CHAPTER **14**

Areas Related to Circles

EXERCISE 14A

For Basic and Standard Levels

1. (*i*) Let r be the radius of the circle,

then $r = 3.5$ cm

Circumference $= 2\pi r = 2 \times \frac{22}{7} \times 3.5$ cm $=$ **22 cm.**

(*ii*) Let r be the radius of the circle, then $r = 98$ cm.

Circumference $= 2\pi r = 2 \times \frac{22}{7} \times 98$ cm $=$ **616 cm.**

2. (*i*) Let r be the radius of the wheel.

$$r = \frac{21}{2} \text{ cm}$$

Circumference $= 2\pi r = 2 \times \frac{22}{7} \times \frac{21}{2}$ cm $=$ **66 cm.**

(*ii*) Diameter of planet venus = 12278 km.

$\therefore$ radius $= \frac{12278}{2}$ km $= 6139$ km

Circumference $= 2\pi r = 2 \times \frac{22}{7} \times 6139 =$ **38588 km.**

3. Let r be the radius of the circle, its circumference = 8.8 cm

$$2\pi r = 8.8 \text{ cm}$$

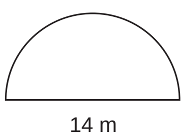
$$2 \times \frac{22}{7} \times r = 8.8$$

$$r = \frac{8.8 \times 7}{2 \times 22} = 1.4 \text{ cm}$$

$\therefore$ Length of the diameter $= 1.4 \times 2 =$ **2.8 cm.**

4. Diameter of the plot = 14 m.

Let r be the radius of the plot.



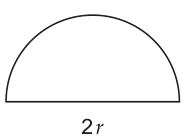
Then $r = \frac{14}{2} = 7$ m.

Perimeter of the plot

$$= \frac{1}{2} \times 2\pi r + 14 \text{ m}$$

$$= \frac{1}{2} \times 2 \times \frac{22}{7} \times 7 + 14 = \textbf{36 m.}$$

5. Perimeter of the protractor = 36 cm.



$$\frac{1}{2} \times 2\pi r + 2r = 36$$

$$\pi r + 2r = 36$$

$$r\left(\frac{22}{7} + 2\right) = 36$$

$$r \times \frac{36}{7} = 36$$

$$r = \frac{36 \times 7}{36} = 7 \text{ cm}$$

$\therefore$ diameter $= 2r = 2 \times 7 =$ **14 cm.**

6. Let r_1 and r_2 be the radii of the circles.

Then $r_1 = 14$ cm.

$\therefore$ $2 \times \pi \times 14 + 2\pi r_2 = 132$ cm

$$2\pi r_2 = (132 - 88) \text{ cm}$$

$$\pi r_2 = \frac{44}{2} = 22 \text{ cm}$$

$$r_2 = \frac{22 \times 7}{22} = 7 \text{ cm.}$$

Radius of the other circle = **7 cm.**

7. The wheel makes 5000 revolutions in moving 11 km.

One revolution $= 2\pi r$.

5000 revolutions in moving 11 km

1 revolution in moving $\frac{11}{5000}$ km

$\therefore$ $2\pi r = \frac{11}{5000}$ km

$$r = \frac{11 \times 1000 \times 100 \times 7}{5000 \times 2 \times 22} = 35 \text{ cm}$$

$\therefore$ diameter $= 35 \times 2 =$ **70 cm.**

8. (*i*) Diameter of the wheel = 84 cm.

$\therefore$ radius = 42 cm

one revolution $= 2\pi r$

$$= 2 \times \frac{22}{7} \times 42$$

$$= 264 \text{ cm}$$

We know that 396 m = 39600 cm.

39600 cm covers in $\frac{39600}{264} =$ **150 revolutions.**

(*ii*) Circumference of the roller = 3 m = 1 revolution

$$2\pi r = 3 \text{ m.}$$

$$r = \frac{3 \times 7}{2 \times 22} \text{ m.}$$

3 m in 1 revolution.

21 m in $\frac{1}{2} \times 21 =$ **7 revolutions.**

9. Let r cm be the radius of the circular wheel.

Then, its area $= \pi r^2$ cm^2 $= \frac{\pi r^2}{10000}$ m^2

$\therefore$ $\frac{\pi r^2}{10000} = 1.54$

$\Rightarrow$ $\pi r^2 = 15400$

$\Rightarrow$ $r^2 = 15400 \times \frac{7}{22} = 4900$

$\Rightarrow$ $r = \sqrt{4900} = 70$

Hence, the radius of the circle = 70 cm.

Let n be the number of revolutions.

Then $$n = \frac{\text{Distance}}{\text{Length of circumference}}$$

$$= \frac{17600}{2\pi r}$$

$$= \frac{17600}{2 \times \frac{22}{7} \times 70}$$

$$= \frac{1760}{44} = 40$$

Hence, the required number of revolutions = **40**.

10. (*i*) Radius of the wheel = 70 cm

$$\text{Circumference} = 2\pi r$$

$$= 2 \times \frac{22}{7} \times 70 \text{ cm}$$

$$= 440 \text{ cm}$$

The wheel goes 440 cm in 1 revolution.

The wheel goes 66 × 1000 × 100 cm in

$\frac{66 \times 1000 \times 100}{440}$ revolutions = 15000 revolutions.

In 60 minutes 15000 revolutions.

In 1 minute $\frac{15000}{60}$ revolutions = **250 revolutions.**

(*ii*) Radius of the wheel = 35 cm

$$\text{Circumference} = 2\pi r$$

$$= 2 \times \frac{22}{7} \times 35 \text{ cm} = 220 \text{ cm}$$

The wheel goes 220 cm in 1 revolution.

The wheel goes 66 × 1000 × 100 cm in

$\frac{66 \times 1000 \times 100}{220}$ revolutions = 30000 revolutions

In 60 minutes 30000 revolutions.

In 1 minute $\frac{30000}{60}$ revolutions = **500 revolutions**.

(*iii*) Radius of the wheel = 35 cm.

$$\text{Circumference} = 2 \times \frac{22}{7} \times 35 = 220 \text{ cm.}$$

In 1 revolution bicycle goes 220 cm.

In 25 revolutions bicycle goes 220 × 25 cm

= 5500 cm = 55 m.

∴ In 10 seconds bicycle goes 55 m.

In 1 second bicycle goes $\frac{55}{10}$ m/second .

$$= \frac{55}{10} \times \frac{18}{5} \text{ km/h.}$$

= **19.8 km/h.**

(*iv*) Diameter of the wheel = 42 cm,

then radius = 21 cm.

$$\text{One revolution} = 2\pi r = 2 \times \frac{22}{7} \times 21$$

= 132 cm.

6 revolutions = 132 × 6 = 792 cm.

Then in 1 second cart goes 792 cm.

Speed of the cart = **792 cm/s or 7.92 m/s**.

11. Sum of the radii of 2 wheels of the tractor = 98 cm.

Let r_1 and r_2 are the radii of 2 wheels.

∴ $r_1 = x$ cm, 50 $r_2 = 98 - x$ cm.

According to the condition

$$2\pi r_1 - 2\pi r_2 = 176 \text{ cm}$$

$$2\pi(x - 98 + x) = 176$$

$$2\pi(2x - 98) = 176$$

$$2x - 98 = \frac{176 \times 7}{2 \times 22} = 28 \text{ cm.}$$

$$2x = 28 + 98 = 126 \text{ cm}$$

$$x = 63 \text{ cm}$$

Radius of the 2nd wheel = 98 – 63 = 35 cm.

∴ Diameters = **126 cm and 70 cm.**

12. (*i*) Long hand of the clock = 6 cm

Short hand of the clock = 4 cm

In 24 hours, the minute hand will complete a full circle 24 times

∴ Distance travelled by long hand = $2\pi R \times 24$

$= 2 \times \pi \times 6 \times 24$

$= 288\pi$ cm

In 24 hours the hour hand will cover complete circle twice.

∴ Distance travelled by short hand = $2\pi r \times 2$

$= 2 \times \pi \times 4 \times 2$

$= 8\pi \times 2$

$= 16\pi$ cm

Total distance travelled by short and long hand

$= (288\pi + 16\pi)$ cm

$= 304\pi$ cm = **954.56 cm**

(*ii*) In 48 hours, the minute hand, i.e. longer hand of the clock will complete 48 revolutions and the hour hand, i.e. the shorter hand of the clock will complete 4 revolution. In 48 revolutions, the longer hand (i.e. the minute hand) of the clock will cover a distance of $2\pi \times 6 \times 48$ cm = 576π cm and in 4 revolutions, the shorter hand (i.e. the hour hand) of the clock will cover a distance of $2\pi \times 4 \times 4 = 32\pi$ cm.

∴ Total distance covered by both the hands = $(576\pi + 32\pi)$ cm = **608π cm,** which is the required distance.

13. Diameter the circle = 2.8 cm

∴ Radius = 1.4 cm

$$\text{Area} = \pi r^2 = \frac{22}{7} \times (1.4)^2 = \mathbf{6.16\ cm^2}$$

14. Radius of the 1st pond = 14 m.

Area of the pond = $\frac{22}{7} \times (1.4)^2$ m^2 = 616 m^2

According to the condition given

Area of the 2nd pond = $\frac{1}{4} \times 616$ = 154 m^2 = πr^2

Radius of the 2nd pond:

$$r^2 = \frac{154 \times 7}{22} = 49 \text{ m}$$

$\boldsymbol{r = 7}$ **m.**

15. (*i*) Area of the circle = πr^2 = 24.64 cm^2

$$\therefore \quad r^2 = \frac{24.64 \times 7}{22} = 7.84 \text{ cm.}$$

$$r = 2.8 \text{ cm}$$

Circumference = $2\pi r$ = $2 \times \frac{22}{7} \times 2.8$ = **17.6 cm.**

(*ii*) Diameter of the circle = 7 cm.

$$\text{Radius} = \frac{7}{2} \text{ cm.}$$

$$\text{Area} = \pi r^2 = \frac{22}{7} \times \left(\frac{7}{2}\right)^2 \text{ cm}^2$$

Area of the 2nd circle, according to given condition

$$= 16 \times \frac{22}{7} \times \left(\frac{7}{2}\right)^2 \text{ cm}^2$$

$$\pi r^2 = 16 \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \text{ cm}^2$$

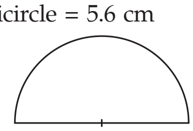
$$r^2 = 16 \times \frac{7}{2} \times \frac{7}{2} \text{ cm}^2$$

$$r^2 = 4 \times 7 \times 7 \text{ cm}^2$$

$$r = 2 \times 7 = 14 \text{ cm}$$

Circumference = $2 \times \frac{22}{7} \times 14$ = **88 cm.**

16. Radius of the semicircle = 5.6 cm



Perimeter of the semicircle = $\frac{1}{2} \times 2\pi r + d$

$= \frac{1}{2} \times 2 \times \frac{22}{7} \times 5.6 + 11.2$ cm

= 17.6 + 11.2

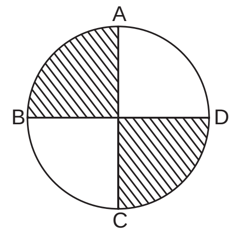
= **28.8 cm**

Area of the semicircle = $\frac{1}{2} \times \pi r^2$

$= \frac{1}{2} \times \frac{22}{7} \times 5.6 \times 5.6$

= **49.28 cm^2**

17. (*i*) Area of the shaded portion of the circle = 308 cm^2



$$\therefore \quad \frac{1}{2}\pi r^2 = 308 \text{ cm}^2$$

$$r^2 = \frac{308 \times 2 \times 7}{22} \text{ cm}^2$$

$$= 196 \text{ cm}^2$$

$$r = 14 \text{ cm.}$$

AC is the diameter.

$\therefore$ d = **28 cm.**

(*ii*) Circumference of the circle = $2\pi r$

$2 \times \frac{22}{7} \times 14$ cm = **88 cm.**

18. Radii of 3 circles are 1 cm, 4 cm, 8 cm.

Sum of the area of 3 circles.

$$\pi 1^2 + \pi 4^2 + \pi\, 8^2 = \pi\,(1 + 16 + 64) \text{ cm}^2$$

$$= \pi\, 81 \text{ cm}^2$$

According to the condition

$$81\pi \text{ cm}^2 = \text{Area of the 4th circle.}$$

$$\therefore \quad \pi\, r^2 = 81\,\pi$$

$$r^2 = 81 \text{ cm}^2,$$

$\boldsymbol{r = 9}$ **cm.**

19. Let d_1 and d_2 are the diameters of 2 circles

$$d_1 : d_2 = 3 : 4 = \frac{3x}{4x}$$

Diameter of 3rd circle = 30 cm.

$\therefore$ Radius = 15 cm.

Now, according to the condition,

$$\pi(3x)^2 + \pi(4x)^2 = \pi(15)^2$$

$$\pi(9x^2 + 16x^2) = \pi(15)^2$$

$$25x^2 = 15^2$$

$$x^2 = \frac{225}{25} = 9 \text{ cm}^2$$

$$x = 3 \text{ cm}$$

Radii of the circle = 3 × 3 cm and 4 × 3 cm

= 9 cm and 12 cm.

$\therefore$ Diameters are **18 cm, 24 cm.**

20. Area of the square formed by the wire = 25 cm^2

$\therefore$ Side of the square = 5 cm

Then length of the wire = 4 × 5 cm = 20 cm

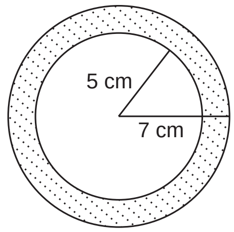
$\therefore$ Circumference of the circle made from the wire = length of the wire = 20 cm.

$$2\pi r = 20, \quad r = \frac{20 \times 7}{2 \times 22} \text{ cm.}$$

$$\therefore \quad \text{Area of the circle} = \pi r^2 = \frac{22}{7} \times \frac{20 \times 7}{2 \times 22} \times \frac{20 \times 7}{2 \times 22}$$

$$= \mathbf{\frac{350}{11}\ cm^2}$$

21. Radii of 2 concentric circles are 7 cm and 5 cm.



Area of the portion between the two circles

$$\pi r_1^2 - \pi r_2^2 = \pi(49 - 25)\text{ cm}^2$$

$$= \frac{22}{7} \times 24\text{ cm}^2$$

$$= \mathbf{\frac{528}{7}\ cm^2}$$

22. Area enclosed between 2 concentric circles = 770 cm^2

Radius of the outer circle = 21 cm, r of the inner circle = r_2

$\therefore$ $$\pi(21)^2 - \pi r_2^2 = 770\text{ cm}^2$$

$$\pi(441 - r_2^2) = 770\text{ cm}^2$$

$$441 - r_2^2 = \frac{770 \times 7}{22} = 245\text{ cm}^2$$

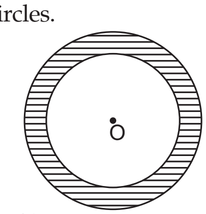
$$441 - 245 = r_2^2$$

$$196\text{ cm} = r_2^2$$

$$\mathbf{14\ cm} = \boldsymbol{r_2}.$$

23. Area of the shaded region in the given figure = 286 cm^2

r_1 and r_2 are radii of the concentric circles.



$\therefore$ $$\pi(r_1^2 - r_2^2) = 286\text{ cm}^2$$

$$\pi(r_1 + r_2)(r_1 - r_2) = 286\text{ cm}^2$$

$$(r_1 + r_2) \times 7 = \frac{286 \times 7}{22}$$

$$r_1 + r_2 = \frac{286 \times 7}{22 \times 7} = 13\text{ cm}$$

$$r_1 + r_2 = \mathbf{13\ cm}$$

24. Radii of inner and outer circle are r_2 and r_1.

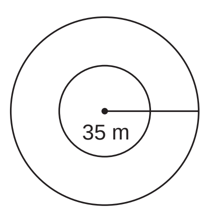
$\therefore$ $$2\pi r_2 = 220\text{ m}$$

$$r_2 = \frac{220 \times 7}{2 \times 22} = 35\text{ m}$$

Width of the track = 7 m

Radius of the outer circle = 35 + 7 = 42 m

Perimeter of the outer circle



$$2\pi r = 2 \times \frac{22}{7} \times 42\text{ m}$$

$$= 264\text{ m}$$

Cost of fencing of outer circle

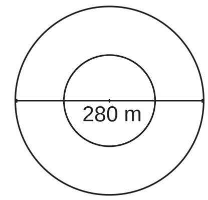
264 × ₹ 20 = **₹ 5280.**

25. Area of the circular road = 22176 m^2

Diameter of the outer circle is 280 m.

$$\pi r_1^2 - \pi r_2^2 = 22176\text{ m}^2$$

$$\pi(140)^2 - \pi r_2^2 = 22176$$



$$\frac{22}{7} \times 19600 - 22176 = \pi r_2^2$$

$$61600 - 22176 = \pi r_2^2$$

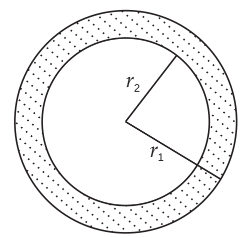
$$\frac{7 \times 39424}{22}\text{ m}^2 = r_2^2$$

$$112\text{ m} = r_2$$

$\therefore$ Width of the road = 140 – 112

= **28 m.**

26. Area of the circular ring between two concentric circles = 286 cm^2



$$\pi(r_1^2 - r_2^2) = 286$$

$$r_1^2 - r_2^2 = \frac{286 \times 7}{22}$$

$$= 91\text{ cm}^2$$

$$d_1 = 2r_1,\ d_2 = 2r_2$$

Difference of the diameters is 14 cm.

$$2r_1 - 2r_2 = 14$$

$$2(r_1 - r_2) = 14\text{ cm}$$

$$r_1 - r_2 = 7\text{ cm} \quad \ldots(1)$$

Again $$r_1^2 - r_2^2 = 91$$

$$(r_1 - r_2)(r_1 + r_2) = 91$$

$$7(r_1 + r_2) = 91$$

$$(r_1 + r_2) = 13 \quad \ldots(2)$$

From (1) and (2) we get, $2r_1 = 20$

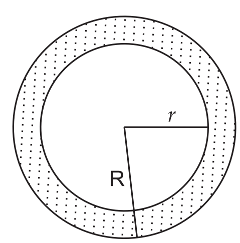
$\therefore$ $r_1 = 10$ cm

d_1 = **20 cm.**

$r_2 = 13 - r_1 = 13 - 10 = 3$

$\therefore$ d_2 = **6 cm.**

27. Let r m be the radius of the park without the road and R m be that of the park along with the road.



Then $r = 105$ m [Given]

and R = (21 + 105) m = 126 m

$\therefore$ Required area of the road

$$= \pi(R^2 - r^2)$$

$$= \pi(R + r)(R - r)$$

$$= \pi(126 + 105)(126 - 105)\text{ m}^2$$

$$= \frac{22}{7} \times 231 \times 21\text{ m}^2$$

$$= \mathbf{15246}\text{ m}^2$$

28. The radius of the pond, $r = \frac{17.5}{2}$ m = 8.75 m.

The radius of the pond along with the road,

$$R = (8.75 + 2) \text{ m} = 10.75 \text{ m}$$

Hence, the area of the road

$$= \pi(R^2 - r^2)$$
$$= 3.14 \times (R + r)(R - r)$$
$$= 3.14 \times (10.75 + 8.75)(10.75 - 8.75) \text{ m}^2$$
$$= 3.14 \times 19.50 \times 2 \text{ m}^2$$
$$= 3.14 \times 39 \text{ m}^2$$

$\therefore$ Required cost = ₹ (3.14 × 39 × 25)

= ₹ (122.46 × 25)

= ₹ **3061.50**.

29. Let r m be the radius of the circular playground.

Then its area is given to be 22176 m^2.

$\therefore \quad \pi r^2 = 22176$

$\Rightarrow \quad r^2 = 22176 \times \frac{7}{22} = 7056$

$\therefore \quad r = \sqrt{7056} = 84$

$\therefore$ The circumference of the playground

$$= 2\pi r$$
$$= 2 \times \frac{22}{7} \times 84$$
$$= 528$$

Hence, the length of the circumference = 528 cm

$\therefore$ Required cost = ₹ 50 × 528

= ₹ **26400.**

30. Circumference of the circular field = $\frac{3960}{21}$ = 188.57 m.

$$2\pi r = 188.57 \text{ m}$$
$$r = \frac{188.57 \times 7}{2 \times 22} = 30 \text{ m}$$

Area of the field = πr^2

$$= \frac{22}{7} \times 30 \times 30 \text{ m}^2$$

Cost of ploughing the field at ₹ 0.77 per m^2

$$= \frac{22}{7} \times 30 \times 30 \times \frac{77}{100} = ₹\ \mathbf{2178}$$

31. (*i*) Diameter of pond = 21 m

Radius of Pond = $\frac{21}{2}$ = 10.5 m

Width of path = 3.5 m

Radius of the circle including pond and path

= 10.5 + 3.5

= 14 m

Area of the outer circle = πR^2

$= \pi \times (14)^2$

$= 196\pi$ m^2

Area of pond = πr^2

$= \pi(10.5)^2$

$= \pi \times 110.25$ m^2

$= 110.25\ \pi$ m^2

Area of path = Area of outer circle − Area of pond

$= 196\pi - 110.25\pi$

$= 85.75\pi$ m^2

Cost of constructing 1 m^2 path = ₹ 25

Cost of constructing 85.75π m^2 path = 85.75π × ₹ 25

= ₹ 2143.75π

= ₹ **6737.5**

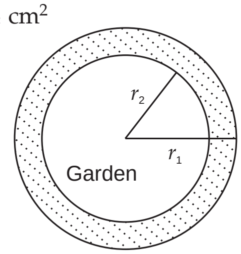
(*ii*) Total cost of turfing the circular road at 20 paise/m^2 is ₹ 215.60.

$\therefore$ Area of the circular road = $\frac{215.60}{0.20}$

= 1078 m^2

Area of the garden is 1386 sq m.

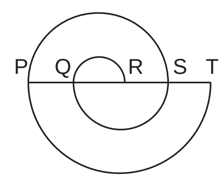
$$\pi r_1^2 - \pi r_2^2 = 1078 \text{ m}^2$$
$$\pi r_1^2 - 1386 = 1078 \text{ m}^2$$
$$\pi r_1^2 = 1078 + 1386 \text{ m}^2 = 2464 \text{ m}^2$$
$$r_1^2 = \frac{2464 \times 7}{22} = 784 \text{ cm}^2$$
$$r_1 = 28 \text{ m}$$
$$\pi r_2^2 = 1386$$
$$r_2^2 = \frac{1386 \times 7}{22} = 441$$
$$r_2^2 = 21 \text{ m.}$$



$\therefore$ Width of the road = 28 − 21 = **7 m.**

For Standard Level

32. Length of the spiral

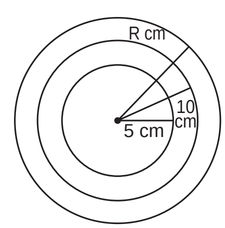


$$= \left(\frac{1}{2} \times 2\pi \times 1 + \frac{1}{2} \times 2\pi \times 2 + \frac{1}{2} \times 2\pi \times 3 + \frac{1}{2} \times 2\pi \times 4\right)$$
$$= \pi \times 1 + \pi \times 2 + \pi \times 3 + \pi \times 4$$
$$= \pi\,(1 + 2 + 3 + 4) = \mathbf{10\pi\ cm.}$$

33. Radii of 1st and 2nd circles are 5 cm and 10 cm.

Area between 1st and 2nd circle

$$\pi r_2^2 - \pi r_1^2 = \pi 100 - \pi 25$$
$$= \pi(100 - 25) = \pi\ 75.$$



Again, area between 2nd and 3rd circle

$$\pi r_3^2 - \pi r_2^2 = 75\pi$$

$$\pi(r_3^2 - r_2^2) = 75\pi$$
$$r_3^2 - r_2^2 = 75$$
$$r_3^2 - 100 = 75$$
$$r_3^2 = 75$$

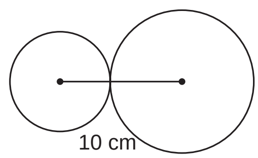
Total area of the 3rd circle.

$$\pi r_3^2 = \frac{22}{7} \times 175 = \mathbf{550\ cm^2}$$

34. Sum of the areas of two circles which touch each other externally = 58 π cm^2

Distance between their centres is 10 cm.

