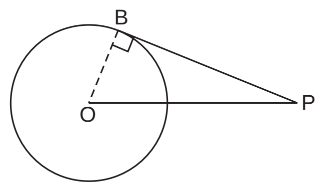
CHAPTER **12**

Circles

EXERCISE 12

1. (*i*) Let O be the centre of the circle and let P be a point 20 cm away from the centre and PB be a tangent to the circle at point B.

Join OB.



Then, radius OB = 5 cm and OP = 20 cm.

Since the tangent at any point of a circle is perpendicular to the radius through the point of contact and PB is a tangent at B and OB is the radius through B, therefore OB $\perp$ PB.

In right ΔOBP, we have

$OB^2 + PB^2 = OP^2$ [By Pythagoras' Theorem]

$\Rightarrow \quad (5 \text{ cm})^2 + PB^2 = (20 \text{ cm})^2$

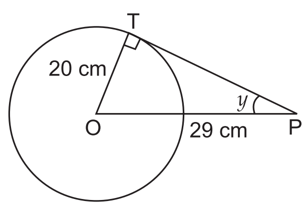
$\Rightarrow \quad PB^2 = (400 - 25) \text{ cm}^2$

$\Rightarrow \quad PB^2 = 375 \text{ cm}^2$

$\Rightarrow \quad PB = \mathbf{5\sqrt{15}\ cm}$.

(*ii*) Let PT be the tangent to the circle with centre at 0. We join OT. We have

OT = radius of the circle = 20 cm



Also, $\angle OTP = 90°$

[$\because$ PT is the tangent and OT is the radius]

We have OP = 29 cm [Given]

$\therefore$ From ΔOPT, by Pythagoras' theorem, we have

$OP^2 = OT^2 + TP^2$

$\Rightarrow \quad TP^2 = OP^2 - OT^2$

$= (OP + OT)(OP - OT)$

$= (29 + 20)(29 - 20)$

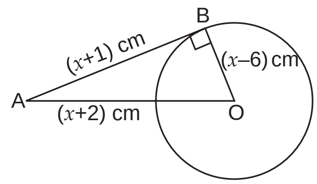
$= 49 \times 9$

$\therefore \quad TP = \sqrt{49 \times 9}$

$= 7 \times 3 = 21$

$\therefore$ Required length of the tangent from P to the circle is **21 cm**.

2. Since the tangent at any point of a circle is perpendicular to the radius through the point of contact and AB is a tangent at B and OB is the radius through B, therefore OB $\perp$ AB.



In right ΔOBA, we have

$OB^2 + AB^2 = AO^2$ [By Pythagoras' Theorem]

$\Rightarrow \quad (x - 6)^2 + (x + 1)^2 = (x + 2)^2$

$\Rightarrow \quad x^2 - 12x + 36 + x^2 + 2x + 1 = x^2 + 4x + 4$

$\Rightarrow \quad x^2 - 12x + 2x - 4x + 36 + 1 - 4 = 0$

$\Rightarrow \quad x^2 - 14x + 33 = 0$

$\Rightarrow \quad x^2 - 3x - 11x + 33 = 0$

$\Rightarrow \quad x(x - 3) - 11(x - 3) = 0$

$\Rightarrow \quad (x - 3)(x - 11) = 0$

$\Rightarrow$ Either $(x - 3) = 0$ or $(x - 11) = 0$

$\Rightarrow$ $x = 3$ (Rejected) or $x = 11$

$AB = (x + 1)$ cm $= (11 + 1)$ cm $= 12$ cm

$OB = (x - 6)$ cm $= (11 - 6)$ cm $= 5$ cm

$OA = (x + 2)$ cm $= (11 + 2)$ cm $= 13$ cm

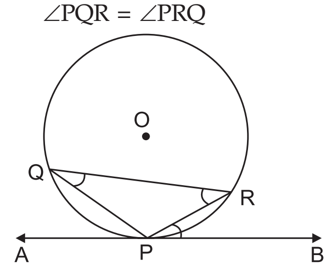
Hence, **AB = 12 cm, OB = 5 cm and OA = 13 cm**

3. Given that arc PQ = arc PR

$\therefore$ Chord PQ = Chord PR

Let AB be a tangent to the circle with centre at O at the point P.

$\therefore \quad \angle PQR = \angle PRQ$



But $\angle PQR = \angle RPB$

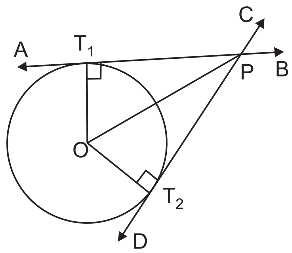
But these are alternate angles.

$\therefore$ QR $\parallel$ PB

4. Given that lines AB and CD are the two tangents to the circle with centre at O, through an external point P.

Let these two lines touch the circle at T_1 and T_2. To prove that OP is the internal bisector of $\angle APD$, i.e.

$\angle APO = \angle DPO$



Construction: We join OT_1 and OT_2.

In ΔOPT_1 and ΔOPT_2, we have

$$\angle OT_1P = \angle OT_2P = 90°$$

$$OT_1 = OT_2 \qquad \text{[Radii of the same circle]}$$

and the hypotenuse OP is common.

$\therefore$ By RHS congruence criterion, we have

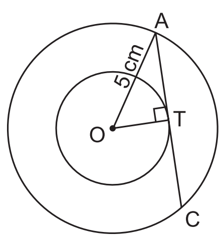
$$\Delta OPT_1 \cong \Delta OPT_2$$

$$\angle T_1PO = \angle T_2PO \qquad \text{[By CPCT]}$$

i.e. $\angle APO = \angle DPO$

Hence, proved.

5. (*i*) Let O be the common centre of the two concentric circles. Let the chord AC of length 8 cm touch the smaller circle at T. Then T is the mid-point of the chord AC. Hence, AT = $\frac{1}{2} \times 8$ cm = 4 cm.



Since AC is a tangent to the smaller circle at T,

$\therefore$ $\angle OTA = 90°$.

$\therefore$ In ΔOTA, we have OA = radius of the larger circle = 5 cm and AT = 4 cm.

$\therefore$ By Pythagoras' theorem, we have

$$OT = \sqrt{OA^2 - AT^2}$$

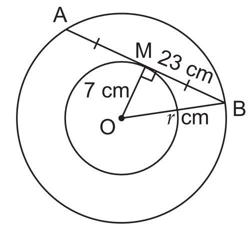
$$= \sqrt{25 - 16}$$

$$= \sqrt{9}$$

$$= 3$$

Hence, the required radius of the smaller circle is **3 cm**.

(*ii*) Let O be the common centre of two concentric circles. Let the chord AB of length 46 cm touch the smaller circle of radius r cm at the point M. Then M is the mid-point of the chord AB. We join OM and OB. Then, OM = 7 cm, OB = r cm, MB = $\frac{1}{2}AB = \frac{1}{2} \times 46$ cm = 23 cm and $\angle OMB = 90°$.



$\therefore$ In OBM, we have by Pythagoras' theorem,

$$OB^2 = OM^2 + MB^2$$

$\Rightarrow$ $r^2 = 7^2 + 23^2$

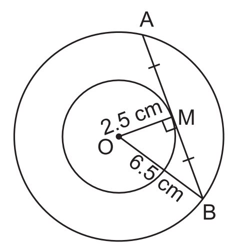
$= 49 + 529$

$= 578$

$\therefore$ $r = \sqrt{578} = 17\sqrt{2}$

Hence, the required value of r is $\mathbf{17\sqrt{2}}$ cm.

6. (*i*) Let O be the common centre of two concentric circles. Let AB be a chord of the bigger circle, which touches the smaller circle at M. Then M is the mid-point of AB and $\angle OMB = 90°$.



We join OM and OB. Then,

OM = radius of the smaller circle = 2.5 cm and

OB = radius of the bigger circle = 6.5 cm.

Let AB = x cm

Then $MB = \frac{1}{2}AB = \frac{1}{2} \times x = \frac{x}{2}$ ...(1)

$\therefore$ From ΔOMB, we have, by Pythagoras' theorem,

$$OB^2 = OM^2 + MB^2$$

$\Rightarrow$ $6.5^2 = 2.5^2 + \frac{x^2}{4}$ [From (1)]

$\Rightarrow$ $\frac{x^2}{4} = 6.5^2 - 2.5^2$

$= (6.5 + 2.5)(6.5 - 2.5)$

$= 9 \times 4$

$= 36$

$\Rightarrow$ $x^2 = 36 \times 4$

$= 144$

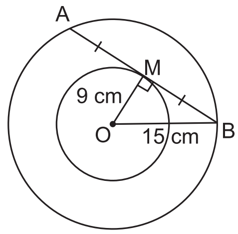
$\Rightarrow$ $x = \sqrt{144}$

$= 12$

Hence, the required length of the chord of the larger circle is **12 cm**.

(*ii*) Let O be the common centre of two concentric circle of radius is $\frac{18 \text{ cm}}{2}$ = 9 cm and $\frac{30 \text{ cm}}{2}$ = 15 cm.

Let AB be a chord of the bigger circle, touching the smaller circle at M. Then M is the mid-point of AB and ∠OMB = 90°.



We join OM and OB.

Then OM = 9 cm and OB = 15 cm [Given]

Let AB = x cm

Then $\quad MB = \frac{x}{2}$ cm ...(1)

∴ From ΔOMB, by Pythagoras' theorem, we get

$$OB^2 = OM^2 + MB^2$$

$$\Rightarrow \quad 15^2 = 9^2 + \frac{x^2}{4} \qquad \text{[From (1)]}$$

$$\Rightarrow \quad \frac{x^2}{4} = 15^2 - 9^2$$

$$= (15 + 9)(15 - 9)$$

$$= 24 \times 6$$

$$= 144$$

$$\therefore \quad \frac{x}{2} = 12$$

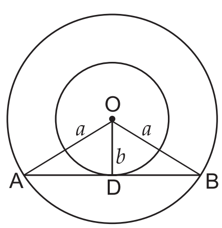
$$\Rightarrow \quad x = 24$$

Hence, the required length of the chord AB is **24 cm**.

(*iii*) Let AB be a chord of the larger of the two concentric circles with radius a and b respectively such that $a > b$

Here, radius of bigger circle = a

Radius of smaller circle = b



In ΔOAD $\quad$ OD ⊥ AD

$$\therefore \quad OD^2 + AD^2 = OA^2$$

[By Pythagoras' Theorem]

$$b^2 + AD^2 = a^2$$

$$AD^2 = a^2 - b^2$$

$$AD = \sqrt{a^2 - b^2}$$

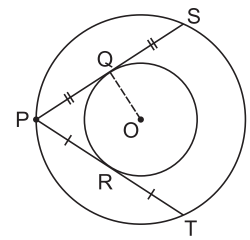
Similarly from ΔOBD

We get $\quad BD = \sqrt{a^2 - b^2}$

Now $\quad AB = AD + BD$

$$= 2\sqrt{a^2 - b^2}$$

7. Given that O is the common centre of two concentric circles. PS and PT are two tangents to the smaller circle drawn from an external point P on the bigger circle, touching the smaller circle at the points Q and R respectively. Given that PR = 5 cm.



Since, PR and PQ are two tangents to the smaller circle, drawn from an outside point P, we have

$$PQ = PR = 5 \text{ cm}$$

$$\therefore \quad 2PQ = 2PR$$

$$= 2 \times 5 \text{ cm}$$

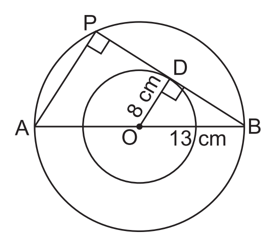
$$= 10 \text{ cm}$$

$$\Rightarrow \quad PS = 10 \text{ cm}$$

Since Q and R are two mid-points of the chords PS and PT respectively.

Hence, the required length of the chord PS is **10 cm**.

8. Given that O is the common centre of two concentric circles of radii 13 cm and 8 cm. AB is a diameter of the bigger circle and BD is a tangent to the smaller circle touching it at D. Let BD produced intersect the bigger circle at P. To find the length of AP.



We join OD.

Then D is the mid-point of PB and ∠ODB = 90°.

Now, OD = radius of the smaller circle = 8 cm [Given]

OB = radius of the bigger circle = 13 cm [Given]

∴ From ΔODB, we have by Pythagoras' theorem,

$$OB^2 = OD^2 + DB^2$$

$$\Rightarrow \quad 13^2 = 8^2 + DB^2$$

$$\Rightarrow \quad DB^2 = 169 - 64 = 105 \qquad ...(1)$$

$$\therefore \quad PB^2 = (2DB)^2 = 4DB^2$$

$$= 4 \times 105 \qquad \text{[From (1)]}$$

$$= 420 \qquad ...(2)$$

Now, $\angle APB = 90°$ [$\because$ angle in a semi-circle is 90°]

$\therefore$ From ΔAPB, we have by Pythagoras' theorem,

$$AB^2 = AP^2 + PB^2$$

$\Rightarrow$ $(2OB)^2 = AP^2 + 420$ [From (2)]

$\Rightarrow$ $(2 \times 13)^2 = AP^2 + 420$

$\Rightarrow$ $4 \times 169 - 420 = AP^2$

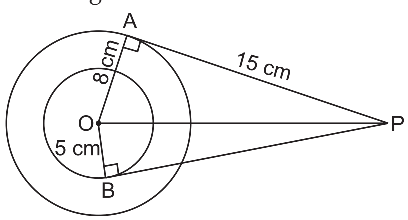
$\Rightarrow$ $AP^2 = 676 - 420 = 256$

$\therefore$ $AP = \sqrt{256} = 16$

$\therefore$ The required length of AP is **16 cm**.

9. Given that O is the centre of two concentric circles of radii 8 cm and 5 cm. From an external point P, two tangents PA and PB are drawn to the circle, touching them at A and B respectively. Given that AP = 15 cm.

To find the length of BP.



We join OP.

Clearly, $\angle OAP = 90° = \angle OBP$.

$\therefore$ From ΔOAP, we have by Pythagoras' theorem,

$$OP^2 = AP^2 + OA^2$$
$$= 15^2 + 8^2 = 225 + 64$$
$$= 289 \quad \ldots(1)$$

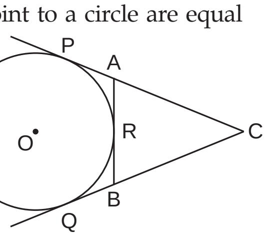
Again, from ΔOBP, we have by Pythagoras' theorem,

$$BP^2 = OP^2 - OB^2$$
$$= 289 - 5^2 \quad \text{[From (1)]}$$
$$= 289 - 25$$
$$= 264$$

$\therefore$ $BP = \sqrt{264} \simeq 16.25$

Hence, the required length of BP is **16.25 cm (approx.)**.

10. (*i*) We know that the lengths of the tangents drawn from an external point to a circle are equal



$\therefore$ $CP = CQ$ [Tangent from C] ...(1)
$BQ = BR$ [Tangent from B]

Now $CQ = CP$ [From (1)]

$\Rightarrow$ $BC + BQ = CP$

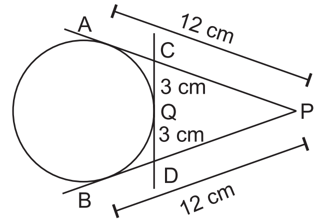
$\Rightarrow$ $BC + BR = CP$ [From (1)]

$\Rightarrow$ $BC + 4 \text{ cm} = 11 \text{ cm}$

$\Rightarrow$ $BC = 7 \text{ cm}$

Hence, **BC = 7 cm.**

(*ii*) Given that PA and PB are tangents to a circle from an external point P. CD is another tangent touching the circle at Q and cutting PA and PB at C and D respectively.



Given that PA = 12 cm,
QC = QD = 3 cm.

To find PC + PD

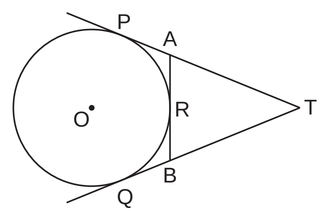
We have PC = PA – CA
= 12 – QC
= 12 – 3
= 9

Similarly, PD = PB – BD
= 12 – QD
= 12 – 3
= 9

Hence, PC + PD = 9 + 9 = 18

Hence, the required length of PC + PD is **18 cm**.

11. We know that the lengths of tangents drawn from an external point to a circle are equal.



$\therefore$ $TP = TQ$ [Tangents from T]
$AP = AR$ [Tangents from A] ...(1)
$BQ = BR$ [Tangents from B]

Now, $TP = TQ$ [From (1)]

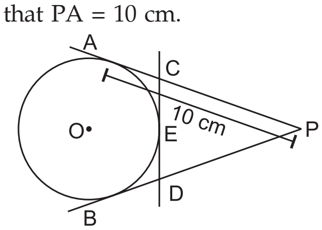
$\Rightarrow$ $TA + AP = TB + BQ$

$\Rightarrow$ $TA + AR = TB + BR$ [Using (1)]

Hence, **TA + AR = TB + BR**

12. (*i*) Given that PA and PB are two tangents drawn from an external point P to a circle with centre at O, touching it at A and B respectively. At another point E on the same circle, a third tangent CD is drawn cutting PA and PB at C and D respectively.

Given that PA = 10 cm.



To find the perimeter CP + DP + CD of ΔPCD.

Required perimeter of ΔPCD

$= CP + DP + CD$

$= (PA - CA) + (PB - DB) + CE + ED$

$= PA + PB - CA - DB + CA + DB$

[$\because$ CE = CA and ED = DB]

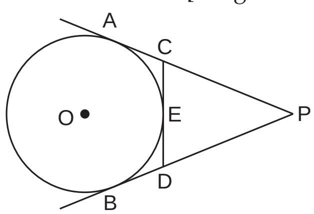
$= 2PA$ [$\because$ PA = PB]

$= 2 \times 10$ cm

= **20 cm**

(*ii*) We know that the lengths of tangents drawn from an external point to a circle are equal.

$\therefore$ PA = PB [Tangents from P]
CE = CA [Tangents from C] ...(1)
DE = DB [Tangents from D]



Perimeter of ΔPCD

$= PC + CD + PD$

$= PC + CE + DE + PD$

$= PC + CA + DB + PD$ [Using (1)]

$= PA + PB$

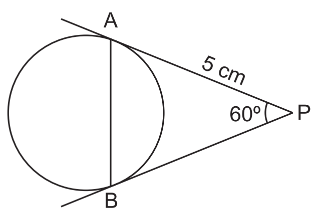
$= PA + PA$ [Using (1)]

$= 2PA$

$= 2 \times 14$ cm = 28 cm

Hence, perimeter of ΔPCD = **28 cm**

(*iii*) In ΔPAB, we have



PA = PB [Tangents from an external point to a circle are equal]

$\therefore$ $\angle PBA = \angle PAB = x$ (say) [Angles opposite equal sides of a triangle]

Also, $\angle APB + \angle PBA + \angle PAB = 180°$ [Sum of angles of a triangle]

$\Rightarrow$ $60° + x + x = 180°$

$\Rightarrow$ $2x = 180° - 60°$

$\Rightarrow$ $2x = 120°$

$\Rightarrow$ $x = 60°$

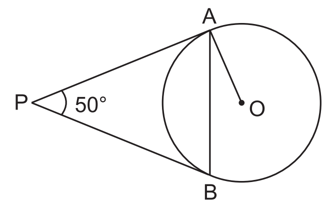
$\therefore$ $\angle PAB = \angle PBA = \angle APB = 60°$

$\Rightarrow$ ΔPAB is an equilateral triangle.

$\therefore$ AB = PA = PB = 5 cm

Hence, AB = **5 cm.**

13. Since the tangent at any point of a circle is perpendicular to the radius through the point of contact and PA is a tangent at A and OA is the radius through A, therefore OA $\perp$ PA.



$\Rightarrow$ $\angle OAP = 90°$ ...(1)

We know that tangents from an external point to a circle are equal.

So, PB = PA

$\angle PAB = \angle PBA$ [Angles opposite equal sides PA and PB of ΔPAB] ...(2)

In ΔPAB, we have

$\angle PAB + \angle PBA + \angle APB = 180°$ [Sum of angles of a triangle]

$\Rightarrow$ $2\angle PAB + \angle APB = 180°$ [Using (2)]

$\Rightarrow$ $\angle APB = 180° - 2\angle PAB$

$\Rightarrow$ $\angle APB = 2(90° - \angle PAB)$

$\Rightarrow$ $\angle APB = 2[\angle OAP - \angle PAB]$ [Using (1)]

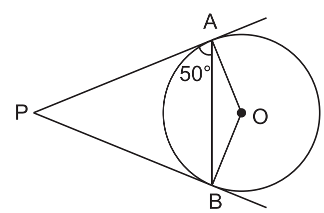
$\Rightarrow$ $\angle APB = 2\angle OAB$

$\therefore$ $50° = 2\angle OAB$

$\Rightarrow$ **$\angle$OAB = 25°**

14. $\angle PAB = 50°$

$\angle AOB = ?$



Radius of a circle is perpendicular to the tangent at the point of contact

OA $\perp$ PA

$\angle OAP = 90°$

$\angle PAB + \angle OAB = 90°$

$\angle OAB = 90° - 50° = 40°$

Now In ΔOAB

OA = OB (radius)

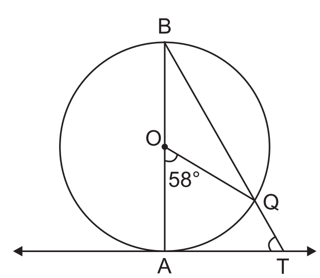
$\therefore$ $\angle OAB = \angle OBA = 40°$

$\angle AOB + \angle OAB + \angle OBA = 180°$

$\angle AOB + 40° + 40° = 180°$

$\angle$AOB = 100°

15.



AB is the diameter

$\angle AOQ = 58°$

$\angle ATQ = ?$

$\angle AOQ + \angle BOQ = 180°$ (Linear pair)

$58° + \angle BOQ = 180°$

$\angle BOQ = 122°$

In ΔBOQ

$OB = OQ$ (radius)

$\therefore \quad \angle OBQ = OQB$

Now

$\angle BOQ + \angle OQB + \angle OBQ = 180°$

$122° + 2\angle OQB = 180°$

$$\angle OQB = \frac{180° - 122°}{2} = \frac{58°}{2} = 29°$$

$\angle OQB + \angle OQT = 180°$ (Linear pair)

$\angle OQT = 180° - 29° = 151°$

In quadrilateral OATQ

$\angle OAT + \angle ATQ + \angle OQT + \angle AOQ = 360°$

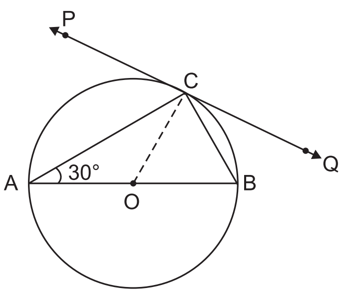
$90° + \angle ATQ + 151° + 58° = 360°$

$\angle ATQ + 299° = 360°$

$\mathbf{\angle ATQ = 61°}$

16. $\angle CAB = 30°$

$\angle PCA = ?$



In ΔOAC $\quad OA = OC$

$\therefore \quad \angle OAC = \angle OCA = 30°$

Radius of a circle is perpendicular to the tangent at the point of contact

$\therefore \quad OC \perp PQ$

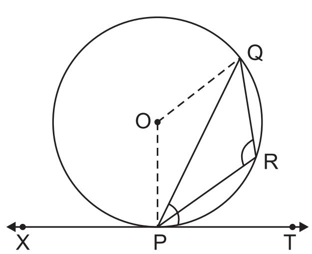
$\therefore \quad \angle OCP = 90°$

$\angle OCP = \angle OCA + \angle PCA = 90°$

$30° + \angle PCA = 90°$

$\mathbf{\angle PCA = 60°}$

17. We are given $\angle QPT = 60°$



$\angle QPT + \angle QPX = 180°$ (Linear Pair)

$\angle QPX = 180° - \angle QPT$

$= 180° - 60°$

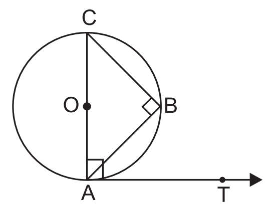
$= 120°$

Now $\quad \angle PRQ = \angle QPX = \mathbf{120°}$

(Alternate Segment Theorm)

18. Given that AB is a chord of a circle with centre at O and AOC is a diameter of the circle. AT is a tangent to the circle at A.

