TEACHER'S HANDBOOK

STELLAR LEARNING

Mathematics

9

On Board! BOOKS

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Contents

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Number Systems

Checkpoint (Page 6)

1. Find a rational number between $-\frac{2}{5}$ and $\frac{1}{4}$ with the denominator as 40.

Sol. We have

$$-\frac{2}{5} = \frac{-2 \times 8}{5 \times 8} = -\frac{16}{40}$$

and

$$\frac{1}{4} = \frac{1 \times 10}{4 \times 10} = \frac{10}{40}$$

Any rational number between $-\frac{2}{5}$ and $\frac{1}{4}$ is $\frac{-3}{40}$.

Note: Here the answer is not unique. So, there may be many other rational numbers between $-\frac{2}{5}$ and $\frac{1}{4}$.

2. Will the rational number $\frac{4}{5}$ lie on the left or the right of the rational number $\frac{5}{6}$ on the number line? Explain why?

Sol. We have

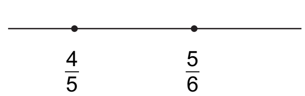
$$\frac{4}{5} = \frac{4 \times 6}{5 \times 6} = \frac{24}{30}$$

and

$$\frac{5}{6} = \frac{5 \times 5}{6 \times 5} = \frac{25}{30}$$

$\because \frac{24}{30} < \frac{25}{30}$

$\therefore \frac{4}{5} < \frac{5}{6}$



$\therefore \frac{4}{5}$ will lie on the left of the rational number $\frac{5}{6}$ on the number line.

3. Write the multiplicative inverse of $-\frac{13}{17}$.

Sol. The required multiplicative inverse of $-\frac{13}{17}$ is

$$\frac{1}{-\frac{13}{17}} = -\frac{17}{13}.$$

4. Evaluate $5\sqrt[5]{32} - 3\sqrt[4]{81}$.

Sol. We have

$$5\sqrt[5]{32} - 3\sqrt[4]{81} = 5 \times \sqrt[5]{2^5} - 3 \times \sqrt[4]{3^4}$$

$$= 5 \times 2^{\frac{5}{5}} - 3 \times 3^{\frac{4}{4}}$$

$$= 5 \times 2 - 3 \times 3$$

$$= 10 - 9$$

$$= 1$$

5. If $\frac{n5^{n+1} - 7 \times 5^{n+2}}{5^{n-1} + 7 \times 5^n} = 5$, then find the value of n, where n is a rational number.

Sol. We have

$$\frac{n5^{n+1} - 7 \times 5^{n+2}}{5^{n-1} + 7 \times 5^n} = 5$$

$$\Rightarrow \quad n5^{n+1} - 7 \times 5^{n+2} = 5(5^{n-1} + 7 \times 5^n)$$

$$= 5^n + 7 \times 5^{n+1}$$

$$\Rightarrow \quad n5^n \times 5 - 7 \times 5^n \times 25 = 5^n (1 + 7 \times 5)$$

$$5^n (5n - 175) = 5^n \times 36$$

$$\Rightarrow \quad 5n = 36 + 175$$

$$\therefore \quad n = \frac{211}{5}$$

$$= 42.2$$

$\therefore$ The required value of n is 42.2.

6. Evaluate $\left(\frac{1024}{3125}\right)^{-\frac{3}{5}}$.

Sol. We have

$$\left(\frac{1024}{3125}\right)^{-\frac{3}{5}} = \left(\frac{3125}{1024}\right)^{\frac{3}{5}}$$

$$= \frac{\left(5^5\right)^{\frac{3}{5}}}{\left(2^{10}\right)^{\frac{3}{5}}} = \frac{5^{5\times\frac{3}{5}}}{2^{10\times\frac{3}{5}}}$$

$$= \frac{5^3}{2^6} = \frac{125}{64}$$

7. A drum full of rice weighs $31\frac{1}{6}$ kg. If the weight of the empty drum is $11\frac{3}{4}$ kg, then find the weight of rice in the drum.

Sol. The required weight of the rice in the drum = Weight of drum full of rice – Weight of empty drum

$$= \left(31\frac{1}{6} - 11\frac{3}{4}\right) \text{ kg}$$

$$= \left(\frac{187}{6} - \frac{47}{4}\right) \text{ kg}$$

$$= \frac{374 - 141}{12} \text{ kg}$$

$$= \frac{233}{12} \text{ kg}$$

$$= 19\frac{5}{12} \text{ kg}$$

Hence, the weight of rice in the drum is $19\frac{5}{12}$ kg.

8. Solve for x:

$$25^{2x+1} \times 125^{-x+2} = 5$$

Sol. We have

$$25^{2x+1} \times 125^{-x+2} = 5$$

$\Rightarrow$ $(5^2)^{2x+1} \times (5^3)^{-x+2} = 5$

$\Rightarrow$ $5^{4x+2} \times 5^{-3x+6} = 5$

$\Rightarrow$ $5^{4x+2-3x+6} = 5$

$\Rightarrow$ $5^{x+8} = 5^1$

$\Rightarrow$ $x + 8 = 1$

$\Rightarrow$ $x = 1 - 8$

$= -7$

$\therefore$ The required value of x is –7.

9. Find the value of x, if $\frac{8}{13} + x = \frac{16}{3}$.

Sol. We have

$$\frac{8}{13} + x = \frac{16}{3}$$

$$x = \frac{16}{3} - \frac{8}{13}$$

$$= \frac{208 - 24}{39} = \frac{184}{39}$$

$\therefore$ The required value of x is $\frac{184}{39}$.

10. What is the value of $\sqrt[3]{\sqrt[4]{x}}$ when $x = 4096$?

Sol. We have

$$\sqrt[3]{\sqrt[4]{x}} = x^{\frac{1}{3}\times\frac{1}{4}}$$

$$= x^{\frac{1}{12}} = 4096^{\frac{1}{12}}$$

$$= \left(2^{12}\right)^{\frac{1}{12}} = 2$$

$\therefore$ The required value of the expression is 2.

Milestone

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Multiple-Choice Questions

1. A rational number between 2 and 3 is

(*a*) 2.010010001 . . . (*b*) $\sqrt{6}$

(*c*) $\frac{5}{2}$ (*d*) $4 - \sqrt{2}$ **[CBSE SP 2013]**

Sol. (*c*) $\frac{5}{2}$

A rational number between 2 and 3 is $\frac{2+3}{2} = \frac{5}{2}$.

2. If $x(5 + \sqrt{7})$ is a rational number, then x must be equal to

(*a*) $5+\sqrt{7}$ (*b*) $\sqrt{7}-5$

(*c*) $\sqrt{7}+\sqrt{5}$ (*d*) $7+\sqrt{5}$

Sol. (*b*) $\sqrt{7} - 5$

We know that

$$\left(\sqrt{7}+5\right)\left(\sqrt{7}-5\right) = 7 - 25 = -18$$

which is a rational number.

$\therefore$ The required value of x is $\sqrt{7} - 5$.

Very Short Answer Type Questions

3. If $a = b^x$, $b = c^y$ and $a = c^z$, then show that $z = xy$.

Sol. We have

$$a = b^x = (c^y)^x = c^{xy} \quad \ldots(1)$$

Also, $$a = c^z \quad \ldots(2)$$

$\therefore$ $$c^z = c^{xy} \quad \text{[From (1) and (2)]}$$

$\Rightarrow$ $$z = xy$$

Hence, proved.

4. If $(\sqrt{a^3})^{\frac{2}{3}} \div \sqrt[12]{(a^4)^{\frac{1}{3}}} = x$, then prove that $\sqrt[8]{x^9} = a$.

Sol. We have

$$\Rightarrow \left(\sqrt{a^3}\right)^{\frac{2}{3}} \div \sqrt[12]{\left(a^4\right)^{\frac{1}{3}}} = x$$

$$\Rightarrow a^{\frac{3}{2}\times\frac{2}{3}} \div a^{\frac{4}{3}\times\frac{1}{12}} = x$$

$$\Rightarrow a \div a^{\frac{1}{9}} = x$$

$$\Rightarrow a^{\frac{8}{9}} = x$$

$$\therefore a = x^{\frac{9}{8}} = \sqrt[8]{x^9}$$

Hence, proved.

Short Answer Type-I Questions

5. If $\sqrt{3} = 1.732$, find the approximate value of $\frac{2+\sqrt{3}}{\sqrt{3}}$.

Sol. We have

$$\frac{2+\sqrt{3}}{\sqrt{3}} = \frac{\left(2+\sqrt{3}\right)\sqrt{3}}{\sqrt{3}\times\sqrt{3}}$$

$$= \frac{3+2\sqrt{3}}{3}$$

$$= 1 + \frac{2}{3}\times\sqrt{3}$$

$$= 1 + \frac{2\times 1.732}{3}$$

$$= 1 + \frac{3.464}{3}$$

$$= 1 + 1.155$$

$$= 2.155 \text{ (approx.)}$$

6. If $\frac{5+2\sqrt{3}}{7+4\sqrt{3}} = x + y\sqrt{3}$, where x and y are rational numbers, prove that $\frac{x}{y} = -\frac{11}{6}$.

Sol. We have

$$\frac{5+2\sqrt{3}}{7+4\sqrt{3}} = x + y\sqrt{3}$$

$$\Rightarrow \frac{\left(5+2\sqrt{3}\right)\left(7-4\sqrt{3}\right)}{7^2-\left(4\sqrt{3}\right)^2} = x + y\sqrt{3}$$

$$\Rightarrow \frac{35-24+14\sqrt{3}-20\sqrt{3}}{49-48} = x + y\sqrt{3}$$

$$\Rightarrow 11 - 6\sqrt{3} = x + y\sqrt{3}$$

$$\therefore x = 11 \text{ and } y = -6$$

$$\therefore \frac{x}{y} = -\frac{11}{6}$$

Hence, proved.

Short Answer Type-II Questions

7. Express $\frac{\sqrt{7}+\sqrt{2}}{9+2\sqrt{14}}$ with a rational denominator.

Sol. We have

$$\frac{\sqrt{7}+\sqrt{2}}{9+2\sqrt{14}} = \frac{\left(\sqrt{7}+\sqrt{2}\right)\left(9-2\sqrt{14}\right)}{9^2-\left(2\sqrt{14}\right)^2}$$

$$= \frac{9\sqrt{7}+9\sqrt{2}-2\sqrt{98}-2\sqrt{28}}{81-56}$$

$$= \frac{9\sqrt{7}+9\sqrt{2}-14\sqrt{2}-4\sqrt{7}}{25}$$

$$= \frac{5\sqrt{7}-5\sqrt{2}}{25}$$

$$= \frac{5\left(\sqrt{7}-\sqrt{2}\right)}{25}$$

$$= \frac{\sqrt{7}-\sqrt{2}}{5}$$

8. Simplify:

$$\frac{1}{2+\sqrt{5}} + \frac{1}{\sqrt{5}+\sqrt{6}} + \frac{1}{\sqrt{6}+\sqrt{7}} + \frac{1}{\sqrt{7}+\sqrt{8}}$$

[CBSE SP 2011]

Sol. We have

$$\frac{1}{2+\sqrt{5}} = \frac{2-\sqrt{5}}{2^2-\left(\sqrt{5}\right)^2}$$

$$= \frac{2-\sqrt{5}}{4-5} = \sqrt{5}-2 \quad \ldots(1)$$

$$\frac{1}{\sqrt{5}+\sqrt{6}} = \frac{\sqrt{5}-\sqrt{6}}{\left(\sqrt{5}\right)^2-\left(\sqrt{6}\right)^2}$$

$$= \frac{\sqrt{5}-\sqrt{6}}{5-6} = \sqrt{6}-\sqrt{5} \quad \ldots(2)$$

$$\frac{1}{\sqrt{6}+\sqrt{7}} = \frac{\sqrt{6}-\sqrt{7}}{(\sqrt{6})^2-(\sqrt{7})^2}$$

$$= \frac{\sqrt{6}-\sqrt{7}}{6-7} = \sqrt{7}-\sqrt{6} \quad \ldots(3)$$

$$\frac{1}{\sqrt{7}+\sqrt{8}} = \frac{\sqrt{7}-\sqrt{8}}{(\sqrt{7})^2-(\sqrt{8})^2}$$

$$= \frac{\sqrt{7}-\sqrt{8}}{7-8} = \sqrt{8}-\sqrt{7} \quad \ldots(4)$$

$$\frac{1}{2+\sqrt{5}}+\frac{1}{\sqrt{5}+\sqrt{6}}+\frac{1}{\sqrt{6}+\sqrt{7}}+\frac{1}{\sqrt{7}+\sqrt{8}}$$

$$= \sqrt{5}-2+\sqrt{6}-\sqrt{5}+\sqrt{7}-\sqrt{6}+\sqrt{8}-\sqrt{7}$$

[From (1), (2), (3) and (4)]

$$= \sqrt{8}-2 = 2\sqrt{2}-2$$

$$= 2(\sqrt{2}-1)$$

Long Answer Type Questions

9. If $a=\frac{\sqrt{5}+\sqrt{10}}{\sqrt{10}-\sqrt{5}}$ and $b = \frac{\sqrt{10}-\sqrt{5}}{\sqrt{10}+\sqrt{5}}$, show that $\sqrt{a}-\sqrt{b}-2\sqrt{ab}=0.$ **[CBSE SP 2013]**

Sol. We have

$$a = \frac{(\sqrt{10}+\sqrt{5})^2}{(\sqrt{10})^2-(\sqrt{5})^2}$$

$$= \frac{(\sqrt{10}+\sqrt{5})^2}{10-5}$$

$$= \frac{(\sqrt{10}+\sqrt{5})^2}{5} \quad \ldots(1)$$

$$b = \frac{(\sqrt{10}-\sqrt{5})^2}{(\sqrt{10})^2-(\sqrt{5})^2}$$

$$= \frac{(\sqrt{10}-\sqrt{5})^2}{10-5}$$

$$= \frac{(\sqrt{10}-\sqrt{5})^2}{5} \quad \ldots(2)$$

$$\therefore \quad \sqrt{a} = \frac{\sqrt{10}+\sqrt{5}}{\sqrt{5}}$$

and $$\sqrt{b} = \frac{\sqrt{10}-\sqrt{5}}{\sqrt{5}}$$

$$\therefore \quad \sqrt{ab} = \frac{(\sqrt{10})^2-(\sqrt{5})^2}{5}$$

$$= \frac{10-5}{5} = 1 \quad \ldots(3)$$

$\therefore \quad \sqrt{a}-\sqrt{b}-2\sqrt{ab}$

$$= \frac{\sqrt{10}+\sqrt{5}-\sqrt{10}+\sqrt{5}}{\sqrt{5}} - 2\times 1$$

[From (1), (2) and (3)]

$$= \frac{2\sqrt{5}}{\sqrt{5}} - 2\times 1$$

$$= 2-2 = 0$$

Hence, proved.

10. If $\frac{3+\sqrt{7}}{3-4\sqrt{7}} = a+b\sqrt{7}$, where a and b are rational numbers, find the values of a and b.

[CBSE SP 2013]

Sol. We have

$$\frac{3+\sqrt{7}}{3-4\sqrt{7}} = a+b\sqrt{7}$$

$$\Rightarrow \quad \frac{(3+\sqrt{7})(3+4\sqrt{7})}{3^2-(4\sqrt{7})^2} = a+b\sqrt{7}$$

$$\Rightarrow \quad \frac{9+28+12\sqrt{7}+3\sqrt{7}}{9-112} = a+b\sqrt{7}$$

$$\Rightarrow \quad \frac{37+15\sqrt{7}}{-103} = a+b\sqrt{7}$$

$$\therefore \quad a = -\frac{37}{103}$$

and $$b = -\frac{15}{103}$$

$\therefore$ The required value of a is $-\frac{37}{103}$ and b is $-\frac{15}{103}$.

11. Find the value of

$$\frac{\sqrt{\sqrt{5}+2}+\sqrt{\sqrt{5}-2}}{\sqrt{\sqrt{5}+1}}.$$ **[CBSE SP 2011]**

Sol. Let $x = \frac{\sqrt{\sqrt{5}+2}+\sqrt{\sqrt{5}-2}}{\sqrt{\sqrt{5}+1}}$

Then,

$$x^2 = \frac{\sqrt{5}+2+\sqrt{5}-2+2\sqrt{(\sqrt{5}+2)(\sqrt{5}-2)}}{\sqrt{5}+1}$$

$$= \frac{2\sqrt{5} + 2\sqrt{5-4}}{\sqrt{5}+1}$$

$$= \frac{2\left(\sqrt{5}+1\right)}{\sqrt{5}+1} = 2$$

$\therefore$ $x = \sqrt{2}$ (Taking the positive value only)

$\therefore$ The required value of the given expression is $\sqrt{2}$.

12. If $x = 7 + \sqrt{40}$, find the value of $\sqrt{x} + \frac{1}{\sqrt{x}}$.

[CBSE SP 2011]

Sol. We have

$$\begin{aligned} x &= 7 + \sqrt{40} \\ &= 5 + 2 + 2\sqrt{10} \\ &= \left(\sqrt{5}\right)^2 + \left(\sqrt{2}\right)^2 + 2\sqrt{5} \times \sqrt{2} \\ &= \left(\sqrt{5} + \sqrt{2}\right)^2 \end{aligned}$$

$$\therefore \quad \sqrt{x} = \sqrt{5} + \sqrt{2}$$

and

$$\begin{aligned} \frac{1}{\sqrt{x}} &= \frac{1}{\sqrt{5}+\sqrt{2}} \\ &= \frac{\sqrt{5}-\sqrt{2}}{\left(\sqrt{5}\right)^2 - \left(\sqrt{2}\right)^2} \\ &= \frac{\sqrt{5}-\sqrt{2}}{5-2} \end{aligned}$$

$$\begin{aligned} \therefore \quad \sqrt{x} + \frac{1}{\sqrt{x}} &= \sqrt{5} + \sqrt{2} + \frac{\sqrt{5}-\sqrt{2}}{3} \\ &= \frac{4\sqrt{5} + 2\sqrt{2}}{3} \\ &= \frac{2\left(2\sqrt{5} + \sqrt{2}\right)}{3} \\ &= \frac{2}{3}\left(2\sqrt{5} + \sqrt{2}\right) \end{aligned}$$

$\therefore$ The required value of $\sqrt{x} + \frac{1}{\sqrt{x}}$ is $\frac{2}{3}\left(2\sqrt{5} + \sqrt{2}\right)$.

Higher Order Thinking Skills (HOTS) Questions

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1. If $x = \frac{2\sqrt{5}+3\sqrt{2}}{2\sqrt{5}-3\sqrt{2}}$, find the value of $x^3 - \frac{1}{x^3}$.

Sol. We have

$$\begin{aligned} x &= \frac{2\sqrt{5}+3\sqrt{2}}{2\sqrt{5}-3\sqrt{2}} \\ &= \frac{\left(2\sqrt{5}+3\sqrt{2}\right)\left(2\sqrt{5}+3\sqrt{2}\right)}{\left(2\sqrt{5}-3\sqrt{2}\right)\left(2\sqrt{5}+3\sqrt{2}\right)} \\ &= \frac{\left(2\sqrt{5}+3\sqrt{2}\right)^2}{\left(2\sqrt{5}\right)^2 - \left(3\sqrt{2}\right)^2} \\ &= \frac{20 + 18 + 2 \times 2\sqrt{5} \times 3\sqrt{2}}{20 - 18} \\ &= \frac{38 + 12\sqrt{10}}{2} \end{aligned}$$

$$\begin{aligned} \therefore \quad x - \frac{1}{x} &= 19 + 6\sqrt{10} - \frac{1}{19 + 6\sqrt{10}} \\ &= 19 + 6\sqrt{10} - \frac{19 - 6\sqrt{10}}{19^2 - \left(6\sqrt{10}\right)^2} \\ &= 19 + 6\sqrt{10} - \frac{19 - 6\sqrt{10}}{361 - 360} \\ &= 19 + 6\sqrt{10} - 19 + 6\sqrt{10} \\ &= 12\sqrt{10} \qquad \ldots(1) \end{aligned}$$

$$\begin{aligned} \therefore \quad x^3 - \frac{1}{x^3} &= \left(x - \frac{1}{x}\right)^3 + 3\left(x - \frac{1}{x}\right) \\ &= \left(12\sqrt{10}\right)^3 + 3 \times 12\sqrt{10} \qquad \text{[From (1)]} \\ &= 17280\sqrt{10} + 36\sqrt{10} \\ &= 17316\sqrt{10} \end{aligned}$$

$\therefore$ The required value of $x^3 - \frac{1}{x^3}$ is $17316\sqrt{10}$.

2. If $x = \frac{1}{4+\sqrt{15}}$, find the value of

$x^4 - 16x^3 + 66x^2 - 16x + 72$.

Sol. We have

$$\begin{aligned} x &= \frac{1}{4+\sqrt{15}} \\ &= \frac{4-\sqrt{15}}{4^2 - \left(\sqrt{15}\right)^2} \\ &= \frac{4-\sqrt{15}}{16-15} \\ &= 4 - \sqrt{15} \end{aligned}$$

$$\therefore \quad (x-4)^2 = \left(-\sqrt{15}\right)^2 = 15$$

$\Rightarrow \quad x^2 - 8x + 16 - 15 = 0$

$\Rightarrow \quad x^2 = 8x - 1 \quad \ldots(1)$

$\therefore\ x^4 - 16x^3 + 66x^2 - 16x + 72$

$= (x^2)^2 - 16x^2.\ x + 66x^2 - 16x + 72$

$= (8x - 1)^2 - 16(8x - 1)x + 66\ (8x - 1) - 16x + 72$ [From (1)]

$= 64x^2 + 1 - 16x - 128x^2 + 16x + 528x - 66 - 16x + 72$

$= -64x^2 + 512x + 7$

$= -64\ (8x - 1) + 512x + 7$ [From (1)]

$= -512x + 64 + 512x + 7 = 71$

$\therefore$ The required value of the given expression is 71.

3. If $x = \dfrac{1}{5+3\sqrt{3}}$ and $y = \dfrac{1}{5-3\sqrt{3}}$, show that $xy^2 + x^2y = \dfrac{5}{2}$.

Sol. We have

$$x = \frac{1}{5+3\sqrt{3}} = \frac{5-3\sqrt{3}}{5^2-\left(3\sqrt{3}\right)^2} = \frac{5-3\sqrt{3}}{25-27} = \frac{3\sqrt{3}-5}{2}$$

and

$$y = \frac{1}{5-3\sqrt{3}} = \frac{5+3\sqrt{3}}{5^2-\left(3\sqrt{3}\right)^2} = \frac{5+3\sqrt{3}}{25-27} = \frac{-5-3\sqrt{3}}{2}$$

$$\therefore \quad x + y = \frac{3\sqrt{3}-5-5-3\sqrt{3}}{2} = -5 \quad \ldots(1)$$

Also,

$$xy = \frac{5-3\sqrt{3}}{2}\times\frac{5+3\sqrt{3}}{2} = \frac{5^2-\left(3\sqrt{3}\right)^2}{4} = \frac{25-27}{4} = -\frac{1}{2} \quad \ldots(2)$$

$$\therefore \quad xy^2 + x^2y = xy(x+y) = 5\times\frac{1}{2} \quad \text{[From (1) and (2)]}$$

$$= \frac{5}{2}$$

Hence, proved.

4. $\dfrac{\sqrt{a+2b}+\sqrt{a-2b}}{\sqrt{a+2b}-\sqrt{a-2b}} = x$, prove that $bx^2 - ax + b = 0$.

Sol. We have

$$\frac{x}{1} = \frac{\sqrt{a+2b}+\sqrt{a-2b}}{\sqrt{a+2b}-\sqrt{a-2b}}$$

$$\Rightarrow \frac{x+1}{x-1} = \frac{\sqrt{a+2b}+\sqrt{a-2b}+\sqrt{a+2b}-\sqrt{a-2b}}{\sqrt{a+2b}+\sqrt{a-2b}-\sqrt{a+2b}+\sqrt{a-2b}}$$

[By applying componendo and dividendo on both sides]

$$\Rightarrow \frac{x+1}{x-1} = \frac{2\sqrt{a+2b}}{2\sqrt{a-2b}} = \frac{\sqrt{a+2b}}{\sqrt{a-2b}}$$

$$\Rightarrow \frac{(x+1)^2}{(x-1)^2} = \frac{\left(\sqrt{a+2b}\right)^2}{\left(\sqrt{a-2b}\right)^2} = \frac{a+2b}{a-2b}$$

$$\Rightarrow \frac{(x+1)^2+(x-1)^2}{(x+1)^2-(x-1)^2} = \frac{a+2b+a-2b}{a+2b-a+2b}$$

[By applying componendo and dividendo on both sides]

$$\Rightarrow \frac{2\left(x^2+1\right)}{4x} = \frac{2a}{4b}$$

$$\Rightarrow \frac{x^2+1}{x} = \frac{a}{b}$$

$$\Rightarrow bx^2 + b = ax$$

$$\Rightarrow bx^2 - ax + b = 0$$

Hence, proved.

5. Find the value of

$$\frac{1+x}{1+\sqrt{1+x}} + \frac{1-x}{1-\sqrt{1-x}} \text{ when } x = \frac{\sqrt{3}}{2}.$$

Sol. We have

$$1 + x = 1 + \frac{\sqrt{3}}{2} = \frac{2+\sqrt{3}}{2} = \frac{4+2\sqrt{3}}{4} = \frac{3+1+2\sqrt{3}}{4} = \frac{\left(\sqrt{3}\right)^2 + 1^2 + 2\sqrt{3}\times 1}{2^2}$$

$$= \left(\frac{\sqrt{3}+1}{2}\right)^2 \quad \ldots(1)$$

Similarly, $1 - x = \left(\frac{\sqrt{3}-1}{2}\right)^2 \quad \ldots(2)$

$$\therefore \frac{1+x}{1+\sqrt{1+x}} + \frac{1-x}{1-\sqrt{1-x}}$$

$$= \frac{\frac{\left(\sqrt{3}+1\right)^2}{4}}{1+\frac{\sqrt{3}+1}{2}} + \frac{\frac{\left(\sqrt{3}-1\right)^2}{4}}{1-\frac{\sqrt{3}-1}{2}} \quad \text{[From (1) and (2)]}$$

$$= \frac{\left(\sqrt{3}+1\right)^2}{4+2\left(\sqrt{3}+1\right)} + \frac{\left(\sqrt{3}-1\right)^2}{4-2\left(\sqrt{3}-1\right)}$$

$$= \frac{\left(\sqrt{3}\right)^2+1+2\sqrt{3}}{6+2\sqrt{3}} + \frac{\left(\sqrt{3}\right)^2+1-2\sqrt{3}}{6-2\sqrt{3}}$$

$$= \frac{4+2\sqrt{3}}{6+2\sqrt{3}} + \frac{4-2\sqrt{3}}{6-2\sqrt{3}}$$

$$= \frac{2+\sqrt{3}}{3+\sqrt{3}} + \frac{2-\sqrt{3}}{3-\sqrt{3}}$$

$$= \frac{\left(2+\sqrt{3}\right)\left(3-\sqrt{3}\right)+\left(2-\sqrt{3}\right)\left(3+\sqrt{3}\right)}{\left(3+\sqrt{3}\right)\left(3-\sqrt{3}\right)}$$

$$= \frac{6-2\sqrt{3}+3\sqrt{3}-3+6+2\sqrt{3}-3\sqrt{3}-3}{3^2-\left(\sqrt{3}\right)^2}$$

$$= \frac{6}{9-3} = \frac{6}{6} = 1$$

Hence, the required value of the given expression is 1.

Self-Assessment

(Page 12)

Multiple-Choice Questions

1. A rational number which is equivalent to $\frac{2}{13}$ is

(*a*) $\frac{4}{13}$ (*b*) $\frac{4}{26}$

(*c*) $\frac{13}{2}$ (*d*) $\frac{2}{26}$

Sol. (*b*) $\frac{4}{26}$

We have $\frac{2}{13} = \frac{2 \times 2}{13 \times 2} = \frac{4}{26}$

2. A number which is irrational is

(*a*) $0.\overline{142857}$ (*b*) 2.3785

(*c*) $0.0\overline{53}$ (*d*) 3.050050005 …

Sol. (*d*) 3.050050005….

We know that any non-recurring and non-terminating number is an irrational number.

Fill in the Blanks

3. π is an irrational number because its decimal expansion is **non-terminating non-repeating**.

4. Every point on a number line represents **a unique real** number.

5. The value of $\sqrt{(3^{-2})}$ is $\mathbf{\frac{1}{3}}$.

Sol. $\sqrt{(3^{-2})} = \sqrt{\frac{1}{3^2}} = \frac{1}{3}$

6. The value of $\frac{2^0+7^0}{5^0}$ is **2**.

Sol. $\frac{2^0+7^0}{5^0} = \frac{1+1}{1} = 2$

Assertion-Reason Type Questions

Directions (Q. Nos. 7 to 10): Each of these questions contains an assertion followed by reason. Read them carefully, and answer the question on the basis of the following options, select the one that best describes the two statements.

(a) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.

(b) Both Assertion and Reason are true but Reason is not the correct explanation of Assertion.

(c) Assertion is true but Reason is false.

(d) Assertion is false but Reason is true.

7. **Assertion:** 2/6 and 3/9 are equivalent rational numbers.

Reason: Both 2/6 and 3/9 are equal to 1/3 in their lowest form.

Sol. The correct answer is (a). Both the Assertion and Reason are correct and Reason is correct explanation of Assertion, since all the fractions are equivalent.

8. **Assertion:** 7 is a rational number.

Reason: The square roots of all positive integers are irrationals.

Sol. The correct answer is (c). 7 is a rational number, as it can be written as $\frac{7}{1}$, which is a rational number.

The square roots of all positive integers are not irrational. For example , $\sqrt{4} = \pm 2$, where 2 and –2 are both rational numbers.

Thus, Assertion is correct but Reason is incorrect.

9. **Assertion:** $\frac{13^5}{13^2} = 13^7$

Reason: If $a > 0$ be real number and m and n be rational numbers, then $\frac{a^m}{a^n} = a^{m-n}$

Sol. The correct answer is (d). Assertion is incorrect, as $\frac{13^5}{13^2} \neq 13^7$, it is $\frac{13^5}{13^2} = 13^{5-2} = 13^3$.

Reason is correct, as it explains the concept.

Thus, Assertion is incorrect but Reason is correct.

10. **Assertion:** $\sqrt{3}$ is an irrational number.

Reason: A number which is not in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$ is called an irrational number.

Sol. The correct answer is (a).

$\sqrt{3}$ is an irrational number because it cannot be written in the form $\frac{p}{q}$. Thus, both Assertion and Reason are correct and Reason is the correct explanation of Assertion.

Case Study Based Questions

11. The Mathematics teacher of class IX of a certain school wrote two different expressions on the blackboard. She selects two students Anu and Mayank to solve two different expressions on the blackboard and asks them to explain their simplifications. Anu explains simplification of $\frac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}}$ by rationalising the denominator and Mayank explains simplification of $(3\sqrt{5}-2\sqrt{3})$ $(3\sqrt{5}+2\sqrt{3})$ by using the identity $(a - b)$ $(a + b)$. Now, answer the following questions.

(*a*) What is the conjugate of $\sqrt{10}+\sqrt{3}$?

(*i*) $\sqrt{10}+\sqrt{3}$

(*ii*) $\sqrt{10}-\sqrt{3}$

(*iii*) $\sqrt{10}\times\sqrt{3}$

(*iv*) $\frac{\sqrt{10}}{\sqrt{3}}$

Ans. (*ii*) $\sqrt{10}-\sqrt{3}$

(*b*) What is the answer got by Anu by rationalising the denominator of $\frac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}}$?

(*i*) $\frac{7\sqrt{30}-20}{7}$

(*ii*) $\sqrt{30}-3$

(*iii*) $\frac{7\sqrt{30}-20}{13}$

(*iv*) $\sqrt{30}+3$

Ans. (*ii*) $\sqrt{30}-3$

(*c*) The product of two irrational numbers is

(*i*) always rational.

(*ii*) always irrational.

(*iii*) always integer.

(*iv*) sometimes rational and sometimes irrational.

Ans. (*iv*) sometimes rational and sometimes irrational.

(*d*) What is the answer got by Mayank by simplifying $(3\sqrt{5}-2\sqrt{3})\,(3\sqrt{5}+2\sqrt{3})$?

(*i*) 12 (*ii*) 33

(*iii*) 45 (*iv*) 57

Ans. (*ii*) 33

(*e*) The sum of a rational and irrational numbers is

(*i*) irrational. (*ii*) rational.

(*iii*) Both of these (*iv*) None of these

Ans. (*i*) irrational.

12. In today's fast-paced world, everyone wants to raise their living standards. They are always working hard to give themselves and their loved ones a better life. But in this race of life, they do not get the time for social service. Hence they donate money to NGOs that are dedicated to helping the people in need. A survey was conducted, it was found that 7 out of every 11 households are donating some amount of their income to NGOs. Based on the above information, answer the following questions.

(*a*) What is the fraction of households which are donating?

(*i*) $\frac{6}{11}$ (*ii*) $\frac{4}{11}$

(*iii*) $\frac{7}{11}$ (*iv*) $\frac{9}{11}$

Ans. (*iii*) $\frac{7}{11}$

(*b*) What is the fraction of households which are not donating?

(*i*) $\frac{5}{11}$ (*ii*) $\frac{4}{11}$

(*iii*) $\frac{7}{11}$ (*iv*) $\frac{10}{11}$

Ans. (*ii*) $\frac{4}{11}$

(*c*) What is the decimal form of the fraction of households, which are donating?

(*i*) $0.\overline{07}$ (*ii*) $0.\overline{32}$

(*iii*) $0.\overline{63}$ (*iv*) $0.\overline{62}$

Ans. (*iii*) $0.\overline{63}$

(*d*) What is the decimal form of the fraction of households, which are not donating?

(*i*) $0.\overline{34}$ (*ii*) $0.\overline{36}$

(*iii*) $0.\overline{65}$ (*iv*) $0.\overline{70}$

Ans. (*ii*) $0.\overline{36}$

(*e*) What is the type of decimal expansion of $\frac{4}{11}$?

(*i*) Terminating

(*ii*) Non-terminating

(*iii*) Non-terminating repeating

(*iv*) Non-terminating non-repeating

Ans. (*iii*) Non-terminating repeating

Very Short Answer Type Questions

13. What is the simplest rationalisation factor of $\sqrt[3]{40}$ among the numbers $\sqrt[3]{5}, \sqrt{40}, \sqrt[3]{40}, \sqrt{5}, \sqrt[3]{25}$?

Sol. We know that

$$\sqrt[3]{40} = \sqrt[3]{8 \times 5} = \sqrt[3]{2^3 \times 5} = 2 \times \sqrt[3]{5} = 2\sqrt[3]{5}$$

Also, the simplest rationalisation factor of $\sqrt[3]{5}$ is $\sqrt[3]{5^2} = \sqrt[3]{25}$.

Hence, $\sqrt[3]{25}$ is the only simplest rationalisation factor of $\sqrt[3]{40}$.

14. Write any two irrational numbers between 0.21 and 0.22.

Sol. We know that $\sqrt{5} \simeq 2.236$

$\therefore \quad \frac{\sqrt{5}}{10} \simeq 0.224$

Also, any two rational numbers between 0.210 and 0.220 are 0.214 and 0.215 which are less than 0.224 by 0.010 and 0.009 respectively. Hence, any two irrational numbers between 0.21 and 0.22 can be taken as $\frac{\sqrt{5}}{10} - 0.010$ and $\frac{\sqrt{5}}{10} - 0.009$.

Alternatively, any two non-terminating and non-recurring numbers between 0.21 and 0.22 can be taken as 0.2101001000100001… and 0.212122122212222…

Note: Here the answer is not unique and so there may be many such irrational numbers between 0.21 and 0.22.

Short Answer Type-I Questions

15. Express $0.\overline{31}$ in the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

Sol. Let $x = 0.\overline{31} = 0.313131\ldots$

$\therefore \quad 100x = 31.313131\ldots$

$\therefore \quad 100x - x = 31$

$\Rightarrow \quad 99x = 31$

$\Rightarrow \quad x = \frac{31}{99}$

which is the required expression.

16. Express $\sqrt{5}$ correct upto three places of decimals.

Sol. We find $\sqrt{5}$ as follows:

	2.236…
2	5.$\overline{00}\,\overline{00}\,\overline{00}$
	4
42	100
	84
443	1600
	1329
4466	27100
	26796
	304

$\therefore\ \sqrt{5} \simeq 2.236$

Short Answer Type-II Questions

17. Simplify: $\sqrt{432} - \frac{5}{2}\sqrt{\frac{1}{3}} + 4\sqrt{3}$

Sol. We have

$$\sqrt{432} - \frac{5}{2}\sqrt{\frac{1}{3}} + 4\sqrt{3}$$

$$= \sqrt{2^2 \times 2^2 \times 3^2 \times 3} - \frac{5}{2} \times \frac{\sqrt{3}}{3} + 4\sqrt{3}$$

$$= 12\sqrt{3} - \frac{5\sqrt{3}}{6} + 4\sqrt{3}$$

$$= \frac{72\sqrt{3} - 5\sqrt{3} + 24\sqrt{3}}{6} = \frac{91\sqrt{3}}{6}$$

18. Write $\frac{2-\sqrt{3}}{3-7\sqrt{2}}$ in the form of an expression with a rational denominator.

Sol. We have
$$\frac{2-\sqrt{3}}{3-7\sqrt{2}} = \frac{\left(2-\sqrt{3}\right)\left(3+7\sqrt{2}\right)}{\left(3-7\sqrt{2}\right)\left(3+7\sqrt{2}\right)}$$

$$= \frac{6+14\sqrt{2}-3\sqrt{3}-7\sqrt{6}}{3^2-\left(7\sqrt{2}\right)^2}$$

$$= \frac{6+14\sqrt{2}-3\sqrt{3}-7\sqrt{6}}{9-98}$$

$$= \frac{3\sqrt{3}+7\sqrt{6}-6-14\sqrt{2}}{89}$$

which is the required expression.

Long Answer Type Questions

19. Simplify:

$$\frac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}} - \frac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}} - \frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}}$$

Sol. We have

$$\frac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}} = \frac{7\sqrt{3}\left(\sqrt{10}-\sqrt{3}\right)}{\left(\sqrt{10}\right)^2-\left(\sqrt{3}\right)^2}$$

$$= \frac{7\sqrt{30}-21}{10-3}$$

$$= \frac{7\sqrt{30}-21}{7}$$

$$= \sqrt{30}-3 \qquad \text{...(1)}$$

$$\frac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}} = \frac{2\sqrt{5}\left(\sqrt{6}-\sqrt{5}\right)}{\left(\sqrt{6}\right)^2-\left(\sqrt{5}\right)^2}$$

$$= \frac{2\sqrt{30}-10}{6-5}$$

$$= 2\sqrt{30}-10 \qquad \text{...(2)}$$

$$\frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}} = \frac{3\sqrt{2}\left(\sqrt{15}-3\sqrt{2}\right)}{\left(\sqrt{15}\right)^2-\left(3\sqrt{2}\right)^2}$$

$$= \frac{3\sqrt{30}-18}{15-18}$$

$$= \frac{18-3\sqrt{30}}{3}$$

$$= 6-\sqrt{30} \qquad \text{...(3)}$$

$$\therefore\ \frac{7\sqrt{3}}{\sqrt{10}+\sqrt{3}} - \frac{2\sqrt{5}}{\sqrt{6}+\sqrt{5}} - \frac{3\sqrt{2}}{\sqrt{15}+3\sqrt{2}}$$

$$= \sqrt{30}-3-2\sqrt{30}+10-6+\sqrt{30}$$

$$= 10-9 = 1 \qquad \text{[From (1), (2) and (3)]}$$

20. If $x = \frac{1}{2-\sqrt{3}}$, find the value of $x^3 - 2x^2 - 7x + 5$.

[CBSE SP 2011]

Sol. We have
$$x = \frac{1}{2-\sqrt{3}}$$

$$= \frac{2+\sqrt{3}}{2^2-\left(\sqrt{3}\right)^2}$$

$$= \frac{2+\sqrt{3}}{1}$$

$$= 2+\sqrt{3}$$

$$\therefore\ (x-2)^2 = 3$$

$$\Rightarrow\ x^2 - 4x + 4 = 3$$

$$\Rightarrow\ x^2 = 4x - 1 \qquad \text{...(1)}$$

$$\therefore\ x^3 - 2x^2 - 7x + 5$$

$$= x \times (4x-1) - 2(4x-1) - 7x + 5 \qquad \text{[From (1)]}$$

$$= 4x^2 - x - 8x + 2 - 7x + 5$$

$$= 4(4x-1) - 16x + 7 \qquad \text{[From (1)]}$$

$$= 16x - 4 - 16x + 7 = 3$$

Hence, the value of $x^3 - 2x^2 - 7x + 5$ is 3.

Let's Compete

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Multiple-Choice Questions

1. A rational number between $\sqrt{2}$ and $\sqrt{3}$ is

(*a*) 1.31 (*b*) 1.42

(*c*) 1.41 (*d*) 1.75

Sol. (*b*) 1.42

We know that $\sqrt{2} \simeq 1.414$ and $\sqrt{3} \simeq 1.732$

We see that out of four choices, only 1.42 lies between 1.414 and 1.732.

2. An irrational number between two rational numbers $2.\overline{19}$ and $2.\overline{33}$ is

(*a*) $\sqrt{\frac{11}{2}}$ (*b*) $\frac{\sqrt{17}}{3}$

(*c*) $\frac{\sqrt{15}}{2}$ (*d*) $\sqrt{5}$

Sol. (*d*) $\sqrt{5}$

We see by actual calculation that

$$\sqrt{\frac{11}{2}} = \frac{\sqrt{22}}{2} \simeq 2.345,$$

$$\frac{\sqrt{17}}{3} \simeq 1.37,\ \frac{\sqrt{15}}{2} \simeq 1.9 \text{ and } \sqrt{5} \simeq 2.236$$

Out of these numbers only 2.236 i.e. $\sqrt{5}$ lies between two given numbers 2.191919… and 2.33333…

3. The real number $2.1\overline{06}$ when expressed in the form $\frac{p}{q}$ where p and q are integers and $q \neq 0$ is

(*a*) $\frac{15}{7}$ (*b*) $\frac{13}{6}$

(*c*) $\frac{139}{66}$ (*d*) $\frac{141}{67}$

Sol. (*c*) $\frac{139}{66}$

Let $x = 2.1\overline{06}$

$= 2.1060606\ldots$

$\therefore \quad 100x = 210.606060 \quad \ldots(1)$

$\therefore \quad 10000x = 21060.606060 \quad \ldots(2)$

Subtracting (1) from (2), we get

$$9900\,x = 20850$$

$$\Rightarrow \quad x = \frac{20850}{9900} = \frac{139}{66}$$

4. If $a + b\sqrt{7} = \frac{3+\sqrt{7}}{3-\sqrt{7}}$, where a and b are rational numbers, then the values of a and b are respectively

(*a*) 8 and 3 (*b*) 3 and 8

(*c*) 3 and 7 (*d*) 7 and 3

Sol. (*a*) 8 and 3

We have

$$a + b\sqrt{7} = \frac{3+\sqrt{7}}{3-\sqrt{7}}$$

$$= \frac{\left(3+\sqrt{7}\right)^2}{\left(3-\sqrt{7}\right)\left(3+\sqrt{7}\right)}$$

$$= \frac{9+7+6\sqrt{7}}{3^2-\left(\sqrt{7}\right)^2}$$

$$= \frac{16+6\sqrt{7}}{9-7}$$

$$= 8 + 3\sqrt{7}$$

$\therefore \quad a = 8$ and $b = 3$

5. The value of $\dfrac{5^{\frac{1}{3}} \times 25^{\frac{-2}{3}} \times 625^{\frac{1}{3}}}{\sqrt[3]{125}}$ is equal to

(*a*) $\sqrt[3]{25}$ (*b*) $5^{\frac{4}{3}}$

(*c*) $\frac{1}{\sqrt[3]{25}}$ (*d*) $5^{-\frac{4}{3}}$

Sol. (*c*) $\frac{1}{\sqrt[3]{25}}$

$$\frac{5^{\frac{1}{3}} \times 25^{-\frac{2}{3}} \times 625^{\frac{1}{3}}}{\sqrt[3]{125}} = \frac{5^{\frac{1}{3}} \times \left(5^2\right)^{-\frac{2}{3}} \times \left(5^4\right)^{\frac{1}{3}}}{\left(5^3\right)^{\frac{1}{3}}}$$

$$= \frac{5^{\frac{1}{3}} \times 5^{-\frac{4}{3}} \times 5^{\frac{4}{3}}}{5}$$

$$= 5^{\frac{1}{3}-\frac{4}{3}+\frac{4}{3}-1}$$

$$= 5^{-\frac{2}{3}}$$

$$= \frac{1}{5^{\frac{2}{3}}} = \frac{1}{\sqrt[3]{25}}$$

6. The value of $\dfrac{4}{(216)^{-\frac{2}{3}}} - \dfrac{1}{(256)^{-\frac{3}{4}}}$ is equal to

(*a*) –60 (*b*) 60

(*c*) –80 (*d*) 80

Sol. (*d*) 80

$$\frac{4}{(216)^{-\frac{2}{3}}}-\frac{1}{(256)^{-\frac{3}{4}}} = \frac{4}{\left(2^3\times 3^3\right)^{-\frac{2}{3}}}-\frac{1}{\left(2^8\right)^{-\frac{3}{4}}}$$

$$= \frac{4}{2^{-3\times\frac{2}{3}}\times 3^{-3\times\frac{2}{3}}}-\frac{1}{2^{-8\times\frac{3}{4}}}$$

$$= 4\times 2^2\times 3^2-2^6$$

$$= 144-64$$

$$= 80$$

7. If $a = 2$ and $b = 3$, then the value of $(a^a + b^b)^{-1}$ is equal to

(*a*) 31 (*b*) $\frac{1}{31}$

(*c*) $\frac{1}{13}$ (*d*) 13

Sol. (*b*) $\frac{1}{31}$

We have

$$(a^a + b^b)^{-1} = (2^2 + 3^3)^{-1}$$

$$= (4 + 27)^{-1} = \frac{1}{31}$$

8. If $\left(\frac{2}{7}\right)\times\left(\frac{7}{5}\right)^{3x}=\frac{98}{125}$, then the value of x is equal to

(*a*) 1 (*b*) 2

(*c*) –1 (*d*) –2

Sol. (*a*) 1

We have

$$\left(\frac{2}{7}\right)\times\left(\frac{7}{5}\right)^{3x} = \frac{98}{125}$$

$$\left(\frac{7}{5}\right)^{3x} = \frac{98}{125}\times\frac{7}{2}$$

$$= \frac{49\times 7}{125}$$

$$= \frac{343}{125}$$

$$\Rightarrow \left(\frac{7}{5}\right)^{3x} = \left(\frac{7}{5}\right)^{3}$$

On comparing, we get

$$3x = 3$$

$$\Rightarrow x = 1$$

∴ The required value of x is 1.

9. The value of $\sqrt{5-\sqrt{24}}$ is equal to

(*a*) $\sqrt{5}-1$ (*b*) $\sqrt{6}-\sqrt{5}$

(*c*) $\sqrt{6}-1$ (*d*) $\sqrt{3}-\sqrt{2}$

Sol. (*d*) $\sqrt{3}-\sqrt{2}$

We have

$$5-\sqrt{24} = 3+2-2\sqrt{6}$$

$$= \left(\sqrt{3}\right)^2+\left(\sqrt{2}\right)^2-2\times\sqrt{2}\times\sqrt{3}$$

$$= \left(\sqrt{3}-\sqrt{2}\right)^2$$

$$\therefore \sqrt{5-\sqrt{24}} = \sqrt{3}-\sqrt{2}$$

10. The simplified value of $\frac{5^{89}+5^{90}}{5^{92}-5^{91}}$ is equal to

(*a*) $\frac{3}{25}$ (*b*) $\frac{2}{25}$

(*c*) 0.06 (*d*) 0.05

Sol. (*c*) 0.06

Let $5^{89} = a$

∴ $5^{90} = 5a$, $5^{92} = 125a$ and $5^{91} = 25a$

$$\frac{5^{89}+5^{90}}{5^{92}-5^{91}} = \frac{a+5a}{125a-25a}$$

$$= \frac{6}{100}$$

$$= 0.06$$

Value-based Questions (Optional)

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1. Two classmates Salma and Anil simplified two different expressions during the revision hour and explained to each other their simplifications.

Salma explains simplification of $\frac{\sqrt{2}}{\sqrt{5}+\sqrt{3}}$ and

Anil explains simplification of $\sqrt{28}+\sqrt{98}+\sqrt{147}$.

Write both the simplifications. What value does it depict? **[CBSE SP 2013]**

Sol. We have

$$\frac{\sqrt{2}}{\sqrt{5}+\sqrt{3}} = \frac{\sqrt{2}\left(\sqrt{5}-\sqrt{3}\right)}{\left(\sqrt{5}\right)^2-\left(\sqrt{3}\right)^2}$$

$$= \frac{\sqrt{10}-\sqrt{6}}{5-3}$$

$$= \frac{\sqrt{10}-\sqrt{6}}{2}$$

and $\sqrt{28}+\sqrt{98}+\sqrt{147}$

$$= \sqrt{4\times 7}+\sqrt{49\times 2}+\sqrt{49\times 3}$$

$$= 2\sqrt{7}+7\sqrt{2}+7\sqrt{3}$$

Values: Concern for classmates who faces difficulty in simplifying the given expression and helpfulness.

2. Varun was facing some difficulty in simplifying $\frac{1}{\sqrt{7}-\sqrt{3}}$. His classmate Priya gave him a clue to rationalise the denominator for simplification. Varun simplified the expression and thanked Priya for this goodwill. How did Varun simplify $\frac{1}{\sqrt{7}-\sqrt{3}}$? What value does it indicate?

[CBSE SP 2013]

Sol. Varun's simplification

$$\frac{1}{\sqrt{7}-\sqrt{3}} = \frac{\sqrt{7}+\sqrt{3}}{\left(\sqrt{7}-\sqrt{3}\right)\left(\sqrt{7}+\sqrt{3}\right)}$$

$$= \frac{\sqrt{7}+\sqrt{3}}{\left(\sqrt{7}\right)^2-\left(\sqrt{3}\right)^2}$$

$$= \frac{\sqrt{7}+\sqrt{3}}{7-3}$$

$$= \frac{\sqrt{7}+\sqrt{3}}{4}$$

Values: Concern for classmates who faces difficulty in simplifying the given expression and helpfulness.

2

Polynomials

Checkpoint (Page 17)

1. Simplify $-3b(ab + b^2) + 300$ and find its values for $a = 3$ and $b = 4$.

Sol. We have

$-3b(ab + b^2) + 300 = -3ab^2 - 3b^3 + 300$

When $a = 3$ and $b = 4$, the given expression

$$= -3 \times 3 \times 4^2 - 3 \times 4^3 + 300$$
$$= -144 - 192 + 300$$
$$= -336 + 300$$
$$= -36$$

$\therefore$ The required value is -36.

2. Find the product: $x^3y^2(x^2 + y^2 + z^2)$.

Sol. $x^3y^2(x^2 + y^2 + z^2) = x^3y^2 \times x^2 + x^3y^2 \times y^2 + x^3y^2 \times z^2$

$$= x^5y^2 + x^3y^4 + x^3y^2z^2$$

3. Using a suitable identity, determine the product: $\left(x+\frac{3}{4}\right)\left(x+\frac{3}{4}\right)$.

Sol. $$\left(x+\frac{3}{4}\right)\left(x+\frac{3}{4}\right) = \left(x+\frac{3}{4}\right)^2$$
$$= x^2 + 2x \times \frac{3}{4} + \left(\frac{3}{4}\right)^2$$
$$= x^2 + \frac{3}{2}x + \frac{9}{16}$$

4. By using suitable identity, simplify $(xy - yz)^2$.

Sol. $$(xy - yz)^2 = (xy)^2 + (yz)^2 - 2xy \times yz$$
$$= x^2y^2 + y^2z^2 - 2xy^2z$$

5. Using the identity

$$(x + a)(x + b) = x^2 + (a + b)x + ab,$$

find the product: $(2x + 3)(2x - 5)$.

Sol. $$(2x + 3)(2x - 5) = 2\left(x+\frac{3}{2}\right) \times 2\left(x-\frac{5}{2}\right)$$
$$= 2 \times 2\left(x+\frac{3}{2}\right)\left(x-\frac{5}{2}\right)$$
$$= 4\left[x^2 + \left(\frac{3}{2}-\frac{5}{2}\right)x + \left(\frac{3}{2}\right)\left(\frac{-5}{2}\right)\right]$$
$$= 4\left[x^2 - x - \frac{15}{4}\right]$$
$$= 4x^2 - 4x - 15$$

6. Evaluate $(91)^2$ by using identity.

Sol. We have $91^2 = (90 + 1)^2$

$$= 90^2 + 2 \times 90 \times 1 + 1^2$$
$$= 8100 + 180 + 1$$
$$= 8281$$

7. Simplify $(4x + 3)^2 - (4x - 3)^2$.

Sol. $(4x + 3)^2 - (4x - 3)^2$

$= (4x)^2 + 2 \times 4x \times 3 + 3^2 - \{(4x)^2 - 2 \times 4x \times 3 + 3^2\}$

$= 16x^2 + 24x + 9 - 16x^2 + 24x - 9$

$= 48x$

8. Find the continued product: $(x + 2)(x - 2)(x^2 + 4)$.

Sol. $(x + 2)(x - 2)(x^2 + 4)$

$$= (x^2 - 2^2)(x^2 + 4)$$
$$= (x^2 - 4)(x^2 + 4)$$
$$= (x^2)^2 - 4^2$$
$$= x^4 - 16$$

9. If $2x + 3y = 11$ and $xy = 8$, find the value of $4x^2 + 9y^2$.

Sol. $$4x^2 + 9y^2 = (2x)^2 + (3y)^2$$
$$= (2x + 3y)^2 - 2 \times 2x \times 3y$$

$= 11^2 - 12 \times 8$

$= 121 - 96 = 25$

10. If $x - \frac{1}{x} = 3$, find the value of $x^2 + \frac{1}{x^2}$.

Sol. We have

$$x^2 + \frac{1}{x^2} = \left(x - \frac{1}{x}\right)^2 + 2x \times \frac{1}{x}$$

$$= 3^2 + 2$$

$$= 11$$

Milestone 1

(Page 19)

Multiple-Choice Questions

1. $\sqrt{3}$ is a polynomial of degree

(*a*) 1 (*b*) 2

(*c*) 0 (*d*) $\frac{1}{2}$

Sol. (*c*) 0

We know that any constant is a polynomial of degree 0.

Since $\sqrt{3}$ is a constant, hence it is a polynomial of degree 0.

2. Which of the following is a zero of the polynomial $x^3 + 2x^2 - 2x - 1$?

(*a*) –1 (*b*) 1

(*c*) –2 (*d*) 2

Sol. (*b*) 1

We see that when $x = 1$, then

$$x^3 + 2x^2 - 2x - 1 = 1^3 + 2 \times 1^2 - 2 \times 1 - 1$$

$$= 3 - 3 = 0$$

Very Short Answer Type Questions

3. What is the coefficient of x^3 in the polynomial $9x^3 + \frac{x}{8} + 6x^5 + \frac{3}{5}$?

Sol. The coefficient of x^3 in the given polynomial is 9.

4. Find the value of the polynomial $5x^2 - 4$ at $x = 2$.

Sol. The required value is $5 \times 2^2 - 4 = 20 - 4 = 16$.

Short Answer Type-I Questions

5. Classify the following as constant, linear, quadratic and cubic polynomials:

(*a*) $x^2 + \sqrt{7}x + 9$ (*b*) –200

(*c*) $t - 5t^3$ (*d*) $4x - \sqrt{3}$

Sol. (*a*) Quadratic (degree 2)

(*b*) Constant (degree 0)

(*c*) Cubic (degree 3)

(*d*) Linear (degree 1)

6. Which of the following expressions are polynomials? Justify your answers.

(*a*) $\sqrt{3}x^3 - \frac{1}{5}x + \frac{\sqrt{3}}{\sqrt{7}}$

(*b*) $9\sqrt{x} - 7$

(*c*) $\frac{5}{x^2} - 8$

(*d*) $7x^{-2} - 3x^2 + 5$

(*e*) $\sqrt{8}x^3 - \frac{3}{\sqrt{5}}x + \frac{1}{\sqrt{7}}$

(*f*) $\sqrt{5}x^2 + 9x + 3$

Sol. We see that all exponents of x in each of the expressions in (*a*), (*e*) and (*f*) are non-negative integer, and those in the rest of the expression are not. Hence, only expressions in (*a*), (*e*) and (*f*) are polynomials.

Short Answer Type-II Questions

7. If $p(x) = ax^2 - 3x + 7b$ and 1, –2 are zeroes of $p(x)$, then find the value of a and b.

Sol. We see that $p(1) = 0$

$\therefore$ $a - 3 + 7b = 0$ …(1)

and $p(-2) = 0$

$\Rightarrow$ $a(-2)^2 + 3 \times 2 + 7b = 0$

$4a + 6 + 7b = 0$ …(2)

Subtracting (1) from (2), we get

$3a + 9 = 0$

$\therefore$ $a = -3$

$\therefore$ From (1), $7b = 3 - a$

$= 3 + 3$

$= 6$

$\Rightarrow$ $b = \frac{6}{7}$

$\therefore$ The required values of a and b are respectively –3 and $\frac{6}{7}$.

8. If $x = \frac{-1}{3}$ is a zero of a polynomial $p(x) = 27x^3 - ax^2 - x + 3$, then find the value of a.

Sol. We have $p\left(-\frac{1}{3}\right) = 0$

$$\Rightarrow 27\left(-\frac{1}{3}\right)^3 - a\left(-\frac{1}{3}\right)^2 + \frac{1}{3} + 3 = 0$$

$\Rightarrow \quad -1 - \frac{a}{9} + \frac{1}{3} + 3 = 0$

$\Rightarrow \quad 2 + \frac{1}{3} = \frac{a}{9}$

$\Rightarrow \quad a = 18 + \frac{1}{3} \times 9$

$= 18 + 3$

$= 21$

$\therefore$ The required value of a is 21.

Long Answer Type Questions

9. If $p(x) = x^3 + 3x - 2x^2 - 6$ and $q(x) = 3x^2 - 7x - 8$, then prove that $q(-1)$ is a zero of $p(x)$.

Sol. We have $\quad q(-1) = 3(-1)^2 + 7 \times 1 - 8$

$= 3 + 7 - 8$

$= 2$

$\therefore \quad p\{q(-1)\} = p(2)$

$= 2^3 + 3 \times 2 - 2 \times 2^2 - 6$

$= 8 + 6 - 8 - 6$

$= 0$

$\therefore \quad p\{q(-1)\} = 0$

$\therefore$ $q(-1)$ is a zero of $p(x)$.

Hence, proved.

10. If 2 and –2 are the zeroes of the polynomial $p(x) = ax^4 + 2x^3 - 3x^2 + bx - 4$, find the values of a and b and hence $p(-3)$.

Sol. We have $\quad p(2) = 0$

$\Rightarrow \quad a \times 2^4 + 2 \times 2^3 - 3 \times 2^2 + b \times 2 - 4 = 0$

$\Rightarrow \quad 16a + 16 - 12 + 2b - 4 = 0$

$\Rightarrow \quad 16a + 2b = 0$

$\Rightarrow \quad 8a + b = 0 \quad \ldots(1)$

Again, $\quad p(-2) = 0$

$\therefore \quad a \times (-2)^4 + 2 \times (-2)^3 - 3 \times (-2)^2 + b \times (-2) - 4 = 0$

$\Rightarrow \quad 16a - 16 - 12 - 2b - 4 = 0$

$\Rightarrow \quad 16a - 2b = 32$

$\Rightarrow \quad 8a - b = 16 \quad \ldots(2)$

Adding (1) and (2), we get

$16a = 16$

$\Rightarrow \quad a = 1$

$\therefore$ From (1), $\quad 8 + b = 0$

$\Rightarrow \quad b = -8$

$\therefore$ The required values of a and b are respectively 1 and –8.

Now, $\quad p(-3) = a \times (-3)^4 + 2 \times (-3)^3 - 3(-3)^2 + b \times (-3) - 4$

$= 81a - 54 - 27 - 3b - 4$

$= 81a - 3b - 85$

$= 81 \times 1 - 3 \times (-8) - 85$

[Putting the values of a and b]

$= 81 + 24 - 85$

$= 20$

Hence, the required values of a, b and $p(-3)$ are respectively 1, –8 and 20.

Milestone 2

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Multiple-Choice Questions

1. The zero of the polynomial $p(x)$ where $p(x) = ax + 1$ where $a \neq 0$, is

(*a*) 1 (*b*) $-a$

(*c*) 0 (*d*) $-\frac{1}{a}$ **[CBSE SP 2010]**

Sol. (*d*) $\frac{-1}{a}$

Putting $p(x) = 0$, we get

$ax + 1 = 0$

$\Rightarrow \quad x = \frac{-1}{a}$

$\therefore$ The zero of $p(x)$ is $-\frac{1}{a}$

2. The value of k for which the polynomial $x^3 + 3x^2 - 3x + k$ has –3 as its zero, is

(*a*) –9 (*b*) –3

(*c*) 9 (*d*) 12 **[CBSE SP 2011]**

Sol. (*a*) – 9

Since –3 is a zero of the polynomial $p(x) = x^3 + 3x^2 - 3x + k$,

$\therefore \quad p(-3) = 0$

$\Rightarrow \quad (-3)^3 + 3(-3)^2 + 3 \times 3 + k = 0$

$\Rightarrow \quad -27 + 27 + 9 + k = 0$

$\Rightarrow \quad k = -9$

Very Short Answer Type Questions

3. What is the remainder when $x^2 + 2x - 1$ is divided by $x + 1$?

Sol. The required remainder when $p(x) = x^2 + 2x - 1$ is divided by $x + 1$ is $p(-1) = (-1)^2 - 2 \times 1 - 1 = -2$

4. Show that $3x + 2$ is a factor of $3x^3 + x^2 - 20x - \frac{116}{9}$.

Sol. Putting $x = -\frac{2}{3}$ in $p(x) = 3x^3 + x^2 - 20x - \frac{116}{9}$, we get,

$$p\left(-\frac{2}{3}\right) = 3 \times \left(-\frac{2}{3}\right)^3 + \left(-\frac{2}{3}\right)^2 + 20 \times \frac{2}{3} - \frac{116}{9}$$

$$= \frac{-8}{9} + \frac{4}{9} + \frac{40}{3} - \frac{116}{9}$$

$$= \frac{-8 + 4 + 120 - 116}{9}$$

$$= \frac{0}{9} = 0$$

$\therefore$ By factor theorem, $x + \frac{2}{3}$ is a factor of $p(x)$

or, $\frac{3x+2}{3}$ is a factor of $p(x)$ or $3x + 2$ is a factor of $p(x)$.

Hence, proved.

Short Answer Type-I Questions

5. Find the remainder when $2x^2 - x + 1$ is divided by $2x + 1$. **[CBSE SP 2013]**

Sol. Putting $x = -\frac{1}{2}$ in $p(x) = 2x^2 - x + 1$, we get

$$p\left(-\frac{1}{2}\right) = 2 \times \frac{1}{4} + \frac{1}{2} + 1$$

$$= \frac{1}{2} + \frac{1}{2} + 1$$

$$= 2$$

$\therefore$ By remainder theorem, 2 is the required remainder.

6. Using the factor theorem, show that the polynomial $p(x) = x^3 - 2x^2 + 3x - 18$ is a multiple of $x - 3$.

Sol. We see that

$$p(3) = 3^3 - 2 \times 3^2 + 3 \times 3 - 18$$

$$= 27 - 18 + 9 - 18$$

$$= 36 - 36$$

$$= 0$$

$\therefore$ By factor theorem, $x - 3$ is a factor of $p(x)$, i.e. $p(x)$ is a multiple of $x - 3$.

Short Answer Type-II Questions

7. If the polynomial $p(x) = x^4 - 2x^3 + 3x^2 - ax + 8$ is divided by $x - 2$, it leaves a remainder 10. Find the value of a.

Sol. We have $p(2) = 10$

$\Rightarrow$ $2^4 - 2 \times 2^3 + 3 \times 2^2 - 2a + 8 = 10$

$\Rightarrow$ $16 - 16 + 12 - 2a + 8 = 10$

$\Rightarrow$ $2a = 20 - 10 = 10$

$\Rightarrow$ $a = 5$

$\therefore$ The required value of a is 5.

8. The polynomial $p(x) = x^3 + ax^2 + bx - 20$ when divided by $x - 5$ and $x - 3$ leaves the remainders 0 and –2 respectively. Find the values of a and b.

Sol. We have $p(5) = 0$ and $p(3) = -2$

When $p(5) = 0$, we have

$5^3 + a \times 5^2 + 5b - 20 = 0$

$\Rightarrow$ $125 + 25a + 5b - 20 = 0$

$\Rightarrow$ $25a + 5b + 105 = 0$

$5a + b + 21 = 0$ …(1)

When $p(3) = -2$, then

$3^3 + a \times 3^2 + 3b - 20 = -2$

$\Rightarrow$ $27 + 9a + 3b = 18$

$\Rightarrow$ $9 + 3a + b = 6$

$\Rightarrow$ $3a + b + 3 = 0$ …(2)

Subtracting (2) from (1), we get

$2a + 18 = 0$

$\Rightarrow$ $a = -9$

$\therefore$ From (2), $-27 + b + 3 = 0$

$\Rightarrow$ $b = 24$

Hence, the required values of a and b are –9 and 24 respectively.

Long Answer Type Questions

9. If p and q are remainders when the polynomials $x^3 + 2x^2 - 5ax - 7$ and $x^3 + ax^2 - 12x + 6$ are divided by $x + 1$ and $x - 2$ respectively and if $2p + q = 6$, find the value of a. **[CBSE SP 2013]**

Sol. Let $P(x) = x^3 + 2x^2 - 5ax - 7$

and $Q(x) = x^3 + ax^2 - 12x + 6$.

According to the problem, $P(-1) = p$ and $Q(2) = q$

$\therefore$ $-1^3 + 2 \times 1 + 5a - 7 = p$

and $2^3 + 4a - 12 \times 2 + 6 = q$

$\therefore$ $p = 5a - 6$ … (1)

and $8 + 4a - 24 + 6 = q$

$\Rightarrow$ $-10 + 4a = q$ …(2)

Now, $2p + q = 6$

$\therefore$ $2 \times (5a - 6) + 4a - 10 = 6$

[From (1) and (2)]

$\Rightarrow \quad 10a - 12 + 4a - 10 - 6 = 0$

$\Rightarrow \quad 14a = 28$

$\Rightarrow \quad a = \frac{28}{14} = 2$

$\therefore$ The required value of a is 2.

10. If $f(x) = x^3 + mx^2 + nx + 6$ has $x - 2$ as a factor and leaves a remainder 3, when divided by $x - 3$. Find the values of m and n. **[CBSE SP 2013]**

Sol. $f(x) = x^3 + mx^2 + nx + 6$

We have $f(2) = 0$

and $f(3) = 3$

$\because \quad f(2) = 0$

$\Rightarrow 8 + 4m + 2n + 6 = 0$

$\Rightarrow \quad 4m + 2n = -14$

$\Rightarrow \quad 2m + n = -7 \quad \ldots(1)$

Also, $27 + 9m + 3n + 6 = 3$

$\Rightarrow \quad 9m + 3n + 33 = 3$

$\Rightarrow \quad 9m + 3n = -30$

$\Rightarrow \quad 3m + n = -10 \quad \ldots(2)$

Subtracting (1) from (2), we get

$m = -3$

$\therefore$ From (2), $n = -10 - 3m$

$= -10 + 9$

$= -1$

Hence, required values of m and n are -3 and -1 respectively.

Milestone 3

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Multiple-Choice Questions

1. The value of

$1.75 \times 1.75 - 2 \times 0.75 \times 1.75 + 0.75 \times 0.75$

is equal to

(*a*) 2.5 (*b*) 1.0

(*c*) 1.5 (*d*) 2.0

Sol. (*b*) 1.0

$1.75 \times 1.75 - 2 \times 0.75 \times 1.75 + 0.75 \times 0.75$

$= (1.75)^2 - 2 \times 1.75 \times 0.75 + (0.75)^2$

$= (1.75 - 0.75)^2 = 1^2 = 1.0$

2. The product

$$\left(\frac{5y}{2}+\frac{2x}{3}\right)\left(\frac{4x^2}{9}+\frac{25y^2}{4}\right)\left(\frac{2x}{3}-\frac{5y}{2}\right)$$

is equal to

(*a*) $\frac{25y^2}{4}-\frac{4x^2}{9}$ (*b*) $\frac{625y^4}{16}-\frac{16x^4}{81}$

(*c*) $\frac{16x^4}{81}-\frac{625y^4}{16}$ (*d*) $\frac{4x^2}{9}-\frac{25y^2}{4}$ **[CBSE SP 2011]**

Sol. (*c*) $\left(\frac{16x^4}{81}-\frac{625y^4}{16}\right)$

$$\left(\frac{5y}{2}+\frac{2x}{3}\right)\left(\frac{4x^2}{9}+\frac{25y^2}{4}\right)\left(\frac{2x}{3}-\frac{5y}{3}\right)$$

$$= \left(\frac{2x}{3}+\frac{5y}{2}\right)\left(\frac{2x}{3}-\frac{5y}{2}\right)\left(\frac{4x^2}{9}+\frac{25y^2}{4}\right)$$

$$= \left\{\left(\frac{2x}{3}\right)^2-\left(\frac{5y}{2}\right)^2\right\}\left(\frac{4x^2}{9}+\frac{25y^2}{9}\right)$$

$$= \left(\frac{4x^2}{9}-\frac{25y^2}{4}\right)\left(\frac{4x^2}{9}+\frac{25y^2}{4}\right)$$

$$= \left(\frac{4x^2}{9}\right)^2-\left(\frac{25y^2}{4}\right)^2$$

$$= \frac{16x^4}{81}-\frac{625y^4}{16}$$

Very Short Answer Type Questions

3. If $16x^2 + 8y^2 = 9$ and $xy = \frac{-\sqrt{2}}{9}$, find the value of $4x - 2\sqrt{2}y$.

Sol. We have

$$16x^2 + 8y^2 = 9$$

$$\Rightarrow \quad (4x)^2 + \left(2\sqrt{2}y\right)^2 = 9$$

$$\Rightarrow \quad \left(4x - 2\sqrt{2}y\right)^2 + 2 \times 4x \times 2\sqrt{2}y = 9$$

$$\Rightarrow \quad \left(4x - 2\sqrt{2}y\right)^2 + 16\sqrt{2} \times \left(-\frac{\sqrt{2}}{9}\right) = 9$$

$$\left[\because xy = -\frac{\sqrt{2}}{9}\right]$$

$$\Rightarrow \quad \left(4x - 2\sqrt{2}y\right)^2 - \frac{32}{9} = 9$$

$$\Rightarrow \quad \left(4x - 2\sqrt{2}y\right)^2 = \frac{32}{9} + 9$$

$$= \frac{113}{9}$$

$$\therefore \quad 4x - 2\sqrt{2}y = \pm\frac{\sqrt{113}}{3}$$

4. If $a + \frac{1}{a} = 5$, find the value of $a^3 + \frac{1}{a^3}$.

Sol. We have

$$a^3 + \frac{1}{a^3} = \left(a + \frac{1}{a}\right)^3 - 3a \times \frac{1}{a}\left(a + \frac{1}{a}\right)$$

$$= 5^3 - 3 \times 5$$

$$= 125 - 15$$

$$= 110$$

which is the required value.

Short Answer Type-I Questions

5. Express $(x - a - b - c)(a - b + x + c)$ as the difference of two squares.

Sol. $(x - a - b - c)(a - b + x + c)$

$$= \{(x - b) - (a + c)\}\{(x - b) + (a + c)\}$$

$$= (x - b)^2 - (a + c)^2$$

which is the required expression.

6. If $a - b = 3$ and $a + b = 5$, find

(*a*) ab (*b*) $a^2 + b^2$

Sol. (*a*) We have

$$ab = \frac{(a + b)^2 - (a - b)^2}{4}$$

$$= \frac{5^2 - 3^2}{4}$$

$$= \frac{25 - 9}{4}$$

$$= \frac{16}{4} = 4$$

(*b*)
$$a^2 + b^2 = \frac{(a + b)^2 + (a - b)^2}{2}$$

$$= \frac{5^2 + 3^2}{2}$$

$$= \frac{25 + 9}{2}$$

$$= \frac{34}{2} = 17$$

Short Answer Type-II Questions

7. Express $-7.985^3 + 11.861^3 - 3.876^3$ as the product of any four distinct real numbers.

Sol. We see that

$$a + b + c = 11.861 - 7.985 - 3.876$$

$$= 11.861 - 11.861$$

$$= 0$$

where $a = 11.861$, $b = -7.985$ and $c = -3.876$

$\therefore \quad a^3 + b^3 + c^3 = 3abc$

i.e. $11.861^3 - 7.985^3 - 3.876^3$

$$= 3 \times 11.861 \times 7.985 \times 3.876$$

which is the required expression.

8. If $a + b + c = 5$, $ab + ac + bc = 3$ and $abc = -27$, find the value of $a^3 + b^3 + c^3$.

Sol. We have

$$(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + ac + bc)$$

$\Rightarrow \quad 5^2 = a^2 + b^2 + c^2 + 2 \times 3$

$\Rightarrow \quad a^2 + b^2 + c^2 = 25 - 6$

$= 19 \quad \ldots(1)$

Now, $a^3 + b^3 + c^3 - 3abc$

$= (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

$\Rightarrow a^3 + b^3 + c^3 + 3 \times 27 = 5 \times (19 - 3)$ [From (1)]

$= 5 \times 16$

$= 80$

$\therefore \quad a^3 + b^2 + c^3 = 80 - 81$

$= -1$

which is the required value.

Long Answer Type Questions

9. If $x + y + z = 7$ and $x^2 + y^2 + z^2 = 33$, find the value of $x^3 + y^3 + z^3 - 3xyz$.

Sol. We have

$$(x + y + z)^2 = x^2 + y^2 + z^2 + 2(xy + xz + yz)$$

$\Rightarrow \quad 7^2 = 33 + 2(xy + xz + yz)$

$\Rightarrow \quad xy + xz + yz = \dfrac{49 - 33}{2}$

$= \dfrac{16}{2} = 8 \quad \ldots(1)$

$\therefore \quad x^3 + y^3 + z^3 - 3xyz$

$= (x + y + z)(x^2 + y^2 + z^2 - xy - xz - yz)$

$= 7 \times (33 - 8)$ [From (1)]

$= 7 \times 25$

$= 175$

which is the required value.

10. If $x^3 + y^3 + z^3 = 49$ and $x + y + z = 1$, find the value of $xy + yz + zx - xyz$.

Sol. We have $x^3 + y^3 + z^3 - 3xyz$

$= (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

$= [x^2 + y^2 + z^2 - (xy + yz + zx)]$

$= (x + y + z)^2 - 2(xy + yz + zx) - (xy + yz + zx)$

$= 1 - 3(xy + yz + zx)$

$\Rightarrow \quad 49 - 3xyz + 3(xy + yz + zx) = 1$

$\Rightarrow \quad 3[xy + yz + zx - xyz] = 1 - 49$

$= -48$

$$\Rightarrow \qquad xy + yz + zx - xyz = -\frac{48}{3} = -16$$

which is the required value of the given expression.

Milestone 4

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Multiple-Choice Questions

1. If the area of a triangle is $(6x^2 - x - 1)$ square units, then its possible dimensions in appropriate units are

(*a*) $2x + 1$ and $3x + 1$ (*b*) $3x + 1$ and $2x - 1$

(*c*) $3x - 1$ and $2x - 1$ (*d*) $3x - 1$ and $2x + 1$

Sol. (*b*) $3x + 1$ and $2x - 1$

We see that

$$6x^2 - x - 1 = 6x^2 + 2x - 3x - 1$$
$$= 2x(3x + 1) - 1(3x + 1)$$
$$= (2x - 1)(3x + 1)$$

Since the area of a triangle is $\frac{1}{2} \times \text{base} \times \text{altitude}$, hence the possible dimensions of the triangle are $3x + 1$ and $2x - 1$.

2. The factors of $\frac{1}{12} - \frac{x^2}{48}$ are

(*a*) $\frac{1}{48}(4+x)(4-x)$ (*b*) $\frac{1}{24}(4+x)(4-x)$

(*c*) $\frac{1}{12}\left(1-\frac{x}{2}\right)\left(1+\frac{x}{2}\right)$ (*d*) $\frac{1}{12}\left(1+\frac{x}{4}\right)\left(1-\frac{x}{4}\right)$

Sol. (*c*) $\frac{1}{12}\left(1-\frac{x}{2}\right)\left(1+\frac{x}{2}\right)$

We have

$$\frac{1}{12} - \frac{x^2}{48} = \frac{1}{12}\left(1 - \frac{x^2}{4}\right)$$
$$= \frac{1}{12}\left(1 + \frac{x}{2}\right)\left(1 - \frac{x}{2}\right)$$

Very Short Answer Type Questions

3. Find all the real factors of $1 - 16x^4$.

Sol. We have

$$1 - 16x^4 = 1^2 - (4x^2)^2$$
$$= (1 + 4x^2)(1 - 4x^2)$$
$$= (1 + 4x^2)\{1^2 - (2x)^2\}$$
$$= (1 + 4x^2)(1 + 2x)(1 - 2x)$$

which are the required factors.

4. Resolve $27a^3 + 8b^3$ into two real factors.

Sol. We have

$$27a^3 + 8b^3 = (3a)^3 + (2b)^3$$
$$= (3a + 2b)\{(3a)^2 - 3a \times 2b + (2b)^2\}$$
$$= (3a + 2b)(9a^2 - 6ab + 4b^2).$$

Short Answer Type-I Questions

Factorise:

5. $3 - 12(a - b)^2$ **[CBSE SP 2011]**

Sol. We have

$$3 - 12(a - b)^2 = 3 - 12a^2 - 12b^2 + 24ab$$
$$= 3(1 - 4a^2 - 4b^2 + 8ab)$$
$$= 3[1^2 - \{(2a)^2 + (2b)^2 - 2 \times 2a \times 2b\}]$$
$$= 3[1^2 - (2a - 2b)^2]$$
$$= 3(1 + 2a - 2b)(1 - 2a + 2b)$$

which are the required factors.

6. $x^{12} - y^{12}$ **[CBSE 2010]**

Sol. We have

$$x^{12} - y^{12}$$
$$= (x^4)^3 - (y^4)^3$$
$$= (x^4 - y^4)(x^8 + y^8 + x^4y^4)$$
$$= (x^2 + y^2)(x^2 - y^2)\{(x^4 + y^4)^2 - (x^2y^2)^2\}$$
$$= (x^2 + y^2)(x + y)(x - y)(x^4 + y^4 + x^2y^2)(x^4 + y^4 - x^2y^2)$$
$$= (x^2 + y^2)(x + y)(x - y)(x^4 + y^4 - x^2y^2)\{(x^2)^2 + (y^2)^2 + 2x^2y^2 - (xy)^2\}$$
$$= (x^2 + y^2)(x + y)(x - y)(x^4 + y^4 - x^2y^2)\{(x^2 + y^2)^2 - (xy)^2\}$$
$$= (x^2 + y^2)(x + y)(x - y)(x^4 + y^4 - x^2y^2)(x^2 + y^2 + xy)(x^2 + y^2 - xy)$$

which are the required factors.

Short Answer Type-II Questions

Factorise into linear factors:

7. $18x^2 + 41x - 10$

Sol. We have

$$18x^2 + 41x - 10 = 18x^2 + 45x - 4x - 10$$
$$= 9x(2x + 5) - 2(2x + 5)$$
$$= (2x + 5)(9x - 2)$$

which are the required factors.

8. $(a^2 - 2a)^2 - 23(a^2 - 2a) + 120$ **[CBSE SP 2013]**

Sol. We have $(a^2 - 2a)^2 - 23(a^2 - 2a) + 120$

Let $\qquad a^2 - 2a = b \qquad \ldots (1)$

Then $(a^2 - 2a)^2 - 23(a^2 - 2a) + 120$
$= b^2 - 23b + 120$
$= b^2 - 8b - 15b + 120$
$= b(b - 8) - 15(b - 8)$
$= (b - 8)(b - 15)$
$= (a^2 - 2a - 8)(a^2 - 2a - 15)$ [From (1)]
$= (a^2 + 2a - 4a - 8)(a^2 + 3a - 5a - 15)$
$= \{a(a + 2) - 4(a + 2)\}\{a(a + 3) - 5(a + 3)\}$
$= (a + 2)(a - 4)(a + 3)(a - 5)$
which are the required factors.

Long Answer Type Questions

9. Simplify:

$$\frac{(25y^2 - 16z^2)^3 + (16z^2 - 9x^2)^3 + (9x^2 - 25y^2)^3}{(5y - 4z)^3 + (4z - 3x)^3 + (3x - 5y)^3}$$

Sol. Since,
$(25y^2 - 16z^2) + (16z^2 - 9x^2) + (9x^2 - 25y^2) = 0$
$\therefore (25y^2 - 16z^2)^3 + (16z^2 - 9x^2)^3 + (9x^2 - 25y^2)^3$
$= 3(25y^2 - 16z^2)(16z^2 - 9x^2)(9x^2 - 25y^2)$
$= 3\{(5y)^2 - (4z)^2\}\{(4z)^2 - (3x)^2\}\{(3x)^2 - (5y)^2\}$
$= 3(5y + 4z)(5y - 4z)(4z + 3x)(4z - 3x)(3x + 5y)(3x - 5y)$

Again, since
$(5y - 4z) + (4z - 3x) + (3x - 5y) = 0$
$\therefore (5y - 4z)^3 + (4z - 3x)^3 + (3x - 5y)^3$
$= 3(5y - 4z)(4z - 3x)(3x - 5y)$

$$\therefore \frac{(25y^2 - 16z^2)^3 + (16z^2 - 9x^2)^3 + (9x^2 - 25y^2)^3}{(5y - 4z)^3 + (4z - 3x)^3 + (3x - 5y)^3}$$

$$= \frac{3(5y + 4z)(4z + 3x)(3x + 5y)(5y - 4z)(4z - 3x)(3x - 5y)}{3(5y - 4z)(4z - 3x)(3x - 55y)}$$

$= (5y + 4z)(4z + 3x)(3x + 5y)$
which is the required simplified expression.

10. Prove that
$(x + y)^3 + (y + z)^3 + (z + x)^3 - 3(x + y)(y + z)(z + x)$
$= 2(x^3 + y^3 + z^3 - 3xyz)$ **[CBSE SP 2010]**

Sol. We have,
LHS
$= (x + y)^3 + (y + z)^3 + (z + x)^3 - 3(x + y)(y + z)(x + x)$
$= (x + y + y + z + z + x)\{(x + y)^2 + (y + z)^2 + (z + x)^2 - (x + y)(y + z) - (x + y)(z + x) - (y + z)(z + y)\}$
$= 2(x + y + z)(x^2 + y^2 + y^2 + z^2 + z^2 + x^2 + 2xy + 2yz + 2zx - x^2 - y^2 - z^2 - xy - yz - zx - xz - yz - xy - yz - xy - zx)$
$= 2(x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$
$= 2(x^3 + y^3 + z^3 - 3xyz)$
Hence, proved.

11. The volume of a cuboid is given by the algebraic expression $2x^3 - x^2 - 13x - 6$ in appropriate unit. Find the possible expressions for the dimensions of the cuboid in appropriate units.

Sol. Since the volume of a cuboid = Length × Breadth × Height
$\therefore$ The possible dimensions of the cuboid will be three factors of the given expression. So, we factorize $2x^3 - x^2 - 13x - 6$.
We see that this expression is zero when $x = -2$. Hence, $x + 2$ will be a factor of this expression.
$\therefore$ We rewrite this expression as follows:
$2x^3 - x^2 - 13x - 6$
$= 2x^3 + 4x^2 - 5x^2 - 10x - 3x - 6$
$= 2x^2(x + 2) - 5x(x + 2) - 3(x + 2)$
$= (x + 2)(2x^2 - 5x - 3)$
$= (x + 2)(2x^2 + x - 6x - 3)$
$= (x + 2)\{x(2x + 1) - 3(2x + 1)\}$
$= (x + 2)(2x + 1)(x - 3)$
Hence, the required dimensions of the cuboid in appropriate units are $x + 2$, $2x + 1$ and $x - 3$

12. If $3x + y + z = 0$, show that $27x^3 + y^3 + z^3 = 9xyz$. **[CBSE SP 2011]**

Sol. We have
$27x^3 + y^3 + z^3 - 9xyz$
$= (3x)^3 + y^3 + z^3 - 3 \times 3xyz$
$= (3x + y + z)(9x^2 + y^2 + z^2 - 3xy - 3xz - yz)$
$= 0 \times (9x^2 + y^2 + z^2 - 3xy - 3xz - yz)$
$= 0$
$\therefore 27x^3 + y^3 + z^3 = 9xyz$

Higher Order Thinking Skills (HOTS) Questions

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Factorise the following:

1. $x^2 + 3y^2 - z^2 + 2yz - 4xy$

Sol. We have
$x^2 + 3y^2 - z^2 + 2yz - 4xy$

$= x^2 - 4xy + 4y^2 - y^2 - z^2 + 2yz$
$= \{x^2 - 2x \times 2y + (2y)^2\} - (y^2 + z^2 - 2yz)$
$= (x - 2y)^2 - (y - z)^2$
$= (x - 2y + y - z)\ (x - 2y - y + z)$
$= (x - y - z)\ (x - 3y + z)$
which are the required factors.

2. $a^2 + 2ab - ac - 3b^2 + 5bc - 2c^2$

Sol. We have

$a^2 + 2ab - ac - 3b^2 + 5bc - 2c^2$

$$= a^2 + a(2b - c) + \left(\frac{2b-c}{2}\right)^2 - \left(\frac{2b-c}{2}\right)^2 - 3b^2 + 5bc - 2c^2$$

$$= \left(a + \frac{2b-c}{2}\right)^2 - \frac{(2b-c)^2}{4} - 3b^2 + 5bc - 2c^2$$

$$= \frac{(2a+2b-c)^2}{4} - \frac{(2b-c)^2 + 12b^2 - 20bc + 8c^2}{4}$$

$$= \frac{(2a+2b-c)^2}{4} - \frac{4b^2 + c^2 - 4bc + 12b^2 - 20bc + 8c^2}{4}$$

$$= \frac{(2a+2b-c)^2}{4} - \frac{(4b)^2 + (3c)^2 - 2 \times 4b \times 3c}{4}$$

$$= \frac{(2a+2b-c)^2 - (4b-3c)^2}{4}$$

$$= \frac{(2a+2b-c+4b-3c)(2a+2b-c-4b+3c)}{4}$$

$$= \frac{(2a+6b-4c)(2a-2b+2c)}{4}$$

$= (a + 3b - 2c)\ (a - b + c)$

which are the required factors.

3. $y^4 + y^2 - 2ay + 1 - a^2$

Sol. $y^4 + y^2 - 2ay + 1 - a^2$

$= y^4 + 1^2 + 2y^2 - y^2 - 2ay - a^2$
$= (y^2 + 1)^2 - (y + a)^2$
$= (y^2 + 1 + y + a)\ (y^2 + 1 - y - a)$

which are the required factors.

4. $a^4 + b^4 + c^4 - 2b^2c^2 - 2a^2c^2 - 2a^2b^2$

Sol. $a^4 + b^4 + c^4 - 2b^2c^2 - 2a^2c^2 - 2a^2b^2$

$= a^4 + b^4 + c^4 + 2b^2c^2 - 2a^2c^2 - 2a^2b^2 - 4b^2c^2$
$= (a^2 - b^2 - c^2)^2 - (2bc)^2$
$= (a^2 - b^2 - c^2 + 2bc)\ (a^2 - b^2 - c^2 - 2bc)$
$= \{a^2 - (b - c)^2\}\ \{a^2 - (b + c)^2\}$
$= (a + b - c)\ (a - b + c)\ (a + b + c)\ (a - b - c)$

which are the required factors.

5. $(x + 1)(x + 3)(x + 5)(x + 7) + 15$

Sol. $(x + 1)\ (x + 3)\ (x + 5)\ (x + 7) + 15$

$= \{(x + 1)\ (x + 7)\}\ \{(x + 3)\ (x + 5)\} + 15$
$= (x^2 + 8x + 7)\ (x^2 + 8x + 15) + 15$
$= (a + 7)\ (a + 15) + 15,$ where $x^2 + 8x = a$ … (1)
$= a^2 + 22a + 105 + 15$
$= a^2 + 22a + 120$
$= a^2 + 12a + 10a + 120$
$= a(a + 12) + 10\ (a + 12)$
$= (a + 12)\ (a + 10)$
$= (x^2 + 8x + 12)\ (x^2 + 8x + 10)$ [From (1)]
$= (x^2 + 6x + 2x + 12)\ (x^2 + 8x + 10)$
$= \{x(x + 6) + 2\ (x + 6)\}\ (x^2 + 8x + 10)$
$= (x + 6)\ (x + 2)\ (x^2 + 8x + 10)$

which are the required factors.

6. Prove that

$(y + z - x)^3 + (z + x - y)^3 + (x + y - z)^3$
$= 3(y + z - x)(z + x - y)(x + y - z)$
$= -24xyz$, if $x + y + z = 0$

Sol. We see that

$(y + z - x) + (z + x - y) + (x + y - z)$
$= x + y + z = 0$ [Given]
$\because\ (y + z - x)^3 + (z + x - y)^3 + (x + y - z)^3$
$= 3(y + z - x)\ (z + x - y)\ (x + y - z)$
$= 3(-x - x)\ (-y - y)\ (-z - z)$ $[\because x + y + z = 0]$
$= 3(-2x)\ (-2y)\ (-2z)$
$= -24\ xyz$

Hence, proved.

7. If $x + y = a$, $x^2 + y^2 = b^2$ and $x^3 + y^3 = c^3$, prove that $a^3 + 2c^3 = 3ab^2$.

Sol. We have

$x + y = a$ …(1)
$x^2 + y^2 = b^2$ …(2)
$\Rightarrow\ (x + y)^2 - 2xy = b^2$
$\Rightarrow\ a^2 - 2xy = b^2$ [From (1)]
$\Rightarrow\ xy = \dfrac{a^2 - b^2}{2}$ …(3)

Also, $x^3 + y^3 = c^3$
$\Rightarrow\ (x + y)\ (x^2 + y^2 - xy) = c^3$
$\Rightarrow\ a\left(b^2 - \dfrac{a^2 - b^2}{2}\right) = c^3$ [From (2) and (3)]
$\Rightarrow\ a(3b^2 - a^2) = 2c^3$

$\Rightarrow \quad 3ab^2 - a^3 = 2c^3$

$\Rightarrow \quad a^3 + 2c^3 = 3ab^2$

Hence, proved.

8. If $3s = a + b + c$, show that

$(s - a)^3 + (s - b)^3 + (s - c)^3 = 3(s - a)(s - b)(s - c)$

Sol. We have $s + s + s = a + b + c$

$\Rightarrow \; (s - a) + (s - b) + (s - c) = 0$

$\therefore \; (s - a)^3 + (s - b)^3 + (s - c)^3$

$= 3(s - a)\,(s - b)\,(s - c)$

Hence, proved.

9. If $2s = a + b + c$, show that

$(s - a)^3 + (s - b)^3 + (s - c)^3 - 3(s - a)(s - b)(s - c)$

$= \frac{1}{2}(a^3 + b^3 + c^3 - 3abc)$

Sol. We have

$(s - a)^3 + (s - b)^3 + (s - c)^3 - 3(s - a)\,(s - b)\,(s - c)$

$= (s - a + s - b + s - c)\,[(s - a)^2 + (s - b)^2 + (s - c)^2$

$- (s - a)\,(s - b) - (s - b)\,(s - c) - (s - c)\,(s - a)]$

$= [3s - (a + b + c)]\,[3s^2 + a^2 + b^2 + c^2 - 2s(a + b + c)$

$- \{s^2 - (a + b)\,s + ab\} - \{s^2 - (b + c)\,s + bc\}$

$- \{s^2 - (c + a)\,s + ca\}]$

$= (3s - 2s)\,[a^2 + b^2 + c^2 - 2s \times 2s + 2s\,(a + b + c)$

$- ab - bc - ca]$

$= s[a^2 + b^2 + c^2 - 4s^2 + 4s^2 - ab - bc - ca]$

$= \frac{a + b + c}{2}(a^2 + b^2 + c^2 - ab - bc - ca)$

$= \frac{1}{2}\left(a^3 + b^3 + c^3 - 3abc\right)$

Hence, proved.

10. Factorise:

$-7x^2 - 3y^2 + 22xy - 44x + 12y - 12$

Sol. We have

$-7x^2 - 3y^2 + 22xy - 44x + 12y - 12$

$= -7x^2 + 22x\,(y - 2) - 3y^2 + 12y - 12$

$= -7x^2 + 22x\,(y - 2) - 3\,(y - 2)^2$

$= -7x^2 + 21x\,(y - 2) + (y - 2)x - 3(y - 2)^2$

$= -7x\{x - 3\,(y - 2)\} + (y - 2)\,\{x - 3\,(y - 2)\}$

$= (x - 3y + 6)\,(y - 2 - 7x)$

which are the required factors.

Self-Assessment

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Multiple-Choice Questions

1. $\sqrt{2}$ is a polynomial of degree

(*a*) 2 (*b*) 0

(*c*) 1 (*d*) $\frac{1}{2}$ **[CBSE SP 2012]**

Sol. (*b*) 0

Since any constant is a polynomial of degree 0 and since $\sqrt{2}$ is a constant, hence, it is a polynomial of degree 0.

2. Degree of polynomial $(x^3 - 2)(x^2 + 11)$ is

(*a*) 0 (*b*) 3

(*c*) 5 (*d*) 2 **[CBSE SP 2012]**

Sol. (*c*) 5

We have $(x^3 - 2)\,(x^2 + 11) = x^5 + 11x^3 - 2x^2 - 22$ which is a polynomial of degree 5.

Fill in the Blanks

3. Degree of zero polynomial is **not defined**.

4. The coefficient of x^2 in $(2x^2 - 5)\,(4 + 3x^2)$ is **–7**.

5. Zeroes of the polynomial $p(x) = (x + 2)\,(x + 5)$ are **–2** and **–5**.

6. A polynomial of degree 5 in x has at most **6** terms.

Assertion-Reason Type Questions

Directions (Q. Nos. 7 to 10): Each of these questions contains an assertion followed by reason. Read them carefully, and answer the question on the basis of the following options, select the one that best describes the two statements.

(a) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.

(b) Both Assertion and Reason are true but Reason is not the correct explanation of Assertion.

(c) Assertion is true but Reason is false.

(d) Assertion is false but Reason is true.

7. Assertion: $x + 8 = 0$ is a linear polynomial.

Reason: A polynomial of degree 1 is called a linear polynomial.

Sol. The correct answer is (a).

Both Assertion and Reason are correct and Reason is the correct explanation of Assertion.

8. **Assertion:** 2 is a zero of the polynomial

$p(x) = x^2 - 3x - 4$

Reason: Putting $x = 2$, we get $p(x) = -6$

Sol. The correct answer is (d).

Putting $x = 2$, we get $p(x) = -6$.

Thus, Assertion is incorrect and Reason is correct.

9. **Assertion:** $99^3 = 100^3 - 3 \times 100 \times 99 + 99^3$

Reason: $(x - y)^3 = x^3 - 3xy(x - y) - y^3$

Sol. The correct answer is (d).

Assertion is incorrect and Reason is correct as it is a direct formula.

10. **Assertion:** $(a + b)^2 = a^2 + 2ab + b^2$ is an algebraic identity.

Reason: $(a + b)^2 = a^2 + 2ab + b^2$ holds true only for a particular pair of a and b.

Sol. The correct answer is (c).

$(a + b)^2 = a^2 + 2ab + b^2$ is an algebraic identity and it holds true for any pair of a and b. Thus, Assertion is correct but Reason is incorrect.

Case Study Based Questions

11. CBSE Udaan is a project started by Central Board of Secondary Education under the guidance of the Ministry of Human Resource Development, Government of India. The main objective of this program is to increase the enrolment rate of girls in the engineering colleges of the country.

In a college, a group of $(x + y)$ lecturers, $(x^2 + y^2)$ girls and $(x^3 + y^3)$ boys organised a campaign on CBSE Udaan. Based on the above information, answer the following questions.

(*a*) Which of the following mathematical concepts is used here?

(*i*) Polynomial (*ii*) Linear equations

(*iii*) Heron's formula (*iv*) None of these

Ans. (*i*) Polynomial

(*b*) The expansion of $(x + y)^3$ is

(*i*) $x^3 + y^3 + 3x^2y + 3xy^2$

(*ii*) $x^3 - y^3 + 3x^2y + 3xy^2$

(*iii*) $x^3 + y^3 - 3x^2y + 3xy^2$

(*iv*) $x^3 + y^3 + 3x^2y - 3xy^2$

Ans. (*i*) $x^3 + y^3 + 3x^2y + 3xy^2$

(*c*) Which of the following is the correct identity?

(*i*) $x^3 + y^3 = (x + y)(x^2 + xy + y^2)$

(*ii*) $x^3 + y^3 = (x - y)(x^2 + xy + y^2)$

(*iii*) $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

(*iv*) None of these

Ans. (*iii*) $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

(*d*) If in the group, there are 12 lecturers and 80 girls, then what is the number of boys?

(*i*) 80 (*ii*) 240

(*iii*) 576 (*iv*) 600

Ans. (*iii*) 576

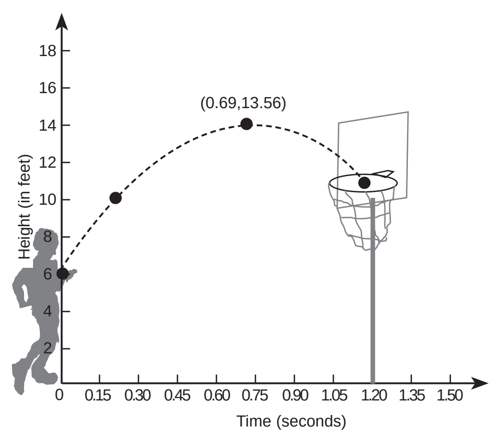
(*e*) If in the group, there are 8 lecturers and the value of xy is equal to 11, then what is the number of boys?

(*i*) 60 (*ii*) 180

(*iii*) 248 (*iv*) 260

Ans. (*iii*) 248

12. Two teams from a certain school were playing basketball match in the school playground. One of the team members, Varun observes that the path followed by the basketball is parabolic in shape. Then the path of the basketball was placed on a graph.



If the equation of the height of a ball (in feet) at a given time (t) is

$$h(t) = -16t^2 + 22t + 6$$

then answer the following questions.

(*a*) What is the degree of the polynomial?

(*i*) 0 (*ii*) 1

(*iii*) 2 (*iv*) 3

Ans. (*iii*) 2

(*b*) The polynomial is classified as ____________ on the basis of number of terms.

(*i*) linear polynomial

(*ii*) monomial

(*iii*) binomial

(*iv*) trinomial

Ans. (*iv*) trinomial

(*c*) What is the name of the given polynomial on the basis of degree?

(*i*) Constant polynomial

(*ii*) Linear polynomial

(*iii*) Quadratic polynomial

(*iv*) Cubic polynomial

Ans. (*iii*) Quadratic polynomial

(*d*) At what times (in sec) does the ball reach 10 feet?

(*i*) 0.22 sec, 0.32 sec

(*ii*) 0.24 sec, 0.95 sec

(*iii*) 0.22 sec, 1.16 sec

(*iv*) 0.24 sec, 1.26 sec

Ans. (*iii*) 0.22 sec, 1.16 sec

(*e*) From the graph, at what time does the ball reach its maximum height?

(*i*) 0.32 sec (*ii*) 0.45 sec

(*iii*) 0.69 sec (*iv*) 1.01 sec

Ans. (*iii*) 0.69 sec

Very Short Answer Type Questions

13. Write the algebraic expression $\frac{2}{x^{-5}} + \frac{3}{x^{-1}} - \frac{\sqrt{3}}{4}x^6 + \sqrt{8} - 7x^3$ in the standard form of a polynomial.

Sol. $\frac{2}{x^{-5}} + \frac{3}{x^{-1}} - \frac{\sqrt{3}}{4}x^6 + \sqrt{8} - 7x^3$

$$= 2x^5 + 3x - \frac{\sqrt{3}}{4}x^6 + \sqrt{8} - 7x^3$$

$\therefore$ The required polynomial in standard form is

$$-\frac{\sqrt{3}}{4}x^6 + 2x^5 - 7x^3 + 3x + \sqrt{8}$$

14. If $f(x) = 3x^2 + 5x - 7$, show that $\frac{f(1) - f(0)}{f(2)} = \frac{8}{15}$.

Sol. We have $f(1) = 3 + 5 - 7 = 1$

$f(0) = -7$, $f(2) = 3 \times 4 + 5 \times 2 - 7 = 15$

$$\therefore \frac{f(1) - f(0)}{f(2)} = \frac{1 + 7}{15} = \frac{8}{15}$$

Hence, proved.

Short Answer Type-I Questions

15. Simplify by using an algebraic identity:

$$\frac{18.31 \times 18.31 - 8.31 \times 8.31}{26.62}$$

Sol. $\frac{18.31 \times 18.31 - 8.31 \times 8.31}{26.62}$

$$= \frac{18.31^2 - 8.31^2}{26.62}$$

$$= \frac{(18.31 + 8.31)(18.31 - 8.31)}{26.62}$$

$$= \frac{26.62 \times 10}{26.62}$$

$$= 10$$

16. If $x + \frac{1}{x} = 2$, find the value of $x^3 + \frac{1}{x^3}$.

Sol. We have

$$x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x}\right)^3 - 3x \times \frac{1}{x}\left(x + \frac{1}{x}\right)$$

$$= 2^3 - 3 \times 2$$

$$= 8 - 6$$

$$= 2$$

which is the required value.

Short Answer Type-II Questions

17. Express $3a^2 + 2b^2 + c^2 - 2\sqrt{6}\,ab + 2\sqrt{3}\,ac - 2\sqrt{2}bc$ as the square of a trinomial.

Sol. $3a^2 + 2b^2 + c^2 - 2\sqrt{6}\,ab + 2\sqrt{3}\,ac - 2\sqrt{2}\,bc$

$$= \left(\sqrt{3}a\right)^2 + \left(\sqrt{2}b\right)^2 + c^2 - 2\sqrt{2} \times \sqrt{3}\,ab$$

$$+ 2 \times \sqrt{3}\,ac - 2 \times \sqrt{2}\,bc$$

$$= \left(\sqrt{3}a - \sqrt{2}b + c\right)^2$$

which is the required expression.

18. If $x^2 + \frac{9}{x^2} = 12$, find the value of $x^3 + \frac{27}{x^3}$.

Sol. We have

$$x^2 + \frac{9}{x^2} = 12$$

$$\Rightarrow \quad x^2 + \left(\frac{3}{x}\right)^2 = 12$$

$$\Rightarrow \quad \left(x + \frac{3}{x}\right)^2 - 2 \times x \times \frac{3}{x} = 12$$

$$\Rightarrow \quad \left(x + \frac{3}{x}\right)^2 = 18$$

$$\Rightarrow \qquad x + \frac{3}{x} = \pm 3\sqrt{2} \qquad \ldots(1)$$

When $x + \frac{3}{x} = 3\sqrt{2}$, we have

$$x^3 + \frac{27}{x^3} = x^3 + \left(\frac{3}{x}\right)^3$$

$$= \left(x + \frac{3}{x}\right)^3 - 3 \times x \times \frac{3}{x}\left(x + \frac{3}{x}\right)$$

$$= \left(3\sqrt{2}\right)^3 - 9 \times 3\sqrt{2}$$

$$= 54\sqrt{2} - 27\sqrt{2}$$

$$= 27\sqrt{2}$$

When $x + \frac{3}{x} = -3\sqrt{2}$, we have

$$x^3 + \frac{27}{23} = \left(-3\sqrt{2}\right)^3 - 9 \times \left(-3\sqrt{2}\right)$$

$$= -54\sqrt{2} + 27\sqrt{2}$$

$$= -27\sqrt{2}$$

$\therefore$ The required value of the given expression is $\pm 27\sqrt{2}$.

Long Answer Type Questions

19. Show that

$$x^{18} - y^{18} = (x + y)(x^2 - xy + y^2)(x - y)(x^2 + xy + y^2) \times (x^6 - x^3y^3 + y^6)(x^6 + x^3y^3 + y^6).$$

Sol. We have

$$x^{18} - y^{18} = (x^9)^2 - (y^9)^2$$

$$= (x^9 + y^9)(x^9 - y^9)$$

$$= \{(x^3)^3 + (y^3)^3\}\{(x^3)^3 - (y^3)^3\}$$

$$= (x^3 + y^3)(x^6 - x^3y^3 + y^6)(x^3 - y^3)(x^6 + x^3y^3 + y^6)$$

$$= (x + y)(x^2 - xy + y^2)(x - y)(x^2 + xy + y^2)(x^6 - x^3y^3 + y^6)(x^6 + x^3y^3 + y^6)$$

Hence, proved.

20. Find all the real factors of

$2x^5 - 6x^4 + 13x^3 - 39x^2 + 20x - 60.$

Sol. We have

$2x^5 - 6x^4 + 13x^3 - 39x^2 + 20x - 60$

$= 2x^4(x - 3) + 13x^2(x - 3) + 20(x - 3)$

$= (x - 3)(2x^4 + 13x^2 + 20) \qquad \ldots(1)$

Now, $2x^4 + 13x^2 + 20$

$$= 2\left(x^4 + \frac{13}{2}x^2 + 10\right)$$

$$= 2\left[\left(x^2\right)^2 + 2x^2 \times \frac{13}{4} + \left(\frac{13}{4}\right)^2 - \left(\frac{13}{4}\right)^2 + 10\right]$$

$$= 2\left[\left(x^2 + \frac{13}{4}\right)^2 + 10 - \frac{169}{16}\right]$$

$$= 2\left[\left(x^2 + \frac{13}{4}\right)^2 - \left(\frac{3}{4}\right)^2\right]$$

$$= 2\left[\left(x^2 + \frac{13}{4} + \frac{3}{4}\right)\left(x^2 + \frac{13}{4} - \frac{3}{4}\right)\right]$$

$$= 2\left[\left(x^2 + 4\right)\left(x^2 + \frac{5}{2}\right)\right]$$

$= (x^2 + 4)(2x^2 + 5)$

$\therefore\ 2x^5 - 6x^4 + 13x^3 - 39x^2 + 20x - 60$

$= (x - 3)(x^2 + 4)(2x^2 + 5)$ [From (1)]

which are the required factors.

Let's Compete

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Multiple-Choice Questions

1. A polynomial $f(x)$ has degree 10. Then the maximum and minimum numbers of terms that $f(x)$ may have are

(*a*) 11 and 1 (*b*) 10 and 1

(*c*) 10 and 2 (*d*) 11 and 10

Sol. (*a*) 11 and 1

A polynomial $f(x)$ of degree 10 with maximum number of term is as follows:

$$f(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5 + a_6x^6 + a_7x^7 + a_8x^8 + a_9x^9 + a_{10}x^{10}$$

$\therefore$ It has at most 11 terms and at least 1 term *viz.* $f(x) = a_0$.

2. If each of the last two terms of the polynomial $2x^3 + 5x^2 - 9x + 10$ is increased by d so that the resulting polynomial has 1 as its zero, then the value of d is equal to

(*a*) 1 (*b*) –1

(*c*) 2 (*d*) –4

Sol. (*d*) –4

Let the resulting polynomial be

$$f(x) = 2x^3 + 5x^2 - 9x + d + 10 + d$$

$$= 2x^3 + 5x^2 - 9x + 10 + 2d$$

Now, $\qquad f(1) = 0$

$\Rightarrow \quad 2 + 5 - 9 + 10 + 2d = 0$

$\Rightarrow \quad 8 + 2d = 0$

$\Rightarrow \quad d = -4$

3. The number to be subtracted from the polynomial $x^4 + 2x^3 - 3x^2 + 5$ so that –3 becomes its zero, is

(a) –5 (b) 5

(c) 1 (d) 4

Sol. (b) 5

Let the required number to be subtracted from the given polynomial be n. Then the new polynomial $p(x)$ is given by

$$p(x) = x^4 + 2x^3 - 3x^2 + 5 - n$$

Since –3 is a zero of this polynomial,

$$\therefore \quad p(-3) = 0$$

$$\Rightarrow \quad (-3)^4 + 2(-3)^3 - 3(-3)^2 + 5 - n = 0$$

$$\Rightarrow \quad 81 - 54 - 27 + 5 - n = 0$$

$$\Rightarrow \quad 86 - 81 - n = 0$$

$$\Rightarrow \quad n = 5$$

4. When $x^3 + 4x^2 - 3x + b$ is divided by $x - 2$, then the remainder is the zero of another polynomial $x^3 - 19x^2 + x - 19$. Then the value of b is equal to

(a) –2 (b) –1

(c) 3 (d) 1

Sol. (d) 1

Let $f(x) = x^3 + 4x^2 - 3x + b$

and $g(x) = x^3 - 19x^2 + x - 19$

The zero of the polynomial $g(x)$ is given by the equation $g(x) = 0$

$$\Rightarrow \quad x^2(x - 19) + 1(x - 19) = 0$$

$$\Rightarrow \quad (x - 19)(x^2 + 1) = 0$$

$$\Rightarrow \quad x = 19 \qquad (\because x^2 + 1 \neq 0)$$

$\therefore$ Zero of $g(x)$ is 19.

Hence, 19 is the remainder when $f(x)$ is divided by $x - 2$.

$$\therefore \quad f(2) = 2^3 + 4 \times 2^2 - 3 \times 2 + b = 19$$

$$\Rightarrow \quad 8 + 16 - 6 + b = 19$$

$$\Rightarrow \quad 18 + b = 19$$

$$\Rightarrow \quad b = 1$$

5. $x + 2$ is a factor of the polynomial

(a) $x^3 - 2x^2 + 3x - 6$ (b) $x^3 + 2x^2 - 3x - 6$

(c) $x^3 + 2x^2 + 3x - 6$ (d) $x^3 + 2x^2 + 3x + 6$

Sol. (b) $x^2 + 2x^2 - 3x - 6$

We see that the polynomial in only (b) is 0 when $x = -2$, since,

$$(-2)^3 + 2(-2)^2 - 3(-2) - 6 = -8 + 8 + 6 - 6 = 0$$

6. The product of two expressions is $9x^2 - 16y^2 - 12x + 16y$. If factor is $3x + 4(y - 1)$, then the other factor is

(a) $3x - 4y$ (b) $4y - 3x$

(c) $3x + 4y$ (d) $-3x - 4y$

Sol. (a) $3x - 4y$

We have

$$9x^2 - 16y^2 - 12x + 16y$$
$$= (3x)^2 - (4y)^2 - 12x + 16y$$
$$= (3x + 4y)(3x - 4y) - 4(3x - 4y)$$
$$= (3x - 4y)(3x + 4y - 4)$$
$$= (3x - 4y)\{3x + 4(y - 1)\}$$

$\therefore$ If one factor is $3x + 4(y - 1)$, then the other factor is $3x - 4y$.

7. The linear polynomial in 'a' which must be added with the polynomial $a^4 + 2a^3 - 2a^2 + a - 1$ so that the resulting polynomial is exactly divisible by $a^2 + 2a + 3$, is

(a) $11a - 14$ (b) $14 - 11a$

(c) $11a + 14$ (d) $-11a - 14$

Sol. (d) $-11a - 14$

Let the required linear polynomial in 'a' be $k_1a + k_2$ where k_1 and k_2 are some constants.

$\therefore$ The new polynomial is $a^4 + 2a^3 - 2a^2 + (k_1 + 1)a + k_2 - 1$. If we divide this polynomial by $a^2 + 2a + 3$ by long division method, then the remainder becomes $(k_1 + 11)a + k_2 + 14$. If the remainder is zero, then

$$k_1 = -11$$

and $k_2 = -14$.

$\therefore$ The required linear polynomial is $-11a - 14$.

8. The zeroes of the polynomial $-6x^3 - 23x^2 + 5x + 4$ are $-\frac{1}{3}, \frac{1}{2}$ and -4. Then the factors of the polynomial are

(a) $3x - 1$, $1 + 2x$, $4 + x$ and 6

(b) $3x + 1$, $1 - 2x$, $4 + x$ and -6

(c) $3x - 2$, $2 + x$, $4x + 1$ and 24

(d) $1 + 3x$, $1 + 2x$, $4 - x$ and 24

Sol. (b) $3x + 1$, $1 - 2x$, $4 + x$ and -6

We have $\left(x + \frac{1}{3}\right)\left(x - \frac{1}{2}\right)(x + 4)$

$$= \frac{(3x + 1)(2x - 1)(x + 4)}{6}$$

$$= \frac{(6x^2 - x - 1)(x + 4)}{6}$$

$$= \frac{6x^3 - x^2 - x + 24x^2 - 4x - 4}{6}$$

$$= \frac{6x^3 + 23x^2 - 5x - 4}{6}$$

$$= \frac{-6x^3 - 23x^2 + 5x + 4}{-6}$$

Hence, the required factors of the given polynomial are $3x + 1$, $1 - 2x$, $4 + x$ and -6.

9. If $x^2 + y^2 + z^2 = 16$ and $x + y + z = 12$, then the value of $xy + yz + zx$ is

(*a*) 128 (*b*) ±8

(*c*) 64 (*d*) ±16

Sol. (*c*) 64

We have $x^2 + y^2 + z^2 = 16$

$\Rightarrow \quad (x + y + z)^2 - (xy + yz + zx) = 16$

$\Rightarrow \quad 12^2 - 2(xy + yz + zx) = 16$

$\Rightarrow \quad xy + yz + zx = \frac{144 - 16}{2}$

$= \frac{128}{2} = 64$

10. If $x^3 + y^3 + z^3 = 42$, $x^2 + y^2 + z^2 = 16$, and $x + y + z = 6$, then the value of xyz is

(*a*) –2 (*b*) 2

(*c*) 6 (*d*) –6

Sol. (*b*) 2

We have

$x^2 + y^2 + z^2 = 16$

$\Rightarrow \quad (x + y + z)^2 - 2(xy + yz + zx) = 16$

$\Rightarrow \quad 6^2 - 2(xy + yz + zx) = 16$

$\Rightarrow \quad xy + yz + zx = \frac{36 - 16}{2}$

$= 10 \quad \ldots(1)$

Now, $x^3 + y^3 + z^3 - 3xyz$

$= (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$

$\Rightarrow \quad 42 - 3xyz = 6 \times (16 - 10) = 36$ [From (1)]

$\Rightarrow \quad xyz = \frac{42 - 36}{3} = \frac{6}{3} = 2$

Value-based Question (Optional)

(Page 30)

1. Two brothers were studying in a school in classes X and XII. The younger brother, a student of class X could not find the factors of the polynomial $x^3 + 4x^2 + x - 6$. He asked his elder brother, a student of class XII to help him to solve the problem. The elder brother did not remember theorems to solve the problem. Hence, out of affection to his younger brother, the elder brother promised to help his younger brother to solve the problem. The elder brother had to study the remainder and factor theorems in order to explain the solution of the problem to his younger brother. Ultimately, he solved the problem and made the younger brother happy.

(*a*) Find all the factors of the polynomial.

(*b*) What values are depicted in this problem?

Sol. (*a*) We have

$$f(x) = x^3 + 4x^2 + x - 6$$

$f(x) = 0$ when $x = 1$

$\therefore$ $x - 1$ is a factor of $f(x)$. So, we write

$f(x) = x^3 - x^2 + 5x^2 - 5x + 6x - 6$

$= x^2(x - 1) + 5x(x - 1) + 6(x - 1)$

$= (x^2 + 5x + 6)(x - 1)$

$= (x^2 + 3x + 2x + 6)\, 6(x - 1)$

$= \{x(x + 3) + 2(x + 3)\}(x - 1)$

$= (x + 3)(x + 2)(x - 1)$

which are the required factors of the given polynomial.

(*b*) Helpfulness, kindness and affection to younger brother.

3

Coordinate Geometry

Checkpoint (Page 32)

1. The x-coordinate of every point on y-axis is

(*a*) 0 (*b*) 1

(*c*) 2 (*d*) None of these

Sol. (*a*) 0

Since $x = 0$ is the equation of y-axis, hence the x-coordinate of every point on the y-axis is 0.

2. A point whose y-coordinate is zero, will lie on

(*a*) y-axis

(*b*) x-axis

(*c*) line parallel to x-axis

(*d*) line parallel to y-axis

Sol. (*b*) x-axis

Since the equation of x-axis is $y = 0$, hence the y-coordinate of every point on the x-axis is zero.

3. Points whose x and y coordinates are equal, lie on a line

(*a*) x-axis

(*b*) y-axis

(*c*) $y = x$

(*d*) None of these

Sol. (*c*) $y = x$

We know that the equation of a line whose x-coordinate and y-coordinate are equal is $y = x$.

4. Write the coordinates of origin.

Sol. The coordinates of the origin are clearly (0, 0).

5. Plot the following points on the graph sheet.

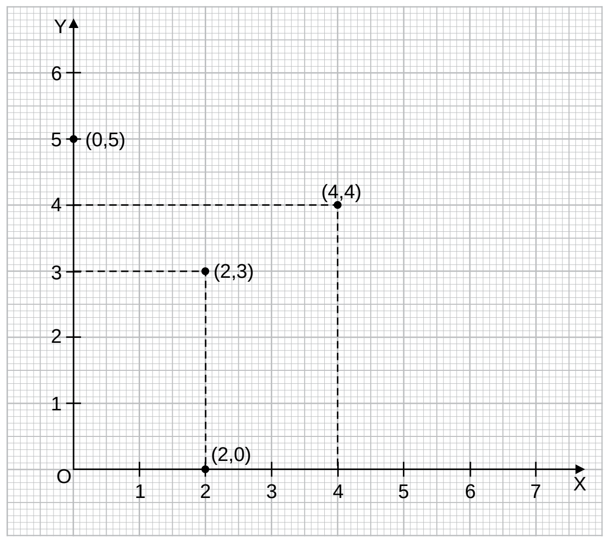
(*a*) (0, 5)

(*b*) (2, 0)

(*c*) (4, 4)

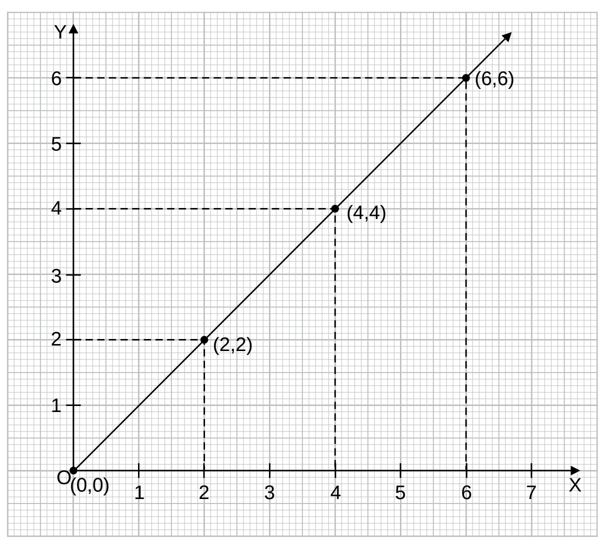
(*d*) (2, 3)

Sol. The given points are shown in the graph paper.