Let x be radius of one circle radius of the other circle = $(10 - x)$ cm



$\therefore \quad \pi x^2 + \pi(10 - x)^2 = 58\pi$

$\Rightarrow \quad \pi[x^2 + (10 - x)^2] = 58\pi$

$\Rightarrow \quad x^2 + 100 + x^2 - 20x = 58$

$\Rightarrow \quad 2x^2 - 20x + 100 = 58$

$\Rightarrow \quad x^2 - 10x + 50 = 29$

$\Rightarrow \quad x^2 - 10x + 21 = 0$

$\Rightarrow \quad (x - 7)(x - 3) = 0$

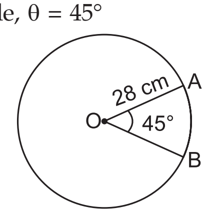
$\therefore \quad x =$ **3 cm, 7 cm**

EXERCISE 14B

For Basic and Standard Levels

1. (*i*) The radius, r of the circle = 28 cm

Central angle, $\theta = 45°$



$\therefore$ Required area of the sector

$$= \frac{\pi r^2 \theta}{360°}$$
$$= \frac{22}{7} \times 28 \times 28 \times \frac{45}{360}\ \text{cm}^2$$
$$= \mathbf{308\ cm^2}$$

(*ii*) We have radius of the circle, r = 21 cm

Central angle, $\theta = 60°$.

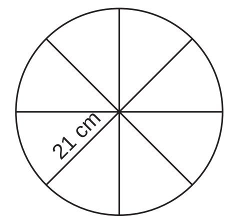
(*a*) Required length of the arc

$$= 2\pi r \times \frac{\theta}{360°}$$
$$= 2 \times \frac{22}{7} \times 21 \times \frac{60°}{360°}\ \text{cm}$$
$$= \mathbf{22\ cm}.$$

(*b*) Required area of the sector

$$= \pi r^2 \times \frac{\theta}{360°}$$
$$= \frac{22}{7} \times 21 \times 21 \times \frac{60}{360}\ \text{cm}^2$$
$$= \mathbf{231\ cm^2}$$

2. (*i*) Diameter of the pizza, divided in eight equal sectors = 21 cm, radius = $\frac{21}{2}$ cm.

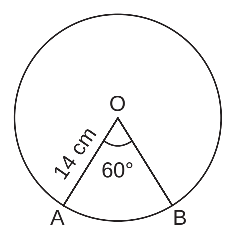


(*a*) Angle of each sector = $\frac{360}{8}$ = **45°**

(*b*) Area of each sector = $\frac{\theta}{360} \times \pi r^2$

$$= \frac{45}{360} \times \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2}\ \text{cm}^2$$
$$= \mathbf{43.3125\ cm^2}$$

(*ii*) Radius of the circle is 14 cm and angle of the sector = 60°.



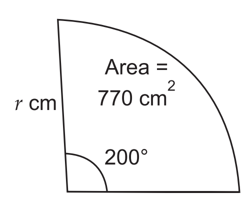
(*a*) Length of its arc = $\frac{\theta}{360} \times 2\pi r$

$$= \frac{60}{360} \times 2 \times \frac{22}{7} \times 14\ \text{cm}$$
$$= \mathbf{\frac{44}{3}\ cm}$$

(*b*) Area of the sector = $\frac{\theta}{360} \times \pi r^2$ cm^2

$$= \frac{60}{360} \times \frac{22}{7} \times 14 \times 14\ \text{cm}^2$$
$$= \mathbf{\frac{308}{3}\ cm^2}$$

3. (*i*) Let r cm be the radius of the sector. Let θ be the central angle.



Then area of the sector

$$= \pi r^2 \frac{\theta}{360}$$

$$= \frac{22}{7} \times r^2 \times \frac{200}{300}$$

$$= 770 \qquad \text{[Given]}$$

$$\Rightarrow \quad r^2 = 770 \times \frac{7}{22} \times \frac{36}{20}$$

$$\therefore \quad r = \sqrt{7 \times 7 \times 9} = 21$$

$\therefore$ Required length of the arc of the above sector

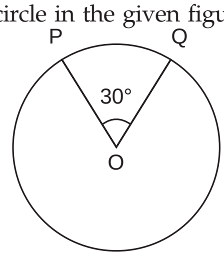
$$= 2\pi r \frac{\theta}{360}$$

$$= 2 \times \frac{22}{7} \times 21 \times \frac{200}{360} \text{ cm}$$

$$= \frac{220}{3} \text{ cm}$$

$$= \mathbf{73\frac{1}{3} \text{ cm}}$$

(*ii*) Area of the circle in the given figure = π cm^2



Length of minor arc PQ = $\frac{\theta}{360} \times 2\pi$

$$\text{Area} = \pi r^2 = \pi \text{ cm}^2 \qquad \text{[Given]}$$

$$r^2 = 1$$

$$r = 1 \text{ cm.}$$

$$\text{Arc PQ} = \frac{30}{360} \times 2 \times 1 = \mathbf{\frac{\pi}{6} \text{ cm}}$$

4. Radius of the circle = 36 cm

$$\text{Area} = 54\pi \text{ cm}^2$$

$$\pi r^2 = 54\pi$$

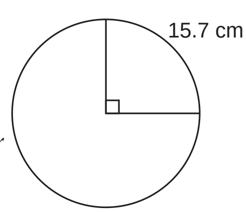
$$54\pi = \frac{\theta}{360} \times \pi \times 36 \times 36$$

$$\theta = \frac{54 \times 360}{36 \times 36} = 15°$$

$$\therefore \quad \text{Length} = \frac{\theta}{360} \times 2 \times \frac{22}{7} \times 36$$

$$= \mathbf{\frac{66}{7} \text{ cm}}.$$

5. (*i*) Length of the arc = 15.7 cm



$$\text{Arc} = \frac{\theta}{360} \times 2\pi r$$

$$15.7 = \frac{90}{360} \times 2 \times \frac{22}{7} \times r$$

$$r = \frac{15.7 \times 360 \times 7}{2 \times 22 \times 90}$$

$$= 9.99 \text{ cm} = \mathbf{10 \text{ cm.}}$$

(*ii*) Length of the arc of the circle

$$= 2\pi r \times \frac{\theta}{360}$$

where r is the radius of the arc and $\theta = 60°$.

$\therefore$ Required length of the arc

$$= 2\pi r \times \frac{60}{360} \text{ cm}$$

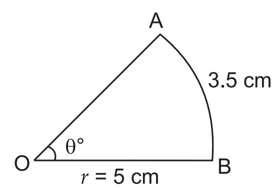
$$= \frac{\pi r}{3}$$

$$\therefore \quad \frac{\pi r}{3} = 20 \qquad \text{[Given]}$$

$$\Rightarrow \quad r = \frac{60}{\pi}$$

$\therefore$ Required length of the radius of the circle is $\mathbf{\frac{60}{\pi}}$ cm.

(*iii*) Let OAB be the sector of radius, r = OA = 5 cm and arc AB = 3.5 cm.



Then the length of the arc

$$= 2\pi \times 5 \times \frac{\theta}{360}$$

$$= \frac{\pi\theta}{36} = 3.5 \qquad \text{[Given]}$$

$$\therefore \quad \pi\theta = 3.5 \times 36 = 126 \qquad \ldots(1)$$

$\therefore$ Required area of the sector OAB

$$= \pi r^2 \frac{\theta}{360} \text{ cm}^2$$

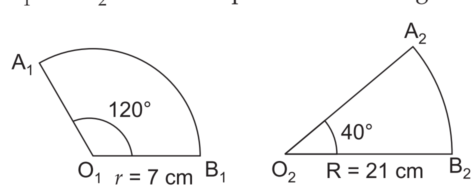
$$= \frac{25\pi\theta}{360} \text{ cm}^2$$

$$= \frac{5\pi\theta}{72} \text{ cm}^2$$

$$= \frac{5}{72} \times 126 \text{ cm}^2 \qquad \text{[From (1)]}$$

$$= \frac{35}{4} \text{ cm}^2 = \mathbf{8.75 \text{ cm}^2}$$

6. Let r cm and R cm be the radii of two circular sectors $O_1A_1B_1$ and $O_2A_2B_2$ such that r = 7 cm and R = 21 cm. Let θ_1 and θ_2 be their respective central angles.



Then the area, A_1 of the circular sector $O_1A_1B_1$ is

$$A_1 = \pi r^2 \times \frac{\theta_1}{360} \text{ cm}^2$$

$$= \pi \times 7^2 \times \frac{120°}{360°} \text{ cm}^2$$

$$= \frac{49\pi}{3} \text{ cm}^2$$

$$= \frac{49}{3} \times \frac{22}{7} \text{ cm}^2$$

$$= \frac{7}{3} \times 22 \text{ cm}^2$$

$$= \frac{154}{3} \text{ cm}^2$$

and the area, A_2 of the circular sector $O_2 A_2 B_2$ is

$$A_2 = \pi R^2 \frac{\theta_2}{360}$$

$$= \pi \times 21 \times 21 \times \frac{40°}{360°} \text{ cm}^2$$

$$= 49\pi \text{ cm}^2$$

$$= 49 \times \frac{22}{7} \text{ cm}^2$$

$$= 7 \times 22 \text{ cm}^2$$

$$= 154 \text{ cm}^2$$

Again, if L_1 and L_2 be the lengths of the arcs of two sectors $O_1A_1B_1$ and $O_2A_2B_2$ respectively, then

$$L_1 = 2\pi r_1 \times \frac{\theta_1}{360}$$

$$= \frac{2\pi \times 7 \times 120°}{360°} \text{ cm}$$

$$= \frac{14\pi}{3} \text{ cm}$$

$$= \frac{14}{3} \times \frac{22}{7} \text{ cm}$$

$$= \frac{44}{3} \text{ cm}$$

$$L_2 = 2\pi R \times \frac{\theta_2}{360}$$

$$= \frac{2\pi \times 21 \times 40°}{360°} \text{ cm}$$

$$= \frac{14\pi}{3} \text{ cm}$$

$$= \frac{14}{3} \times \frac{22}{7} \text{ cm}$$

$$= \frac{44}{3} \text{ cm}$$

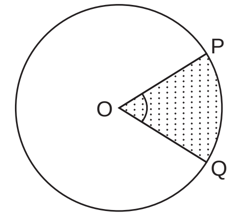
From the above, we observe that two sectors of two different circles of different radii and different central angles may have equal arc lengths, but their areas need not be equal.

The required areas of two sectors are $\mathbf{\frac{154}{3}}$ **cm²** and **154 cm²**.

7. Area of the circle = πr^2

Area of the sector = $\frac{\theta}{360} \times \pi r^2$

According to given condition,



Area of the sector = $\frac{3}{20}$ of πr^2

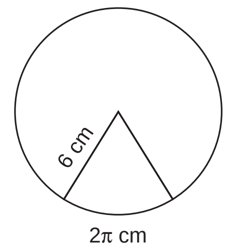
$$\therefore \quad \frac{\theta}{360} \times \pi r^2 = \frac{3}{20} \times \pi r^2$$

$$\theta = \frac{3 \times 360}{20}$$

$$\theta = \mathbf{54°.}$$

8. Radius of the circle = 6 cm

Length of an arc = 2π cm



$$\therefore \quad 2\pi = \frac{\theta}{360} \times 2\pi r$$

$$\theta = \frac{360°}{6} = \mathbf{60°}$$

9. Radius of the chain wheel = 7 cm

Portion (Arc) of the chain in contact with the wheel = 11 cm.

$$\therefore \quad 11 = \frac{\theta}{360} \times 2 \times \frac{22}{7} \times 7$$

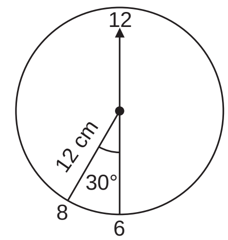
$$\theta = \frac{11 \times 360 \times 7}{2 \times 22 \times 7}$$

$$\theta = \mathbf{90°}$$

10. (*i*) Minute hand (radius) = 12 cm

area covered from 8:00 am to 8:35 am

$$= \frac{1}{2}\pi r^2 + \frac{\theta}{360}\pi r^2$$



1 min = 6°

$\therefore$ 5 min = 30°

$$\text{Area} = \frac{1}{2}\pi r^2 + \frac{\theta}{360}\pi r^2$$

$$= \pi r^2\left(\frac{1}{2} + \frac{30}{360}\right)$$

$$= \frac{22}{7} \times 12 \times 12 \times \frac{7}{12} = \mathbf{264 \text{ cm}^2}$$

(*ii*) The time from 6:05 am to 6:40 am is 35 minutes. We know that in 1 minute, the hand describes an angle of 6°.

$\therefore$ In 35 min, the minute hand describes an angle of 6° × 35 = 210.

Hence, the required area swept by the minute hand of length 5 cm = the area of the sector of a circle of radius, r = 5 cm and

Central angle, θ = 210°

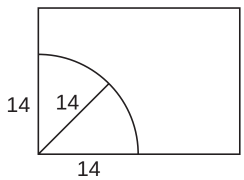
$$= \pi r^2 \frac{\theta}{360} \text{ cm}^2$$

$$= \frac{22}{7} \times 5^2 \times \frac{210}{360} \text{ cm}^2$$

$$= \frac{22}{7} \times 25 \times \frac{7}{12} \text{ cm}^2$$

$$= \frac{275}{6} \text{ cm}^2 = \mathbf{45\frac{5}{6} \text{ cm}^2}.$$

11. Length (radius) of the rope = 14 cm.



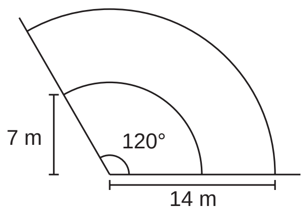
Area on which the goat can graze

$$= \frac{1}{4} \times \pi r^2$$

$$= \frac{1}{4} \times \frac{22}{7} \times 14 \times 14$$

$$= \mathbf{154\ m^2}$$

12. (*i*) Area of the sector the cow can graze with 7 m (radius) rope



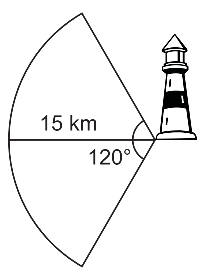
$$= \frac{120}{360} \times \frac{22}{7} \times 7 \times 7 = \mathbf{\frac{154}{3}\ m^2}$$

(*ii*) Area of the sector, if the rope was 14 m long

$$= \frac{120}{360} \times \frac{22}{7} \times 14 \times 14\ \text{m}^2 = \frac{616}{3}\ \text{m}^2$$

$$\text{Increase in area} = \left(\frac{616}{3} - \frac{154}{3}\right)\text{m}^2 = \mathbf{154\ m^2}$$

13. Sector of an angle, a lighthouse spreads the light = 120°, radius = 15 km



$$\text{Area of the sector} = \frac{120}{360} \times \frac{22}{7} \times 15 \times 15\ \text{km}^2 = \mathbf{235.5\ km^2}$$

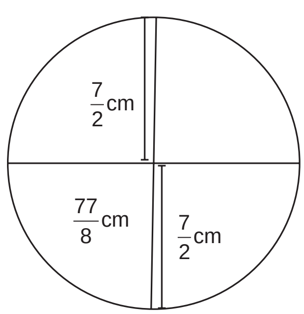
14. (*i*) Area of the quadrant $= \frac{77}{8}\ \text{cm}^2$

$$\frac{1}{4}\pi r^2 = \frac{77}{8}\ \text{cm}^2$$

$$r^2 = \frac{77}{8} \times \frac{4 \times 7}{22}$$

$$r^2 = \frac{49}{4}\ \text{cm}^2$$

$$r = \frac{7}{2}\ \text{cm.}$$



$$\text{Perimeter of the quadrant} = \frac{1}{4} \times 2\pi r + \frac{7}{2} + \frac{7}{2}$$

$$= \frac{1}{4} \times 2 \times \frac{22}{7} \times \frac{7}{2} + \frac{14}{2}$$

$$= \frac{11}{2} + \frac{14}{2} = \mathbf{12.5\ cm.}$$

(*ii*) Perimeter of the quadrant $= \frac{1}{4}$ of $2\pi r + 2r = 22.5$ cm

$$\frac{11}{7}r + 2r = 22.5\ \text{cm}$$

$$r = \frac{22.5 \times 7}{25} = 6.3\ \text{cm.}$$

$$\text{Area of the quadrant} = \frac{1}{4}\pi r^2$$

$$= \frac{1}{4} \times \frac{22}{7} \times 6.3 \times 6.3$$

$$= \mathbf{31.185\ cm^2}$$

15. Radius of the sector of the circle = 11.2 cm

Perimeter of the sector = 54.4 cm

$$\therefore \quad \frac{\theta}{360} \times 2\pi r + 2r = 54.4\ \text{cm}$$

$$\Rightarrow \quad \frac{\theta}{360} \times 2 \times \frac{22}{7} \times 11.2 + 22.4 = 54.4\ \text{cm}$$

$$\Rightarrow \quad \frac{\theta}{360} \times 2 \times \frac{22}{7} \times 11.2 = 32\ \text{cm}$$

$$\Rightarrow \quad \theta = \frac{32 \times 360 \times 7}{2 \times 22 \times 11.2}$$

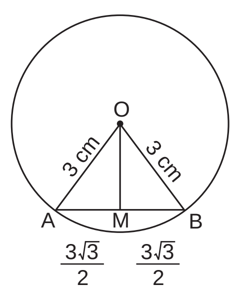
$$= 163.636° = 164°$$

$$\therefore \quad \text{Area of the sector} = \frac{164}{360} \times \frac{22}{7} \times 11.2 \times 11.2$$

$$= 179.598\ \text{cm}^2\ \text{(approx.)}$$

$$= \mathbf{179.6\ cm^2}\ \text{(approx.)}$$

16. Radius of the circle = 3 cm.



Length of the chord $= 3\sqrt{3}$ cm.

Draw OM ⊥ AB. AM = BM $= \frac{3\sqrt{3}}{2}$ cm.

$$\sin \angle AOM = \frac{3\sqrt{3}}{2} \times \frac{1}{3} = \frac{\sqrt{3}}{2}$$

$$\angle AOM = 60°,$$

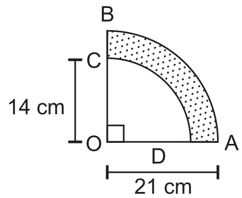
Similarly $\angle BOM = 60°$

$$\angle AOB = 120°$$

Area of the sector AOB $= \frac{120}{360} \times \pi \times 3 \times 3 \text{ cm}^2$

$= \mathbf{3\pi \text{ cm}^2}$

17. ABCD is the flower bed. OA = 21 cm



OC = 14 cm.

$\therefore \quad \frac{1}{4}\pi\, 21^2 - \frac{1}{4}\pi\, 14^2 = \frac{1}{4}\pi\,(441 - 196) \text{ cm}^2$

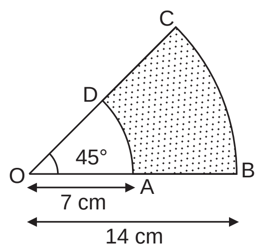
$= \mathbf{192.5 \text{ cm}^2}$

18. In the given figure,

OA = 7 m,

OB = 14 m

$\angle$BOC = 45°

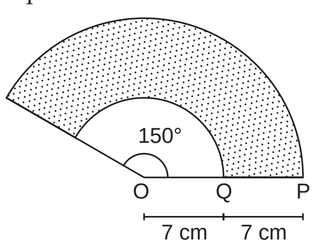


Area of the shaded portion

$= \frac{45}{360}\pi\, 14^2 - \frac{45}{360}\pi\, 7^2$

$= \frac{45}{360}\pi\,(196 - 49) \text{ m}^2 = \mathbf{57.75 \text{ m}^2.}$

19. Angle of the wiper = 150°



Again OQ = PQ = 7 cm.

Area swept clean by the wiper

$= \frac{150}{360}\pi\, 14^2 - \frac{150}{360}\pi 7^2$

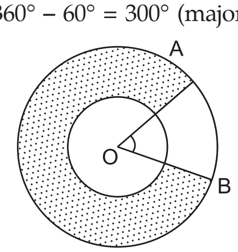
$= \frac{150}{360}\pi\,(14^2 - 7^2)$

$= \frac{150}{360}\pi\,(196 - 49) = \mathbf{192.5 \text{ cm}^2}$

20. Radii of 2 concentric circles are 21 cm and 42 cm.

Here $\angle$AOB = 60° (minor)

and $\angle$AOB = 360° – 60° = 300° (major)



Now, Area of shaded portion

$= \left[\frac{300}{360}\pi(42)^2\right] - \left[\frac{300}{360}\pi(21)^2\right]$

$= \left[\frac{5}{6} \times \frac{22}{7} \times 42 \times 42\right] - \left[\frac{5}{6} \times \frac{22}{7} \times 21 \times 21\right]$

$= \frac{5}{6} \times \frac{22}{7}\left(42^2 - 21^2\right)$

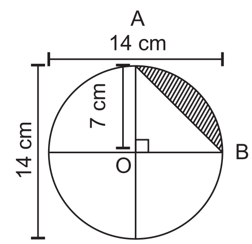
$= \frac{5}{6} \times \frac{22}{7} \times 21 \times 63$

$= \mathbf{3465 \text{ cm}^2}$

21. Area of the shaded region

= Area of the quadrant

– Area of ΔAOB



$= \frac{1}{4} \times \frac{22}{7} \times 7^2 - \frac{1}{2} \times 7 \times 7 \text{ cm}^2$

$= \frac{77}{2} - \frac{49}{2} \text{ cm}^2 = \mathbf{14 \text{ cm}^2}$

22. (*i*) Radius = 15 cm = OA = OB. Chord AB subtends an angle of 60° at the centre.

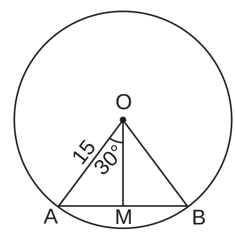
$\angle$AOB = 60°

Draw OM $\perp$ AB

AM = MB [$\because$ AOB is an isosceles Δ]

$\because \quad \Delta AOM \cong \Delta BOM$

$\angle$AOM = $\angle$BOM = 30°



$\sin 30° = \frac{AM}{AO}$

$\frac{1}{2} = \frac{AM}{15}$

$AM = \frac{15}{2} = 7.5 \text{ cm}$

$\therefore \quad AB = 15 \text{ cm}$

$\cos 30° = \frac{OM}{OA}$

$\frac{\sqrt{3}}{2} = \frac{OM}{15}$

$\frac{15\sqrt{3}}{2} = OM, \ OM = 7.5\sqrt{3}$

Area of $\Delta AOB = \frac{1}{2} \times 15 \times 7.5\sqrt{3} = 97.3125$ cm.

