Trigonometric Ratios of Complementary Angles

EXERCISE 10

For Basic and Standard Levels

1. (*i*) $\frac{\sin 35°}{\cos 55°} = \frac{\sin(90° - 55°)}{\cos 55°} = \frac{\cos 55°}{\cos 55°} = \mathbf{1}$

[∵ $\sin(90° - \theta) = \cos\theta$]

(*ii*) $\frac{\cos 59°}{\sin 31°} = \frac{\cos(90° - 31°)}{\sin 31°} = \frac{\sin 31°}{\sin 31°} = \mathbf{1}$

[∵ $\cos(90° - \theta) = \sin\theta$]

(*iii*) $\frac{\sec 43°}{\text{cosec } 47°} = \frac{\sec(90° - 47°)}{\text{cosec } 47°} = \frac{\text{cosec } 47°}{\text{cosec } 47°} = \mathbf{1}$

[∵ $\sec(90° - \theta) = \text{cosec }\theta$]

(*iv*) $\frac{\tan 80°}{\cot 10°} = \frac{\tan(90° - 10°)}{\cot 10°} = \frac{\cot 10°}{\cot 10°} = \mathbf{1}$

[∵ $\tan(90° - \theta) = \cot\theta$]

(*v*) $\frac{\text{cosec } 28°}{\sec 62°} = \frac{\text{cosec}(90° - 62°)}{\sec 62°} = \frac{\sec 62°}{\sec 62°} = \mathbf{1}$

[∵ $\text{cosec}(90° - \theta) = \sec\theta$]

(*vi*) $\frac{\cot 46°}{\tan 44°} = \frac{\cot(90° - 44°)}{\tan 44°} = \frac{\tan 44°}{\tan 44°} = \mathbf{1}$

[∵ $\cot(90° - \theta) = \tan\theta$]

2. (*i*) $\frac{\cos 80°}{\sin 10°} + \cos 59° \text{ cosec } 31°$

$= \frac{\cos(90° - 10°)}{\sin 10°} + \cos(90° - 31°) \text{ cosec } 31°$

$= \frac{\sin 10°}{\sin 10°} + \sin 31° \text{ cosec } 31°$

[∵ $\cos(90° - \theta) = \sin\theta$]

$= 1 + 1$ [∵ $\text{cosec }\theta = \frac{1}{\sin\theta}$]

$= \mathbf{2}$

(*ii*) $\frac{\sin 55°}{\cos 35°} + \frac{\text{cosec } 35°}{\sec 55°} - 5\cos 55° \text{ cosec } 35°$

$= \frac{\sin(90° - 35°)}{\cos 35°} + \frac{\text{cosec}(90° - 55°)}{\sec 55°} - 5\cos 55° \text{ cosec}(90° - 55°)$

$= \frac{\cos 35°}{\cos 35°} + \frac{\sec 55°}{\sec 55°} - 5\cos 55° \sec 55°$

[∵ $\sin(90° - \theta) = \cos\theta$ and $\text{cosec}(90 - \theta) = \sec\theta$]

$= 1 + 1 - 5\cos 55° \times \frac{1}{\cos 55°}$

$= 2 - 5$

$= \mathbf{-3}$

(*iii*) $\frac{2\sin 43°}{\cos 47°} - \frac{\cos 30°}{\tan 60°} - \sqrt{2}\sin 45°$

$= \frac{2\sin(90° - 47°)}{\cos 47°} - \frac{\cot(90° - 60°)}{\tan 60°} - \sqrt{2}\sin 45°$

$= \frac{2\cos 47°}{\cos 47°} - \frac{\tan 60°}{\tan 60°} - \sqrt{2} \times \frac{1}{\sqrt{2}}$

[∵ $\sin(90° - \theta) = \cos\theta$, $\cot(90° - \theta) = \tan\theta$ and $\sin 45° = \frac{1}{\sqrt{2}}$]

$= 2 - 1 - 1$

$= \mathbf{0}$

(*iv*) $\left(\frac{\cos 80°}{\sin 10°}\right)\left(\frac{\cot 59°}{\tan 31°}\right) + 2\left(\frac{\text{cosec } 51°}{\sec 39°}\right)$

$= \left[\frac{\cos(90° - 10°)}{\sin 10°}\right]\frac{\cot(90° - 31°)}{\tan 31°} + 2\left[\frac{\text{cosec}(90° - 39°)}{\sec 39°}\right]$

$= \left(\frac{\sin 10°}{\sin 10°}\right)\left(\frac{\tan 31°}{\tan 31°}\right) + 2\left(\frac{\sec 39°}{\sec 39°}\right)$

[∵ $\cos(90° - \theta) = \sin\theta$, $\cot(90° - \theta) = \tan\theta$ and $\text{cosec}(90° - \theta) = \sec\theta$]

$= (1)(1) + 2(1)$

$= 1 + 2$

$= \mathbf{3}$

(*v*) $\frac{\tan 50° + \sec 50°}{\cot 40° + \text{cosec } 40°} + \cos 40° \text{ cosec } 50°$

$= \frac{\tan(90° - 40°) + \sec(90° - 40°)}{\cot 40° + \text{cosec } 40°} + \cos(90° - 50°) \text{ cosec } 50°$

$= \frac{(\cot 40° + \text{cosec } 40°)}{(\cot 40° + \text{cosec } 40°)} + \sin 50° \text{ cosec } 50°$

[∵ $\tan(90° - \theta) = \cot\theta$, $\sec(90° - \theta) = \text{cosec }\theta$ and $\cos(90° - \theta) = \sin\theta$]

$= 1 + 1$ [∵ $\text{cosec }\theta = \frac{1}{\sin\theta}$]

$= \mathbf{2}$

(*vi*) $\frac{5\sin 87° + \tan 72° - \sec 5°}{\cot 18° - \text{cosec } 85° + 5\cos 3°}$

$= \frac{5\sin(90° - 3°) + \tan(90° - 18°) - \sec(90° - 85°)}{\cot 18° - \text{cosec } 85° + 5\cos 3°}$

$= \frac{(5\cos 3° + \cot 18° - \text{cosec } 85°)}{(\cot 18° - \text{cosec } 85° + 5\cos 3°)} = \mathbf{1}$

[∵ $\sin(90° - \theta) = \cos\theta$, $\tan(90° - \theta) = \cot\theta$ and $\sec(90° - \theta) = \text{cosec }\theta$]

3. (*i*) LHS = $\mathbf{\cos 67° - \sin 23°}$

$= \cos(90° - 23°) - \sin 23°$

$= \sin 23° - \sin 23°$

[∵ $\cos(90° - \theta) = \sin\theta$]

$= \mathbf{0}$

= RHS

(*ii*) LHS = $\mathbf{\cot 79° - \tan 11°}$

$= \cot(90° - 11°) - \tan 11°$

$= \tan 11° - \tan 11°$

$[\because \cot (90° - \theta) = \tan \theta]$

$= \mathbf{0}$

= RHS

(*iii*) LHS = $\mathbf{cosec^2 67° - tan^2 23°}$

$= \text{cosec}^2 67° - \tan^2 (90° - 67°)$

$= \text{cosec}^2 67° - \cot^2 67°$

$[\because \tan (90° - \theta) = \cot \theta]$

$= \mathbf{1}$ $[\because \text{cosec}^2 \theta - \cot^2 \theta = 1]$

= RHS

(*iv*) LHS = $\mathbf{sec^2 43° - cot^2 47°}$

$= \sec^2 (90° - 47) - \cot^2 47°$

$= \text{cosec}^2 47° - \cot^2 47°$

$[\because \sec (90° - \theta) = \text{cosec}\, \theta]$

$= \mathbf{1}$ $[\because \text{cosec}^2 \theta - \cot^2 \theta = 1]$

= RHS

(*v*) LHS = $\mathbf{(sin\ 25° + cos\ 65°)\ (sin\ 25° - cos\ 65°)}$

$= \sin^2 25° - \cos^2 65°$

$[\because (a + b)(a - b) = a^2 - b^2]$

$= \sin^2 (90° - 65°) - \cos^2 65°$

$= \cos^2 65° - \cos^2 65°$

$[\because \sin (90° - \theta) = \cos \theta]$

$= \mathbf{0}$ = RHS

4. (*i*) $\mathbf{sin^2 20° + sin^2 70° - tan^2 45°}$

$= \sin^2 20° + \sin^2 (90° - 20°) - \tan^2 45°$

$= \sin^2 20° + \cos^2 20° - \tan^2 45°$

$[\because \sin (90° - \theta) = \cos \theta]$

$= 1 - (1)^2$

$[\because \sin^2 \theta + \cos^2 \theta = 1 \text{ and } \tan 45° = 1]$

$= 1 - 1$

$= \mathbf{0}$

(*ii*) $\mathbf{\dfrac{2\sin^2 63° + 1 + 2\sin^2 27°}{3\cos^2 17° - 2 + 3\cos^2 73°}}$

$$= \frac{2\sin^2 (90° - 27°) + 1 + 2\sin^2 27°}{3\cos^2 17° - 2 + 3\cos^2 (90° - 17°)}$$

$$= \frac{2\cos^2 27° + 2\sin^2 27° + 1}{3\cos^2 17° + 3\sin^2 17° - 2}$$

$[\because \sin (90° - \theta) = \cos \theta \text{ and } \cos (90° - \theta) = \sin \theta]$

$$= \frac{2(\cos^2 27° + \sin^2 27°) + 1}{3(\cos^2 17° + 3\sin^2 17°) - 2}$$

$$= \frac{2 + 1}{3 - 2} \quad [\because \sin^2 \theta + \cos^2 \theta = 1]$$

$$= \frac{3}{1}$$

$= \mathbf{3}$

(*iii*) $\mathbf{\left(\dfrac{\sin 35°}{\cos 55°}\right)^2 + \left(\dfrac{\cos 55°}{\sin 35°}\right) - 2\cos 60°}$

$$= \frac{\sin^2 35°}{\cos^2 55°} + \frac{\cos (90° - 35°)}{\sin 35°} - 2\cos 60°$$

$$= \frac{\sin^2 35°}{\cos^2 (90° - 35°)} + \frac{\cos (90° - 35°)}{\sin 35°} - 2\cos 60°$$

$$= \frac{\sin^2 35°}{\sin^2 35°} + \frac{\sin 35°}{\sin 35°} - 2\cos 60°$$

$[\because \cos (90° - \theta) = \sin \theta]$

$$= 1 + 1 - 2\left(\frac{1}{2}\right) \quad [\because \cos 60° = \tfrac{1}{2}]$$

$= 2 - 1$

$= \mathbf{1}$

(*iv*) $\mathbf{\left(\dfrac{\sin 47°}{\cos 43°}\right)^3 + \left(\dfrac{\cos 43°}{\sin 47°}\right)^3 - 4\cos^2 45°}$

$$= \left[\frac{\sin (90° - 43°)}{\cos 43°}\right]^3 + \left[\frac{\cos 43°}{\sin (90° - 43°)}\right]^3 - 4\cos^2 45°$$

$$= \left(\frac{\cos 43°}{\cos 43°}\right)^3 + \left(\frac{\cos 43°}{\cos 43°}\right)^3 - 4\left(\frac{1}{\sqrt{2}}\right)^2$$

$[\because \sin (90° - \theta) = \cos \theta \text{ and } \cos 45° = \tfrac{1}{\sqrt{2}}]$

$$= (1)^3 + (1)^3 - 4 \times \frac{1}{2}$$

$= 1 + 1 - 2$

$= \mathbf{0}$

(*v*) $\mathbf{\dfrac{\cos^2 20° + \cos^2 70°}{\sin^2 59° + \sin^2 31°} + \sin 35° \sec 55°}$

$$= \frac{\cos^2 (90° - 70°) + \cos^2 70°}{\sin^2 59° + \sin^2 (90° - 59°)} + \sin 35° \sec (90° - 35°)$$

$$= \frac{\sin^2 70° + \cos^2 70°}{\sin^2 59° + \cos^2 59°} + \sin 35° \text{ cosec } 35°$$

$[\because \cos (90° - \theta) = \sin \theta, \sin (90° - \theta) = \cos \theta$
and $\sec (90° - \theta) = \text{cosec}\, \theta]$

$$= \frac{1}{1} + \sin 35° \times \frac{1}{\sin 35°} \quad [\because \sin^2 \theta + \cos^2 \theta = 1]$$

$= 1 + 1$

$= \mathbf{2}$

(*vi*) $\mathbf{\dfrac{\sec^2 54° - \cot^2 36°}{\text{cosec}^2 57° - \tan^2 33°} + 2\sin^2 38° \sec^2 52° - \sin^2 45°}$

$$= \frac{\sec^2 54° - \cot^2 (90° - 54°)}{\text{cosec}^2 57° - \tan^2 (90° - 57°)} + 2\sin^2 38° \sec^2 (90° - 38°) - \sin^2 45°$$

$$= \frac{\sec^2 54° - \tan^2 54°}{\text{cosec}^2 57° - \cot^2 57°} + 2\sin^2 38° \text{ cosec}^2 38° - \sin^2 45°$$

$[\because \cot (90° - \theta) = \tan \theta, \tan (90° - \theta) = \cot \theta$
and $\sec (90° - \theta) = \text{cosec}\, \theta]$

$$= \frac{1}{1} + 2\sin^2 38° \times \frac{1}{\sin^2 38°} - \left(\frac{1}{\sqrt{2}}\right)^2$$

$[\because \sec^2 \theta - \tan^2 \theta = 1, \text{cosec}^2 \theta - \cot^2 \theta = 1$
and $\sin 45° = \tfrac{1}{\sqrt{2}}]$

$$= 1 + 2 - \frac{1}{2} = 3 - \frac{1}{2} = \mathbf{\frac{5}{2}}$$

(*vii*) $\mathbf{\dfrac{\text{cosec}^2 63° + \tan^2 24°}{\cot^2 66° + \sec^2 27°} + \dfrac{\sin^2 63° + \cos 63° \sin 27° + \sin 27° \sec 63°}{2(\text{cosec}^2 65° - \tan^2 25°)}}$

$$\Rightarrow \frac{\text{cosec}^2(90°-27°)+\tan^2(90°-66°)}{\cot^2 66°+\sec^2 27°} +$$

$$\frac{\sin^2 63°+\cos 63° \sin(90°-63°)+\sin(90°-63°)\times\frac{1}{\cos 63°}}{2\left(\text{cosec}^2 65°-\tan^2(90°-65°)\right)}$$

$$\Rightarrow \frac{\sec^2 27°+\cot^2 66°}{\cot^2 66°+\sec^2 27} +$$

$$\frac{\sin^2 63°+\cos 63° \cos 63°+\cos 63°\times\frac{1}{\cos 63°}}{2\left(\text{cosec}^2 65°-\cot^2 65°\right)}$$

$$\Rightarrow 1+\frac{\sin^2 63°+\cos^2 63°+1}{2(1)}$$

$$\Rightarrow 1+\frac{1+1}{2}$$

$$\Rightarrow 1+\frac{2}{2}$$

$\Rightarrow$ 1 + 1

$\Rightarrow$ **2**

5. (*i*) **cosec 61° + cot 61°**

= cosec (90° – 29°) + cot (90° – 29°)

= sec 29° + tan 29°

[∵ cosec (90° – θ) = sec θ and cot (90° – θ) = tan θ]

(*ii*) **tan 67° + sec 89°**

= tan (90° – 23°) + sec (90° – 1°)

= cot 23° + cosec 1°

[∵ tan (90° – θ) = cot θ and sec (90° – θ) = cosec θ]

(*iii*) **cos 83° + tan 78°**

= cos (90° – 7°) + tan (90° – 12°)

= sin 7° + cot 12°

[∵ cos (90° – θ) = sin θ and tan (90° – θ) = cot θ]