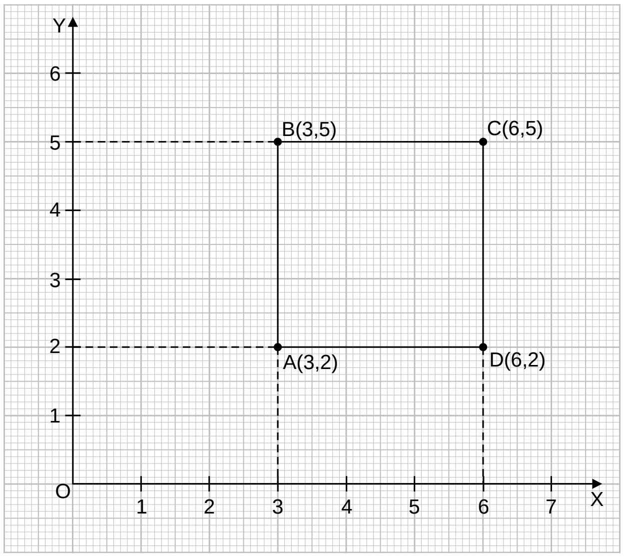
6. Plot the points (0, 0), (2, 2), (4, 4) and (6, 6) on the graph sheet. Do they lie on a line?

Sol. The given points are plotted in the given graph paper. Yes, all these points lie on a line.



7. Plot the points A(3, 2), B(3, 5), C(6, 5) and D(6, 2). Name the figure formed by joining points A, B, C and D. Find its area.

Sol. From the graph



We see that AD = (6 – 3) units = 3 units

and AB = (5 – 2) units = 3 units

$\therefore$ AB = AD

Similarly, BC = (6 – 3) units = 3 units

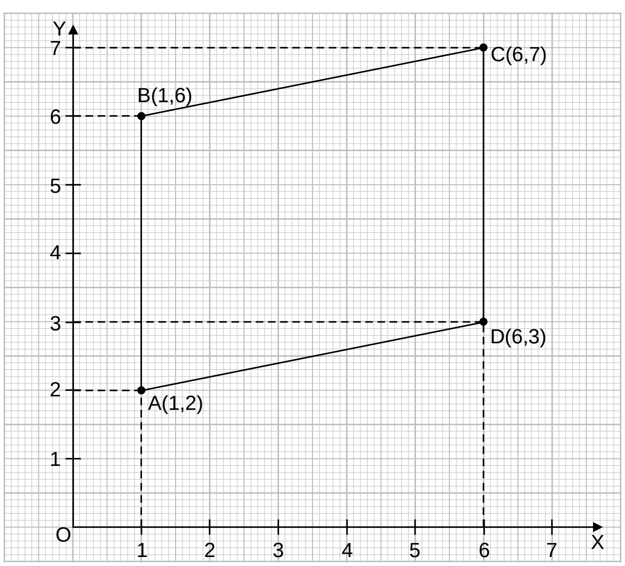
and CD = (5 – 2) units = 3 units

$\therefore$ AB = BC = CD = AD = 3 units and each of the angles A, B, C and D is 90°.

$\therefore$ The figure is a square with area 3 × 3 sq units i.e. 9 sq units.

8. Name the figure formed by joining points A(1, 2), B(1, 6), C(6, 7) and D(6, 3).

Sol. The given points are plotted in the graph paper.

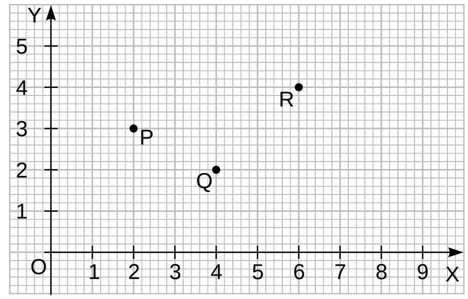


From the figure we see that

AB ∥ CD and BC ∥ AD.

$\therefore$ The figure ABCD is a parallelogram.

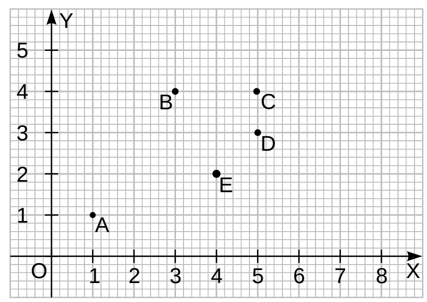
9. From the graph, write coordinates of points P and Q.



Sol. From the graph, we see that x-coordinate of P is 2 and its y-coordinate is 3. Hence, coordinates of P are (2, 3).

Similarly, the coordinates of Q are (4, 2).

10. From the graph, choose the letters that correspond to points whose coordinates are (1, 1) and (5, 4).



Sol. We see from the graph that the x and y coordinates of the point A are 1 and 1 respectively. Hence, A is the point (1, 1). Similarly, C is the point (5, 4).

Milestone

(Page 36)

Multiple-Choice Questions

1. Equation of x-axis is

(*a*) $y = 0$ (*b*) $x = 0$

(*c*) both (*a*) and (*b*) (*d*) None of these

Sol. (*a*) $y = 0$

We know that the y-coordinate of every point on the x-axis is zero. Hence, the equation of x-axis is $y = 0$.

2. In which quadrant does the point (–3, –4) lies?

(*a*) Quadrant I (*b*) Quadrant II

(*c*) Quadrant III (*d*) Quadrant IV

Sol. (*c*) Quadrant III

We know that in the quadrant III, both x and y coordinates are negative.

∴ (–3, –4) lies in the quadrant III.

3. If the ordinate of a point is 0, it will lie on

(*a*) x-axis (*b*) y-axis

(*c*) Quadrant III (*d*) Quadrant IV

Sol. (*a*) x-axis

We know that the ordinate of every point on the x-axis is 0.

4. The distance of point (–3, –5) from y-axis is

(*a*) –3 units (*b*) 5 units

(*c*) 3 units (*d*) –5 units

Sol. (*c*) 3 units

We know that the point (–3, –5) lies in the quadrant III and its distance from the y-axis is 3. (Since distance cannot be negative)

5. If abscissa of a point is 7 and ordinate is –8, the point is

(*a*) (–8, 7) (*b*) (8, –7)

(*c*) (–7, 8) (*d*) (7, –8)

Sol. (*d*) (7, –8)

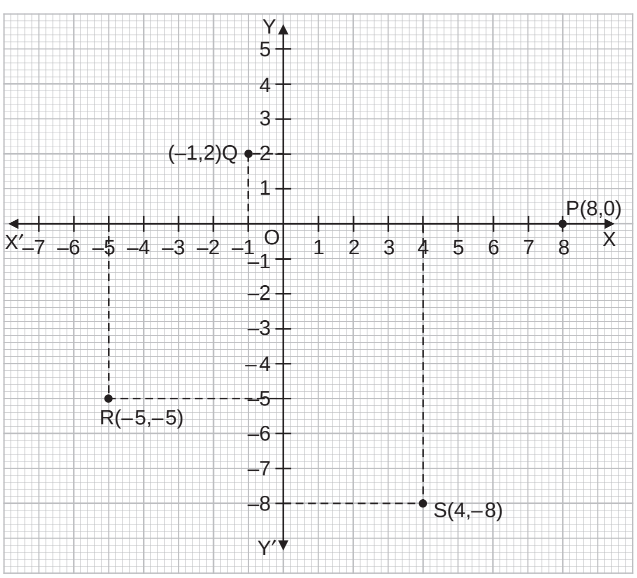
We know that if (*a*, *b*) is a point, then '*a*' is called its abscissa and '*b*' in called its ordinate.

Hence, the coordinates of the given point will be (7, –8)

Very Short Answer Type Questions

6. Plot the points P(8, 0), Q(–1, 2), R(–5, –5) and S(4, –8) on graph paper.

Sol. The given points are plotted on the graph paper below.



7. Write the sign of coordinates of a point in

(*a*) Quadrant III (*b*) Quadrant IV

Sol. (*a*) We know that in quadrant III both x-coordinate and y-coordinate are negative. Hence, the required sign of coordinates of a point is (–, –).

(*b*) In quadrant IV, x-coordinate is positive and y-coordinate is negative. Hence, the required sign of coordinates of a point is (+, –).

8. Locate two points on x-axis whose distance from origin is 10 units?

Sol. We know that the y-coordinate of every point on the x-axis is zero. So, the two points on the x-axis, which are at a distance of 10 units from the origin are (10, 0) and (–10, 0).

9. Name the axis on which (0, –5) lie?

Sol. Since the x-coordinate of every point on the y-axis is 0, hence, (0, –5) will lie on the y-axis in the negative direction of this axis.

10. Find the distance of P(–3, 8) from x-axis?

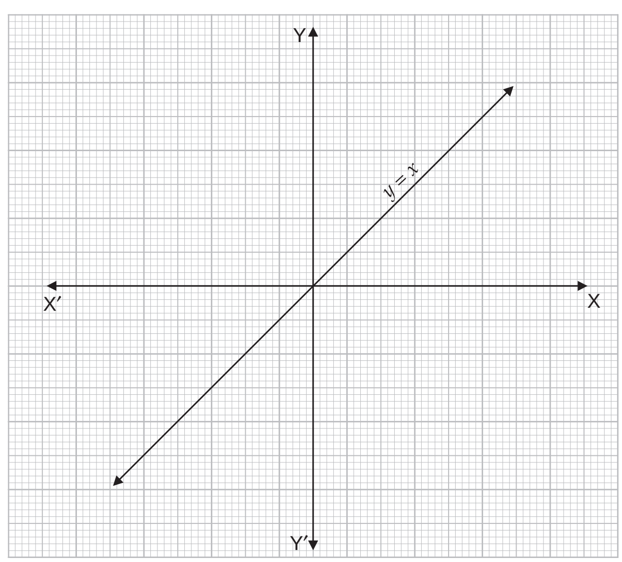
Sol. We know that the distance of a point P(a, b) from the x and y axes are respectively $|b|$ and $|a|$ units.

Hence, the required distance of the point (–3, 8) from the x-axis is 8 units.

Short Answer Type-I Questions

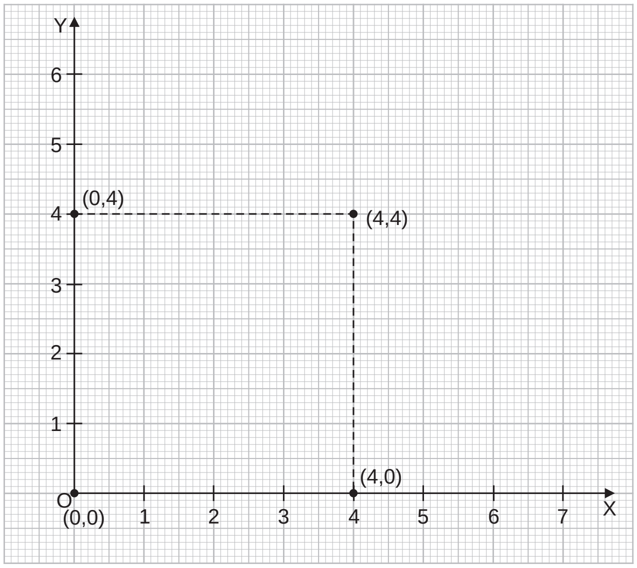
11. Draw the line $x = y$. Name the point at which the line drawn cuts x-axis and y-axis.

Sol. We know that the line $x = y$ will cut the x-axis at the point where $y = 0$ and cut the y-axis at the point where $x = 0$. So, this line will cut both x and y axes at the point where both $x = 0$ and $y = 0$ i.e. at the origin (0, 0).



12. Plot the points (0, 0), (0, 4), (4, 4), (4, 0). Name the figure obtained on joining them and find its area.

Sol. The given points are plotted in the adjoining graph paper. Clearly, the figure is a square of side 4 units. Hence, its area is 4 × 4 sq units i.e. 16 sq units.



13. In which quadrant do the following points lie?

(*a*) (5, 3) (*b*) (–5, 4)

(*c*) (–6, –7) (*d*) (8, –3)

Sol. (*a*) Here both abscissa and ordinate are positive. Hence, this point lies in the quadrant I.

(*b*) Here the abscissa is negative and ordinate is positive. Hence, this point lies in the quadrant II.

(*c*) Here both the abscissa and ordinate are negative. Hence, this point lies in the quadrant III.

(*d*) Here the abscissa is positive and ordinate is negative. Hence, this point lies in the quadrant IV.

Short Answer Type-II Questions

14. Find the area of the triangle whose vertices are (0, 4), (0, 0), (2, 0) by plotting them on graph.

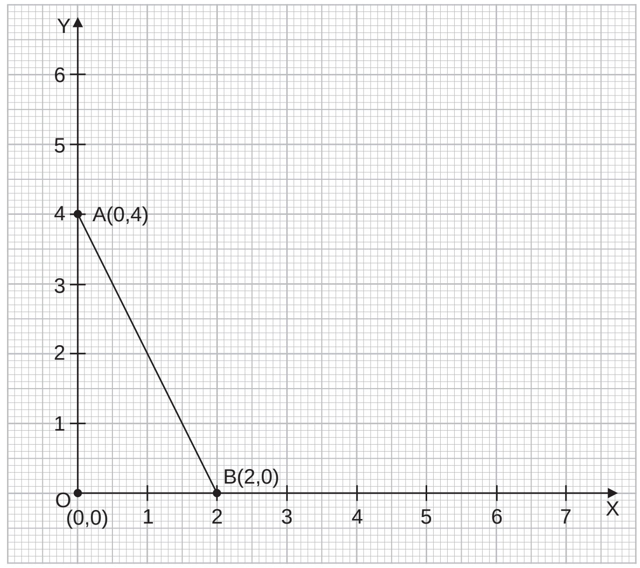
Sol. From the graph, where A(0, 4), O(0, 0), B(2, 0).

We see that the figure AOB is a triangle with ∠AOB = 90°

$\therefore$ Its area = $\frac{1}{2}$ OB × OA

= $\frac{1}{2}$ × 2 × 4 sq units

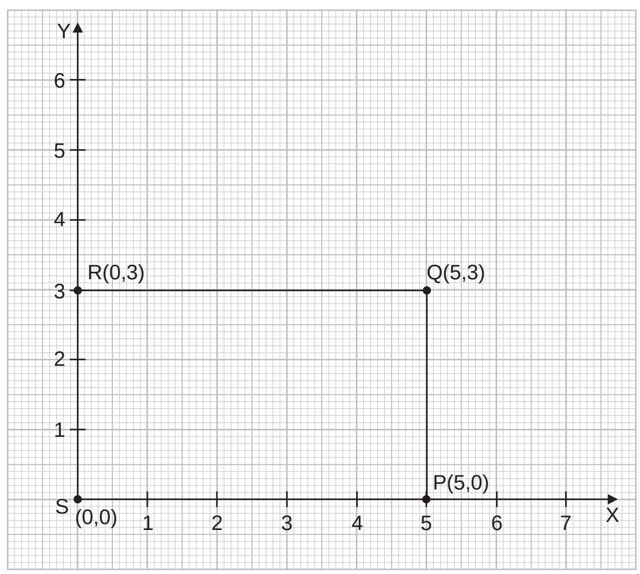
= 4 sq units



Hence, area of triangle whose vertices are (0, 4), (0, 0) and (2, 0) is 4 sq units.

15. Draw a rectangle PQRS with vertices P(5, 0), Q(5, 3), R(0, 3) and S(0, 0).

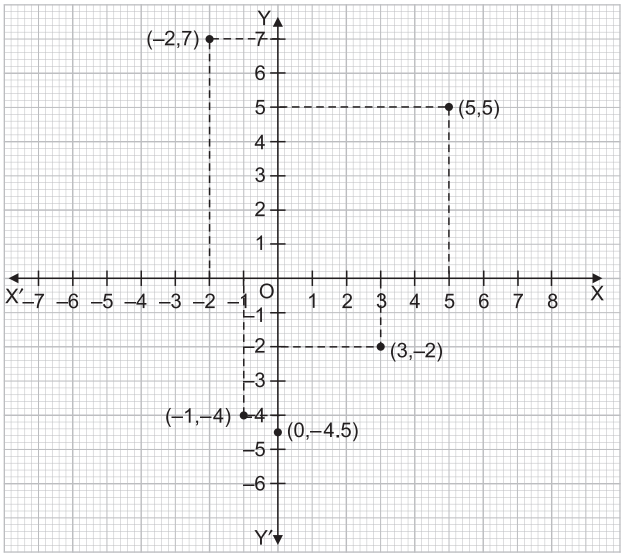
Sol. The given points P, Q, R and S are shown in the adjoining graph. Clearly, the figure PQRS is a rectangle.



16. Plot the points (x, y) given in the following table by using a graph paper.

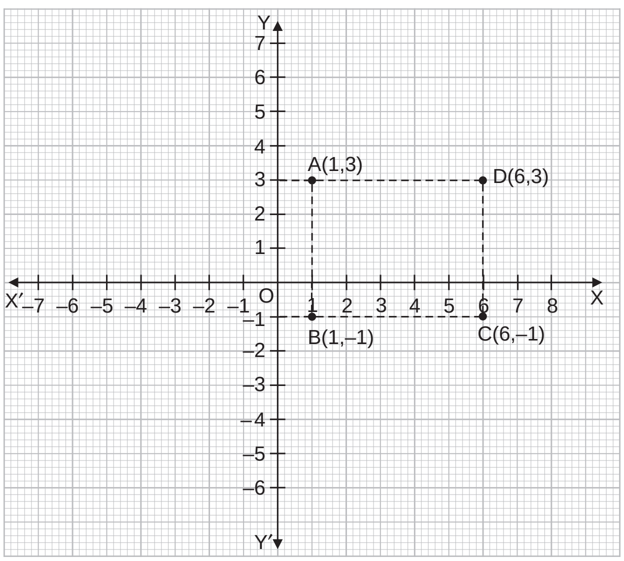
x	–2	0	–1	5	3
y	7	–4.5	–4	5	–2

Sol. The given points are shown in the adjoining graph.



17. Plot the points A(1, 3), B(1, –1), C(6, –1), D(6, 3). Join them respectively and name the type of figure so formed and find its area.

Sol. The given points A, B, C and D are plotted in the adjoining graph. From the graph we see that ABCD is a rectangle of length AD = (6 – 1) units i.e. 5 units and breadth AB = (3 + 1) units i.e. 4 units.

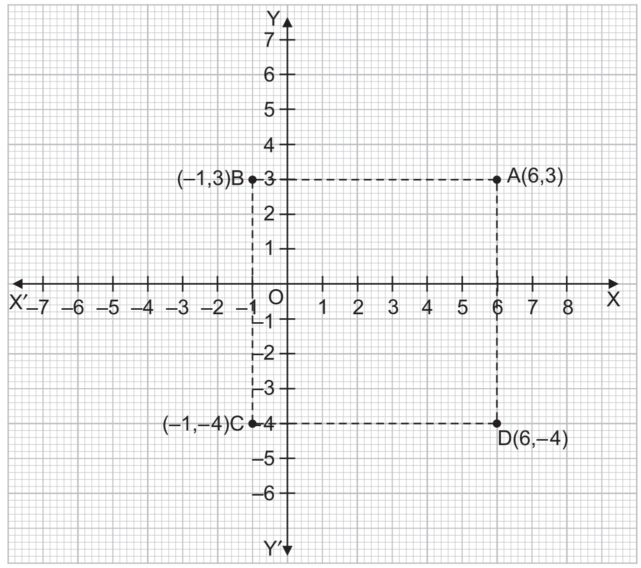


Hence, the required area of the rectangle is 5 × 4 sq units i.e. 20 sq units.

Long Answer Type Questions

18. Points A(6, 3), B(–1, 3) and D(6, –4) are three vertices of a square ABCD. Plot these on a graph paper and hence find the coordinates of point C. Also, find its area.

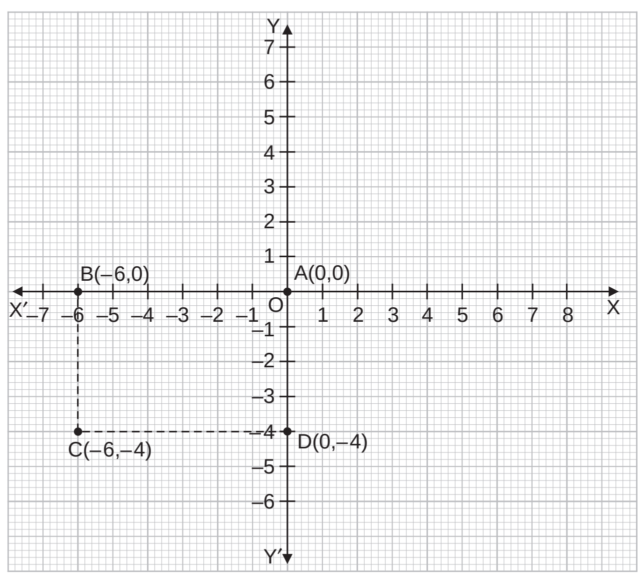
Sol. If we join the points B, A; A, D and then draw a line CD through D parallel to AB and another line BC parallel to AD so that ABCD is a square, then the coordinates of C will be (–1, –4). Also, the area of the square of side 7 units is 7 × 7 sq units i.e. 49 sq units.



∴ The coordinates of point *c* is (–1, –4) and the area of square ABCD is 49 sq units.

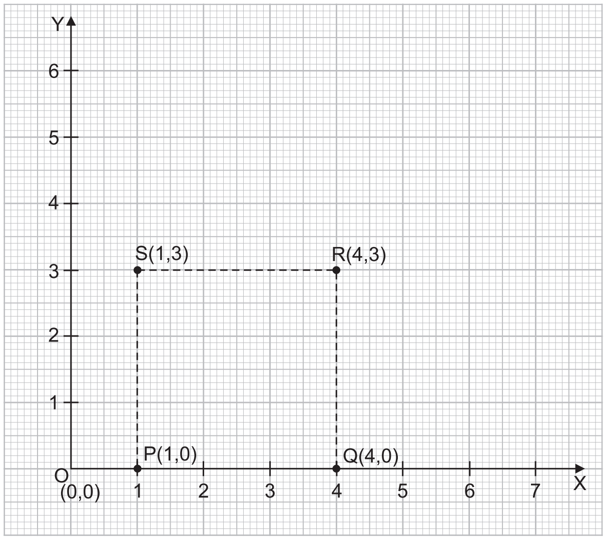
19. Write the coordinates of the vertices of a rectangle whose length is 6 units and breadth is 4 units and one vertex is at the origin and longer side lie on *x*-axis and one of the vertices lie in the third quadrant. Also find its area.

Sol. A rectangle ABCD with A (0, 0), B(–6, 0), C(–6, –4) and D(0, –4) is shown in the adjoining graph. Its length and breadth are AB = 6 units and BC = 4 units respectively. Hence, the required area of the rectangle is 6 × 4 sq units i.e. 24 sq units.



20. Plot the points P(1, 0), Q(4, 0) and S(1, 3). Find the coordinates of the point R such that PQRS is a square.

Sol. The points P, Q and S are plotted on the graph paper. If we draw two line segments through S parallel to x-axis and through Q parallel to PS, then these two line segments will intersect at R.



Hence, PQRS is a square.

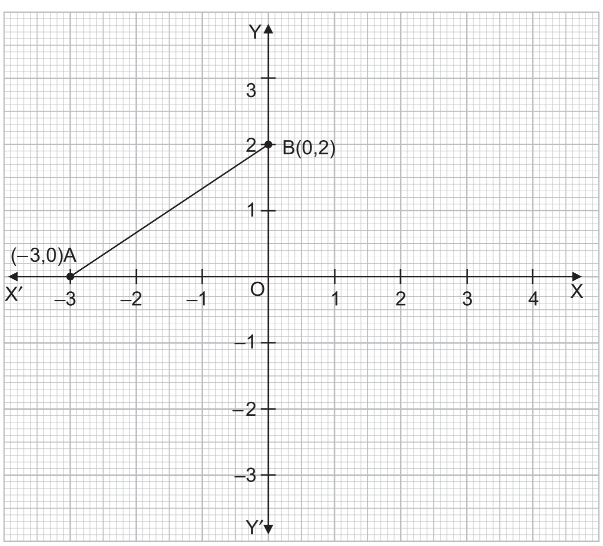
∴ The coordinates of the point R is (4, 3).

Higher Order Thinking Skills (HOTS) Questions

(Page 38)

1. Find the area of the triangle formed between the line $x = 0$, $y = 0$ and $2x - 3y + 6 = 0$ by using graph method.

Sol. In the given equation, if we put $y = 0$, then we get $x = -3$ and if we put $x = 0$, then we get $y = 2$.



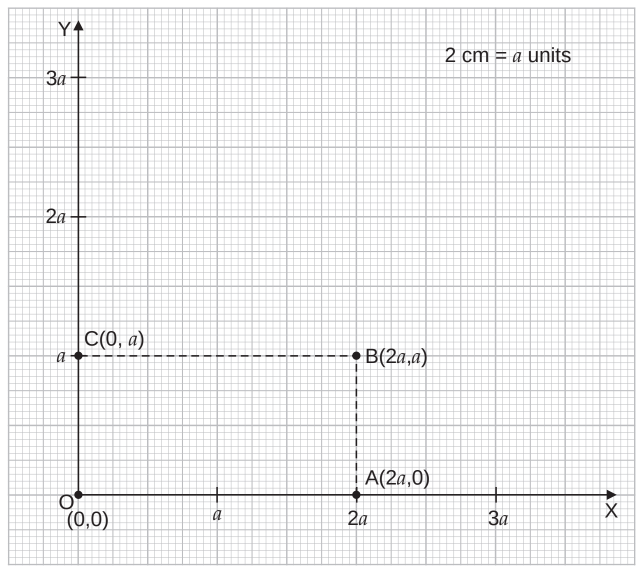
∴ In the figure, ΔOAB is the required triangle, O being the origin. The required area of ΔOAB is $\frac{1}{2} \times 3 \times 2$ sq units i.e. 3 sq units.

2. Find the equation of line parallel to x-axis and at a distance of 3 units below x-axis.

Sol. The given line is parallel to x-axis and cuts the y-axis at the point (0, –3). Hence, the equation of a line is $y = -3$.

3. Draw a rectangle of length $2a$ units and breadth a units taking length and breadth on positive direction of x-axis and y-axis respectively. Also, write the coordinates of its vertices with one vertex at origin.

Sol. The coordinates of the vertices of a rectangle is O(0, 0), A($2a$, 0), B($2a$, a) and C(0, a).



Self-Assessment

(Page 38)

Multiple-Choice Questions

1. The point which lie on y-axis at a distance of 6 units in the negative direction of y-axis is

(*a*) (0, 6) (*b*) (6, 0)

(*c*) (0, –6) (*d*) (–6, 0)

Sol. (*c*) (0, –6)

The point which lie on y-axis at a distance of 6 units in the negative direction of y-axis is (0, –6).

2. If the coordinates of the two points are P(–3, 2) and Q(–5, 3), then abscissa of P – abscissa of Q is,

(*a*) –3 (*b*) 2

(*c*) –2 (*d*) –1

Sol. (*b*) 2

We see that the abscissa of P(–3, 2) is –3, and abscissa of Q (–5, 3) is –5. Abscissa of P – Abscissa of Q = –3 – (–5) = 5 – 3 = 2.

3. Signs of abscissa and ordinate of a point in the second quadrant are

(*a*) (+, +) (*b*) (–, –)

(*c*) (+, –) (*d*) (–, +)

Sol. (*d*) (–, +)

We know that in quadrant II, abscissa is negative and the ordinate is positive. Hence, signs of abscissa and ordinate of a point in quadrant II will be (–, +) respectively.

Fill in the Blanks

4. The point of intersection of the coordinate axes is called the **origin**.

5. The coordinate axes divide the Cartesian plane into four parts known as the **quadrants**.

6. The coordinates of the origin are **(0, 0)**.

7. The y-coordinate of every point on the x-axis is **zero** and the x-coordinate of every point on the y-axis is **zero**.

Assertion-Reason Type Questions

Directions (Q. Nos. 8 to 11): Each of these questions contains an assertion followed by reason. Read them carefully, and answer the question on the basis of the following options, select the one that best describes the two statements.

(a) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.

(b) Both Assertion and Reason are true but Reason is not the correct explanation of Assertion.

(c) Assertion is true but Reason is false.

(d) Assertion is false but Reason is true.

8. **Assertion:** (2, –3) belongs to the third quadrant.

Reason: In the third quadrant, both x and y are negative.

Sol. (*d*)

In 3rd quadrant, both x and y are negative. So, (2, –3) does not belong to 3rd quadrant. Hence assertion is incorrect but reason is correct.

9. **Assertion:** For the point (6, 7), ordinate is 7.

Reason: The coordinates of a point is written as (abscissa, ordinate)

Sol. (*a*)

The coordinates of a point is written as (abscissa, ordinate). Thus 7 is ordinate of point (6, 7).

Therefore, both assertion and reason are correct and reason is correct explanation of assertion.

10. **Assertion:** The point (3, 0) lies on the x-axis.

Reason: On y-axis, the value of abscissa is 0.

Sol. (*b*)

y-coordinate of point (3, 0) is zero. So, (3, 0) lies on x-axis. Also, on y-axis abscissa is 0.

∴ Both assertion and reason are correct and the reason is not a proper explanation of the assertion.

11. **Assertion:** (–3, 4) and (3, –4) lie on adjacent quadrants.

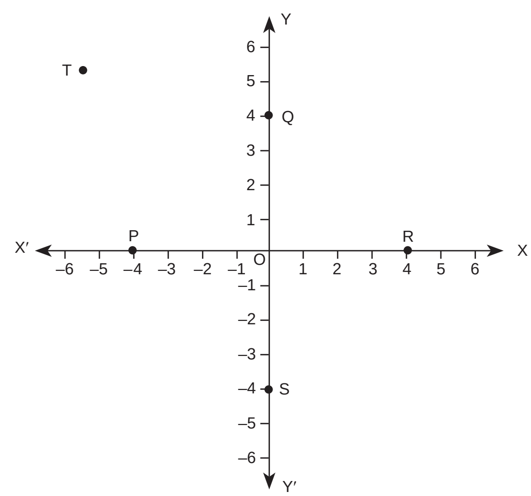
Reason: (–3, 4) lies on 2nd quadrant and (3, –4) lies on 4th quadrant.

Sol. (*d*)

(–3, 4) lies on 2nd quadrant and (3, –4) lies on 4th quadrant which are diagonally opposite. Therefore, assertion is wrong but reason is correct.

Case Study Based Questions

12. Four students Aditya, Ankit, Gaurav and Pranav are standing at different positions P, Q, R, and S in their school playground for a drill practice as shown on the graph. Teacher is standing at position T.



Observe the given graph and answer the following questions.

(*a*) What are the coordinates of P?

(*i*) (–4, 0) (*ii*) (0, –4)

(*iii*) (0, 4) (*iv*) (4, 0)

Ans. (*i*) (–4, 0)

(*b*) Name the points whose y-coordinate is zero.

(*i*) R and S (*ii*) Q and S

(*iii*) P and R (*iv*) P and Q

Ans. (*iii*) P and R

(*c*) Point T lies in the

(*i*) first quadrant. (*ii*) second quadrant.

(*iii*) third quadrant. (*iv*) fourth quadrant.

Ans. (*ii*) second quadrant.

(*d*) Name the closed figure obtained by joining the points P, Q, R and S.

(*i*) Triangle (*ii*) Quadrilateral

(*iii*) Pentagon (*iv*) Hexagon

Ans. (*ii*) Quadrilateral

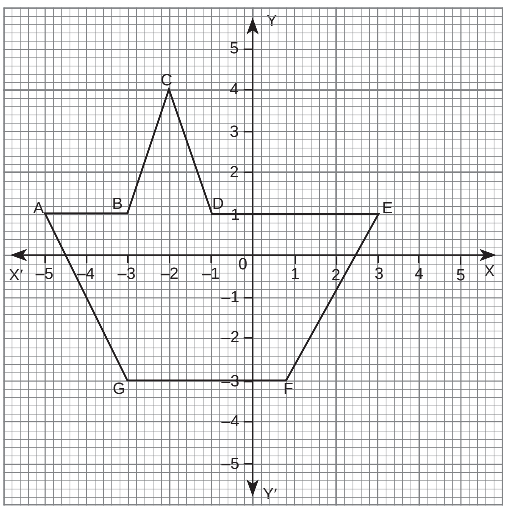
(*e*) What are the coordinates of the point of intersection of PR and QS?

(*i*) (–5, 5) (*ii*) (–2, 2)

(*iii*) (0, 0) (*iv*) (–1, 1)

Ans. (*iii*) (0, 0)

13. Rohit draws a boat on the graph paper. A, B, C, D, E, F and G are the points of the boat drawn.



Observe the given graph and answer the following questions.

(*a*) In which quadrant does the point G lie?

(*i*) First quadrant (*ii*) Second quadrant

(*iii*) Third quadrant (*iv*) Fourth quadrant

Ans. (*iii*) Third quadrant

(*b*) What are the coordinates of D?

(*i*) (–1, 1) (*ii*) (–1, 0)

(*iii*) (0, 1) (*iv*) (3, 1)

Ans. (*i*) (–1, 1)

(*c*) What is the ordinate of point B?

(*i*) 1 (*ii*) –3

(*iii*) 0 (*iv*) –1

Ans. (*i*) 1

(*d*) What is the abscissa of point C?

(*i*) –1 (*ii*) 4

(*iii*) –2 (*iv*) 3

Ans. (*iii*) –2

(*e*) Which point lies in first quadrant?

(*i*) A (*ii*) E

(*iii*) F (*iv*) G

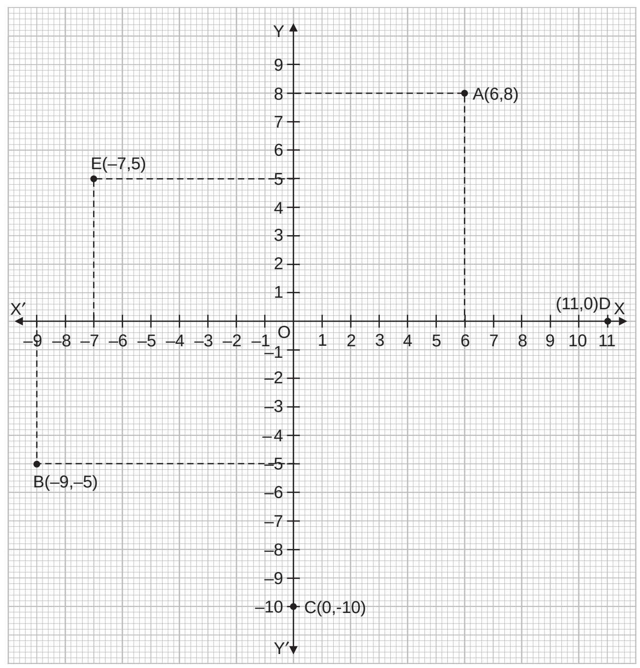
Ans. (*ii*) E

Very Short Answer Type Questions

14. Plot the points (x, y) given in the following table on the plane choosing suitable units of distances on the axes.

x	6	–9	0	11	–7
y	8	–5	–10	0	5

Sol. The points A(6, 8), B(–9, –5), C(0, –10), D(11, 0), E(–7, 5) are plotted in the adjoining graph.



15. Without plotting the points, find the quadrant in which they lie?

(*a*) (5, 4) (*b*) (–3, –5)

(*c*) (6, –8) (*d*) (–10, 5)

Sol. (*a*) We know that in the quadrant I, x is positive and y is positive.

$\therefore$ The point (5, 4) lie is the quadrant I.

(*b*) In the quadrant III, both x and y are negative. Hence, the point (–3, –5) lie in the quadrant III.

(*c*) In the quadrant IV, x is positive and y is negative. Hence, the point (6, –8) lie in the quadrant IV.

(*d*) In the quadrant II, x is negative and y is positive.

$\therefore$ The point (–10, 5) lie in the quadrant II.

16. Which of the following points lie on x-axis?

(*a*) (6, 0) (*b*) (8, 5)

(*c*) (9, 0) (*d*) (–5, 5)

(*e*) (0, 10)

Sol. We know that the ordinate of every point on the x-axis is zero. Hence, the points in (*a*) and (*c*) only lie on the x-axis.

Short Answer Type-I Questions

17. Find the coordinates of the point

(*a*) which lies on x and y axes both.

(*b*) whose ordinate is –10 and which lies on y-axis.

(*c*) whose abscissa is 6 and lies on x-axis.

Sol. (*a*) We know that only the origin lies in both x and y axes.

$\therefore$ The required coordinates of such a point are (0, 0).

(*b*) We know that the abscissa of every point on the y-axis is zero. Hence, the required coordinates of the point in this case will be (0, –10).

(*c*) We know that the ordinate of every point on the x-axis is zero. Hence, the required coordinates of the point in this case will be (6, 0).

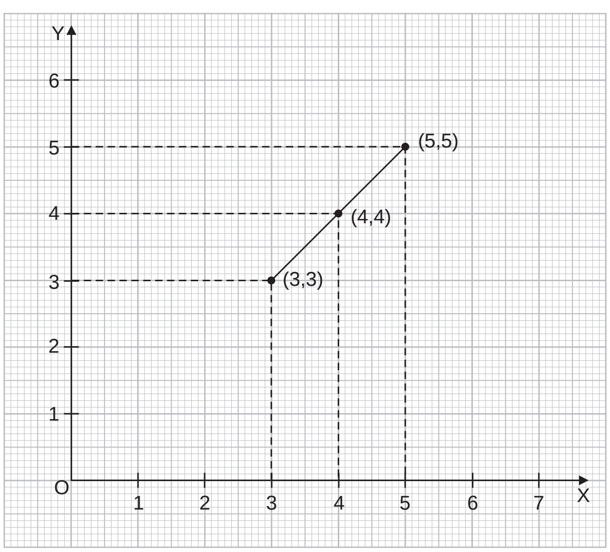
18. Plot the points and check whether they are collinear.

(*a*) (3, 3), (4, 4), (5, 5)

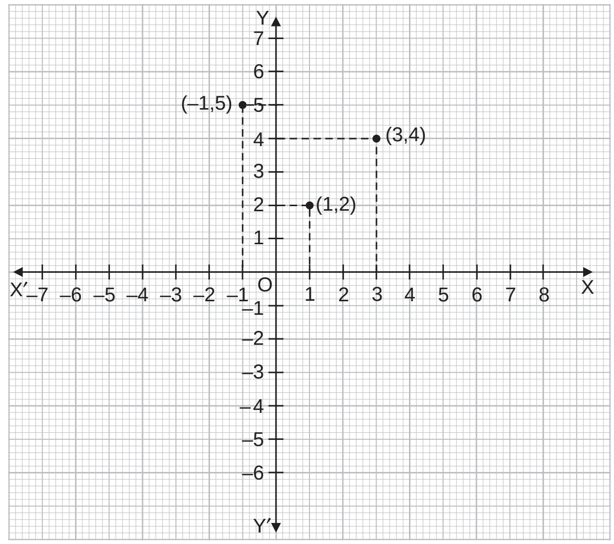
(*b*) (1, 2), (3, 4), (–1, 5)

(*c*) (–2, 1), (4, 1), (2, 1)

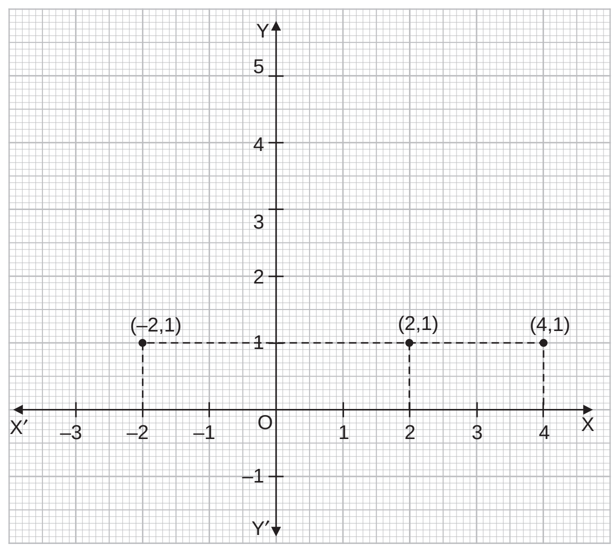
Sol. (*a*) From the graph, we see that the given point. (3, 3), (4, 4) and (5, 5) are collinear.



(*b*) From the graph we see that the given point (1, 2), (3, 4) and (–1, 5) are not collinear.



(*c*) From the graph, we see that the given points (–2, 1), (4, 1) and (2, 1) and collinear.



Short Answer Type-II Questions

19. A point lies on y-axis at a distance of 5 units from x-axis. What are its coordinates? What will be its coordinates if it lies on x-axis at a distance of 5 units from the y-axis?

Sol. On y-axis, the abscissa of every point is zero. Hence, the coordinates of the point lying on the y-axis and at a distance of 5 units from the x-axis are (0, 5) or (0, –5). We also know that the ordinate of every point on the x-axis is zero. Hence, the coordinates of the point lying on the

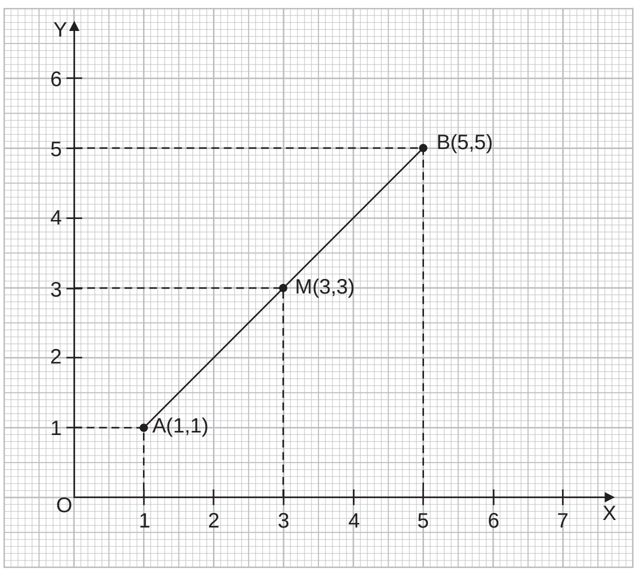
x-axis and at a distance of 5 units from the y-axis will be (5, 0) or (–5, 0).

20. Write the coordinates of a point whose abscissa is 8, and ordinate is $\frac{3}{4}$ times of abscissa.

Sol. Abscissa of the point is 8 and its ordinate is $\frac{3}{4} \times 8$ $= \frac{24}{4} = 6$. Hence, the required coordinates of this point are (8, 6).

21. Plot the points A(1, 1) and B(5, 5). Write the coordinate of mid-point of AB.

Sol. The points A(1, 1) and B(5, 5) are plotted on a graph paper. We now join the points A and B with a scale. We marked the middle point M of AB with a compass and a scale. From the graph, we see that the coordinates of M are (3, 3).

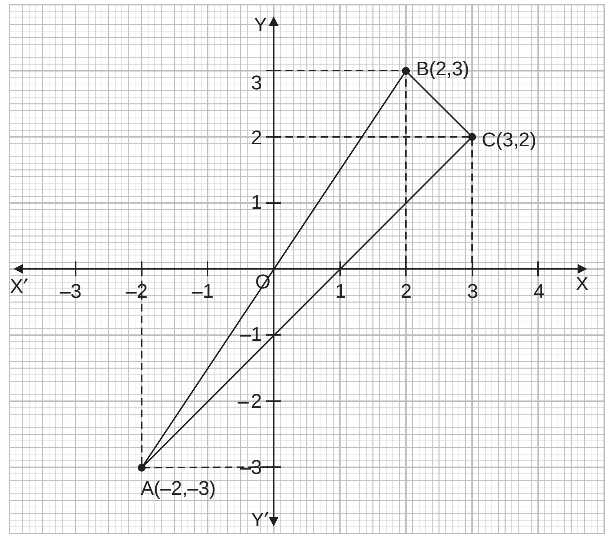


22. Rohit wants to travel from Amritsar (A) to Bareilly (B) by bus for business purpose. There are two routes to travel from A to B. First bus reaches at B via Chandigarh (C) and second bus reaches from A to B directly. If the coordinates of A, B and C are (–2,–3), (2,3) and (3,2) respectively then by which bus would he like to travel from A to B, assuming both buses have same speed?

Sol. The points A (–2, –3), B(2, 3) and C(3, 2) are plotted on a graph paper. We see that the figure ABC is a triangle. Also, AC + CB > AB

Hence, the direct route from A to B is shorter than the route AC and CB.

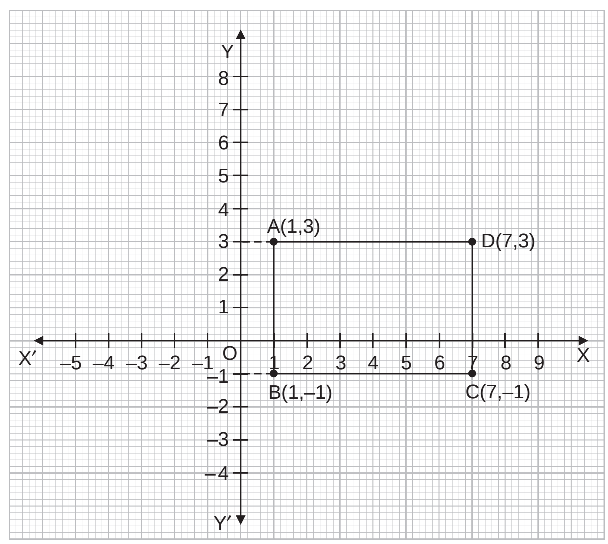
So, travelling by the direct route from A to B should be preferred by him.



Long Answer Type Questions

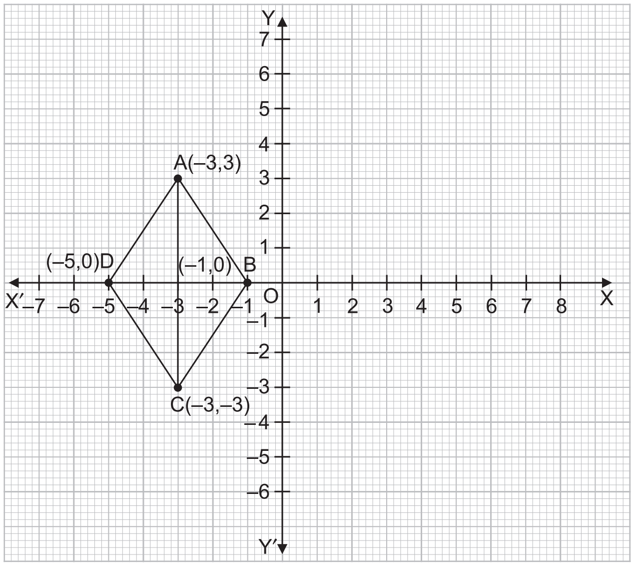
23. Plot the points A(1, 3), B(1, –1), C(7, –1) and D(7, 3) in the cartesian plane. Join them in order and name the figure so obtained.

Sol. The points A(1, 3), B(1, –1), C(7, –1) and D(7, 3) are plotted on the graph paper. We join these points by a scale in order. We see that the figure ABCD, thus formed is a rectangle.



24. Draw the quadrilateral with vertices (–3, 3), (–1, 0), (–3, –3) and (–5, 0). Also, name the type of quadrilateral formed and find its area.

Sol. We plot the points A(–3, 3), B(–1, 0), C(–3, –3) and D(–5, 0) on a graph paper and join them in order by a pencil and scale. We join the diagonals AC and BD. We see that the two diagonals AC and BD perpendicular to each other. Also AC = (3 + 3) units i.e. 6 units and BD = (5 – 1) units i.e. 4 units.



∵ The two diagonals of the quadrilateral ABCD are perpendicular to each other and they are unequal in length, hence the figure ABCD must be a rhombus of area

$$= \frac{1}{2}(\text{Product of two diagonals})$$

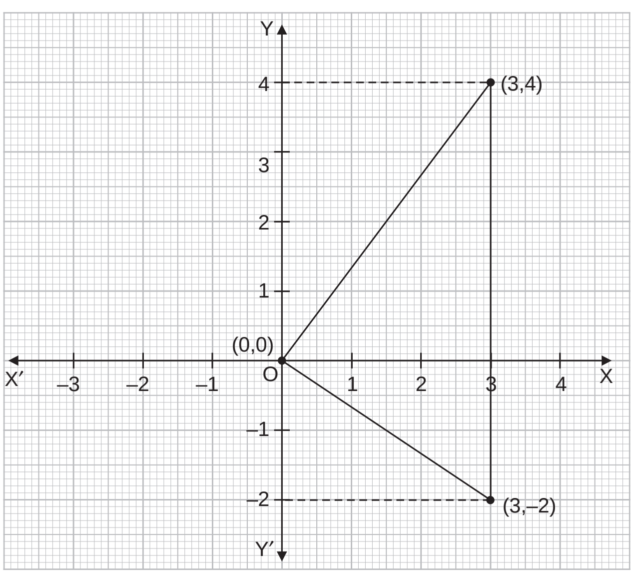
$$= \frac{1}{2} \times 6 \times 4 \text{ sq units}$$

$$= 12 \text{ sq units}$$

∴ Area of the rhombus is 12 sq units.

25. Plot the points (3, 4), (3, –2) and (0, 0). Check whether they are collinear or not. If not, find the area of the figure.

Sol. We plot the points A(3, 4), B(3, –2) and O(0, 0) on a graph paper. We see that the three points are not collinear.



If we join them by a scale and a pencil, we see that these three points form a triangle AOB with base AB of length (4 + 2) units and height OD of length 3 units, where D is the points of intersection of AB and the x-axis.

$$\therefore \text{ Area of } \Delta AOB = \frac{1}{2} AB \times OD$$

$$= \frac{1}{2} \times 6 \times 3 \text{ sq units}$$

$$= 9 \text{ sq units}$$

Hence, points (3, 4), (3, –2) and (0, 0) are not collinear and the area of triangle AOB is 9 sq units.

Let's Compete

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Multiple-Choice Questions

1. The point (–10, b) lies on x-axis, when

(a) $b = 0$ (b) $b = -10$

(c) $b = 10$ (d) $b < 10$

Sol. (a) $b = 0$

We know that the ordinate of every point on the x-axis is zero.

∴ $b = 0$

2. The distance between (−8, 10) and (−8, 5) is

(a) 0 (b) 5

(c) 16 (d) 15

Sol. (b) 5

The required distance between the points A(−8, 10) and B(−8, 5) is (10 – 5) units i.e. 5 units, since AB is parallel to y-axis and the two points are lying on the line AB at a distance of 10 and 5 units respectively from the x-axis.

3. The line segment joining (–3, 4) and (–3, −4) lies completely in

(a) quadrant I (b) quadrant II

(c) quadrant I and II (d) quadrant II and III

Sol. (d) quadrant II and III

The point (−8, 10) lies in the quadrant II and the point (–3, −4) lies is the quadrant III. Also, the line-segment joining these two points is parallel to y-axis and lies wholly in quadrant II and III.

4. Line $x = y$ passes through

(a) quadrant I

(b) origin

(*c*) quadrant III

(*d*) All of these

Sol. (*d*) All of these

If we put $x = 0$ and $y = 0$, then the given equation $y = x$ is satisfied. Hence, the line $x = y$ passes through the origin (0, 0).

Also, some points on this line lie in quadrant I and quadrant III. For example (1, 1), (2, 2), (3, 3), (–1, –1), (–2, –2), (–3, –3), etc. are such points. Hence, this line $x = y$ passes through the origin and lie in both quadrant I and III.

5. The measure between the coordinate axes is

(*a*) 90° (*b*) 180°

(*c*) 0° (*d*) None of these

Sol. (*a*) 90°

We know that the angle between x and y axes i.e. the coordinate axes is 90°.

6. Two points with different abscissa but same ordinate lie on

(*a*) x-axis

(*b*) y-axis

(*c*) a line parallel to y-axis

(*d*) a line parallel to x-axis

Sol. (*d*) a line parallel to x-axis

One such point will lie on the positive side of the x-axis and the other point will lie on the negative side of the x-axis.

Since these two points have the same ordinate, hence the line joining these two points will be parallel to x-axis.

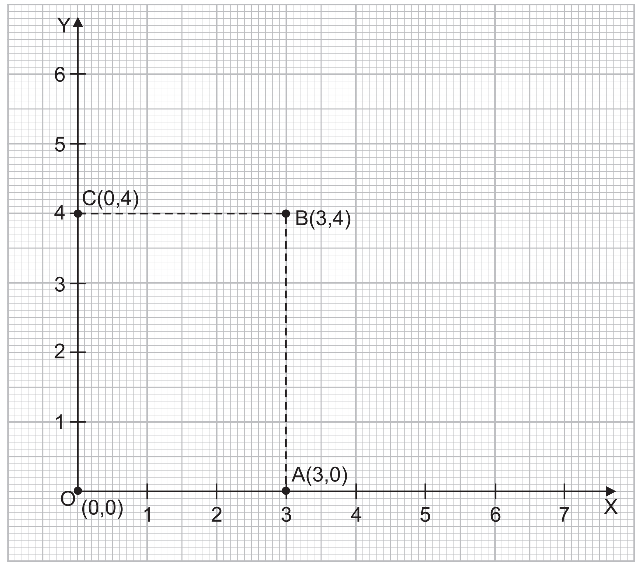
7. On plotting the points O(0, 0), A(3, 0), B(3, 4) and C(0, 4) and joining OA, AB, BC and CO, which of the following figure is obtained?

(*a*) Square (*b*) Rectangle

(*c*) Rhombus (*d*) Trapezium

Sol. (*b*) Rectangle

We first plot the points O(0, 0), A(3, 0), B(3, 4) and C(0, 4) on a graph paper and join them in order by a pencil and a scale. We see that the figure OABC is a rectangle of length OC = 4 units and breadth OA = 3 units.



8. Abscissa of a point is positive in

(*a*) quadrant I and II

(*b*) quadrant I only

(*c*) quadrant I and IV

(*d*) quadrant II only

Sol. (*c*) quadrant I and IV

We know that the abscissa of a point in the quadrant I and quadrant IV is positive.

9. If y-coordinate of a point is zero, then this point lies

(*a*) in quadrant I (*b*) on y-axis

(*c*) on x-axis (*d*) None of these

Sol. (*c*) on x-axis

We know that the y-coordinate of every point on the x-axis is 0.

10. The values of x and y for which two ordered pairs $(x - 2, 10)$ and $(5, x + y)$ are equal is

(*a*) $x = 7, y = 3$ (*b*) $x = 3, y = 7$

(*c*) $x = -3, y = 7$ (*d*) $x = 7, y = -3$

Sol. (*a*) $x = 7, y = 3$

If the two ordered pairs are equal, then

$$x - 2 = 5 \quad \ldots(1)$$

$$x + y = 10 \quad \ldots(2)$$

From (1), we get

$$x = 7 \quad \ldots(3)$$

Putting (3) in (2), we get

$$\Rightarrow \quad 7 + y = 10$$

$$\Rightarrow \quad y = 3$$

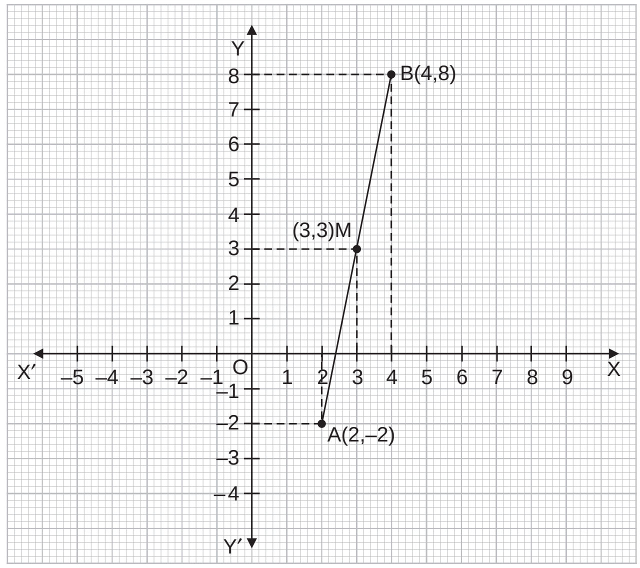
Hence, the values of x and y are 7 and 3 respectively.

Value-based Question (Optional)

(Page 40)

1. In a sports day celebration, Rohan and Sohan are standing at positions A and B whose coordinates are (2, –2) and (4, 8) respectively. The teacher asked Meghna to fix the country flag at the mid-point of the line joining the points A and B.
   (*a*) Find the coordinates of the mid-point?
   (*b*) Which type of value would you infer from the question?

Sol. (*a*) We plot the points A(2, –2) and B(4, 8) on a graph paper and join them by a pencil and a scale. We find the middle point M of the line segment AB by a scale and a compass. From the graph we see that the coordinates of the middle point M of AB are (3, 3).



(*b*) Enjoyment and reasoning.

4

Linear Equations in Two Variables

Checkpoint (Page 43)

1. The value of x which satisfies the equation $4x - 8 = 0$ is
   (*a*) $x = -2$ (*b*) $x = 2$
   (*c*) $x = 4$ (*d*) $x = -8$

Sol. (*b*) $x = 2$

We have $4x - 8 = 0$

$$\Rightarrow \quad x = \frac{8}{2} = 2$$

which is the required value of x.

2. Three-fourth of a certain number is 27. The number is
   (*a*) 36 (*b*) 54
   (*c*) 18 (*d*) 20

Sol. (*a*) 36

Let the number be x. Then according to the problem, we have

$$\frac{3}{4}x = 27$$

$$\Rightarrow \quad 3x = 4 \times 27$$

$$\Rightarrow \quad x = \frac{4 \times 27}{3} = 36$$

3. The sum of two consecutive multiples of 6 is 78. One of them is
   (*a*) 18 (*b*) 24
   (*c*) 36 (*d*) 54

Sol. (*c*) 36

Let the lower multiple be x. Then the next higher multiple is $x + 6$. According to the problem,

$$x + (x + 6) = 78$$

$$\Rightarrow \quad 2x = 78 - 6 = 72$$

$$\Rightarrow \quad x = \frac{72}{2} = 36$$

4. If $3(x - 4) = 9$, then the value of x is
   (*a*) 6 (*b*) 8
   (*c*) 7 (*d*) 9

Sol. (*c*) 7

We have $3(x - 4) = 9$

$$\Rightarrow \quad x - 4 = \frac{9}{3} = 3$$

$$\Rightarrow \quad x = 3 + 4 = 7$$

5. The value of y for which $\frac{3y+2}{y+3} = 1$ is
   (*a*) $y = 1$ (*b*) $y = \frac{1}{2}$
   (*c*) $y = 2$ (*d*) $y = 3$

Sol. (*b*) $y = \frac{1}{2}$

We have $\frac{3y + 2}{y + 3} = 1$

$$\Rightarrow \quad 3y + 2 = y + 3$$

$$\Rightarrow \quad 3y - y = 3 - 2$$

$$\Rightarrow \quad 2y = 1$$

$$\Rightarrow \quad y = \frac{1}{2}$$

6. Solve the following equation

$$\frac{(2x+1)-(3x+1)}{(3x-2)-(4x+1)} = \frac{1}{2}$$

Sol. We have

$$\frac{(2x+1)-(3x+1)}{(3x-2)-(4x+1)} = \frac{1}{2}$$

$$\Rightarrow \quad \frac{2x+1-3x-1}{3x-2-4x-1} = \frac{1}{2}$$

$$\Rightarrow \quad \frac{-x}{-x-3} = \frac{1}{2}$$

$\Rightarrow \quad -x - 3 = -2x$

$\Rightarrow \quad 2x - x = 3$

$\Rightarrow \quad x = 3$

which is the required solution.

7. The sum of the digits of two-digit number is 15. If 9 is added to the number, the digits are interchanged. Find the number.

Sol. Let the digit in the unit's place be x. Then the digit in the ten's place is $15 - x$.

$\therefore$ According to the problem, we have

$$9 + x + 10(15 - x) = 15 - x + 10x$$

$\Rightarrow \quad 9 + x + 150 - 10x = 15 + 9x$

$\Rightarrow \quad 10x + 9x - x = 159 - 15$

$\Rightarrow \quad 18x = 144$

$\Rightarrow \quad x = \frac{144}{18} = 8$

The digit in the unit's place is 8 and that in the ten's place is $15 - 8 = 7$

$\therefore$ The required number is 78.

8. If $x = \frac{-3}{2}$ and $y = \frac{-5}{7}$ is a solution of the equations $2x + 3k_1 = 0$ and $7y - 5k_2 = 0$, find the values of k_1 and k_2.

Sol. Since $x = \frac{-3}{2}$ and $y = -\frac{5}{7}$ is the solution of the equations $2x + 3k_1 = 0$ and $7y - 5k_2 = 0$ respectively, hence, we have

$$2 \times \left(-\frac{3}{2}\right) + 3k_1 = 0$$

$\Rightarrow \quad -3 + 3k_1 = 0$

$\Rightarrow \quad k_1 = 1$

and $\quad 7 \times \left(-\frac{5}{7}\right) - 5k_2 = 0$

$\Rightarrow \quad -5 - 5k_2 = 0$

$\Rightarrow \quad k_2 = -1$

Hence, the required values of k_1 and k_2 are 1 and –1 respectively.

9. The width of a rectangle is $\frac{2}{3}$ of its length. If the perimeter is 100 units, find the dimensions of the rectangle.

Sol. Let the length of the rectangle be l units. Then its width is $\frac{2l}{3}$ units.

$\therefore$ Perimeter of the rectangle $= 2\left(l + \frac{2l}{3}\right)$ units

$= 2 \times \frac{5l}{3}$ units.

$\therefore$ According to the problem,

$$2 \times \frac{5l}{3} = 100 \text{ units}$$

$$l = \frac{100 \times 3}{10} \text{ units} = 30 \text{ units}$$

$\therefore \quad b = \frac{2l}{3}$

$= \frac{2 \times 30}{3}$ units = 20 units

$\therefore$ The required length is 30 units and breadth is 20 units

10. I have collected coins in a piggy bank. The day I opened it, I found I had 3 times as many 50 paise coins as one rupee coins. The total amount in the piggy bank is ₹ 150. How many coins of each kind are there?

Sol. Let the number of ₹1 coins be x. Then the number of 50 paisa coins be $3x$.

$\therefore$ According to the problem

$$x + 3x \times \frac{50}{100} = 150$$

$\Rightarrow \quad x + \frac{3x}{2} = 150$

$\Rightarrow \quad \frac{5x}{2} = 150$

$\Rightarrow \quad x = 150 \times \frac{2}{5} = 60$

The required number of ₹ 1 coins is 60 and that of 50 paisa coins is 60 × 3, i.e. 180.

Milestone

(Page 50)

Multiple-Choice Questions

1. The condition that the equation $ax + by + c = 0$ represents the linear equation in two variables is

(*a*) $a \neq 0, b = 0$ (*b*) $b \neq 0, a = 0$

(*c*) $a = 0, b = 0$ (*d*) $a \neq 0, b \neq 0$

[CBSE SP 2011]

Sol. (*d*) $a \neq 0, b \neq 0$

We know that a linear equation in two variables contain both x and y.

Hence, in $ax + by + c = 0$, we must have $a \neq 0$ and $b \neq 0$.

2. If the line $2x + 3y = 2k$ where k is a constant, passes through the point (–1, 2), then the value of k is

(*a*) 4 (*b*) 2

(*c*) 1 (*d*) $\frac{1}{2}$

Sol. (*b*) 2

If the line $2x + 3y = 2k$ passes through the point (–1, 2), then $x = -1$ and $y = 2$ must satisfy this equation.

$\therefore\ 2 \times (-1) + 3 \times 2 = 2k$

$\Rightarrow \qquad k = 2$

Very Short Answer Type Questions

3. Show that the point (3, –1) does not lie on the line $3x - 5y = 9$.

Sol. Putting $x = 3$ any $y = -1$ in the equation $3x - 5y = 9$. We see that LHS = $3 \times 3 - 5 \times (-1) = 14 \neq$ RHS

$\therefore$ The point (3, –1) does not lie on the given line.

4. Write any two different solutions of the equation $2x + 3y = 0$.

Sol. We see that $x = 0$ and $y = 0$ or $x = 3$, $y = -2$ satisfy the given equation. Hence, any two solutions of the given equation are $x = 0$, $y = 0$ and $x = 3$, $y = -2$.

(Note that there may be many other solutions also).

Short Answer Type-I Questions

5. Verify that $x = \frac{22}{7}$, $y = \frac{127}{35}$ is a solution of the equation $-8x + 5y + 7 = 0$.

Sol.
$$\text{LHS} = -8x + 5y + 7 = -8 \times \frac{22}{7} + 5 \times \frac{127}{35} + 7$$
$$= -\frac{176}{7} + \frac{127}{7} + 7$$
$$= \frac{-176 + 127 + 49}{7}$$
$$= \frac{-176 + 176}{7}$$
$$= 0$$
$$= \text{RHS}$$

$\therefore\ x = \frac{22}{7}$ and $y = \frac{127}{35}$ is a solution of the given equation.

6. Without drawing the graph of the line $2x - 3y = 6$, say whether it will cut the x-axis at a distance of 3 units from the origin or not.

Sol. Yes, if the line cuts the x-axis, then the co-ordinates of the point of intersection will be (3, 0). Putting $x = 3$ and $y = 0$ on the left hand side of the given equation, we see that LHS = RHS. Hence, the given line will cut the x-axis at the given point.

Short Answer Type-II Questions

7. Draw the graph of equation $5x - 2y = 1$ and from the graph, determine whether (*a*) $x = 1$, $y = 2$ and (*b*) $x = -1$, $y = 3$ are solutions or not.

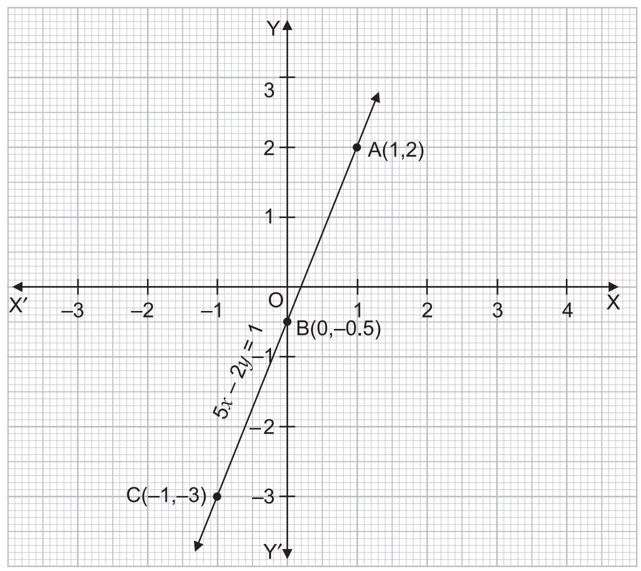
Sol. We have $\qquad 2y = 5x - 1$

$$\Rightarrow \qquad y = \frac{5x - 1}{2}$$

We now tabulate some values of x and y as follows:

x	1	–1	0
y	2	–3	–0.5

We now plot the points A (1, 2), B(0, –0.5) and C(–1, –3) on a graph paper and join them by a straight line AC. Then line AC is the required graph of the given line.



From the graph, we see that

(*a*) (1, 2) is a point on the line but

(*b*) (–1, 3) is not a point on the line.

8. Draw the graph of equation $5x + 6y = 30$ and hence find the area of the triangle formed by this line and the coordinate axes.

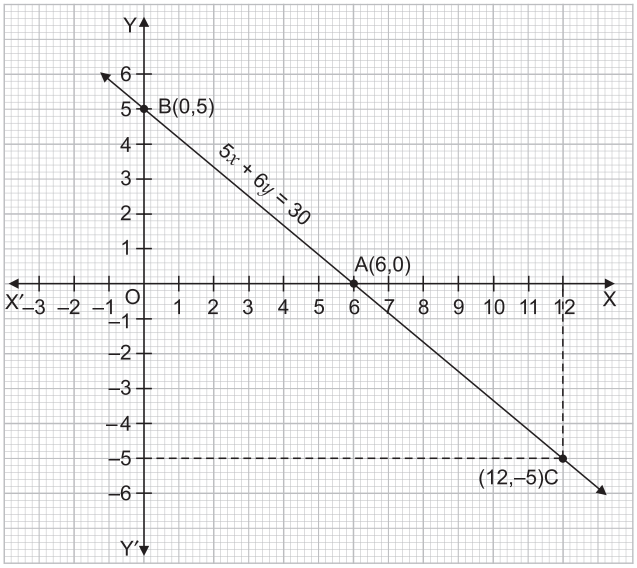
Sol. From the given equation, we have

$$x = \frac{30 - 6y}{5}$$

We now tabulate some values of x and y as follows:

x	6	0	12
y	0	5	–5

we now plot the points A(6, 0), B(0, 5) and C(12, –5) on a graph paper and join them by a straight line BC. Then BC is the required line of the given equation.



Let O be the origin (0, 0). Then in ΔBOA, base OA = 6 units and height OB = 5 units.

$\therefore$ Area of ΔBOA $= \frac{1}{2} \times$ base $\times$ height

$= \frac{1}{2} \times 6 \times 5$ sq units

= 15 sq units

$\therefore$ The required area of the triangle is 15 sq units.

Long Answer Type Questions

9. In an election 60% of the voters cast their votes. Form an equation and draw the graph with this data. From the graph, find

(*a*) the total number of voters if 2,100 voters cast their votes and

(*b*) the number of votes cast if the total number of voters are 10,000. **[CBSE SP 2012]**

Sol. Let x be the total number of voters. According to the problem, $\frac{60x}{100}$ voters $= \frac{3x}{5}$ voters cast their votes. Let y be the number of voters who cast their votes.

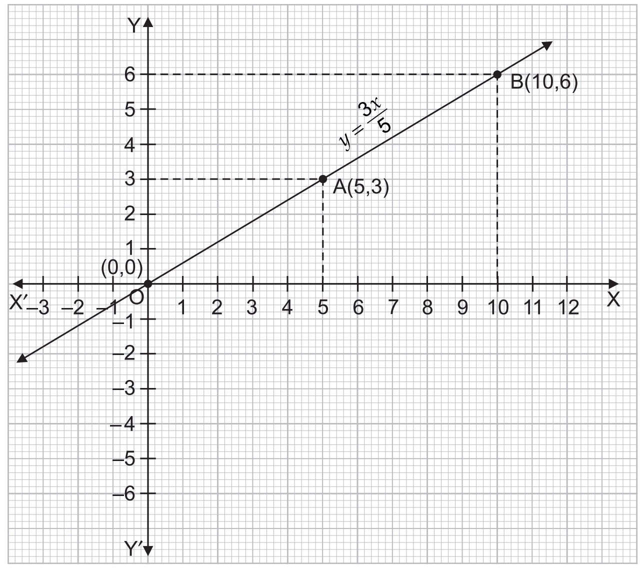
$\therefore$ We have $y = \frac{3x}{5}$

$\Rightarrow$ $5y - 3x = 0$

We now tabulate some values of x and y as follows:

x	0	5	10
y	0	3	6

We now plot the point O(0, 0), A(5, 3) and B (10, 6) on a graph paper and join them by a straight line OB. Then OB is the required line.



(*a*) If $y = 2100$, then $x = \frac{5y}{3}$

$= \frac{5}{3} \times 2100 = 3500$

We see that $y : x = 2100 : 3500$

$= 3 : 5$

Since (5, 3) is a point on the graph, hence, out of 3500 votes, 2100 cast their votes.

(*b*) If $x = 10000$

then $y = \frac{3x}{5} \times 10000 = 6000$

We see that $y : x = 6000 : 10000$

$= 3 : 5$

Since (5, 3) is a point on the graph, hence, out of 10000 voters, only 6000 cast their votes in this case.

10. The population of a small city is 12,000. The ratio of the female and male is 5 : 7. Set up an equation between the population and females. Then, draw the graph with the help of this equation. By reading the graph, find the number of females in the city. **[CBSE SP 2012]**

Sol. Let x be the number of population of the city and y be the number of females. Then the number of male $= x - y$.

$\therefore$ According to the problem,

$y : x - y = 5 : 7$

$\Rightarrow$ $\frac{y}{x-y} = \frac{5}{7}$

$\Rightarrow \quad 7y = 5x - 5y$

$\Rightarrow \quad 12y = 5x$

$$\Rightarrow \quad y = \frac{5x}{12} \quad \ldots(1)$$

(1) is the required equation between the number of population and the number of females.

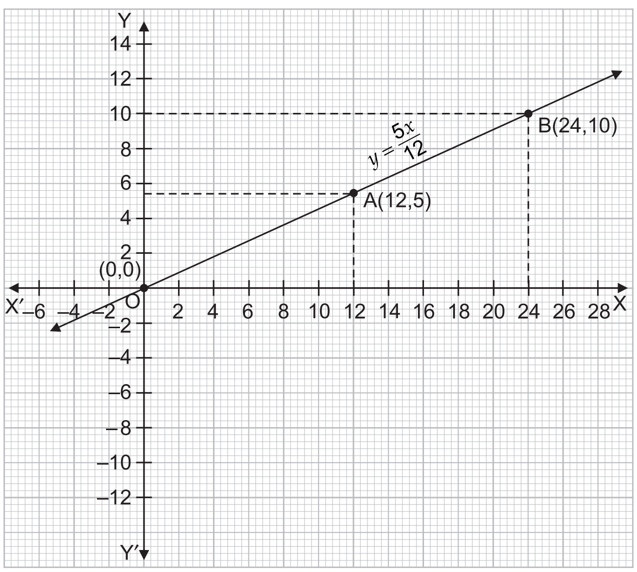
From (1), we have

$$x = \frac{12y}{5} \quad \ldots(2)$$

We now tabulate some values of x and y as follows:

x	12	0	24
y	5	0	10

We now plot the points O(0, 0), A(12, 5) and B(24, 10) on a graph paper and join them by a straight line OAB.



Now, when $x = 12000$, then from (1), we see that

$$y = \frac{5}{12} \times 12000 = 5000$$

Hence, the required number of females = 5000.

Higher Order Thinking Skills (HOTS) Questions

(Page 51)

1. The ratio of the ages of a father and his son, 10 years ago, was 4 : 1 and that after 20 years was 7 : 4. Form two simultaneous linear equations and hence solve these two equations algebraically to find the present ages of the father and his son.

Sol. Let the present ages of the father and his son be x years and y years respectively. Then according to the problem,

$$(x - 10) : (y - 10) = 4 : 1$$

$$\Rightarrow \quad \frac{x-10}{y-10} = \frac{4}{1}$$

$\Rightarrow \quad 4y - 40 = x - 10$

$\Rightarrow \quad x - 4y + 30 = 0 \quad \ldots(1)$

and $\quad (x + 20) : (y + 20) = 7 : 4$

$$\Rightarrow \quad \frac{x+20}{y+20} = \frac{7}{4}$$

$\Rightarrow \quad 7y + 140 = 4x + 80$

$\Rightarrow \quad 4x - 7y - 60 = 0 \quad \ldots(2)$

From (1), we have

$x = 4y - 30 \quad \ldots(3)$

$\therefore$ From (2) and (3), we get

$4(4y - 30) - 7y - 60 = 0$

$\Rightarrow \quad 16y - 120 - 7y = 60$

$\Rightarrow \quad 9y = 60 + 120$

$= 180$

$$\Rightarrow \quad y = \frac{180}{9}$$

$= 20$

$\therefore$ From (3), $x = 4 \times 20 - 30 = 80 - 30 = 50$

Hence, the required present ages of the father and the son are 50 years and 20 years respectively.

2. Draw the graph of the line $3x + 4y = 24$. Hence, find the length of the hypotenuse of the right-angled triangle formed by this line with the coordinate axes. Also, find the area of this triangle.

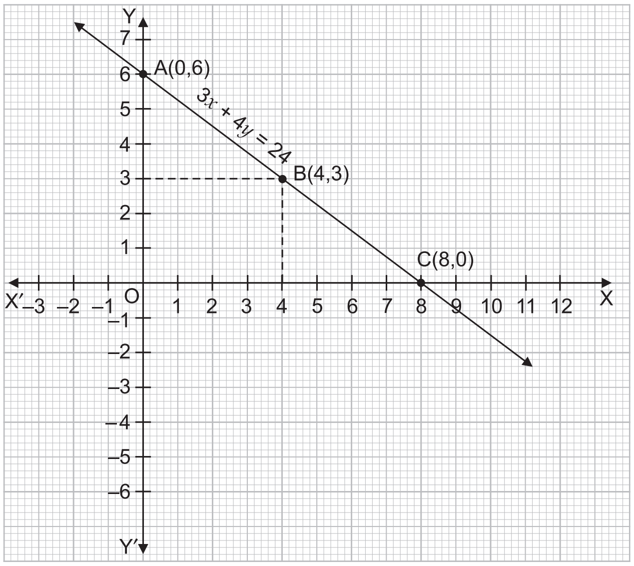
Sol. From the given equation, we have

$$y = \frac{24 - 3x}{4}$$

We now tabulate some values of x and y as follows:

x	0	4	8
y	6	3	0

We now plot the points A(0, 6), B(4, 3) and C(8, 0) on a graph paper and join them by a straight line AC. This is the required graph. If O is the origin, then AOC is a right-angled triangle formed by the given line and the coordinates axes.



$\therefore$ Area of the triangle AOC

$$= \frac{1}{2} \times CO \times AO$$

$$= \frac{1}{2} \times 8 \times 6 \text{ sq units}$$

$$= 24 \text{ sq units}$$

The required length of the hypotenuse AC of the right angled $\Delta AOC = \sqrt{AO^2 + OC^2}$

$$= \sqrt{6^2 + 8^2} \text{ units}$$

$$= \sqrt{36 + 64} \text{ units}$$

$$= \sqrt{100} \text{ units} = 10 \text{ units}$$

Hence, the required area of this triangle is 10 units.

Self-Assessment

(Page 51)

Multiple-Choice Questions

1. If three times the abscissa of a point is subtracted from four times its ordinate and then the whole expression is divided by 12, then the result becomes 3. When this statement is expressed in term of an equation it becomes

(*a*) $4y - 3x = 36$

(*b*) $3x - 4y = 36$

(*c*) $3x + 4y - 36 = 0$

(*d*) $4y + 3x + 36 = 0$

Sol. (*a*) $4y - 3x = 36$

According to the problem, we have

$$\frac{4y - 3x}{12} = 3$$

$$\Rightarrow \quad 4y - 3x = 3 \times 12 = 36$$

which is same as (*a*).

2. Which of the following is a solution of the equation $x + 2y = 7$?

(*a*) $x = 3, y = -5$ (*b*) $x = 3, y = 5$

(*c*) $x = 0, y = 7$ (*d*) $x = 3, y = 2$

Sol. (*d*) $x = 3, y = 2$

We see that when $x = 3$ and $y = 2$, then LHS $= 3 + 2 \times 2 = 7 =$ RHS and the other values, in (*a*), (*b*) and (*c*) do not satisfy this equation.

Fill in the Blanks

3. The equation of x-axis is of the form $\boldsymbol{y = 0}$.

4. The linear equation of the type $y = mx$, $m \neq 0$ has **infinitely many** solutions.

5. The negative solutions of the equation $ax + by + c = 0$ always lie in the **III** quadrant.

6. If (2,0) is a solution of the linear equation $2x + 3y - k = 0$, then the value of k is **4**.

Sol. $2x + 3y - k = 0 \Rightarrow 2(2) + 3(0) - k = 0 \Rightarrow 4 - k = 0$
$\Rightarrow k = 4$

Assertion-Reason Type Questions

Directions (Q. Nos. 7 to 10): Each of these questions contains an assertion followed by reason. Read them carefully, and answer the question on the basis of the following options, select the one that best describes the two statements.

(a) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.

(b) Both Assertion and Reason are true but Reason is not the correct explanation of Assertion.

(c) Assertion is true but Reason is false.

(d) Assertion is false but Reason is true.

7. Assertion: $x + y = 0$ is a linear equation in two variables.

Reason: The variables x and y have non-zero coefficients.

Sol. (*a*)

$x + y = 0$ is a linear equation in two variables with $a \neq 0$ and $b \neq 0$.

Thus assertion and reason are correct and reason is correct explanation of assertion.

8. Assertion: $x = 2y$ represents the statement "ordinate of a point is twice its abscissa".

Reason: x-coordinate is called abscissa and y-coordinate is called ordinate.

Sol. (*d*)

x-coordinate is called abscissa and y-coordinate is called ordinate. Therefore reason is correct but assertion is incorrect as $y = 2x$ represents the statement 'ordinate of a point is twice its abscissa'.

9. **Assertion:** A linear equation in two variables has infinitely many solutions.

Reason: A linear equation in two variables should have non-zero coefficient for two variables.

Sol. (*b*)

Both reason and assertion are correct but reason is not the correct explanation of assertion.

10. **Assertion:** (1, 1) is a solution of the equation $x + y = 1$.

Reason: Replacing (1, 1) in the equation, we get 2.

Sol. (*d*)

Assertion is wrong as replacing (1, 1) in equation $x + y$, we get 1 + 1 = 2 which is not equal to RHS.

The reason is correct as when we replace (1,1) in the equation $x + y$ we get 2.

Case Study Based Questions

11. Two friends Rahul and Anmol decided to travel the historical monuments in Delhi city. So, they book a cab for travelling. The taxi fare in the city is as follows.

Fare for the first kilometre is ₹ 10 and ₹ 6 for every subsequent kilometre.

Based on the above information, answer the following questions.

(*a*) Consider the distance covered as x km and the total fare as ₹ y, which of the following linear equation represents the above situation?

(*i*) $6x + y + 4 = 0$ (*ii*) $6x - y + 4 = 0$

(*iii*) $6x - y - 4 = 0$ (*iv*) $6x + 2y + 4 = 0$

Ans. (*ii*) $6x - y + 4 = 0$

(*b*) Which mathematical concept is used here?

(*i*) Polynomial

(*ii*) Probability

(*iii*) Heron's formula

(*iv*) Linear equations in two variables

Ans. (*iv*) Linear equations in two variables

(*c*) If they paid total fare ₹ 724, then find the total distance travelled.

(*i*) 110 km (*ii*) 120 km

(*iii*) 130 km (*iv*) 140 km

Ans. (*ii*) 120 km

(*d*) If the distance covered by them is 94 km, then find the total fare they have to pay.

(*i*) ₹ 564 (*ii*) ₹ 566

(*iii*) ₹ 568 (*iv*) ₹ 570

Ans. (*iii*) ₹ 568

(*e*) If fare for first kilometre is ₹ 10 and ₹ 4.50 for every subsequent kilometre, then what amount they have to pay for 111 km?

(*i*) ₹ 404 (*ii*) ₹ 445

(*iii*) ₹ 505 (*iv*) ₹ 555

Ans. (*iii*) ₹ 505

12. A boy walks across an x metre wide road at the speed of 1.5 m/s and crosses it in y_1 seconds. Next day, the boy is just about to cross the same road, when he spots a visually impaired man who also wants to cross that road. He helps that man to go across by reducing his walking speed to 0.5 m/s and crosses it in y_2 seconds. Based on this situation, answer the following questions.

(*a*) Write a linear equation in two variables to express relationship between x and y_1.

(*i*) $x - y_1 = 1.5$ (*ii*) $x/1.5 = y_1$

(*iii*) $x + y_1 = 1.5$ (*iv*) $y_1/1.5 = x$

Ans. (*ii*) $x/1.5 = y_1$

(*b*) Write the above equation in standard form.

(*i*) $x - y_1 - 1.5 = 0$ (*ii*) $2y_1 - 3x = 0$

(*iii*) $x + y_1 - 1.5 = 0$ (*iv*) $2x - 3y_1 = 0$

Ans. (*iv*) $2x - 3y_1 = 0$

(*c*) Write a linear equation in two variables to express relationship between x and y_2.

(*i*) $x - y_2 = 0.5$ (*ii*) $x/0.5 = y_2$

(*iii*) $x + y_2 = 1.5$ (*iv*) $y_2/0.5 = x$

Ans. (*ii*) $x/0.5 = y_2$

(*d*) Find the ratio between the times taken by the boy to cross the road on the first day and the second day.

	(i) 1 : 2	(ii) 2 : 3
	(iii) 1 : 3	(iv) 3 : 2

Ans. (*iii*) 1 : 3

(*e*) What value did the boy show?

(*i*) Charity (*ii*) Compassion

(*iii*) Cheating (*iv*) None of these

Ans. (*ii*) Compassion

Very Short Answer Type Questions

13. What is the distance of the line $2x + 1 = 0$ from the origin?

Sol. From the given equation, we see that $2x = -1$

$\Rightarrow x = -\frac{1}{2}$. Hence, the required distance of the line from the origin, i.e. from the y-axis is $\frac{1}{2}$.

14. How many linear equations in x and y can be satisfied by $x = -5$ and $y = \frac{1}{3}$?

Sol. We know that infinitely many straight lines can pass through a single point $(-5, \frac{1}{3})$. Hence, the number of linear equations satisfied by $x = -5$ and $y = \frac{1}{3}$ is infinitely many.

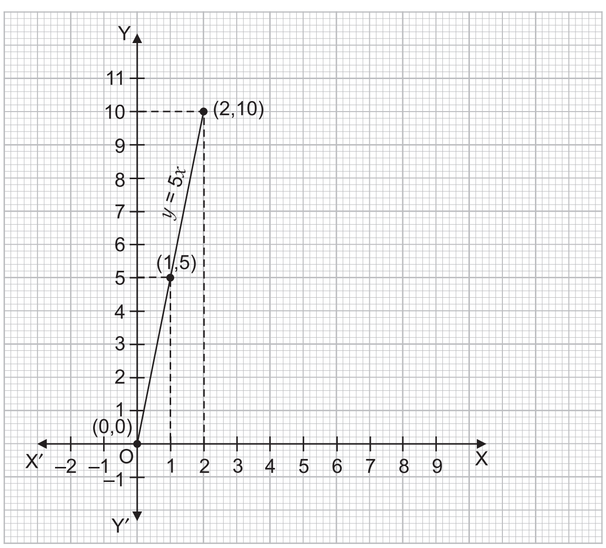
Short Answer Type-I Questions

15. If 5 pens cost ₹ 25, find graphically how many pens can be bought for ₹ 35. Also, find the cost of 3 pens.

Sol. Let x pens cost ₹ y. Since 5 pens cost ₹25,

$\therefore$ Each pen costs ₹25 ÷ 5 = ₹5

$\therefore \quad y = 5x \quad \ldots(1)$



x	0	1	2
y	0	5	10

When $y = 35$, then $x = 7$ [From (1)]

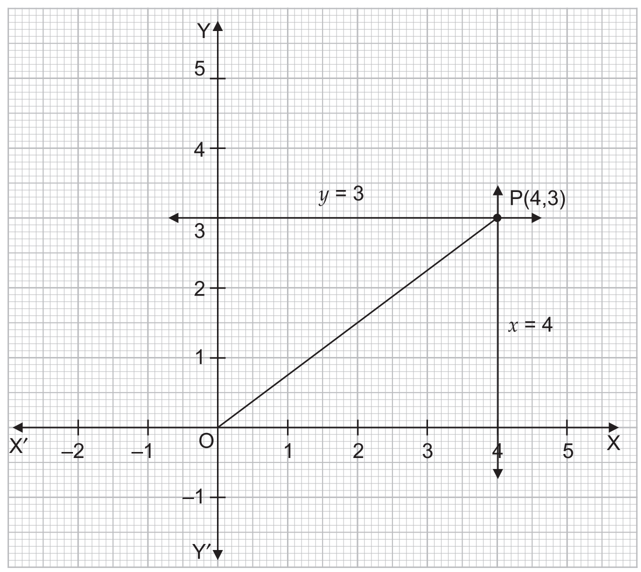
and when $x = 3$, then $y = 15$ [From (1)]

Hence, 7 pens can be bought for ₹35 and the required cost of 3 pens is ₹15.

16. From the rough graphs of the lines $x = 4$ and $y = 3$ with the same coordinate axes, find the distance of their point of intersection from the origin by using Pythagoras' theorem.

Sol. The rough graphs of $x = 4$ and $y = 3$ are shown in the adjoining figure. $x = 4$ is a line parallel to y-axis and $y = 3$ is a line parallel to x-axis. If M is the point where $x = 4$ cuts the x-axis, then OM = 4 and PM = 3 where P is the point of intersection of the line $x = 4$ and $y = 3$.

$\therefore$ P is the point (4, 3).



If O be the origin, then by Pythagoras' theorem,

we have $\quad OP = \sqrt{OM^2 + PM^2}$

$= \sqrt{4^2 + 3^2}$ units

$= \sqrt{16 + 9}$ units

$= \sqrt{25}$ units = 5 units.

Short Answer Type-II Questions

17. Draw the graphs of $3x + 2y = 0$ and $2x - 3y = 0$. What is the point of intersection of the two lines representing the above equations. **[CBSE SP 2013]**

Sol. From the given equations, we have

$$3x + 2y = 0$$

and $$2x - 3y = 0$$

$$\Rightarrow \qquad y = \frac{-3x}{2} \qquad \ldots(1)$$

$$\text{and} \qquad x = \frac{3y}{2} \qquad \ldots(2)$$

We now tabulate some values of x and y from the equations (1) and (2) separately as follows:

Table 1

x	0	2	4
y	0	–3	–6

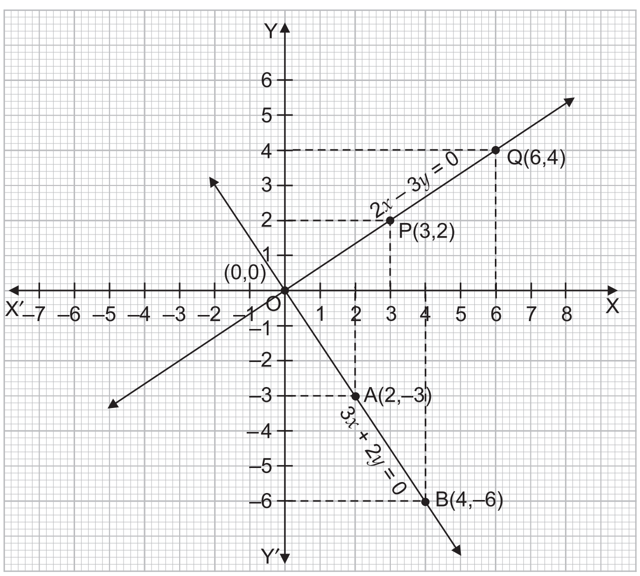
Table 2

x	0	3	6
y	0	2	4

We now plot the points

O(0, 0), A(2, –3), B(4, –6) and P(3, 2), Q(6, 4) on the same graph paper as follows:

We join the points O, A and B by a line OB and also join the points O, P and Q by another line OQ. Then OB and OQ are the required two lines.



The required points of intersection of these two lines is (0, 0), i.e. the origin.

18. Draw the graph of two lines whose equations are $3x - 2y + 6 = 0$ and $x + 2y - 6 = 0$ on the same graph paper. Find the area of the triangle formed by the two lines and the x-axis. **[CBSE SP 2011]**

Sol. From the given equation, we have

$$3x - 2y + 6 = 0$$

$$\text{and} \qquad x + 2y - 6 = 0$$

$$\Rightarrow \qquad y = \frac{3x + 6}{2} \qquad \ldots(1)$$

$$\text{and} \qquad x = 6 - 2y \qquad \ldots(2)$$

We now tabulate some values of x and y from the equation (1) and (2) in separate tables as follows:

Table 1

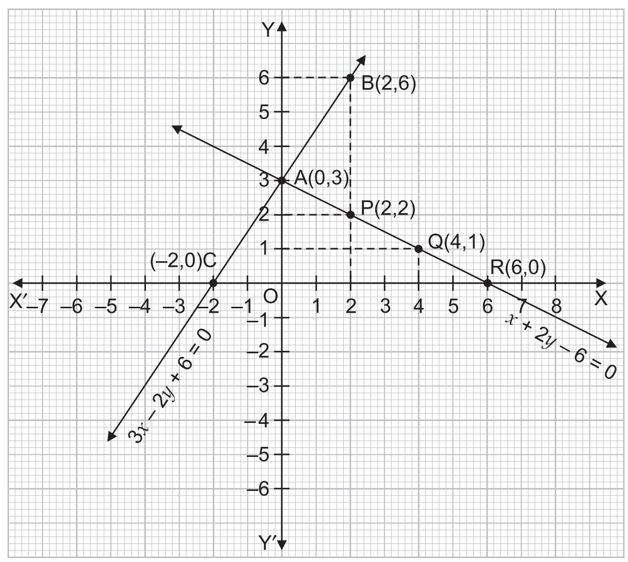
x	0	2	–2
y	3	6	0

Table 2

x	0	2	4
y	3	2	1

We now plot the points A(0, 3), B(2, 6) and C(–2, 0) and join them by a line CB.

We plot the points A(0, 3), P(2, 2) and Q(4, 1) and join them by another line AQ.



These two lines form a triangle ACR with the x-axis, where R is the point (6, 0) where the second line AQ intersect the x-axis. The base of ΔACR is RC and altitude is OA.

From the graph, we see that CR = (6 + 2) units = 8 units and OA = 3 units.

$\therefore$ Required area of ΔACR

$$= \frac{1}{2} \times CR \times OA$$

$$= \frac{1}{2} \times 8 \times 3 \text{ sq units}$$

$$= 12 \text{ sq units}$$

19. Read the following passage and answer the questions that follows:

The residents of a housing colony are facing problem of water shortage. All residents decided to do rainwater harvesting as it helps in preserving rainwater for different purposes and for the future needs as well. One of the ways to preserve rainwater is in underground tank. The rainwater was collected in an underground tank at the rate of 0.12 m^3/hour.

(*a*) If x cm^3 of rainwater is collected in the underground tank in y minutes, then write the linear equation in two variables to express this situation.

(*b*) What will be the volume of rainwater in the underground tank collected in 5 minutes?

Sol. (*a*) In 60 minutes, the amount of water collected

$$= 0.12 \text{ m}^3$$
$$= 0.12 \times 100 \times 100 \times 100 \text{ cm}^3$$
$$= 120000 \text{ cm}^3$$

$\therefore$ In y minutes, the amount of water collected

$$= \frac{120000}{60} y \text{ cm}^3$$
$$= 2000y \text{ cm}^3.$$

$\therefore$ According to the problem, $x = 2000y$, which is the required equation.

(*b*) In 5 minutes, the amount of water collected

$$= 2000 \times 5 \text{ cm}^3 \text{ [From part (}a\text{)]}$$
$$= 10000 \text{ cm}^3$$

Long Answer Type Questions

20. Solve the equation $3y - 2 = 10 - y$, and represent the solution(s) on

(*a*) the number line

(*b*) the Cartesian plane.

Sol. From the given equation, we have

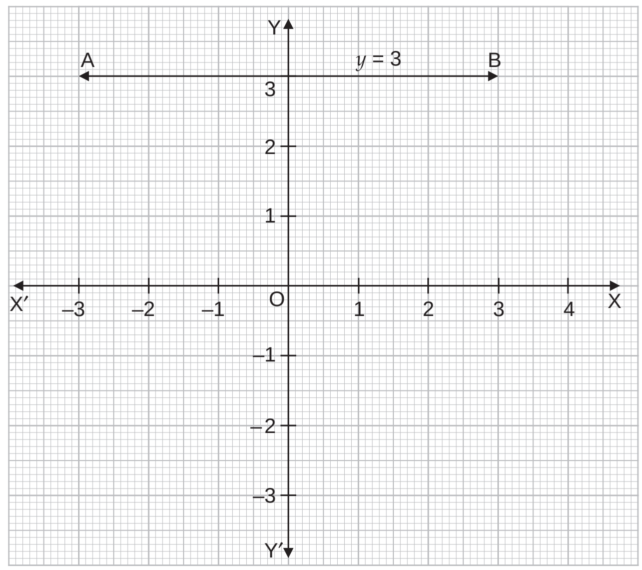
$$3y - 2 = 10 - y$$
$$\Rightarrow \quad 3y + y = 10 + 2$$
$$\Rightarrow \quad 4y = 12$$
$$\Rightarrow \quad y = \frac{12}{4} = 3$$

which is the required solution.

(*a*) If we represent this solution on a number line, then it will be as follows:

Clearly, on a number line it represents a point P.

(*b*) On a Cartesian xy-plane, it will represent a straight line AB parallel to x-axis, as shown in the figure.



21. If the work done by a body on application of a constant force is directly proportional to the distance travelled by the body, express this in the form of an equation in two variables and draw the graph of the same by taking the constant force as 5 units. Also read from the graph, the work done when the distance travelled by the body is

(*a*) 2 units

(*b*) 0 unit. **[CBSE SP 2011, 2012]**

Sol. Let x be the displacement and y be the force.

Then $\quad y \propto x$

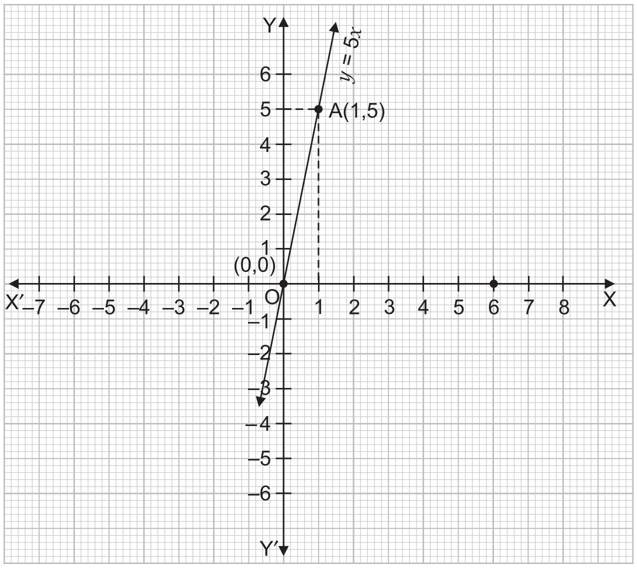
$\Rightarrow \quad y = kx$

where k is a constant force.

It is given that $k = 5$ units

$\therefore \quad y = 5x \quad \ldots(1)$

x	0	1
y	0	5



The graph of $y = 5x$ is OA as shown in the figure.

(*a*) When $x = 2$ units,

then $y = 2 \times 5$ units [From (1)]

$= 10$ units

(*b*) When $x = 0$ unit,

then $y = 5 \times 0$ unit

$= 0$ unit.

Let's Compete

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Multiple-Choice Questions

1. If the linear equation has solutions (–7, 7), (0, 0), (7, –7), then equation is

(*a*) $x + y = 0$ (*b*) $x - y = 0$

(*c*) $2x - y = 0$ (*d*) $x + 2y = 0$

Sol. (*a*) $x + y = 0$

From the given points, (–7, 7), (0, 0), (7, –7) satisfies the equation $y = -x$, i.e. $y + x = 0$.

2. Any point on the line $y = x$ is of the form

(*a*) $(k, 0)$ (*b*) $(0, k)$

(*c*) (k, k) (*d*) $(k, -k)$

Sol. (*c*) (k, k)

From the equation $y = x$, we see that abscissa = ordinate.

Hence, the point (k, k) where k is any real number is the required point.

3. The graph of the linear equation $2x - y = 3$ cuts the y-axis at the point

(*a*) (0, –3) (*b*) (0, 3)

(*c*) (3, 0) (*d*) (–3, 0)

Sol. (*a*) (0, –3)

At any point where the given line cuts the y-axis, we have $x = 0$.

$\therefore$ Putting $x = 0$ in the given equation, we see that $y = -3$. Hence, the required point is (0, –3).

4. At what point does the graph of the linear equation $x + y = 10$ meet a line which is parallel to y-axis at a distance of 6 units from the origin and in the positive direction of x-axis.

(*a*) (0, 6) (*b*) (0, 4)

(*c*) (4, 6) (*d*) (6, 4)

Sol. (*d*) (6, 4)

The equation of the line which is parallel to y-axis and lies at a distance of 6 units from the origin towards the positive direction of x-axis is $x = 6$.

Putting $x = 6$ is the given equation $x + y = 10$, we get $y = 10 - 6 = 4$.

$\therefore$ The required point of intersection of the two lines $x + y = 10$ and $x = 6$ is (6, 4).

5. The coordinates of the point on the graph of the equation $8x - 7y = 14$, whose ordinate is $\frac{6}{7}$ times its abscissa are

(*a*) (–7, 6) (*b*) (–6, 7)

(*c*) (6, 7) (*d*) (7, 6)

Sol. (*d*) (7, 6)

The ordinate $y = \frac{6}{7}$ times the abscissa x.

$$\therefore \quad y = \frac{6x}{7}$$

Putting $y = \frac{6x}{7}$ in the second equation $8x - 7y = 14$, we get

$$8x - 7 \times \frac{6x}{7} = 14$$

$$\Rightarrow \quad 2x = 14$$

$$\Rightarrow \quad x = \frac{14}{2} = 7$$

$$\therefore \quad y = \frac{6}{7} \times 7 = 6$$

$\therefore$ The required point is (7, 6).

6. The measure of angle between the graph of the equations $y = 2$ and $x = 6$ is

(*a*) 0° (*b*) 45°

(*c*) 90° (*d*) None of these

Sol. (*c*) 90°

We know that the line $y = 2$ is parallel to x-axis and the line $x = 6$ is parallel to y-axis. Since the angle between x and y axes is 90°, hence, the angle between the lines $x = 6$ and $y = 2$ is also 90°.

7. Thrice the ordinate of point when decreased by twice the abscissa is 4. The given statement is

(*a*) $3x - 2y = 4$ (*b*) $3y - 2x = 4$

(*c*) $2x - 4 = 3y$ (*d*) None of these

Sol. (*b*) $3y - 2x = 4$

According to the problem $3y - 2x = 4$.

8. The x and y intercepts made by the graph of the linear equation $7x + 8y = 5$ on the x-axis and y-axis respectively are in the ratio

(*a*) 25 : 56 (*b*) 56 : 25

(*c*) 8 : 7 (*d*) 7 : 5

Sol. (*c*) 8 : 7

x-intercept is obtained from the given equation by putting $y = 0$ and the y-intercept is obtained by putting $x = 0$ in the given equation.

$\therefore$ x-intercept $= \frac{5}{7}$ and y-intercept $= \frac{5}{8}$

$\therefore$ Ratio of x-intercept and y-intercept

$$= \frac{5}{7} : \frac{5}{8}$$
$$= \frac{1}{7} : \frac{1}{8}$$
$$= \frac{1}{7} \times 56 : \frac{1}{8} \times 56$$
$$= 8 : 7$$

9. The graph of linear equation $3x - y = 6$ cuts x-axis at

(*a*) (2, 0) (*b*) (3, 0)

(*c*) (0, –2) (*d*) (0, –3)

Sol. (*a*) (2, 0)

Putting $y = 0$ in the given equation, we get

$$x = \frac{6}{3} = 2$$

$\therefore$ The line $3x - y = 6$ cuts the x-axis at the point (2, 0).

10. The number of solution(s) of the equation $3x - 2 = -2x + 1$ on the number line and on the Cartesian plane respectively is

(*a*) one and two.

(*b*) one and infinitely many solutions.

(*c*) infinitely many solutions and one.

(*d*) two and one.

Sol. (*b*) one and infinitely many solutions

Solving the given equation, we get

$$3x + 2x = 1 + 2$$
$$\Rightarrow \quad 5x = 3$$
$$\Rightarrow \quad x = \frac{3}{5}$$

This represent only one point on the number line at a distance $\frac{3}{5}$ from the number 0 on the night side, but on a Cartesian xy-plane, it will represent a line parallel to y-axis and lying at a distance of $\frac{3}{5}$ units from the origin towards the positive direction of x-axis. We know that a line consists of infinitely many points representing the solution of the equation.

Hence, in the first case, $x = \frac{3}{5}$ will represent only one solution and in the second case, it will represent infinite number of solutions. Hence, (*b*) is the correct choice.

Value-based Question (Optional)

(Page 53)

1. A boy walks across an x metre wide road at a speed of 1.75 m/s and crosses it in t_1 seconds.

(*a*) Write a linear equation in two variables x and t_1 to express this situation.

(*b*) Next day, the boy is just about to cross the same road, when he spots a visually impaired old man who also wants to cross the road. The boy then helps that old man to go across the road by reducing his walking speed to 0.75 m/s. If t_2 seconds represents the time taken by them to cross the road, write a relationship between x and t_2.

(*c*) Hence, find the ratio between the time taken by the boy to cross the road on the first day and the second day.

(*d*) What values did the boy show?

Sol. (*a*) We know that

$$\text{Distance} = \text{Speed} \times \text{Time}$$
$$x = 1.75\, t_1$$
$$= 1\frac{3}{4}\, t_1$$

$$= \frac{7t_1}{4}$$

$\Rightarrow \quad 4x = 7t_1$

$\Rightarrow \quad 4x - 7t_1 = 0$

which is the required equation.

(*b*) In this case, speed = 0.75 m/s = $\frac{3}{4}$ m/s .

$\therefore \quad x = 0.75t_2$

$$= \frac{3}{4}t_2$$

$\Rightarrow \quad 4x - 3t_2 = 0$

which is the required equation.

(*c*) From (*a*), we have $t_1 = \frac{4x}{7}$ and

from (*b*), $t_2 = \frac{4x}{3}$

$\therefore \quad t_1 : t_2 = \frac{4x}{7} : \frac{4x}{3}$

$$= \frac{1}{7} : \frac{1}{3}$$

$$= \frac{1}{7} \times 21 : \frac{1}{3} \times 21$$

$$= 3 : 7$$

which is the required ratio.

(*d*) Empathy, helpfulness, compassion, sensitivity and responsibility.

5

Introduction to Euclid's Geometry

Checkpoint (Page 56)

1. How many lines can pass through
   (*a*) one point
   (*b*) two distinct points?

Sol. We know from Euclid's postulates that
(*a*) infinitely many lines can pass through one point.
(*b*) only one line can pass through two distinct points.

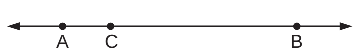
2. What is the least number of distinct points which determines a unique line?

Sol. From Euclid's postulates, we know that the least number of distinct points which determines a unique line is two.

3. Identify the portions of parallel and intersecting lines in the following:
   (*a*) Two opposite edges of a rectangular door
   (*b*) The adjacent edges of your geometry box

Sol. (*a*) We know that two opposite sides of a rectangle are parallel. Hence, two opposite edges of a rectangular door are parallel.
(*b*) We know that two adjacent sides of a rectangle represent portion of two intersecting lines meeting at a point. Hence, the adjacent edges of our geometry box which is rectangular in shape represent portion of two intersecting lines.

4. How many line(s) and line segments are there in the given figure? Name them.



Sol. From definitions of a line and line segment, we see that in the given figure, there is only one line AB and three line segments AC, CB and AB.

5. How many diameters can you draw in a circle?

Sol. We know that all diameters of a circle are line segments passing through the unique centre of a circle. Since, infinite number of line segments can be drawn through one point, hence there are infinitely many diameters which can be drawn in a circle.

6. Find the minimum and maximum number of points of intersection of three distinct lines in a plane?

Sol. We know that three lines can either be parallel to each other or they can form a triangle with three vertices. Hence, the minimum number of points of intersection of three distinct lines is 0, Since parallel lines can never meet together, and the maximum number of points of intersection of three lines is 3, in this case they can form a triangle with three vertices.

7. Define equilateral triangle and right triangle.

Sol. An equilateral triangle is a triangle with three equal sides and a right-angled triangle is a triangle with one angle measuring 90°.

8. If the distance between two lines is the same everywhere, what kind of lines are they?

Sol. We know that the distance between two parallel lines is same everywhere. Hence, in this case, the two lines are parallel.

Milestone

(Page 60)

Multiple-Choice Questions

1. Which of the following needs a proof?

(*a*) an axiom (*b*) a theorem

(*c*) a definition (*d*) a postulate

[CBSE SP 2010]

Sol. (*b*) a theorem

We know that only a theorem needs a proof.

2. If the area of a triangle is equal to that of a rectangle and the area of a rectangle is equal to that of a square, then the area of the triangle is equal to the area of the square. This follows from Euclid's

(*a*) fourth axiom (*b*) second axiom

(*c*) third axiom (*d*) first axiom

Sol. (*d*) first axiom

From Euclid's first axiom, we know that things which are equal to the same thing are equal to one another. Here the area of a triangle and that of a rectangle are each equal to that of a square. Hence, the area of the triangle is equal to the area of the square.

Very Short Answer Type Questions

3. What is the maximum number of point(s) of intersection of two distinct lines?

Sol. We know that two distinct lines can either intersect each other at only one point or else they may be parallel to each other. In the second case they cannot intersect each other at all. Hence, the required maximum number of point(s) of intersection will be only one.

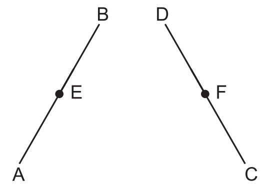
4. How many maximum number of distinct lines can be drawn through three non-collinear points in a plane?

Sol. We know that a triangle can be drawn through three non-collinear points by three distinct lines. Hence, the maximum number of distinct lines that can be drawn through three non-collinear points is three.

Short Answer Type-I Questions

5. In the given figure, AE = DF. E is the mid-point of AB and F is the mid-point of DC. Using Euclid's axiom, show that

AB = DC **[CBSE SP 2012]**



Sol. We have AE = DF [Given]

Also, AE × 2 = DE × 2

[By Euclid's axiom, i.e. if equals are multiplied by equals then their products are equal]

$\Rightarrow$ AB = DC

[E and F are mid-point of AB and CD respectively]

Hence, proved.

6. If four line segments PQ, PR, PS and PT are parallel to a line *l*, then prove that the points P, Q, R, S and T are collinear.

Sol. Given that four line segments PQ, PR, PS and PT are parallel to a line *l*.

To prove that points P, Q, R, S and T are collinear.

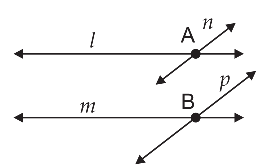
Two distinct lines PQ and PR intersect at P. So, these two lines cannot be parallel to the same line *l*, outside the point P by Playfairs axiom. Hence, PQ and PR must coincide. Similarly PR, PS and PT will coincide. In other words, points P, Q, R, S and T must lie on the same line segment PT i.e. these points must be collinear.

Hence, proved.

Short Answer Type-II Questions

7. Four lines *l*, *m*, *n* and *p* are such that $l \parallel m$ and $n \parallel p$. If *l* and *n* intersect at a point, prove that *m* and *p* also intersect at a point. State the axiom(s) used to prove it.

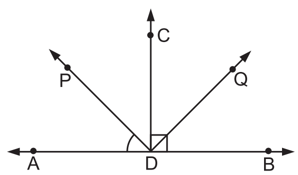
Sol. Given that four lines *l*, *m*, *n* and *p* are such that $l \parallel m$ and $n \parallel p$. Also, line *l* and *n* intersect each other at a point, say A. To show that *m* and *p* also intersect each other at another point, say B.



Since two distinct lines *l* and *n* intersect at a point A, hence by Playfairs Axiom, both *l* and *n* cannot be parallel to the same line m or p. Since, $m \parallel l$ and $p \parallel n$ and l and n intersect each other at A, hence m and p cannot be parallel to each other. Therefore, they must intersect at another point, say B.

Hence, proved.

8. In the given figure, AB $\perp$ CD. If $\angle$PDA = $\angle$QDB, prove that $\angle$PDC = $\angle$QDC.



State the axiom(s) used to prove this.

Sol. Given that line AB is perpendicular to the ray DC. Also, DQ and DP are two rays such that

$$\angle PDA = \angle QDB$$

To prove that,

$$\angle PDC = \angle QDC$$

We have $\quad \angle CDB = \angle CDA = 90°$ $\qquad$ [Given]

Now, subtract $\angle$QDB from both sides, we get

$$\angle CDB - \angle QDB = \angle CDA - \angle QDB$$
$$= \angle CDA - \angle PDA$$
$$[\because \angle QDB = \angle PDA]$$

$$\Rightarrow \quad \angle QDC = \angle PDC$$

Hence, proved.

Euclid's axiom used here is the following:

"If equals are subtracted from equals, the remainders are equal".

We have also used Euclid's postulate: "All right angles are equal to one another".

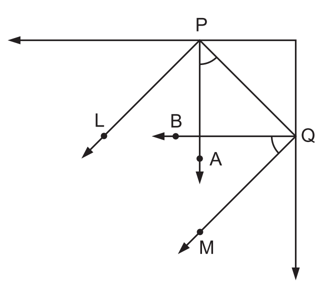
Long Answer Type Questions

9. In the given figure, $\angle$APQ = $\angle$BQM. PA bisects $\angle$LPQ and QB bisects $\angle$PQM.

Show that

$$\angle LPQ = \angle PQM.$$

State the axiom(s) used.



Sol. Given that $\angle$APQ = $\angle$BQM

$$\angle LPQ = 2\angle APQ$$
$$[\because \text{PA bisects } \angle LPQ] \quad \ldots(1)$$

and $\quad \angle PQM = 2\angle BQM$
$$[\because \text{QB bisects } \angle PQM] \quad \ldots(2)$$

To prove that

$$\angle LPQ = \angle PQM$$

We have

$$\angle APQ = \angle BQM \qquad \text{[Given]}$$
$$\Rightarrow \quad 2\angle APQ = 2\angle BQM$$
$$\Rightarrow \quad \angle LPQ = \angle PQM \qquad \text{[From (1) and (2)]}$$

Hence, proved.

Euclid's axiom used here is the following:

"It equals are multiplied by equals, then their products are equal".

or

"Things which are double of equal things are equal to one another".

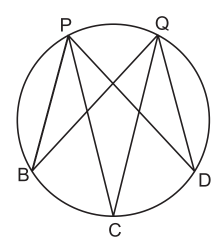
10. In the given figure,

$$\angle BPD = \angle BQD$$

and $\quad \angle BPC = \angle BQC$

Prove that

$$\angle CPD = \angle CQD$$

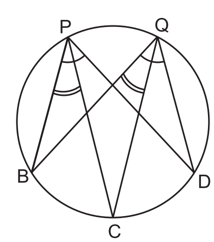


State the axiom(s) used.

Sol. In the given figure,

$$\angle BPD = \angle BQD \qquad \text{[Given]}$$

and $\quad \angle BPC = \angle BQC$ $\qquad$ [Given]



To prove that

$$\angle CPD = \angle CQD$$

We have $\angle BPD = \angle BQD$ [Given]

$\Rightarrow \angle BPD - \angle BPC = \angle BQD - \angle BQC$

[$\because$ $\angle BPC = \angle BQC$]

$\Rightarrow \angle CPD = \angle CQD$

Hence, proved.

Euclid's axiom used here is the following:

"If equals are subtracted from equals, then the remainders are equals".

Higher Order Thinking Skills (HOTS) Question

(Page 60)

1. In finding each angle of a regular polygon of n sides by using the formula:

Sum of all interior angles of a polygon of n sides $= (n - 2) \times 180°$, it was found that each angle of a regular polygon was 168°. Find the number of sides of this polygon mentioning the Euclid's axiom(s) used.

Sol. A regular polygon of n sides has n equal angles. Since each angle measures 168°, hence, n such angles will measure $(168\ n)°$.

$\therefore$ According to the problem,

$(n - 2) \times 180 = 168n$

$180n - 168n = 360$

$\Rightarrow 12n = 360$

$\Rightarrow n = \frac{360}{12} = 30$

Hence, the required number of sides of the regular polygon is 30.

Euclid's axiom used here is the following:

"If equals are added to equals, then the whole are equal".

Note: In the above solution, we have added n equal angles, each of measures 168°.

$\therefore$ The total angle of the polygon measure $(168n)°$.

Self-Assessment

(Page 60)

Multiple-Choice Questions

1. Which of the following are known as the boundaries of solids?

(*a*) Curves (*b*) Lines

(*c*) Points (*d*) Surfaces

Sol. (*d*) Surfaces

We know that any solid is bounded by a surface and not by a curve, or a line or a point.

2. The number of dimensions, a cuboid has

(*a*) 1 (*b*) 2

(*c*) 3 (*d*) 4

Sol. (*c*) 3

We know that a cuboid has length, breadth and height. Hence, it has 3 dimensions.

Assertion-Reason Type Questions

Directions (Q. Nos. 3 to 6): Each of these questions contains an assertion followed by reason. Read them carefully, and answer the question on the basis of the following options, select the one that best describes the two statements.

(a) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.

(b) Both Assertion and Reason are true but Reason is not the correct explanation of Assertion.

(c) Assertion is true but Reason is false.

(d) Assertion is false but Reason is true.

3. Assertion: A straight line may be drawn from any one point to any other point.

Reason: A straight line joins all the points on a plane.

Sol. (*c*) Assertion is correct but reason is incorrect.

A straight line may be drawn from any one point to any other point but is not true that straight line joins all the points on a plane.

4. Assertion: Only one circle can be drawn from a given center.

Reason: From a point many circles can be drawn with different radii.

Sol. (*d*) Assertion is incorrect but reason is correct.

From a point many circles with different radii can be drawn (concentric circles).

5. Assertion: Two distinct lines cannot have more than one point in common.

Reason: Only parallel lines can have more than one common point.

Sol. (*c*) Assertion is correct but reason is incorrect.

Parallel lines do not intersect so they do not have any point in common.

6. Assertion: The two lines which are parallel to the same line, are perpendicular to each other.

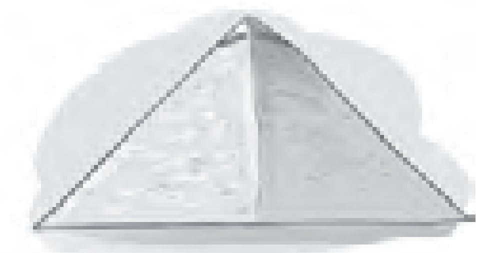
Reason: The two lines which are parallel to the same line, are parallel to each other.

Sol. (*d*) Assertion is wrong but reason is correct.

The two lines parallel to the same line are parallel to each other.

Case Study Based Questions

7. Roshni and her family had their last summer vacation in Egypt. They visited the iconic pyramids of Giza, as well as other tourist attractions in and around. The Great Pyramid of Giza is the oldest and the largest of the pyramids in the Giza pyramid complex. Roshni was astounded to discover that the ancient Egyptians were adept at geometrical calculations. They built pyramids using their understanding of geometry. Now, answer the following questions.



(*i*) A pyramid is a solid figure, the base of which is

(*a*) a triangle. (*b*) a square.

(*c*) a rectangle. (*d*) any polygon.

Sol. (*d*) any polygon.

Base of a pyramid is a triangle or square or some other polygon.

(*ii*) The side faces of a pyramid are

(*a*) triangles. (*b*) squares.

(*c*) polygons. (*d*) trapeziums.

Sol. (*a*) triangles.

(*iii*) The number of dimensions, a surface has is

(*a*) 0 (*b*) 1

(*c*) 2 (*d*) 3

Sol. (*c*) 2

Surface has only two dimensions length and breadth.

(*iv*) The number of dimensions, a solid has is

(*a*) 1 (*b*) 2

(*c*) 3 (*d*) 0

Sol. (*c*) 3

Every solid has 3 dimensions length breadth and height.

(*v*) The boundaries of solids are

(*a*) surfaces. (*b*) curves.

(*c*) lines. (*d*) points.

Sol. (*a*) surfaces.

Boundaries of solids are called surfaces.

8. Kavya is passionate to know about India's ancient heritage and culture. She had recently studied about the magnificent Indus Valley civilization in her history classes. She had also studied two of the Indus Valley civilization's major settlements, Harappa and Mohenjodaro. Geometry was used extensively in the Indus Valley culture, as evident by the excavations at these locations. Now, answer the following questions.

(*i*) In the Indus Valley civilization, the bricks used for the construction work had the dimensions in the ratio

(*a*) 1 : 3 : 4 (*b*) 4 : 2 : 1

(*c*) 4 : 4 : 1 (*d*) 4 : 3 : 2

Sol. (*b*) 4 : 2 : 1

(*ii*) In ancient India, the shapes of altars used for household rituals were

(*a*) squares and circles.

(*b*) triangles and rectangles.

(*c*) trapeziums and pyramids.

(*d*) rectangles and squares.

Sol. (*a*) squares and circles.

(*iii*) The number of dimensions, a line has is

(*a*) 1 (*b*) 3

(*c*) 0 (*d*) 2

Sol. (*a*) 1

(*iv*) The number of interwoven isosceles triangles in Sriyantras (in the Atharva Veda) is

(*a*) 7 (*b*) 8

(*c*) 9 (*d*) 11

Sol. (*c*) 9

(*v*) In Sriyantras (from the Atharva Veda), the number of subsidiary triangles formed by the arrangement of isosceles triangles is

(*a*) 43 (*b*) 55

(*c*) 68 (*d*) 39

Sol. (*a*) 43

Very Short Answer Type Questions

9. Two distinct lines in a plane have x common point(s). What is the value of x?

Sol. We know that two distinct lines in a plane can intersect each other at only one point. In other words, only one point may be common for two distinct lines in a plane. Hence, the required value of x is 1.

