Sample Question Paper

Standard (Code 041)

ANSWERS

Section - A

1. (*b*)	2. (*c*)	3. (*c*)
4. (*b*)	5. (*a*)	6. (*a*)
7. (*b*)	8. (*d*)	9. (*c*)
10. (*a*)	11. (*d*)	12. (*c*)
13. (*b*)	14. (*a*)	15. (*d*)
16. (*b*)	17. (*a*)	18. (*c*)
19. (*d*)	20. (*b*)	

Section - B

21. Let us assume on the contrary that $\sqrt{7}-\sqrt{2}$ is a rational number. Then, there exist coprime a and b ($b \neq 0$), such that

$$\sqrt{7}-\sqrt{2} = \frac{a}{b} \quad \Rightarrow \quad \frac{a}{b}+\sqrt{2}=\sqrt{7}$$

$$\Rightarrow \quad \left(\frac{a}{b}+\sqrt{2}\right)^2 = (\sqrt{17})^2 \quad \Rightarrow \quad \frac{a^2}{b^2}+\frac{2a\sqrt{2}}{b}+2 = 7$$

$$\Rightarrow \quad \frac{a^2}{b^2}-5 = \frac{-2a\sqrt{2}}{b} \quad \Rightarrow \quad \frac{a^2-5b^2}{b^2} = \frac{-2a\sqrt{2}}{b}$$

$$\Rightarrow \quad \frac{a^2-5b^2}{2ab} = -\sqrt{2} \quad \Rightarrow \quad \sqrt{2} \text{ is a rational number.}$$

This contradicts the fact that $\sqrt{2}$ is irrational.

So, our assumption is wrong.

Hence, $\sqrt{7}-\sqrt{2}$ is irrational.

22. In ΔPST and ΔRQT,

$$\angle PTS = \angle QTR \qquad \text{[Vertically opp. } \angle s]$$

$$\angle PST = \angle RQT \qquad \text{[PS || QR, alternate angles]}$$

$$\therefore \quad \Delta PST \sim \Delta RQT \qquad \text{[By AA criterion of similarity]}$$

$$\Rightarrow \quad \frac{PS}{ST} = \frac{RQ}{TQ} \quad \Rightarrow \quad \frac{18}{13} = \frac{36}{TQ} \quad \Rightarrow \quad TQ = 26 \text{ cm}$$

In ΔQUT and ΔQPS,

$$\angle QUT = \angle QPS \quad \text{[Corresponding angles, TU || PS]}$$

$$\angle QTU = \angle QSP \quad \text{[Corresponding angles, TU || PS]}$$

$$\therefore \quad \Delta QUT \sim \Delta QPS \quad \text{[By AA criterion of similarity]}$$

$$\therefore \quad \frac{QT}{QS} = \frac{UT}{PS} \Rightarrow \frac{26}{39} = \frac{UT}{18} \quad \text{[QS = ST + QT]}$$

$$\Rightarrow \quad UT = 12 \text{ cm}$$

Hence, TQ = 26 cm and UT = 12 cm.

23. Since the tangent at any point of a circle is perpendicular to the radius through the point of contact, therefore $CQ \perp XZ$ and $CP \perp XY$

$$\Rightarrow \quad \angle X = \angle CPX = \angle CQX = 90^\circ$$

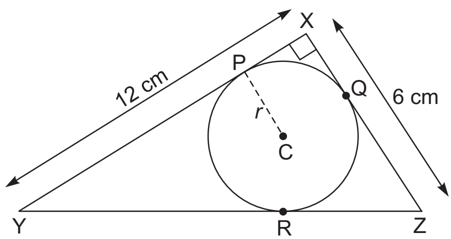
$$\therefore \quad CP = XQ = XP = r \quad \text{[Radius of circle]}$$

Now, $$ZQ = ZR = XZ - XQ = (16 \text{ cm} - r) \quad ...(1)$$

$$YR = YP = XY - XP = (12 \text{ cm} - r) \quad ...(2)$$

$$\therefore \quad YZ = ZR + YR = 16 \text{ cm} - r + 12 \text{ cm} - r = 28 \text{ cm} - 2r$$

[Using (1) and (2)] ...(3)



Now, $$YZ^2 = XZ^2 + XY^2$$

$$\Rightarrow \quad (28 \text{ cm} - 2r)^2 = 16^2 \text{ cm}^2 + 12^2 \text{ cm}^2 \quad \text{[Using (3)]}$$

