CHAPTER 9

Trigonometric Identities

EXERCISE 9

For Basic and Standard Levels

1. (*i*) (*a*) $\sec\theta - \tan\theta\sin\theta = \dfrac{1}{\cos\theta} - \dfrac{\sin\theta}{\cos\theta}\cdot\sin\theta$

$= \dfrac{1-\sin^2\theta}{\cos\theta}$

$= \dfrac{\cos^2\theta}{\cos\theta}$

$[\because \sin^2\theta + \cos^2\theta = 1 \Rightarrow 1 - \sin^2\theta = \cos^2\theta]$

$= \mathbf{\cos}\,\theta$

(*b*) $\dfrac{1-\tan^2\theta}{\cot^2\theta - 1} = \dfrac{1-\dfrac{\sin^2\theta}{\cos^2\theta}}{\dfrac{\cos^2\theta}{\sin^2\theta}-1}$

$= \dfrac{\dfrac{\cos^2\theta-\sin^2\theta}{\cos^2\theta}}{\dfrac{\cos^2\theta-\sin^2\theta}{\sin^2\theta}}$

$= \dfrac{\cos^2\theta-\sin^2\theta}{\cos^2\theta}\times\dfrac{\sin^2\theta}{\cos^2\theta-\sin^2\theta}$

$= \dfrac{\sin^2\theta}{\cos^2\theta}$

$= \mathbf{\tan^2}\,\theta$

(*c*) $\dfrac{2+\tan\theta}{\sec\theta + 2\operatorname{cosec}\theta} = \dfrac{2+\dfrac{\sin\theta}{\cos\theta}}{\dfrac{1}{\cos\theta}+\dfrac{2}{\sin\theta}}$

$= \dfrac{\dfrac{(2\cos\theta+\sin\theta)}{\cos\theta}}{\dfrac{(\sin\theta+2\cos\theta)}{\sin\theta\cos\theta}}$

$= \dfrac{(2\cos\theta+\sin\theta)}{\cos\theta}\times\dfrac{\sin\theta\cos\theta}{(\sin\theta+2\cos\theta)}$

$= \mathbf{\sin}\,\theta$

(*d*) $\dfrac{2+\cot\theta}{2\sec\theta + \operatorname{cosec}\theta} = \dfrac{2+\dfrac{\cos\theta}{\sin\theta}}{\dfrac{2}{\cos\theta}+\dfrac{1}{\sin\theta}}$

$= \dfrac{\dfrac{(2\sin\theta+\cos\theta)}{\sin\theta}}{\dfrac{(2\sin\theta+\cos\theta)}{\sin\theta\cos\theta}}$

$= \dfrac{(2\sin\theta+\cos\theta)}{\sin\theta}\times\dfrac{\sin\theta\cos\theta}{(2\sin\theta+\cos\theta)}$

$= \mathbf{\cos}\,\theta$

(*e*) $\dfrac{2-\tan\theta}{2\operatorname{cosec}\theta - \sec\theta} = \dfrac{2-\dfrac{\sin\theta}{\cos\theta}}{\dfrac{2}{\sin\theta}-\dfrac{1}{\cos\theta}}$

$= \dfrac{\dfrac{(2\cos\theta-\sin\theta)}{\cos\theta}}{\dfrac{(2\cos\theta-\sin\theta)}{\sin\theta\cos\theta}}$

$= \dfrac{(2\cos\theta-\sin\theta)}{\cos\theta}\times\dfrac{\sin\theta\cos\theta}{(2\cos\theta-\sin\theta)}$

$= \mathbf{\sin}\,\theta$

(*f*) $\dfrac{\sec^2\theta-\tan^2\theta}{\cot\theta+\tan\theta} = \dfrac{\dfrac{1}{\cos^2\theta}-\dfrac{\sin^2\theta}{\cos^2\theta}}{\dfrac{\cos\theta}{\sin\theta}+\dfrac{\sin\theta}{\cos\theta}}$

