CHAPTER **17**

Probability

EXERCISE 17A

For Basic and Standard Levels

1. (*i*) 0 (*ii*) 1 (*iii*) 1 (*iv*) E

(*v*) 0 and 1.

2. (*i*) (*a*) The six faces of a die have six different numbers 1, 2, 3, 4, 5 and 6 out of which only 3 numbers, i.e. 2, 3 and 5 are prime numbers.

$\therefore$ Required probability = $\frac{3}{6} = \frac{1}{2}$.

(*b*) Only 3 numbers *viz.* 3, 4 and 5 lie between 2 and 6.

Hence, the required probability = $\frac{3}{6} = \frac{1}{2}$.

(*ii*) (*a*) Total number of possible outcomes = 6.

The number of favourable outcome = 1.

Hence, P(number 3) = $\frac{1}{6}$.

(*b*) Total number of possible outcomes = 6.

Outcomes favourable to the event 'an even number' are 2, 4 and 6.

Thus, the number of favourable outcomes = 3.

P(an even number) = $\frac{3}{6} = \frac{1}{2}$.

(*c*) Total number of possible outcomes = 6.

Number between 2 and 5 = 3 and 4.

Number of favourable outcomes = 2.

P(a number between 2 and 5) = $\frac{2}{6} = \frac{1}{3}$.

(*d*) Total number of possible outcomes = 6.

An odd number in a die = 1, 3, 5.

Number of favourable outcomes = 3.

P(an odd number) = $\frac{3}{6} = \frac{1}{2}$.

(*e*) Total number of possible outcomes = 6.

An odd prime number = 3, 5.

Number of favourable outcomes = 2.

P(an odd prime number) = $\frac{2}{6} = \frac{1}{3}$.

3. PROBABILITY

Consonants → P, R, B, B, L, T, Y

$$\text{P (not a vowel)} = \frac{\text{No. of consonants}}{\text{Total number of letters}} = \frac{7}{11}$$

4. (*i*) Total number of possible outcomes = 26.

Number of vowels = 5 (*a*, *e*, *i*, *o*, *u*)

Number of favourable outcomes = 5.

P(a vowel) = $\frac{5}{26}$.

(*ii*) Total number of possible outcomes = 26.

Number of consonants = 21.

i.e., number of favourable outcomes = 21.

P(a consonant) = $\frac{21}{26}$.

(*iii*) Total number of possible outcomes = 26.

Number of letter in word noble = 5.

Number of favourable outcomes = 5.

P(a letter of work noble) = $\frac{5}{26}$.

5. (*i*) Total number of possible outcomes = 940.

Number of tickets Prema bought = 5.

So, number of favourable outcomes = 5.

P(winning ticket for Prema) = $\frac{5}{940} = \frac{1}{188}$.

(*ii*) Jayant bought 4 tickets.

Number of favourable outcomes = 4.

P(winning ticket for Jayant) = $\frac{4}{940} = \frac{1}{235}$.

6. (*i*) Between 0 to 100, there are 99 integers out of which the numbers divisible by 7 are

7, 14, 21, 28, 35, 42, 49, 56, 63, 70, 77, 84, 91 and 98.

$\therefore$ There are 14 numbers between 0 and 100, which are divisible by 7.

Hence, the required probability = $\mathbf{\frac{14}{99}}$.

(*ii*) Required probability that 14 numbers between 0 and 100 which are divisible by 7. Hence, the number of integers between 0 and 100 which are not divisible by 7 is 99 – 14 = 85.

$\therefore$ Required probability is this case is $\mathbf{\frac{85}{99}}$.

7. The squares of the given numbers are 9, 4, 1 and 0 out of all the given 7 numbers, there are 3 numbers *viz* –1, 0 and 1 which are less than or equal to 1.

Hence, the required probability = $\mathbf{\frac{3}{7}}$.

8. (*i*) (*a*) Total number of possible outcomes = 52.

There are 4 aces in a deck of 52 cards.

Hence, P(an ace) = $\frac{4}{52} = \frac{1}{13}$.

(*b*) Number of the 2 of spades = 1.

$P(\text{2 of spades}) = \frac{1}{52}$.

(*c*) Number of the 10 of a black card = 2.

$P(\text{10 of a black card}) = \frac{2}{52} = \frac{1}{26}$.

(*ii*) (*a*) Number of queens in cards = 4.

$P(\text{a queen}) = \frac{4}{52} = \frac{1}{13}$.

(*b*) Number of diamond in 52 cards = 13.

$P(\text{a diamond}) = \frac{13}{52} = \frac{1}{4}$.

(*c*) Number of a king or an ace in cards = 4 + 4 = 8.

$P(\text{a king or an ace}) = \frac{8}{52} = \frac{2}{13}$.

(*d*) Number of red ace = 2.

$P(\text{a red ace}) = \frac{2}{52} = \frac{1}{26}$.

(*iii*) Total number of cards = 52

Face cards = 12 black cards = 26

Non-face cards = 40 red cards = 26

(*a*) $P(\text{non-face card}) = \frac{40}{52} = \frac{10}{13}$

(*b*) $P(\text{black king or red queen}) = \frac{4}{52} = \frac{1}{13}$

(*c*) $P(\text{a card of spade or an ace}) = \frac{16}{52} = \frac{4}{13}$

(*d*) $P(\text{neither a king or a queen}) = \frac{44}{52} = \frac{11}{13}$

(*iv*) Total number of possible outcomes = 52.

King, queen and jack are called face cards.

(2 × 3 = 6)

Favourable outcomes = 6.

$P(\text{a red face card}) = \frac{6}{52} = \frac{3}{26}$.

9. Total number of possible outcomes = 52.

There are 26 red cards (including 2 queens) and 2 more queens = 26 + 2 = 28.

Then number of favourable outcomes = (52 – 28) = 24.

$P(\text{neither red card nor queen}) = \frac{24}{52} = \frac{6}{13}$.

10. Total possible outcomes = 5.

(*i*) Favourable outcomes = 3.

$P(\text{getting a 3}) = \frac{3}{5}$.

(*ii*) $P(\text{a diamond}) = \frac{2}{5}$. (diamonds = 3 and 4)

(*iii*) $P(\text{a 4}) = \frac{1}{5}$.

(*iv*) P(a black 4) = 0. (it is not in the five cards)

(*v*) Favourable outcomes = 2.

$P(\text{a club}) = \frac{2}{5}$.

(*vi*) As 4 of hearts is not in the five cards.

$P(\text{a 4 of hearts}) = \frac{0}{5} = 0$.

(*vii*) As no cards in 5 so,

P(a 5) = 0.

(*viii*) As a 7 of club is in the 5 cards, so favourable outcome = 1.

$P(\text{a 7 of club}) = \frac{1}{5}$.

(*ix*) Red cards = (3 of hearts, 3 of diamonds, 4 of diamonds) = 3.

Favourable outcomes = 3,

$P(\text{a red card}) = \frac{3}{5}$.

(*x*) As there is no spade in the five cards

P(a spade) = 0.

11. Total possible outcomes = (20 + 40 + 60 + 100) sq m

= 220 sq m.

(*i*) Sand hazards measuring 20 sq m.

$P(\text{sand hazards measuring 20 sq m}) = \frac{20}{220} = \frac{1}{11}$.

(*ii*) $P(\text{water hazards measuring 40 sq m}) = \frac{40}{220} = \frac{4}{22}$

$= \frac{2}{11}$.

(*iii*) $P(\text{rough hazards measuring 60 sq m}) = \frac{60}{220} = \frac{3}{11}$.

(*iv*) $P(\text{green measuring 100 sq m}) = \frac{100}{220} = \frac{5}{11}$.

12. Total possible outcomes = 10.

(*i*) $P(5) = \frac{1}{10}$.

(*ii*) Odd numbers = 1, 3, 5, 7, 9 = favourble outcomes

= 5.

$P(\text{odd number}) = \frac{5}{10} = \frac{1}{2}$.

(*iii*) Number greater than 7 = 8, 9.

So, number of favourable outcomes = 2.

$P(\text{greater than 7}) = \frac{2}{10} = \frac{1}{5}$.

(*iv*) Natural numbers = favourable outcomes = 9 (1 to 9).

$P(\text{a natural number}) = \frac{9}{10}$.

13. (*i*) Percentage of people with white hair

= (100 – 65 – 25) = 10%.

∴ $P(\text{white}) = \frac{10}{100} = \frac{1}{10}$.

(*ii*) Percentage of people with brown or black hair

$= (65 + 25) = 90\%$.

$\therefore$ P(brown or black) $= \frac{90}{100} = \frac{9}{10}$.

(*iii*) Percentage of people with white or black hair

$= (10 + 65) = 75\%$.

$\therefore$ P(white or black) $= \frac{75}{100} = \frac{3}{4}$.

(*iv*) Percentage of people with neither brown nor white hair = 65%.

$\therefore$ P(neither brown nor white) $= \frac{65}{100} = \frac{13}{20}$.

14. Total number of patients are 12 out of whom 3 patients are extremely patient, other 6 are extremely honest and the remaining 13 are extremely kind. Hence, total number of outcomes of selecting a patient is 12.

The number of possible outcomes in favour of extremely patient = 3.

(*i*) $\therefore$ The required probability of selecting a patient who is extremely patient $= \frac{3}{12} = \mathbf{\frac{1}{4}}$.

(*ii*) There are remaining 3 + 6 = 9 patients who are either extremely kind or honest.

Hence, the required probability of selecting a patient who is either extremely kind or honest $= \frac{9}{12} = \mathbf{\frac{3}{4}}$.

15. S = {1, 2, 3, 4, 5, 6, 7, 8}

(*i*) Odd numbers = 1, 3, 5, 7

P(odd numbers) $= \frac{4}{8} = \frac{1}{2}$

(*ii*) Numbers greater than 3 = 4, 5, 6, 7, 8

P(number greater than 3) $= \frac{5}{8}$

(*iii*) Numbers less than 9 = 1, 2, 3, 4, 5, 6, 7, 8

P(number less than 9) $= \frac{8}{8} = 1$

16. Total number of possible outcomes = 6.

(*i*) Number of interior angles = 3.