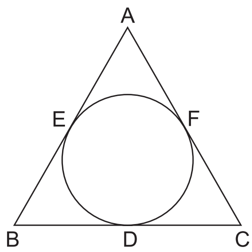
$$\Rightarrow \quad (28 \text{ cm} - 2r)^2 = 20^2 \text{ cm}^2$$

$$\Rightarrow \quad 28 \text{ cm} - 2r = 20 \text{ cm} \Rightarrow r = 4 \text{ cm}$$

Hence, r = 4 cm.

or

Let side BC of isosceles ΔABC touch circle at D.



Given: AB = AC

To prove: BD = DC

Proof: Since the lengths of tangents drawn from an external point to circle are equal,

$\therefore$ AE = AF

Similarly, BE = BD and CF = CD

Now, AB – AE = AC – AF

$\Rightarrow$ BE = CF

$\therefore$ BD = DC [$\because$ BE = BD and CF = DC]

Hence, BC is bisected at point of contact.

24. $\operatorname{cosec}\theta - \sin\theta = p$ and $\sec\theta - \cos\theta = q$ [Given]

$$\Rightarrow \quad \frac{1}{\sin\theta} - \sin\theta = p \quad \text{and} \quad \frac{1}{\cos\theta} - \cos\theta = q$$

$$\Rightarrow \quad \frac{1-\sin^2\theta}{\sin\theta} = p \quad \text{and} \quad \frac{1-\cos^2\theta}{\cos\theta} = q$$

$$\Rightarrow \quad \frac{\cos^2\theta}{\sin\theta} = p \quad \text{and} \quad \frac{\sin^2\theta}{\cos\theta} = q$$

$$\text{LHS} = p^{4/3}q^{2/3} + p^{2/3}q^{4/3}$$

$$= \left(\frac{\cos^2\theta}{\sin\theta}\right)^{4/3}\left(\frac{\sin^2\theta}{\cos\theta}\right)^{2/3} + \left(\frac{\cos^2\theta}{\sin\theta}\right)^{2/3}\left(\frac{\sin^2\theta}{\cos\theta}\right)^{4/3}$$

$$= \frac{(\cos\theta)^{8/3-2/3}}{(\sin\theta)^{4/3-4/3}} + \frac{(\cos\theta)^{4/3-4/3}}{(\sin\theta)^{2/3-8/3}}$$

$$= \frac{(\cos\theta)^{6/3}}{1} + \frac{1}{(\sin\theta)^{-6/3}}$$

$$= \cos^2\theta + \sin^2\theta = 1 = \text{RHS}$$

Hence, $p^{4/3}q^{2/3} + p^{2/3}q^{4/3} = 1$

or

$$\text{LHS} = \sin^6\theta + \cos^6\theta$$

$$= (\sin^2\theta)^3 + (\cos^2\theta)^3$$

$$= (\sin^2\theta + \cos^2\theta)(\sin^4\theta + \cos^4\theta - \sin^2\theta\cos^2\theta)$$

$$= \sin^4\theta + \cos^4\theta - \sin^2\theta\cos^2\theta \quad [\sin^2\theta + \cos^2\theta = 1]$$

$$= \sin^4\theta + \cos^4\theta + 2\sin^2\theta\cos^2\theta - 2\sin^2\theta\cos^2\theta - \sin^2\theta\cos^2\theta$$

$$= (\sin^2\theta + \cos^2\theta)^2 - 3\sin^2\theta\cos^2\theta$$

$$= 1^2 - 3\sin^2\theta\cos^2\theta$$

$$= 1 - 3\sin^2\theta\cos^2\theta = \text{RHS}$$

25. Let the diameters of concentric circle be $4d$ and $5d$.

$\therefore$ Radii are $2d$ and $\frac{5}{2}d$.

Ratio of areas of the two regions

$$\begin{aligned}
&= \text{Area of region I : Area of region II} \\
&= \pi(2d)^2 : \pi\left(\frac{5}{2}d\right)^2 - \pi(2d)^2 \\
&= 4d^2 : \frac{25}{4}d^2 - 4d^2 \\
&= 4d^2 : \frac{25-16}{4}d^2 \\
&= 4d^2 : \frac{9}{4}d^2 \\
&= 16 : 9
\end{aligned}$$

Hence, the ratio of the areas of these regions is 16 : 9.

