CHAPTER **15**

Surface Areas and Volumes

EXERCISE 15A

For Basic and Standard Levels

1. (*i*) Let the length of each edge of cube of volume 2197 cm^3 be x cm. Then,

volume = 2197 cm^3

$\Rightarrow \quad x^3 = 2197$

$\Rightarrow \quad x^3 = 13^3$

$\Rightarrow \quad x = 13$ cm

Hence, length of edge of cube is **13 cm**.

(*ii*) Surface area of the cube = 6 $(\text{side})^2 = 6x^2$

$= 6\,(13 \text{ cm})^2 = 1014 \text{ cm}^2$

Hence, surface area of the cube is **1014 cm^2**.

2. It is given that the length of diagonal of the cube is $7\sqrt{3}$ cm.

Diagonal of cube is $a\sqrt{3}$

Now, $\quad a\sqrt{3} = 7\sqrt{3} \Rightarrow a = 7$

Then, $\quad$ volume of cube = $a^3 = 7^3$ = **343 cm^3**

Surface area of cube = $6a^2 = 6 \times 7^2 = 294$ **cm^2**

3. $\quad$ Volume = 135 cm^3

and $\quad$ Area of base = 30 cm^2

$$\therefore \quad \text{Height} = \frac{\text{Volume}}{\text{Area of base}}$$

$$h = \frac{135 \text{ cm}^3}{30 \text{ cm}^2} = 4.5 \text{ cm}$$

Hence, height of the cuboid is **4.5 cm**.

4. Surface area = 1216 cm^2.

Since, dimensions of cuboid are in the ratio 5 : 4 : 2, let the dimensions be $5x$, $4x$ and $2x$.

Given, $\quad$ Area = $2\,(lb + bh + lh)$

$= 1216 \text{ cm}^2$

$\Rightarrow 2\,(5x \times 4x + 4x \times 2x + 5x \times 2x) = 1216$

$\Rightarrow \quad 2\,(20x^2 + 8x^2 + 10x^2) = 1216$

$\Rightarrow \quad 2\,(38x^2) = 1216$

$$\Rightarrow \quad x^2 = \frac{1216}{2 \times 38} = 16$$

$\Rightarrow \quad x = \pm 4 \quad$ (Reject $x = -4$)

Hence, we have $\quad l = 5x = 5 \times 4$ cm = **20 cm**

$b = 4x = 4 \times 4$ cm = **16 cm**

$h = 2x = 2 \times 4$ cm = **8 cm**

5. Let a_1 units and a_2 units be the sides of the two cubes. Then their volumes are a_1^3 units and a_2^3 units respectively,

$\therefore$ According to the problem

$$\frac{a_1^3}{a_2^3} = \frac{1}{27}$$

$$\therefore \quad \frac{a_1}{a_2} = \sqrt[3]{\frac{1}{27}} = \frac{1}{3}$$

$$\therefore \quad \frac{6a_1^2}{6a_2^2} = \frac{1}{9}$$

i.e. the surface area of the 1st cube : the surface area of the second cube = 1 : 9

$\therefore$ Required ratio is **1 : 9**.

6. (*i*) Diameter of cylinder = 42 cm

Now, we have

$$\text{radius, } r = \frac{42 \text{ cm}}{2} = 21 \text{ cm}$$

height of cylinder, h = 10 cm

(*a*) Curved surface area = $2\pi rh = 2\pi \times 21 \text{ cm} \times 10 \text{ cm}$

= **1320 cm^2**

(*b*) Total surface area = $(2\pi rh + 2\pi r^2)$

$= [1320 + 2\pi \times (21)^2]$

$= (1320 + 2772)$ = **4092 cm^2**

(*c*) Volume of cylinder $= \pi r^2 h = \pi \times (21)^2 \times 10$

$$= \frac{22}{7} \times 21 \times 21 \times 10$$

= **13860 cm^3**

(*ii*) Length of the rectangular paper, l = 88 cm

Width of the rectangular paper, b = 10 cm

Height of the cylinder = width of the rectangular paper

$\Rightarrow \quad h = 10$ cm

Circumference of the base of the cylinder

= length of the rectangular paper

$\Rightarrow \quad 2\pi r = 88$ cm

$$r = \frac{88 \text{ cm}}{2\pi}$$

Now, volume of the cylinder = $\pi r^2 h$

$$= \pi \times \left(\frac{88}{2\pi}\right)^2 \times 10 \text{ cm}$$

$$= \frac{88^2 \times 10}{\left(\frac{22}{7}\right) \times 4}$$

$$= \frac{88^2 \times 10 \times 7}{22 \times 4} = \mathbf{6160 \text{ cm}^3}$$

7. Volume of right circular cylinder = 832π cm^3

Then, $\quad$ radius of its base, $\quad r = 8$ cm

Now, $\quad$ volume = 832π cm^3

$\Rightarrow \quad \pi r^2 h = 832\pi$

$$\Rightarrow \quad h = \frac{832}{r^2} = \frac{832}{64} = \mathbf{13 \text{ cm}}$$

$\therefore$ Lateral surface area = $2\pi rh$

$$= 2\times\frac{22}{7}\times 8\times 13$$

$= 654\text{ cm}^2$ (approx.)

Total surface area = $(2\pi rh + 2\pi r^2)$

$= (654 + 2\pi \times 8^2)$

$= (654 + 402)$

$= \mathbf{1056\ cm^2}$

8. We have, height of wooden pole,

$h = 8.4$ m

Radius, $r = 15$ cm $= 0.15$ m

Now, density of wood,

$\rho = 375$ kg per m^3

As we know, $\rho = \frac{m}{v}$

Volume of cylinder $= \pi r^2 h = \frac{22}{7}\times(0.15)^2\times 8.4$

$= 0.594\text{ m}^3$

Now, mass of the wooden pole,

$m = \rho \times v = 375 \times 0.594$

$= \mathbf{222.75\ kg}$

9. Height of the cylindrical well, $h = 22.5$ m

Radius of the cylinder well, $r = \frac{7\text{ m}}{2} = 3.5$ m

$\therefore$ Volume $= \pi r^2 h$

$$= \frac{22}{7}\times(3.5)^2\times 22.5 = \mathbf{866.25\ m^3}$$

Total surface area to be polished $= (2\pi rh + \pi r^2)$

$$= \left(2\times\frac{22}{7}\times 3.5\times 22.5 + \frac{22}{7}\times 3.5\times 3.5\right)$$

$= (495 + 38.5) = 533.5\text{ m}^2$

Now, cost of plastering the well = 533.5 × 3 = **₹ 1600.50**

10. We have, height, $h = 7$ cm

Curved surface area = 211.2 cm^2 …(1)

As we know,

Curved surface area = $2\pi rh$

From (1), $2\pi rh = 211.2$

$$r = \frac{211.2}{2\pi h} = \frac{211.2\times 7}{2\times 22\times 7} = \frac{24}{5}\text{ cm}$$

Now, Volume, V = $\pi r^2 h$

$$V = \frac{22}{7}\times\left(\frac{24}{5}\right)^2\times 7$$

$$= \mathbf{\frac{12672}{25}\ cm^3}$$

11. (*i*) We have, $r = 2x$, $h = 3x$

Volume of cylinder, V = $\pi r^2 h$ = 1617 cm^3

$\Rightarrow$ $\frac{22}{7}\times(2x)^2\times 3x = 1617$

$\Rightarrow$ $x^3 = \frac{1617\times 7}{22\times 4\times 3} = 42.875$

$\Rightarrow$ $x^3 = (3.5)^3$

$\Rightarrow$ $x = 3.5$ cm …(1)

Using (1), $r = 2x = 2 \times 3.5$ cm = 7 cm, $h = 10.5$ cm

Total surface area $= 2\pi r\,(r + h)$

$$= 2\times\frac{22}{7}\times 7\times(7+10.5)$$

$$= 2\times\frac{22}{7}\times 7\times 17.5 = \mathbf{770\ cm^2}$$

(*ii*) Total surface area = 231 cm^2

Curved surface area $= \frac{2}{3}\times$(Total surface area)

$= \frac{2}{3}\times 231 = 154\text{ cm}^2$

Total surface area $= (2\pi rh + 2\pi r^2)$

$2\pi r^2 + 154 = 231$

$2\pi r^2 = 77\text{ cm}^2$ …(1)

and $2\pi rh = 154\text{ cm}^2$ …(2)

From (1), $2\pi r^2 = 77$

$\Rightarrow$ $r^2 = \frac{77\times 7}{44}$

$\Rightarrow$ $r^2 = \frac{49}{4}$

$\Rightarrow$ $r = \frac{7}{2}$ cm = **3.5 cm**

From (2), $2\pi rh = 154$

$$h = \frac{154}{2\pi r} = \frac{154\times 7}{2\times 22\times 3.5} = \mathbf{7\ cm}$$

12. Let r be the radius of cylinder and h be its height.

Then, $r + h = 37$ cm …(1)

Total surface area, $2\pi r\,(h + r) = 1628\text{ cm}^2$

$\Rightarrow$ $2\pi r\,(37) = 1628$

$\Rightarrow$ $r = \frac{1628\times 7}{37\times 2\times 22} = 7$ cm

From (1), $r + h = 37$

$h = 37 - r$

$= 37 - 7 = 30$ cm

Now, Volume $= \pi r^2 h$

$$= \frac{22}{7}\times 7\times 7\times 30$$

$= \mathbf{4620\ cm^3}$

13. Let h be height of garden roller and r be its radius.

Given, $h = 2$ m

$r = 0.7$ m

Now, curved surface area = $2\pi rh$

$$= 2\times\frac{22}{7}\times 0.7\times 2 = 8.8\text{ m}^2$$

Hence, area curved under 5 revolutions.

$= 5 \times 8.8\text{ cm}^2 = 44\text{ m}^2$

14. It is given that ₹ 100 is the cost of boring 1 m^3 of cylindrical hole.

$\therefore$ ₹ 2200 is the cost of boring $\frac{2200}{100}$ m^3

$= 22\ m^3$ of cylindrical hole

Radius, $r = \frac{1}{2}$ m

Now, Volume, $\pi r^2 h = 22\ m^3$

$\Rightarrow \frac{22}{7} \times \frac{1}{2} \times \frac{1}{2} \times h = 22\ m^3$

$\Rightarrow h = \mathbf{28\ m}$

15. (*i*) Given, $h = 1120$ cm

$r = \frac{5.8}{2} = 2.9$ cm

External radius = Internal radius + Thickness

$= (2.9 + 0.2) = 3.1$ cm

Volume of the metal

= External volume – Internal volume

$= \pi R^2 h - \pi r^2 h$

$= \pi h\,(R^2 - r^2)$

$= \pi \times 1120 \times [(3.1)^2 - (2.9)^2]$

$= \frac{22}{7} \times 1120 \times [9.61 - 8.41]$

$= \frac{22}{7} \times 1120 \times 1.2 = \mathbf{4224\ cm^3}$

(*ii*) We have, external radius of pipe, R = 9 cm

Length of pipe, $h = 14$ cm

Volume of pipe, V = 748 cm^3.

Let r be internal radius, then

volume = 748 cm^3

$\Rightarrow \pi\,(R^2 - r^2)\,14 = 748$

$\Rightarrow \frac{22}{7}(9^2 - r^2)14 = 748$

$\Rightarrow (81 - r^2) = \frac{748 \times 7}{14 \times 22}$

$\Rightarrow 81 - r^2 = \frac{748}{44} = 17$

$\Rightarrow r^2 = 81 - 17 = 64\ cm^2$

$\Rightarrow r = 8$ cm

Hence, thickness of pipe = (R – r) = (9 – 8) = **1 cm**

16. Given, $r = 6$ m

$h = 8$ m

Now, slant height, $l^2 = r^2 + h^2$

$\Rightarrow l = \sqrt{r^2 + h^2}$

$= \sqrt{36 + 64} = 10$ m

Curved surface area $= \pi r l$

$= \frac{22}{7} \times 6 \times 10 = \mathbf{\frac{1320}{7}\ m^2}$

Volume of conical tent $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 6 \times 6 \times 8$

$= \mathbf{\frac{2112}{7}\ m^3}$

17. Circumference of the base of a cone = 44 cm

Slant high = 25 cm

Circumference = 44

$2\pi r = 44$

$2 \times \frac{22}{7} \times r = 44$

$r = \frac{44 \times 7}{2 \times 22} = 7$ cm

Curved surface area $= \pi r l$

$= \frac{22}{7} \times 7 \times 25 = \mathbf{550\ cm^2}$

18. Let r be the radius of the conical tent, h be its vertical height and l its slant height.

Then, $r = \frac{14}{2}$ m = 7 m, $h = 24$ m

$\therefore l = \sqrt{r^2 + h^2}$

$= \sqrt{49 + 24 \times 24}$ m

$= \sqrt{49 + 576}$ m

$= \sqrt{625}$ m

= 25 m

Area of the cloth required

= Curved surface area of the cone

$= \pi r l$

$= \frac{22}{7} \times 7 \times 25\ m^2 = 550\ m^2$

$\therefore$ Length of the cloth required

$= \frac{\text{Area of the cloth}}{\text{Width of the cloth}}$

$= \frac{550}{5}$ m = 110 m

Hence, the required cost of the cloth = ₹ 25 × 110 = ₹ 2750

19. Radius, r of the cone $\frac{9}{2}$ m = 4.5 m

Height h of the cone = 3.5 m

$\therefore$ Required volume of the rice

= Volume of the cone

$= \frac{1}{3} \times \pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 4.5 \times 4.5 \times 3.5\ m^3$

$= \mathbf{74.25\ m^3}$

Slant height, l of the cone

$= \sqrt{r^2 + h^2}$

$= \sqrt{4.5^2 + 3.5^2}$ m

$= \sqrt{20.25 + 12.25}$ m

$= \sqrt{32.50}$ m = 5.7 m

$\therefore$ Required area of the canvas cloth

= Curved surface area of the cone

$= \pi rl$

$= \frac{22}{7} \times 4.5 \times 5.7\ \text{m}^2$

$= \frac{22}{7} \times 25.65\ \text{m}^2$

$= \frac{564.30}{7}\ \text{m}^2$

= 80.61 m²

20. When semicircular sheet is bent into an open conical cup, the radius of sheet becomes the slant height of cup. The circumference of semicircular sheet becomes the circumference of the base of cone.

$\therefore$ Slant height of conical cup $= \frac{28\ \text{cm}}{2} = 14$ cm

Let r be radius and h be the height of the conical cup

Then, circumference of conical cup

= circumference of semicircular sheet

$\Rightarrow \quad 2\pi r = \pi \times 14$

$\Rightarrow \quad r = 7$ cm

Now, $\quad l^2 = r^2 + h^2$

$\Rightarrow \quad h = \sqrt{l^2 - r^2}$

$= \sqrt{14^2 - 7^2}$

$= 7\sqrt{3}$ cm $\qquad (\because \sqrt{3} = 1.732)$

= 7 × 1.732 = 12.124 cm

$\therefore$ Depth of the cup,

h = **12.124 cm**

Now, capacity of the cup = Volume of cup

Capacity of the cup $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times (7)^2 \times 12.124$ cm

= 622.365 cm³

21. Let the radius and the height of the cone be r and h respectively.

Then $r = 5x$ cm and $h = 12x$ cm, where x is any non-zero constant.

$\therefore$ Volume of the cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times 3.14 \times 25x^2 \times 12x\ \text{cm}^3$

$= 314x^3\ \text{cm}^3$

Given that $\quad$ Volume = 314 cm³

$\therefore \quad 314x^3 = 314$

$\Rightarrow \quad x^3 = 1$

$\Rightarrow \quad x = 1$

Hence, $r = 5$ cm and $h = 12$ cm.

$\therefore$ The slant height, $l = \sqrt{r^2 + h^2}$

$= \sqrt{5^2 + 12^2}$ cm

$= \sqrt{169}$ cm

= 13 cm

$\therefore$ The required total surface area of the cone

$= \pi rl + \pi r^2$

= 3.14 × (5 × 13 + 25) cm²

= 3.14 × 90 cm²

= 282.60 cm²

22. Let r and l be the radius and slant height of the cone respectively.

Thus, $\quad r = 5x,\ l = 7x.$

Then, curved surface area of the cone

= 2750 cm²

$\Rightarrow \quad \pi rl = 2750$

$\Rightarrow \quad \pi \times 5x \times 7x = 2750$

$\Rightarrow \quad x^2 = \frac{2750 \times 7}{35 \times 22} = \frac{19250}{770}$

$\Rightarrow \quad x^2 = 25$

$\Rightarrow \quad x = \pm 5 \qquad$ (Reject $x = -5$)

$\therefore \quad r = 5x = 5 \times 5 = 25$ cm

Hence, radius is **25 cm**.

23. (*i*) Let the radius of circular cone be r cm.

Slant height of cone, $l = 13$ cm.

Given, surface area of cone = 90π cm²

$\Rightarrow \quad \pi r (l + r) = 90\pi$

$\Rightarrow \quad r(13 + r) = 90$

$\Rightarrow \quad r^2 + 13r - 90 = 0$

$\Rightarrow \quad r^2 + 18r - 5r - 90 = 0$

$\Rightarrow \quad r(r + 18) - 5(r + 18) = 0$

$\Rightarrow \quad r = 5,\ r = -18 \qquad$ (Reject $x = -18$)

Radius of cone is **5 cm**.

Now, height of cone, $h^2 = l^2 - r^2$

$\Rightarrow \quad h^2 = \sqrt{(13)^2 - (5)^2} = \sqrt{169 - 25}$

$\Rightarrow \quad h^2 = \sqrt{144}$

$\Rightarrow \quad h = \pm 12$ cm(Reject $x = -12$ cm)

(*ii*) Then, volume of cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi \times (5)^2 \times 12$

$= \frac{1}{3}\pi \times 25 \times 12 =$ **100π cm³**

24. Base area, $\quad \pi r^2 = 3850 \qquad \ldots(1)$

Height, $h = 84$ cm

$\Rightarrow \quad r^2 = \frac{3850}{22} \times 7 = 1225$

$\Rightarrow \quad r = 35$ cm

Now, volume of the cone = capacity of the cone

$= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \frac{22}{7} \times 35 \times 35 \times 84$

$= \mathbf{107800\ cm^3}$

25. Circumference of the base of a cone = 44 cm

Slant height = 25 cm

Circumference = 44

$2\pi r = 44$

$2 \times \frac{22}{7} \times r = 44$

$r = \frac{44 \times 7}{2 \times 22} = 7$ cm

Curved surface area = πrl

$= \frac{22}{7} \times 7 \times 25$

$= \mathbf{550\ cm^2}$

26. (*i*) Given, $V = 314\frac{2}{7}\ \text{cm}^3 = \frac{2200}{7}\ \text{cm}^3$

Volume of the cone = $\frac{1}{3}\pi r^2 h$

$\Rightarrow \quad \frac{1}{3}\pi r^2 h = \frac{2200}{7}$

$\Rightarrow \quad r^2 = \frac{2200}{7} \times \frac{3 \times 7}{22 \times 12}$

$\Rightarrow \quad r^2 = 25$

$\Rightarrow \quad r = 5$ cm

Surface area of cone = $\pi rl + \pi r^2$ …(2)

Slant height, $l^2 = \sqrt{r^2 + h^2} = \sqrt{(5)^2 + (12)^2}$

$\Rightarrow \quad l^2 = \sqrt{25 + 144}$

$\Rightarrow \quad = \sqrt{169}$

$\Rightarrow \quad l = 13$ cm

From (2), $\pi rl + \pi r^2 = \frac{22}{7} \times 5 \times 13 + \frac{22}{7} \times 25$

$= \frac{1430}{7} + \frac{550}{7} = \mathbf{\frac{1980}{7}\ cm^2}$

(*ii*) Radius of cone = 3 cm

Curved surface area = 47.1 cm^2

$\pi rl = 47.1$

$3.14 \times 3 \times l = 47.1$

$l = \frac{47.1}{9.42}$

$l = 5$ cm

$l = \sqrt{r^2 + h^2}$

$5 = \sqrt{9 + h^2}$

$25 = 9 + h^2$

$h^2 = 16$

$h = 4$ cm

Volume of cone = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times 3.14 \times 9 \times 4 = \mathbf{37.68\ cm^3}$

27. Let r be the radius of circle, $r = 10.5$ cm

Surface area of solid sphere = $4\pi r^2$

$= 4 \times \frac{22}{7} \times (10.5)^2$

$= \frac{9702}{7}$

$= \mathbf{1386\ cm^2}$

Volume of solid generated = $\frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times (10.5)^3$

$= \mathbf{4851\ cm^3}$

28. Diameter of sphere = 8.4 cm

Now, radius of sphere, $r = \frac{8.4\text{ cm}}{2} = 4.2$ cm

Surface area of sphere = $4\pi r^2$

$= 4 \times \frac{22}{7} \times (4.2)^2$

$= \mathbf{221.76\ cm^2}$

Volume of sphere = $\frac{4}{3} \times \frac{22}{7} \times (4.2)^3$

$= \mathbf{310.464\ cm^3}$

29. Surface area of the sphere = 5544 cm^2

$\Rightarrow \quad 4\pi r^2 = 5544$

$\Rightarrow \quad r^2 = \frac{5544 \times 7}{4 \times 22} = 441$

$\Rightarrow \quad r = 21$ cm

Now, volume of sphere = $\frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times 21 \times 21 \times 21$

$= \mathbf{38808\ cm^3}$

30. We have, surface area of sphere = volume of sphere

$\Rightarrow \quad 4\pi r^2 = \frac{4}{3}\pi r^3$

$\Rightarrow \quad r = 3$ units

Hence, radius of sphere is **3 units**

31. Let the internal and external diameters of the hollow sphere of metal be 20 cm and 22 cm respectively.

Then, internal radius of sphere, $r = \frac{20}{2} = 10$ cm

external radius of sphere, $R = \frac{22}{2} = 11$ cm

Now, volume of sphere $= \frac{4}{3}\pi(R^3 - r^3)$

$= \frac{4}{3}\pi[(11)^3 - (10)^3]$

$= \frac{4}{3} \times \frac{22}{7}[1331 - 1000]$

$= \frac{88}{21}[331] = 1387.048 \text{ cm}^3$

Now, weight of hollow sphere

$= 1387.048 \times 21\text{g} = 29128 \text{ g}$

$= \frac{29128}{1000}\text{ kg} = \mathbf{29.128 \text{ kg}}$ (1 kg = 1000 g)

32. (*i*) Radius of the hemisphere,

$r = 10$ cm

Volume of the hemisphere $= \frac{2}{3}\pi r^3$

$= \frac{2}{3} \times 3.14 \times (10 \text{ cm})^3$

$= \mathbf{2093.33 \text{ cm}^3}$

Total surface area of the hemisphere $= 3\pi r^2$

$= 3 \times 3.14 \times (10 \text{ cm})^2$

$= \mathbf{942 \text{ cm}^2}$

(*ii*) Total surface area of hemisphere = 462 cm^2

$3\pi r^2 = 462$

$3 \times \frac{22}{7} \times r^2 = 462$

$r^2 = \frac{462 \times 7}{22 \times 3}$

$r^2 = 49$

$r = 7$ cm

Volume $= \frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \frac{22}{7} \times 7 \times 7 \times 7$

$= \frac{44 \times 49}{3}$

$= \mathbf{\frac{2156}{3} \text{ cm}^3}$

33. Let r cm be the radius of the hemisphere.

$\therefore$ Volume of the hemisphere $= \frac{2}{3}\pi r^3 = \frac{2}{3} \times \frac{22}{7} \times r^3 \text{ cm}^3$

$\therefore$ According to the problem, we have

$\frac{2}{3} \times \frac{22}{7} \times r^3 = 2425\frac{1}{2} = \frac{4851}{2}$

$\therefore$ $r^3 = \frac{4851}{2} \times \frac{3}{2} \times \frac{7}{22}$

$= \frac{7 \times 7 \times 3^2 \times 3 \times 7}{2^3}$

$= \frac{7^3 \times 3^3}{2^3}$

$\therefore$ $r = \frac{7 \times 3}{2} = \frac{21}{2} = 10.5$

$\therefore$ Required curved surface area of the hemisphere

$= 2\pi r^2$

$= 2 \times \frac{22}{7} \times 10.5 \times 10.5 \text{ cm}^2$

$= 66 \times 10.5 \text{ cm}^2$

$= 693 \text{ cm}^2$

34. Circumference of hemispherical edge = 66 cm

Then, $2\pi r = 66$ cm

$\Rightarrow$ $2 \times \frac{22}{7} \times r = 66$ cm

$\Rightarrow$ $r = \frac{66 \times 7}{22 \times 2} = \frac{21}{2}$ cm

Now, the capacity of bowl = volume of the bowl

$= \frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \frac{22}{7} \times \left(\frac{21}{2}\right)^3$

$= \frac{4851}{2} = \mathbf{2425.5 \text{ cm}^3}$

35. Let r unit be the radius of the hemisphere.

Then volume of the hemisphere $= \frac{2}{3}\pi r^3$ cube units

Also, total surface area of the hemisphere $= 3\pi r^2$ cube units.

$\therefore$ According to the problem,

$\frac{2}{3}\pi r^3 = 3\pi r^2$

$\Rightarrow$ $r = 3 \times \frac{3}{2} = \frac{9}{2}$

Hence, the required diameter of the hemisphere

$= 2r = 2 \times \frac{9}{2}$ units

= 9 units

36. Internal radius of hemisphere,

$r_1 = \frac{d_1}{2} = \frac{24}{2} \text{ cm} = 12$ cm

External radius of hemisphere,

$r_2 = \frac{d_2}{2} = \frac{25}{2} = 12.5$ cm

Surface area of hemispherical vessel

$= 2\pi r_2^2 + 2\pi r_1^2 + \pi(r_2^2 - r_1^2)$

$= 2 \times \frac{22}{7} \times (12.5)^2 + 2 \times \frac{22}{7} \times (12)^2$

$+ \frac{22}{7}[12.5 \times 12.5 - 12 \times 12]$

$= 982.14 + 905.14 + 38.5$

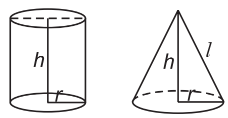
$= 1925.78 \text{ cm}^2$

Hence, cost of painting 1 cm^2 = ₹ 5.25

Now, cost of painting 1925.78 cm^2 = ₹ 5.25 × 1925.78

= **₹ 10110.345 (approx.)**

37. Let r units be the radius of their equal bases and h units be their equal heights. It l units be the slant height of the cone, then $l = \sqrt{r^2 + h^2}$ units.



Now, the curved surface area of the cylinder

$= 2\pi rh$ sq units

And the curved surface area of the cone

$= \pi rl$ sq units

$= \pi r\sqrt{r^2 + h^2}$ sq units

Given that

$$\frac{2\pi rh}{\pi r\sqrt{r^2 + h^2}} = \frac{8}{5}$$

$\Rightarrow \quad 10h = 8\sqrt{r^2 + h^2}$

$\Rightarrow \quad 5h = 4\sqrt{r^2 + h^2}$

$\Rightarrow \quad 25h^2 = 16(r^2 + h^2)$

$\Rightarrow \quad 9h^2 = 16r^2$

$\Rightarrow \quad 3h = 4r$

$\Rightarrow \quad r : h = 3 : 4$

Hence, the required ratio between the radius of their bases to their heights is **3 : 4**.

38. Let r be the radius of sphere, r = 5 cm

l be the slant height of cone, l = 25 cm

Now, curved surface area of cone

= curved surface area of sphere

$\Rightarrow \quad \pi rl = 4\pi r^2$

$\Rightarrow \quad 25r = 4\ (5)^2$

$\Rightarrow \quad r = \frac{100}{25} = \mathbf{4\ cm}$

39. Radius of sphere = 5 cm

Radius of cone = 4 cm

Surface area of sphere = $4\pi r^2$

$= 4 \times \pi \times 25$

$= 100\pi$ cm^2

Curved surface area of cone = πrl

$= \pi \times 4 \times l$

$= 4\pi l$ cm^2

Surface area of sphere = 5 × curved surface area of cone

$100\pi = 5 \times 4\pi l$

$100\pi = 20\pi l$

$l = \frac{100\pi}{20\pi} = 5$ cm

$l^2 = r^2 + h^2$

$25 = 16 + h^2$

$h^2 = 9$

$h = 3$ cm

Volume of cone = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 4 \times 4 \times 3$

$= \mathbf{\frac{352}{7}\ cm^3}$

40. Let r units be the radius of the sphere and a units be the side of the cube.

Then the surface areas of the sphere and the cube are respectively 45 πr^2 and $6a^2$.

∴ According to the problem,

$6a^2 = 4\pi r^2$

$\Rightarrow \quad a = \sqrt{\frac{2\pi}{3}}\ r \quad \ldots(1)$

Let V_1 cube units and V_2 cube units be the volumes of the sphere and the cube respectively.

Then $V_1 = \frac{4}{3}\pi r^3$ and $V_2 = a^3$

$$\therefore \quad \frac{V_1}{V_2} = \frac{\frac{4}{3}\pi r^3}{\left(\frac{2\pi}{3}\right)^{3/2} r^3} \quad \text{[From (1)]}$$

$$= \frac{2^2}{3}\pi \times \frac{3^{3/2}}{2^{3/2}\pi^{3/2}}$$

$$= \frac{3^{\frac{3}{2}-1} \times 2^{2-\frac{3}{2}}}{\pi^{3/2-1}}$$

$$= \frac{3^{1/2} \times 2^{1/2}}{\pi^{1/2}}$$

$$= \frac{\sqrt{6}}{\sqrt{\pi}}$$

$\therefore \quad V_1 : V_2 = \sqrt{6} : \sqrt{\pi}$

$\therefore \quad$ Required ratio = $\mathbf{\sqrt{6}} : \sqrt{\pi}$.

For Standard Level

41. Let r be the radius of each pencil.

Then the length of circumference of the base of the cylindrical pencil is $2\pi r$.

$\therefore \quad 2\pi r = 1.5 \quad$ [Given] …(1)

Now, the curved surface area of each cylindrical pencil

$= 2\pi rh$

$= 1.5 \times 25$ cm^2 [From (1)]

$= 37.5$ cm^2

Now, the cost of colouring 1 dm^2 of pencil = ₹0.05

i.e. the cost of colouring 100 cm^2 of pencil = ₹0.05

$\therefore$ The cost of colouring 37.5 cm^2 of 1 pencil = ₹ $\frac{0.05}{100} \times 37.5$

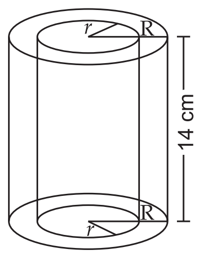
Hence, the required cost of colouring 120000 such pencils

$$= ₹\ \frac{0.05 \times 37.5 \times 120000}{100}$$

= ₹ 1.875 × 1200

= ₹ **2250**

42. Let r cm and R cm be the inner and outer radii of the cylinder of height h = 14 cm.



Then according to the problem, we have

$$2\pi h(R - r) = 88$$

$\Rightarrow \quad 2 \times \frac{22}{7} \times 14 \times (R - r) = 88$

$\Rightarrow \quad R - r = 1 \quad \ldots(1)$

The volume of the metal used in making the cylinder

$= \pi \times 14(R^2 - r^2)$

$= 176$ [Given]

Also, $\pi \times 14 \times (R^2 - r^2) = 176$

$\Rightarrow \quad \frac{22}{7} \times 14 \times (R^2 - r^2) = 176$

$\Rightarrow \quad R^2 - r^2 = \frac{176}{144} = 4 \quad \ldots(2)$

From (1), $R = r + 1 \quad \ldots(3)$

$\therefore$ From (2) and (3), we have

$(r + 1)^2 - r^2 = 4$

$\Rightarrow \quad 2r = 3$

$\Rightarrow \quad r = \frac{3}{2}$

$\therefore$ From (3), $R = \frac{3}{2} + 1 = \frac{5}{2}$

$\therefore$ Inner diameter = $2r$ = 3 cm

and outer diameter = 2R = 5 cm

Hence, the required outer and inner diameters of the cylinder are **5 cm** and **3 cm** respectively.

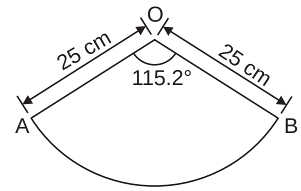
43. Given, Radius of the sector of circle, R = 25 cm.

Angle of sector, $\theta = 115.2°$

Length of arc of the sector $= \frac{\theta}{360°} \times 2\pi r$

$= \frac{115.2°}{360°} \times 2\pi \times 25$

$= 16\pi$ cm



Let r cm be the base radius of cone.

$\therefore$ Circumference of base of the cone

= Length of arc of sector

$\Rightarrow \quad 2\pi r = 16\pi$

$\Rightarrow \quad r = 8$ cm

Slant height of the cone,

l = Radius OA of the given sector

$\Rightarrow \quad l = 25$ cm

Now, height of cone,

$$h = \sqrt{l^2 - r^2} = \sqrt{(25)^2 - (8)^2}$$

$= \sqrt{625 - 64} = \sqrt{561}$

= 23.7 cm (approx.)

Volume of cone $= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times 3.14 \times 8 \times 8 \times 23.7\ cm^3$

= 1587.5 cm = **1588 cm^3 (approx.)**

44. Let r be the radius of sphere.

Then, height of the circumscribed cylinder, $h = 2r$

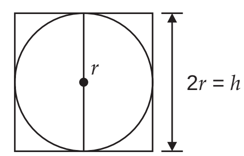
Surface area of the sphere, $A_s = 4\pi r^2$

Surface area of the circumscribed cylinder,

$A_c = 2\pi rh$

$= 2\pi r\ (2r)$

$= 4\pi r^2$



Thus, $A_s = A_c$

Hence, the surface area of a sphere is equal to the curved surface area of the circumscribed cylinder.

45. Radius of hemisphere

= height of the cylinder

= height of the cone.

Let the height of cone be h.

Then, radius of cone (r) = h = height of the cylinder.

Now, volume of cone : volume of hemisphere : volume of cylinder

$\Rightarrow \quad \frac{1}{3}\pi r^2 h : \frac{2}{3}\pi r^3 : \pi r^2 h$

$\Rightarrow \quad \frac{1}{3}\pi r^2 \times r : \frac{2}{3}\pi r^3 : \pi r^2 \times r$

$\Rightarrow \quad \frac{1}{3} : \frac{2}{3} : 1$

$\Rightarrow \quad 1 : 2 : 3$ (On multiplying by 3)

EXERCISE 15B

For Basic and Standard Levels

1. The dimensions of the cuboid so formed are as under

l = length = 100 cm, b = breadth = 80 cm

h = height = 64 cm.

Volume of cuboid = Volume of cube

$l \times b \times h = \text{side}^3$

$100 \text{ cm} \times 80 \text{ cm} \times 64 \text{ cm} = a^3$

$\therefore \quad a = 80 \text{ cm}$

Surface area of cube $= 6a^2$

$= 6 \times (80 \text{ cm})^2$

$= \mathbf{38400 \text{ cm}^2}$

2. Volume of the cuboid $= 9 \times 8 \times 2 \text{ m}^3 = 144 \text{ m}^3$

Volume of each cube of edge 2 m $= 2^3 \text{ m}^3 = 8 \text{ m}^3$

$\therefore$ Required number of cubes $= \frac{144}{8} = \mathbf{18}$

3. The volumes of three given smaller cubes are 3^3 cm^3, 4^3 cm^3 and 5^3 cm^3, i.e. 27 cm^3, 64 cm^3 and 125 cm^3.

$\therefore$ Sum of these volumes $= (27 + 64 + 125) \text{ cm}^2 = 216 \text{ cm}^3$

Let a cm be the edge of the larger cube formed.

Then $\quad a^3 = 216 = 6^3$

$\Rightarrow \quad a = 6$

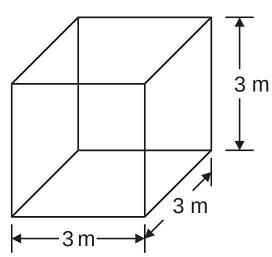
Hence, the required edge of the larger cube formed is **6 cm**.

