find the circumcentre of the triangle whose vertices are (3, 1), (0, –2) and (–3, 1).

Sol. Let A, B, C be the points (3, 1), (0, –2) and (–3, 1) respectively and let O(x, y) be the circumcentre of this triangle.

Then $\quad OA = OB = OC$

$\Rightarrow \quad OA^2 = OB^2 = OC^2$



By distance formula

$$(3-x)^2 + (1-y)^2 = (0-x)^2 + (-2-y)^2$$

$$= (-3-x)^2 + (1-y)^2$$

$\Rightarrow$ $x^2 + y^2 - 6x - 2y + 9 + 1$

$$= x^2 + y^2 + 4y + 4$$

$$= x^2 + y^2 + 6x - 2y + 9 + 1$$

$\Rightarrow \quad -6x - 2y + 10 = 4y + 4$

$$= 6x - 2y + 10$$

From the first and the second expressions, we have

$6x + 4y + 2y + 4 - 10 = 0$

$\Rightarrow \quad 6x + 6y - 6 = 0$

$\Rightarrow \quad x + y - 1 = 0 \quad \ldots(1)$

From the first and the last expressions, we have

$$12x = 0$$

$\Rightarrow \quad x = 0$

$\therefore$ From (1), $\quad y = 1$

$\therefore$ The required coordinates of the circumcentre are (0, 1).

12. Find the distance between the points

(a) (a, b) and ($-a$, $-b$)

(b) (p sin 55°, 0) and (0, p sin 35°)

Sol. (a) The distance between the points (a, b) and ($-a$, $-b$)

$$= \sqrt{(-a-a)^2 + (-b-b)^2}$$

$$= \sqrt{(-2a)^2 + (-2b)^2}$$

$$= \sqrt{4a^2 + 4b^2}$$

$$= 2\sqrt{a^2 + b^2}$$

$\therefore$ The required distance is $2\sqrt{a^2 + b^2}$.

(b) We have sin 35° = cos (90° – 35°) = cos 55°

$\therefore$ The distance between the points (p sin 55°, 0) and (0, p cos 55°)

$$= \sqrt{(0 - p\sin 55°)^2 + (p\cos 55° - 0)^2}$$

$$= \sqrt{(-p\sin 55°)^2 + (p\cos 55°)^2}$$

$$= \sqrt{p^2(\sin^2 55° + \cos^2 55°)}$$

$$= \sqrt{p^2} = p$$

$\therefore$ The required distance is p.

Long Answer Type Questions

13. If the points (3, 7) and (x, y) are equidistant from point (9, 10), prove $x^2 + y^2 - 18x - 20y + 136 = 0$.

Sol. Let A, B and C be the points (3, 7), (x, y) and (9, 10) respectively. It is given that A and B are equidistant from the point C.

i.e. AC = BC

$\therefore$ By distance formula,

$$\sqrt{(9-3)^2 + (10-7)^2} = \sqrt{(9-x)^2 + (10-y)^2}$$

$$\Rightarrow \sqrt{6^2 + 3^2} = \sqrt{(9-x)^2 + (10-y)^2}$$

$$\Rightarrow \sqrt{36 + 9} = \sqrt{x^2 + y^2 - 18x - 20y + 81 + 100}$$

On squaring both sides,

$\Rightarrow \; 45 = x^2 + y^2 - 18x - 20y + 181$

$\Rightarrow \; x^2 + y^2 - 18x - 20y + 181 - 45 = 0$

$\Rightarrow \; x^2 + y^2 - 18x - 20y + 136 = 0$

Hence, proved.

14. Show (5, 1) is the centre of circle circumscribing the triangle whose vertices are (3, 3), (7, 3) and (3, –1).

Sol. Let O be the point (5, 1) and A(3, 3), B(7, 3) and C(3, –1) be the vertices of ΔABC.

$\therefore$ By distance formula,

$$OA = \sqrt{(3-5)^2 + (3-1)^2} = \sqrt{(-2)^2 + 2^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$$

$$OB = \sqrt{(7-5)^2 + (3-1)^2} = \sqrt{2^2 + 2^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$$

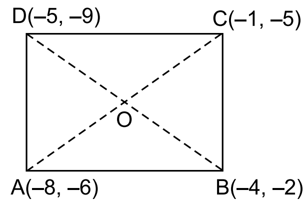
$$OC = \sqrt{(3-5)^2 + (-1-1)^2} = \sqrt{(-2)^2 + (-2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$$

$\therefore \quad OA = OB = OC$

$\therefore$ O(5, 1) is the centre of the circle circumscribing the triangle whose vertices are (3, 3), (7, 3) and (–3, –1). Hence, proved.

15. Show that points (–8, –6), (–4, –2), (–1, –5) and (–5, –9) are vertices of a rectangle.

Sol. Let A(–8, –6), B(–4, –2), C(–1, –5) and D(–5, –9) be the vertices of a quadrilateral ABCD. To prove that ABCD is a rectangle. We join AC and BD.



By distance formula,

$$AB = \sqrt{(-4+8)^2 + (-2+6)^2} = \sqrt{4^2 + 4^2} = \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$$

$$BC = \sqrt{(-1+4)^2 + (-5+2)^2} = \sqrt{3^2 + (-3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$$

$$CD = \sqrt{(-5+1)^2 + (-9+5)^2} = \sqrt{(-4)^2 + (-4)^2} = \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$$

$$DA = \sqrt{(-8+5)^2 + (-6+9)^2} = \sqrt{(-3)^2 + 3^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$$

$$BD = \sqrt{(-5+4)^2 + (-9+2)^2} = \sqrt{(-1)^2 + (-7)^2} = \sqrt{1+49} = \sqrt{50} = 5\sqrt{2}$$

$$AC = \sqrt{(-1+8)^2 + (-5+6)^2} = \sqrt{7^2 + 1^2}$$

$$= \sqrt{49+1}$$
$$= \sqrt{50}$$
$$= 5\sqrt{2}$$

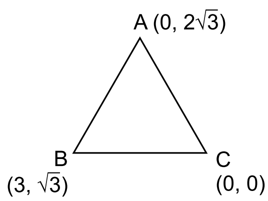
$\therefore$ AB = CD and BC = DA

i.e. each pair of opposite sides of the quadrilateral is equal. Also, diagonals AC and BD are equal to each other. Hence, the quadrilateral ABCD is a rectangle.

Hence, proved.

16. Show that points $\left(0, 2\sqrt{3}\right)$, $\left(3, \sqrt{3}\right)$ and (0, 0) are vertices of an equilateral triangle.

Sol. Let $A\left(0, 2\sqrt{3}\right), B\left(3, \sqrt{3}\right)$ and C(0, 0) be the vertices of a triangle ABC. To prove that ΔABC is an equilateral triangle.



We have by distance formula

$$AB = \sqrt{(3-a)^2 + \left(\sqrt{3} - 2\sqrt{3}\right)^2}$$
$$= \sqrt{3^2 + \left(-\sqrt{3}\right)^2}$$
$$= \sqrt{9+3}$$
$$= \sqrt{12}$$
$$= 2\sqrt{3}$$
$$AC = \sqrt{0^2 + \left(2\sqrt{3}\right)^2}$$
$$= \sqrt{12}$$
$$= 2\sqrt{3}$$

and $$BC = \sqrt{3^2 + \left(\sqrt{3}\right)^2}$$
$$= \sqrt{12}$$
$$= 2\sqrt{3}$$

$\therefore$ AB = BC = AC

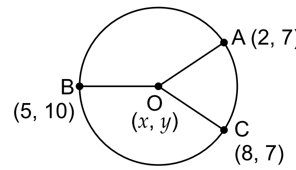
$\therefore$ ΔABC is an equilateral triangle.

$\therefore$ The points $(0,\ 2\sqrt{3})$, $\left(3, \sqrt{3}\right)$ and (0, 0) are vertices of an equilateral triangle.

Hence, proved.

17. Find the coordinates of the centre of circle passing through points (2, 7), (5, 10) and (8, 7). Also find its radius.

Sol. Let the coordinate of the centre O of the circle be (x, y) and be A(2, 7), B(5, 10) and C(8, 7) be three given points. A circle passes through the points A, B and C. To find the centre (x, y) and the radius OA of the circle.



We see that OA = OB = OC = radius of the circle.

$\therefore$ By distance formula

$$OA = \sqrt{(2-x)^2 + (7-y)^2}$$
$$= \sqrt{x^2 + y^2 + 4 + 49 - 4x - 14y}$$
$$= \sqrt{x^2 + y^2 - 4x - 14y + 53} \quad \ldots(1)$$
$$OB = \sqrt{(5-x)^2 + (10-y)^2}$$
$$= \sqrt{x^2 + y^2 + 25 + 100 - 10x - 20y}$$
$$= \sqrt{x^2 + y^2 - 10x - 20y + 125}$$
$$OC = \sqrt{(8-x)^2 + (7-y)^2}$$
$$= \sqrt{x^2 + y^2 + 64 + 49 - 16x - 14y}$$
$$= \sqrt{x^2 + y^2 - 16x - 14y + 113}$$

$\therefore \quad OA^2 = OB^2 = OC^2$

$$\therefore\ x^2 + y^2 - 4x - 14y + 53$$
$$= x^2 + y^2 - 10x - 20y + 125$$
$$= x^2 + y^2 - 16x + 14y + 113$$
$$\Rightarrow \quad -4x - 14y + 53 = -10x - 20y + 125$$
$$= -16x - 14y + 113$$

Hence, from the first and the second expressions, we have

$$-4x + 10x - 14y + 20y + 53 - 125 = 0$$
$$\Rightarrow \quad 6x + 6y - 72 = 0$$
$$\Rightarrow \quad x + y - 12 = 0 \quad \ldots(2)$$

Also, from the first and the last expressions, we have

$$-4x - 14y + 53 + 16x + 14y - 113 = 0$$
$$\Rightarrow \quad 12x - 60 = 0$$
$$\Rightarrow \quad x = \frac{60}{12} = 5$$

$\therefore$ From (2), $\quad y = 12 - x$

$= 12 - 5$

$= 7$

Hence, the required coordinates of the centre O of the circle are (5, 7).

Also, radius = OA = $\sqrt{x^2 + y^2 - 4x - 14y + 53}$ [From (1)]

$= \sqrt{25 + 49 - 4 \times 5 - 14 \times 7 + 53}$

$= \sqrt{74 - 20 - 98 + 53}$

$= \sqrt{127 - 118}$

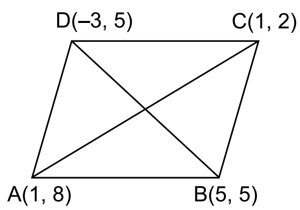
$= \sqrt{9} = 3$

Hence, the required coordinates are (5, 7) and the required radius is 3 units.

18. If A(1, 8), B(5, 5), C(1, 2), and D(–3, 5) are the four points in a plane. Show that quadrilateral formed by joining them, ABCD is a rhombus but not a square. Also find its area.

Sol. Here A(1, 8), B(5, 5), C(1, 2) and D(–3, 5) are four points in a plane forming a quadrilateral ABCD.

To prove that ABCD is a rhombus.



We have, by distance formula

$$AB = \sqrt{(5-1)^2 + (5-8)^2}$$

$$= \sqrt{4^2 + (-3)^2}$$

$$= \sqrt{16 + 9}$$

$$= \sqrt{25}$$

$$= 5$$

$$BC = \sqrt{(1-5)^2 + (2-5)^2}$$

$$= \sqrt{(-4)^2 + (-3)^2}$$

$$= \sqrt{16 + 9}$$

$$= \sqrt{25}$$

$$= 5$$

$$CD = \sqrt{(-3-1)^2 + (5-2)^2}$$

$$= \sqrt{(-4)^2 + 3^2}$$

$$= \sqrt{16 + 9}$$

$$= \sqrt{25}$$

$$= 5$$

$$DA = \sqrt{(1+3)^2 + (8-5)^2}$$

$$= \sqrt{4^2 + 3^2}$$

$$= \sqrt{16 + 9}$$

$$= \sqrt{25}$$

$$= 5$$

$$AC = \sqrt{(1-1)^2 + (2-8)^2}$$

$$= \sqrt{0^2 + (-6)^2}$$

$$= \sqrt{36}$$

$$= 6$$

$$BD = \sqrt{(-3-5)^2 + (5-5)^2}$$

$$= \sqrt{(-8)^2 + 0^2}$$

$$= \sqrt{64}$$

$$= 8$$

$\therefore$ We see that AB = BC = CD = DA.

But the diagonals AC and BD are not equal.

Hence, the quadrilateral ABCD is a rhombus and not a square.

Also, the area of the rhombus

$$= \frac{1}{2} \times AC \times BD$$

$$= \frac{1}{2} \times 6 \times 8 \text{ sq units}$$

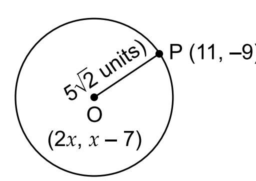
$$= 24 \text{ sq units}$$

$\therefore$ The required area is 24 sq units.

19. The radius of a circle passing through (11, –9) is $5\sqrt{2}$ units. Find x if the centre of circle is $(2x, x - 7)$.

Sol. Let P(11, –9) be any point on the circle with centre at O$(2x, x - 7)$ and radius OP = $5\sqrt{2}$ units.

To find x.



We have, by distance formula

$$OP = \sqrt{(11 - 2x)^2 + (-9 - x + 7)^2}$$

$$\Rightarrow \quad 5\sqrt{2} = \sqrt{(11-2x)^2 + (-2-x)^2}$$

$$= \sqrt{4x^2 + 11^2 - 44x + 4x + 4 + x^2}$$

$$\Rightarrow \quad 5\sqrt{2} = \sqrt{5x^2 - 40x + 125}$$

On squaring both sides

$$5x^2 - 40x + 125 = \left(5\sqrt{2}\right)^2 = 50$$

$$\Rightarrow \quad 5x^2 - 40x + 75 = 0$$

$$\Rightarrow \quad x^2 - 8x + 15 = 0$$

$$\Rightarrow \quad x^2 - 3x - 5x + 15 = 0$$

$$\Rightarrow \quad x(x-3) - 5(x-3) = 0$$

$$\Rightarrow \quad (x-3)(x-5) = 0$$

$\therefore$ Either $\quad x - 3 = 0 \quad \Rightarrow \quad x = 3$

or, $\quad x - 5 = 0 \quad \Rightarrow \quad x = 5$

Hence, the required values of x is 3 or 5.

Milestone 2

(Page 136)

Multiple-Choice Questions

1. The mid-point of the line segment joining points A(–2, 8) and B(–6, –4) is

(*a*) (–4, –6) (*b*) (–4, 2)

(*c*) (2, 6) (*d*) (4, 2)

Sol. (*b*) (–4, 2)

If $P(x_1, y_1)$ be the mid-point of AB, then

$$x_1 = \frac{-2-6}{2} = -4, \quad y_1 = \frac{8-4}{2} = 2$$

$\therefore$ (–4, 2) is the required mid-point.

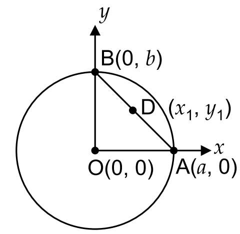
2. The coordinates of the circumcentre of the triangle formed by the points O(0, 0), A(a, 0) and B(0, b) are

(*a*) (a, b) (*b*) $\left(\frac{a}{2}, \frac{b}{2}\right)$

(*c*) $\left(\frac{b}{2}, \frac{a}{2}\right)$ (*d*) (b, a)

Sol. (*b*) $\left(\frac{a}{2}, \frac{b}{2}\right)$

Clearly, O is the origin. A(a, 0) is a point on the x-axis and B(0, b) is a point on the y-axis. A circle with centre at O passes through A and B as shown is the figure. Then $\angle AOB = 90°$.



$\therefore$ AB is the hypotenuse of the right-angled triangle AOB. Hence, the circumcentre of ΔAOB will be the mid-point of the hypotenuse AB. If $D(x_1, y_1)$ be the mid-point of AB, then

$$x_1 = \frac{1}{2}(0 + a) = \frac{a}{2} \text{ and } y_1 = \frac{1}{2}(b + 0) = \frac{b}{2}.$$

$\therefore$ The required coordinates of the circumcentre are $\left(\frac{a}{2}, \frac{b}{2}\right)$.

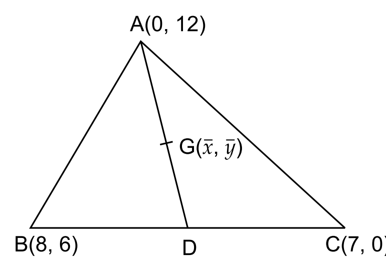
Very Short Answer Type Questions

3. Find the coordinates of the centroid of a triangle whose vertices are (0, 12), (8, 6) and (7, 0).

Sol. Let $G(\bar{x}, \bar{y})$ be the centroid of ΔABC where A(0, 12), B(8,6) and C(7, 0) are the vertices of ΔABC.

Then, $$\bar{x} = \frac{0+8+7}{3} = \frac{15}{3} = 5$$

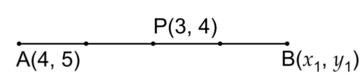
$$\bar{y} = \frac{12+6+0}{3} = \frac{18}{3} = 6$$



$\therefore$ The required coordinates of the centroid are (5, 6).

4. If line segment AB is bisected at P(3, 4) where coordinates of point A are (4, 5). Then find the coordinates of point B.

Sol. Let the coordinates of B be (x_1, y_1). Then, since P(3, 4) is the mid-point of AB.



$\therefore$ By mid-point formula,

$$3 = \frac{4 + x_1}{2} \quad \Rightarrow \quad x_1 = 6 - 4 = 2$$

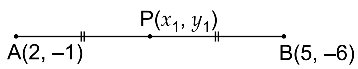
$$4 = \frac{5 + y_1}{2} \qquad \Rightarrow \quad y_1 = 8 - 5 = 3$$

$\therefore$ The required coordinates of B are (2, 3).

Short Answer Type-I Questions

5. The line joining the points (2, –1) and (5, –6) is bisected at P. If P lies on the line $2x + 4y + k = 0$, find the value of k.

Sol. Let A(2, –1) and B(5, –6) be the given points. Let P(x_1, y_1) be the mid-point of AB.



Then $\qquad x_1 = \dfrac{2+5}{2} = \dfrac{7}{2} \qquad$ ...(1)

and $\qquad y_1 = \dfrac{-1-6}{2} = -\dfrac{7}{2} \qquad$ ...(2)

$\because$ (x_1, y_1) lies on the line $2x + 4y + k = 0$

$\therefore \quad 2x_1 + 4y_1 + k = 0$

$\Rightarrow \quad 2 \times \dfrac{7}{2} - 4 \times \dfrac{7}{2} + k = 0 \qquad$ [From (1) and (2)]

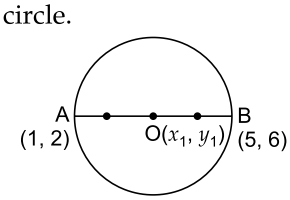
$\Rightarrow \quad 7 - 14 + k = 0$

$\Rightarrow \quad k = 7$

$\therefore$ The required value of k is 7.

6. If the coordinates of the end points of a diameter of a circle are A(1, 2) and B(5, 6). Find the coordinates of the centre of circle.

Sol. Let AB be a diameter of the circle with centre at O on AB. Let (x_1, y_1) be the coordinate of the centre O of the circle.



$\because$ (x_1, y_1) is the mid-point of AB.

$\therefore$ By mid-point formula,

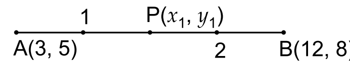
$$x_1 = \frac{1+5}{2} = \frac{6}{2} = 3, \; y_1 = \frac{2+6}{2} = \frac{8}{2} = 4$$

$\therefore$ The required coordinates of the centre O of the circle are (3, 4).

Short Answer Type-II Questions

7. Find the coordinates of the points P which divides the line segment joining points A(3, 5) and B(12, 8) internally in the ratio 1 : 2.

Sol. Let P be the point (x_1, y_1). Now, P divides AB in the ratio 1 : 2.



$\therefore$ PA : PB = 1 : 2

$\therefore$ By section formula,

$$x_1 = \frac{12 \times 1 + 3 \times 2}{1+2} = \frac{12+6}{3} = \frac{18}{3} = 6$$

$$y_1 = \frac{1 \times 8 + 5 \times 2}{2} = \frac{8 \times 10}{3} = \frac{18}{3} = 6$$

$\therefore$ The required coordinates of P are (6, 6).

8. In what ratio does the point P(–7, –5) divides the line segment joining points A(–8, –3) and B(–5, –9).

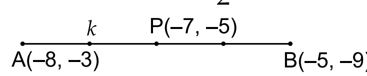
Sol. Let P divide AB in the ratio AP : PB = k : 1 where k is same non-zero constant. Then by section formula, we have

$$-7 = \frac{-5 \times k - 8 \times 1}{k+1} = \frac{-5k - 8}{k+1}$$

$\Rightarrow \quad 7k + 7 = 5k + 8$

$\Rightarrow \quad 2k = 1$

$\Rightarrow \quad k = \dfrac{1}{2}$

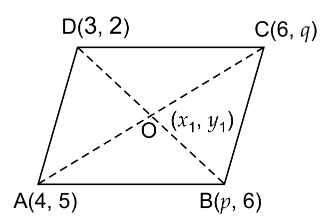


$\therefore$ P divides AB in the ratio $\dfrac{1}{2} : 1$, i.e. 1 : 2.

$\therefore$ The required ratio is 1 : 2.

9. If A(4, 5), B(p, 6), C(6, q) and D(3, 2) are the vertices of a parallelogram, find the value of p and q.

Sol. Let O be the point of intersection of the two diagonals AC and BD of the parallelogram ABCD. Then O will be the common middle point of the two diagonals AC and BD. Let (x_1, y_1) be the coordinates of O.



Since, O(x_1, y_1) is the mid-point of BD

$\therefore \qquad x_1 = \dfrac{3+p}{2}$

$\Rightarrow \qquad 3 + p = 2x_1 \qquad$ ...(1)

and $\qquad y_1 = \dfrac{2+6}{2} = \dfrac{8}{2} = 4 \qquad$ ...(2)

Again, since $O(x_1, y_1)$ is the mid-point of AC

$\therefore \quad x_1 = \frac{4+6}{2} = \frac{10}{2} = 5 \quad \ldots(3)$

and $\quad y_1 = \frac{5+q}{2}$

$\Rightarrow \quad 5 + q = 2y_1 \quad \ldots(4)$

Now, from (1) and (2), we have

$3 + p = 2 \times 5$

$\Rightarrow \quad p = 10 - 3 = 7$

Also, from (2) and (4), we have

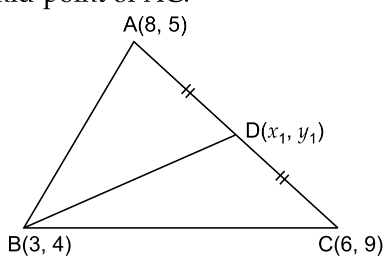
$5 + q = 2 \times 4$

$\Rightarrow \quad q = 8 - 5 = 3$

$\therefore$ The required values of p and q are 7 and 3 respectively.

10. Find the lengths of medians of the triangle through vertex B to AC whose vertices are A(8, 5), B(3, 4) and C(6, 9).

Sol. Let BD be a median of ΔABC, where D (x_1, y_1) is the mid-point of AC.



$\therefore$ By mid-point formula,

$$x_1 = \frac{1}{2}(8+6) = \frac{14}{2} = 7$$

$$y_1 = \frac{1}{2}(9+5) = \frac{14}{2} = 7$$

$\therefore$ D is the point (7, 7).

$\therefore$ The required length of the median BD

$$= \sqrt{(7-3)^2 + (7-4)^2} \text{ units}$$

$$= \sqrt{4^2 + 3^2} \text{ units}$$

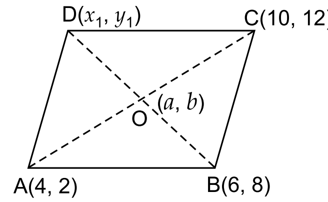
$$= \sqrt{16+9} \text{ units}$$

$$= \sqrt{25} \text{ units}$$

$$= 5 \text{ units.}$$

11. Three consecutive vertices of a parallelogram ABCD are A(4, 2), B(6, 8) and C(10, 12). Find the fourth vertex D.

Sol. Let (x_1, y_1) be the coordinates of the fourth vertex D of the parallelogram and let (a, b) be the coordinates of the point O, where two diagonals. AC and BD intersect each other. Since, the diagonals of a parallelogram bisect each other at O.



$\because$ (a, b) is the mid-point at DB

$\therefore \quad a = \frac{1}{2}(x_1 + 6)$

$\Rightarrow \quad x_1 + 6 = 2a \quad \ldots(1)$

$\quad b = \frac{1}{2}(y_1 + 8)$

$\Rightarrow \quad y_1 + 8 = 2b \quad \ldots(2)$

Also, (a, b) is the mid-point of AC

$\therefore$ By mid-point formula,

$$a = \frac{1}{2}(4+10) = \frac{14}{2} = 7 \quad \ldots(3)$$

$$b = \frac{1}{2}(2+12) = \frac{14}{2} = 7 \quad \ldots(4)$$

$\therefore$ From (1) and (3), we have

$x_1 = 2a - 6$
$= 2 \times 7 - 6$
$= 14 - 6$
$= 8$

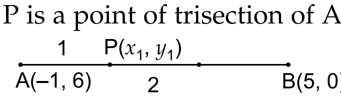
and from (2) and (4), we have

$y_1 = 2b - 8$
$= 2 \times 7 - 8$
$= 14 - 8$
$= 6$

$\therefore$ The required coordinates of D are (8, 6).

12. The line joining points A(–1, 6) and B(5, 0) is trisected at points P and Q. If point P also lies on the line $x + y - k = 0$, find the value of k.

Sol. Since, P is a point of trisection of AB



$\therefore$ AP : PB = 1 : 2

If P is the point (x_1, y_1), then by section formula,

$$x_1 = \frac{1 \times 5 - 2 \times 1}{2+1}$$

$$= \frac{5-2}{3} = \frac{3}{3} = 1$$

$$y_1 = \frac{1 \times 0 + 2 \times 6}{2 + 1}$$

$$= \frac{12}{3} = 4$$

Now, (x_1, y_1) i.e. (1, 4) lies on the line $x + y - k = 0$

$\therefore \quad x_1 + y_1 - k = 0$

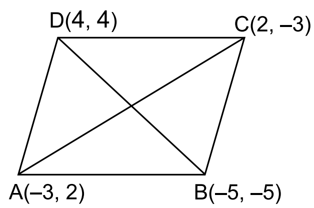
$\Rightarrow \quad 1 + 4 - k = 0$

$\Rightarrow \quad k = 5$

$\therefore$ The required value of k is 5.

13. Show that A(–3, 2), B(–5, –5), C(2, –3) and D(4, 4) are the vertices of a rhombus.

Sol.



We have

$$AB = \sqrt{(-5+3)^2 + (-5-2)^2}$$

$$= \sqrt{(-2)^2 + (-7)^2}$$

$$= \sqrt{4 + 49}$$

$$= \sqrt{53}$$

$$BC = \sqrt{(2+5)^2 + (-3+5)^2}$$

$$= \sqrt{7^2 + 2^2}$$

$$= \sqrt{49 + 4}$$

$$= \sqrt{53}$$

$$CD = \sqrt{(4-2)^2 + (4+3)^2}$$

$$= \sqrt{2^2 + 7^2}$$

$$= \sqrt{4 + 49}$$

$$= \sqrt{53}$$

$$DA = \sqrt{(-3-4)^2 + (2-4)^2}$$

$$= \sqrt{(-7)^2 + (-2)^2}$$

$$= \sqrt{49 + 4}$$

$$= \sqrt{53}$$

$$AC = \sqrt{(2+3)^2 + (-3-2)^2}$$

$$= \sqrt{5^2 + (-5)^2}$$

$$= \sqrt{25 + 25}$$

$$= \sqrt{50}$$

$$= 5\sqrt{2}$$

$$BD = \sqrt{(4+5)^2 + (4+5)^2}$$

$$= \sqrt{81 + 81}$$

$$= 9\sqrt{2}$$

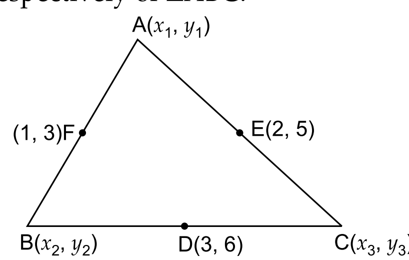
$\therefore$ AB = BC = CD = DA, but AC $\neq$ BD.

i.e. all sides of a quadrilateral ABCD are of equal length but the two diagonals are not of equal length. Hence, ABCD is a rhombus i.e. the given points are the vertices of a rhombus.

Long Answer Type Questions

14. If the mid-points of sides of ΔABC are (3, 6), (2, 5) and (1, 3), then find the coordinates of three vertices of ΔABC.

Sol. Let D(3, 6), E(2, 5) and F(1, 3) be the coordinates of the mid-points of the sides BC, CA and AB respectively of ΔABC. Let (x_1, y_1), (x_2, y_2) and (x_3, y_3) be the coordinates of the vertices A, B and C respectively of ΔABC.



Since, D is the mid-point of BC.

$\therefore$ By mid-point formula,

$$3 = \frac{x_2 + x_3}{2} \quad \Rightarrow \quad x_2 + x_3 = 6 \qquad \ldots(1)$$

and $$6 = \frac{y_2 + y_3}{2} \quad \Rightarrow \quad y_2 + y_3 = 12 \qquad \ldots(2)$$

E is the mid-point of AC.

$\therefore$ By mid-point formula,

$$2 = \frac{x_1 + x_3}{2} \quad \Rightarrow \quad x_1 + x_3 = 4 \qquad \ldots(3)$$

and $$5 = \frac{y_1 + y_3}{2} \quad \Rightarrow \quad y_1 + y_3 = 10 \qquad \ldots(4)$$

Lastly, F is the mid-point of AB.

$\therefore$ By mid-point formula,

$$1 = \frac{x_1 + x_2}{2} \quad \Rightarrow \quad x_1 + x_2 = 2 \qquad \ldots(5)$$

and $$3 = \frac{y_1 + y_2}{2} \quad \Rightarrow \quad y + y_2 = 6 \qquad \ldots(6)$$

Adding (1), (3) and (5), we have

$$2(x_1 + x_2 + x_3) = 6 + 4 + 2 = 12$$

$$\Rightarrow \quad x_1 + x_2 + x_3 = 6 \quad \ldots(7)$$

Subtracting (1), (3) and (5) successively from (7), we get $x_1 = 0$, $x_2 = 2$ and $x_3 = 4$

Again, adding (2), (4) and (6), we get

$$2(y_1 + y_2 + y_3) = 28$$

$$\Rightarrow \quad y_1 + y_2 + y_3 = 14 \quad \ldots(8)$$

Subtracting (2), (4) and (6) successively from (8), we get $y_1 = 2$, $y_2 = 4$ and $y_3 = 8$.

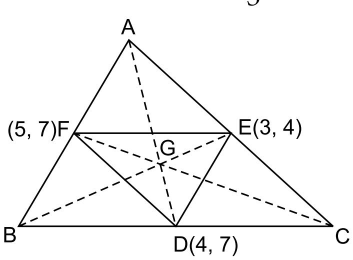
$\therefore$ The required coordinates of A, B and C are (0, 2), (2, 4) and (4, 8) respectively.

15. If the coordinates of the mid-points of the sides of the triangle are (4, 7), (3, 4) and (5, 7), find its centroid.

Sol. Let D(4, 7), E(3, 4) and F(5, 7) be the mid-points of the sides BC, CA and AB respectively of ΔABC. Then we know that the coordinates of the centroid of ΔABC will be identical with the coordinates of the centroid of ΔDEF. If $(\bar{x}, \bar{y})$ be the centroid of ΔDEF, then

$$\bar{x} = \frac{4+3+5}{3} = \frac{12}{3} = 4$$

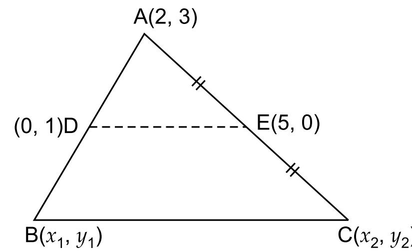
$$\bar{y} = \frac{7+4+7}{3} = \frac{18}{3} = 6$$



Hence, the required coordinates of the centroid of ΔABC are (4, 6).

16. If a vertex of a triangle is (2, 3) and the middle points of the sides through it are (0, 1) and (5, 0), find other vertices.

Sol. Let A(2, 3) be the vertex of ΔABC and D and E be the mid-points of AB and AC respectively such that D(0, 1) and E(5, 0). Let (x_1, y_1) and (x_2, y_2) be respectively the coordinates of the two other vertices B and C of ΔABC.



Then since (0, 1) is the mid-point of AB and (5, 0) is the mid-point of AC.

$\therefore$ By mid-point formula,

$$\frac{2 + x_1}{2} = 0$$

$$\Rightarrow \quad x_1 = -2$$

and $$\frac{y_1 + 3}{2} = 1$$

$$\Rightarrow \quad y_1 = 2 - 3 = -1$$

$\therefore$ The coordinates of B are (–2, –1)

Again, $$\frac{2 + x_2}{2} = 5$$

$$\Rightarrow \quad x_2 = 10 - 2 = 8$$

and $$\frac{y_2 + 3}{2} = 0$$

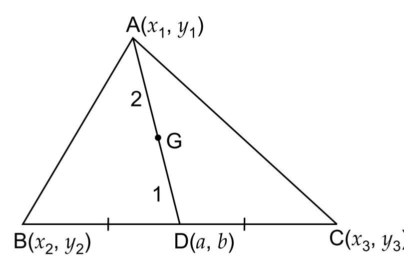
$$\Rightarrow \quad y_2 = -3$$

$\therefore$ The coordinates of C are (8, –3).

Hence, the required coordinates of other vertices are (–2, –1) and (8, –3).

17. Prove that coordinates of the centroid of a triangle whose vertices are (x_1, y_1), (x_2, y_2) and (x_3, y_3) are given by $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$.

Sol. Let A(x_1, y_1), B(x_2, y_2) and C(x_3, y_3) be the vertices of ΔABC and AD be a median of ΔABC, where D is the mid-point of BC. Let D be the point (a, b).



$\therefore$ By mid-point formula,

$$a = \frac{x_2 + x_3}{2} \quad \Rightarrow \quad x_2 + x_3 = 2a \quad \ldots(1)$$

and $$b = \frac{y_2 + y_3}{2} \quad \Rightarrow \quad y_2 + y_3 = 2b \quad \ldots(2)$$

Now, let the centroid be $G(\bar{x}, \bar{y})$. Since G divides the median AD is the ratio AG : GD = 2 : 1.

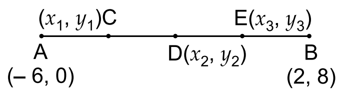
$$\therefore \quad \bar{x} = \frac{2a + 1 \times x_1}{2+1} = \frac{x_2 + x_3 + x_1}{3} \quad \text{[From (1)]}$$

$$\text{and} \quad \bar{y} = \frac{2b + y_1}{3} = \frac{y_2 + y_3 + y_1}{3} \quad \text{[From (2)]}$$

Hence, the required coordinates of the centroid are $\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$.

18. Find the coordinates of the points which divides the line segment joining A(–6, 0) and B(2, 8) in four equal parts.

Sol. Let $C(x_1, y_1)$, $D(x_2, y_2)$ and $E(x_3, y_3)$ divide AB into four equal parts so that AC = CD = DE = EB.



Clearly, C divides AB in the ratio AC : CB = 1 : 3

$$\therefore \quad x_1 = \frac{1 \times 2 + 3 \times (-6)}{1+3}$$

$$= \frac{2 - 18}{4}$$

$$= \frac{-16}{4}$$

$$= -4$$

$$y_1 = \frac{1 \times 8 + 3 \times 0}{1+3}$$

$$= \frac{8}{4}$$

$$= 2$$

Finally, E divides AB in the ratio AE : EB = 3 : 1

$$\therefore \quad x_3 = \frac{3 \times 2 + 1 \times (-6)}{3+1}$$

$$= \frac{6-6}{4} = 0$$

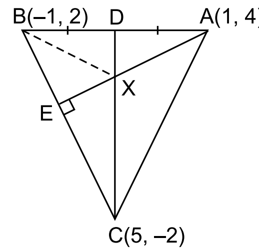
$$y_3 = \frac{3 \times 8 + 1 \times 0}{3+1}$$

$$= \frac{24}{4} = 6$$

$\therefore$ The required coordinates are (–4, 2), (–2, 4) and (0, 6).

19. A(1, 4), B(–1, 2) and C(5, –2) are the vertices of a ΔABC. Find the coordinates of the point where the right bisector of BC intersects the median through C.

Sol.



We have

$$AC = \sqrt{(5-1)^2 + (-2-4)^2}$$

$$= \sqrt{4^2 + (-6)^2}$$

$$= \sqrt{16 + 36}$$

$$= \sqrt{52}$$

$$= 3\sqrt{6}$$

$$BC = \sqrt{(5+1)^2 + (-2-2)^2}$$

$$= \sqrt{6^2 + (-4)^2}$$

$$= \sqrt{36 + 16}$$

$$= \sqrt{52}$$

$$= 3\sqrt{6}$$

$$BA = \sqrt{(1+1)^2 + (4-2)^2}$$

$$= \sqrt{2^2 + 2^2}$$

$$= \sqrt{4 + 4}$$

$$= \sqrt{8}$$

$$= 2\sqrt{2}$$

$$\therefore \quad AC = BC \neq BA$$

$\therefore$ ΔABC is an isosceles triangle with D and E as the mid-points of AB and BC respectively.

$$\therefore \quad \angle CDA = \angle CDB = 90°.$$

$\therefore$ CD is the perpendicular bisector of AB. Let the perpendicular bisector of CB meet that of AB at X. To find the coordinates of X. Let (x_1, y_1) be the coordinates of x. Clearly X will be the circumcentre of ΔABC.

$$\therefore \quad AX = BX = CX$$

$$\Rightarrow \quad (x_1 - 1)^2 + (y_1 - 4)^2$$

$$= (x_1 + 1)^2 + (y_1 - 2)^2$$

$$= (x_1 - 5)^2 + (y_1 + 2)^2$$

$$\Rightarrow \quad x_1^2 + y_1^2 - 2x_1 - 8y_1 + 17$$

$$= x_1^2 + y_1^2 - 2x_1 - 4y_1 + 5$$

$$= x_1^2 + y_1^2 - 10x_1 + 4y_1 + 29$$

$\Rightarrow \quad -2x_1 - 8y_1 + 17 = 2x_1 - 4y_1 + 5$

$= -10x_1 + 4y_1 + 29$

From the first and the second expressions, we have

$4x_1 + 4y_1 - 12 = 0$

$\Rightarrow \quad x_1 + y_1 - 3 = 0$

$\Rightarrow \quad y_1 = 3 - x_1 \quad \ldots(1)$

Also, from the first and the last expressions, we have

$8x_1 - 12y_1 - 12 = 0$

$\Rightarrow \quad 2x_1 - 3y_1 - 3 = 0$

$\Rightarrow \quad 2x_1 - 3(3 - x_1) - 3 = 0 \quad$ [From (1)]

$\Rightarrow \quad 2x_1 - 9 + 3x_1 - 3 = 0$

$\Rightarrow \quad 5x_1 = 12$

$\Rightarrow \quad x_1 = \frac{12}{5}$

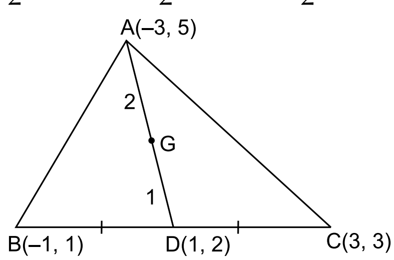
$\therefore$ From (1), we have

$$y_1 = 3 - \frac{12}{5} = \frac{15-12}{5} = \frac{3}{5}$$

$\therefore$ The required coordinates of X are $\left(\frac{12}{5}, \frac{3}{5}\right)$.

20. If A(–3, 5), B(–1, 1), C(3, 3) are the vertices of triangle ΔABC, find the length of median AD. Also find the coordinates of the point which divides the median in the ratio 2 : 1. **(CBSE 2013)**

Sol. Since D is the mid-point of BC, if (x_1, y_1) are the coordinates of D then by mid-point formula, $x_1 = \frac{1}{2}(-1+3) = \frac{2}{2} = 1, y_1 = \frac{1}{2}(1+3) = \frac{4}{2} = 2$



$\therefore$ D is the point (1, 2).

Let G be a point on AD such that

AG : GD = 2 : 1.

If $(\bar{x}, \bar{y})$ be the coordinates of G, then

$$\bar{x} = \frac{2 \times 1 + 1 \times (-3)}{2+1}$$

$$= \frac{2-3}{3} = \frac{-1}{3}$$

$$\bar{y} = \frac{2 \times 2 + 1 \times 5}{2+1}$$

$$= \frac{4+5}{3} = \frac{9}{3} = 3$$

$\therefore$ The required coordinates of G are $\left(-\frac{1}{3}, 3\right)$ and length of median AD is

$\sqrt{(1+3)^2 + (2-5)^2}$ units $= \sqrt{4^2 + (-3)^2}$ units

$= \sqrt{16+9}$ units

$= \sqrt{25}$ units

$= 5$ units

Milestone 3

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Multiple-Choice Questions

1. Three points $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are collinear if

(*a*) $x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$

(*b*) $x_1(y_3 - y_1) + x_2(y_2 - y_3) + x_3(y_2 - y_1) = 0$

(*c*) $x_1(y_2 + y_3) + x_2(y_3 + y_1) + x_3(y_1 + y_2) = 0$

(*d*) $x_2(y_3 - y_2) + x_3(y_1 - y_3) + x_1(y_2 - y_1) = 0$

Sol. (*a*) $x_1(y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2) = 0$.

The area of ΔABC is

$\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$.

If the three points A, B and C are collinear, then the area of ΔABC is 0.

Hence, $x_1(y_2 - y_3) + x_2 (y_3 - y_1) + x_3 (y_1 - y_2) = 0$ is the required condition that the three points are collinear.

2. The area of the triangle with vertices (3, 0), (7, 0) and (8, 4) is

(*a*) 14 sq units (*b*) 28 sq units

(*c*) 8 sq units (*d*) 6 sq units

Sol. (*c*) 8 sq units

If (x_1, y_1), (x_2, y_2) and (x_3, y_3) are the vertices of the triangle, then the area of this triangle is $\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$ where (x_1, y_1) is the point (3, 0), (x_2, y_2) is the point (7, 0) and (x_3, y_3) is the point (8, 4).

$\therefore$ Area of the triangle

$= \left|\frac{1}{2}[3(0-4) + 7(4-0) + 8(0-0)]\right|$ sq units

$= \left|\frac{1}{2}(-12+28)\right|$ sq units

$= \frac{16}{2}$ sq units

$= 8$ sq units

Hence, the required area of the triangle is 8 sq units.

Very Short Answer Type Questions

3. Show that the points (–3, –7), (4, 7) and (5, 9) are collinear.

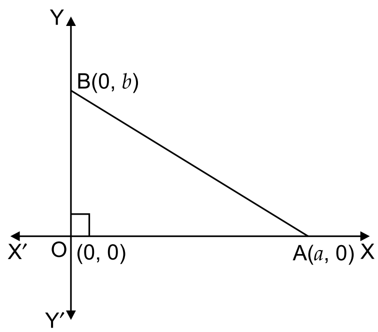
Sol. We know that three points $(x_1, y_1) = (-3, -7)$, $(x_2, y_2) = (4, 7)$ and $(x_3, y_3) = (5, 9)$ are collinear if $x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$

Now, $\text{LHS} = -3\,(7 - 9) + 4\,(9 + 7) + 5\,(-7 - 7)$
$= (-3) \times (-2) + 4 \times 16 + 5 \times (-14)$
$= 6 + 64 - 70$
$= 70 - 70$
$= 0$
$= \text{RHS}$

Hence, the three given points are collinear.

4. Find the area of the triangle with vertices (0, 0), $(a, 0)$ and $(0, b)$.

Sol. Let $A(a, 0)$, $B(0, b)$ be two point on x and y axes respectively and O(0, 0) is the origin. $\angle AOB = 90°$, $OA = a$ and $OB = b$.



$\therefore$ The required area of $\Delta AOB = \frac{1}{2}ab$ sq units.

Short Answer Type-I Questions

5. Find the value of k for which $A(-6, 0)$, $B(-4, k)$ and $C(-2, -2)$ are collinear.

Sol. $A(-6, 0)$, $B(-4, k)$ and $C(-2, -2)$ are collinear if

$-6(k + 2) - 4\,(-2 - 0) - 2(0 - k) = 0$
$\Rightarrow \quad -6k - 12 + 8 + 2k = 0$
$\Rightarrow \quad -4k - 4 = 0$
$\Rightarrow \quad k = -1$

Hence, the req value of k is –1.

6. Find the value of k so that area of triangle with vertices $(k, -4)$, $(-3, -2k)$ and $(7, -8)$ is 20 sq units.

Sol. We know that the area of the triangle with vertices $(k, -4)$, $(-3, -2k)$ and $(7, -8)$

$= \frac{1}{2}[k(-2k + 8) - 3\,(-8 + 4) + 7\,(-4 + 2k)]$
$= \frac{1}{2}[-2k^2 + 8k + 24 - 12 - 28 + 14k]$
$= \frac{1}{2}[22k - 2k^2 - 16]$
$= 11k - k^2 - 8$

Given that

$11k - k^2 - 8 = 20$
$\Rightarrow \quad k^2 - 11k + 28 = 0$
$\Rightarrow \quad k^2 - 7k - 4k + 28 = 0$
$\Rightarrow \quad k(k - 7) - 4(k - 7) = 0$
$\Rightarrow \quad (k - 7)\,(k - 4) = 0$
$\therefore$ Either $\quad k - 7 = 0 \quad \Rightarrow \quad k = 7$
or $\quad k - 4 = 0 \quad \Rightarrow \quad k = 4$

Hence, the required value of k is 7 or 4.

Short Answer Type-II Questions

7. If the points (x, y), $[(x - a), (y - b)]$ and (a, b) are collinear, show that $bx = ay$. **[CBSE 2010]**

Sol. Since the point (x, y), $[(x - a), (y - b)]$ and (a, b) are collinear

$\therefore\; x(y - b - b) + (x - a)\,(b - y) + a\,(y - y + b) = 0$
$\Rightarrow \quad x(y - 2b) + bx - xy + ay - ay + ab = 0$
$\Rightarrow \quad xy - 2bx + bx - xy + ay = 0$
$\Rightarrow \quad -bx + ay = 0$
$\Rightarrow \quad bx = ay$

Hence, proved.

8. Find the condition that the point $p(x, y)$ lies on the line segment joining A(7, – 4) and B(3, – 8).

Sol. If (x, y) lies on the line AB where A and B are the points (7, –4) and (3, –8) respectively, then the three points (x, y), (7, –4) and (3, –8) will be collinear.

$\therefore\; x(-4 + 8) + 7\,(-8 - y) + 3\,(y + 4) = 0$
$\Rightarrow \quad 4x - 56 - 7y + 3y + 12 = 0$
$\Rightarrow \quad 4x - 4y - 44 = 0$
$\Rightarrow \quad x - y - 11 = 0$

Hence, proved.

9. If $P(x, y)$ is any point on the line joining points $M(a, 0)$ and $N(0, b)$ then show that $\frac{x}{a} + \frac{y}{b} = 1$. **[CBSE 2009]**

Sol. Here the points (x, y), $(a, 0)$ and $(0, b)$ are collinear.

$\therefore \quad x(0 - b) + a(b - y) + 0\,(y - 0) = 0$

$\Rightarrow \quad -bx + ab - ay = 0$

$\Rightarrow \quad ay + bx = ab$

$\Rightarrow \quad \dfrac{ay}{ab} + \dfrac{bx}{ab} = 1$

$\Rightarrow \quad \dfrac{y}{b} + \dfrac{x}{a} = 1$

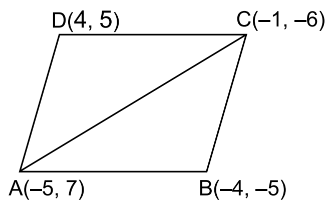
$\Rightarrow \quad \dfrac{x}{a} + \dfrac{y}{b} = 1$

Hence, proved.

Long Answer Type Questions

10. If A(–5, 7), B(–4, –5), C(–1, –6) and D(4, 5) are the vertices of a quadrilateral, find the area of the quadrilateral ABCD. **[CBSE 2018]**

Sol. Join AC. The quadrilateral ABCD forms two triangles ABC and ACD.



Area of a triangle

$$= \frac{1}{2}\,[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$\therefore$ Area of ΔABC

$$= \left|\frac{1}{2}[-5(-5+6) - 4(-6-7) - 1(7+5)]\right| \text{ sq units}$$

$$= \left|\frac{1}{2}[-5 + 52 - 12]\right| \text{ sq units}$$

$$= \left|\frac{1}{2}[52 - 17]\right| \text{ sq units}$$

$$= \frac{1}{2} \times 35 \text{ sq units}$$

$$= \frac{35}{2} \text{ sq units}$$

and area of ΔACD

$$= \left|\frac{1}{2}[4(7+6) - 5(-6-5) - 1(5-7)]\right| \text{ sq units}$$

$$= \left|\frac{1}{2}[52 + 55 + 2]\right| \text{ sq units}$$

$$= \frac{1}{2} \times 109 \text{ sq units}$$

$$= \frac{109}{2} \text{ sq units}$$

$\therefore$ Area of the quadrilateral ABCD

= area of ΔABC + area of ΔACD

$$= \left(\frac{35}{2} + \frac{109}{2}\right) \text{ sq units}$$

$$= \frac{144}{2} \text{ sq units}$$

= 72 sq units.

Hence, area of the quadrilateral ABCD is 72 sq units.

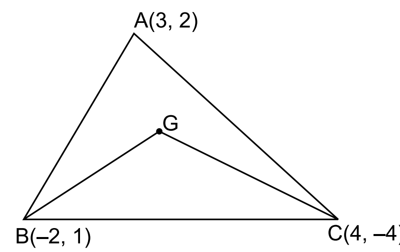
11. Find the coordinates of centroid G of ΔABC whose vertices are A(3, 2), B(–2, 1) and C(4, –4). Determine ar(ΔGBC) : ar(ΔABC).

Sol. If $(\bar{x}, \bar{y})$ be the coordinates of the centroid G of ΔABC, then

$$\bar{x} = \frac{3 + 4 - 2}{3} = \frac{5}{3}$$

and $$\bar{y} = \frac{2 - 4 + 1}{3} = -\frac{1}{3}$$

$\therefore$ The coordinates of centroid G of ΔABC is $\left(\frac{5}{3}, -\frac{1}{3}\right)$.



Also, ar(ΔABC)

$$= \left|\frac{1}{2}[3(1+4) - 2(-4-2) + 4(2-1)]\right| \text{ sq units}$$

$$= \left|\frac{1}{2}[15 + 12 + 4]\right| \text{ sq units}$$

$$= \frac{31}{2} \text{ sq units}$$

and ar(ΔGBC)

$$= \left|\frac{1}{2}\left[\frac{5}{3}(1+4) - 2\left(-4 + \frac{1}{3}\right) + 4\left(\frac{1}{-3} - 1\right)\right]\right| \text{ sq units}$$

$$= \left|\frac{1}{2}\left[\frac{25}{3} + \frac{22}{3} - \frac{16}{3}\right]\right| \text{ sq units}$$

$$= \frac{1}{2} \times \frac{25 + 22 - 16}{3} \text{ sq units}$$

$$= \frac{47 - 16}{6} \text{ sq units}$$

$$= \frac{31}{6} \text{ sq units}$$

$$\therefore \text{ar}(\Delta GBC) : \text{ar}(\Delta ABC) = \frac{31}{6} : \frac{31}{2} = \frac{1}{6} : \frac{1}{2} = 1 : 3$$

$\therefore$ The required ratio is 1 : 3.

Higher Order Thinking Skills (HOTS) Questions

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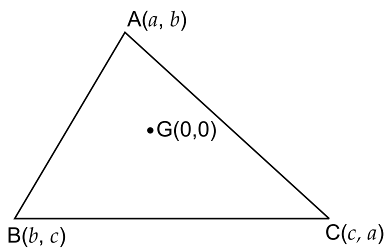
1. If the centroid of the triangle formed by A(a, b), B(b, c) and C(c, a) is at the origin, what is the value of $\frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab}$.

Sol. If G(0, 0) be the centroid of ΔABC, then

$$0 = \frac{a+b+c}{3}$$

$$\Rightarrow \quad a + b + c = 0 \quad \ldots(1)$$

Now, $\frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab} = \frac{a^3 + b^3 + c^3}{abc}$ ...(2)



Now, if $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$ ...(3)

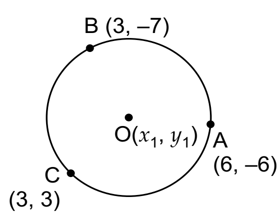
$\therefore$ From (2) and (3), we get

$$\frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab} = \frac{3abc}{abc} = 3$$

$\therefore$ The required value of $\frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab}$ is 3.

2. Find the centre of a circle passing through the points (6, – 6), (3, – 7) and (3, 3).

Sol. Let O(x_1, y_1) be the centre of the circle passing through the points A(6, –6), B(3, –7) and C(3, 3).



Then $AO^2 = BO^2 = CO^2$

$$\Rightarrow (x_1 - 6)^2 + (y_1 + 6)^2 = (x_1 - 3)^2 + (y_1 + 7)^2$$
$$= (x_1 - 3)^2 + (y_1 - 3)^2$$

$$\Rightarrow x_1^2 + y_1^2 - 12x_1 + 12y_1 + 72$$
$$= x_1^2 + y_1^2 - 6x_1 + 14y_1 + 58$$
$$= x_1^2 + y_1^2 - 6x_1 - 6y_1 + 18$$

$$\Rightarrow -12x_1 + 12y_1 + 72 = -6x_1 + 14y_1 + 58$$
$$= -6x_1 - 6y_1 + 18$$

From the first and the last expressions, we have

$$6x_1 - 18y - 54 = 0$$
$$\Rightarrow \quad x_1 - 3y_1 - 9 = 0$$
$$\Rightarrow \quad x_1 = 3y_1 + 9 \quad \ldots(1)$$

From the first and the second expressions, we have

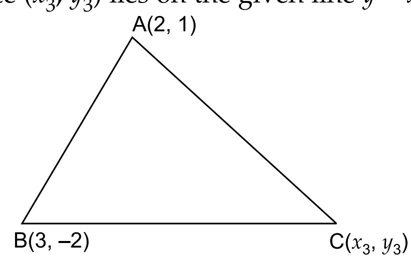
$$6x_1 + 2y_1 - 14 = 0$$
$$\Rightarrow \quad 3x_1 + y_1 - 7 = 0$$
$$\Rightarrow \quad 3(3y_1 + 9) + y_1 - 7 = 0 \quad \text{[From (1)]}$$
$$\Rightarrow \quad 10y_1 + 20 = 0$$
$$\Rightarrow \quad y_1 = -2$$

$\therefore$ From (1), $x_1 = -3 \times 2 + 9$
$= -6 + 9$
$= 3$

$\therefore$ The required centre of the circle is (3, –2).

3. Two vertices of a triangle are (2, 1) and (3, –2). The area of the triangle is 5 sq units. Find the third vertex if it lies on $y = x + 3$.

Sol. Let A(2, 1) and B(3, –2) be the given vertices of the triangle and let the vertex be C(x_3, y_3). Now, since (x_3, y_3) lies on the given line $y = x + 3$,



$$\therefore \quad y_3 = x_3 + 3 \quad \ldots(1)$$

Also, area of ΔABC

$$= \frac{1}{2}\left|2(-2 - y_3) + 3(y_3 - 1) + x_3(1 + 2)\right|$$

$$= \pm 5 \quad \text{[Given]}$$

$$\Rightarrow \quad -4 - 2y_3 + 3y_3 - 3 + 3y = \pm 10$$
$$\Rightarrow \quad y_3 + 3x_3 = 17 \quad \ldots(2)$$
$$\text{or,} \quad y_3 + 3x_3 = -3 \quad \ldots(3)$$

From (1) and (2), we have

$$x_3 + 3 + 3x_3 = 17$$
$$\Rightarrow \quad 4x_3 = -14$$
$$\Rightarrow \quad x_3 = \frac{7}{2}$$

$\therefore$ From (1), $y_3 = \frac{7}{2} + 3 = \frac{13}{2}$

and from (1) and (3), we have

$$x_3 + 3 + 3x_3 = -3$$

$$\Rightarrow \quad 4x_3 = -6$$

$$\Rightarrow \quad x_3 = -\frac{3}{2}$$

$\therefore$ From (1), $\quad y_3 = 3 - \frac{3}{2} = \frac{3}{2}$

Hence, the required third vertex is $\left(\frac{7}{2}, \frac{13}{2}\right)$ or $\left(\frac{3}{2}, -\frac{3}{2}\right)$.

4. ABCD is a rectangle formed by A(–1, – 1), B(–1, 4), C(5, 4) and D(5, –1). P, Q, R and S are the mid-points of AB, BC, CD, DA respectively. Is the quadrilateral PQRS a square, a rectangle or a rhombus? Justify your answer.

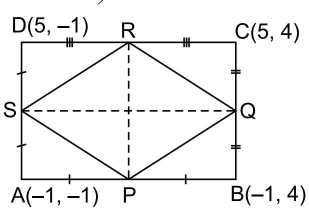
Sol. Since P, Q, R and S are the mid-point of AB, BC, CD and DA respectively, hence the coordinates of P, Q, R and S are

$$P : \left(\frac{-1-1}{2}, \frac{-1+4}{2}\right) = \left(-1, \frac{3}{2}\right)$$

$$Q : \left(\frac{5-1}{2}, \frac{4+4}{2}\right) = (2, 4)$$

$$R : \left(\frac{5+5}{2}, \frac{-1+4}{2}\right) = \left(5, \frac{3}{2}\right)$$

$$S : \left(\frac{5-1}{2}, \frac{-1-1}{2}\right) = (2, -1)$$



Now $\quad PQ = \sqrt{(2+1)^2 + \left(4 - \frac{3}{2}\right)^2}$

$$= \sqrt{3^2 + \left(\frac{5}{2}\right)^2}$$

$$= \sqrt{9 + \frac{25}{4}}$$

$$= \sqrt{\frac{36+25}{4}}$$

$$= \sqrt{\frac{61}{2}}$$

$$QR = \sqrt{(2-5)^2 + \left(4 - \frac{3}{2}\right)^2}$$

$$= \sqrt{3^2 + \left(\frac{5}{2}\right)^2}$$

$$= \sqrt{9 + \frac{25}{4}}$$

$$= \sqrt{\frac{36+25}{4}}$$

$$= \sqrt{\frac{61}{2}}$$

$$RS = \sqrt{(5-2)^2 + \left(\frac{3}{2} + 1\right)^2}$$

$$= \sqrt{3^2 + \left(\frac{5}{2}\right)^2}$$

$$= \sqrt{9 + \frac{25}{4}}$$

$$= \sqrt{\frac{36+25}{4}}$$

$$= \sqrt{\frac{61}{2}}$$

$$SP = \sqrt{(-1-2)^2 + \left(\frac{3}{2} + 1\right)^2}$$

$$= \sqrt{3^2 + \left(\frac{5}{2}\right)^2}$$

$$= \sqrt{9 + \frac{25}{4}}$$

$$= \sqrt{\frac{36+25}{4}}$$

$$= \sqrt{\frac{61}{2}}$$

Also, $\quad SQ = \sqrt{(2-2)^2 + (4+1)^2}$

$$= \sqrt{0 + 5^2}$$

$$= \sqrt{25}$$

$$= 5$$

and $\quad PR = \sqrt{(5+1)^2 + \left(\frac{3}{2} - \frac{3}{2}\right)^2}$

$$= \sqrt{6^2 + 0}$$

$$= \sqrt{36}$$

$$= 6$$

$\therefore$ PQ = QR = RS = SP, but diagonals SQ $\neq$ PR

Hence, the quadrilateral PQRS is a rhombus and not a square.

Self-Assessment

(Page 140)

Multiple-Choice Questions

1. The distance of a point P(–8, 6) from origin is

(*a*) 5 units (*b*) 8 units

(*c*) 10 units (*d*) 12 units

Sol. (*c*) 10 units

The required distance $= \sqrt{(-8)^2 + 6^2}$ units

$= \sqrt{64 + 36}$ units

$= \sqrt{100}$ units

$= 10$ units

2. The value of a if A(0, 0), B(3, $\sqrt{3}$) and C(3, a) form an equilateral triangle is

(*a*) 2 (*b*) –3

(*c*) –4 (*d*) $-\sqrt{3}$

Sol. (*d*) $-\sqrt{3}$

We have $AB^2 = 3^2 + \left(\sqrt{3}\right)^2$

$= 9 + 3$

$= 12$

$AC^2 = 3^2 + a^2$

and $BC^2 = (3 - 3)^2 + (a - \sqrt{3})^2$

$= 0 + 3 + a^2 - 2\sqrt{3}\,a$

$\because$ $AB^2 = AC^2 = BC^2$

$\therefore$ $9 + a^2 = 12$

$\Rightarrow$ $a = \pm\sqrt{3}$

When $a = \sqrt{3}$, B and C become identical point.

$a \neq \sqrt{3}$

$\therefore$ $a = -\sqrt{3}$

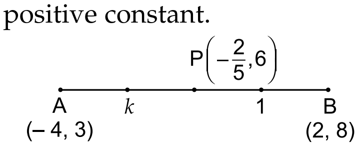
3. The ratio in which the point $P\left(\frac{-2}{5}, 6\right)$ divides the line segment joining the points A(–4, 3) and B(2, 8) is

(*a*) 2 : 3 (*b*) 1 : 3

(*c*) 3 : 2 (*d*) 3 : 1

Sol. (*c*) 3 : 2

Let P divide AB is the ratio PA : PB = k : 1 where k is a positive constant.



$\therefore$ $-\frac{2}{5} = \frac{2 \times k - 4 \times 1}{k + 1} = \frac{2k - 4}{k + 1}$

$\Rightarrow$ $-2k - 2 = 10k - 20$

$\Rightarrow$ $12k = 18$

$\Rightarrow$ $k = \frac{18}{12} = \frac{3}{2}$

$\therefore$ $k : 1 = \frac{3}{2} : 1 = 3 : 2$

4. If the points (0, 0), (1, 2) and (x, y) are collinear, then

(*a*) $x = y$ (*b*) $2x = y$

(*c*) $x = 2y$ (*d*) $2x = -4y$ **[CBSE 2012]**

Sol. (*b*) $2x = y$

Area of the triangle with vertices A(0, 0), B(1, 2) and C(x, y) is

$\frac{1}{2}\left[0(2 - y) + 1(y - 0) + x(0 - 2)\right]$

$= \frac{1}{2}\left[y - 2x\right]$

$= \frac{y - 2x}{2}$

$\therefore$ The three points are collinear, hence, this area is zero.

$\therefore$ $y - 2x = 0$

$\Rightarrow$ $y = 2x$

$\Rightarrow$ $2x = y$

Fill in the Blanks

5. The distance between the points P(6, 0) and Q(–2, 0) is **8 units**.

Sol. $PQ = \sqrt{(-2-6)^2 + (0-0)^2}$

$= \sqrt{(-8)^2} = 8$ units

6. The coordinates of a point P dividing the line segment joining the points A(1, 3) and B(4, 6) in the ratio 2 : 1 are **(3, 5)**.

Sol. Let the coordinates of the point P be (x, y). As the point P divides the line segment AB in the ratio 2 : 1.

Then, $x = \frac{1 \times 1 + 2 \times 4}{1 + 2} = \frac{1 + 8}{3} = 3$

and $y = \frac{1 \times 3 + 2 \times 6}{1 + 2} = \frac{3 + 12}{3} = 5$

Hence, the coordinates of the point P are (3, 5).

7. The ratio in which the line segment joining A(6, 3) and B(–2, –5) is divided by the x–axis is **3 : 5**.

Sol. Let P(x, 0) be the coordinates of x-axis. The point P divides the line segment A(6, 3) and B(–2, –5) in the ratio k : 1. By using section formula,

$$0 = \frac{k \times (-5) + 1 \times 3}{k+1}$$

$$\Rightarrow \quad 5k = 3$$

$$\Rightarrow \quad k = \frac{3}{5}$$

Hence, the ratio is 3 : 5.

8. If points A(3, 2), B(4, k) and C(5, 3) are collinear, then the value of k is $\frac{\mathbf{5}}{\mathbf{2}}$.

Sol. The given points are collinear if the area of the triangle formed by the points A, B and C is equal to zero.

No, area of triangle ABC = 0

$$\Rightarrow \frac{1}{2}\{3(k-3) + 4(3-2) + 5(2-k)\} = 0$$

$$\Rightarrow 3k - 9 + 4 + 10 - 5k = 0$$

$$\Rightarrow -2k + 5 = 0$$

$$\Rightarrow k = \frac{5}{2}$$

Assertion-Reason Type Questions

Directions (Q. Nos. 9 to 11): Each of these questions contains an assertion followed by reason. Read them carefully, and answer the question on the basis of the following options, select the one that best describes the two statements.

(a) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.

(b) Both Assertion and Reason are true but Reason is not the correct explanation of Assertion.

(c) If Assertion is true but Reason is false.

(d) If Assertion is false but Reason is true.

9. Assertion: Distance of the point (1, 1) from the origin is 2 units.

Reason: Distance between two points is given by root of sum of square of the differences in coordinates.

Sol. The correct answer is (*d*) as the Assertion is wrong as the correct distance is $\sqrt{(1-0)^2 + (1-0)^2}$ $= \sqrt{2}$.

As per distance formula, distance between two points is given by root of sum of square of the differences in coordinates.

10. Assertion: Points (4, 5), (4, 7) and (4, 9) cannot form a triangle.

Reason: They fall on a single line.

Sol. The correct answer is (*a*).

Points (4, 5), (4, 7) and (4, 9) cannot form a triangle as they fall in a straight line. Thus both statements are correct and reason is correct explanation of the assertion.

11. Assertion: Area of a triangle whose vertices are (1, –1), (–4, 6) and (–3, –5) is 24 square units.

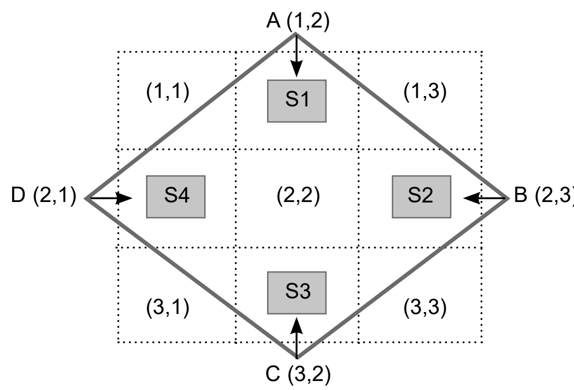
Reason: Area is given by half of the sum of the product of x-coordinate and difference of y-coordinates.

Sol. The correct answer is (*a*).

According to the formula of area of triangle, for vertices (1, –1), (–4, 6) and (–3, –5), area = 24 sq units. Thus both statements are correct and reason is correct explanation of the assertion.

Case Study Based Questions

12. Nikhil and Ayesha are playing a game on a coordinate grid. Nikhil has written some coordinates on some cells of the grid. Nikhil told Ayesha that the coordinates are written in a



pattern. Some cells are written as S1, S2, S3 and S4. Looking at the given pattern, Ayesha should fill the cells. Ayesha enjoyed and filled the cells with correct coordinates. Ayesha joined the cells and find the shape formed. Based on this situation, answer the following questions.

(*a*) Name the shape formed by joining the points.

(*i*) Square (*ii*) Rhombus

(*iii*) Rectangle (*iv*) Parallelogram

Ans. (*i*) Square

(*b*) Which one of the following is the distance formula?

(*i*) $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

(*ii*) $\sqrt{(x_2 - x_1)^2 + (y_1 - y_2)^2}$

(*iii*) $\sqrt{(x_2 - x_1)^2 - (y_2 - y_1)^2}$

(*iv*) $\sqrt{(x_1 - x_2)^2 + (y_2 - y_1)^2}$

Ans. (*i*) $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

(*c*) Join A and C, B and D. What is the relation between two diagonals AC and BD?

(*i*) AC = BD (*ii*) AC > BD

(*iii*) AC < BD (*iv*) AC = 1/2 BD

Ans. (*i*) AC = BD

(*d*) The point where the two diagonals meet is

(*i*) (1, 2) (*ii*) (2, 2)

(*iii*) (–1, 2) (*iv*) (2, –1)

Ans. (*ii*) (2, 2)

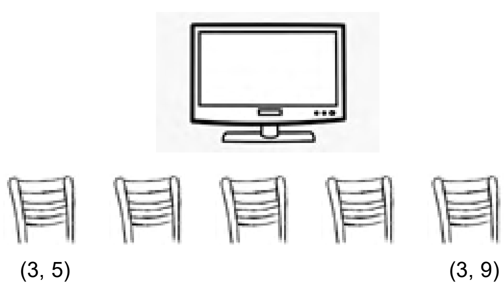
(*e*) Find the side of the square.

(*i*) 1.41 units (*ii*) 1.73 units

(*iii*) 1.52 units (*iv*) 1.84 units

Ans. (*i*) 1.41 units

13. Five friends are watching TV in a room. In a row, there are five chairs. The first chair's position is (3, 5). The last chair's position is (3, 9). The students sitting on the first chair and the last chair have some difficulty in watching TV. Based on the given situation, answer the following questions.



(*a*) What is the formula for finding the position of the chair which is placed in the middle?

(*i*) $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

(*ii*) $\left(\frac{x_1 - x_2}{2}, \frac{y_1 - y_2}{2}\right)$

(*iii*) $\left(\frac{x_1 + x_2}{2}, \frac{y_1 - y_2}{2}\right)$

(*iv*) $\left(\frac{x_1 - x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Ans. (*i*) $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

(*b*) All the chairs are in a row. What will be the middle position of the row?

(*i*) (2, 7) (*ii*) (3, 7)

(*iii*) (4, 7) (*iv*) (5, 7)

Ans. (*ii*) (3, 7)

(*c*) The mid-point of the line segment joining the points A($2a$, p) and B(a, $2p$) is

(*i*) $\left(\frac{3p}{2}, \frac{3a}{2}\right)$ (*ii*) $\left(\frac{3a}{2}, \frac{3p}{2}\right)$

(*iii*) $\left(\frac{p}{2}, \frac{3a}{2}\right)$ (*iv*) $\left(\frac{3a}{2}, \frac{p}{2}\right)$

Ans. (*ii*) $\left(\frac{3a}{2}, \frac{3p}{2}\right)$

(*d*) If the mid-point of the line segment joining the points A(2, 3) and B(a, 5) is P(x, y) and $x - y = 6$, find the value of a.

(*i*) 20 (*ii*) 16

(*iii*) 18 (*iv*) 15

Ans. (*iii*) 18

(*e*) Is the point P(–4, 6) lies on the line segment joining the points A(–5, 3) and B(–3, 9)?

(*i*) Yes (*ii*) No

(*iii*) Can't say (*iv*) May be

Ans. (*i*) Yes

Very Short Answer Type Questions

14. Find the distance 2AB, if A and B are points (–4, 2) and (–8, –1) respectively.

Sol. The required distance

$$= 2\sqrt{(-8+4)^2 + (-1-2)^2} \text{ units}$$
$$= 2\sqrt{(-4)^2 + (-3)^2}$$
$$= 2\sqrt{16+9} \text{ units}$$
$$= 2 \times 5 \text{ units}$$
$$= 10 \text{ units}$$

15. Write the condition of collinearity of three points.

Sol. The required condition for the collinearity of three points (x_1, y_1), (x_2, y_2), and (x_3, y_3) is $x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$, since, the area of the triangle with vertices (x_1, y_1), (x_2, y_2) and (x_3, y_3) is $\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$.

Short Answer Type-I Questions

16. Find x so that the point (3, x) lies on the line represented by $2x - 3y + 5 = 0$.

Sol. If the point (3, x) lies on the line $2x - 3y + 5 = 0$, then $x = 3$ and $y = x$ will satisfy this equation.

$\therefore \quad 2 \times 3 - 3 \times x + 5 = 0$

$\Rightarrow \quad 6 - 3x + 5 = 0$

$\Rightarrow \qquad 3x = 11$

$\Rightarrow \qquad x = \dfrac{11}{3}$

17. If the mid-point of a line segment joining $A\left(\dfrac{x}{2}, \dfrac{y+1}{2}\right)$ and $B(x + 1, y - 3)$ is $C(5, -2)$. Find x and y.

Sol. We have

$$\frac{\frac{x}{2} + x + 1}{2} = 5 \qquad \ldots(1)$$

and
$$\frac{\frac{y+1}{2} + y - 3}{2} = -2 \qquad \ldots(2)$$

From (1), $\dfrac{3x}{2} + 1 = 10$

$\Rightarrow \qquad \dfrac{3x}{2} = 10 - 1 = 9$

$\therefore \qquad x = 9 \times \dfrac{2}{3} = 6$

From (2) $\dfrac{3y+1}{2} - 3 = -4$

$\Rightarrow \qquad \dfrac{3y+1}{2} = -1$

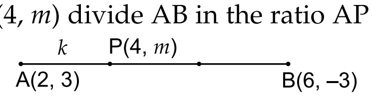
$\Rightarrow \qquad 3y = -2 - 1 = -3$

$\therefore \qquad y = -1$

$\therefore$ The required values of x and y are 6 and -1 respectively.

18. Find the ratio in which $P(4, m)$ divides the line segment joining the points $A(2, 3)$ and $B(6, -3)$. Hence, find m. **[CBSE 2018]**

Sol. Let $P(4, m)$ divide AB in the ratio $AP : PB = k : 1$



$\therefore \qquad 4 = \dfrac{6k + 1 \times 2}{k + 1}$

$\Rightarrow \qquad 4k + 4 = 6k + 2$

$\Rightarrow \qquad 2k = 2$

$\Rightarrow \qquad k = 1$

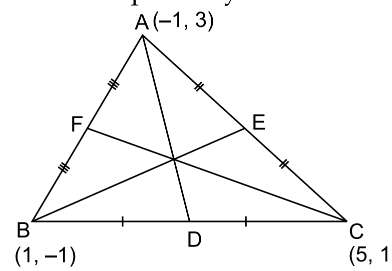
$\therefore$ Required ratio is $k : 1$, i.e. $1 : 1$.

Also, $m = \dfrac{1}{2}(3 - 3) = 0$ which is the required value of m.

Short Answer Type-II Questions

19. Find the length of median of a triangle whose vertices are $A(-1, 3)$, $B(1, -1)$ and $C(5, 1)$.

Sol. Let D, E and F be the mid-points of the sides BC, CA and AB respectively.



Then, D : $\left(\dfrac{5+1}{2}, \dfrac{-1+1}{2}\right) = (3, 0)$

$\therefore$ Length of the median AD

$= \sqrt{(3+1)^2 + (0-3)^2}$ units

$= \sqrt{16 + 9}$ units

$= 5$ units

E : $\left(\dfrac{-1+5}{2}, \dfrac{3+1}{2}\right) = (2, 2)$

$\therefore$ Length of the median BE

$= \sqrt{(2-1)^2 + (2+1)^2}$ units

$= \sqrt{1 + 9}$ units $= \sqrt{10}$ units

F : $\left(\dfrac{1-1}{2}, \dfrac{-1+3}{2}\right) = (0, 1)$

$\therefore$ Length of the median CF

$= \sqrt{(5-0)^2 + (1-1)^2}$ units

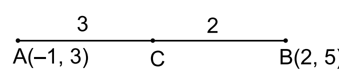
$= \sqrt{25}$ units

$= 5$ units.

Hence, the required length of the three medians are AD = 5 units, BE = $\sqrt{10}$ units and CF = 5 units respectively.

20. Find the coordinates of the point C on the line segment joining points $A(-1, 3)$ and $B(2, 5)$ such that $AC = \dfrac{3}{5} AB$.

Sol.



We have

$$\frac{AC}{AB} = \frac{3}{5}$$

$\Rightarrow \qquad \dfrac{AB}{AC} = \dfrac{5}{3}$

$\Rightarrow \qquad \dfrac{AC + CB}{AC} = \dfrac{5}{3}$

$\Rightarrow \qquad 1 + \dfrac{CB}{AC} = \dfrac{5}{3}$

$\Rightarrow \qquad \dfrac{BC}{AC} = \dfrac{5}{3} - 1 = \dfrac{2}{3}$

$\Rightarrow \qquad AC : CB = 3 : 2$

Let C be the point (x_1, y_1)

$$\therefore \qquad x_1 = \frac{3 \times 2 + 2 \times (-1)}{3 + 2}$$

$$= \frac{6 - 2}{5} = \frac{4}{5}$$

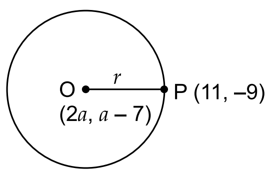
$$y_1 = \frac{3 \times 5 + 2 \times 3}{3 + 2}$$

$$= \frac{15 + 6}{5} = \frac{21}{5}$$

Hence, the required coordinates of C are $\left(\frac{4}{5}, \frac{21}{5}\right)$.

21. The centre of the circle is $(2a, a - 7)$. Find the value of a of the circle which passes through the point $(11, -9)$ has diameter $10\sqrt{2}$ units.

Sol. Let O be the centre, P(11, –9) be a point on the circle. Let r be its radius.



Then $\qquad r = OP$

$$= \sqrt{(2a - 11)^2 + (a - 7 + 9)^2}$$

$$= \sqrt{(2a - 11)^2 + (a + 2)^2}$$

But $\qquad r = \frac{10\sqrt{2}}{2} = 5\sqrt{2}$

$\therefore$ We have

$(2a - 11)^2 + (a + 2)^2 = (5\sqrt{2})^2 = 50$

$\Rightarrow \quad 4a^2 + 121 - 44a + a^2 + 4a + 4 = 50$

$\Rightarrow \quad 5a^2 - 40a + 75 = 0$

$\Rightarrow \quad a^2 - 8a + 15 = 0$

$\Rightarrow \quad (a - 5)(a - 3) = 0$

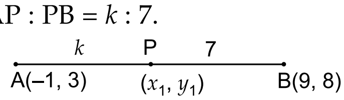
$\therefore$ Either $\quad a - 5 = 0 \Rightarrow a = 5$

or $\quad a - 3 = 0 \Rightarrow a = 3$

Hence, the required value of a is 5 or 3.

22. Point P divides the line segment joining the points A(–1, 3) and B(9, 8) such that $\frac{AP}{PB} = \frac{k}{7}$. If P lies on line $x - y + 2 = 0$, find the value of k.

Sol. Let P be the point (x_1, y_1) and P divides AB in the ratio AP : PB = k : 7.



$$\therefore \qquad x_1 = \frac{9k - 7}{k + 7}$$

and $\qquad y_1 = \frac{8k + 7 \times 3}{k + 7} = \frac{8k + 21}{k + 7}$

But $P(x_1, y_1)$ lies on the line $x - y + 2 = 0$.

$\therefore \qquad x_1 - y_1 + 2 = 0$

$\Rightarrow \qquad \frac{9k - 7}{k + 7} - \frac{8k + 21}{k + 7} + 2 = 0$

$\Rightarrow \qquad 9k - 7 - 8k - 21 + 2(k + 7) = 0$

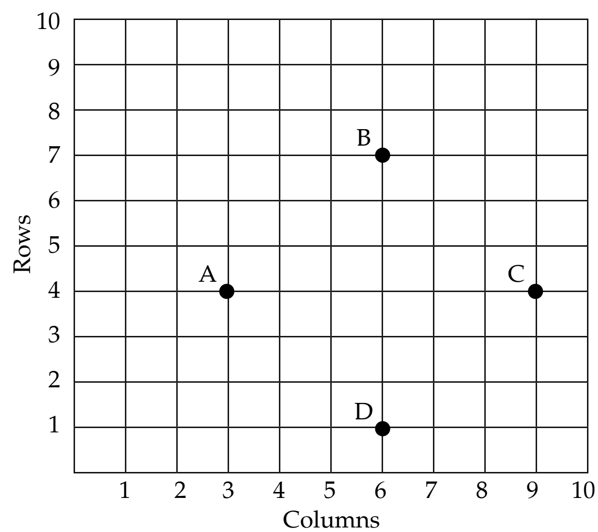
$\Rightarrow \qquad k - 28 + 2k + 14 = 0$

$\Rightarrow \qquad 3k = 14$

$\Rightarrow \; k = \frac{14}{3}$ which is the required value of k.

23. Read the following passage and answer the questions that follows:

In a classroom, four students Sita, Gita, Rita and Anita are sitting at A(3,4), B(6,7), C(9,4), D(6,1) respectively. Then a new student Anjali joins the class.



(*a*) Teacher tells Anjali to sit in the middle of the four students. Find the coordinates of the position where she can sit.

(*b*) Calculate the distance between Sita and Anita.

(*c*) Which two students are equidistant from Gita. **[CBSE SP(Basic) 2019]**

Sol. (*a*) (6, 4)

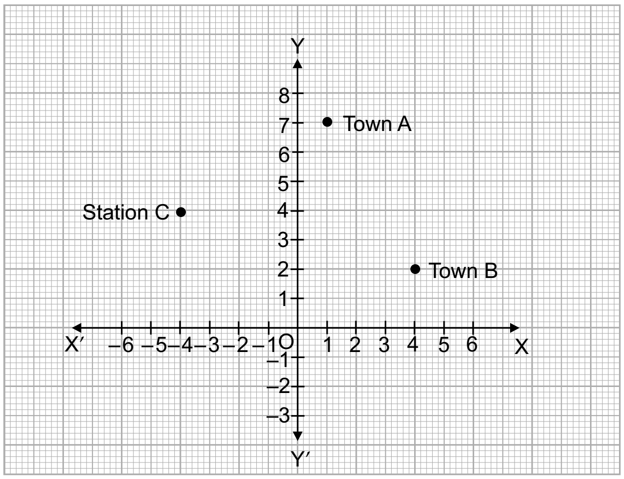
(*b*) $AD = \sqrt{(6 - 3)^2 + (1 - 4)^2}$

$$= \sqrt{(3)^2 + (-3)^2} = \sqrt{9 + 9} = \sqrt{18}$$

(*c*) Sita and Rita

24. Two friends Seema and Aditya work in the same office at Delhi. In the Christmas vacations, both decided to go to their hometowns represented by

Town A and Town B respectively in the figure given below. Town A and Town B are connected by trains from the same station C (in the given figure) in Delhi. Based on the given situation, answer the following questions:



(*a*) Who will travel more distance, Seema or Aditya, to reach to their hometown?

(*b*) Seema and Aditya planned to meet at a location D situated at a point D represented by the mid-point of the line joining the points represented by Town A and Town B. Find the coordinates of the point represented by the point D.

(*c*) Find the area of the triangle formed by joining the points represented by A, B and C.

[CBSE SP(Standard) 2019]

Sol. (*a*) A(1, 7), B(4, 2), C(–4, 4)

Distance travelled by Seema

$= AC = \sqrt{(-4-1)^2 + (4-7)^2} = \sqrt{34}$ units

Distance travelled by Aditya

$= BC = \sqrt{(-4-4)^2 + (4-2)^2} = \sqrt{68}$ units

$\therefore$ Aditya travels more distance

(*b*) Coordinates of D are $\left(\frac{1+4}{2}, \frac{7+2}{2}\right) = \left(\frac{5}{2}, \frac{9}{2}\right)$

(*c*) $ar(\Delta ABC) = \left|\frac{1}{2}[1(2-4) + 4(4-7) - 4(7-2)]\right|$

$= \left|\frac{1}{2}[(-2 + 4(-3) - 4(5)]\right|$

$= \left|\frac{1}{2}[-2 - 12 - 20]\right|$

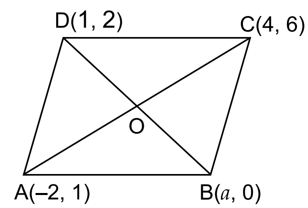
$= \left|\frac{1}{2}[-34]\right| = 17$ sq units

Long Answer Type Questions

25. If A(–2, 1), B(*a*, 0), C(4, *b*) and D(1, 2) are the vertices of a parallelogram ABCD, find the values of *a* and *b*. Hence find the lengths of its sides.

[CBSE 2018]

Sol. Let O be the point of intersection of the two diagonals AC and BD of the parallelogram. Then O is the mid-point of both BD and AC. Now, the mid-point of BD is the point $\left(\frac{a+1}{2}, \frac{2}{2}\right) = \left(\frac{a+1}{2}, 1\right)$ and the mid-point of AC is the point $\left(\frac{4-2}{2}, \frac{b+2}{2}\right) = \left(1, \frac{b+1}{2}\right)$.



$\therefore \quad \frac{a+1}{2} = 1$

$\Rightarrow \quad a = 1$

and $\quad \frac{b+1}{2} = 2$

$\Rightarrow \quad b = 1$

$\therefore$ AB = distance between (–2, 1) and (1, 0)

$= \sqrt{(1+2)^2 + 1^2}$ units

$= \sqrt{10}$ units

DC = distance between (1, 2) and (4, 1)

$= \sqrt{3^2 + 1^2}$ units

$= \sqrt{10}$ units

Also, BC = distance between (1, 0) and (4, 1)

$= \sqrt{(4-1)^2 + 1^2}$ units

$= \sqrt{10}$ units

AD = distance between (–2, 1) and (1, 2)

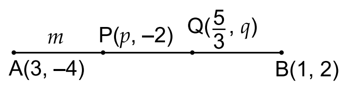
$= \sqrt{(1+2)^2 + (2-1)^2}$ units

$= \sqrt{10}$ units

$\therefore$ Each side is of length $\sqrt{10}$ units and $a = 1$, $b = 1$.

26. The line segment joining the points (3, −4) and (1, 2) is trisected at the points P and Q. If the coordinates of P and Q are $(p, -2)$ and $\left(\frac{5}{3}, q\right)$ respectively, find the value of p. **[CBSE 2005]**

Sol.



P divides AB in the ratio AP : PB = 1 : 2

and Q divides AB is the ratio AQ : QB = 2 : 1.

$\therefore \quad p = \dfrac{1 \times 1 + 2 \times 3}{2+1} = \dfrac{7}{3}$

$q = \dfrac{2 \times 2 - 4 \times 1}{2+1} = \dfrac{0}{3} = 0$

$\therefore$ The required values of p and q are $\frac{7}{3}$ and 0 respectively.

27. Find the circumcentre of the triangle whose vertices are (–2, –3), (–1, 0) and (7, –6).

Sol. Let $P(x_1, y_1)$ be the coordinates of the circumcentre of the triangle. Let A, B and C be the points (–2, –3), (–1, 0) and (7, –6) respectively. Then, PA = PB = PC

$\Rightarrow \quad PA^2 = PB^2 = PC^2$

$\Rightarrow \quad (x_1 + 2)^2 + (y_1 + 3)^2 = (x_1 + 1)^2 + y_1^2$

$= (x_1 - 7)^2 + (y_1 + 6)^2$

$\Rightarrow \quad x_1^2 + y_1^2 + 4x_1 + 6y_1 + 13 = x_1^2 + y_1^2 + 2x_1 + 1$

$= x_1^2 + y_1^2 - 14x_1 + 12y_1 + 85$

$\Rightarrow \quad 4x_1 + 6y_1 + 13 = 2x_1 + 1 = 12y_1 - 14x_1 + 85.$

$\therefore$ We have

$4x_1 + 6y_1 + 13 = 2x_1 + 1$

$\Rightarrow \quad 2x_1 + 6y_1 + 12 = 0$

$\Rightarrow \quad x_1 + 3y_1 + 6 = 0 \qquad \ldots(1)$

and $\quad 12y_1 - 14x_1 + 85 - 2x_1 - 1 = 0$

$\Rightarrow \quad -16x_1 + 12y_1 + 84 = 0$

$\Rightarrow \quad 4x_1 - 3y_1 - 21 = 0 \qquad \ldots(2)$

Adding (1) and (2), we get

$5x_1 - 15 = 0$

$\Rightarrow \quad x_1 = 3$

$\therefore$ From (1), $\quad 3 + 3y_1 + 6 = 0$

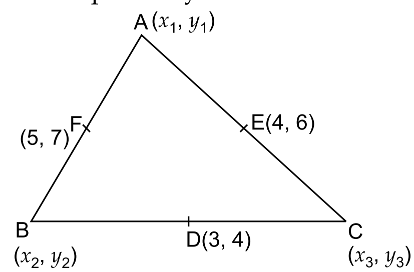
$\Rightarrow \quad 3y_1 = -9$

$\Rightarrow \quad y_1 = -3$

$\therefore$ The required coordinates of the circumcentre are (3, –3).

28. If the coordinates of the mid-points of the sides of a triangle are (3, 4), (4, 6) and (5, 7), find its vertices. **[CBSE 2008]**

Sol. Let (x_1, y_1), (x_2, y_2) and (x_3, y_3) be the coordinates of the vertices A, B and C of ΔABC respectively and let D, E and F be the mid-points of BC, CA and AB respectively with coordinates.



D : (3, 4), E : (4, 6) and F : (5, 7)

$\therefore \quad 3 = \dfrac{x_2 + x_3}{2}$

$\Rightarrow \quad x_2 + x_3 = 6 \qquad \ldots(1)$

Similarly, $\quad 4 = \dfrac{y_2 + y_3}{2}$

$\Rightarrow \quad y_2 + y_3 = 8 \qquad \ldots(2)$

Similarly, $\quad x_3 + x_1 = 8 \qquad \ldots(3)$

$y_3 + y_1 = 12 \qquad \ldots(4)$

$x_1 + x_2 = 10 \qquad \ldots(5)$

$y_1 + y_2 = 14 \qquad \ldots(6)$

Adding (1), (3) and (5), we get $2(x_1 + x_2 + x_3) = 24$

Subtracting (1), (3) and (5) successively from (7), we get $x_1 = 6$, $x_2 = 4$ and $x_3 = 2$.

Similarly adding (2), (4) and (6), we get

$2(y_1 + y_2 + y_3) = 34$

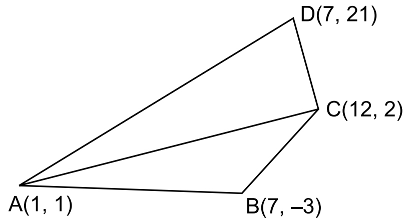
$\Rightarrow \quad y_1 + y_2 + y_3 = 17 \qquad \ldots(8)$

Subtracting (2), (4) and (6) successively from (8), we get $y_1 = 9$, $y_2 = 5$, $y_3 = 3$.

Hence, the required coordinates of D, E and F are respectively (6, 9), (4, 5) and (2, 3).

29. Find the area of the quadrilateral ABCD whose vertices are A(1, 1), B(7, –3), C(12, 2) and D(7, 21) respectively.

Sol. We divide the quadrilateral ABCD into two triangles ABC and DAC by joining one diagonal AC of the quadrilateral.



Now, area of ΔABC

$= \frac{1}{2}\left|\left[(-3-2)+7(2-1)+12(1+3)\right]\right|$ sq units

$= \frac{1}{2}\left|[-5+7+48]\right|$ sq units

= 25 sq units.

area of ΔACD

$= \frac{1}{2}\left|1\times(2-21)+12(21-1)+7(1-2)\right|$ sq units

$= \frac{1}{2}\left|19+240-7\right|$ sq units

$= \frac{1}{2}(240-26)$ sq units

= 107 sq units.

∴ Required area of the quadrilateral ABCD

= (25 + 107) sq units

= 132 sq units.

Let's Compete

(Page 142)

Multiple-Choice Questions

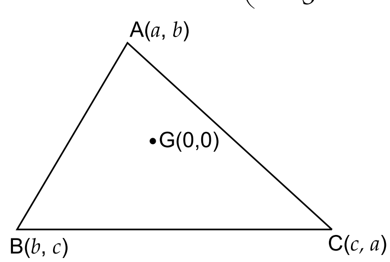
1. If centroid of a triangle formed by points (a, b), (b, c) and (c, a) is at (0, 0), then $a^3 + b^3 + c^3$ equals to

(*a*) abc (*b*) $a + b + c$

(*c*) $3abc$ (*d*) 0

Sol. (*c*) $3abc$

The centroid of ΔABC is $\left(\frac{a+b+c}{3}, \frac{b+c+a}{3}\right)$



$\therefore \quad a + b + c = 0$

$\therefore \quad a^3 + b^3 + c^3 - 3abc$

$= (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

$= 0$

$\therefore \quad a^3 + b^3 + c^3 = 3abc$

∴ (*c*) is correct.

2. The point which lies on circle of radius 6 cm and centre (3, 5) is

(*a*) (2, 4) (*b*) (3, 4)

(*c*) (–3, 5) (*d*) (–2, 4)

Sol. (*c*) (–3, 5)

If (x, y) be any points on the circle, then

$$(x-3)^2 + (y-5)^2 = 6^2$$

which is clearly satisfied by $x = -3$, $y = 5$

∴ (*c*) is the correct solution.

3. If points $(a, 0)$, $(0, b)$, (1, 1) are collinear, then $\frac{1}{a}+\frac{1}{b}$ equals to

(*a*) –1 (*b*) 0

(*c*) 1 (*d*) 2

Sol. (*c*) 1

The three points $(a, 0)$, $(0, b)$ and (1, 1) are collinear if $a(b - 1) + 0(1 - 0) + 1(0 - b) = 0$

$\Rightarrow \quad ab - a - b = 0$

$\Rightarrow \quad a + b = ab$

$\Rightarrow \quad \frac{a+b}{ab} = 1$

$\Rightarrow \quad \frac{1}{a}+\frac{1}{b} = 1$

∴ (c) is correct.

4. If $k\left(\frac{a}{3}, 4\right)$ is the mid-point of the line segment joining the points Q(–6, 5) and R(–2, 3), then the value of a is

(*a*) –8 (*b*) –12

(*c*) 12 (*d*) –4

Sol. (*b*) –12

We have

$$\frac{a}{3} = \frac{-6-2}{2} = -4$$

$\therefore \quad a = -12$

5. The ratio in which the line segment joining the points $A(a_1, b_1)$ and $B(a_2, b_2)$ is divided by y-axis is

(*a*) $-b_1 : b_2$ (*b*) $b_1 : b_2$

(*c*) $a_1 : a_2$ (*d*) $-a_1 : a_2$

Sol. (*d*) $-a_1 : a_2$

The coordinates of the point which divides. AB in the ratio $k : 1$ are $\left(\frac{ka_2 + a_1}{k+1}, \frac{kb_2 + b_1}{k+1}\right)$.

$\because$ This points lies on the y-axis its x-coordinate $= 0$

$$ka_2 + a_1 = 0$$

$$\Rightarrow \quad k = \frac{-a_1}{a_2}$$

$$\therefore \quad k : 1 = -a_1 : a_2$$

$\therefore$ The required ratio is $-a_1 : a_2$

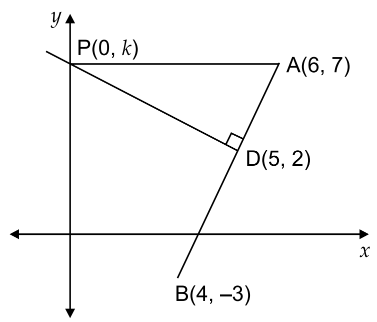
6. The coordinates of a point on y-axis which lies on the perpendicular bisector of line segment joining the points (6, 7) and (4, –3) are

(*a*) (0, 3) (*b*) (0, –3)

(*c*) (0, 2) (*d*) (0, –2)

Sol. (*a*) (0, 3)

Let A (6, 7) and B(4, –3) be the given points and P(0, k) be a point on the y-axis such that PD is the perpendicular bisector of AB. Hence, D is a point on AB such that AD = BD and $\angle$PDA = 90°.



Now, D is the mid-point of AB.

$\therefore$ The coordinates of D are $\left(\frac{6+4}{2}, \frac{7-3}{2}\right) = (5, 2)$

Now, from ΔPDA,

$$\because \quad \angle PDA = 90°$$

$\therefore$ By Pythagoras' theorem, we have

$$PA^2 = PD^2 + AD^2$$

$$\Rightarrow \quad 6^2 + (7-k)^2 = 5^2 + (2-k)^2 + (5-6)^2 + (2-7)^2$$

$$\Rightarrow \quad 36 + 49 + k^2 - 14k = 25 + 4 + k^2 - 4k + 1 + 25$$

$$\Rightarrow \quad 10k + 55 - 85 = 0$$

$$\Rightarrow \quad 10k - 30 = 0$$

$$\Rightarrow \quad k = 3$$

$\therefore$ The required coordinates are (0, 3).

7. The opposite vertices of a square are (–1, 2) and (3, 2), then the coordinates of other two vertices are

(*a*) (0, 1) and (1, 4) (*b*) (1, 0) and (1, 4)

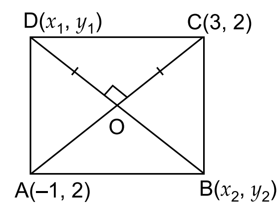
(*c*) (0, 1) and (4, 1) (*d*) (1, 0) and (4, 1)

Sol. (*b*) (1, 0) and (1, 4)

Let A(–1, 2) and C(3, 2) be the opposite vertices of the square ABCD. Then the two diagonals AC and BD will bisect each other at a point, say O. Then the coordinates of O are $\left(\frac{1}{2}(3-1), \frac{1}{2}(2+2)\right)$

$= (1, 2)$

Let the coordinates of the other two vertices be D(x_1, y_1) and B(x_2, y_2).



$$\text{Then} \quad \frac{1}{2}(x_1 + x_2) = 1$$

$$\Rightarrow \quad x_1 + x_2 = 2 \qquad \ldots(1)$$

$$\text{Also,} \quad CD^2 = AD^2$$

$$\Rightarrow \quad (x_1 - 3)^2 + (y_1 - 2)^2 = (x_1 + 1)^2 + (y_1 - 2)^2$$

$$\Rightarrow \quad x_1^2 - 6x_1 + 9 = x_1^2 + 2x_1 + 1$$

$$\Rightarrow \quad 8x_1 - 8 = 0$$

$$\Rightarrow \quad x_1 = 1 \qquad \ldots(2)$$

$$\therefore \text{ From (2),} \quad x_2 = 2 - 1 = 1 \qquad \ldots(3)$$

$$\text{Now} \quad OC^2 = OD^2$$

$$\Rightarrow \quad (3-1)^2 + (2-2)^2 = (x_1 - 1)^2 + (y_1 - 2)^2$$

$$\Rightarrow \quad 4 = (y_1 - 2)^2 \qquad \text{[From (2)]}$$

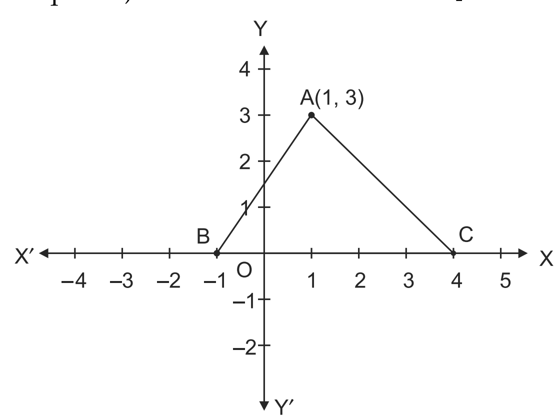
$$\therefore \quad y_1 - 2 = \pm 2$$

$$\Rightarrow \quad y_1 = 2 \pm 2 = 4, 0$$

$$\therefore \quad y_1 = 4, y_2 = 0$$

$\therefore$ The required points are (1, 4) and (1, 0).

8. In the given figure, the area of triangle ABC (in sq units) **[CBSE 2013]**



COORDINATE GEOMETRY

(*a*) 15 (*b*) 10
(*c*) 7.5 (*d*) 2.5

Sol. (*c*) 7.5

From the given figure, we see that A(1, 3), B(–1, 0) and C(4, 0).

