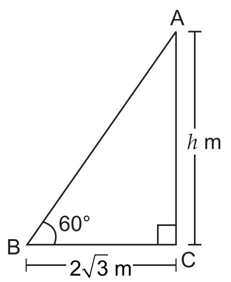
CHAPTER 11 Some Applications of Trigonometry

EXERCISE 11

1. Let AC (= h m) be the height of the pole and let B be a point on the horizontal ground at a distance of $2\sqrt{3}$ m from the foot of the tower. The angle of elevation of the top A of the tower AC at point B is 60°, i.e. ∠ABC = 60°.

Also, ∠ACB = 90°

and BC = $2\sqrt{3}$ m



In right ΔACB,

$$\tan 60° = \frac{AC}{BC}$$

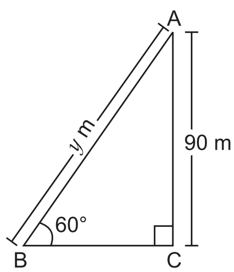
$$\Rightarrow \quad \sqrt{3} = \frac{h}{2\sqrt{3}}$$

$$\Rightarrow \quad h = 6 \text{ m}$$

Hence, the height of the pole is 6 m.

2. (*i*) Let AB (= y m) be the length of the string and let B be a point on the horizontal ground. Let AC be the height of the kite from the ground.

The angle of elevation of the top A of the kite from the point B is 60°.



i.e. ∠ABC = 60°.

Also, ∠ACB = 90° and AC = 90 m

In right ΔACB,

$$\sin 60° = \frac{AC}{AB}$$

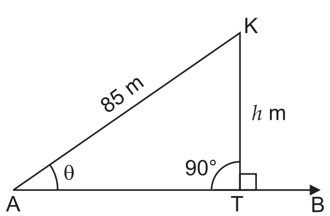
$$\Rightarrow \quad AB = \frac{AC}{\sin 60°}$$

$$\Rightarrow \quad y = \frac{90 \text{ m}}{\left(\frac{\sqrt{3}}{2}\right)} = \frac{2 \times 90 \text{ m}}{\sqrt{3}}$$

$$= \frac{180 \text{ m}}{1.732} = 103.92 \text{ m}$$

Hence, the length of the string is 103.92 m

(*ii*) Let AB be the horizontal ground, KT the position of the kite and KT = h m is the vertical height of the kite so that ∠KTA = 90° and ∠KAT = θ. Given that the length of the string is 85 m from a point A on the ground so that AK = 85 m.



Now, from ΔAKT, we have

$$\tan \theta = \frac{h}{AT} = \frac{15}{8} = \frac{h}{\sqrt{85^2 - h^2}}$$

where by Pythagoras' theorem,

$$AT^2 = AK^2 - KT^2$$
$$= 85^2 - h^2$$

$$\Rightarrow \quad 64h^2 = 15^2\,(85^2 - h^2) = 225(85^2 - h^2)$$

$$\Rightarrow \quad (64 + 225)h^2 = 225 \times 85^2$$

$$\Rightarrow \quad 289\,h^2 = 15^2 \times 85^2$$

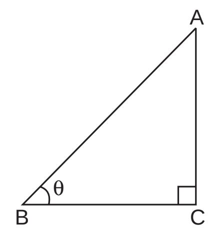
$$\Rightarrow \quad 17h = 15 \times 85$$

$$\Rightarrow \quad h = \frac{15 \times 85}{17} = 75$$

Hence, the required height of the kite is 75 m.

3. (*i*) Let AC be the height of the vertical tower and let BC be the length of the shadow of the vertical tower.

Let the angle of elevation of the Sun be θ.



Then, ∠ABC = θ.

∠ACB = 90°

and BC = $\sqrt{3}$ AC.

In right ΔACB, we have

$$\tan \theta = \frac{AC}{BC}$$

$$\Rightarrow \quad \tan \theta = \frac{AC}{\sqrt{3}\,AC}$$

$$\Rightarrow \quad \tan \theta = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad \tan \theta = \tan 30° \qquad \left[\text{Using } \tan 30° = \frac{1}{\sqrt{3}}\right]$$

$$\Rightarrow \quad \theta = 30°$$

Hence, the angle of elevation of the Sun is 30°.

(*ii*) Height of pole = 6 m

Length of shadow = $2\sqrt{3}$ m

Let the angle of elevation = x

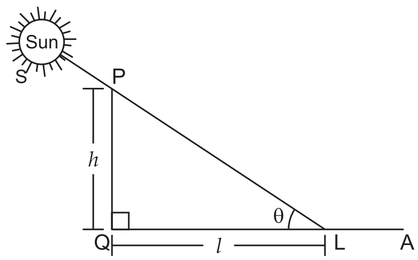
$$\tan x = \frac{P}{B}$$

$$\tan x = \frac{6}{2\sqrt{3}}$$

$$\tan x = \sqrt{3}$$

$$x = 60°$$

(*iii*) Let PQ be the vertical pole of height h m standing on the horizontal ground QA. Let QL = l m be the length of the shadow of the pole on the ground.



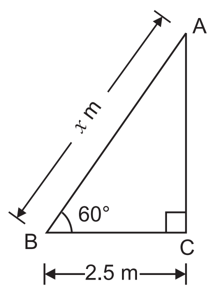
Let $\angle PLQ = \theta$ be the angle of elevation of the sun S.

It is given that $\tan\theta = \frac{h}{l} = \frac{\sqrt{3}}{1} = \sqrt{3} = \tan 60°$

$\therefore\ \theta = 60°$ which is the required angle of elevation of the sun.

4. (*i*) Let BC be the distance of the foot of the ladder from the wall and AB (= x) be the length of the ladder.

The angle of elevation of the top A from the point B is 60°.



i.e. $\angle ABC = 60°$

Also, $AB = x$ m

$\angle ACB = 90°$

and BC = 2.5 metres.

In right ΔACB, we have

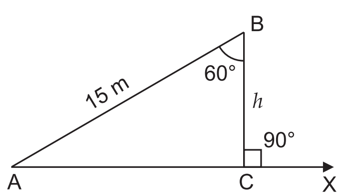
$$\cos 60° = \frac{BC}{AB}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{2.5}{x}$$

$$\Rightarrow \quad x = 2.5 \times 2 = 5 \text{ m}$$

Hence, the length of the ladder is 5 m.

(*ii*) Let AB be the ladder, B be the top of the vertical wall BC standing on the horizontal ground AX. Then $\angle BCX = 90°$.



Here AB = 15 m [given] and $\angle ABC = 60°$

Let the height of the wall be h m.

Now, from ΔABC, we have

$$\cos\angle ABC = \frac{h}{15}$$

$$\Rightarrow \quad \cos 60° = \frac{h}{15}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{h}{15}$$

$$\Rightarrow \quad h = \frac{15}{2} = 7.5$$

Hence, the required height of the wall is 7.5 m.

5. Let AB (= x metres) be the height of the tree.

Suppose the tree breaks at point P and then part AP assumes the position CP, meeting the ground at point C.

Let $PB = y$ metres

Then, $AP = AB - PB$

$= (x - y)$ metres.

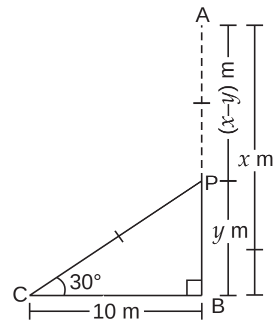
and $PC = AP$

$= (x - y)$ metres

$\angle PCB = 30°$,

BC = 10 m

and $\angle PBC = 90°$



In right ΔPBC, we have

$$\tan 30° = \frac{PB}{BC}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{y}{10 \text{ m}}$$

$$\Rightarrow \quad y = \frac{10}{\sqrt{3}} \text{ m} \qquad \dots (1)$$

and
$$\sin 30° = \frac{PB}{PC}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{y}{x - y}$$

$$\Rightarrow \quad x - y = 2y$$

$$\Rightarrow \quad x = 3y \qquad \dots (2)$$

From equation (1) and equation (2), we have

$$x = 3y = 3 \times \frac{10}{\sqrt{3}} \text{ m}$$

$$= 10\sqrt{3} \text{ m} = 10 \times 1.732 \text{ m} = 17.32 \text{ m}$$

Hence, the height of the tree is 17.32 m.

6. Let AB (= x metres) be the height of the tree. Suppose the tree breaks at point P and then part AP assumes the position CP, meeting the ground at point C.

Let $PB = y$ metres.

Then, $AP = AB - PB$

$= (x - y)$ metres.

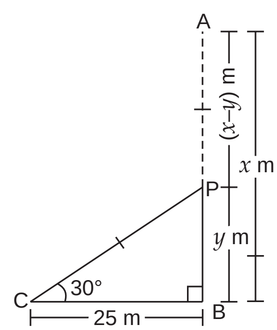
and $PC = AP$

$= (x - y)$ metres

$\angle PCB = 30°$

$\angle PBC = 90°$

and BC = 25 m



In right ΔPBC, we have

$$\tan 30° = \frac{PB}{BC}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{y}{25 \text{ m}}$$

$$\Rightarrow \quad y = \frac{25 \text{ m}}{\sqrt{3}} \qquad \dots (1)$$

and
$$\sin 30° = \frac{PB}{PC}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{y}{x - y} \quad \Rightarrow \quad x - y = 2y$$

$$\Rightarrow \quad x = 3y \qquad \dots (2)$$

From equation (1) and equation (2), we have

$$x = 3y = 3 \times \frac{25 \text{ m}}{\sqrt{3}} = 25\sqrt{3} \text{ m}$$

$$= 25 \times 1.732 \text{ m} = 43.3 \text{ m}$$

Hence, the height of the tree is 43.3 m.

7. Let AC be the length of the bridge and AB be the width of the river. Now, A and C are the ends of the bridge.

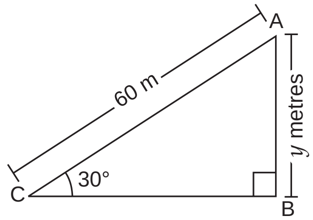
Then, $\angle ACB = 30°$

$AC = 60$ m

$AB = y$ metres

and $\angle ABC = 90°$.

In right ΔABC,



$$\sin 30° = \frac{AB}{AC}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{y}{60 \text{ m}}$$

$$\Rightarrow \quad y = \frac{60 \text{ m}}{2} = 30 \text{ m}$$

Hence, the width of the river is 30 m.

8. Let AB be the tower and CB be its shadow when the Sun's elevation is 45°.

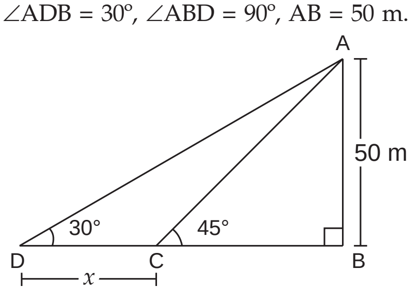
Then, $\angle ACB = 45°$.

Let D be a point x metres away from C.

Then, the length of the shadow of the tower when the Sun's elevation is 30° is

$$DB = x + CB$$

Also, $\angle ADB = 30°$, $\angle ABD = 90°$, AB = 50 m.



In right ΔABC, we have

$$\tan 45° = \frac{AB}{CB} \Rightarrow 1 = \frac{AB}{CB} \Rightarrow CB = 50 \text{ m}$$

In right ΔABD, we have

$$\tan 30° = \frac{AB}{DB} \Rightarrow \frac{1}{\sqrt{3}} = \frac{50 \text{ m}}{x + CB}$$

$$\Rightarrow \quad x = 50\sqrt{3} \text{ m} - CB = 50\sqrt{3} \text{ m} - 50 \text{ m}$$

$$= (1.732 - 1)\, 50 \text{ m} = 0.732 \times 50 \text{ m}$$

$$= 36.6 \text{ m} = 36 \text{ m } 60 \text{ cm}$$

Hence, the value of x is 36 m 60 cm.

9. (*i*) Let AB (= h metres) be the height of the tower and let CB (= x metres) be its shadow when the sun's elevation is 60°.

Then, $\angle ACB = 60°$.

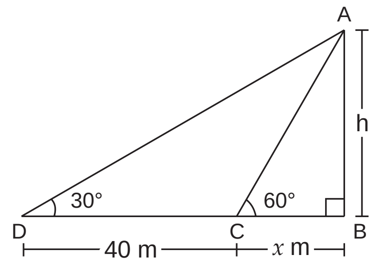
Let D be a point 40 m away from C.

Now, the length of the shadow of the tower when the Sun's elevation is 30° is

$$DB = (40 + x) \text{ m}$$

and $\angle ADB = 30°$

Also, $\angle ABC = \angle ABD = 90°$



In right ΔABC, we have

$$\tan 60° = \frac{AB}{CB}$$

$$\Rightarrow \quad \sqrt{3} = \frac{h}{x}$$

$$\Rightarrow \quad x = \frac{h}{\sqrt{3}} \qquad \ldots (1)$$

In right ΔABD, we have

$$\tan 30° = \frac{AB}{DB}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{40 + x}$$

$$\Rightarrow \quad 40 + x = \sqrt{3}\, h$$

$$\Rightarrow \quad 40 + \frac{h}{\sqrt{3}} = \sqrt{3}\, h \qquad \text{[Using equation (1)]}$$

$$\Rightarrow h\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right) = 40$$

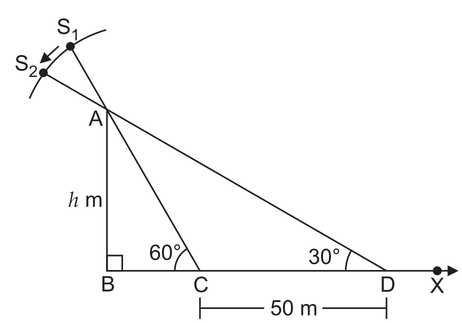
$$\Rightarrow \quad h = \frac{40\sqrt{3}}{2} \text{ m} = 20\sqrt{3} \text{ m}$$

Hence, the height of the tower is $20\sqrt{3}$ m.

(*ii*) Let AB be the vertical tower standing on the horizontal ground BX so that $\angle ABD = 90°$.

Let BC be the shadow of the tower when the Sun's angle of elevation is 60° so that $\angle ACB = 60°$.

Let BD be the shadow of the tower when the Sun's angle of elevation is 30°.



So that $\angle ADB = 30°$. Let S_1 and S_2 be the two positions of the Sun. It is given that

$$CD = BD - BC = 50 \text{ m} \quad \ldots(1)$$

Let h m be the height of the tower.

Then from ΔABC, we have

$$\frac{h}{BC} = \tan 60° = \sqrt{3}$$

$$\Rightarrow \quad BC = \frac{h}{\sqrt{3}} \quad \ldots(2)$$

Also from ΔABD, we have

$$\frac{h}{BD} = \tan 30° = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad BD = h\sqrt{3} \quad \ldots(3)$$

$\therefore$ From (1), (2) and (3) we have,

$$50 = h\sqrt{3} - \frac{h}{\sqrt{3}} = \frac{h(3-1)}{\sqrt{3}} = \frac{2h}{\sqrt{3}}$$

$$\Rightarrow \quad h = 25\sqrt{3}$$

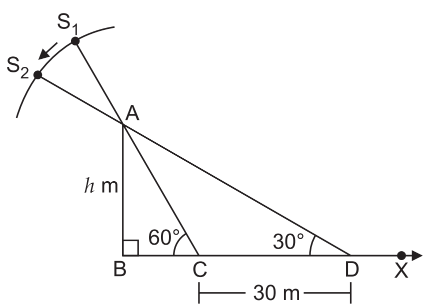
$\therefore$ The required height of the tower is $25\sqrt{3}$ m.

(*iii*) Let AB be the vertical tower standing on the horizontal ground BX, so that $\angle ABD = 90°$.

Let BC be the shadow of the tower when the position of the Sun is S_1 and BD be the shadow of the tower when the Sun's position is at S_2.

It is given that $\angle ACB = 60°$, $\angle ADB = 30°$

and CD = 30 m $\quad \ldots(1)$



Let h m be the height of the tower.

Then from ΔABC, we have

$$\frac{h}{BC} = \tan 60° = \sqrt{3}$$

$$\Rightarrow \quad BC = \frac{h}{\sqrt{3}} \quad \ldots(2)$$

Also from ΔABD, we have

$$\frac{h}{BD} = \tan 30° = \frac{1}{\sqrt{3}}$$

$$\therefore \quad BD = h\sqrt{3} \quad \ldots(3)$$

$\therefore$ From (1), (2) and (3), we have

$$30 = BD - BC$$

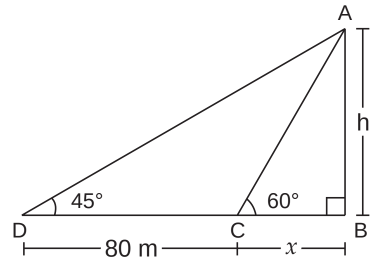
$$= h\sqrt{3} - \frac{h}{\sqrt{3}} = h\left(\frac{3-1}{\sqrt{3}}\right) = \frac{2h}{\sqrt{3}}$$

$$\therefore \quad h = \frac{30\sqrt{3}}{2} = 15\sqrt{3}$$

$$= 15 \times 1.732 = 25.98.$$

Hence, the required height of the tower is 25.98 m.

10. (*i*) Let AB (= h metres) be the tower and let BC (= x metres) be the distance from B when the angle of elevation is 60º.



Then, $\angle ACB = 60°$ and $CB = x$.

Let D be the point when the angle of elevation is 45º.

Then, $\angle ADB = 45°$, $DB = (80 + x)$ m

and $\angle ABC = \angle ABD = 90°$

In right ΔABC, we have

$$\tan 60° = \frac{AB}{CB} \Rightarrow \sqrt{3} = \frac{h}{x} \Rightarrow x = \frac{h}{\sqrt{3}}$$

In right ΔABD, we have

$$\tan 45° = \frac{AB}{DB}$$

$$\Rightarrow \quad 1 = \frac{AB}{DB}$$

$$\Rightarrow \quad DB = AB$$

$$\Rightarrow \quad 80 + x = h$$

$$\Rightarrow \quad 80 \text{ m} + \frac{h}{\sqrt{3}} = h$$

$$\Rightarrow \quad h\left(\frac{\sqrt{3}-1}{\sqrt{3}}\right) = 80 \text{ m}$$

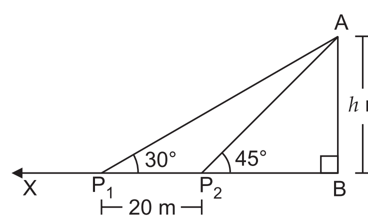
$$\Rightarrow \quad h = \frac{80\sqrt{3}}{\sqrt{3}-1} \text{ m} = \frac{80 \times 1.732}{1.732 - 1} \text{ m}$$

$$= \frac{138.56}{0.732} \text{ m} = 189.28 \text{ m}$$

Hence, the height of the tower is 189.28 m.

(*ii*) Let AB of height h m be the vertical tower standing on the horizontal base BX. Hence, $\angle ABX = 90°$.

Let P_1 be the 1st position of the observer and P_2 be the 2nd position of the observer so that $\angle AP_1B = 30°$, $\angle AP_2B = 30° + 15° = 45°$ and $P_1P_2 = 20$ m.



Then from ΔAP_1B, we have

$$\frac{h}{P_1B} = \tan 30° = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \qquad P_1B = h\sqrt{3} \qquad \ldots(1)$$

Also, from ΔAP_2B, we have

$$\frac{h}{P_2B} = \tan 45° = 1$$

$$\Rightarrow \qquad P_2B = h \qquad \ldots(2)$$

$\therefore$ From (1) and (2), we have

$$P_1B - P_2B = h\left(\sqrt{3}-1\right)$$

$$\Rightarrow \qquad P_1P_2 = h\left(\sqrt{3}-1\right)$$

$$\Rightarrow \qquad 20 = h\left(\sqrt{3}-1\right)$$

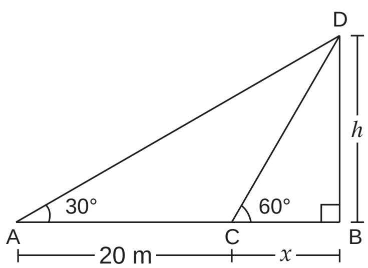
$$\Rightarrow \qquad h = \frac{20}{\sqrt{3}-1}$$

$$= \frac{20\left(\sqrt{3}+1\right)}{3-1}$$

$$= 10\left(\sqrt{3}+1\right)$$

Hence, the required height is $10\left(\sqrt{3}+1\right)$ m.

(*iii*) Let DB (= h metres) be the height of the tower.



Let A be the point of observation of the top D when the angle of elevation is 30°.

Then, $\angle DAB = 30°$.

Let C be the point on moving a distance 20 m from A towards the foot of the tower.

Then, CB = x, AC = 20 m

AB = $(20 + x)$m

$\angle ABD = \angle CBD = 90°$,

$\angle DCB = 60°$

In right ΔDBC, we have

$$\tan 60° = \frac{DB}{CB}$$

$$\Rightarrow \qquad \sqrt{3} = \frac{h}{x} \quad \Rightarrow \quad x = \frac{h}{\sqrt{3}}$$

In right ΔABD, we have

$$\tan 30° = \frac{DB}{AB} \quad \Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{20+x}$$

$$\Rightarrow \qquad 20 \text{ m} + x = h\sqrt{3}$$

$$\Rightarrow 20 \text{ m} + \frac{h}{\sqrt{3}} = h\sqrt{3} \qquad \left[\text{Using } x = \frac{h}{\sqrt{3}}\right]$$

$$\Rightarrow \qquad 20 \text{ m} = h\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right)$$

$$\Rightarrow \qquad h = \frac{20\sqrt{3}}{2} \text{ m} = 10 \times 1.732 \text{ m} = 17.32 \text{ m}$$

Now, $\quad x = \frac{h}{\sqrt{3}} = \frac{20\sqrt{3}}{2\times\sqrt{3}} = 10$ m

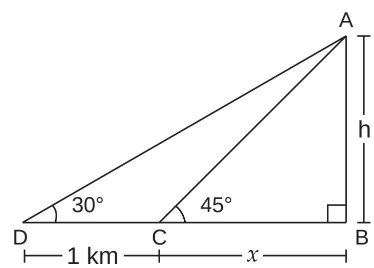
Thus, $\quad$ AB = AC + BC = 20 m + x

= 20 m + 10 m

= 30 m

Hence, the height of the tower is 17.32 m and the distance of the tower from the point A is 30 m.

11. Let AB be the height of the hill and let C and D be the positions of the kilometre stones.



Then, $\quad$ CD = 1 km, AB = h

The angles of elevation of A at D and C are 30° and 45° respectively.

Then, $\angle ADB = 30°$ and $\angle ACB = 45°$.

CB = x, $\angle ABC = 90° = \angle ABD = 90°$.

In right ΔABC, we have

$$\tan 45° = \frac{AB}{BC}$$

$$\Rightarrow \qquad 1 = \frac{h}{x} \quad \Rightarrow \quad x = h \qquad \ldots (1)$$

In right ΔABD, we have

$$\tan 30° = \frac{AB}{BD}$$

$$\Rightarrow \qquad \frac{1}{\sqrt{3}} = \frac{h}{x+1}$$

$$\Rightarrow \qquad \frac{1}{\sqrt{3}} = \frac{h}{h+1} \qquad \text{[Using equation (1)]}$$

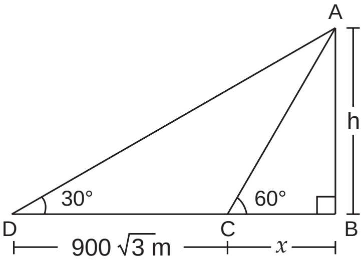
$$\Rightarrow \qquad h + 1 = \sqrt{3}h$$

$$\Rightarrow \qquad h = \frac{1}{\sqrt{3}-1}$$

$$= \frac{1}{1.732-1} = 1.366 \text{ km} = 1366 \text{ m}$$

Hence, the height of the hill is 1366 m.

12. (*i*) Let AB be the height of the mountain. Let D be the point from the foot of the mountain at which the angle of elevation is 30°.



Then, $AB = h$ and $\angle ADB = 30^\circ$.

Let C be the point from D at which the angle of elevation to the point A is 60°.

Then, $BC = x$, $\angle ACB = 60^\circ$

and $\angle ABC = \angle ABD = 90^\circ$.

In right ΔABC, we have

$$\tan 60^\circ = \frac{h}{x}$$

$$\Rightarrow \quad \sqrt{3} = \frac{h}{x}$$

$$\Rightarrow \quad x = \frac{h}{\sqrt{3}}$$

In right ΔABD, we have

$$\tan 30^\circ = \frac{AB}{DB}$$

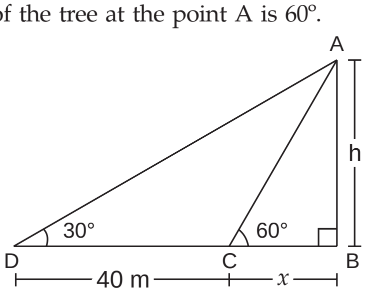
$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{900\sqrt{3}+x}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{900\sqrt{3}+\frac{h}{\sqrt{3}}}$$

$$\Rightarrow \quad h - \frac{h}{3} = 900 \quad \Rightarrow \quad h = 1350 \text{ m}$$

Hence, the height of the mountain is 1350 m.

(*ii*) Let AB be the height of the tree and C be the point on the bank of the river. Thus, the width of the river is x metres. The angle of elevation at the point C to the top of the tree at the point A is 60°.



Then, $CB = x$, $AB = h$, $\angle ACB = 60^\circ$.

On moving 40 m away from the point C, the angle of elevation at the point D to the top of the tree at A is 30°.

Then, $ADB = 30^\circ$, $DB = 40 + x$,

$\angle ABC = \angle ABD = 90^\circ$

In right ΔABC, we have

$$\frac{AB}{CB} = \tan 60^\circ$$

$$\Rightarrow \quad \frac{h}{x} = \sqrt{3}$$

$$\Rightarrow \quad x = \frac{h}{\sqrt{3}} \qquad \dots (1)$$

In right ΔABD, we have

$$\frac{AB}{DB} = \tan 30^\circ$$

$$\Rightarrow \quad \frac{h}{40+x} = \frac{1}{\sqrt{3}} \qquad \text{[Using (1)]}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{40+\frac{h}{\sqrt{3}}}$$

$$\Rightarrow \quad \frac{40}{\sqrt{3}} + \frac{h}{3} = h$$

$$\Rightarrow \quad h - \frac{h}{3} = \frac{40}{\sqrt{3}}$$

$$\Rightarrow \quad 2h = \frac{40 \times 3}{\sqrt{3}}$$

$$\Rightarrow \quad h = 20\sqrt{3} = 20 \times 1.732 = 34.64 \text{ m}$$

Hence, the height of the tree is 34.64 m.

From equation (1),

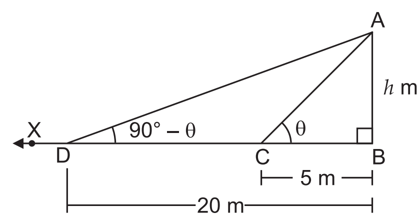
$$x = \frac{h}{\sqrt{3}} = \frac{20\sqrt{3}}{\sqrt{3}} = 20 \text{ m}$$

Hence, the width of the river is 20 m.

(*iii*) Let B be the base of a tower AB and BX be the horizontal ground so that $\angle ABX = 90^\circ$.

D and C are two positions of the observer on the ground so that BD = 20 m and CB = 5 m.

Let $AB = h$ m, $BC = 5$ m, $BD = 20$ m, $\angle ACB = \theta$ and $\angle ADB = 90^\circ - \theta$.



Now, from ΔABC, we have

$$\tan\theta = \frac{AB}{BC} = \frac{h}{5} \qquad \dots(1)$$

and from ΔADB, we have

$$\tan(90^\circ - \theta) = \frac{AB}{BD} = \frac{h}{20}$$

$$\Rightarrow \quad \cot\theta = \frac{h}{20} \qquad \dots(2)$$

$\therefore$ From (1) and (2), we have

$$\frac{h}{5} \times \frac{h}{20} = \tan\theta \times \cot\theta = \frac{\tan\theta}{\tan\theta} = 1$$

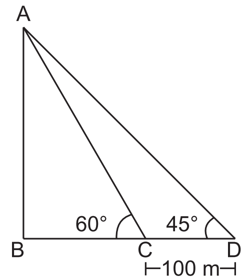
$$\therefore \quad h^2 = 100$$

$$\Rightarrow \quad h = 10$$

$\therefore$ The required height of the tower is 10 m.