(*iv*) **sin 85° + cosec 85°**

= sin (90° – 5°) + cosec (90° – 5°)

= cos 5° + sec 5°

[∵ sin (90° – θ) = cos θ and cosec (90° – θ) = sec θ]

6. (*i*) **tan 3θ = cot 7θ** [Given]

$\Rightarrow$ tan 3θ = tan (90° – 7θ) [∵ cot θ = tan (90° – θ)]

$\Rightarrow$ 3θ = 90° – 7θ

$\Rightarrow$ 3θ + 7θ = 90°

$\Rightarrow$ 10θ = 90°

$\Rightarrow$ θ = 9°

Hence, θ = **9°**

(*ii*) **cos 2θ = sin 4θ** [Given]

$\Rightarrow$ sin (90° – 2θ) = sin 4θ [∵ cos θ = sin (90° – θ)]

$\Rightarrow$ 90° – 2θ = 4θ

$\Rightarrow$ 6θ = 90°

$$\Rightarrow \theta = \left(\frac{90}{6}\right)^\circ = 15°$$

Hence, θ = **15°**

(*iii*) sin θ = cos θ

$\Rightarrow$ cos (90° – θ) = cos θ [∵ sin θ = cos (90° – θ)]

$\Rightarrow$ 90° – θ = θ

$\Rightarrow$ 2θ = 90°

$\Rightarrow$ θ = 45°

Hence, θ = **45°**

7. (*i*) LHS = **sec 50° sin 40° + cos 40° cosec 50°**

= sec (90° – 40°) sin 40° + cos 40° cosec (90° – 40°)

= cosec 40° sin 40° + cos 40° sec 40°

[∵ sec (90° – θ) = cosec θ and cosec (90° – θ) = sec θ]

$$= \frac{1}{\sin 40°} \times \sin 40° + \cos 40° \times \frac{1}{\cos 40°}$$

= 1 + 1 = **2** = RHS

(*ii*) LHS = **sin 63° cos 27° + cos 63° sin 27°**

= sin 63° cos (90° – 63°) + cos 63° sin (90° – 63°)

= sin 63° sin 63° + cos 63° cos 63°

[∵ cos (90° – θ) = sin θ and sin (90° – θ) = cos θ]

= $\sin^2 63° + \cos^2 63°$

= **1** [∵ $\sin^2 \theta + \cos^2 \theta = 1$] = RHS

(*iii*) LHS = **2(sin 42° sec 48°) –** $\frac{1}{2}$ **(cos 42° cosec 48°)**

= 2[sin 42° sec (90° – 42°)] – $\frac{1}{2}$ [cos 42° cosec (90° – 42°)]

= 2(sin 42° cosec 42°) – $\frac{1}{2}$ (cos 42° sec 42°)

[∵ sec (90° – θ) = cosec θ and cosec (90° – θ) = sec θ]

$$= 2\left(\sin 42° \times \frac{1}{\sin 42°}\right) - \frac{1}{2}\left(\cos 42° \times \frac{1}{\cos 42°}\right)$$

$$= 2(1) - \frac{1}{2}(1) = 2 - \frac{1}{2} = \mathbf{\frac{3}{2}} = \text{RHS}$$

8. (*i*) **tan 1° tan 2° tan 3°… tan 89°**

= (tan 1° tan 89°) (tan 2° tan 88°) (tan 3° tan 87°) … (tan 44° tan 46°) tan 45°

= [tan 1° tan (90° – 1°)][tan 2° tan(90° – 2°)] [tan 3° tan (90° – 3°)]…[tan 44° tan(90° – 44°)]tan 45°

= (tan 1° cot 1°) (tan 2° cot 2) (tan 3° cot 3°) … (tan 44° cot 44°) tan 45° [∵ tan (90° – θ) = cot θ]

= 1 × 1 × 1 … 1 × 1

= **1**

(*ii*) **tan 15° tan 20° tan 70° tan 75°**

= (tan 15° tan 75°) (tan 20° tan 70°)

= [tan 15° tan (90° – 15°)] [tan 20° tan (90° – 20°)]

= (tan 15° cot 15°) (tan 20° cot 20°) [∵ tan (90° – θ) = cot θ]

= 1 × 1 = **1**

(*iii*) **cot 18° cot 39° cot 51° cot 60° cot 72°**

= (cot 18° cot 72°) (cot 39° cot 51°) cot 60°

= [cot 18° cot (90° – 18°)] [cot 39° cot (90° – 39°)] cot 60°

= (cot 18° tan 18°) (cot 39° tan 39°) cot 60° [∵ cot (90° – θ) = tan θ]

$$= (1)(1)\left(\frac{1}{\sqrt{3}}\right) = \mathbf{\frac{1}{\sqrt{3}}}$$

(*iv*) **cot 12° cot 38° cot 52° cot 60° cot 78°**

$\Rightarrow$ (cot 12° cot 78°) (cot 38° cot 52°) cot 60°

$\Rightarrow$ (cot (90° – 78°) cot 78°) (cot (90° – 52°) cot 52°) cot 60°

$\Rightarrow$ (tan 78° cot 78°) (tan 52° cot 52°) cot 60°

$$\Rightarrow (1)(1)\left(\frac{1}{\sqrt{3}}\right)$$

$$\Rightarrow \mathbf{\frac{1}{\sqrt{3}}}$$

(*v*) **tan 35° tan 40° tan 45° tan 50° tan 55°**

= (tan 35° tan 55°) (tan 40° tan 50°) tan 45°

= [tan 35° tan (90° – 35°)] [tan 40° tan (90° – 40°)] tan 45°

= (tan 35° cot 35°) (tan 40° cot 40°) tan 45° [∵ tan (90° – θ) = cot θ]

= (1) (1) (1) = **1**

(*vi*) **tan 5° tan 25° tan 30° tan 65° tan 85°**

⇒ (tan 5° tan 85°) (tan 25° tan 65°) tan 30°

⇒ (tan (90° – 85°) tan 85°) (tan (90° – 65°) tan 65°) tan 30°

⇒ (cot 85° tan 85°) (cot 65° tan 65°) tan 30°

⇒ $(1)\,(1)\left(\frac{1}{\sqrt{3}}\right)$

⇒ $\mathbf{\frac{1}{\sqrt{3}}}$

9. (*i*) $\mathbf{\frac{\cos 70°}{\sin 20°} + \frac{\cos 55° \operatorname{cosec} 35°}{\tan 5° \tan 25° \tan 45° \tan 65° \tan 85°}}$

$$\Rightarrow \frac{\cos(90°-20°)}{\sin 20°} + \frac{\cos(90°-35°)\operatorname{cosec} 35°}{(\tan(90°-85°)\tan 85°)(\tan(90°-65°)\tan 65°)\tan 45°}$$

$$\Rightarrow \frac{\sin 20°}{\sin 20°} + \frac{\sin 35° \times \frac{1}{\sin 35°}}{(\cot 85° \tan 85°)(\cot 65° \tan 65°)\tan 45°}$$

$$\Rightarrow 1 + \frac{1}{(1)(1)(1)}$$

⇒ 1 + 1

⇒ **2**

(*ii*) $\mathbf{\frac{\sin 18°}{\cos 72°} + \sqrt{3}\,[\tan 10° \tan 30° \tan 40° \tan 50° \tan 80°]}$

$$= \frac{\sin(90° - 72°)}{\cos 72°} + \sqrt{3}\ [(\tan 10° \tan 80°)(\tan 40° \tan 50°) \tan 30°]$$

$$= \frac{\cos 72°}{\cos 72°} + \sqrt{3}\,[\tan 10° \tan(90° - 10°)][\tan 40° \tan(90° - 40°)] \tan 30°$$

= 1 + $\sqrt{3}$ (tan 10° cot 10°) (tan 40° cot 40°) tan 30°

= 1 + $\sqrt{3}\,(1)(1)\left(\frac{1}{\sqrt{3}}\right)$

= 1 + 1

= **2**

(*iii*) $\mathbf{2\left(\frac{\cos 65°}{\sin 25°}\right) - \frac{\tan 20°}{\cot 70°} - \sin 90° + \tan 5° \tan 35° \tan 60° \tan 55° \tan 85°}$

$$= \frac{2\cos(90° - 25°)}{\sin 25°} - \frac{\tan(90° - 70°)}{\cot 70°} - 1 + (\tan 5° \tan 85°)(\tan 35° \tan 55°) \tan 60°$$

[∵ sin 90° = 1]

$$= \frac{2\sin 25°}{\sin 25°} - \frac{\cot 70°}{\cot 70°} - 1 + [\tan 5° \tan(90° - 5°)][\tan 35° \tan(90° - 35°)] \tan 60°$$

[∵ cos (90° – θ) = sin θ, tan (90° – θ) = cot θ]

= 2 – 1 – 1 + (tan 5° cot 5°) (tan 35° cot 35°) $\sqrt{3}$

[∵ tan (90° – θ) = cot θ and tan 60° = $\sqrt{3}$]

= (1)(1)($\sqrt{3}$)

= $\mathbf{\sqrt{3}}$

(*iv*) $\mathbf{\frac{\sin 15° \cos 75° + \cos 15° \sin 75°}{\tan 5° \tan 30° \tan 35° \tan 55° \tan 85°}}$

$$= \frac{\sin 15° \cos(90° - 15°) + \cos 15° \sin(90° - 15°)}{(\tan 5° \tan 85°)(\tan 35° \tan 55°) \tan 30°}$$

$$= \frac{\sin 15° \sin 15° + \cos 15° \cos 15°}{\tan 5° \tan(90° - 5)(\tan 35° \tan(90° - 35°) \times \frac{1}{\sqrt{3}}}$$

[∵ cos (90° – θ) = sin θ and sin (90° – θ) = cos θ]

$$= \frac{(\sin^2\theta + \cos^2\theta)\sqrt{3}}{(\tan 5° \cot 5°)(\tan 35° \cot 35°)}$$

[∵ tan (90° – θ) = cot θ]

$$= \frac{\sqrt{3}}{(1)(1)}$$ [∵ $\sin^2\theta + \cos^2\theta = 1$]

= $\mathbf{\sqrt{3}}$

(*v*) $\mathbf{2\left(\frac{\cos 58°}{\sin 32°}\right) - \sqrt{3}\left(\frac{\cos 38° \operatorname{cosec} 52°}{\tan 15° \tan 60° \tan 75°}\right)}$

$$= 2\,\frac{\cos(90° - 32°)}{\sin 32°} - \sqrt{3}\left[\frac{\cos 38° \operatorname{cosec}(90° - 38°)}{\tan 15° \tan 75° \sqrt{3}}\right]$$

[∵ tan 60° = $\sqrt{3}$]

$$= 2\,\frac{\sin 32°}{\sin 32°} - \frac{\cos 38° \sec 38°}{\tan 15° \tan(90° - 15°)}$$

[∵ cos (90° – θ) = sin θ]

$$= 2 - \frac{1}{\tan 15° \cot 15°}$$ [∵ tan (90° – θ) = cot θ]

$$= 2 - \frac{1}{1}$$

= **1**

(*vi*) $\mathbf{\frac{3\cos 55°}{7\sin 35°} - \frac{4(\cos 70° \operatorname{cosec} 20°)}{7(\tan 5° \tan 25° \tan 45° \tan 65° \tan 85°)}}$

$$= \frac{3\cos(90° - 35°)}{7\sin 35°} - \frac{4[\cos 70° \operatorname{cosec}(90° - 70°)]}{7[(\tan 5° \tan 85°)(\tan 25° \tan 65°)\tan 45°]}$$

$$= \frac{3\sin 35°}{7\sin 35°} - \frac{4(\cos 70° \sec 70°)}{7[\tan 5° \tan(90° - 5°) \tan 25° \tan(90° - 25°)(1)]}$$

[∵ cos (90° – θ) = sin θ, cosec (90° – θ) = sec θ and tan 45° = 1]

$$= \frac{3}{7} - \frac{4(1)}{7(\tan 5° \cot 5°)(\tan 25° \cot 25°)}$$

$[\because \tan (90° - \theta) = \cot \theta]$

$$= \frac{3}{7} - \frac{4}{7(1)(1)} = \frac{3}{7} - \frac{4}{7} = \frac{3-4}{7} = \frac{-1}{7}$$

(*vii*) $\mathbf{2\frac{\sin 68°}{\cos 22°} - \frac{2\cot 15°}{5\tan 75°} - \frac{3\tan 45° \tan 20° \tan 40° \tan 50° \tan 70°}{5}}$

$$\frac{2\sin(90°-22°)}{\cos 22°} - \frac{2\cot(90°-75°)}{5\tan 75°} - \frac{3\tan 45°(\tan(90°-70°)\tan 70°)(\tan(90°-50°)\tan 50°)}{5}$$

$$\Rightarrow \frac{2\cos 22°}{\cos 22°} - \frac{2\tan 75°}{5\tan 75°} - \frac{3\tan 45°(\cot 70° \tan 70°)(\cot 50° \tan 50°)}{5}$$

$$\Rightarrow 2 - \frac{2}{5} - \frac{3(1)(1)(1)}{5}$$

$$\Rightarrow 2 - \frac{2}{5} - \frac{3}{5} \Rightarrow \frac{10-2-3}{5} \Rightarrow \frac{10-5}{5} \Rightarrow \frac{5}{5} \Rightarrow \mathbf{1}$$

(*viii*) $\mathbf{(\sin^2 25° + \sin^2 65°) + \sqrt{3}(\tan 5° \tan 15° \tan 30° \tan 75° \tan 85°)}$

$\Rightarrow [\sin^2 25° + \sin^2(90° - 25°)] + \sqrt{3}\,[\tan (90° - 85°) \tan 85°][\tan (90° - 75°) \tan 75°] \tan 30°$

$\Rightarrow (\sin^2 25° + \cos^2 25°) + \sqrt{3}\,[(\cot 85° \tan 85°)(\cot 75° \tan 75°) \tan 30°]$

$$\Rightarrow 1 + \sqrt{3}\left((1)(1)\left(\frac{1}{\sqrt{3}}\right)\right)$$

$\Rightarrow 1 + 1$

$\Rightarrow \mathbf{2}$

(*ix*) $\mathbf{\frac{2}{3}\text{cosec}^2 58° - \frac{2}{3} \cot 58° \tan 32° - \frac{5}{3} \tan 13° \tan 37° \tan 45° \tan 53° \tan 77°}$

$$= \frac{2}{3} \text{cosec}^2 58° - \frac{2}{3} \cot 58° \tan (90° - 58°) - \frac{5}{3} (\tan 13° \tan 77°)(\tan 37° \tan 53°) \tan 45°$$

$$= \frac{2}{3} \text{cosec}^2 58° - \frac{2}{3} \cot 58° \cot 58° - \frac{5}{3} [\tan 13° \tan (90° - 13°)] [\tan 37° \tan (90° - 37°)](1)$$

$[\because \tan (90° - \theta) = \cot \theta \text{ and } \tan 45° = 1]$

$$= \frac{2}{3} (\text{cosec}^2 58° - \cot^2 58°) - \frac{5}{3} (\tan 13° \cot 13°)(\tan 37° \cot 37°)$$

$[\because \tan (90° - \theta) = \cot \theta]$

$$= \frac{2}{3} (1) - \frac{5}{3} (1)(1)$$ $[\because \text{cosec}^2 \theta - \cot^2 \theta = 1]$

$$= \frac{2}{3} - \frac{5}{3} = \frac{2-5}{3} = \frac{-3}{3}$$

$= \mathbf{-1}$

(*x*) $\mathbf{\frac{\sec 39°}{\text{cosec } 51°} + \frac{2}{\sqrt{3}} \tan 17° \tan 38° \tan 60° \tan 52° \tan 73° - 3(\sin^2 31° + \sin^2 59°)}$

$$= \frac{\sec (90° - 51°)}{\text{cosec } 51°} + \frac{2}{\sqrt{3}} (\tan 17° \tan 73°)(\tan 38° \tan 52°) \tan 60° - 3 [\sin^2 31° + \sin^2 (90° - 31°)]$$

$$= \frac{\text{cosec } 51°}{\text{cosec } 51°} + \frac{2}{\sqrt{3}} [\tan 17° \tan (90° - 17°)][\tan 38° \tan (90° - 38°)] \sqrt{3} - 3[\sin^2 31° + \cos^2 31°]$$

