CHAPTER **16**

Statistics

EXERCISE 16A

For Basic and Standard Levels

1. Let $\alpha - \beta$ be the class whose class mark is $\frac{\alpha+\beta}{2} = 16$

$\Rightarrow \quad \alpha + \beta = 2 \times 16 = 32 \quad \ldots(1)$

Also, $\quad \beta - \alpha$ = class interval

= difference between two consecutive

Class marks $= 14 - 12 = 2 \quad \ldots(2)$

Adding (1) and (2), we get

$2\beta = 34$

$\Rightarrow \quad \beta = 17$

$\therefore \quad \alpha = 32 - \beta = 32 - 17 = 15$

$\therefore$ The required class whose class mark is 16 is 15–17.

Again, let $\gamma - \delta$ be the class whose class mark is 22.

$\therefore \quad \frac{\gamma+\delta}{2} = 22$

$\Rightarrow \quad \gamma + \delta = 44 \quad \ldots(3)$

Also, class interval $\delta - \gamma$ = difference between two consecutive classes

$= 14 - 12$

$= 2 \quad \ldots(4)$

Adding (3) and (4), we get

$2\delta = 44 + 2 = 46$

$\Rightarrow \quad \delta = 23$

$\therefore \quad \gamma = 44 - \delta$

$= 44 - 23 = 21$

Hence, the class whose class mark is 22 is 21–23.

Hence, the required class-intervals are **15–17** and **21–23** respectively.

2.

Mid-value x_i	Frequency f_i	$f_i x_i$
15	4	60
25	28	700
35	15	525
45	20	900
55	17	935
65	16	1040
	$\Sigma f_i = 100$	$\Sigma f_i x_i = 4160$

Mean $\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{4160}{100} = \mathbf{41.6}$

3.

Class interval	Mid-value x_i	Frequency f_i	$x_i f_i$
1–3	2	12	24
3–5	4	22	88
5–7	6	27	162
7–9	8	19	152
		$\Sigma f_i = 80$	$\Sigma x_i f_i = 426$

$\therefore$ Mean $= \frac{\Sigma x_i f_i}{\Sigma f_i} = \frac{426}{80} = \mathbf{5.325}$ which is the required mean.

4.

Class interval	Mid-value x_i	Frequency f_i	$x_i f_i$
0–10	5	8	40
10–20	15	12	180
20–30	25	10	250
30–40	35	11	385
40–50	45	9	405
		$\Sigma f_i = 50$	$\Sigma x_i f_i = 1260$

$\therefore$ Required mean $= \frac{\Sigma x_i f_i}{\Sigma f_i} = \frac{1260}{50} = \mathbf{25.2}$

5.

Class interval	Mid-value x_i	Frequency f_i	$f_i x_i$
0–6	3	7	21
6–12	9	5	45
12–18	15	10	150
18–24	21	12	252
24–30	27	6	162
		$\Sigma f_i = 40$	$\Sigma f_i x_i = 630$

Mean $\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{630}{40} = \mathbf{15.75}$

6.

Class interval	*Mid-value* x_i	*Frequency* f_i	f_ix_i
0 – 10	5	12	60
10 – 20	15	16	240
20 – 30	25	6	150
30 – 40	35	7	245
40 – 50	45	9	405
		$\Sigma f_i = 50$	$\Sigma f_ix_i = 1100$

Mean $= \bar{x} = \dfrac{\Sigma f_i x_i}{\Sigma f_i} = \dfrac{1100}{50} = \mathbf{22}$

7.

Daily expenditure (in ₹)	*Mid-value* x_i	*Frequency* f_i	x_if_i
100 – 150	125	4	500
150 – 200	175	5	875
200 – 250	225	12	2700
250 – 300	275	2	550
300 – 350	325	2	650
		$\Sigma f_i = 25$	$\Sigma x_if_i = 5275$

$\therefore$ Required mean $= \dfrac{\Sigma x_i f_i}{\Sigma f_i} = \dfrac{5275}{25} = 211.$

Hence, the mean daily expenditure on food is ₹ **211**.

8.

Number of days absent	*Mid-value* x_i	*Number of students* f_i	$f_i\,x_i$
0 – 8	4	5	20
8 – 16	12	9	108
16 – 24	20	10	200
24 – 32	28	8	224
32 – 40	36	8	288
		$\Sigma f_i = 40$	$\Sigma f_i\,x_i = 840$

Mean $= \bar{x} = \dfrac{\Sigma f_i x_i}{\Sigma f_i} = \dfrac{840}{40} = 21$

Hence, the mean number of days a student was absent = **21 days**

9.

Class interval	*Mid-value* x_i	*Frequency* f_i	x_if_i
11 – 13	12	3	36
13 – 15	14	6	84
15 – 17	16	9	144
17 – 19	18	13	234
19 – 21	20	f	$20f$
21 – 23	22	5	110
23 – 25	24	4	96
		$\Sigma f_i = 40 + f$	$\Sigma x_if_i = 704 + 20f$

$\therefore$ Mean $= \dfrac{\Sigma x_i f_i}{\Sigma f_i} = \dfrac{704 + 20f}{40 + f}$

$\therefore$ According to the problem, we have

$$\frac{704 + 20f}{40 + f} = 18$$

$\Rightarrow \quad 704 + 20f = 720 + 18f$

$\Rightarrow \quad 2f = 720 - 704 = 16$

$\therefore \quad f = \dfrac{16}{2} = 8$

Hence, the required frequency of the class 19 – 21 is **8**.

10.

Monthly pocket allowance (in ₹)	*Mid-value* x_i	*Frequency* f_i	f_ix_i
0 – 20	10	12	120
20 – 40	30	15	450
40 – 60	50	32	1600
60 – 80	70	p	$70p$
80 – 100	90	13	1170
		$\Sigma f_i = 72 + p$	$\Sigma f_i\,x_i = 3340 + 70p$

$$\text{Mean} = \bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

$\Rightarrow \quad 53 = \dfrac{3340 + 70p}{72 + p}$

$\Rightarrow \quad 3816 + 53p = 3340 + 70p$

$\Rightarrow \quad 476 = 17p$

$\Rightarrow \quad p = \dfrac{476}{17} = 28$

Hence, $\boldsymbol{p}$ **= 28.**

11.

Class interval	Mid-value x_i	Frequency f_i	$f_i x_i$
7.5 – 12.5	10	3	30
12.5 – 17.5	15	10	150
17.5 – 22.5	20	p	$20p$
22.5 – 27.5	25	8	200
27.5 – 32.5	30	2	60
		$\Sigma f_i = 23 + p$	$\Sigma f_i x_i = 440 + 20p$

$$\text{Mean} = \bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

$$\Rightarrow \quad 19\frac{3}{8} = \frac{440 + 20p}{23 + p}$$

$$\Rightarrow \quad \frac{155}{8} = \frac{440 + 20p}{23 + p}$$

$$\Rightarrow \quad 155(23 + p) = 8(440 + 20p)$$

$$\Rightarrow \quad 3565 + 155p = 3520 + 160p$$

$$\Rightarrow \quad 5p = 45$$

$$\Rightarrow \quad p = 9$$

Hence, $\boldsymbol{p = 9}$.

12.

Class interval	Mid-value x_i	Frequency f_i	$f_i x_i$
0 – 20	10	34	340
20 – 40	30	$3x - 4$	$90x - 120$
40 – 60	50	64	3200
60 – 80	70	48	3360
80 – 100	90	$2(x - 1)$	$180x - 180$
		$\Sigma f_i = 140 + 5x$	$\Sigma f_i x_i = 6600 + 270x$

$$\text{Mean} = \bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

$$\Rightarrow \quad 50 = \frac{6600 + 270x}{140 + 5x}$$

$$\Rightarrow \quad 7000 + 250x = 6600 + 270x$$

$$\Rightarrow \quad 400 = 20x$$

$$\Rightarrow \quad x = \frac{400}{20} = 20$$

Hence, $\boldsymbol{x = 20}$.

For Standard Level

13.

Class interval	Mid-value x_i	Frequency f_i	$x_i f_i$
0 – 10	5	7	35
10 – 20	15	10	150
20 – 30	25	x	$25x$
30 – 40	35	13	455
40 – 50	45	y	$45y$
50 – 60	55	10	550
60 – 70	65	14	910
70 – 80	75	9	675
		$\Sigma f_i = 63 + x + y$	$\Sigma x_i f_i = 2775 + 25x + 45y$

Given that

$$63 + x + y = 100$$

$$\Rightarrow \quad x + y = 100 - 63 = 37 \qquad \ldots(1)$$

and

$$\text{Mean} = \frac{\sum x_i f_i}{\sum f_i}$$

$$= \frac{2775 + 25x + 45y}{63 + x + y} = 42$$

$$\therefore \quad 2775 + 25x + 25y + 20y = 42 \times (63 + x + y)$$

$$\Rightarrow \quad 2775 + 25 \times 37 + 20y = 42 \times (63 + 37) \quad \text{[From (1)]}$$

$$= 42 \times 100$$

$$= 4200$$

$$\Rightarrow \quad 2775 + 25 \times 37 + 20y = 4200$$

$$\Rightarrow \quad 2775 + 925 + 20y = 4200$$

$$\Rightarrow \quad 3700 + 20y = 4200$$

$$\Rightarrow \quad 20y = 4200 - 3700 = 500$$

$$\therefore \quad y = \frac{500}{20} = 25$$

$\therefore$ From (1), $\quad x = 37 - 25 = 12$

$\therefore$ Required value of x and y are **12** and **25** respectively.

14.

Class interval	Mid-value x_i	Frequency f_i	$f_i x_i$
0 – 20	10	7	70
20 – 40	30	f_1	$30f_1$
40 – 60	50	12	600
60 – 80	70	f_2	$70f_2$

80–100	90	8	720
100–120	110	5	550
		$\Sigma f_i = 32 + f_1 + f_2 = 50$	$\Sigma f_i x_i = 1940 + 30f_1 + 70f_2$

$$\Sigma f_i = 32 + f_1 + f_2 = 50 \qquad \text{[Given]}$$

$$\Rightarrow \quad f_1 + f_2 = 18 \qquad \dots (1)$$

$$\text{Mean} = \bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

$$\Rightarrow \quad 57.6 = \frac{1940 + 30f_1 + 70f_2}{50}$$

$$\Rightarrow \quad 2880 - 1940 = 30f_1 + 70f_2$$

$$\Rightarrow \quad 940 = 30f_1 + 70f_2$$

$$\Rightarrow \quad 94 = 3f_1 + 7f_2 \qquad \dots (2)$$

Multiplying equation (1) by 7 we get

$$7f_1 + 7f_2 = 126 \qquad \dots (3)$$

Subtracting equation (2) from equation (3), we get

$$4f_1 = 32$$

$$\Rightarrow \quad f_1 = 8$$

Substituting $f_1 = 8$ in equation (1), we get

$$f_2 = 10$$

Hence, $\boldsymbol{f_1 = 8, f_2 = 10}$

15.

Class interval	*Mid-value* x_i	*Frequency* f_i	$x_i f_i$
0–20	10	17	170
20–40	30	f_1	$30f_1$
40–60	50	32	1600
60–80	70	f_2	$70f_2$
80–100	90	19	1710
		$\Sigma f_i = 68 + f_1 + f_2$	$\Sigma x_i f_i = 3480 + 30f_1 + 70f_2$

Now, given that mean

$$\frac{\Sigma x_i f_i}{\Sigma f_i} = 50$$

$$\Rightarrow \quad \frac{3480 + 30f_1 + 70f_2}{68 + f_1 + f_2} = 50 \qquad \dots(1)$$

Also, $\quad 68 + f_1 + f_2 = 120$

$$\Rightarrow \quad f_1 + f_2 = 120 - 68 = 52 \qquad \dots(2)$$

From (2), $\quad f_2 = 52 - f_1 \qquad \dots(3)$

$\therefore$ From (1), (2) and (3), we have

$$\frac{3480 + 30f_1 + 70(52 - f_1)}{68 + 52} = 50$$

$$\Rightarrow 3480 + 30f_1 + 3640 - 70f_1 = 50 \times 120 = 6000$$

$$\Rightarrow \quad 40f_1 = 7120 - 6000 = 1120$$

$$\Rightarrow \quad f_1 = \frac{1120}{40} = \frac{112}{4} = 28$$

$\therefore$ From (3), $\quad f_2 = 52 - 28 = 24$

$\therefore$ Required values of f_1 and f_2 are respectively **28** and **24**.

16.

Expenditure (in ₹)	*Mid-value* x_i	*Frequency* f_i	$x_i f_i$
140–160	150	5	750
160–180	170	25	4250
180–200	190	f_1	$190f_1$
200–220	210	f_2	$210f_2$
220–240	230	5	1150
		$\Sigma f_i = 35 + f_1 + f_2$	$\Sigma x_i f_i = 6150 + 190f_1 + 210f_2$

Given that

$$f_1 + f_2 + 35 = 100$$

$$\Rightarrow \quad f_1 + f_2 = 65 \qquad \dots(1)$$

$$\text{Also, mean} = \frac{\Sigma x_i f_i}{\Sigma f_i} = \frac{6150 + 190f_1 + 210f_2}{35 + f_1 + f_2} = 188 \qquad \dots(2)$$

From (1) and (2), we have

$$\frac{6150 + 190f_1 + 210(65 - f_1)}{35 + 65} = 188$$

$$\Rightarrow 6150 + 190f_1 - 210f_1 + 210 \times 65 = 188 \times 100 = 18800$$

$$\Rightarrow \quad 190f_1 - 210f_1 = 18800 - 6150 - 210 \times 65$$

$$\Rightarrow \quad -20f_1 = 12650 - 13650 = -1000$$

$$\therefore \quad f_1 = \frac{1000}{20} = 50$$

$\therefore$ From (1), $\quad f_2 = 65 - f_1 = 65 - 50 = 15$

$\therefore$ The required values of f_1 and f_2 are respectively **50** and **15**.

17.

Class interval	*Mid-value* x_i	*Frequency* f_i	$f_i x_i$
50–60	55	8	440
60–70	65	6	390
70–80	75	12	900

80–90	85	$4x-1$	$340x-85$
90–100	95	$2y+3$	$190y+285$
		$\Sigma f_i = 28 + 4x + 2y = 50$	$\Sigma f_i x_i = 1930 + 340x + 190y$

$$\Sigma f_i = 28 + 4x + 2y = 50 \quad \text{[Given]}$$

$$\Rightarrow \quad 4x + 2y = 22$$

$$\Rightarrow \quad 2x + y = 11 \quad \ldots (1)$$

$$\text{Mean} = \bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

$$\Rightarrow \quad 78 = \frac{1930 + 340x + 190y}{50}$$

$$\Rightarrow \quad 3900 = 1930 + 340x + 190y$$

$$\Rightarrow \quad 1970 = 340x + 190y$$

$$\Rightarrow \quad 197 = 34x + 19y \quad \ldots (2)$$

Multiplying equation (1) by 19, we get

$$38x + 19y = 209 \quad \ldots (3)$$

Subtracting equation (2) from equation (3), we get

$$4x = 12$$

$$\Rightarrow \quad x = 3$$

Substituting $x = 3$ in equation (1), we get

$$2(3) + y = 11$$

$$\Rightarrow \quad y = 11-6$$

$$= 5$$

Frequency corresponding to class 80–90 is $4(3)-1 = 11$ and the frequency corresponding to class 90–100 is $2(5) + 3 = 13$.

Hence, the frequency corresponding to class 80–90 is **11** and the frequency corresponding to class 90–100 is **13**.

18. **Distribution I**

Class interval	*Mid-value* x_i	*Frequency* f_i	$f_i x_i$
0–10	5	5	25
10–20	15	8	120
20–30	25	$2x+3$	$50x+75$
30–40	35	16	560
40–50	45	$5y+1$	$225y+45$
		$\Sigma f_i = 33 + 2x + 5y$	$825 + 50x + 225y$

$$\text{Mean} = \bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

$$30 = \frac{825 + 50x + 225y}{33 + 2x + 5y}$$

$$= \quad 990 + 60x + 150y = 825 + 50x + 225y$$

$$= \quad 165 + 10x - 75y = 0$$

$$= \quad 33 + 2x - 15y = 0 \quad \ldots (1)$$

Distribution II

Class interval	*Mid-value* x_i	*Frequency* f_i	$f_i x_i$
0–10	5	$x-1$	$5x-5$
10–20	15	9	135
20–30	25	$3x$	$75x$
30–40	35	$4y$	$140y$
40–50	45	6	270
		$\Sigma f_i = 14 + 4x + 4y$	$\Sigma f_i x_i = 400 + 80x + 140y$

$$\text{Mean} = \bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

$$\Rightarrow \quad 26 = \frac{400 + 80x + 140y}{14 + 4x + 4y}$$

$$\Rightarrow \quad 364 + 104x + 104y = 400 + 80x + 140y$$

$$\Rightarrow \quad 36 - 24x + 36y = 0$$

$$\Rightarrow \quad 3 - 2x + 3y = 0 \quad \ldots (2)$$

Adding equations (1) and (2), we get

$$36 - 12y = 0$$

$$\Rightarrow \quad 12y = 36$$

$$\Rightarrow \quad y = 3$$

Substituting $y = 3$ in equation (2), we get

$$3 - 2x + 3 \times 3 = 0$$

$$\Rightarrow \quad 2x = 12$$

$$\Rightarrow \quad x = 6$$

Hence, $\boldsymbol{x = 6, y = 3}$.

19. We have from the frequency and cumulative columns

$$a = 12 \quad \ldots(1)$$

$$b + a = 25$$

$$\therefore \quad b = 25 - a = 25 - 12 = 13 \quad \ldots(2)$$

$$10 + 25 = c$$

$$\Rightarrow \quad c = 35 \quad \ldots(3)$$

$$d + c = 43$$

$$\Rightarrow \quad d = 43 - 35 = 8 \quad \ldots(4) \text{ [Using (3)]}$$

$$e + 43 = 48$$

$\therefore \quad e = 48 - 43 = 5 \quad \ldots(5)$

$2 + 48 = f$

$\Rightarrow \quad f = 50 \quad \ldots(6)$

Hence, the required value of a, b, c, d, e and f are respectively **12, 13, 35, 8, 5** and **50**.