Section - C

26. Time required for the bells to ring together is the LCM of 2, 4, 6, 8, 10 and 12 (in minutes).

LCM of 2, 4, 6, 8, 10, 12 = 120

$\therefore$ After every 120 minutes or 2 hours bells toll together

Required number of times = $\frac{12}{2} + 1$ = 7 times

Hence, they rang together between 9 a.m. and 9 p.m. 7 times.

27. Since α and β are the zeroes of the polynomial

$$f(x) = px^2 + qx + r$$

$$\alpha + \beta = \frac{-q}{p}, \alpha\beta = \frac{r}{p} \quad \text{...(1)}$$

$$\therefore \quad (\alpha+\beta)^2 = \frac{q^2}{p^2} \Rightarrow \alpha^2 + \beta^2 + 2\alpha\beta = \frac{q^2}{p^2}$$

$$\Rightarrow \quad \alpha^2 + \beta^2 + \frac{2r}{p} = \frac{q^2}{p^2}$$

$$\Rightarrow \quad \alpha^2 + \beta^2 = \frac{q^2}{p^2} - \frac{2r}{p} \quad \text{...(2)}$$

Now, $p\left(\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}\right) + q\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right) = p\left(\frac{\alpha^3 + \beta^3}{\alpha\beta}\right) + q\left(\frac{\alpha^2 + \beta^2}{\alpha\beta}\right)$

$$= p\,\frac{(\alpha+\beta)(\alpha^2+\beta^2-\alpha\beta)}{\alpha\beta} + q\,\frac{(\alpha^2+\beta^2)}{\alpha\beta}$$

$$= \frac{p\left(\frac{-q}{p}\right)\left(\frac{q^2}{p^2}-\frac{2r}{p}-\frac{r}{p}\right)}{r/p} + \frac{q\left(\frac{q^2}{p^2}-\frac{2r}{p}\right)}{r/p}$$

[Using (1) and (2)]

$$= \frac{(-q)(q^2-3rp)\times p}{p^2\times r} + \frac{q(q^2-2pr)\times p}{p^2\times r}$$

$$= \frac{-q^3+3prq+q^3-2prq}{pr}$$

$$= \frac{prq}{pr} = q$$

28. Let the total number of students be x and number of rows be y.

Number of students in each row = $\frac{x}{y}$

Total number of students = No. of rows × No. of students in each row

$$\Rightarrow \quad x = (y-1)\left(\frac{x}{y}+2\right)$$

$$\Rightarrow \quad x = x + 2y - \frac{x}{y} - 2 \quad \Rightarrow \quad 2y - \frac{x}{y} - 2 = 0 \qquad ...(1)$$

Now, 2 students are removed.

$$x = (y+2)\left(\frac{x}{y}-2\right)$$

$$\Rightarrow \quad x = x - 2y + \frac{2x}{y} - 4 \quad \Rightarrow \quad 2y - \frac{2x}{y} + 4 = 0 \qquad ...(2)$$

Multiplying equation (1) by 2 and then subtracting the result from equation (2), we get,

$$\Rightarrow \quad 2y - \frac{2x}{y} + 4 - \left(4y - \frac{2x}{y} - 4\right) = 0$$

$$\Rightarrow \quad -2y + 8 = 0 \quad \Rightarrow \quad y = 4$$

Substituting $y = 4$ in equation (1), we get

$$2 \times 4 - \frac{x}{4} - 2 = 0$$

$$\Rightarrow \quad 6 - \frac{x}{4} = 0 \quad \Rightarrow \quad x = 24$$

Hence, the total number of students in parade is 24.

or

Suppose 1 man can finish the work in x days and 1 woman can finish it in y days.