Area of minor segment

= Area of the minor sector – Area of ΔAOB

$= \frac{60}{360} \times 3.14 \times (15)^2 - 97.3125$

$= \mathbf{20.4375 \text{ cm}^2}$

Area of the major segment

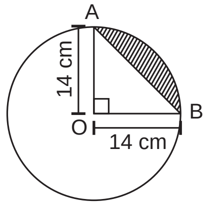
= Area of the circle – Area of minor segment.

$= 3.14 \times (15)^2 - 20.4375 = \mathbf{686.0625 \text{ cm}^2}$

(*ii*) Area of the minor sector – Area of ΔAOB

$$= \frac{90}{360} \times \frac{22}{7} \times 14 \times 14 - \frac{1}{2} \times 14 \times 14$$

$$= \mathbf{56\ cm^2}$$



Area of the major sector

$$= \frac{22}{7} \times 14 \times 14 - 56 = \mathbf{560\ cm^2}$$

(*iii*) Area of the sector OACB of a circle with centre, O and radius, r = 12 cm and the central angle, $\theta = 60°$

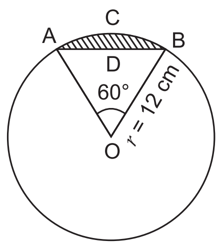
$$= \pi r^2 \cdot \frac{\theta}{360}$$

$$= 3.14 \times 12^2 \times \frac{60}{360}\ cm^2$$

$$= 3.14 \times 144 \times \frac{1}{6}\ cm^2$$

$$= 3.14 \times 24\ cm^2$$

$$= 75.36\ cm^2$$



Area of equilateral triangle OAB of side 12 cm is $\frac{\sqrt{3}}{4} \times 12^2\ cm^2 = 36\sqrt{3}\ cm^2$.

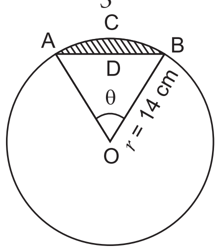
Hence, the required area of the segment ACBD (i.e., the shaded region) is $\mathbf{(75.36 - 36\sqrt{3})\ cm^2}$.

(*iv*) Area of the sector OACB of a circle with centre O and radius, r = 14 cm and the central angle, $\theta = 60°$

$$= \pi r^2 \times \frac{\theta}{360}$$

$$= \frac{22}{7} \times 14 \times 14 \times \frac{60}{360}\ cm^2$$

$$= \frac{308}{3}\ cm^2$$



Also, the area of the equilateral ΔOAB is

$$= \frac{\sqrt{3}}{4} \times 14^2\ cm^2$$

$$= \frac{\sqrt{3}}{4} \times 196\ cm^2$$

$$= 49\sqrt{3}\ cm^2$$

Hence, the required area of the minor segment of the wide is $\left(\mathbf{\frac{308}{3} - 49\sqrt{3}}\right)\ \mathbf{cm^2}$.

23. Radius of the circle = 42 cm

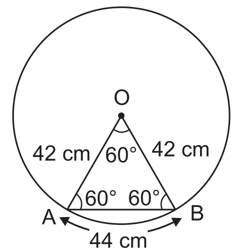
Length of the arc = 44 cm

Circumference of the circle = $2\pi r$

$$= 2 \times \frac{22}{7} \times 42 = 264\ cm$$

264 cm subtends 360° at the centre

44 cm subtends $\left(\frac{360}{264} \times 44\right)^\circ = 60°$ at the centre



Area of minor sector $= \frac{\theta}{360°} \times \pi r^2$

$$= \frac{60°}{360°} \times \frac{22}{7} \times 42 \times 42$$

$$= 924\ cm^2$$

Now the arc subtends 60° at the centre

OA = OB

$\therefore$ $\angle A = \angle B = 60°$

$\therefore$ ΔOAB is an equilateral triangle.

Area of $\Delta OAB = \frac{\sqrt{3}}{4} a^2$

$$= \frac{\sqrt{3}}{4} \times 42 \times 42$$

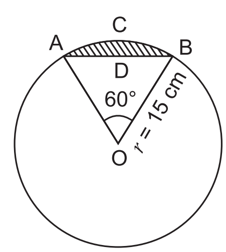
$$= 441\sqrt{3}\ cm^2$$

Area of minor segment = Area of sector – Area of ΔOAB

$$= (924 - 441\sqrt{3})\ cm^2$$

$$= \mathbf{21\left(44 - 21\sqrt{3}\right)\ cm^2}$$

24. Let r cm be the radius of the circle with centre at O, so that r = 15. The chord AB subtends an angle, $\theta = 60°$ at the centre O.



$\therefore$ Area of the sector OACB of the circle

$$= \pi r^2 \times \frac{\theta}{360}$$

$$= 3.14 \times 15^2 \times \frac{60°}{360°} \text{ cm}^2$$

$$= 3.14 \times 225 \times \frac{1}{6} \text{ cm}^2$$

$$= 117.75 \text{ cm}^2$$

Now, area of the equilateral ΔAOB

$$= \frac{\sqrt{3}}{4} \times 15 \times 15 \text{ cm}^2$$

$$= \frac{225 \times 1.73}{4} \text{ cm}^2$$

$$= 97.32 \text{ cm}^2$$

$\therefore$ Required area of the minor segment of the circle, ACBD (the shaded region)

$$= (117.75 - 97.32) \text{ cm}^2$$

$$= 20.43 \text{ cm}^2 \quad \ldots(1)$$

Now, area of the whole circle

$$= \pi r^2$$

$$= 3.14 \times 225 \text{ cm}^2$$

$$= 706.50 \text{ cm}^2 \quad \ldots(2)$$

$\therefore$ Required area of the major circle = Area of the whole circle – Area of the minor circle

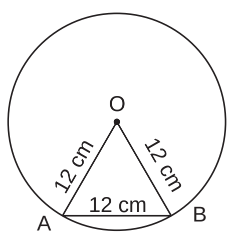
$$= (706.50 - 20.43) \text{ cm}^2 \quad \text{[From (1) and (2)]}$$

$$= \mathbf{686.07\ cm^2}.$$

25. Radius of the circle = 12 cm.

Chord AB = 12 cm.

$\because$ ΔAOB is an equilateral triangle, therefore, all angles of ΔAOB is 60°.



Area of Minor segment = Area of Minor sector – Area of ΔAOB

$$\left[\text{Area of an equilateral triangle} = \frac{\sqrt{3}}{4} \times (\text{side})^2\right]$$

$$= \frac{60}{360} \times 3.14 \times 144 - \frac{\sqrt{3}}{4} \times 144$$

$$= \mathbf{13.08\ cm^2\ (approx.)}$$

Length of the minor arc AB

$$= \frac{60}{360} \times 2 \times 3.14 \times 12$$

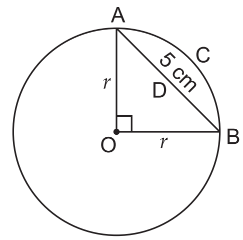
$$= \mathbf{12.56\ cm\ (approx.)}$$

$\therefore$ Major arc $= 2\pi r - 12.56$

$$= 75.36 - 12.56$$

$$= \mathbf{62.8\ cm^2\ (approx.)}$$

26. Let AB be the chord of length 5 cm, of a circle with centre at O such that the chord AB subtends an angle of 90° at O.



To find the difference of the area of the major segment and the minor segment of the circle.

Let r cm be the radius of the circle.

Then, from ΔAOB, we have by Pythagoras' theorem,

$$OA^2 + OB^2 = AB^2$$

$$\Rightarrow \quad r^2 + r^2 = 5^2$$

$$\therefore \quad r^2 = \frac{25}{2} \quad \ldots(1)$$

$\therefore$ Area of the whole circle $= \pi r^2 = \frac{25\pi}{2}$ cm² [From (1)] …(A)

Also area of $\Delta AOB = \frac{1}{2} r \times r \text{ cm}^2 = \frac{r^2}{2} \text{ cm} = \frac{25}{4} \text{ cm}$ [From (1)]…(2)

Now, area of the sector AOBC

$$= \pi r^2 \times \frac{90}{360} \text{ cm}^2$$

$$= \frac{25\pi}{2 \times 4} \text{ cm}^2$$

$$= \frac{25\pi}{8} \text{ cm}^2 \quad \ldots(3)$$

$\therefore$ Area of the minor segment of the circle ACBD

$$= \left(\frac{25\pi}{8} - \frac{25}{4}\right) \text{cm}^2 \quad \text{[From (2) and (3)]} \ldots(4)$$

$\therefore$ Area of the major segment of the circle

= Area of the whole circle – Area of the minor segment of the circle

$$= \left(\frac{25\pi}{2} - \frac{25\pi}{8} + \frac{25}{4}\right) \text{cm}^2 \quad \text{[From (A) and (4)]}$$

$$= \left(\frac{75\pi}{8} + \frac{25}{4}\right) \text{cm}^2 \quad \ldots(5)$$

$\therefore$ Required difference of the areas of major and minor segments of the circle

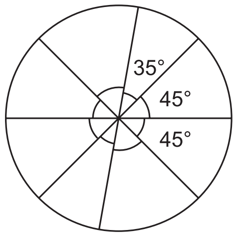
$$= \left(\frac{75\pi}{8} + \frac{25}{4} - \frac{25\pi}{8} + \frac{25}{4}\right) \text{cm}^2 \quad \text{[From (4) and (5)]}$$

$$= \left(\frac{\mathbf{25\pi}}{\mathbf{4}} + \frac{\mathbf{25}}{\mathbf{2}}\right)\mathbf{cm^2}$$

$$= \frac{\mathbf{25}}{\mathbf{4}}(\pi + 2)\ \mathbf{cm^2}$$

For Standard Level

27. Let r = 35 cm be the radius of the circle. If the circle is divided in 8 equal parts by 4 diameters, equally inclined to each other, then the entire circle will be divided equally by 8 arcs, each substending an angle $\frac{360°}{8}$ = 45° at the centre.



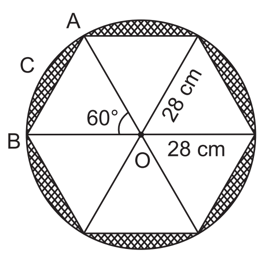
$\therefore$ Required area between two consecutive ribs of the umbrella = The area of each small sector of the circle with central angle 45° at the centre

$$= \pi r^2 \frac{45°}{360°}$$

$$= \frac{22}{7} \times 35 \times 35 \times \frac{1}{8} \text{ cm}^2$$

$$= \frac{1925}{4} \text{ cm}^2$$

28. Each of 6 circular design is the segment of a circle with central angle $\frac{360°}{6}$ = 60°.



The areas of all these 6 designs are equal. Let ACBA is one such circular segment which subtends an angle 60° at the centre O of the circle radius r, of the circle is 28 cm.

$\therefore$ Area of the segment ACBA of the circle

= Area of the AOBCA

– area of equilateral triangle AOB of side 28 cm.

$$= \left(\pi r^2 \cdot \frac{60}{360} - \frac{\sqrt{3}}{4} \times 28 \times 28\right) \text{ cm}^2$$

$$= \left(\frac{22}{7} \times 28 \times 28 \times \frac{1}{6} - \sqrt{3} \times 196\right) \text{ cm}^2$$

$$= \left(\frac{1232}{3} - 1.7 \times 196\right) \text{cm}^2$$

$$= \left(\frac{1232}{3} - \frac{17 \times 196}{10}\right)^2$$

$$= \left(\frac{1232}{3} - \frac{1666}{5}\right) \text{ cm}^2$$

$$= \frac{6160 - 4998}{15} \text{ cm}^2$$

$$= \frac{1162}{15} \text{ cm}^2.$$

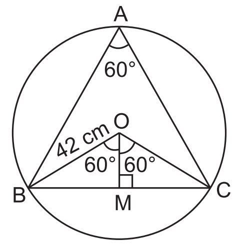
$\therefore$ Area of 6 such sector each of the same area

$$= \frac{1162}{15} \times 6 \text{ cm}^2$$

$$= \frac{2324}{5} \text{ cm}^2$$

$\therefore$ Required cost = ₹ $\frac{35}{100} \times \frac{2324}{5}$ = ₹ **162.68**

29. Let O be the centre of the circle of radius r = OB = 42 cm.



Then, $\angle BOC = 2\angle BAC$ [Angle at the centre]

$= 2 \times 60° = 120°$

We now draw OM $\perp$ BC.

Then M will be the middle point of BC.

Now, in ΔOBM,

$$BM = OB \sin 60°$$

$$= 42 \times \frac{\sqrt{3}}{2}$$

$$= 21\sqrt{3}$$

$\therefore$ BC = 2BM = $42\sqrt{3}$

Hence, the length of each side of the equilateral triangle ABC = $21\sqrt{3}$ cm.

$\therefore$ Area of $\Delta ABC = \frac{\sqrt{3}}{4} \times 42\sqrt{3} \times 42\sqrt{3}$ cm²

$= 1323\sqrt{3}$ cm²

= 2288.79 cm …(1)

Area of the whole circle

$= 2\pi r^2$

$= \frac{22}{7} \times 42 \times 42 \text{ cm}^2$

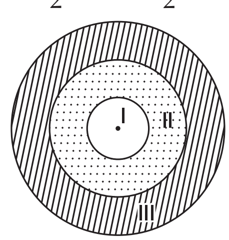
= 5544 cm² …(2)

$\therefore$ Required area of the design

= (5544 – 2288.79) cm² [(2) – (1)]

= **3255.21 cm**

30. We mark the three concentric circular regions by I, II and III. Let the radii of the three concentric circles be $\frac{d}{2}$, $\frac{2d}{2}$ and $\frac{3d}{2}$ units, i.e. $\frac{d}{2}$, d and $\frac{3d}{2}$ units.



Then the area of the smallest circle I is $\frac{\pi d^2}{4}$, the area of the next circular region II is $\pi d^2 - \frac{\pi d^2}{4} = \frac{3\pi d^2}{4}$ and the area of the last circular region III is $\pi\left(\frac{3d}{2}\right)^2 - \pi d^2 = \frac{9\pi d^2}{4} - \pi d^2 = \frac{5\pi d^2}{4}$.

Hence, the required ratio of the areas of three regions are

$\frac{\pi d^2}{4} : \frac{3\pi d^2}{4} : \frac{5\pi d^2}{4}$ = **1 : 3 : 5**.

EXERCISE 14C

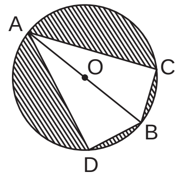
For Basic and Standard Levels

1. In the figure BC = BD = 8 cm.

(*i*) AC = AD = 15 cm.

$AB^2 = AC^2 + BC^2 = 225 + 64$

$\angle D = 90°$, $AB^2 = 289$ cm (Angle of the diameter)



Similarly, $\angle C = 90°$

$\because$ $r = 8.5$ cm.

Area of the shaded region = Area of the circle – Area of 2 triangles

$= \frac{22}{7} \times (8.5)^2 - 2 \times \frac{1}{2} \times 15 \times 8$

$= \mathbf{\frac{1499}{14}}$ **cm²**

$= 106.865\ cm^2 \approx 107\ cm^2$

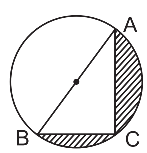
(*ii*) AC = 12 cm and AB = 13 cm

Now $\angle ACB = 90°$ [Angle in a semicircle]

$\therefore$ $AB^2 = AC^2 + BC^2$ [Pythagoras' theorem]

$\Rightarrow$ $BC^2 = 169 - 144 = 25$

$\Rightarrow$ BC = 5 cm



$\therefore$ Area of shaded portion

$= \frac{1}{2} \times$ Area of circle – Area of ΔABC

$= \frac{1}{2} \times \pi \times \left(\frac{13}{2}\right)^2 - \frac{1}{2} \times 12 \times 5$

$= 66.3325 - 30$

= **36.3325** cm^2

2. Radius of the circle = 7 cm

Area of the semicircle $= \frac{1}{2} \times \frac{22}{7} \times 7 \times 7$ = **77 cm²**

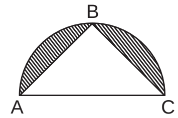
Perimeter of the semicircle $= \frac{1}{2} \times 2 \times \frac{22}{7} \times 7 + 14$ = **36 cm**

$\because$ ΔABC is an isosceles right triangle

$14^2 = AB^2 + BC^2$

$196 = 2AB^2$

$\therefore$ $AB = 7\sqrt{2}$ cm.



Area of the shaded region = Area of the semicircle – Area of the triangle

$= 77 - \frac{1}{2} \times 7\sqrt{2} \times 7\sqrt{2}$ = **28 cm²**

3. FDE is a quadrant. ABCD is a rectangle.

BC = 6 cm, CE = 2 cm

Join BD.

Then BD = DE = r

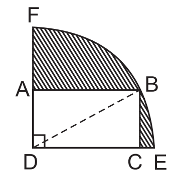
and DC = DE – CE = $r - 2$

In rt. ΔBCD we have,

$BD^2 = BC^2 + DC^2$

$r^2 = 6^2 + (r - 2)^2$

$r = 10$ cm.



Area of the shaded region

$= \left[\frac{1}{4} \times 3.14 \times 10 \times 10 - 6 \times (10 - 2)\right] cm^2$

= **30.5 cm²**

4. AOB is a quadrant

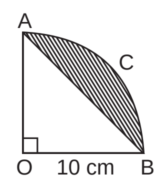
OB = 10 cm.

Area of the shaded portion

= Quadrant AOB – ΔAOB

$= \frac{1}{4} \times \pi \times 100 - \frac{1}{2} \times 10 \times 10$

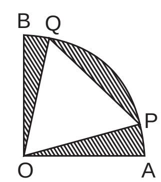
$= \mathbf{\frac{200}{7}}$ **cm²**



5. AOB is a quadrant.

radius of the circle = 14 cm = OA = OP = OQ

ΔOPQ is an equilateral triangle.



Area of the shaded part = Area of the quadrant – the equilateral triangle.

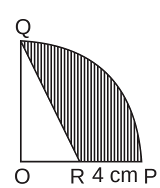
$= \frac{1}{4} \times \pi \times 14 \times 14 - \frac{\sqrt{3}}{4} \times 14 \times 14$

= **69.132 cm²**

6. OPQ is a quadrant.

Radius = OQ = OP = 7 cm

and PR = 4 cm.



(*i*) Area of $\Delta OQR = \frac{1}{2} \times 7 \times 3$

= **10.5 cm²**

(*ii*) Area of the quadrant $= \frac{1}{4} \times \frac{22}{7} \times 7 \times 7 = \frac{77}{2}\ cm^2$

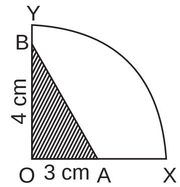
= **38.5 cm²**

(*iii*) Area of the shaded part = Area of the quadrant – ΔOQR

$= 38.5 - 10.5 = \mathbf{28\ cm^2}$

7. YOX is a quadrant of radius 7 cm.

$\angle AOB$ is right-angled triangle.



$OB = 4$ cm,

$OA = 3$ cm

$$\text{Area of } \Delta AOB = \frac{1}{2} \times 4 \times 3$$

$$= 6 \text{ cm}^2$$

Area of the quadrant

$$\frac{1}{4} \times \frac{22}{7} \times 7 \times 7 = 38.5 \text{ cm}^2$$

Area of the remaining part

$$38.5 - 6 = \mathbf{32.5\ cm^2}$$

In ΔAOB, $AB^2 = OB^2 + OA^2$

$= 16 + 9 = 25$

$AB = 5$ cm.

$$\text{Length of arc XY} = \frac{1}{4} \times 2 \times \frac{22}{7} \times 7$$

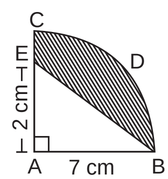
$$= 11 \text{ cm}$$

Perimeter of the unshaded part

= Length of arc XY + XA + AB + BY

= 11 cm + 4 cm + 5 cm + 3 cm = **23 cm.**

8. ABDCA is a quadrant of a circle of radius = 7 cm.



Area of the shaded portion

= Area of the quadrant – Area of ΔABE

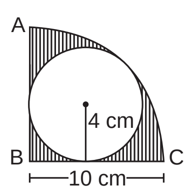
$$= \frac{1}{4} \times \frac{22}{7} \times 7 \times 7 - \frac{1}{2} \times 2 \times 7$$

$= 38.5 - 7$

$= \mathbf{31.5\ cm^2}$

9. Radius of the circle = 4 cm.

Radius of the quadrant = 10 cm.



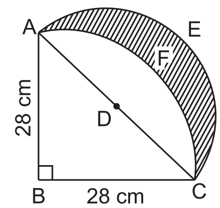
Area of the shaded region

= Area of the quadrant – Area of circle

$$= \frac{1}{4} \times \frac{22}{7} \times 10^2 - \frac{22}{7} \times 4^2$$

$$= \mathbf{\frac{198}{7}\ cm^2}$$

10. In the given figure, B is the centre of the quadrant of the circle, ABCFA where $\angle ABC = 90°$, and the radius, *r* of this circle is 28 cm.



$\therefore$ Area of this quadrant of a circle

$$= \frac{1}{4}\pi r^2$$

$$= \frac{1}{4} \times \frac{22}{7} \times 28 \times 28 \text{ cm}^2$$

$= 616 \text{ cm}^2$ …(1)

$$\text{Area of } \Delta ABC = \frac{1}{2} \times 28 \times 28 \text{ cm}^2$$

$= 392 \text{ cm}^2$ …(2)

Now, from ΔABC, by Pythagoras' theorem, we have

$$AC = \sqrt{28^2 + 28^2} \text{ cm}$$

$$= 28\sqrt{2} \text{ cm}$$

$\therefore$ If D is the mid-point of AC, then

$$AD = \frac{1}{2} \times 28\sqrt{2} \text{ cm}$$

$$= 14\sqrt{2} \text{ cm}$$

$\therefore$ Radius of the semicircle with AD as the radius is $14\sqrt{2}$ cm.

$\therefore$ Area of the semicircle AECA

$= \pi \times AD^2$

$$= \frac{22}{7} \times 14 \times 14 \times 2 \text{ cm}^2$$

$= 616 \text{ cm}^2$ …(3)

$\therefore$ Area of the quadrant of the circle with centre at B – Area of ΔABC = Area of the segment ACFA of this quadrant

$\Rightarrow$ Area of the segment ACFA of the quadrant of the circle

$= (616 - 392) \text{ cm}^2$

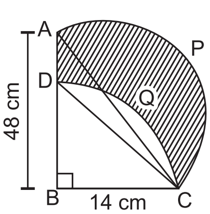
$= 224 \text{ cm}^2$ [From (1) and (2)] …(4)

$\therefore$ Required area of the shaded region

$= (616 - 224) \text{ cm}^2$ [From (3) and (4)]

$= \mathbf{392\ cm^2}$

11. Given that ΔABC is a right-angled triangle with $\angle ABC = 90°$, BC = 14 cm and AB = 48 cm. With AC as diameter, a semicircle APCA is drawn and with BC as radius, a quadrant of a circle is drawn. To find the area of the shaded region.