4. Volume of big cube $= lbh = 18 \times 12 \times 9$

$\Rightarrow \quad a^3 = 1944 \text{ m}^3$

Now, volume of small cube $= a_1^3$

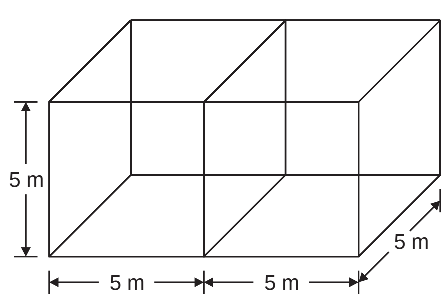
Length of each edge = 3 m



Then, $\quad a_1^3 = 27 \text{ m}^3$

Hence, $\quad$ noumber of cubes $= \frac{a^3}{a_1^3} = \frac{1944}{27} = \mathbf{72}$.

5. Let the length of each edge of the cube of volume be a.



Volume of cube $= 125 \text{ m}^3 = a^3$

$\Rightarrow \quad a = 5 \text{ m}$

Now, dimensions of the cuboid are:

length = 5 + 5 = 10 m

breadth = 5 m, height = 5 m

Surface area of resulting cuboid

$= 2 (lb + bh + hl)$

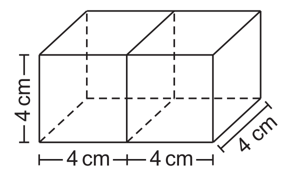
$= 2 (10 \times 5 + 5 \times 5 + 5 \times 10)$

$= 2 (50 + 25 + 50) = \mathbf{250 \text{ m}^2}$

6. If the side of the cube is a cm, then

$a^3 = 64 \text{ cm}^3$

$\Rightarrow \quad a = 4 \text{ cm}$



If two such cubes are joined together end to end as shown in the figure, then the dimension of the resulting cuboid will be 4 + 4 cm, 4 cm and 4 cm, i.e. 8 cm, 4 cm, 4 cm.

Hence, the required surface area of the resulting cuboid is $2(8 \times 4 + 4 \times 4 + 8 \times 4) \text{ cm}^2 = 2 \times (32 + 16 + 32) \text{ cm}^2$

$= \mathbf{160 \text{ cm}^2}$

7. Let the depth of cylindrical cone be H

Radius of cylindrical tank, r = 3 m

The dimensions of rectangular tank are

length, l = 18 m,

breadth, b = 11 m,

height, h = 4 m.

Volume of cylindrical tank

= Volume of rectangular water tank

$\Rightarrow \quad \pi r^2 H = l \times b \times h$

$\Rightarrow \quad H = \frac{l \times b \times h}{\pi r^2} = \frac{18 \times 11 \times 4 \times 7}{22 \times 9}$

$\Rightarrow \quad H = 28 \text{ m}$

Hence, the depth of cylindrical tank is **28 m**.

8. Diameter of coins = 1.5 cm

Thickness of coin = 0.2 cm

Volume of 1 coin $= \pi r^2 h$

$= \pi \times \frac{15}{20} \times \frac{15}{20} \times \frac{2}{10}$

$= \frac{405}{8} \text{ cm}^3$

Height of cylinder = 10 cm

Diameter of cylinder = 4.5 cm

Volume of cylinder $= \pi r^2 h$

$= \pi \times \frac{45}{20} \times \frac{45}{20} \times 10$

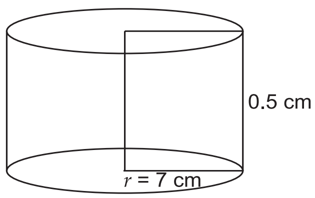
$= \frac{405}{8} \text{ cm}^3$

$$\text{No. of coins} = \frac{\text{volume of cylinder}}{\text{volume of 1 coin}}$$

$$= \frac{405\pi}{8} \times \frac{80}{9\pi}$$

$$= \mathbf{450}$$

9. Thickness of each circular plate = 0.5 cm and its radius = 7 cm.



Then 50 plates are placed one above another to form a cylinder of height, h = 0.5 cm × 50 = 25 cm and radius r = 7 cm.

∴ Required total surface area of this cylinder is

$$2\pi rh + 2\pi r^2 = 2\pi r\,(h + r)$$

$$= 2 \times \frac{22}{7} \times 7 \times (25 + 7)\ \text{cm}^2$$

$$= 44 \times 32\ \text{cm}^2$$

$$= \mathbf{1408\ cm^2}$$

10. Let the thickness of the wire be d.

$$\text{Radius of wire} = \frac{d}{2}$$

Length of wire, h = 1800 cm

The diameter of the cylindrical copper rod = 6 cm

Its radius, $r = \dfrac{6\text{ cm}}{2} = 3$ cm, length = 8 cm

Now, volume of copper wire

= volume of cylindrical copper rod

$$\Rightarrow \quad \pi\left(\frac{d}{2}\right)^2 h = \pi r^2 L$$

$$\Rightarrow \quad \frac{d^2}{4} \times 1800 = 3^2 \times 8$$

$$\Rightarrow \quad d^2 = \frac{9 \times 8 \times 4}{1800} = \frac{4}{25} = \frac{2}{5} = 0.4\ \text{cm}$$

Hence, thickness of wire is 0.4 cm and radius = **0.2 cm**

11. Radius, R of the cylindrical copper rod = $\dfrac{2}{2}$ cm = 1 cm

Length of the copper rod or height of the cylinder

= 10 cm

∴ Volume of the copper rod

$$= \pi \times 1^2 \times 10\ \text{cm}^3$$

$$= 10\pi\ \text{cm}^3 \qquad \ldots(1)$$

Length of the wire = 10 m = 1000 cm

Let the thickness of the wire be d cm.

This wire is a very thin cylinder or radius, $r = \dfrac{d}{2}$ cm and height h = 1000 cm.

∴ Volume of the wire = $\pi r^2 h$

$$= \pi^2 \cdot \frac{d^2}{4} \times 1000\ \text{cm}^3$$

$$= 250\pi d^2\ \text{cm}^3 \qquad \ldots(2)$$

We see that the volume of the copper rod

= volume of the wire

∴ From (1) and (2), we have $10\pi = 250\pi d^2$

$$\Rightarrow \quad d^2 = \frac{1}{25}$$

$$\Rightarrow \quad d = \frac{1}{5}$$

∴ Required thickness of the wire

$$= \frac{1}{5}\ \text{cm} = 0.2\ \text{cm} = \mathbf{2\ mm}$$

12. Diameter of well = 4 m

Height of well = 14 m

Volume of earth taken out = $\pi r^2 h$

$$= \pi \times 2 \times 2 \times 14$$

$$= 56\pi\ \text{m}^3$$

Let the width of embankment be w

Height of embankment = 0.4 m

$$\text{Volume} = \pi(w + 2)^2 \times 0.4 - \pi\,(2)^2 0.4$$

$$= \pi \times 0.4\,[(w + 2)^2 - (2)^2]$$

$$= 0.4\pi\,[w^2 + 4w + 4 - 4]$$

$$= 0.4\pi\,[w^2 + 4w]\ \text{m}^2$$

Volume of embankment = Volume of earth taken out

$$0.4\pi\,(w^2 + 4w) = 56\pi$$

$$w^2 + 4w = 140$$

$$w^2 + 4w - 140 = 0$$

$$w^2 + 14w^2 - 10w^2 - 140 = 0$$

$$w(w + 14) - 10(w + 14) = 0$$

$$(w - 10)\,(w + 14)$$

$$w = 10, -14$$

Since width cannot be negative therefore it is **10 m**.

13. Volume of the earth inside the cylindrical well of radius 3m and height 21 m is

$$\pi \times 3^2 \times 21\ \text{m}^3 = \frac{22}{7} \times 9 \times 21\ \text{m}^3 = 594\ \text{m}^3$$

Also, if h m be the height of the platform of size 27 m × 11 m in the shape of a cuboid, then its volume is $27 \times 11 \times h$ m^3.

$$\therefore \quad 27 \times 11 \times h = 594$$

$$\Rightarrow \quad h = \frac{594}{27 \times 11} = \frac{594}{297} = 2\ \text{m}$$

Hence, the required height of the platform is **2 m**.

14. Volume of the earth inside the cylindrical well of radius $\dfrac{3}{2}$ m and height 14 m is

$$\pi\left(\frac{3}{2}\right)^2 \times 14\ \text{m}^3 = \frac{22}{7} \times \frac{9}{4} \times 14\ \text{m}^3 = 99\ \text{m}^3 \qquad \ldots(1)$$

Let r and R be the internal and external radii and h be the height of the embankment.

Then, $r = \frac{3}{2}$ m and R $= \left(\frac{3}{2}+4\right)$m $= \frac{11}{2}$ m

∴ Volume of the embankment

$$= \pi(R^2 - r^2)h$$
$$= \frac{22}{7}\left(\frac{11^2}{4} - \frac{3^2}{4}\right)\times h \text{ m}^3$$
$$= \frac{22}{7}\times\frac{(11+3)(11-3)}{4}\times h \text{ m}^3$$
$$= \frac{22}{7}\times\frac{14\times 8}{4} h \text{ m}^3$$
$$= 88h \text{ m}^3 \qquad \ldots(2)$$

∴ From (1) and (2),

$$88h = 99$$
$$\Rightarrow \qquad h = \frac{99}{88} = \frac{9}{8}$$

Hence, the required height = $\mathbf{\frac{9}{8}}$ **m**

15. Let height of well, h = 13 m

diameter, d = 8 m

radius, $r = \frac{d}{2}$ = 4 m

Given, length of the rectangular field, l = 32 m

breadth of the rectangular field, b = 22 m

Now, surface area of field = $l \times b$

= 32 × 22

= 704 m²

Surface area of rectangular field

$$= \pi r^2 h + \pi r^2$$
$$= \frac{22}{7}\times(4)^2\times 13 + \frac{22}{7}\times(4)^2$$
$$= 704 \text{ m}^2$$

Now, rise in level of water $= \frac{704}{704}$ = **1 m**

16. Height of hollow cylinder, h = 3 cm

Inner radius, r_1 = 1.1 cm

External radius, r_2 = 4.3 cm

Height of the solid cylinder, h_1 = 9 cm

Let r be the radius of the solid cylinder.

Then, volume of solid cylinder

= volume of hollow cylinder

$$\Rightarrow \qquad \pi r^2 h_1 = \pi(r_2^2 - r_1^2)h$$
$$\Rightarrow \qquad r^2 \times 9 = [(4.3)^2 - (1.1)^2] \times 3$$
$$\Rightarrow \qquad r^2 = \frac{3.2\times 5.4}{3}$$
$$= \frac{17.28}{3} = 5.76$$
$$\Rightarrow \qquad r = 2.4 \text{ cm}$$

Hence, radius of solid cylinder is **2.4 cm**.

17. Let the required rainfall be x cm $= \frac{x}{100}$ m

Then the volume of rain water collected on the roof of the building

$$= \frac{(22\times 20\times x)}{100} \text{ m}^3 = 4.4x \text{ m}^3 \ldots(1)$$

Volume of the cylindrical vessel of radius, $\frac{2}{2}$ m = 1 m and height 3.5 m is $\pi \times (1)^2 \times 3.5$ m³.

$$= \frac{22}{7}\times 3.5 \text{ m}^3$$
$$= 11 \text{ m}^3 \qquad \ldots(2)$$

∴ From (1) and (2),

$$4.4x = 11$$
$$\Rightarrow \qquad x = \frac{11}{4.4} = \frac{110}{44} = 2.5 \text{ cm}$$

Hence, the required rainfall is **2.5 cm**.

18. Volume of the solid rectangular block

$$= 4.4 \times 2.6 \times 1 \text{ m}^3$$
$$= 11.44 \text{ m}^3 \qquad \ldots(1)$$

Let the length of the hollow cylindrical pipe be x m and let r m and R m be the internal radius and the external radius respectively of the pipe.

Then $\qquad r = \frac{30}{100}$ m = 0.3 m

And $\qquad R = \frac{30+5}{100}$ m = 0.35 m

∴ Volume of the pipe = $\pi(R^2 - r^2)x$ m³

$$= \frac{22}{7}\times\left(0.35^2 - 0.3^2\right)x \text{ m}^3$$
$$= \frac{22}{7}\times(0.35+0.3)(0.35-0.3)\,x \text{ m}^3$$
$$= \frac{22}{7}\times 0.65\times 0.05\,x \text{ m}^3 \qquad \ldots(2)$$

∴ From (1) and (2), we have

$$\frac{22}{7}\times 0.65\times 0.05\,x = 11.44$$
$$\Rightarrow \qquad x = \frac{7\times 11.44}{22\times 0.0325}$$
$$= \frac{7.28}{0.0650}$$
$$= \frac{7280}{65}$$
$$= 112$$

Hence, the required length of the pipe is **112 m**.

19. Volumes of the three small spheres are $\frac{4}{3}\pi(6)^3$ cm³, $\frac{4}{3}\pi(8)^3$ cm³ and $\frac{4}{3}\pi(10)^3$ cm³

$\therefore$ The sum of these three volumes

$$= \frac{4}{3}\pi\left(6^3 + 8^3 + 10^3\right)^3 \text{ cm}^3 \quad \ldots(1)$$

If R cm be the radius of the bigger resulting sphere, then its volume

$$= \frac{4}{3}\pi R^3 \text{ cm}^3 \quad \ldots(2)$$

$\therefore$ From (1) and (2), we have

$$\frac{4}{3}\pi R^3 = \frac{4}{3}\pi\left(6^3 + 8^3 + 10^3\right)$$

$\Rightarrow$ $R^3 = 6^3 + 8^3 + 10^3$

$= (6 + 8 + 10)(6^2 + 8^2 + 10^2 - 6 \times 8 - 6 \times 10 - 8 \times 10 + 3 \times 6 \times 8 \times 10)$

[Using the formula: $a^3 + b^3 + c^3 = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca) + 3abc$]

$= 24(36 + 64 + 100 - 48 - 60 - 80) + 1440$

$= 24 \times 12 + 1440$

$= 1728$

$= 3^3 \times 4^3$

$= 12^3$

$\therefore$ R = 12

Hence, the required radius of the resulting sphere is **12 cm**.

20. Let the radius of the third small ball be r cm. The radii of two given smaller balls are $\frac{1}{2}$ cm and $\frac{3}{4}$ cm.

$\therefore$ The volumes of these three small balls are

$$\frac{4}{3}\pi r^3 \text{ cm}^3, \frac{4}{3}\pi\left(\frac{1}{2}\right)^3 \text{ cm}^3 \text{ and } \frac{4}{3}\pi\left(\frac{3}{4}\right)^3 \text{ cm}^3.$$

$\therefore$ Sum of the volumes of these three balls

$$= \frac{4}{3}\pi\left[r^3 + \frac{1}{8} + \frac{27}{64}\right] \text{ cm}^3$$

$$= \frac{4}{3}\pi\left[\frac{64r^3 + 8 + 27}{64}\right] \text{ cm}^3$$

$$= \frac{4}{3}\pi\left[\frac{64r^3 + 35}{64}\right] \text{cm}^3$$

Also, the volume of the big spherical ball

$$= \frac{4}{3}\pi \times \left(\frac{3}{2}\right)^3 \text{cm}^3 = \frac{4}{3}\pi \times \frac{27}{8} \text{ cm}^3$$

$$\therefore \frac{4}{3}\pi \times \frac{\left(64r^3 + 35\right)}{64} = \frac{4}{3}\pi\left(\frac{27}{8}\right)$$

$$\Rightarrow r^3 = \frac{27}{8} - \frac{35}{64}$$

$$= \frac{216 - 35}{64} = \frac{181}{4^3}$$

$$\therefore r = \frac{(181)^{1/3}}{4}$$

$$\therefore \text{Required diameter} = \frac{2 \times (181)^{1/3}}{4} \text{ cm}$$

$$= \mathbf{\frac{(181)^{1/3}}{2} \text{ cm}}$$

21. Weight of spheres = 1 kg and 7 kg

Radius of smaller sphere = 3 cm

Volume of smaller sphere $= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \pi \times 3 \times 3 \times 3$

$= 36\pi \text{ cm}^3$

Density of metal $= \frac{m}{v} = \frac{1000}{36\pi}$ gm/cm^3

Volume of sphere $= \frac{\text{Mass}}{\text{Density of metal}}$

$= \frac{7000}{1000} \times 36\pi$

$= 252\pi \text{ cm}^3$

Volume of new sphere = volume of two older sphere

$\frac{4}{3}\pi r^3 = 36\pi + 252\pi$

$= 288\,\pi \text{ cm}^3$

$r^3 = \frac{288 \times 3}{4} = 216$

$r = 6$ cm

$\Rightarrow$ Diameter = **12 cm**

22. Volume of the big solid sphere of radius 3 cm is

$$V = \frac{4}{3}\pi 3^3 \text{ cm}^3 = \frac{4}{3}\pi(27) \text{ cm}^3 \quad \ldots(1)$$

Volume of each small spherical both of radius $\frac{0.6}{2}$ cm = 0.3 cm is

$$V_1 = \frac{4}{3}\pi(0.3)^3 \text{ cm}^3$$

$$= \frac{4}{3}\pi \times 0.027 \text{ cm}^3 \quad \ldots(2)$$

$\therefore$ Required numbers of small balls

$$= \frac{V}{V_1}$$

$$= \frac{\frac{4}{3}\pi \times 2.7}{\frac{4}{3}\pi \times 0.027} \quad \text{[From (1) and (2)]}$$

$$= \frac{27000}{27}$$

$= \mathbf{1000}$

23. Let, radius of sphere be r.

Then, radius of small ball $= \frac{1}{8}r$

Now, Volume of sphere = n (volume of small balls)

$$n = \frac{\text{Volume of sphere}}{\text{Volume of small ball}}$$

$$= \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi\left(\frac{1}{8}r\right)^3} = \textbf{512 balls}$$

24. Volume of the solid cube of edge 44 cm is $V = 44^3$ cm^3 …(1)

Volume of each small spherical bullet of radius $\frac{4}{2}$ cm = 2 cm is

$$V_1 = \frac{4}{3}\pi \times 2^3 \text{ cm}^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times 8 \text{ cm}^3$$

$$= \frac{88 \times 8}{21} \text{ cm}^3 \quad \text{…(2)}$$

∴ Required no. of bullets

$$= \frac{V}{V_1}$$

$$= \frac{44^3 \times 21}{88 \times 8}$$

$$= \frac{44 \times 44 \times 44 \times 21}{88 \times 8}$$

= **2541**

25. Volume of the cuboidal lead solid, $V = 9 \times 11 \times 12$ cm^3

Volume of each small spherical shots of radius $\frac{3}{2}$ cm is

$$V_1 = \frac{4}{3}\pi\left(\frac{3}{2}\right)^3 \text{ cm}^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times \frac{27}{8} \text{ cm}^3$$

$$= \frac{99}{7} \text{ cm}^3$$

∴ Required number of shots

$$= \frac{V}{V_1} = \frac{9 \times 11 \times 12 \times 7}{99} = \textbf{84}.$$

26. Diameter of spherical lead shots = 6 cm

Volume of 1 lead shot $= \frac{4}{3}\pi r^3$

$$= \frac{4}{3} \times \pi \times 3 \times 3 \times 3$$

$$= 36\pi \text{ cm}^3$$

Dimensions of cuboid = 24 cm × 22 cm × 12 cm

Volume of cuboid = 24 × 22 × 12 = 6336 cm^3

$$\text{No. of lead shots} = \frac{\text{Volume of cuboid}}{\text{Volume of 1 lead shot}}$$

$$= \frac{6336}{36\pi}$$

$$= \frac{6336 \times 7}{36 \times 22}$$

$$= \frac{176 \times 7}{22} = \textbf{56}$$

27. Volume of the solid rectangular lead piece of dimensions 66 cm, 42 cm and 21 cm is $V = 66 \times 42$ cm $\times 21$ cm^3.

Volume of each small spherical lead shot of radius $\frac{4.2}{2}$ cm = 2.1 cm is

$$V_1 = \frac{4}{3}\pi \times (2.1)^3 \text{ cm}^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times 2.1 \times 2.1 \times 2.1 \text{ cm}^3$$

$$= 88 \times 0.441 \text{ cm}^3$$

∴ Required no. of spherical lead shots

$$= \frac{V}{V_1}$$

$$= \frac{66 \times 42 \times 21}{88 \times 0.441}$$

$$= \frac{6 \times 42000}{8 \times 21} = \textbf{1500}$$

28. Let h be the height of cylinder, $h = 2.5$ mm $= 0.25$ cm

Radius of cylinder, $r = 12$ cm

Radius of sphere be r_1.

Then, volume of cylinder = volume of sphere

$$\Rightarrow \quad \pi r^2 h = \frac{4}{3}\pi r_1^3$$

$$\Rightarrow \quad r_1^3 = \frac{3r^2 h}{4}$$

$$\Rightarrow \quad r_1^3 = \frac{(12)^2 \times 0.25 \times 3}{4}$$

$$\Rightarrow \quad r_1^3 = 27$$

$$\Rightarrow \quad r_1 = \textbf{3 cm}$$

29. Let diameter of cylinder be d.

radius of cylinder, $r = \frac{d}{2}$ cm

height of cylinder, $h = \frac{2}{3} \times d = \frac{2d}{3}$

radius of sphere, $r_1 = 4$ cm

Then, volume of cylinder = volume of sphere

$$\Rightarrow \quad \pi r^2 h = \frac{4}{3}\pi r_1^3$$

$$\Rightarrow \quad \left(\frac{d}{2}\right)^2 \times \frac{2d}{3} = \frac{4}{3} \times (4)^3$$

$$\Rightarrow \quad d^3 = 4 \times 4 \times 4 \times 4 \times 2 = 512$$

$$\Rightarrow \quad d = 8 \text{ m}$$

Now, radius of the base of cylinder,

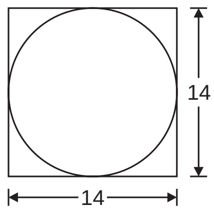
$$r = \frac{d}{2} = \frac{8}{2} = \textbf{4 cm}$$

30. Let, h be the height of cylinder.

Now, radius of cylinder, $r = 7$ cm.

Diameter of sphere, $d = 14$ cm

$$\text{Radius of sphere } r = \frac{d}{2} = 7 \text{ cm.}$$



$$\text{Volume of sphere} = \frac{4}{3}\pi r^3$$
$$= \frac{4}{3} \times \frac{22}{7} \times (7)^3$$
$$= 1437.33 \text{ cm}^3.$$

Hence, volume of sphere is **1437.33 cm³**.

31. Let, diameter of ball = 21 cm

$$\text{Radius of ball, } r = \frac{21}{2} \text{ cm}$$

$$\text{Now, Volume of spherical ball} = \frac{4}{3}\pi r^3$$
$$= \frac{4}{3} \times \frac{22}{7} \times \left(\frac{21}{2}\right)^3$$
$$= 4851 \text{ cm}^3$$

Edge of the cube = 1 cm.

Then, volume of cube = (1) (1) (1) = 1 cm³.

Now, number of cube formed

$$= \frac{\text{Volume of spherical ball}}{\text{Volume of cube}}$$
$$= \frac{4851}{1} = 4851$$

Hence, number of cubes formed are **4851**.

32. Radius of copper sphere, $r = \frac{18 \text{ cm}}{2} = 9$ cm.

Let R be the radius of the copper cylindrical wire.

Height of the cylindrical copper wire, h = 108 m = 10800 cm.

Then, volume of copper sphere = volume of cylindrical copper wire

$$\frac{4}{3}\pi r^3 = \pi R^2 h$$
$$\frac{4}{3} \times (9 \text{ cm})^3 = R^2 (10800 \text{ cm})$$
$$R^2 = \frac{4}{3} \times \frac{(9 \text{ cm})^3}{10800 \text{ cm}}$$
$$= \sqrt{\frac{2916}{32400}}$$
$$R = 0.3 \text{ cm}$$

Now, diameter of the cylindrical copper wire

$$= 2R = 2 \times 0.3 \text{ cm} = \mathbf{0.6 \text{ cm}}$$

33. Radius of the metallic sphere $= \frac{6}{2}$ cm = 3 cm

$$\therefore \text{ Volume of the sphere} = \frac{4}{3}\pi(3)^3 \text{ cm}^3 = 36\pi \text{ cm}^3 \quad \ldots(1)$$

Length of the cylindrical wire = 36 m = 3600 cm

Let the radius of the wire be r cm

∴ Volume of the cylindrical wire

$$= \pi r^2 (3600) \text{ cm}^3 \quad \ldots(2)$$

∴ From (1) and (2),

$$\pi r^2(3600) = 36\pi$$
$$\Rightarrow \quad r^2 = \frac{36}{3600} = \frac{1}{100}$$
$$\therefore \quad r = \frac{1}{10} = 0.1$$

Hence, the required radius of the wire is 0.1 cm or **1 mm**.

34. (*i*) Radius of the solid sphere, r = 6 cm.

External radius of the hollow cylinder, r_1 = 5 cm.

Height of the hollow cylinder, h = 32 cm.

Let r_2 be the internal radius of the hollow cylinder.

Volume of the solid sphere = Volume of the hollow cylinder

$$\frac{4}{3}\pi r^3 = \pi h(r_1^2 - r_2^2)$$
$$\Rightarrow \quad \frac{4}{3}(6 \text{ cm})^3 = 32 \text{ cm}(5^2 - r_2^2)$$
$$\Rightarrow \quad r_2^2 = 5^2 - \frac{4 \times 6^3}{3 \times 32 \text{ cm}}$$
$$\Rightarrow \quad r_2^2 = 25 - 9 = 16$$
$$\Rightarrow \quad r_2 = 4 \text{ cm}$$

Thus, the thickness of the hollow cylinder is

$$d = r_1 - r_2$$
$$= 5 \text{ cm} - 4 \text{ cm} = \mathbf{1 \text{ cm}}$$

(*ii*) External radius of the hollow cylinder, r_1 = 4 cm

Height of the hollow cylinder, h = 24 cm

Thickness of the hollow cylinder, d = 2 cm

Let, r_2 be the internal radius of the hollow cylinder Then,

$$r_2 = r_1 - d = 4 \text{ cm} - 2 \text{ cm} = 2 \text{ cm}$$

Let r be the radius of the solid sphere.

Now, volume of the solid sphere = volume of the hollow cylinder

$$\Rightarrow \quad \frac{4}{3}\pi r^3 = \pi h(r_1^2 - r_2^2)$$
$$\Rightarrow \quad r^3 = \frac{3h}{4}(r_1^2 - r_2^2) = \frac{3 \times 24 \text{ cm} \times (4^2 - 2^2)}{4}$$
$$\Rightarrow \quad r^3 = 216$$
$$\Rightarrow \quad r = 6 \text{ cm}$$

Hence, the radius of the solid sphere is **6 cm**.

35. Volume of the hollow spherical shell of internal and external radii 3 cm and 5 cm respectively is

$$\frac{4}{3}\pi(5^3 - 3^3)\text{ cm}^3 = \frac{4\pi}{3}\times 98\text{ cm}^3 \qquad \ldots(1)$$

The radius of the solid cylinder = $\frac{14}{2}$ cm = 7 cm

∴ Volume of the solid cylinder = $\pi \times 7^2 \times h$

$= 49\pi h\text{ cm}^3 \qquad \ldots(2)$

where h cm is the height of the cylinder.

∴ From (1) and (2), we have

$$49\pi h = 98\pi \times \frac{4}{3}$$

$$\Rightarrow \quad h = \frac{8}{3}$$

∴ Required height of the cylinder is $\mathbf{\frac{8}{3}}$ **cm**.

36. Given, diameter of external sphere = 10 cm

Diameter of internal sphere = 6 cm

Now, external radius, $r_1 = \frac{10}{2} = 5$ cm

Internal radius, $r_2 = \frac{6}{2} = 3$ cm

Also, length of cylinder, $h = 2\frac{2}{3} = \frac{8}{3}$ cm

Let r be radius of cylinder.

Now,volume of cylinder

= volume of sphere

$$\Rightarrow \quad \pi r^2 h = \frac{4}{3}\pi(r_1^3 - r_2^3)$$

$$\Rightarrow \quad r^2 \times \frac{8}{3} = \frac{4}{3}(5^3 - 3^3)$$

$$\Rightarrow \quad r^2 = \frac{4}{8}(125 - 27) = 49\text{ cm}$$

$$\Rightarrow \quad r = 7\text{ cm}$$

Hence, diameter of cylinder = $2r = 2 \times 7$ cm = **14 cm**

37. Let r cm and R cm be respectively the internal and external radii of the spherical shell. Then R = $\frac{18}{2}$ cm = 9 cm.

∴ Volume of the spherical shell

$$= \frac{4}{3}\pi\left(9^3 - r^3\right)\text{ cm}^3$$

$$= \frac{4\pi}{3}\left(729 - r^3\right)\text{ cm}^3 \qquad \ldots(1)$$

Also, volume of the cylinder

$= \pi \times 6^2 \times 19\text{ cm}^3$

$= 36 \times 19\pi\text{ cm}^3 \qquad \ldots(2)$

$$\therefore \quad \frac{4\pi}{3}\left(729 - r^3\right) = 36 \times 19\pi \qquad \text{[From (1) and (2)]}$$

$$\Rightarrow \quad \frac{4}{3}\times 729 - \frac{4}{3}r^3 = 36 \times 19$$

$$\Rightarrow \quad 4 \times 729 - 108 \times 19 = 4r^3$$

$$\Rightarrow \quad r^3 = \frac{2916 - 2052}{4}$$

$$= \frac{864}{4}$$

$$= 213 = 6^3$$

$$\therefore \quad r = 6$$

∴ Required diameter is 6 × 2 cm = **12 cm**

38. Inner radius of hemisphere, $r_1 = 9$ cm

Radius of cylinder, $r = 6$ cm

Let, height of cylinder be h.

Now, volume of cylinder = volume of hemisphere

$$\Rightarrow \quad \pi r^2 h = \frac{2}{3}\pi r_1^3$$

$$\Rightarrow \quad (6)^2 h = \frac{2}{3}\times(9)^3$$

$$\Rightarrow \quad h = \frac{2\times 729}{36\times 3} = \frac{27}{2} = 13.5\text{ cm.}$$

Hence, the height to which the water rises in the cylindrical vessel is **13.5 cm.**

39. Volume of the hemispherical bowl of internal radius 9 cm is $\frac{2}{3}\pi 9^3\text{ cm}^3 = \frac{2}{3}\times 729\,\pi\text{ cm}^3 = 486\pi\text{ cm}^3$

Volume of each small cylindrical bottles of radius $\frac{3}{2}$ cm and height 4 cm is $\pi\left(\frac{3}{2}\right)^2\times 4\text{ cm}^3 = 9\pi\text{ cm}^3$

∴ Required. no of small bottles = $\frac{486\pi}{9\pi}$ = **54**

40. Volume of the hemispherical bowl of internal radius $\frac{30}{2}$ cm = 15 cm is $\frac{2}{3}\pi\times 15^3\text{ cm}^3 = 2250\pi\text{ cm}^3$

Volume of each small cylindrical bottles of radius $\frac{5}{2}$ cm and height 6 cm is

$$\pi\left(\frac{5}{2}\right)^2 6\text{ cm}^3 = \frac{75}{2}\pi\text{ cm}^3$$

Hence, the required no. of small bottles

$$= \frac{2250\pi\times 2}{75\pi} = \mathbf{60}$$

41. Diameter of hemispherical bowl = 36 cm

Volume of bowl = $2\pi r^3$

$= \frac{2}{3} \times \pi \times 18 \times 18 \times 18$

$= 3888\pi\text{ cm}^3$

Volume of liquid wasted = $\frac{10}{100} \times 3888\pi$

$= 388.8\,\pi\text{ cm}^3$

Liquid remaining after wastage = $3499.2\pi\text{ cm}^3$

Diameter of bottles = 6 cm

No. of bottles = 72

Height of bottles = h

Volume of 72 bottles = $72 \times \pi r^2 h$

$= 72 \times \pi \times 3 \times 3 \times h$

$= 648h\pi \text{ cm}^3$

Volume of 72 bottles = volume of liquid

$648\, h\pi = 3499.2\pi$

$h = \frac{3499.2}{648} = \mathbf{5.4\ cm}$

42. Height of cylinder = 10 cm

Radius of cylinder = 4.2 cm

Radius of hemisphere scooped out = 4.2 cm

Volume of cylinder = $\pi r^2 h$

$= \pi \times 4.2 \times 4.2 \times 10$

$= 176.4\, \pi \text{ cm}^3$

Volume of hemisphere scooped out = $\frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \pi \times 4.2 \times 4.2 \times 4.2$

$= 49.392\, \pi \text{ cm}^3$

Volume of remaining structure = Volume of cylinder − 2(Volume of hemisphere)

$= 176.4\pi - 2 \times 49.392\pi$

$= 77.616\pi \text{ cm}^3$

Thickness of wire = 1.4 cm

Length of wire = h

Volume of wire = $\pi r^2 h$

$= \pi \times 0.7 \times 0.7 \times h$

$= 0.49\pi\, h \text{ cm}^3$

Volume of wire = Volume of remaining structure

$0.49\pi\, h = 77.616\pi$

$h = \frac{77.616}{0.49}$

$h = \mathbf{158.4\ cm}$

43. Let h be height of cylindrical bucket, h = 32 cm

r be radius of bucket, r = 16 cm

Height of conical heap, h_1 = 24 cm.

Let r_1 be the height of the cone.

(*i*) Volume of cone = Volume of cylinder

$\Rightarrow \quad \frac{1}{3}\pi r_1^2 h_1 = \pi r^2 h$

$\Rightarrow \quad r_1^2 = \frac{3 \times (16)^2 \times 32}{24} = 1024$

$\Rightarrow \quad r_1 = \mathbf{32\ cm}$

(*ii*) Slant height, $l = \sqrt{r_1^2 + h_1^2}$

$= \sqrt{(32)^2 + (24)^2}$

$= \sqrt{1024 + 576} = \sqrt{1600}$

$= \mathbf{40\ cm}$

44. Let, r be radius of cylinder, $r = \frac{21}{2}$ cm

Also, height of cylindrical vessel, h = 38 cm

Radius of cone, $r_1 = \frac{7}{2}$ cm

Height of cone, h_1 = 12 cm

Now, no. of ice cream

$$= \frac{\text{Volume of 4 cylindrical vessel}}{\text{Volume of ice cream (cone + hemisphere)}}$$

$$= \frac{4 \times \pi r^2 h}{\frac{1}{3}\pi r_1^2 h_1 + \frac{2}{3}\pi r_1^3} = \frac{4\pi r^2 h}{\frac{1}{3}[h_1 + 2r_1]\pi r_1^2}$$

$$= \frac{4\left(\frac{21}{2}\right)^2 \times 38}{\frac{1}{3}\left(\frac{7}{2}\right)^2\left(12 + \frac{2 \times 7}{2}\right)} = \mathbf{216}$$

45. Radius of the spherical cannon ball = $\frac{28}{2}$ cm = 14 cm

∴ Volume of the spherical cannon ball

$= \frac{4}{3}\pi \times 14^3 \text{ cm}^3$ …(1)

Radius of the base of the conical mould = $\frac{35}{2}$ cm

Let h cm be the height of the conical mould

∴ Volume of the conical mould = $\frac{1}{3}\pi\left(\frac{35}{2}\right)^2 h \text{ cm}^3$ …(2)

∴ From (1) and (2), we have

$$\frac{\pi}{3} \times \frac{35^2}{4} h = \frac{4\pi}{3} \times 14 \times 14 \times 14$$

$$\Rightarrow \quad h = \frac{16 \times 14 \times 14 \times 14}{35 \times 35}$$

$$= \frac{56 \times 16}{25} = 35.84$$

Hence, the required height of the cone is **35.84 cm**.

46. Volume of the sphere = $\frac{4}{3}\pi(5.6)^3 \text{ cm}^3$ …(1)

Volume of each small solid cone

$= \frac{1}{3}\pi(2.8)^2 \times 3.2 \text{ cm}^3$ …(2)

∴ Required no. of cones

$$= \frac{\frac{4}{3}\pi(5.6)^3}{\frac{\pi}{3}(2.8)^2 \times 3.2}$$ [Dividing (1) by (2)]

$$= \frac{4 \times 5.6 \times 5.6 \times 5.6}{2.8 \times 2.8 \times 3.2}$$

$$= \frac{4 \times 56 \times 56 \times 56}{28 \times 28 \times 32} = 28$$

∴ Required no. of cones = **28**

47. Let r cm be the radius of the solid sphere. Then its surface area = $4\pi r^2$ cm^2

$$\therefore \quad 4\pi r^2 = 616 \quad \text{[Given]}$$

$$\Rightarrow \quad 4 \times \frac{22}{7} \times r^2 = 616$$

$$\Rightarrow \quad r^2 = \frac{7 \times 616}{88} = 7^2$$

$$\therefore \quad r = 7$$

$\therefore$ Radius of the sphere is 7 cm.

