Sample Question Paper

Basic (Code 241)

ANSWERS

Section - A

1. (*a*)	2. (*c*)	3. (*b*)
4. (*d*)	5. (*b*)	6. (*c*)
7. (*a*)	8. (*b*)	9. (*b*)
10. (*d*)	11. (*a*)	12. (*b*)
13. (*b*)	14. (*d*)	15. (*c*)
16. (*d*)	17. (*a*)	18. (*a*)
19. (*b*)	20. (*c*)	

Section - B

21. Here $a_1 = 4$, $a_2 = 2$, $b_1 = p$, $b_2 = 2$.

Now for the given pair of equations to have unique solution

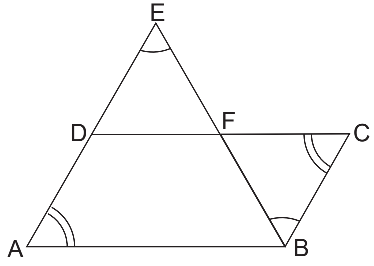
$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

i.e. $$\frac{4}{2} \neq \frac{p}{2}$$

$\Rightarrow$ $$p \neq 4$$

Therefore, all the values for p, except 4, the given pair of equations will have a unique solution.

22. Given, E is a point on the side AD produced of a parallelogram ABCD and BE intersects CD at F. Consider the figure below.



In ΔABE and ΔCFB,

$$\angle A = \angle C \quad \text{[Opposite angles of a parallelogram]}$$

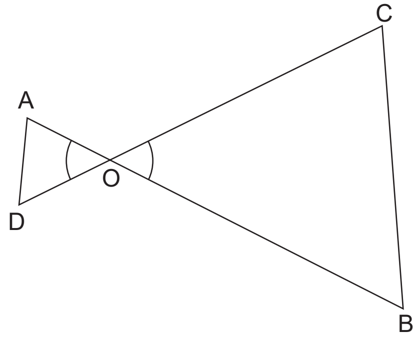
$$\angle AEB = \angle CBF \quad \text{[Alternate interior angles as AE || BC]}$$

$$\therefore \quad \Delta ABE \sim \Delta CFB \quad \text{[AA similarity criterion]}$$

or

22. Given $OA \times OB = OC \times OD$

So, $$\frac{OA}{OC} = \frac{OD}{OB} \quad \ldots(1)$$



Also, we have $\angle AOD = \angle COB$ [Vertically opposite angles] ...(2)

Therefore, from (1) and (2), $\Delta AOD \sim \Delta COB$ [SAS similarity criterion]

So, $\angle A = \angle C$ and $\angle D = \angle B$

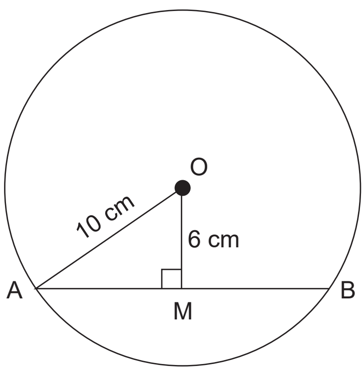
[Corresponding angles of similar triangles]

23. AB is the chord of the circle with centre O and radius OA and OM $\perp$ AB.

Diameter of the circle = 20 cm

$$\text{Radius} = \frac{20}{2} = 10 \text{ cm}$$

$$OA = 10 \text{ cm}, OM = 6 \text{ cm}$$



Now in right ΔOAM,

$$OA^2 = AM^2 + OM^2 \quad \text{[By Pythagoras' Theorem]}$$

$$\Rightarrow \quad (10)^2 = AM^2 + 6^2$$

$\Rightarrow \quad AM^2 = 100 - 36$

$\Rightarrow \quad AM^2 = 64$

$\therefore \quad AM = 8$ cm

Since, the perpendicular drawn from the centre of a circle to a chord bisects the chord.

M is the mid-point of AB.

$$AB = 2AM = 2 \times 8 = 16 \text{ cm}$$

Hence, the length of the chord is 16 cm.