From ΔABC, we have by Pythagoras' theorem,

$$AC = \sqrt{48^2 + 14^2} \text{ cm}$$
$$= \sqrt{2500} \text{ cm}$$
$$= 50 \text{ cm.}$$

$\therefore$ Radius of the semicircle APCA is 25 cm.

$\therefore$ Area of the semicircle APCA

$$= \frac{\pi}{2} \times 25 \times 25 \text{ cm}^2 \quad \ldots(1)$$

$$\text{Area of } \Delta ABC = \frac{1}{2} \times 14 \times 48 \text{ cm}^2$$
$$= 336 \text{ cm}^2 \quad \ldots(2)$$

Area of the quadrant of the circle BCQD

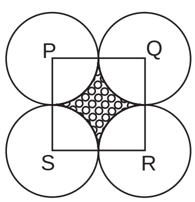
$$= \frac{1}{4} \pi \times 14 \times 14 \text{ cm}^2$$
$$= 49\pi \text{ cm}^2 \quad \ldots(3)$$

Hence, the required area of the shaded region

= Area of the semicircle APCA + Area of ΔABC – Area of the quadrant BCQD

$$= \left(\frac{\pi}{2} \times 625 + 336 - 49\pi\right) \text{ cm}^2 \quad \text{[From (1), (2) and (3)]}$$
$$= \left(\frac{22 \times 625}{7 \times 2} + 336 + 49 \times \frac{22}{7}\right) \text{ cm}^2$$
$$= \left(\frac{6875}{7} + 336 - 154\right) \text{ cm}^2$$
$$= (982.14 + 182) \text{ cm}^2$$
$$= \mathbf{1164.14 \text{ cm}^2}$$

12. PQRS is a square, joining the centres of 4 circles.



Let the radius of each circle = r cm

$\therefore$ Side of the square = 2 × radius = $2r$

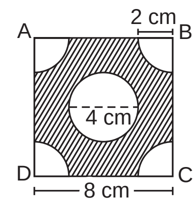
Area of square = $(\text{side})^2 = (4r^2)$ cm²

$$\text{Area of 4 quarters of circles} = 4\left[\frac{\pi r^2}{4}\right]$$
$$= \pi \times r^2$$
$$= \pi r^2$$

Shaded area = Area of square – Area of quarter circles

$$4r^2 - \pi r^2 = \frac{24}{7}$$
$$r^2(4 - \pi) = \frac{24}{7}$$
$$r^2 \times \left(4 - \frac{22}{7}\right) = \frac{24}{7}$$
$$r^2 \times \frac{6}{7} = \frac{24}{7}$$
$$r^2 = \frac{24}{7} \times \frac{7}{6}$$
$$r^2 = 4$$
$$\mathbf{r = 2 \text{ cm}}$$

13. Each side of the square ABCD = 8 cm.



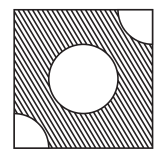
Radius of each quadrant = 2 cm.

Area of the shaded portion

= Area of the square – Area of 4 quadrants

$$= 64 - \frac{22}{7} \times 2 \times 2 = \mathbf{\frac{272}{7} \text{ cm}^2}$$

14. Each side of the square = 8 cm.



Radius of each quadrant = 1.4 cm

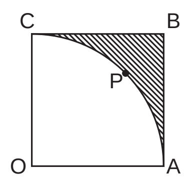
Radius of the inner circle = 4.2 cm

Area of the shaded portion

= Area of the square – Area of 2 quadrants – Area of the circle.

$$= 64 - 2 \times \frac{1}{4} \times \frac{22}{7} \times (1.4)^2 - \frac{22}{7} \times (4.2)^2$$
$$= \mathbf{5.48 \text{ cm}^2} \text{ (approx.)}$$

15. OABC is a square of side 7 cm.

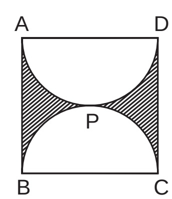


OAPC is a quadrant with radius = 7 cm.

Area of the shaded region = Square – Quadrant.

$$49 - \frac{1}{4} \times \frac{22}{7} \times 7 \times 7 = 49 - 38.5 = \mathbf{10.5 \text{ cm}^2}$$

16. ABCD is a square of side = 28 m



Area of the square = 28 × 28

= 784 cm²

Diameter of the square = 28 cm

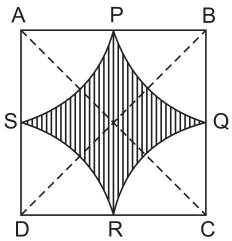
Radius of the circle = 14 cm

$$\text{Area of 2 semicircles} = 2 \times \frac{1}{2} \times \frac{22}{7} \times 14 \times 14$$
$$= 616 \text{ cm}^2$$

Area of the shaded region = 784 – 616

= **168 cm²**

17. P, Q, R, S are the mid-points of the sides AB, BC, CD and DA respectively of a square ABCD of side 12 cm.



PSA, PQB, QRC and SRD are 4 quadrants of four circles with centres at A, B, C and D and each with radius 6 cm.

$\therefore$ Sum of the areas of these 4 quadrants

$$= 4 \times \frac{1}{4}\pi \times 6^2 \text{ cm}^2$$

$$= 36\pi \text{ cm}^2$$

$$= 36 \times 3.14 \text{ cm}^2$$

$$= 113.04 \text{ cm}^2$$

Now, the area of the square

$$= 12 \times 12 \text{ cm}^2$$

$$= 144 \text{ cm}^2$$

Hence, the required area of the shaded region = area of the square – sum of the areas of 4 quadrants.

$$= (144 - 113.04) \text{ cm}^2$$

$$= \mathbf{30.96 \text{ cm}^2}$$

18. (*i*) Perimeter of the circular plot = 660 m

$\therefore$ $2\pi r = 660$ m

(r = radius of the circular plot)

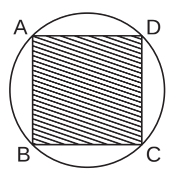
$$r = \frac{660 \times 7}{2 \times 22}$$

$$= 105 \text{ m}$$

diameter of the circle = 210 m

$\therefore$ diagonal of the square = 210 m

Side of the square = a



$\therefore$ $a^2 + a^2 = (210 \text{ m})^2$

$$2a^2 = 44100 \text{ m}^2$$

$$a^2 = \frac{44100}{2} \text{ m}^2$$

Area of the square = a^2 = **22050 m^2**

(*ii*) Side of the square inscribed in the circle = 4 cm.

Area of the square = 16 cm^2

$\therefore$ $4^2 + 4^2 = BD^2$

$$32 = BD^2$$

$$BD = 4\sqrt{2} \text{ cm}$$

BD is also the diameter of the circle.

$\therefore$ Radius = $2\sqrt{2}$ cm

$$\text{Area of the circle} = \frac{22}{7} \times (2\sqrt{2})^2$$

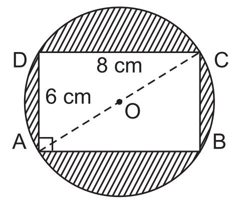
$$= \frac{176}{7} \text{ cm}^2$$

Area enclosed between circle and square

$$= \frac{176}{7} - 16$$

$$= \mathbf{\frac{64}{7} \text{ cm}^2}$$

19. ABCD is a rectangle of length and breadth 8 cm and 6 cm respectively. This rectangle is inscribed in a circle of centre O which is the point of intersection of the two diagonals of the rectangle.



The length of the diagonal AC of the rectangle

$$= \sqrt{8^2 + 6^2} \text{ cm}$$

$$= 10 \text{ cm}$$

$\therefore$ The radius of the circle = $\frac{10}{2}$ cm = 5 cm

To find the area of the shaded region.

Required area of the shaded region

= Area of the circle – Area of the rectangle

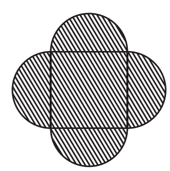
$$= (\pi \times 5^2 - 6 \times 8) \text{ cm}^2$$

$$= (3.14 \times 25 - 48) \text{ cm}^2$$

$$= (78.50 - 48) \text{ cm}^2$$

$$= \mathbf{30.5 \text{ cm}^2}$$

20. Side of the square (in the centre) = 21 m.



Area of the square = 441 m^2

$\therefore$ Diameter of the circles = 21 m

Radius = 10.5 m

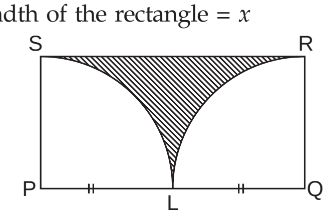
$$\text{Area of 4 half circles} = 4 \times \frac{1}{2} \times \frac{22}{7} \times (10.5)^2 = 693 \text{ m}^2$$

Total area = 441 + 693

= 1134 m^2 each plant needs 6 m^2 of space.

$\therefore$ Number of plants = $\frac{1134}{6}$ = **189**.

21. Let the breadth of the rectangle = x



Areas Related to Circles

$\therefore$ Length = $2x$

Area of the rectangle PQRS = $2x^2$.

L is the mid-point of PQ

$\therefore$ Radius of each quadrant = x

Area of 2 quadrants = $2 \times \frac{1}{4} \times \pi \times x^2 = \frac{11}{7}x^2$

Area of the shaded region = $2x^2 - \frac{11}{7}x^2 = \frac{3}{7}x^2$

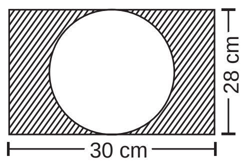
Ratio of rectangle : shaded region

$= 2x^2 \times \frac{7}{3x^2}$

$= \frac{14}{3}$ = **14 : 3**

22. Area of the rectangle 30 cm long and 28 cm wide

$= 30 \times 28 = 840 \text{ cm}^2$



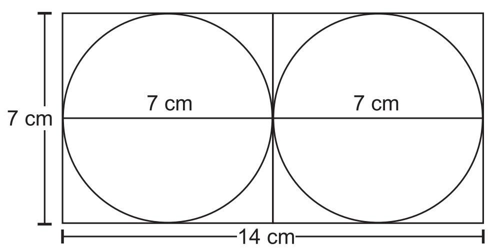
Radius of the circle = 14 cm

Area of the circle = $\frac{22}{7} \times 196 \text{ cm}^2$ = **616 cm²**

Area of the paper left after cutting

the circle = 840 – 616 = **224 cm².**

23. Area of rectangle = $14 \times 7 \text{ cm}^2$.



Area of two circles

= 2 × (Area of 1 circle with diameter 7 cm)

$= 2 \times \pi \times \left(\frac{7}{2}\right)^2$

$= \frac{22}{7} \times 2 \times \frac{7}{2} \times \frac{7}{2} = 77 \text{ cm}^2$

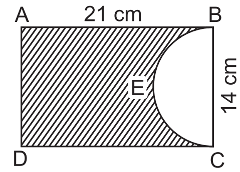
Now area of primary cardboard

= Area of rectangular cardboard – Area of two circles

$= 98 \text{ cm}^2 - 77 \text{ cm}^2$

= **21 cm²**

24. ABCD is a rectangle of length AB = 21 cm and breadth BC = 14 cm. BDC is a semicircle with BC as a diameter. To find the area and the perimeter of the shaded region.



Required area of the shaded region

= Area of the rectangle

– Area of the semicircle with BC as a diameter

$= 14 \times 21 \text{ cm}^2 - \frac{\pi}{2} \times 7^2 \text{ cm}^2$

$= 294 \text{ cm}^2 - \frac{22}{7} \times \frac{7 \times 7}{2} \text{ cm}^2$

$= 294 \text{ cm}^2 - 77 \text{ cm}^2$

$= 217 \text{ cm}^2$

Semi-perimeter of the circle with BC as a diameter

= Length of the semicircle BDC

$= \frac{1}{2}\pi \times r$

$= \frac{22}{7} \times 7 \text{ cm}$

= 22 cm

$\therefore$ Required perimeter of the shaded region

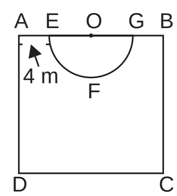
= AB + AD + CD + BDC

= (21 + 14 + 21 + 22) cm

= **78 cm**

25. Area of the square ABCD = 576 m^2

AE = 4 m



Side of the square = $\sqrt{576}$ = 24 m.

O is the mid-point of AB = CD.

AO = $\frac{1}{2}$AB = 12 m

$\therefore$ OE = 12 – 4

OE = Radius = 8 m.

Area for the circular pillar = $\frac{1}{2} \times \frac{22}{7} \times 64$

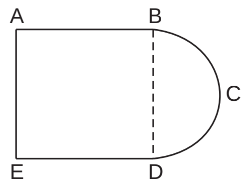
$= \frac{22 \times 32}{7} \text{ m}^2$

Area for the carpet = $576 - \frac{22 \times 32}{7}$

$= 475.429 \text{ cm}^2$

$= \mathbf{\frac{3328}{7}\ m^2}$

26. Width of the hall = AE = 3.5 m.



Length of hall with balcony = 11.75 m

Diameter of semicircular balcony = 3.5 m

$\therefore$ Radius = 1.75 m

Length of the hall (without balcony)

= 11.75 – 1.75 = 10 m.

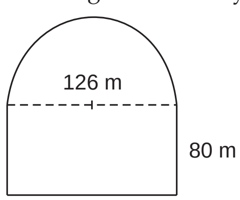
Area of the rectangle = 3.5 × 10 = 35 m^2

Area of the semicircle = $\frac{1}{2} \times \frac{22}{7} \times (1.75)^2$ m^2

= 4.8125 m^2

Total area = 35 + 4.8125 = **39.8125 m^2.**

27. Dimensions of the rectangle = 126 m by 80 m.



Diameter of the semicircle = 126 m.

Radius = 63 m.

$\therefore$ Total area of rectangle and half circle

= 126 × 80 + $\frac{1}{2} \times \frac{22}{7} \times (63)^2$

= (10080 + 6237) m^2 = **16317 m^2**

Perimeter of the total area = 80 + 80 + 126 + $\frac{1}{2} \times 2 \times \frac{22}{7} \times 63$

= **484 m**.

28. Perimeter of the figure = 400 m

Perimeter of the semicircular region = 400 – 260 = 140 m.

$\therefore$ $2\pi r = 140 = \frac{140 \times 7}{2 \times 22} = \frac{35 \times 7}{11}$

$\therefore$ Area = $\pi r^2 + 130 \times 2r$

$= \frac{22}{7} \times \frac{35 \times 7}{11} \times \frac{35 \times 7}{11} + 130 \times 2 \times \frac{35 \times 7}{11}$

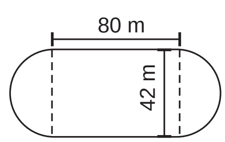
$= \frac{(70 \times 35 \times 7) + (130 \times 70 \times 7)}{11}$

$= \frac{70 \times 35 \times 33}{11}$ = **7350 m^2**

29. Width of rectangle

= diameter of semicircle

= 42 m



Radius of circle = 21 cm

Area of the given figure

= $l \times b + 2 \times \frac{1}{2}\pi r^2$

= 80 m × 42 m + $\frac{22}{7}$ × 21 m × 21 m

= 3360 m^2 + 1386 m^2 = **4746 m^2**

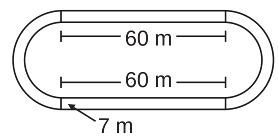
30. Inside perimeter of the track

$340 = 60 + 60 + 2 \times \frac{1}{2} \times 2 \times \frac{22}{7} \times r$

$340 - 120 = 2 \times \frac{22}{7} \times r$

$\frac{220 \times 7}{2 \times 22} = r$

r = 35 m, width of the track = 7 m.



Radius of the outer boundary = 35 + 7 = 42 m.

Outer perimeter of the track

$= 60 + 60 + 2 \times \frac{1}{2} \times 2\pi r$

$= 120 + 2 \times \frac{22}{7} \times 4^2$ = 120 + 264 = **384 m**

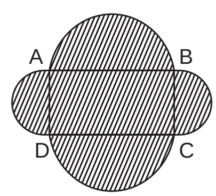
Area of the track

= Area of the figure with track
– Area of the figure without track

$= \left[60 \times 84 + \frac{22}{7} \times 42 \times 42\right] - \left[60 \times 70 + 2 \times \frac{22}{7} \times 35 \times 35\right]$

= (10584 – 8050)m^2 = **2534 m^2.**

31. (*i*) In the figure ABCD is a rectangle



AB = 7 cm, BC = $\frac{7}{2}$ cm

Radius of the circles on the longer side = $\frac{7}{2}$ cm.

Radius of the circles on the shorter side = $\frac{7}{4}$ cm.

Perimeter of the figure = $2\pi\frac{7}{2}$ cm $+ 2\pi\frac{7}{4}$ cm

$= 2\pi\left(\frac{7}{2} + \frac{7}{4}\right)$ cm = **33 cm**

(*ii*) Area of the figure = Area of the rectangle
+ Area of 4 half circles

$$= 7 \times \frac{7}{2} + \frac{22}{7} \times \left(\frac{7}{2}\right)^2 + \frac{22}{7}\left(\frac{7}{4}\right)^2$$

$= \mathbf{72.625\ cm^2}$ (approx.)

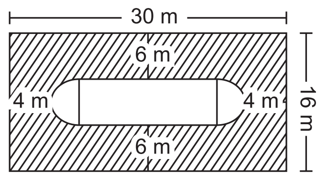
32. Area of the rectangle = 30 × 16 = 480 cm^2

Diameter of the semicircular portion = (16 – 6 – 6) = 4m

Let r = radius of the semicircular portion = 2 m

Area of two semicircular portion

$$= 2 \times \frac{\pi r^2}{2} = \pi r^2 = \pi\ (2\ m)^2 = 4\pi\ m^2$$



Length of the rectangular portion

= 30 – 2 × 4 – 2r

= (30 – 8 – 2 × 2) m

= 18 m

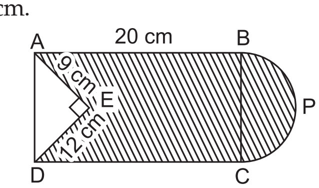
Width of the rectangular portion = 4 m.

Area of rectangular portion = 18 m × 4 m = 72 m^2

Area of the shaded region

= 480 – 72 – 4π m^2 = **(408 – 4π) m^2**.

33. ABCD is a rectangle with AB = 20 cm and ΔAED is a right-angled triangle with ∠AED = 90°, AE = 9 cm and DE = 12 cm.



BPC is a semicircle with BC as a diameter.

To find the area of the shaded region.

Now, from ΔAED, by Pythagoras' theorem, we have

$$AD = \sqrt{AE^2 + ED^2}$$

$$= \sqrt{9^2 + 12^2}\ cm$$

= 15 cm

∴ BC = AD = 15 cm

∴ Area of the rectangle ABCD

= 20 × 15 cm^2 = 300 cm^2 …(1)

$$\text{Area of } \Delta ADE = \frac{1}{2} \times 12 \times 9\ cm^2 = 54\ cm^2 \quad \ldots(2)$$

Area of the semicircle with BC as a diameter

$$= \frac{1}{2} \times 3.14 \times \left(\frac{15}{2}\right)^2 cm^2$$

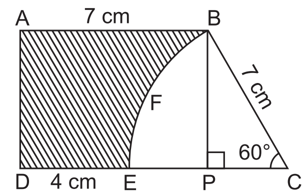
= 88.31 cm^2 (approx.) …(3)

∴ Required area of the shaded region = Area of the rectangle + Area of the semicircle – Area of the ΔADE

= (300 + 88.31 – 54) cm^2

= **334.31 cm^2**

34. ABCD is a trapezium with AB ∥ CD and AB = BC = 7 cm. BFE is an arc of a circle with centre at C and with a central angle ∠BCE = 60°.



We draw BP ⊥ DC. Then from ΔBPC, we have

BP = BC sin 60°

= 7 sin 60°

$$= \frac{7\sqrt{3}}{2} \quad \ldots(1)$$

Also, DE = 4 cm and EC = CB = 7 cm

∴ DC = DE + EC

= (4 + 7) cm

= 11 cm …(2)

Now, the area of the whole trapezium ABCD

$$= \frac{1}{2}(\text{Sum of two parallel sides}) \times \text{Distance between them}$$

$$= \frac{1}{2}(AB + DC) \times BP$$

$$= \frac{1}{2}(7 + 11) \times \frac{7\sqrt{3}}{2} \quad \text{[From (1) and (2)]}$$

$$= \frac{63\sqrt{3}}{2}\ cm^2$$

$$= \frac{63 \times 1.73}{2}\ cm^2$$

$$= \frac{108.09}{2}\ cm^2$$

= 54.50 cm^2 …(3)

Also, area of the sector CEFB

$$= \pi \times 7^2 \times \frac{60}{360}\ cm^2$$

$$= \frac{22}{7} \times 7 \times 7 \times \frac{1}{6}\ cm^2$$

$$= \frac{77}{3}\ cm^2$$

= 25.67 cm^2 (approx.) …(4)

Hence, the required area of the shaded region

= Area of the trapezium ABCD – Area of the sector CEFB

= (54.50 – 25.67) cm^2 [From (3) and (4)]

= **28.83 cm^2 (approx.)**

35. Area of trapezium = 24.5 sq cm

AD = 10 cm

BC = 4 cm

Area of trapezium = 24.5

$$\frac{1}{2}(AD + BC) \times AB = 24.5$$

$$\frac{1}{2} \times 14 \times AB = 24.5$$

$$AB = 3.5 \text{ cm}$$

Radius of quadrant = 3.5 cm

Area of shaded region = Area of trapezium – Area of quadrant

$$= 24.5 - \frac{\pi r^2}{4}$$

$$= 24.5 - \frac{\pi}{4} \times (3.5)^2$$

$$= 24.5 - \frac{12.25\pi}{4}$$

$$= 24.5 - 9.625$$

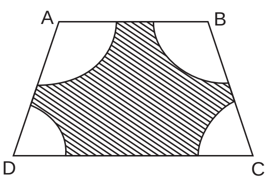
$$= \mathbf{14.875\ cm^2}$$

36. AB = 18 cm

CD = 32 cm

Distance between AB and CD = 14 cm

Radius of arcs = 7 cm



Area of trapezium

$$= \frac{1}{2}(AB + CD) \times h$$

$$= \frac{1}{2} \times (18 + 32) \times 14$$

$$= 50 \times 7$$

$$= 350 \text{ cm}^2$$

Since ∠A and ∠D are supplementary, therefore the arcs with centre A and D for a semicircle together. Similarly arcs with centre B and D for a semicircle.

Area of non-shaded region $= 2\left(\frac{\pi r^2}{2}\right)$

$$= \pi \times 49$$

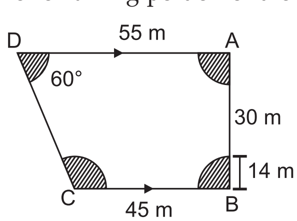
$$= \frac{22}{7} \times 49$$

$$= 154 \text{ cm}^2$$

Shaded area = 350 – 154

$$= \mathbf{196\ cm^2}$$

37. ABCD is a trapezium with AD ∥ BC, AD = 55 m, CB = 45 m, AB = 30 m, ∠ABC = 90° and ∠ADC = 60°. Taking A, B, C and D as centres, four sectors are formed with the same radius 14 m, at the four corners of the trapezium. We shall find the (*i*) total area of the four sectors and (*ii*) area of the remaining portion of the trapezium.