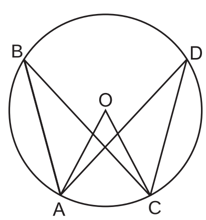
10. Can you draw an equilateral triangle with one side equal to the length of the line? If not, state why?

Sol. We know that the three sides of an equilateral triangle are line segment and not lines since a line does not have a definite length. Hence, the required answer to this problem is "no".

Short Answer Type-I Questions

11. In the given figure,

$$\angle ABC = \frac{1}{2}\angle AOC \text{ and } \angle AOC = 2\angle ADC$$



Show that $\angle ABC = \angle ADC$. State the axiom(s) used.

Sol. In the given figure, it is given that

$$\angle ABC = \frac{1}{2}\angle AOC$$

and $\angle AOC = 2\angle ADC$

To prove that

$$\angle ABC = \angle ADC$$

We have

$$2\angle ADC = \angle AOC \quad \text{[Given]}$$

$$\Rightarrow \quad \angle ADC = \frac{1}{2}\angle AOC \quad \ldots(1)$$

Also, $\angle ABC = \frac{1}{2}\angle AOC$ …(2)

∴ From (1) and (2),

$$\angle ABC = \angle ADC$$

Hence, proved.

Euclid's axiom used here is the following:

(*i*) "If equals are divided by equals, then their quotients are equal" or "Things which are halves of equal things are equal".

(*ii*) "Things which are equal to the same thing are equal to one another".

12. Show that an obtuse angle is greater than an acute angle. State the axiom(s) used.

Sol. We know that an obtuse angle is greater than 90° and an acute angle is less than 90°. Hence, an acute angle is a part of an obtuse angle.

Euclid's axiom used here is the following:

"The whole is greater than the part".

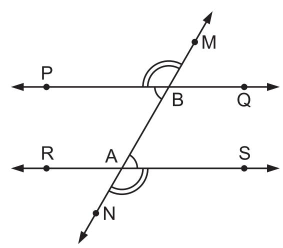
Short Answer Type-II Questions

13. In the given figure,

$$\angle BAS = \angle PBA \text{ and } \angle PBM = \angle SAN$$

Prove that $\angle MBA = \angle BAN$

State the axiom(s) used.



Sol. In the given figure, it is given that

$$\angle BAS = \angle PBA$$

and $\angle PBM = \angle SAN$

To prove that

$$\angle MBA = \angle BAN$$

We have

$$\angle PBA = \angle BAS \quad \ldots(1)$$

and $\angle PBM = \angle SAN$ …(2)

Adding (1) and (2), we get

$$\angle PBA + \angle PBM = \angle BAS + \angle SAN$$

$$\Rightarrow \quad \angle MBA = \angle BAN$$

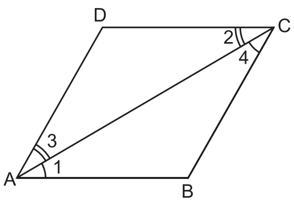
Hence, proved.

Euclid's axiom used here is the following:

"If equal are added to equals, then the wholes are equal".

14. In the given figure,
$\angle 1 = \angle 4$, $\angle 3 = \angle 2$
and $\angle 2 = \angle 4$
Prove that $\angle 1 = \angle 3$.
State the axiom(s) used. **[CBSE SP 2010]**

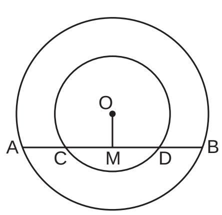
Sol. In the given figure, it is given that $\angle 1 = \angle 4$, $\angle 3 = \angle 2$ and $\angle 2 = \angle 4$.



To prove that
$$\angle 1 = \angle 3$$
We have
$$\angle 1 = \angle 4 \quad \ldots(1)$$
and $$\angle 2 = \angle 4 \quad \ldots(2)$$
∴ From (1) and (2), we have
$$\angle 1 = \angle 2 \quad \ldots(3)$$
Also, $$\angle 2 = \angle 3 \quad \ldots(4)$$
∴ From (3) and (4), we have,
$$\angle 1 = \angle 3$$
Hence, proved.
Euclid's axiom used here is the following:
"Things which are equal to the same thing are equal to one another".

Long Answer Type Questions

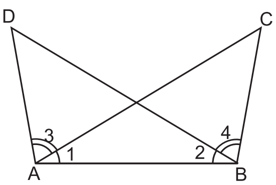
15. In the given figure, O is the centre of two concentric circles. It is given that AM = BM and CM = DM.
Show that AC = BD.



State the axiom(s) used.

Sol. In the given figure, O is the centre of two concentric circles. It is given that
$$AM = BM \quad \ldots(1)$$
and $$CM = DM \quad \ldots(2)$$
To prove that AC = BD.
We have
$$AB = BM$$
Subtract CM from both sides, we get
$$\Rightarrow \quad AM - CM = BM - CM$$
$$= BM - DM \quad \text{[From (2)]}$$
$$\Rightarrow \quad AC = BD$$
Hence, proved.
Euclid's axiom used here is the following:
"If equals are subtracted from equals, then the remainders are equal".

16. In the given figure,
(*a*) If $\angle 1 = \angle 2$
and $\angle 3 = \angle 4$.



Show that
$$\angle DAB = \angle ABC$$
State the axiom(s) used.
(*b*) If $\angle DAB = \angle ABX$ and $\angle 3 = \angle 4$
Show that $\angle 1 = \angle 2$
State the axiom(s) used.

Sol. (*a*) In the given figure, it is given that
$$\angle 1 = \angle 2$$
and $$\angle 3 = \angle 4$$
To show that
$$\angle DAB = \angle ABC$$
We have
$$\angle 1 = \angle 2 \quad \ldots(1)$$
$$\angle 3 = \angle 4 \quad \ldots(2)$$
Adding $\angle 3$ both side in equation (1), we get
$$\angle 1 + \angle 3 = \angle 2 + \angle 3$$
$$= \angle 2 + \angle 4 \quad \text{[From (2)]}$$
$$\angle DAB = \angle ABC$$
Hence, proved.
Euclid's axiom used here is the following:
"If equals are added to equals, then the wholes are equal".

(*b*) In this case, it is given that
$$\angle DAB = \angle ABC \quad \ldots(3)$$
and $$\angle 3 = \angle 4 \quad \ldots(4)$$
To show that
$$\angle 1 = \angle 2$$

From (3) and (4), we have

$$\angle DAB = \angle ABC$$

Subtracting $\angle 3$ from both sides, we get

$$\angle DAB - \angle 3 = \angle ABC - \angle 3$$

$$= \angle ABC - \angle 4 \quad \text{[From (4)]}$$

$$\angle 1 = \angle 2$$

Hence, proved.

Euclid's axiom used here is the following:

"If equals are subtracted from equals, then the remainders are equals".

Let's Compete

(Page 62)

Multiple-Choice Questions

1. "Lines are parallel if they do not intersect" is stated in the form of

(*a*) an axiom. (*b*) a postulate.

(*c*) a definition. (*d*) a proof.

Sol. (*c*) a definition.

We know from the definition of two parallel lines that they do not intersect each other.

2. If $x = y + z$ where $y, z > 0$, then $x > y$ and $x > z$. In this statement, which of the following axioms of Euclid is used?

(*a*) Things which are equal to the same thing are equal to one another.

(*b*) The whole is greater than the part.

(*c*) If equals are subtracted from equals, the remainders are equal.

(*d*) If equals are added to equals, the whole are equal.

Sol. (*b*) The whole is greater than the part.

We see from the given equation $x = y + z$ where $y, z > 0$ that $y + z$ is greater than both y and z.

In other words,

Each of y and z is a part of the whole $y + z$ i.e. x.

$\therefore$ $x > y$ and $x > z$ follows from Euclid's axiom: "The whole is greater than the part".

3. One of Euclid's axioms is "Things which are double of the same things are equal to one another".

An example of this axiom is

(*a*) If $2x = 4y$, then $2x - 4y = 0$.

(*b*) If AB = CD, then AB + x = CD + x.

(*c*) An angle subtended by an arc of a circle at its centre is double the angle subtended by the arc at a point on its circumference.

(*d*) All diameters of a circle are equal.

Sol. (*d*) All diameters of a circle are equal.

We know that a diameter of a circle is twice the length of its radius. Also, all radii of a circle are equal in length. Hence, all diameters are also equal.

4. Trichotomy law or axiom is followed in which of the following example.

(*a*) If x and y are two numbers, then only one of the following statements is true: $x = y$, $x > y$ and $x < y$.

(*b*) If $x = y$, then $\frac{x}{a} = \frac{x}{b}$ where $a, b \neq 0$.

(*c*) The lengths of all sides of an equilateral triangle are equal.

(*d*) The sum of three angles of any triangle is 180°.

Sol. (*a*) If x and y are two numbers, then only one of the following statements is true: $x = y$, $x > y$ and $x < y$.

Euclid's trichotomy law states that of two quantities of the same kind, the first is greater than, equal to or less than the second.

5. Assuming that the sum of three angles of a triangle is 180°, we can prove, by using some axiom(s) of Euclid that the sum of four angles of a quadrilateral is

(*a*) 190° (*b*) 200°

(*c*) 360° (*d*) 300°

Sol. (*c*) 360°

A quadrilateral can be divided into two triangles by drawing a diagonal. We know that the sum of three angles of a quadrilateral is 180°. Hence, the sum of three angles of each of two triangles within the quadrilateral is 180°. We can prove that the sum of four angles of the quadrilateral is 360°, by using suitable axioms of Euclid.

6. The sum of interior angles of any regular polygon of n sides is given by $(n - 2) \times 180°$. By using same axiom(s) of Euclid we can show that each interior angle of a regular polygon of 60 sides is

(*a*) 164° (*b*) 174°

(*c*) 175° (*d*) 100°

Sol. (*b*) 174°

In this case, the sum of all angles of a regular polygon of 60 sides = $(60 - 2) \times 180° = 58 \times 180°$. Since all angles of 60 sided regular polygon are equal, hence each angle will measure

$$\frac{58 \times 180°}{60} = 174°$$

7. The ratio of the radii of two circles is equal to the ratio of their circumferences. This result can be proved by using the result $C = 2\pi r$ where r is the radius of a circle and C is its circumference, by using the Euclid's axiom.

(*a*) If equals are added to equals, the whole are equal.

(*b*) If equals are subtracted from equals, the remainders are equal.

(*c*) The whole is the greater than the part.

(*d*) Things which are equal to the same thing are equal to one another.

Sol. (*d*) Things which are equal to the same things are equal to one another.

Let r_1 and r_2 be the radii of the two circles and C_1 and C_2 be their respective circumferences. Then we have

$$C_1 = 2\pi r_1 \quad \ldots(1)$$

and $$C_2 = 2\pi r_2 \quad \ldots(2)$$

From (1), $$\frac{C_1}{r_1} = 2\pi$$

[Dividing both sides by r_1] …(3)

and From (2),

$$\frac{C_2}{r_2} = 2\pi$$

[Dividing both sides by r_2] …(4)

From (3) and (4), we have

$$\frac{C_1}{r_1} = \frac{C_2}{r_2} \quad \ldots(5)$$

Multiplying both sides of equation (5) by $\frac{r_2}{C_1}$, we get

$$\frac{C_1}{r_1} \times \frac{r_2}{C_1} = \frac{C_2}{r_2} \times \frac{r_2}{C_1}$$

$$\Rightarrow \quad \frac{r_2}{r_1} = \frac{C_2}{C_1}$$

Hence, proved.

Euclid's axiom used here is the following:

"Things which are equal to the same thing are equal to one another".

8. Euclid stated that all right angles are equal to one another in the form of

(*a*) a postulate. (*b*) a definition.

(*c*) a proof. (*d*) an axiom.

Sol. (*a*) a postulate.

The given statement is a universal truth which is used in geometry only. Hence, it is a postulate.

9. "If equals are multiplied by equals then their products are equal". An example of this axiom is

(*a*) The area and the circumference of a circle of radius r are respectively πr^2 and $2\pi r$.

(*b*) If $2x = 3y$, then $2x \pm k = 3y \pm k$.

(*c*) If A > B, then $-\frac{A}{2} < \frac{B}{2}$.

(*d*) If AC is an arc of a circle with centre at O and if D and B are two points on the remaining part of the circumference, and if $\angle AOB = 2\angle ADC$ and $\angle ABC = \frac{1}{2}\angle AOB$, then $\angle ABC = \angle ADC$.

Sol. (*c*) If A > B, then $\frac{-A}{2} < \frac{B}{2}$

Let A, B > 0 and $A = B + x$ where $x > 0$. Clearly A > B, by axiom: "The whole is greater than the part".

$$\therefore \quad -\frac{A}{2} = -\frac{B}{2} - \frac{x}{2}$$

By using axiom: "If equals are multiplied by equals, then their products are equal". But by definition, $-\frac{B}{2} - \frac{x}{2} < \frac{B}{2}$, since a negative number is always less than a positive number, by definition. Hence, $-\frac{A}{2} < \frac{B}{2}$.

10. If $\Delta ABC \cong \Delta DEF$, then AB = DE, AC = DF, BC = EF, $\angle A = \angle D$, $\angle B = \angle E$ and $\angle C = \angle F$. This important result follows Euclid's axiom stated as

(*a*) Things which are equal to the same thing are equal to one another.

(*b*) Things which coincide with one another are equal in all respects.

(*c*) If equals are subtracted from equals, the remainders are equal.

(*d*) Things which are halves to the same things are equal to one another.

Sol. (*b*) Things which coincide with one another are equal in all respects.

We know that three angles and three sides are six components of a triangle. If these six components of each triangle coincide respectively with those of another triangle, then the two triangles will be congruent to each other. This clearly follows from

Euclid's axiom: "Things which coincide with one another are equal in all respects".

Value-based Question (Optional)

(Page 63)

1. Two businessmen, Mahesh and Badal, were rich people in a village. Mahesh's income was more than that of Badal. They decided to donate 25% of their income for the education of poor students in a school in the village. After donation, it was found that the balance of Mahesh's income was double that of Badal.

(*a*) Prove that Mahesh's income was double the income of Badal.

(*b*) In proving this result, which axiom(s) of Euclid will you use?

(*c*) What values are depicted by Mahesh and Badal in this problem?

Sol. (*a*) Let the incomes of Mahesh and Badal be ₹x and ₹y respectively, where $x > y > 0$.

According to the problem,

$$x - \frac{x}{4} = 2\left(y - \frac{y}{4}\right)$$

$$\Rightarrow \quad \frac{3x}{4} = \frac{3y}{2}$$

$$\Rightarrow \quad \frac{3x}{4} \div \frac{3}{4} = \frac{3y}{2} \div \frac{3}{4} \quad \ldots(1)$$

$$\Rightarrow \quad \frac{3x}{4} \times \frac{4}{3} = \frac{3y}{2} \times \frac{4}{3}$$

$$\Rightarrow \quad x = 2y \quad \ldots(2)$$

From (2), we see that Mahesh's income was double the income of Badal.

Hence, proved.

(*b*) We have obtained equation (2) from (1) by dividing both sides by the same number $\frac{3}{4}$.

Hence, we have used the following axiom of Euclid's:

"If equals are divided by equals, then the quotients are equal".

(*c*) Kindness towards the poor students and interest for the improvement of education.

6

Lines and Angles

Checkpoint (Page 66)

1. State why it is not possible to divide a line into two equal parts.

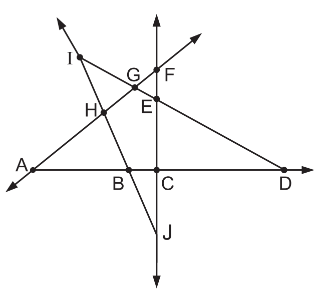
Sol. It is not possible to divide a line into two equal parts, because a line does not have a finite length.

2. In the figure given along side, find

(*a*) the maximum number of distinct closed regions bounded by line segments such that no part of a region may lie within the other region and the minimum number of such closed regions.

(*b*) any three rays and two lines.

(*c*) the total number of distinct points common to each pair of line segments among all the pairs. Name any ten pairs of line segments passing through these points.



Sol. (*a*) The maximum number of distinct closed regions is six, *viz*. IGH, ABH, GEF, CDE, BCJ and BCEGH and the minimum number of such regions is only one, *viz*. ABJCDEFGIH.

(*b*) Ray CD, ray HI and ray GF are any three rays, and AF, FJ are two lines.

(*c*) The total number of distinct points common to each pair of line segments among all the pairs are A, B, C, D, E, F, G, H, I, J.

Any ten pairs of line segments passing through the above points are:

AH, AB; IH, IG; GI, GH; FE, FG; EF, EG; HG, HI; BC, BJ; JC, JB; CD, CE and DE, DC.

3. Three distinct points are given in a plane. How many lines can be drawn through them?

Sol. If the three distinct points are collinear as in Fig. (*i*), then only one line can be drawn through those points. If the three points are not collinear, then three distinct lines can be drawn through them as in Fig. (*ii*).

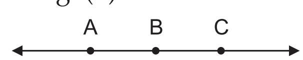


Fig. (*i*)

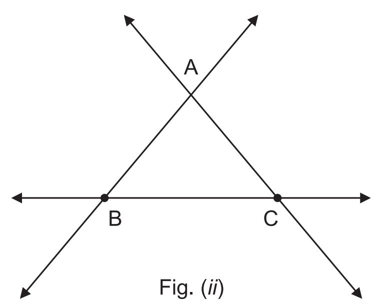


Fig. (*ii*)

4. A line and a point, not on the line, are given. How many planes can be made to pass through the line and the point?

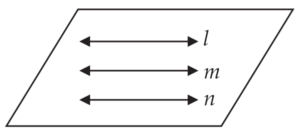
Sol. We know that only one plane can be drawn through a line and a point not lying on the line.

5. What is the least number of distinct points which determines a unique line?

Sol. The least number of distinct points which determines a unique line is 2.

6. Draw three lines in a plane so that you may get the minimum number of points of intersection. What is the number of such points of intersection?

Sol. Three parallel lines *l*, *m* and *n* are drawn on a plane shown in the figure. The lines do not intersect each other at all. Hence, the required minimum number of points of intersection of these three lines on a plane is zero.



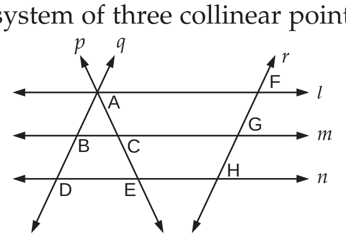
7. In the given figure, write the name(s) of

(*a*) all pairs of parallel lines.

(*b*) all pairs of intersecting lines.

(*c*) the point through which three lines pass and also name these lines.

(*d*) all system of three collinear points.



Sol. (*a*) Pairs of parallel lines are : *l*, *m*; *l*, *n*; *m*, *n*; *q*, *r*.

(*b*) Pairs of intersecting lines are : *p*, *q*; *p*, *l*; *p*, *m*; *p*, *n*; *q*, *l*; *q*, *m*; *q*, *n*; *r*, *l*; *r*, *m*; *r*, *n* and *p*, *r* when extended.

(*c*) The three lines *p*, *q* and *l* passes through a single point A.

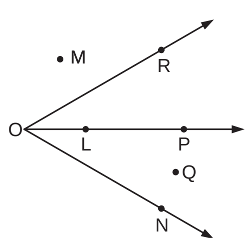
(*d*) All systems of three collinear points are A, B and D; A, C and E; F, G and H; B, C and G; D, E and H.

8. In the given diagram, name the point(s)

(*a*) in the exterior of ∠POR.

(*b*) in the interior of ∠NOP.

(*c*) on ∠POR.



Sol. (*a*) The points M, Q and N lie in the exterior of ∠POR.

(*b*) The only point Q lies in the interior of ∠NOP.

(*c*) The points P, L, O and R lie on ∠POR.

Milestone 1

(Page 70)

Multiple-Choice Questions

1. The supplement of an angle is less than three times its complement by 20°. Then the angle is

(*a*) 50° (*b*) 30°

(*c*) 35° (*d*) 45°

Sol. (*c*) 35°

Let the required angle be of measure *x* in degrees. Then the supplement and complement of *x* are 180° – *x* and 90° – *x* respectively.

∴ According to the problem, we have

$$3(90° - x) - (180° - x) = 20°$$

$$\Rightarrow \quad 270° - 3x - 180° + x = 20°$$

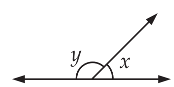
$$\Rightarrow \quad 2x = 270° - 180° - 20° = 70°$$

$$\therefore \quad x = \frac{70°}{2} = 35°$$

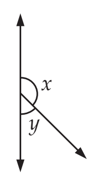
∴ The angle is 35°.

2. In the given figures, which pairs of angles represent a linear pair?

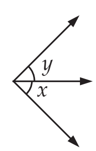
(*i*)



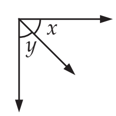
(*ii*)



(*iii*)



(*iv*)



(*a*) (*i*) and (*iv*) (*b*) (*iii*) and (*iv*)

(*c*) (*i*) and (*iii*) (*d*) (*i*) and (*ii*)

Sol. (*d*) (*i*) and (*ii*)

In Fig. (*i*) and (*ii*), we see that $x + y = 180°$, but in Fig. (*iii*) and (*iv*) $x + y < 180°$. Hence, the pair of angles only in Fig. (*i*) and (*ii*) represents a linear pair.

Very Short Answer Type Questions

3. Find the measure of an angle which is three times its supplement.

Sol. Let the required measure of the angle be x in degrees.

$\therefore$ According to the problem, we have

$$x = 3(180° - x)$$

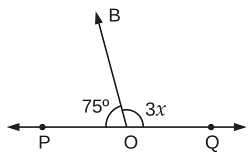
$\Rightarrow$ $3x + x = 540°$

$\Rightarrow$ $4x = 540°$

$\Rightarrow$ $x = \frac{540°}{4} = 135°$

Hence, the required measure of the angle is 135°.

4. In the given figure, POQ is a straight line. Find x.



Sol. From the figure, we see that

$$3x + 75° = 180° \qquad \text{[Linear pair]}$$

$\Rightarrow$ $3x = 180° - 75° = 105°$

$\Rightarrow$ $x = \frac{105°}{3} = 35°$

Hence, the required value of x is 35°.

Short Answer Type-I Questions

5. Two supplementary angles are in the ratio 5 : 4. Find the angles.

Sol. Let the two supplementary angles measure x and $180 - x$ in degrees.

$\therefore$ According to the problem, we have

$$\frac{x}{180 - x} = \frac{5}{4}$$

$\Rightarrow$ $900 - 5x = 4x$

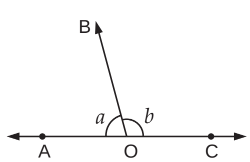
$\Rightarrow$ $5x + 4x = 900$

$\Rightarrow$ $9x = 900$

$\therefore$ $x = \frac{900}{9} = 100$

$\therefore$ The required angles measure 100° and 180° – 100° = 80°.

6. In the given figure, ∠AOB and ∠COB form a linear pair. Find the values of a and b if $2a = b - 30°$.



Sol. According to the problem, we have

$$a + b = 180° \qquad \ldots(1)$$

and $2a = b - 30°$

$$b = 2a + 30° \qquad \ldots(2)$$

$\therefore$ From (1) and (2), we have

$$a + 2a + 30° = 180° \qquad \text{[Linear pair]}$$

$\Rightarrow$ $3a = 180° - 30° = 150°$

$\Rightarrow$ $a = 50°$

$\therefore$ From (2), $b = 2 \times 50° + 30° = 130°$

$\therefore$ The required values of a and b are 50° and 130° respectively.

Short Answer Type-II Questions

7. If the complement of an angle is one-sixth of its supplement, find the angle.

Sol. Let the required angle measure x in degrees.

Then according to the problem, we have

$$90° - x = \frac{1}{6}(180° - x)$$

$\Rightarrow$ $540° - 6x = 180° - x$

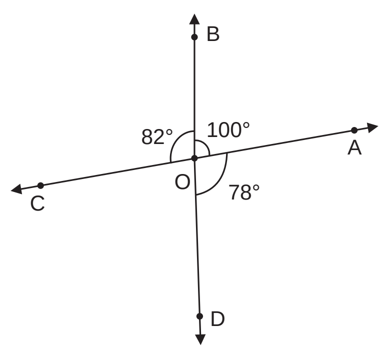
$\Rightarrow$ $5x = 540° - 180° = 360°$

$\Rightarrow$ $x = \frac{360°}{5} = 72°$

Hence, the required angle is 72°.

8. Let OA, OB, OC and OD be rays in anticlockwise direction starting from OA such that ∠AOB = ∠COD = 100°, ∠BOC = 82° and ∠AOD = 78°. Is it true that AOC and BOD are straight lines? Justify your answer. **[CBSE SP 2012]**

Sol. From the figure,



We have

$$\angle AOC = \angle AOB + \angle BOC$$
$$= 100° + 82°$$
$$= 182° \neq 180°$$

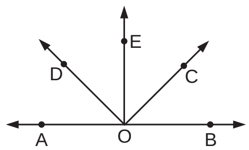
and

$$\angle BOD = \angle BOA + \angle AOD$$
$$= 100° + 78°$$
$$= 178° \neq 180°$$

Since, neither ∠AOC nor ∠BOD is 180°, hence, AOC and BOD are not straight lines.

Long Answer Type Questions

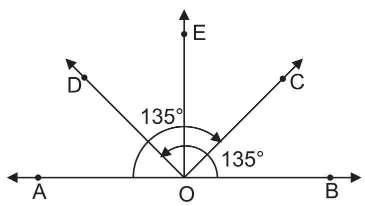
9. In the given figure, $\angle AOC = \angle BOD = 135°$ and $\angle DOE = \angle COE$. Find the measures of $\angle AOD$, $\angle DOC$ and $\angle EOC$.



Sol. Given that

$$\angle AOC = \angle BOD = 135° \quad \ldots(1)$$

Let $\quad \angle DOE = \angle COE = x \quad \ldots(2)$



Now, $\quad \angle AOD = \angle AOC - (\angle DOE + \angle COE)$

$= 135° - (x + x)$ [From (1) and (2)]

$\Rightarrow \quad \angle AOD = 135° - 2x \quad \ldots(3)$

Similarly, $\quad \angle BOC = 135° - 2x \quad \ldots(4)$

$\angle AOD + \angle DOE + \angle COE + \angle BOC = 180°$

[$\because$ AOB is a line]

$\Rightarrow 135° - 2x + x + x + 135° - 2x = 180°$

[From (1), (2), (3) and (4)]

$\Rightarrow \quad 270° - 2x = 180°$

$\Rightarrow \quad 270° - 180° = 2x$

$\Rightarrow \quad 90° = 2x$

$\Rightarrow \quad x = \dfrac{90°}{2}$

$\Rightarrow \quad x = 45°$

$\Rightarrow \quad \angle AOD = 135° - 2(45°)$

$= 135° - 90°$

$= 45°$

$\Rightarrow \quad \angle EOC = 45°$

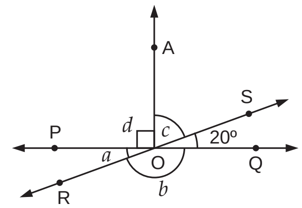
$\Rightarrow \quad \angle DOC = \angle DOE + \angle COE$

$= 45° + 45°$

$= 90°$

$\therefore$ The measure of $\angle AOD = 45°$, $\angle DOC = 90°$ and $\angle EOC = 45°$.

10. In the given figure, lines PQ and RS intersect at O. AO $\perp$ PQ. If $\angle SOQ = 20°$, find the angles a, b, c and d.



Sol. Given that POQ and ROS are two straight lines which intersect each other at O. The ray OA is perpendicular to line PQ.

Also, $\angle AOP = d$, $\angle POR = a$, $\angle QOR = b$, $\angle AOS = c$ and $\angle SOQ = 20°$.

To find the angles a, b, c and d, we have

$a = \angle POR = \angle SOQ$

[Vertically opposite angles]

$\Rightarrow \quad a = 20° \quad \ldots(1)$

$c = \angle AOS$

$= \angle AOQ - \angle SOQ$

$= 90° - 20°$

$\Rightarrow \quad c = 70°$

$b = \angle QOR$

$= \angle POQ - a$

$= 180° - 20°$ [From (1)]

$\Rightarrow \quad b = 160°$

$d = \angle AOP = 90°$ [$\because$ OA $\perp$ PQ]

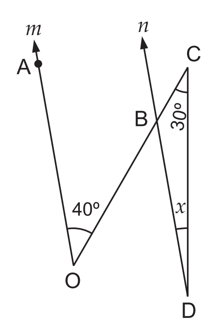
Hence, the required values of a, b, c and d are 20°, 160°, 70° and 90° respectively.

Milestone 2

(Page 78)

Multiple-Choice Questions

1. In the given figure, if $m \parallel n$, then the measure of x is



(*a*) 10° (*b*) 40°

(*c*) 70° (*d*) 30°

Sol. (*a*) 10°

Given that rays m = OA and n = DB are parallel.

Also, ∠AOB = 40°, ∠BCD = 30° and ∠BDC = x

∵ OA ∥ DB and OBC is a transversal,

∴ ∠OBD = ∠AOB = 40°

[Alternate angles]

∵ ∠OBD is an exterior angle of ΔBCD.

∴ ∠OBD = 40° = ∠BCD + ∠BDC

[Property of exterior angle of a triangle]

⇒ 40° = 30° + x

⇒ x = 40° – 30° = 10°

∴ The measure of x is 10°.

2. The angles of triangle are in the ratio 4 : 5 : 9. The triangle is

(*a*) an isosceles triangle.

(*b*) an obtuse-angled triangle.

(*c*) an acute-angled triangle.

(*d*) a right triangle.

Sol. (*d*) a right triangle.

Let the three angles of the triangle be 4x, 5x and 9x in degrees, where x is any unknown variable.

∴ $4x + 5x + 9x = 180°$

[Angles sum property of a triangle]

⇒ $18x = 180°$

∴ $x = \frac{180°}{18} = 10°$

∴ The three angles of the triangle are 4 × 10°, 5 × 10°, 9 × 10°, i.e. 40°, 50° and 90°.

Since one angle of the triangle is 90°, i.e. a right angle, hence, the triangle is a right triangle.

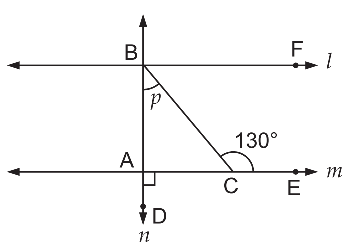
3. In the given figure, if l ∥ m and n ⊥ m, then the measure of angle p is

(*a*) 30°

(*b*) 40°

(*c*) 50°

(*d*) 60°



Sol. (*b*) 40°

Given that lines l and m are parallel to each other and n is perpendicular to m. Also, ∠BCE = 130°, ∠ABC = p and ∠CAD = 90°.

∴ ∠BCE is an exterior angle of a ΔBAC

∴ ∠BCE = ∠ABC + ∠BAC

[Property of exterior angle of a triangle]

⇒ 130° = p + 90°

⇒ p = 130° – 90° = 40°

Very Short Answer Type Questions

4. An exterior angle of a triangle is 130° and its two opposite interior angles are equal. Then what is the measure of each of these two equal angles?

Sol. Let x be the measure of two opposite interior angles.

∴ According to the question, we have

$130° = x + x$

[Property of exterior angle of a triangle]

⇒ $130° = 2x$

⇒ $x = \frac{130°}{2}$

⇒ $x = 65°$

∴ The measure of each of the two opposite interior angles is 65°.

5. If the measure of each of two equal base angles of an isosceles triangle is 56°, what is the measure of its vertical angle?

Sol. Let the measure of the vertical angle be x in degrees.

∴ By using angle sum property of a triangle, we have

$56° + 56° + x = 180°$

⇒ $x = 180° – 112° = 68°$

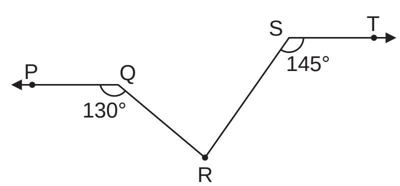
∴ The required measure the vertical angle is 68°.

Short Answer Type-I Questions

6. In the given figure, PQ ∥ ST.

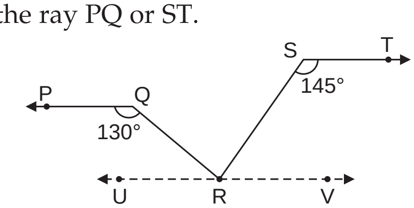
∠PQR = 130° and ∠RST = 145°.

Find the measure of ∠QRS.



Sol. Given that PQ ∥ ST, ∠PQR = 130° and ∠RST = 145°.

Construction: Draw a line UV through R parallel to the ray PQ or ST.



$\because$ PQ $\parallel$ UR and QR is a transversal,

$\therefore \quad \angle PQR + \angle QRU = 180°$

[Cointerior $\angle$s, PQ $\parallel$ UV]

$\Rightarrow \quad 130° + \angle QRU = 180°$

$\Rightarrow \quad QRU = 180° - 130° = 50° \quad \ldots(1)$

Similarly, since ST $\parallel$ RV,

$\therefore \quad \angle TSR + \angle SRV = 180°$

[Cointerior $\angle$s, ST $\parallel$ UV]

$\Rightarrow \quad 145° + \angle SRV = 180°$

$\angle SRV = 180° - 145 = 35° \quad \ldots(2)$

Hence, $\quad \angle QRS = \angle URV - \angle QRU - \angle SRV$

$= 180° - 50° - 35°$

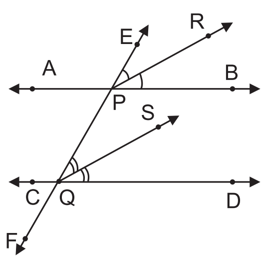
[From (1) and (2)]

$= 180° - 85° = 95°$

$\therefore$ The measure of $\angle QRS$ is 95°.

7. Prove that if two parallel lines are intersected by a transversal, then the bisectors of any two corresponding angles are parallel.

Sol. Given that AB and CD are two parallel lines and EF is their transversal. PR and QS are the bisectors of the two corresponding angles, $\angle EPB$ and $\angle PQD$ respectively.



To prove that PR $\parallel$ QS.

We have

$\angle EPB = \angle PQD$

[Corresponding $\angle$s, AB $\parallel$ CD]

$\Rightarrow \quad \frac{1}{2}\angle EPB = \frac{1}{2}\angle PQD$

$\Rightarrow \quad \angle EPR = \angle PQS$

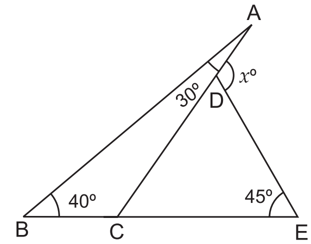
[$\because$ PR and QS are the bisectors of $\angle EPB$ and $\angle PQD$ respectively]

But these two are corresponding angles with respect to rays PR and QS and their transversal EPQ.

$\therefore$ PR $\parallel$ QS

Hence, proved.

8. Find the value of x in the given figure.



[CBSE SP 2011]

Sol. Given that $\angle BAC = 30°$, $\angle ABC = 40°$, $\angle DEC = 45°$ and $\angle ADE = x°$

$\because \quad \angle ACE$ is an exterior angle of ΔABC,

$\therefore \quad \angle ACE = \angle ABC + \angle BAC$

[Property of exterior angle of a triangle]

$\Rightarrow \quad \angle ACE = 40° + 30°$

$\Rightarrow \quad \angle ACE = 70° \quad \ldots(1)$

Again, $\angle ADE = x°$ is an exterior angle of ΔDCE.

$\therefore \quad \angle ADE = x° = \angle DCE + \angle DEC$

[Property of exterior angle a triangle]

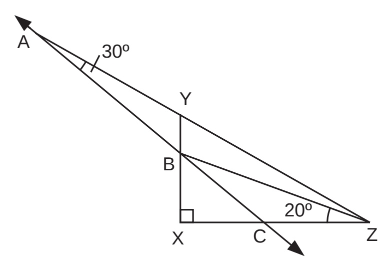
$\Rightarrow \quad \angle ADE = 70° + 45° \quad$ [From (1)]

$\Rightarrow \quad \angle ADE = 115°$

$\therefore$ The value of x is 115°.

Short Answer Type-II Questions

9. In ΔYXZ, $\angle X = 90°$, and YX = XZ. A is a point on ZY produced and the line ABC cuts XY at B and XZ at C. If $\angle BAY = 30°$ and $\angle CZB = 20°$, find the measures of $\angle CBZ$ and $\angle BCZ$.



Sol. Given that ΔYXZ is a right-angled triangle with $\angle X = 90°$ and YX = XZ.

Hence, $\angle XYZ = \angle XZY = 45°$

Also, A is a point on ZY produced and the line ABC cuts XY at B and XZ at C such that $\angle CZB = 20°$ and $\angle BAY = 30°$.

We have

$\angle BZY = \angle XZY - \angle CZB$

$= 45° - 20° = 25°$

Now, $\angle CBZ$ is an exterior angle of ΔABZ.

$\therefore \quad \angle CBZ = \angle BAZ + \angle BZY$

[Property of exterior angle of a triangle]

$\Rightarrow \quad \angle CBZ = 30° + 25°$

$\Rightarrow \quad \angle CBZ = 55°$

In ΔACZ, we have

$\angle ACZ + \angle CAZ + \angle AZC = 180°$

[Angle sum property of a triangle]

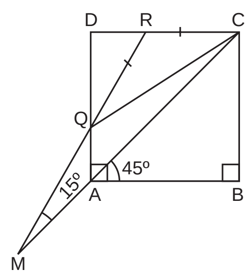
$\Rightarrow \quad \angle BCZ + 30° + 45° = 180°$

[$\because \angle ACZ = \angle BCZ$]

$\Rightarrow \quad \angle BCZ = 180° - 75° = 105°$

Hence, measures of $\angle CBZ$ and $\angle BCZ$ are 55° and 105° respectively.

10. In the figure given below, ABCD is a square. R is a point on CD and Q is a point on AD such that CR = QR. CA and RQ produced meet at M such that $\angle AMQ = 15°$. Find the measures of $\angle ACQ$ and $\angle QCR$. Also find the relation between QC and QM.



Sol. Given that ABCD is a square. R is a point on CD and Q in a point on AD such that CR = QR. CA and RQ produced meet at M such that $\angle AMQ = 15°$.

We have

$\angle CAQ = 45°$

[$\because$ ABCD is a square and AC is a diagonal]

Now, $\angle CAQ$ is an exterior angle of a ΔAMQ.

$\therefore \quad \angle CAQ = \angle AMQ + \angle MQA$

[Property of exterior angle of a triangle]

$\Rightarrow \quad 45° = 15° + \angle MQA$

$\Rightarrow \quad \angle MQA = 45° - 15° = 30°$

But $\quad \angle MQA = \angle DQR$

[Vertically opposite $\angle$s]

$\therefore \quad \angle DQR = 30°$

$\therefore \quad \angle QRD = 90° - 30°$

[$\because$ QDR is a right-angled triangle]

$\Rightarrow \quad \angle QRD = 60°$

$\Rightarrow \quad \angle QRC = 180° - 60° = 120°$

Now, in ΔRQC,

$\angle RQC + \angle RCQ + \angle QRC = 180°$

[Angle sum property of a triangle]

$\Rightarrow \quad 2\angle RQC + 120° = 180°$

[$\because RQ = RC, \therefore \angle RQC = \angle RCQ$]

$\Rightarrow \quad \angle RQC = \dfrac{180° - 120°}{2} = \dfrac{60°}{2} = 30°$

$\therefore \quad \angle QCR = \angle RQC = 30°$

Now, $\angle RQC$ is an exterior angle of ΔMQC.

$\therefore \quad \angle RQC = \angle QMC + \angle MCQ$

$\Rightarrow \quad 30° = 15° + \angle MCQ$

$\Rightarrow \quad \angle MCQ = 30° - 15° = 15°$

In ΔQMC,

$\angle QMC = \angle MCQ = 15°$

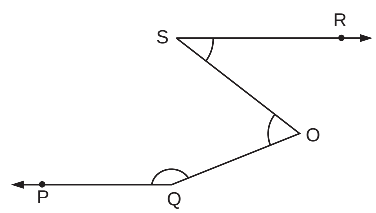
$\therefore \quad QM = QC$

$\therefore \quad \angle ACQ = \angle MCQ = 15°$

Hence, the measure of $\angle ACQ$ is 15° and $\angle QCR$ is 30°.

Also, QC = QM.

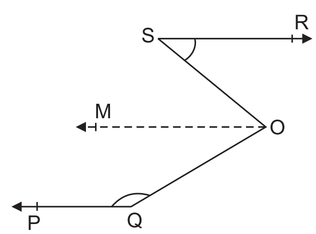
11. In the given figure, $PQ \parallel RS$. Prove that $\angle PQO + \angle QOS = 180° + \angle OSR$.



Sol. Given that $PQ \parallel RS$

To prove that $\angle PQO + \angle QOS = 180° + \angle OSR$.

Construction: We draw a ray OM through O parallel to SR or PQ.



$\because PQ \parallel OM$ and OQ is a transversal,

$\therefore \quad \angle PQO + \angle QOM = 180°$

[Cointerior $\angle$s]

$\Rightarrow \quad \angle PQO + \angle QOS - \angle MOS = 180°$

$\Rightarrow \quad \angle PQO + \angle QOS = 180° + \angle MOS$

$= 180° + \angle OSR$

[Alternate ∠s, OM ∥ RS]

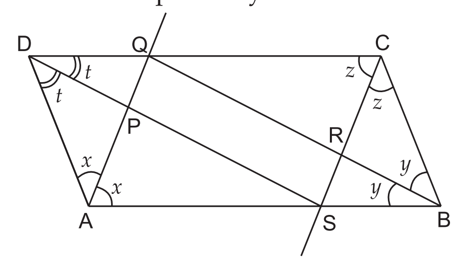
$\Rightarrow \quad \angle PQO + \angle QOS = 180° + \angle OSR$

Hence, proved.

Long Answer Type Questions

12. The four internal bisectors of the angles of a parallelogram form a quadrilateral within this parallelogram. Prove that each angle of this quadrilateral is 90°.

Sol. Let ABCD be a parallelogram. AP, BR, CR and DP be internal bisectors of ∠DAB, ∠ABC, ∠BCD and ∠CDA respectively.



Let the measures of these angles be $2x$, $2y$, $2z$ and $2t$ respectively in degrees.

To prove that the quadrilateral PQRS is such that each of its angles P, Q, R and S is 90°.

Since, ABCD is a parallelogram,

$\therefore \quad \angle DAB + \angle ADC = 180°$

$\Rightarrow \quad 2x + 2t = 180°$

$\Rightarrow \quad x + t = 90°$

i.e. $\quad \angle PAD + \angle PDA = 90° \quad \ldots(1)$

$\Rightarrow \quad \angle APD + \angle PAD + \angle PDA = 180°$

[Angle sum property of a triangle]

$\Rightarrow \quad \angle APD + 90° = 180°$ [From (1)]

$\Rightarrow \quad \angle APD = 90°$

$\Rightarrow \quad \angle APD = \angle SPQ = 90°$

[Vertically opposite ∠s]

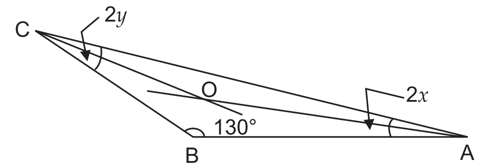
$\Rightarrow$ ∠P is 90°.

Similarly, we can prove that $\angle Q = \angle R = \angle S = 90°$

Hence, proved.

13. The bisectors OA and OC of two acute angles of an obtuse-angled triangle ABC with ∠B = 130° meet at O. Prove that ΔOAC is an obtuse-angled triangle and find the measure of this obtuse angle.

Sol. Let ABC be an obtuse-angled triangle with ∠ABC = 130°.



Each of ∠BAC and ∠BCA is an acute angle.

Let the measures of these two acute angles be $2x$ and $2y$ respectively, so that $\angle BAC = 2x < 90°$ and $\angle BCA = 2y < 90°$.

Let OA and OC be the bisector of ∠BAC and ∠BCA respectively. Let them meet at O.

To prove that ΔOAC is an obtuse-angled triangle and to find the measure of ∠AOC.

In ΔABC,

$\angle ABC + \angle BCA + \angle BAC = 180°$

[Angle sum property of a triangle]

$\Rightarrow \quad 130° + 2y + 2x = 180°$

$\Rightarrow \quad x + y = \dfrac{180° - 130°}{2} = 25° \quad \ldots(1)$

Now, in ΔAOC, we have

$\angle AOC + \angle OAC + \angle OCA = 180°$

[Angle sum property of a triangle]

$\Rightarrow \quad \angle AOC + x + y = 180°$

[∵ AO and CO are the bisectors of ∠BAC and ∠BCA respectively]

$\Rightarrow \quad \angle AOC + 25° = 180°$ [From (1)]

$\Rightarrow \quad \angle AOC = 180° - 25°$

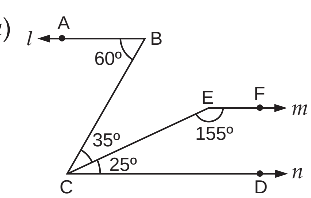
$\Rightarrow \quad \angle AOC = 155°$

which is an obtuse angle.

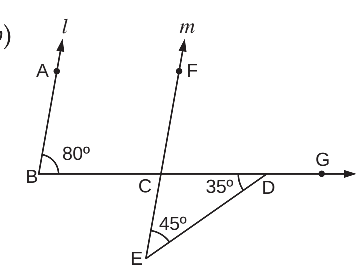
Hence ΔOAC is an obtuse-angled triangle and measure of an obtuse angle is 155°.

14. In the given figures, show that $l \parallel m$.

(*a*)



(*b*)



[CBSE SP 2013]

Sol. (*a*) We have

$$\angle BCD = \angle BCE + \angle ECD = 35° + 25° = 60° = \angle ABC$$

But these two are alternate angles with respect to the pair of lines AB and CD and their transversal BC.

$\therefore$ AB $\parallel$ CD, i.e. $l \parallel n$ ...(1)

Again,

$\angle ECD + \angle CEF = 25° + 155° = 180°$

$\because$ The sum of two interior angles is 180°,

$\therefore$ EF $\parallel$ CD $\Rightarrow$ $m \parallel n$ ...(2)

$\therefore$ From (1) and (2), we see that $l \parallel m$.

(*b*) In ΔECD, we have

$\angle CED + \angle EDC + \angle ECD = 180°$

[Angle sum property of a triangle]

$\Rightarrow$ $45° + 35° + \angle ECD = 180°$

$\Rightarrow$ $\angle ECD = 180° - 80° = 100°$

$\therefore$ $\angle FCD = \angle ECF - \angle ECD = 180° - 100° = 80° = \angle ABC$

$\therefore$ $\angle FCD = \angle ABC$

But these two angles are corresponding angles with respect to the pair of rays BA and CF and the transversal BCD.

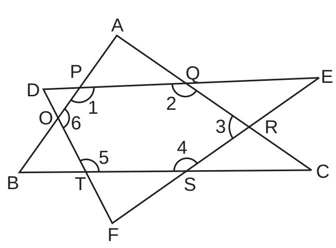
Hence, AB $\parallel$ CF, i.e $l \parallel m$.

Hence, proved.

Higher Order Thinking Skills (HOTS) Questions

(Page 79)

1. In the given figure,



find $\angle OPQ + \angle PQR + \angle QRS + \angle RST + \angle STO + \angle TOP$.

Sol. Given that ABC and DEF are two triangles. The pair of sides of these two triangles intersect at P, Q, R, S, T and O forming angles $\angle 1, \angle 2, \angle 3, \angle 4, \angle 5$ and $\angle 6$ as shown in the figure.

To find the measure of the sum of all these angles.

We have $\angle APQ = \angle APB - \angle BPQ = 180° - \angle 1$ ...(1)

Similarly, $\angle AQP = 180° - \angle 2$ ...(2)

$\therefore$ In ΔAPQ, we have

$\angle A = 180° - \angle APQ - \angle AQP$

[Angle sum property of a triangle]

$= 180° - 180° + \angle 1 - 180° + \angle 2$

[From (1) and (2)]

$\Rightarrow$ $\angle A = \angle 1 + \angle 2 - 180°$ ...(3)

Similarly,

$\angle E = \angle DEF = \angle 2 + \angle 3 - 180°$ ...(4)

$\angle C = \angle ACB = \angle 3 + \angle 4 - 180°$ ...(5)

$\angle F = \angle EFD = \angle 4 + \angle 5 - 180°$ ...(6)

$\angle B = \angle CBA = \angle 5 + \angle 6 - 180°$ ...(7)

$\angle D = \angle FDE = \angle 1 + \angle 6 - 180°$ ...(8)

Adding (3) to (8), we get

$(\angle A + \angle B + \angle C) + (\angle D + \angle E + \angle F)$

$= 2(\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6) - 1080°$

$\Rightarrow$ $180° + 180°$

$= 2(\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6) - 1080°$

[Angle sum property of a triangle]

$\Rightarrow$ $2(\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6)$

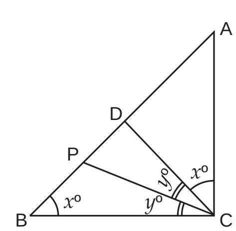
$= 1080° + 360°$

$= 1440°$

$\therefore$ $\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 = \frac{1440°}{2} = 720°$,

which is the required measure of the sum of the angles.

2. In the figure given below, $\angle ACD = \angle ABC = x°$ and CP bisects $\angle BCD$. Prove that $\angle APC = \angle ACP$.



Sol. Given that $\angle ACD = \angle ABC = x°$ and CP bisects $\angle BCD$

$\therefore$ $\angle BCP = \angle DCP = y°$, say

To prove that

$$\angle APC = \angle ACP$$

In ΔBPC,

$\angle APC$ is an exterior angle.

$\therefore \quad \angle APC = \angle PBC + \angle PCB$

[Property of exterior angle of a triangle]

$= x° + y° \quad \ldots(1)$

Also, $\quad \angle ACP = \angle ACD + \angle PCD$

$= x° + y° \quad \ldots(2)$

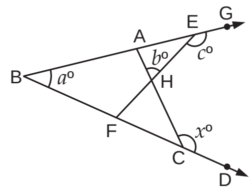
From (1) and (2), we have

$$\angle APC = \angle ACP$$

Hence, proved.

3. In the figure given below,

show that $x° = 180° - c° + a° + b°$



Sol. In ΔABC,

$\angle ACD$ is an exterior angle.

$\therefore \quad \angle ACD = \angle ABC + \angle BAC$

[Property of exterior angle of a triangle]

$\Rightarrow \quad x° = a° + \angle BAC \quad \ldots(1)$

Now, $\quad \angle BAC = 180° - \angle HAE \quad \ldots(2)$

Now, in ΔHAE,

$\angle HEG$ is an exterior angle.

$\therefore \quad \angle HEG = \angle HAE + \angle AHE$

[Property of exterior angle of a triangle]

$\Rightarrow \quad c° = \angle HAE + b°$

$\Rightarrow \quad \angle HAE = c° - b° \quad \ldots(3)$

$\therefore$ From (2) and (3), we have

$\angle BAC = 180° - (c° - b°)$

$= 180° + b° - c° \quad \ldots(4)$

$\therefore$ From (1) and (4), we have

$x° = a° + 180° + b° - c°$

$= 180° - c° + a° + b°$

Hence, proved.

Self-Assessment

(Page 80)

Multiple-Choice Questions

1. The supplement of an angle is equal to three times its complement. Then the measure of the angle is

(*a*) 60° (*b*) 50°

(*c*) 45° (*d*) 30°

Sol. (*c*) 45°

Let the measure of the angle be x in degrees.

$\therefore$ According to the problem, we have

$180° - x = 3(90° - x)$

$= 270° - 3x$

$\Rightarrow \quad 3x - x = 270° - 180°$

$\Rightarrow \quad 2x = 90°$

$\Rightarrow \quad x = 45°$

2. If an exterior angle of a triangle is acute, then the triangle must be

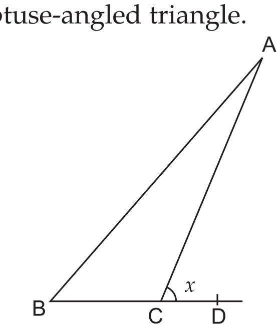
(*a*) a right-angled triangle

(*b*) an obtuse-angled triangle

(*c*) an acute-angled triangle

(*d*) an equilateral triangle

Sol. (*b*) an obtuse-angled triangle.



Let x be the measure of the exterior $\angle ACD$ of a ΔABC, where $x < 90°$.

$\therefore \quad \angle ACB = 180° - \angle ACD$

$= 180° - x \quad \ldots(1)$

Now, since $\quad x < 90°$

$\therefore \quad -x > -90°$

$\Rightarrow \quad 180° - x > 180° - 90° = 90°$

$\Rightarrow \quad \angle ACB > 90° \quad$ [From (1)]

$\therefore$ ΔACB is an obtuse-angled triangle.

Fill in the Blanks

3. If two angles of a triangle are complementary, then it is **a right** triangle.

4. The measure of an angle which is four times its complement is **72°**.

Sol. Angle = 4 (Its complement)

$\Rightarrow \quad x = 4\,(90° - x)$

$\Rightarrow \quad 5x = 360°$

$\Rightarrow \quad x = 72°$

5. If two complementary angles are in the ratio 2 : 3, then the angles are **36°** and **54°**.

Sol. $2x + 3x = 90°$

$\Rightarrow$ $5x = 90°$

$\Rightarrow$ $x = 18°$

Then, $2x = 2 \times 18° = 36°$

and $3x = 3 \times 18° = 54°$

6. The measure of an angle which is 24° more than its complement is **57°**.

Sol. Angle = Its complement + 24°

$x = (90° - x) + 24°$

$\Rightarrow$ $2x = 114°$

$\Rightarrow$ $x = 57°$

Assertion-Reason Type Questions

Directions (Q. Nos. 7 to 10): Each of these questions contains an assertion followed by reason. Read them carefully, and answer the question on the basis of the following options, select the one that best describes the two statements.

(a) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.

(b) Both Assertion and Reason are true but Reason is not the correct explanation of Assertion.

(c) Assertion is true but Reason is false.

(d) Assertion is false but Reason is true.

7. **Assertion:** Three collinear points will always form a triangle.

Reason: A triangle needs at least 3 non-collinear points.

Sol. (*d*) Assertion is incorrect but reason is correct.

A triangle needs at least 3 non-collinear points.

8. **Assertion:** Two intersecting lines have only one point in common.

Reason: Intersecting lines have two common points.

Sol. (*c*) Assertion is correct but reason is incorrect as two intersecting lines have only one point in common.

9. **Assertion:** Three lines are concurrent if they form a triangle.

Reason: Concurrent lines have only one common point.

Sol. (*d*) Assertion is incorrect but reason is correct. Concurrent lines have only one point in common so they can never form a triangle.

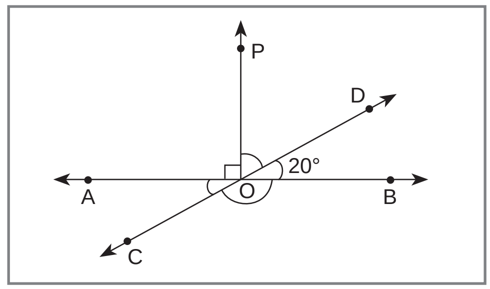
10. **Assertion:** Two adjacent angles have a common vertex.

Reason: Adjacent angles originate from the same vertex and have one common arm.

Sol. (*a*) Assertion and reason are correct and reason is correct explanation of assertion as adjacent angles have a common vertex and one common arm.

Case Study Based Questions

11. To judge the preparation of students of class IX on the topic 'Lines and angles', Mathematics teacher draws the figure on the whiteboard such that lines AB and CD intersect at O. If PO ⊥ AB and ∠DOB = 20°, then answer the following questions.



(*a*) What is the measure of ∠AOC?

(*i*) 20° (*ii*) 60°

(*iii*) 70° (*iv*) 30°

Ans. (*i*) 20°

(*b*) What is the measure of ∠AOP?

(*i*) 45° (*ii*) 50°

(*iii*) 90° (*iv*) 100°

Ans. (*iii*) 90°

(*c*) What is the measure of ∠POD?

(*i*) 50° (*ii*) 60°

(*iii*) 70° (*iv*) 80°

Ans. (*iii*) 70°

(*d*) What is the measure of ∠COB?

(*i*) 140° (*ii*) 160°

(*iii*) 180° (*iv*) 200°

Ans. (*ii*) 160°

(*e*) Two angles whose sum is 90° are called

(*i*) Right angle.

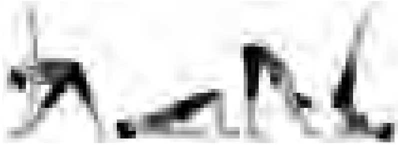
(*ii*) Consecutive interior angles.

(*iii*) Complementary angles.

(*iv*) Supplementary angles.

Ans. (*iii*) Complementary angles.

12. Our Maths teacher knows yoga. She asked Nitya to show yoga postures. She made marks on angles. She showed acute, obtuse and right angles through yoga postures.



Based on the above information, answer the following questions.

(*a*) The exterior angle of a triangle is equal to the

(*i*) sum of the two interior opposite angles.

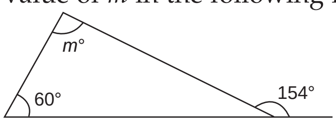
(*ii*) sum of the three interior angles.

(*iii*) difference of two interior angles.

(*iv*) opposite of the interior angle.

Ans. (*i*) sum of the two interior opposite angles.

(*b*) The value of *m* in the following figure is

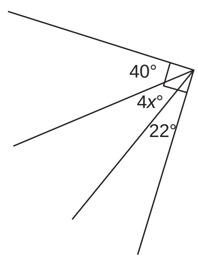


(*i*) 94° (*ii*) 214°

(*iii*) 84° (*iv*) 26°

Ans. (*i*) 94°

(*c*) The value of *x* in the following figure is

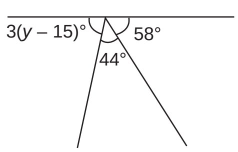


(*i*) 7° (*ii*) 9°

(*iii*) 6° (*iv*) 4°

Ans. (*i*) 7°

(*d*) The value of *y* in the following figure is

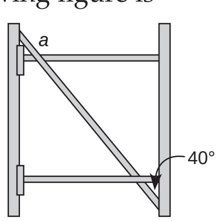


(*i*) 123° (*ii*) 41°

(*iii*) 45° (*iv*) 43°

Ans. (*ii*) 41°

(*e*) Two gates consist of vertical posts, horizontal struts and diagonal beams. The angle *a* from the following figure is



(*i*) 140° (*ii*) 40°

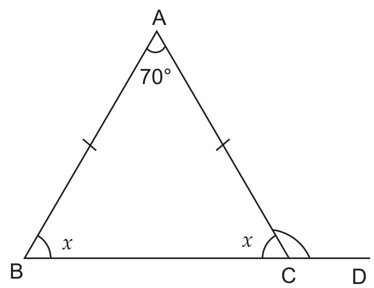
(*iii*) 120° (*iv*) 50°

Ans. (*i*) 140°

Very Short Answer Type Questions

13. If the vertex angle of an isosceles triangle is 70°, then what is the measure of the exterior angle to one of the base angles of this triangle?

Sol. Let ΔABC be an isosceles triangle with vertical angle ∠BAC = 70°.



To find the measure of the exterior angle ACD.

$\because \quad AB = AC$

$\therefore \quad \angle ABC = \angle ACB = x$, say

$\therefore \quad x + x + 70° = 180°$

[Angle sum property of a triangle]

$\Rightarrow \quad 2x = 180° - 70° = 110°$

$\Rightarrow \quad x = \dfrac{110°}{2} = 55°$

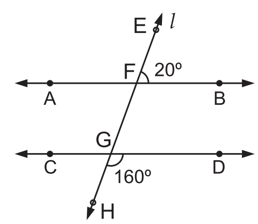
$\therefore \quad \angle ACD = 180° - x$

$= 180° - 55°$

$= 125°$

∴ The measure of exterior ∠ACD is 125°.

14. In the given figure, show that AB ∥ CD.



Sol. In the given figure, it is given that ∠EFB = 20° and ∠HGD = 160°.

To prove that AB ∥ CD.

We have $\angle FGD = 180° - \angle HGD$

$= 180° - 160°$

$= 20° = \angle EFB$

$\therefore \quad \angle FGD = \angle EFB$

But these two angles are corresponding angles with respect to the pair of lines AB and CD and the transversal l.

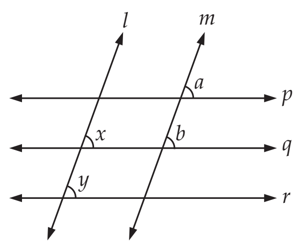
Since, these corresponding angles are equal.

$\therefore$ AB $\parallel$ CD

Hence, proved.

Short Answer Type-I Questions

15. In the given figure, if $\angle a = \angle b$ and $\angle x = \angle y$, prove that $p \parallel r$.



Sol. Given that x and y, the corresponding angles with respect to the pair of lines q and r are equal. Also, a and b, the corresponding angles with respect to the pair of lines p and q are equal.

To prove that $p \parallel r$.

$\because$ The corresponding angles with respect to q and r are equal.

$\therefore \quad q \parallel r \quad$ ...(1)

Again, the corresponding angles with respect to p and q are equal.

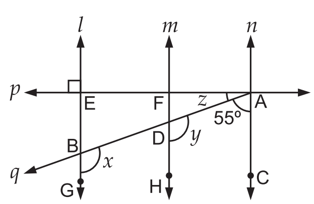
$\therefore \quad p \parallel q \quad$ ...(2)

From (1) and (2), we have

$$p \parallel r$$

Hence, proved.

16. In the given figure, $l \parallel m \parallel n$. If $p \perp l$, $\angle BAC = 55°$, find the values of x, y and z. **[CBSE SP 2012]**



Sol. Given that $l \parallel m \parallel n$, $p \perp l$ and $\angle BAC = 55°$

Also, $\angle FAD = z$, $\angle ADH = y$ and $\angle DBG = x$

To find the values of x, y and z.

$\because p \perp l$

$\therefore p \perp m$ and $p \perp n$

$\therefore \quad \angle FAC = 90°$

$\therefore \quad z = \angle FAD$

$= \angle FAC - \angle DAC$

$= 90° - 55°$

$= 35°$

$\angle HDA + \angle DAC = 180° \quad [\because m \parallel n]$

$\Rightarrow \quad y + 55° = 180°$

$\Rightarrow \quad y = 180° - 55°$

$= 125°$

Also, $\quad \angle DBG = \angle ADH$

[Corresponding $\angle$s, $l \parallel m$]

$\Rightarrow \quad x = y = 125°$

Hence, the required values of x, y, and z are 125°, 125° and 35° respectively.

Short Answer Type-II Questions

17. In ΔPQR, $\angle P + \angle Q = 120°$, $\angle Q + \angle R = 140°$. Find $\angle P$, $\angle Q$ and $\angle R$.

Sol. Given that PQR is a triangle, where

$\angle P + \angle Q = 120° \quad$ ...(1)

and $\quad \angle Q + \angle R = 140° \quad$ ...(2)

To find $\angle P$, $\angle Q$ and $\angle R$.

We have

$\angle P + \angle Q + \angle R = 180° \quad$ ...(3)

[Angle sum property of a triangle]

Subtracting (1) from (3), we get

$\angle R = 180° - 120° = 60° \quad$ ...(4)

Subtracting (2) from (3), we get

$\angle P = 180° - 140° = 40° \quad$ ...(5)

Adding (4) and (5), we get

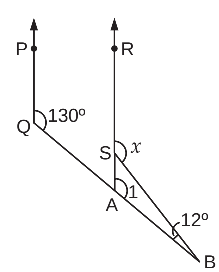
$\angle R + \angle P = 60° + 40° = 100° \quad$ ...(6)

Subtracting (6) from (3), we get

$\angle Q = 180° - 100° = 80°$

Hence, the required values of $\angle P$, $\angle Q$ and $\angle R$ are 40°, 80° and 60° respectively.

18. In the given figure, PQ $\parallel$ RS. Find the values of x if RSA is the straight line.



Sol. Given that PQ ∥ RS and RSA is a straight line, where A is a point on QB.

Also, ∠ABS = 12°, ∠PQA = 130°, ∠RSB = x and ∠SAB = ∠1

To find x.

We have PQ ∥ RA

∴ ∠RAB = ∠PQA

[Corresponding ∠s, PQ ∥ RA]

⇒ ∠1 = 130° …(1)

Again, ∠RSB is an exterior angle of ΔABS.

∴ ∠RSB = ∠SAB + ∠ABS

[Property of exterior angle of a triangle]

⇒ x = ∠1 + 12°

= 130° + 12° [From (1)]

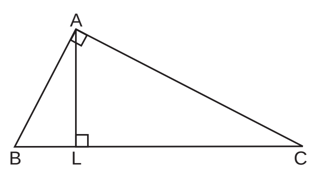
= 142°

Hence, the required value of x is 142°.

Long Answer Type Questions

19. In ΔABC, ∠A = 90°. AL is drawn perpendicular to the hypotenuse BC. Prove that ∠BAL = ∠ACB.

[CBSE SP 2010]



Sol. Given that ABC is a triangle with ∠A = 90° and L is a point on the hypotenuse BC such that ∠ALC = 90°.

To prove that ∠BAL = ∠ACB.

In ΔBAL,

∠BAL + ∠ABL = 90° [∵ ∠BLA = 90°]

∴ ∠BAL = 90° – ∠ABC …(1)

[∵ ∠ABL = ∠ABC]

Now, in ΔABC,

∠ABC + ∠ACB = 90° [∵ ∠BAC = 90°]

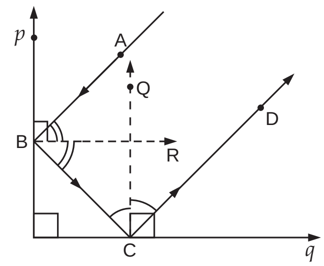
∴ ∠ACB = 90° – ∠ABC …(2)

∴ From (1) and (2), we have

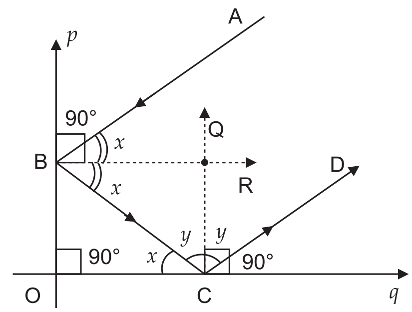
∠BAL = ∠ACB

Hence, proved.

20. In the given figure, p and q are two plane mirrors perpendicular to each other. An incident ray AB strikes the mirror p at B, gets reflected along BC and then strikes the mirror q at C and finally gets reflected along CD. Prove that AB ∥ CD.



Sol. Given that two mirrors p and q along OB and OC respectively are perpendicular to each other and intersect each other at O.



An incident ray AB strikes the mirror p at B and gets reflected along BC and then strikes the mirror q at C.

Finally, this ray gets reflected along CD.

Also, BR is drawn perpendicular to OB and CQ is drawn perpendicular to OC.

To prove that AB ∥ CD.

We know that angle of incidence ∠ABR = angle of reflection ∠CBR = x, say

and similarly, ∠BCQ = ∠DCQ = y, say.

∵ ∠RBO + ∠QCO = 90° + 90° = 180°

∴ BR ∥ OC

Similarly, CQ ∥ OB

∴ ∠CBR = ∠OCB

[Alternate ∠s, BR ∥ OC]

⇒ x = ∠OCB

But $x + y = 90°$ …(1)

∴ ∠ABC + ∠BCD = $2x + 2y$

= $2(x + y)$

= 2 × 90° = 180° [From (1)]

∴ Sum of the interior angles with respect to the pair of lines BA and CD is 180°.

∴ AB ∥ CD

Hence, proved.

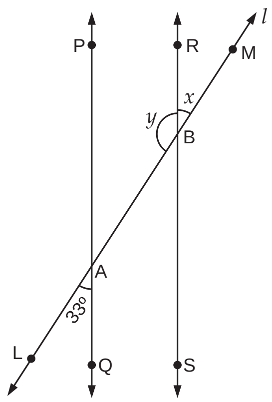
Let's Compete

(Page 82)

Multiple-Choice Questions

1. In the given figure, PQ ∥ RS. A transversal l intersects PQ and RS at the points A and B respectively such that $\angle RBM = x$ and $\angle RBA = y$. Then the values of x and y are respectively

(*a*) 23° and 157°
(*b*) 33° and 147°
(*c*) 33° and 157°
(*d*) 23° and 147°



Sol. (*b*) 33° and 147°

Given that PQ and RS are two parallel lines and the transversal l intersect these two lines at the points A and B respectively such that $\angle RBM = x$ and $\angle RBA = y$.

To find the values of x and y, we have

$\angle ABS = \angle LAQ$ [Corresponding ∠s, PQ ∥ RS]

⇒ $\angle ABS = 33°$

$\angle ABS = \angle RBM = 33°$ [Vertically opposite ∠s]

⇒ $x = 33°$

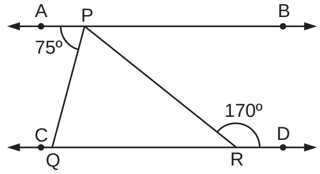
∴ $y = 180° - x$

$= 180° - 33° = 147°$

∴ The values of x and y are 33° and 147° respectively.

2. In the given figure, AB ∥ CD. If $\angle APQ = 75°$ and $\angle PRD = 170°$, then $\angle QPR$ is equal to

(*a*) 100°
(*b*) 75°
(*c*) 85°
(*d*) 95°



Sol. (*d*) 95°

Given that AB and CD are two parallel lines. $\angle APQ = 75°$ and $\angle PRD = 170°$.

To find $\angle QPR$, we have

$\angle PQR = \angle APQ$ [Alternate ∠s, AB ∥ CD]

⇒ $\angle PQR = 75°$

Also, $\angle PRQ = 180° - \angle PRD$

⇒ $\angle PQR = 180° - 170°$

⇒ $\angle PQR = 10°$

∴ In ΔPQR,

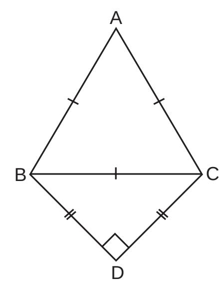
$\angle QPR = 180° - \angle PQR - \angle PRQ$ [Angle sum property of a triangle]

⇒ $\angle QPR = 180° - 75° - 10°$

$= 95°$

∴ The measure of $\angle QPR$ is 95°.

3. In the given figure, ABC is an equilateral triangle and BDC is an isosceles triangle, right-angled at D.



Then $\angle ABD$ is equal to

(*a*) 90° (*b*) 95°
(*c*) 105° (*d*) 100° **[CBSE SP 2011]**

Sol. (*c*) 105°

Given that ΔABC is an equilateral triangle and ΔBDC is an isosceles right-angled triangle with BD = DC and $\angle BDC = 90°$. To find $\angle ABD$, we have

∵ ΔABC is an equilateral triangle,

∴ $\angle ABC = 60°$ …(1)

Also, in ΔBDC,

BD = DC and $\angle BDC = 90°$

∴ $\angle DBC = \angle DCB = 45°$ …(2)

Adding (1) and (2), we get

$\angle ABC + \angle DBC = 60° + 45°$

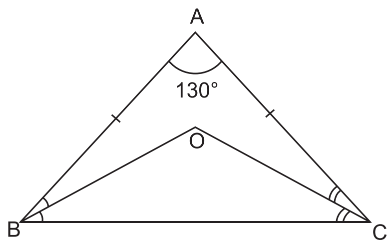
⇒ $\angle ABD = 105°$

4. If one angle of an isosceles triangle is 130°, then the angle between the bisectors of the other two angles is

(*a*) 155°   (*b*) 90°
(*c*) 150°   (*d*) 145°

Sol. (*a*) 155°

Let ABC be an isosceles triangle with AB = AC and ∠BAC = 130°. (Note that each base angle of an isosceles triangle cannot be obtuse, for, in that case, the sum of only two base angles will be more than 180°, which is absurd). Let BO and CO be the bisectors of ∠ABC and ∠ACB respectively, meeting at a point O.



To find the measure of ∠BOC.

Let the measure of ∠BOC be x in degrees.

∵ $\angle ABC = \angle ACB$

∴ $\frac{1}{2}\angle ABC = \frac{1}{2}\angle ACB$

⇒ $\angle OBC = \angle OCB = y$ in degrees, say

Then $2y + x = 180°$

[Angle sum property of a triangle]

⇒ $2y = 180° - x$ …(1)

∴ $\angle ABC = 2y = \angle ACB$

∴ From (1),

$\angle ABC = \angle ACB = 2y = 180° - x$

Now, in ΔABC, we have

$\angle ABC + \angle ACB + \angle BAC = 180°$

[Angle sum property of a triangle]

⇒ $2y + 2y + 130° = 180°$

⇒ $2(180° - x) + 130° = 180°$

⇒ $360° - 2x + 130° = 180°$

⇒ $2x = 360° + 130° - 180°$

$= 310°$

⇒ $x = \frac{310°}{2} = 155°$

∴ The measure of ∠BOC is 155°.

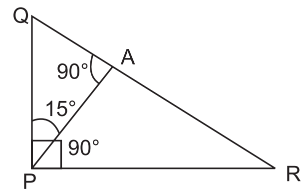
5. ΔPQR is a right-angled triangle where ∠QPR = 90°. PA is drawn perpendicular to the hypotenuse QR. If ∠QPA = 15°, then ∠PRQ is equal to

(*a*) 65°   (*b*) 25°
(*c*) 75°   (*d*) 15°

Sol. (*d*) 15°

Given that in ΔPQR, ∠QPR = 90°.

A is a point on the hypotenuse QR such that PA ⊥ QR, ∠QPA = 15°.



To find the measure of ∠PRQ, we have

In ΔPQA,

$\angle QPA + \angle PAQ + \angle PQA = 180°$

[Angle sum property of a triangle]

⇒ $15° + 90° + \angle PQA = 180°$

⇒ $\angle PQA = 180° - 105°$

$\angle PQA = \angle PQR$

⇒ $\angle PQR = 75°$ …(1)

Now, in ΔPQR,

$\angle PQR + \angle QPR + \angle PRQ = 180°$

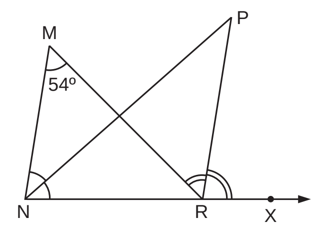
[Angle sum property of a triangle]

⇒ $75° + 90° + \angle PRQ = 180°$ [From (1)]

⇒ $165° + \angle PRQ = 180°$

⇒ $\angle PRQ = 180° - 165° = 15°$

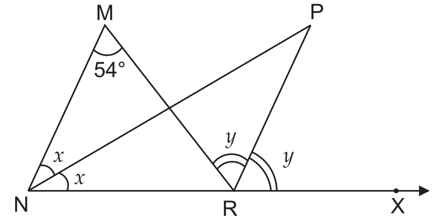
6. The side NR of ΔMNR is produced to X. The bisectors of ∠MNR and ∠MRX meet at a point P. If ∠NMR = 54°, then the measure of ∠NPR is



(*a*) 36°   (*b*) 25°
(*c*) 27°   (*d*) 30°

Sol. (*c*) 27°

Given that MNR is a triangle with ∠NMR = 54°, the side NR is produced is X. NP and RP, the bisectors of ∠MNR and ∠MRX respectively, meets at P.



To find the measure of ∠NPR.

Let the measure of ∠MNP (= ∠PNR) and ∠MRP (= ∠PRX) be x and y respectively. Now, in ΔMNR, ∠MRX is an exterior angle.

∴ ∠MRX = ∠NMR + ∠MNR

[Property of exterior angle of a triangle]

⇒ $2y = 54° + 2x$

$y - x = 27°$ …(1)

Now, ∠PRX is an exterior angle of ΔPNR.

∴ ∠PRX = ∠PNR + ∠NPR

[Property of exterior angle of triangle]

⇒ $y = x$ + ∠NPR

∠NPR = $y - x = 27°$ [From (1)]

∴ The measure of ∠NPR is 27°.

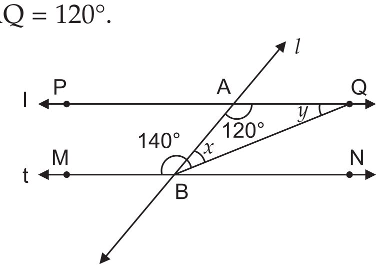
7. Lines PQ and MN are parallel to each other. A transversal l intersects PQ and MN at the points A and B respectively such that ∠BAQ = 120°. If ∠MBQ = 140°, ∠ABQ = x and ∠AQB = y, then the values of x and y are respectively

(*a*) 30° and 50° (*b*) 20° and 40°

(*c*) 40° and 20° (*d*) 50° and 30°

Sol. (*b*) 20° and 40°

Given that PQ and MN are two parallel lines and l is a transversal which intersects PQ and MN at the points A and B respectively such that ∠BAQ = 120°.



Also, ∠MBQ = 140°

∠ABQ = x

and ∠AQB = y

To find the values of x and y, we have

∠MBA = ∠BAQ

[Alternate ∠s, PQ ∥ MN]

⇒ ∠MBQ – ∠ABQ = 120°

⇒ $140° - x = 120°$

⇒ $x = 140° - 120° = 20°$ …(1)

Now, in ΔABQ, we have

∠ABQ + ∠AQB + ∠BAQ = 180°

[Angle sum property of a triangle]

⇒ $x + y + 120° = 180°$

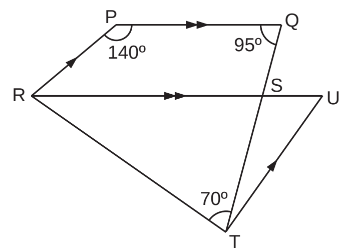
$y = 180° - 120° - x$

$= 60° - x$

$= 60° - 20° = 40°$ [From (1)]

Hence, the required values of x and y are 20° and 40° respectively.

8. In the given figure, PQTR is a quadrilateral in which ∠RTQ = 70°, ∠PQT = 95° and ∠RPQ = 140°. If RU ∥ PQ and TU ∥ RP and RU intersects QT at S, then the measure of ∠UTS is



(*a*) 55° (*b*) 60°

(*c*) 70° (*d*) 65°

Sol. (*a*) 55°

Given that PQTR is a quadrilateral in which ∠RTQ = 70°, ∠PQT = 90° and ∠RPQ = 140°.

Also, RU ∥ PQ, TU ∥ RP and RU intersects QT at S. To find the measure of ∠UTS, we have

∠QSU = ∠PQS

[Alternate ∠s, PQ ∥ RU]

= 95°

∴ ∠TSU = 180° – ∠QSU

= 180° – 95° = 85° …(1)

∠PRU = 180° – ∠RPQ

[Cointerior ∠s, PQ ∥ RU]

= 180° – 140° = 40°

∴ ∠RUT = ∠PRU

[Alternate ∠s, TU ∥ RP]

= 40°

∴ ∠SUT = 40° …(2)

Now, in ΔSTU, we have

∠UTS + ∠TSU + ∠SUT = 180°

[Angle sum property of a triangle]

⇒ ∠UTS + 85° + 40° = 180° [From (1) and (2)]

⇒ ∠UTS = 180° – 85° – 40°

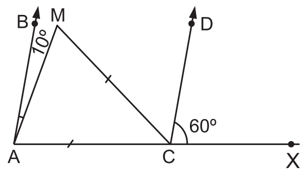
= 180° – 125°

= 55°

∴ The measure of ∠UTS is 55°.

9. In the given figure, AB ∥ CD and ∠MAB = 10°, CM = CA. Then ∠MCD is equal to

(*a*) 45° (*b*) 60°
(*c*) 50° (*d*) 40°



Sol. (*d*) 40°

Given that AB ∥ CD, ∠MAB = 10° and CM = CA.

AC is produced to X where X is a point on AC produced. To find the measure of ∠MCD.

Let ∠CAM = ∠CMA = y

and ∠MCD = x, both x and y in degrees

Then, since AB ∥ CD

∠BAC = ∠DCX

[Corresponding ∠s, AB ∥ CD]

⇒ ∠BAM + ∠CAM = 60°

⇒ $10° + y = 60°$

⇒ $y = 60° - 10°$

⇒ $y = 50°$ …(1)

Now, ∠MCX is an exterior angle of ΔMAC.

∴ ∠MCX = ∠CAM + ∠CMA

[Property of exterior angle of a triangle]

⇒ $x + 60° = 2y = 100°$ [From (1)]

∴ $x = 100° - 60° = 40°$

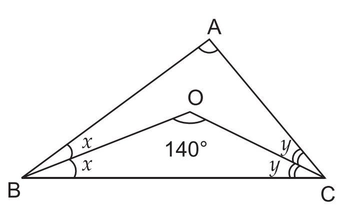
10. If the bisectors of the base angles of a triangle enclose an angle of 140°, then the triangle is

(*a*) an acute-angled triangle
(*b*) an obtuse-angled triangle
(*c*) an equilateral triangle
(*d*) a right-angled triangle

Sol. (*b*) an obtuse-angled triangle

Given that ABC is any triangle, BO and CO are the bisectors of ∠ABC and ∠BCA respectively, intersecting each other at O so that ∠BOC = 140°.

To find the measure of ∠BAC.



Let ∠OBC = ∠ABO = x and ∠OCB = ∠ACO = y, where both x and y are in degrees.

Now, in ΔOBC,

$x + y + 140° = 180°$

[Angle sum property of a triangle]

$x + y = 180° - 140° = 40°$ …(1)

In ΔABC, we have

∠ABC + ∠ACB + ∠BAC = 180°

⇒ $2x + 2y$ + ∠BAC = 180°

⇒ $2 \times 40°$ + ∠BAC = 180° [From (1)]

∠BAC = 180° – 80°

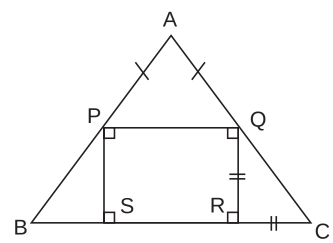
= 100° > 90°

∴ ΔABC is an obtuse-angled triangle.

Value-based Question (Optional)

(Page 83)

1. An old man is the owner of a big triangular plot of land ABC. He wants to build an old age home during his lifetime, in a rectangle PQRS, where P, Q, R and S are points on the sides AB, AC and BC of the triangle, as shown in the figure.



He also wants to build a nursery for plants in the triangular area PBS and a yoga centre in the triangular area QRC. He reserves the triangular area APQ for rainwater harvesting. He has also planted trees along AB and AC. It is given that AP = AQ and CR = RQ.

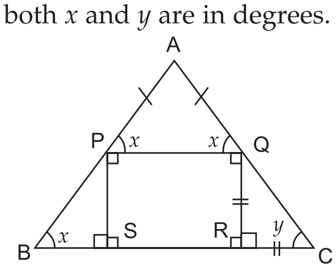
(*a*) Based on the angles of ΔABC, what type of triangle is it?

(*b*) Write the values exhibited by the old man.

Sol. (*a*) Let ∠APQ = ∠AQP = x

and ∠RQC = ∠RCQ = y

where both x and y are in degrees.



To find ∠BAC, we have

$\angle APQ + \angle BPS = 90°$ [$\because \angle QPS = 90°$]

$\Rightarrow \quad x + \angle BPS = 90°$

$\Rightarrow \quad \angle BPS = 90° - x$ …(1)

$\therefore \quad \angle PBS = 90° - \angle BPS$ [$\because \angle PSB = 90°$]

$= 90° - 90° + x$ [From (1)]

$= x$

$\therefore \quad \angle ABC = \angle PBS = x$ …(2)

Similarly, $\angle RQC = 90° - x$ [$\because \angle PQR = 90°$]

$\Rightarrow \quad \angle QCR = \angle RQC = y$ [$\because$ QR = RC]

$= 90° - x$

$\therefore \quad \angle ACB = y = 90° - x$

$\Rightarrow \quad x + y = 90°$ …(3)

Now, in ΔABC,

$\angle ABC + \angle ACB = x + y = 90°$ [From (3)]

$\therefore \quad \angle BAC = 180° - 90° = 90°$

$\therefore$ ΔABC is a right-angled triangle.

(*b*) Empathy, kindness, social responsibility and environmental protection.

7

Triangles

Checkpoint (Page 86)

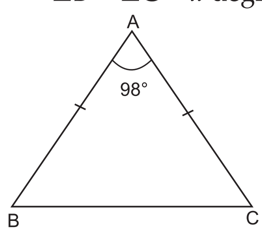
1. In ΔABC, $AB = AC$ and $\angle BAC = 98°$. Find the measure of $\angle ABC$ and $\angle ACB$.

Sol. In ΔABC, given that $AB = AC$ and $\angle BAC = 98°$.

To find the measure of $\angle ABC$ and $\angle ACB$.

$\because$ $AB = AC$

$\therefore$ $\angle B = \angle C = x$ degree (say)



In ΔABC, we have

$\angle A + \angle B + \angle C = 180°$

[Angle sum property of a triangle]

$\Rightarrow$ $98° + x + x = 180°$

$\Rightarrow$ $2x = 180° - 98° = 82°$

$\Rightarrow$ $x = \dfrac{82°}{2} = 41°$

$\therefore$ $\angle ABC = \angle ACB = 41°$ are the required measures.

2. If one of the angles $(30 - a)°$ and $(125 + 2a)°$ is the supplement of the other, then find the value of a.

Sol. According to the condition of the problem,

$(125 + 2a)° + (30 - a)° = 180°$

$\Rightarrow$ $125° + 2a + 30° - a = 180°$

$\Rightarrow$ $a = 180° - (30° + 125°)$

$= 180° - 155°$

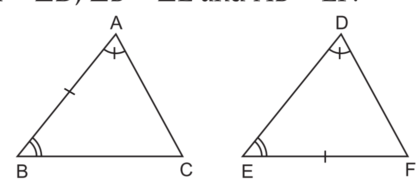
$= 25°$

which is the required value of a.

3. In triangles ABC and DEF, $\angle A = \angle D$, $\angle B = \angle E$ and $AB = EF$. Are the two triangles congruent? If yes, by which congruent criterion?

Sol. Given that in ΔABC and ΔDEF,

$\angle A = \angle D$, $\angle B = \angle E$ and $AB = EF$.



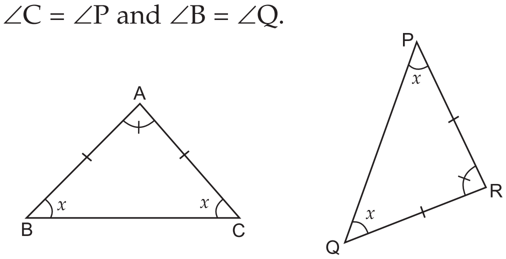
In this case the two triangles are not congruent for in the first triangle, the congruence criteria should be ASA and in the second triangle it is AAS.

4. In ΔABC and ΔPQR, $AB = AC$, $\angle C = \angle P$ and $\angle B = \angle Q$. Are the two triangles always

(*a*) congruent (*b*) isosceles?

Explain why?

Sol. Given that in ΔABC and ΔPQR, $AB = AC$, $\angle C = \angle P$ and $\angle B = \angle Q$.



(*a*) In this case, we see that ΔABC is not congruent to ΔPQR but $\Delta ABC \cong \Delta RQP$, since

$\angle A = \angle R$

$\angle B = \angle Q$ (Given)

$AB = RQ$

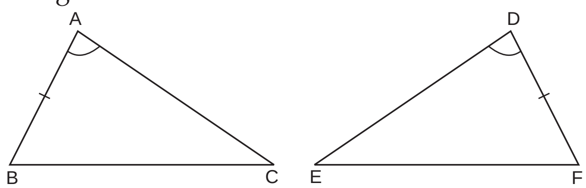
[Side opposite to equal angle, are equal]

By ASA, $\Delta ABC \cong \Delta RQP$.

(*b*) Yes; the two triangle are always isosceles. ΔABC is isosceles, since $AB = AC$, $\angle C = \angle B$ and ΔPQR is isosceles since $\angle P = \angle Q$.

5. In ΔABC and ΔDEF, $AB = DF$ and $\angle A = \angle D$. If the two triangles are congruent by SAS congruent axiom, then which additional condition is needed? Explain why?

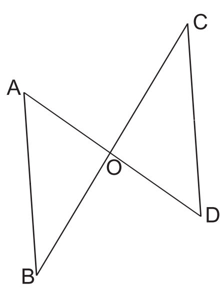
Sol. Given that in ΔABC and ΔDEF, $\angle A = \angle D$, $AB = DF$. Hence, if it is also given that $AC = DE$, then the two triangles will be congruent by SAS congruence criteria.



The required additional condition is $AC = DE$.

6. Line segments AD and BC intersect each other at a point O forming ΔAOB and ΔCOD on opposite sides of the point O such that $\angle B < \angle A$ and $\angle D > \angle C$. Which of the line segments AD and BC is greater and why?

Sol. Given that AD and BC are line segments which intersect each other at O and form ΔAOB and ΔCOD when we join AB and CD. It is also given that $\angle B < \angle A$ and $\angle D > \angle C$.



To determine whether $AD > BC$ or $AD < BC$.

In ΔAOB, since $\angle B < \angle A$

$\therefore$ side $OB >$ side OA

[By triangular inequality] …(1)

Again, in ΔCOD, since $\angle D > \angle C$

$\therefore$ side $OC >$ side OD

[By triangular inequality] …(2)

Adding (1) and (2), we get

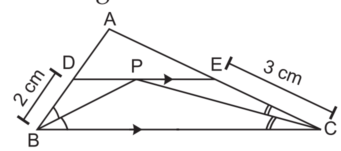
side OB + side OC > side OA + side OD

$\Rightarrow \quad BC > AD$.

$\therefore$ The line segment AD is greater than BC.

7. ABC is a triangle and BP and CP are bisectors of $\angle ABC$ and $\angle ACB$ respectively meeting at P within ΔABC. If D and E are points on AB and AC respectively such that the line segment DPE is parallel to BC and if $BD = 2$ cm and $CE = 3$ cm, find the length of DE.

Sol. Given that ABC is a triangle and BP and CP are bisectors of $\angle ABC$ and $\angle ACB$ respectively meeting at P within ΔABC. The line segment DPE is drawn parallel to BC, where D and E are points on sides AB and AC respectively. It is also given that length of $BD = 2$ cm and length of $CE = 3$ cm. To find the length of DE.



We have $\quad \angle PBC$ = alternate$\angle DPB$ [$\because DP \parallel BC$]

Also, $\quad \angle PBC = \angle PBD$

[$\because$ BP is the bisector of $\angle DBC$]

$\therefore \quad \angle DPB = \angle PBD$

$\therefore \quad DP = DB = 2$ cm …(1)

Similarly, $\quad \angle EPC = \angle PCB = \angle ECP$

$\therefore \quad EP = EC = 3$ cm

Adding (1) and (2), we get

$DE = DP + EP$

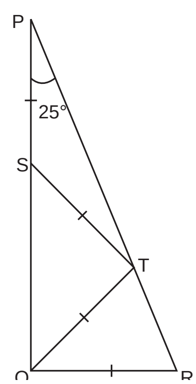
$= (2 + 3)$ cm

[From (1) and (2)]

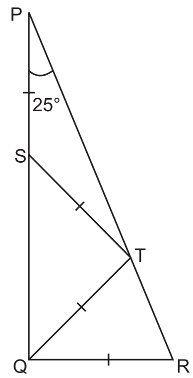
$= 5$ cm

$\therefore$ Required length of DE is 5 cm.

8. In the given figure, PQR is a triangle. S and T are points on PQ and PR respectively such that $PS = ST = TQ = QR$. If $\angle QPR = 25°$, what is the measure of $\angle PRQ$?



Sol. Given that PQR is a triangle. S and T are points on PQ and PR respectively such that $PS = ST = TQ = QR$ and $\angle QPR = 25°$. To find the measure of $\angle PRQ$.



We have $\angle STP = \angle SPT$ [$\because$ SP = ST]

$= 25°$ ...(1)

In ΔPST, ∠QST is an exterior angle.

$\therefore$ $\angle QST = \angle SPT + \angle STP$

[By property exterior angle of a triangle]

$= 25° + 25°$

$= 50°$ ...(1)

$\therefore$ $\angle SQT = \angle QST$ [$\because$ ST = QT]

$= 50°$ [From (1)]

$\therefore$ From ΔSTQ, by using angle sum property, we get

$\angle STQ + \angle QST + \angle SQT = 180°$

$\Rightarrow$ $\angle STQ + 50° + 50° = 180°$

$\therefore$ $\angle STQ = 180° - 100°$

$= 80°$ ...(2)

$\angle QTP = \angle STQ + \angle STP$

$= 80° + 25°$

[From (1) and (2)]

$= 105°$ ...(3)

$\therefore$ $\angle QTR = 180° - \angle QTP$

$= 180° - 105°$ [From (3)]

$= 75°$ ...(4)

$\therefore$ $\angle PRQ = \angle QRT = \angle QTR$

[$\because$ QT = QR]

$= 75°$ [From (4)]

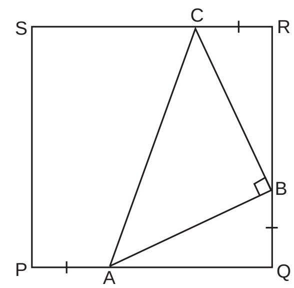
Hence, the required measure of ∠PRQ is 75°.

Milestone 1

(Page 88)

Multiple-Choice Questions

1. In the given figure, PQRS is a square. A, B and C are points on PQ, QR and RS respectively such that PA = QB = RC and ∠ABC = 90°.



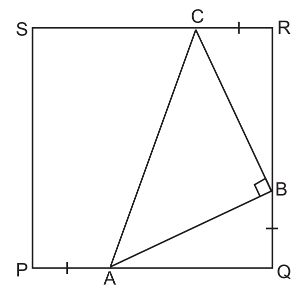
Then the measure of ∠CAB is

(*a*) 60° (*b*) 30°

(*c*) 45° (*d*) 75°

Sol. (*c*) 45°

Given that PQRS is a square, A, B, C are points on PQ, QR and RS respectively such that PA = QB = RC and ∠ABC = 90°.



To find the measure of ∠CAB.

In ΔBQA and ΔCRB, we have

BQ = CR (Given)

AQ = PQ – PA = RQ – QB

[$\because$ PQRS is a square, $\therefore$ PQ = RQ and also PA = QB (given)]

= BR.

$\angle AQB = \angle BRC = 90°$

[$\because$ PQRS is a square]

$\therefore$ By SAS congruence criterion, ΔBQA ≅ ΔCRB.

$\therefore$ BA = CB.

Now, in right angled ΔABC,

BA = CB

and $\angle ABC = 90°$

$\therefore$ Each of ∠BAC and ∠BCA = 45°

$\therefore$ $\angle CAB = 45°$

2. In triangles ABC and PQR, AB = PR and ∠A = ∠P. The two triangles will be congruent by SAS axiom if

(*a*) BC = PQ (*b*) AC = QR

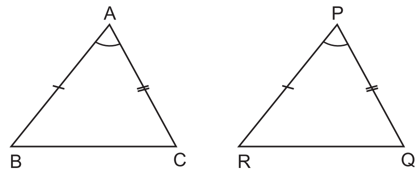
(*c*) BC = QR (*d*) AC = PQ

Sol. (*d*) AC = PQ

Given that in ΔABC and ΔPQR,

$$AB = PR$$

and $\angle A = \angle P$

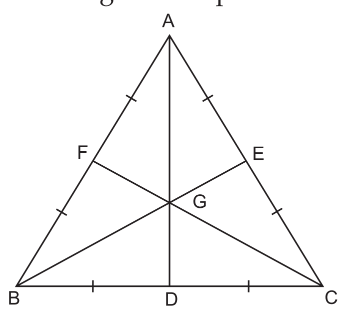


Also, given that ΔABC and ΔPRQ are congruent by SAS axiom. Hence, another condition viz. AC = PQ must be given to satisfy this axiom.

Very Short Answer Type Questions

3. Find a relation between the medians of an equilateral triangle. **[CBSE SP 2010, 2011]**

Sol. Given that ΔABC is an equilateral triangle and AD, BE and CF are its three medians, meeting inside the triangle at the point G.



To find a relation between the three medians AD, BE and CF.

In ΔBEC and ΔADB, we have

$$BC = AB,\ EC = \frac{1}{2} BC = DB$$

$$\angle BCE = \angle ABD = 60^\circ$$

[$\because$ ABC is an equilateral Δ]

$\therefore$ By SAS congruence criterion, $\Delta BEC \cong \Delta ADB$.

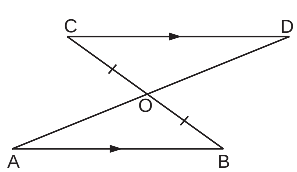
$\therefore$ $BE = AD$ [By CPCT] …(1)

Similarly, we can show that $\Delta CFA \cong \Delta ADB$.

$\therefore$ $CF = AD$ (By CPCT) …(2)

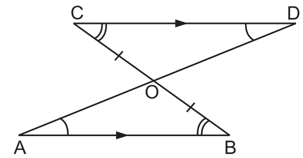
From (1) and (2), we see that BE = AD = CF, i.e. three medians are of equal length.

4. In the given figure, AB | | CD and O is the mid-point of BC. What is the ratio of AO and OD?



Sol. Given that AB ∥ CD and AD and BC intersect each other at O where O is the mid-point of BC.

To find the ratio of AO and OD.



In ΔABO and ΔDCO, we have

$$\angle ABO = \text{alternate } \angle DCO$$

[$\because$ AB ∥ CD]

$$\angle OAB = \text{alternate} \angle ODC$$

and $OB = OC$

$\therefore$ By AAS congruence criterion, we have

$$\Delta ABO \cong \Delta DCO$$

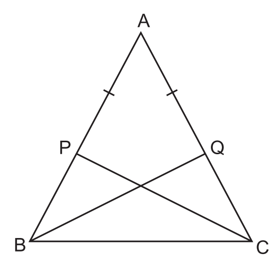
$\therefore$ $AO = DO$ [By CPCT]

AO : OD = 1 : 1 which is the required ratio.

Short Answer Type-I Questions

5. ABC is an isosceles triangle with AB = AC. P and Q are points on AB and AC respectively such that AP = AQ. Prove that $\angle ACP = \angle ABQ$ and CP = BQ.

Sol. Given that ABC is an isosceles triangle with AB = AC. P and Q are points on AB and AC respectively such that AP = AQ. To prove that $\angle ACP = \angle ABQ$ and CP = BQ.



In ΔAPC and ΔAQB, we have

AP = AQ, AC = AB and $\angle PAC = \angle QAB$

[Common angle]

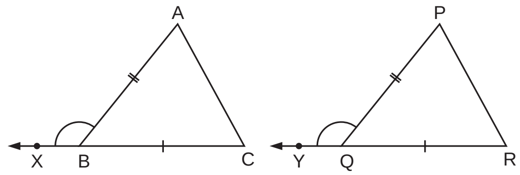
$\therefore$ By SAS, $\Delta APC \cong \Delta AQB$.

$\therefore$ $\angle ACP = \angle ABQ$

and $CP = BQ$ [By CPCT]

Hence, proved.

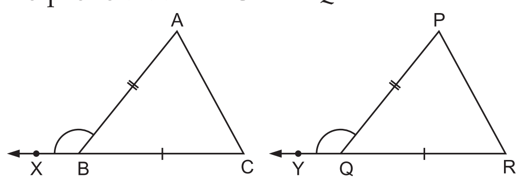
6. In ΔABC and ΔPQR, AB = PQ, BC = QR. CB and RQ are extended to X and Y respectively such that $\angle ABX = \angle PQY$.



Prove that $\Delta ABC \cong \Delta PQR$.

Sol. Given that in ΔABC and ΔPQR, $AB = PQ$, $BC = QR$ and $\angle ABX = \angle AQY$ where X and Y are points on CB produced and RQ produced respectively.

To prove that $\Delta ABC \cong \Delta PQR$.



In ABC and PQR, we have

$$AB = PQ$$
$$BC = QR$$

and $\angle ABC = 180° - \angle ABX$

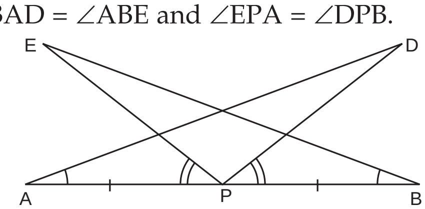
$$= 180° - \angle AQY$$
$$= \angle PQR.$$

Hence, by SAS congruence criterion, $\Delta ABC \cong \Delta PQR$.

Hence, proved.

Short Answer Type-II Questions

7. AB is a line segment and P is its mid-point. D and E are points on the same side of AB such that $\angle BAD = \angle ABE$ and $\angle EPA = \angle DPB$.

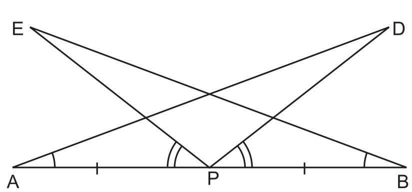


Show that

(*a*) $\Delta DAP \cong \Delta EBP$ and

(*b*) $AD = BE$ **[CBSE SP 2012]**

Sol. Given that AB is a line segment and P is its mid-point, so that $AP = BP$. D and E are points on the same side of AB such that $\angle BAD = \angle ABE$ and $\angle EPA = \angle DPB$.



To prove that (*a*) $\Delta DAP \cong \Delta EBP$ and (*b*) $AD = BE$.

(*a*) We have

$$\angle EPA = \angle DPB$$
$$\Rightarrow \quad \angle EPA + \angle EPD = \angle DPB + \angle EPD$$
$$\angle DPA = \angle EPB \qquad \ldots(1)$$

Now, in ΔDAP and ΔEBP, we have

$$\angle PAD = \angle PBE$$
$$\angle DPA = \angle EPB$$
$$AP = BP$$

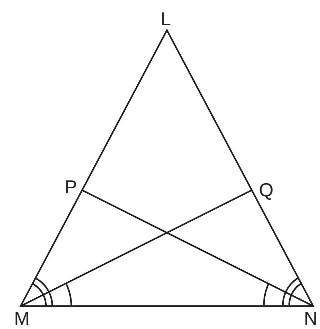
By ASA congruence criterion, we have

$$\Delta DAP \cong \Delta EBP$$

(*b*) Also, $AD = BE$ [By CPCT]

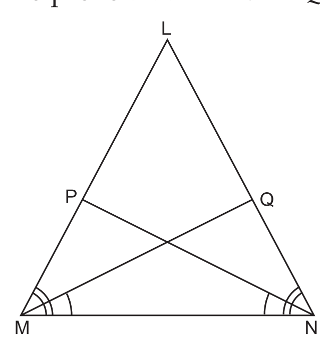
8. Two triangles PMN and QNM are on the same side of their common base MN such that $\angle QMN = \angle PNM$ and $\angle QNM = \angle PMN$.

Also, MP and NQ produced, if necessary, intersect each other at L. Prove that $\Delta PLN \cong \Delta QLM$.



Sol. Given that two triangles PMN and QNM are on the same side of their common base MN such that $\angle QMN = \angle PNM$ and $\angle QNM = \angle PMN$.

MP and NQ are produced to meet together at the point L. To prove that $\Delta PLN \cong \Delta QLM$.



In ΔPMN and ΔQNM, we have

$$\angle PMN \cong \angle QNM$$
$$\angle PNM = \angle QMN$$

and $MN = NM$ [Common side]

Hence, by ASA congruence criterion,

$\Delta PMN \cong \Delta QNM$.

$\Rightarrow$ $PN = QM$ [By CPCT] ...(1)

Again, in ΔPLN and ΔQLM we have

$\angle PLN = \angle QLM$ [Common angle]

$\angle PNL = \angle LNM - \angle PNM$ ...(2)

and $\angle QML = \angle LMN - \angle NMQ$

$= \angle LNM - \angle PNM$ ...(3)

[$\because$ $\angle PMN = \angle QNM$]

$\therefore$ From (2) and (3)

$\Delta PLN \approx \Delta QLM$ ...(4)

Also, we have,

$PN = QM$ [By (1)] ...(5)

From (2), (4) and (5), we have

$\Delta PLN \cong \Delta QLM$

[By ASA congruence criterion]

Hence, proved.

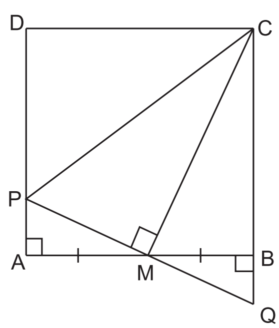
Long Answer Type Questions

9. ABCD is a square. M is the mid-point of AB and $PQ \perp CM$ meets AD at P and CB produced at Q where PQ passes through M. Prove that

(*a*) PA = BQ

(*b*) CP = AB + PA **[CBSE SP 2010]**

Sol. Given that ABCD is a square and M is the mid-point of AB. Also. $PQ \perp CM$ meets AD at P and CB produced at Q, where PQ passes through M. To prove that (*a*) PA = BQ and (*b*) CP = AB + PA.



In ΔPAM and ΔQBM, we have

$AM = BM$ [Given]

$\angle PAM = \angle QBM = 90°$

$\angle PMA = \angle QMB$

[$\because$ Vertically opposite angle]

$\therefore$ By ASA congruence criterion, we have

$\Delta PAM \cong \Delta QBM$

(*a*) $PA = QB$ [By CPCT]

and $PM = QM$ [By CPCT]

(*b*) In ΔCPM and ΔCQM, we have

$PM = QM$ [By (*a*)]

$\angle PMC = \angle QMC = 90°$

$MC = MC$

By SAS congruence criterion, we have

$\Delta CPM \cong \Delta CQM$

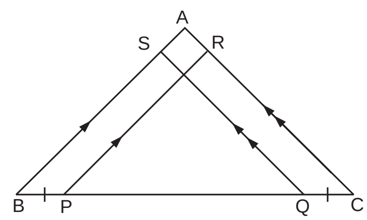
$\Rightarrow$ $CP = CQ$ [By CPCT]

$CB + QB = AB + PA$

[By (*a*) and since ABCD is a square]

Hence, proved.

10. In the given figure, BA || PR, CA || QS and BP = QC.



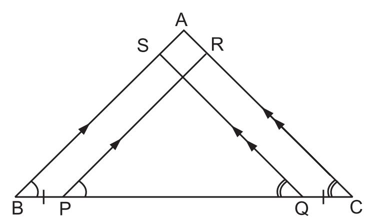
Prove that

(*a*) BS = PR

(*b*) QS = CR

Sol. Given that $BA \parallel PR$, $CA \parallel QS$ where P and Q are two points on the sides BC of ΔABC such that BP = QC.

To prove that (*a*) BS = PR and (*b*) QS = CR.



In ΔSBQ and ΔRPC, we have

$BQ = BP + PQ$

$= QC + PQ$

$= PC$

$\angle SBQ = \angle RPC$

[Corresponding angle, $BA \parallel PR$]

and $\angle SQB = \angle RCP$

[Corresponding angle, $CA \parallel QS$]

$\therefore$ By ASA congruence criterion, $\Delta SBQ \cong \Delta RPC$.

$\therefore$ (*a*) BS = PR and (*b*) QS = CR [By CPCT]

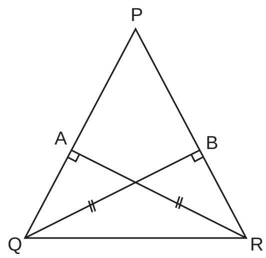
Hence, proved.

Milestone 2

(Page 91)

Multiple-Choice Questions

1. In ΔPQR, $RA \perp PQ$ and $QB \perp PR$ where A and B are two points on PQ and PR respectively. If BQ = AR and PQ = 7 cm, then the measure of PR is

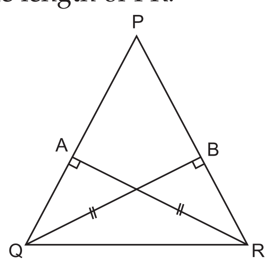


(*a*) 6 cm
(*b*) 5 cm
(*c*) 7 cm
(*d*) 8 cm

Sol. (*c*) 7 cm

Given that in ΔPQR, $RA \perp PQ$ and $QB \perp PR$, where A and B are two points on PQ and PR respectively. Also, BQ = AR and PQ = 7 cm.

To find the length of PR.



In ΔBQP and ΔARP, we have

$$BQ = AR$$

$$\angle PBQ = \angle PAR = 90°$$

$$\angle QPB = \angle RPA \qquad \text{[Common angle]}$$

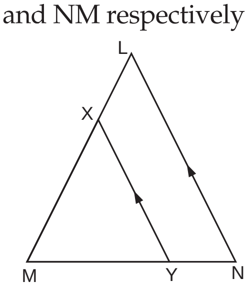
$\therefore$ By AAS congruence criterion, we have

$$\Delta BQP \cong \Delta ARP$$

$\Rightarrow \qquad PQ = PQ \qquad$ [By CPCT]

$\Rightarrow \qquad PR = PQ = 7$ cm

2. In ΔLMN, LM = LN and XM = 5.7 cm, where X and Y are two points on LM and NM respectively such that XY || LN. Then the length of XY is equal to

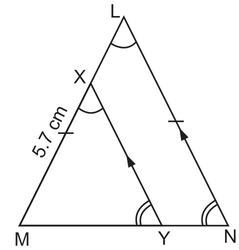


(*a*) 5.6 cm
(*b*) 5.7 cm
(*c*) 5.8 cm
(*d*) 5.9 cm

Sol. (*b*) 5.7 cm

Given that in ΔLMN, LM = LN and XM = 5.7 cm, where X and Y are two points on LM and NM respectively such that XY || LN.

To find the length of XY.



In ΔLMN, we have

$$LM = LN$$

$\therefore \qquad \angle LNM = \angle LMN$

But $\qquad \angle LNM = \angle XYM$

[$\because$ Corresponding angles, XY || LN]

$\therefore \qquad \angle XYM = \angle LNM = \angle LMN = \angle XMY$

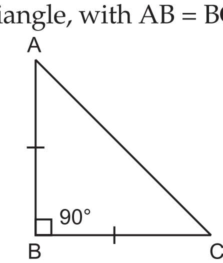
$\therefore \qquad XM = XY$

$\Rightarrow \qquad XY = 5.7$ cm

Very Short Answer Type Questions

3. Can an isosceles right-angled triangle have one obtuse angle? Explain why.

Sol. No, an isosceles right-angled triangle cannot have one obtuse angle. Let ΔABC be a right-angled isosceles triangle, with AB = BC and $\angle ABC = 90°$.



$\because \quad \angle A + \angle B + \angle C = 180°$

[Angle sum property of a triangle]

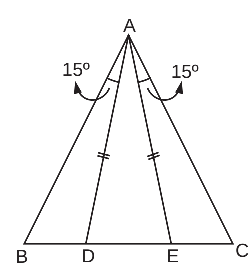
$\therefore \qquad \angle A + \angle C = 180° - \angle B$

$$= 180° - 90° = 90°$$

$\therefore$ Each of $\angle A$ and $\angle C$ will be less than 90°. Also $\angle B = 90°$.

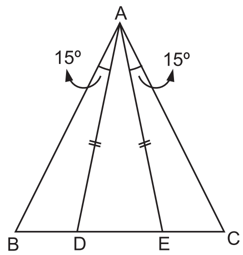
$\therefore$ No angle of ABC can be more than 90°, i.e. obtuse.

4. Within an equilateral triangle ABC, an isosceles triangle ADE is drawn, where D and E are points on BC such that AD = AE. If $\angle BAD = \angle CAE = 15°$, find the angles of the isosceles triangle.



Sol. Given that ΔABC is an equilateral triangle and, D and E are points on BC such that ΔADE is an isosceles triangle with AD = AE.

Also, ∠BAD = ∠CAE = 15°. To find, ∠DAE, ∠ADE and ∠AED.



We have $\angle DAE = \angle BAC - \angle BAD - \angle CAE$

$= 60° - 15° - 15°$

$= 30°$

[∵ ΔABC is an equilateral Δ]

Again, in ΔADE,

$AD = AE$

∴ $\angle ADE = \angle AED = x°$, say

∴ $x° + x° + \angle DAE = 180°$

[Angle sum property of a triangle]

⇒ $2x° + 30° = 180°$

⇒ $x° = \dfrac{150°}{2} = 75°.$

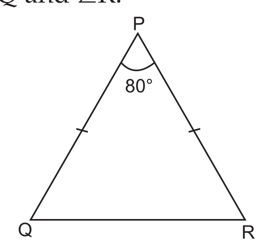
Hence, ∠ADE = ∠AED = 75° and ∠DAE = 30°.

Short Answer Type-I Questions

5. In ΔPQR, ∠P = 80°. If PQ = PR, find ∠Q and ∠R.

Sol. Given that in ΔPQR, PQ = PR and ∠QPR = 80°.

To find ∠Q and ∠R.



∵ $PQ = QR$

∴ $\angle Q = \angle R$

[Angles opposite to equal sides are equal]

Now, $\angle Q + \angle R + \angle P = 180°$

[Angle sum property of a triangle]

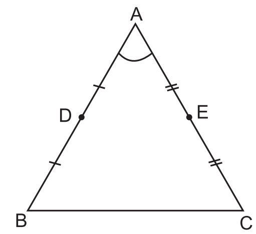
⇒ $2\angle Q + 80° = 180°$ [From (1)]

⇒ $\angle Q = \dfrac{180° - 80°}{2} = 50°$

∴ $\angle Q = \angle R = 50°$

6. In an isosceles triangle, prove that the line segments joining the end points of the base to the mid-points of the opposite sides are equal.

Sol. Given that ΔABC is an isosceles triangle with AB = AC. D and E are the mid-points of AB and AC respectively. To show that BD = CE.



We have

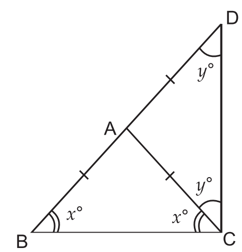
$$BD = \frac{1}{2}AB = \frac{1}{2}AC = CE.$$

Hence, proved.

Short Answer Type-II Questions

7. ABC is an isosceles triangle in which AB = AC, side BA is produced to D such that AD = AB. Show that ∠BCD is a right angle. **[CBSE SP 2012]**

Sol. Given that ΔABC is an isosceles triangle with AB = AC. BA is produced to D such that AB = AD. We join DC.



To prove that ∠BCD = 90°.

∵ $AB = AC$

∴ $\angle ABC = \angle ACB = x°$ (say)

Ext. angle $\angle CAD = \angle ABC + \angle ACB$

$= x° + x°$

$= 2x°$ …(1)

In ΔADC, $AD = AC$

∴ $\angle ACD = \angle ADC = y°$ (say)

Now, in ΔACD, we have

$\angle CAD + \angle ACD + \angle ADC = 180°$

[Angle sum property of a triangle]

⇒ $2x° + 2y° = 180°$ [From (1)]

⇒ $x° + y° = \dfrac{180°}{2} = 90°$

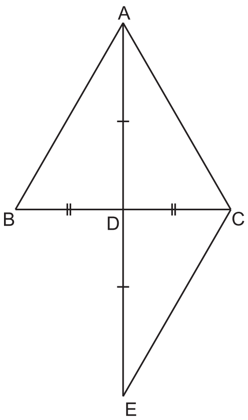
⇒ $\angle ACB + \angle ACD = 90°$

⇒ $\angle BCD = 90°$

Hence, proved.

8. If the bisector of the vertical angle of a triangle bisects the base, prove that the triangle is isosceles.

Sol. Let AD be the bisector of the vertical angle ∠BAC of ΔABC, where D is a point on BC such that BD = CD.



To prove that AB = AC i.e. ΔABC is an isosceles triangle.

Construction: We produce AD to E such that AD = DE. We join EC.

In ΔABD and ΔECD, we have

BD = CD [Given]

∠ADB = ∠EDC [Vertically opposite angles]

AD = ED [By construction]

∴ By SAS congruence criterion

ΔABD ≅ ΔECD

⇒ AB = EC [By CPCT] …(1)

and ∠BAD = ∠CED [By CPCT] …(2)

But ∠BAD = ∠CAD [∵ AD is the bisector of ∠BAC]

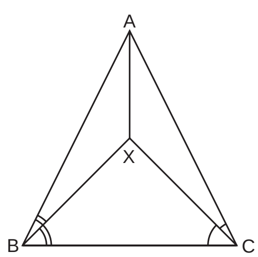
∴ ∠CAD = ∠CED

∴ AC = EC = AB [From (1)]

∴ ΔABC is an isosceles triangle.

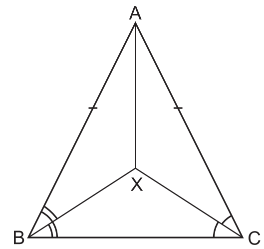
Long Answer Type Questions

9. ABC is an isosceles triangle in which AB = AC. The bisectors of ∠ABC and ∠ACB meet at X. Prove that XA bisects ∠BAC.



Sol. Given that ΔABC is an isosceles triangle with AB = AC. The bisector BX and CX of ∠ABC and ∠ACB respectively meet with each other at a point X. XA is joined.

To prove that XA bisects ∠BAC, i.e. ∠BAX = ∠CAX.



In ΔXBC, we have

$\angle XBC = \frac{1}{2}\angle ABC$

[∵ BX is the bisector of ∠ABC]

$= \frac{1}{2}\angle ACB$

[∵ ∠ABC = ∠ACB, for AB = AC]

= ∠XCB

∴ XB = XC …(1)

(Sides opposite to equal angles are equal)

Now, in ΔABX and ΔACX, we have

AB = AC

BX = CX [from 1]

$\angle ABX = \frac{1}{2}\angle ABC$

$= \frac{1}{2}\angle ACB$

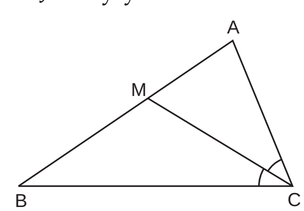
= ∠ACX

∴ By SAS congruence criterion, we have

ΔABX ≅ ΔACX

⇒ ∠BAX = ∠CAX [By CPCT]

10. In the given figure, ∠BAC = 2∠ABC = ∠ACB. CM bisects ∠ACB, where M is a point on AB. Name the line segments which are all equal to each other. Justify your answer.

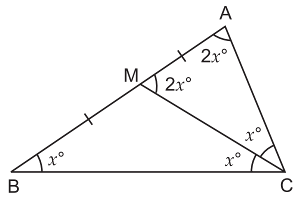


Sol. Given that in ΔBAC, ∠BAC = ∠BAC = 2∠ABC = ∠ACB. CM bisects ∠ACB, where M is a point on AB. To find all line segments whose lengths are all equal.

Let ∠ABC = $x°$.

Then, ∠BAC = ∠ACB = $2x°$

$\therefore \quad \angle MCB = \angle MCA = x°$



Now, in ΔBMC,

∠AMC is an exterior angle.

$\angle AMC = \angle MBC + \angle MCB$

$= x° + x° = 2x°$

Now, in ΔMBC,

$\angle MBC = \angle MCB$

$\therefore \quad BM = MC \quad \ldots(1)$

In ΔABC, $\angle BAC = \angle BCA = 2x°$

$\therefore \quad BC = AB \quad \ldots(2)$

In ΔAMC, $\angle CMA = \angle CAM = 2x°$

$\therefore \quad MC = AC \quad \ldots(3)$

From (1) and (3), we have

$AC = BM \quad \ldots(4)$

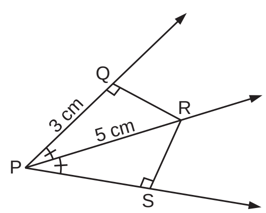
Hence, (1), (2), (3) and (4) are all line segments which are equal.