24. Given $\quad 13 \sin A = 5 \quad \Rightarrow \quad \sin A = \frac{5}{13}$.

$$\cos^2 A = 1 - \left(\frac{5}{13}\right)^2 = 1 - \frac{25}{169} = \frac{144}{169}$$

[$\cos^2 A = 1 - \sin^2 A$]

$$\therefore \quad \cos A = \frac{12}{13}$$

$$\tan A = \frac{\sin A}{\cos A} = \frac{5}{13} \times \frac{13}{12} = \frac{5}{12}$$

$$\therefore \quad \frac{5 \sin A - 2 \cos A}{\tan A} = \frac{5 \times \frac{5}{13} - 2 \times \frac{12}{13}}{\frac{5}{12}} = \frac{\frac{1}{13}}{\frac{5}{12}} = \frac{12}{65}$$

25. Circumference of circle = 22 cm = $2\pi r$

$$\therefore \quad 2 \times \frac{22}{7} \times r = 22$$

$$\Rightarrow \quad r = \frac{22 \times 7}{22 \times 2} = \frac{7}{2} \text{ cm}$$

$$\therefore \quad \text{Area of quadrant} = \frac{1}{4}\pi r^2$$

$$= \frac{1}{4} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} = \frac{77}{8} \text{ cm}^2$$

or

25. $\quad$ Area of sector $= 154 \text{ cm}^2$

$$\Rightarrow \quad \frac{1}{2} lr = 154$$

$$\Rightarrow \quad \frac{1}{2} l \times 14 = 154$$

$$\therefore \quad l = 22$$

Length of the corresponding arc, $l = 22$ cm

Section - C

26. Suppose that $\sqrt{3} + \sqrt{5}$ is rational, say r.

Then, $\quad \sqrt{3} + \sqrt{5} = r \quad$ [Where $r \neq 0$]

$$\Rightarrow \quad \sqrt{5} = r - \sqrt{3}$$

$$\Rightarrow \quad \left(\sqrt{5}\right)^2 = (r - \sqrt{3})^2$$

$$\Rightarrow \quad 5 = r^2 + 3 - 2\sqrt{3}\, r$$

$$\Rightarrow \quad 2\sqrt{3}\, r = r^2 - 2$$

$$\sqrt{3} = \frac{r^2 - 2}{2r}$$

As r is rational and $r \neq 0$, So $\frac{r^2-2}{2r}$ is rational.

$\Rightarrow \quad \sqrt{3}$ is rational.

But this contradicts that $\sqrt{3}$ is irrational. Hence, our supposition is wrong.

Therefore, $\sqrt{3} + \sqrt{5}$ is an irrational number.

27. The given equation is $x^2 + x - (a + 2)(a + 1) = 0$

Composing with $Ax^2 + Bx + C = 0$, we get

$$A = 1, B = 1, C = -(a + 2)(a + 1) = -(a^2 + 3a + 2).$$

$$x = \frac{-1 \pm \sqrt{1^2 - 4 \times 1\left[-(a^2 + 3a + 2)\right]}}{2 \times 1}$$

$$\left[x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\right]$$

$$= \frac{-1 \pm \sqrt{1 + 4\left(a^2 + 3a + 2\right)}}{2}$$

$$= \frac{-1 \pm \sqrt{4a^2 + 12a + 9}}{2}$$

$$= \frac{-1 \pm \sqrt{(2a+3)^2}}{2} = \frac{-1 \pm (2a+3)}{2}$$

$$= \frac{-1 + 2a + 3}{2}, \frac{-1 - 2a - 3}{2}$$

$$= \frac{2a+2}{2}, \frac{-(2a+4)}{2}$$

$$x = a + 1, -(a + 2)$$

$\therefore$ Roots of the given equation are $(a + 1), -(a + 2)$.

28. Let the incomes per month of two persons be ₹ x and ₹ y respectively. As each person saves ₹ 2000 per month, so their expenditures are ₹ $(x - 2000)$ and ₹ $(y - 2000)$.

According to question, we have

$$\frac{x}{y} = \frac{9}{7} \qquad \text{i.e. } 72 - 9y = 0$$

and $\dfrac{x-2000}{y-2000} = \dfrac{4}{3}$ i.e. $3x - 4y + 2000 = 0$

$\Rightarrow$ $7x - 9y = 0$ ...(1)

and $3x - 4y = -2000$ ...(2)

On multiplying (1) by 3 and (2) by 7 respectively,

$21x - 27y = 0$ ...(3)

and $21x - 28y = -14000$ ...(4)

Subtracting (4) from (3), we get

$y = 14000$

Substituting $y = 14000$ in (1), we get

$7x - (9 \times 14000) = 0$

$\Rightarrow$ $x = 18000$

$\therefore$ their monthly incomes are ₹ 14000 and ₹ 18000.

or

28. Let x be the digit at ten's place and y be the digit at unit's place.

The number = $10x + y$

The digit obtained by increasing the digit at ten's place by unity = $x + 1$

According to the question,

$x + y = 5$ ...(1)

and $x + 1 = \dfrac{1}{8}(10x + y)$

$\Rightarrow$ $8(x + 1) = 10x + y$

$\Rightarrow$ $8x + 8 = 10x + y$

$\Rightarrow$ $2x + y = 8$ ...(2)

Subtracting (1) from (2), we get $x = 3$.