$= \dfrac{\dfrac{1-\sin^2\theta}{\cos^2\theta}}{\dfrac{(\cos^2\theta+\sin^2\theta)}{\sin\theta\cos\theta}}$

$= \dfrac{\dfrac{\cos^2\theta}{\cos^2\theta}}{\dfrac{1}{\sin\theta\cos\theta}}$

$[\because \sin^2\theta + \cos^2\theta = 1]$

$= \dfrac{\cos^2\theta}{\cos^2\theta}\times\dfrac{\sin\theta\cos\theta}{1}$

$= \mathbf{\sin}\,\theta\,\mathbf{\cos}\,\theta$

(*g*) $\dfrac{\sin\theta-2\sin^3\theta}{2\cos^3\theta-\cos\theta} = \dfrac{\sin\theta\,(1-2\sin^2\theta)}{\cos\theta\,(2\cos^2\theta-1)}$

$= \dfrac{\sin\theta\,[(\sin^2\theta+\cos^2\theta-2\sin^2\theta)]}{\cos\theta\,[2\cos^2\theta-(\sin^2\theta+\cos^2\theta)]}$

$[\because \sin^2\theta + \cos^2\theta = 1]$

$= \dfrac{\sin\theta(\cos^2\theta-\sin^2\theta)}{\cos\theta(2\cos^2\theta-\sin^2\theta-\cos^2\theta)}$

$= \dfrac{\sin\theta(\cos^2\theta-\sin^2\theta)}{\cos\theta(\cos^2\theta-\sin^2\theta)}$

$= \dfrac{\sin\theta}{\cos\theta}$

$= \mathbf{\tan}\,\theta$

(*h*) $\dfrac{\sin^3\theta-\cos^3\theta}{\sin\theta-\cos\theta}$

$= \dfrac{(\sin\theta-\cos\theta)(\sin^2\theta+\cos^2\theta+\sin\theta\cos\theta)}{(\sin\theta-\cos\theta)}$

$= \mathbf{(1 + \sin\,\theta\,\cos\,\theta)}$

(*ii*) (*a*) $\text{cosec}\,\theta = 2x$

$\Rightarrow \quad x = \dfrac{\text{cosec}\,\theta}{2}$

and $\quad \cot\theta = \dfrac{2}{x}$

$\Rightarrow \quad \dfrac{1}{x} = \dfrac{\cot\theta}{2}$

$$2\left(x^2 - \frac{1}{x^2}\right) = 2\left[\left(\frac{\text{cosec}\,\theta}{2}\right)^2 - \left(\frac{\cot\theta}{2}\right)^2\right]$$

$$= 2\left(\frac{\text{cosec}^2\theta - \cot^2\theta}{4}\right)$$

$$= 2\left(\frac{1}{4}\right)$$

$[\because \cot^2\theta + 1 = \text{cosec}^2\theta]$

$$= \mathbf{\frac{1}{2}}$$

(*b*) $2x = \sec A$

$\Rightarrow \quad x = \dfrac{\sec A}{2}$

and $\quad \dfrac{2}{x} = \tan A$

$\Rightarrow \quad \dfrac{1}{x} = \dfrac{\tan A}{2}$

$$2\left(x^2 - \frac{1}{x^2}\right) = 2\left[\left(\frac{\sec A}{2}\right)^2 - \left(\frac{\tan A}{2}\right)^2\right]$$

$$= 2\left(\frac{\sec^2 A - \tan^2 A}{4}\right)$$

$$= 2\left(\frac{1}{4}\right) [\because \tan^2 A + 1 = \sec^2 A]$$

$$= \mathbf{\frac{1}{2}}$$

2. (*i*) $5\,\text{cosec}^2 A - 5\cot^2 A = 5(\text{cosec}^2 A - \cot^2 A)$

$= 5(1 + \cot^2 A - \cot^2 A)$

$= \mathbf{5}$ $[\because \text{cosec}^2 A = 1 + \cot^2 A]$

(*ii*) $7\sin^2\theta + \dfrac{7}{\sec^2\theta} = 7\left(\sin^2\theta + \dfrac{1}{\sec^2\theta}\right)$