13. (*i*) Angle of depression of two cars 45° and 60°

Distance between the cars = 100 m



In ΔABC, we have

$$\tan 60° = \frac{AB}{BC}$$

$$\sqrt{3} = \frac{AB}{BC}$$

$$AB = \sqrt{3}\,BC$$

Now, in ΔABD

$$\tan 45° = \frac{AB}{BD}$$

$$1 = \frac{AB}{BC+100} \qquad [\because BD = BC + 100]$$

$$BC + 100 = AB$$

$$BC + 100 = \sqrt{3}\,BC \qquad [\because AB = \sqrt{3}\,BC]$$

$$(\sqrt{3} - 1)BC = 100$$

$$BC = \frac{100}{\sqrt{3}-1}$$

$$= \frac{100}{(\sqrt{3}-1)} \times \frac{(\sqrt{3}+1)}{(\sqrt{3}+1)}$$

$$= \frac{100(\sqrt{3}+1)}{2}$$

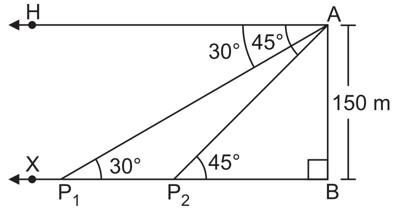
$$= 50(\sqrt{3}+1)$$

Height of balloon = $AB = \sqrt{3}\,BC$

$$= 50(\sqrt{3}+1) \times \sqrt{3}$$

$$= 50\,(3+\sqrt{3})\ \text{m}$$

(*ii*) Let A be the top of the light house AB standing on the horizontal sea level BX. Let P_1 and P_2 be the two positions of two ships on the sea level on the same side of the light house such that $\angle HAP_1 = 30° = \angle AP_1B$ and $\angle HAP_2 = 45° = \angle AP_2B$, AH being a horizontal ray through A.



Given that AB = 150 m.

Now, from $\angle AP_1B$, we have, $\tan 30° = \dfrac{AB}{P_1B}$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{150}{P_1B}$$

$$\Rightarrow \quad P_1B = 150\sqrt{3} \qquad \ldots(1)$$

Also, from ΔAP_2B, we have

$$\tan 45° = \frac{AB}{P_2B}$$

$$\Rightarrow \quad 1 = \frac{150}{P_2B}$$

$$\Rightarrow \quad P_2B = 150 \qquad \ldots(2)$$

$$P_1B - P_2B = 150\sqrt{3} - 150$$

$$= (\sqrt{3}-1)150$$

$$= 150 \times (1.732 - 1)$$

$$= 150 \times 0.732 \simeq 109.8$$

Hence, the required distance between the two ships is 109.8 m.

(*iii*) Let AB be the height of the lighthouse. The angle of depression of a ship as observed from the top of the lighthouse at the point D and C are respectively 30° and 60°. Then, CD is the distance travelled by the ship during the period of observation.

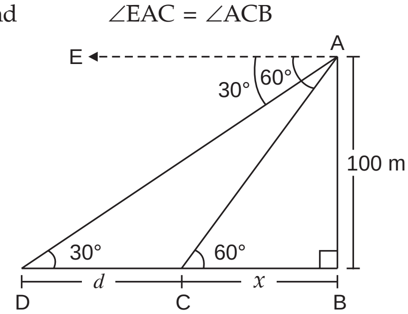
Then, $\angle EAD = 30°$ and $\angle EAC = 60°$.

Now, $AE \parallel BC$.

Thus, $\angle EAD = \angle ADB$

$\Rightarrow$ $\angle ADB = 30°$

and $\angle EAC = \angle ACB$



$\Rightarrow$ $\angle ACB = 60°$

Then, $AB = 100$ m, $CD = d$, $CB = x$.

$\angle ABC = \angle ABD = 90°$

$DB = d + x$

In right ΔABC, we have

$$\tan 60° = \frac{AB}{CB}$$

$$\Rightarrow \quad \sqrt{3} = \frac{100\text{ m}}{x}$$

$$\Rightarrow \quad x = \frac{100\text{ m}}{\sqrt{3}} \qquad \ldots(1)$$

In right ΔABD, we have

$$\tan 30° = \frac{AB}{DB}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{100 \text{ m}}{d+x}$$

$$\Rightarrow \quad d + x = 100\sqrt{3}$$

$$\Rightarrow \quad d = 100\sqrt{3} - x \quad \text{[Using equation (1)]}$$

$$= 100\sqrt{3} - \frac{100}{\sqrt{3}}$$

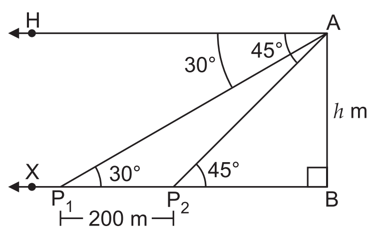
$$= \frac{100 \times 2}{\sqrt{3}} = \frac{200}{\sqrt{3}}$$

$$= \frac{200}{1.732}$$

$$= 115.47 \text{ (approx.)}$$

Hence, the distance travelled by the ship is 115.47 (approx.).

(*iv*) Let A be the top of a light house AB standing on the horizontal sea level BX. Let P_1 and P_2 be the two positions of two ships on the same side of the light house such that $\angle HAP_1 = 30° = \angle AP_1B$ and $\angle HAP_2 = 45° = \angle AP_2B$, AH being horizontal ray through A. Given that P_1P_2 = 200 m. Let h m be the height of the light house.



Then from ΔAP_1B, we have

$$\tan 30° = \frac{AB}{P_1B}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{P_1B}$$

$$\Rightarrow \quad P_1B = \sqrt{3}\,h \qquad \ldots(1)$$

Also, from ΔAP_2B, we have

$$\tan 45° = \frac{h}{P_2B}$$

$$\Rightarrow \quad 1 = \frac{h}{P_2B}$$

$$\therefore \quad P_2B = h \qquad \ldots(2)$$

$\therefore$ From (1) and (2), we get

$$P_1B - P_2B = \left(\sqrt{3}-1\right)h$$

$$\Rightarrow \quad 200 = \left(\sqrt{3}-1\right)h$$

$$\Rightarrow \quad h = \frac{200\left(\sqrt{3}+1\right)}{3-1}$$

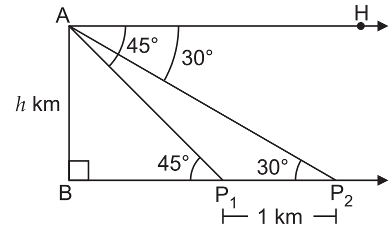
$$= 100\left(\sqrt{3}+1\right)$$

$$= 100(1.732 + 1)$$

$$= 273.2$$

Hence, the required height of the light house is 273.2 m.

(*v*) Let A be the top of a hill AB and let P_1 and P_2 be the positions of two consecutive kilometre stones due east of the hill such that $\angle HAP_1 = 45° = \angle AP_1B$ and $\angle HAP_2 = 30° = \angle AP_2B$, AH being a horizontal ray through A. Given that P_1P_2 = 1 km.



Now, from ΔAP_1B, we have

$$\tan 45° = \frac{AB}{BP_1}$$

$$\Rightarrow \quad 1 = \frac{h}{BP_1}$$

$$\Rightarrow \quad BP_1 = h \qquad \ldots(1)$$

Also, from ΔAP_2B, we have

$$\tan 30° = \frac{AB}{BP_2}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{BP_2}$$

$$\Rightarrow \quad BP_2 = h\sqrt{3} \qquad \ldots(2)$$

$\therefore$ From (1) and (2), we have

$$BP_2 - BP_1 = h\sqrt{3} - h$$

$$\Rightarrow \quad P_1P_2 = h\left(\sqrt{3}-1\right)$$

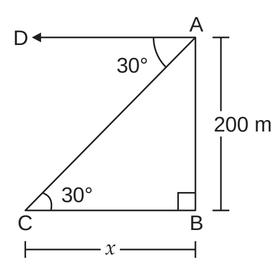
$$\therefore \quad h = \frac{P_1P_2}{\sqrt{3}-1}$$

$$= \frac{1}{\sqrt{3}-1}$$

$$= \frac{\sqrt{3}+1}{2}$$

Hence, the required height of the hill is $\frac{\sqrt{3}+1}{2}$ km.

14. (*i*) Let AB be the height of the tower and C be the point at which the enemy boat is observed. The angle of depression made at the point C from the point A is 30°. Let CB be the distance of the boat from the point B.



Then, $\angle CAD = 30°$.

Now, $AD \parallel BC$.

Then, $\angle DAC = \angle BCA = 30°$

AB = 200 m, BC = x metres.

$\angle ABC = 90°$

In right ΔABC, we have

$$\frac{AB}{BC} = \tan 30^\circ$$

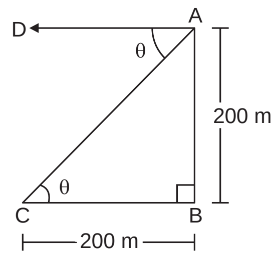
$$\Rightarrow \quad \frac{200 \text{ m}}{x} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad x = 200\sqrt{3} \text{ m}$$

$$= 200 \times 1.732 \text{ m} = 346.4 \text{ m}$$

Hence, the distance of the boat from the foot of the observation tower is 346.4 m.

(*ii*) Let θ be the angle of depression at which the boat is 200 m from the foot of the observation tower.



Then, $\angle DAC = \theta$

Now, $AD \parallel BC$.

Then, $\angle DAC = \angle ACB = \theta$,

$BC = 200$ m,

$AB = 200$ m,

$\angle ABC = 90^\circ$.

In right ΔABC, we have

$$\tan\theta = \frac{AB}{BC}$$

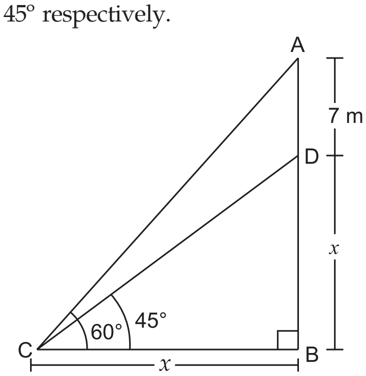
$$\Rightarrow \quad \tan\theta = \frac{200 \text{ m}}{200 \text{ m}}$$

$$\Rightarrow \quad \tan\theta = 1$$

$$\Rightarrow \quad \theta = 45^\circ.$$

Hence, the angle of depression is 45°.

15. (*i*) Let AD be the height of the vertical flag staff and x be the height of the tower. The angles of elevation from the point C to the top and bottom of the staff are 60° and 45° respectively.



Then, $AD = 7\text{m}$, $BD = x$ m

$$\therefore \quad \tan 45^\circ = \frac{x}{BC}$$

$$1 = \frac{x}{BC}$$

$$BC = x$$

Now, $$\tan 60^\circ = \frac{AB}{BC}$$

$$\sqrt{3} = \frac{x+7}{x}$$

$$\sqrt{3}x = x + 7$$

$$x\left(\sqrt{3}-1\right) = 7$$

$$x = \frac{7}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$= \frac{7\left(\sqrt{3}+1\right)}{2}$$

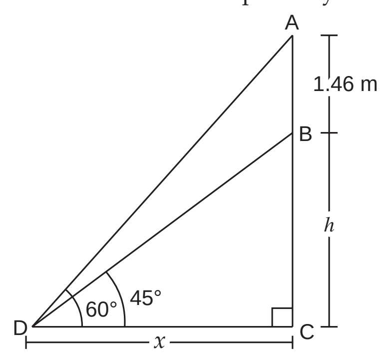
$$= \frac{7 \times (1.732+1)}{2}$$

$$= \frac{7 \times 2.732}{2}$$

$$= 9.562 \text{ m}$$

$$\approx 9.6 \text{ m}$$

(*ii*) Let AB be the height of the statue and BC be the height of the pedestal. Let D be the point on the ground at which the angles of elevation at the top of the statue and pedestal is 60° and 45° respectively.



Then, $AB = 1.46$ m, $BC = h$, $CD = x$,

$\angle ADC = 60^\circ$, $\angle BDC = 45^\circ$,

$\angle ACD = 90^\circ = \angle BCD$.

In right ΔBCD, we have

$$\tan 45^\circ = \frac{BC}{CD}$$

$$\Rightarrow \quad 1 = \frac{h}{x}$$

$$\Rightarrow \quad x = h$$

In right ΔACD, we have

$$\tan 60^\circ = \frac{AC}{CD}$$

$$\Rightarrow \quad \sqrt{3} = \frac{1.46 + h}{x}$$

$$\Rightarrow \quad x\sqrt{3} = 1.46 + h$$

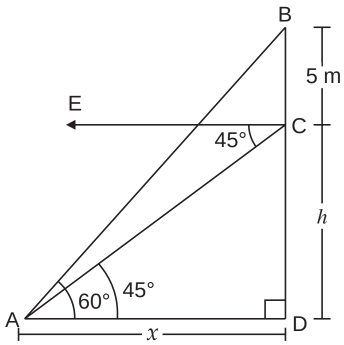
$\Rightarrow \quad \sqrt{3}\,h = 1.46 + h$

$$\Rightarrow \quad h = \frac{1.46}{\sqrt{3}-1} = \frac{1.46}{1.732-1}$$

$= 1.994$ m $= 2$ m (approx.)

Hence, the height of the pedestal is 2 m.

16. Let BC be the height of the pole and CD be the height of the tower. From the point A, the angle of elevation of the top of the pole is 60° and the angle of depression from the point C to the ground at the point A is 45°.



Then, $\angle ECA = 45°$

Now, $CE \parallel DA$.

Thus, $\angle ECA = \angle CAD = 45°$

Also, $BC = 5$ m, $CD = h$,

$AD = x$, $\angle BAD = 60°$,

$\angle BDA = \angle CDA = 90°$.

In right ΔCDA, we have

$$\tan 45° = \frac{CD}{AD}$$

$$\Rightarrow \quad 1 = \frac{h}{x}$$

$\Rightarrow \quad h = x$ ... (1)

In right ΔBDA, we have

$$\tan 60° = \frac{BD}{AD}$$

$$\Rightarrow \quad \sqrt{3} = \frac{5+h}{x}$$

$\Rightarrow \quad x\sqrt{3} = 5 + h$

$\Rightarrow \quad h\sqrt{3} = 5 + h$ [Using equation (1)]

$$\Rightarrow \quad h = \frac{5}{\sqrt{3}-1}$$

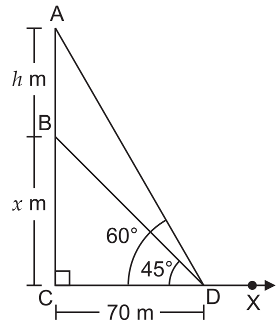
$$= \frac{5}{1.732-1} = 6.83 \text{ m}$$

Hence, the height of the tower is 6.83 m.

17. (*i*) Let BC be the vertical tower standing on a horizontal plane CX, C being the foot of the tower.

The tower is surmounted by a vertical flag staff AB of height h m. Let D be a point on the horizontal plane such that CD = 70 m.

It is given that $\angle BDC = 45°$ and $\angle ADC = 60°$.



Let BC = x m.

Now, from ΔADC, we have

$$\tan 60° = \frac{AC}{CD} = \frac{h+x}{70}$$

$$\Rightarrow \quad \sqrt{3} = \frac{h+x}{70}$$

$\Rightarrow \quad h + x = 70\sqrt{3}$ ...(1)

Also, from ΔBCD, we have

$$\tan 45° = \frac{BC}{CD} = \frac{x}{70}$$

$$\Rightarrow \quad 1 = \frac{x}{70}$$

$\Rightarrow \quad x = 70$ ...(2)

From (1) and (2), we have, $x = 70$ m

and $h + 70 = 70\sqrt{3}$

$\Rightarrow \quad h = 70\sqrt{3} - 70$

$= 70\left(\sqrt{3}-1\right)$

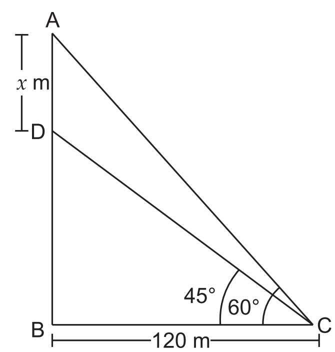
$= 70 \times (1.732 - 1)$

$= 70 \times 0.732$

$= 51.24$

Hence, the required height of the flagstaff is 51.24 m and that of the tower is 70 m.

(*ii*) Let AD be the height of the flagstaff say x and BD be the height of the tower. Tower is at a distance of 120 m from a point C. The angle of elevation of the top and bottom of the flagstaff is 60° and 45° respectively.



Then, $AD = x$, $BC = 120$ m

$\therefore$ In ΔDBC, we have

$$\tan 45° = \frac{BD}{BC}$$

$$1 = \frac{BD}{120}$$

$$BD = 120 \text{ m}$$

In ΔABC,

$$\tan 60^\circ = \frac{AB}{BC}$$

$$\sqrt{3} = \frac{x + 120}{120}$$

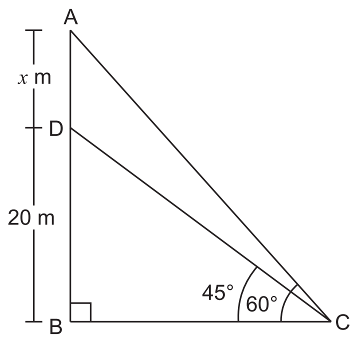
$$120\sqrt{3} = x + 120$$

$$x = 120\left(\sqrt{3} - 1\right)$$

$$= 120 \times 0.732$$

$$= 87.84 \text{ m}$$

(*iii*) Let AD (= x m) be the height of the tower. The angle of elevation of the bottom and top of a tower fixed at the 20 m high building are 45° and 60° respectively.



Then AD = x m, BD = 20 m

In ΔDBC, we have

$$\tan 45^\circ = \frac{BD}{BC}$$

$$1 = \frac{20}{BC}$$

$$BC = 20 \text{ m}$$

In ΔABC,

$$\tan 60^\circ = \frac{AB}{BC}$$

$$\sqrt{3} = \frac{x + 20}{20}$$

$$20\sqrt{3} = x + 20$$

$$x = 20\left(\sqrt{3} - 1\right)$$

$$= 20 \times 0.732$$

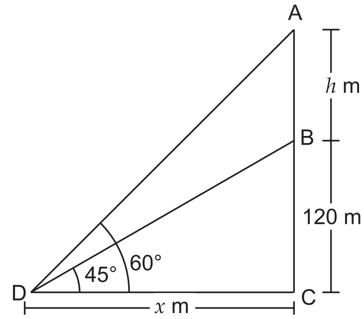
$$= 14.64 \text{ m}$$

18. Let BC be the height of the unfinished tower and AB be the remaining part of the building need to be raised. The angle of elevation at the points B and A are respectively 45° and 60°.

Then, AB = h, BC = 120 m,

$\angle ADC = 60^\circ$, $\angle BDC = 45^\circ$,

CD = x, $\angle ACD = \angle BCD = 90^\circ$



In right ΔBCD, we have

$$\tan 45^\circ = \frac{BC}{CD}$$

$$\Rightarrow \quad 1 = \frac{120 \text{ m}}{x}$$

$$\Rightarrow \quad x = 120 \text{ m} \qquad \dots (1)$$

In right ΔACD, we have,

$$\tan 60^\circ = \frac{AC}{CD} \Rightarrow \sqrt{3} = \frac{h + 120}{x}$$

$$\Rightarrow \quad x\sqrt{3} = h + 120$$

$$\Rightarrow \quad h = x\sqrt{3} - 120$$

$$= 120\sqrt{3} - 120 \qquad \text{[Using (1)]}$$

$$= 120(\sqrt{3} - 1)$$

$$= 120 \times 0.732 = 87.84 \text{ m}$$

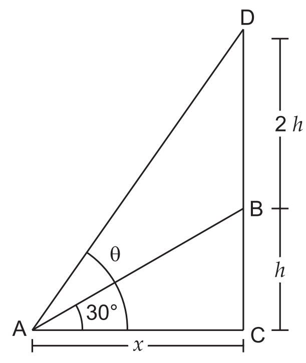
Hence, the height of the building should be raised to 87.84 m from the unfinished part.

19. Let BC be the height of the tower.

Then, BC = h.

It is given that the height of the flagstaff fixed on the tower is DB.

or DB = $2h$.



A is the observation point on the ground having a distance x from the foot of the tower.

The angle of elevation of the point B at A is 30°.

Let θ be the angle of elevation of the top of the flagstaff at A.

Then, $\angle DAC = \theta$, $\angle BAC = 30^\circ$,

DB = $2h$, BC = h, CA = x,

$\angle DCA = \angle BCA = 90^\circ$.

In right ΔBCA, we have

$$\tan 30° = \frac{BC}{CA}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{x}$$

$$\Rightarrow \quad x = \sqrt{3}\,h \qquad \dots (1)$$

In right ΔDCA, we have

$$\tan \theta = \frac{DC}{CA}$$

$$\Rightarrow \quad \tan \theta = \frac{2h+h}{x}$$

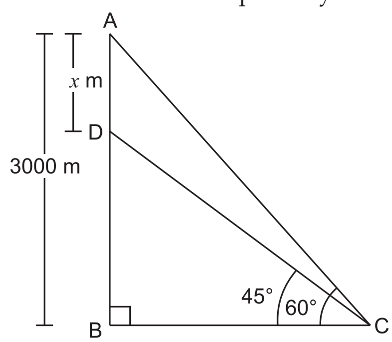
$$\Rightarrow \quad \tan \theta = \frac{3h}{\sqrt{3}\,h} \qquad \text{[Using (1)]}$$

$$\Rightarrow \quad \tan \theta = \sqrt{3}$$

$$\Rightarrow \quad \tan \theta = \tan 60°$$

Hence, the angle of elevation of the top of the flagstaff at the point A is 60°.

20. (*i*) Let A and B be the positions of the two aircraft from the ground. The angles of elevation of the points A and B at D are 60° and 45° respectively.



Then, AD = x m, BD = (3000 – x) m

∴ In ΔBDC

$$\tan 45° = \frac{BD}{BC}$$

$$BD = BC$$

In ΔABC

$$\tan 60° = \frac{AB}{BC}$$

$$\sqrt{3} = \frac{3000}{BD} \qquad (\because BC = BD)$$

$$\sqrt{3}\,BD = 3000$$

$$\sqrt{3}\,(3000 - x) = 3000$$

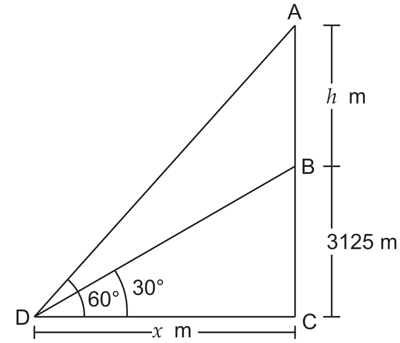
$$3000\sqrt{3} - \sqrt{3}\,x = 3000$$

$$\sqrt{3}\,x = 3000\left(\sqrt{3}-1\right)$$

$$x = \frac{3000\left(\sqrt{3}-1\right)}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{3000\left(\sqrt{3}-1\right)\sqrt{3}}{3} = 1262.9 \text{ m}$$

(*ii*) Let A and B be the positions of the two aeroplanes from the point C. The point of observation of the aeroplane is the point D. The angles of elevation of the two aeroplanes at D are 60° and 30° respectively.



Then, AB = h, BC = 3125 m, ∠ADC = 60°,

∠BDC = 30°, CD = x

∠ACD = 90° = ∠BCD.

The distance between the two aeroplanes is h or AB.

In right ΔBCD, we have

$$\tan 30° = \frac{BC}{CD}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{3125}{x}$$

$$\Rightarrow \quad x = 3125\sqrt{3}$$

In right ΔACD, we have

$$\tan 60° = \frac{AC}{CD}$$

$$\Rightarrow \quad \sqrt{3} = \frac{h+3125}{x}$$

$$\Rightarrow \quad \sqrt{3}\,x = h + 3125$$

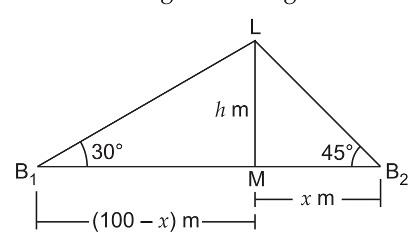
$$\Rightarrow \quad h = \sqrt{3}\,x - 3215$$

$$= \sqrt{3} \times 3125\sqrt{3} - 3125$$

$$= 6250 \text{ m}$$

Hence, the distance between the two aeroplanes at that instant is 6250 m.

21. (*i*) Let the two boats be at B_1 and B_2 in the horizontal sea B_1B_2. Let LM be the light house in mid-sea and let h m be the height of the light house.



Let MB = x m. Then B_1M = (100 – x) m.

$$\because \quad B_1B_2 = 100 \text{ m} \qquad \text{[Given]}$$

Now, from ΔLB_1M, we have

$$\tan 30° = \frac{LM}{B_1M} = \frac{h}{100-x}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{100-x}$$

$$\Rightarrow \quad 100 - x = \sqrt{3}\,h \qquad \ldots(1)$$

Again, from ΔLB_2M, we have

$$\tan 45° = \frac{LM}{MB_2}$$

$$\Rightarrow \quad 1 = \frac{h}{x}$$

$$\Rightarrow \quad x = h \qquad \ldots(2)$$

$\therefore$ From (1) and (2), we have

$$100 - h = \sqrt{3}h$$

$$\Rightarrow \quad h\left(\sqrt{3}+1\right) = 100$$

$$\Rightarrow \quad h = \frac{100}{\sqrt{3}+1}$$

$$= \frac{100\left(\sqrt{3}-1\right)}{3-1}$$

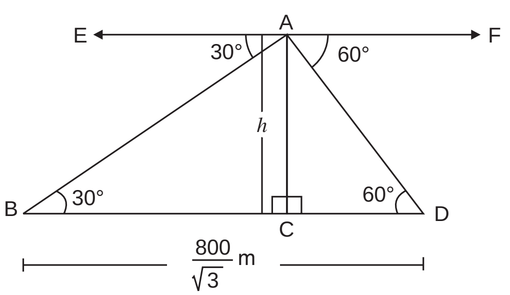
$$= 50(1.732 - 1)$$

$$= 50 \times 0.732$$

$$= 36.6$$

Hence, the required height of the light house is 36.6 m.

(*ii*) Let AC be the height of the lighthouse. Let the positions of the two ships on the opposite sides of the lighthouse be B and D. The angles of depression of the points B and D are 30° and 60° respectively.



Then, $\angle EAB = 30°, \angle FAD = 60°$

Now, $EF \parallel BD, \angle EAB = \angle ABC$

And $\angle FAD = \angle ADC$

Then, $AC = h, \angle ABC = 30°, \angle ADC = 60°,$

$\angle ACB = \angle ACD = 90°.$

Thus, the distance between the two ships is BD which is $\frac{800}{\sqrt{3}}$ m.

In right ΔACB, we have

$$\tan 30° = \frac{AC}{BC} \quad \Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{BC} \quad \Rightarrow \quad BC = \sqrt{3}\,h$$

In right ΔACD, we have

$$\tan 60° = \frac{AC}{CD} \quad \Rightarrow \quad \sqrt{3} = \frac{h}{CD} \quad \Rightarrow \quad CD = \frac{h}{\sqrt{3}}.$$

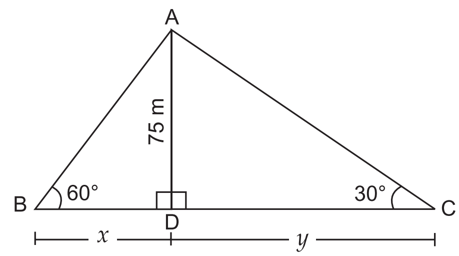
Now, $BD = BC + CD$

$$\Rightarrow \quad \frac{800}{\sqrt{3}} = \sqrt{3}\,h + \frac{h}{\sqrt{3}}$$

$$\Rightarrow \quad \frac{800}{\sqrt{3}} = \frac{3h+h}{\sqrt{3}} \quad \Rightarrow \quad h = \frac{800}{4} = 200 \text{ m}$$

Hence, the height of the lighthouse is 200 m.

22. Let AD be the height of the building. The positions of the two men on the opposite sides of the building are B and C respectively. The angles of elevation of the top of the building A at points B and C are 60° and 30° respectively.



Then, $\angle ABD = 60°, \angle ACD = 30°,$

$AD = 75$ m, $\angle ADB = \angle ADC = 90°.$

$BD = x$ and $CD = y$

In ΔABD, we have

$$\tan 60° = \frac{AD}{BD}$$

$$\sqrt{3} = \frac{75}{BD}$$

$$BD = \frac{75}{\sqrt{3}} = 25\sqrt{3}$$

In ΔACD,

$$\tan 30° = \frac{AD}{CD}$$

$$\frac{1}{\sqrt{3}} = \frac{75}{CD}$$

$$CD = 75\sqrt{3}$$

Distance between two men $= BD + CD$

$$= 25\sqrt{3} + 75\sqrt{3}$$

$$= 100\sqrt{3}$$

$$= 100 \times 1.73$$

$$= 173 \text{ m}$$

23. (*i*) Let AC be the height of the lighthouse. Let B and D be the positions of the two ships on the opposites sides of the lighthouse. The angles of depression of the points B and D at A are 30° and 45° respectively.