20. From the given table of cumulative frequencies, we construct a table of simple frequencies as shown below:

Class interval	Mid-value x_i	Frequency f_i	$x_i f_i$
0–20	10	6	60
20–40	30	18 – 6 = 12	360
40–60	50	40 – 18 = 22	1100
60–80	70	47 – 40 = 7	490
80–100	90	50 – 47 = 3	270
		$\Sigma f_i = 50$	$\Sigma f_i x_i = 2280$

$\therefore$ Required mean marks $= \dfrac{\Sigma x_i f_i}{\Sigma f_i} = \dfrac{2280}{50} = 45.6.$

EXERCISE 16B

For Basic and Standard Levels

1. Let the assumed mean be $a = 9$

Class interval (amount of milk in litres)	Mid-value x_i	Number of cows f_i	$d_i = x_i - a$	$f_i d_i$	
0–2	1	0	1–9 = –8	0	
2–4	3	4	3–9 = –6	–24	
4–6	5	14	5–9 = –4	–56	} – 114
6–8	7	17	7–9 = –2	–34	
8–10	9	20	9–9 = 0	0	
10–12	11	10	11–9 = 2	+20	
12–14	13	13	13–9 = 4	+52	} + 224
14–16	15	12	15–9 = 6	+72	
16–18	17	10	17–9 = 8	+80	
		$\Sigma f_i = 100$		$\Sigma f_i d_i = +110$	

$$\text{Mean} = \bar{x} = a + \frac{\Sigma f_i d_i}{\Sigma f_i} = 9 + \frac{110}{100} = 9 + 1.1 = 10.1$$

Hence, the mean litres of milk given by the cows = **10.1 litres.**

2. The given series is an inclusive series, converting it into exclusive series, we get

Class interval (marks)	Mid-value x_i	Number of students f_i	$d_i = x_i - a$ (let $a = 54.5$) $= x_i - 54.5$	$f_i d_i$	
–0.5–9.5	4.5	2	4.5–54.5 = –50	–100	
9.5–19.5	14.5	5	14.5–54.5 = –40	–200	
19.5–29.5	24.5	7	24.5–54.5 = –30	–210	}–780
29.5–39.5	34.5	8	34.5–54.5 = –20	–160	
39.5–49.5	44.5	11	44.5–54.5 = –10	–110	
49.5–59.5	54.5	12	54.5–54.5 = 0	0	
59.5–69.5	64.5	9	64.5–54.5 = 10	+90	
69.5–79.5	74.5	7	74.5–4.5 = 20	+140	}+540
79.5–89.5	84.5	5	84.5–54.5 = 30	+150	
89.5–99.5	94.5	4	94.5–54.5 = 40	+160	
		$\Sigma f_i = 70$		$\Sigma f_i d_i = -240$	

$$\text{Mean} = \bar{x} = a + \frac{\Sigma f_i d_i}{\Sigma f_i}$$

$$= 54.5 - \frac{240}{70} = \frac{109}{2} - \frac{24}{7}$$

$$= \frac{763 - 48}{14} = \frac{715}{14} = \mathbf{51.07}$$

3. Here $h = 10$. Let the assumed mean be $a = 25$

Class interval	Mid-value x_i	Frequency f_i	$u_i = \dfrac{x_i - a}{h}$	$\Sigma f_i u_i$	
0–10	5	7	$\dfrac{5-25}{10} = -2$	–14	} – 24
10–20	15	10	$\dfrac{15-25}{10} = -1$	–10	
20–30	25	15	$\dfrac{25-25}{10} = 0$	0	
30–40	35	8	$\dfrac{35-25}{10} = 1$	8	} + 28
40–50	45	10	$\dfrac{45-25}{10} = 2$	20	
		$\Sigma f_i = 50$		$\Sigma f_i u_i = 4$	

$$\text{Mean} = \bar{x} = a + h\left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right)$$

$$= 25 + 10\left(\frac{4}{50}\right) = 25 + 0.8 = \mathbf{25.8}$$

4. Here $h = 7$. Let the assumed mean be $a = 87.5$

Class-interval	*Mid-value* x_i	*Frequency* f_i	$u_i = \dfrac{x_i - a}{h}$	$f_i u_i$	
63–70	66.5	9	$\dfrac{66.5 - 87.5}{7} = -3$	–27	
70–77	73.5	13	$\dfrac{73.5 - 87.5}{7} = -2$	–26	-80
77–84	80.5	27	$\dfrac{80.5 - 87.5}{7} = -1$	–27	
84–91	87.5	38	$\dfrac{87.5 - 87.5}{7} = 0$	0	
91–98	94.5	32	$\dfrac{94.5 - 87.5}{7} = 1$	+32	
98–105	101.5	16	$\dfrac{101.5 - 87.5}{7} = 2$	+32	$+109$
105–112	108.5	15	$\dfrac{108.5 - 87.5}{7} = 3$	+45	
		$\Sigma f_i = 150$		$\Sigma f_i u_i = +29$	

$$\text{Mean} = \bar{x} = a + h\left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right)$$

$$= 87.5 + 7\left(\frac{29}{150}\right)$$

$$= 87.5 + 1.353 = \mathbf{88.853}$$

5. Let the assumed mean be $a = 40$.

Here h = length of each class interval, $h = 10$.

Class interval	*Class marks* x_i	*Frequency* f_i	$u_i = \dfrac{x_i - a}{h}$	$u_i f_i$
5–15	10	6	–3	–18
15–25	20	10	–2	–20
25–35	30	16	–1	–16
35–45	40	15	0	0
45–55	50	24	1	24
55–65	60	8	2	16
65–75	70	7	3	21
		$\Sigma f_i = 86$		$\Sigma u_i f_i = 7$

$\therefore$ Required mean $= a + h\dfrac{\Sigma u_i f_i}{\Sigma f_i}$

$$= 40 + 10 \times \frac{7}{86}$$

$$= 40 + \frac{35}{43}$$

$$= \mathbf{40.81}$$

6. Here $h = 50$. Let the assumed mean be $a = 125$

Class interval	*Mid-value* x_i	*Frequency* f_i	$u_i = \dfrac{x_i - a}{h}$	$f_i u_i$	
0–50	25	17	$\dfrac{25 - 125}{50} = -2$	–34	-69
50–100	75	35	$\dfrac{75 - 125}{50} = -1$	–35	
100–150	125	43	$\dfrac{125 - 125}{50} = 0$	0	
150–200	175	40	$\dfrac{175 - 125}{50} = 1$	+40	
200–250	225	21	$\dfrac{225 - 125}{50} = 2$	+42	$+154$
250–300	275	24	$\dfrac{275 - 125}{50} = 3$	+72	
		$\Sigma f_i = 180$		$\Sigma f_i u_i = 85$	

$$\text{Mean} = \bar{x} = a + h\left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right)$$

$$= 125 + 50\left(\frac{85}{180}\right)$$

$$= 125 + 23.611$$

$$= \mathbf{148.611}$$

7. Here we apply the stop deviation method to calculate the mean. The assumed mean, $a = 45$ and length of each class interval, $h = 10$.

Class interval	*Class marks* x_i	*Frequency* f_i	$u_i = \dfrac{x_i - 45}{10}$	$u_i f_i$
20–30	25	25	–2	–50
30–40	35	40	–1	–40
40–50	45	42	0	0
50–60	55	33	1	33
60–70	65	10	2	20
		$\Sigma f_i = 150$		$\Sigma u_i f_i = -37$

$\therefore$ Required mean $= \bar{x} = a + h\dfrac{\Sigma u_i f_i}{\Sigma f_i}$

$$= 45 - 10 \times \frac{37}{150}$$

$= 45 - \dfrac{37}{15}$

$= 45 - 2.467$

$= \mathbf{42.533}$

8. Here, we apply the step derivation method to calculate the mean.

Here, the assumed mean, $a = 70$ (in %) and the length of each class interval, $h = 10$.

Literacy rate (in %)	*Class marks x_i (in %)*	*Frequency f_i*	$u_i = \dfrac{x_i - 70}{10}$	$u_i f_i$
45–55	50	4	–2	–8
55–65	60	11	–1	–11
65–75	70	12	0	0
75–85	80	9	1	9
85–95	90	4	2	8
		$\Sigma f_i = 40$		$\Sigma u_i f_i = -2$

$\therefore$ Required mean $= \bar{x} = a + h\dfrac{\Sigma u_i f_i}{\Sigma f_i}$

$= \left(70 - 10 \times \dfrac{2}{40}\right)\%$

$= (70 - 0.5)\%$

$= \mathbf{69.5\%}$.

9.

Class interval (total pulses)	*Mid-value x_i*	*Number of calls f_i*	$f_i x_i$
0–10	5	5	25
10–20	15	10	150
20–30	25	25	625
30–40	35	30	1050
40–50	45	20	900
50–60	55	10	550
		$\Sigma f_i = 100$	3300

$$\text{Mean} = \bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{3300}{100} = 33$$

Hence, the mean number of pulses per call = 33
Mean = **33 pulses**

10.

Marks	*Mid-value x_i*	*Number of students f_i*	$f_i x_i$
0–10	5	15	75
10–20	15	20	300
20–30	25	35	875
30–40	35	20	700
40–50	45	10	450
		$\Sigma f_i = 100$	$\Sigma f_i x_i = 2400$

$$\text{Mean} = \bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{2400}{100} = 24$$

Hence, **mean = 24 marks**

11. The given series in an inclusive series. Converting it into an exclusive series, we get

Marks	*Mid-value x_i*	*Number of students f_i*	*Let $a = 425.5$, $u_i = \dfrac{x_i - a}{h}$*	$f_i u_i$
200.5–250.5	225.5	62	$\dfrac{225.5 - 425.5}{50} = -4$	–248
250.5–300.5	275.5	120	$\dfrac{275.5 - 425.5}{50} = -3$	–360
300.5–350.5	325.5	412	$\dfrac{325.5 - 425.5}{50} = -2$	–824
350.5–400.5	375.5	100	$\dfrac{375.5 - 425.5}{50} = -1$	–100
400.5–450.5	425.5	479	$\dfrac{425.5 - 425.5}{50} = 0$	0
450.5–500.5	475.5	87	$\dfrac{475.5 - 425.5}{50} = 1$	+87
500.5–550.5	525.5	23	$\dfrac{525.5 - 425.5}{50} = 2$	+46
550.5–600.5	575.5	12	$\dfrac{575.5 - 425.5}{50} = 3$	+36
600.5–650.5	625.5	3	$\dfrac{625.5 - 425.5}{50} = 4$	+12
650.5–700.5	675.5	2	$\dfrac{675.5 - 425.5}{50} = 5$	+10
		$\Sigma f_i = 1300$		$\Sigma f_i u_i = -1341$

–1532

+191

$$\text{Mean} = \bar{x} = a + h\left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right)$$

$$= 425.5 + 50\left(\frac{-1341}{1300}\right)$$

$$= 425.5 - 51.576 = 373.924$$

Hence, mean = **373.924 marks**.

12. Here, $h = 80$. Let the assumed mean $a = 200$

Class interval	*Mid-value* x_i	*Frequency* f_i	$u_i = \frac{x_i - a}{h}$	$f_i u_i$
0–80	40	f_1	$\frac{40-200}{80} = -2$	$-2f_1$
80–160	120	35	$\frac{120-200}{80} = -1$	–35
160–240	200	f_2	$\frac{200-200}{80} = 0$	0
240–320	280	25	$\frac{280-200}{80} = 1$	+25
320–400	360	24	$\frac{360-200}{80} = 2$	+48
		$\Sigma f_i = 84 + f_1 + f_2 = 150$		$\Sigma f_i u_i = 38 - 2f_1$

$$84 + f_1 + f_2 = 150$$

$$\Rightarrow \quad f_1 + f_2 = 66 \qquad \ldots (1)$$

$$\text{Mean} = \bar{x} = a + h\left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right)$$

$$= 200 + 80\left(\frac{38 - 2f_1}{150}\right)$$

$$\frac{984}{5} = 200 + \frac{8(38 - 2f_1)}{15}$$

$$\Rightarrow \quad 2952 = 3000 + (304 - 16f_1)$$

$$\Rightarrow \quad 16f_1 = 3304 - 2952 = 352$$

$$\Rightarrow \quad f_1 = \frac{352}{16} = 22$$

Substituting $f_1 = 22$ in equation (1), we get

$$22 + f_2 = 66$$

$$= \quad f_2 = 44$$

Hence, $\mathbf{f_1 = 22, f_2 = 44}$.

13. Here $h = 80$. Let the assumed mean $a = 200$

Class	*Mid-value* x_i	*Frequency* f_i	$u_i = \frac{x_i - a}{h}$	$f_i u_i$
0–80	40	20	$\frac{40-200}{80} = -2$	–40
80–160	120	25	$\frac{120-200}{80} = -1$	–25
160–240	200	f_1	$\frac{200-200}{80} = 0$	0
240–320	280	f_2	$\frac{280-200}{80} = 1$	f_2
320–400	360	10	$\frac{360-200}{80} = 2$	+20
		$\Sigma f_i = 55 + f_1 + f_2 = 100$		$\Sigma f_i u_i = -45 + f_2$

$$55 + f_1 + f_2 = 100$$

$$\Rightarrow \quad f_1 + f_2 = 45 \qquad \ldots (1)$$

$$\text{Mean} = \bar{x} = a + h\left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right)$$

$$\Rightarrow \quad 188 = 200 + 80\left(\frac{-45 + f_2}{100}\right)$$

$$\Rightarrow \quad 188 - 200 = \frac{4}{5}(-45 + f_2)$$

$$\Rightarrow \quad (-12)5 = 4(-45 + f_2)$$

$$\Rightarrow \quad -60 = -180 + 4f_2$$

$$\Rightarrow \quad 4f_2 = 180 - 60$$

$$\Rightarrow \quad f_2 = \frac{120}{4} = 30$$

Substituting $f_2 = 30$ in equation (1), we get

$$f_1 + 30 = 45$$

$$\Rightarrow \quad f_1 = 15$$

Hence, $\mathbf{f_1 = 15, f_2 = 30}$.

14. Here $h = 20$. Let the assumed mean $a = 50$

Class interval	*Mid-value* x_i	*Frequency* f_i	$u_i = \frac{x_i - a}{h}$	$f_i u_i$
0–20	10	5	$\frac{10-50}{20} = -2$	–10
20–40	30	f_1	$\frac{30-50}{20} = -1$	$-f_1$
40–60	50	10	$\frac{50-50}{20} = 0$	0
60–80	70	f_2	$\frac{70-50}{20} = 1$	f_2
80–100	90	7	$\frac{90-50}{20} = 2$	+14
100–120	110	8	$\frac{110-50}{20} = 3$	+24
		$\Sigma f_i = 30 + f_1 + f_2 = 50$		$\Sigma f_i u_i = 28 - f_1 + f_2$

$$30 + f_1 + f_2 = 50$$

$$\Rightarrow \quad f_1 + f_2 = 20 \qquad \ldots (1)$$

$$\text{Mean} = \bar{x} = a + h\left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right)$$

$$62.8 = 50 + \frac{20(28 - f_1 + f_2)}{50}$$

$\Rightarrow \quad 62.8 - 50 = \frac{2}{5}(28 - f_1 + f_2)$

$\Rightarrow \quad 6.4 \times 5 = 28 - f_1 + f_2$

$\Rightarrow \quad -f_1 + f_2 = 4 \qquad \ldots (2)$

Adding (1) and (2), we get

$2f_2 = 24$

$\Rightarrow \quad f_2 = 12$

Substituting $f_2 = 12$ in (1), we get

$f_1 = 8$

Hence, $f_1 = \mathbf{8}$, $f_2 = \mathbf{12}$.

15. Here $h = 20$. Let the assumed mean $a = 50$

Class	*Mid-value* x_i	*Frequency* f_i	$u_i = \frac{x_i - a}{h}$	$f_i u_i$	
0–20	10	15	$\frac{10-50}{20} = -2$	–30	– 46
20–40	30	16	$\frac{30-50}{20} = -1$	– 16	
40–60	50	24	$\frac{50-50}{20} = 0$	0	
60–80	70	15	$\frac{70-50}{20} = +1$	15	+ 35
80–100	90	10	$\frac{90-50}{20} = +2$	20	
		$\Sigma f_i = 80$		$\Sigma f_i u_i = -11$	

$$\text{Mean} = \bar{x} = a + h\left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right)$$

$$= 50 + 20\left(\frac{-11}{80}\right)$$

$$= 50 - 2.75$$

$$= 47.25$$

Hence, mean = **47.25 marks.**

16. Here $h = 1000$. Let the assumed mean $a = 3500$

Savings in ₹	*Mid-value* x_i	*Frequency* f_i	$u_i = \frac{x_i - a}{h}$	$f_i u_i$	
0–1000	500	15	$\frac{500-3500}{1000} = -3$	–45	–114
1000–2000	1500	20	$\frac{1500-3500}{1000} = -2$	–40	
2000–3000	2500	29	$\frac{2500-3500}{1000} = -1$	–29	
3000–4000	3500	20	$\frac{3500-3500}{1000} = 0$	0	
4000–5000	4500	12	$\frac{4500-3500}{1000} = +1$	+12	+532
5000–6000	5500	24	$\frac{5500-3500}{1000} = +2$	+48	
6000–7000	6500	72	$\frac{6500-3500}{1000} = +3$	+216	
7000–8000	7500	64	$\frac{7500-3500}{1000} = +4$	+256	
		$\Sigma f_i = 256$		$\Sigma f_i u_i = +418$	

$$\text{Mean} = \bar{x} = a + h\left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right)$$

$$= 3500 + 1000\left(\frac{418}{256}\right)$$

$$= 3500 + \frac{26125}{16}$$

$$= 3500 + 1632.812$$

$$= 5132.81$$

Hence, mean savings = ₹ **5132.81**

17. Here $h = 10$. Let the assumed mean $a = 25$

Marks	*Mid-value* x_i	*Frequency* f_i	$u_i = \frac{x_i - a}{h}$	$f_i u_i$	
0–10	5	4	$\frac{5-25}{10} = -2$	–8	– 24
10–20	15	16	$\frac{15-25}{10} = -1$	–16	
20–30	25	20	$\frac{25-25}{10} = 0$	0	
30–40	35	10	$\frac{35-25}{10} = 1$	+10	+ 33
40–50	45	7	$\frac{45-25}{10} = 2$	+14	
50–60	55	3	$\frac{55-25}{10} = 3$	+9	
		$\Sigma f_i = 60$		$\Sigma f_i u_i = +9$	

$$\text{Mean} = \bar{x} = a + h\left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right)$$

$$= 25 + 10\left(\frac{9}{60}\right)$$

$$= 25 + \frac{3}{2}$$

$$= 26.5$$

Hence, mean **= 26.5 marks.**

EXERCISE 16C

For Basic and Standard Levels

1.

Weight (in kg)	60–62	63–65	66–68	69–71	72–74
Number of workers	5	18	42	27	8

Converting the given inclusive series to an exclusive form, we get

Weight (in kg)	59.5–62.5	62.5–65.5	65.5–68.5	68.5–71.5	71.5–74.5
Number of workers	5	18	42	27	8

SOLUTION: Here the maximum class frequency is 42 and the class corresponding to this frequency is 65.5–68.5.
So, the modal class is 65.5–68.5.
$\therefore$ $l = 65.5, f_1 = 42, f_0 = 18, f_2 = 27$ and $h = 3$

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)$$

$$= 65.5 + \left(\frac{42 - 18}{2 \times 42 - 18 - 27}\right) \times 3$$

$$= 65.5 + \frac{24}{(84 - 45)} \times 3$$

$$= 65.5 + \frac{24}{39} \times 3$$

$$= 65.5 + \frac{24}{13}$$

$$= 65.5 + 1.846$$
$$= 67.346$$

Hence, modal class **65.5–68.5** and mode = **67.346 kg.**

2.

Marks	0–10	10–20	20–30	30–40	40–50
Number of students	6	10	12	32	20

SOLUTION: Here the maximum class frequency is 32 and the class corresponding to this frequency is 30–40.
So the modal class is 30–40.
$\therefore$ $l = 30, f_1 = 32, f_0 = 12, f_2 = 20$ and $h = 10$

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 30 + \left(\frac{32 - 12}{2 \times 32 - 12 - 20}\right) \times 10$$

$$= 30 + \left(\frac{20}{64 - 32}\right) \times 10$$

$$= 30 + \frac{200}{32}$$

$$= 30 + 6.25$$
$$= 36.25$$

Hence, mode = **36.25 marks.**

3. Here the maximum frequency is 28 and so the class corresponding to this frequency is the modal class.

So, the modal class is 40–60.

$\therefore$ l = lower limit of the modal class = 40

h = size of each class interval = 20

f_1 = frequency of the modal class = 28

f_0 = frequency of the class preceding the modal class = 16

and f_2 = frequency of the class succeeding the modal class = 20.

$$\therefore \quad \text{Required mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 40 + \left(\frac{28 - 16}{2 \times 28 - 16 - 20}\right) \times 20$$

$$= 40 + \frac{12 \times 20}{56 - 36}$$

$$= 40 + \frac{12 \times 20}{20}$$

$$= \mathbf{52}.$$

4. Here the maximum frequency is 45 and so the class corresponding to this frequency is the modal class.

$\therefore$ The modal class is 30–40.