(*i*) ∵ AD ∥ CB and DC is a transversal

∴ ∠DCB + ∠ADC = 180°

⇒ ∠DCB + 60° = 180°

⇒ DCB = 180° – 60° = 120° …(1)

Similarly,

∠DAB + ∠ABC = 180°

⇒ ∠DAB + 90° = 180°

⇒ ∠DAB = 180° – 90° = 90° …(2)

∴ Area of the sector at D

$$= \pi \times 14^2 \times \frac{60}{360} \text{ m}^2$$

$$= \frac{22}{7} \times 14 \times 14 \times \frac{1}{6} \text{ m}^2$$

$$= \frac{308}{3} \text{ m}^2 \quad \text{…(3)}$$

Area of the sector at C $= \frac{22}{7} \times 14^2 \times \frac{120}{360}$

$$= \frac{22 \times 28}{3} \text{ m}^2$$

$$= \frac{616}{3} \text{ m}^2 \quad \text{…(4)}$$

Area of the sector at B

$$= \frac{22}{7} \times 14 \times 14 \times \frac{90}{360} \text{ m}^2 \quad \text{…(5)}$$

$$= 154 \text{ m}^2$$

Area of the sector at A

$$= \frac{22}{7} \times 14 \times 14 \times \frac{90}{4} \text{ m}^2$$

$$= 154 \text{ m}^2 \quad \text{…(6)}$$

∴ Total area of four sectors at A, B, C and D

$$= \left(\frac{308 + 616}{3} + 154 + 154\right) \text{m}^2$$

[From (3), (4), (5) & (6)] …(7)

$$= \left(\frac{924}{3} + 308\right) \text{m}^2$$

= **616 m²** which is the required area.

(*ii*) Area of the trapezium ABCD

$$= \frac{1}{2}(AD + BC) \times AB$$

$$= \frac{1}{2}(55 + 45) \times 30 \text{ m}^2$$

$$= 15 \times 100 \text{ m}^2$$

$$= 1500 \text{ m}^2 \quad \text{…(8)}$$

∴ Remaining area of the trapezium excluding the sum of areas of four sectors at A, B, C and D

= (1500 – 616) m² [From (7) and (8)]

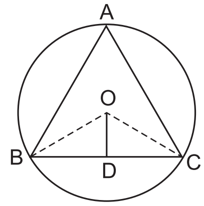
= **884 m²** which is the required area.

38. ΔABC is an equilateral triangle.

∴ All angles are 60°.

∴ Angle ∠BOC = 120°

Radius of the circle = 70 cm.



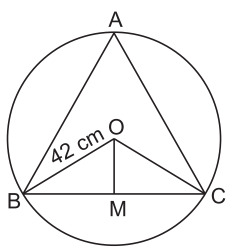
Area of the circle $= \frac{22}{7} \times 70 \times 70 = 15400 \text{ cm}^2$

$$OD \perp BC$$
$$\angle BOC = 120°$$
$$\because \quad \Delta BOD \cong \Delta COD$$
$$\therefore \quad BD = CD$$
$$\angle BOD = 60°$$
$$\sin 60° = \frac{BD}{OB}$$
$$\frac{\sqrt{3}}{2} = \frac{BD}{70}$$
$$BD = 35\sqrt{3} \text{ cm}$$
$$\therefore \quad BC = 70\sqrt{3} \text{ cm}$$

Area of the shaded region

$$= \text{Area of the circle} - \text{Area of the } \Delta ABC$$
$$= 15400 - \frac{\sqrt{3}}{4} \times 70\sqrt{3} \times 70\sqrt{3}$$
$$= 15400 - 6357.75 = \mathbf{9042.25 \text{ cm}^2} \text{ (approx.)}$$

39. An equilateral triangle ABC is inscribed in a circle with centre at O and radius 42 cm.



To find the area of the shaded region.

We have required area

= Area of the circle of radius 42 cm

– Area of equilateral ΔABC …(1)

Now, we join OB and draw OM ⊥ BC. Then M will be the mid-point of BC.

Now, ∠BOC = angle subtended by the arc BC at the centre O

$$= 2\angle BAC$$
$$= 2 \times 60°$$
$$= 120°$$
$$\therefore \quad \angle BQM = \frac{1}{2}\angle BOC$$
$$= \frac{120°}{2} = 60°$$

∴ From ΔBQM, we have

$$BM = BQ \sin 60°$$
$$= 42 \times \frac{\sqrt{3}}{2} \text{ cm}$$
$$= 21\sqrt{3} \text{ cm}$$
$$\therefore \quad BC = 2BM = 42\sqrt{3} \text{ cm}$$
$$\therefore \quad \text{Area of } \Delta ABC = \frac{\sqrt{3}}{4} \times 42 \times 42 \times 3 \text{ cm}^2$$
$$= 1.73 \times 21 \times 21 \times 3 \text{ cm}^2$$
$$= 2288.79 \text{ cm}^2 \quad \text{…(2)}$$

Area of the whole circle

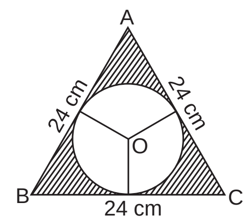
$$= \frac{22}{7} \times 42 \times 42 \text{ cm}^2$$
$$= 5544 \text{ cm}^2 \quad \text{…(3)}$$

∴ From (1), (2) and (3), we get

Required area of the shaded region

$$= (5544 - 2288.79) \text{ cm}^2$$
$$= \mathbf{3255.21 \text{ cm}^2}$$

40. ABC is an equilateral triangle of side 24 cm.



Area of the triangle $= \frac{\sqrt{3}}{4}a^2 = \frac{\sqrt{3}}{4} \times 24 \times 24$

$$= 249.4153 \text{ cm}^2$$

Now, $249.4153 = \Delta AOC + \Delta BOC + \Delta BOA$

$$249.4153 = \frac{1}{2} \times 24 \times r + \frac{1}{2} \times 24 \times r + \frac{1}{2} \times 24 \times r$$
$$249.4153 = \frac{1}{2} \times 24 \times 3r; \quad r = 6.9282 \text{ cm} = 7 \text{ cm.}$$

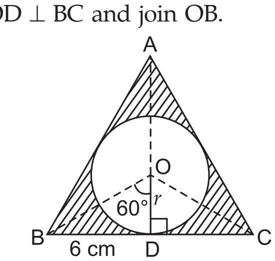
Area of the circle $= \frac{22}{7} \times 6.9282 \times 6.9282 = 150.857 \text{ cm}^2$

Area of the remaining part $= 249.4153 - 150.857$

$$= 98.5583 \text{ cm}^2$$
$$= \mathbf{98.55 \text{ cm}^2} \text{ (approx.)}$$

41. A circle with centre at O is inscribed in an equilateral triangle ABC of side of length 12 cm.

We draw OD ⊥ BC and join OB.



Now, $\angle BOC = 2\angle BAC$

$$= 2 \times 60°$$
$$= 120°$$

$\therefore \qquad \angle BOD = \frac{1}{2} \times 120° = 60°$

Let r be the radius of the circle. Then r = OD

Now, from right-angled triangle OBD, we have

$$\tan(\angle BOD) = \frac{6}{r}$$

$$\Rightarrow \qquad \tan 60° = \frac{6}{r}$$

$$\Rightarrow \qquad \sqrt{3} = \frac{6}{r}$$

$$\Rightarrow \qquad r = \frac{6}{\sqrt{3}} = \frac{6\sqrt{3}}{3} = 2\sqrt{3}$$

Hence, the required length of the radius of the circle is $2\sqrt{3}$ cm.

$\therefore$ Area of the circle = $\pi r^2 = 3.14 \times 12 \text{ cm}^2$

$= 37.68 \text{ cm}^2$ …(1)

Also, area of $\Delta ABC = \frac{\sqrt{3}}{4} \times 12 \times 12 \text{ cm}^2$

$= 36\sqrt{3} \text{ cm}^2$

$= 36 \times 1.73 \text{ cm}^2$

$= 62.28 \text{ cm}^2$ …(2)

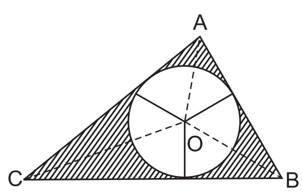
$\therefore$ Required area of the shaded region

= Area of ΔABC – Area of the circle

$= (62.28 - 37.68) \text{ cm}^2$

= 24.6 cm²

42. ΔABC is right-angled at A.



$\therefore \qquad AC^2 = BC^2 - AB^2$

$AC^2 = 100 - 36$

$= 64 \text{ cm}^2$

AC = 8 cm

Area of $\Delta ABC = \frac{1}{2} \times 8 \times 6 = 24 \text{ cm}^2$

Now, $\qquad 24 = ar(\Delta AOC) + ar(\Delta AOB) + ar(\Delta BOC)$

$24 = \frac{1}{2} r(10 + 6 + 8)$

$48 = 24r \quad r = 2$ cm.

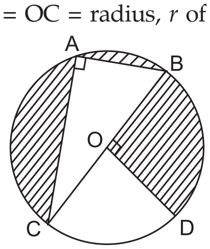
Area of the shaded region

$$= 24 - \frac{22}{7} \times 2 \times 2 = \mathbf{11.44 \text{ cm}^2}$$

43. O is the centre of a circle, BOC is a diameter of the circle such that $\angle$BAC = angle in a semicircle = 90°.

OD is drawn $\perp$ BC.

Then OB = OD = OC = radius, r of the circle.



Given that AB = 7 cm and DC = 24 cm.

$\therefore$ From ΔABC, by Pythagoras' theorem, we have

$$BC = \sqrt{7^2 + 24^2} \text{ cm}$$

$$= \sqrt{576 + 49}$$

$$= \sqrt{625} = 25$$

$$\Rightarrow \qquad 2r = 25$$

$$\Rightarrow \qquad r = \frac{25}{2}$$

$\therefore$ Area of the sector COD

$$= \pi r^2 + \frac{90}{360} \text{ cm}^2$$

$$= 3.14 \times \frac{625}{4} \times \frac{1}{4} \text{ cm}^2$$

$$= \frac{1962.5}{16} \text{ cm}^2$$

$= 122.66 \text{ cm}^2$ …(1)

Area of the whole circle $= 3.14 \times \left(\frac{25}{2}\right)^2 \text{ cm}^2$

$$= \frac{3.14 \times 625}{4} \text{ cm}^2$$

$= 490.62 \text{ cm}^2$ …(2)

Area of $\Delta ABC = \frac{1}{2} \times AB \times AC$

$$= \frac{1}{2} \times 7 \times 24 \text{ cm}^2$$

$= 84 \text{ cm}^2$ …(3)

Hence, required area of the shaded region

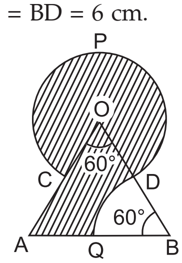
= Area of the circle – Area of the sector COD – Area of ΔABC

$= (490.62 - 122.66 - 84) \text{ cm}^2$ [From (1), (2) and (3)]

$= (490.62 - 206.66) \text{ cm}^2$

= 283.96 cm²

44. Let O be the centre of a circle of radius 6 cm such that O is the vertex of an equilateral triangle OAB of side 12 cm. BQD is the sector of another circle with centre at B and radius BQ = BD = 6 cm.



To find the area of the shaded region.

We see from the figure that

Area of the shaded region = Area of the major sector CPD of the circle with centre at O + Area of ΔOAB – Area of the sector BQD of another circle with centre at B …(1)

Now, the area of the major sector CPD of the circle with centre at O

$$= \pi \times 6^2 \times \frac{300}{360} \text{ cm}^2$$

[$\because$ Reflex – $\angle$COD = 360° – 60° = 300°]

$$= 3.14 \times 36 \times \frac{5}{6} \text{ cm}^2$$

$$= 3.14 \times 30 \text{ cm}^2$$

$$= 94.2 \text{ cm}^2 \qquad \ldots(2)$$

Area of equilateral ΔOAB of side 12 cm

$$= \frac{\sqrt{3}}{4} \times 12 \times 12 \text{ cm}^2$$

$$= 36\sqrt{3} \text{ cm}^2$$

$$= 36 \times 1.73 \text{ cm}^2$$

$$= 62.28 \text{ cm}^2 \qquad \ldots(3)$$

Area of the sector BQD with radius 6 cm and centre at B

$$= 3.14 \times 6^2 \times \frac{60}{360} \text{ cm}^2$$

$$= 3.14 \times 6 \text{ cm}^2$$

$$= 18.84 \text{ cm}^2 \qquad \ldots(4)$$

Hence, from (1), (2), (3), and (4), the required area of the shaded region

$$= (94.2 + 62.28 - 18.84) \text{ cm}^2$$

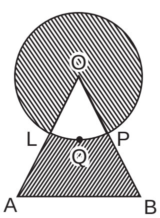
$$= (156.48 - 18.84) \text{ cm}^2$$

$$= \mathbf{137.64 \text{ cm}^2}$$

45. Radius of circle = 6 cm

Side of the triangle = 12 cm

Area of circle $= \pi r^2 = \pi \times 36 = 36\pi \text{ cm}^2$



$$\text{Area of the triangle} = \frac{\sqrt{3}}{4} a^2$$

$$= \frac{\sqrt{3}}{4} \times 12 \times 12$$

$$= 36\sqrt{3} \text{ cm}^2$$

Since OAB is an equilateral triangle, therefore $\angle AOB = 60°$

$$\text{Area of sector OLQP} = \frac{\theta}{360} \times \pi r^2$$

$$= \frac{60}{360} \times \pi \times 36$$

$$= 6\pi \text{ cm}^2$$

Area of shaded region = Area of circle + Area of ΔOAB – 2(Area of sector OLQP)

$$= 36\pi + 36\sqrt{3} - 12\pi$$

$$= 12(3\pi + 3\sqrt{3} - \pi)$$

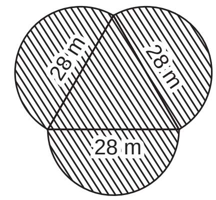
$$= 12(9.42 + 5.19 - 3.14)$$

$$= 12(11.47)$$

$$= \mathbf{137.64 \text{ cm}^2}$$

46. $$\text{Area of the triangle} = \frac{\sqrt{3}}{4} \times 28 \times 28$$

$$= 196\sqrt{3} \text{ m}^2 = 339.08 \text{ m}^2$$



$$\text{Area of the 3 half circles} = 3 \times \frac{1}{2} \times \frac{22}{7} \times 14 \times 14$$

$$= 923.16 \text{ m}^2$$

$$\text{Total area} = 923.16 + 339.08 \text{ m}^2$$

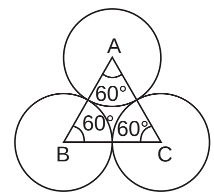
$$= 1262.24 \text{ m}^2$$

Total cost = 1262.24 × ₹2.50

= **₹ 3155.60**

47. Let each side of triangle be a cm.

Then area of of ΔABC



$$= 49\sqrt{3} \text{ cm}^2$$

$$\Rightarrow \quad \frac{\sqrt{3}}{4} a^2 = 49\sqrt{3}$$

$$\Rightarrow \quad a = 49 \times 4$$

$$\Rightarrow \quad a = 14 \text{ cm}$$

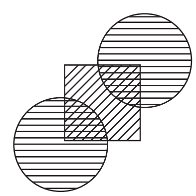
So radius of each circle is 7 cm.

$$\therefore \quad \text{Required area} = \left[49\sqrt{3} - 3\left(\frac{60}{360} \times \frac{22}{7} \times 7^2\right)\right] \text{cm}^2$$

$$= (49\sqrt{3} - 77) \text{ cm}^2$$

$$= \mathbf{7.77 \text{ cm}^2.}$$

48. Side of the square = 28 cm.



Radius of each sector is half $\frac{1}{2}$ side of the square

= 14 cm.

$$\text{Area of the square} = (28)^2 \text{ cm}^2 = 784 \text{ cm}^2$$

$$\text{Area of 2 circles} = 2 \times \pi \times (14)^2 = 1232 \text{ cm}^2$$

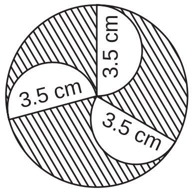
$$\text{Area of each sector} = \frac{1}{4} \times \pi \times 14^2 = 154 \text{ cm}^2$$

$$\text{Area of shaded region} = 784 + 1232 - (154 \times 2)$$

$$= \mathbf{1708 \text{ cm}^2}$$

49. Radius of the big circle $= \frac{7}{2}$ cm.

$$\text{Area of the big circle} = \pi\left(\frac{7}{2}\right)^2$$



Area of 3 small semicircles = $3\times\frac{1}{2}\times\pi\left(\frac{7}{4}\right)^2$

Area of the shaded portion = $\pi\left(\frac{7}{2}\right)^2-3\times\frac{1}{2}\times\pi\left(\frac{7}{4}\right)^2$ cm^2

$$= \pi\left[\left(\frac{7}{2}\right)^2-\frac{3}{2}\left(\frac{7}{4}\right)^2\right]\text{cm}^2$$

$$= \pi\left[\frac{49}{4}-\frac{3}{2}\times\frac{49}{16}\right]\text{ cm}^2$$

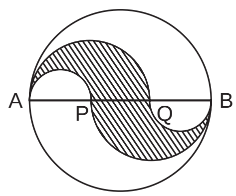
$$= \frac{22}{7}\left[\frac{49}{4}\times\frac{5}{8}\right] = \mathbf{\frac{385}{16}}\text{ cm}^2$$

50. Diameter of the big circle = 12 cm.

AP = PQ = QB = 4 cm

AQ = 8 cm = PB

Area of the big circle = $\frac{22}{7}\times 6^2$ = 113.04 cm^2



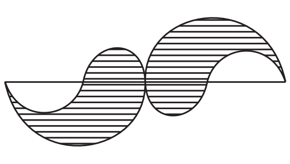
Area of the shaded portion with radius AQ and PB

$$= 2\left[\frac{1}{2}\pi\, 4^2-\frac{1}{2}\pi 2^2\right]\text{cm}^2$$

$$= 2\times\frac{1}{2}\pi\,[16-4]\text{ cm}^2$$

$$= \pi\times 12 = \mathbf{\frac{264}{7}\ cm^2}$$

51. Radius of the bigger semicircle = 42 cm.



Area of the shaded region = $\frac{1}{2}\pi\, 42^2+\frac{1}{2}\pi\, 42^2$

$$= \frac{1}{2}\pi\,(1764+1764)$$

$$= \mathbf{5544\ cm^2}$$

Perimeter of the shaded region = Perimeter of 2 bigger semicircles + Perimeter of 4 smaller semicircles

$$= 2\pi\left(42+\frac{1}{2}\times 4\times 21\right) = \mathbf{528\ cm}$$

52. Side of square PQRS = 42 m

Since diagonals intersect at 90°

∴ $SO^2 + PO^2 = SP^2$

$x^2 + x^2 = (42)^2$

$2x^2 = 42 \times 42$

$x = 21\sqrt{2}$ m

Radius of circular flower bed = $21\sqrt{2}$ m

Area of shaded region = 2 × (Area of segment)

= 2(Area of sector – Area of triangle)

$$= 2\left(\frac{90}{360}\times\pi\times 882-\frac{1}{2}\times 21\sqrt{2}\times 21\sqrt{2}\right)$$

$$= 2\left(\frac{441\pi}{2}-441\right)$$

$$= 2\times 441\left(\frac{\pi}{2}-1\right)$$

$$= \mathbf{504\ m^2}$$

53. (*i*) Perimeter of figure = 40 cm

$\pi R + \pi r + OB = 40$

$\pi R + \pi r + R = 40$

$\pi R + \pi\frac{R}{2} + R = 40$

$R\left(\pi+\frac{\pi}{2}+1\right) = 40$

$R = \frac{40}{5.714} = 7$ cm

Radius of semicircle APB = 7 cm

Radius of semicircle AQO = $\frac{7}{2}$ cm

Shaded area = $\frac{1}{2}(\pi R^2 + \pi r^2)$

$$= \frac{\pi}{2}(R^2 + r^2)$$

$$= \frac{\pi}{2}\left(49+\frac{49}{4}\right)$$

$$= \frac{49\pi}{2}\times\frac{5}{4}$$

$$= \frac{49}{2}\times\frac{22}{7}\times\frac{5}{4}$$

$$= \frac{35\times 11}{4}$$

$$= \frac{385}{4}$$

$$= \mathbf{96.25\ cm^2}$$

(*ii*) Shaded area = $\widehat{APD}+\widehat{BRC}-\widehat{AQB}-\widehat{CSD}$

$$= \frac{\pi r_1^2}{2}+\frac{\pi r_2^2}{2}-\frac{\pi r_3^2}{2}-\frac{\pi r_4^2}{2}$$

$$= \frac{\pi}{2}(r_1^2 + r_2^2 - r_3^2 - r_4^2)$$

$$= \frac{\pi}{2}\left(49 + \frac{49}{4} - \frac{12.25}{4} - \frac{12.25}{4}\right)$$

$$= \frac{\pi}{2}\left(49 + \frac{49}{4} - \frac{12.25}{2}\right)$$

$$= \frac{\pi}{2}(49 + 12.25 - 6.125)$$

$$= 27.5625 \times \frac{22}{7}$$

$$= \mathbf{86.625\ cm^2}$$

54. Area of the circle = 1256 cm^2

$$\frac{22}{7}r^2 = 1256$$

$$r^2 = \frac{1256 \times 7}{22} = 400$$

$$r = 20 \text{ cm.}$$

Diagonal of the rhombus = 40 cm.

$\because$ Diameter of the circle = 40 cm.

$$a^2 + a^2 = 1600$$

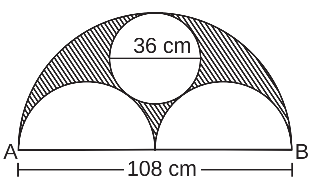
$$2a^2 = 1600$$

$$a^2 = 800$$

$$a = 28.28 \text{ cm.}$$

$$a^2 = \mathbf{799.87\ cm^2 \approx 800\ cm^2}$$

55. Let the radii of the small circle, smaller semicircles and large semicircle be r_1, r_2 and r_3 respectively.



Then $r_1 = \frac{36}{2} = 18$ cm

$$r_2 = \frac{1}{2}\left(\frac{108}{2}\right) = 27 \text{ cm}$$

$$r_3 = \frac{108}{2} = 54 \text{ cm}$$

Area of the shaded region

$$= \frac{\pi r_3^2}{2} - \frac{2 \times \pi r_2^2}{2} - \pi r_1^2$$

$$= \pi\left(\frac{54 \times 54}{3} - 27 \times 27 - 18 \times 18\right) \text{cm}^2$$

$$= \mathbf{\frac{8910}{7}\ cm^2}$$

For Standard Level

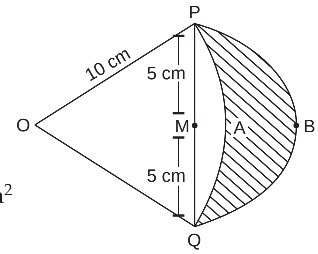
56. Area of the minor segment

= Area of sector – Area of ΔOPQ

$$= \frac{\theta}{360} \times \pi r^2 - \frac{\sqrt{3}}{4} r^2$$

$$= \frac{60}{360} \times \pi \times 100 - \frac{\sqrt{3}}{4} 100$$

$$= 100\left(\frac{\pi}{6} - \frac{\sqrt{3}}{4}\right) \text{cm}^2$$



Area of semicircle $= \frac{\pi r^2}{2}$

$$= \frac{\pi \times (5)^2}{2}$$

$$= \frac{25\pi}{2} \text{ cm}^2$$

Area of shaded region

= Area of semicircle – Area of segment

$$= \frac{25\pi}{2} - 100\left(\frac{\pi}{6} - \frac{\sqrt{3}}{4}\right)$$

$$= 25\left[\frac{\pi}{2} - \frac{4\pi}{6} + \sqrt{3}\right]$$

$$= 25\left(\sqrt{3} - \frac{\pi}{6}\right) \text{cm}^2$$

Hence proved.