Then, $\angle EAB = 30°$ and $\angle FAD = 45°$

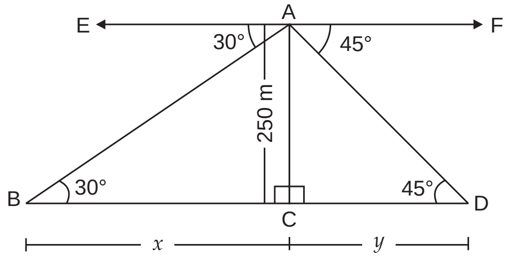
Now, $EF \parallel BD.$

Then, $\angle EAB = \angle ABC$ and $\angle FAD = \angle ADC$

Thus, $AC = 250$ m, $\angle ABC = 30°,$

$\angle ADC = 45°, BC = x, CD = y,$

$\angle ACB = \angle ACD = 90°$



In right ΔACB, we have

$$\tan 30° = \frac{AC}{BC}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{250 \text{ m}}{x}$$

$$\Rightarrow \quad x = 250\sqrt{3} \text{ m}$$

In right ΔACD, we have

$$\tan 45° = \frac{AC}{CD}$$

$$\Rightarrow \quad 1 = \frac{250 \text{ m}}{y}$$

$$\Rightarrow \quad y = 250 \text{ m}$$

Now, the distance between B and D is

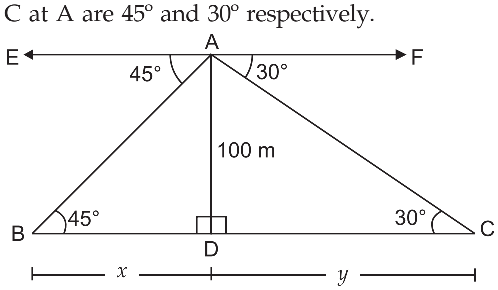
$$BD = x + y = 250\sqrt{3} \text{ m} + 250 \text{ m}$$

$$= 250(\sqrt{3} + 1) \text{ m}$$

$$= 250 \times 2.732 = 683 \text{ m}$$

Hence, the distance between the two ships on the opposite side of the lighthouse is 683 m.

(*ii*) Let AD be the altitude of the tower from the ground. Let B and C be the points on the opposite side of the tower. The angles of depression of the points B and C at A are 45° and 30° respectively.



Then, $\angle EAB = 45°$ and $\angle FAC = 30°$.

Now, $EF \parallel BC$.

Thus, $\angle EAB = \angle ABD$

and $\angle FAC = \angle ACD$

Now, $AD = 100$ m, $BD = x$, $DC = y$,

$\angle ABD = 45°$, $\angle ACD = 30°$.

$\angle ADB = \angle ADC = 90°$.

In ΔABD, we have

$$\tan 45° = \frac{AD}{BD}$$

$$1 = \frac{100}{BD} \Rightarrow BD = 100 \text{ m}$$

In ΔACD, we have

$$\tan 30° = \frac{AD}{CD}$$

$$\frac{1}{\sqrt{3}} = \frac{100}{CD}$$

$$CD = 100\sqrt{3}$$

$$BC = BD + CD$$

$$= 100 + 100\sqrt{3}$$

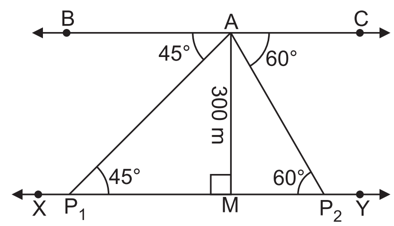
$$= 100\left(1+\sqrt{3}\right)$$

$$= 100 \times 2.73$$

$$= 273 \text{ m}$$

(*iii*) Let A, the aeroplane, flying along the horizontal line BC through A, P_1 and P_2 be two points on two banks of a horizontal river.

The angles of depression of the points P_1 and P_2 from A are 45° and 60° respectively.



$\therefore \quad \angle BAP_1 = \angle AP_1M = 45°$

and $\quad \angle CAP_2 = \angle AP_2M = 60°$

Let the height of the aeroplane at A be $AM \perp P_1P_2$.

$\therefore \quad AM = 300$ m [Given]

Now, from ΔAP_1M, we have

$$\tan 45° = \frac{AM}{P_1M} = \frac{300}{P_1M}$$

$$\Rightarrow \quad 1 = \frac{300}{P_1M}$$

$$\Rightarrow \quad P_1M = 300 \quad \ldots(1)$$

From ΔAP_2M, we have

$$\tan 60° = \frac{AM}{P_2M}$$

$$\Rightarrow \quad \sqrt{3} = \frac{300}{P_2M}$$

$$\Rightarrow \quad P_2M = \frac{300}{\sqrt{3}}$$

$$= \frac{300\sqrt{3}}{3}$$

$$= 100\sqrt{3}$$

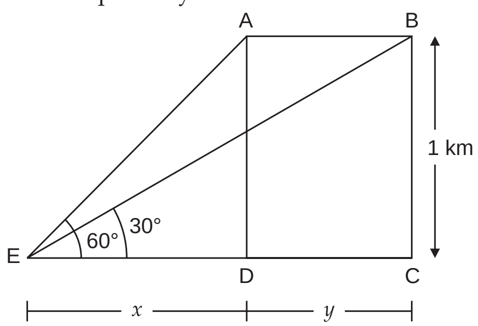
$$= 100 \times 1.732$$

$$= 173.2 \quad \ldots(2)$$

Hence, the required width of the river = $P_1M + P_2M$ = (300 + 173.2) m = 473.2 m.

24. (*i*) Let A be the point at which the aeroplane is above the ground. Then, AD is the altitude of the aeroplane.

The angles of elevation of points A and B at E are 60° and 30° respectively.



Thus, the aeroplane moves from A to B in 10 seconds.

Then, $BC = 1$ km, $\angle AEC = 60°$, $\angle BEC = 30°$,

$ED = x$, $CD = y$

$\angle ADE = \angle BCE = 90°$

In right ΔADE, we have

$$\tan 60° = \frac{AD}{ED}$$

$$\Rightarrow \quad \sqrt{3} = \frac{1\text{ km}}{x}$$

$$\Rightarrow \quad x = \frac{1}{\sqrt{3}}\text{ km}$$

In right ΔBCE, we have

$$\tan 30° = \frac{BC}{EC}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{1\text{ km}}{x+y}$$

$$\Rightarrow \quad x + y = \sqrt{3}$$

$$\Rightarrow \quad y = \sqrt{3} - x = \sqrt{3} - \frac{1}{\sqrt{3}} = \frac{2}{\sqrt{3}}\text{ km}$$

Let t be the time taken by the aeroplane to move from A to B. Then, the uniform speed of the aeroplane is

$$v = \frac{y}{t} = \frac{\left(\frac{2}{\sqrt{3}}\right)\text{km}}{10s}$$

$$= \frac{2 \times 60 \times 60}{\sqrt{3} \times 10}\text{ km/hour}$$

$$= 415.70\text{ km/hour}$$

Hence, the uniform speed of the aeroplane is 415.70 km/hour.

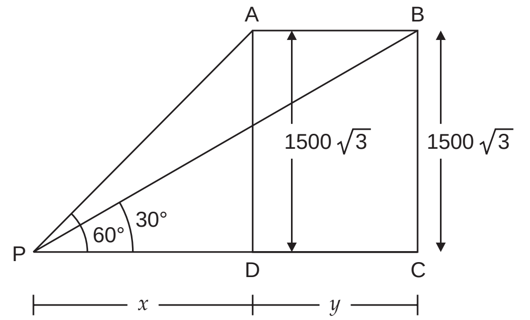
(*ii*) Let A and B be the positions of the jet plane from the ground. The angles of elevation of A and B at P are 60° and 30° respectively. Let AD = BC be the altitude of the jet plane.

Then, $\angle APD = 60°$, $\angle BPC = 30°$,

$AD = BC = 1500\sqrt{3}$ m,

$PD = x$, $CD = AB = y$.

$\angle ADP = \angle BCP = 90°$.



In right ΔADP, we have

$$\tan 60° = \frac{AD}{PD}$$

$$\Rightarrow \quad \sqrt{3} = \frac{1500\sqrt{3}}{x}$$

$$\Rightarrow \quad x = 1500\text{ m} \qquad \ldots (1)$$

In right ΔBCP, we have

$$\tan 30° = \frac{BC}{PC}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{1500\sqrt{3}}{x+y}$$

$$\Rightarrow \quad x + y = 1500 \times 3$$

$$\Rightarrow \quad y = 1500 \times 3 - 1500 \qquad \text{[Using (1)]}$$

$$= 3000\text{ m}$$

$$= 3\text{ km}$$

∴ Distance covered by the jet plane from A to B, $y = 3$ km and the time taken by the jet plane, $t = 15$ seconds.

∴ Speed of the jet plane $= \dfrac{\text{Distance}}{\text{time}}$

$$= \frac{y}{t}$$

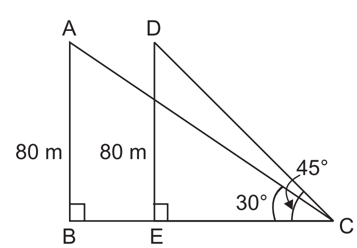
$$= \frac{3\text{ km}}{15\text{ seconds}}$$

$$= \frac{3 \times 60 \times 60}{15}\text{ km/hour}$$

$$= 720\text{ km/hour}$$

Hence, the speed of the jet plane is 720 km/hour.

(*iii*) Let A and D be the positions of the bird from the ground. The angle of elevation of A and D at C are 30° and 45° respectively.



In ΔABC, we have

$$\tan 30° = \frac{AB}{BC}$$

$$\frac{1}{\sqrt{3}} = \frac{80}{BC}$$

$$BC = 80\sqrt{3} \text{ m}$$

In ΔDEC, we have

$$\tan 45° = \frac{DE}{EC}$$

$$1 = \frac{80}{EC}$$

$$EC = 80 \text{ m}$$

Distance travelled by bird = BC – EC

$$= 80\sqrt{3} - 80$$

$$= 80\left(\sqrt{3}-1\right)$$

$$= 80 \times 0.732 = 58.56 \text{ m}$$

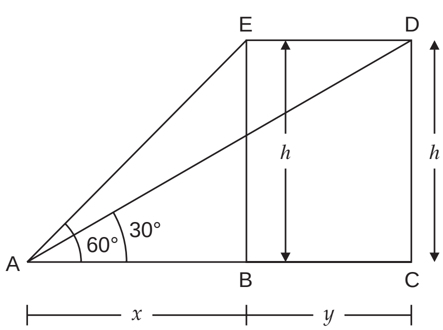
We know

$$\text{Speed} = \frac{\text{distance}}{\text{time}}$$

$$= \frac{58.56}{2}$$

$$= 29.28 \text{ m/s}$$

25. (*i*) Let E and D be the positions of the jet plane from the ground. The angles of elevation of E and D at A are 60° and 30° respectively. Let BE and CD are the altitude of the aeroplane which is a constant.



Then, $\angle EAB = 60°$, $\angle DAC = 30°$, BE = h = CD,

$\angle EBA = 90°$, $\angle DCA = 90°$,

AB = x, BC = ED = y.

In right ΔEBA, we have

$$\tan 60° = \frac{BE}{AB}$$

$$\Rightarrow \quad \sqrt{3} = \frac{h}{x}$$

$$\Rightarrow \quad x = \frac{h}{\sqrt{3}} \qquad \ldots (1)$$

In right ΔDCA, we have

$$\tan 30° = \frac{CD}{AC}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{x+y}$$

$$\Rightarrow \quad x + y = \sqrt{3}\,h$$

$$\Rightarrow \quad y = \sqrt{3}\,h - x$$

$$= \sqrt{3}\,h - \frac{h}{\sqrt{3}} \qquad \text{[Using (1)]}$$

$$= \frac{2}{\sqrt{3}}h$$

Distance covered by the aeroplane from E to D is $y = \frac{2}{\sqrt{3}}h$ and time taken by the aeroplane to move from E to D is t = 10 seconds. It is given that speed of the aeroplane is 900 km per hour.

Now, $$\text{Speed} = \frac{\text{Distance}}{\text{time}}$$

$$\Rightarrow \quad 900 \text{ km/hour} = \frac{\left(\frac{2}{\sqrt{3}}h\right)}{10 \text{ seconds}}$$

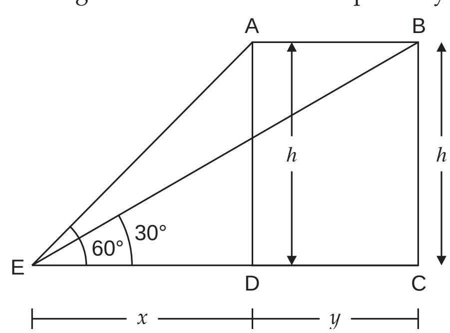
$$\Rightarrow \quad \frac{2}{\sqrt{3}}h = 900 \text{ km/hour} \times \frac{10}{60 \times 60} \text{ hour}$$

$$\Rightarrow \quad h = \frac{\sqrt{3} \times 900 \times 10}{2 \times 60 \times 60} \text{ km}$$

$$= 2.165 \text{ km} = 2165 \text{ m}.$$

Hence, the constant height at which the jet is flying is 2165 m.

(*ii*) Let E be the point of observation of the position of the jet fighter. Let A and B be the positions of the jet fighter from the ground which is at a constant height. The angles of elevation of the two positions A and B at E on the ground are 60° and 30° respectively.



Then, $\angle AED = 60°$, $\angle BEC = 30°$, AD = BC = h

ED = x, CD = y, CE = $x + y$,

$\angle ADE = 90°$, $\angle BCE = 90°$.

In right ΔADE, we have

$$\tan 60° = \frac{AD}{ED}$$

$$\Rightarrow \quad \sqrt{3} = \frac{h}{x}$$

$$\Rightarrow \quad x = \frac{h}{\sqrt{3}} \qquad \ldots (1)$$

In right ΔBCE, we have

$$\tan 30^\circ = \frac{BC}{CE}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{x+y}$$

$$\Rightarrow \quad x + y = \sqrt{3}\,h$$

$$\Rightarrow \quad y = \sqrt{3}\,h - x$$

$$= \sqrt{3}\,h - \frac{h}{\sqrt{3}} \qquad \text{[Using (1)]}$$

$$= \frac{2h}{\sqrt{3}}$$

Now, the distance covered by the jet fighter in moving from A to B is y.

Then, $$y = \frac{2h}{\sqrt{3}}.$$

Also, the time taken in moving the distance y is t. Then, $t = 15$ seconds. It is given that the speed of the jet fighter is 720 km/h.

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$\Rightarrow \quad 720 \text{ km/h} = \frac{\left(\frac{2h}{\sqrt{3}}\right)}{15 \text{ seconds}}$$

$$\Rightarrow \quad \frac{2h}{\sqrt{3}} = 720 \text{ km/h} \times 15 \text{ seconds}$$

$$\Rightarrow \quad h = \frac{\sqrt{3}}{2} \times 720 \times \frac{15}{60 \times 60} \text{ km}$$

$$= 1.732 \times \frac{15}{10} \text{ km}$$

$$= 2.598 \text{ km} = 2598 \text{ m}$$

Hence, the constant height at which the jet fighter is flying is 2598 m.

26. (*i*) Let AB and DC be the two vertical poles of different heights. Let the height of pole AB is greater than pole DC. Then angle of elevation of the point A at the point C is 60°. Also, the angle of elevation of the point D at the point B is 45°.

Also, $\angle ACB = 60^\circ$, $\angle DBC = 45^\circ$,

$\angle ABC = 90^\circ = \angle DCB$

Distance between the two poles,

$BC = 25$ m

Let AE (= h) be the difference between the heights of the two vertical poles.

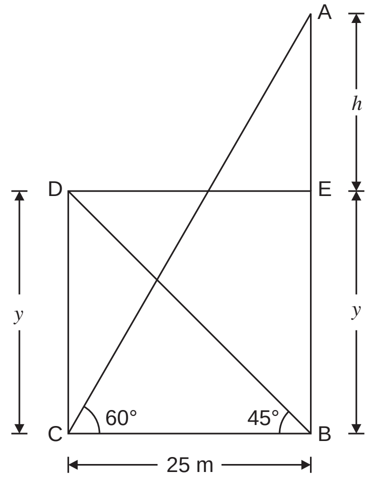
Then, $DC = y = BE$

$AB = h + y$

In right ΔDBC, we have

$$\tan 45^\circ = \frac{DC}{BC} \quad \Rightarrow \quad 1 = \frac{y}{25 \text{ m}}$$

$$\Rightarrow \quad y = 25 \text{ m} \qquad \ldots (1)$$



In right ΔABC, we have

$$\tan 60^\circ = \frac{AB}{BC} \quad \Rightarrow \quad \sqrt{3} = \frac{h+y}{25 \text{ m}}$$

$$\Rightarrow \quad h + y = 25\sqrt{3}$$

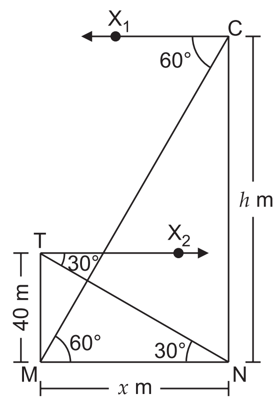
$$\Rightarrow \quad h = 25\sqrt{3} - y$$

$$= 25\sqrt{3} - 25 \qquad \text{[Using (1)]}$$

$$= (\sqrt{3} - 1)\, 25 \text{ m} = 18.3 \text{ m}$$

Hence, the difference between the heights of the two vertical poles is 18.3 m.

(*ii*) Let TM be the vertical tower and CN, the vertical chimney, M and N being their feet on the horizontal ground MN. CX_1 and TX_2 are horizontal rays from C and T respectively so that



$\angle X_1CM = \angle CMN$

$= 60^\circ$

and $\angle X_2TN = \angle TNM$

$= 30^\circ$

$TM = 40$ m [Given]

Let $CN = h$ m and $MN = x$ m

Now, from ΔTMN, we have

$$\tan 30^\circ = \frac{TM}{MN} = \frac{40}{x} = \frac{1}{\sqrt{3}} = \frac{40}{x}$$

$$\Rightarrow \quad x = 40\sqrt{3} \qquad \ldots(1)$$

Also, from ΔMNC, we have

$$\tan 60^\circ = \frac{CN}{MN}$$

$$\Rightarrow \quad \sqrt{3} = \frac{h}{x}$$

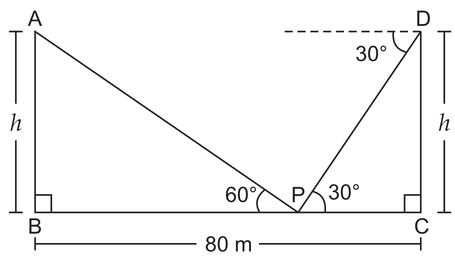
$$\Rightarrow \quad h = \sqrt{3}\,x = \sqrt{3} \times 40\sqrt{3} \qquad \text{[From (1)]}$$

$$= 120$$

Hence, the required height of the chimney is 120 m. Since the minimum height of a smoke-emitting chimney is 100 m according to the pollution control norms and the height of the present chimney to 120 m > 100 m, hence, this chimney meets the pollution norms.

***Value*: Responsibility towards environment.**

27. (*i*) Let AB and DC be the height of the two poles. Let BC be the width of the road. P is the point on the road way between the poles. The angle of elevation of A is 60° and the angle of depression from the top of another pole is 30°.



Then, AB = DC = h, BC = 80 m,

$\angle$APB = 60°, $\angle$DPC = 30°

$\angle$ABP = $\angle$DCP = 90°, BP = x, CP = y.

In right ΔABP, we have

$$\tan 60° = \frac{h}{BP} \Rightarrow \sqrt{3} = \frac{h}{x}$$

$$\Rightarrow \quad x = \frac{h}{\sqrt{3}} \quad \ldots (1)$$

In right ΔDCP, we have

$$\tan 30° = \frac{DC}{CP}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{y}$$

$$\Rightarrow \quad y = \sqrt{3}\,h \quad \ldots (2)$$

Now, the distance between the two poles is

BC = $x + y$

$$\Rightarrow \quad 80 \text{ m} = \frac{h}{\sqrt{3}} + \sqrt{3}\,h \quad \text{[Using (1) and (2)]}$$

$$\Rightarrow \quad 80 \text{ m} = \frac{h + 3h}{\sqrt{3}}$$

$$\Rightarrow \quad h = \frac{80\sqrt{3}}{4} = 20\sqrt{3}$$

Hence, the height of the poles is $20\sqrt{3}$ m

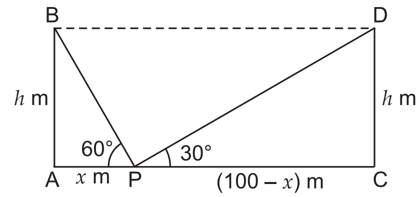
Now, $$x = \frac{h}{\sqrt{3}} = \frac{20\sqrt{3}}{\sqrt{3}} = 20 \text{ m}$$

and $$y = \sqrt{3}\,h = \sqrt{3} \times 20\sqrt{3} = 60 \text{ m}$$

Hence, the point P is 20 m from the pillar AB and 60 m from the pillar DC.

(*ii*) Let AB and CD be two pillars each of height h m, standing on the horizontal ground AC.

Let P be the position of a point from where the angles of elevation of B and D are 60° and 30° respectively. Let AP = x m. Then PC = (100 – x) m.



From ΔABP, we have

$$\tan 60° = \frac{h}{x}$$

$$\Rightarrow \quad \sqrt{3} = \frac{h}{x}$$

$$\Rightarrow \quad h = \sqrt{3}\,x \quad \ldots(1)$$

From ΔPDC, we have

$$\tan 30° = \frac{h}{100 - x}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{100 - x}$$

$$\Rightarrow \quad 100 - x = h\sqrt{3}$$

$$\Rightarrow \quad 100 - x = \sqrt{3}\,x\left(\sqrt{3}\right) \quad \text{[From (1)]}$$

$$\Rightarrow \quad 100 - x = 3x$$

$$\Rightarrow \quad 4x = 100$$

$$\Rightarrow \quad x = \frac{100}{4} = 25$$

∴ From (1),

$$h = \sqrt{3} \times 25 = 25\sqrt{3}$$

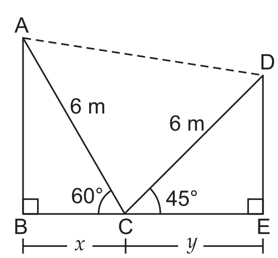
$$= 25 \times 1.732$$

$$= 43.3.$$

Hence, the required distance of the point from the 1st pillar AB is 25 m and (100 – 25) m = 75 m from 2nd pillar.

Also, the required height of each pillar is 43.3 m.

28. (*i*) Let AB and DE be two opposite walls of the same room. So, the heights of the two walls will be the same. BE is the horizontal ground and C is a point on it between the two walls. A ladder standing at C leans against the wall ED making an angle 45° with the ground. The same ladder standing at C leans against the wall BA making an angle 60° with the ground.



Hence, $\angle$DCE = 45°, $\angle$ACB = 60° and $\angle$ABC = 90° = $\angle$DEC. Also, AC = CD = 6 m (given).

Let BC = x m and CE = y m.

Now, from ΔABC, we have

$$\cos 60° = \frac{BC}{AC}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{x}{6} = x = 3 \qquad \ldots(1)$$

From ΔDCE, we have

$$\cos 45° = \frac{CE}{CD} = \frac{y}{6}$$

$$\Rightarrow \quad \frac{1}{\sqrt{2}} = \frac{y}{6}$$

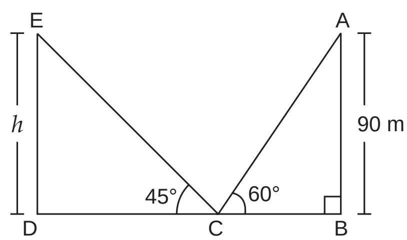
$$\Rightarrow \quad y = \frac{6}{\sqrt{2}} = 3\sqrt{2} \qquad \ldots(2)$$

From (1) and (2), we get

$$x + y = 3 + 3\sqrt{2}$$
$$= 3\left(1+\sqrt{2}\right) \simeq 3(1 + 1.414)$$
$$= 3 \times 2.414$$
$$= 7.242 \text{ (approx.)}$$

Hence, the required distance between the walls is 7.242 m approximately.

(*ii*) Let CA be the length of the ladder which is leaning against the wall AB. Let C be the point between the two walls AB and ED on the ground. The angle of elevation of A at C is 60º.



Then, ∠ACB = 60º.

The angle of elevation of E at point C is 45º.

Then, ∠ECD = 45º.

Let ED (= h) be the length of the other wall.

Now, ED = h, AB = 90 m,

∠ABC = ∠EDC = 90º, AC = EC = l.

In right ΔABC, we have

$$\sin 60° = \frac{AB}{AC} \quad \Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{90}{l} \quad \Rightarrow \quad l = \frac{180}{\sqrt{3}} \text{ m}$$

In right ΔEDC, we have

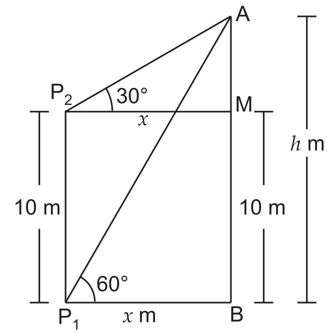
$$\sin 45° = \frac{ED}{EC}$$

$$\Rightarrow \quad \frac{1}{\sqrt{2}} = \frac{h}{\left(\frac{180}{\sqrt{3}}\right)} \quad \Rightarrow \quad h = \frac{180}{\sqrt{6}} = 73.47 \text{ m}$$

Hence, the height the ladder have reached on the second wall is 73.47 m.

29. (*i*) Let AB be a vertical tower standing on the horizontal ground P_1B, P_1 being a point on the ground such that $\angle BP_1A = 60°$.

P_2 is a point 10 m vertically above P_1 such that $\angle AP_2M = 30°$ where P_2M is a horizontal line segment through P_2 cutting AB at M.



It is given that $P_1P_2 = 10$ m

∴ BM = 10 m

Let AB = h m.

Then AM = (h – 10) m. Let $P_1B = P_2M = x$ m.

Now, from ΔAP_1B, we have

$$\tan 60° = \frac{h}{x}$$

$$\Rightarrow \quad \sqrt{3} = \frac{h}{x}$$

$$\Rightarrow \quad h = \sqrt{3}x$$

$$\Rightarrow \quad x = \frac{h}{\sqrt{3}} \qquad \ldots(1)$$

From ΔAP_2M,

$$\tan 30° = \frac{AM}{P_2M} = \frac{1}{\sqrt{3}} = \frac{h-10}{x}$$

$$\Rightarrow \quad x = \sqrt{3}(h-10) \qquad \ldots(2)$$

From (1) and (2), we have

$$\frac{h}{\sqrt{3}} = \sqrt{3}(h-10)$$

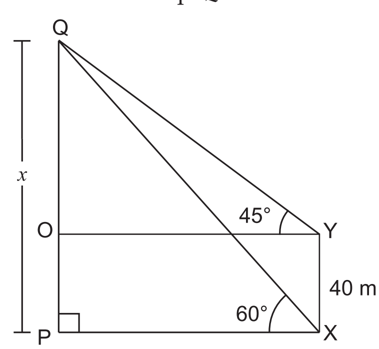
$$\Rightarrow \quad h = 3h - 30$$

$$\Rightarrow \quad 2h = 30$$

$$\Rightarrow \quad h = 15$$

Hence, the required height of the tower is 15 m.

(*ii*) Let height of the tower PQ be x m. The angle of elevation of the top Q of tower PQ from a point X is 60º. From a point Y, 40 m vertically above X, the angle of elevation of the top Q of tower is 45º.



Then OP = XY = 40 m, PQ = x m

∴ In right ΔQPX, we have

$$\tan 60° = \frac{QP}{PX}$$

$$\sqrt{3} = \frac{x}{PX}$$

$$PX = \frac{x}{\sqrt{3}}$$

In ΔQOY, we have

$$\tan 45° = \frac{QO}{OY}$$

$$1 = \frac{QO}{OY}$$

$$QO = OY$$

Since Y is vertically above.