We join BC.



To prove that $\angle BAT = \angle ACB$.

Let $\quad \angle BAT = \theta$ …(A)

Then $\quad \angle BAC = 90° - \theta$ … (1) [$\because \angle CAT = 90°$]

Also, $\quad \angle ABC = 90°$ [$\because$ Angle is a semicircle is 90°]

$\therefore \quad \angle ACB + \angle BAC = 90°$

[Angle-sum property of a triangle]

$\therefore \quad \angle ACB = 90° - \angle BAC = 90° - (90° - \theta)$

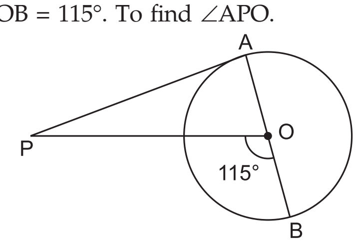
[From (1)]

$\therefore$ From (A) and (B),

$\angle ACB = \angle BAT = \theta$ …(3)

Hence, proved.

19. Given that PA is a tangent to a circle with centre at O, touching the circle at A. AO is joined and produced to cut the circle at B. Then AB is diameter of the circle. Given that $\angle POB = 115°$. To find $\angle APO$.



Since PA is a tangent and OA is a radius of the circle,

$\therefore \quad \angle PAO = 90°$ …(1)

Also, $\quad \angle POA = \angle AOB - \angle POB$

$= 180° - 115°$

$= 65°$ …(2)

Now, in ΔAPO, we have

$\angle APO + \angle AOP + PAO = 180°$

[By angle sum property of a triangle]

$\Rightarrow \quad \angle APO + 65° + 90° = 180°$ [From (1) and (2)]

$\Rightarrow \quad \angle APO = 180° - 90° - 65°$

$= 180° - 155°$

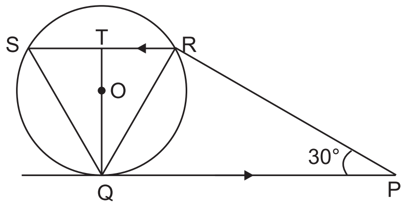
$= \mathbf{25°}$

which is the required measure of $\angle APO$.

20. Given that PQ and PR are two tangents drawn from an external point P, to a circle with centre at O such that ∠RPQ = 30°.

RS is a chord drawn parallel to the tangent PQ.

SQ is joined. To find ∠RQS.



Construction: We join QO and produce it to cut SR at T.

Then QOT ⊥ SR and T is the mid-point of the chord SR.

Now, since the lengths of two tangents drawn from an external point P to a circle are equal.

∴ PQ = PR

∴ ∠PQR = ∠PRQ

Since ∠QPR = 30°

∴ ∠PQR + ∠PRQ = 180° – 30° = 150°

∴ ∠PQR = ∠PRQ

= 75°

Now, since OQ is a radius and QP is a tangent through Q on the circle, ∠TQP = 90°.

∴ ∠TQR = ∠TQP – ∠PQR

= 90° – 75°

= 15° …(1)

Now, in ΔSQT and ΔRQT, we have QT ⊥ SR.

∠QTS = ∠QTR = 90°

TS = TR and TQ is common.

∴ By SAS congruence criterion

ΔSQT ≅ ΔRQT

∴ ∠TQS = ∠TQR [By CPCT]

∴ ∠TQS = 15° [From (1)] …(2)

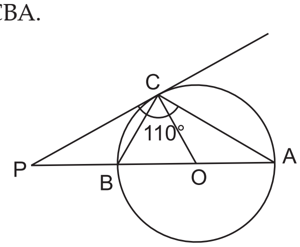
Hence, ∠RQS = 2 ∠TQR = 2 × 15° [From (1)]

= 30°

Hence, the required measure of ∠RQS is 30°.

21. Given that P is an external point on the diameter AOB produced of a circle with centre at O, such that the tangent PC to the circle at a point C on it makes an angle of 110° with the line segment AC. Hence, ∠PCA = 110°.

To find ∠CBA.



Construction: We join CO.

Now, since AB is a diameter of the circle, hence

∠ACB = 90°

∴ ∠PCB = ∠PCA – ∠ACB

= 110° – 90°

= 20°

∴ ∠CAB = ∠PCB = 20°

[∵ ∠PCB is the angle between the tangent PC to the circle and its chord CB]

Now, in ΔABC, we have

∠ACB = 90° and ∠CAB = 20°

∴ ∠CBA = 180° – (∠AOB + ∠CAB)

[By angle sum property of ΔABC]

= 180° – (90° + 20°)

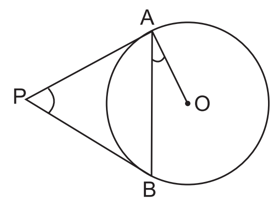
= 180° – 110° = **70°**

which is the required measure of ∠CBA.

22. Given that PA and PB are two tangents to a circle with centre at O. These two tangents touch the circle at A and B. AO and AB are joined.

To prove that ∠APB = 2∠OAB

Let ∠PAB = θ



Then since PA and PB are two tangents to the circle with centre at O, drawn from an external point P.

∴ PA = PB

∠PBA = ∠PAB = θ

∴ In ΔPAB,

∠APB = 180° – 2θ

[Angle sum property of ΔPAB]

= 2(90° – θ) …(1)

Now, since OA is a radius of the circle and PA is a tangent at A from an outside point P of the circle,

∴ ∠OAP = 90°

∴ ∠OAB = ∠OAP – ∠PAB = 90° – θ …(2)

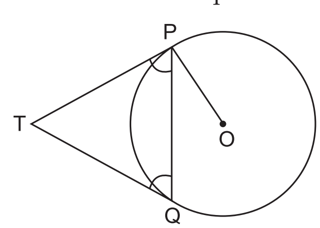
From (1) and (2), we see that

∠APB = 2∠OAB

Hence, proved.

or

We know that the lengths of tangents drawn from an external point to a circle are equal.



∴ TP = TQ

In ΔTPQ, $TP = TQ$

$\Rightarrow \quad \angle TQP = TPQ \quad \ldots(1)$

[Angles opposite to equal sides]

$\angle TQP + \angle TPQ + \angle PTQ = 180°$ [Angle sum property]

$\Rightarrow \quad 2\angle TPQ + \angle PTQ = 180°$ [Using (1)]

$\Rightarrow \quad \angle PTQ = 180° - 2\angle TPQ \quad \ldots(2)$

We know that, a tangent to a circle is perpendicular to the radius through the point of contact,

$OP \perp PT,$

$\therefore \quad \angle OPT = 90°$

$\Rightarrow \quad \angle OPQ + \angle TPQ = 90°$

$\Rightarrow \quad \angle OPQ = 90° - \angle TPQ$

$\Rightarrow \quad 2\angle OPQ = 2(90° - \angle TPQ)$

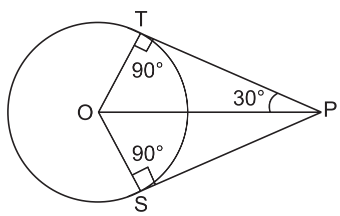
$= 180° - 2\angle TPQ \quad \ldots(3)$

From (2) and (3), we get

$\angle PTQ = 2\angle OPQ$

Hence, proved.

23. Given that PT and PS are two tangents to a circle with centre at 0, drawn from an external point P. We join PO, OT and OS. Given that $\angle OPT = 30°$.



To find reflex $\angle TOS$.

Now, in ΔOPT and ΔOPS,

we have $TP = PS$, $OT = OS$ [Radii of the same circle]

and OP is common.

$\therefore$ By SSS congruence criterion,

$\Delta OPT \cong \Delta OPS$

$\therefore \quad \angle OPS = \angle OPT = 30°$

$\therefore \quad \angle SPT = 2\angle OPT$

$= 2 \times 30° = 60°$

Now, since

$\angle OTP + \angle OSP = 90° + 90° = 180°,$

$\therefore \quad \angle TOS + \angle SPT = 180°$

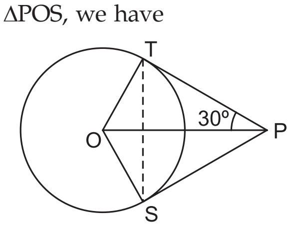
$\Rightarrow \quad \angle TOS = 180° - 60° = 120°$

$\therefore \quad$ Reflex $\angle TOS = 360° - 120$

$= \mathbf{240°}$

which is the required measure of reflex $\angle TOS$.

24. In ΔPOT and ΔPOS, we have



$PT = PS$ [Length of tangents drawn from an external point to a circle are equal]

$PO = PO$ [Common]

$OT = OS$ [Radii of a circle]

$\therefore \quad \Delta POT \cong \Delta POS$ [By SSS congruence]

$\Rightarrow \quad \angle OPT = \angle OPS$

$\Rightarrow \quad 30° = \angle OPS$

Since the tangent at any point of a circle is perpendicular to the radius through the point of contact

$\therefore \quad \angle OTP = \angle OSP = 90° \quad \ldots(1)$

In ΔOTP, we have

$\angle OTP + \angle TOP + \angle OPT = 180°$ [Sum of angles of a triangle]

$\Rightarrow \quad 90° + TOP + 30° = 180°$ [Using (1)]

$\Rightarrow \quad \angle TOP = 180° - (90° + 30°)$

$= 180° - 120° = 60° \ldots(2)$

Similarly $\angle SOP = 60° \quad \ldots(3)$

Now, $\angle TOS = \angle TOP + \angle SOP$

$= 60° + 60°$ [Using (2) and (3)]

$\Rightarrow \quad \angle TOS = 120°$

Now, In ΔSOT

$\angle OST + \angle OTS + \angle TOS = 180°$

$\angle OST + \angle OTS = 180° - \angle TOS$

$= 180° - 120 = 60°$

Now $\angle OST = \angle OTS$

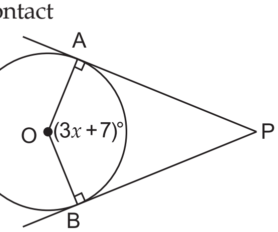
($\because$ OT = OS isosceles triangle)

$2\angle OST = 2\angle OTS = 60°$

$\angle OST = \angle OTS = 30°$

Hence proved.

25. Since radius of a circle is perpendicular to the tangent at the point of contact



$\therefore$ $OA \perp AP$ and $OB \perp PB$

$\therefore$ $\angle OAP = \angle OBP = 90°$

Now in quadrilateral PAOB

$\angle P + \angle O + \angle A + \angle B = 360°$

$(2x + 3)° + (3x + 7)° + (90° + 90°) = 360°$

$5x + 10 = 360 - 180$

$5x = 180 - 10$

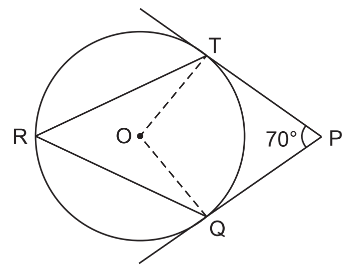
$5x = 170$

$\boldsymbol{x = 34}$

26. $\angle TPQ = 70°$

Join OT and OQ.

Radius of a circle is perpendicular to the tangent at the point of contact



$\therefore$ OT $\perp$ PT and OQ $\perp$ PQ

$$\angle OTP = \angle OQP = 90°$$

In quadrilateral PTOQ

$$\angle TPQ + \angle PQO + \angle QOT + \angle OTP = 360°$$
$$70° + 90° + \angle QOT + 90° = 360°$$
$$\angle QOT = 360° - 250°$$
$$\angle QOT = 110°$$

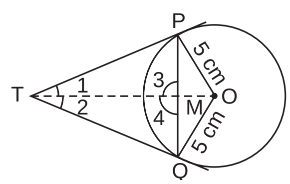
We know that angle subtended at the centre is twice the angle subtended at the circle

$\therefore$ $\angle QOT = 2\angle TRQ$

$$\angle TRQ = \frac{\angle QOT}{2} = \frac{110°}{2}$$

$$\mathbf{\angle TRQ = 55°}$$

27. Join OT and let it intersect PQ at M.



In ΔOPT and ΔOQT, we have

OP = OQ [Radii of a circle]

OT = OT [Common]

TP = TQ [Lengths of tangents from an external point to a circle are equal]

$\therefore$ $\Delta OPT \cong \Delta OQT$ [By SSS congruence]

$\Rightarrow$ $\angle 1 = \angle 2$ [By CPCT] ...(1)

In ΔMPT and ΔMQT, we have

TP = TQ [Lengths of tangents from an external point to a circle are equal]

$\angle 1 = \angle 2$ [From (1)]

TM = TM [Common]

$\therefore$ $\Delta MPT \cong \Delta MQT$ [By SAS congruence]

$\Rightarrow$ MP = MQ

and $\angle 3 = \angle 4$ [CPCT] ...(2)

Also, $\angle 3 + \angle 4 = 180°$ [Linear pair] ...(3)

From (2) and (3), we get

TM is the perpendicular bisector of PQ.

$\therefore$ $MP = MQ = \frac{1}{2}PQ$

$= \frac{1}{2} \times 8 \text{ cm} = 4 \text{ cm}$ ...(4)

In right ΔPMO, we have

$MP^2 + OM^2 = OP^2$ [By Pythagoras' Theorem]

$\Rightarrow$ $(4 \text{ cm})^2 + (OM)^2 = (5 \text{ cm})^2$

$\Rightarrow$ $OM^2 = (25 - 16) \text{ cm}^2 = 9 \text{ cm}^2$

$\Rightarrow$ $OM = 3$ cm

In right ΔPMT, we have

$TP^2 = MP^2 + MT^2$ [By Pythagoras' Theorem]

$\Rightarrow$ $TP^2 = (4 \text{ cm})^2 + MT^2$ [Using (4)] ...(5)

Since the tangent at any point of a circle is perpendicular to the radius through the point of contact and TP is a tangent at P and OP is the radius through P, therefore OP $\perp$ TP $\Rightarrow$ $\angle OPT = 90°$.

In right ΔOPT, we have

$(OT)^2 = OP^2 + TP^2$ [By Pythagoras' Theorem]

$(MO + MT)^2 = OP^2 + TP^2$

$\Rightarrow$ $OM^2 + MT^2 + 2MO\,(MT) = OP^2 + TP^2$

$\Rightarrow$ $9 \text{ cm}^2 + MT^2 + 2(3 \text{ cm})\,(MT)$

$= (5 \text{ cm})^2 + 16 \text{ cm}^2 + MT^2$ [Using (5)]

$\Rightarrow$ $MT = \frac{(25+16-9)}{6}$ cm

$= \frac{41-9}{6}$ cm $= \frac{32}{6}$ cm $= \frac{16}{3}$ cm

Substituting $MT = \frac{16}{3}$ cm in (5), we get

$$TP^2 = (4 \text{ cm})^2 + \left(\frac{16}{3}\text{ cm}\right)^2$$

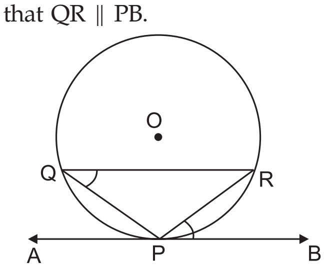
$$= \left(16 + \frac{256}{9}\right)\text{cm}^2$$

$\Rightarrow$ $TP^2 = \frac{144+256}{9} \text{ cm}^2 = \frac{400}{9} \text{ cm}^2$

$\Rightarrow$ $\mathbf{TP = \frac{20}{3}\ cm}$

28. Given that P is the mid-point of arc QR of a circle with centre at O and AB is a tangent to the circle at P.

To prove that QR $\parallel$ PB.



Since arc PQ = arc PR

$\therefore$ chord PQ = chord PR

$\therefore$ In ΔPQR, PQ = PR

$\therefore$ $\angle PQR = \angle PRQ$

But $\angle PQR = \angle RPB$

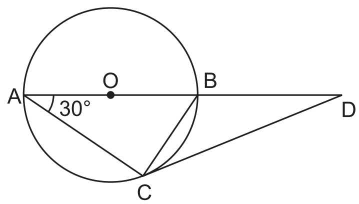
$\therefore$ $\angle RPB = \angle PRQ$

But these two angles are alternate angles between the line AB and chord QR. Hence, QR || PB.

Hence, proved.

29. Given that AOB is a diameter of a circle with centre at O. C is a point on the circle such that the chord AC makes an angle of 30° with the diameter AB, i.e. ∠BAC = 30°.

CD is a tangent to the circle at C, which cuts AB produced at D. To prove that BC = BD.



Since, BC is a chord of the circle and CD is a tangent to the circle at C,

∴ ∠BCD = ∠BAC = 30°.

Also, ∠ACB = 90°

[∴ Angle is a semicircle is 90°]

∴ ∠ABC = 180° – (∠BAC + ∠ACB)

= 180° – (30° + 90°)

= 180° – 120°

= 60°

∴ ∠CBD = 180° – ∠ABC

= 180° – 60°

= 120°

∴ In ΔBCD, we have ∠BCD = 30° and ∠CBD = 120°

∴ ∠BDC = 180° – (30° + 120°)

[Angle sum property of a triangle]

= 180° – 150°

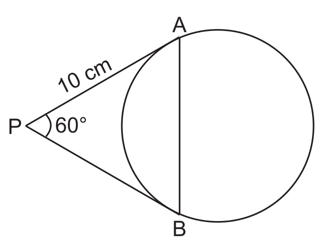
= 30°

∴ ∠BCD = ∠BDC

∴ BC = BD

Hence, proved.

30. Given that PA and PB are tangents to a circle from an outside point P such that PA = 10 cm and ∠APB = 60°. To find the length of the chord AB.



We know that PB = PA = 10 cm

∴ In ΔPAB,

$$\angle PAB = \angle PBA = \frac{180° - 60°}{2} = 60°$$

∴ ΔPAB is an equilateral triangle.

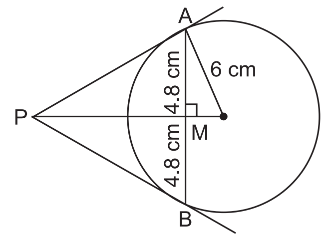
∴ AB = PB = PA = **10 cm** which is the required length of AB.

31. (*i*) Given that two tangents PA and PB are drawn to a circle with centre O from an external point P. OP, AB and OA are joined.

OA = radius of the circle = 6 cm [Given]

Also, AM = MB = 4.8 cm [Given]

To find the length of PA.



Since M is the mid-point of the chord AB,

∴ OM ⊥ AB.

Also, since OA is a radius and AP is a tangent to the circle,

∴ ∠PAO = 90°

Let AP = x and PM = y

From ΔOAM, we have by Pythagoras' theorem,

$$OA^2 = OM^2 + AM^2$$

$$\Rightarrow \quad 36 = OM^2 + 4.8^2$$

$$\Rightarrow \quad OM^2 = 36 - 4.8^2$$

$$= 36 - 23.04$$

$$= 12.96$$

$$\therefore \quad OM = \sqrt{12.96} = 3.6 \quad \ldots(1)$$

Now, from ΔAPM, we have

$$AP^2 = AM^2 + PM^2$$

$$\Rightarrow \quad x^2 = 4.8^2 + y^2 \quad \ldots(2)$$

Also, from ΔAPO, we have

$$PO^2 = OA^2 + AP^2$$

$$\Rightarrow \quad (PM + OM)^2 = OA^2 + AP^2$$

$$\Rightarrow \quad (y + 3.6)^2 = 36 + x^2$$

$$\Rightarrow \quad y^2 + 7.2y + 12.96 = 36 + 4.8^2 + y^2 \quad \text{[From (1)]}$$

$$7.2y + 12.96 = 36 + 23.04$$

$$\Rightarrow \quad 7.2y = 36 + 23.04 - 12.96$$

$$= 36 + 10.08$$

$$= 46.08$$

$$y = \frac{46.08}{7.2} = \frac{4608}{72} = 6.4 \quad \ldots(3)$$

∴ From (2) and (3), we have

$$x^2 = 4.8^2 + 6.4^2$$

$$= 23.04 + 40.96 = 64$$

$$\therefore \quad x = \sqrt{64} = 8$$

Hence, the required length of PA is 8 cm.

(*ii*) Given that PQ is a chord of length 8 cm of a circle with centre at O and radius = 5 cm. Tangents at P and Q intersect each other at T. Let OT intersect PQ at M. OP and OQ are joined. Given that OP = OQ = 5 cm.

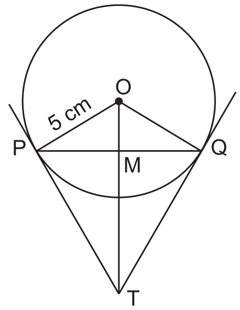
Now, OP is a radius and PT is a tangents, at P.

$\therefore$ PO ⊥ PT. Similarly, OQ ⊥ QT.

Now, in ΔOPT and ΔOQT, we have

$$OP = OQ,$$
$$\angle OPT = \angle OQT = 90°$$

and OT is common



Hence, by RHS congruence criterion,

$$\Delta OPT \cong \Delta OQT$$

$\therefore$ $\angle POM = \angle QOM$ [by CPCT]

Now, in ΔOPM and ΔOQM, we have

$$OP = OQ,$$
$$\angle POM = \angle QOM$$

and OT is common.

Hence, by SAS congruence criterion,

$$\Delta OPM \cong \Delta OQM$$

$\therefore$ $PM = QM$ [By CPCT]

i.e. M is the mid-point of the chord PQ.

$\therefore$ $PM = MQ = 4$ cm

$\therefore$ From ΔOPM, we have by Pythagoras' theorem,

$$OM^2 = OP^2 - PM^2$$
$$= 5^2 - 4^2$$
$$= 25 - 16 = 9$$

$\therefore$ $OM = 3$ …(1)

Let MT = x cm and PT = y cm.