$$1 \text{ man's } 1 \text{ day work} = \frac{1}{x}$$

$$1 \text{ woman's } 1 \text{ day work} = \frac{1}{y}$$

8 men and 12 women can finish the work in 10 days

$$\therefore \quad \frac{8}{x} + \frac{12}{y} = \frac{1}{10} \quad ...(1)$$

6 men and 8 women can finish the work in 14 days

$$\therefore \quad \frac{6}{x} + \frac{8}{y} = \frac{1}{14} \quad ...(2)$$

Multiply equation (1) by 2 and equation (2) by 3

$$\Rightarrow \quad \frac{16}{x} + \frac{24}{y} = \frac{1}{5} \quad ...(3)$$

$$\Rightarrow \quad \frac{18}{x} + \frac{24}{y} = \frac{3}{14} \quad ...(4)$$

Subtracting equation (3) from equation (4)

$$\Rightarrow \quad \frac{18}{x} - \frac{16}{x} = \frac{3}{14} - \frac{1}{5}$$

$$\Rightarrow \quad \frac{2}{x} = \frac{1}{70} \Rightarrow x = 140$$

Substituting $x = 140$ in equation (2), we get

$$\frac{6}{140} + \frac{8}{y} = \frac{1}{14} \Rightarrow \frac{8}{y} = \frac{1}{14} - \frac{6}{140} \Rightarrow \frac{8}{y} = \frac{4}{140}$$

$$\Rightarrow \quad y = 280$$

Hence, time taken by one man alone to finish the work is 140 days.

And time taken by one woman alone to finish the work is 280 days.

29. Suppose point of intersection of chord QR and PC is M. Let tangent PQ be x and PM be y.

$$\therefore \quad QM = \frac{QR}{2} = \frac{16}{2} \text{ cm} = 8 \text{ cm} \quad ...(1)$$

[PC is perpendicular bisector of QR]

In ΔQCM, $CM^2 = QC^2 - QM^2$ [By Pythagoras theorem]

$\Rightarrow$ $CM = \sqrt{10^2 - 8^2} \text{ cm} = 6 \text{ cm}$ [Using (1)]

In right ΔPMQ, $PQ^2 = MP^2 + QM^2$

$$\Rightarrow \quad x^2 = y^2 + 64 \text{ cm}^2 \quad ...(2)$$

$$\angle PQC = 90°$$

[Radius through the point of contact is perpendicular to the tangent]

In right ΔPQC, $PC^2 = PQ^2 + QC^2$

$$\Rightarrow \quad (PM + CM)^2 = x^2 + 10^2 \text{ cm}^2$$

$$\Rightarrow \quad (y + 6)^2 = x^2 + 100 \quad \Rightarrow \quad y^2 + 36 + 12y = y^2 + 64 + 100$$

[Using (2)]

$$\Rightarrow \quad 12y = 164 - 36 \quad \Rightarrow \quad y = 10.67$$

Putting value of y in equation (2)

$$x^2 = (10.67)^2 + 64 = 177.85$$

$$\Rightarrow \quad x = 13.33$$

Hence, PQ = 13.3 cm.

30. **Given:** $\sec\theta + \tan\theta = a$ ...(1)

Since $\sec^2\theta - \tan^2\theta = 1 \quad \Rightarrow \quad (\sec\theta - \tan\theta)(\sec\theta + \tan\theta) = 1$

$$\Rightarrow \quad \sec\theta - \tan\theta = \frac{1}{a} \qquad \text{...(2)}$$

Adding equations (1) and (2), we get,

$$2\sec\theta = a + \frac{1}{a} \quad \Rightarrow \quad \sec\theta = \frac{a^2+1}{2a} \qquad \text{...(3)}$$

Subtracting equation (2) from (1), we get,

$$2\tan\theta = a - \frac{1}{a} \quad \Rightarrow \quad \tan\theta = \frac{a^2-1}{2a} \qquad \text{...(4)}$$

Now, $$\operatorname{cosec}\theta = \frac{\sec\theta}{\tan\theta} = \frac{\frac{a^2+1}{2a}}{\frac{a^2-1}{2a}} \qquad \text{[Using (3) and (4)]}$$

$$= \frac{a^2+1}{a^2-1}$$

31. Maximum class frequency 15 is of class 60–70 and mode is 67. So, modal class is 60–70.

$$\therefore \quad l = 60, h = 10, f_1 = 15, f_2 = 12, f_0 = x$$

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$\Rightarrow \quad 67 = 60 + \left(\frac{15 - x}{2 \times 15 - x - 12}\right) \times 10$$

$$\Rightarrow \quad 7(18 - x) = (15 - x) \times 10$$

$$\Rightarrow \quad 126 - 7x = 150 - 10x$$

$$\Rightarrow \quad 126 - 150 = -10x + 7x$$

$$\Rightarrow \quad -24 = -3x$$

$$\Rightarrow \quad x = 8$$

Hence, value of x is 8.