Therefore, OY = PX

$$\therefore \quad QO = \frac{x}{\sqrt{3}}$$

$$QP - OP = \frac{x}{\sqrt{3}}$$

$$x - 40 = \frac{x}{\sqrt{3}}$$

$$x - \frac{x}{\sqrt{3}} = 40$$

$$x\left(1 - \frac{1}{\sqrt{3}}\right) = 40$$

$$x = \frac{40\sqrt{3}}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$$

$$= \frac{40\sqrt{3}\left(\sqrt{3}+1\right)}{2}$$

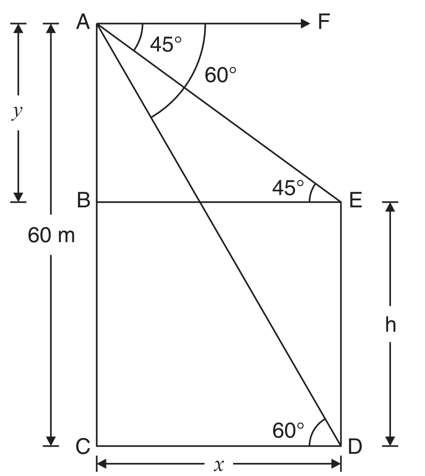
$$= 20\left(3+\sqrt{3}\right)$$

$$= 20 \times 4.732 = 94.64 \text{ m}$$

Height of tower = 94.64 m

Now $\quad PX = \frac{x}{\sqrt{3}} = \frac{94.64}{1.732} = 54.64$ m

30. (*i*) Let AC be the height of the building and ED be the height of the tower. The angle of depression of E at A is 45° and the angle of depression of D at A is 60°.



Then, $\angle FAE = 45°$ and $\angle FAD = 60°$.

Now, AF ∥ BE.

Then, $\angle FAE = \angle AEB = 45°$

Again, AF ∥ CD.

Then, $\angle FAD = \angle ADC = 60°$.

Let CD be the distance between the tower and the building.

Then, $BE = CD = x$, AC = 60 m,

$AB = y$, $\angle ABE = 90° = \angle ACD$

$ED = h$

In right ΔABE, we have

$$\tan 45° = \frac{AB}{BE}$$

$$\Rightarrow \quad 1 = \frac{y}{x}$$

$$\Rightarrow \quad x = y \qquad \ldots (1)$$

In right ΔACD, we have

$$\tan 60° = \frac{AC}{CD}$$

$$\Rightarrow \quad \sqrt{3} = \frac{60}{x}$$

$$\Rightarrow \quad x = \frac{60}{\sqrt{3}}$$

$$\Rightarrow \quad y = \frac{60}{\sqrt{3}} \qquad \text{[Using (1)]}$$

$$\Rightarrow \quad y = 20\sqrt{3} \text{ m}$$

Now, the height of the tower is

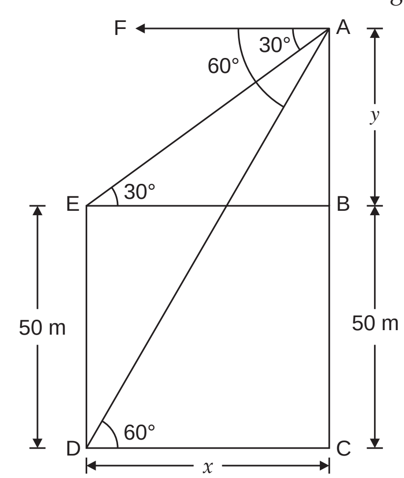
$$h = 60 \text{ m} - 20\sqrt{3}$$

$$= 20(3-\sqrt{3})$$

$$= 25.36 \text{ m}$$

Hence, the height of the tower is 25.36 m.

(*ii*) Let ED (= 50 m) be the height of the building and AC be the height of the tower. Let CD be the horizontal distance between the tower and the building.



The angle of depression of E at A is 30° and the angle of depression of D at A is 60°.

Then, $\angle FAE = 30^\circ$ and $\angle FAD = 60^\circ$.

Now, AF || BE.

Thus, $\angle FAE = \angle AEB = 30^\circ$

Also, AF || CD.

Thus, $\angle FAD = \angle ADC = 60^\circ$

Then, $\angle ABE = 90^\circ = \angle ACD$,

ED = BC = 50 m, AB = y.

CD = BE = x

In right ΔABE, we have

$$\tan 30^\circ = \frac{AB}{BE}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{y}{x} \Rightarrow x = \sqrt{3}\,y \quad \dots (1)$$

In right ΔACD, we have

$$\tan 60^\circ = \frac{AC}{CD}$$

$$\Rightarrow \quad \sqrt{3} = \frac{y+50}{x}$$

$$\Rightarrow \quad \sqrt{3}\,x = y + 50$$

$$\Rightarrow \sqrt{3} \times \sqrt{3}\,y = y + 50 \quad \text{[Using (1)]}$$

$$\Rightarrow \quad y = 25 \text{ m}$$

Now, height of the tower is

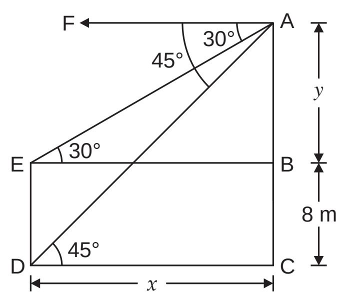
$$AC = AB + BC = y + 50$$
$$= 25 + 50 = 75 \text{ m}.$$

and distance between the building and the tower is

$$x = \sqrt{3}\,y = \sqrt{3} \times 25 \text{ m} = 43.3 \text{ m}$$

Hence, the height of the tower is 75 m and the horizontal distance between the building and the tower is 43.3 m.

(*iii*) Let ED be the height of the building and AC be the height of the multi-storeyed building. Let CD be the distance between the building and the multi-storeyed building.



The angle of depression of E at A is 30°.

Then, $\angle FAE = 30^\circ$

The angle of depression of D at A is 45°.

Then, $\angle FAD = 45^\circ$

Now, AF || BE.

Then, $\angle FAE = \angle AEB = 30^\circ$

Also, AF || CD.

Then, $\angle FAD = \angle ADC = 45^\circ$

Thus, ED = BC = 8 m, AB = y,

CD = BE = x,

$\angle ABE = \angle ACD = 90^\circ$.

In right ΔABE, we have

$$\tan 30^\circ = \frac{AB}{BE}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{y}{x}$$

$$\Rightarrow \quad x = \sqrt{3}\,y$$

In right ΔACD, we have

$$\tan 45^\circ = \frac{AC}{CD}$$

$$\Rightarrow \quad 1 = \frac{y+8}{x}$$

$$\Rightarrow \quad x = y + 8$$

$$\Rightarrow \quad \sqrt{3}\,y = y + 8$$

$$\Rightarrow \quad y = \frac{8}{\sqrt{3}-1} = \frac{8(\sqrt{3}+1)}{2}$$

$$= 4(\sqrt{3} + 1) \text{ m}$$

Now, $AC = AB + BC = y + 8 \text{ m}$

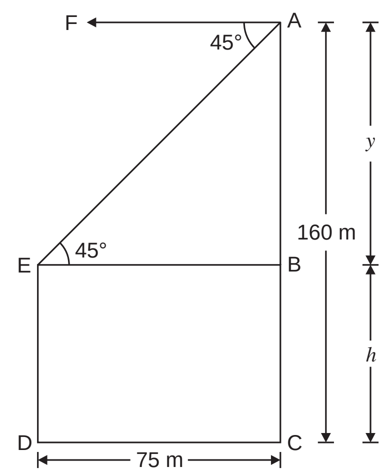
$$= 4(\sqrt{3} + 1) + 8 = 4(\sqrt{3} + 1 + 2)$$
$$= 4(3 + \sqrt{3}) \text{ m}$$

and $x = \sqrt{3}\,y = \sqrt{3} \times 4(\sqrt{3} + 1)$

$$= 4(3 + \sqrt{3})$$

Hence, the height of the multi-storeyed building is $4(3 + \sqrt{3})$ m and the distance between the buildings is $4(3 + \sqrt{3})$ m.

31. (*i*) Let ED be the height of the first tower and AC (= 160 m) be the height of the second tower. Let C and D be the horizontal distance between the two towers.



The angle of depression of E at A is 45°.

Then, $\angle FAE = 45^\circ$.

Now, AF || BE.

Thus, $\angle FAE = \angle AEB = 45^\circ$

Also, $AB = y$, $AC = 160$ m,

$BC = h = ED$, $CD = BE = 75$ m

$\angle ABE = 90°$

In right ΔABE, we have

$$\tan 45° = \frac{AB}{BE}$$

$$\Rightarrow \quad 1 = \frac{y}{75 \text{ m}}$$

$$\Rightarrow \quad y = 75 \text{ m}$$

Now, $ED = BC$

$$\Rightarrow \quad h = AC - AB = 160 \text{ m} - y$$
$$= 160 \text{ m} - 75 \text{ m}$$
$$= 85 \text{ m}$$

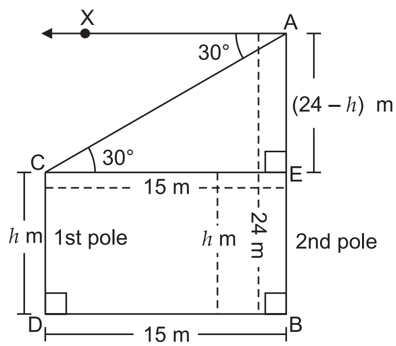
Hence, the height of the first tower is 85 m.

(*ii*) Let CD and AB be two poles standing vertically on the horizontal ground DB. The distance between these two poles is DB = 15 m.

Let AX be the horizontal ray through A so that the angle of depression of the top of the smaller first pole CD is $\angle CAX = \angle ACE = 30°$ where CE is $\perp$ to AB.

$\therefore \quad DB = CE = 15$ m

Let the height of the 1st smaller pole be h m so that $CD = h$ m and that of the taller pole AB be 24 m.



$\therefore \quad AE = AB - EB = AB - CD = (24 - h)$ m.

$\therefore$ From ΔACE, we have

$$\tan 30° = \frac{AE}{CE} = \frac{24-h}{15}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{24-h}{15}$$

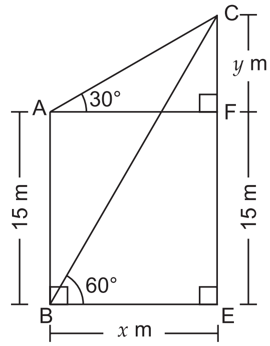
$$\Rightarrow \quad 24 - h = \frac{15}{\sqrt{3}} = 5\sqrt{3}.$$

$$\Rightarrow \quad h = 24 - 5\sqrt{3}$$
$$= 24 - 5 \times 1.732$$
$$= 24 - 8.66$$
$$= 15.34$$

Hence, the required height of the 1st tower is 15.34 m.

32. Let AB be the vertical building of height 15 m and CE be the vertical tower, both standing on the horizontal ground BE so that $\angle ABE = \angle CEB = 90°$. Let AF be the horizontal line through A cutting CE at F so that $\angle CFA = 90°$. Let $BE = x$ m and $CF = y$ m.

Given that $\angle CBE = 60°$ and $\angle CAF = 30°$.



Now, from ΔCAF, we have

$$\tan 30° = \frac{CF}{AF}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{y}{x}$$

$$\Rightarrow \quad x = \sqrt{3}\,y \quad \ldots(1)$$

From ΔCBE, we have

$$\tan 60° = \frac{CE}{BE}$$

$$\Rightarrow \quad \sqrt{3} = \frac{15+y}{x}$$

$$\Rightarrow \quad 15 + y = \sqrt{3}\,x$$

$$\Rightarrow \quad 15 + y = \sqrt{3} \cdot \sqrt{3}y \qquad \text{[From (1)]}$$

$$\Rightarrow \quad 15 + y = 3y$$

$$\Rightarrow \quad 2y = 15$$

$$\Rightarrow \quad y = 7.5 \qquad \ldots(2)$$

$\therefore$ From (1) and (2), we have

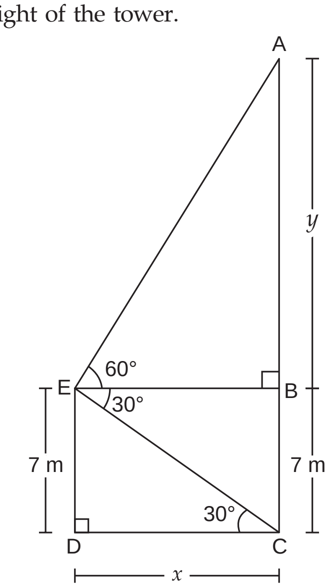
$$x = \sqrt{3} \times 7.5$$
$$= 1.732 \times 7.5$$
$$= 12.99$$

and

$$CE = CF + FE$$
$$= y + 15$$
$$= 7.5 + 15$$
$$= 22.5 \text{ m}$$

Hence, the required height of the tower is CE = 22.5 m and the distance between the building and the tower is 12.99 m.

33. (*i*) Let ED (= 7 m) be the height of the building and AC be the height of the tower.



The angle of elevation of the point A at E is 60°.

Then, $\angle AEB = 60°$

The angle of depression of the point C at E is 30°.

Then, $\angle BEC = 30°$

Now, $EB \parallel CD$.

Thus, $\angle BEC = \angle ECD = 30°$.

Also, $AB = y$, $\angle ABE = \angle EDC = 90°$,

$CD = BE = x$, $ED = BC = 7$ m

In right ΔABE, we have

$$\tan 60° = \frac{AB}{BE}$$

$$\Rightarrow \quad \sqrt{3} = \frac{y}{x} \Rightarrow x = \frac{y}{\sqrt{3}} \quad \ldots (1)$$

In right ΔEDC, we have

$$\tan 30° = \frac{ED}{CD}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{7 \text{ m}}{x}$$

$$\Rightarrow \quad x = 7\sqrt{3}$$

$$\Rightarrow \quad \frac{y}{\sqrt{3}} = 7\sqrt{3}$$

$$\Rightarrow \quad y = 21 \text{ m} \quad \ldots (2)$$

Now, AC = height of the tower

$\Rightarrow$ AC = AB + BC

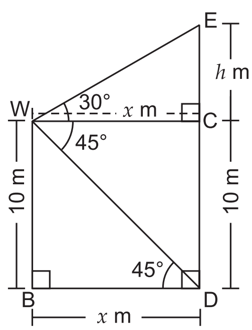
= y + 7 m

= 21 m + 7 m [Using (2)]

= 28 m

Hence, the height of the tower is 28 m.

(*ii*) Let W be the window, 10 m high, of a building, above the horizontal ground BD, in a street.



Let ED be the height of another building on the other side of the street BD. Let WC be the horizontal line through W, cutting ED at C.

Then WB = CD = 10 m.

Let BD = WC = x m.

It is given that $\angle EWC = 30°$ and $\angle DWC = 45°$.

Let EC = h m.

Also, $\angle WBD = \angle BDC = \angle ECW = 90°$.

To find the height ED of the 2nd building.

Now, from ΔEWC, we have

$$\tan 30° = \frac{EC}{WC}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{x}$$

$$\therefore \quad x = h\sqrt{3} \quad \ldots (1)$$

Again, from ΔWBD, we have

$$\tan 45° = \frac{WB}{BD}$$

$$\Rightarrow \quad 1 = \frac{10}{x}$$

$$\Rightarrow \quad x = 10 \quad \ldots (2)$$

$\therefore$ From (1) and (2),

$$h = \frac{10}{\sqrt{3}} = \frac{10\sqrt{3}}{3}$$

$$= \frac{10 \times 1.732}{3}$$

$$\simeq 10 \times 0.5773$$

$$= 5.773$$

Hence, the required height of the 2nd building is

DE = DC + CE = 10 m + 5.773 m = 15.773 m.

(*iii*) (*a*) Let ED (= 60 m) be the height of the building and AC be the height of the lighthouse. The distance between the building and the lighthouse is x metres.

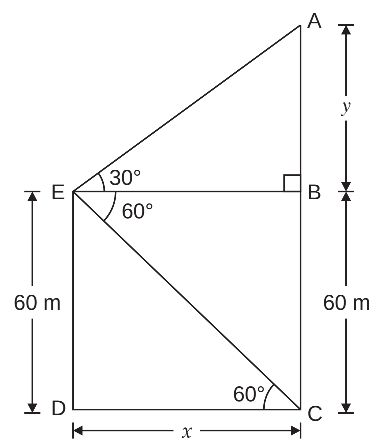
The angle of elevation of the top of the lighthouse A at E is 30°.

Then, $\angle AEB = 30°$

The angle of depression of the bottom of the lighthouse C at E is 60°.

Then, $\angle BEC = 60°$

Now, $BE \parallel CD$.



Then, $\angle BEC = \angle ECD = 60°$.

Also, $ED = BC = 60$ m, $CD = BE = x$,

$\angle ABE = \angle EDC = 90°$

Let AB (= y) be the difference of height between the building and the lighthouse.

In right ΔABE, we have

$$\tan 30° = \frac{AB}{BE}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{y}{x}$$

$$\Rightarrow \quad x = \sqrt{3}\,y \quad \ldots (1)$$

In right ΔEDC, we have

$$\tan 60° = \frac{ED}{CD}$$

$$\Rightarrow \quad \sqrt{3} = \frac{60 \text{ m}}{x} \quad \Rightarrow \quad x = \frac{60 \text{ m}}{\sqrt{3}}$$

$$\Rightarrow \quad \sqrt{3}\,y = \frac{60 \text{ m}}{\sqrt{3}} \quad \Rightarrow \quad y = 20 \text{ m}$$

Hence, the difference between the height of the lighthouse and the building is 20 m.

(*b*) From equation (1),

$$x = \sqrt{3}\,y = \sqrt{3} \times 20 \text{ m}$$
$$= 1.732 \times 20 \text{ m} = 34.64 \text{ m}$$

Hence, the distance between the lighthouse and the building is 34.64 m.

(*iv*) Let CD be the water level. Let ED (= 12 m) be the height of the deck of the ship above the water level.

Let AC be the height of the cliff. The distance between the man and the cliff is x metres.

The angle of elevation of the top of the cliff A at E is 60°.

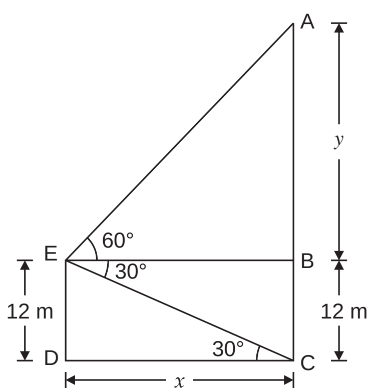
$\therefore \quad \angle AEB = 60°$

The angle of depression of the bottom of the cliff C at E is 30°.

$\therefore \quad \angle BEC = 60°$

Now, BE ∥ CD.

Then, $\angle BEC = \angle ECD = 30°$.



Also, $BE = CD = x$, $ED = BC = 12$ m,

$AB = y$, $\angle ABE = \angle EDC = 90°$

In right ΔABE, we have

$$\tan 60° = \frac{AB}{BE} \quad \Rightarrow \quad \sqrt{3} = \frac{y}{x}$$

$$\Rightarrow \quad x = \frac{y}{\sqrt{3}} \qquad \dots (1)$$

In right ΔEDC, we have

$$\tan 30° = \frac{ED}{CD}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{12 \text{ m}}{x}$$

$$\Rightarrow \quad x = 12\sqrt{3}$$

$$\Rightarrow \quad \frac{y}{\sqrt{3}} = 12\sqrt{3} \qquad \text{[Using equation (1)]}$$

$$\Rightarrow \quad y = 36 \text{ m}$$

Now, height of the cliff = AC

= AB + BC

= y + 12 m

= 36 m + 12 m

= 48 m

In equation (1), the distance between the ship and the cliff is

$$x = \frac{y}{\sqrt{3}} = \frac{36}{\sqrt{3}} \text{ m} \quad [\text{Using } y = 36 \text{ m}]$$

$$= \frac{36 \text{ m}}{1.732} = 20.785 \text{ m}$$

Hence, the distance of the cliff from the ship is 20.785 m and the height of the cliff is 48 m.

34. (*i*) Let EF be the surface of the lake and A be the point of observation 20 m above the lake such that AB = 20 m.

The distance of the cloud above sea level is equal to the reflection of cloud below sea level.

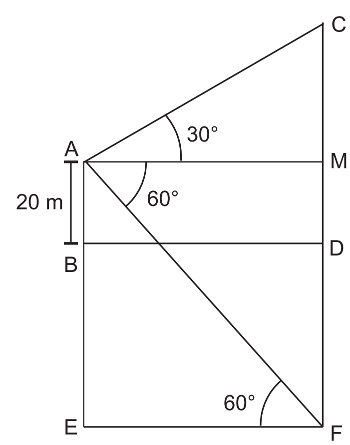
$\therefore \quad CD = DF$

Now, In ΔAEF

$$\tan 60° = \frac{AE}{EF}$$

$$\sqrt{3} = \frac{20+x}{EF}$$

$$EF = \frac{20+x}{\sqrt{3}}$$



In ΔACM, we have

$$\tan 30° = \frac{CM}{AM}$$

$$\frac{1}{\sqrt{3}} = \frac{CM}{EF} \qquad (\because AM = EF)$$

$$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}CM}{20+x}$$

$$3CM = 20 + x$$

We know $CD = DF$

$\therefore$ $CD = x$

$$CM + MD = x$$

$$\frac{20+x}{3} + 20 = x$$

$$20 + x + 60 = 3x$$

$$80 = 2x$$

$$x = 40 \text{ m}$$

$\Rightarrow$ $CM = \frac{20+x}{3} = 20$

Now we have

In ΔACM,

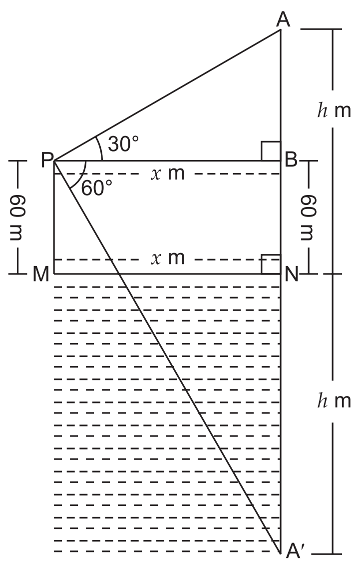
$$\sin 30^\circ = \frac{CM}{AC}$$

$$\frac{1}{2} = \frac{20}{AC}$$

$$AC = 40 \text{ m}$$

Distance of cloud from point A is 40 m.

(*ii*) Let MN be the surface of the lake and let P be a point of observation, 60 m above the lake vertically above M such that PM = 60 m. Let A be the position of the cloud and A′ be its reflection in the lake. Let the height of the cloud above the lake be h m.



Then AN = A′N = h m [$\because$ object distance = image distance on a plane reflecting surface]

We draw $PB \perp AA'$.

Then $\angle ABP = \angle A'BP = \angle ANM = \angle A'NM = 90^\circ$,

$\angle APB = 30^\circ$ and $\angle BPA' = 60^\circ$ [Given]

Now, Let PB = MN = x m

Then, from ΔAPB, we have

$$\tan 30^\circ = \frac{AB}{PB} = \frac{1}{\sqrt{3}} = \frac{h-60}{x}$$

$\Rightarrow$ $x = (h - 60)\sqrt{3}$ …(1)

From ΔPBN, we have

$$\tan 60^\circ = \frac{A'B}{PB} = \frac{h+60}{x}$$

$\Rightarrow$ $\sqrt{3}\,x = h + 60$ …(2)

$\therefore$ From (1) and (2), we have

$$(h - 60)3 = h + 60$$

$\Rightarrow$ $3h - h = 60 + 180$

$\Rightarrow$ $2h = 240$

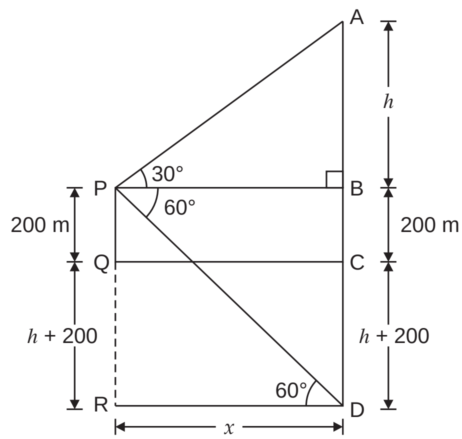
$\Rightarrow$ $h = \frac{240}{2} = 120$

Hence, the required height of the cloud from the surface of water is 120 m.

(*iii*) Let QC be the surface of the lake and P be the point of observation 200 m above the lake such that PQ = 200 m.

Let A be the position of the cloud and D be its reflection in the lake. Let AC be the height of the cloud above the lake.

Then, $CD = AC = AB + BC = h + 200$



Draw $PB \perp AD$.

Then, $\angle ABP = \angle PBD = 90^\circ$.

The angle of elevation of the point A at the point P is 30°.

Then, $\angle APB = 30^\circ$.

The angle of depression of the point D at the point P is 60°.

Thus, $\angle BPD = 60^\circ$.

Now, $BP \parallel DR$.

Then, $\angle BPD = \angle PDR = 60^\circ$.

Also, $AB = h$, $BC = 200$ m, $CD = h + 200$,

$PQ = 200$, $QR = h + 200$

$DR = CQ = BP = x$, $\angle DBP = 90^\circ = \angle PRD$.

In right ΔABP, we have

$$\tan 30^\circ = \frac{AB}{BP} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{x} \Rightarrow x = \sqrt{3}\,h$$

In right ΔPRD, we have

$$\tan 60^\circ = \frac{PR}{DR}$$

$$\Rightarrow \quad \sqrt{3} = \frac{PQ + QR}{x}$$

$$\Rightarrow \quad \sqrt{3}\,x = 200 + h + 200$$

$$\Rightarrow \quad \sqrt{3}\,x = h + 400$$

$$\Rightarrow \sqrt{3} \times \sqrt{3}\,h = h + 400$$

$$\Rightarrow \quad h = 200 \text{ m}$$

Now, $\quad AC = AB + BC$

$= h + 200$ m

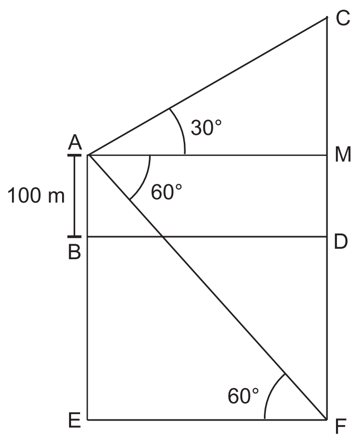
$= 200$ m $+ 200$ m

$= 400$ m

Hence, the height of the could from the surface of the lake is 400 m.

(*iv*) Let EF be the surface of the lake. From a point 100 m above the lake (A), angle of elevation of helicopter in 30° and angle of depression of reflection of the helicopter in the lake is 60°.

Let shadow be x m below sea level



$DF = x$

$BE = DF = x$ m

$CD = DF$

$CD = x$

$\therefore$ In ΔAEF, we have

$$\tan 60° = \frac{AE}{EF}$$

$$\sqrt{3} = \frac{100 + x}{EF}$$

$$EF = \frac{100 + x}{\sqrt{3}}$$

In ΔACM, we have

$$\tan 30° = \frac{CM}{AM}$$

$$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}CM}{100 + x} \qquad [\because AM = EF]$$

$$3CM = 100 + x$$

We know

$CD = DF$

$CM + MD = DF$

$$\frac{100 + x}{3} + 100 = x$$

$$100 + x + 300 = 3x$$

$$2x = 400$$

$$x = 200$$

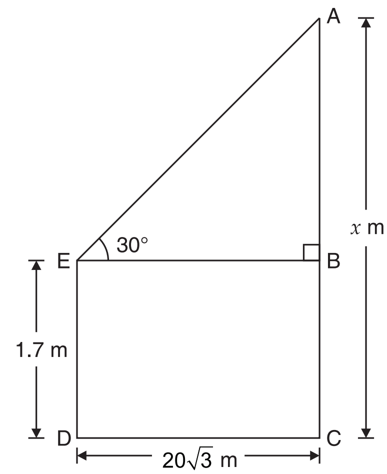
$$CM = \frac{100 + x}{3} = 100$$

Height of helicopter above lake = CM + MD

= 100 + 100

= 200 m

35. (*i*) Let ED be the height of the man and AC be the height of the tower. C is the foot of the tower. The distance of the man from the tower is $20\sqrt{3}$ m.



The AB (= h) be the difference in the height of the man and the tower.

The angle of elevation of the top of the tower A at the point E be 30°.

Then, $\quad \angle AEB = 30°$

Now, $\quad AC = x$ m, $BC = 1.7$ m,

$CD = 20\sqrt{3}$ m, $\angle ABE = 90°$.

$BE = CD = 20\sqrt{3}$ m

$AB = x$ m $- 1.7$ m $= (x - 1.7)$ m

In right ΔABE, we have

$$\tan 30° = \frac{AB}{20\sqrt{3}}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{AB}{20\sqrt{3}}$$

$$\Rightarrow \quad AB = 20 \text{ m}$$

$$\Rightarrow \quad x = (20 + 1.7) \text{ m}$$

$$\Rightarrow \quad x = 21.7 \text{ m}$$

Hence, the height of the tower is 21.7 m.