On substituting $x = 3$ in (1), we get

$3 + y = 5$

$\Rightarrow$ $y = 2$

Hence, the required number is 32.

29. Since the tangent to a circle is perpendicular to the radius through the point of contact.

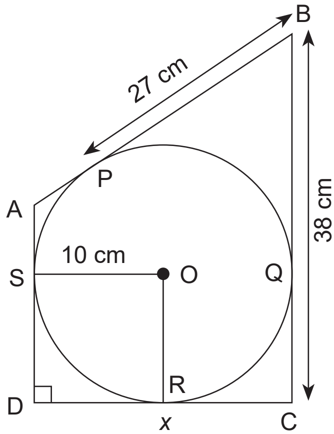
$\therefore$ $\angle QSD = \angle OXD = 90°$ ...(1)

$\angle D = 90°$ [Given] ...(2)

OS = OR [Radii of a circle] ...(3)

$\therefore$ OSDR is a square. [Using (1), (2) and (3)] ...(4)

$\therefore$ OS = SD = DR = OR = 10 cm ...(5)



Since the tangents from an external point to a circle are equal.

$\therefore$ AB = BQ = 27 cm

QC = RC

QC = 38 cm – 27 cm = 11 cm

$\therefore$ RC = 11 cm ...(6)

x = CD = DR + RC

= 10 cm + 11 cm [From (5) & (6)]

= 21 cm.

30. To prove:

$$\sqrt{\frac{\sec A - 1}{\sec A + 1}} + \sqrt{\frac{\sec A + 1}{\sec A - 1}} = 2 \text{ cosec } A$$

L.H.S: $\sqrt{\frac{\sec A - 1}{\sec A + 1}} + \sqrt{\frac{\sec A + 1}{\sec A - 1}}$

$$= \sqrt{\frac{(\sec A - 1)(\sec A - 1)}{(\sec A + 1)(\sec A - 1)}} + \sqrt{\frac{(\sec A + 1)(\sec A + 1)}{(\sec A - 1)(\sec A + 1)}}$$

$$= \sqrt{\frac{(\sec A + 1)^2}{\sec^2 A - 1}} + \sqrt{\frac{(\sec A + 1)^2}{\sec^2 A - 1}}$$

$$= \sqrt{\frac{(\sec A - 1)^2}{\tan^2 A}} + \sqrt{\frac{(\sec A + 1)^2}{\tan^2 A}}$$

[$\because$ $1 + \tan^2 A = \sec^2 A \Rightarrow \sec^2 A - 1 = \tan^2 A$]

$$= \frac{\sec A - 1}{\tan A} + \frac{\sec A + 1}{\tan A}$$

$$= \frac{\sec A - 1 + \sec A + 1}{\tan A}$$

$$= \frac{2\sec A}{\tan A}$$

$$= \frac{\frac{2}{\cos A}}{\frac{\sin A}{\cos A}} = \frac{2}{\sin A}$$

$$= 2 \text{ cosec } A = \text{RHS}$$

Hence, proved.

or

30. Given $\tan^4\theta + \tan^2\theta = 1$

$\Rightarrow \quad \tan^2\theta(\tan^2\theta + 1) = 1$

$\Rightarrow \quad 1 + \tan^2\theta = \dfrac{1}{\tan^2\theta}$

$\Rightarrow \quad \sec^2\theta = \cot^2\theta \qquad [\because \sec^2\theta = 1 + \tan^2\theta]$

$\Rightarrow \quad \dfrac{1}{\cos^2\theta} = \dfrac{\cos^2\theta}{\sin^2\theta}$

$\Rightarrow \quad \sin^2\theta = \cos^4\theta$

$\Rightarrow \quad 1 - \cos^2\theta = \cos^4\theta \qquad [\sin^2\theta + \cos^2\theta = 1]$

$\therefore \quad \cos^4\theta + \cos^2\theta = 1$

Hence, proved.