P(an interior angle) $= \frac{3}{6} = \frac{1}{2}$.

(*ii*) As there is no right angle so,

P(a right angle) = 0.

(*iii*) Acute angle= 3 (favourable outcome)

P(an acute angle) $= \frac{3}{6} = \frac{1}{2}$.

(*iv*) Straight angle = 0.

P(a straight angle) = 0.

(*v*) 60° angle = 1.

P(a 60° angle) $= \frac{1}{6}$.

(*vi*) No acute exterior angle is given.

$\therefore$ P(an acute exterior angle) = 0.

(*vii*) Angle whose measure is < 180° = 6.

P(an angle < 180°) $= \frac{6}{6} = 1$.

17. (*i*) (*a*) Total possible outcomes = 15.

Favourable outcomes = 6.

P(white) $= \frac{6}{15} = \frac{2}{5}$.

(*b*) Favourable outcomes = 4.

P(red) $= \frac{4}{15}$.

(*c*) Balls i.e., not black = 10 = favourable outcomes

P(not black) $= \frac{10}{15} = \frac{2}{3}$.

(*d*) Favourable outcomes for red or white = 10.

P(red or white) $= \frac{10}{15} = \frac{2}{3}$.

(*ii*) Total possible outcomes = 5 + 8 + 4 + 7 = 24.

(*a*) Favourable outcomes for black ball = 7.

P(black) $= \frac{7}{24}$.

(*b*) Balls not green = 5 + 8 + 7 = 20.

P(not green) $= \frac{20}{24} = \frac{10}{12} = \frac{5}{6}$.

(*iii*) Total possible outcomes = 5 + 8 + 7 = 20.

Neither green nor a red ball = 7.

P(neither a green nor red ball) $= \frac{7}{20}$.

(*iv*) Total number of balls = 5 + 7 + 4 + 2 = 18.

(*a*) Number of white or blue balls = 5 + 2 = 7.

$\therefore$ P(white or blue) $= \frac{7}{18}$.

(*b*) Number of red or black balls = 7 + 4 = 11.

$\therefore$ P(red or black) $= \frac{11}{18}$.

(*c*) Number of balls which are not white

= 7 + 4 + 2 = 13.

$\therefore$ P(not white) $= \frac{13}{18}$.

(*d*) Number of balls which are neither white nor black

= 7 + 2 = 9.

$\therefore$ P(neither white nor black) $= \frac{9}{18} = \frac{1}{2}$.

18. There are 24 balls of which x are red, $2x$ are white and $3x$ are blue.

$\therefore$ The probability of getting red ball, R, is

$$P(R) = \frac{x}{24} \quad \ldots(1)$$

The probability of getting a white ball, W, is

$$P(W) = \frac{2x}{24} \quad \ldots(2)$$

and the probability of getting a blue ball, B is

$$P(B) = \frac{3x}{24} \quad \dots(3)$$

Also, total number of possible outcomes = 24.

∴ The total probability is $\frac{x}{24} + \frac{2x}{24} + \frac{3x}{24} = 1$

$$\Rightarrow \quad \frac{6x}{24} = 1$$

$$\Rightarrow \quad x = 4 \quad \dots(4)$$

(*i*) Here $P(R) = \frac{4}{24}$ [From (1) and (4)]

$$= \frac{1}{6}$$

$$\therefore \quad P(\bar{R}) = 1 - \frac{1}{6} = \mathbf{\frac{5}{6}}$$

which is the required probability in this case.

(*ii*) Here $P(W) = \frac{2 \times 4}{24}$ [From (2) and (4)]

$$= \mathbf{\frac{1}{3}}$$

which is the required probability in this case.

19. Total number of possible outcomes = 5 + 6 + 3 + 7 + 9 = 30.

(*i*) Number of red pencils = 3.

$$P(\text{red}) = \frac{3}{30} = \frac{1}{10}.$$

(*ii*) Number of green pencils = 6.

$$P(\text{green}) = \frac{6}{30} = \frac{1}{5}.$$

(*iii*) Number of pencils not black = 30 – 7 = 23.

$$P(\text{not black}) = \frac{23}{30}.$$

(*iv*) Number of pencils not yellow = 30 – 9 = 21.

$$P(\text{not yellow}) = \frac{21}{30} = \frac{7}{10}.$$

(*v*) Number of pencils (brown) = 0.

$$P(\text{brown}) = \frac{0}{30} = 0.$$

20. Here there are a total of cards, 100 + 200 + 50 = 350 out of which there are 100 red cards, 200 yellow cards and 50 blue cards.

∴ The total number of outcomes of drawing a card from the box = 350.

Let P(B) = Probability of drawing a blue card

P(Y) = Probability of drawing a yellow card and

$P(\overline{BY})$ = Probability of drawing neither a blue card nor a yellow card

= P(R) …(1)

(*i*) Here $P(B) = \frac{50}{350} = \mathbf{\frac{1}{7}}$

(*ii*) Here $P(Y) = \frac{200}{350} = \frac{4}{7}$

$$\therefore \quad P(\bar{Y}) = 1 - \frac{4}{7} = \mathbf{\frac{3}{7}}$$

(*iii*) Here $P(R) = \frac{100}{350} = \mathbf{\frac{2}{7}}$

$$= P(\overline{BY})$$

Hence, the required probability drawing a blue card, not a yellow card and neither a yellow nor a blue card are respectively $\frac{1}{7}$, $\frac{3}{7}$ and $\frac{2}{7}$.

21. (*i*) (*a*) There are 20 cards from 11 to 30.

Total possible outcomes = 20.

Multiples of 7 = 14, 21, 28

So favourable outcomes = 3.

$$P(\text{multiple of 7}) = \frac{3}{20}.$$

(*b*) Greater than 15 and multiple of 5 = 20, 25, 30.

Favourable outcomes = 3.

$$P(\text{greater than 15 and a multiple of 5}) = \frac{3}{20}.$$

(*ii*) S = {3, 4, 5, 6, …, 49, 50}

(*a*) Divisible by 11 = 11, 22, 33, 44

$$P(\text{divisible by 11}) = \frac{4}{48} = \frac{1}{12}$$

(*b*) Perfect squares = 4, 9, 16, 25, 36, 49

$$P(\text{a perfect square}) = \frac{6}{48} = \frac{1}{8}$$

(*c*) Multiple of 6 = 6, 12, 18, 24, 30, 36, 42, 48

$$P(\text{multiple of 6}) = \frac{8}{48} = \frac{1}{6}$$

(*iii*) Favourable outcomes = 20.

Total possible outcomes = 30.

$$P(\text{not divisible by 3}) = \frac{20}{30} = \frac{2}{3}.$$

(*iv*) (*a*) Total number of possible outcomes = 18.

Prime numbers in the bag less than 15 are 3, 5, 7, 11 and 13.

∴ Number of favourable outcomes = 5.

$$P(\text{prime number less than 15}) = \frac{5}{18}.$$

(*b*) In the bag only card with number 15 is divisible by 3 and 5.

$$P(\text{a number divisible by 3 and 5}) = \frac{1}{18}.$$

(*v*) (*a*) Total possible outcomes = 15.

An even number = 7.

$$P(\text{an even number}) = \frac{7}{15}.$$

(*b*) Numbers divisible by 3 or 5 = 7.

$$P(\text{a number divisible by 3 or 5}) = \frac{7}{15}.$$

(*vi*) S = {6, 7, 8, 9, 10, …, 69, 70}

(*a*) One digit numbers = {6, 7, 8, 9}

P (one digit number) = $\frac{4}{65}$

(*b*) Number divisible by 5 = {10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70}

P (number divisible by 5) = $\frac{13}{65} = \frac{1}{5}$

(*c*) Odd number less than 30 = {7, 9, 11, 13, …, 29}

P(odd number less than 30 = $\frac{12}{65}$

(*d*) Composite number between 50 and 70 = 15

P(composite numbers between 50 and 70) = $\frac{15}{65}$

$= \frac{3}{13}$

(*vii*) Sample space = {1, 3, 5, 7, …, 49}

(*a*) Divisible by 3 = {3, 9, 15, 21, 27, 33, 39, 45}

P(divisible by 3) = $\frac{8}{25}$

(*b*) Composite numbers = Total – Prime numbers
= 25 – 15 = 10

P(composite number) = $\frac{10}{25} = \frac{2}{5}$

(*c*) Not a perfect square = Total – Perfect squares
= 25 – 4 = 21

P(not a perfect square) = $\frac{21}{25}$

(*d*) Multiple of 3 and 5 = {15, 45}

P(multiple of 3 and 5) = $\frac{2}{25}$

(*viii*) Numbers in all the cards are as follows:

11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60.

The total number of such numbers is 50.

∴ The total number of outcomes of drawing a card is 50.

Out of these numbers, the total number of odd numbers is 25, the perfect square numbers are 16, 25, 36 and 49.

∴ The number of these perfect numbers is 4. The numbers which are divisible by 5 are 15, 20, 25, 30, 35, 40, 45, 50, 55 and 60.

∴ The number of numbers which are divisible by 5 is 10.

Finally, out of these, the prime numbers are 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53 and 59.

∴ The prime numbers which are less than 20 are 11, 13, 17, 19.

∴ The number of such prime numbers is 4.

∴ The number of prime numbers is 13.

Hence,

(*a*) Required probability of getting an odd number = $\frac{25}{50} = \mathbf{\frac{1}{2}}$.

(*b*) Required probability of getting a perfect square number = $\frac{4}{50} = \mathbf{\frac{2}{25}}$.