(*ii*) Let E be the eye of the observer EG, 1.5 m tall, standing vertically on the horizontal ground GB, 30 m away from a verticaly chimney AB, A being the top of the chimney. We draw EC $\perp$ AB, so that EC = GB = 30 m. Also, EG = 1.5 m, the height of the observer.

$\angle$AEC = 60° (given).

Let AC = h m.

From ΔAEC, we have

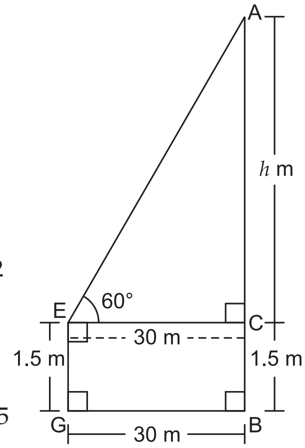
$$\tan 60° = \frac{AC}{EC}$$

$$\Rightarrow \quad \sqrt{3} = \frac{h}{30}$$

$$\Rightarrow \quad h = 30\sqrt{3}$$

$$= 30 \times 1.732$$

$$= 51.96$$



$$\therefore \quad AB = AC + CB$$

$$= h + 1.5$$

$$= 51.96 + 1.5$$

$$= 53.46$$

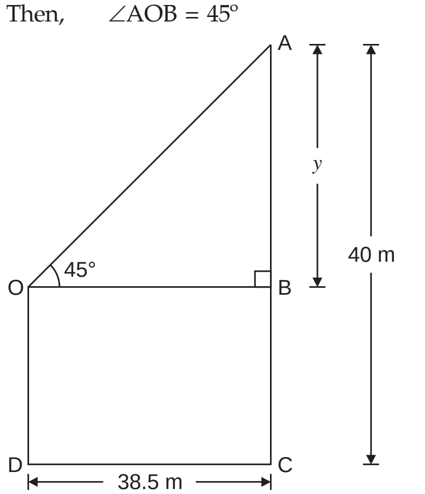
Hence, the required height of the chimney is 53.46 m.

(*iii*) Let OD be the height of the man and AC be the height of the tower. Let C be the foot of the tower. Then, CD is the distance between the man and the tower.

Let AB (= y) be the difference between the height of the man and the tower.

The angle of elevation of the top of the building A at the point O is 45º.

Then, $\angle$AOB = 45º



Now, CD = OB = 38.5 m, AB = y,

AC = 40 m, $\angle$ABO = 90º.

In right ΔABO, we have

$$\tan 45º = \frac{AB}{BO}$$

$$\Rightarrow \quad 1 = \frac{y}{38.5 \text{ m}}$$

$$\Rightarrow \quad y = 38.5 \text{ m}$$

Thus, the height of the man is

$$OD = BC = AC - AB$$

$$= 40 \text{ m} - y = 40 \text{ m} - 38.5 \text{ m} = 1.5 \text{ m}$$

Hence, the man is 1.5 m tall from the ground.

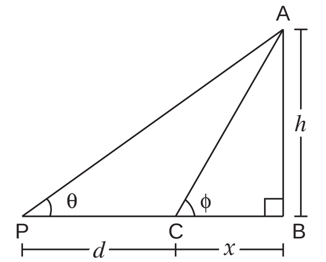
For Standard Level

36. Let AB (= h metres) be the height of the tower.

Let P be the point from the foot of the tower when the angle of elevation is θ.

Then, AB = h, CB = x

$\angle$APB = θ.



Let C be the point when a distance of d is moved from the point P. It is the point at which the angle of elevation is ϕ.

Then, PB = d + CB = $d + x$,

$\angle$ACB = ϕ

and $\angle$ABC = $\angle$ABP = 90º.

In right ΔABC, we have

$$\tan \phi = \frac{h}{x}$$

$$\Rightarrow \quad x = \frac{h}{\tan \phi}$$

$$\Rightarrow \quad x = h \cot \phi \quad \dots (1)$$

In right ΔABP, we have

$$\tan \theta = \frac{h}{PB}$$

$$h = PB \tan \theta$$

$$\Rightarrow \quad h = (d + x) \tan \theta$$

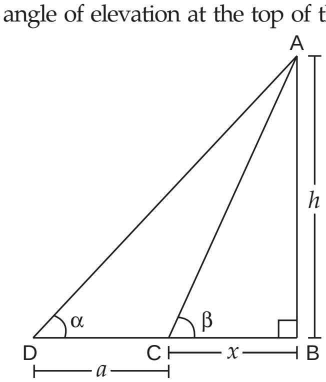
$$\Rightarrow \quad h \cot \theta = d + h \cot \phi$$

$$\Rightarrow \quad h(\cot \theta - \cot \phi) = d \quad \text{[Using (1)]}$$

$$\Rightarrow \quad h = \frac{d}{\cot \theta - \cot \phi}$$

Hence, the height of the tower is $\dfrac{d}{\cot \theta - \cot \phi}$.

37. Let AB be the height of the tower. At the point D on the ground, the angle of elevation at the top of the tower is α.



Then, $AB = h$, $\angle ADB = \alpha$.

Let C be the point from D such that the angle of elevation at the top of the tower is β.

Then, $CD = a$, $CB = x$, $\angle ACB = \beta$,

$\angle ABC = \angle ABD = 90°$

and $DB = a + x$

In right ΔABC, we have

$$\frac{AB}{BC} = \tan\beta$$

$$\Rightarrow \quad \frac{h}{x} = \tan\beta$$

$$\Rightarrow \quad x = \frac{h}{\tan\beta} \quad \ldots (1)$$

In right ΔABD, we have

$$\frac{AB}{BD} = \tan\alpha \quad \Rightarrow \quad \frac{h}{a+x} = \tan\alpha$$

$$\Rightarrow \quad \frac{h}{a + \frac{h}{\tan\beta}} = \tan\alpha \quad \text{[Using (1)]}$$

$$\Rightarrow \quad h = \tan\alpha\left(a + \frac{h}{\tan\beta}\right)$$

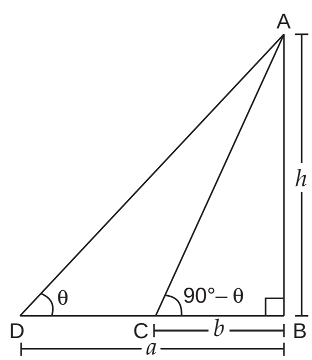
$$\Rightarrow \quad h = a\tan\alpha + \frac{h\tan\alpha}{\tan\beta}$$

$$\Rightarrow \quad h = \frac{a\tan\alpha\tan\beta + h\tan\alpha}{\tan\beta}$$

$$\Rightarrow \quad h(\tan\beta - \tan\alpha) = a\tan\alpha\tan\beta$$

$$\Rightarrow \quad h = \frac{a\tan\alpha\tan\beta}{\tan\beta - \tan\alpha}$$

38. Let AB be the height of the tower. Let C be the point from the foot of the tower at which the angle of elevation is $90° - \theta$.



Then, $AB = h$, $BC = b$, $\angle ACB = 90° - \theta$.

Let D be the point from the foot of the tower at which the angle of elevation is θ.

Then, $\angle ADB = \theta$, $\angle ABC = \angle ABD = 90°$, $BD = a$

In right ΔABC, we have

$$\frac{AB}{BC} = \tan(90° - \theta)$$

$$\Rightarrow \quad \frac{h}{b} = \cot\theta \ldots (1) \text{[Using } \tan(90° - \theta) = \cot\theta\text{]}$$

In right ΔABD, we have

$$\frac{AB}{BD} = \tan\theta \quad \Rightarrow \quad \frac{h}{a} = \tan\theta \quad \ldots (2)$$

From equation (1) and equation (2), we have

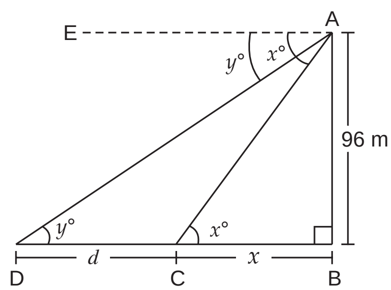
$$b\cot\theta = a\tan\theta$$

$$\Rightarrow \quad \frac{b}{\tan\theta} = a\tan\theta \quad \left[\text{Using } \tan\theta = \frac{1}{\cot\theta}\right]$$

$$\Rightarrow \quad \tan^2\theta = \frac{b}{a}$$

$$\Rightarrow \quad \tan\theta = \sqrt{\frac{b}{a}}$$

39. Let AB be the height of the building. The angles of depressions of the two vehicles at the point D and C are $y°$ and $x°$ respectively.



$\angle EAC = x°$ and $\angle EAD = y°$.

Now, $AE \parallel BC$.

Then, $\angle ACB = \angle EAC = x°$

and $\angle EAD = \angle ADB = y°$.

Thus, $AB = 96$ m

$CD = d$

$BC = x$

$\angle ABC = \angle ABD = 90°$

In right ΔABC, we have

$$\tan x° = \frac{AB}{CB}$$

$$\Rightarrow \quad \frac{3}{4} = \frac{96 \text{ m}}{CB}$$

$$\Rightarrow \quad CB = 128 \text{ m}$$

$$\Rightarrow \quad x = 128 \text{ m}$$

In right ΔABD, we have

$$\tan y° = \frac{AB}{DB}$$

$$\Rightarrow \quad \frac{1}{3} = \frac{96 \text{ m}}{d + x}$$

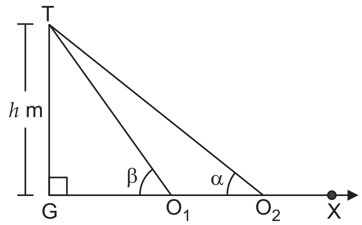
$$\Rightarrow \quad \frac{1}{3}(d + x) = 96 \text{ m}$$

$$\Rightarrow \quad d = 96 \text{ m} \times 3 - x$$

$$= 96 \text{ m} \times 3 - 128 \text{ m} = 160 \text{ m}.$$

Hence, the distance between the two vehicles is 160 m.

40. Let T be the top of the vertical tower TG of height h m and standing on the horizontal ground GX.



Let O_1 and O_2 be two objects on GX, such that $\angle TO_1G = \beta$ and $\angle TO_2G = \alpha$ where $\beta > \alpha$. We have $\angle TGX = 90°$

$\therefore$ From ΔTGO_1, we have

$$\frac{h}{GO_1} = \tan\beta \Rightarrow GO_1 = h\cot\beta \quad \ldots(1)$$

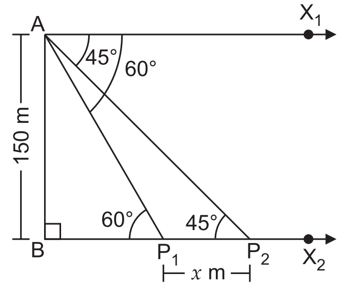
Also, from ΔTGO_2, we have

$$\frac{h}{GO_2} = \tan\alpha \Rightarrow GO_2 = h\cot\alpha \quad \ldots(2)$$

$$\therefore \quad O_1O_2 = GO_2 - GO_1$$
$$= h(\cot\alpha - \cot\beta) \quad \text{[From (1) and (2)]}$$

Hence, the required distance between two objects O_1 and O_2 is $h(\cot\alpha - \cot\beta)$ metres.

41. Let A be the top of the vertical cliff AB standing on the horizontal ground BX_2. Let P_1 and P_2 be the two positions of the boat moving from P_1 to P_2 away from the point B. Let AX_1 be the horizontal through A, so that $\angle X_1AP_1 = \angle AP_1B = 60°$ and $\angle X_1AP_2 = \angle AP_2B = 45°$.



Now, given that $\angle ABX_2 = 90°$ and AB = 150 m.

Let $P_1P_2 = x$ m.

Now, from ΔABP_1, we have

$$\tan 60° = \frac{AB}{BP_1}$$

$$\Rightarrow \quad \sqrt{3} = \frac{150}{BP_1}$$

$$\Rightarrow \quad BP_1 = \frac{150}{\sqrt{3}}$$
$$= 50\sqrt{3}$$
$$= 50 \times 1.732$$
$$= 86.6 \quad \ldots(1)$$

Again, from ΔABP_2, we have

$$\tan 45° = \frac{AB}{BP_2} = \frac{150}{BP_2}$$

$$\Rightarrow \quad 1 = \frac{150}{BP_2}$$

$$\Rightarrow \quad BP_2 = 150 \quad \ldots(2)$$

$$\therefore \quad x = P_1P_2$$
$$= BP_2 - BP_1$$
$$= 150 - 86.6 \quad \text{[From (1) and (2)]}$$
$$= 63.4 \text{ m}$$

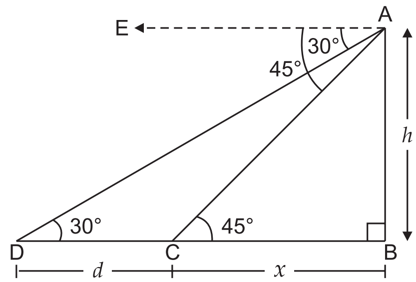
The boat describes a distance of 63.4 m in 2 min, i.e., in $\frac{1}{30}$ h.

$\therefore$ Speed of the boat is $\dfrac{63.4}{\frac{1}{30}}$ m/h = 63.4 × 30 m/h

$$= 1902 \text{ m/h}.$$

Hence, the required speed of the boat is 1902 m/h.

42. (*i*) Let AB be the height of the tower. From the top of the tower at point A, the angle of depression is 30° at the point D and 45° at the point C.



Then, $\angle EAD = 30°$ and $\angle EAC = 45°$.

Now, AE ∥ BD.

Thus, $\angle ADB = \angle EAD = 30°$.

and $\angle ACB = \angle EAC = 45°$.

Let t be the time taken to travel from D to C.

Then, $t = 12$ min.

CD = d, DB = $d + x$,

$\angle ABC = \angle ABD = 90°$,

AB = h = BC = x

Let v be the uniform speed at which the car moves from D to C.

$$\text{Then,} \quad v = \frac{d}{t} \quad \ldots(1)$$

In right ΔABC, we have

$$\frac{AB}{BC} = \tan 45°$$

$$\Rightarrow \quad \frac{AB}{BC} = 1$$

$$\Rightarrow \quad AB = BC$$

$$\Rightarrow \quad h = x \quad \ldots(2)$$

In right ΔABD, we have

$$\frac{AB}{DB} = \tan 30°$$

$$\Rightarrow \quad \frac{h}{d+x} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad \frac{h}{d+h} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad \sqrt{3}\,h = d + h$$

$$\Rightarrow \quad d = (\sqrt{3} - 1)\,h \qquad \dots (3)$$

Let T be the time taken by the car to move from C to B.

$$\text{Then, } T = \frac{x}{v} = \frac{x}{\left(\frac{d}{t}\right)} \qquad \text{[Using (1)]}$$

$$= \frac{xt}{d} = \frac{ht}{(\sqrt{3}-1)\,h} \qquad \text{[Using (2) and (3)]}$$

$$= \frac{12 \text{ min}}{1.732 - 1} = \frac{12 \text{ min}}{0.732}$$

$$= 16.39 \text{ min (approx.)}$$

Hence, the time taken by the car to move from the point C is 16.39 min (approx.).

(*ii*) Let AB be the height of the multi-storeyed building. The angle of depression as observed from A to the point D is α.

Then, ∠EAD = α and AB = h metres.

The angle of depression of C at A is β.

∠EAC = β

Thus, the car moves from D to C as the angle of depression changes.

Now, AE ∥ BD.

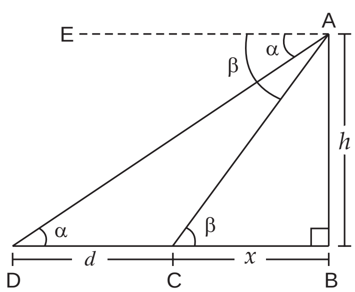
Then, ∠ADB = ∠EAD = α

and ∠ACB = ∠EAC = β

Also, CB = x, DC = d,

∠ABC = ∠ABD = 90°,

DB = $d + x$.



Let t be the time taken by the car to reach from D to C. Thus, the speed of the car is

$$v = \frac{d}{t} \qquad \dots (1)$$

In right ΔABC, we have

$$\tan \beta = \frac{AB}{CB}$$

$$\Rightarrow \quad \sqrt{5} = \frac{h}{x}$$

$$\Rightarrow \quad x = \frac{h}{\sqrt{5}} \qquad \dots (2)$$

In right ΔABD, we have

$$\tan \alpha = \frac{AB}{DB}$$

$$\Rightarrow \quad \frac{1}{\sqrt{5}} = \frac{h}{d+x}$$

$$\Rightarrow \quad d + x = \sqrt{5}\,h$$

$$\Rightarrow \quad d = \sqrt{5}\,h - x = \sqrt{5}\,h - \frac{h}{\sqrt{5}}$$

$$= \frac{4}{\sqrt{5}}\,h \qquad \dots (3)$$

Let T be the time taken by the car to move from C to B.

$$\text{Thus, } T = \frac{x}{v}$$

$$= \frac{x}{\left(\frac{d}{t}\right)} \qquad \text{[Using (1)]}$$

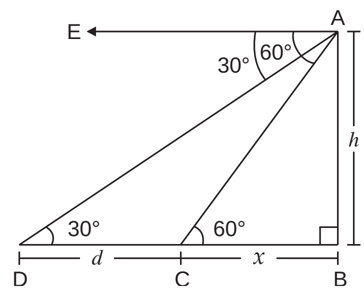
$$= \frac{xt}{d} = \frac{\left(\frac{h}{\sqrt{5}}\right)t}{\frac{4}{\sqrt{5}}h}$$

[Using (2) and (3)]

$$= \frac{t}{4} = \frac{10 \text{ min}}{4} = 2.5 \text{ min.}$$

Hence, the time taken by the car to reach the base of the building is 2.5 minutes.

43. (*i*) Let AB be the height of the cliff. The angle of depression observed by the man from the point A to the boat at the point D is 30°.



Then, AB = h, ∠EAD = 30°.

After 3 minutes, the boat moves from D to C and the angle of depression is observed to be 60°.

Then, ∠EAC = 60°.

Now, AE ∥ BD.

Then, ∠ADB = ∠EAD = 30°

and ∠ACB = ∠EAC = 60°

Also, BC = x, CD = d,

∠ABC = ∠ABD = 90°.

Let t be the time taken by the boat to move from D to C. Then, the speed of the boat is

$$v = \frac{d}{t} \quad \ldots (1)$$

In right ΔABC, we have

$$\tan 60° = \frac{AB}{CB}$$

$$\Rightarrow \quad \sqrt{3} = \frac{h}{x}$$

$$\Rightarrow \quad x = \frac{h}{\sqrt{3}} \quad \ldots (2)$$

In right ΔABD, we have

$$\tan 30° = \frac{AB}{DB}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{d+x}$$

$$\Rightarrow \quad d + x = \sqrt{3}\,h$$

$$\Rightarrow \quad d = \sqrt{3}\,h - \frac{h}{\sqrt{3}}h = \frac{2}{\sqrt{3}}h \quad \ldots (3)$$

Let T be the time taken by the boat to move from C to B.

$$\text{Then,} \quad T = \frac{x}{v} = \frac{x}{\left(\frac{d}{t}\right)} \quad \text{[Using (1)]}$$

$$= \frac{xt}{d} = \frac{\left(\frac{h}{\sqrt{3}}\right)t}{\frac{2}{\sqrt{3}}h} \quad \text{[Using (2) and (3)]}$$

$$= \frac{t}{2} = \frac{3\text{ min}}{2} = 1.5\text{ min}$$

Hence, the time taken by the boat to move from C to B is 1.5 minutes.

(*ii*) It is given that the height of the cliff is 500 m.

Then, $h = 500$ m.

Using equation (3), we have

$$d = \frac{2}{\sqrt{3}} \times h = \frac{2}{\sqrt{3}} \times 500\text{ m} = \frac{1000\text{ m}}{\sqrt{3}}$$

Now, the speed of the boat is

$$v = \frac{d}{t} = \frac{\left(\frac{1000\text{ m}}{\sqrt{3}}\right)}{3\text{ min}} = \frac{1000}{180 \times \sqrt{3}}\text{ m/s}$$

$$= 3.207\text{ m/s (approx.)}$$

Hence, the uniform speed of the boat is 3.207 m/s (approx.)

44. Let AC (=h) be the height of the lighthouse. Let B and D be the positions of the ships on the opposite sides of the lighthouse. The angles of depression of the points B and D at A are α and β respectively.

Then, ∠EAB = α and ∠FAD = β.

Now, EF ∥ BD.

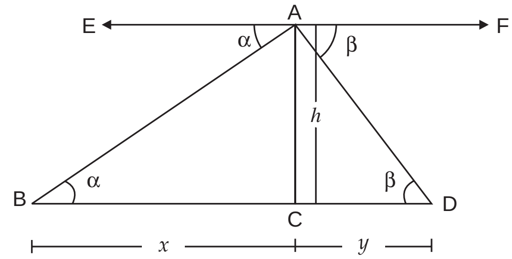
Then, ∠ABC = ∠EAB

⇒ ∠ABC = α

and ∠ADC = ∠FAD

⇒ ∠ADC = β.

Also, AC = h, BC = x and CD = y.



In right ΔACB, we have

$$\tan \alpha = \frac{AC}{BC}$$

$$\Rightarrow \quad \tan \alpha = \frac{h}{x}$$

$$\Rightarrow \quad x = \frac{h}{\tan \alpha}$$

In right ΔACB, we have

$$\tan \beta = \frac{AC}{CD}$$

$$\Rightarrow \quad \tan \beta = \frac{h}{y}$$

$$\Rightarrow \quad y = \frac{h}{\tan \beta}$$

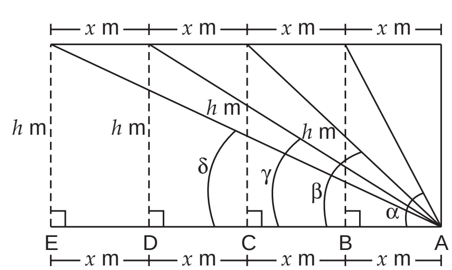
Now, the distance between the two ships is BD.

Thus, BD = $x + y$

$$= \frac{h}{\tan \alpha} + \frac{h}{\tan \beta} = \frac{h(\tan \alpha + \tan \beta)}{\tan \alpha \tan \beta}$$

Hence, the distance between the ships is $\frac{h(\tan \alpha + \tan \beta)}{\tan \alpha \tan \beta}$.

45. Let A be the point of observation. Since the aeroplane is flying with a uniform speed, it will cover equal distances in equal intervals of time. Let it cover x metres after each observation. Suppose the aeroplane is flying at a height of h metres.



Then, $\cot \alpha = \frac{x}{h}$, $\cot \beta = \frac{2x}{h}$,

$$\cot \gamma = \frac{3x}{h}, \cot \delta = \frac{4x}{h}$$

$$\text{LHS} = 3(\cot^2 \beta - \cot^2 \gamma)$$

$$= 3\left[\frac{4x^2}{h^2} - \frac{9x^2}{h^2}\right] = \frac{-15x^2}{h^2} \quad \ldots (1)$$

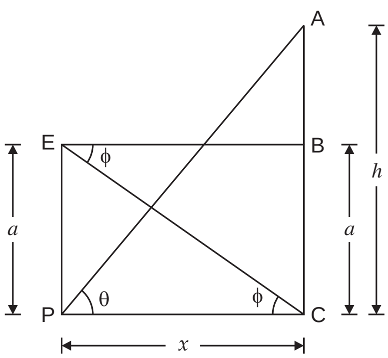
$$\text{RHS} = \cot^2 \alpha - \cot^2 \delta$$

$$= \frac{x^2}{h^2} - \frac{16x^2}{h^2} = \frac{-15x^2}{h^2} \quad \ldots (2)$$

From (1) and (2), we get

$$3(\cot^2 \beta - \cot^2 \gamma) = \cot^2 \alpha - \cot^2 \delta$$

46. Let AC be the height of the tower. The angle of elevation of the top of the tower at P on the ground is θ.



Then, $\angle APC = \theta$.

E is the point which is 'a metres' vertically from P. The angle of depression of C at E is ϕ.

Then, $\angle BEC = \phi$.

Now, EB ∥ CP.

Then, $\angle BEC = \angle ECP = \phi$.

Also, $EP = BC = a$, $AC = h$,

$\angle ACP = \angle EPC = 90°$, $CP = x$

In right ΔACP, we have

$$\tan \theta = \frac{AC}{CP}$$

$$\Rightarrow \quad \tan \theta = \frac{h}{x}$$

$$\Rightarrow \quad x = \frac{h}{\tan \theta} \quad \ldots (2)$$

In right ΔEPC, we have

$$\tan \phi = \frac{EP}{CP} \Rightarrow \tan \phi = \frac{a}{x}$$

$$\Rightarrow \quad x = \frac{a}{\tan \phi}$$

$$\Rightarrow \quad x = a \cot \phi \quad \left[\text{Using } \cot \phi = \frac{1}{\tan \phi}\right]$$

$$\Rightarrow \quad \frac{h}{\tan \theta} = a \cot \phi \quad [\text{Using (1)}]$$

$$\Rightarrow \quad h = a \tan \theta \cot \phi$$

Hence, the height of the tower is $a \tan \theta \cot \phi$ metres.

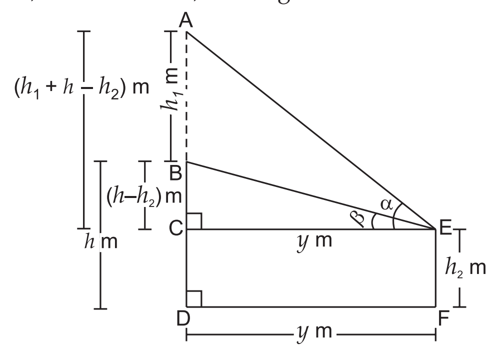
47. Let AB (= h_1 m) be the marble statue on pedestal BD.

Let BD, the height of pedestal = h m.

Let E be the point of observation at a height of h_2 m above the ground.

Then, $EF = h_2$ m.

From E, draw EC ∥ FD, meeting AD at C.



The angle of elevation of the top A of the statue AB at E is α and the angle of elevation of the bottom B of the statue AB at E is β.

i.e. $\angle AEC = \alpha$ and $\angle BEC = \beta$.

Let $DF = y$ metres.

Then, $CE = DF = y$ metres

$BC = BD - CD = BD - EF = (h - h_2)$ m

and $AC = AB + BC = (h_1 + h - h_2)$ m

In rt. ΔACE, $\tan \alpha = \dfrac{AC}{CE}$

$$\Rightarrow \quad \tan \alpha = \frac{h_1 + h - h_2}{y}$$

$$\Rightarrow \quad y = \frac{h_1 + h - h_2}{\tan \alpha} \quad \ldots (1)$$

In rt. ΔBCE, $\tan \beta = \dfrac{BC}{CE}$

$$\Rightarrow \quad \tan \beta = \frac{h - h_2}{y}$$

$$\Rightarrow \quad y = \frac{h - h_2}{\tan \beta} \quad \ldots (2)$$

On equating the values of y from (1) and (2), we get

$$\frac{h_1 + h - h_2}{\tan \alpha} = \frac{h - h_2}{\tan \beta}$$

Solving this equation, we get

$$h = \frac{(h_1 - h_2)\tan \beta + h_2 \tan \alpha}{\tan \alpha - \tan \beta}$$

Hence, the height of the pedestal is

$$\frac{(h_1 - h_2)\tan \beta + h_2 \tan \alpha}{\tan \alpha - \tan \beta}.$$

48. Let ED be the height of the window from the ground in the street.

Let AC be the height of the another house. Then, the distance between the two houses is x.

The angle of elevation of A at E is 60º.

Then, $\angle AEB = 60°$

Also, the angle of depression of C at E is 45º.

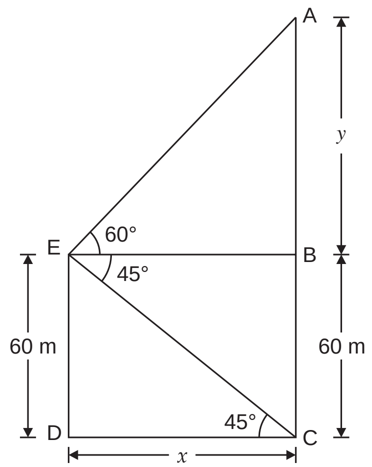
Then, $\angle BEC = 45°$.

Now, $BE \parallel CD$.

Thus, $\angle BEC = \angle DCE = 45°$.

Also, $ED = BC = 60$ m, $CD = BE = x$,

$AB = y$, $\angle ABE = \angle EDC = 90°$.