Milestone 3

(Page 93)

Multiple-Choice Questions

1. In the given figure, if PQ = 3 cm, PR = 5 cm and ∠QPR = ∠SPR, then RS is equal to



(*a*) 2 cm
(*b*) 3 cm
(*c*) 4 cm
(*d*) 5 cm

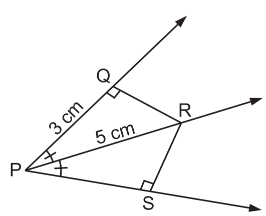
Sol. (*c*) 4 cm

Given that PR is the bisector of ∠QPS so that

$\angle QPR = \angle SPR$

$RQ \perp PQ$ and $RS \perp PS$, PQ = 3 cm and PR = 5 cm

To find the length of RS.



In ΔPQR and ΔPSR, we have

$\angle QPR = \angle SPR$

$\angle PQR = \angle PSR = 90°$

$PR = PR \quad$ [Common]

By AAS congruence criterion,

$\Delta PQR \cong \Delta PSR.$

$\therefore \quad PQ = PS \quad$ [By CPCT]

$PS = PQ = 3$ cm

∴ From ΔPSR, by Pythagoras' theorem,

$RS = \sqrt{PR^2 - PS^2}$

$= \sqrt{5^2 - 3^2}$ cm

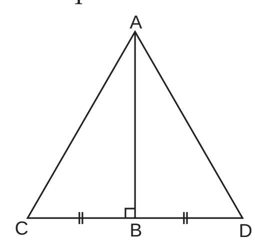
$= \sqrt{25 - 9}$ cm

$= \sqrt{16}$ cm

$= 4$ cm

2. In ΔABC, ∠ABC = 90°, ∠ACB = 2∠CAB.

Produce CB to a point D such that CB = BD as shown in the given figure. If BC = 3 cm, then the length of the hypotenuse AC is equal to



(*a*) 4 cm
(*b*) 3 cm
(*c*) 5 cm
(*d*) 6 cm

Sol. (*d*) 6 cm

Given that in ΔABC,

$\angle ABC = 90°$

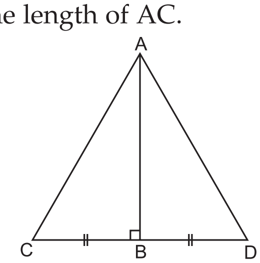
and $\angle ACB = 2\angle CAB$

CB is produced to D such that

CB = BD

BC = 3 cm

To find the length of AC.



From ΔACB, we have

$\angle ACB + \angle CAB = 90° \quad [\because \angle ABC = 90°]$

$\Rightarrow \quad 2\angle CAB + \angle CAB = 90° \quad [\because \angle ACB = 2\angle CAB]$

$\Rightarrow \quad 3\angle CAB = 90°$

$\angle CAB = \frac{90°}{3} = 30°$

$\therefore \quad \angle ACB = 30° \times 2 = 60° \quad \ldots(1)$

Now, in ΔABC and ΔABD, we have

$$CB = DB$$

$$AB = AB \quad \text{[Common]}$$

$$\angle ABC = \angle ABD = 90°$$

By SAS congruence criterion

$$\Delta ABC \cong \Delta ABD$$

$\Rightarrow$ $AC = AD$

and $\angle ADB = \angle ACB$ (By CPCT)

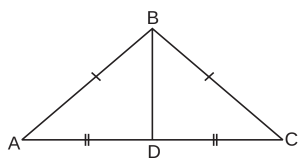
$\therefore$ $\angle ADB = \angle ACB = 60°$ ...(1)

$\therefore$ ACD is an equilateral triangle.

$\therefore$ $AC = CD = BC + BD = (3 + 3)$ cm $= 6$ cm.

Very Short Answer Type Questions

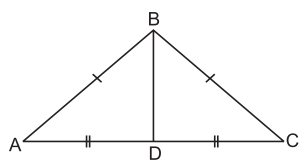
3. In the given figure, AB = CB and AD = CD. Prove that $\Delta ADB \cong \Delta CDB$.



Sol. Given that in ΔABC,

$$AB = BC$$

and $AD = CD$



To prove that

$$\Delta ADB \cong \Delta CDB$$

In ΔADB and ΔCDB, we have

$$AB = BC$$

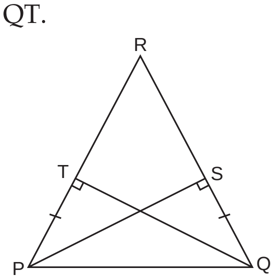
$$AD = CD$$

and $DB = DB$ (Common)

By SSS congruence criterion, we have

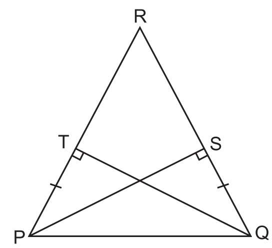
$$\Delta ADB \cong \Delta CDB$$

4. In the given figure, PS and QT are respectively the altitudes of ΔPQR such that PT = QS. Prove that PS = QT.



Sol. Given that in PQR, T and S are points on PR and QR respectively such that $QT \perp PR$ and $PS \perp QR$. Also, PT = QS.

To prove that PS = QT.



In ΔPTQ and ΔQSP we have

$$PT = QS$$

$$PQ = QP \quad \text{[Common]}$$

$$\angle PTQ = \angle QSP = 90°$$

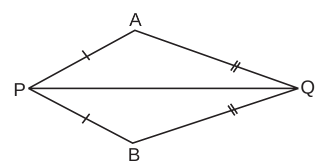
$\therefore$ By RHS congruence criterion.

$$\Delta PTQ \cong \Delta QSP$$

$$QT = PS \quad \text{[By CPCT]}$$

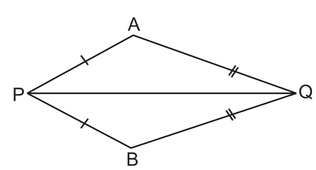
Short Answer Type-I Questions

5. In the given figure, PA = PB, QA = QB. Prove that PQ is the bisector of $\angle APB$ and $\angle AQB$.



Sol. Given that ΔAPQ and ΔBPQ stand on the same base PQ on opposite sides of PQ such that PA = PB and QA = QB.

To prove that PQ is the bisector of $\angle APB$ and $\angle AQB$.



In ΔAPQ and ΔBPQ, we have

$$PA = PB$$

$$QA = QB$$

$$PQ = PQ \quad \text{[Common]}$$

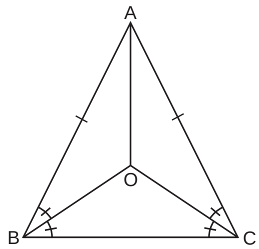
$\therefore$ By SSS congruence criterion,

$\Rightarrow$ $\angle APQ = \angle BQP$

and $\angle AQP = \angle BQP$

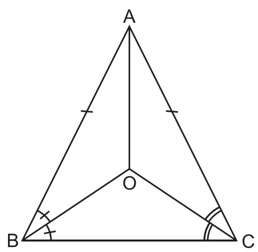
i.e. PQ is the bisector of both $\angle APB$ and $\angle AQB$.

6. In ΔABC, AB = AC and the bisectors of angles B and C intersect at point O as shown in the given figure. Prove that BO = CO and AO is the bisector of angle BAC.



Sol. Given that in ΔABC, AB = AC. BO and CO, the bisectors of $\angle ABC$ and $\angle ACB$ respectively meet each other at O.

To prove that BO = CO and AO is the bisector of $\angle BAC$.



In ΔOBC, we have

$$\angle OBC = \frac{1}{2}\angle ABC$$

$$= \frac{1}{2}\angle ACB = \angle OCB$$

$\therefore$ OB = OC

Now, in ΔAOB and ΔAOC, we have

AB = AC

OB = OC

AO = AO [Common]

$\therefore$ By SSS congruence criterion, we have

$\Delta AOB \cong \Delta AOC$

$\Rightarrow$ $\angle OAB = \angle OAC$ [By CPCT]

i.e. OA is the bisector of $\angle BAC$.

Short Answer Type-II Questions

7. In an equilateral ΔPQR, if PS is the median, then prove that $\angle PSR$ is a right angle.

Sol. Given that ΔPQR is an equilateral triangle and PS is a median.

$\therefore$ QS = SR

To prove that

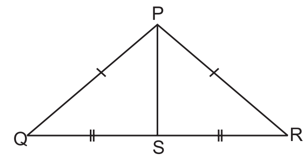
$\angle PSR = 90°$

In ΔPSQ and ΔPSR, we have

PQ = PR

QS = RS

PS = PS [Common]



$\therefore$ By SSS congruence criterion, we have

$\Delta PSQ \cong \Delta PSR$

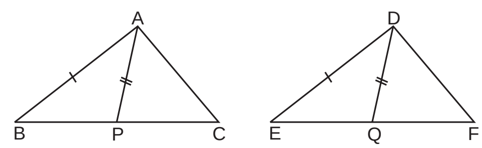
$\Rightarrow$ $\angle PSQ = \angle PSR$.

Also, $\angle PSQ + \angle PSR = 180°$ [By CPCT]

$\therefore$ $\angle PSQ = \angle PSR = \frac{180°}{2} = 90°$.

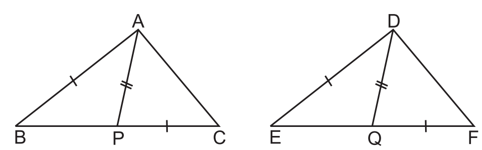
Hence, proved.

8. In the given figure, AB = DE, BC = EF and median AP = median DQ. Prove that $\angle B = \angle E$.



Sol. Given that AP and DQ are medians through A and D respectively of ΔABC and ΔDEF respectively. Also, AB = DE, BC = EF and AP = DQ.

To prove that $\angle B = \angle E$.



In ΔABP and ΔDEQ, we have

AB = DE

AP = DQ

$$BP = \frac{1}{2}BC = \frac{1}{2}EF = EQ$$

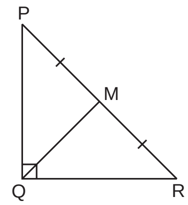
$\therefore$ By SSS congruence criterion, $\Delta ABP \cong \Delta DEQ$.

$\Rightarrow$ $\angle ABP = \angle DEQ$ [By CPCT]

$\angle B = \angle E$

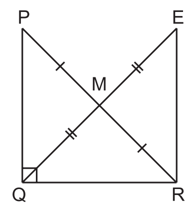
Long Answer Type Questions

9. In the given figure, ΔPQR is a right-angled triangle with $\angle PQR = 90°$. If M is the mid-point of the hypotenuse PR, prove that QM = PM = MR.



Sol. Given that ΔPQR is a right-angled triangle with PR as the hypotenuse and $\angle PQR = 90°$. M is the mid-point of PR so that PM = RM.

To prove that QM = PM = MR.



Construction: We produce QM to E such that

$$QM = ME$$

We join ER.

In ΔPMQ and ΔRME, we have

$PM = RM$ [Given]

$QM = EM$ [By construction]

$\angle PMQ = \angle RME$ [Vertically opposite angles]

$\therefore$ By SAS congruence criterion, we have

$$\Delta PMQ \cong \Delta RME$$

$\Rightarrow$ $PQ = RE$ [By CPCT]

$\angle PQM = \angle REM$ [By CPCT]

But these two are alternate angles with respect to PQ and ER and transversal QE.

$\therefore$ QP $\parallel$ RE.

$\therefore$ $\angle ERQ + \angle PQR = 180°$

$\Rightarrow$ $\angle ERQ + 90° = 180°$

$\Rightarrow$ $\angle ERQ = 180° - 90°$

$= 90°$

Now, in ΔPQR and ΔERQ, we have

$PQ = ER$

$QR = RQ$ [Common side]

$\angle PQR = \angle ERQ = 90°$.

By SAS congruence criterion, we have

$$\Delta PQR \cong \Delta ERQ$$

$\Rightarrow$ $PR = EQ$ [By CPCT]

$\Rightarrow$ $\frac{1}{2} PR = \frac{1}{2} EQ$

$\Rightarrow$ $PM = QM$

$QM = PM = MR$

Hence, proved.

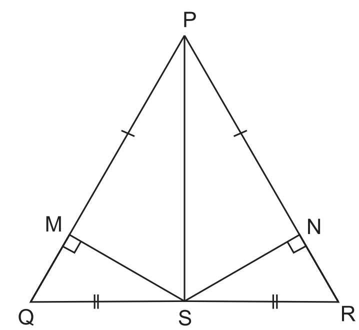
10. ΔPQR is an isosceles triangle with PQ = PR. S is the mid-point of QR. From S, SM and SN are drawn perpendicular to PQ and PR respectively. Prove that

(*a*) SM = SN

(*b*) PM = PN and

(*c*) SP bisects $\angle QPR$.

Sol. Given that ΔPQR is an isosceles triangle with PQ = PR. S is the mid-point of QR so that QS = RS. M and N are points on PQ and PR respectively such that SM $\perp$ PQ and SN $\perp$ PR.



To prove that

(*a*) SM = SN

(*b*) PM = PN and

(*c*) SP bisects $\angle QPR$.

In ΔSMQ and ΔSNR, we have

$SQ = SR$

$\angle MQS = \angle NRS$ [$\because$ PQ = PR]

$\angle QMS = \angle RNS = 90°$.

$\therefore$ By AAS congruence criterion, we have

$$\Delta SMQ \cong \Delta SNR$$

(*a*) SM = SN [By CPCT] and MQ = NR [By CPCT]

(*b*) $\therefore$ PM = PQ – MQ = PR – NR = PN.

(*c*) In ΔPMS and ΔPNS, we have

$PM = PN$

$SM = SN$

$PS = PS$ [Common]

$\therefore$ By SSS congruence criterion, we have

$$\Delta PMS \cong \Delta PNS$$

$\Rightarrow$ $\angle MPS = \angle NPS$ [By CPCT]

$\therefore$ SP is the bisector of $\angle QPR$.

Hence, proved.

Milestone 4

(Page 95)

Multiple-Choice Questions

1. In ΔPQR if $\angle R > \angle Q$, then

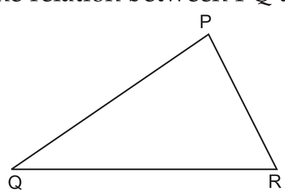
(*a*) QR > PR (*b*) PQ > PR

(*c*) PQ < PR (*d*) QR < PR

Sol. (*b*) PQ > PR

It is given that PQR is a triangle in which $\angle R > \angle Q$.

To find the relation between PQ and PR.

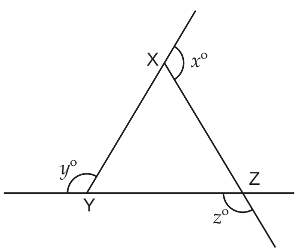


$\because \quad \angle R > \angle Q$

$\therefore \quad PQ > PR$

[$\because$ PQ is the side opposite to the greater angle]

2. In ΔXYZ, the exterior angles at X, Y and Z are of measures $x°$, $y°$ and $z°$ respectively as shown in the given figure.



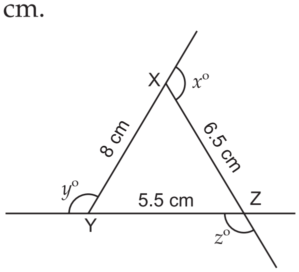
If XY = 8 cm, YZ = 5.5 cm and ZX = 6.8 cm, then

(*a*) $x < z < y$ (*b*) $y < x < z$

(*c*) $x < y < z$ (*d*) $z < y < x$

Sol. (*d*) $z < y < x$

Given that in ΔXYZ, the exterior angles at X, Y and Z are respectively $x°$, $y°$ and $z°$ as shown in the figure. Also, XY = 8 cm, YZ = 5.5 cm and ZX = 6.8 cm.



To find a relation between x, y and z.

Since XY > XZ, $\angle XZY > \angle XYZ$

$\Rightarrow \quad 180° - z° > 180° - y°$

$\Rightarrow \quad - z° > - y°$

$\Rightarrow \quad z° < y°$

$\Rightarrow \quad z < y$ ...(1)

Again since XZ > YZ

$\therefore \quad \angle XYZ > \angle YXZ$

$\Rightarrow \quad 180° - y° > 180° - x°$

$\Rightarrow \quad -y° > -x°$

$\Rightarrow \quad y° < x°$

$\Rightarrow \quad y < x$ ...(2)

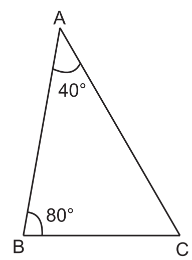
$\therefore$ From (1) and (2), we have $z < y < x$.

Very Short Answer Type Questions

3. In ΔABC, if $\angle A = 40°$ and $\angle B = 80°$ then arrange the sides of the triangle in ascending order.

Sol. Given that in ΔABC, $\angle A = 40°$, $\angle B = 80°$.

To arrange the sides of the triangle in ascending order.



We have $\angle A + \angle B + \angle C = 180°$

[By angle sum property of a triangle]

$\Rightarrow \quad 40° + 80° + \angle C = 180°$

$\Rightarrow \quad \angle C = 180° - 120° = 60°$

$\because \quad 40° < 60° < 80°$

$\therefore \quad \angle A < \angle C < \angle B$

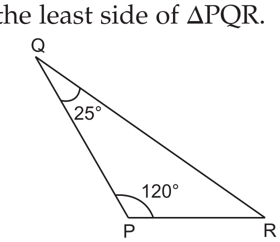
$\therefore \quad BC < AB < AC$

which is the required result.

4. Which side of ΔPQR will be least if $\angle P = 120°$ and $\angle Q = 25°$?

Sol. Given that in ΔPQR, $\angle P = 120°$ and $\angle Q = 25°$.

To find the least side of ΔPQR.



In ΔPQR, we have

$\angle P + \angle Q + \angle R = 180°$

[By angle sum property of a triangle]

$\Rightarrow \quad 120° + 25° + \angle R = 180°$

$\Rightarrow \quad \angle R = 180° - 145°$

$= 35°$

$\because \quad 120° > 35° > 25°$

$\therefore \quad \angle P > \angle R > \angle Q$

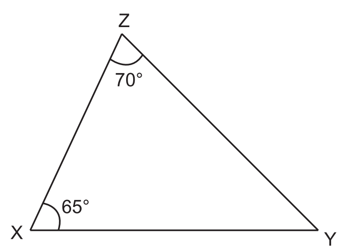
$\therefore \quad QR > PQ > PR.$

$\therefore$ The side RP of ΔPQR is the least side.

Short Answer Type-I Questions

5. In ΔXYZ, $\angle X = 65°$ and $\angle Z = 70°$. Then find the shortest and longest sides of the triangle.

Sol. Given that in ΔXYZ, $\angle X = 65°$ and $\angle Z = 70°$. To find the shortest and the longest sides of the triangle.



In ΔXYZ, we have

$\angle X + \angle Z + \angle Y = 180°$

[By angle sum property of a triangle]

$\Rightarrow \quad 65° + 70° + \angle Y = 180°$

$\Rightarrow \quad \angle Y = 180° - 135°$

$= 45°$

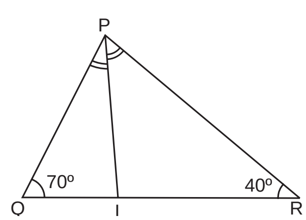
$\because \quad 70° > 65° > 45°$

$\therefore \quad \angle Z > \angle X > \angle Y$

$\therefore \quad XY > ZY > ZX.$

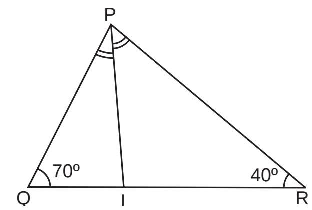
$\therefore$ The required shortest side is XZ and the longest side is XY.

6. In the given figure, PL bisects $\angle QPR$. If $\angle PQL = 70°$ and $\angle PRL = 40°$, arrange PR, LR and PL in descending order.



Sol. Given that in ΔPQR, PL bisects $\angle QPR$, where L is a point on QR and $\angle PQR = 70°$ and $\angle PRQ = 40°$.

To arrange PR, LR and PL in descending order.



In ΔPQR, we have,

$\angle Q + \angle R + \angle QPR = 180°$

[By angle sum property of a triangle]

$\Rightarrow \quad 70° + 40° + \angle QPR = 180°$

$\therefore \quad \angle QPR = 180° - 110° = 70°$

$\therefore \quad \angle QPL = \frac{1}{2} \angle QPR$

$= \frac{1}{2} \times 70° = 35°$

$\therefore \quad \angle LPR = \angle QPL = 35°$

$\therefore \quad \angle PLR = 180° - \angle LPR - \angle PRL$

$= 180° - 35° - 40°$

$= 180° - 75°$

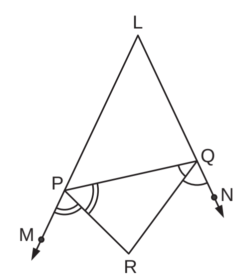
$= 105°$

Now, in ΔLPR, we have, $\angle PLR > \angle LRP > \angle LPR$

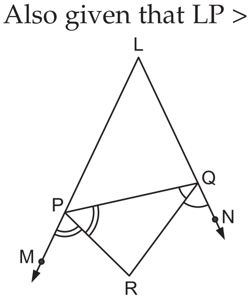
$\therefore \quad PR > PL > LR$, which is the required result.

Short Answer Type-II Questions

7. The sides LP and LQ of ΔLPQ are produced to M and N respectively and the bisectors of $\angle MPQ$ and $\angle NQP$ meet at R. If LP > LQ, prove that RQ > RP.



Sol. Given that sides LP and LQ of ΔLPQ are produced to M and N respectively and the bisectors PR and QR of $\angle MPQ$ and $\angle PQN$ respectively meet each other at R. Also given that LP > LQ.



To prove that RQ > RP.

Since $\quad LP > LQ$

$\therefore \quad \angle LQP > \angle LPQ$

$\Rightarrow \quad 180° - \angle PQN > 180° - \angle MPQ.$

$\Rightarrow \quad \angle PQN < \angle MPQ$

$\Rightarrow \quad 2\angle PQR < \angle 2RPQ$

$\Rightarrow \quad \angle PQR < \angle RPQ$

$\Rightarrow \quad PR < RQ$

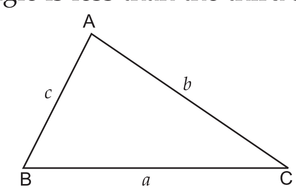
$\Rightarrow \quad RQ > RP.$

Hence, proved.

8. Show that the difference of any two sides of a triangle is less than the third side.

Sol. Let a, b and c be the sides, of the triangle ABC as in the figure.

To prove that the difference of any two sides of the triangle is less than the third side.



We know that $b + c > a$

$\Rightarrow \quad b > a - c$

$\Rightarrow \quad a - c < b \quad \ldots(1)$

Also, $\quad c + a > b$

$\Rightarrow \quad c > b - a$

$\Rightarrow \quad b - a < c \quad \ldots(2)$

Finally, $\quad a + b > c$

$a > c - b$

$\Rightarrow \quad c - b < a \quad \ldots(3)$

From (1), (2) and (3), we see that $a - c < b$, $b - a < c$ and $c - b < a$.

Thus, the difference of any two sides of a triangle is less than the third side.

Hence, proved.

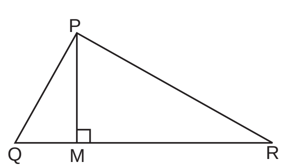
Long Answer Type Questions

9. In the given figure, PM ⊥ QR.

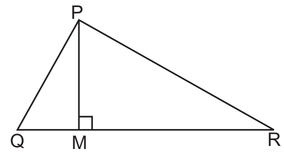
Prove that

(*a*) PQ > QM (*b*) PR > MR

(*c*) PQ + PR > QR (*d*) PQ + PR > 2 PM



Sol. Given that in ΔPQR, PM ⊥ QR, where M is a point on QR.



To prove that

(a) PQ > QM (b) PR > MR

(c) PQ + PR > QR (d) PQ + PR > 2PM.

In right-angled triangle PMQ, we have

$\angle PMQ = 90°$

$\therefore \quad \angle PQM < \angle PMQ$

Also, $\quad \angle QPM < \angle PMQ$

$\therefore \quad PQ > PM$

and $\quad PQ > QM$ proving (a)

Similarly in ΔPMR,

$PR > MR$ proving (b)

Adding (a) and (b), we see that

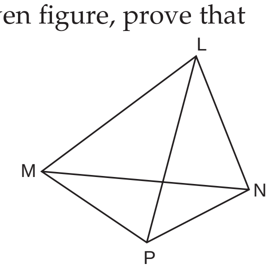
$PQ + PR > QM + MR = QR$ proving (c)

Finally, since PQ > PM and PR > PM

$\therefore \quad PQ + PR > PM + PM = 2\ PM$ proving (d).

Hence, proved.

10. In the given figure, prove that



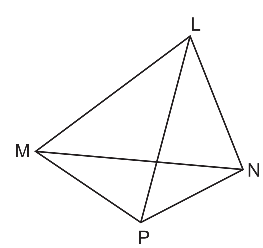
(*a*) PN + NL + LM > MP

(*b*) PN + NL + LM + MP > 2LP

(*c*) PN + NL + LM + MP > 2MN

(*d*) PN + NL + LM + MP > LP + MN

Sol. In the given figure, LMN and MPN are two triangles with common base MN, lying on the opposite sides of the base MN. LP is joined.



To prove that

(*a*) PN + NL + LM > MP

(*b*) PN + NL + LM + MP > 2LP

(*c*) PN + NL + LM + MP > 2MN and

(*d*) PN + NL + M + MP > LP + MN

In ΔLNP, we have PN + NL > LP ...(1)

and in ΔLMP, we have

LP + LM > MP

⇒ LP > MP – LM ...(2)

(*a*) From (1) and (2), we get PN + NL > MP – LM

⇒ PN + NL + LM > MP.

(*b*) In ΔLMP, we have LM + MP > LP ...(3)

Adding (1) and (3), we get

PN + NL + LM + MP > LP + LP = 2LP

(*c*) In ΔLMN, we have ML + LN > MN ...(4)

and in ΔPMN, we have MP + PN > MN ...(5)

Adding (4) and (5), we get

PN + NL + LM + MP > MN + MN = 2MN.

(*d*) Adding the results in (b) and (c), we get

PN + NL + LM + MP + PN + NL + LM + MP > 2(LP + MN)

⇒ 2(PN + NL + LM + MP) > 2(LP + MN)

⇒ PN + NL + LM + MP > LP + MN.

Hence, proved.

Higher Order Thinking Skills (HOTS) Questions

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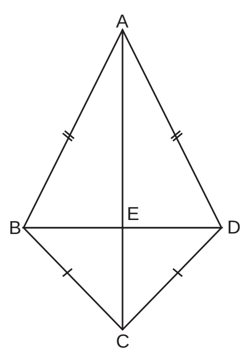
1. In the given figure, ABCD is a quadrilateral in which AB = AD and BC = DC.



Prove that

(*a*) AC bisects ∠BAD and ∠BCD.

(*b*) AC is perpendicular bisector of BD.

(*c*) ∠ABC = ∠ADC.

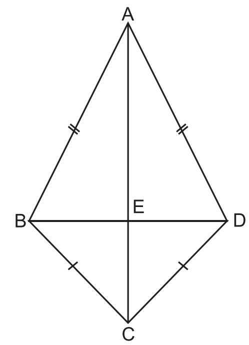
Sol. Given that ABCD is a quadrilateral in which AB = AD and BC = DC.

To prove that

(*a*) AC bisects ∠BAD and ∠BCD.

(*b*) AC is perpendicular bisector of BD and

(*c*) ∠ABC = ∠ADC.



In ΔABC and ΔADC, we have

AB = AD

BC = DC

AC = AC [Common]

∵ By SSS congruence criterion, we have

ΔABC ≅ ΔADC

(*a*) ∵ ΔABC ≅ ΔADC

∴ ∠BAC = ∠DAC and ∠BCA = ∠DCA [By CPCT]

∴ AC bisects both ∠BAD and ∠BCD.

(*b*) In ΔABE and ΔADE, we have

AB = AD

∠BAE = ∠DAE

AE = AE [Common]

∴ By SAS congruence criterion, we have

ΔABE ≅ ΔADE

∴ ∠AEB = ∠AED [By CPCT]

and BE = DE [By CPCT] ...(1)

Also, ∠ABE = ∠ADE ...(2)

But ∠AEB + ∠AED = 180°

∴ ∠AEB = ∠AED = 90° ...(3)

∴ AC is the perpendicular bisector of BD [From (1) and (3)]

(*c*) In ΔBEC and ΔDEC, we have

BE = DE [From (1)]

EC = EC [Common]

∠BEC = ∠DEC = 90°

∴ By SAS congruence criterion,

ΔBEC ≅ ΔDEC

∴ ∠CBE = ∠CDE [By CPCT]...(4)

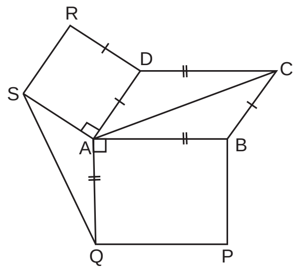
Adding (2) and (4), we get

∠AEB + ∠CBE = ∠ADE + ∠CDE

⇒ ∠ABC = ∠ADC

Hence, proved.

2. In the given figure, squares ABPQ and ADRS are drawn on the sides AB and AD of a parallelogram ABCD.



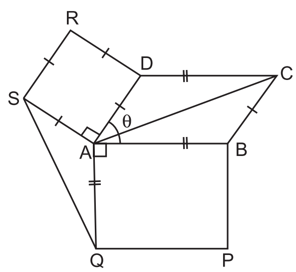
Prove that

(*a*) $\angle SAQ = \angle ABC$

(*b*) SQ = AC

Sol. Given that two squares ABPQ and ADRS are drawn on two adjacent side, AB and AD respectively of a parallelogram ABCD. To prove that (*a*) $\angle SAQ = \angle ABC$ and (*b*) SQ = AC.

Let $\angle BAD = \theta$



Now, in ΔSAQ and ΔCBA, we have

$$\angle SAQ = 360° - \angle SAD - \angle BAD - \angle BAQ$$
$$= 360° - 90° - \theta - 90°$$
$$= 180° - \theta.$$
$$= \angle CBA$$

[$\because$ $\angle DAB + \angle CBA = 180°$

$\Rightarrow$ $\angle CBA = 180° - \angle DAB = 180° - \theta$].

SA = AD = CB

[$\because$ ADRS is a square and ABCD is a parallelogram]

AQ = BA [$\because$ ABPQ is a square]

$\therefore$ By SAS congruence criterion, we have

$$\Delta SAQ \cong \Delta CBA.$$

(*a*) $\angle SAQ = \angle CBA = \angle ABC$ [By CPCT]

(*b*) SQ = CA = AC [By CPCT]

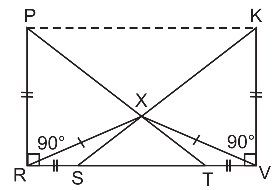
Hence, proved.

3. Let $PR \perp RV$, $KV \perp RV$, PR = KV, S and T are two points on RV such that RS : SV = 1 : 2 and VT : TR = 1 : 2. If PT and KS intersect at X such that RX = VX, then prove that

(*a*) $\Delta PRX \cong \Delta KVX$ and

(*b*) SX = TX = RS = ST = TV

Sol. Given that RV is a line segment, $RP \perp RV$ and $VK \perp RV$ where P and K lie on the same side RV, such that PR = KV. S and T are two points on RV such that RS : SV = 1 : 2 and VT : TR = 1 : 2.



PT and KS intersect each other at X such that RX = VX. To prove that

(*a*) $\Delta PRX \cong \Delta KVX$ and

(*b*) SX = TX = RS = ST = TV.

Let RV = 3*a*

$\therefore$ RS + SV = 3*a*

Also, RS : SV = 1 : 2

$\therefore$ $RS = \frac{1}{3} \times 3a = a$ …(1)

and $SV = \frac{2}{3} \times 3a = 2a$ …(2)

Again, VT + TR = 3*a*

Also, VT : TR = 1 : 2

$\therefore$ $VT = \frac{1}{3} \times 3a = a$ …(3)

and $TR = \frac{2}{3} \times 3a = 2a$ …(4)

$\therefore$ From (1) and (3),

RS = TV = *a* …(5)

and from (2) and (4),

SV = TR = 2*a* …(6)

Now, SV = 2*a*

$\Rightarrow$ ST + TV = 2*a*

$\Rightarrow$ ST + *a* = 2*a*

$\Rightarrow$ ST = *a* [From (5)] …(7)

$\therefore$ From (5) and (7).

RS = ST = TV = *a* …(8)

(*a*) Now, in ΔPRX and ΔKVX, we have

PR = KV, RX = VX

In ΔRXV,

since RX = VX,

$\therefore$ $\angle XRV = \angle XVR = \theta$ (say)

$\therefore$ $\angle PRX = \angle PRV - \angle XRV$

$= 90° - \theta$

and $\angle KVX = \angle KVR - \angle XVR$

$= 90° - \theta$

$\therefore \quad \angle PRX = \angle KVX$

$\therefore$ By SAS congruence criterion, we have

$\Delta PRX \cong \Delta KVX$

(*b*) Let the length of RV = 3*a* units

Now, $RS + SV = 1 : 2$

Also, $RS + SV = RV = 3a$

$\therefore \quad RS = \frac{1}{3} \times 3a = a$ …(A)

and $SV = \frac{2}{3} \times 3a = 2a$ …(B)

Also, $VT + TR = 1 : 2$

Also, $VT + TR = RV = 3a$

$\therefore \quad VT = \frac{1}{3} \times 3a$ …(C)

and $TR = \frac{2}{3} \times 3a = 2a$ …(D)

Now, from (B), $SV = 2a$

$\Rightarrow \quad ST + TV = 2a$

$ST = 2a - a$ [From (C)]

$= a$ …(E)

$\therefore$ From (A), (B), (C), (D) and (E), we have

$RS = ST = TV$ …(9)

Now, in ΔXRT and ΔXVS, we have

$XR = XV$

$RT = VS$ [From (B) and (D)]

$\angle XRT = \angle XVS = \theta$

$\therefore$ By SAS congruence criterion, we have

$\Delta XRT \cong \Delta XVS$

$\therefore \quad TX = SX$ …(10) [By CPCT]

From (9) and (10), we have

$SX = TX = RS = ST = TV.$

Hence, proved.

Self-Assessment

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Multiple-Choice Questions

1. Which of the following is not a criterion for congruence of triangles?

(*a*) SAS (*b*) SSA

(*c*) ASA (*d*) SSS

Sol. (*b*) SSA

It is well known result that SSA is not a criterion for congruence of triangles.

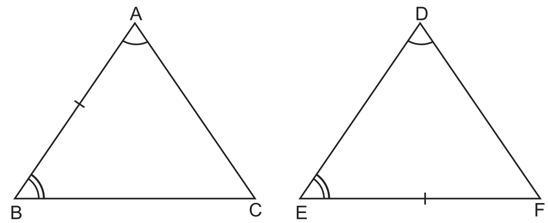
2. In triangles ABC and DEF, ∠A = ∠D, ∠B = ∠E and AB = EF, then are the two triangles congruent? If yes, by which congruency criterion?

(*a*) Yes, by AAS (*b*) Yes by ASA

(*c*) No (*d*) Yes, by RHS

Sol. (*c*) No

In ΔABC, two angles A and B and the included side AB are given to be equal to the corresponding angle D and E of ΔDEF are given, but the corresponding sides are not equal.



Hence, ΔABC cannot be congruent to ΔDEF.

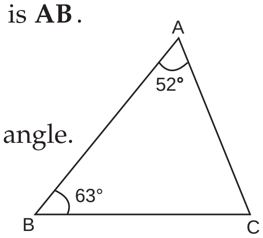
Fill in the Blanks

3. In ΔABC, if ∠A = 52° and ∠B = 63°, then the greatest side of the triangle is **AB**.

Sol. $\angle C = 180° - (52° + 63°)$

$= 65°$

$\Rightarrow \angle C = 65°$ is the greatest angle.



$\therefore$ Side opposite to ∠C,

i.e. AB is the greatest side

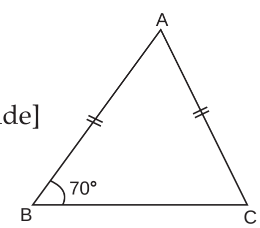
[Greatest angle has greatest side opp. to it]

4. In ΔABC, if AB = AC and ∠B = 70°, then the measure of ∠A is **40°**.

Sol. $\because \quad AB = AC$

$\Rightarrow \angle B = \angle C = 70°$

[∠s opp. equal side]



$\therefore \quad \angle A = 180° - (70° + 70°)$

$= 180° - 140°$

$= 40°$

5. The measure of each of the base angles of an isosceles triangle whose base angle is double the vertical angle is **72°**.

Sol. $x + 2x + 2x = 180°$

$\Rightarrow \quad 5x = 180°$

$\Rightarrow \quad x = 36°$

$\therefore$ Each base angle

$= 2x = 2 \times 36° = 72°$

6. In ΔABC, ∠A = 50° and ∠B = 60°. Arranging the sides of the triangle in ascending order we get **BC < CA < AB**.

Sol. $x = 180° - (60° + 50°)$

$= 180° - 110°$

$= 70°$

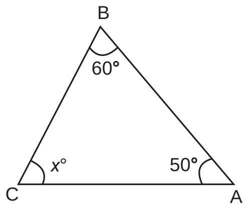
$\because$ 70° > 60° > 50°

$\therefore$ AB > AC > BC

[Greater angle has greater side opp. to it]

$\Rightarrow$ Required ascending order is

BC < AC < AB



Assertion-Reason Type Questions

Directions (Q. Nos. 7 to 10): Each of these questions contains an assertion followed by reason. Read them carefully, and answer the question on the basis of the following options, select the one that best describes the two statements.

(a) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.

(b) Both Assertion and Reason are true but Reason is not the correct explanation of Assertion.

(c) Assertion is true but Reason is false.

(d) Assertion is false but Reason is true.

7. Assertion: Two triangles are congruent if they have same angles.

Reason: Two congruent triangles will have same angles.

Sol. (*d*)

Assertion is incorrect as two triangles are not congruent if they have only same angles but reason is correct.

8. Assertion: Two triangles are congruent if they satisfy SAS condition.

Reason: If two triangles satisfy SAS condition then the third side will be equal.

Sol. (*a*)

Both assertion and reason are correct and reason is correct explanation of assertion.

9. Assertion: In any triangle, the side opposite to the greater angle is longer.

Reason: The side opposite to the smaller angle is shorter.

Sol. (*b*)

Both assertion and reasoning are correct but reason is not the correct explanation of assertion.

10. Assertion: The sum of any two sides of a triangle is less than the third side.

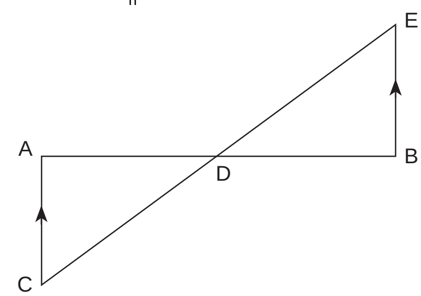
Reason: Sum of angles in a triangle is 180°.

Sol. (*d*)

Assertion is incorrect as sum of two sides will always be greater than third side but reason is correct as sum of all angles of a triangle is 180°.

Case Study Based Questions

11. Five friends Kanika, Divya, Atul, Himanshi and Prerna are standing at the positions A, B, C, D and E as shown in the given figure such that BE = AC and BE || AC.



Based on the above situation, answer the following questions.

(*a*) Which of the following properties is valid for $\angle DAC = \angle DBE$?

(*i*) Vertically opposite angles

(*ii*) Alternate interior angles

(*iii*) Corresponding angles

(*iv*) None of these

Ans. (*ii*) Alternate interior angles

(*b*) What is the congruency criterion by which ΔADC is congruent to ΔBDE?

(*i*) SAS (*ii*) ASA

(*iii*) SSS (*iv*) None of these

Ans. (*ii*) ASA

(*c*) Which of the following options is incorrect for the given figure?

(*i*) DA = DB

(*ii*) DC = DE

(*iii*) $\angle DCA = \angle DEB$

(*iv*) None of these

Ans. (*iv*) None of these

(*d*) For two triangles, if two angles and the included side of one triangle are equal to the corresponding two angles and the included side of the other triangle, then the congruency rule is

(*i*) SAS. (*ii*) ASA.

(*iii*) SSS. (*iv*) RHS.

Ans. (*ii*) ASA.

(*e*) The angles opposite to equal sides of a triangle are

(*i*) equal.

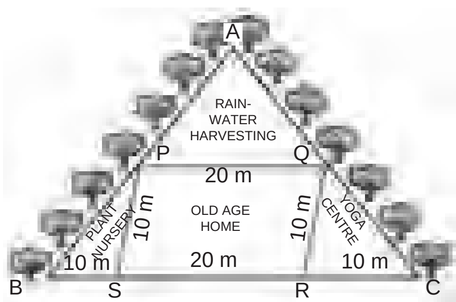
(*ii*) unequal.

(*iii*) complementary angles.

(*iv*) supplementary angles.

Ans. (*i*) equal.

12. BAC is a triangular plot of land owned by Mrs Sachdeva. She gets an old age home built in parallelogram PQRS, a nursery for plants in triangular area PBS and a yoga centre in triangular area QRC. She reserves the triangular area APQ for rainwater harvesting. She also has trees planted along BA and AC. Based on the above information, answer the following questions.



(*a*) Based on angles of triangle BAC, what type of triangle is this?

(*i*) Scalene

(*ii*) Isosceles

(*iii*) Right-angled

(*iv*) Equilateral

Ans. (*iii*) Right-angled

(*b*) What is the measure of ∠PSR if ∠PBS = x?

(*i*) x (*ii*) $2x$

(*iii*) $3x$ (*iv*) $4x$

Ans. (*ii*) $2x$

(*c*) Choose the correct option.

(*i*) A triangle has two right angles.

(*ii*) All angles of a triangle are more than 60°.

(*iii*) An exterior angle of a triangle is always greater than opposite interior angle.

(*iv*) All the angles of a triangle are less than 60°.

Ans. (*iii*) An exterior angle of a triangle is always greater than opposite interior angle.

(*d*) If ∠QCR= 50°, then measure of ∠QRS is

(*i*) 50° (*ii*) 60°

(*iii*) 90° (*iv*) 100°

Ans. (*iv*) 100°

(*e*) In ΔABC, the value of (∠ABC + ∠ACB) is

(*i*) 45° (*ii*) 60°

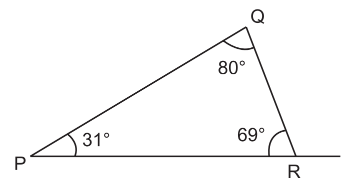
(*iii*) 90° (*iv*) 180°

Ans. (*iii*) 90°

Very Short Answer Type Questions

13. In ΔPQR, ∠P = 31° and ∠Q = 80°. Name the greatest side of ΔPQR.

Sol. Given that in ΔPQR, ∠P = 31°, ∠Q = 80° and ∠R = 180°– (31° + 80°) = 69°.



To find the greatest side.

∵ ∠Q > ∠R > ∠P

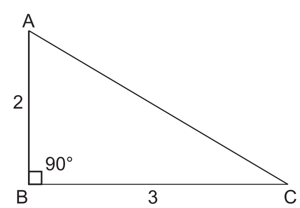
∴ PR > PQ > QR.

∴ PR is the greatest side.

14. In ΔABC, ∠B = 90° and AB : BC = 2 : 3. Which angle is the least angle?

Sol. In ΔABC, it is given that ∠ABC = 90° and AB : BC = 2 : 3.

To determine the least angle.



∵ AB : BC = 2 : 3

∴ ∠C : ∠A = 2 : 3

[∵ ∠C is opposite to the side of length 2 units and ∠A is opposite to the side of length 3 units.]

But ∠A + ∠C = 90° [∵ ∠B = 90°]

∴ $\angle C = \frac{2}{5} \times 90° = 36°$

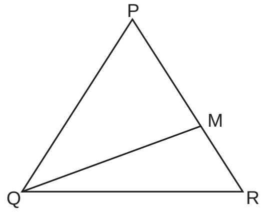
and $\angle A = \frac{3}{5} \times 90° = 54°$

∴ ∠A = 54°, ∠C = 36° and ∠B = 90°.

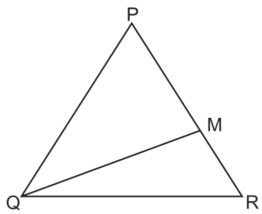
∴ ∠C is the least angle.

Short Answer Type-I Questions

15. ΔPQR is an equilateral triangle, and M is any point on PR. Prove that QM > PM.



Sol. Given that ΔPQR is an equilateral triangle and M is any point on PR. QM is joined. To prove that $QM > PM$.



We see that $\angle PQM < \angle PQR = \angle QPR$

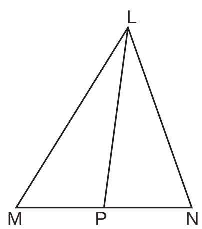
[$\because$ Each of these two angles is 60°]

$\therefore$ $QM > PM$

[QM is the side opposite to bigger angle $\angle QPR$]

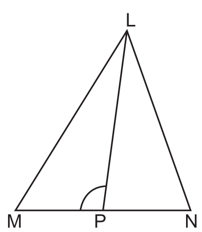
Hence, proved.

16. In the given figure, $LM > LN$. Prove that $LM > LP$.



Sol. Given that in ΔLMN, $LM > LN$.

To prove that $LM > LP$ where P is a point on MN.



In ΔLPN, $\angle LPN$ is an exterior angle.

$\therefore$ $\angle LPM = \angle LNP + \angle PLN$

$\therefore$ $\angle LPM > \angle LNP$ …(1)

But $\because$ $LM > LN$

$\therefore$ $\angle LNM > \angle LMN$.

$\Rightarrow$ $\angle LNP > \angle LMP$ …(2)

$\therefore$ From (1) and (2), we have

$\angle LPM > \angle LMP$

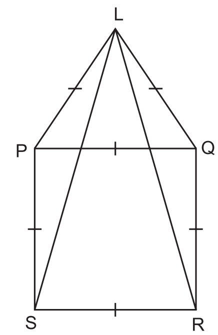
$\therefore$ $LM > LP$

Hence, proved.

Short Answer Type-II Questions

17. An equilateral triangle PQL is drawn outside the square PQRS with PQ as the base. Show that $\Delta PLS \cong \Delta QLR$.

Sol. Given that ΔPQL is an equilateral triangle drawn outside the square PQRS with PQ as the base. To prove that $\Delta PLS \cong \Delta QLR$.



In ΔPLS and ΔQLR, we have

$PL = QL$

[$\because$ PQL is an equilateral triangle]

$PS = QR$ [$\because$ PQRS is a square]

and $\angle LPS = \angle LPQ + \angle QPS$

$= 60° + 90°$

$= 150°$

and $\angle LQR = \angle LQP + \angle PQR$

$= 60° + 90°$

$= 150°$

$\therefore$ $\angle LPS = \angle LQR$

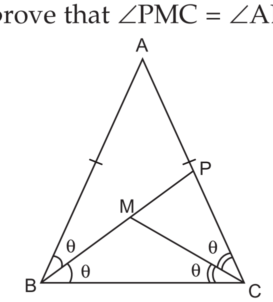
$\therefore$ By SAS congruence criterion, we have

$\Delta PLS \cong \Delta QLR$

Hence, proved.

18. The bisectors of angles ABC and ACB of an isosceles triangle ABC with $AB = AC$, intersect each other at M within the triangle. BM is now produced to a point P on AC. Prove that $\angle PMC = \angle ABC$.

Sol. Given that ΔABC is an isosceles triangle with $AB = AC$. BM and CM are two bisectors of $\angle ABC$ and $\angle ACB$ respectively, meeting with each other at a point M. BM is produced to cut the side AC at P. To prove that $\angle PMC = \angle ABC$.



Let $\angle MBC = \theta$

Then $\angle ABM = \theta$

$\because$ $\angle ABC = \angle ACB$

$\therefore$ $\angle BCM = \angle PCM = \theta$.

Now, in ΔBMC, $\angle PMC$ is an exterior angle.

$\therefore$ $\angle PMC = \angle MBC + \angle MCB$

$= \theta + \theta = 2\theta$

Also, $\angle ABC = \angle ABM + \angle MBC$

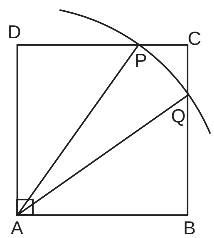
$= \theta + \theta = 2\theta$

$\angle PMC = \angle ABC$

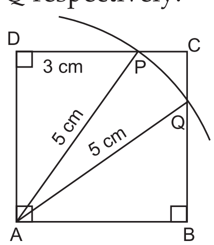
Hence, proved.

Long Answer Type Questions

19. ABCD is a square. With centre A and radius 5 cm, an arc is drawn to cut CD at P and BC at Q. If PD = 3 cm, then what is the length of QC?



Sol. Given that ABCD is a square. Arc PQ is drawn with centre at A and radius 5 cm cutting CD and CB at P and Q respectively.



$\therefore$ AP = AQ = 5 cm and PD = 3 cm.

To find the length of QC.

In ΔADP, we have AP = 5 cm, DP = 3 cm and $\angle ADP = 90°$.

$\therefore$ By Pythagoras' theorem, we have

$$AP^2 = AD^2 + PD^2$$

$\Rightarrow$ $25 = AD^2 + 9$

$\Rightarrow$ $AD = \sqrt{25 - 9} = 4$

$\because$ ABCD is a square

$\therefore$ BC = AD = 4 cm

$\therefore$ QC = BC – BQ = AD – BQ

$= 4 \text{ cm} - \sqrt{AQ^2 - AB^2} \text{ cm}$

$= 4 \text{ cm} - \sqrt{25 - 16} \text{ cm}$

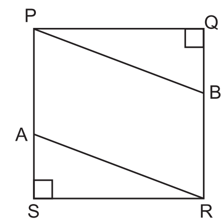
[$\because$ AB = AD = 4 cm]

= 4 cm – 3 cm = 1 cm

Hence, the required length of QC is 1 cm.

20. PQRS is a square and A and B are the mid-points of PS and QR respectively. Prove that PB = RA.

Sol. Given that PQRS is a square and A, B are the mid-points of the sides, PS and QR respectively of the square. PB and RA are joined. To prove that PB = RA.



Let a unit be the side of the square.

Then from ΔASR, we have SR = a, AS = $\frac{1}{2} \times PS$

$= \frac{a}{2}$.

$\therefore$ By Pythagoras' theorem,

$$RA = \sqrt{SR^2 + AS^2}$$

$$= \sqrt{a^2 + \frac{a^2}{4}} = \frac{\sqrt{5}}{2}a$$

Similarly, for ΔPQB, we have

$$PB = \sqrt{PQ^2 + QB^2}$$

$$= \sqrt{a^2 + \frac{a^2}{4}} = \frac{\sqrt{5}}{2}a$$

$\therefore$ PB = RA.

Hence, proved.

Let's Compete

(Page 99)

Multiple-Choice Questions

1. It is given that $\Delta PQR \cong \Delta LMN$ and PQ = 3 cm, $\angle P = 75°$ and $\angle Q = 60°$. Then which of the following is true?

(*a*) ML = 3 cm and $\angle L = 45°$

(*b*) MN = 3 cm and $\angle N = 45°$

(*c*) ML = 3 cm and $\angle N = 45°$

(*d*) ML = 3 cm and $\angle M = 45°$

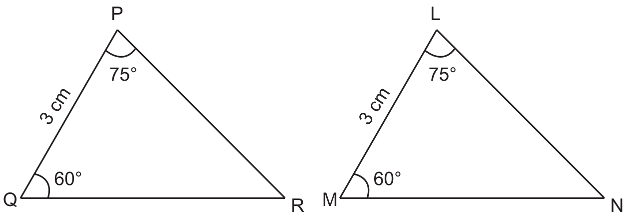
Sol. (*c*) ML = 3 cm and $\angle N = 45°$.

Given that $\Delta PQR \cong \Delta LMN$,

$\angle P = 75°$, $\angle Q = 60°$,

$\therefore \quad \angle L = \angle P = 75°$

and $\angle M = \angle Q = 60°$ and LM = PQ = 3 cm.



Also, $\quad \angle R = 180° - \angle P - \angle Q$

[By angle sum property of a triangle]

$= 180° - 75° - 60°$

$= 45°$

$\therefore \quad \angle N = \angle R = 45°$

Then ML = 3 cm, $\angle N = 45°$.

2. Different parts of two triangles ABC and PQR are given below. In which of the following cases, the two triangles will be congruent?

(*a*) $\angle A = 60°$, $\angle B = 75°$, $\angle Q = 75°$, $\angle R = 60°$ and AB = PQ

(*b*) $\angle B = 60°$, $\angle C = 70°$, $\angle P = 50°$, $\angle R = 70°$ and BC = QR

(*c*) $\angle B = 90°$, $\angle C = 65°$, $\angle Q = 50°$, $\angle R = 90°$ and AB = QR

(*d*) $\angle A = 35°$, $\angle B = 90°$, $\angle R = 55°$, $\angle Q = 90°$ and BC = PQ

Sol. (*b*) $\angle B = 60°$, $\angle C = 70°$, $\angle P = 50°$, $\angle R = 70°$ and BC = QR.

In case (*a*), we see that AB = PQ.

$\therefore \quad \angle A = \angle P$, $\angle B = \angle Q$ and $\angle C = \angle R$,

If $\Delta ABC \cong \Delta PQR$

Now, $\angle A = 60°$, $\angle B = 75°$,

$\therefore \quad \angle C = 180° - 60° - 75°$

$= 45° \neq \angle R$

$\therefore$ In this case, two triangles are not congruent.

In case (*b*), we see that

BC = QR

$\therefore$ $\angle B = \angle Q$, $\angle C = \angle R$ and $\angle A = \angle P$.

If $\Delta ABC \cong \Delta PQR$.

Now, $\angle B = 60°$, $\angle C = 70°$

$\therefore \quad \angle A = 180° - 60° - 70° = 50°$

$\angle P = 50°$, $\angle R = 70°$

$\therefore \quad \angle Q = 180° - 50° - 70° = 60°$.

$\therefore \quad \angle A = \angle P$, $\angle B = \angle Q$, $\angle C = \angle R$.

$\therefore$ In this case $\Delta ABC \cong \Delta PQR$.

In case (*c*), we see that AB = QR.

$\therefore \quad \angle A = \angle Q$ and $\angle B = \angle R$

if $\Delta ABC \cong \Delta QRP$.

Now, $\quad \angle B = 90°$, $\angle C = 65°$

$\therefore \quad \angle A = 180° - 90° - 65° = 25°$

$\angle Q = 50°$, $\angle R = 90°$

$\therefore \quad \angle P = 180° - 90° - 50° = 40°$.

$\therefore$ We see that $\angle A \neq \angle Q$.

$\therefore$ In this case, two triangles are not congruent.

In case (*d*), we see that BC = PQ.

$\angle B = \angle P$ and $\angle C = \angle Q$ if $\Delta ABC \cong \Delta RPQ$.

Now, $\angle A = 35°$, $\angle B = 90°$.

$\therefore \quad \angle C = 180° - 35° - 90° = 45°$

and $\quad \angle R = 55°$, $\angle Q = 90°$.

$\therefore \quad \angle P = 180° - 90° - 55° = 35°$.

We see that $\angle B \neq \angle P$

$\therefore$ In this case also, the two triangles are not congruent.

$\therefore$ The two triangle are congruent only in case (*b*).

3. Two line segments AB and CD intersect each other at a point P. Then P will be the middle point of both AB and CD if

(*a*) $\angle DAB = \angle CBA$ and AD = BC

(*b*) $\angle DAB = \angle CBA$ and AD = 2BC

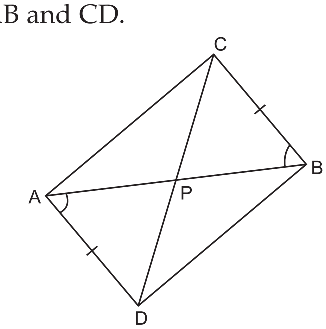
(*c*) $\angle ADC = \angle DCB$ and BC = 2AD

(*d*) $\angle ADC = \angle DCB$ and BC = $\frac{1}{2}$ AD

Sol. (*a*) $\quad \angle DAB = \angle CBA$ and AD = BC

Given that AB and CD are two line segments intersecting each other at P.

To find the condition that P is the mid-point of both AB and CD.



Construction: We join AC, DB, AD and BC to form a quadrilateral.

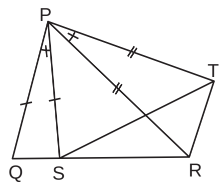
We see that AB and CD are the two diagonals of the quadrilateral which will bisect each other at P only if the quadrilateral is a parallelogram. We also know that a quadrilateral is a parallelogram if a pair of opposite sides is equal and parallel. Now, if ∠DAB = alternate ∠CBA, the AD ∥ CB.

Also, AD is given to be equal is CB.

∴ The figure ACBD must be a parallelogram.

∴ In this case only, the given line segments will bisect each other at P. Hence, (a) is the only correct choice.

4. In the given figure, PR = PT, PQ = PS and ∠QPS = ∠TPR. If ∠PRQ = 40°, then the measure of ∠PTS is

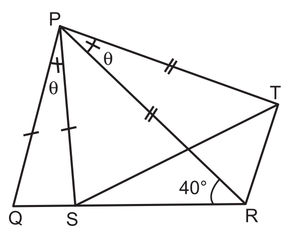


(*a*) 80° (*b*) 20°

(*c*) 60° (*d*) 40°

Sol. (*d*) 40°

Given that PQ = PS, PR = PT and ∠QPS = ∠TPR. Also, ∠PRQ = 40°. Q, S and R lie on the same line segment. RT is joined and TS are joined. To find the measure of ∠PTS.



Let ∠QPS = ∠TPR = θ.

∴ ∠QPR = ∠QPS + ∠SPR

= θ + ∠SPR

= ∠SPT …(1)

Now, in ΔPQR and ΔPST, we have

PQ = PS

PR = PT

∠QPR = ∠SPT [From (1)]

Hence, by SAS congruence criterion, we have

ΔPQR ≅ ΔPST

∠PTS = ∠PRQ [By CPCT]

= 40°

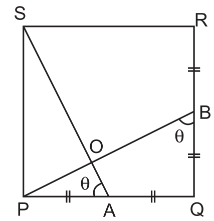
5. A is the mid-point of the side PQ and B is the mid-point of the side QR of a square PQRS. If PB and SA intersect each other at a point O within the square, then the measure of ∠AOB is

(*a*) 100° (*b*) 90°

(*c*) 75° (*d*) 60°

Sol. (*b*) 90°

Given that A is the mid-point of PQ and B is the mid-point of QR, where PQRS is a square. PB and SA intersect each other at the point O. To find the measure of ∠AOB.



In ΔSPA and ΔPQB, we have

SP = PQ

[∵ Sides of a square are equal]

PA = QB

[∵ A is the mid-point of PQ]

∠SPA = ∠PQB = 90°

∴ By SAS congruence criterion, we have

ΔSPA ≅ ΔPQB

∠PAS = ∠QBP [By CPCT]

= θ (say)

and ∠PSA = ∠QPB [By CPCT]

= 90° – θ

Now, in ΔPOA, exterior angle AOB

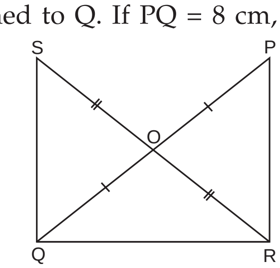
= ∠OPA + ∠OAP

= ∠QPB + ∠PAS

= 90° – θ + θ = 90°

∴ ∠AOB = 90°.

6. In the given figure, ΔPRQ is a right triangle in which ∠R = 90°. O is the mid-point of PQ. R is joined to O and produced to a point S such that OS = OR. Point S is joined to Q. If PQ = 8 cm, then OS is equal to



(*a*) 8 cm

(*b*) 6 cm

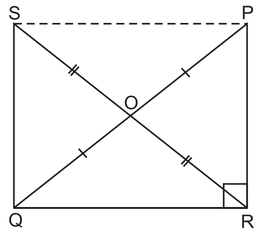
(*c*) 4 cm

(*d*) 2 cm

Sol. (*c*) 4 cm

Given that PRQ is a right triangle with ∠PRQ = 90°. O is the middle point of PQ. R is joined is O and produced to a point S such that OS = OR. SQ is joined. It is given that PQ = 8 cm.

To find the length of OS.



Construction: We join SP.

In ΔOQS and ΔOPR, we have

$$OQ = OP$$
$$OS = OR$$
$$\angle SOQ = \angle ROP$$

[Vertically opposite angles]

By SAS congruence criterion,

$$\Delta OQS \cong \Delta OPR$$

⇒ QS = PR [By CPCT]

and ∠SQO = ∠RPO [By CPCT]

But these two are alternate angles.

∴ SQ ∥ PR

$$\angle SQR = 180° - \angle PRQ$$
$$= 180° - 90°$$
$$= 90°.$$

Hence, the figure PRQS is a rectangle or a square with diagonals QP and RS bisecting each other at O.

∴ PQ = RS

⇒ $\frac{1}{2}$ PQ = $\frac{1}{2}$ RS

⇒ 4 cm = OS.

∴ The required length of OS is 4 cm.

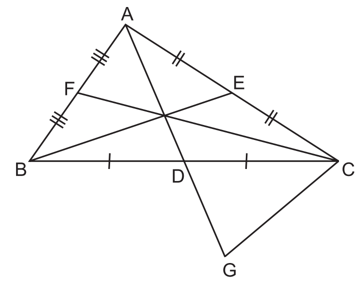
7. In any triangle, if S denotes the sum of three medians and P denotes the perimeter of the triangle, then the true statement is

(*a*) $\frac{1}{4} < \frac{P}{S} < 1$ (*b*) $\frac{1}{2} < \frac{P}{S} < \frac{3}{4}$

(*c*) $\frac{P}{S} = \frac{3}{4}$ (*d*) $\frac{P}{S} > 1$

Sol. (*d*) $\frac{P}{S} > 1$

Let ABC be any triangle with BC = *a*, CA = *b* and AB = *c* units. Let D, E and F be the mid-points of BC, CA and AB respectively so that AD, BE and CF are their medians. Given that S = AD + BE + CF and P = BC + CA + AB = *a* + *b* + *c*.



To find a relation between P and S.

Construction: We produce AD to G such that AD = DG.

We join GC.

In ΔADB and ΔGDC, we have

$$AD = GD$$
$$\sqrt{BD} = CD$$
$$\angle ADB = \angle GDC$$

[Vertically opposite angles]

∴ By SAS congruence criterion, ΔADB ≅ ΔGDC.

AB = GC [By CPCT]

⇒ GC = AB = *c*

Now, in ΔACG, we have

$$AC + CG > AG$$

⇒ $b + c > 2AD$ …(1)

Similarly, we can show that

$c + a > 2BE$ …(2)

and $a + b > 2CF$ …(3)

Adding (1), (2) and (3), we get

$$2(a + b + c) > 2(AD + BE + CF)$$

⇒ $a + b + c > AD + BE + CF$

⇒ P > S

⇒ $\frac{P}{S} > 1$

8. Q is a point on side MN of ΔLMN such that LQ bisects ∠MLN. Then

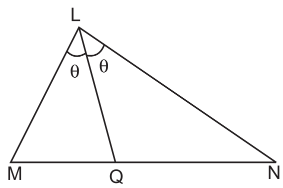
(*a*) ML > MQ (*b*) MQ = NQ
(*c*) MQ > ML (*d*) NQ > LN

Sol. (*a*) ML > MQ

Given that Q is a point on side MN of ΔLMN such that LQ bisects ∠MLN.

To find relations between the pairs ML and MQ; MQ and NQ and NQ and LN.

Let ∠MLQ = ∠NLQ = θ.



Then in ΔLNQ,

the exterior ∠LQM = θ + ∠N > θ or ∠N …(1)

and in LMQ,

the exterior ∠LQN = θ + ∠M > θ or ∠M …(2)

From (1), ∠LQM > ∠MLQ

∴ ML > MQ

∴ (*a*) is correct and (*c*) is wrong, and

From (2), ∠LQN > ∠QLN

∴ LN > NQ

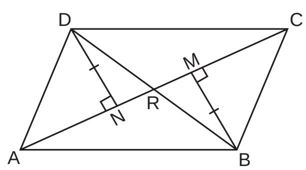
∴ (*d*) is wrong.

(*b*) is clearly not correct, since if MQ = NQ, then ΔMLN will be either an isosceles triangle or an equilateral triangle.

Hence, (*a*) is the only correct choice.

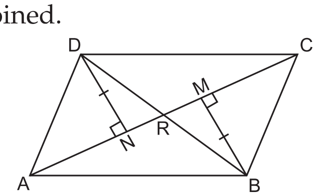
9. In quadrilateral ABCD, BM and DN are drawn perpendicular to AC such that BM = DN. If BR = 8 cm, then the length of BD is

(*a*) 8 cm
(*b*) 12 cm
(*c*) 16 cm
(*d*) 6 cm



Sol. (*c*) 16 cm

Given that ABCD is a quadrilateral, AC is a diagonal of the quadrilateral, M and N are points on AC such that BM ⊥ AC and DN ⊥ AC and BM = DN. To find the length of BD if BR = 8 cm. BD is joined.



In ΔDNR and ΔBMR, we have

DN = BM

∠DNR = BRM = 90°

ΔDRN = ΔBRM

[Vertically opposite angles]

∴ By AAS, ΔDNR ≅ ΔBMR.

⇒ DR = BR [By CPCT]

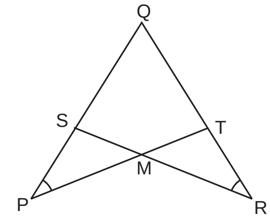
∴ BD = 2BR

= 2 × 8 cm

= 16 cm

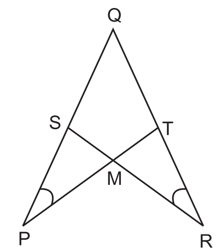
10. In the given figure, ∠QPM = ∠QRM and QP = QR. Then

(*a*) ΔPQT ≅ ΔQRS
(*b*) ΔPQT ≅ ΔRQS
(*c*) ΔPQT ≅ ΔSQR
(*d*) ΔPQT ≅ ΔSRQ



Sol. (*b*) ΔPQT ≅ ΔQRS

Given that PQ and RQ are two line segments such that PQ = RQ and they meet each other at Q. S and T are two points on PQ and RQ respectively such that ∠QPT = ∠QRS. Let PT and RS intersect each other at M.



To find the triangle out of ΔRQS, ΔQRS, ΔSQR and ΔSRQ which is congruent to ΔPQT.

Since it is given that PQ = RQ, it is expected that

ΔPQT ≅ ΔRQS

Now, in ΔPQT and ΔRQS, we see that

PQ = RQ

∠QPT = ∠QRS

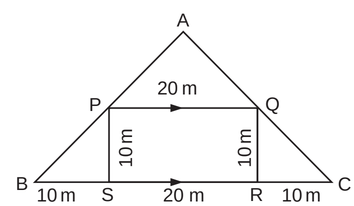
∠TQP = ∠SQR [Common]

∴ ΔPQT ≅ ΔRQS

Value-based Question (Optional)

(Page 99)

1. An old man is the owner of a triangular plot of land ABC as shown in the given figure. He divided the whole triangular plot into four parts. The first two parts are two triangles PBS and QRC with dimensions shown in the figure.



The third part is another triangle APQ where PQ ∥ BC and the fourth part is a rectangle PQRS with dimensions shown in the figure.

The old man had one son and one daughter. He donated the triangular part PBS to his daughter and another part ΔQRC to his son. He kept the upper triangular part APQ for himself.

He reserved the space PQRS in the shape of a rectangle for building an old age home, a nursery for plants and a Yoga centre.

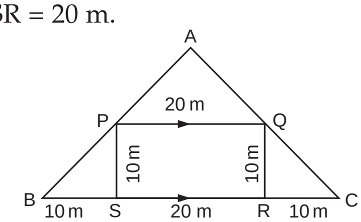
(*a*) Is he justified in donating his two children two triangular parts equally? Explain why.

(*b*) What type of triangle is ΔABC with respect to angles?

(*c*) What is the total perimeter of the entire triangle ABC?

(*d*) Write the values exhibited by the old man.

Sol. We see that in ΔABC, P and Q are two points on AB and AC respectively, PQ ∥ BC, PQ = 20 m, QR = 10 m where S and R are two points on BC and SR = 20 m.



BS = 10 m = RC, and ∠PSB = 90°.

$\therefore \quad \angle PBC = \theta \text{(say)}$

$= \angle SBP$

$\therefore \quad 2\theta = 90°$

$\Rightarrow \quad \theta = 45°$

Similarly, $\because \quad RC = RQ$

$\therefore \quad \angle C = 45° = \angle B$

$\therefore \quad \angle B + \angle C = 90°$

$\therefore \quad \angle A = 90°$

(*b*) ΔABC is a right-angled triangle with ∠BAC = 90° and AB = AC

(*a*) We see that the area of each of triangle PBS and QRC is $\frac{1}{2} \times 10 \times 10 \text{ m}^2 = 50 \text{ m}^2$.

Hence, the old man donated equal amount of land to his two children.

So, in this respect he is justified.

(*c*) Now, since AB = AC and ∠BAC = 90°,

$\therefore$ By Pythagoras' theorem we have

$$AB^2 + AC^2 = BC^2$$

$$\Rightarrow \quad 2AB = 40^2$$

$$\Rightarrow \quad AB^2 = 800$$

$$\therefore \quad AB = AC = 20\sqrt{2}$$

$\therefore$ Required perimeter of ΔABC

$$= AB + AC + BC$$

$$= \left(20\sqrt{2} + 20\sqrt{2} + 40\right) \text{m}$$

$$= 40\left(\sqrt{2} + 1\right) \text{m.}$$

(*d*) Empathy, caring, social responsibility, environmental protection and kindness to his son and daughter equally.

8

Quadrilaterals

Checkpoint (Page 102)

1. Check whether the following groups of four angles represent the angles of a quadrilateral or not.
   (*a*) 80°, 100°, 95°, 85°
   (*b*) 47°, 86°, 92°, 95°
   (*c*) 31°, 79°, 118°, 132°

Sol. We know that the sum of four angles of a quadrilateral is 360°.

(*a*) We see that

$$80° + 100° + 95° + 85° = 180° + 180° = 360°$$

Hence, the given angles are the angles of a quadrilateral.

(*b*) We see that

$$47° + 86° + 92° + 95° = 320° < 360°$$

Hence, the given angles are not the angles of any quadrilateral.

(*c*) We see that

$$31° + 79° + 118° + 132° = 110° + 250° = 360°$$

Hence, the given angles are the angles of a quadrilateral.

2. Three angles of quadrilateral are 57°, 120° and 73°. Find the measure of the fourth angle.

Sol. Let the fourth angle be $x°$.

$\therefore \quad 57° + 120° + 73° + x° = 360°$

$\Rightarrow \quad 250° + x° = 360°$

$\Rightarrow \quad x° = 360° - 250° = 110°$

Hence, the required angle is 110°.

3. The angles of a quadrilateral are in the ratio 3 : 4 : 5 : 6. Find the measures of the angles of the quadrilateral.

Sol. Let the angles of the quadrilateral be $(3x)°$, $(4x)°$, $(5x)°$ and $(6x)°$.

$\therefore \quad 3x + 4x + 5x + 6x = 360$

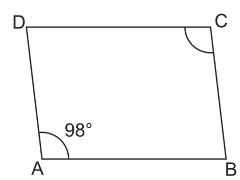
$\Rightarrow \quad 18x = 360$

$\Rightarrow \quad x = \frac{360}{18} = 20$

Hence, the required angles are $(3 \times 20)° = 60°$, $(4 \times 20)° = 80°$, $(5 \times 20)° = 100°$ and $(6 \times 20)° = 120°$.

4. In a parallelogram ABCD, ∠A = 98°. Find ∠B, ∠C and ∠D.

Sol. Let ABCD be a parallelogram and ∠A = 98°.



$\because \quad \angle A = \angle C$

$\therefore \quad \angle C = 98°$

Also, $\quad \angle A + \angle B = 180°$

$\Rightarrow \quad \angle B = 180° - \angle A = 180° - 98° = 82°$

$\therefore \quad \angle D = \angle B = 82°$

Hence, ∠B = 82°, ∠C = 98° and ∠D = 82°.

5. Find the measures of angles of a parallelogram if one angle is 40° less than thrice the smallest angle.

Sol. Let the smaller angle be $x°$. Then another angle is $(3x - 40)°$.

$\therefore$ $(2x)^\circ + 2(3x - 40)^\circ$ = Sum of four angles of the parallelogram

$= 360^\circ$

$\Rightarrow \quad x + 3x - 40 = 180$

$\Rightarrow \quad 4x = 180 + 40$

$\Rightarrow \quad 4x = 220$

$\therefore \quad x = \frac{220}{4} = 55$

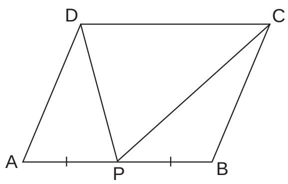
Also, $(3x - 40)^\circ = (3 \times 55 - 40)^\circ$

$= (165 - 40)^\circ$

$= 125^\circ$

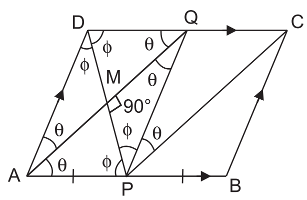
Hence, the required angles are 55° and 125°.

6. In the given figure, ABCD is a parallelogram. If AB = 2AD and P is the mid-point of AB, then find $\angle$CPD.



Sol. Given that ABCD is a parallelogram and P is a point on AB such that AB = 2AD and P is a mid-point of AB.

To find the measure of $\angle$CPD.



Construction: We draw PQ $\parallel$ AD to cut DC at Q.

We see that APQD is a rhombus and we know that the two diagonals AQ and PD of the rhombus bisect each other at right angles.

$\therefore \quad \angle PMQ = 90^\circ$

Let PAD = 2θ and $\angle ADQ = 2\phi$.

$\therefore \quad 2\theta + 2\phi = 180^\circ$

$\Rightarrow \quad \phi + \theta = 90^\circ \quad \ldots(1)$

$\therefore \quad \angle PAQ = \angle QAD = \theta$

and $\quad \angle AQD = \angle AQP = \theta$

Also, $\quad \angle QPC = \angle AQP = \theta$

Also, $\quad \angle ADP = \angle QDP = \phi$

and $\quad \angle APD = \angle DPQ = \phi$

Now, in ΔPMQ, we have

$\angle MQP + \angle MPQ = \theta + \phi$

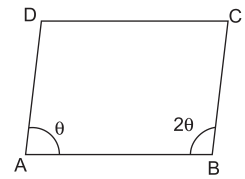
$\Rightarrow \quad \angle QPC + \angle MPQ = \theta + \phi$

$\Rightarrow \quad \angle CPD = \theta + \phi = 90^\circ \quad$ [From (1)]

Hence, the required measure of $\angle$CPD is 90°.

7. If an angle of a parallelogram is half of its adjacent angle, find the angles of the parallelogram.

Sol. Let ABCD be a parallelogram with $\angle A = \theta$ and $\angle B = 2\theta$.



Now, $\quad \angle A + \angle B = 180^\circ$

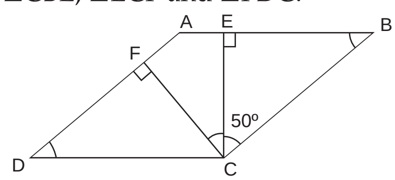
$\Rightarrow \quad 2\theta + \theta = 180^\circ$

$\Rightarrow \quad 3\theta = 180^\circ$

$\Rightarrow \quad \theta = \frac{180^\circ}{3} = 60^\circ$

Hence, the required angles are 60°, 120°, 60° and 120°.

8. In the given figure, ABCD is a parallelogram. If $\angle BCE = 50^\circ$, $\angle BEC = 90^\circ$ and $\angle CFD = 90^\circ$, then find $\angle$CBE, $\angle$ECF and $\angle$FDC.



Sol. Given that ABCD is a parallelogram, CE $\perp$ AB, CF $\perp$ AD and $\angle BCE = 50^\circ$.

To find the measures of $\angle$CBE, $\angle$ECF and $\angle$FDC.

In ΔECB, we have

$\angle CBE = 90^\circ - 50^\circ = 40^\circ$

[$\because$ $\angle CBE + \angle ECB = 90^\circ$]

Also, $\quad \angle FDC = \angle CBE$

[$\because$ ABCD is a parallelogram]

$= 40^\circ$

$\therefore$ In ΔFDC, $\angle FCD = 90^\circ - \angle FDC$

$= 90^\circ - 40^\circ = 50^\circ$

Now, $\quad \angle DAB + \angle ADC = 180^\circ$

$\Rightarrow \quad \angle DAB + 40^\circ = 180^\circ$

$\therefore \quad \angle DAB = 180^\circ - 40^\circ$

$= 140^\circ$

$\therefore \quad \angle DCB = \angle DAB$

$= 140^\circ$

$\Rightarrow \quad \angle FCD + \angle ECF + \angle ECB = 140^\circ$

$\Rightarrow \quad 50° + \angle ECF + 50° = 140°$

$\Rightarrow \quad \angle ECF = 140° - 100°$

$= 40°$

Hence, the required measures of ∠CBE, ∠ECF and ∠FDC are 40°, 40° and 40° respectively.

Milestone 1

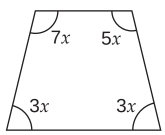
(Page 108)

Multiple-Choice Questions

1. The value of x in the given figure is

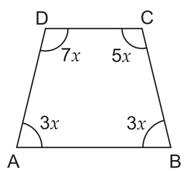
(*a*) 10° (*b*) 20°

(*c*) 30° (*d*) 40°



Sol. (*b*) 20°

Given that four angles A, B, C and D of a quadrilateral ABCD are $3x$, $3x$, $5x$ and $7x$.



To find the value of x.

We have

$$3x + 3x + 5x + 7x = 360°$$

$$\Rightarrow \quad 18x = 360°$$

$$\therefore \quad x = \frac{360°}{18} = 20°$$

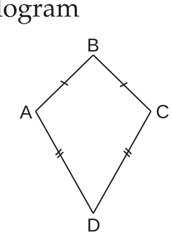
2. In a quadrilateral ABCD, if AB = BC and CD = DA, then quadrilateral ABCD is a

(*a*) trapezium (*b*) rhombus

(*c*) kite (*d*) parallelogram

Sol. (*c*) kite

Quadrilateral ABCD is a kite, because AB = BC and AD = CD.

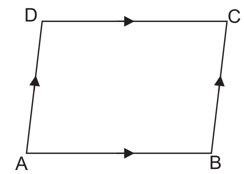


Very Short Answer Type Questions

3. In a parallelogram ABCD, determine the sum of the angles of C and D.

Sol. Let ABCD is a parallelogram. Then AB ∥ DC and AD ∥ BC.

To find the measure of ∠C + ∠D.



∵ AD ∥ BC and DC is their transversal,

∴ sum of the interior angles, i.e.

$\angle ADC + \angle BCD = 180°$

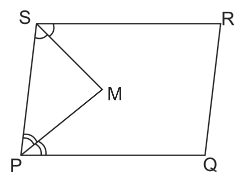
$\Rightarrow \quad \angle C + \angle D = 180°$

Hence, the sum of the angles of C and D is 180°.

4. In a parallelogram PQRS, the bisectors of ∠P and ∠S intersect at M. What is the measure of ∠PMS?

Sol. Given that PQRS is a parallelogram. PM and SM, the bisectors of ∠SPQ and ∠PSR respectively intersect each other at M.

To find the measure of ∠PMS.



We know that

$$\angle SPQ + \angle PSR = 180°$$

$$\Rightarrow \quad \frac{1}{2}\angle SPQ + \frac{1}{2}\angle PSR = 90°$$

$$\Rightarrow \quad \angle SPM + \angle PSM = 90°$$

In ΔPMS, we have

$$\angle PMS = 180° - (\angle SPM + \angle PSM)$$

$$= 180° - 90°$$

$$= 90°$$

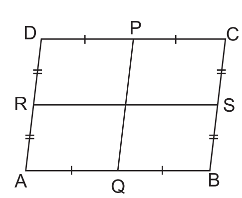
∴ The required measure of ∠PMS is 90°.

Short Answer Type-I Questions

5. Prove that the line segment joining the mid-points of the opposite sides of a parallelogram is parallel to the other pair of parallel sides.

Sol. Given that ABCD is a parallelogram and Q, S, P, R are respectively the mid-points of AB, BC, CD and DA. PQ and RS are joined.

To prove that PQ ∥ AD and RS ∥ AB.



$\because$ ABCD is a parallelogram,

$\therefore \quad AB \parallel CD$

and $\quad AB = DC$

$\therefore \quad AQ \parallel DP$

and $\quad \frac{1}{2} AB = \frac{1}{2} DC$

$\Rightarrow \quad AQ = DP$

$\therefore$ AQPD must be a parallelogram.

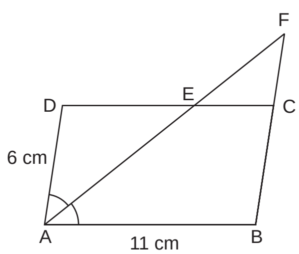
$\therefore \quad PQ \parallel AD$

Similarly, we can show that RS $\parallel$ AB.

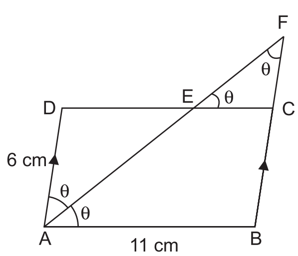
Hence, proved.

6. In a parallelogram ABCD, AB = 11 cm and AD = 6 cm. The bisector of $\angle$A meets DC at E. AE and BC produced meet at the point F.

Find the ratio $\frac{CF}{CB}$.



Sol. Given that ABCD is a parallelogram, AB = 11 cm, AD = 6 cm and E is a point on DC such that AE is the bisector of $\angle$DAB. AE and BC produced meet each other at F.



To find the ratio $\frac{CF}{CB}$.

Let $\quad \angle DAE = \angle EAB = \theta$

Then $\quad \angle FEC = \angle BAE$

[Corresponding angles]

$= \theta$

Also, $\quad \angle EFC = \angle DAE$ [Alternate angles]

$= \theta$

$\therefore$ In ΔABF, $\quad BF = AB = 11$ cm

$\therefore \quad CF = BF - BC$

$= BF - AD \quad [\because BC = AD]$

$= (11 - 6)$ cm

$= 5$ cm

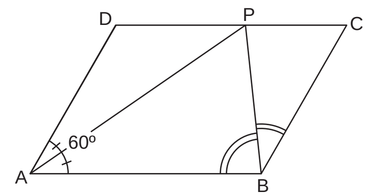
and $\quad CB = DA$

$= 6$ cm

$\therefore$ Required ratio of $\frac{CF}{CB} = \frac{5}{6}$.

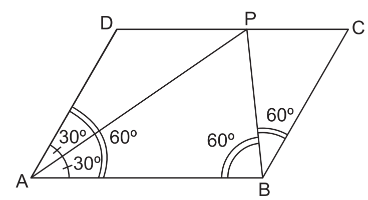
Short Answer Type-II Questions

7. In the given figure, ABCD is a parallelogram and $\angle A = 60°$. If the bisectors of $\angle$A and $\angle$B meet at P, prove that AD = DP, PC = BC and DC = 2AD.



Sol. Given that ABCD is a parallelogram and $\angle A = 60°$. P is a point on DC such that PA and PB are respectively the bisectors of $\angle$DAB and $\angle$ABC.

To prove that AD = DP, PC = BC and DC = 2AD.



We have $\quad \angle DAP = \angle PAB$

$= \frac{60°}{2} = 30° \quad \ldots(1)$

Also, $\quad \angle APD = \angle PAB$

[Alternate angles, $\because$ AB $\parallel$ DC]

$\therefore \quad \angle APD = 30°$

$\therefore \quad \angle APD = \angle DAP \quad$ [From (1)]

$\therefore \quad AD = DP$

Now, $\quad \angle A + \angle B = 180°$

$\therefore \quad \angle B = 180° - \angle A$

$= 180° - 60° = 120°$

$\therefore \quad \angle PBC = \angle PBA$

$=$ alternate $\angle CPB$

[$\because$ AB $\parallel$ DC]

$\therefore \quad PC = BC$

Also, $\quad DC = DP + PC$

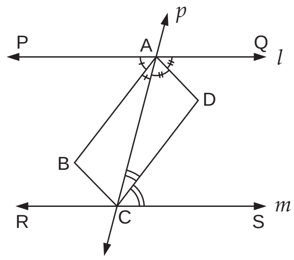
$= AD + BC$

$= AD + AD = 2AD$

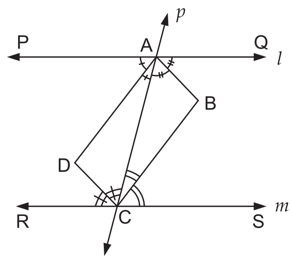
$\therefore \quad DC = 2AD$

Hence, proved.

8. Two parallel lines l and m are intersected by a transversal p as shown in the figure. Show that the quadrilateral formed by the bisectors of interior angles is a rectangle. **[CBSE SP 2013]**



Sol. Given that PQ and RS denoted by l and m respectively are two parallel lines and AC denoted by p is a transversal. AD and CD are the bisectors of ∠PAC and ∠RCA respectively.



Also, AB and CB are the bisectors of ∠QAC and ∠SCA respectively forming a quadrilateral ABCD.

To prove that ABCD is a rectangle.

We have

$$\angle PAC = \angle SCA \quad \text{[Alternate angles, } \because PQ \parallel RS]$$

$$\Rightarrow \quad \frac{1}{2}\angle PAC = \frac{1}{2}\angle SCA$$

$$\Rightarrow \quad \angle DAC = \angle BCA$$

$$\therefore \quad AD \parallel BC \quad \ldots(1)$$

Also, $$\angle QAC = \angle RCA \quad \text{[Alternate angles, } \because PQ \parallel RS]$$

$$\Rightarrow \quad \frac{1}{2}\angle QAC = \frac{1}{2}\angle RCA$$

$$\Rightarrow \quad \angle BAC = \angle DCA$$

$$\therefore \quad AB \parallel DC \quad \ldots(2)$$

∴ From (1) and (2), we see that ABCD is a parallelogram.

Again, $$\angle PAC + \angle RCA = 180° \quad \text{[Sum of interior angles between two parallel lines]}$$

$$\Rightarrow \quad \frac{1}{2}\angle PAC + \frac{1}{2}\angle RCA = \frac{180°}{2} = 90°$$

$$\Rightarrow \quad \angle DAC + \angle DCA = 90°$$

∴ In ΔADC,

$$\angle ADC = 180° - (\angle DAC + \angle DCA)$$
$$= 180° - 90°$$
$$= 90°$$

∴ The parallelogram ABCD is a rectangle.

Hence, proved.

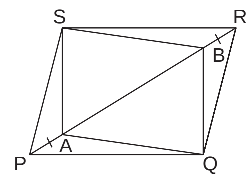
Long Answer Type Questions

9. In parallelogram PQRS, two points A and B are taken on the diagonal PR such that RB = PA as shown in the given figure. Show that

(*a*) ΔSBR ≅ ΔQAP (*b*) SB = QA

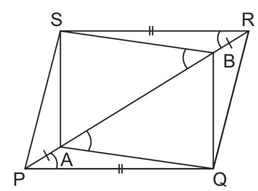
(*c*) ΔSAP ≅ ΔQBR (*d*) SA = QB

(*e*) SBQA is a parallelogram



Sol. Given that PQRS is a parallelogram and PR is a diagonal. A and B are two points on PR such that PA = RB.

To prove that (*a*) ΔSBR ≅ ΔQAP (*b*) SB = QA (*c*) ΔSAP ≅ ΔQBR (*d*) SA = QB (*e*) SBQA is a parallelogram.



(*a*) In ΔSBR and ΔQAP, we have

RB = PA [Given]

SR = QP [∵ PQRS is a parallelogram]

∠SRB = alternate ∠QPA.

∴ By SAS congruence criterion, we have

ΔSBR ≅ ΔQAP

(*b*) ∴ SB = QA [By CPCT]

(*c*) In ΔSAP and ΔQBR, we have

PA = RB [Given]

SP = QR [∵ PQRS is a parallelogram]

∠SPA = alternate ∠QRB

$\therefore$ By SAS congruence criterion,

$$\Delta SAP \cong \Delta QBR$$

(*d*) $\therefore$ $SA = QB$ [By CPCT]

(*e*) $\because$ $\Delta SBR \cong \Delta QAP$,

$\therefore$ $\angle SBR = \angle QAP$

$\Rightarrow$ $180° - \angle SBA = 180° - \angle QAB$

$\Rightarrow$ $\angle SBA = \angle QAB$

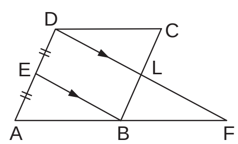
But these are alternate angles.

$\therefore$ $SB \parallel AQ$

Similarly, from the congruency of ΔSAP and ΔQBR we can show that $AS \parallel BQ$.

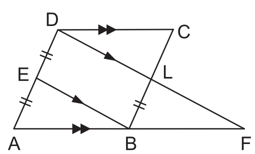
$\therefore$ The figure SBQA is a parallelogram.

10. In the given figure, ABCD is a parallelogram and E is mid-point of AD. DL || BE meets AB produced at F. Prove that B is the mid-point of AF and EB = FL. **[CBSE SP 2012]**



Sol. Given that ABCD is a parallelogram and E is the mid-point of AD. L is a point on BC such that $DL \parallel BE$. DL and AB produced meet each other at the point F.

To prove that B is the mid-point of AF and EB = FL.



$\because$ EBLD is a parallelogram,

$\therefore$ $ED = BL$

Also, since E is the mid-point of AD,

$\therefore$ $AE = ED$

$\therefore$ $AE = BL$

Now, in ΔABE and ΔBFL, we have

$AE = BL$

$\angle EAB =$ corresponding $\angle LBF$

[$\because AD \parallel BC$]

$\angle EBA = \angle LFB$ [$\because EB \parallel DF$]

$\therefore$ By AAS congruence criterion,

$$\Delta ABE \cong \Delta BFL$$

$\therefore$ $EB = LF = FL$

and $AB = BF$ [By CPCT]

$\therefore$ B is the mid-point of AF.

Milestone 2

(Page 111)

Multiple-Choice Questions

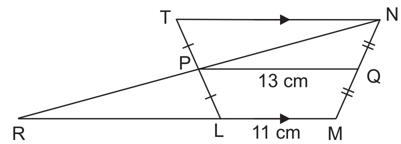
1. LMNT is a trapezium in which LM || TN. P and Q are the mid-points of TL and NM respectively. If LM = 11 cm, PQ = 13 cm, then the length of TN is

(*a*) 18 cm (*b*) 17 cm

(*c*) 16 cm (*d*) 15 cm

Sol. (*d*) 15 cm

Given that P and Q are respectively the mid-points of non parallel sides TL and MN of a trapezium LMNT, where $LM \parallel TN$, LM = 11 cm and PQ = 13 cm.



To prove that $PQ \parallel TN$ or LM and $PQ = \frac{1}{2}(LM + TN)$

and hence to find the length of TN.

Construction: We join NP and produce it to cut ML produced at the point R.

In ΔTPN and ΔLPR, we have

$TP = LP$

$\angle TNP =$ alternate $\angle LRP$

[$\because RM \parallel TN$]

$\angle TPN = \angle LPR$

[Vertically opposite angles]

$\therefore$ By AAS congruence criterion, we have

$$\Delta TPN \cong \Delta LPR$$

$\Rightarrow$ $NP = RP$ [By CPCT]

and $TN = LR$ ...(1)

i.e., P is the mid-point of RN.

Now, in ΔRNM, P and Q are the mid-points of RN and MN respectively.

$\therefore$ By the mid-point theorem of a triangle,

$PQ \parallel RM$ or TN

and $PQ = \frac{1}{2} \times RM$

$= \frac{1}{2}(RL + LM)$

$= \frac{1}{2}(TN + LM)$ [From (1)]

$\Rightarrow$ $13 = \frac{1}{2}(TN + 11)$

$\Rightarrow \quad 26 = TN + 11$

$\Rightarrow \quad TN = 26 - 11$

$= 15$

Hence, the required length of TN is 15 cm.

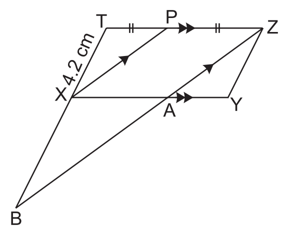
2. P is the mid-point of the side TZ of a parallelogram XYZT. A line through Z parallel to PX intersects XY at A and TX produced at B. If TX = 4.2 cm, then the length of TB is

(*a*) 4.2 cm (*b*) 8.4 cm

(*c*) 12.6 cm (*d*) 6.3 cm

Sol. (*b*) 8.4 cm

Given that P is the mid-point of the side TZ of the parallelogram XYZT. A is a point on XY such that ZA ∥ PX. ZA produced intersects TX produced at the point B.



To find the measure of TB if TX = 4.2 cm.

In ΔTBZ, P is the mid-point of TZ and PX ∥ ZB.

∴ By the converse of mid-point theorem for a triangle, X is the mid-point of TB.

$\therefore \quad TB = TX + XB$

$= TX + TX$

$= 2TX$

$= 2 \times 4.2$

$= 8.4$

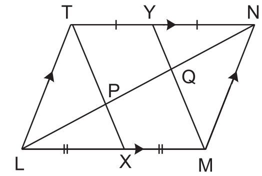
∴ The required length of TB is 8.4 cm.

Very Short Answer Type Questions

3. LMNT is a parallelogram. X and Y are respectively the mid-points of the sides LM and TN. TX and MY meet the diagonal LN at P and Q respectively. If the length of the diagonal LN of the parallelogram is 15 cm, what is the length of PQ?

Sol. Given that LMNT is a parallelogram, X and Y are the mid-points of the sides LM and TN respectively. LN is a diagonal of the parallelogram. TX and YM cut LN at the points P and Q respectively. It is also given that LN = 15 cm.

To find the length of PQ.



We see that $XM = \frac{1}{2} LM = \frac{1}{2} TN$

Also, XM ∥ TY.

∴ The figure TXMY is a parallelogram.

$\therefore \quad TX \parallel MY$

$\Rightarrow \quad TP \parallel QY$

Now, in ΔTNP, Y is the mid-point of TN and TP ∥ QY.

∴ Q is the mid-point of PN

[By mid-point theorem for a triangle]

$\therefore \quad PQ = \frac{1}{2} PN \quad \ldots(1)$

Similarly, in ΔLMQ, ∵ X is the mid-point of LM and PX ∥ QM,

$\therefore \quad PQ = \frac{1}{2} LQ \quad \ldots(2)$

Adding (1) and (2), we get

$2PQ = \frac{1}{2}(PN + LQ)$

$= \frac{1}{2}(PQ + QN + PQ + LP)$

$= \frac{1}{2}(PQ + QN + LP) + \frac{1}{2} PQ$

$= \frac{1}{2} LN + \frac{1}{2} PQ$

$\Rightarrow \quad \left(2 - \frac{1}{2}\right) PQ = \frac{1}{2} LN$

$\Rightarrow \quad 3PQ = LN$

$\Rightarrow \quad PQ = \frac{LN}{3}$

$= \frac{15}{3}$

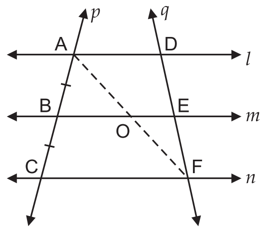
$= 5$

Hence, the required length of PQ is 5 cm.

4. If there are three or more parallel lines and the intercepts made by them on one transversal are equal, then prove that the intercepts made on any other transversal are also equal.

[CBSE SP 2011, 2012]

Sol. Given that *l*, *m*, *n* are three parallel lines cutting the two transversals *p* and *q* at A, B, C and D, E, F respectively.



It is given that intercept AB = intercept BC.

To prove that intercept DE = intercept EF.

Construction: We join AF to cut the line m or BE at O.

In ΔACF, B is the mid-point of AC and BO ∥ CF.

∴ By converse of mid-point theorem for a triangle, O is the mid-point of AF.

Now, in ΔAFD, O is the mid point of AF and OE ∥ AD.

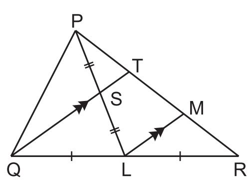
∴ E is the mid-point of DF, i.e. DE = EF.

Hence, proved.

Short Answer Type-I Questions

5. In ΔPQR, PL is the median through P and S is the mid-point of PL. QS produced meets PR at T. Prove that PT = $\frac{1}{3}$ PR.

Sol. Given that PL is a median of a triangle PQR and S is the mid-point of PL. QS produced meets PR at T.



To prove that PT = $\frac{1}{3}$ PR

Construction: We draw LM ∥ QT where M is a point on PR.

In ΔTQR, L is the mid-point of QR and LM ∥ QT.

∴ By converse of mid-point theorem for a triangle, M is the mid-point of TR.

i.e. TM = MR …(1)

Again, in ΔPLM, S is the mid-point of PL and ST ∥ LM.

∴ By converse of mid-point theorem for a triangle, T is the mid-point of PM.

i.e. TM = PT …(2)

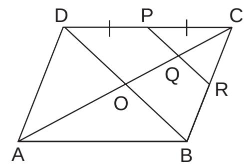
∴ From (1) and (2), we have

PT = TM = MR

i.e. PT = $\frac{1}{3}$ PR

Hence, proved.

6. In the given figure, ABCD is a parallelogram. P is the mid-point of the side DC and Q is a point on AC such that CQ = $\frac{1}{4}$ AC. If PQ produced meets BC at R, prove that R is the mid-point of BC.



Sol. Given that ABCD is a parallelogram, P is the mid-point of DC and Q is a point on the diagonal AC such that CQ = $\frac{1}{4}$ AC. BD is another diagonal of the parallelogram and let the two diagonals meet each other at the point O.

Let PQ produced meet BC at R.

To prove that R is the mid-point of BC.

Since diagonals of a parallelogram bisect each other,

∴ AC = 2 OC

∴ CQ = $\frac{1}{4}$ AC

= $\frac{1}{4}$ × 2 OC

= $\frac{1}{2}$ OC,

i.e. Q is the mid-point of CO.

∴ By mid-point theorem for a triangle, PQ ∥ DO

⇒ PR ∥ DB

Now, in ΔCOB, QR ∥ OB and Q is the mid-point of CO.

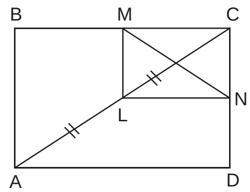
∴ By converse of mid-point theorem of a triangle, R is the mid-point of BC.

Hence, proved.

Short Answer Type-II Questions

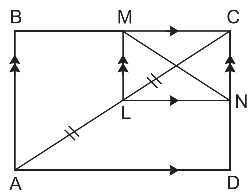
7. In the given figure, ABCD is a rectangle and L is the mid-point of the diagonal AC. LM and LN are respectively parallel to AB and AD. Prove that N is the mid-point of DC. Also prove that

MN = $\frac{1}{2}$ AC.



Sol. Given that ABCD is a rectangle and L is the mid-point of the diagonal AC. LM and LN are respectively parallel to AB and AD, where M and N are points on BC and CD respectively.

To prove that N is the mid-point of DC and $MN = \frac{1}{2} AC$.



In ΔADC, since L is the mid-point of AC and LN $\parallel$ AD, hence, by converse of mid-point theorem, N is also the mid-point of DC.

Again, in ΔABC,

$\because$ L is the mid-point of AC and LM $\parallel$ AB, hence, by converse of mid-point theorem for a triangle, M is also the mid-point of BC.

Now, since ML $\parallel$ CN and LN $\parallel$ MC,

$\therefore$ MLNC is a rectangle with diagonals LC = MN.

Since L is the mid-point of AC,

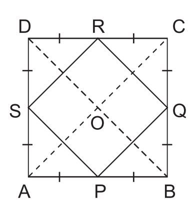
$$\therefore \qquad MN = LC = \frac{1}{2} AC$$

Hence, proved.

8. Quadrilateral ABCD is a rhombus and P, Q, R and S are the mid-points of AB, BC, CD and DA respectively. Prove that PQRS is a rectangle.

Sol. Given that ABCD is a rhombus and P, Q, R, S are the mid-points of AB, BC, CD and DA respectively.

To prove that the quadrilateral PQRS is a rectangle.



Construction: We join the diagonals AC and BD of the rhombus ABCD. Let them intersect each other at the point O.

By mid-point theorem of a triangle, we have

In ΔADC or ΔABC, RS or QP $\parallel$ AC and $RS = \frac{1}{2} AC = QP$

Similarly, in ΔADB or ΔDCB, SP or RQ $\parallel$ DB and $SP = \frac{1}{2} DB = RQ$

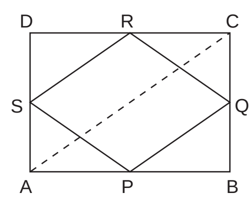
Now, we know that in a rhombus two diagonals are at right angles to each other and unequal in length. Hence, in rhombus ABCD, $AC \perp DB$ and $AC \neq DB$.

$\therefore$ In the quadrilateral PQRS, $RS \perp RQ$, $RS \neq RQ$, but RQ $\parallel$ SP and RQ = SP.

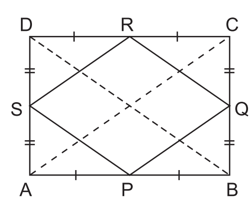
$\therefore$ PQRS is a parallelogram and this parallelogram is a rectangle, since $RS \perp RQ$ and $RS \neq RQ$.

Long Answer Type Questions

9. In the given figure, ABCD is a rectangle and P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively. Prove that quadrilateral PQRS is a rhombus.



Sol. Given that ABCD is a rectangle with AC and DB as its two diagonals. P, Q, R, S are the mid-points of the sides AB, BC, CD and DA respectively. To prove that the quadrilateral PQRS is a rhombus.



In ΔADC, since S and R are the mid-points of AD and DC respectively, hence, by mid-point theorem for a triangle, SR $\parallel$ AC

and $\qquad SR = \frac{1}{2} AC \qquad$ ...(1)

Similarly, from ΔADB,

$$SP = \frac{1}{2} DB = \frac{1}{2} AC \qquad ...(2)$$

[$\because$ Two diagonals AC and DB of a rectangle are equal]

∴ From (1) and (2),

SP = SR

Also, SP ∥ RQ [∵ SP or RQ ∥ DB]

and SR ∥ PQ [∵ SR or PQ ∥ AC]

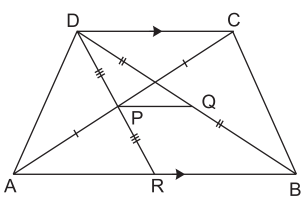
∴ The quadrilateral PQRS is a parallelogram with adjacent sides SP and RS equal. Hence, PQRS is either a rhombus or a square.

But since angle between two diagonals AC and BD of a rectangle is not 90°, hence, angle between SR (∥ AC) and SP (∥ DB) is not 90°. Hence, the parallelogram PQRS must be a rhombus.

10. Prove that the line segment joining the mid-points of the diagonals of a trapezium is parallel to each of the parallel sides and is equal to half the difference of these sides.

Sol. Given that AD and BC are two non parallel sides of a trapezium ABCD where AB ∥ DC. AC and BD are its two diagonals. P and Q are the mid-points of AC and BD respectively.

To prove that PQ ∥ AB or DC and PQ = $\frac{1}{2}$ (AB – DC).



Construction: We join DP and produce it to cut AB at R.

In ΔAPR and ΔCPD, we have

AP = CP

[∵ P is the mid-point of AC]

∠PAR = alternate ∠PCD

[∵ AB ∥ DC and AC is a transversal]

∠APR = ∠CPD

[Vertically opposite angles]

∴ By ASA congruence criterion, we have

ΔAPR ≅ ΔCPD …(1)

∴ AR = CD [By CPCT]

and RP = DP [By CPCT]

Now, in ΔDRB, P and Q are the mid-points of DR and DB respectively. Hence, by mid-point theorem for a triangle, PQ ∥ AB or DC.

Also, PQ = $\frac{1}{2}$ RB

= $\frac{1}{2}$ (AB – AR)

= $\frac{1}{2}$ (AB – CD) [From (1)]

Hence, proved.

Higher Order Thinking Skills (HOTS) Questions

(Page 112)

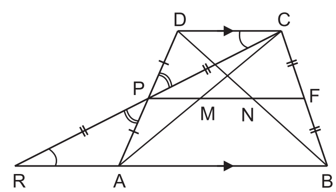
1. Prove that the line segment joining the mid-points of two non parallel sides of a trapezium

(*a*) is parallel to each of the parallel sides of the trapezium,

(*b*) is equal to half of their sum and

(*c*) bisects the two diagonals of the trapezium.

Sol. Given that ABCD is a trapezium, AD and BC are its non parallel sides and AB ∥ DC. P and Q are the mid-points of the sides AD and BC respectively. The line segment PQ intersects the two diagonals AC and BD of the trapezium at the points M and N respectively.



To prove that

(*a*) PQ ∥ AB and DC

(*b*) PQ = $\frac{1}{2}$ (AB + DC)

(*c*) M and N are the mid-points of AC and BD respectively.

Construction: We join CP and produce it to cut BA produced at R.

(*a*) In ΔDPC and ΔAPR, we have

PD = PA [Given]

∠DCP = alternate ∠ARP

[∵ AB ∥ DC and RC is a transversal]

∠DPC = ∠APR

[Vertically opposite angles]

∴ By AAS congruence criterion,

ΔDPC ≅ ΔAPR

∴ CP = RP [By CPCT] …(1)

and DC = AR [By CPCT] …(2)

Now, in ΔCRB, PQ ∥ AB or DC by mid-point theorem for a triangle, since P and Q are the mid-points of CR and CB.

(*b*) Since, P and Q are the mid-points of the sides CR and CB of ΔCRB, hence by mid-point theorem for a triangle,

$$PQ = \frac{1}{2} RB$$
$$= \frac{1}{2} (AR + AB)$$
$$= \frac{1}{2} (DC + AB)$$

[From (2)]

(*c*) In ΔADC, P is the mid-point of AD and PM ∥ DC.

∴ By converse of mid-point theorem for a triangle, M is the mid-point of AC.

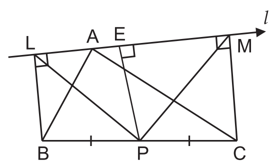
Again, in ΔBDC, Q is the mid-point of BC and QN ∥ DC.

∴ By converse of mid-point theorem for a triangle, N is the mid-point of BD.

Hence, proved.

2. In a ΔABC, BL and CM are drawn perpendiculars to a line segment through A, from B and C respectively. If P is the mid-point of BC, prove that PL = PM.

Sol. Given that ABC is a triangle and LM is a line segment through the point A. BL and CM are drawn perpendiculars to a line segment through A, from B and C respectively. P is the mid-point of BC.



To prove that PL = PM

Construction: Draw PE ⊥ LM

Since BL, PE and CM are perpendiculars to the same line segment LM, they are parallel to one another. Now, BC and LM are transversals between these three parallel line segments. Since intercepts BP and PC between these three parallel line segments are equal, hence, intercepts LE and ME on the other transversal LM will also be equal.

Hence, LE = ME ...(1)

Now, in ΔLEP and ΔMEP, we have

LE = ME [From (1)]

∠LEP = ∠MEP = 90° [By construction]

PE = PE

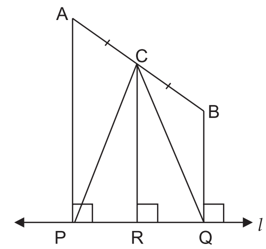
∴ By SAS congruence criterion,

ΔLEP ≅ ΔMEP

∴ PL = PM [By CPCT]

Hence, proved.

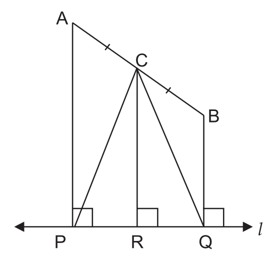
3. In the given figure, AP and BQ are both perpendiculars to the line *l*. C is the mid-point of line segment AB. Prove that CP = CQ.



Sol. Given that AB is a line segment and C is its mid-point. P, R, Q are three points on a line *l* such that AP, CR and BQ are perpendicular to the line *l*. CP and CQ are joined.

To prove that CP = CQ

We see that AP ∥ CR ∥ BQ and PR, RQ are the intercepts on *l* and AC and CB are intercepts on AB.



∵ AC = CB,

∴ PR = RQ ...(1)

Now, in ΔCRP and ΔCRQ, we have

PR = RQ

CR = CR

∠CRP = ∠CRQ

= 90°

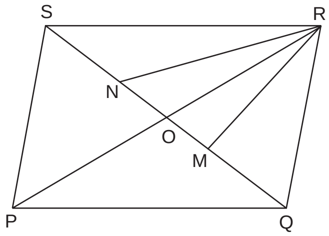
∴ By SAS congruence criterion, we have

ΔCRP ≅ ΔCRQ

∴ CP = CQ [By CPCT]

Hence, proved.

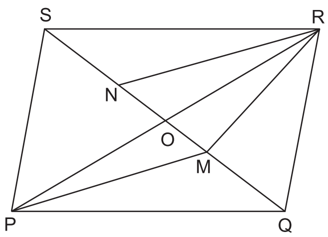
4. In the given figure, PQRS is a parallelogram. The diagonal QS is trisected at M and N. Prove that PM = RN, PM || RN and the line segment MN is bisected by PR.



Sol. Given that PQRS is a parallelogram. The diagonals PR and QS bisect each other at O. N and M are the points of trisection on SQ, so that

$$SN = NM = MQ = \frac{1}{3} SQ \quad \ldots(1)$$

To prove that PM = RN, PM ∥ RN and the line segment MN is bisected by PR.



Construction: We join PM.

We have $SO = OQ = \frac{1}{2} SQ$ …(1)

and PO = RO …(2)

since the diagonals of a parallelogram bisect each other.

$$SN = NM = MQ = \frac{1}{3} SQ$$

[Given] …(3)

$\therefore$ $SO = \frac{1}{2} SQ = OQ$

$\therefore$ $ON = OS - SN = \frac{1}{2} SQ - \frac{1}{3} SQ$ [From (1) and (3)]

$= \frac{SQ}{6}$

$OM = OQ - MQ$

$= \frac{1}{2} SQ - \frac{1}{3} SQ$

$= \frac{SQ}{6}$ [From (1) and (3)]

$\therefore$ ON = OM …(4)

Hence, MN is bisected by PR at O.

Now, in ΔRON and ΔPOM, we have

RO = PO [From (2)]

ON = OM [From (4)]

∠RON = ∠POM [Vertically opposite angles]

$\therefore$ By SAS congruence criterion, we have

ΔRON ≅ ΔPOM

⇒ RN = PM [By CPCT]

and ∠MPO = alternate ∠NRO

$\therefore$ PM ∥ RN

Hence, proved.

Self-Assessment

(Page 113)

Multiple-Choice Questions

1. Which of the following may not be true for any parallelogram?

(*a*) Opposite sides are equal.
(*b*) Opposite angles are equal.
(*c*) Opposite angles are always bisected by the diagonals.
(*d*) Diagonals bisect each other.

Sol. (*c*) Opposite angles are always bisected by the diagonals.

We know that for any parallelogram, opposite sides are always equal, opposite angles are also always equal and the two diagonals always bisect each other. The only property which does not hold good for all parallelograms is that the diagonals bisect opposite angles.

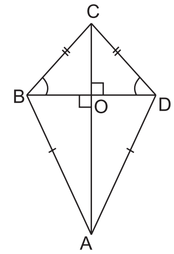
Only in the case of a square or a rhombus, the diagonals bisect the opposite angles, but not in the case of a rectangle.

2. Given a quadrilateral ABCD such that AB = AD and BC = CD, but AD ≠ CD. Also AC ⊥ BD, then the quadrilateral is a

(*a*) rhombus. (*b*) square.
(*c*) rectangle. (*d*) kite.

Sol. (*d*) kite.

Given that ABCD is a quadrilateral such that AB = AD and BC = CD but AD ≠ CD.



Also, diagonals AC and BD intersect each other at O at right angles. These are the properties of a kite only and not a square or a rhombus or a rectangle.

Fill in the Blanks

3. Three angles of a quadrilateral are 60°, 86°, 110°, then its fourth angle is **104°**.

Sol. $60° + 86° + 110° + x = 360°$ [∠s of a quadrilateral]

$$x = 360° - 256° \quad 104°$$

4. In a parallelogram ABCD, if ∠A = 60°, then ∠D is equal to **120°**.

Sol. $\angle A + \angle D = 180°$ [Coint. ∠s, AB ∥ CD]

$\Rightarrow 60° + \angle D = 180°$

$\Rightarrow \angle D = 180° - 60° = 120°$

5. If four angles of a quadrilateral are $(2x + 20)°$, $(3x - 30)°$, $(x + 10)°$ and $(2x)°$, then value of x is **45°**.

Sol. $(2x + 20)° + (3x - 30)° + (x + 10)° + (2x)° = 360°$

$\Rightarrow 8x = 360$

$\Rightarrow x = 45$

6. If the diagonals of a rhombus are 18 cm and 24 cm respectively, then its side is equal to **15 cm**.

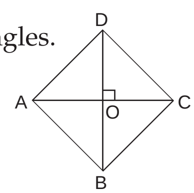
Sol. ABCD is a rhombus.

∴ Its diagonal bisects at right angles.

∴ In ΔCOD, OC = 12 cm

and OD = 9 cm

∴ Using Pythagoras' Theorem



$$DC = \sqrt{12^2 + 9^2} = \sqrt{144 + 81} = \sqrt{225} = 15 \text{ cm}$$

Assertion-Reason Type Questions

Directions (Q. Nos. 7 to 10): Each of these questions contains an assertion followed by reason. Read them carefully, and answer the question on the basis of the following options, select the one that best describes the two statements.

(a) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.

(b) Both Assertion and Reason are true but Reason is not the correct explanation of Assertion.

(c) Assertion is true but Reason is false.

(d) Assertion is false but Reason is true.

7. Assertion: A parallelogram has only one pair of parallel sides.

Reason: A trapezium has only one pair of parallel sides.

Sol. (*d*)

A parallelogram has two pair of parallel sides whereas a trapezium has only one pair of parallel sides.

∴ Assertion is incorrect but reason is correct.

8. Assertion: All rectangles are squares.

Reason: Squares are rectangles with equal sides.

Sol. (*d*)

All rectangles are not squares whereas squares are rectangles with equal sides.

∴ Assertion is incorrect but reason is correct.

9. Assertion: All rhombuses are kites.

Reason: In kites, pairs of adjacent sides are equal.

Sol. (*a*)

All rhombuses are kites since in kites pair of adjacent sides are equal.

∴ Assertion and reason are correct and reason is correct explanation of assertion.

10. Assertion: In a parallelogram, opposite angles are equal.

Reason: In a parallelogram, sum of angles is 360°.

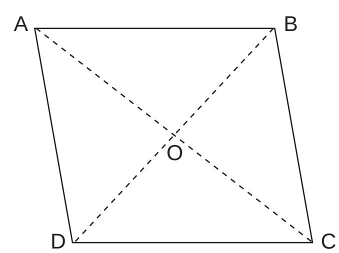
Sol. (*b*)

In parallelogram opposite angles are equal and sum of angles 360°.

∴ Assertion and reason are correct but reason is not the correct explanation of assertion.

Case Study Based Questions

11. Ankur and Shubham are two friends live in a same city. To help out the senior citizens, they go to the old age home every Saturday which is located at point O. Ankur's home is located at point A and Shubham's house is located at point B. ABCD is a rhombus in which ∠ABC = 110°.



Based on the above situation, answer the following questions.

(*a*) The diagonals of a rhombus

(*i*) are equal. (*ii*) are unequal.

(*iii*) bisect each other. (*iv*) have no relation.

Ans. (*iii*) bisect each other.

(*b*) ∠AOB is equal to

(*i*) 60° (*ii*) 80°

(*iii*) 90° (*iv*) 100°

Ans. (*iii*) 90°

(*c*) Measure of ∠OAB is

(*i*) 35° (*ii*) 70°

(*iii*) 140° (*iv*) 60°

Ans. (*i*) 35°

(*d*) If Ankur and Shubham go along AO and BO respectively to help out the senior citizens living in that old age home, then which of them has to cover shorter distance to reach there?

(*i*) Ankur

(*ii*) Shubham

(*iii*) Both of them will reach at same time

(*iv*) None of them

Ans. (*ii*) Shubham

(*e*) What value are shown by Ankur and Shubham?

(*i*) Empathy

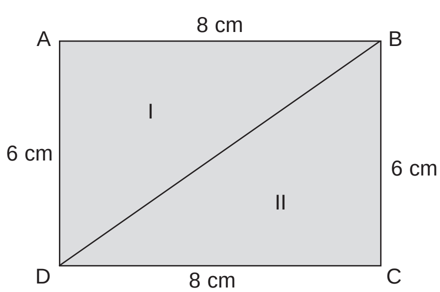
(*ii*) To show off money

(*iii*) Cheating

(*iv*) None of these

Ans. (*i*) Empathy

12. Biscuit that is in the form of quadrilateral with sides 8 cm, 6 cm, 8 cm and 6 cm as shown in the given figure. A mother divides it into two parts on one of it's diagonal. She gives part I to her daughter and part II to her son. It is given that one of the angles of this quadrilateral is a right angle. She asks following questions from her children.



(*a*) The sum of all the angles of a quadrilateral is equal to

(*i*) 90° (*ii*) 180°

(*iii*) 270° (*iv*) 360°

Ans. (*iv*) 360°

(*b*) Which type of quadrilateral is formed in the given figure?

(*i*) Square (*ii*) Rectangle

(*iii*) Rhombus (*iv*) Trapezium

Ans. (*ii*) Rectangle

(*c*) What is the length of the diagonal BD?

(*i*) 6 cm (*ii*) 8 cm

(*iii*) 10 cm (*iv*) 12 cm

Ans. (*iii*) 10 cm

(*d*) A diagonal of a parallelogram divides it into two congruent

(*i*) parallelograms. (*ii*) rectangles.

(*iii*) squares. (*iv*) triangles.

Ans. (*iv*) triangles.

(*e*) Each angle of the rectangle is

(*i*) less than 90° (*ii*) more than 90°

(*iii*) equal to 45° (*iv*) equal to 90°

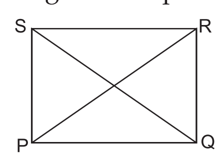
Ans. (*iv*) equal to 90°

Very Short Answer Type Questions

13. PQRS is a parallelogram. If the two diagonals PR and QS are of equal length, what will be the measure of each angle of the parallelogram?

Sol. Given that PQRS is a parallelogram and PR, QS are its diagonals such that PR = QS.

To find each angle of the parallelogram.



In ΔPQR and ΔQPS, we have

PR = QS [Given]

QR = PS

[∵ Opposite sides of a parallelogram]

PQ = QP [Common]

∴ By SSS congruence criterion, ΔPQR ≅ ΔQPS.

⇒ ∠RQP = ∠SPQ [By CPCT] …(1)

Also, since PS ∥ QR,

∴ ∠RQP + ∠SPQ = 180°

∴ From (1), ∠RQP = ∠SPQ = 90°

∴ ∠PSR = ∠RQP = 90°

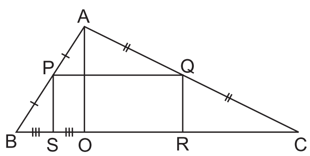
and ∠SRQ = ∠SPQ = 90°

∴ Each angle of the parallelogram is 90°.

14. Let P and Q be the mid-point of the sides AB and AC of ΔABC and O be any point on the side BC. O is joined to A. If S and R are the mid-points of OB and OC respectively, then what special name of the quadrilateral PQRS will you give so that the statement is always true?

Sol. Given that P, Q are the mid-points of the sides AB and AC respectively of ΔABC. O is any point on BC. BO and CO are bisected by the points S and R respectively on BC. PQ, QR and PS are joined.