(*c*) Required probability of getting a number divisible by 5 = $\frac{10}{50} = \mathbf{\frac{1}{5}}$.

(*d*) Required probability of getting a prime number which is less than 20 is $\frac{4}{50} = \mathbf{\frac{2}{25}}$.

(*ix*) The total number of cards in the box is (123 – 11) + 1 = 112 + 1 = 113.

∴ The total number of outcomes of drawing a card from the box is 113.

(*a*) There are 8 squared numbers which are 16, 25, 36, 49, 64, 81, 100 and 121. Hence, the required probability of drawing a squared number is $\mathbf{\frac{8}{113}}$.

(*b*) There are 16 numbers between the given numbers which are multiples of 7 *viz.* 14, 21, 28, 35, 42, 49, 56, 63, 70, 77, 84, 91, 98, 105, 112 and 119.

Hence, the required probability in this case is $\mathbf{\frac{16}{113}}$.

(*x*) From 1 to 90, there are 90 numbers.

∴ The total number of possible outcomes of drawing a disc numbered from 1 to 90, is 90.

(*a*) From 1 to 90, there are (90 – 10) + 1 = 81 two-digit numbers.

∴ Required probability of drawing disc numbered with two-digit numbers = $\frac{81}{90} = \mathbf{\frac{9}{10}}$.

(*b*) From 1 to 90, the perfect square numbers are 1, 4, 9, 16, 25, 36, 49, 64 and 81.

∴ There are 9 such perfect square numbers.

∴ Required probability in this case = $\frac{9}{90} = \mathbf{\frac{1}{10}}$.

(*c*) From 1 to 90, the numbers which are divisible by 5 are 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85 and 90.

∴ There are 18 such numbers. Hence, the required probability of drawing discs numbered with numbers which are divisible by 5 is $\frac{18}{90} = \frac{1}{5}$.

(*xi*) The total number of cards is (20 – 1) + 1 = 20.

∴ Possible number of total outcomes of drawing a box containing cards which are numbered from 1 to 20 is 20.

(*a*) The total number of prime numbers from 1 to 20 is 8 *viz.* 2, 3, 5, 7, 11, 13, 17 and 19.

∴ Required probability = $\frac{8}{20} = \mathbf{\frac{2}{5}}$.

(*b*) The total number of composite numbers from 1 to 20 is 11 *viz.* 4, 6, 8, 9, 10, 12, 14, 15, 16, 18 and 20.

∴ Required probability = $\mathbf{\frac{11}{20}}$.

(*c*) The total number of numbers which are divisible by 3, from 1 to 20 is 6 *viz.* 3, 6, 9, 12, 15 and 18.

∴ Required probability = $\frac{6}{20} = \mathbf{\frac{3}{10}}$.

22. (*i*) When 3 coins are tossed simultaneously, all possible outcomes are HHH, HHT, HTH, THH, HTT, THT, TTH, TTT.

Total number of favorable outcomes = 8.

(*a*) Favourable outcome for getting 3 tails = 1.

P(3 tails) = $\frac{1}{8}$.

(*b*) Favourable outcomes for getting 2 tails = 3.

P(2 tails) = $\frac{3}{8}$.

(*c*) Favourable outcome for no tail = 1 (HHH).

P(no tail) = $\frac{1}{8}$.

(*d*) Favourable outcomes for 2 heads and one tail = 3.

P(2 heads and 1 tail) = $\frac{3}{8}$.

(*e*) Favourable outcome for at least one head = 7.

P(at less one head) = $\frac{7}{8}$.

(*ii*) When 2 coins are tossed simultaneously, all possible outcomes are HH, HT, TH, TT.

Total number of favourable outcomes = 4.

Favourable outcome of getting atleast 1 tail = 3.

∴ P(getting at least one tail) = $\frac{3}{4}$.

23. Total possible outcomes = 8.

Favourable outcome for getting head and tail alternately = 2.

∴ P(getting head and tail alternately) = $\frac{2}{8} = \frac{1}{4}$.

24. We use the symbols: G for a girl and B for boy.

Then, possible outcomes are GGG, BBB, BBG, BGB, GBB, BGG, GBG and GGB.

∴ Number of possible outcomes = 8.

Let E be the event of "having at least one boy".

Then the outcomes favourable to E are BBB, BBG, BGB, GBB, BGG, GBG and GGB.

∴ Number of outcomes favourable to E is 7.

∴ Required probability = Probability (at least one boy) = $\mathbf{\frac{7}{8}}$.

25. Year 1990 is a non-leap year, so it is a normal year i.e., 365 days.

Possible outcomes = 365 × 365 = 133225.

Favourable outcomes when the birthdays will be same = 365.

(*i*) P(having same birthday) = $\frac{365}{133225} = \frac{1}{365}$.

(*ii*) P(having different birthdays) = $\frac{133225 - 365}{133225} = \frac{364}{365}$.

26. (*i*) P(rotten apples) = 0.18

Total apples = 900

$$P(E) = \frac{\text{Favourable outcomes}}{\text{Total outcomes}}$$

$$0.18 = \frac{\text{rotten apples}}{900}$$

Rotten apples = 900 × 0.18 = 162

(*ii*) P(losing) = 1 – P(winning)

= 1 – 0.6 = 0.4.

27. Number of envelopes which contain cash prizes = (10 + 100 + 200) = 310.

∴ Total number of envelopes = 1000 [Given]

∴ Number of envelopes which contain no cash prize at all = 1000 – 310 = 690.

∴ 690 cases are favourable in picking up the envelopes.

Hence, the required probability = $\frac{690}{1000} = \frac{69}{100}$ = **0.69.**

28. Number of ₹1 notes in the first box = 19 and number of ₹5 notes = 25 – 19 = 6.

Number of ₹1 notes in the second box = 45 and number of ₹5 notes = 5.

Out of these notes, the total number of ₹1 notes = 19 + 45 = 64. and number of ₹5 notes = 6 + 5 = 11.

Also, total number of ₹1 and ₹5 notes in two boxes together = 25 + 50 = 75.

∴ Required probability = $\mathbf{\frac{11}{75}}$.

For Standard Level

29. Total number of possible outcomes = 20.

(*i*) Favourable outcomes = 3 (31, 33, 32).

P(31 nails) = $\frac{3}{20}$.

(*ii*) Favourable outcomes for less than 33 nails = (20 – 6) = 14.

P(< 33 nails) = $\frac{14}{20} = \frac{7}{10}$.

(*iii*) Favourable outcomes for at least 28 nails = 20.

P(at least 28 nails) = $\frac{20}{20}$ = 1.

(*iv*) More than 33 nails = 0 boxes.
P(more than 33 nails) = 0.

30. (*i*) Total possible outcomes = 36.
Favourable outcomes = 3{(2, 5), (4, 5), (6, 5)}
P(an even number on the first and 5 on the 2nd die)
$= \frac{3}{36} = \frac{1}{12}$.

(*ii*) Favourable outcomes for number > 4 on each die = 4 {(5, 5), (5, 6), (6, 5), (6, 6)}.
P(number > 4 on each die) $= \frac{4}{36} = \frac{1}{9}$.

31. S = {(1, 1), (1, 2), (1, 3) …, (1, 6), (2,1) (2, 2) …, (6, 6)}
(*i*) Number on each die is even = {(2, 2), (2, 4) …, (6, 6)}
P (Number on each die is even) $= \frac{9}{36} = \frac{1}{4}$

(*ii*) Sum of numbers appearing on two die is 5
= {(1, 4), (2, 3), (3, 2), (4, 1)}
P (Sum of numbers is 5) $= \frac{4}{36} = \frac{1}{9}$

32. Total number of outcomes = 36.
Outcome favourable to the event of getting a total of 12 is (6, 6).
∴ Number of outcomes of getting a total of 12 = 1.
∴ Number of outcomes of getting a total less than 12
= 36 – 1 = 35.
∴ P(a total of less than 12) $= \frac{35}{36}$.

33. (*i*) A die has 6 faces each being marked with six numbers from 1, 2, 3, 4, 5 and 6. Hence, the two dice when thrown together will show the faces 1, 2, 3 …6 pair-wise like 11, 12, 13, …16, 21, 22, …26, 31, 32, 33, … 36, 41, 42, 43, …46, 51, 52, 53, 54, …56, 61, 62, 63 …66.
∴ The total number of outcomes of obtaining a pair of such numbers is 6 × 6 = 36.
Out of these outcomes, the outcomes of the digits getting a pair of numbers with even sums are 11, 13, 31, 15, 51, 22, 24, 42, 26, 62, 44, 46, 64, 51, 53, 35, 55 and 66.
∴ There are 18 such numbers.
Hence, required probability of getting even sum $= \frac{18}{36}$
$= \mathbf{\frac{1}{2}}$.

(*ii*) In this case, the outcomes of getting a pair of numbers with even products of its digits are 12, 21, 14, 41, 16, 61, 22, 23, 32, 24, 42, 25, 52, 26, 62, 34, 43, 36, 63, 44, 45, 54, 46, 64, 56, 65 and 66.
∴ There are 27 such numbers.
Hence, required probability of getting even product
$= \frac{27}{36} = \mathbf{\frac{3}{4}}$.

(*iii*) In this case, the outcomes of getting a pair of numbers, the sum of whose digits is less than 7, are 11, 12, 21, 13, 31, 14, 41, 15, 51, 22, 23, 32, 33, 24 and 42.
∴ There are 15 such numbers.
Hence, the required probability getting such numbers
$= \frac{15}{36} = \mathbf{\frac{5}{12}}$.

(*iv*) In this case, the outcomes of getting a pair of numbers, the product of whose digits are less than 16, are 11, 12, 21, 13, 31, 14, 41, 15, 51, 16, 61, 22, 23, 32, 24, 42, 25, 52, 26, 62, 33, 34, 43, 35 and 53.
∴ There are 25 such numbers.
Hence, the required probability of getting such numbers
$= \mathbf{\frac{25}{36}}$.