$\therefore$ The required area of ΔABC

$$= \frac{1}{2}[1(0-0) - 1(0-3) + 4(3-0)] \text{ sq units}$$

$$= \frac{1}{2}(3 + 12) \text{ sq units} = \frac{15}{2} \text{ sq units}$$

$$= 7.5 \text{ sq units}$$

9. The point A which divides the line segment joining the points P(1, –4) and Q(3, 5) in ratio 3 : 2 internally lies in the

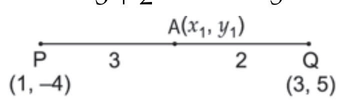
(*a*) I quadrant (*b*) II quadrant
(*c*) III quadrant (*d*) IV quadrant

Sol. (*a*) I quadrant

Since A divides PQ internally in the ratio. PA : AQ = 3 : 2, hence if (x_1, y_1) be the coordinates of A,

then $$x_1 = \frac{3 \times 3 + 2 \times 1}{3 + 2} = \frac{9 + 2}{5} = \frac{11}{5}$$

and $$y_1 = \frac{3 \times 5 - 4 \times 2}{3 + 2} = \frac{15 - 8}{5} = \frac{7}{5}$$

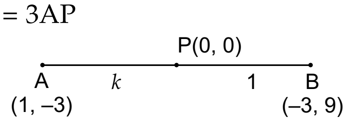


$\therefore$ The point A$\left(\frac{11}{5}, \frac{7}{5}\right)$ lies in the first quadrant.

10. If the point P(0, 0) lies on the line segment joining the points A(1, –3) and B(–3, 9), then

(*a*) PB = $\frac{1}{3}$AP (*b*) PB = 2AP
(*c*) PB = $\frac{1}{2}$AP (*d*) PB = 3AP

Sol. (*d*) PB = 3AP



Let P(0, 0) divides AB in the ratio AP : PB = *k* : 1

$$\therefore \quad 0 = \frac{-3k + 1}{k + 1}$$

$$\Rightarrow \quad k = \frac{1}{3}$$

$$\therefore \quad AP : PB = \frac{1}{3} : 1 = 1 : 3$$

$$\therefore \quad \frac{AP}{PB} = \frac{1}{3}$$

$$\Rightarrow \quad PB = 3AP$$

Value-based Questions (Optional)

(Page 143)

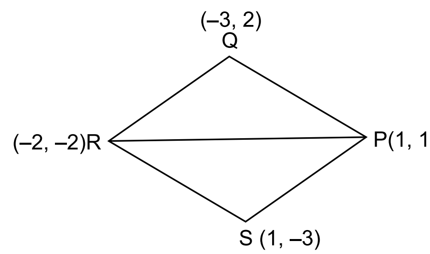
1. The students of class X of a school undertake to work for the campaign "Say no to Plastic" in a city. They took the map and form coordinate plane on it to divide the areas. Group A took the region covered between the coordinates (1, 1), (–3, 2), (–2, –2) and (1, –3) taken in order. Find the area of the region covered by group A.

(*a*) What are the harmful effects of using plastic?

(*b*) How can you contribute in spreading awareness for such campaign?

Sol. Let P(1, 1), Q(–3, 2), R(–2, –2) and S(1, –3) be the vertices of a quadrilateral PQRS.

We join RP.



$\therefore$ Area of ΔPRS

$$= \left|\frac{1}{2}[1(-2+3) - 2(-3-1) + 1(1+2)]\right| \text{ sq units}$$

$$= \frac{1 + 8 + 3}{2} \text{ sq units}$$

$$= \frac{12}{2} \text{ sq units}$$

$$= 6 \text{ sq units}$$

and Area of (ΔPQR)

$$= \left|\frac{1}{2}[1(2+2) - 3(-2-1) - 2(1-2)]\right| \text{ sq units}$$

$$= \left|\frac{1}{2}(4 + 9 + 2)\right| \text{ sq units}$$

$$= \frac{15}{2} \text{ sq units}$$

$\therefore$ Total area of the quadrilateral PQRS

= Area of ΔPRS + Area of ΔPQR

$$= \left(6 + \frac{15}{2}\right) \text{ sq units}$$

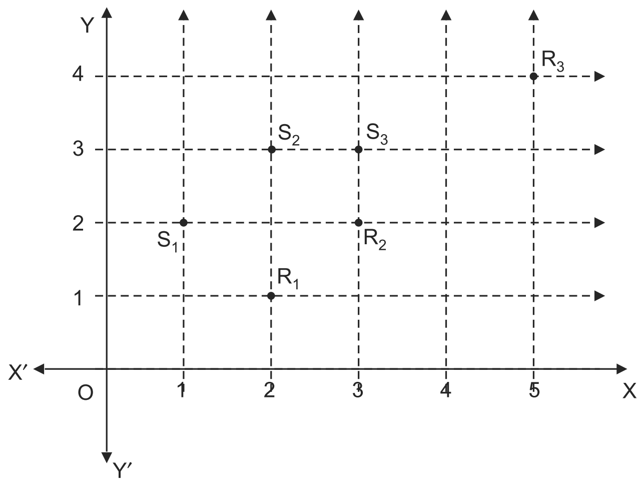
$$= \frac{27}{2} \text{ sq units}$$

Hence, the required area of the region is $\frac{27}{2}$ sq units.

(*a*) Plastic is non-biodegradable and so it causes pollution.

(*b*) By performing plays and preparing posters. we can spread awareness in the society.

2. Riya and Sohan planted some trees in their garden as shown in the figure and both arguing that they planted them in a straight line.



(*a*) Find who is correct? 'R' stands for Riya, 'S' for Sohan.

(*b*) Which social value is depicted?

Sol. (*a*) We see that the coordinates of R_1, R_2 and R_3 are (2, 1), (3, 2) and (5, 4) respectively and the coordinates of S_1, S_2 and S_3 are (1, 2), (2, 3) and (3, 3) respectively.

$\therefore$ Area of $\Delta R_1R_2R_3$

$= \left|\frac{1}{2}\left[2(2-4)+3(4-1)+5(1-2)\right]\right|$ sq units

$= \left|\frac{1}{2}(-4+9-5)\right|$ sq units

$= 0$ sq units

and Area of $\Delta S_1S_2S_3$

$= \left|\frac{1}{2}\left[1(3-3)+2(3-2)+3(2-3)\right]\right|$ sq units

$= \left|\frac{1}{2}(2-3)\right|$ sq units

$= \frac{1}{2}$ sq units

$\therefore$ R_1, R_2 and R_3 are in a straight line, but S_1, S_2 and S_3 are not in a straight line.

Hence, only Riya planted the trees in a straight line, but not Sohan.

(*b*) Planting trees help is making the environment clean. So, by planting trees, children are giving healthy environment to the society.

8

Introduction to Trigonometry

Checkpoint (Page 146)

1. Can a triangle have two obtuse angles? Why?

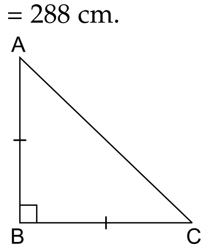
Sol. No; since in this case, the sum of two angles of the triangle will be greater than 180°. This is impossible, since, we know that the sum of three angles of any triangle is 180°.

2. Is it possible to construct a triangle such that each of three angles is less than 60°? Why?

Sol. No; since in this case, the sum of three angles of the triangle will be less than 180°. This is impossible.

3. The length of the hypotenuse of an isosceles right-angled triangle is 2.88 m. What are the lengths of the three sides of this triangle in cm?

Sol. Let ΔABC be an isosceles right-angled triangle with $\angle ABC = 90°$ and $AB = BC$ and the hypotenuse $AC = 2.88$ m = 288 cm.



Let a cm be the length of the sides BC and AB.

Then, since $\angle ABC = 90°$, hence, by Pythagoras' theorem, we have

$$AC^2 = BC^2 + AB^2$$

$$\Rightarrow \quad 288^2 = a^2 + a^2 = 2a^2$$

$$\Rightarrow \quad a^2 = \frac{288 \times 288}{2}$$

$$= 144 \times 288$$

$$= 144 \times 2 \times 144$$

$$\therefore \quad a = 144\sqrt{2}$$

Hence, the required sides of the triangle are $144\sqrt{2}$ cm, $144\sqrt{2}$ cm and 288 cm.

4. If x cm, $(x + 1)$ cm and $(x + 2)$ cm are the lengths of the three sides of a right-angled triangle, then find the value of x.

Sol. The lengths of the three sides of the right-angled triangle are x cm, $(x + 1)$ cm and $(x + 2)$ cm. Clearly, $(x + 2)$ cm is the greatest side and so the length of the hypotenuse is $(x + 2)$ cm.

$\therefore$ By Pythagoras' theorem, we have

$$(x + 2)^2 = x^2 + (x + 1)^2$$

$$\Rightarrow \quad x^2 + 4x + 4 = x^2 + x^2 + 2x + 1$$

$$\Rightarrow \quad x^2 - 2x - 3 = 0$$

$$\Rightarrow \quad x^2 + x - 3x - 3 = 0$$

$$\Rightarrow \quad x(x + 1) - 3(x + 1) = 0$$

$$\Rightarrow \quad (x + 1)(x - 3) = 0$$

$\therefore$ Either $\quad x + 1 = 0$

$$\Rightarrow \quad x = -1$$

which is neglected, since x cannot be negative.

or, $\quad x - 3 = 0$

$$\Rightarrow \quad x = 3$$

Hence, the required value of x is 3.

5. If the area of a triangle whose all three sides are of equal length is $4\sqrt{3}$ cm^2, what is the length of its each side?

Sol. Clearly, the triangle is an equilateral triangle. Hence, the area of the equilateral triangle of side a cm is $\frac{\sqrt{3}}{4}a^2$ cm^2.

$$\therefore \quad \frac{\sqrt{3}}{4}a^2 = 4\sqrt{3}$$

$$\Rightarrow \quad a^2 = 16$$

$$\Rightarrow \quad a = 4$$

[Taking positive value only, since the length of a side cannot be negative]

Hence, the required length is 4 cm.

6. In which of the following cases where the lengths of three sides are given, can you construct a triangle?

(*a*) 3 cm, 5 cm and 9 cm

(*b*) 7 cm, 6 cm and 8 cm

(*c*) 12 cm, 12 cm and 13 cm

Sol. We know that the sum of any two sides of any triangle is greater than its third side.

(*a*) We see that 3 + 5 < 9.

Hence, we cannot construct any triangle in this case.

(*b*) We see that 7 + 6 > 8, 7 + 8 > 6 and 6 + 8 > 7

So, we can construct a triangle in this case.

(*c*) We see that 12 + 12 > 13, 12 + 13 > 12 and so, we can construct a triangle in this case also.

So, we can construct a triangle only in cases (*b*) and (*c*), but not in case of (*a*).

7. Three angles 90°, 45° and 45° are given. How many different triangles can you construct with these three angles?

Sol. In this case, we can draw infinitely many triangles with different lengths of sides but all the triangles will be similar.

8. State two points by which an equation can be distinguished from an identity.

Sol. (*i*) An equation is a statement which is true for a limited number of values of the variable(s) involved, but an identity is a statement which is true for all arbitrary values of the variable(s) involved.

(*ii*) We solve an equation, whereas we prove an identity.

(I) Trigonometric Ratios

Milestone 1

(Page 149)

Multiple-Choice Questions

1. If the value of sec A = $\frac{5}{4}$, then the value of cot A is equal to

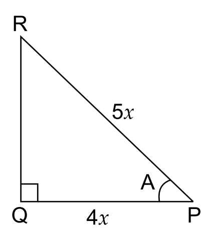
(*a*) $\frac{5}{3}$ (*b*) $\frac{3}{4}$

(*c*) $\frac{3}{5}$ (*d*) $\frac{4}{3}$

Sol. (*d*) $\frac{4}{3}$

Let ΔABC be a right-angled triangle where ∠B = 90° and ∠QPR = A. Clearly, QP is the base, QR is the perpendicular and RP is the hypotenuse of the triangle.

Now, we have sec A = $\frac{5}{4}$.



Let QP = $4x$ units and RP = $5x$ units, where x is a non-zero positive number. Then by Pythagoras' theorem, we have

$$RP^2 = QP^2 + QR^2$$

$$\Rightarrow \quad (5x)^2 = (4x)^2 + QR^2$$

$$\Rightarrow \quad QR^2 = 25x^2 - 16x^2 = 9x^2$$

$$\therefore \quad QR = 3x$$

$$\therefore \quad \cot A = \frac{QP}{QR} = \frac{4x}{3x} = \frac{4}{3}$$

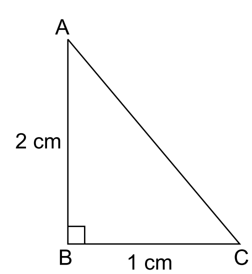
∴ The value of cot A is $\frac{4}{3}$.

2. In a triangle ABC, if ∠B = 90°, AB = 2 cm and BC = 1 cm, then the value of cosec A is equal to

(*a*) $\sqrt{3}$ (*b*) $\sqrt{5}$

(*c*) $\frac{1}{2}$ (*d*) 2

Sol. (*b*) $\sqrt{5}$



From ΔABC, we have by Pythagoras' theorem,

$$AC = \sqrt{BC^2 + AB^2}$$
$$= \sqrt{1^2 + 2^2} \text{ cm}$$
$$= \sqrt{5} \text{ cm}$$

$$\therefore \qquad \text{cosec A} = \frac{AC}{BC} = \frac{\sqrt{5}}{1} = \sqrt{5}$$

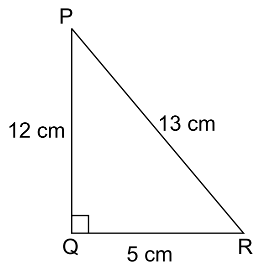
$\therefore$ The value of cosec A is $\sqrt{5}$.

Very Short Answer Type Questions

3. In ΔPQR, $\angle Q = 90°$. If PQ = 12 cm, QR = 5 cm and PR = 13 cm, what is the value of $\sec \angle R$?

Sol. We have

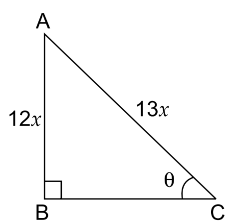
$$\sec \angle R = \frac{PR}{QR} = \frac{13}{5}$$



$\therefore$ The value of $\sec \angle R$ is $\frac{13}{5}$.

4. If $\sin \theta = \frac{12}{13}$, what is the value of $\cos \theta$?

Sol. Let ABC be a right-angled triangle with BC as base, AB as perpendicular, $\angle B = 90°$ and AC as the hypotenuse.



Let AC = $13x$ and AB = $12x$, where x is a non-zero positive number.

$\therefore$ By Pythagoras' theorem, we have

$$AC^2 = AB^2 + BC^2$$
$$\Rightarrow \quad (13x)^2 = (12x)^2 + BC^2$$
$$\Rightarrow \quad 169x^2 = 144x^2 + BC^2$$
$$\Rightarrow \quad BC^2 = 169x^2 - 144x^2$$
$$= 25x^2$$
$$\therefore \quad BC = 5x$$
$$\therefore \quad \cos \theta = \frac{BC}{AC} = \frac{5x}{13x} = \frac{5}{13}$$

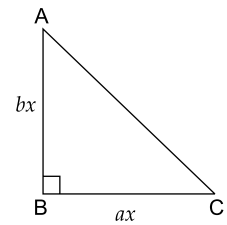
$\therefore$ The value of $\cos \theta$ is $\frac{5}{13}$.

Short Answer Type-I Questions

5. If $\cot A = \frac{b}{a}$, prove that $\frac{2\sec A + 1}{\cos A + 2} = \frac{\sqrt{a^2 + b^2}}{b}$.

[CBSE SP 2011]

Sol. Let ABC be a right-angled triangle with $\angle ABC = 90°$, BC = ax units, AB = bx units, where x is a non-zero positive constant.



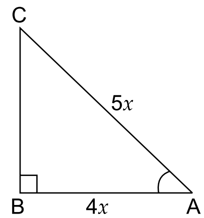
$\therefore$ By Pythagoras' theorem, we have

$$AC^2 = AB^2 + BC^2 = b^2x^2 + a^2x^2$$
$$= (a^2 + b^2)x^2$$
$$\therefore \quad AC = \sqrt{\left(a^2 + b^2\right)}\, x$$
$$\therefore \quad \sec A = \frac{AC}{AB}$$
$$= \frac{\sqrt{a^2 + b^2}\, x}{bx}$$
$$= \frac{\sqrt{a^2 + b^2}}{b}$$
$$\cos A = \frac{AB}{AC}$$
$$= \frac{bx}{\sqrt{a^2 + b^2}\, x}$$
$$= \frac{b}{\sqrt{a^2 + b^2}}$$
$$\therefore \quad \text{LHS} = \frac{2\sec A + 1}{\cos A + 2}$$
$$= \frac{2 \times \frac{\sqrt{a^2 + b^2}}{b} + 1}{\frac{b}{\sqrt{a^2 + b^2}} + 2}$$
$$= \frac{2\sqrt{a^2 + b^2} + b}{b} \times \frac{\sqrt{a^2 + b^2}}{b + 2\sqrt{a^2 + b^2}}$$
$$= \frac{\sqrt{a^2 + b^2}}{b}$$
$$= \text{RHS}$$

Hence, proved.

6. If $\sec A = \dfrac{5}{4}$, prove that $\tan A + \dfrac{1}{\cos A} = 2$.

Sol. Let ABC be a right-angled triangle with $\angle B = 90°$, $AB = 4x$ units and $AC = 5x$ units, where x is a non-zero positive number.



Now, by Pythagoras' theorem, we have

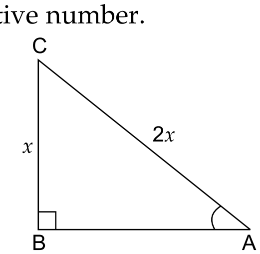
$$AC^2 = AB^2 + BC^2$$
$$\Rightarrow \quad (5x)^2 = (4x)^2 + BC^2$$
$$\Rightarrow \quad 25x^2 = 16x^2 + BC^2$$
$$\Rightarrow \quad BC^2 = 25x^2 - 16x^2$$
$$\Rightarrow \quad BC^2 = 9x^2$$
$$\Rightarrow \quad BC = 3x \text{ units}$$
$$\therefore \quad \tan A = \frac{BC}{AB} = \frac{3x}{4x} = \frac{3}{4}$$
$$\text{and} \quad \cos A = \frac{AB}{AC} = \frac{4x}{5x} = \frac{4}{5}$$
$$\therefore \quad \text{LHS} = \tan A + \frac{1}{\cos A}$$
$$= \frac{3}{4} + \frac{5}{4}$$
$$= \frac{8}{4} = 2 = \text{RHS}$$

Hence, proved.

Short Answer Type-II Questions

7. If cosec A = 2, find the value of $\dfrac{1}{\tan A} + \dfrac{\sin A}{1 + \cos A}$. **[CBSE SP 2011]**

Sol. Let ABC be a right-angled triangle with $\angle B = 90°$, $BC = x$ units and $AC = 2x$ units, where x is a non-zero positive number.



$\therefore$ By Pythagoras' theorem, we have

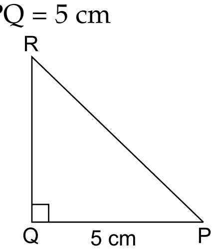
$$AC^2 = AB^2 + BC^2$$
$$\Rightarrow \quad (2x)^2 = AB^2 + x^2$$
$$\Rightarrow \quad AB^2 = 4x^2 - x^2 = 3x^2$$
$$\therefore \quad AB = \sqrt{3}\,x$$
$$\therefore \quad \tan A = \frac{BC}{AB} = \frac{x}{\sqrt{3}x} = \frac{1}{\sqrt{3}},$$
$$\sin A = \frac{BC}{AC} = \frac{x}{2x} = \frac{1}{2}$$
$$\text{and} \quad \cos A = \frac{AB}{AC} = \frac{\sqrt{3}x}{2x} = \frac{\sqrt{3}}{2}$$
$$\therefore \quad \frac{1}{\tan A} + \frac{\sin A}{1 + \cos A} = \sqrt{3} + \frac{\frac{1}{2}}{1 + \frac{\sqrt{3}}{2}}$$
$$= \sqrt{3} + \frac{1}{2 + \sqrt{3}}$$
$$= \frac{2\sqrt{3} + 3 + 1}{2 + \sqrt{3}}$$
$$= \frac{2\sqrt{3} + 4}{2 + \sqrt{3}}$$
$$= \frac{2\left(2 + \sqrt{3}\right)}{2 + \sqrt{3}} = 2$$

Hence, the required value is 2.

8. In ΔPQR, right angled at Q, $PR + QR = 25$ cm and $PQ = 5$ cm. Determine the values of sin P, cos P and tan P. **[CBSE SP 2011]**

Sol. In ΔPQR, we have

$\angle Q = 90°$, $PQ = 5$ cm



Let $QR = x$ cm

$$\because \quad PR + QR = 25 \text{ cm}$$
$$\therefore \quad PR = 25 \text{ cm} - x \text{ cm}$$
$$\Rightarrow \quad PR = (25 - x) \text{ cm} \qquad \ldots(1)$$

Now, by Pythagoras' theorem, we have

$$PR^2 = QR^2 + PQ^2 = x^2 + 5^2$$
$$\Rightarrow \quad (25 - x)^2 = x^2 + 25 \qquad \text{[From (1)]}$$
$$\Rightarrow \quad 625 + x^2 - 50x = x^2 + 25$$
$$\Rightarrow \quad 50x = 600$$
$$\Rightarrow \quad x = 12$$
$$\therefore \quad QR = 12 \text{ cm}$$
$$\text{and} \quad PR = (25 - 12) \text{ cm}$$
$$= 13 \text{ cm} \qquad \text{[From (1)]}$$

Now, $\sin P = \dfrac{QR}{PR} = \dfrac{12}{13}$

$\cos P = \dfrac{PQ}{PR} = \dfrac{5}{13}$

and $\tan P = \dfrac{QR}{PQ} = \dfrac{12}{5}$

Hence, the required values of sin P, cos P and tan P are $\dfrac{12}{13}$, $\dfrac{5}{13}$ and $\dfrac{12}{5}$ respectively.

Long Answer Type Questions

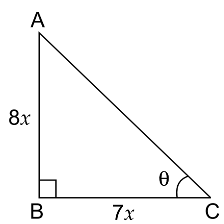
9. If $\cot\theta = \dfrac{7}{8}$, evaluate

(*a*) $\dfrac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$

(*b*) $\cot^2\theta$ **[CBSE SP 2010, 2011]**

(*c*) $\dfrac{\operatorname{cosec}^2\theta-\cot^2\theta+\sec\theta}{\sec^2\theta-\tan^2\theta+\operatorname{cosec}\theta}$

Sol. Let ABC be a right-angled triangle with $\angle B = 90°$, $BC = 7x$ units and $AB = 8x$ units, where x is a non-zero positive number.



$\therefore$ By Pythagoras' theorem, we have

$$AC^2 = AB^2 + BC^2$$
$$= (8x)^2 + (7x)^2 = 113x^2$$

$\therefore$ $AC = \sqrt{113}\,x$ units

(*a*) Let $\angle ACB = \theta$

$\therefore$ $\sin\theta = \dfrac{AB}{AC} = \dfrac{8x}{\sqrt{113}x} = \dfrac{8}{\sqrt{113}}$

$\cos\theta = \dfrac{BC}{AC} = \dfrac{7x}{\sqrt{113}x} = \dfrac{7}{\sqrt{113}}$

$$\therefore\ \frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)} = \frac{\left(1+\frac{8}{\sqrt{113}}\right)\left(1-\frac{8}{\sqrt{113}}\right)}{\left(1+\frac{7}{\sqrt{113}}\right)\left(1-\frac{7}{\sqrt{113}}\right)}$$

$$= \frac{1^2-\left(\frac{8}{\sqrt{113}}\right)^2}{1^2-\left(\frac{7}{\sqrt{113}}\right)^2}$$

$$= \frac{1-\frac{64}{113}}{1-\frac{49}{113}}$$

$$= \frac{49}{113}\times\frac{113}{64}$$

$$= \frac{49}{64}$$

$\therefore$ The required value is $\dfrac{49}{64}$.

(*b*) We have

$$\cot\theta = \frac{7}{8}$$

On squaring both sides,

$$\cot^2\theta = \left(\frac{7}{8}\right)^2 = \frac{49}{64}$$

$\therefore$ The required value is $\dfrac{49}{64}$.

(*c*) We have

$\operatorname{cosec}\theta = \dfrac{AC}{AB} = \dfrac{\sqrt{113}}{8}$, $\sec\theta = \dfrac{AC}{BC} = \dfrac{\sqrt{113}}{7}$

and $\tan\theta = \dfrac{8}{7}$

$$\therefore\ \frac{\operatorname{cosec}^2\theta-\cot^2\theta+\sec\theta}{\sec^2\theta-\tan^2\theta+\operatorname{cosec}\theta} = \frac{\frac{113}{64}-\frac{49}{64}+\frac{\sqrt{113}}{7}}{\frac{113}{49}-\frac{64}{49}+\frac{\sqrt{113}}{8}}$$

$$= \frac{\frac{64}{64}+\frac{\sqrt{113}}{7}}{\frac{49}{49}+\frac{\sqrt{113}}{8}}$$

$$= \frac{1+\frac{\sqrt{113}}{7}}{1+\frac{\sqrt{113}}{8}}$$

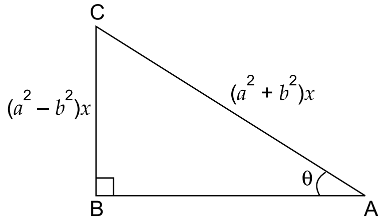
$$= \frac{8\left(7+\sqrt{113}\right)}{7\left(8+\sqrt{113}\right)}$$

$\therefore$ The required value is $\dfrac{8\left(7+\sqrt{113}\right)}{7\left(8+\sqrt{113}\right)}$.

10. If $\sin\theta = \dfrac{a^2-b^2}{a^2+b^2}$, show that

$$\sin\theta\cos\theta\tan\theta = \left(\frac{a^2-b^2}{a^2+b^2}\right)^2$$

Sol. Let ΔABC be a right-angled triangle with $\angle B = 90°$, $BC = (a^2 - b^2)x$ units and $AC = (a^2 + b^2)x$ units, where x is a non-zero positive number.



Let $\angle BAC = \theta$

$\therefore$ By Pythagoras' theorem, we have

$$AC^2 = BC^2 + AB^2$$

$$\Rightarrow \quad (a^2 + b^2)^2x^2 = (a^2 - b^2)^2x^2 + AB^2$$

$$\Rightarrow \quad AB^2 = \{(a^2 + b^2)^2 - (a^2 - b^2)^2\}x^2$$

$$= 4a^2b^2x^2$$

$$\therefore \quad AB = 2abx$$

$\therefore$ LHS = $\sin\theta \cos\theta \tan\theta$

$$= \frac{BC}{AC} \times \frac{AB}{AC} \times \frac{BC}{AB}$$

$$= \frac{(a^2 - b^2)x}{(a^2 + b^2)x} \times \frac{2abx}{(a^2 + b^2)x} \times \frac{(a^2 - b^2)x}{2abx}$$

$$= \left(\frac{a^2 - b^2}{a^2 + b^2}\right)^2$$

= RHS

Hence, proved.

Milestone 2

(Page 152)

Multiple-Choice Questions

1. The value of $\sin^2 30° - \cos^2 30°$ is equal to

(a) $-\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$

(c) $\frac{3}{2}$ (d) $\frac{2}{3}$ **[CBSE SP 2011]**

Sol. (a) $-\frac{1}{2}$

We have

$$\sin^2 30° - \cos^2 30° = \left(\frac{1}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2$$

$$= \frac{1}{4} - \frac{3}{4}$$

$$= -\frac{2}{4} = \frac{-1}{2}$$

2. If $\sin A = \frac{1}{2}$ and $\cos B = \frac{1}{2}$, then the value of A + B is equal to

(a) 0° (b) 60°

(c) 90° (d) 30° **[CBSE SP 2011]**

Sol. (c) 90°

We have $\sin A = \frac{1}{2} = \sin 30°$

$$\Rightarrow \quad A = 30°$$

and $\cos B = \frac{1}{2} = \cos 60°$

$$\Rightarrow \quad B = 60°$$

$$\therefore \quad A + B = 30° + 60°$$

$$= 90°$$

Very Short Answer Type Questions

3. What is the value of $4\cos^3\theta - 3\cos\theta$ when $\theta = 30°$? Is it equal to $\cos 3\theta$?

Sol. We have

$$4\cos^3\theta - 3\cos\theta = 4\cos^3 30° - 3\cos 30°$$

$$= 4 \times \left(\frac{\sqrt{3}}{2}\right)^3 - 3 \times \frac{\sqrt{3}}{2}$$

$$= 4 \times \frac{3}{4}\frac{\sqrt{3}}{2} - \frac{3}{2}\sqrt{3}$$

$$= \frac{3\sqrt{3}}{2} - \frac{3\sqrt{3}}{2}$$

$$= 0$$

Also, $\cos 3\theta = \cos(3 \times 30°)$

$$= \cos 90° = 0$$

Hence, $4\cos^3\theta - 3\cos\theta = \cos 3\theta$ when $\theta = 30°$.

$\therefore$ Yes, the given expression is equal to $\cos 3\theta$.

4. You are given an identity:

$\sin(A - B) = \sin A \cos B - \cos A \sin B$

Is it possible to find the value of sin 15° by putting A = 45° and B = 30° in the above formula? If so, what is the value of sin 15°?

Sol. We have

$$\sin(A - B) = \sin(45° - 30°) = \sin 15°$$

Also, $\sin A \cos B - \cos A \sin B$

$$= \sin 45° \cos 30° - \cos 45° \sin 30°$$

$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2}$$

$$= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}$$

$$= \frac{\sqrt{3}-1}{2\sqrt{2}}$$

$$\Rightarrow \quad \sin 15° = \frac{\sqrt{3}-1}{2\sqrt{2}}$$

$\therefore$ Yes, it is possible to find the value of sin 15° by putting A = 45° and B = 30° in the given formula. Also, the value of sin 15° is $\frac{\sqrt{3}-1}{2\sqrt{2}}$.

Short Answer Type-I Questions

5. If cos $(x + 40°)$ = sin 30°, find the value of x if $0° \le x \le 90°$.

Sol. We have

$$\cos(x + 40°) = \sin 30°$$
$$= \frac{1}{2} = \cos 60°$$
$$\therefore \quad x + 40° = 60°$$
$$\Rightarrow \quad x = 60° - 40°$$
$$= 20°$$

Hence, the required value of x is 20°.

6. Verify the identity $\cos A = \frac{1}{\sqrt{1+\tan^2 A}}$ by taking A = 30°.

Sol. We have, for A = 30°

$$\text{LHS} = \cos A = \cos 30° = \frac{\sqrt{3}}{2}$$

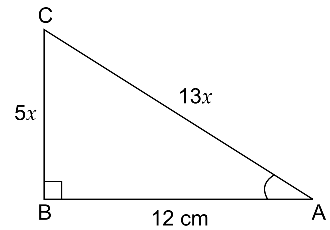
$$\text{RHS} = \frac{1}{\sqrt{1+\tan^2 A}}$$
$$= \frac{1}{\sqrt{1+\tan^2 30°}}$$
$$= \frac{1}{\sqrt{1+\frac{1}{3}}}$$
$$= \frac{\sqrt{3}}{2}$$
$$= \text{LHS}$$

Hence, proved.

Short Answer Type-II Questions

7. In ΔABC, $\angle ABC = 90°$. If AB = 12 cm and $\sin A = \frac{5}{13}$, find the lengths of the hypotenuse and the other side of the triangle. Also, find tan C.

Sol. Let BC = $5x$ cm and AC = $13x$ cm, where x is a non-zero positive number.



Now, $\because \angle B = 90°$,

$\therefore$ By Pythagoras' theorem, we have

$$AC^2 = AB^2 + BC^2$$
$$\Rightarrow \quad (13x)^2 = 12^2 + (5x)^2$$
$$\Rightarrow \quad 169x^2 = 144 + 25x^2$$
$$\Rightarrow \quad (169 - 25)x^2 = 144$$
$$\Rightarrow \quad 144x^2 = 144$$
$$\Rightarrow \quad x^2 = 1$$
$$\Rightarrow \quad x = 1$$

[Taking positive value only since the length of a side cannot be negative]

$$\therefore \quad BC = 5 \text{ cm}$$
$$\text{and} \quad AC = 13 \text{ cm}$$
$$\therefore \quad \tan C = \frac{AB}{BC} = \frac{12}{5}$$

Hence, the required length of the hypotenuse is 13 cm and the other side is 5 cm. Also, the value of tan C is $\frac{12}{5}$.

8. If $\tan(A - B) = \frac{1}{\sqrt{3}}$ and $\sin(A + B) = \frac{\sqrt{3}}{2}$, where $0° \le A + B \le 90°$, then find A and B. Also, find the value of $\sin^2(A + B) - \cos^2(A - B)$.

[CBSE SP 2011]

Sol. We have

$$\tan(A - B) = \frac{1}{\sqrt{3}} = \tan 30°$$

$$\therefore \quad A - B = 30° \qquad \ldots(1)$$

$[\because 0° < A - B < 90°]$

$$\text{Also,} \quad \sin(A + B) = \frac{\sqrt{3}}{2} = \sin 60°$$

$$\therefore \quad A + B = 60° \qquad \ldots(2)$$

$[\because 0° < A + B < 90°]$

Adding (1) and (2), we get

$$2A = 30° + 60° = 90°$$
$$\Rightarrow \quad A = \frac{90°}{2} = 45°$$

Subtracting (1) from (2), we get

$$2B = 60° - 30°$$

$$\Rightarrow \quad B = \frac{30°}{2} = 15°$$

Hence, the required value of A and B are 45° and 15° respectively.

Now, to find the value of $\sin^2(A + B) - \cos^2(A - B)$,

We have $\quad A + B = 45° + 15° = 60°$

and $\quad A - B = 45° - 15° = 30°$

$$\therefore \quad \sin^2(A + B) - \cos^2(A - B)$$

$$= \sin^2 60° - \cos^2 30°$$

$$= \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2$$

$$= 0$$

Hence, the required value of the given expression is 0.

Long Answer Type Questions

9. In ΔABC, $\angle ABC = 90°$, $\angle BAC = 60°$ and BC = 7 cm. Find the lengths of the hypotenuse, the other side and also all trigonometric ratios of the remaining third angle.

Sol. We have

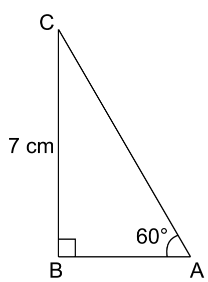
$$\sin 60° = \sin A$$

$$= \frac{BC}{AC}$$

$$= \frac{7}{AC}$$

$$\Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{7}{AC}$$

$$\Rightarrow \quad AC = \frac{14}{\sqrt{3}}$$



$\therefore$ The length of the hypotenuse AC is $\frac{14}{\sqrt{3}}$ cm.

Also, $\quad \cos 60° = \cos A = \frac{AB}{AC}$

$$\Rightarrow \quad \frac{1}{2} = \frac{AB\sqrt{3}}{14}$$

$$\Rightarrow \quad AB = \frac{7}{\sqrt{3}}$$

$\therefore$ The length of the other side AB is $\frac{7}{\sqrt{3}}$ cm.

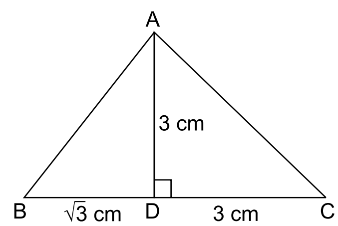
Now, since the third angle of the triangle is $180° - (60° + 90°) = 30°$.

$\therefore \quad \sin 30° = \frac{1}{2}$, $\cos 30° = \frac{\sqrt{3}}{2}$, $\tan 30° = \frac{1}{\sqrt{3}}$,

$\cot 30° = \sqrt{3}$, $\sec 30° = \frac{2}{\sqrt{3}}$ and $\operatorname{cosec} 30° = 2$.

10. In ΔABC, AD is the altitude of ΔABC where D is a point on BC such that AD = 3 cm, BD = $\sqrt{3}$ cm and DC = 3 cm. Determine $\angle BAC$ and $\sin \angle DAC + \tan \angle ACD + \cos \angle ABD$.

Sol. Given that AD is the altitude of ΔABC from A, where D is a point on BC such that BD = $\sqrt{3}$ cm, AD = 3 cm and DC = 3 cm.



Now, in ΔABD,

$\because \quad \angle ADB = 90°$,

$$\therefore \quad \tan B = \frac{AD}{BD}$$

$$= \frac{3}{\sqrt{3}}$$

$$= \sqrt{3}$$

$$= \tan 60°$$

$\therefore \quad B = 60° \qquad \ldots(1)$

Again, in ΔADC,

$\because \quad \angle ADC = 90°$,

$$\therefore \quad \tan C = \frac{AD}{DC}$$

$$= \frac{3}{3} = 1$$

$$= \tan 45°$$

$\therefore \quad C = 45° \qquad \ldots(2)$

$\therefore \quad \angle BAC = 180° - (\angle B + \angle C)$

[Angle sum property of a triangle]

$$= 180° - (60° + 45°)$$

$$= 75°$$

Hence, the required value of $\angle BAC$ is 75°.

Now, in ΔDAC, we have

$\angle DAC + \angle ADC + \angle ACD = 180°$

[Angle sum property of a triangle]

$\Rightarrow \quad \angle DAC + 90° + 45° = 180°$

$\therefore \quad \angle DAC = 180° - 135° = 45° \quad \ldots(3)$

Hence, we have

$\sin \angle DAC + \tan \angle ACD + \cos \angle ABD$

$= \sin 45° + \tan 45° + \cos 60°$

[From (1), (2) and (3)]

$$= \frac{1}{\sqrt{2}} + 1 + \frac{1}{2}$$

$$= \frac{2 + 2\sqrt{2} + \sqrt{2}}{2\sqrt{2}}$$

$$= \frac{3\sqrt{2} + 2}{2\sqrt{2}}$$

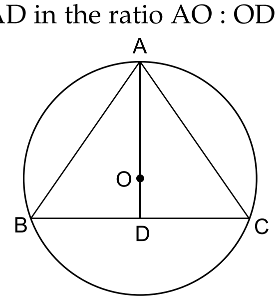
Hence, the required value of the given expression is $\frac{3\sqrt{2}+2}{2\sqrt{2}}$.

Higher Order Thinking Skills (HOTS) Questions

(Page 153)

1. An equilateral triangle is inscribed in a circle of radius 4 cm. Find its side.

Sol. Let O be the centre of the circle circumscribing ΔABC. Then O is the circumcentre of the equilateral triangle ABC. We draw AOD ⊥ BC. Then AD is a median of ΔABC, D being the mid-point of BC. For an equilateral triangle, the centroid and the circumcentre are identical point. Hence, O is the centroid of ΔABC dividing the median AD in the ratio AO : OD = 2 : 1.



$$\therefore \quad AO = \frac{2}{3} AD$$

$$\Rightarrow \quad 4 \text{ cm} = \frac{2}{3} AD$$

$$\Rightarrow \quad AD = \frac{3 \times 4}{2} \text{ cm} = 6 \text{ cm}$$

Hence, the length of the median AD is 6 cm.

Now, in ΔABD, since $\angle ADB = 90°$

$$\therefore \quad \tan \angle ABD = \frac{AD}{BD} = \frac{AD}{\frac{1}{2}BC}$$

$$\Rightarrow \quad \frac{2AD}{BC} = \frac{2 \times 6}{BC} = \frac{12}{BC}$$

$$\Rightarrow \quad \tan 60° = \frac{12}{BC}$$

[∵ In an equilateral triangle, each angle is 60°]

$$\Rightarrow \quad \sqrt{3} = \frac{12}{BC}$$

$$\Rightarrow \quad BC = \frac{12}{\sqrt{3}} = \frac{12\sqrt{3}}{3} = 4\sqrt{3}$$

$$\Rightarrow \quad BC = 4\sqrt{3} \text{ cm}$$

Hence, the required length of each side of ΔABC is $4\sqrt{3}$ cm.

2. Given that $\cos\frac{\theta}{2} + \sin\frac{\theta}{2} = \sqrt{1+\sin\theta}$ and $\cos\frac{\theta}{2} - \sin\frac{\theta}{2} = \sqrt{1-\sin\theta}$. Using these formulae, find the values of cos 15° and sin 15°.

Sol. Given that

$$\cos\frac{\theta}{2} + \sin\frac{\theta}{2} = \sqrt{1+\sin\theta} \quad \ldots(1)$$

$$\text{and} \quad \cos\frac{\theta}{2} - \sin\frac{\theta}{2} = \sqrt{1-\sin\theta} \quad \ldots(2)$$

Adding equation (1) and (2), we get

$$2\cos\frac{\theta}{2} = \sqrt{1+\sin\theta} + \sqrt{1-\sin\theta} \quad \ldots(3)$$

Subtracting equation (2) from (1), we get

$$2\sin\frac{\theta}{2} = \sqrt{1+\sin\theta} - \sqrt{1-\sin\theta} \quad \ldots(4)$$

Putting θ = 30° on both sides of each of (3) and (4), we get

$$2\cos 15° = \sqrt{1+\sin 30°} + \sqrt{1-\sin 30°}$$

$$= \sqrt{1+\frac{1}{2}} + \sqrt{1-\frac{1}{2}}$$

$$= \frac{\sqrt{3}}{\sqrt{2}} + \frac{1}{\sqrt{2}}$$

$$= \frac{\sqrt{3}+1}{\sqrt{2}}$$

$$\Rightarrow \quad \cos 15° = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

$$\text{and} \quad 2\sin 15° = \sqrt{1+\sin 30°} - \sqrt{1-\sin 30°}$$

$$= \frac{\sqrt{3}}{\sqrt{2}} - \frac{1}{\sqrt{2}} = \frac{\sqrt{3}-1}{\sqrt{2}}$$

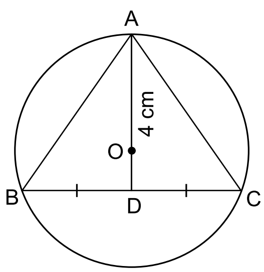
$$\Rightarrow \qquad \sin 15° = \frac{\sqrt{3}-1}{2\sqrt{2}}$$

Hence, the required values of cos 15° and sin 15° are $\frac{\sqrt{3}+1}{2\sqrt{2}}$ and $\frac{\sqrt{3}-1}{2\sqrt{2}}$ respectively.

3. If O is the circumcentre of a circle circumscribing an equilateral triangle ABC and AOD is an altitude of ΔABC through A on the side BC, then find the length of OD, if the radius of the circumscribed circle is 4 cm.

Sol. Given that OA = circumradius of the equilateral triangle ABC = 4 cm.

We know that for an equilateral triangle, the circumcentre is identical with the centroid of ΔABC.



$\therefore$ O is the centroid and if AOD ⊥ BC, then AOD is a median of ΔABC.

$$\therefore \qquad OA : OD = 2 : 1$$

$$\therefore \qquad OA = \frac{2}{3} AD$$

$$\Rightarrow \qquad 4 \text{ cm} = \frac{2}{3} AD$$

$$\Rightarrow \qquad AD = \frac{3 \times 4}{2} = 6 \text{ cm}$$

$$\therefore \qquad OD = \frac{1}{3} AD$$

$$= \frac{1}{3} \times 6 \text{ cm} = 2 \text{ cm}$$

Hence, the required length of OD is 2 cm.

Self-Assessment

(Page 153)

Multiple-Choice Questions

1. Which of the following is not defined?

(*a*) cos 0° (*b*) tan 45°

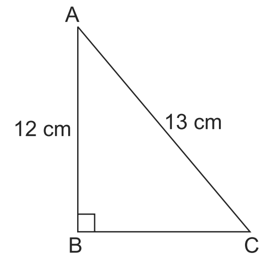
(*c*) sec 90° (*d*) sin 90°

[CBSE SP 2011]

Sol. (*c*) sec 90°

We know that out of cos 0°, tan 45°, sin 90°, and sec 90°, only sec 90° is not defined.

2. In the given figure, tan A – cot C is equal to



(*a*) $\frac{7}{13}$ (*b*) $\frac{-7}{13}$

(*c*) $\frac{5}{12}$ (*d*) 0 **[CBSE SP 2011]**

Sol. (*d*) 0

In Δ ABC, ∠B = 90°, AB = 12 cm and AC = 13 cm.

$\therefore$ By Pythagoras' theorem, we have

$$AC^2 = AB^2 + BC^2$$

$$\Rightarrow \qquad (13 \text{ cm})^2 = (12 \text{ cm})^2 + BC^2$$

$$\Rightarrow \qquad BC^2 = 169 \text{ cm}^2 - 144 \text{ cm}^2$$

$$= 25 \text{ cm}^2$$

$$\therefore \qquad BC^2 = 25 \text{ cm}^2$$

$$\Rightarrow \qquad BC = 5 \text{ cm}$$

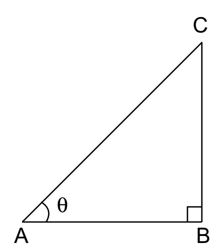
$$\therefore \qquad \tan A - \cot C = \frac{BC}{AB} - \frac{BC}{AB}$$

$$= \frac{5}{12} - \frac{5}{12} = 0$$

Fill in the Blanks

3. If sec θ = $\frac{5}{3}$, then the value of sin θ + tan θ is $\mathbf{\frac{32}{15}}$.

Sol. Draw a right ΔABC in which ∠B = 90°, ∠CAB = θ,



such that $\sec \theta = \frac{5}{3}$

$$\Rightarrow \qquad \frac{AC}{AB} = \frac{5}{3}$$

Let AC = 5*k*. Then, AB = 3*k*

In right ΔABC, we have

$$AC^2 = AB^2 + BC^2$$

$$\Rightarrow \qquad (5k)^2 = (3k)^2 + BC^2$$

$$\Rightarrow \qquad BC^2 = 25k^2 - 9k^2 = 16k^2$$

$$\Rightarrow \qquad BC = 4k$$

$$\sin \theta = \frac{BC}{AC} = \frac{4k}{5k} = \frac{4}{5}$$

and $\tan\theta = \dfrac{BC}{AB} = \dfrac{4k}{3k} = \dfrac{4}{3}$

$\therefore$ $\sin\theta + \tan\theta = \dfrac{4}{5} + \dfrac{4}{3} = \dfrac{12+20}{15} = \dfrac{32}{15}$

4. The value of cosec 30° + tan 45° is **3**.

Sol. cosec 30° + tan 45° = 2 + 1 = 3

5. If 2 cos 3A = 1, then the value of A is **20°**.

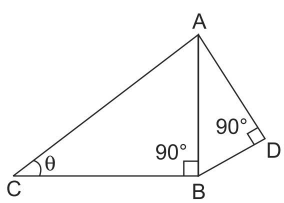
Sol.
$$2\cos 3A = 1$$
$$\Rightarrow \cos 3A = \frac{1}{2}$$
$$\Rightarrow \cos 3A = \cos 60^\circ$$
$$\Rightarrow 3A = 60^\circ$$
$$\Rightarrow A = 20^\circ$$

6. The value of $3\sin^2 30^\circ + 2\tan^2 60^\circ - 5\cos^2 45^\circ$ is $\mathbf{\dfrac{17}{4}}$.

Sol. $3\sin^2 30^\circ + 2\tan^2 60^\circ - 5\cos^2 45^\circ$
$$= 3\left(\frac{1}{2}\right)^2 + 2\left(\sqrt{3}\right)^2 - 5\left(\frac{1}{\sqrt{2}}\right)^2$$
$$= \frac{3}{4} + 6 - \frac{5}{2}$$
$$= \frac{3+24-10}{4}$$
$$= \frac{17}{4}$$

Very Short Answer Type Questions

7. In the given figure, AD = 4 cm, BD = 3 cm and CB = 12 cm. Then what is the value of cot θ?



[CBSE 2008, CBSE SP 2010, 2011]

Sol. In ΔADB,

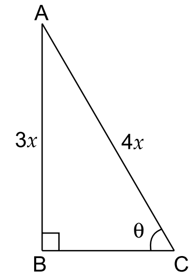
By Pythagoras' theorem, we have
$$AB^2 = AD^2 + BD^2$$
$$\Rightarrow AB^2 = (4\text{ cm})^2 + (3\text{ cm})^2$$
$$= 16\text{ cm}^2 + 9\text{ cm}^2$$
$$= 25\text{ cm}^2$$
$$\Rightarrow AB = 5\text{ cm}$$

$\therefore$ From ΔABC, $\cot\theta = \cot C = \dfrac{CB}{AB} = \dfrac{12}{5}$

$\therefore$ The required value of cot θ is $\dfrac{12}{5}$.

8. If $\text{cosec}\,\theta = \dfrac{4}{3}$, what is the value of $\dfrac{1-\tan^2\theta}{1+\tan^2\theta}$?

Sol. Let ΔABC be a right-angled triangle where ∠B = 90°, AC = $4x$ units and AB = $3x$ units, where x is a non-zero positive number.



Then, by Pythagoras' theorem, we have
$$AC^2 = AB^2 + BC^2$$
$$\Rightarrow (4x)^2 = (3x)^2 + BC^2$$
$$\Rightarrow BC^2 = 16x^2 - 9x^2$$
$$\Rightarrow BC^2 = 7x^2$$
$$\Rightarrow BC = \sqrt{7}x$$

If ∠ACB = θ, then
$$\frac{1-\tan^2\theta}{1+\tan^2\theta} = \frac{1-\dfrac{AB^2}{BC^2}}{1+\dfrac{AB^2}{BC^2}}$$
$$= \frac{BC^2 - AB^2}{AB^2 + BC^2}$$
$$= \frac{7x^2 - 9x^2}{9x^2 + 7x^2}$$
$$= \frac{-2x^2}{16x^2}$$
$$= \frac{-2}{16}$$
$$= -\frac{1}{8}$$

Short Answer Type-I Questions

9. Solve the equation (sec A – 2) (tan 3A – 1) = 0, when 0° < A < 90°.

Sol. We have
$$(\sec A - 2)(\tan 3A - 1) = 0$$

$\therefore$ Either $\sec A - 2 = 0$ …(1)

or, $\tan 3A - 1 = 0$ …(2)

From (1), $\sec A = 2 = \sec 60^\circ$

$\therefore$ $A = 60^\circ$

From (2), $\tan 3A = 1 = \tan 45^\circ$

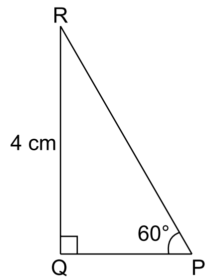
$\therefore$ $3A = 45^\circ$

$\Rightarrow \quad A = \dfrac{45°}{3} = 15°$

$\therefore$ The required solution is A = 60° or 15°.

10. In a right triangle PQR, $\angle Q = 90°$, $\angle QPR = 60°$ and QR = 4 cm. Find the lengths of PQ and PR.

Sol. In ΔPQR, we have



$$\sin P = \frac{QR}{PR}$$

$$\Rightarrow \quad \sin 60° = \frac{4}{PR} \text{ cm}$$

$$\Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{4}{PR} \text{ cm}$$

$$\therefore \quad PR = \frac{8}{\sqrt{3}} \text{ cm}$$

Again, $\tan P = \dfrac{QR}{PQ}$

$$\Rightarrow \quad \tan 60° = \frac{4}{PQ} \text{ cm}$$

$$\Rightarrow \quad \sqrt{3} = \frac{4}{PQ} \text{ cm}$$

$$\Rightarrow \quad PQ = \frac{4}{\sqrt{3}} \text{ cm}$$

Hence, the required lengths of PQ and PR are $\dfrac{4}{\sqrt{3}}$ cm and $\dfrac{8}{\sqrt{3}}$ cm respectively.

Short Answer Type-II Questions

11. Find the value of the expression

$$\frac{23\,\text{cosec}^3 30° - 47\sec^2 45° + 57\cot^2 60°}{31\sec^4 0° + 59\tan^2 60° - 136\cos^3 60°}$$

Sol. We have

$$\frac{23\,\text{cosec}^3 30° - 47\sec^2 45° + 57\cot^2 60°}{31\sec^4 0° + 59\tan^2 60° - 136\cos^3 60°}$$

$$= \frac{23 \times 2^3 - 47 \times \left(\sqrt{2}\right)^2 + 57 \times \left(\frac{1}{\sqrt{3}}\right)^2}{31 \times 1^4 + 59 \times \left(\sqrt{3}\right)^2 - 136 \times \left(\frac{1}{2}\right)^3}$$

$$= \frac{23 \times 8 - 47 \times 2 + 57 \times \frac{1}{3}}{31 + 59 \times 3 - 136 \times \frac{1}{8}}$$

$$= \frac{184 - 94 + 19}{31 + 177 - 17}$$

$$= \frac{203 - 94}{208 - 17}$$

$$= \frac{109}{191}$$

$\therefore$ The required value of the expression is $\dfrac{109}{191}$.

12. Verify the following for $\theta = 60°$ and $\theta = 45°$ separately:

$$\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} - \sec\theta = \sec\theta - \sqrt{\frac{1-\sin\theta}{1+\sin\theta}}.$$

Sol. For $\theta = 60°$, we have

$$\text{LHS} = \sqrt{\frac{1+\sin\theta}{1-\sin\theta}} - \sec\theta$$

$$= \sqrt{\frac{1+\sin 60°}{1-\sin 60°}} - \sec 60°$$

$$= \sqrt{\frac{1+\frac{\sqrt{3}}{2}}{1-\frac{\sqrt{3}}{2}}} - 2$$

$$= \sqrt{\frac{2+\sqrt{3}}{2-\sqrt{3}}} - 2$$

$$= \frac{\sqrt{\left(2+\sqrt{3}\right)^2}}{\sqrt{4-3}} - 2$$

$$= 2 + \sqrt{3} - 2 = \sqrt{3}$$

$$\text{RHS} = \sec\theta - \sqrt{\frac{1-\sin\theta}{1+\sin\theta}}$$

$$= \sec 60° - \sqrt{\frac{1-\sin 60°}{1+\sin 60°}}$$

$$= 2 - \sqrt{\frac{1-\frac{\sqrt{3}}{2}}{1+\frac{\sqrt{3}}{2}}}$$

$$= 2 - \sqrt{\frac{2-\sqrt{3}}{2+\sqrt{3}}}$$

$$= 2 - \frac{\sqrt{\left(2-\sqrt{3}\right)^2}}{\sqrt{4-3}}$$

$$= 2 - 2 + \sqrt{3}$$

$$= \sqrt{3}$$

$$\therefore \quad \text{LHS} = \text{RHS}$$

Again, for $\theta = 45°$, we have

$$\text{LHS} = \sqrt{\frac{1+\sin\theta}{1-\sin\theta}} - \sec\theta$$

$$= \sqrt{\frac{1+\sin 45°}{1-\sin 45°}} - \sec 45°$$

$$= \sqrt{\frac{1+\frac{1}{\sqrt{2}}}{1-\frac{1}{\sqrt{2}}}} - \sqrt{2}$$

$$= \sqrt{\frac{\sqrt{2}+1}{\sqrt{2}-1}} - \sqrt{2}$$

$$= \frac{\sqrt{\left(\sqrt{2}+1\right)^2}}{\sqrt{\left(\sqrt{2}\right)^2 - 1}} - \sqrt{2}$$

$$= \sqrt{2} + 1 - \sqrt{2}$$

$$= 1$$

$$\text{RHS} = \sec\theta - \sqrt{\frac{1-\sin\theta}{1+\sin\theta}}$$

$$= \sec 45° - \sqrt{\frac{1-\sin 45°}{1+\sin 45°}}$$

$$= \sqrt{2} - \sqrt{\frac{1-\frac{1}{\sqrt{2}}}{1+\frac{1}{\sqrt{2}}}}$$

$$= \sqrt{2} - \sqrt{\frac{\sqrt{2}-1}{\sqrt{2}+1}}$$

$$= \sqrt{2} - \frac{\sqrt{\left(\sqrt{2}-1\right)^2}}{\sqrt{\left(\sqrt{2}\right)^2 - 1}}$$

$$= \sqrt{2} - \sqrt{2} + 1$$

$$= 1$$

$\therefore$ LHS = RHS

Hence, proved.

Long Answer Type Questions

13. (*a*) Find the values of A and B if $\sin(A + 2B) = \frac{\sqrt{3}}{2}$, $\cos(A + 4B) = 0$ and $0° < A, B < 90°$.

(*b*) Verify the relation, $\sin 2x = \frac{2\cot x}{1+\cot^2 x}$

where $x = A + B$, where A and B are obtained in (*a*).

Sol. (*a*) We have

$$\sin(A + 2B) = \frac{\sqrt{3}}{2} = \sin 60°$$

$$\therefore \quad A + 2B = 60° \qquad \ldots(1)$$

Again,

$$\cos(A + 4B) = 0 = \cos 90°$$

$$\therefore \quad A + 4B = 90° \qquad \ldots(2)$$

Subtracting (1) from (2), we get

$$2B = 90° - 60°$$

$$= 30°$$

$$\Rightarrow \quad B = 15°$$

$\therefore$ From (1), $A = 60° - 2 \times 15°$

$$= 60° - 30°$$

$$= 30°$$

$\therefore$ The required values of A and B are 30° and 15° respectively.

(*b*) We have

A = 30° and B = 15°

$$\therefore \quad A + B = 30° + 15° = 45°$$

$$\therefore \quad x = 45°$$

Now, $\text{LHS} = \sin 2x$

$$= \sin 2 \times 45°$$

$$= \sin 90°$$

$$= 1$$

$$\text{RHS} = \frac{2\times\cot x}{1+\cot^2 x}$$

$$= \frac{2\times\cot 45°}{1+\cot^2 45°}$$

$$= \frac{2\times 1}{1+1}$$

$$= \frac{2}{2}$$

$$= 1$$

$\therefore$ LHS = RHS

Hence, proved.

14. Using the identities:

$$\cos\theta = \sqrt{\frac{1+\cos 2\theta}{2}} \text{ and } \sin\theta = \sqrt{\frac{1-\cos 2\theta}{2}},$$

find the value of $\cos 22\frac{1°}{2} \times \sin 22\frac{1°}{2}$.

Sol. Given that

$$\cos\theta = \sqrt{\frac{1+\cos 2\theta}{2}} \qquad \ldots(1)$$

and

$$\sin\theta = \sqrt{\frac{1-\cos 2\theta}{2}} \qquad \ldots(2)$$

Putting $\theta = 22\frac{1°}{2}$ so that $2\theta = 22\frac{1°}{2} \times 2 = 45°$ in equation (1) and (2), we get

$$\cos 22\frac{1°}{2} = \sqrt{\frac{1+\cos 45°}{2}}$$

$$= \sqrt{\frac{1+\frac{1}{\sqrt{2}}}{2}} = \sqrt{\frac{\sqrt{2}+1}{2\sqrt{2}}}$$

and

$$\sin 22\frac{1°}{2} = \sqrt{\frac{1-\cos 45°}{2}}$$

$$= \sqrt{\frac{1-\frac{1}{\sqrt{2}}}{2}} = \sqrt{\frac{\sqrt{2}-1}{2\sqrt{2}}}$$

$$\therefore \cos 22\frac{1^\circ}{2} \times \sin 22\frac{1^\circ}{2} = \frac{\sqrt{\sqrt{2}+1}}{\sqrt{2\sqrt{2}}} \times \frac{\sqrt{\sqrt{2}-1}}{\sqrt{2\sqrt{2}}}$$

$$= \frac{\sqrt{\left(\sqrt{2}+1\right)\left(\sqrt{2}-1\right)}}{2\sqrt{2}}$$

$$= \frac{\sqrt{2-1}}{2\sqrt{2}} = \frac{1}{2\sqrt{2}}$$

$\therefore$ The required value of the expression is $\frac{1}{2\sqrt{2}}$.

Let's Compete

(Page 154)

Multiple-Choice Questions

1. If $\sin(2\theta - \phi) = 1$ and $\cos(\theta + \phi) = \frac{1}{2}$, θ and ϕ being positive acute angles, then θ and ϕ are respectively equal to

(*a*) 20° and 70° (*b*) 70° and 20°
(*c*) 10° and 50° (*d*) 50° and 10°

Sol. (*d*) 50° and 10°

We have

$$\sin(2\theta - \phi) = 1 = \sin 90^\circ$$

$$\therefore \quad 2\theta - \phi = 90^\circ \quad \ldots(1)$$

and

$$\cos(\theta + \phi) = \frac{1}{2} = \cos 60^\circ$$

$$\theta + \phi = 60^\circ \quad \ldots(2)$$

Adding (1) and (2), we get

$$3\theta = 90^\circ + 60^\circ = 150^\circ$$

$$\therefore \quad \theta = \frac{150^\circ}{3} = 50^\circ$$

$\therefore$ From (2), we get

$$\phi = 60^\circ - \theta$$
$$= 60^\circ - 50^\circ$$
$$= 10^\circ$$

$\therefore$ The required values of θ and ϕ are 50° and 10° respectively.

2. Using the identity: $\cot(A + B) = \frac{\cot A \cot B - 1}{\cot B + \cot A}$ and choosing suitable standard values of A and B, we can find cot 75° as

(*a*) $2 + \sqrt{3}$ (*b*) $2 - \sqrt{3}$
(*c*) $3 - \sqrt{2}$ (*d*) $3 + \sqrt{2}$

Sol. (*b*) $2 - \sqrt{3}$

Putting A = 30° and B = 45°, so that

$$A + B = 30^\circ + 45^\circ = 75^\circ$$

We have

$$\cot(A + B) = \frac{\cot A \cot B - 1}{\cot B + \cot A}$$

$$\Rightarrow \quad \cot(30^\circ + 45^\circ) = \frac{\cot 30^\circ \cot 45^\circ - 1}{\cot 45^\circ + \cot 30^\circ}$$

$$\Rightarrow \quad \cot 75^\circ = \frac{\sqrt{3} \times 1 - 1}{1 + \sqrt{3}}$$

$$= \frac{\sqrt{3}-1}{\sqrt{3}+1}$$

$$= \frac{\left(\sqrt{3}-1\right)^2}{\left(\sqrt{3}\right)^2 - 1^2}$$

$$= \frac{3 + 1 - 2\sqrt{3}}{2}$$

$$= \frac{4 - 2\sqrt{3}}{2}$$

$$= 2 - \sqrt{3}$$

3. The value of $\frac{\tan 60^\circ - \tan 30^\circ}{1 + \tan 60^\circ \tan 30^\circ}$ is equal to

(*a*) $\frac{\sin 60^\circ}{\cos 30^\circ}$ (*b*) tan 45° tan 60°
(*c*) $\frac{\tan 30^\circ}{\sin 90^\circ}$ (*d*) sin 60° cosec 30°

Sol. (*c*) $\frac{\tan 30^\circ}{\sin 90^\circ}$

We have

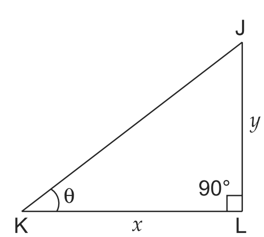
$$\frac{\tan 60^\circ - \tan 30^\circ}{1 + \tan 60^\circ \tan 30^\circ} = \frac{\sqrt{3} - \frac{1}{\sqrt{3}}}{1 + \sqrt{3} \times \frac{1}{\sqrt{3}}}$$

$$= \frac{3 - 1}{(1+1)\sqrt{3}}$$

$$= \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}}$$

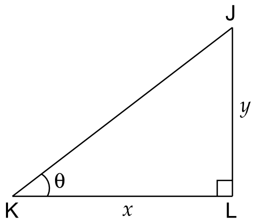
$$= \frac{\tan 30^\circ}{\sin 90^\circ}$$

4. In the adjoining figure, if $\angle JKL = \theta$, KL = x and JL = y, then



(*a*) $\cot\theta < \operatorname{cosec}\theta$
(*b*) $\cot\theta > \operatorname{cosec}\theta$
(*c*) $\cot\theta = \operatorname{cosec}\theta$
(*d*) $\cot\theta$ and $\operatorname{cosec}\theta$ are both undefined

Sol. (*a*) $\cot\theta < \operatorname{cosec}\theta$



In ΔJKL,

$\because \quad \angle L = 90°$

$\therefore$ By Pythagoras' theorem, we have

$$JK^2 = KL^2 + JL^2$$

$$\Rightarrow \quad JK^2 = x^2 + y^2$$

$$\therefore \quad JK = \sqrt{x^2+y^2}$$

Now, in ΔJKL,

$$\cot\theta = \frac{KL}{JL} = \frac{x}{y} \qquad \ldots(1)$$

and
$$\operatorname{cosec}\theta = \frac{JK}{JL} = \frac{\sqrt{x^2+y^2}}{y} \qquad \ldots(2)$$

Now,
$$x < \sqrt{x^2+y^2}$$

[$\because$ Length of the hypotenuse is always greater than the other two sides]

$$\Rightarrow \quad \frac{x}{y} < \frac{\sqrt{x^2+y^2}}{y}$$

$$\Rightarrow \quad \cot\theta < \operatorname{cosec}\theta \quad \text{[From (1) and (2)]}$$

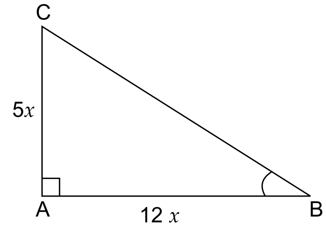
5. In ΔABC, $\angle A = 90°$ and $\cot B = \frac{12}{5}$. Then $\tan^2 B - \sin^2 B$ is equal to

(*a*) $\sin^2 B + \tan^2 B$ (*b*) $\frac{\sin^2 B}{\tan^2 B}$
(*c*) $\frac{\tan^2 B}{\sin^2 B}$ (*d*) $\sin^2 B \tan^2 B$

Sol. (*d*) $\sin^2 B \tan^2 B$

In ΔABC,

Let AB = $12x$ units and AC = $5x$ units, where x is a non-zero positive number.



Now, by Pythagoras' theorem, we have

$$BC^2 = AB^2 + AC^2 = (12x)^2 + (5x)^2 = 169x^2$$

Now,
$$\tan^2 B - \sin^2 B = \frac{AC^2}{AB^2} - \frac{AC^2}{BC^2}$$
$$= AC^2 \times \frac{BC^2 - AB^2}{AB^2 \times BC^2}$$
$$= AC^2 \times \frac{AC^2}{AB^2 \times BC^2}$$
[$\because BC^2 = AC^2 + AB^2$]
$$= \frac{AC^2}{AB^2} \times \frac{AC^2}{BC^2}$$
$$= \tan^2 B \times \sin^2 B$$
$$= \sin^2 B \tan^2 B$$

6. In a right triangle ABC, $\angle C = 90°$, $\tan A = \frac{1}{\sqrt{3}}$, $\tan B = \sqrt{3}$. Then the value of $\cos A \cos B - \sin A \sin B$ is equal to

(*a*) 1 (*b*) 0
(*c*) –1 (*d*) 2

Sol. (*b*) 0

We have

$$\tan A = \frac{1}{\sqrt{3}} = \tan 30°$$

$$\therefore \quad A = 30° \qquad \ldots(1)$$

$$\tan B = \sqrt{3} = \tan 60°$$

$$\therefore \quad B = 60° \qquad \ldots(2)$$

$\therefore$ $\cos A \cos B - \sin A \sin B$
$$= \cos 30° \cos 60° - \sin 30° \sin 60°$$
$$= \frac{\sqrt{3}}{2} \times \frac{1}{2} - \frac{1}{2} \times \frac{\sqrt{3}}{2}$$
$$= \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = 0$$

7. If $45° < \theta < 90°$, then

(*a*) $\cos\theta > \sin\theta$ (*b*) $\sin\theta > \tan\theta$
(*c*) $\sin\theta > \cos\theta$ (*d*) $\sin\theta = \tan\theta$

Sol. (*c*) $\sin\theta > \cos\theta$

We see that $\cos 45° = \frac{1}{\sqrt{2}}$, $\cos 60° = \frac{1}{2}$ and $\cos 90° = 0$ and $\sin 45° = \frac{1}{\sqrt{2}}$, $\sin 60° = \frac{\sqrt{3}}{2}$ and $\sin 90° = 1$.

So, when θ increases from 45° to 90°, then $\cos\theta$ decreases from $\frac{1}{\sqrt{2}}$ to 0, but $\sin\theta$ increases from $\frac{1}{\sqrt{2}}$ to 1. Hence, $\sin\theta > \cos\theta$ when $45° < \theta < 90°$.

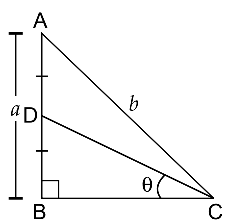
8. ABC is a right triangle such that $\angle ABC = 90°$, AB = a units, D is the mid-point of AB, AC = b units and $\angle DCB = \theta$. Then the value of $\tan\theta$ is equal to

(*a*) $\dfrac{a}{\sqrt{4b^2-3a^2}}$ (*b*) $\dfrac{2\sqrt{b^2-a^2}}{\sqrt{4b^2-3a^2}}$

(*c*) $\dfrac{\sqrt{4b^2-3a^2}}{2\sqrt{b^2-a^2}}$ (*d*) $\dfrac{a}{2\sqrt{b^2-a^2}}$

Sol. (*d*) $\dfrac{a}{2\sqrt{b^2-a^2}}$

In ΔABC, $\angle B = 90°$, D is the mid-point of AB, AB = a units, AC = b units and $\angle DCB = \theta$.



From right-angled triangle DBC,

$$DB = \frac{1}{2} AB = \frac{a}{2} \quad \ldots(1)$$

$\therefore$ In ΔABC, by Pythagoras' theorem, we have

$$AC^2 = AB^2 + BC^2$$

$$\Rightarrow \quad b^2 = a^2 + BC^2$$

$$\therefore \quad BC = \sqrt{b^2 - a^2} \quad \ldots(2)$$

Now, in ΔDBC, we have

$$\tan\theta = \tan \angle DCB = \frac{BD}{BC} = \frac{\frac{a}{2}}{\sqrt{b^2-a^2}}$$

[From (1) and (2)]

$$= \frac{a}{2\sqrt{b^2-a^2}}$$

$\therefore$ The required value of $\tan\theta$ is $\dfrac{a}{2\sqrt{b^2-a^2}}$.

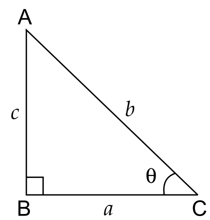
9. If $5\sin\theta - 3\cos\theta = \tan\theta\cos\theta$, then the value of $\tan\theta$ is equal to

(*a*) $\frac{3}{4}$ (*b*) $\frac{4}{3}$

(*c*) $\frac{5}{3}$ (*d*) $\frac{3}{5}$

Sol. (*a*) $\frac{3}{4}$

Let ABC be a right-angled triangle, where $\angle B = 90°$.



Let $\angle ACB = \theta$, BC = a units, AC = b units and AB = c units

Now, from the given equation

$5\sin\theta - 3\cos\theta = \tan\theta\cos\theta$, we have

$$5 \times \frac{c}{b} - 3 \times \frac{a}{b} = \frac{c}{a} \times \frac{a}{b}$$

$$\Rightarrow \quad \frac{5c}{b} - \frac{3a}{b} = \frac{c}{b}$$

$$\Rightarrow \quad 5c - 3a = c$$

$$\Rightarrow \quad 4c = 3a$$

$$\Rightarrow \quad \frac{c}{a} = \frac{3}{4}$$

$$\Rightarrow \quad \tan\theta = \frac{3}{4}$$

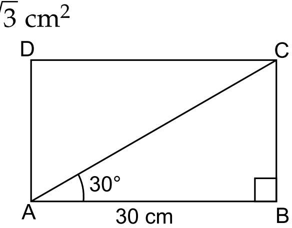
$\therefore$ The required value of $\tan\theta$ is $\frac{3}{4}$.

10. In a rectangle ABCD, AB = 30 cm and $\angle BAC = 30°$ where AC is a diagonal of the rectangle. Then the area of the rectangle ABCD is

(*a*) 300 cm^2 (*b*) $300\sqrt{3}$ cm^2

(*c*) $310\sqrt{3}$ cm^2 (*d*) $300\sqrt{2}$ cm^2

Sol. (*b*) $300\sqrt{3}$ cm^2



In ΔABC, we have

$$\tan 30° = \tan \angle BAC = \frac{BC}{AB}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{BC}{30}$$

$$\Rightarrow \quad BC = \frac{30}{\sqrt{3}} = \frac{30\sqrt{3}}{3} = 10\sqrt{3}$$

$\therefore$ Area of rectangle ABCD

$$= AB \times BC = 30 \times 10\sqrt{3} \text{ cm}^2 = 300\sqrt{3} \text{ cm}^2$$

Value-based Question (Optional)

(Page 155)

1. Two poor young brothers in a village were living separately with their respective families in their paternal property after the death of their parents. They had a common orchard where four vertical fruit trees of different heights were present. Their parents told them before their death that their younger son will be the owner of those trees whose heights were less than or equal to 10 m and the elder son will be the owner of trees whose heights were more than 10 m. But they could not measure the heights of the trees due to the fact that they were not climbable. So, there was frequent quarrel between the two brothers about the ownership of the fruit trees. There was a surveyor in the village who had an instrument known as Sextant to measure the angle of elevation of the top of any object (i.e. the angle made by an object) at a known point. He agreed to solve the problem of measuring the heights of the four trees with the help of his Sextant instrument. He measured the angles of elevation of the top of the trees from their feet say A, B, C and D, from four suitable different points in the orchard, say P, Q, R and S respectively. These angles were 30°, 45°, 60° and 30° respectively. If PA = 12 m, QB = 11 m, RC = 5 m and SD = 24 m,

   (*a*) Find the trees which would be owned by the younger brother and the elder brother. Find their heights also.

   (*b*) Which value of the surveyor is depicted in this problem?

1. (*a*)

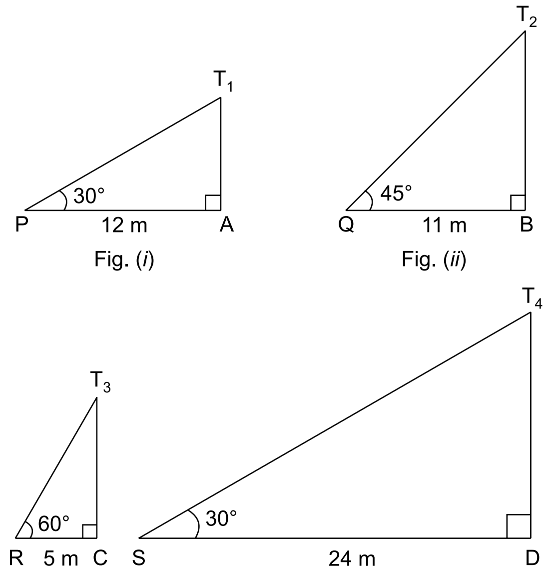


Fig. (*i*) Fig. (*ii*) Fig. (*iii*) Fig. (*iv*)

Let T_1, T_2, T_3 and T_4 be the trees at A, B, C and D respectively. $\angle T_1AP = \angle T_2BQ = \angle T_3CR = \angle T_4DS = 90°$

$\therefore$ From Fig. (*i*) to (*iv*), we have

$$\tan 30° = \frac{AT_1}{PA} = \frac{AT_1}{12}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{AT_1}{12}$$

$$\Rightarrow \quad AT_1 = \frac{12}{\sqrt{3}} = 4\sqrt{3} = 6.92 \text{ m (approx.)}$$

$$\tan 45° = \frac{BT_2}{QB} = \frac{BT_2}{11}$$

$$\Rightarrow \quad 1 = \frac{BT_2}{11}$$

$$\Rightarrow \quad BT_2 = 11 \text{ m}$$

$$\tan 60° = \frac{CT_3}{RC} = \frac{CT_3}{5}$$

$$\Rightarrow \quad \sqrt{3} = \frac{CT_3}{5}$$

$$\Rightarrow \quad CT_3 = 5\sqrt{3} = 8.65 \text{ m (approx.)}$$

$$\tan 30° = \frac{DT_4}{SD} = \frac{DT_4}{24}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{DT_4}{24}$$

$$\Rightarrow \quad DT_4 = \frac{24}{\sqrt{3}}$$

$$= \frac{24\sqrt{3}}{3} = 8\sqrt{3}$$

$$= 13.85 \text{ m (approx.)}$$

Hence, trees with feet A and C of heights 6.92 m and 8.65 m (approx.) respectively will be owned by the younger brother and those with feet B and D of heights 11 m and 13.85 m (approx.) respectively will be owned by the elder brother.

(*b*) Empathy, sympathy and kindness of the surveyor.

(II) Trigonometric Identities

Milestone

(Page 160)

Multiple-Choice Questions

1. $\sin\theta\,(\text{cosec}\,\theta - \sin\theta)$ is equal to

(*a*) $\sin^2\theta$ (*b*) $\sin\theta\cos\theta$

(*c*) $\cos^2\theta$ (*d*) $\cos\theta - \sin^2\theta$

Sol. (*c*) $\cos^2\theta$

We have

$$\sin\theta\,(\text{cosec}\,\theta - \sin\theta) = \sin\theta \times \frac{1}{\sin\theta} - \sin^2\theta$$

$$= 1 - \sin^2\theta = \cos^2\theta$$

2. The value of $\sec A - \frac{\tan^2 A}{1+\sec A}$ is equal to

(*a*) 1 (*b*) 2

(*c*) $\sec^2 A$ (*d*) $\tan^2 A$

Sol. (*a*) 1

We have

$$\sec A - \frac{\tan^2 A}{1+\sec A} = \frac{1}{\cos A} - \frac{\frac{\sin^2 A}{\cos^2 A}}{1+\frac{1}{\cos A}}$$

$$= \frac{1}{\cos A} - \frac{\sin^2 A}{\cos^2 A} \times \frac{\cos A}{1+\cos A}$$

$$= \frac{1}{\cos A} - \frac{\sin^2 A}{\cos A(1+\cos A)}$$

$$= \frac{1+\cos A - \sin^2 A}{\cos A(1+\cos A)}$$

$$= \frac{(1-\sin^2 A) + \cos A}{\cos A(1+\cos A)}$$

$$= \frac{\cos^2 A + \cos A}{\cos^2 A + \cos A} = 1$$

Very Short Answer Type Questions

3. What is the constant value of the expression $2\cos^2\theta\,(1+\tan^2\theta)$?

Sol. We have

$$2\cos^2\theta\,(1+\tan^2\theta) = 2\cos^2\theta \times \sec^2\theta$$

$$= 2 \times \cos^2\theta \times \frac{1}{\cos^2\theta} = 2$$

which is the required value.

4. What is the result of elimination of θ between the equations $x = a\cos\theta$ and $y = a\sin\theta$?

Sol. We have

$$x^2 + y^2 = a^2\cos^2\theta + a^2\sin^2\theta$$

$$= a^2(\cos^2\theta + \sin^2\theta)$$

$$= a^2 \times 1 = a^2$$

$\therefore$ The required result of elimination of θ is $x^2 + y^2 = a^2$.

Short Answer Type-I Questions

5. Prove that $\frac{\sec^2\theta - \sin^2\theta}{\tan^2\theta} = \text{cosec}^2\theta - \cos^2\theta$ $(0° < \theta < 90°)$.

Sol. We have

$$\text{LHS} = \frac{\sec^2\theta - \sin^2\theta}{\tan^2\theta}$$

$$= \frac{\frac{1}{\cos^2\theta} - \sin^2\theta}{\frac{\sin^2\theta}{\cos^2\theta}}$$

$$= \frac{1-\cos^2\theta\sin^2\theta}{\cos^2\theta} \times \frac{\cos^2\theta}{\sin^2\theta}$$

$$= \frac{1-\cos^2\theta\sin^2\theta}{\sin^2\theta}$$

$$= \frac{1}{\sin^2\theta} - \cos^2\theta$$

$$= \text{cosec}^2\theta - \cos^2\theta = \text{RHS}$$

Hence, proved.

6. Prove that $\frac{\cos A}{1-\tan A} - \frac{\sin^2 A}{\cos A - \sin A} = \cos A + \sin A$. $(A \neq 45°)$

Sol. $$\text{LHS} = \frac{\cos A}{1 - \tan A} - \frac{\sin^2 A}{\cos A - \sin A}$$
$$= \frac{\cos A}{1 - \frac{\sin A}{\cos A}} - \frac{\sin^2 A}{\cos A - \sin A}$$
$$= \frac{\cos^2 A}{\cos A - \sin A} - \frac{\sin^2 A}{\cos A - \sin A}$$
$$= \frac{(\cos A + \sin A)(\cos A - \sin A)}{\cos A - \sin A}$$
$$= \cos A + \sin A \qquad [\because A \neq 45°]$$
$$= \text{RHS}$$
Hence, proved.

Short Answer Type-II Questions

7. If $x \sin^3 \theta + y \cos^3 \theta = \sin \theta \cos \theta$, and $x \sin \theta = y \cos \theta$, show that $x^2 + y^2 = 1$.

Sol. We have $x \sin \theta = y \cos \theta$
$$\Rightarrow \quad \frac{\sin\theta}{y} = \frac{\cos\theta}{x}$$
Squaring both sides
$$\Rightarrow \quad \frac{\sin^2\theta}{y^2} = \frac{\cos^2\theta}{x^2}$$
$$= \frac{\sin^2\theta + \cos^2\theta}{x^2 + y^2}$$
$$= \frac{1}{x^2 + y^2}$$
$$\therefore \quad \sin^2 \theta = \frac{y^2}{x^2 + y^2} \qquad \ldots(1)$$
and
$$\cos^2 \theta = \frac{x^2}{x^2 + y^2} \qquad \ldots(2)$$
Now, from the first given equation, we have
$$\frac{x\sin^3\theta}{\sin\theta\cos\theta} + \frac{y\cos^3\theta}{\sin\theta\cos\theta} = 1$$
$$\Rightarrow \quad \frac{x\sin^2\theta}{\cos\theta} + \frac{y\cos^2\theta}{\sin\theta} = 1$$
$$\Rightarrow \quad x \times \frac{y^2}{x^2 + y^2} \times \frac{\sqrt{x^2 + y^2}}{x} + y \times \frac{x^2}{x^2 + y^2} \times \frac{\sqrt{x^2 + y^2}}{y} = 1$$
[From (1) and (2)]
$$\Rightarrow \quad \frac{y^2}{\sqrt{x^2 + y^2}} + \frac{x^2}{\sqrt{x^2 + y^2}} = 1$$
$$\Rightarrow \quad \frac{x^2 + y^2}{\sqrt{x^2 + y^2}} = 1$$
$$\Rightarrow \quad \sqrt{x^2 + y^2} = 1$$
$$\Rightarrow \quad x^2 + y^2 = 1$$
Hence, proved.

8. If $\cot \theta + \tan \theta = m$ and $\operatorname{cosec} \theta - \sin \theta = n$, show that $(m^2 n)^{\frac{2}{3}} - (mn^2)^{\frac{2}{3}} = 1$.

Sol. We have
$$\cot \theta + \tan \theta = m$$
$$\Rightarrow \quad \frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta} = m$$
$$\Rightarrow \quad \frac{\cos^2\theta + \sin^2\theta}{\sin\theta\cos\theta} = m$$
$$\Rightarrow \quad m = \frac{1}{\sin\theta\cos\theta} \qquad \ldots(1)$$
Again, $\operatorname{cosec} \theta - \sin \theta = n$
$$\Rightarrow \quad \frac{1}{\sin\theta} - \sin\theta = n$$
$$\Rightarrow \quad \frac{1 - \sin^2\theta}{\sin\theta} = n$$
$$\Rightarrow \quad n = \frac{\cos^2\theta}{\sin\theta} \qquad \ldots(2)$$
Now, LHS $= (m^2 n)^{\frac{2}{3}} - (mn^2)^{\frac{2}{3}}$
$$= m^{\frac{4}{3}} n^{\frac{2}{3}} - m^{\frac{2}{3}} n^{\frac{4}{3}}$$
$$= \frac{1}{(\sin\theta)^{\frac{4}{3}} (\cos\theta)^{\frac{4}{3}}} \times \frac{(\cos\theta)^{\frac{4}{3}}}{(\sin\theta)^{\frac{2}{3}}} - \frac{1}{(\sin\theta)^{\frac{2}{3}} (\cos\theta)^{\frac{2}{3}}} \times \frac{(\cos\theta)^{\frac{8}{3}}}{(\sin\theta)^{\frac{4}{3}}}$$
$$= \frac{1}{\sin^2\theta} - \frac{\cos^2\theta}{\sin^2\theta}$$
$$= \frac{1 - \cos^2\theta}{\sin^2\theta}$$
$$= \frac{\sin^2\theta}{\sin^2\theta}$$
$$= 1 = \text{RHS}$$
Hence, proved.

Long Answer Type Questions

9. If $\sec \theta = x + \frac{1}{4x}$, prove that

$\sec\theta + \tan\theta = 2x$ or $\frac{1}{2x}$. **[CBSE 2001]**

Sol. We have $\sec\theta = x + \frac{1}{4x}$ ...(1)

Squaring both sides, we get

$$\therefore \quad \sec^2\theta = \left(x + \frac{1}{4x}\right)^2$$

$$= x^2 + \frac{1}{16x^2} + 2x \times \frac{1}{4x}$$

$$= x^2 + \frac{1}{16x^2} + \frac{1}{2}$$

$$\therefore \quad 1 + \tan^2\theta = x^2 + \frac{1}{16x^2} + \frac{1}{2}$$

$$\Rightarrow \quad \tan^2\theta = x^2 + \frac{1}{16x^2} - \frac{1}{2}$$

$$= \left(x - \frac{1}{4x}\right)^2$$

$$\therefore \quad \tan\theta = \pm\left(x - \frac{1}{4x}\right) \quad ...(2)$$

$\therefore$ Adding (1) and (2), we get

$$\sec\theta + \tan\theta = x + \frac{1}{4x} \pm \left(x - \frac{1}{4x}\right)$$

$$= 2x \text{ or } \frac{1}{2x}$$

Hence, proved.

10. Prove that

$$\frac{\cos x}{\sin x + \cos y} + \frac{\cos y}{\sin y - \cos x} = \frac{\cos x}{\sin x - \cos y} + \frac{\cos y}{\sin y + \cos x}$$

Sol. We have

$$\frac{\cos x}{\sin x + \cos y} - \frac{\cos x}{\sin x - \cos y}$$

$$= \frac{\cos x(\sin x - \cos y) - \cos x(\sin x + \cos y)}{\sin^2 x - \cos^2 y}$$

$$= \frac{-2\cos x \cos y}{\sin^2 x - \cos^2 y} \quad ...(1)$$

Also,

$$\frac{\cos y}{\sin y + \cos x} - \frac{\cos y}{\sin y - \cos x}$$

$$= \frac{\cos y(\sin y - \cos x) - \cos y(\sin y + \cos x)}{\sin^2 y - \cos^2 x}$$

$$= \frac{-2\cos x \cos y}{1 - \cos^2 y - 1 + \sin^2 x}$$

$$= \frac{-2\cos x \cos y}{\sin^2 x - \cos^2 y} \quad ...(2)$$

From (1) and (2), we have

$$\frac{\cos x}{\sin x + \cos y} - \frac{\cos x}{\sin x - \cos y} = \frac{\cos y}{\sin y + \cos x} - \frac{\cos y}{\sin y - \cos x}$$

$$\Rightarrow \quad \frac{\cos x}{\sin x + \cos y} + \frac{\cos y}{\sin y - \cos x} = \frac{\cos x}{\sin x - \cos y} + \frac{\cos y}{\sin y + \cos x}$$

$\therefore$ LHS = RHS

Hence, proved.

Higher Order Thinking Skills (HOTS) Questions

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1. Prove that

$$1 + \tan\theta + \sec\theta = \frac{2}{1 + \cot\theta - \operatorname{cosec}\theta}$$

Sol. We have

$(1 + \tan\theta + \sec\theta)(1 + \cot\theta - \operatorname{cosec}\theta)$

$$= \left(1 + \frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta}\right)\left(1 + \frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta}\right)$$

$$= \frac{\cos\theta + \sin\theta + 1}{\cos\theta} \times \frac{\sin\theta + \cos\theta - 1}{\sin\theta}$$

$$= \frac{(\sin\theta + \cos\theta)^2 - 1^2}{\cos\theta\sin\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta - 1}{\cos\theta\sin\theta}$$

$$= \frac{1 - 1 + 2\sin\theta\cos\theta}{\cos\theta\sin\theta}$$

$$= 2$$

Hence, we have

$$1 + \tan\theta + \sec\theta = \frac{2}{1 + \cot\theta - \operatorname{cosec}\theta}$$

Hence, proved.

2. If x and y are two unequal real numbers, show that the equations

(*a*) $\sin^2\theta = \frac{(x+y)^2}{4xy}$ and

(*b*) $\cos\theta = x + \frac{1}{x}$ are both impossible.

Sol. (*a*) We have

$$(x+y)^2 - 4xy = (x-y)^2 > 0, \text{ since } x \neq y$$

$$\therefore \quad (x+y)^2 > 4xy$$

$$\Rightarrow \quad \frac{(x+y)^2}{4xy} > 1$$

$$\Rightarrow \quad \sin^2\theta > 1$$

which is impossible, since we know that $0 \leq \sin^2\theta \leq 1$.

(*b*) We have

$$x + \frac{1}{x} = \frac{x^2+1}{x}$$

Now, $$x^2 + 1 - x = x^2 - 2 \times \frac{1}{2} \times x + \frac{1}{4} + \frac{3}{4}$$

$$= \left(x - \frac{1}{2}\right)^2 + \frac{3}{4} > 0$$

$$\therefore \quad x^2 + 1 > x$$

$$\Rightarrow \quad \frac{x^2+1}{x} > 1$$

$$\Rightarrow \quad x + \frac{1}{x} > 1$$

$$\Rightarrow \quad \cos\theta > 1$$

which is also impossible, since we know that $-1 \leq \cos\theta \leq 1$.

Hence, both (*a*) and (*b*) are impossible.

3. If $\dfrac{\cos^4 x}{\cos^2 y} + \dfrac{\sin^4 x}{\sin^2 y} = 1$, then show that

$$\frac{\cos^4 y}{\cos^2 x} + \frac{\sin^4 y}{\sin^2 x} = 1.$$

Sol. From the given equation, we have

$$\cos^4 x \sin^2 y + \sin^4 x \cos^2 y = \cos^2 y \sin^2 y$$

$$\Rightarrow \quad \cos^4 x (1 - \cos^2 y) + (1 - \cos^2 x)^2 \cos^2 y - \cos^2 y (1 - \cos^2 y) = 0$$

$$\Rightarrow \quad \cos^4 x - \cos^4 x \cos^2 y + \cos^2 y + \cos^4 x \cos^2 y - 2\cos^2 x \cos^2 y - \cos^2 y + \cos^4 y = 0$$

$$\Rightarrow \quad \cos^4 x + \cos^4 y - 2\cos^2 x \cos^2 y = 0$$

$$\Rightarrow \quad (\cos^2 x - \cos^2 y)^2 = 0$$

$$\Rightarrow \quad \cos^2 x = \cos^2 y \qquad \ldots(1)$$

$$\Rightarrow \quad 1 - \sin^2 x = 1 - \sin^2 y$$

$$\Rightarrow \quad \sin^2 x = \sin^2 y \qquad \ldots(2)$$

Now, $$\text{LHS} = \frac{\cos^4 y}{\cos^2 x} + \frac{\sin^4 y}{\sin^2 x}$$

$$= \frac{\cos^4 y}{\cos^2 y} + \frac{\sin^4 y}{\sin^2 y}$$

$$= \cos^2 y + \sin^2 y$$

$$= 1 = \text{RHS}$$

Hence, proved.

4. If $\sec x = \sec y \sec z + \tan y \tan z$, then show that $\sec y = \sec z \sec x \pm \tan z \tan x$.

Sol. We have

$$(\sec x - \sec y \sec z)^2 - (\sec y - \sec z \sec x)^2$$

$$= \sec^2 x + \sec^2 y \sec^2 z - 2\sec x \sec y \sec z - (\sec^2 y + \sec^2 z \sec^2 x - 2\sec x \sec y \sec z)$$

$$= \sec^2 x + \sec^2 y \sec^2 z - \sec^2 y - \sec^2 z \sec^2 x$$

$$= -\sec^2 x (\sec^2 z - 1) + \sec^2 y (\sec^2 z - 1)$$

$$= -\sec^2 x \tan^2 z + \sec^2 y \tan^2 z$$

$$= -\tan^2 z (1 + \tan^2 x) + \tan^2 z (1 + \tan^2 y)$$

$$= \tan^2 z \tan^2 y - \tan^2 z \tan^2 x \qquad \ldots(1)$$

Now, given that $\sec x = \sec y \sec z + \tan y \tan z$

$$\therefore \quad (\sec x - \sec y \sec z)^2 = \tan^2 y \tan^2 z \qquad \ldots(2)$$

From (1) and (2), we have

$$\tan^2 y \tan^2 z - (\sec y - \sec z \sec x)^2 = \tan^2 z \tan^2 y - \tan^2 z \tan^2 x$$

$$\Rightarrow \quad (\sec y - \sec z \sec x)^2 = \tan^2 z \tan^2 x$$

$$\Rightarrow \quad \sec y - \sec z \sec x = \pm \tan z \tan x$$

$$\Rightarrow \quad \sec y = \sec z \sec x \pm \tan z \tan x$$

Hence, proved.

Self-Assessment

(Page 161)

Multiple-Choice Questions

1. The value of $5\tan^2\theta - 5\sec^2\theta$ is

(*a*) 1 (*b*) 5
(*c*) 0 (*d*) –5 **[CBSE SP 2011]**

Sol. (*d*) –5

We have

$$5\tan^2\theta - 5\sec^2\theta = 5\tan^2\theta - 5(1 + \tan^2\theta)$$

$$= -5$$

2. If $0° < \theta < 90°$, then $\sqrt{\dfrac{1+\cos\theta}{1-\cos\theta}}$ is equal to

(*a*) $\operatorname{cosec}^2\theta + \cot^2\theta$ (*b*) $\operatorname{cosec}\theta + \cot\theta$
(*c*) $\cot\theta - \operatorname{cosec}\theta$ (*d*) $\operatorname{cosec}^2\theta - \cot^2\theta$

Sol. (*b*) $\operatorname{cosec}\theta + \cot\theta$

We have $$\sqrt{\frac{1+\cos\theta}{1-\cos\theta}} = \frac{\sqrt{1+\cos\theta}}{\sqrt{1-\cos\theta}} \times \frac{\sqrt{1+\cos\theta}}{\sqrt{1+\cos\theta}}$$

$$= \frac{\sqrt{(1+\cos\theta)^2}}{\sqrt{1-\cos^2\theta}}$$

$$= \frac{1+\cos\theta}{\sqrt{\sin^2\theta}}$$

$$= \frac{1+\cos\theta}{\sin\theta}$$

$$= \frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}$$

$$= \text{cosec}\,\theta + \cot\theta$$

Fill in the Blanks

3. If $\sin\theta = \frac{p}{q}$, then the value of $\tan\theta + \sec\theta$ is $\sqrt{\frac{p+q}{q-p}}$.

Sol. $$\tan\theta + \sec\theta = \frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta}$$

$$= \frac{\sin\theta + 1}{\cos\theta} = \frac{\sin\theta+1}{\sqrt{1-\sin^2\theta}}$$

$$= \frac{\frac{p}{q}+1}{\sqrt{1-\frac{p^2}{q^2}}} \qquad \left[\because \sin\theta = \frac{p}{q}, \text{ given}\right]$$

$$= \frac{p+q}{\sqrt{q^2-p^2}} = \frac{\sqrt{p+q}\,\sqrt{p+q}}{\sqrt{q-p}\,\sqrt{q+p}}$$

$$= \frac{\sqrt{p+q}}{\sqrt{q-p}}$$

4. If $x = 3\sec^2\theta - 1$ and $y = 3\tan^2\theta - 2$, then $x - y$ is equal to **4**.

Sol. $$x - y = 3\sec^2\theta - 1 - 3\tan^2\theta + 2$$

$$= 3(\sec^2\theta - \tan^2\theta) + 1$$

$$= 3(1) + 1$$

$$= 4$$

5. If $x = m\sin\theta$ and $y = n\cos\theta$, then the value of $n^2x^2 + m^2y^2$ is $\boldsymbol{m^2n^2}$.

Sol. $$n^2x^2 + m^2y^2 = n^2(m\sin\theta)^2 + m^2(n\cos\theta)^2$$

$$= n^2m^2\sin^2\theta + m^2n^2\cos^2\theta$$

$$= m^2n^2(\sin^2\theta + \cos^2\theta)$$

$$= m^2n^2$$

6. If $\cos\theta + \cos^2\theta = 1$, then the value of $\sin^2\theta + \sin^4\theta$ is **1**.

Sol. $$\cos\theta + \cos^2\theta = 1$$

$$\cos\theta = 1 - \cos^2\theta$$

$$\cos\theta = \sin^2\theta$$

$$\sin^2\theta + \sin^4\theta = \sin^2\theta(1 + \sin^2\theta)$$

$$= \cos\theta(1 + \cos\theta)$$

$$= \cos\theta + \cos^2\theta$$

$$= 1 \quad [\because \cos\theta + \cos^2\theta = 1, \text{ given}]$$

Very Short Answer Type Questions

7. What is the value of $\frac{1-\tan^2\theta}{1+\tan^2\theta}$ in terms of $\sin^2\theta$ and $\cos^2\theta$?

Sol. We have

$$\frac{1-\tan^2\theta}{1+\tan^2\theta} = \frac{1-\frac{\sin^2\theta}{\cos^2\theta}}{\sec^2\theta}$$

$$= \frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta \times \frac{1}{\cos^2\theta}}$$

$$= \cos^2\theta - \sin^2\theta$$

which is the required value.

8. If $\sin A + \sin^2 A = 1$, then show that $\cos^2 A + \cos^4 A = 1$.

Sol. We have

$$\sin A = 1 - \sin^2 A = \cos^2 A \qquad \ldots(1)$$

$$\therefore \quad \cos^2 A + \cos^4 A = \cos^2 A + \sin^2 A \quad [\text{From (1)}]$$

$$= 1$$

Hence, proved.

Short Answer Type-I Questions

9. Prove that

$$\cos^2\theta + \frac{1}{1+\cot^2\theta} = 1 \qquad (0° < \theta < 90°)$$

Sol. We have

$$\text{LHS} = \cos^2\theta + \frac{1}{1+\cot^2\theta}$$

$$= \cos^2\theta + \frac{1}{\text{cosec}^2\theta}$$

$$= \cos^2\theta + \sin^2\theta$$

$$= 1 = \text{RHS}$$

Hence, proved.

10. Prove that

$$\sin^2\theta(1 + \cot^2\theta) = 1 \qquad (0° < \theta < 90°)$$

Sol. $$\text{LHS} = \sin^2\theta(1 + \cot^2\theta)$$

$$= \sin^2\theta \times \text{cosec}^2\theta$$

$$= \sin^2\theta \times \frac{1}{\sin^2\theta}$$

$$= 1 = \text{RHS}$$

Hence, proved.

Short Answer Type-II Questions

11. Prove that

$(1 + \cot\theta - \operatorname{cosec}\theta)(1 + \tan\theta + \sec\theta) = 2$

[CBSE 2000]

Sol. $\text{LHS} = \left(1 + \frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta}\right)\left(1 + \frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta}\right)$

$$= \frac{\sin\theta + \cos\theta - 1}{\sin\theta} \times \frac{\cos\theta + \sin\theta + 1}{\cos\theta}$$

$$= \frac{(\sin\theta + \cos\theta)^2 - 1^2}{\sin\theta\cos\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta - 1 + 2\sin\theta\cos\theta}{\sin\theta\cos\theta}$$

$$= \frac{1 - 1 + 2\sin\theta\cos\theta}{\sin\theta\cos\theta}$$

$$= \frac{2\sin\theta\cos\theta}{\sin\theta\cos\theta}$$

$$= 2$$

Hence, proved.