$= 7(\sin^2\theta + \cos^2\theta)$

$= \mathbf{7}$ $[\because \sin^2\theta + \cos^2\theta = 1]$

(*iii*) $\dfrac{3}{\cos^2\theta} - 3\tan^2\theta = 3\left(\dfrac{1}{\cos^2\theta} - \tan^2\theta\right)$

$= 3(\sec^2\theta - \tan^2\theta)$

$= 3(1 + \tan^2\theta - \tan^2\theta)$

$[\because \sec^2\theta = 1 + \tan^2\theta]$

$= \mathbf{3}$

(*iv*) $\cos^2\theta + \dfrac{1}{1+\cot^2\theta} = \cos^2\theta + \dfrac{1}{\text{cosec}^2\theta}$

$= \cos^2\theta + \sin^2\theta$

$= \mathbf{1}$ $[\because \cos^2\theta + \sin^2\theta = 1]$

(*v*) $(\sec^2\theta - 1)(1 - \text{cosec}^2\theta) = (1 + \tan^2\theta - 1)$

$[1 - (1 + \cot^2\theta)]$

$[\because \sec^2\theta = 1 + \tan^2\theta$ and $\text{cosec}^2\theta = 1 + \cot^2\theta]$

$= (\tan^2\theta)(1 - 1 - \cot^2\theta)$

$= (\tan^2\theta)(-\cot^2\theta)$

$= (\tan^2\theta)\left(-\dfrac{1}{\tan^2\theta}\right)$

$= \mathbf{-1}$

(*vi*) $4\sec^2 A - 4\sin^2 A\sec^2 A = 4\sec^2 A(1 - \sin^2 A)$

$= 4\sec^2 A\cos^2 A$

$[\because \sin^2 A + \cos^2 A = 1 \Rightarrow 1 - \sin^2 A = \cos^2 A]$

$= 4\ \dfrac{1}{\cos^2 A} \times \cos^2 A$

$= \mathbf{4}$

3. (*i*) LHS $= \dfrac{\mathbf{\cos\theta - \sin\theta}}{\mathbf{\cos\theta}}$

$= \dfrac{\cos\theta}{\cos\theta} - \dfrac{\sin\theta}{\cos\theta}$

$= \mathbf{1 - \tan\theta} =$ RHS

(*ii*) LHS $= \mathbf{(1 + \cos\theta)(1 - \cos\theta)}$

$= 1 - \cos^2\theta$

$= \sin^2\theta$ $[\because \sin^2\theta + \cos^2\theta = 1]$

$= \dfrac{\mathbf{1}}{\mathbf{cosec^2\theta}}$

= RHS

(*iii*) LHS $= \mathbf{(1 - \sin A)(1 + \sin A)(1 + \tan^2 A)}$

$= (1 - \sin^2 A)(\sec^2 A)$

$[\because 1 + \tan^2 A = \sec^2 A]$

$= \cos^2 A \times \dfrac{1}{\cos^2 A}$

$[\because \sin^2 A + \cos^2 A = 1]$

$= \mathbf{1}$

= RHS

(*iv*) LHS $= \dfrac{\mathbf{\cot^2\theta}}{\mathbf{cosec\,\theta + 1}}$

$= \dfrac{\text{cosec}^2\theta - 1}{\text{cosec}\,\theta + 1}$

$[\because 1 + \cot^2\theta = \text{cosec}^2\theta]$

$= \dfrac{(\text{cosec}\,\theta + 1)(\text{cosec}\,\theta - 1)}{(\text{cosec}\,\theta + 1)}$