In right ΔEDC, we have

$$\tan 45° = \frac{ED}{CD}$$

$$\Rightarrow \quad 1 = \frac{60 \text{ m}}{x}$$

$$\Rightarrow \quad x = 60 \text{ m} \qquad \ldots (1)$$

In right ΔABE, we have

$$\tan 60° = \frac{AB}{BE}$$

$$\Rightarrow \quad \sqrt{3} = \frac{y}{x}$$

$$\Rightarrow \quad y = \sqrt{3}\,x$$

$$= \sqrt{3} \times 60 \text{ m} \qquad \text{[Using (1)]}$$

$$= 60\sqrt{3} \text{ m}$$

Now, height of the opposite house

$$= AC = AB + BC$$

$$= y + 60 \text{ m}$$

$$= 60\sqrt{3} \text{ m} + 60 \text{ m}$$

$$= 60(1 + \sqrt{3}) \text{ m}$$

Hence, the height of the opposite house is $60(1 + \sqrt{3})$ metres.

49. Let W_1 and W_2 be the tower and upper windows of a vertical house standing on the horizontal ground G_1G_2. The heights of W_1 and W_2 from G_1 and W_1 respectively are 2 m and 4 m respectively, so that $G_1W_1 = 2$ m and $W_1W_2 = 4$ m.

Let B be the position of the balloon and let W_2X_1 and W_1X_2 be horizontal rays from W_2 and W_1 respectively cutting the vertical line-segment BG_2 at C and D respectively.

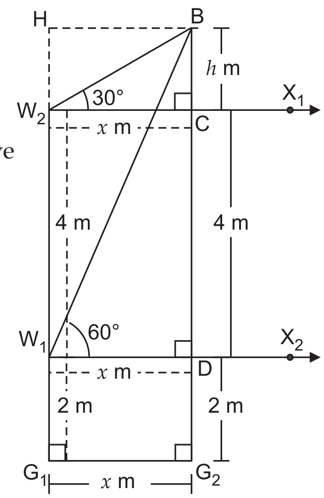
Then $CD = W_2W_1 = 4$ m and $G_2D = W_1G_1 = 2$ m,

$\angle BW_2X_1 = 30°$ and $\angle BW_1X_2 = 60°$. Let $BC = h$ m.

Now, $\angle W_1G_1G_2 = \angle DG_2G_1 = \angle CDW_1 = \angle BCW_2 = 90°$.

Let $G_1G_2 = W_1D = W_2C = x$ m.

$\therefore$ From ΔBW_2C, we have



$$\tan 30° = \frac{BC}{W_2C}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{x}$$

$$\Rightarrow \quad x = h\sqrt{3} \quad \ldots(1)$$

Also, from ΔBW_1D, we have

$$\tan 60° = \frac{BD}{W_1D}$$

$$\Rightarrow \quad \sqrt{3} = \frac{h+4}{x}$$

$$\Rightarrow \quad x = \frac{h+4}{\sqrt{3}} \quad \ldots(2)$$

From (1) and (2), we have

$$h\sqrt{3} = \frac{h+4}{\sqrt{3}}$$

$$\Rightarrow \quad 3h = h + 4$$

$$\Rightarrow \quad 2h = 4$$

$$\Rightarrow \quad h = 2$$

Hence, BC = 2 m.

$$\therefore \quad BG_2 = BC + CD + DG_2$$

$$= (2 + 4 + 2) \text{ m} = 8 \text{ m}$$

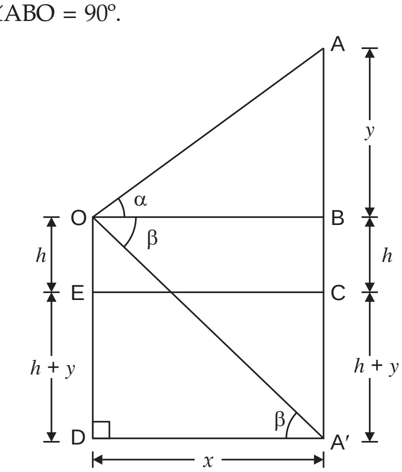
Hence, the required height of the balloon is 8 m.

50. Let EC be the surface of the lake and O be the point of observation vertically above the lake such that $EO = h$ metres. Let A be the position of the cloud and A′ be its reflection in the lake. Let AC be the height of the cloud above the surface of the lake.

Then, $AC = A'C = AB + BC = y + h$

Draw $OB \perp AA'$.

Then, $\angle ABO = 90°$.



Then angle of elevation of point A at the observation point O is α.

Then, $\angle AOB = \alpha$

The angle of depression of point A′ at the observation point O is β.

Then, $\angle BOA' = \beta$

Now, $BO \parallel A'D$.

Then, $\angle BOA' = \angle OA'D = \beta$

Also, $AB = y,\ BC = h,$

$CA' = ED = h + y,\ A'D = BO = x$

$\angle ODA' = 90^\circ$

$$OD = OE + ED = h + h + y = 2h + y$$

In right ΔABO, we have

$$\tan\alpha = \frac{AB}{BO}$$

$$\Rightarrow \tan\alpha = \frac{y}{x}$$

$$\Rightarrow x = \frac{y}{\tan\alpha} \quad \dots (1)$$

In right ODA′, we have

$$\tan\beta = \frac{OD}{A'D}$$

$$\Rightarrow \tan\beta = \frac{2h+y}{x}$$

$$\Rightarrow x\tan\beta = 2h + y$$

$$\Rightarrow \frac{y\tan\beta}{\tan\alpha} = 2h + y \quad \text{[Using (1)]}$$

$$\Rightarrow y\left(\frac{\tan\beta - \tan\alpha}{\tan\alpha}\right) = 2h$$

$$\Rightarrow y = \frac{2h\tan\alpha}{\tan\beta - \tan\alpha}$$

Now, the height of the could above the surface of the lake is

$$AC = AB + BC = h + y$$

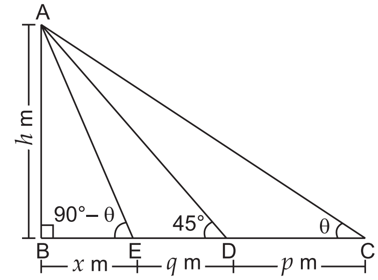
$$= h + \frac{2h\tan\alpha}{\tan\beta - \tan\alpha}$$

$$= \frac{h\tan\beta - h\tan\alpha + 2h\tan\alpha}{\tan\beta - \tan\alpha}$$

$$= \frac{h(\tan\beta + \tan\alpha)}{\tan\beta - \tan\alpha}$$

Hence, the height of the cloud above the surface of the lake is $\dfrac{h(\tan\alpha + \tan\beta)}{\tan\beta - \tan\alpha}$.

51. Let AB (= h metres) be the tower and let C be the point of observation.



Then, $\angle ACB = \theta$

Let D be the second point of observation.

Then, DC = p metres and $\angle ADB = 45^\circ$.

Let E be the third point of observation.

Then, DE = q metres and $\angle AEB = (90^\circ - \theta)$.

Let BE = x metres.

In right ΔABC, we have

$$\tan\theta = \frac{h}{x+q+p} \quad \dots (1)$$

In right ΔABD, we have

$$\tan 45^\circ = \frac{h}{x+q}$$

$$\Rightarrow x + q = h$$

$$\Rightarrow x = h - q \quad \dots (2)$$

From (1) and (2), we get,

$$\tan\theta = \frac{h}{h+p} \quad \dots (3)$$

In right ΔABE,

$$\tan(90^\circ - \theta) = \frac{h}{x}$$

$$\Rightarrow \cot\theta = \frac{h}{x} = \frac{h}{h-q} \quad \text{[Using (2)]} \dots (4)$$

Multiplying equation (3) and equation (4), we get

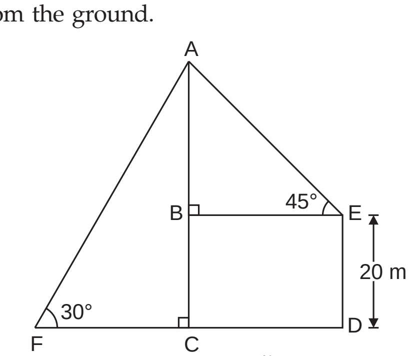
$$\tan\theta \times \cot\theta = \frac{h}{h+p} \times \frac{h}{h-q}$$

$$\Rightarrow (h+p)(h-q) = h^2$$

$$\Rightarrow h^2 + ph - qh - pq = h^2$$

$$\Rightarrow h = \frac{pq}{p-q}$$

52. Let A be the position of the bird from the ground. Let F be the position of the boy and AC be the altitude of the bird from the ground.



The angle of elevation of A at F is 30°.

Thus, $\angle AFC = 30°$ and $\angle ACF = 90°$.

Let E be the position of the girl and ED be the height of the roof. The angle of elevation of A at E is 45°.

Then, $\angle AEB = 45°$ and $\angle ABE = 90°$.

Also, $CD = BE = x$,

$ED = BC = 20$ m,

$AF = 100$ m

In right ΔACF, we have

$$\sin 30° = \frac{AC}{AF}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{AB + BC}{100 \text{ m}}$$

$$\Rightarrow \quad AB = 50 \text{ m} - BC = 50 \text{ m} - 20 \text{ m} = 30 \text{ m}.$$

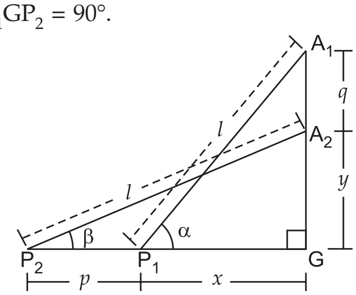
In right ΔABE, we have

$$\sin 45° = \frac{AB}{AE} \quad \Rightarrow \quad AE = 30\sqrt{2} \text{ m} = 42.42 \text{ m}$$

Hence, the distance of the bird from the girl is 42.42 m.

53. Let A_1P_1 be the initial position of the ladder of length l standing at an indication α with the horizontal ground P_1G. Let A_2P_2 be the second position of the ladder making an angle β with P_2G. When the foot of the ladder sides on the ground from the position P_1 to P_2, let its top A_1 slides down the vertical wall A_1G to the position A_2 so that $A_1A_2 = q$ and $P_1P_2 = p$ (given).

It is given that $\angle A_1P_1G = \alpha$, $\angle A_2P_2G = \beta$ and $\angle A_1GP_2 = 90°$.



Let $A_2G = y$ and $P_1G = x$

Now, from ΔA_1P_1G, we have

$$\cos \alpha = \frac{P_1G}{A_1P_1} = \frac{x}{l} \qquad \ldots(1)$$

$$\sin \alpha = \frac{A_1G}{A_1P_1} = \frac{q+y}{l} \qquad \ldots(2)$$

and from ΔA_2P_2G, we have

$$\cos \beta = \frac{P_2G}{A_2P_2} = \frac{x+p}{l} \qquad \ldots(3)$$

and $$\sin \beta = \frac{A_2G}{A_2P_2} = \frac{y}{l} \qquad \ldots(4)$$

$\therefore$ From (1), (2), (3) and (4), we have

$$\frac{\cos\beta - \cos\alpha}{\sin\alpha - \sin\beta} = \frac{\frac{x+p-x}{l}}{\frac{q+y-y}{l}} = \frac{p}{q}$$

Hence, proved.

CHECK YOUR UNDERSTANDING

MULTIPLE-CHOICE QUESTIONS

For Basic and Standard Levels

1. (*c*) **angle of elevation.**

2. (*b*) $\mathbf{20\sqrt{3}}$ **m**

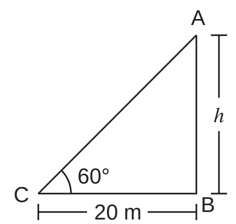
Let AB (= h) be the height of the tower and CB be the distance of the point of observation.

The angle of elevation of A at the point C is 60°.

Then, $\angle ACB = 60°$, $BC = 20$ m

and $\angle ABX = 90°$

In right triangle ABC, we have



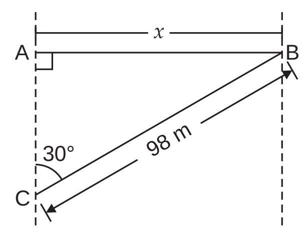
$$\tan 60° = \frac{AB}{BC}$$

$$\Rightarrow \quad \sqrt{3} = \frac{h}{20 \text{ m}}$$

$$\Rightarrow \quad h = 20\sqrt{3} \text{ m}$$

3. (*a*) **49 m**

Let BC be the length of the bridge and AB be the width of the river.



Then, $\angle ACB = 30°$, $\angle BAC = 90°$

$AB = x$ and $BC = 98$ m

In right ΔBAC, we have

$$\sin 30° = \frac{x}{98 \text{ m}}$$

$$\Rightarrow \quad x = \frac{1}{2} \times 98 \text{ m} = 49 \text{ m}$$

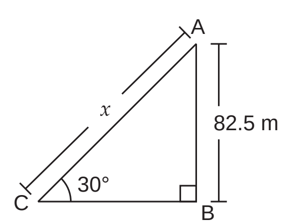
4. (*d*) **165 m**

Let A be the position of the kite from the level ground and AC be the length of string of the kite.

Then, $AC = x$ metres, $AB = 82.5$ m,

$\angle ACB = 30°$, $\angle ABC = 90°$

In right ΔABC, we have



$$\sin 30° = \frac{AB}{AC}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{82.5}{x}$$

$$\Rightarrow \quad x = 165 \text{ m}$$

5. (*a*) **45°**

Let AB be the length of the vertical pole and BC be the length of its shadow.

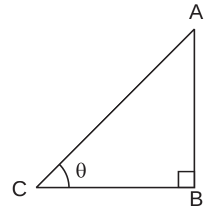
Then, $AB = BC$

and $\angle ABC = 90°$

Let θ be the angle of elevation of the Sun's altitude.

Thus, $\angle ACB = \theta$

In right ΔABC, we have



$$\tan\theta = \frac{AB}{BC}$$

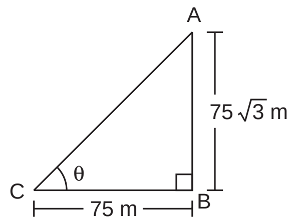
$\Rightarrow$ $\tan\theta = 1$

$\Rightarrow$ $\tan\theta = \tan 45°$

$\Rightarrow$ $\theta = 45°$

6. (*b*) **60°**

Let AB the height of the tower and C be the point which is at a distance of 75 m from the foot of the tower.



Let θ be the angle of elevation of the top of tower A at the point C is θ.

Then, $\angle ACB = \theta$, $\angle ABC = 90°$,

$AB = 75\sqrt{3}$ m and BC = 75 m

In right ΔABC, we have

$$\tan\theta = \frac{AB}{BC}$$

$$\Rightarrow \tan\theta = \frac{75\sqrt{3}\text{ m}}{75\text{ m}}$$

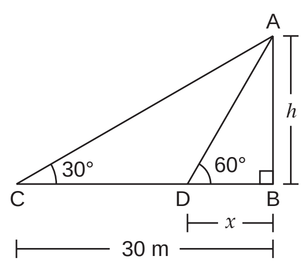
$\Rightarrow$ $\tan\theta = \sqrt{3}$

$\Rightarrow$ $\tan\theta = \tan 60°$

$\Rightarrow$ $\theta = 60°$

7. (*c*) **10 m**

Let AB (= *h*) be the height of the tower and BC be the length of its shadow on the level ground. The angle of elevation of A at C is 30°.



Then, $\angle ACB = 30°$, BC = 30 m, $\angle ABC = 90°$

and AB = *h* metres.

In right ΔABC, we have

$$\tan 30° = \frac{AB}{BC}$$

$$\Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{30\text{ m}}$$

$$\Rightarrow h = \frac{30}{\sqrt{3}}\text{ m}$$

Now, let D be the point on the level ground and is correspond when the Sun's altitude is 60°. Then, the angle of elevation of A at D is 60°. Let BD (= *x*) be the length of the shadow.

Then, $\angle ADB = 60°$, BD = *x* metres

and $\angle ABD = 90°$

In right ΔABD, we have

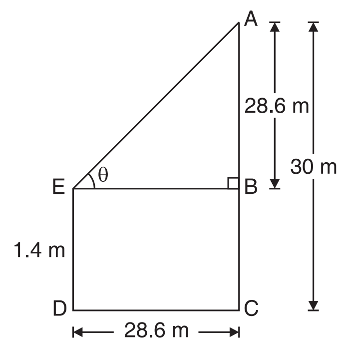
$$\tan 60° = \frac{AB}{BD}$$

$$\Rightarrow \sqrt{3} = \frac{h}{x}$$

$$\Rightarrow x = \frac{h}{\sqrt{3}} = \frac{30}{\sqrt{3}\times\sqrt{3}}\text{ m} = 10\text{ m}$$

8. (*b*) **45°**

Let ED (= 1.4 m) be the height of the observer and AC (= 30 m) be the height of the tower. Let CD = BE be the distance of the observer from the tower. Let θ be the angle of elevation of A at the point E.



Then, CD = BE = 28.6 m,

$\angle AEB = \theta$, ED = BC = 1.4 m

AB = AC – BC = 30 m – 1.4 m = 28.6 m

In right ΔABE, we have

$$\tan\theta = \frac{AB}{BE}$$

$$\Rightarrow \tan\theta = \frac{28.6\text{ m}}{28.6\text{ m}}$$

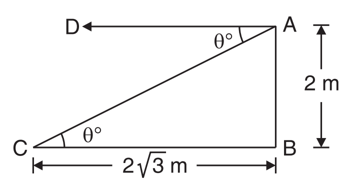
$\Rightarrow$ $\tan\theta = 1$

$\Rightarrow$ $\tan\theta = \tan 45°$

$\Rightarrow$ $\theta = 45°$

9. (*b*) **30°**

Let AD be the line extending from A to D such that AD is parallel to BC.



Let θ be the angle of depression of C at A.

Then, $\angle DAC = \theta$.

Now, $AD \parallel BC$.

Thus, $\angle DAC = \angle ACB = \theta$

Also, $AB = 2$ m,

$BC = 2\sqrt{3}$ m

In right ΔABC, we have

$$\tan\theta = \frac{AB}{BC}$$

$$\Rightarrow \quad \tan\theta = \frac{2 \text{ m}}{2\sqrt{3} \text{ m}}$$

$$\Rightarrow \quad \tan\theta = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad \tan\theta = \tan 30°$$

$$\Rightarrow \quad \theta = 30°$$

10. (*c*) **1 m**

Let AD be the line extending from A to D such that AD is parallel to BC. The angle of depression of point C at A is 45°.

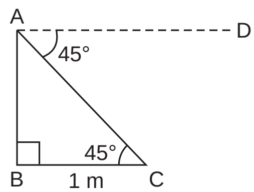
Then, $\angle DAC = 45°$

Now, $AD \parallel BC$.

Thus, $\angle DAC = \angle ACB = 45°$

Also, $BC = 1$ m

In right ΔABC, we have



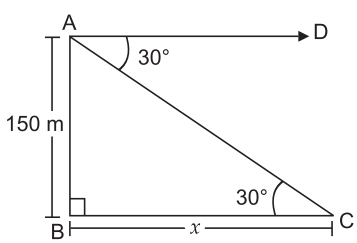
$$\tan 45° = \frac{AB}{BC}$$

$$\Rightarrow \quad 1 = \frac{AB}{1 \text{ m}}$$

$$\Rightarrow \quad AB = 1 \text{ m}$$

11. (*b*) **$150\sqrt{3}$**

Let AB be the height of the tower and C be the position of car parked. The angle of depression of C at A is 30°.



Then, $\angle DAC = 30°$

Now, $AD \parallel BC$

Thus, $\angle DAC = \angle ACB = 30°$

Also, $\angle ABC = 90°$, $AB = 150$ m, $BC = x$ m

In right ΔABC, we have

$$\tan 30° = \frac{AB}{BC}$$

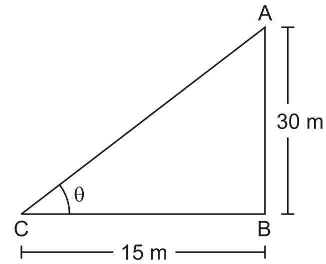
$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{150}{x}$$

$$\Rightarrow \quad x = 150\sqrt{3} \text{ m}$$

Hence, the distance of the car from the tower (in m) is $150\sqrt{3}$.

12. (*a*) **150 m**

Let AB be the height of the vertical stick and BC be the length of its shadow.



Let θ be the angle of elevation of point A at C.

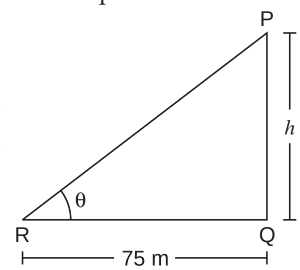
Then, $\angle ACB = \theta$

Also, $\angle ABC = 90°$

$AB = 30$ m

and $BC = 15$ m

In right ΔABC, we have



$$\tan\theta = \frac{AB}{BC}$$

$$\Rightarrow \quad \tan\theta = \frac{30 \text{ m}}{15 \text{ m}}$$

$$\Rightarrow \quad \tan\theta = 2 \qquad \dots (1)$$

At the same time, a tower casts a shadow on the level ground. Let PQ be the height of the tower and QR be the length of its shadow.

Then, $PQ = h$ meters,

$QR = 75$, $\angle PQR = 90°$

The angle of elevation of the point P at R is θ.

Thus, $\angle PRQ = \theta$.

In right ΔPQR, we have

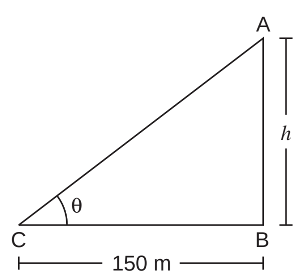
$$\tan\theta = \frac{PQ}{QR}$$

$$\Rightarrow \quad 2 = \frac{h}{75 \text{ m}} \qquad \text{[Using (1)]}$$

$$\Rightarrow \quad h = 150 \text{ m}$$

13. (*a*) **100 m**

Let AB (= *h* metres) be the height of the Qutub Minar and BC be the length of its shadow on the level ground. Let θ be the angle of elevation of the point A at C.



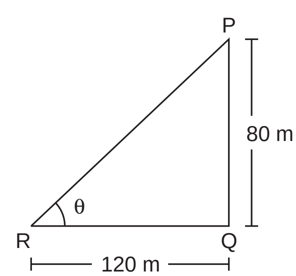
Then, $\angle ACB = \theta$,

$AB = h$,

$BC = 150$ m

and $\angle ABC = 90°$

Let PQ (= 80 m) be the height of the other minar and QR be the length of its shadow



on the level ground. As the measurement is done at the same time, the angle of elevation of P at R is also equal to θ.

Then, $\angle PRQ = \theta$, $\angle PQR = 90°$, PQ = 80 m

and QR = 120 m

In right ΔABC, we have

$$\tan \theta = \frac{AB}{BC}$$

$$\Rightarrow \quad \tan \theta = \frac{h}{150 \text{ m}} \qquad \dots (1)$$

In right ΔPQR, we have

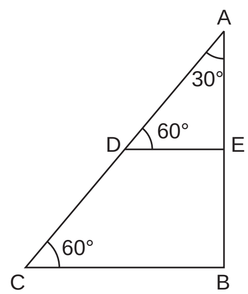
$$\tan \theta = \frac{PQ}{QR}$$

$$\Rightarrow \quad \frac{h}{150 \text{ m}} = \frac{80 \text{ m}}{120 \text{ m}} \qquad \text{[Using (1)]}$$

$$\Rightarrow \quad h = 100 \text{ m}$$

For Standard Level

14. (*c*) **1 m/s**



Let AC be the length of the ladder and AB be the height of the wall. It is given that the ladder is inclined to the wall at an angle of 30°.

Then, $\angle CAB = 30°$

and $\angle ABC = 90°$

In right ΔABC, we have

$\angle ABC + \angle CAB + \angle ACB = 180°$

$\Rightarrow \quad 90° + 30° + \angle ACB = 180° \quad \Rightarrow \quad \angle ACB = 60°$

Let us draw a line DE which is parallel to BC and cuts AC and AB at D and E respectively. Also, let DA be the distance covered in 1 second.

Then, DA = 2 m

Now, DE ∥ CB.

Thus, $\angle ADE = \angle ACB = 60°$

and $\angle AED = 90°$

In right ΔAED, we have

$$\cos 60° = \frac{DE}{DA}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{DE}{2 \text{ m}} \quad \Rightarrow \quad DE = 1 \text{ m}$$

Thus, he approaches the wall at the rate of 1 m/s.

15. (*b*) **2.5 m**

Let EC be the height of the girl and AB be the height of the lamp post. The distance between the girl and the lamp post is BC (= 3 m). Let CD be the shadow of the girl.

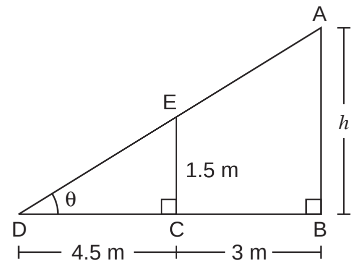
Let θ be the angle of elevation of the point E and A at D.

Then, $\angle EDC = \theta = \angle ADB$.

Also, EC = 1.5 m, AB = h metres,

BC = 3 m, CD = 4.5 m,

$\angle ABD = \angle ECD = 90°$



In right ΔECD, we have

$$\tan \theta = \frac{EC}{CD}$$

$$\Rightarrow \quad \tan \theta = \frac{1.5 \text{ m}}{4.5 \text{ m}} \qquad \dots (1)$$

In right ΔABD, we have

$$\tan \theta = \frac{AB}{BD}$$

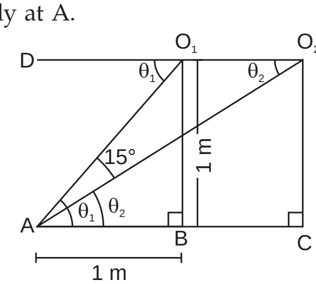
$$\Rightarrow \quad \frac{1.5 \text{ m}}{4.5 \text{ m}} = \frac{AB}{4.5 \text{ m} + 3 \text{ m}}$$

$$\Rightarrow \quad AB = \frac{1.5 \text{ m}}{4.5 \text{ m}} \times 7.5 \text{ m}$$

$$= 2.5 \text{ m}$$

16. (*a*) **45°, 30°**

Let O_2D be parallel to ABC passing through O_1. Let θ_1 and θ_2 be the angles of depression from O_1 and O_2 respectively at A.



Now, $DO_2 \parallel AC$.

Then, $\angle DO_1A = \angle BAO_1 = \theta_1$

and $\angle DO_2A = \angle CAO_2 = \theta_2 = \theta_1 - 15°$.

Also, $\angle O_1AO_2 = 15°$, AB = 1 m, O_1B = 1 m

In right ΔO_1BA, we have

$$\tan \theta_1 = \frac{O_1B}{AB}$$

$$\Rightarrow \quad \tan \theta_1 = \frac{1 \text{ m}}{1 \text{ m}}$$

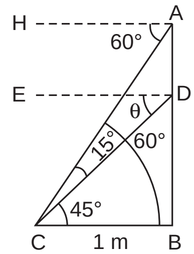
$$\Rightarrow \quad \tan \theta_1 = \tan 45°$$

$$\Rightarrow \quad \theta_1 = 45°$$

Now, $\theta_2 = \theta_1 - 15°$

$= 45° - 15° = 30°$

17. (*a*) **1 m, 45°**



In right ΔDBC, we have

$$\tan 45° = \frac{DB}{BC}$$

$$\Rightarrow \quad 1 = \frac{DB}{1\text{ m}}$$

$$\Rightarrow \quad DB = 1\text{ m}$$

From the figure,

$$\angle HAC = \angle ACB$$

$$\Rightarrow \quad \angle ACB = 60°$$

$$\Rightarrow \quad \angle ACD + \angle DCB = 60°$$

$$\Rightarrow \quad 15° + \angle DCB = 60°$$

$$\Rightarrow \quad \angle DCB = 60° - 15° = 45°$$

Thus, the angle of depression of D at C is

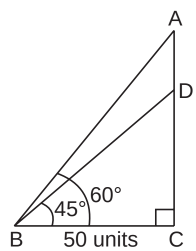
$$\angle EDC = \theta$$

Now, $ED \parallel BC$.