34. Here the total number of pairs of numbers = 6 × 6 = 36.
(*i*) The doublets of odd number in a pair are 11, 33 and 55.
∴ There are 3 such numbers. Hence, the required probability $= \frac{3}{36} = \frac{1}{12}$.

(*ii*) The number of doublets is 6 *viz.* 11, 22, 33, 44, 55 and 66.
Hence, the required probability $= \frac{6}{36} = \mathbf{\frac{1}{6}}$.

(*iii*) Here the numbers shown by the two dice when thrown together such that their sum is 10, are 46, 64 and 55.
∴ The number of such numbers is 3.
Hence, the required probability $= \frac{3}{36} = \mathbf{\frac{1}{12}}$.

(*iv*) Numbers, the products of whose digits are less than 9 are 11, 12, 21, 13, 31, 14, 41, 15, 51, 16, 61, 22, 23, 32, 24 and 42.
There are 16 such numbers.
Hence, the required probability $= \frac{16}{36} = \mathbf{\frac{4}{9}}$.

(*v*) Two-digit numbers, the sums of whose digits are each equal to 7, are 16, 61, 25, 52, 34, 43. The number of such numbers is 6.
∴ Required probability $= \frac{6}{36} = \mathbf{\frac{1}{6}}$.

(*vi*) Prime numbers from 1 to 12 are 2, 3, 5, 7 and 11.
Two-digit numbers, the sums of whose digits are each equal to the prime numbers: 2, 3, 5, 7 or 11, are 11, 12, 21, 32, 23, 41, 14, 43, 34, 25, 52, 61, 16, 65 and 56.
There are 15 such numbers.
Hence, the required probability $= \frac{15}{36} = \mathbf{\frac{5}{12}}$.

(*vii*) We know that there is no two numbers on a pair of dice such that their sum is 1.
Hence, the required probability $= \frac{0}{36} = \mathbf{0}$.

(*viii*) We know that in each dice, there are three prime number *viz.* 2, 3, and 5.

Numbers in a pair of dice, which are such that the product of the digits of each number is equal to 2, 3 or 5, are 12, 21, 31, 13, 15 and 51. Hence, there are 6 such numbers.

$\therefore$ Required probability $= \frac{6}{36} = \mathbf{\frac{1}{6}}$.

(*ix*) Numbers in a pair of dice, which are such that the product of the digits of each number is less than 18, are 12, 13, 14, 15, 16, 21, 22, 23, 24, 25, 26, 31, 32, 33, 34, 35, 36, 41, 42, 43, 44, 51, 52, 53, 61 and 62.
$\therefore$ There are 26 such numbers.

$\therefore$ Required probability $= \frac{26}{36} = \mathbf{\frac{13}{18}}$.

(*x*) Two-digit numbers whose two digits are the same and are the digits of a pair of dice are 11, 22, 33, 44, 55 and 66 which are 6 in number.

$\therefore$ Number of those numbers whose digits are all different and are the digits of a pair of dice is

$$36 - 6 = 30.$$

Hence, the required probability $= \frac{30}{36} = \mathbf{\frac{5}{6}}$.

35. If x is the digit on the dice which comes up in the 1st role and y is the digit on the dice which comes up in the 2nd role, then we shall denote the outcome by the symbol (x, y).

Now, total number of possible outcomes is $6 \times 6 = 36$.

(*i*) Outcomes where the digit 5 will not come up either time are

(1, 1), (1, 2), (1, 3), (1, 4), (1, 6)
(2, 1), (2, 2), (2, 3), (2, 4), (2, 6)
(3, 1), (3, 2), (3, 3), (3, 4), (3, 6)
(4, 1), (4, 2), (4, 3), (4, 4), (4, 6)
(6, 1), (6, 2), (6, 3), (6, 4), (6, 6)

$\therefore$ Number of favourable outcomes = 25

$\therefore$ Required probability that 5 will not come up either time $= \mathbf{\frac{25}{36}}$.

(*ii*) In this case, outcomes where 5 will come up exactly 1 time are (1, 5), (2, 5), (3, 5), (4, 5), (6, 5), (5, 1), (5, 2), (5, 3), (5, 4), (5, 6).

Hence, in this case, the number of favourable outcomes = 10.

$\therefore$ Required probability $= \frac{10}{36} = \mathbf{\frac{5}{18}}$.

(*iii*) Here, the outcomes where 2 will come up at least once, are (1, 2), (2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6), (3, 2), (4, 2), (5, 2) and (6, 2).

$\therefore$ In this case, the number of favourable outcomes = 11.

Hence, the required probability $= \mathbf{\frac{11}{36}}$.

(*iv*) Here, the outcomes where 2 will not come up either times are

(1, 1), (1, 3), (1, 4), (1, 5), (1, 6)
(3, 1), (3, 3), (3, 4), (3, 5), (3, 6)
(4, 1), (4, 3), (4, 4), (4, 5), (4, 6)
(5, 1), (5, 3), (5, 4), (5, 5), (5, 6)
(6, 1), (6, 3), (6, 4), (6, 5), (6, 6)

$\therefore$ Total number of favourable outcomes = 25.

$\therefore$ Required probability $= \mathbf{\frac{25}{36}}$.

36. Let (xy) denote the outcome where x is a number in the 1st dice and y is a number in the 2nd dice.

Then all possible outcomes when two dice are thrown simultaneously, are as follows:

(11), (12), (13), (14), (15), (16),
(21), (22), (23), (24), (25), (26),
(31), (32), (33), (34), (35), (36),
(41), (42), (43), (44), (45), (46),
(51), (52), (53), (54), (55), (56),
(61), (62), (63), (64), (65), (66).

Hence, the total number of possible outcomes is $6 \times 6 = 36$.

(*i*) Outcomes where the sum of the digits is equal to 6, are (15), (51), (24), (42), (33).

$\therefore$ The number of favourable cases is 5.

Hence, the required probability $= \mathbf{\frac{5}{36}}$.

(*ii*) Outcomes where the sum of the digits is equal to 10 are (46), (64) and (55).

$\therefore$ The number of favourable cases in the case is 3.

$\therefore$ Required probability $= \frac{3}{36} = \mathbf{\frac{1}{12}}$.

37. Distinct numbers in two dice are 1, 2, 3, 4, 5 and 6, and 1, 2, 3 respectively.

$\therefore$ Total number of possible distinct outcomes are 11, 12, 13, 21, 22, 23, 31, 32, 33, 41, 42, 43, 51, 52, 53, 61, 62, and 63.

Favourable distinct outcomes for sums 2, 3, 4, 5, 6, 7, 8 and 9 for two numbers of two dice are respectively 11; 12 and 21; 13, 31 and 22; 23, 32 and 41; 15, 51 and 42; 52, 61 and 43; 62 and 53; 63.

$\therefore$ $P(2) = \mathbf{\frac{1}{18}}$; $P(3) = \frac{2}{18} = \mathbf{\frac{1}{9}}$; $P(4) = \frac{3}{18} = \mathbf{\frac{1}{6}}$;

$P(5) = \frac{3}{18} = \mathbf{\frac{1}{6}}$; $P(6) = \frac{3}{18} = \mathbf{\frac{1}{6}}$; $P(7) = \frac{3}{18} = \mathbf{\frac{1}{6}}$,

$P(8) = \frac{2}{18} = \mathbf{\frac{1}{9}}$ and $P(9) = \mathbf{\frac{1}{18}}$,

which are the required probabilities.

38. Two dice have the numbers as follows:

0, 1, 1, 1, 6, 6 and 0, 1, 1, 1, 6, 6

∴ Total scores are

$$0 + 0 = 0, 0 + 1 = 1, 0 + 6 = 6$$

$$1 + 1 = 2, 1 + 6 = 7, 6 + 6 = 12$$

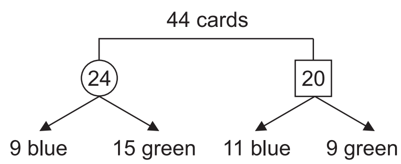
Hence, (*i*) the required different scores are **6** in number *viz.* **0, 1, 2, 6, 7** and **12**.

(*ii*) Total of 7 occurs twice, *viz.* 1 + 6 = 7, 6 + 1 = 7.

∴ Out of 6 total scores, a total of 7 occurs twice.

∴ Required probability of getting a total of 7 = $\frac{2}{6}$ = $\mathbf{\frac{1}{3}}$.

39.



(*i*) Total number of cards = 44.

Total number of squares = 20.

∴ P(square cards) = $\frac{20}{44}$ = $\mathbf{\frac{5}{11}}$.

(*ii*) Total number of green cards = 15 + 9 = 24.

∴ P(Green cards) = $\frac{24}{44}$ = $\mathbf{\frac{6}{11}}$.

(*iii*) Total number of blue circles = 9.

∴ Required probability = $\mathbf{\frac{9}{44}}$.

(*iv*) Total number of green square = 9.

∴ Required probability = $\mathbf{\frac{9}{44}}$.

40. Out of 52 cards, 1 King, 1 Queen and 1 jack of clubs are removed. Hence, there are 52 – 3 = 49 remaining cards.

(*i*) There are 13 cards of spade.

∴ Required probability of a spade card = $\mathbf{\frac{13}{49}}$.

(*ii*) Out of a total of 2 black Kings, 1 black King is removed. Hence, 1 black is left. Hence, required probability of a black King = $\mathbf{\frac{1}{49}}$.

(*iii*) 3 cards (*viz.* 1 King, 1 Queen and 1 Jack of 13 Club cards) are removed. Hence, out of 13 Club cards, 13 – 3 = 10 cards are left. Hence, the required probability of a Club card = $\mathbf{\frac{10}{49}}$.

(*iv*) 1 Jack has been removed. Hence, out of a total of 4 Jacks, only 3 are left. Hence, the required probability of Jack cards = $\mathbf{\frac{3}{49}}$.