Now, from ΔOPT, we have by Pythagoras' theorem,

$$OT^2 = OP^2 + PT^2$$

$\Rightarrow$ $(OM + MT)^2 = OP^2 + PT^2$

$\Rightarrow$ $(x + 3)^2 = 5^2 + y^2$ [From (1)] …(2)

Again, from ΔPMT, we have by Pythagoras' theorem,

$$PT^2 = PM^2 + MT^2$$

$\Rightarrow$ $y^2 = 4^2 + x^2 = 16 + x^2$ …(3)

$\therefore$ From (2) and (3), we get

$\Rightarrow$ $(x + 3)^2 = 25 + 16 + x^2$

$\Rightarrow$ $x^2 + 6x + 9 = 25 + 16 + x^2$

$\Rightarrow$ $6x = 32$

$\therefore$ $x = \frac{16}{3}$ …(4)

$\therefore$ From (3) and (4), we get

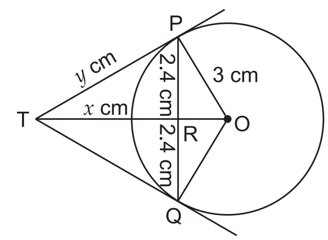
$$y^2 = 16 + \left(\frac{16}{3}\right)^2$$

$$= \frac{144 + 256}{9} = \frac{400}{9}$$

$\therefore$ $y = \sqrt{\frac{400}{9}} = \frac{20}{3}$

Hence, the required length of PT is $\mathbf{\frac{20}{3}}$ **cm**.

(*iii*) Given that PQ is a chord of a circle with centre at O. PT and QT are two tangents to the circle intersecting each other at an outside point T. OP and OT are joined. Let OT intersect PQ at R. Then R will be the mid-point of the chord PQ and OR ⊥ PQ.



Given that PQ = 4.8 cm, radius OP = 3 cm

Let TP = TQ = y cm and RT = x cm

Then, from ΔPOT, since ∠OPT = 90°, hence by Pythagoras' theorem, we have

$$OT^2 = OP^2 + PT^2$$

$\Rightarrow$ $(RT + OR)^2 = 3^2 + y^2$

$\Rightarrow$ $(x + OR)^2 = 9 + y^2$ …(1)

Now, from ΔOPR, we have

$\angle PRO = 90°$, OP = 3 cm and PR = $\frac{4.8}{2}$ cm = 2.4 cm

and RT = x cm.

$\therefore$ By Pythagoras' theorem, we have

$$TP^2 = RT^2 + PR^2$$

$\Rightarrow$ $y^2 = x^2 + 2.4^2$

$= x^2 + 5.76$ …(4)

$\therefore$ From (3) & (4), we get

$$(x + 1.8)^2 = 9 + x^2 + 5.76$$

$\Rightarrow$ $x^2 + 3.6x + 3.24 = 14.76 + x^2$

$\Rightarrow$ $3.6x = 14.76 - 3.24 = 11.52$

$\Rightarrow$ $x = \frac{11.52}{3.6} = 3.2$ …(5)

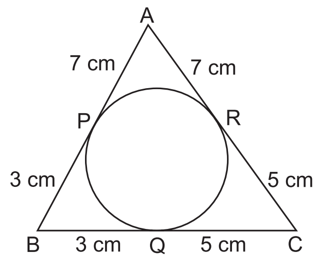
$\therefore$ From (4) and (5), we get

$$y^2 = 3.2^2 + 5.76$$
$$= 10.24 + 5.76$$
$$= 16$$

$\therefore$ $y = \sqrt{16} = 4$

Hence, the required length of the tangent TP is **4 cm**.

32. Given that a circle is inscribed in a triangle ABC touching AB, BC and AC at P, Q and R respectively such that AB = 10 cm, AR = 7 cm and CR = 5 cm.



To find the length of BC.

Since from an external point A, two tangents AP and AR are drawn, hence, we have

$$AP = AR = 7 \text{ cm} \qquad \text{[Given]}$$

Similarly, we have

$$BQ = BP = AB - AP$$
$$= (10 - 7) \text{ cm} = 3 \text{ cm} \qquad \ldots(1)$$

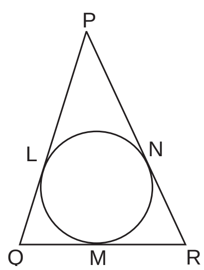
Also, $\quad CQ = CR = 5$ cm $\qquad$ [Given] …(2)

Hence, $\quad BC = BQ + CQ$

$$= (3 + 5) \text{ cm} = 8 \text{ cm}$$

Hence, the required length of BC is **8 cm**.

33. Since the lengths of the tangents from an external point to a circle are equal



$\therefore$ $\quad PL = PN$ $\quad$ [Tangents from P]
$\quad QL = QM$ $\quad$ [Tangents from Q] $\quad$ …(1)
$\quad RM = RN$ $\quad$ [Tangents from R]

Let $\quad QM = x$ cm $\qquad$ …(2)

Then, $\quad RM = QR - QM = (8 - x)$ cm

$\Rightarrow$ $\quad RN = (8 - x)$ cm $\qquad$ [Using (1)] …(3)

$\quad PN = PR - RN = [12 - (8 - x)]$ cm

$\quad = (4 + x)$ cm

$\Rightarrow$ $\quad PL = (4 + x)$ cm $\qquad$ [Using (1)] …(4)

Now $\quad PQ = PL + QL = PL + QM$ $\qquad$ [Using (1)]

$\Rightarrow$ $\quad 10$ cm $= (4 + x + x)$ cm $\qquad$ [Using (2) and (4)]

$\Rightarrow$ $\quad 10$ cm $= (4 + 2x)$ cm

$\Rightarrow$ $\quad 2x = 6 \Rightarrow x = 3$

$\therefore$ $\quad QM = x$ cm $= 3$ cm

$\quad RN = (8 - x)$ cm

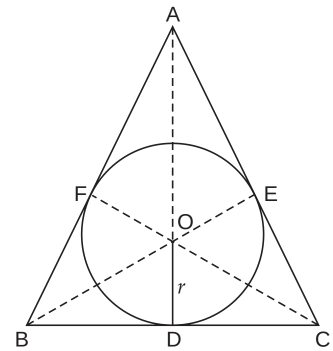
$\quad = (8 - 3)$ cm $= 5$ cm $\qquad$ [Using (3)]

and $\quad PL = (4 + x)$ cm $= (4 + 3)$ cm

$\quad = 7$ cm $\qquad$ [Using (4)]

Hence, $\quad$ **QM = 3 cm, RN = 5 cm and PL = 7 cm.**

34. (*i*) Since, the lengths of tangents from an exterior point to a circle are equal.



$\therefore$ $\quad AF = AE$ $\qquad$ [Tangents from A] …(1)
$\quad BD = BF$ $\qquad$ [Tangents from B] …(2)
$\quad CE = CD$ $\qquad$ [Tangents from C] …(3)

$\therefore$ Adding the corresponding sides of (1), (2) and (3), we get

$$AF + BD + CE = AE + BF + CD \qquad \ldots(4)$$

Now perimeter of ΔABC

$= AB + BC + CA$
$= AF + BF + BD + CD + CE + AE$
$= (AF + BD + CE) + (AE + BF + CD)$
$= 2(AF + BD + CE)$
$= 2 (AE + BF + CD)$ $\qquad$ [Using (4)]

$\Rightarrow$ $\quad \frac{1}{2}$ (Perimeter of ΔABC)

$= AF + BD + CE = AE + BF + CD$

Hence, $\quad$ **AF + BD + CE = AE + BF + CD**

$\quad = \frac{1}{2}$ **(perimeter of ΔABC)**

(*ii*) $\quad AB + CD = AF + BF + CD$

$\quad = AE + BD + CE$ [Using (1), (2) and (3)]

$\Rightarrow$ $\quad AB + CD = (AE + CE) + BD$

$\Rightarrow$ $\quad$ **AB + CD = AC + BD**

(*iii*) Join OA, OB and OC.

Join OE and OF.

Then, $\quad OD = OE = OF = r$ $\qquad$ [radii of a circle] …(4)

Since the tangents at any point of a circle is perpendicular to the radius through the point of contact

$\therefore$ $\quad OF \perp AB$, $OD \perp BC$ and $OE \perp AC$

$\therefore$ $\quad$ OF, OD and OE are altitudes of ΔAOB, ΔBOC and ΔCOA respectively.

$$ar(\Delta ABC) = ar(AOB) + ar(\Delta BOC) + ar(\Delta COA)$$

$$= \frac{1}{2} AB \times OF + \frac{1}{2} BC \times OD + \frac{1}{2} AC \times OE$$

$$= \frac{1}{2} AB \times r + \frac{1}{2} BC \times r + \frac{1}{2} AC \times r \qquad \text{[Using (4)]}$$

$= \frac{1}{2}(\text{AB} + \text{BC} + \text{AC}) + r$

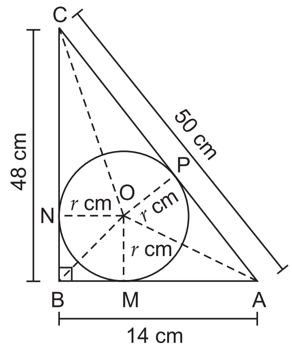
$= \frac{1}{2}(\text{Perimeter of } \Delta\text{ABC}) \times r$

Hence, **area (ΔABC) =** $\mathbf{\frac{1}{2}}$ **(Perimeter of ΔABC) ×** $\boldsymbol{r}$

35. (*i*) Given that ABC is a triangle in which ∠B = 90°, BC = 4.8 cm and AB = 14 cm. A circle with centre at O is inscribed in the triangle. Let the radius of the circle be r cm.

To find r.

Construction: We join OA, OB and OC. We draw OM ⊥ AB, ON ⊥ BC and OP ⊥ AC where OM = ON = OP = r cm.



From ΔABC, we have by Pythagoras' theorem,

$$\text{AC} = \sqrt{\text{AB}^2 + \text{BC}^2}$$
$$= \sqrt{14^2 + 15^2} \text{ cm}$$
$$= \sqrt{196 + 2304} \text{ cm}$$
$$= \sqrt{2500} \text{ cm}$$
$$= 50 \text{ cm}$$

Now, area of $\Delta\text{ABC} = \frac{1}{2}\text{AB} \times \text{BC}$

$= \frac{1}{2} \times 14 \times 48 \text{ cm}^2$

$= 336 \text{ cm}^2$ …(1)

Area of $\Delta\text{OAB} = \frac{1}{2} \times \text{AB} \times \text{OM}$

$= \frac{1}{2} \times 14 \times r = 7r \text{ cm}^2$ …(2)

Area of $\Delta\text{OBC} = \frac{1}{2} \times \text{BC} \times \text{ON}$

$= \frac{1}{2} \times 48 \times r \text{ cm}^2$

$= 24r \text{ cm}^2$ …(3)

and area of $\Delta\text{AOC} = \frac{1}{2} \times \text{AC} \times \text{OP}$

$= \frac{1}{2} \times 50 \times r \text{ cm}^2$

$= 25r \text{ cm}^2$ …(4)

Now, ar(ΔABC) = ar(ΔOAB) + ar(ΔOBC) + ar(AOC)

⇒ $336 = (7r + 24r + 25r)$

[From (1), (2), (3) and (4)]

⇒ $56r = 336$

⇒ $r = \frac{336}{56} = 6$

Hence, the required value of r is **6 cm**.

(*ii*) In right ΔABC, we have

$\text{AC}^2 = \text{AB}^2 + \text{BC}^2$ [By Pythagoras' Theorem]

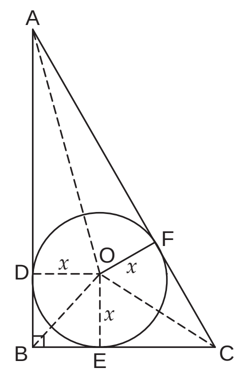
⇒ $\text{AC}^2 = (24 \text{ cm})^2 + (10 \text{ cm})^2$

$= 676 \text{ cm}^2$

⇒ AC = 26 cm …(1)

Join OA, OB and OC.

Let the tangents AB, BC and CA touch the circle at D, E and F respectively.



Since the tangent at any point of a circle is perpendicular to the radius through the point of contact

∴ OD ⊥ AB, OE ⊥ BC and OF ⊥ AC.

⇒ OD, OE and OF are the altitudes of ΔABO, ΔBOC, ΔCOA respectively.

Now,ar(ΔABC) = ar(ΔAOB) + ar(ΔBOC) + ar(ΔCOA)

⇒ $\frac{1}{2}\text{BC} \times \text{AB} = \frac{1}{2}\text{AB} \times \text{OD} + \frac{1}{2}\text{BC} \times \text{OE} + \frac{1}{2}\text{AC} \times \text{OF}$

⇒ $\frac{1}{2}\text{BC} \times \text{AB} = \frac{1}{2}\text{AB} \times x + \frac{1}{2}\text{BC} \times x + \frac{1}{2}\text{CA} \times x$ [OD = OE = OF = x, radii of inscribed circle]

⇒ $\frac{1}{2} \times 10 \text{ cm} \times 24 \text{ cm} = \frac{1}{2} \times 24 \text{ cm} \times x + \frac{1}{2} \times 10 \text{ cm} \times x + \frac{1}{2} \times 26 \text{ cm} \times x$ [Using (1)]

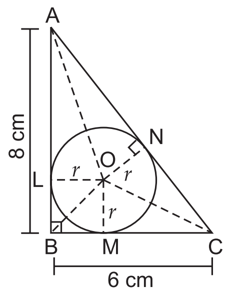
⇒ $120 \text{ cm}^2 = x\,(12 + 5 + 13) \text{ cm}$

⇒ $120 \text{ cm}^2 = 30x \text{ cm}$

⇒ $x = \frac{120 \text{ cm}^2}{30 \text{ cm}} = 4 \text{ cm}$

Hence, $\boldsymbol{x}$ **= 4 cm.**

(*iii*) Given that ABC is a triangle such that $\angle ABC = 90°$, BC = 6 cm and AB = 8 cm.



A circle with centre at O and radius r cm is inscribed in ΔABC.

OL, OM and ON are drawn perpendicular to AB, BC and CA respectively.

$\therefore$ OL = OM = ON = r cm.

OA, OB and OC are joined.

Now, from ΔABC, we have by Pythagoras' theorem,

$$AC = \sqrt{AB^2 + BC^2}$$
$$= \sqrt{8^2 + 6^2} \text{ cm}$$
$$= \sqrt{64 + 36} \text{ cm}$$
$$= \sqrt{100} \text{ cm}^2$$
$$= 10 \text{ cm}$$

$\therefore$ Area of $\Delta ABC = \frac{1}{2} BC \times AB$

$$= \frac{1}{2} \times 6 \times 8 \text{ cm}^2$$
$$= 24 \text{ cm}^2 \quad \ldots(1)$$

Also, $ar(\Delta OAB) = \frac{1}{2} AB \times r$

$$= \frac{1}{2} \times 8 \times r \text{ cm}^2$$
$$= 4r \text{ cm}^2 \quad \ldots(2)$$

$ar(\Delta OBC) = \frac{1}{2} \times BC \times r$

$$= \frac{1}{2} \times 6r \text{ cm}^2$$
$$= 3r \text{ cm}^2 \quad \ldots(3)$$

and $ar(\Delta OCA) = \frac{1}{2} AC \times r$

$$= \frac{1}{2} \times 10r \text{ cm}^2$$
$$= 5r \text{ cm}^2 \quad \ldots(4)$$

Now, $ar(\Delta ABC) = ar(\Delta OAB) + ar(\Delta OBC) + ar(OCA)$

$\Rightarrow$ $24 = 4r + 3r + 5r$

[From (1), (2), (3) and (4)]

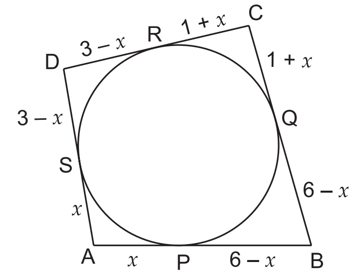
$\Rightarrow$ $12r = 24$

$\Rightarrow$ $r = \frac{24}{12} = 2$

Hence, the required value of r is **2 cm**.

36. Given that ABCD is a quadrilateral such that AB = 6 cm, BC = 7 cm and CD = 4 cm. A circle is inscribed within this quadrilateral touching its sides AB, BC, CD and DA at P, Q, R and S respectively. To find the length of AD.

Since A is an external point to the circle and AP and AS are two tangents to the circle from A, hence AS = AP.



Let AP = x.

$\therefore$ AS = x

Similarly, BP = BQ …(1)

CQ = CR …(2)

and DR = DS …(3)

$\therefore$ From (1), BP = AB – AP

= $6 - x$ = BQ

From (2), CQ = BC – BQ

= $7 - (6 - x)$

= $1 + x$ = CR

and from (3), DR = CD – CR

= $4 - (1 + x)$

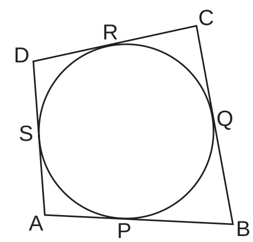
= $3 - x$ = DS

Now, DS = $3 - x$ and AS = x

$\therefore$ AD = AS + DS = $x + 3 - x = 3$

Hence, the required length of AD is **3 cm**.

37. Since the length of tangents from an external point to a circle are equal



$\therefore$ AP = AS = x (say) [Tangents from A] …(1)

BP = BQ [Tangents from B] …(2)

CR = CQ [Tangents from C] …(3)

DR = DS [Tangents from D] …(4)

BP = AB – AP = $(18 - x)$ cm

BQ = $(18 - x)$ cm [Using (2)] …(5)

CQ = BC – BQ

= $[27 - (18 - x)]$ cm [Using (5)]

= $(27 - 18 + x)$ cm

= $(9 + x)$ cm …(6)

$CR = (9 + x)$ cm [Using (3) and (6)] …(7)

$DR = CD - CR$

$= [12 - (9 + x)]$ cm [Using (7)]

$= (12 - 9 - x)$ cm

$= (3 - x)$ cm …(8)

$DS = (3 - x)$ cm [Using (4) and (8)]

$AD = AS + DS$

$= [x + (3 - x)]$ cm [Using (1)]

$\Rightarrow$ $AD = 3$ cm

Hence, **AD = 3 cm**.

38. Since the lengths of tangents from an external point to a circle are equal

$\therefore$ $AP = AS$ [Tangents from A] …(1)

$BQ = BP = 27$ cm [Tangents from B] …(2)

$CQ = CR$ [Tangents from C] …(3)

$DS = DR$ [Tangents from D] …(4)

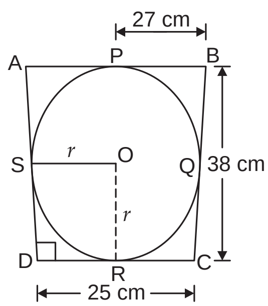
$CR = CQ = CB - BQ$

$= (38 - 27)$ cm $= 11$ cm [Using (2)] …(5)

$DS = DR = DC - CR$

$= (25 - 11)$ cm [Using (5)]

$= 14$ cm …(6)



Since, the tangent at any point on a circle is perpendicular to the radius through the point of contact

$\therefore$ $\angle OSD = \angle ORD = 90°$ …(5)

In quadrilateral OSDR, we have

$\angle OSD = \angle ORD = \angle SDR = 90°$

$\therefore$ $\angle SOR = 90°$ [Sum of angles of a quadrilateral is 360°]

$\Rightarrow$ Each angle of quadrilateral OSDR is a right angle.

Also adjacent sides DR and DS are equal. [From (4)]

$\Rightarrow$ Quadrilateral OSDR is a square

$\Rightarrow$ $OS = OR = DS = DR$ [Sides of a square]

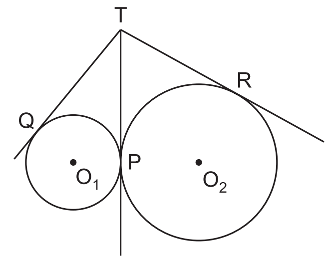
$\Rightarrow$ $r = 14$ cm [Using (6)]

$\Rightarrow$ $r = 14$ cm

Hence, r = **14 cm.**

39. Given that from an external point T, three tangents TP, TQ and TR are drawn to two circles with centres O_1 and O_2, touching each other externally at the point P so that TP is a common tangent to the two circles.

To prove that TQ = TR



We know that the lengths of two tangents drawn from an external point to a circle are equal.

Hence, $TQ = TP$ …(1)

Since, these are two tangents drawn from an external point T to the circle with centre O_1.

Similarly, $TP = TR$ …(2)

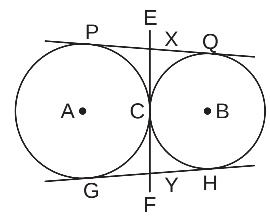
Since these are two tangents drawn from T to the circle with centre O_2.

$\therefore$ From (1) and (2), we have

$TQ = TR$

Hence, proved.

40. Let EF intersect PQ and GH at X and Y respectively. Since the lengths of tangents from an external point to a circle are equal



$\therefore$ $XP = XC$ [Tangents from X to the circle with centre A] …(1)

$XQ = XC$ [Tangents from X to the circle with centre B] …(2)

$YG = YC$ [Tangents from Y to the circle with centre A] …(3)

$YH = YC$ [Tangents from Y to the circle with centre B] …(4)

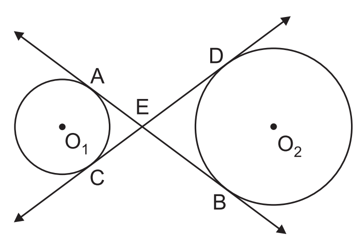
From (1) and (2), we get XP = XQ.

and from (3) and (4), we get YG = YH.

Hence, the common tangent at C bisects the common tangents PQ and GH.

41. Given that AB and CD are two common tangents to two circles with centres at O_1 and O_2 respectively, intersecting each other at E.

To prove that AB = CD.



Since EA and EC are two tangents drawn from an external point E to the circle with centre O_1.

Hence, we have $EA = EC$ …(1)

Similarly, $EB = ED$ …(2)

Adding (1) and (2), we have

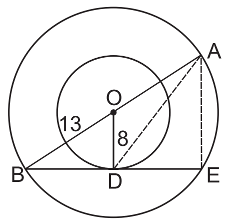
$$EA + EB = EC + ED$$

$\Rightarrow$ $AB = CD$

Hence, proved.