12. If $\operatorname{cosec}\theta = \frac{\sqrt{10}}{3}$, find the value of

$\frac{1}{\sec\theta + \tan\theta} - \frac{1}{\cos\theta}$ analytically.

Sol. We have $\quad \operatorname{cosec}\theta = \frac{\sqrt{10}}{3}$

$$\therefore \quad \sin\theta = \frac{3}{\sqrt{10}} \qquad \ldots(1)$$

$$\therefore \quad \cos\theta = \sqrt{1 - \sin^2\theta}$$

$$= \sqrt{1 - \frac{9}{10}}$$

$$= \frac{1}{\sqrt{10}} \qquad \ldots(2)$$

$$\therefore \quad \frac{1}{\sec\theta + \tan\theta} - \frac{1}{\cos\theta} = \frac{1}{\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}} - \frac{1}{\cos\theta}$$

$$= \frac{\cos\theta}{1 + \sin\theta} - \frac{1}{\cos\theta}$$

$$= \frac{\frac{1}{\sqrt{10}}}{1 + \frac{3}{\sqrt{10}}} - \sqrt{10}$$

[From (1) and (2)]

$$= \frac{1}{\sqrt{10} + 3} - \sqrt{10}$$

$$= \frac{\sqrt{10} - 3}{10 - 9} - \sqrt{10}$$

$$= \sqrt{10} - 3 - \sqrt{10}$$

$$= -3$$

which is the required value.

Long Answer Type Questions

13. If $\sin^2\theta - \cos^2\theta = \cot^2\phi$, prove that

(*a*) $\sin^2\phi - \cos^2\phi = \cot^2\theta$ and

(*b*) $\sqrt{2}\sin\theta\sin\phi = 1$

Sol. (*a*) We have

$$\sin^2\theta - \cos^2\theta = \cot^2\phi$$

$$\Rightarrow \quad 1 - 2\cos^2\theta = \cot^2\phi$$

$$\Rightarrow \quad 2\cos^2\theta = 1 - \cot^2\phi$$

$$\Rightarrow \quad \frac{2}{\sec^2\theta} = 1 - \cot^2\phi$$

$$\Rightarrow \quad \sec^2\theta = \frac{2}{1 - \cot^2\phi}$$

$$\Rightarrow \quad 1 + \tan^2\theta = \frac{2}{1 - \cot^2\phi}$$

$$\Rightarrow \quad \tan^2\theta = \frac{2}{1 - \cot^2\phi} - 1$$

$$= \frac{2 - 1 + \cot^2\phi}{1 - \cot^2\phi}$$

$$= \frac{1 + \cot^2\phi}{1 - \cot^2\phi}$$

$$\Rightarrow \quad \cot^2\theta = \frac{1 - \cot^2\phi}{1 + \cot^2\phi}$$

$$= \frac{1 - \frac{\cos^2\phi}{\sin^2\phi}}{1 + \frac{\cos^2\phi}{\sin^2\phi}}$$

$$= \frac{\sin^2\phi - \cos^2\phi}{\sin^2\phi + \cos^2\phi}$$

$$= \sin^2\phi - \cos^2\phi$$

Hence, proved.

(*b*) From the given equation, we have

$$\sin^2\theta - \cos^2\theta = \cot^2\phi = \frac{\cos^2\phi}{\sin^2\phi}$$

$$\Rightarrow \quad \sin^2\theta - 1 + \sin^2\theta = \frac{1 - \sin^2\phi}{\sin^2\phi}$$

$$\Rightarrow \quad (2\sin^2\theta - 1)\sin^2\phi = 1 - \sin^2\phi$$

$$\Rightarrow \quad 2\sin^2\theta\sin^2\phi = 1$$

$$\Rightarrow \quad \sqrt{2}\sin\theta\sin\phi = 1$$

Hence, proved.

14. If $\tan^2\theta = 1 - e^2$, show that

$\sec\theta + \tan^3\theta \operatorname{cosec}\theta = (2-e^2)^{\frac{3}{2}}$.

Sol. We have $\tan^2\theta = 1 - e^2$

$\Rightarrow \quad 1 + \tan^2\theta = 2 - e^2$

$\Rightarrow \quad \sec^2\theta = 2 - e^2$

$\Rightarrow \quad \sec\theta = \left(2-e^2\right)^{\frac{1}{2}}$

$\Rightarrow \quad \sec^3\theta = \left(2-e^2\right)^{\frac{3}{2}} \qquad \ldots(1)$

Now, $\sec\theta + \tan^3\theta \operatorname{cosec}\theta$

$$= \frac{1}{\cos\theta} + \frac{\sin^3\theta}{\cos^3\theta}\cdot\frac{1}{\sin\theta}$$

$$= \frac{1}{\cos\theta} + \frac{\sin^2\theta}{\cos^3\theta}$$

$$= \frac{\cos^2\theta + \sin^2\theta}{\cos^3\theta}$$

$$= \frac{1}{\cos^3\theta}$$

$$= \sec^3\theta$$

$$= \left(2-e^2\right)^{\frac{3}{2}} \qquad \text{[From (1)]}$$

Hence, proved.

Let's Compete

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Multiple-Choice Questions

1. If $0° < \theta < 90°$, then $\tan^2\theta + \cot^2\theta + 2$ is equal to

(*a*) $\dfrac{\sec^2\theta}{\operatorname{cosec}^2\theta}$ (*b*) $\dfrac{\operatorname{cosec}^2\theta}{\sec^2\theta}$

(*c*) $\sec^2\theta - \operatorname{cosec}^2\theta$ (*d*) $\sec^2\theta + \operatorname{cosec}^2\theta$

Sol. (*d*) $\sec^2\theta + \operatorname{cosec}^2\theta$

We have

$\tan^2\theta + \cot^2\theta + 2$

$$= \frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} + 2$$

$$= \frac{\sin^4\theta + \cos^4\theta + 2\sin^2\theta\cos^2\theta}{\sin^2\theta\cos^2\theta}$$

$$= \frac{\left(\sin^2\theta + \cos^2\theta\right)^2}{\sin^2\theta\cos^2\theta}$$

$$= \frac{1}{\sin^2\theta\cos^2\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\sin^2\theta\cos^2\theta}$$

$$= \frac{1}{\cos^2\theta} + \frac{1}{\sin^2\theta}$$

$$= \sec^2\theta + \operatorname{cosec}^2\theta$$

2. If $\sin A + \sin^2 A = 1$, then the value of $\cos^{12} A + 3\cos^{10} A + 3\cos^8 A + \cos^6 A - 1$ is equal to

(*a*) 2 (*b*) -1

(*c*) 1 (*d*) 0

Sol. (*d*) 0

We have $\sin A = 1 - \sin^2 A = \cos^2 A$

$\therefore \cos^{12} A = \sin^6 A$, $\cos^{10} A = \sin^5 A$, $\cos^8 A = \sin^4 A$, $\cos^6 A = \sin^3 A$

$\therefore \cos^{12} A + 3\cos^{10} A + 3\cos^8 A + \cos^6 A - 1$

$= \sin^6 A + 3\sin^5 A + 3\sin^4 A + \sin^3 A - 1$

$= (\sin^2 A + \sin A)^3 - 1$

$= 1^3 - 1 = 0 \qquad [\because \sin A + \sin^2 A = 1]$

3. If $7\sin^2\theta + 3\cos^2\theta = 4$, then $\tan\theta$ is equal to

(*a*) $\pm\dfrac{1}{2}$ (*b*) $\pm\dfrac{1}{\sqrt{3}}$

(*c*) $\pm\dfrac{1}{\sqrt{2}}$ (*d*) $\pm\dfrac{1}{3}$

Sol. (*b*) $\pm\dfrac{1}{\sqrt{3}}$

We have

$7\sin^2\theta + 3\cos^2\theta = 4$

$\Rightarrow \quad 7\tan^2\theta + 3 = 4\sec^2\theta$

[Dividing both sides by $\cos^2\theta$, since $\cos\theta \neq 0$]

$\Rightarrow \quad 7\tan^2\theta + 3 - 4(1 + \tan^2\theta) = 0$

$\Rightarrow \quad 7\tan^2\theta + 3 - 4 - 4\tan^2\theta = 0$

$\Rightarrow \quad 3\tan^2\theta = 1$

$\Rightarrow \quad \tan^2\theta = \dfrac{1}{3}$

$\Rightarrow \quad \tan\theta = \pm\dfrac{1}{\sqrt{3}}$

4. If $\tan\theta + \sec\theta = x$, then $\sin\theta$ is equal to

(*a*) $\dfrac{x^2-1}{x^2+1}$ (*b*) $\dfrac{x^2+1}{x^2-1}$

(*c*) $x^2 - 1$ (*d*) $x - 1$

Sol. (*a*) $\dfrac{x^2-1}{x^2+1}$

We have

$$\tan\theta + \sec\theta = x$$

$$\Rightarrow \quad (\tan\theta + \sec\theta)^2 = x^2$$

$$\Rightarrow \quad \tan^2\theta + \sec^2\theta + 2\tan\theta\sec\theta = x^2$$

$$\Rightarrow \quad \frac{\sin^2\theta}{\cos^2\theta} + \frac{1}{\cos^2\theta} + 2\times\frac{\sin\theta}{\cos\theta}\times\frac{1}{\cos\theta} = x^2$$

$$\Rightarrow \quad \frac{\sin^2\theta + 1 + 2\sin\theta}{\cos^2\theta} = x^2$$

$$\Rightarrow \quad \frac{(\sin\theta+1)^2}{\cos^2\theta} = x^2$$

$$\Rightarrow \quad \frac{(1+\sin\theta)^2}{1-\sin^2\theta} = x^2$$

$$\Rightarrow \quad \frac{1+\sin\theta}{1-\sin\theta} = x^2$$

$$\Rightarrow \quad \frac{x^2+1}{x^2-1} = \frac{1+\sin\theta+1-\sin\theta}{1+\sin\theta-1+\sin\theta}$$

[By componendo and dividendo]

$$= \frac{2}{2\sin\theta} = \frac{1}{\sin\theta}$$

$$\therefore \quad \sin\theta = \frac{x^2-1}{x^2+1}$$

5. If $\tan\theta = \frac{a}{b}$, then the value of

$\frac{a^2\sin\theta - b^2\cos\theta}{a^2\sin\theta + b^2\cos\theta}$ is equal to

(*a*) $\frac{a^2-b^2}{a^2+b^2}$ (*b*) $\frac{a^3-b^3}{a^3+b^3}$

(*c*) $\frac{a-b}{a+b}$ (*d*) $\frac{a-b}{a^2+b^2}$

Sol. (*b*) $\frac{a^3-b^3}{a^3+b^3}$

We have $\tan\theta = \frac{a}{b}$

$$\frac{\sin\theta}{\cos\theta} = \frac{a}{b}$$

$$\Rightarrow \quad \frac{a^2\sin\theta}{b^2\cos\theta} = \frac{a^3}{b^3}$$

[Multiplying both sides by $\frac{a^2}{b^2}$]

$$\Rightarrow \quad \frac{a^2\sin\theta - b^2\cos\theta}{a^2\sin\theta + b^2\cos\theta} = \frac{a^3-b^3}{a^3+b^3}$$

[By componendo and dividendo]

6. The result of elimination of θ between the equations $x = c(\text{cosec}\,\theta + \cot\theta)$ and $y = c(\text{cosec}\,\theta - \cot\theta)$ is

(*a*) $xy = c$ (*b*) $x + y = c$

(*c*) $x + y = c^2$ (*d*) $xy = c^2$

Sol. (*d*) $xy = c^2$

We have

$$x = c(\text{cosec}\,\theta + \cot\theta) \quad \ldots(1)$$

and

$$y = c(\text{cosec}\,\theta - \cot\theta) \quad \ldots(2)$$

Multiplying (1) by (2), we get

$$xy = c^2(\text{cosec}^2\theta - \cot^2\theta)$$

$$= c^2 \times 1$$

$$= c^2$$

7. If $0° < \theta < 90°$, then the value of

$\frac{(\sin\theta+\cos\theta)^2}{\sin\theta\cos\theta}$ will

(*a*) lie between –1 and 2

(*b*) be less than 2

(*c*) be greater than 2

(*d*) be equal to 2

Sol. (*c*) be greater than 2

We have

$$\frac{(\sin\theta+\cos\theta)^2}{\sin\theta\cos\theta} = \frac{\sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta}{\sin\theta\cos\theta}$$

$$= 2 + \frac{1}{\sin\theta\cos\theta} > 2 \qquad [\because 0° < \theta < 90°]$$

8. If $\sec^2 A\tan^2 A = 1$, then the value of $\sec^4 A - \sec^2 A$ is equal to

(*a*) 1 (*b*) 0

(*c*) –1 (*d*) 2

Sol. (*a*) 1

We have

$$\sec^4 A - \sec^2 A = \sec^2 A(\sec^2 A - 1)$$

$$= \sec^2 A\tan^2 A = 1 \quad \text{[Given]}$$

9. If $0° < \theta < 90°$, then the value of

$\frac{1}{\cos^2 A} - \frac{1}{\text{cosec}^2 A - 1}$ is equal to

(*a*) 2 (*b*) 1

(*c*) –1 (*d*) 0

Sol. (*b*) 1

We have

$$\frac{1}{\cos^2 A} - \frac{1}{\text{cosec}^2 A - 1} = \frac{1}{\cos^2 A} - \frac{1}{\frac{1}{\sin^2 A} - 1}$$

$$= \frac{1}{\cos^2 A} - \frac{\sin^2 A}{1-\sin^2 A}$$

$$= \frac{1}{\cos^2 A} - \frac{\sin^2 A}{\cos^2 A}$$

$$= \frac{1 - \sin^2 A}{\cos^2 A}$$

$$= \frac{\cos^2 A}{\cos^2 A}$$

$$= 1$$

10. $\frac{\sec\theta - 1}{\sec\theta + 1}$ is equal to

(*a*) $\frac{\tan\theta - 1}{\tan\theta + 1}$ (*b*) $\frac{1 - \tan\theta}{1 + \tan\theta}$

(*c*) $\frac{1 - \sin\theta}{1 + \sin\theta}$ (*d*) $\frac{1 - \cos\theta}{1 + \cos\theta}$

Sol. (*d*) $\frac{1 - \cos\theta}{1 + \cos\theta}$

We have

$$\frac{\sec\theta - 1}{\sec\theta + 1} = \frac{\frac{1}{\cos\theta} - 1}{\frac{1}{\cos\theta} + 1}$$

$$= \frac{1 - \cos\theta}{1 + \cos\theta}$$

(III) Trigonometric Ratios of Complementary Angles

Milestone

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Multiple-Choice Questions

1. The value of $\frac{\sin 18°}{\cos 72°} + \frac{\tan 26°}{\cot 64°}$ is

(*a*) 1 (*b*) 2

(*c*) $\frac{3}{2}$ (*d*) $\frac{2}{3}$

Sol. (*b*) 2

We have

$$\frac{\sin 18°}{\cos 72°} + \frac{\tan 26°}{\cot 64°}$$

$$= \frac{\sin 18°}{\cos(90° - 18°)} + \frac{\tan 26°}{\cot(90° - 26°)}$$

$$= \frac{\sin 18°}{\sin 18°} + \frac{\tan 26°}{\tan 26°}$$

$$= 1 + 1 = 2$$

2. If $\theta + \phi = 90°$, then $\sqrt{\cos\theta \operatorname{cosec}\phi - \cos\theta \sin\phi}$ is equal to

(*a*) $\cos\theta$ (*b*) $\sec\theta$

(*c*) $\operatorname{cosec}\theta$ (*d*) $\sin\theta$

Sol. (*d*) $\sin\theta$

We have

$$\sqrt{\cos\theta \operatorname{cosec}\phi - \cos\theta \sin\phi}$$

$$= \sqrt{\cos\theta \operatorname{cosec}(90° - \theta) - \cos\theta \sin(90° - \theta)}$$

$$[\because \theta + \phi = 90°]$$

$$= \sqrt{\cos\theta \sec\theta - \cos\theta \cos\theta}$$

$$= \sqrt{\cos\theta \frac{1}{\cos\theta} - \cos^2\theta}$$

$$= \sqrt{1 - \cos^2\theta}$$

$$= \sqrt{\sin^2\theta}$$

$$= \sin\theta$$

Very Short Answer Type Questions

3. Is $\cos^2 27° > \sin^2 64°$ or $\cos^2 27° < \sin^2 64°$? Why?

Sol. We have $\cos 27° = \sin(90° - 27°) = \sin 63°$

$\therefore$ $\cos^2 27° = \sin^2 63° < \sin^2 64°$

$\therefore$ $\cos^2 27° < \sin^2 64°$

4. If $\sin 5\theta = \cos\theta$ and $5\theta < 90°$, then what is the value of $\cot 3\theta$?

Sol. Given that $\sin 5\theta = \cos\theta$

$\Rightarrow$ $\sin 5\theta = \sin(90° - \theta)$

$\Rightarrow$ $5\theta = 90° - \theta$

$\Rightarrow$ $6\theta = 90°$

$\therefore$ $\theta = \frac{90°}{6}$

$= 15°$

$\therefore$ $\cot 3\theta = \cot 3 \times 15°$

$= \cot 45° = 1$

which is the required value.

Short Answer Type-I Questions

5. Express $\cos 75° + \cot 65°$ in terms of trigonometric ratios of angles between 0° and 45°.

Sol. We have

$\cos 75° + \cot 65° = \sin(90° - 75°) + \tan(90° - 65°)$

$= \sin 15° + \tan 25°$

which is the required expression.

6. If sec 4A = cosec (A – 20°) where 4A is an acute angle, find the value of A. **[CBSE 2008, SP 2011]**

Sol. We have

$$\sec 4A = \text{cosec}\,(A - 20°)$$
$$\Rightarrow \quad \text{cosec}\,(90° - 4A) = \text{cosec}\,(A - 20°)$$
$$\Rightarrow \quad 90° - 4A = A - 20°$$
$$\Rightarrow \quad 5A = 90° + 20° = 110°$$
$$\therefore \quad A = \frac{110°}{5} = 22°$$

which is the required value.

Short Answer Type-II Questions

7. Without using trigonometric tables, evaluate

$$\frac{\cos 58°}{\sin 32°} + \frac{\sin 22°}{\cos 68°} - \frac{\cos 38° \,\text{cosec}\, 52°}{\tan 18° \tan 35° \tan 60° \tan 72° \tan 55°}$$

[CBSE 2003, 2009]

Sol. We have

$$\cos 58° = \cos (90° - 32°) = \sin 32°$$
$$\cos 68° = \cos (90° - 22°) = \sin 22°$$
$$\text{cosec}\, 52° = \text{cosec}\,(90° - 38°) = \sec 38° = \frac{1}{\cos 38°}$$
$$\tan 72° = \tan (90° - 18°) = \cot 18° = \frac{1}{\tan 18°}$$
$$\tan 55° = \tan (90° - 35°) = \cot 35° = \frac{1}{\tan 35°}$$

$$\therefore \quad \frac{\cos 58°}{\sin 32°} + \frac{\sin 22°}{\cos 68°} - \frac{\cos 38° \,\text{cosec}\, 52°}{\tan 18° \tan 35° \tan 60° \tan 72° \tan 55°}$$
$$= \frac{\sin 32°}{\sin 32°} + \frac{\sin 22°}{\sin 22°} - \frac{\cos 38° \times \frac{1}{\cos 38°}}{\tan 18° \tan 35° \times \sqrt{3} \times \frac{1}{\tan 18°} \times \frac{1}{\tan 35°}}$$
$$= 1 + 1 - \frac{1}{1 \times 1 \times \sqrt{3}}$$
$$= 2 - \frac{1}{\sqrt{3}}$$
$$= \frac{2\sqrt{3} - 1}{\sqrt{3}} = \frac{2 \times 3 - \sqrt{3}}{3}$$
$$= \frac{6 - \sqrt{3}}{3}$$

which is the required value.

8. Prove that

$$\sin\theta\cos\theta - \frac{\sin\theta\cos(90° - \theta)\cos\theta}{\sec(90° - \theta)} - \frac{\cos\theta\sin(90° - \theta)\sin\theta}{\text{cosec}\,(90° - \theta)} = 0$$

[CBSE 2003, SP 2011]

Sol. We have

$$\text{LHS} = \sin\theta\cos\theta - \frac{\sin\theta\cos(90° - \theta)\cos\theta}{\sec(90° - \theta)} - \frac{\cos\theta\sin(90° - \theta)\sin\theta}{\text{cosec}\,(90° - \theta)}$$
$$= \sin\theta\cos\theta - \frac{\sin\theta\sin\theta\cos\theta}{\text{cosec}\,\theta} - \frac{\cos\theta\cos\theta\sin\theta}{\sec\theta}$$
$$= \sin\theta\cos\theta - \frac{\sin^2\theta\cos\theta}{\frac{1}{\sin\theta}} - \frac{\cos^2\theta\sin\theta}{\frac{1}{\cos\theta}}$$
$$= \sin\theta\cos\theta - \sin^3\theta\cos\theta - \cos^3\theta\sin\theta$$
$$= \sin\theta\cos\theta - \sin\theta\cos\theta\,(\sin^2\theta + \cos^2\theta)$$
$$= \sin\theta\cos\theta - \sin\theta\cos\theta$$
$$= 0 = \text{RHS}$$

Hence, proved.

Long Answer Type Questions

9. If A, B and C are the angles of a triangle ABC, prove that

(*a*) $\sin \frac{B+C}{2} = \cos \frac{A}{2}$

(*b*) $\tan \frac{B+C}{2} = \cot \frac{A}{2}$

(*c*) $\sec \frac{B+C}{2} = \text{cosec}\, \frac{A}{2}$

Sol. We have

$$A + B + C = 180°$$

[By angle sum property of a triangle]

$$\therefore \quad \frac{B+C}{2} = \frac{180° - A}{2} = 90° - \frac{A}{2}$$

$$\therefore \quad (a) \quad \sin\frac{B+C}{2} = \sin\left(90° - \frac{A}{2}\right) = \cos\frac{A}{2}$$

(*b*) $\tan\dfrac{B+C}{2} = \tan\left(90° - \dfrac{A}{2}\right) = \cot\dfrac{A}{2}$

(*c*) $\sec\dfrac{B+C}{2} = \sec\left(90° - \dfrac{A}{2}\right) = \text{cosec}\dfrac{A}{2}$

Hence, proved.

10. In ΔABC, if $\angle C = 90°$, prove that

(*a*) $\sin^2 A + \sin^2 B = 1$

(*b*) $\sec^2 A - \cot^2 B = 1$ and

(*c*) $\text{cosec}^2 A - \tan^2 B = 1$

Sol. We have

$$\angle A + \angle B + \angle C = 180°$$

[By angle sum property of a triangle]

$$\therefore \quad \angle A + \angle B = 180° - \angle C = 180° - 90° = 90° \qquad \ldots(1)$$

(*a*) LHS $= \sin^2 A + \sin^2 B$

$= \sin^2 A + \sin^2 (90° - A)$ [From (1)]

$= \sin^2 A + \cos^2 A$

$= 1 =$ RHS

(*b*) LHS $= \sec^2 A - \cot^2 B$

$= \sec^2 A - \cot^2 (90° - A)$ [From (1)]

$= \sec^2 A - \tan^2 A$

$= 1 =$ RHS

(*c*) LHS $= \text{cosec}^2 A - \tan^2 B$

$= \text{cosec}^2 A - \tan^2 (90° - A)$ [From (1)]

$= \text{cosec}^2 A - \cot^2 A$

$= 1 =$ RHS

Higher Order Thinking Skills (HOTS) Questions

(Page 164)

1. Prove that

$$\sin^2 5° + \sin^2 10° + \ldots + \sin^2 85° + \sin^2 90° = 9\frac{1}{2}$$

Sol. LHS $= (\sin^2 5° + \sin^2 85°) + (\sin^2 10° + \sin^2 80°) + \ldots + (\sin^2 40° + \sin^2 50°) + \sin^2 45° + \sin^2 90°$

$= \{\sin^2 5° + \sin^2 (90° - 5°)\} + \{\sin^2 10° + \sin^2 (90° - 10°)\} + \ldots$ to 8 pairs $+ \dfrac{1}{2} + 1$

$= (\sin^2 5° + \cos^2 5°) + (\sin^2 10° + \cos^2 10°) + \ldots$ to 8 pairs $+ \dfrac{1}{2} + 1$

$= 1 + 1 + 1 + \ldots$ to 8 terms $+ 1 + \dfrac{1}{2}$

$= 8 \times 1 + 1 + \dfrac{1}{2}$

$= 9\dfrac{1}{2} =$ RHS

2. If $\sec\theta + \text{cosec}\,\theta = a\,\text{cosec}\,(90° - \theta)$, find $\sin\theta$ in terms of a. Hence, find the value of a so that $\sin\theta = \dfrac{1}{\sqrt{2}}$. ($a \neq 0$ and θ is acute)

Sol. We have

$$\sec\theta + \text{cosec}\,\theta = a\,\text{cosec}\,(90° - \theta) = a\sec\theta$$

$$\Rightarrow \quad \text{cosec}\,\theta = (a-1)\sec\theta$$

$$\Rightarrow \quad \frac{1}{\sin\theta} = \frac{a-1}{\cos\theta}$$

$$\Rightarrow \quad \frac{1}{\sin^2\theta} = \frac{(a-1)^2}{\cos^2\theta}$$

$$\Rightarrow \quad \cos^2\theta = (a-1)^2\sin^2\theta$$

$$\Rightarrow \quad 1 - \sin^2\theta = (a^2 - 2a + 1)\sin^2\theta$$

$$\Rightarrow \quad \sin^2\theta(a^2 - 2a + 2) = 1$$

$$\therefore \quad \sin\theta = \frac{1}{\sqrt{a^2 - 2a + 2}}$$

which is the required value of $\sin\theta$ in terms of a.

When $\quad \sin\theta = \dfrac{1}{\sqrt{2}}$

then $\quad \dfrac{1}{\sqrt{2}} = \dfrac{1}{\sqrt{a^2 - 2a + 2}}$

$$\Rightarrow \quad \frac{1}{2} = \frac{1}{a^2 - 2a + 2}$$

$$\Rightarrow \quad a^2 - 2a + 2 = 2$$

$$\Rightarrow \quad a(a-2) = 0$$

$$\therefore \quad a - 2 = 0 \qquad [\because a \neq 0]$$

$$\Rightarrow \quad a = 2$$

which is the required value of a.

3. If $\sin\theta + \cos\theta = \sqrt{2}\sin(90° - \theta)$, determine $\cot\theta$.

Sol. We have

$$\sin\theta + \cos\theta = \sqrt{2}\sin(90° - \theta) = \sqrt{2}\cos\theta$$

$$\therefore \quad \sin\theta = \left(\sqrt{2} - 1\right)\cos\theta$$

$$\Rightarrow \quad \cot\theta = \frac{1}{\sqrt{2} - 1}$$

$$= \frac{\sqrt{2}+1}{2-1}$$

$$= \sqrt{2}+1$$

which is the required value of $\cot \theta$.

4. Prove that

$\sin (50° + \theta) - \cos (40° - \theta) = 0$

Sol. $\text{LHS} = \sin (50° + \theta) - \cos (40° - \theta)$

$= \sin \{90° - (40° - \theta)\} - \cos (40° - \theta)$

$= \cos (40° - \theta) - \cos (40° - \theta)$

$= 0 = \text{RHS}$

Hence, proved.

Self-Assessment

(Page 164)

Multiple-Choice Questions

1. The value of $\cos^2 38° - \sin^2 52°$ is equal to

(*a*) 1 (*b*) –1

(*c*) 2 (*d*) 0

Sol. (*d*) 0

We have

$\cos^2 38° - \sin^2 52° = \cos^2 38° - \sin^2 (90° - 38°)$

$= \cos^2 38° - \cos^2 38°$

$= 0$

2. If $0° < 3A < 90°$, then the value of A from the equation $\cos 2A = \sin 3A$ is

(*a*) 9° (*b*) 18°

(*c*) 36° (*d*) 30°

Sol. (*b*) 18°

We have $\cos 2A = \sin 3A$

$= \cos (90° - 3A)$

$\therefore \quad 2A = 90° - 3A$

$\Rightarrow \quad 5A = 90°$

$\therefore \quad A = \frac{90°}{5}$

$= 18°$

Fill in the Blanks

3. If $\sin \theta = \cos \theta$ then the value of θ is **45°**.

Sol. $\sin \theta = \cos \theta$ [Given]

$\Rightarrow \quad \sin \theta = \sin (90° - \theta)$

$\Rightarrow$ $[\because \cos \theta = \sin (90° - \theta)]$

$\Rightarrow \quad \theta = 90° - \theta$

$\Rightarrow \quad 2\theta = 90°$

$\Rightarrow \quad \theta = 45°$

4. The value of $\cos 36° \cos 54° - \sin 36° \sin 54°$ is **0**.

Sol. $\cos 36° \cos 54° - \sin 36° \sin 54°$

$= \cos (90° - 54°) \cos 54° - \sin (90° - 54°) \sin 54°$

$= \sin 54° \cos 54° - \cos 54° \sin 54°$

$[\because \cos (90° - \theta) = \sin \theta, \sin (90° - \theta) = \cos \theta]$

$= 0$

5. The value of $\sin (60° + \theta) - \cos (30° + \theta)$ is **0**.

Sol. $\sin (60° + \theta) - \cos (30° + \theta)$

$= \sin (60° + \theta) - \sin [90° - (30° - \theta)]$

$[\because \cos \theta = \sin (90° - \theta)]$

$= \sin (60° + \theta) - \sin (60° + \theta)$

$= 0$

6. The value of $17 \sec^2 29° - 17 \cot^2 61°$ is **17**.

Sol. $17 \sec^2 29° - 17 \cot^2 61°$

$= 17 \sec^2 29° - 17 \tan^2 (90° - 61°)$

$[\because \cot \theta = \tan (90° - \theta)]$

$= 17 \sec^2 29° - 17 \tan^2 29°$

$= 17 (\sec^2 29° - \tan^2 29°)$

$= 17 (1)$ $[\because \sec^2 \theta - \tan^2 \theta = 1]$

$= 17$

Assertion-Reason Type Questions

Directions (Q. Nos. 7 to 9): Each of these questions contains an assertion followed by reason. Read them carefully, and answer the question on the basis of the following options, select the one that best describes the two statements.

(a) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.

(b) Both Assertion and Reason are true but Reason is not the correct explanation of Assertion.

(c) If Assertion is true but Reason is false.

(d) If Assertion is false but Reason is true.

7. **Assertion:** The value of sin A is always greater than 1.

Reason: Hypotenuse is the longest side in a right triangle.

Sol. The correct answer is (*d*).

Since, sin A is the ratio of Perpendicular and Hypotenuse and, Hypotenuse is the longest side in a right-angled triangle, hence sin A will always be less than or equal to 1.

Thus, assertion is wrong and reason is correct.

8. Assertion: cot 0° is not defined.

Reason: tan 0° is not defined and cot is inverse of tan.

Sol. The correct answer is (*c*).

tan 0° is 0, cot is inverse of tan so cot 0° will have division by 0 which is not defined.

Thus, reason is wrong.

9. Assertion: $(\sin A - \cos A)^2 + 2 \sin A \cos A = 1$

Reason: $\sin^2 A + \cos^2 A = 1$

Sol. The correct answer is (*a*).

$(\sin A - \cos A)^2 + 2 \sin A \cos A$

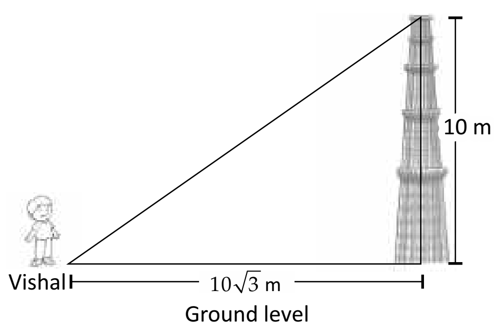
$= \sin^2 A + \cos^2 A - 2 \sin A \cos A + 2 \sin A \cos A$

$= 1$

Thus, both statements are correct and reason is correct explanation of the assertion.

Case Study Based Questions

10. Vishal went to Delhi with his parents during his summer vacation. One day, they visited a Minar. They took photographs of this place. The height of the Minar was found to be 10 m. They were standing at a distance of $10\sqrt{3}$ m from the base of the Minar. Based on the above information, answer the following questions.



(*i*) What will be the tangent of the angle made by the line segment connecting top of the minar and position of Vishal with the base line?

(*a*) $\frac{1}{\sqrt{3}}$ (*b*) $\sqrt{3}$

(*c*) $\frac{2}{\sqrt{3}}$ (*d*) 0

Sol. (*a*) $\frac{1}{\sqrt{3}}$

P = 10 m, B = $10\sqrt{3}$ m

$$\tan\theta = \frac{P}{B} = \frac{10}{10\sqrt{3}} = \frac{1}{\sqrt{3}}$$

(*ii*) What will be the cotangent of the angle made by the line segment connecting top of the minar and position of Vishal with the base line?

(*a*) $\frac{2}{\sqrt{3}}$ (*b*) $\frac{1}{\sqrt{3}}$

(*c*) $\sqrt{3}$ (*d*) 1

Sol. (*c*) $\sqrt{3}$

$$\cot\theta = \frac{B}{P} = \frac{10\sqrt{3}}{10} = \sqrt{3}$$

(*iii*) What will be the secant of the angle made by the line segment connecting top of the minar and position of Vishal with the base line?

(*a*) $\frac{1}{\sqrt{3}}$ (*b*) $\frac{2}{\sqrt{3}}$

(*c*) 1 (*d*) $\sqrt{3}$

Sol. (*b*) $\frac{2}{\sqrt{3}}$

$$\sec\theta = \frac{H}{B}$$

$$H = \sqrt{P^2 + B^2}$$

$$= \sqrt{(10)^2 + (10\sqrt{3})^2}$$

$$= \sqrt{100 + 100 \times 3} = \sqrt{400}$$

$$H = 20$$

$$\therefore \quad \sec\theta = \frac{H}{B} = \frac{20}{10\sqrt{3}} = \frac{2}{\sqrt{3}}$$

(*iv*) What will be the cosine of the angle made by the line segment connecting top of the minar and position of Vishal with the base line?

(*a*) $\frac{\sqrt{3}}{2}$ (*b*) $\frac{2}{\sqrt{3}}$

(*c*) $\sqrt{3}$ (*d*) 1

Sol. (*a*) $\frac{\sqrt{3}}{2}$

$$\cos\theta = \frac{B}{H} = \frac{10\sqrt{3}}{20} = \frac{\sqrt{3}}{2}$$

(*v*) What will be the sine of the angle made by the line segment connecting top of the minar and position of Vishal with the base line?

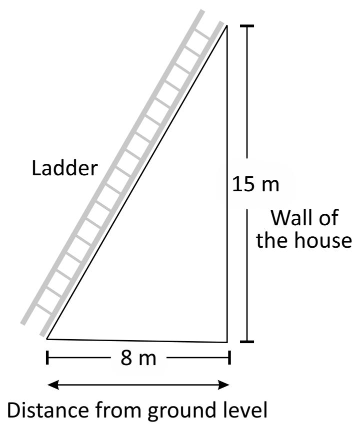
(*a*) 2 (*b*) 1

(c) $\frac{1}{2}$ (d) 0

Sol. (c) $\frac{1}{2}$

$$\sin\theta = \frac{P}{H} = \frac{10}{20} = \frac{1}{2}$$

11. The roof of a house was made up of galvanized steel sheets. One day due to a heavy storm, some sheets were damaged. To repair the sheets, the workers climbed the roof using a ladder. The foot of the ladder was at a distance of 8 m from the wall of the house and the top of the ladder was at a height of 15 m from the ground. Based on the above situation, answer the following questions.



(i) What was the length of the ladder?

(a) 17 m (b) 20 m

(c) 23 m (d) 7 m

Sol. (a) 17 m

P = 15 m, B = 8 m, H = ?

$$H = \sqrt{(P)^2 + (B)^2} = \sqrt{(15)^2 + (8)^2} = \sqrt{225+64} = \sqrt{289}$$

$$H = 17 \text{ m}$$

(ii) If θ be the angle made by the ladder with the ground, then value of the sine of the angle made by the ladder with the ground is given by

(a) $\frac{15}{8}$ (b) $\frac{15}{17}$

(c) $\frac{17}{15}$ (d) $\frac{8}{15}$

Sol. (b) $\frac{15}{17}$

$$\sin\theta = \frac{P}{H} = \frac{15}{17}$$

(iii) If θ be the angle made by the ladder with the ground, then the value of the cosine of the angle made by the ladder with the base level is given by

(a) $\frac{17}{15}$ (b) $\frac{15}{17}$

(c) $\frac{8}{15}$ (d) $\frac{8}{17}$

Sol. (d) $\frac{8}{17}$

$$\cos\theta = \frac{B}{H} = \frac{8}{17}$$

(iv) The value of the $\sin^2\theta + \cos^2\theta$ is equal to

(a) 0 (b) 1

(c) 2 (d) None of these

Sol. (b) 1

$\sin^2\theta + \cos^2\theta = 1$

(v) The value of the $\sin^2\theta - \cos^2\theta$ is equal to

(a) $\frac{160}{289}$ (b) $\frac{180}{289}$

(c) $\frac{161}{289}$ (d) $\frac{170}{289}$

Sol. (c) $\frac{161}{289}$

$\sin^2\theta - \cos^2\theta$

$$= \left(\frac{P}{H}\right)^2 - \left(\frac{B}{H}\right)^2 = \left(\frac{15}{17}\right)^2 - \left(\frac{8}{17}\right)^2$$

$$= \frac{225-64}{289} = \frac{161}{289}$$

Very Short Answer Type Questions

12. If tan 2A = cot (A – 18°), where 2A is an acute angle, what is the value of A?

Sol. We have

$$\tan 2A = \cot(A - 18°)$$

$$\Rightarrow \quad \cot(90° - 2A) = \cot(A - 18°)$$

$$\Rightarrow \quad 90° - 2A = A - 18°$$

$$\Rightarrow \quad 3A = 108°$$

$$\Rightarrow \quad A = \frac{108°}{3} = 36°$$

which is the required value of A.

13. What is the numerical value of

sin 26° sin 64° – cos 26° cos 64°?

Sol. sin 26° sin 64° – cos 26° cos 64°

= sin 26° sin (90° – 26°) – cos 26° cos (90° – 26°)

= sin 26° cos 26° – cos 26° sin 26° = 0

which is the required value.

Short Answer Type-I Questions

14. Prove that

$$\frac{\cos 70^\circ}{\sin 20^\circ}+\frac{\cos 59^\circ}{\sin 31^\circ}=2$$

Sol.
$$\begin{aligned}\text{LHS} &= \frac{\cos 70^\circ}{\sin 20^\circ}+\frac{\cos 59^\circ}{\sin 31^\circ}\\ &= \frac{\cos(90^\circ-20^\circ)}{\sin 20^\circ}+\frac{\cos(90^\circ-31^\circ)}{\sin 31^\circ}\\ &= \frac{\sin 20^\circ}{\sin 20^\circ}+\frac{\sin 31^\circ}{\sin 31^\circ}\\ &= 1+1\\ &= 2 = \text{RHS}\end{aligned}$$

Hence, proved.

15. Prove that

$$1+\sin^2\theta\,\text{cosec}^2(90^\circ-\theta)=\text{cosec}^2(90^\circ-\theta)$$

Sol.
$$\begin{aligned}\text{LHS} &= 1+\sin^2\theta\,\text{cosec}^2(90^\circ-\theta)\\ &= 1+\sin^2\theta\sec^2\theta\\ &= 1+\frac{\sin^2\theta}{\cos^2\theta}\\ &= \frac{\cos^2\theta+\sin^2\theta}{\cos^2\theta}\\ &= \frac{1}{\cos^2\theta}\\ &= \sec^2\theta\\ &= \text{cosec}^2(90^\circ-\theta)\\ &= \text{RHS}\end{aligned}$$

Hence, proved.

Short Answer Type-II Questions

16. If A, B and C are the interior angles of ΔABC, show that $\tan\frac{A+B-C}{2}=\cot C$

Sol. By angle sum property of a triangle ABC, we have

$$A+B+C=180^\circ$$

$$\Rightarrow \quad A+B=180^\circ-C$$

$$\Rightarrow \quad \frac{A+B}{2}=90^\circ-\frac{C}{2} \qquad \ldots(1)$$

Now,
$$\begin{aligned}\text{LHS} &= \tan\frac{A+B-C}{2}\\ &= \tan\left(\frac{A+B}{2}-\frac{C}{2}\right)\\ &= \tan\left(90^\circ-\frac{C}{2}-\frac{C}{2}\right) \qquad \text{[From (1)]}\\ &= \tan(90^\circ-C)\\ &= \cot C\\ &= \text{RHS}\end{aligned}$$

Hence, proved.

17. Without using trigonometric tables, evaluate:

$$\frac{2}{3}\text{cosec}^2 58^\circ-\frac{2}{3}\cot 58^\circ\tan 32^\circ-\frac{5}{3}\tan 13^\circ\tan 37^\circ\tan 53^\circ\tan 77^\circ$$

[CBSE 2009]

Sol.
$$\begin{aligned}&\frac{2}{3}\text{cosec}^2 58^\circ-\frac{2}{3}\cot 58^\circ\tan 32^\circ-\frac{5}{3}\tan 13^\circ\tan 37^\circ\tan 53^\circ\tan 77^\circ\\ &= \frac{2}{3}\text{cosec}^2 58^\circ-\frac{2}{3}\cot 58^\circ\tan(90^\circ-58^\circ)-\frac{5}{3}\tan 13^\circ\tan 37^\circ\tan(90^\circ-37^\circ)\tan(90^\circ-13^\circ)\\ &= \frac{2}{3}\text{cosec}^2 58^\circ-\frac{2}{3}\cot 58^\circ\times\cot 58^\circ-\frac{5}{3}\tan 13^\circ\tan 37^\circ\cot 37^\circ\cot 13^\circ\\ &= \frac{2}{3}\text{cosec}^2 58^\circ-\frac{2}{3}\cot^2 58^\circ-\frac{5}{3}\tan 13^\circ\times\frac{1}{\tan 13^\circ}\times\tan 37^\circ\times\frac{1}{\tan 37^\circ}\\ &= \frac{2}{3}(\text{cosec}^2 58^\circ-\cot^2 58^\circ)-\frac{5}{3}\times 1\times 1\\ &= \frac{2}{3}\times 1-\frac{5}{3}\\ &= \frac{-3}{3}=-1\end{aligned}$$

which is the required value of the given expression.

Long Answer Type Questions

18. Prove that

$$\frac{1+3\cos\theta-4\sin^3(90^\circ-\theta)}{1-\sin(90^\circ-\theta)}=\{1+2\sin(90^\circ-\theta)\}^2$$

Sol. We have

$$\begin{aligned}\text{LHS} &= \frac{1+3\cos\theta-4\sin^3(90^\circ-\theta)}{1-\sin(90^\circ-\theta)}\\ &= \frac{1+3\cos\theta-4\cos^3\theta}{1-\cos\theta}\\ &= \frac{-4\cos^3\theta+3\cos\theta+1}{1-\cos\theta}\\ &= \frac{-4\cos^3\theta+4\cos^2\theta-4\cos^2\theta+4\cos\theta-\cos\theta+1}{1-\cos\theta}\end{aligned}$$

$$= \frac{4\cos^2\theta(1-\cos\theta)+4\cos\theta(1-\cos\theta)+1(1-\cos\theta)}{1-\cos\theta}$$

$$= \frac{(1-\cos\theta)\left(4\cos^2\theta+4\cos\theta+1\right)}{1-\cos\theta}$$

$= 4\cos^2\theta + 4\cos\theta + 1$

$= (2\cos\theta + 1)^2$

$= (1 + 2\cos\theta)^2$

$= \{1 + 2\sin(90° - \theta)\}^2$

= RHS

Hence, proved.

19. If $\sin\theta + \cos\theta = \sqrt{2}\cos(90° - \theta)$, find the values of $\sin\theta$, $\cos\theta$ and $\tan\theta$.

Sol. We have

$$\sin\theta + \cos\theta = \sqrt{2}\cos(90° - \theta)$$

$$\sin\theta + \cos\theta = \sqrt{2}\sin\theta$$

$$\Rightarrow \quad (\sqrt{2} - 1)\sin\theta = \cos\theta \quad \ldots(1)$$

$$\Rightarrow \quad \tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{1}{\sqrt{2}-1}$$

Again, from (1), we have

$$(\sqrt{2} - 1)^2\sin^2\theta = \cos^2\theta$$

$$\Rightarrow \quad (\sqrt{2} - 1)^2(1 - \cos^2\theta) = \cos^2\theta$$

$$\Rightarrow \quad (\sqrt{2} - 1)^2 = \cos^2\theta\,\{1 + (\sqrt{2} - 1)^2\}$$

$$= \cos^2\theta(1 + 2 + 1 - 2\sqrt{2})$$

$$= \cos^2\theta\,(4 - 2\sqrt{2})$$

$$\therefore \quad \cos\theta = \sqrt{\frac{\left(\sqrt{2}-1\right)^2}{4-2\sqrt{2}}}$$

$$= \frac{\sqrt{2}-1}{\sqrt{4-2\sqrt{2}}}$$

Lastly, from (1)

$$(\sqrt{2} - 1)^2\sin^2\theta = \cos^2\theta = 1 - \sin^2\theta$$

$$\Rightarrow \quad \sin^2\theta\,[(\sqrt{2} - 1)^2 + 1] = 1$$

$$\Rightarrow \quad \sin^2\theta = \frac{1}{\left(\sqrt{2}-1\right)^2+1}$$

$$= \frac{1}{2+1-2\sqrt{2}+1}$$

$$= \frac{1}{4-2\sqrt{2}}$$

$$\therefore \quad \sin\theta = \frac{1}{\sqrt{4-2\sqrt{2}}}$$

Hence, the required values of $\sin\theta$, $\cos\theta$ and $\tan\theta$ are respectively $\frac{1}{\sqrt{4-2\sqrt{2}}}$, $\frac{\sqrt{2}-1}{\sqrt{4-2\sqrt{2}}}$ and $\frac{1}{\sqrt{2}-1}$.

Let's Compete

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Multiple-Choice Questions

1. The value of $\tan 7° \tan 23° \tan 67° \tan 83°$ is equal to

(*a*) $\sqrt{3}$ (*b*) 1

(*c*) $2\sqrt{3}$ (*d*) $\frac{\sqrt{3}}{2}$

Sol. (*b*) 1

We have

$\tan 7° \tan 23° \tan 67° \tan 83°$

$= \tan 7° \tan 23° \tan(90° - 23°) \tan(90° - 7°)$

$= \tan 7° \tan 23° \cot 23° \cot 7°$

$= \tan 7° \tan 23° \dfrac{1}{\tan 23°} \times \dfrac{1}{\tan 7°}$

$= 1 \times 1$

$= 1$

2. The value of $\sin(22° + \theta) - \cos(68° - \theta)$ is equal to

(*a*) –1 (*b*) 1

(*c*) 2 (*d*) 0

Sol. (*d*) 0

$\sin(22° + \theta) - \cos(68° - \theta)$

$= \sin(22° + \theta) - \cos\{90° - (22° + \theta)\}$

$= \sin(22° + \theta) - \sin(22° + \theta)$

$= 0$

3. If $\sec 4\theta = \operatorname{cosec}(\theta - 30°)$, where 4θ is an acute angle, then the measure of θ is

(*a*) 10° (*b*) 55°

(*c*) 24° (*d*) 40°

Sol. (*c*) 24°

We have

$$\sec 4\theta = \operatorname{cosec}(\theta - 30°)$$

$$\Rightarrow \quad \operatorname{cosec}(90° - 4\theta) = \operatorname{cosec}(\theta - 30°)$$

$$\therefore \quad 90° - 4\theta = \theta - 30°$$

$$\Rightarrow \quad 5\theta = 90° + 30° = 120°$$

$$\therefore \quad \theta = \frac{120°}{5} = 24°$$

4. If $\cos(40° + A) = \sin 30°$, then the value of $\tan 3A$ is equal to

(*a*) $\sqrt{3}$ (*b*) $\frac{1}{\sqrt{3}}$

(*c*) $\frac{\sqrt{3}}{2}$ (*d*) $\frac{2}{\sqrt{3}}$

Sol. (*a*) $\sqrt{3}$

We have

$$\cos(40° + A) = \sin 30°$$
$$\Rightarrow \quad \cos(40° + A) = \sin(90° - 60°)$$
$$\Rightarrow \quad \cos(40° + A) = \cos 60°$$
$$\Rightarrow \quad 40° + A = 60°$$
$$\Rightarrow \quad A = 60° - 40° = 20°$$
$$\therefore \quad \tan 3A = \tan 3 \times 20° = \tan 60° = \sqrt{3}$$

5. If $\cos(A + B) = 0$, then $\sin(A - B)$ can be reduced to

(*a*) $\cos B$ (*b*) $\cos 2B$

(*c*) $\sin A$ (*d*) $\sin 2A$

Sol. (*b*) $\cos 2B$

We have

$$\cos(A + B) = 0 = \cos 90°$$
$$\therefore \quad A + B = 90°$$
$$\Rightarrow \quad A = 90° - B \qquad \ldots(1)$$

Now, $\sin(A - B) = \sin(90° - B - B)$ [From (1)]

$$= \sin(90° - 2B)$$
$$= \cos 2B$$

6. If $\sin 3\theta = \cos(\theta - 6°)$ and 3θ, $\theta - 6°$ are both acute angles, then the value of $\sin(2\theta - 3°)$ is equal to

(*a*) $\frac{1}{\sqrt{3}}$ (*b*) $\frac{\sqrt{2}}{3}$

(*c*) $\frac{1}{\sqrt{2}}$ (*d*) $\sqrt{2} - 1$

Sol. (*c*) $\frac{1}{\sqrt{2}}$

We have

$$\sin 3\theta = \cos(\theta - 6°)$$
$$\Rightarrow \quad \cos(90° - 3\theta) = \cos(\theta - 6°)$$
$$\therefore \quad 90° - 3\theta = \theta - 6°$$
$$\Rightarrow \quad 4\theta = 96°$$
$$\Rightarrow \quad \theta = \frac{96°}{4} = 24°$$
$$\therefore \quad \sin(2\theta - 3°) = \sin(2 \times 24° - 3°) = \sin(48° - 3°) = \sin 45° = \frac{1}{\sqrt{2}}$$

7. ABCD is a quadrilateral. Then $\cos\frac{A+B}{4}$ is equal to

(*a*) $\cot\frac{C+D}{4}$ (*b*) $\tan\frac{C+D}{4}$

(*c*) $\cos\frac{C+D}{4}$ (*d*) $\sin\frac{C+D}{4}$

Sol. (*d*) $\sin\frac{C+D}{4}$

For a quadrilateral ABCD, we know that

$$A + B + C + D = 360°$$
$$\therefore \quad \frac{A+B}{4} = \frac{360° - (C+D)}{4} = 90° - \frac{C+D}{4} \qquad \ldots(1)$$
$$\therefore \quad \cos\frac{A+B}{4} = \cos\left(90° - \frac{C+D}{4}\right)$$

[From (1)]

$$= \sin\frac{C+D}{4}$$

8. If $x \sin(90° - \theta)\cot(90° - \theta) = \cos(90° - \theta)$, then the value of x is

(*a*) 1 (*b*) –1

(*c*) 2 (*d*) $\frac{1}{2}$

Sol. (*a*) 1

We have

$$x \sin(90° - \theta)\cot(90° - \theta) = \cos(90° - \theta)$$
$$\Rightarrow \quad x\cos\theta\tan\theta = \sin\theta$$
$$\Rightarrow \quad x\cos\theta\frac{\sin\theta}{\cos\theta} = \sin\theta$$
$$\Rightarrow \quad x\sin\theta = \sin\theta$$
$$\Rightarrow \quad x = 1$$

9. The value of

$\operatorname{cosec}(65° + \theta) - \sec(25° - \theta) + \tan(55° - \theta) - \cot(35° + \theta)$ is

(*a*) –1 (*b*) 1

(*c*) 0 (*d*) 2

Sol. (*c*) 0

We have

$\operatorname{cosec}(65° + \theta) - \sec(25° - \theta) + \tan(55° - \theta) - \cot(35° + \theta)$

$= \text{cosec}\,\{90° – (25° – \theta)\} – \sec\,(25° – \theta)$
$+ \tan\,(55° – \theta) – \cot\,\{90° – (55° – \theta)\}$

$= \sec\,(25° – \theta) – \sec\,(25° – \theta) + \tan\,(55° – \theta)$
$– \tan\,(55° – \theta)$

$= 0 + 0 = 0$

10. If $\dfrac{\cos^2 20° + \cos^2 70°}{2\left(\sin^2 59° + \sin^2 31°\right)} = \dfrac{2}{k}$, then k is equal to

(*a*) 3 (*b*) 4
(*c*) 2 (*d*) 1

Sol. (*b*) 4

We have

$$\frac{\cos^2 20° + \cos^2 70°}{2\left(\sin^2 59° + \sin^2 31°\right)} = \frac{2}{k}$$

$$\Rightarrow \quad \frac{\cos^2 20° + \cos^2 (90° – 20°)}{2\left\{\sin^2 (90° – 31°) + \sin^2 31°\right\}} = \frac{2}{k}$$

$$\Rightarrow \quad \frac{\cos^2 20° + \sin^2 20°}{2\left(\cos^2 31° + \sin^2 31°\right)} = \frac{2}{k}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{2}{k}$$

$$\Rightarrow \quad k = 4$$

9

Some Applications of Trigonometry

Checkpoint (Page 170)

1. If $\sec A = \frac{41}{9}$, find cot A when $0° < A < 90°$.

Sol. We have

$$\sec A = \frac{41}{9}$$

$$\therefore \quad \sec^2 A = \frac{41^2}{9^2} = \frac{1681}{81}$$

$$\Rightarrow \quad \tan^2 A + 1 = \frac{1681}{81}$$

$$\Rightarrow \quad \tan^2 A = \frac{1681}{81} - 1 = \frac{1600}{81}$$

$$\therefore \quad \tan A = \sqrt{\frac{1600}{81}} = \frac{40}{9}$$

$$\therefore \quad \cot A = \frac{1}{\tan A} = \frac{9}{40}$$

which is the required value.

2. Express cosec θ in terms of sec θ where $0° < \theta < 90°$.

Sol. We have

$$\operatorname{cosec}\theta = \sqrt{1+\cot^2\theta}$$

$$= \sqrt{1+\frac{1}{\tan^2\theta}}$$

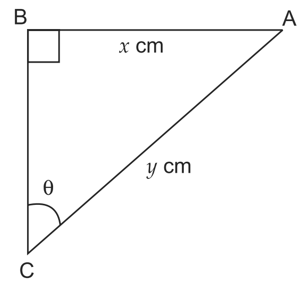
$$= \sqrt{1+\frac{1}{\sec^2\theta - 1}}$$

$$= \sqrt{\frac{\sec^2\theta}{\sec^2\theta - 1}}$$

$$= \frac{\sec\theta}{\sqrt{\sec^2\theta - 1}}$$

which is the required expression.

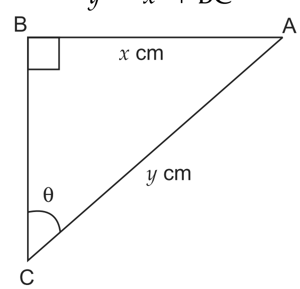
3. In the following figure, find all trigonometric ratios of the angle θ of ΔABC, where ∠B = 90°, AB = x cm, AC = y cm and ∠ACB = θ.



Sol. From ΔABC, we have by Pythagoras' theorem,

$$AC^2 = AB^2 + BC^2$$

$$\Rightarrow \quad y^2 = x^2 + BC^2$$



$$\therefore \quad BC = \sqrt{y^2 - x^2}$$

$$\therefore \quad \sin A = \sin \angle BCA = \frac{AB}{AC} = \frac{x}{y}$$

$$\cos\theta = \frac{BC}{AC} = \frac{\sqrt{y^2 - x^2}}{y}$$

$$\tan\theta = \frac{AB}{BC} = \frac{x}{\sqrt{y^2 - x^2}}$$

$$\therefore \quad \cot\theta = \frac{1}{\tan\theta} = \frac{\sqrt{y^2 - x^2}}{x}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{y}{\sqrt{y^2 - x^2}}$$

$$\text{cosec}\,\theta = \frac{1}{\sin\theta} = \frac{y}{x}$$

Hence, the required values of sin θ, cos θ, tan θ, cot θ, sec θ, and cosec θ are

$$\frac{x}{y}, \frac{\sqrt{y^2 - x^2}}{y}, \frac{x}{\sqrt{y^2 - x^2}}, \frac{\sqrt{y^2 - x^2}}{x}, \frac{y}{\sqrt{y^2 - x^2}}$$

and $\frac{y}{x}$ respectively.

4. If $\sin A = \frac{3}{5}$, find the values of all other trigonometric ratios of angle A.

Sol. We have $\quad \sin A = \frac{3}{5}$

$$\therefore \quad \cos A = \sqrt{1 - \sin^2 A}$$
$$= \sqrt{1 - \frac{9}{25}}$$
$$= \sqrt{\frac{16}{25}}$$
$$= \frac{4}{5}$$

$$\therefore \quad \tan A = \frac{\sin A}{\cos A} = \frac{3}{4}$$

$$\cot A = \frac{1}{\tan A} = \frac{4}{3}$$

$$\sec A = \frac{1}{\cos A} = \frac{5}{4}$$

and $\quad \text{cosec}\,A = \frac{1}{\sin A} = \frac{5}{3}$

Hence, the required values of cos A, tan A, cot A, sec A and cosec A are $\frac{4}{5}, \frac{3}{4}, \frac{4}{3}, \frac{5}{4}$ and $\frac{5}{3}$ respectively.

5. If $\tan A = \sqrt{2} - 1$, show that $\frac{\tan A}{1 + \tan^2 A} = \frac{\sqrt{2}}{4}$.

Sol. We have

$$\tan A = \sqrt{2} - 1$$

$$\therefore \quad \tan^2 A = \left(\sqrt{2} - 1\right)^2$$
$$= 2 + 1 - 2\sqrt{2}$$
$$= 3 - 2\sqrt{2}$$

$$\therefore \quad \text{LHS} = \frac{\tan A}{1 + \tan^2 A}$$
$$= \frac{\sqrt{2} - 1}{3 - 2\sqrt{2} + 1}$$
$$= \frac{\sqrt{2} - 1}{4 - 2\sqrt{2}}$$
$$= \frac{\sqrt{2} - 1}{2\left(2 - \sqrt{2}\right)}$$
$$= \frac{\left(\sqrt{2} - 1\right)\left(2 + \sqrt{2}\right)}{2\left(2^2 - 2\right)}$$
$$= \frac{2\sqrt{2} + 2 - 2 - \sqrt{2}}{2(2)}$$
$$= \frac{\sqrt{2}}{4}$$
$$= \text{RHS}$$

6. If $\cot B = \frac{12}{5}$, show that

$\tan^2 B - \sin^2 B = \sin^2 B \tan^2 B$.

Sol. We have, $\quad \cot B = \frac{12}{15}$

$$\therefore \quad \tan B = \frac{1}{\cot B} = \frac{5}{12}$$

$$\therefore \quad \tan^2 B = \frac{25}{144} \quad \ldots(1)$$

$$\therefore \quad \cot^2 B = \frac{144}{25}$$

$$\therefore \quad \text{cosec}^2 B = 1 + \cot^2 B$$
$$= 1 + \frac{144}{25} = \frac{169}{25}$$

$$\therefore \quad \sin^2 B = \frac{1}{\text{cosec}^2 B} = \frac{25}{169} \quad \ldots(2)$$

Now, $\quad \text{LHS} = \tan^2 B - \sin^2 B$

$$= \frac{25}{144} - \frac{25}{169}$$
$$= 25 \times \frac{169 - 144}{144 \times 169}$$
$$= \frac{25 \times 25}{144 \times 169}$$
$$= \frac{25}{144} \times \frac{25}{169}$$
$$= \tan^2 B \,.\, \sin^2 B$$

[From (1) and (2)]

$$= \text{RHS}$$

Hence, proved.

7. If $\tan\theta = 2$, evaluate

$\sin\theta \sec\theta + \tan^2\theta - \text{cosec}\,\theta$.

Sol. We have

$$\tan\theta = 2$$

$$\therefore \quad \sec^2\theta = 1 + \tan^2\theta$$
$$= 1 + (2)^2$$

$= 1 + 4 = 5$

$\therefore \quad \cos^2\theta = \dfrac{1}{\sec^2\theta} = \dfrac{1}{5}$

$\therefore \quad \sin^2\theta = 1 - \cos^2\theta$

$= 1 - \dfrac{1}{5} = \dfrac{4}{5}$

$\therefore \quad \text{cosec}^2\theta = \dfrac{1}{\sin^2\theta} = \dfrac{5}{4}$

$\therefore \quad \text{cosec}\ \theta = \dfrac{\sqrt{5}}{2}$

Also, $\sec\theta = \sqrt{5}$, $\sin\theta = \sqrt{\dfrac{4}{5}}$

Now, $\sin\theta\sec\theta + \tan^2\theta - \text{cosec}\ \theta$

$= \sqrt{\dfrac{4}{5}} \times \sqrt{5} + 4 - \dfrac{\sqrt{5}}{2}$

$= \dfrac{2}{\sqrt{5}} \times \sqrt{5} + 4 - \dfrac{\sqrt{5}}{2}$

$= 6 - \dfrac{\sqrt{5}}{2}$

$= \dfrac{12 - \sqrt{5}}{2}$

which is the required value.

8. If $\tan\theta = \dfrac{1}{\sqrt{7}}$, show that

$$\frac{\text{cosec}^2\theta - \sec^2\theta}{\text{cosec}^2\theta + \sec^2\theta} = \frac{3}{4}$$

Sol. We have

$\tan\theta = \dfrac{1}{\sqrt{7}}$

$\therefore \quad \sec^2\theta = 1 + \tan^2\theta$

$= 1 + \dfrac{1}{7} = \dfrac{8}{7}$

$\Rightarrow \quad \cot\theta = \dfrac{1}{\tan\theta} = \sqrt{7}$

$\therefore \quad \text{cosec}^2 = 1 + \cot^2\theta$

$= 1 + 7 = 8$

$\therefore \quad \text{LHS} = \dfrac{\text{cosec}^2\theta - \sec^2\theta}{\text{cosec}^2\theta + \sec^2\theta}$

$= \dfrac{8 - \dfrac{8}{7}}{8 + \dfrac{8}{7}}$

$= \dfrac{48}{64}$

$= \dfrac{3}{4} = \text{RHS}$

Hence, proved.

9. Without using trigonometric tables, evaluate

$$\left(\frac{\sin 49^\circ}{\cos 41^\circ}\right)^2 + \left(\frac{\cos 41^\circ}{\sin 49^\circ}\right)^2$$

Sol. We have, $\left(\dfrac{\sin 49^\circ}{\cos 41^\circ}\right) + \left(\dfrac{\cos 41^\circ}{\sin 49^\circ}\right)$

$= \dfrac{\{\sin(90^\circ - 41^\circ)\}^2}{\cos^2 41^\circ} + \dfrac{\cos^2 41^\circ}{\{\sin(90^\circ - 41^\circ)\}^2}$

$= \dfrac{\cos^2 41^\circ}{\cos^2 41^\circ} + \dfrac{\cos^2 41^\circ}{\cos^2 41^\circ}$

$= 1 + 1 = 2$ which is the required value.

10. Find the numerical value of the expression:

$$\frac{4\sin^2 30^\circ + 5\cos^2 45^\circ - 6\tan^2 60^\circ}{\text{cosec}^2 60^\circ \cot^2 30^\circ + \sec^2 45^\circ}$$

Sol. We have,

$\dfrac{4\sin^2 30^\circ + 5\cos^2 45^\circ - 6\tan^2 60^\circ}{\text{cosec}^2 60^\circ \cot^2 30^\circ + \sec^2 45^\circ}$

$= \dfrac{4 \times \left(\dfrac{1^2}{2}\right) + 5 \times \left(\dfrac{1}{\sqrt{2}}\right)^2 - 6 \times \left(\sqrt{3}\right)^2}{\left(\dfrac{2}{\sqrt{3}}\right)^2 \times \left(\sqrt{3}\right)^2 + \left(\sqrt{2}\right)^2}$

$= \dfrac{1 + \dfrac{5}{2} - 18}{4 + 2}$

$= \dfrac{-\dfrac{29}{2}}{6}$

$= -\dfrac{29}{12}$

which is the required value of the given expression.

Milestone

(Page 176)

Multiple-Choice Questions

1. The angle formed by the line of sight with the horizontal when the point being viewed lies above the horizontal level, is called

(*a*) vertical angle

(*b*) angle of depression

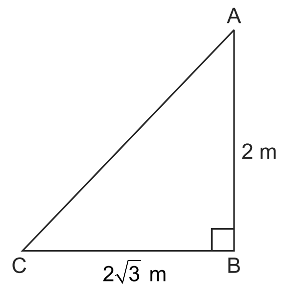
(*c*) angle of elevation

(*d*) obtuse angle **[CBSE SP 2012]**

Sol. (*c*) angle of elevation

Clearly, it is called the angle of elevation.

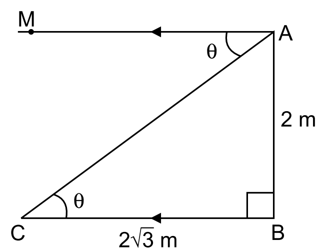
2. The given figure shows the observation of the point C from the point A. The angle of depression from A is



(*a*) 60° (*b*) 30°
(*c*) 45° (*d*) 75° **[CBSE SP 2012]**

Sol. (*b*) 30°

We draw AM as a horizontal ray through A. Hence, AM ∥ BC and AB ⊥ BC.



$$\therefore \quad \angle MAC = \text{angle of depression}$$
$$= \angle ACB \qquad [\because AM \parallel BC]$$
$$= \theta$$

Now, in ΔABC, we have

$$\tan\theta = \tan\angle ACB$$
$$= \frac{AB}{BC}$$
$$= \frac{2}{2\sqrt{3}}$$
$$= \frac{1}{\sqrt{3}}$$
$$= \tan 30°$$
$$\therefore \quad \theta = 30°$$

Very Short Answer Type Questions

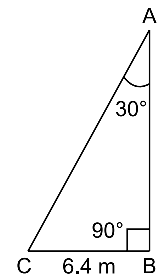
3. The angle between a ladder and a vertical wall, when the ladder is leaning against the wall, is 30° and the foot of the ladder is 6.4 m away from the wall. Find the length of the ladder.

Sol. Let AB be the vertical wall, BC, the horizontal ground and AC, the ladder such that ∠CAB = 30°. Clearly, ∠ABC = 90°.

It is given that CB = 6.4 m.

Now, in ΔABC, we have

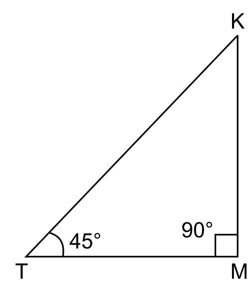
$$\sin 30° = \sin(\angle CAB) = \frac{CB}{AC}$$
$$\Rightarrow \quad \frac{1}{2} = \frac{6.4}{AC}$$
$$\Rightarrow \quad AC = 6.4 \times 2 = 12.8 \text{ m}$$



Hence, the required length of the ladder is 12.8 m.

4. A kite is flying with a thread 300 m long. If the thread is assumed to be stretched straight and make an angle of 45° with the horizontal, find the height of the kite above the ground.

Sol. Let K be the position of the kite in air, KT be the thread 300 m long and TM be the horizontal ground. Clearly, KM is the height of the ground so that ∠KMT = 90°. Given that ∠KTM = angle of elevation of the kite K from the ground T = 45°.



Now, in ΔKTM, we have

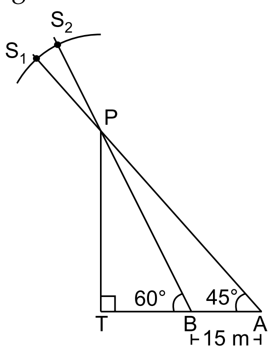
$$\sin 45° = \sin(\angle KTM) = \frac{KM}{KT}$$
$$\Rightarrow \quad \frac{1}{\sqrt{2}} = \frac{KM}{300}$$
$$\Rightarrow \quad KM = \frac{300}{\sqrt{2}} = \frac{300 \times \sqrt{2}}{2} = 150\sqrt{2}$$

∴ The required height of the kite above the ground is $150\sqrt{2}$ m.

Short Answer Type-I Questions

5. The shadow of a vertical tower on level ground decreases by 15 m when the altitude of the Sun changes from angle of elevation of 45° to 60°. Find the height of the tower.

Sol. Let PT be the vertical tower and AT be the horizontal ground so that ∠PTA = 90°.



Let TA be the length of the shadow of the tower when the position of the sun is S_1 such that S_1 PA is a straight line and ∠PAT = 45°. Let TB be the length of the shadow when the position of the sun is at S_2 such that S_2 PB is a straight line and ∠PBT = 60°.

It is given that AB = 15 m.

To find the height PT of the tower.

Now, in ΔPTA, we have

$$\tan(\angle PAT) = \frac{PT}{AT}$$

$$\Rightarrow \quad \tan 45° = \frac{PT}{AT}$$

$$\Rightarrow \quad 1 = \frac{PT}{AT}$$

$$\Rightarrow \quad \frac{AT}{PT} = 1 \qquad \ldots(1)$$

Again, in ΔPTB, we have

$$\tan(\angle PBT) = \frac{PT}{TB}$$

$$\Rightarrow \quad \tan 60° = \frac{PT}{TB}$$

$$\Rightarrow \quad \sqrt{3} = \frac{PT}{TB}$$

$$\Rightarrow \quad \frac{TB}{PT} = \frac{1}{\sqrt{3}} \qquad \ldots(2)$$

Subtracting (2) from (1), we get

$$\frac{AT - TB}{PT} = 1 - \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad \frac{AB}{PT} = \frac{\sqrt{3}-1}{\sqrt{3}}$$

$$\Rightarrow \quad \frac{15}{PT} = \frac{\sqrt{3}-1}{\sqrt{3}}$$

$$\Rightarrow \quad PT = \frac{15\sqrt{3}}{\sqrt{3}-1}$$

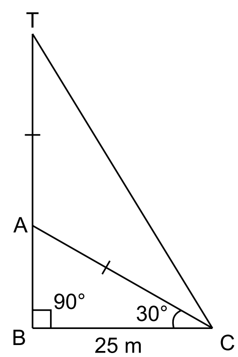
$$= \frac{15\sqrt{3}\left(\sqrt{3}+1\right)}{\left(\sqrt{3}\right)^2 - 1^2}$$

$$= \frac{15\left(3+\sqrt{3}\right)}{2}$$

Hence, the required height of the tower is $\frac{15\left(3+\sqrt{3}\right)}{2}$ m.

6. The upper part of a tree, broken by the wind makes an angle of 30° with the ground and the horizontal distance from the root of the tree to the point where the top of the tree meets the ground is 25 m. Find the height of the tree before it was broken. [Use $\sqrt{3}$ = 1.73] **[CBSE SP 2012]**

Sol. Let BT be the height of the tree where T is its top and B, the foot is on the horizontal ground BC. Let A be the point on the upper part of the tree where the tree broke and the top T of the tree has touched the ground at C such that ∠ACB = 30°.



Clearly, AC = AT and ∠ABC = 90°.

We join TC.

Given that BC = 25 m.

Now, in ΔABC, we have

$$\tan(\angle ACB) = \frac{AB}{BC}$$

$$\Rightarrow \quad \tan 30° = \frac{AB}{25}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{AB}{25}$$

$$\therefore \quad AB = \frac{25}{\sqrt{3}}$$

Now, since ∠ABC = 90°.

∴ By Pythagoras' theorem, we have

$$AC^2 = AB^2 + BC^2$$

$$= \left(\frac{25}{\sqrt{3}}\right)^2 + 25^2$$

$$= 25^2\left(1 + \frac{1}{3}\right)$$

$$= 25^2 \times \frac{4}{3}$$

$\therefore$ $AC = \frac{50}{\sqrt{3}}$

$\therefore$ Required height of BT = AB + AT

$$= AB + AC$$

$$= \frac{25}{\sqrt{3}} + \frac{50}{\sqrt{3}}$$

$$= \frac{75}{\sqrt{3}}$$

$$= \frac{75\sqrt{3}}{3}$$

$$= 25\sqrt{3}$$

$$= 25 \times 1.73$$

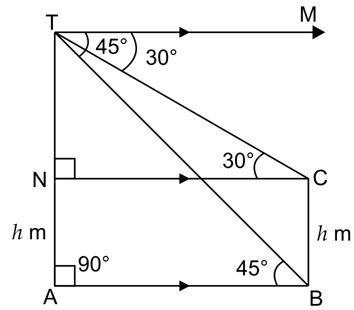
$$= 43.3$$

Hence, the required height of the tree is 43.3 m.

Short Answer Type-II Questions

7. From the top of a 50 m high tower, the angles of depression of the top and bottom of a pole are observed to be 30° and 45°, respectively. Find the height of the pole. [Use $\sqrt{3} = 1.73$] **[CBSE SP 2011]**

Sol. Let T be the top of the vertical tower AT and A be the foot of the tower on the horizontal ground AB so that ∠TAB = 90°.



Let C be the top and B be the bottom of a vertical pole, so that ∠ABC = 90°.

Let TM be a horizontal through T. The angle of depression of C from T = 30°.

$\therefore$ ∠MTC = 30°

Also, the angle of depression of B from T = 45°

$\therefore$ ∠MTB = 45°

$\therefore$ ∠ABT = ∠MTB = 45°

and ∠TCN = ∠MTC = 30°

where CN is drawn horizontal and N is a point on AT.

Given that AT = 50 m. Let BC = h m be the height of the pole.

Now, in ΔTAB, we have

$$\tan(\angle ABT) = \frac{AT}{AB}$$

$$\Rightarrow \quad \tan 45° = \frac{50}{AB}$$

$$\Rightarrow \quad 1 = \frac{50}{AB}$$

$$\Rightarrow \quad AB = 50 \quad \ldots(1).$$

Now, in ΔTNC, we have

$$\tan(\angle TCN) = \frac{TN}{CN}$$

$$\Rightarrow \quad \tan 30° = \frac{AT - AN}{CN}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{50 - h}{50}$$

[∵ ABCN is a rectangle, ∴ AN = BC = h and CN = AB = 50 [From (1)]]

$$\Rightarrow \quad (50 - h)\sqrt{3} = 50$$

$$\Rightarrow \quad h\sqrt{3} = 50\sqrt{3} - 50$$

$$= 50(\sqrt{3} - 1)$$

$$\Rightarrow \quad h = \frac{50\left(\sqrt{3} - 1\right)}{\sqrt{3}}$$

$$= \frac{50\sqrt{3}\left(\sqrt{3} - 1\right)}{3}$$

$$= \frac{50\left(3 - \sqrt{3}\right)}{3}$$

$$= \frac{50 \times (3 - 1.73)}{3}$$

$$= \frac{50 \times 1.27}{3}$$

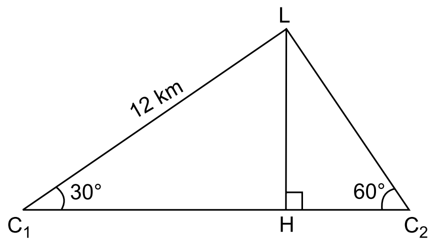
$$= 21.2 \text{ (approx.)}$$

Hence, the height of the pole is 21.2 m (approx.)

8. The angles of elevation of the top of a hill at the city centres of two towns on either side of the hill are observed to be 30° and 60°. If the distance uphill from the first city centre is 12 km, then find (in km) the distance uphill from the other city centre correct upto two decimal places.

[Use $\sqrt{3} = 1.732$]

Sol. Let L be the top and H be the bottom of a hill standing on the horizontal ground $C_1 C_2$ where C_1 and C_2 are two city centres such that $\angle LC_1H = 30°$ and $\angle LC_2H = 60°$. It is given that $C_1L = 12$ km.



To find C_2L.

Now, in ΔLC_1H, since $\angle LHC_2 = 90°$, we have

$$\sin(\angle LC_1H) = \frac{LH}{C_1L}$$

$$\Rightarrow \quad \sin 30° = \frac{LH}{12}$$

From ΔLC_2H, we have

$$\Rightarrow \quad \frac{1}{2} = \frac{LH}{12}$$

$$\therefore \quad LH = 6 \quad \ldots(1)$$

$$\sin(\angle LC_2H) = \frac{LH}{C_2L}$$

$$\Rightarrow \quad \sin 60° = \frac{6}{C_2L} \quad \text{[From (1)]}$$

$$\Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{6}{C_2L}$$

$$\Rightarrow \quad C_2L = \frac{12}{\sqrt{3}}$$

$$= \frac{12\sqrt{3}}{3} = 4\sqrt{3}$$

$$= 4 \times 1.732$$

$$= 6.93 \text{ (approx.)}$$

Hence, the required distance is 6.93 km (approx.)

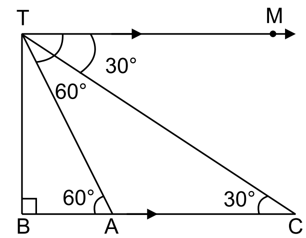
Long Answer Type Questions

9. A man on the top of a vertical tower observes a car coming towards the tower moving with a uniform speed. If it takes 20 minutes for the car to change the angle of depression from 30° to 60°, how soon after this will the car reach the foot of the tower?

Sol. Let T be the top and B be the bottom of the vertical tower TB standing on a horizontal ground BAC so that $\angle TBA = 90°$. Let TM be the horizontal ray through T.

$\therefore$ TM $\parallel$ BC.

The car comes from C to A on the ground. When the car is at C, its angle of depression = $\angle MTC = \angle TCB = 30°$ and when it is at A, then $\angle MTA = \angle TAB = 60°$.



Let v m/min be the uniform speed of the car.

Then AC = $20v$ metres.

$\because$ It takes 20 minutes for the car to come from C to A.

Now, in ΔTBC, we have

$$\tan(\angle TCB) = \frac{TB}{BC}$$

$$\Rightarrow \quad \tan 30° = \frac{TB}{BC}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{TB}{BC}$$

$$\Rightarrow \quad BC = \sqrt{3}\,TB \quad \ldots(1)$$

Again, in ΔTBA, we have

$$\tan(\angle TAB) = \frac{TB}{AB}$$

$$\Rightarrow \quad \tan 60° = \frac{TB}{AB}$$

$$\Rightarrow \quad AB = \frac{TB}{\sqrt{3}} \quad \ldots(2)$$

From (1) and (2), we have

$$\frac{AB}{BC} = \frac{1}{\sqrt{3}} \times \frac{1}{\sqrt{3}} = \frac{1}{3}$$

$$\therefore \quad BC = 3AB$$

$$\therefore \quad AB = BC - AC = 3AB - AC$$

$$\Rightarrow \quad 2AB = AC = 20\,v$$

$$\therefore \quad AB = 10\,v$$

$$\therefore \quad \text{Required time from A to B} = \frac{AB}{v} = \frac{10v}{v} \text{ min.}$$

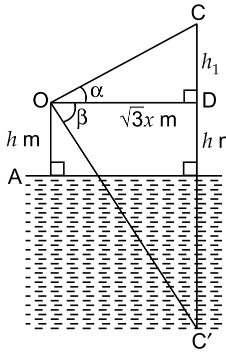
$$= 10 \text{ min.}$$

10. If the angle of elevation of a cloud from a point h metres above a lake is α and the angle of depression of its reflection in the lake is β, prove that the distance of the cloud from the point of observation is $\dfrac{2h\sec\alpha}{\tan\beta - \tan\alpha}$ metres.

[CBSE 2004, CBSE SP 2003, 2011]

Sol. Let AB be the horizontal surface of the lake, AO be the vertical tower, O being the top of the tower, from where a man observes the cloud C in the sky at an angle of elevation $\angle COD = \alpha$

where OD is horizontal. Let C′ be the reflection of the cloud in the lake so that BC = BC′ and ∠CBA = 90°, AB ∥ OD.



Angle of depression of C′ in ∠C′OD = β.

Let CD = h_1 m. Then BD = AO = h m

Now, BC′ = BC = $(h + h_1)$ m

[∵ ABDO is a rectangle]

Now in ΔOCD, we have

$$\tan(\angle COD) = \frac{CD}{OD}$$

$$\Rightarrow \quad \tan\alpha = \frac{h_1}{OD}$$

$$\Rightarrow \quad OD\tan\alpha = h_1$$

$$\therefore \quad OD = h_1\frac{1}{\tan\alpha} = h_1\cot\alpha \qquad \ldots(1)$$

In ΔOC′D, we have

$$\tan(\angle C'OD) = \frac{C'D}{OD}$$

$$\Rightarrow \quad \tan\beta = \frac{C'B + BD}{OD}$$

$$= \frac{CB + BD}{OD}$$

$$= \frac{h + h_1 + h}{h_1\cot + \alpha} \qquad \text{[From (1)]}$$

$$= \frac{2h + h_1}{h_1\cot a} \qquad \ldots(2)$$

Now, in ΔOCD, we have

$$\sin(\angle COD) = \frac{CD}{OC}$$

$$\Rightarrow \quad \sin\alpha = \frac{h_1}{OC}$$

$$\therefore \quad OC = \frac{h_1}{\sin\alpha} = h_1\operatorname{cosec}\alpha \qquad \ldots(3)$$

From (2), we have

$$2h + h_1 = h_1\cot\alpha\tan\beta$$

$$\Rightarrow \quad 2h = h_1\left(\frac{\tan\beta}{\tan\alpha} - 1\right)$$

$$= h_1\frac{\tan\beta - \tan\alpha}{\tan\alpha}$$

$$\therefore \quad h_1 = \frac{2h\tan\alpha}{\tan\beta - \tan\alpha}$$

∴ From (3),

$$OC = \frac{h_1}{\sin\alpha}$$

$$= \frac{2h\tan\alpha}{\tan\beta - \tan\alpha} \times \frac{1}{\sin\alpha}$$

$$= 2h\frac{\sin\alpha}{\cos\alpha} \times \frac{1}{\tan\beta - \tan\alpha} \times \frac{1}{\sin.\alpha}$$

$$= \frac{2h\sec\alpha}{\tan\beta - \tan\alpha}$$

∴ The required distance of the cloud is $\dfrac{2h\sec\alpha}{\tan\beta - \tan\alpha}$ m.

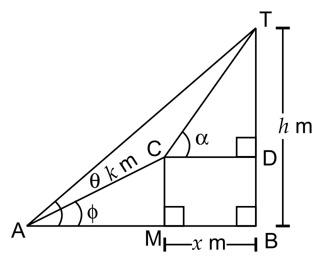
Higher Order Thinking Skills (HOTS) Questions

(Page 177)

1. The angle of elevation of a cliff from a fixed point is θ. After going up a distance of k metres towards the top of the cliff at an angle of inclination ϕ, it is found that the angle of elevation is α. Show that the height of the cliff, in metres, is

$$\frac{k(\cos\phi - \sin\phi\cot\alpha)}{\cot\theta - \cot\alpha}$$

Sol. Let T be the top and B be the bottom of a cliff and A is a fixed point on the ground AB such that ∠TAB = angle of elevation of T = θ. AC = k m such that ∠CAB = ϕ. We draw CM ⊥ AB.



From C, the angle of elevation of T is α so that ∠TCD = α where CD is horizontal and D is a point on TB.

We have ∠TDC = ∠TBA = ∠CMB = 90°.

Let BT = h m, MB = CD = x m. It is given that AC = k m.

Now, in ∠CAM we have, $\dfrac{CM}{AC} = \sin\phi$

$$\Rightarrow \qquad \frac{CM}{k} = \sin \phi$$

$$\Rightarrow \qquad CM = k \sin \phi \qquad \ldots(1)$$

Similarity, $$\frac{AM}{AC} = \cos \phi$$

$$\Rightarrow \qquad \frac{AM}{k} = \cos \phi$$

$$\Rightarrow \qquad AM = k \cos \phi \qquad \ldots(2)$$

Now, ΔCDT, we have

$$\tan \alpha = \frac{TD}{CD}$$
$$= \frac{BT - BD}{CD}$$
$$= \frac{BT - CM}{CD}$$
$$= \frac{h - k\sin\phi}{MB} \qquad \text{[From (1)]}$$
$$= \frac{h - k\sin\phi}{AB - AM}$$
$$= \frac{h - k\sin\phi}{AB - k\cos\phi} \quad \text{[From (2)]} \ldots(3)$$

Now, in ΔABT, we have

$$\tan \theta = \tan (TAB) = \frac{TB}{AB} = \frac{h}{AB}$$

$$\therefore \qquad AB = \frac{h}{\tan\theta} = h \cot \theta \qquad \ldots(4)$$

$\therefore$ From (3) and (4), we have

$$\tan \alpha = \frac{h - k\sin\phi}{h\cot\theta - k\cos\phi}$$

$$\Rightarrow h \cot \theta \tan \alpha - k \cos \phi \tan \alpha = h - k \sin \phi$$

$$\Rightarrow h (\cot \theta \tan \alpha - 1) = k (\cos \phi \tan \alpha - \sin \phi)$$

$$\Rightarrow \quad h\left[\frac{\cot\theta}{\cot\alpha} - 1\right] = k [\cos \phi \tan \alpha - \sin \phi]$$

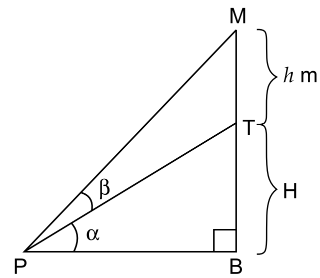
$$\Rightarrow h(\cot \theta - \cot \alpha) = k[\cos \phi \tan \alpha - \sin \phi] \times \frac{\cos\alpha}{\sin\alpha}$$
$$= k\left[\cos\phi\frac{\sin\alpha}{\cos\alpha} - \sin\phi\right] \times \frac{\cos\alpha}{\sin\alpha}$$
$$= k[\cos \phi - \sin \phi \cot \alpha]$$

$$\Rightarrow h = \frac{k[\cos\phi - \sin\phi\cot\alpha]}{\cot\theta - \cot\alpha}$$

Hence, proved.

2. At a point on a level plane, a tower subtends an angle α and a man who is h m tall standing on its top subtends an angle β at the same point. Prove that the height of the tower is $\dfrac{h\tan\alpha}{\tan(\alpha+\beta)-\tan\alpha}$ in metres.

Sol. Let T be the top and B, the bottom of the tower, standing vertically at B on a horizontal ground PB. TM is the man of height h m standing at T such that MTB is a straight line and $\angle MBP = 90°$, where P is a point on the ground PB such that $\angle TPB = \alpha$ and $\angle MPT = \beta$.



To find the height BT of the tower. Let H m be the height of the tower.

Now, in ΔPMB, we have

$$\tan (\angle MPB) = \frac{MB}{PB}$$

$$\Rightarrow \qquad \tan (\alpha + \beta) = \frac{H + h}{PB}$$

$$\Rightarrow \qquad PB = \frac{H + h}{\tan(\alpha + \beta)} \qquad \ldots(1)$$

Again, in ΔTPB, we have

$$\tan (\Delta TPB) = \frac{BT}{PB}$$

$$\Rightarrow \qquad \tan \alpha = \frac{H\tan(\alpha + \beta)}{H + h} \qquad \text{[From (2)]}$$

$$\Rightarrow \quad (H + h) \tan \alpha = H \tan (\alpha + \beta)$$

$$\Rightarrow \quad H[\tan (\alpha + \beta) - \tan \alpha] = h \tan \alpha$$

$$\Rightarrow \qquad H = \frac{h\tan\alpha}{\tan(\alpha + \beta) - \tan\alpha}$$

Hence, the required height of the tower is $\dfrac{h\tan\alpha}{\tan(\alpha + \beta) - \tan\alpha}$ m.

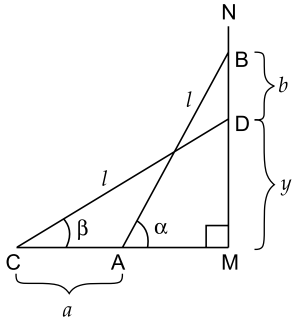
3. A ladder rests against a wall at an angle α to the horizontal. Its foot is pulled away from the wall through a distance a so that it slides a distance b down the wall making an angle β with the horizontal. Show that $\dfrac{a}{b} = \dfrac{\cos\alpha - \cos\beta}{\sin\beta - \sin\alpha}$.

Sol. Let MN be the vertical wall standing on a horizontal ground CM, M being on the ground so that $\angle NMC = 90°$.

Let AB or CD be the ladder of length l m, so that AB = CD = l m.

Given that $\angle BAM = \alpha$ and $\angle DCM = \beta$,

BD = b m and CA = a m.



Let AM = x m and MD = y m.

In ΔABM, we have

$$\sin(\angle BAM) = \frac{BM}{AB}$$

$$\Rightarrow \quad \sin\alpha = \frac{b+y}{l}$$

$$\Rightarrow \quad b + y = l\sin\alpha \quad \ldots(1)$$

and $$\cos\alpha = \frac{x}{l} \quad \ldots(2)$$

Also, in ΔDCM, we have

$$\cos(\angle DCM) = \frac{CM}{CD}$$

$$\Rightarrow \quad \cos\beta = \frac{a+x}{l}$$

$$\Rightarrow \quad a + x = l\cos\beta \quad \ldots(3)$$

and $$\sin\beta = \frac{y}{l} \quad \ldots(4)$$

From (1) and (4), we have

$$b + l\sin\beta = l\sin\alpha$$

$$\Rightarrow \quad b = l(\sin\alpha - \sin\beta) \quad \ldots(5)$$

From (2) and (3), we have

$$a + l\cos\alpha = l\cos\beta$$

$$\Rightarrow \quad a = l(\cos\beta - \cos\alpha) \quad \ldots(6)$$

Dividing (6) by (5), we get

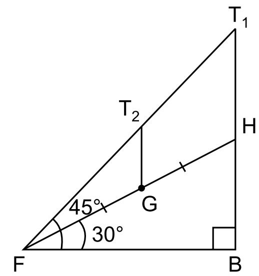
$$\frac{a}{b} = \frac{\cos\beta - \cos\alpha}{\sin\alpha - \sin\beta}$$

$$= \frac{\cos\alpha - \cos\beta}{\sin\beta - \sin\alpha}$$

Hence, proved.

4. The line joining the top of a hill to the foot of the hill makes an angle of 30° with the horizontal through the foot of the hill. There is one temple at the top of the hill and a guest house halfway from the foot to the top of the hill. The top of the temple and the top of the guest house both make an elevation of 45° at the foot of the hill. If the guest house is 100 m away from the foot of the hill, show that the height of the guest house and the height of the temple from the surface of the hill are respectively $50(\sqrt{3}-1)$ m and $100(\sqrt{3}-1)$ m.

Sol. Let H be the top and B be the bottom of the hill HB standing vertically on the horizontal ground FB, where F is a point on the ground such that $\angle$HFB = 30°.



Let HT_1 be the vertical temple at the top H of the hill such that T_1HB is a straight line and $\angle T_1BF$ = 90°. Let G T_2 be the vertical guesthouse at the mid-point G of FH, the line segment joining the foot and the top of the hill. Hence, $GT_2 \parallel HT_1$. Given that $\angle T_1FB$ = 45° and FG = GH = 100 m.

We see by the basic proportionality theorem in ΔT_1FH, T_2 is the mid-point of FT_1 and $HT_1 = 2GT_2$.

Now, in ΔFBT_1, we have

$$\tan(\angle T_1FB) = \frac{T_1B}{FB}$$

$$\Rightarrow \quad \tan 45° = \frac{T_1B}{FB}$$

$$\Rightarrow \quad 1 = \frac{T_1B}{FB}$$

$$\Rightarrow \quad T_1B = FB \quad \ldots(1)$$

Now, in ΔFBH, we have

$$\tan(\angle HFB) = \frac{BH}{FB}$$

$$\Rightarrow \quad \tan 30° = \frac{BH}{FB}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{BH}{T_1B} \quad \text{[From (1)]}$$

$$\Rightarrow \quad T_1B = \sqrt{3}\,BH$$

$$\Rightarrow \quad T_1H + BH = \sqrt{3}\,BH$$

$$\Rightarrow \quad T_1H = (\sqrt{3}-1)\,BH \quad \ldots(2)$$

Now, in ΔFBH

$$\sin(\angle HFB) = \frac{BH}{FH}$$

$$\Rightarrow \quad \sin 30° = \frac{BH}{200}$$

[$\because$ FH = FG + GH = (100 + 100) m = 200 m]

$$\Rightarrow \quad \frac{1}{2} = \frac{BH}{200}$$

$$\Rightarrow \quad BH = 100$$

$\therefore$ From (2), we have

$$HT_1 = (\sqrt{3} - 1)\ 100 \text{ m}$$

and $$GT_2 = \frac{1}{2} HT_1 = (\sqrt{3} - 1) \text{ m}$$

Hence, the required height of the guesthouse and the temple are respectively $50\left(\sqrt{3} - 1\right)$ m and $100\left(\sqrt{3} - 1\right)$ m.

Self-Assessment

(Page 178)

Multiple-Choice Questions

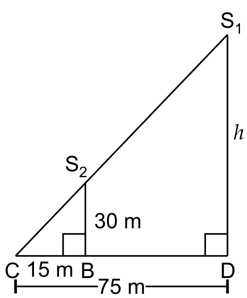
1. A vertical stick, 30 m long casts a shadow 15 m long on the ground. At the same time, a tower casts a shadow 75 m long on the ground. Then the height of the tower is

(*a*) 150 m (*b*) 100 m

(*c*) 25 m (*d*) 200 m **[CBSE SP 2012]**

Sol. (*a*) 150 m

Let S_1D and S_2B be the vertical tower and vertical stick respectively standing on the horizontal ground CD



so that $\angle CBS_2 = \angle CDS_1 = 90°$

The sun rays fall on the ground at C along the line S_1S_2 C forming shadows CB of length 15 m and CD of length 75 m. Let h m be the height of the tower. Given that $BS_2 = 30$ m.

Now, $\Delta S_2CB \sim S_1CD$.

$$\therefore \text{ We have } \frac{30}{h} = \frac{CB}{CD}$$

$$\Rightarrow \quad \frac{30}{h} = \frac{15}{75}$$

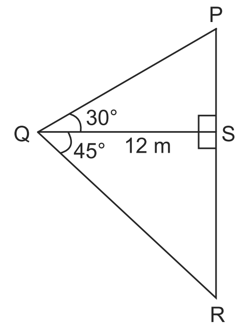
$$\Rightarrow \quad \frac{30}{h} = \frac{1}{5}$$

$$\Rightarrow \quad h = 30 \times 5$$

$$\Rightarrow \quad h = 150$$

Hence, the height of the tower is 150 m.

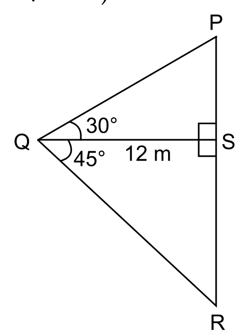
2. In the figure given below the perimeter of ΔPQR where $QS \perp PR$, QS = 12 m, $\angle PQS = 30°$ and $\angle RQS = 45°$ is



(*a*) $12\left(\sqrt{3}+\sqrt{2}-1\right)$ m (*b*) $12\left(\sqrt{3}-\sqrt{2}+1\right)$ m

(*c*) $12\left(\sqrt{3}+\sqrt{2}+1\right)$ m (*d*) $12\left(\sqrt{3}-\sqrt{2}-1\right)$ m

Sol. (*c*) $12\ (\sqrt{3} + \sqrt{2} + 1)$ m



In ΔPQS, we have

$$\cos(\angle PQS) = \frac{QS}{PQ}$$

$$\cos 30° = \frac{12}{PQ}$$

$$\Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{12}{PQ}$$

$$\Rightarrow \quad PQ = \frac{24}{\sqrt{3}}$$

$$= \frac{24\sqrt{3}}{3}$$

$$\Rightarrow \quad PQ = 8\sqrt{3} \qquad \ldots(1)$$

Also, $$\sin(\angle PQS) = \frac{PS}{PQ}$$

$$\Rightarrow \quad \sin 30° = \frac{PS}{8\sqrt{3}} \qquad \text{[From (1)]}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{PS}{8\sqrt{3}}$$

$$\Rightarrow \quad PS = 4\sqrt{3} \qquad \ldots(2)$$

In ΔQRS, we have

$$\cos(\angle RQS) = \frac{QS}{QR}$$

$$\Rightarrow \quad \cos 45° = \frac{12}{QR}$$

$$\Rightarrow \quad \frac{1}{\sqrt{2}} = \frac{12}{QR}$$

$$\Rightarrow \quad QR = 12\sqrt{2}$$

Also, $\sin(\angle RQS) = \dfrac{RS}{QR}$

$$\Rightarrow \quad \sin 45° = \frac{RS}{12\sqrt{2}} \qquad \text{[From (3)]}$$

$$\Rightarrow \quad \frac{1}{\sqrt{2}} = \frac{RS}{12\sqrt{2}}$$

$$\Rightarrow \quad RS = 12 \qquad \ldots(4)$$

From (1), (2), (3) and (4),

Perimeter of $\Delta PQR = PQ + PS + SR + QR$

$$= (8\sqrt{3} + 4\sqrt{3} + 12 + 12\sqrt{2}) \text{ m}$$

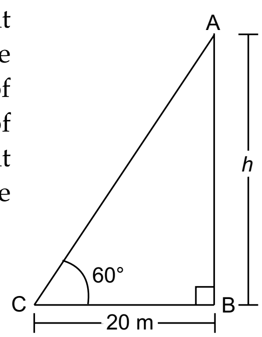
$$= 12(\sqrt{3} + \sqrt{2} + 1) \text{ m}$$

Fill in the Blanks

3. The angle of **elevation** of an object viewed, is the angle formed by the line of sight with the horizontal when it is above the horizontal level.

4. The angle of elevation of the top of a tower from a point on the ground, 20 m away from the foot of the tower is 60°. Then, the height of the tower is **$20\sqrt{3}$ m**.

Sol. Let AB (= h) be the height of the tower and CB be the distance from the point of observation. The angle of elevation of A at the point C is 60°. In right ΔABC, we have



$$\tan 60° = \frac{AB}{BC}$$

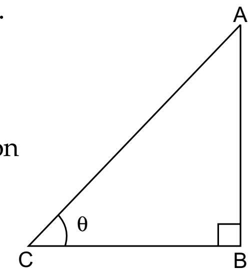
$$\Rightarrow \quad \sqrt{3} = \frac{h}{20 \text{ m}}$$

$$\Rightarrow \quad h = 20\sqrt{3} \text{ m}$$

5. The **line of sight** is the line drawn from the eye of an observer to the point in the object viewed by the observer.

6. If the length of the shadow of a vertical pole is equal to its height, the angle of elevation of Sun's altitude is **45°**.

Sol. Let AB be the length of the vertical pole and BC be the length of its shadow.



Then, $AB = BC$

and $\angle ABC = 90°$

Let θ be the angle of elevation of the Sun's altitude.

Thus,

$$\angle ACB = \theta$$

In right ΔABC, we have

$$\tan\theta = \frac{AB}{BC}$$

$$\Rightarrow \quad \tan\theta = 1 \qquad [\because AB = BC]$$

$$\Rightarrow \quad \tan\theta = \tan 45°$$

$$\Rightarrow \quad \theta = 45°$$

Assertion-Reason Type Questions

Directions (Q. Nos. 7 to 9): Each of these questions contains an assertion followed by reason. Read them carefully, and answer the question on the basis of the following options, select the one that best describes the two statements.

(a) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.

(b) Both Assertion and Reason are true but Reason is not the correct explanation of Assertion.

(c) If Assertion is true but Reason is false.

(d) If Assertion is false but Reason is true.

7. **Assertion:** If angle of elevation is 60°, distance from base is equal to the height.

Reason: The value of tan 45° is 1.

Sol. The correct answer is (*d*).

If tan 45° is 1 which means if angle of elevation is 45°, then distance from base is equal to height. Thus, Reason is correct but Assertion is wrong.

8. **Assertion:** The height of a pole is 6 m, then angle of depression from the top will be 45° at a distance of 6 m from the base.