41. We know that in a deck of 52 cards, there are 2 red Queens and 2 black Jacks. (Note that all 26 cards of Diamonds and Hearts are red and all 26 cards of Clubs and Spades are black).

∴ When 2 red Queens and 2 red Jacks are removed, 52 – 4 = 48 cards are left.

(*i*) Out of all these 48 cards, 4 are Kings. Hence, the required probability of drawing a King card = $\frac{4}{48}$ = $\mathbf{\frac{1}{12}}$.

(*ii*) We know that out of a deck of 52 cards, 26 are red. If two red Queens and 2 black Jacks are removed, 52 – 4 = 48 cards are left out of which 26 – 2 = 24 cards are red. Hence, in this case the required probability of getting red cards = $\frac{24}{48}$ = $\mathbf{\frac{1}{2}}$.

(*iii*) We know that there are 4 Kings, 4 Queens and 4 Jacks in face cards. Out of these 12 face cards, there are 2 red Queens and 2 black Jacks which have been removed. Hence, the number of remaining face cards = 12 – 4 = 8.

Also, total number of remaining cards out of a deck of 52 cards = 48.

Hence, the required probability of getting a face card = $\frac{8}{48}$ = $\mathbf{\frac{1}{6}}$.

(*iv*) Since 2 red Queens out of a total of 4 Queens have been removed, only 4 – 2 = 2 Queens are left out of the remaining total 48 cards. Hence, the required probability of getting a Queens card = $\frac{2}{48}$ = $\mathbf{\frac{1}{24}}$.

42. A total of 3 cards *viz.* 1 King, 1 Jack and 1 ten of Spade cards are lost out of a deck of 52 cards. Hence, the total number of the remaining cards = 52 – 3 = 49.

(*i*) Now, we know that all Spade cards are black. Hence, all the 26 red cards remained in the total number of remaining 49 cards.

∴ Required probability of getting a red card = $\mathbf{\frac{26}{49}}$.

(*ii*) We know that in a deck of 52 cards, there are 4 Jacks. Since, black Jack of Spade cards in lost, only 1 black Jack of Club cards remained. Hence, the required probability of getting 1 black Jack out of the remaining total 49 cards = $\mathbf{\frac{1}{49}}$.

(*iii*) Since all 13 cards of Spade are black, 2 red Kings out a deck of 52 cards remained as it is. Hence, the required probability of getting a red King out of the remaining total 49 cards, is $\mathbf{\frac{2}{49}}$.

(*iv*) There is only 1 ten of Hearts in the remaining total 49 cards.

Hence, the required probability of getting ten of Hearts is $\mathbf{\frac{1}{49}}$.

43. 1 King, 1 Queen and 1 Jack of Clubs cards removed from a deck of 52 cards.

∴ The total number of remaining cards = 52 – 3 = 49.

(*i*) We know that there are 13 Heart cards. Hence, the required probability of getting a card of Heart from the total of the remaining 49 cards is $\mathbf{\frac{13}{49}}$.

(*ii*) Out of 4 Queens in a deck of 52 cards. 1 Queen of a Heart card is removed and so 3 remaining Queens out of a deck of 52 cards remained.

Hence, the required probability of getting a Queen card = $\mathbf{\frac{3}{49}}$.

(*iii*) Out of 13 cards of Clubs, 3 cards (*viz.* 1 King, 1 Queen and 1 Jack) are removed. Hence, only 13 – 3 = 10 cards of Clubs remained.

Hence, the required probability of getting the remaining 10 cards of clubs out of the remaining total 49 cards is $\mathbf{\frac{10}{49}}$.

(*iv*) Out of a total of 12 face cards, 3 face cards (*viz.* 1 King, 1 Queen and 1 Jack of Clubs cards) have been removed.

∴ Number of remaining face cards = 12 – 3 = 9.

Hence, the required probability of getting 9 face cards out of the remaining total 49 cards is $\mathbf{\frac{9}{49}}$.

(*v*) There is only one Queen of Diamond card. Hence, the required probability of getting a Queen of Diamond card out of the remaining total 49 cards is $\mathbf{\frac{1}{49}}$.

44. (*i*) Total cards = 52

Cards removed = 3 face card remaining = 9

Remaining cards = 49

(*a*) P (face card) = $\frac{9}{49}$

(*b*) P (a red card) = $\frac{23}{49}$

(*c*) P (a king) = $\frac{3}{49}$

(*d*) P (a diamond) = $\frac{10}{49}$

(*e*) P (a spade) = $\frac{13}{49}$

(*f*) P (the '7' of clubs) = $\frac{1}{49}$

(*ii*) Number of cards removed = (2 + 2 + 2 + 2) = 8.

Total number of remaining cards = (52 – 8) = 44.

Now, there are 2 jacks, 2 queens, 2 kings and 2 aces of black colour only.

(*a*) Number of black queen = 2.

∴ P(getting a black queen) = $\frac{2}{44} = \frac{1}{22}$.

(*b*) Number of red cards = (26 – 8) = 18.

∴ P(getting a red card) = $\frac{18}{44} = \frac{9}{22}$.

(*c*) Number of tens = 4.

∴ P(getting a ten) = $\frac{4}{44} = \frac{1}{11}$.

(*d*) Jacks, queens and kings are picture cards.

∴ Remaining number of picture cards

= (2 + 2 + 2) = 6.

∴ P(getting a picture card) = $\frac{6}{44} = \frac{3}{22}$.

(*iii*) Red face cards removed

Face cards remaining = 6

Total cards remaining = 46

(*a*) P (a red card) = $\frac{20}{46} = \frac{10}{23}$

(*b*) P (a face card) = $\frac{6}{46} = \frac{3}{23}$

(*c*) P (a card of clubs) = $\frac{13}{46}$

45. We know that in a deck of 52 cards, there are 4 Jacks, 4 Kings and 4 Queens.

∴ Total number of cards which are removed = 4 × 3 = 12.

∴ Total number of remaining cards from a deck of 52 cards = 52 – 12 = 40.

(*i*) We know that all 13 cards of Diamonds are red.

∴ Number of black cards in Diamond cards is 0.

Hence, the required probability of getting a black face card = $\frac{0}{40}$ = **0**.

(*ii*) We know that there are altogether 26 red cards in a deck of 52 cards. Out of these red cards, 3 red face cards of Diamonds (*viz.* 1 red Kings, 1 red Queen and 1 red Jack) and 3 red face cards of Hearts (*viz.* 1 red King, 1 red Queen and 1 red Jack) are removed.

Hence, a total of 6 red face cards are removed from Diamond and Hearts cards.

∴ Number of the remaining red cards = 26 – 6 = 20. Hence, the required probability of getting a red card = $\frac{20}{40} = \mathbf{\frac{1}{2}}$.

46. We know that in a deck of 52 cards, there are 4 Kings and 4 Queens. If these 8 cards are removed from the deck, then the total number of remaining cards is 52 – 8 = 44.

(*i*) We know that after removing all red Kings and Queens of Diamond and Heart cards, only 1 red Jack of Diamond and 1 red Jack of Heart among all face cards remained, i.e. only 2 red face cards remained. Hence, the required probability of getting a red face card = $\frac{2}{44} = \mathbf{\frac{1}{22}}$.

(*ii*) Out of 26 black cards, only 4 black face cards, (*viz.* 1 black King of Spade and 1 black King of Club and 1 black Queen of Spade and 1 black Queen of Club) are removed.

∴ 26 – 4 = 22 black cards remained. Hence, the required probability of getting a black card in this case = $\frac{22}{44} = \mathbf{\frac{1}{2}}$.

47. Let n be the required number of red balls in the bag. Then the total number of red balls in the box after 6 more red balls are put in the box is $n + 6$.

$\therefore$ According to the problem,

Probability of getting a red ball in the first case = $\frac{n}{12}$

and the probability of getting a red ball in the second case = $\frac{n+6}{18}$, since the total number of balls in this case is 12 + 6 = 18.

$\therefore$ According to the problem, we have

$$\frac{n+6}{18} = 2 \times \frac{n}{12}$$

$$\Rightarrow \quad \frac{n+6}{18} = \frac{n}{6}$$

$$\Rightarrow \quad 6n + 36 = 18n$$

$$\Rightarrow \quad 12n = 36$$

$$\Rightarrow \quad n = \frac{36}{12} = 3$$

Hence, the required number of red balls = **3**.

48.

$$\text{P (Red ball)} = \frac{1}{4}$$

$$\text{P(Blue ball)} = \frac{1}{3}$$

Orange balls = 10

P(red ball) + P (blue ball) + P(Orange ball) = 1

$$\frac{1}{4} + \frac{1}{3} + \frac{10}{\text{Total balls}} = 1$$

$$\frac{7}{12} + \frac{10}{\text{Total balls}} = 1$$

$$\frac{10}{\text{Total balls}} = \frac{5}{12}$$

$$\text{Total balls} = \frac{12 \times 10}{5} = 24$$

49. Number of red balls = 3.

Let the number of black balls = x.

P(black ball) = 2 × P(red ball)

$$\Rightarrow \quad \frac{x}{x+3} = 2 \times \frac{3}{x+3}$$

$$\Rightarrow \quad x = 6.$$

Hence, black balls = 6.

50. (*i*) Total number of outcomes = 20.