or

31. Here, $h = 10$. Let the assumed mean be $a = 55$.

Class Marks	*Mid value (x_i)*	*Number of students (f_i)*	$u_i = \dfrac{x_i - a}{h}$	$f_i u_i$
0 – 10	5	5	$\dfrac{5-55}{10} = -5$	– 25
10 – 20	15	9 – 5 = 4	$\dfrac{15-55}{10} = -4$	– 16
20 – 30	25	17 – 9 = 8	$\dfrac{25-55}{10} = -3$	– 24
30 – 40	35	29 – 17 = 12	$\dfrac{35-55}{10} = -2$	– 24
40 – 50	45	44 – 29 = 15	$\dfrac{45-55}{10} = -1$	– 15
50 – 60	55 = a	60 – 44 = 16	$\dfrac{55-55}{10} = 0$	0
60 – 70	65	70 – 60 = 10	$\dfrac{65-55}{10} = 1$	10
70 – 80	75	78 – 70 = 8	$\dfrac{75-55}{10} = 2$	16
80 – 90	85	83 – 78 = 5	$\dfrac{85-55}{10} = 3$	15
90 – 100	95	85 – 83 = 2	$\dfrac{95-55}{10} = 4$	8
		$\Sigma f_i = 85$		$\Sigma f_i u_i = -55$

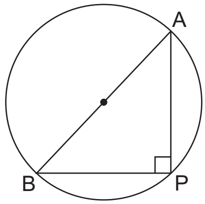
$$\text{Mean, } \bar{x} = a + h\bar{u} = a + h\left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right)$$

$$= 55 + 10 \times \left(\frac{-55}{85}\right) = 48.53$$

Hence, mean marks scored by the students is 48.53.

Section - D

32. Let P be the required location of pole such that its distance from gate B is x.



$\therefore$ $BP = x \Rightarrow AP = x + 7$

[Difference of the two distances is 7 m]

In right ΔAPB, $AB^2 = BP^2 + AP^2 \Rightarrow 17^2 = x^2 + (x+7)^2$

$\Rightarrow 289 = x^2 + x^2 + 49 + 14x$

$\Rightarrow 2x^2 + 14x - 240 = 0$

$\Rightarrow x^2 + 7x - 120 = 0$

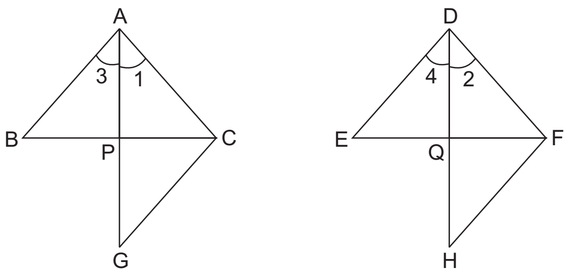
$\Rightarrow (x + 15)(x - 8) = 0$

$\Rightarrow x = -15$ or $x = 8$

Distance can't be negative.

Hence, pole is to be erected at 8 m from gates.

33. **Given:** ΔABC and ΔDEF in which AP and DQ are the medians respectively, such that $\frac{AB}{DE} = \frac{AC}{DF} = \frac{AP}{DQ}$



To prove: $\Delta ABC \sim \Delta DEF$

Proof: Produce AP to G so that PG = AP and join CG.

Similarly, DQ = QH and join FH.

In ΔAPB and ΔGPC, we have

$BP = CP$ [AP is median]

$AP = GP$

$\angle APB = \angle CPG$ [Vertically opposite angles]

$\therefore \quad \Delta APB \cong \Delta GPC$ [By SAS criterion of congruence]

$\therefore \quad AB = CQ$

[Corresponding sides of congruent triangle are equal]

Similarly, $\Delta DQE \cong \Delta HQF$ [BY SAS criterion of congruence]

$\therefore \quad DE = HF$

[Corresponding sides of congruent triangle are equal]

Now, $\dfrac{AB}{DE} = \dfrac{AC}{DF} = \dfrac{AP}{DQ}$

$\Rightarrow \quad \dfrac{GC}{HF} = \dfrac{AC}{DF} = \dfrac{AP}{DQ}$

$\Rightarrow \quad \dfrac{GC}{HF} = \dfrac{AC}{DF} = \dfrac{2AP}{2DQ} \quad \Rightarrow \quad \dfrac{GC}{HF} = \dfrac{AC}{DF} = \dfrac{AG}{DH}$