Reason: The angle formed by the line of sight with the vertical is called the angle of depression.

Sol. The correct answer is (*b*).

Both statements are correct. However, the Reason does not tell anything about measurement.

Thus it is not a proper explanation of Assertion.

9. **Assertion:** As the distance from the base increases, the angle of elevation decreases.

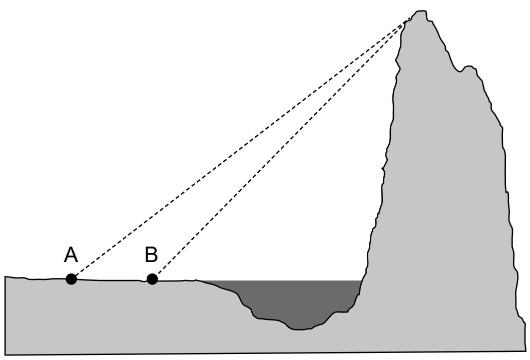
Reason: Angle of elevation does not depend on distance from base.

Sol. The correct answer is (*c*).

Assertion is correct, however reason is wrong as one moves away, the angle of elevation will keep decreasing.

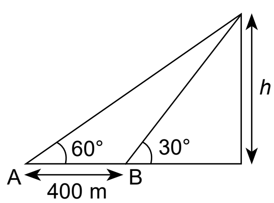
Case Study Based Questions

10. A team went to Nainital to survey mountains. The team members A and B were standing on the ground and wanted to find the height of the mountain some distance away from the other side of the lake. One team member was standing on the top of the mountain. The angle between the horizontal ground at A and the line of sight to the top of the mountain to be 30°. The angle between the horizontal ground at B and the line of sight to the top of the mountain be 60°. The distance between A and B is 400 m. Based on the given situation, answer the following questions.

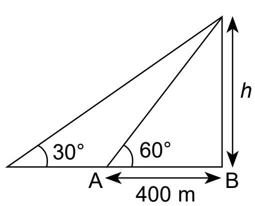


(*a*) Choose the correct option from the following labelled diagrams showing above situation.

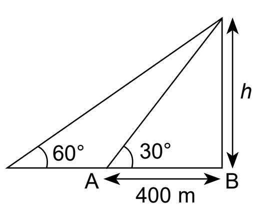
(*i*)



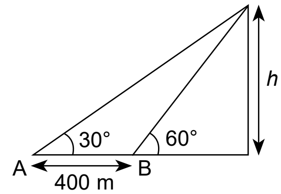
(*ii*)



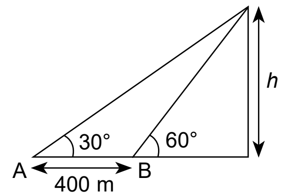
(*iii*)



(*iv*)



Ans. (*iv*)



(*b*) Find the horizontal distance from B to mountain.

(*i*) 400 m (*ii*) 200 m
(*iii*) 600 m (*iv*) 100 m

Ans. (*ii*) 200 m

(*c*) If you increase the distance from the mountain, the angle with the mountain will be

(*i*) increased. (*ii*) decreased.
(*iii*) same. (*iv*) can't say.

Ans. (*ii*) decreased.

(*d*) If the height of the mountain is increased, then the angle from A will be

(*i*) increased. (*ii*) decreased.
(*iii*) same. (*iv*) can't say.

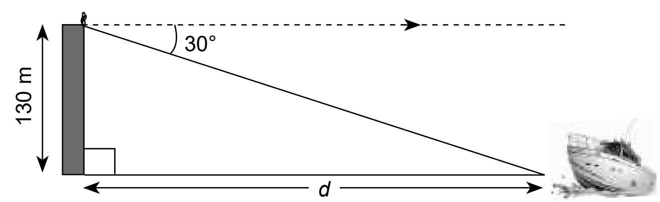
Ans. (*i*) increased.

(*e*) Find the height of the mountain.

(*i*) 346 m (*ii*) 400 m
(*iii*) 560 m (*iv*) 600 m

Ans. (*i*) 346 m

11. Rishi was sitting on a roof from a height of 130 m above sea level. He saw a motor boat at sea at an angle of depression of 30°. Based on the above situation, answer the following questions.



(*a*) What is the angle made if a person sitting on boat and looking at Rishi?

(*i*) 30° (*ii*) 60°
(*iii*) 45° (*iv*) 90°

Ans. (*i*) 30°

(*b*) If you draw a dotted line as shown in the diagram, it will be

(*i*) always parallel to *d*.

(*ii*) sometimes parallel to *d*.

(*iii*) never parallel to *d*.

(*iv*) can't say.

Ans. (*i*) always parallel to *d*.

(*c*) What is the horizontal distance from boat to the roof?

(*i*) 67.5 m (*ii*) 124.9 m

(*iii*) 224.9 m (*iv*) 260 m

Ans. (*iii*) 224.9 m

(*d*) What is the angle made by the roof and the sea level?

(*i*) 30° (*ii*) 45°

(*iii*) 60° (*iv*) 90°

Ans. (*iv*) 90°

(*e*) If Rishi was standing on the roof and he is 1.74 m tall, then what is the total height from where Rishi was looking at the boat?

(*i*) 120 m (*ii*) 128.26 m

(*iii*) 131.74 m (*iv*) 132.5 m

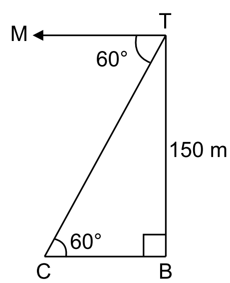
Ans. (*iii*) 131.74 m

Very Short Answer Type Questions

12. What is the distance of a car which is parked on the road, from a tower 150 m high, if the angle of depression of the car from the top of the tower is 60°?

Sol. Let TB be the tower standing vertically at B of the horizontal ground CB. C is the position of the car.

TM is drawn horizontal through T so that TM ∥ CB. Also, ∠TBC = 90°, ∠MTC = ∠TCB = 60° and BT = 150 m.



Now, in ΔCBT, we have

$$\tan(\angle TCB) = \frac{TB}{CB}$$

$$\Rightarrow \quad \tan 60° = \frac{150}{CB}$$

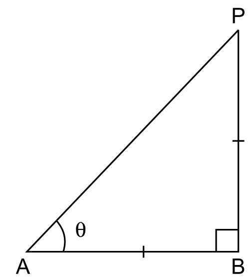
$$\Rightarrow \quad \sqrt{3} = \frac{150}{CB}$$

$$\therefore \quad CB = \frac{150}{\sqrt{3}} = \frac{150\sqrt{3}}{3} = 50\sqrt{3}.$$

∴ The required distance = $50\sqrt{3}$ m.

13. What is the angle of elevation of the Sun's altitude if the length of the shadow of a vertical pole is equal to its height?

Sol. Let PB be the vertical pole standing on the horizontal ground AB. The sun rays are falling along PA so that AB is the length of the shadow of the pole such that AB = BP. Let θ be required angle of elevation of the Sun's altitude so that ∠PAB = θ.



In ΔPAB, we have

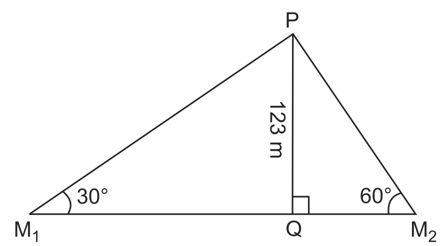
$$\tan\theta = \frac{PB}{AB} = \frac{PB}{PB} = 1 = \tan 45°$$

$$\therefore \quad \theta = 45°$$

Hence, the required angle of elevation is 45°.

Short Answer Type-I Questions

14. In the given figure, two men M_1 and M_2 are standing on opposite sides of a tower PQ of height 123 m. Find the distance between the men.



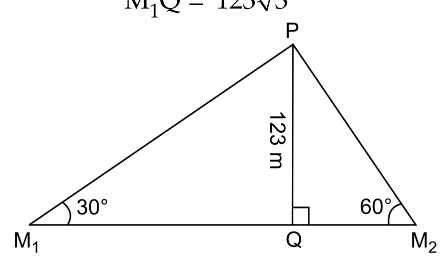
Sol. In PM_1Q, we have

$$\tan(\angle PM_1Q) = \frac{PQ}{M_1Q}$$

$$\Rightarrow \quad \tan 30° = \frac{123}{M_1Q}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{123}{M_1Q}$$

$$\Rightarrow \quad M_1Q = 123\sqrt{3} \qquad \ldots(1)$$



In, ΔPQM_2, we have

$$\tan(\angle PM_2Q) = \frac{PQ}{M_2Q}$$

$$\Rightarrow \quad \tan 60° = \frac{123}{M_2Q}$$

$$\Rightarrow \quad \sqrt{3} = \frac{123}{M_2Q}$$

$$\Rightarrow \quad M_2Q = \frac{123}{\sqrt{3}} = \frac{123\sqrt{3}}{3}$$

$$= 41\sqrt{3} \qquad \ldots(2)$$

Hence, $M_1M_2 = M_1Q + M_2Q$

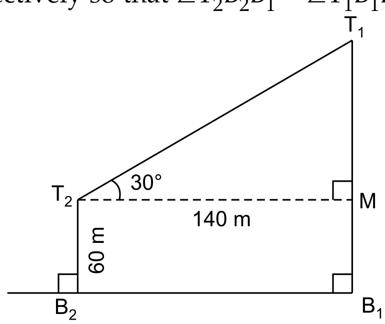
$$= 123\sqrt{3} + 41\sqrt{3}$$

$$= 164\sqrt{3}$$

Hence, the required distance between the men is $164\sqrt{3}$ m.

15. The horizontal distance between two towers of different heights is 140 m. The angle of elevation of the top of the first tower when seen from the top of the second tower is 30°. If the height of the second tower is 60 m, show that the height of the first tower is $\frac{20(7\sqrt{3}+9)}{3}$ m. **[CBSE SP 2011]**

Sol. Let T_1B_1 and T_2B_2 be two vertical towers standing on the horizontal ground B_2B_1 at B_1 and B_2 respectively so that $\angle T_2B_2B_1 = \angle T_1B_1B_2 = 90°$.



We draw T_2M horizontal, where M is a point on T_1B_1.

$\therefore$ $T_2M \parallel B_2B_1$.

It is given that

$$\angle T_1T_2M = 30°,$$

$$T_2M = 140 \text{ m}$$

and $T_2B_2 = 60$ m $\qquad \ldots(1)$

Since, $T_1MT_2 = 90°$,

$\therefore$ In ΔT_1T_2M, we have

$$\tan(T_1T_2M) = \frac{T_1M}{T_2M}$$

$$\Rightarrow \quad \tan 30° = \frac{T_1M}{140}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{T_1M}{140}$$

$$\therefore \quad T_1M = \frac{140}{\sqrt{3}} \qquad \ldots(2)$$

$\therefore$ Required height of the tower T_1B_1 is

$$T_1B_1 = T_1M + MB_1$$

$$= T_1M + T_2B_2$$

$[\because T_2MB_1B_2$ is a rectangle$]$

$$= \left(\frac{140}{\sqrt{3}} + 60\right) \text{m}$$

[From (1) and (2)]

$$= \frac{140 + 60\sqrt{3}}{\sqrt{3}} \text{ m}$$

$$= \frac{\left(140 + 60\sqrt{3}\right)\sqrt{3}}{3} \text{ m}$$

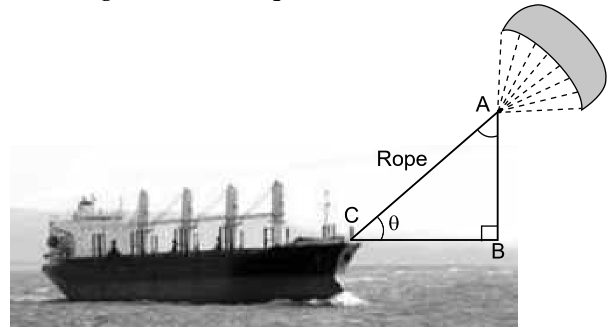
$$= \frac{140\sqrt{3} + 180}{3} \text{ m}$$

$$= \frac{20\left(7\sqrt{3} + 9\right)}{3} \text{ m}$$

Hence, the height of the first tower is $\frac{20\left(7\sqrt{3}+9\right)}{3}$ m.

16. 'Skysails' is that genre of engineering science that uses extensive utilization of wind energy to move a vessel in the sea water. The 'Skysails' technology allows the towing kite to gain a height of anything between 100 metres – 300 metres. The sailing kite is made in such a way that it can be raised to its proper elevation and then brought back with the help of a 'telescopic mast' that enables the kite to be raised properly and effectively.

Based on the following figure related to sky sailing, answer the questions:



(*a*) In the given figure, if $\sin \theta = \cos(3\theta - 30°)$, where θ and $3\theta - 30°$ are acute angles, then find the value of θ.

(*b*) What should be the length of the rope of the kite sail in order to pull the ship at the angle θ

(calculated above) and be at a vertical height of 200 m? **[CBSE SP(Standard) 2019]**

Sol. (*a*) $\sin\theta = \cos(3\theta - 30°)$

$\Rightarrow \quad \cos(90° - \theta) = \cos(3\theta - 30°)$

$[\because \sin\theta = \cos(90° - \theta)]$

$\Rightarrow \quad 90° - \theta = 3\theta - 30° \Rightarrow \theta = 30°$

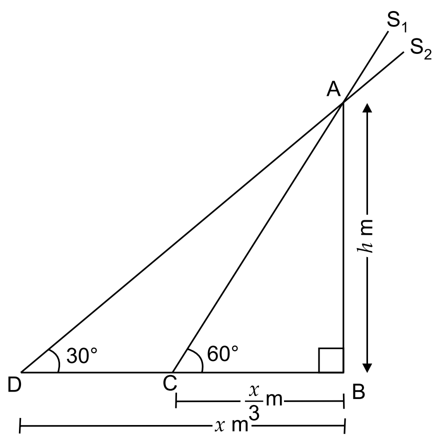
(*b*) $\dfrac{AB}{AC} = \sin 30°$

$\therefore$ Length of rope = AC = 400 m

Short Answer Type-II Questions

17. The length of the shadow of a tower at a particular time is one-third of its shadow, when the Sun's rays meet the ground at an angle of 30°. Find the angle between the Sun's rays and the ground at the time of shorter shadow.

Sol. Let AB be the vertical tower of height h m, BC is the length of the shadow when the position of the Sun is at S_1 and the Sun's ray marks an angle θ with the horizontal ground CB. BD is the length of the shadow when the position of the Sun is at S_2 and the Sun's ray AD makes an angle 30° with the ground DB.



Hence, $\angle ADB = 30°$ and $\angle ACB = \theta$.

Also, $\angle ABC = \angle ABD = 90°$. Let BD = x m.

Then BC = $\dfrac{x}{3}$ m.

Now, in ΔACB, we have

$$\tan(\angle ACB) = \frac{AB}{BC}$$

$$\Rightarrow \quad \tan\theta = \frac{h}{\frac{x}{3}} = \frac{3h}{x} \qquad \ldots(1)$$

Also, in ΔADB, we have,

$$\tan(\angle ADB) = \frac{AB}{DB}$$

$$\Rightarrow \quad \tan 30° = \frac{h}{x}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{x} \qquad \ldots(2)$$

$\therefore$ From (1) and (2), we have

$$\tan\theta = \frac{3}{\sqrt{3}} = \sqrt{3} = \tan 60°$$

$$\therefore \quad \theta = 60°$$

Hence, the required angle is 60°.

18. A man on the roof of a house which is 10 m high observes the angle of elevation of the top of a building as 45° and the angle of depression of the base of the building as 30°. Find the height of the building and its distance from the house. **[CBSE 2000]**

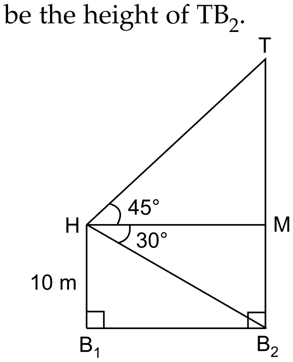
Sol. Let HB_1 be the house standing vertically on the horizontal ground B_1B_2 at the point B_1 such that

$$HB_1B_2 = 90°$$

and $\quad HB_1 = 10$ m

Let TB_2 be the building vertically on the ground at the point B_2 such that $\angle TB_2B_1 = 90°$, $\angle THM = 45°$ $\angle MHB_2 = 30°$, where HM is horizontal through H and M is a point on TB_2.

Let h m be the height of TB_2.



Now, in ΔTHM, we have

$$\tan(\angle THM) = \frac{TM}{HM}$$

$$\Rightarrow \quad \tan 45° = \frac{TM}{HM}$$

$$\Rightarrow \quad 1 = \frac{TM}{B_1B_2}$$

$[\because B_1B_2MH$ is a rectangle$]$

$$\therefore \quad B_1B_2 = TM \qquad \ldots(1)$$

Again, in HMB_2, we have

$$\tan(\angle MHB_2) = \frac{MB_2}{HM}$$

$$\Rightarrow \quad \tan 30° = \frac{MB_2}{B_1B_2}$$

$$\Rightarrow \qquad \frac{1}{\sqrt{3}} = \frac{MB_2}{B_1B_2} = \frac{MB_2}{TM} \qquad \text{[From (1)]}$$

$$= \frac{10}{TM} \qquad [\because MB_2 = HB_1 = 10]$$

Hence, $TM = 10\sqrt{3}$.

$$\Rightarrow \qquad B_1B_2 = TM = 10\sqrt{3}$$

Now, $\qquad TB_2 = TM + MB_2 = TM + HB_1$

$$= 10\sqrt{3} + 10$$

$$= 10\left(\sqrt{3} + 1\right)$$

Hence, the required height of the building is $10\left(\sqrt{3} + 1\right)$ m and its distance from the house is $10\sqrt{3}$ m.

Long Answer Type Questions

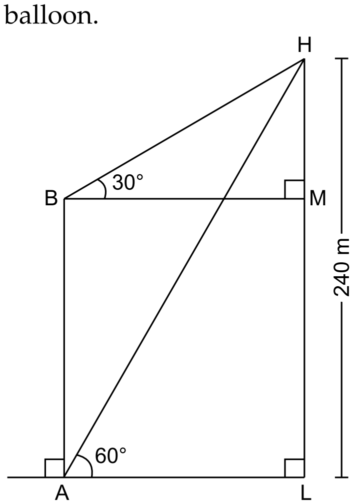
19. The height of a hill is 240 m above the level of a horizontal plane. From a point A on this plane, the angular elevation of the top of the hill is 60°. A balloon rises from A and ascends vertically upwards at a uniform rate; after $2\frac{1}{12}$ minutes, the angular elevation of the top of the hill to an observer in the balloon is 30°. Find the speed of ascent of the balloon in m/s.

Sol. Let HL be the vertical hill standing on the horizontal ground AL such that ∠HAL = 60° and ∠ALH = 90°.

A balloon ascends vertically upwards along AB at a uniform speed and comes to B after $2\frac{1}{12}$ minutes where ∠BAL = 90°. We draw BM horizontal through B where M is a point on HL.

Given that ∠HBM = 30°, HL = 240 m.

Let HM = h m and v m/min be the uniform speed of the balloon.



Since, ABML is a rectangle, hence, ML = AB = $2\frac{1}{12}$ metres.

Now, in ΔHBM, we have,

$$\tan(\angle HBM) = \frac{HM}{BM}$$

$$\Rightarrow \qquad \tan 30° = \frac{h}{BM}$$

$$\Rightarrow \qquad \frac{1}{\sqrt{3}} = \frac{h}{BM}$$

$$\therefore \qquad BM = h\sqrt{3} \qquad \ldots(1)$$

Also, in ΔHAL, we have

$$\tan(\angle HAL) = \frac{HL}{AL}$$

$$\Rightarrow \qquad \tan 60° = \frac{HL}{BM} \qquad [\because AL = BM]$$

$$\Rightarrow \qquad \sqrt{3} = \frac{HL}{h\sqrt{3}} \qquad \text{[From (1)]}$$

$$\therefore \qquad HL = 3h$$

$$\Rightarrow \qquad 240 = 3h$$

$$\Rightarrow \qquad h = 80 \qquad \ldots(2)$$

$$\therefore \qquad AB = ML$$

$$= HL - HM$$

$$= 240 - h$$

$$= 240 - 80 = 160 \qquad \text{[From (2)]}$$

$$\therefore \qquad v = \frac{AB}{2\frac{1}{12} \times 60} \text{ m/s}$$

$$= \frac{160}{\frac{25}{12} \times 60} \text{ m/s}$$

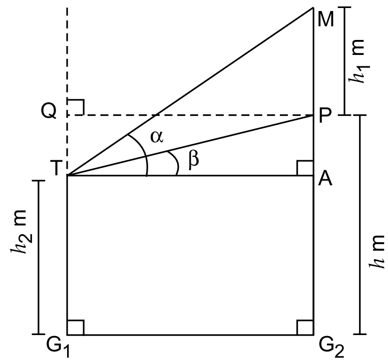
$$= \frac{160 \times 12}{25 \times 60} \text{ m/s}$$

$$= 1.28 \text{ m/s}$$

Hence, the required speed is 1.28 m/s.

20. A marble statue of height h_1 metres is mounted on a pedestal. The angles of elevation of the top and bottom of the statue from a point h_2 metres above the ground level are α and β respectively. Show that the height of the pedestal in metres is $\dfrac{(h_1 - h_2)\tan\beta + h_2\tan\alpha}{\tan\alpha - \tan\beta}$.

Sol. Let PQ be the horizontal pedestal and PM be the marble statue of height h_1 on it so that PM = h_1 m.



M is the top and P is the bottom of the statue. G_1G_2 is the horizontal ground and $TG_1 \perp G_1G_2$ and $TG_1 = h_2$ m. Given that the angles of elevation of M and P of the temple from the point T are α and β respectively so that $\angle MTA = \alpha$ and $\angle PTA = \beta$ where $MPAG_2$ is a straight line and $MG_2 \perp G_1G_2$. Also, $\angle PAT = 90°$. Now let the height of the pedestal from the ground be h m so that $PG_2 = h$ m.

Now, since G_1G_2 AT is a rectangle,

$$\therefore \quad AG_2 = TG_1 = h_2$$

$$\therefore \quad AP = PG_2 - AG_2 = h - h_2$$

$$MA = MP + AP$$

$$= h_1 + h - h_2$$

$$= (h_1 - h_2) + h$$

Now, in ΔMTA, we have

$$\tan(\angle MTA) = \frac{MA}{AT}$$

$$\Rightarrow \quad \tan\alpha = \frac{(h_1 - h_2) + h}{AT} \quad \ldots(1)$$

Also, in ΔPTA, we have $\tan(\angle PTA) = \dfrac{PA}{AT}$

$$\Rightarrow \quad \tan\beta = \frac{h - h_2}{AT} \quad \ldots(2)$$

$\therefore$ Dividing (1) by (2), we get

$$\frac{\tan\alpha}{\tan\beta} = \frac{(h_1 - h_2) + h}{h - h_2}$$

$$\Rightarrow h\tan\alpha - h_2\tan\alpha = (h_1 - h_2)\tan\beta + h\tan\beta$$

$$\Rightarrow h(\tan\alpha - \tan\beta) = h_2\tan\alpha + (h_1 - h_2)\tan\beta$$

$$\Rightarrow h = \frac{h_2\tan\alpha + (h_1 - h_2)\tan\beta}{\tan\alpha - \tan\beta}$$

Hence, proved.

Let's Compete

(Page 181)

Multiple-Choice Questions

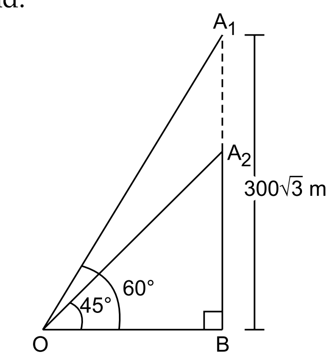
1. An aeroplane when $300\sqrt{3}$ m high passes vertically above another aeroplane at an instant when the angles of elevation of the two aeroplanes from the same point on the ground are 60° and 45° respectively. Then the vertical distance between the two aeroplanes in metres is

(*a*) $300(\sqrt{3}-1)$ m (*b*) $300(\sqrt{2}-1)$ m

(*c*) $200\sqrt{3}$ m (*d*) $200(\sqrt{3}-1)$ m

Sol. (*a*) $300(\sqrt{3}-1)$ m

Let A_1 and A_2 be the positions of two aeroplanes, A_1 being vertically above A_2 and the straight line A_1A_2B is perpendicular to OB, the horizontal ground.



It is given that $A_1B = 300\sqrt{3}$ m, $\angle A_1OB = 60°$ and $\angle A_2OB = 45°$.

Now, in ΔA_1OB, we have

$$\tan(\angle A_1OB) = \frac{A_1B}{OB}$$

$$\Rightarrow \quad \tan 60° = \frac{300\sqrt{3}}{OB}$$

$$\Rightarrow \quad \sqrt{3} = \frac{300\sqrt{3}}{OB}$$

$$\Rightarrow \quad OB = 300 \quad \ldots(1)$$

Now, in ΔA_2OB, we have

$$\tan(\angle A_2OB) = \frac{A_2B}{OB} = \frac{A_2B}{300} \quad \text{[From (1)]}$$

$$\Rightarrow \quad \tan 45° = \frac{A_2B}{300}$$

$$\Rightarrow \quad 1 = \frac{A_2B}{300}$$

$$\therefore \quad A_2B = 300 \quad \ldots(2)$$

Now, $\quad A_1A_2 = A_1B - A_2B$

$$= \left(300\sqrt{3} - 300\right) \text{ m} \quad [\text{From (2)}]$$

$$= 300\left(\sqrt{3} - 1\right) \text{ m}$$

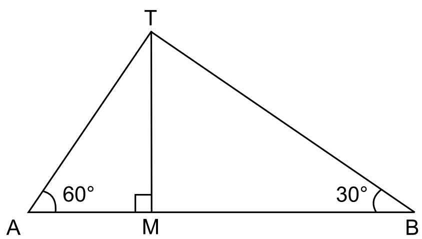
2. There is a small island in the middle of a river, h m wide. A tall tree stands on the island. A and B are points directly opposite to each other on the two banks, and in line with the tree. If the angles of elevation of the top of the tree from A and B are 60° and 30° respectively, then the height of the tree in metres is

(*a*) $\dfrac{2}{\sqrt{3}}h$ (*b*) $\dfrac{4}{\sqrt{3}}h$

(*c*) $\dfrac{\sqrt{3}}{4}h$ (*d*) $\dfrac{\sqrt{2}}{4}h$

Sol. (*c*) $\dfrac{\sqrt{3}}{4}h$

Let A and B be two points on the same line directly opposite to each other on the two banks of the river such that AB = h m. TM is the vertical tree on an island in the river such that ∠TAM = 60° and ∠TBM = 30°.



Now, in ΔATM, we have

$$\tan(\angle TAM) = \frac{TM}{AM}$$

$$\Rightarrow \quad \tan 60° = \frac{TM}{AM}$$

$$\Rightarrow \quad \sqrt{3} = \frac{TM}{AM}$$

$$\Rightarrow \quad AM = \frac{TM}{\sqrt{3}} \qquad \ldots(1)$$

In ΔBTM, we have

$$\tan(\angle TBM) = \frac{TM}{BM}$$

$$\Rightarrow \quad \tan 30° = \frac{TM}{BM}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{TM}{BM}$$

$$\therefore \quad BM = TM\sqrt{3} \qquad \ldots(2)$$

Adding (1) and (2) we get

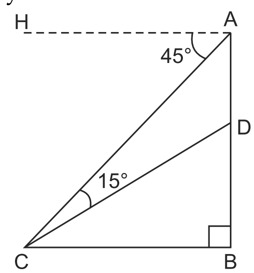
$$AM + BM = TM\left(\frac{1}{\sqrt{3}} + \sqrt{3}\right) = \frac{4TM}{\sqrt{3}}$$

$$\Rightarrow \quad AB = h = \frac{4TM}{\sqrt{3}}$$

$$\Rightarrow \quad TM = \frac{\sqrt{3}h}{4}$$

Hence, the height of the tree in $\dfrac{\sqrt{3}h}{4}$ m.

3. In the given figure, if DB = $\sqrt{3}$ m, then the measure of BC and the angle of depression of the point C when observed from the point D are respectively



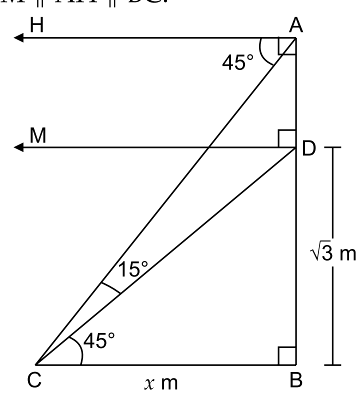
(*a*) $\sqrt{3}$ m, 45° (*b*) 3 m, 30°

(*c*) $3\sqrt{3}$ m, 30° (*d*) $\dfrac{\sqrt{3}}{3}$ m, 45°

Sol. (*b*) 3 m, 30°

We draw DM horizontal through D, a point on AB such that DB = $\sqrt{3}$ m.

∴ DM ∥ AH ∥ BC.



The angle of depression of the point C when observed from D = ∠MDC = ∠DCB

$$= \angle ACB - \angle ACD$$

$$= 45° - 15° = 30°$$

Let $\quad BC = x$ m

Now, in ΔDCB, we have

$$\tan(\angle DCB) = \frac{DB}{BC}$$

$$\Rightarrow \qquad \tan 30° = \frac{\sqrt{3}}{x}$$

$$\Rightarrow \qquad \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{x}$$

$$\Rightarrow \qquad x = 3$$

$\therefore$ BC = 3 m and angle of depression = 30°.

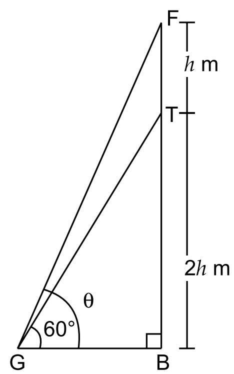
4. If the height of a flagstaff is half the height of the tower on which it is fixed and the angle of elevation of the top of the tower as seen from a point on the ground is 60°, then the angle of elevation θ of the top of the flagstaff as seen from the same point is given by the equation cosec θ = a where a is equal to

(*a*) $\frac{3\sqrt{3}}{\sqrt{31}}$ (*b*) $\sqrt{\frac{3}{31}}$

(*c*) $\frac{31}{3\sqrt{3}}$ (*d*) $\frac{\sqrt{31}}{3\sqrt{3}}$

Sol. (*d*) $\frac{\sqrt{31}}{3\sqrt{3}}$

Let TB be the vertical tower of height $2h$ m standing at a point B on the horizontal ground GB so that ∠TBG = 90°.



TF is the flagstaff on the top of the tower of height h m such that FTB is a straight line.

Given that ∠TGB = 60°.

Let ∠FGB = θ.

Now, in ΔTGB, we have

$$\tan(\angle TGB) = \frac{TB}{GB}$$

$$\Rightarrow \qquad \tan 60° = \frac{2h}{GB}$$

$$\Rightarrow \qquad \sqrt{3} = \frac{2h}{GB} \qquad \ldots(1)$$

Also, in ΔFGB, we have

$$\tan(\angle FGB) = \frac{FB}{GB}$$

$$\Rightarrow \qquad \tan\theta = \frac{3h}{GB} \qquad \ldots(2)$$

Dividing (1) by (2), we get

$$\frac{\sqrt{3}}{\tan\theta} = \frac{2}{3}$$

$$\Rightarrow \qquad \tan\theta = \frac{3\sqrt{3}}{2}$$

$$\therefore \qquad \cot\theta = \frac{2}{3\sqrt{3}} \qquad \ldots(3)$$

$$\therefore \qquad \text{cosec}\,\theta = \sqrt{1+\cot^2\theta}$$

$$= \sqrt{1+\frac{4}{27}} \qquad \text{[From (3)]}$$

$$= \sqrt{\frac{31}{27}}$$

$$= \frac{\sqrt{31}}{3\sqrt{3}}$$

$$\therefore \qquad a = \frac{\sqrt{31}}{3\sqrt{3}} \qquad [\because a = \text{cosec}\,\theta]$$

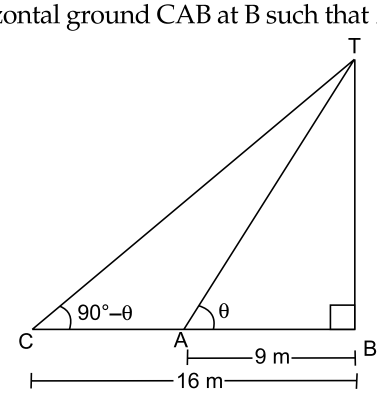
5. If the angles of elevation of the top of a tower from two points at distances of 9 m and 16 m from the base of the tower and in the same line and in the same direction of the tower, are complementary, then the height of the tower is

(*a*) 15 m (*b*) 14 m

(*c*) 12 m (*d*) 20 m

Sol. (*c*) 12 m

Let TB be the vertical tower standing on the horizontal ground CAB at B such that ∠CBT = 90°.



A and B are two points on the ground lying on the same line CAB such that ∠TAB = θ and ∠TCB = 90° – θ.

Given that AB = 9 m and BC = 16 m.

In ΔTAB, we have

$$\tan(\angle TAB) = \frac{TB}{AB}$$

$$\Rightarrow \qquad \tan\theta = \frac{TB}{9} \qquad \ldots(1)$$

and in ΔTCB we have

$$\tan(\angle TCB) = \frac{TB}{BC}$$

$$\Rightarrow \qquad \tan(90° - \theta) = \frac{TB}{16}$$

$$\Rightarrow \qquad \cot\theta = \frac{TB}{16}$$

$$\Rightarrow \qquad \frac{1}{\tan\theta} = \frac{TB}{16} \qquad \ldots(2)$$

Multiplying (1) and (2), we get

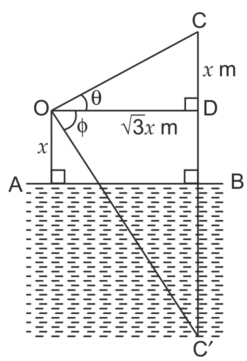
$$1 = \frac{TB^2}{16 \times 9}$$

$$\Rightarrow \qquad TB = \sqrt{16 \times 9}$$

$$= 4 \times 3 = 12$$

$\therefore$ The height of the tower is 12 m.

6. In the figure given below, C′ is reflection of the cloud C in the lake with surface of water along the horizontal ground AB. If the angle of elevation of C and the angle of depression of the point C′ from the same point of observation O be θ and ϕ respectively, and if CD = x m, OD = $\sqrt{3}\,x$ m, then $\theta + \phi$ is equal to



(*a*) 45° (*b*) 60°

(*c*) 30° (*d*) 90°

Sol. (*d*) 90°

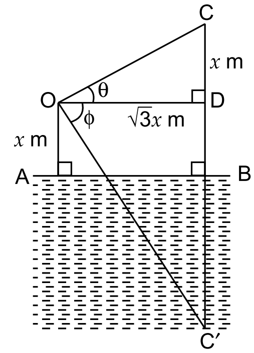
In ΔCOD, we have

$$\tan(\angle COD) = \frac{CD}{OD}$$

$$\Rightarrow \qquad \tan\theta = \frac{x}{\sqrt{3}x}$$

$$= \frac{1}{\sqrt{3}} = \tan 30°$$

$$\therefore \qquad \theta = 30° \qquad \ldots(1)$$



In $\Delta OC'D$, we have

$$\tan(\angle DOC') = \frac{DC'}{OD} = \frac{DC'}{\sqrt{3}x} \qquad \ldots(2)$$

Now, $\quad DC' = CC' - CD$

$= 2BC - CD$

$= 2 \times 2x - x$

$= 3x$

$\therefore$ From (2),

$$\tan\phi = \frac{3x}{\sqrt{3}x} = \sqrt{3} = \tan 60°$$

$$\therefore \qquad \phi = 60°$$

$$\therefore \qquad \theta + \phi = 30° + 60° = 90°$$

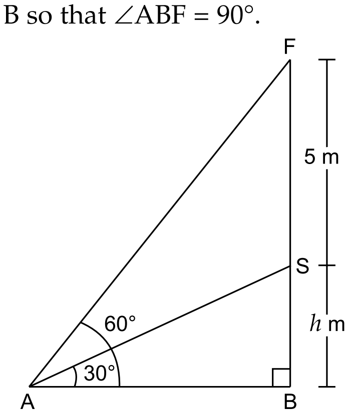
7. A vertical steel rod stands on a horizontal plane and is surmounted by a vertical flagstaff of height 5 m. At a point on the plane the angles of elevation of the bottom and the top of the flagstaff are 30° and 60° respectively. Then the height of the steel rod is

(*a*) 1.7 m (*b*) 2.5 m

(*c*) 2 m (*d*) 1.5 m

Sol. (*b*) 2.5 m

Let h m be the height of the vertical steel rod BS standing on the horizontal ground AB at the point B so that $\angle ABF = 90°$.



Let SF be the height of the flagstaff so that SF = 5 m and FSB is a straight line. A is a point on the ground such that $\angle SAB = 30°$ and $\angle FAB = 60°$.

Now, in ΔSAB, we have

$$\tan(\angle SAB) = \frac{SB}{AB}$$

$$\Rightarrow \quad \tan 30° = \frac{h}{AB}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{AB}$$

$$\therefore \quad AB = h\sqrt{3} \quad \ldots(1)$$

Again, in ΔFAB, we have

$$\tan(\angle FAB) = \frac{BF}{AB}$$

$$\Rightarrow \quad \tan 60° = \frac{5+h}{\sqrt{3}h} \quad \text{[From (1)]}$$

$$\Rightarrow \quad \sqrt{3} = \frac{5+h}{\sqrt{3}h}$$

$$\Rightarrow \quad 3h = 5 + h$$

$$\Rightarrow \quad 2h = 5$$

$$\Rightarrow \quad h = \frac{5}{2} = 2.5$$

Hence, the required height of the steel rod is 2.5 m.

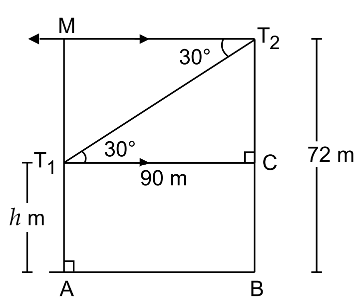
8. The horizontal distance between two trees of different heights is 90 m. The angle of depression of the top of the first tree when seen from the top of the second tree is 30°. If the height of the second tree is 72 m, then the height of the first tree will be

(*a*) $\left(72 - 30\sqrt{3}\right)$ m (*b*) 30 m

(*c*) $\frac{30}{\sqrt{3}}$ m (*d*) $25\sqrt{3}$ m

Sol. (*a*) $\left(72 - 30\sqrt{3}\right)$ m

Let T_1A be the first and T_2B be the second of the two vertical trees standing on the horizontal ground AB at A and B respectively. T_1C is drawn horizontal so that $T_1C \parallel AB$, C being a point on BT_2.



Clearly, $\angle ABC = \angle T_1CT_2 = \angle T_1AB$
$= \angle MT_1C = 90°$

where T_2M is drawn parallel to AB.

$\Rightarrow$ $T_2M \parallel CT_1 \parallel AB$.

Given that $T_1C = 90$ m, $T_2B = 72$ m, $\angle MT_2T_1 = \angle T_2T_1C = 30°$.

Let $T_1A = h$ m.

Now, in ΔT_1T_2C, we have

$$\tan(\angle T_2T_1C) = \frac{T_2C}{T_1C}$$

$$\Rightarrow \quad \tan 30° = \frac{72-h}{90}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{72-h}{90}$$

$$\Rightarrow \quad 72\sqrt{3} - h\sqrt{3} = 90$$

$$\Rightarrow \quad h = \frac{72\sqrt{3} - 90}{\sqrt{3}}$$

$$= 72 - \frac{90\sqrt{3}}{3}$$

$$= 72 - 30\sqrt{3}$$

Hence, the height of the first tree is $\left(72 - 30\sqrt{3}\right)$ m.

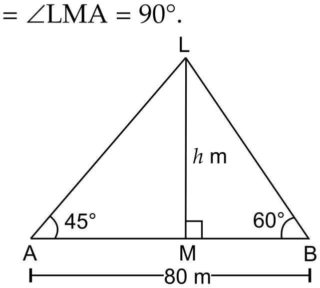
9. Two boats approach a lighthouse in mid sea from opposite directions. The angles of elevation of the top of the lighthouse from two boats are 45° and 60° respectively. If the distance between the two boats is 80 m, then the height of the lighthouse is

(*a*) 41.7 m (*b*) 40 m

(*c*) $40\left(3+\sqrt{3}\right)$ m (*d*) $40\left(3-\sqrt{3}\right)$ m

Sol. (*d*) $40(3 - \sqrt{3})$ m

Let LM be the vertical lighthouse in mid sea and A, B are two boats on the opposite sides of the lighthouse. The two boats are on the two opposite sides of the lighthouse and these two boats are approaching towards the lighthouse so that A, M and B may lie on the same straight line. Given that AB = 80 m, $\angle LAM = 45°$ and $\angle LBM = 60°$. Let h m be the height of the lighthouse. We have $\angle LMB = \angle LMA = 90°$.



Now, in ΔLAM, we have

$$\tan(\angle LAM) = \frac{LM}{AM}$$

$$\Rightarrow \quad \tan 45^\circ = \frac{h}{AM}$$

$$\Rightarrow \quad 1 = \frac{h}{AM}$$

$$\therefore \quad h = AM \quad \ldots(1)$$

Again, in ΔLMB, we have

$$\tan(\angle LMB) = \frac{LM}{MB}$$

$$\Rightarrow \quad \tan 60^\circ = \frac{h}{MB}$$

$$\Rightarrow \quad \sqrt{3} = \frac{h}{MB}$$

$$\Rightarrow \quad MB = \frac{h}{\sqrt{3}} \quad \ldots(2)$$

$\therefore$ From (1) and (2) we have

$$AM + MB = AB = 80$$

$$\Rightarrow \quad h + \frac{h}{\sqrt{3}} = 80$$

$$\Rightarrow \quad h.\frac{1+\sqrt{3}}{\sqrt{3}} = 80$$

$$\Rightarrow \quad h = \frac{80\sqrt{3}}{\sqrt{3}+1}$$

$$= \frac{80\sqrt{3}\left(\sqrt{3}-1\right)}{3-1}$$

$$= 40\sqrt{3}\left(\sqrt{3}-1\right)$$

$$= 40\left(3-\sqrt{3}\right)$$

$\therefore$ The required height of the lighthouse is $40\left(3-\sqrt{3}\right)$ m.

10. Two poles of equal heights are standing opposite to each other on either side of a road which is 100 m wide. From a point between them on the road, the angles of elevation of their tops are 30° and 60°. Then the height of each pole is

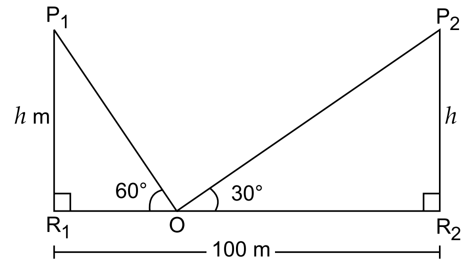
(*a*) $\left(25-\sqrt{3}\right)$ m (*b*) $\left(25+\sqrt{3}\right)$ m

(*c*) $25\sqrt{3}$ m (*d*) $\frac{25}{\sqrt{3}}$ m

Sol. (*c*) $25\sqrt{3}$ m

Let R_1R_2 be the width of the horizontal road such that $R_1R_2 = 100$ m.

R_1P_1 and R_2P_2 are two vertical poles of equal height h m standing opposite to each other on the two sides of the road. Let O be a point on R_1R_2 such that $\angle P_1OR_1 = 60^\circ$ and $\angle P_2OR_2 = 30^\circ$. Also, $\angle P_1R_1O = \angle P_2R_2O = 90^\circ$.



Now, in ΔP_1R_1O, we have

$$\tan(\angle P_1OR_1) = \frac{P_1R_1}{R_1O}$$

$$\Rightarrow \quad \tan 60^\circ = \frac{h}{R_1O}$$

$$\Rightarrow \quad \sqrt{3} = \frac{h}{R_1O}$$

$$\Rightarrow \quad R_1O = \frac{h}{\sqrt{3}} \quad \ldots(1)$$

Again, in ΔP_2R_2O, we have

$$\tan(\angle P_2OR_2) = \frac{P_2R_2}{OR_2}$$

$$\Rightarrow \quad \tan 30^\circ = \frac{h}{OR_2}$$

$$\Rightarrow \quad OR_2 = h\sqrt{3} \quad \ldots(2)$$

Adding (1) and (2), we get

$$R_1O + OR_2 = R_1R_2 = 100$$

$$\Rightarrow \quad \frac{h}{\sqrt{3}} + h\sqrt{3} = 100$$

$$\Rightarrow \quad h\left(\frac{1+3}{\sqrt{3}}\right) = 100$$

$$\Rightarrow \quad h = \frac{100\sqrt{3}}{4}$$

$$= 25\sqrt{3}$$

Hence, the required height of each pole is $25\sqrt{3}$ m.

Value-based Questions (Optional)

(Page 182)

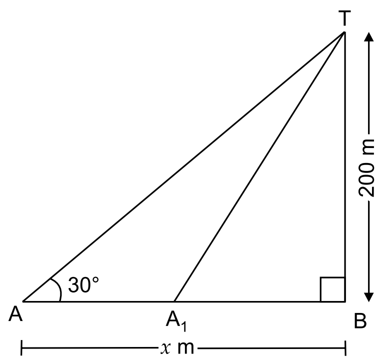
1. A guard observes an enemy boat from the top of a 200 m high observation tower. He finds the angle of depression of the boat to be 30°.

(*a*) Calculate the distance of the boat from the foot of the observation tower.

(*b*) He raises an alarm when the distance of the enemy boat from the foot of the tower reduces by $200\left(\sqrt{3}-1\right)$ m. What is the new angle of depression of the boat from the top of the observation tower?

(*c*) What is the value shown by the guard?

Sol. (*a*) Let BT be the vertical observation tower in the mid sea and AB is the horizontal surface of the sea. Let A be the initial position of the enemy boat such that $\angle TAB = 30°$. Also, $\angle TBA = 90°$ and TB = 200 m.



Let AB = x m.

In ΔTAB, we have

$$\tan(\angle TAB) = \frac{TB}{AB}$$

$$\Rightarrow \quad \tan 30° = \frac{TB}{x}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{TB}{x}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{200}{x}$$

$$\Rightarrow \quad x = 200\sqrt{3}$$

Hence, the required distance of the boat A from B is $200\sqrt{3}$ m.

(*b*) The guard raises an alarm, when the boat comes to A_1 when $AA_1 = \left(\sqrt{3}-1\right)$ m

So that $\quad A_1B = AB - AA_1$

$$= \left\{200\sqrt{3} - 200\left(\sqrt{3}-1\right)\right\} \text{ m}$$

$$= 200 \text{ m} \qquad \ldots(1)$$

$\therefore$ In ΔTA_1B, we have

$$\tan(\angle TA_1B) = \frac{TB}{A_1B} = \frac{200}{200} \qquad \text{[From (1)]}$$

$$= 1 = \tan 45°$$

$$\therefore \quad \angle TA_1B = 45°$$

Hence, the required new angle of depression of the boat from the top of the observation tower is 45°.

(*c*) Responsibility and decision-making

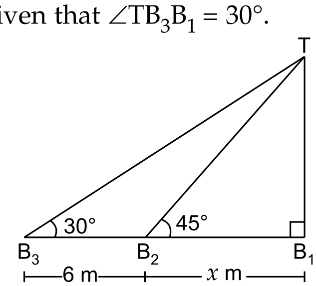
2. Two villages were connected by a small canal full of water. There was only one school in one of the villages. The children of another village could not attend this school, because there was no bridge over the canal. So, the poor children were facing tremendous difficulties, to get education. An NGO, after knowing about this difficulty, came to visit the place. They decided to construct a temporary bridge at a cheap rate with bamboo sticks of shortest possible lengths for the convenience of the children. So, they wanted to find the breadth of the canal cleverly. An expert in the NGO observed the top of a tall tree just on the opposite bank of the canal from one side of the canal where all other people were standing. He measured the angle of elevation of the top of the tree with his sextant instrument to be 45°. On receding 6 m from the bank, perpendicular to its edge, he found the angle of elevation of the top of the same tree to be 30°. Finally, he found the breadth of the canal and hence, the minimum length of each bamboo pole to build the temporary bridge. Finally, the NGO was successful in constructing the bridge at a minimum cost.

(*a*) Find the breadth of the river and hence the minimum length of each bamboo pole.

(*b*) Which values of the NGO were depicted in this problem?

Sol. Let B_1 and B_2 be two points on two opposite sides of the canal such that B_1B_2 is perpendicular to the flow of water in the canal. Let B_1T be the vertical height of a tree on one side of the bank. B_2 is a point on just opposite side of the canal such that $\angle TB_2B_1 = 45°$. B_3 is a point on B_1B_2 produced such that $B_2B_3 = 6$ m. Let B_1B_2 = breadth of the river = x m.

Also, given that $\angle TB_3B_1 = 30°$.



(*a*) Now, in ΔTB_1B_2, we have

$$\tan(\angle TB_2B_1) = \frac{TB_1}{B_1B_2}$$

$$\Rightarrow \quad \tan 45° = \frac{TB_1}{x}$$

$$\Rightarrow \quad 1 = \frac{TB_1}{x}$$

$$\therefore \quad TB_1 = x \qquad \ldots(1)$$

In ΔTB_3B_1, we have

$$\tan(\angle TB_3B_1) = \frac{TB_1}{B_3B_1}$$

$$\Rightarrow \quad \tan 30° = \frac{x}{x+6}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{x}{x+6}$$

$$\Rightarrow \quad x + 6 = \sqrt{3}\,x$$

$$\therefore \quad x = \frac{6}{\sqrt{3}-1}$$

$$= \frac{6\left(\sqrt{3}+1\right)}{3-1}$$

$$= 3\left(\sqrt{3}+1\right)$$

$$= 3(1.73 + 1)$$

$$= 3 \times 2.73 = 8.19 \text{ (approx.)}$$

∴ The required breadth of the river = 8.19 m (approx.)

∴ The minimum length of each bamboo pole = 8.19 m(approx.)

(*b*) Kindness, helpfulness and interest in education for the poor children.

10

Circles

Checkpoint (Page 184)

1. The distance of a chord of a circle of radius 5 cm from its centre is 3 cm. What is the length of the chord?

Sol. Let AB be the chord of a circle with centre at O.

$\because$ OM $\perp$ AB

$\therefore$ M is the mid-point of AB. OA is the radius, OA = 5 cm.

In ΔOAM, by using Pythagoras' theorem, we have

$$OA^2 = OM^2 + AM^2$$

$$\Rightarrow \quad (5)^2 = (3)^2 + AM^2$$

$$\Rightarrow \quad AM^2 = 25 \text{ cm}^2 - 9 \text{ cm}^2$$

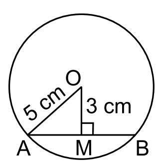
$$\Rightarrow \quad AM^2 = 16 \text{ cm}^2$$

$$\Rightarrow \quad AM = 4 \text{ cm}$$

$$\therefore \quad AB = 2\,AM$$

$$= 2 \times 4 \text{ cm}$$

$$= 8 \text{ cm}$$



Hence, the required length of the chord is 8 cm.

2. A chord of a circle subtends an angle of 60° at the centre of the circle. If the radius of the circle is 4 cm, find the area of the triangle formed by the chord and the two radii of the circle.

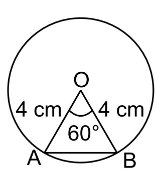
Sol. Let the chord AB subtend an angle of 60° at the centre O, i.e. $\angle AOB = 60°$

$$\text{Radius} = OA = OB$$

$$= 4 \text{ cm}$$

$$\therefore \quad \angle A = \angle B$$

$$= 60°$$



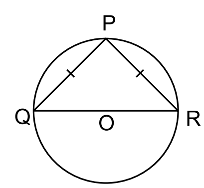
$\therefore$ Area of the equilateral triangle OAB

$$= \frac{\sqrt{3}}{4} \times 4^2 \text{ cm}^2$$

$$= 4\sqrt{3} \text{ cm}^2$$

3. P is a point on the circumference of a circle with diameter QR such that PQ = PR. Then find the angles of the triangle PQR.

Sol. Let the diameter QR through the centre O of the circle form a ΔPQR at a point P on the circle such that PQ = PR.



$$\therefore \quad \angle QPR = 90° \text{ [Angle in a semicircle]}$$

$$\because \quad PQ = PR$$

$$\therefore \quad \angle PQR = \angle PRQ$$

$$= 45°$$

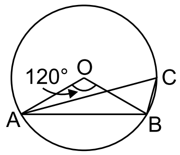
Hence, the angles of the triangle PQR are $\angle P = 90°$, $\angle Q = 45°$ and $\angle R = 45°$.

4. The arc AB of a circle with centre at O subtends an angle 120° at O. If C be a point on the remaining part of the circumference, what is the measure of $\angle ACB$?

Sol. We have $\angle AOB = 120°$

$$\angle AOB = 2\angle ACB$$

[$\because$ The angle subtended by an arc of a circle at the centre is double the angle subtended by it at any point on the remaining part of the circle.]



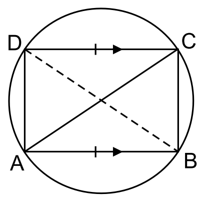
$$\Rightarrow \qquad 120° = 2\,\angle ACB$$

$$\Rightarrow \qquad \angle ACB = \frac{120°}{2} = 60°$$

$\therefore$ The measure of $\angle ACB$ is 60°.

5. AB and CD are two opposite sides of a cyclic quadrilateral such that AB = CD and AB ∥ CD. If one diagonal of the quadrilateral is of length 5 cm, what is the length of the other diagonal? Give reasons.

Sol. Given that AB and CD are two opposite sides of a cyclic quadrilateral such that AB = CD. Let AC and BD be two diagonals of ABCD and AC = 5 cm. To find the length of BD.



We have

$$\angle B + \angle D = 180°$$

[∵ Opposite ∠s of cyclic quadrilateral] …(1)

$$\angle B + \angle C = 180°$$

[∵ AB ∥ CD, Interior ∠s] …(2)

From (1) and (2), we have

$$\angle B + \angle D = \angle B + \angle C$$

$$\Rightarrow \qquad \angle D = \angle C$$

$$\Rightarrow \qquad \angle ADC = \angle BCD \qquad \ldots(3)$$

In ΔADC and ΔBCD

$$\angle ADC = \angle BCD \qquad \text{[From (3)]}$$

$$\angle DAC = \angle CBD$$

[Angles in the same segment of a circle are equal]

$$CD = CD \qquad \text{[Common]}$$

$$\therefore \qquad \Delta ADC \cong \Delta BCD$$

[By AAS criterion of similarity]

$$\therefore \qquad AC = BD \qquad \text{[By CPCT]}$$

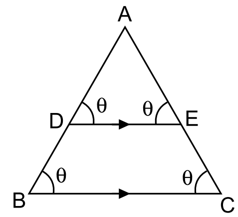
$$\Rightarrow \qquad BD = 5 \text{ cm}$$

$\therefore$ The required length is 5 cm.

6. If a line is drawn parallel to the base of an isosceles triangle to intersect its equal sides, prove that the quadrilateral so formed is cyclic.

Sol. Let ΔABC be an isosceles triangle with AB = AC. D and E are two points on AB and AC respectively such that DE ∥ BC.

To prove that the quadrilateral BCED is cyclic.



We have

$$\angle ABC = \angle ADE \qquad [\because DE \parallel BC]$$

$$= \theta, \text{ say}$$

$$\therefore \qquad \angle ACB = \angle ABC = \theta \qquad \ldots(1)$$

$$\text{But} \qquad \angle ACB = \angle AED \qquad [\because DE \parallel BC]$$

$$\therefore \qquad \angle AED = \theta$$

$$\Rightarrow \qquad \angle DEC = 180° - \theta \qquad \ldots(2)$$

Now, $\angle DBC + \angle DEC$

$$= \angle ABC + \angle DEC$$

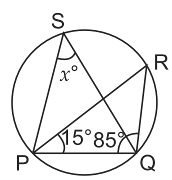
$$= \theta + 180° - \theta \quad \text{[From (1) and (2)]}$$

$$= 180°$$

$\therefore$ DBCE is a cyclic quadrilateral, since, a pair of opposite angles is supplementary.

Hence, proved.

7. In the given figure, find the value of x, if $\angle RPQ = 15°$ and $\angle PQR = 85°$.



Sol. Given that PQ is a chord of a circle and R, S are two points on the circle such that $\angle PSQ = x°$, $\angle PQR = 85°$ and $\angle RPQ = 15°$. To find the value of x.

In ΔPQR, we have

$$\angle PRQ + \angle PQR + \angle RPQ = 180°$$

[Angle sum property of a triangle]

$$\angle PRQ = 180° - (\angle PQR + \angle RPQ)$$

$$= 180° - (85° + 15°)$$

$$= 180° - 100°$$

$$= 80°$$

Now, since $\angle PRQ$ and $\angle PSQ$ stand on the same chord PQ of the circle.

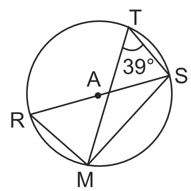
$\therefore \quad \angle PSQ = \angle PRQ$

$\Rightarrow \quad x° = 80°$

$\Rightarrow \quad x = 80$

Hence, the required value of x is 80.

8. In the given figure, A is the centre of a circle and $\angle MTS = 39°$. What is the measure of $\angle MSR$?



Sol. Given that A is the centre of a circle and RAS is a diameter of the circle. M is a point on the circle, such that $\angle MTS = 39°$, where T is a point on the circle on the other side of the diameter RAS.

To find $\angle MSR$.

We have

$\angle RMS = 90°$ ...(1)

[$\because$ RS is a diameter and angle in a semi-circle is 90°]

Now, $\angle MRS$ and $\angle MTS$ stand on the same chord MS of the circle, hence, we have

$\angle MRS = \angle MTS = 39°$ ...(2)

Now, in ΔMSR, we have

$\angle RMS + \angle MRS + \angle MSR = 180°$

[Angle sum property of a triangle]

$\Rightarrow \quad 90° + 39° + \angle MSR = 180°$

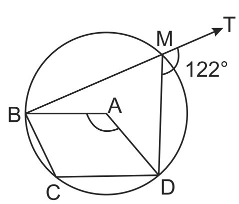
[From (1) and (2)]

$\Rightarrow \quad \angle MSR = 180° - 129°$

$= 51°$

$\therefore$ The measure of $\angle MSR$ is 51°.

9. In the given figure, A is the centre of a circle and $\angle DMT = 122°$. Find the measures of $\angle BAD$ and $\angle BCD$.



Sol. Given that B, C, D and M are points on a circle of centre at A such that ABCD is a quadrilateral. BM is produced to T such that $\angle TMD = 122°$. To find $\angle BAD$ and $\angle BCD$.

We have

$\because$ BCDM is a cyclic quadrilateral,

$\therefore \quad \angle BCD + \angle BMD = 180°$ ...(1)

Also, $\quad \angle BMD + \angle TMD = 180°$

[Linear pair]...(2)

$\therefore$ From (1) and (2), we have

$\angle BCD = \angle TMD$

$= 122°$

$\therefore \quad \angle BMD = 180° - 122°$

[From (1)]

$= 58°$ ...(3)

Now, $\quad \angle BAD = 2\angle BMD$

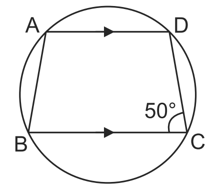
[$\because$ Angle at the centre of a circle is twice the angle on the circumference on the other side of the arc BD]

$= 2 \times 58°$ [From (3)]

$= 116°$

Hence, the required measures of $\angle BAD$ and $\angle BCD$ are 116° and 122° respectively.

10. ABCD is a cyclic quadrilateral with AD $\parallel$ BC. If $\angle C = 50°$, determine the remaining angles.



Sol. Given that ABCD is a cyclic quadrilateral with AD $\parallel$ BC and $\angle C = 50°$. To find $\angle B$, $\angle D$ and $\angle A$.

We have

$\angle C + \angle D = 180°$

[$\because$ AD $\parallel$ BC and CD is a transversal]

$\Rightarrow \quad 50° + \angle D = 180°$

$\Rightarrow \quad \angle D = 180° - 50° = 130°$

Again, $\quad \angle C + \angle A = 180°$

[$\because$ ABCD is a cyclic quadrilateral]

$\Rightarrow \quad 50° + \angle A = 180°$

$\therefore \quad \angle A = 180° - 50° = 130°$

Also, $\quad \angle B + \angle D = 180°$

[$\because$ ABCD is a cyclic quadrilateral]

$\Rightarrow \quad \angle B + 130° = 180°$

$\Rightarrow \quad \angle B = 180° - 130° = 50°$

Hence, the required measures of $\angle A$, $\angle B$ and $\angle D$ are 130°, 50° and 130° respectively.

Milestone

(Page 188)

Multiple-Choice Questions

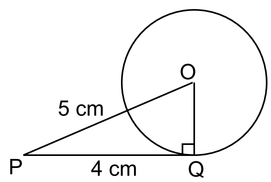
1. The length of a tangent PQ from an external point P is 4 cm. If the distance of P from the centre is 5 cm, then the diameter of the circle is

(*a*) $\sqrt{7}$ cm (*b*) 3 cm

(*c*) 6 cm (*d*) $2\sqrt{2}$ cm

Sol. (*c*) 6 cm

Given that PQ is a tangent to a circle with centre O at a point Q.



Also, OP = 5 cm and PQ = 4 cm.

To find the diameter of the circle.

We have

PQ is a tangent at a point Q on the circle and OQ is a radius, hence, $\angle OQP = 90°$.

$\therefore$ In ΔOPQ, by using Pythagoras' theorem, we have

$$OP^2 = OQ^2 + PQ^2$$

$$\Rightarrow \quad OQ^2 = OP^2 - PQ^2$$

$$= 5^2 - 4^2$$

$$= 25 - 16$$

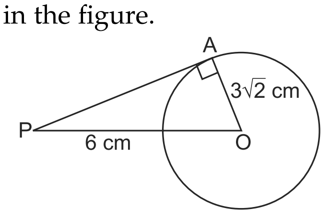
$$= 9$$

$$\therefore \quad OQ = 3 \text{ cm}$$

$$\therefore \quad 2OQ = 2 \times 3 = 6 \text{ cm}$$

Hence, the required length of the diameter of the circle is 6 cm.

2. A tangent PA is drawn from an external point P to a circle of radius $3\sqrt{2}$ cm such that the distance of the point P from the centre at O is 6 cm as shown in the figure.



The value of $\angle APO$ is

(*a*) 45° (*b*) 50°

(*c*) 60° (*d*) 30° **[CBSE SP 2012]**

Sol. (*a*) 45°

Since PA is a tangent to a circle with centre at O and radius, OA = $3\sqrt{2}$ cm such that PO = 6 cm and $\angle PAO = 90°$

Now, in ΔAPO, we have

$$\sin \angle APO = \frac{OA}{PO}$$

$$= \frac{3\sqrt{2}}{6}$$

$$= \frac{\sqrt{2}}{2}$$

$$= \frac{1}{\sqrt{2}}$$

$$= \sin 45°$$

$$\therefore \quad \angle APO = 45°$$

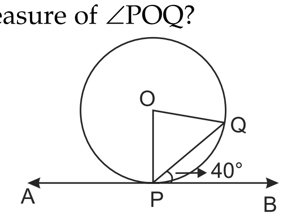
Very Short Answer Type Questions

3. How many parallel tangents can a circle have?

[CBSE SP 2012]

Sol. We know each pair of tangents at the two ends of any diameter of a circle is parallel to each other. Now, a circle has infinite number of diameters. Hence, infinite number of parallel tangents can be drawn to a circle.

4. In the given figure, APB is a tangent to a circle with centre O at a point P. If $\angle QPB = 40°$, what is the measure of $\angle POQ$?



Sol. Given that APB is a tangent to a circle with centre O at a point P on it and PQ is a chord of the circle such that $\angle QPB = 40°$.

To find $\angle POQ$.

We have

$\because$ OP is a radius at a point P on the tangent APB,

$$\therefore \quad \angle OPB = 90°$$

$$\therefore \quad \angle OPQ = \angle OPB - \angle QPB$$

$$= 90° - 40° = 50°$$

Now, in ΔOPQ, we have

$$OP = OQ \quad \text{[Radius of the circle]}$$

$$\therefore \quad \angle OPQ = \angle OQP = 50°$$

$\therefore$ In ΔOPQ, we have

$$\angle OPQ + \angle OQP + \angle POQ = 180°$$

[Angle sum property of a triangle]

$\Rightarrow$ $50° + 50° + \angle POQ = 180°$

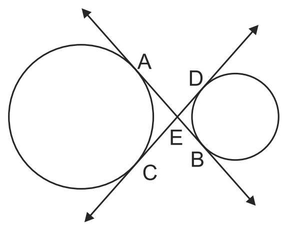
$\Rightarrow$ $100° + \angle POQ = 180°$

$\Rightarrow$ $\angle POQ = 180° - 100° = 80°$

Hence, the required measure of $\angle$POQ is 80°.

Short Answer Type-I Questions

5. In the given figure, common tangents AB and CD to the two circles intersect at E. Prove that AB = CD. **[CBSE 2014, SP 2016]**



Sol. Given that AB and CD are two common tangents to the two circles and these two tangents intersect each other at a point E.

To prove that AB = CD

We have

$\because$ E is an external point. Also EA and EC are two tangents to a circle from this point E,

$\therefore$ EA = EC ...(1)

Similarly, from the second circle,

EB = ED ...(2)

Adding (1) and (2), we get

EA + EB = EC + ED

$\Rightarrow$ AB = CD

Hence, proved.

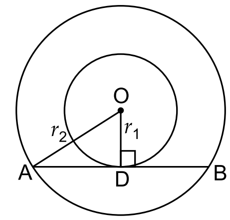
6. If r_1 and r_2 where $r_2 > r_1$ be the radii of two concentric circles and if p be the length of a chord of the bigger circle touching the smaller circle at a point, prove that $4r_2^2 = p^2 + 4r_1^2$.

Sol. Given that r_1 and r_2 ($r_2 > r_1$) are the radii of two concentric circles with common centre O and AB be a chord of the bigger circle touching the smaller circle at a point D. OD and OA are joined. Then OD = r_1, OA = r_2 and AB = p. To prove that $4r_2^2 = p^2 + 4r_1^2$

We have

$\because$ OD is the radius of the smaller circle and ADB is a tangent to this circle at a point D on it.

$\therefore$ OD $\perp$ AB



Now, in ΔOAD, by using Pythagoras' theorem, we have

$$OA^2 = OD^2 + AD^2$$

$\Rightarrow$ $r_2^2 = r_1^2 + \left(\frac{1}{2}AB\right)^2$

[Perpendicular from the centre of the circle to the chord bisects the chord]

$\Rightarrow$ $r_2^2 = r_1^2 + \frac{1}{4}AB^2$

$\Rightarrow$ $4r_2^2 = p^2 + 4r_1^2$

Hence, proved.

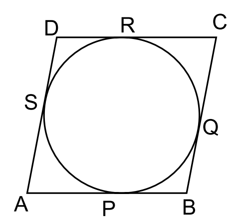
Short Answer Type-II Questions

7. Prove that the parallelogram circumscribing a circle is a rhombus.

[CBSE 2001, 2002C, 2008, 2009, 2012, SP 2011, 2013]

Sol. Given that ABCD is a parallelogram circumscribing a circle. To prove that ABCD is a rhombus.

We know that the lengths of tangents drawn from an external point to a circle are equal.



$\because$ A is an external point to the circle and AS and AP touch the circle at S and P respectively, hence

AP = AS [Tangents from A] ...(1)

Similarly, since BP and BQ are two tangents to the circle,

$\therefore$ BP = BQ [Tangents from B] ...(2)

Similarly, since CR and CQ are two tangents to the circle,

$\therefore$ CQ = CR [Tangents from C] ...(3)

Finally, DS and DR are two tangents to the circle,

$\therefore$ DR = DS [Tangents from D] ...(4)

Adding (1) and (2), we get

$$AP + BP = AS + BQ$$

$\Rightarrow$ $AB = AS + BQ$ …(5)

Similarly, adding (3) and (4), we get

$$DR + CR = CQ + DS$$

$\Rightarrow$ $DC = CQ + DS$ …(6)

Now, adding (5) and (6), we get

$$AB + DC = (AS + DS) + (BQ + CQ)$$

$\Rightarrow$ $AB + DC = AD + BC$

$\Rightarrow$ $2AB = 2AD$

[$\because$ ABCD is a parallelogram,

$\therefore$ AB = DC and AD = BC]

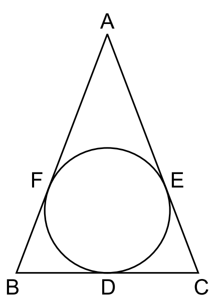
$\Rightarrow$ $AB = AD$

Hence, two adjacent sides AB and AD of the parallelogram ABCD are equal. Hence, ABCD is a rhombus.

Hence, proved.

8. ABC is an isosceles triangle in which AB = AC, circumscribed about a circle. Show that BC is bisected at the point of contact. **[CBSE 2008, 2012]**

Sol. Given that ABC is an isosceles triangle with AB = AC. A circle is inscribed within this triangle touching the sides BC, CA and AB at D, E and F respectively.



To prove that BC is bisected at D.

We know that the lengths of tangents drawn from an external point to a circle are equal.

$\because$ A is an external point and AF and AE are two tangents to the circle at F and E respectively,

$\therefore$ $AF = AE$ [Tangents from A] …(1)

Also, $AB = AC$ [Given] …(2)

$\therefore$ Subtracting (1) from (2), we get

$$AB - AF = AC - AE$$

$\Rightarrow$ $BF = CE$ …(3)

Again, BF and BD are tangents at F and D respectively.

$\therefore$ $BF = BD$ [Tangents from B] …(4)

Also, CD and CE are tangents at D and E respectively.

$\therefore$ $CE = CD$ [Tangents from C] …(5)

$\therefore$ From (3) and (4), we have

$BD = CE$ …(6)

$\therefore$ From (5) and (6), we get

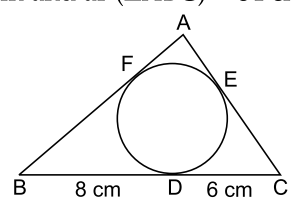
BD = CD, i.e., D is the mid-point of the tangent BC. Hence, BC is bisected at D.

Hence, proved.

Long Answer Type Questions

9. ΔABC is drawn to circumscribe a circle of radius 4 cm such that segments BD and DC into which BC is divided by the point of contact are of lengths 8 cm and 6 cm respectively. If the area of ΔABC is 84 cm^2, find the sides AB and AC. **[CBSE SP 2012]**

Sol. Let a circle is inscribed in a ΔABC whose sides BC, CA and AB touch the circle at the points D, E and F respectively, such that BD = 8 cm, DC = 6 cm and ar (ΔABC) = 84 cm^2.



To find the lengths of the sides AB and AC.

Let BC = a cm, CA = b cm and AB = c cm.

Then $a = BC = BD + DC$

$= (8 + 6)$ cm

$= 14$ cm …(1)

We know that the lengths of tangents, drawn from an external point to a circle are equal.

$\therefore$ $BF = BD = 8$ cm

$\therefore$ $AF = AB - BF$

$= (c - 8)$ cm

Also, $CE = CD = 6$ cm

$\therefore$ $AE = AC - CE = (b - 6)$ cm

Now, since $AF = AE$

$\therefore$ $c - 8 = b - 6$

$\Rightarrow$ $c = b + 2$ …(2)

Now, by Heron's formula, we have

$$\text{ar}(\Delta ABC) = \sqrt{s(s-a)(s-b)(s-c)}$$

where $s = \dfrac{1}{2}(a + b + c)$

$\therefore \quad s = \frac{1}{2}(14 + b + 2 + b)$

[From (1) and (2)]

$\Rightarrow \quad s = 8 + b \quad \ldots(3)$

$\therefore \quad s - a = 8 + b - 14$

$= b - 6$

$s - b = 8 + b - b = 8$

$s - c = 8 + b - 2 - b = 6$

$\therefore \quad ar(\Delta ABC) = \sqrt{s(s-a)(s-b)(s-c)}$

$\Rightarrow \quad ar(\Delta ABC) = \sqrt{(8+b)(b-6)(8 \times 6)}$

$\Rightarrow \quad ar(\Delta ABC) = \sqrt{48\left(b^2 + 2b - 48\right)}$

$\Rightarrow \quad 84 \times 84 = 48(b^2 + 2b - 48)$

$[\because ar(\Delta ABC) = 84 \text{ cm}^2]$

$\Rightarrow \quad b^2 + 2b - 48 = \frac{84 \times 84}{48} = 147$

$\Rightarrow \quad b^2 + 2b - 48 = 147$

$\Rightarrow \quad b^2 + 2b - 195 = 0$

$\Rightarrow \quad b^2 + 15b - 13b - 195 = 0$

$\Rightarrow \quad b(b + 15) - 13(b + 15) = 0$

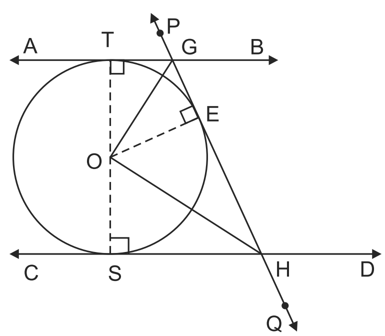
$\Rightarrow \quad (b + 15)(b - 13) = 0$

$\therefore \quad b = 13 \quad [\because b \neq -15]$

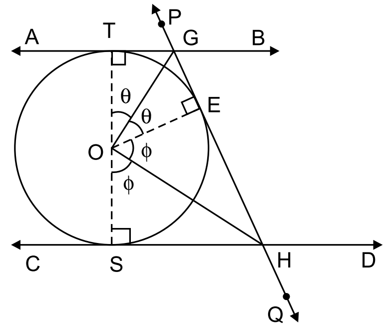
Hence, from (2), $c = 13 + 2 = 15$

Hence, the required lengths of AB and AC are 15 cm and 13 cm respectively.

10. In the given figure, AB and CD are two parallel tangents to a circle with centre O. Another tangent PQ touching the circle E intersects the tangents AB and CD at G and H respectively. Prove that OG ⊥ OH.



Sol. TOS is a diameter of a circle with centre at O. AB and CD are two parallel tangents at T and S respectively. PQ is another tangent to the circle at E intersecting the tangents AB and CD at G and H respectively. To prove that OG ⊥ OH.



We know that two tangents GT and GE of equal lengths subtend equal angle, say θ, at the centre O. Similarly, two tangents HE and HS of equal length subtend equal angle, say ϕ, at the centre O.

$\therefore \quad \angle TOG = \angle GOE = \theta$

and $\quad \angle SOH = \angle HOE = \phi$

$\therefore \quad \angle GOE + \angle HOE = \theta + \phi$

$\because$ TS is a diameter of the circle,

$\therefore \quad \angle TOS = 180°$

$\Rightarrow \quad 2(\theta + \phi) = 180°$

$\Rightarrow \quad \theta + \phi = 90°$

$\therefore \quad \angle GOE + \angle HOE = \angle GOH = 90°$

$\therefore \quad OG \perp OH$

Hence, proved.

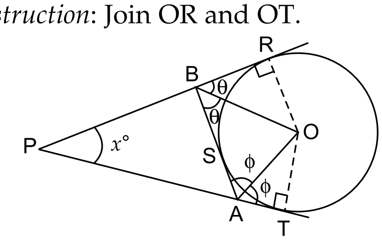
Higher Order Thinking Skills (HOTS) Questions

(Page 190)

1. Two tangents PT and PR are drawn from a point P outside a circle with centre O touching it at T and R respectively. A third tangent is drawn at a point S on the minor arc TR of the circle cutting PT and PR at A and B respectively such that OA and OB bisect ∠TAB and ∠RBA respectively. Show that

$\angle AOB = 90° - \frac{x°}{2}$, where $\angle RPT = x°$.

Sol. *Construction*: Join OR and OT.



$\therefore \quad \angle ORP = \angle OTP = 90°$

Let $\quad \angle OBR = \angle OBA = \theta$

and $\angle OAT = \angle OAB = \phi$

$\therefore$ $\angle ROB = 90° - \theta$

and $\angle AOT = 90° - \phi$

Now, in ΔAOB, we have

$\angle OBA + \angle OAB + \angle AOB = 180°$

[Angle sum property of a triangle]

$\Rightarrow$ $\theta + \phi + \angle AOB = 180°$

$\therefore$ $\angle AOB = 180° - (\theta + \phi)$ …(1)

Now, since $\angle PRO + \angle PTO = 90° + 90° = 180°$,

$\therefore$ PTOR is a cyclic quadrilateral.

$\therefore$ $\angle ROT + \angle RPT = 180°$

$\Rightarrow$ $\angle AOB + \angle AOT + \angle BOR + \angle RPT = 180°$

$\Rightarrow$ $180° - (\theta + \phi) + 90° - \phi + 90° - \theta + x° = 180°$

$\Rightarrow$ $180° - 2(\theta + \phi) + x° = 0$

$\Rightarrow$ $\theta + \phi = 90° + \dfrac{x°}{2}$ …(2)

$\therefore$ From (1) and (2), we get

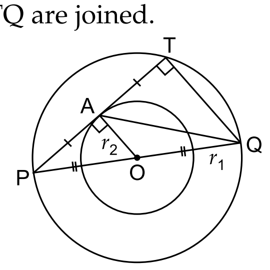
$$\angle AOB = 180° - 90° - \frac{x°}{2}$$

$$= 90° - \frac{x°}{2}$$

Hence, proved.

2. The radii of two concentric circles are r_1 cm and r_2 cm ($r_1 > r_2$). PQ is a diameter of the bigger circle and PA is a tangent to the smaller circle touching it at A. Show that the length of QA is $\sqrt{r_1^2 + 3r_2^2}$ cm.

Sol. Given that POQ is the diameter of the bigger of two concentric circles with common centre at O and radii r_1 cm and r_2 cm where $r_1 > r_2$. A and T are two points on the smaller and bigger circles respectively such that PAT is a tangent to the smaller circle and a chord of the bigger circle. AQ and TQ are joined.



To prove that QA = $\sqrt{r_1^2 + 3r_2^2}$ cm.

Since PQ is a diameter of the bigger circle,

$\therefore$ $\angle PTQ = 90°$

$\therefore$ In ΔATQ, by Pythagoras' theorem, we have

$$AQ^2 = AT^2 + TQ^2 \quad …(1)$$

Now, $OP = OQ = r_1$

Also, $OA \perp PT$

$\therefore$ $\angle PAO = \angle PTQ = 90°$

$\therefore$ $AO \parallel TQ$

$\therefore$ $PA : AT = PO : OQ = 1 : 1$

$\therefore$ A is the mid-point of PT and $AO = \dfrac{1}{2}TQ$

i.e. $TQ = 2AO = 2r_2$ …(2)

and $AT = PA = \sqrt{OP^2 - r_2^2}$

[From right-angled triangle PAO]

$= \sqrt{r_1^2 - r_2^2}$ …(3)

$\therefore$ From (1), $QA = \sqrt{AT^2 + TQ^2}$

$= \sqrt{r_1^2 - r_2^2 + 4r_2^2}$

[From (2) and (3)]

$\Rightarrow$ $QA = \sqrt{r_1^2 + 3r_2^2}$

Hence, the length of QA is $\sqrt{r_1^2 + 3r_2^2}$ cm.

Hence, proved.

Self-Assessment

(Page 190)

Multiple-Choice Questions

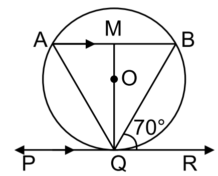
1. PQR is a tangent to a circle at Q, whose centre is at O. AB is a chord parallel to PR and $\angle BQR = 70°$, then $\angle AQB$ is equal to

(*a*) 20° (*b*) 40°

(*c*) 35° (*d*) 45° **[CBSE SP 2012]**

Sol. (*b*) 40°

Given that PQR is a tangent to the circle with centre at O.



OQ is joined and OM is drawn perpendicular to the chord AB which is parallel to the tangent PQR. Given that $\angle BQR = 70°$. To find $\angle AQB$.

Since OQ is a radius of the circle and PQR is a tangent at Q,

$\angle MQR = 90°$

$\therefore$ $\angle MQB = 90° - \angle BQR$

$= 90° – 70°$

$= 20°$

Now, since QOM $\perp$ AB,

$\therefore$ M is the mid-point of AB.

$\therefore$ In ΔQMA and QMB, we have

AM = BM [M is the mid-point of AB]

MQ = MQ [Common]

$\angle$QMA = $\angle$QMB [Each equal to 90°]

$\therefore$ $\Delta QMA \cong \Delta QMB$ [By SAS similarity criterion]

$\therefore$ $\angle$MQA = $\angle$MQB [By CPCT]

$= 20°$

$\therefore$ $\angle AQB = 2\angle MQA$

$= 2 \times 20° = 40°$

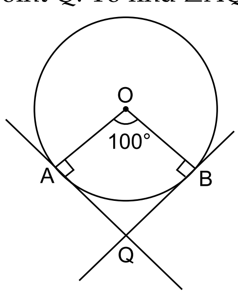
2. If the angle between the radii of a circle is 100°, then the angle between the tangents at the ends of those two radii is

(*a*) 50° (*b*) 60°

(*c*) 80° (*d*) 90° **[CBSE 2012]**

Sol. (*c*) 80°

Given that A and B are two points on a circle with centre at O such that $\angle$AOB = 100°. AQ and BQ are tangents to the circle at A and B, meeting each other at a point Q. To find $\angle$AQB.



Since AQ and BQ are tangents to the circle at A and B respectively and OA and OB are the radii of the circle,

$\therefore$ $\angle OAQ = \angle OBQ = 90°$

$\therefore$ $\angle OAQ + \angle OBQ = 90° + 90° = 180°$

Hence, two opposite angles of the quadrilateral AQBO are supplementary. Hence, the quadrilateral AQBO is cyclic.

$\therefore$ $\angle AQB + \angle AOB = 180°$

$\Rightarrow$ $\angle AQB + 100° = 180°$

$\Rightarrow$ $\angle AQB = 180° – 100° = 80°$

Fill in the Blanks

3. The distance between two parallel tangents of a circle of radius 3 cm is **6 cm.**

Sol. Tangents at the end of a diameter of a circle are parallel. So, the distance between them is equal to the diamter or 2*r*.

Hence, distance = 2 × 3 cm = 6 cm

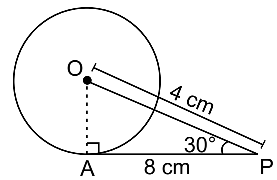
4. AP is a tangent to the circle with centre O such that OP = 4 cm and $\angle$OPA = 30°. Then, AP is equal to **$2\sqrt{3}$ cm.**

Sol. $\because$ The tangent to a circle at any point is perpendicular to the radius through the point of contact

$\therefore$ OA $\perp$ AP

$\Rightarrow$ $\angle$OAP = 90°

In right ΔOAP, we have



$$\cos 30° = \frac{AP}{OP}$$

$$\Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{AP}{4 \text{ cm}}$$

$$\Rightarrow \quad AP = 2\sqrt{3} \text{ cm}$$

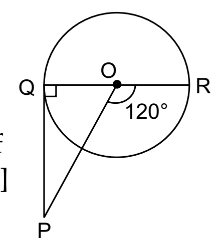
5. PQ is a tangent drawn from a point P to a circle with centre O and QOR is a diameter of the circle such that $\angle$POR = 120°, then $\angle$OPQ is **30°**.

Sol. OQ $\perp$ QP

$\Rightarrow$ $\angle$OQP = 90°

$\angle OQP + \angle OPQ = 120°$

[Exterior angle = Sum of interior opposite angles]



$\Rightarrow$ $90° + \angle OPQ = 120°$

$\Rightarrow$ $\angle OPQ = 120° – 90°$

$\Rightarrow$ $\angle OPQ = 30°$

6. The maximum number of common tangents that can be drawn to two circles intersecting at two distinct points is **2**.

Assertion-Reason Type Questions

Directions (Q. Nos. 7 to 9): Each of these questions contains an assertion followed by reason. Read them carefully, and answer the question on the basis of the following options, select the one that best describes the two statements.

(a) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.

(b) Both Assertion and Reason are true but Reason is not the correct explanation of Assertion.

(c) If Assertion is true but Reason is false.

(d) If Assertion is false but Reason is true.

7. **Assertion:** From an external point P, two tangents are drawn to a circle with points of contact A and B, then PA = 2 PB.

Reason: The lengths of tangents drawn from an external point to a circle are equal.

Sol. The correct answer is (*d*).

The lengths of tangents drawn from an external point to a circle are equal.

Thus, PA = PB.

$\therefore$ Assertion is wrong but reason is correct.

8. **Assertion:** : Diameter of a circle represents the largest secant inside the circle.

Reason: Two ends of a diameter are the farthest apart pair of points in a circle.

Sol. The correct answer is (*a*).

Two ends of a diameter are the farthest apart pair of points in a circle. Thus diameter represents the largest possible secant inside a circle.

Thus reason is the correct explanation of the assertion.

9. **Assertion:** A normal to a circle is perpendicular to the tangent at that point.

Reason: Normal is the line containing the radius through the point of contact of a tangent in a circle.

Sol. The correct answer is (*a*).

The line containing the radius through the point of contact is also sometimes called the normal to the circle at the point.

Thus reason is a correct explanation of the assertion.

Case Study Based Questions

10. Rain water harvesting is the collection and storage of rain water, rather than allowing it to run off. To save rain water, Sushant presented a project for rain water harvesting. Diagrammatic representation of the project is given below. Through pipes PQ and PR, each of 15 m long are bringing water from the terrace of a building (as shown in the figure). Based on the above information, answer the following questions.

(*a*) If the radius of circle is 8 m, then find the length of OP.

(*i*) 13 m (*ii*) 15 m

(*iii*) 17 m (*iv*) 19 m

Ans. (*iii*) 17 m

(*b*) If two circles intersect each other at two points, then the number of common tangents is

(*i*) 0 (*ii*) 1

(*iii*) 2 (*iv*) 3

Ans. (*iii*) 2

(*c*) The length of the tangent from an external point P on a circle with centre O

(*i*) is always greater than OP.

(*ii*) is equal to OP.

(*iii*) is always less than OP.

(*iv*) cannot be estimated.

Ans. (*i*) is always greater than OP.

(*d*) If a line OP makes an angle 60° from one of the tangent line, then the length of OP is

(*i*) 5 m (*ii*) 15 m

(*iii*) 20 m (*iv*) 30 m

Ans. (*iv*) 30 m

(*e*) If we draw angle bisector line from intersection point of pair of tangent to the circle, then it always passes through

(*i*) centre.

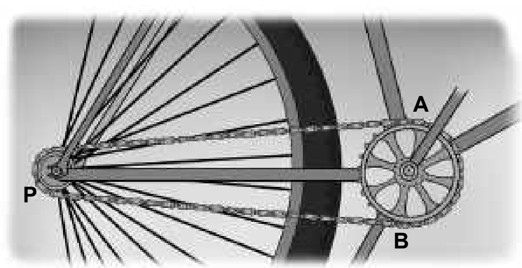
(*ii*) left side of centre.

(*iii*) right side of centre.

(*iv*) none of these

Ans. (*i*) centre.

11. The chain and gears of bicycles or motorcycles or belt around two pulleys are some real life examples of tangents and circles.



In the given figure, the big gear represents the circle while the smaller one represents the exterior point P of the intersection of tangent lines. PA and PB are two tangents intersecting outside the circle at the point P.

Based on the above situation, answer the following questions.

(*a*) If PB is equal to 16 inches, then the measure of PA is equal to

(*i*) 16 inches. (*ii*) 8 inches.

(*iii*) 32 inches. (*iv*) 15 inches.

Ans. (*i*) 16 inches.

(*b*) If a point is inside the circle, how many tangents can be drawn from that point?

(*i*) 0 (*ii*) 1

(*iii*) 2 (*iv*) 3

Ans. (*i*) 0

(*c*) If the angle made by the tangents PA and PB at the centre O of the circle is 120°, then measure of $\angle APB$ is equal to

(*i*) 120° (*ii*) 20°

(*iii*) 60° (*iv*) 80°

Ans. (*iii*) 60°

(*d*) If we draw two tangents at the end of the diameter of a circle, then these tangents are always

(*i*) parallel. (*ii*) perpendicular.

(*iii*) coincident. (*iv*) None of these

Ans. (*i*) parallel.

(*e*) If PA is equal to 12 inches and radius of bigger circle is 5 inches, then the distance of P from the centre O of larger circle is

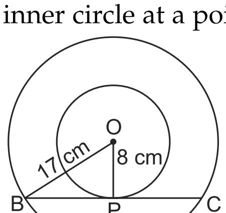
(*i*) 7 inches. (*ii*) 13 inches.

(*iii*) 17 inches. (*iv*) 169 inches.

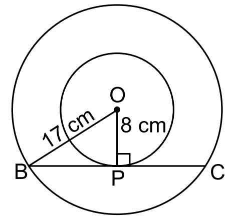
Ans. (*ii*) 13 inches.

Very Short Answer Type Questions

12. Two concentric circles of radii 17 cm and 8 cm are given. What is the length of the chord BC which touches the inner circle at a point P?



Sol. Given that B and C are two points on a circle with centre at O and radius 17 cm such that the chord BC of this circle is a tangent to a smaller concentric circle of centre at O and radius 8 cm at the point P.



To find the length of the chord BC.

Since BPC is a tangent to the smaller circle at P,

$\therefore \quad \angle OPB = 90°$

Also, OP = 8 cm and OB = 17 cm.

$\therefore$ In ΔOBP, by using Pythagoras' theorem, we have

$$OB^2 = OP^2 + BP^2$$

$$\Rightarrow \quad 17^2 = 8^2 + BP^2$$

$$\Rightarrow \quad BP = \sqrt{17^2 - 8^2}$$

$$= \sqrt{(17+8)(17-8)}$$

$$= \sqrt{25 \times 9}$$

$$= 5 \times 3 = 15$$

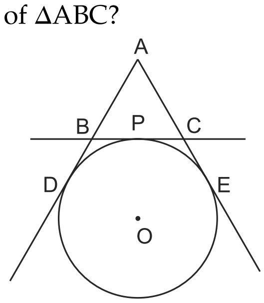
Now, since BC is a chord of the bigger circle and $OP \perp BC$,

$\therefore$ P is the mid-point of BC.

$\therefore$ BC = 2BP = 2 × 15 = 30

Hence, the required length of the chord BC is 30 cm.

13. In the given figure, the side BC of ΔABC touches the circle with centre O, at the point P. If AB and AC produced touch the same circle at D and E respectively and if AD = 10.5 cm, then what is the perimeter of ΔABC?



Sol. Given that ABC is a triangle. Also, AB and AC produced touch a circle with centre at O at the points D and E respectively. Let BC touch the circle at P. It is given that AD = 10.5 cm. To find the perimeter of ΔABC.

Since A is an external point to the circle, Also, AD and AE are two tangents from A,

$\therefore \quad AD = AE \quad \ldots(1)$

Now, $AB + AC = AD - BD + AE - CE$

$= AD - BP + AD - CP$

$[\because BD = BP \text{ and } CE = CP]$

$= 2\,AD - (BP + CP)$

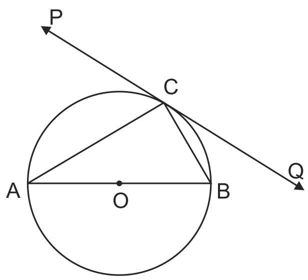
$= 2 \times 10.5 - BC$

$\Rightarrow \; AB + AC + BC = 21$

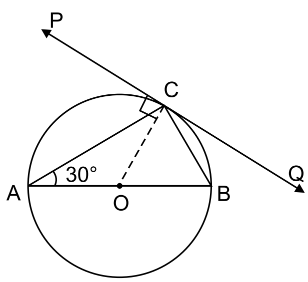
Hence, the required perimeter is 21 cm.

Short Answer Type-I Questions

14. In the given figure, PQ is a tangent at a point C to a circle with centre O. If AB is a diameter and $\angle CAB = 30°$, find $\angle PCA$. **[CBSE 2016]**



Sol. Given that AOB is a diameter of a circle with centre at O and C is a point on the circle such that $\angle CAB = 30°$.



PCQ is a tangent to the circle at C.

To find $\angle PCA$.

Construction: Join OC.

Since OC is a radius of the circle and PCQ is a tangent at C,

$\therefore \quad \angle OCP = 90°$

Since AB is a diameter of the circle,

$\therefore \quad \angle ACB = 90°$

$\therefore \quad \angle PCA = \angle OCP - \angle OCA$

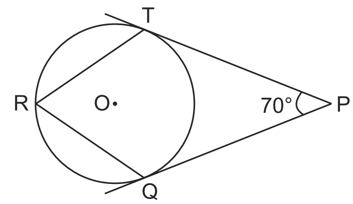
$= 90° - 30°$

$[\because OA = OC = \text{radius of the same circle}, \therefore \angle OCA = \angle OAC = 30°]$

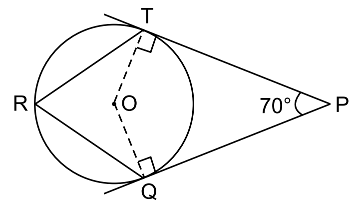
$= 60°$

Hence, the required angle is 60°.

15. In the given figure, O is the centre of the circle. PT and PQ are tangents to the circle from an external point P. If $\angle TPQ = 70°$, find $\angle TRQ$. **[CBSE 2015]**



Sol. *Construction*: Join OT and OQ. To find $\angle TRQ$.



Since OT, OQ are the radii of the same circle and TP, PQ are tangents to the circle, hence, we have

$$\angle OTP = \angle OQP = 90°$$

$\therefore \quad \angle OTP + \angle OQP = 90° + 90° = 180°$

$\therefore$ TPQO is cyclic quadrilateral.

$\therefore \quad \angle TOQ + \angle TPQ = 180°$

$\Rightarrow \quad \angle TOQ = 180° - \angle TPQ$

$= 180° - 70°$

$= 110°$

But $\quad \angle TOQ = 2\angle TRQ$

[Angle subtended by the arc TQ at O]

$\Rightarrow \quad 110° = 2\angle TRQ$

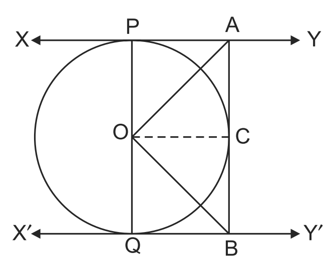
$\Rightarrow \quad \angle TRQ = \dfrac{110°}{2} = 55°$

$\therefore$ The required angle is 55°.

Short Answer Type-II Questions

16. In the given figure, XY and X′ Y′ are two parallel tangents to a circle with centre O and another tangent AB, with point of contact C intersects XY at A and X′Y′ at B. Prove that $\angle AOB = 90°$.

[CBSE SP 2012, 2013, 2016, 2017]



Sol. Given that XY and X′Y′ are two parallel tangents to a circle with centre at O. Clearly, POQ is a diameter of the circle, C is another point on the circle such that ACB is a tangent to the circle where A and B are points on XY and X′Y′ respectively.

To prove that ∠AOB = 90°.

We have AP = AC and BQ = BC. Hence, AP and AC will subtend equal angles at O. Similarly, BQ and BC will subtend another pair of equal angles at O.

Let ∠AOP = ∠AOC = θ and ∠BOQ = ∠BOC = ϕ.

$\therefore \quad \angle POC = \angle AOP + \angle AOC = 2\theta$

and $\quad \angle COQ = \angle BOQ + \angle BOC = 2\phi$

$\therefore \quad \angle POC + \angle COQ = 2(\theta + \phi)$

$\Rightarrow \quad \angle POQ = 2(\theta + \phi)$

$\Rightarrow \quad 180° = 2(\theta + \phi)$

[∵ PQ is a diameter]

$\Rightarrow \quad \theta + \phi = \frac{180°}{2} = 90° \quad \ldots(1)$

Now, $\quad \angle AOB = \angle AOC + \angle COB = \theta + \phi = 90°$ [From (1)]

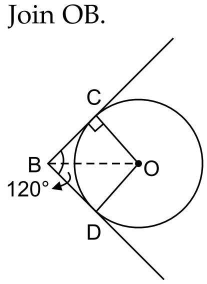
Hence, proved.

17. From an external point B of a circle with centre at O, two tangents BC and BD are drawn to the circle. If ∠DBC = 120°, prove that OB = 2BC.

Sol. Given that B is an external point to a circle with centre at O and two tangents BC and BD touching the circle at C and D respectively such that ∠DBC = 120°.

To prove that OB = 2BC.

Construction: Join OB.



We know that OB is the bisector of ∠DBC.

$\therefore \quad \angle OBC = \frac{120°}{2} = 60°$

Now, in ΔOBC,

since $\quad \angle OCB = 90°$,

$\therefore \quad \cos \angle OBC = \frac{BC}{OB}$

$\Rightarrow \quad \cos 60° = \frac{BC}{OB}$

$\Rightarrow \quad \frac{1}{2} = \frac{BC}{OB}$

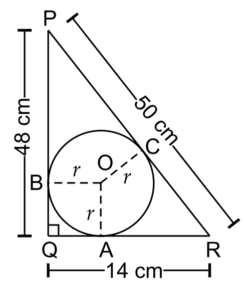
$\Rightarrow \quad OB = 2BC$

Hence, proved.

Long Answer Type Questions

18. PQR is a right triangle with PQ = 48 cm, QR = 14 cm and PR = 50 cm. A circle with centre at O and radius *r* is inscribed in ΔPQR. Find the value of *r*.

Sol. Given that PQR is a right-angled triangle with ∠PQR = 90°. A circle with centre at O is inscribed within this triangle touching the sides QR, PQ and PR at the points A, B and C respectively. It is given that PQ = 48 cm, QR = 14 cm and PR = 50 cm. Let *r* cm be the radius of the circle. To find the value of *r*.



Construction: Join OA, OB and OC.

We know that the length of tangents drawn from an external point to a circle are equal.

Since, OA = OB = *r* and OB ⊥ PQ, hence OBQA is a square of side *r* cm.

$\therefore \quad QA = QB = r$

$\therefore \quad PB = (48 - r)\text{ cm} = PC$

$\Rightarrow \quad PC = (48 - r)\text{ cm} \quad \ldots(1)$

and $\quad AR = (14 - r)\text{ cm} = CR$

$\Rightarrow \quad CR = (14 - r)\text{ cm} \quad \ldots(2)$

Now, $\quad PC + CR = PR = 50\text{ cm}$

$\Rightarrow \quad (48 - r)\text{ cm} + (14 - r)\text{ cm} = 50\text{ cm}$

[From (1) and (2)]

$\Rightarrow \quad 62\text{ cm} - 2r = 50\text{ cm}$

$\Rightarrow \quad 2r = 12\text{ cm}$

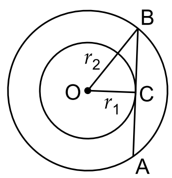
$\Rightarrow \quad r = \frac{12}{2}\text{ cm}$

$\Rightarrow \quad r = 6\text{ cm}$

Hence, the required length of *r* is 6 cm.

19. Two concentric circles have radii $a^2 + b^2$ and $a^2 - b^2$. Show that the length of the chord of the larger circle which touches the smaller circle is $4ab$.

Sol. Let r_1 cm and r_2 cm where $r_2 > r_1$ be the radii of two concentric circles with common centre O and let C be a point on the smaller circle such that the chord AB of the bigger circle is a tangent to the smaller circle at C.



It is given that $r_1 = a^2 - b^2$ and $r_2 = a^2 + b^2$.

To prove that AB = $4ab$.

Construction: Join OC and OB.