31. (*a*) $\text{P(E)} = \dfrac{\text{Number of favourable outcomes}}{\text{Number of possible outcomes}}$

$\text{P(Red)} = \dfrac{7}{15}$

(*b*) $\text{P(Black or white)} = \dfrac{5+3}{15} = \dfrac{8}{15}$

(*c*) Probability of not black = Probability of red and white

$\text{P(not black)} = \dfrac{7+3}{15} = \dfrac{10}{15} = \dfrac{2}{3}$

Section - D

32. Let side of 1st square be x metre and side of 2nd square be y metre, and $x > y$. Then

area of 1st square $= x^2 \text{ m}^2$

area of 2nd square $= y^2 \text{ m}^2$,

perimeter of 1st square $= 4x$ m

and perimeter of 2nd square $= 4y$ m

According to given conditions,

$x^2 + y^2 = 640 \qquad \ldots(1)$

and $4x - 4y = 64$

$\Rightarrow x - y = 16$

$\Rightarrow x = y + 16$ ...(2)

Substituting the value of x from (2) in (1), we get

$$(y + 16)^2 + y^2 = 640$$

$$\Rightarrow y^2 + 32y + 256 + y^2 = 640$$

$$\Rightarrow 2y^2 + 32y - 384 = 0$$

$$\Rightarrow y^2 + 16y - 192 = 0$$

$$\Rightarrow y^2 + 24y - 8y - 192 = 0$$

$$\Rightarrow y(y + 24) - 8(y + 24) = 0$$

$$\Rightarrow (y - 8)(y + 24) = 0$$

$$\Rightarrow y - 8 = 0 \quad \text{or} \quad y + 24 = 0$$

$$y = 8, \quad \text{or} \quad y = -24$$

But y being side of a square cannot be negative.

$$\therefore \quad y = 8$$

From (2), when $y = 8$, $x = 8 + 16 = 24$.

Hence, the sides of the two squares are 24 m and 8 m.

or

32. Let the speed of the stream be x km/h, $0 < x < 5$.

Speed of the boat upstream = $(5 - x)$ km/h

Speed of the boat downstream = $(5 + x)$ km/h

Time taken for going 5.25 km upstream = $\frac{5.25}{5-x}$ h

Time taken for returning 5.25 km downstream = $\frac{5.25}{5+x}$ h

According to given information,

$$\frac{5.25}{5-x} - \frac{5.25}{5+x} = 1$$

$$\Rightarrow 5.25\left(\frac{1}{5-x} - \frac{1}{5+x}\right) = 1$$

$$\Rightarrow \frac{525}{100}\left(\frac{1}{5-x} - \frac{1}{5+x}\right) = 1$$

$$\Rightarrow \frac{21}{4}\left(\frac{1}{5-x} - \frac{1}{5+x}\right) = 1$$

$$\Rightarrow \frac{21}{4}\left[\frac{5+x-5+x}{(5-x)(5+x)}\right] = 1$$

$$\Rightarrow \frac{21}{4}\left[\frac{2x}{25-x^2}\right] = 1$$

$$\Rightarrow \quad 42x = 4(25 - x^2)$$
$$\Rightarrow \quad 4x^2 + 42x - 100 = 0$$
$$\Rightarrow \quad 2x^2 + 21x - 50 = 0$$
$$\Rightarrow \quad 2x^2 + 25x - 4x - 50 = 0$$
$$\Rightarrow \quad x(2x + 25) - 2(2x + 25) = 0$$
$$\Rightarrow \quad (x - 2)(2x + 25) = 0$$
$$\Rightarrow \quad x - 2 = 0 \text{ or } 2x + 25 = 0$$
$$\Rightarrow \quad x = 2 \text{ or } x = \frac{-25}{2} \text{ but } 0 < x < 5$$
$$\Rightarrow \quad x = 2$$

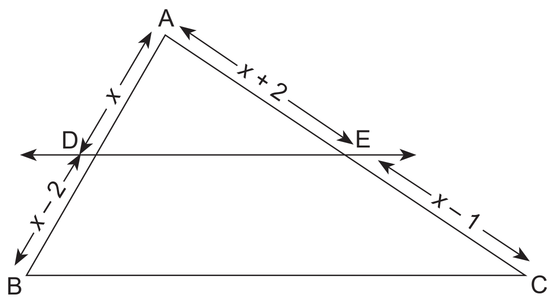
Hence, the speed of the stream = 2 km/h

33. (*a*) It is given that,

$$DE \parallel BC$$

and $AD = x$, $DB = x - 2$, $AE = x + 2$

and $EC = x - 1$



Consider ΔABC,

$$\frac{AD}{DB} = \frac{AE}{EC}$$

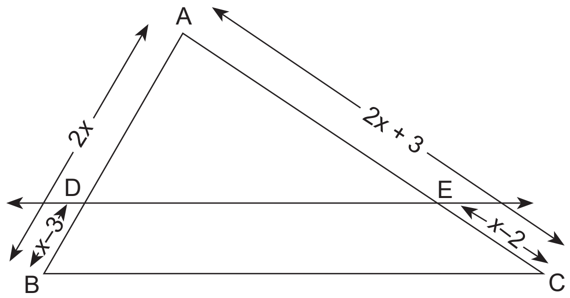
[Theorem - If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other 2 sides are divided in the same ratio]