$\therefore$ l = lower limit of the modal class = 30

h = size of each class interval = 10

f_1 = frequency of the modal class = 45

f_0 = frequency of the class preceding the modal class = 35

and f_2 = frequency of the class succeeding the modal class = 25.

$$\therefore \quad \text{Required mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 30 + \left(\frac{45 - 35}{2 \times 45 - 35 - 25}\right) \times 10$$

$$= 30 + \left(\frac{10}{90 - 60}\right) \times 10$$

$$= 30 + \frac{10 \times 10}{30}$$

$$= 30 + 3.33$$

$$= \mathbf{33.33}$$

5. Here the maximum frequency is 16 and so the class corresponding to this frequency is the modal class.

$\therefore$ The modal class is 30–40.

$\therefore$ l = lower limit of the modal class = 30

h = size of each class interval = 10

f_1 = frequency of the modal class = 16

f_0 = frequency of the class preceding the modal class = 10

and f_2 = frequency of the class succeeding the modal class = 12

$$\therefore \quad \text{Required mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 30 + \left(\frac{16 - 10}{2 \times 16 - (10 + 12)}\right) \times 10$$

$$= 30 + \left(\frac{6}{32 - 22}\right) \times 10$$

$$= 30 + \frac{6}{10} \times 10$$

$$= \mathbf{36}.$$

6.

Marks (percentage)	0–10	10–20	20–30	30–40	40–50	50–60	60–70	70–80	80–90	90–100
Number of candidates	5	15	40	70	90	100	80	35	10	5

SOLUTION: Here the maximum class frequency is 100 and the class corresponding to this frequency is 50–60 So, the modal class is 50–60.

$\therefore \quad l = 50, f_1 = 100, f_0 = 90, f_2 = 80$ and $h = 10$

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 50 + \left(\frac{100 - 90}{2 \times 100 - 90 - 80}\right) \times 10$$

$$= 50 + \left(\frac{100}{200 - 170}\right)$$

$$= 50 + \frac{100}{30}$$

$$= 50 + 3.33$$

$$= 53.33$$

Hence, mode = **53.33% marks.**

7.

IQ Score	80–90	90–100	100–110	110–120	120–130	130–140
Number of stundents	5	10	16	15	3	1

SOLUTION: Here the maximum class frequency is 16 and the class corresponding to this frequency is 100–110 So, the modal class is 100–110.

$\therefore \quad l = 100, f_1 = 16, f_0 = 10, f_2 = 15$ and $h = 10$

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 100 + \left(\frac{16 - 10}{2 \times 16 - 10 - 15}\right) \times 10$$

$$= 100 + \frac{60}{32 - 25}$$

$$= 100 + \frac{60}{7}$$

$$= 100 + 8.751$$

$$= 108.571$$

Hence, mode = **108.571**.

8.

Family size	1–3	3–5	5–7	7–9	9–11
Number of families	7	8	3	1	1

SOLUTION: Here the maximum class frequency is 8 and the class corresponding to this frequency is 3–5. So, the modal class is 3–5.

$\therefore \quad l = 3, f_1 = 8, f_0 = 7, f_2 = 3$ and $h = 2$

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 3 + \left(\frac{8 - 7}{2 \times 8 - 7 - 3}\right) \times 2$$

$$= 3 + \frac{2}{6}$$

$$= 3 + \frac{1}{3}$$

$$= 3 + 0.333$$

$$= 3.333$$

Hence, mode = **3.333.**

9. Here the maximum frequency is 26 and so the class corresponding to this frequency is 201–202.

$\therefore$ The modal class is 201–202.

$\therefore$ l = lower limit of the modal class = 201

h = size of each class interval = 1

f_1 = frequency of the modal class = 26

f_0 = frequency of the class preceding the modal class = 12

and f_2 = frequency of the class succeeding the modal class = 20

$$\therefore \quad \text{Required mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 201 + \left(\frac{26 - 12}{2 \times 26 - (12 + 20)}\right) \times 1$$

$$= 201 + \frac{14}{52 - 32}$$

$$= 201 + \frac{14}{20}$$

$$= 201 + \frac{7}{10}$$

$$= 201.7$$

Hence, the modal weight is **201.7 g**.

10. Here the maximum frequency is 41 and so the class corresponding to this frequency is 10000–15000.

∴ The modal class is 10000–15000.

∴ l = lower limit of the modal class = 10000

h = size of each class interval = 5000

f_1 = frequency of the modal class = 41

f_0 = frequency of the class preceding the modal class = 26

and f_2 = frequency of the class succeeding the modal class = 16

∴ Required mode $= l + \left(\dfrac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$

$$= 10000 + \left(\frac{41 - 26}{2 \times 41 - (26 + 16)}\right) \times 5000$$

$$= 10000 + \left(\frac{15}{82 - 42}\right) \times 5000$$

$$= 10000 + \frac{15}{40} \times 5000$$

$= 10000 + 15 \times 125$

$= 10000 + 1875$

$= 11875.$

Hence, the modal income is ₹ **11875**.

11.

Ages (in years)	10–19	20–29	30–39	40–49	50–59	60–69
Number of patients	19	21	27	21	22	20

SOLUTION: Converting the given inclusive series into an exclusive form, we get

Ages (in years)	9.5–19.5	19.5–29.5	29.5–39.5	39.5–49.5	49.5–59.5	59.5–69.5
Number of patients	19	21	27	21	22	20

Here maximum class frequency is 27 and the class corresponding to this frequency is 29.5–39.5.
So, the modal class is 29.5–39.5.

∴ $l = 29.5, f_1 = 27, f_0 = 21, f_2 = 21$ and $h = 10$

Mode $= l + \left(\dfrac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$

$$= 29.5 + \left(\frac{27 - 21}{2 \times 27 - 21 - 21}\right) \times 10$$

$$= 29.5 + \frac{60}{54 - 42}$$

$$= 29.5 + \frac{60}{12}$$

$= 29.5 + 5$

$= 34.5$

Hence, mode = **34.5 years.**

12. Here the maximum frequency is 12 and so the class corresponding to this frequency is 200–250.

∴ The modal class is 200–250.

∴ l = lower limit of the modal class = 200

h = size of each class interval = 50

f_1 = frequency of the modal class = 12

f_0 = frequency of the class preceding the modal class = 5

and f_2 = frequency of the class succeeding the modal class = 2

∴ Required mode $= l + \dfrac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$

$$= 200 + \left(\frac{12 - 5}{2 \times 12 - (5 + 2)}\right) \times 50$$

$$= 200 + \left(\frac{7}{24 - 7}\right) \times 50$$

$$= 200 + \frac{7}{17} \times 50$$

$$= 200 + \frac{350}{17}$$

$= 220.59.$

Hence, the modal daily expenditure on grocery is ₹ **220.59**.

13. **Modal height**

Here the maximum frequency is 20 and so the class corresponding to this frequency is 160–165.

∴ The modal class is 160–165.

∴ l = lower limit of the modal class = 160

h = size of each class interval = 5

f_1 = frequency of the modal class = 20

f_0 = frequency of the class preceding the modal class = 8

and f_2 = frequency of the class succeeding the modal class = 12

∴ Required mode $= l + \left(\dfrac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$

$$= \left\{160 + \left(\frac{20 - 8}{2 \times 20 - (8 + 12)}\right) \times 5\right\} \text{ cm}$$

$$= \left(160 + \frac{12 \times 5}{40 - 20}\right) \text{cm}$$

$$= \left(160 + \frac{60}{20}\right) \text{cm}$$

$= 163$ cm.

Mean height

For this, we form the following table:

Height (in cm)	*Class mark (in cm)* x_i	*Number of students* f_i	$x_i f_i$
150–155	152.5	15	2287.5
155–160	157.5	8	1260.0
160–165	162.5	20	3250.0
165–170	167.5	12	2010.0
170–175	172.5	5	862.5
		$\Sigma f_i = 60$	$\Sigma f_i x_i =$ 9670.0

$\therefore$ Required mean $= \dfrac{\Sigma f_i x_i}{\Sigma f_i} = \dfrac{9670.0}{60.0} = 161.17$ cm

We have thus got the modal height of the students as **163 cm** and the mean height of the students = **161.17 cm**. This means that the height of maximum number of students in the class is 163 cm and also the average height of each student is 161.17 cm. We also see that the average height of each student is almost equal to the height of the maximum number of students in the class.

14. Mode:

Tomatoes per plant	1–5	6–10	11–15	16–20
Number of plants	20	50	46	22

SOLUTION: Converting the given inclusive series to an exclusive form, we get

Tomatoes per plant	0.5–5.5	5.5–10.5	10.5–15.5	15.5–20.5
Number of plants	20	50	46	22

Here maximum class frequency is 50 and the class corresponding to this frequency is 5.5–10.5.
So, the class is 5.5–10.5.

$\therefore \quad l = 5.5, f_1 = 50, f_0 = 20, f_2 = 46$ and $h = 5$

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 5.5 + \left(\frac{50 - 20}{2 \times 50 - 20 - 46}\right) \times 5$$

$$= 5.5 + \left(\frac{30}{100 - 66}\right) \times 5$$

$$= 5.5 + \frac{150}{34}$$

$$= 5.5 + 4.411$$

$$= \mathbf{9.911}$$

Mean: Here $h = 5$. Let the assumed mean $a = 8$

Class interval	*Mid-value* x_i	*Frequency* f_i	$u_i = \dfrac{x_i - a}{h}$	$f_i u_i$	
0.5–5.5	3	20	$\frac{3-8}{5} = -1$	–20	– 20
5.5–10.5	8	50	$\frac{8-8}{5} = 0$	0	
10.5–15.5	13	46	$\frac{13-8}{5} = +1$	+46	+ 90
15.5–20.5	18	22	$\frac{18-3}{5} = +2$	+44	
		$\Sigma f_i = 138$		$\Sigma f_i u_i = +70$	

$$\text{Mean} = \bar{x} = a + h\left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right)$$

$$= 8 + 5 \times \left(\frac{70}{138}\right)$$

$$= 8 + 2.536$$

$$= \mathbf{10.536}$$

Interpretation: On an average the number of tomatoes per plant is 10.536, while maximum number of plant yield 9.91 tomatoes per plant.

15. Mode:

Life time (in hours)	0–10	10–20	20–30	30–40	40–50	50–60	60–70	70–80
Frequency	15	20	25	24	12	31	71	52

SOLUTION: Here the maximum class frequency is 71 and the class corresponding to this frequency is 60–70. So the modal class is 60–70.

$\therefore \quad l = 60, f_1 = 71, f_0 = 31, f_2 = 52$ and $h = 10$

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)$$

$$= 60 + \left(\frac{71 - 31}{2 \times 71 - 31 - 52}\right) \times 10$$

$$= 60 + \frac{40}{(142 - 83)} \times 10$$

$$= 60 + \frac{400}{59}$$

$$= 60 + 6.779$$

$$= 66.779$$

Mean: Here, $h = 10$. Let the assumed mean $a = 35$

Let time in hours	*Mid-value* x_i	*Frequency* f_i	$u_i = \dfrac{x_i - a}{h}$	$f_i u_i$	
0–10	5	15	$\dfrac{5-35}{10} = -3$	–45	
10–20	15	20	$\dfrac{15-35}{10} = -2$	–40	– 110
20–30	25	25	$\dfrac{25-35}{10} = -1$	–25	
30–40	35	24	$\dfrac{35-35}{10} = 0$	0	
40–50	45	12	$\dfrac{45-35}{10} = +1$	+12	
50–60	55	31	$\dfrac{55-35}{10} = +2$	+62	+495
60–70	65	71	$\dfrac{65-35}{10} = +3$	+213	
70–80	75	52	$\dfrac{75-35}{10} = +4$	+208	
		$\Sigma f_i = 250$		$\Sigma f_i u_i = +385$	

$$\text{Mean} = \bar{x} = a + h\left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right)$$

$$= 35 + 10\left(\frac{385}{250}\right)$$

$$= 35 + 15.4$$
$$= 50.4$$

Hence, mode = **66.779 hours**
mean = **50.4 hours**

For Standard Level

16.

Daily allowance (in ₹)	16–18	18–20	20–22	22–24	24–26
Group A	50	78	46	11	40
Group B	54	89	40	29	13

Group A:
Here the maximum frequency is 78 and the class corresponding to this frequency is 18–20.
So, the modal class is 18–20.
$\therefore \quad l = 18, f_1 = 78, f_0 = 50, f_2 = 46$ and $h = 2$

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 18 + \left(\frac{78-50}{2 \times 78 - 50 - 46}\right) \times 2$$

$$= 18 + \left(\frac{28}{156 - 96}\right) \times 2$$

$$= 18 + \left(\frac{28}{60}\right) \times 2$$

$$= 18 + 0.93$$
$$= 18.93$$

Group B:
Here the maximum frequency is 89 and the class corresponding to this frequency is 18–20.
So, the modal class is 18–20.
$\therefore \quad l = 18, f_1 = 89, f_0 = 54, f_2 = 40$ and $h = 2$

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 18 + \left(\frac{89-54}{2 \times 89 - 54 - 40}\right) \times 2$$

$$= 18 + \left(\frac{35}{178 - 94}\right) \times 2$$

$$= 18 + \frac{35}{84} \times 2$$

$$= 18 + 0.83$$
$$= 18.83$$

Modal daily allowance of group A **= ₹ 18.93**
Modal daily allowance of group B **= ₹ 18.83**

17.

Marks	0–10	10–20	20–30	30–40	40–50	50–60	60–70	70–80	80–90	90–100
Number of students	3	5	7	10	12	15	12	6	2	8

Here the maximum class frequency is 15 and the class corresponding to this frequency is 50–60.
So, the modal class is 50–60.
$\therefore \quad l = 50, f_1 = 15, f_0 = 12, f_2 = 12$ and $h = 10$

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 50 + \left(\frac{15 - 12}{2 \times 15 - 12 - 12}\right) \times 10$$

$$= 50 + \frac{30}{(30 - 24)}$$

$$= 50 + \frac{30}{6}$$

$$= 50 + 5$$
$$= 55$$

Hence, mode = **55 marks.**

EXERCISE 16D

For Basic and Standard Levels

1.

Class interval	*Frequency, f_i*	*Cumulative frequency*
0–8	10	10
8–16	8	18
16–24	16	34
24–32	24	58
32–40	8	66
40–48	14	80

$$n = \Sigma f_i = 80$$

Here, $n = \Sigma f_i = 80$, so, $\frac{n}{2} = \frac{80}{2} = 40$

Cumulative frequency just greater than 40 is 58 and the corresponding class is 24–32.
So, the median class is 24–32.

$\therefore$ $l = 24$, $cf = 34$, $f = 24$ and $h = 8$

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

$$= 24 + \left(\frac{40 - 34}{24}\right) \times 8$$

$$= 24 + \left(\frac{6}{24}\right) \times 8$$

$$= 24 + 2$$
$$= 26$$

Median = **26**

2. We first form the cumulative frequency table for the given data as follows:

Class interval	*Frequency f_i*	*Cumulative frequency cf*
20–30	5	5
30–40	15	20
40–50	25	45
50–60	20	65
60–70	7	72
70–80	8	80
80–90	10	90
	$n = \Sigma f_i = 90$	

Here $n = \Sigma f_i = 90$.

So, $\frac{n}{2} = \frac{90}{2} = 45$.

The cumulative frequency just greater than 45 is 65 and so, the corresponding class is 50–60. So, the median class is 50–60.

$\therefore$ Here l = lower limit of the median class = 50

cf_{-1} = cumulative frequency of the class preceding the median class = 45

f_m = frequency of the median class = 20

and h = class size = 10.

Hence, the required median $= l + \left(\frac{\frac{n}{2} - cf_{-1}}{f_m}\right) \times h$

$$= 50 + \left(\frac{45 - 45}{20}\right) \times 10$$

$$= 50 + 0 = \mathbf{50}.$$

3. Converting the given inclusive series to an exclusive series and preparing the cumulative frequency table, we get

Height (in cm)	*Frequency, fi*	*Cumulative freqnecy*
159.5–162.5	15	15
162.5–165.5	118	133
165.5–168.5	142	275
168.5–171.5	127	402
171.5–174.5	18	420
	$n = \Sigma f_i = 420$	

Here, $n = \Sigma f_i = 420$. So, $\frac{n}{2} = \frac{420}{2} = 210$

Cumulative frequency just greater than 210 is 275 and the corresponding class is 165.5–168.5.
So, the median class is 165.5–168.5.

$\therefore$ $l = 165.5$, $cf = 133$, $f = 142$ and $h = 3$

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

$$= 165.5 + \left(\frac{210 - 133}{142}\right) \times 3$$

$$= 165.5 + \left(\frac{77}{142}\right) \times 3$$

$$= 165.5 + 1.626$$
$$= 167.126$$

Hence, median = **167.126**

4. Class size = Difference between any two consecutive mid-values = 350–250 = 100.
Mid-value 250 corresponds to class

$\left(250 - \frac{100}{2}\right) - \left(250 + \frac{100}{2}\right)$, i.e. 200–300.

Thus, we have the following cumulative frequency table for the given data

Class interval	Frequency	Cumulative frequency
200–300	3	3
300–400	5	8
400–500	20	28
500–600	10	38
600–700	6	44

$$n = \Sigma f_i = 44$$

Here, $n = \Sigma f_i = 44$. So, $\frac{n}{2} = \frac{44}{2} = 22$

Cumulative frequency just greater than 22 is 28 and the corresponding class is 400–500.
So, the median class is 400–500.
$\therefore \quad l = 400, cf = 8, f = 20$ and $h = 100$

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

$$= 400 + \left(\frac{22 - 8}{20}\right) \times 100$$

$$= 400 + \left(\frac{14}{20}\right) \times 100$$

$$= 400 + 70$$
$$= 470$$

Hence, **Median = 470**

5. We first form the cumulative frequency table for the given data as follows:

Class interval of speed (in km/h)	Frequency f_i	Cumulative frequency cf
85–100	11	11
100–115	9	20
115–130	8	28
130–145	5	33
	$n = \sum f_i = 33$	

Here $n = \Sigma f_i = 33$.

So, $\frac{n}{2} = \frac{33}{2} = 16.5$.

The cumulative frequency just greater than 16.5 is 20 and so, the corresponding class is 100–115. So, the median class is 100–115.

$\therefore$ Here, l = lower limit of the median class = 100
cf_{-1} = cumulative frequency of the class preceding the median class = 11
f_m = frequency of the median class = 9
and h = class size = 15

Hence, the required median $= l + \left(\frac{\frac{n}{2} - cf_{-1}}{f_m}\right) \times h$

$$= \left(100 + \left\{\frac{16.5 - 11}{9}\right\} \times 15\right) \text{km/h}$$

$$= \left(100 + \frac{5.5 \times 15}{9}\right) \text{km/h}$$

$$= (100 + 9.17) \text{ km/h}$$

$$= \mathbf{109.17 \text{ km/h}}$$

6. We first form the cumulative frequency table for the given data as follows:

Class interval of wages (in ₹)	Frequency f_i	Cumulative frequency cf
800–820	7	7
820–840	14	21
840–860	19	40
860–880	25	65
880–900	20	85
900–920	10	95
920–940	5	100
	$n = \sum f_i = 100$	

Here $n = \Sigma f_i = 100$.

So, $\frac{n}{2} = \frac{100}{2} = 50$.

The cumulative frequency just greater than 50 is 65, so the corresponding class is 860–880.

$\therefore$ The median class is 860–880.