We have $\angle OCB = 90°$, $OB = r_2 = a^2 + b^2$ and $OC = r_1 = a^2 - b^2$

$\therefore$ In ΔOCB, by using Pythagoras' theorem, we have

$$BC^2 + OC^2 = OB^2$$

$$\Rightarrow \quad BC^2 + r_1^2 = r_2^2$$

$$\Rightarrow \quad BC = \sqrt{r_2^2 - r_1^2}$$

$$= \sqrt{\left(a^2 + b^2\right)^2 - \left(a^2 - b^2\right)^2}$$

$$= \sqrt{4a^2b^2} = 2ab$$

$$\therefore \quad AB = 2BC$$

$$= 2 \times 2ab$$

$$= 4ab$$

Hence, proved.

Let's Compete

(Page 192)

Multiple-Choice Questions

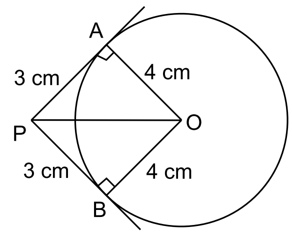
1. The area of the quadrilateral formed by two tangents PA and PB each of length 3 cm, drawn from an external point P to a circle with centre at O and two radii, each of length 4 cm, is

(*a*) 6 cm^2 (*b*) 10 cm^2

(*c*) 15 cm^2 (*d*) 12 cm^2

Sol. (*d*) 12 cm^2

Let P be an external point to a circle with centre at O and radius 4 cm. A and B are two points on the circle such that PA and PB are two tangents to the circle at A and B respectively such that PA = PB = 3 cm.



To find the area of the quadrilateral PAOB.

Construction: Join PO, OA and OB.

We see that

$\Delta PAO \cong \Delta PBO$, by SSS congruence property, since PA = PB, OA = OB and OP is common.

$$\therefore \quad ar(\Delta PAO) = ar(\Delta PBO)$$

$$= \frac{1}{2} \times AP \times AO$$

$$= \frac{1}{2} \times 3 \times 4 \text{ cm}^2$$

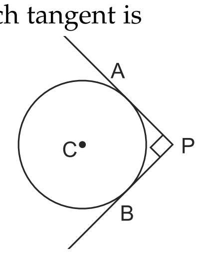
$$= 6 \text{ cm}^2$$

$\therefore$ Area of quadrilateral APBO

$$= 2 \times 6 \text{ cm}^2$$

$$= 12 \text{ cm}^2$$

2. In the given figure, PA and PB are two tangents drawn from an external point to a circle with centre C, and radius 4 cm. If PA $\perp$ PB, then the length of each tangent is **[CBSE SP 2013]**



(*a*) 3 cm (*b*) 4 cm

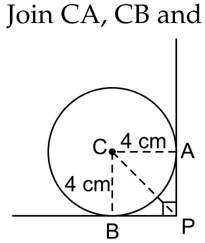
(*c*) 5 cm (*d*) 6 cm

Sol. (*b*) 4 cm

Given that PA and PB are two tangents drawn from an external point P to a circle with centre at C such that $\angle APB = 90°$.

To find the length of PA or PB.

Construction: Join CA, CB and CP.



We have

$$\angle CAP = 90° = \angle APB$$

Also, $\angle CBP = 90° = \angle APB$

Hence, APBC is a square.

$\therefore$ $PA = PB = AC = 4$ cm

$\therefore$ The required length of each tangent is 4 cm.

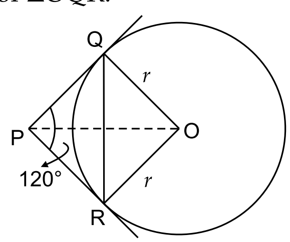
3. If PQ and PR are tangents drawn from an external point P to the circle with centre O and radius r cm at the points of contact Q and R such that $\angle QPR = 120°$, then the area of the triangle OQR is

(*a*) $\frac{\sqrt{3}}{4}r^2$ cm^2 (*b*) $\frac{\sqrt{3}}{2}r^2$ cm^2

(*c*) $\sqrt{3}r^2$ cm^2 (*d*) $\frac{4r^2}{\sqrt{3}}$ cm^2

Sol. (*a*) $\frac{\sqrt{3}}{4}r^2$ cm^2

Given that PQ and PR are two tangents drawn from an external point P to a circle with centre at O such that $\angle QPR = 120°$. QR, OQ and OR are joined. Let r cm be the radius of the circle. To find the area of ΔOQR.



Since $\angle OQP = \angle ORP = 90°$,

$\therefore$ $\angle OQP + \angle ORP = 90° + 90° = 180°$

$\therefore$ The quadrilateral PQOR is cyclic.

$\therefore$ $\angle QPR + \angle QOR = 180°$

$\Rightarrow$ $120° + \angle QOR = 180°$

$\Rightarrow$ $\angle QOR = 180° - 120° = 60°$

Now, in ΔOQR, $OQ = OR = r$ cm

$\therefore$ $\angle OQR = \angle ORQ$

$\therefore$ $\angle OQR + \angle ORQ + \angle QOR = 180°$

[Angle sum property of a triangle]

$\Rightarrow$ $\angle OQR + \angle ORQ + 60° = 180°$

$\Rightarrow$ $2\angle OQR = 180° - 60°$

$\Rightarrow$ $2\angle OQR = 120°$

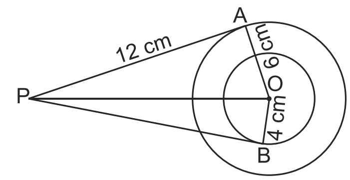
$\Rightarrow$ $\angle OQR = 60°$

$\Rightarrow$ $\angle OQR = \angle ORQ = \angle QOR = 60°$

$\therefore$ ΔOQR is an equilateral triangle of side r cm.

$\therefore$ Area of ΔOQR is $\frac{\sqrt{3}}{4}r^2$ cm^2.

4. Two concentric circles with centre O are of radii 6 cm and 4 cm. From an external point P, tangents PA and PB are drawn to these two circles as shown in the figure. If AP = 12 cm, then BP is equal to



(*a*) 12 cm (*b*) $2\sqrt{41}$ cm

(*c*) $4\sqrt{7}$ cm (*d*) 11 cm

Sol. (*b*) $2\sqrt{41}$ cm

Given that A and B are two points on two concentric circles with centre at O and radii 6 cm and 4 cm respectively. P is an external point to both the circles and PA and PB are tangents to the two circles from P at the points A and B respectively such that AP = 12 cm. To find the length of PB.

We have

In ΔPAO,

$\because$ $\angle PAO = 90°$

$\therefore$ By using Pythagoras' theorem, we have

$$OP^2 = AP^2 + AO^2 = 12^2 + 6^2 = 144 + 36 = 180$$

$\therefore$ $OP^2 = 180$ ...(1)

Now, in ΔPBO, since $\angle PBO = 90°$,

$\therefore$ By using Pythagoras' theorem, we have

$$BP = \sqrt{OP^2 - OB^2} = \sqrt{180 - 16} = \sqrt{164}$$

[$\because$ From (1), $OP^2 = 180$ and OB = 4]

$$= \sqrt{4 \times 41} = 2\sqrt{41}$$

Hence, the required length of BP is $2\sqrt{41}$ cm.

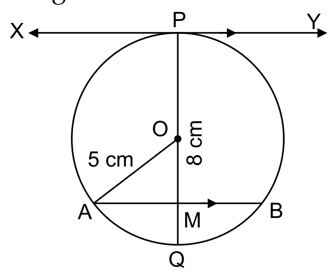
5. At one end of a diameter PQ of a circle of radius 5 cm, a tangent XPY is drawn to the circle. Then the length of the chord AB parallel to XY and at a distance of 8 cm from P is

(*a*) 8 cm (*b*) 10 cm

(*c*) 11 cm (*d*) 12 cm

Sol. (*a*) 8 cm

Let PQ be a diameter of a circle with centre at O and radius 5 cm. XY is a tangent to the circle at P and AB is a chord of the circle parallel to XY such that PM = 8 cm where PM is drawn through O perpendicular to AB and M is a point on AB. To find the length of the chord AB.



Construction: Join OA.

Since OM $\perp$ AB,

$\therefore$ M is the mid-point of AB.

Now, in ΔAOM, we have

$$AO = 5 \text{ cm}$$

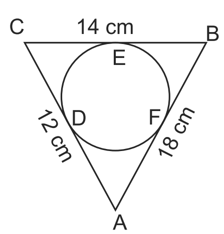
and $$OM = PM - PO$$
$$= (8 - 5)\text{cm}$$
$$= 3 \text{ cm}$$

$\therefore$ By using Pythagoras' theorem, we have

$$AM = \sqrt{AO^2 - OM^2}$$
$$= \sqrt{5^2 - 3^2} \text{ cm}$$
$$= \sqrt{16} \text{ cm}$$
$$= 4 \text{ cm}$$

$\therefore$ $$AB = 2\,AM$$
$$= 2 \times 4 \text{ cm}$$
$$= 8 \text{ cm}$$

6. A circle is inscribed in a triangle ABC having sides BC = 14 cm, AB = 18 cm and AC = 12 cm as shown in the figure. If AC, CB and AB touch the circle at the points D, E and F respectively, then



(*a*) AD = 8 cm and BE = 4 cm
(*b*) AD = 10 cm and BE = 8 cm
(*c*) AD = 8 cm and BE = 10 cm
(*d*) AD = 10 cm and BE = 4 cm

Sol. (*c*) AD = 8 cm and BE = 10 cm

Given that ABC is a triangle of sides AB = 18 cm, BC = 14 cm and AC = 12 cm. A circle is inscribed within this triangle touching the sides AC, CB and AB at the point D, E and F respectively. To find the lengths of AD and BE.

Let $AD = AF = x$ cm

and $BE = BF = y$ cm

Now, $CD = CE$

$\Rightarrow$ $AC - AD = CB - BE$

$\Rightarrow$ $12 - x = 14 - y$

$\Rightarrow$ $y - x = 14 - 12 = 2$

$\Rightarrow$ $y - x = 2$ …(1)

Also, $AF + FB = 18$

$\Rightarrow$ $x + y = 18$ …(2)

Adding (1) and (2), we get

$$2y = 20$$

$\Rightarrow$ $y = 10$

$\therefore$ From (1), $x = 10 - 2 = 8$

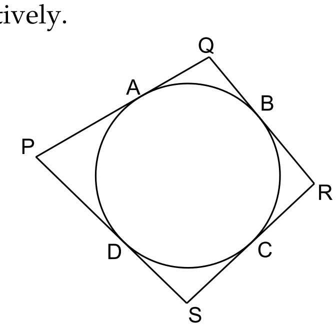
Hence, AD = 8 cm and BE = 10 cm

7. A quadrilateral PQRS circumscribes a circle such that the sides PQ, QR, RS and SP touch the circle at A, B, C and D respectively. Then PD QB is equal to

(*a*) PS (*b*) QR
(*c*) RS (*d*) PQ

Sol. (*d*) PQ

Let PQRS be a quadrilateral and a circle is inscribed within this quadrilateral touching the sides PQ, QR, RS and SP at the points A, B, C and D respectively.



To find PD + QB

Let PD = x and QB = y

We know that the lengths of tangents drawn from an external point to a circle are equal.

$$PA = PD = x \quad \text{[Tangents from P]}$$

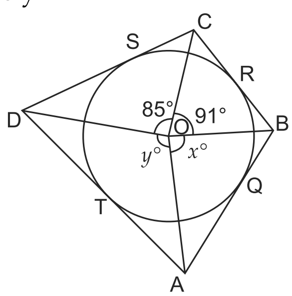
and $$QA = QB = y \quad \text{[Tangents from Q]}$$

$\therefore$ $$PQ = PA + QA = x + y$$

$\Rightarrow$ $$PQ = x + y \qquad \ldots(1)$$

Hence, $PD + QB = x + y = PQ$ [From (1)]

8. A quadrilateral ABCD circumscribes a circle with centre O as shown in the figure. The sides AB, BC, CD and DA of the quadrilateral touch the circle at the points Q, R, S and T respectively. If $\angle DOC = 85°$ and $\angle BOC = 91°$, $\angle AOB = x°$ and $\angle AOD = y°$, then the values of x and y are respectively



(*a*) 95° and 89° (*b*) 89° and 95°
(*c*) 97° and 79° (*d*) 97° and 89°

Sol. (*a*) 95° and 89°

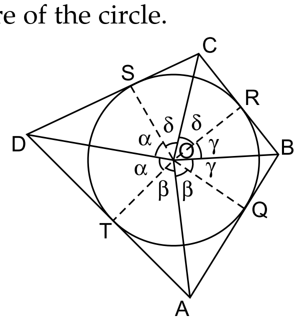
Given that ABCD is a quadrilateral which circumscribes a circle with centre at O such that

$\angle DOC = 85°$, $\angle COB = 91°$, $\angle AOB = x°$ and $\angle AOD = y°$.

To find the values of $x°$ and $y°$.

Construction: Join OQ, OR OS and OT.

We know that two equal tangents drawn from an external point to a circle subtend equal angles at the centre of the circle.



$\therefore$ $$\angle TOD = \angle SOD = \alpha, \text{ say}$$

$$\angle TOA = \angle QOA = \beta, \text{ say}$$

$$\angle QOB = \angle ROB = \gamma, \text{ say}$$

and $$\angle ROC = \angle SOC = \delta, \text{ say}$$

$\therefore$ $$2\alpha + 2\beta + 2\gamma + 2\delta = 360°$$

$\Rightarrow$ $$\alpha + \beta + \gamma + \delta = \frac{360°}{2} = 180° \qquad \ldots(1)$$

Now, given that

$$\angle DOC = 85°$$

$\Rightarrow$ $$\alpha + \delta = 85° \qquad \ldots(2)$$

$$\angle BOC = 91°$$

$\Rightarrow$ $$\gamma + \delta = 91° \qquad \ldots(3)$$

$$\angle AOB = x°$$

$$\beta + \gamma = x° \qquad \ldots(4)$$

and $$\angle AOD = y°$$

$\Rightarrow$ $$\alpha + \beta = y° \qquad \ldots(5)$$

From (2) and (3), we have

$$\gamma - \alpha = 91° - 85° = 6° \qquad \ldots(6)$$

Also, from (4) and (5),

$$\gamma - \alpha = x° - y° \qquad \ldots(7)$$

From (6) and (7),

$$x° - y° = 6° \qquad \ldots(8)$$

Adding (2), (3), (4) and (5), we get

$$2(\alpha + \beta + \gamma + \delta) = 85° + 91° + x° + y°$$

$\Rightarrow$ $$2 \times 180° - 176° = x° + y° \quad \text{[From (1)]}$$

$\Rightarrow$ $$360° - 176° = x° + y°$$

$\Rightarrow$ $$x° + y° = 184° \qquad \ldots(9)$$

Adding (8) and (9), we get

$$2x° = 190°$$

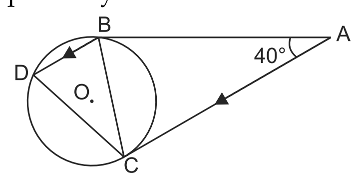
$\Rightarrow$ $$x° = \frac{190°}{2} = 95°$$

$\therefore$ From (8), we have

$$y° = 95° - 6° = 89°$$

Hence, the required values of $x°$ and $y°$ are 95° and 89° respectively.

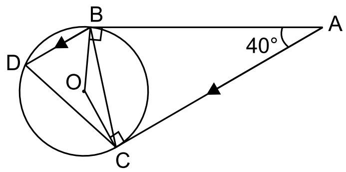
9. In the given figure, AB and AC are tangents to a circle with centre O from an external point A. If $\angle BAC = 40°$, AC || BD where D is a point on the circle and B and C are the points of contact of the tangents with the circle, then the measures of $\angle DBC$, $\angle BDC$ and $\angle BCD$ of ΔBDC are respectively



(*a*) 40°, 70° and 70°
(*b*) 75°, 40° and 65°
(*c*) 65° 40° and 75°
(*d*) 70°, 70° and 40°

Sol. (*d*) 70°, 70° and 40°

Given that B and D are two points on a circle with center at O. C is another point on the circle such that tangents AB and AC at B and C meet each other at an external point A and $\angle BAC = 40°$. Also, AC $\parallel$ BD.



BC is joined. To find the angles of ΔBDC.

Construction: Join OB and OC.

$\because$ OB $\perp$ AB and OC $\perp$ AC,

$\therefore$ OBAC is a cyclic quadrilateral.

$\therefore \quad \angle BOC + \angle BAC = 180°$

$\Rightarrow \quad \angle BOC + 40° = 180°$

$\Rightarrow \quad \angle BOC = 180° - 40° = 140°$

But $\quad \angle BOC = 2\angle BDC$

$\Rightarrow \quad 2\angle BDC = 140°$

$\Rightarrow \quad \angle BDC = \dfrac{140°}{2} = 70°$

Since AB = AC,

$$\therefore \quad \angle ABC = \angle ACB = \frac{180° - 40°}{2} = \frac{140°}{2} = 70°$$

$\therefore \quad \angle DBC = \angle DBA - \angle ABC$

$= (180° - 40°) - 70°$

$[\because$ AC $\parallel$ BD,

$\therefore \angle DBA + \angle BAC = 180°$

$\Rightarrow \angle DBA = 180° - 40°]$

$= 140 - 70° = 70°$

$\therefore \quad \angle DCB = 180° - (70° + 70°) = 40°$

$\therefore \quad \angle DBC = \angle BDC = 70°$

and $\quad \angle DCB = 40°$

$\therefore$ The required angles are 70°, 70° and 40° respectively.

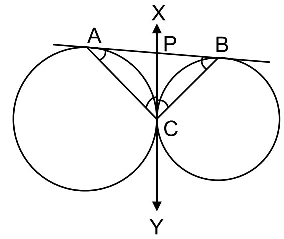
10. Two circles touch each other externally at P. AB is a common tangent to the circles touching them at A and B. Then the value of $\angle APB$ is

(*a*) 90° (*b*) 30°

(*c*) 45° (*d*) 60° **[CBSE 2014]**

Sol. (*a*) 90°

Draw XY the common tangent at C to the externally touching circles and let it intersect AB at P.



Since the lengths of tangents drawn from an external point to a circle are equal

$\therefore$ PA = PC and PB = PC

$\therefore \quad \angle PCA = \angle PAC = x$, say

and $\quad \angle PCB = \angle PBC = y$, say ...(1)

[Angles opposite to equal sides]

In ΔABC, we have

$\angle BAC + \angle ACB + \angle ABC = 180°$

[Angle sum property of a triangle]

$\Rightarrow \angle PAC + (\angle PCA + \angle PCB) + \angle PBC = 180°$

$\Rightarrow \quad x + (x + y) + y = 180°$ [From (1)]

$\Rightarrow \quad 2x + 2y = 180°$

$\Rightarrow \quad x + y = 90°$

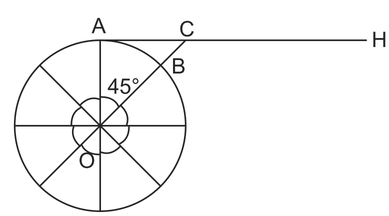
$\Rightarrow \quad \angle PCA + \angle PCB = 90°$

$\Rightarrow \quad \angle ACB = 90°$

Value-based Questions (Optional)

(Page 193)

1. A circular park has 8 radial roads equally inclined to each other. A girl decides to take her grandfather on a wheel chair to the centre O of the park. She starts from her house at H and follows the road along HA which is a tangent to the circular park at the point A and then takes a turn at A to reach the centre O. On her return trip she opts to go along the radial road OB and continues along the path BC in the same direction to be able to reach the point C on the road AH. She then follows the path CH to reach home.



(*a*) If the radius of the circular park is $16\sqrt{2}$ m, find the length of the path BC.

(*b*) What values are exhibited by the girl?

Sol. (*a*) Angle between any two consecutive radii of the circle = $\frac{360°}{8} = 45°$

$\therefore$ In ΔAOC,

Since $\angle AOC = 45°$ and $\angle OAC = 90°$,

$\therefore \quad \angle OCA = 90° - 45° = 45°$

$\therefore \quad AC = AO = 16\sqrt{2}$ m

$\therefore$ By using Pythagoras' theorem in ΔOAC, we have

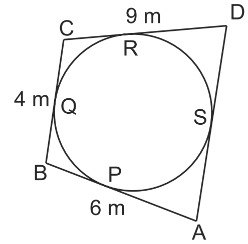
$$OC = \sqrt{OA^2 + AC^2}$$
$$= \sqrt{2OA^2}$$
$$= \sqrt{2}\ OA$$
$$= 16\sqrt{2} \times \sqrt{2}\ \text{m}$$
$$= 32\ \text{m}$$

$\therefore \quad BC = OC - OB = (32 - 16\sqrt{2})$ m

which is the required length.

(*b*) Empathy and interpersonal relationship.

2. Mr Joshi constructed a circular pond for the ducks somewhere within his rectangular garden. He also got a fencing made in the form of a quadrilateral ABCD (as shown in the figure) whose sides AB, BC, CD and DA touch the circular pond at points P, Q, R and S respectively.



(*a*) If AB = 6 m, BC = 4 m and CD = 9 m, find the length of the side AD.

(*b*) What values are exhibited by Mr Joshi by his decision to make pond for the ducks in his garden?

Sol. (*a*) Let DR = DS = x m

$\therefore \quad AS = (AD - x)$ m ...(1)

CR = CQ [Tangents from C]

$\Rightarrow \quad 9\text{ m} - x\text{ m} = 4\text{ m} - BQ$

$\Rightarrow \quad 9\text{ m} - x\text{ m} = 4\text{ m} - BP \quad [\because BQ = BP]$

$\Rightarrow \quad 9\text{ m} - x\text{ m} = 4\text{ m} - (6\text{ m} - AP)$

$\Rightarrow \quad 9\text{ m} - x\text{ m} = -2\text{ m} + AP$

$= -2\text{ m} + AS$

$= -2\text{ m} + AD - x\text{ m}$

[From (1)]

$\Rightarrow \quad 9\text{ m} + 2\text{ m} = AD$

$\Rightarrow \quad AD = 11$ m

Hence, the required length of AB is 11 m.

(*b*) Empathy and environmental awareness.

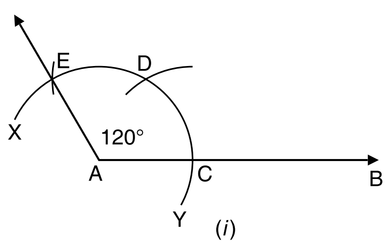
11

Constructions

Checkpoint (Page 197)

1. Draw angles of 120°, 90° and $7\frac{1}{2}^\circ$ with the help of a compass and a ruler only.

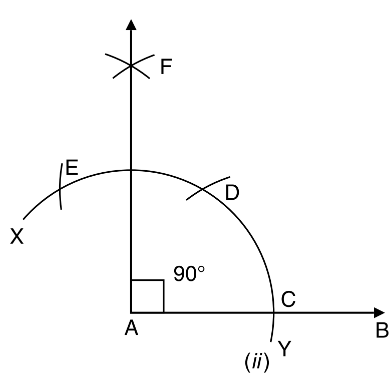
Sol. (*i*) To draw an angle of 120°:



(*i*)

Let AB be any ray. With A as centre and any suitable length as radius, we draw an arc XY cutting the ray at C. With C as centre and the same length as radius, we draw an arc to cut the arc XY at D. With D as centre and the same length as radius, we draw another arc to cut the arc XY at E. We join AE. Then ∠EAB= 120°, the required angle.

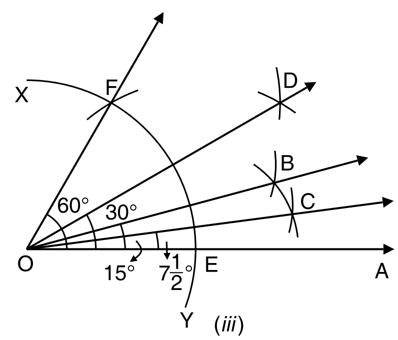
(*ii*) To draw an angle of 90°.



(*ii*)

Let AB be any ray, with A as centre and any suitable length as radius, we draw an arc XY cutting the ray at C. With C as centre and the same length as radius, we draw an arc to cut the arc XY at D. With D as centre and the same length as radius, we draw another arc to cut the arc XY at E. With E as centre and the same length as radius, we draw an arc. With D as centre and the same length as radius, we draw another arc to cut the former arc at the point F. We join AF. Then ∠FAB = 90°, the required angle.

(*iii*) To draw an angle of $7\frac{1}{2}^\circ$:

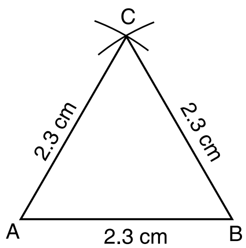


(*iii*)

Let OA be any ray. With O as centre and a suitable length as radius, we draw an arc XY to cut the ray OA at E. With E as centre and the same length as radius, we draw an arc to cut the arc XY at F. We join OF. The ∠FOE = 60°. We now bisect ∠FOE by the ray OD. Then ∠DOE = 30°. We next bisect ∠DOE by ray OB. Then ∠BOE = 15°. Finally, we bisect ∠BOE by the ray OC. Then ∠COE = $7\frac{1}{2}^\circ$, the required angle.

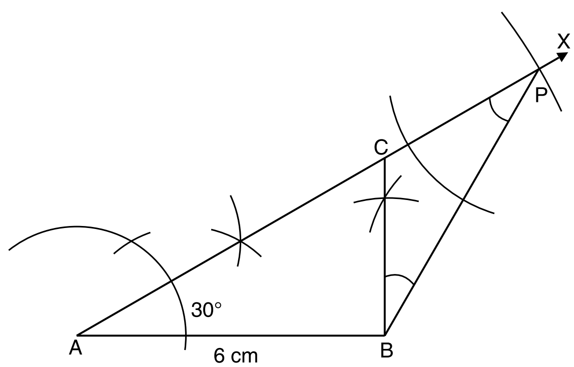
2. Draw an equilateral triangle with side 2.3 cm.

Sol. Let AB be a line segment of length 2.3 cm. With A as centre and a length of 2.3 cm as radius we draw an arc. With B as centre and the same length as the radius, we draw another arc to cut the former arc at C. We join AC and BC. Then, ΔABC is the required equilateral triangle of side 2.3 cm.



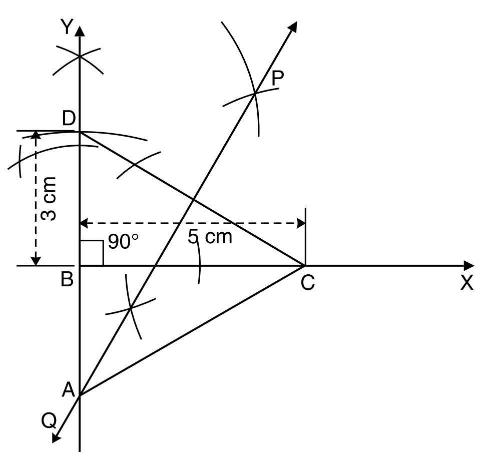
3. Construct a triangle whose base is 6 cm, the sum of other two sides is 10 cm and one base angle is 30°.

Sol. We take a line segment AB = 6 cm and draw an angle of 30° at A, i.e. ∠XAB = 30°. From the ray AX, we cut a length AP = 20 cm. We join PB. At B on BP, we draw an ∠CBP = ∠CPB and then join CB. Then, ΔABC is the required triangle.



4. Construct a right triangle ABC with BC = 5 cm and AC – AB = 3 cm where AC is the hypotenuse.

Sol. We draw a line segment BC = 5 cm. At B, we construct ∠YBC = 90°.



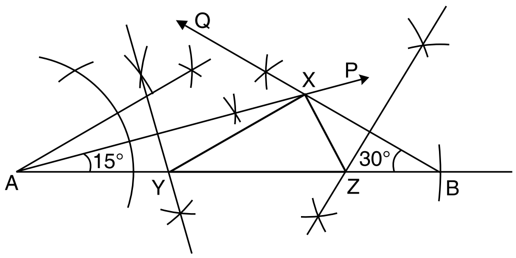
With B as centre and radius equal to a length 3 cm, we draw an arc cutting YB at D. We join DC.

We now draw PQ, the perpendicular bisector of DC and let it intersect DB produced at A. We join AC.

Then, ΔABC is the required triangle.

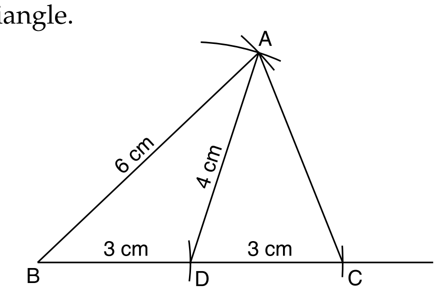
5. Construct a triangle with perimeter 12 cm and base angles 30° and 60°.

Sol. We draw a line segment AB = 12 cm. At A, we construct ∠PAB = 15° and at B, we construct ∠QBA = 30°. Let PA and QB intersect each other at X. We now draw perpendicular bisectors of XA and XB meeting AB at Y and Z respectively. We now join XY and XZ to obtain a ΔXYZ. Then, ΔXYZ is the required triangle.



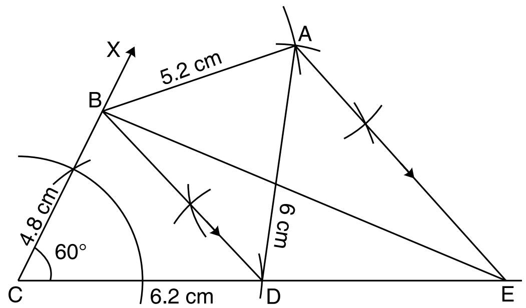
6. Construct a triangle ABC such that BC = 6 cm, AB = 6 cm and the median AD = 4 cm.

Sol. We draw a line segment BD = 3 cm. Taking B as centre and a length of 6 cm as radius, we draw an arc. Taking D as centre and a length of 4 cm as radius, we draw another arc cutting the previous arc at a point A. We now join AB and AD. Then AB = 6 cm, AD = 4 cm and BD = 3 cm. We now produce BD to a point C such that BD = DC = 3 cm. We join AC. Then, ΔABC is the required triangle.



7. Given a quadrilateral ABCD in which AB = 5.2 cm, BC = 4.8 cm, CD = 6.2 cm, DA = 6 cm and ∠C = 60°. Now, construct a triangle equal in area to this quadrilateral.

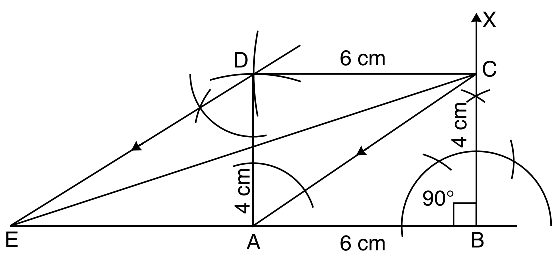
Sol. We draw a line segment CD = 6.2 cm. At C, we draw ∠XCD = 60°. With C as centre and radius equal to a length of 4.8 cm, we draw an arc to cut XC at B. With B as centre and a length equal to 5.2 cm as radius, we draw an arc. With D as centre and a length equal to 6 cm as radius, we draw another arc to cut the pervious arc at A. We join AB and AD. Then, ABCD is the required quadrilateral.



To construct a triangle whose area is equal to the area of the quadrilateral ABCD, we join the diagonal BD of the quadrilateral. Through A, we draw AE ∥ BD to cut CD produced at E. We join BE. Then, ΔBCE is the required triangle whose area is equal to the area of the quadrilateral ABCD.

8. Construct a rectangle ABCD such that AB = 6 cm and BC = 4 cm. Now, construct a triangle with base AB and area equal to the area of the rectangle.

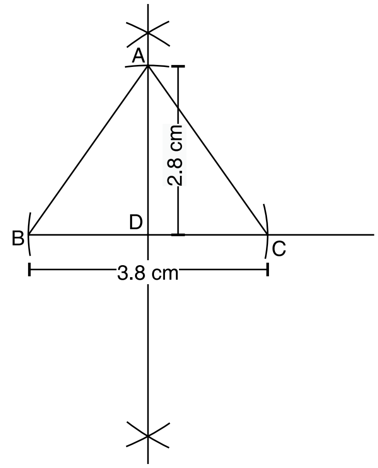
Sol. We draw a line segment AB = 6 cm. At B, we draw BX ⊥ AB. We then cut a length BC = 4 cm from BX and taking A as centre and BC as radius, we draw another arc to cut the previous arc at D.



We now join AD and CD. Then, ABCD is the required rectangle. Now, to draw a triangle equal in area to this rectangle, we first join a diagonal AC and through D, we draw a line segment DE ∥ AC to cut BA produced at E. We then join EC. Then, ΔBEC is the required triangle.

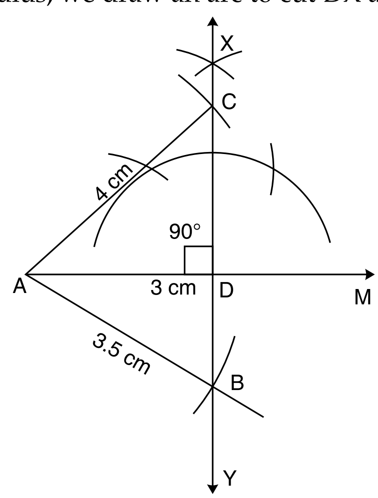
9. Construct an isosceles triangle in which base BC = 3.8 cm and altitude from A on BC is 2.8 cm.

Sol. We draw a line segment BC = 3.8 cm. We then draw AC as the perpendicular bisector of BC. Taking D as the centre and DA = 2.8 cm as radius, we cut a length AD. We then join AB and AC. Then, ΔABC is the required isosceles triangle.



10. Construct a triangle ABC in which AB = 3.5 cm, AC = 4 cm and the length of perpendicular from A on BC is 3 cm.

Sol. We draw a line segment AD = 3 cm from a ray AM. We now draw a line XY through D perpendicular to AM. Taking A as centre and a length of 4 cm as radius, we draw an arc to cut DX at C.



Again, taking A as centre and a length of 3.5 cm as radius, we draw an arc to cut DY at B. We now join AB and AC. Then, ΔABC is the required triangle.

Milestone 1

(Page 199)

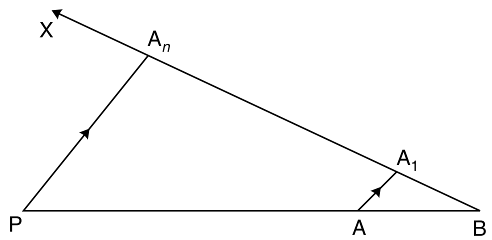
Multiple-Choice Questions

1. To divide a line segment AB = 5.3 cm externally in the ratio 3 : 5, we first draw a ray BX making an acute angle ∠XBA with AB. We then take a minimum n points on BX at equal distances where n is equal to

(*a*) 3 (*b*) 5

(*c*) 2 (*d*) 8

Sol. (*b*) 5



Given that

$$\frac{PA}{PB} = \frac{3}{5} \quad \ldots(1)$$

If AA_1 is drawn parallel to PA_n, then we have

$$\frac{PA}{PB} = \frac{A_nA_1}{A_nB}$$

$$\therefore \quad \frac{A_nA_1}{A_nB} = \frac{3}{5}$$

$$\Rightarrow \quad 3A_nB = 5A_nA_1$$

$$= 5(A_nB - A_1B)$$

$$= 5A_nB - 5A_1B$$

$$\Rightarrow \quad 2A_nB = 5A_1B$$

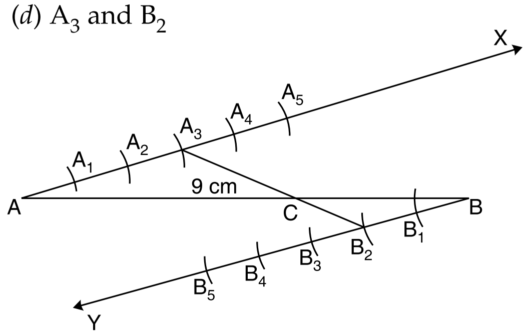
$$\Rightarrow \quad A_1B = \frac{1}{5} \times (2A_nB)$$

Hence, we should divide BX equally into n parts where $n = 5$.

2. To divide a line segment AB = 9 cm internally in the ratio 2 : 3, we draw a ray AX such that $\angle BAX$ is an acute angle and then locate points A_1, A_2, A_3, A_4 and A_5 on AX such that all these lengths are equal. Then draw another ray BY at B, such that BY $\parallel$ AX and then locate points B_1, B_2 B_3, B_4 and B_5 on BY such that all these lengths are equal. Then the points to be joined are

(*a*) A_1 and B_1 (*b*) A_3 and B_1
(*c*) B_3 and A_1 (*d*) A_3 and B_2

Sol. (*d*) A_3 and B_2



From similar triangles A_3AC and B_2BC, we have $\frac{AC}{BC} = \frac{A_3A}{B_2B} = \frac{3}{2}$. Hence, we must join A_3 and B_2.

Very Short Answer Type Questions

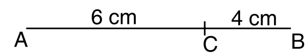
3. If you divide a line segment AB of length 10 cm in the ratio 3 : 2 by means of the point C, can you construct a triangle with sides AC, CB and another side of length 2 cm? Give reason.

Sol. If AC : CB = 3 : 2 and AB = 10 cm, then

$$AC = \frac{3}{5} \times 10 \text{ cm} = 6 \text{ cm}$$

and

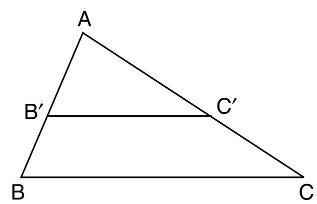
$$BC = \frac{2}{5} \times 10 \text{ cm} = 4 \text{ cm}$$



Hence, the three sides are of lengths 4 cm, 6 cm and 2 cm. Hence, no triangle can be constructed with these three sides since, 4 + 2 = 6, i.e. the sum of two sides is equal to the third side, i.e. the three points are collinear.

4. You are given a triangle ABC. You have constructed another triangle AB′C′ and B′ and C′ are points on AB and AC respectively such that $AB' = \frac{2}{3}AB$ and $AC' = \frac{2}{3}AC$. Find the relation between the sides BC and B′C′.

Sol.



Given that $\quad AB' = \frac{2}{3}AB$

and $\quad AC' = \frac{2}{3}AC$

$$\therefore \quad \frac{AB'}{AB} = \frac{AC'}{AC} = \frac{2}{3}$$

$$\therefore \quad \Delta AB'C' \sim \Delta ABC$$

$$\therefore \quad \frac{B'C'}{BC} = \frac{AB'}{AB} = \frac{AC'}{AC} = \frac{2}{3}$$

$$\Rightarrow \quad 2BC = 3B'C'$$

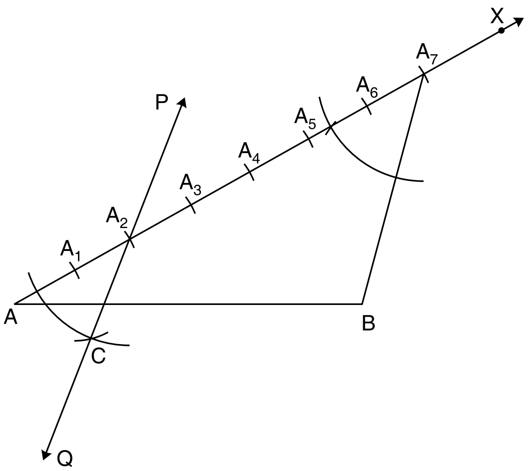
which is the required relation.

Short Answer Type-I Questions

5. Draw a line segment of length 6.3 cm and divide it internally in the ratio 2 : 5.

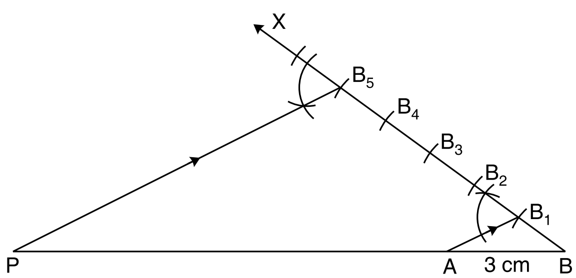
Sol. We draw a line segment AB of length 6.3 cm and draw any acute angle XAB at A. On AX, we mark 7 points A_1, A_2, A_3, A_4, A_5, A_6 and A_7 such that $AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5 = A_5A_6 =$

A_6A_7. We join A_7 and B. Through A_2, we draw PQ parallel to A_7B cutting AB at C. Then AC : CB = 2 : 5. Hence, C is the required point of division in the given ration 2 : 5.



6. Divide a given line segment of 3 cm externally in the ratio 4 : 5. Write the steps of construction.

Sol. We draw a line segment AB of length 3 cm. At B, we draw a ray BX making an acute angle $\angle XBA$ with AB. Along BX, we mark 5 points B_1, B_2, B_3, B_4, and B_5 at equal distances such that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4 = B_4B_5$. We join B_1A. Through B_5, we draw $B_5P \parallel B_1A$ intersecting BA produced at P. Then the point P divides AB externally in the ratio PA : PB = 4 : 5.

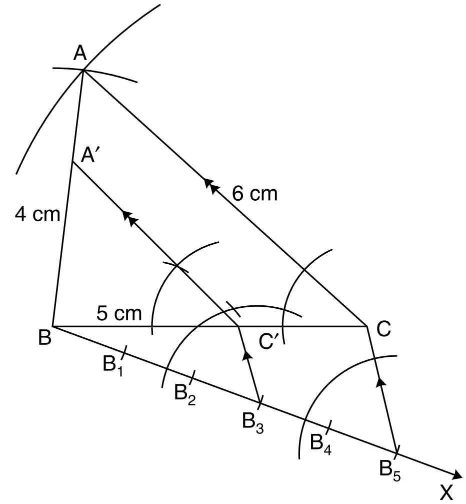


Short Answer Type-II Questions

7. Draw a triangle ABC in which AB = 4 cm, BC = 5 cm and AC = 6 cm. Then construct another triangle whose sides are $\frac{3}{5}$ of the corresponding sides of ΔABC. **[CBSE 2016]**

Sol. We draw a line segment BC = 5 cm.

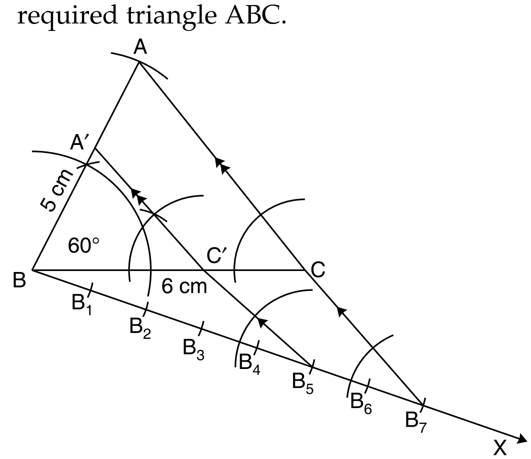
With B as centre and a length of 4 cm as radius, we draw an arc. With C as centre and a length of 6 cm as radius, we draw another arc cutting the previous arc at A. We now join AB and AC. Then, ΔABC is the required triangle.



Now, we draw a ray BX making an acute angle with BC on opposite side of the vertex A of ΔABC. Starting from B, we cut off 5 equal line segments BB_1, B_1B_2, B_2B_3, B_3B_4, and B_4B_5. We now join B_5C. From B_3, we draw $B_3C \parallel B_5C$ to meet BC at C′. From C′, we draw $C'A' \parallel CA$ to meet BA at A′. Then, $\Delta A'BC'$ is the required triangle whose sides are $\frac{3}{5}$ of the corresponding sides of ΔABC.

8. Construct a triangle ABC in which AB = 5 cm, BC = 6 cm and $\angle ABC = 60°$. Now construct another triangle whose sides are $\frac{5}{7}$ times the corresponding sides of ΔABC. **[CBSE 2015]**

Sol. We draw a line segment BC = 6 cm and at B, we construct $\Delta ABC = 60°$. From BA, we cut a length BA = 5 cm. We then join AC to obtain the required triangle ABC.



Now, we draw a ray BX making an acute angle with BC on opposite side of the vertex A of ΔABC. Starting from B, we cut off 7 equal line segments BB_1, B_1B_2, B_2B_3, B_3B_4, B_4B_5, B_5B_6, and B_6B_7. We now join B_7C. From B_5, we draw

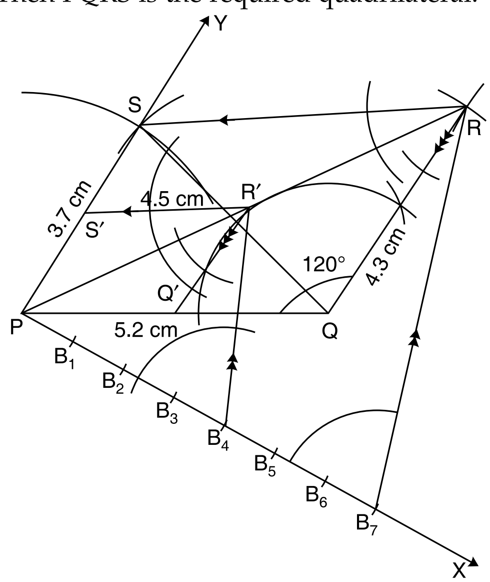
$B_5C' \parallel B_7C$ to meet BC at C′. From C′, we draw $C'A' \parallel CA$ to meet BA at A′.

Then, ΔA′BC′ is the required triangle whose sides are $\frac{5}{7}$ times the corresponding sides of ΔABC.

Long Answer Type Questions

9. Construct a quadrilateral PQRS with PQ = 5.2 cm, QS = 4.5 cm, PS = 3.7 cm, ∠Q = 120° and QR = 4.3 cm. Construct another quadrilateral similar to PQRS such that the sides of the new quadrilateral is $\frac{4}{7}$ th of the corresponding sides of the quadrilateral PQRS.

Sol. We draw a line segment PQ = 5.2 cm. At Q, we construct ∠PQR = 120°. With centre P and a length of 3.7 cm as radius, we draw an arc. With centre at Q and a length of 4.5 cm as radius we draw another arc to cut the previous arc at S. With centre at Q and a length of 4.3 cm as radius we cut a length QR. We now join RS and PR. Then PQRS is the required quadrilateral.

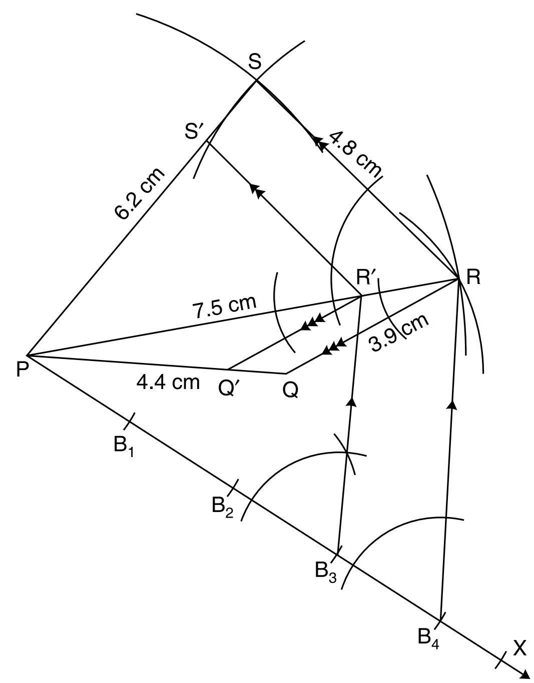


We now draw a ray PX inclined at a certain acute angle with PQ on the opposite side of R or S. Starting from P, we cut off 7 equal line segments PB_1, B_1B_2, B_2B_3, B_3B_4, B_4B_5, B_5B_6 and B_6B_7 on PX. We join B_7R and draw a line segment $B_4R' \parallel B_7R$ to meet PR at R′. From R′, we draw $R'S' \parallel RS$ to meet PS at S′. Finally, from R′, we draw $R'Q' \parallel RQ$ to meet PQ at Q′. Then, PQ′R′S′ is the required quadrilateral whose sides are $\frac{4}{7}$ th of the corresponding sides of the quadrilateral PQRS.

10. Construct a quadrilateral PQRS, where PQ = 4.4 cm, QR = 3.9 cm, RS = 4.8 cm, PS = 6.2 cm and PR = 7.5 cm. Construct another quadrilateral similar to PQRS such that the sides of the new quadrilateral is $\frac{3}{4}$th of the corresponding sides of the quadrilateral PQRS.

Sol. We draw a line segment PQ = 4.4 cm. With P as centre and a length of 7.5 cm as radius, we draw an arc. With Q as centre and a length of 3.9 cm as radius, we draw another arc to cut the previous arc at R. With P as centre and a length 6.2 cm as radius we draw an arc. With R as centre and a length of 4.8 cm as radius, we draw another arc to cut the previous arc at S. We join PS, RS and PR. Then, PQRS is the required quadrilateral.

We now draw a ray PX inclined at a certain acute angle with PQ on opposite side of R or S. Starting from P, we cut off 4 equal line segments PB_1, B_1B_2, B_2B_3, and B_3B_4 on PX. We now join B_4R and draw a line segment $B_3R'_1 \parallel B_4R$ to meet PR at R′. From R′, we draw R′S′ parallel to RS to meet PS at S′. Finally, from R′, we draw $R'Q' \parallel RQ$ to meet PQ at Q′. Then, PQ′R′S′ is the required quadrilateral whose sides are $\frac{3}{4}$ th of the corresponding sides of the quadrilateral PQRS.



Milestone 2

(Page 201)

Multiple-Choice Questions

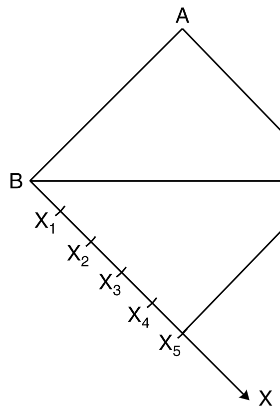
1. To construct a triangle similar to a given triangle ABC, draw a ray BX such that ∠CBX is an acute angle and X is on the opposite side of A with respect to BC. Then locate points X_1, X_2, X_3, X_4 and X_5 at equal distances on BX. The points to be joined in the next step are

(*a*) X_7 and C (*b*) X_2 and C

(*c*) X_5 and C (*d*) X_{12} and C

Sol. (*c*) X_5 and C

There are 5 points X_1, X_2, X_3, X_4, X_5 on BX at equal distances from B.



∴ We must join the last point X_5 with C.

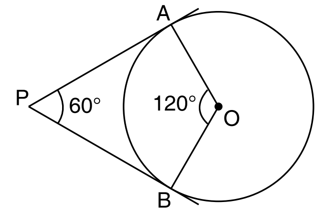
2. To draw a pair of tangents to a circle which are inclined to each other at an angle of 60°, it is required to draw tangents at the end points of the two radii of the circle, which are inclined at an angle of

(*a*) 135° (*b*) 120°

(*c*) 60° (*d*) 90° **[CBSE SP 2012]**

Sol. (*b*) 120°

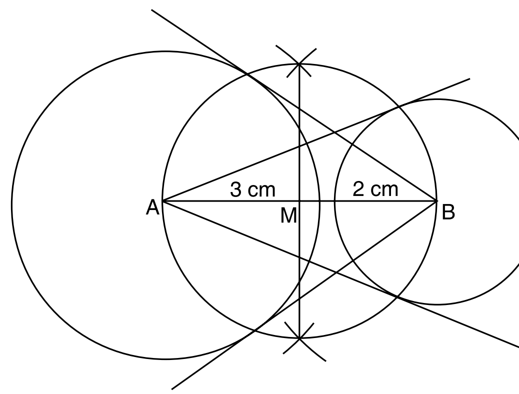
We know that the quadrilateral formed by two radii OA and OB of a circle with centre at O, inclined at certain angle with each other, other than 180° or a reflex angle and the two tangents PA and PB at its two ends A and B, meeting with each other at P, form a cyclic quadrilateral PAOB. Hence, any pair of opposite angles is supplementary. So, if the angle between the two tangents is 60°, then the opposite angle between the pair of radii OA and OB is the supplement of 60°, i.e. 180° – 60° = 120°.



Very Short Answer Type Questions

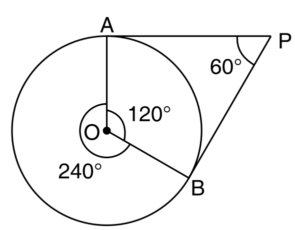
3. To draw tangents to each of the two circles with radii 3 cm and 2 cm from the centre of the other circle such that the distance between their centres A and B is 6 cm, a perpendicular bisector of AB is drawn intersecting AB at M. The next step of construction is to draw a third circle cutting the two given circles. What is the diameter and centre of this third circle?

Sol. The diameter of the third circle should be AB and the centre of this circle should be at the point M, the mid-point of AB as shown in the figure.



4. Two tangents are drawn at the end points of two radii of a circle. If the reflex angle between these two radii of the circle is 240°, then find the angle between the two tangents.

Sol. Let O be the centre of the circle and OA and OB are two radii of this circle such that reflex ∠AOB = 240°. Two tangents at A and B meet each other at P outside the circle. Then AOBP is a cyclic quadrilateral.



Here, ∠AOB = 360° – 240° = 120°

Since, $\angle AOB + \angle APB = 180°$

$\therefore \quad 120° + \angle APB = 180°$

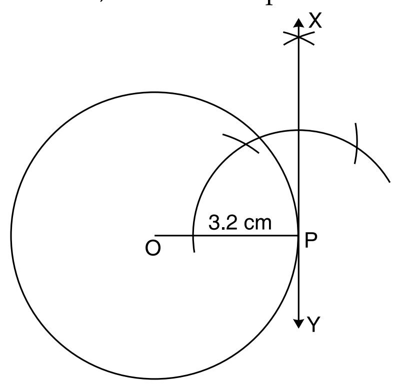
$\Rightarrow \quad \angle APB = 180° - 120° = 60°$

Hence, the required angle between the two tangents is 60°.

Short Answer Type-I Questions

5. From a point P on the circle of radius 3.2 cm, draw a tangent to the circle. Write the steps of construction.

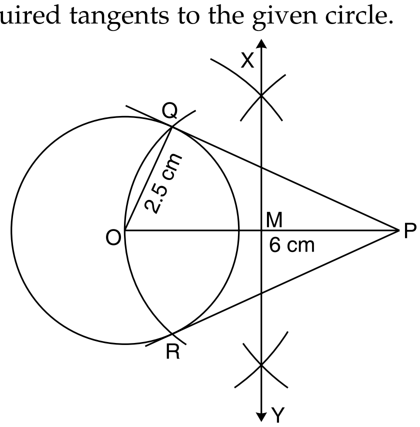
Sol. Taking O as centre and OP = 3.2 cm as radius, we draw a circle, where P is a point on the circle.



We join OP and at P, we draw a line XY ⊥ OP. Then, XPY is the required tangent at P to the circle.

6. Construct a pair of tangents from a point 6 cm away from the centre of a circle of radius 2.5 cm. Measure the lengths of the tangents.

Sol. Let O be the centre of the circle with radius OQ = 2.5 cm, where Q is a point on the circle. P is a point outside this circle at a distance of 6 cm from O so that OP = 6 cm. We now draw a perpendicular bisector XY of OP. Let it intersect OP at M. Then M is the mid-point of OP. With M as centre and OM as radius, we draw a circle cutting the given circle at the points Q and R. We now join PQ and PR. Then PQ and PR are two required tangents to the given circle.



We join OQ. Then $\angle OQP = 90°$.

$\therefore$ By using Pythagoras' Theorem, we have

$$PQ = \sqrt{OP^2 - OQ^2}$$

$$= \sqrt{6^2 - 2.5^2} \text{ cm}$$

$$= \sqrt{29.75} \text{ cm}$$

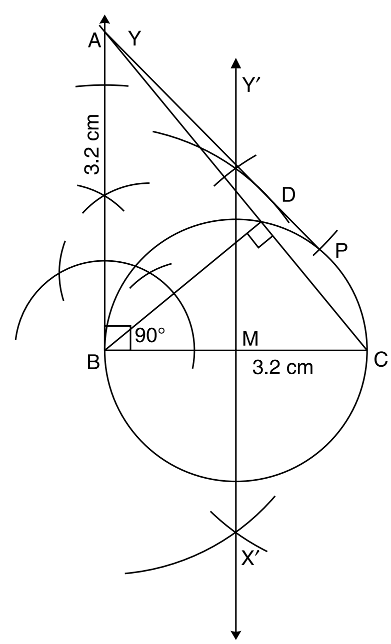
$$= 5.45 \text{ cm (approx.)}$$

Hence, PQ = PR = 5.45 cm (approx.)

Short Answer Type-II Questions

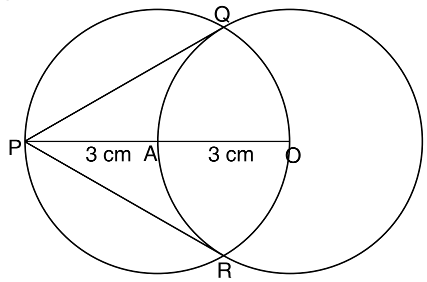
7. Let ABC be a right triangle in which AB = 3.2 cm, BC = 3.2 cm and $\angle B = 90°$. BD is the perpendicular from B on AC. The circle through B, C and D is drawn. Construct the tangents from A to this circle. **[CBSE SP 2011]**

Sol. We draw a line segment BC of length 3.2 cm. We now draw $\angle YBC = 90°$ at B and cut a length AC = 3.2 cm from BY. We join AC. We now draw BD ⊥ AC. We now draw X′Y′ as the perpendicular bisector of BC. Let X′Y′ cut BC at M. Then M is the mid-point of BC. With M as centre and MB as a radius, we draw a circle. This circle will pass through the point D, since $\angle BDC = 90°$. Since, $\angle ABC = 90°$, $\therefore$ AB is a tangent to this circle from A. Taking A as centre and a length AB = 3.2 cm as radius, we draw an arc to cut the circle through D at P. We join AP. Then, AP and AB are the required two tangents to the circle passing through the points B, C and D.



8. Draw a pair of tangents to a circle of radius 3 cm which are inclined to each other at an angle of 60°. Write the steps of construction. **[CBSE SP 2015]**

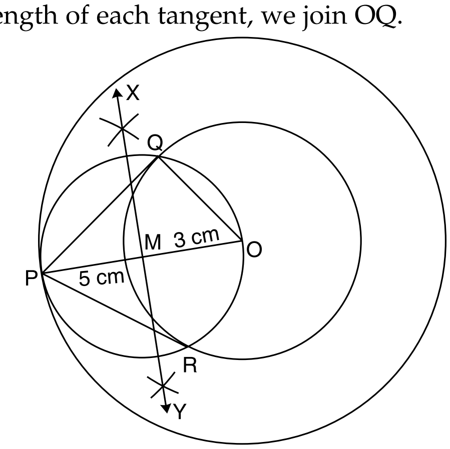
Sol. Taking O as centre and OA =3 cm as radius, we draw a circle. We produce OA to P such that OA = AP = 3 cm. With A as centre and radius equal to 3 cm, we draw another circle intersecting the first circle at the points Q and R. We join PQ and PR. Then, PQ and PR are the required tangents.



Long Answer Type Questions

9. Draw two concentric circles of radii 3 cm and 5 cm. Construct a tangent to the smaller circle from a point on the larger circle. Also, measure the length of this tangent. **[CBSE SP 2016]**

Sol. Taking a point O as the centre, we draw two concentric circles of radii 3 cm and 5 cm. We take a point P on the bigger circle and join OP. We draw XY as perpendicular bisectors of OP and let it intersect OP at M. Then M is the mid-point of PO. Taking M as centre and MO as radius, we draw a circle intersecting the smaller circle of radius 3 cm at the points Q and R. We now join PQ and PR. Then, PQ and PR are the required tangents from P to the smaller circle. To find the length of each tangent, we join OQ.



Since $\angle PQO = 90°$, hence, by using Pythagoras' Theorem in ΔPQO, we get

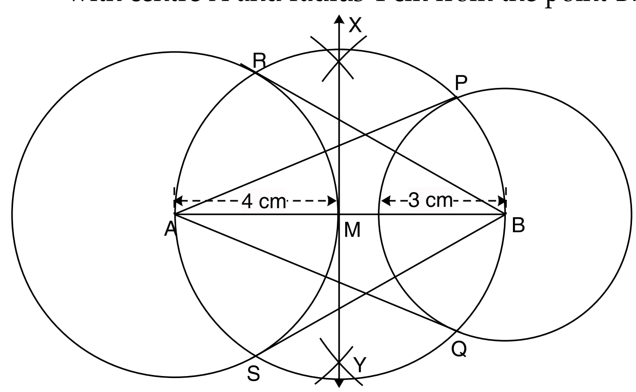
$$PQ = \sqrt{PO^2 - OQ^2} = \sqrt{5^2 - 3^2} \text{ cm}$$

$$= \sqrt{25 - 9} \text{ cm} = \sqrt{16} \text{ cm} = 4 \text{ cm}$$

Hence, the required length is 4 cm.

10. Draw a line segment AB of length 8 cm. Taking A as centre, draw a circle of radius 4 cm and taking B as centre, draw another circle of radius 3 cm. Construct tangents to each circle from the centre of the other circle. Write the steps of construction also. **[CBSE 2014]**

Sol. We draw a line segment AB = 8 cm. With A as centre and radius = 4 cm, we draw a circle. With B as centre and radius = 3 cm, we draw another circle. We now draw XY as the perpendicular bisector of AB and let it cut AB at M. Then M is the mid-point of AB. With M as centre and AM as radius, we draw a circle intersecting the circle of radius 3 cm at P and Q. We now join AP, AQ, BR and BS. Then, AP and AQ are tangents to the circle with centre B and radius 3 cm from the point A. Also, BR and BS are tangents to the circle with centre A and radius 4 cm from the point B.

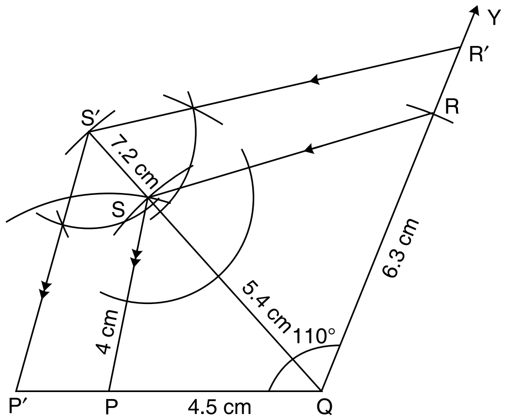


Higher Order Thinking Skills (HOTS) Questions

(Page 202)

1. Construct a quadrilateral PQRS such that PQ = 4.5 cm, PS = 4 cm, SQ = 5.4 cm, QR = 6.3 cm and $\angle Q = 110°$. Construct another quadrilateral P′QR′S′ similar to the quadrilateral PQRS so that the diagonal S′Q = 7.2 cm.

Sol. We draw a line segment PQ = 4.5 cm and construct $\angle YQP = 110°$ with the help of a protractor. From QY, we mark QR = 6.3 cm. Taking P as centre and a length of 4 cm as radius, we draw an arc. Again, taking Q as centre and a length of 5.4 cm as radius, we draw another arc to cut the previous arc at a point S. We join SP and SR to obtain a quadrilateral PQRS. To draw another quadrilateral similar to the quadrilateral PQRS, we join QS and produce it to S′ such that QS′ = 7.2 cm. We see that $\frac{SQ}{S'Q} = \frac{5.4}{7.2} = \frac{3}{4}$.



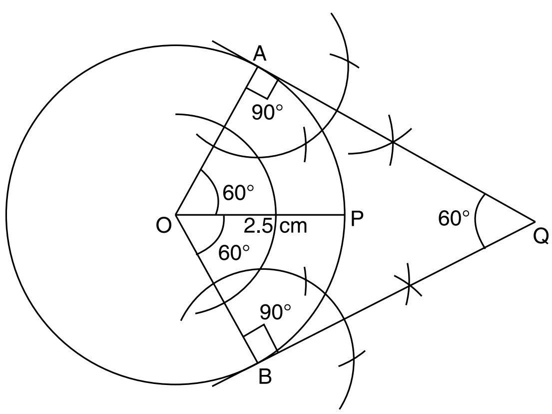
We now draw S′R′ ∥ SR to intersect QR produced at R′. Again, we draw S′P′ ∥ SP to intersect QP produced at P′. Then we can see that

$$\frac{QR}{QR'} = \frac{SQ}{S'Q} = \frac{3}{4} \text{ and } \frac{QP}{QP'} = \frac{SQ}{S'Q} = \frac{3}{4}.$$

Hence, the quadrilateral P′QR′S′ is the required quadrilateral similar to the quadrilateral PQRS.

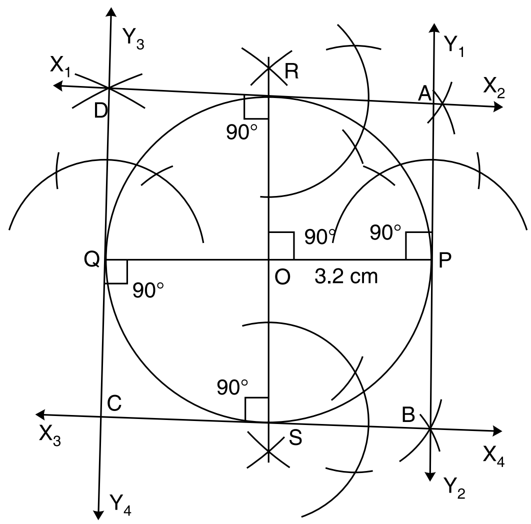
2. Draw a circle of radius 2.5 cm. Draw two tangents to it inclined at an angle of 60° to each other.

Sol. Let O be the centre of a circle of radius OP = 2.5 cm, P being a point on the circle. We join OP. We now construct ∠AOP = ∠BOP = 60° where A and B are two points on the circle on opposite sides of OP. At A, we draw ∠OAQ = 90°. and at B, we draw ∠OBQ = 90°, such that AQ and BQ are the required tangents to the circle at A and B respectively such that ∠AQB = 180° – 120° = 60°.



3. Draw a circle of radius 3.2 cm. Construct a square circumscribing the circle.

Sol. We draw a circle with centre at O and radius OP = 3.2 cm, where P is a point on the circle. We produce PO to cut the circle at Q so that POQ is diameter of the circle. We now draw ROS ⊥ POQ, where R and S are two points on the circle. We now draw lines Y_1PY_2 at P and Y_3QY_4 at Q each perpendicular to PQ. Also, we draw lines X_1RX_2 at R and X_3SX_4 at S, each perpendicular to RS. Let X_1X_2 intersect the lines Y_1Y_2 and Y_3Y_4 at A and D respectively and the lines X_3X_4 intersect the lines Y_1Y_2 and Y_3Y_4 at B and C respectively. Then, ABCD is the required square circumscribing the given circle.



Self-Assessment

(Page 202)

Multiple-Choice Questions

1. To construct a cyclic quadrilateral PQRS in which ∠Q = 90°, if a circle on which points P, Q, R and S lie, has to be drawn, then the centre of this circle is

(*a*) the mid-point of the diagonal QS.

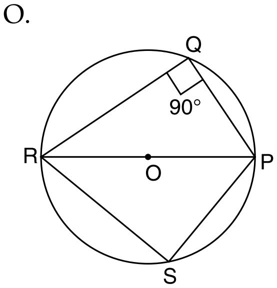
(*b*) the mid-point of the diagonal PR.

(*c*) the point of intersection of diagonals PR and QS.

(*d*) a point which lies neither on PR nor on QS.

Sol. (*b*) the mid-point of the diagonal PR.

Let P, Q, R and S be four points on a circle with centre at O.



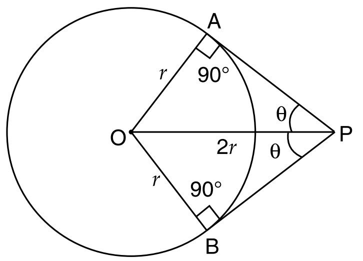
Then, PQRS is a cyclic quadrilateral. It is given that $\angle PQR = 90°$. Hence, the diagonal PR must be a diameter of the circle, since we know that angle in a semi-circle is 90°. Hence, the centre of the circle will be at O, the mid-point of the diagonal PR.

2. If you draw a pair of tangents to a circle C(O, r) from a point P such that $OP = 2r$, then the angle between the two tangents is

(*a*) 90° (*b*) 45°

(*c*) 60° (*d*) 30°

Sol. (*c*) 60°



Let the two tangents from P touch the circle C(O, r) at A and B. Then $\angle OAP = \angle OBP = 90°$. If $\angle OPA = \angle OPB = \theta$, then in ΔOAP,

$$\sin\theta = \frac{OA}{OP} = \frac{r}{2r} = \frac{1}{2}$$

$$= \sin 30°$$

$$\therefore \quad \theta = 30°$$

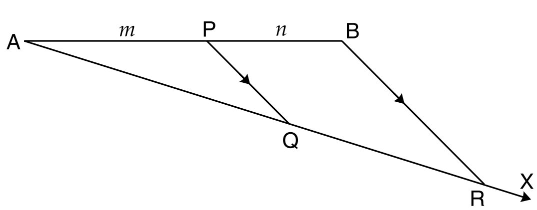
$$\therefore \quad \angle APB = 2\theta = 2 \times 30° = 60°$$

Hence, the required angle between the two tangents is 60°.

Very Short Answer Type Questions

3. To divide a line segment AB internally in the ratio $m : n$ (where m and n are positive integers), draw a ray AX so that $\angle BAX$ is an acute angle and then mark certain points on ray AX at equal distances. Then what will be the minimum number of these points?

Sol. Let P be the point on AB such that $AP : PB = m : n$, we take a point R on AX and join BR.



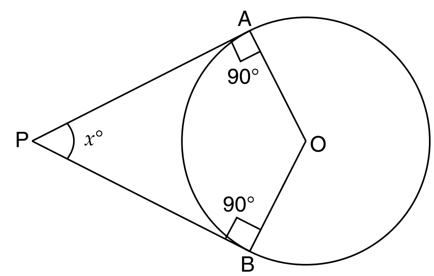
If Q is a point on AR such that $\frac{AP}{AB} = \frac{AQ}{AR}$, then PQ ∥ BR.

$$\therefore \quad \frac{AQ}{AR} = \frac{m}{AB + PB} = \frac{m}{m+n}$$

Hence, it follows that we should divide AR at least into $m + n$ equal parts. Hence, the minimum number of these points of division will be $m + n$.

4. To draw a pair of tangents to a circle, which are inclined to each other at an angle $x°$, it is required to draw tangents at the end points of those two radii of the circle. What will be the angle between these two radii?

Sol. Let PA and PB be a pair of tangents to a circle with centre at O, A and B being two points on the circle and P is an outside point. Clearly, PAOB will be a cyclic quadrilateral, since $\angle OAP + \angle OBP = 90° + 90° = 180°$.



$\therefore$ $\angle APB$ and $\angle AOB$ of the cyclic quadrilateral APBO will be supplementary angles.

$$\therefore \quad \angle APB + \angle AOB = 180°$$

$$\Rightarrow \quad x° + \angle AOB = 180°$$

$$\Rightarrow \quad \angle AOB = 180° - x°$$

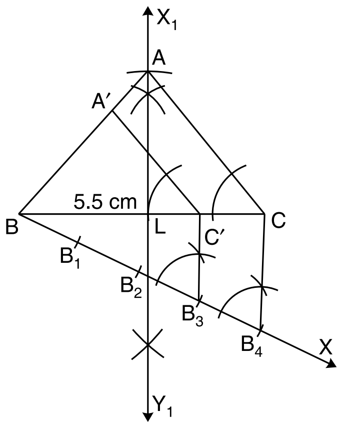
i.e. the required angle between the two radii OA and OB of the circle will be $180° - x°$.

Short Answer Type-I Questions

5. Draw an isosceles triangle ABC in which BC = 5.5 cm and altitude AL = 3 cm. Then construct another triangle whose sides are $\frac{3}{4}$ of the corresponding sides of ΔABC. **[CBSE 2016]**

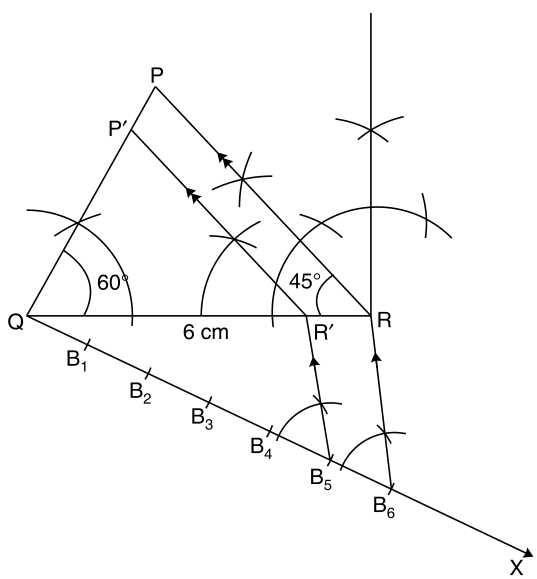
Sol. We draw a line segment BC = 5.5 cm. Let X_1Y_1 be the perpendicular bisector of BC intersecting BC at L. Then L is the mid-point of BC. Taking L as the center and a length of 3 cm as the radius, we draw an arc cutting X_1L at A. We now join AB and AC. Then, ΔABC with AB = AC is the required isosceles triangle. Now, we draw a ray BX such that $\angle XBC$ is an acute angle. From B, we mark 4 points B_1, B_2, B_3 and B_4 at equal distances. We join B_4C and from B_3, draw $B_3C' \parallel B_4C$ cutting BC at C′. Then BC′ will be $\frac{3}{4}$ BC. Now, from C′, we draw $C'A' \parallel CA$ cutting BA at A′. Then,

$\Delta A'BC'$ is the required triangle whose sides are $\frac{3}{4}$ of the corresponding sides of ΔABC.



6. Construct a triangle PQR in which QR = 6 cm, $\angle Q = 60°$ and $\angle R = 45°$. Construct another triangle similar to ΔPQR such that its sides are $\frac{5}{6}$ of the corresponding sides of ΔPQR. **[CBSE 2010]**

Sol. We draw a line segment QR = 6 cm. At Q and R, we draw angles of 60° and 45° respectively on the same side of QR so that $\angle PQR = 60°$ and $\angle PRQ = 45°$ where P is the point where the two arms of two angles meet each other. Then, ΔPQR is the required triangle. At Q, we draw any acute angle XQR. From Q, we mark 6 points B_1, B_2, B_3, B_4, B_5 and B_6 on BX such that $QB_1 = B_1B_2 = B_2B_3 = B_3B_4 = B_4B_5 = B_5B_6$.

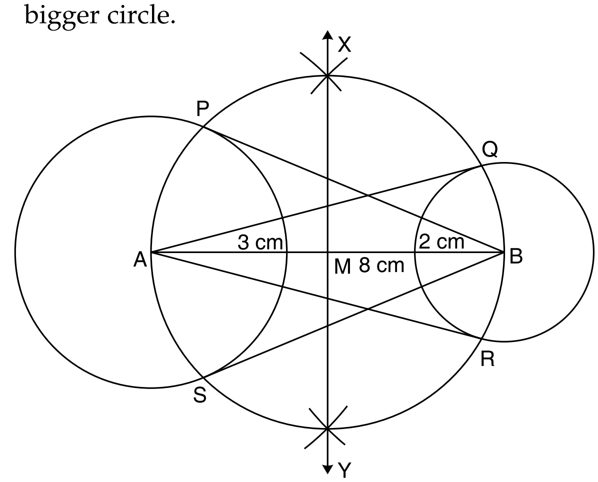


We now join B_6R and through B_5, we draw $B_5R' \parallel B_6R$ intersecting QR at R′. Through R′, we draw $R'P' \parallel RP$ intersecting QP at P′. Then, $\Delta P'QR'$ is the required triangle similar to ΔPQR such that its sides are $\frac{5}{6}$th of the corresponding sides of ΔPQR.

Short Answer Type-II Questions

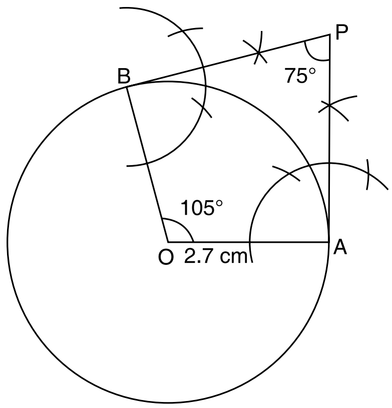
7. Draw a line segment AB of length 8 cm. Taking A as centre, draw a circle of radius 3 cm and taking B as centre draw another circle of radius 2 cm. Construct tangents to each circle from the centre of the other circle. Write the steps of construction also.

Sol. We draw a line segment AB = 8 cm. With A as centre and a length 3 cm as radius, we draw a circle. Again, with B as centre and a length of 2 cm as radius, we draw another circle. We now draw XY as the perpendicular bisector of AB. Let it intersect AB at M. Then, M is the mid-point of AB. Taking M as the centre and a length MA as radius, we draw another circle intersecting the bigger circle of radius 3 cm at P and S and the smaller circle of radius 2 cm at Q and R. We now join AQ, AR and BP, BS. Then AQ and AR are the required tangents from A to the smaller circle and BP, BS are the tangents from B to the bigger circle.



8. Draw a circle of radius 2.7 cm and then draw two tangents to this circle so that the angle between them is 75°.

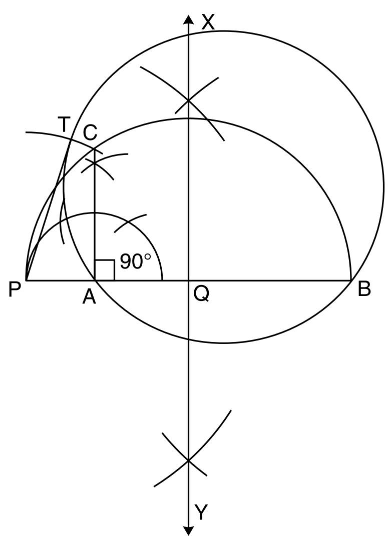
Sol. We draw a circle with centre at O and radius OA = 2.7 cm where A is a point on the circle. We construct an angle $\angle AOB = 180° - 75° = 105°$ at the centre O with the help of a protector, where B is another point on the circle. At A and B. We draw two rays perpendicular to OA and OB respectively. Let these two rays meet each other at P. Then PA and PB are the required two tangents such that $\angle APB = 180° - \angle AOB = 180° - 105° = 75°$, the given angle.



Long Answer Type Questions

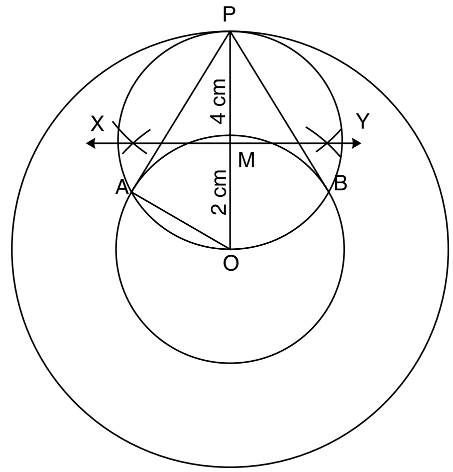
9. Draw a circle of radius 2.5 cm. P is a point outside this circle. Draw two tangents to the circle from the point assuming that the centre of the circle is not known. Write the steps of construction.

Sol. We draw a circle with radius 2.5 cm and take any point P outside the circle. We now draw a secant PAB to the circle intersecting it at A and B, we draw a perpendicular bisector XY of PB bisecting it at a point Q. Taking Q as centre, and PQ as radius, we draw a semi-circle. We now draw a line segment AC perpendicular to PB through A to intersect the semi-circle at C. Taking P as centre and PC as radius, we draw an arc intersecting the given circle at T. We now join PT. Then, PT is the required tangent.



10. Construct a tangent to a circle of radius 2 cm from a point on the concentric circle of radius 4 cm and measure its length. Also, verify the measurements by actual calculation.

Sol. We draw two concentric circles with common centre O and the radius 2 cm and 4 cm. We take any point P on the bigger circle and join OP. Let XY be the perpendicular bisector of OP intersecting it at a point M. Then M is the mid-point of OP. Taking M as centre and OM as radius, we draw a circle intersecting the smaller circle at A and B. We join AP and BP. Then AP or BP is the required tangent to the smaller circle from P.



To find the length of OP, we join AO.

Then $\angle OAP = 90°$.

$\therefore$ Using Pythagoras' theorem in ΔAPO, we have

$$\begin{aligned} AP &= \sqrt{OP^2 - OA^2} \\ &= \sqrt{4^2 - 2^2} \text{ cm} \\ &= \sqrt{16 - 4} \text{ cm} \\ &= \sqrt{12} \text{ cm} \\ &= 2\sqrt{3} \text{ cm} \\ &= 2 \times 1.73 \text{ cm} \\ &= 3.46 \text{ cm (approx.)} \end{aligned}$$

By actual measurement, we see that AP = 3.5 cm (approx.)

Let's Compete

(Page 202)

Multiple-Choice Questions

1. By geometrical construction, it is possible to divide a line segment exactly in the ratio

(*a*) $\sqrt{5} : \dfrac{1}{\sqrt{10}}$ (*b*) $\sqrt{10} : \sqrt{5}$

(*c*) $\sqrt{5} : \dfrac{1}{\sqrt{5}}$ (*d*) $\sqrt{5} : \sqrt{10}$

Sol. (*c*) $\sqrt{5} : \dfrac{1}{\sqrt{5}}$

We observe that among all the given ratios, only $\sqrt{5} : \frac{1}{\sqrt{5}}$ which is the same as 5 : 1 can be represented by a rational number. Other ratios represent irrational numbers. Hence, by geometrical construction, it is possible to divide a line segment exactly in the ratio $\sqrt{5} : \frac{1}{\sqrt{5}}$, i.e. 5 : 1.

2. A pair of tangents can be constructed to a circle from an external point only if the angle between these two tangents is

(*a*) 150° (*b*) 180°

(*c*) 190° (*d*) 210°

Sol. (*a*) 150°

We know that angle between two tangents drawn from an external point, which is the supplement of the angle between two radii of a circle, is less than 180°. Now, only 150° in (*a*) is less than 180°.

3. Construct a triangle with sides 3 cm, 4 cm and 5 cm. Draw a circle inside the triangle such that the three sides of the triangle are tangents to the circle. Then the centre of the circle will be the point of intersection of

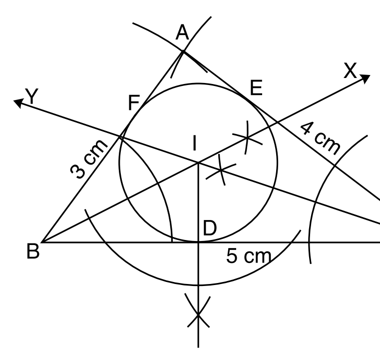
(*a*) three altitudes of the triangle.

(*b*) three medians of the triangle.

(*c*) three perpendicular bisectors of the sides of the triangle.

(*d*) three bisectors of the angles of the triangle.

Sol. (*d*) three bisectors of the angles of the triangle.



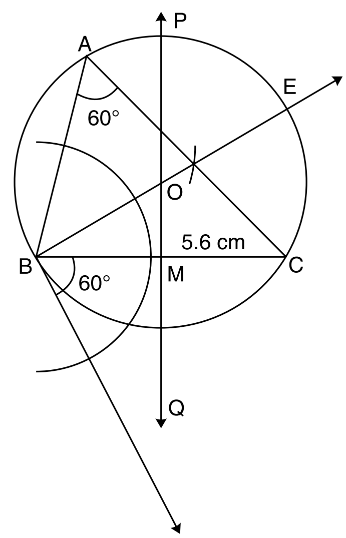
Taking BC = 5 cm, we draw a line segment: Taking B as centre and a length of 3 cm as radius, we draw an arc. Taking C as centre and a length of 4 cm as radius, we draw another arc intersecting the previous arc at A. We join AB and AC. Then, ΔABC is the required triangle. To draw a circle within this triangle such that the three sides AB, BC and CA are tangents to this circle, we shall find the in-centre of this triangle. We know that the in-centre is the point of intersection of three bisectors of the angles of a triangle and the three bisectors are always concurrent. We now bisect any two angles, say, ∠B and ∠C. Let I be the point of intersection of these two bisectors. Then I is the incentre of ΔABC. We now draw ID perpendicular to BC where D is a point on BC. Taking I as centre and ID as radius, we draw a circle touching the three sides BC, CA and AB at D, E and F respectively.

4. Construct a triangle ABC with the base BC = 5.6 cm and the vertex A such that ∠BAC = 60°. For this, draw a circle circumscribing ΔABC and then draw a tangent to the circle at B such that this tangent makes an angle 60° with the side BC of ΔABC. The total number of such triangles will be

(*a*) one only (*b*) infinitely many

(*c*) two (*d*) three

Sol. (*b*) infinitely many



We draw the line segment BC = 5.6 cm.

Below BC, we make ∠CBY = 60° and draw BE ⊥ BY.

We now draw perpendicular bisector PQ of BC cutting BC at M. Let this bisector intersect BE at O. Taking O as centre and OB as radius, we draw a circle which will pass through B and C. We take any point A on the circumference of the circle on the same side of BC on which O lies. We finally join AB and AC. Then, ΔABC will be the required triangle.

Now, we can choose infinite number of points on the arc of the circle through B and C, above BC like the point A. Hence, an infinite number of triangle can be drawn with the given base and the vertical angle.

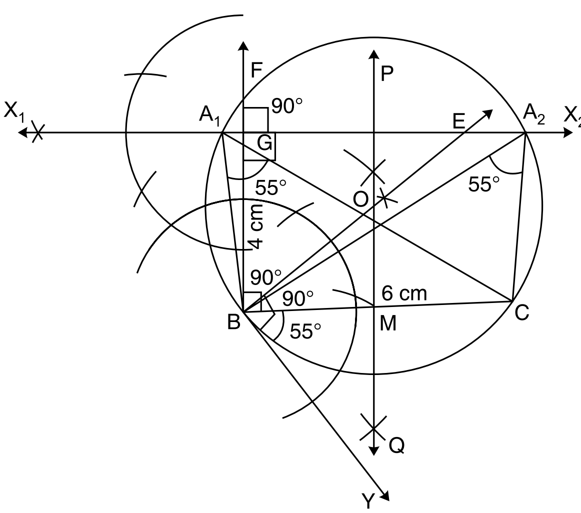
5. Construct a triangle ABC with the base BC = 6 cm, and the vertex A such that $\angle BAC = 55°$ and the altitude through A of the triangle is 4 cm. For this, draw a circle circumscribing ΔABC and then draw a tangent to the circle at B such that this tangent makes an angle 55° with the side BC of ΔABC. The total number of such triangles will be

(*a*) two (*b*) one

(*c*) three (*d*) four

Sol. (*a*) two

We draw the segment BC = 6 cm. Below BC, we make $\angle YBC = 55°$ with the help of a protector and draw $BE \perp BY$.



We now draw perpendicular bisector PQ of BC cutting BC at M. Let this bisector intersect BE at O. Taking O as centre and OB as radius, we draw a circle which will pass through B and C. We now draw $BF \perp BC$ and from B, we cut off BG = 4 cm where G is a point on BF.

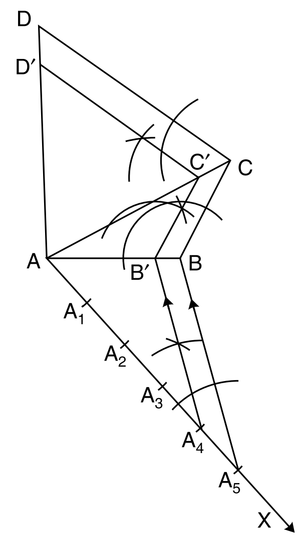
At G, we draw a line $X_1GX_2 \perp BG$. Let the line X_1GX_2 intersect the circle through B and C at the point A_1 and A_2. We join A_1B and A_2B. Then ΔA_1BC and ΔA_2BC will be two required triangles. Since the line X_1X_2 cut the circle at only two points A_1 and A_2, hence the total number of such triangles will be only two.

6. To construct a quadrilateral AB′C′D′ similar to a given quadrilateral ABCD with its sides $\frac{4}{5}$th of the corresponding sides of ABCD, join AC and make an acute angle CAX below AC. Now, mark off *n* points $A_1, A_2, A_3, \ldots, A_n$ along AX such that $AA_1 = A_1A_2 = A_2A_3 \ldots$ Then the number *n* must be equal to

(*a*) 9 (*b*) 4

(*c*) 5 (*d*) 10

Sol. (*c*) 5



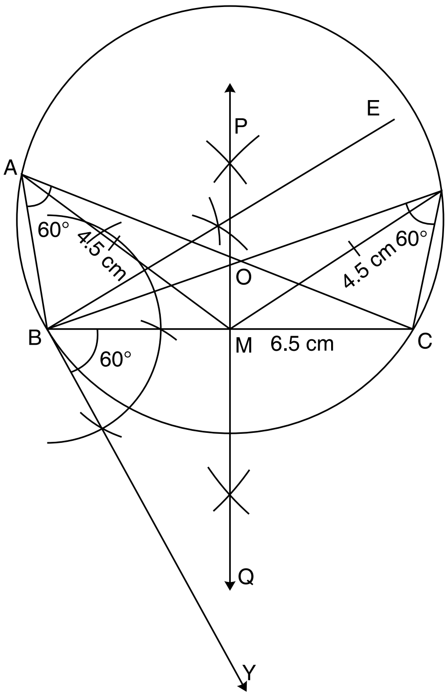
Since each side of the quadrilateral AB′C′D′ will be $\frac{4}{5}$th of the corresponding sides of the quadrilateral ABCD, hence, the total number of points $A_1, A_2, A_3, \ldots$ along AX such that $AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5$ must be 5 and the last point A_5 should be joined with B and through A_4 we shall draw $A_4B' \parallel A_5B$ where B′ is a point on AB, similarly, we shall draw $B'C' \parallel BC$ where C′ is a point on AC. Finally, we should draw $C'D' \parallel CD$ where D′ is a point on AD.

7. To construct a ΔABC in which base BC = 6.5 cm, $\angle BAC = 60°$ and the median through A is 4.5 cm long, first of all draw BC = 6.5 cm and $\angle CBY = 60°$ below BC. Now, draw a circle such that BY is a tangent to the circle at B. Choose a point A on the circle such that the median of ΔABC through A is of length 4.5 cm. Then the total number of such triangles ABC that can be drawn is

(*a*) four (*b*) three

(*c*) two (*d*) one

Sol. (*d*) one



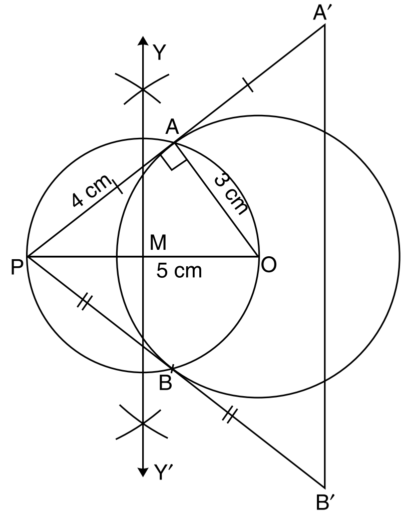
We draw the line segment BC = 6.5 cm. Below BC, we make $\angle$CBY = 60° and draw BE $\perp$ BY. We now draw perpendicular bisector PQ of BC cutting BC at M. Let this bisector intersect BE at O. Taking O as centre and OB as radius, we draw a circle which will pass through B and C. Taking M as centre and a length 4.5 cm as radius, we draw two arcs cutting the circle through B and C at two points A and A_1 only. We now join AB, AC and A_1B, A_1C to obtain two triangles ΔABC and ΔA_1BC with BC = 6.5 cm, $\angle$BAC = $\angle BA_1C$ = $\angle$CBY = 60° and BY as tangent and AM and A_1M as medians.

8. Draw a circle of radius 3 cm and take a point P outside the circle at a distance of 5 cm from its centre. Draw two tangents, PA and PB at the points A and B respectively on the circle. Produce PA and PB to PA′ and PB′ respectively such that PA = AA′ and PB = BB′. Join A′B′. Then PA′B′ will be an isosceles triangle with equal side

(*a*) 6 cm (*b*) 8 cm
(*c*) 10 cm (*d*) 12 cm

Sol. (*b*) 8 cm

We draw a circle with centre O and radius = 3 cm. We take a point P outside the circle such that OP = 5 cm. Let YY′ be the perpendicular bisector of OP intersecting OP at M. With M as centre and PM as radius, we draw a circle passing through P and O. Let this circle intersect the given circle at A and B. We join PA and PB. Then, PA and PB will be two tangents to the given circle of radius 3 cm.



Now, if PA = AA′ and PB = BB′, then

$$PA + AA' = 4 \times 2 \text{ cm} = 8 \text{ cm}$$

Similarly,

$$PB + BB' = 4 \times 2 \text{ cm} = 8 \text{ cm}$$

$\therefore$ ΔPA′B′ is an isosceles triangle with PA′ = PB′ = 8 cm.

It cannot be an equilateral triangle, since

$$\tan(\angle APO) = \frac{3}{4}$$

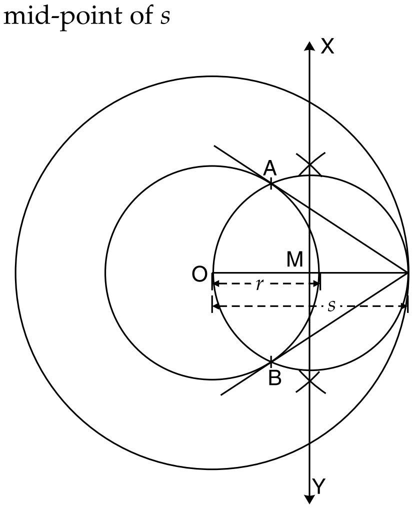
$\therefore$ $\angle APO \neq 30°$

and so $\angle A'PB' \neq 60°$

9. To draw tangents to a circle of radius *r* from a point on the concentric circle of radius *s*, the first step is to find

(*a*) mid-point of *s* (*b*) mid-point of *r*
(*c*) mid-point of *s* – *t* (*d*) mid-point of *r* + *s*

Sol. (*a*) mid-point of *s*



Clearly, in this case $s > r$. We take a point P on the bigger circle and join OP, where O is the common centre of two concentric circles. To construct tangents from P to the smaller circle, we should find the perpendicular bisector XY of OP to intersect it at M. Taking M as centre and MO as radius, we shall then draw a circle passing through O and P. Let this circle intersect the given smaller circle at A and B. We join PA and PB. Then PA and PB will be the required two tangents.

Hence, to find the perpendicular bisector of OP, we should first of all find the mid-point of s.

10. To draw a tangent at a point Q to the circumcircle of an isosceles right triangle PQR, right-angled at Q, we need to draw through Q

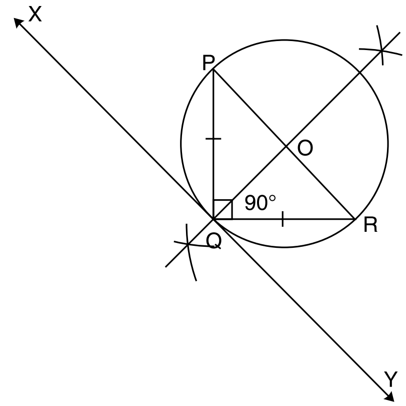
(*a*) a line inclined at 60° to PQ.

(*b*) a line perpendicular to PQ.

(*c*) a line perpendicular to QR.

(*d*) a line parallel to PR.

Sol. (*a*) a line parallel to PR.



If ΔPQR is a triangle with $PQ = QR$ and $\angle PQR = 90°$, then $\angle QPR = \angle QRP = 45°$.

Now, if a line XY through Q is a tangent to the circle at Q, then $\angle RQY = \angle QPR = \angle QRP$.

But $\angle QRP$ and $\angle RQY$ are alternate angles with respect to the transversal QR. Hence, $XY \parallel PR$.

12

Areas Related to Circles

Checkpoint (Page 206)

1. What is the circumference of a circle of radius 7 cm?

Sol. Circumference = $2\pi r = 2 \times \frac{22}{7} \times 7$ cm = 44 cm

Hence, the required circumference of a circle is 44 cm.

2. The perimeter of a square of side 11 cm is equal to the circumference of a circle of radius r. What is the area of this circle?

Sol. The perimeter of the square = 4 × 11 cm = 44 cm

The circumference of the circle = $2\pi r$

∴ According to the problem, we have

$$2\pi r = 44 \text{ cm}$$

$$\Rightarrow \quad 2 \times \frac{22}{7} \times r = 44 \text{ cm}$$

$$\Rightarrow \quad r = \frac{44 \times 7}{2 \times 22} \text{ cm}$$

$$= \frac{44 \times 7}{44} \text{ cm} = 7 \text{ cm}$$

∴ Area of the circle = πr^2

$$= \frac{22}{7} \times 7 \times 7 \text{ cm}^2$$

$$= 154 \text{ cm}^2$$

Hence, the required area of the circle is 154 cm^2.

3. A wheel rotates 50000 times to cover a distance of 176 km. Find the radius of the wheel in cm.

Sol. If r cm be the radius of the wheel, then

$$2\pi r \times 50000 = 176 \times 1000 \times 100 \text{ cm}$$

$$\Rightarrow \quad r = \frac{17600000}{100000} \times \frac{7}{22} \text{ cm}$$

$$\Rightarrow \quad r = 8 \times 7 \text{ cm}$$

$$= 56 \text{ cm}$$

Hence, the required radius of the wheel is 56 cm.

4. What is the perimeter of a semicircular wire of radius 14 cm?

Sol. Perimeter of a semicircle = Circumference of a semicircle + Diameter

$$= \pi r + 2r$$

$$= r(\pi + 2)$$

$$= 14 \times \left(\frac{22}{7} + 2\right) \text{ cm}$$

$$= 14 \times \frac{36}{7} \text{ cm}$$

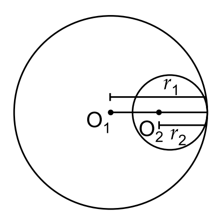
$$= 2 \times 36 \text{ cm}$$

$$= 72 \text{ cm}$$

Hence, the required perimeter of a semicircular wire is 72 cm.

5. Two circles of radii 8 cm and 6 cm have centres at O_1 and O_2 respectively. If O_1O_2 = 2 cm, will the two circles touch each other internally or externally?

Sol. If r_1 cm and r_2 cm be the radii of two circles with centres at O_1 and O_2 respectively, then r_1 = 8 cm, r_2 = 6 cm.

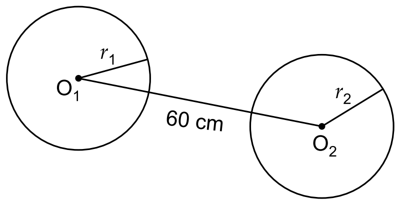


Now, $O_1O_2 = 2 \text{ cm} = r_1 - r_2$

So, the two circles touch each other internally.

6. Two circles with centres at O_1 and O_2 have radii 25 cm and 31 cm respectively. If $O_1O_2 = 60$ cm, can the two circles intersect each other or not?

Sol. If r_1 cm and r_2 cm be the radii of two circles with centres at O_1 and O_2 respectively, then $r_1 = 25$ cm, $r_2 = 31$ cm.



Now, $O_1O_2 = 60$ cm

and $r_1 + r_2 = (25 + 31)$ cm $= 56$ cm

$\therefore$ $r_1 + r_2 < O_1O_2$

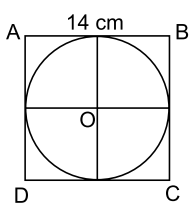
Hence, the two circles will not intersect each other.

7. In the above problem what is the area of the common region between the two circles?

Sol. Since the two circles do not intersect each other at all, there is no common area between the two circles. Hence, the common area is zero.

8. A circle is drawn inside a square of side 14 cm such that it touches all the sides of the square. What is the area of the circle?

Sol. If a circle with centre at O is inscribed within a square of side 14 cm, then the radius r of the circle is $\frac{14}{2}$ cm = 7 cm.



$\therefore$ Area of the circle $= \pi r^2$

$$= \frac{22}{7} \times 7 \times 7 \text{ cm}^2$$

$$= 154 \text{ cm}^2$$

Hence, the required area of the circle is 154 cm^2.

9. A circular wire of diameter 56 cm is cut into 4 equal pieces and a square of maximum area is formed with these 4 pieces. What is the area of this square?