$[\because \sec (90° - \theta) = \text{cosec } \theta \text{ and } \sin (90° - \theta) = \cos \theta]$

$$= 1 + \frac{2}{\sqrt{3}} (\tan 17° \cot 17°)(\tan 38° \cot 38°) \sqrt{3} - 3(1)$$

$[\because \tan (90° - \theta) = \cot \theta \text{ and } \sin^2 \theta + \cos^2 \theta = 1]$

$$= 1 + \frac{2}{\sqrt{3}} (1)(1)(\sqrt{3}) - 3$$

$= 1 + 2 - 3$

$= \mathbf{0}$

(*xi*) $\mathbf{3\left(\frac{\sin 36°}{\cos 54°}\right)^2 - 2\left(\frac{\tan 18°}{\cot 72°}\right)^3 + 2 \tan 13° \tan 21° \tan 69° \tan 77°}$

$$= 3\left[\frac{\sin (90° - 54°)}{\cos 54°}\right]^2 - 2\left[\frac{\tan (90° - 72°)}{\cot 72°}\right]^3 + 2(\tan 13° \tan 77°) (\tan 21° \tan 69°)$$

$$= 3\left(\frac{\cos 54°}{\cos 54°}\right)^2 - 2\left(\frac{\cot 72°}{\cot 72°}\right)^3 + 2[\tan 13° \tan (90° - 13°) \tan 21° \tan (90° - 21°)]$$

$[\because \sin (90° - \theta) = \cos \theta, \tan (90° - \theta) = \cot \theta]$

$= 3(1)^2 - 2(1)^3 + 2 (\tan 13° \cot 13°) (\tan 21° \cot 21°)$

$[\because \tan (90° - \theta) = \cot \theta]$

$= 3(1) - 2(1) + 2(1) (1)$

$= 3 - 2 + 2$

$= \mathbf{3}$

(*xii*) $$\mathbf{\frac{3\tan 25° \tan 40° \tan 50° \tan 65° - \frac{1}{2}\tan^2 60°}{4(\cos^2 29° + \cos^2 61°)}}$$

$$= \frac{3(\tan 25° \tan 65°)(\tan 40° \tan 50°) - \frac{1}{2}(\sqrt{3})^2}{4[\cos^2 (90° - 61°) + \cos 61°]}$$

$[\because \tan 60° = \sqrt{3}]$

$$= \frac{3[\tan 25° \tan (90° - 25°)][\tan 40° \tan (90° - 40°)] - \frac{3}{2}}{4(\sin^2 61° + \cos^2 61°)}$$

$[\because \cos (90° - \theta) = \sin \theta]$

$$= \frac{3(\tan 25° \cot 25°)(\tan 40° \cot 40°) - \frac{3}{2}}{4(1)}$$

$[\because \tan (90° - \theta) = \cot \theta \text{ and } \sin^2 \theta + \cos^2 \theta = 1]$

$= \dfrac{3(1)(1) - \frac{3}{2}}{4} = \dfrac{3 - \frac{3}{2}}{4} = \dfrac{\frac{6-3}{2}}{4}$

$= \dfrac{3}{2} \times \dfrac{1}{4} = \mathbf{\dfrac{3}{8}}$

(*xiii*) $\mathbf{\dfrac{\sin^2 40° + \sin^2 50°}{\cos^2 20° + \cos^2 70°} + \tan 10° \tan 20° \tan 60° \tan 70° \tan 80°}$

$= \dfrac{\sin^2 40° + \sin^2 (90° - 40°)}{\cos^2 (90° - 70°) + \cos^2 70°} + (\tan 10° \tan 80°)(\tan 20° \tan 70°) \tan 60°$

$= \dfrac{\sin^2 40° + \cos^2 40°}{\sin^2 70° + \cos^2 70°} + [\tan 10° \tan (90° - 10°)][\tan 20° \tan (90° - 20°)] \sqrt{3}$

[$\because \sin (90° - \theta) = \cos \theta$ and $\cos (90° - \theta) = \sin \theta$ and $\tan 60° = \sqrt{3}$]

$= \dfrac{1}{1} + (\tan 10° \cot 10°)(\tan 20° \cot 20°)\sqrt{3}$

[$\because \tan (90° - \theta) = \cot \theta$]

$= 1 + (1)(1)(\sqrt{3}) = \mathbf{1 + \sqrt{3}}$

(*xiv*) $\mathbf{\dfrac{\cos^2 20° + \cos^2 70°}{\sec^2 50° - \cot^2 40°} + 2 \operatorname{cosec}^2 58° - 2 \cot 58° \tan 32° - 4 \tan 13° \tan 37° \tan 45° \tan 53° \tan 77°}$

$\Rightarrow \dfrac{\cos^2 (90° - 70°) + \cos^2 70°}{\sec^2 (90° - 40°) - \cot^2 40°} + 2 \operatorname{cosec}^2 58° - 2 \cot 58° \tan (90° - 58°) - 4[\tan (90° - 77°) \tan 77°][\tan (90° - 53°) \tan 53°] \tan 45°$

$\Rightarrow \dfrac{\sin^2 70° + \cos^2 70°}{\operatorname{cosec}^2 40° - \cot^2 40°} + 2 \operatorname{cosec}^2 58° - 2 \cot^2 58° - 4 (\cot 77° \tan 77°)(\cot 53° \tan 53°) \tan 45°$

$\Rightarrow \dfrac{1}{1} + 2 - 4(1)(1)(1)$

$\Rightarrow 1 + 2 - 4$

$\Rightarrow 3 - 4$

$\Rightarrow \mathbf{-1}$

10. (*i*) LHS $= \mathbf{\cos (90° - \theta) \operatorname{cosec} (90° - \theta)}$

$= \sin \theta \sec \theta$

$= \dfrac{\sin \theta}{\cos \theta} = \mathbf{\tan}\ \theta =$ RHS

(*ii*) LHS $= \mathbf{\tan \theta + \tan (90° - \theta)}$

$= \tan \theta + \cot \theta$

$= \tan \theta + \dfrac{1}{\tan \theta}$

$= \dfrac{\tan^2 \theta + 1}{\tan \theta}$

[Using identity : $1 + \tan^2 \theta = \sec^2 \theta$]

$= \dfrac{\sec^2 \theta}{\tan \theta}$

$= \sec \theta \times \dfrac{1}{\sin \theta}$

$= \sec \theta \operatorname{cosec} \theta$

$= \sec \theta \sec (90° - \theta)$

[$\because \sec (90° - \theta) = \operatorname{cosec} \theta$]

= RHS

(*iii*) LHS $= \mathbf{\dfrac{\sin (90° - \theta) \tan (90° - \theta)}{\sec (90° - \theta) \cos \theta}}$

$= \dfrac{\cos \theta \cot \theta}{\operatorname{cosec} \theta \cos \theta}$

$= \dfrac{\cot \theta}{\operatorname{cosec} \theta} = \cot \theta \sin \theta$

$= \dfrac{\cos \theta}{\sin \theta} \sin \theta = \mathbf{\cos}\ \theta =$ RHS

(*iv*) LHS $= \mathbf{\dfrac{\cos \theta}{\sin (90° - \theta)} + \dfrac{\sin \theta}{\cos (90° - \theta)}}$

$= \dfrac{\cos \theta}{\cos \theta} + \dfrac{\sin \theta}{\sin \theta} = 1 + 1$

$= \mathbf{2} =$ RHS

(*v*) LHS $= \mathbf{\dfrac{\cos (90° - \theta)}{1 + \sin (90° - \theta)} + \dfrac{1 + \sin (90° - \theta)}{\cos (90° - \theta)}}$

$= \dfrac{\sin \theta}{1 + \cos \theta} + \dfrac{1 + \cos \theta}{\sin \theta}$

$= \dfrac{\sin^2 \theta + 1 + \cos^2 \theta + 2 \cos \theta}{\sin \theta (1 + \cos \theta)}$

$= \dfrac{1 + (\sin^2 \theta + \cos^2 \theta) + 2 \cos \theta}{\sin \theta (1 + \cos \theta)}$

$= \dfrac{2 + 2 \cos \theta}{\sin \theta (1 + \cos \theta)}$ [$\because \sin^2 \theta + \cos^2 \theta = 1$]

$= \dfrac{2(1 + \cos \theta)}{\sin \theta (1 + \cos \theta)}$

$= \dfrac{2}{\sin \theta} = \mathbf{2 \operatorname{cosec}}\ \theta$

= RHS

(*vi*) LHS $= \mathbf{\dfrac{\sin \theta \cos (90° - \theta) \cos \theta}{\sin (90° - \theta)} + \dfrac{\cos \theta \sin (90° - \theta) \sin \theta}{\cos (90° - \theta)}}$

$= \dfrac{\sin \theta \sin \theta \cos \theta}{\cos \theta} + \dfrac{\cos \theta \cos \theta \sin \theta}{\sin \theta}$

$= \sin^2 \theta + \cos^2 \theta$

$= \mathbf{1}$

= RHS

(*vii*) LHS $= \mathbf{\cot \theta \tan (90° - \theta) - \sec (90° - \theta) \operatorname{cosec} \theta + \sqrt{3} \tan 12° \tan 60° \tan 78°}$

$= \cot \theta \cot \theta - \operatorname{cosec} \theta \operatorname{cosec} \theta + \sqrt{3} (\tan (90° - 78°) \tan 78°) \tan 60°$

$= \cot^2 \theta - \operatorname{cosec}^2 \theta + \sqrt{3}(\cot 78° \tan 78°) \times \sqrt{3}$

$= -1 \times \sqrt{3} \times \sqrt{3}$ [$\because 1 + \cot^2 \theta = \operatorname{cosec}^2 \theta$]

$= -1 + 3$

$= \mathbf{2}$

= RHS

11. (*i*) $\sin\theta \cot\theta \cot(90° - \theta) \sec(90° - \theta)$

$= \sin\theta \cot\theta \tan\theta \operatorname{cosec}\theta$

$= (\sin\theta \operatorname{cosec}\theta)(\cot\theta \tan\theta)$

$= \left(\sin\theta \times \frac{1}{\sin\theta}\right)\left(\frac{1}{\tan\theta} \times \tan\theta\right)$

$= (1)(1)$

$= \mathbf{1}$

(*ii*) $\sin\theta \cos(90° - \theta) + \cos\theta \sin(90° - \theta)$

$= \sin\theta \sin\theta + \cos\theta \cos\theta$

$= \sin^2\theta + \cos^2\theta$

$= \mathbf{1}$

12. (*i*) LHS = $\tan^2 A \sec^2(90° - A) - \sin^2 A \operatorname{cosec}^2(90° - A)$

$= \tan^2 A \operatorname{cosec}^2 A - \sin^2 A \sec^2 A$

[$\because \sec(90° - A) = \operatorname{cosec} A$ and $\operatorname{cosec}(90° - A) = \sec A$]

$= \frac{\sin^2 A}{\cos^2 A}\left(\frac{1}{\sin^2 A}\right) - \sin^2 A\left(\frac{1}{\cos^2 A}\right)$

$= \frac{1}{\cos^2 A} - \tan^2 A$

$= \sec^2 A - \tan^2 A$

$= \mathbf{1}$ = RHS

(*ii*) LHS = $\frac{\operatorname{cosec}^2\theta \tan^2\theta \cot\theta}{\cot(90° - \theta)\sec^2\theta}$

$= \frac{\left(\frac{1}{\sin^2\theta}\right)\left(\frac{\sin^2\theta}{\cos^2\theta}\right)\left(\frac{\cos\theta}{\sin\theta}\right)}{\tan\theta\left(\frac{1}{\cos^2\theta}\right)}$

$= \frac{\frac{1}{\cos\theta\sin\theta}}{\left(\frac{\sin\theta}{\cos\theta}\right)\left(\frac{1}{\cos^2\theta}\right)}$

$= \frac{1}{(\cos\theta\sin\theta)} \frac{(\cos\theta)(\cos^2\theta)}{(\sin\theta)}$

$= \frac{\cos^2\theta}{\sin^2\theta} = \cot^2\theta$

$= \operatorname{cosec}^2\theta - 1$ [$\because 1 + \cot^2\theta = \operatorname{cosec}^2\theta$]

$= \mathbf{\sec^2(90° - \theta) - 1}$

[$\because \operatorname{cosec}\theta = \sec(90° - \theta)$]

= RHS

(*iii*) LHS = $\frac{\sec(90° - \theta)\operatorname{cosec}\theta - \tan(90° - \theta)\cot\theta + (\cos^2 35° + \cos^2 55°)}{\tan 5° \tan 15° \tan 45° \tan 75° \tan 85°}$

$= \frac{\operatorname{cosec}\theta\operatorname{cosec}\theta - \cot\theta\cot\theta + [\cos^2(90° - 55°) + \cos^2 55°]}{(\tan 5° \tan 85°)(\tan 15° \tan 75°)\tan 45°}$

[$\because \sec(90° - \theta) = \operatorname{cosec}\theta$ and $\tan(90° - \theta) = \cot\theta$]

$= \frac{\operatorname{cosec}^2\theta - \cot^2\theta + (\sin^2 55° + \cos^2 55°)}{[\tan 5° \tan(90° - 5°)][\tan 15° \tan(90° - 15°)]\tan 45°}$

$= \frac{1+1}{(\tan 5° \cot 5°)(\tan 15° \cot 15°)(1)}$

[$\because \operatorname{cosec}^2\theta - \cot^2\theta = 1$, $\sin^2\theta + \cos^2\theta = 1$ and $\tan 45° = 1$]

$= \frac{2}{1} = \mathbf{2}$ = RHS

13. (*i*) $(\cos^2 25° + \cos^2 65°) + \operatorname{cosec}\theta \sec(90° - \theta) - \cot\theta \tan(90° - \theta)$

$= \cos^2(90° - 65°) + \cos^2 65 + \operatorname{cosec}\theta \operatorname{cosec}\theta - \cot\theta \cot\theta$

[$\because \sec(90° - \theta) = \operatorname{cosec}\theta$ and $\tan(90° - \theta) = \cot\theta$]

$= (\sin^2 65° + \cos^2 65°) + (\operatorname{cosec}^2\theta - \cot^2\theta)$

$= 1 + 1$

[$\because \sin^2\theta + \cos^2\theta = 1$ and $\operatorname{cosec}^2\theta - \cot^2\theta = 1$]

$= \mathbf{2}$

(*ii*) $\cot\theta \tan(90° - \theta) - \sec(90° - \theta)\operatorname{cosec}\theta + \sqrt{3}(\tan 5° \tan 15° \tan 30° \tan 75° \tan 85°) + \sin^2 25° + \sin^2 65°$

$= \cot\theta \cot\theta - \operatorname{cosec}\theta \operatorname{cosec}\theta + \sqrt{3}\,[(\tan 5° \tan 85°)(\tan 15° \tan 75°)\tan 30°] + \sin^2 25° + \sin^2(90° - 25°)$

$= \cot^2\theta - \operatorname{cosec}^2\theta + \sqrt{3}\,[\tan 5° \tan(90° - 5°)]$

$[\tan 15° \tan(90° - 15°)\frac{1}{\sqrt{3}}] + \sin^2 25° + \cos^2 25°$

[$\because \sin(90° - \theta) = \cos\theta$]