$\therefore$ Volume of the sphere = $\frac{4}{3} \times \pi \times 7^3$ cm^3 ...(1)

Let r_1 cm be the radius of base of the cone.

Then the volume of the cone = $\frac{1}{3}\pi r_1^2 \times 28$ cm^3 ...(2)

$\therefore$ From (1) and (2), we have

$$\frac{28\pi}{3} r_1^2 = \frac{4\pi}{3} \times 7^3$$

$$\therefore \quad r_1^2 = \frac{4 \times 7 \times 7 \times 7}{28} = 7^2$$

$$\Rightarrow \quad r_1 = 7$$

$\therefore$ Required diameter of the base of the cone is 2×7 cm = **14 cm**.

48. Let radius of the sphere, r = 10.5 cm

Diameter of small cone = 3.5 cm

Radius of each cone, $r_1 = \frac{3.5}{2}$ cm

Height of each cone, h = 3 cm

Now, number of cones = $\frac{\text{Volume of sphere}}{\text{Volume of each cone}}$

$$= \frac{\frac{4}{3}\pi r^3}{\frac{1}{3}\pi r_1^2 h} = \frac{\frac{4}{3}\pi (10.5)^3}{\frac{1}{3}\pi\left(\frac{3.5}{2}\right)^2 \times 3} = 504$$

Hence, number of cones is **504**.

49. Radius of three spheres are

$$r_1 = 6 \text{ cm}, r_2 = 8 \text{ cm}, r_3 = 10 \text{ cm}$$

Radius of cone, r = 12 cm

Volume of cone = Volume of three spheres

$$\frac{1}{3}\pi r^2 h = \frac{4}{3}\pi(r_1^3 + r_2^3 + r_3^3)$$

$$(12)^2 h = 4\,(6^3 + 8^3 + 10^3)$$

$$h = \frac{4 \times (216 + 512 + 1000)}{12 \times 12} = 48 \text{ cm}$$

Hence, height of cone is **48 cm**.

50. The internal and external radii of the hollow sphere are respectively $\frac{4}{2}$ cm = 2 cm and $\frac{8}{2}$ cm = 4 cm.

$\therefore$ Volume of the hollow sphere

$$= \frac{4}{3} \times \pi\left(4^3 - 2^3\right) \text{ cm}^3$$

$$= \frac{4 \times 56\pi}{3} \text{ cm}^3 \quad ...(1)$$

Let the radius of the base of the cone be r_1 cm, and the vertical height and slant height of the cone be h cm and l cm respectively.

Then $\quad r_1 = \frac{8}{2}$ cm = 4 cm

$\therefore$ Volume of the cone = $\frac{1}{3}\pi \times 4^2 h$ cm^3

$$= \frac{16\pi h}{3} \text{ cm}^3 \quad ...(2)$$

$\therefore$ From (1) and (2), we have

$$\frac{16\pi h}{3} = 4 \times \frac{56\pi}{3}$$

$$\Rightarrow \quad h = \frac{4 \times 56}{16} = 14$$

$\therefore$ Required height of the cone is **14 cm**.

Now, $\quad l = \sqrt{r_1^2 + h^2}$

$$= \sqrt{4^2 + 14^2}$$

$$= \sqrt{16 + 96}$$

$$= \sqrt{212}$$

$$= \sqrt{4 \times 53}$$

$$= 2\sqrt{53}$$

Hence, the required slant height of the cone is $\mathbf{2\sqrt{53}}$ **cm**.

51. External diameter of spherical shell = 18 cm

External radius of spherical shell, $r_1 = \frac{18 \text{ cm}}{2}$ = 9 cm

Radius of cone, r = 14 cm

Height of cone, $h = 4\frac{3}{7} = \frac{31}{7}$ cm

Let r_2 be the internal radius of the spherical shell.

Now, volume of spherical shell = volume of cone

$$\Rightarrow \quad \frac{4}{3}\pi(r_1^3 - r_2^3) = \frac{1}{3}\pi r^2 h$$

$$\Rightarrow \quad \frac{4}{3}(9^3 - r_2^3) = \frac{1}{3} \times (14)^2 \times \frac{31}{7}$$

$$\Rightarrow \quad 729 - r_2^3 = \frac{14 \times 14 \times 31}{4 \times 7} = 217$$

$$\Rightarrow \quad r_2^3 = 729 - 217 = 512$$

$$\Rightarrow \quad r_2 = 8 \text{ cm}$$

Hence, inner diameter of shell = 8 × 2 = **16 cm**

52. The internal and external radii of the hollow hemisphere are respectively $\frac{6}{3}$ cm = 3 cm and $\frac{10}{2}$ cm = 5 cm.

$\therefore$ Volume of this hollow hemisphere = $\frac{2}{3}\pi\,(5^3 - 3^3)$ cm^3

$$= \frac{2}{3}\pi\,(125 - 27) \text{ cm}^3$$

$$= \frac{2\pi}{3} \times 98 \text{ cm}^3 \quad ...(1)$$

The radius of the base of the solid cone = $\frac{14}{2}$ cm = 7 cm

Let h cm be the height of this cone.

Then volume of this cone

$$= \frac{1}{3}\pi \times 7^2 \times h \text{ cm}^3$$

$$= \frac{49\pi}{3} h \text{ cm}^3 \quad \ldots(2)$$

$\therefore$ From (1) and (2), we have

$$\frac{49\pi}{3} h = \frac{2\pi}{3} \times 98$$

$$\Rightarrow \quad h = \frac{2 \times 98}{49} = 4$$

Hence, the required height of the cone is **4 cm**.

53. Radius of hemisphere, $r_1 = 8$ cm

Radius of cone, $r = 6$ cm

Let, height of cone be h.

Now, volume of cone = volume of hemisphere

$$\Rightarrow \quad \frac{1}{3}\pi r^2 h = \frac{2}{3}\pi r_1^3$$

$$\Rightarrow \quad (6)^2 h = 2\,(8)^3$$

$$\Rightarrow \quad h = \frac{2 \times 8 \times 8 \times 8}{6 \times 6} = 28.44 \text{ cm}$$

Hence, height of the cone is **28.44 cm.**

54. Volume of the cone with base radius 5 cm and height 20 cm is

$$\frac{1}{3}\pi \times 5^2 \times 20 \text{ cm}^3 = \frac{\pi}{3} \times 500 \text{ cm}^3 \quad \ldots(1)$$

If this cone is converted to a sphere of radius r cm, then the volume of the sphere will be equal to that of the cone.

Now, volume of the sphere $= \frac{4}{3}\pi r^3 \quad \ldots(2)$

$$\frac{4}{3}\pi r^3 = \frac{\pi}{3} \times 500$$

$$\Rightarrow \quad r^3 = \frac{500}{4} = 125 = 5^3$$

$$\therefore \quad r = 5$$

$\therefore$ Required diameter of the sphere is 5×2 cm = **10 cm**.

55. (*i*) Height of cone, $h = 24$ cm

Base radius of cone, $r = 6$ cm

Let the radius of sphere be r_1

Volume of sphere = Volume of cone

$$\Rightarrow \quad \frac{4}{3}\pi r_1^3 = \frac{1}{3}\pi r^2 h$$

$$\Rightarrow \quad r_1^3 = \frac{(6)^2 \times 24}{4} = 6^3$$

$$\Rightarrow \quad r_1 = 6 \text{ cm}$$

Hence, radius of sphere is **6 cm**.

(*ii*) No. of cones = 504

Diameter of cone = 3.5 cm

Height of cone = 3 cm

Volume of 1 cone $= \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times \pi \times \frac{35}{20} \times \frac{35}{20} \times 3$$

$$= \frac{49\pi}{16} \text{ cm}^3$$

Volume of 504 cone $= 504 \times \frac{49\pi}{16}$

Volume of sphere = Volume of 504 cones

$$\frac{4}{3}\pi r^3 = 504 \times \frac{49\pi}{16}$$

$$r^3 = \frac{504 \times 49 \times 3}{16 \times 4}$$

$$r = \sqrt[3]{1157.625}$$

$$r = 10.5 \text{ cm}$$

Diameter of sphere $= 2 \times r = 2 \times 10.5 =$ **21 cm**

Surface area of sphere $= 4\pi r^2$

$$= 4 \times \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2}$$

$$= 22 \times 63$$

$$= \mathbf{1386\ cm^2}$$

56. Volume of the solid cone

$$= \frac{1}{3}\pi \times 12^2 \times 24 \text{ cm}^3$$

$$= \frac{\pi}{3} \times 144 \times 24 \text{ cm}^3 \quad \ldots(1)$$

Radius of each spherical ball $= \frac{6}{2}$ cm = 3 cm

$\therefore$ Volume of each small spherical ball

$$= \frac{4}{3}\pi 3^3 \text{ cm}^3$$

$$= \frac{\pi}{3} \times 108 \text{ cm}^3 \quad \ldots(2)$$

$\therefore$ From (1) and (2), we get

The required no. of small balls $= \dfrac{\frac{\pi}{3} \times 144 \times 24}{\frac{\pi}{3} \times 108} =$ **32.**

57. We see that the volume of $\frac{3}{4}$th part of the conical vessel with internal radius 5 cm and height 24 cm is equal to the volume of a cylindrical vessel with internal radius 10 cm and height h cm.

Now, volume V_1 of conical vessel is given by

$$V_1 = \frac{1}{3}\pi \times 5^2 \times 24 \text{ cm}^3$$

$$= \frac{\pi}{3} \times 25 \times 24 \text{ cm}^3 \quad \ldots(1)$$

Also, volume V_2 of the cylindrical vessel of height h cm is given by

$$V_2 = \pi \times 10^2 h \text{ cm}^3$$

$$= 100\pi h \text{ cm}^3 \quad \ldots(2)$$

$\therefore$ From (1) and (2), we have

$$\frac{3}{4} V_1 = V_2$$

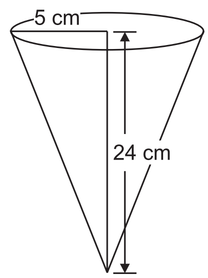
$$\frac{3}{4} \times \frac{\pi}{3} \times 25 \times 24 = \pi \times 100\,h$$

$$\Rightarrow \quad h = \frac{25 \times 24}{400} = \frac{3}{2}$$

$\therefore$ Required height of the cylindrical vessel in $\frac{3}{2}$ cm = **1.5 cm**.

58. Radius of the vessel = 5 cm

Height of the vessel is 24 cm



Since the vessel is full of water.

Therefore volume of vessel = Volume of water

$$\text{Volume} = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3} \times \frac{22}{7} \times 5^2 \times 24$$

$$= \frac{4400}{7} \text{ cm}^3$$

Now, this volume is emptied in a cylindrical vessel.

Let the height of the cylindrical vessel is h and radius = 10 cm

$\therefore$ Volume of water in cylindrical vessel

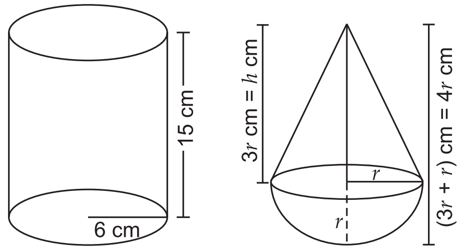
$$= \pi r^2 h$$

$$= \frac{22}{7} \times 10^2 \times h$$

$$\frac{4400}{7} = \frac{22}{7} \times 10^2 h$$

$$h = \mathbf{2\ cm}$$

59. Radius of the cylinder = $\frac{12}{2}$ cm = 6 cm and the height of the cylinder 15 cm.



(*i*) Cylinder (*ii*) Toy

$\therefore$ Volume of the cylinder

$$= \pi \times 6^2 \times 15 \text{ cm}^3$$

$$= 36 \times 15\pi \text{ cm}^3 \qquad \ldots(1)$$

Let r cm be the common radius of the base of the cone and the hemisphere as shown in Figure (*ii*).

The toy consists of a cone of height, $h = 3r$ cm

$\therefore$ Total height of the toy is

$$(h + r) \text{ cm} = (3r + r) \text{ cm} = 4r \text{ cm}$$

Now, volume of the cone

$$= \frac{1}{3}\pi r^2 \times 3r \text{ cm}^3$$

$$= \pi r^3 \text{ cm}^3 \qquad \ldots(2)$$

Also, volume of the hemisphere

$$= \frac{2}{3}\pi r^3 \text{ cm}^3 \qquad \ldots(3)$$

Since each toy consists of the cone and the hemisphere, hence, the volume of each toy

$$= \left(\pi r^3 + \frac{2}{3}\pi r^3\right) \text{cm}^3$$

$$= \frac{5\pi}{3} r^3 \text{ cm}^3$$

And so volume of 12 toys

$$= \frac{5\pi}{3} r^3 \times 12 \text{ cm}^3$$

$$= 20\pi r^3 \text{ cm}^3 \qquad \ldots(4)$$

Since, the volume of the cylinder is equal to the volume of 12 toys, hence,

$$20\pi r^3 = 36 \times 15\pi \qquad \text{[From (1) and (4)]}$$

$$\Rightarrow \quad r^3 = \frac{36 \times 15}{20} = 3^3$$

$$\Rightarrow \quad r = 3$$

$\therefore$ Total height of the toy = Height of the cone + Radius of the hemisphere

$$= (r + 3r) \text{ cm}$$

$$= 4r \text{ cm}$$

$$= 4 \times 3 \text{ cm} = 12 \text{ cm}$$

Hence, the required radius of the hemisphere and the total height of the toy are respectively **3 cm** and **12 cm**.

60. (*i*) We have,

Volume of water flowing in 30 minutes

= Volume of water collected in 30 minutes

$$\Rightarrow \quad 6 \times 1.5 \times \frac{10000}{60} \times 30 \text{ m}^3 = A \times \frac{8}{100} \text{ m}^3$$

Where A denotes the area (in m^2) which is irrigated.

$$\Rightarrow \quad \frac{270 \times 10000}{60} = \frac{8A}{100}$$

$$\Rightarrow \quad A = \frac{27000000}{8 \times 6} = 562500$$

Hence, the required area is **562500 m²**.

(*ii*) Let width of canal = 30 m

depth of canal = 12 m

Speed of water in canal = 20 km/h

$$= \frac{20 \times 5}{18} = \frac{50}{9} \text{ m/s}$$

Height of standing water = 9 cm = 9×10^{-2} m

Area irrigated in 30 min

$$= \frac{\text{Area of canal} \times \text{Speed of water} \times \text{Time}}{\text{Height of standing water level}}$$

$$= \frac{(30 \times 12) \times \frac{50}{9} \times (30 \times 60)}{9 \times 10^{-2}}$$

$$= \frac{30 \times 12 \times 50 \times 30 \times 60 \times 100}{9 \times 9}$$

$= 40000000 \text{ m}^2$

Hence, area irrigated is **40000000 m²**.

61. (*i*) The radius of the cylindrical pipe

$$= \frac{14}{2 \times 100} \text{ m} = 0.07 \text{ m}$$

∴ Volume of water flowing through the pipe in 1 hour

$$= \pi \times 0.07^2 \times 5000 \text{ m}^3 \quad \ldots(1)$$

Let the rectangular tank, 50 m long and 44 m wide be filled up by water through a height of $\frac{7}{100}$ m or 0.07 m in t hours.

Then the volume of water in the tank collected in t hours

$$= 50 \times 44 \times 0.07 \text{ m}^3 \quad \ldots(2)$$

From (1), the volume of water flowing through the pipe in t hours

$$= \pi \times 0.07^2 \times 5000\, t \text{ m}^3$$

$$= \frac{22}{7} \times 0.0049 \times 5000\, t \text{ m}^3$$

$$= \frac{22 \times 4.9 \times 5t}{7} \text{ m}^3 = 77\, t \text{ m}^3$$

∴ $77t = 50 \times 44 \times 0.07$

⇒ $t = \frac{154}{77} = 2$

∴ Required time = **2 hours**

(*ii*) Inner diameter = 20 cm = 0.2 m

Inner radius of circular pipe, $r_1 = \frac{0.2 \text{ m}}{2} = 0.1$ m

Depth of cylindrical tank, h = 2 m

Radius of cylindrical tank, $r = \frac{10}{2} = 5$ m

Time required to the cistern to be filled

$$= \frac{\text{Volume of cylindrical tank}}{\text{Speed of water} \times \text{Cross sectional area of pipe}}$$

$$= \frac{\pi r^2 h}{\frac{5}{6} \times \pi r_1^2} = \frac{\pi \times 5^2 \times 2}{\frac{5}{6} \times \pi (0.1)^2}$$

= 600

= **1 hour 40 min**

62. Diameter of pipe = 2 cm

Radius of base of tank = 40 cm

Rate of flow of water = 0.7 m/s

Time = 30 min

= 30 × 60 s

= 1800 s

1 s → 0.7 m

1800 s → 1800 × 0.7 = 1260.0 m

Volume of water flowing through pipe = $\pi r^2 h$

$= \pi \times 1 \times 1 \times 126000$

$= 126000\pi \text{ cm}^3$

Let increase in water level in tank = h cm

Volume of tank = Volume of water

$\pi r^2 h = 126000\pi$

$\pi \times 40 \times 40 \times h = 126000\, \pi$

$16h = 1260$

$h = \frac{1260}{16}$

h = **78.75 cm**

63. Diameter of tank = 3 m

Volume of hemispherical tank = $\frac{2}{3}\pi r^3$

$$= \frac{2}{3} \times \frac{22}{7} \times \frac{3}{2} \times \frac{3}{2} \times \frac{3}{2}$$

$$= \frac{99}{14} \text{ m}^3$$

$1 \text{ m}^3 = 1000$ litres

$\frac{99}{14} \text{ m}^3 = \frac{99000}{14}$ litres

Rate of emptying tank = $3\frac{4}{7}$ litre/second

1 second → $\frac{25}{7}$ litre

1 litre in $\frac{7}{25}$ second

$\frac{99000}{14}$ litre in $\frac{7}{25} \times \frac{99000}{14}$

= 1980 seconds

The time taken to empty half tank = $\frac{1980}{2}$ seconds

= **990 seconds**

64. Diameter of pipe = 4 cm

Rate of flowing water = 20 m/min

Radius of conical tank = 40 cm

Height of conical tank = 72 cm

Volume of conical tank = $\frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times \pi \times 40 \times 40 \times 72$$

$= 38400\, \pi \text{ cm}^3$

Rate of flowing water = 20 m/min

Let it take x min to fill the tank.

$$\text{Length of pipe} = 20x \text{ m}$$
$$= 20x \times 100 \text{ cm}$$
$$\text{Volume of pipe} = \pi r^2 h$$
$$= \pi \times 2 \times 2 \times 20x \times 100$$
$$= 8000\,\pi\, x \text{ cm}^3$$

Volume conical tank = Volume of pipe

$$8000\pi x = 38400\pi$$
$$x = \frac{384}{80} \text{ min}$$
$$= \frac{384}{80} \times 60 \text{ sec}$$
$$= \mathbf{288\ seconds}$$

65. Let the internal radius of the pipe be r metres.

The radius of the cylindrical tank

$$= 40 \text{ cm} = \frac{40}{100} \text{ m} = 0.4 \text{ m}$$

Length of water flowing through the pipe in $\frac{1}{2}$ hour

$$= \frac{2520}{2} = 1260 \text{ m}$$

$\therefore$ Volume of water flowing through the pipe in $\frac{1}{2}$ hour

= Volume of water collected in the cylindrical tank in $\frac{1}{2}$ hour

Now, the volume of water flowing through the pipe in $\frac{1}{2}$ hour $= \pi r^2 \times 1260 \text{ m}^3$ ...(1)

Also, the volume of water collected in the cylindrical tank in $\frac{1}{2}$ hour

$$= \pi \times (0.4)^2 \times 3.15 \text{ m}^3$$
$$= \pi \times \frac{16}{100} \times \frac{315}{100} \text{ m}^3$$
$$= \pi \times \frac{5040}{10000} \text{ m}^3 \quad \text{...(2)}$$

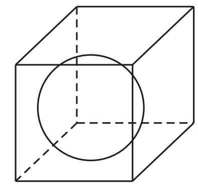
$\therefore$ From (1) and (2), we have

$$\pi r^2 \times 1260 = \pi \times \frac{5040}{10000}$$
$$\Rightarrow \quad r^2 = \frac{504}{1260 \times 1000} = \frac{1}{2500}$$
$$\therefore \quad r = \sqrt{\frac{1}{2500}} = \frac{1}{50}$$

Hence, the required diameter $= \frac{2}{50}$ m = **4 cm**

For Standard Level

66. The volume of the sphere will be largest if the sphere touches all the four faces of the cube internally. In this case, the diameter of the sphere will be equal to the edge of the cube = 7 cm.



(*i*) Hence, the radius of the sphere $= \frac{7}{2}$ cm

$\therefore$ Required volume of the sphere

$$= \frac{4}{3}\pi\left(\frac{7}{2}\right)^3 \text{ cm}^3$$
$$= \frac{4}{3} \times \frac{22}{7} \times \frac{7 \times 7 \times 7}{8} \text{ cm}^3$$
$$= \frac{539}{3} \text{ cm}^3 = \mathbf{179.67\ cm^3\ (approx.)}$$

(*ii*) After the largest sphere is carved out, the remaining volume of the cube = Original volume of the cube of side 7 cm – Volume of the sphere carved out

$$= 7 \times 7 \times 7 \text{ cm}^3 - \frac{539}{3} \text{ cm}^3$$
$$= \left(343 - \frac{539}{3}\right) \text{ cm}^3$$
$$= \frac{1029 - 539}{3} \text{ cm}^3$$
$$= \frac{490}{3} \text{ cm}^3$$

$\therefore$ Required percentage of wood wasted in the process

$$= \frac{\frac{490}{3}}{343} \times 100\%$$
$$= \frac{490 \times 100}{1029}\%$$
$$= \mathbf{\frac{1000}{24}\%}$$

67. Volume of the rectangular wall

$$= 24 \times 0.4 \times 6 \text{ m}^3$$
$$= 9.6 \times 6 \text{ m}^3$$
$$= 57.6 \text{ m}^3$$

Volume of the space occupied by the mortar

$$= \frac{1}{10} \times 57.6 \text{ m}^3$$
$$= 5.76 \text{ m}^3$$

$\therefore$ Volume of the remaining space to be occupied by the bricks

$$= (57.60 - 5.76) \text{ m}^3$$
$$= 51.84 \text{ m}^3 \quad \text{...(1)}$$

Volume of each brick $= 25 \times 16 \times 10 \text{ cm}^3$

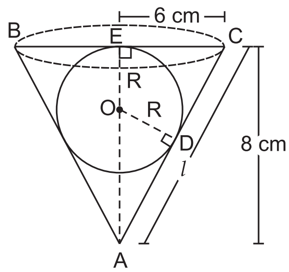
$$= 4000 \text{ cm}^3$$
$$= \frac{4000}{1000000} \text{ m}^3$$
$$= \frac{4}{1000} \text{ m}^3 \quad \text{...(2)}$$

$\therefore$ Required no. of bricks = $\dfrac{\frac{51.84}{4}}{1000}$ [From (1) and (2)]

$= \dfrac{51840}{4}$

= **12960**

68. Let r cm be the radius of the base of the cone, h cm be its vertical height and l cm be its slant height.



Then $r = 6$, $h = 8$.

$\therefore \quad l = \sqrt{r^2 + h^2}$

$= \sqrt{6^2 + 8^2}$

$= \sqrt{100}$

$= 10$

Let R cm be the radius of the sphere.

Let ABC be the cone. BC is the horizontal diameter of the base of the cone, A is its vertex and AC = AB, its slant height l cm.

This cone is full of water, BC being the horizontal level of water inside the cone.

Let O be the centre of the sphere of radius R cm, AOE $\perp$ BC and OD $\perp$ AC. This sphere is completely immersed in the water so that the upper surface BC of the water touches the sphere at E. Also, the sphere touches slant lengths AC and AB of the cone.

In this Figure, EC = r cm = 6 cm

We see that in ΔAEC and ΔADO,

We have $\angle AEC = \angle ADO = 90°$

And $\angle CAE = \angle OAD$ [Common angle]

$\therefore$ By AA similarity criterion, we have

$\Delta AEC \sim \Delta ADO$

$\therefore \quad \dfrac{AC}{AO} = \dfrac{EC}{DO}$

$\Rightarrow \quad \dfrac{l}{AE - EO} = \dfrac{r}{R}$

$\Rightarrow \quad \dfrac{10}{8 - R} = \dfrac{6}{R}$

$\Rightarrow \quad 48 - 6R = 10R$

$\Rightarrow \quad 16R = 49$

$\Rightarrow \quad R = \dfrac{48}{16} = 3$

$\therefore$ The radius of the sphere = 3 cm

$\therefore$ Volume of water that overflows = Volume of water displaced by the complete sphere = Volume of the sphere

$\therefore$ Required fraction of water that overflows

$= \dfrac{\text{Volume of the sphere}}{\text{Total volume of water in the cone}}$

$= \dfrac{\text{Volume of the sphere}}{\text{Volume of the cone}}$

$= \dfrac{\frac{4}{3}\pi R^3}{\frac{1}{3}\pi r^2 h}$

$= \dfrac{4R^3}{r^2 h}$

$= \dfrac{4 \times 3^3}{6^2 \times 8}$

$= \dfrac{4 \times 27}{36 \times 8} = \mathbf{\dfrac{3}{8}}$

EXERCISE 15C

For Basic and Standard Levels

1. Let r be the radius and h be the height of the cone.

Then, $r = 18$ m, $h = 33$ dm = 3.3 m

Slant height, $l = \sqrt{r^2 + h^2}$

$= \sqrt{(18)^2 + (3.3)^2}$

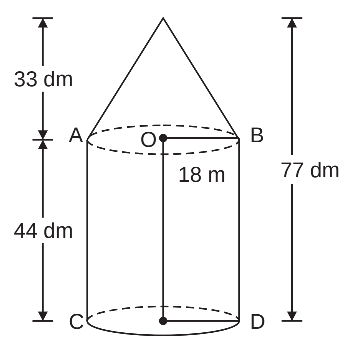
$= \sqrt{324 + 10.89}$

$= \sqrt{334.89} = 18.3$ m

S_1 = Curved surface area of cone = πrl

$= \dfrac{22}{7} \times 18 \times 18.3$ [$\because$ $l = 18.3$ m]

$= 1035.26$ m^2



Height of the cylinder, $h_1 = 44$ dm = 4.4 m

S_2 = Curved surface area of cylinder

$= 2\pi rh = 2 \times \dfrac{22}{7} \times 18 \times 4.4$

$= 497.83$ m^2

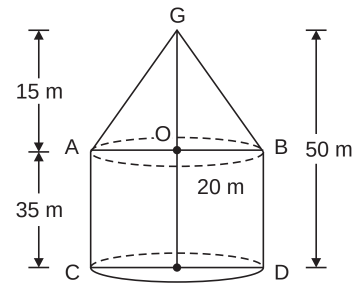
So, total surface area = curved surface area of cone + curved surface area of cylinder

$= S_1 + S_2 = 1035.26 + 497.83$

$= 1533.09$ m^2

Hence, cost of painting = 1533.09 × ₹ 3.50 = ₹ **5365.80**

2. Let r be radius and h be the height of the cone.



Height of the cylinder, $h_1 = 35$ m

Then, $r = 20$ m, $h = 15$ m

$\therefore$ Slant height, $l = \sqrt{r^2 + h^2}$

$= \sqrt{(20)^2 + (15)^2}$

$= \sqrt{400 + 225}$

$= \sqrt{625} = 25$ m

Now, S_1 = curved surface area of cone.

$= \pi rl = \frac{22}{7} \times 20 \times 25$

$= 1571.43 \text{ m}^2$

S_2 = Curved surface area of cylinder

$= 2\pi rh_1 = 2 \times \frac{22}{7} \times 20 \times 35$

$= 4400 \text{ m}^2$

So, total surface area = curved surface area of cone + curved surface area of cylinder

$= S_1 + S_2 = 1571.43 + 4400$

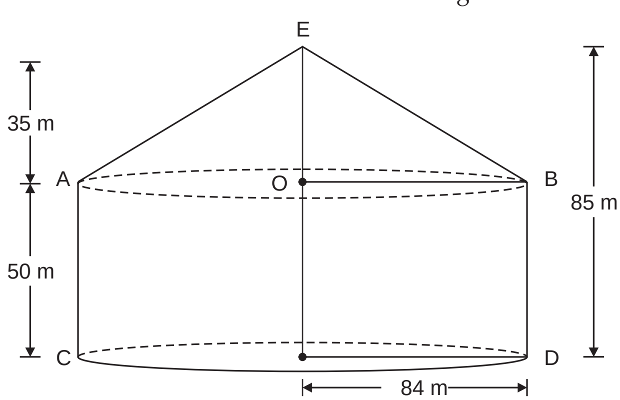
$= 5971.43 \text{ m}^2$

Hence, cost of cloth required

$= 5971.43 \times$ ₹ 14

= **₹ 83600** [approx.]

3. Let r be radius of cone and h be the height of the cone.



Then, $r = 84$ m, $h = 35$ m.

$\therefore$ Slant height $= l^2 = r^2 + h^2$

$= \sqrt{(84)^2 + (35)^2}$

$= \sqrt{8281} = 91$ m.

Now, curved surface area of cone

$$S_1 = \pi rl = \frac{22}{7} \times 84 \times 91 = 24024 \text{ m}^2.$$

Height of the cylinder, $h_1 = 50$ m.

Curved surface area of cylinder

$$S_2 = 2\pi rh_1 = 2 \times \frac{22}{7} \times 84 \times 50$$

$= 26400 \text{ m}^2$.

Hence, total curved surface area

$= S_1 + S_2$

$= 24024 + 26400 = 50424 \text{ m}^2$.

Required 20% extra cloth for stitching $= 50424 \times 20\%$

$= 50424 \times \frac{20}{100}$

$= 10084.80 \text{ m}^2$.

Now, total cloth required for stitching $= 50424 + 10084.80$

$= 60508.80$

$\approx$ **60509 m²**

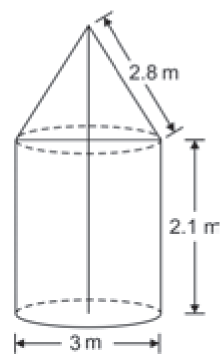
4. Given, diameter of cylinder = 3 m

Radius of cylinder $= r_1 = 1.5$ m

Height of cylinder $= h_1 = 2.1$ m

Also, radius of cone $= r_2 = 1.5$ m

Slant height of cone $= l = 2.8$ m



Now, surface area of tent = curved surface area of cone + curved surface area of cylinder

$= \pi r_2 l + 2\pi r_1 h_1$

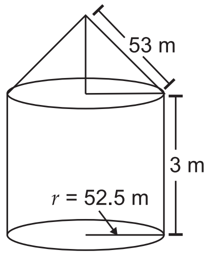
$= \frac{22}{7} \times 1.5 \times 2.8 + 2 \times \frac{22}{7} \times 1.5 \times 2.1$

$= 13.2 + 19.8 = 33 \text{ m}^2$

Now, cost of canvas needed to make tent = ₹ 33 × 500

= **₹ 16500**

5. Let r be the common radius of the cone and the cylinder. Let l be the slant height of the cone and h be the vertical height of the cylinder.



Then $r = 52.5$ m

$h = 3$ m

$l = 53$ m

Now, the curved surface area of the cone

$= \pi rl$

$= \frac{22}{7} \times 52.5 \times 53 \text{ m}^2$

$= 8745 \text{ m}^2$

Also, the curved surface area of the cylinder

$= 2\pi rh$

$= 2 \times \frac{22}{7} \times 52.5 \times 3 \text{ m}^2$

$= 990 \text{ m}^2$

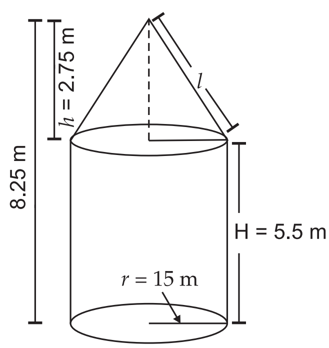
Hence, total surface area of the cone and the cylinder

$= (8745 + 990) \text{ m}^2$

$= 9735 \text{ m}^2$

Hence, the required area of the canvas needed = **9735 m²**

6. Let r be the common radius of the base of the cone and that of the cylinder, l be the slant height of the cone, h be the vertical height of the cone and H be the height of the cylinder.



Then $r = 15$ m, H = 5.5 m = $\frac{11}{2}$ m

$h = (8.24 - 5.5)$ m

$= 2.75$ m

$= 2\frac{3}{4}$ m

$= \frac{11}{4}$ m

and $l = \sqrt{r^2 + h^2}$

$= \sqrt{15^2 + \frac{11^2}{16}}$ m

$= \sqrt{225 + \frac{121}{16}}$ m

$= \frac{61}{4}$ m

Hence, the curved surface area of the cone

$= \pi rl$

$= \frac{22}{7} \times 15 \times \frac{61}{4} \text{ m}^2$ …(1)

And the curved surface area of the cylinder

$= 2\pi r\text{H}$

$= 2 \times \frac{22}{7} \times 15 \times \frac{11}{2} \text{ m}^2$ …(2)

∴ Total surface area of the cone and the cylinder

$= \frac{22}{7} \times 15\left(\frac{61}{4} + 11\right)$ m [From (1) and (2)]

$= \frac{22}{7} \times 15 \times \frac{105}{4} \text{ m}^2$

∴ Total surface area of the canvas used in making the tent

$= \frac{22 \times 15 \times 105}{7 \times 4} \text{ m}^2$

∴ Required length of the canvas

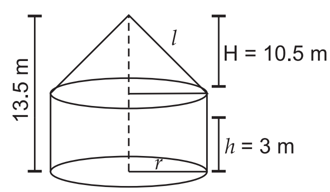
$= \left(\frac{22 \times 15 \times 105}{28} \div 1.5\right)$ m

$= \frac{22 \times 15 \times 105}{28} \times \frac{2}{3}$ m

$= 11 \times 75$ m

= **825 m**

7. Let l be the slant height H be the vertical height and r be the common radius of the base of the cone and the cylinder. Let h be the height of the cylinder.



Then,

$r = 14$ m, H = (13.5 – 3) m = 10.5 m = $\frac{21}{2}$ m, H =3 m

∴ $l = \sqrt{r^2 + H^2}$

$= \sqrt{14^2 + \frac{21^2}{4}}$ m

$= \sqrt{196 + \frac{441}{4}}$ m

$= \sqrt{\frac{1225}{4}}$ m $= \frac{35}{2}$ m

Now, curved surface area of the cone

$= \pi rl$

$$= \frac{22}{7} \times 14 \times \frac{35}{2} \text{ m}^2$$

$= 770 \text{ m}^2$ ...(1)

Curved surface area of the cylinder

$= 2\pi rh$

$$= 2 \times \frac{22}{7} \times 14 \times 3 \text{ m}^2$$

$= 264 \text{ m}^2$ ...(2)

∴ Total curved surface area

$= (770 + 264) \text{ m}^2$ [From (1) and (2)]

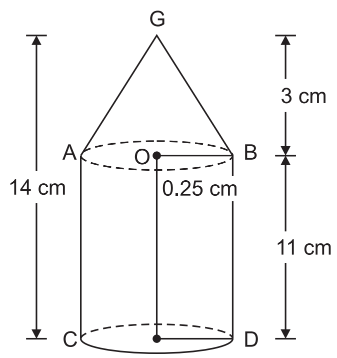
$= 1034 \text{ m}^2$

∴ Total area of the cloth required to make the tent $= 1034 \text{ m}^2$

Hence, the required cost of the cloth = ₹80 × 1034

= **₹82720**

8. Let r be the radius and h be the height of the cone.



Then, $r = 0.25$ cm, $h = 3$ cm

Slant height, $l = \sqrt{r^2 + h^2}$

$$= \sqrt{(0.25)^2 + (3)^2}$$

$$= \sqrt{0.0625 + 9} = \sqrt{9.0625}$$

$= 3.0104$ cm

Volume of the cone, $V_1 = \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times \frac{22}{7} \times (0.25) \times (0.25) \times 3$$

$= 0.196 \text{ cm}^3$

Base radius of the cylinder, $r = 0.25$ cm

Height of the cylinder, $h_1 = 11$ cm

V_2 = volume of cylinder

$= \pi r^2 h_1$

$$= \frac{22}{7} \times (0.25)^2 \times 11$$

$= 2.16 \text{ cm}^3$

Total volume $= V_1 + V_2 = 0.196 + 2.16$

$= 2.356 \text{ cm}^3$

Hence, mass of needle = volume × density

$= 2.356 \times 7$

= **16.5 g**

9. Radius of cylindrical part, $r = 3$ m

Height of cylindrical part, $h = 7$ m

Height of conical part, $h_1 = 11 - 7 = 4$ m

Radius of conical part, $r = 3$ m

Let the numbers of bags be N.