$\therefore \quad \Delta AGC \sim \Delta DHF$ [By SSS criterion of similarity]

$\therefore \quad \angle 1 = \angle 2$ [Corresponding angles of similar triangles]

Similarly, $\angle 3 = \angle 4$

$\Rightarrow \quad \angle 1 + \angle 3 = \angle 2 + \angle 4$

$\Rightarrow \quad \angle A = \angle D$

In ΔABC and ΔDEF

$\angle A = \angle D$

$\dfrac{AB}{DE} = \dfrac{AC}{DF}$

$\therefore \quad \Delta ABC \sim \Delta DEF$ [By SAS criterion of similarity]

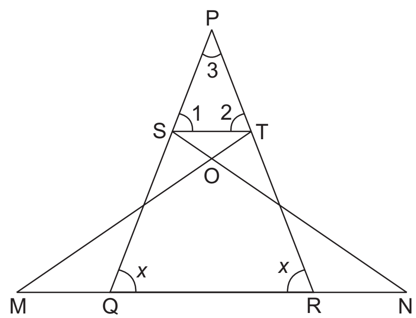
or

$\Delta NSQ \cong \Delta MTR$ [Given]

$\therefore \quad \angle SQN = \angle TRM$

[Corresponding angles of congruent triangles]

$\Rightarrow \quad \angle PQR = \angle PRQ = x$ ...(1)



In ΔPQR

$$\angle PQR + \angle PRQ + \angle QPR = 180^\circ$$
$$x + x + \angle 3 = 180^\circ$$
$$2x = 180^\circ - \angle 3 \Rightarrow x = \frac{180^\circ - \angle 3}{2} \quad ...(2)$$

In ΔPST

$$\angle 1 + \angle 2 + \angle 3 = 180^\circ$$
$$2\angle 1 = 180^\circ - \angle 3 \Rightarrow \angle 1 = \frac{180^\circ - \angle 3}{2} \quad ...(3)$$

From (1), (2) and (3)

$$\angle 1 = x$$
$$\therefore \quad \angle 1 = \angle 2 = x = \angle PQR = \angle PRQ$$

In ΔPTS and ΔPRQ

$$\angle 1 = \angle PQR \quad \text{[Proved above]}$$
$$\angle 2 = \angle PRQ$$
$$\angle SPT = \angle QPR \quad \text{[Common]}$$

Hence, $\Delta PTS \sim \Delta PRQ$ [By AAA criterion of similarity]

34. Volume of object = Volume of cone + Volume of hemisphere

$$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$$
$$= \left(\frac{1}{3}\times\pi\times 5^2\times 5 + \frac{2}{3}\times\pi\times 5^3\right)\text{cm}^3$$
$$= \frac{1}{3}\pi\times 5^3\times 3 \text{ cm}^3 = \pi(5)^3 \text{ cm}^3$$

Volume of object = Volume of water displaced in the cylinder

$$\pi(5)^3 \text{ cm}^3 = \pi R^2 H$$

[R is radius of cylinder and H is the increase in water level]

$$\Rightarrow \quad (5)^3 \text{ cm}^3 = R^2 \times 4 \text{ cm}^3$$
$$\Rightarrow \quad R = 5.1 \text{ cm}$$

Hence, diameter of container is 11.2 cm.

or

(*i*) Let r_1 be the radius of cylinder and h_1 be the height of cylinder. Then, radius of cone be r_2 and height of cone be h_2.

Then $r_1 = \frac{7}{2}$ cm, $h_1 = 15$ cm, $r_2 = 3$ cm, $h_2 = 4$ cm.

$$\text{Slant height} = \sqrt{r_2^2 + h_2^2} = \sqrt{3^2 + 4^2} \text{ cm} = \sqrt{25} \text{ cm} = 5 \text{ cm}$$
$$\therefore \quad l = 5 \text{ cm}$$

