WORKSHEET ①

1. Evaluate the following:

   (i) $\dfrac{\sin 20°}{\cos 70°}$ (ii) $\dfrac{\cos 19°}{\sin 71°}$ (iii) $\dfrac{\tan 10°}{\cot 80°}$

2. Evaluate the following:

   (i) $\left(\dfrac{\sin 49°}{\cos 41°}\right)^2 + \left(\dfrac{\cos 41°}{\sin 49°}\right)^2$

   (ii) $\dfrac{\cot 40°}{\tan 50°} - \dfrac{1}{2}\left(\dfrac{\cos 35°}{\sin 55°}\right)$

   (iii) $\dfrac{\tan 35°}{\cot 55°} + \dfrac{\cot 78°}{\tan 12°} - 1$

3. Prove that:

$$\frac{\cos(90° - \theta)\ \sec(90° - \theta)\ \tan \theta}{\text{cosec}(90° - \theta)\ \sin(90° - \theta)\ \cot(90° - \theta)} + \frac{\tan(90° - \theta)}{\cot \theta} = 2.$$

4. Without using trigonometric tables, evaluate

$$\frac{\cos 58°}{\sin 32°} + \frac{\sin 22°}{\cos 68°} - \frac{\cos 38°\ \text{cosec}\ 52°}{\tan 18°\ \tan 35°\ \tan 60°\ \tan 72°\ \tan 55°}$$

5. Without using trigonometric tables, prove that:

   (i) $\tan 7°\ \tan 23°\ \tan 60°\ \tan 67°\ \tan 83° = \sqrt{3}$

   (ii) $\cot 12°\ \cot 38°\ \cot 52°\ \cot 60°\ \cot 78° = \dfrac{1}{\sqrt{3}}$

6. Without using trigonometric tables, evaluate each of the following:

   (i) $\sin^2 65° + \sin^2 25°$ (ii) $\cos^2 17° - \sin^2 73°$

   (iii) $\sec^2 36° - \cot^2 54°$

7. Without using trigonometric tables, evaluate the following:

$$\frac{\cot(90° - \theta)\ \sin(90° - \theta)}{\sin \theta} + \frac{\cot 40°}{\tan 50°} - (\cos^2 20° + \cos^2 70°)$$

8. If A, B, C are the angles of a $\triangle ABC$, show that

$$\sin\left(\frac{B + C}{2}\right) = \cos\frac{A}{2}.$$

9. If $\sin 3A = \cos(A - 10°)$, where 3A is an acute angle, then find the value of A.

10. Prove that:

    (i) $\sin(70° + \theta) - \cos(20° - \theta) = 0$
    (ii) $\tan(55° - \theta) - \cot(35° + \theta) = 0$
    (iii) $\text{cosec}(67° + \theta) - \sec(23° - \theta) = 0$

11. Prove that:

    (i) $\text{cosec}(65° + \theta) - \sec(25° - \theta) - \tan(55° - \theta) + \cot(35° + \theta) = 0$
    (ii) $\sin(50° + \theta) - \cos(40° - \theta) + \tan 1°\ \tan 10°\ \tan 80°\ \tan 89° = 1$

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12. Express each of the following in terms of trigonometric ratios of angles lying between 0° and 45°.
    (*i*) sin 59° + cos 56°
    (*ii*) tan 65° + cot 49°
    (*iii*) sec 76° + cosec 52°
    (*iv*) cot 85° + cos 75°

13. Prove that:
    (*i*) tan 20° tan 35° tan 45° tan 55° tan 70° = 1
    (*ii*) sin 48° sec 42° + cos 48° cosec 42° = 2.

14. If sec 2A = cosec (A − 42°), where 2A is an acute angle, find the value of A.

15. If cos 2θ = sin 4θ, where 2θ and 4θ are acute angles, find the value of θ.

16. If tan A = cot B, prove that A + B = 90°.

17. If A + B = 90°, prove that

$$\sqrt{\frac{\tan A \tan B + \tan A \cot B}{\sin A \ \sec B} - \frac{\sin^2 B}{\cos^2 A}} = \tan A$$

18. Evaluate the following:
    (*i*) $\dfrac{2}{3}$ (cos^4 30° − sin^4 45°) − 3 (sin^2 60° − sec^2 45°) + $\dfrac{1}{4}$ cot^2 30°

    (*ii*) 4 (sin^4 30° + cos^4 60°) − $\dfrac{2}{3}$ (sin^2 60° − cos^2 45°) + $\dfrac{1}{2}$ tan^2 60°.

19. Without using trigonometric tables, prove that:
    (*i*) cos 81° − sin 9° = 0
    (*ii*) tan 71° − cot 19° = 0
    (*iii*) tan^2 66° − cot^2 24° = 0
    (*iv*) sin^2 48° + sin^2 42° = 1

20. Without using trigonometric table, evaluate:

$$\frac{\sec^2 54° - \cot^2 36°}{\cosec^2 57° - \tan^2 33°} + 2 \sin^2 38° \sec^2 52° - \sin^2 45°.$$

ANSWERS

1. (*i*) 1   (*ii*) 1   (*iii*) 1

2. (*i*) 2   (*ii*) $\dfrac{1}{2}$ (*iii*) 1

4. $2 - \dfrac{1}{\sqrt{3}}$

6. (*i*) 1   (*ii*) 0   (*iii*) 1

7. 1

9. A = 25°

12. (*i*) cos 31° + sin 34°   (*ii*) cot 25° + tan 41°
   (*iii*) cosec 14° + sec 38°   (*iv*) tan 5° + sin 15°

14. 44°

15. 15°

18. (*i*) $\dfrac{113}{24}$ (*ii*) $\dfrac{11}{6}$

20. $\dfrac{5}{2}$