$= \mathbf{cosec\,\theta - 1}$

= RHS

(*v*) LHS $= \mathbf{2\cos^2\theta + \dfrac{2}{1 + \cot^2\theta}}$

$= 2\left(\cos^2\theta + \dfrac{1}{\text{cosec}^2\theta}\right)$

$= 2(\cos^2\theta + \sin^2\theta)$

$= \mathbf{2}$ $[\because \sin^2\theta + \cos^2\theta = 1]$

= RHS

(*vi*) LHS $= \mathbf{\dfrac{1}{1 + \sin\theta} + \dfrac{1}{1 - \sin\theta}}$

$= \dfrac{1 - \sin\theta + 1 + \sin\theta}{1 - \sin^2\theta}$

$= \dfrac{2}{\cos^2\theta}$ $[\because \sin^2\theta + \cos^2\theta = 1]$

$= \mathbf{2\sec^2\theta}$

= RHS

4. *(i)* LHS = $\mathbf{2\cos^2\theta\,(1+\tan^2\theta)}$
$= 2\cos^2\theta\sec^2\theta$ $[\because 1+\tan^2\theta=\sec^2\theta]$
$= 2\cos^2\theta\times\dfrac{1}{\cos^2\theta}$
$= \mathbf{2}$
= RHS

(ii) LHS = $\mathbf{(1-\sin^2 A)\sec^2 A - 1}$
$=\cos^2 A\sec^2 A-1$ $[\because \sin^2\theta+\cos^2\theta=1]$
$=\cos^2 A\times\dfrac{1}{\cos^2 A}-1$
$=1-1$
$=\mathbf{0}$
= RHS

(iii) LHS = $\mathbf{(1+\cot^2\theta)(1+\cos\theta)(1-\cos\theta)}$
$=(\text{cosec}^2\theta)(1-\cos^2\theta)$
$[\because 1+\cot^2\theta=\text{cosec}^2\theta]$
$=\text{cosec}^2\theta\sin^2\theta$ $[\because \sin^2\theta+\cos^2\theta=1]$
$=\dfrac{1}{\sin^2\theta}\times\sin^2\theta$
$=\mathbf{1}$
= RHS

(iv) LHS = $\mathbf{(\cos^2\theta-1)(\cot^2\theta+1)+1}$
$=[\cos^2\theta-(\sin^2\theta+\cos^2\theta)]\,\text{cosec}^2\theta+1$
$[\because \sin^2\theta+\cos^2\theta=1 \text{ and } 1+\cot^2\theta=\text{cosec}^2\theta]$
$=(\cos^2\theta-\sin^2\theta-\cos^2\theta)\,\text{cosec}^2\theta+1$
$=-\sin^2\theta\times\dfrac{1}{\sin^2\theta}+1$
$=-1+1$
$=\mathbf{0}$
= RHS

(v) LHS = $\dfrac{(\mathbf{1}+\tan^2\theta)\cot\theta}{\text{cosec}^2\theta}$
$=\dfrac{\sec^2\theta\cot\theta}{\text{cosec}^2\theta}$ $[\because 1+\tan^2\theta=\sec^2\theta]$
$=\dfrac{\sin^2\theta}{\cos^2\theta}\,\dfrac{\cos\theta}{\sin\theta}=\dfrac{\sin\theta}{\cos\theta}$
$=\mathbf{\tan}\,\theta$
= RHS

(vi) LHS = $\mathbf{\sin^2 A\cot^2 A+\cos^2 A\tan^2 A}$
$=\sin^2 A\,\dfrac{\cos^2 A}{\sin^2 A}+\cos^2 A\,\dfrac{\sin^2 A}{\cos^2 A}$
$=\cos^2 A+\sin^2 A$
$=\mathbf{1}$ $[\because \sin^2 A+\cos^2 A=1]$
= RHS