Surface area of remaining solid

$$= 2\pi r_1 h_1 + 2\pi r_1^2 - 2\pi r_2^2 + 2\pi r_2 l$$

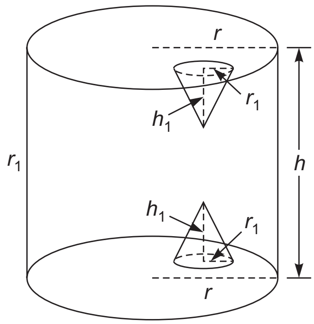
$$= 2 \times \pi(r_1 h_1 + r_1^2 - r_2^2 + r_2 l)$$

$$= 2 \times \frac{22}{7}\left[\frac{7}{2} \times 15 + \frac{49}{4} - 9 + 15\right] \text{ cm}^2$$

$$= 444.7 \text{ cm}^2 \text{ (approx.)}$$

(*ii*) Let r and h be respectively the radius and the height of the solid cylinder and let r_1 and h_1 be the radius of the base and the height respectively of each of two identical conical holes at the two ends of the cylinder so that

$$r = \frac{7}{2} \text{ cm}, h = 15 \text{ cm}, r_1 = 3 \text{ cm and } h_1 = 4 \text{ cm.}$$



Now, volume of the cylinder

$$= \pi r^2 h$$

$$= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 15 \text{ cm}^3$$

$$= 577.5 \text{ cm}^3 \quad ...(1)$$

Sum of the volumes of two identical conical holes at the two ends of the cylinder

$$= \frac{2}{3} \times \pi r_1^2 h_1$$

$$= \frac{2}{3} \times \frac{22}{7} \times 3 \times 3 \times 4 \text{ cm}^3$$

$$= 75.43 \text{ cm}^3 \quad ...(2)$$

∴ Required volume of the cylinder excluding the two conical holes

$$= (577.5 - 75.43) \text{ cm}^3 \quad \text{[From (1) and (2)]}$$

$$= 502.07 \text{ cm}^3$$

35. Here, $h = 3$. Let the assumed mean be $a = 11.5$.

Class	*Mid-value x_i*	*Frequency f_i*	*Cumulative frequency*	$u_i = \frac{x_i - a}{h}$	$f_i u_i$
1 – 4	2.5	6	6	– 3	– 18
4 – 7	5.5	30	36	– 2	– 60
7 – 10	8.5	40	76	– 1	– 40
10 – 13	11.5 = a	16	92	0	0
13 – 16	14.5	4	96	1	4
16 – 19	17.5	4	100	2	8
		$\Sigma f_i = 100$			$\Sigma f_i u_i = -106$

$$\text{Mean, } \bar{x} = a + h \times \left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right) = 11.5 + 3 \times \left(\frac{-106}{100}\right) = 8.32$$

Now, $n = 100 \Rightarrow \frac{n}{2} = 50$

Cumulative frequency greater than 50 is 76 and corresponding class is 7 – 10. Thus, median class is 7 – 10.

$$l = 7, h = 3, f = 40, cf = 36, \frac{n}{2} = 50$$

$$\text{Median} = l + h \times \frac{\left(\frac{n}{2} - cf\right)}{f}$$

$$= 7 + 3 \times \frac{(50 - 36)}{40}$$

$$= 7 + \frac{21}{20} = 8.05$$

$$\text{Mode} = 3 \text{ Median} - 2 \text{ Mean}$$

$$= 3 \times 8.05 - 2 \times 8.32 = 24.15 - 16.64 = 7.51$$

Hence, the mean number of alphabets in the names is 8.32, median is 8.05 and modal size is 7.51.

Section - E

36. (*i*) Number of passengers in first coach, $a_1 = 115$

Number of passengers in second coach, $a_2 = 130$

Difference of passengers between 2 coaches, $d = 130 - 115 = 15$

Number of passengers in tenth coach,

$$a_n = a_1 + (n - 1)d$$

$$\Rightarrow \quad a_{10} = 115 + (10 - 1) \times 15$$

$\Rightarrow \qquad a_{10} = 115 + 9 \times 15 \quad \Rightarrow \quad a_{10} = 250$

Hence, 250 passengers are seated in the tenth coach.