Here l = lower limit of the median class = 860
cf_{-1} = cumulative frequency of the class preceding the median class = 40
f_m = frequency of the median class = 25
and h = class size = 20

Hence, the required median $= l + \frac{\frac{n}{2} - cf_{-1}}{f_m} \times h$

$$= ₹\left(860 + \frac{50 - 40}{25} \times 20\right)$$

$$= ₹\left(860 + \frac{10 \times 20}{25}\right)$$

$$= \mathbf{₹\ 868.}$$

7. We first form the cumulative frequency table for the given data as follows:

Class interval (size of the agricultural holdings in ha)	Frequency f_i	Cumulative frequency (cf)
0–5	10	10
5–10	15	25
10–15	30	55
15–20	80	135
20–25	40	175
25–30	20	195
30–35	5	200
	$n = \Sigma f_i = 200$	

Here $n = \Sigma f_i = 200$

$\therefore \quad \frac{n}{2} = \frac{200}{2} = 100.$

So, the cumulative frequency just greater than 100 is 135 and so the corresponding class is 15–20.

$\therefore$ The median class is 15–20.

$\therefore \quad l$ = the lower limit of the median class = 15

cf_{-1} = cumulative frequency of the class preceding the median class = 55

f_m = frequency of the median class = 80

and h = class size = 5

Hence, the required median $= l + \dfrac{\frac{n}{2} - cf_{-1}}{f_m} \times h$

$$= \left(15 + \frac{100 - 55}{80} \times 5\right) \text{ ha}$$

$$= \left(15 + \frac{45 \times 5}{80}\right) \text{ ha}$$

$$= (15 + 2.81) \text{ ha}$$

$$= \mathbf{17.81 \text{ ha}}$$

Calculation of mode

Here the maximum frequency is 80 and so the class corresponding to this frequency is the modal class. So, here the modal class is 15–20.

$\therefore \quad l_1$ = lower limit of the modal class = 15

h = size of each class interval = 5

f'_1 = frequency of the modal class = 80

f'_0 = frequency of the class preceding the modal class = 30

and f'_2 = frequency of the class succeeding the modal class = 40

$\therefore$ Required mode $= l_1 + \dfrac{f'_1 - f'_0}{2f'_1 - (f'_0 + f'_2)} \times h$

$$= \left(15 + \frac{80 - 30}{2 \times 80 - (30 + 40)} \times 5\right) \text{ ha}$$

$$= \left(15 + \frac{50 \times 5}{160 - 70}\right) \text{ ha}$$

$$= \left(15 + \frac{250}{90}\right) \text{ ha}$$

$= (15 + 2.78)$ ha = **17.78 ha**.

8.

Temperature (in °C)	Number of days f_i	Cumulative frequency
6.5–7.5	5	5
7.5–8.5	12	17
8.5–9.5	25	42
9.5–10.5	48	90
10.5–11.5	32	122
11.5–12.5	6	128
12.5–13.5	1	129

$$n = \Sigma f_i = 129$$

Here, $n = \Sigma f_i = 129$. So, $\frac{n}{2} = \frac{129}{2} = 64.5$

Cumulative frequency just greater than 64.5 is 90 and the corresponding class is 9.5–10.5.

So, the median class is 9.5–10.5.

$\therefore \quad l = 9.5$, $cf = 42$, $f = 48$ and $h = 1$

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

$$= 9.5 + \left(\frac{64.5 - 42}{48}\right) \times 1$$

$$= 9.5 + \frac{22.5}{48} = 9.5 + 0.468$$

$$= 9.968$$

Hence, median minimum temperature = **9.968 °C**

9. Converting the given inclusive series to an exclusive series and preparing the cumulative frequency table, we get

Wages/week (in ₹)	Number of workers f_i	Cumulative frequency
49.5–59.5	15	15
59.5–69.5	40	55
69.5–79.5	50	105
79.5–89.5	60	165
89.5–99.5	45	210
99.5–109.5	40	250
109.–119.5	15	265
	$n = \Sigma f_i = 265$	

Here, $n = \Sigma f_i = 265$. So, $\frac{n}{2} = \frac{265}{2} = 132.5$.

Cumulative frequency just greater than 132.5 is 165 and the corresponding class is 79.5–89.5.

So, the median class is 79.5–89.5.

$\therefore \quad l = 79.5, cf = 105, f = 60$ and $h = 10$

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

$$= 79.5 + \left(\frac{132.5 - 105}{60}\right) \times 10$$

$$= 79.5 + \frac{27.5}{6}$$

$$= 79.5 + 4.58$$

$$= 84.08$$

Hence, median = ₹ **84.08**

10. We first form the cumulative frequency table for the given data as follows:

Class interval Salary (in thousand ₹)	*Number of persons f_i*	*Cumulative frequency (cf)*
5–10	49	49
10–15	133	182
15–20	63	245
20–25	15	260
25–30	6	266
30–35	7	273
35–40	4	277
40–45	2	279
45–50	1	280
	$n = \Sigma f_i = 280$	

Here $n = \Sigma f_i = 280$. $\therefore \frac{n}{2} = 140$.

So, the cumulative frequency just greater than 140 is 182 and so the corresponding class is 10–15.

$\therefore$ The median class is 10–15.

$\therefore$ l = the lower limit of the median class = 10

cf_{-1} = cumulative frequency of the class preceding the median class = 49

f_m = frequency of the median class = 133

and h = class size = 5

Hence, the required median $= l + \frac{\frac{n}{2} - cf_{-1}}{f_m} \times h$

$$= ₹ \left(10 + \frac{140 - 49}{133} \times 5\right) \text{ thousand}$$

$$= ₹ \left(10 + \frac{91 \times 5}{133}\right) \text{ thousand}$$

$$= ₹ \left(10 + \frac{455}{133}\right) \text{ thousand}$$

= ₹ 13.42 thousand

= ₹ 13420.

Calculation of mode: We see that the class 10–15 has the maximum frequency, i.e. 133.

Hence, the modal class is 10–15.

Here, l_1 = lower limit of the modal class = 10

h = class size = 5

f'_1 = frequency of the modal class = 133

f'_0 = frequency of the class preceding the modal class = 49

f'_2 = frequency of the class succeeding the modal class = 63

$$\therefore \quad \text{Required mode} = l_1 + \frac{f'_1 - f'_0}{2f'_1 - (f'_0 + f'_2)} \times h$$

$$= ₹ \left(10 + \frac{133 - 49}{2 \times 133 - (49 + 63)} \times 5\right) \text{ thousand}$$

$$= ₹ \left(10 + \frac{84 \times 5}{266 - 112}\right) \text{ thousand}$$

$$= ₹ \left(10 + \frac{420}{154}\right) \text{ thousand}$$

= ₹ (10 + 2.73) thousand

= ₹ (12.73 × 1000) = ₹ **12730**.

11.

Daily income (in ₹)	*Mid-value x_i*	*Number of Workers f_i*	*Cumulative frequency cf*	$f_i x_i$
100–120	110	12	12	1320
120–140	130	14	26	1820
140–160	150	8	34	1200
160–180	170	6	40	1020
180–200	190	10	50	1900
		$n = \Sigma f_i = 50$		$\Sigma f_i x_i = 7260$

$$\text{Mean} = \bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{7260}{50} = 145.2$$

Mode: Here the maximum class frequency is 14 and the class corresponding to this frequency is 120–140. So, the modal class is 120–140.

$\therefore \quad l = 120, f_1 = 14, f_0 = 12, f_2 = 8$ and $h = 20$

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 120 + \left(\frac{14 - 12}{2 \times 14 - 12 - 8}\right) \times 20$$

$$= 120 + \left(\frac{2}{28 - 20}\right) \times 20$$

$$= 120 + \frac{40}{8}$$

$$= 120 + 5$$

$$= 125$$

Median: Here, $n = \Sigma f_i = 50$. So, $\frac{n}{2} = \frac{50}{2} = 25$.

Cumulative frequency just greater than 25 is 26 and the corresponding class is 120–140.

So, the median class is 120–140.

$\therefore \quad l = 120, cf = 12, f = 14$ and $h = 20$

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

$$= 120 + \left(\frac{25 - 12}{14}\right) \times 20$$

$$= 120 + \left(\frac{13}{14}\right) \times 20$$

$$= 120 + 18.57$$

$$= 138.57$$

Hence, Mean income = ₹ **145.20,**

Modal income = ₹ **125,**

Median income = ₹ **138.57.**

12.

Class interval	*Mid-value* x_i	*Frequency* f_i	*Cumulative frequency* cf	$f_i x_i$
0–10	5	8	8	40
10–20	15	7	15	105
20–30	25	15	30	375
30–40	35	20	50	700
40–50	45	12	62	540
50–60	55	8	70	440
60–70	65	10	80	650
		$n = \Sigma f_i = 80$		$\Sigma f_i x_i = 2850$

$$\text{Mean} = \bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{2850}{80} = 35.625$$

Mode: Here the maximum frequency is 20 and the class corresponding to this frequency is 30–40.

So, the modal class is 30–40.

$\therefore \quad l = 30, f_1 = 20, f_0 = 15, f_2 = 12$ and $h = 10$

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 30 + \left(\frac{20 - 15}{2 \times 20 - 15 - 12}\right) \times 10$$

$$= 30 + \frac{50}{40 - 27}$$

$$= 30 + \frac{50}{13}$$

$$= 30 + 3.846$$

$$= 33.846$$

Median: Here, $n = \Sigma f_i = 80$. So, $\frac{n}{2} = \frac{80}{2} = 40$.

Cumulative frequency just greater than 40 is 50 and the corresponding class is 30–40.

So, the median class is 30–40.

$\therefore \quad l = 30, cf = 30, f = 20$ and $h = 10$

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

$$= 30 + \left(\frac{40 - 30}{20}\right) \times 10$$

$$= 30 + \left(\frac{10}{20}\right) \times 10$$

$$= 30 + 5$$

$$= 35$$

Hence, Mean = **35.625,** Mode = **33.846,** Median = **35.**

13.

Monthly consumption (in units)	*Mid-value* x_i	*Number of consumers* f_i	*Cumulative frequency* cf	$f_i x_i$
65–85	75	3	3	225
85–105	95	6	9	570
105–125	115	13	22	1495
125–145	135	20	42	2700
145–165	155	14	56	2170
165–185	175	6	62	1050
185–205	195	5	67	975
		$n = \Sigma f_i = 67$		$\Sigma f_i x_i = 9185$

$$\text{Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{9185}{67} = 137.089$$

Median: Here $n = \Sigma f_i = 67$. So, $\frac{n}{2} = \frac{67}{2} = 33.5$.

Cumulative frequency just greater than 33.5 is 42 and the corresponding class is 125–145.

So, the median class is 125–145.

$\therefore \quad l = 125, cf = 22, f = 20$ and $h = 20$

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

$$= 125 + \left(\frac{33.5 - 22}{20}\right) \times 20$$

$$= 125 + 11.5$$
$$= 136.5$$

Mode: Here the maximum frequency is 20 and the class corresponding to it is 125–145.
So, the modal class is 125–145.
$\therefore \quad l = 125, f_1 = 20, f_0 = 13, f_2 = 14$ and $h = 20$

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 125 + \left(\frac{20 - 13}{2 \times 20 - 13 - 14}\right) \times 20$$

$$= 125 + \left(\frac{7}{40 - 27}\right) \times 20$$

$$= 125 + \left(\frac{7}{13}\right) \times 20$$

$$= 125 + 10.769$$
$$= 135.769$$

Hence, Mean = **137.089 units,** Median = **136.5 units,** Mode = **135.769 units.**

14.

Class interval	Mid-value x_i	Frequency f_i	$u_i = \frac{x_i - a}{h}$	$f_i u_i$	
10–20	15	4	$\frac{15-45}{10} = -3$	–12	– 38
20–30	25	8	$\frac{25-45}{10} = -2$	–16	
30–40	35	10	$\frac{35-45}{10} = -1$	–10	
40–50	45	12	$\frac{45-45}{10} = 0$	0	
50–60	55	10	$\frac{55-45}{10} = +1$	+10	+ 24
60–70	65	4	$\frac{65-45}{10} = +2$	+8	
70–80	75	2	$\frac{75-45}{10} = +3$	+6	
		$\Sigma f_i = 50$		$\Sigma f_i u_i = -14$	

$$\text{Mean} = a + h\left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right)$$

$$= 45 + 10\left(\frac{-14}{50}\right)$$

$$= 45 - \frac{14}{5}$$

$$= 45 - 2.8$$
$$= 42.2$$

Median = 42.5 [Given]

Now, 3 Median = Mode + 2 Mean
$\therefore \quad 3 \times 42.5 = \text{Mode} + 2 \times 42.2$
$\Rightarrow \quad \text{Mode} = 127.5 - 84.4 = 43.1$
Hence, Mean = **42.2,** Mode = **43.1.**

15.

Class interval	Frequency (f_i)	Cumulative frequency
0–10	2	2
10–20	5	7
20–30	x	$7 + x$
30–40	12	$19 + x$
40–50	17	$36 + x$
50–60	20	$56 + x$
60–70	y	$56 + x + y$
70–80	9	$65 + x + y$
80–90	7	$72 + x + y$
90–100	4	$76 + x + y$
	$n = \Sigma f_i = 76 + x + y$	

Since, the total frequency is 100.
$\therefore \quad 76 + x + y = 100$
$\Rightarrow \quad x + y = 24 \quad \ldots (1)$
Median is 52.5, so it lies in the class 50–60.
So the median class is 50–60.
$\therefore \quad l = 50, cf = 36 + x, f = 20$ and $h = 10$

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

$$\Rightarrow \quad 52.5 = 50 + \left[\frac{\frac{100}{2} - (36 + x)}{20}\right] \times 10$$

$$\Rightarrow \quad 2.5 = \left(\frac{50 - 36 - x}{20}\right) \times 10$$

$$\Rightarrow \quad 2.5 = \left(\frac{14 - x}{2}\right)$$

$\Rightarrow \quad 5 = 14 - x$
$\Rightarrow \quad x = 14 - 5$
$\Rightarrow \quad x = 9$
Substituting $x = 9$ in equation (1), we get
$9 + y = 24$
$\Rightarrow \quad y = 24 - 9$
$\Rightarrow \quad y = 15$
Hence, x **= 9,** y **= 15**

16. From the given date, we first find the cumulative frequency table as follows:

Class interval	*Frequency* f_i	*Cumulative frequency (cf)*
0–5	12	12
5–10	a	$12 + a$
10–15	12	$24 + a$
15–20	15	$39 + a$
20–25	6	$39 + a + b$
25–30	6	$45 + a + b$
30–35	6	$51 + a + b$
35–40	4	$55 + a + b$
	$n = \Sigma f_i = 55 + a + b$ …(1)	

It is given that the total frequencies, $n = 70$.

$\therefore \quad \frac{n}{2} = \frac{70}{2} = 35 \quad$ …(2)

$\therefore$ From (1), $\quad 55 + a + b = 70$

$\therefore \quad a + b = 70 - 55 = 15 \quad$ …(3)

It is also given that the median of the data is 16 which lies in the class interval 15–20.

$\therefore \quad l$ = the lower limit of the median class = 15

cf_{-1} = cumulative frequency of the class preceding the median class = $24 + a$

f_m = frequency of the median class = 15

h = class size = 5

Hence, the median $= l + \frac{\frac{n}{2} - cf_{-1}}{f_m} \times h$

$$= 15 + \frac{35 - 24 - a}{15} \times 5$$

$$= 15 + \frac{11 - a}{3} = 16 \qquad \text{[Given]}$$

$\Rightarrow \quad \frac{11 - a}{3} = 16 - 15 = 1$

$\Rightarrow \quad 11 - a = 3$

$\Rightarrow \quad a = 11 - 3 = 8$

$\therefore$ From (3), $b = 15 - a = 15 - 8 = 7$.

$\therefore$ The required values of missing data, a and b are respectively **8** and **7**.

17. From the given data, we construct the cumulative frequency table as follows:

Marks	*Frequency* f_i	*Cumulative frequency (cf)*
20–30	p	p
30–40	15	$p + 15$
40–50	25	$p + 40$
50–60	20	$p + 60$
60–70	q	$p + q + 60$
70–80	8	$p + q + 68$
80–90	10	$p + q + 78$
	$n = \Sigma f_i = p + q + 78$	

Given that $\quad \Sigma f_i = 90$

$\therefore \quad p + q + 78 = 90$

$\Rightarrow \quad p + q = 90 - 78 = 12 \quad$ …(1)

Also, median class is 50–60, since, given that median = 50.

$\therefore$ l = lower limit of the median class = 50

cf_{-1} = cumulative frequency of the class preceding the median class = $p + 40$, h = class size = 10.

$$\frac{n}{2} = \frac{90}{2} = 45$$

[$\because$ Given that total frequency = 90]

So f_m = frequency of the median class = 20

$\therefore \quad$ Median $= l + \frac{\frac{n}{2} - cf_{-1}}{f_m} \times h$

$$= 50 + \frac{45 - p - 40}{20} \times 10$$

$$= 50 + \frac{5 - p}{2} = 50 \qquad \text{[Given]}$$

$\therefore \quad p = 5$

$\therefore$ From (1), $\quad q = 12 - 5 = 7$.

Hence, required values of p and q are respectively **5** and **7**.

18. From the given data, we construct the cumulative frequency table as follows:

Marks	*Frequency* f_i	*Cumulative frequency (cf)*
0–10	10	10
10–20	f_1	$10 + f_1$
20–30	25	$35 + f_1$
30–40	30	$65 + f_1$
40–50	f_2	$65 + f_1 + f_2$
50–60	10	$75 + f_1 + f_2$
	$n = \Sigma f_i = 75 + f_1 + f_2$	

Since total frequencies are given to be 100.

$\therefore \quad 75 + f_1 + f_2 = 100$

$\Rightarrow \quad f_1 + f_2 = 100 - 75 = 25 \quad \ldots(1)$

Since the median is given to be 32.

$\therefore$ Median class is 30–40 in which 32 lies.

Hence, l = the lower limit of the median class = 30

f_m = the frequency of the median class = 30

cf_{-1} = cumulative frequency of the class preceding the median class = $35 + f_1$

$$\frac{n}{2} = \frac{75 + f_1 + f_2}{2} = \frac{75 + 25}{2} \quad \text{[From (1)]}$$

$= 50$

h = class size = 10

$$\therefore \quad \text{Median} = l + \frac{\frac{n}{2} - cf_{-1}}{f_m} \times h$$

$$= 30 + \frac{50 - 35 - f_1}{30} \times 10$$

$$\Rightarrow \quad 30 + \frac{15 - f_1}{3} = 32 \quad \text{(Given)}$$

$$\Rightarrow \quad \frac{15 - f_1}{3} = 32 - 30 = 2$$

$\Rightarrow \quad 15 - f_1 = 6$

$\Rightarrow \quad f_1 = 9$

$\therefore$ From (1), $\quad f_2 = 25 - 9 = 16$

Hence, the required values of f_1 and f_2 are respectively **9** and **16**.

19.

Class interval	*Frequency* f_i	*Cumulative frequency*
0–5	6	6
5–10	12	18
10–15	f_1	$18 + f_1$
15–20	120	$138 + f_1$
20–25	225	$363 + f_1$
25–30	250	$613 + f_1$
30–35	185	$798 + f_1$
35–40	110	$908 + f_1$
40–45	32	$940 + f_1$
45–50	f_2	$940 + f_1 + f_2$
	$n = \Sigma f_i = 940 + f_1 + f_2$	

Since, the total number of observations = 1000

$\therefore \quad 940 + f_1 + f_2 = 1000$

$\Rightarrow \quad f_1 + f_2 = 60 \quad \ldots(1)$

Median is 26.74, so it lies in the class 25–30.

So, the median class is 25–30.