Now, $$N = \frac{\text{Volume of cone + Volume of cylinder}}{\text{Volume of a bag}}$$

$$= \frac{\frac{1}{3}\pi r^2 h_1 + \pi r^2 h}{0.628}$$

$$= \frac{\frac{1}{3} \times \frac{22}{7} \times (3)^2 \times 4 + \frac{22}{7} \times (3)^2 \times 7}{0.628}$$

$$= \frac{37.714 + 198}{0.628} = 375$$

Hence, number of bags is **375**.

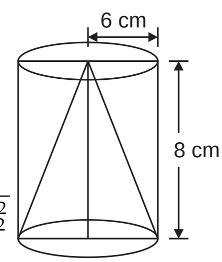
10. (*i*) We have,

Radius of cylinder, $r_1 = 6$ cm

Height of cylinder, $h_1 = 8$ cm

Radius of cone, $r_2 = 6$ cm

Height of cone, $h_2 = 8$ cm



Then, slant height, $l_2 = \sqrt{r_2^2 + h_2^2}$

$$= \sqrt{(6)^2 + (8)^2}$$

$$= \sqrt{36 + 64} = \sqrt{100}$$

$= 10$ cm

Total surface area of remaining solid

= Curved surface area of cylinder + curved surface area of cone + area of the base of cylinder

$$= 2\pi r_1 h_1 + \pi r_2 l_2 + \pi r_1^2$$

$$= \pi(2r_1 h_1 + r_2 l_2 + \pi r_1^2) \quad [\because \quad r_1 = r_2]$$

$= 3.14 (2 \times 6 \times 8 + 6 \times 10 + 6^2)$

$= 3.14 (96 + 60 + 36) = 602.88 \text{ cm}^2$

Volume of remaining solid = Volume of the cylinder − volume of the cone

$$= \pi r_1^2 h_1 - \frac{1}{3}\pi r_2^2 h_2$$

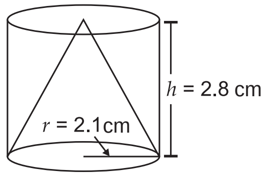
$$= \pi\left(r_1^2 h_1 - \frac{1}{3} r_2^2 h_2\right)$$

$$= 3.14\left((6)^2 \times 8 - \frac{1}{3}(6)^2 \times 8\right)$$

$= 3.14 (288 - 96)$

= **602.88 cm³**

(*ii*) Let r be the common radius of the base of the cone and the cylinder.



Let h be the common height of the cone and the cylinder and l be the slant height of the cone.

Then $r = 2.1$ cm, $h = 2.8$ cm

$$\therefore \quad l = \sqrt{r^2 + h^2}$$
$$= \sqrt{2.1^2 + 2.8^2} \text{ cm}$$
$$= \sqrt{4.41 + 7.84} \text{ cm}$$
$$= \sqrt{12.25} \text{ cm}$$
$$= 3.5 \text{ cm}$$

Then the outer curved surface of the cylinder

$$= 2\pi rh$$
$$= 2 \times \frac{22}{7} \times 2.1 \times 2.8 \text{ cm}^2$$
$$= 44 \times 0.84 \text{ cm}^2$$
$$= 36.96 \text{ cm}^2 \qquad \ldots(1)$$

Also, the curved surface area of the cone

$$= \pi rl$$
$$= \frac{22}{7} \times 2.1 \times 3.5 \text{ cm}^2$$
$$= 23.10 \text{ cm}^2 \qquad \ldots(2)$$

Finally, the area of the upper circular end of the cylinder $= \pi r^2$

$$= \frac{22}{7} \times 2.1 \times 2.1 \text{ cm}^2$$
$$= 13.86 \text{ cm}^2 \qquad \ldots(3)$$

Adding (1), (2) and (3), we get

The total surface area of the remaining solid

$$= (36.96 + 23.10 + 13.86) \text{ cm}^2$$

$= \mathbf{73.92\ cm^2}$ which is the required area.

11. Given, radius of the cylinder, $r = 3$ cm.

Height of the cylinder, $h = 5$ cm.

Volume of the cylinder before a conical hole is drilled out

$$= \pi r^2 h$$
$$= \pi \times (3)^2 \times 5$$
$$= 45\pi \text{ cm}^3.$$

$$\text{Volume of the conical hole} = \frac{1}{3}\pi r^2 h$$
$$= \frac{1}{3} \times \pi \times \left(\frac{3}{2}\right)^2 \times \left(\frac{8}{9}\right)$$
$$= \frac{2}{3}\pi \text{cm}^3$$

$$\text{Volume of metal B} = \frac{2}{3}\pi \text{ cm}^3$$

$$\text{Volume of metal A} = \left(45\pi - \frac{2}{3}\pi\right)$$
$$= \frac{133}{3}\pi$$

$$\frac{\text{Volume of metal A}}{\text{Volume of metal B}} = \frac{\frac{133\pi}{3}}{\frac{2}{3}\pi}$$
$$= \frac{133}{2}$$

$\Rightarrow$ Ratio = **133 : 2**

12. (*i*) Let r_1 be the radius of cylinder and h_1 be the height of cylinder. Then, radius of cone be r_2 and height of cone be h_2.

Then, $r_1 = \frac{7}{2}$ cm, $h_1 = 15$ cm, $r_2 = 3$ m, $h_2 = 4$ cm.

$$\text{Slant height} = \sqrt{r_2^2 + h_2^2} = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$$

$$\therefore \quad l = 5 \text{ cm}$$

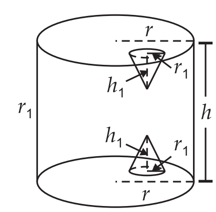
Surface area of remaining solid

$$= 2\pi r_1 h_1 + 2\pi r_1^2 - 2\pi r_2^2 + 2\pi r_2 l$$
$$= 2 \times \pi(r_1 h_1 + r_1^2 - r_2^2 + r_2 l)$$
$$= 2 \times \frac{22}{7}\left[\frac{7}{2} \times 15 + \frac{49}{4} - 9 + 15\right]$$

$= \mathbf{444.7\ cm^2}$ **(approx.)**

(*ii*) Let r and h be respectively the radius and the height of the solid cylinder and let r_1 and h_1 be the radius of the base and the height respectively of each of two identical conical holes at the two ends of the cylinder so that

$r = \frac{7}{2}$ cm, $h = 14$ cm, $r_1 = 2.1$ cm and $h_1 = 4$ cm.



Now, volume of the cylinder

$$= \pi r^2 h$$
$$= \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 14 \text{ cm}^3$$
$$= 539 \text{ cm}^3 \qquad \ldots(1)$$

Sum of the volumes of two identical conical holes at the two ends of the cylinder

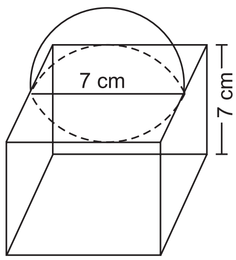
$$= \frac{2}{3} \times \pi r_1^2 h_1$$
$$= \frac{2}{3} \times \frac{22}{7} \times 2.1 \times 2.1 \times 4 \text{ cm}^3$$
$$= 36.96 \text{ cm}^3 \qquad \ldots(2)$$

$\therefore$ Required volume of the cylinder excluding the two conical holes

$= (539 - 36.96)\ cm^3$ [From (1) and (2)]

$= \mathbf{502.04\ cm^3}$

13. (*i*) The size of the hemisphere will be maximum when its base touches all the sides of the upper square face of the cube. In this case, the diameter of the hemisphere will be equal to the side of the cubical block.



Let r be the radius of the hemisphere and a be the side of the cube.

Then $r = \frac{7}{2}$ cm and $a = 7$ cm

Now, the surface area of the hemisphere

$= 2\pi r^2$

$= 2 \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}\ cm^2$

$= 77\ cm^2$ ...(1)

Sum of the surface areas of 5 faces of the cube excluding the topmost face of the cube

$= 5a^2$

$= 5 \times 7^2\ cm^2$

$= 245\ cm^2$ ...(2)

Area of the top face of the cube excluding the circular base of the hemisphere

$= \left(7^2 - \pi \times \frac{7}{2} \times \frac{7}{2}\right) cm^2$

$= \left(49 - \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}\right) cm^2$

$= \left(49 - \frac{77}{2}\right) cm^2$

$= \frac{21}{2}\ cm^2$ ...(3)

$\therefore$ Required total surface area

$= (77 + 245 + \frac{21}{2})\ cm^2$

[From (1), (2) and (3)]

$= \left(322 + \frac{21}{2}\right) cm^2$

$= \frac{665}{2}\ cm^2$

$= \mathbf{332.5\ cm^2}$

(*ii*) Side of the cubical block = 10 cm

Largest diameter the hemisphere can have = side of cubical block = **10 cm**

Surface area of the solid = surface area of cube + curved surface area of hemisphere – surface area of base of hemisphere

$= 6a^2 + 2\pi r^2 - \pi r^2$

$= 6a^2 + \pi r^2$

$= 6(10)^2 + \pi\ (5)^2$

$= 600 + 25 \times 3.14 = 678.5\ cm^2$

Cost of painting

$100\ cm^2$ = ₹5

$1\ cm^2$ = ₹ $\frac{5}{100}$

$678.5\ cm^2 = \frac{5}{100} \times 678.5$

= ₹33.925 = **₹33.93** (approx.)

(*iii*) Side of cube = 6 cm

Diameter of hemisphere = 3.5 cm

Total surface area of block = surface area of cube + curved surface area of hemisphere – surface area of base of hemisphere

$= 6(a)^2 + 2\pi r^2 - \pi r^2$

$= 6a^2 + \pi r^2$

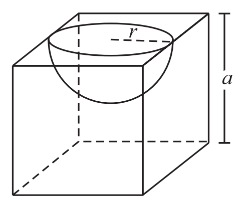
$= 6(6)^2 + \frac{22}{7} \times \frac{35}{20} \times \frac{35}{20}$

$= 216 + \frac{77}{8}$

$= 216 + 9.625$

$= \mathbf{225.625\ cm^2}$

14. Let r be the radius of the hemisphere and a be the side of the cube.



Then $a = 21$ cm and $r = \frac{21}{2}$ cm.

The sum of the surface areas of 5 equal square faces of the cube excluding the upper face where a hemisphere is carved out $= 5a^2 = 5 \times 21^2\ cm^2 = 2205\ cm^2$ ...(1)

The area of the upper square face of the cube excluding the area of the circular base of the carved out hemisphere

$= a^2 - \pi r^2$

$= \left(21 \times 21 - \frac{22}{7} \times \frac{21 \times 21}{4}\right) cm^2$

$= \left(441 - \frac{693}{2}\right) cm^2$

$= \frac{189}{2}\ cm^2$

$= 94.5\ cm^2$ ...(2)

Curved surface area of the inner part of the hemisphere

$= 2\pi r^2$

$= 2 \times \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2}\ \text{cm}^2$

$= 693\ \text{cm}^2$ …(3)

$\therefore$ Required total surface area of the remaining piece

$= (2205 + 94.5 + 693)\ \text{cm}^2$

$=$ **$2992.5\ \text{cm}^2$** [From (1), (2) and (3)]

The required volume of the remaining part of the cube excluding the volume of the hemisphere

$= a^3 - \frac{2}{3}\pi r^3$

$= \left(21^3 - \frac{2}{3} \times \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} \times \frac{21}{2}\right)\text{cm}^3$

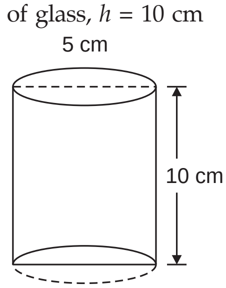
$= \left(9261 - \frac{4851}{2}\right)\ \text{cm}^3$

$= (9261 - 2425.5)\ \text{cm}^3$

$=$ **$6835.5\ \text{cm}^3$**

15. We have, inner diameter of glass = 5 cm

Height of glass, $h = 10$ cm



Radius of the glass, $r = \frac{5}{2}$ cm

$\therefore$ Apparent capacity of glass $= \pi r^2 h$

$= \frac{22}{7} \times \left(\frac{5}{2}\right)^2 \times 10$

$=$ **$196.25\ \text{cm}^3$**

Volume of hemispherical part $= \frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \frac{22}{7} \times \left(\frac{5}{2}\right)^3$

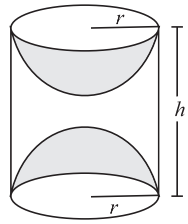
$= 32.71\ \text{cm}^3$

$\therefore$ Actual capacity of glass = Apparent capacity of glass – Volume of hemispherical part.

$= (196.25 - 32.71)$

$=$ **163.54** cm^3

16. (*i*) Let h be the height of the cylinder and r be the radius of each of the two identical hemispheres at the two ends of the cylinder.



Then $h = 20$ cm and $r = \frac{7}{2}$ cm.

$\therefore$ The curved surface area of the cylinder

$= 2\pi rh$

$= 2 \times \frac{22}{7} \times \frac{7}{2} \times 20\ \text{cm}^2$

$= 440\ \text{cm}^2$ …(1)

Sum of the curved surface areas of the two hemispheres

$= 4\pi r^2$

$= 4 \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}\ \text{cm}^2$

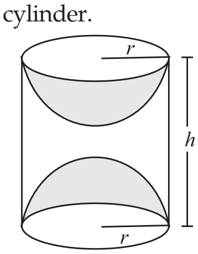
$= 154\ \text{cm}^2$ …(2)

$\therefore$ Required total surface area

$= (440 + 154)\ \text{cm}^2$ [From (1) and (2)]

$=$ **$594\ \text{cm}^2$**

(*ii*) Let h be the height of the cylinder and r be the radius of each of the two identical hemispheres at the two ends of the cylinder.



Then $h = 10$ cm and $r = 3.5$ cm

$\therefore$ The curved surface area of the cylinder

$= 2\pi rh$

$= 2 \times \frac{22}{7} \times 3.5 \times 10\ \text{cm}^2$

$= 220\ \text{cm}^2$ …(1)

The sum of the curved surface areas of two hemispheres

$= 4\pi r^2$

$= 4 \times \frac{22}{7} \times 3.5 \times 3.5\ \text{cm}^2$

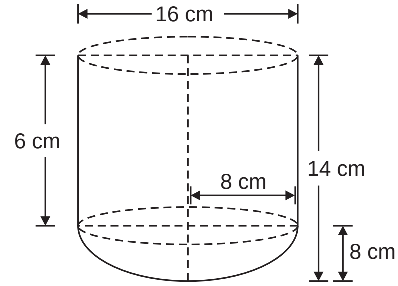
$= 154\ \text{cm}^2$ …(2)

$\therefore$ Required total surface area

$= (220 + 154)\ \text{cm}^2$ [From (1) and (2)]

$=$ **$374\ \text{cm}^2$**

17. Let r be radius of hemispherical bowl and h be height of cylinder.



Then, $r = 8$ cm, $h = 14$ cm

Total capacity of the bowl

= Volume of cylinder + Volume of hemisphere

$= \pi r^2 h + \frac{2}{3}\pi r^3 = \pi r^2\left[h + \frac{2}{3}r\right]$

$= \frac{22}{7} \times (8)^2\left[6 + \frac{2}{3} \times 8\right]$

= **2279.619 cm^3 (approx.)**.

18. The height of empty portion of test tube is 9 cm, when 396 cm^3 of water is poured into it.

Let h be the height of the empty portion of the test tube and r be the radius.

Now, Volume of empty portion of the test tube

= Volume of whole test tube – Volume of water in the test tube when 9 cm of the test tube remains empty.

$\Rightarrow \quad \pi r^2 h = \frac{4554}{7} - 396$

$\Rightarrow \quad \pi r^2\, 9 = 254$

$\Rightarrow \quad r^2 = \frac{254}{\pi \times 9} = \frac{254 \times 7}{22 \times 9} = \frac{1778}{198} = 9$

$\Rightarrow \quad r =$ **3 cm**

Volume of water in full test tube $= \frac{2}{3}\pi r^3 + \pi r^2 h$

$\Rightarrow \quad \frac{4554}{7} = \pi r^2\left[\frac{2}{3}r + h\right]$

$\Rightarrow \quad = \frac{22}{7} \times (3)^2\left[\frac{2}{3} \times 3 + h\right]$

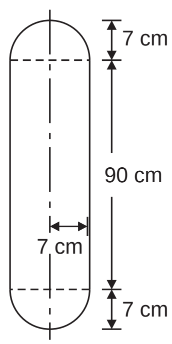
$\Rightarrow \quad 2 + h = \frac{4554}{7} \times \frac{7}{22} \times \frac{1}{3} \times \frac{1}{3}$

$\Rightarrow \quad 2 + h = 23$

$\Rightarrow \quad h = 23 - 2 =$ **21 cm**

19. We have, radius of cylinder, $r = 7$ cm.

Height of cylinder, $h = 104 - 2 \times 7 = 90$ cm



$\therefore$ Total surface area = curved surface area of cylinder + 2 × curved surface area of hemispherical ends

$= (2\pi rh + 2 \times 2\pi r^2)$

$= (2\pi rh + 4\pi r^2) = 2\pi r\,(h + 2r)$

$= 2 \times 3.14 \times 7 \times (90 + 2 \times 7)$

$= 2 \times 3.14 \times 7 \times 104 = 4571.84$ cm^2

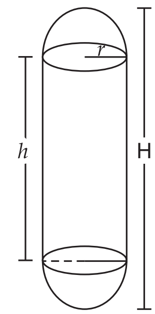
$= 45.7184$ dm^2

Rate of polishing = ₹ 10 per dm^2

Cost of polishing = (45.7184 dm^2) × (₹ 10 per dm^2)

= **₹ 457.184 (approx.)**

20. Let r be the radius of each of the two identical hemispheres at the two ends of the cylinder and let H be the total height of the toy consisting of the cylinder and the two hemispheres.



Then H = 90 cm and $r = 21$ cm.

If h be the height of the cylinder only,

then $h = (90 - 21 \times 2)$ cm = 48 cm

Now, the curved surface area of the cylinder

$= 2\pi rh$

$= 2 \times \frac{22}{7} \times 21 \times 48$ cm^2

$= 6336$ cm^2 …(1)

The sum of the surface areas of two identical hemispheres at the two ends of the cylinder

$= 4\pi r^2$

$= 4 \times \frac{22}{7} \times 21 \times 21$ cm^2

$= 5544$ cm^2 …(2)

$\therefore$ Total areas = (6336 + 5544) cm^2 [From (1) and (2)]

$= 11880$ cm^2

Hence, the required total cost of painting the toy

= ₹(0.70 × 11880)

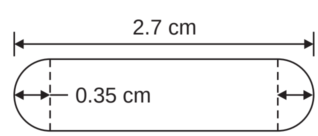
= ₹8316

21. We have, length of cylindrical part of gulab jamun,

$h = (2.7 - 0.35 - 0.35) = 2$ cm

Radius of cylindrical part of gulab jamun,

$r = 0.35$ cm



Also, radius of hemispherical part of gulab jamun,

$r = 0.35$ cm.

Volume of gulab jamun = Volume of two hemispherical part + Volume of cylindrical part

$$= 2\left(\frac{2}{3}\pi r^3\right) + \pi r^2 h$$

$$= \frac{4}{3}\pi r^3 + \pi r^2 h \;=\; \pi r^2\left(\frac{4}{3}r + h\right)$$

$$= \frac{22}{7}\times 0.35\times 0.35\times\left[\left(\frac{4}{3}\right)\times 0.35 + 2\right]$$

$$= \frac{2.695}{7}\times\left[\frac{1.4}{3}+2\right]$$

$$= \frac{2.695}{7}\times\left[\frac{1.4+6}{3}\right] = \textbf{0.95 cm}^3\textbf{ (approx.)}.$$

22. Let tennis balls are packed in a cylindrical container that contains 3 balls. Let r be the radius of ball and cylinder's height be $6r$.

Radius of ball and cylinder = r

Height of cylinder, $h = 6r$

$$\text{Then, fraction} = \frac{3\times\text{Volume of a ball}}{\text{Volume of cylinder}}$$

$$= \frac{3\times\frac{4}{3}\pi r^3}{\pi r^2 h} = \frac{3\times\frac{4}{3}\pi r^3}{\pi r^2(6r)}$$

$$= \frac{4\pi r^3}{6r^3} = \frac{4}{6} = \frac{2}{3}$$

Now, fraction of the container which is not occupied by the balls

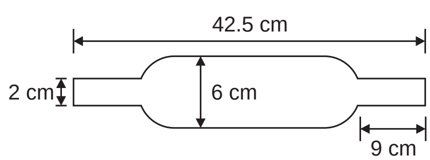
$$= 1-\frac{2}{3} = \mathbf{\frac{1}{3}}$$

23. Length of middle cylindrical portion,

h = Total length – length of 2 cylindrical rods – 2 × radius of hemisphere.

$= (42.5 - 2\times 9 - 2\times 3)$

$= (42.5 - 24) = 18.5$ cm



Radius of the middle cylindrical portion, $r = \dfrac{6\text{ cm}}{2}$

$= 3$ cm.

Radius of hemispherical portion, $r = 3$ cm.

Radius of cylindrical rods, $r_1 = \dfrac{2\text{ cm}}{2} = 1$ cm.

Volume of rolling pin = Volume of middle cylindrical portion + Volume of 2 hemispherical portions + Volume of 2 cylindrical rods.

$$= \pi r^2 h + 2\times\frac{2}{3}\pi r^3 + 2\times\pi r_1^2 h_1$$

$$= \left[\pi(3)^2\times 18.5 + \frac{4}{3}\pi(3)^3 + 2\pi(1)^2\times 9\right]$$

$$= 9\times\frac{22}{7}\times 24.5$$

$$= \textbf{693 cm}^3$$

Mass of rolling pin = 693 × 0.9 g = **623.7 g**

24. The base of the largest right circular cone will be the circle inscribed in a face of the cube and its height will be equal to an edge of the cube.

∴ Radius of the base of cone,

$$r = \frac{9}{2}\text{ cm}$$

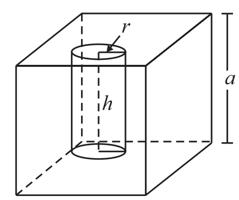
Height of cone, $h = 9$ cm.

$$\text{Hence, volume of cone} = \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3}\times\frac{22}{7}\times\left(\frac{9}{2}\right)^2\times 9$$

$$= \textbf{190.93 cm}^3\textbf{ (approx.)}$$

25. Let the side of the cube be a and the radius and the height of the conical cavity be r and h respectively.



Then $a = 7$ cm, $r = 3$ cm and $h = 7$ cm.

Now, the volume of the cube

$= a^3 = 7^3$ cm^3

$= 343$ cm^3 …(1)

The volume of the conical cavity is

$$\frac{1}{3}\pi r^2 h = \frac{1}{3}\times\frac{22}{7}\times 3\times 3\times 7\ \text{cm}^3$$

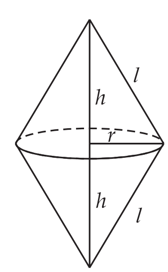
$= 66$ cm^3 …(2)

Hence, the required volume of the remaining solid

$= (343 - 66)$ cm^3 [From (1) and (2)]

$= \textbf{277 cm}^3$

26. Let r be the radius of each of two identical cones and h be its height. Let l be their common slant height.



Then $r = 8$ cm, $h = 15$ cm

And $l = \sqrt{r^2 + h^2}$

$= \sqrt{64 + 15 \times 15}$ cm

$= \sqrt{64 + 225}$ cm

$= \sqrt{289}$ cm

= 17 cm

The curved surface area of each cone = πrl

$= \frac{22}{7} \times 8 \times 17$ cm^2 ...(1)

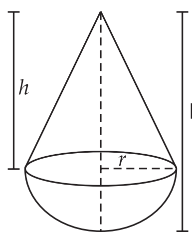
$\therefore$ Required total surface area of the two identical cones

$= 2 \times \frac{22}{7} \times 8 \times 17$ cm^2 [From (1)]

$= \frac{44 \times 8 \times 17}{7}$ cm^2

= 855 cm^2 (approx.)

27. Let r be the common radius of the hemisphere and the cone, H be the complete height of the whole toy consisting of the cone and the hemisphere and let h be the height of the cone only.



Then $r = \frac{7}{2}$ cm

$\text{H} = \left(h + \frac{7}{2}\right)$ cm

Now, the volume of the cone

$= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times \frac{7 \times 7}{4} h$ cm^3

$= \frac{77}{6} h$ cm^3 ...(1)

Also, volume of the hemisphere

$= \frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times \frac{7}{2}$ cm^3

$= \frac{539}{6}$ cm^3 ...(2)

$\therefore$ Total volume of toy $= \frac{77h + 539}{6}$ cm^3.

$\therefore$ According to the problem, we have

$\frac{77h + 539}{6} = 231$

$\Rightarrow$ $77h = 231 \times 6 - 539$

$= 1386 - 539$

$= 847$

$\Rightarrow$ $h = \frac{847}{77} = 11$ ...(1)

Hence, the height of the cone is 11 cm and so the required height of the toy consisting of the cone and the hemisphere is

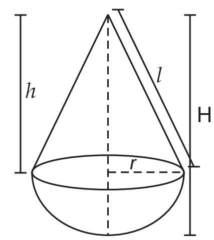
$\text{H} = h + r$

$= \left(11 + \frac{7}{2}\right)$ cm

= (11 + 3.5) cm

= **14.5 cm**

28. (*i*) Let r be the common radius of the cone and the hemisphere, h be the height of the cone only and let H be the height of the toy consisting of the cone and the hemisphere.



Then r = 3.5 cm, H = 15.5 cm

And H = $h + r$ = (h + 3.5) cm

$\Rightarrow$ $15.5 = h + 3.5$

$\Rightarrow$ $h = 15.5 - 3.5$

$= 12$

Hence, the height of cone, h = 12 cm.

If l be the slant height of the cone,

Then $l = \sqrt{r^2 + h^2}$

$= \sqrt{3.5^2 + 12^2}$ cm

$= \sqrt{12.25 + 144}$ cm

$= \sqrt{156.25}$ cm

= 12.5 cm

Now, curved surface area of the cone = πrl and the curved surface area of the hemisphere = $2\pi r^2$

$\therefore$ Total surface area of the toy

$= \pi rl + 2\pi r^2$

$= \pi r(l + 2r)$

$= \frac{22}{7} \times 3.5 \times (12.5 + 7)$ cm^2

= 11 × 19.5 cm^2

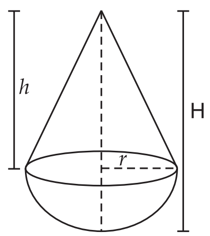
= 214.5 cm^2

Hence, the required total surface area of the toy **= 214.5 cm^2**

(*ii*) Let r be the common radius of the cone and the hemisphere, h is the vertical height of the cone only and H is the total height of the solid.

$\therefore$ r = 3.5 cm, H = 9.5 cm

And $h = H - r = (9.5 - 3.5)$ cm = 6 cm



Now, volume of the cone $= \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times \frac{22}{7} \times 3.5 \times 3.5 \times 6 \text{ cm}^3$$

$$= 77 \text{ cm}^3 \quad \ldots(1)$$

Volume of the hemisphere

$$= \frac{2}{3}\pi r^3$$

$$= \frac{2}{3} \times \frac{22}{7} \times 3.5 \times 3.5 \times 3.5 \text{ cm}^3$$

$$= \frac{269.50}{3} \text{ cm}^3 \quad \ldots(2)$$

$\therefore$ Total volume of the whole toy

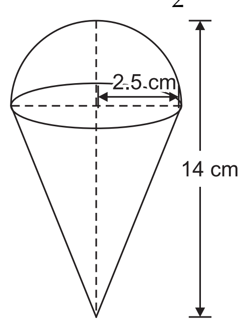
$$= \left(77 + \frac{269.50}{3}\right) \text{ cm}^3$$

$$= (77 + 89.83) \text{ cm}^3$$

$$= \mathbf{166.83 \text{ cm}^3}.$$

which is the required volume.

29. Given, radius of the cone, $r = \frac{5 \text{ cm}}{2} = 2.5$ cm.



Volume of ice cream

= Volume of cone + Volume of hemispherical top of ice cream cone

$$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3 = \frac{1}{3}\pi r[rh + 2r^2]$$

$$= \frac{1}{3} \times \frac{22}{7} \times 2.5[2.5 \times 14 + 2 \times (2.5)^2]$$

$$= 2.62 \, [35 + 12.5]$$

$$= \mathbf{124.4 \text{ cm}^3 \text{ (approx.)}}$$

30. Radius of cone, $r = 21$ cm

Let h be the height of the cone.

According to question,

$$\text{Volume of cone} = \frac{2}{3} \times \text{Volume of hemisphere}$$

$$\frac{1}{3}\pi r^2 h = \frac{2}{3} \times \frac{2}{3} \times \pi r^3$$

$$h = \frac{2 \times 2 \times r}{3}$$

$$= \frac{4 \times 21}{3} \quad (\because r = 21 \text{ cm})$$

$$= 28 \text{ cm}$$

Hence, height of cone is **28 cm**.

Slant height of the cone,

$$l = \sqrt{r^2 + h^2}$$

$$= \sqrt{(21 \text{ cm})^2 + (28 \text{ cm})^2} = 35 \text{ cm}$$

Surface area of the toy = Curved surface area of hemisphere + curved surface area of the cone

$$= 2\pi r^2 + \pi rl$$

$$= \pi r \, (2r + l)$$

$$= \frac{22}{7} \times 21 \text{ cm } (2 \times 21 \text{ cm} + 35 \text{ cm})$$

$$= \mathbf{5082 \text{ cm}^2}$$

31. Radius of hemisphere = 3.5 cm

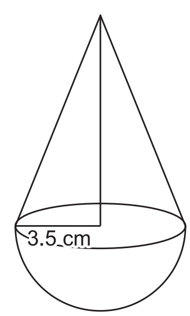
Radius of cone = 3.5 cm

Volume of toy = volume of hemisphere + volume of cone

$$= \frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h$$

$$= \frac{\pi r^2}{3}(2r + h)$$

$$\text{Volume of toy} = 166\frac{5}{6} \text{ cm}^3$$



$$\frac{\pi r^2}{3}(2r + h) = \frac{1001}{6}$$

$$\frac{1}{3} \times \frac{22}{7} \times \frac{35}{10} \times \frac{35}{10} (7 + h) = \frac{1001}{6}$$

$$7 + h = \frac{91}{7}$$

$$h = 13 - 7$$

$$h = 6 \text{ cm}$$

Height of cone = 6 cm

Height of toy = (radius of hemisphere) + height of cone

$$= 3.5 + 6 = \mathbf{9.5 \text{ cm}}$$

Surface area of hemisphere $= 2 \times \pi r^2$

$$= 2 \times \frac{22}{7} \times \frac{35}{10} \times \frac{35}{10}$$

$$= 77 \text{ cm}^2$$

Cost of painting hemisphere = surface area × 10

$$= 77 \times 10$$

= ₹770

32. Let radius of the cone and hemisphere be r.

Let slant height of the cone be l.

Curved surface area of cone = πrl

Curved surface area of hemisphere = $2\pi r^2$

Given, Curved surface area of cone = Curved surface area of hemisphere

$$\Rightarrow \quad \pi rl = 2\pi r^2$$

$$\Rightarrow \quad l = 2r \quad \ldots(1)$$

Now, height of cone, $h = \sqrt{l^2 - r^2}$

$= \sqrt{(2r)^2 - r^2}$ [Using (1)]

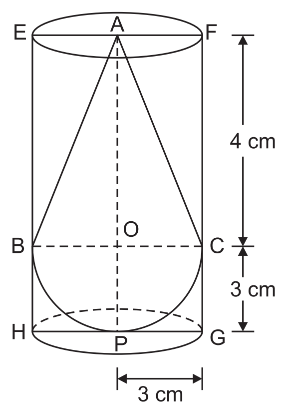
$= \sqrt{4r^2 - r^2}$

$= \sqrt{3r^2} = \sqrt{3}\,r$

$$\Rightarrow \quad \frac{r}{h} = \frac{1}{\sqrt{3}}$$

Hence, the ratio $r : h = \mathbf{1 : \sqrt{3}}$

33. Let BPC be the hemisphere and ABC be the cone mounted on the base of the hemisphere. Let EFGH be the right circular cylinder circumscribing the given toy.



We have, h = OA = height of cone = 4 cm.

BC = Diameter of the base of cone = 6 cm.

$\therefore$ BO = Radius of the hemisphere

$= \frac{1}{2}\text{BC} = 3 \text{ cm.} \quad \left(\because \frac{1}{2}\text{BC} = \frac{1}{2}(6) = 3 \text{ cm}\right)$

$\Rightarrow$ OP = 3 cm

Height of the cylinder,

h_1 = AP = OP + OA = (3 + 4) = 7 cm.

Now, Volume of right circular cylinder

$= \pi r^2 h_1 = \pi (3)^2 \times 7$ cm

$= 63\pi \text{ cm}^3.$ ...(1)

Volume of solid toy $= \frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h$

$= \frac{2}{3}\pi(3)^3 + \frac{1}{3}\pi(3)^2 \times 4$

$= \pi(3)[2 \times 3 + 4]$

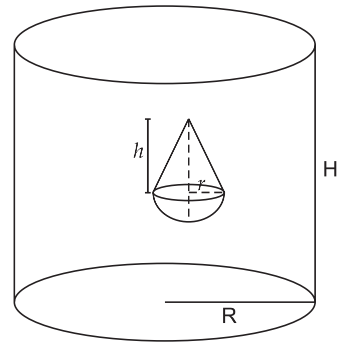
$= 3\pi \times 10 = 30\pi \text{ cm}^3$

Required space = Volume of right circular cylinder – Volume of toy

$= 63\pi - 30\pi = \mathbf{33\pi \text{ cm}^3}$

34. Let r be the common radius of the solid cone and the hemisphere and h be the height of the cone only.

Let H be the height of the cylinder containing water and let R be its radius.



Then r = 2.1 cm, h = 4 cm, H = 9.8 cm and R = 5 cm.

When the solid consisting of the cone and the hemisphere is completely immersed in water inside a big cylindrical tub full of water, then the solid will displace the volume of water, which is equal to the volume of the combined solid.

Now, volume of the cone $= \frac{1}{3}\pi r^2 h$

and the volume of the hemisphere $= \frac{2}{3}\pi r^3$.

Hence, the total volume of the solid consisting of the cone and the hemisphere $= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$

$= \frac{\pi r^2}{3}(h + 2r)$

$= \frac{22}{7} \times \frac{1}{3} \times (2.1)^2 (4 + 2 \times 2.1) \text{ cm}^3$

$= \frac{22}{7} \times \frac{1}{3} \times 2.1 \times 2.1 \times 8.2 \text{ cm}^3$

$= 37.884 \text{ cm}^3$ ...(1)

Again, volume of the whole cylinder

$= \pi R^2 H$

$= \frac{22}{7} \times 5^2 \times 9.8 \text{ cm}^3$

$= 770 \text{ cm}^3$ ...(2)

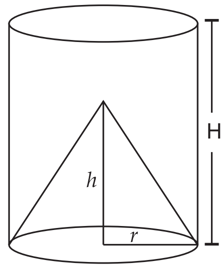
$\therefore$ Volume of water left in the cylindrical tub

= Volume of the cylinder – total volume of solid consisting of the cone and the hemisphere

$= (770 - 37.884) \text{ cm}^3$

= $\mathbf{732.116 \text{ cm}^3}$ which is the required volume.

35. Let r be the common radius of the base of the cone and the cylinder h, the height of the cone and H, the height of the cylinder full of water.



Then $r = 60$ cm, $h = 120$ cm

And H = 180 cm.

When the solid cone is fully immersed in the water within the cylinder, then the cone will displace the volume of water which is equal to its own volume. Hence, the volume of the remaining water in the cylinder which was initially full of water will be equal to the volume of the cylinder – volume of the cone.