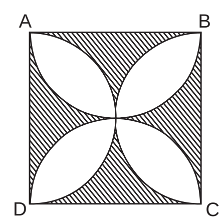
57. Side of square ABCD = 14 cm

Radius of semicircle = 7 cm

Area of square = $(\text{side})^2 = (14)^2 = 196$ cm^2

Area of each semicircle $= \frac{\pi r^2}{2} = \frac{\pi \times (7)^2}{2} = \frac{49\pi}{2}$ cm^2

Shaded area = 2[area of square – 2(area of semicircle)]



$$= 2\left[196 - 2 \times \frac{49\pi}{2}\right]$$

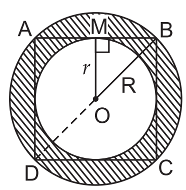
$$= 2\left[196 - 49 \times \frac{22}{7}\right]$$

$$= 2[196 - 154]$$

$$= 2 \times 42$$

$$= \mathbf{84\ cm^2}$$

58. Let ABCD be a square of side of length 10 cm and let O be the common centre of two concentric circles, one inscribed in the square and the other circumscribes the square ABCD. Let r cm be the radius of the inscribed circle and R cm be the radius of the circumscribed circle.



Then OB = R cm and OM = r cm, where OM $\perp$ AB and BOD is a diagonal of the square.

Now, $BM = \frac{1}{2} AB = \frac{1}{2} \times 10$ cm = 5 cm

$\therefore$ From ΔOMB, we have

$$OM^2 + MB^2 = R^2$$

$$\Rightarrow \quad r^2 + 5^2 = R^2$$

$$\Rightarrow \quad R^2 - r^2 = 25 \quad \ldots(1)$$

Now, the area of the region included between the two circles

= The area of the shaded region

$= \pi R^2 - \pi r^2 = \pi(R^2 - r^2)$

$= 25\pi$ cm^2 [From (1)]

Hence, the required area is **25π cm^2**.

59. $AC^2 = (2a)^2 + (2a)^2 = 8a^2$

$AC = 2\sqrt{2}a$

Radius of the circumcircle $= \dfrac{2\sqrt{2}a}{2} = \sqrt{2}a$

Radius of the incircle $= a$

Area of the circumcircle $= \pi\sqrt{2}a = 2\pi a^2$

Area of the big circle $= \pi a^2$

$$\text{Ratio} = \frac{2\pi a^2}{\pi a^2} = \frac{2}{1}$$

$\therefore$ The area of the circumcircle of any square is twice the area of its incircle.

60. Let the radius of the semicircle $= r$

Side of the square $= a$

According to given condition,

$$\frac{1}{2} \times 2\pi r + 2r = 4a$$

$$2r\left(\frac{\pi}{2}+1\right) = 4a$$

$$2r \times \frac{18}{7} = 4a$$

$$18r = 14a$$

$$a = \frac{18}{14}r$$

Again, $a^2 = \dfrac{1}{2}\pi r^2 + 4$

$$\frac{81}{49}r^2 = \frac{1}{2}\pi r^2 + 4$$

$$\frac{81}{49}r^2 = \frac{22}{7\times 2}r^2 + 4$$

$$\frac{81}{49}r^2 - \frac{11}{7}r^2 = 4$$

$$r^2 = 7^2$$

$$r = 7$$

$$\therefore \quad a = \frac{9}{7}\times r = \frac{9}{7}\times 7 = 9$$

Perimeter of the semicircle $= \dfrac{22}{7}\times 49$

$= \dfrac{1}{2}\times 2\times\dfrac{22}{7}\times 7 + 2\times 7$

$= 22 + 14 = 36$ cm

Perimeter of the square = **36 cm**

Area of the semicircle $= \dfrac{1}{2}\times\dfrac{22}{7}\times 7\times 7$

$= \dfrac{1}{2}\times 154 =$ **77 cm^2**

Area of the square $= 9 \times 9 =$ **81 cm^2**

61. ABCD is a square.

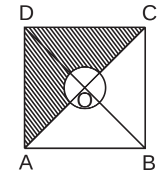
AB = 10.5 cm

Area of the square $= 10.5 \times 10.5$ cm^2

$= 110.25$ cm^2

Area of the circle $= \dfrac{1}{5} \times$ Area of the square

$= \dfrac{1}{5}\times 110.25 = 22.05$ cm^2



Area of the shaded region

$= \text{Area of } \Delta ADC - \dfrac{1}{2}\times \text{Area of circle}$

$= \dfrac{1}{2}\times 10.5\times 10.5 - \dfrac{1}{2}\times 22.05$

$= 55.125 - 11.025 =$ **44.1 cm^2**

Perimeter of the shaded region

$= 10.5 + 10.5 + 15 - 2\sqrt{7} + \dfrac{1}{2}\times 2\times\dfrac{22}{7}\sqrt{7}$

$= 36 - \left(\dfrac{14\sqrt{7} - 22\sqrt{7}}{7}\right)$

= **38.87 cm (approx.)**

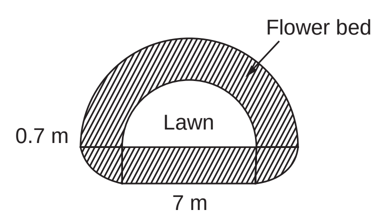
AC = 14.83 cm

= 15 cm [$\because$ $AC^2 = (10.5)^2 + (10.5)^2 = 220.50$]

62. Diameter of the semicircle = 7 m.

Radius $= \dfrac{7}{2}$ m

Area of the lawn $= \left(\dfrac{1}{2}\times\dfrac{22}{7}\times\dfrac{7}{2}\times\dfrac{7}{2}\right)$ m^2



Area of the rectangle $= (7 \times 0.7)$ m^2 $= 4.9$ m^2

($\because$ Width of the lawn = 0.7 m)

Area of the 2 quadrants with radius = 0.7 m

$= 2\times\dfrac{1}{4}\times\dfrac{22}{7}\times(0.7)^2 = \dfrac{77}{100}$ m^2

$$\text{Area of the border} = \frac{1}{2} \times \pi \times (4.2)^2 - \frac{1}{2}\pi\left(\frac{7}{2}\right)^2$$

$$= \frac{1}{2}\pi\left[(4.2)^2 - \left(\frac{7}{2}\right)^2\right]$$

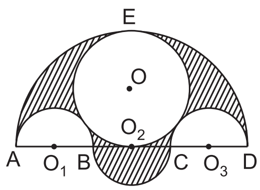
$$= 27.72 - 19.25 = 8.47 \text{ m}^2$$

Total area of the flower bed

= Border + rectangle + 2 quadrants

= 8.47 + 4.9 + 0.77 = **14.14 m²**.

63. Radius of each of the semicircles with centres at O_1, O_2, and O_3 is 1.5 cm, the radius of the complete circles with centre O is $\frac{4.5}{2}$ cm = 2.25 cm and the radius of the semi-circle with diameter AD is 4.5 cm.



Now, area of the semicircle with diameter AD is

$$\frac{1}{2}\pi \times 4.5^2 \text{ cm}^2 = \frac{3.14}{2} \times 4.5 \times 4.5 \text{ cm}^2 \quad \ldots(1)$$

Area of the semicircle with centre O_2 is

$$\frac{1}{2}\pi \times 1.5^2 \text{ cm}^2 = \frac{3.14}{2} \times 2.25 \text{ cm}^2 \quad \ldots(2)$$

Area of the circle with centre O

$$= \pi \times 2.25^2 \text{ cm}^2$$

$$= 3.14 \times 2.25 \times 2.25 \text{ cm}^2 \quad \ldots(3)$$

Sum of the area of two semicircles with centres at O_1 and O_3 is

$$= \pi \times 1.5^2$$

$$= 3.14 \times 2.25 \text{ cm}^2 \quad \ldots(4)$$

∴ Required area of the shaded region

= Area of the semicircle with diameter AD + Area of the semicircle with centre O_2 – [Area of the circle with centre O + Sum of the areas of two semicircles with centres O_1 and O_3]

$$= \left\{\frac{3.14}{2} \times 4.5 \times 4.5 + \frac{3.14}{2} \times 2.25 - [3.14 \times 2.25 \times 2.25 + 3.14 \times 2.25]\right\} \text{ cm}^2$$

[From (1), (2), (3) and (4)]

$$= 3.14 \times \left[\frac{4.5 \times 4.5}{2} + \frac{2.25}{2} - 2.25 \times 2.25 - 2.25\right] \text{cm}^2$$

$$= 3.14 \times \left[\frac{20.25}{2} + 2.25\left\{\frac{1}{2} - 2.25 - 1\right\}\right] \text{cm}^2$$

$$= 3.14 \times [10.125 - 2.25 \times 2.75] \text{ cm}^2$$

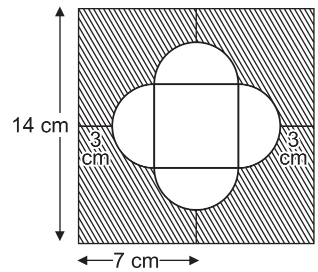
$$= 3.14 \times [10.125 - 6.1875] \text{ cm}^2$$

$$= 3.14 \times 3.9375 \text{ cm}^2$$

$$= 12.364 \text{ cm}^2 \text{ (approx.)}$$

64. Radius of semicircles = 2 cm

Side of square = 4 cm



Shaded area = Area of square – Area of smaller square – 4(Area of semicircles)

$$= (14)^2 - (4)^2 - 4\left(\frac{\pi r^2}{2}\right)$$

$$= 196 - 16 - 2(\pi \times 4)$$

$$= 180 - 8\pi$$

$$= 180 - 8 \times 3.14$$

$$= 180 - 25.12$$

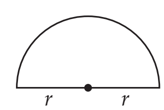
$$= \mathbf{154.88 \text{ cm}^2}$$

CHECK YOUR UNDERSTANDING

MULTIPLE-CHOICE QUESTIONS

For Basic and Standard Levels

1. (*c*) **14 cm**



$$\frac{1}{2} \times 2 \times \frac{22}{7} \times r + 2r = 36$$

$$\frac{22r + 14r}{7} = 36$$

$$36r = 36 \times 7$$

$$r = 7 \text{ cm}$$

∴ Diameter = 14 cm.

2. (*a*) **3.92 cm**

According to given condition

$$2\pi r = 2r + 16.8$$

$$44r = 14r + 16.8 \times 7$$

$$30r = 117.6$$

$$r = \frac{117.6}{30} = 3.92 \text{ cm.}$$

3. (*a*) **56 cm**

$$2\pi\, 19 + 2\pi\, 9 = 2\pi r$$

$$2\pi\,(19 + 9) = 2\pi r$$

$$28 = r$$

∴ Diameter of the third circle = 56 cm.

4. (*b*) **3.5 cm**

Circumference of a circle = 44 cm = $2\pi r$

According to given condtion, $2 \times \frac{22}{7} \times r = 44$

$$r = \frac{44 \times 7}{44} = 7 \text{ cm}$$

$$44 + 22 = 66 \text{ cm}$$

$$66 = 2\pi r_1$$

$$r_1 = \frac{66 \times 7}{22 \times 2} = 10.5 \text{ cm}$$

Increase in radius = 10.5 – 7

= 3.5 cm.

5. (*c*) **18 cm**

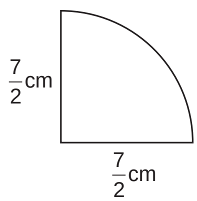
Radius = 3.5 cm

Perimeter of the semicircle = $\frac{1}{2} \times 2 \times \frac{22}{7} \times 3.5 + 2 \times 3.5$

= 18 cm.

6. (*b*) **12.5 cm**

Radius = $\frac{7}{2}$ cm



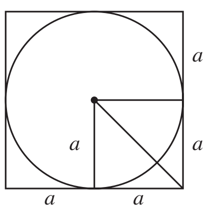
Perimeter of the quadrant

$= \frac{1}{4} \times 2 \times \frac{22}{7} \times \frac{7}{2} + \frac{7}{2} + \frac{7}{2}$

= 12.5 cm.

7. (*a*) **8*a* cm**

Radius = *a* cm



∴ Side of the square = *a* + *a* = 2*a* cm.

Perimeter of the square = 4 × 2*a* = 8*a* cm.

8. (*b*) **44**

$$2\pi r - r = 37 \text{ cm.}$$

$$r\left(2 \times \frac{22}{7} - 1\right) = 37$$

$$r \times \frac{37}{7} = 37$$

$$r = 7$$

∴ Circumference = $2 \times \frac{22}{7} \times 7$ = 44 cm.

9. (*d*) **44 cm**

Area = πr^2 = 154 cm^2

$r^2 = 49$, $r = 7$ cm.

Perimeter of the circle = $2 \times \frac{22}{7} \times 7 = 44$ cm.

10. (*b*) **1.1**

$2\pi r$ = one revolution, diameter = 35 cm.

∴ Radius = 17.5 cm

∴ $2\pi r = 2 \times \frac{22}{7} \times 17.5$

= 110 cm = 1.1 m.

11. (*d*) **154 cm²**

$$2\pi r = 44 \text{ cm}$$

$$r = \frac{44 \times 7}{2 \times 22} = 7 \text{ cm}$$

∴ Area = $\pi r^2 = \frac{22}{7} \times 7 \times 7 = 154$ cm^2

12. (*d*) **Four times**

Circumference of a circle = $2\pi r$

According to given condition,

$2\pi r = 2\pi$ ∴ $\pi r^2 = \frac{22}{7} \times 1^2 = \pi$

$r = 1$

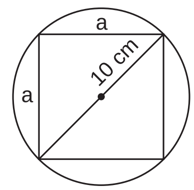
Now when $2\pi r = 4\pi$

$r = 2$

$\pi r^2 = \frac{22}{7} \times 4 = 4\pi$

∴ Area increases four times.

13. (*b*) **200 cm²**



$a^2 + a^2 = (20)^2$

$= 400$

$a^2 = 200$

$a = 10\sqrt{2}$

∴ Area of the square = $(10\sqrt{2})^2$ = 200 cm^2.

14. (*a*) **3 : 4**

r_1 and r_2 being the radius of 2 circles.

$$\frac{\pi r_1^2}{\pi r_2^2} = \frac{9}{16}, \quad \frac{r_1}{r_2} = \frac{3}{4}$$

∴ $\frac{2\pi r_1}{2\pi r_2} = \frac{3}{4} = 3 : 4.$

15. (*b*) **14 : 11**

Circumference of a circle = Perimeter of a square.

∴ $2\pi r = 4a$ [*r* = radius of the circle
a = side of a square]

$$\frac{r}{a} = \frac{4}{2\pi} = \frac{2}{\pi}$$

Now, ratio of their area = $\frac{\pi r^2}{a^2} = \pi\left(\frac{2}{\pi}\right)^2 = \frac{4}{\pi} = \frac{14}{11}$

$\left[\text{Taking } \pi = \frac{22}{7}\right]$.

16. (*b*) **26**

$$\pi r^2 = \pi 5^2 + \pi 12^2$$

$$\pi r^2 = \pi(25 + 144)$$

$r^2 = 169$, $r = 13$,

∴ Diameter = 26 cm.

17. (*c*) **8 units**

According to condition.

$\pi r^2 = 2 \times 2\pi r$ (*r* = radius of a circle)

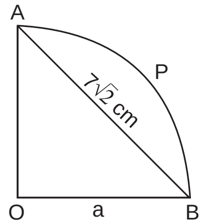
∴ $r = 4$,

∴ Diameter = 8

18. (*a*) **25 cm.**

Length of the chord

$$AB = 7\sqrt{2} \text{ cm},$$



Let $AO = OB = \text{radius} = a$

$\therefore \quad a^2 + a^2 = (7\sqrt{2})^2$

$2a^2 = 49 \times 2$

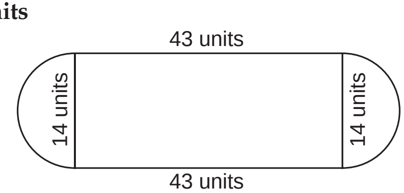
$a^2 = 49,\ a = 7$ cm.

Now, perimeter of the quadrant BPAO

$$= 7 + 7 + \frac{1}{4} \times 2 \times \frac{22}{7} \times 7$$

$$= 14 + 11 = 25 \text{ cm.}$$

19. (*c*) **130 units**

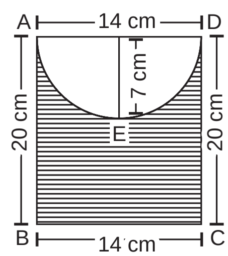


Perimeter of the given plot

$$= 43 + 43 + \frac{1}{2} \times 2\pi r + \frac{1}{2} \times 2\pi r$$

$$= 86 + 2\pi r = 86 + 40 = 130 \text{ units}$$

20. (*d*) **76 cm.**



Perimeter of the shaded region

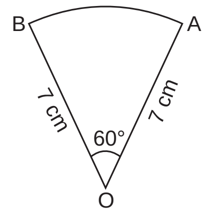
$$= 20 + 14 + 20 + \frac{1}{2} \times 2 \times \frac{22}{7} \times 7 = 76 \text{ cm.}$$

21. (*a*) $\mathbf{\frac{64}{3}}$ **cm**

Perimeter of sector AOB

$$= 7 + 7 + \frac{60}{360} \times 2 \times \frac{22}{7} \times 7$$

$$= 14 + \frac{22}{3} = \frac{64}{3} \text{ cm.}$$

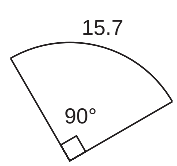


22. (*b*) **10 cm.**

Length of an arc = 15.7 cm

$$\therefore \quad 15.7 = \frac{90}{360} \times 2 \times \frac{22}{7} \times r$$

$$r = \frac{109.9}{11} = 9.99 = 10 \text{ cm.}$$

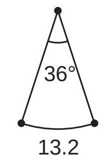


23. (*c*) **1 : 4**

$$\frac{\text{Arc}}{\text{Circumference}} = \frac{90}{360} \times 2\pi r \div 2\pi r = \frac{1}{4} = 1:4$$

24. (*a*) **21 cm**

$$13.2 = \frac{36}{360} \times 2 \times \frac{22}{7} \times r \quad [r = \text{radius or length of the pendulum}]$$



$$r = \frac{13.2 \times 360 \times 7}{32 \times 2 \times 22}$$

$$= 6 \times 5 \times 7 = 21 \text{ cm}$$

25. (*c*) **5.5 cm²**

Minute hand = $\sqrt{21}$ cm = radius

1 minute = 6° $\quad \therefore$ 7 am to 7:05 am = 5 minutes

5 minutes = 30°

$$\text{Area} = \frac{30}{360} \times \frac{22}{7} \times (\sqrt{21})^2 = \frac{11}{2} = 5.5 \text{ cm}^2$$

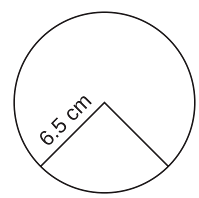
26. (*b*) **231 cm²**

Radius = 21 cm, angle at the centre = 60°

$$\therefore \quad \text{Area of the sector} = \frac{60}{360} \times \frac{22}{7} \times 21 \times 21$$

$$= 231 \text{ cm}^2$$

27. (*b*) **52 cm²**



Perimeter of the sector =

$$29 = \frac{\theta}{360} \times 2 \times \frac{22}{7} \times 6.5 + 6.5 + 6.5$$

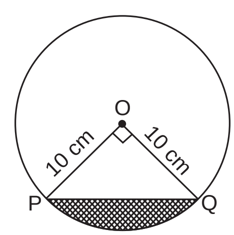
$$16 = \frac{\theta}{360} \times 2 \times \frac{22}{7} \times 6.5$$

$$\theta = \frac{2016}{14.3} = 140.97 = 141°$$

$$\text{Area of the sector} = \frac{141}{360} \times \frac{22}{7} \times (6.5)^2$$

$$= 51.96 \text{ cm}^2 = 52 \text{ cm}^2.$$

28. (*d*) **28.5 cm²**



$$\text{Area of sector} = \frac{90}{360} \times \frac{22}{7} \times 10^2 = \frac{550}{7} \text{ cm}^2$$

$$\sin 45° = \frac{BA}{BO} = \frac{1}{\sqrt{2}},$$

$$BA = \frac{10}{\sqrt{2}},\ BC = \frac{20}{\sqrt{2}}$$

$$\cos 45° = \frac{OA}{OB} = \frac{1}{\sqrt{2}}, \; OA = \frac{10}{\sqrt{2}}$$

$$\text{Area of the } \Delta OBC = \frac{1}{2} \times \frac{10}{\sqrt{2}} \times \frac{20}{\sqrt{2}}$$

$$= 50$$

∴ Area of the minor segment

$$= \text{Sector} - \Delta OBC$$

$$= \frac{550}{7} - 50 = 28.5 \text{ cm}^2$$

29. (*c*) **1 : 4**

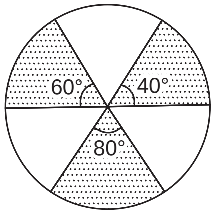
Let r unit be the radius of the circle.

Then length of the arc $= \frac{1}{4} \times 2\pi r$

Length of circumference $= 2\pi r$

∴ Required ratio $= \frac{1}{4} \times 2\pi r : 2\pi r = \frac{1}{4} = 1 : 4$

30. (*a*) **77**



Area of the shaded sectors

$$= \pi r^2 \left(\frac{60}{360} + \frac{80}{360} + \frac{40}{360} \right)$$

$$= \frac{22}{7} \times 7 \times 7 \left(\frac{1}{6} + \frac{2}{9} + \frac{1}{9} \right) \text{cm}^2$$

$$= 22 \times 7 \left(\frac{9}{18} \right) \text{cm}^2 = 77 \text{ cm}^2.$$

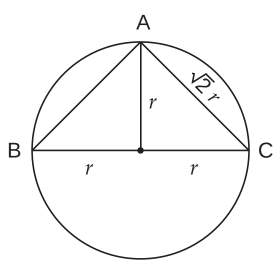
31. (*b*) r^2 **cm²**

$$r^2 + r^2 = AC^2$$

$$2r^2 = AC^2$$

$$\sqrt{2}r = AC$$

Similarly, $AB = \sqrt{2}r$



∴ Area of the largest triangle

$$= \frac{1}{2} \times \sqrt{2}r \times \sqrt{2}r$$

$$= r^2 \text{ cm}^2.$$

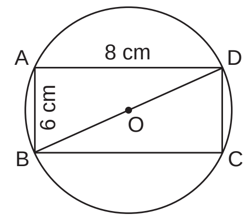
32. (*c*) **25π : 48**

Area of the rectangle = 6 × 8 = 48 cm²

$$BD^2 = 64 + 36 = 100$$

$$BD = 10 \text{ cm}$$

∴ Radius = 5 cm.



$$\text{Area of the circle} = \pi r^2 = \frac{22}{7} \times 25 = \frac{550}{7} \text{cm}^2 = 25\pi$$

$$\text{Ratio of the areas} = \frac{25\pi}{48} = 25\pi : 48$$

33. (*a*) **2 : 3**

Let r_1 units and r_2 units be the radii of the two circles.