42. We have radius of bigger circle = 13 cm

and radius of smaller circle = 8 cm



Join AE

Also, $AE \perp BE$ [since angle in a semicircle is 90°]

$\therefore$ $BD^2 = OB^2 - OD^2$

[By Pythagoras' Theorem]

$= 169 - 64$

$BD^2 = 105$

$BD = \sqrt{105}$

$\therefore$ $BE = 2BD = 2\sqrt{105}$

Now in ΔAED

$AE^2 + DE^2 = AD^2$ …(1)

and in ΔAEB

$AE^2 = AB^2 - BE^2$

($\because$ AB = AOB = 2 × 13 = 26)

$= (26)^2 - \left(2\sqrt{105}\right)^2$

$= (676) - (4 \times 105)$

$= 676 - 420$

$= 256$

$\therefore$ $AE = 16$

Putting the value of AE in eq. (1)

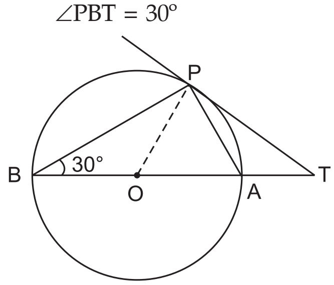
$AE^2 + DE^2 = AD^2$

$256 + 105 = AD^2$

$AD^2 = 361$

AD = **19 cm**

43. $\angle PBT = 30°$



Prove BA : AT = 2 : 1

Join OP

Let the radius of the circle be r

$OP = OB = OA = r$ (radius)

$\angle POA = 2\angle PBA$

(angle subtended at the centre is twice the angle subtended at the circle)

$\therefore$ $\angle POA = 2 \times 30° = 60°$

Radius of a circle is perpendicular to the tangent at the point of contact

$\therefore$ $OP \perp PT$

$\angle OPT = 90°$

In ΔOPT

$\angle OPT + \angle PTO + \angle POA = 180°$

$90° + \angle PTO + 60° = 180°$

$\angle PTO = 30°$

In ΔOPA

$\angle POA = 60°$

$\therefore$ $OA = OP$

$\therefore$ $\angle OPA = \angle OAP$

$\angle OAP + \angle OPA + \angle POA = 180°$

$2\angle OAP + 60° = 180°$

$\angle OAP = 60°$

$\angle OAP = \angle OPA = 60°$

Now In ΔBPA and ΔTPO

$\angle PBA = \angle PTO$ (30°)

$PA = PO$

$\angle PAO = \angle POT$ (60°)

$\therefore$ $\Delta BPA \cong \Delta TPO$

$\therefore$ $BA = OT$ [By CPCT]

$OT = BA = 2r$

$OT = OA + AT$

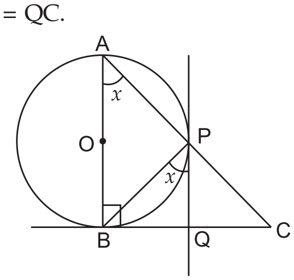
$2r = r + AT$

$AT = r$

$$\frac{BA}{AT} = \frac{2r}{r} = \frac{2}{1}$$

BA : AT = 2 : 1

44. Given that ABC is a triangle with $\angle ABC = 90°$. A circle with centre at O is drawn with AB as a diameter intersecting the hypotenuse AC at P. A tangent PQ is drawn at P intersecting BC at Q. To prove that Q is the mid-point of BC, i.e. BQ = QC.



Construction: We join BP.

Let ∠BAC = x.

Then ∠ACB = 90° – x [∵ ∠ABC = 90°]

Then ∠BPQ = ∠BAC = x

[∵ Angles in alternate segments are equal]

Now, ∠ACB = 90° – x …(1)

Also, ∠APB = ∠BPC = 90°

[∴ Angle in a semicircle is 90°]

∴ ∠QPC = 90° – ∠BPQ = 90° – x …(2)

∴ From (1) and (2), we have

∠QPC = ∠ACB = ∠PCQ

∴ PQ = QC …(3)

But QP = BQ …(4)

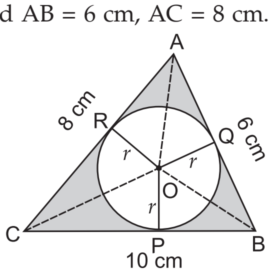
[∵ Tangents drawn from an external point of a circle are equal]

∴ From (3) and (4), we get

BQ = QC

Hence, proved.

45. Given that ΔABC is a right-angled triangle in which ∠A = 90° and AB = 6 cm, AC = 8 cm.



∴ By Pythagoras' theorem,

We have $BC = \sqrt{AC^2 + AB^2}$

$= \sqrt{8^2 + 6^2}$ cm

$= \sqrt{64 + 36}$ cm

$= \sqrt{100}$ cm

= 10 cm

Let r cm be the radius of the incircle with centre at O, touching the sides BC, AB and AC at P, Q and R. To find the area of the shaded region.

Now, area of ΔABC $= \frac{1}{2} AB \times AC$

$= \frac{1}{2} \times 6 \times 8$ cm^2

= 24 cm^2 …(1)

Again, ar(ΔOBC) $= \frac{1}{2} BC \times r$

$= \frac{1}{2} \times 10r$ cm^2

= 5r cm^2 …(2)

ar(ΔOBA) $= \frac{1}{2} AB \times r$

$= \frac{1}{2} \times 6r$ cm^2

= $3r$ cm^2 … (3)

and ar(ΔOAC) $= \frac{1}{2} AC \times r$

$= \frac{1}{2} \times 8r$ cm^2

= $4r$ cm^2 …(4)

∴ From (1), (2) and (3), we have

ar(ΔOBC) + ar(ΔOBA) + ar(ΔOAC) = 24

⇒ $(5r + 3r + 4r) = 24$

⇒ $12r = 24$

⇒ $r = \frac{24}{12} = 2$

Hence, the radius of the in circle is 2 cm.

∴ Required area of the shaded region

= area of ΔABC – area of the circle

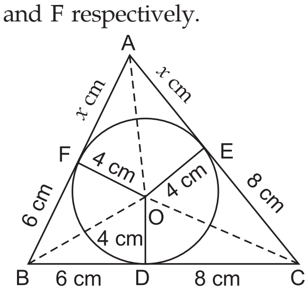
= $(24 - \pi 2^2)$ cm^2

= (24 – 4 × 3.14) cm^2

= (24 – 12.56) cm^2

= **11.44 cm²**

46. (*i*) Given that ABC is a triangle which circumscribes a circle with centre at O and radius 4 cm such that it touches the sides BC, CA and AB of the triangle ABC at D, E and F respectively.



Given that BD = 6 cm and CD = 8 cm

Construction: We join OD, OE and OF.

Also, we join OA, OB and OC.

Then OD = OE = OF = 4 cm, BF = BD = 6 cm and CE = CD = 8 cm.

Let AF = AE = x cm.

To find the length of the sides AB and AC of ΔABC.

We have a = BC = (6 + 8) cm = 14 cm …(1)

b = AC = (8 + x) cm …(2)

and c = AB = (6 + x) cm …(3)

∴ Semi-perimeter of the triangle is given by

$S = \frac{1}{2}(14 + 8 + x + 6 + x)$ cm

= (14 + x) cm

∴ Area of ΔABC

$= \sqrt{s(s-a)(s-b)(s-c)}$ [By Heron's formula]

$= \sqrt{(14+x)(14+x-14)(14+x-8-x)(14+x-6-x)}$

[From (1), (2) and (3)]

$= \sqrt{(14+x)x \times 6 \times 8}$

$= 4\sqrt{42x + 3x^2}$ …(1)

Also, area of ΔABC

= ar(ΔOAB) + ar(ΔOAC) + ar(ΔOBC)

$= \left\{\frac{1}{2}\times(6+x)\times 4 + \frac{1}{2}(8+x)\times 4 + \frac{1}{2}(6+8)\times 4\right\}$ cm^2

= $(12 + 2x + 16 + 2x + 28)$ cm^2

= $4x + 56$

= $4(x + 14)$

∴ From (1) and (2), we have

$$4\sqrt{42x + 3x^2} = 4(x + 14)$$

⇒ $42x + 3x^2 = (x + 14)^2$

$= x^2 + 28x + 196$

⇒ $2x^2 + 14x - 196 = 0$

⇒ $x^2 + 7x - 98 = 0$

⇒ $x^2 + 14x - 7x - 98 = 0$

⇒ $x(x + 14) - 7(x + 14) = 0$

⇒ $(x - 7)(x + 14) = 0$

∴ Either $x - 7 = 0$ ⇒ $x = 7$

Or $x + 14 = 0$ ⇒ $x = -14$

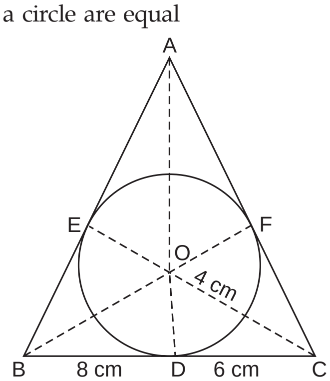
which is absurd, since x cannot be negative.

∴ We have $x = 7$

Hence, the required length of the sides AB and AC are respectively (6 + 7) cm = **13 cm** and (8 + 7) cm = **15 cm**.

(*ii*) Let AB and AC touch the circle at E and F respectively.

Since the length of tangents drawn from an external point to a circle are equal



∴ BE = BD = 8 cm [Tangents from B] …(1)

CF = DC = 6 cm [Tangents from C]

AE = AF = x cm (say) [Tangents from A]

∴ AB = AE + BE = $(x + 8)$ cm,

AC = AF + CF

= $(x + 6)$ cm [Using (1)] …(2)

Join OE and OF.

Then OD = OE = OF = 4 cm [radii of incircle] …(3)

Join OA, OB and OC.

Since the tangent at any point of a circle is perpendicular to the radius through the point of contact

∴ OE ⊥ AB, OD ⊥ BC and OF ⊥ AC

⇒ OE, OD and OF are altitudes of ΔAOB, ΔBOC and ΔAOC respectively.

Now, ar(ΔABC) = ar(ΔAOB) + ar(ΔBOC) + ar(ΔAOC)

⇒ 84 cm^2 = $\frac{1}{2}$ AB × OE + $\frac{1}{2}$ BC × OD + $\frac{1}{2}$ AC × OF

⇒ 84 cm^2 = $\frac{1}{2}$ AB × 4 cm + $\frac{1}{2}$ BC × 4 cm + $\frac{1}{2}$ AC × 4 cm [Using (3)]

⇒ 84 cm^2 = $\frac{1}{2}$ × 4 (AB + BC + CA)

⇒ 84 cm^2 = $\frac{1}{2}$ × 4 $[(x + 8) + (8 + 6) + (6 + x)]$ cm [Using (2)]

⇒ 84 cm^2 = 2 $(2x + 28)$ cm

⇒ 84 cm^2 = 4 $(x + 14)$ cm^2

⇒ $21 = x + 14$

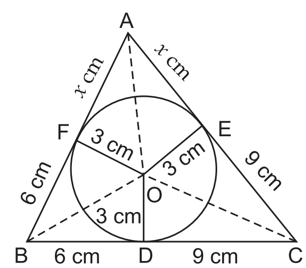
⇒ $x = 21 - 14 = 7$

AB = $(x + 8)$ cm = (7 + 8) cm = 15 cm

AC = $(x + 6)$ cm = (7 + 6) cm = 13 cm

Hence, **AB = 15 cm, AC = 13 cm.**

(*iii*) Given that ΔABC circumscribes a circle with centre at O and radius 3 cm, touching the sides BC, CA and AB of ΔABC at the points D, E and F respectively such that BD = 6 cm and DC = 9 cm. Given that ar(ΔABC) = 54 cm^2.



Construction: We join OA, OB, OC, OD, OE and OF.

To find the lengths of AB and AC.

We have BF = BD = 6 cm

and CE = CD = 9 cm …(1)

Let AF = AE = x cm

∴ Lengths of AB and AC are respectively $(6 + x)$ cm and $(9 + x)$ cm.

Now,

ar(ΔABC) = ar(ΔOBC) + ar(ΔOAC) + ar(ΔOAB)

⇒ $54 = \frac{1}{2}BC\times3+\frac{1}{2}AC\times3+\frac{1}{2}AB\times3$

$= \frac{1}{2}\times(6+9)\times3+\frac{1}{2}\times(9+x)\times3+\frac{1}{2}\times(6+x)\times3$

$54 \times \frac{2}{3} = 15 + 9 + x + 6 + x$

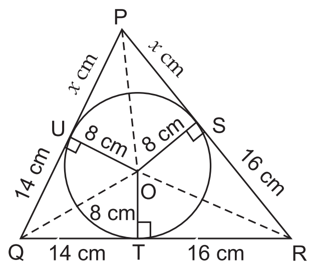
$\Rightarrow \quad 36 = 2x + 30$

$\Rightarrow \quad 2x = 6$

$\Rightarrow \quad x = 3$

$\therefore$ The required lengths of AB and AC are respectively (6 + 3) cm and (9 + 3) cm, i.e., **9 cm** and **12 cm**.

(*iv*) Given that ΔPQR circumscribes a circle with centre at O and radius 8 cm, touching the sides QR, RP and PQ at the points T, S and U respectively such that QU = QT = 14 cm and RS = RT = 16 cm.



Let PU = PS = x cm.

Given that ar(ΔPQR) = 336 cm^2 ...(1)

To find the lengths of PQ and PR.

Construction: We join OQ, OR, OP, OT, OS and OU

We have

QR = (14 + 16) cm = 30 cm,

PQ = (14 + x) cm

and PR = (x + 16) cm.

Now, ar(ΔPQR) = ar(ΔOQR) + ar(ΔOPR) + ar(ΔOPQ)

$\Rightarrow \quad 336 = \frac{1}{2} \times QR \times 8 + \frac{1}{2} \times PR \times 8 + \frac{1}{2} \times PQ \times 8$

$= 4(QT + RT) + 4(RS + PS) + 4(QU + PU)$

$= 4(14 + 16) + 4(16 + x) + 4(14 + x)$

$\Rightarrow \quad 84 = 30 + 16 + x + 14 + x$

$= 60 + 2x$

$\Rightarrow \quad 84 - 60 = 2x$

$\Rightarrow \quad x = \frac{24}{2} = 12$

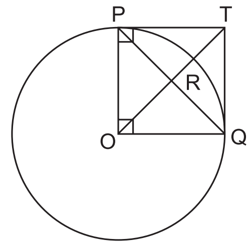
$\therefore$ PQ = 14 + 12 = 26

and PR = 16 + 12 = 28

Hence, the required length of PQ and PR are respectively **26 cm** and **26 cm**.

47. Given that P and Q are two points on a circle with centre at O such that OP ⊥ OQ. Two tangents at P and Q meet each other at an external points T. PQ and OT are joined.

To prove that PQ and OT intersect each other at a point R such that PQ and OT bisect each other at R at right angles.



Since, OP is the radius and PT is a tangent to the circle at P,

$\therefore$ ∠OPT = 90°

Since ∠POQ = 90°

$\therefore$ ∠POQ + ∠OPT = 180°

$\therefore$ OQ ∥ PT.

Similarly, OP ∥ TQ.

Hence, OPTQ is a ∥gm

Also, TP = TQ and ∠OPT = 90°.

i.e. two adjacent sides TP and TQ are equal.

Hence, the ∥gm OPTQ is a square with diagonals OT and PQ.

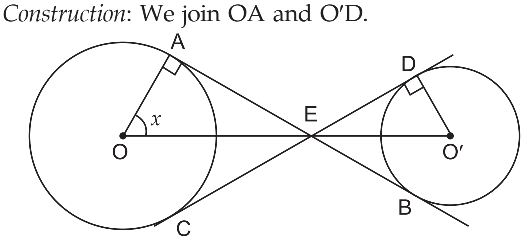
We know that two diagonals of a square bisect each other at right angles. Hence, OT and PQ bisect each other at R at right angles, i.e. RO = RT and PR = RQ.

Also, ∠PRO = ∠ORQ = 90°

Hence, proved.

48. Given that AB and CD are two common tangents to two circles with centres at O and O′, intersecting each other at E. To prove that O, E and O′ are collinear.

Construction: We join OA and O′D.



Let ∠AOE = x

$\because$ ∠OAE = 90°

$\therefore$ ∠AEO = 90° − x ...(1)

In ΔAOE, AEO′ is an exterior angle.

$\therefore$ ∠AEO′ = 180 − ∠AEO

= 180° − 90° + x

= 90° + x ...(2)

$\therefore$ From (1) and (2), we have

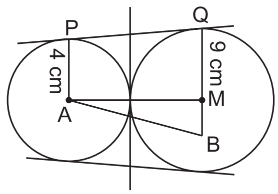
∠AEO + ∠AEO′ = 90° + 90° = 180°

But E is the point of intersection of two tangents.

$\therefore$ O, E and O^1 lie on the same line, i.e. these three points are collinear.

49. Given that two circles with centres A and B and radii 4 cm and 9 cm respectively touch each other externally. Let

PQ be a common tangent to the two circles where P and Q are the points of contact on the two circles respectively. We join AP and BQ. Then, AP = 4 cm and AP ⊥ PQ and BQ = 9 cm and BQ ⊥ PQ.



Also, ∠AMQ = 90°.

Hence, AM ∥ PQ and AP ∥ QM.

∴ The opposite sides of the quadrilateral are parallel and each of its angles is 90°.

∴ The quadrilateral is a rectangle or a square.

Now, BM = BQ – MQ

= (9 – 4) cm = 5 cm ...(1)

Now, since the two circles touch each other externally, hence, distance between their centre = sum of their radii

AB = (4 + 9) cm = 13 cm ...(2)

∴ In right-angled triangle AMB, we have by Pythagoras' theorem,

$$AM = \sqrt{AB^2 - BM^2}$$

$$= \sqrt{13^2 - 5^2} \text{ cm} \quad \text{[From (1) and (2)]}$$

$$= \sqrt{144}$$

= 12 cm ...(3)

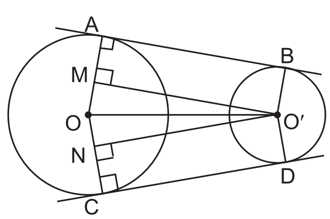
Hence, the adjacent sides of the quadrilateral are 12 cm and 14 cm. Since these sides are unequal hence, the figure APQM is a rectangle.

(i) From (1) BM = 5 cm and from (3), we have

(ii) PQ = AM = 12 cm.

Hence, the required lengths of BM and PQ are respectively **5 cm** and **12 cm**.

50. Given that AB and CD are two common tangents to two circles of unequal radii. Let the centres of these circles be O and O′.



To prove that AB = CD

Construction: We join, OA, OC, O′B and O′D. We now draw O′M ⊥ OA and O′N ⊥ OC.

∴ The figures ABO′M and CDO′N are rectangles.

∴ MO′ = AB ...(1)

and NO′ = CD ...(2)

But MO′ = AB ...(3)

and NO′ = CD ...(4)

[∵ ABCD is a rectangle]

∴ AB = CD [From (3) and (4)]

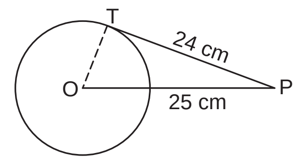
Hence, proved.

CHECK YOUR UNDERSTANDING

MULTIPLE-CHOICE QUESTIONS

For Basic and Standard Levels

1. (*b*) **14 cm**



Since the tangent at any point on the circle is perpendicular to the radius through the point of contact and PT is a tangent T and OT is the radius through T , ∴ OT ⊥ PT.

In right ΔOTP, we have

$OT^2 + PT^2 = OP^2$ [By Pythagoras' Theorem]

⇒ $OT^2 = OP^2 - PT^2$

$= (25 \text{ cm})^2 - (24 \text{ cm})^2$

$= (625 - 576) \text{ cm}^2 = 49 \text{ cm}^2$

⇒ OT = 7 cm

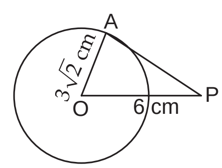
⇒ radius = 7 cm

Diameter = 2 × radius

= 2 × 7 cm = 14 cm.

2. (*b*) **45°**

PA ⊥ OA (Refer to MCQ 1)



In right ΔOAP, we have

$OA^2 + PA^2 = OP^2$ [By Pythagoras' Theorem]

⇒ $(3\sqrt{2} \text{ cm})^2 + PA^2 = (6 \text{ cm})^2$

⇒ $PA^2 = (36 - 18) \text{ cm}^2$

$= 18 \text{ cm}^2$

⇒ $PA = \sqrt{18}$ cm

$= 3\sqrt{2}$ cm

Now, in ΔOAP, we have OA = PA

∴ ∠APO = ∠AOP = x° (say)

Also ∠APO + ∠AOP + ∠OAP = 180° [Sum of angles of a triangle]

⇒ ∠APO + ∠APO + 90° = 180°

⇒ 2∠APO = 180° – 90°

⇒ 2∠APO = 90°

⇒ ∠APO = 45°

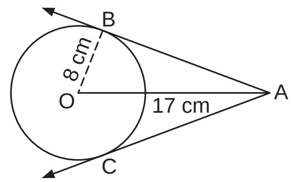
3. (*c*) **Infinite**

A circle can have infinite number of tangents because there are infinite number of points on a circle. Each of these tangents has a parallel tangent at the end of the diameter drawn through the point of contact.

So, a circle can have **infinite** parallel tangents.

4. (*b*) **15 cm**

$OB \perp AB$ [Refer to MCQ 1]



In right ΔOBA, we have

$OB^2 + AB^2 = OA^2$ [By Pythagoras' Theorem]

$\Rightarrow \quad AB^2 = (17 \text{ cm})^2 - (8 \text{ cm})^2$

$= (289 - 64) \text{ cm}^2$

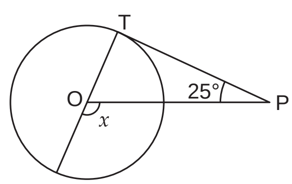
$= 225 \text{ cm}^2$

$\Rightarrow \quad AB = 15 \text{ cm}$ ...(1)

$\Rightarrow \quad AC = AB$ [Lengths of tangents drawn from an external point to a circle are equal]

$\therefore \quad AC = 15 \text{ cm}$ [Using (1)]

5. (*d*) **115°**



$OT \perp PT$ [Refer to MCQ 1]

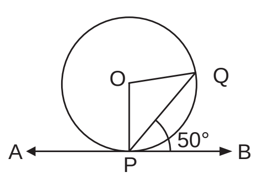
$\Rightarrow \quad \angle OTP = 90°$ ...(1)

$x = \angle OTP + \angle TPO$ [Exterior angle = Sum of interior opposite angles]

$\Rightarrow \quad x = 90° + 25°$ [Using (1)]

$\Rightarrow \quad x = \mathbf{115°}$

6. (*b*) **100°**



$OP \perp APB$ [Refer to MCQ 1]

$\Rightarrow \quad \angle OPB = 90°$

$\angle OPQ = \angle OPB - \angle QPB$

$\Rightarrow \quad \angle OPQ = 90° - 50° = 40°$

$\angle OQP = \angle OPQ = 40°$ [Angles opposite equal sides OQ and OP of ΔOPQ]

In ΔOPQ, we have

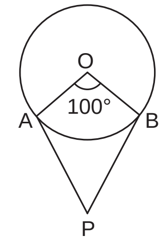
$\angle OQP + \angle OPQ + \angle POQ = 180°$ [Sum of angles of a triangle]

$\Rightarrow \quad 40° + 40° + \angle POQ = 180°$

$\Rightarrow \quad \angle POQ = 180° - (40° + 40°)$

$= 100°$

7. (*c*) **80°**



PA and PB are tangents at the end of radii OA and OB such that $\angle AOB = 100°$.