$= (-1) + (\tan 5° \cot 5°)(\tan 15° \cot 15°) + (1)$

[$\because \cot^2\theta - \operatorname{cosec}^2\theta = -1$, $\tan(90° - \theta) = \cot\theta$ and $\sin^2\theta + \cos^2\theta = 1$]

$= -1 + (1)(1) + 1 = \mathbf{1}$

(*iii*) $[\cos(90° - \theta) + \sin(90° - \theta)]^2 + [\sin(90° - \theta) - \cos(90° - \theta)]^2$

$= (\sin\theta + \cos\theta)^2 + (\cos\theta - \sin\theta)^2$

$= (\sin^2\theta + \cos^2\theta) + 2\sin\theta\cos\theta + (\cos^2\theta + \sin^2\theta) - 2\sin\theta\cos\theta$

$= 1 + 1$ [$\because \sin^2\theta + \cos^2\theta = 1$]

$= \mathbf{2}$

(*iv*) $\frac{\sec(90° - \theta)\operatorname{cosec}\theta - \tan(90° - \theta)\cot\theta + \cos^2 25° + \sin^2 25°}{3\tan 27° \tan 63°}$

$= \frac{\operatorname{cosec}\theta\operatorname{cosec}\theta - \cot\theta\cot\theta + 1}{3\tan 27° \tan(90° - 27°)}$

[$\because \sin^2\theta + \cos^2\theta = 1$]

$= \frac{\operatorname{cosec}^2\theta - \cot^2\theta + 1}{3\tan 27° \cot 27°}$ [$\because \tan(90° - \theta) = \cot\theta$]

$= \frac{1+1}{3(1)}$ [$\because \operatorname{cosec}^2\theta - \cot^2\theta = 1$]

$= \mathbf{\frac{2}{3}}$

(*v*) $\frac{-\tan\theta \cot(90° - \theta) + \sec\theta \operatorname{cosec}(90° - \theta) + \sin^2 35° + \sin^2 55°}{\tan 10° \tan 20° \tan 45° \tan 70° \tan 80°}$

$$= \frac{-\tan\theta\tan\theta + \sec\theta\sec\theta + \sin^2 35° + \sin^2(90° - 35°)}{(\tan 10°\tan 80°)(\tan 20°\tan 70°)\tan 45°}$$

$$= \frac{(-\tan^2\theta + \sec^2\theta) + (\sin^2 35° + \cos^2 35°)}{[\tan 10°\tan(90° - 10°)][\tan 20°\tan(90° - 20°)]\tan 45°}$$

$[\because \sin(90° - \theta) = \cos\theta]$

$$= \frac{1 + 1}{(\tan 10°\cot 10°)(\tan 20°\cot 20°)(1)}$$

$[\because \sec^2\theta - \tan^2\theta = 1, \sin^2\theta + \cos^2\theta = 1 \text{ and } \tan 45° = 1]$

$$= \frac{2}{(1)(1)(1)}$$

$= \mathbf{2}$

(*vi*) $\dfrac{\sec^2(90° - \theta) - \cot^2\theta}{2(\sin^2 25° + \sin^2 65°)} + \dfrac{2\cos^2 60°\tan^2 28°\tan^2 62°}{3(\sec^2 43° - \cot^2 47°)}$

$$= \frac{\text{cosec}^2\theta - \cot^2\theta}{2[\sin^2 25° + \sin^2(90° - 25°)]} + \frac{2\cos^2 60°\tan^2 28°\tan^2(90° - 28°)}{3[\sec^2 43° - \cot^2(90° - 43°)]}$$

$$= \frac{1}{2(\sin^2 25° + \cos^2 25°)} + \frac{2\left(\frac{1}{2}\right)^2\tan^2 28°\cot^2 28°}{3[\sec^2 43° - \cot^2(90° - 43°)]}$$

$[\because \sin(90° - \theta) = \cos\theta, \cos 60° = \frac{1}{2},$
$\tan(90° - \theta) = \cot\theta \text{ and } \cot(90° - \theta) = \tan\theta]$

$$= \frac{1}{2(1)} + \frac{\frac{2}{4}(1)}{3(1)}$$

$[\because \sin^2\theta + \cos^2\theta = 1 \text{ and } \sec^2\theta - \tan^2\theta = 1]$

$$= \left(\frac{1}{2}\right) + \left(\frac{1}{2} \times \frac{1}{3}\right) = \frac{1}{2} + \frac{1}{6}$$

$$= \frac{3 + 1}{6} = \frac{4}{6}$$

$$= \mathbf{\frac{2}{3}}$$

(*vii*) $\dfrac{\sin^2 20° + \sin^2 70°}{\cos^2 20° + \cos^2 70°} + \dfrac{\sin(90° - \theta)\sin\theta}{\tan\theta} + \dfrac{\cos(90° - \theta)\cos\theta}{\cot\theta}$

$$= \frac{\sin^2 20° + \sin^2(90° - 20°)}{\cos^2(90° - 70°) + \cos^2 70°} + \frac{\cos\theta\sin\theta}{\frac{\sin\theta}{\cos\theta}} + \frac{\sin\theta\cos\theta}{\frac{\cos\theta}{\sin\theta}}$$

$$= \frac{\sin^2 20° + \cos^2 20°}{\sin^2 70° + \cos^2 70°} + \frac{\cos\theta\sin\theta\cos\theta}{\sin\theta} + \frac{\cos\theta\sin\theta\sin\theta}{\cos\theta}$$

$[\because \sin(90° - \theta) = \cos\theta \text{ and } \cos(90° - \theta) = \sin\theta]$

$= \frac{1}{1} + (\cos^2\theta + \sin^2\theta)$ $\quad[\because \sin^2\theta + \cos^2\theta = 1]$

$= 1 + 1$ $\quad[\because \sin^2\theta + \cos^2\theta = 1]$

$= \mathbf{2}$

(*viii*) $\dfrac{2\sin 68°}{\cos 22°} - \dfrac{2\tan(90° - 15°)}{5\cot 15°} - \dfrac{3\tan 40°\tan 20°\tan 50°\tan 70°}{5(\sin^2 70° + \sin^2 20°)}$

$$= \frac{2\sin(90° - 22°)}{\cos 22°} - \frac{2\cot 15°}{5\cot 15°} - \frac{3(\tan 20°\tan 70°)(\tan 40°\tan 50°)}{5[\sin^2 70° + \sin^2(90° - 70°)]}$$

$$= \frac{2\cos 22°}{\cos 22°} - \frac{2}{5} - \frac{3[\tan 20°\tan(90° - 20°)][\tan 40°\tan(90° - 40°)]}{5(\sin^2 70° + \cos^2 70°)}$$

$[\because \sin(90° - \theta) = \cos\theta]$

$$= 2 - \frac{2}{5} - \frac{3(\tan 20°\cot 20°)(\tan 40°\cot 40°)}{5(1)}$$

$[\because \tan(90° - \theta) = \cot\theta \text{ and } \sin^2\theta + \cos^2\theta = 1]$

$$= 2 - \frac{2}{5} - \frac{3(1)(1)}{5}$$

$$= \frac{10 - 2 - 3}{5} = \frac{5}{5}$$

$= \mathbf{1}$

14. (*i*) LHS = $\mathbf{\cos(73° + \theta)}$

$= \sin[90° - (73° + \theta)]$

$= \sin[90° - (73° + \theta)]$

$[\because \cos\theta = \sin(90° - \theta)]$

$= \sin(90° - 73° - \theta)$

$= \mathbf{\sin(17° - \theta)}$

= RHS

(*ii*) LHS = $\mathbf{\tan(30° - \theta)} = \cot[90° - (30° - \theta)]$

$[\because \tan\theta = \cot(90° - \theta)]$

$= \cot(90° - 30° + \theta)$

$= \mathbf{\cot(60° + \theta)}$

= RHS

15. (*i*) $\tan\theta = \cot(30° + \theta)$

$\Rightarrow \cot(90° - \theta) = \cot(30° + \theta)$

$[\because \cot(90° - \theta) = \tan\theta]$

$\Rightarrow 90° - \theta = 30° + \theta$

$\Rightarrow 2\theta = 60°$

$\Rightarrow \theta = \mathbf{30°}$

(*ii*) $\sec 2A = \text{cosec}(A - 42°)$

$\Rightarrow \text{cosec}(90° - 2A) = \text{cosec}(A - 42°)$

$[\because \text{cosec}(90° - \theta) = \sec\theta]$

$\Rightarrow 90° - 2A = A - 42°$

$\Rightarrow 3A = 90° + 42°$

$\Rightarrow 3A = 132°$

$\Rightarrow A = \mathbf{44°}$

(*iii*) $\tan 2\theta = \cot(\theta - 24°)$

$\Rightarrow \cot(90° - 2\theta) = \cot(\theta - 24°)$

$[\because \tan(90° - A) = \cot A]$

$\Rightarrow 90° - 2\theta = \theta - 24°$

$\Rightarrow 3\theta = 90° + 24°$

$\Rightarrow 3\theta = 114°$

$\Rightarrow \theta = \mathbf{38°}$

(*iv*) $\sin 3\theta = \cos(\theta - 6°)$ [Given]

$\Rightarrow \cos(90° - 3\theta) = \cos(\theta - 6°)$ $[\because \sin\theta = \cos(90° - \theta)]$

$\Rightarrow 90° - 3\theta = \theta - 6°$

$\Rightarrow 96° = 4\theta$

$\Rightarrow \theta = \frac{96°}{4} = 24°$

Hence, $\theta =$ **24°**

(*v*) $\tan 3\theta = \cot(\theta - 6°)$ [Given]

$\Rightarrow \cot(90° - 3\theta) = \cot(\theta - 6°)$ $[\because \tan\theta = \cot(90° - \theta)]$

$\Rightarrow 90° - 3\theta = \theta - 6°$

$\Rightarrow 96° = 4\theta$

$\Rightarrow \theta = \frac{96°}{4} = 24°$

Hence, $\theta =$ **24°**

(*vi*) $\sec 4A = \text{cosec}(A - 20°)$ [Given]

$\Rightarrow \text{cosec}(90° - 4A) = \text{cosec}(A - 20°)$ $[\because \sec\theta = \text{cosec}(90° - \theta)]$

$\Rightarrow 90° - 4A = A - 20°$

$\Rightarrow 110° = 5A$

$\Rightarrow A = \frac{110°}{5} = 22°$

Hence, A = **22°**

(*vii*) $\sin(\theta + 24°) = \cos\theta$ [Given]

$\Rightarrow \sin(\theta + 24°) = \sin(90° - \theta)$ $[\because \cos\theta = \sin(90° - \theta)]$

$\Rightarrow \theta + 24° = 90° - \theta$

$\Rightarrow 2\theta = 90° - 24°$

$\Rightarrow 2\theta = 66°$

$\Rightarrow \theta = 33°$

Hence, $\theta =$ **33°**

(*viii*) $\cot(14\theta - 15°) = \tan\theta$ [Given]

$\Rightarrow \cot(14\theta - 15°) = \cot(90° - \theta)$

$\Rightarrow 14\theta - 15° = 90° - \theta$

$\Rightarrow 15\theta = 105°$

$\Rightarrow \theta = \frac{105°}{15} = 7°$

Hence, $\theta =$ **7°**

(*ix*) $\sec(4\theta + 40°) = \text{cosec}\,\theta$ $[\because \text{cosec}\,\theta = \sec(90° - \theta)]$

$\Rightarrow \sec(4\theta + 40°) = \sec(90° - \theta)$

$\Rightarrow 4\theta + 40° = 90° - \theta$

$\Rightarrow 5\theta = 50°$

$\Rightarrow \theta = 10°$

Hence, $\theta =$ **10°**

16. (*i*) $\mathbf{\tan(55° - \theta) - \cot(35° + \theta)}$

$= \cot[90° - (55° - \theta)] - \cot(35° + \theta)$ $[\because \tan\theta = \cot(90° - \theta)]$

$= \cot(90° - 55° + \theta) - \cot(35° + \theta)$

$= \cot(35° + \theta) - \cot(35° + \theta) = \mathbf{0}$

(*ii*) $\mathbf{\text{cosec}(65° + \theta) - \sec(25° - \theta)}$

$= \sec[90° - (65° + \theta)] - \sec(25° - \theta)]$ $[\because \text{cosec}\,\theta = \sec(90° - \theta)]$

$= \sec(90° - 65° - \theta) - \sec(25° - \theta)$

$= \sec(25° - \theta) - \sec(25° - \theta)$

$= \mathbf{0}$

(*iii*) $\mathbf{\sin(42° + \theta) - \cos(48° - \theta) + \text{cosec}(11° + \theta) - \sec(79° - \theta)}$

$= \cos[90° - (42° + \theta)] - \cos(48° - \theta) + \sec[90° - (11° + \theta)] - \sec(79° - \theta)$

$[\sin\theta = \cos(90° - \theta)$ and $\text{cosec}\,\theta = \sec(90° - \theta)]$

$= \cos(90° - 42° - \theta) - \cos(48° - \theta) + \sec(90° - 11 - \theta) - \sec(79° - \theta)$

$= \cos(48° - \theta) - \cos(48° - \theta) + \sec(79° - \theta) - \sec(79° - \theta)$

$= \mathbf{0}$

(*iv*) $\mathbf{\text{cosec}(65° + \theta) - \sec(25° - \theta) + \tan(55° - \theta) - \cot(35° + \theta)}$

$= \sec[90° - (65° + \theta)] - \sec(25° - \theta) + \cot[90° - (55° - \theta)] - \cot(35° + \theta)$

$[\because \text{cosec}\,\theta = \sec(90° - \theta)$ and $\tan\theta = \cot(90° - \theta)]$

$= \sec(90° - 65° - \theta) - \sec(25° - \theta) + \cot(90° - 55° + \theta) - \cot(35° + \theta)$

$= \sec(25° - \theta) - \sec(25° - \theta) + \cot(35° + \theta) - \cot(35° + \theta)$

$= \mathbf{0}$

17. (*i*) $\mathbf{\cos(40° - \theta) - \sin(50° + \theta) + \frac{\cos^2 40° + \cos^2 50°}{\sin^2 40° + \sin^2 50°}}$

$\Rightarrow \cos(90° - (50° + \theta)) - \sin(50° + \theta) + \frac{\cos^2(90° - 50°) + \cos^2 50°}{\sin^2 40° + \sin^2(90° - 40°)}$

$\Rightarrow \sin(50° + \theta) - \sin(50° + \theta) + \frac{\sin^2 50° + \cos^2 50°}{\sin^2 40° + \cos^2 40°}$

$\Rightarrow 0 + \frac{1}{1}$

$\Rightarrow$ **1**

(*ii*) $\mathbf{\cos(50° + \theta) - \sin(40° - \theta) + \cot 1° \cot 10° \cot 20° \cot 70° \cot 80° \cot 89°}$

$= \sin[90° - (50° + \theta)] - \sin(40° - \theta) + (\cot 1° \cot 89°)(\cot 10° \cot 80°)(\cot 20° \cot 70°)$ $[\because \cos\theta = \sin(90° - \theta)]$

$= \sin(90° - 50° - \theta) - \sin(40° - \theta) + [\cot 1° \cot(90° - 1°)][\cot 10° \cot(90° - 10°)][\cot 20° \cot(90° - 20°)]$

$= \sin(40° - \theta) - \sin(40° - \theta) + (\cot 1° \tan 1°)(\cot 10° \tan 10°)(\cot 20° \tan 20°)$ $[\because \cot(90° - \theta) = \tan\theta]$

$= 0 + (1)(1)(1)$

$= 0 + 1$

$=$ **1**

(*iii*) $\mathbf{\tan 12° \tan 38° \tan 52° \tan 60° \tan 78° + \cot(55° - \theta) - \tan(35° + \theta) + \sin(40° + \theta) - \cos(50° - \theta) + \frac{\sin^2 40° + \sin^2 50°}{\cos^2 40° + \cos^2 50°}}$