Then the perimeter of the 1st semicircle $= \pi r_1$

and the perimeter of the 2nd semicircle $= \pi r_2$

Now, areas of the 1st and 2nd circles are πr_1^2 and πr_2^2 respectively.

∴ Their ratio $= r_1^2 : r_2^2 = 4 : 9$ [Given]

∴ $r_1 : r_2 = 2 : 3$ …(1)

∴ Required ratio of the two perimeters = 2 : 3 [From (1)]

34. (*d*) **550 cm²**

Area between 2 concentric circles

$$= \pi \times 20^2 - \pi \times 15^2$$

$$= \pi(400 - 225)$$

$$= \frac{22}{7} \times 175 = 550 \text{ cm}^2$$

35. (*c*) **126**

Let ∠POQ = θ°

$$\therefore \text{ Area of the sector POQ} = \frac{\theta}{360} \times \pi r^2$$

where r units are the radius of the circle.

Area of the whole circle $= \pi r^2$

∴ According to the problem,

$$\frac{\frac{\theta}{360} \times \pi r^2}{\pi r^2} = \frac{\theta}{360} = \frac{7}{20}$$

$$\therefore \quad \theta = \frac{7}{20} \times 360 = 126$$

Hence, ∠POQ = 126°.

36. (*b*) $\mathbf{\frac{600}{7}}$ cm²

Area of the square = 20 × 20 cm² = 400 cm²

Area of two semicircles each of radius 10 cm

$= \text{Area of the whole circle of radius 10 cm}$

$= \pi \times 10^2 \text{ cm}^2$

$= \frac{22}{7} \times 100 \text{ cm}^2$

$= \frac{2200}{7} \text{ cm}^2$

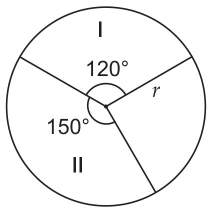
$\therefore$ Area of the shaded region

$= \left(400 - \frac{2200}{7}\right) \text{cm}^2$

$= \frac{600}{7} \text{ cm}^2$

37. (*d*) **4 : 5**

Let r units be the radius of the circle.



Then the area of sector I

$= \pi r^2 \times \frac{120}{360}$ sq units

$= \frac{\pi r^2}{3}$ sq units

The area of sector II

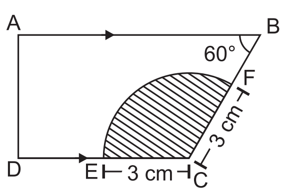
$= \pi r^2 \times \frac{150}{360}$ sq units

$= \frac{5\pi r^2}{12}$ square units

$\therefore$ Required ratio $= \dfrac{\frac{\pi r^2}{3}}{\frac{5\pi r^2}{12}} = \frac{1}{3} \times \frac{12}{5} = \frac{4}{5} = 4 : 5$

38. (*a*) **3π cm²**

Let ABCD be the trapezium where AD ⊥ CD, DA ⊥ AB, ∠ABC = 60° and AB ∥ CD. Also EF is the arc of a circle with centre at C such that CE = CF = 3 cm, the radius of the circle.



$\therefore$ Area of the sector CEFC $= \pi \times 3^2 \times \frac{120°}{360°} \text{ cm}^2$ …(1)

[∵ AB ∥ CD, ∠ABC + ∠BCD = 180°

⇒ 60° + ∠BCD = 180°

∴ ∠BCD = 180° – 60° = 120°]

$\therefore$ From (1), Area of sector CEFC = 3π cm²

39. (*b*) **2 : $\sqrt{\pi}$**

Let a cm be the side of the square and r unit be the radius of the circle. Then the area of the square = a^2 sq units and the area of the circle is πr^2 sq units.

Given that $a^2 = \pi r^2$

$\therefore \quad a = \sqrt{\pi}\, r$ …(1)

$\therefore$ Perimeter of the square : Perimeter of the circle

$= 4a : 2\pi r = 2a : \pi r = 2\sqrt{\pi}\, r : \pi r$ [From (1)]

$= 2 : \sqrt{\pi}$.

40. (*c*) **20 cm**

Let r cm and R cm be the radii of the two given circles

Then $2r = 12$ and $2R = 16$

$\Rightarrow \quad r = 6$ and $R = 8$ …(1)

Let r' cm be the radius of the required circle

Then its area $= \pi r'^2$

$= \pi r^2 + \pi R^2$ [Given]

$= 36\pi + 64\pi$ [From (1)]

$= 100\,\pi$

$\therefore \quad r'^2 = 100$

$\Rightarrow \quad r' = \sqrt{100} = 10$

$\therefore$ The required length of the diameter

$= 2 \times 10$ cm = 20 cm.

41. (*b*) **π : $\sqrt{3}$**

Let a cm be the side of the equilateral triangle. Then according to the problem, the diameter of the circle is a cm and so its radius is $\frac{a}{2}$ cm.

$\therefore$ Area of the circle $= \pi\left(\frac{a}{2}\right)^2 \text{cm}^2 = \frac{\pi a^2}{4} \text{ cm}^2$

and area of the equilateral triangle

$= \frac{\sqrt{3}}{4} \times a^2 \text{ cm}^2 = \frac{\sqrt{3}a^2}{4} \text{ cm}^2$.

$\therefore$ The required ratio of these two areas

$= \frac{\pi a^2}{4} : \frac{\sqrt{3}a^2}{4}$

$= \pi : \sqrt{3}$

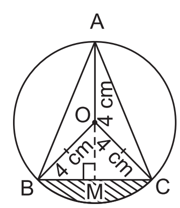
42. (*b*) $r_1^2 + r_2^2 = r^2$

According to the problem, we have

$\pi r_1^2 + \pi r_2^2 = \pi r^2$

$\Rightarrow \quad r^2 = r_1^2 + r_2^2$

43. (*a*) $\frac{4}{3}\left(4\pi - 3\sqrt{3}\right)$ **cm²**



We have $\angle BOC = 2\angle BAC$

$= 2 \times 60°$

$= 120°$

Let OM ⊥ BC.

Then $\angle BOM = \frac{1}{2}\angle BOC$

$= \frac{1}{2} \times 120°$

$= 60°$

$\therefore$ From ΔOBC, we have

$$BM = OB \sin 60°$$
$$= 4 \times \frac{\sqrt{3}}{2} \text{ cm}$$
$$= 2\sqrt{3} \text{ cm}$$

and $OM = OB \cos 60°$

$$= 4 \times \frac{1}{2} \text{ cm} = 2 \text{ cm}$$

$\therefore$ $ar(\Delta OBC) = BM \times OM = 4\sqrt{3}\ \text{cm}^2$

Also, $ar(\text{sector } OBC) = \pi \times 4^2 \times \frac{120}{360}\ \text{cm}^2$

$$= \frac{16\pi}{3}\ \text{cm}^2$$

$\therefore$ Required area of the segment of the circle, BCM, i.e., area of the shaded region

$$= \left(\frac{16\pi}{3} - 4\sqrt{3}\right)\text{cm}^2$$
$$= \frac{4}{3}\left(4\pi - 3\sqrt{3}\right)\ \text{cm}^2$$

44. (*c*) **area of the square < area of the circle**

Let a units be the side of the square and r units be the radius of the circle. Then according to the problem

$$2\pi r = 4a$$

$\Rightarrow$ $a = \frac{\pi r}{2}$

$\therefore$ Area of the square $= a^2 = \frac{\pi^2 r^2}{2} = S$ (say)

and area of the circle is $\pi r^2 = C$ (say)

$$\frac{S}{C} = \frac{\frac{\pi^2 r^2}{4}}{\pi r^2} = \frac{\pi}{4} = \frac{22}{28} < 1$$

$\therefore$ S < C

i.e. the area of the square < the area of the circle

45. (*d*) **11 : 14**

Let a units be the side of the square and r units be the radius of the circle.

$\therefore$ According to the problem

$$4a = 2\pi r$$

$\Rightarrow$ $a = \frac{\pi r}{2}$ ...(1)

Let A_1 units^2 be the area of the square and A_2 units^2 be the area of the circle. Then $A_1 = a^2$ and $A_2 = \pi r^2$

$\therefore$ $\frac{A_1}{A_2} = \frac{a^2}{\pi r^2} = \frac{\pi^2 r^2}{4\pi r^2} = \frac{\pi}{4} = \frac{22}{28} = \frac{11}{14}$

$\therefore$ $A_1 : A_2 = 11 : 14$

46. (*a*) **16π cm**

Let r cm be the radius of the circle.

According to the problem, we have

$$\pi r^2 = 64\pi$$

$\Rightarrow$ $r^2 = 64$

$\Rightarrow$ $r = \sqrt{64} = 8$

$\therefore$ Required length of circumference of the circle

$$= 2\pi r$$
$$= 2\pi \times 8 \text{ cm}$$
$$= 16\pi \text{ cm}$$

47. (*a*) **10 m**

Let r_1 m and r_2 m be the radii of the two given circular parks, where $2r_1 = 16$ and $2r_2 = 12$.

$\Rightarrow$ $r_1 = 8$ and $r_2 = 6$

Then the sum of the areas of these two parks

$$= (\pi 8^2 + \pi 6^2)\ \text{m}^2$$
$$= (64 + 36)\pi\ \text{m}^2$$
$$= 100\pi\ \text{m}^2$$

If r' be the radius of the new park, then according to the condition of the problem, we have

$$\pi r'^2 = 100\pi$$

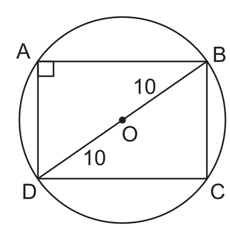
$\Rightarrow$ $r'^2 = 100$

$\Rightarrow$ $r' = 10$

Hence, the radius of the new park would be 10 m.

48. (*b*) **200 cm^2**

Let ABCD be a square inscribed in a circle with centre at O and radius OB = OD = 10 cm.



$\therefore$ $OB + OD = DB = 20$ cm

$\angle DAB = 90°$

$\therefore$ From ΔABD, we have by Pythagoras' theorem,

$$AB^2 + AD^2 = 20^2$$

$\Rightarrow$ $a^2 + a^2 = 400$

where a cm is the side of the square

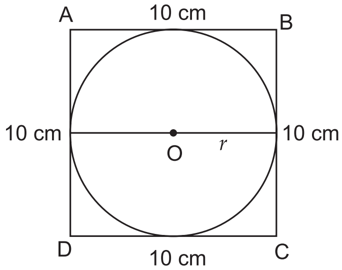
$\therefore$ $2a^2 = 400$

$\Rightarrow$ $a^2 = 200$

Hence, the required area of the square = 200 cm^2.

49. (*d*) **25π cm^2**

Let r cm be radius of the circle.



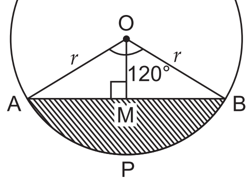
Then since the circle is inscribed within the square.

$\therefore \quad 2r = 10$

$\Rightarrow \quad r = \frac{10}{2} = 5$

Hence, the required area of the circle is $\pi r^2 = 25\pi$ cm^2

50. (*b*) $\left(\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right)r^2$



We draw OM ⊥ AB.

Then $\quad \angle AOM = \frac{1}{2}\angle AOB = \frac{1}{2} \times 120° = 60°.$

$\therefore$ From ΔAOM, we have

$$AM = r \sin 60° = \frac{r\sqrt{3}}{2}$$

and $\quad OM = r \cos 60° = \frac{r}{2}$

$\therefore \quad$ Area of $\Delta AOB = OM \times AM$

$$= \frac{r\sqrt{3}}{2} \times \frac{r}{2} = \frac{r^2\sqrt{3}}{4}$$

Also, area of the sector OAB $= \pi r^2 \times \frac{120}{360} = \frac{\pi r^2}{3}$

$\therefore \quad$ Required area of the segment APB

$$= \frac{\pi r^2}{3} - \frac{r^2\sqrt{3}}{4}$$

$$= \left(\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right) r^2$$

51. (*a*) **96%**

Let d cm be the original diameter of the circle. If it is increased by 40%, then the new diameter D cm of the circle becomes

$$D \text{ cm} = \left(d + \frac{40d}{100}\right) \text{cm} = \frac{7d}{5} \text{ cm}$$

Then the original area of the circle $= \frac{\pi d^2}{4}$ cm^2

and the new area of the circle $= \pi\left(\frac{7d}{10}\right)^2 = \frac{49\pi d^2}{100}$

$\therefore \quad$ Increase in area $= \left(\frac{49\pi d^2}{100} - \frac{\pi d^2}{4}\right)$ cm^2

$$= \frac{49\pi d^2 - 25\pi d^2}{100}$$

$$= \frac{24\pi d^2}{100}$$

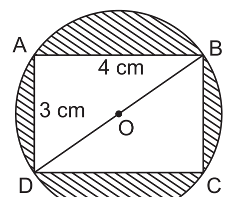
$\therefore$ Required percentage increase in area

$$= \frac{24\pi d^2}{100} \times \frac{4}{\pi d^2} \times 100\%$$

$$= 96\%$$

52. (*a*) **7.625 cm^2**

Let ABCD be the rectangle inscribed in a circle of centre O and diameter DOB. Given that AB = 4 cm and AD = 3 cm.



$\therefore \quad DB = \sqrt{4^2 + 3^2}$ cm = 5 cm

$\therefore$ Radius of the circle $= \frac{5}{2}$ cm

$\therefore$ Area of the circle $= \pi\left(\frac{5}{2}\right)^2$ cm^2 $= \frac{25\pi}{4}$ cm^2

$$= \frac{25 \times 3.14}{4} \text{ cm}^2$$

$$= \frac{78.50}{4} \text{ cm}^2$$

$$= 19.625 \text{ cm}^2$$

Area of the rectangle ABCD

$$= 4 \times 3 \text{ cm}^2 = 12 \text{ cm}^2$$

$\therefore$ Required area enclosed between the rectangle and the circle = Area of the shaded region

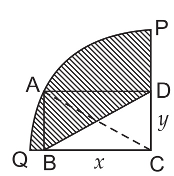
$$= (19.625 - 12) \text{ cm}^2$$

$$= 7.625 \text{ cm}^2$$

53. (*d*) **(5π + 17) units**

Let ABCD be a rectangle and PAQC is a quadrant of a circle of radius CQ = CP = 10 units. The perimeter of the rectangle ABCD is given to be 26 units.

To find the perimeter of the shaded region.



Let x units and y units be the length and breadth respectively of the rectangle. Then

$$2(x + y) = 26$$

$$\Rightarrow \quad x + y = 13 \quad \ldots(1)$$

Length of the diagonal of the rectangle

$$= \sqrt{x^2 + y^2} \text{ units}$$

$$\therefore \quad \sqrt{x^2 + y^2} = 10$$

[∵ In the figure, AC = QC = 10]

$$\Rightarrow \quad x^2 + y^2 = 100 \quad \ldots(2)$$

Now, from (1), we have

$$y = 13 - x \quad \ldots(3)$$

∴ From (2), we have

$$x^2 + (13 - x)^2 = 100$$

$$\Rightarrow \quad 2x^2 - 26x + 169 - 100 = 0$$

$$\Rightarrow \quad 2x^2 - 26x + 69 = 0$$

$$\therefore \quad x = \frac{26 \pm \sqrt{26^2 - 8 \times 69}}{8}$$

$$= \frac{26 \pm \sqrt{676 - 552}}{4}$$

$$= \frac{26 \pm \sqrt{124}}{4}$$

$$= \frac{26 \pm 11.135}{4}$$

$$= \frac{37.135}{4} \text{ or } \frac{14.865}{4}$$

$$= 9.3 \text{ or } 3.7 \text{ (approx.)}$$

∴ When $x = 9.3$, then $y = 3.7$ or when $x = 3.7$, then $x = 9.3$

∵ $x > y$ in one figure.

∴ $x = 9.3$ and $y = 3.7$

∴ BC = 9.3 units and DC = 3.7 units ...(4)

∴ BQ = 10 − 9.3 = 0.7 units ...(5)

and DP = (10 − 3.7) units

= 6.3 units [From, (2)] ...(6)

$$\therefore \quad BD = \sqrt{x^2 + y^2} = 10 \quad \ldots(7)$$

$$\text{Also,} \quad \text{arc PAQ} = \frac{2\pi r}{4} = \frac{\pi \times 20}{4} \text{ units}$$

$$= 5\pi \text{ units} \quad \ldots(8)$$

Hence, the required perimeter of the shaded region

= BD + DP + BQ + arc QAP

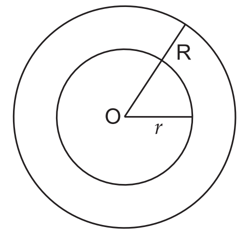
= (10 + 6.3 + 0.7 + 5π) units

[From (5), (6), (7) and (8)]

= (5π + 17) units

54. (*c*) **3.5 cm**

Let r cm and R cm be the radii of two concentric circles with the centre at O.



Then given that

$$\pi r^2 = 962.5 \quad \ldots(1)$$

$$\text{and} \quad \pi R^2 = 1386 \quad \ldots(2)$$

$$\text{From (1),} \quad r^2 = \frac{962.5}{3.14} = 306.53$$

$$\Rightarrow \quad r = \sqrt{306.53}$$

$$= 17.5 \text{ (approx.)}$$

$$\text{From (2),} \quad R^2 = \frac{1386}{3.14} = 441 \text{ (approx.)}$$

$$\therefore \quad R = \sqrt{441} = 21$$

$$\therefore \quad R - r = 21 - 17.5 = 3.5$$

Hence, width of the ring is 3.5 cm.

55. (*c*) **8 cm**

Length of the arc = 6π cm.

Area of the sector = 24π cm²

$$\frac{\theta}{360} \times 2\pi r = 6\pi$$

$$\theta = \frac{360 \times 3}{r}$$

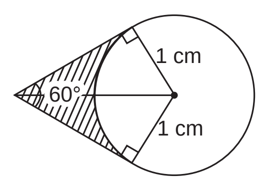
$$\text{and} \quad \frac{\theta}{360} \times \pi r^2 = 24\pi$$

$$\theta = \frac{24 \times 360}{r^2}$$

$$\therefore \quad \frac{360 \times 3}{r} = \frac{24 \times 360}{r^2}$$

$$r = 8 \text{ cm.}$$

56. (*a*) $\left(\sqrt{3} - \dfrac{\pi}{3}\right)$ **cm²**



$$\tan 30° = \frac{OA}{AC} = \frac{1}{\sqrt{3}}$$

$$AC = \sqrt{3}$$

∴ Quadrilateral BCA = ΔAOC + ΔBOC

$$= \frac{1}{2} \times 1 \times \sqrt{3} + \frac{1}{2} \times \sqrt{3} \times 1$$

$$= 2 \times \frac{\sqrt{3}}{2} = \sqrt{3}$$

∠A = 90°, ∠ACO = 30°,

∴ ∠AOC = 60°

∴ ∠BOC = 60°

$$\text{Area of the sector} = \frac{120}{360}\pi 1^2 = \frac{\pi}{3} \text{ cm}^2$$

Area of the shaded region = $\left(\sqrt{3} - \frac{\pi}{3}\right) \text{cm}^2$

57. (*b*) $\mathbf{10\sqrt{3}}$ **cm**

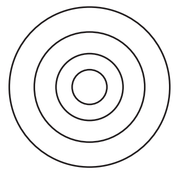
Radius of the biggest circle = 20 cm = r_1

Area = $\pi \times 20 \times 20$

Radius of the next circle = r_2

Equal area between the four circles

$$= \frac{\pi \times 20 \times 20}{4} = 100\,\pi$$



$$100\pi = \pi r_1^2 - \pi r_2^2$$

$$100\pi = \pi(400 - r_2^2)$$

$$100 = 400 - r_2^2$$

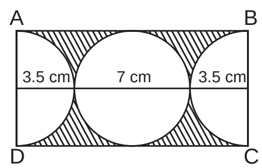
$$r_2^2 = 300, \; r_2 = 10\sqrt{3} \text{ cm}$$

SHORT ANSWER QUESTIONS

For Basic and Standard Levels

1. Length of the rectangle = 3.5 + 7 + 3.5 = 14 cm

Width of the rectangle = 3.5 + 3.5 = 7 cm



Area of the rectangle = 14 × 7 = 98 cm^2

Area of the unshaded portion

$$= \pi(3.5)^2 + \frac{1}{2}\pi(3.5)^2 + \frac{1}{2}\pi(3.5)^2 \text{ cm}^2$$

$$= \pi(12.25 + 12.25) \text{ cm}^2$$

$$= \frac{22}{7} \times 24.50 = 77 \text{ cm}^2$$

Area of the unshaded region

= rectangle – unshaded portion

= (98 – 77) cm^2 = **21 cm^2**

2. (*i*) Radius of the circle = 2*a* cm.

∴ Side of the square = 4*a* cm.

So the perimeter of the square = 4 × 4*a* = 16*a* cm.

Yes, because the side of the square is 4*a*.

(*ii*) $\frac{\theta}{360} \times 2\pi r = \frac{\phi}{360} \times 2\pi \times 2r$

$$\theta = 2\phi$$

Hence proved.

3. (*i*) $2\pi r_1 = 2\pi r_2$

∴ $r_1 = r_2$

Yes, areas are equal, because their radii are equal.

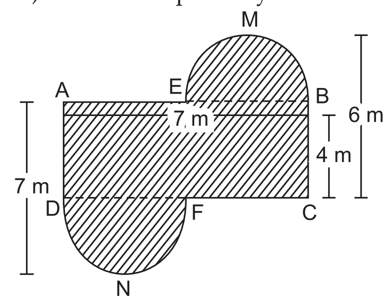
(*ii*) $\pi r_1^2 = \pi r_2^2$

$$r_1^2 = r_2^2$$

$$r_1 = r_2$$

Yes, circumference are equal, because their radii are equal.

4. ABCD is a rectangle with AB = 7 m and BC = 4 m. EMB and DNF are two semicircles of radius (6 – 4) m = 2 m and (7 – 4) m = 3 m respectively.



Area of the semicircle EMB

$$= \frac{\pi \times 2^2}{2} = 2\pi \text{ m}^2 \qquad \ldots(1)$$

Area of the semicircle DNF

$$= \frac{\pi \times 3^2}{2} \text{ m}^2 = \frac{9\pi}{2} \text{ m}^2 \qquad \ldots(2)$$

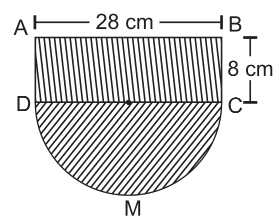
Area of the rectangle ABCD = (7 × 4) m^2 = 28 m^2 …(3)

∴ Required total area = Area of the shaded region

$$= \left(2\pi + \frac{9\pi}{2} + 28\right) \text{m}^2$$

$$= \left(\mathbf{\frac{13\pi}{2} + 28}\right) \text{m}^2$$

5. Let ABCD be a rectangle with sides AB = 28 cm and BC = 8 cm. A semicircle DMC is drawn with DC as a diameter.



To find the area and the perimeter of the total region ABCMD.