To find the special name of the quadrilateral PQRS.



Since, P and Q are the mid-points of the sides AB and AC of ΔABC, respectively, hence by mid-point theorem for a triangle, PQ ∥ BC

and $\quad PQ = \frac{1}{2} BC \quad$ ...(1)

Now, $\quad SR = SO + OR$

$= \frac{1}{2} OB + \frac{1}{2} OC$

[∵ S and R are the mid-points of OB and OC respectively]

$= \frac{1}{2}(OB + OC)$

$\Rightarrow \quad SR = \frac{1}{2} BC \quad$ ...(2)

∴ From (1) and (2),

PQ = SR and PQ ∥ SR

Hence, the quadrilateral PQRS is a parallelogram.

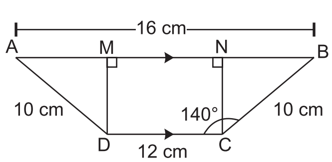
Short Answer Type-I Questions

15. ABCD is a trapezium such that AB = 16 cm, CD = 12 cm, BC = AD = 10 cm, ∠DCB = 140° and AB || DC. Find the measure of ∠BAD.

Sol. Given that ABCD is a trapezium in which AB ∥ DC, ∠DCB = 140°, AB = 16 cm, CD = 12 cm, BC = AD = 10 cm.

To find the measure of ∠BAD.

Construction: We draw DM ⊥ AB and CN ⊥ AB where M and N are points on AB.



In ΔAMD and ΔBNC, we have

∠AMD = ∠BNC = 90°

AD = BC = 10 cm

DM = CN

[∵ AB ∥ CD, DM ⊥ AB and CN ⊥ AB]

∴ By RHS congruence criterion, we have

ΔAMD ≅ ΔBNC

⇒ ∠ADM = ∠BCN [By CPCT] ...(1)

and ∠DAM = ∠CBN [By CPCT]

Now, ∠BCN = ∠BCD – ∠NCD

= 140° – 90°

= 50°

∴ From (1), ∠ADM = ∠BCN = 50°

∴ ∠DAM = 90° – 50°

= 40°

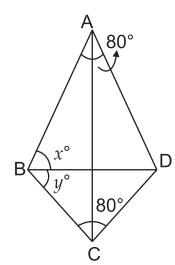
⇒ ∠BAD = 40°

which is the required measure of the angle.

16. A diagonal BD of a rhombus ABCD makes angle of measure $x°$ and $y°$ respectively with the sides BA and BC. If ∠BCD = 80°, then what is the value of $x° + 2y°$?

Sol. Given that ABCD is a rhombus, BD and AC are its two diagonals and ∠BCD = 80°, ∠ABD = $x°$ and ∠CBD = $y°$.

To find the value of $x° + 2y°$.



In ΔABD,

∵ AB = AD

∴ ∠ABD = ∠ADB

∴ ∠ADB = $x°$

∴ From ΔABD,

∠ABD + ∠ADB + ∠BAD = 180°

[Angle sum property of a triangle]

⇒ $x° + x° + 80° = 180°$

⇒ $2x° = 100°$

∴ $x° = 50°$

Similarly, from ΔBCD, we can show that

$2y° + 80° = 180°$

$\therefore \quad y° = 50°$

$\therefore \quad x° + 2y° = 50° + 2 \times 50°$

$= 150°$

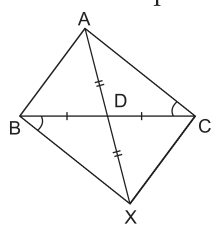
which is the required value.

Short Answer Type-II Questions

17. In a triangle ABC, median AD is produced to X such that AD = DX. Prove that ABXC is a parallelogram.

Sol. Given that AD is a median of a triangle ABC. The median AD is produced to a point X such that AD = DX. XC and XB are joined.

To prove that ABXC is a parallelogram.



In ΔADC and ΔXDB, we have

AD = XD [Given]

CD = BD [∵ AD is a median]

∠ADC = ∠XDB

[Vertically opposite angles]

∴ By SAS congruence criterion, we have

ΔADC ≅ ΔXDB

⇒ AC = XB [By CPCT] …(1)

and ∠ACD = ∠XBD [By CPCT]

But ∠ACD and ∠XBD are alternate angles.

∴ BX || AC

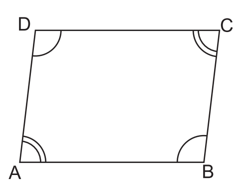
Also, AC = BX [By (1)]

∴ The quadrilateral ABXC is a parallelogram.

18. Prove that if each pair of opposite angles of a quadrilateral is equal, then it is a parallelogram.

Sol. Given that ABCD is a quadrilateral in which ∠A = ∠C and ∠B = ∠D.

To prove that ABCD is a parallelogram.



We have ∠A = ∠C and ∠B = ∠D.

∴ Adding, we get

∠A + ∠B = ∠C + ∠D …(1)

Also, ∠A + ∠B + ∠C + ∠D = 360°

[Angle sum property of a quadrilateral]

⇒ 2(∠A + ∠B) = 360° [From (1)]

⇒ $\angle A + \angle B = \frac{360°}{2} = 180°$ …(2)

Now, AB intersects AD and BC at A and B respectively such that the sum of the consecutive interior angles ∠A and ∠B is 180°.

∴ AD || BC …(3)

Again, from (2), we have

∠C + ∠B = 180° [∵ ∠A = ∠C]

Now, BC intersects DC and AB at C and B respectively such that the sum of the consecutive interior angles ∠B and ∠C is 180°.

∴ AB || DC …(4)

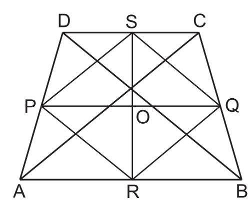
From (3) and (4), we conclude that the opposite sides of a quadrilateral ABCD are parallel. Hence, ABCD is a parallelogram.

Long Answer Type Questions

19. Show that the line segments joining the mid-points of the opposite sides of a quadrilateral bisect each other. **[CBSE SP 2012]**

Sol. Given that ABCD is a quadrilateral and P, Q, R, S are the mid-point of the sides AD, BC, AB and DC respectively. PQ and RS are joined.

To prove that PQ and RS bisect each other at a point O.



Construction: We join PS, SQ, RQ and PR to form a quadrilateral PSQR with diagonals PQ and RS. We also join AC and DB.

We first prove that the quadrilateral PSQR is a parallelogram.

In ΔABD, P and R are the mid-points of AD and AB respectively.

∴ By mid-point theorem for a triangle, PR || DB

and $PR = \frac{1}{2} DB$ …(1)

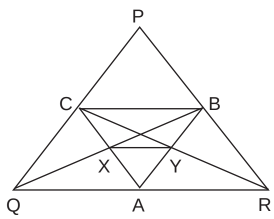
Similarly, in ΔDCB, S and Q are the mid-points of DC and CB respectively.

∴ By mid-point theorem for a triangle, SQ || DB

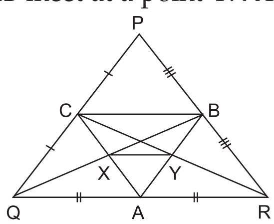
and $SQ = \frac{1}{2} DB$ …(2)

From (1) and (2), we see that PR ∥ SQ and PR = SQ, i.e. one pair of opposite sides of PRQS is equal and parallel and so PRQS is a parallelogram with PQ and SR as two diagonals. Since two diagonals of a parallelogram bisect each other, hence, it follows that PQ and RS bisect each other at a point O.

20. In the given figure, A, B and C are respectively the mid-points of sides QR, RP and PQ of ΔPQR. AC and QB meet at X. RC and AB meet at Y. Prove that XY = $\frac{1}{4}$ QR.



Sol. Given that A, B and C are the mid-points of the sides QR, RP and PQ of a triangle PQR, respectively. QB and AC meet at a point X and RC and AB meet at a point Y. XY is joined.



To prove that $XY = \frac{1}{4} QR$

In ΔPQR, C is the mid-point of PQ and B is the mid-point of PR.

∴ CB ∥ QR and $CB = \frac{1}{2} QR = QA$ …(1)

[By mid-point theorem for a triangle]

From (1), we see that CB ∥ QA and CB = QA.

∴ The quadrilateral QABC is a parallelogram. Since the diagonals of a parallelogram bisect each other, hence X is the mid-point of the diagonal AC of the parallelogram QABC.

Similarly, Y is the mid-point of the diagonal AB of the parallelogram ARBC.

∴ In ΔABC, X is the mid-point of AC and Y is the mid-point of AB.

∴ $XY = \frac{1}{2} BC$

[By mid-point theorem for a triangle]

$= \frac{1}{2} \cdot \frac{1}{2} QR$ [From (1)]

$= \frac{1}{4} QR$

Hence, proved.

Let's Compete

(Page 114)

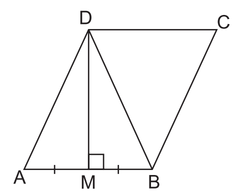
Multiple-Choice Questions

1. ABCD is a rhombus in which the altitude from D to side AB bisects AB. Then the angles of the rhombus are

(*a*) 100°, 80°, 100°, 80°
(*b*) 110°, 70°, 110°, 70°
(*c*) 120°, 60°, 120°, 60°
(*d*) 130°, 50°, 130°, 50°

Sol. (*c*) 120°, 60°, 120°, 60°

Given that ABCD is a rhombus, DM ⊥ AB where M is a point on AB and AM = MB.



To find the angles of the rhombus.

Construction: We join DM.

In ΔMAD and ΔMBD, we have

MA = MB [Given]

MD = MD [Common]

∠DMA = ∠DMB = 90° [Given]

∴ By SAS congruence criterion,

ΔMAD ≅ ΔMBD

⇒ BD = AD [By CPCT]

= AB

[∵ ABCD is a rhombus]

∴ ΔABD is an equilateral triangle.

∴ ∠DAB = 60°

∴ ∠BCD = ∠DAB = 60°

Also, ∠ABC = 180° – ∠DAB

= 180° – 60°

= 120°

∴ ∠ADC = ∠ABC

= 120°

Hence, the required angles of the rhombus are 120°, 60°, 120° and 60°.

2. PQRS is a parallelogram and M is the mid-point of the side QR and ∠SPM = ∠QPM. If PS = 14 cm, then the length of SR is

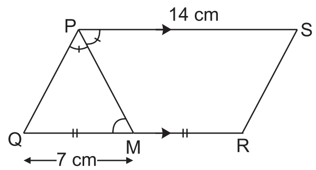
(*a*) 7 cm (*b*) 14 cm

(*c*) 6 cm (*d*) 15 cm

Sol. (*a*) 7 cm

Given that PQRS is a parallelogram and M is the mid-point of the side QR. Also, ∠SPM = ∠QPM and PS = 14 cm.

To find the length of SR.



We have

∠PMQ = alternate ∠SPM

[∵ PS ∥ QR and PM is a transversal]

= ∠QPM [Given]

∴ In ΔPQM, ∠PMQ = ∠QPM

∴ PQ = QM

$= \frac{1}{2}$ QR

$= \frac{1}{2}$ PS

$= \frac{1}{2} \times 14$ cm

= 7 cm

But PQ = SR

∴ SR = 7 cm

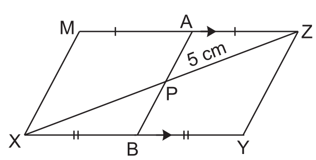
3. XYZM is a parallelogram. A and B are respectively the mid-points of MZ and XY respectively. If the diagonal XZ intersect AB at P and if PZ = 5 cm then the length of XZ is

(*a*) 5 cm (*b*) 10 cm

(*c*) 6 cm (*d*) 12 cm

Sol. (*b*) 10 cm

Given that XYZM is a parallelogram, A and B are the mid-points of MZ and XY respectively. XZ is a diagonal of the parallelogram. The line segment AB intersects XZ at P. Also, PZ = 5 cm.



To find the length of XZ.

In ΔMZX, A and B are the mid-points of MZ and XY respectively.

∴ $MA = \frac{1}{2} MZ = \frac{1}{2} XY = BX$

∴ MABX is a parallelogram.

∴ AB ∥ MX

⇒ AP ∥ MX

∴ In ΔMZX, AP ∥ MX and A is the mid-point of the side MZ.

∴ By the converse of mid-point theorem for a triangle, P is the mid-point of XZ.

∴ XZ = 2PZ

= 2 × 5 cm

= 10 cm

4. Two diagonals PR and QS of a parallelogram PQRS intersect each other at A. If ∠SPA = 25° and ∠PAQ = 68°, then ∠SQR is equal to

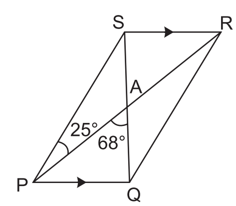
(*a*) 40° (*b*) 68°

(*c*) 25° (*d*) 43°

Sol. (*d*) 43°

Given that PQRS is a parallelogram and the two diagonals PR and QS intersect each other at a point A. Also, ∠SPA = 25° and ∠PAQ = 68°.

To find the measure of ∠SQR.



We have

∠SQR = alternate ∠PSQ …(1)

[∵ PS ∥ QR and SQ is a transversal]

Now, ∠PAQ + ∠PAS = 180°

⇒ 68° + ∠PAS = 180°

⇒ ∠PAS = 180° – 68° = 112° …(2)

∴ From ΔPAS,

∠PAS + ∠SPA + ∠PSA = 180°

[Angle sum property of a triangle]

⇒ 112° + 25° + ∠PSA = 180°

⇒ ∠PSA = 180° – 112° – 25°

= 180° – 137°

= 43°

⇒ ∠PSQ = 43° …(3)

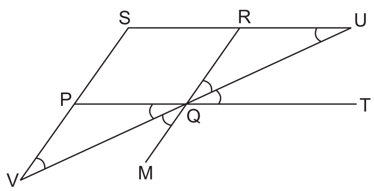
∴ From (1), ∠SQR = ∠PSQ = 43° [From (3)]

5. In parallelogram PQRS, the side PQ is produced to the point T. If the bisector of $\angle RQT$ meets SR produced and SP produced at U and V respectively and if SV = 30 cm, then the length of SU is

(*a*) 30 cm (*b*) 15 cm
(*c*) 7.5 cm (*d*) 25 cm

Sol. (*a*) 30 cm

Given that PQRS is a parallelogram. The side PQ is produced to a point T. The bisector of $\angle RQT$ meets SR produced and SP produced at the points U and V respectively.



It is given that SV = 30 cm.

To find the length of SU.

Construction: We produce RQ to M.

We have

$\angle RQU = \angle UQT$ [Given]

$= \text{alternate } \angle RUQ$

[$\because$ PT $\parallel$ SR and UV is a transversal]

$\therefore$ In ΔRQU, we have

$\angle RQU = \angle RUQ$

$\therefore$ RU = RQ = PS …(1)

$\therefore$ SU = SR + RU

= SR + RQ

= SR + SP …(2)

Now, $\angle PVQ = \text{alternate } \angle VQM$

[$\because$ SV $\parallel$ RM]

$= \angle PQV$

$\therefore$ In ΔPQV,

PV = PQ = SR

$\therefore$ SV = SP + PV

= RQ + SR = SU [From (2)]

$\therefore$ 30 cm = SU

$\therefore$ Required length of SU is 30 cm.

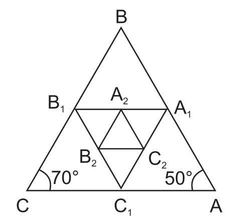
6. The angles of a triangle ABC are 50°, 60° and 70°. Let the triangle formed by joining the mid-points of the sides of ΔABC be called $\Delta A_1B_1C_1$. Then the angles of the triangle formed by joining the mid-points of the sides of $\Delta A_1B_1C_1$ are

(*a*) 70°, 70° and 40°
(*b*) 60°, 40° and 80°
(*c*) 50°, 60° and 70°
(*d*) 40°, 90° and 50°

Sol. (*c*) 50°, 60° and 70°

Given that in ΔABC, A_1, B_1, and C_1, are the mid-points of the sides AB, BC and CA respectively. Also, A_2, B_2 and C_2 are the mid-points of the sides A_1B_1, B_1C_1 and C_1A_1 respectively.

To find the angles of $\Delta\, A_2B_2C_2$ if $\angle A = 50°$, $\angle B = 60°$ and $\angle C = 70°$.



Since A_1, B_1, C_1 are the mid-points of the sides AB, BC and CA of ΔABC,

$\therefore$ By mid-point theorem, we have

$A_1B_1 \parallel AC$, $B_1C_1 \parallel AB$ and $C_1A_1 \parallel BC$ …(1)

Again, since A_2, B_2, C_2 are the mid-points of the sides A_1B_1, B_1C_1, and C_1A_1 of $\Delta A_1B_1C_1$

$\therefore$ By mid-point theorem, we have

$A_2B_2 \parallel A_1C_1$, $B_2C_2 \parallel A_1B_1$ and $C_2A_2 \parallel B_1C_1$ …(2)

$\therefore$ From (1) and (2), we have

$A_2B_2 \parallel BC$, $B_2C_2 \parallel AC$ and $C_2A_2 \parallel AB$

$\therefore$ Angle between A_2B_2 and A_2C_2 will be the same as that between BC and AB, i.e. 60°, the angle between A_2B_2 and B_2C_2 will be the same as that between BC and AC, i.e. 70° and the angle between B_2C_2 and C_2A_2 will be the same as that between AC and AB, i.e. 50°.

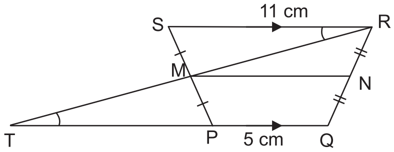
Hence, the angles of $\Delta A_2B_2C_2$ will be 50°, 60° and 70°.

7. PQRS is a trapezium with PQ || SR and M, N are the mid-points of the sides PS and QR respectively. If PQ = 5 cm and SR = 11 cm, then the length of MN is

(*a*) 9 cm (*b*) 10 cm
(*c*) 7 cm (*d*) 8 cm

Sol. (*d*) 8 cm

Given that PQRS is a trapezium with PQ $\parallel$ SR; M and N are the mid-points of SP and RQ respectively. PQ = 5 cm and SR = 11 cm. To find the length of MN.



Construction: We join RM and produce it to cut QP produced at T.

In ΔSMR and ΔPMT, we have

$$SM = PM \quad \text{[Given]}$$

$$\angle SRM = \text{alternate } \angle PTM$$

[∵ SR ∥ PQ and TR is a transversal]

$$\angle SMR = \angle PMT$$

[Vertically opposite angles]

∴ By AAS congruence criterion,

$$\Delta SMR \cong \Delta PMT$$

⇒ $TM = RM$ [By CPCT] …(1)

and $SR = PT$ [By CPCT] …(2)

Now, in ΔRTQ, M and N are the mid-points of the sides RT and RQ respectively.

∴ By mid-point theorem for a triangle, MN ∥ TQ

and $MN = \frac{1}{2} TQ$ …(3)

Now, $TQ = PQ + PT$

$= PQ + SR$ [From (2)]

$= (5 + 11)$ cm

$= 16$ cm

∴ From (3), $MN = \frac{1}{2} TQ$

$= \frac{1}{2} \times 16$ cm

$= 8$ cm

8. The mid-points A, B, C and D of the sides MQ, QP, PN and NM respectively of a rhombus MNPQ whose diagonals MP and NQ are of lengths 6 cm and 8 cm respectively are the vertices of a parallelogram ABCD whose two diagonals are of length

(*a*) 5 cm and 5 cm

(*b*) 6 cm and 5 cm

(*c*) 6 cm and 6 cm

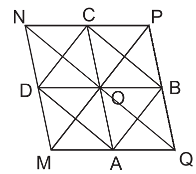
(*d*) 8 cm and 6 cm

Sol. (*a*) 5 cm and 5 cm

Given that MNPQ is a rhombus and A, B, C, D are the mid-points of the sides MQ, QP, PN and NM respectively. MP and NQ are the diagonals of the rhombus, intersecting each other at a point O. AB, BC, CD and DA are joined to form a parallelogram ABCD. The diagonals AC and DB of this parallelogram pass through O.

To find the length of AC and DB.

We shall first prove that the parallelogram ABCD is a rectangle.



Since, C and D are the mid-points of the sides NP and NM respectively of ΔNMP, hence by mid-point theorem for a triangle,

$DC \parallel MP$ and $DC = \frac{1}{2} MP$ …(1)

Similarly, from ΔQMP,

$AB \parallel MP$ and $AB = \frac{1}{2} MP$ …(2)

From (1) and (2), we see that

$DC = AB$ and $DC \parallel AB$.

∴ ABCD is a parallelogram.

Now, AB ∥ MP and BC ∥ NQ.

∴ Angle between AB and BC will be the same as that between MP and NQ.

But we know that angle between two diagonals MP and NQ of a rhombus is 90°.

∴ $AB \perp BC$

∴ The parallelogram ABCD is a rectangle.

∴ From (1) and (2),

$DC = AB$

$= \frac{1}{2} \times MP$ [From 2]

$= \frac{1}{2} \times 6$ cm

$= 3$ cm

and $AD = BC$

$= \frac{1}{2} \times NQ$

$= \frac{1}{2} \times 8$ cm

$= 4$ cm

∴ By Pythagoras' theorem, we get

$AC^2 = AD^2 + DC^2$

[From right-angled ΔADC]

$= 4^2 + 3^2$

$= 25$

$\therefore \quad AC = 5$

Also, $\quad AC = BD$

[$\because$ For a rectangle, two diagonals are of equal length]

$\therefore \quad BD = 5$

Hence, the required lengths of two diagonals of the rectangle ABCD are 5 cm and 5 cm.

9. The two diagonals of a quadrilateral ABCD are equal and perpendicular to each other. Another quadrilateral PQRS is formed by joining the mid-points of the sides of the former quadrilateral ABCD. Then the diagonals of the quadrilateral PQRS are

(*a*) equal but not perpendicular to each other.

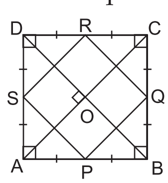
(*b*) equal and perpendicular to each other.

(*c*) neither equal nor perpendicular to each other.

(*d*) not equal but perpendicular to each other.

Sol. (*b*) equal and perpendicular to each other.

Given that ABCD is a quadrilateral such that the two diagonals AC and BD are of equal length and perpendicular to each other. Hence, the quadrilateral ABCD is a square. Let the two diagonals meet together at O. P, Q, R, S are the mid-points of the sides AB, BC, CD and DA of the square ABCD. We join PQ, QR, RS and SP to form a quadrilateral PQRS. We shall first prove that PQRS is another square.



$\because$ P, Q, R, S are the mid-points of the sides AB, BC, CD and DA of the square ABCD, hence by mid-point theorem for a triangle, we get

$SR = \frac{1}{2}AC$ and $SR \parallel AC$ from ΔADC,

$SP \parallel BD$ and $SP = \frac{1}{2}BD$ from ΔADB,

$PQ = \frac{1}{2}AC$ and $PQ \parallel AC$ from ΔABC

and $QR = \frac{1}{2}BD$ and $QR \parallel BD$ from ΔBCD.

Also, angle between SR and SP is equal to that between AC and BD, i.e. 90°.

Hence, the quadrilateral PQRS is such that $PQ = SR = \frac{1}{2}AC$, $SP = QR = \frac{1}{2}BD$.

$\therefore \quad PQ = SP \quad [\because AC = BD]$

and $\quad \angle RSP = \angle DOA = 90°$

$\therefore$ The figure PQRS is also a square. Now, we know that the two diagonals of a square are perpendicular to each other. Also, they are equal to each other.

10. The quadrilateral PQRS formed by joining the mid-points P, Q, R and S of sides AB, BC, CD and DA respectively of another quadrilateral ABCD such that $PR \perp QS$, PR = 8 cm and QS = 6 cm. Then the two diagonals of the original quadrilateral ABCD are always

(*a*) equal and perpendicular to each other.

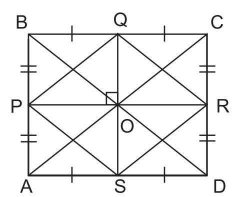
(*b*) equal but not perpendicular to each other.

(*c*) not equal but perpendicular to each other.

(*d*) neither equal nor perpendicular to each other.

Sol. (*b*) equal but not perpendicular to each other.

Given that P, Q, R, S are the mid-points of the sides AB, BC, CD and DA respectively of the original quadrilateral ABCD. We join PQ, QR, RS, SP, PR and QS to form a quadrilateral PQRS with diagonals PR and QS, where PR = 8 cm, QS = 6 cm and $PR \perp QS$. Since $PR \neq QS$ and $PR \perp QS$, hence, the second quadrilateral PQRS must be a rhombus.



We join AC and BD. We shall now determine the special name of the quadrilateral ABCD.

Since P, Q, R, S are the mid-points of AB, BC, CD and DA respectively of the quadrilateral ABCD, hence by mid-point theorem for a triangle, we have

$PQ = \frac{1}{2}AC$ and $PQ \parallel AC$ from ΔABC,

$QR = \frac{1}{2}BD$ and $QR \parallel BD$ from ΔBCD,

$RS = \frac{1}{2}AC$ and $RS \parallel AC$ from ΔADC,

and $PS = \frac{1}{2}BD$ and $PS \parallel BD$ from ΔBAD.

$\therefore$ $AC = 2PQ = 2RS$ and $BD = 2QR = 2PS$, $PQ \parallel RS$ and $QR \parallel PS$.

Now, since PQRS is a rhombus, hence RS = PS and so AC = BD.

∴ The quadrilateral ABCD is a parallelogram with diagonals AC and BD such that AC = BD.

∴ ABCD may be either a rectangle or a square.

Now, we see in ΔBCD where ∠BCD = 90°, that

$$BC^2 = BD^2 - CD^2 \quad \text{[By Pythagoras' theorem]}$$

$$= BD^2 - QS^2 \quad \ldots(1)$$

and in ΔABD, $AB^2 = BD^2 - AD^2$

$$= BD^2 - PR^2 \quad \ldots(2)$$

∵ PR = 8 cm

and QS = 6 cm

∴ PR ≠ QS

∴ From (1) and (2), we see that AB ≠ BC.

∴ ABCD is not a square. So, finally we see that ABCD is a rectangle. Since two diagonals of a rectangle are equal but not perpendicular to each other, hence choice (*b*) is correct.

Value-based Questions (Optional)

(Page 115)

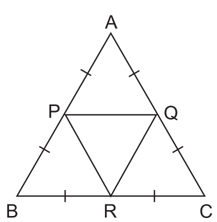
1. The sports teacher of a school drew an equilateral triangle ABC by putting white powder on the school grassy field. A few students were asked to stand along the sides of this triangle. He suggested all students to draw another equilateral triangle within ΔABC such that the vertices P, Q and R of this triangle lie in the mid-points of sides AB, AC and BC respectively and then join PQ, QR and PR.

(*a*) Was the suggestion of the sports teacher right? Justify your answer.

(*b*) Write two values exhibited by the sports teacher.

Sol. (*a*) Yes, the suggestion of the sports teacher was correct. This is due to the reasons as follows:

Since P, Q, R are the mid-points of AB, AC and BC.



∴ By mid-point theorem for a triangle,

$PQ = \frac{1}{2}BC$, $QR = \frac{1}{2}AB$ and $PR = \frac{1}{2}AC$.

∵ AB = BC = AC,

∴ PQ = QR = PR

∴ ΔPQR is also an equilateral triangle.

(*b*) Problem-solving and helpfulness.

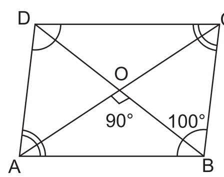
2. ABCD is a rhombus in which ∠ABC = 100°. Arvinda's house is at A and Parimal's house is at B. There is an old age home at O, the point of intersection of two diagonals of the rhombus ABCD. Both the men have to come to the old age home at O from their respective houses everyday to help the old people there. They have to walk along AO and BO respectively.

(*a*) Who used to cover the shorter distance to reach to old age home?

(*b*) What values were shown by the two men?

Sol. (*a*) We shall determine whether AO > BO or AO < BO.

It is given that ABCD is a rhombus and its two diagonals AC and BD bisect each other at a point O and ∠AOB = 90°, ∠ABC = 100°.



∵ ∠DAB + ∠ABC = 180°

∴ ∠DAB = 180° – ∠ABC

= 180° – 100° = 80°

Also, since $\angle OAB = \frac{1}{2}\angle DAB$

$= \frac{1}{2} \times 80° = 40°$

and $\angle OBA = \frac{1}{2} \times \angle ABC$

$= \frac{1}{2} \times 100°$

= 50°

∴ ∠OBA > ∠OAB

∴ AO > BO

[By triangular inequality]

Hence, Parimal whose house is at B used to cover the shorter distance to reach the old age home at O.

(*b*) Empathy, concern for the old people, helpfulness and caring for others.

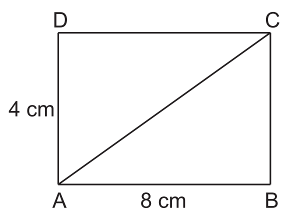
9

Areas of Parallelograms and Triangles

Checkpoint (Page 119)

1. A rectangle ABCD is of dimensions 8 cm × 4 cm. What is the area of ΔABC?

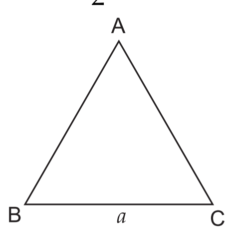
Sol. Required area of $\Delta ABC = \frac{1}{2} \times 8 \times 4\ \text{cm}^2 = 16\ \text{cm}^2$.



2. Find the area of an equilateral triangle of side a metre.

Sol. Let a cm be the side of an equilateral triangle.

Then the semi-perimeter of the triangle, $s = \frac{a+a+a}{2}\ \text{m} = \frac{3a}{2}\ \text{m}$



$\therefore$ By Heron's formula, the required area of the triangle is

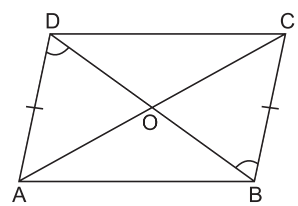
$$\sqrt{s(s-a)(s-a)(s-a)} = \sqrt{\frac{3a}{2} \times \left(\frac{3a}{2} - a\right)^3}\ \text{m}^2$$

$$= \sqrt{\frac{3a}{2} \times \frac{a^3}{8}}\ \text{m}^2$$

$$= \frac{\sqrt{3}}{4} a^2\ \text{m}^2$$

3. Prove that the diagonals of a parallelogram bisect each other.

Sol. Given that ABCD is a parallelogram, AC and BD are its two diagonals which intersect each other at O.



To prove that AO = OC and DO = OB.

In ΔAOD and ΔCOB, we have

AD = CB [Opposite sides of a ∥gm are equal]

$\angle ADO = \angle CBO$ [$\because$ ABCD is a ∥gm, $\therefore$ AB ∥ CD and AD ∥ BC]

and $\angle DAO = \angle BCO$ [$\because$ AD ∥ BC]

$\therefore$ By ASA congruence criterion,

$\Delta AOD \cong \Delta COB$

$\therefore$ AO = CO

and DO = BO [By CPCT]

Hence, the diagonals bisect each other.

4. Find the area of a rhombus, whose diagonals are 60 cm and 80 cm respectively.

Sol. Required area of the rhombus

$= \frac{1}{2} \times$ Product of two diagonals

$= \frac{1}{2} \times 60 \times 80\ cm^2$

$= 2400\ cm^2$

5. The area of a trapezium is $480\ cm^2$. If the parallel sides are 48 cm and 72 cm long, find the distance between them.

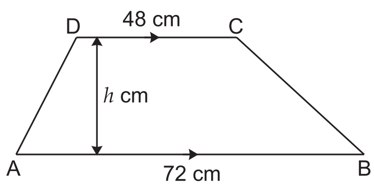
Sol. If h cm be the distance between the parallel sides AB and CD of the trapezium ABCD, where

AB = 72 cm and DC = 48 cm, then the required area of the trapezium

$= \frac{1}{2}(AB + DC) \times h$

$= \frac{1}{2}(72 + 48) \times h\ cm^2$

$= 60 \times h\ cm^2$



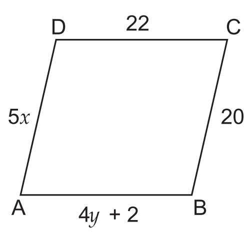
But given that area of the trapezium = $480\ cm^2$.

$\therefore \quad 60h = 480$

$\Rightarrow \quad h = 8$

Hence, the required distance between two parallel sides is 8 cm.

6. In the given figure, ABCD is a parallelogram. Find x and y (lengths are in cm).



Sol. In a parallelogram, two opposite sides are of equal length.

$\therefore \quad AB = CD$

$\Rightarrow \quad 4y + 2 = 22$

$\Rightarrow \quad y = \frac{20}{4} = 5$

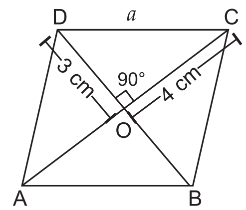
and $\quad 5x = 20$

$\Rightarrow \quad x = \frac{20}{5} = 4$

$\therefore$ $x = 4$ and $y = 5$ are the required values of x and y.

7. The diagonals of a rhombus are 6 cm and 8 cm. Find the length of a side of the rhombus.

Sol. We know that the diagonals AC and DB of a rhombus are perpendicular to each other.



$\therefore \quad \angle DOC = 90°$

Also, $\quad DO = \frac{6}{2}$ cm = 3 cm

and $\quad OC = \frac{8}{2}$ cm = 4 cm

$\therefore$ From ΔDOC, by Pythagoras' Theorem if a be the side of the rhombus, then

$a^2 = (4\ cm)^2 + (3\ cm)^2$

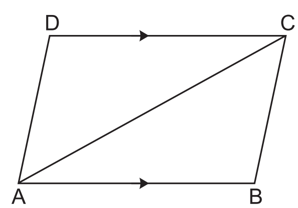
$\Rightarrow \quad a^2 = 25\ cm^2$

$\Rightarrow \quad a = 5$ cm

$\therefore$ The required length of the side of the rhombus is 5 cm.

8. If a pair of opposite sides of a quadrilateral are equal and parallel, prove that it is a parallelogram.

Sol. Given that ABCD is a quadrilateral. The two opposite sides AB and CD are such that AB = CD and AB ∥ CD. To prove that ABCD is parallelogram.



Construction: We join AC.

In ΔABC and ΔCDA, we have

AB = CD [Given]

$\angle BAC = \angle DCA$

[Alternate $\angle$s, $\because$ AB ∥ DC]

AC = CA [Common]

$\therefore$ By SAS congruence criterion,

$\Delta ABC \cong \Delta CDA$

$\therefore \quad \angle BCA = \angle DAC$ [By CPCT]

But $\angle BCA$ and $\angle DAC$ are alternate angles with respect to the transversal CA.

$\therefore \quad$ BC ∥ AD

∴ AB ∥ DC [Given]

and BC ∥ AD [Already proved]

Hence, by definition, ABCD is a parallelogram.

Milestone

(Page 125)

Multiple-Choice Questions

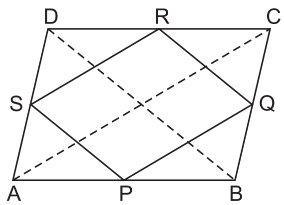
1. The area of the figure formed by joining the mid-points of the adjacent sides of a rhombus with diagonals of lengths 12 cm and 14 cm is

(*a*) 28 cm^2 (*b*) 42 cm^2

(*c*) 168 cm^2 (*d*) 112 cm^2

Sol. (*b*) 42 cm^2

Let ABCD be a rhombus and P, Q, R and S are respectively the mid-points of the sides AB, BC, CD and DA. Then the figure PQRS is a rectangle. Let AC and BD be the two diagonals of the rhombus.



Then, AC = 14 cm and DB = 12 cm.

$$\therefore \quad PQ = \frac{1}{2} AC = \frac{1}{2} \times 14 \text{ cm} = 7 \text{ cm}$$

$$\text{and} \quad QR = \frac{1}{2} DB = \frac{1}{2} \times 12 \text{ cm} = 6 \text{ cm}$$

∴ Required area of the rectangle PQRS $= 7 \times 6 \, cm^2$ $= 42 \, cm^2$

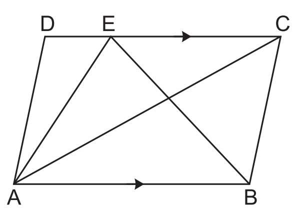
2. If a triangle and a parallelogram stand on the same base and between the same parallels, then the ratio of the area of the triangle to the area of the parallelogram is

(*a*) 1 : 4 (*b*) 3 : 1

(*c*) 4 : 1 (*d*) 1 : 2

Sol. (*d*) 1 : 2

Let the parallelogram ABCD and the triangle AEB are on the same base AB and lie between two parallels AB and DC. We join the diagonal AC.



Now, $ar(\Delta ABC) = \frac{1}{2} ar(\parallel gm \ ABCD)$

[∵ Each diagonal of a ∥gm divide its area into two equal parts]

But $ar(\Delta ABC) = ar(\Delta AEB)$

[∵ They stand on the same base AB and lie between two parallels AB and DC]

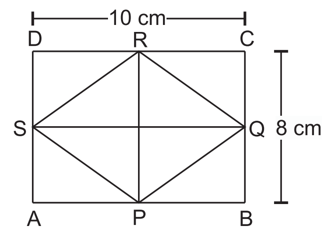
∴ $ar(\Delta AEB) = \frac{1}{2} ar(\parallel gm \ ABCD)$

∴ The required ratio of the area of triangle AEB and the area of parallelogram ABCD is 1 : 2.

Very Short Answer Type Questions

3. Find the area of the quadrilateral obtained by joining the mid-points of the adjacent sides of a rectangle of sides 10 cm and 8 cm.

Sol. Let ABCD be a rectangle and P, Q, R, S be the mid-points of the sides AB, BC, CD and DA respectively. Then PQRS will be a rhombus with diagonals PR = BC = 8 cm and SQ = DC = 10 cm.



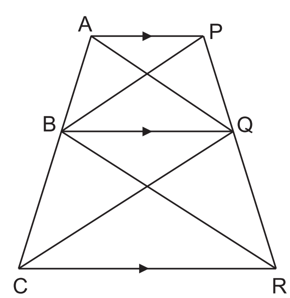
Hence, the required area of the rhombus PQRS $= \frac{1}{2} \times 8 \times 10 \, cm^2 = 40 \, cm^2$

4. ABQP and BQRC are trapeziums on the two opposite sides of BQ such that AP ∥ BQ ∥ CR. If $ar(\Delta AQC) = 32 \, cm^2$, then find the area of ΔPBR.

Sol. Given that

$$ar(\Delta AQC) = 32 \, cm^2$$

To find ar(ΔPBR).



We have $ar(\Delta PBR) = ar(\Delta PBQ) + ar(\Delta QBR)$ …(1)

But $ar(\Delta PBQ) = ar(\Delta ABQ)$ …(2)

[∵ They stand on the same base BQ and between two parallels BQ and AP]

and $ar(\Delta QBR) = ar(\Delta QBC)$ …(3)

[∵ They stand on the same base BQ and between two parallels BQ and CR]

∴ From (1), (2) and (3), we have

$ar(\Delta PBR) = ar(\Delta ABQ) + ar(\Delta QBC)$

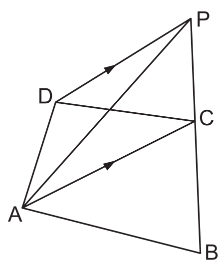
$= ar(AQC) = 32\ cm^2$ [Given]

Hence, the required area of ΔPBR is $32\ cm^2$.

Short Answer Type-I Questions

5. In the given figure, ABCD is a quadrilateral. A line through D parallel to AC meets BC produced at P. Prove that

$ar(\Delta ABP) = ar(\text{quad } ABCD)$



Sol. Given that ABCD is a quadrilateral, P is a point on BC produced such that DP ∥ AC. AP is joined.

To prove that $ar(\Delta ABP) = ar(\text{quad } ABCD)$

We have

$ar(\Delta APC) = ar(\Delta ADC)$

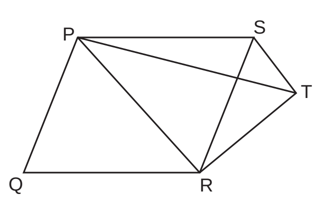
[∵ They stand on the same base AC and between two parallels AC and DP]

∴ $ar(\Delta APC) + ar(\Delta ABC) = ar(\Delta ADC) + ar(\Delta ABC)$

⇒ $ar(\Delta ABP) = ar(\text{quad } ABCD)$

Hence, proved.

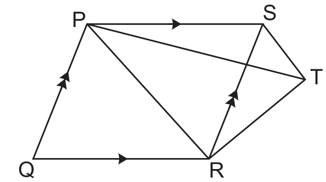
6. PQRT is a quadrilateral and PQRS is a parallelogram. Prove that if the diagonal PR of the quadrilateral PQRT bisects it, then ST ∥ PR.



Sol. Given that PQRT is a quadrilateral and PQRS is a parallelogram. Now, the diagonal PR of the quadrilateral PQRT bisects its area, i.e.

$ar(\Delta PQR) = ar(\Delta PRT)$

ST is joined. To prove that ST ∥ PR.



We see that PR is diagonal of the parallelogram PQRS.

∴ $ar(\Delta PQR) = ar(\Delta PSR)$ …(1)

But it is given that

$ar(\Delta PQR) = ar(\Delta PRT)$ …(2)

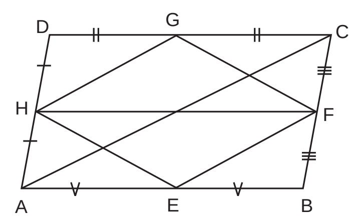
∴ From (1) and (2),

$ar(\Delta PSR) = ar(\Delta PRT)$ …(3)

But these two triangles stand on the same base PR. Hence, they must lie between two parallels PR and ST so that (3) may be true. Hence, it follows that ST ∥ PR.

Short Answer Type-II Questions

7. If E, F, G and H are respectively the mid-points of the sides AB, BC, CD and DA of the parallelogram ABCD, show that the quadrilateral EFGH is a parallelogram and its area is half the area of the parallelogram ABCD.



Sol. Given that ABCD is a parallelogram, E, F, G and H be the mid-points of the sides AB, BC CD and DA respectively. To prove that the quadrilateral EFGH is a parallelogram and its area is half the area of the whole parallelogram ABCD.

We see that EF ∥ AC and $EF = \frac{1}{2} AC$

[∵ E and F are the mid-points of the sides AB and BC respectively of ΔABC]

Similarly, HG ∥ AC and $HG = \frac{1}{2} AC$

[∵ H and G are the mid-points of the sides DA and CD respectively of ΔDAC]

∴ In quadrilateral EFGH,

$EF \parallel HG$

and $EF = HG$

∴ The figure EFGH is a parallelogram

Again, $HD = FC$

and $HD \parallel FC$

∴ The quadrilateral HFCD is a parallelogram.

∴ $ar(\Delta HGF) = \frac{1}{2} ar(\parallel gm\ HFCD)$

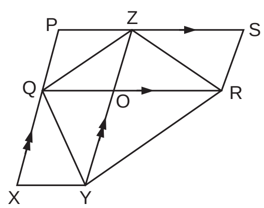
[∵ They stand on the same base HF and between two parallels HF and DC]

∴ $ar(\parallel gm\ EFGH) = \frac{1}{2} ar(\parallel gm\ ABCD)$

[∵ $ar(\Delta HGF) = ar(\Delta HEF)$ and $ar(\parallel gm\ HFCD) = ar(\parallel gm\ HFBA)$]

Hence, proved.

8. In the given figure, PQRS and PXYZ are two parallelograms of equal area. Prove that $YR \parallel QZ$.



Sol. Given that PQRS and PXYZ are two parallelograms of equal area. YR and QZ are joined. To prove that $YR \parallel QZ$.

We have

$ar(\parallel gm\ PXYZ) = ar(\parallel gm\ PQRS)$ [Given]

Subtracting the common area of parallelogram PZOQ from both sides, we get

$\Rightarrow ar(\parallel gm\ QXYO) = ar(\parallel gm\ ZORS)$

$\Rightarrow 2\ ar(\Delta QOY) = 2\ ar(\Delta ZOR)$

[∵ Each diagonal of a $\parallel$gm bisects its area into two equal area]

$\Rightarrow ar(\Delta QOY) = ar(\Delta ZOR)$

$\Rightarrow ar(\Delta QOY) + ar(\Delta OQZ)$

$= ar(\Delta ZOR) + ar(\Delta OQZ)$

$\Rightarrow ar(\Delta QYZ) = ar(\Delta QRZ)$ …(1)

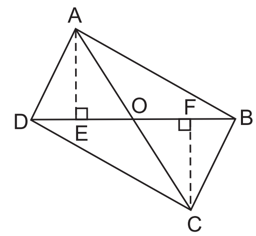
But these two triangles stand on the same base QZ. Hence, they must be between two parallels YR and QZ in order that (1) may be true. It follows that $YR \parallel QZ$.

Hence, proved.

9. A quadrilateral ABCD is such that the diagonal BD divides its area into two equal parts. Prove that BD bisects AC.

Sol. Given that ABCD is a quadrilateral and its diagonal BD divides its area into two equal parts.

Hence, $ar(\Delta ABD) = ar(\Delta BDC)$.



Also, AC is another diagonal of the quadrilateral such that it intersects the diagonal BD at O.

To prove that AO = CO.

Construction: We draw $AE \perp DB$ and $CF \perp DB$, where E and F are two points on DB.

We have

$ar(\Delta ABD) = \frac{1}{2} DB \times AE$

and $ar(\Delta DBC) = \frac{1}{2} DB \times CF$

Since these two areas are equal to each other,

∴ $AE = CF$ …(1)

∴ In ΔAEO and ΔCFO, we have

$\angle AEO = \angle CFO = 90°$ [By construction]

$\angle AOE = \angle COF$ [Vertically opposite $\angle$s]

and $AE = CF$ [From (1)]

∴ By AAS congruence criterion, we have

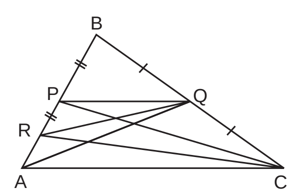
$\Delta AEO \cong \Delta CFO$

∴ $AO = CO$ [By CPCT]

Hence, BD bisect AC.

Long Answer Type Questions

10. In ΔABC, P and Q are respectively the mid-points of AB and BC and R is the mid-point of AP. Prove that



(*a*) $ar(\Delta PQR) = \frac{1}{2} ar(\Delta ARC)$

(*b*) $ar(\Delta RQC) = \frac{3}{8} ar(\Delta ABC)$

(c) ar(ΔPBQ) = ar(ΔARC)

Sol. Given that in ΔABC, P and Q are mid-points of AB and BC respectively and R is the mid-point of AP. To prove that

(a) ar(ΔPQR) = $\frac{1}{2}$ ar(ΔARC)

(b) ar(ΔRQC) = $\frac{3}{8}$ ar(ΔABC)

(c) ar(ΔPBQ) = ar(ΔARC)

We know that the median of any triangle divides the triangle into two parts of equal area.

∴ ar(ΔPQR) = $\frac{1}{2}$ ar(ΔPQA)

[∵ RQ is a median of ΔPQA]

= $\frac{1}{4}$ ar(ΔABQ)

[∵ PQ is a median of ΔABQ,
∴ ar(ΔPQA) = $\frac{1}{2}$ ar(ΔABQ)]

= $\frac{1}{8}$ ar(ΔABC) ...(1)

[∵ AQ is a median of ΔABC,
∴ ar(ΔABQ) = $\frac{1}{2}$ (ΔABC)]

Again, ar(ΔARC) = $\frac{1}{2}$ ar(ΔAPC)

[∵ RC is a median of ΔAPC]

= $\frac{1}{4}$ ar(ΔABC) ...(2)

[∵ CP is a median of ΔABC,
∴ ar(ΔAPC) = $\frac{1}{2}$ ar(ΔABC)]

(a) From (1) and (2), we get

ar(ΔPQR) = $\frac{1}{2}$ ar(ΔARC)

(b) We have

ar(ΔRQC) = $\frac{1}{2}$ ar(ΔRBC)

[∵ RQ is a median of ΔRBC]

= $\frac{1}{2}$ [ar(ΔABC) – ar(ΔARC)]

= $\frac{1}{2}$ [ar(ΔABC) – $\frac{1}{4}$ ar(ΔABC)]

[From (2)]

= $\frac{3}{8}$ ar(ΔABC)

(c) Finally,

ar(ΔPBQ) = $\frac{1}{2}$ ar(ΔPBC)

[∵ PQ is a median of ΔPBC]

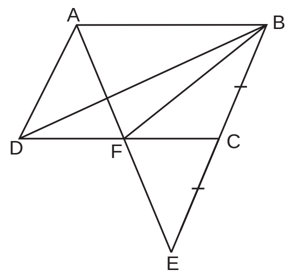
= $\frac{1}{4}$ ar(ΔABC) ...(3)

[∵ PC is a median of ΔABC,
∴ ar(ΔPBC) = $\frac{1}{2}$ ar(ΔABC)]

∴ From (2) and (3), we get

ar(ΔPBQ) = ar(ΔARC)

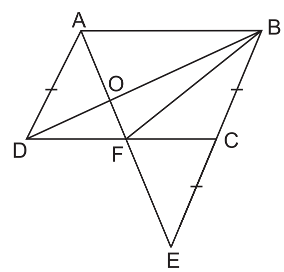
11. ABCD is a parallelogram in which BC is produced to E such that CE = BC. AE intersects CD at F.



(a) Prove that ar(ΔADF) = ar(ΔECF)

(b) If ar(ΔDFB) = 3 cm^2, find the area of parallelogram ABCD.

Sol. Given that ABCD is a parallelogram and BC is produced to E such that CE = BC. AE is joined. F is the point of intersection of AE and DC.



Also, ar(ΔDFB) = 3 cm^2

(a) To prove that

ar(ΔADF) = ar(ΔECF)

(b) To calculate ar(||gm ABCD) if ar(ΔDFB) = 3 cm^2

Let AF and BD intersect each other at O.

(a) In ΔAFD and ΔEFC, we have

AD = BC = EC

∠ADE = ∠ECF

[Alternate ∠s, ∵ BE || AD]

∠DAE = ∠CEF

[Alternate ∠s, ∵ BE || AD]

∴ By ASA congruence criterion,

ΔAFD ≅ ΔEFC

∴ DF = CF [By CPCT] ...(1)

and ar(ΔADF) = ar(ΔECF)

(*b*) Area of parallelogram ABCD = 2 ar (ΔDBC)

[∵ DB is a diagonal of the ∥gm]

= 2 × 2 ar(ΔDBF)

[∵ BF is a median of ΔDBC]

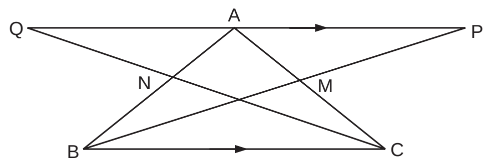
= 4 × 3 cm^2 [Given]

= 12 cm^2

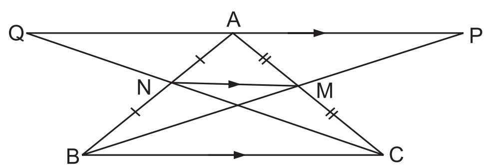
∴ The required area of parallologram ABCD is 12 cm^2.

12. M and N are the mid-points of AC and AB respectively. QP ∥ BC and CNQ and BMP are straight lines. Prove that

ar(ΔABP) = ar(ΔACQ)



Sol. Given that N and M are the mid-points of AB and AC of a ΔABC. CNQ and BMP are two straight lines and QP ∥ BC.



To prove that

ar(ΔABP) = ar(ΔACQ)

Construction: We join NM.

∵ N and M are the mid-points of AB and AC respectively of ΔABC,

∴ NM ∥ BC ∥ QP and $\frac{NM}{BC} = \frac{1}{2}$

Again, since N is the mid-points of AB and NM ∥ AP,

∴ In ΔABP,

M is also the mid-point of BP and $\frac{AP}{NM} = 2$

∴ $$\frac{ar(\Delta ABP)}{ar(\Delta NBM)} = \frac{AP^2}{NM^2} = 4$$

∴ ar(ΔABP) = 4 ar(ΔNBM) …(1)

Similarly, in ΔACQ, N and M are the mid-points of CQ and CA respectively and $\frac{AQ}{NM} = 2$

∴ $$\frac{ar(\Delta ACQ)}{ar(\Delta NCM)} = \frac{AQ^2}{NM^2} = 4$$

∴ ar(ΔACQ) = 4 ar(ΔNCM) …(2)

Now, ΔNBM and ΔNCM stand on the same base NM and between two parallels NM and BC.

∴ ar(ΔNBM) = ar(ΔNCM)

∴ From (1) and (2),

ar(ΔABP) = ar(ΔACQ)

Hence, proved.

Higher Order Thinking Skills (HOTS) Questions

(Page 126)

1. ABCD is a trapezium in which DC ∥ AB, DC = 40 cm and AB = 60 cm. If X and Y are respectively the mid-points of AD and BC, prove that

(*a*) DCYX and ABYX are trapeziums.

(*b*) XY = 50 cm and

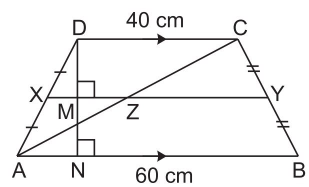
(*c*) ar(trap DCYX) = $\frac{9}{11}$ ar(trap XYBA)

Sol. Given that ABCD is a trapezium in which DC ∥ AB, DC = 40 cm and AB = 60 cm. X and Y are the mid-points of AD and BC respectively. To prove that

(*a*) DCYX and ABYX are trapeziums

(*b*) XY = 50 cm and

(*c*) ar(trap DCYX) = $\frac{9}{11}$ ar(trap XYBA)



Construction: We join AC. Let Z be the point of intersection of AC and XY. We draw DM ⊥ XY and DMN ⊥ AB.

(*a*) Since X and Y are the mid-points of the non-parallel sides AD and BC respectively of the trapezium ABCD, hence XY ∥ DC ∥ AB. Hence, the quadrilaterals DCYX and ABYX are also trapeziums.

(*b*) In ΔADC, since X is the mid-point of AD and XZ ∥ DC,

∴ Z is also the mid-point of AC

and XZ = $\frac{1}{2}$ × DC

= $\frac{1}{2}$ × 40 cm

= 20 cm

Now, in ΔABC, since Z and Y are the mid-points of AC and BC respectively, hence $ZY = \frac{1}{2} AB$

$= \frac{1}{2} \times 60$ cm = 30 cm.

$$\therefore \quad XY = XZ + ZY$$
$$= (20 + 30) \text{ cm}$$
$$= 50 \text{ cm}$$

(*c*) Since, $\quad \Delta DXM \sim \Delta DAN,$

$$\therefore \quad \frac{DM}{DN} = \frac{DX}{DA} = \frac{1}{2}$$

$$\therefore \quad DM = \frac{1}{2} DN \quad \ldots(1)$$

Also, $\quad MN = DN - DM$

$$= DN - \frac{1}{2} DN \quad \text{[From (1)]}$$

$$= \frac{1}{2} DN \quad \ldots(2)$$

Now, $\text{ar(trap DCYX)} = \frac{1}{2}(DC + XY) \times DM$

$$= \frac{1}{2}(40 + 50) \times \frac{1}{2} DN$$

$$= \frac{45}{2} DN \quad \ldots(3)$$

$$\text{ar(trap XYBA)} = \frac{1}{2}(XY + BA) \times MN$$

$$= \frac{1}{2}(50 + 60) \times \frac{1}{2} DN \quad \text{[From (2)]}$$

$$= \frac{55}{2} DN \quad \ldots(4)$$

$$\frac{\text{ar}(\text{trapDCYX})}{\text{ar}(\text{trapXYBA})} = \frac{45}{2} \times \frac{2}{55} \quad \text{[From (3) and (4)]}$$

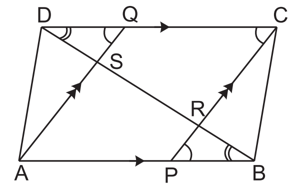
$$= \frac{9}{11}$$

$$\therefore \quad \text{ar(trap DCYX)} = \frac{9}{11} \text{ar(trap XYBA)}$$

2. In a parallelogram ABCD, P is any point on the side AB. A line segment AQ, drawn parallel to PC, intersects DC at Q. If BD intersects AQ at S and PC at R, prove that

$$\text{ar}(\Delta PRB) = \text{ar}(\Delta QSD)$$

Sol. Given that ABCD is a parallelogram P and Q are two points on AB and DC respectively such that AQ ∥ PC. The diagonal DB intersects AQ at S and PC at R.



To prove that ar(ΔPRB) = ar(ΔQSD).

We see that AP ∥ QC and AQ ∥ PC

$\therefore$ The figure APCQ is a parallelogram.

$$\therefore \quad AP = QC \quad \ldots(1)$$

Hence, $\quad PB = AB - AP$

$$= DC - QC$$
$$= QD \quad \ldots(2)$$

Now, in ΔPRB and ΔQSD, we have

$$PB = QD \quad \text{[From (2)]}$$
$$\angle RPB = \angle QCR \quad [\text{Alternate } \angle s]$$
$$= \angle SQD \quad [\text{Corresponding } \angle s]$$
$$\angle PBR = \angle QDS \quad [\text{Alternate } \angle s]$$

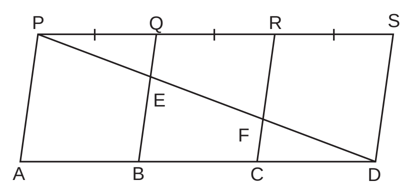
Hence, by ASA congruence criterion,

$$\Delta PRB \cong \Delta QSD$$

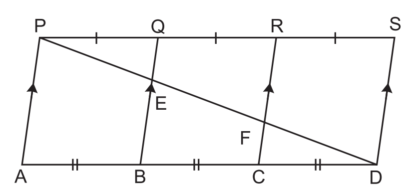
$$\therefore \quad \text{ar}(\Delta PRB) = \text{ar}(\Delta QSD)$$

Hence, proved.

3. In the given figure, PSDA is a parallelogram in which PQ = QR = RS and AP ∥ BQ ∥ CR. Prove that ar(ΔPQE) = ar(ΔDCF).



Sol. Given that PSDA is a parallelogram. Q, R are points on PS and B, C are points on AD such that PQ = QR = RS and AP ∥ BQ ∥ CR. PD is joined. Let PD cuts QB and RC at E and F respectively.



To prove that ar(ΔPQE) = ar(ΔDCF)

Let a and b be the length and breadth respectively of the parallelogram PSDA.

Then PS = a and PA = b.

$\therefore \quad PQ = \frac{a}{3} = \frac{PS}{3}$

$\Rightarrow \quad \frac{PQ}{PS} = \frac{1}{3}$

$\therefore \quad \frac{ar(\Delta PQE)}{ar(\Delta SPD)} = \frac{PQ^2}{PS^2} = \frac{1}{9}$ …(1)

Similarly,

$\frac{ar(\Delta DCF)}{ar(\Delta APD)} = \frac{1}{9}$ …(2)

But $ar(\Delta SPD) = ar(\Delta APD)$

$= \frac{1}{2} ar (\| gm\ PSDA)$

$\therefore$ From (1) and (2) it follows that

$ar(\Delta PQE) = ar(\Delta DCF)$

Hence, proved.

Self-Assessment

(Page 126)

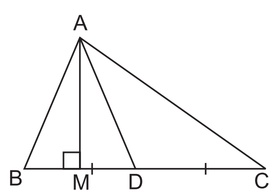
Multiple-Choice Questions

1. The median of a triangle divides it into two
   (*a*) equilateral triangles
   (*b*) isosceles triangles
   (*c*) right triangles
   (*d*) triangles of equal areas

[CBSE SP 2012]

Sol. (*d*) triangles of equal areas

We know that the median AD of any triangle ABC divides the triangle into two parts of equal area.



This is because of the following:

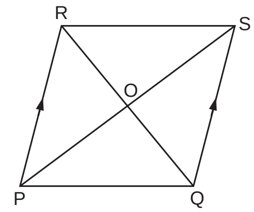
Let AM ⊥ BC.

Then area of $\Delta ABD = \frac{1}{4} BC \times AM$ and the area of $\Delta ADC = \frac{1}{4} BC \times AM$

$\therefore \quad ar(\Delta ABD) = ar(\Delta ADC)$

$\therefore$ The median of a triangle divides it into two triangles of equal areas.

2. In the given figure, if PR ∥ QS, then the triangle which is equal in area to ΔROS is
   (*a*) ΔPSR (*b*) ΔQOP
   (*c*) ΔPQS (*d*) ΔRSQ



Sol. (*b*) ΔQOP

Given that PQSR is a quadrilateral such that PR ∥ QS.

To find the triangle whose area is equal to the area of ΔROS.

We have

$ar(\Delta PQR) = ar(\Delta PSR)$

[∵ These two triangles lie on the same base PR and between two parallels PR and QS]

$\Rightarrow ar(\Delta PQR) - ar(\Delta POR) = ar(\Delta PSR) - ar(\Delta POR)$

$\Rightarrow \quad ar(\Delta QOP) = ar(\Delta ROS)$

Hence, the required area is ar(ΔQOP).

Fill in the Blanks

3. Two triangles of equal area lying between same parallels have equal **bases**.
4. Area of a rhombus is equal to $\mathbf{\frac{1}{2}}$ **× product of diagonals**.
5. Median of a triangle divides it into two triangles having equal **area**.
6. Area of a parallelogram is equal to **base × corresponding height**.

Assertion-Reason Type Questions

Directions (Q. Nos. 7 to 10): Each of these questions contains an assertion followed by reason. Read them carefully, and answer the question on the basis of the following options, select the one that best describes the two statements.

(a) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.
(b) Both Assertion and Reason are true but Reason is not the correct explanation of Assertion.
(c) Assertion is true but Reason is false.
(d) Assertion is false but Reason is true.

7. Assertion: A diagonal of a parallelogram divides it into two triangles of equal areas.

Reason: A diagonal divides a parallelogram into two triangles on same base and between same parallel lines.

Sol. (*c*)

A diagonal of a parallelogram divides it into two triangles of equal area.

∴ Assertion is correct but reason is incorrect.

8. Assertion: Parallelograms on the same base and having equal areas lie between the same parallels.

Reason: Area of parallelogram = Base × Distance between parallel sides

Sol. (*a*)

Parallelograms on the same base and between same parallel have equal areas as area of parallelogram is equal to base × distance.

∴ Both assertion and reasoning are correct and reason is correct explanation of assertion.

9. Assertion: If a triangle and a parallelogram are on the same base and between the same parallels, then their areas are equal.

Reason: Area of a triangle is half the product of its base and height.

Sol. (*d*)

Area of triangle is half the product of its base and height but area of triangle is half area of parallelogram if they lie on same base and between the same parallel.

∴ Assertion is incorrect and reason is correct.

10. Assertion: Two triangles on the same base and between the same parallels are equal in area.

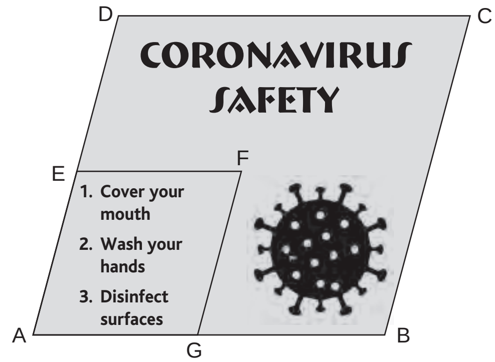
Reason: Area of a triangle is double the product of its base and height.

Sol. (*c*)

Assertion is correct but reason is wrong since area of $\Delta = \frac{1}{2} \times b \times h$.

Case Study Based Questions

11. Anita makes a poster on the parallelogram sheet on the topic 'Coronavirus Safety' for her society named Grand Square Apartments. So she draws image and writes Coronavirus Safety on the poster and it covers $\frac{3}{4}$ area of parallelogram sheet. Based on above situation, answer the following questions.



(*a*) EFGA is a parallelogram if EF ∥ _____ and FG ∥ _____.

(*i*) CB; EA (*ii*) DA; AB

(*iii*) AG; DC (*iv*) DC; CB

Ans. (*iv*) DC; CB

(*b*) If area of sheet is 16 cm^2, then what is the area of EFGA?

(*i*) 3 cm^2 (*ii*) 4 cm^2

(*iii*) 6 cm^2 (*iv*) 8 cm^2

Ans. (*ii*) 4 cm^2

(*c*) If EF is equal to 4 cm, then AG is equal to

(*i*) 2 cm (*ii*) 3 cm

(*iii*) 4 cm (*iv*) 5 cm

Ans. (*iii*) 4 cm

(*d*) If the measure of ∠DEF is equal to 50°, then the measure of ∠EAG is equal to

(*i*) 40° (*ii*) 50°

(*iii*) 100° (*iv*) 130°

Ans. (*ii*) 50°

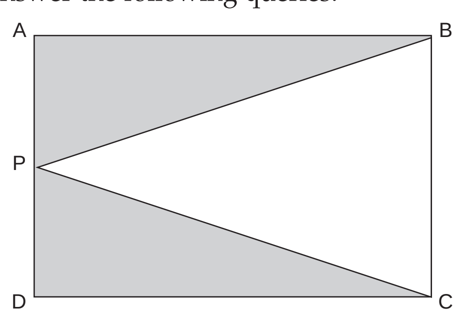
(*e*) If ∠DEF is equal to 50°, then ∠AEF is equal to

(*i*) 110° (*ii*) 120°

(*iii*) 50° (*iv*) 130°

Ans. (*iv*) 130°

12. ABCD is a village in the shape of parallelogram. The Panchayat has decided to use the triangular area BPC for farming as shown in the given figure. Farming is a part of agriculture. But members of Panchayat have some queries. Answer the following queries.



(*a*) If area of triangle BPC is 200 m^2, then what is the remaining area of the village?

(*i*) 50 m^2 (*ii*) 100 m^2

(*iii*) 200 m^2 (*iv*) 300 m^2

Ans. (*iii*) 200 m^2

(*b*) If area of triangle BPC is 200 m^2, then what is the area of the village?

(*i*) 200 m^2 (*ii*) 300 m^2

(*iii*) 400 m^2 (*iv*) 500 m^2

Ans. (*iii*) 400 m^2

(*c*) If a triangle and a parallelogram are on the same base and between the same parallels, then the ratio of the area of the triangle to the area of the parallelogram will be

(*i*) 1 : 2 (*ii*) 1 : 3

(*iii*) 1 : 4 (*iv*) 2 : 3

Ans. (*i*) 1 : 2

(*d*) What is the area of the parallelogram?

(*i*) base × height

(*ii*) $\frac{1}{2}$ × base × height

(*iii*) 2 × base × height

(*iv*) $\frac{1}{4}$ × base × height

Ans. (*i*) base × height

(*e*) If we shift the point P little upwards and ar(ΔBAP) is equal to 50 m^2 and ar(ΔCPD) is equal to 150 m^2, then ar(|| gm ABCD) is equal to

(*i*) 200 m^2 (*ii*) 400 m^2

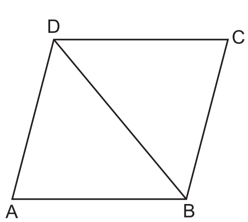
(*iii*) 600 m^2 (*iv*) 800 m^2

Ans. (*ii*) 400 m^2

Very Short Answer Type Questions

13. What is the ratio of the areas of a rhombus and an equilateral triangle standing on the same base and between the same parallels?

Sol. Let ABCD be a rhombus and BD is its one of the diagonals, such that BD = AB = AD so that ΔABD is an equilateral triangle.

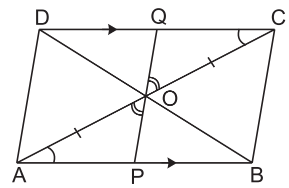


Now, ar(ΔABD) = $\frac{1}{2}$ ar (rhom ABCD), since BD is a diagonal of the rhombus.

∴ ar(rhom ABCD) : ar(ΔABD) = 2 : 1.

14. ABCD is a parallelogram whose diagonals AC and BD intersect each other at O. A line segment through O meets AB and DC at P and Q respectively. Prove that ar(ΔPOA) = ar(ΔQOC).

Sol. Given that ABCD is a parallelogram, the diagonals AB and AC intersect each other at O. P and Q are two points on AB and CD respectively such that POQ is a straight line.



To prove that ar(ΔPOA) = ar(ΔQOC)

In ΔPOA and ΔQOC, we have

AO = CO

[∵ Diagonals of a ||gm bisect each other]

∠AOP = ∠COQ

[Vertically opposite ∠s]

∠PAO = ∠QCO

[Alternate ∠s, ∵ DC || AB]

∴ By ASA congruence criterion,

ΔPOA ≅ ΔQOC

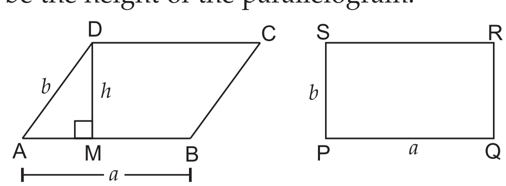
∴ ar(ΔPOA) = ar(ΔQOC)

Hence, proved.

Short Answer Type-I Questions

15. Prove that, of all the parallelograms of which the sides are given, the parallelogram which is a rectangle has the greatest area.

Sol. Let a and b be the length and breadth of the rectangle PQRS or a parallelogram ABCD. Let h be the height of the parallelogram.



Then the area of the rectangle = ab

and the area of the parallelogram = ah

But since b is the hypotenuse of ΔADM, where DM ⊥ AB

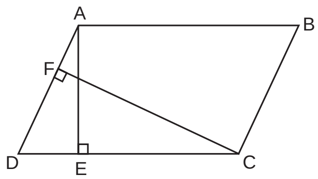
$\therefore \quad b > h$

$\Rightarrow \quad h < b$

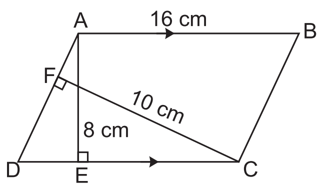
$\therefore \quad ah < ab$

Hence, the area of the rectangle is greater than that of the parallelogram. In other words, the parallelogram which is a rectangle has the greatest area.

16. In the given figure, ABCD is a parallelogram, AE ⊥ DC and CF ⊥ AD. If AB = 16 cm, AE = 8 cm and CF = 10 cm, find AD. **[CBSE SP 2010]**



Sol. Given that ABCD is a parallelogram, E and F are points on DC and AD respectively such that AE ⊥ DC and CF ⊥ AD. Also, given that AB = 16 cm, AE = 8 cm and CF = 10 cm.



To find AD.

$$\text{Area of } \| \text{gm ABCD} = \frac{1}{2} \times AB \times AE$$

$$= \frac{1}{2} \times 16 \times 8 \text{ cm}^2$$

$$= 64 \text{ cm}^2 \quad \ldots(1)$$

Also,

$$\text{area of } \| \text{gm ABCD} = \frac{1}{2} \times AD \times CF$$

$$= \frac{1}{2} \times AD \times 10$$

$$= 5\,AD \quad \ldots(2)$$

$\therefore$ From (1) and (2), we have

$$5 \times AD = 64$$

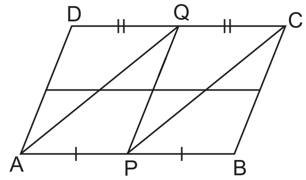
$$\Rightarrow \quad AD = \frac{64}{5} = 12.8$$

Hence, the required length of AD is 12.8 cm.

Short Answer Type-II Questions

17. Show that the line segment joining the mid-points of a pair of opposite sides of a parallelogram divides it into two parallelograms of equal area.

Sol. Given that ABCD is a parallelogram. P and Q are mid-points of AB and DC respectively. PQ is joined.



To prove that

(*a*) quadrilaterals APQD and PBCQ are parallelogram and

(*b*) ar(∥gm APQD) = ar(∥gm PBCQ)

(*a*) We have

$$AP = \frac{1}{2} AB$$

$$= \frac{1}{2} DC$$

$$= DQ$$

Also, AP ∥ DQ

$\therefore$ The quadrilateral APQD is a parallelogram.

Similarly, the quadrilateral PBCQ is a parallelogram.

(*b*) We join AQ and PC.

Now, ar(∥gm APQD) = 2 ar(ΔAPQ) …(1)

and ar(∥gm PBCQ) = 2 ar(ΔPBC) …(2)

Now, ΔAPQ and ΔPBQ stand on equal bases AP and PB respectively and they lie between the same parallels AB and DC.

$\therefore$ ar(ΔAPQ) = ar(ΔPBC)

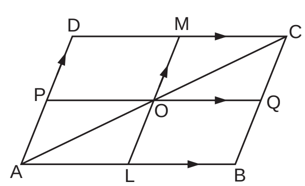
$\therefore$ From (1) and (2),

ar(∥gm APQD) = ar(∥gm PBCQ)

Hence, proved.

18. In the figure, ABCD is a parallelogram and O is any point on the diagonal AC. If PQ ∥ AB and LM ∥ AD where P, Q, L, M are points on AD, BC, AB and DC respectively. Prove that

ar(∥gm DPOM) = ar(∥gm BLOQ)



Sol. Given that ABCD is a parallelogram and O is a point on the diagonal AC such that POQ ∥ MOL, where P, Q, L and M are points on AD, CB, AB and CD respectively.

To prove that

ar(∥gm DPOM) = ar(∥gm BLOQ)

We see that the each of the quadrilaterals APOL, OLBQ, DPOM and MOQC is a parallelogram.

Since, each diagonal of a parallelogram divides it into two triangles of equal area,

∴ ar(ΔADC) = ar(ΔCBA)

⇒ ar(ΔAPO) + ar(∥gm DPOM) + ar(ΔOMC)

= ar(ΔOLA) + ar(∥gm OLBQ) + ar(ΔCQO) …(1)

Now, ar(ΔAPO) = ar(ΔOLA)

[∵ AO is a diagonal of ∥gm APOL]

ar(ΔOMC) = ar(ΔCQO)

[∵ OC is a diagonal of ∥gm NOQC]

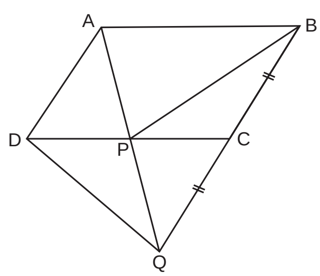
∴ From (1), we have

ar(∥gm DPOM) = ar(∥gm BLOQ)

Hence, proved.

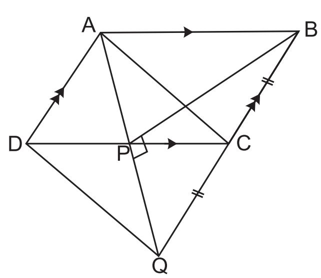
19. In the given figure, ABCD is a parallelogram. BC is produced to Q such that BC = CQ. AQ is joined to intersect DC at P. Prove that

ar(ΔBCP) = ar(ΔDPQ)



Sol. Given that ABCD is a parallelogram. BC is produced to Q such that BC = CQ. AQ is joined to intersect DC at P. BP is joined. To prove that

ar(ΔBCP) = ar(DPQ)



Construction: We join DQ.

Since PC is a median of ΔPBQ,

∴ ar(ΔBCP) = ar(ΔPCQ) …(1)

Again, the quadrilateral ADQC is a parallelogram and so its diagonals AQ and DC bisect each other at P.

∴ DP = PC

∴ QP is a median of ΔDQC.

∴ ar(ΔPCQ) = ar(ΔDPQ) …(2)

∴ From (1) and (2), we have

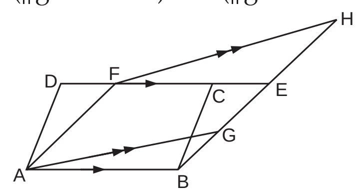
ar(ΔBCP) = ar(ΔDPQ)

Hence, proved.

Long Answer Type Questions

20. In the given figure, ABCD, ABEF and AGHF are parallelograms. Prove that

ar(∥gm ABCD) = ar(∥gm AGHF)



Sol. Given that ABCD, ABEF and AGHF are parallelograms, where F and E are points on DC and DC produced. G is a point on BE and H is a point on BE produced such that AG ∥ FH. To prove that

ar (∥gm ABCD) = ar(∥gm AGHF)

We have

ar(∥gm ABCD) = ar(∥gm ABEF)

(∵ They stand on the same base AB and lie between two parallels AB and DE)

= ar(ΔABG) + ar(quad AGEF) …(1)

Now, in ΔABG and ΔFEH, we have

AB = FE [∵ ABEF is a ∥gm]

AG = FH [∵ AGHF is a ∥gm]

and ∠GAB = ∠HFE

[∵ FH ∥ AG and FE ∥ AB]

∴ By ASA congruence criterion,

ΔABG ≅ ΔFEH

∴ ar(ΔABG) = ar(ΔFEH)

∴ From (1),

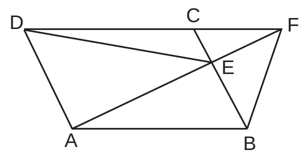
ar(∥gm ABCD) = ar(ΔFEH) + ar(quad AGEF)

= ar(∥gm AGHF)

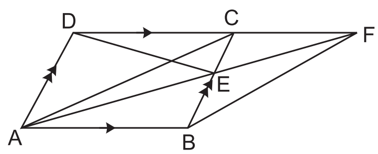
Hence, proved.

21. Through the vertex A of the parallelogram ABCD in the figure, a line AEF is drawn to meet BC at E and DC produced at F. Show that

ar(ΔBEF) = ar(ΔDCE)



Sol. Given that ABCD is a parallelogram. Through the vertex A, a line-segment AEF is drawn to meet BC at E and DC produced at F. BF and DE are joined. To prove that ar(ΔBEF) = ar(ΔDCE).



Construction: We join AC.

∵ ΔABF and ΔABC stand on the same base AB and lie between two parallels AB and DF,

⇒ ar(ΔABF) – ar(ΔAEB) = ar(ΔABC) – ar(ΔAEB)

⇒ ar(ΔBEF) = ar(ΔCEA) = ar(ΔDCE)

[∵ ΔCEA and ΔCED stand on the same base CE and they lie between two parallels CE and AD]

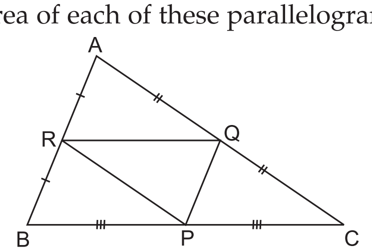
Hence, proved.

22. Prove that the mid-points of the sides of a triangle ABC along with any of the vertices as the fourth point make a parallelogram. Find the relation between the area of the triangle ABC and the area of the parallelogram.

Sol. Given that P, Q and R are the mid-points of the sides BC, CA and AB respectively of ΔABC. To prove that

(*a*) Each of the quadrilateral PQAR, QRBP and RPCQ is a parallelogram

(*b*) to find a relation between the ar(ΔABC) and the area of each of these parallelogram.



(*a*) ∵ P, Q and R are the mid-points of BC, CA and AB respectively,

∴ PQ ∥ AR, RQ ∥ BP and RP∥ AQ

Also, $PQ = \frac{1}{2}AB = AR,$

$RQ = \frac{1}{2}BC = BP$

and $RP = \frac{1}{2}AC = AQ$

∴ Each of the quadrilaterals PQAR, QRBP and RPCQ is a parallelogram.

Now, $\frac{ar(\Delta ARQ)}{ar(\Delta ABC)} = \frac{AR^2}{AB^2} = \frac{1}{4}$

∴ $ar(\Delta ARQ) = \frac{1}{4}ar(\Delta ABC)$ …(1)

Similarly, $ar(\Delta BPR) = \frac{1}{4}ar(\Delta ABC)$

and $ar(\Delta CPQ) = \frac{1}{4}ar(\Delta ABC)$

Also, $ar(\Delta ARQ) = \frac{1}{2}ar(\parallel gm\ ARPQ)$ …(2)

[∵ RQ is a diagonal of ∥gm ARPQ]

∴ From (1) and (2),

$ar(\parallel gm\ ARPQ) = 2\,ar(\Delta ARQ)$

$= \frac{1}{2}ar(\Delta ABC)$

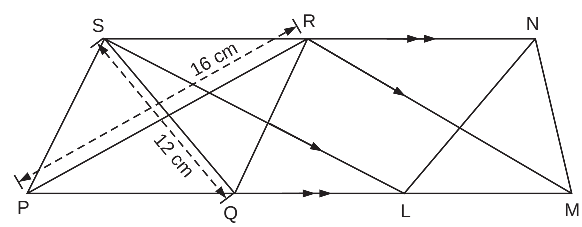
Hence, area of each parallelogram = $\frac{1}{2}$ ar(ΔABC).

Let's Compete

(Page 129)

Multiple-Choice Questions

1. In the given figure, PQRS is a rhombus and SRML is a parallelogram. N is any point on the line SRN. If PR = 16 cm and SQ = 12 cm then the area of ΔLMN is



(*a*) 144 cm^2 (*b*) 96 cm^2

(*c*) 192 cm^2 (*d*) 48 cm^2

Sol. (*d*) 48 cm^2

Given that PQRS is a rhombus and SRML is a parallelogram. N is a point on SR produced. NL and NM are joined. Diagonals PR and SQ of the rhombus are PR = 16 cm and SQ = 12 cm. To find the area of ΔLMN.

We have

$$ar(\Delta LMN) = \frac{1}{2} ar(\| gm\ LMRS)$$

[∵ They stand on the same base LM and lie between two parallels LM and SRN]

$$= \frac{1}{2} ar(rhom\ RSPQ)$$

[∵ They stand on the same base RS and lie between two parallels SR and PQLM]

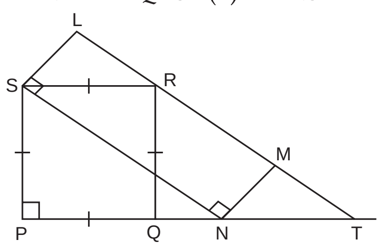
$$= \frac{1}{2} \times \frac{1}{2} \times 12 \times 16\ cm^2$$

$$= 48\ cm^2$$

2. In the given figure, PQRS is a square and SNML a rectangle. Also, LRMT || SN, PQNT || SR, SL || MN and SP || RQ. Then the pair of quadrilaterals whose areas are equal is

(*a*) PQRS and SLMN (*b*) SLMN and SRTN

(*c*) SRMN and PQRS (*d*) LTNS and PQRS

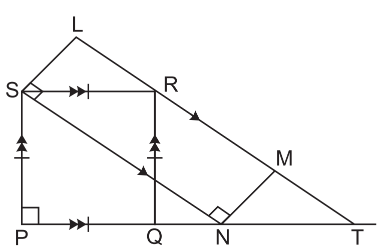


Sol. (*b*) SLMN and SRTN

We see that

In (*a*) PQRS and SLMN do not stand on the same base and lie between two parallels. Hence, their areas are not equal to each other.

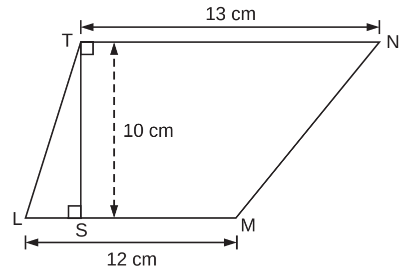
In (*c*), quadrilateral SRMN is not a parallelogram, but PQRS is a square and so their areas are not equal to each other.



In (*d*), the quadrilateral LTNS is not a parallelogram but PQRS is a square and so their areas are not equal to each other.

In (*b*), both the parallelogram SLMN and SRTN stand on the same base SN and lie between two parallels SN and LRMT. Hence, their areas are equal.

3. In the given figure, the area of the quadrilateral LMNT is



(*a*) 130 cm^2 (*b*) 60 cm^2

(*c*) 125 cm^2 (*d*) 120 cm^2

Sol. (*c*) 125 cm^2

Given that LMNT is a quadrilateral and TN = 13 cm, LM =12 cm, TS ⊥ LM and TS ⊥ TN.

∴ TN || LM and TS = 10 cm

To find the area of the quadrilateral LMNT.

We see that LMNT is a trapezium,

∴ LM || TN

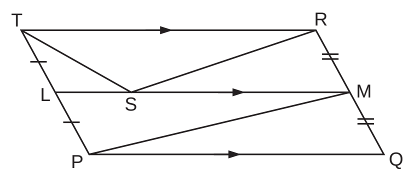
∴ Area of the trapezium LMNT

$$= \frac{1}{2}(LM + TN) \times TS$$

$$= \frac{1}{2}(13 + 12) \times 10\ cm^2$$

$$= 125\ cm^2$$

4. PQRT is a parallelogram and L and M are the mid-points of PT and QR respectively. S is any point on LM and area of ΔPMQ = 30 cm^2. Then the area of ΔTSL + the area of ΔRSM is equal to



(*a*) 30 cm^2 (*b*) 15 cm^2

(*c*) 60 cm^2 (*d*) 45 cm^2

Sol. (*a*) 30 cm^2

Given that PQRT is a parallelogram, L and M are mid-points of PT and QR respectively. S is any point on LM and ar(ΔPMQ) = 30 cm^2. To find ar(ΔTSL) + ar(ΔRSM).

We have

ar(ΔTSL) + ar(ΔRSM)

$$= ar(\| gm\ TLMR) - ar(\Delta TSR)$$

$$= \text{ar}(\|\text{gm TLMR}) - \frac{1}{2}\text{ar}(\|\text{gm TLMR})$$

[$\because$ ΔTSR and ∥gm TLMR stand on the same base TR and between the same parallels TR and LM]

$$= \frac{1}{2}\text{ar}(\|\text{gm TLMR})$$

$$= \frac{1}{2}\text{ar}(\|\text{gm PQML})$$

[$\because$ ar(∥gm TLMR) = ar(∥gm PQML)]

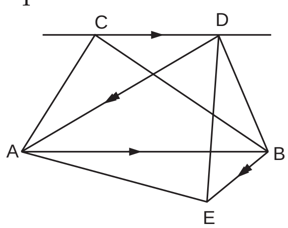
$$= \frac{1}{2} \times 2\,\text{ar}(\Delta PMQ)$$

[$\because$ PM is a diagonal of ∥gm PQML]

$$= \text{ar}(\Delta PMQ)$$

$$= 30 \text{ cm}^2$$

5. In the figure, CAB is a triangle. D is any point on a line through C parallel to AB. If E is any point on a line through B parallel to DA, then the three triangles equal in area are



(*a*) ΔADB, ΔEDB and ΔCAB
(*b*) ΔCAB, ΔADB and ΔABE
(*c*) ΔABE, ΔDBE and ΔADB
(*d*) ΔCAB, ΔADB and ΔADE

Sol. (*d*) ΔCAB, ΔADB and ΔADE

Given that ΔCAB is a triangle and D is a point on a line through C parallel to AB. E is a point on a line through B parallel to DA.

We see that ar(ΔADB) = ar(ΔCAB), since they stand on the same base AB and lie between two parallels AB and CD.

In (*a*), ar(ΔEDB) is not equal to the area of any of the triangle ADB and CAB.

In (*b*), ar(ΔABE) is not equal to the area of any of the triangle ADB and CAB.

In (*c*) ar(ΔDBE) = ar(ΔABE), since, they stand on the same base BE and lie between two parallels BE and AD, but ar(ΔADB) is not equal to the area of any of these two triangles.

Finally in (*d*), we see that

$$\text{ar}(\Delta CAB) = \text{ar}(\Delta ADB) = \text{ar}(\Delta ADE)$$

$\because$ ΔADB and ΔADE stand on the same base AD and lie between two parallels AD and BE.

Hence, their areas are equal.

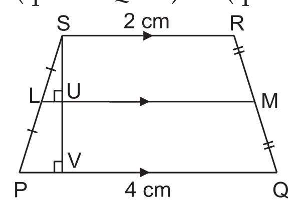
6. PQRS is a trapezium with PQ ∥ SR. L and M are the mid-points of non-parallel sides SP and RQ respectively. If PQ = 4 cm and SR = 2 cm, then the ratio of ar(quad PQML) to ar(quad LMRS) is

(*a*) 3 : 7 (*b*) 5 : 7
(*c*) 7 : 3 (*d*) 7 : 5

Sol. (*d*) 7 : 5

Given that PQRS is a trapezium with PQ ∥ SR. L and M are the mid-points of SP and RQ respectively. Also, SR = 2 cm and PQ = 4 cm.

To find ar(quad PQML) : ar(quad LMRS).



Construction: We draw SUV perpendicular to both LM and PQ, where U and V are points on LM and PQ respectively.

We see that both LMRS and PQML are trapeziums, since SR ∥ LM ∥ PQ.

$$LM = \frac{1}{2}(SR + PQ)$$

$$= \frac{1}{2}(2 + 4) \text{ cm} = 3 \text{ cm}$$

Now,

$$\text{ar(trap LMRS)} = \frac{1}{2}(LM + SR) \times SU$$

$$= \frac{1}{2}(3 + 2)\,SU \text{ cm}^2$$

$$= \frac{5}{2}\,SU \text{ cm}^2 \quad \ldots(1)$$

$$\text{Also, ar(trap PQML)} = \frac{1}{2}(PQ + LM) \times UV$$

$$= \frac{1}{2}(4 + 3)UV \text{ cm}^2$$

[From (1)]

$$= \frac{7}{2}\,UV \text{ cm}^2 \quad \ldots(2)$$

Again, from ΔSLU and ΔSPV, we have

$$\frac{SU}{SV} = \frac{LS}{SP} = \frac{1}{2}$$

$$\Rightarrow \quad \frac{SU}{SU + UV} = \frac{1}{2}$$

$$\Rightarrow \quad \frac{SU + UV}{SU} = 2$$

$$\Rightarrow \quad \frac{UV}{SU} = 2 - 1 = 1 \quad \ldots(3)$$

$\therefore$ From (1), (2) and (3), we get

$$\frac{ar(trapPQML)}{ar(trapLMRS)} = \frac{7}{5} \times \frac{UV}{SU} = \frac{7}{5} \quad \text{[From (3)]}$$

$\therefore$ Required ratio = 7 : 5

7. P is a fixed point within a triangle ABC such that ar(ΔPAB) = ar(ΔPAC) = ar(ΔPBC). Then P must be the

(*a*) centroid (*b*) orthocentre

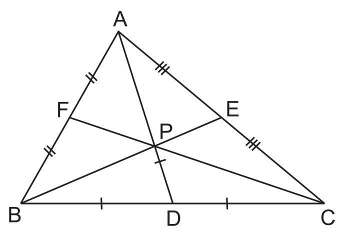
(*c*) incentre (*d*) circumcentre

Sol. (*a*) centroid

Given that P is a fixed point within a triangle ABC such that ar(ΔPAB) = ar(ΔPAC) = ar(ΔPBC).

To determine the geometrical position of the point.

Construction: Let D, E and F be the mid-points of BC, CA and AB respectively. We join PD, PE and PF.



We have

ar(ΔPAB) = ar(ΔPAC) [Given]

$\therefore$ ar(ΔPAB) + ar(ΔPBD) = ar(ΔPAC) + ar(ΔPCD)

[$\because$ PD is a median of ΔPBC,
$\therefore$ ar(ΔPBD) = ar(ΔPCD)]

$\Rightarrow$ ar(quad PABD) = ar(quad PACD) ...(1)

(1) will be true only when PA and PD are the same line so that the line segment AD becomes a median of ΔABC.

Similarly, ar(ΔPAC) = ar(ΔPBC), we have

ar(ΔPAC) + ar(ΔPAF) = ar(ΔPBC) + ar(ΔPBF)

[$\because$ FP is a median of ΔAPB,
$\therefore$ ar(ΔPAF) = ar(ΔPBF)]

$\Rightarrow$ ar(quad PACF) = ar(quad PBCF) ...(2)

(2) will be true only when PF and PC are the same line so that the line segment CF becomes another median of ΔABC. Exactly, in the same way, we can prove that PB and PE will also become the same line segment BE. So, BE will be the third median of ΔABC. Hence, the three medians AD, BE and CF will pass through the point P. So, P must be the centroid of ΔABC.

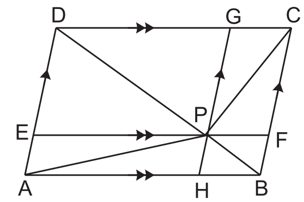
8. The area of a parallelogram ABCD is equal to that of a trapezium, the lengths of whose parallel sides are 32 cm and 16 cm and the distance between them is 6 cm. If P is any point within the parallelogram ABCD, then the sum of the areas of ΔAPD and ΔPBC is equal to

(*a*) 48 cm^2 (*b*) 72 cm^2

(*c*) 80 cm^2 (*d*) 96 cm^2

Sol. (*b*) 72 cm^2

Given that ABCD is a parallelogram. PA, PB, PC and PD are joined.



It is given that the area of parallelogram ABCD is equal to the area of a trapezium whose parallel sides are 32 cm and 16 cm and the distance between them is 6 cm.

To find ar(ΔAPD) + ar(ΔPBC).

Construction: We draw a line segment GPH through P parallel to AD and BC to cut AB and DC at H and G respectively. Similarly, we draw another line segment EPF through P parallel to CD or AB to cut AD and BC at E and F respectively.

We see that quadrilateral EPGD, PFCG, PFBH and AHPE are parallelograms.

$\therefore$ ar(ΔAPD) = ar(ΔDPE) + ar(ΔEPA)

$= \frac{1}{2}$ ar(‖gm EPGD) + $\frac{1}{2}$ ar(‖gm EPHA)

$= \frac{1}{2}$ [ar(‖gm EPGD) + ar(‖gm EPHA)]

$= \frac{1}{2}$ ar(‖gm DGHA) ...(1)

Similarly,

ar(ΔPBC) = $\frac{1}{2}$ ar(‖gm GCBH) ...(2)

$\therefore$ ar(ΔAPD) + ar(ΔPBC)

$= \frac{1}{2}$ [ar(‖gm DGHA) + ar(‖gm GCBH)]

[From (1) and (2)]

$= \frac{1}{2}$ ar(‖gm ABCD) ...(3)

Now, ar(‖gm ABCD)

= Area of the given trapezium

$= \frac{1}{2}(32 + 16) \times 6\ cm^2$

$= 24 \times 6 \text{ cm}^2$

$= 144 \text{ cm}^2$

$\therefore$ From (3),

$$ar(\Delta APD) + ar(\Delta PBC) = \frac{1}{2} \times 144 \text{ cm}^2 = 72 \text{ cm}^2$$

9. P is the point of intersection of the bisectors of internal angles of a triangle ABC, such that the area of each of ΔPAB and ΔPAC is equal to the area of a square of perimeter 24 cm and the area of ΔPBC is equal to the area of a rhombus whose diagonals are of lengths 10 cm and 6 cm then ΔABC can never be

(*a*) a scalene right- angled triangle

(*b*) a scalene obtuse-angled triangle

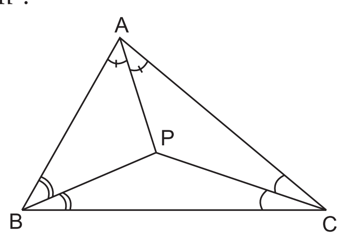
(*c*) an equilateral triangle

(*d*) a scalene acute-angled triangle

Sol. (*c*) an equilateral triangle

Given that ABC is a triangle and P is a point within the triangle such that PA, PB and PC are internal bisectors of the triangle meeting at O. Also, it is given that ar(ΔPAB) = ar(ΔPAC) but ar(ΔPBC) is different from this common area. Area of the square $= \frac{24}{4} \times \frac{24}{4} \text{ cm}^2 = 36 \text{ cm}^2$ and the area of the rhombus $= \frac{1}{2} \times 10 \times 6 \text{ cm}^2$

$= 30 \text{ cm}^2$.



$\therefore \quad ar(\Delta PAB) = ar(\Delta PAC) = 36 \text{ cm}^2$

but $\quad ar(\Delta PBC) = 30 \text{ cm}^2$

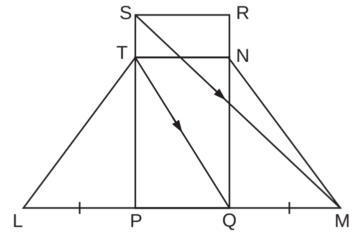
To determine the name of special type of the triangle ABC.

We see that P is the incentre of ΔABC. Out of all triangles, only for an equilateral triangle, the incentre is identical with the centroid or circum centre. So, if ΔABC is an equilateral triangle, then P must be the centroid also and so P will divide the area of ΔABC into three parts of equal area, i.e. if ABC is an equilateral triangle, then

$$ar(\Delta PAB) = ar(\Delta PBC) = ar(\Delta PCA)$$

which is not true.

10. In the given figure, PQRS is a rectangle and LTNM is a trapezium with PL = QM, LM ∥ TN. If TQ ∥ SM and ar(rect PQRS) = 36 cm², then the area of the trapezium LTNM is

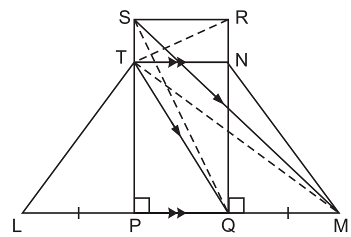


(*a*) 72 cm² (*b*) 36 cm²

(*c*) 54 cm² (*d*) 40 cm²

Sol. (*b*) 36 cm²

Given that PQRS is a rectangle and LTMN is a trapezium with PL = QM, LM ∥ TN, TQ ∥ SM and ar(rect PQRS) = 36 cm².



To find the area of the trapezium LTNM.

Construction: We join TM, SQ and TR.

In ΔNQM and ΔTPL, we have

QM = PL [Given]

NQ = TP

[∵ TNQP is a rectangle]

and ∠NQM = ∠TPL = 90°

[∵ SPQR is a rectangle]

∴ By SAS congruence criterion,

ΔNQM ≅ ΔTPL

∴ ar(ΔNQM) = ar(ΔTPL) …(1)

Now, ar(ΔNQM) = ar(ΔQTM)

[∵ ΔNQM and ΔQTM lie on the same base QM and between two parallels QM and TN]

= ar(ΔSTQ)

[∵ ΔQTM and ΔSTQ lie on the same base QT and between two parallels QT and SM]

= ar(ΔSTR)

[∵ ΔSTQ and ΔSTR lie on the same base ST and between two parallels ST and RNQ].

$= \frac{1}{2}$ ar(rect STNR) …(2)

[∵ TR is a diagonal of the rect STNR]

$\therefore$ From (1) and (2), we have

ar(ΔNQM) + ar(ΔTPL) = ar(rect STNR) …(3)

Hence, required ar(trapezium LTNM)

= ar(rect PQNT) + ar(ΔNQM) + ar(ΔTPL)

= ar(rect PQNT) + ar(rect STNR) [From (2)]

= ar(rect PQRS) = 36 cm^2 [Given]

Value-based Question (Optional)

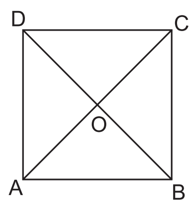
(Page 130)

1. For cleanliness campaign, a group of students of a school decided to prepare four triangular badges of equal areas, cut out from a big square shaped badge, with slogans written in four triangular parts of the badges.

(*a*) How can the students divide the square badge into four triangular parts of equal areas?

(*b*) Write some values which were exhibited by the students through this activity.

Sol. (*a*) We know that two diagonals of any square divide the square into four triangles of equal area.



Hence, students can divide the square badge, say ABCD into four triangles OAB, OBC, OCD and ODA by joining the two diagonals AC and BD of the square ABCD, O being the point of intersection of the two diagonals. The areas of the four triangles are equal because of the following reasons:

ar(ΔABD) = ar(ΔDBC)

[$\because$ DB is a diagonal of the square ABCD]

$= \frac{1}{2}$ ar(square ABCD)

Also, ar(ΔOAB) = ar(ΔOAD)

[$\because$ AO is a median of ΔABD, $\because$ OD = OB]

$= \frac{1}{4}$ ar(square ABCD)

Hence, it follows that ar(ΔOAB) = ar(ΔOBC) = ar(ΔOCD) = ar(ΔOAD)

(*b*) Environmental cleaning, awareness and leadership.

10

Circles

Checkpoint (Page 133)

1. The diameter of planet Venus is 12278 km. Find its circumference.

Sol. Diameter = 12278 km

$$\text{Radius } (R) = \frac{\text{Diameter}}{2}$$

$$= \frac{12278}{2} \text{ km} = 6139 \text{ km}$$

$$\text{Radius } (R) = 6139 \text{ km}$$

$$\text{Circumference} = 2\pi R$$

$$= 2 \times \frac{22}{7} \times 6139 \text{ km}$$

$$= 2 \times 22 \times 877 \text{ km}$$

$$= 38588 \text{ km}$$

Hence, circumference of planet venus is 38588 km.

2. Two circles having the same centre have radii 350 m and 490 m. What is the difference between their circumferences?

Sol. Radius of one circle (R_1) = 350 m

Radius of other circle (R_2) = 490 m

$$\text{Circumference } (C_1) = 2\pi R_1$$

$$= 2 \times \frac{22}{7} \times 350 \text{ m}$$

$$= 2 \times 22 \times 50 \text{ m} = 2200 \text{ m}$$

$$\text{Circumference } (C_2) = 2\pi R_2$$

$$= 2 \times \frac{22}{7} \times 490 \text{ m}$$

$$= 2 \times 22 \times 70 \text{ m} = 3080 \text{ m}$$

Difference between the circumference

$$= C_2 - C_1$$

$$= (3080 - 2200) \text{ m} = 880 \text{ m}$$

Hence, difference between the circumference of two circles is 880 m.

3. Find the perimeter of a semicircular plate of radius 3.85 cm.

Sol. Radius (R) = 3.85 cm

Perimeter of semi-circular plate = $\pi R + 2R$

$$= \left(\frac{22}{7} \times 3.85 + 2 \times 3.85\right) \text{cm}$$

$$= (22 \times 0.55 + 2 \times 3.85) \text{ cm}$$

$$= (12.1 + 7.7) \text{ cm}$$

$$= 19.8 \text{ cm}$$

Hence, perimeter of semi-circular plate is 19.8 cm.

4. The area of a circle is 55.44 m^2. Find its radius.

Sol. Area of a circle (A) = 55.44 m^2

$$A = \pi R^2$$

$$55.44 \text{ m}^2 = \frac{22}{7} \times R^2$$

$$(55.44 \times 7) \text{ m}^2 = 22 \times R^2$$

$$\left(\frac{55.44 \times 7}{22}\right) \text{m}^2 = R^2$$

$$R^2 = \left(\frac{55.44 \times 7}{22}\right) \text{m}^2$$

$$= (2.52 \times 7) \text{ m}^2$$

$$= (17.64) \text{ m}^2$$

$$R^2 = \left(\frac{1764}{100}\right) \text{m}^2$$

$$R^2 = \left(\frac{42}{10}\right)^2 \text{m}^2$$

$$R = 4.2 \text{ m}$$

Hence, the radius of a circle is 4.2 m.

5. The area of two circles are in the ratio 16 : 25. Find the ratio of their circumferences.

Sol. Let area of one circle be A_1

Let area of other circle be A_2

Ratio of area of two circles = 16 : 25

$$\frac{A_1}{A_2} = \frac{16}{25}$$

$$\frac{\pi R_1^2}{\pi R_2^2} = \frac{16}{25}$$

$$\frac{R_1^2}{R_2^2} = \frac{16}{25}$$

$$\left(\frac{R_1}{R_2}\right)^2 = \left(\frac{4}{5}\right)^2$$

$$\frac{R_1}{R_2} = \frac{4}{5}$$

Ratio of circumference of two circles are

$$\frac{C_1}{C_2} = \frac{2\pi R_1}{2\pi R_2} = \frac{R_1}{R_2} = \frac{4}{5}$$

Hence ratio of circumference of two circles is 4 : 5.

6. The minute hand of a circular clock is 11 cm long. How far does the tip of the minute hand move in 2 hours? [Take $\pi = 3.14$]

Sol. The minute hand of a circular clock (Radius) = 11 cm. We have to find circumference of a circular clock.

$$\text{Circumference (C)} = 2\pi R = 2 \times 3.14 \times 11 \text{ cm} = 69.08 \text{ cm}$$

2 hours = 2 complete revolution

Distance covered by tip of minute hand in

2 hours = 2 × 69.08 cm

= 138.16 cm

Hence, the tip of the minute hand move 138.16 cm in 2 hours.

7. If the area of a circle is 24.64 cm^2, then find its circumference.

Sol. $$\text{Area (A)} = 24.64 \text{ cm}^2$$

$$A = \pi R^2$$

$$\Rightarrow \quad 24.64 \text{ cm}^2 = \frac{22}{7} \times R^2$$

$$\Rightarrow \quad (24.64 \times 7) \text{ cm}^2 = 22 \times R^2$$

$$\Rightarrow \quad \left(\frac{24.64 \times 7}{22}\right) \text{ cm}^2 = R^2$$

$$\Rightarrow \quad R^2 = \left(\frac{2.24 \times 7}{2}\right) \text{ cm}^2$$

$$\Rightarrow \quad R^2 = (1.12 \times 7) \text{ cm}^2$$

$$\Rightarrow \quad R^2 = 7.84 \text{ cm}^2$$

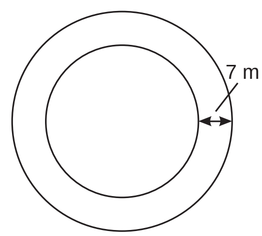
$$\Rightarrow \quad R = 2.8 \text{ cm}$$

$$\text{Circumference (C)} = 2\pi R = 2 \times \frac{22}{7} \times 2.8 \text{ cm} = (2 \times 22 \times 0.4) \text{ cm} = 17.6 \text{ cm}$$

Hence, circumference of a circle is 17.6 cm.

8. The circumference of a circular park is 660 m. A 7 m wide path surrounds it. Find the cost of fencing the outer boundary at the rate of ₹ 60 per metre.

Sol. Circumference of a circular park (C) = 660 m



$$C = 2\pi R$$

[R is the radius of a circle]

$$\Rightarrow \quad 660 \text{ m} = 2 \times \frac{22}{7} \times R$$

$$\Rightarrow \quad (660 \times 7) \text{ m} = 2 \times 22 \times R$$

$$\Rightarrow \quad \left(\frac{660 \times 7}{2 \times 22}\right) \text{ m} = R$$

$$\Rightarrow \quad R = \left(\frac{30 \times 7}{2}\right) \text{ m} = (15 \times 7) \text{ m}$$

$$\Rightarrow \quad R = 105 \text{ m}$$

∴ Radius of circular park = 105 m

We need to find the circumference of the circular path around the park.

$$\text{Circumference } (C_1) = 2\pi R_1$$

[$R_1 = R + 7 = (105 + 7)$ m = 112 m]

$$= 2 \times \frac{22}{7} \times 112 \text{ m} = (2 \times 22 \times 16) \text{ m} = 704 \text{ m}$$

Cost of fencing 1 m outer boundary = ₹60

Cost of fencing 704 m outer boundary

= ₹60 × 704

= ₹42240

Hence, cost of fencing outer boundary is ₹42240.

9. Find the radius of a circle whose area is twice the area of a circle of radius 14 cm.

Sol. Radius of a circle (R) = 14 cm

Area of a circle (A) = πR^2

$$= \left(\frac{22}{7} \times 14 \times 14\right) \text{ cm}^2$$

$$= (22 \times 2 \times 14) \text{ cm}^2$$

$$= 616 \text{ cm}^2$$

Let area of the other circle be A_1 and radius be R_1.

Given that

Area of the other circle = 2 × area of a circle

$$A_1 = (2 \times 616) \text{ cm}^2$$

$$\pi R_1^2 = (2 \times 616) \text{ cm}^2$$

$$R_1^2 = \frac{2 \times 616 \times 7}{22} \text{cm}^2$$

$$= \frac{8624}{22} \text{cm}^2$$

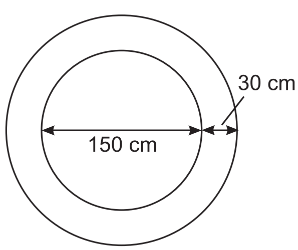
$$R_1^2 = 392 \text{ cm}^2$$

$$R_1 = 14\sqrt{2} \text{ cm}$$

Hence, radius of the other circle is $14\sqrt{2}$ cm.

10. A well of diameter 150 cm has a 30 cm wide parapet running around it. Find the area of the parapet.

Sol.



Radius of inner circle (R) = $\frac{\text{Diameter}}{2}$

$$= \frac{150}{2} \text{ cm} = 75 \text{ cm}$$

Area of inner circle = πR^2

$$= \left(\frac{22}{7} \times 75 \times 75\right) \text{cm}^2$$

Radius of outer circle (R_1) = R + 30

= (75 + 30) cm

= 105 cm

Area of outer circle = πR_1^2

$$= \left(\frac{22}{7} \times 105 \times 105\right) \text{cm}^2$$

Area of ring = Area of outer circle – Area of inner circle

$$= \pi R_1^2 - \pi R^2$$

$$= \pi\left(R_1^2 - R^2\right)$$

$$= \frac{22}{7}[105 \times 105 - 75 \times 75] \text{cm}^2$$

$$= \frac{22}{7}\left[(105)^2 - (75)^2\right] \text{cm}^2$$

$$= \frac{22}{7}(105 + 75)(105 - 75) \text{ cm}^2$$

[Using identity $a^2 - b^2 = (a + b)(a - b)$]

$$= \left(\frac{22}{7} \times 180 \times 30\right) \text{ cm}^2$$

$$= \frac{118800}{7} \text{ cm}^2$$

$$= 16971.42 \text{ cm}^2$$

Hence, area of parapet is 16971 cm² (approx.)

Milestone 1

(Page 140)

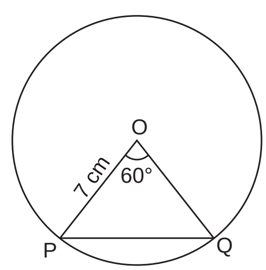
Multiple-Choice Questions

1. PQ is a chord of a circle with centre O and radius equal to 7 cm. If ∠POQ = 60°, then the length of the chord PQ is

(*a*) $\frac{7}{2}$ cm (*b*) $\frac{7\sqrt{3}}{2}$ cm

(*c*) 7 cm (*d*) $\frac{7\sqrt{3}}{4}$ cm

Sol. (*c*) 7 cm



Given,

Radius = 7 cm, OP = OQ = 7 cm

Also, $\angle POQ = 60°$

In ΔPOQ, we have

$\angle POQ + \angle OPQ + \angle OQP = 180°$

[Angle sum property of a triangle]

$\Rightarrow \quad 60° + \angle OPQ + \angle OPQ = 180°$

[$\angle OPQ = \angle OQP$, $\because$ OP = OQ, ΔPOQ is an isosceles triangle]

$\Rightarrow \quad 60° + 2\angle OPQ = 180°$

$\Rightarrow \quad 2\angle OPQ = 180° - 60°$

$\Rightarrow \quad 2\angle OPQ = 120°$

$\Rightarrow \quad \angle OPQ = 60°$

$\Rightarrow \quad \angle OQP = 60°$

Hence, ΔPOQ is an equilateral triangle as

$\angle POQ = \angle OPQ = \angle OQP = 60°$.

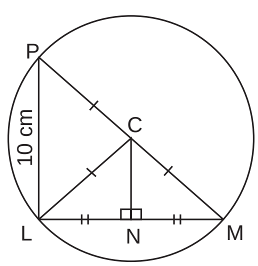
$\therefore$ Length of chord PQ is 7 cm.

2. LM is a chord of a circle with centre C. If CN $\perp$ LM, MC produced intersects the circle at P and if PL = 10 cm, then the length of CN will be

(*a*) 8 cm (*b*) 5 cm

(*c*) 6 cm (*d*) 9 cm

Sol. (*b*) 5 cm



Given,

C is the centre of a circle and let

$PC = MC = LC = R$

[Radius of a circle]

$\because$ ΔPCL and ΔCLM is an isosceles triangle

[PC = LC = MC]

Let $\angle CPL = \angle CLP = \angle CLM = \angle CML = x$

In ΔPLM, we have

$\angle LPM + \angle LMP + \angle PLM = 180°$

[Angle sum property of a triangle]

$\Rightarrow \angle LPM + \angle LMP + \angle CLP + \angle CLM = 180°$

$\Rightarrow \quad x + x + x + x = 180°$

$\Rightarrow \quad 4x = 180°$

$\Rightarrow \quad x = 45°$

$\therefore$ ΔPCL is a right-angled triangle.

By Pythagoras' Theorem, we have

$(PL)^2 = (LC)^2 + (PC)^2$

$\Rightarrow \quad (10 \text{ cm})^2 = R^2 + R^2$

$\Rightarrow \quad 100 \text{ cm}^2 = 2R^2$

$\Rightarrow \quad R^2 = 50 \text{ cm}^2$

$\Rightarrow \quad R = 5\sqrt{2}$ cm

$PM = PC + MC$

$= 5\sqrt{2} + 5\sqrt{2}$

$= 10\sqrt{2}$

$\because$ ΔPLM is a right-angled triangle,

$\therefore$ By Pythagoras' Theorem, we have

$(PM)^2 = (PL)^2 + (LM)^2$

$\Rightarrow \quad \left(10\sqrt{2} \text{ cm}\right)^2 = (10 \text{ cm})^2 + (LM)^2$

$\Rightarrow \quad 200 \text{ cm}^2 = 100 \text{ cm}^2 + (LM)^2$

$\Rightarrow \quad (LM)^2 = 100 \text{ cm}^2$

$\Rightarrow \quad LM = 10$ cm

CN is perpendicular to LM, so it will bisect LM. [The $\perp$ from the centre of a circle to a chord bisects the chord]

LN = NM = 5 cm

$\because$ ΔCNL is a right-angled triangle,

$\therefore$ By Pythagoras' Theorem, we have

$(LC)^2 = (CN)^2 + (LN)^2$

$\Rightarrow \quad \left(5\sqrt{2} \text{ cm}\right)^2 = (CN)^2 + (5 \text{ cm})^2$

$\Rightarrow \quad 50 \text{ cm}^2 = (CN)^2 + 25 \text{ cm}^2$

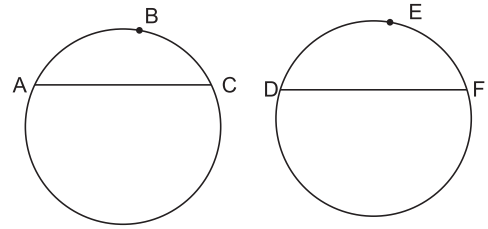
$\Rightarrow \quad (CN)^2 = 25 \text{ cm}^2$

$\Rightarrow \quad CN = 5$ cm

Very Short Answer Type Questions

3. The arcs ABC and DEF of two circles of equal radius are equal. If the chord AC of the first circle is 4.5 cm, find the length of the chord DF of the second circle.

Sol. Given, radius of both the circles are equal and $\overset{\frown}{ABC} = \overset{\frown}{DEF}$



$\because$ Chord AC = 4.5 cm

∴ Chord DF = Chord AC

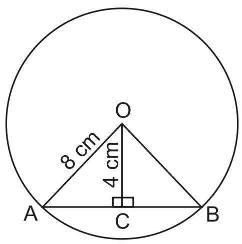
= 4.5 cm

[Two circles of equal radius and equal arcs have equal chords]

Hence, length of the chord DF is 4.5 cm.

4. Find the length of the chord of a circle of radius 8 cm, if its distance from the centre of the circle is 4 cm.

Sol. Let O is the centre of a circle and AB be the chord.



Given,

Radius(OA = OB) = 8 cm and OC = 4 cm is the distance of the chord from the centre of a circle.

∵ ΔACO is a right-angled triangle,

∴ By Pythagoras' Theorem, we have

$$(OA)^2 = (AC)^2 + (OC)^2$$

$$(8\text{ cm})^2 = (AC)^2 + (4)^2$$

$$\Rightarrow \quad 64\text{ cm}^2 = (AC)^2 + 16\text{ cm}^2$$

$$\Rightarrow \quad (AC)^2 = (64 - 16)\text{ cm}^2$$

$$\Rightarrow \quad (AC)^2 = 48\text{ cm}^2$$

$$\Rightarrow \quad AC = 4\sqrt{3}\text{ cm}$$

$$\because \quad AB = AC + CB$$

[AC = CB, ∵ The ⊥ from the centre of a circle to a chord bisects the chord]

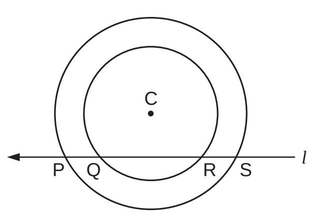
$$= \left(4\sqrt{3} + 4\sqrt{3}\right)\text{cm}$$

$$= 8\sqrt{3}\text{ cm}$$

Hence, length of the chord of a circle is $8\sqrt{3}$ cm.

Short Answer Type-I Questions

5. Two concentric circles with centre C are cut by a line *l* at P, Q, R and S as shown in the figure. Prove that PQ = RS.



Sol.

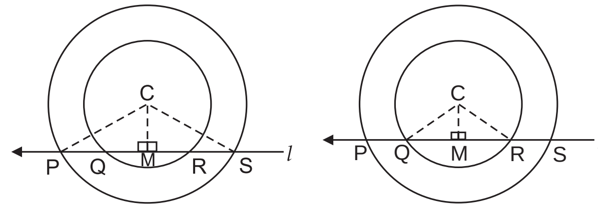


Fig (*i*) Fig (*ii*)

Construction: Draw a line CM ⊥ PS.

Also, join CP and CS in Fig (*i*)

∵ PS is a chord,

∴ PM = MS …(1)

[The ⊥ from the centre of a circle to a chord bisects the chord]

Construction: Draw CM ⊥ PS.

Also, join CQ and CR in Fig (*ii*)

∵ CM ⊥ QR

∴ QM = MR …(2)

[∵ The ⊥ from the centre of a circle to a chord bisects the chord]

Subtract (2) from (1), we get

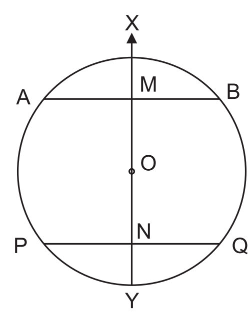
PM – QM = MS – MR

⇒ PQ = RS

Hence, proved.

6. If a diameter of a circle bisects each of the two chords of the circle, prove that the chords are parallel.

Sol.



Let XOY bisect the chord AB at M and chord PQ at N.

∵ M is mid-point of AB,

∴ OM ⊥ AB

[The line drawn joining the centre of a circle to the mid-point of a chord is perpendicular to the chord]

∴ ∠BMO is a right angle. …(1)

Similarly, ON ⊥ PQ

$\therefore$ $\angle PNO$ is a right angle. ...(2)

$\therefore$ $\angle BMO = \angle PNO$ [From (1) and (2)]

$\Rightarrow$ $\angle BMN = \angle PNM$ [Each 90°]

But $\angle BMN$ and $\angle PNM$ are alternate angles.

$\therefore$ AB ∥ PQ

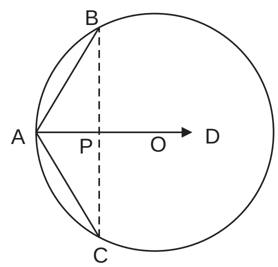
Hence, proved.

Short Answer Type-II Questions

7. Two chords AB and AC of a circle are equal. Prove that the centre of the circle lies on the bisector of $\angle BAC$.

Sol. Let AD be the bisector of $\angle BAC$.

Join BC and let it intersect AD at P.



In ΔBAP and ΔCAP, we have

$AB = AC$ [Given]

$\angle BAP = \angle CAP$

[$\because$ AD is the bisector of $\angle BAC$]

$AP = AP$ [Common]

$\therefore$ $\Delta BAP \cong \Delta CAP$

[By SAS congruence]

$\Rightarrow$ $PB = PC$ [By CPCT] ...(1)

$\Rightarrow$ $\angle APB = \angle APC$ [By CPCT] ...(2)

Now, $\angle APB + \angle APC = 180°$ [Linear pair] ...(3)

$\therefore$ $\angle APB = \angle APC = 90°$

[From (2) and (3)] ...(4)

$\therefore$ AP is perpendicular bisector of chord BC

[From (1) and (4)]

$\Rightarrow$ AD is the perpendicular bisector of chord BC. But the perpendicular bisector of a chord always passes through the centre of the circle.