5. *(i)* LHS = $\dfrac{\mathbf{1}}{\mathbf{\tan\theta+\cot\theta}}$
$=\dfrac{1}{\dfrac{\sin\theta}{\cos\theta}+\dfrac{\cos\theta}{\sin\theta}}$
$=\dfrac{1}{\dfrac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}}$
$=\dfrac{\sin\theta\cos\theta}{(\sin^2\theta+\cos^2\theta)}$
$=\mathbf{\sin}\,\theta\,\mathbf{\cos}\,\theta$ $[\because \sin^2\theta+\cos^2\theta=1]$
= RHS

(ii) RHS = $\dfrac{\mathbf{2\cos^2\theta-1}}{\mathbf{\sin\theta\cos\theta}}$
$=\dfrac{2\cos^2\theta-(\sin^2\theta+\cos^2\theta)}{\sin\theta\cos\theta}$
$[\because \sin^2\theta+\cos^2\theta=1]$
$=\dfrac{2\cos^2\theta-\sin^2\theta-\cos^2\theta}{\sin\theta\cos\theta}$
$=\dfrac{\cos^2\theta-\sin^2\theta}{\sin\theta\cos\theta}$
$=\dfrac{\cos^2\theta}{\sin\theta\cos\theta}-\dfrac{\sin^2\theta}{\sin\theta\cos\theta}$
$=\dfrac{\cos\theta}{\sin\theta}-\dfrac{\sin\theta}{\cos\theta}$
$=\mathbf{\cot}\,\theta-\mathbf{\tan}\,\theta$
= LHS

(iii) LHS = $\dfrac{\mathbf{1-\tan^2\theta}}{\mathbf{\cot^2\theta-1}}$
$=\dfrac{1-\dfrac{\sin^2\theta}{\cos^2\theta}}{\dfrac{\cos^2\theta}{\sin^2\theta}-1}$
$=\dfrac{\dfrac{\cos^2\theta-\sin^2\theta}{\cos^2}}{\dfrac{\cos^2\theta-\sin^2\theta}{\sin^2\theta}}$
$=\dfrac{\cos^2\theta-\sin^2\theta}{\cos^2\theta}\times\dfrac{\sin^2\theta}{\cos^2\theta-\sin^2\theta}$
$=\dfrac{\sin^2\theta}{\cos^2\theta}$
$=\mathbf{\tan^2}\,\theta$
= RHS

(iv) LHS = $\mathbf{\sec^2}\,\theta+\mathbf{\text{cosec}^2}\,\theta$
$=\dfrac{1}{\cos^2\theta}+\dfrac{1}{\sin^2\theta}$
$=\dfrac{\sin^2\theta+\cos^2\theta}{\cos^2\theta\sin^2\theta}$
$=\dfrac{1}{\cos^2\theta\sin^2\theta}$ $[\because \sin^2\theta+\cos^2\theta=1]$
$=\mathbf{\sec^2}\,\theta\,\mathbf{\text{cosec}^2}\,\theta$
= RHS

(v) LHS = $\dfrac{\mathbf{\sin\theta-2\sin^3\theta}}{\mathbf{2\cos^3\theta-\cos\theta}}$
$=\dfrac{\sin\theta(1-2\sin^2\theta)}{\cos\theta(2\cos^2\theta-1)}$
$=\dfrac{\sin\theta[1-2(1-\cos^2\theta)]}{\cos\theta(2\cos^2\theta-1)}$
$[\because \sin^2\theta+\cos^2\theta=1 \Rightarrow \sin^2\theta=1-\cos^2\theta]$
$=\dfrac{\sin\theta}{\cos\theta}\;\dfrac{(1-2+2\cos^2\theta)}{(2\cos^2\theta-1)}$