$\therefore \quad l = 25,\ cf = 363 + f_1,\ f = 250$ and $h = 5$

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

$$\Rightarrow \quad 26.74 = 25 + \left[\frac{\frac{1000}{2} - (363 + f_1)}{250}\right] \times 5$$

$$\Rightarrow \quad 26.74 - 25 = \left(\frac{500 - 363 - f_1}{50}\right)$$

$\Rightarrow \quad 1.74 \times 50 = 137 - f_1$

$\Rightarrow \quad 87 = 137 - f_1$

$\Rightarrow \quad f_1 = 137 - 87$

$\Rightarrow \quad f_1 = 50$

Substituting $f_1 = 50$ in equation (1), we get

$50 + f_2 = 60$

$\Rightarrow \quad f_2 = 60 - 50$

$\Rightarrow \quad f_2 = 10$

Hence, f **= 50,** f_2 **= 10**

20. From the given data, we construct the cumulative frequency table as follows:

Class interval	*Frequency* f_i	*Cumulative frequency (cf)*
0–10	f_1	f_1
10–20	5	$5 + f_1$
20–30	9	$14 + f_1$
30–40	12	$26 + f_1$
40–50	f_2	$26 + f_1 + f_2$
50–60	3	$29 + f_1 + f_2$
60–70	2	$31 + f_1 + f_2$
	$\Sigma f_i = 31 + f_1 + f_2$	

Since the total frequency n, is given to be 40.

$\therefore \quad 31 + f_1 + f_2 = 40$

$\Rightarrow \quad f_1 + f_2 = 40 - 31 = 9 \quad \ldots(1)$

$$\therefore \quad \frac{n}{2} = \frac{40}{2} = 20$$

Since the given median 32.5 lies in the class 30–40, hence the median class is 30–40.

$\therefore \quad l$ = the lower limit of the median class = 30

cf_1 = cumulative frequency of the class preceding the median class = $14 + f_1$

f_m = frequency of the median class = 12

h = class size = 10

$\therefore$ $$\text{Median} = l + \frac{\frac{n}{2} - cf_{-1}}{f_m} \times h$$

$$= 30 + \frac{20 - 14 - f_1}{12} \times 10$$

$$= 30 + \frac{(6 - f_1)5}{6}$$

$$= 30 + \frac{30 - 5f_1}{3} = 32.5$$

[$\because$ Given that median = 32.5]

$\Rightarrow$ $\frac{30 - 5f_1}{6} = 32.5 - 30 = 2.5$

$\Rightarrow$ $30 - 5f_1 = 15$

$\Rightarrow$ $5f_1 = 30 - 15 = 15$

$\Rightarrow$ $f_1 = 3$

$\therefore$ From (1), $f_2 = 9 - 3 = 6$

Hence, the required values of f_1 and f_2 are respectively **3** and **6**.

21.

Class interval	*Frequency* f_i	*Cumulative frequency*
20–40	6	6
40–60	9	15
60–80	$2x + 3$	$18 + 2x$
80–100	14	$32 + 2x$
100–120	20	$52 + 2x$
120–140	$4y - 5$	$47 + 2x + 4y$
140–160	10	$57 + 2x + 4y$
160–180	8	$65 + 2x + 4y$
180–200	7	$72 + 2x + 4y$
	$n = \Sigma f_i = 72 + 2x + 4y$	

Since, the total frequency is 100.

$\therefore$ $72 + 2x + 4y = 100$

$\Rightarrow$ $2x + 4y = 28$

$\Rightarrow$ $x + 2y = 14$ … (1)

Median is 110, so it lies in the class 100–120.

So, the median class is 100–120.

$\therefore$ $l = 100$, $cf = 32 + 2x$, $f = 20$ and $h = 20$

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

$\Rightarrow$ $$110 = 100 + \left[\frac{\frac{100}{2} - (32 + 2x)}{20}\right] \times 20$$

$\Rightarrow$ $110 - 100 = 50 - 32 - 2x$

$\Rightarrow$ $10 = 18 - 2x$

$\Rightarrow$ $2x = 18 - 10$

$\Rightarrow$ $2x = 8$

$\Rightarrow$ $x = 4$

Substituting $x = 4$ in equation (1), we get

$4 + 2y = 14$

$\Rightarrow$ $2y = 14 - 4$

$\Rightarrow$ $2y = 10$

$\Rightarrow$ $y = 5$

Hence, x **= 4,** y **= 5.**

22.

Marks	*Frequency* f_i	*Cumulative frequency*
0–10	3	3
10–20	5	8
20–30	8	16
30–40	6	22
40–50	3	25
	$\Sigma f_i = 25$	

Here, $n = \Sigma f_i = 25$. So, $\frac{n}{2} = \frac{25}{2} = 12.5$.

Cumulative frequency just greater than 12.5 is 16 and the corresponding class is 20–30.

So, the median class is 20–30.

$\therefore$ $l = 20$, $cf = 8$, $f = 8$ and $h = 10$

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

$$= 20 + \left(\frac{\frac{25}{2} - 8}{8}\right) \times 10$$

$$= 20 + \left(\frac{12.5 - 8}{8}\right) \times 10$$

$$= 20 + \frac{45}{8}$$

$$= 20 + 5.625 = 25.625$$

Hence, Median = **25.625 marks.**

23.

Weight (in kg)	*Number of persons*	*Cumulative frequency*
50–60	3	3
60–70	5	8
70–80	9	17
80–90	12	29
90–100	5	34
100–110	4	38
110–120	2	40
	$n = \Sigma f_i = 40$	

Less than table

Weight in kg	*Number of persons*
Less than 50	0
Less than 60	3
Less than 70	8
Less than 80	17
Less than 90	29
Less than 100	34
Less than 110	38
Less than 120	40

Here, $n = \Sigma f_i = 40$. So, $\frac{n}{2} = \frac{40}{2} = 20$.

Cumulative frequency just greater than 20 is 29 and the corresponding class is 80–90.
So the median class is 80–90.

$\therefore \quad l = 80,\ cf = 17,\ f = 12$ and $h = 10$

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

$$= 80 + \left(\frac{20 - 17}{12}\right) \times 10$$

$$= 80 + \frac{30}{12} = 80 + 2.5 = 82.5$$

Hence, Median = **82.5 kg.**

24.

Marks	*Number of students* f_i	*Cumulative frequency*
0–10	4	4
10–20	6	10
20–30	20	30
30–40	10	40
40–50	7	47
50–60	3	50
	$n = \Sigma f_i = 50$	

Here, $n = \Sigma f_i = 50$. So, $\frac{n}{2} = \frac{50}{2} = 25$.

Cumulative frequency just greater than 25 is 30 and the corresponding class is 20–30.

$\therefore \quad l = 20,\ cf = 10,\ f = 20$ and $h = 10$

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

$$= 20 + \left(\frac{25 - 10}{20}\right) \times 10$$

$$= 20 + \frac{150}{20} = 20 + 7.5 = 27.5$$

Hence, Median = **27.5 marks.**

EXERCISE 16E

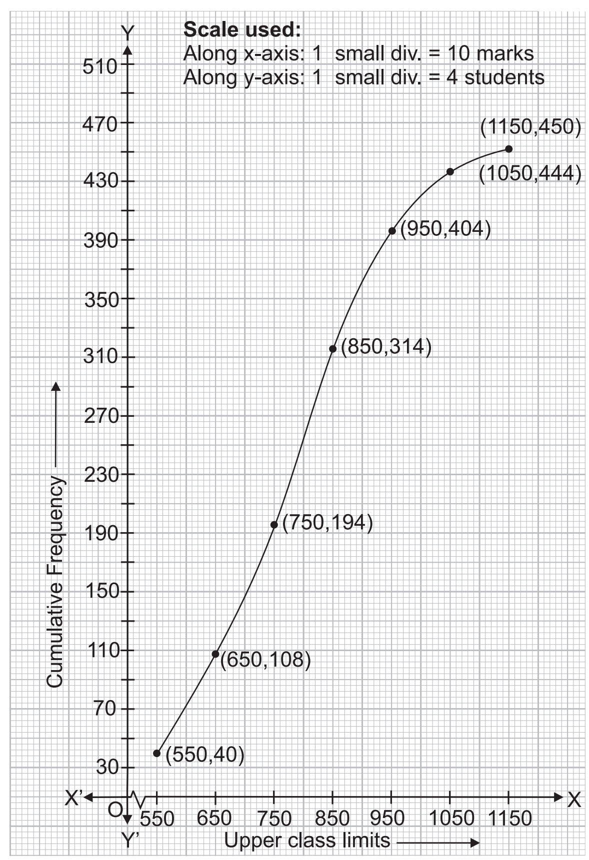
For Basic and Standard Levels

1. From the given distribution prepare the (less than type) cumulative frequency distribution as shown below:

Marks	*Number of students*
Less than 550	40
Less than 650	108
Less than 750	194
Less than 850	314
Less than 950	404
Less than 1050	444
Less than 1150	450

Upper class limit	550	650	750	850	950	1050	1150
Cumulative frequency	40	108	194	314	404	444	450

Plotting points **(550, 40), (650, 108), (750, 194), (850, 314), (950, 404), (1050, 444), (1150, 450)** and joining them by a free hand smooth curve, we get 'less than ogive' as shown in the following graph.



2. From the given table, we prepare a cumulative frequency distribution of the "less than type" as follows:

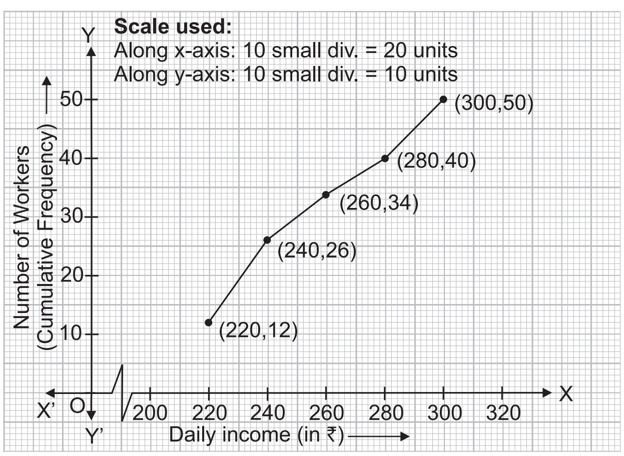
Daily income (in ₹)	*Number of workers (cumulative frequency)*
Less than 220	12
Less than 240	12 + 14 = 26
Less than 260	26 + 8 = 34
Less than 280	34 + 6 = 40
Less than 300	40 + 10 = 50

From the given data, we first prepare a cumulative frequency distribution of "the less than type".

Upper class limit	*Cumulative frequency*
220	12
240	26
260	34
280	40
300	50

$n = 50$

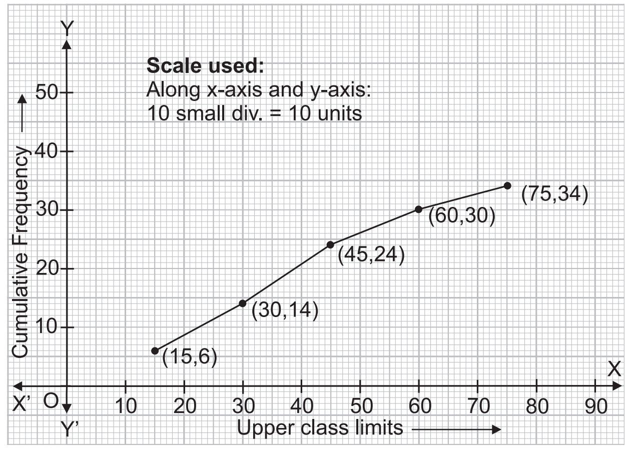
We now mark the upper class limit along the x-axis and the cumulative frequency along the y-axis on a suitable scale. Then, we plot the points (**220, 12**), (**240, 26**), (**260, 34**), (**280, 40**) and (**300, 50**) on a graph paper and join them by a freehand smooth curve to obtain ogive by 'less than' method.



3. From the given table, we prepare a cumulative frequency distribution of the 'less than type' as follows:

Upper class limit	*Cumulative frequency*
15	6
30	6 + 8 = 14
45	14 + 10 = 24
60	24 + 6 = 30
75	30 + 4 = 34 = n

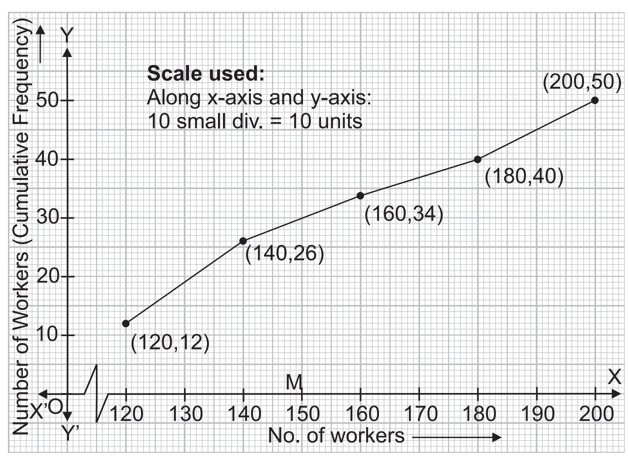
We now mark the upper class limit along the x-axis and the cumulative frequency along the y-axis on a suitable scale. Then, we plot the points **(15, 6), (30, 14), (45, 24), (60, 30)** and **(75, 34)** on a graph paper and join them by a freehand smooth curve to obtain ogive by 'less than' method.



4. From the given data, we prepare a cumulative frequency distribution of the 'less than type' as follows:

Upper class limit	*Cumulative frequency*
120	12
140	12 + 14 = 26
160	26 + 8 = 34
180	34 + 6 = 40
200	40 + 10 = 50 = n

We now mark the upper class limits along the x-axis and the cumulative frequencies along the y-axis on a suitable scale. Then, we plot the points **(120, 12), (140, 26), (160, 34), (180, 40)** and **(200, 50)** on a graph paper and join them by a freehand, smooth curve to obtain ogive by 'less than' method.

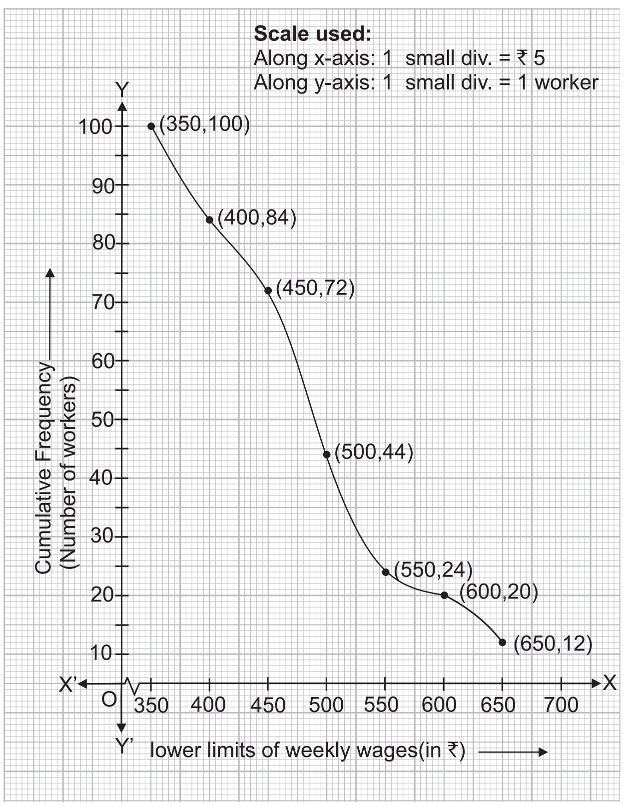


5. Converting the given distribution to a more than type distribution, we get

Weekly wages (in ₹)	*Number of workers (cumulative frequency)*
More than or equal to 350	100
More than or equal to 400	84
More than or equal to 450	72
More than or equal to 500	44
More than or equal to 550	24
More than or equal to 600	20
More than or equal to 650	12

Lower class limit	350	400	450	500	550	600	650
Cumulative frequency	100	84	72	44	24	20	12

Plotting the points **(350, 100), (400, 84), (450, 72), (500, 44), (550, 24), (600, 20), (650, 12)** and joining them by a a free hand curve, we get 'more than ogive' as shown in the following graph.



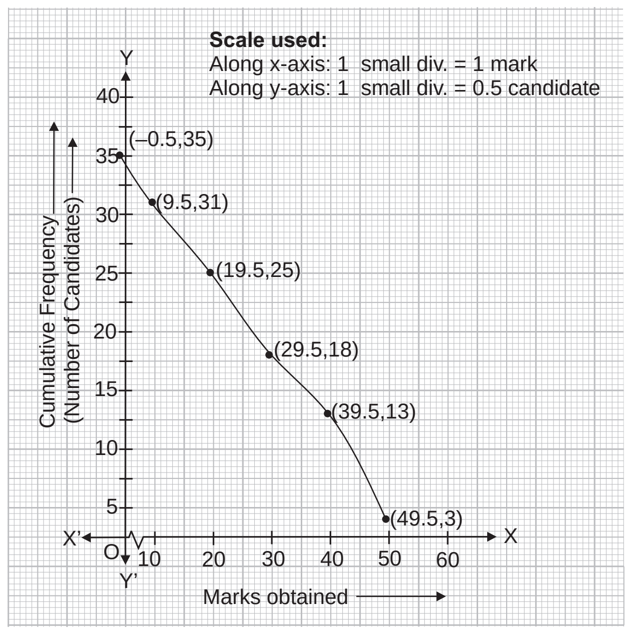
6. Converting the given distribution to an exclusive series and preparing the more than type cumulative frequency distribution, we get

Marks obtained	*Frequency*	*Marks more than or equal to*	*Cumulative frequency*
−0.5−9.5	4	−0.5	35
9.5−19.5	6	9.5	31
19.5−29.5	7	19.5	25
29.5−39.5	5	29.5	18
39.5−49.5	10	39.5	13
49.5−59.5	3	49.5	3

$\Sigma f_i = 35$

Lower class limit	−0.5	9.5	19.5	29.5	39.5	49.5
Cumulative frequency	35	31	25	18	13	3

Now, we plot the points **(−0.5, 35), (9.5, 31), (19.5, 25), (29.5, 18), (39.5, 13) (49.5, 3)** and join them by a free hand smooth curve to obtain ogive by more than method.



7. Less than type ogive
From the given distribution, we have

Upper class limits	150	155	160	165	170	175
Cumulative frequency	8	18	27	42	52	60

Plotting the points **(150, 8), (155, 18), (160, 27), (165, 42), (170, 52), (175, 60)** and joining them by a free hand curve, we get 'less than type ogive'.

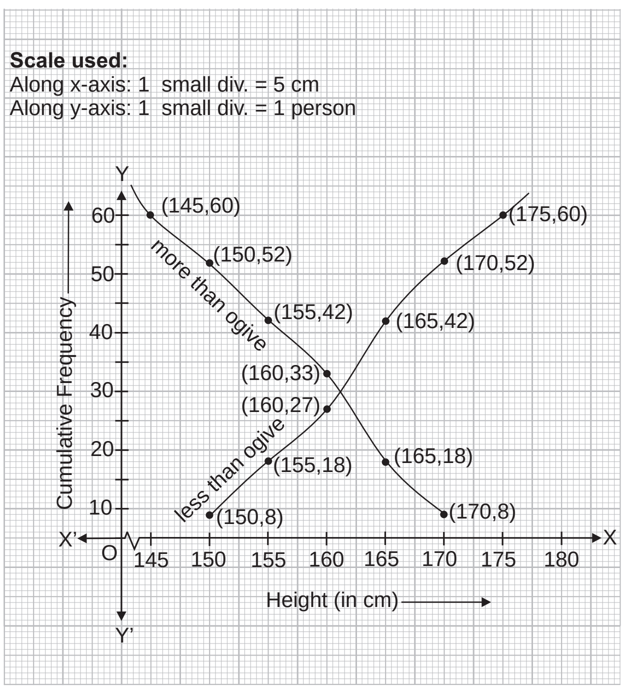
More than type ogive

From the given less than type 'distribution, prepare the more than type cumulative frequency distribution, as shown below:

Class interval	*Frequency*	*More than or equal to*	*Cumulative Frequency*
145 – 150	8	145	60
150 – 155	10	150	52
155 – 160	9	155	42
160 – 165	15	160	33
165 – 170	10	165	18
170 – 180	8	170	8

Lower class limits	145	150	155	160	165	170
Cumulative frequency	60	52	42	33	18	8

Plotting the points **(145, 60), (150, 52), (155, 42), (160, 33), (165, 18), (170, 8)** and joining them by a free hand smooth curve, we get 'more than type ogive'.