Thus, $\angle DCB = \angle EDC$

$$\Rightarrow \quad \angle EDC = 45°$$

18. (*b*) **$50(\sqrt{3}-1)$** units



In right ΔDCB, we have

$$\tan 45° = \frac{DC}{BC}$$

$$\Rightarrow \quad 1 = \frac{DC}{50\text{ units}}$$

$$\Rightarrow \quad DC = 50\text{ units}$$

In right ΔACB, we have

$$\tan 60° = \frac{AC}{BC}$$

$$\Rightarrow \quad \sqrt{3} = \frac{AC}{50\text{ units}}$$

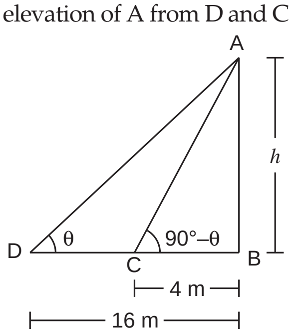
$$\Rightarrow \quad AC = 50\sqrt{3}\text{ units}$$

$$\Rightarrow \quad AD = AC - DC = 50\sqrt{3}\text{ units} - 50\text{ units}$$

$$= 50(\sqrt{3} - 1)\text{ units}$$

19. (*c*) **8 m**

Let AB be the height of the tower. Let θ and 90° – θ be the angles of elevation of A from D and C respectively.



Now, θ and 90° – θ are the complimentary angles.

Then, $AB = h$, $\angle ADB = \theta$,

$$\angle ACB = 90° - \theta$$

$$BD = 16\text{ m},\ BC = 4\text{ m},$$

$$\angle ABC = 90° = \angle ABD$$

In right ΔABD, we have

$$\tan\theta = \frac{AB}{BD} \quad \Rightarrow \quad \tan\theta = \frac{h}{16\text{ m}} \qquad \dots (1)$$

In right ΔABC, we have

$$\tan(90° - \theta) = \frac{AB}{BC}$$

$$\Rightarrow \quad \cot\theta = \frac{h}{4\text{ m}} \quad [\text{Using } \tan(90° - \theta) = \cot\theta] \qquad \dots (2)$$

Multiplying equation (1) and equation (2), we have

$$\tan\theta \times \cot\theta = \frac{h}{16\text{ m}} \times \frac{h}{4\text{ m}}$$

$$\Rightarrow \quad h^2 = 64$$

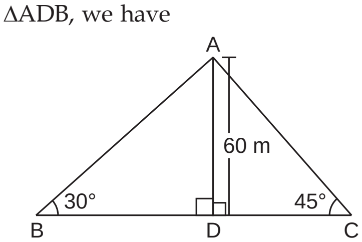
$$\Rightarrow \quad h = \pm 8\text{ m}$$

$$= 8\text{ m} \qquad [\text{Neglecting } - 8\text{ m}]$$

Hence, the height of the tower is 8 m.

20. (*d*) **$60(\sqrt{3}+1)$ m**

In right ΔADB, we have



$$\tan 30° = \frac{AD}{BD}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{60\text{ m}}{BD}$$

$$\Rightarrow \quad BD = 60\sqrt{3}\text{ m}$$

In right ΔADC, we have

$$\tan 45° = \frac{AD}{CD} \quad \Rightarrow \quad 1 = \frac{60\text{ m}}{CD}$$

$$\Rightarrow \quad CD = 60\text{ m}$$

Now, the distance between the two men is

$$BC = BD + CD = 60\sqrt{3}\text{ m} + 60\text{ m}$$

$$= 60(\sqrt{3} + 1)\text{ m}$$

21. (*b*) **60°**

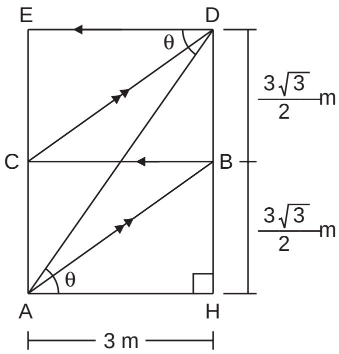
In the figure, we have

$$DH = BD + HB$$

$$= \frac{3\sqrt{3}}{2}\text{ m} + \frac{3\sqrt{3}}{2} = 3\sqrt{3}\text{ m}$$

$$AH = 3\text{ m}$$

Now, the angle of depression of point A when observed from point D is $\angle EDA = \theta$.



Also, ED ∥ AH.

Thus, ∠EDA = ∠DAH

In right ΔDHA, we have

$$\tan\theta = \frac{DH}{AH}$$

$$\Rightarrow \quad \tan\theta = \frac{3\sqrt{3}}{3}$$

$$\Rightarrow \quad \tan\theta = \sqrt{3}$$

$$\Rightarrow \quad \tan\theta = \tan 60^\circ$$

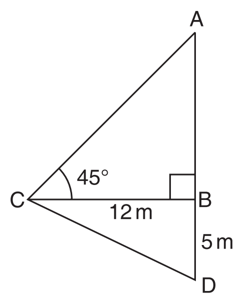
$$\Rightarrow \quad \theta = 60^\circ$$

22. (*b*) **$6(5+2\sqrt{2})$** m

In the figure, we have

CB = 12 m, ∠ACB = 45°, BD = 5 m

In right ΔABC, we have



$$\tan 45^\circ = \frac{AB}{CB}$$

$$\Rightarrow \quad 1 = \frac{AB}{12\text{ m}}$$

$$\Rightarrow \quad AB = 12\text{ m}$$

Also, $\sin 45^\circ = \frac{AB}{AC}$

$$\Rightarrow \quad \frac{1}{\sqrt{2}} = \frac{12\text{ m}}{AC}$$

$$\Rightarrow \quad AC = 12\sqrt{2}\text{ m}$$

In right ΔCBD, we have

$$CD = \sqrt{CB^2 + BD^2}$$

$$= \sqrt{(12\text{ m})^2 + (5\text{ m})^2}$$

$$= \sqrt{144+25}\text{ m} = 13\text{ m}$$

$$AD = AB + BD = 12\text{ m} + 5\text{ m} = 17\text{ m}$$

Now, the perimeter of ΔACD

$$= AC + CD + AD$$

$$= 12\sqrt{2}\text{ m} + 13\text{ m} + 17\text{ m}$$

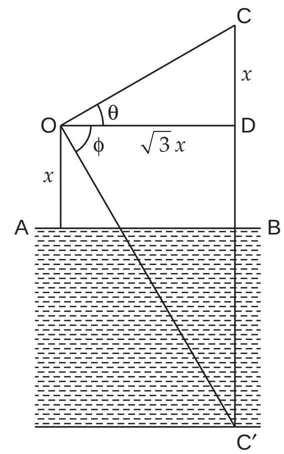
$$= (30 + 12\sqrt{2})\text{ m} = 6(5 + 2\sqrt{2})\text{ m}$$

23. (*c*) **90°**

In the figure, we have

$$CD = x,\ DO = \sqrt{3}\,x$$

$$\angle COD = \theta,\ \angle DOC = \phi,\ OA = x$$



In right ΔCDO, we have

$$\tan\theta = \frac{CD}{DO}$$

$$\Rightarrow \quad \tan\theta = \frac{x}{\sqrt{3}\,x}$$

$$\Rightarrow \quad \tan\theta = \tan 30^\circ$$

$$\Rightarrow \quad \theta = 30^\circ$$

Now, $CB = CD + DB = CD + OA$

$= x + x = 2x$

Also, $CB = C'B$

$\Rightarrow \quad C'B = 2x$

Thus, $C'D = DB + C'B$

$= OA + C'B = x + 2x = 3x$

In right ΔODC′, we have

$$\tan\phi = \frac{C'D}{OD}$$

$$\Rightarrow \quad \tan\phi = \frac{3x}{\sqrt{3}\,x}$$

$$\Rightarrow \quad \tan\phi = \sqrt{3}$$

$$\Rightarrow \quad \tan\phi = \tan 60^\circ$$

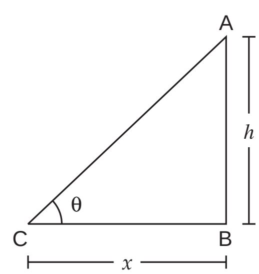
$$\Rightarrow \quad \phi = 60^\circ$$

Now, the sum of the angle of elevation (θ) and angle of depression (ϕ) is

$$\theta + \phi = 30^\circ + 60^\circ = 90^\circ$$

24. (*b*) **remains unchanged**

Let AB be the height of the tower and CB be the distance of the point of observation and foot of the tower.



Then, AB = h and CB = x

The angle of elevation of the point A at C is θ.

Then, ∠ACB = θ, ∠ABC = 90°

In right ΔABC, we have

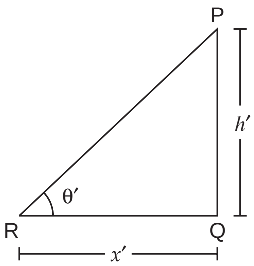
$$\tan \theta = \frac{h}{x} \quad \dots (1)$$

Now, if the height and the distance of the point of observation from its foot are increased by 10%, then the height and distance are respectively given as

$$h' = h + 10\% \text{ of } h = h + \frac{10}{100}h = 1.1\,h$$

and $$x' = x + 10\% \text{ of } x = x + \frac{10}{100}x = 1.1\,x$$

In right ΔPQR,



$$\tan \theta' = \frac{PQ}{QR}$$

$$\Rightarrow \quad \tan \theta' = \frac{h'}{x'}$$

$$\Rightarrow \quad \tan \theta' = \frac{1.1\,h}{1.1\,x}$$

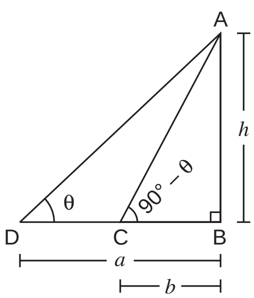
$$\Rightarrow \quad \tan \theta' = \tan \theta \quad \text{[Using (1)]}$$

$$\Rightarrow \quad \theta' = \theta$$

Thus, the angle of elevation remains unchanged.

25. (*c*) $\sqrt{ab}$

Let AB be the height of the tower. Let D and C be the observation points. The angle of elevation of A at D and C are θ and 90° – θ respectively.



Then, ∠ADB = θ,

∠ACB = 90° – θ

Also, AB = h,

DB = a,

CB = b,

∠ABC = ∠ABD = 90°

In right ΔABC, we have

$$\tan (90° - \theta) = \frac{AB}{CB}$$

$$\Rightarrow \quad \cot \theta = \frac{h}{b} \quad [\text{Using } \tan (90° - \theta) = \cot \theta] \quad \dots(1)$$

In right ΔABD, we have

$$\tan \theta = \frac{AB}{DB}$$

$$\Rightarrow \quad \tan \theta = \frac{h}{a} \quad \dots (2)$$

Multiplying equation (1) and equation (2), we have

$$\cot \theta \times \tan \theta = \frac{h}{b} \times \frac{h}{a}$$

$$\Rightarrow \quad 1 = \frac{h^2}{ab}$$

$$\Rightarrow \quad h = \sqrt{ab}$$

26. (*b*) **2*x***

Let AC be the height of the tower and ED (= x metres) be the height of the cliff. The angle of elevation of A at E is θ.

Then, ∠AEB = θ.

Also, the angle of depression of C at E is also θ.

Then, ∠BEC = θ.

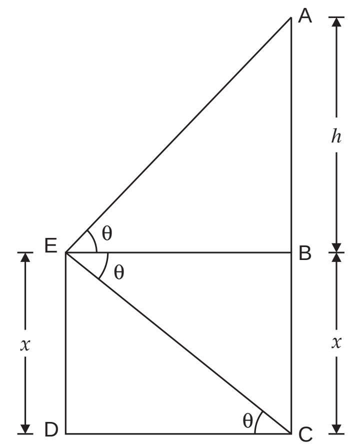
Now, EB ∥ CD.

Thus, ∠BEC = ∠ECD.

Also, ED = BC = x metres,

AB = h metres,

∠ABE = 90° = ∠EDC.



In right ΔABE, we have

$$\tan \theta = \frac{AB}{EB}$$

$$\Rightarrow \quad \tan \theta = \frac{h}{EB} \quad \dots (1)$$

In right ΔEDC, we have

$$\tan \theta = \frac{ED}{DC}$$

$$\Rightarrow \quad \tan \theta = \frac{x}{DC}$$

$$\Rightarrow \quad \frac{h}{EB} = \frac{x}{CD} \quad \text{[Using (1)]}$$

$$\Rightarrow \quad h = x$$

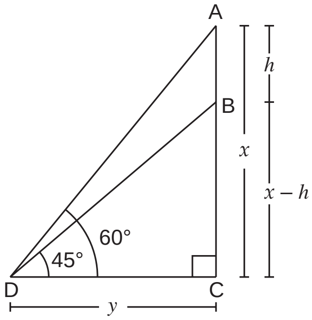
Now, height of the tower

= AC = AB + BC

= $h + x = x + x = 2x$

27. (*b*) $\dfrac{(\sqrt{3}-1)x}{\sqrt{3}}$

Let A and B be the positions of the two aeroplanes from the ground at point C. Then, AB is the vertical distance between the aeroplanes.



Let D be the point of observation on the level ground. The angles of elevation of A and B at D are 60° and 45° respectively.

Then, $\angle ADC = 60°, \angle BDC = 45°,$

$AC = x$ metres, $AB = h$ metres,

$BC = x - h$, $CD = y$ metres,

$\angle BCD = \angle ACD = 90°.$

In right ΔBCD, we have

$$\tan 45° = \frac{x-h}{y} \Rightarrow 1 = \frac{x-h}{y}$$

$$\Rightarrow \quad y = x - h \quad \ldots (1)$$

In right ΔACD, we have

$$\tan 60° = \frac{AC}{CD}$$

$$\Rightarrow \quad \sqrt{3} = \frac{x}{y}$$

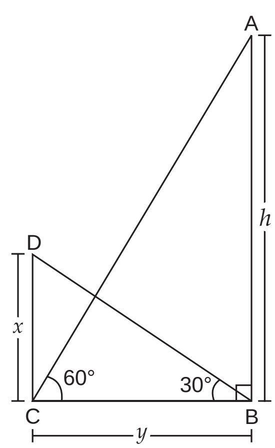
$$\Rightarrow \quad y = \frac{x}{\sqrt{3}}$$

$$\Rightarrow \quad x - h = \frac{x}{\sqrt{3}} \quad \text{[Using (1)]}$$

$$\Rightarrow \quad h = \frac{(\sqrt{3}-1)}{\sqrt{3}}x$$

28. (*b*) **3*x***

Let CD (= x) be the height of the tower and AB (= h) be the height of the hill. The angle of elevation of A at C is 60°.



Then, $\angle ACB = 60°$

and the angle of elevation of the point D at B is 30°.

Then, $\angle DBC = 30°$

Also, $\angle DCB = \angle ABC = 90°$,

$CD = x$, $AB = h$, $BC = y$

In right ΔABC, we have

$$\tan 60° = \frac{AB}{BC}$$

$$\Rightarrow \quad \sqrt{3} = \frac{h}{y}$$

$$\Rightarrow \quad y = \frac{h}{\sqrt{3}} \quad \ldots (1)$$

In right ΔDCB, we have

$$\tan 30° = \frac{CD}{BC}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{x}{y}$$

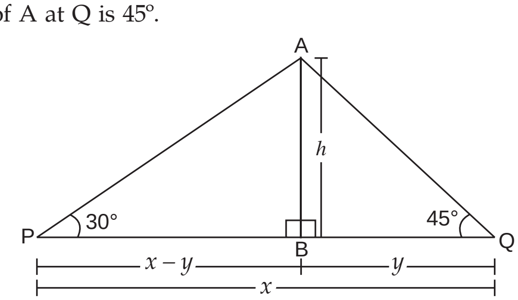
$$\Rightarrow \quad y = \sqrt{3}\,x$$

$$\Rightarrow \quad \frac{h}{\sqrt{3}} = \sqrt{3}\,x \quad \text{[Using (1)]}$$

$$\Rightarrow \quad h = 3x$$

29. (*c*) $\mathbf{\frac{(\sqrt{3}-1)}{2}x}$

Let AB be the height of the tall tree. The angle of elevation of A at P is 30° and the angle of elevation of A at Q is 45°.



Then, $\angle APB = 30°$ and $\angle AQB = 45°$,

$\angle ABP = \angle ABQ = 90°$

$PQ = x$ metres, $BQ = y$,

$AB = h$ metres, $PB = x - y$.

In right ΔABQ, we have

$$\tan 45° = \frac{AB}{BQ}$$

$$\Rightarrow \quad 1 = \frac{h}{y}$$

$$\Rightarrow \quad h = y$$

In right ΔABP, we have

$$\tan 30° = \frac{AB}{PB}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{x-y}$$

$$\Rightarrow \quad x - y = \sqrt{3}\,h$$

$$\Rightarrow \quad x - h = \sqrt{3}\,h$$

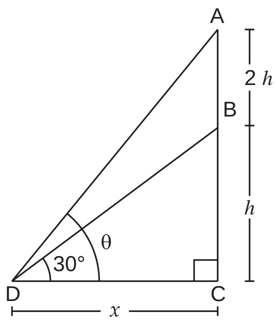
$$\Rightarrow \quad h = \frac{x}{\sqrt{3}+1} \text{ metres}$$

$$= \frac{x(\sqrt{3}-1)}{2} \text{ metres}$$

30. (*c*) **60°**

Let BC be the height of the tower and AB be the height of the flagstaff.

Then, BC = h and AB = $2h$



Let D be the point of observation on the ground and x metres be the length of the distance from the foot of the tower.

The angle of elevation of B at D is 30°.

Then, ∠BDC = 30°

Also, CD = x metres,

∠ACD = ∠BCD

= 90°, AC = $3h$

In right ΔBCD, we have

$$\tan 30° = \frac{BC}{CD}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{x}$$

$$\Rightarrow \quad x = \sqrt{3}\,h \qquad \ldots (1)$$

In right ΔACD, we have

$$\tan\theta = \frac{AC}{CD}$$

$$\Rightarrow \quad \tan\theta = \frac{3h}{x}$$

$$\Rightarrow \quad \tan\theta = \frac{3h}{\sqrt{3}\,h} \qquad \text{[Using (1)]}$$

$$\Rightarrow \quad \tan\theta = \sqrt{3}$$

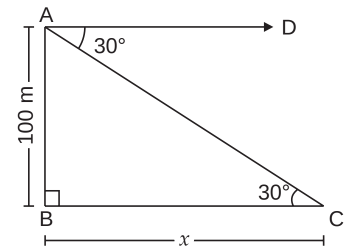
$$\Rightarrow \quad \tan\theta = \tan 60°$$

$$\Rightarrow \quad \theta = 60°$$

SHORT ANSWER QUESTIONS

For Basic and Standard Levels

1. Let BC be the level of the water. Let AB be the height of the road and C be the position of the boat. The angle of depression of C at A is 30°.



Then, ∠DAC = 30°

Now, AD ∥ BC.

Thus, ∠DAC = ∠ACB = 30°

Also, ∠ABC = 90°, AB = 100 m,

BC = x metres

In right ΔABC, we have

$$\tan 30° = \frac{AB}{BC}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{100 \text{ m}}{x}$$

$$\Rightarrow \quad x = 100 \text{ m} \times 1.732 = 173.2 \text{ m}$$

Hence, the distance of the boat from the base of the rock is **173.2 m**.

2. AD = 2.54 m

AB = 6 m

AB = AD + DB

DB = AB – AD

= 6 – 2.54 = 3.46 m

Now, in ΔDBC

$$\sin 60° = \frac{DB}{DC}$$

$$\frac{\sqrt{3}}{2} = \frac{3.46}{DC}$$

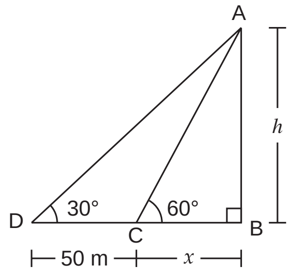
$$DC = \frac{6.92}{\sqrt{3}}$$

$$= \frac{6.92}{1.73}$$

$$= 4 \text{ m}$$

Length of ladder = **4 m**

3. Let AB (= h) be the height of the tower. Let D and C be the point of observation of the Sun's elevation. The angle of elevation of A at D is 30° and the angle of elevation of A at C is 60°.



Then, $\angle ADB = 30^\circ$, $\angle ACB = 60^\circ$,

$\angle ABC = \angle ABD = 90^\circ$

CD = 50 m, BC = x, AB = h

In right ΔABC, we have

$$\tan 60^\circ = \frac{AB}{BC} \Rightarrow \sqrt{3} = \frac{h}{x}$$

$$\Rightarrow \quad x = \frac{h}{\sqrt{3}} \quad \ldots (1)$$

In right ΔABD, we have

$$\tan 30^\circ = \frac{AB}{BD} \Rightarrow \frac{1}{\sqrt{3}} = \frac{h}{CD + BC}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{50 + x}$$

$$\Rightarrow \quad 50 + \frac{h}{\sqrt{3}} = \sqrt{3}\,h \quad \text{[Using (1)]}$$

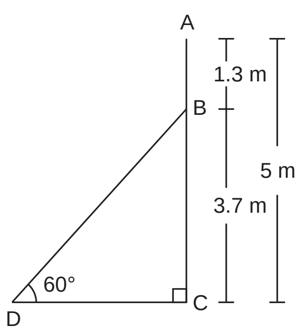
$$\Rightarrow \quad h\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right) = 50$$

$$\Rightarrow \quad h = 25\sqrt{3} \text{ m}$$

Hence, the height of the tower is **$25\sqrt{3}$ m**.

For Standard Level

4. Let AC be the height of the pole and let B be the position of the electric fault down from the top of the pole.



Then, AC = 5 m

AB = 1.3 m

Now, the distance of the electric fault on the pole from the ground is

BC = AC – AB

= 5 m – 1.3 m = 3.7 m

Let D be the observation point on the ground. The angle of elevation of B at D is 60°.

Then, $\angle BDC = 60^\circ$

Also, $\angle BCD = 90^\circ$.

Let BD (= l) be the length of the ladder.

In right ΔBCD, we have

$$\sin 60^\circ = \frac{BC}{BD}$$

$$\Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{3.7 \text{ m}}{l}$$

$$\Rightarrow \quad l = \frac{3.7 \times 2}{\sqrt{3}} \text{ m} = 4.27 \text{ m (approx.)}$$

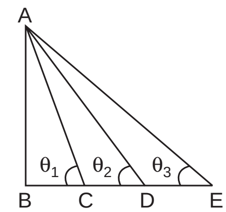
Hence, the length of the ladder is **4.27 m**.

5. (*a*) True. As E is moved closer to B along EB, it is observed

(*i*) θ increases (as $\theta_2 > \theta_3$, $\theta_1 > \theta_2$ …) and

(*ii*) BE decreases (as BD < BE, BC < BD…)

Thus, as θ increases, the perpendicular AB remains constant but the base BE decreases.



So, $\tan\theta\left(= \frac{\text{perpendicular}}{\text{base}}\right)$ increases as θ increases.

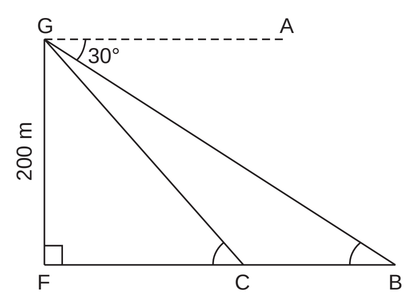
(*b*) True. $\tan\theta = \frac{\sin\theta}{\cos\theta}$

We know that $\sin\theta$ increases as θ increases but $\cos\theta$ decreases as θ increases.

So, as θ increases, the numerator increases and the denominator decreases. Hence, $\tan\theta$ increases. But in case of $\sin\theta$ which can be seen as $\frac{\sin\theta}{1}$, only the numerator increases but the denominator remains fixed at 1. Hence, $\tan\theta$ increases faster than $\sin\theta$ as θ increases.

VALUE-BASED QUESTION

1. (*i*) Let GF represents the tower and B the position of the boat. Let $\angle AGB$ be the angle of depression of the boat from the top of the tower.



Then, GF = 200 m

and $\angle GBF = \angle AGB = 30^\circ$

In right triangle GBF, we have

$$\tan 30^\circ = \frac{GF}{FB}$$

$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{200}{FB}$

$\Rightarrow \quad FB = 200\sqrt{3}$ … (1)

Hence, the distance of the boat from the foot of the tower is **$200\sqrt{3}$ m**.

(*ii*) Let C be the new position of the boat.

Then, BC = $200(\sqrt{3} - 1)$ m

Now, FC = FB – BC

$= [200\sqrt{3} - 200(\sqrt{3} - 1)]$ m [Using (1)]

$= (200\sqrt{3} - 200\sqrt{3} + 200)$ m

= 200 m … (2)

In ΔGFC, we have

$\tan \angle GCF = \frac{GF}{FC} = \frac{200 \text{ m}}{200 \text{ m}} = 1$ [Using (2)]

$\Rightarrow \quad \angle GCF = 45°$ [∵ tan 45° = 1]

∴ New angle of depression = ∠AGC = ∠GCF = **45°**

(*iii*) Responsibility and decision-making.

UNIT TEST 1

For Basic Level

1. (*d*) **45°**

Let AB be the height of the tower and BC be the length of its shadow.

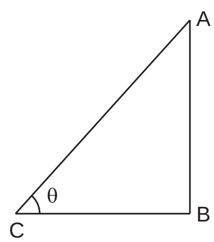
Let θ be the Sun's altitude at the time of observation.

Then, ∠ACB = θ

Also, ∠ABC = 90°

The length of the shadow is equal to the height of the tower.

Thus, AB = BC



In right ΔABC, we have

$\tan\theta = \frac{AB}{BC}$

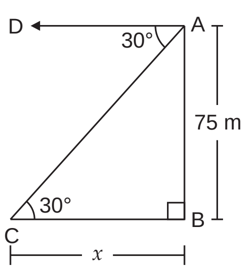
$\Rightarrow \quad \tan\theta = 1$

$\Rightarrow \quad \tan\theta = \tan 45°$ [Using tan 45° = 1]

$\Rightarrow \quad \theta = 45°$

2. (*c*) **$75\sqrt{3}$**

Let AB be the height of the car and C be the position of the car on the ground. The angle of depression of C at A is 30°.



Then, ∠DAC = 30°.

Now, AD ∥ BC.

Thus, ∠DAC = ∠ACB = 30°.

Also, AB = 75 m,

BC = x metres,

∠ABC = 90°.

In right ΔABC, we have

$\tan 30° = \frac{AB}{BC}$

$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{75 \text{ m}}{x}$

$\Rightarrow \quad x = 75\sqrt{3}$ m

3. (*b*) **30°**

Let A be the position of the kite vertically from the ground at point C.

Let AB be the length of the string of the kite.

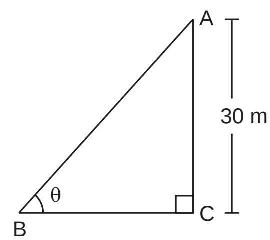
Let θ be the angle of elevation of A at B.

Then, ∠ABC = θ,

∠ACB = 90°,

AB = 60 m

and AC = 30 m



In right ΔACB, we have

$\sin\theta = \frac{AC}{AB}$

$\Rightarrow \quad \sin\theta = \frac{30 \text{ m}}{60 \text{ m}}$

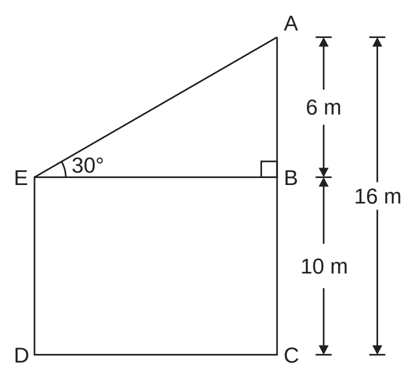
$\Rightarrow \quad \sin\theta = \frac{1}{2}$

$\Rightarrow \quad \sin\theta = \sin 30°$

$\Rightarrow \quad \theta = 30°$

4. (*c*) **12 m**

Let ED and AC be the heights of the two towers respectively. Let CD = EB be the distance between the two towers. Let AE be the length of the string.



Then, ED = 10 m, AC = 16 m

Now, AB = AC – BC

= AC – ED

= 16 m – 10 m = 6 m

$\angle AEB = 30°,$

$\angle ABE = 90°$

In right ΔABE, we have

$$\sin 30° = \frac{AB}{AE}$$

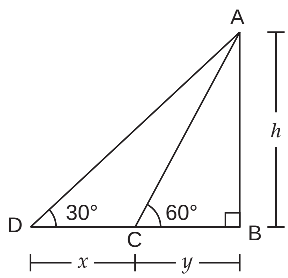
$$\Rightarrow \quad \frac{1}{2} = \frac{6 \text{ m}}{AE}$$

$$\Rightarrow \quad AE = 12 \text{ m}$$

Hence, the length of the string is 12 m.

5. (*d*) $\frac{\sqrt{3}}{2}x$

Let AB be the height of the chimney. Let D and C be the points of observation. The angles of elevation of A at D and C respectively are 30° and 60°.