Now, the volume of the cylinder = $\pi r^2 \text{H}$ cm^3 ...(1)

and the volume of the cone = $\frac{1}{3}\pi r^2 h$ cm^3 ...(2)

Hence, the required volume of the remaining water in the cylinder

$$= \left(\pi r^2 H - \frac{1}{3}\pi r^2 h\right) \text{ cm}^3$$

$$= \frac{\pi r^2}{3}(3H - h) \text{ cm}^3$$

$$= \frac{22}{7} \times \frac{60 \times 60}{3} (3 \times 180 - 120) \text{ cm}^3$$

$$= \frac{22 \times 3600 \times (540 - 120)}{7 \times 3} \text{ cm}^3$$

$$= \frac{22 \times 3600 \times 420}{7 \times 3} \text{ cm}^3$$

$$= 1584000 \text{ cm}^3$$

$$= \frac{1584000}{100 \times 100 \times 100} \text{ m}^3$$

$$= \mathbf{1.584 \text{ m}^3}$$

36. Height of cone = 60 cm

Radius of cone = 30 cm

Height of cylinder = 180 cm

Radius of cylinder = 60 cm

Volume of cone = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \pi \times 30 \times 30 \times 60$

$= 18000\pi$ cm^3

Volume of water flown out = Volume of cone

$= 18000\pi$ cm^3

Volume of water in cylinder = $\pi r^2 h$

$= \pi \times 60 \times 60 \times 180$

$= 648000\pi$ cm^3

Volume of water lift in cylinder = volume of cylinder – volume of cone

$= 648000\pi - 18000\pi$

$= 630000\pi$ cm^3

$= 630000 \times \frac{22}{7}$

$= 1980000$ cm^3

$= \mathbf{1.98 \text{ m}^3}$

37. We have, radius of the hemisphere,

Cylinder and cone $r = 2.1$ cm

Height of the cylinder, $h = 12$ cm

Height of the cone, $h_1 = 7$ cm.

Volume of solid toy = Volume of conical portion + Volume of the cylindrical portion + Volume of the hemispherical portion

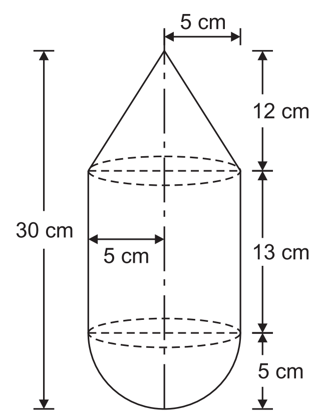
$$= \left[\frac{1}{3}\pi r^2 h_1 + \pi r^2 h + \frac{2}{3}\pi r^3\right]$$

$$= \frac{1}{3} \times \frac{22}{7} \times (2.1)^2 \times 7 + \frac{22}{7} \times (2.1)^2 \times 12 + \frac{2}{3} \times \frac{22}{7} \times (2.1)^3$$

$$= 32.34 + 166.32 + 19.404$$

$$= \mathbf{218.064 \text{ cm}^3}$$

38. Let r be radius and h be height of cylindrical part. Then, $r = 5$ cm and $h = 13$ cm.



Radii of the spherical part and base of conical part are also r. Let h_1 be the height, l be slant height of conical part. Then,

$$l^2 = r^2 + h_1^2$$

$$\Rightarrow \quad l = \sqrt{r^2 + h_1^2} = \sqrt{(5)^2 + (12)^2}$$

$$\Rightarrow \quad l = \sqrt{25 + 144} = \sqrt{169} \Rightarrow l = 13 \text{ cm}$$

Now, Surface area of toy = Curved surface area of cylindrical part + Curved surface area of hemispherical part + Curved surface area of conical part

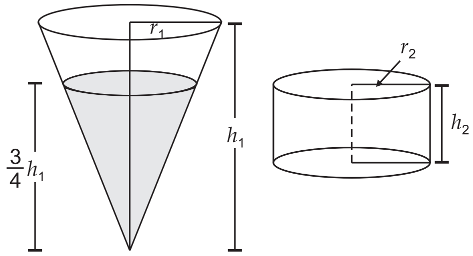
$$= (2\pi rh + 2\pi r^2 + \pi rl)$$

$$= \pi r(2h + 2r + l)$$

$$= \left[\frac{22}{7} \times 5(2 \times 13 + 2 \times 5 + 13)\right]$$

$$= \frac{5390}{7} = \mathbf{770 \text{ cm}^2}$$

39. Let r_1 and h_1 be respectively the radius of the base and the height of the cone and let r_2 and h_2 be respectively those of the cylinder.



Then r_1 = 5 cm, r_2 = 10 cm and h_1 = 24 cm

Then clearly, $\frac{3}{4}$th part of the volume of water in the cone will be equal to the volume of water in the whole cylinder.

Now, the volume of $\frac{3}{4}$th part of the cone

$$= \frac{3}{4} \times \frac{1}{3} \pi r_1^2 h_1$$

$$= \frac{\pi}{4} r_1^2 h_1 \quad \text{...(1)}$$

and the volume of the cylinder $= \pi r_2^2 h_2$ ...(2)

$\therefore$ From (1) and (2), we have

$$\pi r_2^2 h_2 = \frac{\pi}{4} r_1^2 h_1$$

$$\Rightarrow \quad 10^2 h_2 = \frac{1}{4} \times 5^2 \times 24$$

$$\Rightarrow \quad h_2 = \frac{25 \times 24}{100 \times 4} = \frac{3}{2} = 1.5$$

Hence, the required height of the cylindrical vessel is **1.5 cm**.

40. Diameter of lead sphere is 6 cm

Radius of lead sphere, R = 3 cm

Height of water level rise, h = 40 cm

Diameter of beaker is 18 cm.

Radius of beaker, r = 9 cm

Now, Volume of sphere $= \frac{4}{3}\pi R^3$

$$= \frac{4}{3}\pi (3)^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times 27 = 113.14 \text{ cm}^3.$$

Volume of rise water $= \pi r^2 h$

$$= \frac{22}{7} \times (9)^2 \times 40$$

$$= 10182.85 \text{ cm}^3$$

Now, number of lead spheres

$$= \frac{\text{Volume of rise water}}{\text{Volume of sphere}}$$

$$= \frac{10182.85}{113.14} = 90$$

Hence, number of lead spheres dropped are **90**.

41. Height of cone = 8 cm

Radius of cone = 5 cm

Volume of water in cone $= \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times \pi \times 5 \times 5 \times 8$$

$$= \frac{200}{3} \pi \text{ cm}^3$$

Volume of water flown out $= \frac{1}{4} \times$ volume of water

$$= \frac{1}{4} \times \frac{200\pi}{3} \text{ cm}^3$$

$$= \frac{50\pi}{3} \text{ cm}^3$$

Volume of 1 spherical ball $= \frac{4}{3}\pi r^3$

Volume of 100 spherical balls $= 100 \times \frac{4}{3}\pi r^3$

$$= \frac{400}{3}\pi r^3$$

Volume of 100 spherical balls = volume of water flown out

$$\frac{400}{3}\pi r^3 = \frac{50}{3}\pi$$

$$r^3 = \frac{50}{400}$$

$$r = \frac{1}{2} \text{ cm}$$

r = **0.5 cm**

42. Diameter of spherical marbles = 1.4 cm

No. of marbles = 150

Diameter of cylindrical vessel = 7 cm

Volume of 1 spherical marble $= \frac{4}{3}\pi r^3$

$$= \frac{4}{3} \times \pi \times \frac{14}{20} \times \frac{14}{20} \times \frac{14}{20}$$

$$= \frac{343\pi}{750} \text{ cm}^3$$

Volume of 150 spherical marbles $= 150 \times \frac{343\pi}{750}$

$$= \frac{343\pi}{5} \text{ cm}^3$$

Volume of 150 marbles = Volume of water risen in vessel

$$\frac{343\pi}{5} = \pi \times r^2 h$$

$$\frac{343\pi}{5} = \pi \times \frac{7}{2} \times \frac{7}{2} \times h$$

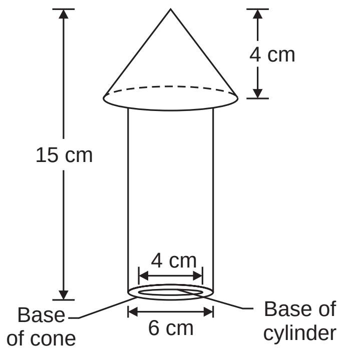
$$h = \frac{7 \times 2 \times 2}{5} = \frac{28}{5}$$

h = **5.6 cm**

For Standard Level

43. Let r_1 be radius and h_1 be height of cylinder.

Given, h_1 = 11 cm, r_1 = 2 cm, radius of the cone, r = 3 cm.



Now, Slant height,

$$l = \sqrt{r^2 + h^2} = \sqrt{(3)^2 + (4)^2}$$

$$= \sqrt{9+16} = 5 \text{ cm.}$$

Now, area to be painted green

= curved surface area of cone + base area of cone – base area of cylinder.

$$= \pi r l + \pi r^2 - \pi r_1^2$$

$$= \pi[r l + r^2 - r_1^2]$$

$$= 3.14\ [3 \times 5 + (3)^2 - (2)^2]$$

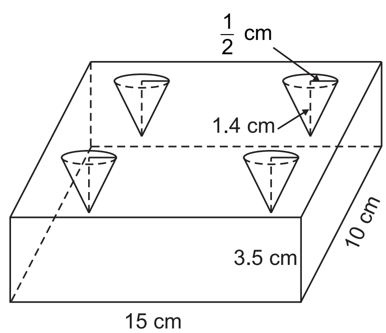
$$= 3.14\ [15 + 9 - 4] = \mathbf{62.8\ cm^2}$$

Area to be painted blue = curved surface area of cylinder + area of the base of cylinder

$$= 2\pi r_1 h_1 + \pi r_1^2 = \pi r_1 (2h_1 + r_1)$$

$$= 3.14 \times 2\ [2 \times 11 + 2] = \mathbf{150.72\ cm^2}$$

44. The dimensions of the cuboid are 15 cm by 10 cm by 3.5 cm. Let the radius of the base of each of the four identical conical depressions be r and its height be h.



Then $r = \frac{1}{2}$ cm and h = 1.4 cm.

Now, the volume of the wooden cuboid

$$= 15 \times 10 \times 3.5 \text{ cm}^3$$

$$= 525 \text{ cm}^3 \qquad \ldots(1)$$

Total volume of the 4 identical conical depressions

$$= 4 \times \frac{1}{3}\pi r^2 h$$

$$= 4 \times \frac{\pi}{3} \times \frac{1}{2} \times \frac{1}{2} \times 1.4 \text{ cm}^3$$

$$= \frac{1}{3} \times \frac{22}{7} \times 1.4 \text{ cm}^3$$

$$= \frac{4.4}{3} \text{ cm}^3 \qquad \ldots(2)$$

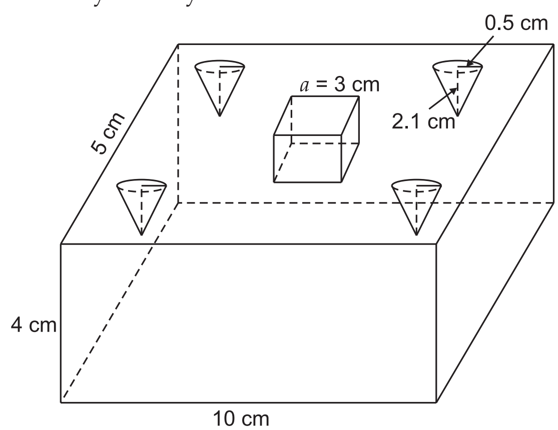
∴ Required volume of the wood in the entire stand

$$= \left(525 - \frac{4.4}{3}\right) \text{cm}^3 \qquad \text{[From (1) and (2)]}$$

$$= \frac{1575 - 4.4}{3} \text{ cm}^3$$

$$= \mathbf{523.53\ cm^3}$$

45. Let r be the radius of the base of each of the four identical conical depressions and h be its height. Let a be the edge of the cubical depression on the entire cuboid of dimensions 10 cm by 5 cm by 4 cm.



Here r = 0.5 cm and h = 2.1 cm

The volume of in the entire stand = Volume of the entire cuboid – Sum of the volumes of the 4 conical depressions – the volume of the cubical depression on the stand.

Now, the volume of the entire cuboid

$$= 10 \times 5 \times 4 \text{ cm}^3 = 200 \text{ cm}^3 \qquad \ldots(1)$$

Sum of the volumes of 4 identical conical depressions

$$= 4 \times \frac{1}{3}\pi r^2 h$$

$$= \frac{4}{3} \times \frac{22}{7} \times 0.5 \times 0.5 \times 2.1 \text{ cm}^3$$

$$= 2.2 \text{ cm}^3 \qquad \ldots(2)$$

Volumes of the cubical depression

$$= 3^3 \text{ cm}^3 = 27 \text{ cm}^3 \qquad \ldots(3)$$

∴ Required volume of the wood in the entire stand

$$= (200 - 2.2 - 27) \text{ cm}^3$$

[From (1), (2) and (3)]

$$= \mathbf{170.8\ cm^3}$$

46. Let ABC be the right-angled triangle such that

AB = 15 cm and AC = 20 cm.

Using Pythagoras' theorem, we have

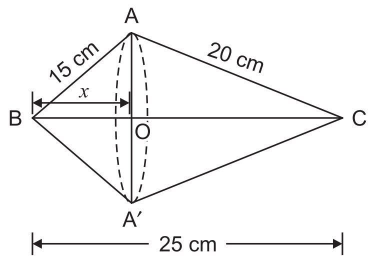
⇒ $BC^2 = AB^2 + AC^2$

$= 15^2 + 20^2$

$= 225 + 400$

$= 625$

$\Rightarrow \quad BC = 25$ cm



Let, OB = x, OA = y.

Applying Pythagoras' theorem in triangles OAB and OAC, we have

$$AB^2 = OB^2 + OA^2$$

and $\quad AC^2 = OA^2 + OC^2$

$\Rightarrow \quad 15^2 = x^2 + y^2$

and $\quad 20^2 = y^2 + (25 - x)^2$

$\Rightarrow \quad x^2 + y^2 = 225$

and $\quad (25 - x)^2 + y^2 = 400$

$\Rightarrow \quad [(25 - x)^2 + y^2] - [x^2 + y^2] = 400 - 225$

$\Rightarrow \quad [(25 - x)^2 - x^2] = 175$

$\Rightarrow \quad [(25 - x - x)(25 - x + x) = 175$

$\Rightarrow \quad 25 - 2x = 7$

$\Rightarrow \quad 2x = 18$

$\Rightarrow \quad x = 9$

Putting $x = 9$ in $x^2 + y^2 = 225$, we get

$81 + y^2 = 225$

$\Rightarrow \quad y^2 = 144$

$\Rightarrow \quad y = 12$

Thus, we have OA = 12 cm and OB = 9 cm.

Now, Volume of double cone

= Volume of cone CAA′ + Volume of cone BAA′

$$= \frac{1}{3}\pi(OA^2) \times OC + \frac{1}{3}\pi(OA^2) \times OB$$

$$= \frac{1}{3}\pi(12^2) \times 16 + \frac{1}{3}\pi(12^2) \times 9$$

$$= \frac{1}{3}\pi(12)^2[16 + 9] = \frac{1}{3} \times \frac{22}{7} \times 12 \times 12 \times 25$$

= **3768 cm³**

Surface area of double cone

= Curved surface area of cone CAA′ + Curved surface area of cone BAA′

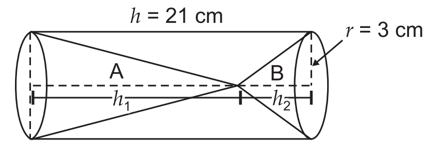
$= \pi \times OA \times AC + \pi \times OA \times AB$

$= (\pi \times 12 \times 20 + \pi \times 12 \times 15)$

$= \pi \times 420$ cm²

$$= \frac{22}{7} \times 420 = \mathbf{1318.8\ cm^2}$$

47. Let h_1 and h_2 be the height of the two cones A and B respectively and let r be the common radius of the two cones and the cylinder of height h.



Then $r = 3$ cm and $h = 21$ cm.

We see that $h_1 + h_2 = 21$ cm …(1)

Now, ratio of the capacities (or volume) of the two cones is

$$\frac{\frac{1}{3}\pi r^2 h_1}{\frac{1}{3}\pi r^2 h_2} = \frac{h_1}{h_2} = \frac{2}{1} \qquad \text{[Given]}$$

$\therefore \quad h_1 = 2h_2$ …(2)

$\therefore$ From (1) and (2), we have $3h_2 = 21$

$$\Rightarrow \quad h_2 = \frac{21}{3} = 7$$

$\therefore$ The required heights of the cones A and B are **14 cm** and **7 cm** respectively.

$\therefore$ The required volume of the cone A is

$$\frac{1}{3}\pi \times r^2 h_1 = \frac{1}{3} \times \frac{22}{7} \times 9 \times 14 \text{ cm}^3$$

= **132 cm³** …(1)

The required volume of the cone B is

$$\frac{1}{3}\pi \times r^2 h_2 = \frac{1}{3} \times \frac{22}{7} \times 9 \times 7 \text{ cm}^3$$

= **66 cm³** …(3)

$\therefore$ Sum of the volumes of the cones A and B is

(132 + 66) cm³ [From (1) and (2)]

= 198 cm³ …(3)

Also, the volume of the cylinder

$= \pi r^2 h$

$$= \frac{22}{7} \times 9 \times 21 \text{ cm}^3$$

= 594 cm³ …(4)

$\therefore$ The required volume of the remaining portion of the cylinder

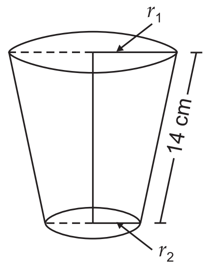
= (594 – 198) cm³ [From (3) and (4)]

= **396 cm³**

EXERCISE 15D

For Basic and Standard Levels

1. Let the radii of two circular ends of the frustum of cone be r_1 and r_2 where $r_1 > r_2$ and let h be the height of the frustum. Then $r_1 = 8$ cm, $r_2 = 6$ cm and $h = 14$ cm.



$\therefore$ The volume (or capacity) of the frustum

$$= \frac{\pi h}{3}\left(r_1^2 + r_1 r_2 + r_2^2\right)\ \text{cm}^3$$

$$= \frac{22}{7} \times \frac{1}{3} \times 14 \times \left(8^2 + 8 \times 6 + 6^2\right) \text{cm}^3$$

$$= \frac{44}{3} \times (64 + 48 + 36)\ \text{cm}^3$$

$$= \frac{44}{3} \times 148\ \text{cm}^3$$

$$= \frac{6512}{3}\ \text{cm}^3$$

$$= \mathbf{2170.67\ cm^3}$$

2. Let bucket forms a frustum of a cone such that the radii of its circular ends are $r_1 = 28$ cm, $r_2 = 7$ cm and height $h = 45$ cm.

Therefore,

Capacity of bucket = Volume of frustum

$$= \frac{1}{3}\pi h[r_1^2 + r_2^2 + r_1 r_2]$$

$$= \frac{1}{3} \times \frac{22}{7} \times 45[(28)^2 + (7)^2 + 28 \times 7]$$

$$= 48510\ \text{cm}^3$$

3. Let, radius of cone, R = 3 m.

Height of cone, H = 7 m.

Radius of frustum, $r_1 = 3$ m.

$r_2 = 2$ m

Height, $h = 10.5 - 7 = 3.5$ m.

Now, volume of haystack

= Volume of cone + Volume of frustum

$$= \left[\frac{1}{3}\pi R^2 H + \frac{1}{3}\pi h(r_1^2 + r_2^2 + r_1 r_2)\right]$$

$$= \frac{1}{3}\pi[R^2 H + h(r_1^2 + r_2^2 + r_1 r_2)]$$

$$= \frac{1}{3} \times \frac{22}{7}[9 \times 7 + 3.5(9 + 4 + 6)]$$

$$= \frac{1}{3} \times \frac{22}{7}[63 + 3.5(19)]$$

$$= \frac{1}{3} \times \frac{22}{7}[63 + 66.5]$$

$$= \frac{1}{3} \times \frac{22}{7}[129.5] = \mathbf{135.66\ m^3\ (approx.)}.$$

4. Let r_1 and r_2 be radii of circular ends of frustum and h be its height.

Then, $2\pi r_1 = 207.24$,
$2\pi r_2 = 169.56$

$$r_1 = \frac{207.24}{2\pi},\quad r_2 = \frac{169.56}{2\pi}$$

$$r_1 = \frac{103.62}{\pi},\quad r_2 = \frac{84.78}{\pi}$$

$\therefore$ Curved surface area $= \pi(r_1 + r_2)\, l$

$$= \pi\left[\frac{103.62}{\pi} + \frac{84.78}{\pi}\right]10$$

$$= \mathbf{1884\ cm^2}$$

5. Let r_1 and r_2 be radii of circular ends of frustum and h be its height.

Then, $2\pi r_1 = 96$, $h = 20$ cm.

$$r_1 = \frac{96}{2\pi} = \frac{48}{\pi} = \mathbf{15.27\ cm}$$

and $2\pi r_2 = 68$

$$r_2 = \frac{68}{2\pi} = \frac{34}{\pi} = \mathbf{10.82\ cm}$$

Let V be volume of frustum, then

$$V = \frac{1}{3}\pi[r_1^2 + r_2^2 + r_1 r_2]h$$

$$= \frac{1}{3}\pi\left[\left(\frac{48}{\pi}\right)^2 + \left(\frac{34}{\pi}\right)^2 + \frac{48}{\pi} \times \frac{34}{\pi}\right]20$$

$$= \frac{1}{3}\pi\left[\frac{2304 + 1156 + 1632}{\pi^2}\right]20$$

$$= \frac{1}{3}\left[\frac{5092 \times 7}{22}\right]20$$

$$= \mathbf{10801.21\ cm^3}$$

Slant height, $l^2 = h^2 + (r_1 - r_2)^2$
$= (20)^2 + (15.27 - 10.82)^2$
$= (20)^2 + (4.45)^2$

$\Rightarrow$ $l = \sqrt{419.84}$ cm

$= \mathbf{20.49\ cm}$

Total surface area

$$= \pi[(r_1 + r_2)l + r_1^2 + r_2^2]$$

$$= \frac{22}{7}[(15.27 + 10.82)20.49 + (15.27)^2 + (10.82)^2]$$

$$= \frac{22}{7}[534.58 + 233.17 + 117.07]$$

$$= \mathbf{2780.89\ cm^2}$$

6. Let r_1 and r_2 be the radii of frustum of cone and h be the height. Thus, $h = 9$ m.

Areas of ends are given 40 sq metres and 10 sq metres.

Now, Volume of frustum of cone

$$= \frac{1}{3}\pi h[r_1^2 + r_1 r_2 + r_2^2]$$

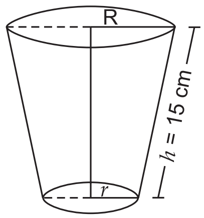
$$= \frac{1}{3}h[\pi r_1^2 + \pi r_1 r_2 + \pi r_2^2]$$

$$= \frac{h}{3}\left[A_1 + A_2 + \sqrt{A_1 A_2}\right]$$

$= \frac{9}{3}[40 + 10 + \sqrt{40 \times 10}] = 3[50 + 20]$

$= \mathbf{210\ m^3}$.

7. Let R and r (R > r) be the radii of two ends of the bucket in the shape of a frustum of a cone and let h be its height. Then R = 14 cm and h = 15 cm.



$\therefore$ The volume of the bucket

$$= \frac{\pi h}{3}\left(R^2 + Rr + r^2\right) \text{ cm}^3$$

$$= \frac{22 \times 15}{7 \times 3} \times \left(14^2 + 14r + r^2\right) \text{ cm}^3$$

$$= \frac{110}{7}\left(196 + 14r + r^2\right) \text{ cm}^3$$

$\therefore$ According to the problem, we have

$$\frac{110}{7}\left(196 + 14r + r^2\right) = 5390$$

$$\Rightarrow \quad 196 + 14r + r^2 = \frac{5390 \times 7}{110} = 343$$

$$\Rightarrow \quad r^2 + 14r - 147 = 0$$

$$\Rightarrow \quad r = \frac{-14 \pm \sqrt{14^2 + 4 \times 147}}{2}$$

$$= \frac{-14 \pm \sqrt{196 + 588}}{2}$$

$$= \frac{-14 \pm \sqrt{784}}{2}$$

$$= \frac{-14 \pm 28}{2}$$

$$= 7 \qquad [\because r > 0]$$

$\therefore$ Required value of r is **7 cm**.

8. (*i*) Here, $r_1 = 28$ cm, $r_2 = 21$ cm
and V = 28.490 litres
= 28490 cm^3 [$\because$ 1 litre = 1000 cm^3]

Then, volume of bucket $= \frac{1}{3}\pi h[r_1^2 + r_2^2 + r_1 r_2]$

$$\Rightarrow \quad 28490 = \frac{1}{3}\pi h[(28)^2 + (21)^2 + 28 \times 21]$$

$$\Rightarrow \quad 28490 = \frac{1}{3} \times \frac{22}{7} \times h[784 + 441 + 588]$$

$$\Rightarrow \quad 28490 = (1899)\, h$$

$$\Rightarrow \quad h = \frac{28490}{1899} = 15 \text{ cm}$$

Hence, height of bucket is **15 cm**.

(*ii*) Here, $r_1 = 20$ cm, $r_2 = 12$ cm
and V = 12308.8 cm^3

Then, Volume $= \frac{1}{3}\pi h[r_1^2 + r_2^2 + r_1 r_2]$

$$\Rightarrow \quad 12308.8 = \frac{1}{3}\pi h[(20)^2 + (12)^2 + 20 \times 12]$$

$$\Rightarrow \quad 12308.8 = \frac{1}{3} \times \frac{22}{7} \times h[784]$$

$$\Rightarrow \quad 12308.8 = 821.3\, h$$

$$\Rightarrow \quad h = 15 \text{ cm}$$

Now, whole surface area = Lateral surface area + Area of circular base

$$= \pi(r_1 + r_2)l + \pi r_2^2$$

$$= \pi(r_1 + r_2)\sqrt{h^2 + (r_1 - r_2)^2} + \pi r_2^2$$

$$= \pi[(20 + 12)\sqrt{(15)^2 + (20 - 12)^2} + (12)^2]$$

$$= 3.14[(32)\sqrt{289} + 144]$$

$$= 3.14\ [(32)\ (17) + 144]$$

$$= 2160.32 \text{ cm}^2$$

Hence, height of bucket is **15 cm** and area of metal sheet used in its making **2160.32 cm^2**.

9. Let radii of top and bottom of reservoir are 50 m and 100 m respectively. Then,

R = 50 m, r = 100 m.

Let the height of reservoir be H.

Volume of reservoir $= 44 \times 10^7$ litres $= \frac{44 \times 10^7}{10^3}$ m^3

$= 44 \times 10^4$ m^3

$$\Rightarrow \quad \frac{1}{3}\pi H[R^2 + r^2 + Rr] = 44 \times 10^4$$

$$\Rightarrow \quad \frac{1}{3} \times \frac{22}{7} \times H[(50)^2 + (100)^2 + 50 \times 100] = 44 \times 10^4$$

$$\Rightarrow \quad H\ [2500 + 10000 + 5000] = \frac{44 \times 10^4 \times 3 \times 7}{22}$$

$$\Rightarrow \quad H \times 17500 = 2 \times 3 \times 7 \times 10^4$$

$$\Rightarrow \quad H = \frac{2 \times 3 \times 7 \times 10^4}{17500} = \mathbf{24\ m}$$

Now, slant height,

$$l = \sqrt{(R - r)^2 + H^2} = \sqrt{(100 - 50)^2 + (24)^2}$$

$$= \sqrt{(50)^2 + (24)^2} = \sqrt{3076}$$

$$= 55.46 \text{ m}$$

Now, the lateral surface area $= \pi l\ (R + r)$

$$= \frac{22}{7} \times 55.46(100 + 50)$$

$$= \frac{22}{7} \times 55.46 \times 150$$

$$= \frac{183018}{7}$$

$$= \mathbf{26145.43\ m}$$

10. Let l be the slant height of frustum of the cone.

Thus, $l = 5$ cm.

Given, $R - r = 4$ cm.

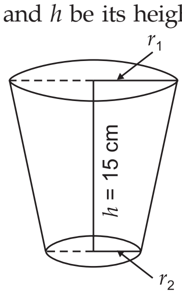
As we know, height of frustum,

$$h^2 = l^2 - (R - r)^2 = l^2 - (R - r)^2$$

$$\Rightarrow \quad h = \sqrt{(5)^2 - (4)^2}$$

$$= \sqrt{25 - 16} = \sqrt{9} = \mathbf{3\ cm}$$

11. (*i*) Let r_1 and r_2 be the radii of the two ends of the frustum, where $r_1 > r_2$ and h be its height.



Then $r_1 = 28$ cm, $r_2 = 21$ cm and $h = 15$ cm.

∴ Volume of the frustum

$$= \frac{\pi h}{3}\left[r_1^2 + r_1r_2 + r_2^2\right]$$

$$= \frac{22 \times 15}{7 \times 3}\left[28^2 + 28 \times 21 + 21^2\right] \text{cm}^3$$

$$= \frac{110}{7}(784 + 588 + 441) \text{ cm}^3$$

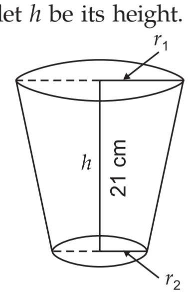
$$= \frac{110}{7} \times 1813 \text{ cm}^3$$

$$= 28490 \text{ cm}^3$$

$$= 28.48 \text{ litres}$$

∴ Required volume of water in the bucket is **28.49 litres**.

(*ii*) Let r_1 and r_2 be the radii (where $r_1 > r_2$) of the two circular ends of the bucket in the shape of a frustum of cone and let h be its height.



Then $r_1 = 20$ cm, $r_2 = 10$ cm and $h = 21$ cm.

∴ Volume of the bucket

$$= \frac{\pi h}{3}\left[r_1^2 + r_1r_2 + r_2^2\right]$$

$$= \frac{22 \times 21}{7 \times 3}\left[20^2 + 20 \times 10 + 10^2\right] \text{cm}^3$$

$$= 22 \times (400 + 200 + 100) \text{ cm}^3$$

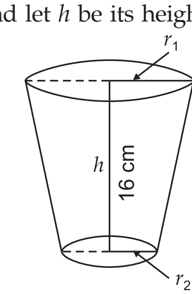
$$= 22 \times 700 \text{ cm}^3$$

$$= 15400 \text{ cm}^3$$

∴ Volume of milk = 15.45 litres

∴ Required cost of the milk = ₹30 × 15.5 = ₹**462**

(*iii*) Let r_1 and r_2 (where $r_1 > r_2$) be the radii of the two circular ends of the container in the form of a frustum of a cone and let h be its height.



Then $r_1 = 20$ cm, $r_2 = 8$ cm and $h = 16$ cm.

∴ Volume of the container

$$= \frac{\pi h}{3}\left[r_1^2 + r_1r_2 + r_2^2\right]$$

$$= \frac{22 \times 16}{7 \times 3}\left[20^2 + 20 \times 8 + 8^2\right] \text{cm}^3$$

$$= \frac{22 \times 16 \times 624}{3 \times 7} \text{ cm}^3$$

$$= 10459.43 \text{ cm}^3$$

$$= 10.45943 \text{ litre}$$

∴ Volume of the milk in the container = 10.45943 litres

∴ Required cost of the milk = ₹22 × 10.45943 = ₹**230.11**

(*iv*) Height of frustum = 21 cm

Radius of upper end = 8 cm

Radius of lower end = 20 cm

$$\text{Volume of frustum} = \frac{\pi h}{3}(R^2 + r^2 + rR)$$

$$= \frac{22}{7} \times \frac{21}{3}(400 + 64 + 160)$$

$$= 22 \times 624 = 13728 \text{ cm}^3$$

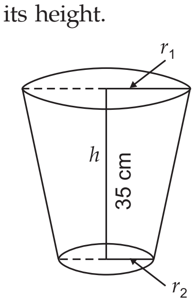
$1 \text{ cm}^3 = 0.001$ litre

$13728 \text{ cm}^3 = 13.728$ litres

Cost of milk, 1 litre = ₹35

13.728 litres = ₹35 × 13.728 = ₹**480.48**

(*v*) Let r_1 and r_2 (where $r_1 > r_2$) be the radii of two circular ends of the bucket in the form of a frustum of a cone and let h be its height.



Then $r_1 = 30$ cm, $r_2 = 12$ cm and $h = 35$ cm.

$\therefore$ Volume of the bucket

$$= \frac{\pi h}{3}\left[r_1^2 + r_1 r_2 + r_2^2\right]$$

$$= \frac{22 \times 35}{7 \times 3}\left[30^2 + 30 \times 12 + 12^2\right] \text{cm}^3$$

$$= \frac{110}{3} \times [90 + 360 + 144] \text{ cm}^3$$

$$= \frac{110 \times 1404}{3} \text{ cm}^3$$

$$= \frac{154440}{3} \text{ cm}^3$$

$= 51480$ cm^3

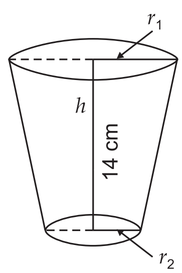
$= 51.48$ litres

$\therefore$ Required volume of milk = 51.48 litres

And required cost of the milk = ₹40 × 51.48

= **₹2059.20**

(*vi*) Let r_1 and r_2 (where $r_1 > r_2$) be the radii of two circular ends of the container is the shape of a frustum of a cone and h be its height.



Then $r_1 = 17.5$ cm, $r_2 = 15$ cm and $h = 14$ cm.

$\therefore$ Volume of the container

$$= \frac{\pi h}{3}\left[r_1^2 + r_1 r_2 + r_2^2\right]$$

$$= \frac{22 \times 14}{7 \times 3} \times \left[17.5^2 + 17.5 \times 15 + 15^2\right] \text{cm}^3$$

$$= \frac{44}{3} \times (306.25 + 262.5 + 225) \text{ cm}^3$$

$$= \frac{44}{3} \times 793.75 \text{ cm}^3$$

$$= \frac{34925}{3} \text{ cm}^3$$

$\therefore$ Volume of the oil in the container

$$= \frac{34925}{3} \text{ cm}^3$$

$\therefore$ Mass of the oil $= \frac{1.2 \times 34925}{3}$ g

= 13970 g

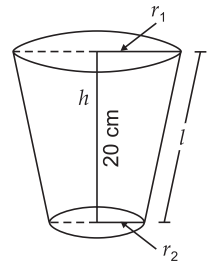
= 13.97 kg

$\therefore$ Required cost of the oil = ₹40 × 13.97 = **₹558.80**

12. (*i*) Let r_1 and r_2 be the radii of the circular ends of the bucket in the shape of a frustum of a cone and h be its height where $r_1 > r_2$.

Then $r_1 = 25$ cm, $r_2 = 10$ cm and $h = 20$ cm

Let l be the slant height of the bucket.



Then $l = \sqrt{h^2 + (r_1 - r_2)^2}$

$= \sqrt{20^2 + (25-10)^2}$ cm

$= \sqrt{400 + 225}$ cm

$= \sqrt{625}$ cm

= 25 cm

Now, the curved surface area of the frustum

$= \pi l(r_1 + r_2)$ cm^2

= 3.14 × 25 × 35 cm^2

= 3.14 × 875 cm^2 …(1)

Also, area of the circle at the bottom of the bucket

$= \pi r_2^2$

= 3.14 × 100 cm^2 …(2)

$\therefore$ Total area of the aluminium sheet required to make the bucket open at the top is

3.14(25 × 35 + 100) cm^2 = 3.14 × 975 cm^2

[From (1) and (2)]

= 3061.5 cm^2

$\therefore$ Required cost of the aluminium sheet

= ₹ $\frac{70}{100} \times 3061.5$ = **₹2143.05**

(*ii*) Let r_1 and r_2 are the radii of bucket and h be the height. Then,

$r_1 = 20$ cm, $r_2 = 8$ cm, $h = 16$ cm

Slant height, $l = \sqrt{h^2 + (r_1 - r_2)^2}$

$= \sqrt{16^2 + (20-8)^2}$ = 20 cm

Surface area of bucket (excluding the upper end)

$= \pi l\,(r + R) + \pi r^2$

$= \pi\,[l(r + R) + r^2]$

= 3.14 [20 (8 + 20) + 8^2]

= 1959.36 cm^2

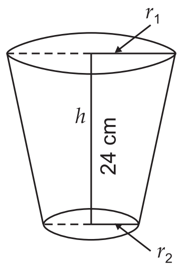
Cost of the metal sheet used $= \frac{1959.36 \times ₹\,15}{100}$

= **₹ 293.90**

(*iii*) Let r_1 and r_2 (where $r_1 > r_2$) be the radii of two circular ends of the bucket in the shape of a frustum of a cone and let h be its height.