$$\Rightarrow \quad \frac{x}{x-2} = \frac{x+2}{x-1}$$

By cross-multiplication we get,

$$\Rightarrow \quad x(x - 1) = (x - 2)(x + 2)$$
$$\Rightarrow \quad x^2 - x = x^2 - 4$$
$$\Rightarrow \quad -x = -4$$
$$\Rightarrow \quad x = 4$$

(*b*) DB = $x - 3$, AB = $2x$, EC = $x - 2$ and AC = $2x + 3$



Consider ΔABC,

$$\frac{AB}{DB} = \frac{AC}{EC}$$

$$\Rightarrow \quad \frac{2x}{x-3} = \frac{2x+3}{x-2}$$

By cross-multiplication,

$$\Rightarrow \quad 2x(x - 2) = (2x + 3)(x - 3)$$

$$\Rightarrow \quad 2x^2 - 4x = 2x^2 - 6x + 3x - 9$$

$$\Rightarrow \quad 2x^2 - 4x - 2x^2 + 6x - 3x = -9$$

$$\Rightarrow \quad -x = -9$$

$$\Rightarrow \quad x = 9$$

34. Radius of cylinder = $\frac{1}{2} \times 4$ cm = 2 cm, and its height is 9 cm.

$\therefore$ Volume of the cylinder $= \pi r^2 h = \pi \times 2^2 \times 9 \text{ cm}^3$

$= 36\,\pi \text{ cm}^3$

Radius of the cone $= \frac{1}{2} \times 6$ cm = 3 cm

Let h be the height of the cone.

$\therefore$ Volume of the cone $= \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \times 3^2 \text{ cm}^2 \times h = 3\,\pi h \text{ cm}^2$

Since the cylinder is melted and recasted into a cone,

Volume of the cone = Volume of cylinder

$3\,\pi h \text{ cm}^2 = 36\pi \text{ cm}^3 \quad \Rightarrow \quad h = 12$ cm

$\therefore$ The height of the cone = 12 cm

Slant height of the cone $= \sqrt{r^2 + h^2} = \sqrt{3^2 + 12^2}$

$= \sqrt{153}$ cm = 12.37 cm

Total surface area of the cone $= \pi r(l + r)$

$= 3.14 \times 3(12.37 + 3)$

$= 144.8 \text{ cm}^2$

or

34. Radius of sphere = $\frac{12}{2}$ cm = 6 cm

$$\text{Volume of sphere} = \frac{4}{3}\pi \times (6\text{ cm})^3$$

Rise of water level in cylindrical vessel = $\frac{32}{9}$ cm

Let r cm be the radius of the base of the cylindrical vessel.

Volume of raised level of water in cylindrical vessel = Volume of sphere

Volume of raised level of water in cylindrical vessel = $\pi r^2 \times \frac{32}{9}$

$$\Rightarrow \quad \pi r^2 \times \frac{32}{9} = \frac{4}{3}\pi \times 6^3$$

$$\Rightarrow \quad r^2 = \frac{4}{3} \times 216 \times \frac{9}{32}$$

$$\Rightarrow \quad r^2 = 81$$

$$\therefore \quad r = 9$$

Hence, diameter of cylindrical vessel = 2 × 9 = 18 cm

35. Let the assumed mean be a = 13

Class interval	f_i	*Mid-value* x_i	$u_i = \frac{x_i - a}{h}$	$f_i u_i$
1–5	20	3	–2	–40
6–10	50	8	–1	–50
11–15	46	13	0	0
16–20	22	18	1	22
21–25	12	23	2	24
	150			–44