Then, $\angle ADB = 30°$ and $\angle ACB = 60°$

Also, $AB = h, BC = y, CD = x,$

$\angle ABC = \angle ABD = 90°$

In right ΔABC, we have

$$\tan 60° = \frac{AB}{BC}$$

$$\Rightarrow \quad \sqrt{3} = \frac{h}{y}$$

$$\Rightarrow \quad y = \frac{h}{\sqrt{3}} \qquad \ldots (1)$$

In right ΔABD, we have

$$\tan 30° = \frac{AB}{BD}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{BC + CD}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{x+y}$$

$$\Rightarrow \quad x + y = \sqrt{3}h$$

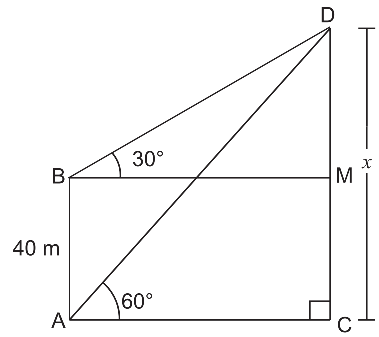
$$\Rightarrow \quad x + \frac{h}{\sqrt{3}} = \sqrt{3}h$$

$$\Rightarrow \quad h\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right) = x$$

$$\Rightarrow \quad h = \frac{\sqrt{3}}{2}x \text{ metres}$$

Hence, the height of the chimney is $\frac{\sqrt{3}}{2}x$ metres.

6. Let CD (= x) be the height of the tower and let AC be the horizontal to distance between the first point of observation A and tower CD.



AB = 40 m = CM, CD = x m.

$\therefore$ In ΔADC, we have

$$\tan 60° = \frac{DC}{AC}$$

$$\sqrt{3} = \frac{x}{AC}$$

$$AC = \frac{x}{\sqrt{3}} \qquad \ldots(1)$$

In ΔBDM, $\quad \tan 30° = \frac{DM}{BM}$

$$\frac{1}{\sqrt{3}} = \frac{DM}{AC} \qquad (\because BM = AC)$$

$$AC = \sqrt{3}\,DM \qquad \ldots(2)$$

From equations (1) and (2), we get

$$\frac{x}{\sqrt{3}} = \sqrt{3}\,DM$$

$$x = 3\,DM$$

$$x = 3(DC - MC)$$

$$x = 3(x - AB) \qquad [\because MC = AB]$$

$$x = 3(x - 40)$$

$$x = 3x - 120$$

$$+2x = +120$$

$$x = 60 \text{ m}$$

Height of tower = 60 m

Horizontal distance from the point of observation

$$AC = \frac{x}{\sqrt{3}}$$

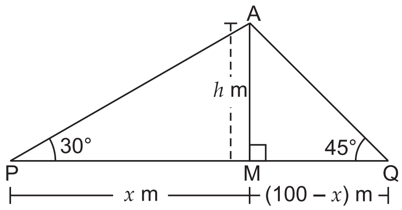
$$= \frac{60}{\sqrt{3}}$$

$$= 20\sqrt{3} \text{ m}$$

Hence, height of the tower is **60 m** and horizontal distance from the point of observation is $\mathbf{20\sqrt{3}}$ **m**.

7. Let P and Q are two points on the two banks of a river of width 100 m. AM is a vertical tree standing on a small island in the middle of the river such that $\angle APM = 30°$ and $\angle AQM = 45°$.

Also, $\angle AMP = \angle AMQ = 90°$.



Let PM = x m.

Then MQ = $(100 - x)$ m

Let AM = h m be the height of the tree.

Then from ΔAPM, we have

$$\tan 30° = \frac{AM}{PM}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{x}$$

$$\Rightarrow \quad x = \sqrt{3}\ h \qquad ...(1)$$

From ΔAQM, we have

$$\tan 45° = \frac{AM}{MQ}$$

$$\Rightarrow \quad 1 = \frac{h}{100 - x}$$

$$\Rightarrow \quad h = 100 - x \qquad ...(2)$$

$\therefore$ From (1) and (2), we get

$$h = 100 - \sqrt{3}\,h$$

$$\Rightarrow \quad h(1 + \sqrt{3}) = 100$$

$$\Rightarrow \quad h = \frac{100}{\sqrt{3}+1} = \frac{100\left(\sqrt{3}-1\right)}{\left(\sqrt{3}\right)^2 - 1^2}$$

$$= 50\left(\sqrt{3}-1\right)$$

$$\simeq 50(1.73 - 1)$$

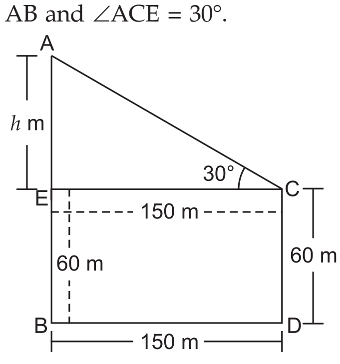
$$= 50 \times 0.73$$

$$= 36.5$$

Hence, the required height of the tree is **36.5 m**.

8. Let AB and CD be two towers standing vertically on the horizontal ground BD where BD = 150 m.

Let CE $\perp$ AB and $\angle ACE = 30°$.



Then CE = BD = 150 m.

Given that the height of the smaller tower CD is 60 m. We have $\angle AEC = \angle ABD = \angle CDB = 90°$.

Let AE = h m.

Then from ΔAEC, we have

$$\tan 30° = \frac{AE}{EC}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{150}$$

$$\Rightarrow \quad h = \frac{150}{\sqrt{3}}$$

$$= \frac{150\sqrt{3}}{3}$$

$$= 50\sqrt{3}$$

$$\simeq 50 \times 1.732$$

$$= 86.6 \qquad ...(1)$$

Hence, AB = AE + EB

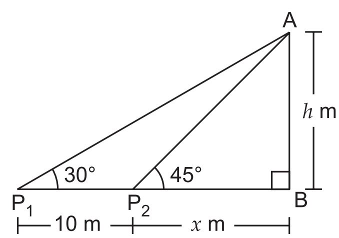
$$= h + CD = 86.6 + 60$$

$$= 146.6$$

Hence, the required height of the 1st tower is **146.6 m**.

9. Let BP_2 be the horizontal road, B is the foot of the vertical lamp post AB standing on the one side B of the road such that $\angle AP_2B = 45°$, P_1 is a point 10 m away from the point P_2 such that BP_2P_1 is on the same line and $\angle AP_1B = 30°$. Let AB = h m and $P_2B = x$ m.

Now, given that $P_1P_2 = 10$ m, $\angle ABP_1 = 90°$.



Now, from ΔAP_2B, we have

$$\tan 45° = \frac{AB}{P_2B}$$

$$\Rightarrow \quad 1 = \frac{h}{x}$$

$$\Rightarrow \quad x = h \qquad ...(1)$$

Also, from ΔAP_1B, we have

$$\tan 30° = \frac{AB}{P_1B}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{h}{x+10}$$

$$\Rightarrow \quad x + 10 = \sqrt{3}\,h$$

$$\Rightarrow \quad x + 10 = \sqrt{3}\,x \qquad \text{[From (1)]}$$

$$\Rightarrow \quad x\left(\sqrt{3}-1\right) = 10$$

$$\Rightarrow \quad x = \frac{10}{\sqrt{3}-1}$$

$= 5\left(\sqrt{3}+1\right)$

$= 5 \times (1 + 1.732)$

$= 5 \times 2.732$

$= 13.66.$

(*i*) Hence, the required width of the road P_2B is **13.66 m**.

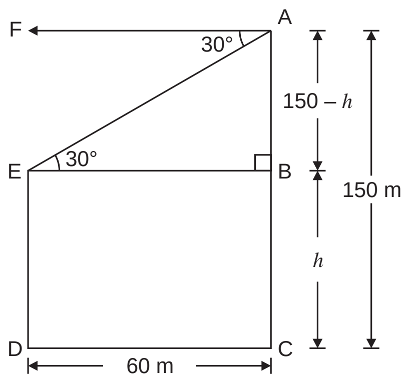
(*ii*) If the speed of the hedgehog is 1 m/s, then the required time taken by the hedgehog to cross the road of width 13.66 m is

$$\text{Time} = \frac{\text{Distance}}{\text{Speed}} = \frac{13.66}{1} \text{ s}$$

= **13.66 seconds**

10. Let ED (= h) and AC (= 150 m) be the heights of the first and second vertical poles respectively. Then angle of depression of E at A is 30º.

Then, $\angle FAE = 30°$



Now, AF || BE.

Then, $\angle AEB = \angle FAE = 30°$

Also, ED = h metres,

AB = AC – BC

= 150 – h

CD = EB = 60 m, $\angle ABE = 90°$.

In right ΔABE, we have

$$\tan 30° = \frac{AB}{EB}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{150-h}{60}$$

$$\Rightarrow \quad 60 = 150\sqrt{3} - h\sqrt{3}$$

$$\Rightarrow \quad h = \frac{150\sqrt{3}-60}{\sqrt{3}} = \frac{10\,(15\sqrt{3}-6)}{\sqrt{3}}$$

$$= \frac{10\,(15\times 1.732 - 6)}{1.732}$$

= 115.36 m (approx.)

Hence, the height of the first vertical tower is **115.36 m**.

UNIT TEST 2

For Standard Level

1. (*b*) **4.64 m**

Let AB be the vertical tree of height 10 m standing on the horizontal ground BG. Hence, AB = 10 m and $\angle ABG = 90°$. Let C be the point on the tree where it broke and touched the ground at a point G so that $\angle CGB = 60°$.

Let AC = x m. Let BC = h m and CG = x m.

Now, from ΔCBG, we have

$$\sin 60° = \frac{CB}{CG} = \frac{h}{x}$$

$$\Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{h}{x}$$

$$\Rightarrow \quad \sqrt{3}\,x = 2h \qquad \ldots(1)$$

Also, AB = AC + BC

$$\Rightarrow \quad 10 = x + h = \frac{2h}{\sqrt{3}} + h \qquad \text{[From (1)]}$$

$$\Rightarrow \quad 10 = \frac{\left(2+\sqrt{3}\right)}{\sqrt{3}}h$$

$$\Rightarrow \quad h = \frac{10\sqrt{3}}{2+\sqrt{3}}$$

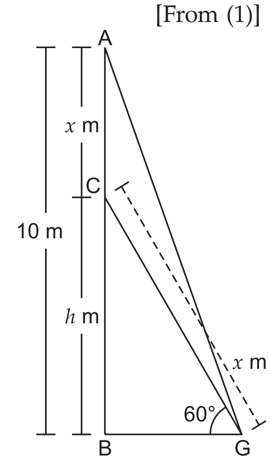
$$= \frac{10\sqrt{3}\left(2-\sqrt{3}\right)}{4-3}$$

$= 20\sqrt{3} - 30$

$= 20 \times 1.732 - 30$

$= 34.64 - 30$

$= 4.64.$



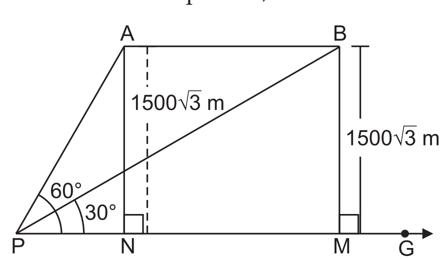
Hence, the required height from the ground at which the tree got bent is 4.64 m .

2. (*a*) **720 km/h**

Let A be the initial position of the jet plane at a constant height of $1500\sqrt{3}$ m from the horizontal ground PG so that the angle of devotion of A from an observer P on the ground is 60° so that $\angle APG = 60°$.

Let AN $\perp$ PG and BM $\perp$ PG.

Then AN = BM = $1500\sqrt{3}$ m. After 15 s, let the jet plane come to the point B, so that $\angle BPM = 30°$.



Now, from ΔAPN, we have

$$\tan 60° = \frac{AN}{PN}$$

$$\Rightarrow \quad \sqrt{3} = \frac{1500\sqrt{3}}{PN}$$

$$\therefore \quad PN = 1500 \quad \ldots(1)$$

Again, from ΔBPM, we have

$$\tan 30° = \frac{BM}{PM}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{1500\sqrt{3}}{PM}$$

$$\therefore \quad PM = 1500\sqrt{3}\times\sqrt{3} = 4500 \quad \ldots(2)$$

$$\therefore \quad AB = NM = PM - PN$$

$$= 4500 - 1500 \quad \text{[From (1) and (2)]}$$

$$= 3000$$

Now, 3000 m = 3 km

and $15 \text{ s} = \frac{15}{60\times 60} \text{ h} = \frac{1}{240} \text{ h}$

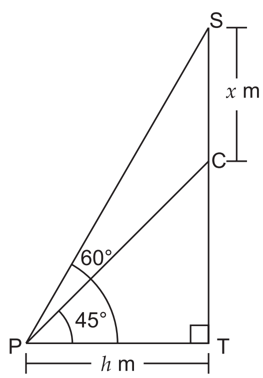
Hence, required speed = $\dfrac{3}{\frac{1}{240}}$ km/h = 720 km/h.

3. (*d*) $\dfrac{(\sqrt{3}+1)x}{2}$

Let CS be the vertical statue which stands on the top of a pedestal, TC standing vertically on the horizontal ground PT, so that ∠PTC = 90°.

Given that ∠CPT = 45° and ∠SPT = 60°. Let PT = *h* m.

Then from ΔPTC, we have



$$\tan 45° = \frac{CT}{PT}$$

$$\Rightarrow \quad 1 = \frac{CT}{h}$$

$$\Rightarrow \quad CT = h \quad \ldots(1)$$

∴ From ΔPTS, we have

$$\tan 60° = \frac{ST}{PT}$$

$$\Rightarrow \quad \sqrt{3} = \frac{SC+CT}{h}$$

$$= \frac{x+h}{h} \quad \text{[From (1)]}$$

$$= \frac{x}{h}+1$$

$$\therefore \quad \frac{x}{h} = \sqrt{3}-1$$

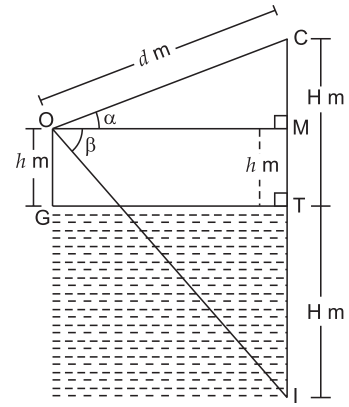
$$\therefore \quad h = \frac{x}{\sqrt{3}-1} = \frac{x(\sqrt{3}+1)}{3-1} = \frac{x(\sqrt{3}+1)}{2}$$

∴ Required height of the pedestal is $\dfrac{x(\sqrt{3}+1)}{2}$ m.

4. (*d*) **40 m**

Let O be the point of observation of the cloud at C, at a height of *h* m from the horizontal ground GT and let the distance of the cloud from the point of observation be *d* m, then OC = *d* m.

Let I be the image of the cloud so that CT = TI.



Now, from ΔCMO, we have

$$\cos\alpha = \frac{OM}{CO} = \frac{OM}{d}$$

$$\Rightarrow \quad OM = d\cos\alpha \quad \ldots(1)$$

Also, $\sin\alpha = \dfrac{CM}{d}$

$$\Rightarrow \quad CM = d\sin\alpha \quad \ldots(2)$$

$$\therefore \quad MI = MT + TI$$

$$= h + TC$$

$$= h + CM + MT$$

$$= 2h + d\sin\alpha \quad \text{[From (2)]} \ldots(3)$$

Now, from ΔMOI, we have

$$\tan\beta = \frac{MI}{OM}$$

$$= \frac{2h + d\sin\alpha}{d\cos\alpha} \quad \text{[From (1) and (3)]}$$

$$= \frac{2h}{d}\sec\alpha + \tan\alpha$$

$$\Rightarrow \tan\beta - \tan\alpha = \frac{2h}{d}\sec\alpha$$

$$\Rightarrow \quad d = \frac{2h\sec\alpha}{\tan\beta - \tan\alpha} \quad \ldots(4)$$

Now, when α = 30°, β = 60° and *d* = 80 m,

then $d = \dfrac{2h\sec 30°}{\tan 60° - \tan 30°}$

$$\Rightarrow \quad 80 = \frac{2\times\frac{2}{\sqrt{3}}h}{\sqrt{3}-\frac{1}{\sqrt{3}}} = \frac{4h}{2} = 2h.$$

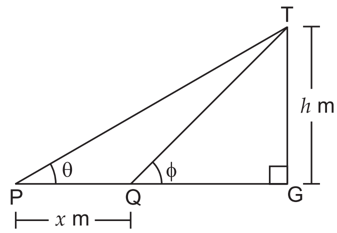
$$\therefore \quad h = \frac{80}{2} = 40$$

∴ The required height of the point of observation above the lake is 40 m.

5. (*c*) $\dfrac{x}{\cot\theta - \cot\phi}$

Let T be the top of the vertical pole TG standing on the horizontal ground PG. Let P and Q be two points on the ground such that $\angle TPG = \theta$ and $TQG = \phi$

Let TG = h m where TG ⊥ PG.



Now, from ΔTPG, we have

$$\tan\theta = \frac{h}{PG}$$

$$\Rightarrow \quad PG = h\cot\theta \quad \ldots(1)$$

and from ΔTQG, we have,

$$\tan\phi = \frac{h}{QG}$$

$$\Rightarrow \quad QG = h\cot\phi \quad \ldots(2)$$

∴ From (1) and (2), we have

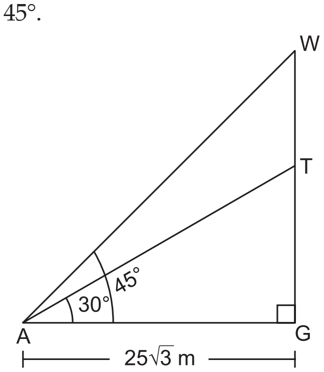
$$PG - QG = h(\cot\theta - \cot\phi)$$

$$\Rightarrow \quad x = h(\cot\theta - \cot\phi)$$

$$\Rightarrow \quad h = \frac{x}{\cot\theta - \cot\phi}$$

6. Let TG be the vertical tower standing on the horizontal ground AG at a distance $25\sqrt{3}$ m from a point A on the ground so that AG = $25\sqrt{3}$ m, $\angle AGT = 90°$ and $\angle TAG = 30°$.

Let WT be the water tank lying on the top of the tower TG so that WT is the depth of the water tank and $\angle WAG = 45°$.



Now, from ΔATG, we have

$$\tan 30° = \frac{TG}{AG}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{TG}{25\sqrt{3}}$$

$$\Rightarrow \quad TG = 25 \quad \ldots(1)$$

Hence, the required height of the tower is **25 m**.

Again, from ΔWAG, we have

$$\tan 45° = \frac{WG}{AG} = \frac{WG}{25\sqrt{3}}$$

$$\Rightarrow \quad 1 = \frac{WG}{25\sqrt{3}}$$

$$\therefore \quad WG = 25 \times 1.732 = 43.3 \quad \ldots(2)$$

∴ The required depth of the tank is

$$WT = WG - TG$$

$$= (43.3 - 25) \text{ m} \quad \text{[From (1) and (2)]}$$

$$= \mathbf{18.3\ m}$$

7. Let FE and GE be the heights of windows of the third floor and fourth floor respectively. Let AD be the height of the building where the pigeon is sitting.

The angles of elevations of A at the point G and F are respectively 30° and 60°.

Then, $\angle AGB = 30°$ and $\angle AFC = 60°$

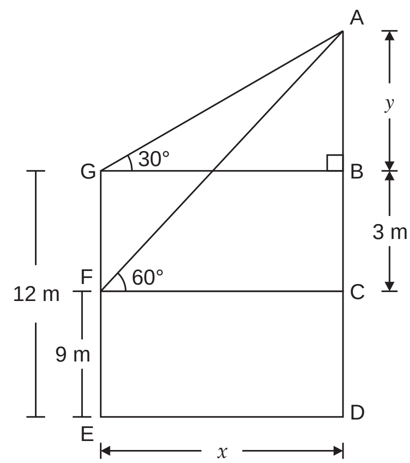
Let x be the distance between the two building.

Also, FE = 9 m, GE = 12 m,

ED = FC = GB = x metres,

AB = y, BC = 3 m,

CD = 9 m, $\angle ABG = \angle ACF = 90°$.



Now, AC = AB + BC = y + 3.

In right ΔABG, we have

$$\tan 30° = \frac{AB}{GB}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{y}{x}$$

$$\Rightarrow \quad x = \sqrt{3}\,y \quad \ldots(1)$$

In right ΔACF, we have

$$\tan 60° = \frac{AC}{CF}$$

$$\Rightarrow \quad \sqrt{3} = \frac{y+3}{x}$$

$$\Rightarrow \quad \sqrt{3} \times \sqrt{3}\,y = y + 3 \quad \text{[Using (1)]}$$

$$\Rightarrow \quad y = \frac{3}{2} = 1.5 \text{ m}$$

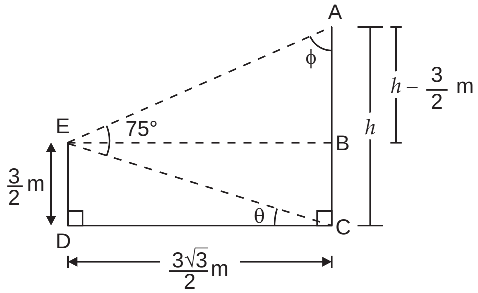
∴ Height of the pigeon above the ground = AD

= AB + BC + CD

= 1.5 m + 3 m + 9 m = 13.5 m

Hence, the height of the pigeon above the ground is **13.5 m**.

8. The given figure is shown below:



Let $\angle ECD = \theta$

In right ΔEDC, we have

$$\tan\theta = \frac{\left(\frac{3}{2}\text{ m}\right)}{\left(\frac{3\sqrt{3}}{2}\text{ m}\right)}$$

$$\Rightarrow \quad \tan\theta = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad \tan\theta = \tan 30°$$

$$\Rightarrow \quad \theta = 30°$$

In ΔEDC, we have

$$\angle CED + \angle EDC + \theta = 180°$$

$$\Rightarrow \quad \angle CED + 90° + 30° = 180°$$

$$\Rightarrow \quad \angle CED = 60°$$

Now, $\quad ED \parallel BC$

Then, $\quad \angle CED = \angle ECB$

$$\Rightarrow \quad \angle ECB = 60°$$

$$\Rightarrow \quad \angle ECA = 60°$$

Let $\quad \angle EAC = \phi$

In ΔAEC,

$$\angle AEC + \angle ECA + \angle CAE = 180°$$

$$\Rightarrow \quad 75° + 60° + \phi = 180°$$

$$\Rightarrow \quad \phi = 180° - 135° = 45°$$

In right ΔABE, we have

$$\tan\phi = \frac{CD}{AB} \qquad [\because \quad EB = CD]$$

$$\Rightarrow \quad \tan 45° = \frac{\left(\frac{3\sqrt{3}}{2}\text{ m}\right)}{h - \frac{3}{2}}$$

$$\Rightarrow \quad 1 = \frac{\left(\frac{3\sqrt{3}}{2}\text{ m}\right)}{h - \frac{3}{2}}$$

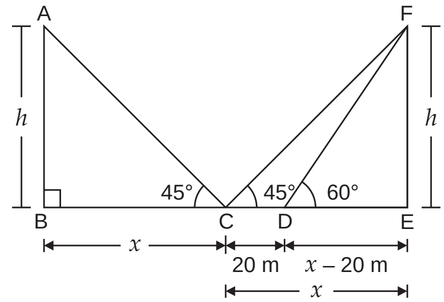
$$\Rightarrow \quad h - \frac{3}{2} = \frac{3\sqrt{3}}{2}$$

$$\Rightarrow \quad h = \frac{3\sqrt{3}}{2} + \frac{3}{2}$$

$$= \frac{3}{2}(1 + \sqrt{3})\text{ m}$$

Hence, the height of the building is $\mathbf{\frac{3}{2}(1 + \sqrt{3})\text{ m}}$.

9. Let AB and FE be the heights of the two lamp posts which are equal in heights.



Let C be the position of the boy midway between the lamp posts. The angle of elevation of A and F at C are 45° each.

Then, $\quad \angle ACB = \angle FCE = 45°$.

After walking 20 m closer to the lamp post EF, the angle of elevation of F at D is 60°.

Then, $\quad \angle FDE = 60°$

Now, $\quad AB = EF = h$, $CD = 20$ m, $BC = CE = x$,

$ED = x - 20$, $\angle ABC = \angle FEC = 90° = \angle FED$

In right ΔFEC, we have

$$\tan 45° = \frac{FE}{CE}$$

$$\Rightarrow \quad 1 = \frac{h}{x}$$

$$\Rightarrow \quad h = x \qquad \ldots (1)$$

In right ΔFED, we have

$$\tan 60° = \frac{FE}{DE}$$

$$\Rightarrow \quad \sqrt{3} = \frac{h}{x - 20}$$

$$\Rightarrow \quad \sqrt{3}(h - 20) = h \qquad [\text{Using (1)}]$$

$$\Rightarrow \quad \sqrt{3}h - 20\sqrt{3} = h$$

$$\Rightarrow \quad h = \frac{20\sqrt{3}}{\sqrt{3} - 1} = \frac{20\sqrt{3}(\sqrt{3} + 1)}{2}$$

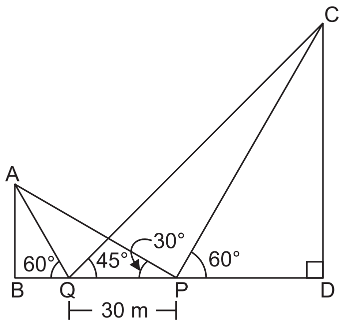
$$= 10(3 + \sqrt{3})$$

$$= 10(3 + 1.732)$$

$$= \mathbf{47.32\text{ m}}$$

Hence, the heights of the lamp posts are **47.32 m**.

10. Let AB and CD be two vertical poles standing on the horizontal ground BD where PQ = 30 m, $\angle AQB = 60°$, $\angle APB = 30°$, $\angle CQD = 45°$, $\angle CPD = 60°$, $\angle ABQ = \angle CDP = 90°$.



Now, from ΔABQ, we have

$$\tan 60° = \frac{AB}{BQ}$$

$$\Rightarrow \quad \sqrt{3} = \frac{AB}{BQ}$$

$$\therefore \quad BQ = \frac{AB}{\sqrt{3}} \quad \ldots(1)$$

From ΔABP, we have

$$\tan 30° = \frac{AB}{BP}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{AB}{BQ+PQ} = \frac{AB}{BQ+30}$$

$$\Rightarrow \quad BQ + 30 = AB\sqrt{3} \quad \ldots(2)$$

From ΔCPD, we have

$$\tan 60° = \frac{CD}{PD}$$

$$\Rightarrow \quad \sqrt{3} = \frac{CD}{PD}$$

$$\Rightarrow \quad PD = \frac{CD}{\sqrt{3}} \quad \ldots(3)$$

and from ΔQCD, we have

$$\tan 45° = \frac{CD}{QD}$$

$$\Rightarrow \quad 1 = \frac{CD}{PQ+PD} = \frac{CD}{30+PD} = 1$$

$$\Rightarrow \quad PD + 30 = CD \quad \ldots(4)$$

From (1) and (2), we have

$$\frac{AB}{\sqrt{3}}+30 = AB\sqrt{3}$$

$$\Rightarrow \quad AB\left(\sqrt{3}-\frac{1}{\sqrt{3}}\right) = 30$$

$$\Rightarrow \quad AB = \frac{30\sqrt{3}}{3-1} = 15\sqrt{3} \quad \ldots(5)$$

$\therefore$ From (1) and (5), we have

$$BQ = 15 \quad \ldots(6)$$

Again, from (3) and (4), we get

$$\frac{CD}{\sqrt{3}}+30 = CD$$

$$\Rightarrow \quad CD\left(1-\frac{1}{\sqrt{3}}\right) = 30$$

$$\Rightarrow \quad CD\left(\sqrt{3}-1\right) = 30\sqrt{3}$$

$$\Rightarrow \quad CD = \frac{30\sqrt{3}\left(\sqrt{3}+1\right)}{\left(\sqrt{3}-1\right)\left(\sqrt{3}+1\right)}$$

$$= \frac{30\left(3+\sqrt{3}\right)}{2}$$

$$= 15\left(3+\sqrt{3}\right)$$

$\therefore$ From (3) and (7), we get

$$PD = \frac{15\left(3+\sqrt{3}\right)}{\sqrt{3}}$$

$$= \frac{45+15\sqrt{3}}{\sqrt{3}}$$

$$= \frac{45\sqrt{3}}{3}+15$$

$$= 15 + 15\sqrt{3} \quad \ldots(7)$$

$$\therefore \quad BD = BQ + QP + PD$$

$$= (15 + 30 + 15 + 15\sqrt{3}) \text{ m}$$

[From (1), (6) and (7)]

$$= (60 + 15\sqrt{3}) \text{ m.}$$

Hence, the required height of the poles are $\mathbf{15\sqrt{3}}$ and $\mathbf{15\left(3+\sqrt{3}\right)}$ and the required distance between the pole is BD = **(60 + 15$\sqrt{3}$**) m.