Then $r_1 = 15$ cm, $r_2 = 5$ cm and $h = 24$ cm.

Let l be the slant height of the frustum of the cone.



Then $l = \sqrt{h^2 + (r_1 - r_2)^2}$

$= \sqrt{24^2 + 10^2}$ cm

$= \sqrt{676}$ cm

= 26 cm

∴ Curved surface area of the bucket

$= \pi(r_1 + r_2)l$

$= 3.14 \times (15 + 5) \times 26 \text{ cm}^2$

$= 3.14 \times 520 \text{ cm}^2$ …(1)

Also, the lower circular area of the bucket

$= \pi r_2^2$

$= 3.14 \times 25 \text{ cm}^2$ …(2)

∴ Total surface area of the metal sheet required to make the bucket

$= 3.14 \times (520 + 25) \text{ cm}^2$ [From (1) and (2)]

$= 3.14 \times 545 \text{ cm}^2$

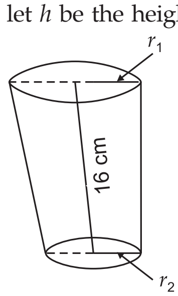
$= 1711.3 \text{ cm}^2$

∴ Required total cost of the metal sheet

$= ₹ \frac{10}{100} \times 1711.3$

= ₹1711.3

13. (*i*) Let r_1 and r_2 (where $r_1 > r_2$) be the radii of the two circular ends of the container in the shape of a frustum of a cone and let h be the height of the frustum.



Then $r_1 = 20$ cm, $r_2 = 8$ cm and $h = 16$ cm.

Then the volume of the container

$= \frac{\pi h}{3}\left[r_1^2 + r_1 r_2 + r_2^2\right]$

$= \frac{3.14 \times 16}{3}\left(20^2 + 20 \times 8 + 8^2\right) \text{ cm}^3$

$= \frac{3.14 \times 16}{3}(400 + 160 + 64) \text{ cm}^3$

$= \frac{3.14 \times 16}{3} \times 624 \text{ cm}^3$

$= 10449.92 \text{ cm}^3$

= 10.44992 litres

∴ Volume of the milk in the container

= Volume of the container

= 10.44992 litres

∴ Required cost of the milk

$= ₹15 \times 10.44992$

= **₹156.75**

Let l be the slant height of the frustum.

Then $l = \sqrt{h^2 + (r_1 - r_2)^2}$

$= \sqrt{16^2 + 12^2} \text{ cm}^2$

$= \sqrt{400}$ cm = 20 cm

∴ Curved surface area of the frustum

$= \pi l(r_1 + r_2)$

$= 3.14 \times 20 \times 28 \text{ cm}^2$

$= 3.14 \times 560 \text{ cm}^2$ …(1)

Also, area of the circle at the lower end of the frustum

$= \pi r_2^2$

$= 3.14 \times 64$ …(2)

∴ Total area of the frustum open at the top

$= 3.14 \times (560 + 64) \text{ cm}^2$

$= 3.14 \times 624 \text{ cm}^2$ [From (1) and (2)]

$= 1959.36 \text{ cm}^2$

Hence, required cost of the metal sheet used to make the container $= ₹ \frac{5}{100} \times 1959.36$ = **₹97.97**

(*ii*) Let, $r_1 = 16$ cm, $r_2 = 10$ cm, $h = 21$ cm.

Slant height, $l = \sqrt{(r_1 - r_2)^2 + (21)^2}$

$= \sqrt{(6)^2 + (21)^2} = \sqrt{36 + 441}$

= 21.84 cm

Volume of container

$= \frac{\pi}{3}[r_1^2 + r_2^2 + r_1 r_2]h$

$= \frac{22}{3 \times 7}[(16)^2 + (10)^2 + (16)(10)]21$

$= 11352 \text{ cm}^3 = 11.352$ litres

Cost of milk at ₹ 17.50 per litre $= 11.352 \times 17.50$

= **₹ 198.66**

Surface area of the metal sheet

$= \pi(r_1 + r_2)l + \pi r_2^2$

$= \frac{22}{7}[(16 + 10)(21.84) + (10)^2]$

$= 2098.92 \text{ cm}^2$

Cost of metal sheet at ₹ 7 per 100 cm² $= \frac{2098.92 \times 7}{100}$

= **₹146.92**

(*iii*) Volume of frustum = $10459\frac{3}{7} = \frac{73216}{7}\text{ cm}^3$

Let r_1 and r_2 be the lower and upper radii of frustum.

$r_1 = 20$ cm, $r_2 = 8$ cm.

Now, Volume of container

$$= \frac{\pi}{3}[r_1^2 + r_2^2 + r_1 r_2]h$$

$$\Rightarrow \frac{73216}{7} = \frac{\pi}{3}[(20)^2 + (8)^2 + 20 \times 8]h$$

$$\Rightarrow \frac{73216}{7} = \frac{22}{7 \times 3}[400 + 64 + 160]h$$

$$\Rightarrow \frac{73216}{7} = 653.71\ h$$

$$\Rightarrow h = \frac{73216}{653.71 \times 7} = 16 \text{ cm}$$

Slant height, $l = \sqrt{16^2 + (r_1 - r_2)^2}$

$$= \sqrt{16^2 + (20 - 8)^2} = 20 \text{ cm}$$

Now, Surface area $= \pi(r_1 + r_2)l + \pi r_2^2$

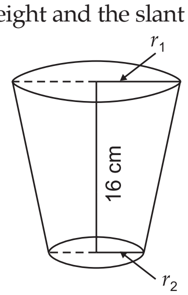
$$= \frac{22}{7}\ [(20 + 8)\ 20 + 64]$$

$$= 1961.14 \text{ cm}^2$$

Hence, cost of metal used in making the container at rate of ₹ 1.40 per sq. cm

$= 1961.14 \times 1.40 =$ **₹ 2745.60**

(*iv*) Let r_1 and r_2 be the radii of the circular ends of the frustum, where $r_1 > r_2$ and let h and l be respectively the vertical height and the slant height of the frustum.



Then $r_1 = 20$ cm, $r_2 = 8$ cm and $h = 16$ cm

$$\therefore \quad l = \sqrt{h^2 + (r_1 - r_2)^2}$$

$$= \sqrt{16^2 + (20 - 8)^2} \text{ cm}$$

$$= \sqrt{256 + 144} \text{ cm}$$

$$= \sqrt{400} \text{ cm}$$

$$= 20 \text{ cm}$$

(*a*) Sum of the curved surface area and the area of the circular base of the frustum

$$= \pi(r_1 + r_2)l + \pi r_2^2$$

$$= \pi\left[(r_1 + r_2)l + r_2^2\right]$$

$$= 3.14 \times \{(20 + 8) \times 20 + 8^2\} \text{ cm}^2$$

$$= 3.14 \times (560 + 64) \text{ cm}^2$$

$$= 3.14 \times 624 \text{ cm}^2$$

$$= 1959.36 \text{ cm}^2$$

∴ Required cost of the metal sheet

$$= ₹\ \frac{10}{100} \times 1959.36$$

= **₹195.94**.

(*b*) Now, the volume of the container

$$= \frac{\pi h}{3}\left(r_1^2 + r_1 r_2 + r_2^2\right)$$

$$= \frac{3.14 \times 16}{3}\left(20^2 + 20 \times 8 + 8^2\right) \text{ cm}^3$$

$$= \frac{3.14 \times 16}{3}(400 + 160 + 64) \text{ cm}^3$$

$$= \frac{3.14 \times 16 \times 624}{3} \text{ cm}^3$$

$$= 3.14 \times 16 \times 208 \text{ cm}^3$$

$$= 10449.92 \text{ cm}^3$$

= 10.44992 litres

= 10.45 litres

∴ Required cost of the milk

= volume of the milk in the container

= ₹35 × 10.45

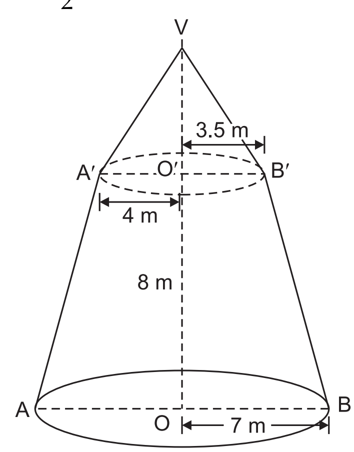
= **₹365.75**

14. (*i*) Let h be the height of the frustum and r_1 and r_2 be the radii of its circular bases.

Then, $h = 8$ m,

$$r_1 = \frac{14}{2} = 7 \text{ m},$$

$$r_2 = \frac{7}{2} = 3.5 \text{ m}.$$



Slant height of frustum,

$$l = \sqrt{(r_2 - r_1)^2 + h^2} = \sqrt{\left(7 - \frac{7}{2}\right)^2 + (8)^2}$$

$$= \sqrt{\left(\frac{7}{2}\right)^2 + (8)^2}$$

$$= \sqrt{(3.5)^2 + 64} = 8.7 \text{ m}$$

For cone VA′B′, we have

l_2 = slant height

$= \sqrt{O'B'^2 + VO'^2}$

$= \sqrt{r_2^2 + h^2}$

$= \sqrt{(3.5)^2 + (4)^2}$

$= \sqrt{28.25} = 5.36$ m

∴ Quantity of canvas required = Lateral surface area of frustum + Lateral surface area of cone VA′B′

$= [\pi (r_1 + r_2)\, l + \pi\, r_2\, l_2]$

$= \pi\, [(7 + 3.5)\, 8.7 + 3.5 \times 5.36]$

= **346.06 m²**

(*ii*) Let, r_1 be the radius of lower end of frustum,

$r_1 = \dfrac{26}{2} = 13$ m

r_2 be radius of upper end of frustum,

$r_2 = \dfrac{14}{2} = 7$ m and height of the frustum,

$h = 8$ m.

Then, slant height of the frustum,

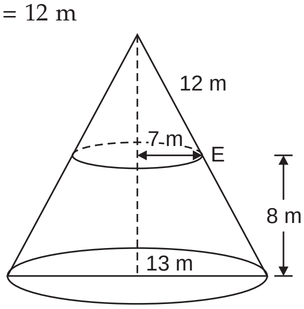
$l_1 = \sqrt{(r_2 - r_1)^2 + h^2} = \sqrt{(6)^2 + (8)^2}$

$= \sqrt{36 + 64}$

$= \sqrt{100} = 10$ m

Slant height of the cone,

$l_2 = 12$ m



∴ Quantity of canvas required to make the tent

= Lateral surface area of frustum + Lateral surface area of the cone surmounted

$= [\pi (r_1 + r_2)\, l_1 + \pi\, r_2\, l_2]$

$= \pi[(13 + 7) \times 10 + 7 \times 12]$

$= \dfrac{22}{7} \times 284 = \mathbf{\dfrac{6248}{7}}$ **m²**

15. External radius of bucket, $R = 14$ cm

Internal radius of bucket, $r = 7$ cm

Height, H = 16 cm.

Let the radius of hemisphere be x.

Volume of hemisphere = Volume of frustum

$\Rightarrow \quad \dfrac{2}{3}\pi x^3 = \dfrac{1}{3}\pi H[R^2 + r^2 + Rr]$

$\Rightarrow \quad 2x^3 = 16\, [(14)^2 + (7)^2 + 14 \times 7]$

$\Rightarrow \quad x^3 = 8\, [196 + 49 + 98]$

$\Rightarrow \quad = 8 \times 343$

$\Rightarrow \quad x^3 = (2 \times 7)^3$

$\Rightarrow \quad x = 2 \times 7 = 14$ cm

∴ Diameter of hemisphere = $2x = 2 \times 14 =$ **28 cm**

16. External radius (R) of frustum $= \dfrac{40 \text{ cm}}{2} = 20$ cm

Internal radius (r) of frustum $= \dfrac{16 \text{ cm}}{2} = 8$ cm

Height, H = 16 cm.

Now, Volume of frustum

$= \dfrac{1}{3}\pi[R^2 + r^2 + Rr]H$

$= \dfrac{1}{3} \times 3.14[(20)^2 + (8)^2 + (20)(8)]16$

$= \dfrac{1}{3} \times 3.14[400 + 64 + 160]16$

$\Rightarrow \quad V = 10449.92 \text{ cm}^3$

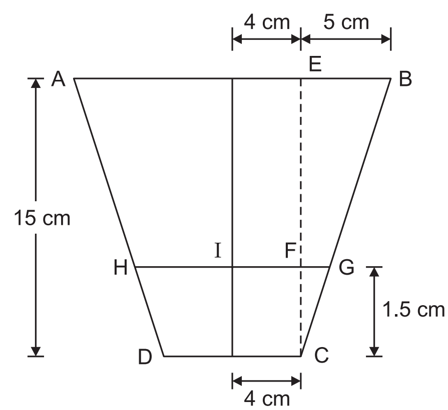
Now, weight of juice is 980 g-wt

$= \dfrac{10449.92 \times 980}{1000}$

= 10240.92 g

= **10.24 kg** (W = V × ρ)

17. Let, height of frustum be = 15 cm.



Let r_1 and r_2 be the radii of frustum HGCD.

Then, $r_2 = 4$ cm.

Draw CE ⊥ AB, Δ FGC ~ Δ EBC [By AA similarity]

$\therefore \quad \dfrac{FG}{EB} = \dfrac{FC}{EC}$

$\Rightarrow \quad \dfrac{FG}{5 \text{ cm}} = \dfrac{1.5 \text{ cm}}{15 \text{ cm}}$

$\Rightarrow \quad FG = 0.5$ cm

Now, $r_1 = IG = 4 + 0.5 = 4.5$ cm

Volume of frustum HGCD

$= \dfrac{1}{3}\pi h[r_1^2 + r_2^2 + r_1 r_2]$

$= \frac{22}{7} \times \frac{1}{3} \times 1.5[(4.5)^2 + (4)^2 + (4 \times 4.5)] \text{ cm}^3$

$= 85.25 \text{ cm}^3$

= Volume of water collected in 30 min

Volume of water collected in 1 hr = (85.25 × 2)

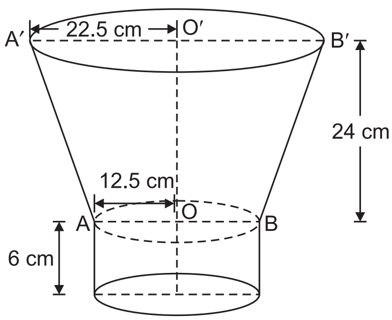
$= 170.5 \text{ cm}^3$

Hence, the rainfall = **170.5 cm³/h**

18. We have,

h = height of the frustum of the cone

= (30 – 6) = 24 cm.



Radii of circular ends are,

$r_1 = 22.5$ cm.

and $r_2 = 12.5$ cm.

Height the cylinder,

$h_2 = 6$ cm.

∴ l = slant height of the frustum

$\Rightarrow \quad l = \sqrt{h^2 + (r_1 - r_2)^2}$

$= \sqrt{(24)^2 + (22.5 - 12.5)^2}$

$= \sqrt{576 + 100} = \sqrt{676}$

$\Rightarrow \quad l = 26$ cm

Area of metallic sheet used = Curved surface area of the frustum of cone + Area of circular base + curved surface area of cylinder

$= \pi(r_1 + r_2)l + \pi r_2^2 + 2\pi r_2 h_2$

$= \frac{22}{7}[(22.5 + 12.5) \times 26 + (12.5)^2 + 2 \times 12.5 \times 6]$

$= \frac{22}{7}(910 + 156.25 + 150)$ = **3822.5 cm²**

Volume of water in the bucket

$= \frac{\pi}{3} \times h \times [r_1^2 + r_2^2 + r_1 r_2]$

$= \frac{22}{7} \times \frac{24}{3} \times [(22.5)^2 + (12.5)^2 + (22.5 \times 12.5)] \text{ cm}^3$

$= 23728.57 \text{ cm}^3$

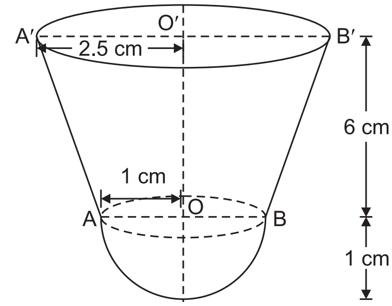
= 23.73 litres (approx.)

19. We have, r_1 = radius of the lower end of the frustum = 1 cm.

r_2 = radius of the upper end of the frustum = 2.5 cm.

r_3 = height of the frustum = 6 cm.

l = slant height of the frustum,



$\Rightarrow \quad l = \sqrt{h^2 + (r_2 - r_1)^2}$

$= \sqrt{36 + (2.5 - 1)^2} = 6.18$ cm

∴ External surface area of shuttle cock

= Curved surface area of frustum + Surface area of hemisphere

$= \pi(r_1 + r_2)l + 2\pi r_1^2$

$= [\pi(1 + 2.5) \times 6.18 + 2 \times \pi \times (1)^2]$

$= \left[\frac{22}{7}(3.5) \times 6.18 + 2 \times \frac{22}{7}\right]$

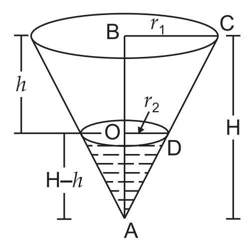
= [67.98 + 6.28] = 74.26 cm² (approx.)

For Standard Level

20. Let r_1 and r_2 be the radii of the upper and lower circular bases of the frustum of a cone, where $r_1 > r_2$.

Let h be the height of the frustum and H be the height of the whole cone. Then the height of the lower part of the cone (shown by shaded region) is H – h.

Hence, r_1 = 9 cm, r_2 = 3 cm, h = 8 cm, H – h = (H – 8) cm = AO in the figure.



Now, from similar triangles ABC and AOD, we have

$\frac{OD}{BC} = \frac{AO}{AB}$

$\Rightarrow \quad \frac{r_2}{r_1} = \frac{H - 8}{H}$

$\Rightarrow \quad \frac{3}{9} = \frac{1}{3} = \frac{H - 8}{H}$

$\Rightarrow \quad H = 3H - 24$

$\Rightarrow \quad 2H = 24$

$\Rightarrow \qquad H = \frac{24}{2} = 12$

(*i*) $\therefore$ The required height of the whole cone of which the bucket in the form of the frustum is a part is 12 cm.

(*ii*) The volume of the bucket completely filled with water

$$= \frac{\pi h}{3}\left(r_1^2 + r_1 r_2 + r_2^2\right) \text{ cm}^3$$

$$= \frac{\pi \times 8}{3}\left(9^2 + 9 \times 3 + 3^2\right) \text{ cm}^3$$

$$= \frac{8\pi}{3}(81 + 27 + 9) \text{ cm}^3$$

$$= \frac{8}{3} \times \pi \times 117 \text{ cm}^3 = \mathbf{312\pi \text{ cm}^3}$$

Required volume of water in the bucket.

(*iii*) Let l be the slant height of the frustum of cone.

Then, $l = \sqrt{h^2 + (r_1 - r_2)^2}$

$$= \sqrt{8^2 + (9-3)^2} \text{ cm}$$

$$= \sqrt{64 + 36} \text{ cm}$$

$$= \sqrt{100} \text{ cm}$$

$$= 10 \text{ cm}$$

$\therefore$ Total surface area of the frustum

$$= \pi[(r_1 + r_2)l + r_2^2]$$

$$= \pi[12 \times 10 + 9] \text{ cm}^2$$

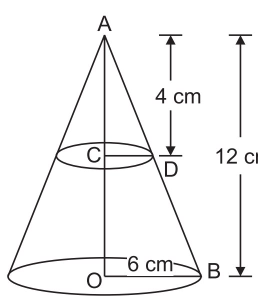
$$= 129\pi \text{ cm}^2$$

Hence, the required area of the copper sheet required to make the bucket in the shape of the above frustum $= \mathbf{129\pi \text{ cm}^2}$

21. Height of cone = 12 cm

Radius of cone = 6 cm

Height of cone removed = 4 cm



$$\Delta ACD \sim \Delta AOB$$

$$\therefore \quad \frac{AC}{AO} = \frac{CD}{OB}$$

$$\frac{4}{12} = \frac{CD}{6}$$

$$CD = \frac{6 \times 4}{12}$$

Radius of cone removed = 2 cm

Curved surface area of cone = πrl

$$= \pi \times 6 \times \sqrt{(6)^2 + (12)^2}$$

$$= \pi \times 6 \times \sqrt{36 + 144}$$

$$= 6\pi \times \sqrt{180}$$

$$= 36\sqrt{5}\,\pi \text{ cm}^2$$

Curved surface area of cone removed = πrl

$$= \pi \times 2 \times \sqrt{(2)^2 + (4)^2}$$

$$= 2\pi \sqrt{4 + 16}$$

$$= 2\pi \sqrt{20}$$

$$= 4\sqrt{5}\,\pi$$

Total surface area of remaining solid = curved surface area of cone – curved surface area of cone removed + area of base + area of top

$$= 36\sqrt{5}\,\pi - 4\sqrt{5}\,\pi + \pi\,(2)^2 + \pi\,(6)^2$$

$$= 32\sqrt{5}\,\pi + 4\pi + 36\pi$$

$$= 32\sqrt{5}\,\pi + 40\pi$$

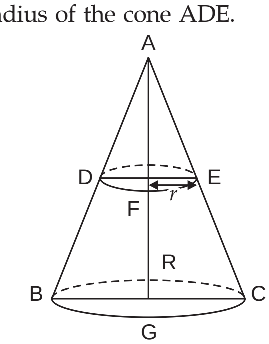
$$= \mathbf{350.59 \text{ cm}^2 \text{ (approx.)}}$$

22. Given, Radius of cone, r_1 = 10 cm

Let the height of cone be H.

Then, height of two parts = $\frac{H}{2}$

Let r be the radius of the cone ADE.



In ΔAEF and ΔACG,

$$\frac{\frac{H}{2}}{r} = \frac{H}{10}$$

$$\Rightarrow \qquad r = \frac{10}{2}$$

$$\Rightarrow \qquad r = 5 \text{ cm}$$

Volume of cone ADE $= \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times \pi \times (5)^2 \times \frac{H}{2} \qquad \ldots(1)$$

Volume of frustum $= \frac{1}{3}\pi \frac{H}{2}[r_1^2 + r^2 + r_1 r]$

$$= \frac{1}{3}\pi[(10)^2 + (5)^2 + 10 \times 5] \times \frac{H}{2}$$

$$= \frac{1}{3}\pi[100 + 25 + 50] \times \frac{H}{2}$$

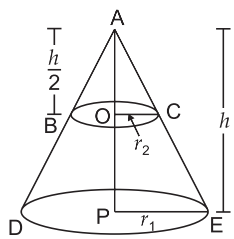
$$= \frac{1}{3}\pi \times 175 \times \frac{H}{2} \quad \ldots(2)$$

Solving (1) and (2),

$$\frac{\text{Volume of cone}}{\text{Volume of frustum}} = \frac{\frac{1}{3}\pi \times 5 \times 5 \times \frac{H}{2}}{\frac{1}{3}\pi \times 175 \times \frac{H}{2}} = \frac{1}{7}$$

Hence, the ratio between volume of two parts are **1 : 7**.

23. Let r_1 and r_2 be the radii of the bases of two cones ADE and ABC respectively. Let h be the height of the whole cone. The cone is divided into two parts by a plane through the mid-point O of the axis AP of the cone.



Then r_1 = 8 cm

Then $\quad AO = OP = \frac{h}{2} = \frac{12}{2} = 6$

Now, from similar triangles AOC and APE, we have

$$\frac{r_2}{r_1} = \frac{AO}{AP} = \frac{h/2}{h} = \frac{1}{2}$$

$\therefore \quad r_1 = 2r_2$

$\Rightarrow \quad r_2 = \frac{r_1}{2} = \frac{8}{2}$ cm = 4 cm $\quad \ldots(1)$

r_1 = 8 cm $\quad \ldots(2)$

and $\quad h$ = 12 cm $\quad \ldots(3)$

Now, volume of the cut conical part

$$= \frac{1}{3}\pi r_2^2 \frac{h}{2}$$

$$= \frac{\pi}{6} \times 16 \times 12 \text{ cm}^3 = 32\pi \quad \ldots(4)$$

Also, volume of the frustum BDEC

$$= \frac{\frac{\pi h}{2}}{3}\left(r_1^2 + r_1 r_2 + r_2^2\right)$$

$$= \frac{\pi}{6} \times 12 \times \left(8^2 + 4 \times 8 + 16\right) \text{ cm}^3$$

[From (1), (2) and (3)]

$= 2\pi \times (64 + 32 + 16)$ cm^3

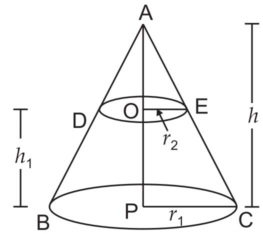
$= 224\pi$ cm^3 $\quad \ldots(5)$

$\therefore$ Required ratio of volume of the frustum and the volume of cut conical part

$= 224\pi : 32\pi \quad$ [From (4) and (5)]

$=$ **7 : 1**

24. Let the radius of the base of the whole cone ABC be r_1 and let the radius of the cut off smaller cone from the top be r_2. Let h be the height of the whole cone ABC and h_1 be the height of the base of the smaller cone ADE.



Then $\quad h$ = 20 cm $\quad$ [Given] …(1)

Let the height of the upper cone ADE be h' so that

$$h' = 20 - h_1 \quad \ldots(2)$$

Now, volume of the whole cone $= \frac{1}{3}\pi r_1^2 h$

$$= \frac{20}{3}\pi r_1^2 \quad \ldots(3)$$

Now, form two similar triangles AOE and APC, we have

$$\frac{r_1}{r_2} = \frac{20}{20 - h_1} = \frac{20}{h'} \quad \ldots(4)$$

$$\therefore \quad r_2 = \frac{(20 - h_1)r_1}{20} \quad \ldots(5)$$

Now, volume of the smaller cone AOE

$$= \frac{\pi}{3}h' r_2^2$$

$$= \frac{\pi}{3} \times (20 - h_1)\frac{(20 - h_1)}{400}r_1^2 \quad \ldots(6)$$

[From (4)]

$$= \frac{1}{8} \times \frac{\pi}{3}r_1^2 \times 20 \quad \text{[Given]}$$

$$= \frac{5\pi}{6}r_1^2 \quad \ldots(7)$$

$\therefore$ From (6) and (7), we have

$$\frac{1}{3}\frac{(20 - h_1)^3}{400} = \frac{5}{6}$$

$$\Rightarrow \quad (20 - h_1)^3 = \frac{5}{6} \times 3 \times 400$$

$$= \frac{5}{2} \times 400 = 1000 = 10^3$$

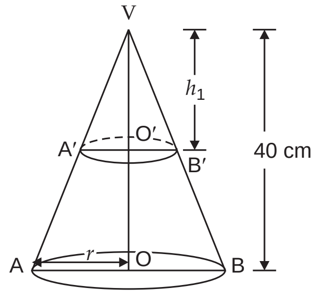
$$\Rightarrow \quad 20 - h_1 = 10$$

$$\Rightarrow \quad h_1 = 20 - 10 = 10$$

Hence, the required height of the base of the smaller cone cut off is **10 cm**.

25. Let VAB be a cone of height 40 cm and base radius r. Suppose it is cut off by a plane parallel to the base at a height h from the base of the cone. Let r_1 be the radius of the cone VA′B′.

$$\therefore \quad = \frac{VO}{VO'} = \frac{OA}{O'A'} = \frac{40}{h_1} = \frac{r}{r_1} \quad \ldots(1)$$



Given, Volume of cone VA′B′

$$= \frac{1}{64} \text{ Volume of cone VAB}$$

$$\Rightarrow \quad \frac{1}{3}\pi r_1^2 h_1 = \frac{1}{64} \times \frac{1}{3}\pi r^2 \times 40$$

$$\Rightarrow \quad \left(\frac{r_1}{r}\right)^2 h_1 = \frac{5}{8}$$

$$\Rightarrow \quad \left(\frac{h_1}{40}\right)^2 h_1 = \frac{5}{8} \qquad \text{[From (1)]}$$

$$\Rightarrow \quad (h_1)^3 = \frac{1600 \times 5}{8} = 1000$$

$$\Rightarrow \quad h_1 = 10 \text{ cm}$$

and $\quad h = 40 - h_1 = 40 - 10 =$ **30 cm**.

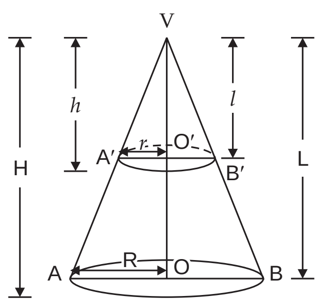
26. ΔVOA ~ Δ VO′A′

$$\therefore \quad \frac{r}{R} = \frac{h}{H} = \frac{l}{L} \qquad \ldots(1)$$

According to question,

Curved surface area of frustum ABB′A′

$$= \frac{15}{16} \times \text{Curved surface area of cone VAB}$$



$$\Rightarrow \quad \pi\,(R + r)\,(L - l) = \frac{15}{16}\pi RL$$

$$\Rightarrow \quad \left(\frac{R+r}{R}\right)\left(\frac{L-l}{L}\right) = \frac{15}{16}$$

$$\Rightarrow \quad \left(1+\frac{r}{R}\right)\left(1-\frac{l}{L}\right) = \frac{15}{16}$$

$$\Rightarrow \quad \left(1+\frac{h}{H}\right)\left(1-\frac{h}{H}\right) = \frac{15}{16} \qquad \text{[Using (1)]}$$

$$\Rightarrow \quad \left(1-\frac{h^2}{H^2}\right) = \frac{15}{16}$$

$$\Rightarrow \quad \frac{h^2}{H^2} = 1 - \frac{15}{16}$$

$$\Rightarrow \quad \frac{h^2}{H^2} = \frac{1}{16}$$

$$\Rightarrow \quad \frac{h}{H} = \frac{1}{4}$$

$$\Rightarrow \quad H = 4\,h$$

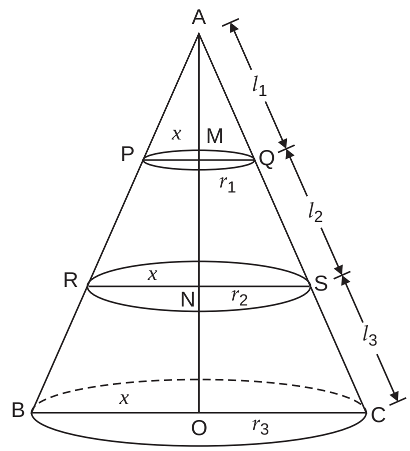
Then, the required ratio is

$$\frac{VO'}{O'O} = \frac{h}{H-h}$$

$$= \frac{h}{4h-h} = \frac{1}{3}.$$

Hence, the ratio = **1 : 3**.

27. Let ABC be a cone of height h and base radius r_3. Suppose, it is cut by a plane parallel to base of cone at two point at N or M. Let MQ = r_1, NS = r_2, OC = r_3. Plane PQ and RS trisect the height AO at M and N, i.e. AM = MN = NO = x.



The cone gets divided into three portions namely cone APQ, frustum PQSR and frustum RSCB.

Let $\quad$ MQ = r_1, NS = r_2, OC = r_3,

AQ = l_1, QS = l_2, SC = l_3.

ΔAMQ ~ ΔANS,

$$\therefore \quad \frac{AM}{AN} = \frac{MQ}{NS} = \frac{AQ}{AS}$$

$$\Rightarrow \quad \frac{x}{2x} = \frac{r_1}{r_2} = \frac{l_1}{l_1 + l_2}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{r_1}{r_2} = \frac{l_1}{l_1 + l_2}$$

$$\Rightarrow \quad r_2 = 2r_1 \text{ and } l_2 = l_1 \qquad \ldots(1)$$

Similarly, by proving ΔAMQ ~ Δ AOC, we can prove that $r_3 = 3r_1$ and $l_3 = l_1$.

Curved surface area of cone APQ : Curved surface area of frustum PQSR : Curved surface area of frustum RSCB.

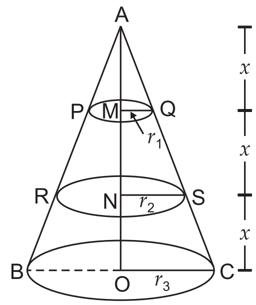
$$\Rightarrow \quad \pi r_1 l_1 : \pi l_2 (r_1 + r_2) : \pi l_3 (r_2 + r_3)$$

$$\Rightarrow \quad r_1 l_1 : l_1 (r_1 + 2r_1) : l_1 (2r_1 + 3r_1)$$

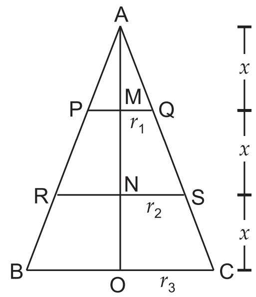
$\Rightarrow \quad r_1 l_1 : 3r_1 l_1 : 5r_1 l_1 =$ **1 : 3 : 5**.

28. Let the cone ABC be divided into three parts by trisecting the axis AO by two planes PMQ and RNS parallel to the base BOC of the whole cone. Let x cm be the length of each equal part of the axis, AM, MN and NO so that

$$AM = MN = NO = x \text{ cm}$$



Let MQ = r_1, NS = r_2 and OC = r_3 be the radii of three circles.



Now, $\Delta AMQ \sim \Delta ANS$

$$\therefore \quad \frac{AM}{AN} = \frac{MQ}{NS}$$

$$\Rightarrow \quad \frac{x}{2x} = \frac{r_1}{r_2}$$

$$\Rightarrow \quad \frac{r_1}{r_2} = \frac{1}{2}$$

$$\Rightarrow \quad r_2 = 2r_1 \qquad \ldots(1)$$

Similarly, $\Delta AMQ \sim \Delta AOC$

$$\therefore \quad \frac{AM}{AO} = \frac{MQ}{OC}$$

$$\Rightarrow \quad \frac{x}{3x} = \frac{r_1}{r_3}$$

$$\Rightarrow \quad r_3 = 3r_1 \qquad \ldots(2)$$

$\therefore$ Volume the cone APQ = $\frac{1}{3}\pi r_1^2 x$ cm^3

Volume of the frustum PQSR

$$= \frac{\pi}{3} \times x\left(r_1^2 + r_2 r_3 + r_2^2\right) \text{ cm}^3$$

Volume of the frustum RSCB = $\frac{\pi}{3} \times x\left(r_2^2 + r_1 r_2 + r_3^2\right)$ cm^2

$\therefore$ Volume of the cone APQ : Volume of frustum PQSR : Volume of frustum RSCB

$$= \frac{1}{3}\pi r_1^2 x \ : \ \frac{1}{3}\pi x\left(r_1^2 + r_1 r_2 + r_2^2\right) \ : \ \frac{1}{3}\pi x\left(r_2^2 + r_3^2 + r_2 r_3\right)$$

$$= r_1^2 \ : \ \left(r_1^2 + r_1 r_2 + r_2^2\right) \ : \ \left(r_2^2 + r_2 r_3 + r_3^2\right)$$

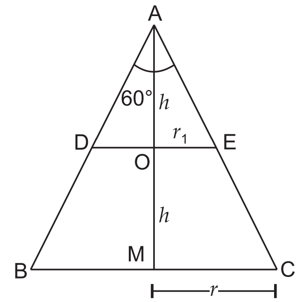
$$= r_1^2 \ : \ \left(r_1^2 + 2r_1^2 + 4r_1^2\right) \ : \ \left(4r_1^2 + 6r_1^2 + 9r_1^2\right)$$

[Using (1) and (2)]

$$= r_1^2 \ : \ 7r_1^2 \ : \ 19r_1^2$$

= 1 : 7 : 19 which is the required ratio.

29. Let the radius of the whole cone ABC be r and h be the height of the cone ADE and the frustum DECB. Let the radii of the bases of the two cones ADE and ABC be r_1 and r_2 respectively. Since the vertical angle $\angle BAC$ is given to be 60° and AB = AC.



$\therefore$ Each of the three angles $\angle BAC$, $\angle ACB$ and $\angle ABC$ is 60°.

$\therefore$ The ΔABC is an equilateral triangle.