Sol. Circumference of the circle $= 2\pi r$

$$= 2 \times \frac{22}{7} \times \frac{56}{2} \text{ cm}$$

$$= 22 \times 8 \text{ cm}$$

$$= 176 \text{ cm}$$

Length of each piece of the wire

$$= \frac{176}{4} \text{ cm}$$

$$= 44 \text{ cm}$$

$\therefore$ Maximum area of the square $= 44 \times 44$ cm^2 $= 1936$ cm^2

Hence, the required area of the square is 1936 cm^2.

10. What is the maximum length of a rod that can be kept inside a circular enclosure of area 346.5 m^2? [Use $\pi = \frac{22}{7}$]

Sol. If r m be the radius of the circle, then

$$\pi r^2 = 346.5 \text{ m}^2$$

$$\Rightarrow \quad \frac{22}{7} \times r^2 = 346.5 \text{ m}^2$$

$$\Rightarrow \quad r^2 = \frac{346.5 \times 7}{22} \text{ m}^2$$

$$= \frac{2425.5}{22} \text{ m}^2$$

$$= 110.25 \text{ m}^2$$

$$\Rightarrow \quad r = \sqrt{110.25} \text{ m}$$

$$= 10.5 \text{ m}$$

$\therefore$ Radius of the circle is 10.5 m.

$\therefore$ The maximum length of the rod

= diameter of the circle

$= 2r$

$= 10.5 \times 2$ m

$= 21$ m

Hence, the maximum length of a rod is 21 m.

Milestone 1

(Page 209)

Multiple-Choice Questions

1. If the perimeter of a semicircular protractor is 36 cm, then its diameter is

(*a*) 10 cm (*b*) 12 cm
(*c*) 14 cm (*d*) 16 cm

[CBSE SP 2012]

Sol. (*c*) 14 cm

Let r be the radius of the semicircle.

Then, perimeter $= (\pi r + 2r)$

$$= \left(\frac{22}{7} \times r + 2r\right)$$

$$= \frac{22r + 14r}{7}$$

$$= \frac{36}{7} r$$

$$\frac{36r}{7} = 36 \text{ cm}$$

$$\Rightarrow \quad r = 7 \text{ cm}$$

$\therefore$ Diameter $= 2r = 2 \times 7$ cm $= 14$ cm.

Hence, the required diameter is 14 cm.

2. The radii of two circles are 14 cm and 20 cm. Then, the diameter of the circle having circumference equal to the sum of the circumferences of the two circles is

(*a*) 68 cm (*b*) 56 cm

(*c*) 60 cm (*d*) 55 cm

Sol. (*a*) 68 cm

Let r_1 and r_2 be the radii of two circles, where $r_2 > r_1$.

Then $r_2 = 20$ cm and $r_1 = 14$ cm

Then the sum of the circumferences of the two circles $= 2\pi r_1 + 2\pi r_2$

$$= 2\pi(r_1 + r_2)$$

$$= 2\pi(20 + 14) \text{ cm}$$

$$= 68\pi \text{ cm}$$

If R cm be the radius of the required circle, then

$$2\pi R = 68\pi \text{ cm}$$

$$\Rightarrow \quad 2R = 68 \text{ cm}$$

Hence, the required diameter is 68 cm.

Very Short Answer Type Questions

3. What is the radius of the circle whose circumference is 39.6 cm?

Sol. Let r be the radius of the circle, then

$$2\pi r = 39.6 \text{ cm}$$

$$\Rightarrow \quad 2 \times \frac{22}{7} \times r = 39.6 \text{ cm}$$

$$\Rightarrow \quad r = \frac{39.6 \times 7}{2 \times 22} \text{ cm}$$

$$= \frac{277.2}{44} \text{ cm} = 6.3 \text{ cm}$$

Hence, the required length of the radius is 6.3 cm.

4. A circular wire of radius 28 cm is bent into a square. What is the length of a side of the square?

Sol. Let r be the radius of the circle, then

$$2\pi r = 2 \times \frac{22}{7} \times 28 \text{ cm}$$

$$= 2 \times 22 \times 4 \text{ cm}$$

$$= 176 \text{ cm}$$

$\therefore$ The perimeter of the square = 176 cm

$\therefore$ Side of the square $= \frac{176}{4}$ cm $= 44$ cm

Hence, the required length of the side of square is 44 cm.

Short Answer Type-I Questions

5. A circle with centre C is formed by adding the areas of three other circles with radii 3 cm, 4 cm and 12 cm. A square is now drawn circumscribing the circle with centre C which coincides with the point of intersection of two diagonals of the square. Determine the length of each diagonal of the square.

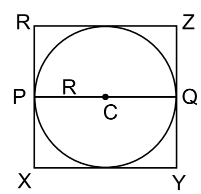
Sol. Let R be the radius of the circle with centre C, then

$$\pi R^2 = \pi \times 3^2 \text{ cm}^2 + \pi \times 4^2 \text{ cm}^2 + \pi \times 12^2 \text{ cm}^2$$

$$\Rightarrow \quad R^2 = 9 \text{ cm}^2 + 16 \text{ cm}^2 + 144 \text{ cm}^2$$

$$\Rightarrow \quad R^2 = 169 \text{ cm}^2$$

$$\Rightarrow \quad R = 13 \text{ cm}$$



$\therefore$ The radius of the circle with centre at C is 13 cm. If the square circumscribes this circle, then

Each side of the square = diameter of the circle

$$= 13 \times 2 \text{ cm}$$

$$= 26 \text{ cm}$$

$\therefore$ Length of the diagonal of the square

$$= \sqrt{26^2 + 26^2} \text{ cm}$$

$$= \sqrt{26^2} \times \sqrt{2} \text{ cm}$$

$$= 26\sqrt{2} \text{ cm}$$

Hence, the length of each diagonal of the square is $26\sqrt{2}$ cm.

6. The diameters of two circles are in the ratio 3 : 4 and the sum of the areas of the circles is equal to the area of a circle whose diameter measures 30 cm. Find the diameters of the given circles.

[CBSE SP 2012]

Sol. Let the diameters of the two given circles be $6x$ cm and $8x$ cm so that these are in the given ratio 3 : 4.

$\therefore$ Radii of the two circles are $3x$ cm and $4x$ cm, where $x > 0$.

The area of the circle whose diameter is 30 cm

$$= \pi \times 15^2 \text{ cm}^2$$
$$= 225\,\pi \text{ cm}^2$$

$\therefore$ According to the problem, we have

$$\pi \times (3x)^2 + \pi \times (4x)^2 = 225\pi \text{ cm}^2$$
$$\Rightarrow \quad 9\pi x^2 + 16\pi x^2 = 225\pi \text{ cm}^2$$
$$\Rightarrow \quad 25x^2 = 225 \text{ cm}^2$$
$$\Rightarrow \quad x^2 = 9 \text{ cm}^2$$
$$\Rightarrow \quad x = 3 \text{ cm} \qquad [\because x > 0]$$

$\therefore$ The required diameters of the given circles are 6×3 cm and 8×3 cm, i.e. 18 cm and 24 cm.

Short Answer Type-II Questions

7. Mohan is driving a cart. A wheel of the cart is making 6 revolutions per second. If the diameter of the wheel is 42 cm, find the speed of the cart in km/h. **[CBSE SP 2011]**

Sol. Let r be the radius of the wheel.

Then the circumference of the wheel

$$= 2\pi r$$
$$= 2 \times \frac{22}{7} \times 21 \text{ cm}$$
$$= 2 \times 22 \times 3 \text{ cm}$$
$$= 132 \text{ cm}$$

$\therefore$ In 1 revolution the wheel covers a distance of 132 cm

$\therefore$ In 6 revolutions the wheel covers a distance of 132×6 cm = 792 cm

$\therefore$ Distance covered by the cart in 1 second = 792 cm

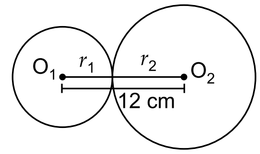
$\therefore$ Speed of the cart in km/h

$$= \frac{792}{100 \times 1000} \text{ km/s}$$
$$= \frac{792}{100000} \times 60 \times 60 \text{ km/h}$$
$$= \frac{792 \times 36}{1000} \text{ km/h}$$
$$= 28.512 \text{ km/h}$$

Hence, the speed of the cart is 28.512 km/h.

8. Two circles touch each other externally. If the distance between their centres is 12 cm and the sum of their areas is 80π cm^2, find the radii of the two circles.

Sol. Let r_1 and r_2 be the radii of the two circles with centres at O_1 and O_2 respectively, where $r_2 > r_1$.



Now, since the two circles touch each other externally,

$$\therefore \quad r_1 + r_2 = 12 \qquad \ldots(1)$$

Also, $\quad \pi\left(r_1^2 + r_2^2\right) = 80\,\pi \text{ cm}^2$

$$\Rightarrow \quad r_1^2 + r_2^2 = 80 \text{ cm}^2 \qquad \ldots(2)$$

From (1), $\quad r_2 = 12 - r_1 \qquad \ldots(3)$

$\therefore$ From (2) and (3), we get

$$r_1^2 + (12 - r_1)^2 = 80$$
$$\Rightarrow \quad r_1^2 + 144 + r_1^2 - 24r_1 = 80$$
$$\Rightarrow \quad 2r_1^2 - 24r_1 + 144 - 80 = 0$$
$$\Rightarrow \quad r_1^2 - 12r_1 + 32 = 0$$
$$\Rightarrow \quad r_1^2 - 8r_1 - 4r_1 + 32 = 0$$
$$\Rightarrow \quad r_1(r_1 - 8) - 4(r_1 - 8) = 0$$
$$\Rightarrow \quad (r_1 - 8)(r_1 - 4) = 0$$
$$\therefore \quad r_1 = 8 \text{ or } 4$$

$\therefore$ From (3) $\quad r_2 = 4$ or 8

$\because r_2 > r_1$,

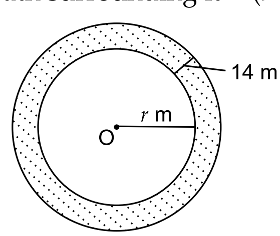
$\therefore r_2 = 8$ cm and $r_1 = 4$ cm

Hence, the radii of the two circles are 4 cm and 8 cm.

Long Answer Type Questions

9. The circumference of a circular lawn is 440 m. A 14 m wide path surrounds it. Find the cost of levelling the path at the rate of ₹10 per m^2. Also find the cost of fencing the outer boundary of the circular lawn at the rate of ₹7 per metre.

Sol. Let r be the radius of the circular lawn with centre at O. Then the radius of the lawn with the circular path surrounding it = $(r + 14)$ m.



Now, it is given that

$$2\pi r = 440 \text{ m}$$

$$\Rightarrow \quad 2 \times \frac{22}{7} \times r = 440 \text{ m}$$

$$\Rightarrow \quad r = \frac{440 \times 7}{2 \times 22} \text{ m}$$

$$= 70 \text{ m}$$

Hence, the radius of the lawn is 70 m.

$\therefore$ The radius of the lawn together with the circular path surrounding it is (70 + 14) m, i.e. 84 m.

$\therefore$ Area of the path only

$$= \pi(84^2 - 70^2) \text{ m}^2$$

$$= \frac{22}{7} \times (84 + 70)(84 - 70) \text{ m}^2$$

$$= \frac{22}{7} \times 154 \times 14 \text{ m}^2$$

$$= 22 \times 22 \times 14 \text{ m}^2$$

$$= 6776 \text{ m}^2$$

$\therefore$ Cost of levelling the path

= ₹10 × 6776

= ₹67760

The circumference of the outer boundary of the lawn together with the circular path surrounding it $= 2\pi \times 84$ m

$$= 2 \times \frac{22}{7} \times 84 \text{ m}$$

$$= 2 \times 22 \times 12 \text{ m}$$

$$= 528 \text{ m}$$

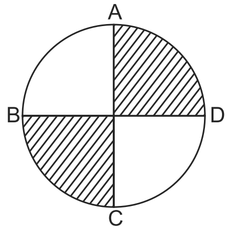
$\therefore$ Cost of fencing the boundary

= ₹ 7 × 528

= ₹ 3696

Hence, the cost of levelling the path is ₹ 67760 and cost of fencing the outer boundary is ₹ 3696.

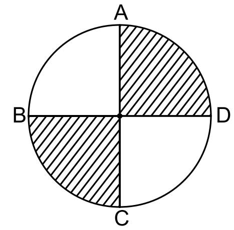
10. AC and BD are two perpendicular diameters of a circle as shown in the figure. If the area of the shaded part is 616 cm^2, calculate



(*a*) the length of AC and

(*b*) the circumference of the circle.

Sol. Let r be the radius of the circle. The two shaded parts together are equivalent to the semicircle.



$\therefore$ Sum of the areas of two shaded parts

$$= \text{area of the semicircle} = \frac{\pi r^2}{2}$$

$$\therefore \quad \frac{\pi r^2}{2} = 616 \text{ cm}^2$$

$$\Rightarrow \quad \frac{22}{7} \times \frac{1}{2} \times r^2 = 616 \text{ cm}^2$$

$$\Rightarrow \quad r^2 = \frac{616 \times 7 \times 2}{22} \text{ cm}^2$$

$$= 56 \times 7 \text{ cm}^2$$

$$\Rightarrow \quad r = \sqrt{56 \times 7} \text{ cm}$$

$$= \sqrt{7 \times 4 \times 2 \times 7} \text{ cm}$$

$$= 7 \times 2\sqrt{2} \text{ cm}$$

$$= 14\sqrt{2} \text{ cm}$$

$\therefore$ Radius of the circle is $14\sqrt{2}$ cm.

(*a*) Length of AC = diameter of the circle

$$= 2 \times r = 2 \times 14\sqrt{2} \text{ cm}$$

$$= 28\sqrt{2} \text{ cm}$$

Hence, the length of AC is $28\sqrt{2}$ cm.

(*b*) Circumference of the circle

$$= 2\pi r$$

$$= 2 \times \frac{22}{7} \times 14\sqrt{2} \text{ cm}$$

$$= 2 \times 22 \times 2\sqrt{2} \text{ cm}$$

$$= 88\sqrt{2} \text{ cm}$$

Hence, the circumference of the circle is $88\sqrt{2}$ cm.

Milestone 2

(Page 212)

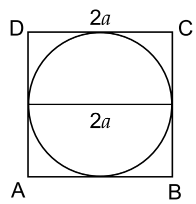
Multiple-Choice Questions

1. The perimeter (in cm) of a square circumscribing a circle of radius a cm is

(*a*) $8a$ (*b*) $4a$

(*c*) $2a$ (*d*) $16a$ **[CBSE 2011]**

Sol. (*a*) $8a$



Side of the square = diameter of the circle

$= 2a$ cm

$\therefore$ Perimeter of the square $= 4 \times 2a$ cm $= 8a$ cm

2. If the circumference of a circle increases from 4π cm to 5π cm, then the area of the smaller circle is increased by

(*a*) 50% (*b*) 56%

(*c*) 56.25% (*d*) 52%

Sol. (*c*) 56.25%

Let the radii of the original circle and the increased circle be r cm and R cm respectively.

Then $2\pi r = 4\pi$ cm

$\Rightarrow$ $r = 2$ cm

and $2\pi R = 5\pi$ cm

$\Rightarrow$ $R = \frac{5}{2}$ cm

$\therefore$ The radii of the original circle and the increased circle are 2 cm and $\frac{5}{2}$ cm respectively.

Then the area of the original circle $= \pi r^2 = 4\pi$ cm^2 and the area of the increased circle $= \pi R^2 = \pi \frac{25}{4}$ cm^2.

$\therefore$ Increase in area $= \left(\frac{25\pi}{4} - 4\pi\right) = \frac{9\pi}{4}$ cm^2

$\therefore$ Percentage increase in the area

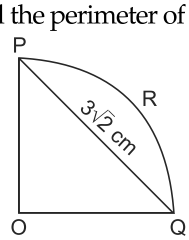
$$= \left(\frac{9\pi}{4} \times \frac{1}{4\pi}\right) \times 100\%$$

$$= \frac{9 \times 100}{16}\%$$

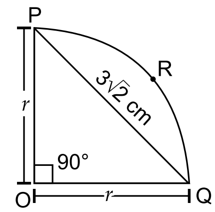
$$= \frac{225}{4}\% = 56.25\%$$

Very Short Answer Type Questions

3. In the given figure, the length of the chord PQ is $3\sqrt{2}$ cm. Find the perimeter of the quadrant OPRQ.



Sol. Let r be the radius of the quadrant of the circle, OPRQ.



Then since $\angle POQ = 90°$.

$\therefore$ By using Pythagoras' Theorem in ΔPOQ, we have

$$OP^2 + OQ^2 = PQ^2$$

$$2r^2 = \left(3\sqrt{2}\text{ cm}\right)^2$$

$\Rightarrow$ $2r^2 = 18$ cm^2

$\Rightarrow$ $r^2 = 9$ cm^2

$\Rightarrow$ $r = 3$ cm

$\therefore$ Radius of the quadrant = 3 cm

$\therefore$ Perimeter of the quadrant OPRQ

$= 2r +$ arc PRQ

$$= \left(2 \times 3 + \frac{1}{4} \times 2\pi \times 3\right)\text{ cm}$$

$$= \left(6 + \frac{1}{2} \times \frac{22}{7} \times 3\right)\text{ cm}$$

$$= \left(6 + \frac{33}{7}\right)\text{ cm}$$

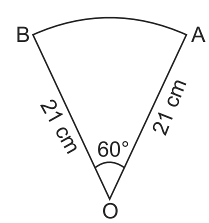
$$= \frac{42 + 33}{7}\text{ cm}$$

$$= \frac{75}{7}\text{ cm}$$

$= 10.71$ cm (approx.)

Hence, the perimeter of the quadrant OPRQ is 10.71 cm (approx.)

4. Find the perimeter of the sector OAB as shown in the figure. [Use $\pi = \frac{22}{7}$]



Sol. Radius of the sector, $r = OA = OB = 21$ cm

$\therefore$ Perimeter of the sector OAB

$$= 2\pi r \times \frac{60°}{360°} + 2r$$

$$= \left(2 \times \frac{22}{7} \times 21 \times \frac{1}{6} + 2 \times 21\right) \text{ cm}$$

$$= (22 + 42) \text{ cm}$$

$$= 64 \text{ cm}$$

Hence, the perimeter of the sector OAB is 64 cm.

5. If the area of a circle is numerically equal to twice its circumference, then what is the diameter of the circle? **[CBSE 2011]**

Sol. Let r be the radius of the circle.

Then according to the problem, we have

$$\pi r^2 = 2 \times 2\pi r$$

$$= 4\pi r$$

$$\Rightarrow \quad r = 4 \text{ units}$$

$$\therefore \quad \text{Diameter} = 2r = 2 \times 4 \text{ units}$$

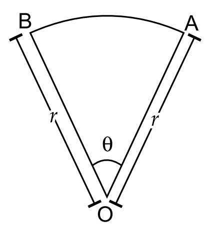
$$= 8 \text{ units}$$

Hence, the required diameter of the circle is 8 units.

Short Answer Type-I Questions

6. If the area of a sector of angle θ of a circle is $\frac{5}{18}$ times the area of the whole circle, then find the value of θ.

Sol. Let r be the radius of the sector.



Then area of the sector $= \pi r^2 \times \frac{\theta}{360°}$

$\therefore$ According to the problem, we have

$$\pi r^2 \times \frac{\theta}{360°} = \frac{5}{18} \times \pi r^2$$

$$\Rightarrow \quad \theta = \frac{5}{18} \times 360°$$

$$= 5 \times 20°$$

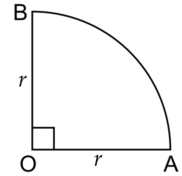
$$= 100°$$

Hence, the required value of θ is 100°.

7. If the area of a quadrant of a circle is $\frac{77}{8}$ cm^2, what is its perimeter?

Sol. Let r be the radius of the quadrant of the circle.

Then $\quad$ OA = OB = r



$\therefore$ According to the problem, we have

$$\frac{1}{4}\pi r^2 = \frac{77}{8} \text{ cm}^2$$

$$\Rightarrow \quad \frac{1}{4} \times \frac{22}{7} \times r^2 = \frac{77}{8} \text{ cm}^2$$

$$\Rightarrow \quad r^2 = \frac{77 \times 7 \times 4}{22 \times 8} \text{ cm}^2$$

$$\Rightarrow \quad r^2 = \frac{49}{4} \text{ cm}^2$$

$$\Rightarrow \quad r^2 = \left(\frac{7}{2} \text{ cm}\right)^2$$

$$\Rightarrow \quad r = \frac{7}{2} \text{ cm}$$

$\therefore$ Radius of the quadrant of the circle $= \frac{7}{2}$ cm

$\therefore$ Perimeter of the quadrant

$$= \frac{1}{4} \times 2\pi r + 2r$$

$$= r\left(\frac{\pi}{2} + 2\right)$$

$$= \frac{7}{2} \times \left(\frac{22}{7 \times 2} + 2\right) \text{ cm}$$

$$= \left(\frac{11}{2} + 7\right) \text{ cm}$$

$$= \frac{25}{2} \text{ cm}$$

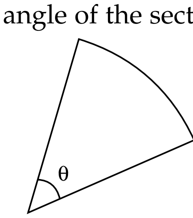
$$= 12.5 \text{ cm}$$

Hence, the required perimeter is 12.5 cm.

Short Answer Type-II Questions

8. If the perimeter of a sector of a circle of radius 5.6 cm is 27.2 cm, find the area of the sector. **[CBSE SP 2011]**

Sol. Let r be the radius of the sector. Then $r = 5.6$ cm. Let θ be the angle of the sector.



$\therefore$ Perimeter of the sector

$$= 2r + 2\pi r \times \frac{\theta}{360°}$$

$$\Rightarrow \quad 2 \times 5.6 + 11.2 \times \frac{\pi\theta}{360°} = 27.2 \qquad \text{[Given]}$$

$$\Rightarrow \quad 11.2 \times \frac{\pi\theta}{360°} = 27.2 - 11.2 = 16$$

$$\Rightarrow \quad \frac{\pi\theta}{360°} = \frac{16}{11.2} = \frac{10}{7} \qquad \ldots(1)$$

$\therefore$ Area of the sector $= \pi r^2 \times \frac{\theta}{360°}$ cm^2

$$= 5.6^2 \times \frac{\pi\theta}{360°} \text{ cm}^2$$

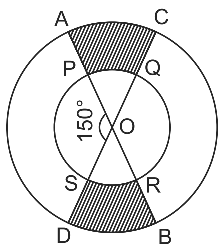
$$= 5.6^2 \times \frac{10}{7} \text{ cm}^2 \qquad \text{[From (1)]}$$

$$= \frac{5.6 \times 56}{7} \text{ cm}^2$$

$$= 44.8 \text{ cm}^2$$

Hence, the required area of the sector is 44.8 cm^2.

9. In the given figure, O is the centre of two concentric circles. The radius of the smaller circle is half the radius of the bigger circle. If OP = the radius of the smaller circle = 7 cm, $\angle$AOD = 150° where AOB and COD are diameters of the bigger circle. Calculate the area of the shaded region.



Sol. Given that $\angle AOD = 150°$

$\therefore \quad \angle AOD = \angle BOC = 150°$ [Vertically opposite $\angle$s]

Now $\quad \angle AOD + \angle BOD = 180°$ [Linear pair]

$\Rightarrow \quad 150° + \angle BOD = 180°$

$\Rightarrow \quad \angle BOD = 180° - 150° = 30°$

$\therefore \quad \angle BOD = \angle AOC = 30°$ [Vertically opposite $\angle$s]

Also, OP = 7 cm and OA = 14 cm.

$\therefore$ Area of the sector OAC $= \pi \times OA^2 \times \frac{\theta}{360°}$

$$= \pi \times 14^2 \times \frac{30°}{360°} \text{ cm}^2$$

$$= \frac{14^2\pi}{12} \text{ cm}^2$$

Area of the sector OPQ $= \pi \times OP^2 \times \frac{\theta}{360°}$

$$= \pi \times 7^2 \times \frac{30°}{360°} \text{ cm}^2$$

$$= \frac{7^2\pi}{12} \text{ cm}^2$$

$\therefore$ Area of the shaded region APQC

$$= \pi \times \left(\frac{14^2}{12} - \frac{7^2}{12}\right) \text{cm}^2$$

$$= \pi \times \left(\frac{14^2 - 7^2}{12}\right) \text{cm}^2$$

$$= \frac{22}{7} \times \frac{(14+7)(14-7)}{12} \text{cm}^2$$

$$= \frac{22}{7} \times \frac{21 \times 7}{12} \text{ cm}^2$$

$$= \frac{77}{2} \text{ cm}^2$$

= Area of the shaded region BDSR.

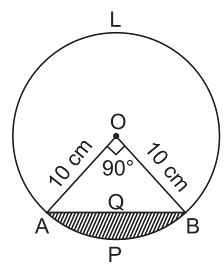
$\therefore$ Area of the sum of the two shaded regions

$$= \frac{77}{2} \times 2 \text{ cm}^2 = 77 \text{ cm}^2$$

Hence, the required area of the shaded region is 77 cm^2.

Long Answer Type Questions

10. In the given figure, AB is a chord of a circle with centre O and radius 10 cm, that subtends a right angle at the centre of the circle. Find the area of the minor segment AQBP. Hence, find the area of the major segment ALBQA. [Use π = 3.14]



[CBSE 2016]

Sol. Area of the sector OAB

$$= \pi r^2 \times \frac{\theta}{360°}$$

$$= \pi \times 10^2 \times \frac{90°}{360°} \text{ cm}^2$$

$$= 3.14 \times 100 \times \frac{1}{4} \text{ cm}^2$$

$$= 3.14 \times 25 \text{ cm}^2$$

$$= 78.5 \text{ cm}^2$$

Area of $\Delta OAB = \frac{1}{2} \times 10 \times 10 \text{ cm}^2 = 50 \text{ cm}^2$

$\therefore$ Area of the minor segment AQBP

(78.5 – 50) cm^2 = 28.5 cm^2

Area of the entire circle = πr^2

$= 3.14 \times 10^2$ cm^2

$= 3.14 \times 100$ cm^2

$= 314$ cm^2

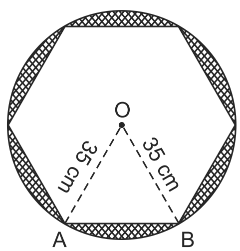
$\therefore$ Area of the major segment ALBQA

$= (314 - 28.5)$ cm^2

$= 285.5$ cm^2

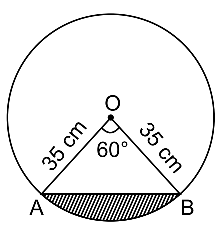
Hence, the required area of the minor segment AQBP is 28.5 cm^2 and the area of the major segment is 285.5 cm^2.

11. A round table cover has six equal designs as shown in the figure below. If the radius of the cover is 35 cm, then find the total area of the design. $\left[\text{Use } \pi = 3.14 \text{ and } \sqrt{3} = 1.732\right]$



[CBSE 2009]

Sol. Each design is in the shape of a minor segment of a circle subtending an angle of $\frac{360°}{6} = 60°$ at the centre O.



$\therefore$ Area of each segment

= Area of the sector OAB – area of the equilateral triangle OAB

$$= \left(\pi \times 35^2 \times \frac{60°}{360°} - \frac{\sqrt{3}}{4} \times 35^2\right) \text{ cm}^2$$

$$= 35^2 \times \left(\frac{3.14}{6} - \frac{1.732}{4}\right) \text{ cm}^2$$

$$= 35^2 \times \frac{6.28 - 5.196}{12} \text{ cm}^2$$

$$= \frac{35^2 \times 1.084}{12} \text{ cm}^2$$

$\therefore$ Area of the six designs

$$= \frac{35^2 \times 1.084}{12} \times 6 \text{ cm}^2$$

$= 35^2 \times 0.542$ cm^2

$= 663.95$ cm^2 (approx.)

Hence, the required total area of the design is 663.95 cm^2 (approx.)

Milestone 3

(Page 216)

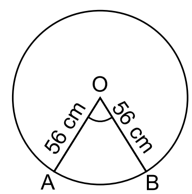
Multiple-Choice Questions

1. In a circle of radius 56 cm, if the angle subtended by an arc at the centre is 45°, then the area of the sector is

(*a*) 1200 cm^2 (*b*) 1232 cm^2

(*c*) 1240 cm^2 (*d*) 1250 cm^2

Sol. (*b*) 1232 cm^2



Area of the sector $= \pi \times 56^2 \times \frac{45°}{360°}$ cm^2

$$= \frac{22}{7} \times 56 \times 56 \times \frac{45°}{360°} \text{ cm}^2$$

$= 1232$ cm^2

$= 22 \times 8 \times 7 \times$ cm^2

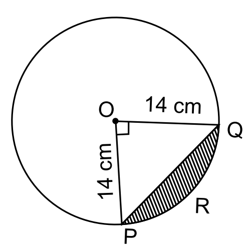
$= 1232$ cm^2

Hence, the area of the sector is 1232 cm^2.

2. If the chord PQ of a circle of radius 14 cm makes a right angle at the centre of the circle, then the area of the minor segment is

(*a*) 54 cm^2 (*b*) 60 cm^2 (*c*) 50 cm^2 (*d*) 56 cm^2

Sol. (*d*) 56 cm^2



Area of the minor segment PRQ

= area of the sector POQ – area of ΔQOP

$$= \left(\pi \times 14^2 \times \frac{90°}{360°} - \frac{1}{2} \times 14 \times 14\right) \text{ cm}^2$$

$$= \left(\frac{22}{7} \times 14 \times 14 \times \frac{1}{4} - 7 \times 14\right) \text{ cm}^2$$

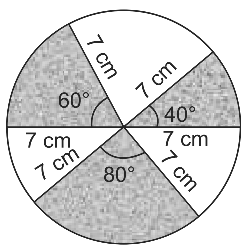
$$= (154 - 98) \text{ cm}^2$$

$$= 56 \text{ cm}^2$$

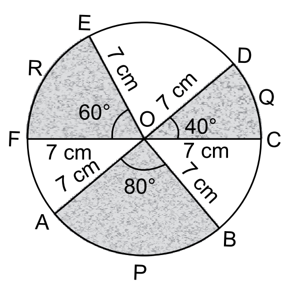
Hence, the required of the minor segment is 56 cm^2.

Very Short Answer Type Questions

3. In the given figure, three sectors of a circle of radius 7 cm, making angles of 60°, 80° and 40° at the centre are shaded. What is the area of the shaded region in cm^2?



Sol.



Area of the sum of the sectors

AOBP, DOCQ and EOFR

$$= \left(\pi \times 7^2 \times \frac{80°}{360°} + \pi \times 7^2 \times \frac{40°}{360°} + \pi \times 7^2 \times \frac{60°}{360°}\right) \text{ cm}^2$$

$$= \frac{22}{7} \times 49\left(\frac{2}{9} + \frac{1}{9} + \frac{1}{6}\right) \text{ cm}^2$$

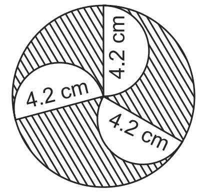
$$= 154 \times \frac{4 + 2 + 3}{18} \text{ cm}^2$$

$$= 77 \times \frac{9}{9} \text{ cm}^2$$

$$= 77 \text{ cm}^2$$

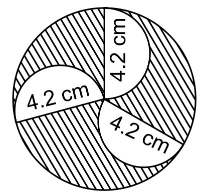
Hence, the required area of the shaded region is 77 cm^2.

4. The adjoining figure is made up of one circle and three semicircles. Find the area of the shaded part.



Sol. If r be the radius of each semicircle, then

$$r = \frac{4.2}{2} \text{ cm} = 2.1 \text{ cm}$$



$\therefore$ Radius of the whole circle = $2r$ cm

$\therefore$ Sum of the areas of 3 semicircles = $3 \times \frac{1}{2}\pi r^2$

Also, area of the whole circle = $\pi \times (2r)^2 = 4\pi r^2$

$\therefore$ Area of the shaded region

$$= 4\pi r^2 - \frac{3}{2}\pi r^2$$

$$= \frac{5}{2}\pi r^2$$

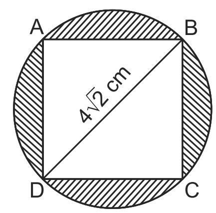
$$= \frac{5}{2} \times \frac{22}{7} \times 2.1 \times 2.1 \text{ cm}^2$$

$$= 34.65 \text{ cm}^2$$

Hence, the required area of the shaded part is 34.65 cm^2.

Short Answer Type-I Questions

5. Calculate the area of the shaded part in the given figure, where ABCD is a square with diagonal of length $4\sqrt{2}$ cm.



Sol. Let a cm be the side of the square ABCD.

Since, $\angle DAB = 90°$,

$\therefore$ From ΔDAB, we have

by Pythagoras' theorem,

$$a^2 + a^2 = \left(4\sqrt{2} \text{ cm}\right)^2$$

$$\Rightarrow \quad 2a^2 = 32 \text{ cm}^2$$

$$\Rightarrow \quad a^2 = 16 \text{ cm}^2$$

$\therefore$ Area of the square = 16 cm^2

Also, radius of the circle = $\frac{4\sqrt{2}}{2}$ cm = $2\sqrt{2}$ cm

$\therefore$ Area of the circle = $\pi \times \left(2\sqrt{2}\right)^2$ cm^2

$$= \frac{22}{7} \times 8 \text{ cm}^2$$

$$= \frac{176}{7} \text{ cm}^2$$

$\therefore$ Area of the shaded region

= Area of the circle – Area of the square

$$= \left(\frac{176}{7} - 16\right) \text{ cm}^2$$

$$= \frac{176 - 112}{7} \text{ cm}^2$$

$$= \frac{64}{7} \text{ cm}^2$$

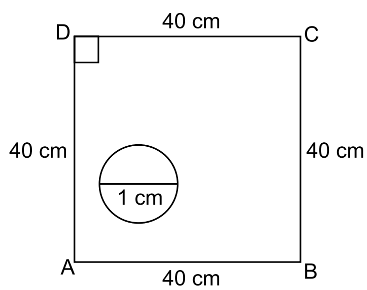
= 9.14 cm^2 (approx.)

Hence, the required area of the shaded region is 9.14 cm^2 (approx.).

6. A drain cover is made from a square metal plate of side 40 cm by having 441 holes of diameter 1 cm each drilled in it. Find the area of the remaining square plate. **[CBSE SP 2011]**

Sol. Area of the whole square ABCD = 40^2 cm^2

= 1600 cm^2



Area of each circular hole of radius $\frac{1}{2}$ cm

$$= \pi \times \frac{1}{4} \text{ cm}^2$$

$$= \frac{22}{7} \times \frac{1}{4} \text{ cm}^2$$

$$= \frac{11}{14} \text{ cm}^2$$

$\therefore$ Total area of 441 such holes $\frac{11}{14} \times 441$ cm^2

$$= \frac{693}{2} \text{ cm}^2$$

$\therefore$ Area of the remaining part of the square

$$= \left(1600 - \frac{693}{2}\right) \text{ cm}^2$$

$$= \frac{3200 - 693}{2} \text{ cm}^2$$

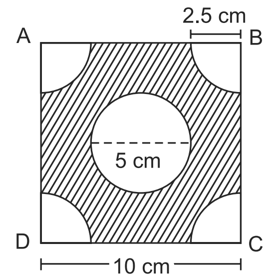
$$= \frac{2507}{2} \text{ cm}^2$$

= 1253.5 cm^2

Hence, the required area of the remaining square plate is 1253.5 cm^2.

Short Answer Type-II Questions

7. In the given figure, ABCD is a square of side 10 cm. Find the area of the shaded portion.



Sol. Area of the square ABCD

= 10 × 10 cm^2 = 100 cm^2 ...(1)

There are 4 quadrants of a circle at the 4 corners of the square, each of radius 2.5 cm.

The sum of the areas of these 4 quadrants

= Area of the whole circle of radius 2.5 cm

= $\pi \times 2.5^2$ cm^2

$$= \frac{22}{7} \times 6.25 \text{ cm}^2$$

$$= \frac{137.5}{7} \text{ cm}^2 \quad \text{...(2)}$$

Finally, area of the circle of radius $\frac{5}{2}$ cm at the centre of the square

$$= \pi \times \left(\frac{5}{2}\right)^2 \text{ cm}^2 = \frac{22}{7} \times \frac{24}{4} \text{ cm}^2 = \frac{275}{14} \text{ cm}^2 \quad \text{...(3)}$$

$\therefore$ Area of the shaded region

= Area of the square – Sum of the areas of the 4 quadrants of a circle – Area of the circle at the centre of the square

$$= \left(100 - \frac{137.5}{7} - \frac{275}{14}\right) \text{ cm}^2 \text{ [From (1), (2) and (3)]}$$

$$= \frac{1400 - 275 - 275}{14} \text{ cm}^2$$

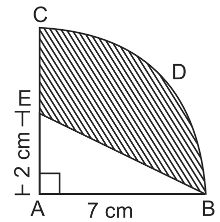
$$= \frac{1400 - 550}{14} \text{ cm}^2$$

$$= \frac{850}{14} \text{ cm}^2$$

$= \dfrac{425}{7}$ cm^2

Hence, the required area of the shaded portion is $\dfrac{425}{7}$ cm^2.

8. In the figure, ABDCA represents a quadrant of a circle of radius 7 cm with centre at A. Calculate the area of the shaded region EBDC with sides AE = 2 cm and AB = 7 cm.



[CBSE SP 2011]

Sol. Area of the quadrant of the circle

$$= \frac{1}{4} \times \pi \times 7^2 \text{ cm}^2$$

$$= \frac{1}{4} \times \frac{22}{7} \times 49 \text{ cm}^2$$

$$= \frac{77}{2} \text{ cm}^2$$

Area of $\Delta ABE = \dfrac{1}{2} \times 7 \times 2 \text{ cm}^2 = 7 \text{ cm}^2$

$\therefore$ Area of the shaded region

= Area of the quadrant of the circle − Area of ΔABE

$$= \frac{77}{2} \text{ cm}^2 - 7 \text{ cm}^2$$

$$= \frac{77-14}{2} \text{ cm}^2$$

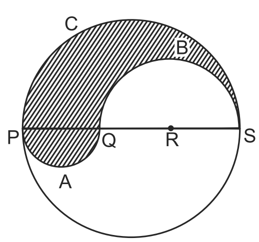
$$= \frac{63}{2} \text{ cm}^2$$

$$= 31.5 \text{ cm}^2$$

Hence, the required area of the shaded region EBDC is 31.5 cm^2.

Long Answer Type Questions

9. PQRS is a diameter of a circle of radius 6 cm. The lengths PQ, QR and RS are equal. Semicircles are drawn on PQ and QS as diameters as shown in the figure. Find the perimeter and the area of the shaded portion.



[CBSE SP 2011]

Sol. Radius of the whole circle, r = 6 cm.

Now, given that

$$PQ = QR = RS$$

$$= \frac{1}{3} \times PS$$

$$= \frac{1}{3} \times 12 \text{ cm}$$

$$= 4 \text{ cm}$$

Radius of the semicircle PAQ,

$$r_1 = \frac{PQ}{2} = 2 \text{ cm}$$

Radius of the semicircle QBS,

$$r_2 = \frac{QS}{2} = \frac{QR+RS}{2}$$

$$= \frac{8}{2} \text{ cm}$$

$$= 4 \text{ cm}$$

$\therefore$ Length of circumference of the semicircle PAQ

$$= \pi r_1 = 2\pi \text{ cm}$$

Length of circumference of the semicircle QBS

$$= \pi r_2 = 4\pi \text{ cm}$$

Length of circumference of the semicircle PCS

$$= \pi r = 6\pi \text{ cm}$$

$\therefore$ Perimeter of the shaded region

= Sum of the lengths of circumferences of the above three semicircles,

$$= (2\pi + 4\pi + 6\pi) \text{ cm}$$

$$= 12 \times \frac{22}{7} \text{ cm}$$

$$= \frac{264}{7} \text{ cm}$$

$\therefore$ Area of the shaded portion =

Area of the semicircle PAQ + Area of the semicircle PCS − Area of the semicircle QBS

$$= \left(\frac{\pi \times 2^2}{2} + \frac{\pi 6^2}{2} - \pi \times \frac{4^2}{2} \right) \text{cm}^2$$

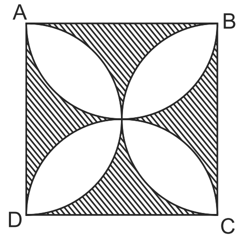
$$= \pi(2 + 18 - 8) \text{ cm}^2$$

$$= \frac{22}{7} \times 12 \text{ cm}^2$$

$$= \frac{264}{7} \text{ cm}^2$$

Hence, the required perimeter and the area of the shaded portion is $\dfrac{264}{7}$ cm and $\dfrac{264}{7}$ cm^2 respectively.

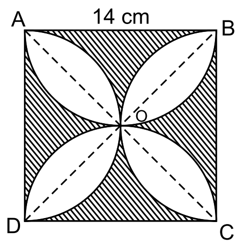
10. In the given figure, ABCD is a square of side 14 cm. Semicircles are drawn with each side of square as a diameter. Find the area of the shaded region. $\left[\text{Use } \pi = \frac{22}{7}\right]$



[CBSE 2016]

Sol. Let O be the point of intersection of two diagonals of the square. Then the four semicircles will pass through the point O and the radius of each semicircle will be $\frac{14}{2}$ cm = 7 cm and the area of each semicircle will be

$$\frac{1}{2}\pi \times 7^2 \text{ cm}^2 = \frac{1}{2} \times \frac{22}{7} \times 49 \text{ cm}^2$$
$$= 77 \text{ cm}^2$$



Now, sum of the areas of the shaded regions on the upper and lower parts of the square ABCD

= Area of the square ABCD – Sum of the areas of two semicircles AOB and COD

$$= (14 \times 14 - 77 \times 2) \text{ cm}^2$$
$$= (196 - 154) \text{ cm}^2$$
$$= 42 \text{ cm}^2$$

Similarly, the sum of the areas of the two shaded regions on the left hand side and the right hand side of the square = 42 cm^2

$\therefore$ Total area of the whole shaded region

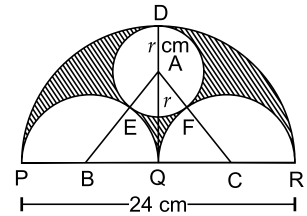
$$= 42 \times 2 \text{ cm}^2$$
$$= 84 \text{ cm}^2$$

Hence, the required area of the shaded region is 84 cm^2.

Higher Order Thinking Skills (HOTS) Questions

(Page 217)

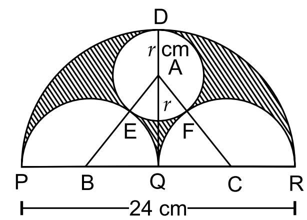
1. In the given figure, a semicircle is drawn with line segment PR as a diameter. Q is the midpoint of line segment PR. Two semicircles with line segments PQ and QR as diameters are drawn. A circle is drawn which touches the three semicircles. If PR = 24 cm, find the area of the shaded region.



Sol. Radius of the complete semicircle PDR = $\frac{24}{2}$ cm

= 12 cm

Let r be the radius of the complete circle with centre at A and passing through the points E and F on the two semicircles, PEQ and QFR.



Then since $\angle AQB = 90°$,

$$AQ = DQ - DA$$
$$= (12 - r) \text{ cm}$$

and
$$AB = BE + EA$$
$$= (6 + r) \text{ cm},$$

$\therefore$ By using Pythagoras' Theorem in ΔABQ, we have

$$AB^2 = AQ^2 + BQ^2$$
$$\Rightarrow \quad (6 + r)^2 = (12 - r)^2 + 36$$
$$\Rightarrow \quad 36 + r^2 + 12r - 144 - r^2 + 24r = 36$$
$$\Rightarrow \quad 36r = 144$$
$$\Rightarrow \quad r = \frac{144}{36} = 4$$

$\therefore$ The radius of the circle with centre A is 4 cm.

$\therefore$ Its area = $\pi \times 4^2$ cm^2 = 16 π cm^2 ...(1)

Sum of the areas of the two semicircles PEQ and QFR

$$= \frac{\pi \times 6^2}{2} \times 2 \text{ cm}^2$$
$$= 36\pi \text{ cm}^2 \quad ...(2)$$

Area of the complete semicircle PDR

$$= \frac{\pi \times 12^2}{2} \text{ cm}^2$$
$$= 72\pi \text{ cm}^2 \quad ...(3)$$

∴ Area of the shaded region

$= (72\pi - 36\pi - 16\pi)$ cm^2 [From (1), (2) and (3)]

$= (72 - 52)\pi$ cm^2

$= 20 \times \dfrac{22}{7}$ cm^2

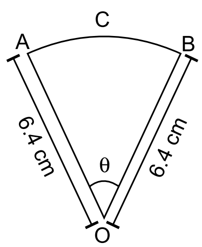
$= \dfrac{440}{7}$ cm^2

$= 62.86$ cm^2 (approx.)

Hence, the required area of the shaded region is 62.86 cm^2 (approx.)

2. The perimeter of a sector of a circle of radius 6.4 cm is 18.8 cm. Find the area of the sector.

Sol. Let the arc ACB of the sector OACD makes an angle θ at the centre O. Here radius *r* of the sector is 6.4 cm.



∴ Perimeter of the sector

$$= 2\pi r \times \frac{\theta}{360^\circ} + 2r$$

$$= 2\pi \times 6.4 \times \frac{\theta}{360^\circ} + 2 \times 6.4$$

∴ According to the problem, we have

$$\frac{\pi\theta}{360^\circ} \times 12.8 + 12.8 = 18.8$$

$$\Rightarrow \quad \frac{\pi\theta}{360^\circ} = \frac{18.8 - 12.8}{12.8} = \frac{6}{12.8} = \frac{60}{128} = \frac{15}{32} \quad \ldots(1)$$

∴ Area of the sector OACB

$$= \pi r^2 \frac{\theta}{360^\circ}$$

$$= \frac{\pi\theta}{360^\circ} \times 6.4 \times 6.4 \text{ cm}^2$$

$$= \frac{15}{32} \times 6.4 \times 6.4 \text{ cm}^2 \quad \text{[From (1)]}$$

$$= \frac{15}{32} \times 40.96 \text{ cm}^2$$

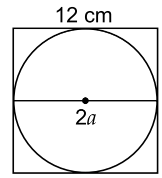
$$= \frac{61.44}{32} \text{ cm}^2$$

$$= 19.2 \text{ cm}^2$$

Hence, the required area of the sector is 19.2 cm^2.

3. Find the radius of the greatest circle (i.e. the circle with the greatest area) which can be inscribed within a square of side 12 cm.

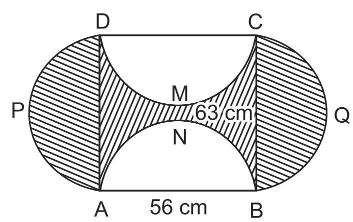
Sol. Let a be the radius of the circle. For the area of the circle to be greatest, $2a = 12$ cm.



$\Rightarrow \quad a = 6$ cm

Hence, the required radius of the circle is 6 cm.

4. Find the area of the shaded region in the given figure where ABCD is a rectangle with AB = 56 cm, BC = 63 cm, $\overset{\frown}{DMC}$, $\overset{\frown}{ANB}$, $\overset{\frown}{BQC}$ and $\overset{\frown}{APD}$ are semicircles on DC, AB, BC and AD respectively.



Sol. Area of the shaded region

= Sum of the areas of two semicircles APD and BQD, each with radius $\dfrac{63}{2}$ cm + Area of the rectangle ABCD – Sum of the areas of two semicircles ANB and DMC, each with radius 28 cm

$$= \left[\pi \times \left(\frac{63}{2}\right)^2 \times \frac{1}{2} \times 2 + 56 \times 63 - \frac{1}{2} \times \pi \times 28^2 \times 2\right] \text{cm}^2$$

$$= \left(\frac{22}{7} \times \frac{63 \times 63}{4} + 3528 - \frac{22}{7} \times 28 \times 28\right) \text{cm}^2$$

$$= \left(\frac{11 \times 9 \times 63}{2} + 3528 - 22 \times 4 \times 28\right) \text{cm}^2$$

$$= \left(\frac{6237}{2} + 3528 - 2464\right) \text{cm}^2$$

$= (3118.5 + 1064)$ cm^2

$= 4182.5$ cm^2

Hence, the required area of the shaded region is 4182.5 cm^2.

Self-Assessment

(Page 217)

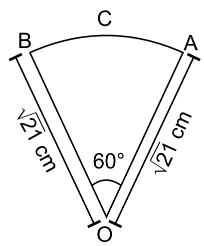
Multiple-Choice Questions

1. The minute hand of a clock is $\sqrt{21}$ cm long. Then the area described by the minute hand on the face of the clock between 7 : 00 am and 7 : 10 am is

(*a*) 11.5 cm^2 (*b*) 10.5 cm^2
(*c*) 10 cm^2 (*d*) 11 cm^2

Sol. (*d*) 11 cm^2

We know that the angle described by a minute hand of a clock in 1 minute $= \frac{360°}{60} = 6°$



$\therefore$ In 10 minutes from 7 : 00 a.m. to 7 : 10 am, the minute hand OA described an angle of $6° \times 10 = 60°$. Hence, in 10 minutes the minute hand describes an area equal to the area of the sector OACB of radius $\sqrt{21}$ cm.

$$\therefore \quad \text{Area} = \pi\left(\sqrt{21}\right)^2 \times \frac{60°}{360°} \text{ cm}^2$$

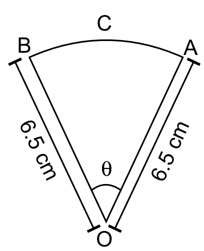
$$= \frac{22}{7} \times 21 \times \frac{1}{6} \text{ cm}^2 = 11 \text{ cm}^2$$

Hence, the required area is 11 cm^2.

2. If the perimeter of a sector of a circle of radius 6.5 cm is 21 cm, then the area of the sector is

(*a*) 52 cm^2 (*b*) 26 cm^2
(*c*) 25 cm^2 (*d*) 56 cm^2

Sol. (*b*) 26 cm^2



The perimeter of the sector OACB with $\angle AOB = \theta$

$$= \left(2 \times 6.5 + \frac{\pi\theta}{360°} \times 2 \times 6.5\right) \text{ cm}$$

$$= 13 + \frac{13\pi\theta}{360}$$

$\therefore$ According to the problem, we have

$$13 + \frac{13\pi\theta}{360} = 21$$

$$\Rightarrow \quad \frac{\pi\theta}{360°} = \frac{21-13}{13} = \frac{8}{13} \quad \ldots(1)$$

$\therefore$ Area of the sector OACB

$$= \pi \times 6.5^2 \times \frac{\theta}{360°} \text{ cm}^2$$

$$= 6.5 \times 6.5 \times \frac{8}{13} \text{ cm}^2 \quad \text{[From (1)]}$$

$$= 6.5 \times 0.5 \times 8 \text{ cm}^2$$

$$= 26 \text{ cm}^2$$

Hence, the required area of the sector is 26 cm^2.

Fill in the Blanks

3. If the circumference of a circle exceeds its diameter by 16.8 cm, then the radius of the circle is **3.92 cm**.

Sol.
$$2\pi r = 2r + 16.8$$
$$44r = 14r + 16.8 \times 7$$
$$30r = 117.6$$
$$r = \frac{117.6}{30} = 3.92 \text{ cm}$$

4. If the radius of a circle is 3.5 cm, then the perimeter of the semicircle is **18 cm**.

Sol. Perimeter of the semicircle

$$= \left(\frac{1}{2} \times 2 \times \frac{22}{7} \times 3.5 + 2 \times 3.5\right) \text{ cm}$$
$$= (22 \times 0.5 + 2 \times 3.5) \text{ cm}$$
$$= (11 + 7) \text{ cm}$$
$$= 18 \text{ cm}$$

5. If the areas of two circles are in the ratio 9 : 16, then the ratio of the perimeters of the circles is **3 : 4**.

Sol. r_1 and r_2 being the radius of two circles

$$\frac{\pi r_1^2}{\pi r_2^2} = \frac{9}{16}, \frac{r_1}{r_2} = \frac{3}{4}$$

$$\therefore \quad \frac{2\pi r_1}{2\pi r_2} = \frac{3}{4} = 3 : 4$$

6. If the area of a circle is equal to the sum of areas of circles of diameters 10 cm and 24 cm, then the diameter of the larger circle (in cm) is **26 cm**.

Sol.
$$\pi r^2 = \pi(5)^2 + \pi(12)^2$$
$$\pi r^2 = \pi(25 + 144)$$
$$r^2 = 169$$
$$r = 13$$
$$\therefore \quad \text{Diameter} = 2 \times 13 \text{ cm} = 26 \text{ cm}$$

Assertion-Reason Type Questions

Directions (Q. Nos. 7 to 9): Each of these questions contains an assertion followed by reason. Read them carefully, and answer the question on the basis of the following options, select the one that best describes the two statements.

(a) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.

(b) Both Assertion and Reason are true but Reason is not the correct explanation of Assertion.

(c) If Assertion is true but Reason is false.

(d) If Assertion is false but Reason is true.

7. Assertion: The circumference of a circle is an exact multiple of its diameter.

Reason: The ratio of circumference and diameter of a circle is always π.

Sol. (*d*) The ratio of circumference and diameter of a circle is π which is non-recurring non-terminating decimal. Thus circumference is not an exact multiple of diameter.

8. Assertion: If the angle of a sector is doubled, its area will be doubled.

Reason: Area of a sector is directly proportional to the angle.

Sol. (*a*) Both the statements are correct. Since the area of sector is directly proportional to its angle, hence doubling the angle will make the area double. Thus Reason is proper explanation of the Assertion.

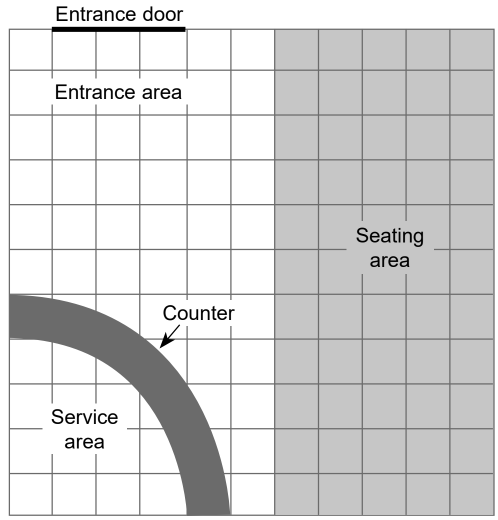
9. Assertion: A cake is cut into two halves along the diameter. The two segments thus formed are equal.

Reason: Diameter divides the circle into two equal halves.

Sol. (*a*) Both statements are correct. Reason is correct explanation of the Assertion.

Case Study Based Questions

10. Sanjana plans to open a coffee shop. So, she has created a floor plan for it. Floor plan for Sanjana's coffee shop is given below on the grid. The service area is surrounded by the service counter. Each square on the grid represents 1 metre × 1 metre.



Based on the above situation, answer the following questions.

(*a*) The radius of the inner edge of the counter is

(*i*) 2 m (*ii*) 2.5 m

(*iii*) 3 m (*iv*) 4 m

Ans. (*iv*) 4 m

(*b*) The service area is in the shape of

(*i*) quadrant. (*ii*) semicircle.

(*iii*) circle. (*iv*) square.

Ans. (*i*) quadrant.

(*c*) The area of the outer edge of the counter is

(*i*) $\frac{55}{7}$ m^2 (*ii*) $\frac{275}{7}$ m^2

(*iii*) $\frac{275}{14}$ m^2 (*iv*) $\frac{550}{7}$ m^2

Ans. (*iii*) $\frac{275}{14}$ m^2

(*d*) The area of the counter is

(*i*) $\frac{99}{14}$ m^2 (*ii*) $\frac{121}{56}$ m^2

(*iii*) $\frac{121}{2}$ m^2 (*iv*) $\frac{55}{28}$ m^2

Ans. (*i*) $\frac{99}{14}$ m^2

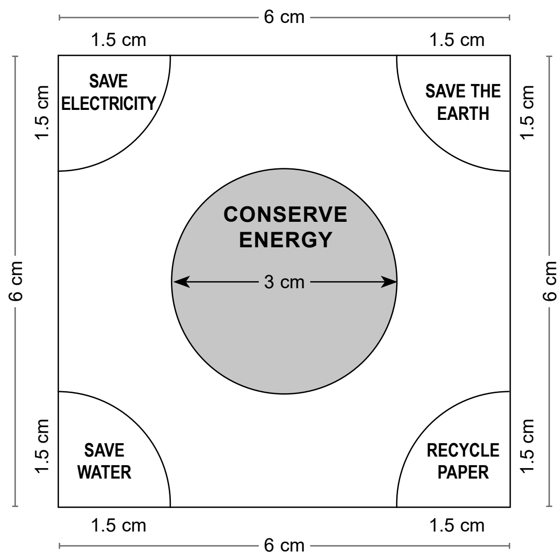
(*e*) The area of the floor of the coffee shop excluding the counter area and the service area is

(*i*) $\frac{99}{14}$ m^2 (*ii*) $\frac{275}{14}$ m^2

(*iii*) $\frac{795}{14}$ m^2 (*iv*) $\frac{1419}{14}$ m^2

Ans. (*iv*) $\frac{1419}{14}$ m^2

11. The Principal of a school asked each student to prepare a badge to be worn on every Friday of the week, so as to spread awareness about 'conserving energy'. Each badge has to be made on a square cardboard of side 6 cm with a quadrant of a circle of radius 1.5 cm drawn at each vertex of the square and a circle of diameter 3 cm drawn in its centre with the slogan 'CONSERVE ENERGY' written on it. Also, write the message on 4 quadrants as shown in the figure below.



Based on the given situation, answer the following questions.

(*a*) What is the area of each quadrant?

(*i*) $\dfrac{99}{14}$ cm^2 (*ii*) $\dfrac{99}{56}$ cm^2

(*iii*) $\dfrac{99}{7}$ cm^2 (*iv*) $\dfrac{99}{28}$ cm^2

Ans. (*ii*) $\dfrac{99}{56}$ cm^2

(*b*) The given problem is based on which mathematical concept?

(*i*) Area related to Circles

(*ii*) Triangles

(*iii*) Construction

(*iv*) None of these

Ans. (*i*) Area related to Circles

(*c*) What is the area of the circle?

(*i*) $\dfrac{99}{7}$ cm^2 (*ii*) $\dfrac{99}{14}$ cm^2

(*iii*) $\dfrac{99}{28}$ cm^2 (*iv*) $\dfrac{99}{56}$ cm^2

Ans. (*ii*) $\dfrac{99}{14}$ cm^2

(*d*) What is the area of the remaining portion of the badge?

(*i*) $\dfrac{253}{7}$ cm^2 (*ii*) $\dfrac{257}{14}$ cm^2

(*iii*) $\dfrac{153}{7}$ cm^2 (*iv*) $\dfrac{153}{14}$ cm^2

Ans. (*iii*) $\dfrac{153}{7}$ cm^2

(*e*) A "pie-slice" part of a circle is known as

(*i*) sector.

(*ii*) segment.

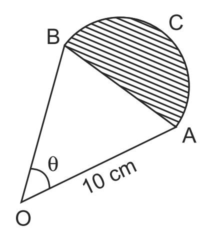
(*iii*) arc.

(*iv*) none of these

Ans. (*i*) sector.

Very Short Answer Type Questions

12. If the area of the segment ACBA of a circle of radius 10 cm and angle θ be $\dfrac{200}{7}$ cm^2, then what is the value of θ in degrees?



Sol. Area of the segment ACBA of the circle

$$= \pi r^2 \frac{\theta}{360^\circ} - \frac{1}{2} r^2 \sin\theta$$

$$= \left(\frac{22}{7} \times 10^2 \times \frac{\theta}{360^\circ} - \frac{1}{2} \times 10^2 \times \sin\theta\right) \text{ cm}^2$$

$$= \left(\frac{11}{7} \times 5 \times \frac{\theta}{9} - 50\sin\theta\right) \text{ cm}^2$$

$$= \left(\frac{55\theta}{63} - 50\sin\theta\right) \text{ cm}^2$$

∴ According to the problem, we have

$$\frac{55\theta}{63} - 50\sin\theta = \frac{200}{7}$$

$$\Rightarrow \quad 50\sin\theta = \frac{55\theta}{63} - \frac{200}{7} = \frac{55\theta - 1800}{63}$$

$$\Rightarrow \quad \sin\theta = \frac{55\theta - 1800}{63 \times 50}$$

$$= \frac{11\theta}{630} - \frac{36}{63}$$

$$= \frac{11\theta - 360}{630}$$

We see from above that among the standard values 30°, 45°, 60° and 90° of θ, when θ = 30° and 90°, both LHS and RHS are rational numbers.

But for θ = 30°,

$$\text{LHS} = \frac{1}{2}$$

and $$\text{RHS} = \frac{11 \times 30° - 360}{630}$$

$$= \frac{330 - 360}{63} < 0$$

and for θ = 90°,

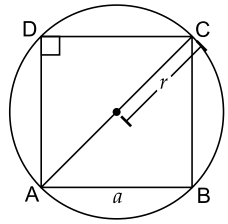
$$\text{LHS} = \text{RHS} = 1$$

Hence, the required value of θ is 90°.

13. A circle has a perimeter of 660 m. A square is inscribed within the circle such that the four vertices of the square lie on the circumference of the circle. Find the area of the square.

[CBSE SP 2011]

Sol. Let r be the radius of the circle.



Then $2\pi r = 660$ m [Given]

$\therefore \quad 2 \times \frac{22}{7} \times r = 600$ m

$\Rightarrow \quad r = \frac{660 \times 7}{2 \times 22}$ m

$\Rightarrow \quad r = 105$ m

$\therefore$ Radius of the circle is 105 m.

If a be the side of the square, then by Pythagoras' Theorem, we have

$$a^2 + a^2 = 4r^2$$

$\Rightarrow \quad a^2 = 2 \times 105 \times 105 \text{ m}^2$

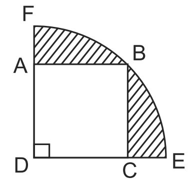
$$= 22050 \text{ m}^2$$

$\therefore$ Area of the square is 22050 m^2.

Hence, the required area of the square is 22050 m^2.

Short Answer Type-I Questions

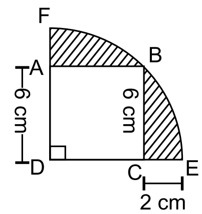
14. In the given figure, FDE is a quadrant of a circle and ABCD is a rectangle of area 48 cm^2 where BC = 6 cm, CE = 2 cm and ∠ADC = 90°. Calculate the area of the shaded region. [Use π = 3.14]



Sol. Area of the rectangle ABCD

$$= \text{AD} \times \text{DC}$$

$$= 6 \text{ cm} \times \text{DC}$$



$\therefore \quad 6 \text{ cm} \times \text{DC} = 48 \text{ cm}^2$ [Given]

$\Rightarrow \quad \text{DC} = \frac{48}{6} \text{ cm} = 8 \text{ cm}$

Hence, the length of DC is 8 cm.

$\therefore$ Length of DE = DC + CE

$$= 8 \text{ cm} + 2 \text{ cm} = 10 \text{ cm}$$

$\therefore$ Radius of the quarter circle = r = 10 cm

Hence, the area of the quadrant DEBF of the circle

$$= \frac{1}{4}\pi r^2$$

$$= \frac{1}{4} \times \frac{22}{7} \times 100 \text{ cm}^2$$

$$= \frac{550}{7} \text{ cm}^2$$

Also, area of the rectangle ABCD = 48 cm^2

$\therefore$ Area of the shaded region

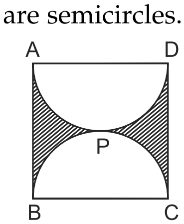
$$= \left(\frac{550}{7} - 48\right) \text{ cm}^2$$

$$= \frac{550 - 336}{7} \text{ cm}^2$$

$$= \frac{214}{7} \text{ cm}^2$$

$$= 30.6 \text{ cm}^2 \text{ (approx.)}$$

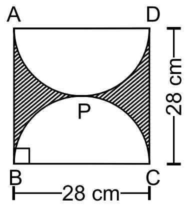
15. Find the area of the shaded region in the given figure, if ABCD is a square of side 28 cm and APD and BPC are semicircles.



[CBSE SP 2012]

Sol. Area of the square

$$= 28^2 \text{ cm}^2 = 784 \text{ cm}^2 \quad \ldots(1)$$



Sum of the areas of the two semicircles

$$\text{BPC and APD} = \frac{1}{2}\pi r^2 \times 2$$

$$= \frac{1}{2} \times \frac{22}{7} \times 14^2 \times 2 \text{ cm}^2$$

$$= 616 \text{ cm}^2 \quad \ldots(2)$$

$\therefore$ Area of the shaded region

$$= (784 - 616) \text{ cm}^2$$

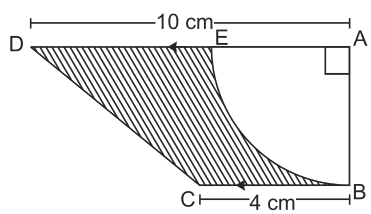
[From (1) and (2)]

$$= 168 \text{ cm}^2$$

Hence, the required area of the shaded region is 168 cm^2.

Short Answer Type-II Questions

16. In the given figure, ABCD is a trapezium of area 24.5 cm^2. In it, AD $\parallel$ BC, $\angle$DAB = 90°, AD = 10 cm and BC = 4 cm. If ABE is a quadrant of a circle, find the area of the shaded region. $\left[\text{Use } \pi = \frac{22}{7}\right]$



[CBSE 2014]

Sol. Area of the trapezium ABCD

$$= \frac{1}{2}(\text{AD} + \text{BC}) \times \text{AB}$$

$$= \frac{1}{2}(10 + 4) \text{ cm} \times \text{AB}$$

$$= 7 \text{ AB cm}$$

$\therefore$ 7AB = 24.5 cm

$\Rightarrow$ AB = 3.5 cm

$\therefore$ Length of AB = 3.5 cm

Area of the quadrant of a circle, AEB

$$= \frac{1}{4} \times \frac{22}{7} \times 3.5^2 \text{ cm}^2$$

$$= \frac{11}{14} \times 3.5 \times 3.5 \text{ cm}^2$$

$$= \frac{11}{2} \times 0.5 \times 3.5 \text{ cm}^2$$

$$= \frac{19.25}{2} \text{ cm}^2$$

$$= 9.625 \text{ cm}^2$$

$\therefore$ Area of the shaded region

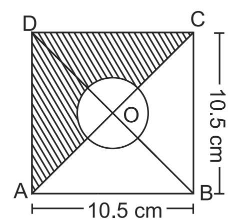
= Area of the trapezium – Area of the quadrant of the circle

$$= 24.5 \text{ cm}^2 - 9.625 \text{ cm}^2$$

$$= 14.875 \text{ cm}^2$$

Hence, the required area of the shaded region is 14.875 cm^2.

17. In the given figure, ABCD is a square of side 10.5 cm. A circle with centre at O is constructed such that its centre is the point of intersection of the two diagonals of the square and the area of the circle is one-fifth of the area of the square. Calculate the area and perimeter of the shaded region.



Sol. Area of the square ABCD = 10.5^2 cm^2

= 110.25 cm^2

Hence, the area of half portion of the square

$$= \text{area of } \Delta\text{ADC}$$

$$= \frac{110.25}{2} \text{ cm}^2$$

$$= 55.125 \text{ cm}^2 \quad \ldots(1)$$

$\therefore$ Area of the circle with centre O

$$= \frac{110.25}{5} \text{ cm}^2$$

$$= 22.05 \text{ cm}^2 \quad \ldots(2)$$

$\therefore$ Area of the semicircle

$$= \frac{22.05}{2} \text{ cm}^2$$

$$= 11.025 \text{ cm}^2 \quad \ldots(3)$$

$\therefore$ Area of the shaded region

$$= (55.125 - 11.025) \text{ cm}^2$$

[From (1) and (3)]

$$= 44.1 \text{ cm}^2$$

To determine the perimeter of the shaded region, we first find the radius, r of the circle with centre O.

Now, we have from (2),

$$\pi r^2 = 22.05 \text{ cm}^2$$

$$\Rightarrow \quad r^2 = 22.05 \times \frac{7}{22} \text{ cm}^2$$

$$= \frac{154.35}{22} \text{ cm}^2$$

$$= 7.0159 \text{ cm (approx.)}$$

$$\therefore \quad r = 2.648 \text{ cm (approx.)} \quad \ldots(3)$$

Now, the diagonal AC of the square

$$= \sqrt{10.5^2 + 10.5^2} \text{ cm}$$

[By Pythagoras' Theorem in ΔADC]

$$= 10.5\sqrt{2} \text{ cm}$$

$$= 10.5 \times 1.414$$

$$= 14.8470 \text{ (approx.)}$$

$$\therefore \quad AC - 2r = 14.8470 - 2 \times 2.648 \text{ [From (3)]}$$

$$= 14.8470 - 5.296 = 9.551 \ldots(4)$$

Now, semi-circumference of the circle with centre O

$$= \frac{22}{7} \times 2.648 \text{ cm}$$

$$= 8.322 \text{ cm} \quad \ldots(5)$$

$\therefore$ Perimeter of the shaded region

= AD + DC + (AC – $2r$) + 8.322 [From (5)]

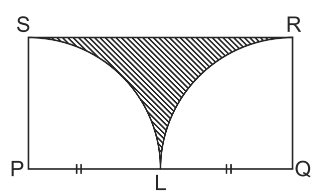
= (21 + 9.551 + 8.322) cm [From (4)]

= 38.87 cm (approx.)

Hence, the required area and perimeter of the shaded region is 44.1 cm^2 and 38.87 cm (approx.) respectively.

Long Answer Type Questions

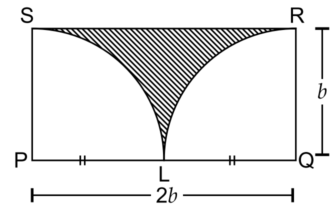
18. In the given figure, PQRS is a rectangle in which the length is twice the breadth and L is the mid-point of PQ. With P and Q as centres, draw two quadrants as shown in the figure. Find the ratios of



(*a*) the area of the rectangle PQRS to the area of the shaded region. **[CBSE SP 2012]**

(*b*) Also, find the ratio of the perimeter of the rectangle PQRS to that of the shaded region.

Sol. Let $2b$ and b units be the length and breadth of the rectangle PQRS respectively.



(*a*) Area of the rectangle PQRS

$$= 2b^2 \text{ sq units} \quad \ldots(1)$$

Sum of the areas of two quadrant with centres P and Q, of a circle $= 2 \times \frac{1}{4}\pi b^2$ sq units

$$= \frac{\pi b^2}{2} \text{ sq units}$$

$$= \frac{22}{14} b^2 \text{ sq units}$$

$$= \frac{11b^2}{7} \text{ sq units.}$$

$\therefore$ Area of the shaded region

$$= \left(2b^2 - \frac{11b^2}{7}\right) \text{ sq units}$$

$$= \frac{3b^2}{7} \text{ sq units} \quad \ldots(2)$$

$$\therefore \quad \text{Required ratio} = \frac{2b^2}{\frac{3b^2}{7}} \text{ [From (1) and (2)]}$$

$$= \frac{14}{3}$$

Hence, the required ratio is 14 : 3.

(*b*) Perimeter of the rectangle PQRS

$$= 2 \times (2b + b) \text{ units}$$

$$= 6b \text{ units} \quad \ldots(3)$$

Sum of the quarter circumferences of the two quadrants of a circle, PLS and QLR

$$= \frac{1}{4} \times 2\pi \times b \times 2 \text{ units}$$

$$= \frac{22}{7} b \text{ units}$$

$\therefore$ Perimeter of the shaded region

$$= \left(2b + \frac{22b}{7}\right) \text{ units}$$

$$= \frac{36b}{7} \text{ units} \quad \ldots(4)$$

$$\therefore \quad \text{Required ratio} = \frac{6b}{\frac{36b}{7}} \text{ [From (3) and (4)]}$$

$$= \frac{7}{6}$$

Hence, the required ratio is 7 : 6.

19. All four vertices of a rhombus are on a circle. Find the area of the rhombus if the area of the circle is 5024 cm^2. [Use π = 3.14]

Sol. Let r be the radius of the square.

Then $\pi r^2 = 5024 \text{ cm}^2$

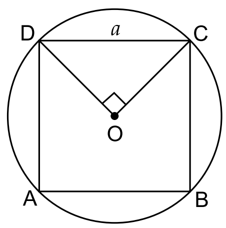
$\Rightarrow$ $3.14 \times r^2 = 5024 \text{ cm}^2$

$$r^2 = \frac{5024}{3.14} \text{ cm}^2$$

$$= 1600 \text{ cm}^2 \quad \ldots(1)$$

$\Rightarrow$ $r = 40$ cm

$\therefore$ Radius of the circle = 40 cm



Now, we know that a cyclic rhombus is a square. Let a be the side of the square. We know that diagonals of a square are of equal length and they bisect each other at right angles.

Hence, by using Pythagoras' Theorem, we have

$$a^2 = r^2 + r^2$$

$$= 2r^2 = 3200 \quad \text{[From (1)]}$$

Hence, the required area of the rhombus is 3200 cm^2.

Let's Compete

(Page 220)

Multiple-Choice Questions

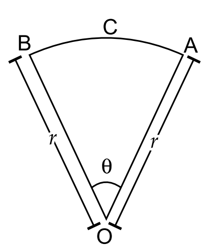
1. The area of a sector of a circle bounded by an arc of length 12π cm is equal to 48π cm^2. Then the radius of the circle is

(*a*) 16 cm (*b*) 8 cm

(*c*) 12 cm (*d*) 10 cm

Sol. (*b*) 8 cm

Let r be the radius of the sector and θ be the $\angle$AOB of the sector.



Given that

$$\pi r^2 \times \frac{\theta}{360°} = 48\pi \text{ cm}^2$$

$$\Rightarrow \quad \frac{\theta r^2}{360°} = 48 \text{ cm}^2 \quad \ldots(1)$$

Also, $$2\pi r \frac{\theta}{360°} = 12\pi \text{ cm}$$

$$\Rightarrow \quad \frac{\theta r}{360°} = 6 \text{ cm} \quad \ldots(2)$$

Dividing (1) by (2), we have

$$r = \frac{48}{6} \text{ cm} = 8 \text{ cm}$$

Hence, the required radius of the circle is 8 cm.

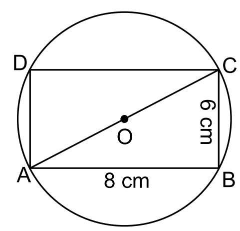
2. A rectangle whose length is $\frac{4}{3}$ times its breadth is inscribed inside a circle with centre at O. If the breadth of the rectangle is 6 cm, then the ratio of the area of the circle to the area of the rectangle is

(*a*) 163 : 270 (*b*) 168 : 275

(*c*) 275 : 168 (*d*) 270 : 163

Sol. (*c*) 275 : 168.

Let ABCD be a rectangle inscribed in a circle with centre at O and radius = r.



Now, breadth BC = AD of the rectangle is 6 cm and so the length AB = DC of the rectangle is $\frac{4 \times 6}{3}$ cm, i.e. 8 cm.

Now, $AC^2 = AD^2 + DC^2$

[By using Pythagoras' Theorem in ΔADC, $\because \angle ADC = 90°$]

$$\Rightarrow \quad 4r^2 = (6 \text{ cm})^2 + (8 \text{ cm})^2$$

$$= 36 \text{ cm}^2 + 64 \text{ cm}^2 = 100 \text{ cm}^2$$

$$\Rightarrow \quad r^2 = 25 \text{ cm}^2$$

$$\Rightarrow \quad r = 5 \text{ cm}$$

$$\therefore \quad \frac{\text{Area of the circle}}{\text{Area of the rectangle}} = \frac{\pi \times 25}{8 \times 6}$$

$$= \frac{22}{7} \times \frac{25}{48}$$

$$= \frac{275}{168}$$

$\therefore$ Required ratio = 275 : 168

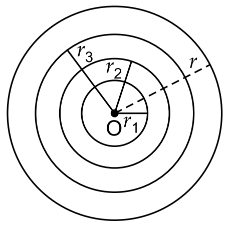
3. The radius of a circle is 20 cm. It is divided into four parts of equal area by drawing three concentric circles inside it. Then the radius of the largest of the three concentric circles drawn is

(*a*) $10\sqrt{3}$ cm (*b*) $10\sqrt{5}$ cm

(*c*) 10 cm (*d*) 20 cm

Sol. (*a*) $10\sqrt{3}$ cm

Let r_1, r_2 and r_3 be the radii of concentric circles of common centre O and r be the radius of the original circle such that $r_3 > r_2 > r_1$ and $r = 20$ cm.



According to the problem, we have

Area of each of three concentric circles

$$= \frac{1}{4} \times \pi \times 20^2 \text{ cm}^2 = 100 \text{ cm}^2$$

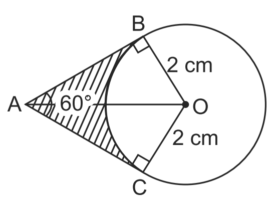
$$\therefore \quad \pi r_1^2 = 100\pi, \pi r_2^2 = 200\pi, \pi r_3^2 = 300\pi$$

$$\therefore \quad r_3^2 = 300$$

$$\therefore \quad r_3 = \sqrt{300} = 10\sqrt{3}$$

Hence, the required radius of the largest concentric circle is $10\sqrt{3}$ cm.

4. In the given figure, if the radius of the circle is 2 cm and $\angle A = 60°$, then the area of the shaded region is



(*a*) $4\left(\sqrt{3}+\frac{\pi}{3}\right)\text{cm}^2$ (*b*) $4\left(\frac{\pi}{\sqrt{3}}+3\right)\text{cm}^2$

(*c*) $4\left(\frac{\pi}{\sqrt{3}}-3\right)\text{cm}^2$ (*d*) $4\left(\sqrt{3}-\frac{\pi}{3}\right)\text{cm}^2$

Sol. (*d*) $4\left(\sqrt{3}-\frac{\pi}{3}\right)$ cm^2

We see that ABOC is a cyclic quadrilateral,

$$\therefore \quad \angle COB = 180° - 60° = 120°$$

$\therefore$ Area of the sector BOC

$$= \pi \times 2^2 \times \frac{120°}{360°} \text{ cm}^2$$

$$= \frac{4}{3}\pi \text{ cm}^2 \qquad \ldots(1)$$

In ΔAOB,

$$\because \quad \angle ABO = 90°,$$

$$\therefore \quad \frac{2}{AB} = \tan \angle OAB$$

$$= \tan 30° = \frac{1}{\sqrt{3}}$$

$$\therefore \quad AB = 2\sqrt{3}$$

$\therefore$ Length of AB = $2\sqrt{3}$ cm

$\therefore$ Area of ΔAOB $= \frac{1}{2} \times 2 \times 2\sqrt{3}$ cm^2

$$= 2\sqrt{3} \text{ cm}^2$$

Area of the quadrilateral ABOC

$$= 2 \times \text{area of } \Delta AOB$$

$$= 4\sqrt{3} \text{ cm}^2 \qquad \ldots(2)$$

$\therefore$ Area of the shaded region

= Area of quadrilateral ABOC

– Area of the sector BOC

$$= \left(4\sqrt{3} - \frac{4\pi}{3}\right) \text{cm}^2$$

[From (1) and (2)]

$$= 4\left(\sqrt{3} - \frac{\pi}{3}\right) \text{cm}^2$$

5. The area of the largest triangle, i.e. triangle of the greatest area, that can be inscribed in a semicircle of radius 4 cm is

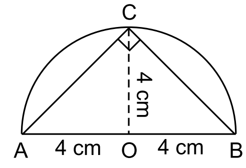
(*a*) 8 cm^2 (*b*) 16 cm^2

(*c*) 4 cm^2 (*d*) 2 cm^2

Sol. (*b*) 16 cm^2

Area of the largest triangle

= Area of ΔABC, where OC $\perp$ AB and O is the centre of the circle.



Now, $\quad$ AC = BC

$$= \sqrt{AO^2 + CO^2}$$

$$= \sqrt{4^2 + 4^2} \text{ cm}$$

$$= \sqrt{32} \text{ cm}$$

$\therefore$ Required area $= \frac{1}{2} \times \sqrt{32} \times \sqrt{32}$ cm^2

$$= 16 \text{ cm}^2$$

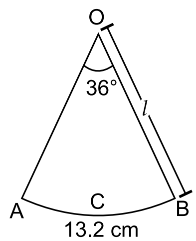
6. A pendulum swings through an angle of 36° and describes an arc of length 13.2 cm. Then the length of the pendulum sweeps an area equal to

(*a*) 135.8 cm^2 (*b*) 130.6 cm^2
(*c*) 138.6 cm^2 (*d*) 136 cm^2

Sol. (*c*) 138.6 cm^2

The pendulum describes an arc ACB of a sector OAB of a circle with centre at O and radius = l, (say).

$$\text{Length of arc ACB} = \frac{36}{360°} \times 2\pi l$$
$$= \frac{36}{360°} \times 2 \times \frac{22}{7} \times l$$
$$= \frac{1}{10} \times \frac{2}{7} \times 22\, l = \frac{22l}{35}$$



$$\therefore \quad \frac{22l}{35} = 13.2 \text{ cm}$$
$$\Rightarrow \quad l = \frac{13.2 \times 35}{22} \text{ cm} = 21 \text{ cm}$$

$\therefore$ Radius of the sector = 21 cm

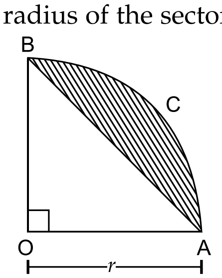
$$\therefore \quad \text{Area of the sector} = \frac{36}{360°} \times \pi l^2$$
$$= \frac{1}{10} \times \frac{22}{7} \times 21 \times 21 \text{ cm}^2$$
$$= \frac{1}{10} \times 22 \times 3 \times 21 \text{ cm}^2$$
$$= 138.6 \text{ cm}^2$$

7. An arc ACB of a circle forms an angle of 90° at the centre O of the circle. Then the ratio of the area of the sector OAB to the area of the minor segment ACBA is

(*a*) $\pi : (\pi - 2)$ (*b*) $\pi : (\pi - 1)$
(*c*) $(\pi + 1) : (\pi - 1)$ (*d*) $(\pi + 1) : \pi$

Sol. (*a*) $\pi : (\pi - 2)$

Let r be the radius of the sector.



Then area of sector OAB = $\frac{1}{4}\pi r^2$ cm^2 ...(1)

$\therefore$ Area of the minor segment ACBA

$$= \left(\frac{\pi r^2}{4} - \frac{1}{2}r^2\right) \text{cm}^2$$
$$= \frac{(\pi - 2)r^2}{4} \quad \text{...(2)}$$

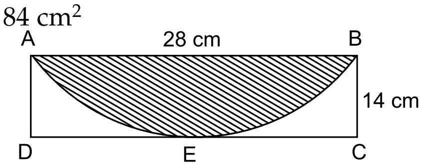
$\therefore$ Required ratio = $\frac{\pi}{\pi - 2}$ [From (1) and (2)]

$= \pi : (\pi - 2)$

8. A semicircle of maximum area is cut off from a rectangular piece of paper of length 28 cm and breadth 14 cm. Then the area of the remaining portion of the rectangle is

(*a*) 96 cm^2 (*b*) 90 cm^2
(*c*) 100 cm^2 (*d*) 84 cm^2

Sol. (*d*) 84 cm^2



Area of the rectangle ABCD

$= 28 \times 14$ cm^2 = 392 cm^2 ...(1)

Area of the semicircle of maximum area

= Area of the semicircle AEB

$$= \frac{\pi \times 14^2}{2} \text{ cm}^2$$
$$= \frac{22}{14} \times 14 \times 14 \text{ cm}^2$$
$$= 308 \text{ cm}^2 \quad \text{...(2)}$$

$\therefore$ Required area of the remaining portion

= (392 – 308) cm^2

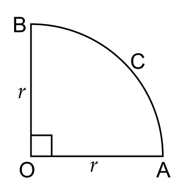
[From (1) and (2)]

= 84 cm^2

9. If the perimeter of a quadrant of a circle is 25 cm, then the area of the quadrant is

(*a*) 77 cm^2 (*b*) 38.5 cm^2
(*c*) 12.5 cm^2 (*d*) 20 cm^2

Sol. (*b*) 38.5 cm^2



If r be the radius of the quadrant OACB of a circle, then

$$2r + \frac{2\pi r}{4} = 25 \text{ cm}$$

$$\Rightarrow \quad 2r\left(1 + \frac{22}{7} \times \frac{1}{4}\right) = 25 \text{ cm}$$

$$\Rightarrow \quad 2r\left(1 + \frac{11}{14}\right) = 25 \text{ cm}$$

$$\Rightarrow \quad 2r \times \frac{25}{14} = 25 \text{ cm}$$

$$\Rightarrow \quad r = \frac{25 \times 14}{25 \times 2} \text{ cm}$$

$$\Rightarrow \quad r = 7 \text{ cm}$$

$$\therefore \quad \text{Required area} = \frac{\pi r^2}{4}$$

$$= \frac{22}{7} \times \frac{1}{4} \times 7 \times 7 \text{ cm}^2$$

$$= 38.5 \text{ cm}^2$$

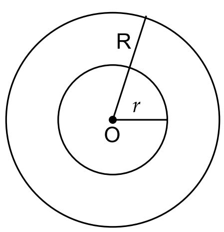
10. If the areas of two concentric circles are in the ratio 9 : 16, then the ratio of the perimeter of the ring formed by the two concentric circles and the difference of the perimeters of the two circles is

(*a*) 2 : 7 (*b*) 7 : 2

(*c*) 7 : 1 (*d*) 1 : 7

Sol. (*c*) 7 : 1

Let r and R be the radii of two concentric circles with centre at O, where $R > r$.



Given that $\dfrac{\pi r^2}{\pi R^2} = \dfrac{9}{16}$

$$\Rightarrow \quad \frac{r}{R} = \frac{3}{4} \qquad \ldots(1)$$

Now, perimeter of the ring $= 2\pi(r + R)$ …(2)

Difference of the perimeter of two circles

$$= 2\pi(R - r) \qquad \ldots(3)$$

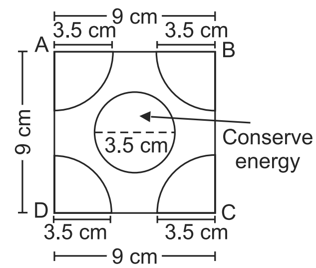
$$\therefore \text{ Required ratio} = \frac{R + r}{R - r} = \frac{7}{1} \qquad \text{[From (1)]}$$

$$= 7 : 1$$

Value-based Questions (Optional)

(Page 221)

1. The Principal of a school asked each student to prepare a badge to be worn on every Monday of the week, so as to spread awareness about 'Conserving energy'. Each badge had to be made on a square cardboard of side 9 cm with a quadrant of a circle of radius 3.5 cm drawn at each vertex of the square and a circle of diameter 3.5 cm drawn in its centre with the slogan 'Conserve energy' written on it.



(*a*) Find the area of the remaining portion of the badge.

(*b*) Suggest the writings which students can provide in the four quadrants.

(*c*) What value did the Principal of the school exhibit?

Sol. Sum of the area of 4 quadrants at the four corners of the square ABCD

= Area of the circle with radius 3.5 cm

$= \pi \times 3.5^2 \text{ cm}^2$

$= \dfrac{22}{7} \times 3.5 \times 3.5 \text{ cm}^2$

$= 38.5 \text{ cm}^2$ …(1)

Area of the circle of radius $\dfrac{3.5}{2}$ cm, at the centre of the square

$$= \pi \times \frac{3.5^2}{4} \text{ cm}^2$$

$$= \frac{1}{4} \times 38.5 \text{ cm}^2 \qquad \text{[From (1)]}$$

$$= 9.625 \text{ cm}^2 \qquad \ldots(2)$$

$\therefore$ Area of the square $= 9 \times 9 \text{ cm}^2 = 81 \text{ cm}^2$ …(3)

(*a*) Area of the remaining portion of the badge

$= (81 - 38.5 - 9.625) \text{ cm}^2$ [From (1), (2) and (3)]

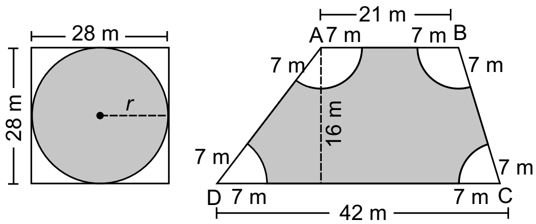
$= (81 - 48.125) \text{ cm}^2$

$= 32.875 \text{ cm}^2$

(*b*) Save water, make car pools, switch off electrical appliances when not in use, repair leaking taps, use LED, use solar energy, etc. are some phrases which the students can write in the quadrants.

(*c*) Awareness about energy conservation, creative, thinking decision-making and effective communication.

2. Farmer A owns a square field whereas a farmer B holds a field in the form of a trapezium as shown below:



On Van Mahotsav Day, farmers A and B plant trees in the shaded regions of their respective fields.

(*a*) Which of them uses more area to plant trees and by how much?

(*b*) What are the values exhibited by the farmers?

Sol. From the square field, for farmer A

$$2r = 28 \text{ m}$$

$$\Rightarrow \quad r = 14 \text{ m}$$

$\therefore$ Area of the circle inscribed in the square

$$= \pi \times 14^2 \text{ m}^2$$

$$= \frac{22}{7} \times 14 \times 14 \text{ m}^2$$

$$= 22 \times 2 \times 14 \text{ m}^2$$

$$= 616 \text{ m}^2 \qquad \ldots(1)$$

Now,

Area of the trapezium ABCD

$$= \frac{1}{2}(21 + 42) \times 16 \text{ m}^2$$

$$= 8 \times 63 \text{ m}^2$$

$$= 504 \text{ m}^2 \qquad \ldots(2)$$

Let θ_1, θ_2, θ_3 and θ_4 be the angles of arcs at A, B, C and D respectively.

Then $\theta_1 + \theta_2 + \theta_3 + \theta_4 = 360°$ ...(3)

Sum of the areas of the four sectors of circles with centres at A, B, C and D

$$= \frac{\pi 7^2}{360°}(\theta_1 + \theta_2 + \theta_3 + \theta_4) \text{ m}^2$$

$$= \frac{22}{7} \times 49 \times \frac{36°}{360°} \text{ m}^2 \text{ [From 3]}$$

$$= 154 \text{ m}^2 \qquad \ldots(4)$$

$\therefore$ Area of the shaded region in trapezium ABCD

$$= (504 - 154) \text{ m}^2$$

[From (2) and (4)]

$$= 350 \text{ m}^2 \qquad \ldots(5)$$

(*a*) From (1) and (5), we see that Farmer A uses (616 – 350) m^2, i.e. 266 m^2 more area than B.

(*b*) Awareness about the environment.

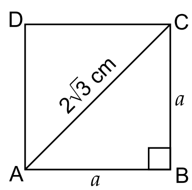
13

Surface Areas and Volumes

Checkpoint (Page 224)

1. The length of the diagonal of a face of a cube is $2\sqrt{3}$ cm. What are the total surface area and the volume of the cube?

Sol. Let a be the side of the cube with a diagonal AC = $2\sqrt{3}$ cm.



Hence, by using Pythagoras' Theorem, we have

$$a^2 + a^2 = \left(2\sqrt{3}\text{ cm}\right)^2 = 12\text{ cm}^2$$

$\Rightarrow$ $2a^2 = 12\text{ cm}^2$

$\Rightarrow$ $a^2 = 6\text{ cm}^2$

$\Rightarrow$ $a = \sqrt{6}$ cm

$\therefore$ Total surface area of the cube

$= 6a^2$

$= 6 \times 6\text{ cm}^2$

$= 36\text{ cm}^2$

and the total volume

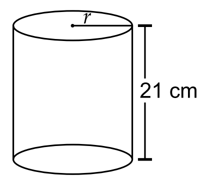
$= a^3$

$= \left(\sqrt{6}\right)^3\text{ cm}^3$

$= 6\sqrt{6}\text{ cm}^3$

2. The curved surface area of an open right circular cylinder is 924 cm^2. If the height of the cylinder is 21 cm, what is its volume?

Sol. Let r be the radius of the base of the cylinder and h be its height. Then h = 21 cm.



Now, curved surface area of the cylinder

$= 2\pi rh$

$= 2 \times \frac{22}{7} \times r \times 21$ cm

$= 132r$ cm

$\therefore$ $132r\text{ cm} = 924\text{ cm}^2$

$\Rightarrow$ $r = \frac{924}{132}$ cm

$\Rightarrow$ $r = 7$ cm

$\therefore$ Required volume $= \pi r^2 h$

$= \frac{22}{7} \times 7 \times 7 \times 21\text{ cm}^3$

$= 22 \times 7 \times 21\text{ cm}^3$

$= 3234\text{ cm}^3$

Hence, the required volume is 3234 cm^3.

3. The volume of a cube is 1728 cm^3. Find its total surface area.

Sol. Let a be the side of the cube.

Then $a^3 = 1728\text{ cm}^3$

$= 4^3 \times 3^3\text{ cm}^3$

$\Rightarrow$ $a = 4 \times 3\text{ cm} = 12$ cm

$\therefore$ Total surface area of the cube

$= 6a^2$

$= 6 \times 12^2\text{ cm}^2$

$= 864\text{ cm}^2$

Hence, the required total surface area is 864 cm^2.

4. The length, breadth and height of a rectangular solid object are in the ratio 2 : 3 : 1. Find the length, breadth and height of the solid if the total surface area of the solid is 88 cm^2.

Sol. Let the length, $l = 2x$, breadth, $b = 3x$ and height, $h = x$, where x is a non-zero positive number.

Then the total surface area of the solid

$$= 2(lb + lh + bh)$$
$$= 2(2x \times 3x + 2x \times x + 3x \times x)$$
$$= 2(6x^2 + 2x^2 + 3x^2)$$
$$= 2 \times 11x^2$$
$$= 22x^2$$

$\therefore \quad 22x^2 = 88 \text{ cm}^2$

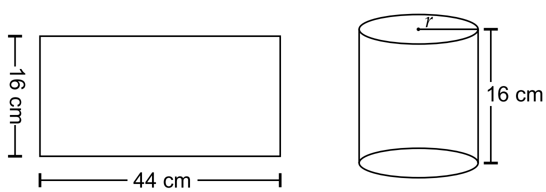
$\Rightarrow \quad x^2 = 4 \text{ cm}^2$

$\Rightarrow \quad x = 2 \text{ cm}$

Hence, the required length = 2 × 2 cm = 4 cm, breadth = 3 × 2 cm = 6 cm and height = 2 cm.

5. A rectangular piece of paper is 44 cm long and 16 cm wide. A cylinder is formed by rolling this rectangular piece of paper along its length. Find the volume of the cylinder so formed.

Sol. Let l be the length of the rectangular paper and b be its breadth. Then the length of the circumference of the cylinder = l and height of the cylinder = b. Let r be the radius of the base of the cylinder.



Then, $\quad l = 2\pi r$

$\Rightarrow \quad 44 \text{ cm} = 2 \times \frac{22}{7} \times r$

$\Rightarrow \quad r = \frac{44 \times 7}{2 \times 22} \text{ cm}$

$\Rightarrow \quad r = 7 \text{ cm}$

$\therefore$ Volume of the cylinder

$$= \pi r^2 b$$
$$= \frac{22}{7} \times 7 \times 7 \times 16 \text{ cm}^3$$
$$= 22 \times 7 \times 16 \text{ cm}^3$$
$$= 2464 \text{ cm}^3$$

Hence, the required volume of the cylinder is 2464 cm^3.

6. The diameter of a garden roller is 1.4 m and it is 2.5 m long. How much area will it cover in 5 revolutions?

Sol. Given, diameter = 1.4 m

$\therefore \quad$ Radius, $r = \frac{1.4}{2} \text{ m} = 0.7 \text{ m}$

Height, $h = 2.5$ m

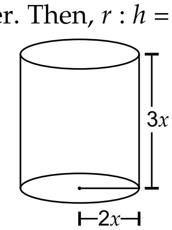
Curved surface area of garden roller

$$= 2\pi rh$$
$$= 2 \times \frac{22}{7} \times 0.7 \times 2.5 \text{ m}^2$$
$$= 11 \text{ m}^2$$

Hence, area covered by garden roller in 5 revolutions = 5 × 11 m^2 = 55 m^2.

7. The radius of the base and the height of a solid right circular cylinder are in the ratio 2 : 3 and its volume is 1617 cm^3. Find the total surface area of the cylinder.

Sol. Let r be the radius of the base and h be the height of the cylinder. Then, $r : h = 2 : 3$



Let $r = 2x$ and $h = 3x$, where x is a non-zero positive number.

Now, volume of the cylinder

$$= \pi r^2 h$$
$$= \frac{22}{7} \times 4x^2 \times 3x$$

$\therefore \quad \frac{22}{7} \times 4x^2 \times 3x = 1617 \text{ cm}^3$

$\Rightarrow \quad x^3 = \frac{1617 \times 7}{22 \times 4 \times 3} \text{ cm}^3$

$$= \left(\frac{7}{2}\text{cm}\right)^3$$

$\therefore \quad x = \frac{7}{2} \text{ cm}$

$\therefore \quad r = 2 \times \frac{7}{2} \text{cm} = 7 \text{ cm}$

$h = 3 \times \frac{7}{2} \text{cm} = \frac{21}{2} \text{ cm}$

$\therefore$ Total surface area of the cylinder

$$= 2\pi rh + 2\pi r^2$$
$$= 2\pi r(h + r)$$

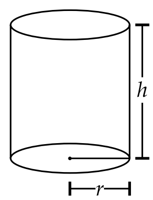
$$= 2 \times \frac{22}{7} \times 7 \times \left(\frac{21}{2} + 7\right) \text{ cm}^2$$

$$= 44 \times \frac{35}{2} \text{ cm}^2 = 770 \text{ cm}^2$$

Hence, the required total surface area of the cylinder is 770 cm^2.

8. The total surface area of a solid cylinder is 231 cm^2. If the curved surface area of this cylinder is $\frac{2}{3}$ of its total surface area, then find its radius and height.

Sol. Let r be the radius of the base of the cylinder and h be its height. Then the total surface area of the solid cylinder = $2\pi rh + 2\pi r^2$ and the curved surface area = $2\pi rh$.



It is given that

$$2\pi rh = \frac{2}{3} \times (2\pi rh + 2\pi r^2) \qquad \ldots(1)$$

and $\quad 2\pi rh + 2\pi r^2 = 231 \text{ cm}^2 \qquad \ldots(2)$

From (1), we have

$$h = \frac{2}{3}h + \frac{2}{3}r$$

$$\Rightarrow \quad \frac{h}{3} = \frac{2}{3}r$$

$$\Rightarrow \quad h = 2r \qquad \ldots(3)$$

$\therefore$ From (2)

$$2\pi r(h + r) = 231 \text{ cm}^2$$

$$\Rightarrow \quad 2 \times \frac{22}{7} \times r \times 3r = 231 \text{ cm}^2$$

$$\Rightarrow \quad r^2 = \frac{49}{4} \text{ cm}^2$$

$$\therefore \quad r = \frac{7}{2} \text{ cm} = 3.5 \text{ cm}$$

From (3), $\quad h = 2 \times 3.5 \text{ cm} = 7 \text{ cm}$

Hence, the required radius is 3.5 cm and the height is 7 cm.

9. Find the radius of a sphere whose surface area is numerically equal to its volume.

Sol. If r be the radius of the sphere.

Then its volume = $\frac{4}{3}\pi r^3$

and the surface area = $4\pi r^2$

It is given that

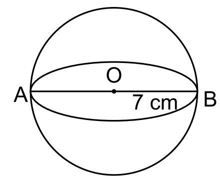
$$4\pi r^2 = \frac{4}{3}\pi r^3$$

$$\Rightarrow \quad r = 3$$

Hence, the required radius is 3 units.

10. A circle of radius 7 cm is rotated about its diameter. Find the surface area of the solid thus generated.

Sol. The circle of radius, r = 7 cm will generate a solid sphere with radius, r = 7 cm when rotated about its diameter AB.