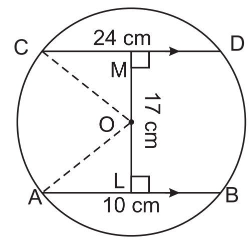
$\therefore$ AD passes through the center O of the circle.

$\Rightarrow$ O lies on AD.

Hence, the centre of the circle lies on the angle bisector of $\angle BAC$.

8. AB and CD are two parallel chords of a circle lying on the opposite sides of the centre such that AB = 10 cm and CD = 24 cm. If the distance between AB and CD is 17 cm, determine the radius of the circle.

Sol. Given, AB and CD are two chords of a circle such that AB ∥ CD, AB = 10 cm, CD = 24 cm and distance between AB and CD = 17 cm.



Draw OL ⊥ AB and OM ⊥ CD. Join OA and OC.

Then, $OA = OC = r$

[Radius of a circle]

$\because$ OL ⊥ AB, OM ⊥ CD and AB ∥ CD,

$\therefore$ The points L, O and M are collinear and LM = 17 cm

Let OL = x cm, OM = $(17 - x)$ cm

$\because$ Perpendicular from the centre of a circle to a chord bisects the chord.

$\therefore$ $AL = \frac{1}{2}AB$

$= \frac{1}{2} \times 10 = 5$ cm

and $CM = \frac{1}{2}CD$

$= \frac{1}{2} \times 24 = 12$ cm

$\because$ ΔOLA is a right-angled triangle,

$\therefore$ By Pythagoras' theorem, we have

$(OA)^2 = (OL)^2 + (AL)^2$

$\Rightarrow$ $r^2 = x^2 + (5)^2$ ...(1)

$\because$ ΔOMC is a right-angled triangle,

$\therefore$ By Pythagoras' Theorem, we have

$(OC)^2 = (OM)^2 + (CM)^2$

$\Rightarrow$ $r^2 = (17 - x)^2 + (12)^2$ ...(2)

From (1) and (2), we have

$x^2 + (5)^2 = (17 - x)^2 + (12)^2$

$\Rightarrow$ $x^2 + 25 = 289 + x^2 - 34x + 144$

$\Rightarrow$ $34x = 408$

$\Rightarrow$ $x = 12$

Now, substituting $x = 12$ in (1), we get

$r^2 = (12)^2 + (5)^2$

$\Rightarrow$ $r^2 = 144 + 25$

$\Rightarrow \qquad r^2 = 169$

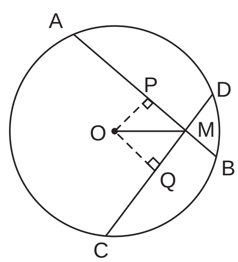
$\Rightarrow \qquad r = 13$ cm

Hence, the radius of a circle is 13 cm.

9. If two equal chords of a circle intersect within the circle, prove that the line joining the point of intersection to the centre makes equal angles with the chords.

Sol. Let AB and CD be two equal chords of a circle with centre O, such that they intersect at point M within the circle.

Draw OP ⊥ AB and OQ ⊥ CD. Also, join OM.



In ΔOPM and ΔOQM, we have

OP = OQ

[Equal chords are equidistant from the centre]

OM = OM [Common]

∴ ΔOPM ≅ ΔOQM

[By RHS congruence]

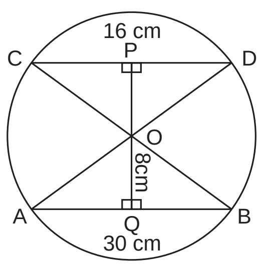
$\Rightarrow$ ∠OMP = ∠OMQ [By CPCT]

Thus, if two equal chords of a circle intersect within the circle, then the line joining the point of intersection to the centre makes equal angles with the chords.

Long Answer Type Questions

10. If the length of a chord of a circle 8 cm away from the centre of the circle, is 30 cm, find the distance of a chord of length 16 cm, from the centre of the circle.

Sol. O is the centre of a circle.



Chord AB = 30 cm

Distance of a chord AB from centre, OQ = 8 cm

∵ OQ ⊥ AB

[The ⊥ from the centre of a circle to a chord bisects the chord]

∴ AQ = 15 cm

∵ ΔAQO is a right-angled triangle,

∵ By Pythagoras' Theorem, we have

$$(AO)^2 = (AQ)^2 + (OQ)^2$$
$$= (15 \text{ cm})^2 + (8 \text{ cm})^2$$
$$= (225 + 64) \text{ cm}^2$$

$\Rightarrow (AO)^2 = 289 \text{ cm}^2$

$\Rightarrow$ AO = 17 cm

$\Rightarrow$ AO = BO = CO = DO = 17 cm

[Radius of a circle]

∵ OP ⊥ CD

[The ⊥ from the centre of a circle to a chord bisects the chord]

∴ CP = 8 cm

∵ ΔCPO is a right-angled triangle,

∴ By Pythagoras' Theorem, we have

$$(CO)^2 = (CP)^2 + (OP)^2$$

$\Rightarrow (17 \text{ cm})^2 = (8 \text{ cm})^2 + (OP)^2$

$\Rightarrow 289 \text{ cm}^2 = 64 \text{ cm}^2 + (OP)^2$

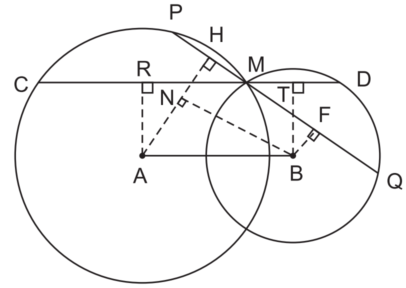
$\Rightarrow (OP)^2 = 225 \text{ cm}^2$

$\Rightarrow$ OP = 15 cm

Hence, the distance of a chord of length 16 cm from the centre is 15 cm.

11. Prove that of all the line segments drawn through a point of intersection of two circles and terminated by them, the one which is parallel to the line of centres is the greatest.

Sol. Given that two circles with centres A and B intersect and M is one of the points of intersection. CD and PQ are two line-segments through M, terminated by the two circles such that CD ∥ AB.



To prove that CD > PQ.

Construction: We draw AR ⊥ CD, BT ⊥ CD, AH ⊥ PQ, BF ⊥ PQ and BN ⊥ AH.

∵ AH is perpendicular to the chord PM.

∴ H is the mid-point of PM

∴ PM = 2HM …(1)

Again, since BF is perpendicular to the chord MQ,

∴ F is the mid-point of MQ

∴ MQ = 2MF …(2)

Adding (1) and (2), we have

PM + MQ = 2(HM + MF)

⇒ PQ = 2HF …(3)

Now, since BF ⊥ HF, NH ⊥ HF, BN ⊥ HN and FH ⊥ NH, the figure NBFH is a rectangle.

∴ HF = NB

∴ From (3), PQ = 2NB …(4)

Now, in right-angled triangle ANB,

AB > NB

∴ 2AB > 2NB = PQ [From (4)] …(5)

Now, AR is perpendicular to the chord CM. Hence, R is the mid-point of CM.

∴ CM = 2RM …(6)

Again, BT is perpendicular to the chord MD. Hence, T is the mid-point of MD.

∴ MD = 2MT …(7)

Adding (6) and (7), we get

CM + MD = 2(RM + MT)

⇒ CD = 2RT = 2AB …(6)

From (5) and (6), we have

CD > PQ

Hence, proved.

Milestone 2

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Multiple-Choice Questions

1. ABCD is a quadrilateral with A as the centre of a circle passing through the points B, C and D such that AB = AC = AD, ∠CBD = 20° and ∠CDB = 30°. Then ∠BAD is equal to

(*a*) 80° (*b*) 70°

(*c*) 140° (*d*) 100°

Sol. (*d*) 100°

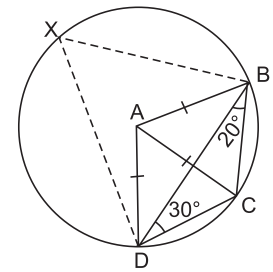
In ΔDCB,

∠BDC + ∠DBC + ∠DCB = 180°

30° + 20° + ∠DCB = 180°

50° + ∠DCB = 180°

∠DCB = 130°



Mark a point X on the circle and join XD and XB.

∠BAD = 2∠DXB …(1)

∵ XBCD is a cyclic quadrilateral [X, B, C, D lie on the circle]

∴ ∠DCB + ∠DXB = 180°

[Opposite angles of a cyclic quadrilateral is supplementary]

130° + ∠DXB = 180°

∠DXB = 50° …(2)

Now, substitute (2) in (1),

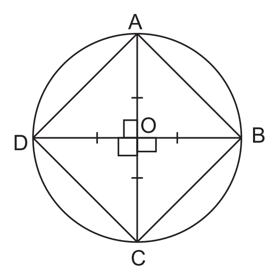
∠BAD = 2 × 50° = 100°

2. Two diameters of a circle intersect each other at right angles. Then the quadrilateral formed by joining their end points is

(*a*) a rhombus (*b*) a square

(*c*) a rectangle (*d*) a kite

Sol. (*b*) a square



In ΔAOD and ΔAOB, we have

AO = AO [Common]

OD = OB [Radius of a circle]

∠AOD = ∠AOB [Each 90°]

∴ ΔAOD ≅ ΔAOB

[By SAS congruence]

⇒ AD = AB [By CPCT]

Similarly, DC = BC

AD = DC

AB = BC

∴ All sides of a quadrilateral are equal. …(1)

∵ ΔAOD is a right-angled isosceles triangle.

∴ $\angle ODA = \angle OAD = 45°$

$\angle OAB = 45° = \angle OAD$

[ΔAOD ≅ ΔAOB]

∴ $\angle DAB = \angle OAD + \angle OAB$

$= 45° + 45°$

$= 90°$

Similarly, $\angle ADC = 90°$

∵ ABCD is a cyclic quadrilateral. So, opposite angles of a cyclic quadrilateral are supplementary.

∴ $\angle DCB = 90°$

and $\angle ABC = 90°$

∵ All angles of a quadrilateral are of 90°. …(2)

∴ From (1) and (2),

Quadrilateral is a square.

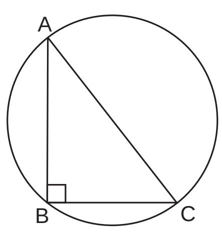
Hence, proved.

Very Short Answer Type Questions

3. ΔABC is a right-angled triangle with ∠B = 90°. A circle is drawn circumscribing the ΔABC. State with reason, what will be the position of the centre of this circle.

Sol. Given, ΔABC is a right-angled triangle.

We know, the arc of a circle subtending a right-angle at any point on the remaining part of the circle is a semi-circle.



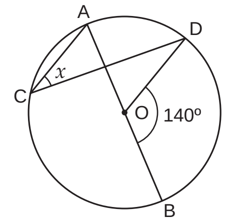
∴ $\overset{\frown}{AC}$ is a semi-circle.

⇒ AC is the diameter of a circle.

⇒ Centre of a circle will lie on AC, i.e hypotenuse of a right-angled ΔABC.

∴ Position of the centre of this circle is mid-point of the hypotenuse AC, since angle in a semicircle is 90°.

4. In the given figure, if O is the centre of a circle, AB is a diameter, CD is a chord, and ∠DOB = 140°, what is the value of x where ∠ACD = x?



Sol. $\angle DOB + \angle AOD = 180°$ [Linear pair]

⇒ $140° + \angle AOD = 180°$

⇒ $\angle AOD = 40°$

$\angle AOD = 2\angle ACD$

[The angle subtended by an arc of a circle at the centre is double the angle subtended by it at any point on the remaining part of the circle]

⇒ $40° = 2\angle ACD$

⇒ $\angle ACD = 20°$

$\angle ACD = x = 20°$

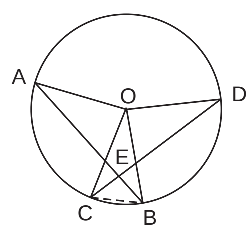
Hence, value of x, where ∠ACD = x is 20°.

Short Answer Type-I Questions

5. In a circle with centre O, chords AB and CD intersect inside the circumference at E. Prove that ∠AOC + ∠BOD = 2∠AEC **[CBSE SP 2011]**

Sol. *Construction*: Join CB.

Since the angle made by an arc of a circle at the centre is double the angle made by it at a point on the remaining part of the circle.



∴ $\angle AOC = 2\angle ABC$ …(1)

and $\angle BOD = 2\angle BCD$ …(2)

Adding (1) and (2), we have

$\angle AOC + \angle BOD = 2\angle ABC + 2\angle BCD$

$= 2\,(\angle ABC + \angle BCD)$

$= 2\,(\angle EBC + \angle BCE)$

[∵ ∠ABC = ∠EBC and ∠BCD = ∠BCE]

$= 2\angle AEC$

[Exterior angle of ΔBEC = Sum of interior opposite angles]

∴ ∠AOC + ∠BOD = 2 ∠AEC

Hence, proved.

6. A right triangle PQR, right-angled at R, in which a circle is drawn on the hypotenuse PQ as a diameter. Prove that the circle passes through the point R.

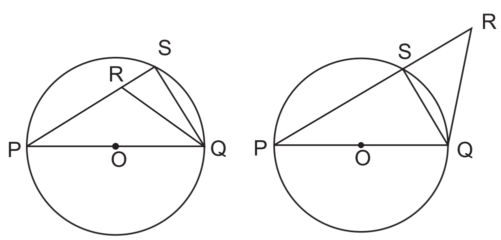
Sol. Given, a triangle PQR, right-angled at R. A circle is drawn on PQ as diameter.

Suppose the circle does not pass through R and cuts PR produced in S (Fig (*i*)) or PR in S (Fig (*ii*)).

∠PRQ = 90° [Given]

∠PSQ = 90° [Angle in a semi-circle]

⇒ ∠PRQ = ∠PSQ



Exterior angle of a triangle cannot be equal to its interior opposite angle.

∴ We reach a contradiction. So, our supposition is wrong.

Hence, the required result is proved.

Short Answer Type-II Questions

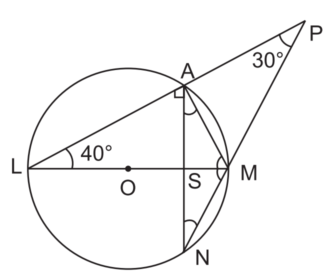
7. LAM is a right-angled triangle with ∠LAM = 90°. A circle drawn with the centre at O, the mid-point of LM. AN is a chord of the circle cutting LM at S and the circle at N. NM produced cuts LA produced at P. If ∠LPM = 30° and ∠ALM = 40°, find ∠ANM, ∠AMN and ∠NAM.

Sol. Given, ∠LPM = 30°,

∠ALM = 40°,

∠LAM = 90°

and ∠PAM = ∠LAM = 90° [Linear pair]



∵ ∠AMN is exterior angle of ΔPAM,

∴ ∠PAM + ∠APM = ∠AMN

[Exterior angle is equal to sum of its two opposite interior angles]

⇒ 90° + 30° = ∠AMN

⇒ ∠AMN = 120°

∵ ∠ALM = ∠ANM

[Angles in the same segment]

∴ ∠ANM = 40°

In ΔAMN, we have

∠ANM + ∠AMN + ∠NAM = 180°

40° + 120° + ∠NAM = 180°

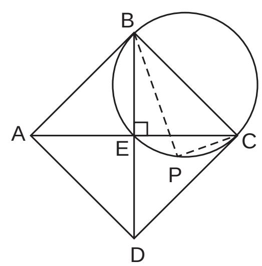
160° + ∠NAM = 180°

∠NAM = 20°

Hence, ∠ANM = 40°, ∠AMN = 120° and ∠NAM = 20°.

8. Prove that the circle drawn with any side of a rhombus as a diameter, passes through the point of intersection of its diagonals.

Sol. Let ABCD be the rhombus with its diagonal AC and BD intersecting at E.



Draw a circle with BC as diameter. Take a point P on the circle and join PB and PC.

∠BPC = 90° [Angle in a semi-circle] …(1)

But ∠BEC = 90° [Diagonals of a rhombus are ⊥ to each other] …(2)

∴ ∠BEC = ∠BPC [From (1) and (2)]

⇒ BC subtends equal angles at points E and P which are on the same side of it.

We know that if a line segment joining two points subtends equal angles on the same side of the line containing the line segment, the four points lie on a circle.

∴ Points B, E, P and C are concyclic.

∴ E lies on the circle with BC as a diameter.

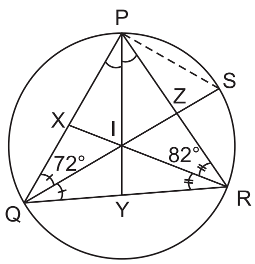
Similarly, it can be proved that E lies on circle with AB, AD and CD as diameters.

Hence, the circle drawn with any side of rhombus as diameter passes through the point of intersection of its diagonals.

Long Answer Type Questions

9. In $\angle PQR$, $\angle PQR = 72^\circ$ and $\angle QRP = 82^\circ$ and I is the incentre of ΔPQR. When QI is produced, it meets the circumcircle of ΔPQR at S. SQ is joined. Calculate $\angle SPR$, $\angle IPR$ and $\angle PIS$.

Sol. The incentre of a triangle is the intersection of the angle bisectors of the triangle.



Given, $\angle PQR = 72^\circ$, $\angle QRP = 82^\circ$

In ΔPQR, we have

$$\angle PQR + \angle QRP + \angle QPR = 180^\circ$$

$$\Rightarrow \quad 72^\circ + 82^\circ + \angle QPR = 180^\circ$$

$$\Rightarrow \quad 154^\circ + \angle QPR = 180^\circ$$

$$\Rightarrow \quad \angle QPR = 26^\circ$$

$$\angle IPR = \frac{1}{2}\angle QPR$$

[PY is the angle bisector of $\angle QPR$]

$$\Rightarrow \quad \angle IPR = \frac{1}{2} \times 26^\circ = 13$$

$$\Rightarrow \quad \angle IPR = 13^\circ$$

$$\angle SQR = \frac{1}{2}\angle PQR$$

[QZ is the angle bisector of $\angle PQR$]

$$\Rightarrow \quad \angle SQR = \frac{1}{2} \times 72^\circ$$

$$\Rightarrow \quad \angle SQR = 36^\circ$$

$$\angle SQR = \angle SPR$$

[Angles in the same segment of a circle are equal]

$$\Rightarrow \quad \angle SPR = 36^\circ$$

In ΔQZR, we have

$$\angle ZQR + \angle ZRQ = \angle QZP$$

[Exterior angle of a triangle is equal to sum of its two opposite interior angles]

$$\Rightarrow \quad 36^\circ + 82^\circ = \angle QZP$$

$$\Rightarrow \quad \angle QZP = 118^\circ$$

In ΔPIZ, we have

$$\angle IZP + \angle PIZ + \angle IPZ = 180^\circ$$

[Angle sum property of a triangle]

$$118^\circ + \angle PIZ + 13^\circ = 180^\circ$$

[$\because \angle QZP = \angle IZP$ and $\angle IPR = \angle IPZ$]

$$131^\circ + \angle PIZ = 180^\circ$$

$$\angle PIZ = 49^\circ$$

$$\angle PIS = 49^\circ \qquad [\because \angle PIS = \angle PIZ]$$

Hence, measure of $\angle PIS = 49^\circ$, $\angle SPR = 36^\circ$ and $\angle IPR = 13^\circ$.

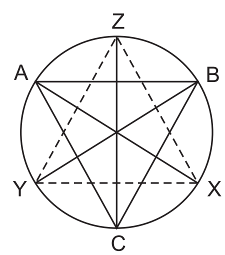
10. ΔABC is inscribed in a circle and the bisectors of $\angle A$, $\angle B$ and $\angle C$ meet the circumference of the circle at X, Y and Z respectively. Prove that

$$\angle X = 90^\circ - \frac{\angle A}{2},$$

$$\angle Y = 90^\circ - \frac{\angle B}{2}$$

and $$\angle Z = 90^\circ - \frac{\angle C}{2}.$$

Sol. Let bisectors of $\angle A$, $\angle B$ and $\angle C$ of a triangle ABC intersect the circumference of the circle at X, Y and Z respectively.



Now, from figure

$$\angle X = \angle YXZ$$

$$\angle X = \angle YXA + \angle AXZ$$

$\because$ $\angle YXA$ and $\angle YBA$ are the angles in the same segment of the circle.

$\therefore \quad \angle YXA = \angle YBA$

Hence, $\quad \angle X = \angle YBA + \angle AXZ$

Again, $\angle AXZ$ and $\angle YCA$ are the angles in the same segment of the circle.

Hence, $\quad \angle AXZ = \angle YCA$

Again, $\because$ BY is the bisector of $\angle B$ and CZ is the bisector of $\angle C$

So, $\quad \angle X = \frac{1}{2}\angle B + \frac{1}{2}\angle C$

Similarly, $\quad \angle Y = \frac{1}{2}\angle C + \frac{1}{2}\angle A$

and $\angle Z = \frac{1}{2}\angle A + \frac{1}{2}\angle B$

Now, $\angle X = \frac{1}{2}\angle B + \frac{1}{2}\angle C$

$= \frac{1}{2}(\angle B + \angle C)$

$\Rightarrow \angle X = \frac{1}{2}(180° - \angle A)$

$[\angle A + \angle B + \angle C = 180°]$

$\Rightarrow \angle X = 90° - \frac{1}{2}\angle A$

Similarly, $\angle Y = \frac{1}{2}(180° - \angle B)$

$= 90° - \frac{1}{2}\angle B$

$\angle Y = 90° - \frac{1}{2}\angle B$

and

$\angle Z = \frac{1}{2}(180° - \angle C)$

$= 90° - \frac{1}{2}\angle C$

$\angle Z = 90° - \frac{1}{2}\angle C$

Hence, proved.

Milestone 3

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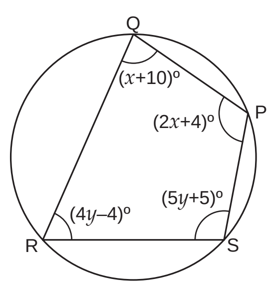
Multiple-Choice Questions

1. In the given figure, PQRS is a cyclic quadrilateral. If $\angle P = (2x + 4)^o$, $\angle Q = (x + 10)^o$, $\angle R = (4y - 4)^o$ and $\angle S = (5y + 5)^o$, then the values of x and y are respectively



(*a*) 25 and 40 (*b*) 40 and 25
(*c*) 27 and 38 (*d*) 38 and 27

Sol. (*b*) 40 and 25

Given, PQRS is a cyclic quadrilateral.

So, opposite angles of a cyclic quadrilateral are supplementary.

$\angle P + \angle R = 180°$

$2x + 4 + 4y - 4 = 180°$

$2x + 4y = 180°$

$x + 2y = 90°$ …(1)

$\angle Q + \angle S = 180°$

$x + 10 + 5y + 5 = 180°$

$x + 5y + 15 = 180°$

$x + 5y = 165°$ …(2)

Solving (1) and (2), we get

$x + 2y = 90°$

$\underset{-}{x} + \underset{-}{5y} = \underset{-}{165°}$

$- 3y = -75°$

$\Rightarrow y = 25°$

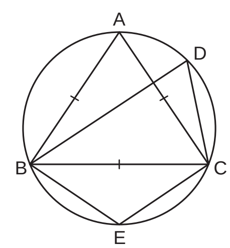
Putting value of 'y' in (1), we get

$x + 2 \times 25° = 90°$

$x + 50° = 90°$

$x = 40°$

2. If ABC is an equilateral triangle, then the measure of $\angle BDC$ and $\angle BEC$ are respectively **[CBSE SP 2011]**



(*a*) 120° and 60° (*b*) 60° and 100°
(*c*) 120° and 50° (*d*) 60° and 120°

Sol. (*d*) 60° and 120°

Given, ΔABC is an equilateral triangle. So, all the angles of ΔABC will be equal to 60°.

$\angle ABC = \angle BAC = \angle ACB = 60°$

$\angle BAC = \angle BDC = 60°$

[Angles in the same segment of a circle are equal]

$\because$ BDCE is a cyclic quadrilateral.

[B, D, C and E lie on the circle]

$\therefore \angle BDC + \angle BEC = 180°$

[Opposite angles of a cyclic quadrilateral are supplementary]

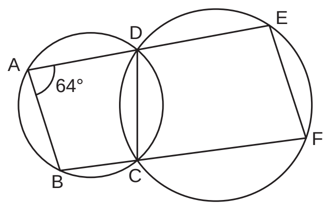
$\Rightarrow 60° + \angle BEC = 180°$

$\Rightarrow \angle BEC = 180° - 60°$

$\therefore \angle BEC = 120°$

Very Short Answer Type Questions

3. In the given figure, if $\angle BAD = 64^\circ$, find the measures of $\angle DCF$ and $\angle DEF$.



Sol. Given, $\angle BAD = 64^\circ$

ABCD is a cyclic quadrilateral [A, B, C, D lie on the circle]

$\angle BAD + \angle BCD = 180^\circ$

[Opposite angles of a cyclic quadrilateral are supplementary]

$\Rightarrow \quad 64^\circ + \angle BCD = 180^\circ$

$\Rightarrow \quad \angle BCD = 180^\circ - 64^\circ$

$= 116^\circ$

$\angle BCD + \angle DCF = 180^\circ$ [Linear pair]

$\Rightarrow \quad 116^\circ + \angle DCF = 180^\circ$

$\Rightarrow \quad \angle DCF = 64^\circ$

DEFC is a cyclic quadrilateral (D, E, F, C lie on the circle)

$\angle DCF + \angle DEF = 180^\circ$

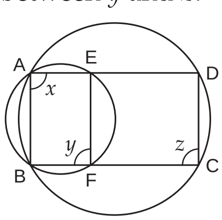
[Opposite angles of a cyclic quadrilateral are supplementary]

$\Rightarrow \quad 64^\circ + \angle DEF = 180^\circ$

$\Rightarrow \quad \angle DEF = 116^\circ$

Hence, measure of $\angle DCF = 64^\circ$ and $\angle DEF = 116^\circ$.

4. In the given figure below, ABCD is a cyclic quadrilateral. A second circle passing through A and B meets AD and BC at E and F respectively. If $\angle BAD = x$, $\angle EFB = y$ and $\angle BCD = z$, what is the relation between y and z?



Sol. Given, $\angle BAD = x$

$\angle EFB = y$

$\angle BCD = z$

ABCD is a cyclic quadrilateral

$x + z = 180^\circ$

$x = 180^\circ - z \quad \ldots(1)$

AEFB is a cyclic quadrilateral

$x + y = 180^\circ \quad \ldots(2)$

Put the value of x from (1) in (2), we get

$180^\circ - z + y = 180^\circ$

$-z + y = 0$

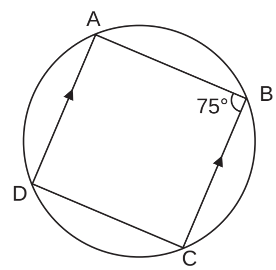
$y = z$

Hence, relation between y and z is $y = z$.

Short Answer Type-I Questions

5. ABCD is a cyclic quadrilateral with AD ∥ BC if $\angle B = 75^\circ$, determine the remaining angles.

Sol. Given, ABCD is a cyclic quadrilateral with AD ∥ BC and $\angle B = 75^\circ$



$\angle A + \angle B = 180^\circ$

[Cointerior $\angle$s, AD ∥ BC]

$\Rightarrow \quad \angle A + 75^\circ = 180^\circ$

$\Rightarrow \quad \angle A = 105^\circ$

$\angle B + \angle D = 180^\circ$

[Opposite angles of a cyclic quadrilateral are supplementary]

$75^\circ + \angle D = 180^\circ$

$\angle D = 105^\circ$

Similarly, $\angle A + \angle C = 180^\circ$

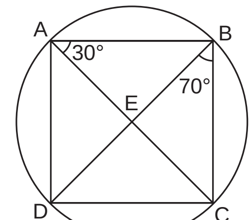
$\Rightarrow \quad 105^\circ + \angle C = 180^\circ$

$\Rightarrow \quad \angle C = 75^\circ$

Hence, the remaining angles are $\angle A = 105^\circ$, $\angle C = 75^\circ$ and $\angle D = 105^\circ$.

6. ABCD is a cyclic quadrilateral whose diagonals intersect each other at a point E. If $\angle DBC = 70^\circ$, $\angle BAC = 30^\circ$, find $\angle BCD$. Further, if AB = BC, find $\angle ECD$. **[CBSE SP 2011]**

Sol. Given, $\angle DBC = 70^\circ$ and $\angle BAC = 30^\circ$



$\angle DBC = \angle DAC$

[Angles in the same segment of a circle are equal]

$\angle DAC = 70°$

$\angle DAB = \angle DAC + \angle BAC$

$= 70° + 30°$

$= 100°$

$\Rightarrow \quad \angle DAB = 100°$

$\angle DAB + \angle BCD = 180°$

[Opposite angles of a cyclic quadrilateral are supplementary]

$100° + \angle BCD = 180°$

$\Rightarrow \quad \angle BCD = 80°$

If AB = BC, then ΔABC is an isosceles triangle

$\angle BAC = \angle BCA = 30°$

$\angle ECD = \angle BCD - \angle BCA$

$= 80° - 30°$

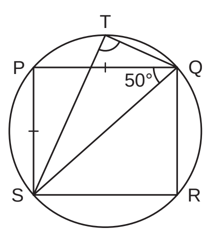
$= 50°$

$\Rightarrow \quad \angle ECD = 50°$

Hence, measure of $\angle BCD = 80°$ and $\angle ECD = 50°$.

Short Answer Type-II Questions

7. PQRS is a cyclic quadrilateral such that PQ = PS and $\angle PQS = 50°$. T is a point on the circle such that P and T lie on the same side of QS. TQ and TS are joined. Find $m\angle QTS$ and $m\angle QRS$.



Sol. Given, $\angle PQS = 50°$ and ΔPSQ is an isosceles triangle such that PS = PQ

$\angle PQS = \angle PSQ = 50°$

In ΔPSQ, we have

$\angle PQS + \angle PSQ + \angle SPQ = 180°$

$\Rightarrow \quad 50° + 50° + \angle SPQ = 180°$

$\Rightarrow \quad 100° + \angle SPQ = 180°$

$\Rightarrow \quad \angle SPQ = 80°$

$\angle SPQ = \angle QTS$

[Angles in the same segment of a circle are equal]

$\Rightarrow \quad \angle QTS = 80°$

PQRS is a cyclic quadrilateral

[P, Q, R, S lie on a circle]

$\angle SPQ + \angle QRS = 180°$

[Opposite $\angle$s of a cyclic quadrilateral are supplementary]

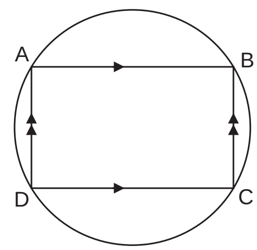
$80° + \angle QRS = 180°$

$\angle QRS = 100°$

Hence, measure of $\angle QTS = 80°$ and $\angle QRS = 100°$, i.e. $m\angle QTS = 80°$ and $m\angle QRS = 100°$.

8. Prove that a cyclic parallelogram is a rectangle.

Sol. Given, ABCD is a cyclic parallelogram.



$\because$ Opposite angles of a cyclic parallelogram are supplementary,

$\therefore \quad \angle ABC + \angle ADC = 180° \qquad \ldots(1)$

$\angle ABC = \angle ADC$

[Opposite angles of a parallelogram are equal) …(2)

$\therefore \quad 2\angle ABC = 180° \qquad$ [From (1) and (2)]

$\Rightarrow \quad \angle ABC = 90°$

Also, $\quad \angle ADC = 90° \qquad$ [From (2)]

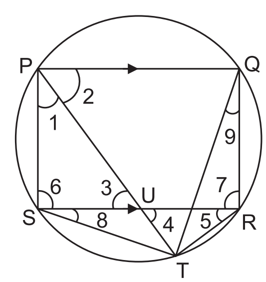
$\therefore$ Each angle of a parallelogram ABCD is a right angle.

Hence, ABCD is a rectangle.

Long Answer Type Questions

9. In a cyclic quadrilateral PQRS, PQ ∥ SR. The internal bisector of $\angle P$ meets RS at U and the circle at T. TR, TS and TQ are joined. Prove that (*a*) UT = TR and (*b*) ΔQRT ≅ ΔSUT.

Sol.



(*a*) $\quad \angle 1 = \angle 2$ [PT is the bisector of $\angle P$] …(1)

$\angle 2 = \angle 3 \quad$ [Alternate $\angle$s, PQ ∥ SR] …(2)

$\angle 3 = \angle 4 \quad$ [Vertically opposite $\angle$s] …(3)

$\angle 1 = \angle 5$

[Angles in the same segment] ...(4)

From equation (1), (2), (3) and (4), we have

$\angle 5 = \angle 4$

$\Rightarrow$ UT = TR

[Sides opposite to equal angles]

Hence, proved.

(*b*) $\angle QPS + \angle PSR = 180°$

[Cointerior $\angle$s, PQ $\parallel$ SR]

$\angle 1 + \angle 2 + \angle 6 = 180°$ ...(5)

$\because$ PQRS is a cyclic quadrilateral. So, opposite $\angle$s of a cyclic quadrilateral are supplementary,

$\therefore$ $\angle QPS + \angle QRS = 180°$

$\Rightarrow$ $\angle 1 + \angle 2 + \angle 7 = 180°$ ...(6)

$\angle 6 = \angle 7$

[From (5) and (6)] ...(7)

Adding equation (4) and (7), we get

$\angle 1 + \angle 6 = \angle 5 + \angle 7$

$\Rightarrow$ $\angle SUT = \angle QRT$

[Ext. $\angle$SUT of ΔPSU = sum of int. opp. $\angle$s] ...(8)

In ΔQRT and ΔSUT, we have

$\angle 9 = \angle 8$

[Angles in the same segment]

$\angle QRT = \angle SUT$ [From (8)]

UT = TR [Proved in part (*a*)]

$\therefore$ $\Delta QRT \cong \Delta SUT$

[By AAS congruence]

Hence, proved.

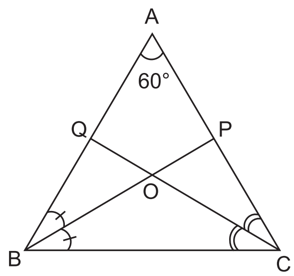
10. In a ΔABC, if $\angle A = 60°$ and the bisectors of $\angle$B and $\angle$C meet AC and AB at P and Q respectively and intersect each other at O. Prove that APOQ is a cyclic quadrilateral.

Sol. Let $\angle ABP = \angle PBC = x$

[$\because$ BP is the bisector of $\angle$B] ...(1)

Let $\angle ACQ = \angle QCB = y$

[$\because$ QC is the bisector of $\angle$C] ...(2)



In ΔOBC, we have

$\angle OBC + \angle OCB + \angle BOC = 180°$

[Angle sum property of a triangle]

$\Rightarrow$ $\angle BOC + \angle PBC + \angle QCB = 180°$

[$\angle OBC = \angle PBC$ and $\angle OCB = \angle QCB$]

$\Rightarrow$ $\angle BOC + x + y = 180°$

$\Rightarrow$ $\angle QOP + x + y = 180°$

[$\angle BOC = \angle QOP$, vertically opp. $\angle$s]

$\angle QOP = 180° - (x + y)$ ...(3)

In ΔABC, we have

$\angle ABC + \angle ACB + \angle BAC = 180°$

$2x + 2y + 60° = 180°$

$2x + 2y = 120°$

$x + y = 60°$ ...(4)

From (3) and (4), we have

$\angle QOP = 180° - 60° = 120°$

In quadrilateral APOQ, we have

$\angle QAP + \angle QOP = 60° + 120°$

$\Rightarrow$ $\angle QAP + \angle QOP = 180°$

But $\angle$QAP and $\angle$QOP are opposite angles of a quadrilateral APOQ.

Hence, APOQ is a cyclic quadrilateral.

Hence, proved.

Higher Order Thinking Skills (HOTS) Questions

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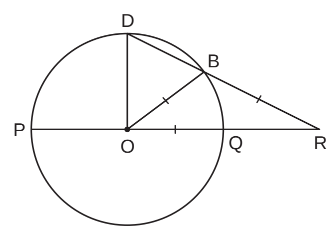
1. From a point R outside a circle with centre O, a line segment RQP is drawn through O cutting the circle at Q and P as shown in the figure. Another line segment RBD is drawn cutting the circle at B and D. If RB = OQ, show that $\overset{\frown}{PD} = 3\overset{\frown}{QB}$.



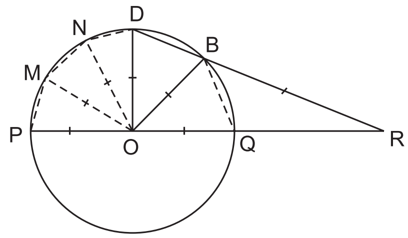
Sol. Join BQ, ND, MN and PM.

In ΔQOB and ΔBOD, we have

OQ = OD [Radius of a circle]

$\angle OBQ = \angle ODB$

[OQ = OB = OD; angle opposite to same sides are equal]



$\angle OQB = \angle OBD$ [OQ = OB = OD]

$\therefore \quad \Delta QOB \cong \Delta BOD$

[By AAS congruence]

$\angle QOB = \angle BOD$ [By CPCT]

$\Rightarrow \quad \overset{\frown}{QB} = \overset{\frown}{DB}$ …(1)

Similarly, $\Delta ODB \cong \Delta ODN$

$\Rightarrow \quad \overset{\frown}{DB} = \overset{\frown}{ND}$ …(2)

$\Delta ODN \cong \Delta ONM$

$\Rightarrow \quad \overset{\frown}{ND} = \overset{\frown}{MN}$ …(3)

$\Delta ONM \cong \Delta OMP$

$\Rightarrow \quad \overset{\frown}{MN} = \overset{\frown}{PM}$ …(4)

Adding (2), (3) and (4), we get

$$\overset{\frown}{DB} + \overset{\frown}{ND} + \overset{\frown}{MN} = \overset{\frown}{ND} + \overset{\frown}{MN} + \overset{\frown}{PM}$$

$$\Rightarrow \overset{\frown}{DB} + \overset{\frown}{ND} + \overset{\frown}{MN} = \overset{\frown}{PD}$$

$$\Rightarrow \overset{\frown}{QB} + \overset{\frown}{QB} + \overset{\frown}{QB} = \overset{\frown}{PD}$$

$[\overset{\frown}{QB} = \overset{\frown}{DB} = \overset{\frown}{ND} = \overset{\frown}{MN} = \overset{\frown}{PM}]$

$\therefore \quad \overset{\frown}{PD} = 3\overset{\frown}{QB}$

Hence, proved.

2. A is the centre of a circle and B, C, D are points on the circle forming a quadrilateral ABCD in which AB = AC = AD. Prove that

$$\angle BAD = 2(\angle CBD + \angle CDB)$$

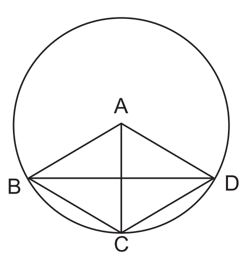
Sol. We know that the angle subtended by an arc of a circle at the centre is double the angle subtended by it at any point on the remaining part of the circle.

$\therefore \quad \angle CAD = 2\angle CBD$

[Angles subtended by $\overset{\frown}{CD}$] …(1)

Also, $\angle BAC = 2\angle CDB$

[Angles subtended by $\overset{\frown}{BC}$] …(2)



Adding (1) and (2), we get

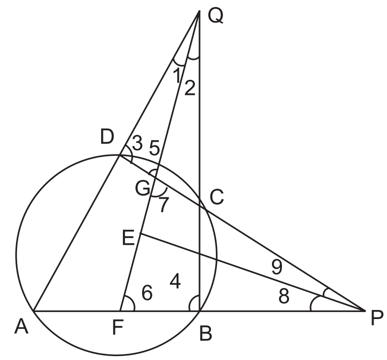
$$\angle CAD + \angle BAC = 2\angle CBD + 2\angle CDB$$

$$\angle BAD = 2(\angle CBD + \angle CDB)$$

Hence, proved.

3. Prove that the bisectors of the angles formed by producing the opposite sides of a cyclic quadrilateral intersect at right angles.

Sol. ABCD is a cyclic quadrilateral whose opposite sides when produced intersect at the points P and Q respectively. The bisectors PE and QF of $\angle P$ and $\angle Q$ meet at E and F respectively.



In ΔQDG and ΔQBF, we have

$\angle 1 = \angle 2$

[QE is bisector of $\angle DQC$]

$\angle 3 = \angle 4$

[Exterior $\angle$s of cyclic quadrilateral is equal to its interior opp. $\angle$s]

$\angle 5 = \angle 6$ …(1)

But, $\angle 5 = \angle 7$

[Vertically opposite $\angle$s] …(2)

From (1) and (2), we have

$\angle 6 = \angle 7$

Now, in ΔPGE and ΔPFE,

$\angle 9 = \angle 8$

[PE is bisector of $\angle CPB$]

$\angle 6 = \angle 7$ [Already proved]

$\therefore \quad \angle PEG = \angle PEF$

But $\quad \angle PEG + PEF = 180°$

$\therefore \quad \angle PEG = \angle PEF = 90°$

So, $\quad \angle PEG = 90°$

Hence, proved.

4. If the bisectors of the opposite angles of a cyclic quadrilateral intersect the corresponding circle at two points, then prove that the line segment joining these two points is a diameter of the circle.

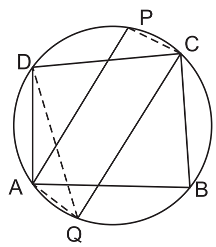
Sol. ∵ ABCD is a cyclic quadrilateral,

∴ $\angle A + \angle C = 180°$

[Opposite ∠s of a cyclic quadrilateral are supplementary]

⇒ $\frac{1}{2}\angle A + \frac{1}{2}\angle C = 90°$

⇒ $\angle PAB + \angle BCQ = 90°$



But $\angle BCQ = \angle BAQ$

[Angles in same segment of a circle are equal]

∴ $\angle PAB + \angle BAQ = 90°$

⇒ $\angle PAQ = 90°$

⇒ ∠PAQ is in a semi-circle.

⇒ PQ is a diameter of a circle.

Hence, proved.

Self-Assessment

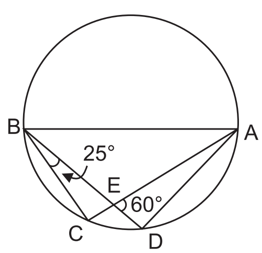
(Page 148)

Multiple-Choice Questions

1. Two chords BD and AC of a circle intersect each other at E. If A and B are the ends of a diameter of the circle and if ∠CBE = 25° and ∠DEA = 60°, then the measure of ∠ADB is

(*a*) 90° (*b*) 85°
(*c*) 95° (*d*) 120°

Sol. (*c*) 95°



Given, ∠CBE =25° and ∠DEA = 60°

Also, AB is a diameter of a circle.

$\angle DEA = \angle CEB = 60°$ [Vertically opposite ∠s]

In ΔBCE, we have

$\angle CEB + \angle CBE + \angle BCE = 180°$

[Angle sum property of a triangle]

⇒ $60° + 25° + \angle BCE = 180°$

⇒ $85° + \angle BCE = 180°$

⇒ $\angle BCE = 95°$

$\angle ADB = \angle BCE$

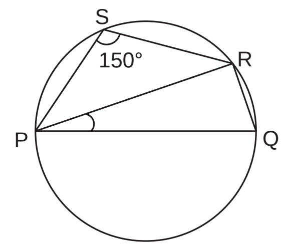
[Angle in a same segment of a circle are equal]

∴ $\angle ADB = 95°$

2. PQRS is a cyclic quadrilateral such that PQ is a diameter of the circle circumscribing the quadrilateral and ∠PSR = 150°, then the measure of ∠QPR is

(*a*) 60° (*b*) 50°
(*c*) 40° (*d*) 30°

Sol. (*a*) 60°



∵ PQRS is a cyclic quadrilateral,

∴ $\angle PSR + \angle RQP = 180°$

[Opposite ∠s of a cyclic quadrilateral are supplementary]

⇒ $150° + \angle RQP = 180°$

∴ $\angle RQP = 30°$

∵ PQ is a diameter,

∴ $\angle PRQ = 90°$

[Angle in a semi-circle is a right angle]

In ΔPRQ, we have

$\angle RQP + \angle PRQ + \angle QPR = 180°$

[Angle sum property of a triangle]

⇒ $30° + 90° + \angle QPR = 180°$

⇒ $120° + \angle QPR = 180°$

∴ $\angle QPR = 60°$

Hence, measure of ∠QPR is 60°.

Fill in the Blanks

3. Angle formed in minor segment of a circle is **an obtuse angle**.
4. Number of circles that can be drawn through three non-collinear points is **1**.
5. Greatest chord of a circle is called its **diameter**.
6. The region between a chord and either of the arc is called **a segment**.

Assertion-Reason Type Questions

Directions (Q. Nos. 7 to 10): Each of these questions contains an assertion followed by reason. Read them carefully, and answer the question on the basis of the following options, select the one that best describes the two statements.

(a) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.

(b) Both Assertion and Reason are true but Reason is not the correct explanation of Assertion.

(c) Assertion is true but Reason is false.

(d) Assertion is false but Reason is true.

7. **Assertion:** Two conjugate arcs of a circle will have same end points.

Reason: Two conjugate arcs of a circle complete the circle.

Sol. (*a*)

The assertion and reason both are correct and reason is correct explanation of assertion.

8. **Assertion:** The degree measure of a semicircle is 90.

Reason: Semicircle is half of a circle.

Sol. (*d*)

Semicircle is half of a circle and its degree measure is 180°.

$\therefore$ Reason is correct but assertion is incorrect.

9. **Assertion:** Diagonal of a circle is its greatest chord.

Reason: A chord divides a circle into segments.

Sol. (*b*)

Both assertion and reason are correct but reason is not correct explanation of assertion.

10. **Assertion:** There is one and only one circle passing through three given non-collinear points.

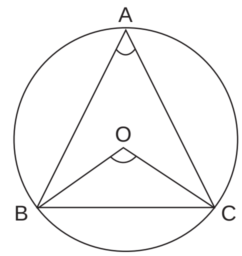
Reason: Circles with same area have equal radii.

Sol. (*b*)

Both assertion and reason are correct but reason is not the correct explanation of assertion.

Case Study Based Questions

11. Coronavirus disease (COVID-19) is an infectious disease caused by a newly discovered coronavirus. The COVID-19 pandemic has led to a dramatic loss of human life and greatest challenge we have faced since World War II. In order to protect people against severe COVID-19 disease, Government of India is working regularly in speeding up Covid-19 vaccination. For this, there are three vaccination centres in a village situated at A, B and C as shown in the figure. These three vaccination centres are equidistant from each other as shown in the figure. Based on the above situation, answer the following questions.



(*a*) Which type of ΔABC is in the given figure?

(*i*) Right-angled triangle
(*ii*) Scalene triangle
(*iii*) Isosceles triangle
(*iv*) Equilateral triangle

Ans. (*iv*) Equilateral triangle

(*b*) What is the measure of $\angle ABC$?

(*i*) 30° (*ii*) 45°
(*iii*) 60° (*iv*) 90°

Ans. (*iii*) 60°

(*c*) If the length of AB is 6 km, then value of BC + CA is

(*i*) 6 km (*ii*) 10 km
(*iii*) 12 km (*iv*) 14 km

Ans. (*iii*) 12 km

(*d*) What is the measure of $\angle BOC$?

(*i*) 50° (*ii*) 90°
(*iii*) 100° (*iv*) 120°

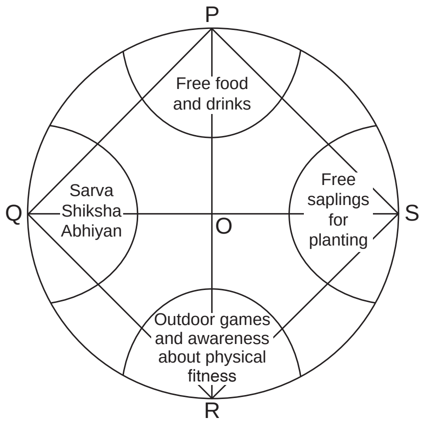
Ans. (*iv*) 120°

(*e*) What is the value of ($\angle OBC + \angle OCB$)?

(*i*) 30° (*ii*) 45°
(*iii*) 60° (*iv*) 90°

Ans. (*iii*) 60°

12. A group of social workers organised a mela for slum children in a circular park. They set-up four equidistant stalls P, Q, R and S along the boundary of the park. Stall P provided free food and drinks. Stall Q provided the awareness about the importance of education, while stalls R and S dealt with physical fitness and environment protection respectively.



If PR and QS intersect at right angle, then answer the following questions.

(*a*) What is the measure of ∠PSO?

(*i*) 30° (*ii*) 45°
(*iii*) 90° (*iv*) 120°

Ans. (*ii*) 45°

(*b*) What is the measure of ∠RSQ?

(*i*) 30° (*ii*) 45°
(*iii*) 90° (*iv*) 120°

Ans. (*ii*) 45°

(*c*) What is the measure of ∠PSR?

(*i*) 30° (*ii*) 45°
(*iii*) 90° (*iv*) 120°

Ans. (*iii*) 90°

(*d*) What is the property of cyclic quadrilateral?

(*i*) Adjacent angles are equal.
(*ii*) Opposite angles are equal.
(*iii*) Opposite angles are supplementary.
(*iv*) Adjacent angles are supplementary.

Ans. (*iii*) Opposite angles are supplementary.

(*e*) What is the special type of quadrilateral PQRS?

(*i*) Square
(*ii*) Rectangle
(*iii*) Kite
(*iv*) Trapezium

Ans. (*i*) Square

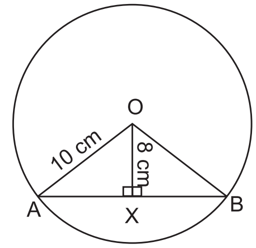
Very Short Answer Type Questions

13. The distance of a chord from the centre of a circle of radius 10 cm is 8 cm. What is the length of length of this chord?

Sol. Given, OA = OB = 10 cm [Radius of a circle]

OX = 8 cm

[Distance of a chord from the centre of a circle]



∵ OX ⊥ AX

∴ ΔOXA is a right-angled triangle.

∴ By Pythagoras' Theorem, we have

$$(OA)^2 = (AX)^2 + (OX)^2$$

$$\Rightarrow (10)^2 = (AX)^2 + (8)^2$$

$$\Rightarrow 100 = (AX)^2 + 64$$

$$\Rightarrow (AX)^2 = 36$$

$$\Rightarrow AX = 6 \text{ cm}$$

∴ AB = 2 AX

[OX ⊥ AB, The ⊥ from the centre of a circle to a chord bisects the chord]

= 2 × 6

= 12

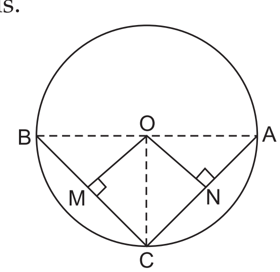
AB = 12 cm

Hence, length of the chord AB is 12 cm.

14. The two perpendicular bisectors of two chords BC and CA of a circle meet at a point O. Then O will be the centre of the circle and the radius of the circle will be OA, OB or OC. Is this statement true or false?

Sol. The statement is true.

To prove that O is the centre and OA, OB and OC are radius.



In ΔBOM and ΔCOM, we have

BM = MC

[OM is perpendicular bisector of BC]

OM = OM [Common]

∠OMB = ∠OMC [Each 90°]

⇒ ΔBOM ≅ ΔCOM

[By SAS congruence]

⇒ OB = OC [By CPCT]…(1)

In ΔCON and ΔNOA

$CN = NA$ [ON is perpendicular bisector of CA]

$ON = ON$ [Common]

$\angle CNO = \angle ANO$ [Each 90°]

$\Rightarrow$ $\Delta CON \cong \Delta NOA$ [By SAS congruence]

$\Rightarrow$ $OC = OA$ [By CPCT] ...(2)

From (1) and (2), we have

$OA = OB = OC$

$\Rightarrow$ O is the centre of a circle

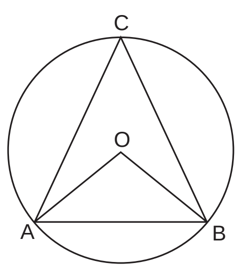
The centre of the circle is the only point within the circle that has points on the circumference which are equal in distance from it.

Hence, O is the centre of a circle and OA, OB and OC are the radius.

Short Answer Type-I Questions

15. Prove that an angle in a major segment of a circle is acute.

Sol. Let $\angle ACB$ be an angle formed in the major segment of a circle with centre O.



$$\angle ACB = \frac{1}{2}\angle AOB \quad ...(1)$$

[Angle subtended by an arc of a circle at the centre is twice the angle subtented by it on the remaining part of the circle]

But $\angle AOB < 180°$

$\Rightarrow$ $\frac{1}{2}\angle AOB < 90°$

$\Rightarrow$ $\angle ACB < 90°$ [From (1)]

$\therefore$ $\angle ACB$ is an acute angle in the major segment.

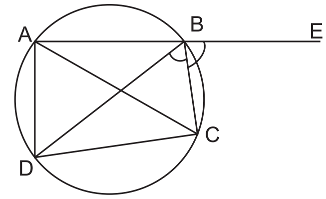
Hence, an angle in a measure segment of a circle is acute.

16. ABCD is a cyclic quadrilateral. The side AB is produced to E outside the circle such that BC is the internal bisector of $\angle DBE$. Prove that AC = CD.

Sol. We have $\angle EBC = \angle ADC$

[Exterior $\angle$s of cyclic quadrilateral is equal to interior opposite $\angle$s] ...(1)

$\angle EBC = \angle DBC$ [Given] ...(2)



From (1) and (2), we have

$\angle ADC = \angle DBC$ ...(3)

But $\angle DAC = \angle DBC$

[Angles in the same segment] ...(4)

From (3) and (4), we have

$\angle ADC = \angle DAC$

$\Rightarrow$ $AC = CD$

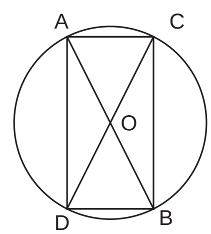
[Sides opposite to equal angles]

Hence, proved.

Short Answer Type-II Questions

17. Prove that if two chords of a circle bisect each other at a point within the circle, then these chords are the diameters of the circle.

Sol. Let AB and CD be two chords.



In ΔAOD and ΔCOB

$AO = OB$

$DO = OC$

[Given, chords are bisecting each other]

$\angle AOD = \angle COB$ [Vertically opp. $\angle$s]

$\therefore$ $\Delta AOD \cong \Delta COB$

[By SAS congruence]

$\Rightarrow$ $AD = CB$ [By CPCT]

Also, $\overset{\frown}{AD} \cong \overset{\frown}{CB}$

[If two chords are equal then their corresponding arcs are congruent] ...(1)

Similarly, $\Delta AOC \cong \Delta BOD$

[By SAS congruence]

$AC = BD$ [By CPCT]

$\overset{\frown}{AC} \cong \overset{\frown}{BD}$ ...(2)

Adding (1) and (2), we get

$$\overset{\frown}{AC} + \overset{\frown}{AD} \cong \overset{\frown}{BD} + \overset{\frown}{CB}$$

$\Rightarrow \qquad \overset{\frown}{CAD} \cong \overset{\frown}{CBD}$

CD divides the circle into two equal parts. So, CD and AB are diameters of a circle.

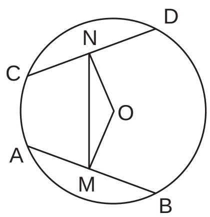
Hence, proved.

18. Prove that the line joining the mid-points of two equal chords of a circle subtends equal angles with the chords.

Sol. To Prove that $\angle AMN = \angle CNM$ and $\angle BMN = \angle DNM$

Construction: Join OM and ON

$\because$ The line segment joining the centre of a circle to the mid-point of a chord is perpendicular to the chord.



Given, AB and CD are equal chords.

$\therefore$ They are equidistant from the centre,

i.e. $\qquad OM = ON$

In ΔOMN, we have

$$OM = ON$$

$$\angle OMN = \angle ONM$$

[Angles opposite to equal sides] ...(1)

$\angle OMA = \angle ONC$ [Each 90°] ...(2)

$\angle OMB = \angle OND$ [Each 90°] ...(3)

Subtracting (2) from (1), we have

$$\angle OMA - \angle OMN = \angle ONC - \angle ONM$$

$\Rightarrow \qquad \angle AMN = \angle CNM$

Adding (1) and (3), we have

$$\angle OMB + \angle OMN = \angle OND + \angle ONM$$

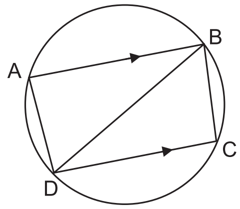
$\Rightarrow \qquad \angle BMN = \angle DNM$

Hence, proved.

19. If two sides of a cyclic quadrilateral are parallel, prove that the remaining two sides are equal.

[CBSE SP 2011]

Sol. Let ABCD be the cyclic quadrilateral with AB $\parallel$ DC. Join BD.



Now, since AB $\parallel$ DC and BD is the transversal.

$\angle ABD = \angle CDB$ [Alternate $\angle$s]

$\angle ABD$ is subtented by chord AD on the circumference and $\angle CBD$ is subtented by chord BC.

We know, if the angles subtended by two chords on the circumference of the circle are equal, then the length of the chords is also equal.

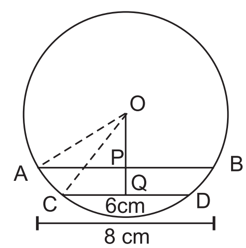
Hence, AD = BC, i.e the remaining two sides of the cyclic quadrilateral are equal.

Long Answer Type Questions

20. The lengths of two parallel chords on the same side of the centre are 6 cm and 8 cm. If the smaller chord is at a distance of 4 cm from the centre, what is the distance of the other chord from the centre? **[CBSE SP 2011]**

Sol. Let AB and CD be two parallel chords of a circle with centre O such that AB = 8 cm and CD = 6 cm.

Draw OQ $\perp$ CD and OP $\perp$ AB.



$\because$ AB $\parallel$ CD and OQ $\perp$ CD, OP $\perp$ AB,

$\therefore$ Points O, P, and Q are collinear.

$$CQ = \frac{1}{2}CD = \frac{1}{2} \times 6 = 3 \text{ cm}$$

[As the $\perp$ from the centre of a circle to the chord bisects the chord)

$$AP = \frac{1}{2}AB = \frac{1}{2} \times 8 = 4 \text{ cm}$$

$\because$ ΔOQC is a right-angled triangle,

$\therefore$ By Pythagoras' Theorem, we have

$$(OC)^2 = (CQ)^2 + (OQ)^2$$

$\Rightarrow \qquad (OC)^2 = (3)^2 + (4)^2$

$\Rightarrow \qquad (OC)^2 = 9 + 16$

$\Rightarrow \qquad (OC)^2 = 25$

$\Rightarrow \qquad OC = 5$ cm

$\Rightarrow \qquad OA = OC$ [Radii of the circle]

$\Rightarrow \qquad OA = 5$ cm

$\because$ ΔOPA is a right-angled triangle,

$\therefore$ By Pythagoras' Theorem, we have

$$(OA)^2 = (AP)^2 + (OP)^2$$

$\Rightarrow$ $(5)^2 = (4)^2 + (OP)^2$

$\Rightarrow$ $25 = 16 + (OP)^2$

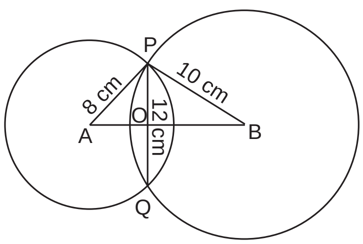
$\Rightarrow$ $(OP)^2 = 9$

$\Rightarrow$ $OP = 3$ cm

Hence, distance of the bigger chord from the centre is 3 cm.

21. Two circles of radii 10 cm and 8 cm intersect each other at P and Q and the length of their common chord is 12 cm. Find the distance between their centres. **[CBSE SP 2011]**

Sol. Let A and B are the centres of two circles. PQ is the common chord of length 12 cm.



Given, $AP = 8$ cm

$BP = 10$ cm

$PO = \frac{1}{2}PQ$

[Perpendicular from the centre of the circle to a chord bisects the chord]

$= \frac{1}{2} \times 12 = 6$

$PO = 6$ cm

$\because$ ΔPOA is a right-angled triangle,

$\therefore$ By Pythagoras' Therorem, we have

$$(AP)^2 = (AO)^2 + (PO)^2$$

$\Rightarrow$ $(8 \text{ cm})^2 = (AO)^2 + (6 \text{ cm})^2$

$\Rightarrow$ $64 \text{ cm}^2 = (AO)^2 + 36 \text{ cm}^2$

$\Rightarrow$ $(AO)^2 = 28 \text{ cm}^2$

$\Rightarrow$ $AO = 2\sqrt{7}$ cm

$= 2 \times 2.64$ cm

$= 5.29$ cm

$\Rightarrow$ $AO = 5.29$ cm

$\because$ ΔPOB is a right-angled triangle,

$\therefore$ By Pythagoras' Theorem, we have

$$(BP)^2 = (BO)^2 + (PO)^2$$

$\Rightarrow$ $(10 \text{ cm})^2 = (BO)^2 + (6 \text{ cm})^2$

$\Rightarrow$ $100 \text{ cm}^2 = (BO)^2 + 36 \text{ cm}^2$

$\Rightarrow$ $(BO)^2 = 64 \text{ cm}^2$

$\Rightarrow$ $BO = 8$ cm

$\because$ $AB = AO + BO$

$= (5.29 + 8)$ cm

$= 13.29$ cm

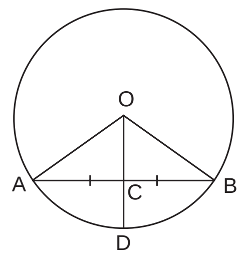
$\therefore$ $AB = 13.29$ cm

Hence, distance between the centres of two circles is 13.29 cm.

22. Prove that the line joining the mid-point of a chord to the centre of a circle passes through the mid-point of the corresponding minor arc.

Sol. Given, C is the mid-point of chord AB.

To prove that D is the mid-point of arc AB.



In ΔOAC and ΔOBC, we have

$OA = OB$ [Radius of a circle]

$OC = OC$ [Common]

$AC = BC$ [C is the mid-point of AB]

$\therefore$ $\Delta OAC \cong \Delta OBC$

[By SSS congruence]

$\Rightarrow$ $\angle AOC = \angle BOC$ [By CPCT]

$\Rightarrow$ $\overset{\frown}{AD} \cong \overset{\frown}{BD}$

Hence, D is the mid-point of arc AB.

Let's Compete

(Page 150)

Multiple-Choice Questions

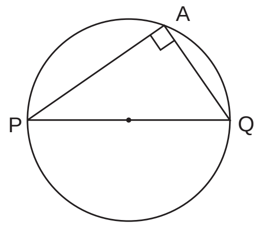
1. PQ is a chord of a circle with radius *r*. If A is any point on the circle such that $\angle PAQ = 90°$, then PQ is equal to

(*a*) r (*b*) $2r$

(*c*) $3r$ (*d*) $4r$

Sol. (*b*) $2r$

We know, angle in a semi-circle is a right angle.



Given, radius = r

Diameter = $2r$

PQ = Diameter = $2r$

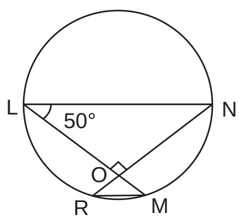
Hence, PQ is $2r$.

2. Chords LM and NR intersect each other within a circle at right angles. If ∠NLM = 50°, then ∠LMR is equal to

(*a*) 60° (*b*) 50°
(*c*) 40° (*d*) 70°

Sol. (*c*) 40°

Given, ∠NLM = 50°



In ΔLON, we have

$\angle LNO + \angle NLO + \angle LON = 180°$

$\Rightarrow \quad \angle LNO + 50° + 90° = 180°$

$\Rightarrow \quad \angle LNO + 140° = 180°$

$\angle LNO = 40°$

$\angle LMR = \angle LNO$

[Angle in a same segment of a circle are equal]

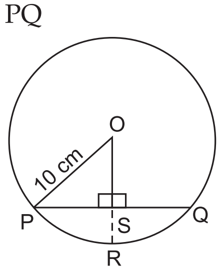
$\therefore \quad \angle LMR = 40°$

3. PQ is a chord of a circle with centre at O. If OP = 10 cm, PQ = 16 cm and OS ⊥ PQ, then the length of SR is equal to

(*a*) 4 cm (*b*) 6 cm
(*c*) 3 cm (*d*) 5 cm

Sol. (*a*) 4 cm

Given OS ⊥ PQ



∵ The perpendicular from the centre of a circle to a chord bisects the chord.

$\Rightarrow \quad PS = 8$ cm

∵ ΔPSO is a right-angled triangle,

∴ By Pythagoras' Theorem, we have

$(PO)^2 = (PS)^2 + (SO)^2$

$\Rightarrow \quad (10)^2 = (8)^2 + (SO)^2$

$\Rightarrow \quad 100 = 64 + (SO)^2$

$\Rightarrow \quad (SO)^2 = 36$

$\Rightarrow \quad SO = 6$ cm

Since, OP = OR = 10 cm [Radius]

$\therefore \quad SR = OR - SO$

$= 10 \text{ cm} - 6 \text{ cm}$

$= 4$ cm

Hence, the length of SR is 4 cm.

4. PQRS is a cyclic trapezium in which PS || QR and ∠Q = 70°. Then ∠QRS is equal to

(*a*) 50° (*b*) 60°
(*c*) 80° (*d*) 70°

Sol. (*d*) 70°

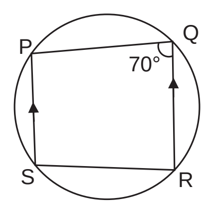
Given, ∠Q = 70°

$\angle P + \angle Q = 180°$

[Cointerior ∠s, PS || QR]

$\Rightarrow \quad \angle P + 70° = 180°$

$\angle P = 110°$



∵ PQRS is a cyclic trapezium,

$\therefore \quad \angle P + \angle R = 180°$

[Opposite ∠s of cyclic quadrilateral are supplementary]

$\Rightarrow \quad 110° + \angle R = 180°$

$\angle R = 70°$

$\angle QRS = 70°$

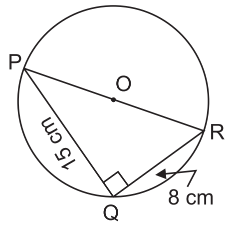
5. If PQ = 15 cm and QR = 8 cm are two line segments intersecting each other at Q at right angles. Then the radius of the circle passing through the points P, Q and R is

(*a*) 8 cm (*b*) 8.5 cm
(*c*) 9 cm (*d*) 9.5 cm

Sol. (*b*) 8.5 cm

Given, PQ = 15 cm

QR = 8 cm



$\because$ The arc of a circle subtending a right angle at any point on the remaining part of the circle is a semi-circle.

$\therefore$ PR is the diameter of a circle.

$\because$ ΔPQR is a right-angled triangle,

$\therefore$ By Pythagoras' Theorem, we have

$$(PR)^2 = (QP)^2 + (QR)^2$$
$$= (15 \text{ cm})^2 + (8 \text{ cm})^2$$
$$= 225 \text{ cm}^2 + 64 \text{ cm}^2$$
$$= 289 \text{ cm}^2$$
$$PR = 17 \text{ cm}$$
$$\text{Radius} = \frac{PR}{2}$$
$$= \frac{17}{2} \text{ cm} = 8.5 \text{ cm}$$
$$\text{Radius} = 8.5 \text{ cm}$$

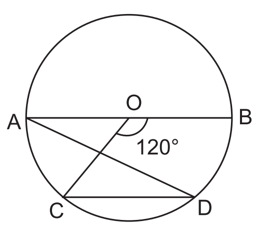
Hence, radius of a circle is 8.5 cm.

6. AOB is a diameter of a circle with centre at O. CD is a chord of the circle such that $\angle COB = 120^\circ$. Then the measure of $\angle ADC$ is equal to

(*a*) 50° (*b*) 40°
(*c*) 30° (*d*) 60°

Sol. (*c*) 30°

Given, $\angle COB = 120^\circ$



$\because$ AOB is a line,

$\therefore$ $\angle AOC + \angle COB = 180^\circ$ [Linear pair]

$\Rightarrow$ $\angle AOC + 120^\circ = 180^\circ$

$\angle AOC = 60^\circ$

$\angle AOC = 2\,\angle ADC$

[Angle subtended by an arc of a circle at the centre is double the angle subtented by it at any point on the remaining part of the circle]

$\Rightarrow$ $60^\circ = 2\angle ADC$

$\Rightarrow$ $\angle ADC = 30^\circ$

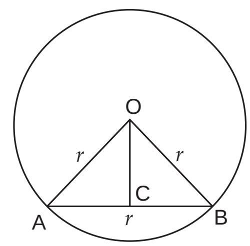
Hence, measure of $\angle ADC$ is 30°

7. If a chord of a circle is equal to its radius *r*, then the distance of this chord from the centre of the circle is

(*a*) $\frac{\sqrt{2}}{3}r$ (*b*) $\frac{2}{3}r$

(*c*) $\frac{\sqrt{3}}{2}r$ (*d*) $\frac{2}{\sqrt{3}}r$

Sol. (*c*) $\frac{\sqrt{3}}{2}r$



Given, radius (*r*) = chord AB

$\therefore$ OA = OB = AB = *r*

OC $\perp$ AB

[$\perp$ from the centre to the chord bisects the chord]

$$AC = CB = \frac{r}{2}$$

$\because$ ΔACO is a right-angled triangle.

$\therefore$ By Pythagoras' theorem, we have

$$(AO)^2 = (AC)^2 + (CO)^2$$

$\Rightarrow$ $r^2 = \left(\frac{r}{2}\right)^2 + (CO)^2$

$\Rightarrow$ $(CO)^2 = r^2 - \frac{r^2}{4} = \frac{3r^2}{4}$

$\Rightarrow$ $CO = \frac{r\sqrt{3}}{2}$

Hence, distance of the chord from the centre of a circle is $\frac{\sqrt{3}}{2}r$.

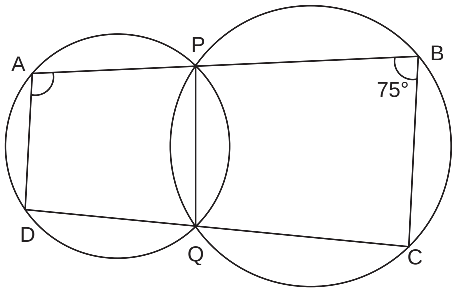
8. PQ is the common chord of two circles intersecting each other at P and Q. If AD and BC are two chords of the smaller and bigger circles respectively such that A, P, B lie on a line and D, Q, C lie on the second line and if $\angle ABC = 75^\circ$, then the measure of $\angle PAD$ is equal to

(*a*) 125° (*b*) 150°
(*c*) 75° (*d*) 105°

Sol. (*d*) 105°

PBCQ is a cyclic quadrilateral

[P, B, C, Q lie on circle]



$\angle PBC + \angle PQC = 180°$

[Opposite $\angle$s of a cyclic quadrilateral are supplementary]

$\Rightarrow \quad 75° + \angle PQC = 180°$

$\angle PQC = 105°$

$\because$ DQC is a line,

$\therefore \quad \angle DQP + \angle PQC = 180°$ [Linear pair]

$\Rightarrow \quad \angle DQP + 105° = 180°$

$\angle DQP = 75°$

$\because$ APQD is a cyclic quadrilateral,

$\therefore \quad \angle PAD + \angle DQP = 180°$

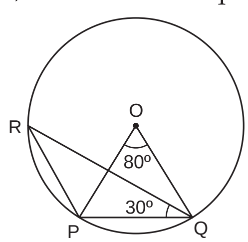
[Opposite $\angle$s of a cyclic quadrilateral are supplementary]

$\Rightarrow \quad \angle PAD + 75° = 180°$

$\angle PAD = 105°$

Hence, measure of $\angle PAD$ is 105°.

9. In the given figure, if $\angle POQ = 80°$ and $\angle PQR = 30°$, then $\angle RPO$ is equal to



(*a*) 30° (*b*) 60°

(*c*) 80° (*d*) 40°

Sol. (*b*) 60°

$\angle POQ = 2\angle PRQ$

[Angle subtended by an arc of a circle at the centre is double the angle subtended by it at any point on the remaining part of the circle]

$\Rightarrow \quad 80° = 2\angle PRQ$

$\angle PRQ = 40°$

In ΔOPQ, we have

$\angle OPQ + \angle OQP + \angle POQ = 180°$

[Angle sum property of a triangle]

$\Rightarrow \quad 2\angle OPQ + 80° = 180°$

[$\because$ $\angle OPQ = \angle OQP$, angles opposite to equal sides (radii) OP and OQ of ΔOPQ]

$\Rightarrow \quad 2\angle OPQ = 100°$

$\angle OPQ = 50°$

In ΔQRP, we have

$\angle RQP + \angle RPQ + \angle PRQ = 180°$

$30° + \angle RPO + \angle OPQ + 40° = 180°$

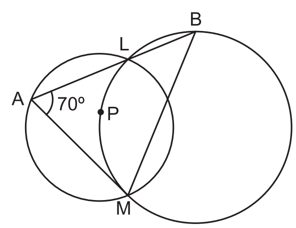
$\Rightarrow \quad 70° + \angle RPO + 50° = 180°$

$\Rightarrow \quad 120° + \angle RPO = 180°$

$\angle RPO = 60°$

Hence, measure of $\angle RPO$ is 60°.

10. In the figure, two circles intersect each other at L and M. The centre P of the smaller circle lies on the circumference of the larger circle. If $\angle LAM = 70°$, then the measure of $\angle LBM$ is



(*a*) 40° (*b*) 70°

(*c*) 60° (*d*) 50°

Sol. (*a*) 40°

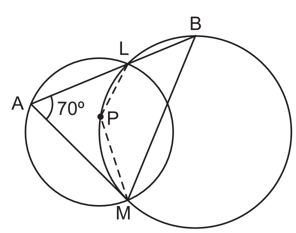
$\angle LPM = 2\angle LAM$

[Angle subtended by the arc of a circle at the centre is double the angle subtended by it at any point on the remaining part of the circle]

$= 2 \times 70°$

$= 140°$

$\angle LPM = 140°$ …(1)



$\because$ Since the opposite $\angle$s of a cyclic quadrilateral are supplementary and BMPL is a cyclic quadrilateral,

$\therefore$ $\angle LPM + \angle LBM = 180°$

$\Rightarrow$ $140° + \angle LBM = 180°$ [From (1)]

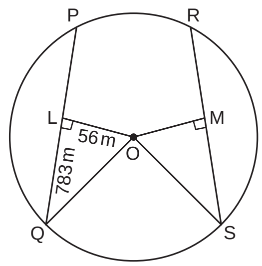
$\angle LBM = 40°$

Hence, measure of $\angle LBM$ is 40°.

Value-based Questions (Optional)

(Page 151)

1. Two roads PQ and RS each of length 1566 m, run through a circular park as shown in the figure.



(*a*) If the road PQ is at a distance of 56 m from the centre O of the circular park, then find the distance of the road RS from the centre.

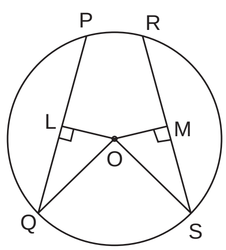
(*b*) Find the radius of the circular park.

(*c*) On Van Mahotsav Day, some students planted a few trees at equal distance of nearly 6.3 m from each other. Find the approximate number of trees planted around the park.

(*d*) What values are depicted by the students who planted the trees?

Sol. Given, length of road PQ = 1566 m.

Length of road RS = 1566 m.



(*a*) Distance of PQ from centre O = 56 m [Given]

We know equal chords of a circle are equidistant from the centre of the circle.

$\because$ PQ and RS are of equal length,

$\therefore$ They are equidistant from the centre.

$\therefore$ Distance of RS from centre = 56 m

(*b*) Let $OL \perp PQ$ and $OM \perp RS$

$PQ = 2\angle Q$

[Perpendicular drawn from the centre of the circle to the chord bisects the chord]

$$\angle Q = \frac{1}{2} PQ$$

$$= \frac{1}{2} \times 1566 \text{ m}$$

$$= 783 \text{ m}$$

$$\angle Q = 783 \text{ m}$$

$\because$ ΔOLQ is a right-angled triangle,

$\therefore$ By Pythagoras' Theorem, we have

$$(OQ)^2 = (OL)^2 + (LQ)^2$$

$$= (56 \text{ m})^2 + (783 \text{ m})^2$$

$$= (3136 + 613089) \text{ m}^2$$

$$(OQ)^2 = 616225 \text{ m}^2$$

$$OQ = 785 \text{ m}$$

Hence, radius of the circular park is 785 m.

(*c*) Circumference of circular park = $2\pi r$

$$= 2 \times \frac{22}{7} \times 785 \text{ m}$$

$$= \frac{34540}{7} \text{ m}$$

Given, students planted few trees at equal distance of nearly 6.3 m from each other along the circumference of a circular park.

Number of trees planted

$$= \frac{\text{Circumference of park}}{\text{Distance between one tree from other}}$$

$$= \frac{34540}{7 \times 6.3}$$

$$= \frac{34540 \times 10}{7 \times 63}$$

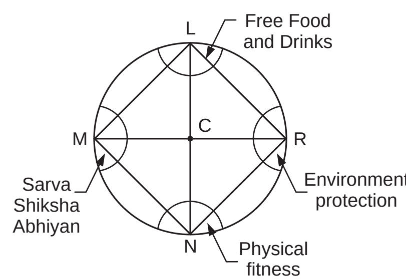
$$= \frac{345400}{441}$$

$$= 783 \text{ (approx.)}$$

Hence, approximate number of trees planted around the park is 783.

(*d*) Environmental protection, leadership and responsible citizenship.

2. A group of social workers organised a mela for slum children in a circular park. They set-up four equidistant stalls at L, M, N and R along the boundary of the park. Stall L provided free food and drinks. Stall M provided awareness about the importance of education, while the stalls N and R dealt with physical fitness and environment protection respectively.



(*a*) If LN and MR intersect each other at C at right angles to each other, prove that the quadrilateral LMNR is a square.

(*b*) What values were inculcated in the underprivileged children via this mela?

Sol. (*a*) $\angle LCM = 2\angle LRM$

$\Rightarrow \quad 90° = 2\angle LRM$

$\Rightarrow \quad \angle LRM = 45°$ ...(1)

$\angle MCN = 2\angle MRN$

$\Rightarrow \quad 90° = 2\angle MRN$

$\Rightarrow \quad \angle MRN = 45°$ ...(2)

Adding (1) and (2), we get

$\angle LRM + \angle MRN = 45° + 45°$

$\Rightarrow \quad LRN = 90°$

Similarly,

$\angle LMN = \angle MNR = \angle RLM = 90°$ ...(3)

In ΔLCR and ΔNCR, we have

$\angle LCR = \angle NCR$ [Each is 90°]

$CR = CR$ [Common]

$\angle LRC = \angle NRC$ [Each is 45°]

$\therefore \quad \Delta LCR \cong \Delta NCR$

[By ASA congruence]

$\Rightarrow \quad LR = RN$ [By CPCT]

Similarly, $LM = MN = LR = RN$ ...(4)

$\Rightarrow$ LMNR is a quadrilateral in which all sides are equal and each angle is 90°.

[From (3) and (4)]

Hence, LMNR is a square.

(*b*) Awareness about environmental protection and importance of education and physical fitness.

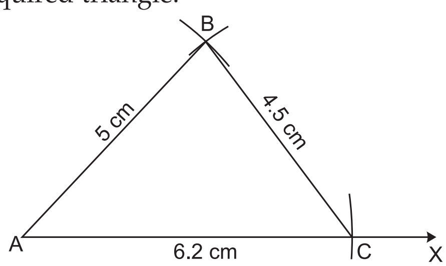
11

Constructions

Checkpoint (Page 154)

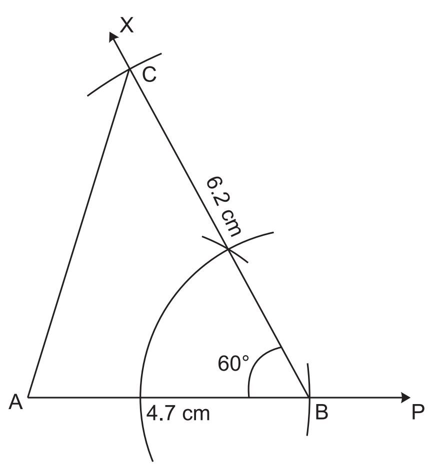
1. Construct ΔABC, given that AB = 5 cm, BC = 4.5 cm and AC = 6.2 cm.

Sol. We draw any ray AX. Taking A as centre and AC = 6.2 cm as radius, we draw an arc cutting the ray AX at C. Taking A as centre and AB = 5 cm as radius, we draw an arc. Taking C as centre and a length of 4.5 cm as radius, we draw another arc to cut the previous arc at the point B. We now join BA and BC. Then ΔABC is the required triangle.



2. Construct ΔABC, given that AB = 4.7 cm, BC = 6.2 cm and $\angle ABC = 60°$.

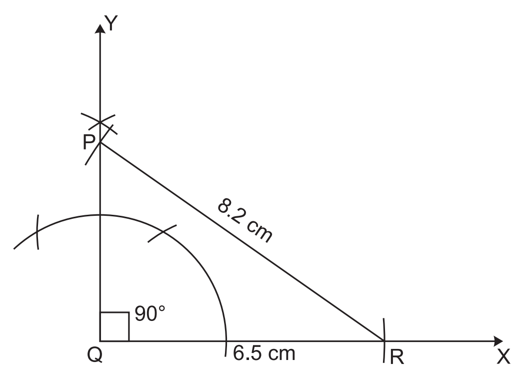
Sol.



We draw any ray AP. Taking A as centre and a length of 4.7 cm as radius, we draw an arc cutting the ray AP at B. We draw $\angle ABX = 60°$ at B with a compass, ruler and pencil. From BX, we cut a length BC = 6.2 cm with a compass. We now join AC. Then ΔABC is the required triangle.

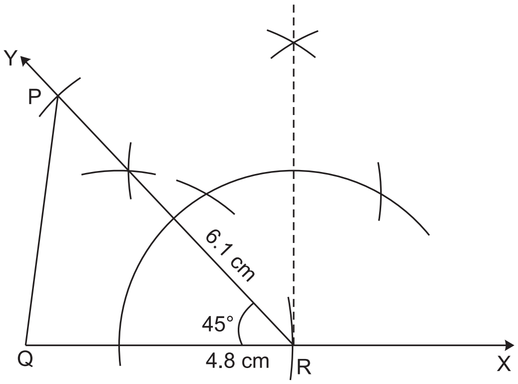
3. Construct ΔPQR right-angled at Q, given that PR = 8.2 cm and QR = 6.5 cm.

Sol. We draw a ray QX and cut a length QR = 6.5 cm from it. We construct $\angle YQR = 90°$ at Q with the help of a compass, ruler and a pencil. Taking R as centre and a length 8.2 cm as radius we draw an arc to cut QY at P. We join PR. Then ΔPQR is the required triangle.



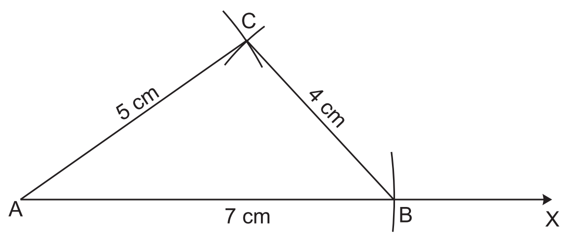
4. Construct ΔPQR with QR = 4.8 cm, PR = 6.1 cm and $\angle QRP = 45°$.

Sol. We draw a ray QX and cut a length QR = 4.8 cm from it. At R, we draw an angle of 90° first by a compass, a ruler and a pencil and then bisect it by a ray RY. From RY, we cut a length RP = 8.2 cm and then join PQ. Then ΔPQR with $\angle PRQ = 45°$ is the required triangle.



5. Construct a triangle of sides 4 cm, 5 cm, and 7 cm.

Sol. We draw a ray AX and cut a length AB = 7 cm from it. Taking A as centre and a length 5 cm as radius, we draw an arc. Taking B as centre and a length of 4 cm as radius, we draw another arc to cut the previous arc at C. We now join AC and BC. Then ΔABC is the required triangle.

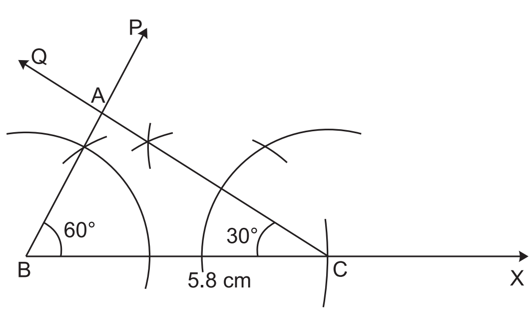


6. Can you construct a triangle of sides 2 cm, 8 cm and 4 cm? Explain why?

Sol. No, the sum of any sides of a triangle is always greater than the third side. In the given data, we see that $4 + 2 < 8$, i.e. the sum of two sides of lengths 4 cm and 2 cm is less than 8 cm. Hence, no triangle can be drawn in this case.

7. Construct a triangle ABC with side BC = 5.8 cm, ∠B = 60° and ∠C = 30°.

Sol. We draw a ray BX and cut a length BC = 5.8 cm from it. At B, we construct an angle ∠PBC = 60° and at C, we construct another angle ∠QCB = 30° with the help of a compass, a ruler and a pencil. Let the rays BP and CQ intersect each other at A. Then ΔABC is the required triangle.



8. Can you construct a right-angled triangle ABC with ∠A = 90° and ∠B = an obtuse angle? Explain why?

Sol. No, because the sum of ∠A and ∠B will be greater then 180° in this case, which is not possible for a triangle, for we know that the sum of three angles of a triangle is always 180°.

9. Can you construct a unique triangle ABC when ∠A = 65°, ∠B = 35° and ∠C = 80°? Explain why?

Sol. No, in this case although ∠A + ∠B + ∠C = 65° + 35° + 80° = 180°, an infinite number of similar triangles with different lengths of sides can be constructed. So, a unique triangle is not possible.

10. How many triangles can you construct when two base angles are given as 60° and 40° and the side opposite to one angle is given? Explain why?

Sol. We assume that ∠B = 60°, ∠C = 40° of a triangle ABC and the side AB opposite to 40° are given. Then by angle sum property of a triangle, ∠A = 180° – (60° + 40°) = 80°. Thus, we know ∠B = 60°, ∠A = 80° and AB are given. So, in this case, we can drawn a unique triangle ABC.

Milestone

(Page 158)

Multiple-Choice Questions

1. With the help of a ruler, compass and pencil only, it is possible to construct an angle of

(*a*) 8.5° (*b*) 37.5°
(*c*) 55° (*d*) 70°

Sol. (*b*) 37.5°

We know that with help of a ruler, compass and pencil only, we can draw an angle of 60°, 30° and 15°. By adding 60° and 15°, we get 75°. Thus, we can construct an angle which is half of 75°, i.e. 37.5°. Other angles *viz*. 8.5°, 55° and 70° are not half, one-fourth etc. of these angles. Hence, we can only construct 37.5° with a compass, ruler and pencil.

2. We cannot construct any triangle ABC when BC = 3 cm, ∠B = 30°, if (AB – AC) is equal to

(*a*) 2.5 cm (*b*) 1 cm
(*c*) 1.8 cm (*d*) 3.5 cm

Sol. (*d*) 3.5 cm

We know that construction of a triangle is possible only when the difference of the lengths of any two sides of the triangle is less than the length of its third side. Now, in (*a*), (*b*) and (*c*), (AB – AC) is less than BC = 3 cm, but in case (*d*) only, (AB – AC) is more than BC = 3 cm. Hence, in case (*d*) only, construction of a triangle is not possible.

Very Short Answer Type Questions

3. Can you construct a triangle with sides of length 5 cm, 7 cm and 8 cm? State the reason.

Sol. In this case, since the sum of the lengths of any two sides of a triangle is greater than the length of its third side, we can construct a triangle.

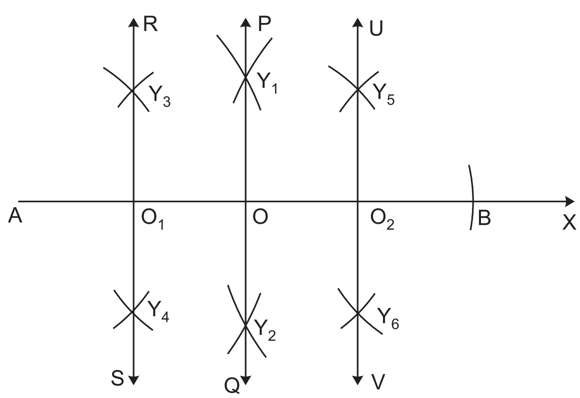
4. How many triangles can you draw through three collinear points? Explain the reason.

Sol. No triangle can be drawn through three collinear points, since a triangle has three sides which cannot be constructed with the help of three collinear points.

Short Answer Type-I Questions

5. Draw a line segment 8.9 cm long. Divide this line segment into four equal parts using bisector construction.

Sol. We take a ray AX and cut a length of 8.9 cm from it. Taking A as centre and a length equal to more than half the length of AB as radius, we draw two arcs on two sides of the line segment AB. Taking B as center and with the same length as radius, we draw two other arcs on both sides of AB cutting the former two arcs at the points Y_1 and Y_2. We now join Y_1Y_2 by a line PQ cutting AB at a point O. Then O is the mid-point of AB. In the same way, we can divide AO and OB successively into two equal parts AO_1, O_1O, OO_2 and O_2B.

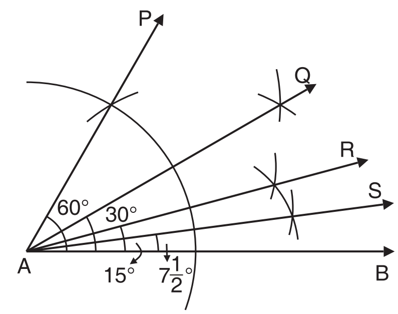


Thus, the line segment AB has been divided into four equal parts by means of the points O_1,O and O_2 so that

$$AO_1 = O_1O = OO_2 = O_2B$$

6. Using a ruler, a pencil and a compass, draw an angle of 7.5°.

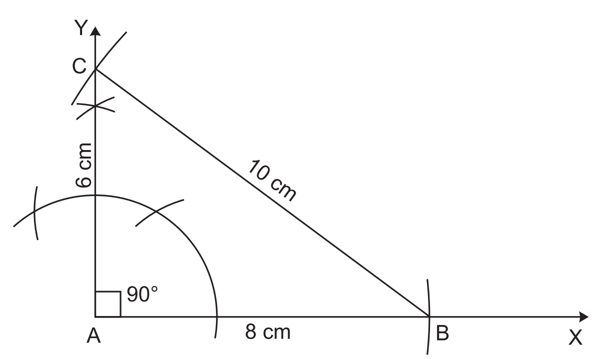
Sol. We draw a line segment AB of any suitable length. At A, we construct ∠PAB = 60° with the help of a compass, a ruler and a pencil. We now bisect ∠PAB by a ray AQ so that ∠QAB = 30°. We bisect this angle QAB again by a ray AR so that ∠RAB = 15°. Finally, we bisect ∠RAB again by another ray AS so that ∠SAB = 7.5°. Hence, ∠SAB is the required angle of measure 7.5°.



Short Answer Type-II Questions

7. Construct a right-angled triangle whose hypotenuse measures 10 cm and the length of one of its side containing the right angle measures 8 cm. Measure the third side of the triangle and verify it using Pythagoras' theorem.

Sol. We first draw a ray AX and cut a length AB = 8 cm from it. We construct ∠YAB = 90° with the help of a compass, a ruler and a pencil. Taking B as centre and a length of 10 cm as radius, we draw an arc cutting AY at a point C. We join BC. Then ΔBAC is the required right-angled triangle with AB = 8 cm and ∠BAC = 90°. With a scale, we measure the length AC = 6 cm.



Verification: Since ∠BAC = 90°,

∴ By Pythagoras' Theorem, we have

$$BC^2 = AB^2 + AC^2$$

$\because \quad AC^2 = BC^2 - AB^2$

$= (10 \text{ cm})^2 - (8 \text{ cm})^2$

$= (100 - 64) \text{ cm}^2$

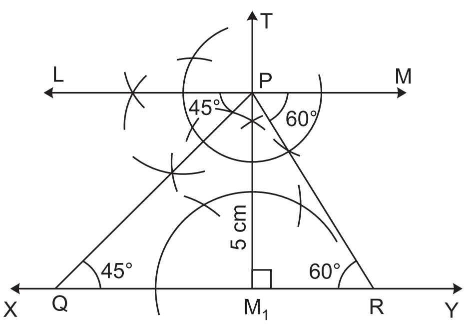
$= 36 \text{ cm}^2$

$\therefore \quad AC = \sqrt{36} \text{ cm} = 6 \text{ cm}$

Hence, the required length of AC is 6 cm which we expected.

8. Construct a triangle PQR in which $\angle Q = 45°$, $\angle R = 60°$ and the perpendicular from the vertex P to the base QR is of length 5 cm.

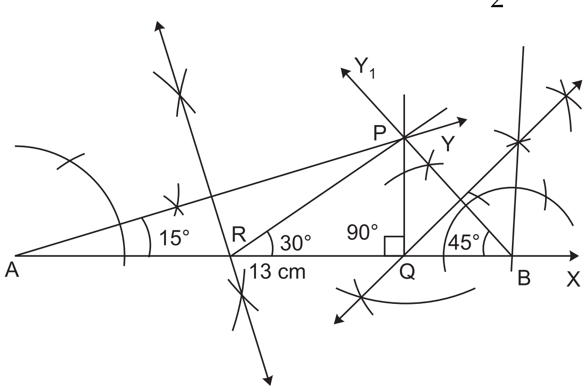
Sol. We first draw any line XY and take any point M_1 on it. We draw $TM_1 \perp XY$. Taking M_1 as centre and a length of 5 cm as radius, we cut a length $PM_1 = 5$ cm from TM_1. Through the point P, we draw a line LM $\parallel$ XY by constructing $\angle LPM_1 = 90°$. At P, we construct $\angle LPQ = 45°$ cutting XY at Q. We construct $\angle MPQ = 60°$ at P cutting XY at R. Then $\angle PQR$ is the required triangle with $\angle Q = 45°$, $\angle R = 60°$ and height $PM_1 = 5$ cm.



Long Answer Type Questions

9. Draw a triangle whose angles at the base are 30° and 90° and the perimeter is of length 13 cm.

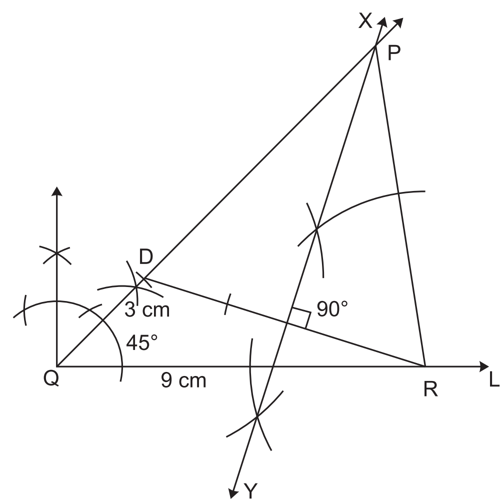
Sol. We cut AB = 13 cm from a ray AX. At A, we construct an angle $\frac{30°}{2} = 15°$. Let this angle be $\angle YAB$. At B, we construct $\angle Y_1BA = \frac{90°}{2} = 45°$.



Let AY and BY_1 intersect each other at P. We now draw two perpendicular bisectors of AP and BP intersecting AB at R and Q respectively. We now join PR and PQ to obtain the required triangle PQR.

10. Construct a triangle PQR in which QR = 9 cm, $\angle Q = 45°$ and PQ – PR = 3 cm. Write the steps of construction and justification.

Sol. We first draw a ray QL and cut a length QR = 9 cm from it. At Q, we draw $\angle Y_1QR = 45°$. Taking Q as centre and a radius equal to 3 cm, we draw an arc cutting QY_1 at D. We join DR. We now draw a perpendicular bisector XY of the line segment DR, cutting QD produced at P. We now join PR. Then ΔPQR is the required triangle.



Justification: Since P lies on the perpendicular bisector of DR,

$\therefore \quad PD = PR$

$\therefore \quad PQ - PR = PD + QD - PR$

$= PR + 3 - PR$

$= 3$

Hence, ΔPQR is the required triangle.

Higher Order Thinking Skills (HOTS) Questions

(Page 159)

1. Construct a triangle ABC such that the ratio of its two sides is 1 : 3. The difference of these two sides is 10 cm and one base angle between these two sides is 45°. Write the steps of construction and proof.

Sol. Let $\frac{AB}{AC} = \frac{1}{3}$

$\Rightarrow$ $AC = 3AB$ …(1)

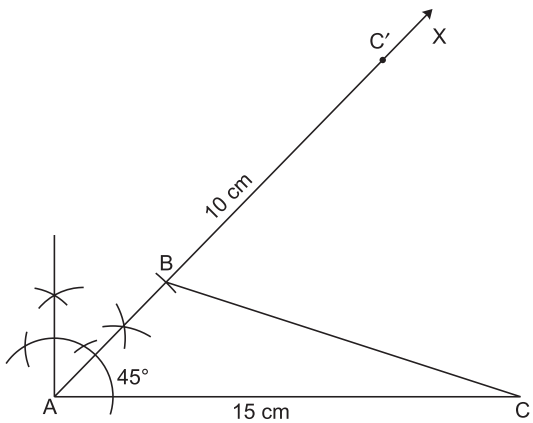
Given that $AC - AB = 10$ cm …(2)

From (1) and (2), we have

$2AB = 10$ cm

$\Rightarrow$ $AB = 5$ cm

$\therefore$ From (1), $AC = 3 \times 5$ cm $= 15$ cm



Let one of the base angles, say $\angle A = 45°$.

We shall now construct a ΔABC such that $AC = 15$ cm, $AC - AB = 10$ cm and $\angle A = 45°$.

We draw a line segment AC of length 15 cm. We now draw $\angle XAC = 45°$ and then cut a length $AC' = AC = 15$ cm from it. From C′, we mark a point B on it such that $C'B = 10$ cm. We now join BC. Then ΔABC is the required triangle.

Now, we prove that the difference of the two sides is 10 cm.

We have

$$AC - AB = AC' - AB$$
$$= BC'$$
$$= 10 \text{ cm}$$

Hence, ΔABC is the required triangle.

2. Consider a triangle ABC with sides $BC = a$, $CA = b$ and $AB = c$. We denote the angles of the triangle ABC by the corresponding capital letters. State in which of the following cases, you can construct a unique triangle, two triangles and no triangle at all.

(*a*) $b = 3$ cm, $c = 8$ cm and $\angle B = 30°$

(*b*) $b = 4$ cm, $c = 8$ cm and $\angle B = 30°$

(*c*) $b = 7$ cm and $c = 4$ cm and $\angle B = 30°$ and $\angle C$ is any acute angle

(*d*) $b = c = 8$ cm and $\angle B$ is any acute angle

(*e*) $b = 3$ cm, $c = 4$ cm and $\angle B = 30°$

Sol. (*a*), (*b*), (*c*) and (*d*)

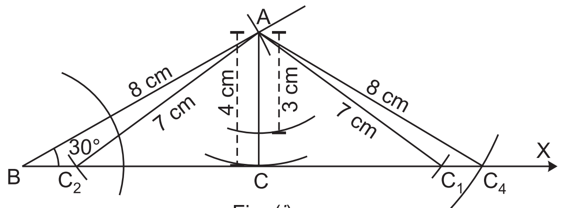


Fig. (*i*)

(*e*)

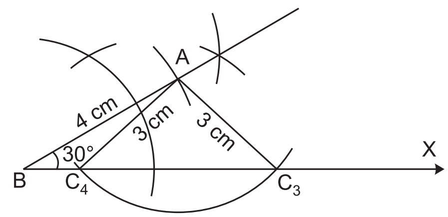


Fig. (*ii*)

In (*a*), we see from *Fig. (ii)* that if $AB = c = 8$ cm, $\angle B = 30°$, then if we draw an arc with A as centre and a length $b = 3$ cm as radius, then this arc does not cut the side BC at any point. Hence, in this case no triangle can be drawn.

In case *(b)*, we see from *Fig (i)* that if $AB = 8$ cm $= c$, $\angle B = 30°$, then if we draw an arc with A as centre and a length $b = 4$ cm as radius, then this arc touches the ray BX at C and so $\angle ACB = 90°$.

$\therefore$ In this case, a right-angled triangle ABC with $\angle C = 90°$can be drawn.

In case (*c*), we see from *Fig. (i)* that if $AB = c = 8$ cm, $\angle B = 30°$, then if we draw an arc with A as centre and a length $b = 7$ cm as radius, then this arc cuts the ray BX at C_1 and C_2 where $\angle BC_1A$ is acute, but $\angle BC_2A$ is obtuse. So, if $\angle C$ is also acute, then only one triangle ABC can be draw, otherwise two triangles ΔABC_1 and ΔABC_2 can be drawn.

In case (*d*), we see from *Fig. (i)* that if $b = c = 8$ cm and $\angle B$ is any acute angles, then only a unique triangle ABC_4 with $AB = AC_4 = 8$ cm can be drawn.

Finally, in case (*e*), we see from *Fig. (ii)* that if $b = 3$ cm, $c = 4$ cm and $\angle B = 30°$, then if we draw an arc with A as center and a length $b = 3$ cm as radius, this arc will intersect the ray BX at two points C_4 and C_3 with $AB = 4$ cm, $AC_4 = AC_3 = 3$ cm and $\angle B = 30°$. Hence, this case is ambiguous case, i.e we can draw two triangles *viz.* ΔABC_4 and ΔABC_3.

Self-Assessment

(Page 159)

Multiple-Choice Questions

1. The angle which cannot be constructed using a ruler and compass only is

(*a*) 7.5° (*b*) 44°

(*c*) 75° (*d*) 105°

Sol. (*b*) 44°

We know that angles of 60°, 30°, 15°, 45°, 90°, 120°, etc. can be drawn with a compass, ruler and a pencil. Also, angles with their different combination like 30° + 45° = 75°, 90° + 15° = 105°, $\frac{15°}{2}$ = 7.5° etc. can be constructed in this way. Among the given angles, 7.5° in (*a*) is $\frac{15°}{2}$, 75° in (*c*) is 30° + 45°, 105° in (*d*) is 90° + 15° but 44° in (*b*) can not be obtained by any combination of these angles.

2. We cannot construct any ΔABC when AB = 4 cm, ∠B = 40°, if (BC – AC) is equal to

(*a*) 3.5 cm (*b*) 3 cm

(*c*) 4 cm (*d*) 2.5 cm

Sol. (*c*) 4 cm

We know that for any triangle, the difference of the lengths of any two sides is always less than the length of its third side. Now, among the given lengths, only 4 cm in (*c*) is not less than the length 4 cm of AB.

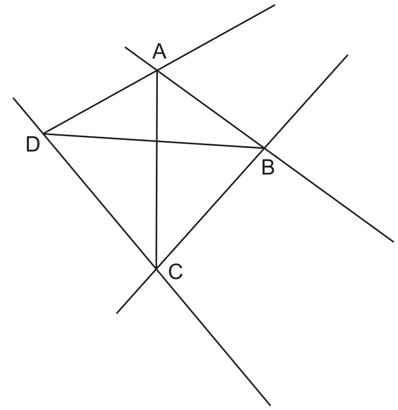
Very Short Answer Type Questions

3. With the help of a pencil, a ruler and a compass only, it is not possible to construct an angle of 35°. Is this statement true or false?

Sol. We see that 35° cannot be obtained by any combination of angles 60°, 30°, 15°, 7.5°, 45°, 90° etc. Hence, this statement is true.

4. How many distinct triangles can be drawn through four non-collinear points? Name them.

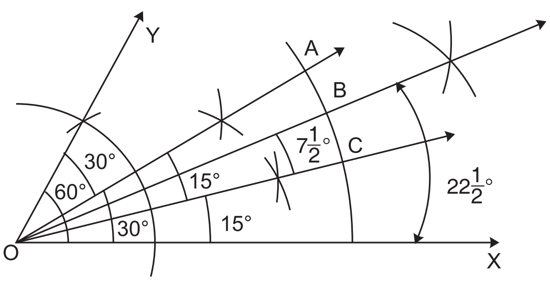
Sol. Let A, B, C, D be four district non-collinear points. By joining two consecutive points successively, we can construct four line segments AB, BC, CD and DA forming a quadrilateral ABCD. By joining two diagonals, AC and BD of the quadrilaterals, we obtain four triangles *viz*. ΔADC, ΔDCB, ΔCBA and ΔABD.



Short Answer Type-I Questions

5. Construct an angle of $22\frac{1}{2}°$ by using a pencil, a ruler and a compass only.

Sol. We first construct an ∠XOY = 60° with the help of a compass, a ruler and a pencil. We then bisect it by a ray OA to get ∠AOX = ∠AOY = 30°. We bisect ∠AOX by a ray OC to get ∠COX = ∠COA = 15°. We finally bisect ∠COA by the ray OB to get ∠COB = ∠BOA = $7\frac{1}{2}°$.



Then ∠BOX = ∠COB + ∠COX

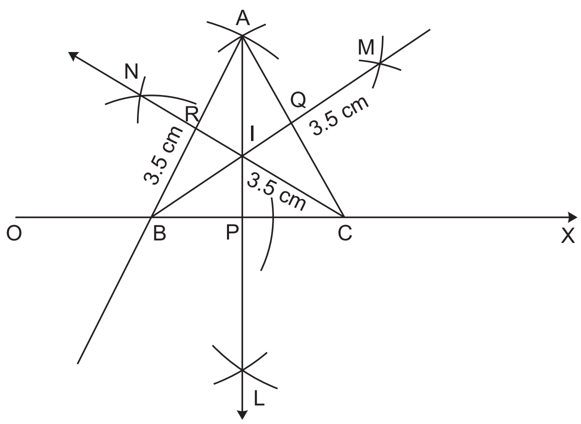
$= 15° + 7\frac{1}{2}°$

$= 22\frac{1}{2}°$

∴ ∠BOX is the required angle of measure $22\frac{1}{2}°$.

6. Construct an equilateral triangle of side 3.5 cm. Draw three perpendicular bisectors of its three sides. What do you observe?

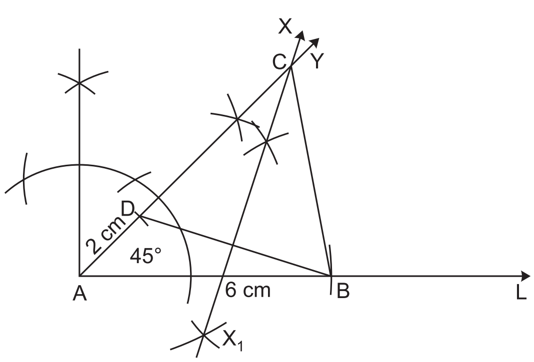
Sol. We draw a line segment BC = 3.5 cm from a ray OX. Taking B as centre and a length equal to 3.5 cm as radius, we draw an arc. With C as centre and the same length as radius, we draw another arc to cut the previous arc at a point A. We join AB and AC. Then ΔABC is the required equilateral triangle of side 3.5 cm. We next draw three perpendicular bisectors AL, BM and CN of the sides BC, CA and AB respectively. Let these three perpendicular bisectors meet at a point I. We observe that these three perpendicular will meet at a point I. We can also observe that if AL, BM and CN meet the sides BC, CA and AB of ΔABC at P, Q and R respectively, then AP, BQ and CR will be the medians of the ΔABC, meeting each other at the same point I within the triangle. So, we see that for an equilateral triangle, the centroid and the incentre are the same point. Also, we can see by actual measurement that AP = BQ = CR, i.e. the length of three medians are equal to each other.



Short Answer Type-II Questions

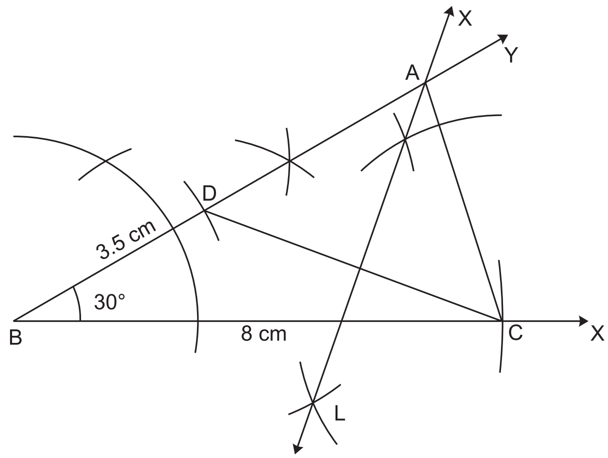
7. Construct a triangle ABC in which AB = 6 cm, ∠A = 45° and AC – BC = 2 cm. **[CBSE SP 2011]**

Sol. From a ray AL, we cut a length AB = 6 cm. At A, we draw an ∠YAB = 45°. Taking A as centre and a length of 2 cm as radius, we cut a length AD = 2 cm. We join DB. We now draw a perpendicular bisector XX_1 of DB. Let XX_1 intersect AY at C. We join CB. Then ΔABC is the required triangle.



8. Construct a triangle ABC in which BC = 8 cm, ∠B = 30° and AB – AC = 3.5 cm **[CBSE SP 2011]**

Sol.

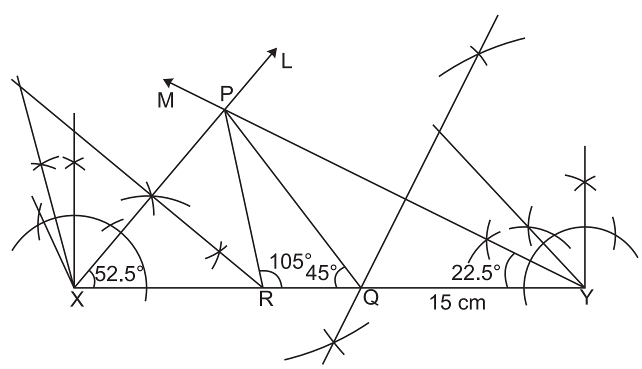


From a ray BX, we cut a length BC = 8 cm and at the point B, we draw ∠YBX = 30° with the help of a compass, ruler and a pencil. We cut a length BD = 3.5 cm from the ray BY. We join DC. We draw a perpendicular bisector XL of DC. Let XL intersect the ray BY at A. We join AC. Then ΔABC is the required triangle.

Long Answer Type Questions

9. Construct a triangle PQR in which ∠Q = 45°, ∠R = 105° and PQ + QR + RP = 15 cm.

Sol. We draw a line segment XY = 15 cm. At X, we construct $\angle LXY = \frac{105°}{2} = 52.5°$ and at Y, we construct $\angle MYX = \frac{45°}{2} = 22.5°$ with the help of a compass, a ruler and a pencil. Let XL and YM intersect each other at P. We draw two perpendicular bisectors of XP and YP intersecting XY at R and Q respectively. We join PR and PQ. Then ΔPQR is the required triangle.



10. The sides of a triangle are in the ratio 2 : 3 : 4. If the perimeter of the triangle is 12.6 cm, construct the triangle by taking help of the lengths of the three sides only.

Sol. We first calculate the three lengths of the sides of the triangle. Let the sides be $2x$, $3x$ and $4x$ in cm, where x is a non-zero positive number. Then

$$2x + 3x + 4x = 12.6 \text{ cm}$$

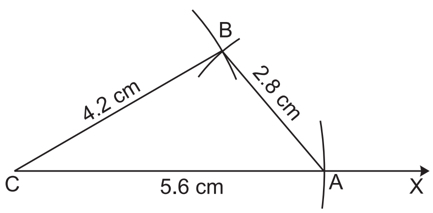
$$\Rightarrow \quad 9x = 12.6 \text{ cm}$$

$$\Rightarrow \quad x = \frac{12.6}{9} \text{ cm}$$

$$= 1.4 \text{ cm}$$

Hence, the sides of the triangle are of lengths AB = 1.4 × 2 cm = 2.8 cm, BC = 1.4 × 3 cm = 4.2 cm and CA = 1.4 × 4 cm = 5.6 cm.

We now construct ΔABC.



From a ray CX, we cut a length CA = 5.6 cm. Taking C as centre and a length of 4.2 cm as radius, we construct an arc. Again, taking A as centre and a length of 2.8 cm as radius, we construct another arc to cut the previous arc at a point B. We now join BC and BA. Then ΔABC is the required triangle.

Let's Compete

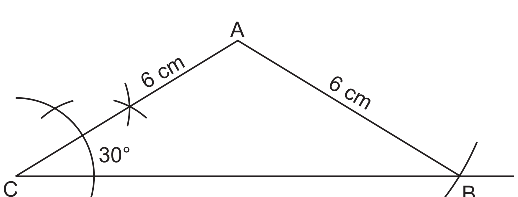
(Page 159)

Multiple-Choice Questions

1. We cannot construct a unique triangle ABC when AC, AB and ∠B are respectively equal to
   (*a*) 6 cm, 6 cm and 30°
   (*b*) 4 cm, 6 cm and 30°
   (*c*) 8 cm, 3 cm and 30°
   (*d*) 5 cm, 10 cm and 30°

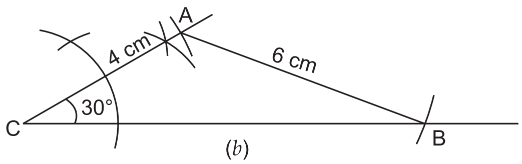
Sol. (*c*) 8 cm, 3 cm and 30°

We see that in case (*a*), construction of a triangle ACB is possible as shown in the figure.



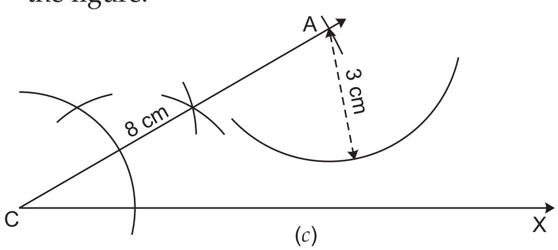
(*a*)

We see that in case (*b*), construction of a triangle ACB is possible as shown in the figure.



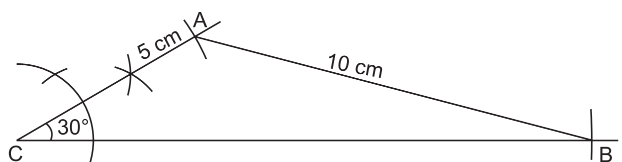
(*b*)

In this case (*c*), the construction is not at all possible as shown in the figure. With A as centre and a length of 3 cm as radius, if we construct an arc, it does not cut CX at any point as shown in the figure.



(*c*)

In case (*d*), the construction of ΔACB is possible as shown in the figure.



(*d*)

2. The number of triangles which can be constructed when its three angles are 30°, 65° and 85° is
   (*a*) one (*b*) two
   (*c*) three (*d*) infinitely many

Sol. (*d*) infinitely many

We know that when three appropriate angles are given so that their sum is 180°, then we can draw infinite number of triangles which will be similar to each other.

3. Construction of a ΔABC is not at all possible when the following are given.
   (*a*) ∠A = 30°, ∠B = 70° and ∠C = 80°
   (*b*) ∠A = 18°, ∠B = 62° and ∠C = any suitable obtuse angle
   (*c*) ∠A = 30°, ∠B = 42° and ∠C = any suitable acute angles less than 60°
   (*d*) ∠A = 72°, ∠B = 80° and ∠C = any suitable acute angle

Sol. (*c*) ∠A = 30°, ∠B = 42° and ∠C = any suitable acute angles less than 60°

Here, in (*a*) ∠A + ∠B + ∠C = 30° + 70° + 80° = 180°. So, we can construct any triangle, but the number of such triangles will be infinite and similar to each other.

In (b), $\angle A + \angle B = 18° + 62° = 80°$. Now, $180° - 80° = 100°$ which is an obtuse angle. So, if $\angle C = 100°$, then we can construct a triangle.

In (c), $\angle A + \angle B = 30° + 42° = 72°$

$\therefore$ If $\angle C = 180° - 72° = 108°$, i.e. an obtuse angle, then it is possible to construct a triangle, but if $\angle C$ is any acute angle less 60°, then it is not at all possible to construct a triangle, for the sum of three angles of the triangle cannot be 180° in this case.

In (d), $\angle A + \angle B = 72° + 80° = 152°$

$\therefore$ If $C = 180° - 152° = 28°$, i.e. an acute angle, then we can construct a triangle.

Hence, we conclude that only in case (c), it is not at all possible to construct any triangle.

4. Construction of a triangle ABC is possible only the following are given

(a) AB = AC = 3 cm and $\angle BAC = 170°$

(b) AB = 8 cm, AC = 7 cm, and $\angle B = 90°$

(c) AB = 3 cm, AC = 2 cm and BC = 6 cm

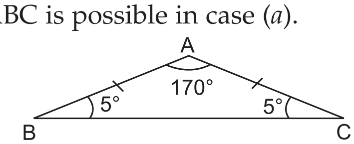
(d) AB = 4 cm = AC, $\angle A = 30°$ and $\angle B = 70°$

Sol. (a) AB = AC = 3 cm and $\angle BAC = 170°$

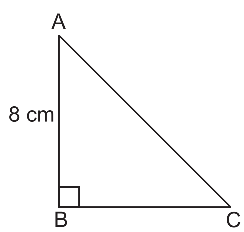
In (a), if $\angle BAC = 170°$,

then $\angle ABC + \angle ACB = 180° - 170° = 10°$.

$\therefore$ With some suitable length of BC, construction of ΔABC is possible in case (a).



In (b), if $\angle B = 90°$, then AC is the hypotenuse of the right-angled triangle. We know that the hypotenuse of a right-angled triangle is the greatest side. So, AC cannot be 7 cm, for in that case AC < AB which is absurd.



Hence, in this case construction of ΔABC is not possible at all.

In (c), we see that AB + AC = (3 + 2) cm = 5 cm < BC = 6 cm. Since, the sum of any two sides of a triangle is greater than the third side, hence, in this case it is not at all possible to construct a triangle.

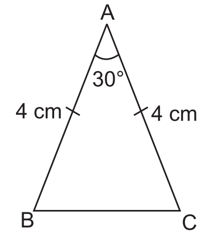
In (d), since AB = AC = 4 cm,

$\therefore \quad \angle ABC = \angle ACB$

But here $\angle A = 30°$, $\angle B = 70°$

$\therefore \quad \angle C = 70°$

$\therefore \quad \angle A + \angle B + \angle C = 170° < 180°$



Hence, the sum of three angles is less than 180°. So, it is not at all possible to construct any triangle in this case.

5. We can construct a unique triangle T_2 by joining the mid-points of the sides of the original triangle T_1 whose sides are 5 cm, 7 cm, and 8 cm, if the perimeter of triangle T_2 is

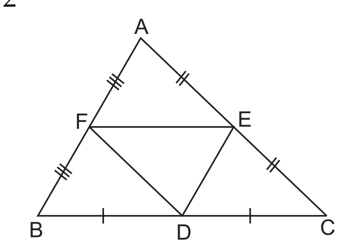
(a) 20 cm (b) 10 cm

(c) 12 cm (d) 15 cm

Sol. (b) 10 cm

If ΔABC be the original triangle and D, E, F are the mid-points of the sides BC, CA and AB respectively. Since $FE = \frac{1}{2} BC$, $ED = \frac{1}{2} AB$ and

$FD = \frac{1}{2} AC$,



$\therefore \quad FE + ED + FD = \frac{1}{2}(BC + CA + AB)$

i.e. the perimeter of the original triangle is always twice that of the new triangle FDE. Here in the present problem, perimeter of the original triangle is (5 + 7 + 8) cm = 20 cm.