$\therefore$ From ΔAMC, we have

$$\frac{AM}{r} = \tan 60° = \sqrt{3}$$

$$\Rightarrow \quad \frac{20}{r} = \sqrt{3}$$

$$\Rightarrow \quad r = \frac{20}{\sqrt{3}}$$

$$\Rightarrow \quad r^2 = \frac{400}{3} \qquad \ldots(1)$$

Now, volume of the frustum DECB of the cone

$$= \frac{\pi}{3} \times 10\left(r^2 + r_1^2 + r r_1\right) \text{ cm}^3$$

$$= \frac{10\pi}{3}\left(\frac{400}{3} + r_1^2 + \frac{20}{\sqrt{3}} r_1\right) \text{ cm}^3 \ldots(2)$$

Now, from similar triangles AOE and AMC, we have

$$\frac{OE}{MC} = \frac{AO}{AM}$$

$$\Rightarrow \quad \frac{r_1}{r} = \frac{10}{20}$$

$$\Rightarrow \quad r_1 = \frac{1}{2}r = \frac{10}{\sqrt{3}} \qquad \ldots(3) \text{ [From (1)]}$$

$\therefore$ From (2) and (3), we have volume of the frustum DECB

$$= \frac{10\pi}{3}\left(\frac{400}{3} + \frac{100}{3} + \frac{200}{3}\right)$$

$$= \frac{10\pi}{3} \times \frac{700}{3} = \frac{7000}{9}\pi \qquad \ldots(4)$$

Now, the volume of the cylindrical wire of length, say l cm and radius = $\frac{1}{24}$ cm is

$$\pi \times \left(\frac{1}{24}\right)^2 \times l \text{ cm}^3 = \frac{l\pi}{576} \qquad \ldots(5)$$

∴ From (4) and (5), we have

$$\frac{l\pi}{576} = \frac{7000\pi}{9}$$

$$\therefore \quad l = \frac{7000 \times 576}{9} \text{ cm}$$

$$= 448000 \text{ cm}$$

$$= 4480 \text{ m}$$

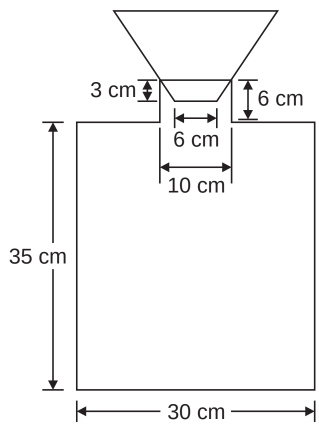
∴ Required length of the wire is **4480 m**.

30. Let, height of cylinder, H = 35 cm

Radius of cylinder, $R = \frac{30}{2} = 15$ cm.

Radius of small cylinder, $r = 5$ cm

Height of small cylinder, $h = 6$ cm



Now, radii (r_1) of frustum = 5 cm.

radii (r_2) of frustum = 3 cm.

Now, capacity of closed jar

= volume of cylinder + volume of small cylinder − volume of frustum

$$= \pi R^2 H + \pi r^2 h - \frac{1}{3}\pi[r_1^2 + r_2^2 + r_1 r_2]h$$

$$= \pi\left\{15 \times 15 \times 35 + 5 \times 5 \times 6 - \frac{1}{3}[(5)^2 + (3)^2 + 5 \times 3]3\right\}$$

$= \pi\,[225 \times 35 + 25 \times 6 - 49]$

$= \pi[7875 + 150 - 49]$

$= \pi \times 7976$

$= 3.14 \times 7976$

$=$ **25044.64 cm³**.

CHECK YOUR UNDERSTANDING

MULTIPLE-CHOICE QUESTIONS

For Basic and Standard Levels

1. (*a*) **three cylinders and two hemispheres.**

2. (*b*) **2*x***

$$a^3 = (2x)^3 = 8x^3$$

$$\Rightarrow \quad a = 2x$$

3. (*d*) **216 cm³**

$6a^2 = 216$ cm²

$\Rightarrow \quad a = 6$ cm

$\Rightarrow \quad 6\,(6)^2 = 216$ cm²

$\Rightarrow \quad V = a^3 = (6)^3$

$= 216$ cm³

4. (*a*) **1000 cm³**

Diagonal of cube $= \sqrt{3}\,a$

$17.32 \text{ cm} = \sqrt{3}\,a$

$$a = \frac{17.32 \text{ cm}}{\sqrt{3}} = \frac{17.32 \text{ cm}}{1.732} = 10 \text{ cm}$$

Now, Volume $= a^3 = (10)^3 = 1000$ cm³

5. (*b*) **4 cm**

Volume of cube $= l\,b\,h = 8 \times 4 \times 2 = 64$ cm³.

$\Rightarrow \quad V = a^3 = 64 = (4)^3$

$\Rightarrow \quad V = 4$ cm.

6. (*c*) **1000000**

Dimension of small boxes

$= 8 \text{ cm} \times 7 \text{ cm} \times 6 \text{ cm}$

$= 336$ cm³.

Dimension of large boxes

$= 8 \text{ m} \times 7 \text{ m} \times 6 \text{ m}$

$= 8 \times 10^2 \text{ cm} \times 7 \times 10^2 \text{ cm} \times 6 \times 10^2 \text{ cm}$

$= 336 \times 10^6$ cm³.

$$\text{Number of boxes} = \frac{\text{Dimension of large boxes}}{\text{Dimension of small boxes}}$$

$$= \frac{336 \times 10^6 \text{ cm}^3}{336 \text{ cm}^3}$$

$= 1000000$

7. (*d*) **8 cm**

Volume of cylinder, $\pi r^2 h = 448\,\pi$

$$\Rightarrow \quad r^2 = \frac{448}{7} = 64$$

$\Rightarrow \quad r = 8$ cm

8. (*d*) **1.25 cm**

According to question, curved surface area of solid cylinder is equal to one to third of its total surface area.

$$2\pi rh = \frac{1}{3}2\pi r(r + h)$$

$$h = \frac{1}{3}(r + h)$$

$$3h = r + h$$

$$h = \frac{r}{2} = \frac{2.5 \text{ cm}}{2} = 1.25 \text{ cm}$$

9. (*a*) **25**

Number of circular plates

$$= \frac{\text{Volume of cylinder}}{\text{Volume of circular plates}}$$

$$= \frac{1925 \text{ cm}^3}{\pi r^2 h}$$

$$= \frac{1925 \text{ cm}^3}{\frac{22}{7} \times (7 \text{ cm})^2 \times (0.5 \text{ cm})} = 25 \text{ plates.}$$

10. (*b*) **126 cm**

Volume of cylindrical wire = 440 cm^3.

$\Rightarrow \quad \pi r^2 H = 440 \qquad (\because \ H = 14 + h)$

$\Rightarrow \ (14 + h) = \frac{440 \times 7}{22} = 140$

$\Rightarrow \quad h = 140 - 14 = 126$ cm.

11. (*b*) **1 : 2**

Let radii of cylinders are in ratio of $\sqrt{2} : 1$.

Now, $\quad V_1 = V_2$

$\Rightarrow \quad \pi r_1^2 h_1 = \pi r_2^2 h_2$

$\Rightarrow \quad (\sqrt{2})^2 h_1 = (1)^2 h_2$

$\Rightarrow \quad \frac{2}{1} = \frac{h_2}{h_1}$

$\Rightarrow \quad h_1 : h_2 = 1 : 2$

12. (*a*) **6 cm**

$4\pi r^2 = 144\,\pi$

$\Rightarrow \quad r^2 = \frac{144}{4} = 36$

$\Rightarrow \quad r = 6$ cm

13. (*d*) **8 : 27**

$\frac{S_1}{S_2} = \frac{4}{9}$

$\Rightarrow \quad \frac{4\pi r^2}{4\pi R^2} = \frac{4}{9}$

$\Rightarrow \quad \frac{r}{R} = \frac{2}{3}$

$\Rightarrow \quad V_1 : V_2 = \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi R^3} = \left(\frac{r}{R}\right)^3 = \left(\frac{2}{3}\right)^3 = \frac{8}{27}$

$\Rightarrow \quad V_1 : V_2 = 8 : 27$

14. (*b*) **3 cm**

Volume of hemisphere = 18π

$\Rightarrow \quad \frac{2}{3}\pi r^3 = 18\pi$

$\Rightarrow \quad r^3 = \frac{18 \times 3}{2} = 3^3$

$\Rightarrow \quad r = 3$ cm

15. (*d*) **15 cm**

$S_1 = \pi r^2 = 314 \text{ cm}^2$

$\Rightarrow \quad r^2 = \frac{314}{3.14} = 100$

$\Rightarrow \quad r = 10$ cm

$\Rightarrow \quad V_1 = \frac{1}{3}\pi r^2 h = 1570 \text{ cm}^3$

$\Rightarrow \quad h = \frac{1570 \times 3 \times 7}{22 \times 10 \times 10} = 15$ cm

16. (*b*) **4.5 cm**

$a^3 = 729 \text{ cm}^3$

$\Rightarrow \quad a = 9$ cm

Radius = $\frac{a}{2} = \frac{9}{2} = 4.5$ cm

17. (*b*) **272π cm²**

2 × curved surface area of cone = $2\pi rl$

$= 2 \times \pi \times 8 \times 17 \qquad \left(\begin{aligned} \therefore\ l &= \sqrt{r^2 + h^2} \\ &= \sqrt{(8)^2 + (15)^2} \\ &= 17 \text{ cm} \end{aligned}\right)$

$= 272\pi \text{ cm}^2$.

18. (*c*) **25 : 64**

$\frac{V_1}{V_2} = \frac{\frac{1}{3}\pi r_1^2 h_1}{\frac{1}{3}\pi r_2^2 h_2} \Rightarrow \frac{1}{4} = \left(\frac{r_1}{r_2}\right)^2 \left(\frac{h_1}{h_2}\right)$

$\Rightarrow \quad \frac{1}{4} = \left(\frac{4}{5}\right)^2 \left(\frac{h_1}{h_2}\right)$

$\Rightarrow \quad \frac{h_1}{h_2} = \frac{25}{64}$

Hence, $h_1 : h_2 = 25 : 64$

19. (*a*) **4 : 1**

$\pi r_1 l_1 = 2(\pi r_2 l_2)$

$\Rightarrow \quad \pi r_1 l_1 = 2(\pi r_2 2 l_1)$

$\Rightarrow \quad \frac{r_1}{r_2} = \frac{4}{1}$

Hence, $r_1 : r_2 = 4 : 1$.

20. (*c*) **14a²**

Surface area of cube = 2 ($lb + bh + hl$)

$= 2\,(3a \times a + a \times a + a \times 3a)$

$= 2\,(3a^2 + a^2 + 3a^2) = 14a^2$.

21. (*d*) **4851 cm³**

Length of edge of cube = a = 21 cm.

Radius, $r = \frac{a}{2} = 10.5$ cm.

Volume of sphere = $\frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times (10.5)^3$

$= 4851 \text{ cm}^3$.

22. (*a*) **πr²**

Volume of cuboid = Volume of right circular cylinder

$\Rightarrow \quad l\,b\,h = \pi r^2 H$

Height of cuboid and right circular cylinder are equal

$h = H$

$\Rightarrow \quad l\,b\,h = \pi r^2 h$

$\Rightarrow \quad l\,b = \pi r^2$

Hence, area of cuboid is πr^2.

23. (*b*) **9 cm**

V_1 = Volume of cone = $\frac{1}{3}\pi r^2 h_1$

$\Rightarrow \quad V_2$ = Volume of cylinder = $\pi r^2 h_2$

$\Rightarrow \quad V_1 = V_2$

$\Rightarrow \frac{1}{3}\pi r^2 h_1 = \pi r^2 h_2$

$\Rightarrow \quad \frac{1}{3}\pi r^2 h_1 = \pi r^2 (3)$

$\Rightarrow \quad h_1 = 9$ cm

24. (*d*) **9 : 8**

$r_1 = 3, r_2 = 4, h_1 = 2, h_2 = 3.$

$$\frac{\text{Volume of cylinder}}{\text{Volume of cone}} = \frac{\pi r_1^2 h_1}{\frac{1}{3}\pi r_2^2 h_2} = \frac{(3)^2 \cdot 2}{\frac{1}{3}\cdot(4)^2\cdot 3} = \frac{9}{8}$$

25. (*c*) **64**

$r_1 = 8$ cm, $r_2 = 2$ cm.

Number of spherical balls

$$= \frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \frac{(8)^3}{(2)^3} = \frac{512}{8} = 64.$$

Hence, number of balls are 64.

26. (*d*) $\mathbf{\frac{4}{3}\pi m^3}$

$r = 1$ m, $h = 4$ m,

Volume of sphere = $\frac{4}{3}\pi r^3$

$= \frac{4}{3}\pi(1)^3 = \frac{4}{3}\pi \text{ m}^3$

27. (*d*) **36 cm²**

Total surface area of sphere, $4\pi r^2 = 48$

$\Rightarrow \quad r^2 = \frac{12}{\pi}$ cm

Now, total surface area of hemisphere

$= 3\pi r^2 = 3\pi \times \frac{12}{\pi}$

$= 36 \text{ cm}^2.$

28. (*a*) **2**

$\frac{2}{3}\pi r^3 = \frac{1}{3}\pi r^2 h$

Volume of hemisphere = Volume of cone

$\Rightarrow \quad 2 = \frac{h}{r} \quad \Rightarrow \quad \frac{h}{r} = 2$

29. (*d*) $\mathbf{\frac{1}{3}\pi h[r_1^2 + r_2^2 + r_1 r_2]}$

30. (*b*) **Remain unaltered.**

31. (*d*) **1 : 2**

Surface area of hemisphere = Surface area of cone

$\Rightarrow \quad 2\pi r^2 = \pi r l$

$\Rightarrow \quad \frac{r}{l} = \frac{1}{2} = 1 : 2$

32. (*a*) $\mathbf{\frac{\pi r^2}{3}[3h - 2r]}$

Volume of cylinder – Volume of hemisphere

$$= \pi r^2 h - \frac{2}{3}\pi r^3 = \frac{\pi r^2}{3}[3h - 2r]$$

33. (*a*) **6 cm**

We have, volume of the cone = volume of the sphere

$\Rightarrow \quad \frac{1}{3}\pi r_1^2 h = \frac{4}{3}\pi r_2^3$

$\Rightarrow \quad r_2^3 = \frac{r_1^2 h}{4} = \frac{(6)^2 \times 24}{4}$

$\Rightarrow \quad r_2 = 6$ cm

34. (*c*) **41**

Let r_1 and r_2 are radii of bucket and h be the height.

$r_1 = 40$ cm, $r_2 = 24$ cm, $h = 15$ cm.

Slant height of bucket, $l = \sqrt{(r_1 - r_2)^2 + h^2}$

$= \sqrt{(24-15)^2 + (40)^2}$

$l = 41$ cm

35. (*a*) **6 cm**

Let $\quad r_1 = 14$ cm, $r_2 = 6$ m, $l = 10$ cm.

Then, $\quad h = \sqrt{l^2 - (r_1 - r_2)^2} = \sqrt{(10)^2 - (14-6)^2}$

$= \sqrt{100 - 64} = \sqrt{36}$

$h = 6$ cm

36. (*c*) **1056 cm³**

$h = 21$ cm, $r_1 = 3$ cm, $r_2 = 5$ cm

Now, Volume of metal used in making the pipe.

$(\pi r_2^2 - \pi r_1^2)h = \pi h(r_2^2 - r_1^2)$

$= \frac{22}{7} \times 21 \times (25 - 9) = 1056 \text{ cm}^3$

For Standard Level

37. (*c*) **28 cm**

CSA of cone = $\pi r l = 2310 \text{ cm}^2$

$\Rightarrow \quad r = \frac{2310 \times 7}{35 \times 22} = 21$ cm

$\Rightarrow \quad h^2 = l^2 - r^2 = (35)^2 - (21)^2$

$= 1225 - 441$

$= 784$

$\Rightarrow \quad h = 28$ cm

38. (*a*) **8 : 27**

$r_1 = \left(1 + \frac{1}{2}\right) r = \frac{3}{2} r$ = , $h_1 = \left(1 + \frac{1}{2}\right) h = \frac{3}{2} h$

Volume of cone = $\frac{1}{3}\pi r_1^2 h_1 = \frac{1}{3}\times\frac{22}{7}\times\left(\frac{3}{2}r\right)^2\left(\frac{3}{2}h\right)$

Now, ratio of volume of cone

$$= \frac{\text{Volume of given cone}}{\text{Volume of new cone}}$$

$$= \frac{\frac{1}{3}\pi r^2 h}{\frac{1}{3}\pi\left(\frac{3}{2}r\right)^2\left(\frac{3}{2}h\right)} = \frac{8}{27} = 8 : 27.$$

39. (*b*) **7 : 4**

Volume of cone = $\frac{1}{3}\pi r^2 h$

$r = 4$, $h = 9$ cm.

Now, $r_1 = 6$, $h_1 = 7$ cm.

$$= \frac{\text{Volume of new cone}}{\text{Volume of original cone}}$$

$$= \frac{\frac{1}{3}\pi(6)^2\times 7}{\frac{1}{3}\pi(4)^2\times 9} = \frac{252}{144} = \frac{7}{4} = 7 : 4.$$

40. (*c*) **1 : 2**

$$\frac{2\pi r_1}{2\pi r_2} = \frac{3}{4}$$

$$\frac{r_1}{r_2} = \frac{3}{4}$$

Also, $\frac{\frac{1}{3}\pi r_1^2 h_1}{\frac{1}{3}\pi r_2^2 h_2} = \frac{9}{32}$

$\Rightarrow \left(\frac{r_1}{r_2}\right)^2 \frac{h_1}{h_2} = \frac{9}{32}$

$\Rightarrow \frac{h_1}{h_2} = \frac{9}{32}\left(\frac{r_2}{r_1}\right)^2 = \frac{9}{32}\times\left(\frac{4}{3}\right)^2$

$$= \frac{1}{2}$$

$h_1 : h_2 = 1 : 2$

41. (*b*) **240**

Volume of cuboid= $lbh = 22 \times 20 \times 16 = 7040$ cm^3.

Volume of cone = $\frac{1}{3}\pi r^2 h$

$$= \frac{1}{3}\pi(2)^2(7) = \frac{1}{3}\pi\times 4\times 7$$

Now, no. of ice cream cone = $\frac{7040\times 3\times 7}{22\times 4\times 7} = 240.$

42. (*d*) **603.4 cm³**

Volume of cone = $\frac{1}{3}\pi r^2 h$

$$= \frac{1}{3}\times\frac{22}{7}\times(6)^2\times 8 = 301 \text{ cm}^3.$$

Volume of cylinder = $\pi r^2 h = \frac{22}{7}\times(6)^2\times 8 = 905.4$ cm^3.

Then, volume of remaining solid

$= 905.4 - 301 = 603.4$ cm^3.

43. (*d*) **14 cm**

Let $r_1 = 3$ cm, $r_2 = 5$ cm

Volume of spherical shell = $\frac{4}{3}\pi(r_2^3 - r_1^3)$

$= \frac{4}{3}\pi(5^3 - 3^3)$

$= \frac{4}{3}\pi(125 - 27)$

$= \frac{4}{3}\pi(98)$

$= \frac{98\times 4}{3}\pi$ cm^3

Volume of cylinder = $\pi r^2 h$

$$\frac{98\times 4}{3}\pi = \pi\times\frac{8}{3}\times r^2$$

$$49 = r^2$$

$$7 = r$$

Now, diameter = 14 cm.

44. (*c*) **2 cm**

Let, $r_1 = 1$ cm, $h_1 = 20$ cm.

Volume of cylinder = $\pi r_1^2 h_1 = \frac{22}{7}\times 1\times 20$

$= 62.85$ cm^3.

Now, volume of 15 solid spheres

= volume of cylinder

$\Rightarrow 15\left(\frac{4}{3}\pi r^3\right) = 62.85$ cm^3.

$\Rightarrow r^3 = \frac{62.85\times 3\times 7}{15\times 4\times 22} = 1$

$\Rightarrow r = 1$ cm

Hence, diameter is 2 cm.

45. (*d*) **179.7 cm³**

Volume of cube = $lbh = 7 \times 7 \times 7 = 343$ cm^3.

$\Rightarrow 2r$ = side of cube

$\Rightarrow 2r = \frac{7}{2} = 3.5$ cm

Now,Volume of sphere = $\frac{4}{3}\pi r^3$

$= \frac{4}{3}\times\frac{22}{7}\times 3.5\times 3.5\times 3.5$

$= 179.7$ cm^3.

46. (*a*) **48 cm²**

Now, Curved surface area of cone = $\pi(r_1 + r_2)\, l$

$\Rightarrow 2\pi r_1 = 18$

$2\pi r_2 = 6$

$\Rightarrow r_1 = \frac{18}{2\pi} = \frac{9}{\pi}$ cm,

$$r_2 = \frac{6}{2\pi} = \frac{3}{\pi}\text{ cm}$$

Now, $\pi(r_1 + r_2)\, l = \pi\left(\frac{9}{\pi}+\frac{3}{\pi}\right)4 = 48\text{ cm}^2.$

47. (*b*) **110 m**

Surface area of cone = πrl

$$= \frac{22}{7}\times 7\times\sqrt{(7)^2+(24)^2}$$

$$= 22\times\sqrt{625}$$

$$= 22 \times 25 = 550\text{ cm}^2.$$

Now, tent is made of 5 m wide canvas = $\frac{1}{5}\times 550$

$$= 110\text{ m.}$$

48. (*d*) **4158 cm²**

Volume of hemisphere = $\frac{2}{3}\pi r^3 = 19404.$

$$\Rightarrow \quad r^3 = \frac{19404\times 3\times 7}{2\times 22} = 9261$$

$$\Rightarrow \quad r = 21\text{ cm}$$

Surface area of hemisphere = $3\pi r^2$

$$3\pi r^2 = 3\times\frac{22}{7}\times(21)^2$$

$$= 4158\text{ cm}^3.$$

49. (*c*) **1 : 4**

$$r_1 = \frac{1}{2}\text{ cm},\ r_2 = 1\text{ cm}$$

$$\frac{\text{Volume of cylinder}}{\text{Volume of original cylinder}} = \frac{\pi r_1^2 h}{\pi r_2^2 h}$$

$$= \frac{\pi\times\left(\frac{1}{2}\right)^2\times h}{\pi(1)^2 h}$$

$$= \frac{\pi\left(\frac{1}{4}\right)h}{\pi h}$$

$$= \frac{1}{4} = 1 : 4.$$

50. (*d*) **329π m²**

Slant height of the cone, $l = \sqrt{r^2+h^2}$

$$= \sqrt{(14)^2+(10.5)^2}$$

$$= 17.5\text{ m.}$$

Curved surface area of cone + curved surface area of cylinder

$$= 2\pi rh + \pi rl$$

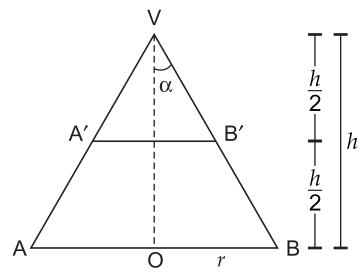
$$= \pi r(2h + l)$$

$$= \frac{22}{7}\times 14(2\times 3.5+17.5) = 329\pi\text{ m}^2.$$

51. **1 : 8**

Let height of cone VAB = h and radius of cone VAB = r.

As horizontal plane cuts the cone VAB into two parts as it passes through the mid-point of its axis.



$\therefore$ Height of cone VA′B′ = $\frac{h}{2}$

Let r' be radius of cone VA′B′

Now, in ΔVCB′ and ΔVDB,

$$\tan\alpha = \frac{r}{h} = \frac{r'}{\left(\frac{h}{2}\right)}$$

$$\Rightarrow \quad r' = \frac{r}{2}$$

Now, ratio of volume of cone VA′B′ and cone VAB

$$= \frac{\frac{1}{3}\pi\left(\frac{r}{2}\right)^2\left(\frac{h}{2}\right)}{\frac{1}{3}\pi r^2 h} = \frac{1}{8} = 1 : 8$$

52. (*c*) **30 hectares**

Width of canal = 300 cm = 3 m

Depth of canal = 120 cm = 1.2 m

Water is flowing with speed = 20 km/h

Length of water column formed in $\frac{20}{60}$ min = $\frac{1}{3}$ h

$$= \frac{1}{3}\times 20 = \frac{20}{3}\text{ km}$$

Volume of water flowing in $\frac{1}{3}$ h

= Volume of cuboid with length $\frac{20000}{3}$ m

$$= \frac{20000}{3}\times 3\times 1.2 = 24000\text{ m}$$

$$\Rightarrow \quad \frac{x\times 8}{100} = 24000\text{ m}$$

$$\Rightarrow \quad x = \frac{24000\times 100}{8} = 300000\text{ m}^2$$

= 30 hectares

53. (*d*) **75**

Radius of cylinder, $r = 3.5$ cm

$h = 2.8$ cm

Radius of marbles, $r_1 = 0.7$ cm

Then, volume of cylinder $= \pi r^2 h$

$$= \frac{22}{7} \times (3.5)^2 \times 2.8$$

$$= 107.8 \text{ cm}^3.$$

Volume of marbles $= \frac{4}{3}\pi r_1^3$

$$= \frac{4}{3} \times \frac{22}{7} \times (0.7)^3 = 1.43 \text{ cm}^3.$$

No. of marbles dropped $= \frac{107.8}{1.43} = 75.$

54. (*c*) **1 : $\sqrt{3}$**

Curved surface area of the hemispherical part $= 2\pi r^2$.

Curved surface area of the conical part $= \pi rl$

Height of the conical part is $h = \sqrt{l^2 - r^2}$

Now, ratio of the curved surface areas is

$$\frac{2\pi r^2}{\pi r l} = 1$$

$$\Rightarrow \quad \frac{r}{l} = \frac{1}{2} \Rightarrow l = 2r$$

The ratio of the radius to the height is

$$\frac{r}{h} = \frac{r}{\sqrt{l^2 - r^2}} = \frac{r}{\sqrt{(2r)^2 - r^2}} = \frac{1}{\sqrt{3}}$$

Hence, $r : h = 1 : \sqrt{3}$.

55. (*b*) **1 : 5**

Radius of the cylinder, $r = \frac{1.6 \text{ m}}{2} = 0.8$ m

Height of the cylinder, $h = 20$ cm $= 0.2$ m

LSA of the cylinder $= 2\pi rh$

CSA of the cylinder $= 2\pi r(r + h)$

Now,

$$\frac{2\pi rh}{2\pi r(r+h)} = \frac{h}{r+h} = \frac{0.2 \text{ m}}{(0.8 + 0.2) \text{ m}}$$

$$= \frac{0.2 \text{ m}}{1 \text{ m}} = \frac{1}{5}$$

Hence, ratio of CSA and LSA = 1 : 5

56. (*c*) **$12\sqrt{3}$ cm**

Sum of the volumes of three cubes of edges 6 cm, 8 cm and 10 cm is

$$(6^3 + 8^3 + 10^3) \text{ cm}^3$$

$$= 216 \text{ cm}^3 + 512 \text{ cm}^3 + 1000 \text{ cm}^3$$

$$= 1728 \text{ cm}^3$$

$\therefore$ Volume of the single cube

$$a^3 = 1728 \text{ cm}^3$$

$$= 4^3 \times 3^3$$

$\therefore \quad a = 4 \times 3 = 12 \quad \ldots(1)$

$\therefore$ Length of the diagonal of the single cube

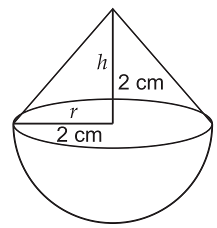
$$= \sqrt{a^2 + a^2 + a^2}$$

$$= \sqrt{3}a$$

$= 12\sqrt{3}$ cm [From (1)]

57. (*a*) **8π cm^3**

Let r be the radius of the base of the cone and h be the height of the cone.



Then r is the radius of the hemisphere also.

$\therefore$ $r = 2$ cm, $h = r = 2$ cm.

$\therefore$ Volume of the cone $= \frac{1}{3}\pi r^2 \times h$

$$= \frac{1}{3}\pi \times 2^2 \times 2 \text{ cm}^3$$

$= \frac{8\pi}{3}$ cm^3 …(1)

Also, volume of the hemisphere $= \frac{2}{3}\pi r^3$ cm^3

$$= \frac{2}{3}\pi \times 2^3 \text{ cm}^3$$

$= \frac{16\pi}{3}$ …(2)

$\therefore$ Required volume of the solid

$$= \left(\frac{8\pi}{3} + \frac{16\pi}{3}\right) \text{cm}^2$$

$$= \frac{24\pi}{3} \text{ cm}^3$$

$= 8\pi$ cm^3 [From (1) & (2)]

58. (*a*) **36 m**

Let r be the radius of the sphere and R be the radius of the cylindrical wire. Also, let l be the length of the wire.

Then $r = \frac{6}{2}$ cm $= 3$ cm

$R = \frac{2}{2}$ mm $= 1$ mm $= \frac{1}{10}$ cm

$\therefore$ Volume of the sphere $= \frac{4}{3}\pi r^3$

$$= \frac{4\pi}{3} \times 27 \text{ cm}^3$$

$= 36\pi$ cm^3 …(1)

Volume of the wire $= \pi R^2 l$

$= \pi \times \frac{1}{100} l$ cm^3 …(2)

$\therefore$ From (1) and (2),

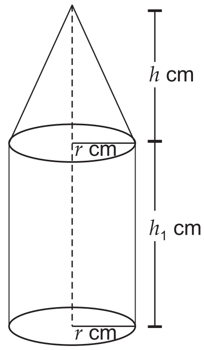
$$\frac{\pi}{100} l = 36\pi$$

$\therefore \quad l = 3600$

$\therefore$ Length of the wire is 3600 cm, i.e. 36 m.

59. (*d*) $\mathbf{\frac{2h}{3}}$

Let r be the common radius of the bases of the cylinder and the cone, h be the vertical height of the cone and h_1, the height of the cylinder.



Then the total volume of the cone and the cylinder is

$$\left(\frac{1}{3}\pi r^2 h + \pi r^2 h_1\right) \text{ cm}^3 = \frac{1}{3}\pi r^2 (h + 3h_1) \text{ cm}^3$$

According to the problem

$$\frac{1}{3}\pi r^2 (h + 3h_1) = 3 \times \frac{\pi h r^2}{3}$$

$\Rightarrow \quad h + 3h_1 = 3h$

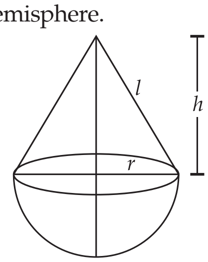
$\Rightarrow \quad 2h = 3h_1$

$\therefore \quad h_1 = \frac{2h}{3}$

$\therefore$ Height of the cylinder $= \frac{2h}{3}$.

60. (*c*) $\mathbf{1 : \sqrt{3}}$

Let l be the slant height of the cone and h be its vertical height. Let r be the common radius of the base of the cone and the hemisphere.



$\therefore$ The surface area of the hemisphere $= 2\pi r^2$ …(1)

and the curved surface area of the cone

$= \pi r l$

$= \pi r\sqrt{r^2 + h^2}$ …(2)

$\therefore$ According to the problem, we have

$2\pi r^2 = \pi r\sqrt{r^2 + h^2}$ [From (1) and (2)]

$\Rightarrow \quad 4r^2 = r^2 + h^2$

$\Rightarrow \quad 3r^2 = h^2$

$\Rightarrow \quad \sqrt{3}\, r = h$

$\therefore \quad r : h = 1 : \sqrt{3}$

SHORT ANSWER QUESTIONS

For Basic and Standard Levels

1. Let diameter of cylinder = 60 cm = 0·6 m

Radius of cylinder, $r = \frac{0 \cdot 6}{2}$ m = 0.3 m

Height of cylinder, h = 1.45 m

Now, total surface area of bird bath $= 2\pi rh + 2\pi r^2$

$= 2\pi r\,[r + h]$

$= 2 \times \frac{22}{7} \times 0.3\,[0.3 + 1.45] = \mathbf{3.3\ m^2}$

2. Let r be radius of pencil = 0.5 cm

Height of cylinder, h = 2 cm

Now, volume of pencil $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times (0.5 \text{ cm})^2 \times (2 \text{ cm})$

$= \frac{11}{21} \text{cm}^3$

Volume of cylindrical pencil $= \pi r^2 h$

$= \frac{22}{7} \times (0.5 \text{ cm})^2 \times (2 \text{ cm})$

$= \frac{22}{7} \times 0.25 \times 2 = \frac{11}{7} \text{cm}^3$

Volume of pencil shavings $= \frac{11}{7} - \frac{11}{21} = \mathbf{\frac{22}{21}}$ **cm³**.

3. Let h be the common height of the two cones and the cylinder. Then,

Sum of volumes of the two cones

$= \frac{1}{3}\pi r_1^2 h + \frac{1}{3}\pi r_2^2 h$

$= \frac{1}{3}\pi\,(r_1^2 + r_2^2)\,h$

Now, volume of the cylinder $= \pi r^2 h$,

where r is the radius of its base

According to question,

$\Rightarrow \quad \pi r^2 h = \frac{1}{3}\pi(r_1^2 + r_2^2)\,h$

$\Rightarrow \quad r^2 = \frac{r_1^2 + r_2^2}{3}$

$\Rightarrow \quad r = \sqrt{\mathbf{\frac{r_1^2 + r_2^2}{3}}}$

4. Let r be the radius of the sphere.

Radius of the 2nd sphere, $r_1 = 2r$

$$\frac{\text{Volume of 2nd sphere}}{\text{Volume of sphere}} = \frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r^3}$$

$$= \frac{(2r)^3}{r^3} = \frac{8}{1}$$

Hence, the ratio between the volume of the 2nd sphere to the volume of the sphere is **8 : 1**.

$$\frac{\text{Surface area of the 2nd sphere}}{\text{Surface area of the sphere}} = \frac{4\pi r_1^2}{4\pi r^2} = \frac{r_1^2}{r^2}$$

$$= \frac{(2r)^2}{r^2} = \frac{4}{1}$$

Hence, the ratio of the surface area of the 2nd sphere to the surface area of the sphere is **4 : 1**.

5. Height of cylinder = 10 cm

Radius of cylinder = 3.5 cm

Surface area of the souvenir = curved surface area of cylinder + 2 (curved surface area of hemisphere)

$= 2\pi rh + 2(2\pi r^2)$

$= 2 \times \frac{22}{7} \times \frac{35}{10} \times 10 + 4 \times \frac{22}{7} \times \frac{35}{10} \times \frac{35}{10}$

$= 220 + 154 = 374 \text{ cm}^2$

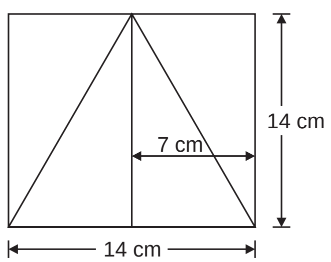
Cost of polishing souvenir

$1 \text{ cm}^2 = ₹10$

$374 \text{ cm}^2 = ₹374 \times 10 =$ **₹3740**

For Standard Level

6. Edge of cube (a) = 14 cm



Height of cone = 14 cm

Diameter of cone = 14 cm

Radius of cone = 7 cm

$\therefore$ Slant height, $l = \sqrt{(7)^2 + (14)^2} = 7\sqrt{5}$ cm

Surface area of the cone $= \pi rl$

$= \frac{22}{7} \times 7 \text{ cm} \times (7\sqrt{5})$

$=$ **344.344 cm²**.