$= (\tan 12° \tan 78°)(\tan 38° \tan 52°) \tan 60° + \tan[90° - (55° - \theta)] - \tan(35° + \theta) + \cos[90° - (40° + \theta)] - \cos(50° - \theta) + \frac{\sin^2 40° + \cos^2(90° - 50°)}{\sin^2(90° - 40°) + \cos^2 50°}$

[$\because$ cot θ = tan (90° – θ), sin θ = cos (90° – θ) and cos θ = sin (90° – θ)]

$= [\tan 12° \tan (90° - 12°)] [\tan 38° \tan (90° - 38°)] \tan 60° + \tan (35° + \theta) - \tan (35° + \theta) + \cos (50° - \theta) - \cos (50° - \theta) + \dfrac{\sin^2 40° + \cos^2 40°}{\sin^2 50° + \cos^2 50°}$

$= (\tan 12° \cot 12°)(\tan 38° \cot 38°)\tan 60° + 0 + 0 + \dfrac{1}{1}$

[$\because \sin^2 \theta + \cos^2 \theta = 1$]

$= (1)(1)(\sqrt{3}) + 1$ [Putting $\tan 60° = \sqrt{3}$]

$= \mathbf{\sqrt{3} + 1}$

(*iv*) $\dfrac{\mathbf{\cos^2 (45° + \theta) + \cos^2 (45° - \theta)}}{\mathbf{\tan (60° + \theta) \tan (30° - \theta)}}$

$= \dfrac{\sin^2 [90° - (45° + \theta)] + \cos^2 (45° - \theta)}{\cot [90° - (60° + \theta)] \tan (30° - \theta)}$

[$\because$ cos θ = sin (90° – θ) and tan θ = cot (90° – θ)]

$= \dfrac{\sin^2 (45° - \theta) + \cos^2 (45° - \theta)}{\cot (30° - \theta) \tan (30° - \theta)}$

$= \dfrac{1}{1}$ [$\because \sin^2 \theta + \cos^2 \theta = 1$]

$= \mathbf{1}$

18. **tan A = cot B** [Given]

$\Rightarrow$ cot (90° – A) = cot B [$\because$ tan A = cot (90° – A)]

$\Rightarrow$ 90° – A = B

$\Rightarrow$ **A + B = 90°**

Hence, proved.

19. (*i*) A + B + C = 180°

[Sum of angles of a triangle is 180°]

$\Rightarrow$ A + B = 180° – C

$\Rightarrow \dfrac{A + B}{2} = \dfrac{180° - C}{2} = 90° - \dfrac{C}{2}$

$\Rightarrow \tan \left(\dfrac{A + B}{2}\right) = \tan \left(90° - \dfrac{C}{2}\right) = \cot \dfrac{C}{2}$

[$\because$ tan (90° – θ) = cot θ]

Hence, $\mathbf{\tan\left(\dfrac{A + B}{2}\right) = \cot \dfrac{C}{2}}$ proved.

(*ii*) A + B + C = 180°

[Sum of angles of a triangle is 180°]

$\Rightarrow$ B + C = 180° – A

$\Rightarrow \dfrac{B + C}{2} = \dfrac{180° - A}{2}$

$= 90° - \dfrac{A}{2}$

$\Rightarrow \sin \left(\dfrac{B + C}{2}\right) = \sin\left(90° - \dfrac{A}{2}\right)$

$= \cos \dfrac{A}{2}$

[$\because$ sin (90° – θ) = cos θ]

Hence, $\mathbf{\sin \left(\dfrac{B + C}{2}\right) = \cos \dfrac{A}{2}}$ proved.

(*iii*) A + B + C = 180°

[Sum of angles of a triangle is 180°]

$\Rightarrow \dfrac{A+B+C}{2} = \dfrac{180°}{2}$

$\Rightarrow \dfrac{A+B}{2} + \dfrac{C}{2} = 90°$

$\Rightarrow \dfrac{C}{2} = 90° - \left(\dfrac{A+B}{2}\right)$

$\Rightarrow \sec \dfrac{C}{2} = \sec\left(90° - \left(\dfrac{A+B}{2}\right)\right)$

$\Rightarrow \sec \dfrac{C}{2} = \operatorname{cosec}\left(\dfrac{A+B}{2}\right)$

Hence, $\mathbf{\operatorname{cosec} \dfrac{A+B}{2} = \sec \dfrac{C}{2}}$ proved.

20. A + B + C = 180°

[Sum of angles of a triangle is 180°]

$\Rightarrow$ A + B + 90° = 180°

$\Rightarrow$ A + B = 90°

$\Rightarrow$ B = 90° – A ... (1)

(*i*) LHS = tan A tan B

= tan A tan (90° – A) [Using (1)]

= tan A cot A [$\because$ tan (90° – A) = cot A]

= **1** = RHS

(*ii*) LHS = **sin A cos B + cos A sin B**

= sin A cos (90° – A) + cos A sin (90° – A) [Using (1)]

= sin A sin A + cos A cos A

[$\because$ cos (90° – A) = sin A and sin (90° – A) = cos A]

= $\sin^2 A + \cos^2 A$

= **1** = RHS

21. A + B + C = 180°

[Sum of angles of a triangle is 180°]

$\Rightarrow$ A + B + 90° = 180°

$\Rightarrow$ A + B = 90°

$\Rightarrow$ B = (90° – A)

and A = (90° – B) ...(1)

(*i*) LHS = $\mathbf{\sin^2 A + \sin^2 B}$

= $\sin^2 A + \sin^2 (90° - A)$ [Using (1)]

= $\sin^2 A + \cos^2 A$

[$\because$ sin (90° – A) = cos A]

= **1** = RHS

(*ii*) LHS = $1 + \cot^2 A = 1 + \cot^2 (90° - B)$ [Using (1)]

= $1 + \tan^2 B$ [$\because$ cot (90° – B) = tan B]

= $\mathbf{\sec^2 B}$

= RHS

22. (*i*) $x \sin (90° - \theta) \cot (90° - \theta) = \cos (90° - \theta)$

$\Rightarrow x \cos \theta \tan \theta = \sin \theta$

$\Rightarrow x \cos \theta \dfrac{\sin\theta}{\cos\theta} = \sin \theta$

$\Rightarrow x = 1$

Hence, $x = \mathbf{1}$

(*ii*) $\sec\theta \operatorname{cosec}(90° - \theta) - x\cot(90° - \theta) = 1$

$\Rightarrow \sec\theta\sec\theta - x\tan\theta = 1$

$\Rightarrow \sec^2\theta - 1 = x\tan\theta$

$\Rightarrow \tan^2\theta = x\tan\theta$

$[\because 1 + \tan^2\theta = \sec^2\theta \Rightarrow \sec^2\theta - 1 = \tan^2\theta]$

$\Rightarrow \tan\theta = x$

Hence, x= **tan** θ

For Standard Level

23. (*i*) LHS = $\mathbf{\tan^2 A \sec^2(90° - A) - \sin^2 A \operatorname{cosec}^2(90° - A)}$

$= \tan^2 A \operatorname{cosec}^2 A - \sin^2 A \sec^2 A$

$= \tan^2 A \operatorname{cosec}^2 A - \dfrac{\sin^2 A}{\cos^2 A}$

$= \tan^2 A \operatorname{cosec}^2 A - \tan^2 A$

$= \tan^2 A(\operatorname{cosec}^2 A - 1)$

$= \tan^2 A \cot^2 A$

$[\because 1 + \cot^2 A = \operatorname{cosec}^2 A]$

$= \tan^2 A \times \dfrac{1}{\tan^2 A}$

= **1**

= RHS

(*ii*) LHS $= \sqrt{\mathbf{1-\sin^2(90°-\theta)}} + \sqrt{\mathbf{4-4\sin^2(90°-\theta)}}$

$= \sqrt{1-\cos^2\theta} + \sqrt{4-4\cos^2\theta}$

$= \sqrt{\sin^2\theta} + \sqrt{4\sin^2\theta}$

$= \sin\theta + 2\sin\theta$

= **3 sin** θ

= RHS

(*iii*) LHS $= \dfrac{\mathbf{\sec(90°-\theta)\operatorname{cosec}\theta - \tan(90°-\theta)\cot\theta + (\cos^2 35° + \cos^2 55°)}}{\mathbf{\tan 5° \tan 15° \tan 45° \tan 75° \tan 85°}}$

$= \dfrac{\operatorname{cosec}\theta\operatorname{cosec}\theta - \cot\theta\cot\theta + (\cos^2(90°-55°)+\cos^2 55°)}{(\tan(90°-85°)\tan 85°)(\tan(90°-75°)\tan 75°)\tan 45°}$

$= \dfrac{\operatorname{cosec}^2\theta - \cot^2\theta + (\sin^2 55° + \cos^2 55°)}{(\cot 85° \tan 85°)(\cot 75° \tan 75°)(1)}$

$= \dfrac{1+1}{(1)(1)(1)}$

$[\because \operatorname{cosec}^2\theta - \cot^2\theta = 1$ and $\sin^2\theta + \cos^2\theta = 1]$

= **2**

= RHS

24. (*i*) $\dfrac{\mathbf{\sec^2(90°-\theta) - \cot^2\theta}}{\mathbf{2(\sin^2 65° + \sin^2 65°)}} + \dfrac{\mathbf{2\sin^2 30° \tan^2 32° \tan^2 58°}}{\mathbf{3(\sec^2 33° - \cot^2 57°)}}$

$\Rightarrow \dfrac{\operatorname{cosec}^2\theta - \cot^2\theta}{2(\sin^2(90°-65°)+\sin^2 65°)} + \dfrac{2\sin^2 30° \tan^2(90°-58°)\tan^2 58°}{3(\sec^2(90°-57°) - \cot^2 57°)}$

$\Rightarrow \dfrac{1}{2(\cos^2 65° + \sin^2 65°)} + \dfrac{2\sin^2 30° \cot^2 58° \tan^2 58°}{3(\operatorname{cosec}^2 57° - \cot^2 57°)}$

$\Rightarrow \dfrac{1}{2} + \dfrac{2\times\left(\frac{1}{2}\right)^2}{3(1)}$ $[\because \sin^2\theta + \cos^2\theta = 1,$ $\operatorname{cosec}^2\theta - \cot^2\theta = 1$ and $\sin 30° = \frac{1}{2}]$

$\Rightarrow \dfrac{1}{2} + \dfrac{1}{6}$

$\Rightarrow \dfrac{3+1}{6}$

$\Rightarrow \dfrac{4}{6}$

$\Rightarrow \mathbf{\dfrac{2}{3}}$

(*ii*) $\dfrac{\mathbf{\sin(90°-\theta)\operatorname{cosec}(90°-\theta)\cot\theta}}{\mathbf{\sec(90°-\theta)\cos(90°-\theta)\tan(90°-\theta)}} + \dfrac{\mathbf{\cot(90°-\theta)}}{\mathbf{\tan\theta}} + \dfrac{\mathbf{\cos^2 25° + \cos^2 65°}}{\mathbf{\tan 17° \tan 42° \tan 45° \tan 48° \tan 73°}}$

$\Rightarrow \dfrac{\cos\theta\sec\theta\cot\theta}{\operatorname{cosec}\theta\sin\theta\cot\theta} + \dfrac{\tan\theta}{\tan\theta} + \dfrac{\cos^2(90°-65°)+\cos^2 65°}{(\tan(90°-73°)\tan 73°)(\tan(90°-48°)\tan 48°)\tan 45°}$

$\Rightarrow 1 + 1 + \dfrac{\sin^2 65° + \cos^2 65°}{(\cot 73° \tan 73°)(\cot 48° \tan 48°)\tan 45°}$

$\Rightarrow 1 + 1 + \dfrac{1}{(1)(1)(1)}$

$\Rightarrow 1 + 1 + 1$

$\Rightarrow$ **3**

25. $\mathbf{\cos(70° + \theta) - \sin(20° - \theta) + \cos 1° \cos 2° \cos 3° \ldots \cos 90° + [\tan 11° \tan 32° \tan 45° \tan 58° \tan 79° + \sin 58° \sec 32°] + \cos 58° \operatorname{cosec} 32°}$

$\Rightarrow \cos(90° - (20° - \theta)) - \sin(20° - \theta) + \cos 1° \cos 2° \cos 3° \ldots \times 0 + [(\tan(90° - 79°)\tan 79°)(\tan(90° - 58°)\tan 58°)\tan 45° + \sin 58° \times \dfrac{1}{\cos(90°-58°)}] + \cos 58° \times \dfrac{1}{\sin(90°-58°)}$

$\Rightarrow \sin(20° - \theta) - \sin(20° - \theta) + 0 + [(\cot 79° \tan 79°)(\cot 58° \tan 58°) + \dfrac{\sin 58°}{\sin 58°}] + \dfrac{\cos 58°}{\cos 58°}$

$\Rightarrow 0 + 0 + [(1)(1) + 1] + 1$

$\Rightarrow 1 + 1 + 1$

$\Rightarrow$ **3**

26. (*i*) $A + B + C = 180°$

[Sum of angles of a triangle is 180°]

$\Rightarrow A + B + C - 2B = 180° - 2B$

$\Rightarrow A - B + C = 180° - 2B$

$\Rightarrow \dfrac{A - B + C}{2} = \dfrac{180° - 2B}{2}$

$= 90° - B$

$$\Rightarrow \sec\left(\frac{A - B + C}{2}\right) = \sec(90° - B) = \text{cosec } B$$

[∵ sec (90° – θ) = cosec θ]

Hence, $\mathbf{\sec\left(\frac{A - B + C}{2}\right) = cosec\ B}$ proved.

(*ii*) $A + B + C = 180°$

[Sum of angles of a triangle is 180°]

$$\Rightarrow B + C = 180° - A$$

$$\Rightarrow \frac{B + C}{2} = \frac{180° - A}{2} = 90° - \frac{A}{2}$$

$$\Rightarrow \text{cosec}^2\left(\frac{B + C}{2}\right) = \text{cosec}^2\left(90° - \frac{A}{2}\right)$$

$$\Rightarrow \text{cosec}^2\left(\frac{B + C}{2}\right) = \sec^2\frac{A}{2}$$

[∵ cosec (90° – θ) = sec θ]

$$\Rightarrow \text{cosec}^2\left(\frac{B + C}{2}\right) = 1 + \tan^2\frac{A}{2}$$

[$\sec^2\theta = 1 + \tan^2\theta$]

$$\Rightarrow \text{cosec}^2\left(\frac{B + C}{2}\right) - \tan^2\frac{A}{2} = 1$$

Hence, $\mathbf{cosec^2\left(\frac{B + C}{2}\right) - tan^2\frac{A}{2} = 1}$ proved.

27. $A + B + C + D = 360°$

[Sum of angles of a quadrilateral is 360°]

$$\Rightarrow A + B = 360° - (C + D)$$

$$\Rightarrow \left(\frac{A + B}{4}\right) = \frac{360° - (C + D)}{4}$$

$$\Rightarrow \left(\frac{A + B}{4}\right) = 90° - \left(\frac{C + D}{4}\right)$$

$$\Rightarrow \cos\left(\frac{A + B}{4}\right) = \cos\left[90° - \left(\frac{C + D}{4}\right)\right]$$

$$\Rightarrow \cos\left(\frac{A + B}{4}\right) = \sin\left(\frac{C + D}{4}\right)$$

[∵ cos (90° – θ) = sin θ]

Hence, $\mathbf{\cos\left(\frac{A + B}{4}\right) = \sin\left(\frac{C + D}{4}\right)}$ proved.