$OA \perp PA$ and $OB \perp PB$ [Refer to MCQ 1]

In quadrilateral OAPB, we have

$\angle PAO + \angle AOB + \angle OBP + \angle APB$

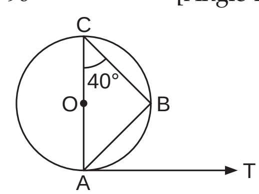
$= 360°$ [Sum of angles of a quadrilateral]

$\Rightarrow \quad 90° + 100° + 90° + \angle APB = 360°$

$\Rightarrow \quad \angle APB = 360° - (90° + 100° + 90°) = 80°$

8. (*c*) **40°**

$\angle CBA = 90°$ [Angle in a semicircle]



In ΔABC, we have

$\angle ACB + \angle CBA + \angle CAB$

$= 180°$ [Sum of angles of triangle]

$\Rightarrow \quad 40° + 90° + \angle CAB = 180°$

$\Rightarrow \quad \angle CAB = 180° - (40° + 90°)$

$= 50°$ ...(1)

$OA \perp AT$ [Refer to MCQ 1]

$\Rightarrow \quad \angle OAT = 90°$

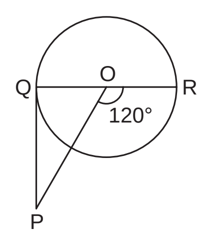
$\angle OAT = \angle OAB + \angle BAT$

$= \angle CAB + \angle BAT$

$\Rightarrow \quad 90° = 50° + \angle BAT$ [Using (1)]

$\Rightarrow \quad \angle BAT = 90° - 50° = 40°$

9. (*a*) **30°**



$OQ \perp QP$ [Refer to MCQ 1]

$\Rightarrow \quad OQP = 90°$ ...(1)

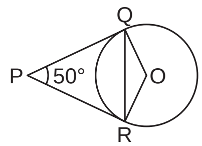
$\angle OQP + \angle OPQ = 120°$ [Exterior angle = Sum of interior opposite angles]

$\Rightarrow \quad 90° + \angle OPQ = 120°$ [Using (1)]

$\Rightarrow \quad \angle OPQ = 120° - 90°$

$\Rightarrow \quad \angle OPQ = \mathbf{30°}$

10. (*a*) **25°**



$OQ \perp PQ$

and $OR \perp PR$ [Tangent at any point of a circle is perpendicular to the radius through the point of contact]

$\Rightarrow$ $\angle OQP = 90°$ and $\angle ORP = 90°$ …(1)

In ΔPQR, we have

$PQ = PR$ [Lengths of tangents drawn from an external point to a circle are equal]

$\Rightarrow$ $\angle PQR = \angle PRQ = x$ (say) [Angles opposite to equal sides] …(2)

In ΔPQR, we have

$\angle QPR + \angle PQR + \angle PRQ = 180°$

[Sum of angles of a triangle]

$\Rightarrow$ $50° + x + x = 180°$ [Using (2)]

$\Rightarrow$ $50° + 2x = 180°$

$\Rightarrow$ $2x = 180° - 50° = 130°$

$\Rightarrow$ $x = 65°$

$\Rightarrow$ $\angle PQR = 65°$ [Using (2)] …(3)

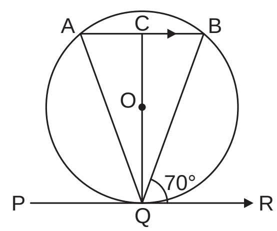
$\angle OQR + \angle PQR = \angle OQP$

$\Rightarrow$ $\angle OQR + 65° = 90°$ [Using (3)]

$\Rightarrow$ $\angle OQR = 90° - 65°$

$= 25°$.

11. (*b*) **40°**



$\angle ABQ = \angle BQR = 70°$ [Alt. $\angle$s, $AB \parallel PQR$] …(1)

Let QO meet AB at C.

Since the tangent at any point of a circle is perpendicular to the radius through the point of contact

$\therefore$ $OQ \perp QR$

$\Rightarrow$ $\angle OQR = 90°$

$\Rightarrow$ $\angle CQR = 90°$ …(2)

$\angle CQB = \angle CQR - \angle BQR$

$= 90° - 70°$ [Using (2)]

$\Rightarrow$ $\angle CQB = 20°$ …(3)

In ΔBCQ, we have

$\angle CQB + \angle CBQ + \angle QCB = 180°$ [Sum of angles of a triangle]

$\Rightarrow$ $20° + 70° + \angle QCB = 180°$[Using (1) and (3)]

$\Rightarrow$ $\angle QCB = 90°$

$\Rightarrow$ $OC \perp AB$

Since perpendicular drawn from the centre of the circle to a chord bisects the chord

$\therefore$ OC bisects AB

$\Rightarrow$ $AC = BC$ …(4)

In right ΔQCA and right ΔQCB, we have

$AC = BC$ [Using (4)]

$CQ = CQ$ [Common]

$\therefore$ $\Delta QCA \cong \Delta QCB$ [By SAS congruence]

$\Rightarrow$ $\angle CQA = \angle CQB$ [CPCT]

$\Rightarrow$ $\angle CQA = 20°$ [Using (3)] …(5)

$\angle AQB = \angle CQA + \angle CQB$

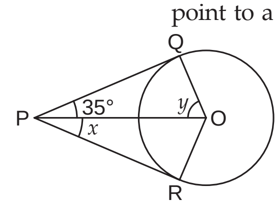
$= 20° + 20°$ [Using (5) and (3)]

$\Rightarrow$ $\angle AQB = 40°$

12. (*b*) $x = 35°, y = 55°$

In ΔPQO and ΔPRO, we have

$PQ = PR$ [Lengths of tangents from an external point to a circle are equal]



$OP = OP$ [Common]

$OQ = OR$ [Radii of a circle]

$\therefore$ $\Delta PQO \cong \Delta PRO$ [By SSS congruence]

$\Rightarrow$ $\angle QPO = \angle RPO$ [CPCT]

$\Rightarrow$ $35° = x$

Since the tangent at any point of a circle is perpendicular to the radius through the point of contact

$\therefore$ $\angle OQP = 90°$ …(1)

In ΔOQP, we have

$\angle OQP + \angle QPO + \angle POQ = 180°$ [Sum of angles of a triangle]

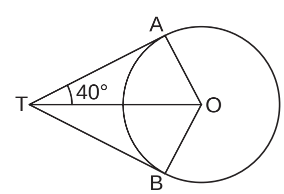
$\Rightarrow$ $90° + 35° + y = 180°$ [Using (1)]

$\Rightarrow$ $y = 180° - (90° + 35°)$

$= 180° - 125° = 55°$

Hence, $x = 35°, y = 55°$.

13. (*b*) **100°**



Since the tangent at any point of a circle is perpendicular to the radius through the point of contact

$\therefore$ $\angle OAT = 90°$ and $\angle OBT = 90°$ …(1)

$\Delta OTB \cong \Delta OTA$ [By SSS congruence]

[Refer to solution of Q. 14]

$\therefore \quad \angle OTB = \angle OTA$ [CPCT] …(2)

$\Rightarrow \quad \angle OTB = 40°$

In quadrilateral OATB, we have

$\angle OAT + \angle ATB + \angle OBT + \angle AOB = 360°$

[Sum of angles of a quadrilateral]

$\Rightarrow \quad 90° + (\angle OTA + \angle OTB) + 90° + \angle AOB = 360°$

$\Rightarrow \quad 90° + (40° + 40°) + 90° + \angle AOB = 360°$ [Using (2)]

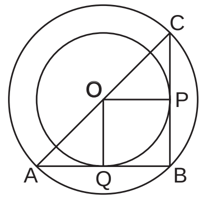
$\Rightarrow \quad \angle AOB = 360° - (90° + 40° + 40° + 90°)$

$= 360° - 260° = 100°$

14. (*d*) **8 cm**

Join OP and OC.

Then, OP = 3 cm and OC = 5 cm.



Since, the tangent at any point on a circle is perpendicular to the radius through the point of contact and BPC is tangent to the smaller circle at P and OP is the radius through the point of contact P.

$\therefore \quad OP \perp BPC \Rightarrow \angle OPC = 90°$

In right ΔOPC, we have

$OC^2 = OP^2 + PC^2$ [By Pythagoras' Theorem]

$\Rightarrow \quad (5 \text{ cm})^2 = (3 \text{ cm})^2 + PC^2$

$\Rightarrow \quad PC^2 = (25 - 9) \text{ cm}^2 = 16 \text{ cm}^2$

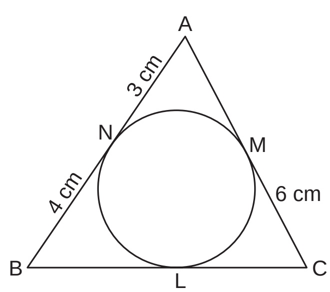
$\Rightarrow \quad PC = 4 \text{ cm}$

Since a perpendicular from the centre of a circle to a chord bisects it

$\therefore$ In the larger circle OP bisects BPC

$\therefore \quad BC = 2\ PC = 2 \times 4 \text{ cm} = 8 \text{ cm}$

15. (*d*) **10 cm**



Since the lengths of tangents drawn from an external point to a circle are equal

$\therefore \quad BL = BN = 4 \text{ cm}$ [Tangents from B] …(1)

$CL = CM = 6 \text{ cm}$ [Tangents from C] …(2)

Now, $BC = BL + CL$

$= 4 \text{ cm} + 6 \text{ cm}$ [Using (1) and (2)]

$\Rightarrow \quad BC = 10 \text{ cm}$

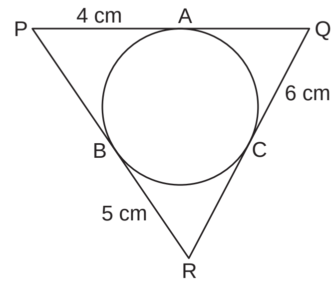
16. (*a*) **30 cm**

Since the lengths of tangents drawn from an external point to a circle are equal

$\therefore \quad PB = PA = 4 \text{ cm}$ [Tangents from P] …(1)

$QA = QC = 6 \text{ cm}$ [Tangents from Q] …(2)

$RC = RB = 5 \text{ cm}$ [Tangents from R] …(3)



Perimeter of ΔPQR

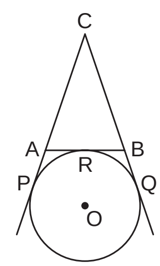
$= PQ + QR + PR$

$= PA + QA + QC + RC + RB + PB$

$= (4 + 6 + 6 + 5 + 5 + 4) \text{ cm}$ [Using (1), (2) and (3)]

$= 30 \text{ cm}$

17. (*c*) **4 cm**



Since the lengths of tangents drawn from an external point to a circle are equal

$\therefore \quad CP = CQ$ [Tangents from C] …(1)

$BR = BQ$ [Tangents from B] …(2)

$CQ = CP$ [From (1)]

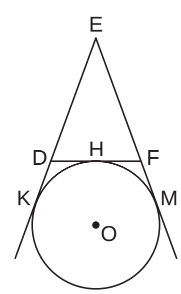
$\Rightarrow \quad BC + BQ = 11 \text{ cm}$

$\Rightarrow \quad 7 \text{ cm} + BR = 11 \text{ cm}$ [Using (2)]

$\Rightarrow \quad BR = (11 - 7) \text{ cm}$

$\Rightarrow \quad BR = 4 \text{ cm}$

18. (*a*) **18**



Since the lengths of tangents drawn from an external point to a circle are equal

$\therefore \quad EK = EM$ [Tangents from E] …(1)

$DH = DK$ [Tangents from D] …(2)

$FH = FM$ [Tangents from F] …(3)

Perimeter of ΔEDF

= ED + DF + EF

= ED + (DH + FH) + EF

= (ED + DK) + (FM + EF) [Using (2) and (3)]

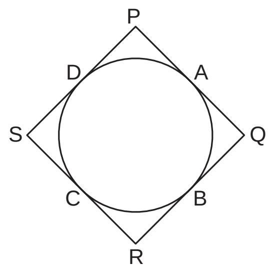
= EK + EM

= EK + EK [Using (1)]

= 2EK = 2× 9 cm

= 18 cm

19. (*c*) **PQ**



Since the lengths of tangents drawn from an external point to a circle are equal

∴ PD = PA [Tangents from P] …(1)

QB = QA [Tangents from Q] …(2)

Adding the corresponding sides of (1) and (2), we get

PD + QB = PA + QA = PQ

20. (*d*) **34 units**

Since the lengths of tangents drawn from an external point to a circle are equal

AS = AP [Tangents from A] …(1)

BP = BQ [Tangents from B] …(2)

CR = CQ [Tangents from C] …(3)

DR = DS [Tangents from D] …(4)

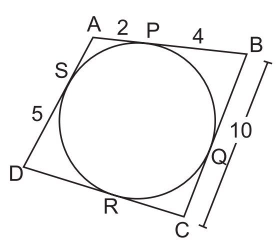
AS = AP = 2 [Using (1)] …(5)

BQ = BP = 4 [Using (2)]

CQ = 10 – 4 = 6

CR = CQ = 6 [Using (3)] …(6)

DR = DS = 5 [Using (4)] …(7)



Perimeter of quadrilateral ABCD

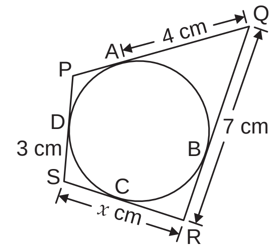
= AB + BC + CD + DA

= (AP + PB) + BC + (CR + DR) + (DS + AS)

= [(2 + 4) + 10 + (6 + 5) + (5 + 2)] units [Using (5), (6) and (7)]

= 34 units

21. (*a*) **6 cm**



Since the lengths of tangents drawn from an external point to a circle are equal

∴ QB = QA = 4 cm [Tangents from Q] …(1)

RC = RB = (7 – 4) cm

= 3 cm [Tangents from R] …(2)

SC = SD = 3 cm [Tangents from S] …(3)

SC + RC = SR

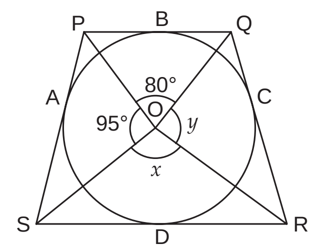
⇒ 3 cm + 3 cm = SR

⇒ 6 cm = x

⇒ x = 6 cm

22. (*b*) $x = 100°$, $y = 85°$

Since, the opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre



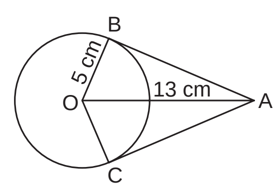
∴ $x + 80° = 180°$ and $y + 95° = 180°$

⇒ $x = 100°$ and $y = 180° – 95° = 85°$

Hence, $x = 100°$, $y = 85°$.

23. (*c*) **60 cm²**

Since the tangent at any point of a circle is perpendicular to the radius through the point of contact.



∴ OB ⊥ AB

⇒ ∠ABO = 90°

In right ΔABO, we have

$AB^2 + OB^2 = AO^2$ [By Pythagoras' Theorem]

⇒ $AB^2 + (5 \text{ cm})^2 = (13 \text{ cm})^2$

⇒ $AB^2 = (169 – 25) \text{ cm}^2$

$= 144 \text{ cm}^2$

⇒ AB = 12 cm …(1)

In ΔABO and ΔACO, we have

AB = AC [Lengths of tangents from an external point to a circle are equal]

$OA = OA$ [Common]

$OB = OC$ [Radii of a circle]

$\therefore \quad \Delta ABO \cong \Delta ACO$

$\Rightarrow \quad ar(\Delta ABO) = ar(\Delta ACO)$ ...(2)

$ar(\Delta ABO) = \frac{1}{2} AB \times OB$

$= \frac{1}{2} \times 12 \text{ cm} \times 5 \text{ cm}$ [Using (1)]

$\Rightarrow \quad ar(\Delta ABO) = 30 \text{ cm}^2$ ...(3)

$\therefore \quad ar(\Delta ACO) = 30 \text{ cm}^2$ [Using (2)] ...(4)

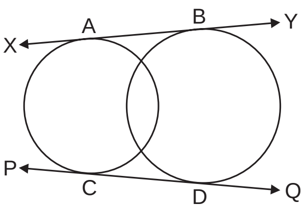
ar quad ABOC = ar(ΔABO) + ar(ΔACO)

$= 30 \text{ cm}^2 + 30 \text{ cm}^2$ [Using (3) and (4)]

$= 60 \text{ cm}^2$

24. (*b*) **2**

XY and PQ are common tangents to two intersecting circles.



For Standard Level

25. (*a*) $3\sqrt{3}$ **cm**

In ΔPAO and ΔPBO, we have

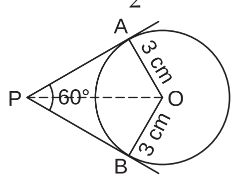
$PA = PB$ [Lengths of tangents drawn from an external point to a circle are equal]

$OA = OB$ [Radii of a circle]

$OP = OP$ [Common]

$\therefore \quad \Delta PAO \cong \Delta PBO$ [By SSS congruence]

$\Rightarrow \quad \angle APO = \angle BPO = \frac{60°}{2} = 30°$



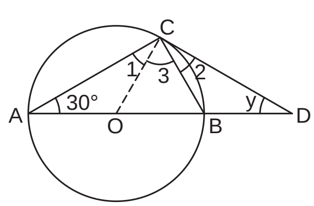
In right ΔOAP, we have

$\tan 30° = \frac{3}{AP}$ cm

$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{3 \text{ cm}}{AP}$

$\Rightarrow \quad AP = 3\sqrt{3}$ cm

26. (*a*) **Isosceles**



$\angle ACB = 90°$ [Angle in a semicircle] ...(1)

In ΔACB, we have

$\angle CAB + \angle ACB + \angle OBC$

$= 180°$ [Sum of angles of a triangle]

$\Rightarrow \quad 30° + 90° + \angle OBC = 180°$ [Using (1)]

$\Rightarrow \quad \angle OBC = 60°$ ...(2)

$\Rightarrow \quad \angle 3 = 60°$ [Angles opposite to equal sides OB and OC of ΔOBC] ...(3)

$\angle 1 + \angle 3 = \angle ACB$

$\Rightarrow \quad \angle 1 + \angle 3 = 90°$ [Using (1)]

$\Rightarrow \quad \angle 1 + 60° = 90°$ [Using (3)]

$\Rightarrow \quad \angle 1 = 30°$ ...(4)

Also $\angle 2 + \angle 3 = \angle OCD$

$\Rightarrow \quad \angle 2 + \angle 3 = 90°$ [OC ⊥ CD, Refer to MCQ 1]

$\therefore \quad \angle 2 + \angle 3 = \angle 1 + \angle 3$

$\Rightarrow \quad \angle 2 = \angle 1 = 30°$ [Using (4)] ...(5)

$\angle 2 + y = \angle OBC$ [Exterior angle = sum of interior opposite angles]

$30° + y = 60°$ [Using (5) and (2)]

$\Rightarrow \quad y = 30°$ ...(6)

$\therefore \quad BC = BD$ [Sides opposite equal angles y and ∠2, using (5) and (6)]

∴ ΔBCD is an isosceles triangle.

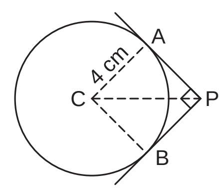
27. (*b*) **4 cm**

In ΔACP and ΔBCP, we have

$CA = CB$ [Radii of a circle]

$CP = CP$ [Common]

$PA = PB$ [Lengths of tangents drawn from an external point to a circle are equal]



$\therefore \quad \Delta ACP \cong \Delta BCP$ [By SSS congruence]

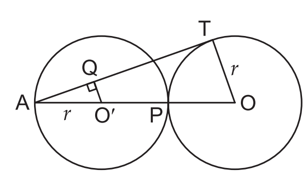
$\Rightarrow \quad \angle APC = \angle BPC = \frac{90°}{2} = 45°$ [CPCT] ...(1)

In ΔACP, we have

$\tan APC = \frac{AC}{AP} \Rightarrow \tan 45° = \frac{4 \text{ cm}}{AP}$ [Using (1)]

$\Rightarrow \quad 1 = \frac{4 \text{ cm}}{AP} \Rightarrow AP = 4$ cm

28. (*c*) $\mathbf{\frac{1}{3}}$



Since the tangent at any point on a circle is perpendicular to the radius through the point of contact

$\therefore$ $\angle ATO = 90°$ ...(1)

$\angle AQO' = 90°$ [$Q'Q \perp AT$, given] ...(2)

From (1) and (2), we get

$\angle ATO = \angle AQO'$.

But these are corresponding angles.

$\therefore$ $O'Q \parallel OT$

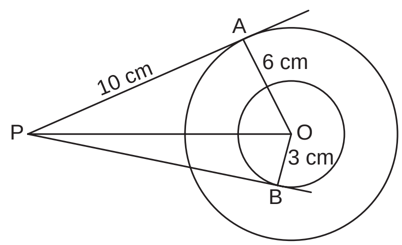
In ΔAOT, we have

$O'Q \parallel OT$

$\therefore$ $\frac{AQ}{AT} = \frac{AO'}{AO}$ [By BPT]

$\Rightarrow$ $\frac{AQ}{AT} = \frac{r}{AP + PO} = \frac{r}{2r + r} = \frac{r}{3r} = \frac{1}{3}$.

29. (*b*) $\sqrt{127}$ **cm**



Since, the tangent at any point on a circle is perpendicular to the radius through the point of contact and PA is a tangent to the bigger circle at A and OA the radius through the point of contact

$\therefore$ $OA \perp PA \Rightarrow \angle OAP = 90°$

In right ΔOAP, we have

$OP^2 = OA^2 + AP^2$ [By Pythagoras' Theorem]

$\Rightarrow$ $OP^2 = (6 \text{ cm})^2 + (10 \text{ cm})^2 = 136 \text{ cm}^2$ ...(1)

PB is a tangent to the smaller circle at B and OB is the radius through the point of contact B.

$\therefore$ $OB \perp BP \Rightarrow \angle OBP = 90°$

In right ΔOBP, we have

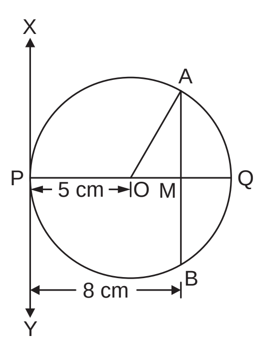
$OP^2 = OB^2 + BP^2$ [By Pythagoras' Theorem]

$\Rightarrow$ $136 \text{ cm}^2 = (3 \text{ cm})^2 + BP^2$ [Using (1)]

$\Rightarrow$ $BP^2 = (136 - 9) \text{ cm}^2 = 127 \text{ cm}^2$

$\Rightarrow$ $BP = \sqrt{127}$ cm

30. (*a*) **8 cm**



Since the tangent at any point of a circle is perpendicular to the radius through the point of contact and XPY is a tangent at P and OP is the radius through P,

$\therefore$ $OP \perp XPY \Rightarrow \angle XPO = 90°$ ...(1)

Let diameter PQ and chord AB intersect at M

$\angle XPO + \angle AMP = 180°$ [Co. int. angles, $XPY \parallel AB$]

$\Rightarrow$ $90° + \angle AMO = 180°$ [Using (1)]

$\Rightarrow$ $\angle AMO = 90°$

In right ΔOMA, we have

$OM^2 + AM^2 = OA^2$ [By Pythagoras' Theorem]

$\Rightarrow$ $(3 \text{ cm})^2 + AM^2 = (5 \text{ cm})^2$ [OM = PM – OP = (8 – 5) cm = 3 cm]

$\Rightarrow$ $AM^2 = (25 - 9) \text{ cm}^2$

$\Rightarrow$ $AM^2 = 16 \text{ cm}^2$

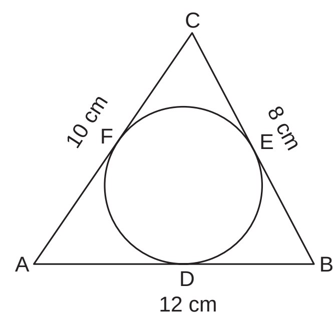
$\Rightarrow$ $AM = 4$ cm

Since the perpendicular from the centre of a circle to a chord bisects the chord

$\therefore$ OM bisects AB

$\Rightarrow$ $AB = 2\ AM = 2 \times 4 \text{ cm} = 8 \text{ cm}$

31. (*d*) **AD = 7 cm, BE = 5 cm**



Since the lengths of tangents drawn from an external point to a circle are equal

$\therefore$ $AF = AD$

$= x$ cm (say) [Tangents from A] ...(1)

$BE = BD$

$= (12 - x)$ cm [Tangents from B] ...(2)

$CE = CF$ [Tangents from C] ...(3)

$\Rightarrow$ $(10 - x) \text{ cm} = [8 - (12 - x)] \text{ cm}$

$\Rightarrow$ $10 - x = 8 - 12 + x$

$\Rightarrow$ $10 + 12 - 8 = 2x$

$\Rightarrow$ $x = 7$

$\Rightarrow$ $AD = 7$ cm

and $BE = (12 - x)$ cm

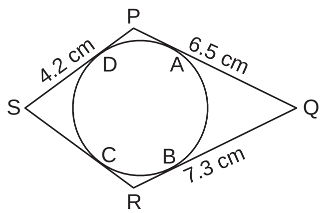
$= (12 - 7)$ cm

$= 5$ cm [Using (1) and (2)]

Hence, AD = 7 cm, BE = 5 cm.