(*ii*) Passengers in fifth coach,

$$a_5 = 115 + (5 - 1) \times 15$$
$$= 115 + 4 \times 15 = 175$$

Extra passengers in tenth coach than fifth coach

$$= a_{10} - a_5$$
$$= 250 - 175 = 75$$

(*iii*) Collection of fare in first coach, $f_1 = 115 \times ₹\ 500 = ₹\ 57{,}500$

Collection of fare in second coach, $f_2 = 130 \times ₹\ 500 = ₹\ 65{,}000$

Difference of fare in 2 consecutive coaches

$$= f_2 - f_1$$
$$= ₹\ 65{,}000 - ₹\ 57{,}500 = ₹\ 7500$$

or

Total passengers seated in train,

$$S_n = \frac{n}{2}[2a_1 + (n-1)d]$$

$$S_{10} = \frac{10}{2}[2 \times 115 + (10 - 1) \times 15]$$

$$S_{10} = 5[230 + 9 \times 15] \quad \Rightarrow \quad S_{10} = 5 \times [230 + 135]$$

$\Rightarrow \qquad S_{10} = 1825$

Hence, total 1825 passengers are seated in the train.

37. (*i*) Lines are along length. So, it will be along x-axis.

Potatoes are placed along breadth. So, it will be along y-axis. $\frac{1}{10}$th of breadth

$$= \frac{1}{10} \times 200 = 20$$

Hence, coordinates of Raju's flag is (5, 20).

(*ii*) Similarly $\frac{1}{8}$th of breadth $= \frac{1}{8} \times 200 = 25$

Hence, coordinates of Sanju's flag is (11, 25).

(*iii*) Coordinates of green flag = mid-point of line joining blue and yellow flag.

By mid-point formula,

$$x = \frac{x_1 + x_2}{2}, \quad y = \frac{y_1 + y_2}{2}$$

$\Rightarrow \qquad x = \frac{5 + 11}{2} \quad y = \frac{25 + 20}{2}$

$\Rightarrow \qquad x = 8, \quad y = \frac{45}{2}$

Hence, coordinates of green flag is $\left(8, \frac{45}{2}\right)$.

or

By distance formula.

Distance between blue and yellow flag

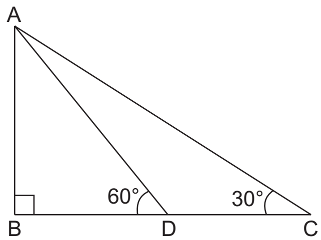
$$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(11 - 5)^2 + (25 - 20)^2}$$

$$= \sqrt{6^2 + 5^2} = \sqrt{36 + 25} = \sqrt{61} \text{ units}$$

38. (*i*) Let AB be height of temple.

D is point on road. DC is the distance moved.



$$DC = 40 \text{ m} \qquad \text{[Given]}$$

In right ΔABD, we have

$$\tan 60° = \frac{AB}{BD}$$

$$\Rightarrow \quad \sqrt{3} = \frac{AB}{BD}$$

$$\Rightarrow \quad BD = \frac{AB}{\sqrt{3}} \qquad ...(1)$$

In right ΔABC, we have

$$\tan 30° = \frac{AB}{BC}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{AB}{BD + 40 \text{ m}}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{AB}{\frac{AB}{\sqrt{3}} + 40 \text{ m}} \qquad \text{[Using (1)]}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{AB\sqrt{3}}{AB + 40\sqrt{3} \text{ m}}$$

$\Rightarrow \quad 3AB = AB + 40\sqrt{3}$ m $\quad \Rightarrow \quad 2AB = 40\sqrt{3}$ m

$\Rightarrow \quad AB = 20\sqrt{3}$ m

Hence, the height of the temple is $20\sqrt{3}$ m.

(*ii*) Area of $\Delta ABD = \frac{1}{2} \times AB \times BD$

$$= \frac{1}{2} \times 20\sqrt{3} \times 20 \text{ m}^2 = 200\sqrt{3} \text{ m}^2$$

Hence, the area of triangle formed is $200\sqrt{3}$ m^2.

(*iii*) Distance between temple and final point = BC

$$BC = BD + 40 \text{ m}$$

As, $\quad BD = \frac{AB}{\sqrt{3}} = \frac{20\sqrt{3} \text{ m}}{\sqrt{3}} = 20$ m

$\therefore \quad BC = (20 + 40)$ m = 60 m

Hence, distance from base of temple to final point of observation is 60 m.

or

Distance between top of temple and final point = AC

As, $\quad \sin 30° = \frac{AB}{AC}$

$\Rightarrow \quad \frac{1}{2} = \frac{20\sqrt{3} \text{ m}}{AC} \quad \Rightarrow \quad AC = 40\sqrt{3}$ m

Hence, distance from top of temple to final point of observation is $40\sqrt{3}$ m.