WORKSHEET 1

CHAPTER 9 – TRIGONOMETRIC IDENTITIES

1. Prove the following identities:
   (*i*) $(1 - \sin^2 \theta) \sec^2 \theta = 1$
   (*ii*) $(1 + \tan^2 \theta)(1 + \sin \theta)(1 - \sin \theta) = 1$
   (*iii*) $\cos^2 \theta + \dfrac{1}{1 + \cot^2 \theta} = 1$

2. Prove the following identities:
   (*i*) $(1 + \cot \theta - \operatorname{cosec} \theta)(1 + \tan \theta + \sec \theta) = 2$
   (*ii*) $\tan^2 \theta + \cot^2 \theta + 2 = \sec^2 \theta \operatorname{cosec}^2 \theta$
   (*iii*) $\sqrt{\sec^2 \theta + \operatorname{cosec}^2 \theta} = \tan \theta + \cot \theta$

3. If $\cos \theta + \sin \theta = \sqrt{2} \cos \theta$, show that $\cos \theta - \sin \theta = \sqrt{2} \sin \theta$.

4. If $\dfrac{\cos \alpha}{\cos \beta} = m$ and $\dfrac{\cos \alpha}{\sin \beta} = n$, show that $(m^2 + n^2) \cos^2 \beta = n^2$.

5. If $\operatorname{cosec} \theta - \sin \theta = l$ and $\sec \theta - \cos \theta = m$, prove that $l^2 m^2 (l^2 + m^2 + 3) = 1$.

6. If $\operatorname{cosec} \theta - \sin \theta = m$ and $\sec \theta - \cos \theta = n$, prove that $(m^2 n)^{2/3} + (mn^2)^{2/3} = 1$.

7. If $x = a \sec \theta + b \tan \theta$ and $y = a \tan \theta + b \sec \theta$, prove that $x^2 - y^2 = a^2 - b^2$.

8. Prove that $\sqrt{\dfrac{\sec \theta - 1}{\sec \theta + 1}} + \sqrt{\dfrac{\sec \theta + 1}{\sec \theta - 1}} = 2 \operatorname{cosec} \theta$.

9. Prove that $\dfrac{1}{(\operatorname{cosec} \theta - \cot \theta)} - \dfrac{1}{\sin \theta} = \dfrac{1}{\sin \theta} - \dfrac{1}{(\operatorname{cosec} \theta + \cot \theta)}$

10. Prove that:
    (*i*) $\dfrac{\tan A + \sin A}{\tan A - \sin A} = \dfrac{\sec A + 1}{\sec A - 1}$
    (*ii*) $\dfrac{\cot A - \cos A}{\cot A + \cos A} = \dfrac{\operatorname{cosec} A - 1}{\operatorname{cosec} A + 1}$

11. Prove that:
    (*i*) $(\sin^2 A \cos^2 B - \cos^2 A \sin^2 B) = \sin^2 A - \sin^2 B$
    (*ii*) $(\tan^2 A \sec^2 B - \sec^2 A \tan^2 B) = \tan^2 A - \tan^2 B$.

12. Prove the following trigonometric identities:
    (*i*) $(\sin \theta + \sec \theta)^2 + (\cos \theta + \operatorname{cosec} \theta)^2 = (1 + \sec \theta \operatorname{cosec} \theta)^2$
    (*ii*) $(\operatorname{cosec} \theta - \cot \theta)^2 = \dfrac{1 - \cos \theta}{1 + \cos \theta}$
    (*iii*) $(\sin \theta - \sec \theta)^2 + (\cos \theta - \operatorname{cosec} \theta)^2 = (1 - \sec \theta \operatorname{cosec} \theta)^2$

13. Prove the following identities:
    (*i*) $\dfrac{\sin \theta}{1 - \cos \theta} + \dfrac{\tan \theta}{1 + \cos \theta} = \sec \theta \operatorname{cosec} \theta + \cot \theta$
    (*ii*) $\dfrac{\tan \theta - \cot \theta}{\sin \theta \cos \theta} = \sec^2 \theta - \operatorname{cosec}^2 \theta = \tan^2 \theta - \cot^2 \theta$
    (*iii*) $\dfrac{\sin \theta - 2 \sin^3 \theta}{2 \cos^3 \theta - \cos \theta} = \tan \theta$

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14. Prove the following identities:

(*i*) $\dfrac{1}{\text{cosec A} - \cot \text{A}} - \dfrac{1}{\sin \text{A}} = \dfrac{1}{\sin \text{A}} - \dfrac{1}{\text{cosec A} + \cot \text{A}}$

(*ii*) $\dfrac{\cos \text{A}}{1 - \tan \text{A}} + \dfrac{\sin \text{A}}{1 - \cot \text{A}} = \cos \text{A} + \sin \text{A}$

(*iii*) $\dfrac{\tan \text{A}}{1 - \cot \text{A}} + \dfrac{\cot \text{A}}{1 - \tan \text{A}} = 1 + \tan \text{A} + \cot \text{A} = 1 + \sec \text{A cosec A}$

15. Prove that $(1 - \sin \theta + \cos \theta)^2 = 2\,(1 + \cos \theta)\,(1 - \sin \theta)$.

16. If $\tan^2 \theta = 1 - a^2$, prove that $\sec \theta + \tan^3 \theta \text{ cosec } \theta = (2 - a^2)^{3/2}$

17. If $\tan \theta + \sin \theta = m$ and $\tan \theta - \sin \theta = n$, show that $m^2 - n^2 = 4\sqrt{mn}$.

18. If $(\sec \theta + \tan \theta) = m$ and $(\sec \theta - \tan \theta) = n$, show that $mn = 1$.

19. If $a \cos \theta - \text{b} \sin \theta = c$, prove that $(a \sin \theta + b \cos \theta) = \pm \sqrt{a^2 + b^2 - c^2}$.

20. Show that,

$$\left\{\frac{1}{(\sec^2 \theta - \cos^2 \theta} + \frac{1}{(\text{cosec}^2 \theta - \sin^2 \theta)}\right\}(\sin^2 \theta \cos^2 \theta) = \frac{1 - \sin^2 \theta \cos^2 \theta}{2 + \sin^2 \theta \cos^2 \theta}.$$