$\therefore$ The perimeter of the new triangle T_2 will be exactly $\frac{20}{2}$ cm = 10 cm.

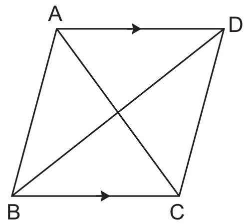
6. Two triangles ABC and DBC of equal areas can be constructed if the angle between BC and AD is

(a) 0° (b) 30°

(c) 90° (d) 60°

Sol. (*a*) 0°

We know that ar(ΔABC) = ar(ΔDBC) if they stand on the same base BC and lie between two parallels BC and AD, i.e. when the angles between BC and AD is zero.

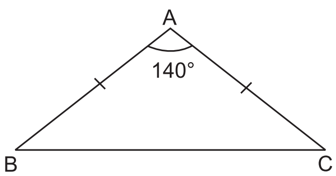


7. The vertical angle of an isosceles triangle is 140°. Then a unique isosceles triangle can be constructed if one of the base angle is

(*a*) 20° (*b*) 90°
(*c*) 15° (*d*) 30°

Sol. (*a*) 20°

If the vertical angle ∠A of an isosceles triangle ABC is 140°, then the sum of base angles ∠B and ∠C will be 180° – 140° = 40°.



∵ ∠B = ∠C

∴ Each of ∠B and ∠C is $\frac{40°}{2}$, i.e. 20°.

8. We can construct a quadrilateral ABCD with vertices lying on a circle if the three angles ∠A, ∠B and ∠C are respectively

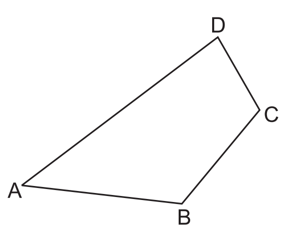
(*a*) 89°, 69°, 81° (*b*) 80°, 50°, 70°
(*c*) 73°, 93°, 107° (*d*) 75°, 82°, 95°

Sol. (*c*) 73°, 93°, 107°

We know that ABCD is a cyclic quadrilateral if

∠A + ∠C = 180°

∠B + ∠D = 180°



In case (*a*), we see that 89° + 81° = 70° ≠ 180°

In case (*b*), 80° + 70° = 150° ≠ 180°

In case (*c*), 73° + 107° = 180°

In case (*d*), 75° + 95° = 170° ≠ 180°

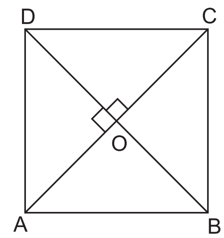
∴ Only in case (*c*), sum of opposite angles of the quadrilateral is 180°.

∴ The quadrilateral is cyclic only in case (*c*).

9. The total number of isosceles and equilateral triangles that can be constructed on the four sides of a square with the point of intersection of two diagonals of the square as one of the vertices of each triangle and the side of the square as one of the sides of each triangle, is

(*a*) 6 (*b*) 4
(*c*) 8 (*d*) 2

Sol. (*b*) 4



Let AC and DB be the two diagonals of the square ABCD, intersecting at a point O. Then each of the triangles AOB, BOC, COD and DOA is an isosceles triangle and since ∠AOB = ∠BOC = ∠COD = ∠AOD = 90°,

∴ No triangle can be equilateral. Hence, the total number of such triangles is four.

10. We cannot construct any triangle at all when its sides are proportional to

(*a*) 1 : 2 : 5 (*b*) 3 : 4 : 5
(*c*) 6 : 7 : 9 (*d*) 1 : 2 : 2

Sol. (*a*) 1 : 2 : 5

For any triangle, the sum of any two sides is greater then the third side.

In the given choices, we see only in case (*a*)

$$1 + 2 < 5$$

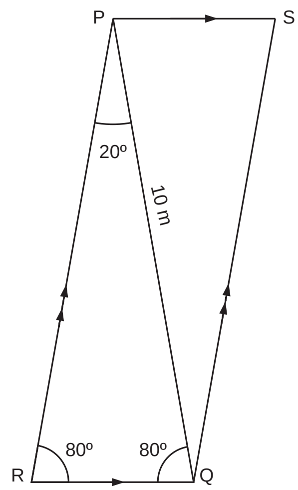
So, we cannot construct any triangle in this case.

Value-based Question (Optional)

(Page 160)

1. A kind-hearted old lady knitted small pieces of wool from the leftover wool and stitched them together to form two blankets, each in the shape of a ΔPQR such that the two triangles are congruent and have the same perimeter of length 24 m. Also, ∠PQR = ∠QRP = 80° and ∠RPQ = 20°.

She then joined the two blankets together along the side PQ so as to form a bigger blanket in the shape of a parallelogram PRQS as shown in the given figure. She gave this blanket to a poor girl child to save her from extreme cold weather.



(*a*) If the length of the diagonal PQ of the parallelogram is 10 m, find the length of the perimeter of the whole blanket in the shape of a parallelogram.

(*b*) What values are exhibited by the lady?

Sol. (*a*) $\because$ $\angle PRQ = \angle PQR = 80°$

$\therefore$ PR = PQ = 10 m

$\because$ PR + RQ + PQ = 24 m

$\therefore$ 10 m + RQ + 10 m = 24

$\therefore$ RQ = 4 m

$\therefore$ Length of RQ = 4 m

$\therefore$ Perimeter of the parallelogram PSQR

= 2(10 + 4)m

= 28 m

(*b*) Kindness, creative skill, empathy, helpfulness, caring and social responsibility.

12

Heron's Formula

Checkpoint (Page 163)

1. Find the area of an equilateral triangle with side 4 cm.

Sol. Here, side = 4 cm

Area of an equilateral triangle = $\frac{\sqrt{3}}{4}(\text{side})^2$

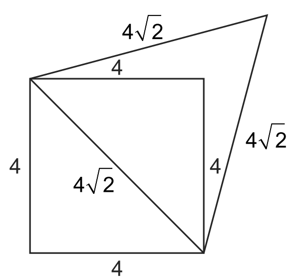
$= \frac{\sqrt{3}}{4}(4)^2 \text{ cm}^2$

$= 4\sqrt{3} \text{ cm}^2$

Hence, area of the equilateral triangle is $4\sqrt{3}$ cm^2.

2. Find the area of an equilateral triangle drawn from the diagonal of a square of side 4 cm.

Sol. Here, side of square = 4 cm



Diagonal of square = $\sqrt{2}$ (side)

$= \sqrt{2}$ (4) cm

$= 4\sqrt{2}$ cm

Area of an equilateral triangle drawn from the Diagonal of a square = $\frac{\sqrt{3}}{4}(\text{side})^2$

$= \frac{\sqrt{3}}{4}(4\sqrt{2})^2 \text{ cm}^2$

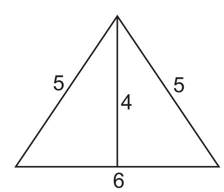
$= \frac{\sqrt{3}}{4}(4 \times 4 \times 2) \text{ cm}^2$

$= 8\sqrt{3} \text{ cm}^2$

Hence, the area of the equilateral triangle drawn from the diagonal of a square is $8\sqrt{3}$ cm^2.

3. Find the area of an isosceles triangle with base of length 6 cm and each equal side of length 5 cm, by using the formula: area = $\frac{1}{2}$ base × altitude.

Sol. Area of an isosceles triangle



$= \frac{1}{2} \times \text{base} \times \text{altitude}$

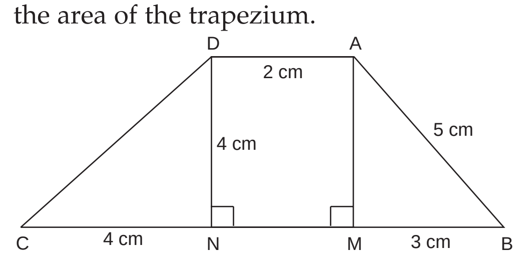
$= \frac{1}{2} \times 6 \times 4 \text{ cm}^2$

[By Pythagoras' Theorem, altitude = 4 cm]

$= 12 \text{ cm}^2$

Hence, the area of an isosceles triangle is 12 cm^2.

4. ABCD is a trapezium with AD || CB and CB > AD. AM ⊥ CB and DN ⊥ CB as shown in the given figure. If CN = 4 cm, DN = 4 cm, AB = 5 cm, BM = 3 cm and AD = 2 cm, then find the area of the trapezium.



Sol. Here, CN = 4 cm, DN = 4 cm, AB = 5 cm, BM = 3 cm, AD = 2 cm

$\therefore$ CB = CN + NM + BM

= (4 + 2 + 3) cm

[$\because$ NM = AD = 2 cm]

= 9 cm

Area of trapezium ABCD = $\frac{1}{2}$ × (Sum of the two parallel sides) × Distance between them

$= \frac{1}{2} \times (AD + CB) \times DN$

$= \frac{1}{2} \times (2 + 9) \times 4 \text{ cm}^2$

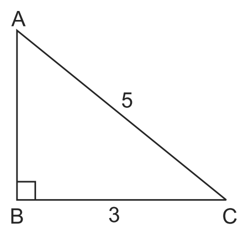
$= \frac{1}{2} \times 11 \times 4 \text{ cm}^2$

$= 22 \text{ cm}^2$

Hence, the area of the trapezium is 22 cm^2.

5. Find the length of the perimeter of a right-angled triangle whose hypotenuse and one side are of lengths 5 cm and 3 cm respectively.

Sol. Let ABC be right-angled triangle whose hypotenuse = AC = 5 cm and one of its side = BC = 3 cm.



By Pythagoras' theorem,

$(AC)^2 = (AB)^2 + (BC)^2$

$\Rightarrow (5 \text{ cm})^2 = (AB)^2 + (3 \text{ cm})^2$

$\Rightarrow (25 - 9) \text{ cm}^2 = (AB)^2$

$\Rightarrow AB = \sqrt{16} \text{ cm}$

$\Rightarrow AB = 4 \text{ cm}$

$\therefore$ Perimeter of a right-angled triangle ABC

= AB + BC + AC

= (4 + 3 + 5) cm

= 12 cm

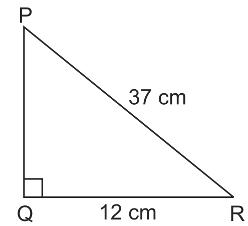
Hence, the length of the perimeter of a right-angled triangle is 12 cm.

6. Taking the length of the perimeter of the right-angled triangle in problem no. 5 above as the length of one side of another right-angled triangle, the second right-angled triangle is constructed with hypotenuse of length 37 cm. Find the area of this second right-angled triangle.

Sol. Perimeter of right-angled triangle in problem number 5 = Length of one side of second right-angled triangle [Given]

= 12 cm

Hypotenuse of second right-angled triangle = 37 cm



By Pythagoras' Theorem,

$(PR)^2 = (PQ)^2 + (QR)^2$

$\Rightarrow (37 \text{ cm})^2 = (PQ)^2 + (12 \text{ cm})^2$

$\Rightarrow 1369 \text{ cm}^2 = (PQ)^2 + 144 \text{ cm}^2$

$\Rightarrow 1225 \text{ cm}^2 = (PQ)^2$

$\Rightarrow PQ = 35 \text{ cm}$

Area of the second right-angled triangle

$= \frac{1}{2} \times \text{base} \times \text{altitude}$

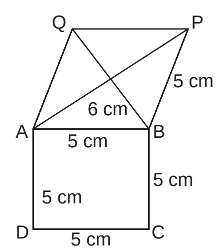
$= \frac{1}{2} \times QR \times PQ$

$= \left(\frac{1}{2} \times 12 \times 35\right) \text{cm}^2$

$= 210 \text{ cm}^2$

Hence, the area of the second right-angled triangle is 210 cm^2.

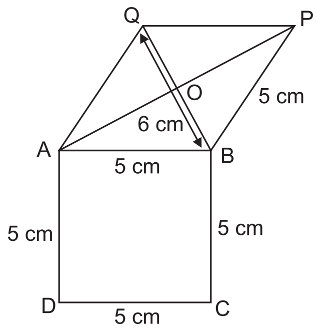
7. The length of a side of a square ABCD is 5 cm. A rhombus ABPQ is constructed on the side AB as shown in the given figure. If the length of the shorter diagonal BQ of the rhombus ABPQ is 6 cm, find the sum of the areas of the square and the rhombus.



Sol. Given AB = BC = CD = DA = 5 cm, BQ = 6 cm

$\therefore$ OB = 3 cm

[$\because$ Diagonals bisect each other at right angles in a rhombus]



In ΔPOB, by Pythagoras' Theorem,

$$(PB)^2 = (OB)^2 + (OP)^2$$

$$\Rightarrow \quad (5 \text{ cm})^2 = (3 \text{ cm})^2 + (OP)^2$$

$$\Rightarrow \quad (25 - 9) \text{ cm}^2 = (OP)^2$$

$$\Rightarrow \quad 16 \text{ cm}^2 = (OP)^2$$

$$\Rightarrow \quad OP = 4 \text{ cm}$$

$$\therefore \quad AP = 2\,(OP) = 2(4 \text{ cm}) = 8 \text{ cm}$$

Area of the rhombus

$$= \frac{1}{2} \times \text{Product of its diagonals}$$

$$= \frac{1}{2} \times (BQ) \times (AP)$$

$$= \left(\frac{1}{2} \times 6 \times 8\right) \text{cm}^2$$

$$= 24 \text{ cm}^2 \qquad \ldots(1)$$

Area of square = $(\text{Side})^2$

$$= (5)^2 \text{ cm}^2$$

$$= 25 \text{ cm}^2 \qquad \ldots(2)$$

Sum of the areas of the square and the rhombus

$$= (24 + 25) \text{ cm}^2$$

[From (1) and (2)]

$$= 49 \text{ cm}^2$$

Hence, the sum of the areas of the square and the rhombus is 49 cm^2.

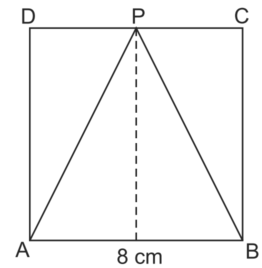
8. On the side AB of length 8 cm of a square ABCD, a triangle ABP is constructed where P is any point on another side CD of the square. Find the area of this triangle.

Sol. Here, altitude = 8 cm, AB = 8 cm.

Area of triangle ABP $= \frac{1}{2} \times \text{base} \times \text{altitude}$

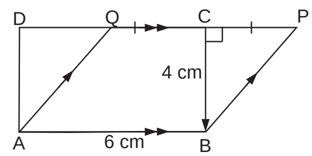
$$= \left(\frac{1}{2} \times 8 \times 8\right) \text{cm}^2$$

$$= 32 \text{ cm}^2$$



Hence, the area of the triangle is 32 cm^2.

9. ABCD is a rectangle of side AB = 6 cm and BC = 4 cm. If ABPQ is a parallelogram, where Q and P are points on DC and DC produced respectively as shown in the given figure and if C is the mid-point of PQ, then find the area of ΔCPB.



Sol. Given AB = 6 cm, BC = 4 cm

Since C is the mid-point of PQ.

$$\Rightarrow \quad CP = CQ$$

As ABCD is a square and ABPQ is a parallelogram

$\therefore$ Q is the mid-point of CD.

$$\Rightarrow \quad CQ = DQ = 3 \text{ cm}$$

[$\because$ CD = AB = 6 cm]

$$\therefore \quad \text{Area of } \Delta CPB = \frac{1}{2} \times \text{base} \times \text{altitude}$$

$$= \frac{1}{2} \times 3 \times 4$$

$$= 6 \text{ cm}^2$$

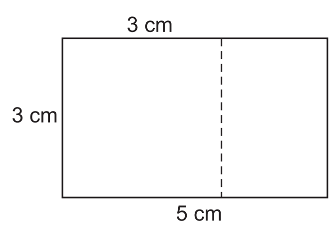
Hence, the area of ΔCPB is 6 cm^2.

10. Find the area of the largest square which can be drawn within a rectangle of dimension 5 cm × 3 cm.

Sol. Area of the largest square = $(\text{Side})^2$

$$= (3)^2 \text{ cm}^2$$

$$= 9 \text{ cm}^2$$



Hence, the area of the largest square which can be drawn within a rectangle of dimension 5 cm × 3 cm is 9 cm^2.

Milestone

(Page 169)

Multiple-Choice Questions

1. The sides of a triangle are 12 cm, 16 cm and 20 cm respectively. Its area is

(*a*) 48 cm^2 (*b*) 120 cm^2

(*c*) 96 cm^2 (*d*) 160 cm^2 **[CBSE SP 2012]**

Sol. (*c*) 96 cm^2

Here, $a = 12$ cm, $b = 16$ cm, $c = 20$ cm

$$\therefore \quad s = \frac{a+b+c}{2}$$

$$= \frac{12+16+20}{2} \text{ cm}$$

$$= \frac{48}{2} \text{ cm} = 24 \text{ cm}$$

Area of the triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{24(24-12)(24-16)(24-20)} \text{ cm}^2$$

$$= \sqrt{24(12)(8)(4)} \text{ cm}^2$$

$$= \sqrt{12 \times 2 \times 12 \times 2 \times 4 \times 4} \text{ cm}^2$$

$$= 12 \times 2 \times 4 \text{ cm}^2$$

$$= 96 \text{ cm}^2$$

2. If the area of an equilateral triangle is $16\sqrt{3}$ cm^2, then the perimeter of the triangle is

(*a*) 12 cm (*b*) 24 cm

(*c*) 48 cm (*d*) 36 cm **[CBSE SP 2013]**

Sol. (*b*) 24 cm

Area of an equilateral triangle = $16\sqrt{3}$ cm^2 [Given]

$$\Rightarrow \quad \frac{\sqrt{3}}{4}a^2 = 16\sqrt{3}$$

$$\Rightarrow \quad a^2 = 64$$

$$\Rightarrow \quad a = \pm 8$$

$$\Rightarrow \quad a = 8 \quad [\because a = -8; \text{ rejected}]$$

$\therefore$ Side of an equilateral triangle = 8 cm

$\therefore$ Perimeter of an equilateral triangle

$$= (8 + 8 + 8) \text{ cm} = 24 \text{ cm}$$

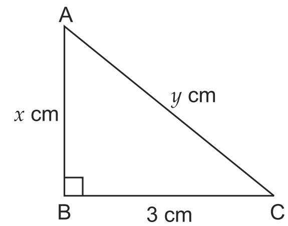
Very Short Answer Type Questions

3. Find the perimeter of a right-angled triangle of area 6 cm^2 and one side 3 cm.

Sol. Given, area = 6 cm^2

One Side = 3 cm

Let other side = x cm



$\because$ Area of right-angled triangle

$$= \frac{1}{2} \times \text{base} \times \text{altitude}$$

$$\Rightarrow \quad 6 \text{ cm}^2 = \frac{1}{2} \times 3 \text{ cm} \times x$$

$$\Rightarrow \quad 4 \text{ cm} = x$$

$$\Rightarrow \quad AB = 4 \text{ cm} \quad \ldots(1)$$

In right triangle ΔABC, we have

$$AC^2 = AB^2 + BC^2$$

$$= (4 \text{ cm})^2 + (3 \text{ cm})^2$$

$$= (16 + 9) \text{ cm}^2$$

$$= 25 \text{ cm}^2 \quad [\text{From (1)}]$$

$$\Rightarrow \quad AC = 5 \text{ cm}$$

$\therefore$ Perimeter of a right-angled triangle

$$= (3 + 4 + 5) \text{ cm}$$

$$= 12 \text{ cm}$$

Hence, the perimeter of a right-angled triangle is 12 cm.

4. The perimeter of a triangle whose sides are in the ratio 3 : 2 : 4 is 72 cm. Find the sum of the lengths of its longest and shortest sides.

Sol. Let the sides of the triangle be $a = 3x$ cm, $b = 2x$ cm and $c = 4x$ cm.

$$\text{Perimeter} = 72 \text{ cm}$$

$$\Rightarrow \quad 3x + 2x + 4x = 72 \text{ cm}$$

$$\Rightarrow \quad 9x = 72 \text{ cm}$$

$$\Rightarrow \quad x = 8 \text{ cm}$$

$$\therefore \quad a = 3x = (3 \times 8) \text{ cm} = 24 \text{ cm}$$

$$b = 2x = (2 \times 8) \text{ cm} = 16 \text{ cm}$$

$$c = 4x = (4 \times 8) \text{ cm} = 32 \text{ cm}$$

$\therefore$ Sum of the lengths of its longest and shortest sides = (32 + 16) cm = 48 cm.

Short Answer Type-I Questions

5. Find the area of a triangle whose perimeter is 68 cm and two of its sides are 25 cm and 26 cm.

Sol. Let $a = 25$ cm, $b = 26$ cm, $c = x$ cm

$$\text{Perimeter} = 68 \text{ cm}$$

$$\Rightarrow \quad 25 + 26 + x = 68 \text{ cm}$$

$\Rightarrow \quad 51 + x = 68$ cm

$\Rightarrow \quad x = 17$ cm

$\therefore \quad c = 17$ cm

$$s = \frac{\text{Perimeter}}{2}$$

$$= \frac{68}{2} \text{ cm} = 34 \text{ cm}$$

Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{34(34-25)(34-26)(34-17)} \text{ cm}^2$$

$$= \sqrt{34(9)(8)(17)} \text{ cm}^2$$

$$= \sqrt{2 \times 17 \times 3 \times 3 \times 2 \times 2 \times 2 \times 17} \text{ cm}^2$$

$$= 2 \times 17 \times 3 \times 2 \text{ cm}^2$$

$$= 204 \text{ cm}^2$$

Hence, the area of a triangle is 204 cm^2.

6. One side of an equilateral triangle measures 8 cm. Find its area and altitude.

Sol. Here, $a = 8$ cm, $b = 8$ cm and $c = 8$ cm

$$s = \frac{a+b+c}{2}$$

$$= \frac{8+8+8}{2} \text{ cm}$$

$$= \frac{24}{2} \text{ cm} = 12 \text{ cm}$$

Area of the given equilateral triangle

$$= \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{12(12-8)(12-8)(12-8)} \text{ cm}^2$$

$$= \sqrt{12 \times 4 \times 4 \times 4} \text{ cm}^2$$

$$= \sqrt{3 \times 4 \times 4 \times 4 \times 4} \text{ cm}^2$$

$$= 4 \times 4\sqrt{3} \text{ cm}^2$$

$$= 16\sqrt{3} \text{ cm}^2$$

Also, $\frac{1}{2} \times$ base $\times$ altitude = Area of the triangle

$\Rightarrow \quad \frac{1}{2} \times 8 \text{ cm} \times \text{altitude} = 16\sqrt{3} \text{ cm}^2$

$\Rightarrow \quad \text{Altitude} = \frac{16\sqrt{3} \times 2}{8} \text{ cm}$

$$= 4\sqrt{3} \text{ cm}$$

Hence, the area of the given equilateral triangle is $16\sqrt{3}$ cm^2 and its altitude is $4\sqrt{3}$ cm.

Short Answer Type-II Questions

7. Find the area of the quadrilateral whose sides measure 9 cm, 40 cm, 28 cm and 15 cm and in which the angle between the first two sides is a right angle.

Sol. Let AB = 40 cm, BC = 15 cm, CD = 28 cm, AD = 9 cm, $\angle BAD = 90°$

In right ΔBAD, we have

$$(BD)^2 = (AD)^2 + (AB)^2$$

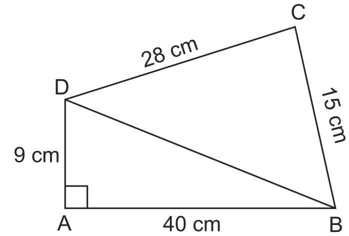
$$= (9 \text{ cm})^2 + (40 \text{ cm})^2$$

$$= 81 \text{ cm}^2 + 1600 \text{ cm}^2$$

$$= 1681 \text{ cm}^2$$

$\Rightarrow \quad BD = \sqrt{1681}$ cm

$\Rightarrow \quad BD = 41$ cm



Area of right $\Delta BAD = \frac{1}{2} \times AB \times AD$

$$= \left(\frac{1}{2} \times 40 \times 9\right) \text{ cm}^2$$

$$= (20 \times 9) \text{ cm}^2$$

$$= 180 \text{ cm}^2 \quad \ldots(1)$$

In ΔBCD, we have

$a = BC = 15$ cm, $b = CD = 28$ cm, $c = BD = 41$ cm

$\therefore \quad s = \frac{a+b+c}{2}$

$$= \left(\frac{15+28+41}{2}\right) \text{ cm}$$

$$= \frac{84}{2} \text{ cm}$$

$$= 42 \text{ cm}$$

Area of $\Delta BCD = \sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{42(42-15)(42-28)(42-41)} \text{ cm}^2$$

$$= \sqrt{(42)(27)(14)(1)} \text{ cm}^2$$

$$= \sqrt{2 \times 7 \times 3 \times 3 \times 3 \times 3 \times 2 \times 7} \text{ cm}^2$$

$$= 2 \times 7 \times 3 \times 3 \text{ cm}^2$$

$$= 126 \text{ cm}^2 \quad \ldots(2)$$

Area of quadrilateral ABCD

= Area of right ΔBAD + Area of ΔBCD

= (180 + 126) cm^2 [From (1) and (2)]

= 306 cm^2

Hence, the area of the quadrilateral ABCD is 306 cm^2.

8. The perimeter of a rhombus is 20 cm and one of its diagonals is 6 cm long. Find the length of the other diagonal.

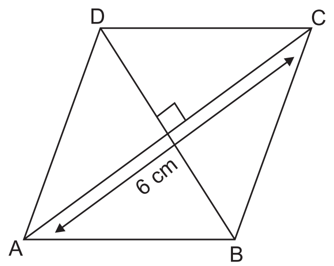
Sol. Let ABCD represent the given rhombus,

$$\text{Perimeter} = 20 \text{ cm}$$

$$\Rightarrow \quad 4x = 20 \text{ cm}$$

$$\Rightarrow \quad x = 5 \text{ cm}$$

$\Rightarrow$ Side of rhombus is 5 cm.



In ABC, we have

a = AB = 5 cm, b = BC = 5 cm, c = 6 cm

$$\therefore \quad s = \frac{a+b+c}{2}$$

$$= \left(\frac{5+5+6}{2}\right)\text{cm}$$

$$= \left(\frac{16}{2}\right)\text{cm}$$

$$= 8 \text{ cm}$$

$$\text{Area of } \Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{8(8-5)(8-5)(8-6)} \text{ cm}^2$$

$$= \sqrt{8(3)(3)(2)} \text{ cm}^2$$

$$= \sqrt{2 \times 2 \times 2 \times 3 \times 3 \times 2} \text{ cm}^2$$

$$= 2 \times 2 \times 3 \text{ cm}^2$$

$$= 12 \text{ cm}^2 \qquad \ldots(1)$$

We know that the diagonal of a rhombus divides it into two congruent triangles.

$$\Rightarrow \quad \Delta ABC \cong \Delta ADC$$

$$\Rightarrow \quad ar(\Delta ABC) = ar(\Delta ADC) \qquad \ldots(2)$$

$$ar(\text{rhombus ABCD}) = ar(\Delta ABC) + ar(\Delta ADC)$$

$$= 2\, ar(\Delta ABC) \qquad [\text{Using (2)}]$$

$$= 2 \times 12 \text{ cm}^2 \qquad [\text{Using (1)}]$$

$$= 24 \text{ cm}^2 \qquad \ldots(3)$$

$$\text{Also, } ar(\text{rhombus ABCD}) = \frac{1}{2} \times AC \times BD \qquad \ldots(4)$$

From (3) and (4), we get

$$\frac{1}{2} \times AC \times BD = 24 \text{ cm}^2$$

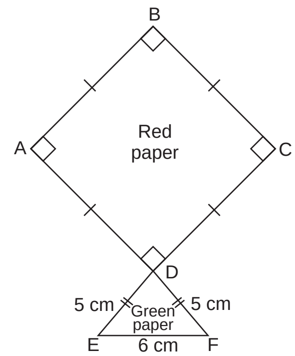
$$\Rightarrow \quad \frac{1}{2} \times 6 \text{ cm} \times BD = 24 \text{ cm}^2$$

$$\Rightarrow \quad BD = 8 \text{ cm}$$

$\therefore$ The length of the other diagonal is 8 cm.

Long Answer Type Questions

9. A kite is made up of square shaped red paper ABCD whose diagonal is $20\sqrt{2}$ cm. It has a green paper tail in the shape of an isosceles triangle DEF of base 6 cm and each side 5 cm as shown in the figure. Find the area of the red and the green paper used to make the kite.

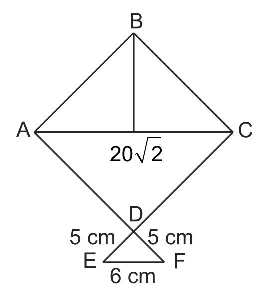


Sol.

$$\sqrt{2} \text{ side} = \text{Diagonal of a square}$$

$$\Rightarrow \quad \sqrt{2} \text{ side} = 20\sqrt{2} \text{ cm}$$

$$\Rightarrow \quad \text{side} = 20 \text{ cm}$$



In ΔABC, let a = BC = 20 cm, b = AC = $20\sqrt{2}$ cm and c = AB = 20 cm

$$\text{Then,} \quad s = \frac{a+b+c}{2}$$

$$= \frac{20 + 20\sqrt{2} + 20}{2} \text{ cm}$$

$$= \frac{20 + 20\sqrt{2}}{2} \text{ cm}$$

$$= (20 + 10\sqrt{2}) \text{ cm}$$

$$ar(\Delta ABC) = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{(20+10\sqrt{2})(20+10\sqrt{2}-20)(20+10\sqrt{2}-20\sqrt{2})(20+10\sqrt{2}-20)} \text{ cm}^2$$

$$= \sqrt{(20+10\sqrt{2})(10\sqrt{2})(20-10\sqrt{2})(10\sqrt{2})} \text{ cm}^2$$

$= 10\sqrt{2}\sqrt{(20)^2 - (10\sqrt{2})^2}$ cm^2

$= 10\sqrt{2}\sqrt{400 - 100 \times 2}$ cm^2

$= 10\sqrt{2} \times \sqrt{200}$ cm^2

$= 10\sqrt{2} \times 10\sqrt{2}$ cm^2

$= 100 \times 2$ cm^2

$= 200$ cm^2 ...(1)

We know that the diagonal of a square divides it into two congruent triangles.

$\Rightarrow$ $\Delta ABC \cong \Delta ADC$

$\Rightarrow$ $ar(\Delta ABC) = ar(\Delta ADC)$ ...(2)

Area of red paper used = ar(sq ABCD)

= ar(ΔABC) + ar(ΔADC)

= 2 ar(ΔABC) [Using (2)]

= 2 × 200 cm^2 [Using (1)]

= 400 cm^2

In ΔDEF, let a = DE = 50 cm, b = DF = 5 cm and c = EF = 6 cm

$\therefore \quad s = \frac{a+b+c}{2}$

$= \frac{5+5+6}{2}$ cm

$= \frac{16}{2}$ cm = 8 cm

Area of green paper used = ar(ΔDEF)

$= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{8(8-5)(8-5)(8-6)}$ cm^2

$= \sqrt{8 \times 3 \times 3 \times 2}$ cm^2

$= \sqrt{3 \times 3 \times 16}$ cm^2

$= 3 \times 4$ cm^2

$= 12$ cm^2

Hence, the area of the red paper used is 400 cm^2 and the area of the green paper used in 12 cm^2.

10. Find the area of a rhombus, one side of which measures 20 cm and one of whose diagonals is 24 cm. **[CBSE 2013]**

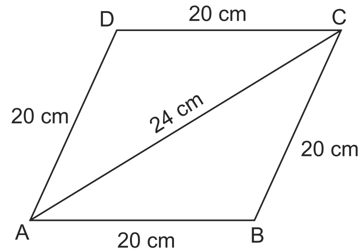
Sol. In ΔABC, we have

a = AB = 20 cm, b = BC = 20 cm, c = AC = 24 cm

$\therefore \quad s = \frac{a+b+c}{2}$

$= \frac{20+20+24}{2}$ cm

$= \frac{64}{2}$ cm = 32 cm



Area of ΔABC $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{32(32-20)(32-20)(32-24)}$ cm^2

$= \sqrt{32 \times 12 \times 12 \times 8}$ cm^2

$= \sqrt{4 \times 4 \times 2 \times 12 \times 12 \times 2 \times 2 \times 2}$ cm^2

$= 4 \times 2 \times 12 \times 2$ cm^2

$= 192$ cm^2 ...(1)

We know that the diagonal of a rhombus divides it into two congruent triangles.

$\Rightarrow$ $\Delta ABC \cong \Delta ADC$

$\Rightarrow$ $ar(\Delta ABC) = ar(\Delta ADC)$ ...(2)

ar(rhombus ABCD) = ar(ΔABC) + ar(ΔADC)

= 2 × ar(ΔABC) [From (1)]

= 2 × 192 cm^2

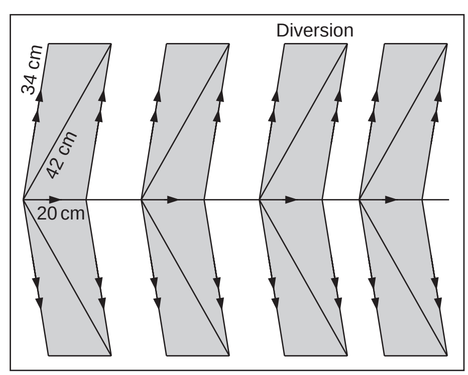
= 384 cm^2

Hence, the area of a rhombus is 384 cm^2.

Higher Order Thinking Skills (HOTS) Questions

(Page 169)

1. The shaded part of the diversion sign consists of 8 parallelograms of exactly the same shapes and sizes as shown in the given figure. If the length, breadth and one diagonal of each parallelogram are 34 cm, 20 cm and 42 cm respectively, find the total area of all these parallelograms which are to be painted red.



Sol. Let ABCD represent one of the 8 congruent parallelograms which have to be painted red.

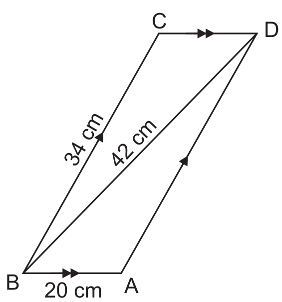
Then, AB = DC = 20 cm [Opp. sides of a ‖gm]

In ΔABD, we have

a = BD = 42 cm, b = AD = 34 cm and c = AB = 20 cm

$$\therefore \quad s = \frac{a+b+c}{2}$$
$$= \frac{42+34+20}{2} \text{ cm}$$
$$= \frac{96}{2} \text{ cm}$$
$$= 48 \text{ cm}$$

$$\text{ar}(\Delta ABD) = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{48(48-42)(48-34)(48-20)} \text{ cm}^2$$
$$= \sqrt{48 \times 6 \times 14 \times 28} \text{ cm}^2$$
$$= \sqrt{2 \times 2 \times 2 \times 6 \times 6 \times 14 \times 2 \times 14} \text{ cm}^2$$
$$= 2 \times 2 \times 6 \times 14 \text{ cm}^2$$
$$= 336 \text{ cm}^2 \quad \ldots(1)$$



Since, the diagonal of a parallelogram divides it into two congruent triangles.

$\therefore$ ΔABD ≅ ΔCDB

$\Rightarrow$ ar(ΔABD) = ar(ΔCDB) ...(2)

ar(‖gm ABCD) = ar(ABD) + ar(ΔCDB)

= 2 ar(ΔABD) [Using (2)]

= 2 × 336 cm^2

= 672 cm^2 ...(3)

Area to the painted red = 8 × ar(‖gm ABC)

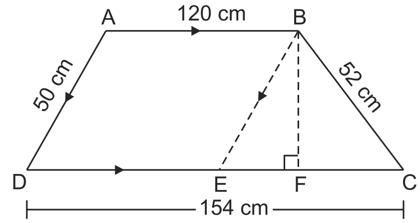
= 8 × 672 cm^2 [Using (3)]

= 5376 cm^2

Hence, the total area to be painted red is 5376 cm^2.

2. Two parallel sides of a trapezium are 120 cm and 154 cm and the other sides are 50 cm and 52 cm. Find the area of the trapezium. **[CBSE SP 2011]**

Sol. Let ABCD be the trapezium in which AB = 120 cm, BC = 52 cm, CD = 154 cm and AD = 50 cm.



Through B draw BE ‖ AD and let it meet DC at E.

Also, draw BF ⊥ DC.

Now, BE = AD = 25 cm

[Opp. sides of a ‖gm]

and EC = DC – DE

= DC – AB

[∵ AB = DE, Opp. sides of a ‖gm]

= (154 – 120) cm

= 34 cm

In ΔBEC, we have

a = BC = 52 cm, b = EC = 34 cm and c = BE = 50 cm

$$\therefore \quad s = \frac{a+b+c}{2}$$
$$= \frac{52+34+50}{2} \text{ cm}$$
$$= \frac{136}{2} \text{ cm} = 68 \text{ cm}$$

$$\text{ar}(\Delta BEC) = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{68(68-52)(68-34)(68-50)} \text{ cm}^2$$
$$= \sqrt{68(16)(34)(18)} \text{ cm}^2$$
$$= \sqrt{2 \times 34 \times 4 \times 4 \times 34 \times 2 \times 3 \times 3} \text{ cm}^2$$
$$= 2 \times 34 \times 4 \times 3 \text{ cm}^2$$
$$= 816 \text{ cm}^2 \quad \ldots(1)$$

Also, $\text{ar}(\Delta BEC) = \frac{1}{2} \times \text{EC} \times \text{BF}$

$= \frac{1}{2} \times 34 \text{ cm} \times \text{BF}$ ...(2)

From (1) and (2), we get

$$\frac{1}{2} \times 34 \text{ cm} \times \text{BF} = 816 \text{ cm}^2$$

$$\Rightarrow \quad \text{BF} = \frac{816 \times 2}{34} \text{ cm}$$

$$\Rightarrow \quad \text{BF} = 48 \text{ cm}$$

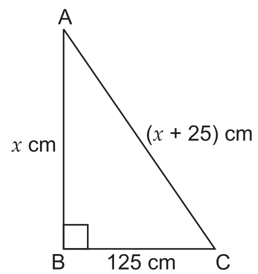
$$\text{ar(trapezium ABCD)} = \frac{1}{2}(\text{AB} + \text{DC}) \times \text{BF}$$
$$= \frac{1}{2}(120 + 154) \times 48 \text{ cm}^2$$
$$= 6576 \text{ cm}^2$$

Hence the area of the trapezium is 6576 cm^2.

3. One side of a right-angled triangle measures 125 cm and the difference in length of its hypotenuse and the other side is 25 cm. Find the lengths of its two unknown sides and calculate its area. Verify the result by Heron's formula.

Sol. Let ΔABC be the right-angled triangle at B.

Let BC = 125 cm, AB = x cm, AC = $(x + 25)$ cm



In ΔABC,

$$(AC)^2 = (AB)^2 + (BC)^2$$

[By Pythagoras' Theorem]

$\Rightarrow \quad (x + 25)^2 = (x)^2 + (125)^2$

$\Rightarrow \quad x^2 + 625 + 50x = x^2 + 15625$

$\Rightarrow \quad 50x = 15000$

$\Rightarrow \quad x = 300$

$\Rightarrow \quad AB = 300$ cm

Now, $\quad AC = (x + 25)$ cm

$= (300 + 25)$ cm

$AC = 325$ cm

Area of right $\Delta ABC = \frac{1}{2} \times BC \times AB$

$= \left(\frac{1}{2} \times 125 \times 300\right) cm^2$

$= 18750\ cm^2$

Verification by using Heron's formula:

In ΔABC, we have

$a = AB = 300$ cm, $b = BC = 125$ cm, $c = AC = 325$ cm

$\therefore \quad s = \frac{a + b + c}{2}$

$= \frac{300 + 125 + 325}{2}$ cm

$= 375$ cm

$ar(\Delta ABC) = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{375(375-300)(375-125)(375-325)}\ cm^2$

$= \sqrt{375(75)(250)(50)}\ cm^2$

$= \sqrt{5 \times 75 \times 75 \times 50 \times 5 \times 50}\ cm^2$

$= 5 \times 75 \times 50\ cm^2$

$= 18750\ cm^2$

Hence, the length of two unknown sides of the ΔABC are 300 cm and 325 cm and its area is 18750 cm^2.

Self-Assessment

(Page 169)

Multiple-Choice Questions

1. A triangle ABC in which AB = AC = 4 cm and $\angle A = 90°$, has an area

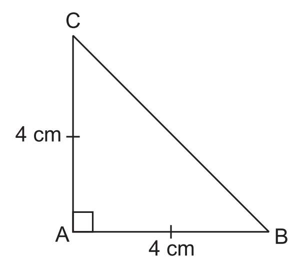
(*a*) 4 cm^2 (*b*) 16 cm^2

(*c*) 8 cm^2 (*d*) 12 cm^2

Sol. (*c*) 8 cm^2

Area of $\Delta ABC = \frac{1}{2} \times \text{base} \times \text{altitude}$

$= \left(\frac{1}{2} \times 4 \times 4\right) cm^2 = 8\ cm^2$



2. The area of a triangle whose sides are 7 cm, 9 cm and 8 cm is

(*a*) $12\sqrt{5}\ cm^2$ (*b*) $15\sqrt{5}\ cm^2$

(*c*) $15\sqrt{3}\ cm^2$ (*d*) $12\sqrt{15}\ cm^2$

Sol. (*a*) $12\sqrt{5}\ cm^2$

Let $a = 7$ cm, $b = 9$ cm, $c = 8$ cm

Semi-perimeter of triangle $= \frac{a + b + c}{2}$

$= \left(\frac{7 + 9 + 8}{2}\right)$ cm

$= \left(\frac{24}{2}\right)$ cm

$= 12$ cm

Area of the triangle

$= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{12(12-7)(12-9)(12-8)}\ cm^2$

$= \sqrt{12 \times 5 \times 3 \times 4}\ cm^2$

$= \sqrt{3 \times 4 \times 5 \times 3 \times 4}\ cm^2$

$= 3 \times 4\sqrt{5}$ cm^2

$= 12\sqrt{5}$ cm^2

Fill in the Blanks

3. The area of a triangle with base 8 cm and height 10 cm is **40 cm²**.

Sol. Area $= \frac{1}{2} \times$ Base $\times$ Height

$\therefore \quad = \frac{1}{2} \times 8 \times 10$ cm^2

$= 40$ cm^2

4. The area of a triangle whose sides are 3 cm, 4 cm and 5 cm is **6 cm²**.

Sol. $\therefore \quad s = \frac{3+4+5}{2} = \frac{12}{2} = 6$

Area $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{6(6-3)(6-4)(6-5)}$ cm^2

$= \sqrt{6(3)(2)(1)}$ cm^2

$= 6$ cm^2

5. If the perimeter and base of an isosceles triangle are 11 cm and 5 cm respectively, then its area is $\mathbf{\frac{5}{4}\sqrt{11}}$ **cm²**.

Sol. Let each equal side be x. Perimeter = 11 cm

$x + x + 5$ cm $= 11$ cm

$\Rightarrow \quad 2x = 6$ cm

$\Rightarrow \quad x = 3$ cm

Here, $a = 3$ cm, $b = 5$ cm and $c = 3$ cm

$s = \frac{a+b+c}{2} = \frac{3+5+3}{2}$ cm $= \frac{11}{2}$ cm

Area $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{\frac{11}{2}\left(\frac{11}{2}-3\right)\left(\frac{11}{2}-5\right)\left(\frac{11}{2}-3\right)}$ cm^2

$= \sqrt{\frac{11}{2}\left(\frac{5}{2}\right)\left(\frac{1}{2}\right)\left(\frac{5}{2}\right)}$ cm^2

$= \frac{5}{4}\sqrt{11}$ cm^2

6. The area of an isosceles triangle having base 24 cm and length of one of the equal sides 20 cm is **192 cm²**.

Sol. Here $a = 20$ cm, $b = 20$ cm and $c = 24$ cm

$s = \frac{a+b+c}{2} = \frac{20+20+24}{2}$ cm $= \frac{64}{2}$ cm $= 32$ cm

Area $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{32(32-20)(32-20)(32-24)}$ cm^2

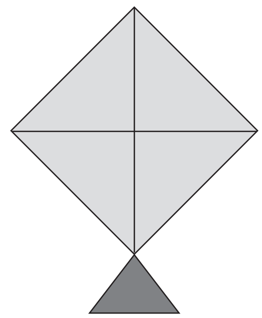
$= \sqrt{32(12)(12)(8)}$ cm^2

$= 8 \times 2 \times 12$ cm^2

$= 192$ cm^2

Case Study Based Questions

7. Karan made a kite to fly it on a Sunday. The kite is made up of square shaped red paper whose diagonal is $20\sqrt{2}$ cm and a green paper tail in the shape of an isosceles triangle of base 6 cm and each side 5 cm. Based on the above situation, answer the following questions.



(*a*) What is the measure of the side of the square shaped red paper?

(*i*) 10 cm (*ii*) 20 cm

(*iii*) 30 cm (*iv*) 40 cm

Ans. (*ii*) 20 cm

(*b*) What is the formula of semi-perimeter of a triangle with sides a, b and c?

(*i*) $a + b + c$ (*ii*) $a + b - c$

(*iii*) $\frac{a+b+c}{2}$ (*iv*) $\frac{a+b+c}{4}$

Ans. (*iii*) $\frac{a+b+c}{2}$

(*c*) What is the formula to find the area of a triangle (by Heron's formula)?

(*i*) $\sqrt{s(s+a)(s+b)(s+c)}$

(*ii*) $\sqrt{\frac{s(s+a)(s+b)(s+c)}{2}}$

(*iii*) $\sqrt{s(s-a)(s-b)(s-c)}$

(*iv*) None of these

Ans. (*i*) $\sqrt{s(s+a)(s+b)(s+c)}$

(*d*) What is the total area of red paper needed?

(*i*) 100 cm^2 (*ii*) 200 cm^2

(*iii*) 300 cm^2 (*iv*) 400 cm^2

Ans. (*iv*) 400 cm^2

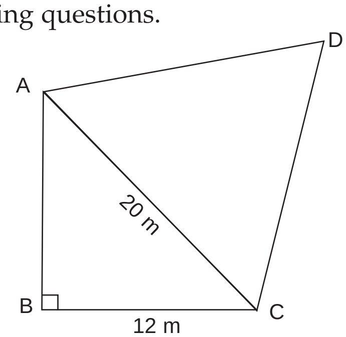
(*e*) What is the total area of green paper needed?

(*i*) 6 cm^2 (*ii*) 12 cm^2

(*iii*) 18 cm^2 (*iv*) 20 cm^2

Ans. (*ii*) 12 cm^2

8. Some students of a school staged a rally for cleanliness in the quadrilateral park ABCD. They walked through the lanes in two groups. First group walked through the lanes AB, BC and CA, while the second group through AC, CD and DA. Then they cleaned the area enclosed within their lanes. If BC = 12 m, AC = 20 m, $\angle ABC = 90°$ and ΔACD is an equilateral triangle, then answer the following questions.



(*a*) What is the length of AB?

(*i*) 12 m (*ii*) 14 m

(*iii*) 16 m (*iv*) 18 m

Ans. (*iii*) 16 m

(*b*) What is the area cleaned by first group (area of ΔABC)?

(*i*) 24 m^2 (*ii*) 48 m^2

(*iii*) 96 m^2 (*iv*) 192 m^2

Ans. (*iii*) 96 m^2

(*c*) What is the area cleaned by second group (area of ΔACD)?

(*i*) 96 m^2 (*ii*) 100 m^2

(*iii*) 167 m^2 (*iv*) 173 m^2

Ans. (*iv*) 173 m^2

(*d*) Which group cleaned more area and by how much?

(*i*) First group, 77 m^2

(*ii*) First group, 71 m^2

(*iii*) Second group, 77 m^2

(*iv*) Second group, 71 m^2

Ans. (*iii*) Second group, 77 m^2

(*e*) What is the total area cleaned by the students (neglecting the width of the lanes)?

(*i*) 263 m^2 (*ii*) 269 m^2

(*iii*) 346 m^2 (*iv*) 365 m^2

Ans. (*ii*) 269 m^2

Very Short Answer Type Questions

9. Find the area of an equilateral triangle whose perimeter is 18 cm. (Use $\sqrt{3}$ = 1.732)

Sol. Perimeter of the given equilateral triangle = 18 cm

$$\therefore \quad s = \frac{\text{Perimeter}}{2}$$

$$= \frac{18}{2} \text{ cm} = 9 \text{ cm}$$

and each side $= \frac{18}{3}$ cm = 6 cm

So, a = 6 cm, b = 6 cm, c = 6 cm

Area of the given equilateral triangle

$$= \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{9(9-6)(9-6)(9-6)} \text{ cm}^2$$

$$= \sqrt{9(3)(3)(3)} \text{ cm}^2$$

$$= \sqrt{3 \times 3 \times 3 \times 3 \times 3} \text{ cm}^2$$

$$= 3 \times 3\sqrt{3} \text{ cm}^2$$

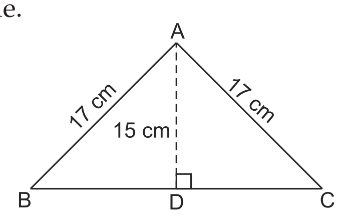
$$= 9 \times 1.732 \text{ cm}^2$$

$$= 15.588 \text{ cm}^2$$

Hence, the area of an equilateral triangle is 15.588 cm^2.

10. Find the area of an isosceles triangle whose altitude is 15 cm and one of the equal sides is 17 cm.

Sol. Let ABC be an isosceles triangle with AD as altitude.



Here, AB = AC = 17 cm, AD = 15 cm

In right ΔADC, we have

$$(AC)^2 = (AD)^2 + (DC)^2$$

[By Pythagoras' Theorem]

$$\Rightarrow (17 \text{ cm})^2 = (15 \text{ cm})^2 + (DC)^2$$

$$\Rightarrow 289 \text{ cm}^2 = 225 \text{ cm}^2 + (DC)^2$$

$$\Rightarrow (289 - 225) \text{ cm}^2 = (DC)^2$$

$\Rightarrow \quad (DC)^2 = 64 \text{ cm}^2$

$\Rightarrow \quad DC = 8 \text{ cm}$

Now, $\quad BC = 2(DC)$

$= 2(8) \text{ cm}$

$= 16 \text{ cm}$

In ΔABC, we have

$a = AB = 17 \text{ cm}, b = AC = 17 \text{ cm}, c = BC = 16 \text{ cm}$

$\therefore \quad s = \frac{a+b+c}{2}$

$= \frac{17+17+16}{2} \text{ cm}$

$= \frac{50}{2} \text{ cm} = 25 \text{ cm}$

Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{25(25-17)(25-17)(25-16)} \text{ cm}^2$

$= \sqrt{25 \times 8 \times 8 \times 9} \text{ cm}^2$

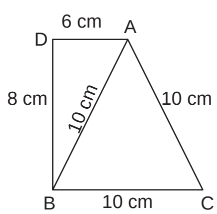
$= \sqrt{5 \times 5 \times 8 \times 8 \times 3 \times 3} \text{ cm}^2$

$= (5 \times 8 \times 3) \text{ cm}^2 = 120 \text{ cm}^2$

Hence, the area of an isosceles triangle is 120 cm^2.

Short Answer Type-I Questions

11. Find the area that is to be added to the area of ΔADB with dimensions shown in the given figure, so that it becomes equal to the area of ΔABC.(Use $\sqrt{3}$ = 1.732)



Sol. In ΔADB, we have

$a = AB = 10 \text{ cm}, b = BD = 8 \text{ cm}, c = AD = 6 \text{ cm}$

$\therefore \quad s = \frac{a+b+c}{2}$

$= \frac{10+8+6}{2} \text{ cm}$

$= \frac{24}{2} \text{ cm} = 12 \text{ cm}$

Area of $\Delta ADB = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{12(12-10)(12-8)(12-6)} \text{ cm}^2$

$= \sqrt{12(2)(4)(6)} \text{ cm}^2$

$= \sqrt{4 \times 3 \times 2 \times 4 \times 3 \times 2} \text{ cm}^2$

$= (4 \times 3 \times 2) \text{ cm}^2$

$= 24 \text{ cm}^2$

In ΔABC,

Area of $\Delta ABC = \frac{\sqrt{3}}{4} (\text{side})^2$

$= \frac{\sqrt{3}}{4} (10)^2 \text{ cm}^2$

$= \frac{\sqrt{3}}{4} \times 100 \text{ cm}^2$

$= 25\sqrt{3} \text{ cm}^2$

$= 25 (1.732) \text{ cm}^2$

$= 43.3 \text{ cm}^2$

$\therefore$ Area that is to be added to the area of ΔADB so that it becomes equal to the area of ΔABC

= Area of ΔABC – Area of ΔADB

$= (43.3 - 24) \text{ cm}^2 = 19.3 \text{ cm}^2$

12. Find the area of a triangle whose sides are 13 cm, 14 cm and 15 cm. **[CBSE SP 2011]**

Sol. Here, $a = 13 \text{ cm}, b = 14 \text{ cm}, c = 15 \text{ cm}$

$\therefore \quad s = \frac{a+b+c}{2}$

$= \left(\frac{13+14+15}{2}\right) \text{ cm}$

$= \frac{42}{2} \text{ cm} = 21 \text{ cm}$

Area of a triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{21(21-13)(21-14)(21-15)} \text{ cm}^2$

$= \sqrt{21 \times 8 \times 7 \times 6} \text{ cm}^2$

$= \sqrt{7 \times 3 \times 2 \times 2 \times 2 \times 7 \times 2 \times 3} \text{ cm}^2$

$= (7 \times 3 \times 2 \times 2) \text{ cm}^2$

$= 84 \text{ cm}^2$

Short Answer Type-II Questions

13. Using Heron's formula, find the area of an equilateral triangle whose perimeter is 24 cm. **[CBSE SP 2013]**

Sol. Perimeter of the given equilateral triangle = 24 cm

$\therefore \quad s = \frac{\text{Perimeter}}{2}$

$= \frac{24}{2} \text{ cm} = 12 \text{ cm}$

and $\quad$ each side $= \frac{24}{3} \text{ cm} = 8 \text{ cm}$

So, $a = 8 \text{ cm}, b = 8 \text{ cm}, c = 8 \text{ cm}$

Area of the given equilateral triangle

$= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{12(12-8)(12-8)(12-8)} \text{ cm}^2$

$= \sqrt{12(4)(4)(4)}\text{ cm}^2$

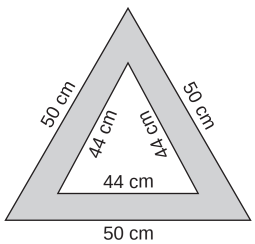
$= \sqrt{3 \times 4 \times 4 \times 4 \times 4}\text{ cm}^2$

$= 4 \times 4\sqrt{3}\text{ cm}^2$

$= 16\sqrt{3}\text{ cm}^2$

Hence, the area of an equilateral triangle is $16\sqrt{3}\text{ cm}^2$.

14. Find the cost of painting the shaded area shown in the given figure at the rate of ₹ 2 per cm^2. (Use $\sqrt{3}$ = 1.73)



Sol. Let a, b, c be sides of the given larger triangle.

Then, $a = b = c = 50$ cm

$\therefore \quad s = \dfrac{a+b+c}{2}$

$= \dfrac{50+50+50}{2}$ cm

$= \dfrac{150}{2}$ cm = 75 cm

Area of the given larger triangle

$= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{75(75-50)(75-50)(75-50)}\text{ cm}^2$

$= \sqrt{75(25)(25)(25)}\text{ cm}^2$

$= \sqrt{3 \times 25 \times 25 \times 25 \times 25}\text{ cm}^2$

$= 25 \times 25\sqrt{3}\text{ cm}^2$

$= 625\sqrt{3}\text{ cm}^2$

$= A_1$ (say)

Let a_1, b_1, c_1, be the side of the given smaller triangle.

Then, $a_1 = b_1 = c_1 = 44$ cm

$\therefore \quad s_1 = \dfrac{a_1+b_1+c_1}{2}$

$= \dfrac{44+44+44}{2}$ cm

$= \dfrac{132}{2}$ cm = 66 cm

Area of the given smaller

$= \sqrt{s_1(s_1-a_1)(s_1-b_1)(s_1-c_1)}$

$= \sqrt{66(66-44)(66-44)(66-44)}\text{ cm}^2$

$= \sqrt{66(22)(22)(22)}\text{ cm}^2$

$= \sqrt{3 \times 22 \times 22 \times 22 \times 22}\text{ cm}^2$

$= 22 \times 22\sqrt{3}\text{ cm}^2$

$= 484\sqrt{3}\text{ cm}^2$

$= A_2$ (say)

Area to be painted $= A_1 - A_2$

$= \left(625\sqrt{3} - 484\sqrt{3}\right)\text{ cm}^2$

$= 141\sqrt{3}\text{ cm}^2$

$= 141 \times 1.73\text{ cm}^2$

$= 243.93\text{ cm}^2$

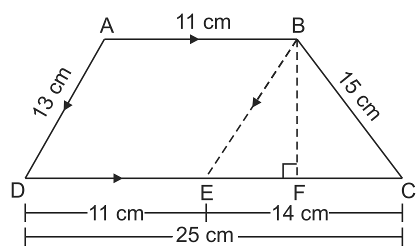
$\therefore$ Cost of painting = 243.93 × ₹2 = ₹487.86

Hence, the cost of painting the shaded area is ₹487.86.

Long Answer Type Questions

15. Find the area of a trapezium whose parallel sides are of lengths 25 cm and 11 cm and non-parallel sides are of lengths 15 cm and 13 cm.

Sol. Let ABCD be the trapezium in which AB = 11 cm, BC = 15 cm, CD = 25 cm and AD = 13 cm.



Through B, draw BE ∥ AD and let it meet DC at E. Also, draw BF ⊥ DC.

Now, BE = AD = 13 cm

[Opp. sides of a ∥gm]

and EC = CD – DE

= CD – AB

[∵ AB = DE, Opp. sides of a ∥gm]

= (25 – 11) cm

= 14 cm

In ΔBEC, we have

a = BC = 15 cm

b = EC = 14 cm

c = BE = 13 cm

$\therefore \quad s = \dfrac{a+b+c}{2}$

$$= \frac{15 + 14 + 13}{2} \text{ cm}$$

$$= \frac{42}{2} \text{ cm} = 21 \text{ cm}$$

$$\text{ar}(\Delta\text{BEC}) = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{21(21-15)(21-14)(21-13)} \text{ cm}^2$$

$$= \sqrt{21 \times 6 \times 7 \times 8} \text{ cm}^2$$

$$= \sqrt{7 \times 3 \times 3 \times 2 \times 7 \times 2 \times 2 \times 2} \text{ cm}^2$$

$$= 7 \times 3 \times 2 \times 2 \text{ cm}^2$$

$$= 84 \text{ cm}^2 \quad \ldots(1)$$

Also, $\text{ar}(\Delta\text{BEC}) = \frac{1}{2} \times \text{EC} \times \text{BF}$

$$= \frac{1}{2} \times 14 \text{ cm} \times \text{BF} \quad \ldots(2)$$

From (1) and (2), we get

$$\frac{1}{2} \times 14 \text{ cm} \times \text{BF} = 84 \text{ cm}^2$$

$$\Rightarrow \quad \text{BF} = \frac{84 \times 2}{14} \text{ cm}^2$$

$$\Rightarrow \quad \text{BF} = 12 \text{ cm}^2 \quad \ldots(3)$$

$$\text{ar(trapezium ABCD)} = \frac{1}{2}(\text{AB} + \text{DC}) \times \text{BF}$$

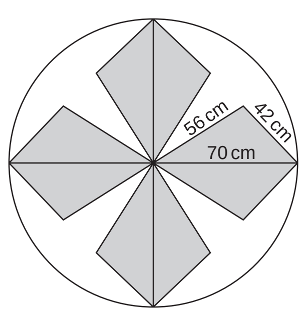
$$= \frac{1}{2}(11 + 25) \times 12 \text{ cm}^2$$

$$= \frac{1}{2} \times 36 \times 12 \text{ cm}^2$$

$$= 216 \text{ cm}^2$$

Hence, the area of a trapezium is 216 cm^2.

16. A design made up of 8 congruent triangles each having sides as 70 cm, 56 cm and 42 cm is painted on a circular glass window of radius 70 cm as shown in the given figure. Find the painted area of the glass window and remaining area.



Sol. Let the sides of each painted triangle be a, b and c, such that

$a = 70$ cm, $b = 56$ cm and $c = 42$ cm

Then, $s = \frac{a + b + c}{2}$

$$= \frac{70 + 56 + 42}{2} \text{ cm}$$

$$= \frac{168}{2} \text{ cm} = 84 \text{ cm}$$

Painted area of the glass window

$$= 8\sqrt{s(s-a)(s-b)(s-c)}$$

$$= 8\sqrt{84(84-70)(84-56)(84-42)} \text{ cm}^2$$

$$= 8\sqrt{84(14 \times 28 \times 42)} \text{ cm}^2$$

$$= 8\sqrt{2 \times 42 \times 14 \times 14 \times 2 \times 42} \text{ cm}^2$$

$$= 8(2 \times 42 \times 14) \text{ cm}^2$$

$$= 9408 \text{ cm}^2$$

Remaining Area = Area of the circular window – Area to be painted

$$= \pi r^2 - 9408 \text{ cm}^2$$

$$= \left(\frac{22}{7} \times 70 \times 70 - 9408\right) \text{ cm}^2$$

$$= (15400 - 9408) \text{ cm}^2$$

$$= 5992 \text{ cm}^2$$

Hence, the painted area of the glass window is 9408 cm^2 and remaining area is 5992 cm^2.

Let's Compete

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Multiple-Choice Questions

1. The edges of a triangular board are 6 cm, 8 cm and 10 cm. The cost of painting it at the rate of 90 paise per cm^2 is

(*a*) ₹ 19.75 (*b*) ₹ 20.50
(*c*) ₹ 21.60 (*d*) ₹ 23.70

Sol. (*c*) ₹ 21.60

Let the edges of a triangular board be a, b, and c such that

$a = 6$ cm, $b = 8$ cm, $c = 10$ cm

Then, $s = \frac{a + b + c}{2}$

$$= \frac{6 + 8 + 10}{2} \text{ cm}$$

$$= \frac{24}{2} \text{ cm} = 12 \text{ cm}$$

Area of the triangular board

$$= \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{12(12-6)(12-8)(12-10)} \text{ cm}^2$$

$$= \sqrt{12(6)(4)(2)} \text{ cm}^2$$

$= \sqrt{6 \times 2 \times 6 \times 2 \times 2 \times 2}$ cm^2

$= (6 \times 2 \times 2)$ cm^2

$= 24$ cm^2

Cost of painting the triangular board

$= 24 \times ₹ \frac{90}{100}$

$= ₹21.60$

Hence, the cost of painting the traingular board is ₹21.60.

2. The perimeter of a rhombus is 40 cm. If one of its diagonals is 12 cm, then its area is

(*a*) 30 cm^2 (*b*) 24 cm^2

(*c*) 48 cm^2 (*d*) 96 cm^2

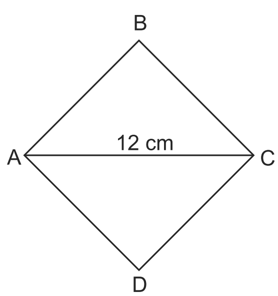
Sol. (*d*) 96 cm^2

Let ABCD be the given rhombus in which

$AB = BC = CD = DA = \frac{\text{Perimeter}}{4}$

$= \frac{40}{4}$ cm $= 10$ cm

and diagonal AC = 12 cm



In ΔABC,

Let $a = 10$ cm, $b = 12$ cm and $c = 10$ cm

Then, $s = \frac{a+b+c}{2}$

$= \frac{10+12+10}{2}$ cm

$= \frac{32}{2}$ cm $= 16$ cm

Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{16(16-10)(16-12)(16-10)}$ cm^2

$= \sqrt{16(6)(4)(6)}$ cm^2

$= \sqrt{4 \times 4 \times 6 \times 2 \times 2 \times 6}$ cm^2

$= 4 \times 6 \times 2$ cm^2

$= 48$ cm^2

Diagonal of a rhombus divides it in two congruent triangles.

$\Rightarrow \quad \Delta ABC \cong \Delta ADC$

$\Rightarrow$ Area of ΔABC = Area of ΔADC

Area of rhombus ABCD = Area of ΔABC + Area of ΔADC

= 2 Area of ΔABC

$= 2\,(48)$ cm^2

$= 96$ cm^2

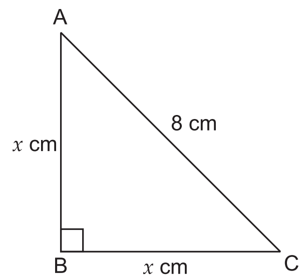
Hence, the area of the rhombus is 96 cm^2.

3. If the length of hypotenuse of a right-angled isosceles triangle is 8 cm, then its area is

(*a*) 16 cm^2 (*b*) 32 cm^2

(*c*) 64 cm^2 (*d*) 40 cm^2

Sol. (*a*) 16 cm^2



Let ABC be a right triangle such that AB = AC = x cm and hypotenuse = AC = 8 cm.

In right ΔABC, we have

$AC^2 = AB^2 + BC^2$

$\Rightarrow \quad (8)^2 = (x)^2 + (x)^2$

$\Rightarrow \quad 64 = 2x^2$

$\Rightarrow \quad 32 = x^2$

$\Rightarrow \quad x = \pm\sqrt{32}$

$\Rightarrow \quad x = 4\sqrt{2}$

So, $AB = AC = 4\sqrt{2}$ cm

Area of $\Delta ABC = \frac{1}{2} \times BC \times AB$

$= \frac{1}{2} \times 4\sqrt{2} \times 4\sqrt{2}$ cm^2

$= \frac{1}{2} \times 16 \times 2$ cm^2

$= 16$ cm^2

Hence, the area of a right-angled isosceles triangle is 16 cm^2.

4. The ratio of the equal side to its base of an isosceles triangle is 5 : 3. If the perimeter of the triangle is 26 cm, then the area of the triangle is

(*a*) 91 cm^2 (*b*) $91\sqrt{3}$ cm^2

(*c*) $3\sqrt{91}$ cm^2 (*d*) 273 cm^2

Sol. (*c*) $3\sqrt{91}$ cm^2

Given ratio of equal sides to its base of an isosceles triangle is 5 : 3.

Let the sides of the triangle be $5x$ cm, $3x$ cm and $5x$ cm.

Then, $5x + 3x + 5x = 26$ cm

$\Rightarrow \quad 13x = 26$ cm

$\Rightarrow \quad x = 2$ cm

So, the sides of the triangle are (5×2) cm, (3×2) cm and (5×2) cm, i.e. 10 cm, 6 cm and 10 cm.

Here, $a = 10$ cm, $b = 6$ cm, $c = 10$ cm

$$\therefore \quad s = \frac{a+b+c}{2} = \frac{10+6+10}{2} \text{ cm} = \frac{26}{2} \text{ cm} = 13 \text{ cm}$$

$$\text{Area of the triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{13(13-10)(13-6)(13-10)} \text{ cm}^2$$
$$= \sqrt{13 \times 3 \times 7 \times 3} \text{ cm}^2$$
$$= 3\sqrt{91} \text{ cm}^2$$

Hence, the area of the triangle is $3\sqrt{91}$ cm^2.

5. If the differences between the semi-perimeter and the sides of a triangle are 12 cm, 8 cm and 4 cm, then the area of the triangle is

(*a*) 48 cm^2 (*b*) 96 cm^2
(*c*) 80 cm^2 (*d*) 90 cm^2

Sol. (*b*) 96 cm^2

Given, $(s - a) = 12$ cm
$(s - b) = 8$ cm
$(s - c) = 4$ cm

$\Rightarrow (s - a + s - b + s - c) = (12 + 8 + 4)$ cm

$\Rightarrow \quad 3s - (a + b + c) = 24$ cm

$\Rightarrow \quad 3s - 2s = 24$ cm $\quad \left[\because \frac{a+b+c}{2} = s\right]$

$\Rightarrow \quad s = 24$ cm

$$\text{Area of the triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{24(12)(8)(4)} \text{ cm}^2$$
$$= \sqrt{12 \times 2 \times 12 \times 2 \times 4 \times 4} \text{ cm}^2$$
$$= (12 \times 2 \times 4) \text{ cm}^2$$
$$= 96 \text{ cm}^2$$

Hence, the area of the triangle is 96 cm^2.

6. If the area of a triangle is 384 cm^2 and its perimeter is 96 cm, then the numerical value of the product of the differences of its semi-perimeter and the lengths of its sides is

(*a*) 3072 (*b*) 1536
(*c*) 8 (*d*) 4

Sol. (*a*) 3072

Area of a triangle = 384 cm^2

Perimeter of a triangle = 96 cm

$\Rightarrow \quad a + b + c = 96$ cm

$$\text{Then,} \quad s = \frac{a+b+c}{2} = \frac{96}{2} \text{ cm} = 48 \text{ cm}$$

$$\text{Area of triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$\Rightarrow \quad 384 = \sqrt{48(s-a)(s-b)(s-c)}$$

Squaring both sides

$$\Rightarrow \quad \frac{384 \times 384}{48} = (s-a)(s-b)(s-c)$$

$$\Rightarrow \quad 3072 = (s-a)(s-b)(s-c)$$

Hence, the numerical value of the product of the differences of its semi-perimeter and the length of its sides is 3072.

7. If the differences of the semi-perimeter and the sides of a triangle are 60 m, 30 m and 10 m and the area of the triangle is 1344 m^2, then the height of the triangle corresponding to the smallest side as the base is

(*a*) 67.2 m (*b*) 38.4 m
(*c*) 29.9 m (*d*) 50.3 m

Sol. (*a*) 67.2 m

Given $(s - a) = 60$ m ...(1)
$(s - b) = 30$ m ...(2)
$(s - c) = 10$ m ...(3)

Area of the triangle = 1344 m^2

Adding (1), (2) and (3), we get

$s - a + s - b + s - c = (60 + 30 + 10)$ m

$\Rightarrow 3s - (a + b + c) = 100$ m

$\Rightarrow \quad 3s - 2s = 100$ m $\quad \left[\because \frac{a+b+c}{2} = s\right]$

$\Rightarrow \quad s = 100$ m

So, the sides of the triangle are $s - a = 60$

$\Rightarrow \quad 100 - a = 60$

$\Rightarrow \quad a = 40$

and $\quad s - b = 30$

$\Rightarrow \quad 100 - b = 30$

$\Rightarrow \quad b = 70$

and $\quad s - c = 10$

$\Rightarrow \quad 100 - c = 10$

$\Rightarrow \quad c = 90$

i.e. $a = 40$ m, $b = 70$ m, $c = 90$ m

$\therefore$ Smallest side as base = 40 m

We know that

Area of the triangle = $\frac{1}{2}$ × base × height

$\Rightarrow \quad 1344 \text{ m}^2 = \frac{1}{2} \times 40 \text{ m} \times \text{height}$

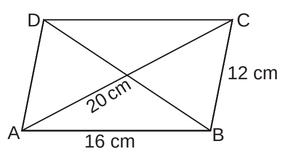
$\Rightarrow \quad \left(\frac{1344 \times 2}{40}\right) \text{m} = \text{height}$

$\Rightarrow \quad \text{height} = 67.2 \text{ m}$

$\therefore$ The height of the triangle corresponding to the smallest side as the base is 67.2 m.

8. The two adjacent sides of a parallelogram ABCD are of lengths AB = 16 cm and BC = 12 cm and one diagonal AC of the parallelogram is of length 20 cm. Then the area of ΔDBC is

(*a*) 240 cm^2 (*b*) 96 cm^2
(*c*) 336 cm^2 (*d*) 192 cm^2



Sol. (*b*) 96 cm^2

Given ABCD is a parallelogram such that AB = 16 cm, BC = 12 cm and AC = 20 cm

$\therefore$ AB = CD = 16 cm, BC = AD = 12 cm

In ΔDBC, we have

a = CD = 16 cm, b = BC = 12 cm, c = AC = 20 cm

Then, $\quad s = \frac{a+b+c}{2}$

$= \left(\frac{16+12+20}{2}\right) \text{cm}$

$= \left(\frac{48}{2}\right) \text{cm} = 24 \text{ cm}$

Area of ΔDBC = $\sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{24(24-16)(24-12)(24-20)} \text{ cm}^2$

$= \sqrt{24(8)(12)(4)} \text{ cm}^2$

$= \sqrt{12 \times 2 \times 2 \times 4 \times 12 \times 4} \text{ cm}^2$

$= (12 \times 2 \times 4) \text{ cm}^2$

$= 96 \text{ cm}^2$

$\therefore$ Area of ΔDBC is 96 cm^2.

9. If the differences of the perimeter and the sides of a triangle are 57 cm, 71 cm and 88 cm, then the area of this triangle is

(*a*) 326 cm^2 (*b*) 300 cm^2
(*c*) 320 cm^2 (*d*) 306 cm^2

Sol. (*d*) 306 cm^2

Let perimeter = P and semi-perimeter = s

Given $\quad (P - a) = 57$ cm

$(P - b) = 71$ cm

$(P - c) = 88$ cm

Adding these, we get

$P - a + P - b + P - c = (57 + 71 + 88)$ cm

$\Rightarrow 3P - (a + b + c) = 216$ cm

$\Rightarrow \quad 3(2s) - (2s) = 216$ cm

[$\because$ Perimeter = 2 semi-perimeter and $\frac{a+b+c}{2} = s$]

$\Rightarrow \quad 6s - 2s = 216$ cm

$\Rightarrow \quad 4s = 216$ cm

$\Rightarrow \quad s = 54$ cm

$\therefore \quad$ Perimeter, P = 2s

= 2(54 cm)

$\Rightarrow \quad$ P = 108 cm

So, the sides of the triangle and $P - a = 57$

$\Rightarrow \quad 108 - a = 57$

$\Rightarrow \quad a = 51$

and $\quad P - b = 71$

$\Rightarrow \quad 108 - b = 71$

$\Rightarrow \quad b = 37$

and $\quad P - c = 88$

$\Rightarrow \quad 108 - c = 88$

$\Rightarrow \quad c = 20$

i.e. $a = 51$ cm, $b = 37$ cm, $c = 20$ cm

Area of the triangle = $\sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{54(54-51)(54-37)(54-20)} \text{ cm}^2$

$= \sqrt{54(3)(17)(34)} \text{ cm}^2$

$= \sqrt{3 \times 3 \times 3 \times 2 \times 3 \times 17 \times 17 \times 2} \text{ cm}^2$

$= (3 \times 3 \times 2 \times 17) \text{ cm}^2$

$= 306 \text{ cm}^2$

Hence, the area of the triangle is 306 cm^2.

10. A tent is made by stitching 4 triangular pieces of canvas, each piece 13 m, 13 m and 24 m. Then the total area of the canvas required to make the tent is

(*a*) 120 m^2 (*b*) 200 m^2

(*c*) 240 m^2 (*d*) 60 m^2

Sol. (*c*) 240 m^2

Here, $a = 13$ m, $b = 13$ m, $c = 24$ m

Then, $s = \dfrac{a+b+c}{2}$

$= \left(\dfrac{13+13+24}{2}\right)$ m

$= \left(\dfrac{50}{2}\right)$ m

$= 25$ m

Area of a triangular piece of canvas

$= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{25(25-13)(25-13)(25-24)}$ m^2

$= \sqrt{25(12)(12)(1)}$ m^2

$= \sqrt{5 \times 5 \times 12 \times 12}$ m^2

$= 5 \times 12$ m^2

$= 60$ m^2

Total area of the canvas required to make the tent

$= 4 \times$ Area of a triangular piece of canvas

$= (4 \times 60)$ m^2

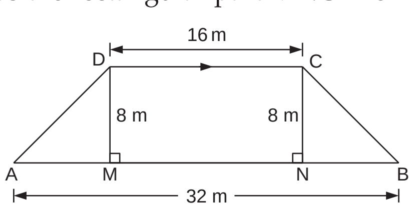
$= 240$ m^2

Hence, the total area of the canvas required to make the tent is 240 m^2.

Value-based Questions (Optional)

(Page 172)

1. ABCD in the shape of an isosceles trapezium with AB || CD represents a plot of land owned by an old man, where AB = 32 m and CD = 16 m. He divides the whole land into three parts by drawing DM ⊥ AB and CN ⊥ AB, where M and N are points on AB and DM = CN = 8 m. He donates both the triangular parts to an orphanage and keeps the rectangular part MNCD for himself.



(*a*) Find the total area of land donated by him to the orphanage.

(*b*) What are the values shown by the old man in this problem?

Sol. (*a*) Given ABCD is an isosceles trapezium with AB || CD where AB = 32 m and CD = 16 m, DM = CN = 8 m.

∴ MN = CD = 16 m

AM = NB …(1)

[∵ ABCD is an isosceles trapezium]

Now, AB = AM + MN + NB

⇒ 32 m = AM + 16 m + NB

⇒ 16 m = AM + NB

⇒ 16 m = 2 AM [∵ NB = AM]

⇒ AM = 8 m [From (1)]

In ΔAMD,

Area of ΔAMD $= \dfrac{1}{2} \times$ base $\times$ height

$= \dfrac{1}{2} \times$ AM $\times$ DM

$= \dfrac{1}{2} \times 8$ m $\times 8$ m

$= 32$ m^2 …(2)

Since ABCD is an isosceles trapezium

∴ ΔAMD ≅ ΔBNC

⇒ Area of ΔAMD = Area of ΔBNC

∴ Area of ΔBNC = 32 m^2 …(3)

Total area of land donated by him to orphanage

= Area of ΔAMD + Area of ΔBNC

= 32 m^2 + 32 m^2 [From (2) and (3)]

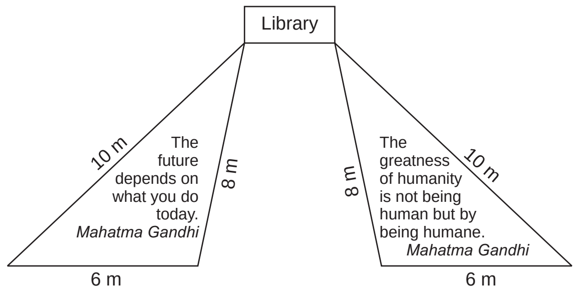
= 64 m^2

Hence, he donated 64 m^2 of land to an orphanage.

(*b*) Empathy and concern for orphans.

2. The entrance to a library consists of two congruent triangular side walls as shown below with quotes of Mahatma Gandhi about the greatness of humanity, hardworking, helpfulness etc.

(*a*) Find the area of each triangle if the sides of each triangle are 10 m, 8 m and 6 m.



(*b*) What values can be inculcated by the quotes of Mahatma Gandhi in the visitors?

Sol. (*a*) For each triangular wall.

$a = 10$ m, $b = 8$ m, $c = 6$ m

Then, $$s = \frac{a+b+c}{2}$$

$$= \frac{10+8+6}{2} \text{ m}$$

$$= \frac{24}{2} \text{ m} = 12 \text{ m}$$

Area of each triangular wall

$$= \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{12(12-10)(12-8)(12-6)} \text{ m}^2$$

$$= \sqrt{12(2)(4)(6)} \text{ m}^2$$

$$= \sqrt{2\times 6\times 2\times 2\times 2\times 6} \text{ m}^2$$

$$= 2 \times 6 \times 2 \text{ m}^2$$

$$= 24 \text{ m}^2$$

Hence, the area of each triangle is 24 m^2.

(*b*) To become hardworking, determined, helpful and humane.

13

Surface Areas and Volumes

Checkpoint (Page 174)

1. Find the lateral surface area and the total surface area of a cuboid whose length = 12 cm, breadth = 10 cm and height = 14 cm.

Sol. Let l, b and h be the length, breadth and height of the cuboid.

Then l = 12 cm, b =10 cm and h = 14 cm.

$\therefore$ Lateral surface area $= 2h(l + b)$

$= 2 \times 14 \times (12 + 10)$ cm^2

$= 28 \times 22$ cm^2

$= 616$ cm^2

Total surface area $= 2(lb + lh + bh)$

$= 2(12 \times 10 + 12 \times 14 + 10 \times 14)$ cm^2

$= 2(120 + 168 + 140)$ cm^2

$= 2 \times 428$ cm^2

$= 856$ cm^2

2. Find the edge of a cube whose surface area is 26.46 cm^2.

Sol. Let a be the side of a cube.

Then $6a^2 = 26.46$ cm^2

$\Rightarrow a^2 = \dfrac{26.46}{6}$ cm^2

$= 4.41$ cm^2

$\therefore a = \sqrt{4.41}$ cm $= 2.1$ cm

Hence, the required edge of the cube is 2.1 cm.

3. The perimeter of each face of a cube is 20 cm. Find its lateral surface area.

Sol. Let a be the length of each square face of the cube.

$\therefore 4a = 20$ cm

$\Rightarrow a = 5$ cm

$\therefore$ Lateral surface area $= 4a^2$

$= 4 \times 5^2$ cm^2

$= 100$ cm^2

4. The surface area of a cuboid is 2350 cm^2. If its length and breadth are 25 cm and 20 cm respectively, find its height.

Sol. Let l, b and h be the length, breadth and height of the cuboid.

$\therefore$ l = 25 cm, b = 20 cm

Then its surface area

$= 2(lb + lh + bh)$

$= [2(25 \times 20) + 2h(l + b)]$ cm^2

$= [2 \times 500 + 2h(25 + 20)]$ cm^2

$= (1000 + 90h)$ cm^2

$\therefore 90h + 1000 = 2350$

$\Rightarrow 90h = 1350$

$\therefore h = \dfrac{1350}{90} = 15$

Hence, the required height is 15 cm.

5. The curved surface area of a right circular cylinder of height 14 cm is 924 cm^2. Find the radius of its base.

Sol. Let r and h be the radius of the base and the height of the cylinder respectively.

Then $h = 14$ cm

$\therefore 2\pi rh = 924$

$\Rightarrow 2 \times \dfrac{22}{7} \times r \times 14 = 924$

$\Rightarrow r = \dfrac{924}{88} = 10.5$

Hence, the required radius of the base is 10.5 cm.

6. The volume of a cuboid is 528 cm^2 and the area of its base is 88 cm^2. Find its height.

Sol. Let l, b and h be the length, breadth and height of the cuboid respectively.

Then its volume = $l \times b \times h = 88h$

$$88h = 528$$

$$\therefore \quad h = \frac{528}{88} = 6$$

Hence, the required height is 6 cm.

7. Find the height of a cylinder whose volume is 3.08 m^3 and diameter of the base is 140 cm.

Sol. Let r be the radius of the base and h be the height of the cylinder.

Then $\quad r = \frac{140}{2}$ cm = 70 cm

Volume of the cylinder = $\pi r^2 h$

$$= \frac{22}{7} \times 70^2 \times h$$

$$\Rightarrow \quad 3.08 \times 100 \times 100 \times 100 = \frac{22}{7} \times 70^2 \times h$$

$$\Rightarrow \quad h = \frac{30800 \times 7 \times 100}{22 \times 70 \times 70} = 200$$

Hence, the required height is 200 cm or 2 m.

8. The circumference of the base of a cylindrical vessel is 132 cm and its height is 25 cm. How many litres of water can it hold.

Sol. Let r and h be the radius of the base and height of the cylinder respectively.

Then $\quad 2\pi r = 132$ cm

$$\Rightarrow \quad r = 66 \times \frac{7}{22} \text{ cm} = 21 \text{ cm}$$

Also, $\quad h = 25$ cm

$\therefore$ Volume of a cylinder = $\pi r^2 h$

$$= \frac{22}{7} \times 21 \times 21 \times 25 \text{ cm}^3$$

$$= 66 \times 525 \text{ cm}^3$$

$$= 34650 \text{ cm}^3$$

= 34.65 litres

9. A road roller is cylindrical in shape. Its circular end has a diameter of 280 cm and its width is 2 m. Find the least number of revolutions that the roller must make in order to level a playground 220 × 50 m.

Sol. Let r and h be the radius of the base and the height of the cylinder respectively.

Then r = 140 cm = 1.4 m and h = 2 m.

In 1 revolution, the roller levels a ground of surface area

$$= 2\pi rh$$

$$= 2 \times \pi \times 1.4 \times 2 \text{ m}^2$$

$$= 5.6\pi \text{ m}^2$$

$$\therefore \text{ Required number of revolution } = \frac{220 \times 50}{5.6 \times \frac{22}{7}}$$

$$= \frac{220 \times 50}{17.6}$$

$$= 625$$

10. A river 2.5 m deep and 30 m wide is flowing at the rate of 2 km/h. How much water will fall into the sea in a minute?

Sol. Speed of the river = 2km/h

$$= \frac{2000}{60} \text{ m/minute}$$

$$= \frac{100}{3} \text{ m/minute}$$

The shape of the river is cuboid of length l, breadth = b and depth = h.

We have

$$h = 2.5 \text{ m}, b = 30 \text{ m}, l = \frac{100}{3} \text{ m}$$

$\therefore$ Required volume of water

$$= l \times b \times h$$

$$= \frac{100}{3} \times 30 \times 2.5 \text{ m}^3$$

$$= 2500 \text{ m}^3$$

Milestone 1

(Page 178)

Multiple-Choice Questions

1. The difference between the total surface area of a cube of side 6 cm and its lateral surface area is

(*a*) 144 cm^2 (*b*) 72 cm^2

(*c*) 36 cm^2 (*d*) 100 cm^2

Sol. (*b*) 72 cm^2

Let a be the side of the cube.

Then $\quad a = 6$ cm

$\therefore$ Total surface area

$$= 6a^2$$

and the lateral surface area

$$= 4a^2$$

∴ Difference of these two areas

$= 6a^2 - 4a^2$

$= 2a^2$

$= 2 \times 6^2 \text{ cm}^2$

$= 72 \text{ cm}^2$

2. The curved surface area of a cylinder is 132 cm^2 and its height is 14 cm. Then its base radius is

(*a*) 2.5 cm (*b*) 1 cm

(*c*) 2 cm (*d*) 1.5 cm

Sol. (*d*) 1.5 cm

Let r and h be the radius of the base and the height of the cylinder respectively.

Then $h = 14$

Now, curved surface area $= 2\pi rh = 132$

$\Rightarrow \quad 2 \times \frac{22}{7} \times 14r = 132$

$\Rightarrow \quad r = \frac{132}{88} = 1.5$

Hence, the required radius is 1.5 cm.

Very Short Answer Type Questions

3. The circumference of the base of a right circular cylinder is 44 cm. If its whole surface area is 968 cm^2, then what is the sum of its height and the radius of its base?

Sol. Let r and h be the radius of the base and the height of the cylinder respectively.

Then $2\pi r = 44$ cm

$\Rightarrow \quad r = \frac{44 \times 7}{2 \times 22} \text{ cm} = 7 \text{ cm}$

Now, whole surface area $= 2\pi r(h + r) = 968$

$\Rightarrow \quad 2 \times \frac{22}{7} \times 7 \times (h + r) = 968$

$\Rightarrow \quad h + r = \frac{968}{44} = 22$

Hence, the required sum of radius and height is 22 cm.

4. If the radius of the base of a hemisphere is doubled, then find the ratio of the total surface area of the new hemisphere to that of the original hemisphere.

Sol. Let r be the radius of the original hemisphere and R be the radius of the new hemisphere.

Then $R = 2r$

Total surface area of the original hemisphere

$= 3\pi r^2$

and that of the new hemisphere

$= 3\pi R^2$

$= 3\pi(2r)^2$

$= 12\pi r^2$

∴ Required ratio $= \frac{12\pi r^2}{3\pi r^2} = \frac{4}{1} = 4 : 1$

Short Answer Type-I Questions

5. If the total surface area of a hemisphere is 462 cm^2, find its diameter.

Sol. If r be the radius of the hemisphere, then its total surface area $= 3\pi r^2$

$\therefore \quad 3\pi r^2 = 462 \text{ cm}^2$

$\Rightarrow \quad r^2 = \frac{462 \times 7}{3 \times 22} \text{ cm}^2 = 49 \text{ cm}^2$

$\therefore \quad r = 7$ cm

∴ Required diameter $= 2r = 2 \times 7$ cm $= 14$ cm

6. The curved surface area of a cone is 4070 cm^2 and its diameter is 70 cm. What is its slant height?

Sol. Let r be the radius of the base and l be the slant height of the cone.

Then $r = \frac{70}{2} \text{ cm} = 35 \text{ cm}$

∴ Curved surface area of the cone $= \pi rl$

Then $\pi rl = 4070$

$\Rightarrow \quad \frac{22}{7} \times 35 \times l = 4070$

$\Rightarrow \quad l = \frac{4070}{22 \times 5} = 37$

Hence, the required slant height is 37 cm.

Short Answer Type-II Questions

7. How many metres of cloth 1.1 m wide will be required to make a conical tent, whose vertical height is 12 m and base radius is 16 m? Also find the cost of cloth used at the rate of ₹ 14 per metre.

Sol. Let r and h be the radius of the base and the vertical height of a cone respectively and l be the slant height of the cone. Then $h = 12$ m, $r = 16$ m.

$\therefore \quad l = \sqrt{h^2 + r^2}$

$= \sqrt{12^2 + 16^2}$ m

$= \sqrt{144 + 256}$ m

$= \sqrt{400}$ m

$= 20$ m

∴ Curved surface area of the cone

$= \pi rl$

$$= \frac{22}{7} \times 16 \times 20 \text{ m}^2$$

$$= \frac{352 \times 20}{7} \text{ m}^2$$

Area of the rectangular cloth = 1.1 L

where L is the length of the cloth.

$$\therefore \quad 1.1 \text{ L} = \frac{352 \times 20}{7}$$

$$\Rightarrow \quad \text{L} = \frac{352 \times 200}{7 \times 11}$$

$$= \frac{6400}{7}$$

Hence, the required length of the cloth is $\frac{6400}{7}$ m.

Also, required cost of the cloth is ₹14 × $\frac{6400}{7}$, i.e. ₹12800.

8. The diameter of a roller, 1 m 40 cm long, is 80 cm. If it takes 600 complete revolutions to level a playground, find the cost of levelling the ground at 75 paise per sq metre.

Sol. Let r be the radius of the roller and h be the length of the roller.

Then, $r = \frac{80}{2}$ cm = 40 cm and h = 140 cm.

In 1 revolution, the roller levels an area of

$$2\pi rh = 2 \times \frac{22}{7} \times 40 \times 140 \text{ cm}^2$$

∴ In 600 revolutions, the roller levels an area of

$$2 \times \frac{22}{7} \times 40 \times 140 \times \frac{600}{100 \times 100} \text{ m}^2$$

$$= 2112 \text{ m}^2$$

∴ Required cost of levelling the ground

= ₹0.75 × 2112

= ₹1584

Long Answer Type Questions

9. A metal pipe is 77 cm long. The inner diameter of the cross section is 4 cm, the outer diameter being 4.4 cm. Find its

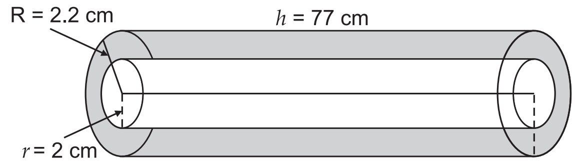
(*a*) inner curved surface area

(*b*) outer curved surface area

(*c*) total surface area.

Sol. Let r and R be the inner and outer radii of the pipe and let h be its length.

Then r = 2 cm, R = 2.2 cm and h = 77 cm.



(*a*) Inner curved surface area of the pipe

$$= 2\pi rh$$

$$= 2 \times \frac{22}{7} \times 2 \times 77 \text{ cm}^2$$

$$= 968 \text{ cm}^2$$

Hence, the required inner surface area is 968 cm^2.

(*b*) Outer curved surface area of the pipe

$$= 2\pi \text{R}h$$

$$= 2 \times \frac{22}{7} \times 2.2 \times 77 \text{ cm}^2$$

$$= 1064.8 \text{ cm}^2$$

Hence, the required curved surface area of the pipe is 1064.8 cm^2.

(*c*) Total surface area

$$= 2\pi \text{R}h + 2\pi rh + 2\pi(\text{R}^2 - r^2)$$

$$= 2\pi[(\text{R} + r)h + \text{R}^2 - r^2]$$

$$= 2 \times \frac{22}{7} \times [(2 + 2.2) \times 77 + (2.2)^2 - 2^2] \text{ cm}^2$$

$$= \frac{44}{7} \times [4.2 \times 77 + (2.2 + 2) \times (2.2 - 2)] \text{ cm}^2$$

$$= \frac{44}{7} \times [4.2 \times 77 + 4.2 \times 0.2] \text{ cm}^2$$

$$= \frac{44}{7} \times 4.2 \times 77 + \frac{44}{7} \times 4.2 \times 0.2 \text{ cm}^2$$

$$= 2032.8 + 5.280 \text{ cm}^2 = 2038.08 \text{ cm}^2$$

Hence, the required total surface area is 2038.08 cm^2.

10. The curved surface area of a 15 cm high cylinder is 2310 cm^2. A wire of diameter 6 mm is wound around it so as to cover it completely. Find the length of the wire.

Sol. Let r be the radius of the wire.

$$\therefore \quad r = \frac{6}{2} \text{ mm} = \frac{3}{10} \text{ cm} = 0.3 \text{ cm}$$

∴ Diameter (thickness) of the wire = 0.6 cm

∴ 0.6 cm of length of the cylinder will be covered by 1 round of wire.

∴ 15 cm of length of the cylinder will be covered by $\frac{15}{0.6}$ = 25 rounds of wire.

Let R be the radius of the cylinder and h be its length.

Then $2\pi Rh = 2310$

$\Rightarrow\quad 2 \times \frac{22}{7} \times R \times 15 = 2310$

$\Rightarrow\quad R = \frac{2310 \times 7}{44 \times 15}$

$= \frac{49}{2}$

$\therefore$ Radius of the base of the cylinder = $\frac{49}{2}$ cm.

Now, the length of 1 round of wire = circumference of the cylinder

$= 2\pi R$

$= 2 \times \frac{22}{7} \times \frac{49}{2}$ cm

= 154 cm

$\therefore$ Length of 1 round of wire = 154 cm = 1.54 m.

$\therefore$ Length of 25 rounds of wire = 1.54 × 25 m

= 38.5 m

Hence, the required length of the wire is 38.5 m.

Milestone 2

(Page 182)

Multiple-Choice Questions

1. The total surface area of a cube is 150 cm^2. Then the volume of the cube is

(*a*) 100 cm^3 (*b*) 125 cm^3

(*c*) 96 cm^3 (*d*) 150 cm^3

Sol. (*b*) 125 cm^3

Let a be the side of the cube.

Then $6a^2 = 150\ cm^2$

$\Rightarrow\quad a^2 = 25\ cm^2$

$\Rightarrow\quad a = 5$ cm

$\therefore$ Required volume of the cube = a^3

$= 5^3\ cm^3$

$= 125\ cm^3$

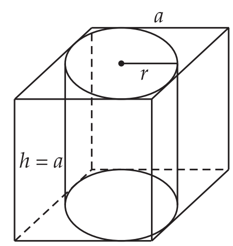
2. A cylindrical piece of maximum volume has to be cut out of an iron cube of edge 6 cm. Then the maximum volume of the iron cylinder is

(*a*) $36\pi\ cm^3$ (*b*) $81\pi\ cm^3$

(*c*) $27\pi\ cm^3$ (*d*) $54\pi\ cm^3$

Sol. (*d*) $54\pi\ cm^3$

Let r be the radius of the cylinder and a be the edge of the cube.



Then for maximum volume of the cylinder

$a = 2r$

$\Rightarrow\quad 6\text{ cm} = 2r$

$\Rightarrow\quad r = 3$ cm

If h be the height of the cylinder, then

$h = a = 6$ cm

$\therefore$ Required volume of the cylinder

$= \pi r^2 h$

$= \pi \times 3^2 \times 6\ cm^3$

$= 54\pi\ cm^3$

Very Short Answer Type Questions

3. If the volume of a sphere is numerically equal to its surface area then what is the diameter of the sphere?

Sol. If r units be the radius of the sphere, then

$\frac{4}{3}\pi r^3 = 4\pi r^2$

$\Rightarrow\quad r = 3$

$\therefore\quad 2r = 6$

Hence, the required diameter of the sphere is 6 units.

4. The radii of a solid sphere and a solid cylinder are respectively 3 cm and 2 cm. How many such spheres can be moulded to form such a cylinder, if the height of the cylinder is 54 cm?

Sol. Let r and R be the radii of the sphere and the cylinder respectively and let h be the height of the cylinder.

Then r = 3 cm, R =2 cm and h = 54 cm.

Then the volume of the cylinder V_1

$= \pi R^2 h$

$= \pi \times 4 \times 54\ cm^3$

$= 216\pi\ cm^3$

Also, the volume of each sphere = $\frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \pi \times 3^3\ cm^3$

$= 36\pi\ cm^3$

$\therefore$ Required number of spheres = $\frac{216\pi}{36\pi} = 6$

Short Answer Type-I Questions

5. A cube and a sphere have equal surface areas. Prove that their volumes are in the ratio $1 : \sqrt{\frac{\pi}{6}}$.

Sol. Let a and r be the side of the cube and the radius of the sphere respectively. Then the total surface area of the cube = $6a^2$ and that of the sphere = $4\pi r^2$

$\therefore$ According to the problem, we have

$$6a^2 = 4\pi r^2$$

$$\Rightarrow \quad \frac{a^2}{r^2} = \frac{4\pi}{6} = \frac{2\pi}{3} \quad \ldots(1)$$

Now, the volume V_1 of the cube = a^3

and the volume V_2 of the sphere = $\frac{4}{3}\pi r^3$.

$$\therefore \quad \frac{V_1}{V_2} = \frac{a^3}{\frac{4}{3}\pi r^3}$$

$$= \frac{3}{4\pi} \times \left(\frac{a}{r}\right)^3$$

$$= \frac{3}{4\pi} \times \frac{a^2}{r^2} \times \frac{a}{r}$$

$$= \frac{3}{4\pi} \times \frac{2\pi}{3} \times \sqrt{\frac{2\pi}{3}} \quad \text{[From (1)]}$$

$$= \sqrt{\frac{2\pi}{3 \times 4}} = \sqrt{\frac{\pi}{6}}$$

$\therefore$ Required ratio = $\sqrt{\frac{\pi}{6}} : 1$.

6. The diameter of a copper sphere is 6 cm. The sphere is melted and drawn into a long wire of uniform thickness. If the length of the wire is 36 cm, find its radius.

Sol. Let R be the radius of the sphere. Then R = 3 cm. Let r be the radius of the cylindrical wire and l be the length of the wire. Then l = 36 cm.

Now, volume of the sphere = $\frac{4}{3}\pi R^3$

$$= \frac{4}{3} \times \pi \times 3^3 \text{ cm}^3$$

$$= 36\pi \text{ cm}^3$$

Volume of the wire = $\pi r^2 l$

$$= \pi \times 36r^2 \text{ cm}^3$$
$$= 36\pi r^2 \text{ cm}^3$$

$$\therefore \quad 36\pi r^2 = 36\pi$$

$$\Rightarrow \quad r^2 = 1$$

$$\Rightarrow \quad r = 1$$

Hence, the required length of the wire is 1 cm.

Short Answer Type-II Questions

7. A conical vessel of base radius 9 cm and height 20 cm is full of water. A part of this water is now poured into a hollow cylinder, closed at one end, till the cylinder is completely filled with water. If the base radius and the height of the cylinder are 6 cm and 10 cm respectively, find the volume of water which is left in the cone. (Take π = 3.14)

Sol. Let R and r be the radii of the bases of the cone and cylinder respectively and let H and h be the corresponding vertical height of the cone and the cylinder respectively.

Then R = 9 cm, H = 20 cm, r = 6 cm and h = 10 cm

$\therefore$ Volume V_1 of the cone = $\frac{1}{3}\pi R^2 H$

$$= \frac{1}{3} \times \pi \times 9^2 \times 20 \text{ cm}^3$$

$$= 540\pi \text{ cm}^3$$

Volume V_2 of the cylinder = $\pi r^2 h$

$$= \pi \times 6^2 \times 10 \text{ cm}^3$$
$$= 360\pi \text{ cm}^3$$

$\therefore$ Required volume of water left in the cone

$$= V_1 - V_2$$
$$= (540 - 360)\pi \text{ cm}^3$$
$$= 180 \times 3.14 \text{ cm}^3$$
$$= 565.2 \text{ cm}^3$$

8. The mass of a spherical ball of radius 2 cm is 8 kg. Find the mass of a spherical shell of the same material whose inner and outer radii are 4 cm and 5 cm respectively.

Sol. Let r, r_1, and r_2 be the radii of the spherical ball, the inner shell and the outer shell respectively, where $r_2 > r_1$. Then r = 2 cm, r_1 = 4 cm and r_2 = 5 cm.

Then the volume of the sphere = $\frac{4}{3}\pi r^3$

$$= \frac{4}{3} \times \frac{22}{7} \times 8 \text{ cm}^3$$

Hence, the density, d of the ball

$$= \frac{\text{mass}}{\text{volume}}$$

$$= \frac{8000 \times 3 \times 7}{4 \times 22 \times 8} \text{ g/cm}^3$$

$$= \frac{21 \times 125}{11} \text{ g/cm}^3$$

Now, volume of the spherical shell

$$= \frac{4}{3}\pi\left(r_2^3 - r_1^3\right)$$

$$= \frac{4}{3} \times \frac{22}{7} \times (5^3 - 4^3) \text{ cm}^3$$

$$= \frac{4 \times 22 \times (125 - 64)}{21} \text{ cm}^3$$

$$= \frac{88 \times 61}{21} \text{ cm}^3$$

∴ Required mass of the spherical shell

= Its volume × Its density

$$= \frac{88 \times 61}{21} \times \frac{21 \times 125}{11} \text{ g}$$

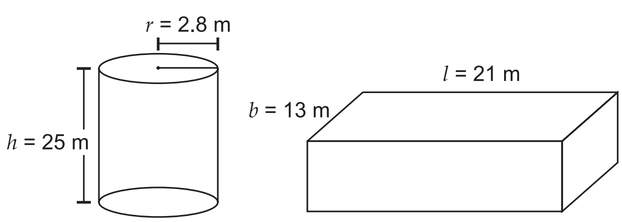
= 61000 g = 61 kg

Long Answer Type Questions

9. A circular well of diameter 5.6 m and depth 25 m is to be dug out and the earth so dug is spread on a rectangular plot of length 21 m and breadth 13 m. Find the volume of the earth dug out, area of the rectangular plot and the height of the platform formed by spreading the earth on the rectangular plot.

Sol. Let r and h be the radius of the cylindrical well so that $r = \frac{5.6}{2}$ m = 2.8 m and h = 25 m.

Let l, b and H be the length, breadth and height respectively of the rectangular plot. Then the volume of the earth dug out = volume of the rectangular plot.



Now, required volume of the earth dug out

$= \pi r^2 h$

$$= \frac{22}{7} \times 2.8 \times 2.8 \times 25 \text{ m}^3$$

$= 8.8 \times 70 \text{ m}^3$

$= 616 \text{ m}^3$ …(1)

Area of the rectangular plot = $l \times b$

$= 21 \times 13 \text{ m}^2$

$= 273 \text{ m}^2$

Volume of the rectangular plot of height H

$= l \times b \times h$

$= 21 \times 13 \times \text{H}$ …(2)

∴ From (1) and (2),

$21 \times 13 \times \text{H} = 616$

$$\Rightarrow \quad \text{H} = \frac{616}{21 \times 31}$$

$$= \frac{616}{273} \simeq 2.26 \text{ (approx.)}$$

Hence, the required volume of the earth dug out is 616 m^3, area of the rectangular plot is 273 m^2 and height of the platform formed is 2.26 m (approx.)

10. The difference between the outside and the inside surfaces of a cylindrical metallic pipe 21 cm long is 66 cm^2. If the pipe is made of 214.5 cm^3 of metal, find the inner and the outer radii of the pipe.

Sol. Let r and R be the inner and the outer radii respectively of the cylindrical pipe of height h. Then h = 21 cm.

Then the total curved surface area of the pipe

$= 2\pi(\text{R} - r)h$

$$= 2 \times \frac{22}{7} \times (\text{R} - r) \times 21 \text{ cm}^2$$

$= 132(\text{R} - r) \text{ cm}^2$

∴ $132(\text{R} - r) = 66$

$$\Rightarrow \quad \text{R} - r = \frac{66}{132} = \frac{1}{2} \quad \text{…(1)}$$

Now, the volume of the pipe

$= \pi(\text{R}^2 - r^2)h$

$= \pi(\text{R} - r)(\text{R} + r)h$

$$= \frac{22}{7} \times (\text{R} + r) \times \frac{1}{2} \times 21 \text{ cm}^3$$

$= 33(\text{R} + r) \text{ cm}^3$

∴ $33(\text{R} + r) = 214.5$

$$\Rightarrow \quad \text{R} + r = \frac{2145}{330} = \frac{13}{2} \quad \text{…(2)}$$

∴ Adding (1) and (2), we get

$$2\text{R} = \frac{14}{2} = 7$$

$$\Rightarrow \quad \text{R} = \frac{7}{2} = 3.5$$

∴ From (2), $r = \frac{13}{2} - \text{R}$

$$= \frac{13}{2} - \frac{7}{2} = 3$$

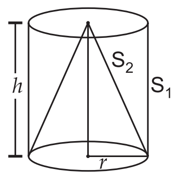
Hence, the required inner and outer radii of the pipe are 3 cm and 3.5 cm respectively.

Higher Order Thinking Skills (HOTS) Questions

(Page 183)

1. A right circular cylinder and a right circular cone have equal bases and equal heights. If their curved surfaces are in the ratio 8 : 5, show that the ratio of the base to the height is 3 : 4 for each of them.

Sol. Let r and h be the common radius of the bases and the height respectively of the cylinder and the cone and let S_1 and S_2 be the curved surface areas of the cylinder and the cone respectively. Let l be the slant height of the cone.



Then $l = \sqrt{r^2 + h^2}$, $S_1 = 2\pi rh$

and $S_2 = \pi rl = \pi r\sqrt{r^2 + h^2}$

It is given that

$$\frac{S_1}{S_2} = \frac{8}{5}$$

$$\therefore \quad \frac{2\pi rh}{\pi r\sqrt{r^2 + h^2}} = \frac{8}{5}$$

$$\Rightarrow \quad \frac{h}{\sqrt{r^2 + h^2}} = \frac{4}{5}$$

$$\Rightarrow \quad \frac{h^2}{r^2 + h^2} = \frac{16}{25}$$

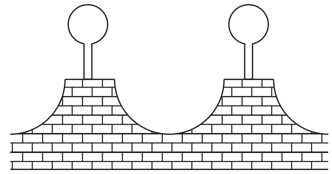
$$\Rightarrow \quad 25h^2 = 16r^2 + 16h^2$$

$$\Rightarrow \quad 9h^2 = 16r^2$$

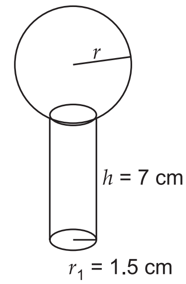
$$\Rightarrow \quad \frac{r}{h} = \frac{3}{4}$$

Hence, the ratio of the base and the height is 3 : 4.

2. The front compound wall of a house is decorated by wooden spheres of diameter 21 cm, placed on small supports as shown in the following figure. Eight such spheres are used for this purpose and are to be painted silver. Each support is a cylinder of radius 1.5 cm and height 7 cm and is to be painted black. Find the cost of paint required if silver paint costs 25 paise per cm^2 and black paint costs 5 paise per cm^2. **[CBSE SP 2012]**



Sol. Let r be the radius of each wooden sphere and r_1, h be the radius and height of each cylinder respectively.



Then $h = 7$ cm, $r_1 = 1.5$ cm, $r = \frac{21}{2}$ cm $= 10.5$ cm

Area of the upper circular end of each cylinder

$$= \pi r_1^2 = \pi \times 1.5^2 \text{ cm}^2$$

$\therefore$ Surface area of each sphere excluding the area of the upper end of the cylinder

$$= (4\pi r^2 - \pi \times 1.5^2) \text{ cm}^2$$
$$= \pi(4 \times 10.5^2 - 1.5^2) \text{ cm}^2$$
$$= \pi(21^2 - 2.25) \text{ cm}^2$$
$$= \frac{22}{7} \times (441 - 2.25) \text{ cm}^2$$
$$= \frac{22}{7} \times 438.75 \text{ cm}^2$$

$\therefore$ Total surface area of 8 such spheres

$$= \frac{22}{7} \times 438.75 \times 8 \text{ cm}^2$$
$$= \frac{22 \times 3510}{7} \text{ cm}^2$$

$\therefore$ The cost of the silver paint

$$= ₹\,0.25 \times \frac{22 \times 3510}{7}$$
$$= ₹\,\frac{5.5 \times 3510}{7}$$
$$= ₹\,\frac{19305}{7}$$
$$= ₹\,2757.86 \qquad \ldots(1)$$

Now, the total curved surface area of 8 cylinders

$$= 8 \times 2\pi r_1 h$$
$$= 16 \times \frac{22}{7} \times \frac{3}{2} \times 7 \text{ cm}^2$$
$$= 528 \text{ cm}^2$$

$\therefore$ The cost of the black paint

$$= ₹\, \frac{5}{100} \times 528$$

$$= ₹\, 26.40 \qquad \ldots(2)$$

$\therefore$ Required total cost = ₹ (2757.86 + 26.40)

[From (1) and (2)]

$$= ₹\, 2784.26 \text{ (approx.)}$$

3. A cylindrical vessel, open at one end, is made of copper sheet. The area of the inner surface is 148.5 dm^2 and the area of the base is 38.5 dm^2. How much cubic dm of milk will the vessel hold?

Sol. Let r and h the radius of the base and the height of the cylinder respectively.

Then $2\pi rh = 148.5$

$$\Rightarrow \quad rh = \frac{148.5 \times 7}{2 \times 22} = \frac{94.5}{4} \qquad \ldots(1)$$

Also, $\pi r^2 = 38.5 \text{ dm}^2$

$$\Rightarrow \quad r^2 = \frac{38.5 \times 7}{22} \text{ dm}^2$$

$$= \frac{24.5}{2} \text{ dm}^2 = 12.25 \text{ dm}^2$$

$$\therefore \quad r = \sqrt{12.25} \text{ dm} = 3.5 \text{ dm} \qquad \ldots(2)$$

$\therefore$ From (1) and (2),

$$h = \frac{94.5}{4} \times \frac{1}{3.5} \text{ dm}$$

$$= \frac{94.5}{14} \text{ dm} = \frac{13.5}{2} \text{ dm}$$

$\therefore$ The required volume of the milk

$$= \pi r^2 h$$

$$= \frac{22}{7} \times 12.25 \times \frac{13.5}{2} \text{ dm}^3$$

$$= 259.875 \text{ dm}^3$$

Self-Assessment

(Page 183)

Multiple-Choice Questions

1. The total surface area of a cone whose radius is $\frac{r}{2}$ and slant height is $2l$ is

(*a*) $2\pi rl$ (*b*) $\pi r(l + r)$

(*c*) $\pi r\left(l + \frac{r}{4}\right)$ (*d*) $2\pi r\,(l + r)$

Sol. (*c*) $\pi r\left(l + \frac{r}{4}\right)$

Let R be the radius of the base of the cone and L be its slant height. Then $R = \frac{r}{2}$ and $L = 2l$.

$\therefore$ Total surface area of the cone

$$= \pi R^2 + \pi RL$$

$$= \pi \times \frac{r^2}{4} + \pi \times \frac{r}{2} \times 2l$$

$$= \pi r\left(l + \frac{r}{4}\right)$$

2. If the slant height and the vertical height of a cone are 5 cm and 4 cm respectively, then the volume of the cone is

(*a*) 12π cm^3 (*b*) 13π cm^3

(*c*) 14π cm^3 (*d*) 15π cm^3

Sol. (*a*) 12π cm^3

Let r be the radius of the base, h be the vertical height and l, the slant height of the cone.

Then $l = 5$ cm and $h = 4$ cm.

Then $l = \sqrt{r^2 + h^2} = \sqrt{r^2 + 16}$

$$\Rightarrow \quad 5^2 = r^2 + 16$$

$$\Rightarrow \quad r^2 = 9$$

$$\Rightarrow \quad r = 3$$

$\therefore$ Required volume of the cone $= \frac{h}{3}\pi r^2$

$$= \frac{4}{3} \times 9\pi \text{ cm}^3$$

$$= 12\pi \text{ cm}^3$$

Fill in the Blanks

3. The volume of a cube whose diagonal is $2\sqrt{3}$ cm is **8 cm³**.

Sol. Diagonal of a cube $= \sqrt{3}$ (Edge)

$$\Rightarrow \quad 2\sqrt{3} \text{ cm} = \sqrt{3} \text{ (Edge)}$$

$$\Rightarrow \quad \text{Edge} = 2 \text{ cm}$$

Volume of a cube = (Edge)^3

$$= (2 \text{ cm})^3$$

$$= 8 \text{ cm}^3$$

4. The surface area of a sphere of radius 3.5 cm is **154 cm²**.

Sol. S.A. of a sphere $= 4\pi r^2$

$$= 4 \times \frac{22}{7} \times 3.5 \times 3.5$$

$$= 4 \times 22 \times 0.5 \times 3.5$$

$$= 154 \text{ cm}^2$$

5. The length of the longest rod that can be placed in a room 12 m long, 9 m broad and 8 m high is **17 m**.

Sol. Length of the longest rod that can be placed in a room = Length of the diagonal of the room

$$= \sqrt{l^2 + b^2 + h^2}$$
$$= \sqrt{(12)^2 + (9)^2 + (8)^2} \text{ m}$$
$$= \sqrt{144 + 81 + 64} \text{ m}$$
$$= \sqrt{289} \text{ m}$$
$$= 17 \text{ m}$$

6. The total surface area of a cone of radius $2r$ and slant height $\frac{1}{2}$ is $\pi r(l + 4r)$.

Sol. Let $R = 2r$ and $L = \frac{l}{2}$

$$\text{T.S.A. of a cone} = \pi R (L + R)$$
$$= \pi(2r)\left(\frac{l}{2} + 2r\right)$$
$$= 2\pi r\left(\frac{l + 4r}{2}\right)$$
$$= \pi r (l + 4r)$$

Assertion-Reason Type Questions

Directions (Q. Nos. 7 to 10): Each of these questions contains an assertion followed by reason. Read them carefully, and answer the question on the basis of the following options, select the one that best describes the two statements.

(a) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.

(b) Both Assertion and Reason are true but Reason is not the correct explanation of Assertion.

(c) Assertion is true but Reason is false.

(d) Assertion is false but Reason is true.

7. **Assertion:** A cube is cut into two halves and volume of each will be exactly half of the original.

Reason: Total volume of the cube remains same.

Sol. (*a*)

Both assertion and reason are correct and reason is correct explanation of assertion.

8. **Assertion:** Total surface area of a cube is 1.5 times the lateral surface area.

Reason: Lateral surface area is half of the remaining surface area.

Sol. (*c*)

Lateral surface area = $4a^2$

and total surface area = $6a^2$

$\therefore \quad 6a^2 = 1.5(4a^2)$

Hence, assertion is correct but reason is incorrect.

9. **Assertion:** The height of a cylinder is doubled, so its curved surface area will also be doubled.

Reason: The radius of a cylinder is half of its diameter.

Sol. (*b*)

Let curved surface area of cylinder with height $h = \pi r^2 h$

If height is doubled, then curved surface area = $2\pi r^2 h$

Also, radius is half of diameter. Thus, both assertion and reason are correct but reason is not correct explanation of assertion.

10. **Assertion:** The surface area of a sphere is doubled if its radius is doubled.

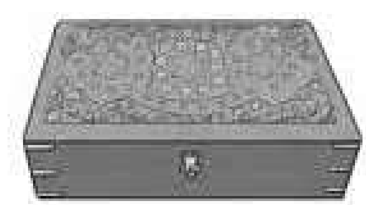
Reason: The surface area of the sphere is $4\pi r^2$.

Sol. (*d*)

Assertion is not correct since if its radius is doubled, then surface area will become four times but reason is correct.

Case Study Based Questions

11. Garima planned to make a makeup box to gift her friend Jyoti on her birthday. She made the wooden makeup box in the shape of a cuboid. The outer dimensions of wooden makeup box are 24 cm by 20 cm by 16 cm. Thickness of the wood is 1 cm. Based on the above information, answer the following questions.



(*a*) What is the volume of the outer wooden box?

(*i*) 768 cm^3 (*ii*) 3840 cm^3
(*iii*) 7680 cm^3 (*iv*) 15360 cm^3

Ans. (*iii*) 7680 cm^3

(*b*) What is the length of the inner wooden box?

(*i*) 22 cm (*ii*) 23 cm
(*iii*) 24 cm (*iv*) 25 cm

Ans. (*i*) 22 cm

(c) What is the height of the inner wooden box?

(i) 14 cm (ii) 15 cm

(iii) 16 cm (iv) 18 cm

Ans. (i) 14 cm

(d) What is the volume of inner wooden box?

(i) 2211 cm^3 (ii) 3322 cm^3

(iii) 4433 cm^3 (iv) 5544 cm^3

Ans. (iv) 5544 cm^3

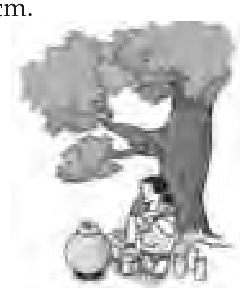
(e) What is the total cost of wood required to make the box if 1 cm^3 of wood cost ₹ 2.

(i) ₹ 2136 (ii) ₹ 4272

(iii) ₹ 11088 (iv) ₹ 15360

Ans. (ii) ₹ 4272

12. On a hot summer day, Sheila set-up a stall near her house in the village under the shade of a tree. She took cold water in a completely full spherical matka (an earthen pot) of radius 39 cm and served this water to thirsty people passing by in cylindrical glasses each of radius 4 cm and height 13 cm.



Based on the above information, answer the following questions.

(a) What is the volume of the spherical matka?

(i) 32000π cm^3 (ii) 55662π cm^3

(iii) 79092π cm^3 (iv) 93521π cm^3

Ans. (iii) 79092π cm^3

(b) Which mathematical concept is used in the above situation?

(i) Probability

(ii) Linear equations in two variables

(iii) Polynomials

(iv) Surface areas and Volumes

Ans. (iv) Surface areas and Volumes

(c) What is the volume of each cylindrical glass?

(i) 158π cm^3 (ii) 198π cm^3

(iii) 208π cm^3 (iv) 258π cm^3

Ans. (iii) 208π cm^3

(d) If she fills each glass up to three-fourths of its height with water, find the number of people she can give water to.

(i) 407 (ii) 427

(iii) 507 (iv) 527

Ans. (iii) 507

(e) What is the formula to calculate the volume of a cylinder?

(i) $\frac{1}{3}\pi r^2 h$ (ii) $\frac{2}{3}\pi r^3$

(iii) $\pi r^2 h$ (iv) $\frac{4}{3}\pi r^3$

Ans. (iii) $\pi r^2 h$

Very Short Answer Type Questions

13. A hollow cylinder open at one end, with base radius r and height h contains the same volume of water which is three times the volume of a hollow cone of the same base radius and same height. Is this statement true or false? State with reason.

Sol. We have

Volume (V) of the cylinder = $\pi r^2 h$ ...(1)

and volume (V_1) of the cone = $\frac{1}{3}\pi r^2 h$

$\therefore$ $3V_1 = \pi r^2 h = V$ [From (1)]

$\therefore$ Volume of the cylinder is three times the volume of the cone. Hence, the given statement is true.

14. The circumference of the base of a 27 m high solid cone is 44 m. Find the volume of the cone.

Sol. Let r be the radius of the base of the cone and h be its vertical height.