32. (*d*) **5 cm**

Let PQ, QR, SR and SP touch the circle at A, B, C and D respectively.



Since the lengths of tangents drawn from an external point to a circle are equal

PA = PD [Tangents from P] …(1)

QA = QB [Tangents from Q] …(2)

RB = RC [Tangents from R] …(3)

SC = SD [Tangents from S] …(4)

Let PA = x cm

Then, QA = QB = (6.5 – x) cm [Using (2)]

⇒ RB = 7.3 – (6.5 – x) cm

= (0.8 + x) cm

⇒ RC = (0.8 + x) cm [Using (3)] …(5)

PD = PA = x cm

⇒ SD = (4.2 – x) cm

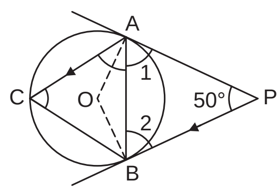
⇒ SC = (4.2 – x) cm [Using (4)] …(6)

RS = RC + SC

= [(0.8 + x) + (4.2 – x)] cm [Using (5) and (6)]

⇒ RS = 5 cm

33. (*d*) **65°, 50°, 65°**



Since the lengths of tangents drawn from an external point to a circle are equal.

∴ PB = PA

⇒ ∠PAB = ∠PBA [Angles opposite to equal sides of the ΔPAB] …(1)

In ΔPAB, we have

∠APB + ∠PAB + ∠PBA

= 180° [Sum of angles of a triangle]

⇒ 50° + ∠PBA + ∠PBA = 180° [Using (1)]

⇒ 2∠PBA = 130°

⇒ ∠PBA = 65°

∠CAB = ∠PBA = 65° [Alternate angles AC ∥ PB] …(2)

Join OA and OB.

In quadrilateral AOBP, we have

∠PAO + ∠PBO + ∠APB + ∠AOB

= 360° [Sum of angles of quadrilateral AOBP]

⇒ 90° + 90° + 50° + ∠AOB = 360°

⇒ ∠AOB = 360° – (90° + 90° + 50°)

ΔAOB = 360° – (230°)

⇒ ∠AOB = 130°

2∠ACB = ∠AOB [Angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle]

⇒ $\angle ACB = \frac{1}{2}\angle AOB = \frac{1}{2} \times 130° = 65°$ …(3)

In ΔABC, we have

∠CAB + ∠ACB + ∠ABC

= 180° [Sum of angles of a triangle]

⇒ 65° + 65° + ∠ABC = 180°

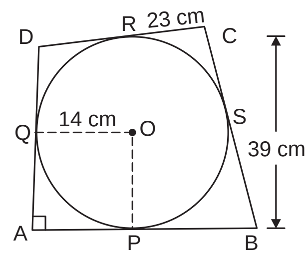
⇒ ∠ABC = 180° – (65° + 65°) = 50° …(4)

So, the angles of the triangle are 65°, 50°, 65° [Using (2), (4) and (3)]

Alternative Method: Use alternate segment theorem.

34. (*c*) **30 cm**

Join OP.



Since the tangent at any point of a circle is perpendicular to the radius through the point of contact

∴ OQ ⊥ AD and OP ⊥ AB

⇒ ∠OQA = 90° and ∠OPA = 90°

Also ∠QAP = 90° [∠A = 90°, given]

So, in quadrilateral AQOP, each angle is 90° and

OQ = adjacent side OP [Radii of a circle]

∴ Quadrilateral AQOP is square

∴ AP = OQ = 14 cm …(1)

Since the lengths of tangents drawn from an external point to a circle are equal.

∴ CS = CR = 23 cm [Tangents from C]

BP = BS = (39 – 23) cm

= 16 cm [Tangents from B] …(2)

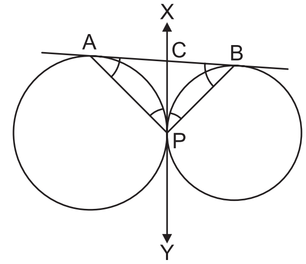
AB = AP + BP

= 14 cm + 16 cm [Using (1) and (2)]

⇒ AB = 30 cm

35. (*d*) **90°**

Draw XY the common tangent at P to the externally touching circles and let it intersect AB at C.



Since the lengths of tangents drawn from an external point to a circle are equal.

$\therefore \quad CA = CP$ and $BC = CP$

$\therefore \quad \angle CPA = \angle CAP = x$ (say)

and $\angle CPB = \angle CBP = y$ (say) …(1)

[Angles opposite to equal sides]

In ΔABP, we have

$\angle BAP + \angle APB + \angle ABP$

$= 180°$ [Sum of angles of a triangle]

$\Rightarrow \quad \angle CAP + (\angle CPA + \angle CPB) + \angle CBP = 180°$

$\Rightarrow \quad x + (x + y) + y = 180°$ [Using (1)]

$\Rightarrow \quad 2x + 2y = 180°$

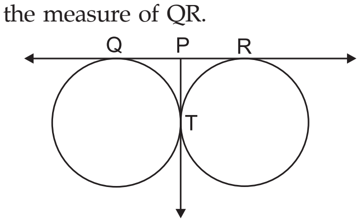
$\Rightarrow \quad x + y = 90°$

$\Rightarrow \quad \angle CPA + \angle CPB = 90°$

$\Rightarrow \quad \angle APB = 90°$

36. (*a*) **9 cm**

Given that two circles touch each other externally at T. QR is a common tangent to the two circles and P is a point on QR such that PT is a tangent to the two circles at T. To find the measure of QR.



We have, $PQ = PT = PR = 4.5$ cm

$\therefore \quad QR = QP + PR$

$= (4.5 + 4.5)$ cm

$= 9$ cm

37. (*c*) **24 cm**

Given that BC and BD are two tangents drawn from an external point B to a circle with centre at O and radius 9 cm. OB and OC are joined.

Given that $OB = 15$ cm

Also, $OC = 9$ cm

To find the measure of BC + BD

$\because \quad \angle OCB = 90°$

$\therefore$ By Pythagoras' theorem, we have

$BC = \sqrt{OB^2 - OC^2}$

$= \sqrt{15^2 - 9^2}$ cm

$= \sqrt{225 - 81}$ cm

Also, $BD = BC = 12$ cm

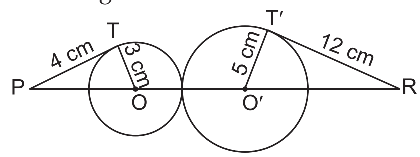
$= \sqrt{144}$ cm $= 12$ cm

$\therefore \quad BC + BD = (12 + 12)$ cm $= 24$ cm

38. (*d*) **26 cm**

Given that two circles, with centres at O and O′ and radii 3 cm and 5 cm touch each other externally. P and R are two external points such that PR passes through O and O′ and PT and RT′ are tangents at T and T′ respectively such that PT = 4 cm and RT′ = 12 cm.

To find the length of PR.



Since the two circles touch each other externally,

$\therefore$ The distance between their centres is equal to the sum of their radii

i.e. $OO' = (3 + 5)$ cm $= 8$ cm …(1)

Now, by Pythagoras' theorem, we have

$PO = \sqrt{PT^2 + OT^2}$

$= \sqrt{4^2 + 3^2}$ cm

$= \sqrt{25}$ cm

$= 5$ cm …(2)

and $O'R = \sqrt{RT'^2 + O'T'^2}$ cm

[$\because \angle O'T'R = 90°$]

$= \sqrt{12^2 + 5^2}$ cm

$= \sqrt{144 + 25}$ cm

$= \sqrt{169}$ cm

$= 13$ cm …(3)

Hence, $PR = PO + OO' + O'R$

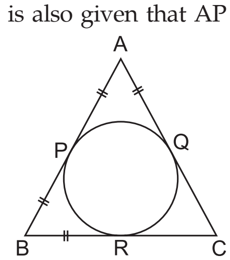
$= (5 + 8 + 13)$ cm

[From (1), (2) and (3)]

$= 26$ cm

39. (*b*) **AC = BC**

Given that a circle is inscribed in a triangle ABC such that the sides AB, BC and CA touch the circle at P, R and Q respectively. It is also given that AP = PB.



To find a relation between two sides of the triangle.

We have $AP = PB = BR = AQ$ …(1)

Now, $CQ = CR$

$\Rightarrow \quad CQ + AQ = CR + AQ = CR + AP$

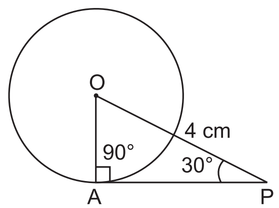
$= CR + PB$

$= CR + BR$ [From (1)]

$\Rightarrow \quad AC = BC$

40. (*c*) $\mathbf{2\sqrt{3}}$ **cm**

Given that P is an external point to a circle with centre at O such that OP = 4 cm. A is a point on the circle such that AP is a tangent to the circle at A. We join OA. Then $\angle OAP = 90°$.



Given that OP = 4 cm and $\angle OPA = 30°$.

To find the length of AP.

From ΔOAP, we have

$$AO = OP \sin 30°$$
$$= 4 \times \frac{1}{2} \text{ cm} = 2 \text{ cm}$$

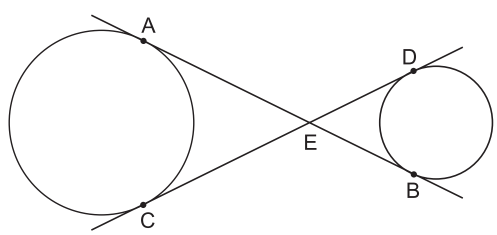
$\therefore$ By Pythagoras' theorem, we have

$$AP = \sqrt{OP^2 - OA^2}$$
$$= \sqrt{4^2 - 2^2} \text{ cm}$$
$$= \sqrt{12} \text{ cm}$$
$$= 2\sqrt{3} \text{ cm}$$

SHORT ANSWER QUESTIONS

For Basic and Standard Levels

1. Prove that AB = CD



Length of tangents drawn from an external point to a circle are equal

$\therefore$ $EA = EC$ (1)

$EB = ED$ (2)

Adding equation (1) and (2), we get

$$EA + EB = EC + CD$$

$\therefore$ $AB = CD$

Hence, proved.

2. In ΔOAP and ΔOBP, we have

$OA = OB$ [Radii of a circle]

$OP = OP$ [Common]

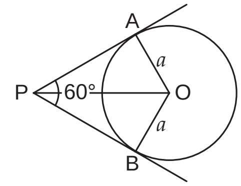
$PA = PB$ [Lengths of tangents drawn from an external point to a circle are equal]

$\therefore$ $\Delta OAP \cong \Delta OBP$ [By SSS congruence]

$\Rightarrow$ $\angle OPA = \angle OPB = \frac{60°}{2} = 30°$ [CPCT] …(1)

Since the tangent at any point of a circle is perpendicular to the radius through the point of contact

$\therefore$ $OA \perp AP \Rightarrow \angle OAP = 90°$



In right ΔOAP, we have

$$\sin \angle OPA = \frac{OA}{OP}$$

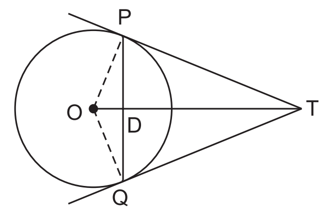
$\Rightarrow$ $\sin 30° = \frac{a}{OP}$ [Using (1)]

$\Rightarrow$ $\frac{1}{2} = \frac{a}{OP}$

$\Rightarrow$ $OP = 2a$

Hence, $OP = \mathbf{2a}$

3. Prove OT is a right bisector of line segment PQ



Join OP and OQ

Now In ΔOPT and ΔOQT

$TP = TQ$ [Tangents from same point]

$OP = OQ$ [Radii of a circle]

$OT = OT$ [Common]

$\therefore$ $\Delta OPT \cong \Delta OQT$ [By SSS congruence]

$\therefore$ $\angle PTO = \angle QTO$ [CPCT]

Now In ΔPDT and ΔQDT

$TP = TQ$

$\angle PTD = \angle QTD$

$DT = DT$

$\therefore$ $\Delta PDT \cong \Delta QDT$ [By SAS congruence]

$\therefore$ $PD = QD$ (CPCT)

$\angle PDT = \angle QDT$ (CPCT)

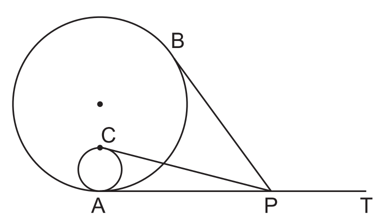
$\angle PDT + \angle QDT = 180°$ (Linear pair)

$2\angle PDT = 2\angle QDT = 180°$

$\angle PDT = \angle QDT = 90°$

Hence OT is the right bisector of line segment PQ

4. Given that two circles touch each other internally at A. P is any point on the tangent AT at the point A of the two circles.



Two tangents PC and PB are drawn from P to the two circles. To prove that PB = PC.

From an external point P, two tangents PA and PC are drawn to the smaller circle.

$\therefore$ PA = PC …(1)

Again, two other tangents PA and PB are drawn from P to the bigger circle.

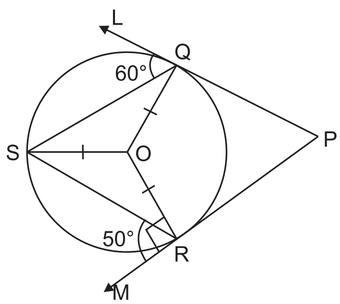
$\therefore$ PB = PA …(2)

$\therefore$ From (1) and (2), we have PB = PC

Hence, proved.

5. Given that PQL and PRM are two tangents to a circle with centre O, drawn from an external point P of the circle.

OQ, OR, OS, SQ and SR are drawn such that $\angle SQL = 60°$ and $\angle SRM = 50°$.



To find the measure of $\angle QSR$.

Since $OQ \perp PL$ and $OR \perp RM$.

We have $\angle OQS = \angle OQL - \angle SQL$

$= 90° - 60° = 30°$ …(1)

Also, $\angle ORS = \angle ORM - \angle SRM$

$= 90° - 50° = 40°$ …(2)

Again, since OQ = OS = OR

= radius of the same circle

$\therefore$ $\angle OSQ = \angle OQS = 30°$ [From (1)] …(3)

and $\angle OSR = \angle ORS = 40°$ [From (2)] …(4)

Hence, $\angle QSR = \angle OSQ + \angle OSR$

$= 30° + 40°$ [From (3) and (4)]

$= 70°$

which is the required measure of $\angle QSR$.

6. In right ΔABC, we have

$BC^2 = AC^2 + AB^2$ [By Pythagoras' theorem]

$\Rightarrow$ $BC^2 = (8 \text{ cm})^2 + (6 \text{ cm})^2$

$\Rightarrow$ $BC^2 = 64 \text{ cm}^2 + 36 \text{ cm}^2$

$= 100 \text{ cm}^2$

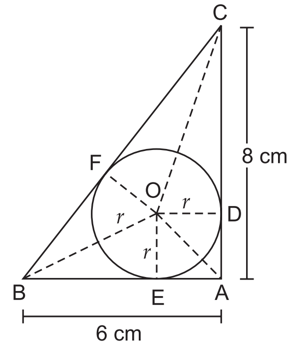
$\Rightarrow$ BC = 10 cm …(1)

Join OA, OB and OC.

Let the tangents AC, AB and BC touch the circle at D, E and F respectively.

Since the tangents at any point of a circle is perpendicular to the radius through the point of contact.

$\therefore$ $OD \perp AC$, $OE \perp AB$ and $OF \perp BC$



$\Rightarrow$ OD, OE and OF are the altitudes of ΔAOC, ΔBOA and ΔBOC respectively.

Now, $ar(\Delta ABC) = ar(\Delta AOC) + ar(\Delta BOA) + ar(\Delta BOC)$

$$\frac{1}{2}\times AB\times AC = \frac{1}{2}\times AC\times OD+\frac{1}{2}\times AB\times OE+\frac{1}{2}\times BC\times OF$$

$$\frac{1}{2}\times AB\times AC = \frac{1}{2}\times AC\times x+\frac{1}{2}\times AB\times x+\frac{1}{2}\times BC\times x$$

[OD = OE = OF = x, radii of inscribed circle]

$$\frac{1}{2}\times 6 \text{ cm}\times 8 \text{ cm}$$

$$= \frac{1}{2}\times 8 \text{ cm}\times x+\frac{1}{2}\times 6 \text{ cm}\times x+\frac{1}{2}\times 10 \text{ cm}\times x$$

[Using (1)]

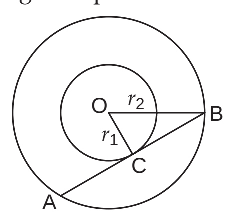
$\Rightarrow$ $24 \text{ cm}^2 = x(4 + 3 + 5)$ cm

$\Rightarrow$ $24 \text{ cm}^2 = 12x$ cm

$\Rightarrow$ $x = \dfrac{24\text{cm}^2}{12\text{cm}} = 2$ cm

Hence, x = **2 cm**.

7. Since the tangent at any point of a circle is perpendicular to the radius through the point of contact



$\therefore$ $OC \perp ACB \Rightarrow \angle OCB = 90°$

In right ΔOCB, we have

$OC^2 + CB^2 = OB^2$ [By Pythagoras' Theorem]

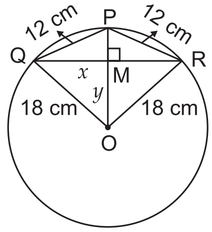
$\Rightarrow$ $r_1^2 + \left(\dfrac{AB}{2}\right)^2 = r_2^2$ [Perpendicular from the centre of the circle to the chord bisects the chord and $OC \perp$ chord ACB of the larger circle]

$\Rightarrow$ $r_1^2 + \left(\dfrac{c}{2}\right)^2 = r_2^2$ [AB = c, given]

$\Rightarrow$ $r_1^2 + \dfrac{c^2}{4} = r_2^2 \Rightarrow 4r_1^2 + c^2 = 4r_2^2$

Hence, $\mathbf{4r_2^2 = 4r_1^2 + c^2}$

8. Given that ΔPQR is an isosceles triangle with equal sides PQ = PR = 12 cm, which is inscribed in a circle with centre at O and radius = 18 cm. QO and RO are joined. Then OQ = OR = OP = 18 cm.



To find the area of ΔPQR.

Let QM = x cm and OM = y cm

Then from ΔOQM, since

$$\angle QMO = 90°$$

∴ By Pythagoras' theorem, we have

$$QM^2 + OM^2 = OQ^2$$

⇒ $$x^2 + y^2 = 18 \quad ...(1)$$

and from ΔPQM, since, QM = x cm

$$PM = (18 - y) \text{ cm}$$

and $$PQ = 12 \text{ cm}$$

Hence, by Pythagoras' theorem, we have

$$PQ^2 = PM^2 + QM^2$$

⇒ $$12^2 = (18 - y)^2 + x^2$$

⇒ $$x^2 + (18 - y^2) = 12^2 \quad ...(2)$$

∴ Subtracting (2) from (1), we get

$$y^2 - (18 - y)^2 = 18^2 - 12^2$$

⇒ $$(y + 18 - y)\ (y - 18 + y) = (18 + 12)\ (18 - 12)$$

⇒ $$18(2y - 18) = 30 \times 6$$

⇒ $$36y - 18^2 = 180$$

⇒ $$y = \frac{18^2 + 180}{36}$$

$$= \frac{324 + 180}{36} = \frac{504}{36} = 14 \quad ...(3)$$

Hence, from (1), $x^2 = 324 - 14^2$

$$= 324 - 196$$

$$= 128$$

∴ $$x = 8\sqrt{2} \quad ...(4)$$

Now, since PM is a median of ΔPQR.

∴ $$ar(\Delta PQR) = 2\ ar(\Delta PQM)$$

$$= 2 \times \frac{1}{2}QM \times PM$$

$$= QM \times PM = x \times (18 - y)$$

$$= 8\sqrt{2} \times (18 - 14) \text{ cm}^2$$

[From (3) and (4)]

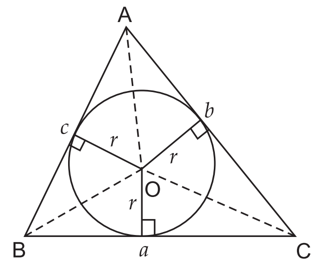
$$= \mathbf{32\sqrt{2}\ cm^2}$$

which is the required area of ΔPQR.

9. Given that ABC is a triangle circumscribing a circle with centre at O and radius r. Let a, b, c be the lengths of the sides of ΔABC opposite to the vertices A, B and C respectively. Given that S is the area of ΔABC and s is the semi-perimeter of ΔABC, i.e.

$$s = \frac{a + b + c}{2} \quad ...(1)$$

To prove that S = rs.



Construction: We join OA, OB and OC.

We have

$$ar(\Delta ABC) = ar(\Delta OBC) + ar(\Delta OAC) + ar(\Delta OAB)$$

$$= \frac{1}{2}ar + \frac{1}{2}br + \frac{1}{2}cr$$

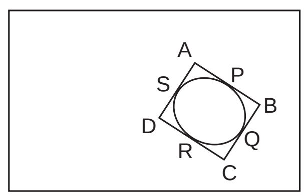
⇒ $$S = \frac{1}{2}(a + b + c)r = sr \quad \text{[From (1)]}$$

Hence, the result.

VALUE-BASED QUESTIONS

For Basic and Standard Levels

1. (*i*) Since the lengths of tangents drawn from an external point to a circle are equal



∴ AS = AP

= x m (say) [Tangents from A] ...(1)

BP = BQ [Tangents from B] ...(2)

CR = CQ [Tangents from C] ...(3)

DR = DS [Tangents from D] ...(4)

BP = AB – AP = 5 m – x m

= (5 – x) m [Using (1)]

⇒ BQ = (5 – x) [Using (2)]

CQ = BC – BQ = [3 – (5 – x)] m

= (x – 2) m

CR = (x – 2) m [Using (3)]

DR = CD – CR = [6.8 – (x – 2)] m

= (8.8 – x) m

DS = (8.8 – x) m [Using (4)] ...(5)

Now, AD = AS + DS

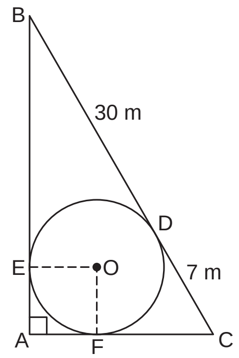
= [x + (8.8 – x)] m [Using (1) and (5)]

$\Rightarrow$ AD = 8.8 m

Hence, AD = **8.8 m**

(*ii*) Empathy and environment awareness.