28. $\sin\theta + \cos\theta = \sqrt{2}\sin(90° - \theta)$

$$\Rightarrow \sin\theta + \cos\theta = \sqrt{2}\cos\theta$$

$$\Rightarrow \sin\theta = \sqrt{2}\cos\theta - \cos\theta$$

$$\Rightarrow \sin\theta = \cos\theta(\sqrt{2} - 1)$$

$$\Rightarrow \frac{1}{\sqrt{2} - 1} = \frac{\cos\theta}{\sin\theta}$$

$$\Rightarrow \frac{1}{\sqrt{2} - 1} \times \frac{\sqrt{2} + 1}{\sqrt{2} + 1} = \cot\theta$$

$$\Rightarrow \frac{\sqrt{2} + 1}{2 - 1} = \cot\theta$$

$$\Rightarrow \sqrt{2} + 1 = \cot\theta$$

Hence, $\cot\theta = \mathbf{\sqrt{2} + 1}$

29. $4\left(\frac{\sec^2 59° - \cot^2 31°}{3}\right) - \frac{2}{3}\sin 90° + 3\tan^2 56°\tan^2 34° = \frac{x}{3}$

$$\Rightarrow 4\left(\frac{\sec^2(90° - 31°) - \cot^2 31°}{3}\right) - \frac{2}{3}(1) + 3\tan^2 56°\tan^2(90° - 56°) = \frac{x}{3}$$

[∵ sin 90° = 1]

$$\Rightarrow 4\left(\frac{\text{cosec}^2 31° - \cot^2 31°}{3}\right) - \frac{2}{3} + 3\tan^2 56°\cot^2 56° = \frac{x}{3}$$

[∵ sec (90° – θ) = cosec θ, tan (90° – θ) = cot θ]

$$\Rightarrow 4\left(\frac{1}{3}\right) - \frac{2}{3} + 3(1) = \frac{x}{3} \quad [\because \text{cosec}^2\theta - \cot^2\theta = 1]$$

$$\Rightarrow \frac{4}{3} - \frac{2}{3} + 3 = \frac{x}{3}$$

$$\Rightarrow 4 - 2 + 9 = x$$

$$\Rightarrow 11 = x$$

Hence, x = **11**

30. LHS $= \sqrt{\dfrac{\tan A\tan B + \tan A\cot B}{\sin A\sec B} - \dfrac{\sin^2 B}{\cos^2 A}}$

$$= \sqrt{\frac{\tan A\tan(90° - A) + \tan A\cot(90° - A)}{\sin A\sec(90° - A)} - \frac{\sin^2(90° - A)}{\cos^2 A}}$$

[∵ A + B = 90° ⇒ B = (90° – A)]

$$= \sqrt{\frac{\tan A\cot A + \tan A\tan A}{\sin A\,\text{cosec} A} - \frac{\cos^2 A}{\cos^2 A}}$$

[∵ tan (90° – A) = cot A, cot (90° – A) = tan A and sin (90° – A) = cos A]

$$= \sqrt{(1 + \tan^2 A) - (1)}$$

$$= \sqrt{\tan^2 A}$$

= **tan A** = RHS

CHECK YOUR UNDERSTANDING

TRUE OR FALSE

For Basic and Standard Levels

1. **False**

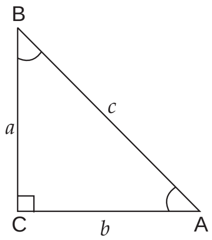
∵ tan has no meaning where as

$\tan A = \frac{\text{perpendicular}}{\text{base}}$ in right ΔABC,

where ∠A is an acute angle.

2. **True**

In right ΔABC, $\sin A = \frac{a}{c}$ and $\sin B = \frac{b}{c}$



Now, $\sin A = \sin B$

$\Rightarrow \quad \dfrac{a}{c} = \dfrac{b}{c}$

$\Rightarrow \quad a = b$

$\Rightarrow \quad \angle A = \angle B$

[Angles opposite to equal sides]

3. **False**

$\because \quad \cos^2 27° - \sin^2 63° = \cos^2 27° - \sin^2 (90° - 27°)$

$= \cos^2 27° - \cos^2 27°$

$[\because \sin (90° - \theta) = \cos \theta]$

$= 0$

4. **False**

$\because \quad \sin 75° - \cos 75° = \sin 75° - \sin (90° - 75°)$

$= \sin 75° - \sin 15° > 0$

$[\because \sin \theta$ increases as θ increases$]$

5. **True**

$\because \quad \sin 5\alpha = \cos \alpha$ and $5\alpha < 90°$ [Given]

$\Rightarrow \quad \sin 5\alpha = \sin (90° - \alpha)$

$[\because \cos \alpha = \sin (90° - \alpha)]$

$\Rightarrow \quad 5\alpha = 90° - \alpha$

$\Rightarrow \quad 5\alpha + \alpha = 90°$

$\Rightarrow \quad 6\alpha = 90°$

$\Rightarrow \quad \alpha = \dfrac{90°}{6} = 15°$

Now, $\tan 3\alpha = \tan (3 \times 15°) = \tan 45° = 1$

MULTIPLE-CHOICE QUESTIONS

For Basic and Standard Levels

1. (*c*) **cos 15° + cosec 15°**

$\sin 75° + \sec 75°$

$= \sin (90° - 15°) + \sec (90° - 15°)$

$= \cos 15° + \text{cosec } 15°$

$[\because \sin (90° - \theta) = \cos \theta, \sec (90° - \theta) = \text{cosec } \theta]$

2. (*b*) **1**

$\text{cosec } A \sec (90° - A) - \cot A \tan (90° - A)$

$= \text{cosec } A \text{ cosec } A - \cot A \cot A$

$[\because \sec (90° - A) = \text{cosec } A, \tan (90° - A) = \cot A]$

$= \text{cosec}^2 A - \cot^2 A = 1 \quad [\because \text{cosec}^2 A - \cot^2 A = 1]$

3. (*a*) **0**

$\cos 36° \cos 54° - \sin 36° \sin 54°$

$= \cos (90° - 54°) \cos 54° - \sin (90° - 54°) \sin 54°$

$= \sin 54° \cos 54° - \cos 54° \sin 54°$

$[\because \cos (90° - \theta) = \sin \theta, \sin (90° - \theta) = \cos \theta]$

$= 0$

4. (*b*) **2**

$\dfrac{\sin 18°}{\cos 72°} + \dfrac{\tan 26°}{\cot 64°} = \dfrac{\sin (90° - 72°)}{\cos 72°} + \dfrac{\tan (90° - 64°)}{\cot 64°}$

$= \dfrac{\cos 72°}{\cos 72°} + \dfrac{\cot 64°}{\cot 64°}$

$[\because \sin (90° - \theta) = \cos \theta, \tan (90° - \theta) = \cot \theta]$

$= 1 + 1 = 2$

5. (*b*) **2**

$\dfrac{\tan 55°}{\cot 35°} + \cot 1° \cot 2° \cot 3° \ldots \cot 89°$

$= \dfrac{\tan (90° - 35°)}{\cot 35°} + (\cot 1° \cot 89°) (\cot 2° \cot 88°)$

$\ldots (\cot 46° \cot 44°) \cot 45°$

$= \dfrac{\cot 35°}{\cot 35°} + [\cot 1° \cot (90° - 1°)] [\cot 2° \cot (90° - 2°)]$

$\ldots [\cot 46° \cot (90° - 46°)] \cot 45°$

$[\because \tan (90° - \theta) = \cot \theta]$

$= 1 + (\cot 1° \tan 1°) (\cot 2° \tan 2°)$

$\ldots (\cot 46° \tan 46°) \cot 45°$

$[\because \cot (90° - \theta) = \tan \theta]$

$= 1 + (1) (1) \ldots (1) (1) \quad [\because \cot 45° = 1]$

$= 1 + 1 = 2$

6. (*c*) **0**

$\sin (60° + \theta) - \cos (30° - \theta)$

$= \sin (60° + \theta) - \sin [90° - (30° - \theta)]$

$[\because \cos \theta = \sin (90° - \theta)]$

$= \sin (60° + \theta) - \sin (60° + \theta) = 0$

7. (*c*) **0**

$\text{cosec } (69° + \theta) - \sec (21° - \theta) - \cot (35° - \theta) + \tan (55° + \theta)$

$= \text{cosec } (69° + \theta) - \text{cosec } [90° - (21° - \theta)] - \cot (35° - \theta)$

$+ \cot [90° - (55° + \theta)]$

$[\because \sec \theta = \text{cosec } (90° - \theta), \tan \theta = \cot (90° - \theta)]$

$= \text{cosec } (69° + \theta) - \text{cosec } (69° + \theta) - \cot (35° - \theta)$

$+ \cot (35° - \theta) = 0$

8. (*c*) **17**

$17 \sec^2 29° - 17 \cot^2 61° = 17 \sec^2 29° - 17 \tan^2 (90° - 61°)$

$[\because \cot \theta = \tan (90° - \theta)]$

$= 17 \sec^2 29° - 17 \tan^2 29°$

$= 17 (\sec^2 29° - \tan^2 29°)$

$= 17 \times 1 \quad [\because \sec^2 \theta - \tan^2 \theta = 1]$

$= 17$

9. (*b*) **sin α**

$\sqrt{\text{cosec}\,\alpha\,\text{cosec}\,\beta - \cos\alpha\,\sin\beta}$

$= \sqrt{\cos\alpha\,\text{cosec}\,(90° - \alpha) - \cos\alpha\,\sin (90° - \alpha)}$

$[\because \alpha + \beta = 90° \Rightarrow \beta = 90° - \alpha]$

$= \sqrt{\cos\alpha\,\sec\alpha - \cos\alpha\,\cos\alpha}$

$[\because \text{cosec } (90° - \alpha) = \sec \alpha, \sin (90° - \alpha) = \cos \alpha]$

$= \sqrt{1 - \cos^2\alpha}$

$= \sqrt{(\sin^2\alpha)}$

$[\because \sin^2 \alpha + \cos^2 \alpha = 1 \Rightarrow 1 - \cos^2 \alpha = \sin^2 \alpha]$

$= \sin \alpha \quad [\because \sin \alpha > 0$ of all acute angles$]$

10. (*c*) **27°**

$$\cos(81° + \theta) = \sin\left(\frac{k}{3} - \theta\right) \quad \text{[Given]}$$

$$\Rightarrow \sin[90° - (81° + \theta)] = \sin\left(\frac{k}{3} - \theta\right)$$

$$[\because \cos\theta = \sin(90° - \theta)]$$

$$\Rightarrow 9° - \theta = \frac{k}{3} - \theta$$

$$\Rightarrow 9° = \frac{k}{3}$$

$$\Rightarrow k = 27°$$

11. (*c*) **6**

$$\frac{\cos 20°}{\sin 70°} + \frac{2\cos\theta}{\sin(90° - \theta)} = \frac{k}{2} \quad \text{[Given]}$$

$$\Rightarrow \frac{\cos(90° - 70°)}{\sin 70°} + \frac{2\cos\theta}{\cos\theta} = \frac{k}{2}$$

$$[\because \sin(90° - \theta) = \cos\theta]$$

$$\Rightarrow \frac{\sin 70°}{\sin 70°} + 2 = \frac{k}{2}$$

$$\Rightarrow 1 + 2 = \frac{k}{2}$$

$$[\because \cos(90° - \theta) = \sin\theta]$$

$$\Rightarrow 3 = \frac{k}{2}$$

$$\Rightarrow k = 6$$

12. (*b*) **45°**

$$\sin\theta = \cos\theta \quad \text{[Given]}$$

$$\Rightarrow \sin\theta = \sin(90° - \theta)$$

$$[\because \cos\theta = \sin(90° - \theta)]$$

$$\Rightarrow \theta = 90° - \theta$$

$$\Rightarrow 2\theta = 90°$$

$$\Rightarrow \theta = 45°$$

13. (*b*) **90°**

$$\tan A = \cot B$$

$$\Rightarrow \tan A = \tan(90° - B)$$

$$[\because \cot B = \tan(90° - B)]$$

$$\Rightarrow A = 90° - B$$

$$\Rightarrow A + B = 90°$$

14. (*d*) **1**

$$\cos 9\theta = \sin\theta \text{ and } 9\theta < 90° \quad \text{[Given]}$$

$$\Rightarrow \cos 9\theta = \cos(90° - \theta)$$

$$[\because \sin\theta = \cos(90° - \theta)]$$

$$\Rightarrow 9\theta = 90° - \theta$$

$$\Rightarrow 10\theta = 90°$$

$$\Rightarrow \theta = 9°$$

Now, $\tan 5\theta = \tan 5(9°) = \tan 45° = 1$

15. (*a*) **36°**

$$\tan 2\theta = \cot(\theta - 18°)$$

$$\Rightarrow \cot(90° - 2\theta) = \cot(\theta - 18°)$$

$$[\because \tan\theta = \cot(90° - \theta)]$$

$$\Rightarrow 90° - 2\theta = \theta - 18°$$

$$\Rightarrow 108° = 3\theta$$

$$\Rightarrow \theta = 36°$$

16. (*c*) **24°**

$$\sec 4\theta = \text{cosec}(\theta - 30°) \quad \text{[Given]}$$

$$\Rightarrow \text{cosec}(90° - 4\theta) = \text{cosec}(\theta - 30°)$$

$$[\because \sec\theta = \text{cosec}(90° - \theta)]$$

$$\Rightarrow 90° - 4\theta = \theta - 30°$$

$$\Rightarrow 5\theta = 120°$$

$$\Rightarrow \theta = 24°$$

17. (*a*) **29°**

$$\sin 3A = \cos(A - 26°) \quad \text{[Given]}$$

$$\Rightarrow \cos(90° - 3A) = \cos(A - 26°)$$

$$[\because \sin\theta = \cos(90° - \theta)]$$

$$\Rightarrow 90° - 3A = A - 26°$$

$$\Rightarrow 116° = 4A$$

$$\Rightarrow A = 29°$$

18. (*d*) **20°**

$$\cos(40° + A) = \sin 30° \quad \text{[Given]}$$

$$\Rightarrow \cos(40° + A) = \cos(90° - 30°) = \cos 60°$$

$$[\because \sin A = \cos(90° - A)]$$

$$\Rightarrow 40° + A = 60°$$

$$\Rightarrow A = 20°$$

ALTERNATIVE METHOD

$$\cos(40° + A) = \sin 30° \quad \text{[Given]}$$

$$\Rightarrow \cos(40° + A) = \frac{1}{2} = \cos 60°$$

$$\Rightarrow 40° + A = 60°$$

$$\Rightarrow A = 20°$$

For Standard Level

19. (*a*) $\mathbf{\frac{5}{2}}$

$$\frac{\sec^2 54° - \cot^2 36°}{\text{cosec}^2 57° - \tan^2 33°} + 2\sin^2 38° \sec^2 52° - \sin^2 45°$$

$$= \frac{\sec^2 54° - \tan^2(90° - 36°)}{\text{cosec}^2 57° - \cot^2(90° - 33°)} + 2\cos^2(90° - 38°) \sec^2 52° - \sin^2 45°$$

$$[\because \cot\theta = \tan(90° - \theta), \tan\theta = \cot(90° - \theta)]$$

$$= \frac{\sec^2 54° - \tan^2 54°}{\text{cosec}^2 57° - \cot^2 57°} + 2\cos^2 52° \sec^2 52° - \sin^2 45°$$

$$= \frac{1}{1} + 2(1) - \left(\frac{1}{\sqrt{2}}\right)^2$$

$$\left[\because \sec^2\theta - \tan^2\theta = 1, \text{cosec}^2\theta - \cot^2\theta = 1, \sin 45° = \frac{1}{\sqrt{2}}\right]$$

$$= 1 + 2 - \frac{1}{2} = 3 - \frac{1}{2} = \frac{5}{2}$$

20. (*d*) **1**

$$\frac{\cos^2(45° - \theta) + \cos^2(45° + \theta)}{\tan^2(30° - \theta)\tan^2(60° + \theta)}$$

$$= \frac{\sin^2[90° - (45° - \theta)] + \cos^2(45° + \theta)}{\tan^2(30° - \theta)\cot^2[90° - (60° + \theta)]}$$

$$[\because \cos\theta = \sin(90° - \theta), \tan\theta = \cot(90° - \theta)]$$

$$= \frac{\sin^2(45° + \theta) + \cos^2(45° + \theta)}{\tan^2(30° - \theta)\cot^2(30° - \theta)}$$

$$= \frac{1}{1} \quad [\because \sin^2\theta + \cos^2\theta = 1]$$

$$= 1$$

21. (*b*) **4**

$$\frac{\cos^2 20° + \cos^2 70°}{2(\sin^2 59° + \sin^2 31°)} = \frac{2}{k} \quad \text{[Given]}$$

$$\Rightarrow \frac{\cos^2(90° - 70°) + \cos^2 70°}{2[\sin^2 59° + \sin^2(90° - 59°)]} = \frac{2}{k}$$

$$\Rightarrow \frac{\sin^2 70° + \cos^2 70°}{2(\sin^2 59° + \cos^2 59°)} = \frac{2}{k}$$