$= \dfrac{\sin\theta(2\cos^2\theta - 1)}{\cos\theta(2\cos^2\theta - 1)}$

$= \dfrac{\sin\theta}{\cos\theta}$

$= \textbf{tan}\ \theta$

= RHS

(*vi*) LHS $= \textbf{sec}^2\ \theta - \dfrac{\mathbf{sin^2\theta - 2sin^4\theta}}{\mathbf{2cos^4\theta - cos^2\theta}}$

$= \dfrac{1}{\cos^2\theta} - \dfrac{\sin^2\theta(1 - 2\sin^2\theta)}{\cos^2\theta(2\cos^2\theta - 1)}$

$= \dfrac{1}{\cos^2\theta} - \dfrac{\sin^2\theta[1 - 2(1 - \cos^2\theta)]}{\cos^2\theta(2\cos^2\theta - 1)}$

$[\because \sin^2\theta + \cos^2\theta = 1 \Rightarrow \sin^2\theta = 1 - \cos^2\theta]$

$= \dfrac{1}{\cos^2\theta} - \dfrac{\sin^2\theta(1 - 2 + 2\cos^2\theta)}{\cos^2\theta(2\cos^2\theta - 1)}$

$= \dfrac{1}{\cos^2\theta} - \dfrac{\sin^2\theta(2\cos^2\theta - 1)}{\cos^2\theta(2\cos^2\theta - 1)}$

$= \dfrac{1}{\cos^2\theta} - \dfrac{\sin^2\theta}{\cos^2\theta}$

$= \dfrac{1 - \sin^2\theta}{\cos^2\theta}$

$= \dfrac{\cos^2\theta}{\cos^2\theta}$

= **1**

= RHS

6. LHS = **cosec θ (1 + cos θ) (cosec θ – cot θ)**

$= \dfrac{1}{\sin\theta}(1 + \cos\theta)\left(\dfrac{1}{\sin\theta} - \dfrac{\cos\theta}{\sin\theta}\right)$

$= \left(\dfrac{1 + \cos\theta}{\sin\theta}\right)\left(\dfrac{1 - \cos\theta}{\sin\theta}\right)$

$= \dfrac{1 - \cos^2\theta}{\sin^2\theta}$

$= \dfrac{\sin^2\theta}{\sin^2\theta}$

$[\because \sin^2\theta + \cos^2\theta = 1 \Rightarrow 1 - \cos^2\theta = \sin^2\theta]$

= **1**

= RHS

7. LHS = **sec A(1 – sin A) (sec A + tan A)**

$= \dfrac{1}{\cos A}(1 - \sin A)\left(\dfrac{1}{\cos A} + \dfrac{\sin A}{\cos A}\right)$

$= \left(\dfrac{1 - \sin A}{\cos A}\right)\left(\dfrac{1 + \sin A}{\cos A}\right)$

$= \dfrac{1 - \sin^2 A}{\cos^2 A}$

$= \dfrac{\cos^2 A}{\cos^2 A}$

$[\because \sin^2 A + \cos^2 A = 1 \Rightarrow 1 - \sin^2 A = \cos^2 A]$

= **1** = RHS

8. LHS = **(sin α + cos α) (tan α + cot α)**

$= (\sin\alpha + \cos\alpha)\left(\dfrac{\sin\alpha}{\cos\alpha} + \dfrac{\cos\alpha}{\sin\alpha}\right)$

$= (\sin\alpha + \cos\alpha)\ \dfrac{(\sin^2\alpha + \cos^2\alpha)}{\cos\alpha\sin\alpha}$

$= \dfrac{\sin\alpha + \cos\alpha}{\cos\alpha\sin\alpha}\ [\because \sin^2\alpha + \cos^2\alpha = 1]$

$= \dfrac{\sin\alpha}{\cos\alpha\sin\alpha} + \dfrac{\cos\alpha}{\cos\alpha\sin\alpha}$