8. From the given table, we first prepare a cumulative frequency distribution of the less than type

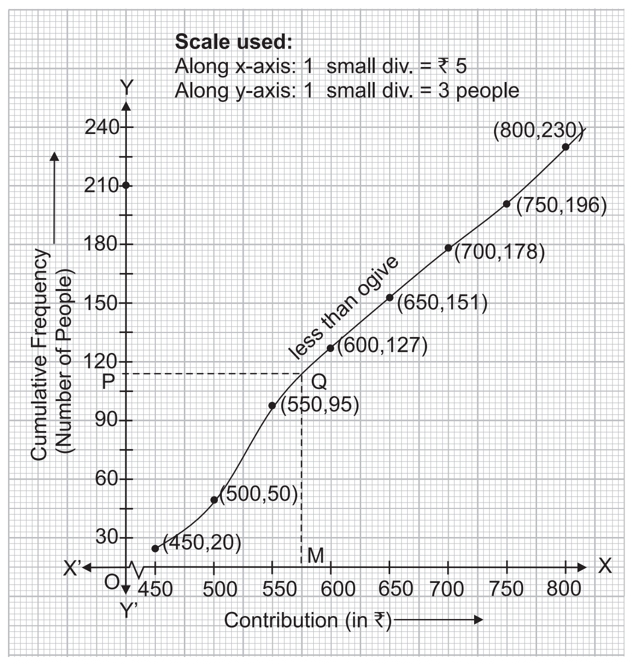
Contribution (in ₹)	*Number of people (cumulative frequency)*
Less than 450	20
Less than 500	55
Less than 550	95
Less than 600	127
Less than 650	151
Less than 700	178
Less than 750	196
Less than 800	230

Upper class limits	450	500	550	600	650	700	750	800
Cumulative frequency	20	55	95	127	151	178	196	230

Plotting the points **(450, 20), (500, 55), (550, 95), (600, 127), (650, 151), (700, 178), (750, 196), (800, 230)** and joining them by a free hand curve, we get less than type ogive.

Here, $n = 230$. So, $\frac{n}{2} = \frac{230}{2} = 115$

Take a point P(0, 115) on the y-axis and draw PQ parallel to the x-axis and let it meet the ogive at Q. Draw QM $\perp$ x-axis, and let it intersect the x-axis at point M.



The x-coordinate of M is 575.

Hence, median = ₹ **575**

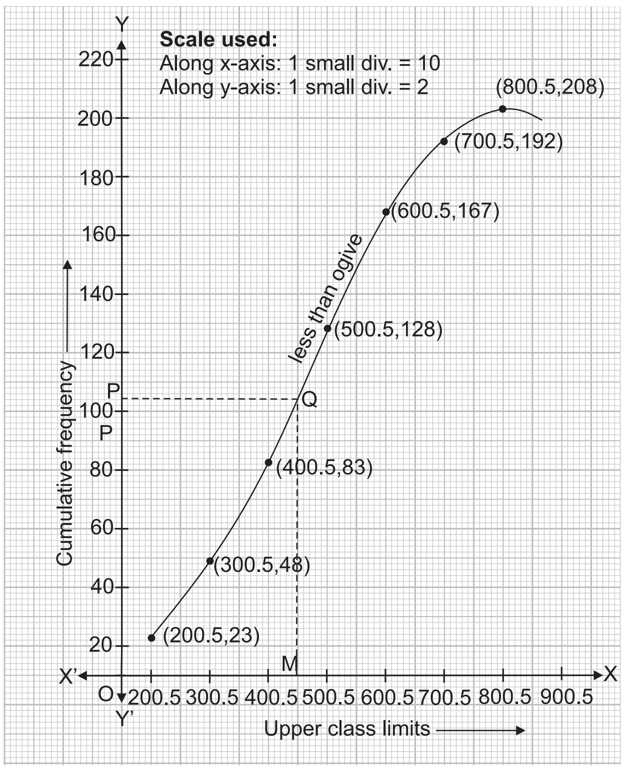
9. From the given inclusive series to an exclusive series and preparing the cumulative frequency table we get

Class interval	*Frequency*		*Cumulative frequency*
100.5 – 200.5	23	Less than 200.5	23
200.5 – 300.5	25	Less than 300.5	48

300.5 – 400.5	35	Less than 400.5	83
400.5 – 500.5	45	Less than 500.5	128
500.5 – 600.5	39	Less than 600.5	167
600.5 – 700.5	25	Less than 700.5	192
700.5 – 800.5	16	Less than 800.5	208

Upper class limit	200.5	300.5	400.5	500.5	600.5	700.5	800.5
Cumulative frequency	23	48	83	128	167	192	208

Plotting the points **(200.5, 23), (300.5, 48), (400.5, 83), (500.5, 128), (600.5, 167), (700.5, 192), (800.5, 208)** and joining them by a free hand smooth curve, we get 'less than ogive' as shown in the graph.



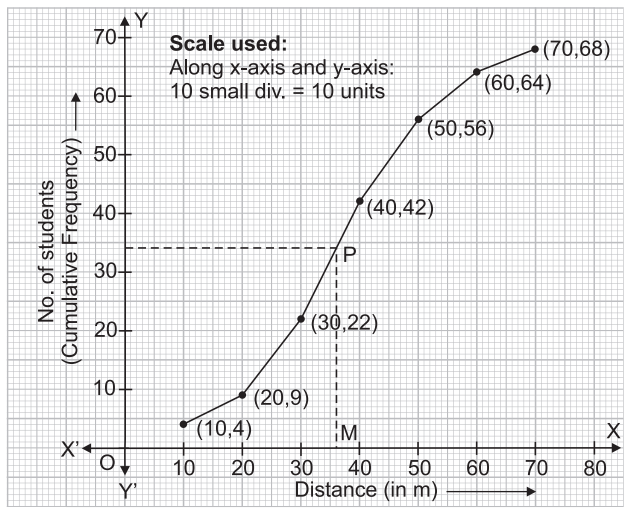
Here, n = 208. So, $\frac{n}{2}$ = 104.

Locate a point P(0, 104) on the y-axis and draw PQ parallel to the x-axis and let it meet the ogive at Q. Draw QM $\perp$ x-axis and let it intersect the x-axis at M. The x-coordinate of M is 447 units.
Hence, Median = **447.**

10. From the given table, we prepare a cumulative frequency distribution of the 'less than type' as follows:

Upper class limit	*Cumulative frequency*
10	4
20	4 + 5 = 9
30	9 + 13 = 22
40	22 + 20 = 42
50	42 + 14 = 56
60	56 + 8 = 64
70	64 + 4 = 68 = n

We now mark the upper class limits along the x-axis and the cumulative frequencies along the y-axis on a suitable scale. Then, we plot the points **(10, 4), (20, 9), (30, 22), (40, 42), (50, 56), (60, 64)** and **(70, 68)** on a graph paper and join them by a freehand smooth curve to obtain ogive by 'less than' method.



Calculation of median from the 'less than' ogive:

From the graph, the total of all frequencies = n = 68.

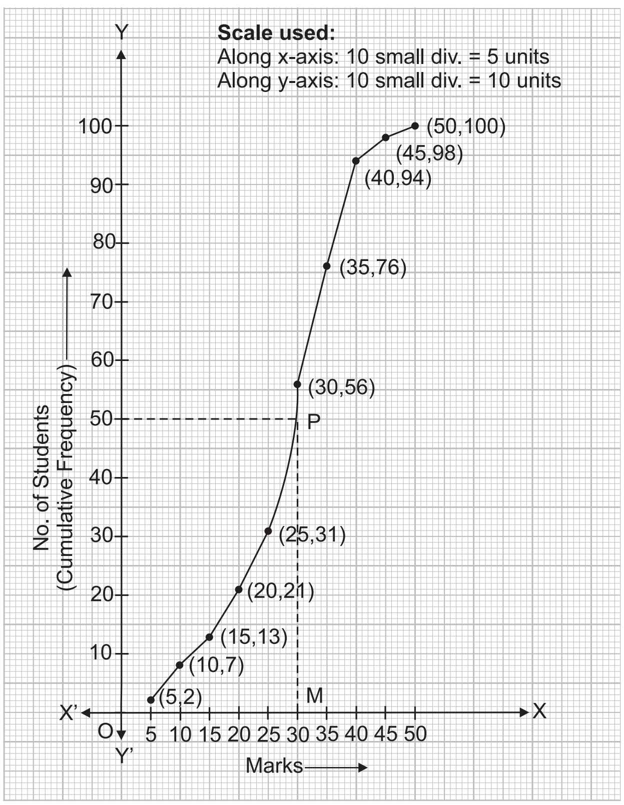
$$\therefore \quad \frac{n}{2} = \frac{68}{2} = 34.$$

From the point 34 on the y-axis, we draw a line parallel to the x-axis to cut the ogive at a point P(*say*). From P, we drawn another line perpendicular to the x-axis at a point M(*say*). The point M represents the median. Since the abscissa of the point M is 36. Hence, the required median is **36 m**.

11. From the given table, we prepare a cumulative frequency distribution of the 'less than type' as follows:

Upper class limit	*Cumulative frequency*
5	2
10	2 + 5 = 7
15	7 + 6 = 13
20	13 + 8 = 21
25	21 + 10 = 31
30	31 + 25 = 56
35	56 + 20 = 76
40	76 + 18 = 94
45	94 + 4 = 98
50	98 + 2 = 100 = n

We now mark the upper class limits along the x-axis and the cumulative frequencies along the y-axis on a suitable scale. Then, we plot the points **(5, 2), (10, 7), (15, 13), (20, 21), (25, 31), (30, 56), (35, 76), (40, 94), (45, 98)** and **(50, 100)** on a graph paper and join them by a freehand smooth curve to obtain ogive by 'less than' method.



Calculation of median from the 'less than ogive':

From the graph, the total of all frequencies = n = 100.

$$\therefore \qquad \frac{n}{2} = \frac{100}{2} = 50.$$

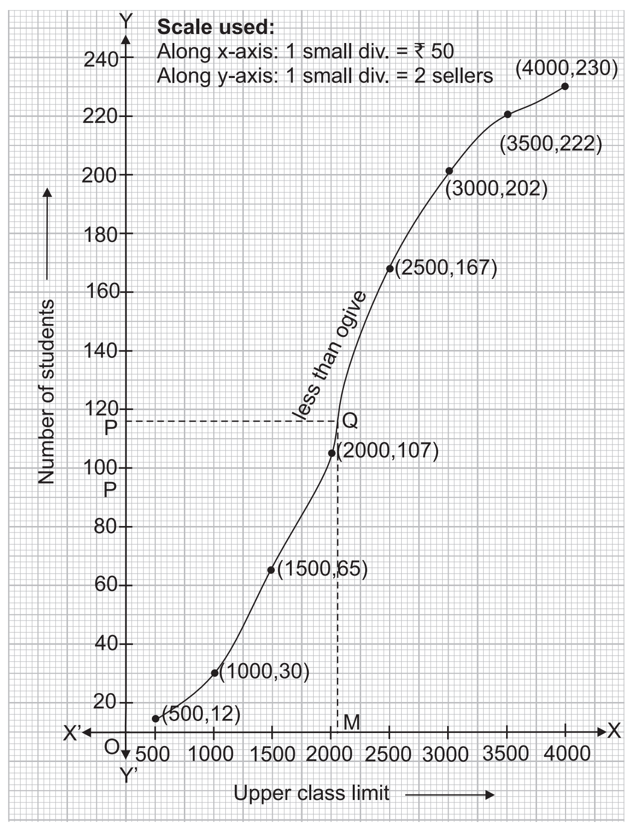
From the point 50 on the y-axis, we draw a line parallel to the x-axis to cut the ogive at a point P(*say*). From P, we draw another line perpendicular to the x-axis at a point M(*say*). The point M represents the median. Since the abscissa of the point M is 29 (approx.), hence the required median is **29 (approx.)**.

12. Converting the given distribution to less than type cumulative frequency distribution as shown below:

Sales in (₹)	*Cumulative frequency*
Less than ₹ 500	12
Less than ₹ 1000	30
Less than ₹ 1500	65
Less than ₹ 2000	107
Less than ₹ 2500	157
Less than ₹ 3000	202
Less than ₹ 3500	222
Less than ₹ 4000	230

Upper class limit	500	1000	1500	2000	2500	3000	3500	4000
Cumulative frequency	12	30	65	107	157	202	222	230

Plotting the points **(500, 12), (1000, 30), (1500, 65), (2000, 107), (2500, 157), (3000, 202), (3500, 222), (4000, 230)** and joining them by a free hand curve, we get 'less than ogive' as shown in the following graph.



Here, $n = 230$. So, $\frac{n}{2} = \frac{230}{2} = 115$.

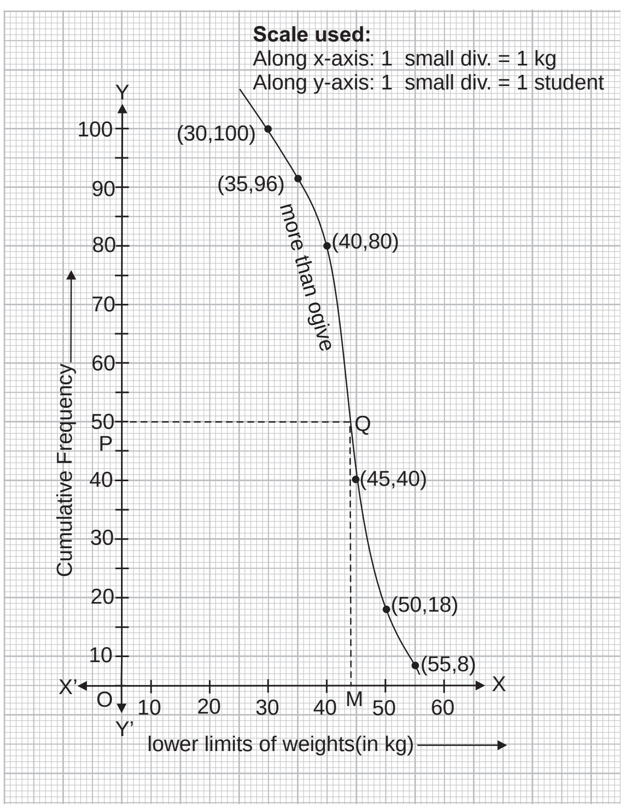
Locate a point P(0, 115) on the y-axis and draw PQ parallel to the x-axis and let it meet the ogive at Q. Draw QM $\perp$ x-axis and let it intersect the x-axis at M. The x-coordinate of M is 2080 units.
Hence, median = **2080**.

13. Converting the given distribution to more than type distribution, we get

Weight (in kg)	*Number of students Cumulative frequency*
More than or equal to 30	100
More than or equal to 35	96
More than or equal to 40	80
More than or equal to 45	40
More than or equal to 50	18
More than or equal to 55	8

Lower class limits	30	35	40	45	50	55
Cumulative frequency	100	96	80	40	18	8

Plotting the points **(30, 100), (35, 96), (40, 80), (45, 40), (50, 18), (55, 8)** and joining them by a free hand curve, we get more than ogive as shown in the following graph.



Locate point P on y-axis corresponding to $\frac{n}{2}$,

i.e. $\frac{100}{2} = 50$.

From P draw a line parallel to the x-axis cutting the curve at Q. Draw QM $\perp$ x-axis and let it intersect the x-axis at M.
The x-coordinate is 44. Hence, the median is **44 kg.**

For Standard Level

14. Less than ogive:
From the given table, we first prepare a cumulative frequency distribution of less than type.

Weekly wages in (₹)	*Cumulative frequency*
Less than 20	41
Less than 40	92
Less than 60	156
Less than 80	194
Less than 100	201

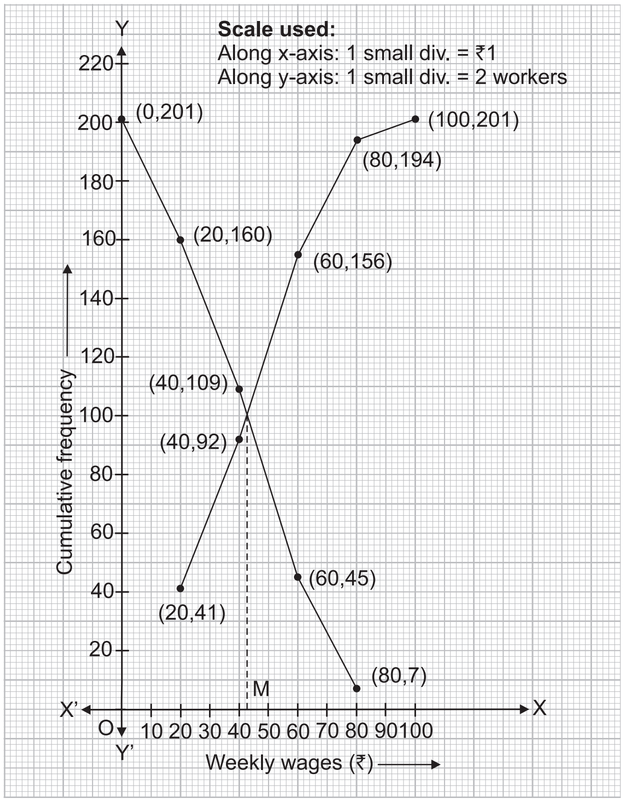
Upper class limits	20	40	60	80	100
Cumulative frequency	41	92	156	194	201

Plotting the points **(20, 41), (40, 92), (60, 156), (80, 194), (100, 201)** and joining them by a free hand curve we get 'less than ogive'.
More than ogive:
From the given table, we prepare a cumulative frequency distribution of more than type.

Weekly wages (in ₹)	*Number of works*
More than or equal to 0	201
More than or equal to 20	160
More than or equal to 40	109
More than or equal to 60	45
More than or equal to 80	7

Lower class limits	0	20	40	60	80
Cumulative frequency	201	160	109	45	7

Plotting the points **(0, 201), (20, 160), (40, 109), (60, 45), (80, 7)** and joining them by a free hand curve, we get 'more than ogive'.
From the point at which the two ogives intersect draw a perpendicular on the x-axis.
The point at which this perpendicular cuts the x-axis, gives the median.
Hence, median weekly wage = **₹ 42.65**



15. More than ogive

Lower class limits	0	5	10	15	20	25	30
Cumulative frequency	80	75	60	40	30	20	5

Plotting the points **(0, 80), (5, 75), (10, 60), (15, 40), (20, 30), (25, 20), (30, 5)** and joining them by a free hand smooth curve we get 'more than ogive'.