$$a = 13,$$

$$\text{Mean} = a + h\left(\frac{\sum f_i u_i}{\sum f_i}\right)$$

$$= 13 + 5 \times \left(\frac{-44}{150}\right)$$

$$= 13 - \frac{44}{30}$$

$$= 13 - 1.466$$

$$= 11.534$$

∴ The mean of the number of tomatoes per plant = 11.53

Section - E

36. Production in 6th year or $a_6 = 16000$

Production in 9th year or $a_9 = 22600$

$$a_9 - a_6 = (9-6)d \qquad (a_m - a_n) = (m-n)d$$

$\Rightarrow$ $22600 - 16000 = 3d$

$\Rightarrow$ $3d = 6600$

$\Rightarrow$ $d = 2200$

$a_9 = a + 8d$

$\Rightarrow$ $22600 = a + 8 \times 2200$

$\Rightarrow$ $22600 = a + 17600$

$\Rightarrow$ $a = 5000$

(*a*) Production in first year = 5000 sets

(*b*) Production in 8th year,

$a_8 = 5000 + (8-1) \times 2200$

$a_8 = 5000 + 15400$

$\Rightarrow$ $a_8 = 20400$

$\therefore$ Production in 8th year is 20400.

(*c*) $S_3 = \frac{3}{2}[2 \times 5000 + (3-1)2200]$

$= \frac{3}{2}[10000 + 4400]$

$\Rightarrow$ $S_3 = \frac{3}{2} \times 14400 = 21600$

or

(*c*) Let the production be 29,200 sets in the *n*th year.

$29200 = 5000 + (n-1) \times 2200$

$24200 = 2200\,n - 2200$

$n = \frac{26400}{2200} = 12$

$\therefore$ In 12th year, production of TV sets is 29200.

37. (*a*) (5, 20)

(*b*) On the 8th line, at a distance of 22.5 m along the breadth

(*c*) Coordinates of blue flag = (11, 25)

Coordinates of yellow flag = (5, 20)

$\therefore$ Distance between blue and yellow flag:

$\because$ $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$$= \sqrt{(5-11)^2 + (20-25)^2}$$

$$= \sqrt{(-6)^2 + (-5)^2}$$

$$= \sqrt{36+25} = \sqrt{61} \text{ units}$$

or

(*c*) Coordinates of yellow flag = (5, 20)

Coordinates of green flag = (8, 22.5)

∴ Distance between yellow and green flag:

$$\because \quad d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(8-5)^2 + (22.5-20)^2}$$

$$= \sqrt{(3)^2 + (2.5)^2}$$

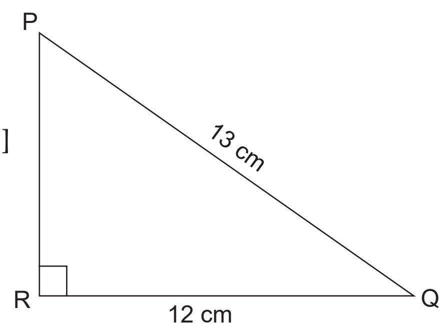
$$= \sqrt{9 + \left(\frac{5}{2}\right)^2}$$

$$= \sqrt{9 + \frac{25}{4}} = \sqrt{\frac{36+25}{4}}$$

$$= \sqrt{\frac{61}{4}} = \frac{\sqrt{61}}{2}$$

38. (*a*) $\angle PQR = \theta$

$$\cos\theta = \frac{RQ}{PQ} = \frac{12}{13}$$



(*b*) $\sec\theta = \frac{13}{12}$ $\left[\sec\theta = \frac{1}{\cos\theta}\right]$

(*c*) $PR^2 = PQ^2 - RQ^2$

$= 13^2 - 12^2$

$= 169 - 144$

$PR^2 = 25$

$PR = 5$ cm

$$\tan\theta = \frac{PR}{RQ}$$

$$\tan\theta = \frac{5}{12}$$

$$\therefore \quad \frac{\tan\theta}{1+\tan^2\theta} = \frac{\tan\theta}{\sec^2\theta} = \frac{5}{12} \div \left(\frac{13}{12}\right)^2$$

$$= \frac{5}{12} \times \frac{12}{13} \times \frac{12}{13} = \frac{60}{169}$$

or

(*c*) $\cot^2\theta - \text{cosec}^2\theta$

$$\cot\theta = \frac{1}{\tan\theta} = \frac{12}{5}$$

$$\text{cosec}\,\theta = \frac{PQ}{PR} = \frac{13}{5}$$

$$\therefore \quad \cot^2\theta - \text{cosec}^2\theta = \left(\frac{12}{5}\right)^2 - \left(\frac{13}{5}\right)^2$$

$$= \frac{144}{25} - \frac{169}{25} = \frac{-25}{25} = -1$$