$\therefore$ Surface area of the sphere, thus generated

$$= 4\pi r^2$$

$$= 4 \times \frac{22}{7} \times 7 \times 7 \text{ cm}^2$$

$$= 616 \text{ cm}^2$$

Hence, the required surface area of the solid thus generated is 616 cm^2.

Milestone 1

(Page 228)

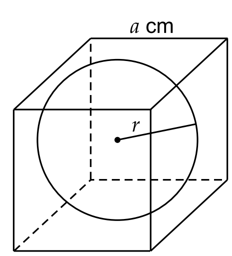
Multiple-Choice Questions

1. The volume of the largest sphere that can be carved out of a cube of side 21 cm is

(*a*) 4410 cm^3 (*b*) 4851 cm^3
(*c*) 6615 cm^3 (*d*) 5292 cm^3

Sol. (*b*) 4851 cm^3

Let a be the side of the cube and r be the radius of the sphere. The largest sphere will clearly touch the four square faces of the cube. Hence, the diameter of the sphere will be equal to the side of the cube in this case.



$$\therefore \quad a = 2r$$

$\Rightarrow \qquad 21 \text{ cm} = 2r$

$\Rightarrow \qquad r = \frac{21}{2} \text{ cm}$

$= 10.5 \text{ cm}$

Hence, the radius of the sphere is 10.5 cm.

$\therefore$ Volume of the sphere

$$= \frac{4}{3}\pi r^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times 10.5 \times 10.5 \times 10.5 \text{ cm}^3$$

$$= 11 \times 42 \times 10.5 \text{ cm}^3$$

$$= 4851 \text{ cm}^3$$

Hence, the required volume is 4851 cm^3.

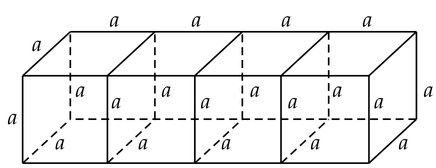
2. If four cubes each of side a units are joined together end to end to form a cuboid, then the ratio of the volume of the cuboid to its surface area is

(*a*) $9 : 2a$ (*b*) $a : 18$

(*c*) $18 : a$ (*d*) $2a : 9$

Sol. (*d*) $2a : 19$

Let a be the side of each cube. Then the four cubes when joined together end to end as in the figure to form a cuboid, then the length, breadth and height of the cuboid will be $4a$ cm, a cm and a cm respectively.



$\therefore$ Volume of the cuboid $= 4a \times a \times a = 4a^3$

and surface area of the cuboid

$$= 2(4a \times a + 4a \times a + a \times a)$$

$$= 18\,a^2$$

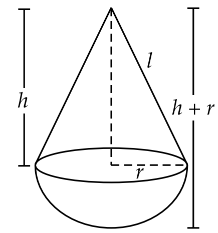
$\therefore$ Required ratio $= 4a^3 : 18a^2$

$$= 2a : 9$$

Very Short Answer Type Questions

3. A solid is hemispherical at the bottom and conical above. If the volumes of the two parts are equal, then find the ratio of the height of the combined solid to the radius of the hemisphere.

Sol. Let r be the common radius of the base of the cone and the hemisphere and let h be the height of the cone. It is given that their volumes are equal to each other.



Now, the volume of the cone $= \frac{1}{3}\pi r^2 h$ and the volume of the hemisphere $= \frac{2}{3}\pi r^3$.

$$\therefore \qquad \frac{1}{3}\pi r^2 h = \frac{2}{3}\pi r^3$$

$$\Rightarrow \qquad h = 2r \qquad \ldots(1)$$

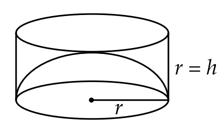
Now, the height of the combined solid $= h + r$

$$\therefore \qquad \frac{h + r}{r} = \frac{2r + r}{r} \qquad \text{[From (1)]}$$

Hence, the required ratio is 3 : 1

4. A hemisphere and a cylinder stand together on the same common circular base on the ground such that their heights are the same. What is the relation between the volume of the cylinder and that of the hemisphere?

Sol. Let r be the common radius of the cylinder and the hemisphere.



Then the height, h of the cylinder be r. If V_1 and V_2 be volumes of the cylinder and the hemisphere respectively, then

$$V_1 = \pi r^2 h = \pi r^2 \times r = \pi r^3$$

and $$V_2 = \frac{2}{3}\pi r^3$$

$$\therefore \qquad \frac{V_1}{V_2} = \frac{\pi r^3}{\frac{2\pi r^3}{3}} = \frac{3}{2}$$

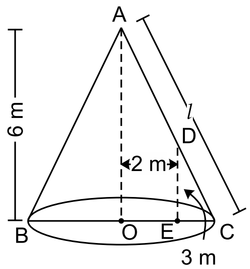
$$\therefore \qquad V_2 = \frac{2}{3} V_1$$

Hence, the required relation is that the volume of the hemisphere is equal to $\frac{2}{3}$ rd of the volume of the cylinder.

Short Answer Type-I Questions

5. Find the area of the canvas required for a 6 m high conical tent in which an object of height 3 m may just be kept at a distance of 2 m from the centre of the base of the tent.

Sol. Let r be the radius of the base of the cone ABC and h be its height. Let l be the slant height of the cone. Let DE be an object standing vertically on the base BOC of the cone such that OE = 2m, DE = 3m, O being the centre of the circular base of the cone. Then, in ΔDEC and ΔAOC, we see that DE $\parallel$ AO.



$$\therefore \quad \frac{DE}{AO} = \frac{EC}{OC}$$

$$\Rightarrow \quad \frac{3}{6} = \frac{OC - OE}{OC} = \frac{r-2}{r}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{r-2}{r}$$

$$\Rightarrow \quad r = 2r - 4$$

$$\Rightarrow \quad r = 4 \text{ m} \qquad \ldots(1)$$

Also, $\quad h = AO = 6$ m

$\therefore$ In ΔAOC, by Pythagoras' Theorem

$$AC = \sqrt{AO^2 + OC^2}$$

$$\Rightarrow \quad l = \sqrt{36 + r^2}$$

$$= \sqrt{36+16} \qquad \text{[From (1)]}$$

$$= \sqrt{52}$$

$$= 2\sqrt{13} \qquad \ldots(2)$$

Hence, the curved surface area of the cone

$$= \pi r l$$

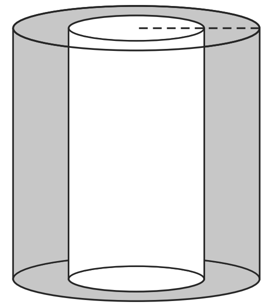
$$= \frac{22}{7} \times 4 \times 2\sqrt{13} \text{ m}^2$$

[From (1) and (2)]

$$= \frac{176\sqrt{3}}{7} \text{ m}^2$$

Hence, the area of the canvas required to cover the tent is $\frac{176\sqrt{3}}{7}$ m^2.

6. Two cylinders of the same height are standing on the same base as shown in the figure such that the radius of the base of the inner cylinder is half that of the outer cylinder. Find the ratio of the volume of the shaded portion to the volume of the outer cylinder.



Sol. Let r and R be the radii of the bases of the two cylinders, where R > r and let h be the common height of the two cylinders. Let V_1 and V_2 be the volumes of the two cylinders, where $V_2 > V_1$.

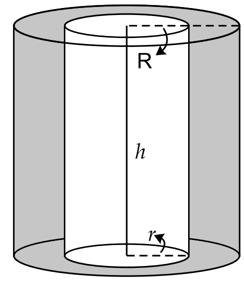
Then $V_1 = \pi r^2 h$, $V_2 = \pi R^2 h = \pi(2r)^2 h = 4\pi r^2 h$

[$\because$ R = $2r$, given]

$\therefore V_2 - V_1$ = Volume of the shaded portion

$= 4\pi r^2 h - \pi r^2 h$

$= 3\pi r^2 h$



$$\therefore \quad \text{Required ratio} = \frac{V_2 - V_1}{V_2}$$

$$= \frac{3\pi r^2 h}{4\pi r^2 h} = \frac{3}{4} = 3 : 4$$

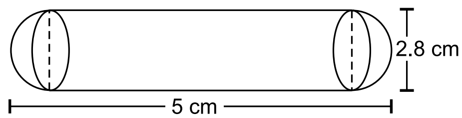
Short Answer Type-II Questions

7. A gulab jamun, when ready for eating, contains sugar syrup of about 30% of its volume. Find approximately how much syrup would be found in 45 such gulab jamuns, each shaped like a cylinder with two hemispherical ends, if the complete length of each of them is 5 cm and its diameter is 2.8 cm. **[CBSE 2008, SP 2011]**

Sol. Let r be the radius of each hemispherical ends.

Then, $\quad r = \frac{2.8}{2}$ cm

$= 1.4$ cm



Let h be the length of the cylindrical part of the gulab jamun.

Then, the total length of the gulab jamun

$$= r + r + h$$
$$= 2r + h$$

$\therefore \quad 2r + h = 5$ cm [Given]

$\Rightarrow \quad 2 \times 1.4$ cm $+ h = 5$ cm

$\Rightarrow \quad h = (5 - 2.8)$ cm

$= 2.2$ cm

Hence, the height of the cylinder is 2.2 cm.

$\therefore$ Volume of the each gulab jamun = Volume of the cylinder + Sum of the volumes of two hemispheres

$$= \pi r^2 h + \frac{2}{3}\pi r^3 \times 2$$

$$= \pi r^2 h + \frac{4}{3}\pi r^3$$

$$= \frac{22}{7} \times [(1.4)^2 \times 2.2 + \frac{4}{3} \times (1.4)^3] \text{ cm}^3$$

$\therefore$ Volumes of 45 gulab jamuns

$$= 45 \times \frac{22}{7} \times 1.4 \times 1.4 \times (2.2 + \frac{4 \times 1.4}{3}) \text{ cm}^3$$

$$= 45 \times \frac{22}{7} \times \frac{14 \times 14}{100} \times \frac{6.6 + 5.6}{3} \text{ cm}^3$$

$$= 45 \times \frac{22}{7} \times \frac{14 \times 14}{100} \times \frac{12.2}{3} \text{ cm}^3$$

$$= \frac{15 \times 22 \times 2 \times 14 \times 122}{1000} \text{ cm}^3$$

$$= \frac{1127280}{1000} \text{ cm}^3$$

$$= 1127.28 \text{ cm}^3$$

$\therefore$ Volume of syrup = 30% of 1127.28 cm^3

$$= \frac{1127.28 \times 30}{100} \text{ cm}^3$$

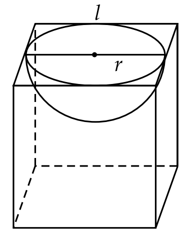
$$= 338.18 \text{ cm}^3$$

$$= 338 \text{ cm}^3 \text{ (approx.)}$$

Hence, the required volume of the syrup is 338 cm^3 (approx.)

8. A hemispherical depression is cut out from one face of a cubical wooden block such that the diameter l of the hemisphere is equal to the edge of the cube. Determine the surface area of the remaining solid. **[CBSE 2012, SP 2011]**

Sol. Let r be the radius of the base of the hemisphere. Then $2r = l$ = edge of the cube.



After the hemispherical depression is cut out, the surface area of the remaining solid

= Total surface area of the cubical block – Area of the base of the hemisphere + Curved surface area of the hemispherical depression

$$= 6l^2 - \pi r^2 + 2\pi r^2$$

$$= 6l^2 + \pi r^2 = 6l^2 + \frac{\pi l^2}{4}$$

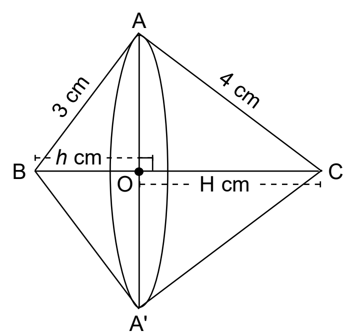
$$= \frac{l^2}{4}(24 + \pi)$$

Hence, the required surface area of the remaining solid is $\frac{l^2}{4}(24 + \pi)$ sq units.

Long Answer Type Questions

9. A right triangle with sides 3 cm and 4 cm is revolved around its hypotenuse. Find the volume of the double cone thus generated. **[CBSE SP 2011]**

Sol. Let ΔBAC be a right-angled triangle with AB = 3 cm, AC = 4 cm and $\angle$BAC = 90°.



$\therefore$ From ΔBAC, by using Pythagoras' Theorem, we have

$$BC^2 = AB^2 + AC^2$$
$$= (3 \text{ cm})^2 + (4 \text{ cm})^2 = 25 \text{ cm}^2$$

$\therefore \quad$ BC = 5 cm

$\therefore$ Length of the hypotenuse is 5 cm.

This triangle ABC is now revolved around its hypotenuse BC. As a result, two cones ABA′OA and ACA′OA with O as the centre of the common base circle of the two cones. Here, O lies on BC and AOA′ ⊥ BC. Let the common radius of the base of two cones be r and the heights BO and CO of the two cones be h and H respectively.

$\therefore \quad h + H = BC = 5$

$\Rightarrow \quad H = 5 - h \quad \ldots(1)$

∴ From ΔABO and ΔACO, by using Pythagoras' Theorem, we have

$$3^2 = h^2 + r^2$$

$\Rightarrow \quad h^2 + r^2 = 9 \quad \ldots(2)$

and $\quad 4^2 = H^2 + r^2$

$\Rightarrow \quad H^2 + r^2 = 16 \quad \ldots(3)$

From (1) and (3), we get

$$(5 - h)^2 + r^2 = 16$$

$\Rightarrow \quad 25 - 10h + h^2 + r^2 - 16 = 0$

$\Rightarrow \quad 9 - 10h + 9 = 0 \quad$ [From (2)]

$\Rightarrow \quad h = \frac{18}{10} = \frac{9}{5} \quad \ldots(4)$

∴ From (2), $\quad r = \sqrt{9 - h^2}$

$$= \sqrt{9 - \frac{81}{25}} \quad \text{[From (4)]}$$

$$= \sqrt{\frac{225 - 81}{25}}$$

$$= \frac{12}{5} \quad \ldots(5)$$

From (1) and (4),

$$H = 5 - \frac{9}{5} = \frac{16}{5} \quad \ldots(6)$$

Now, volume of the cone ABA′OA

$$= \frac{1}{3}\pi r^2 h$$

$$= \frac{\pi}{3} \times \frac{144}{25} \times \frac{9}{5} \text{ cm}^3$$

[From (4) and (5)]

and volume of the cone ACA′OA

$$= \frac{1}{3}\pi r^2 H$$

$$= \frac{\pi}{3} \times \frac{144}{25} \times \frac{16}{5} \text{ cm}^3$$

[From (5) and (6)]

∴ The required sum of the volumes of the above two cones

$$= \frac{\pi}{3} \times \left[\frac{144 \times 9}{125} + \frac{144 \times 16}{125}\right] \text{ cm}^3$$

$$= \frac{22}{21} \times \frac{144}{125} \times 25 \text{ cm}^3$$

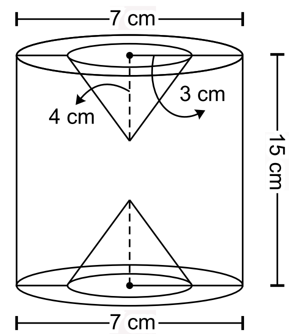
$$= \frac{22 \times 48}{35} \text{ cm}^3$$

$$= \frac{1056}{35} \text{ cm}^3$$

10. The height of a solid cylinder is 15 cm and its diameter is 7 cm. Two equal conical holes, each of radius 3 cm and height 4 cm, are cut off. Find the surface area of the solid. **[CBSE 2015]**

Sol. Let r be the radius of the base of the cylinder and h be its height. Also, let the radius of the base of each cone at the two ends of the solid cylinder be r_1 and the vertical height be h_1. If l_1 be the slant height of each cone, then

$$l_1 = \sqrt{r_1^2 + h_1^2}$$



Here, $r = \frac{7}{2}$ cm, $h = 15$ cm, $r_1 = 3$ cm, $h_1 = 4$ cm

and $l_1 = \sqrt{3^2 + 4^2} = 5$ cm

Now, the total surface area of the cylinder

$$= 2\pi rh + 2\pi r^2$$

$$= 2\pi r(r + h)$$

$$= 2\pi \times \frac{7}{2} \times \left(\frac{7}{2} + 15\right) \text{ cm}^2$$

$$= \frac{259}{2}\pi \text{ cm}^2 \quad \ldots(1)$$

Sum of the areas of the circular bases of two cones

$$= 2\pi r_1^2$$

$$= 2\pi \times 9 \text{ cm}^2$$

$$= 18\pi \text{ cm}^2 \quad \ldots(2)$$

Sum of the curved surface areas of the two cones

$$= 2\pi r_1 l_1$$

$$= 2\pi \times 3 \times 5 \text{ cm}^2$$

$$= 30\pi \text{ cm}^2 \quad \ldots(3)$$

$\therefore$ Surface area of the remaining solid

= Total surface area of the solid cylinder – Sum of the areas of the circular bases of two cones + Sum of the curved surface areas of the two cones

$$= \left(\frac{259}{2} - 18 + 30\right)\pi \text{ cm}^2$$

[From (1), (2) and (3)]

$$= \frac{22}{7} \times \frac{259 + 60 - 36}{2} \text{ cm}^2$$

$$= \frac{11 \times 283}{7} \text{ cm}^2$$

$$= \frac{3113}{7} \text{ cm}^2$$

$$= 444.7 \text{ cm}^2 \text{ (approx.)}$$

Hence, the required surface area of the solid is 444.7 cm^2 (approx.)

Milestone 2

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Multiple-Choice Questions

1. The maximum number of boxes of dimensions 9 cm × 6 cm × 5 cm that can be fitted in a box of dimension 9 m × 6 m × 5 m is

(*a*) 10^5 (*b*) 10^6

(*c*) 10^7 (*d*) 10^4

Sol. (*b*) 10^6

The volume of the bigger box = $9 \times 6 \times 5 \times 10^6$ cm^3

The volume of each smaller box = $9 \times 6 \times 5$ cm^3

$\therefore$ Required number of smaller boxes

$$= \frac{9 \times 6 \times 5 \times 10^6}{9 \times 6 \times 5} = 10^6$$

2. If the curved surface area of a solid cylinder is one-third of its total surface area and if the radius of the cylinder is 4 cm, then the volume of the cylinder is

(*a*) 20π cm^3 (*b*) 24π cm^3

(*c*) 16π cm^3 (*d*) 32π cm^3

Sol. (*d*) 32π cm^3

Let r be the radius of the base of the cylinder and h be its height. Then $r = 4$ cm.

$\therefore$ According to the problem, we have

$$2\pi rh = \frac{1}{3}\left(2\pi rh + 2\pi r^2\right)$$

$$\Rightarrow \quad 3h = h + r$$

$$\Rightarrow \quad 2h = r = 4 \text{ cm}$$

$$\therefore \quad h = \frac{4}{2} \text{ cm} = 2 \text{ cm}$$

Volume of the cylinder = $\pi r^2 h$

$$= \pi \times 16 \times 2 \text{ cm}^3$$

$$= 32\pi \text{ cm}^3$$

Hence, the required volume of the cylinder is 32π cm^3.

Very Short Answer Type Questions

3. The dimensions of a metallic cuboid are 125 cm × 64 cm × 27 cm. It is melted and recast into a cube. Find the surface area of this cube.

Sol. Let a be the side of the cube.

Then the volume of the cuboid

= Volume of the cube.

$$\therefore \quad 125 \times 64 \times 27 \text{ cm}^3 = a^3$$

$$\Rightarrow \quad a^3 = 5^3 \times 4^3 \times 3^3 \text{ cm}^3$$

$$\Rightarrow \quad a = 5 \times 4 \times 3 \text{ cm}$$

$$= 60 \text{ cm}$$

$\therefore$ Surface area of the cube

$$= 6a^2$$

$$= 6 \times 60^2 \text{ cm}^2$$

$$= 21600 \text{ cm}^2$$

Hence, the required surface area of the cube is 21600 cm^2.

4. Find the number of coins of 1.5 cm diameter and 0.2 cm thickness to be melted to form a right circular cylinder of height 10 cm and diameter 4.5 cm. **[CBSE SP 2015]**

Sol. Radius of each coin = $\frac{1.5}{2}$ cm = 0.75 cm

and thickness of each coin = 0.2 cm

$\therefore$ Volume of each cylindrical coin

$$= \pi \times (0.75)^2 \times 0.2 \text{ cm}^3$$

Radius of the base of the cylinder

$$= \frac{4.5}{2} \text{ cm}$$

$$= 2.25 \text{ cm}$$

and the height of the cylinder = 10 cm

$\therefore$ Volume of the cylinder = $\pi \times (2.25)^2 \times 10$ cm^3

$$\text{Number of coins} = \frac{\pi \times (2.25)^2 \times 10}{\pi \times (0.75)^2 \times 0.2}$$

$$= \frac{225 \times 225 \times 100}{75 \times 75 \times 2} = 450$$

Hence, the required number of coins is 450.

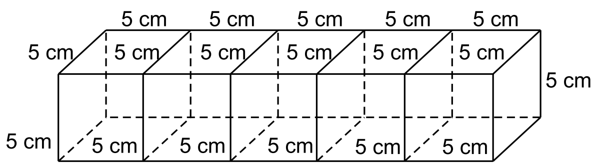
Short Answer Type-I Questions

5. Five cubes of each of side 5 cm are joined end to end. Find the surface area and the volume of the resulting cuboid.

Sol. Length of the cuboid = 5 cm × 5 = 25 cm

Breadth of the cuboid = 5 cm

Height of the cuboid = 5 cm



$\therefore$ Volume of the cuboid = $25 \times 5 \times 5 \text{ cm}^3$

$= 625 \text{ cm}^3$

Surface area of the cuboid

$= 2 \times (25 \times 5 + 25 \times 5 + 5 \times 5) \text{ cm}^2$

$= 2 \times 275 \text{ cm}^2 = 550 \text{ cm}^2$

Hence, the required surface area and volume of the cuboid is 550 cm^2 and 625 cm^3 respectively.

6. How many silver coins, 1.75 cm in diameter and thickness 2 mm must be melted to form a cuboid of dimension 5.5 cm × 10 cm × 3.5 cm? **[CBSE SP 2011]**

Sol. Radius of each silver coin = $\frac{1.75}{2}$ cm = $\frac{7}{8}$ cm

Thickness of the coin = 2 mm = $\frac{1}{5}$ cm

$\therefore$ Volume of each cylindrical silver coin

$$= \pi \times \left(\frac{7}{8}\right)^2 \times \frac{1}{5} \text{ cm}^3$$

$$= \frac{22}{7} \times \frac{49}{64} \times \frac{1}{5} \text{ cm}^3$$

$$= \frac{11 \times 7}{32 \times 5} \text{ cm}^3$$

$$= \frac{77}{160} \text{ cm}^3$$

Also, volume of the cuboid

$$= 5.5 \times 10 \times 3.5 \text{ cm}^3$$

$$= 55 \times \frac{35}{10} \text{ cm}^3$$

$$= \frac{11 \times 35}{2} \text{ cm}^3$$

$$= \frac{385}{2} \text{ cm}^3$$

$\therefore$ Number of coins = $\frac{385}{2} \times \frac{160}{77} = 5 \times 80 = 400$

Hence, the required number of silver coins is 400.

Short Answer Type-II Questions

7. A farmer connects a pipe of internal diameter 20 cm from a canal into a cylindrical tank in his field, which is 10 m in diameter and 2 m deep. If the water flows through the pipe at the rate of 6 km/h, in how much time will the tank be filled completely? **[CBSE 2008, SP 2011]**

Sol. 6 km/h = $\frac{600000}{60}$ cm/minute = 10000 cm/minute

Radius of the cylindrical pipe = $\frac{20}{2}$ cm = 10 cm

Hence, the volume of water flowing through the pipe in 1 minute

$= \pi \times 10^2 \times 10000 \text{ cm}^3$

$= \pi \times 1000000 \text{ cm}^3$

Now, the radius of the cylindrical tank

$= \frac{10}{2}$ m = 5 m

= 500 cm

Height of the cylindrical tank = 2 m = 200 cm

Hence, the volume of the tank

$= \pi \times 500^2 \times 200 \text{ cm}^3$

$= \pi \times 50000000 \text{ cm}^3$

Hence, the required time = $\frac{\pi \times 50000000}{\pi \times 1000000}$ minutes

= 50 minutes

8. If the diameter of the cross section of a wire is decreased by 5%, how much per cent will the length be increased so that the volume remains the same?

Sol. Let the original radius of the wire be l and the length of the wire be l. Now, the diameter $2r$ is decreased by 5%

$\therefore$ Decrease in diameter = $2r \times \frac{5}{100}$ cm = $\frac{r}{10}$ cm.

$\therefore$ Decreased diameter = $\left(2r - \frac{r}{10}\right)$ cm

$= \frac{19r}{10}$ cm

$\therefore$ Decreased radius = $\frac{19r}{20}$ cm.

Hence, the new radius is $\frac{19r}{20}$ cm.

Let the increased length of the wire be L.

Then the volume of the original wire = $\pi r^2 l$

and the volume of the new wire = $\pi\left(\frac{19r}{20}\right)^2$ L.

Since, these two volumes are the same,

$$\therefore \qquad \pi r^2 l = \pi\left(\frac{19r}{20}\right)^2 L$$

$$= \pi r^2\left(\frac{19}{20}\right)^2 L$$

$$\therefore \qquad l = \left(\frac{19}{20}\right)^2 L$$

$$\Rightarrow \qquad L = \left(\frac{20}{19}\right)^2 l$$

$\therefore$ Increase in length of the wire = $L - l$

$$= \left(\frac{20^2}{19^2} - 1\right) l$$

$$= \frac{39}{19 \times 19} l$$

$$= \frac{39}{361} l$$

$\therefore$ Percentage increase in length

$$= \frac{39}{361} \times l \times \frac{1}{l} \times 100\%$$

$$= \frac{3900}{361}\%$$

$$= 10.8\% \text{ (approx.)}$$

Hence, the required increase in length in percentage is 10.8% (approx.)

Long Answer Type Questions

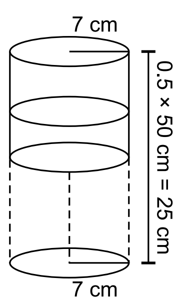
9. 50 circular discs, each of radius 7 cm and thickness 0.5 cm are placed one above the other. Find the total surface area of the solid so formed. Find how much space will be left in a cubical box of side 25 cm, if the solid formed is placed inside it. **[CBSE SP 2016]**

Sol. The curved surface area of each circular disc of thickness 0.5 cm $= 2\pi \times 7 \times 0.5 \text{ cm}^2$

$$= 2 \times \frac{22}{7} \times 7 \times 0.5 \text{ cm}^2$$

$$= 22 \text{ cm}^2$$

$\therefore$ The sum of curved surface areas of 50 circular discs = $22 \times 50 \text{ cm}^2 = 1100 \text{ cm}^2$.



It these circular discs are placed one above the other, then they will form a solid cylinder of two additional circular discs at the top and bottom of the cylinder.

The sum of the areas of these two circular discs

$$= 2\pi \times 7^2 \text{ cm}^2$$

$$= 2 \times \frac{22}{7} \times 49 \text{ cm}^2 = 308 \text{ cm}^2$$

$\therefore$ Total surface area of the solid cylinder thus formed = $1100 \text{ cm}^2 + 308 \text{ cm}^2 = 1408 \text{ cm}^2$.

Now, the volume of the cylinder thus formed

$$= \pi \times 7^2 \times 25 \text{ cm}^3$$

$$= \frac{22}{7} \times 49 \times 25 \text{ cm}^3$$

$$= 3850 \text{ cm}^3$$

Volume of the cube = 25^3 cm^3

$$= 625 \times 25 \text{ cm}^3$$

$$= 15625 \text{ cm}$$

$\therefore$ Volume of the remaing space

$$= (15625 - 3850) \text{ cm}^3$$

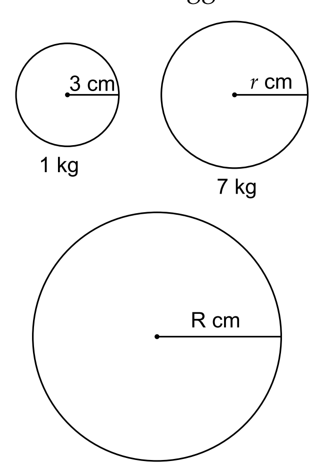
$$= 11775 \text{ cm}^3$$

Hence, the required surface area of the solid cylinder is 1408 cm^2 and the volume of the remaining space left in cubical box is 11775 cm^3.

10. Two spheres of the same metal weigh 1 kg and 7 kg. The radius of the smaller sphere is 3 cm. The two spheres are melted to form a single big sphere. Find the diameter of the new sphere.

Also, find the ratio of the total surface area of the hemisphere formed from the biggest new sphere and the total surface area of the entire smallest sphere.

Sol. The radius of the sphere of weight 1 kg = 3 cm. Let the radius of the sphere of weight 7 kg be r and the radius of the biggest new sphere be R.



If V_1 and V_2 be the volumes of the spheres of weights 1 kg and 7 kg respectively, then

$$V_1 = \frac{4}{3}\pi \times 3^3 \text{ cm}^3$$

$$= 36\pi \text{ cm}^3 \quad \ldots(1)$$

and $$V_2 = \frac{4}{3}\pi r^3 \text{ cm}^3 \quad \ldots(2)$$

If σ kg/cm^3 be the density of the solid metal, then

$$V_1\sigma = 1$$

$$\Rightarrow \quad 36\pi\sigma = 1$$

$$\therefore \quad \sigma = \frac{1}{36\pi}$$

Also, $$V_2\sigma = 7$$

$$\Rightarrow \quad \frac{4}{3}\pi r^3 \times \frac{1}{36\pi} = 7$$

$$\Rightarrow \quad r^3 = 27 \times 7 = 189 \quad \ldots(3)$$

Now, volume of the biggest sphere = sum of the volumes of the two smaller spheres

$$\therefore \quad \frac{4}{3}\pi R^3 = \frac{4}{3}\pi (27 + r^3)$$

[From (1) and (2)]

$$\Rightarrow \quad R^3 = 27 + r^3 = 27 + 189 \text{ [From (3)]}$$

$$= 216 = 6^3$$

$$\therefore \quad R = 6 \text{ cm}$$

Hence, the radius of the biggest new sphere is 6 cm.

$\therefore$ Required diameter of the new sphere

$$= 6 \times 2 \text{ cm}$$

$$= 12 \text{ cm}$$

Now, the total surface area of the biggest hemisphere of radius R

$$= 2\pi R^2 + \pi R^2$$

$$= 3\pi R^2$$

$$= 3\pi \times 36 \text{ cm}^2 = 108\pi \text{ cm}^2$$

$\therefore$ Required ratio = $108\pi : 36\pi = 3 : 1$

Milestone 3

(Page 235)

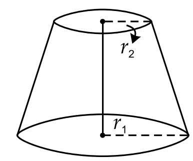
Multiple-Choice Questions

1. The slant height of a frustum of a cone is 5 cm and the perimeters of its circular ends are 20 cm and 8 cm. Then the curved surface area of the frustum is

(*a*) 48 cm^2 (*b*) 70 cm^2
(*c*) 96 cm^2 (*d*) 45 cm^2

Sol. (*b*) 70 cm^2

Let r_1 and r_2 be the radii of the circular ends of the frustum of the cone, where $r_1 > r_2$. Given, perimeters of circular ends are 20 cm and 8 cm. Also, slant height l = 5 cm.



$$\therefore \quad 2\pi r_1 = 20 \text{ cm}$$

and $$2\pi r_2 = 8 \text{ cm}$$

$$\therefore \quad r_1 = \frac{10}{\pi} \quad \ldots(1)$$

and $$r_2 = \frac{4}{\pi} \quad \ldots(2)$$

Then the curved surface area of the frustum

$$= \pi(r_1 + r_2)l$$

$$= \pi \times \frac{14}{\pi} \times 5 \text{ cm}^2$$

[From (1) and (2)]

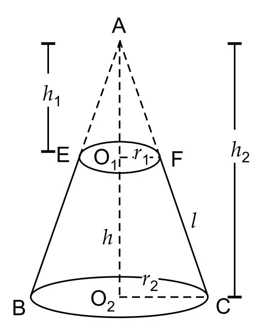
$$= 70 \text{ cm}^2$$

2. A cone is cut into two parts by a horizontal plane passing through the mid-point of its axis. Then the ratio of the volumes of the upper part and the lower frustum of the whole cone is

(*a*) 1 : 7 (*b*) 7 : 1
(*c*) 1 : 8 (*d*) 8 : 1

Sol. (*a*) 1 : 7

Let the cone ABC with axis AO_1O_2 be cut into two parts by a horizontal plane EF such that $AO_1 = O_1O_2$.



Let the radii of the two circles with centres at O_1 and O_2 be r_1 and r_2 respectively and let h_1 and h_2 be the heights of two cones AEF and ABC, so that $O_1F = r_1$, $O_2C = r_2$, $AO_1 = h_1$ and $AO_2 = h_2$. It V_1 and V_2 be the volumes of these two cones respectively, then $V_1 = \frac{1}{3}\pi r_1^2 h_1$ and $V_2 = \frac{1}{3}\pi r_2^2 h_2$

Now, $\dfrac{r_1}{r_2} = \dfrac{h_1}{h_2} = \dfrac{1}{2}$

$\therefore \quad r_2 = 2r_1$ and $h_2 = 2h_1$

$$\therefore \qquad V_2 = \frac{\pi}{3} \times 4r_1^2 \times 2h_1$$

$$= \frac{8}{3}\pi r_1^2 h_1$$

$\therefore$ Volume of the frustum of the cone, EBCF is

$$V_2 - V_1 = \left(\frac{8\pi}{3} - \frac{\pi}{3}\right) r_1^2 h_1$$

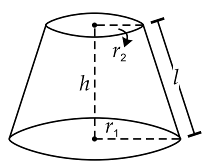
$$= \frac{7\pi}{3} r_1^2 h_1$$

$$\therefore \quad V_1 : (V_2 - V_1) = \frac{\pi}{3} r_1^2 h_1 : \frac{7\pi}{3} r_1^2 h_1 = 1 : 7$$

Very Short Answer Type Questions

3. The perimeters of the ends of a frustum of a right circular cone are 16π metres and 10π metres. If its slant height is 5 metres, what is the vertical height of the frustum?

Sol. Let r_1 and r_2 be the radii of the two circular ends of the frustum of the cone, where $r_1 > r_2$.



$$\therefore \qquad 2\pi r_1 = 16\pi \text{ m}$$

$$\Rightarrow \qquad r_1 = 8 \text{ m}$$

and $\qquad 2\pi r_2 = 10\pi$ m

$$\Rightarrow \qquad r_2 = 5 \text{ m}$$

Let h be the vertical height and l be the slant height of the frustum. Then $l = 5$ m

Also, $\qquad l = \sqrt{h^2 + (r_1 - r_2)^2}$

$$\Rightarrow \qquad 5 = \sqrt{h^2 + (8-5)^2}$$

$$\Rightarrow \qquad 25 = h^2 + 9$$

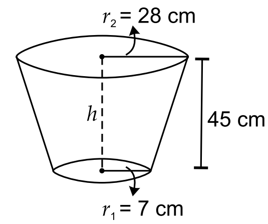
$$\Rightarrow \qquad h^2 = 16$$

$$\Rightarrow \qquad h = 4$$

Hence, the required vertical height is 4 m.

4. If the radii of the circular ends of a conical bucket which is 45 cm high are 28 cm and 7 cm, find the capacity of the bucket. **[CBSE SP 2011]**

Sol. Let r_1 and r_2 be the radii of the circular ends of the frustum, where $r_2 > r_1$.



Then $r_2 = 28$ cm, $r_1 = 7$ cm. If h be the vertical height of the frustum, then $h = 45$ cm.

$\therefore$ Capacity (or volume) of the bucket

$$= \frac{1}{3}\pi\left(r_1^2 + r_1 r_2 + r_2^2\right)h$$

$$= \frac{22}{7 \times 3} \times (7^2 + 7 \times 28 + 28^2) \times 45 \text{ cm}^3$$

$$= \frac{22}{21} \times 1029 \times 45 \text{ cm}^3$$

$$= 22 \times 49 \times 45 \text{ cm}^3$$

$$= 48510 \text{ cm}^3$$

Hence, the required capacity of the bucket is 48510 cm^3.

Short Answer Type-I Questions

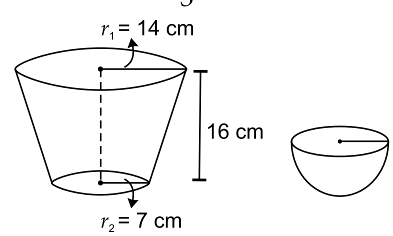
5. The internal radii of the ends of a bucket, full of milk and internal height 16 cm are 14 cm and 7 cm. If this milk is poured into a hemispherical vessel, the vessel is completely filled. Find the internal diameter of the spherical vessel. **[CBSE SP 2006, 2012]**

Sol. Volume of the bucket in the shape of a frustum of a cone

$$= \frac{1}{3}\pi\left(r_1^2 + r_2^2 + r_1 r_2\right)h$$

$$= \frac{1}{3}\pi(14^2 + 7^2 + 14 \times 7) \times 16 \text{ cm}^3$$

$$= \frac{16 \times 343\pi}{3} \text{ cm}^3$$



Also, volume of the hemispherical vessel $= \dfrac{2}{3}\pi r^3$.

$$\therefore \qquad \frac{2}{3}\pi r^3 = \frac{16 \times 343\pi}{3} \text{ cm}^3$$

$$\Rightarrow \qquad r^3 = 343 \times 8 \text{ cm}^3$$

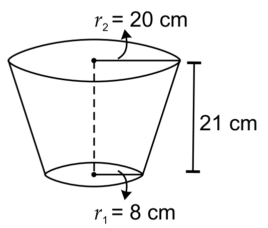
$$= 7^3 \times 2^3 \text{ cm}^3$$

$\therefore \quad r = 7 \times 2 \text{ cm} = 14 \text{ cm}$

Hence, the required diameter of the hemispherical vessel is 14 × 2 cm, i.e. 28 cm.

6. A metal container, open from the top, is in the shape of a frustum of a cone of height 21 cm with radii of its lower and upper circular ends as 8 cm and 20 cm respectively. Find the cost of the milk which can completely fill the container, at the rate of ₹35 per litre. **[CBSE 2016]**

Sol. Let r_1 and r_2 be the radii of the circular ends of the frustum of the cone, where $r_2 > r_1$ and let h be the vertical height of the frustum.



Here, $r_2 = 20$ cm, $r_1 = 8$ cm and $h = 21$ cm.

$\therefore$ Volume of the frustum

$$= \frac{1}{3}\pi\left(r_1^2 + r_2^2 + r_1 r_2\right)h$$

$$= \frac{22}{7} \times \frac{1}{3} \times (8^2 + 20^2 + 8 \times 20) \times 21 \text{ cm}^3$$

$$= 22 \times (64 + 400 + 160) \text{ cm}^3$$

$$= 13728 \text{ cm}^3$$

$$= 13.728 \text{ litres}$$

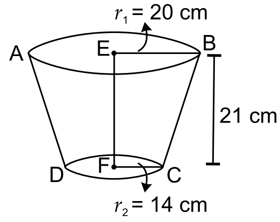
$\therefore$ Cost of the milk = ₹ 35 × 13.728 = ₹ 480.48

Hence, cost of the milk is ₹ 480.48.

Short Answer Type-II Questions

7. A bucket is 40 cm in diameter at the top and 28 cm in diameter at the bottom. Find the capacity of the bucket in litres, if it is 21 cm deep. Also, find the cost of tin sheet used in making the bucket, if the cost of tin is ₹1.50 per sq dm.

Sol. Let ABCD be the bucket. E and F, the centres of the circular ends at the top and bottom respectively are of radii r_1 and r_2 respectively. Let l be the slant height and h be the vertical height of the bucket.



Then, $r_1 = 20$ cm, $r_2 = 14$ cm and $h = 21$ cm.

Now, $\quad l = \sqrt{h^2 + (r_1 - r_2)^2}$ cm

$$= \sqrt{21^2 + 36} \text{ cm}$$

$$= \sqrt{477} \text{ cm}$$

$$= 21.84 \text{ cm (approx.)}$$

Now, the required volume of the bucket

$$= \frac{\pi}{3}\left(r_1^2 + r_2^2 + r_1 r_2\right)h$$

$$= \frac{22}{21} \times (20^2 + 14^2 + 20 \times 14) \times 21 \text{ cm}^3$$

$$= 22 \times 876 \text{ cm}^3$$

$$= 19272 \text{ cm}^3$$

$$= 19.272 \text{ litres}$$

Again, the curved surface area of the bucket

$$= \pi(r_1 + r_2)l$$

$$= \frac{22}{7} \times 34 \times 21.84 \text{ cm}^2$$

$$= 2333.76 \text{ cm}^2$$

$$= 23.3376 \text{ dm}^2$$

Also, the area of the base of the bucket

$$= \pi r_2^2$$

$$= \frac{22}{7} \times 14 \times 14 \text{ cm}^2$$

$$= 616 \text{ cm}^2$$

$$= 6.16 \text{ dm}^2$$

$\therefore$ Total surface area of the bucket (which is open at the top)

$$= (23.3376 + 6.16) \text{ dm}^2$$

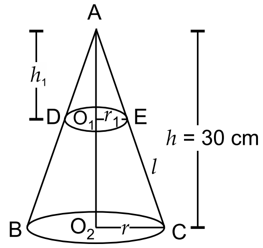
$$= 29.50 \text{ dm}^2$$

Hence, the required cost of the tin sheet is ₹ 1.50 × 29.50 i.e. ₹ 44.25 (approx.).

8. The height of a cone is 30 cm. A small cone is cut off at the top by a plane parallel to the base. If the volume is $\frac{1}{27}$ of the volume of the given cone, at what height above the base is the section made? **[CBSE 2017]**

Sol. Let r, h and V be the radius of the base, vertical height and the volume respectively of the entire cone ABC and let r_1, h_1 and V_1 be the radius of the base, vertical height and the volume respectively of the smaller cone ADE which is cut off.

Then, $h = 30$ cm.



Let O_1 and O_2 be the centres of the circular bases of the smaller and bigger cones respectively.

Then from similar triangles AO_1E and AO_2C, we have

$$\frac{r_1}{r} = \frac{h_1}{h} = \frac{h_1}{30}$$

$$\therefore \quad V_1 = \frac{rh_1}{30} \qquad \ldots(1)$$

$$V_1 = \frac{\pi}{3} r_1^2 h_1$$

and $$V = \frac{\pi}{3} r^2 h = 10\pi r^2 \qquad \ldots(2)$$

Given that $$V_1 = \frac{V}{27}$$

$$\therefore \quad \frac{\pi}{3} r_1^2 h_1 = \frac{1}{27} \times 10\pi r^2 \qquad \text{[From (2)]}$$

$$\Rightarrow \quad 9r_1^2 h_1 = 10r^2$$

$$\Rightarrow \quad 9 \times \frac{r^2 h_1^2}{900} \times h_1 = 10r^2 \qquad \text{[From (1)]}$$

$$\Rightarrow \quad h_1^3 = 1000$$

$$\Rightarrow \quad h_1 = 10$$

Hence, $h - h_1 = O_1O_2 = (30 - 10)$ cm = 20 cm.

Hence, the required height above the base of the bigger cone, where the smaller cone was cut off is 20 cm.

Long Answer Type Questions

9. If the height of a frustum of a right circular cone is twice the mean proportional between the radii of its bases, show that the slant height of the frustum is equal to the sum of their radii.

Sol. Let h be the vertical height, r_1 and r_2 be the radii of the circular bases and l be the slant height of the frustum of the cone. It is given that $h = 2\sqrt{r_1 r_2}$

$$\Rightarrow \quad h^2 = 4r_1 r_2 \qquad \ldots(1)$$

Now, $$l^2 = h^2 + (r_1 - r_2)^2$$

$$= h^2 + (r_1 + r_2)^2 - 4r_1 r_2$$

$$= 4r_1 r_2 + (r_1 + r_2)^2 - 4r_1 r_2$$

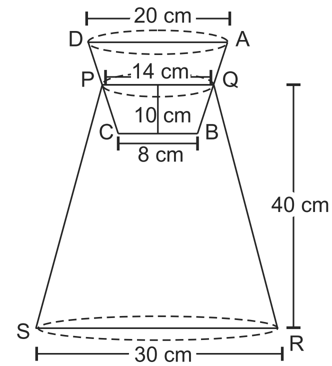
[From (1)]

$$= (r_1 + r_2)^2$$

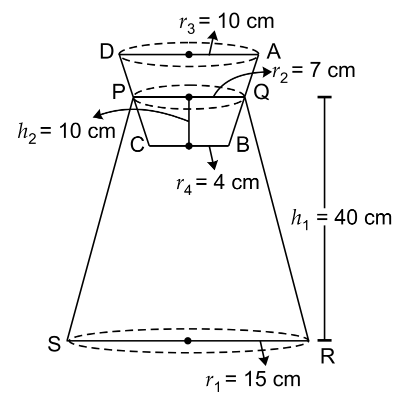
$$\therefore \quad l = r_1 + r_2$$

Hence, proved.

10. The figure given below shows a bottle PQRS closed with a cork ABCD. Find the capacity of the closed bottle. Give your answer in terms of π.



Sol. Let r_1 = 15 cm, r_2 =7 cm, r_3 = 10 cm, r_4 = 4 cm, h_1 = 40 cm and h_2 = 10 cm



Volume of the frustum PQRS

$$= \frac{\pi}{3}\left(r_1^2 + r_2^2 + r_1 r_2\right) \times h_1$$

$$= \frac{\pi}{3}(15^2 + 7^2 + 15 \times 7) \times 40 \text{ cm}^3$$

$$= \frac{\pi}{3}(225 + 49 + 105) \times 40 \text{ cm}^3$$

$$= \frac{\pi}{3} \times 379 \times 40 \text{ cm}^3$$

$$= \frac{15160\pi}{3} \text{ cm}^3$$

Again, volume of the frustum PQBC

$$= \frac{\pi}{3}\left(r_2^2 + r_4^2 + r_2 \times r_4\right) \times h_2$$

$$= \frac{\pi}{3}(49 + 16 + 28) \times 10 \text{ cm}^3$$

$$= \frac{\pi}{3} \times 930 \text{ cm}^3$$

$\therefore$ Volume of the closed bottle

$$= \left(\frac{15160\pi}{3} - \frac{930\pi}{3}\right) \text{cm}^3$$

$$= \frac{14230\pi}{3} \text{cm}^3$$

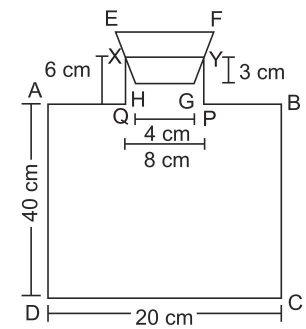
$$= 4743.3\pi \text{ cm}^3$$

Hence, the required capacity of the closed bottle is 4743.3π cm^3.

Higher Order Thinking Skills (HOTS) Questions

(Page 237)

1. A cylindrical glass jar is fitted with a cork as shown in the given figure. The tight fitting cork is in the form of a frustum of a cone. Find the capacity of the closed jar. [Use π = 3.14]



Sol. Let EFGH is the cork in the shape of a frustum of a cone, the portion XYGH of the cork is inserted tightly with a cylindrical glass jar ABCD with a cylindrical narrow mouth.

Let r be the radius of the base of the cylinder ABCD and h be its height.

$\therefore$ r = 10 cm and h = 40 cm.

Let r_1 be the radius of the base of the smaller cylindrical mouth XYPQ and let h_1 be its height.

Then r_1 = 4 cm and h_1 = 6 cm.

Let r_2 and r_3 be the radii of the bases of cork XYGH in the shape of a frustum of a cone and let h_2 be its vertical height.

Then r_2 = 4 cm, r_3 = 2 cm and h_2 = 3 cm.

Now, the volume of the cylinder

$$= \pi r^2 h$$

$$= \pi \times 10^2 \times 40 \text{ cm}^3$$

$$= 4000\pi \text{ cm}^3 \qquad \ldots(1)$$

Volume of the smaller cylindrical mouth XYPQ

$$= \pi \times 4^2 \times 6 \text{ cm}^3$$

$$= 96\pi \text{ cm}^3 \qquad \ldots(2)$$

Volume of the frustum XYGH of the cork

$$= \frac{\pi}{3}(2^2 + 4^2 + 2 \times 4) \times 3 \text{ cm}^3$$

$$= \frac{\pi}{3} \times 28 \times 3 \text{ cm}^3$$

$$= 28\pi \text{ cm}^3 \qquad \ldots(3)$$

$\therefore$ Capacity (or volume) of the closed jar

$$= (4000 + 96 - 28)\pi \text{ cm}^3$$

[From (1), (2) and (3)]

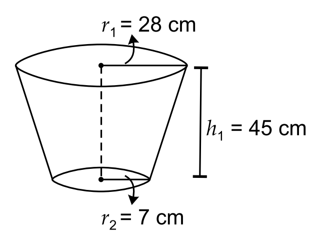
$$= 4068 \times 3.14 \text{ cm}^3$$

$$= 12773.52 \text{ cm}^3$$

Hence, the required capacity of the closed jar is 12773.52 cm^3.

2. The radii of the circular ends of a solid frustum of a cone are 28 cm and 7 cm and its height is 45 cm. This solid is melted and recast into a cylinder and a sphere of the same volume. Find the radii of the cylinder and the sphere, if the height of the cylinder is 15 cm.

Sol. Let r_1 and r_2 be the radii of the circular ends of the frustum where $r_1 > r_2$ and h_1 be the vertical height of the frustum. Then, h_1 = 45 cm, r_1 = 28 cm and r_2 = 7 cm.



$\therefore$ Volume of the frustum,

$$V = \frac{\pi}{3}\left(r_1^2 + r_2^2 + r_1 \times r_2\right)h_1$$

$$= \frac{\pi}{3}(28^2 + 7^2 + 28 \times 7) \times 45 \text{ cm}^3$$

$$= \frac{\pi}{3}(784 + 49 + 196) \times 45 \text{ cm}^3$$

$$= 15 \times 1029\pi \text{ cm}^3$$

$$= 15435\pi \text{ cm}^3 \qquad \ldots(1)$$

Let r_3 be the radius of the base of the cylinder and r_4 be the radius of the sphere. Let h_2 be the height of the cylinder.

Then $\qquad h_2$ = 15 cm

$\therefore$ Volume of the cylinder,

$$V_1 = \pi r_3^2 h_2$$

$$= 15 r_3^2 \pi \text{ cm}^3 \qquad \ldots(2)$$

Volume of the sphere,

$$V_2 = \frac{4}{3}\pi r_4^3 \text{ cm}^3 \qquad \ldots(3)$$

Now, given that

$$V_1 = V_2$$

$$\therefore \quad 15r_3^2 s = \frac{4}{3}\pi r_4^3$$

$$\Rightarrow \quad 4r_4^3 = 45r_3^2 \qquad \ldots(4)$$

[From (2) and (3)]

Also, $V = V_1 + V_2$

$$\Rightarrow \quad 15435\pi = 15r_3^2\pi + \frac{4}{3}\pi r_4^3$$

[From (1), (2) and (3)]

$$\Rightarrow \quad 15435 = 15r_3^2 + \frac{1}{3} \times 45r_3^2 \quad \text{[From (4)]}$$

$$= 30r_3^2$$

$$\therefore \quad r_3^2 = \frac{15435}{30} = 514.5$$

$$\therefore \quad r_3 = \sqrt{514.5} = 22.68 \text{ (approx.)}$$

Hence, the required radius of the base of the cylinder is 22.68 cm (approx.)

Now, $4r_4^3 = 45r_3^2$

$$= 45 \times \frac{5145}{10}$$

$$= 9 \times \frac{5145}{2}$$

$$\Rightarrow \quad r_4^3 = \frac{9 \times 5145}{8}$$

$$= \frac{3 \times 3 \times 3 \times 7 \times 7 \times 7 \times 5}{8}$$

$$= \frac{3^3 \times 7^3 \times 5}{2^3}$$

$$\therefore \quad r^4 = \sqrt[3]{\frac{3^3 \times 7^3 \times 5}{2^3}}$$

$$= \frac{21}{2} \times \sqrt[3]{5}$$

$$= 10.5 \times 1.71$$

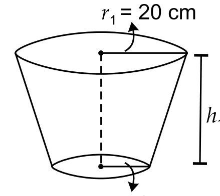
[By log table, $5^{1/3} = 1.71$]

$$= 17.95 \text{ (approx.)}$$

Hence, the required radius of the sphere is 17.95 cm (approx.)

3. A metallic vessel is in the form of a frustum of a cone of height 16 cm. The radii of its lower and upper ends are 8 cm and 20 cm respectively. Find the cost of milk which can completely fill the vessel at the rate of ₹15 per litre and the cost of the metal sheet used, if it costs ₹5 per 100 cm^2. [Use π = 3.14]

Sol. Let r_1 and r_2 be the radii of the circular ends of the frustum of the cone and let h_1 be its vertical height, where $r_1 > r_2$.



Then r_1 = 20 cm, r_2 = 8 cm, h_1 = 16 cm

$\therefore$ Volume of the vessel in the shape of a frustum of a cone

$$= \frac{\pi}{3}\left(r_1^2 + r_2^2 + r_1 r_2\right)h_1$$

$$= \frac{\pi}{3}(20^2 + 8^2 + 20 \times 8) \times 16 \text{ cm}^3$$

$$= \frac{3.14}{3}(400 + 64 + 160) \times 16 \text{ cm}^3$$

$$= \frac{3.14 \times 624 \times 16}{3} \text{ cm}^3$$

$$= 10449.92 \text{ cm}^3$$

$$= 10.4499 \text{ litres}$$

$\therefore$ Cost of the milk

= ₹ 15 × 10.4499

= ₹ 156.75 (approx.)

Let l be the slant height of the frustum of the cone.

Then

$$l = \sqrt{(r_1 - r_2)^2 + h_1^2}$$

$$= \sqrt{12^2 + 16^2} \text{ cm}$$

$$= \sqrt{144 + 256} \text{ cm}$$

$$= \sqrt{400} \text{ cm}$$

$$= 20 \text{ cm}$$

$\therefore$ Curved surface area of the frustum of the cone $= \pi(r_1 + r_2)l$

$$= \pi(20 + 8) \times 20 \text{ cm}^2$$

$$= 3.14 \times 28 \times 20 \text{ cm}^2$$

$$= 3.14 \times 560 \text{ cm}^2$$

Also, area of the circular base of the vessel

$$= \pi r_2^2$$

$$= 3.14 \times 8^2 \text{ cm}^2$$

$$= 3.14 \times 64 \text{ cm}^2$$

$\therefore$ Total surface area of the metal sheet

$$= 3.14 \times (64 + 560)\text{cm}^2$$
$$= 3.14 \times 624 \text{ cm}^2$$
$$= 1959.36 \text{ cm}^2$$

$\therefore$ Cost of the metal sheet

$$= ₹\ \frac{5}{100} \times 1959.36$$
$$= ₹\ \frac{9796.80}{100}$$
$$= ₹\ 97.97 \text{ (approx.)}$$

Hence, the required cost of milk is ₹ 156.75 (approx.) and the cost of the metal sheet is ₹ 97.97 (approx.).

Self-Assessment

(Page 237)

Multiple-Choice Questions

1. If a solid right circular cone of height 28 cm and base radius 7 cm is melted and recast in the shape of a sphere. The radius of the sphere is

(*a*) 8 cm (*b*) 7 cm

(*c*) 14 cm (*d*) 10 cm

Sol. (*b*) 7 cm

Let r be the radius of the base and h be the vertical height of the cone. Then $h = 28$ cm and $r = 7$ cm. Let R be the radius of the sphere.

Then the volume of the cone

$$= \frac{\pi}{3} r^2 h$$
$$= \frac{\pi}{3} \times 7 \times 7 \times 28 \text{ cm}^3$$

According to the problem, we have

$$\frac{\pi}{3} \times 7 \times 7 \times 28 = \frac{4}{3} \pi R^3$$

$\Rightarrow$ $R^3 = 7^3$

$\Rightarrow$ $R = 7$ cm

$\therefore$ The required radius of the sphere is 7 cm.

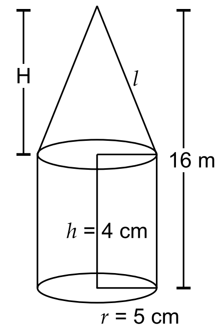
2. A tent is in the shape of a right circular cylinder up to a height of 4 m and conical above it. The total height of the tent is 16 m and radius of the base is 5 m. Then, its total surface area is

(*a*) 100π m^2 (*b*) 110π m^2

(*c*) 105π m^2 (*d*) 210π m^2

Sol. (*c*) 105π m^2

Let r be the radius of the base of the cylinder and h be its height.



Then, $r = 5$ m, $h = 4$ m.

Height of the cone, H = 16 m – 4 m = 12 m

If l be the slant height of the cone, then

$$l = \sqrt{H^2 + r^2}$$
$$= \sqrt{12^2 + 5^2} \text{ m}$$
$$= \sqrt{169} \text{ m}$$
$$= 13 \text{ m}$$

$\therefore$ Surface area of the cone

$$= \pi r l$$
$$= \pi \times 5 \times 13 \text{ m}^2$$
$$= 65\pi \text{ m}^2$$

Curved surface area of the cylinder

$$= 2\pi r h$$
$$= 2\pi \times 5 \times 4 \text{ m}^2$$
$$= 40\pi \text{ m}^2$$

$\therefore$ Total surface area of the cylinder and the cone

$$= (65\pi + 40\pi) \text{ m}^2$$
$$= 105\pi \text{ m}^2$$

Hence, the required total surface area is 105π m^2.

Fill in the Blanks

3. The edge of a cube whose volume is $8x^3$ is $\mathbf{2x}$.

Sol. $a^3 = (2x)^3 = 8x^3$

$\Rightarrow$ $a = 2x$

4. Total surface area of a cube is 216 cm^2, its volume is **216 cm^3**.

Sol. $6a^2 = 216$ cm^2

$\Rightarrow$ $a = 6$ cm

$\Rightarrow$ $V = a^3 = (6)^3 = 216$ cm^3

5. If the surface area of a sphere is 144π, then its radius is **6 cm**.

Sol. $4\pi r^2 = 144\,\pi$

$\Rightarrow$ $r^2 = \frac{144}{4} = 36$

$\Rightarrow$ $r = 6$ cm

6. The curved surface area of one cone is twice that of the other cone. If the slant height of the latter is twice that of the former, then the ratio of their radii is **4 : 1**.

Sol. $\pi r_1 l_1 = 2\ (\pi r_2 l_2)$

$\Rightarrow \quad \pi r_1 l_1 = 2\ (\pi r_2 \times 2l_1)$

$\Rightarrow \quad \dfrac{r_1}{r_2} = \dfrac{4}{1} = 4 : 1$

Assertion-Reason Type Questions

Directions (Q. Nos. 7 to 9): Each of these questions contains an assertion followed by reason. Read them carefully, and answer the question on the basis of the following options, select the one that best describes the two statements.

(a) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.

(b) Both Assertion and Reason are true but Reason is not the correct explanation of Assertion.

(c) If Assertion is true but Reason is false.

(d) If Assertion is false but Reason is true.

7. **Assertion:** A cuboid and a cube of volume 10 units and 15 units are glued together, the new volume would be 25 units.

Reason: Volume of combined solids is equal to the sum of individual solids.

Sol. Volume of combined solids is equal to sum of individual solids. Thus, the combined volume will be 10 + 15 = 25 units. Thus reason is a correct explanation of the assertion.

The answer is (*a*).

8. **Assertion:** When two cubes are joined together by glue, the total surface area increases.

Reason: The number of faces decreases.

Sol. When two cubes are glued, two of the faces are reduced, so total surface area will be less. Thus, assertion is wrong but reason is correct.

The answer is (*d*).

9. **Assertion:** A medicine capsule can be considered as combination of a cylinder bounded by two hemispheres. The total surface area of the capsule will be the exact sum of the individual surface areas.

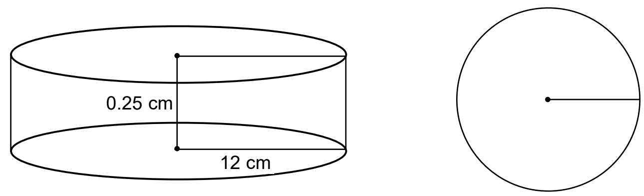
Reason: Joined faces will not contribute to the total surface area.

Sol. When two solids are joined, two of the faces are reduced, so total surface area will be less. Thus, assertion is wrong but reason is correct.

The answer is (*d*).

Case Study Based Questions

10. A father gifted a clay set to his son to indulge him in some different activities. This will help him to stay away from electronics and develop his creative skills. He takes out one colour of clay and started making right circular cylinder. He then converted this into a sphere. The base radius and height of right circular cylinder is 12 cm and 0.25 cm respectively. Based on the above information, answer the following questions.



(*a*) What is the volume of the right circular cylinder?

(*i*) $12\ \pi\ cm^3$ (*ii*) $24\ \pi\ cm^3$

(*iii*) $36\ \pi\ cm^3$ (*iv*) $48\ \pi\ cm^3$

Ans. (*iii*) $36\ \pi\ cm^3$

(*b*) What is the formula for finding the volume of the sphere?

(*i*) $\pi r^2 h$ (*ii*) $\dfrac{4}{3}\pi r^3$

(*iii*) $\dfrac{1}{3}\pi r^2 h$ (*iv*) $\dfrac{2}{3}\pi r^3$

Ans. (*ii*) $\dfrac{4}{3}\pi r^3$

(*c*) What is curved surface area of the right circular cylinder?

(*i*) $\dfrac{132}{7}\ cm^2$ (*ii*) $\dfrac{264}{7}\ cm^2$

(*iii*) $132\ cm^2$ (*iv*) $264\ cm^2$

Ans. (*i*) $\dfrac{132}{7}\ cm^2$

(*d*) What is the radius of the sphere?

(*i*) 2 cm (*ii*) 3 cm

(*iii*) 6 cm (*iv*) 5 cm

Ans. (*ii*) 3 cm

(*e*) If the right circular cylinder is converted into the shape of right circular cone having radius 3 cm, then the height of the cone is

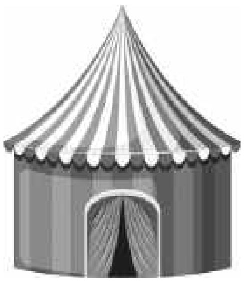
(*i*) 3 cm (*ii*) 6 cm

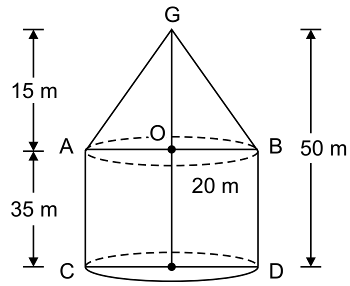
(*iii*) 9 cm (*iv*) 12 cm

Ans. (*iv*) 12 cm

11. The owner of the circus rented the ground in a certain city. A big tent was put up there. A circus

tent of total height 50 metres is to be made in the form of a right circular cylinder surmounted by a right circular cone. If the height and radius of the conical portion of the tent are 15 metres and 20 metres respectively, answer the following questions.





(*a*) What is the slant height of the conical part of the tent?

(*i*) 15 m (*ii*) 20 m

(*iii*) 25 m (*iv*) 35 m

Ans. (*iii*) 25 m

(*b*) What is the curved surface area of the conical part of the tent?

(*i*) 1571.43 m^2 (*ii*) 2024.95 m^2

(*iii*) 3215.65 m^2 (*iv*) 4400 m^2

Ans. (*i*) 1571.43 m^2

(*c*) What is the curved surface area of the cylindrical part of the tent?

(*i*) 1571.43 m^2 (*ii*) 2024.95 m^2

(*iii*) 3215.65 m^2 (*iv*) 4400 m^2

Ans. (*iv*) 4400 m^2

(*d*) Find the area of the canvas used for making the tent.

(*i*) 3548.65 m^2 (*ii*) 4596.95 m^2

(*iii*) 5971.43 m^2 (*iv*) 6547.93 m^2

Ans. (*iii*) 5971.43 m^2

(*e*) Find the cost of the cloth required, at the rate of ₹ 14 per square metre to make the tent. (Note that the base of the tent will not be covered with canvas.)

(*i*) ₹ 26500 (approx.)

(*ii*) ₹ 42600 (approx.)

(*iii*) ₹ 65400 (approx.)

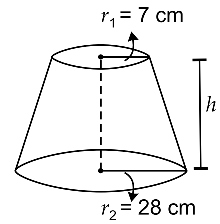
(*iv*) ₹ 83600 (approx.)

Ans. (*iv*) ₹ 83600 (approx.)

Very Short Answer Type Questions

12. If the radii of the circular ends of a solid frustum of a cone are 28 cm and 7 cm and its slant height is $\sqrt{585}$ cm, then what is the vertical height of the frustum?

Sol. Let r_1 and r_2 (where $r_2 > r_1$) be the radii of the ends of the frustum and let h be its vertical height and l be its slant height.



Then $r_1 = 7$ cm, $r_2 = 28$ cm and $l = \sqrt{585}$ cm

Now, $l = \sqrt{585} = \sqrt{h^2 + (r_1 - r_2)^2} = \sqrt{h^2 + 21^2}$

$\Rightarrow \quad 585 - 21^2 = h^2$

$\Rightarrow \quad h^2 = 585 - 441$

$= 144$

$\therefore \quad h = \sqrt{144} = 12$ cm

Hence, the required height of the frustum is 12 cm.

13. The diameter of a copper sphere is 20 cm. The sphere is melted and recast into small spherical ball. Each ball is of diameter 2 cm. Find the number of balls obtained.

Sol. Let R be the radius of the copper sphere and r be the radius of each spherical ball.

Then R = 10 cm and $r = 1$ cm.

$\therefore$ Volume of the copper sphere

$$= \frac{4}{3}\pi R^3$$

$$= \frac{4}{3}\pi \times 10^3 \text{ cm}^3$$

$$= \frac{4000\pi}{3} \text{ cm}^3$$

Volume of each small spherical ball

$$= \frac{4}{3}\pi r^3$$

$$= \frac{4}{3}\pi \text{ cm}^3$$

$\therefore$ Number of balls $= \dfrac{4000\pi}{3} \times \dfrac{3}{4\pi} = 1000$

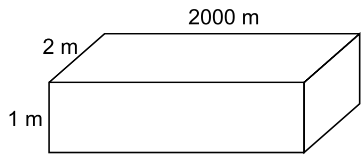
Hence, the required number of balls is 1000.

Short Answer Type-I Questions

14. Water in a canal, 2 m wide and 1 m deep, is flowing with a speed of 6 km/hour. How much area will it irrigate in 20 minutes, if 10 cm of standing water is required for irrigation.

Sol. $$6 \text{ km/h} = \frac{6000}{60} \text{ m/min}$$

$$= 100 \text{ m/min}$$



$\therefore$ In 20 minutes,

Length of water in the canal = 100×20 m

$= 2000$ m

$\therefore$ Area of the base = $2000 \times 2 \text{ m}^2 = 4000 \text{ m}^2$

$\therefore$ Volume of water flowing in 20 minutes

$= 4000 \times 1 \text{ m}^3$

$= 4000 \text{ m}^3$

If the height of the water = 10 cm = $\frac{1}{10}$ m,

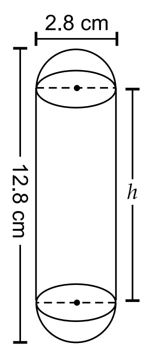
then the required area to be irrigated = $\dfrac{4000}{\frac{1}{10}}$ m^2

$= 40000 \text{ m}^2$

Hence, the area it will irrigate in 20 minutes is 40000 m^2.

15. Find the volume of a solid in the form of a right circular cylinder with hemispherical ends whose total length is 12.8 cm. The diameter of each hemispherical end is 2.8 cm.

Sol. Let h be the height of the cylinder and r be the radius of the base of each hemisphere.



Then $$r = \frac{2.8}{2} \text{ cm} = 1.4 \text{ cm}$$

$$h = (12.8 - 2.8) \text{ cm} = 10 \text{ cm}$$

Volume of the cylinder = $\pi r^2 h$

$= \pi \times (1.4)^2 \times 10 \text{ cm}^3$

Sum of the volumes of two hemispheres

$= 2 \times \frac{2}{3}\pi \times (1.4)^3 \text{ cm}^3$

$= \pi \times (1.4)^2 \left(10 + \frac{56}{30}\right) \text{ cm}^3$

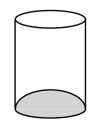
$= \frac{22}{7} \times (1.4)^2 \times \frac{356}{30} \text{ cm}^3$

$= 73.10 \text{ cm}^3$ (approx.)

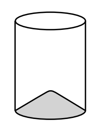
Hence, the required volume of a solid is 73.10 cm^3 (approx.)

16. Isha is 10 years old girl. On the result day, Isha and her father Suresh were very happy as she got first position in the class. While coming back to their home, Isha asked for a treat from her father as a reward for her success. They went to a juice shop and asked for two glasses of juice.

Aisha, a juice seller, was serving juice to her customers in two types of glasses. Both the glasses had inner radius 3 cm. The height of both the glasses was 10 cm.



First type: A glass with hemispherical raised bottom.



Second type: A glass with conical raised bottom of height 1.5 cm.

Isha insisted to have the juice in first type of glass and her father decided to have the juice in second type of glass. Out of the two, Isha or her father Suresh, who got more quantity of juice to drink and by how much? **[CBSE SP(Standard) 2019]**

Sol. Capacity of first glass

$= \pi r^2 H - \frac{2}{3}\pi r^3$

$= \pi \times 9(10 - 2) = \pi \times 9(8)$

$= 72\pi \text{ cm}^3$

Capacity of second glass

$= \pi r^2 H - \frac{1}{3}\pi r^2 h$

$= \pi \times 3 \times 3(10 - 0.5) = \pi \times 9\,(9.5)$

$= 85.5\pi \text{ cm}^3$

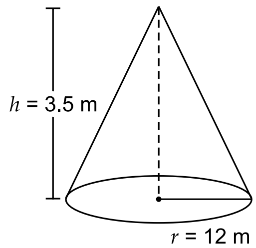
$\therefore$ Suresh got more quantity of juice.

$\therefore$ Suresh got [$(85.5\pi - 72\pi)$ cm^3 =] 13.5π cm^3 more quantity of juice than Isha.

Short Answer Type-II Questions

17. A heap of rice is in the form of a cone of base diameter 24 m and height 3.5 m. Find the volume of the rice. How much canvas cloth is required to just cover the heap? **[CBSE 2018]**

Sol. Let r and h be the radius of the base and the vertical height respectively of the cone and l be its slant height.



Then $r = 12$ m, $h = 3.5$ m

$\therefore$ Volume of the rice $= \frac{\pi}{3} r^2 h$

$$= \frac{\pi}{3} \times 12^2 \times 3.5 \text{ m}^3$$

$$= \frac{22}{7 \times 3} \times 12 \times 12 \times \frac{35}{10} \text{ m}^3$$

$$= 22 \times 2 \times 12 \text{ m}^3$$

$$= 528 \text{ m}^3$$

Also, $l = \sqrt{r^2 + h^2}$

$$= \sqrt{12^2 + (3.5)^2} \text{ m}$$

$$= \sqrt{144 + \frac{1225}{100}} \text{ m}$$

$$= \frac{\sqrt{14400 + 1225}}{10} \text{ m}$$

$$= \frac{\sqrt{15625}}{10} \text{ m}$$

$$= \frac{125}{10} \text{ m}$$

$$= 12.5 \text{ m}$$

$\therefore$ Curved surface area of the cone $= \pi r l$

$$= \frac{22}{7} \times 12 \times 12.5 \text{ m}^2$$

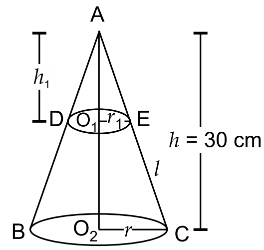
$$= \frac{3300}{7} \text{ m}^2$$

$$= 471.4 \text{ m}^2 \text{ (approx.)}$$

Hence, the required canvas cloth is 471.4 m^2 (approx.)

18. The height of a cone is 30 cm. A small cone is cut off at the top by a plane parallel to the base. If its volume is $\frac{1}{27}$ of the volume of the given cone, at what height above the base is the section made? **[CBSE 2017, 2005 C, SP 2011]**

Sol. Let r, h and V be the radius of the base, vertical height and the volume respectively of the entire cone ABC and let r_1, h_1 and V_1 be the radius of the base, vertical height and the volume respectively of the smaller cone ADE which is cut off.



Then, $h = 30$ cm.

Let O_1 and O_2 be the centres of the circular bases of the smaller and bigger cones respectively.

Then from similar triangles AO_1E and AO_2C, we have

$$\frac{r_1}{r} = \frac{h_1}{h} = \frac{h_1}{30}$$

$$\therefore \quad r_1 = \frac{r^2 h_1^2}{900} \quad \ldots(1)$$

$$V_1 = \frac{\pi}{3} r_1^2 h_1$$

and $$V = \frac{\pi}{3} r^2 h = 10\pi r^2 \quad \ldots(2)$$

Given that $V_1 = \frac{V}{27}$

$$\therefore \quad \frac{\pi}{3} r_1^2 h_1 = \frac{1}{27} \times 10\pi r^2 \quad \text{[From (2)]}$$

$$\Rightarrow \quad 9 r_1^2 h_1 = 10 r^2$$

$$\Rightarrow \quad 9 \times \frac{r^2 h_1^2}{900} \times h_1 = 10 r^2 \quad \text{[From (1)]}$$

$$\Rightarrow \quad h_1^3 = 1000$$

$$\Rightarrow \quad h_1 = 10$$

$\therefore$ $h - h_1 = O_1O_2 = (30 - 10)$ cm $= 20$ cm

Hence, the required height above the base of the bigger cone, where the smaller cone was cut off is 20 cm.

Long Answer Type Questions

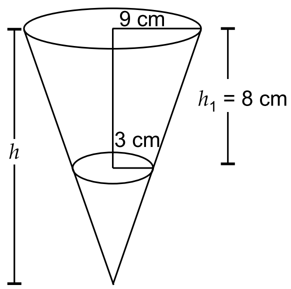
19. A bucket of height 8 cm and made up of copper sheet is in the form of a frustum of right circular cone with radii of its lower and upper end, as 3 cm and 9 cm respectively. Calculate:

(*a*) the ratio of the height of the cone of which the bucket is a part and the height of the frustum of the cone.

(*b*) the volume of water which can be filled completely in the bucket.

(*c*) the area of the copper sheet required to make the bucket. **[CBSE SP 2003]**

Sol. Let r_1 and r_2 be the radii of the circular ends of the frustum of the cone, where $r_1 > r_2$ and let h_1 be the vertical height of the frustum and let h be the height of the entire cone of which this frustum is a part.



Then $r_1 = 9$ cm, $r_2 = 3$ cm and $h_1 = 8$ cm

(*a*) We have $\quad \dfrac{r_1}{r_2} = \dfrac{h}{h_1} = \dfrac{h}{8}$

$$\Rightarrow \quad \frac{9}{3} = \frac{h}{8}$$

$$\Rightarrow \quad h = 24$$

Hence, $\dfrac{h}{h_1} = \dfrac{24}{8} = \dfrac{3}{1}$

$$\therefore \quad h : h_1 = 3 : 1$$

(*b*) Volume of the frustum

$$= \frac{\pi}{3}\left(r_1^2 + r_2^2 + r_1 r_2\right) \times h_1$$

$$= \frac{\pi}{3}(81 + 9 + 27) \times 8 \text{ cm}^3$$

$$= \frac{\pi}{3} \times 117 \times 8 \text{ cm}^3$$

$$= 312\pi \text{ cm}^3$$

(*c*) Let l be the slant height of the frustum of the cone.

$$\therefore \quad l = \sqrt{h_1^2 + (r_1 - r_2)^2}$$

$$= \sqrt{64 + 36} \text{ cm}$$

$$= \sqrt{100} \text{ cm}$$

$$= 10 \text{ cm}$$

$\therefore$ Surface area of the frustum

$$= \pi(r_1 + r_2) \times l$$

$$= \pi \times 12 \times 10 \text{ cm}^2$$

$$= 9\pi \text{ cm}^2$$

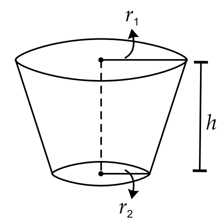
$\therefore$ Area of the copper sheet

$$= (120 + 9)\pi \text{ cm}^2$$

$$= 129\pi \text{ cm}^2$$

20. The perimeters of the ends of frustum of a cone are 96 cm and 68 cm. If the height of the frustum is 20 cm, find its radii, slant height, volume and total surface area. **[CBSE SP 2011]**

Sol. Let r_1 and r_2 be the radii of the two circular ends of the frustum, h be the vertical height and l be the slant height of the frustum of the cone.



Then $\quad 2\pi r_1 = 96$ cm

$$\Rightarrow \quad r_1 = \frac{96}{2} \times \frac{7}{22} \text{ cm}$$

$$= \frac{168}{11} \text{ cm}$$

$$= 15.27 \text{ cm}$$

$$2\pi r_2 = 68 \text{ cm}$$

$$\Rightarrow \quad r_2 = \frac{68}{2} \times \frac{7}{22} \text{ cm}$$

$$= \frac{119}{11} \text{ cm}$$

$$= 10.82 \text{ cm}$$

Hence, the required radii are 15.27 cm and 10.82 cm.

Also, $\quad l = \sqrt{(r_1 - r_2)^2 + h_2}$

$$= \sqrt{\left(\frac{168 - 119}{11}\right)^2 + 20^2} \text{ cm}$$

$$= \sqrt{\left(\frac{49}{11}\right)^2 + 20^2} \text{ cm}$$

$$= \sqrt{\frac{2401}{121} + 400} \text{ cm}$$

$$= \sqrt{\frac{50801}{121}} \text{ cm}$$

$$= \frac{225.39}{11} \text{ cm} = 20.49 \text{ cm}$$

Hence, the required slant height is 20.49 cm.

Volume of the frustum

$$= \frac{\pi \times h}{3}\left(r_1^2 + r_2^2 + r_1 r_2\right)$$

$$= \frac{22 \times 20}{7 \times 3}\,[(15.27)^2 + (10.82)^2 + 15.27 \times 10.82]\ \text{cm}^3$$

$$= \frac{440}{21} \times (233.1729 + 117.0724 + 165.2214)\ \text{cm}^3$$

$$= 10800.2546667\ \text{cm}^3$$

Hence, the required volume is 10800.25 cm^3 (approx.)

Again, curved surface area of the frustum

$$= \pi(r_1 + r_2)l$$

$$= \frac{22}{7} \times (15.27 + 10.82) \times 20.49\ \text{cm}^2$$

$$= \frac{22}{7} \times 26.09 \times 20.49\ \text{cm}^2$$

$$= 1680.12\ \text{cm}^2\ \text{(approx.)}$$

Lastly, the sum of the areas of the two circular ends of the frustum

$$= \pi\left(r_1^2 + r_2^2\right)$$

$$= \frac{22}{7} \times [(15.27)^2 + (10.82)^2]\ \text{cm}^2$$

$$= \frac{22}{7} \times (233.1729 + 117.0724)\ \text{cm}^2$$

$$= 1100.\ 7709\ \text{cm}^2$$

$\therefore$ Required total surface area

$$= (1682.12 + 1100.7709)\ \text{cm}^2$$

$$= 2780.89\ \text{cm}^2\ \text{(approx.)}$$

Let's Compete

(Page 239)

Multiple-Choice Questions

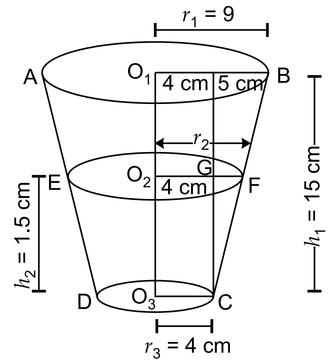
1. A bucket whose upper and lower interior diameters were 18 cm and 8 cm respectively and the depth was 15 cm was placed on a flat surface during a rainfall. After 30 minutes exposure, the depth of the water in the bucket was found to be 1.5 cm. Then the amount of rainfall per hour was

(*a*) 164.5 cm^3 (*b*) 164 cm^3

(*c*) 170.5 cm^3 (*d*) 170 cm^3

Sol. (*c*) 170.5 cm^3

Let ABCD be the bucket in the shape of a frustum of a cone with upper and lower circular ends of radii r_1 and r_3 ($r_1 > r_3$) respectively and let h_1 be the vertical height of this bucket. Then r_1 = 9 cm and r_3 = 4 cm and h_1 = 15 cm.



When the empty bucket is kept in the rain, let the rain water fall on the bucket upto a height h_2 forming another frustum FCDE with upper and lower circular ends of radii r_2 and r_3 respectively so that h_2 = ED = 1.5 cm, where $r_1 > r_2 > r_3$.

Let O_1, O_2 and O_3 be the centres of the circular ends of the two frustums so that $O_1O_2O_3$ is vertical. We draw CGH parallel to $O_1O_2O_3$ cutting O_2F and and O_1B at G and H respectively. Now, O_1B = 9 cm and $O_1H = O_2G = O_2G = O_3C$ = 4 cm and DE = 1.5 cm and AD = 15 cm.

Now, in ΔBCH, we have

$$\frac{GF}{HB} = \frac{CG}{CH}$$

$$\Rightarrow \qquad \frac{r_2 - 4}{9 - 4} = \frac{1.5}{15}$$

$$\Rightarrow \qquad \frac{r_2 - 4}{5} = 0.1$$

$$\Rightarrow \qquad r^2 = 4.5\ \text{cm}$$

We shall now find the volume of water in the lower frustum EFCD with radii of the end circles r_2 = 4.5 cm and r_3 = 4 cm and vertical height h_2 = 1.5 cm.

$\therefore$ Volume of water which is deposited in the bucket in 30 minutes

$$= \frac{\pi h_2}{3}\left(r_2^2 + r_3^2 + r_2 \times r_3\right)$$

$$= \frac{22}{7} \times \frac{1.5}{3}\left[(4.5)^2 + 4^2 + 4.5 \times 4\right]\text{cm}^3$$

$$= \frac{11}{7}(20.25 + 16 + 18)\ \text{cm}^3$$

$$= \frac{11}{7} \times 54.25\ \text{cm}^3$$

$\therefore$ The required amount of rainfall/hour

$$= \frac{11}{7} \times 54.25 \times 3\ \text{cm}^3$$

$$= \frac{11}{7} \times 108.50\ \text{cm}^3$$

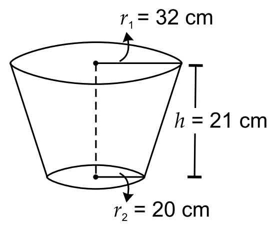
$$= 170.5\ \text{cm}^3$$

2. A solid frustum of a cone is 21 cm high. The radii of its ends are 32 cm and 20 cm. If it is melted and recast into cones of base diameter 7 cm and height 10 cm, then the maximum number of complete cones thus formed is

(*a*) 354 (*b*) 353 (*c*) 360 (*d*) 100

Sol. (*b*) 353

Let the radii of the circular ends of the frustum be r_1 and r_2 where $r_1 > r_2$ and let h be the vertical height.



Then r_1 = 32 cm, r_2 = 20 cm and h = 21 cm.

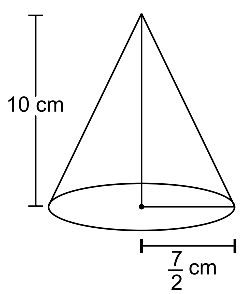
$\therefore$ Volume of the frustum

$$= \frac{\pi h}{3}\left(r_1^2 + r_2^2 + r_1 r_2\right)$$

$$= \frac{\pi}{3} \times 21 \times (32^2 + 20^2 + 32 \times 20) \text{ cm}^3$$

$$= 7\pi \times 2064 \text{ cm}^3$$

Let the radius of the base of the cone be r and H be the vertical height of the cone. Then $r = \frac{7}{2}$ cm and H = 10 cm.



$\therefore$ Volume of each cone $= \dfrac{\pi r^2 H}{3}$

$$= \frac{\pi}{3}\left(\frac{7}{2}\right)^2 \times 10 \text{ cm}^3$$

$$= \frac{\pi}{3} \times \frac{49}{4} \times 10 \text{ cm}^3$$

$\therefore$ Number of cones $= 7\pi \times 2064 \times \dfrac{3 \times 4}{49\pi \times 10}$

$$= \frac{12384}{35}$$

$$= 353.82857$$

Hence, the required number of complete cones are 353.

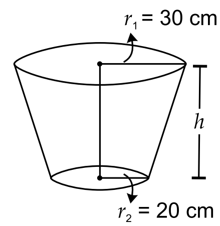
3. A bucket is in the form of a frustum of a right circular cone. It holds 20.9 litres of water. The radii of the top and the bottom ends are respectively 30 cm and 20 cm. Then the slant height of the frustum is

(*a*) 14.5 cm (*b*) 14 cm

(*c*) 10.5 cm (*d*) 10 cm

Sol. (*a*) 14.5 cm

Let r_1 and r_2 be the radii of the circular ends of the frustum where $r_1 > r_2$ and let h be the vertical height of the frustum. Then r_1 = 30 cm, r_2 = 20 cm.



$\therefore$ Volume of the frustum

$$= \frac{\pi h}{3}\left(r_1^2 + r_2^2 + r_1 r_2\right)$$

$$= \frac{22}{21} h(30^2 + 20^2 + 30 \times 20) \text{ cm}^3$$

$$= \frac{22}{21} h(900 + 400 + 600) \text{ cm}^3$$

$$= \frac{22h}{21} \times 1900 \text{ cm}^3$$

$\therefore \quad \dfrac{22h}{21} \times 1900 = 20.9 \times 1000 = 20900$

$\Rightarrow \quad h = 20900 \times \dfrac{21}{22 \times 1900} = 10.5$

$\Rightarrow \quad h = 10.5$

$\therefore$ The vertical height of the frustum 10.5 cm

$\therefore$ The required slant height, l

$$= \sqrt{h^2 + (r_1 - r_2)^2}$$

$$= \sqrt{(10.5)^2 + (30 - 20)^2} \text{ cm}$$

$$= \sqrt{110.25 + 100} \text{ cm}$$

$$= \sqrt{210.25} \text{ cm}$$

$$= 14.5 \text{ cm}$$

4. If the total surface area of a cube is 216 cm^2, then its volume is

(*a*) 144 cm^3 (*b*) 196 cm^3

(*c*) 212 cm^3 (*d*) 216 cm^3

[CBSE SP 2012]

Sol. (*d*) 216 cm^3

Let a be the side of the cube.

Then $6a^2 = 216\ cm^2$

$\Rightarrow$ $a^2 = 36\ cm^2$

$\Rightarrow$ $a = 6$ cm

$\therefore$ Required volume $= a^3 = 6^3\ cm^3 = 216\ cm^3$

5. The number of circular plates each of radius 7 cm and thickness 0.5 cm that should be placed one above the other to form a solid right circular cylinder of volume 7700 cm^3 is

(*a*) 70 (*b*) 100

(*c*) 200 (*d*) 150

Sol. (*b*) 100

Volume of each circular plate of radius 7 cm and thickness 0.5 cm $= \pi \times 7^2 \times 0.5\ cm^3$

$$= \frac{22}{7} \times 49 \times 0.5\ cm^3$$

$$= 77\ cm^3$$

$\therefore$ Required number of circular plates $= \frac{7700}{77}$

$$= 100$$

6. Volume of a cylindrical wire of radius 1 cm is 220 cm^3. It is cut into three unequal segments. If the lengths of two cut segments are 10 cm and 15 cm, then the length of the third segment is

(*a*) 45 cm (*b*) 50 cm

(*c*) 40 cm (*d*) 55 cm

Sol. (*a*) 45 cm

Let r be the radius and h be the length of the cylindrical wire

Then, its volume $= \pi r^2 h = \pi \times h$

Given, volume $= 220\ cm^3$

$\therefore$ $\frac{22}{7} \times h = 220$

$\Rightarrow$ $h = 70$

$\therefore$ Total length of the wire = 70 cm

Total length of two segment of wire

$$= (10 + 15) \text{ cm} = 25 \text{ cm}$$

$\therefore$ Required length of the third segment

$$= (70 - 25) \text{ cm} = 45 \text{ cm}$$

7. If the total surface area of a hemisphere is 9 cm^2, then its volume is

(*a*) $\sqrt{\frac{\pi}{3}}\ cm^3$ (*b*) $2\sqrt{\frac{\pi}{3}}\ cm^3$

(*c*) $\sqrt{\frac{3}{\pi}}\ cm^3$ (*d*) $2\sqrt{\frac{3}{\pi}}\ cm^3$

Sol. (*d*) $2\sqrt{\frac{3}{\pi}}\ cm^3$

We have $3\pi r^2 = 9$ where r is the radius of the hemisphere.

$\therefore$ $$r = \left(\frac{3}{\pi}\right)^{1/2} \qquad \ldots(1)$$

$\therefore$ Required volume of the hemisphere

$$= \frac{2}{3}\pi r^3$$

$$= \frac{2}{3} \times \pi \times \left(\frac{3}{\pi}\right)^{3/2} cm^3 \qquad \text{[From (1)]}$$

$$= 2 \times \frac{3^{3/2 - 1}}{\pi^{3/2 - 1}}\ cm^3$$

$$= 2\sqrt{\frac{3}{\pi}}\ cm^3$$

8. The ratio of the volumes of two cones is 2 : 5. If the ratio of their diameters is 4 : 5, then the ratio of their heights is

(*a*) 25 : 64 (*b*) 16 : 25

(*c*) 5 : 8 (*d*) 4 : 25

Sol. (*c*) 5 : 8

Let r_1 and r_2 be the radii of the bases of two cones, h_1 and h_2 be their respective vertical heights and let V_1 units and V_2 be their respective volumes.

Then, $V_1 = \frac{\pi}{3} r_1^2 h_1$ and $V_2 = \frac{\pi}{3} r_2^2 h_2$

$\therefore$ $$\frac{V_1}{V_2} = \frac{r_1^2 h_1}{r_2^2 h_2}$$

$\Rightarrow$ $$\frac{2}{5} = \frac{r_1^2}{r_2^2} \times \frac{h_1}{h_2} = \frac{4^2}{5^2} \times \frac{h_1}{h_2}$$

$\Rightarrow$ $$\frac{h_1}{h_2} = \frac{2}{5} \times \frac{25}{16} = \frac{5}{8}$$

$\therefore$ $h_1 : h_2 = 5 : 8$

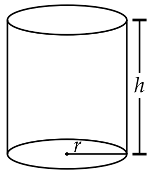
9. A cuboid and a right circular cylinder have equal volumes. Their heights are also equal. If r and h are respectively the radius of the base and the height of the cylinder, then the area of the bottom of the cuboid is

(*a*) $\pi h r^2$ (*b*) πr^2

(*c*) πh^2 (*d*) $\pi r h$

Sol. (*b*) πr^2

The radius of the base and the height of the cylinder are r and h respectively.



$\therefore$ Volume of the cylinder = $\pi r^2 h$

Height of the cuboid = h

Let a and b be the length and breadth of the cuboid.

$\therefore$ Volume of the cuboid = abh

According to the problem, we have

$\therefore$ $abh = \pi r^2 h$

$\Rightarrow$ $ab = \pi r^2$

$\therefore$ Area of the bottom of the cuboid = $ab = \pi r^2$

10. The volume of the largest sphere that can be cut from a cylindrical log of wood of base radius 1 m and height 4 m is

(*a*) $\frac{4}{3}\pi$ m^3 (*b*) $\frac{16\pi}{3}$ m^3

(*c*) $\frac{10}{3}$ m^3 (*d*) $\frac{8\pi}{3}$ m^3

[CBSE SP 2012]

Sol. (*a*) $\frac{4}{3}\pi$ m^3

We see that the volume of the sphere will be largest when the diameter of the sphere = the diameter of the base of the cylinder, i.e. their radii are equal.

$\therefore$ The radius of the sphere will be 1 m.

Hence, the required volume of the sphere

$$= \frac{4}{3}\pi \times 1^3 \text{ m}^3$$

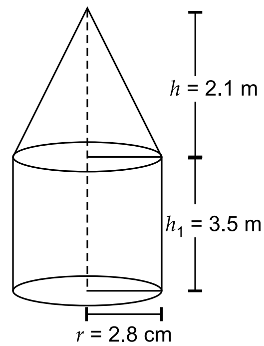
$$= \frac{4}{3}\pi \text{ m}^3$$

Value-based Questions (Optional)

(Page 240)

1. Thousands of people were rendered homeless due to heavy floods in a state. 50 schools collectively offered to the State Government to provide place and the canvas for 1500 tents to be fixed by the Government and decided to share the whole expenditure equally. The lower part of each tent is cylindrical of base radius 2.8 m and height 3.5 m, with conical upper part of the same base radius but of height 2.1 m. If the canvas used to make the tents costs ₹120 per sq m, find the amount shared by each school to set-up the tents. What value is shown by the school authorities and the Government? $\left[\text{Use } \pi = \frac{22}{7}\right]$ **[CBSE 2016]**

Sol. Let r be the common radius of the base of the cylinder and the cone. Then r = 2.8 m. Let h and h_1 be the vertical height of the cone and the cylinder respectively.



Then h = 2.1 m and h_1 = 3.5 m. If l be the slant height of the cone, then

$$l = \sqrt{h^2 + r^2}$$

$$= \sqrt{(2.1)^2 + (2.8)^2} \text{ m}$$

$$= \sqrt{4.41 + 7.84} \text{ m}$$

$$= \sqrt{12.25} \text{ m}$$

$$= 3.5 \text{ m}$$

$\therefore$ Curved surface area of the cone

$$= \pi r l$$

$$= \frac{22}{7} \times 2.8 \times 3.5 \text{ m}^2$$

$$= 30.8 \text{ m}^2 \quad \ldots(1)$$

Also, curved surface area of the cylinder

$$= 2\pi r h_1$$

$$= 2 \times \frac{22}{7} \times 2.8 \times 3.5 \text{ m}^2$$

$$= 61.6 \text{ m}^2 \quad \ldots(2)$$

$\therefore$ Total surface area of the cone and the cylinder

$$= (30.8 + 61.6) \text{ m}^2$$

$$= 92.4 \text{ m}^2 \quad \text{[From (1) and (2)]}$$

$\therefore$ Total surface area of the canvas required for each tent = 92.4 m^2

$\therefore$ Total surface area of 1500 tents

$$= 92.4 \times 1500 \text{ m}^2$$

$$= 138600 \text{ m}^2$$

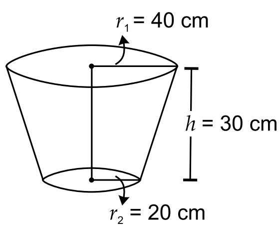
$\therefore$ Total cost required for each of 50 schools is

₹ $\frac{138600 \times 120}{50}$, i.e. ₹ 332640

Values: Empathy, decision making and concern.

2. Milk in a container which is in the form of a frustum of a cone of height 30 cm and the radii of whose lower and upper circular ends are 20 cm and 40 cm respectively, is to be distributed in a camp for flood victims. If this milk is available at the rate of ₹35 per litre and 880 litres of milk is needed daily for a camp, find how many such containers of milk are needed for a camp and what cost will it put on the donor agency for this. What value is indicated through this by the donor agency? **[CBSE 2015, SP 2016]**

Sol. Let r_1 and r_2 (where $r_1 > r_2$) be the radii of the circular ends of the frustum.



Then r_1 = 40 cm and r_2 = 20 cm. Let h be its vertical height.

$\therefore \quad h = 30$ cm

$\therefore$ Volume of the frustum

$$= \frac{\pi h}{3}\left(r_1^2 + r_2^2 + r_1 r_2\right)$$

$$= \frac{22}{7} \times \frac{1}{3} \times 30 \times (40^2 + 20^2 + 40 \times 20) \text{ cm}^3$$

$$= \frac{220}{7} \times (1600 + 400 + 800) \text{ cm}^3$$

$$= \frac{220}{7} \times 2800 \text{ cm}^3$$

$$= 88000 \text{ cm}^3$$

$$= 88 \text{ litres}$$

$\therefore$ Number of containers = 880 ÷ 88 = 10

Also, required cost of milk = ₹ 35 × 880 = ₹ 30800

Value: The donor agency exhibited compassion.

14

Statistics

Checkpoint (Page 243)

1. What is the arithmetic mean of $2x$, $4x - 3$, $5x + 6$ and $5x + 9$?

Sol. Arithmetic mean $= \dfrac{\text{Sum of all observations}}{\text{Number of observations}}$

$$= \frac{2x + 4x - 3 + 5x + 6 + 5x + 9}{4}$$

$$= \frac{16x + 12}{4}$$

$$= \frac{4(4x + 3)}{4}$$

$$= 4x + 3$$

2. The mean of n observations is $\bar{x}$. If the first observation is increased by 1, the second by 2, the third by 3, and so on, then find the new mean.

Sol. Let the observations be $x_1, x_2, x_3, \ldots, x_n$.

New observations are $x_1 + 1$, $x_2 + 2$, $x_3 + 3$, …, $x_n + n$

Sum of new observations

$$= x_1 + 1 + x_2 + 2 + x_3 + 3 + \ldots + x_n + n$$

$$= x_1 + x_2 + x_3 + \ldots + x_n + 1 + 2 + 3 + \ldots + n$$

Mean of new observations

$$= \frac{x_1 + x_2 + \ldots + x_n + 1 + 2 + 3 + \ldots + n}{n}$$

$$= \frac{x_1 + x_2 + \ldots + x_n}{n} + \frac{1 + 2 + 3 + \ldots + n}{n}$$

$$= \bar{x} + \frac{n(n+1)}{2n}$$

$$= \bar{x} + \frac{n+1}{2}$$

3. Find the class marks of the following classes:

10.5 – 20.5 and 20.5 – 30.5

Sol. Class mark

$$= \frac{\text{Upper class limit + Lower class limit}}{2}$$

For the class 10.5 – 20.5

$$\text{Class mark} = \frac{10.5 + 20.5}{2} = \frac{31}{2} = 15.5$$

For the class 20.5 – 30.5

$$\text{Class mark} = \frac{20.5 + 30.5}{2} = \frac{51}{2} = 25.5$$

4. Find the median of the following observations:

5, 13, 7, 4, 19 and 10

Sol. Arranging the terms in an ascending order

4, 5, 7, 10, 13, 19

Here, $n = 6$ which is even

$\therefore$ Median

$$= \frac{\left(\frac{n}{2}\right)^{\text{th}} \text{observation} + \left(\frac{n}{2} + 1\right)^{\text{th}} \text{observation}}{2}$$

$$= \frac{7 + 10}{2}$$

$$= \frac{17}{2} = 8.5$$

5. If the average of three consecutive odd integers is 25, find the integers.

Sol. Let the consecutive odd integers be $x + 1$, $x + 3$, $x + 5$

Average of 3 consecutive odd integers

$$= \frac{x + 1 + x + 3 + x + 5}{3}$$

$$= \frac{3x + 9}{3}$$

$$25 = x + 3$$

$$\Rightarrow \quad x = 22$$

$\therefore$ First integer $= x + 1 = 22 + 1 = 23$

Second integer $= x + 3 = 22 + 3 = 25$

Third integer $= x + 5 = 22 + 5 = 27$

6. The ages (in years) of 9 students are as follows:

12, 6, 11, 13, 11, 12, 6, 7, 6

Find their modal age.

Sol. Arrange the ages in an ascending order

6, 6, 6, 7, 11, 11, 12, 12, 13

Here, 6 occurs most frequently.

$\therefore$ Modal age = 6

7. The points scored by a basketball team in a series of matches are as follows:

17, 2, 7, 27, 25, 5, 14, 18, 10, 24, 10, 8, 7, 10

Find the mean, median and mode for the data.