Number of red balls = x.

$$\therefore \quad \text{P(red ball)} = \frac{x}{20}.$$

$$\text{P (not a red ball)} = \frac{20-x}{20}$$

(*ii*) Red balls = x

Total balls = 20

Case I

$$\text{P (red ball)} = \frac{x}{20}$$

Case II

4 red balls added

Total balls = 24

red balls = $x + 4$

$$\text{P(red ball)} = \frac{x+4}{24}$$

According to the condition given in question

$$\frac{x+4}{24} = \frac{5}{4} \times \frac{x}{20}$$

$$\Rightarrow \quad 4x + 16 = 6x$$

$$\Rightarrow \quad 2x = 16$$

$$\Rightarrow \quad x = 8$$

51. White balls = 15

P(black ball) = 3P(white ball)

Let the black balls be x

$$\text{P(black ball)} = \frac{x}{15+x}$$

$$\text{P(White ball)} = \frac{15}{15+x}$$

P(black ball) = 3 × P (white ball)

$$\Rightarrow \quad \frac{x}{15+x} = 3 \times \frac{15}{15+x}$$

$$\Rightarrow \quad x = 45$$

$\therefore$ No. of black balls = 45

52.

$$\text{P(white ball)} = \frac{3}{10}$$

$$\text{P(black ball)} = \frac{2}{5}$$

P(red ball) = ?

P(white ball) + P(black ball) + P(red ball) = 1

$$\frac{3}{10} + \frac{2}{5} + \text{P(red ball)} = 1$$

$$\text{P(red ball)} = 1 - \frac{7}{10}$$

$$\text{P(red ball)} = \mathbf{\frac{3}{10}}$$

black balls = 20

$$\text{P(black ball)} = \frac{20}{\text{Total balls}}$$

$$\frac{2}{5} = \frac{20}{\text{Total balls}}$$

Total balls = **50**

53. P(asked a girl) = $\frac{2}{3}$.

Number of boys = x.

Number of girls = $x + 2$.

Total possible outcomes = $(x + x + 2) = 2x + 2$.

$$\frac{2}{3} = \frac{x+2}{2x+2}$$

$$3x + 6 = 4x + 4.$$

$$x = 2.$$

Boys = 2, Girls = 4.

54. P(a girl chosen) = $\frac{3}{7}$.

Let the number of girls = x.

Boys = $x + 2$,

Total possible outcomes = $2x + 2$.

So, $\frac{3}{7} = \frac{x}{2x+2}$.

$\Rightarrow$ $7x = 6x + 6$.

$x = 6$.

Girls = 6,

Boys = $x + 2 = 6 + 2 = 8$.

55. P(toffees) = $\frac{3}{8}$.

Let the number of toffees = x.

Number of eclairs = $x + 6$, total outcomes = $2x + 6$.

So, $\frac{3}{8} = \frac{x}{2x+6}$

$\Rightarrow$ $8x = 6x + 18$

$\Rightarrow$ $2x = 18$

$\therefore$ x **= 9 (Toffees)**

Eclairs = $x + 6 = 9 + 6 =$ **15**.

56. When a coin is tossed 3 times, it has the following outcomes:

HHH, HTH, HHT, THH and TTT, THT, TTH, HTT

where, for example, HTH stands for Head (H) on the 1st toss, Tail (T) on the 2nd toss and Head (H) on the third toss, and so on.

$\therefore$ Total number of outcomes = 8.

According to the problem,

Outcomes favourable to get a success are (HHH, TTT) i.e., 2 outcomes.

Hence, probability of getting a success = $\frac{2}{8} = \frac{1}{4}$.

$\therefore$ Probability of getting a failure = $1 - \frac{1}{4} = \frac{3}{4}$

$\therefore$ Required probability of losing the game = $\frac{3}{4}$

57. Leap year has 366 days = 52 weeks + 2 days. Possibility of remaining 2 days can be

(*i*) Monday and Tuesday,

(*ii*) Tuesday and Wednesday,

(*iii*) Wednesday and Thursday

(*iv*) Thursday and Friday,

(*v*) Friday and Saturday,

(*vi*) Saturday and Sunday,

(*vii*) Sunday and Monday.

$\therefore$ Required Probability = $\mathbf{\frac{2}{7}}$.

58. (*i*) For a leap year, there will be 30 days in the month of June.

Number of days in a week = 7.

If 1st June is a Monday then,

1, 8, 15, 22, 29 will be Monday.

If 2nd June is a Monday then,

2, 9, 16, 23 and 30 will be Monday.

So, number of weeks of 5 Mondays = 2.

P(5 Mondays in June) = $\frac{2}{7}$.

(*ii*) Same as above as in a non-leap year June is of 30 days.

59. (*i*) No. of days in a week = 7.

In a leap year, February is of 29 days.

If 1 February is Wednesday then,

1, 8, 15, 22 and 29 will be Wednesday = 1.

So, number of 5 Wednesdays = 1

$$P = \mathbf{\frac{1}{7}}.$$

(*ii*) As in a non-leap year February is of 28 days, so 5 Wednesday won't be possible.

$$P = 0.$$

60. Total possible value of $\frac{a}{b}$ = 36.

Value of $\frac{a}{b} > 1 = 15$

$$P\left(\frac{a}{b} > 1\right) = \frac{15}{36} = \mathbf{\frac{5}{12}}.$$

EXERCISE 17B

For Basic and Standard Levels

1. Angle measure of whole circle = 360°.

Given, $\angle XOZ = 72°$,

So, $\angle ZOY$ (region B) = 180° − 72°
= 108°.

$$P = \frac{\text{Measure of the specified region}}{\text{Measure of whole region}} = \frac{108°}{360°} = \mathbf{\frac{3}{10}}.$$

For Standard Level

2. Total possible outcomes = $\pi r^2 = \pi \times (10 \text{ cm})^2 = 100\pi \text{ cm}^2$.

Area of shaded region = $\pi x^2 - \pi (5 \text{ cm})^2$

$= \pi x^2 - 25\pi \text{ cm}^2$

$= \pi (x^2 - 25)$ = favourable outcome.

$$P = \frac{39}{100}$$

So, $\frac{39}{100} = \frac{\pi(x^2 - 25)}{100\pi}$

$\Rightarrow \quad 39 = x^2 - 25$

$\Rightarrow \quad x^2 = 64$

$\Rightarrow \quad x = \mathbf{8\ cm}.$

3. Area of square = $(14\text{ cm})^2 = 196\text{ cm}^2$ (possible outcome)

$$\text{Area of two semicircles} = 2\times\frac{1}{2}\pi r^2 = \pi r^2 = \pi \times (7)^2 = 49\pi\text{ cm}^2 = 154\text{ cm}^2$$

$$\text{Area of shaded region} = 196\text{ cm}^2 - 154\text{ cm}^2. = 42\text{ cm}^2.$$

$$P = \frac{42}{196} = \mathbf{\frac{3}{14}}.$$

4. Area of square = $(10\text{ cm})^2 = 100\text{ cm}^2$ (possible outcome)

$$\text{Area of circle joining four corners} = \pi r^2 = \pi \times (5\text{ cm})^2 = 25\pi\text{ cm}^2.$$

$$\text{Area of shaded region} = 100\text{ cm}^2 - 25\pi\text{ cm}^2. = 25(4-\pi)\text{ cm}^2.$$

$$P = \frac{\text{Area of shaded region}}{\text{Area of square}} = \frac{25(4-\pi)\text{ cm}^2}{100\text{ cm}^2} = \frac{4-\pi}{4} = \left(\mathbf{1-\frac{\pi}{4}}\right).$$

5.
$$\text{Probability} = \frac{\text{Area of shaded region}}{\text{Area of whole circle}} = \frac{2\left(\frac{45}{360}\right)\pi(14^2-7^2)\text{ cm}^2}{\pi(14)(14)\text{ cm}^2} = \mathbf{\frac{3}{16}}.$$

6. $\angle BAC = 90°.$

$BC^2 = 14^2 = AB^2 + AC^2 = 2AB^2$

$\Rightarrow \quad AB = AC = 7\sqrt{2}\text{ cm}.$

$$\text{Probability} = \frac{(7)(7)\left(\frac{22}{14}-1\right)\text{cm}^2}{\frac{1}{2}\times\frac{22}{7}(7)(7)\text{ cm}^2} = \frac{\frac{8}{14}}{\frac{11}{7}} = \mathbf{\frac{4}{11}}.$$

CHECK YOUR UNDERSTANDING

MULTIPLE-CHOICE QUESTIONS

For Basic and Standard Levels

1. (*a*) **1.5**

As the sum of the probabilities of all elementary events of an experiment is not more than 1.

2. (*b*) **0.0001.**

3. (*a*) $\mathbf{\frac{7}{8}}$.

Favourable outcomes = 7

$$P(\text{atleast one boy}) = \frac{\text{Favourable outcomes}}{\text{Possible outcomes}} = \frac{7}{8}.$$

4. (*a*) $\mathbf{\frac{1}{3}}$

Favourable outcomes = 2 (1, 4)

$$P(\text{perfect square}) = \frac{2}{6} = \frac{1}{3}.$$

5. (*d*) $\mathbf{\frac{1}{26}}$

$$P(\text{black queen}) = \frac{\text{Favourable outcomes}}{\text{Possible outcomes}} = \frac{2}{52} = \frac{1}{26}$$

6. (*b*) **51**

There are only one king of spade = 52 – 1 = 51.

7. (*c*) $\mathbf{\frac{12}{13}}$.

$$P(\text{not an ace}) = \frac{\text{Favourable outcomes}}{\text{Possible outcomes}} = \frac{52-4}{52} = \frac{48}{52} = \frac{12}{13}.$$

8. (*c*) $\mathbf{\frac{2}{13}}$.

$$P(\text{either a king or a queen}) = \frac{\text{Favourable outcome}}{\text{Possible outcome}} = \frac{4+4}{52} = \frac{8}{52} = \frac{2}{13}.$$

9. (*b*) $\mathbf{\frac{7}{13}}$.

$$P(\text{a red card or a king}) = \frac{\text{Favourable outcomes}}{\text{Possible outcomes}} = \frac{13+13+2}{52} = \frac{28}{52} = \frac{7}{13}.$$

10. (*a*) **E and O.**

As E and O are not repeated like R.

11. (*b*) $\mathbf{\frac{1}{5}}$.