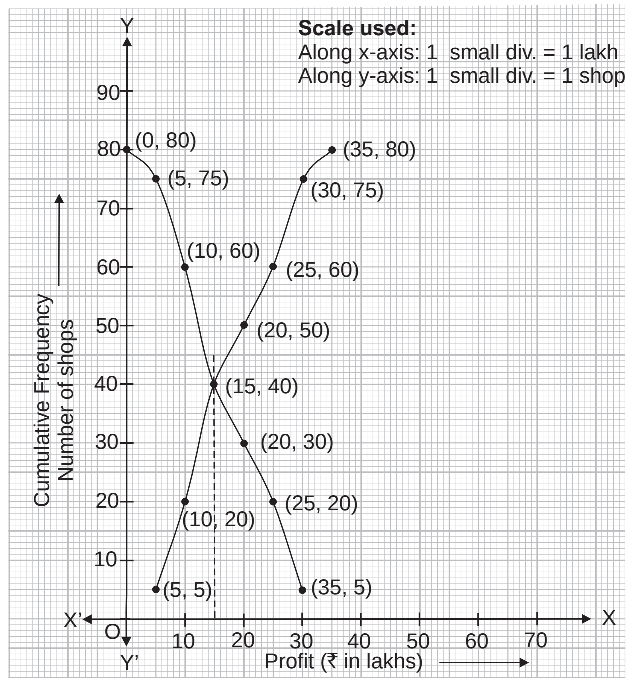
Less than ogive

From the given distribution we construct frequency table consisting of simple frequency and cumulative frequencies as shown below.

Profit (in ₹ lakhs)	*Frequency*	*Cumulative frequency*
0–5	5	5
5–10	15	20
10–15	20	40
15–20	10	50
20–25	10	60
25–30	15	75
30–35	5	80

Upper class limits	5	10	15	20	25	30	35
Cumulative frequency	5	20	40	50	60	75	80

Plotting the points **(5, 5), (10, 20), (15, 40), (20, 50), (25, 60), (30, 75), (35, 80)** and joining them by a free hand smooth curve, we get 'less than ogive'.



From the point at which the two ogives intersect, drop a perpendicular on the x-axis. The point at which this perpendicular cuts the x-axis gives the median

Hence, median profit = ₹ **15 lakhs**

Verification (by using formula)

Profit (₹ in lakhs)	*Number of shops f_1*	*Cumulative frequency*
0–5	5	5
5–10	15	20
10–15	20	40
15–20	10	50
20–25	10	60
25–30	15	75
30–35	5	80

$$n = \Sigma f_i = 80$$

Here, $n = \Sigma f_i = 80$. So, $\frac{n}{2} = \frac{80}{2} = 40$.

Class whose cumulative frequency is greater than (and nearest to) $\frac{n}{2} = 40$ is 15–20.

So the median class is 15–20.

$\therefore$ l = 15, cf = 40, f = 10 and h = 5

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

$$= 15 + \left(\frac{40 - 40}{10}\right) \times 5$$

$$= 15 + 0$$
$$= 15$$

$\Rightarrow$ Median profit = ₹ 15 lakhs

Hence, by using the formula, it is verified that the median profit = ₹ **15 lakhs**.

16. From the given table, we first prepare a cumulative frequency distribution of the 'less than' type as follows:

Upper class limit	*Cumulative frequency*
10	22
20	22 + 10 = 32
30	32 + 8 = 40
40	40 + 15 = 55
50	55 + 5 = 60
60	60 + 6 = 66 = n

We now mark the upper class limits along the x-axis and the cumulative frequencies along the y-axis on a suitable scale. Then, we plot the points **(10, 22), (20, 32), (30, 40), (40, 55), (50, 60)** and **(60, 66)** on a graph paper and join them by a freehand smooth curve to obtain ogive by 'less than' method.

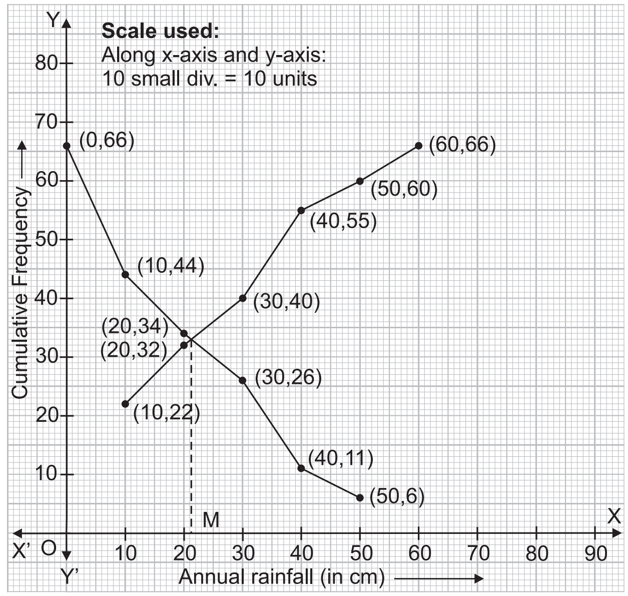
More than ogive:

From the given table, we prepare the 'more than type' cumulative frequency table as shown below:

Lower class limit	*Cumulative frequency*
0	66
10	66 – 22 = 44
20	44 – 10 = 34
30	34 – 8 = 26
40	26 – 15 = 11
50	11 – 5 = 6

More than ogive:

We now mark the lower class limits along the x-axis and the cumulative frequencies along the y-axis on a suitable scale. Then, we plot the points **(0, 66), (10, 44), (20, 34), (30, 26), (40, 11)** and **(50, 6)** on a graph paper and join them by a freehand smooth curve to obtain 'more than type' cumulative frequency curve (also called 'more than type' ogive.)



From the point at which the two ogives intersect, drop a perpendicular on the x-axis. The point at which this perpendicular cuts the x-axis gives the median. Hence, median rainfall is **21.25 cm**.

CHECK YOUR UNDERSTANDING

MULTIPLE-CHOICE QUESTIONS

For Basic and Standard Levels

1. (*d*) **Standard deviation**

Mean, Median and mode are measures of central tendency but standard deviation is not a measure of central tendency.

2. (*a*) $\boldsymbol{x + 6}$

$$\text{Mean} = \frac{\text{sum of observations}}{\text{number of observations}}$$

$$= \frac{x + x + 3 + x + 6 + x + 9 + x + 12}{5}$$

$$= \frac{5x + 30}{5}$$

$$= x + 6$$

3. (*c*) **105**

Here, each observation has been increased by 100.

$\therefore$ Mean will also increase by 100.

Hence, mean = 105

4. (*b*) $\boldsymbol{\frac{n+1}{2}}$

Sum of n natural numbers = $\frac{n(n+1)}{2}$

$\therefore$ Mean of n natural numbers

$$= \frac{n(n+1)}{2} \div n$$

$$= \frac{n(n+1)}{2} \times \frac{1}{n}$$

$$= \frac{n+1}{2}$$

5. (*c*) **17.5, 45**

Class marks of class 10–25 = $\frac{10 + 25}{2} = \frac{35}{2} = 17.5$

Class marks of class 35–55 = $\frac{35 + 55}{2} = \frac{90}{2} = 45$

Hence, class marks of classes 10 – 25 and 35 – 55 respectively are 17.5, 45.

6. (*d*) **centred at the class marks of the classes**

$\because \quad \bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i}$

where x_i represent the class marks (mid-values).

7. (*a*) **mid-points of the classes**

d_i s are deviation of assumed mean a from mid-points of the classes

$$d_i = x_i - a$$

8. (*b*) $\boldsymbol{\frac{x_i - a}{h}}$

In the formula $\bar{x} = a + h\,\frac{\Sigma f_i u_i}{\Sigma f_i}$

where $u_i = \frac{x_i - a}{h}$

9. (*c*) **Maximum frequency**

Mode is the value of the variable which has maximum frequency.

$\therefore$ It is the observation which occurs most number of times in the given data.

10. (*b*) **45**

Since 43 is the mode

$\therefore$ $x = 43$ and $x + 2 = 45$

11. (*c*) **mode**

Since we need to find out the consumer item which is maximum in demand, mode would be best suited to determine its demand.

12. (*a*) **2**

Since observation 2 occurs most number of times (5 times)

$\therefore$ The mode of the data is 2.

13. (*d*) **17**

Here $n = 6$ (even) so the median is average value of $\frac{n}{2}$ th observation and $\left(\frac{n}{2}+1\right)$ th observation

$\Rightarrow$ 16 = average value of 3rd and 4th observation

$\Rightarrow \quad 16 = \frac{x - 2 + x}{2}$

$\Rightarrow \quad 32 = 2x - 2$

$\Rightarrow \quad 2x = 34$

$\Rightarrow \quad x = 17$

14. (*b*) **3.5, 5**

Here $n = 18$ (even). So the median = average value of $\frac{18}{2}$ th (i.e. 9th) term and $\left(\frac{18}{2}+1\right)$ th, i.e. 10th term

$\therefore \quad \text{Median} = \frac{3 + 4}{2} = \frac{7}{2} = 3.5$

Mode = 5

($\because$ 5 occurs maximum number of times, i.e. 4 times)

Thus, the median and mode respectively are 3.5, 5

15. (*b*) **47**

Total number of observations = 8 + 4 + 8 = 20.

The middle 4 terms arranged in increasing order are 39, 43, 51, 69.

$\therefore$ Median = average value of $\frac{20}{2}$ th term, i.e. 10th term and $\left(\frac{20}{2}+1\right)$ th term = 11th term.

$\therefore \quad \text{Median} = \frac{43 + 51}{2} = \frac{94}{2} = 47$

16. (*a*) $\boldsymbol{x - \frac{5}{2}}$

Here $n = 8$ (even)

$\therefore$ Median = average value of $\frac{8}{2}$ = 4th and $\frac{8}{2}$ + 1 = 5th term (of the data arranged in ascending or descending order)

$$= \frac{1}{2}(2x - 5) = x - \frac{5}{2}$$

17. (*c*) **25**

Here $n = 8$

$\therefore$ Median = average value of $\frac{n}{2}$ th, i.e. $\frac{8}{2}$ th = 4th term and $\left(\frac{n}{2}+1\right)$ th, i.e. $\left(\frac{8}{2}+1\right)$ th term = 5th term

$$= \frac{x + 2 + x + 3}{2}$$

$$= \frac{2x + 5}{2}$$

$$= x + 2.5$$

$= 27.5$ [Given]

$\Rightarrow \quad x = 25$

18. (*b*) **30–40**

Class interval	*Frequency*	*Cumulative frequency*
0–10	6	6
10–20	8	14
20–30	7	21
30–40	9	30
40–50	14	44

Here, $n = 44$. So, $\frac{n}{2} = \frac{44}{2} = 22$.

Cumulative frequency just greater than 22 is 30 and the corresponding class is 30–40.

So, the median class is 30–40.

19. (*c*) **51**

Class interval	*Frequency*	*Cumulative frequency*
30–35	14	14
35–40	16	30
40–45	18	48
45–50	23	71
50–55	18	89
55–60	8	97
60–56	3	100

Here n = 100. So, $\frac{n}{2} = \frac{100}{2} = 50$.

Cumulative frequency just greater than 50 is 71 and the corresponding class is 45–50.
Maximum frequency is 23 and the corresponding class is 45–50.
So, the modal class is 45–50

∴ The difference of the upper limit of the median class and the lower limit of the modal class is
50 – 45 = 5

20. (*d*) **an ogive**
Median of a given frequency distribution is found graphically with the help of an ogive

21. (*a*) **55**
Here n = 80.
So, $\frac{n}{2} = \frac{80}{2} = 40$. Take a point P(0, 40) on the y-axis and draw PQ ∥ x-axis, so as to meet the graph at Q.
Draw QM ⊥ x-axis.
Median marks are given by the x-coordinate of M i.e 55
Hence, median = 55

22. (*d*) **7**

$$3 \text{ Median} = \text{Mode} + 2 \text{ Mean}$$
$$3 \text{ Median} = 7 + 2\,(7) = 21$$
$$\Rightarrow \quad \text{Median} = \frac{21}{3} = 7$$

23. (*d*) **51.8**

$$3 \text{ Median} = \text{Mode} + 2 \text{ Mean}$$
$$3\,(52) = 52.4 + 2 \text{ Mean}$$
$$\Rightarrow \quad 2 \text{ Mean} = 156 - 52.4 = 103.6$$
$$\Rightarrow \quad \text{Mean} = \frac{103.6}{2} = 51.8$$

24. (*b*) **6**

$$\text{Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i}$$
$$\Rightarrow \quad 8.1 = \frac{132 + 5p}{20}$$
$$\Rightarrow \quad 162 - 132 = 5p$$
$$\Rightarrow \quad 5p = 30$$
$$\Rightarrow \quad p = 6$$

25. (*c*) **12**
The first ten prime numbers are
2, 3, 5, 7, 11, 13, 17, 19, 23, 29
Here n = 10 (even).

$$\therefore \quad \text{Median} = \text{average value of } \frac{n}{2}\text{th term and } \left(\frac{n}{2}+1\right)\text{th term}$$
$$= \text{average value of 5th term and 6th term}$$
$$= \frac{11 + 13}{2} = 12$$

For Standard Level

26. (*b*) $\bar{x} + \frac{n+1}{2}$

Let the observations be $x_1, x_2, x_3 \ldots x_n$
New observations are $x_1 + 1, x_2 + 2, x_3 + 3 \ldots x_n + n$
Sum of new observations

$$= x_1 + 1 + x_2 + 2, + x_3 + 3 \ldots + x_n + n$$
$$= x_1 + x_2 + x_3 \ldots + x_n + 1 + 2 + 3 \ldots + n$$

Mean of new observations

$$= \frac{x_1 + x_2 + x_3 \ldots + x_n + 1 + 2 + 3 \ldots + n}{n}$$
$$= \frac{x_1 + x_2 + x_3 \ldots + x_n}{n} + \frac{1 + 2 + 3 + \ldots + n}{n}$$
$$= \bar{x} + \frac{n(n+1)}{2n}$$
$$= \bar{x} + \frac{n+1}{2}$$

27. (*a*) ₹ **1450**

Mean monthly salary of 10 members = ₹ 1445
Total monthly salary of 10 members = ₹ 14450
Total monthly salary of 11 members = ₹ (14450 + 1500)
= ₹ 15950

$$\therefore \quad \text{Mean monthly salary of 11 members} = ₹\ \frac{15950}{11}$$
$$= ₹\ 1450$$

28. (*c*) **11**

Mean of 6 numbers = 16
∴ Total of 6 numbers = 16 × 6 = 96
Mean of 5 numbers = 17
∴ Total of 5 numbers = 17 × 5 = 85
Number removed = 96 – 85 = 11

29. (*b*) **7**
Original set of observations is 45, 49, 52, 53, 67, 77, 81, 99
n = 8 (even)

$$\therefore \text{ Median} = \text{average value of } \frac{n}{2}\text{th term and } \left(\frac{n}{2}+1\right)\text{th term}$$
$$\Rightarrow \text{ Average value of } \frac{8}{2}\text{, i.e 4th and } \left(\frac{8}{2}+1\right)\text{, i.e.}$$

5th term

$$\Rightarrow \quad \frac{53 + 67}{2} = \frac{120}{2} = 60$$

New set of observations is 45, 49, 52, 53, 67, 77, 81, 89, 99

$n = 9$ (odd)

$\therefore$ Median = value of $\left(\frac{n+1}{2}\right)$th term

= 5th term

= 67

$\therefore$ Median increases by 67 – 60 = 7

30. (*a*) **37**

The number of students who get less than 30 marks = 2 + 10 + 25 = 37

31. (*d*) **145**

Class interval	*Frequency*
135–140	4
140–145	7
145–150	18
150–155	11
155–160	6
160–165	5

Maximum frequency is 18 and the class corresponding to this frequency is 145–150.

So, the modal class is 145–150.

Thus, the lower limit of the modal class is 145.

32. (*b*) **median**

The *x*-coordinate of the point of intersection of 'more than ogive' and 'less than ogive' gives the median.

33. (*c*) **4**

Median is given by the *x*-coordinate of the point of intersection of more than ogive and less than ogive and in the given graph it is 4.

34. (*c*) **6**

$$p(\text{median} - \text{mean}) = 2\,(\text{mode} - \text{mean})$$

$$\Rightarrow \quad p = \frac{2\,(\text{mode} - \text{mean})}{(\text{median} - \text{mean})}$$

$$= \frac{2\,(3\text{ median} - 2\text{ mean} - \text{mean})}{(\text{median} - \text{mean})}$$

$$= \frac{2\,(3\text{ median} - 3\text{ mean})}{(\text{median} - \text{mean})}$$

$$= \frac{6\,(\text{median} - \text{mean})}{(\text{median} - \text{mean})}$$

$$= 6$$

35. (*c*) **30**

Mean of 11 observations = 30

$\therefore$ Total of 11 observations = 30 × 11 = 330

Mean of first 6 observations = 28

$\therefore$ Total of first 6 observations = 28 × 6 = 168

Mean of last 6 observations = 32

$\therefore$ Total of last 6 observations = 32 × 6 = 192

$\therefore$ 6th observation = 168 + 192 – 330

= 360 – 330

= 30

36. (*b*) **52**

Class interval	*Frequency*
0–20	15
20–40	6
40–60	18
60–80	10

Maximum frequency is 18 and the corresponding class is 40 – 60.

So, the modal class is 40 – 60.

$\therefore$ $l = 40$, $f_1 = 18$, $f_0 = 6$, $f_2 = 10$ and $h = 20$

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 40 + \left(\frac{18 - 6}{2 \times 18 - 6 - 10}\right) \times 20$$

$$= 40 + \left(\frac{12}{20}\right) \times 20$$

$$= 40 + 12$$

$$= 52$$

37. (*c*) **26**

Class-interval	*Frequency*	*Cumulative frequency*
0–8	8	8
8–16	10	18
16–24	16	34
24–32	24	58
32–40	15	73
40–48	7	80

Here, $n = 80$. So, $\frac{n}{2} = \frac{80}{2} = 40$.

Cumulative frequency just greater than 40 is 58 and the class corresponding to this frequency is 24–32.

So, the class is 24–32.

$\therefore$ $l = 24$, $cf = 34$, $f = 24$ and $h = 8$

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

$$= 24 + \left(\frac{40 - 34}{24}\right) \times 8$$

$$= 24 + \left(\frac{6}{24}\right) \times 8$$

$$= 24 + 2$$

$$= 26$$

38. (*d*) **7**

$$\frac{1+3+4+5+7+4}{6} = m$$

$\Rightarrow$ $$\frac{24}{6} = m$$

$\Rightarrow$ $m = 4$ ... (1)

and $$\frac{3+2+2+4+3+3+p}{7} = m - 1$$

$\Rightarrow$ $$\frac{17+p}{7} = 4 - 1$$ [Using (1)]

$\Rightarrow$ $$\frac{17+p}{7} = 3$$

$\Rightarrow$ $17 + p = 21$

$\Rightarrow$ $p = 21 - 17$

$\Rightarrow$ $p = 4$ ... (2)

Median of 2, 2, 3, 3, 4 , 4 is q.

$\Rightarrow$ $q = 3$ (median) ... (3)

$\therefore$ $p + q = 4 + 3 = 7$ [Using (2) and (3)]

39. (*b*) **20**

$(x_1 - 50) + (x_2 - 50) + (x_3 - 50) + \ldots + (x_n - 50) = -10$

$\Rightarrow$ $(x_1 + x_2 + x_3 \ldots + x_n) - 50n = -10$ ... (1)

$(x_1 - 46) + (x_2 - 46) + (x_3 - 46) + \ldots + (x_n - 46) = 70$

$\Rightarrow$ $(x_1 + x_2 + x_3 + \ldots + x_n) - 46n = 70$ ... (2)

Subtracting (1) from (2), we get

$4n = 80$

$\Rightarrow$ $n = 20$

40. (*b*) $\boldsymbol{x = 5, y = 8, z = 60}$

$x = 5$, $y = 58 - 50 = 8$ and $z = 58 + 2 = 60$

Hence, $x = 5$, $y = 8$, $z = 60$.

SHORT ANSWER QUESTIONS

For Basic and Standard Levels

1. (*i*) It is not necessary to always use the formula $\bar{x} = a + \frac{\Sigma f_i d_i}{\Sigma f_i}$ because the mean of the data does not depend on the choice of assumed mean 'a'.