$\therefore$ $h = 27$ m

Also, $2\pi r = 44$ m

$\Rightarrow$ $r = \frac{22}{\pi}$ m ...(1)

$\therefore$ Required volume $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \pi \times \frac{22 \times 22}{\pi \times \pi} \times 27$ m^3

$= \frac{22 \times 22 \times 9 \times 7}{22}$ m^3

$= 1386$ m^3

Short Answer Type-I Questions

15. The ratio of the heights of a cone and a cylinder is 2 : 1 and the ratio of the radii of their bases is 3 : 1 respectively. Find the ratio of their volumes.

Sol. Let H be the vertical height of the cone and h be the height of the cylinder and let R be the radius of the base of the cone and r be the radius of the cylinder.

Then $\quad H : h = 2 : 1 \quad$ [Given] ...(1)

and $\quad R : r = 3 : 1 \quad$ [Given] ...(2)

If V_1 and V_2 be the volumes of the cone and the cylinder respectively then

$$V_1 = \frac{1}{3}\pi R^2 H$$

and $$V_2 = \pi r^2 h$$

$$\therefore \quad \frac{V_1}{V_2} = \frac{\frac{1}{3}\pi R^2 H}{\pi r^2 h}$$

$$= \frac{1}{3} \times \left(\frac{R}{r}\right)^2 \times \frac{H}{h}$$

$$= \frac{1}{3} \times 9 \times 2 \text{ [From (1) and (2)]}$$

$$= \frac{6}{1}$$

$\therefore$ $V_1 : V_2 = 6 : 1$ which is the required ratio.

16. A cone, a hemisphere and a cylinder stand on equal bases and have the same height equal to the radius of the bases of these solids. Show that their volumes are in the ratio 1 : 2 : 3.

Sol. Let the common radius of the bases of the cone, cylinder and hemisphere be r and the common vertical heights of the cone and the cylinder be h. Let V_1, V_2 and V_3 be the volumes of the cone, the hemisphere and the cylinder respectively.

Then $$V_1 = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi r^3$$

$$V_2 = \frac{2}{3}\pi r^3$$

$$V_3 = \pi r^2 \times r = \pi r^3$$

$$\therefore \quad V_1 : V_2 : V_3 = \frac{1}{3} : \frac{2}{3} : 1 = 1 : 2 : 3$$

Hence, proved.

Short Answer Type-II Questions

17. If the height of a right circular cone is double its diameter, then what is the ratio of its slant height to its radius?

Sol. Let r be the radius of the base and h be the vertical height of the cone. Given that $h = 4r$.

Let l be the slant height of the cone.

Then $$l = \sqrt{h^2 + r^2}$$

$$= \sqrt{16r^2 + r^2}$$

$$= \sqrt{17}\, r$$

$\therefore$ $l : r = \sqrt{17} : 1$ which is the required ratio.

18. Find the volume of a sphere whose surface area is 616 cm^2.

Sol. Let r be the radius of the sphere.

Then its surface area = $4\pi r^2$

$\therefore \quad 4\pi r^2 = 616 \text{ cm}^2$ [Given]

$$\Rightarrow \quad r^2 = \frac{616 \times 7}{4 \times 22} \text{ cm}^2 = 49 \text{ cm}^2$$

$\therefore \quad r = 7$ cm

$\therefore$ Required volume of the sphere

$$= \frac{4}{3}\pi r^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times 7 \times 7 \times 7 \text{ cm}^3$$

$$= \frac{4312}{3} \text{ cm}^3$$

19. Read the following passage and answer the questions that follows:

On a hot summer day in Rajasthan, a poor woman named Aasha set-up a stall near her house in the village under the shade of a big Banyan tree. She bought seven earthen pots, each of radius 40 cm. She took cold water in all the seven earthen pots and served this water to thirsty people passing by in cylindrical glasses each of radius 5 cm and height 14 cm. Each earthen pot was full up to three-fourths of its height.

(*a*) If Aasha fills each glass completely with water, find the number of people she can give water to.

(*b*) What is the volume of each cylindrical glasses?

Sol. (*b*) Let r and h be the radius and height of each cylindrical glass. Then $r = 5$ cm and $h = 14$ cm.

$\therefore$ Volume of each glass

$$= \pi r^2 h$$

$$= \frac{22}{7} \times 25 \times 14 \text{ cm}^3$$

$$= 1100 \text{ cm}^3$$

(*a*) Let R be the radius of each spherical earthen pot.

Then R = 40 cm.

$\therefore$ Volume of 7 spherical earthen pots

$$= 7 \times \frac{4}{3}\pi R^3$$

$$= 7 \times \frac{4}{3} \times \frac{22}{7} \times 40^3 \text{ cm}^3$$

$$= \frac{88 \times 40^3}{3} \text{ cm}^3$$

Then the total volume of water in 7 earthen pots each of which is full upto $\frac{3}{4}$ th of its height

$$= \frac{88 \times 40^3}{3} \times \frac{3}{4} \text{ cm}^3$$

$$= 22 \times 40^3 \text{ cm}^3$$

∴ Required number of glasses

$$= \frac{22 \times 40 \times 40 \times 40}{1100} \text{ [Using } (b)]$$

$$= 1280$$

Long Answer Type Questions

20. Find the mass of a hollow sphere of metal having internal and external diameters of 20 cm and 22 cm respectively, given 1 cm^3 of metal has a mass of 21 g.

Sol. Let R and r be the external and internal radii of the hollow sphere. Then R = 11 cm and r = 10 cm.

∴ Volume of the sphere

$$= \frac{4}{3}\pi(R^3 - r^3)$$

$$= \frac{4}{3} \times \frac{22}{7}(11^3 - 10^3) \text{ cm}^3$$

$$= \frac{88}{21} \times (1331 - 1000) \text{ cm}^3$$

$$= \frac{88}{21} \times 331 \text{ cm}^3$$

∴ Required mass $= \frac{88}{21} \times 331 \times 21$ g

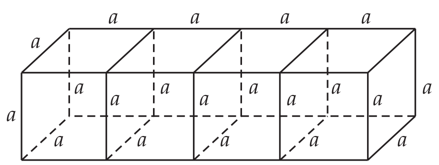
$$= 29128 \text{ g}$$

$$= 29.128 \text{ kg}$$

21. Four equal cubes are placed side by side in a row to form a cuboid. Find the ratio of the total surface area of the cuboid to the total surface area of the four cubes.

Sol. Let a be the side of the cube and l, b and h be the length, breadth and height of the cuboid respectively.

Then $l = 4a$, $b = a$ and $h = a$.



∴ Total surface area of the cuboid

$$= 2(l \times b + l \times h + b \times h)$$

$$= 2(4a \times a + 4a \times a + a \times a)$$

$$= 18a^2$$

Total surface area of the 4 cubes

$$= 4 \times 6a^2$$

$$= 24a^2$$

∴ Required ratio $= 18a^2 : 24a^2 = 3 : 4$

Let's Compete

(Page 185)

Multiple-Choice Questions

1. If the radius, r of a sphere is reduced to its half, then the new volume would be

(*a*) $\frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$ (*b*) $\frac{4\pi}{3}\left(\frac{r^3}{8}\right)$

(*c*) $\frac{4}{3}\pi\frac{r^3}{2}$ (*d*) $\frac{4}{6}\pi\frac{r^3}{8}$

Sol. (*b*) $\frac{4\pi}{3}\left(\frac{r^3}{8}\right)$

If V and V′ be the volumes of the original sphere and the reduced sphere respectively, then

$$V = \frac{4}{3}\pi r^3 \text{ and } V' = \frac{4}{3}\pi\left(\frac{r}{2}\right)^3 = \frac{4}{3}\pi\left(\frac{r^3}{8}\right)$$

2. The difference between the total surface area of a cube of side 5 cm and its lateral surface area is

(*a*) 20 cm^2 (*b*) 25 cm^2

(*c*) 40 cm^2 (*d*) 50 cm^2

Sol. (*d*) 50 cm^2

If a be the side of the cube, then its total surface area = $6a^2$ and the lateral surface area = $4a^2$.

∴ Difference between these two areas

$$= 6a^2 - 4a^2$$

$$= 2a^2$$

$$= 2 \times 25 \text{ cm}^2$$

$$= 50 \text{ cm}^2 \quad [\because a = 5 \text{ cm}]$$

3. The length, breadth and height of a cuboid are 5 cm, 4 cm and 3 cm respectively. Then the total surface area of a cube with side equal to the length of the diagonal of the cuboid is

(*a*) 200 cm^2 (*b*) 250 cm^2

(*c*) 300 cm^2 (*d*) 350 cm^2

Sol. (*c*) 300 cm^2

Let l, b and h be the length, breadth and height of the cuboid respectively. Then l = 5 cm, b = 4 cm and h = 3 cm.

If d be the length of its diagonal then

$$d = \sqrt{l^2 + b^2 + h^2}$$
$$= \sqrt{5^2 + 4^2 + 3^2} \text{ cm}$$
$$= \sqrt{50} \text{ cm}$$
$$= 5\sqrt{2} \text{ cm}$$

$\therefore$ The required total surface area of the cube of side $5\sqrt{2}$ cm is $6 \times \left(5\sqrt{2}\right)^2 \text{ cm}^2 = 6 \times 50 \text{ cm}^2 = 300 \text{ cm}^2$.

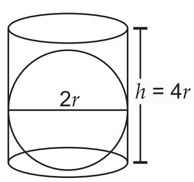
4. From a solid cylinder of base radius r cm, certain mass in the shape of the largest sphere is scooped out. If the height of the cylinder is twice its diameter, then the volume of the remaining part of the cylinder is

(*a*) $\frac{8\pi r^3}{3}$ cm^3 (*b*) $\frac{4\pi r^3}{3}$ cm^3

(*c*) $\frac{16\pi r^3}{3}$ cm^3 (*d*) $4\pi r^3$ cm^3

Sol. (*a*) $\frac{8\pi r^3}{3}$ cm^3

Let h be the height of the cylinder. Now the volume of the sphere which is scooped out from the cylinder will be highest only when the diameter of the cylinder is equal to the diameter of the sphere.



Then $h = 4r$ [Given]

$\therefore$ Volume of the sphere $= \frac{4}{3}\pi r^3$

Volume of the cylinder $= \pi r^2 h$
$= \pi r^2 \times 4r \text{ cm}^3$
$= 4\pi r^3 \text{ cm}^3$

$\therefore$ Required volume of the remaining part of the cylinder is $\left(4\pi r^3 - \frac{4}{3}\pi r^3\right)$cm^3, i.e. $\frac{8}{3}\pi r^3$ cm^3.

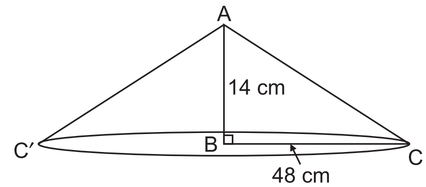
5. ABC is a right-angled triangle with AB = 14 cm, BC = 48 cm and $\angle$ABC = 90°. This triangle is turned around the shorter side so as to form a cone of volume V_1 cm^3. The triangle is now turned around the longer side to form another cone of volume V_2 cm^3. Then the $V_1 : V_2$ is equal to

(*a*) 1 : 1 (*b*) 24 : 7

(*c*) 7 : 24 (*d*) 12 : 7

Sol. (*b*) 24 : 7

When the triangle ABC is revolved about the shorter side AB, then the cone ACC′ is generated. Let r_1 and h_1 be the radius of the base and the vertical height of this cone respectively, then $r_1 = 48$ cm, $h_1 = 14$ cm.

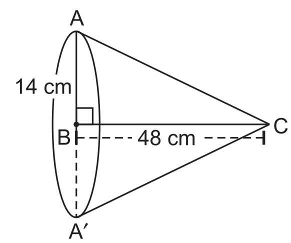


Let V_1 be the volume of this cone.

Then $V_1 = \frac{1}{3}\pi r_1^2 h_1$

$= \frac{1}{3}\pi \times 48^2 \times 14 \text{ cm}^3$

When the triangle ABC is revolved about the longer side BC then the cone ACA′ is generated. Let r and h be the radius of the base and the vertical height of this cone respectively.



Then $r = 14$ cm, $h = 48$ cm.

Let V_2 be the volume of this cone.

Then $V_2 = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi \times 14^2 \times 48 \text{ cm}^3$

$$\therefore \quad \frac{V_1}{V_2} = \frac{\frac{1}{3}\pi \times 48 \times 48 \times 4}{\frac{1}{3}\pi \times 14 \times 14 \times 48} = \frac{24}{7}$$

$\therefore$ $V_1 : V_2 = 24 : 7$

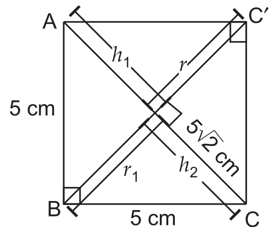
6. An isosceles right-angled triangle, each equal side of length 5 cm, is turned around the hypotenuse so as to form a double cone. The volume of this combined cone is

(*a*) $\frac{125\sqrt{2}}{3}$ cm^3 (*b*) $\frac{125\sqrt{2}\pi}{3}$ cm^3

(*c*) $\frac{125\sqrt{2}}{6}$ cm^3 (*d*) $\frac{125\sqrt{2}\pi}{6}$ cm^3

Sol. (*d*) $\frac{125\sqrt{2}\pi}{6}$ cm^3

Let ABC be an isosceles right-angled triangle with AB = BC = 5 cm and ∠ABC = 90°.



When it is revolved about the hypotenuse AC, double cones AC′B and CC′B arc generated where BC′ ⊥ AC.

Let r be the common radius of the base of the two cones and let h_1 and h_2 be the vertical heights of the cones ABC′ and CBC′ respectively.

Then $\quad h_1 = h_2 = \frac{1}{2}\,AC = \frac{5\sqrt{2}}{2} = \frac{5}{\sqrt{2}}$

If O is the mid-point of AC, then AO = OC = $\frac{5}{\sqrt{2}}$.

∵ $\quad \angle AOB = 90°$

∴ By Pythagoras' Theorem, we have

$$AO^2 + OB^2 = AB^2$$

$$\Rightarrow \quad \frac{25}{2} + r^2 = 5^2 = 25$$

$$\therefore \quad r^2 = 25 - \frac{25}{2} = \frac{50-25}{2} = \frac{25}{2}$$

$$\therefore \quad r = \frac{5}{\sqrt{2}}$$

∴ Volume of each cone = $\frac{1}{3}\pi r^2 h_1$

$$= \frac{1}{3} \times \frac{25}{2} \times \frac{5}{\sqrt{2}} \text{ cm}^3$$

$$= \frac{125}{6\sqrt{2}}\pi \text{ cm}^3$$

∴ The required volume of the combined cone

$$= 2 \times \frac{125\pi}{6\sqrt{2}}$$

$$= \frac{125\sqrt{2}\pi}{6} \text{ cm}^3$$

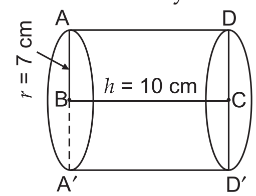
7. A rectangle of dimensions 7 cm × 10 cm is revolved around the longer side so as to form a right circular cylinder. The curved surface area of this cylinder is

(*a*) 440 cm^2 (*b*) 594 cm^2

(*c*) 154 cm^2 (*d*) 400 cm^2

Sol. (*a*) 440 cm^2

Let AB = 7 cm and BC = 10 cm are the sides of the rectangle ABCD. It is revolved about the longer side BC so as to form a cylinder AA′D′D.



Let r and h be the radius and the height of the cylinder respectively.

Then r = 7 cm and h = 10 cm.

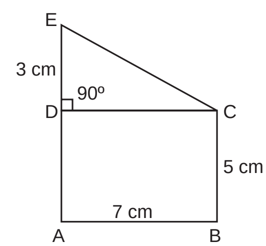
∴ Required curved surface area of the cylinder

$$= 2\pi rh$$

$$= 2 \times \frac{22}{7} \times 7 \times 10 \text{ cm}^2$$

$$= 440 \text{ cm}^2$$

8. ABCD is a rectangle with AB = 7 cm and BC = 5 cm and ΔEDC is a right-angled triangle where E is a point on AD produced such that DE = 3 cm and ∠EDC = 90° as shown in the figure. The combined plane figure ABCE is now revolved around AE so as to form a combined solid figure. The volume of this combined solid is

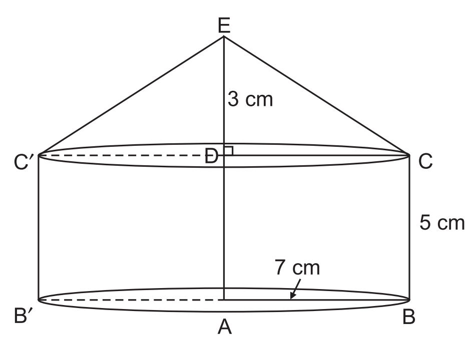


(*a*) 850 cm^3 (*b*) 924 cm^3

(*c*) 900 cm^3 (*d*) 890 cm^3

Sol. (*b*) 924 cm^3

Let ABCD be the rectangle with AB = 7 cm, BC = 5 cm and E is a point on AD produced such that ED = 3 cm and ∠EDC = 90°.



If the combined figure ABCE is revolved about AE, then one cone ECC′ and one cylinder CB B′C′ are generated. It r and R be the radii of this cone and this cylinder respectively, h and H be the vertical height of the cone and the cylinder respectively.

Then $r = 7$ cm, $R = 7$ cm, $h = 3$ cm and $H = 5$ cm

∴ Volume of the cone

$$= \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\pi \times 7^2 \times 3 \text{ cm}^3$$

$$= \frac{1}{3} \times \frac{22}{7} \times 7^2 \times 3 \text{ cm}^3$$

$$= 154 \text{ cm}^3$$

and volume of the cylinder

$$= \pi R^2 H$$

$$= \frac{22}{7} \times 7 \times 7 \times 5 \text{ cm}^3$$

$$= 770 \text{ cm}^3$$

∴ Required volume of the combined solid

$$= (154 + 770) \text{ cm}^3$$

$$= 924 \text{ cm}^3$$

9. If each bag containing wheat occupies 2.2 m^3 of space, then the number of full bags which can be emptied into a cylindrical drum of radius 4 m and height 3.5 m is

(*a*) 90
(*b*) 70
(*c*) 80
(*d*) 50

Sol. (*c*) 80

Let r and h be the radius and the height of the cylinder. Then $r = 4$ m and $h = 3.5$ m.

∴ Volume of the cylinder

$$= \pi r^2 h$$

$$= \frac{22}{7} \times 4 \times 4 \times 3.5 \text{ m}^3$$

$$= 176 \text{ m}^3$$

Volume of wheat in each bag = 2.2 m^3

∴ Required number of bags $= \frac{176}{2.2} = 80$

10. A conical pandal 350 m in radius and 120 m high is made of cloth which is 220 m wide. Then the length of the cloth used to make the pandal is

(*a*) 1850 m
(*b*) 1800 m
(*c*) 1875 m
(*d*) 1900 m

Sol. (*a*) 1850 m

Let r and h be the radius of the base and the vertical height of the cone respectively. Let l be its slant height.

Then $r = 350$ m, $h = 120$ m.

$$\therefore \quad l = \sqrt{r^2 + h^2}$$

$$= \sqrt{350^2 + 120^2} \text{ m}$$

$$= \sqrt{136900} \text{ m}$$

$$= 370 \text{ m}$$

∴ Curved surface area of the cone

$$= \pi r l$$

$$= \frac{22}{7} \times 350 \times 370 \text{ m}^2$$

$$= 22 \times 50 \times 370 \text{ m}^2$$

If L be the length of the cloth, then

$$L \times 220 = 22 \times 50 \times 370$$

$$\therefore \quad L = \frac{22 \times 50 \times 370}{220} = 1850$$

Hence, the required length is 1850 m

Value-based Question (Optional)

(Page 186)

1. A conical tent was set-up to accomodate 30 students for a summer camp in which students participated in activities like planting saplings, yoga, cleaning lakes, testing the water for contaminants and pollutant levels and desilt the lake bed and also using the silt to strengthen bunds under the guidance of two teachers who accompanied them, but were staying in a nearby guest house.

(*a*) Find the height of the tent if each student must have 4 m^2 of the space on the ground and 20 m^3 of air to breathe.

(*b*) What values do the participants learn during their stay in such summer camps?

Sol. (*a*) Let r and h be the radius of the base and the vertical height of the conical tent.

For 30 students, the area of the space needed

$$= 30 \times 4 \text{ m}^2$$

$$= 120 \text{ m}^2$$

$$\therefore \quad \pi r^2 = 120 \text{ m}^2$$

$$\Rightarrow \quad r^2 = \frac{120 \times 7}{22} \text{ m}^2 = \frac{420}{11} \text{ m}^2 \quad \ldots(1)$$

Now, the volume of the conical tent

$$= \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3} \times \frac{22}{7} \times \frac{420}{11} \times h \text{ m}^3$$

[From (1)]

$$= 40h \text{ m}^3$$

$$\therefore \quad 40h = 20 \times 30 \quad \text{[Given]}$$

$$\therefore \quad h = \frac{600}{40} = 15$$

Hence, the required height of the tent is 15 m.

(*b*) Environmental protection, awareness about physical fitness, team work, cooperation, creative thinking, waste management problem solving, decision making and to become a responsible citizen.

14

Statistics

Checkpoint (Page 189)

1. The class mark of the class 10 – 20 is
   (*a*) 10 (*b*) 20
   (*c*) 15 (*d*) 5

Sol. (*c*) 15

Required class mark = $\frac{10 + 20}{2} = 15$

2. In the following set of discrete data

   7, 5, 3, 2, 3, 5, 7, 1, 3, 7, 8, 9, 10

   the frequency of 7 is
   (*a*) 7 (*b*) 3
   (*c*) 2 (*d*) 4

Sol. (*b*) 3

Here 7 occurs 3 times only. Hence, the required frequency is 3.

3. The mean of 25 observations is 34. If each observation is increased by 2, what is the new mean?

Sol. We know that if each observation x is increased by a, then the new mean $\bar{x}$ is increased by a and so the new mean = $\bar{x} + a$

∴ The required new mean = 34 + 2 = 36.

4. What is the class size of a class 35 – 40 of some grouped data?

Sol. The required class size is 40 – 35 = 5.

5. If the mean of six observations is 12, the mean of the first four observations is 13 and that of the last three observations is 14, then what is the fourth observation?

Sol. Let x_1, x_2, x_3, x_4, x_5 and x_6 be six observations.

Given that

$$\frac{x_1 + x_2 + x_3 + x_4 + x_5 + x_6}{6} = 12$$

$$\therefore \quad x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 12 \times 6 = 72 \quad \ldots(1)$$

Also, $$\frac{x_1 + x_2 + x_3 + x_4}{4} = 13$$

$$\therefore \quad x_1 + x_2 + x_3 + x_4 = 13 \times 4 = 52 \quad \ldots(2)$$

∴ Subtracting (2) from (1), we get

$$x_5 + x_6 = 72 - 52 = 20 \quad \ldots(3)$$

Again, $$\frac{x_4 + x_5 + x_6}{3} = 14$$

$$\Rightarrow \quad x_4 + x_5 + x_6 = 14 \times 3 = 42 \quad \ldots(4)$$

∴ Subtracting (3) from (4), we get

$$x_4 = 42 - 20 = 22$$

∴ The required fourth observation is 22.

6. The mean of five numbers is 18. If one number is excluded, then the mean of the remaining numbers becomes 16. What is the excluded number?

Sol. Required excluded number is 18 × 5 – 16 × 4
= 90 – 64 = 26.

7. What is the difference between the mean and median of the first five composite numbers?

Sol. The first five composite numbers are 4, 6, 8, 9 and 10.

The mean of these numbers = $\bar{x}$

$$= \frac{4 + 6 + 8 + 9 + 10}{5} = 7.4$$

and the median = 8

∴ Required difference = 8 – 7.4 = 0.6

8. If the mean of the observations $2x$, $3x + 1$, $4x - 5$ and $x + 8$ is 6, then find the value of x.

Sol. We have

$$\text{Mean} = \frac{2x + (3x + 1) + (4x - 5) + (x + 8)}{4}$$

$$= \frac{10x + 4}{4} = 6 \qquad \text{[Given]}$$

$$\Rightarrow \quad 10x = 24 - 4 = 20$$

$$\therefore \quad x = 2$$

$\therefore$ The required value of x is 2.

9. Find the median of 72, – 30, 59, – 7, 8, – 6.

Sol. Writing the given numbers in decreasing order, we have

72, 59, 8, –6, –7, –30.

The number of observation is 6 which is even.

$\therefore$ The mean of $\frac{6}{2}$ th and $\left(\frac{6}{2} + 1\right)$ th observation

= 3rd and 4th observation is the median

$\therefore$ The required median is $\frac{8 - 6}{2}$, i.e. 1.

10. If the mean of x^2 and $\frac{1}{x^2}$ is 17, where $x > 0$, what is the value of $x + \frac{1}{x}$?

Sol. Given that

$$\frac{1}{2}\left(x^2 + \frac{1}{x^2}\right) = 17$$

$$\Rightarrow \quad x^2 + \frac{1}{x^2} = 34$$

$$\Rightarrow \quad \left(x + \frac{1}{x}\right)^2 - 2x \times \frac{1}{x} = 34$$

$$\Rightarrow \quad \left(x + \frac{1}{x}\right)^2 = 34 + 2 = 36$$

$$\therefore \quad x + \frac{1}{x} = \sqrt{36} = 6$$

which is the required value.

Milestone 1

(Page 196)

Multiple-Choice Questions

1. An observation is such that its maximum value is 83 and the range is 30. Then its minimum value is

(*a*) 50 (*b*) 53

(*c*) 55 (*d*) 113

Sol. (*b*) 53

Let x_2 and x_1 be the maximum and minimum values respectively of a variable x. Then $x_2 - x_1$ = range = 30 and $x_2 = 83$

$$\therefore \quad x_1 = 83 - 30 = 53$$

$\therefore$ The required minimum value is 53.

2. One of the sides of a frequency polygon is

(*a*) both the coordinate axes

(*b*) neither of the coordinate axes

(*c*) x-axis only

(*d*) y-axis only

Sol. (*c*) x-axis only

We know that a frequency polygon is a closed curve and its end points lie on the x-axis only. Hence, one of the sides of this polygon must be x-axis only.

Very Short Answer Type Questions

3. Find any class with class size 5 and class mark 53.5.

Sol. Let x_2 and x_1 be the upper limit and the lower limit of a class respectively.

$$\therefore \quad \text{Class size} = x_2 - x_1$$

$$\text{and} \quad \text{class mark} = \frac{x_1 + x_2}{2}$$

$$\therefore \quad \frac{x_1 + x_2}{2} = 53.5$$

$$\Rightarrow \quad x_1 + x_2 = 107 \qquad \ldots(1)$$

$$\text{and} \quad x_2 - x_1 = 5 \qquad \ldots(2)$$

Adding (1) and (2), we get

$$2x_2 = 112$$

$$\Rightarrow \quad x_2 = 56$$

$\therefore$ From (1), $x_1 = 107 - 56 = 51$

$\therefore$ The required class is 51 – 56.

4. If 40 is subtracted from the sum of 25 observations, then the result is 260. What is the mean of these observations?

Sol. Let the 25 observations be $x_1, x_2, x_3 \ldots x_{25}$.

$\therefore$ According to the problem, we have

$$x_1 + x_2 + x_3 + \ldots + x_{25} - 40 = 260$$

$$\Rightarrow x_1 + x_2 + x_3 + \ldots + x_{25} = 40 + 260 = 300 \quad \ldots(1)$$

$$\therefore \text{Required mean} = \frac{x_1 + x_2 + \ldots + x_{25}}{25}$$

$$= \frac{300}{25} \qquad \text{[From (1)]}$$

$$= 12$$

Short Answer Type-I Questions

5. Prepare a discrete frequency distribution table for the following monthly wages in ₹ of 15 workers in a factory.

2000, 1900, 1200, 2500, 1900, 2000, 3000, 1500, 2500, 1900, 1200, 1900, 1500, 1900, 1500

Answer the following questions from the table:

(*a*) What is the range of the wages?

(*b*) What are the monthly wages of maximum number of workers and how many workers get these wages?

(*c*) What are the monthly wages of minimum number of workers(s) and how many workers(s) gets/get these wages?

(*d*) Find the number of workers whose monthly wages are

(*i*) less than ₹ 2000

(*ii*) more than ₹ 2000

(*iii*) lie between ₹ 1000 and ₹ 2100.

Sol. Required discrete frequency distribution table is as follows:

Monthly wages (₹)	*Tally marks*	*Frequency*
1200	||	2
1500	|||	3
1900	~~||||~~	5
2000	||	2
2500	||	2
3000	|	1

(*a*) Required range = ₹ (3000 – 1200) = ₹ 1800

(*b*) Required maximum number of workers who gets the same monthly wages, i.e. ₹ 1900 is 5.

(*c*) Required number of worker(*s*) who get maximum monthly wages, i.e. ₹ 3000 is 1.

(*d*) (*i*) Required number of workers whose monthly wages are less than ₹ 2000 is 5 + 3 + 2 = 10.

(*ii*) Required number of workers whose monthly wages are more than ₹ 2000 is 2 + 1 = 3.

(*iii*) Required number of workers whose monthly wages lie between ₹ 1000 and ₹ 2100 is 2 + 3 + 5 + 2 = 12.

6. The mean of five observations was found to be 36. Later on, it was detected that one observation 15 was misread is 50. Find the correct mean.

Sol. Let the five observations be x_1, x_2, x_3, x_4 and x_5. We assume that the correct value of x_5 was 15 and its wrong value was 50.

∴ According to the problem,

$$\frac{x_1 + x_2 + x_3 + x_4 + 50}{5} = 36$$

$$\Rightarrow \quad x_1 + x_2 + x_3 + x_4 = 5 \times 36 - 50$$

$$= 180 - 50$$

$$= 130 \qquad \ldots(1)$$

∴ Required correct mean

$$= \frac{x_1 + x_2 + x_3 + x_4 + 15}{5}$$

$$= \frac{130 + 15}{5} \qquad \text{[From (1)]}$$

$$= \frac{145}{5} = 29$$

Short Answer Type-II Questions

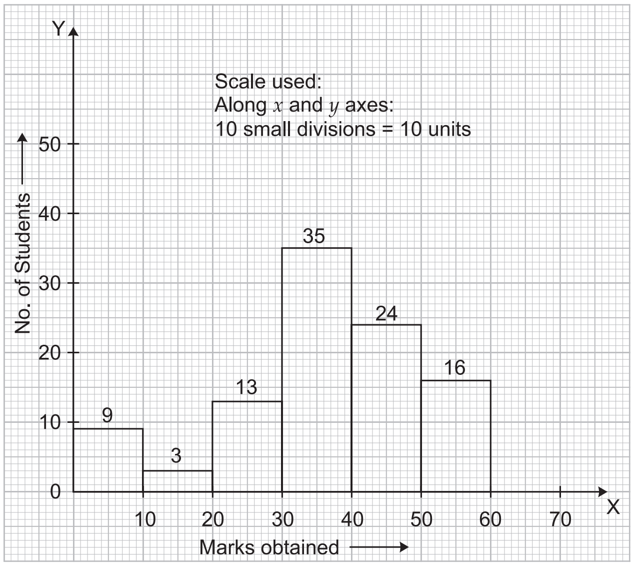
7. Make a normal frequency table from the following and hence draw the histogram.

Marks	*Number of students*
More than 60	0
More than 50	16
More than 40	40
More than 30	75
More than 20	88
More than 10	91
More than 0	100

Sol. The required normal frequency distribution table is as follows:

Marks obtained	*Number of students*
0–10	(100 – 91) = 9
10–20	(91 – 88) = 3
20–30	(88 – 75) = 13
30–40	(75 – 40) = 35
40–50	(40 – 16) = 24
50–60	(16 – 0) = 16
	Total = 100

The required histogram is shown below:



A histogram showing the marks obtained by
a number of students

8. Construct a histogram for the given data and hence, answer the following questions:

Class interval	*Frequency*
1 – 3	24
4 – 6	30
7 – 9	28
10 – 12	41
13 – 15	19
16 – 18	8

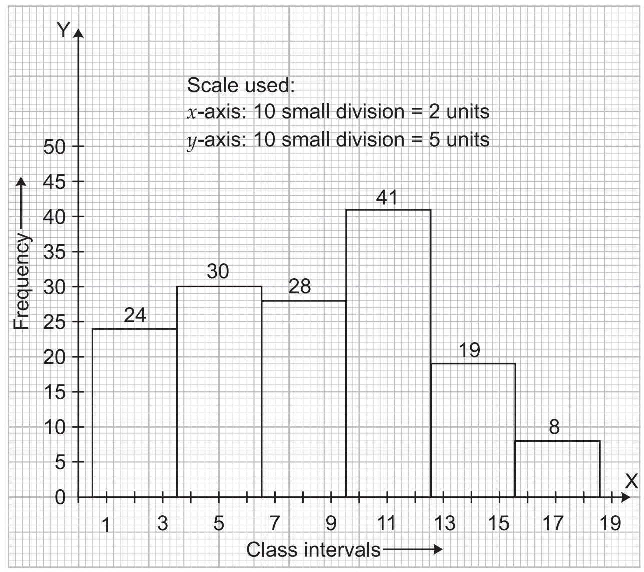
(*a*) What is the total frequency in the table?

(*b*) Which interval of the inclusive form of frequency distribution contains the greatest frequency?

(*c*) How many scores were reported from 10 through 18?

Sol. We see that the data given in the table are in inclusive form. Therefore, we represent the data in exclusive form by adding and subtracting $\frac{4-3}{2} = 0.5$ with the upper limit and the lower limit respectively of each inclusive class. The table shown as follows:

Class interval	*Frequency*
0.5 – 3.5	24
3.5 – 6.5	30
6.5 – 9.5	28
9.5 – 12.5	41
12.5 – 15.5	19
15.5 – 18.5	8
	Total : 150



(*a*) Required total frequency = 150

(*b*) We see that 41 is the greatest frequency corresponding to the interval 10 – 12 of inclusive form of frequency distribution.

∴ 10 – 12 is the required interval.

(*c*) Required number of scores = 41 + 19 + 8 = 68

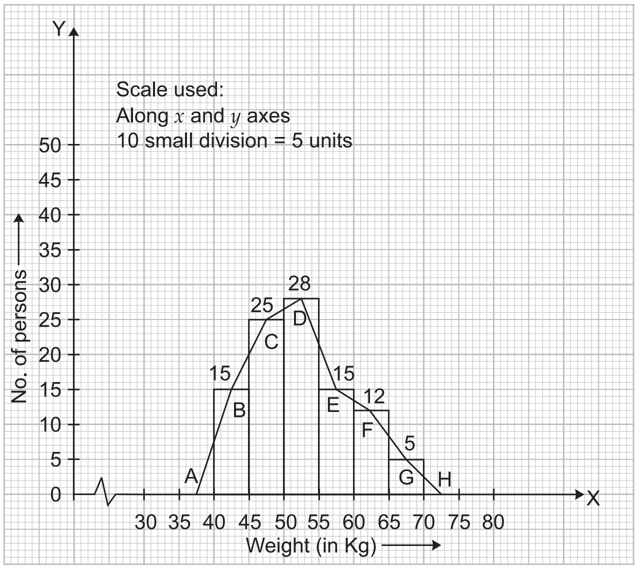
Long Answer Type Questions

9. Construct a histogram and frequency polygon for the following frequency distribution.

Weight (in kg)	*Number of persons*
40 – 45	15
45 – 50	25
50 – 55	28
55 – 60	15
60 – 65	12
65 – 70	5

[CBSE SP 2012]

Sol.



∴ ABCDEFGHA is the required frequency polygon.

10. The following are the scores of two groups of Class VI students in a test of reading ability.

Scores	*Group A*	*Group B*
53 – 55	2	1
50 – 52	3	2
47 – 49	12	4
44 – 46	14	6
41 – 43	18	8
38 – 40	22	12
35 – 37	10	16
32 – 34	13	20
	Total = 94	Total = 69

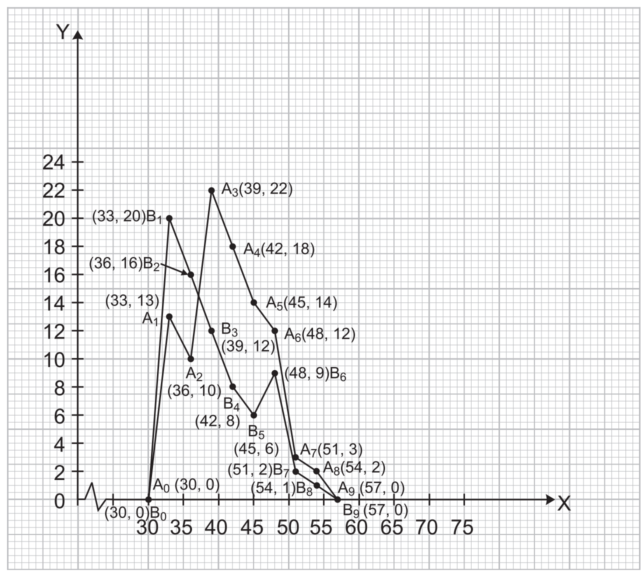
Construct a frequency polygon for each of these two groups of students on the same axes without drawing histograms.

Sol. The exclusive forms of the frequency distribution are as follows:

Scores	*Group A*	*Group B*
31.5 – 34.5	13	20
34.5 – 37.5	10	16
37.5 – 40.5	22	12
40.5 – 43.5	18	8
43.5 – 46.5	14	6
46.5 – 49.5	12	4
49.5 – 52.5	3	2
52.5 – 55.5	2	1
	Total = 94	Total = 69

We see the class marks of different classes of scores are $\frac{31.5 + 34.5}{5} = 33$, $\frac{34.5 + 37.5}{2} = 36$, $\frac{37.5 + 40.5}{2} = 39\ldots$, $\frac{52.5 + 55.5}{2} = 54$, i.e. 33, 36, 39, 42, 45, 48, 51 and 54. Now, to draw the frequency polygon for Group A of students, we plot the points $A_1(33, 13)$, $A_2(36, 10)$, $A_3(39, 22)$, $A_4(42, 18)$, $A_5(45, 14)$, $A_6(48, 12)$, $A_7(51, 3)$, and $A_8(54, 2)$ and join these points pair wires successively by eight line segments.

Similarly, to draw the frequency polygon for Group B of students, we plot the points $B_1(33, 20)$, $B_2(36, 16)$, $B_3(39, 12)$, $B_4(42, 18)$, $B_5(45, 6)$, $B_6(48, 9)$, $B_7(51, 2)$, and $B_8(54, 1)$ and join them pairwise successively by eight other line segments.



The polygon $A_0\, A_1\, A_2\, A_3\, A_4\, A_5\, A_6\, A_7\, A_8\, A_9\, A_0$ is the required frequency polygon for Group A students and $B_0\, B_1\, B_2\, B_3\, B_4\, B_5\, B_6\, B_7\, B_8\, B_9\, B_0$ is the required frequency polygon for Group B students, where $A_1, A_2 \ldots A_8, A_9, A_0$ are the points (33, 13), (36, 10), (39, 22), (42, 18), (45, 14), (48, 12), (51, 3), (54, 2), (57, 0) and (30, 0) respectively and $B_1, B_2, \ldots B_8, B_9, B_0$ are the points (33, 20), (36, 16), (39, 12), (42, 8), (45, 13), (48, 6), (48, 9), (51, 2), (54, 1), (57, 0), and (30, 0) respectively. Scales used are as follows:

Along *x*-axis : 10 small divisions = 5 units

Along *y*-axis :10 small divisions = 2 units

Milestone 2

(Page 200)

Multiple-Choice Questions

1. The value of p for which the mode of the following data

12, 15, 11, 12, 18, 15, 9, p, 12, 16, 17, 15, 18, 11, is 12 is

(*a*) 11 (*b*) 18

(*c*) 15 (*d*) 12

Sol. (*d*) 12

The frequency distribution of the given data excluding the unknown observation p is as follows:

Observations	*Tally marks*	*Frequency*
9	|	1
11	||	2
12	|||	3
15	|||	3
16	|	1
17	|	1
18	||	2
		Total = 13

If the mode is 12, then the observation 12 will have maximum frequency. But the maximum frequency of 12 and 15 excluding p are the same i.e. 3. Hence, we conclude that if the mode of all the data including p is 12, then the value of p must be 12 so that 12 has the maximum frequency i.e. 3 + 1 = 4. Hence, the required value of p is 12.

2. The median of all prime numbers between 70 and 100 is

(*a*) 86 (*b*) 81

(*c*) 89 (*d*) 83

Sol. (*b*) 81

Prime numbers between 70 and 100 are 71, 73, 79, 83, 89 and 97. So, the number of all these observations is 6 which is even. Hence, the average of $\frac{6}{2}$ th and $\left(\frac{6}{2}+1\right)$ th observation, i.e. 3rd and 4th the observation is the median.

$\therefore$ Required median is $\frac{79+83}{2} = 162 = 81.$

Very Short Answer Type Questions

3. The mean of 8 numbers is 25. If each number is multiplied by 3, what will be the new mean?

Sol. Let the 8 observations be $x_1, x_2, x_3, \ldots x_8$.

Given that $\frac{x_1 + x_2 + x_3 + \ldots + x_8}{8} = 25$

$\Rightarrow x_1 + x_2 + x_3 + \ldots + x_8 = 8 \times 25 = 200$ ...(1)

$\therefore$ Required mean of the new numbers $3x_1, 3x_2, 3x_3, \ldots 3x_8$ is $\frac{3(x_1 + x_2 + x_3 + \ldots + x_8)}{8} = \frac{3}{8} \times 200$ [From (1)]

$= 75$

4. The percentage of marks obtained by students of a class in mathematics are 64, 36, 37, 23, 0, 19, 82, 91, 72, 31, 10, 5. Find the median.

Sol. Arranging the mark in decreasing order, we get

91, 82, 72, 64, 37, 36, 31, 23, 19, 10, 5, 0

Total number of these observations is 12 which is even.

$\therefore$ Average of $\left(\frac{12}{2}\right)$th and $\left(\frac{12}{2}+1\right)$th observations = 6th and 7th observations is the median.

$\therefore$ Required median $= \frac{36+31}{2} = \frac{67}{2} = 33.5$

Short Answer Type-I Questions

5. Find the mean of all factors of 210.

Sol. We first find all factors of 210 as follows:

2	210
3	105
5	35
	7

$\therefore$ All factors of 210 are 1, 210, 2, 3, 5, 7, 2 × 3, 2 × 5, 2 × 7, 3 × 5, 3 × 7, 5 × 7, 2 × 3 × 5, 2 × 3 × 7, 2 × 5 × 7 and 3 × 5 × 7, i.e. 1, 210, 3, 5, 7, 6, 10, 14, 15, 21, 35, 30, 42, 70, 105.

Total number of observations is 16.

$\therefore$ Required mean

$$= \frac{\left[\begin{array}{r}1 + 210 + 2 + 3 + 5 + 7 + 6 + 10 + 14 + 15 \\ + 21 + 35 + 30 + 42 + 70 + 105\end{array}\right]}{16}$$

$$= \frac{576}{16} = 36$$

6. The following observations have been arranged in ascending order of magnitude. If the median of the data is 72, find the value of x.

26, 38, 42, 55, x, x + 4, 79, 85, 91, 97

Sol. Total number of observations = 10, which is even.

Hence, the average of $\left(\frac{10}{2}\right)$th and $\left(\frac{10}{2}+1\right)$th observations = 5th and 6th observation is the median.

$$\therefore \quad \frac{x+x+y}{2} = 72$$

$$\Rightarrow \quad 2x + 4 = 144$$

$$\Rightarrow \quad x = \frac{140}{2} = 70$$

Hence, the required value of x is 70.

Short Answer Type-II Questions

7. For the data: 3, 9, $x + 6$, $2x + 3$, 4, 10 and 5, if the mean is 7, find the value of x.

Using this value of x, find the mode of the data.

Sol. There are 7 observations. Their mean is

$$\frac{3+9+x+6+2x+3+4+10+5}{7} = 7$$

$$\Rightarrow \quad 40 + 3x = 49$$

$$\Rightarrow \quad 3x = 49 - 40$$

$$\Rightarrow \quad 3x = 9$$

$$\Rightarrow \quad x = 3$$

$\therefore$ $x = 3$ which is the required value of x.

With the value of x, the given data are 3, 9, 9, 9, 4, 10, 5. Among these, the observation 9 occurs maximum number of times, i.e. 3 times. Hence, the required mode is 9.

8. Numbers 50, 42, 35, $2x + 10$, $2x - 8$, 12, 11, 8 and 6 are written in descending order. Prove that the value of x does not exist, if the median of these data is 25. What is the maximum value of x so that the given numbers are in decreasing order and that the median exists? Find the median of the numbers, when the median exists.

Sol. Number of observation = 9, which is odd. Hence, $\frac{9+1}{2}$ th = 5th observation is the median. Now, the 5th observation is $2x - 8$

$$\therefore \quad 2x - 8 = 25$$

$$\Rightarrow \quad 2x = 8 + 25 = 33$$

4th observation = $2x + 10 = 33 + 10 = 43$ which is more than 35, the 3rd observation, which is absurd, since the numbers are arranged in descending order. Hence, in this case, if the median is 25, then x does not exist.

Now, in order that the median may exist, we must have

$$2x + 10 \leq 35 \quad \ldots(1)$$

$$2x - 8 \leq 2x + 10 \quad \ldots(2)$$

and $\quad 2x - 8 \geq 12 \quad \ldots(3)$

From (1), $\quad x \leq 12.5 \quad \ldots(4)$

$\therefore$ (2) is always true.

From (4), we conclude that the required maximum value of x should be 12.5, so that the given numbers are in descending order. Taking $x = 12.5$, we have the following data : 50, 42, 35, 35, 17, 12, 11, 8, 6.

Number of observations = 9

$\therefore$ The required median is $\frac{9+1}{2}$ th = 5th observation which is 17.

Long Answer Type Questions

9. The mean marks scored by 40 students was found to be 65. Later on, it was discovered that a score of 80 was misread as 50. Find the correct mean.

Sol. Let the marks of 39 students be $x_1, x_2, x_3 \ldots x_{39}$ and the marks of the 40th student were wrongly takes as 50 in place of the correct marks 80 and the wrong mean of 65 was obtained.

$$\therefore \quad \frac{x_1 + x_2 + x_3 + \ldots + x_{39} + 50}{40} = 65$$

$$\Rightarrow \quad x_1 + x_2 + x_3 + \ldots + x_{39} = 65 \times 40 - 50$$

$$= 2600 - 50$$

$$= 2550 \quad \ldots(1)$$

$\therefore$ The required correct mean

$$= \frac{x_1 + x_2 + x_3 + \ldots + x_{39} + 80}{40}$$

$$= \frac{2550 + 80}{40} \quad \text{[From (1)]}$$

$$= \frac{2630}{40} = 65.75$$

10. Find the values of f_1 and f_2 from the following data, if its mean is 25.25:

x_i	12	18	25	$f_1 + 8$	40
f_i	3	f_2	5	6	2

Total frequency = 20.

Sol. From the given data, we have

$$\Sigma x_i f_i = 12 \times 3 + 18f_2 + 25 \times 5 + 6(f_1 + 8) + 40 \times 2$$

$$= 36 + 18f_2 + 125 + 6f_1 + 48 + 80$$

$$= 289 + 18f_2 + 6f_1 \quad \ldots(1)$$

Also, $\quad n = \Sigma f_i = 3 + f_2 + 5 + 6 + 2$

$$= 16 + f_2 = 20 \quad \text{[Given]}$$

$$\therefore \quad f_2 = 4$$

$\therefore$ From (1), $\Sigma x_i f_i = 289 + 18 \times 4 + 6 f_1$

$= 361 + 6f_1$ ...(2)

$\therefore$ Mean $= \dfrac{\Sigma x_i f_i}{\Sigma f_i}$

$= \dfrac{361 + 6 f_1}{20} = 25.25$ [Given]

$\Rightarrow 6f_1 = 25.25 \times 20 - 361$

$= 505 - 361 = 144$

$\Rightarrow f_1 = \dfrac{144}{6} = 24$

Hence, the required value of f_1 and f_2 are 4 and 24 respectively.

Higher Order Thinking Skills (HOTS) Questions

(Page 202)

1. The sum of the deviations of n observations $x_1, x_2, ..., x_n$ measured from 32 is – 5 and the sum of the deviations of these observations measured from 40 is – 101. Find the value of the mean of these observations and n. [Note that the deviation of a number 'a' from another number 'b' is defined as $a - b$].

Sol. According to the problem, we have

$(x_1 - 32) + (x_2 - 32) + (x_3 - 32) + \ldots + (x_n - 32) = -5$

$\Rightarrow -32n + (x_1 + x_2 + x_3 + \ldots + x_n) = -5$

$\Rightarrow x_1 + x_2 + x_3 + \ldots + x_n = 32n - 5$ ...(1)

and $(x_1 - 40) + (x_2 - 40) + (x_3 - 40) + \ldots + (x_n - 40)$

$= -101$

$\Rightarrow -40n + (x_1 + x_2 + x_3 + \ldots + x_n) = -101$

$\Rightarrow x_1 + x_2 + x_3 + \ldots + x_n = 40n - 101$ ...(2)

$\therefore$ From (1) and (2), we have

$32n - 5 = 40n - 101$

$\Rightarrow 8n = 101 - 5 = 96$

$\Rightarrow n = \dfrac{96}{8} = 12$ ...(3)

$\therefore$ From (1),

$x_1 + x_2 + x_3 + \ldots + x_n = 32 \times 12 - 5$

$= 384 - 5 = 379$ ...(4)

$\therefore$ Required mean $= \dfrac{x_1 + x_2 + x_3 + \ldots + x_n}{n}$

$= \dfrac{379}{12}$ [From (3) and (4)]

$= 31.58$ (approx.)

Also, the required value of n is 12.

2. The mean marks of 150 students in a school in an examination were 55%. The mean marks of the boys and girls were 50% and 60% respectively. Find the number of boys and the number of girls who appeared at the examination.

Sol. Let the number of boys and girls be N and n respectively.

$\therefore$ N + n = 150 ...(1)

Total marks obtained by N boys and n girls are $50N + 60n$

$\therefore$ According to the problem, we have

$\dfrac{50N + 60n}{N + n} = 55$

$\Rightarrow 50N + 60n = 55N + 55n$

$\Rightarrow 5N = 5n$

$\Rightarrow N = n$ ...(2)

From (1) and (2), we have

$n = N = 75$

$\therefore$ Required number of boys and girls are 75 and 75.

3. A boy got the following marks in his weekly tests out of 100 marks.

65, 68, 72, 75, 62, 61, 78, 67, 64, 72, 70, 76, 71, 79

What marks should he expect in the next weekly test out of the following intervals of marks?

(*a*) between 60 to 70

(*b*) between 70 to 80.

Sol. (*b*) between 70 to 80

We see that out of the given marks, the observation 72 occurs maximum number of times. Hence, the mode of the data will be 72 which lies between 70 and 80.

Self-Assessment

(Page 202)

Multiple-Choice Questions

1. In a grouped frequency distribution, three classes are as follows:

10.5 – 20.5, 20.5 – 30.5, 30.5 – 40.5

Then the class marks are respectively

(*a*) 10.5, 20.5 and 30.5

(*b*) 15.5, 25.5 and 35.5

(*c*) 20.5, 30.5 and 40.5

(*d*) 10, 10 and 10.

Sol. (*b*) 15.5, 25.5 and 35.5

The class marks of the three given classes are $\frac{10.5 + 20.5}{2} = 15.5$, $\frac{20.5 + 30.5}{2} = 25.5$ and $\frac{30.5 + 40.5}{2} = 35.5$ respectively.

2. The given cumulative frequency distribution shows the class intervals and their corresponding cumulative frequencies:

Class interval	30 – 40	40 – 50	50 – 60
Cumulative frequency	13	19	27

Then the frequency of the class interval 40 – 50 is

(*a*) 13 (*b*) 14

(*c*) 8 (*d*) 6

Sol. (*d*) 6

Let the frequency of the classes be f_1, f_2 and f_3 respectively. We have the following cumulative frequency distribution:

Class intervals	*Frequency*	*Cumulative frequency*
30-40	f_1	$13 = f_1$
40-50	f_2	$19 = 13 + f_2$
50-60	f_3	$27 = 13 + f_2 + f_3$

We have $f_1 = 13, f_2 + 13 = 19$

$\Rightarrow$ $f_2 = 19 - 3 = 6$

and $13 + f_2 + f_3 = 27$

$\Rightarrow$ $13 + 6 + f_3 = 27$

$f_3 = 8$

$\therefore$ Respective frequencies are 13, 6 and 8.

Hence, required frequency of the class 40-50 is $f_2 = 6$.

Fill in the Blanks

3. To analyse the election results, the data is collected from newspapers. The data thus collected is known as **secondary data**.

4. The mode of 4, 6, 7, 6, 4, 2, 4, 8, 6, 4, 3, 4, 6 is **4**.

5. The class mark of the class interval 2.4–6.6 is **4.5**.

Sol.
$$\text{Class mark} = \frac{2.4 + 6.6}{2} = \frac{9}{2} = 4.5$$

6. The lower limit of class interval 40–50 is **40**.

Assertion-Reason Type Questions

Directions (Q. Nos. 7 to 10): Each of these questions contains an assertion followed by reason. Read them carefully, and answer the question on the basis of the following options, select the one that best describes the two statements.

(a) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.

(b) Both Assertion and Reason are true but Reason is not the correct explanation of Assertion.

(c) Assertion is true but Reason is false.

(d) Assertion is false but Reason is true.

7. **Assertion:** Election results obtained from a newspaper is an example of primary data.

Reason: The data collected by the investigator himself for a specific purpose is known as primary data.

Sol. (*d*)

Assertion is incorrect but reason is correct.

8. **Assertion:** In a class, if 5 students obtain exactly 100 marks, then the frequency of 5 is 100.

Reason: The number of times an observation occurs in the given data, is called the frequency of the observation.

Sol. (*d*)

Assertion is incorrect as observation is 100 and frequency is 5 but reason is correct.

9. **Assertion:** 9 – 11, 12 – 14, 15 – 17, … are examples of inclusive group.

Reason: In an exclusive group, upper limit and lower limit are included in the class.

Sol. (*c*)

Assertion is in correct but reason is incorrect as in exclusive group lower limit is included.

10. **Assertion:** In a distribution of the form 0 – 10, 10 – 20, 20 – 30,… the true upper limit is equal to the upper limit of the class.

Reason: The given distribution is exclusive.

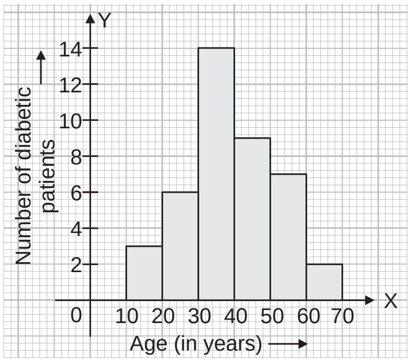
Sol. (*a*)

Both assertion and reason are correct and reason is correct explanation of assertion.

Case Study Based Questions

11. A survey was conducted that shows the ages of diabetic patients admitted in a hospital during a year. The data was recorded in the frequency distribution table and has been represented by the histogram.

Age (in years)	10–20	20–30	30–40	40–50	50–60	60–70
Number of diabetic patients	3	6	14	9	5	2



Study the graph and answer the following questions.

(*a*) Which of the following age group has the minimum number of diabetic patients?

(*i*) 30–40 (*ii*) 10–20

(*iii*) 20–30 (*iv*) 60–70

Ans. (*iv*) 60–70

(*b*) Which of the following age group has the maximum number of diabetic patients?

(*i*) 20–30 (*ii*) 30–40

(*iii*) 40–50 (*iv*) 60–70

Ans. (*ii*) 30–40

(*c*) What is the total number of diabetic patients in the age group of 10–70?

(*i*) 25 (*ii*) 33

(*iii*) 39 (*iv*) 49

Ans. (*iii*) 39

(*d*) In a histogram, each class rectangle is constructed with base as

(*i*) frequency.

(*ii*) class intervals.

(*iii*) range.

(*iv*) size of the class.

Ans. (*ii*) class intervals.

(*e*) How many more diabetic patients are there in the age group 40–50 than in the age group 60–70?

(*i*) 2 (*ii*) 4

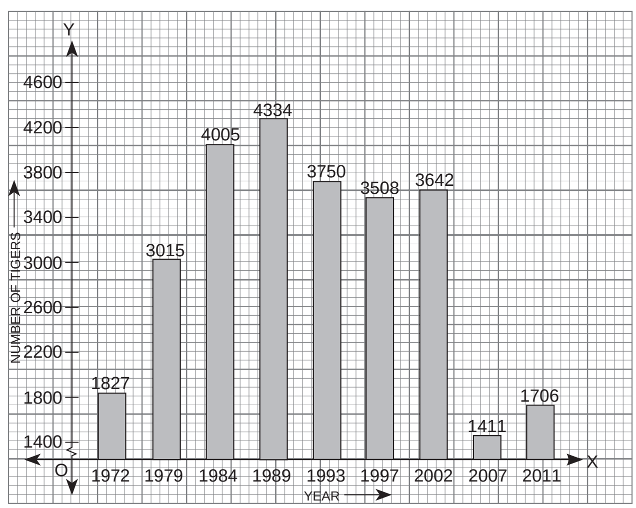
(*iii*) 7 (*iv*) 9

Ans. (*iii*) 7

12. The Government of India has taken a pioneering initiative for conserving its national animal, the Tiger, by launching the 'Project Tiger' in 1973. A survey was conducted by the Ministry of Environment, Forests and Climate change, according to which population of tiger has increased.

The following table gives the population of tigers in India.

Year	*Number of tigers*
1972	1827
1979	3015
1984	4005
1989	4334
1993	3750
1997	3508
2002	3642
2007	1411
2011	1706



Read the above bar graph and answer the following questions:

(*a*) In which year were the number of tigers minimum?

(*i*) 1972 (*ii*) 2002
(*iii*) 2007 (*iv*) 2011

Ans. (*iii*) 2007

(*b*) In which year were the number of tigers maximum?

(*i*) 1979 (*ii*) 1984
(*iii*) 1989 (*iv*) 1993

Ans. (*iii*) 1989

(*c*) In a bar graph, the widths of bars

(*i*) have no significance.
(*ii*) are proportional to the corresponding heights.
(*iii*) are proportional to the corresponding frequencies.
(*iv*) are proportional to the space between two consecutive bars.

Ans. (*i*) have no significance.

(*d*) How much decrease in number of tigers was there between 2002 to 2007?

(*i*) 2950
(*ii*) 3166
(*iii*) 1936
(*iv*) 2231

Ans. (*iv*) 2231

(*e*) How much increase in number of tigers was there between 2007 to 2011?

(*i*) 295
(*ii*) 314
(*iii*) 2231
(*iv*) 3166

Ans. (*i*) 295

Very Short Answer Type Questions

13. What is the significance of the widths of bars in a bar graph?

Sol. We know that the widths of the bars in a bar graph have no significance.

14. What is the mean of four consecutive prime numbers the greatest of which is 67?

Sol. Four consecutive prime numbers less than and equal to 67 are 67, 61, 59 and 53. Hence, the required mean of these 4 numbers

$$= \frac{67 + 61 + 59 + 53}{4} = \frac{240}{4} = 60°$$

Short Answer Type-I Questions

15. The mean of six numbers is 20. If one number is deleted, their mean is 15. Find the deleted number.

Sol. Let the deleted number be x and let the remaining numbers be x_1, x_2, x_3, x_4 and x_5. Then according to the problem, we have

$$\frac{x_1 + x_2 + x_3 + x_4 + x_5}{5} = 15$$

$$\Rightarrow \quad x_1 + x_2 + x_3 + x_4 + x_5 = 75 \quad \ldots(1)$$

$$\therefore \quad \frac{x_1 + x_2 + x_3 + x_4 + x_5 + x}{6} = 20$$

$$\Rightarrow \quad \frac{75 + x}{6} = 20 \quad \text{[From (1)]}$$

$$\Rightarrow \quad x = 120 - 75 = 45$$

∴ Required deleted number is 45.

16. The numbers 7, 6, 5, 5, $2x + 1$, 4, 4, 3, 2, are written in descending order. If their median is 5, find x and hence the mode.

Sol. We see that the number of observations = 9, which is odd.

$\therefore \frac{9+1}{2}$ th = 5th observation is the median.

$$\therefore \quad 2x + 1 = 5$$

$$\Rightarrow \quad x = 2$$

∴ The given data are 7, 6, 5, 5, 5, 4, 4, 3, 2.

Hence, the observation 5 occurs maximum number of times.

Hence, the mode is 5.

Hence, required values of x and the mode are 2 and 5 respectively.

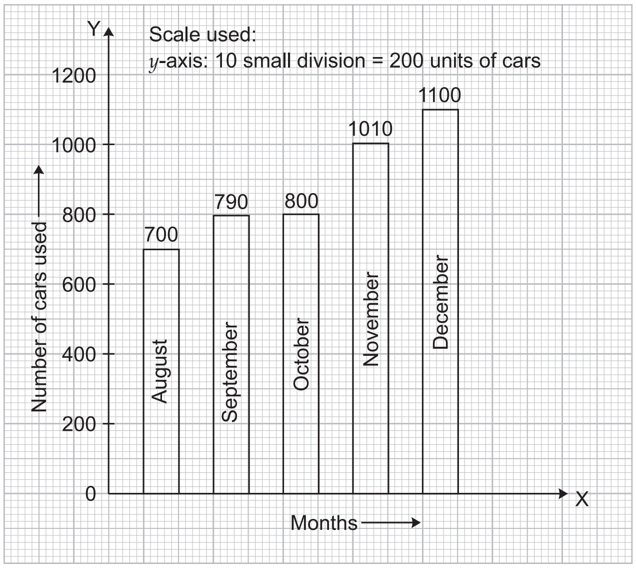
Short Answer Type-II Questions

17. Represent the following data by a bar graph:

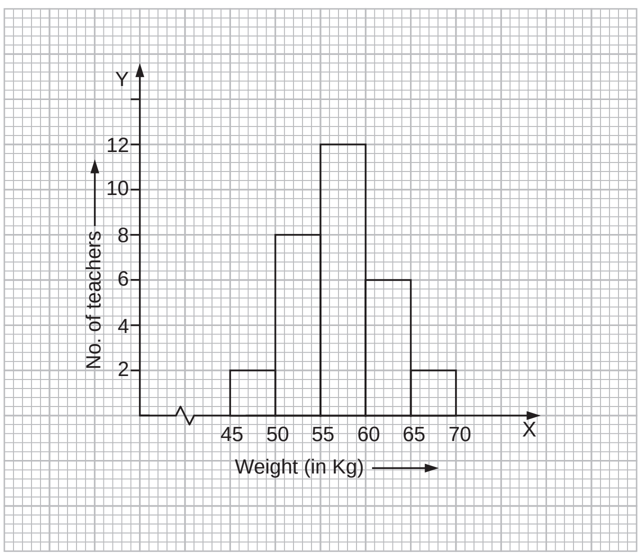
The number of cars sold in Delhi during August, 1998 to December, 1998 was as follows:

Month	*Number of cars sold*
August	700
September	790
October	800
November	1010
December	1100

Sol. Bar graph showing the number of cars sold in different months of a year.



18. Prepare a grouped frequency distribution table for the histogram shown below:



Histogram for the weight of 30 teachers in a school

Sol. The required grouped frequency distribution table for the given histogram is as follows:

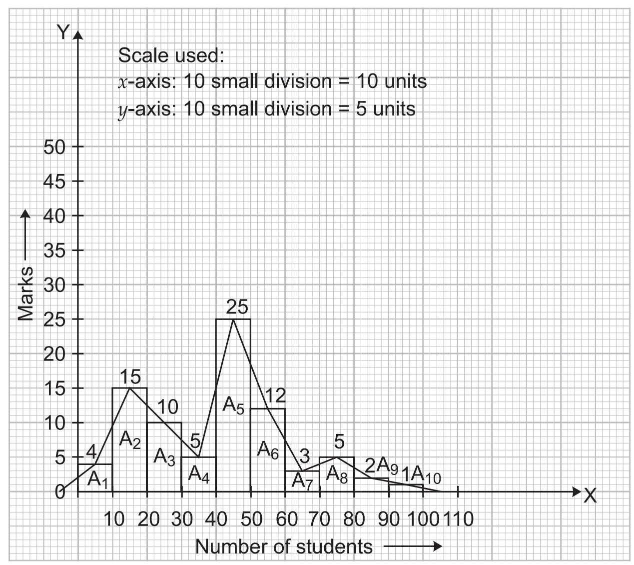
Weight in kg	45–50	50–55	55–60	60–65	65–70
No. of teachers	2	8	12	6	2

Long Answer Type Questions

19. Draw a histogram and a frequency polygon for the following data:

Marks	*Number of students*
0 – 10	4
10 – 20	15
20 – 30	10
30 – 40	5
40 – 50	25
50 – 60	12
60 – 70	3
70 – 80	5
80 – 90	2
90 – 100	1

Sol.



$\therefore$ $A_0A_1A_2A_3A_4A_5A_6A_7A_8A_9A_{10}A_{11}A_0$ is the requried frequency polygon.

20. Using the assumed mean method, find the mean daily earnings of a worker from the following frequency distribution table:

Daily earnings (in ₹)	25	75	125	175	225	275
Number of workers	8	15	32	26	12	7

Sol. We construct the following frequency distribution table for the calculation of mean by the assumed mean method.

Daily earning (in ₹) (x_i)	$d_i = x_i - a$ *where* $a = 125$	*No. of workers* (f_i)	$d_i f_i$
25	–100	8	–800
75	–50	15	–750
125	0	32	0
175	50	26	1300
225	100	12	1200
275	150	7	1050
		Total: $n = 100$	Total: 2000 $= \Sigma f_i d_i$

$\therefore$ Required mean $= a + \dfrac{\sum_{i=1}^{6} f_i d_i}{n}$

$$= ₹\left(125 + \frac{2000}{100}\right) = ₹\ 145$$

Let's Compete

(Page 204)

Multiple-Choice Questions

1. If the class marks of four successive classes in a grouped frequency distribution are 63, 68, 73 and 78, then the class size and the class in which the observation 65.5 lies are respectively

(*a*) 5 and 65.5 – 70.5

(*b*) 10 and 60.5 – 70.5

(*c*) 5 and 63.5 – 68.5

(*d*) 10 and 65.5 – 75.5

Sol. (*a*) 5 and 65.5 – 70.5

Let the four successive classes be $x_1 - x_2$, $x_2 - x_3$, $x_3 - x_4$ and $x_4 - x_5$.

$\therefore$ Their respective class marks are

$$d_1 = \frac{x_1 + x_2}{2} = 63,\ d_2 = \frac{x_2 + x_3}{2} = 68,$$

$$d_3 = \frac{x_3 + x_4}{2} = 73 \text{ and } d_4 = \frac{x_4 + x_5}{2} = 78$$

$\therefore$ $x_1 + x_2 = 126$ …(1)

$x_2 + x_3 = 136$ …(2)

$x_3 + x_4 = 146$ …(3)

and $x_4 + x_5 = 156$ …(4)

Also, $x_2 - x_1 = d_2 - d_1 = 68 - 63 = 5$ …(1′)

$x_3 - x_2 = 5$ …(2′)

$x_4 - x_3 = 5$ …(3′)

$x_5 - x_4 = 5$ …(4′)

From (1) and (1′), we have

$$x_2 = \frac{126 + 5}{2} = \frac{131}{2} = 65.5 \text{ and } x_1 = 60.5$$

From (2) and (2′), we have

$$x_3 = \frac{136 + 5}{2} = \frac{141}{2} = 70.5 \text{ and } x_4 = 75.5$$

From (3) and (3′), we have

$$x_4 = \frac{146 + 5}{2} = \frac{151}{2} = 75.5 \text{ and } x_5 = 80.5$$

From (4) and (4′), we have

$$x_5 = \frac{156 + 5}{2} = \frac{161}{2} = 80.5 \text{ and } x_6 = 85.5$$

Now, the observation 65.5 lies in the class 65.5 – 70.5, i.e. in $x_2 - x_3$.

2. For drawing a frequency polygon of a continuous frequency distribution, we plot the points whose ordinates are the frequency of respective classes and abscissae are respectively

(*a*) lower limits of the classes

(*b*) upper limits of the classes

(*c*) class marks of the classes

(*d*) upper limits of the preceding classes

Sol. (*c*) class marks of the classes

We know that the respective abscissa of the points of frequency polygon are the class marks of the classes.

3. The smallest of four consecutive odd integers is 59. Then the mean of these four integers is

(*a*) 63 (*b*) 62

(*c*) 64 (*d*) 65

Sol. (*b*) 62

Consecutive four odd numbers less than and equal to 59 are 59, 61, 63 and 65.

$\therefore$ Required mean of these numbers

$$= \frac{59 + 61 + 63 + 65}{4}$$

$$= \frac{248}{4} = 62$$

4. Out of fourteen observations arranged in increasing order, the sixth, seventh, and eighth observations are respectively 53, 75 and 81. Then the median is

(*a*) 64 (*b*) 67
(*c*) 75 (*d*) 78

Sol. (*d*) 78

Let the 14 observations in increasing order be x_1, x_2, x_3 … x_{14}, where $x_6 = 53$, $x_7 = 75$ and $x_8 = 81$.

Median of these 14 observations is the average of 7th and 8th observation.

$$\therefore \quad \text{Required median} = \frac{x_7 + x_8}{2}$$

$$= \frac{75 + 81}{2} = \frac{156}{2} = 78$$

5. If the mean of x and $\frac{1}{x}$ is M, then the mean of x^2 and $\frac{1}{x^2}$ is

(*a*) $2M^2 - 1$ (*b*) $2M^2 + 1$
(*c*) $2M + 1$ (*d*) $2M - 1$

Sol. (*a*) $2M^2 - 1$

Given that $\quad M = \frac{1}{2}\left(x + \frac{1}{x}\right)$

$$\Rightarrow \quad x + \frac{1}{x} = 2M \quad \ldots(1)$$

$\therefore$ Mean of x^2 and $\frac{1}{x^2}$ is

$$\frac{1}{2}\left(x^2 + \frac{1}{x^2}\right) = \frac{1}{2}\left\{\left(x + \frac{1}{x}\right)^2 - 2x \times \frac{1}{x}\right\}$$

$$= \frac{1}{2}\left(4M^2 - 2\right) \quad \text{[From (1)]}$$

$$= 2M^2 - 1$$

6. A set of data consists of 10 numbers:

8, 3, 5, 8, 4, 8, 3, 4, x and y

The difference between the modes in the two cases when $x = 8$ and $y = 9$ and when $x = y = 3$, is

(*a*) 3 (*b*) 4
(*c*) 5 (*d*) 6

Sol. (*c*) 5

Taking $x = 8$ and $y = 9$ we obtain the following observation: 8, 3, 5, 8, 4, 8, 3, 4, 8, 9, i.e. 3, 3, 4, 4, 5, 8, 8, 8, 8 and 9 whose mode is clearly 8, since it occurs maximum number of times.

Again, taking $x = y = 3$, we get the following observations: 8, 3, 5, 8, 4, 8, 3, 4, 3, 3, i.e. 3, 3, 3, 3, 4, 4, 5, 8, 8, 8, whose mode is clearly 3, since it occurs maximum number of times.

$\therefore$ Difference between these two modes is $8 - 3 = 5$.

7. For which set of data is the median equal to the mode?

(*a*) 10, 1, 1, 5, 2, 4 (*b*) 2, 7, 2, 5, 42
(*c*) 8, 8, 7, 6, 3, 2 (*d*) 5, 3, 2, 3, 3

Sol. (*d*) 5, 3, 2, 3, 3

In (*a*), the data in decreasing order are 10, 5, 4, 2, 1, 1. The number of observations is 6 which is even. Hence, the average of $\left(\frac{6}{2}\right)$th and $\left(\frac{6}{2} + 1\right)$th observation = 3rd and the 4th observation is the median.

$$\therefore \quad \text{Median} = \frac{4 + 2}{2} = 3$$

Also, mode is 1, since it occurs maximum number of times.

$\therefore$ In this case mode ≠ median.

In (*b*), the data in decreasing order are 42, 7, 5, 2, 2. The number of observations is 5, which is odd.

$$\therefore \quad \frac{5 + 1}{2} = \text{3rd observation is the median.}$$

$\therefore$ Median = 5

But mode = 2, since it occurs maximum number of times.

$\therefore$ In this case also mode ≠ median.

In (*c*), the data in decreasing order are 8, 8, 7, 6, 3, 2. The number of data is 6 which is even.

$\therefore$ The average of $\left(\frac{6}{2}\right)$th and $\left(\frac{6 + 1}{2}\right)$th observation = 3rd and the 4th observation is the median.

$$\therefore \quad \text{Median} = \frac{7 + 6}{2} = 6.5$$

Also, mode = 8, since it occurs maximum number of times.

$\therefore$ In this case also, mode ≠ median.

Finally, in (*d*), the data in decreasing order are 5, 3, 3, 3, 2. Since the number of observation is 5 which is odd, hence, the median is $\frac{5 + 1}{2}$ = 3rd observation which is 3.

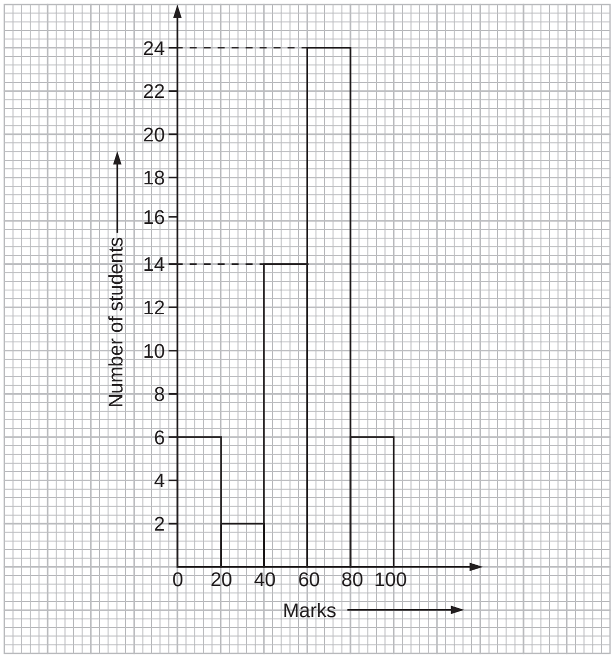
$\therefore$ Median = 3. Also, mode = 3, since it occurs maximum number of times.

$\therefore$ In this case only, we see that mode = median = 3.

8. In the given graph, the number of students who scored 40 and more than 40 marks is

(*a*) 30 (*b*) 44

(*c*) 24 (*d*) 38



Sol. (*b*) 44

From the graph, we see that 14 students marks lie between 40 and 60, 24 students marks lie between 60 and 80 and 6 students marks lie between 80 and 100. Hence, the required number of students whose marks is greater than and equal to 40 is $14 + 24 + 6 = 44$.

9. In which of the following sets of data is the mean not equal to their median?

(*a*) The first 8 even natural numbers

(*b*) The first 10 natural numbers

(*c*) The first 5 consecutive prime numbers

(*d*) The first four natural numbers which are multiples of 3

Sol. (*c*) The first 5 consecutive prime numbers

In (*a*), the mean of the 1st 8 even natural numbers

$$= \frac{2+4+6+8+10+12+14+16}{8}$$

$$= \frac{2(1+2+3+4+\ldots+8)}{8}$$

$$= \frac{8\times 9}{8} = 9$$

Also, median = average of 4th and 5th data

$$= \frac{8+10}{2} = 9$$

$\therefore$ Mean = Median

In (*b*), the mean of 10 natural numbers

$$= \frac{1+2+3+\ldots+10}{10}$$

$$= \frac{10\times 11}{10\times 2}$$

$$= \frac{11}{2}$$

Also, median = average of 5th and 6th observation

$$= \frac{5+6}{2} = \frac{11}{2}$$

$\therefore$ In this case also, mean = medians.

In (*c*), the mean of 1st 5 consecutive prime numbers

$$= \frac{2+3+5+7+11}{5}$$

$$= \frac{28}{5}$$

Also, median $= \frac{5+1}{2}$ = 3rd observation = 5

$\therefore$ In this case, mean $\neq$ median.

Finally, in case (*d*), the mean of the 1st four natural numbers which are multiples of 3 is

$$\frac{3+6+9+12}{4} = \frac{30}{4} = 7.5$$

Also, the median = average of 2nd and 3rd observations

$$= \frac{6+9}{2} = 7.5$$

$\therefore$ In this case, mean = median

10. Out of mean, mode and median of the following set of data:

6, 5, 8, 5, 6, 8, 5,

which statement is true?

(*a*) Mode < Median < Mean

(*b*) Mean = Mode = Median

(*c*) Mean < Mode

(*d*) Median > Mean

Sol. (*a*) Mode < Median < Mean

We first find the mean, mode and median of the given data: 8, 8, 6, 6, 5, 5, 5 which are in decreasing order.

Number of observation = 7, which is odd.

$\therefore$ Median $= \frac{7+1}{2}$th = 4th observation = 6

Mode = 5, since it occurs maximum number of times.

$$\text{Mean} = \frac{8+8+6+6+5+5+5}{7}$$

$$= \frac{43}{7} = 6\frac{1}{7}$$

We see that $5 < 6 < 6\frac{1}{7}$

$\therefore$ Mode < Median < Mean

Value-based Questions (Optional)

(Page 205)

1. Free medical camp was organised by some kind hearted doctors for poor people in a particular month of a year. The following table shows the number of patients treated on various days.

17	14	10	16	24	19	18	19	11	12
4	18	12	5	25	23	7	15	4	18
12	8	27	23	6	17	22	6	27	19

(*a*) Make grouped frequency distribution table for the data, taking the class width as 5 and one of the class intervals as 5 – 10.

(*b*) For how many days, the number of patients treated were 20 or more?

(*c*) What values were depicted by the doctors who organised the medical camp?

Sol. (*a*) The required grouped frequency distribution table is as follows:

No. of patients	*Tally marks*	*No. of days in a month (frequency)*
0 – 5	||	2
5 – 10	卌	5
10 – 15	卌 |	6
15 – 20	卌 卌	10
20 – 25	||||	4
25 – 30	|||	3
		Total: 30

(*b*) Required number of days = 4 + 3 = 7 days

(*c*) Commitment, empathy, concern for unprivileged people, responsible behavior leadership and helpfulness.

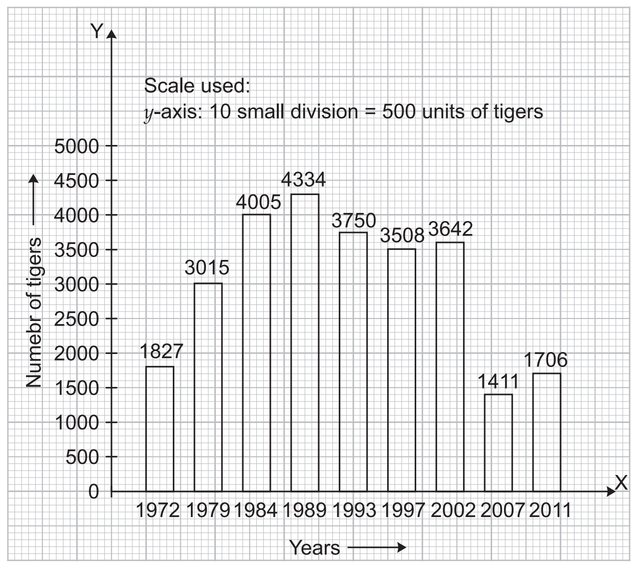
2. The following table gives the population of tigers in India.

Year	*Number of tigers*
1972	1827
1979	3015
1984	4005
1989	4334
1993	3750
1997	3508
2002	3642
2007	1411
2011	1706

(*a*) Represent the above data by a bar graph.

(*b*) What steps do you think helped to stop the decline of tiger population from 2007 to 2011?

Sol. (*a*) The required bar graph is as follows:



A bar graph showing the number of tigers in different years.

(*b*) Creating awareness and educating people about the significance of tigers and discouraging poaching.

3. The percentages of salary spent by 10 households for buying Diwali gifts for the children of an orphanage are 2, 2, 3, 6, 4, 4, 7, 4, 10 and 12.

(*a*) Find the mean, median and mode of the data.

(*b*) What do you infer from the values of mean median and mode?

(*c*) What values are possessed by the members of these households?

Sol. (*a*) The given data in decreasing order are 12, 10, 7, 6, 4, 4, 4, 3, 2, 2.

The total number of data is 10 which is even.

Hence, the average of 5th and 6th data is the median.

$\therefore \quad \text{Median} = \frac{4 + 4}{2} = 4$

Mode = 4, since it occurs maximum number of times.

$$\text{Mean} = \frac{12 + 10 + 7 + 6 + 4 + 4 + 4 + 3 + 2 + 2}{10}$$

$$= \frac{54}{10} = 5.4$$

$\therefore$ Required mean, mode and median are 5.4, 4 and 4 respectively.

(*b*) "The mean is 5.4" means that the average percentage of salary spent for Diwali gifts is 5.5%.

"Median is 4" means 5 families spent more than 4% of salary for Diwali gifts and 5 families spent less than 4% of salary for these gifts.

"Mode is 4" means that out of 10 families most of the families spent more than 4% of the salary for Diwali gifts.

(*c*) Compassion, empathy, caring and generosity.

4. A social organisation decided to build up a clinic in a small village that has no healthcare facilities. They conducted a census to collect information about the ages of people living in that village and recorded the following data.

Age (in years)	*Frequency (number of people)*
0 – 10	12
10 – 20	40
20 – 30	50
30 – 40	28
40 – 50	50
50 – 60	30
60 – 70	15
70 – 80	10

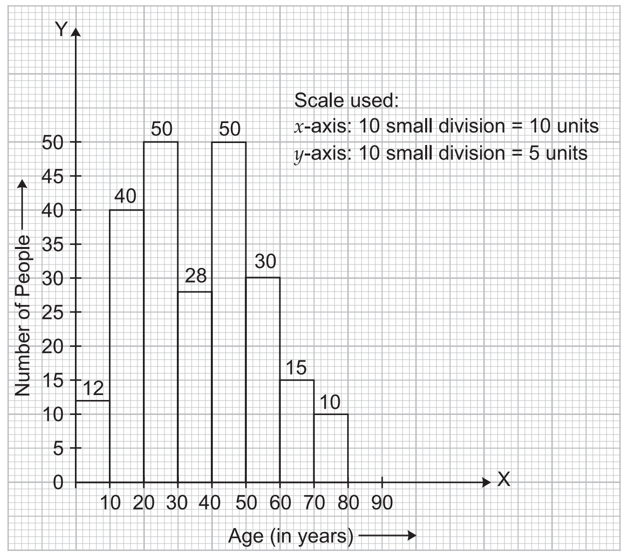
(*a*) Draw a histogram for the above frequency table. From the histogram answer the following questions:

(*b*) What is the total population of the village?

(*c*) How many people are 50 years or more than 50 years old?

(*d*) What values are exhibited here by the social organisation?

Sol. (*a*) The required histogram for the given frequency table is shown below:



(*b*) Required total population of the village is

12 + 40 + 50 + 28 + 50 + 30 + 15 + 10 = 235

(*c*) Required number of people who are more than or equal to 50 years old

= 30 + 15 + 10 = 55

(*d*) Empathy, concern for the welfare of the people of the villagers, caring and helpfulness.

15

Probability

Checkpoint (Page 209)

1. What is the probability of getting
   (*a*) a head and
   (*b*) a tail when a coin is tossed?

Sol. Possible outcomes when a coin is tossed are head or tail.

∴ The number of possible outcomes = 2

(*a*) Number of heads = 1

∴ P(getting a head) = $\frac{1}{2}$

(*b*) Number of tails = 1

∴ P(getting a tail) = $\frac{1}{2}$

2. Find the probability of "getting a six" when a die is rolled.

Sol. All the possible outcomes when a die is rolled are 1, 2, 3, 4, 5 and 6.

∴ The number of all possible outcomes = 6

P(getting a six) = $\frac{1}{6}$

3. Give an example of
   (*a*) an impossible event
   (*b*) a certain event.

Sol. (*a*) Getting a number more than 6 in the throwing of a die is an impossible event.

(*b*) Getting a number less than or equal to 6 in the throwing of a die is a certain event.

4. A bag contains 6 red and 5 white balls. Calculate the probability of getting a white ball.

Sol. Total number of balls in the bag = 6 + 5 = 11

∴ The number of possible outcomes = 11

Number of white balls = 5

Thus, the number of favourable outcomes = 5

∴ P(getting a white ball) = $\frac{5}{11}$

5. Find the probability of getting a number which is a multiple of 3 when a die is rolled.

Sol. All possible outcomes when a die is rolled are 1, 2, 3, 4, 5 and 6.

∴ The number of all possible outcomes = 6

Multiples of 3 are 3 and 6.

Thus, the number of favourable outcomes = 2

∴ P(multiple of 3) = $\frac{2}{6} = \frac{1}{3}$

6. Find the probability of getting a prime number when a die is rolled.

Sol. All possible outcomes when a die is rolled are 1, 2, 3, 4, 5 and 6

∴ The number of all possible outcomes = 6

Prime numbers are 2, 3 and 5.

Thus, the number of favourable outcomes = 3

∴ P(a prime number) = $\frac{3}{6} = \frac{1}{2}$

7. Find the probability of getting two heads when a pair of coins are tossed.

Sol. Possible outcomes are HH, HT, TH, TT.

∴ Total number of possible outcomes = 4

Getting two heads means HH.

Thus, the number of favourable outcomes = 1

∴ P(getting two heads) = $\frac{1}{4}$

8. Which of the following cannot be the probability of any event? State reason.

(a) $\frac{3}{2}$ (b) $\frac{2}{7}$

(c) $-\frac{1}{2}$ (d) 0

(e) 1 (f) $3\frac{1}{2}$

Sol. (a), (c) and (f) cannot be the probability of any event because probability of an event lies between $0 \leq P(E) \leq 1$. Therefore, probability of an event cannot be negative.

9. What is the probability of drawing a king card from a well-shuffled deck of 52 cards?

Sol. Total number of cards = 52

$\therefore$ Total number of possible outcomes = 52

Number of all kings = 4

Thus, the number of favourable outcomes = 4

$\therefore$ P(getting a king) = $\frac{4}{52} = \frac{1}{13}$

10. A card is drawn from a well-shuffled deck of 52 cards. Find the probability of getting a face card.

Sol. Total number of cards = 52

$\therefore$ Total number of possible outcomes = 52

Number of face cards = 12

Thus, the number of favourable outcomes = 12

$\therefore$ P(getting a face card) = $\frac{12}{52} = \frac{3}{13}$

Milestone

(Page 212)

Multiple-Choice Questions

1. In a sample study of 300 students, it was found that 240 students were interested in mathematics. If a student is selected at random, then the probability that the student is not interested in mathematics is

(a) 0.5 (b) 0.8

(c) 0.2 (d) 0.4

Sol. (c) 0.2

Total number of students = 300

Number of students interested in mathematics = 240

P(student is not interested in mathematics)

= 1 – P(students interested in mathematics)

$= 1 - \frac{240}{300} = \frac{60}{300} = \frac{1}{5} = 0.2$

2. Two coins are tossed 50 times and the outcomes are recorded as follows:

Number of heads	2	1	0
Frequency	20	25	5

Then the probability of at least one head is

(a) 0.25 (b) 0.9

(c) 0.20 (d) 0.1

Sol. (b) 0.9

Number of times two coins tossed = 50

P(probability of at least one head)

$$= 1 - P(\text{no head})$$

$$= 1 - \frac{5}{50}$$

$$= 1 - \frac{1}{10}$$

$$= \frac{9}{10} = 0.9$$

Very Short Answer Type Questions

3. Stating reason, which of the following cannot be the probability of any event:

$\frac{4}{9}, \frac{5}{4}, \frac{3}{7}, 0.05, -\frac{2}{3}, 5.5$

Sol. $\frac{5}{4}, \frac{-2}{3}$ and 5.5 cannot be the probability of any event because probability of an event lies between $0 \leq P(E) \leq 1$. Therefore probability of an event cannot be negative.

4. What is the probability of getting an odd number when a die is rolled once?

Sol. All possible outcomes when a die is rolled once are 1, 2, 3, 4, 5 and 6.

$\therefore$ The number of all possible outcomes = 6

Odd numbers are 1, 3 and 5.

Thus, the number of favourable outcomes = 3

$\therefore$ P(getting an odd number) = $\frac{3}{6} = \frac{1}{2}$

Short Answer Type-I Questions

5. Weather forecast from a news channel shows that out of past 200 consecutive days, its weather forecast was correct 180 times.

What is the probability that on a given day the weather forecast

(a) was correct (b) was not correct.

Sol. Total number of days for which the weather forecast was made = 200

(a) Let E_1 be the event that the forecast was correct on the given day.

Number of days for which the forecast was correct = 180

P(the forecast was correct on a given day)

= $P(E_1)$

$$= \frac{\text{Number of days when the forecast was correct}}{\text{Total number of days for which the forecast was made}}$$

$$= \frac{180}{200} = \frac{9}{10} = 0.9$$

(*b*) Let E_2 be the event that the forecast was not correct on the given day

Number of days for which the forecast was not correct = 200 – 180 = 20

P(the forecast was not correct on a given day)

= $P(E_2)$

$$= \frac{\text{Number of days when the forcast was not correct}}{\text{Total number of days for which the forecast was made}}$$

$$= \frac{20}{200} = \frac{1}{10} = 0.1$$

Note that E_1 and E_2 are the possible outcomes and $P(E_1) + P(E_2) = 0.9 + 0.1 = 1$

6. Marks obtained by students of two sections of a class in a school in mathematics out of 100 marks are given in the following table:

Marks (in %)	0–20	20–40	40–60	60–70	70–80	80–100
Number of students	5	7	4	30	2	2

Find the probability that a student obtained

(*a*) less than 60% marks

(*b*) more than or equal to 70% marks.

Sol. Total number of students

= 5 + 7 + 4 + 30 + 2 + 2 = 50

(*a*) Number of students obtained less than 60% marks = 5 + 7 + 4 = 16

P(a student obtained less than 60% marks)

$$= \frac{16}{50} = \frac{8}{25} = 0.32$$

(*b*) Number of students obtained more than or equal to 70% marks = 2 + 2 = 4.

P(a student obtained more than or equal to 70% marks) $= \frac{4}{50} = \frac{2}{25} = 0.08$

Short Answer Type-II Questions

7. The following frequency distribution gives the weights of 30 students of a class:

Weight (in kg)	*Number of students*
30–35	3
35–40	4
40–45	6
45–50	10
50–55	3
55–60	2
60–65	1
65–70	1
Total	30

Find the probability that the weight of a student is at most 60 kg.

Sol. Total number of students = 30

Number of students whose weight is at most 60 kg = 3 + 4 + 6 + 10 + 3 + 2 = 28

∴ P(students whose weight is at most 60 kg)

$$= \frac{28}{30} = \frac{14}{15}$$

8. The distances (in km) of 30 employees from their places of residences to their places of work are as follows:

2	5	3	7	5	9	11	20	8	6
18	16	20	3	2	8	14	13	12	9
23	7	6	5	2	10	4	8	10	3

Find the probability that an employee lives

(*a*) less than or equal to 5 km

(*b*) more than 10 km

(*c*) within 1 km

(*d*) more than 25 km

from his/her place of work.

Sol. Total number of employees = 30

(*a*) Number of employees living at a distance less than or equal to 5 km from their place of work = 10

∴ P(employees living at a distance less than or equal to 5 km from their place of work)

$$= \frac{10}{30} = \frac{1}{3}$$

(*b*) Number of employees living at a distance more than 10 km from their place of work = 9

∴ P(employees living at a distance more than 10 km from their place of work)

$$= \frac{9}{30} = \frac{3}{10}$$

(*c*) Number of employees living within 1 km from their place of work = 0

∴ P(employees living within 1 km from their place of work) = $\frac{0}{30} = 0$

(*d*) Number of employees living at a distance more than 25 km from their place of work = 0

∴ P(employees living at a distance more than 25 km from place of work) = $\frac{0}{30} = 0$

Long Answer Type Questions

9. The number of family members in 30 families were recorded as follows:

2, 1, 3, 4, 5, 2, 3, 1, 6, 2, 3, 2, 4, 3, 5, 6, 7, 2, 3, 4, 5, 4, 2, 2, 3, 5, 6, 4, 3, 2

Find, in percentage, the probability that a randomly chosen family has

(*a*) less than 6 members

(*b*) more than or equal to 4 members

(*c*) only 1 member

(*d*) more than 7 members.

Sol. Total number of families = 30

(*a*) Number of families having 'less than 6 members' = 26

∴ P(chosen family has 'less than 6 members')

$$= \frac{26}{30} \times 100\%$$

$$= \frac{260}{3}\%$$

$$= 86\frac{2}{3}\%$$

(*b*) Number of families having 'more than or equal to 4 members' = 13

∴ P(chosen family has 'more than or equal to 4 members')

$$= \frac{13}{30} \times 100\%$$

$$= \frac{130}{3}\% = 43\frac{1}{3}\%$$

(*c*) Number of families having 'only 1 member' = 2

∴ P(chosen family has 'only 1 member')

$$= \frac{2}{30} \times 100\% = \frac{20}{3}\% = 6\frac{2}{3}\%$$

(*d*) Number of families having 'more than 7 members' = 0

∴ P(chosen family has 'more than 7 members') = $\frac{0}{30} \times 100\% = 0\%$

10. There are 150 telephone numbers on one page of a telephone directory. The frequency distribution of their unit's digit is given below:

Unit's digit	0	1	2	3	4	5	6	7	8	9	Total
Number of telephones	17	13	7	15	27	38	10	11	9	3	150

One of the numbers is chosen at random from this page. What is the probability that the unit's digit of the chosen number is

(*a*) less than or equal to 3

(*b*) greater than or equal to 7.

Sol. Total number of telephone numbers on one page of directory = 150

(*a*) Number of telephone numbers having 'less than or equal to 3' as the unit's digit

= 17 + 13 + 7 + 15 = 52

∴ P(chosen number is 'less than or equal to 3')

$$= \frac{52}{150} = \frac{26}{75}$$

(*b*) Number of telephone numbers having 'greater than or equal to 7' as the unit's digit

= 11 + 9 + 3 = 23

∴ P(chosen number is 'greater than or equal to 7') = $\frac{23}{150}$

Higher Order Thinking Skills (HOTS) Questions

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1. A bag contains 3 red balls each bearing one of the numbers 1, 2 and 3; and 2 black balls each bearing one of the numbers 4 and 5. A ball is drawn, its number is noted and the ball is replaced in the bag. Then another ball is drawn and its number is noted.

Find the probability of drawing

(*a*) a number less than or equal to 4 on the first draw and 5 on the second draw

(*b*) a total of 7.

Sol. The balls are drawn with replacement.

∴ The sample space

$$S = \left\{ \begin{matrix} (1, 1), (1, 2), (1, 3), (1, 4), (1, 5) \\ (2, 1), (2, 2), (2, 3), (2, 4), (2, 5) \\ (3, 1), (3, 2), (3, 3), (3, 4), (3, 5) \\ (4, 1), (4, 2), (4, 3), (4, 4), (4, 5) \\ (5, 1), (5, 2), (5, 3), (5, 4), (5, 5) \end{matrix} \right\}$$

∴ The total number of sample space [n(S)] = 25

(*a*) Let E_1 be the event 'a number ≤ 4 on the first draw and 5 on the second draw'

$\Rightarrow \quad E_1 = \{(1, 5), (2, 5), (3, 5), (4, 5)\}$

$\Rightarrow \quad n(E_1) = 4$

∴ P(number ≤ 4 on the first draw and 5 on the second draw)

$$= P(E_1) = \frac{n(E_1)}{n(S)} = \frac{4}{25}$$

(*b*) Let E_2 be the event getting 'a total of 7'

$\Rightarrow \quad E_2 = \{(3, 4), (4, 3), (2, 5), (5, 2)\}$

$\Rightarrow \quad n(E_2) = 4$

$\therefore \quad P(\text{a total of 7}) = P(E_2) = \frac{n(E_2)}{n(S)} = \frac{4}{25}$

2. Two dice are thrown simultaneously.

Find the probability of getting

(*a*) an odd number on the first die and an even number on the second die.

(*b*) a number greater than or equal to 5 on each die.

(*c*) a total of 10.

(*d*) a total greater than or equal to 4 but less than 7.

Sol. Two dice are thrown simultaneously.

∴ The sample space

$$S = \begin{Bmatrix} (1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6) \\ (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6) \\ (3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6) \\ (4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6) \\ (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6) \\ (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6) \end{Bmatrix}$$

∴ n(S) = 36

(*a*) Let E_1 be the event getting 'an odd number on the first die and an even number on the second die'.

$E_1 = \{(1, 2), (1, 4), (1, 6), (3, 2), (3, 4)$
$(3, 6), (5, 2), (5, 4), (5, 6)\}$

$\Rightarrow \quad n(E_1) = 9$

$$\therefore \quad P(E_1) = \frac{n(E_1)}{n(S)} = \frac{9}{36} = \frac{3}{12} = \frac{1}{4}$$

(*b*) Let E_2 be the event getting 'a number greater than or equal to 5 on each die'.

$E_2 = \{(5, 5), (5, 6), (6, 5), (6, 6)\}$

$\Rightarrow \quad n(E_2) = 4$

$$\therefore \quad P(E_2) = \frac{n(E_2)}{n(S)} = \frac{4}{36} = \frac{1}{9}$$

(*c*) Let E_3 be the event getting 'a total of 10'

$E_3 = \{(4, 6), (5, 5), (6, 4)\}$

$\Rightarrow \quad n(E_3) = 3$

$$\therefore \quad P(E_3) = \frac{n(E_3)}{n(S)} = \frac{3}{36} = \frac{1}{12}$$

(*d*) Let E_4 be the event of getting 'a total greater than or equal to 4 but less than 7'.

$E_4 = \{(1, 3), (1, 4), (1, 5), (2, 2), (2, 3), (2, 4)$
$(3, 1), (3, 2), (3, 3), (4, 1), (4, 2), (5, 1)\}$

$\Rightarrow \quad n(E_4) = 12$

$$\therefore \quad P(E_4) = \frac{n(E_4)}{n(S)} = \frac{12}{36} = \frac{1}{3}$$

Self-Assessment

(Page 214)

Multiple-Choice Questions

1. The sum of the probabilities of all events of a trial is

(*a*) less than 1

(*b*) equal to 1

(*c*) greater than 1

(*d*) equal a number lying between 0 and 1.

Sol. (*b*) equal to 1

The sum of all the probabilities of all events of a trial is equal to 1.

2. A fair coin is tossed 50 times and the tail occurs 37 times. Then the probability of getting a head is

(*a*) 0.6 (*b*) 0.5

(*c*) 0.37 (*d*) 0.26

Sol. (*d*) 0.26

Since the coin is tossed 50 times.

∴ The total number of trials = 50

Let E be the event of 'getting a head'.

Number of times tail occurs = 37

∴ Number of times head occurs = 50 – 37 = 13

∴ The number of trails in which the event E happens = 13

P(getting a head) = P(E)

$$= \frac{\text{Number of trails in which the event E happens}}{\text{Total number of trials}}$$

$$= \frac{13}{50} = 0.26$$

Fill in the Blanks

3. The probability of an impossible event is **0**.

4. The probability of a sure event is **1**.

5. In n trials of a random experiment, if an event E happens m times, then P(E) is equal to $\boldsymbol{\frac{m}{n}}$.

Sol. $P(E) = \frac{\text{Number of trials in which E happened}}{\text{Total number of trials}}$

$$= \frac{m}{n}$$

6. The probability of an event of a trial is always between **0 and 1 (both inclusive)**.

Assertion-Reason Type Questions

Directions (Q. Nos. 7 to 10): Each of these questions contains an assertion followed by reason. Read them carefully, and answer the question on the basis of the following options, select the one that best describes the two statements.

(a) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.

(b) Both Assertion and Reason are true but Reason is not the correct explanation of Assertion.

(c) Assertion is true but Reason is false.

(d) Assertion is false but Reason is true.

7. **Assertion:** Coin tossing is a random experiment.

Reason: The result of coin tossing is not fixed and cannot be predicted.

Sol. (*a*)

Both assertion and reason are correct and reason is the correct explanation of assertion.

8. **Assertion:** Probability of a sure event is 1 and that of an impossible event is –1.

Reason: The probability of an impossible event is 0.

Sol. (*d*)

Probability of impossible event is 0.

∴ Assertion is wrong but reason is correct.

9. **Assertion:** When a die is thrown, probability of getting an even number is 0.5.

Reason: When a die is thrown, out of 6 possible outcomes, 3 are even numbers.

Sol. (*a*)

Both assertion and reason are correct and reason is correct explanation of assertion.

10. **Assertion:** A coin has the heavier head side and that makes probability of head as 0.8, then probability of tail is 0.2.

Reason: Total probability of all outcomes is 1.

Sol. (*a*)

Both assertion and reason are correct and reason is correct explanation of assertion.

Case Study Based Questions

11. Republic Day is celebrated on 26th January in every school. It is celebrated to mark the day when the constitution of India came into effect. In a school, Republic day was celebrated. Out of 1200 students, 480 students took part in this celebration. Based on the above information, answer the following questions.

(*a*) What is the probability of the number of students who participated in the celebration?

(*i*) $\frac{2}{5}$ (*ii*) $\frac{3}{5}$

(*iii*) $\frac{3}{2}$ (*iv*) $\frac{2}{3}$

Ans. (*i*) $\frac{2}{5}$

(*b*) What is the probability of the number of students who did not participate in the celebration?

(*i*) $\frac{2}{5}$ (*ii*) $\frac{3}{5}$

(*iii*) $\frac{1}{3}$ (*iv*) $\frac{3}{2}$

Ans. (*ii*) $\frac{3}{5}$

(*c*) What is the sum of the probabilities of all events of a trial?

(*i*) Greater than 1

(*ii*) Less than 1

(*iii*) 1

(*iv*) Between 0 and 1

Ans. (*iii*) 1

(*d*) What is the empirical probability of an event E?

(*i*) Number of trials in which the event E happened/The total number of trials

(*ii*) The total number of trials – Number of trials in which the event E happened

(*iii*) The total number of trials/Number of trials in which the event E happened

(*iv*) None of these

Ans. (*i*) Number of trials in which the event E happened/The total number of trials

(*e*) What is the probability of the students participated if 880 students took part in the celebration out of 1200 students?

(*i*) $\frac{4}{5}$ (*ii*) $\frac{9}{10}$

(*iii*) $\frac{11}{15}$ (*iv*) $\frac{17}{60}$

Ans. (*iii*) $\frac{11}{15}$

12. A mathematics teacher has recently joined a certain school. She wanted to analyse the performance of the class. So she decided to take a test and recorded the distribution of marks into the following table:

Marks	0–20	20–40	40–60	60–80	80–100
Number of students	7	13	11	10	9

Based on the above information, answer the following questions.

(*a*) What is the number of students who appeared for the test?

(*i*) 37 (*ii*) 39

(*iii*) 50 (*iv*) 60

Ans. (*iii*) 50

(*b*) What is the probability that a student obtained less than 40 marks in the test?

(*i*) $\frac{7}{50}$ (*ii*) $\frac{13}{50}$

(*iii*) $\frac{31}{50}$ (*iv*) $\frac{2}{5}$

Ans. (*iv*) $\frac{2}{5}$

(*c*) The probability of a sure event is

(*i*) more than 1

(*ii*) 1

(*iii*) less than 1

(*iv*) between 0 and 1

Ans. (*ii*) 1

(*d*) What is the probability that a student obtained 40 or more marks?

(*i*) $\frac{2}{5}$ (*ii*) $\frac{3}{5}$

(*iii*) $\frac{19}{50}$ (*iv*) $\frac{31}{50}$

Ans. (*ii*) $\frac{3}{5}$

(*e*) What is the probability that a student obtained 60 or more marks?

(*i*) $\frac{2}{5}$ (*ii*) $\frac{3}{5}$

(*iii*) $\frac{19}{50}$ (*iv*) $\frac{31}{50}$

Ans. (*iii*) $\frac{19}{50}$

Very Short Answer Type Questions

13. A die is thrown 200 times and the odd numbers are obtained 150 times. Then what is the probability of getting an even number?

Sol. Total number of times two dice are thrown simultaneously = 200

Let E be the event of getting 'an even number'.

Number of times odd numbers are obtained = 150

Number of times even numbers are obtained

= 200 – 150 = 50

∴ The number of trials in which the event E happens = 50

P(getting an even number) = P(E)

$$= \frac{\text{Number of trials in which the event E happens}}{\text{Total number of trials}}$$

$$= \frac{50}{200} = \frac{1}{4} = 0.25$$

14. Two coins are tossed simultaneously 100 times. Either 1 head or 2 heads occur 25 times. What is the probability of getting no head?

Sol. Number of times two coins are tossed simultaneously = 100

$\therefore$ Total number of trials = 100

Let E be the event of 'getting no head'.

Number of times either 1 head or 2 heads occur = 25

Then, the number of trials in which the event E happened = 100 – 25 = 75

$\therefore$ P(getting no head) = P(E)

$$= \frac{\text{Number of trials in which the event E happens}}{\text{Total number of trials}}$$

$$= \frac{75}{100} = 0.75$$

Short Answer Type-I Questions

15. Following is the distribution of marks of 50 students in a certain school test:

Marks below	10	20	30	40	50	60	70	80	90	100
Number of students	2	3	5	7	11	13	27	42	48	50

Find the probability that a student scores

(*a*) less than 20 marks

(*b*) marks less than 50 but greater than or equal to 20

(*c*) marks more than 90.

Sol. Frequency table

Marks	*Number of students*
0–10	2
10–20	1
20–30	2
30–40	2
40–50	4
50–60	2
60–70	14
70–80	15
80–90	6
90–100	2

Total number of students = 50

(*a*) Number of students scoring less than 20 marks = 3

$\therefore$ P(a student scoring less than 20 marks)

$$= \frac{3}{50} = 0.06$$

(*b*) Number of students scoring less than 50 marks but greater than or equal to 20 marks = 8

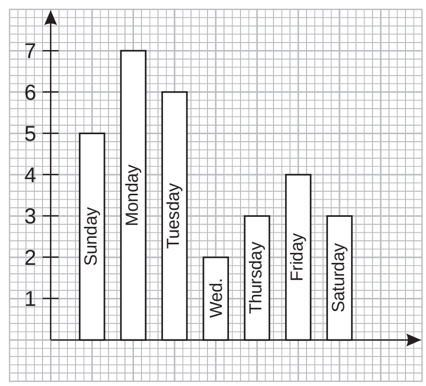
$\therefore$ P(a student scoring less than 50 but greater than or equal to 20 marks) = $\frac{8}{50}$ = 0.16

(*c*) Number of students scoring more than 90 marks = 2

P(a student scoring more than 90 marks)

$$= \frac{2}{50} = 0.04$$

16. The given bar graph provides information on the day of birth of 30 students. Read the bar and find the probability that a student of the class was born on Thursday.



Sol. Total number of students = 30

Number of students born on Thursday = 3

$\therefore$ P(a student was born on Thursday)

$$= \frac{3}{30}$$

$$= \frac{1}{10} = 0.1$$

Short Answer Type-II Questions

17. A die is thrown 20 times and the outcomes are noted as given below:

Outcome	1	2	3	4	5	6
Number of times	3	4	1	3	5	4

Find the probability of happening of each outcome and hence find the sum of all these probabilities.

Sol. Since a die is thrown 20 times.

$\therefore$ Total number of trials = 20

Number of times getting an outcome 1 = 3

P(getting an outcome 1) = $\frac{3}{20}$

Similarly,

P(getting an outcome 2) = $\frac{4}{20}$

P(getting an outcome 3) = $\frac{1}{20}$

P(getting an outcome 4) = $\frac{3}{20}$

P(getting an outcome 5) = $\frac{5}{20}$

P(getting an outcome 6) = $\frac{4}{20}$

Adding all the probabilities, we get

P(sum of all these probabilities)

$= \frac{3}{20} + \frac{4}{20} + \frac{1}{20} + \frac{3}{20} + \frac{5}{20} + \frac{4}{20} = \frac{20}{20} = 1$

18. In a cricket match, a batsman hits a boundary 4 times out of 30 balls he plays. Find the probability that he did not hit a boundary.

[CBSE SP 2012]

Sol. Total number of balls played = 30

Number of times the batsman hits the boundary = 4

Number of times the batsman does not hit the boundary = 30 – 4 = 26

$\therefore$ P(batsman did not hit a boundary) = $\frac{26}{30} = \frac{13}{15}$

19. In a survey of 250 students conducted by a school, it was found that 160 students like to participate in outdoor games, yoga and jogging while the remaining students like to watch TV instead. Based on the given situation, answer the following questions:

(*a*) Find the probability that a student chosen at random likes outdoor games, yoga and jogging.

(*b*) Find the probability that a student chosen at random likes to watch TV.

Sol. Total number of children = 250

(*a*) Number of children, who like outdoor games, yoga and jogging = 160

$\therefore$ P(chosen child likes outdoor games yoga and jogging) = $\frac{160}{250}$ = 0.64

(*b*) Number of children who like to watch TV instead = 250 – 160 = 90

P(chosen child likes to watch TV) = $\frac{90}{250}$

= 0.36

Long Answer Type Questions

20. In a single throw of two dice, what is the probability of getting a prime number on each die?

Sol. In a single throw of two dice, we have 6^2, i.e. 36 chances.

So, n(S) = 36

Let E be the event getting a prime number on each die.

$\Rightarrow$ E = {(2, 2), (2, 3), (2, 5), (3, 2), (3, 3), (3, 5) (5, 2), (5, 3), (5, 5)}

$\Rightarrow$ n(E) = 9

Thus, P(getting a prime number on each die)

= P(E) = $\frac{9}{36} = \frac{1}{4}$ = 0.25

21. The table given below shows the weekly pocket money of 50 students:

Pocket money (in ₹)	*Number of students*
115	10
125	10
140	8
150	5
175	7
200	6
250	4
Total	50

Find the probability that the weekly pocket money is

(*a*) ₹ 250 (*b*) less than ₹ 200

(*c*) less than ₹ 150 (*d*) ₹ 100

(*e*) more than ₹ 175.

Sol. Total number of students = 50

(*a*) Number of students whose weekly pocket money is ₹ 250 = 4

P(weekly pocket money is ₹ 250) = $\frac{4}{50}$ = 0.08

(*b*) Number of students whose weekly pocket money is less than ₹200 = 40

P(weekly pocket money less than ₹200)

$= \frac{40}{50} = \frac{20}{25} = \frac{4}{5} = 0.8$

(*c*) Number of students whose weekly pocket money is less than ₹150 = 28.

P(weekly pocket money less than ₹150)

$= \frac{28}{50} = \frac{14}{25} = 0.56$

(*d*) Number of students whose weekly pocket money is ₹ 100 = 0

P(weekly pocket money is ₹ 100) = $\frac{0}{50}$ = 0

(*e*) Number of students whose weekly pocket money is more than ₹ 175 = 10

P(weekly pocket money more than ₹ 175)

= $\frac{10}{50}$ = $\frac{1}{5}$ = 0.2

Let's Compete

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Multiple-Choice Questions

1. In a single throw of two dice, the probability of obtaining "a total of 7" is

(*a*) $\frac{1}{9}$ (*b*) $\frac{1}{7}$

(*c*) $\frac{1}{6}$ (*d*) $\frac{1}{8}$

Sol. (*c*) $\frac{1}{6}$

In the single throw of two dice, we have 6^2, i.e. 36 chances

So, $n(S) = 36$

Let E be the event of getting 'a total of 7'

⇒ E = {(3, 4), (4, 3), (2, 5), (5, 2), (1, 6), (6, 1)}

⇒ $n(E) = 6$

∴ P(getting a total of 7) = P(E)

$= \frac{6}{36} = \frac{1}{6}$

2. A tyre manufacturing company kept a record of the distance covered before a tyre needed to be replaced. The table below shows the result of 3000 cases.

Distance (in km)	*Frequency (number of cars)*
Less than 2000	360
2000 to 10000	600
10000 to 15000	1000
More than 15000	1040

The probability that the tyre need to be replaced before it has covered 2000 km is

(*a*) 0.2 (*b*) 0.12

(*c*) 0.68 (*d*) 0.53

Sol. (*b*) 0.12

Total number of case studies (of distance covered before a tyre needed to be replaced) = 3000

Number of cases in which tyre needed to be replaced before it covered 2000 km = 360

∴ P(tyre needs to be replaced before it has covered 2000 km) = $\frac{360}{3000} = \frac{6}{50} = \frac{3}{25}$ = 0.12

3. The table given below shows the number of hours of study put in per day, by 40 students of class IX of a school:

Number of hours spent daily for studying	1	2	3	4	More than 4 hours
Number of students	6	8	8	5	3

A student is selected at random. Then the probability that the number of hours spent daily by him/her for studying are less than 4 hours is

(*a*) 0.125 (*b*) 0.2

(*c*) 0.675 (*d*) 0.55

Sol. (*d*) 0.55

Total number of students = 40

Number of hours spent daily by him/her for studying less than 4 hours = 22

∴ P(a student studying less than 4 hours) = $\frac{22}{40}$

= 0.55

4. According to a survey conducted in a school, it was found that the ages of the teachers of that school are distributed as follows:

Age (in years)	20–29	30–39	40–49	50–59	60 and above
Number of teachers	5	30	45	20	0

If a teacher is selected at random, then the probability that the teacher's age (in years) is over 30 but under 50

(*a*) 0.75 (*b*) 0.45

(*c*) 0.30 (*d*) 0.15

Sol. (*a*) 0.75

Total number of teachers = 100

Number of teacher's whose age (in years) is over 30 but under 50 = 75

P(teacher's age is over 30 but under 50) = $\frac{75}{100}$

$= \frac{3}{4}$

= 0.75

5. Three coins are tossed simultaneously 100 times with the following frequencies:

Outcome	*Number of times (frequency)*
3 Heads	40
2 Heads	42
1 Head	8
No Head	10
Total	100

Then the probability of getting at most two heads is

(*a*) 0.18 (*b*) 0.40
(*c*) 0.6 (*d*) 0.42

Sol. (*c*) 0.6

Since the three coins are tossed simultaneously 100 times.

$\therefore$ The total number of trials = 100

Let E be the event of getting at most two heads.

Then, the number of trials in which the event E happens = 60

$\therefore$ P(getting at most two heads) = P(E)

$$= \frac{60}{100} = 0.6$$

6. Four coins are tossed once. There are 16 possible outcomes, for instance, HHHH, HHHT, HHTH, HHTT etc. Then the probability of getting at least 3 tails is

(*a*) $\frac{3}{16}$ (*b*) $\frac{1}{16}$
(*c*) $\frac{4}{16}$ (*d*) $\frac{5}{16}$

Sol. (*d*) $\frac{5}{16}$

Total number of outcomes in tossing four coins once = 16

Let E be the event of getting at least 3 tails.

E = {TTTH, THTT, TTHT, TTHT, HTTT, TTTT}

Number of times the event E happens = 5

$\therefore$ P(getting at least 3 tails) = P(E) = $\frac{5}{16}$

7. 100 seeds were selected at random from each of 5 bags of seeds and were kept under standardised conditions favourable to germination. After 20 days, the number of seeds which had germinated in each collection were counted and recorded as follows:

Bag	1	2	3	4	5
Number of seeds germinated	40	50	30	60	20

Then the probability of germination of more than 15 but less than or equal to 40 seeds in a bag is

(*a*) 0.6 (*b*) 0.5
(*c*) 0.4 (*d*) 0.8

Sol. (*a*) 0.6

Total number of bags of seeds = 5

Number of bags having more than 15 but less than or equal to 40 germinated seeds = 3

$\therefore$ P(germination of more than 15 but less than or equal to 40 seeds in a bag) = $\frac{3}{5}$ = 0.6

8. The probability that a leap year, selected at random, will contain 53 Sundays is

(*a*) $\frac{1}{7}$ (*b*) $\frac{2}{7}$
(*c*) $\frac{53}{366}$ (*d*) $\frac{53}{365}$

Sol. (*b*) $\frac{2}{7}$

Let E be the event 'leap year has 53 Sundays'.

We know that

Leap year has 366 days = 52 weeks + 2 day

Possibility or remaining 2 days can be.

(*i*) Monday and Tuesday
(*ii*) Tuesday and Wednesday
(*iii*) Wednesday and Thursday
(*iv*) Thursday and Friday
(*v*) Friday and Saturday
(*vi*) Saturday and Sunday
(*vii*) Sunday and Monday

$\therefore$ The total number of pairs = 7 out of which only two pairs of days contain Sundays i.e., Sunday and Monday and Saturday and Sunday

$\therefore$ Probability of getting 1 Sunday out of these 7 pairs of days = $\frac{2}{7}$

Hence, P(E) = $\frac{2}{7}$.

9. A bag contains 5 red balls and some blue balls. If the probability of drawing a blue ball is double that of a red ball, then the number of blue balls in the bag is

(*a*) 5 (*b*) 9
(*c*) 10 (*d*) 8

Sol. (*c*) 10

Let the number of blue balls in the bag = x

The number of red balls in the bag = 5

$\therefore$ Total number of balls in the bag = $x + 5$

Out of $(x + 5)$ balls, one ball can be drawn in $(x + 5)$ ways.

$\therefore$ Total number of possible outcomes = $(x + 5)$

The number of favourable ways of 'getting a red ball' = 5

$\therefore$ P(red ball) = $\frac{5}{x+5}$

The number of favourable ways of 'getting a blue ball' = x

$\therefore$ P(blue ball) = $\frac{x}{x+5}$

According to the problem:

P(blue ball) = 2 P(red ball)

$\Rightarrow \quad \frac{x}{x+5} = 2\left(\frac{5}{x+5}\right)$

$\Rightarrow \quad x^2 + 5x = 10(x + 5)$

$\Rightarrow \quad x^2 + 5x = 10x + 50$

$\Rightarrow \quad x^2 - 5x - 50 = 0$

$\Rightarrow \quad x^2 - 10x + 5x - 50 = 0$

$\Rightarrow \quad x(x - 10) + 5(x - 10) = 0$

$\Rightarrow \quad (x - 10)(x + 5) = 0$

$\Rightarrow$ Either $x - 10 = 0$ or $x + 5 = 0$

$\Rightarrow \quad x = 10$ or $x = -5$ [Rejected]

Hence, there are 10 blue balls in the bag.

10. Two dice are tossed once. The probability of getting a multiple of 3 on the first die or total of 10 in both the dice is

(*a*) $\frac{7}{18}$ (*b*) $\frac{5}{18}$
(*c*) $\frac{1}{3}$ (*d*) $\frac{2}{3}$

Sol. (*a*) $\frac{7}{18}$

Total number of possible outcomes = 36

Let E be the event of getting a multiple of 3 on the first die or total of 10 in both the dice.

$\Rightarrow$ E = {(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 5), (3, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6) (4, 6), (5, 5)}

$\Rightarrow n(E) = 14$

$\therefore$ P(getting a multiple of 3 on the first die or total of 10 in both the dice) = $\frac{14}{36} = \frac{7}{18}$

Value-based Question (Optional)

(Page 218)

1. 30 college students offered to donate blood for their fellow student. Their blood groups were as given in the table:

Blood group	*Number of students*
A	7
B	8
O	9
AB	6

(*a*) One student of this group is chosen at random. What is the probability that the chosen student has a blood group

(*i*) A (*ii*) B (*iii*) O and (*iv*) AB?

(*b*) What values are depicted by these students?

Sol. Total number of college students = 30

(*a*) (*i*) Number of students having blood group A = 7

$\therefore$ P(chosen student has A blood group) $= \frac{7}{30}$

(*ii*) Number of students having blood group B = 8

$\therefore$ P(chosen student has B blood group) $= \frac{8}{30} = \frac{4}{15}$

(*iii*) Number of students having blood group O = 9

$\therefore$ P(chosen student has O blood group) $= \frac{9}{30} = \frac{3}{10}$

(*iv*) Number of students having blood group AB = 6

$\therefore$ P(chosen student has AB blood group) $= \frac{6}{30} = \frac{2}{10} = \frac{1}{5}$

(*b*) Compassion, empathy, helpfulness and responsible behavior.