2. (*i*) Since the lengths of tangents drawn from an external point to a circle are equal



$\therefore$ BE = BD = 30 m [Tangents from B] …(1)

CF = CD = 7 m [Tangents from C] …(2)

AE = AF = x m (say)[Tangents from A] …(3)

In right ΔBAC, we have

$AB^2 + AC^2 = BC^2$ [By Pythagoras' Theorem]

$\Rightarrow$ $(30 + x)^2 + (x + 7)^2 = (37)^2$

$\Rightarrow$ $900 + x^2 + 60x + x^2 + 14x + 49 = 1369$

$\Rightarrow$ $2x^2 + 74x + 949 - 1369 = 0$

$\Rightarrow$ $2x^2 + 74x - 420 = 0$

$\Rightarrow$ $x^2 + 37x - 210 = 0$

$\Rightarrow$ $x^2 + 42x - 5x - 210 = 0$

$\Rightarrow$ $x(x + 42) - 5(x + 42) = 0$

$\Rightarrow$ $(x + 42)(x - 5) = 0$

$\Rightarrow$ Either $x + 42 = 0$ or $x - 5 = 0$

$\Rightarrow$ $x = -42$ (Rejected) or $x = 5$ …(4)

AB = (30 + x)m

= (30 + 5)m = 35 m [Using (3) and (4)]

AC = (5 + 7)m = 12 m

and BC = (30 + 7)m = 37 m

In 28 seconds, the person jogs

= (35 + 37 + 12)m = 84 m

$\therefore$ In 1 second the person jogs = $\frac{84}{28}$ m = 3 m

Thus, his average speed of jogging is **3 m/s.**

(*ii*) Join OE and OF.

Since the tangent at any point of the circle is perpendicular to the radius through the point of contact

$\therefore$ OE $\perp$ AB and OF $\perp$ AC

$\Rightarrow$ $\angle$OEA = 90° and $\angle$OFA = 90°

and $\angle$EAF = 90° [$\angle$BAC = 90°, given]

So, OEAF is a quadrilateral in which each angle is 90° and adjacent sides OE = OF.

$\therefore$ Quadrilateral OEAF is a square.

$\therefore$ OE = AE = x m = 5 m [Using (4)]

Hence, the radius of the circular garden is **5 m.**

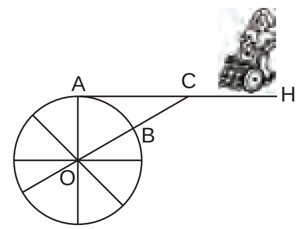
(*iii*) Taking care of physical fitness.

3. (*i*) Angle between two consecutive radial roads

$= \frac{360°}{8} = 45°$

$\Rightarrow$ $\angle$AOC = 45°

Since the tangent at any point of a circle is perpendicular to the radius through the point of contact



$\therefore$ CA $\perp$ OA

$\Rightarrow$ $\angle$CAO = 90°

In right ΔCAO, we have

$\cos 45° = \frac{OA}{OC} = \frac{OA}{OB + BC}$

$\Rightarrow$ $\frac{1}{\sqrt{2}} = \frac{15 \text{ m}}{15 \text{ m} + BC}$

$\Rightarrow$ 15 m + BC = $15\sqrt{2}$ m

$\Rightarrow$ BC = $(15\sqrt{2} - 15)$ m = $15(\sqrt{2} - 1)$ m

= 15 × 0.414 m

$\Rightarrow$ BC = 6.21 m

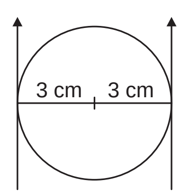
Hence, the length of path BC = **6.21 m.**

(*ii*) Empathy and interpersonal relationship.

UNIT TEST 1

For Basic Level

1. (*a*) **6 cm**

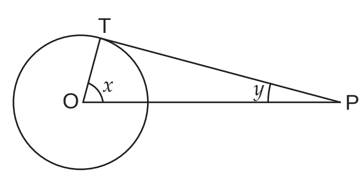


Tangents at the end of a diameter of a circle are parallel.

So the distance between them is equal to the diameter or 2r.

Hence, distance = 2 × 3 cm = 6 cm.

2. (*a*) **90°**



Since the tangent at any point of a circle is perpendicular to the radius through the point of contact

$\therefore$ $\angle$OTP = 90° …(1)

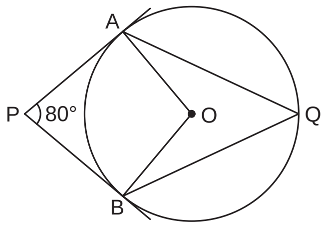
In right ΔOTP, we have

$$\angle OTP + x + y = 180°$$

$\Rightarrow$ $90° + x + y = 180°$ [Using (1)]

$\Rightarrow$ $x + y = 90°$

3. (*d*) **50°**



Since the tangent at any point of a circle is perpendicular to the radius through the point of contact

$\therefore$ $\angle OAP = \angle OBP = 90°$ ...(1)

In quadrilateral APBO, we have

$\angle OAP + \angle APB + \angle PBO + \angle AOB$

$= 360°$ [Sum of angles of a quad]

$\Rightarrow$ $90° + 80° + 90° + \angle AOB = 360°$ [Using (1)]

$\Rightarrow$ $\angle AOB = 360° - (90° + 80° + 90°)$

$\Rightarrow$ $\angle AOB = 360° - 260° = 100°$ ...(2)

Since the angle subtended by the arc of a circle at the centre is double the angle subtended by it at any point on the remaining part of the circle

$\therefore$ $\angle AOB = 2\angle AQB$

$\Rightarrow$ $100° = 2\angle AQB$ [Using (2)]

$\Rightarrow$ $\angle AQB = 50°$

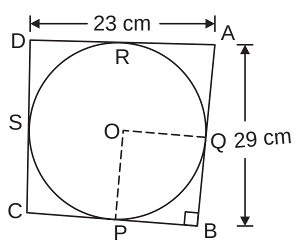
4. (*a*) **11**

Since the lengths of tangents drawn from an external point to a circle are equal

$\therefore$ $AQ = AR$ [Tangents from A] ...(1)

$BQ = BP$ [Tangents from B] ...(2)

$DR = DS$ [Tangents from D] ...(3)



$DR = DS = 5$ cm [Using (3)]

$AR = AD - DR = (23 - 5)$ cm $= 18$ cm

$AQ = 18$ cm [Using (1)]

$\Rightarrow$ $BQ = (29 - 18)$ cm $= 11$ cm ...(4)

$\Rightarrow$ $BP = 11$ cm [Using (2)] ...(4)

Since the tangent at any point of a circle is perpendicular to the radius through the point of contact

$\therefore$ $\angle OQB = \angle OPB = 90°$

Also $\angle QBP = 90°$ [$\angle ABC = 90°$, given]

So, each angle of quadrilateral OQBP is a right angle and its adjacent sides BQ and BP are equal [Using (4)].

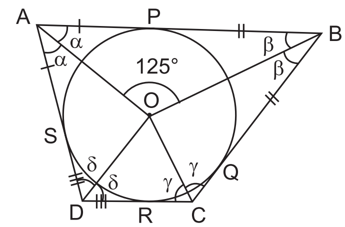
Thus, quadrilateral OQBP is a square

$\therefore$ $OQ = BQ = 11$ cm

Hence, the radius of the circle (in cm) is 11.

5. (*d*) **55°**

Given that a quadrilateral ABCD circumscribes a circle with centre at O such that $\angle AOB = 125°$. OD and OC are joined. To find the measure of $\angle DOC$. Let AB, BC, CD and DA touch the circle at P, Q, R and S respectively.



$\because$ AP and AS are two tangents to the circle from an external point A, hence, AP = AS.

$\angle OAP = \angle OAS = \alpha$ (say)

Similarly, $\angle OBA = \angle OBC = \beta$ (say)

$\angle OCQ = \angle OCD = \gamma$ (say)

and $\angle ODC = \angle ODA = \delta$ (say)

$\therefore$ In ΔOAB, we have

$\alpha + \beta + 125° = 180°$

$\Rightarrow$ $\alpha + \beta = 180° - 125° = 55°$ ...(1)

Now, in quadrilateral ABCD, we have

$\angle ABC + \angle BCD + \angle CDA + \angle DAB$

= sum of all the angles of the quadrilateral

$= 360°$

$\Rightarrow 2\alpha + 2\beta + 2r + 2\delta = 360°$

$\Rightarrow$ $\alpha + \beta + r + \delta = 180°$

$\Rightarrow$ $55° + r + \delta = 180°$ [Using (1)]

$\therefore$ $r + \delta = 180° - 55° = 125°$ ...(2)

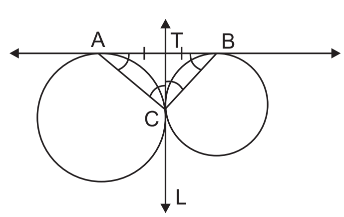
$\therefore$ $\angle COD = 180° - (r + \delta)$

$= 180° - 125°$ [From (2)]

$= 55°$

6. (*c*) **90°**

Given that two circles touch each other externally at C. AB is a common tangent to the two circles. Let TL be a common tangent to the two circles at C where T is a point on AB.



To find ∠ACB,

We have TA = TC = TB …(1)

$\therefore$ In ΔATC, we have ∠TAC = ∠TCA = θ (say) …(2)

$\therefore$ ∠ATC = 180° – 2θ …(3)

$\therefore$ ∠BTC = 180° – ∠ATC

= 180° – (180° – 2θ)

= 2θ …(4) [From (3)]

Now, $\because$ TC = TB

$\therefore$ In ΔTBC, we have

$$\angle TBC = \angle TCB = \frac{180° - \angle BTC}{2}$$

$$= \frac{180° - 2\theta}{2} \quad \text{[From (4)]}$$

$$= 90° - \theta \quad \ldots(5)$$

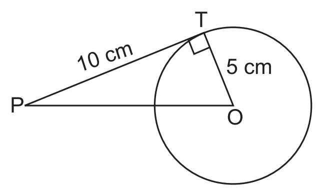
$\therefore$ ∠ACB = ∠TCA + ∠TCB

= θ + 90° – θ [From (3) and (5)]

= 90°

7. (*d*) $\sqrt{125}$ **cm**

Given that PT is a tangent from an external point P to a circle with centre at O, of radius 5 cm such that PT = 10 cm. To find the distance PO.



$\because$ ∠PTO = 90°

$\therefore$ By Pythagoras' theorem, we have

$$OP^2 = OT^2 + PT^2$$

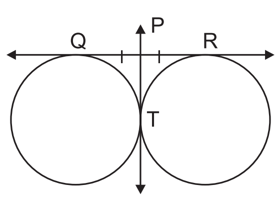
$$= 5^2 + 10^2 = 125$$

$\therefore$ $OP = \sqrt{125}$

Hence, the required distance OP is of measure $\sqrt{125}$ cm.

8. (*d*) **7.6 cm**

Given that two circles touching each other externally at T has a common tangent QR touching the two circles at Q and R. The tangent at T meets QR at P. Given that PT = 3.8 cm.



To find the length of QR.

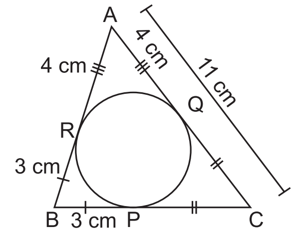
We have PQ = PT = PR = 3.8 cm …(1)

$\therefore$ QR = QP + PR = 2QP = 2 × 3.8 cm

= 7.6 cm [From (1)]

9. (*b*) **10 cm**

Given that a triangle ABC circumscribes a circle which touches the sides BC, CA and AB of the triangle at D, E and F respectively such that AF = 4 cm, BF = 3 cm and AC = 11 cm.



To find the length of BC.

We have BD = BF = 3 cm …(1)

AE = AF = 4 cm …(2)

$\therefore$ CD = CE = AC – AE = (11 – 4) cm [From (2)]

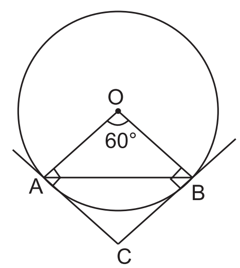
= 7 cm …(3)

$\therefore$ BC = BD + CD = (3 + 7) cm [From (1) and (3)]

= 10 cm

10. (*d*) **120**

Given that a chord AB of a circle with centre at O subtends an angle 60° so that ∠AOB = 60°. The tangents AC and BC to the circle meet each other at a point C outside the circle. To find ∠ACB.



Now, ∠ACB = 360° – ∠AOB – ∠CAO – ∠CBO …(1)

Now, $\because$ OA ⊥ AC and OB ⊥ BC,

∠CAO = 90°, ∠CBO = 90° and ∠AOB = 60° [Given]

Hence, from (1), we have

∠ACB = 360° – 60° – 90° – 90°

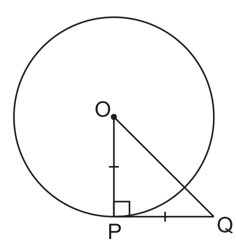
[$\because$ The sum of four angles of the quadrilateral OACB is 360°]

= 360° – 240°

= 120°

11. (*b*) **45°**

Given that PQ is a tangent to a circle with centre at a point O on it such that ΔOPQ is an isosceles triangle. To find the measure of ∠OQP.

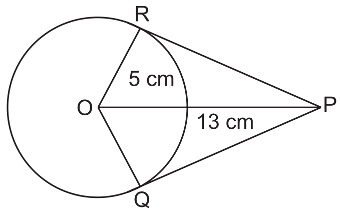


Since, $\angle OPQ = 90°$ and ΔOPQ is an isosceles triangle with $PQ = PO$,

$$\angle PQO = \angle POQ$$
$$= \frac{180° - 90°}{2} = 45°.$$

12. (*a*) **60 cm²**

Given that PQ and PR are two tangents to a circle with centre at O, drawn from an outside point P such that OP = 13 cm.



Given that OQ = OR = radius of the circle = 5 cm

To find the area of the quadrilateral PQOR.

From ΔOPR, we have $\angle ORP = 90°$.

$\therefore$ By Pythagoras' theorem, we have

$$PR = \sqrt{OP^2 - OR^2}$$
$$= \sqrt{13^2 - 5^2} \text{ cm}$$
$$= \sqrt{169 - 25} \text{ cm}$$
$$= \sqrt{144} \text{ cm}$$
$$= 12 \text{ cm} = PQ$$

Now, ar(quadrilateral PQOR) = ar(ΔOPR) + ar(ΔOPQ) …(1)

Now, from ΔPQR, $\because \angle PRQ = 90°$.

Hence, by Pythagoras' theorem, we have

$$PR = \sqrt{PO^2 - OR^2}$$
$$= \sqrt{13^2 - 5^2} \text{ cm}$$
$$= \sqrt{169 - 25} \text{ cm}$$
$$= \sqrt{144} \text{ cm}$$
$$= 12 \text{ cm}$$

$\therefore$ $PQ = PR = 12$ cm

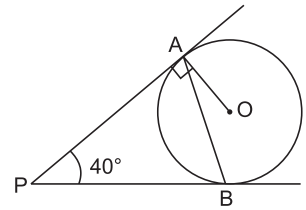
Hence, from (1), we get

ar(quadrilateral OQPR) = ar(ΔOPR) + ar(ΔOPQ)

$$= \frac{1}{2} \times RP \times OR + \frac{1}{2} \times QP \times OQ$$
$$= \left(\frac{1}{2} \times 12 \times 5 + \frac{1}{2} \times 12 \times 5\right) \text{ cm}^2$$
$$= (30 + 30) \text{ cm}^2$$
$$= 60 \text{ cm}^2$$

13. (*a*) **20**

Given that PA and PB are two tangents to a circle with centre at O such that $\angle APB = 40°$, where AB is the line segment joining A and B. OA is joined. To find the measure of $\angle OAB$. Since PA = PB.



$\therefore$ In ΔPAB, we have

$$\angle PAB = \angle PBA$$
$$= \frac{180° - 40°}{2} = 70° \quad \text{…(1)}$$

$\therefore$ $\angle BAO = \angle PAO - \angle PAB$

$= 90° - 70°$

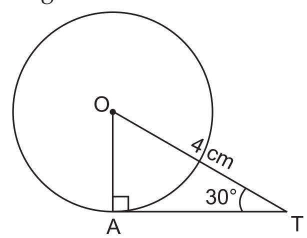
[$\because$ OA is the radius and PA is a tangent through A, hence $\angle PAO = 90°$ and from (1) $\angle PAB = 70°$]

$= 20°$

14. (*c*) $2\sqrt{3}$ **cm.**

Given that AT is a tangent to a circle with centre at O such that OT = 4 cm and $\angle OTA = 30°$.

To find the length of AT.



Construction: We join AO.

From ΔATO, since $\angle OAT = 90°$.

$\therefore$ $\frac{AT}{OT} = \cos 30°$

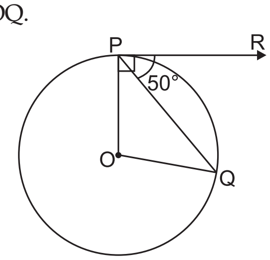
$\Rightarrow$ $AT = OT \cos 30°$

$$= 4 \times \frac{\sqrt{3}}{2} \text{ cm}$$
$$= 2\sqrt{3} \text{ cm}$$

15. (*a*) **100°**

Given that PR is a tangent to a circle with centre at O. PQ is a chord of the circle thorugh P such that $\angle QPR = 50°$. PO and QO are joined.

To find $\angle POQ$.



Since, PR is a tangent and OP is radius of the circle, hence, $\angle OPR = 90°$

$\therefore$ $\angle OPQ = 90° - 50° = 40°$ …(1)

But $\angle OPQ = \angle OQD$ [$\because$ OP = OQ]

$= 40°$ …(2)

$\therefore$ $\angle POQ = 180° - \angle OPQ - \angle OQP$

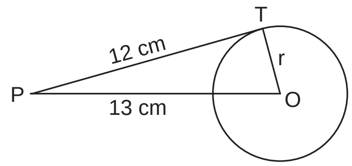
[Angle-sum property of a triangle]

$= 180° - 40° \times 2$ [From (1) and (2)]

$= 180° - 80°$

$= 100°$

16. Let PT be the tangent from P to the circle with centre O. Then, PT = 12 cm



OP = 13 cm. Let r be the radius of the circle.

Then, OT = r.

Since the tangent at any point of a circle is perpendicular to the radius through the point of contact

$\therefore$ $OT \perp TP \Rightarrow \angle OTP = 90°$

In right ΔOTP, we have

$OT^2 + PT^2 = OP^2$

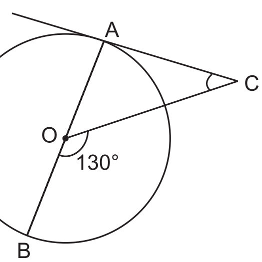
$\Rightarrow$ $r^2 + (12\text{ cm})^2 = (13\text{ cm})^2$

$\Rightarrow$ $r^2 = (169 - 144)\text{ cm}^2 = 25\text{ cm}^2$

$\Rightarrow$ $r = 5$ cm

Hence, the radius of the circle is **5 cm.**

17.



$\angle BOC = 130°$

$\angle ACO = ?$

Radius of a circle is perpendicular to the tangent at the point of contact

$\therefore$ $OA \perp CA$

$\angle OAC = 90°$

$\angle AOC + \angle BOC = 180°$ (Linear pair)

$\angle AOC + 130° = 180°$

$\angle AOC = 50°$

In ΔAOC

$\angle AOC + \angle ACO + \angle CAO = 180°$

$50° + \angle ACO + 90° = 180°$

$\mathbf{\angle ACO = 40°}$

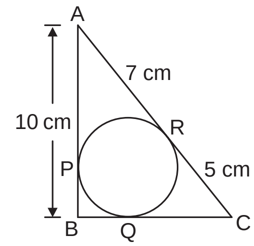
18. Since the lengths of tangents drawn from an external point to a circle are equal

$\therefore$ AP = AR [Tangents from A] …(1)

BQ = BP [Tangents from B] …(2)

CQ = CR [Tangents from C] …(3)

AP = AR = 7 cm [Using (1)]



BP = (10 – 7) cm = 3 cm

BQ = 3 cm [Using (2)] …(4)

CQ = CR = 5 cm [Using (3)] …(5)

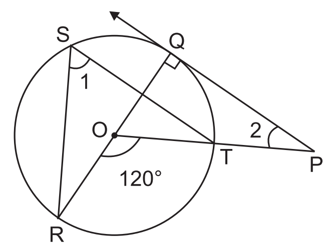
BC = BQ + CQ

= 3 cm + 5 cm [Using (4) and (5)]

$\Rightarrow$ BC = 8 cm

Hence, BC = **8 cm**

19. Given that PQ is a tangent to a circle with centre O, from an external point P. OP cuts the circle at T and $\angle POR = 120°$.



S is a point on the circle. TS and SR are joined.

To find $\angle TSR + \angle QPT$.

We see that $\angle TSR$ is an angle subtended by the arc RT on the circumference and $\angle ROT$ is an angle subtended by the same arc RT on the same side.

$\therefore$ $\angle ROT = 2\angle TSR$

$\Rightarrow$ $120° = 2\angle TSR$

$\therefore$ $\angle 1 = \angle TSR = \dfrac{120°}{2} = 60°$ …(1)

Now, in ΔOPQ, we have

$\angle POQ = 180° - 120°$

[$\because$ $\angle ROT + \angle POQ = 180°$ and $\angle ROP = 120°$]

$= 60°$ …(2)

Also, $\angle OQP = 90°$ … (3)

[$\because$ OQ is a radius and PQ is a tangent at Q]

$\therefore$ $\angle 2 = \angle QPO$

$= 180° - (\angle POQ + \angle OQP)$

[Angle-sum property of ΔPOQ]

$= 180° - (60° + 90°)$

[From (2) and (3)]

$= 180° - 150°$

$= 30°$

$\therefore \quad \angle 2 = \angle QPT = 30°$ …(4)

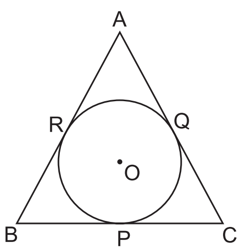
$\therefore$ From (1) and (4), we have

$\angle TSR + \angle QPT = 60° + 30° = \mathbf{90°}$.

which is the required measure of $\angle 1 + \angle 2$, i.e. $\angle TSR + \angle QPT$.

20. Given that an isosceles triangle ABC with AB = AC circumscribes a circle with centre at O. Let BC, AC and AB touch the circle at the points P, Q and R respectively.

To prove that the point P bisects BC, i.e. BP = PC.



We have $\quad AB = AC$ …(1)

[$\because$ ΔABC is an isosceles triangle with AB = AC]

Also, $\quad AR = AQ$ …(2)

[$\because$ AR and AQ are two tangents from an external point A]

$\therefore \quad BR = AB - AR$

$= AC - AQ$ [From (1) and (2)]

$= CQ$ …(3)

Now, $\quad BR = BP$ …(4)

[$\because$ These are tangents from an external point B]

and $\quad CQ = CP$ …(5)

[$\because$ These are tangents from an external point C]

$\therefore$ From (3), (4) and (5), we see that

$BP = CP$

i.e. P bisects BC at P.