Total surface area of remaining solid = Total surface area of cube – Area of circular base + Curved surface area of cone

$= 6a^2 - \pi r^2 + 344.344 \text{ cm}^2$

$= 6(14)^2 - \frac{22}{7} \times (7 \text{ cm})^2 + 344.344 \text{ cm}^2$

$= 6 \times 196 - 154 \text{ cm}^2 + 344.344 \text{ cm}^2$

$= 1176 \text{ cm}^2 - 154 \text{ cm}^2 + 344.344 \text{ cm}^2$

$=$ **1366.344 cm²**

7. Radius of each cone $= \frac{7}{2}$ cm

The ratio of their volume = 3 : 2 : 1

$\Rightarrow \quad \frac{1}{3}\pi r^2 h_1 : \frac{1}{3}\pi r^2 h_2 : \frac{1}{3}\pi r^2 h_3 = 3 : 2 : 1$

$\Rightarrow \quad h_1 : h_2 : h_3 = 3 : 2 : 1$

$h_1 = \frac{3}{6} \times 36 =$ **18 cm**

$h_2 = \frac{2}{6} \times 36 =$ **12 cm**

$h_3 = \frac{1}{6} \times 36 =$ **6 cm**

Now, volume of 1st cone $= \frac{1}{3}\pi r^2 h_1$

$= \frac{1}{3} \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2 \times 18$

$=$ **231 cm³**

Volume of 2nd cone $= \frac{1}{3}\pi r^2 h_2$

$= \frac{1}{3} \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2 \times 12$

$=$ **154 cm³**

Volume of 3rd cone $= \frac{1}{3}\pi r^2 h_3$

$= \frac{1}{3} \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2 \times 6 =$ **77 cm³**

Volume of remaining portion of cylinder = Volume of cylinder – Volume of three cones

$= \pi r^2 h - 462$

$= \frac{22}{7} \times \left(\frac{7}{2}\right)^2 \times 36 - 462$

$= 1386 - 462 =$ **924 cm³**

8. Let h be height of conical tent. Then, $h = 3$ m

By ΔABC and ΔOEC,

$\Rightarrow \quad \frac{AB}{OE} = \frac{BC}{EC}$

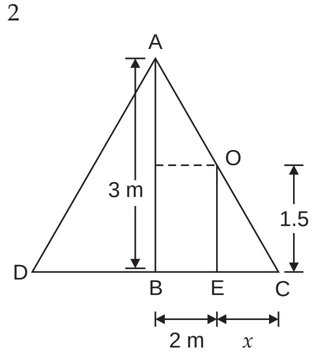
$\Rightarrow \quad \frac{3}{1.5} = \frac{2 + x}{x}$

$\Rightarrow \quad 3x = 3 + 1.5x$

$\Rightarrow \quad 1.5x = 3$

$\Rightarrow \quad x = \frac{3}{1.5} = 2$

$\Rightarrow \quad x = 2$



Let r be the radius of the conical tent.

Now, $r = BC = x + 2 = 2 + 2 = 4$ m

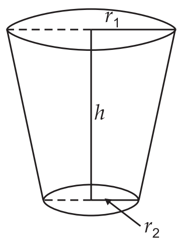
Slant height of the cone ADC

$$l = \sqrt{h^2 + r^2}$$
$$= \sqrt{(3)^2 + (4)^2} = \sqrt{9+16}$$
$$= \sqrt{25} = 5 \text{ m}$$

Curved surface area $= \pi rl$

$$= \frac{22}{7} \times 4 \times 5 = \mathbf{\frac{440}{7}\ m^2}$$

9. Let r_1 and r_2 be the radii of the upper and lower circular bases of the container, where $r_1 > r_2$. The container is in the form of a frustum of a cone, of height h.



Then $r_1 = 20$ cm, $r_2 = 8$ cm, $h = 24$ cm.

$\therefore$ The volume of the frustum

$$= \frac{\pi h}{3}\left(r_1^2 + r_1 r_2 + r_2^2\right)$$
$$= \frac{\pi \times 24}{3}\left(20^2 + 20 \times 8 + 8^2\right) \text{ cm}^3$$
$$= \frac{22}{7} \times 8 \times (400 + 160 + 64) \text{ cm}^2$$
$$= \frac{22 \times 8 \times 624}{7} \text{ cm}^3$$
$$= \frac{109824}{7} \text{ cm}^3$$
$$= 15689 \text{ cm}^3$$
$$= 15.689 \text{ litres}$$

$\therefore$ Volume of the milk in the container completely filled with milk = 15.689 litres

$\therefore$ Required cost of the milk = ₹21 × 15.689 = **₹329.47**.

10. Volume of frustum $= \frac{1}{3}\pi h\,(r_1^2 + r_2 r_1 + r_2^2)$

Internal radius, $(r_1) = 28$ cm

External radius, $(r_2) = 7$ cm

Height, $h = 55$ cm

Now, volume of the frustum

$$= \frac{1}{3} \times \frac{22}{7} \times 55 \left[(28)^2 + 28 \times 7 + 7^2\right]$$
$$= \frac{1}{3} \times \frac{22}{7} \times 55 \left[784 + 196 + 49\right]$$
$$= 59290 \text{ cm}^3$$

Radius of cone/cylinder, $r = \frac{7}{2}$ 3.5 m

height of cone, $h_1 = 6$ cm

height of cylinder, $h_2 = 4$ cm

Volume of cone + Volume of cylinder

$$= \frac{1}{3}\pi r^2 h_1 + \pi r^2 h_2$$
$$= \frac{1}{3} \times \frac{22}{7}\left[\left(\frac{7}{2}\right)^2 \times 6\right] + \frac{22}{7} \times \left[\left(\frac{7}{2}\right)^2 \times 42\right]$$
$$= 77 + 1617 = 1694 \text{ cm}^3$$

Now, number of times he can fill the pichkari

$$= \frac{\text{Volume of frustum}}{\text{Volume of cone + Volume of cylinder}}$$
$$= \frac{59290 \text{ cm}^3}{1694 \text{ cm}^3} = \mathbf{35\ times}$$

VALUE-BASED QUESTIONS

For Basic and Standard Levels

1. (*i*) Radius of cylinder, $r = 3.5$ cm

Height of cylinder, $h = \frac{600}{77}$ cm

Now, volume of cuboid $= lbh = 5 \times 6 \times 11 = 330$ cm^3

Volume of cylinder $= \pi r^2 h$

$$= \frac{22}{7} \times (3.5)^2 \times \frac{600}{77}$$
$$= 300 \text{ cm}^3$$

Number of cuboidal shaped containers $= \frac{3300000}{330}$

$= 10000$

Now, number of cylindrical shaped containers

$$= \frac{3300000}{300} = 11000$$

B sold oil at rate 11000 × 36 = **₹ 396000**

A sold oil at rate 10000 × 36 = **₹ 360000**

(*ii*) **Honesty**.

(*iii*) **No, he was dishonest**.

2. (*i*) Let r be the radius and h the height of each cylindrical containers filled with ice cream.

Then, $r = \frac{12}{2}$ cm = 6 cm and $h = 15$ cm

Let r_1 be the radius and h_1 be the height of each cone.

Then, $r_1 = \frac{6}{2}$ cm = 3 and $h_1 = 2 \times$ diameter

$= 2 \times 6$ cm $= 12$ cm

Let n be the number of children who got the ice cream cones.

Volume of ice cream in 5 cylindrical containers
= n × Volume of each cone

$$\Rightarrow \quad 5(\pi r^2 h) = n\left(\frac{1}{3}\pi r_1^2 h_1 + \frac{2}{3}\pi r_1^3\right)$$

$$\Rightarrow \quad 5r^2 h = n\left(\frac{r_1^2 h_1}{3} + \frac{2}{3} r_1^3\right)$$

$$\Rightarrow \quad 5 \times 6 \times 6 \times 15 = n\left(\frac{3 \times 3 \times 12}{3} + \frac{2}{3} \times 3 \times 3 \times 3\right)$$

$$\Rightarrow \quad 5 \times 6 \times 6 \times 15 = n(36 + 18)$$

$$\Rightarrow \quad 5 \times 6 \times 6 \times 15 = n(54)$$

$$\Rightarrow \quad n = \frac{5 \times 6 \times 6 \times 15}{54}$$

$$\Rightarrow \quad n = 50$$

Hence, **50** children got the ice cream.

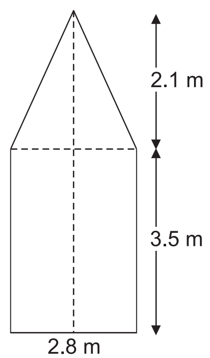
(*ii*) **Empathy and interpersonal relationships**.

3. Radius of cylinder = 2.8 m

Height of cylinder = 3.5 m

Radius of cone = 2.8 m

Height of cone = 2.1 m



Area of canvas required to make 1 tent = Curved surface area of cylinder + Curved surface area of cone

$$= 2\pi rh + \pi rl$$

$$= 2 \times \frac{22}{7} \times \frac{28}{10} \times \frac{35}{10} + \frac{22}{7} \times \frac{28}{10}\sqrt{(2.8)^2 + (2.1)^2}$$

$$= \frac{44 \times 7}{5} + \frac{44}{5}\sqrt{7.84 + 4.41}$$

$$= \frac{308}{5} + \frac{44}{5} \times 3.5$$

$$= 61.6 + 30.8$$

$$= 92.4 \text{ m}^2$$

Cost of 1 tent = 92.4 × 120 = ₹11088

Cost of 1500 tents = 11,088 × 1500 = ₹16632000

$$\text{Amount shared by each school} = \frac{\text{Total cost}}{\text{No. of schools}} = \frac{16632000}{50}$$

= **₹332640**

Values generated are **empathy, decision-making and concern**.

4. (*i*) Radius of cylinder, r = 14 cm

Height of the cylinder, h = 20 cm

Curved surface area of cylinder

$= 2\pi rh$

$= 2 \times 3.14 \times 14 \text{ cm} \times 20 \text{ cm}$

= **1758.4 cm²**

Slant height of the frustum, l = 10 cm

Circular ends of the frustum are

$2\pi r_1 = 207.24$ cm

and $2\pi r_2 = 169.5$ cm

$$r_1 = \frac{207.24 \text{ cm}}{2 \times 3.14} = 33 \text{ cm}$$

$$r_2 = \frac{169.5 \text{ cm}}{2 \times 3.14} = 26.99 \text{ cm}$$

Curved surface area of the frustum

$= \pi l (r_1 + r_2)$

$= 3.14 \times 10 \text{ cm} \times (33 \text{ cm} + 26.99 \text{ cm})$

= **1883.7 cm²**

(*ii*) **Honesty**

5. (*i*) Dimensions of rectangular plot = 28 m × 11 m

Radius of circular pit (r) = 14 m

Let height of circular pit be h

Volume of earth dig in rectangular plot at height 5 m

$= 28 \times 11 \times 5 \text{ m}^3$

Volume of circular pit = $\pi r^2 h$

Now, Volume of earth dig in rectangular plot

= Volume of circular pit

$$\Rightarrow \quad 28 \times 11 \times 5 = \pi r^2 h$$

$$\Rightarrow \quad h = \frac{28 \times 11 \times 5 \times 7}{22 \times 14 \times 14} = \frac{5}{2} = \mathbf{2.5 \text{ m}}$$

(*ii*) **Empathy, concern for children's education and physical fitness, waste recycling, creative thinking and decision-making**.

6. Height of frustum = 30 cm

Radius of lower end = 20 cm

Radius of upper end = 40 cm

$$\text{Volume of container (frustum)} = \frac{\pi h}{3}(R^2 + r^2 + Rr)$$

$$= \frac{22}{7} \times \frac{30}{3}(1600 + 400 + 800)$$

$$= \frac{220}{7} \times 2800 = 88000 \text{ cm}^3$$

1 cm³ = 0.001 litre

88000 cm³ = 88 litres

$$\text{Number of containers} = \frac{\text{Total volume of milk}}{\text{Volume of 1 container}} = \frac{880}{88} = \mathbf{10}$$

Cost of 1 litre milk = ₹35

Cost of 880 litres milk = ₹35 × 880 = **₹30800**

Value indicated by the donor agency is **compassion**.

7. Let r and h be respectively the radius and height of the cylindrical well. Then $r = 5$ m and $h = 14$ m.

$\therefore$ Volume of the earth from the well $= \pi r^2 h$

$\Rightarrow \quad \pi \times 25 \times 14 \text{ m}^3 = 350\pi \text{ m}^3 \quad \ldots(1)$

Let h_1 be the height of the embankment and R be the external radius of the embankment.

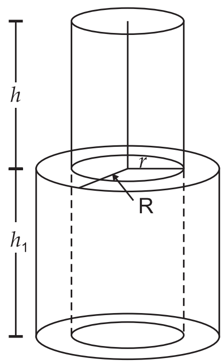
$\therefore \quad R = (5 + 5) \text{ m} = 10 \text{ m}$

Then the volume of the embankment

$= \pi(R^2 - r^2)h$

$= \pi(10^2 - 5^2)h_1 \text{ m}^3$

$= 75\pi h_1 \quad \ldots(2)$



Since the volume of the embankment

= Volume of the earth

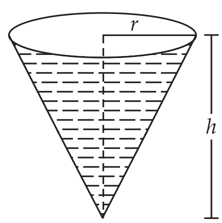
$\therefore \quad 75\pi h_1 = 350\pi$ [From (1) and (2)]

$\therefore \quad h_1 = \frac{350}{75} \text{ m} = \frac{14}{3} \text{ m} = 4.67 \text{ m}$

$\therefore$ Required height of the embankment = **4.67 m**.

Value: Maximise use of available resources.

8. Let r be the radius of circular base of the cone and h be its height.



Then $r = 2.5$ cm and $h = 11$ cm.

The volume of the cone

$= \frac{\pi r^2 h}{3}$

$= \frac{\pi \times 2.5^2 \times 11}{3} \text{ cm}^3$

$= \frac{1}{3}\pi \times 6.25 \times 11 \text{ cm}^3$

$= \frac{\pi}{3} \times 68.75 \text{ cm}^3$

$= \frac{68.75\pi}{3} \text{ cm}^3$

= Volume of water in the cone, completely filled with water ...(1)

The volume of each metallic ball in the shape of a sphere of radius,

$r = \frac{0.5}{2} \text{ cm} = \frac{1}{4} \text{ cm}$, is

$= \frac{4}{3} \times \pi \times \left(\frac{1}{4}\right)^3 \text{ cm}^3$

$\therefore$ Volume of all the metallic balls $= \frac{2}{5}$th of the volume of the cone

$= \frac{4\pi}{3} \times \frac{1}{64} \text{ cm}^3$

$= \frac{\pi}{48} \text{ cm}^3 \quad \ldots(2)$

$\therefore$ Required number of metallic balls

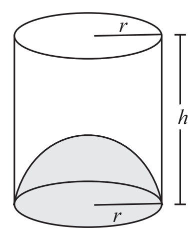
$= \frac{2}{5} \times \frac{68.75\pi}{3} \times \frac{48}{\pi}$ [From (1) and (2)]

$= 27.50 \times 16$

= **440**.

Value: Concern for environment by not wasting water and maximising its use.

9. Let r be the common radius of the bases of the cylinder and the hemisphere. Then $r = \frac{7}{2}$ cm.



Let $h = 14$ cm be the height of the cylinder.

Then the volume of the cylindrical mug $= \pi r^2 h$

$= \frac{22}{7} \times \frac{7 \times 7}{4} \times 14 \text{ cm}^3$

$= 539 \text{ cm}^3$

= 0.539 litres

= Volume of milk according to the dishonest dairy owner A.

Volume of the hemisphere $= \frac{2}{3}\pi r^3$

$= \frac{2}{3} \times \frac{22}{7} \times \frac{7 \times 7 \times 7}{2 \times 2 \times 2} \text{ cm}^3$

$= \frac{539}{6} \text{ cm}^3$

$= 89.833 \text{ cm}^3$

$\therefore$ Volume of the cylinder – Volume of the hemisphere

$= (539 - 89.833) \text{ cm}^3 = 449.167 \text{ cm}^3$

= 0.449167 litres

= actual volume of the milk according to the honest dairy owner B.

$\therefore$ According to the honest dairy owner B, the cost of 0.449167 litres of milk is ₹43.12, but the dishonest dairy owner A took ₹43.12 for 0.539 litres of milk.

$\therefore$ Actual cost of 0.449167 litres of milk

$$= ₹\,\frac{43.12}{0.539}\times 0.449167$$

$$= ₹35.93.$$

$\therefore$ Required cost of the milk according to the honest dairy owner B is ₹35.93.

Value: **Honesty.**

UNIT TEST 1

For Basic Level

1. (*a*) **1 : 27**

Surface area of 2 cubes = $6a_1^2$ and $6a_2^2$

Then, ratio of surface areas are $\frac{S_1}{S_2} = \frac{6a_1^2}{6a_2^2} = \frac{1}{9}$

$$\Rightarrow \quad \frac{a_1^2}{a_2^2} = \frac{1}{9}$$

$$\Rightarrow \quad \frac{a_1}{a_2} = \frac{1}{3}$$

Now, the ratio of the volumes of both cubes are

$$\frac{V_1}{V_2} = \frac{(a_1)^3}{(a_2)^3} = \frac{1^3}{3^3} = \frac{1}{27}$$

Then, ratio between volumes are 1 : 27

2. (*a*) **44 m^3**

Diameter of well = 2 m and $r = \frac{2}{2}$ m = 1 m

Depth = 14 m

Now, volume of cylinder

$$= \pi r^2 h = \frac{22}{7} \times 1 \times 14 = 44 \text{ m}^3$$

3. (*b*) **2 cm**

Volume of cylinder = 88 cm^3

Ratio between their radius and height, $\frac{r}{h} = \frac{2}{7}$

$$r = 2x,\ h = 7x$$

Now, $\pi r^2 h = 88$

$$\Rightarrow \quad x^3 = \frac{88\times 7}{4\times 7\times 22} = 1$$

$$\Rightarrow \quad r = 2 \text{ cm}$$

4. (*c*) **45 cm**

Sum of radius and height, $(r + h)$ = 12 cm

Then, $\quad 2\pi rh + 2\pi r^2 = 540$

$$\Rightarrow \quad 2\pi r\,(h + r) = 540$$

$$\Rightarrow \quad 2\pi r = \frac{540}{12} = 45 \text{ cm}$$

Circumference = 45 cm

5. (*c*) **6 cm**

$$\text{Volume} = 113\frac{1}{7} = \frac{792}{7} \text{ cm}^3$$

$$\text{Volume of sphere} = \frac{4}{3}\pi r^3 = \frac{792}{7}$$

$$\Rightarrow \quad r^3 = \frac{792\times 3\times 7}{7\times 4\times 22} = 27$$

$$\Rightarrow \quad r = 3 \text{ cm}$$

$$\Rightarrow \quad d = 6 \text{ cm}$$

6. (*b*) **21 cm**

Circumference of the circular top of a hemispherical bowl is 132 cm

$$\Rightarrow \quad 2\pi r = 132 \text{ cm}$$

$$\Rightarrow \quad r = \frac{132\times 7}{22\times 2} \Rightarrow r = 21 \text{ cm}$$

7. (*a*) **1 : 9**

Volume of both cubes are $\frac{V_1}{V_2} = \frac{a_1^3}{a_2^3}$

$$\Rightarrow \quad \frac{1}{27} = \left(\frac{a_1}{a_2}\right)^3 \quad \Rightarrow \quad \frac{a_1}{a_2} = \frac{1}{3}$$

Then, surface area of cubes are S_1 and S_2

$$\frac{S_1}{S_2} = \frac{6a_1^2}{6a_2^2}$$

Ratio = 1 : 9

8. (*a*) **3**

$$\text{Volume of cylinder} = \pi r^2 h$$

$$\text{Volume of cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Number of cones required} = \frac{\pi r^2 h}{\frac{1}{3}\pi r^2 h} = 3$$

9. (*a*) **2 : 1**

$$\text{Volume of cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of hemisphere} = \frac{2}{3}\pi r^3$$

$$\text{Ratio} = \frac{\frac{1}{3}\pi r^2 h}{\frac{2}{3}\pi r^3} = 1$$

$$\Rightarrow \quad \frac{h}{2r} = 1 \quad \Rightarrow \quad \frac{h}{r} = \frac{2}{1}$$

Ratio = 2 : 1.

10. (*a*) **Frustum of a cone**

11. Diameter = 14 cm, r = 7 cm

Level of water rises, $h = \frac{32}{77}$ cm

$$\text{Volume} = \pi r^2 h = \frac{22}{7} \times 7 \times 7 \times \frac{32}{77} = 64$$

$\Rightarrow \quad a^3 = 64$

$\Rightarrow \quad a = 4$ cm

Hence, the length of the edge of the cube is **4 cm**.

12. Radius of spherical lead shots, $r_1 = \frac{8}{2}$ cm = 4 cm

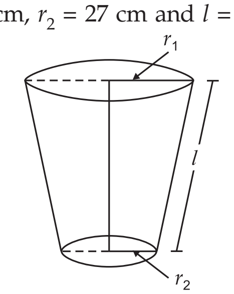
Edge of cube = 88 cm

$$\text{Number of spherical lead} = \frac{\text{Volume of cube}}{\text{Volume of lead shots}}$$

$$= \frac{88 \times 88 \times 88}{\frac{4}{3}\pi (4)^3} = 2541$$

Hence, total no. of spherical lead shots are **2541**.

13. Let r_1 and r_2 (where $r_1 > r_2$) be the radii of two circular ends of the frustum of the cone and let l be its slant height. Then, $r_1 = 33$ cm, $r_2 = 27$ cm and $l = 10$ cm.



$\therefore$ Required total surface area

$$= \pi(r_1 + r_2)l + \pi r_1^2 + \pi r_2^2$$

$$= 3.14\,[(33 + 27) \times 10 + 33^2 + 27^2]\text{ cm}^2$$

$$= 3.14 \times [600 + 1089 + 729]\text{ cm}^2$$

$$= 3.14 \times 2418\text{ cm}^2$$

$$= \mathbf{7592.52\ cm^2}.$$

14. Let r_1 be the radius of each marble in the shape of a sphere.

Then $\quad r_1 = \frac{1.4}{2}$ cm = 0.7 cm.

$$\therefore \text{Volume of each marble} = \frac{4}{3} \times \pi r^3$$

$$= \frac{4}{3} \times \pi \times 0.7^3\ \text{cm}^3 \qquad \ldots(1)$$

Volume of water of height $h = 5.6$ cm inside a cylindrical beaker of radius $R = \frac{7}{2}$ cm is

$$\pi R^2 h = \pi \times \left(\frac{7}{2}\right)^2 \times 5.6\ \text{cm}^3$$

$$= \frac{49}{4} \times 5.6\pi\ \text{cm}^3 \qquad \ldots(2)$$

$$\therefore \text{Required no. of marbles} = \frac{49}{4} \times 5.6 \times \frac{3}{4} \times \frac{1}{(0.7)^3}\ \text{cm}^3$$

$$= \frac{205.8}{1.372} = \mathbf{150}.$$

15. Inner diameter = 6 cm, radius, $r = \frac{6}{2} = 3$ cm

Height of glass, $h = 14$ cm

$\therefore$ Apparent capacity of glass

$$= \pi r^2 h = \frac{22}{7} \times (3)^2 \times 14 = \mathbf{396\ cm^3}$$

$$\text{Actual capacity} = \pi r^2 h - \frac{2}{3}\pi r^3$$

$$= \pi\left(r^2 h - \frac{2}{3} r^3\right) = \frac{22}{7}\left(3^2 \times 14 - \frac{2}{3} \times 3^3\right)$$

$$= \frac{22}{7}\ (126 - 18) = \mathbf{\frac{2376}{7}\ cm^3}$$

16. Radius, $r = 1$ cm = 0.01 m

Rate of water flowing = 6 m/s

Length of water flowing out in 30 min

$$6 \times 100 \times 60 \times 30 \text{ cm} = 1080000 \text{ cm}$$

Volume of water that flows in 30 min

$$= \pi r^2 h = \pi \times 1^2 \times 1080000 \text{ cm}$$

$$= 1080000\pi \text{ cm}^3 \qquad \ldots(1)$$

Radius of base of tank = 60 cm

Volume of water in tank = $\pi R^2 H$

$$= \pi \times 60 \times 60 \times H \qquad \ldots(2)$$

From (1) and (2),

$$3600\,\pi H = 1080000\,\pi$$

$\therefore \quad H = 300$ cm

$\Rightarrow \quad H = 3$ m

Water level rises **3 m** in half an hour.

UNIT TEST 2

For Standard Level

1. (*b*) **6 : π**

Volume of cube = a^3

Volume of sphere = $\frac{4}{3}\pi r^3$

$\Rightarrow \quad a^3 : \frac{4}{3}\pi r^3$

$\Rightarrow \quad a^3 : \frac{4}{3}\pi\left(\frac{a}{2}\right)^3 \qquad (\because r = a/2)$

$\Rightarrow \quad a^3 : \frac{4}{3}\pi \frac{a^3}{8} \Rightarrow 6 : \pi$

Ratio = 6 : π

2. (*b*) **32**

Volume of cone,

$$V_1 = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(12)^2 \times 24 = 3620.57\ \text{cm}^3$$

Volume of spherical balls,

$$V_2 = \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times (3)^3 = 113.14\ \text{cm}^3$$

Number of balls = $\frac{V_1}{V_2} = \frac{3620.57}{113.14} = 32$

3. (*b*) **25 cm, 20 cm**

Given, $R + r = 45$ cm ...(i)

Ratio of volume of two solid spheres, $\frac{V_1}{V_2} = \frac{125}{65}$

$$\Rightarrow \quad \frac{V_1}{V_2} = \frac{\frac{4}{3}\pi R^3}{\frac{4}{3}\pi r^3}$$

$$\Rightarrow \quad \left(\frac{R}{r}\right)^3 = \frac{125}{64}$$

$$\Rightarrow \quad \frac{R}{r} = \frac{5}{4}$$

$$\Rightarrow \quad R = \frac{5r}{4}$$

From (i), $\frac{5r}{4} + r = 45$

$$\Rightarrow \quad \frac{5r+4r}{4} = 45$$

$$\Rightarrow \quad r = \frac{45 \times 4}{9} = 20 \text{ cm}$$

Then, $R = 45 - r = 45 - 20 = 25$ cm

Then, radii be 25 cm and 20 cm.

4. (*a*) **3**

Radius of sphere, $r_1 = \frac{18 \text{ cm}}{2} = 9$ cm

Radius of the cylinder, $r_2 = \frac{36}{2}$ cm = 18 cm

Volume of sphere = Volume of cylinder

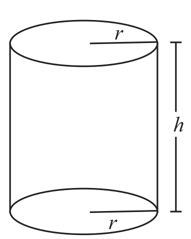
$$\Rightarrow \quad \frac{4}{3}\pi r_1^3 = \pi r_2^2 h$$

$$\Rightarrow \quad h = \frac{4 \times (9 \text{ cm})^3}{3 \times (18 \text{ cm})^2}$$

$$h = \frac{2916}{972} \text{ cm} = 3 \text{ cm}$$

5. (*b*) **1 : 5**

Let *r* and *h* be respectively the radius and height of the cylinder.



Then, $r = \frac{1.6}{2}$ m = 0.8 m

and $h = 20 \text{ cm} = \frac{1}{5}$ m = 0.2 m

∴ Lateral surface area of the cylinder

$= 2\pi rh$

$= 2\pi \times 0.8 \times 0.2 \text{ m}^3$

$= 2\pi \times 0.16 \text{ m}^3$ ...(1)

Also, the total surface area of the cylinder (closed at both ends)

$= 2\pi rh + 2\pi r^2$

$= 2\pi r\,(h + r)$

$= 2\pi + 0.8 \times (0.8 + 0.2) \text{ m}^3$

$= 2\pi \times 0.8 \text{ m}^3$ ...(2)

∴ Required ratio $= 2\pi \times 0.16 : 2\pi \times 0.8$

[From (1) and (2)]

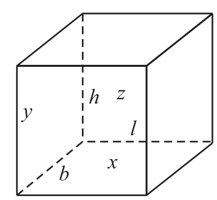
$= 0.16 : 0.8$

$= 16 : 80$

$= 1 : 5.$

6. (*b*) $\sqrt{xyz}$

Let *l*, *b* and *h* be the length, breadth and height respectively of the cuboid. Let *x*, *y* and *z* be respectively the base plane vertical plane one side and that on the back side.



∴ $x = lb,\ y = bh,\ z = lh$ ...(1)

∴ Volume of the cuboid = lbh ...(2)

Now, from (1),

$$xyz = l^2 b^2 h^2$$

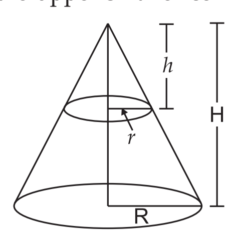
$$\Rightarrow \quad lbh = \sqrt{xyz} \qquad \text{...(3)}$$

∴ From (2) and (3), required volume of the cuboid

$= \sqrt{xyz}$

7. (*a*) **1 : 8**

Let R and H be the radius of the base and height respectively of the whole cone and *h* and *r* be the height and radius of the base of the upper smaller cone.



Then $H = 2h$ ...(1)

and $\frac{r}{R} = \frac{h}{H} = \frac{1}{2}$

∴ $R = 2r$ ...(2)

∴ Volume of the smaller cone = $\frac{1}{3}\pi r^2 h$

and volume of the whole cone = $\frac{1}{3}\pi R^2 H$

∴ Ratio of the volumes of these two cones

$$= \frac{\frac{1}{3}\pi r^2 h}{\frac{1}{3}\pi R^2 H} = \frac{r^2 h}{4r^2 \times 2h} \quad \text{[From (1) and (2)]}$$

$$= \frac{1}{8}$$

∴ Required ratio = 1 : 8

8. Let $3x$, $4x$ and $5x$ are the edges of cube.

Now, Diagonals of cube, $\sqrt{3}\,a = 6\sqrt{3}$

$\Rightarrow \quad a = 6$ cm

Let volume of new cube = Sum of volume of three cubes

$\Rightarrow \quad a^3 = (3x)^3 + (4x)^3 + (5x)^3$

$\Rightarrow \quad 6^3 = (27 + 64 + 125)x^3$

$\Rightarrow \quad 216 = 216x^3$

$\Rightarrow \quad x = 1$

Now, edges of new cubes are **3 cm**, **4 cm** and **5 cm**.

9. Side of cube = 7 cm

Diameter of cone = 7 cm

Radius of cone, $r = \frac{7}{2}$ cm = 3.5 cm

Height of cone, $h = \frac{\sqrt{15}}{2}$ cm

∴ Slant height, $l = \sqrt{h^2 + r^2}$

$$= \sqrt{\left(\frac{\sqrt{15}}{2}\right)^2 + \left(\frac{7}{2}\right)^2} = \sqrt{\frac{15}{4} + \frac{49}{4}}$$

$$= \sqrt{\frac{64}{4}}$$

∴ $l = \frac{8}{2} = 4$ cm

Total surface area of solid exposed to our eyes

= Curved surface area of cone
+ Total surface area of cube – Area of base

$= \pi rl + 6a^2 - \pi r^2$

$= \frac{22}{7} \times \frac{7}{2} \times 4 + 6 \times (7)^2 - \frac{22}{7} \times \left(\frac{7}{2}\right)^2$

= 44 + 294 – 38.5

= **299.5 cm²**

10. Dimensions of cuboidal box are 16 cm × 8 cm × 8 cm

Diameter of sphere = 6 cm

Radius of sphere, $r = \frac{6}{2} = 3$ cm

Then, number of glass spheres

$$= \frac{\text{Volume of cuboid – Volume of empty space}}{\text{Volume of sphere}}$$

$$= \frac{lbh - 119.68}{\frac{4}{3}\pi r^3}$$

$$= \frac{16 \times 8 \times 8 - 119.68}{\frac{4}{3} \times (3.14) \times (3)^3}$$

$$= \frac{1024 - 119.68}{113.04} = 8$$

Hence, total number of balls are **8**.

11. The volume of water in the cylindrical tank of radius $r = \frac{2}{2}$ m = 1 m and height $h = 5$ m is

$\pi r^2 h = \pi \times 1^2 \times 5 \text{ m}^3$

$= 5\pi \text{ m}^3$ …(1)

If h_1 be the height of the standing water in the rectangular park.

Then, volume of water is

$25 \times 20\, h_1 \text{ m}^3 = 500\, h_1 \text{ m}^3$ …(2)

∴ From (1) and (2),

$5\pi = 500h_1$

$\Rightarrow \quad h_1 = \frac{5 \times 3.14}{500} = 0.0314$

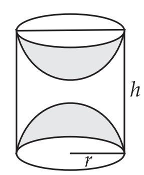
∴ The required height of the standing water

= 0.0314 m

= **3.14 cm**

By recycling of water, we can prevent the wastage of water and use it for the benefit of the public.

12. Let h and r be respectively the height of the cylinder and the common radius of the bases of the hemisphere and the cylinder.



Then, $h = 10$ cm and $r = 4.2$ cm.

Then the volume of the cylinder

$= \pi r^2 h$

$= \frac{22}{7} \times 4.2 \times 4.2 \times 10 \text{ cm}^3$

$= 554.4 \text{ cm}^3$ …(1)

Sum of the volumes of the two hemispheres

$= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times 4.2 \times 4.2 \times 4.2 \text{ cm}^3$

$= 310.464 \text{ cm}^3$

$= 310.46 \text{ cm}^3$ …(2)

∴ Volume of the solid portion of the cylinder excluding the scooped out hemispheres

$= (554.4 - 310.46) \text{ cm}^2$

$= 243.94\ \text{cm}^3$ ...(3)

Let l be the length of the wire in the form of a very thin cylinder of radius $\frac{1.4}{2}$ cm = 0.7 cm

Then the volume of the wire

$= \pi \times (0.7)^2 \times l$

$= \frac{22}{7} \times 0.49 \times l\ \text{cm}^3$

$= 1.54\ l\ \text{cm}^3$ ...(4)

∴ From (3) and (4), we get

$1.54l = 243.94$

∴ $l = \frac{243.94}{1.54} = \frac{24394}{154} = 158.4$

∴ Required length of the wire = **158.4 cm**.

13. (*i*) Radius of the cylinder, $r = \frac{14}{2}$ cm = 7 cm

Height of the cylinder, h = 50 cm

Volume of the cylinder = $\pi r^2 h$

$= \frac{22}{7} \times (7\ \text{cm})^2 \times 50\ \text{cm}$

$= \mathbf{7700\ cm^3}$

(*ii*) Volume of wood wasted

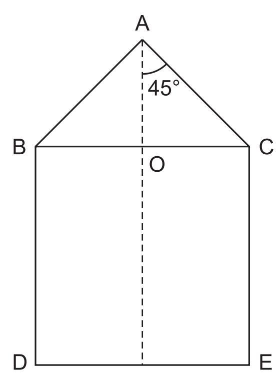
= Volume of the cuboid – Volume of the cylinder

$= 14\ \text{cm} \times 14\ \text{cm} \times 50\ \text{cm} - 7700\ \text{cm}^3$

$= 9800\ \text{cm}^3 - 7700\ \text{cm}^3$

$= \mathbf{2100\ cm^3}$

14. Radius of the cylinder and cone, $r = \frac{4.3}{2}$ m = 2.15 m



Height of the cylinder, h = 3.8 m

In the cone ABC, slant height

$AC = l = \frac{r}{\sin 45°} = \frac{2.15}{0.7072} = 3.04$ m

Surface area of the building = Curved surface area of the cone + Curved surface area of the cylinder

$= 2\pi rh + \pi rl = \pi r\,(2h + l)$

$= 3.14 \times 2.15 \times (2 \times 3.8 + 3.04)\ \text{m}^2$

$= \mathbf{71.83\ m^2}$

15. Radius of iron sphere, r = 8 cm

External radius, $r_1 = \frac{20}{3}$ cm

Internal radius, r_2 = 4 cm

Thickness, h = 3 cm

Volume of sphere $= \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times (8)^3$

$= \frac{45056}{21}\ \text{cm}^3$

Volume of hollow cylinder $= \pi h\,(r_2^2 - r_1^2)$

$= \frac{22}{7} \times 3 \times \left[\left(\frac{20}{3}\right)^2 - (4)^2\right]$

$= \frac{22}{7} \times 3 \times \left[\frac{400}{9} - \frac{16}{1}\right]$

$= \frac{22}{7} \times 3 \times \left[\frac{400 - 144}{9}\right]$

$= \frac{22}{7} \times 3 \times \frac{256}{9}$

$= \frac{5632}{21}\ \text{cm}^3$

Number of rings $= \dfrac{\frac{45056}{21}}{\frac{5632}{21}}$

$= \frac{45056}{21} \times \frac{21}{5632} = \mathbf{8}$

16. In 30 min, the length of water flowing through the cylindrical pipe $= \pi \times 1^2 \times 80 \times 30 \times 60\ \text{cm}^3$ ...(1)

Let the rise of water level in the empty cylindrical tank be h.

The radius of the base of the cylinder = 40 cm

∴ Volume of the water in the tank

$= \pi \times 40^2 \times h\ \text{cm}^3$

$= 1600\pi\ h\ \text{cm}^3$ ...(2)

∴ From (1) and (2), we have

$1600\pi h = 80 \times 30 \times 60\pi$

$\Rightarrow h = \frac{80 \times 30 \times 60}{1600} = 90$

Hence, rise of water level in the tank is **90 cm**.