[∵ cos (90° – θ) = sin θ, sin (90° – θ) = cos θ]

$$\Rightarrow \frac{1}{2} = \frac{2}{k}$$

[∵ $\sin^2 \theta + \cos^2 \theta = 1$]

$$\Rightarrow k = 4$$

22. (*d*) $\mathbf{\frac{1}{4}}$

$$\sin \theta - \cos \theta = 0$$
$$\Rightarrow \sin \theta = \cos \theta$$
$$\Rightarrow \sin \theta = \sin (90° - \theta)$$
$$\Rightarrow \theta = 90° - \theta$$
$$\Rightarrow 2\theta = 90°$$
$$\Rightarrow \theta = 45°$$

Now, $\sin^6 \theta + \cos^6 \theta = \sin^6 45° + \cos^6 45°$

$$= (\sin 45°)^6 + (\cos 45°)^6$$
$$= \left(\frac{1}{\sqrt{2}}\right)^6 + \left(\frac{1}{\sqrt{2}}\right)^6$$
$$= \frac{1}{8} + \frac{1}{8} = \frac{2}{8}$$
$$= \frac{1}{4}$$

23. (*b*) $\mathbf{\sqrt{2} - 1}$

$$\sin \theta + \cos \theta = \sqrt{2} \cos (90° - \theta) \quad \text{[Given]}$$
$$\Rightarrow \sin \theta + \cos \theta = \sqrt{2} \sin \theta$$
$$\Rightarrow \cos \theta = \sqrt{2} \sin \theta - \sin \theta$$
$$\Rightarrow \cos \theta = \sin \theta (\sqrt{2} - 1)$$
$$\Rightarrow \frac{\cos\theta}{\sin\theta} = \sqrt{2} - 1$$
$$\Rightarrow \cot \theta = \sqrt{2} - 1$$

24. (*d*) $\mathbf{\cos \frac{A}{2}}$

$$A + B + C = 180°$$

[Sum of angles of a triangle is 180°]

$$\Rightarrow B + C = 180° - A$$
$$\Rightarrow \frac{B + C}{2} = \frac{180° - A}{2}$$
$$= 90° - \frac{A}{2}$$
$$\Rightarrow \sin\left(\frac{B + C}{2}\right) = \sin\left(90° - \frac{A}{2}\right)$$
$$\Rightarrow \sin\left(\frac{B + C}{2}\right) = \cos \frac{A}{2}$$

25. (*b*) **cos 2β**

$$\cos (\alpha + \beta) = 0 \quad \text{[Given]}$$
$$\Rightarrow \cos (\alpha + \beta) = \cos 90°$$
$$\Rightarrow \alpha + \beta = 90°$$
$$\Rightarrow \alpha + \beta - 2\beta = 90° - 2\beta$$
$$\Rightarrow \alpha - \beta = 90° - 2\beta$$
$$\Rightarrow \sin (\alpha - \beta) = \sin (90° - 2\beta)$$
$$\Rightarrow \sin (\alpha - \beta) = \cos 2\beta$$

[∵ sin (90° – θ) = cos θ]

SHORT ANSWER QUESTIONS

For Basic and Standard Levels

1.
$$\text{LHS} = \frac{\cos\theta \cos (90° - \theta)}{\cot (90° - \theta)}$$
$$= \frac{\cos\theta \sin\theta}{\tan\theta}$$

[∵ cos (90° – θ) = sin θ, cot (90° – θ) = tan θ]

$$= \frac{\cos\theta\sin\theta}{\frac{\sin\theta}{\cos\theta}}$$
$$= \mathbf{\cos^2} \theta$$
$$= \text{RHS}$$

2.
$$\tan 68° + \sec 68° = \tan (90° - 22°) + \sec (90° - 22°)$$
$$= \mathbf{\cot 22° + cosec\ 22°}$$

3.
$$\cos^2 67° - \sin^2 23° = \cos^2 (90° - 23°) - \sin^2 23°$$
$$= \sin^2 23° - \sin^2 23°$$
$$= \mathbf{0}$$

For Standard Level

4. $\cot \theta \tan (90° - \theta) - \sec (90° - \theta) \text{ cosec } \theta + \sin^2 65°$
$+ \sin^2 25° + \sqrt{3} \tan 5° \tan 45° \tan 85°$

$= \cot \theta \cot \theta - \text{cosec } \theta \text{ cosec } \theta + \sin^2 65°$
$+ \sin^2 (90° - 65°) + \sqrt{3} \tan 5° \tan (90° - 5°) \tan 45°$

[∵ tan (90° – θ) = cot θ, sec (90° – θ) = cosec θ]

$= (\cot^2 \theta - \text{cosec}^2 \theta) + \sin^2 65° + \cos^2 65° + \sqrt{3} \tan 5°$
$\cot 5° (1)$

[∵ sin (90° – θ) = cos θ,
tan (90° – θ) = cot θ and tan 45° = 1]

$= -1 + 1 + \sqrt{3} (1)$

[∵ $\text{cosec}^2 \theta - \cot^2 \theta \Rightarrow \cot^2 \theta - \text{cosec}^2 \theta = -1$]

$= \sqrt{3}$

5.
$$A + B + C = 180°$$

[Sum of angles of a triangle is 180°]

$$\Rightarrow B + C = 180° - A$$
$$\Rightarrow \frac{B + C}{2} = 90° - \frac{A}{2}$$
$$\Rightarrow \text{cosec}^2\left(\frac{B + C}{2}\right) = \text{cosec}^2 \left(90° - \frac{A}{2}\right)$$
$$\Rightarrow \text{cosec}^2\left(\frac{B + C}{2}\right) = \sec^2 \frac{A}{2}$$
$$\Rightarrow \text{cosec}^2\left(\frac{B + C}{2}\right) = 1 + \tan^2 \frac{A}{2}$$

$\Rightarrow$ $\mathbf{cosec^2 \left(\frac{B+C}{2}\right) - tan^2 \frac{A}{2} = 1}$

Hence, proved.

6. $A + B + C = 180°$

[Sum of angles of a triangle is 180°]

$\Rightarrow$ $A + B + 90° = 180°$ [$\because$ $\angle C = 1$ right $\angle$]

$\Rightarrow$ $A + B = 90°$

$\Rightarrow$ $\sin (A + B) = \sin 90°$

$\Rightarrow$ $\mathbf{\sin (A + B) = 1}$

Also $A + B = 90°$

$\Rightarrow$ $\cos (A + B) = \cos 90°$

$\Rightarrow$ $\mathbf{\cos (A + B) = 0}$

UNIT TEST 1

For Basic Level

1. (*a*) **1**

$\text{cosec}\ \theta = 2$

$\Rightarrow$ $\text{cosec}^2\ \theta = 4$

$\Rightarrow$ $\text{cosec}^2\ \theta - 1 = 4 - 1$

$\Rightarrow$ $\text{cosec}^2\ \theta - 1 = 3$

$\Rightarrow$ $\cot^2 \theta = 3$

[$\because$ $1 + \cot^2 \theta = \text{cosec}^2\ \theta \Rightarrow \text{cosec}^2\ \theta - 1 = \cot^2 \theta$]

$\Rightarrow$ $\cot \theta = \sqrt{3}$ … (1)

Also $\cot \theta = \sqrt{3}\, p$ [Given] (2)

From (1) and (2), we get

$\sqrt{3}\, p = \sqrt{3}$

$\Rightarrow$ $p = 1$

2. (*b*) **1**

$$\frac{\tan 45°}{\sin 30° + \cos 60°} = \frac{1}{\frac{1}{2} + \frac{1}{2}} = \frac{1}{1} = 1$$

3. (*c*) $\mathbf{\frac{\sqrt{3}+1}{2\sqrt{2}}}$

$\sin (A + B) = \sin A \cos B + \cos A \sin B$

$\Rightarrow$ $\sin 75° = \sin (45° + 30°)$

$= \sin 45° \cos 30° + \cos 45° \sin 30°$

$$= \frac{1}{\sqrt{2}} \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \frac{1}{2}$$

$$= \frac{\sqrt{3}+1}{2\sqrt{2}}$$

4. (*b*) **0**

$\tan A = 1$

and $\sin B = \frac{1}{\sqrt{2}}$ [Given]

$\Rightarrow$ $\tan A = \tan 45°$

and $\sin B = \sin 45°$

$\Rightarrow$ $A = 45°$

and $B = 45°$

$\therefore$ $\cos (A + B) = \cos (45° + 45°)$

$= \cos 90°$

$= 0$

5. (*a*) **30°**

$3 \tan^2 A - 1 = 0, 0° < A < 90°$ [Given]

$\Rightarrow$ $\tan^2 A = \frac{1}{3}$

$\Rightarrow$ $\tan A = \frac{1}{\sqrt{3}}$

$\Rightarrow$ $\tan A = \tan 30°$

$\Rightarrow$ $A = 30°$

6. (*b*) $\mathbf{\frac{\sin A}{\sqrt{1 - \sin^2 A}}}$

$\sin^2 A + \cos^2 A = 1$

$\Rightarrow$ $\cos^2 A = 1 - \sin^2 A$

$\Rightarrow$ $\cos A = \pm\sqrt{1 - \sin^2 A}$

$\Rightarrow$ $\cos A = \sqrt{1 - \sin^2 A}$

[$\because$ $\cos A > 0$ for acute angles]

$\tan A = \frac{\sin A}{\cos A}$

$= \frac{\sin A}{\sqrt{1 - \sin^2 A}}$

7. (*d*) $\mathbf{x = 1}$

$x \sin (90° - \theta) \cot (90° - \theta) = \cos (90° - \theta)$

[$\theta \neq 0$] [Given]

$\Rightarrow$ $x \cos \theta \tan \theta = \sin \theta$

$\Rightarrow$ $x \cos \theta \frac{\sin \theta}{\cos \theta} = \sin \theta$

$\Rightarrow$ $x = 1$

8. (*b*) **0**

$\sin (22° + \theta) - \cos (68° - \theta)$

$= \sin (22° + \theta) - \sin [90° - (68° - \theta)]$

[$\because$ $\cos \theta = \sin (90° - \theta)$]

$= \sin (22° + \theta) - \sin (22° + \theta)$

$= 0$

9. (*b*) **90°**

$\sin 3\theta = \cos 4\theta$ [Given]

$\Rightarrow$ $\sin 3\theta = \sin (90° - 4\theta)$

[$\because$ $\cos \theta = \sin (90° - \theta)$]

$\Rightarrow$ $3\theta = 90° - 4\theta$

$\Rightarrow$ $7\theta = 90°$

10. $\sec 2A = \text{cosec}\ (A - 42°)$

where 2A is an acute angle

$\Rightarrow$ $\text{cosec}\ (90° - 2A) = \text{cosec}\ (A - 42°)$

[$\because$ $\sec \theta = \text{cosec}\ (90° - \theta)$]

$\Rightarrow$ $90° - 2A = A - 42°$

$\Rightarrow$ $90° + 42° = 3A$

$\Rightarrow$ $132° = 3A$

$\Rightarrow$ $A = \mathbf{44°}$

11. $\frac{\cos 75°}{\sin 15°} + \frac{\sin 12°}{\cos 78°} - \cos 18°\ \text{cosec}\ 72°$

$$= \frac{\cos (90° - 15°)}{\sin 15°} + \frac{\sin (90° - 78°)}{\cos 78°}$$

$$- \cos 18° \text{ cosec } (90° - 18°)$$

$$= \frac{\sin 15°}{\sin 15°} + \frac{\cos 78°}{\cos 78°} - \cos 18° \sec 18°$$

[∵ cos (90° – θ) = sin θ, sin (90° – θ) = cos θ, and cosec (90° – θ) = sec θ]

$= 1 + 1 - 1$

$= \mathbf{1}$

12. $A + B + C = 180°$ [Sum of angles of a triangle is 180°]

$\Rightarrow \quad A + B = 180° - C$

$\Rightarrow \quad \frac{A + B}{2} = 90° - \frac{C}{2}$

$\Rightarrow \quad \text{cosec}\left(\frac{A + B}{2}\right) = \text{cosec}\left(90° - \frac{C}{2}\right)$

$\Rightarrow \quad \text{cosec}\left(\frac{A + B}{2}\right) = \sec \frac{C}{2}$

[∵ cosec (90° – θ) = sec θ]

Hence, $\mathbf{cosec}\left(\frac{\mathbf{A + B}}{\mathbf{2}}\right) = \mathbf{sec} \frac{\mathbf{C}}{\mathbf{2}}$ proved.

13. $\cos A + \sin A = \sqrt{3}$ [Given]

$\Rightarrow (\cos^2 A + \sin^2 A) + 2\sin A \cos A = 3$ [Squaring both sides]

$\Rightarrow \quad 1 + 2\sin A \cos A = 3$ [∵ $\sin^2 A + \cos^2 A = 1$]

$\Rightarrow \quad 2\sin A \cos A = 3 - 1 = 2$

$\Rightarrow \quad \sin A \cos A = 1$

$\Rightarrow \quad 1 = \frac{1}{\sin A \cos A}$

$\Rightarrow \quad 1 = \frac{\sin^2 A + \cos^2 A}{\sin A \cos A}$

[Putting $1 = \sin^2 A + \cos^2 A$]

$\Rightarrow \quad 1 = \frac{\sin^2 A}{\sin A \cos A} + \frac{\cos^2 A}{\sin A \cos A}$

$\Rightarrow \quad 1 = \frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}$

$\Rightarrow \quad 1 = \tan A + \cot A$

Hence, **cot A + tan A = 1** proved.