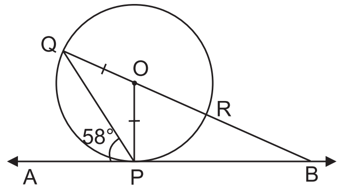
Hence, the result.

UNIT TEST 2

For Standard Level

1. (*b*) **32°**

Given that the line AB is a tangent to a circle with centre at O, at the point P. PQ is a chord of the circle such that $\angle APQ = 58°$. QOR is a diameter of the circle such that QR produced intersect AP produced at B.



To find the measure of $\angle PQB$.

Construction: We join OP.

In ΔOPQ, we have

$\angle OPA = 90°$

[Since OP is a radius and APB is a tangent to the circle]

$\therefore \quad \angle QPO = \angle OPA - \angle QPA$

$= 90° - 58° = 32°$ …(1)

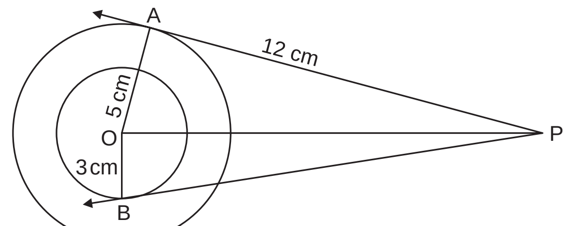
But since $\quad OP = OQ$

[$\because$ Both are radius of the same circle]

$\therefore \quad \angle PQB = \angle PQO = \angle QPO = 32°$

[From (1)]

2. (*b*) $\mathbf{4\sqrt{10}}$



Since the tangent at any point of a circle is perpendicular to the radius through the point of contact

$\therefore \quad OA \perp AP$ and $OB \perp BP$

$\Rightarrow \quad \angle OAP = 90°$ and $\angle OBP = 90°$

In right ΔOAP, we have

$OP^2 = OA^2 + PA^2$ [By Pythagoras' Theorem]

$\Rightarrow \quad OP^2 = (5 \text{ cm})^2 + (12 \text{ cm})^2$

$= (25 + 144) \text{ cm}^2$

$= 169 \text{ cm}^2$ …(1)

In right ΔOBP, we have

$OB^2 + PB^2 = OP^2$ [By Pythagoras' Theorem]

$\Rightarrow \quad (3 \text{ cm})^2 + PB^2 = 169 \text{ cm}^2$ [Using (1)]

$\Rightarrow \quad PB^2 = (169 - 9) \text{ cm}^2$

$\Rightarrow \quad PB^2 = 160 \text{ cm}^2$

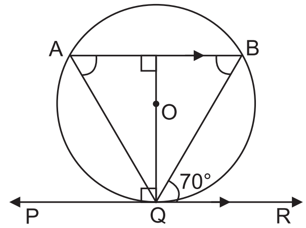
$\Rightarrow \quad PB = \sqrt{160}$ cm

$\Rightarrow \quad PB = 4\sqrt{10}$ cm

Hence, the length of PB (in cm) is $4\sqrt{10}$.

3. (*b*) **40°**

Given that PQR is a tangent to a circle at Q, with centre at O and AB is a chord of the circle parallel to PQR. QA and QB are joined.



Given that $\angle BQR = 70°$.

To find the measure of $\angle AQB$.

We have $\angle QAB = \angle BQR = 70°$

[$\because$ Angle in alternate segments are equal]

Also, $\angle ABQ$ = alternate $\angle BQR$

[$\because$ $AB \parallel PR$ and QB is a transversal]

$= 70°$

$\therefore$ In ΔABQ,

$\angle AQB = 180° - (\angle QAB + \angle QBA)$

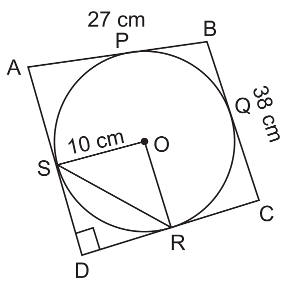
[Angle-sum property of ΔAQB]

$= 180° - (70° + 70°)$

$= 180° - 140° = 40°$

4. (*d*) **21 cm**

Given that a circle with centre O is inscribed in a quadrilateral ABCD touching its sides AB, BC, CD and DA at P, Q, R and S respectively. The radius of the circle is 10 cm, BC = 38 cm, PB = 27 cm and $AD \perp CD$.



To find the length of CD.

Construction: We join OS, OR and SR.

Since PB and BQ are tangents to the circle from an external point B, hence,

$BQ = PB = 27$ cm …(1)

Also since CQ and CR are two tangents to the circle from an external point C,

$\therefore$ $CR = CQ = BC - BQ$

$= (38 - 27)$ cm $= 11$ cm …(2)

Let $CD = x$ cm …(3)

$\therefore$ $DR = CD - CR = (x - 11)$ cm …(4)

[From (2) and (3)]

$\because$ DR and DS are two tangents to the circle from an external point D.

$DS = RD = (x - 11)$ cm [From (4)] …(5)

Now, since $\angle SDR = 90°$, hence, from ΔSDR, we have by Pythagoras' theorem,

$RS^2 = RD^2 + DS^2 = 2(x - 11)^2$ …(6)

[From (4) and (5)]

$\because$ $OS \perp AD$ and $OR \perp RD$.

$\therefore$ In quadrilateral OSDR, we have

$\angle OSD = \angle ORD = 90°$

Also, given that $\angle SDR = 90°$.

$\therefore$ $\angle SOR = 90°$

[Angle-sum property of a quadrilateral]

$\therefore$ In ΔSOR, we have by Pythagoras' theorem,

$SR^2 = OS^2 + OR^2$

$= 10^2 + 10^2 = 200$

$\therefore$ $RS = \sqrt{200} = 10\sqrt{2}$ …(7)

$\therefore$ From (6) and (7), we have

$200 = 2(x - 11)^2$

$\Rightarrow$ $(x - 11)^2 = \frac{200}{2} = 100$

$\therefore$ $x - 11 = \pm\sqrt{100} = \pm 10$

$\Rightarrow$ $x = 11 + 10 = 21$

Or, $x = 11 - 10 = 1$ which is absurd.

Since, $CR = 11$ cm [From (2)]

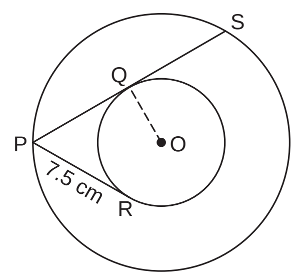
$\therefore$ $x = CD = 1$ cm is absurd.

Hence, $x = 21$

$\therefore$ Length of CD is 21 cm.

5. (*b*) **15 cm**

Join OQ



Since, the tangent at any point of a circle is perpendicular to the radius through the point of contact

$\therefore$ $OQ \perp PQS$

Since the perpendicular from the centre of a circle to a chord bisects the chord

$\therefore$ OQ bisects PS.

$\Rightarrow$ $PQ = QS$ …(1)

Since the lengths of tangents drawn from an external point to a circle are equal.

$\therefore$ $PQ = PR = 7.5$ cm …(2)

Now $PS = PQ + QS = 2PQ$ [Using (1)]

$\Rightarrow$ $PS = 2 \times 7.5$ cm [Using (2)]

$= 15$ cm

6. (*d*) **8 cm**

Given that XY is a tangent to a circle with centre O and radius OA = 5 cm. Let the tangent XY touch the circle at A and AOB is a diameter of the circle. A chord CD at a distance of 8 cm from A is parallel to the tangent XAY and let CD cut AB at M. To find the length of CD.

Construction: We join OD.

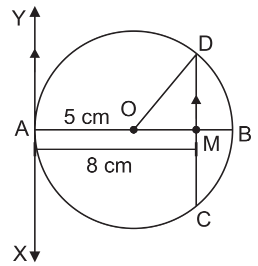
Since XY is a tangent at A and AO is a radius,

$\therefore$ $\angle OAY = 90°$.

Also, since $XY \parallel CD$,

$\therefore$ $\angle OMD = 90°$.

$\therefore$ M is the middle point of CD.



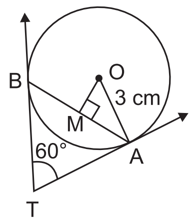
Now, from ΔOMD, we have by Pythagoras' theorem,

$$MD = \sqrt{OD^2 - OM^2}$$
$$= \sqrt{5^2 - (AM - AO)^2} \text{ cm}$$
$$= \sqrt{25 - (8-5)^2} \text{ cm}$$
$$= \sqrt{25 - 9} \text{ cm}$$
$$= \sqrt{16} \text{ cm}$$
$$= 4 \text{ cm}$$

∴ CD = 2MD
= 2 × 4 cm
= 8 cm

7. (*d*) **$3\sqrt{3}$ cm**

Given that TA and TB are two tangents to a circle with centre at O and radius 3 cm, drawn from an external point T such that ∠ATB = 60°.



To find the length of each tangent TA or TB.

Construction: We draw OM ⊥ AB and join OA.

Now, since TA = TB and ∠ATB = 60° …(1)

∴ ∠TAB = ∠TBA = 60° …(2)
[Angle-sum property]

Also, ∠OAT = 90° …(3)
[∵ TA is a tangent at A and OA is a radius of the circle]

∴ ∠OAM = ∠OAT – ∠TAB
= 90° – 60° = 30° …(4)
[From (1) and (2)]

∴ In ΔOAM, we have

AM = OA Cos ∠OAM
= 3 Cos 30°

$$= 3 \cdot \frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{2} \quad \text{…(5) [From (4)]}$$

∵ AB = 2 × AM

$$= 2 \times \frac{3\sqrt{3}}{2} \quad \text{[From (5)]}$$
$$= 3\sqrt{3} \quad \text{…(6)}$$

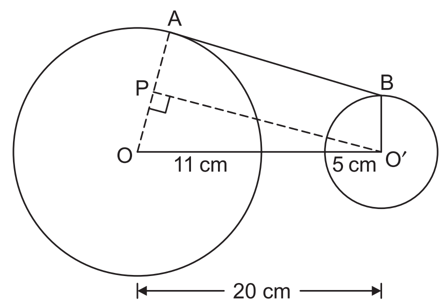
Now, since in ΔTAB, ∠ATB = ∠TBA = ∠TAB = 60°.

∴ ΔTAB is an equilateral triangle

∴ AT = BT = AB = $3\sqrt{3}$ [From (6)]

∴ The length of each tangent is $3\sqrt{3}$ cm.

8. Let C(O, r) and C(O′, r') be the two circles such that r = 11 cm, r' = 5 cm and OO′ = 20 cm. Let AB be one of the external common tangents.



Draw O′P ⊥ AO ⇒ ∠O′PA = ∠O′PO = 90° …(1)

Since, the tangent at any point of the circle is perpendicular to the radius through the point of contact.

∴ OA ⊥ AB and O′B ⊥ AB

⇒ ∠OAB = 90° and ∠O′BA = 90° …(2)

In quadrilateral ABO′P, we have

∠OAB = 90°,
∠O′BA = 90°
and ∠PO′B = 90° [Using (1) and (2)]

∴ Each angle of quad ABO′P is a right angle and its opposite sides are parallel.

∴ Quadrilateral ABO′P is a rectangle

⇒ PO′ = AB …(3)

In right ΔOPO′, we have

$OP^2 + PO'^2 = OO'^2$ [By Pythagoras' Theorem]

⇒ $[(11 - 5) \text{ cm}]^2 + PO'^2 = (20 \text{ cm})^2$

⇒ $PO'^2 = 400 \text{ cm}^2 - 36 \text{ cm}^2$

⇒ $PO'^2 = 364 \text{ cm}^2$

⇒ $PO' = \sqrt{364 \text{ cm}^2}$

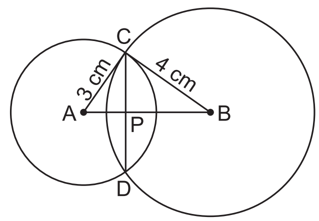
⇒ PO′ = 19.1 cm (approx.)

∴ AB = 19.1 cm (approx.) [Using (3)]

Hence, AB = **19.1 cm (approx.)**

9. Given that two circles with centres at A and B and radii 3 cm and 4 cm respectively intersect at C and D such that AC and BC are tangents to the two circles at C. Centres A and B are joined. Also, CD is joined to cut AB at P.

To find the length of the common chord CD.



AC is a tangent to the circle with centre B and BC is a radius of this circle.

$\therefore \qquad \angle ACB = 90°$

$\therefore$ In ΔABC, we have by Pythagoras' theorem,

$$AB = \sqrt{AC^2 + BC^2}$$

$$= \sqrt{3^2 + 4^2} \text{ cm}$$

$$= \sqrt{25} \text{ cm} = 5 \text{ cm} \qquad \ldots(1)$$

Now, let $\qquad AP = x$ cm $\qquad \ldots(2)$

$\therefore \qquad PB = AB - AP = (5 - x)$ cm $\qquad \ldots(3)$

[From (1) and (2)]

Now, since CD is a common chord of the circles with centre A and B, and AB is the line segment joining their centres,

$\therefore$ AB $\perp$ CD.

$\therefore$ P is the mid-point of CD.

$\therefore \qquad CP = DP \qquad \ldots(4)$

Let $\qquad CP = y$ cm $\qquad \ldots(5)$

$\therefore$ From (2) and (5), and from ΔACP, we have by Pythagoras' theorem,

$$AC^2 = CP^2 + AP^2 = x^2 + y^2$$

[From (2) and (5)]

$\Rightarrow \qquad 9 = x^2 + y^2 \qquad \ldots(6)$

Also, from ΔBCP, we have by Pythagoras' theorem,

$$BC^2 = CP^2 + PB^2$$

$\Rightarrow \qquad 4^2 = y^2 + (5 - x)^2 \qquad$ [From (3) and (5)]

$\Rightarrow \qquad 16 = (5 - x)^2 + y^2 \qquad \ldots(7)$

Subtracting (6) from (7), we get

$$(5 - x)^2 - x^2 = 16 - 9 = 7$$

$\Rightarrow (5 - x + x)(5 - x - 5) = 7$

$\Rightarrow \qquad 5(5 - 2x) = 7$

$\Rightarrow \qquad 10x = 25 - 7 = 18$

$\therefore \qquad x = \frac{9}{5} \qquad \ldots(8)$

$\therefore$ From (6) and (8), we have

$$y^2 = 9 - x^2$$

$$= 9 - \left(\frac{9}{5}\right)^2$$

$$= 9 - \frac{81}{25}$$

$$= \frac{225 - 81}{25}$$

$$= \frac{144}{25}$$

$\therefore \qquad y = \frac{12}{5}$

$\therefore \qquad CD = 2CP$

$$= 2 \times y$$

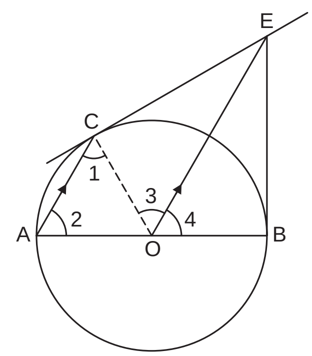
$$= \frac{12}{5} \times 2$$

$$= \frac{24}{5}$$

$$= 4.8$$

$\therefore$ The required length of CD is **4.8 cm**.

10. Join OC.



OA = OC $\qquad$ [Radii of a circle]

$\therefore \qquad \angle 1 = \angle 2 \qquad$ [Angles opposite to equal sides] ...(1)

$\angle 1 = \angle 3 \qquad$ [Alternate angles AC || OE] ...(2)

$\angle 2 = \angle 4 \qquad$ [Corresponding angles AC || OE] ...(3)

From (1), (2) and (3), we get

$\angle 3 = \angle 4 \qquad \ldots(4)$

In ΔOCE and ΔOBE, we have

OC = OB $\qquad$ [Radii of a circle]

$\angle 3 = \angle 4 \qquad$ [From (4)]

OE = OE $\qquad$ [Common]

$\therefore \quad \Delta OCE \cong \Delta OBE \qquad$ [By SAS congruence]

$\Rightarrow \quad \angle OCE = \angle OBE$

$\Rightarrow \qquad 90° = \angle OBE \qquad$ [Since the tangent at any point of a circle is perpendicular to the radius through the point of contact]

$\Rightarrow \qquad OB \perp BE$

Since a line drawn through the end of a radius and perpendicular to it is a tangent to the circle

$\therefore$ BE is tangent to the circle.

Hence, **EB touches the circle.**

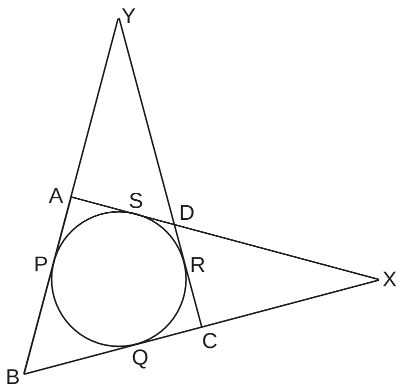
11. Let AB, BC, CD and DA of the quadrilateral ABCD, touch the circle at P, Q, R and S respectively. Since the lengths of tangents drawn from an external point to a circle are equal

$\therefore \qquad$ AS = AP $\qquad$ [Tangents from A] ...(1)

YP = YR $\qquad$ [Tangents from Y] ...(2)

XS = XQ $\qquad$ [Tangents from X] ...(3)

CR = CQ $\qquad$ [Tangents from C] ...(4)



$AY + AX = AY + (AS + DS + DX)$

$\Rightarrow$ $AY + AX = (AY + AP) + XS$ [Using (1)]

$\Rightarrow$ $AY + AX = YP + XS$

$\Rightarrow$ $AY + AX = YR + XQ$ [Using (2) and (3)]

$\Rightarrow$ $AY + AX = (CY - CR) + (CX + CQ)$

$\Rightarrow$ $AY + AX = CY - CQ + CX + CQ$ [Using (4)]

$\Rightarrow$ $AY + AX = CY + CX$

$\Rightarrow$ $AY - CX = CY - AX$

Hence, **the difference between AY and CX is equal to the difference between CY and AX.**

12. Since the tangent at any point of a circle is perpendicular to the radius through the point of contact

$\therefore$ $OD \perp BC$ and $OE \perp AC$

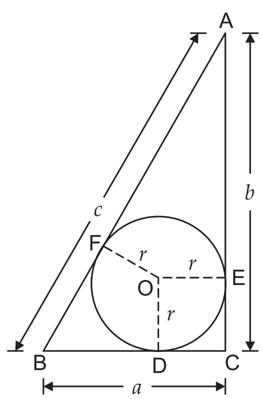
$\Rightarrow$ $\angle ODC = 90°$ and $\angle OEC = 90°$

Also $\angle ECD = 90°$ [Given]

$\therefore$ In quadrilateral OECD, each angle is a right angle and adjacent sides OD and OE are equal (OD and OE are radii of the same circle).

So, quadrilateral OECD is a square.

Thus $CD = CE = OE$ or $OD = r$ ...(1)



Since, the lengths of tangents drawn from an external point to a circle are equal.

$\therefore$ $AE = AF$ [Tangents from A] ...(2)

$BD = BF$ [Tangents from B] ...(3)

$AE = AC - CE = b - r$ [Using (1)]

$AF = b - r$ [Using (2)]

$BF = c - AF = c - b + r$

$BD = c - b + r$ [Using (3)] ...(4)

Now, $BC = CD + BD$

$\Rightarrow$ $a = r + c - b + r$ [Using (1) and (4)]

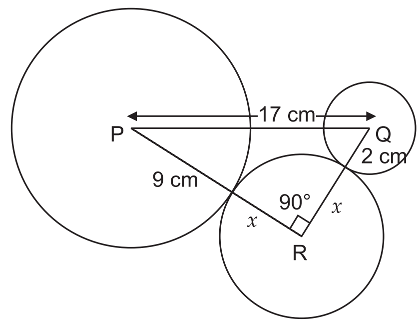
$\Rightarrow$ $a + b = c + 2r$

Hence, $\mathbf{2r + c = a + b}$.

13. $PQ = 17$ cm

$PR = 9 + x$

$RQ = x + 2$



ΔPQR is right-angled triangle

$\therefore$ $PR^2 + RQ^2 = PQ^2$

$(9 + x)^2 + (x + 2)^2 = (17)^2$

$81 + x^2 + 18x + x^2 + 4 + 4x = 289$

$2x^2 + 22x + 85 = 289$

$2x^2 + 22x - 204 = 0$

$\mathbf{x^2 + 11x - 102 = 0}$

$x^2 - 6x + 17x - 102 = 0$

$x(x - 6) + 17(x - 6) = 0$

$(x - 6)(x + 17) = 0$

$x = 6$ or $x = -17$

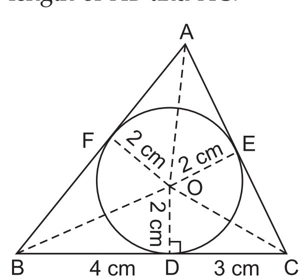
Since the radius of a circle cannot be negative

$\therefore$ $\mathbf{x = 6}$ **cm**

14. Given that a triangle ABC circumscribes a circle with centre O and radius 2 cm such that the line segments BD and DC are of lengths 4 cm and 3 cm respectively.

Given that $ar(\Delta ABC) = 21$ cm^2

To find the length of AB and AC.



Construction: We join AO, BO, CO, OD, OE and OF

We have

$ar(\Delta ABC) = ar(\Delta OBC) + ar(\Delta AOC) + ar(\Delta AOB)$

$$\Rightarrow \quad 21 = \frac{1}{2}BC\times OD+\frac{1}{2}AC\times OE+\frac{1}{2}AB\times OF$$

$$= \frac{1}{2}(4+3)\times 2+\frac{1}{2}y\times 2+\frac{1}{2}x\times 2$$

$$= 7 + x + y,$$

where AB = x, AC = y and BC = (4 + 3) cm = 7 cm

$$\therefore \quad x + y = 21 - 7 = 14 \quad \ldots(1)$$

Now, $\quad$ AF = AB – BF

$$= AB - 4 = x - 4 \quad \ldots(2)$$

And $\quad$ AE = AC – CE

$$= AC - 3 = y - 3 \quad \ldots(3)$$

Now, from ΔAOF, we have

$$AO^2 = AF^2 + OF^2$$

$$= (x-4)^2 + 2^2 \quad \text{[From (2)]}$$

$$= (x-4)^2 + 4 \quad \ldots(4)$$

Also, from ΔAOE, we have

$$AO^2 = AE^2 + OE^2$$

$$= (y-3)^2 + 2^2 \quad \text{[From (3)]}$$

$$= (y-3)^2 + 4 \quad \ldots(5)$$

Subtracting (4) from (5), we get

$$0 = (y-3)^2 - (x-4)^2$$

$$= (y - 3 + x - 4)\ (y - 3 - x + 4)$$

$$= (x + y - 7)\ (y - x + 1)$$

$\therefore$ Either $x + y - 7 = 0 \quad \Rightarrow x + y = 7 \quad \ldots(6)$

Or $\quad y - x + 1 = 0 \quad \Rightarrow x - y = 1 \quad \ldots(7)$

From (1) and (6), we see that 7 = 14 which is absurd.

Hence, we reject equation (6).

From (1) and (7), we get

$$2x = 14 + 1 = 15$$

$$\Rightarrow \quad x = \frac{15}{2} = 7.5$$

and subtracting (7) from (1), we get

$$2y = 14 - 1 = 13$$

$$\therefore \quad y = \frac{13}{2} = 6.5$$

Hence, the required lengths of AB and AC are **7.5 cm** and **6.5 cm** respectively.