(*ii*) The mean of ungrouped data and the mean calculated when the data is grouped are not always the same because in grouped data, it is assumed that the frequency of each class is centred at the mid-point of the class.

2.

Class interval	*Frequency*
0–4	4
4–8	8
8–12	5
12–16	6

Maximum frequency is 8 and the class corresponding this frequency is 4–8.

So, the modal class is 4–8.

$\therefore$ $l = 4, f_1 = 8, f_0 = 4, f_2 = 5$ and $h = 4$

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 4 + \left(\frac{8-4}{2 \times 8 - 4 - 5}\right) \times 4$$

$$= 4 + \frac{4}{7} \times 4$$

$$= 4 + \frac{16}{7}$$

$$= 4 + 2.285 = \mathbf{6.285}$$

3. Here $n = 8$ (even)

$\therefore$ Median = average value of $\frac{n}{2}$th term and $\left(\frac{n}{2}+1\right)$th term

Median = average value of $\frac{8}{2}$ th term and $\left(\frac{8}{2}+1\right)$th

= average value of 4th term and 5th term

$$= \frac{24 - x + 22 + 2x}{2}$$

$$= \frac{46 + x}{2} = 24$$ [Given]

$\Rightarrow$ $46 + x = 48$

$\Rightarrow$ $x = 48 - 46 = 2$

Hence, $\boldsymbol{x = 2}$.

4.

Class interval	*Number of labourers frequency*	*Cumulative frequency*
200–300	3	3
300–400	5	8
400–500	20	28
500–600	10	38
600–700	6	44

Here, $n = 44$.

So, $\frac{n}{2} = \frac{44}{2} = 22$.

Cumulative frequency just greater than 22 is 28 and the corresponding class is 400–500.

So, the median class is 400–500.

$\therefore$ $l = 400$, $cf = 8$, $f = 20$ and $h = 100$

$$\text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right)$$

$$= 400 + \left(\frac{22-8}{20}\right) \times 100$$

$$= 400 + \left(\frac{14}{20}\right) \times 100$$

$$= 400 + 70 = 470$$

Hence, median wage = ₹ **470**

For Standard Level

5. Mean weight of 35 students = 45 kg

$\therefore$ Total weight of 35 students = (45 × 35) kg = 1575 kg

Mean weight of 35 students plus one teacher = 45.5 kg

Total weight of 35 students plus one teacher

= [45.5 × (35 + 1)] kg

= (45.5 × 36) kg = 1638

$\therefore$ Weight of the teacher = (1638 – 1575) kg

= **63 kg**

UNIT TEST 1

For Basic Level

1. (*b*) $\boldsymbol{y = 19 - x}$

$$\frac{6+7+x+8+y+14}{6} = 9$$

$$\Rightarrow \quad 35 + x + y = 54$$
$$\Rightarrow \quad x + y = 54 - 35$$
$$\Rightarrow \quad x + y = 19$$
$$\Rightarrow \quad y = 19 - x$$

2. (*a*) **5**

Mode = 5

$\because$ 5 occurs maximum number of times, i.e. 5 times.

3. (*c*) **21**

Here $n = 9$

$\therefore$ Median = value of $\frac{9+1}{2}$ th term

= value of 5th term = 21

4. (*a*) **not increase**

In a data of 12 numbers arranged in increasing order, the median is the average value of 6th and 7th term. so any change in 9th entry will not affect the median Hence, the median will not increase.

5. (*d*) **0.5**

Median of given observations = $\frac{36+37}{2} = \frac{73}{2} = 36.5$

If 35 is taken out, then median = 37

Increase in median = 37 – 36.5 = 0.5

Hence, the median increases by 0.5.

6. (*a*) **35–40**

35–40 has maximum frequency of 52.

Hence, the modal class for the given distribution is 35–40.

7. (*b*) **median**

Construction of a cumulative frequency table is useful in determining the median

$$\because \quad \text{Median} = l + \left(\frac{\frac{n}{2} - cf}{f}\right) \times h$$

8. **40**

The class 40–50 has maximum frequency.

$\therefore$ The modal class for the given distribution is 40–50.

Hence, the lower limit of the modal class is 40.

9. **13th observation**

Since the number of observations of the data arranged in descending order is 25 which is odd, hence, $\frac{25}{2}$ th, i.e. 13th observation in the required median.

10. **17.5, 45**

The required class marks of the classes 10–25 and 35–55 are respectively $\frac{10+25}{2} = \frac{35}{2} = 17.5$ and $\frac{35+55}{2} = \frac{90}{2} = 45$.

11.

0 or more than 0	**70**
20 or more than 20	70 – 7 = **63**
40 or more than 40	63 – 12 = **51**
60 or more than 60	51 – 23 = **28**
80 or more than 80	28 – 18 = **10**

12. Here 25 is the maximum frequency of the class 40–60.

Hence, the required **modal class is 40–60.**

Again, the total frequency $n = 4 + 6 + 25 + 10 + 5 = 50$

$\therefore \frac{n}{2} = \frac{50}{2} = 25$ which lies in the class is 40–60.

Hence, the required **median class is 40–60**.

13.

Less than 5	**4**
Less than 10	4 + 6 = **10**
Less than 15	10 + 10 = **20**
Less than 20	20 + 10 = **30**
Less than 25	30 + 25 = **55**
Less than 30	55 + 22 = **77**
Less than 35	77 + 18 = **95**
Less than 40	95 + 5 = **100**

14.

Age (in years)	*Frequency*
0–10	6
10–20	11
20–30	21
30–40	23
40–50	14
50–60	5

Maximum frequency is 23 and the class corresponding to this frequency is 30–40.

So the modal class is 30–40.

$\therefore \quad l = 30, f_1 = 23, f_0 = 21, f_2 = 14$ and $h = 10$

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 30 + \left(\frac{23 - 21}{2 \times 23 - 21 - 14}\right) \times 10$$

$$= 30 + \left(\frac{2}{46 - 35}\right) \times 10$$

$$= 30 + \frac{20}{11}$$

$$= 30 + 1.818$$

$$= 31.818$$

Hence, mode = **31.82 approx.**

15. Here $h = 20$ Let the assumed mean $a = 50$.

Class	*Class mark*	*Frequency*	$u_i = \frac{x_i - a}{h}$	$f_i u_i$	
0–20	0	17	$\frac{10-50}{20} = -2$	–34	$\}$ – 62
20–40	30	28	$\frac{30-50}{20} = -1$	–28	
40–60	50	32	$\frac{50-50}{20} = 0$	0	
60–80	70	f_1	$\frac{70-50}{20} = +1$	f_1	$\}$ 38 + f_1
80–100	90	19	$\frac{90-50}{20} = 2$	38	

$$\Sigma f_i = 96 + f_1$$

$$\Sigma f_i u_i = -24 + f_1$$

$$\text{Mean} = a + \left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right) \times h$$

$$\Rightarrow \quad 50 = 50 + \frac{f_1 - 24}{96 + f_1}$$

$$\Rightarrow \quad 50 - 50 = \frac{f_1 - 24}{96 + f_1}$$

$$\Rightarrow \quad 0 = \frac{f_1 - 24}{96 + f_1}$$

$$\Rightarrow \quad 0 = f_1 - 24$$

$$\Rightarrow \quad f_1 = \mathbf{24}$$

UNIT TEST 2

For Standard Level

1. (*i*) The sum of deviations of all observations from the mean is always **zero.**

Justification: Let $x_1, x_2, x_3, \ldots, x_n$ be the observation and let $\bar{x}$ be the mean

Then, sum of deviations of all the observations from the mean

$= (x_1 - \bar{x}) + (x_2 - \bar{x}) + (x_3 - \bar{x}) + (x_4 - \bar{x}) + \ldots + (x_n - \bar{x})$

$= (x_1 + x_2 + x_3 + \ldots + x_n) - n\bar{x}$

$= n\bar{x} - n\bar{x}$

$= 0$

(*ii*) If the mean of 1, 2, 3, …, n is $\frac{8n}{15}$, then the value of n is **15**

Justification:

$$\frac{1 + 2 + 3 \ldots + n}{n} = \frac{8n}{15}$$

$$\Rightarrow \quad \frac{n(n+1)}{2} \times \frac{1}{n} = \frac{8n}{15}$$

$$\left[\because 1 + 2 + 3 + 4 \ldots + n = \frac{n(n+1)}{2}\right]$$

$$\Rightarrow \quad \frac{n+1}{2} = \frac{8n}{15}$$

$$\Rightarrow \quad 15n + 15 = 16n$$

$$\Rightarrow \quad 16n - 15n = 15$$

$$\boldsymbol{n = 15}$$

(*iii*) If the mode of data is $8x$ and its mean is $11x$. Then, the median will be $\mathbf{10x}$.

Justification: 3 Median = Mode + 2 Mean

$\Rightarrow \quad$ 3 Median $= 8x + 2(11x) = 30x$

$\Rightarrow \quad$ Median $= \frac{30x}{3} = 10x$

(*iv*) The mean and median of first eleven numbers is **6.**

Justification:

$$\text{Mean} = \frac{1+2+3+4+5+6+7+8+9+10+11}{11}$$

$$= \frac{66}{11} = 6$$

Here $n = 11$ (odd)

$\therefore$ Median = value of $\left(\frac{11+1}{2}\right)$th term, i.e. value of 6th term = 6

(*v*) The mean of 17 observations is 20. If the mean of first 9 observations is 23 and that of last 9 observations is 18, then the 9th observation is **29**

Justification: Mean of 17 observations = 20

$\therefore$ Total of 17 observations $= 20 \times 17 = 340$

Mean of first 9 observations = 23

$\therefore$ Total of first 9 observations $= 23 \times 9 = 207$

Mean of last 9 observations = 18

$\therefore$ Total of last 9 observations = 18 × 9 = 162

$\therefore$ 9th observation = 207 + 162 − 340

= 369 − 340

= 29

2. (*c*) **27**

$$u_i = \frac{x_i - a}{n} = \frac{x_i - 25}{10}$$

$\Rightarrow$ $a = 25$ and $h = 10$

$$\bar{x} = a + \left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right)$$

$\Rightarrow$ $\bar{x} = 25 + \frac{20}{100} \times 10$

$= 25 + 2 = 27$

3. (*a*) **70**

Class interval	*Frequency*	*Cumulative frequency*
20–25	11	11
25–30	15	26
30–35	11	37
35–40	20	57
40–45	8	65

Frequency of class 35–40 is maximum. So modal class is 35–40.

Here, $n = 65$. So, $\frac{n}{2} = \frac{65}{2} = 32.5$.

Cumulative frequency just greater than 32.5 is 37 and the class corresponding to this frequency is 30–35.

Lower limit of modal class + upper limit of median class =35 + 35 = 70.

4. (*b*) **Mean**

Mean cannot be determined graphically.

5. (*a*) **Histogram**

The mode of a frequency distribution can be determined graphically from histogram.

6. (*d*) **20.5**

The median of a data is given by the *x*-coordinate of the point of intersection of less than ogive and more than ogive.

$\therefore$ The median of the data is 20.5.

7. (*b*) **27.2**

$$\text{Mean} = \bar{x} = a + \left(\frac{\Sigma f_i u_i}{\Sigma f_i}\right) \times h$$

$$= 25 + \left(\frac{11}{50}\right) \times 10$$

$$= 25 + \frac{11}{5}$$

$= 25 + 2.2$
$= 27.2$

8. (*b*) **Mean = Median = Mode**

For a symmetrical distribution, the three measures of central tendency, i.e. mean, median and mode are equal.

9. (*b*) **6**

We know that mean

$$\frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{132 + 5p}{20} \quad \text{[Given]}$$

$\Rightarrow$ $8.1 = \frac{132 + 5p}{20}$

[$\because$ Given mean = 8.1]

$\Rightarrow$ $132 + 5p = 8.1 \times 20 = 162$

$\Rightarrow$ $5p = 162 - 132 = 30$

$\therefore$ $p = \frac{30}{5} = 6$

10. (*c*) **25**

Here the total number of data arranged in ascending order is 8 which is even. Hence, $\frac{8}{2}$ th and $\left(\frac{8}{2} + 1\right)$th, i.e. 4th and 5th observation are respectively $x + 2$ and $x + 3$.

$\therefore$ The median is $\frac{(x+2)+(x+3)}{2} = x + \frac{5}{2} = x + 2.5$

Since the given median = 27.5.

$\therefore$ $x + 2.5 = 27.5$

$\Rightarrow$ $x = 27.5 - 2.5 = 25.$

11.

Income (in ₹)	6000–7000	7000–8000	8000–9000	9000–10000
Number of employees	**40**	108–40 = **68**	194–108 = **86**	314–194 = **120**

12.

Class interval	*Frequency*
0–10	100–95 = **5**
10–20	95–80 = **15**
20–30	80–60 = **20**
30–40	60–37 = **23**
40–50	37–20 = **17**
50–60	20–9 = **11**
60 and above	9–0 = **9**

13. From the given frequency distribution table, we have the following:

Class	*Mid-value of the class* x_i	*Frequency* f_i	$x_i f_i$
0–20	10	17	170
20–40	30	f_1	$30f_1$
40–60	50	32	1600
60–80	70	f_2	$70f_2$
80–100	90	19	1710
		$n = \Sigma f_i = 68 + f_1 + f_2$	$\Sigma f_i x_i = 3480 + 30f_1 + 70f_2$

It is given that $\Sigma f_i = 120$

$\therefore \quad 68 + f_1 + f_2 = 120$

$\Rightarrow \quad f_1 + f_2 = 120 - 68 = 52 \quad \ldots(1)$

$\therefore$ From (1), $\quad f_2 = 52 - f_1 \quad \ldots(2)$

$$\therefore \quad \text{Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

$$= \frac{3480 + 30f_1 + 70f_2}{68 + f_1 + f_2}$$

$$= \frac{3480 + 30f_1 + 70(52 - f_1)}{68 + 52}$$

[From (1) and (2)]

$$= \frac{3480 + 3640 + 30f_1 - 70f_1}{120}$$

$$\Rightarrow \quad 50 = \frac{7120 - 40f_1}{120}$$

[$\because$ Given that mean = 50]

$\Rightarrow \quad 7120 - 40f_1 = 120 \times 50 = 6000$

$\Rightarrow \quad 40f_1 = 7120 - 6000 = 1120$

$$\Rightarrow \quad f_1 = \frac{1120}{40} = 28$$

$\therefore$ From (1), $\quad f_2 = 52 - f_1 = 52 - 28 = 24$

$\therefore$ Required values of f_1 and f_2 are respectively **28** and **24**.

14. Since class 40–60 has the height frequency, i.e. 18, hence, the modal class is 40–60.

$\therefore \quad l$ = lower limit of the modal class = 40

f_1 = frequency of the modal class = 18

f_0 = frequency of the class preceding the modal class = 6

f_2 = frequency of the class succeeding the modal class = 10

h = class size = 20.

Hence, the required mode is

$$l + \frac{f_1 - f_0}{2f_1 - (f_0 + f_2)} \times h = 40 + \frac{18 - 6}{2 \times 18 - (6 + 10)} \times 20$$

$$= 40 + \frac{12}{36 - 16} \times 20$$

$= 40 + 12 = $ **52**.

15. Less than ogive

From the give table, we first prepare a cumulative frequency distribution of less than type.

Age (in years)	*Number of teachers (cumulative frequencies)*
Less than 25	21
30	40
35	90
40	130
45	146
50	166
55	176
60	186
65	195
70	199

Upper class limits	25	30	35	40	45	50	55	60	65	70
Cumulative frequency	21	40	90	130	146	166	176	186	195	199

Plotting the points **(25, 21), (30, 40), (35, 90), (45, 146) (50, 166), (55, 176), (60, 186), (65, 195), (70, 199)** and joining them by a free hand smooth curve we get a 'less than ogive'.

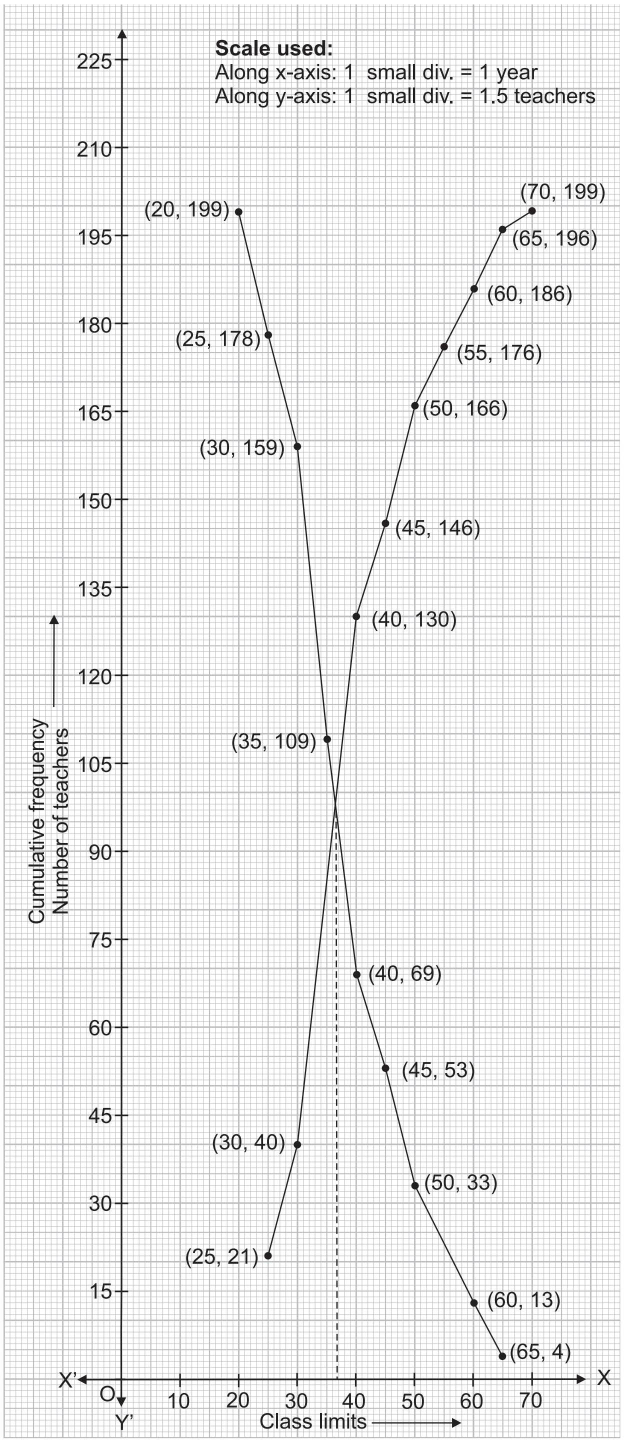
More than ogive

From the given table, we prepare a cumulative frequency distribution of more than type.

Class interval	*Frequency*		
20 – 25	21	More than or equal to 20	199
25 – 30	19	More than or equal to 25	178
30 – 35	50	More than or equal to 30	159
35 – 40	40	More than or equal to 35	109
40 – 45	16	More than or equal to 40	69
45 – 50	20	More than or equal to 45	53
50 – 55	10	More than or equal to 50	33
55 – 60	10	More than or equal to 55	23
60 – 65	9	More than or equal to 60	13
65 – 70	4	More than or equal to 65	4

Lower class limits	20	25	30	35	40	45	50	55	60	65
Cumulative frequency	199	178	159	109	69	53	33	23	13	4

Plotting the points **(20, 199), (25, 178), (30, 159), (35, 109), (40, 69), (45, 53), (50, 33), (55, 23), (60, 13), (65, 4)** and joining by a free hand curve, we get a 'more than ogive' as shown in the following graph:



From the point at which the two ogives intersect, drop a perpendicular on the *x*-axis. The *x*-coordinate of the point at which this perpendicular cuts the *x*-axis gives the median.

Median age = **36.2 years (approx.)**