Sol. Mean $= \dfrac{\text{Sum of all observations}}{\text{Number of observations}}$

$$= \frac{17 + 2 + 7 + 27 + 25 + 5 + 14 + 18 + 10 + 24 + 10 + 8 + 7 + 10}{14}$$

$$= \frac{184}{14}$$

$= 13.142$

Arranging the terms in an ascending order, we get

2, 5, 7, 7, 8, 10, 10, 10, 14, 17, 18, 24, 25, 27

Here, $n = 14$ which is even

$\therefore$ Median

$$= \frac{\left(\frac{n}{2}\right)^{\text{th}} \text{observation} + \left(\frac{n}{2}+1\right)^{\text{th}} \text{observation}}{2}$$

$$= \frac{7^{\text{th}} \text{observation} + 8^{\text{th}} \text{observation}}{2}$$

$$= \frac{10 + 10}{2} = 10$$

$\therefore$ Median = 10

$\therefore$ Mode = 10, which occur most frequently.

8. The minimum value of a set of observations is 63 and its range is 41. Then what is the maximum value?

Sol. The minimum value of a set of observation is 63.

Range = 41

Maximum value = 63 + 41

= 104

9. The class mark of a particular class is 7.5 and the class size is 8. What is the class interval?

Sol. Class mark of a class = 7.5

Class size = 8

$\therefore$ Class interval = 3.5 – 11.5

10. The numbers 5, 8, 13, $4x + 9$, 15, 23 and 30 are written in ascending order. If the median of these data is 13, then what is the mode of these data?

Sol. 5, 8, 13, $4x + 9$, 15, 23 and 30 are in ascending order.

Median = 13 [Given]

Also, median $= 4x + 9$ [since, $n = 7$ is odd]

$\therefore \quad 4x + 9 = 13$

$\Rightarrow \quad 4x = 4$

$\Rightarrow \quad x = 1$

$\therefore \quad 4x + 9 = 4(1) + 9$

$= 13$

$\therefore$ The numbers are 5, 8, 13, 13, 15, 23 and 30.

Hence, 13 occurs most frequently.

$\therefore$ Mode = 13

Milestone

(Page 252)

Multiple-Choice Questions

1. The mean monthly salary of 8 members of a group is ₹2000. If one more member whose monthly salary is ₹2900 joins the group, then the mean monthly salary of 9 members of the group is

(*a*) ₹1800 (*b*) ₹2100

(*c*) ₹1900 (*d*) ₹2200

Sol. (*b*) ₹2100

Mean of monthly salary of 8 members of a group = ₹2000

$$\text{Mean} = \frac{\text{Sum of monthly salaries}}{\text{Number of members}}$$

$$2000 = \frac{\text{Sum of monthly salaries}}{8}$$

$\therefore$ Sum of monthly salary of 8 members

$= ₹2000 \times 8$

$= ₹16000$

Given, the salary of one more member is ₹2900 then, sum of monthly salary of 9 members

$= ₹2900 + ₹16000$

$= ₹18900$

$\therefore$ Mean of monthly salary of 9 members

$= ₹\, \frac{18900}{9}$

$= ₹2100$

2. If the mode of the data

38, 16, 59, 18, 33, 19, 49, 37, $3x + 1$ and 10 is 16, then the value of x is

(*a*) 12 (*b*) 6

(*c*) 16 (*d*) 5

Sol. (*d*) 5

Arranging the data in an ascending order, we have

10, 16, 18, 19, 33, 37, 38, 49, 59, $3x + 1$

Given, mode = 16, i.e. 16 occurs most frequently.

This means we must have $x = 5$ for mode to be 16.

Very Short Answer Type Questions

3. Out of 40 students who appeared in a test, 18 students secured less than 37 marks and 18 students secured more than 65 marks. If the marks secured by the remaining students are 41, 39, 47 and 53, then find the median marks of the whole data.

Sol. Arranging marks in an ascending order, we have,

Marks of 18 students less than 39, 41, 47, 53 and marks of 18 students more than 65.

Here, the number of observation $n = 40$, which is even.

$\therefore$ Median marks = Average of $\left(\frac{n}{2}\right)^{\text{th}}$ observation and $\left(\frac{n}{2}+1\right)^{\text{th}}$ observation

$$= \frac{\left(\frac{40}{2}\right)^{\text{th}} \text{observation} + \left(\frac{40}{2}+1\right)^{\text{th}} \text{observation}}{2}$$

$$= \frac{20^{\text{th}} \text{observation} + 21^{\text{th}} \text{observation}}{2}$$

$$= \frac{41 + 47}{2}$$

$$= \frac{88}{2} = 44$$

Hence, the median is 44.

4. For the following frequency distribution, find the difference between $\frac{1}{2}$ of the lower limit of the modal class and $\frac{1}{4}$th of the upper limit of the median class.

Class	*Number of students*
20 – 30	13
30 – 40	5
40 – 50	8
50 – 60	10
60 – 70	6
70 – 80	17
80 – 90	2
90 – 100	1

Sol.

Class	*Number of students* (f_i)	*Cumulative frequency (cf)*
20 – 30	13	13
30 – 40	5	18
40 – 50	8	26
50 – 60	10	36
60 – 70	6	42
70 – 80	17	59
80 – 90	2	61
90 – 100	1	62

Here, the maximum class frequency is 17 and the class corresponding to this frequency is 70 – 80. So, the modal class is 70 – 80.

$\therefore$ Lower limit of the modal class is 70

Now, to find median class,

$$n = \Sigma f_i = 62$$

So, $\frac{n}{2} = \frac{62}{2} = 31$. The cumulative frequency just greater than 31 is 36. So, the median class is 50 – 60.

$\therefore$ Upper limit of the median class is 60.

Hence, the difference between $\frac{1}{2}$ of the lower limit of the modal class and $\frac{1}{4}$th of the upper limit of the median class

$$= \frac{1}{2} \times 70 - \frac{1 \times 60}{4}$$
$$= 35 - 15$$
$$= 20$$

Short Answer Type-I Questions

5. Find the median of the following distribution:

Height (in cm)	*Frequency*
160 – 162	15
163 – 165	118
166 – 168	142
169 – 171	127
172 – 174	18

[CBSE SP 2010, 2011]

Sol. Converting the given inclusive series to an exclusive series and prepared the cumulative frequency table, we get

Height (in cm)	*Frequency*	*Cumulative frequency (cf)*
159.5 – 162.5	15	15
162.5 – 165.5	118	133
165.5 – 168.5	142	275
168.5 – 171.5	127	402
171.5 – 174.5	18	420

Here, $n = \Sigma f_i = 420$, so $\frac{n}{2} = \frac{420}{2} = 210$. The cumulative frequency just greater than 210 is 275 and the corresponding class is 165.5 – 168.5. So, the median class is 165.5 – 168.5.

$\therefore$ $l = 165.5$, $cf = 133$, $f = 142$ and $h = 3$

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$
$$= 165.5 + \left(\frac{210 - 133}{142}\right) \times 3$$
$$= 165.5 + \frac{77}{142} \times 3$$
$$= 165.5 + 1.626$$
$$= 167.126$$

6. The following table shows the ages of the patients admitted in a hospital during a year.

Age (in years)	5–15	15–25	25–35	35–45	45–55	55–65
Number of patients	6	11	21	23	14	5

Find the mode of the above data. **[CBSE SP 2011]**

Sol. Here, the maximum class frequency is 23 and the class corresponding to this frequency is 35 – 45.

So, the modal class is 35 – 45.

$\therefore$ $l = 35, f_1 = 23, f_0 = 21, f_2 = 14$ and $h = 10$

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$
$$= 35 + \left(\frac{23 - 21}{2 \times 23 - 21 - 14}\right) \times 10$$
$$= 35 + \left(\frac{20}{46 - 35}\right)$$
$$= 35 + \frac{20}{11}$$
$$= 35 + 1.818$$
$$= 36.818$$
$$= 36.8 \text{ year (approx.)}$$

Short Answer Type-II Questions

7. The ages of employees in two factories A and B are given below:

Age of employees (in years)	20–30	30–40	40–50	50–60	60–70
Number of employees in factory A	6	27	79	105	99
Number of employees in factory B	9	41	59	91	84

Compare the modal ages of employees in factory A and factory B.

Sol. Computation of modal age of employees of factory A:

Age group 50 – 60 has the maximum frequency. So, the modal class is 50 – 60.

$\therefore$ $l = 50, h = 10, f_1 = 105, f_0 = 79, f_2 = 99$

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 50 + \left(\frac{105 - 79}{2 \times 105 - 79 - 99}\right) \times 10$$

$$= 50 + \frac{26}{210 - 178} \times 10$$

$$= 50 + \frac{26 \times 10}{32}$$

$$= 50 + \frac{65}{8}$$

$$= 58.125$$

Computational of modal age of employees of Factory B:

Age group 50–60 has maximum frequency, so the modal class is 50–60.

$\therefore$ $l = 50, h = 10, f_1 = 91, f_0 = 59$ and $f_2 = 84$

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 50 + \left(\frac{91 - 59}{2 \times 91 - 59 - 84}\right) \times 10$$

$$= 50 + \frac{32}{182 - 143} \times 10$$

$$= 50 + \frac{32}{182 - 143} \times 10$$

$$= 50 + \frac{32 \times 10}{39}$$

$$= 50 + \frac{320}{39}$$

$$= 58.205$$

$\therefore$ The modal age of employees of factory B > modal age of employees of factory A

8. A survey regarding the height (in cm) of 50 girls of class *X* of a school was conducted and the following data were obtained.

Height (in cm)	*Number of girls*
120 – 130	2
130 – 140	8
140 – 150	12
150 – 160	20
160 – 170	8
Total	50

Find the mean, median and mode of the above data. **[CBSE 2008]**

Sol. Mean

Height (in cm)	x_i	*Number of girls* (f_i)	$f_i x_i$
120 – 130	125	2	250
130 – 140	135	8	1080
140 – 150	145	12	1740
150 – 160	155	20	3100
160 – 170	165	8	1320
Total		$\Sigma fi = 50$	$\Sigma f_i x_i = 7490$

$$\text{Mean} = \bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{7490}{50} = 149.8$$

Median

Height (in cm)	*Number of girls (f)*	*Cumulative frequency (cf)*
120 – 130	2	2
130 – 140	8	10
140 – 150	12	22
150 – 160	20	42
160 – 170	8	50

Here, $n = \Sigma f_i = 50$, so $\frac{n}{2} = \frac{50}{2} = 25$

Cumulative frequency just greater than 25 is 42 and the corresponding class is 150 – 160. So, the median class is 150 – 160.

$l = 150, cf = 22, f = 20$ and $h = 10$

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

$$= 150 + \left(\frac{25 - 22}{20}\right) \times 10$$

$$= 150 + \frac{30}{20}$$

$$= 150 + 1.5$$

$$= 151.5$$

Mode: Here the maximum frequency is 20 and the class corresponding to it is 150 – 160.

So, the modal class is 150 – 160.

$\therefore$ $l = 150, f_1 = 20, f_0 = 12, f_2 = 8$ and $h = 10$

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 150 + \left(\frac{20 - 12}{2 \times 20 - 12 - 8}\right) \times 10$$

$$= 150 + \frac{8}{20} \times 10$$

$$= 150 + 4$$

$$= 154$$

Hence, the mean height of the girls is 149.8 cm, their median height is 151.5 cm and their modal height is 154 cm.

Long Answer Type Questions

9. The following table gives the production yield per hectare of wheat of 100 farms of a village.

Production yield (in kg/ha)	*Number of farms*
50 – 55	2
55 – 60	8
60 – 65	12
65 – 70	24
70 – 75	38
75 – 80	16

Change the distribution to a more than type distribution and draw its ogive.

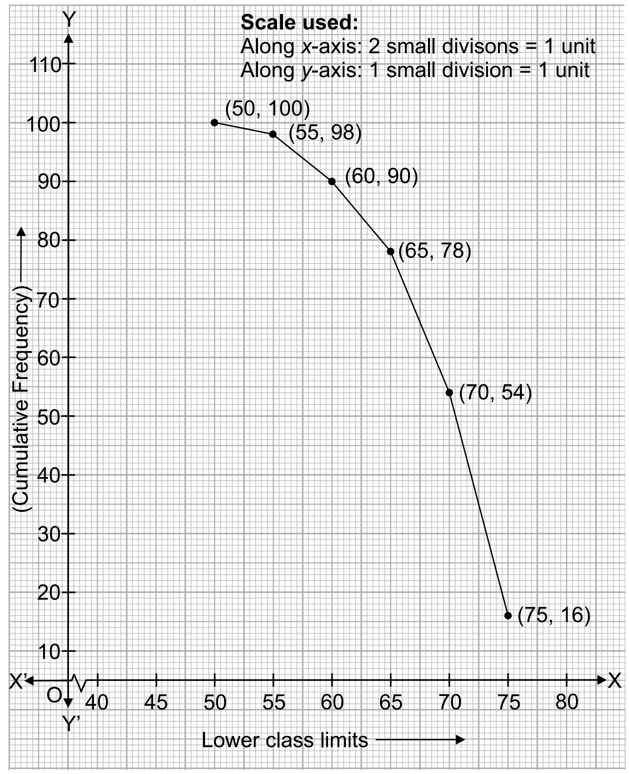
[CBSE SP 2010, 2011]

Sol. Converting the given distribution to more than type distribution, we get

Production yield (in kg/ha)	*Number of farms*
More than or equal to 50	100
More than or equal to 55	98
More than or equal to 60	90
More than or equal to 65	78
More than or equal to 70	54
More than or equal to 75	16

Lower class limit	50	55	60	65	70	75
Cumulative frequency	100	98	90	78	54	16

Plotting the points (50, 100), (55, 98), (60, 90), (65, 78), (70, 54), (75, 16) and joining then by a free hand curve, we get 'more than ogive' as shown in the graph.



10. The annual turnover of 100 companies give rise to the following distribution:

Turnover (₹ in crores)	*Number of companies*
More than or equal to 0	100
More than or equal to 5	92
More than or equal to 10	80
More than or equal to 15	59
More than or equal to 20	29
More than or equal to 25	7

Draw less than and more than type ogives on the same graph and hence obtain the median turnover. Verify the result by using the formula.

Sol. Less than method

From the given table, we first prepare a cumulative frequency distribution of the less than type

Turnover (₹ in crores) less than	*Number of companies (cumulative frequency)*
5	8
10	20
15	41
20	71
25	93
30	100

Upper class limit	5	10	15	20	25	30
Cumulative frequency	8	20	41	71	93	100

Now, we mark the upper class limits along the x-axis and cumulative frequencies along the y-axis on a suitable scale.

Then we plot the points (5, 8), (10, 20), (15, 41), (20, 71), (25, 93), (30, 100) and join them by a free hand smooth curve to obtain ogive by less than method.

From the given table, we prepare the 'more than type' cumulative frequency table, as shown below:

Turnover (₹ in crores)	*Cumulative frequency*
More than or equal to 0	100
More than or equal to 5	92
More than or equal to 10	80
More than or equal to 15	59
More than or equal to 20	29
More than or equal to 25	7

Lower class limit	0	5	10	15	20	25
Cumulative frequency	100	92	80	59	29	7

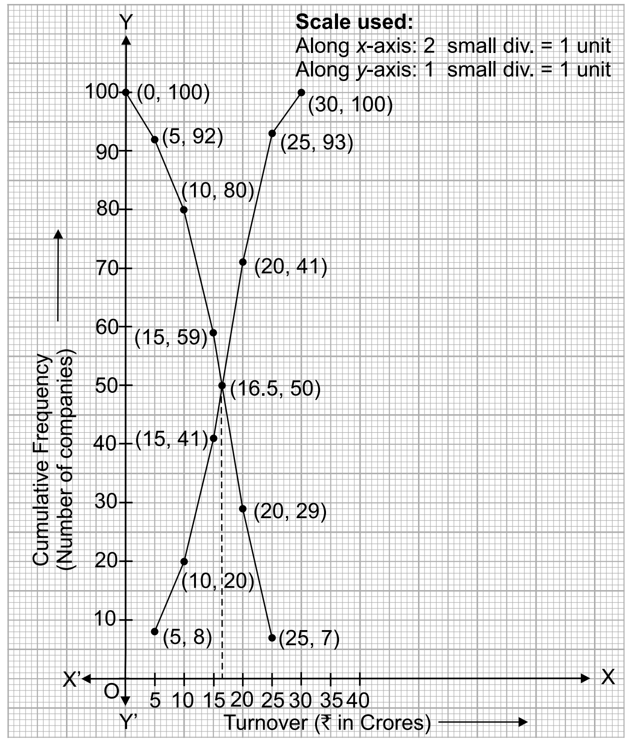
Now, we mark the lower class limits along the x-axis and the cumulative frequency along the y-axis on a suitable scale to plot the points (0, 10), (5, 92), (10, 80), (15, 59), (20, 29), (25, 7) and join these points by a free hand smooth curve to obtain more than type ogive.

The two curves intersect at point P(16.5, 50).

Hence, median turnover is ₹16.5 crores.

Median (by formula)

Turnover (₹ in crores)	*Number of companies*	*Cumulative frequency*
0 – 5	8	8
5 – 10	12	20
10 – 15	21	41
15 – 20	30	71
20 – 25	22	93
25 - 30	7	100



Here, $n = \Sigma f_i = 100$, so $\frac{n}{2} = \frac{100}{2} = 50$

Class whose cumulative frequency is greater than (and nearest to) $\frac{n}{2} = 50$, is 15 – 20.

So, the median class is 15 – 20.

$\therefore \quad l = 15, cf = 41, f = 30$ and $h = 5$

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

$$= 15 + \left(\frac{50 - 41}{30}\right) \times 5$$

$$= 15 + \frac{9 \times 5}{30}$$

$$= 15 + 1.5$$

$$= 16.5$$

Hence, median turnover is ₹16.5 crores.

Higher Order Thinking Skills (HOTS) Questions

(Page 253)

1. Following table gives the cumulative frequency of the age of a group of 199 teachers. Draw the less than ogive and greater than ogive and find the median.

Age (in years)	*Cumulative frequency*
20 – 25	21
25 – 30	40
30 – 35	90
35 – 40	130
40 – 45	146
45 – 50	166
50 – 55	176
55 – 60	186
60 – 65	195
65 – 70	199

[CBSE SP 2011]

Sol. Less than ogive

From the given table, we first prepare a cumulative frequency distribution of less than type.

Age (in years)	*Number of teachers (cumulative frequencies)*
Less than 25	21
30	40
35	90
40	130
45	146
50	166
55	176
60	186
65	195
70	199

Upper class limits	*Cumulative frequency*
25	21
30	40
35	90
40	130
45	146
50	166
55	176
60	186
65	195
70	199

Plotting the points (25, 21), (30, 40), (35, 90), (45, 146), (50, 166), (55, 176), (60, 186), (65, 195), (70, 199) and joining them by a free hand smooth curve we get a 'less than ogive'

More than ogive

From the given table, we prepare a cumulative frequency distribution of more than type.

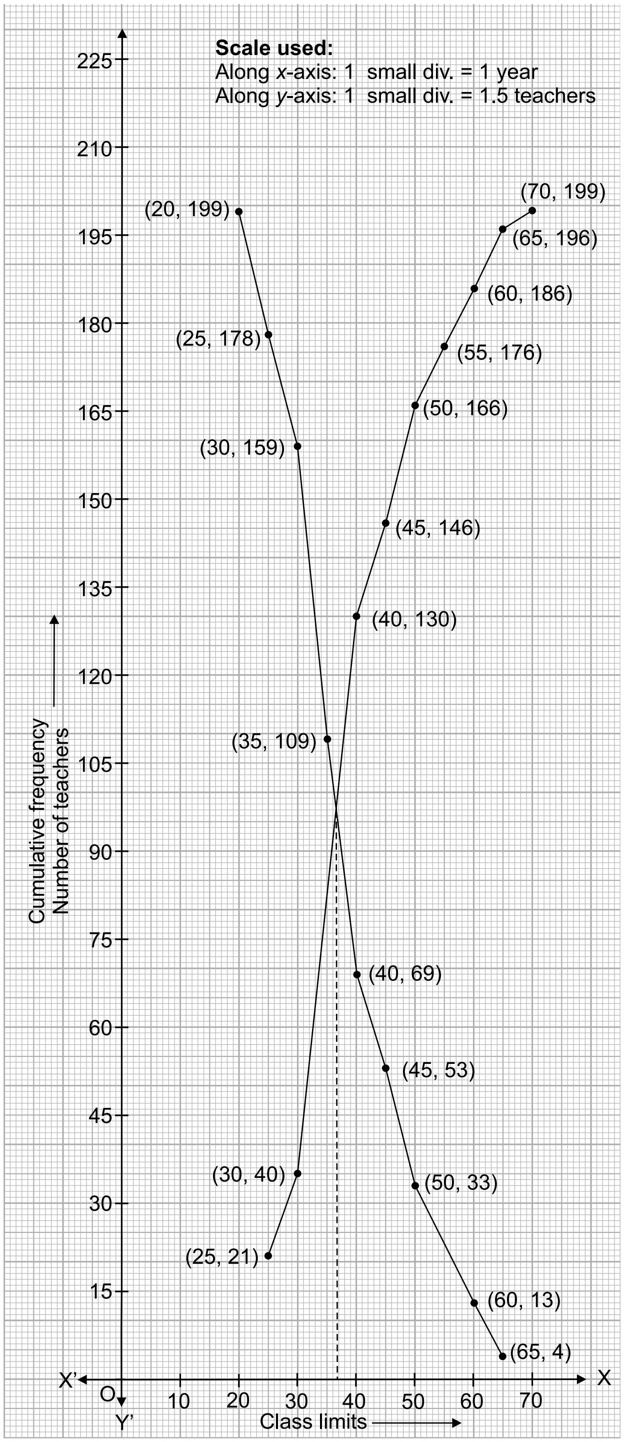
Class interval	*Frequency*		
20-25	21	More than or equal to 20	199
25-30	19	More than or equal to 25	178
30-35	50	More than or equal to 30	159
35-40	40	More than or equal to 35	109
40-45	16	More than or equal to 40	69
45-50	20	More than or equal to 45	53
50-55	10	More than or equal to 50	33
55-60	10	More than or equal to 55	23
60-65	9	More than or equal to 60	13
65-70	4	More than or equal to 65	4

Lower class limits	*Cumulative frequency*
20	199
25	178
30	159
35	109
40	69
45	53
50	33
55	23
60	13
65	4

Plotting the points (20, 199), (25, 178), (30, 159), (35, 109), (40, 69) (45, 53), (50, 33), (55, 23), (60, 13), (65, 4) and joining by a free hand curve, we get a more than ogive' as shown in the following graph:

From the point at which the two ogives intersect, drop a perpendicular on the x-axis. The x-coordinate of the point at which this perpendicular cuts the x-axis gives the median.

Median age = 36.2 years (approx.)



2. If the median of the distribution given below is 28.5, find the values of x and y.

Class interval	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60
Frequency	5	x	20	15	y	5

[CBSE SP 2011]

Sol. We prepare the cumulative frequency table, as given below.

Class interval	*Frequency* f_i	*Cumulative frequency* cf
0 – 10	5	5
10 – 20	x	$5 + x$
20 – 30	20	$(5 + x) + 20 = 25 + x$
30 – 40	15	$(25 + x) + 15 = 40 + x$
40 – 50	y	$(40 + x) + y = 40 + x + y$
50 – 60	5	$(40 + x + y) + 5 = 45 + x + y$
	$n = \Sigma f_i = 45 + x + y = 60$	

Since the total number of observation is 60.

$45 + x + y = 60 \Rightarrow x + y = 15$ ...(1)

Median is 28.5, so it lies in the class 20 – 30. So, the median class is 20 – 30.

$\therefore\ l = 20, cf = 5 + x, f = 20$ and $h = 10$

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

$$\Rightarrow \quad 28.5 = 20 + \left(\frac{\frac{60}{2} - (5 + x)}{20}\right) \times 10$$

$$\Rightarrow \quad 28.5 - 20 = \left(\frac{30 - 5 - x}{20}\right) \times 10$$

$$\Rightarrow \quad 8.5 = \frac{25 - x}{2}$$

$$\Rightarrow \quad 17 = 25 - x$$

$$\Rightarrow \quad x = 25 - 17$$

$$\Rightarrow \quad x = 8$$

$$\therefore \quad 8 + y = 15 \qquad \text{[Using (1)]}$$

$$\Rightarrow \quad y = 7$$

Hence, $x = 8$ and $y = 7$.

3. Given below is a cumulative frequency table showing the monthly savings of a group of people.

Savings in ₹ (less than)	1000	2000	3000	4000	5000	6000	7000
Cumulative frequency	15	35	64	84	96	120	192

Compute the mean savings.

Sol. Here, $h = 1000$. Let the assumed mean $a = 3500$

Savings in ₹	*Mid-value* (x_i)	*Fre-quency* (f_i)	$u_i = \frac{x_i - a}{h}$	f_iu_i	
0 – 1000	500	15	$\frac{500 - 3500}{1000} = -3$	–45	–114
1000 – 2000	1500	20	$\frac{1500 - 3500}{1000} = -2$	–40	
2000 – 3000	2500	29	$\frac{2500 - 3500}{1000} = -1$	–29	
3000 – 4000	3500	20	$\frac{3500 - 3500}{1000} = 0$	0	
4000 – 5000	4500	12	$\frac{4500 - 3500}{1000} = +1$	+12	+276
5000 – 6000	5500	24	$\frac{5500 - 3500}{1000} = +2$	+48	
6000 – 7000	6500	72	$\frac{6500 - 3500}{1000} = +3$	+216	
		$\Sigma f_i =$ 192		$\Sigma f_iu_i =$ +162	

$$\text{Mean} = \bar{x} = a + h\left(\frac{\Sigma f_iu_i}{\Sigma f_i}\right)$$

$$= 3500 + 1000\left(\frac{162}{192}\right)$$

$$= 3500 + \left(\frac{3375}{4}\right)$$

$$= 3500 + 843.75$$

$$= 4343.75$$

Hence, mean savings = ₹ 4343.75

4. Using the step deviation method, calculate the mean of the following distribution:

Class interval	*Number of companies*
0 – 50	17
50 – 100	35
100 – 150	43
150 – 200	40
200 – 250	21
250 – 300	24

[CBSE 2001 C]

Sol. Here, $h = 50$. Let the assumed mean be $a = 125$

Class interval	*Mid-value* x_i	*Frequency* f_i	$u_i = \frac{x_i - a}{h}$	f_iu_i	
0 – 50	25	17	$\frac{25 - 125}{50} = -2$	–34	– 69
50 – 100	75	35	$\frac{75 - 125}{50} = -1$	–35	
100 – 150	125	43	$\frac{125 - 125}{50} = 0$	0	
150 – 200	175	40	$\frac{175 - 125}{50} = 1$	+40	+154
200 – 250	225	21	$\frac{225 - 125}{50} = 2$	+42	
250 – 300	275	24	$\frac{275 - 125}{50} = 3$	+72	
		$\Sigma f_i = 180$		$\Sigma f_iu_i = 85$	

$$\text{Mean} = \bar{x} = a + h\left(\frac{\Sigma f_iu_i}{\Sigma f_i}\right)$$

$$= 125 + 50 \times \left(\frac{85}{180}\right)$$

$$= 125 + 23.611$$

$$= 148.611$$

Self-Assessment

(Page 254)

Multiple-Choice Questions

1. The mean of 6 numbers is 16. With the removal of a number, the mean of the remaining numbers is 17. The number removed is

(*a*) 2 (*b*) 22

(*c*) 11 (*d*) 6 **[CBSE SP 2011]**

Sol. (*c*) 11

Mean of 6 numbers = 16

∴ Total of 6 numbers = 16 × 6 = 96

Mean of 5 numbers = 17

∴ Total of 5 numbers = 17 × 5 = 85

Number removed = 96 – 85 = 11

2. The marks obtained by 60 students are tabulated below:

Marks	0–10	10–20	20–30	30–40	40–50	Total
No. of students	2	10	25	20	3	60

Then the number of students who got marks more than or equal to 40 is

(*a*) 20 (*b*) 3
(*c*) 17 (*d*) 23

Sol. (*b*) 3

The number of students who got marks more than or equal to 40 is 3.

Fill in the Blanks

3. The sum of deviation of all observations from the mean is always **zero**.

Sol. Let $x_1, x_2, x_3, ..., x_n$ be the observations and let x be the mean

Then, sum of deviations of all the observations from the mean

$$= (x_1 - \overline{x}) + (x_2 - \overline{x}) + (x_3 - \overline{x}) + (x_4 - \overline{x}) + ... + (x_n - \overline{x})$$

$$= (x_1 + x_2 + x_3 + ... + x_n) - n\overline{x}$$

$$= n\overline{x} - n\overline{x}$$

$$= 0$$

4. If the mode of data is $8x$ and its mean is $11x$, then the median will be $\mathbf{10x}$.

Sol. 3 Median = Mode + 2 Mean

$\Rightarrow$ 3 Median = $8x + 2\,(11x) = 30x$

$\Rightarrow$ Median = $\dfrac{30x}{3} = 10x$

5. The mean and median of first eleven natural numbers is **6**.

Sol. $\text{Mean} = \dfrac{1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11}{11}$

$$= \frac{66}{11} = 6$$

Here $n = 11$ (odd)

$\therefore$ Median = Value of $\dfrac{11+1}{2}$ th term,

i.e. value of 6th term = 6

6. The mean of 17 observations is 20. If the mean of first 9 observations is 23 and that of last 9 observations is 18, then the 9th observation is **29**.

Sol. Mean of 17 observations = 20

$\therefore$ Total of 17 observations = $20 \times 17 = 340$

Mean of first 9 observations = 23

$\therefore$ Total of first 9 observations = $23 \times 9 = 207$

Mean of last 9 observations = 18

$\therefore$ Total of last 9 observations = $18 \times 9 = 162$

$\therefore$ 9th observations = $207 + 162 - 340$

$369 - 340$

$= 29$

Assertion-Reason Type Questions

Directions (Q. Nos. 7 to 9): Each of these questions contains an assertion followed by reason. Read them carefully, and answer the question on the basis of the following options, select the one that best describes the two statements.

(a) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.

(b) Both Assertion and Reason are true but Reason is not the correct explanation of Assertion.

(c) If Assertion is true but Reason is false.

(d) If Assertion is false but Reason is true.

7. **Assertion:** Step deviation method and assumed mean method will give same result.

Reason: The assumed mean method and step-deviation method are just simplified forms of the direct method.

Sol. The correct answer is (*a*). Both assertion and reason are correct and reason is the correct explanation of the assertion.

8. **Assertion:** Modal class is the class having the maximum frequency.

Reason: Mean is always greater than mode.

Sol. The correct answer is (*c*). As Modal class is the class having the maximum frequency. Mean can be equal to, greater than or less than mode.

Thus, reason is wrong but assertion is correct.

9. **Assertion:** Median of odd number of data is the mid-value.

Reason: Median of even number of observations is the mean of two mid-values.

Sol. The correct answer is (*b*). Median of odd number of data is the mid-value. Median of even number of observations is the mean of two mid values.

Both assertion and reason are correct, but reason is not the correct explanation of assertion.

Case Study Based Questions

10. Vijay Sharma, a life insurance agent is working in HDFC Life Insurance Company. He has to complete his target of 100 policy holders. After completion of his target, he has found the following data for distribution of ages of 100 policy holders. Based on the given data, answer the following questions.

Age (in years)	*Number of policy holders*
Below 20	2
Below 25	6
Below 30	24
Below 35	45
Below 40	78
Below 45	89
Below 50	92
Below 55	98
Below 60	100

(*a*) What is the median class of the policy holder?

(*i*) 25–30 (*ii*) 30–35

(*iii*) 35–40 (*iv*) 40–45

Ans. (*iii*) 35–40

(*b*) Which of the following is not a measure of central tendency?

(*i*) Mean

(*ii*) Median

(*iii*) Mode

(*iv*) Standard deviation

Ans. (*iv*) Standard deviation

(*c*) The median age (in years) of the policy holder, if policies are given only to persons having age 18 years onwards but less than 60 years is

(*i*) 33 years. (*ii*) 35.76 years.

(*iii*) 37.5 years. (*iv*) 41 years.

Ans. (*ii*) 35.76 years.

(*d*) What is the frequency of the median class?

(*i*) 78 (*ii*) 21

(*iii*) 33 (*iv*) 89

Ans. (*iii*) 33

(*e*) The sum of lower limit and upper limit of median class is

(*i*) 65 (*ii*) 75

(*iii*) 85 (*iv*) 95

Ans. (*ii*) 75

11. Women's Day is a global day celebrating the social, economic, cultural and political achievements of women. On this special day, a company has organized a free medical check-up for all women employees. All thirty women in that company were examined by the doctor and the number of heart beats per minute were recorded and summarised as follows. Based on the given data, answer the following questions.

Number of heart beats per minute	*Number of women*
65–68	2
68–71	4
71–74	3
74–77	8
77–80	7
80–83	4
83–86	2

(*a*) How many women recorded more than 74 heart beats per minute?

(*i*) 8 (*ii*) 13 (*iii*) 21 (*iv*) 24

Ans. (*iii*) 21

(*b*) Estimate the modal heart beats per minute of thirty women.

(*i*) 74.5 (*ii*) 75.5

(*iii*) 76.5 (*iv*) 77.5

Ans. (*iii*) 76.5

(*c*) What is the median class of the given data?

(*i*) 71–74 (*ii*) 74–77

(*iii*) 77–80 (*iv*) 80–83

Ans. (*ii*) 74–77

(*d*) The class with the maximum frequency is said to be

(*i*) median class. (*ii*) modal class.

(*iii*) upper class. (*iv*) none of these

Ans. (*ii*) modal class.

(*e*) The difference of upper limits of modal class and median class is

(*i*) 0 (*ii*) 3 (*iii*) 77 (*iv*) 85

Ans. (*i*) 0

Very Short Answer Type Questions

12. The sum of the deviations of a number 85 from each of the observations $x_1, x_2, \ldots, x_n$ is 68 and the sum of the deviations of another number 63 from each of the same observations $x_1, x_2, \ldots, x_n$ is 112. Then find the value of n.

Sol. $(x_1 - 85) + (x_2 - 85) + (x_3 - 85) + \ldots + (x_n - 85) = 68$

$\Rightarrow (x_1 + x_2 + \ldots + x_n) - 85n = 68 \quad \ldots(1)$

$(x_1 - 63) + (x_2 - 63) + (x_3 - 63) + \ldots + (x_n - 63) = 112$

$\Rightarrow (x_1 + x_2 + \ldots + x_n) - 63n = 112 \quad \ldots(2)$

Subtracting (1) from (2), we get

$$-63n + 85n = 112 - 68$$

$$22n = 44$$

$$\Rightarrow \quad n = 2$$

13. Find the mode of the following frequency distribution:

Class interval	0 – 4	4 – 8	8 – 12	12 – 16
Frequency	4	8	5	6

[CBSE SP 2010]

Sol.

Class interval	Frequency
0 – 4	4
4 – 8	8
8 – 12	5
12 – 16	6

Maximum frequency is 8 and the class corresponding this frequency is 4 – 8.

So, the modal class is 4 – 8

$\therefore \; l = 4, f_1 = 8, f_0 = 4, f_2 = 5$ and $h = 4$

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 4 + \left(\frac{8-4}{2 \times 8 - 4 - 5}\right) \times 4$$

$$= 4 + \frac{4}{7} \times 4$$

$$= 4 + \frac{16}{7}$$

$$= 4 + 2.286$$

$$= 6.286$$

$\therefore$ Mode = 6.286 (approx.)

Short Answer Type-I Questions

14. Find the median of the following distribution:

Wages (in ₹)	*No. of labourers*
200 – 300	3
300 – 400	5
400 – 500	20
500 – 600	10
600 – 700	6

[CBSE SP 2010]

Sol. Class size = Difference between any two consecutive mid-values = 350 – 250 = 100

Mid-value 250 corresponds to class

$\left(250 - \frac{100}{2}\right) - \left(250 + \frac{100}{2}\right)$ i.e. 200 – 300

Thus, we have the following cumulative frequency table for the given data

Class interval	*Frequency*	*Cumulative frequency*
200 – 300	3	3
300 – 400	5	8
400 – 500	20	28
500 – 600	10	38
600 - 700	6	44

$$n = \Sigma f_i = 44$$

Here, $n = \Sigma f_i = 44$.

So, $\frac{n}{2} = \frac{44}{2} = 22$

Cumulative frequency just greater than $\frac{n}{2} = 22$ is 28 and the corresponding class is 400 – 500

So, the median class is 400 – 500

$\therefore \; l = 400, cf = 8, f = 20$ and $h = 100$

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

$$= 400 + \left(\frac{200 - 8}{20}\right) \times 100$$

$$= 400 + \left(\frac{14}{20}\right) \times 100$$

$$= 400 + 70 = 470$$

Hence, Median wage = ₹470

15. Prepare a frequency distribution table for the following data:

More than or equal to 70	8
More than or equal to 60	25
More than or equal to 50	40
More than or equal to 40	55
More than or equal to 30	75
More than or equal to 20	90
More than or equal to 10	100

Sol. From the given table, we prepare the frequency distribution table

Class interval	*Frequency*
10 – 20	10
20 – 30	15
30 – 40	20
40 – 50	15
50 – 60	15
60 – 70	17
70 and above	8

Short Answer Type-II Questions

16. The following distribution gives the daily income of 50 workers of a factory:

Daily Income (in ₹)	*Number of workers*
100 – 120	12
120 – 140	14
140 – 160	8
160 – 180	6
180 – 200	10

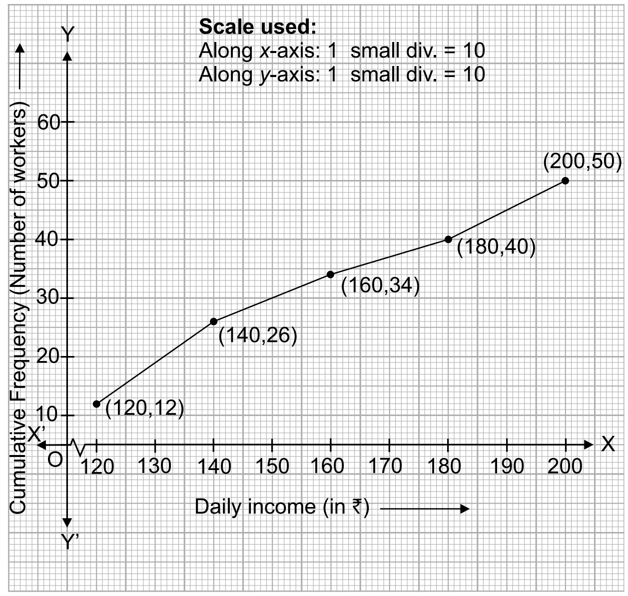
Convert the distribution above to a less than type cumulative frequency distribution and draw its ogive.

Sol. From the given table we first prepare a cumulative frequency distribution of the less than type.

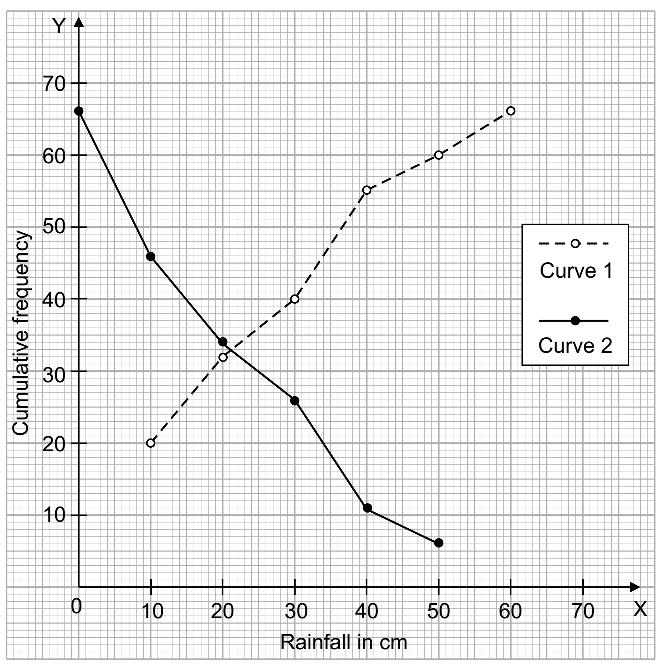
Daily income (in ₹) less than	*Number of workers (cumulative frequency)*
120	12
140	26
160	34
180	40
200	50

Upper class limit	120	140	160	180	200
Cumulative frequency	12	26	34	40	50

Plotting the points (120, 12), (140, 26), (160, 34), (180, 40), (200, 50) and joining them by a free hand curve, we get 'less than ogive'.



17. A TV reporter was given a task to prepare a report on the rainfall of the city Dispur of India in a particular year. After collecting the data, he analyzed the data and prepared a report on the rainfall of the city. Using this report, he drew the following graph for a particular time period of 66 days.



Based on the above graph, answer the following questions:

(*a*) Identify less than type ogive and more than type ogive from the given graph.

(*b*) Find the median rainfall of Dispur.

(*c*) Obtain the Mode of the data if mean rainfall is 23.4 cm. **[CBSE SP(Standard) 2019]**

Sol. (*a*) Curve 1 - Less than ogive, Curve 2 - More than ogive

(*b*) Median Rainfall = 21 cm

(*c*) 3 Median = Mode + 2 Mean

3 (21) = Mode + 2 (23.4)

$\Rightarrow$ 63 = Mode + 46.8

$\therefore$ Mode = 16.2 cm

18. Find the mean age of 100 residents of a colony from the following data:

Age (in years) (greater than or equal to)	0	10	20	30	40	50	60	70
Number of persons	100	90	75	50	25	15	5	0

[CBSE SP 2010]

Sol. Here, h = 10. Let the assumed mean be a = 35.

Class interval (Age in years)	*Mid-value* x_i	*Number of persons* f_i	$u_i = \frac{x_i - a}{h}$	f_iu_i	
0 – 10	5	100 – 90 = 10	$\frac{5-35}{10} = -3$	–30	–85
10 – 20	15	90 – 75 = 15	$\frac{15-35}{10} = -2$	–30	
20 – 30	25	75 – 50 = 25	$\frac{25-35}{10} = -1$	–25	
30 – 40	35	50 – 25 = 25	$\frac{35-35}{10} = 0$	0	
40 – 50	45	25 – 15 = 10	$\frac{45-35}{10} = 1$	10	+45
50 – 60	55	15 – 5 = 10	$\frac{55-35}{10} = 2$	20	
60 – 70	65	5 – 0 = 15	$\frac{65-35}{10} = 3$	15	
Total		$\Sigma f_i = 100$		$f_iu_i = -40$	

$$\text{Mean } \bar{x} = a + h\bar{u} = a + h\left(\frac{\Sigma f_iu_i}{\Sigma f_i}\right)$$

$$= 35 + 10 \times \frac{(-40)}{100}$$

$$= 35 - 4 = 31$$

Hence, the mean age of the resident is 31 years.

Long Answer Type Questions

19. Compute the missing frequencies f_1 and f_2 in the following data if the mean is $166\frac{9}{26}$ and the sum of observations is 52.

Class	*Frequency*
140 – 150	5
150 – 160	f_1
160 – 170	20
170 – 180	f_2
180 – 190	6
190 – 200	2

[CBSE SP 2003]

Sol. Here, h = 10. Let the assumed mean be a = 165.

Class interval	*Mid-value* (x_i)	*Frequency* (f_i)	$u_i = \frac{x_i - a}{h}$	f_iu_i
140 – 150	145	5	$\frac{145-165}{10} = -2$	–10
150 – 160	155	f_1	$\frac{155-165}{10} = -1$	$-f_1$
160 – 170	165	20	$\frac{165-165}{10} = 0$	0
170 – 180	175	f_2	$\frac{175-165}{10} = 1$	f_2
180 – 190	185	6	$\frac{185-165}{10} = 2$	+ 12
190 – 200	195	2	$\frac{195-165}{10} = 3$	+ 6
Total		$\Sigma f_i = 33 + f_1 + f_2 = 52$		$\Sigma f_iu_i = 8 - f_1 + f_2$

$$33 + f_1 + f_2 = 52$$

$$\Rightarrow \quad f_1 + f_2 = 19 \quad \ldots(1)$$

$$\text{Mean} \quad \bar{x} = a + h\bar{u}$$

$$\Rightarrow \quad \bar{x} = a + h\left(\frac{\Sigma f_iu_i}{\Sigma f_i}\right)$$

$$\Rightarrow \quad 166\frac{9}{26} = 165 + 10\left(\frac{8 - f_1 + f_2}{52}\right)$$

$$\Rightarrow \quad \frac{4325}{26} = 165 + \frac{80 - 10f_1 + 10f_2}{52}$$

$$\Rightarrow \quad \frac{4325}{26} - 165 = \frac{80 - 10f_1 + 10f_2}{52}$$

$$\Rightarrow \quad \frac{35}{26} = \frac{80 - 10f_1 + 10f_2}{52}$$

$$\Rightarrow \quad 1820 = 2080 - 260f_1 + 260f_2$$

$$\Rightarrow \quad 260f_1 - 260f_2 = 2080 - 1820$$

$$\Rightarrow \quad 260f_1 - 260f_2 = 260$$

$$\Rightarrow \quad f_1 - f_2 = 1 \quad \ldots(2)$$

Adding equation (1) and equation (2), we get
$2f_1 = 20$

$\Rightarrow \qquad f_1 = 10$

Substituting $f_1 = 10$ in equation (1), we get

$10 + f_2 = 19$

$\Rightarrow \qquad f_2 = 9$

Hence, $f_1 = 10$ and $f_2 = 9$.

20. In a public contribution towards Kerala Chief Minister's Distress Relief Fund, people contributed various amount of money. The following table gives the frequency distribution of the contributions.

Contribution (in ₹)	*Number of People*
200 – 300	12
300 – 400	18
400 – 500	35
500 – 600	42
600 – 700	50
700 – 800	45
800 – 900	20
900 – 1000	8

Locate the median of the above data using only the less than type ogive.

Sol. From the given distribution, we prepare the 'less than type' cumulative frequency distribution as shown below:

Contribution (is ₹) less than	*Number of people (Cumulative frequency)*
300	12
400	30
500	65
600	107
700	157
800	202
900	222
1000	230

Plotting the points (300, 12), (400, 30), (500, 65), (600, 107), (700, 157), (800, 202), (900, 222), (1000, 230) and joining them by a free hand curve, we get 'less than ogive'.

Median (from the graph)

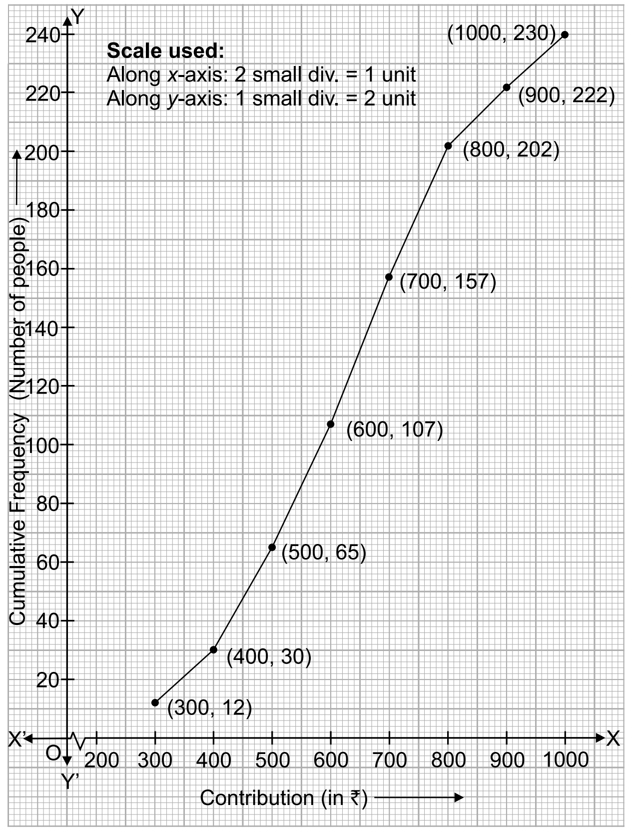
(*i*) Locate $\frac{n}{2} = \frac{230}{2} = 115.5$ on the y-axis

(*ii*) From the point located in step 1, draw a line parallel to the x-axis cutting the curve at a point.

(*iii*) From the point on the curve located in step 2, draw a perpendicular to the x-axis.

(*iv*) The point of intersection of this perpendicular with the x-axis represents the median of the data.

Hence, median contribution = ₹616.



Let's Compete

(Page 257)

Multiple-Choice Questions

1. If $u_i = \frac{x_i - 30}{15}$, $\Sigma f_i u_i = 40$ and $\Sigma f_i = 200$, then the value of $\bar{x}$ is

(*a*) 30   (*b*) 33

(*c*) 27   (*d*) 25

Sol. (*b*) 33

Here, $\qquad u_i = \frac{x_i - 30}{15}$

On comparison with $u_i = \frac{x_i - a}{h}$, we get $a = 30$, $h = 15$

Given, $\qquad \Sigma f_i u_i = 40$, $\Sigma f_i = 200$

then, $\qquad \bar{x} = a + h\left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right)$

Substituting the values, we have

$$\bar{x} = 30 + 15\left(\frac{40}{200}\right)$$

$$= 30 + 3 = 33$$

The value of $\bar{x}$ is 33.

2. Construction of a cumulative frequency table is essential in determining

(*a*) both mean and mode

(*b*) both mean and median

(*c*) mode only

(*d*) median only

Sol. (*d*) median only

3. Consider the following frequency distribution:

Class	20–25	25–30	30–35	35–40	40–45
Frequency	11	15	11	20	8

The sum of the upper limit of the modal class and the lower limit of the median class is

(*a*) 70 (*b*) 65

(*c*) 60 (*d*) 55

Sol. (*a*) 70

Class	*Frequency*	*Cumulative frequency*
20 – 25	11	11
25 – 30	15	26
30 – 35	11	37
35 – 40	20	57
40 – 45	8	65

Here $n = 65$, So, $\frac{n}{2} = \frac{65}{2} = 32.5$

Now, we locate the class whose cumulative frequency is greater than 32.5

$\therefore$ 30 – 35 is the median class.

Now, to determine modal class in the above table, the maximum class frequency is 20 and the class corresponding to this frequency is 35 – 40.

So, the modal class is 35 – 40.

$\therefore$ The upper limit of modal class is 40 and the lower limit of the median class is 30.

Hence, the sum of the upper limit of the modal class and the lower limit of the median class is 40 + 30 = 70

4. For a given set of data with 70 observations the less than ogive and the more than ogive intersect at (20.5, 35). Then the median of the set of data is

(*a*) $\frac{35+20.5}{2}$ (*b*) $\frac{35-20.5}{2}$

(*c*) 20.5 (*d*) 35 **[CBSE SP 2011]**

Sol. (*c*) 20.5

The median of the set of data is 20.5 as the median of grouped data can be obtained graphically as the x-coordinate of the point of intersection of the two ogives.

5. For a set of data consisting of 100 observations distributed into class intervals of width 10, the assumed mean is 27 and $\Sigma u_i f_i = 23$. Then the mean of the set of data is

(*a*) 28.3 (*b*) 29.3

(*c*) 29.5 (*d*) 28.5

Sol. (*b*) 29.3

Here, $a = 27$, $h = 10$

$$\therefore \quad \text{Mean } \bar{x} = a + h\left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right)$$

$$= 27 + 10\left(\frac{23}{100}\right)$$

$$= 27 + 2.3 = 29.3$$

6. If the median of some data is 7 and their mean is 5, then their mode is

(*a*) 31 (*b*) 29

(*c*) 25 (*d*) 11

Sol. (*d*) 11

Median = 7, Mean = 5

We know, 3 Median = Mode + 2 Mean

$$\therefore \quad \text{Mode} = 3\text{ Median} - 2\text{ Mean}$$

$$= 3 \times 7 - 2 \times 5$$

$$= 21 - 10 = 11$$

7. The median of the distribution

Class interval	0–10	10–20	20–30	30–40	40–50	50–60
Frequency	8	10	16	24	15	7

is

(*a*) 35.5 (*b*) 25.5

(*c*) 32.5 (*d*) 20.5

Sol. (*c*) 32.5

Class interval	*Frequency*	*Cumulative frequency*
0 – 10	8	8
10 – 20	10	18
20 – 30	16	34
30 – 40	24	58
40 – 50	15	73
50 – 60	7	80

$$n = \Sigma f_i = 80$$

Here, $n = \Sigma f_i = 80$, So $\frac{n}{2} = \frac{80}{2} = 40.$

The cumulative frequency just greater than 40 is 58 and the corresponding class is 30 – 40. So, the median class is 30 – 40.

$\therefore\ l = 30, cf = 34, f = 24$ and $h = 10$

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

$$= 30 + \left(\frac{40 - 34}{24}\right) \times 10$$

$$= 30 + \frac{6}{24} \times 10$$

$$= 30 + 2.25$$

$$= 32.25$$

8. The unknown entries x, y and z in the distribution given below

Class interval	0–10	10–30	30–60	60–80	80–90
Frequency	5	y	8	x	z
Cumulative frequency	5	26	34	39	49

are

(*a*) $x = 5, y = 21, z = 10$

(*b*) $x = 21, y = 5, z = 10$

(*c*) $x = 10, y = 21, z = 5$

(*d*) $x = 21, y = 10, z = 5$

Sol. (*a*) $x = 5, y = 21, z = 10$

Class interval	*Frequency*	*Cumulative frequency*
0 – 10	5	5
10 – 30	y	26
30 – 60	8	34
60 – 80	x	39
80 – 90	z	49

$$y = 26 - 5 = 21$$

$$x = 39 - 34 = 5$$

$$z = 49 - 39 = 10$$

9. The class mark of the median class of the following frequency distribution

Class interval	*Frequency*	*Cumulative frequency*
0 – 10	7	7
10 – 20	12	19
20 – 30	5	24
30 – 40	3	27
40 – 50	9	36
50 – 60	4	40
60 – 70	10	50

is

(*a*) 25 (*b*) 45

(*c*) 35 (*d*) 55

Sol. (*c*) 35

Here, $n = 50$, So $\frac{n}{2} = 25$

The cumulative frequency just greater than 25 is 27 and the corresponding class is 30 – 40.

So, the median class is 30 – 40.

Class mark $= \frac{30 + 40}{2} = \frac{70}{2} = 35$

10. Consider the following frequency distribution.

Height (in cm)	Less than 140	Less than 145	Less than 150	Less than 155	Less than 160	Less than 165
Number of girls	4	11	29	40	46	51

The lower limit of the modal class of the above distribution is

(*a*) 150 (*b*) 145

(*c*) 140 (*d*) 160

Sol. (*d*) 160

Height (in cm)	*Number of girls*
Below 140	4
140 – 145	11
145 – 150	29
150 – 155	40
155 – 160	46
160 – 165	51

Here, the modal class is 160 – 165.

Hence, lower limit of modal class is 160.

Value-based Questions (Optional)

(Page 258)

1. One month free medical camp was organised by some doctors for poor people. The following table shows the number of patients treated on various days.

Number of patients	0–5	5–10	10–15	15–20	20–25
Number of days	2	6	4	12	6

(*a*) Calculate the median of the above data.

(*b*) What values were depicted by the doctors who organised the medical camp?

Sol. (*a*)

Number of patients	*No. of days i.e. frequency* (f_i)	*Cumulative frequency* (*cf*)
0 – 5	2	2
5 – 10	6	8
10 – 15	4	12
15 – 20	12	24
20 – 25	6	30

Here, $n = \Sigma f_i = 30$, So, $\frac{n}{2} = \frac{30}{2} = 15$.

The cumulative frequency just greater than 15 is 24 and the corresponding class is 15 – 20. So the median class is 15 – 20.

$\therefore \quad l = 15, cf = 12, f = 12$ and $h = 5$

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

$$= 15 + \left(\frac{15 - 12}{12}\right) \times 5$$

$$= 15 + 1.25 = 16.25$$

(*b*) Commitment, empathy, concern for unprivileged people, responsible behavior, leadership and helpfulness.

2. A social organisation decided to build up a clinic in a small village that has no health care facilities. They conducted a census to collect information about the ages of people living in that village and recorded the data shown below:

Age (in years)	*Frequency (Number of people)*
0 – 10	15
10 – 20	35
20 – 30	20
30 – 40	30
40 – 50	45
50 – 60	25
60 – 70	20
70 – 80	15

(*a*) What is the total population of the village?

(*b*) How many people are 50 years or more than 50 years old?

(*c*) Calculate the mode of the above data.

(*d*) What values are exhibited by the social organisation?

Sol. (*a*) The total population of the village is

15 + 35 + 20 + 30 + 45 + 25 + 20 + 15 = 205

(*b*) People who are 50 years or more than 50 years old are 25 + 20 + 15 = 60.

(*c*) Here, the maximum frequency is 45 in the age group 40 – 50.

So, the modal class is 40 – 50.

So, $l = 40, h = 10$

$f_1 = 45, f_0 = 30, f_2 = 25$

Substitute these values in the formula,

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 40 + \left(\frac{45 - 30}{2 \times 45 - 30 - 25}\right) \times 10$$

$$= 40 + \left(\frac{15}{35}\right) \times 10$$

$$= 40 + \frac{30}{7} = 40 + 4.29$$

$$= 44.29 \text{ (approx.)}$$

(*d*) Empathy, concern for welfare of the villagers, caring and helpfulness.

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Probability

Checkpoint (Page 261)

1. If P(A) denotes the probability of an event A, then

(*a*) $P(A) > 1$ (*b*) $P(A) < 0$

(*c*) $-1 \le P(A) \le 1$ (*d*) $0 \le P(A) \le 1$

Sol. (*d*) $0 \le P(A) \le 1$

2. If A is a certain event, then

(*a*) $P(A) = 0$ (*b*) $P(A) = 1$

(*c*) $P(A) = -1$ (*d*) $P(A) > 1$

Sol. (*b*) $P(A) = 1$

3. A die is thrown once. The probability of getting a prime number is

(*a*) $\frac{1}{2}$ (*b*) $\frac{2}{3}$

(*c*) $\frac{1}{3}$ (*d*) $\frac{1}{4}$

Sol. (*a*) Total number of possible outcomes = 6

Number of favourable outcomes = {2, 3, 5} = 3

∴ P(getting a prime number)

$$= \frac{\text{Favourable outcomes}}{\text{Possible outcomes}}$$

$$= \frac{3}{6} = \frac{1}{2}$$

4. A bag contains 5 white and 7 red balls. A ball is drawn at random from the bag. Then the probability of getting a white ball is

(*a*) $\frac{1}{2}$ (*b*) $\frac{2}{3}$

(*c*) $\frac{5}{12}$ (*d*) $\frac{7}{12}$

Sol. (*c*) Total number of possible outcomes = 5 + 7 = 12

Number of white balls = 5

∴ P(white balls) = $\frac{5}{12}$

5. In a lucky draw, there are 5 prizes and 20 blanks. What is the probability of getting a prize?

Sol. Total number of possible outcomes = 5 + 20 = 25

Number of favourable outcomes = 5

∴ P(getting a prize) = $\frac{5}{25} = \frac{1}{5}$

6. If the probability of an event happening is p where $p < 1$, then what is the probability of the event not happening?

Sol. Probability of getting an event = p

Since the probability of a sure event is 1.

∴ Probability of not getting an event = $1 - p$

7. What is the probability of getting a number greater than 6 in the throwing of a die?

Sol. Total number of possible outcomes = 6

Number of favourable outcome = 0

∴ Probability of getting a number greater than 6 in the throwing of a die = $\frac{0}{6} = 0$

8. What is the probability of getting an even number more than 2 in the throwing of a die once?

Sol. Total number of possible outcomes = 6

Number of favourable outcomes = {4, 6} = 2

∴ P (getting an even number more than 2 in the throwing of a die) = $\frac{2}{6} = \frac{1}{3}$

9. In tossing a coin, what is the sum of the probability of getting a head and a tail?

Sol. Total number of possible outcomes = 2

Number of favourable outcomes = {H, T} = 2

P(sum of getting a head and a tail) = $\frac{2}{2} = 1$

10. Out of a deck of 52 cards, a card is drawn at random. What is the probability of getting a card which is king?

Sol. Since there are 4 kings out of deck of 52 cards

$$\therefore \text{P(king)} = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$

$$= \frac{4}{52} = \frac{1}{13}$$

Milestone

(Page 266)

Multiple-Choice Questions

1. If a die is thrown once, the probability of getting an even prime number is

(*a*) $\frac{1}{2}$ (*b*) $\frac{2}{6}$

(*c*) $\frac{1}{6}$ (*d*) $\frac{2}{3}$

Sol. (*c*) $\frac{1}{6}$

Favourable outcomes of event "getting an even prime number" is 2.

$\therefore$ Number of favourable outcome = 1

Total number of possible outcomes = 6

Hence, P (even prime number)

$$= \frac{\text{Number of favourable outcome}}{\text{Total number of possible outcomes}} = \frac{1}{6}$$

2. A card is drawn from a well-shuffled deck of 52 cards. The probability that the card will not be an ace is

(*a*) $\frac{12}{13}$ (*b*) $\frac{1}{13}$

(*c*) $\frac{1}{4}$ (*d*) $\frac{10}{13}$ **[CBSE 2011]**

Sol. (*a*) $\frac{12}{13}$

P(not an ace)

$$= \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$

$$= \frac{52-4}{52} = \frac{48}{52} = \frac{12}{13}$$

Very Short Answer Type Questions

3. A number which is a multiple of 4 is selected at random from the numbers 1, 2, 3, 4, 5, …, 15. Find the probability of selecting this number.

Sol. Total number of possible outcomes = 15

Outcomes favourable to the event "Multiple of 4 from the numbers 1, 2, 3, …, 15" are 4, 8, 12

$\therefore$ Number of favourable outcomes = 3

Hence, P (multiple of 4 from selected numbers)

$$= \frac{3}{15} = \frac{1}{5}$$

4. Find the probability of getting a number which is neither a prime nor a composite from the numbers 1, 4, 7, 13, 16 and 25.

Sol. Total number of possible outcomes = 6

Outcomes favourable to the event = 1

$\therefore$ Number of favourable outcomes = 1

Hence, P (neither a prime nor a composite from selected numbers) = $\frac{1}{6}$

Short Answer Type-I Questions

5. A box contains 90 discs which are numbered from 1 to 90. If one disc is drawn at random from the box, find the probability that it bears a number divisible by 5. **[CBSE 2017, SP 2011]**

Sol. Total number of possible outcomes = 90

Number divisible by 5 from 1 to 90 are 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90.

Thus, the number of favourable ways of "getting a number divisible by 5" = 18

Hence, P (a number divisible by 5) = $\frac{18}{90} = \frac{1}{5}$

6. A bag contains 5 red balls and some blue balls. If the probability of drawing a blue ball is double that of a red ball, find the number of blue balls in the bag.

Sol. Given number of red balls = 5

Let the number of blue balls be x.

Probability of drawing a red ball = $\frac{5}{x+5}$

$\therefore$ As per given condition,

$$\text{P(blue ball)} = 2\left(\frac{5}{x+5}\right)$$

$$\Rightarrow \quad \frac{x}{x+5} = \frac{10}{x+5}$$

$$\Rightarrow \quad x = 10$$

$\therefore$ Number of blue balls in the bag = 10

Short Answer Type-II Questions

7. In a single throw of two dice, find the probability that the total number of dots obtained is a multiple of 3.

Sol. The possible outcomes when two dice are thrown are as follows:

(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)
(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)
(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)
(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)
(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)
(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)

$\therefore$ Total number of possible outcomes = 36

Favourable outcomes to the event = (1, 2), (1, 5), (2, 1), (2, 4), (3, 1), (3, 6), (4, 2), (4, 5), (5, 1), (5, 4), (6, 3), (6, 6)

$\therefore$ Total number of favourable outcomes =12

Hence, P (total number of dots is a multiple of 3)

$$= \frac{12}{36} = \frac{1}{3}$$

8. Five cards – the ten, jack, queen, king and ace of diamonds are well-shuffled with their face downwards. One card is then picked up at random. If the queen is drawn and put aside, what is the probability that the second card picked up is

(*a*) an ace (*b*) a queen?

[CBSE SP 2011]

Sol. If the queen drawn is put aside, then the number of well-shuffled cards left = 4

Out of 4 well-shuffled cards, one card can be drawn in 4 ways.

$\therefore$ Total number of possible outcomes = 4

(*a*) There is only one ace in the remaining 4 cards.

$\therefore$ Number of favourable outcomes = 1

Hence, P(an ace) = $\frac{1}{4}$

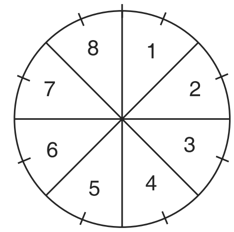
(*b*) If the queen drawn is put aside, then the four remaining well-shuffled cards do not contain any queen

$\therefore$ Number of favourable outcomes = 0

Hence, P(a queen) = $\frac{0}{4}$ = 0

Long Answer Type Questions

9. A game of chance consists of spinning an arrow which comes to rest pointing at one of the numbers 1, 2, 3, 4, 5, 6, 7, 8 and these are equally likely outcomes. What is the probability that it will point at



(*a*) 8
(*b*) an odd number
(*c*) a number greater than 2
(*d*) a number less than 9? **[CBSE SP 2011]**

Sol. The arrow can come to rest pointing at any one of the numbers 1, 2, 3, 4, 5, 6, 7, 8.

$\therefore$ Total number of possible outcomes = 8

(*a*) Number 8 is marked only once.

Thus, the number of favourable ways of 'arrow pointing at 8' is 1.

Hence, P(arrow will point at 8) = $\frac{1}{8}$

(*b*) There are four odd numbers namely 1, 3, 5, 7.

Thus, the number of favourable ways of 'arrow pointing at an odd number' = 4

Hence, P(arrow will point at an odd number)

$$= \frac{4}{8} = \frac{1}{2}$$

(*c*) There are six numbers greater than 2, namely 3, 4, 5, 6, 7, 8

Thus, the number of favourable ways of 'arrow pointing at a number greater than 2' = 6

Hence, P(arrow will point at a number greater than 2) = $\frac{6}{8} = \frac{3}{4}$

(*d*) There are eight numbers less than 9, namely 1, 2, 3, 4, 5, 6, 7, 8.

Thus, the number of favourable ways of 'arrow pointing at a number less than 9' = 8

Hence, P(arrow will point at a number less than 9) = $\frac{8}{8}$ = 1

10. A game consists of tossing a one-rupee coin 3 times and noting its outcome each time. Hanif wins if all the tosses give the same result, i.e. three heads or three tails, and lose otherwise. Calculate the probability that Hanif will lose the game.

[CBSE 2009 C, SP 2011, 2016]

Sol. When a coin is tossed three times, possible outcomes are

HHH, HHT, HTH, THH, HTT, THT, TTH, TTT

$\therefore$ Total number of possible outcomes = 8

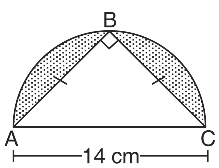
Hanif will lose the game if all the tosses do not give the same result, i.e. three heads or three tails.

So, favourable outcomes are HHT, HTH, THH, HTT, THT, TTH

Thus, the number of favourable outcomes = 6

Hence, P(Hanif will lose the game) = $\frac{6}{8} = \frac{3}{4}$

11. The figure given below shows a semicircle, in which an isosceles right triangle is drawn with its hypotenuse on the diameter of the semicircle. The diameter of the semicircle is 14 cm. A point is selected at random inside the semicircle. Find the probability that the point lies in the shaded region.



Sol. Diameter = 14 cm

$\therefore$ Radius of semicircle = $\frac{14}{2}$ cm = 7 cm

Area of semicircle = $\frac{\pi r^2}{2}$

$= \frac{1}{2} \times \frac{22}{7} \times 7 \times 7 \text{ cm}^2$

$= 77 \text{ cm}^2$

Since ΔABC is a right triangle.

$\therefore$ By Pythagoras' Theorem

$$AC^2 = AB^2 + BC^2$$

$\Rightarrow \quad 14^2 = AB^2 + AB^2 \quad [\because AB = BC]$

$\Rightarrow \quad 14^2 = 2AB^2$

$\Rightarrow \quad AB^2 = \frac{14 \times 14}{2} \text{ cm}^2 = 98 \text{ cm}^2$

$\Rightarrow \quad AB = 7\sqrt{2}$ cm

$\therefore \quad AB = AC = 7\sqrt{2}$ cm

Area of isosceles right triangle

$= \frac{1}{2} \times \text{base} \times \text{height}$

$= \frac{1}{2} \times 7\sqrt{2} \text{ cm} \times 7\sqrt{2} \text{ cm}$

$= 49 \text{ cm}^2$

Area of shaded region

= Area of semicircle – Area of triangle

$= 77 \text{ cm}^2 - 49 \text{ cm}^2$

$= 28 \text{ cm}^2$

Hence, the required probability that the point lies in the shaded region

$= \frac{\text{Area of shaded region}}{\text{Area of semicircle}}$

$= \frac{28 \text{ cm}^2}{77 \text{ cm}^2} = \frac{4}{11}$

Higher Order Thinking Skills (HOTS) Questions

(Page 267)

1. In the simultaneous throw of a pair of dice, find the probability of getting

(*a*) an even number on the first die

(*b*) 7 as the sum

(*c*) a sum greater than 10

(*d*) a sum less than 5

(*e*) an even number on one and a multiple of 3 on the other

(*f*) a doublet

(*g*) a doublet of prime numbers.

Sol. When two dice are throw, the possible outcomes are

(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)
(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)
(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)
(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)
(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)
(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)

$\therefore$ Total number of possible outcomes = 36

(*a*) Outcomes favourable to the event 'an even number on the first die' are (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)

$\therefore$ Number of favourable outcomes = 18

Hence, P(an even number on the first die)

$= \frac{18}{36} = \frac{1}{2}$

(*b*) Outcomes favourable of the event '7 as the sum' are (1, 6), (6, 1), (2, 5), (5, 2), (3, 4), (4, 3)

$\therefore$ Number of favourable outcomes = 6

Hence, P(7 as the sum) = $\frac{6}{36} = \frac{1}{6}$

(*c*) Outcomes favourable to the event 'a sum greater than 10' are (5, 6), (6, 5), (6, 6)

$\therefore$ Number of favourable outcomes = 3

Hence, P (a sum greater than 10) = $\frac{3}{36} = \frac{1}{12}$

(*d*) Outcomes favourable to the event 'a sum less than 5' are (1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (3, 1)

$\therefore$ Number of favourable outcomes = 6

Hence, P (a sum less than 5) = $\frac{6}{36} = \frac{1}{6}$

(*e*) Outcomes favourable to the event 'an even number on one and a multiple of 3 on the other' are (2, 3), (2, 6), (4, 3), (4, 6), (6, 3), (6, 6), (3, 2), (6, 2), (3, 4), (6, 4), (3, 6)

$\therefore$ Number of favourable outcomes = 11

Hence, P(an even number on one and a multiple of 3 on the other) = $\frac{11}{36}$

(*f*) Outcomes favourable to the event 'a doublet' are (1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)

$\therefore$ Number of favourable outcomes = 6

Hence, P (a doublet) = $\frac{6}{36} = \frac{1}{6}$

(*g*) Outcomes favourable to the event 'a doublet of prime numbers' are (2, 2), (3, 3), (5, 5)

$\therefore$ Number of favourable outcomes = 3

Hence, P(a doublet of prime number) = $\frac{3}{36}$

$= \frac{1}{12}$

2. Three identical coins are tossed together. What is the probability of obtaining

(*a*) all heads

(*b*) exactly two heads

(*c*) exactly one head

(*d*) at least one head

(*e*) at least two heads

(*f*) all tails?

Sol. When three identical coins are tossed, possible outcomes are HHH, HHT, HTH, THH, HTT, THT, TTH, TTT.

$\therefore$ Total number of possible outcomes = 8

(*a*) Outcome favourable to the event 'all heads' is HHH

$\therefore$ Number of favourable outcome = 1

Hence, P(all heads) = $\frac{1}{8}$

(*b*) Outcomes favourable to the event 'exactly two heads' are HHT, HTH, THH.

$\therefore$ Number of favourable outcome = 3

Hence, P(exactly two heads) = $\frac{3}{8}$

(*c*) Outcomes favourable to the event "exactly one head" are HTT, THT, TTH

$\therefore$ Number of favourable outcomes = 3

Hence, P(exactly one head) = $\frac{3}{8}$

(*d*) Outcomes favourable to the event 'at least one head 'are HHH, HHT, HTH, THH, HTT, THT, TTH

$\therefore$ Number of favourable outcomes = 7

Hence, P(at least one head) = $\frac{7}{8}$

(*e*) Outcomes favourable to the event 'atleast two heads' are HHH, HHT, HTH, THH

$\therefore$ Number of favourable outcomes = 4

Hence, P(atleast two heads) = $\frac{4}{8} = \frac{1}{2}$

(*f*) Outcome favourable to the event 'all tails" is TTT

$\therefore$ Number of favourable outcome = 1

Hence, P(all tails) = $\frac{1}{8}$

3. What is the probability that a leap year has 53 Sundays and 53 Mondays?

Sol. Leap year has 366 days = 52 weeks + 2 days possibility of remaining 2 days can be

(*i*) Monday and Tuesday,

(*ii*) Tuesday and Wednesday,

(*iii*) Wednesday and Thursday,

(*iv*) Thursday and Friday,

(*v*) Friday and Saturday,

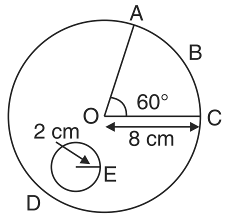
(*vi*) Saturday and Sunday,

(*vii*) Sunday and Monday.

$\therefore$ The total number of pairs = 7 of which only 1 pair of days certain Sunday and Monday

Hence, the required probability that a leap year has 53 Sundays and 53 Mondays = $\frac{1}{7}$.

4. The figure given below shows a circle with centre O and radius 8 cm. The radius of the smaller circle is 2 cm and $\angle$AOC = 60°. A point is selected at random inside the larger circle ABCD. Find the probability that the point lies neither in the sector AOCB nor in the smaller circle E.



Sol. Area of larger circle of radius 8 cm = πr^2

$$= \pi \times 8^2$$
$$= 64\pi$$

Area of sector AOCB = $\frac{\theta}{360} \times \pi r^2$

$$= \frac{60°}{360°} \times \pi \times 8^2$$
$$= \frac{\pi}{6} \times 64$$
$$= \frac{32\pi}{3}$$

Area of smaller circle of radius 2 cm = $\pi \times 2^2 = 4\pi$

$\therefore$ Desired area = Area of larger circle – [Area of sector + Area of smaller circle]

$$= 64\pi - \left[\frac{32\pi}{3} + 4\pi\right]$$
$$= 64\pi - \frac{44\pi}{3}$$
$$= \frac{148\pi}{3}$$

Hence, probability that the point lies neither in the sector AOCB nor in the smaller circle F

$$= \frac{\text{Desired area}}{\text{Total area}}$$
$$= \frac{\frac{148\pi}{3}}{64\pi} = \frac{37}{48}$$

Hence, probability that the point lie neither in the sector AOCB nor in the smaller circle E is $\frac{37}{48}$.

Self-Assessment
(Page 267)

Multiple-Choice Questions

1. The probability of getting a vowel from the word "Mathematics", chosen at random, is

(*a*) $\frac{4}{11}$ (*b*) $\frac{3}{11}$

(*c*) $\frac{3}{10}$ (*d*) $\frac{2}{11}$

Sol. (*a*) $\frac{4}{11}$

Total number of letters in the word "MATHEMATICS" = 11

$\therefore$ Total number of possible outcomes = 11

Outcomes favourable to the event are A, E, A, I.

Thus, the number of favourable outcomes = 4

$$\therefore \text{P(vowel)} = \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$
$$= \frac{4}{11}$$

2. A die is thrown once. Then the probability of getting a number between 2 and 6 is

(*a*) $\frac{1}{6}$ (*b*) $\frac{1}{3}$

(*c*) $\frac{2}{3}$ (*d*) $\frac{1}{2}$

Sol. (*d*) $\frac{1}{2}$

In a throw of a die, any one of the six faces may face upwards.

$\therefore$ The total number of possible outcomes = 6

Outcomes favourable to the event are 3, 4 and 5.

$\therefore$ P (a number between 2 and 6) = $\frac{3}{6} = \frac{1}{2}$

Fill in the Blanks

3. If an event cannot occur, then its probability is **0**.
4. The sum of the probabilities of all the events of an experiment is **1**.
5. If $\overline{E}$ denotes the complementary or negation of an event E, then the value of $P(\overline{E}) + P(E)$ is **1**.
6. If E is an event such that P(E) = $\frac{3}{7}$, then the value of P(not E) is $\mathbf{\frac{4}{7}}$.

Assertion-Reason Type Questions

Directions (Q. Nos. 7 to 9): Each of these questions contains an assertion followed by reason. Read them carefully, and answer the question on the basis of the following options, select the one that best describes the two statements.

(a) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.

(b) Both Assertion and Reason are true but Reason is not the correct explanation of Assertion.

(c) If Assertion is ture but Reason is false.

(d) If Assertion is false but Reason is true.

7. **Assertion:** Probability of getting an odd number when a die is thrown is 1/2.

Reason: There are 3 odd numbers from 1 to 6.

Sol. (a) When a dice is thrown, there are 6 possible outcomes out of which 3 are odd numbers. So, probability is $\frac{3}{6}$ or $\frac{1}{2}$. Thus both Assertion and Reason are correct and Reason is proper explanation of the Assertion.

8. **Assertion:** A card is drawn randomly from a pack of 52; probability that it would be a heart is 1/4.

Reason: Each card has a probability of 1/52 to be drawn.

Sol. (*b*) There are 4 types of cards, so each type will have probability of $\frac{1}{4}$. Individually, each card will have probability of $\frac{1}{52}$, but it does not explain why probability of hearts is $\frac{1}{4}$. Thus both Assertion and Reason are correct but Reason is not a proper explanation of the Assertion.

9. **Assertion:** A ball is drawn randomly from a box containing 4 red and 5 black balls; the drawn ball will certainly be black.

Reason: Probability of drawing a black ball from a mix of 4 red and 5 black balls is $\frac{5}{9}$.

Sol. (*d*) Probability of drawing a back ball from a mix of 4 red and 5 black balls is $\frac{5}{9}$. Thus, probability of drawing a black ball is higher but it is not 100%. Thus Assertion is wrong but Reason is correct.

Case Study Based Questions

10. A teacher conducted a fun activity in the class. She put the cards numbered from 1 to 17 in a box and mixed thoroughly. She randomly selected one student and asked to draw a card from the box.

Based on the above activity, answer the following questions.

(*a*) What is the probability that the number on the card is odd?

(*i*) $\frac{6}{17}$ (*ii*) $\frac{7}{17}$ (*iii*) $\frac{9}{17}$ (*iv*) $\frac{11}{17}$

Ans. (*iii*) $\frac{9}{17}$

(*b*) What is the probability that the number on the card is a prime number?

(*i*) $\frac{5}{17}$ (*ii*) $\frac{7}{17}$ (*iii*) $\frac{9}{17}$ (*iv*) $\frac{11}{17}$

Ans. (*ii*) $\frac{7}{17}$

(*c*) What is the formula for calculating the probability of an event?

(*i*) Number of possible outcomes/Number of favourable outcomes

(*ii*) Number of favourable outcomes × Total number of outcomes

(*iii*) Number of favourable outcomes/Number of possible outcomes

(*iv*) Number of possible outcomes – Number of favourable outcomes

Ans. (*iii*) Number of favourable outcomes/Number of possible outcomes

(*d*) What is the probability that the number on the card is divisible by 3?

(*i*) $\frac{5}{17}$ (*ii*) $\frac{7}{17}$

(*iii*) $\frac{9}{17}$ (*iv*) $\frac{11}{17}$

Ans. (*i*) $\frac{5}{17}$

(*e*) What is the probability that the number on the card is divisible by 3 and 2 both?

(*i*) $\frac{1}{17}$ (*ii*) $\frac{2}{17}$ (*iii*) $\frac{3}{17}$ (*iv*) $\frac{4}{17}$

Ans. (*ii*) $\frac{2}{17}$

11. Three friends Ekta, Ritika and Pooja were fighting to get first chance in a game. Ekta says, "Let us toss two coins. If both tails appear, Ritika will take first chance. If both heads appear, Pooja will take first chance. If one head and one tail appear, I will get the chance." Based on the above situation, answer the following questions.

(*a*) What is the probability of Ekta getting the first chance?

(*i*) $\frac{1}{4}$ (*ii*) $\frac{1}{2}$ (*iii*) $\frac{3}{4}$ (*iv*) $\frac{2}{3}$

Ans. (*ii*) $\frac{1}{2}$

(*b*) What is the probability of Ritika getting the first chance?

(*i*) $\frac{1}{4}$ (*ii*) $\frac{1}{2}$ (*iii*) $\frac{3}{4}$ (*iv*) 1

Ans. (*i*) $\frac{1}{4}$

(*c*) What is the probability of certain event?

(*i*) $\frac{1}{2}$ (*ii*) $\frac{1}{4}$ (*iii*) 1 (*iv*) 0

Ans. (*iii*) 1

(*d*) What is the probability of Pooja getting the first chance?

(*i*) $\frac{1}{2}$ (*ii*) $\frac{1}{3}$ (*iii*) $\frac{1}{4}$ (*iv*) $\frac{3}{4}$

Ans. (*iii*) $\frac{1}{4}$

(*e*) Is her decision fair?

(*i*) Yes
(*ii*) No
(*iii*) Do not say anything
(*iv*) None of these

Ans. (*ii*) No

Very Short Answer Type Questions

12. A bag contains lemon flavoured candies only. A girl takes out one candy without looking into the bag. What is the probability that she takes out

(*a*) an orange-flavoured candy
(*b*) a lemon-flavoured candy?

Sol. Let the number of lemon flavoured candy = x

Since the bag contains lemon flavoured candies only.

$\therefore$ There is no chance of getting orange – flavoured candy from the bag

(*a*) Number of favourable outcome = 0

Total number of possible outcomes = x

Hence, P(an orange flavoured candy)

$$= \frac{0}{x} = 0$$

(*b*) Number of favourable outcomes = x

Total number of possible outcomes = x

Hence, P (a lemon flavoured candy) = $\frac{x}{x}$

$$= 1$$

13. A lot of 20 bulbs contain 4 defective ones. One bulb is drawn at random from the lot once. What is the probability that it is not defective?

Sol. Total number of electric bulbs = 20

Out of 20 bulbs, one bulb can be taken out in 20 ways.

$\therefore$ Total number of possible outcomes = 20

Number of defective bulbs = 4

$\therefore$ Number of non-defective bulbs = 20 – 4 = 16

Thus, the number of favourable ways of 'getting a non-defective bulb' = 16

$\therefore$ P (non-defective bulb) = $\frac{16}{20} = \frac{4}{5}$

Short Answer Type-I Questions

14. A card is drawn at random from a well-shuffled deck of 52 cards. Find the probability that the card drawn is neither a king nor a queen. **[CBSE 2006]**

Sol. Total number of possible outcomes = 52

There are 4 kings and 4 queens in a deck of 52 cards.

So, the number of cards which are neither king nor queen = 52 – 4 – 4 = 44

Hence, P(neither a king nor a queen) = $\frac{44}{52} = \frac{11}{13}$

15. A die is thrown twice. What is the probability that

(*a*) 5 will come up at least once
(*b*) 5 will not come up either time?

[CBSE 2009, SP 2011]

Sol. (*a*) When two dice are thrown, the possible outcomes are

(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)
(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)
(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)
(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)
(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)
(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)

$\therefore$ Total number of possible outcomes = 36

Let E be the event '5 will come up at least once'.

(1, 5), (2, 5), (3, 5), (4, 5), (5, 1), (5, 2)

(5, 3), (5, 4), (5, 5), (5, 6) and (6, 5)

Thus, the number of outcomes favourable to E is 11.

$$P(E) = \frac{\text{Number of outcomes favourable to E}}{\text{Total number of possible outcomes}}$$

Hence, P(5 will come up at least once) = $\frac{11}{36}$

(*b*) $P(\bar{E}) = 1 - P(E)$

$\therefore$ P(5 will not come up either time)

= 1 – P(5 will come up atleast once)

$= 1 - \frac{11}{36} = \frac{25}{36}$

Short Answer Type-II Questions

16. In a single throw of three dice, what is the probability of getting a total of 17 or 18?

Sol. Total number of possible outcomes in a single throw of three dice = 6^3 = 216

Outcomes favourable to the event "getting a total of 17 or 18" are (5, 6, 6), (6, 5, 6) (6, 6, 5) and (6, 6, 6).

Thus, the number of favourable outcomes = 4

$\therefore$ P (a total of 17 or 18)

$$= \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$

$$= \frac{4}{216} = \frac{1}{54}$$

17. A bag contains 24 balls out of which y are blue.

(*a*) If one ball is drawn at random, what is the probability that it will be a blue ball?

(*b*) If 12 more blue balls are put in the bag, the probability of drawing a blue ball will be double that in (*a*). Find y.

Sol. (*a*) Total number of balls in the bag = 24

Out of 24 balls, one ball can be drawn in 24 ways.

$\therefore$ Total number of possible outcomes = 24

There are y blue balls in the bag.

So, the number of favourable outcomes = y

$\therefore$ P(blue ball) = $\frac{y}{24}$

(*b*) Total number of balls in the bag = 24 + 12
= 36

Out of 36 balls, one ball can be drawn in 36 ways.

Total number of possible outcomes = 36

Now, there are (y + 12) blue balls in the bag.

So, the number of favourable ways of 'drawing a blue ball' = y + 12

Hence, P (blue ball) = $\frac{y+12}{36}$

According to the problem;

$$\frac{y+12}{36} = 2\left(\frac{y}{24}\right)$$

$$\Rightarrow \quad \frac{y+12}{36} = \frac{2y}{24}$$

$$\Rightarrow \quad y = 6$$

Thus, y (initial number of blue balls in the bag) = 6

Long Answer Type Questions

18. One card is drawn from a well-shuffled deck of 52 cards. Find the probability of getting

(*a*) a non-face card **[CBSE 2016]**

(*b*) a black king or a red queen **[CBSE 2016]**

(*c*) a card of spade or an ace **[CBSE 2015]**

(*d*) neither a king nor a queen. **[CBSE 2015]**

Sol. (*a*) Total number of cards = 52

Face cards = 12

$\therefore$ Non-face cards = 52 – 12 = 40

Hence, P(non-face card) = $\frac{40}{52} = \frac{10}{13}$

(*b*) There are 2 black kings and 2 red queens in a deck of playing cards.

$\therefore$ Number of favourable outcomes = 4

Hence, P (a black king or a red queen) = $\frac{4}{52}$

$= \frac{1}{13}$

(*c*) There are 13 spade cards and 4 ace cards in a deck of playing cards

$\therefore$ Number of favourable outcomes = 16

Hence, P (a card of spade or an ace) = $\frac{16}{52}$

$= \frac{4}{13}$

(*d*) There are 4 kings and 4 queens is a deck of playing cards.

$\therefore$ Number of favourable outcomes = 8

Hence, P(neither a king nor a queen)

$= 1 - P$ (either a king or a queen)

$= 1 - \frac{8}{52}$

$= 1 - \frac{2}{13} = \frac{11}{13}$

19. A letter is chosen at random from the English alphabet. Find the probability that the letter is

(*a*) a vowel

(*b*) a consonant

(*c*) a letter of the word 'noble'.

[CBSE SP 2011, 2015]

Sol. (*a*) Total number of possible outcome = 26

Number of vowels = 5, i.e. {a, e, i, o, u}

$\therefore$ P (a vowel) $= \frac{5}{26}$

(*b*) Total number of possible outcomes = 26

Number of consonants = 21

Number of favourable outcomes = 21

$\therefore$ P (a consonant) $= \frac{21}{26}$

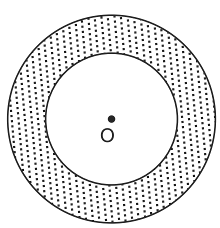
(*c*) Total number of possible outcomes = 26

Number of letter in word noble = 5

Number of favourable outcomes = 5

$\therefore$ P (a letter of word noble) $= \frac{5}{26}$

20. A practice target consists of two concentric circles whose circumferences are 44 cm and 88 cm respectively. A dart is thrown at the target. Find the probability that the dart will land in the shaded region.



Sol. Let '*r*' and '*R*' be the radius of smaller and larger circle respectively.

Given, circumference of smaller circle = 44 cm

$\Rightarrow \quad 2\pi r = 44$ cm

$\Rightarrow \quad r = \frac{44}{2\pi}$ cm

$\Rightarrow \quad r = 7$ cm

Area of smaller circle $= \pi r^2$

$= \frac{22}{7} \times 7 \times 7 \text{ cm}^2$

$= 154 \text{ cm}^2$

Again, circumference of larger circle = 88 cm

$\Rightarrow \quad 2\pi R = 88$ cm

$\Rightarrow \quad R = \frac{88}{2\pi}$ cm

$\Rightarrow \quad R = 14$ cm

$\therefore$ Area of larger circle $= \pi R^2$

$= \frac{22}{7} \times 14 \times 14 \text{ cm}^2$

$= 616 \text{ cm}^2$

$\therefore$ Area of shaded region

= Area of larger circle – Area of smaller circle

$= 616 \text{ cm}^2 - 154 \text{ cm}^2$

$= 462 \text{ cm}^2$

Hence, P(dart land in the shaded region)

$$= \frac{\text{Area of shaded region}}{\text{Area of larger circle}} = \frac{462 \text{ cm}^2}{616 \text{ cm}^2} = \frac{3}{4}$$

Let's Compete

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Multiple-Choice Questions

1. In a family of 8 children, the probability of at least one boy is

(*a*) $\frac{7}{8}$ (*b*) $\frac{1}{8}$

(*c*) $\frac{5}{8}$ (*d*) $\frac{3}{4}$ **[CBSE 2014]**

Sol. (*a*) $\frac{7}{8}$

Number of favourable outcomes = 7

Total number of possible outcomes = 8

Hence, P(at least one boy)

$$= \frac{\text{Number of favourable outcomes}}{\text{Total number of possible outcomes}}$$

$$= \frac{7}{8}$$

2. A bag contains 4 green balls and some white balls. If the probability of drawing a white ball is thrice that of a green ball, then the number of white balls in the bag is

(*a*) 8 (*b*) 6

(*c*) 12 (*d*) 10

Sol. (*c*) 12

Let the number of white ball be x

and the number of green balls = 4

Total number of balls $= x + 4$

$\therefore$ P(green ball) = $\dfrac{4}{x+4}$

According to condition,

$$\text{P(white ball)} = 3\left(\frac{4}{x+4}\right)$$

$$\Rightarrow \quad \frac{x}{x+4} = 3\left(\frac{4}{x+4}\right)$$

$$\Rightarrow \quad x = 12$$

Hence, the number of white balls in the bag = 12

3. Two dice are tossed together. Then the probability that the sum of numbers appearing on the two dice is 5 is

(*a*) $\frac{2}{3}$ (*b*) $\frac{1}{9}$

(*c*) $\frac{5}{9}$ (*d*) $\frac{2}{9}$

Sol. (*b*) $\frac{1}{9}$

When two dice are thrown, the possible outcomes are

(1, 1) (1, 2), (1, 3), (1, 4) (1, 5), (1, 6)
(2, 1) (2, 2), (2, 3), (2, 4) (2, 5), (2, 6)
(3, 1) (3, 2), (3, 3), (3, 4) (3, 5), (3, 6)
(4, 1) (4, 2), (4, 3), (4, 4) (4, 5), (4, 6)
(5, 1) (5, 2), (5, 3), (5, 4) (5, 5), (5, 6)
(6, 1) (6, 2), (6, 3), (6, 4) (6, 5), (6, 6)

$\therefore$ Total number of possible outcomes = 36

Outcomes favourable to the event "sum of number appearing on two dice is 5" are {(1, 4), (2, 3), (3, 2), (4, 1)}.

$\therefore$ Number of favourable outcomes = 4

Hence, P (sum of number is 5) = $\dfrac{4}{36} = \dfrac{1}{9}$

4. The probability that a leap year, selected at random, will have 53 Tuesdays is

(*a*) $\frac{4}{7}$ (*b*) $\frac{3}{7}$

(*c*) $\frac{1}{7}$ (*d*) $\frac{2}{7}$

Sol. (*d*) $\frac{2}{7}$

Leap year has 366 days = 52 weeks + 2 days. Possibility of remaining 2 days can be

(*i*) Monday and Tuesday,
(*ii*) Tuesday and Wednesday,
(*iii*) Wednesday and Thursday
(*iv*) Thursday and Friday,
(*v*) Friday and Saturday,
(*vi*) Saturday and Sunday,
(*vii*) Sunday and Monday,

The total number of pairs = 7 out of which only two pairs of days contain Tuesday, i.e. Monday and Tuesday, and Tuesday and Wednesday.

$\therefore$ Probability of getting 53 Tuesdays = $\dfrac{2}{7}$

5. Ankita and Reema are neighbours. They were both born in 1989. Then the probability that they have the same birthday is

(*a*) $\frac{365}{366}$ (*b*) $\frac{1}{366}$

(*c*) $\frac{1}{365}$ (*d*) $\frac{364}{365}$

Sol. (*c*) $\frac{1}{365}$

Year 1989 is a non-leap year, i.e. 365 days.

$\therefore$ Total number of possible outcomes = 365

If Ankita's birthday is on a same day as Reema's birthday, then number of favourable outcomes = 1

$\therefore$ P (same birthday) = $\dfrac{1}{365}$

6. The probability of selecting a rotten apple randomly from a heap of 900 apples is 0.18. Then the number of rotten apples in the heap is

(*a*) 150 (*b*) 152

(*c*) 160 (*d*) 162

Sol. (*d*) 162

P(rotten apples) = 0.18

Total number of apples = 900

$$\text{P (rotten apples)} = \frac{\text{Number of rotten apples}}{\text{Total number of apples}}$$

$$\Rightarrow \quad 0.18 = \frac{\text{Number of rotten apples}}{900}$$

Hence, number of rotten apples = 0.18 × 900 = 162

7. In a joint family, the number of girls are 3 more than the number of boys. The father asks one of his children at random to go to the market. If he is equally likely to have asked one of his children and the probability that he asked a girl is $\frac{3}{5}$, then the number of boys and the number of girls are respectively

(*a*) 4, 7 (*b*) 6, 9

(c) 3, 6 (d) 2, 5

Sol. (b) 6, 9

Let number of boys = x

$\therefore$ Number of girls = $x + 3$

$\therefore$ Total possible outcomes = $x + x + 3 = 2x + 3$

Given, P (he asked a girl) = $\frac{3}{5}$

$$\Rightarrow \quad \frac{x+3}{2x+3} = \frac{3}{5}$$

$$\Rightarrow \quad 3(2x+3) = 5(x+3)$$

$$\Rightarrow \quad x = 6$$

Hence, number of boys = 6

and number of girls = $x + 3 = 6 + 3 = 9$

8. The probability of selecting a red ball at random from a jar that contains only red, blue and orange balls is $\frac{1}{4}$. The probability of selecting a blue ball at random from the same jar is $\frac{1}{3}$. If the jar contains 10 orange balls, then the total number of all the balls in the jar is

(a) 24 (b) 25

(c) 30 (d) 20

Sol. (a) 24

Let the total number of balls in the jar = x

$$\text{P(red ball)} = \frac{1}{4}$$

$$\text{P(blue ball)} = \frac{1}{3}$$

Number of orange balls = 10

P(red ball) + P(blue ball) + P (orange ball) = 1

$$\Rightarrow \quad \frac{1}{4} + \frac{1}{3} + \frac{10}{\text{Total number of balls}} = 1$$

$$\Rightarrow \quad \frac{7}{12} + \frac{10}{\text{Total number of balls}} = 1$$

$$\Rightarrow \quad \frac{10}{\text{Total number of balls}} = 1 - \frac{7}{12}$$

$$\Rightarrow \quad \frac{10}{\text{Total number of balls}} = \frac{5}{12}$$

$$\Rightarrow \quad \text{Total number of balls} = \frac{10 \times 12}{5}$$

$$= 24$$

Hence, total number of balls = 24.

9. Wooden balls marked with numbers 2 to 101 are placed in a box and is mixed thoroughly. One ball is drawn from the box. The probability that the number on the ball is a perfect cube is

(a) $\frac{5}{100}$ (b) $\frac{11}{100}$

(c) $\frac{3}{100}$ (d) $\frac{9}{100}$

Sol. (c) $\frac{3}{100}$

Numbers from 2 to 101 are 2, 3, 4, …, 101

$\therefore$ Total number of possible outcomes = 100

Favourable outcome to the event "number is a perfect cube" are 8, 27 and 64.

$\therefore$ Number of favourable outcomes = 3

Hence, P (number is perfect cube) = $\frac{3}{100}$

10. Tickets numbered from 1 to 20 are mixed up in a box and then a ticket is picked up at random. The probability that the ticket has a number which is a multiple of 3 or 7 is

(a) $\frac{3}{5}$ (b) $\frac{2}{5}$

(c) $\frac{7}{20}$ (d) $\frac{9}{20}$

Sol. (b) $\frac{2}{5}$

Total number of possible outcome = 20

Numbers which are a multiple of 3 or 7 from 1 to 20 are 3, 6, 7, 9, 12, 14, 15, 18

Number of favourable outcomes = 8

Hence, P (multiple of 3 or 7) = $\frac{8}{20} = \frac{2}{5}$

Value-based Questions (Optional)

(Page 270)

1. A group consists of 12 persons, of which 3 are extremely patient, other 6 are extremely honest and rest are extremely kind. A person from the group is selected at random. Assuming that each person is equally likely to be selected, find the probability of selecting a person who is

(a) extremely patient

(b) extremely kind or honest.

Which of the above values do you prefer more?

[CBSE 2013]

Sol. Total persons = 12

Patient persons = 3

Honest persons = 6

Kind persons = 3

(a) P(extremely patient) = $\frac{3}{12} = \frac{1}{4}$

(*b*) P(extremely kind or honest) = $\frac{9}{12} = \frac{3}{4}$

Answers may vary regarding the preference of the given values.

2. A student holds free adult literacy classes once in a week in a leap year.

(*a*) What is the probability that he holds 53 classes in that year?

(*b*) What value is exhibited by the student?

Sol. (*a*) Number of days in a leap year is 366.

$\therefore$ In a leap year, there are 52 weeks and 2 more days.

For the day over 52 weeks to be a Sunday, the total number of possible outcomes = 7.

Number of favourable outcomes = 2

$$P(E) = \frac{\text{Number outcomes favourable to E}}{\text{Total number of possible outcomes}}$$

$$P(53 \text{ Sundays}) = \frac{2}{7}$$

Hence, P (student holds 53 classes) = $\frac{2}{7}$

(*b*) Empathy

3. In a class, there are 25 boys and 15 girls. One student at random has to be selected to look after the cleanliness of the class.

(*a*) What is the probability that the child selected is a girl?

(*b*) What kind of values are required in the selected child?

Sol. (*a*) Total number of children = (25 + 15) = 40

Number of girls = 15

P(child is a girl) = $\frac{15}{40} = \frac{3}{8}$

(*b*) The selected child should have the following values:

Clean habits, regularity, interpersonal relationship, confidence, critical thinking, decision making and effective communication.

4. 80 senior citizens of a colony become the members of a senior citizen club. 30% of the members go for regular morning walks, 20% go for yoga and 25% go to the gymnasium.

(*a*) Find the number of senior citizens who have not joined any physical fitness programme.

(*b*) If a senior citizen club member is chosen at random, then find the probability that he is a part of physical fitness programme.

(*c*) What is the value shown by the senior citizens who go for morning walks, yoga or gymnasium?

Sol. (*a*) Total number of members of the senior citizen club = 80.

Percentage of senior citizen who participate in any one of the physical fitness programmer = (30 + 20 + 25) % = 75%

$\therefore$ Percentage of senior citizens who do not participate in any one of the physical fitness programme = (100 – 75)% = 25%

$\therefore$ Number of senior citizens who do not participate in any physical fitness programme = 25% of 80 = $\frac{25 \times 80}{100} = 20$

(*b*) Number of senior citizens who participate in any one of the physical fitness programme = 75% of 80 = $\frac{75}{100} \times 80 = 60$

Hence, P (member is part of physical fitness programme) = $\frac{60}{80} = \frac{3}{4}$

(*c*) Awareness about advantages of physical fitness.