Area of the rectangle ABCD = (28 × 8) cm^2 = 224 cm^2

Area of the semicircle DMC

$$= \frac{\pi}{2} \times 14^2 \text{ cm}^2$$

$$= \frac{22}{7} \times \frac{14 \times 14}{2} \text{ cm}^2$$

$$= 308 \text{ cm}^2$$

∴ Total area of the shaded region

= (224 + 308) cm^2

= 532 cm^2.

Total perimeter = (28 + 8 + π × 14 + 8) cm

$$= \left(44 + \frac{22}{7} \times 14\right) \text{cm}$$

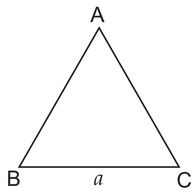
= (44 + 44)cm

= 88 cm

Hence, the required area is **532 cm²** and the required perimeter is **88 cm**.

For Standard Level

6. Let a cm be the side of the equilateral triangle ABC. Then its area is $\frac{\sqrt{3}}{4}a^2$ cm².



$$\therefore \quad \frac{\sqrt{3}}{4}a^2 = 121\sqrt{3} \quad \text{[Given]}$$

$$\Rightarrow \quad a^2 = 484$$

$$\Rightarrow \quad a = \sqrt{484} \text{ cm} = 22 \text{ cm}$$

∴ Perimeter of this triangle = $3a = 22 \times 3$ cm = 66 cm.

Let r cm be the radius of the circle.

Then according to the problem, we have

$$2\pi r = 66$$

$$\Rightarrow \quad r^3 = 66 \times \frac{1}{2} \times \frac{7}{22} \text{ cm}$$

$$= \frac{21}{2} \text{ cm}$$

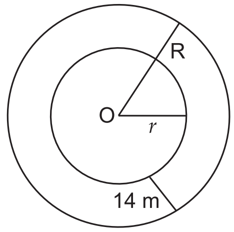
∴ The required area of this circle

$$= \pi r^2 = \frac{\pi \times 21 \times 21}{4} \text{ cm}^2$$

$$= \frac{22}{7} \times \frac{21 \times 21}{4} \text{ cm}^2$$

$$= \frac{693}{2} \text{ cm}^2 = \mathbf{346.5\ cm^2}$$

7. Let O be the common centre of two concentric circles of radii r m and R m where R > r.



Given that

$$R - r = 14 \quad \ldots(1)$$

Also, $\quad 2\pi R = 528$

$$\Rightarrow \quad R = 528 \times \frac{1}{2} \times \frac{7}{22} \text{ m} = 84 \text{ m} \quad \ldots(2)$$

∴ From (1), $\quad r = (84 - 14)$ m = 70 m $\quad \ldots(3)$

∴ Area of the track = $\pi(R^2 - r^2)$

$$= \frac{22}{7} \times \left(84^2 - 70^2\right) \text{ m}^2$$

$$= \frac{22}{7} \times (84 + 70) \times (84 - 70) \text{ m}^2$$

$$= \frac{22}{7} \times 154 \times 14^2 \text{ m}^2$$

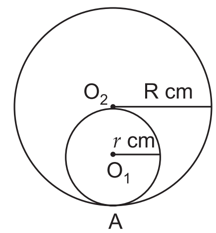
$$= 6776 \text{ m}^2$$

∴ Required cost of leveling the road

$$= ₹\left(\frac{1}{2} \times 6776\right)$$

$$= \mathbf{₹\,3388}$$

8. Let R cm and r cm be the radii of two circles, where R > r, which touch each other internally at A, O_2 and O_1 being their respective centres.



Given that $\quad R - r = 6 \quad \ldots(1)$

and $\quad \pi r^2 + \pi R^2 = 16\pi$

$\Rightarrow \quad R^2 + r^2 = 116 \quad \ldots(2)$

From (1), $\quad R = r + 6 \quad \ldots(3)$

∴ From (2) and (3), we get

$$r^2 + (r + 6)^2 = 116$$

$$\Rightarrow \quad 2r^2 + 12r - 80 = 0$$

$$\Rightarrow \quad r^2 + 6r - 40 = 0$$

$$\Rightarrow \quad r = \frac{-6 \pm \sqrt{6^2 + 160}}{2}$$

$$= \frac{-6 \pm \sqrt{196}}{2}$$

$$= \frac{-6 \pm 14}{2} = 4$$

(Neglecting the negative value of r).

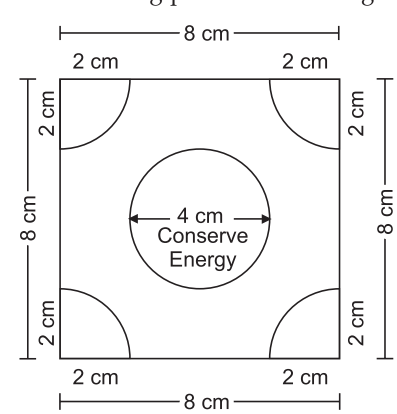
$\therefore \quad R = 6 + 4 = 10 \quad$ [From (3)]

Hence, the required radii of the smaller and larger circles are respectively 4 cm and 10 cm.

VALUE-BASED QUESTIONS

For Basic and Standard Levels

1. (*i*) Area of the remaining portion of the badge:



$$= \text{Square} - \left(\pi r_1^2 + 4 \times \frac{1}{4}\pi r_2^2\right)$$

$$= \pi r_1^2 + 4 \times \frac{1}{4}\pi r_2^2$$

$$= \pi(r_1^2 + r_2^2)$$

$$\pi(4 + 4) = \frac{22}{7} \times 8$$

$$= \frac{176}{7}\text{ cm}^2$$

Remaining portion

$$= 64 - \frac{176}{7} = \mathbf{\frac{272}{7}\ cm^2}$$

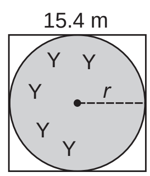
(*ii*) Save water, make car pools, switch off electrical appliances when not in use, repair leaking taps, use LED, use solar energy etc. are some phrases which the students can write in the quadrants.

(*iii*) Awareness about energy conservation, creative thinking, decision-making and effective communicative.

2. (*i*) Area used by farmer A to plant trees

$$= \frac{22}{7} \times 7.7\text{ m} \times 7.7\text{ m} = 186.34\text{ m}^2$$

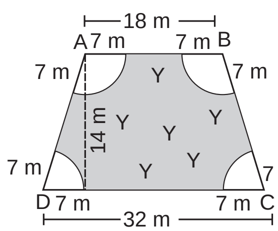
Area used by farmer B to plant trees



$$= \left[\frac{1}{2} \times (18\text{ m} + 32\text{ m}) \times 14\text{ m}\right] - \frac{22}{7} \times 7\text{ m} \times 7\text{m}$$

$$= \left[\frac{1}{2} \times 50\text{ m} \times 14\text{ m}\right] - 154\text{ m}^2$$

$$= 350\text{ m}^2 - 154\text{ m}^2 = 196\text{ m}^2$$



Farmer B uses more area for planting trees.

More area = 196 m^2 – 186.34 m^2 = 9.66 m^2.

(*ii*) Awareness about the environment.

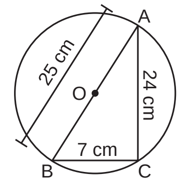
UNIT TEST 1

For Basic Level

1. (*d*) $\boldsymbol{r = r_1 + r_2}$

$$2\pi r_1 + 2\pi r_2 = 2\pi r$$
$$2\pi(r_1 + r_2) = 2\pi r$$
$$r_1 + r_2 = r$$

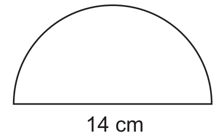
2. (*d*) **25π**



$$AB^2 = 7^2 + 24^2$$
$$AB^2 = 49 + 576 = 625$$
$$AB = 25,\ OA = 12.5$$
$$\therefore\quad 2\pi r = 2 \times \pi \times 12.5 = 25\pi$$

3. (*b*) **36 cm**

Let r cm be the radius of the protractor. Then $r = 7$.



Then the perimeter of the protractor = (πr + 14) cm

$$= \left(\frac{22}{7} \times 7 + 14\right)\text{ cm}$$

$$= 36\text{ cm}$$

4. (*c*) **22 cm**

$$\pi r^2 = 154\text{ cm}^2$$
$$r^2 = \frac{154 \times 7}{22} = 49\text{ cm}$$
$$r = 7\text{ cm}$$

Radius of the first circle = $\frac{7}{2}$ cm

Circumference = $2 \times \frac{22}{7} \times \frac{7}{2} = 22$ cm

5. (*c*) $\mathbf{\frac{77}{8}}$

$$2\pi r = 22$$
$$r = \frac{22 \times 7}{2 \times 22} = \frac{7}{2}\text{ cm}$$

Area of its quadrant is

$$\frac{1}{4} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} = \frac{77}{8}\text{ cm}^2$$

6. (*d*) **16 : 81**

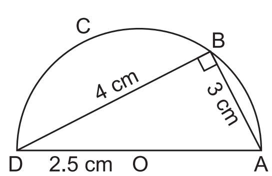
Ratio of circumference of 2 circles

$$\frac{2\pi r_1}{2\pi r_2} = \frac{4}{9}$$

$$\frac{\pi r_1^2}{\pi r_2^2} = \left(\frac{4}{9}\right)^2 = \frac{16}{81}$$

$$\frac{r_1}{r_2} = \frac{4}{9}$$

7. (*a*) $\mathbf{\frac{104}{7}}$



$$AB = 3\text{ cm}$$
$$BD = 4\text{ cm}$$
$$AD^2 = 9 + 16 = 25$$
$$AD = 5\text{ cm}$$

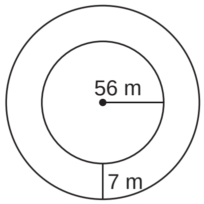
Radius OD = 2.5 cm

Perimeter of the semicircular region without the circle

$= \pi r + 4 + 3$

$= \frac{22}{7} \times 2.5 + 7 = \frac{104}{7}$ cm

8. Area of the road = (Area of the park + road) – (Area of the park)



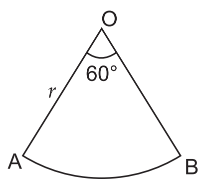
$= \pi \times 63 \times 63 - \pi \times 56 \times 56$

$= \pi\, [(63)^2 - (56)^2]$

$= \pi\, (63 + 56)\, (63 - 56)$ m^2

$= \pi\, (119.7) =$ **2618 m^2**

9. Let AB be the arc of the circle with centre at O where ∠AOB = 60°. Let r be the radius of the circle.



Then length of the arc AB

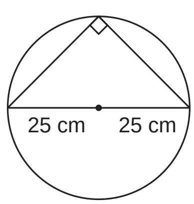
$= 2\pi r \times \frac{60}{360} = \frac{\pi r}{3}$

Given that $\frac{\pi r}{3} = 20$

$\therefore \quad r = \frac{60}{\pi}$

Hence, the required length of the radius is $\frac{60}{\pi}$ cm.

10. Area of the triangle $= \frac{1}{2} \times 5 \times 2.4 =$ **6 cm^2**



11. Area of the sector $= \frac{60}{360} \times \pi \times 6 \times 6 = 6\pi$

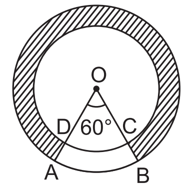
Total area of the circle $= \frac{22}{7} \times 6 =$ **$\frac{132}{7}$ cm^2**

$\therefore$ Area of the major (remaining) sector

$= 36\pi - 6\pi = 30\pi$

$= 30 \times \frac{22}{7} =$ **$\frac{660}{7}$ cm^2**

12. Given that two concentric circles with common centre O and radii 2.1 cm and 4.2 cm enclose a region which is shaded in the figure where ∠AOB = 60°.



To find the area of the shaded region.

$\therefore$ Required area $= \pi(4.2^2 - 2.1^2) \times \frac{300}{360}$ cm^2

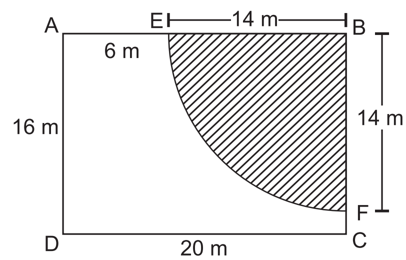
$= \frac{22}{7} \times (4.2 + 2.1)(4.2 - 2.1) \times \frac{5}{6}$ cm^2

$= \frac{22}{7} \times \frac{6.3 \times 2.1 \times 5}{6}$ cm^2

$= \frac{11 \times 6.3 \times 2.1 \times 5}{3 \times 7}$ cm^2

$= 34.65$ cm^2

13. ABCD is a rectangular field where AB = 20 m and BC = 16 m. EBF is the grazing field which is a circular sector of radius 14 m and centre at B.



Area of sector BEF $= \frac{\pi r^2}{4}$

$= \frac{\pi \times 14^2}{4}$ m^2

$= \frac{22}{7 \times 4} \times 14 \times 14$ m$^2 = 154$ m^2

UNIT TEST 2

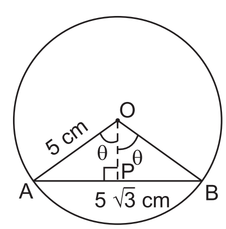
For Standard Level

1. (*d*) **$\frac{25\pi}{3}$ cm^2**

Length of chord AB $= 5\sqrt{3}$ cm

Radius of a circle = 5 cm

In ΔAPO, we have



$\sin\theta = \frac{AP}{AO}$

$\Rightarrow \quad \sin\theta = \frac{5\sqrt{3}}{2 \times 5}$

$\Rightarrow \quad \sin\theta = \frac{\sqrt{3}}{2}$

$\Rightarrow \quad \sin\theta = \sin 60°$

$\Rightarrow \qquad \theta = 60°$

$\therefore \qquad \angle AOB = 120° \ (= 2\theta)$

$$\text{Area of sector OAB} = \frac{120}{360} \times \pi \times (5)^2 \text{ cm}^2$$

$$= \frac{25\pi}{3} \text{ cm}^2$$

2. (*c*) **8 cm**

Length of an arc = 5π cm

Area of a sector = 20π cm^2

$$\text{Now, } \frac{\text{Length of an arc}}{\text{Area of a sector}} = \frac{5\pi}{20\pi}$$

$$\Rightarrow \qquad \frac{2\pi r}{\pi r^2} = \frac{5\pi}{20\pi}$$

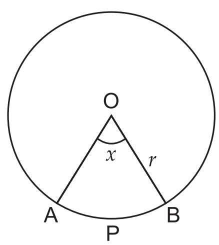
$$\Rightarrow \qquad \frac{2}{r} = \frac{1}{4}$$

$$\Rightarrow \qquad r = 8 \text{ cm}$$

3. (*d*) **100°**

O is the centre of a circle of radius r units, OAPB is a sector of the circle where $\angle AOB = x$ in degrees.

$$\text{Area of sector OAB} = \pi r^2 \times \frac{x}{360} \text{ sq units}$$



According to the problem, we have

$$\frac{\pi r^2 x}{360°} = \frac{5}{18} \times \pi r^2$$

$$\Rightarrow \qquad x = \frac{5}{18} \times 360° = 100°$$

4. (*a*) **1.1**

$$\text{Required distance} = 2\pi \times \frac{35}{2} \text{ cm}$$

$$= 2 \times \frac{22}{7} \times \frac{35}{2} \text{ cm}$$

$$= 110 \text{ cm}$$

$$= 1.1 \text{ m}$$

5. (*d*) **550 cm^2**

$$\text{Required area} = \pi(20^2 - 15^2) \text{ cm}^2$$

$$= \frac{22}{7} \times (20 + 15)(20 - 15) \text{ cm}^2$$

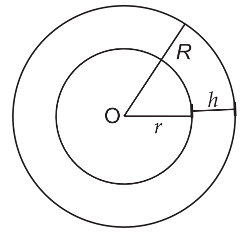
$$= \frac{22}{7} \times 35 \times 5 \text{ cm}^2$$

$$= 550 \text{ cm}^2$$

6. (*b*) πh **(*h* + 2*r*)**

It R be the radius of the bigger circle surrounding the circular path surrounding the circular region of radius r, then

$$R = r + h \qquad \ldots(1)$$



$\therefore$ Required area of the circular path

$$= \pi(R^2 - r^2)$$

$$= \pi[(r + h)^2 - r^2] \qquad \text{[From (1)]}$$

$$= \pi[h^2 + 2hr] = \pi h(h + 2r)$$

7. (*b*) **26**

Let R cm be the radius of the larger circle and let r_1 cm and r_2 cm be the radii of the two smaller circles.

Then $r_1 = 5$ cm and $r_2 = 12$ cm.

Now, according to the condition of the problem, we have

$$\pi R^2 = \pi\left(r_1^2 + r_2^2\right)$$

$$= \pi(5^2 + 12^2)$$

$$= \pi(25 + 144)$$

$$= \pi \times 169$$

$$\therefore \qquad R^2 = 169$$

$$\Rightarrow \qquad R = \sqrt{169} = 13$$

Hence, the required diameter of the larger circle

$$= 13 \times 2 \text{ cm}$$

$$= 26 \text{ cm}$$

8. The wheel goes 19.8 km in 1 hour

1 hour = 60 × 60 seconds

diameter = 70 cm, r = 35 cm.

$$\text{One revolution} = 2 \times \frac{22}{7} \times 35 = 220 \text{ cm} = 2.20 \text{ m}$$

In 60 × 60 seconds wheel goes 19.8 km = 19800 m

$$\text{In 10 seconds, it goes } \frac{19800}{60 \times 60} \times 10 = 55 \text{ m}$$

$$\text{Number of revolutions} = \frac{55 \text{ m}}{2.20 \text{ m}} = \mathbf{25}$$

9. Radius of bigger circle = 14 cm

Radius of smaller circle = 7 cm

Area of the bigger circle = $\pi r^2 = \pi(14)^2 = 196\pi$ cm^2

Area of the smaller circle = $\pi r^2 = \pi(7)^2 = 49\pi$ cm^2

$$\text{Area of sector OBD} = \frac{\theta}{360} \times \pi r^2$$

$$= \frac{40}{360} \times \pi \times 49$$

$$= \frac{49\pi}{9} \text{ cm}^2$$

$$\text{Area of sector OAC} = \frac{40}{360} \times \pi \times 196$$

$$= \frac{196\pi}{9} \text{ cm}^2$$

Shaded area = area of bigger circle – area of smaller circle – (area of sector OAC – area of sector OBD)

$$= 196\pi - 49\pi - \left(\frac{196\pi}{9} - \frac{49\pi}{9}\right)$$

$$= 147\pi - \left(\frac{147\pi}{9}\right)$$

$$= 147\pi\left(1 - \frac{1}{9}\right)$$

$$= 147\pi \times \frac{8}{9}$$

$$= 147 \times \frac{22}{7} \times \frac{8}{9}$$

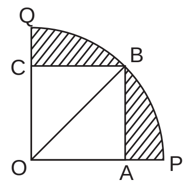
$$= \frac{56 \times 22}{3} = \mathbf{\frac{1232}{3}\ cm^2}$$

10. **Given:** OB = 14 cm. $OB^2 = a^2 + a^2$

$196 = 2a^2$

Area of the square $= a^2 = 98$

$\therefore \quad a = 7\sqrt{2}$ cm.



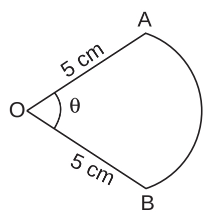
Area of the shaded region

= ar(quadrant) – ar(square)

$$= \left(\frac{1}{4} \times \frac{22}{7} \times 14 \times 14\right) - 98$$

$= 154 - 98 =$ **56 cm²**

11. Perimeter of the sector AOB



$$= \left(\frac{\theta}{360} \times 2 \times \frac{22}{7} \times 5 + 10\right) \text{cm} = 50$$

$$\frac{\theta}{360} \times 2 \times \frac{22}{7} \times 5 = 40$$

$$\theta = \frac{40 \times 360 \times 7}{2 \times 22 \times 5} = \frac{2 \times 7 \times 360}{11}$$

$$\text{Area of the sector} = \frac{\theta}{360} \times \pi r^2$$

$$= \frac{\frac{2 \times 7 \times 360}{11}}{360} \times \frac{22}{7} \times 25$$

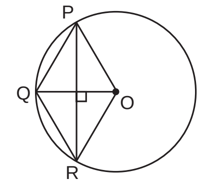
$$= \frac{2 \times 7}{11} \times \frac{22}{7} \times 25 = \mathbf{100\ cm^2}$$

12. OP = OQ = OR = r [Radii of the circle] ...(1)

Also, OP = PQ = QR = OR [Sides of a rhombus] ...(2)

From (1) and (2), we get

ΔOPQ and ΔORQ are equilateral triangles with each side $= r$



Now, ar(ΔOPQ) + ar(ΔORQ) = ar(rhombus OPQR)

$$\Rightarrow \quad \frac{\sqrt{3}}{4}r^2 + \frac{\sqrt{3}}{4}r^2 = 32\sqrt{3}\ \text{cm}^2$$

$$\Rightarrow \quad 2 \times \frac{\sqrt{3}}{4} \times r^2 = 32\sqrt{3}\ \text{cm}^2$$

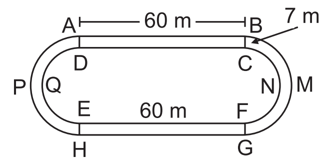
$$r^2 = 32 \times 2\ \text{cm}^2 = 64\ \text{cm}^2$$

$\therefore \quad r =$ **8 cm**

13. Sum of the areas of the two rectangles ABCD and EFGH

$= 2 \times 60 \times 7\ m^2$

$= 840\ m^2$ ...(1)



Let the radius of the inner semicircle CNF or DQE be r m.

Then the radius of the outer semicircle BMG or APH is $(r + 7)$ m.

Then the total inside perimeter of the track

$= (\pi r + \pi r + 60 + 60)$ m

$= (2\pi r + 120)$ m

∴ According to the problem, we have

$2\pi r + 120 = 340$

$\Rightarrow \quad 2\pi r = 340 - 120$

$= 220$

$$\therefore \quad r = 220 \times \frac{1}{2} \times \frac{7}{22} = 35$$

$\therefore \quad R = (r + 7) = 35 + 7 = 42$

∴ Required area of the entire track

$= [\pi(R^2 - r^2) + 840]\ m^2$ [From (1)]

$= \{\pi(42^2 - 35^2) + 840\}\ m^2$

$$= \left\{\frac{22}{7} \times (42 + 35) \times (42 - 35) + 840\right\} m^2$$

$$= \left(\frac{22}{7} \times 77 \times 7 + 840\right) m^2$$

$= (1694 + 840)\ m^2$

$= 2534\ m^2$

Required outer perimeter of the track

$= (2\pi R + 60 + 60)$ m

$$= \left(2 \times \frac{22}{7} \times 42 + 120\right) \text{m}$$

$= (264 + 120)$ m = 384 m