14. $\text{LHS} = \mathbf{sec^2}\ \theta - \left[\frac{\mathbf{\sin^2 \theta - 2\sin^4 \theta}}{\mathbf{2\cos^4 \theta - \cos^2 \theta}}\right]$

$$= \sec^2 \theta - \frac{\sin^2 \theta}{\cos^2 \theta}\left[\frac{1 - 2\sin^2 \theta}{2\cos^2 \theta - 1}\right]$$

$$= \sec^2 \theta - \tan^2 \theta \left[\frac{1 - 2\sin^2 \theta}{2(1 - \sin^2 \theta) - 1}\right]$$

[∵ $\sin^2 \theta + \cos^2 \theta = 1 \Rightarrow \cos^2 \theta = 1 - \sin^2 \theta$]

$$= \sec^2 \theta - \tan^2 \theta \left(\frac{1 - 2\sin^2 \theta}{2 - 2\sin^2 \theta - 1}\right)$$

$$= \sec^2 \theta - \tan^2 \theta \left(\frac{1 - 2\sin^2 \theta}{1 - 2\sin^2 \theta}\right)$$

$= \sec^2 \theta - \tan^2 \theta = \mathbf{1}$

[∵ $1 + \tan^2 \theta = \sec^2 \theta \Rightarrow \sec^2 \theta - \tan^2 \theta = 1$]

= RHS

15. $\text{LHS} = \mathbf{(m^2 + n^2) \cos^2 \beta}$

$$= \left[\left(\frac{\cos \alpha}{\cos \beta}\right)^2 + \left(\frac{\cos \alpha}{\sin \beta}\right)^2\right]\cos^2 \beta$$

$$= \left(\frac{\cos^2 \alpha}{\cos^2 \beta} + \frac{\cos^2 \alpha}{\sin^2 \beta}\right)\cos^2 \beta$$

$$= \cos^2 \alpha \left(\frac{1}{\cos^2 \beta} + \frac{1}{\sin^2 \beta}\right) \cos^2 \beta$$

$$= \cos^2 \alpha \left(\frac{\sin^2 \beta + \cos^2 \beta}{\cos^2 \beta \sin^2 \beta}\right) \cos^2 \beta$$

$$= \cos^2 \alpha \left(\frac{1}{\cos^2 \beta \sin^2 \beta}\right) \cos^2 \beta$$

[∵ $\sin^2 \beta + \cos^2 \beta = 1$]

$$= \frac{\cos^2 \alpha}{\sin^2 \beta}$$

$$= \left(\frac{\cos \alpha}{\sin \beta}\right)^2$$

$= \mathbf{n^2} = \text{RHS}$

UNIT TEST 2

For Standard Level

1. (*b*) **60°, 15°**

(sec A – 2) (tan 3A – 1) = 0 [Given]

Either sec A – 2 = 0 or tan 3A – 1 = 0

⇒ sec A = 2 or tan 3A = 1

⇒ sec A = sec 60° or tan 3A = tan 45°

⇒ A = 60° or 3A = 45°

⇒ A = 15°

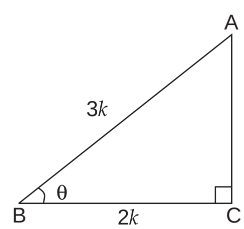
Hence, measure of A is 60° or 15°.

2. (*b*) **0**

$\cos \theta = \frac{2}{3}$ [Given]

Let ΔABC be right Δ in which

∠ABC = θ and C = 90°



Then, $\cos \theta = \frac{BC}{AB} = \frac{2}{3}$

Let BC = 2*k*, then AB = 3*k*

In right ΔACB, we have

$AB^2 = BC^2 + AC^2$

$\Rightarrow \quad (3k)^2 = (2k)^2 + AC^2$

$\Rightarrow \quad AC^2 = (9 - 4)\, k^2 = 5k^2$

$\Rightarrow \quad AC = \sqrt{5}\, k$

Then,

$$\sec\theta = \frac{AB}{BC} = \frac{3k}{2k} = \frac{3}{2}$$

$$\tan\theta = \frac{AC}{BC} = \frac{\sqrt{5}k}{2k} = \frac{\sqrt{5}}{2}$$

$$\therefore\ 2\sec^2\theta + 2\tan^2\theta - 7 = 2\left(\frac{3}{2}\right)^2 + 2\left(\frac{\sqrt{5}}{2}\right)^2 - 7$$

$$= 2\times\frac{9}{4} + 2\times\frac{5}{4} - 7$$

$$= \frac{9}{2} + \frac{5}{2} - 7$$

$$= \frac{9+5-14}{2}$$

$$= \frac{0}{2} = 0$$

3. (*b*) $\boldsymbol{p^2 + q^2}$

$a\cos\theta + b\sin\theta = p$ and $a\sin\theta - b\cos\theta = q$

$\Rightarrow\ a^2\cos^2\theta + b^2\sin^2\theta + 2ab\cos\theta\sin\theta = p^2$ … (1)

and $a^2\sin^2\theta + b^2\cos^2\theta - 2ab\sin\theta\cos\theta = q^2$ … (2)

Adding (1) and (2), we get

$a^2(\sin^2\theta + \cos^2\theta) + b^2(\sin^2\theta + \cos^2\theta) = p^2 + q^2$

$\Rightarrow\ a^2 + b^2 = p^2 + q^2$

[$\because \sin^2\theta + \cos^2\theta = 1$]

4. (*c*) $\boldsymbol{\frac{\sqrt{3}}{2}}$

$$7\sin^2 A + 3\cos^2 A = 4$$

and $0 \le A \le \frac{\pi}{2}$

$$4\sin^2 A + 3\sin^2 A + 3\cos^2 A = 4$$

$\Rightarrow\ 4\sin^2 A + 3(\sin^2 A + \cos^2 A) = 4$

$\Rightarrow\ 4\sin^2 A + 3 = 4$

$\Rightarrow\ 4\sin^2 A = 4 - 3 = 1$

$\Rightarrow\ \sin^2 A = \frac{1}{4}$

$\Rightarrow\ \sin A = \frac{1}{2}$

$\Rightarrow\ A = 30°$

$$\cos A = \cos 30° = \frac{\sqrt{3}}{2}$$

5. (*a*) **2 cosec** θ

$$\frac{\tan\theta}{\sec\theta+1} + \frac{\tan\theta}{\sec\theta-1} = k$$

$\Rightarrow\ \tan\theta\left(\frac{1}{\sec\theta+1} + \frac{1}{\sec\theta-1}\right) = k$

$\Rightarrow\ \tan\theta\left[\frac{\sec\theta-1+\sec\theta+1}{\sec^2\theta-1}\right] = k$

$\Rightarrow\ \tan\theta\left(\frac{2\sec\theta}{\sec^2\theta-1}\right) = k$

$\Rightarrow\ \tan\theta\left(\frac{2\sec\theta}{\tan^2\theta}\right) = k$

[$\because 1 + \tan^2\theta = \sec^2\theta \Rightarrow \sec^2\theta - 1 = \tan^2\theta$]

$\Rightarrow\ \frac{\sin\theta}{\cos\theta}\left(\frac{\frac{2}{\cos\theta}}{\frac{\sin^2\theta}{\cos^2\theta}}\right) = k$

$\Rightarrow\ \frac{\sin\theta}{\cos\theta}\left(\frac{2}{\cos\theta}\right)\left(\frac{\cos^2\theta}{\sin^2\theta}\right) = k$

$\Rightarrow\ \frac{2}{\sin\theta} = k$

$\Rightarrow\ k = \mathbf{2\ cosec}\ \theta$

6. (*b*) $\boldsymbol{\frac{1}{3}}$

$3\cos\theta = 2\sin\theta$ [Given]

$\Rightarrow\ \frac{3}{2} = \frac{\sin\theta}{\cos\theta}$ … (1)

$$\therefore\ \frac{4\sin\theta - 3\cos\theta}{2\sin\theta + 6\cos\theta} = \frac{\frac{4\sin\theta}{\cos\theta} - \frac{3\cos\theta}{\cos\theta}}{\frac{2\sin\theta}{\cos\theta} + \frac{6\cos\theta}{\cos\theta}}$$

[Dividing the num. and denom. by cos θ]

$$= \frac{4\times\frac{3}{2} - 3}{2\times\frac{3}{2} + 6}$$ [Using (1)]

$$= \frac{6-3}{3+6} = \frac{3}{9}$$

$$= \mathbf{\frac{1}{3}}$$

7. $\text{cosec}\ A = \frac{13}{12}$ [Given]

$\Rightarrow\ \text{cosec}^2 A = \frac{169}{144}$

$\Rightarrow\ \text{cosec}^2 A - 1 = \frac{169}{144} - 1$

$$= \frac{169-144}{144} = \frac{25}{144}$$

$\Rightarrow\ \cot^2 A = \frac{25}{144}$

$\Rightarrow\ \cot A = \frac{5}{12}$

$\Rightarrow\ \tan A = \frac{12}{5}$ … (1)

$$\therefore\ \frac{2\sin\theta - 3\cos\theta}{4\sin\theta - 9\cos\theta} = \frac{\frac{2\sin\theta}{\cos\theta} - \frac{3\cos\theta}{\cos\theta}}{\frac{4\sin\theta}{\cos\theta} - \frac{9\cos\theta}{\cos\theta}}$$

[Dividing the num. and denom. by cos θ]

$$= \frac{2\tan\theta - 3}{4\tan\theta - 9}$$

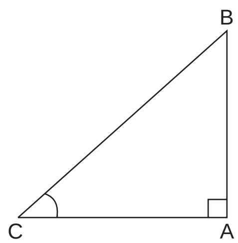
$$= \frac{2\times\frac{12}{5} - 3}{4\times\frac{12}{5} - 9}$$ [Using (1)]

$$= \frac{\frac{24-15}{5}}{\frac{48-45}{5}}$$

$$= \frac{9}{5} \times \frac{5}{3}$$

$$= \mathbf{3}$$

8. Let ABC be a right Δ in which tan C = $\sqrt{3}$



Then, $\tan C = \frac{AB}{AC} = \sqrt{3} = \frac{\sqrt{3}}{1}$

Let AB = $\sqrt{3}\,k$. Then, AC = k

In right ΔABC, we have

$$BC^2 = AB^2 + AC^2$$
$$= (\sqrt{3}\,k)^2 + (k)^2$$
$$= 3k^2 + k^2$$
$$= 4k^2$$

$$\Rightarrow \quad BC = 2k$$

$$\left.\begin{aligned} \sin B &= \frac{AC}{BC} = \frac{k}{2k} = \frac{1}{2} \\ \cos C &= \frac{AC}{BC} = \frac{k}{2k} = \frac{1}{2} \\ \cos B &= \frac{AB}{BC} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2} \\ \sin C &= \frac{AB}{BC} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2} \end{aligned}\right] \ldots(1)$$

$$\therefore \sin B \cos C + \cos B \sin C = \frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$$

[Using (1)]

$$= \frac{1}{4} + \frac{3}{4}$$

$$= \frac{1+3}{4}$$

$$= \frac{4}{4} = \mathbf{1}$$

9.

$$\text{LHS} = \frac{\mathbf{1}}{\mathbf{\sec A + \tan A}} - \frac{\mathbf{1}}{\mathbf{\cos A}}$$

$$= \frac{(\sec A - \tan A)}{(\sec A + \tan A)(\sec A - \tan A)} - \sec A$$

$$= \frac{\sec A - \tan A}{\sec^2 A - \tan^2 A} - \sec A$$

$$= \sec A - \tan A - \sec A$$

$[\because \sec^2 A - \tan^2 A = 1]$

$$= \mathbf{-\tan A}$$

$$\text{RHS} = \frac{1}{\cos A} - \frac{1}{\sec A - \tan A}$$

$$= \sec A - \frac{\sec A + \tan A}{(\sec A - \tan A)(\sec A + \tan A)}$$

$$= \sec A - \frac{\sec A + \tan A}{(\sec^2 A - \tan^2 A)}$$

$$= \sec A - \sec A - \tan A$$

$[\because \sec^2 A - \tan^2 A = 1]$

$$= \mathbf{-\tan A}$$

Hence, $\frac{\mathbf{1}}{\mathbf{\sec A + \tan A}} - \frac{\mathbf{1}}{\mathbf{\cos A}} = \frac{\mathbf{1}}{\mathbf{\cos A}} - \frac{\mathbf{1}}{\mathbf{\sec A - \tan A}}$

[Each is equal to – tan A]

10. LHS = **sin A (1 + tan A) + cos A (1 + cot A)**

$$= \sin A\left(1 + \frac{\sin A}{\cos A}\right) + \cos A\left(1 + \frac{\cos A}{\sin A}\right)$$

$$= \sin A\left(\frac{\cos A + \sin A}{\cos A}\right) + \cos A\left(\frac{\sin A + \cos A}{\sin A}\right)$$

$$= \frac{\sin A \cos A + \sin^2 A}{\cos A} + \frac{\cos A \sin A + \cos^2 A}{\sin A}$$

$$= \frac{\sin^2 A \cos A + \sin^3 A + \cos^2 A \sin A + \cos^3 A}{\sin A \cos A}$$

$$= \frac{\sin^3 A + \cos^3 A + \sin^2 A \cos A + \sin A \cos^2 A}{\sin A \cos A}$$

$$= \frac{(\sin A + \cos A)(\sin^2 A + \cos^2 A - \sin A \cos A) + \sin A \cos A(\sin A + \cos A)}{\sin A \cos A}$$

$$= \frac{(\sin A + \cos A)(1 - \sin A \cos A + \sin A \cos A)}{\sin A \cos A}$$

$$= \frac{\sin A + \cos A}{\sin A \cos A} = \frac{\sin A}{\sin A \cos A} + \frac{\cos A}{\sin A \cos A}$$

$$= \frac{1}{\cos A} + \frac{1}{\sin A}$$

= **sec A + cosec A**

= RHS

11.

$$\frac{3 \tan 25° \tan 40° \tan 50° \tan 65° - \frac{1}{2}\tan^2 60°}{4(\cos^2 29° + \cos^2 61°)}$$

$$= \frac{3 \tan 25° \tan 65° \tan 40° \tan 50° - \frac{1}{2}(\sqrt{3})^2}{4[\cos^2(90° - 61°) + \cos^2 61°]}$$

$[\because \tan 60° = \sqrt{3}]$

$$= \frac{3 \tan 25° \tan (90° - 25°) \tan 40° \tan (90° - 40°) - \frac{3}{2}}{4[\sin^2 61° + \cos^2 61°]}$$

$[\because \cos (90° - \theta) = \sin \theta]$

$$= \frac{3 \tan 25° \cot 25° \tan 40° \cot 40° - \frac{3}{2}}{4(1)}$$

$[\because \tan (90° - \theta) = \cot \theta, \sin^2 \theta + \cos^2 \theta = 1]$

$$= \frac{3(1)(1) - \frac{3}{2}}{4} = \frac{\frac{6-3}{2}}{4}$$

$$= \frac{3}{2} \times \frac{1}{4} = \mathbf{\frac{3}{8}}$$

12. $\sec\theta = x + \frac{1}{4x}$

$\Rightarrow \sec^2\theta = \left(x + \frac{1}{4x}\right)^2$

$\Rightarrow \sec^2\theta - 1 = \left(x + \frac{1}{4x}\right)^2 - 1$

$\Rightarrow \tan^2\theta = x^2 + \frac{1}{16x^2} + 2(x)\left(\frac{1}{4x}\right) - 1$

$[\because 1 + \tan^2\theta = \sec^2\theta \Rightarrow \sec^2\theta - 1 = \tan^2\theta]$

$\Rightarrow \tan^2\theta = x^2 + \frac{1}{16x^2} - \frac{1}{2}$

$\Rightarrow \tan^2\theta = \left(x - \frac{1}{4x}\right)^2$

$\Rightarrow \tan\theta = \pm\left(x - \frac{1}{4x}\right)$

When $\tan\theta = \left(x - \frac{1}{4x}\right)$, then

$\sec\theta + \tan\theta = \left(x + \frac{1}{4x}\right) + \left(x - \frac{1}{4x}\right)$

$= x + \frac{1}{4x} + x - \frac{1}{4x}$

$= \mathbf{2x}$

When $\tan\theta = -\left(x - \frac{1}{4x}\right)$, then

$\sec\theta + \tan\theta = \left(x + \frac{1}{4x}\right) + \left[-\left(x - \frac{1}{4x}\right)\right]$

$= x + \frac{1}{4x} - x + \frac{1}{4x}$

$= \frac{2}{4x}$

$= \mathbf{\frac{1}{2x}}$

Hence, $\mathbf{\sec\theta + \tan\theta = 2x}$ **or** $\mathbf{\frac{1}{2x}}$, proved.