P(neither a prime nor a composite)

$$= \frac{\text{Favourable outcomes}}{\text{Possible outcomes}} = \frac{1}{5}.$$

As 1 is only neither a prime number nor a composite number.

12. (*d*) $\mathbf{\frac{1}{2}}$.

$$P(\text{prime number}) = \frac{\text{Favourable outcomes}}{\text{Possible outcomes}} = \frac{4}{8} = \frac{1}{2}.$$

13. (*a*) **0.009.**

$P(\bar{E}) = 1 - P(E) = 1 - 0.991 = 0.009.$

14. (*b*) **62%.**

$P(F) = 100 - 38 = 62\%.$

15. (*c*) $\mathbf{\frac{2}{3}}$.

$$\text{P(other than rose)} = \frac{\text{Favourable outcomes}}{\text{Possible outcomes}} = \frac{2}{3}.$$

16. (*a*) $\mathbf{\frac{1}{2}}$.

Favourable outcomes = 3(2, 4, 6)

$$\text{P(getting an even number)} = \frac{3}{6} = \frac{1}{2}.$$

17. (*c*) $\mathbf{\frac{6}{7}}$.

$$\text{P(not a sparrow)} = \frac{\text{Favourable outcomes}}{\text{Possible outcomes}} = \frac{6}{7}.$$

18. (*c*) $\mathbf{\frac{1}{9}}$.

Number 6-50, total possible outcomes = 45.
Perfect square between 6-50 = 9, 16, 25, 36, 49 = 5.

$$\text{P(perfect square)} = \frac{5}{45} = \frac{1}{9}.$$

19. (*c*) $\mathbf{\frac{4}{45}}$.

Favourable outcomes = 8(2, 3, 5, 7, 11, 13, 17, 19)

$$\text{P(a prime number less than 23)} = \frac{8}{90} = \frac{4}{45}.$$

20. (*b*) $\mathbf{\frac{1}{4}}$

$$\text{P(getting 3 heads or tails)} = \frac{\text{Favourable outcomes}}{\text{Total possible outcomes}} = \frac{2}{8} = \frac{1}{4}.$$

21. (*b*) $\mathbf{\frac{1}{366}}$.

Total possible outcomes
= 366 × 366 (As 2000 is a leap year)
Favourable outcomes when the birthday will be same
= 366.

$$\text{P(having same birthday)} = \frac{366}{366 \times 366} = \frac{1}{366}.$$

22. (*c*) $\mathbf{\frac{7}{9}}$.

$$\text{P(not white marble)} = \frac{\text{Favourable outcomes}}{\text{Possible outcomes}} = \frac{3+4}{3+2+4} = \frac{7}{9}.$$

23. (*a*) $\mathbf{\frac{3}{5}}$.

$$\text{P(black ball)} = \frac{\text{Favourable outcomes}}{\text{Total possible outcomes}} = \frac{6}{4+6} = \frac{6}{10} = \frac{3}{5}.$$

24. (*b*) **486.**

Let the number of bad eggs = x.

$$0.028 = \frac{x}{500}.$$

$\Rightarrow$ $x = 500 \times 0.028 = 14.$

So, number of good eggs = 500 – 14 = 486.

25. (*c*) **125.**

Let the girl bought x tickets.

So, $0.025 = \frac{x}{5000}.$

For Standard Level

26. (*b*) $\mathbf{\frac{1}{20}}$.

$$\text{P(non-red ace)} = \frac{\text{Favourable outcomes}}{\text{Possible outcomes}} = \frac{2}{52-12} = \frac{2}{40} = \frac{1}{20}.$$

27. (*c*) $\mathbf{\frac{1}{6}}$.

$$\text{P(getting same number)} = \frac{\text{Favourable outcomes}}{\text{Total possible outcomes}} = \frac{6}{36} = \frac{1}{6}.$$

28. (*c*) $\mathbf{\frac{1}{9}}$.

Favourable outcomes = 4{(1, 6), (6, 1), (2, 3), (3, 2)}.

$$\text{P(getting multiple of 6)} = \frac{4}{36} = \frac{1}{9}.$$

29. (*b*) $\mathbf{y = 3x}$.

$$\frac{x}{y} = 1 - \frac{2}{3} = \frac{1}{3}.$$

$\Rightarrow$ $y = 3x.$

30. (*b*) **15.**

P(green ball) = 3 × P(red ball)

$$\frac{n}{5+n} = 3 \times \frac{5}{5+n}$$

$\Rightarrow$ $n = 3 \times 5 = 15.$

31. (*c*) $\mathbf{\frac{2}{3}}$.

$$\text{P(not from D and E)} = \frac{\text{Favourable outcomes}}{\text{Possible outcomes}} = \frac{9+13+10}{48} = \frac{32}{48} = \frac{2}{3}.$$

32. (*a*) $\mathbf{\frac{2}{9}}$.

Total possible outcomes when two dice are thrown
= 36.

Favourable outcomes to get difference of 2 = 8{(6, 4), (5, 3), (4, 2), (3, 1), (4, 6), (3, 5), (2, 4), (1, 3)}

$$\text{P(getting difference 2)} = \frac{8}{36} = \frac{2}{9}.$$

33. (*a*) $\mathbf{\frac{3}{4}}$.

$$\text{P(one head)} = \frac{\text{Favourable outcomes}}{\text{Possible outcomes}} = \frac{3}{4}.$$

34. (*c*) $\mathbf{\frac{7}{8}}$.

Total possible outcomes = 8.

Favourable outcomes = 7.

$$\text{P(getting at most 2 heads)} = \frac{7}{8}.$$

35. (*b*) $\mathbf{\frac{8}{25}}$.

Favourable outcomes= 8 (TW, WT, WTh, ThW, ThF, FTh, FS, SF)

Total possible outcomes = 5 × 5 = 25.

$$\text{P(E)} = \frac{8}{25}.$$

FILL IN THE BLANKS

For Basic and Standard Levels

1. 0 **2.** 1 **3.** 1 **4.** $\frac{4}{7}$.

MATCH THE FOLLOWING

For Basic and Standard Levels

1. (*g*) **2.** (*h*) **3.** (*e*) **4.** (*a*)
5. (*i*) **6.** (*b*) **7.** (*j*) **8.** (*f*)
9. (*c*) **10.** (*d*).

SHORT ANSWER QUESTIONS

For Basic and Standard Levels

1. (*i*) S = {1, 2, 3, 4, 5, 6, 7, 8}

Odd numbers = 1, 3, 5, 7

$$\text{P(odd number)} = \frac{4}{8} = \frac{1}{2}$$

(*ii*) No, the outcomes are not equally likely.

The possible outcomes are *bbb*, *bgg*, *gbg*, *ggb*, *bbg*, *bgb*, *ggg*. For example, 'one boy' means *bgg*, *gbg*, *ggb* and 'three boys' means *bbb* and so on.

2. Yes, the outcomes 'odd number', 'even number' are equally likely in the situation considered as the total number of odd numbers from 1 to 100 are 50 and the total number of even numbers from 1 to 100 are also 50.

3. (*i*) No, outcomes 'getting 1' and 'not getting 1' are not equally likely.

$$\text{P(getting 1)} = \frac{1}{6} \text{ and P(not getting 1)} = \frac{5}{6}.$$

(*ii*) No, the outcomes are not equally likely. All possible outcomes are HHH, HHT, HTH, THH, HTT, THT, TTH and TTT. So probability(no head) = $\frac{1}{8}$.

4. S = {1, 2, 3, 4, 5, …, 18, 19, 20}

(*i*) E = divisible by 2 or 3

Favourable outcomes = 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 3, 9, 15

$$\text{P(E)} = \frac{13}{20}$$

(*ii*) E = a prime numbers

Favourable outcomes = 2, 3, 5, 7, 11, 13, 17, 19

$$\text{P(E)} = \frac{8}{20}$$

For Standard Level

5. We know that the probability of throwing a die once and getting 6 is $\frac{1}{6}$. The square of this probability is $\frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$.

When a pair of dice is thrown, then the product of the probability of getting a six in one die and that of a six in the 2nd die is $\frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$. Hence, these two probabilities are equal.

6. Total outcomes = 36

(*i*) E = Product of two numbers on top of dice is 6

Favourable outcomes = (1, 6), (2, 3), (3, 2), (6, 1)

$$\text{P(E)} = \frac{4}{36} = \frac{1}{9}$$

(*ii*) E = getting a total of 6 or 7

Favourable outcomes = (1, 5), (1, 6), (2, 4), (2, 5), (3, 3), (3, 4), (4, 2), (4, 3), (5, 1), (5, 2), (6, 1)

$$\text{P(E)} = \frac{11}{36}$$

VALUE-BASED QUESTIONS

For Basic and Standard Levels

1. Two coins are tossed simultaneously, then the possible outcomes are HH, HT, TH, TT.

∴ Total number of possible outcomes = 4.

Outcomes favourable to the event of no heads = TT

Thus, the number of favourable outcomes = 1.

$$\text{P(E)} = \frac{\text{Number of outcomes favourable to E}}{\text{Total number of possible outcomes}}$$

∴ P(no head) = $\frac{1}{4}$.

∴ P(A gets a chance to go) = $\mathbf{\frac{1}{4}}$.

Outcomes favourable to the event of no tail = HH.

Thus, the number of favourable outcomes = 1

$\therefore$ P(no tail) = $\frac{1}{4}$

$\therefore$ P(B gets a chance to go) = $\mathbf{\frac{1}{4}}$

Outcomes favourable to the event of a head and a tail = HT and TH.

Thus, the number of favourable outcomes = 2.

$\therefore$ P(a head and a tail) = $\frac{2}{4} = \frac{1}{2}$.

$\therefore$ P(C gets a chance to go) = $\mathbf{\frac{1}{2}}$.

C has not been fair as the probability of going for the exhibition according to his suggestion is higher for him than for his brothers. Thus, he has shown dishonesty.

2. (*i*) Total number of children = (22 + 18) = 40.

Number of girls = 18.

P(selected child not a boy) = $\frac{18}{40} = \mathbf{\frac{9}{20}}$.

(*ii*) The selected child should have the following values:

Clean habits, regularity, interpersonal relationship, confidence, critical thinking, decision making and effective communication.

3. (*i*) Number of days in a non-leap year is 365. Therefore in a non-leap year there are 52 weeks and 1 more day which may be any one the seven days.

For the day over 52 weeks to be a Sunday, the total number of possible outcomes = 7.

Number of favourable outcomes = 1.

$$P(E) = \frac{\text{Number of outcomes favourable to E}}{\text{Total number of possible outcomes}}$$

P(53 Sundays) = $\frac{1}{7}$.

Hence, P(Student holds 53 classes) = $\mathbf{\frac{1}{7}}$.

(*ii*) Empathy.

4. (*i*) Total number of members of the senior citizen club = 75.

Percentage of senior citizens who participate in any one of the physical fitness programme = (35 + 15 + 22)% = 72.

$\therefore$ Percentage of senior citizens who do not participate in any one of the physical fitness programme

= (100 – 72)% = 28%.

$\therefore$ Number of senior citizens who do not participate in any physical fitness programme = 28% of 75.

$= \frac{28}{100} \times 75 = \mathbf{21}$.

(*ii*) Number of senior citizens who participate in any one of the physical fitness programme = 72% of 75.

$= \frac{72}{100} \times 75 = 54$.

Hence, P(member is a part of physical fitness programme)

$= \frac{54}{75} = \mathbf{\frac{18}{25}}$.

(*iii*) Awareness about advantages of physical fitness.

5. Total persons = 12

Patient persons = 3

Honest persons = 6

Kind persons = 3

(*i*) (extremely patient) = $\frac{3}{12} = \mathbf{\frac{1}{4}}$

(*ii*) P(extremely kind or honest) = $\frac{9}{12} = \mathbf{\frac{3}{4}}$

6. (*i*) Total area of the land = 25 km × 5 km = 125 sq km

Area of the specified region

= Total area – Area of canal

$= 125 \text{ sq km} - 25 \text{ km} \times \frac{1}{5} \text{ km}$

= 125 sq km – 5 sq km

= 120 sq km

$$P(E) = \frac{\text{Area of specified region}}{\text{Total area}}$$

$= \frac{120 \text{ sq km}}{125 \text{ sq km}} = \frac{24}{25}$.

(*ii*) Presence of mind, dealing with stress, critical thinking and decision-making.

UNIT TEST 1

For Basic Level

1. (*d*) **0 ≤ P(A) ≤ 1.**

As probability of an event is 1.

2. (*a*) $\mathbf{\frac{1}{3}}$.

P(getting on odd prime number) = $\frac{2}{6} = \frac{1}{3}$.

3. (*b*) $\mathbf{\frac{6}{13}}$.

Neither a jack nor a red card = 52 – 28 = 24.

P(neither a jack nor a red card) = $\frac{24}{52} = \frac{6}{13}$.

4. (*a*) $\mathbf{\frac{3}{13}}$.

Number of face cards = 12.

P(face card) = $\frac{12}{52} = \frac{3}{13}$.

5. (*b*) $\mathbf{\frac{1}{2}}$.

Total possible outcomes from 3 to 20 = 18.

Even numbers = 4, 6, 8, 10, 12, 14, 16, 18, 20 = 9.

$$\text{P(even number)} = \frac{9}{18} = \frac{1}{2}.$$

6. (*b*) $\mathbf{\frac{1}{5}}$.

Total possible outcomes = 5 + 20 = 25

Favourable outcomes = 5.

$$\text{P(getting a prize)} = \frac{2}{25} = \frac{1}{5}.$$

7. (*i*) Total possible outcomes = 100.

Favourable outcomes = 88.

$$\text{P(acceptable to Krishni)} = \frac{88}{100} = \mathbf{\frac{22}{25}}.$$

(*ii*) Favourable outcomes for trader = 88 + 8 = 96.

$$\text{P(acceptable to trader)} = \frac{96}{100} = \mathbf{\frac{24}{25}}.$$

8. Sample space = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

(*i*) E = three heads

Favourable outcome = 1

$$\text{P(E)} = \mathbf{\frac{1}{8}}$$

(*ii*) E = at least two tails

Favourable outcomes = 4

$$\text{P(E)} = \frac{4}{8} = \mathbf{\frac{1}{2}}$$

9. Total possible outcomes = 100.

Favourable outcomes for ₹ 1 = 39 + 31 = 70.

Favourable outcomes for not ₹ 1 = 100 – 70 = 30.

$$\text{P(not ₹ 1)} = \frac{30}{100} = \mathbf{\frac{3}{10}}.$$

UNIT TEST 2

For Standard Level

1. (*a*) $\mathbf{\frac{1}{12}}$.

Total possible outcomes = 6 × 6 = 36

Favourable outcomes = {(2, 2), (4, 4), (6, 6)} = 3.

$$\text{P(even number on both dice)} = \frac{3}{36} = \frac{1}{12}.$$

2. (*b*) $\mathbf{\frac{1}{7}}$.

A non-leap year has 365 days.

365 days = 364 + 1 = (52 × 7) + 1

= 52 weeks + 1 day

The remaining one day can be any day of the week.

Total possible outcomes = 7 (Sunday to Saturday)

Favourable outcome = 1.

$$\text{P} = \frac{1}{7}.$$

3. (*b*) $\mathbf{\frac{1}{2}}$.

Favourable outcomes = HHT, HTH, THH, HHH (4)

Total possible outcomes = 8.

$$\text{P(at least 2 heads)} = \frac{4}{8} = \frac{1}{2}.$$

4. (*a*) **10.**

Let the number of green balls = x.

$$\text{Probability of green balls} = 1-\left(\frac{11}{20}+\frac{1}{5}\right)$$

$$= 1-\frac{3}{4} = \frac{1}{4}.$$

Total possible outcomes = 40.

$$\text{P(green balls)} = \frac{\text{Favourable outcomes}}{\text{Possible outcomes}}$$

$$\Rightarrow \quad \frac{1}{4} = \frac{x}{40}$$

$$\Rightarrow \quad x = 10.$$

5. (*c*) $\mathbf{\frac{5}{9}}$

Here x has three distinct values *viz.* 1, 2, and 3 and y has three distinct values *viz.* 1, 4 and 9.

∴ Probability of choosing each value of x is $\frac{1}{3}$ and that of choosing each value of y is $\frac{1}{3}$.

Now, the total outcomes of choosing the pair of numbers x and y such that their product is less than 9 are 11, 14, 21, 22 and 31.

∴ P(11, 14, 22 or 31)

= P(11) + P(14) + P(22) + P(31) + P(21)

$$= \frac{1}{3} \times \frac{1}{3} + \frac{1}{3} \times \frac{1}{3} + \frac{1}{3} \times \frac{1}{3} + \frac{1}{3} \times \frac{1}{3} + \frac{1}{3} \times \frac{1}{3}$$

$$= \frac{5}{9}$$

$$\therefore \text{Required probability} = \frac{5}{9}$$

6. (*b*) $\mathbf{\frac{6}{23}}$

Required probability = The probability that the selected student is from D or E.

Now, 2 students are selected from house D and the remaining 23 – (4 + 8 + 5 + 2) = 23 – 19 = 4 students are selected from E. Now, probability that 2 out of 23 students are selected from the house D = $\frac{2}{23}$ and the probability that 4 students are selected from the house E = $\frac{4}{23}$. Hence, the required probability that the selected student is from the house D or E is $\frac{2}{23} + \frac{4}{23} = \frac{6}{23}$

7. (*i*) $P = \frac{5}{30} = \mathbf{\frac{1}{6}}$.

(*ii*) Total possible outcomes = 29.

Favourable outcomes = 29 − 5 = 24.

$P = \mathbf{\frac{24}{29}}$.

8. Let the jar contain x blue marbles. Then according to the problem, the number of green marbles = $2x + 5$. Then the probability of drawing a blue marble.

$$= \frac{\text{Number of blue marbles}}{\text{Total number of marbles}} = \frac{x}{x + 2x + 5} = \frac{x}{3x + 5}$$

∴ According to the problem, we have

$$\frac{x}{3x+5} = \frac{2}{7}$$

⇒ $6x + 10 = 7x$

⇒ $7x - 6x = 10$

⇒ $x = 10$.

Hence, the required number of blue marbles = **10** and the number of green marbles = 2 × 10 + 5 = **25**.

9. When a coin is tossed 3 times, the possible outcomes are HTH, HHT, HTT, HHH and THT, TTH, THH, TTT, where HTH, for example, stands for Head (H) is in 1st toss, Tail (T) in the second toss and Head(H) in the third toss, and so on. So, the total number of outcomes = 8 in the tossing of a coin thrice.

Probability of getting a Head (H) or a Tail (T) in a toss = $\frac{1}{2}$.

∴ $P(HTH) = P(H) \times P(T) \times P(H) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$

(*i*) Anjali gets her entry fee back if she throws one or 2 Heads, i.e. her outcomes are HTH, HHT, HTH, THT, TTH, HTT, whose probability are P (HTH, HHT, HTH, THT, TTH or HTT)

$= P(H)\ P(T) \times P(H) \times 6 = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times 6 = \mathbf{\frac{3}{4}}$

(*ii*) She gets thrice the entry fee if she throws all the three Heads whose probability is $P(HHT) = \mathbf{\frac{1}{8}}$

(*iii*) She loses half the entry fee if the probability is neither (*i*) nor (*ii*), i.e. when the probability is

$$1 - \left(\frac{3}{4} + \frac{1}{8}\right) = 1 - \frac{7}{8} = \frac{1}{8}$$

Hence, in this case her probability is $\mathbf{\frac{1}{8}}$.