$= \dfrac{1}{\cos\alpha} + \dfrac{1}{\sin\alpha}$

= **sec α + cosec α**

= RHS

9. LHS = **(sec A + cos A) (sec A – cos A)**

$= \sec^2 A - \cos^2 A$

$= 1 + \tan^2 A - \cos^2 A$

$[\because \sec^2 A = 1 + \tan^2 A]$

$= \tan^2 A + (1 - \cos^2 A)$

$= \mathbf{tan^2 A + sin^2 A}$

$[\because \sin^2 A + \cos^2 A = 1 \Rightarrow 1 - \cos^2 A = \sin^2 A]$

= RHS

10. (*i*) LHS = **(1 + tan θ + sec θ) (1 + cot θ – cosec θ)**

$= \left(1 + \dfrac{\sin\theta}{\cos\theta} + \dfrac{1}{\cos\theta}\right)\left(1 + \dfrac{\cos\theta}{\sin\theta} - \dfrac{1}{\sin\theta}\right)$

$= \left(\dfrac{\cos\theta + \sin\theta + 1}{\cos\theta}\right)\left(\dfrac{\sin\theta + \cos\theta - 1}{\sin\theta}\right)$

$= \dfrac{(\sin\theta + \cos\theta)^2 - (1)^2}{\cos\theta\sin\theta}$

$[\because (a + b)(a - b) = a^2 - b^2]$

$= \dfrac{\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta - 1}{\cos\theta\sin\theta}$

$= \dfrac{1 + 2\sin\theta\cos\theta - 1}{\cos\theta\sin\theta}\ [\because \sin^2\theta + \cos^2\theta = 1]$

$= \dfrac{2\sin\theta\cos\theta}{\sin\theta\cos\theta}$

= **2**

= RHS

(*ii*) LHS = **(1 + cot A – cosec A) (1 + tan A + sec A)**

$= \left(1 + \dfrac{\cos A}{\sin A} - \dfrac{1}{\sin A}\right)\left(1 + \dfrac{\sin A}{\cos A} + \dfrac{1}{\cos A}\right)$

$= \left(\dfrac{\sin A + \cos A - 1}{\sin A}\right)\left(\dfrac{\cos A + \sin A + 1}{\cos A}\right)$

$= \dfrac{(\sin A + \cos A)^2 - (1)^2}{\sin A\cos A}$

$[\because (a - b)(a + b) = a^2 - b^2]$

$= \dfrac{\sin^2 A + \cos^2 A + 2\sin A\cos A - 1}{\sin A\cos A}$

$= \dfrac{1 + 2\sin A\cos A - 1}{\sin A\cos A}$

$[\because \sin^2 A + \cos^2 A = 1]$

$= \dfrac{2\sin A \cos A}{\sin A \cos A}$

$= \mathbf{2}$
= RHS

11. (*i*) LHS = **(sec θ – cosec θ) (1 + tan θ + cot θ)**

$= \left(\dfrac{1}{\cos\theta} - \dfrac{1}{\sin\theta}\right)\left(1 + \dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta}\right)$

$= \left(\dfrac{\sin\theta - \cos\theta}{\cos\theta\sin\theta}\right)\left(\dfrac{\cos\theta\sin\theta + \sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta}\right)$

$= \dfrac{\sin^3\theta - \cos^3\theta}{\cos^2\theta\sin^2\theta}$

[∵ $(a-b)(a^2+ab+b^2) = a^3 - b^3$]

$= \dfrac{\sin^3\theta}{\cos^2\theta\sin^2\theta} - \dfrac{\cos^3\theta}{\cos^2\theta\sin^2\theta}$

$= \dfrac{\sin\theta}{\cos^2\theta} - \dfrac{\cos\theta}{\sin^2\theta}$

$= \left(\dfrac{\sin\theta}{\cos\theta}\right)\left(\dfrac{1}{\cos\theta}\right) - \left(\dfrac{\cos\theta}{\sin\theta}\right)\left(\dfrac{1}{\sin\theta}\right)$

= **tan θ sec θ – cot θ cosec θ**
= RHS

(*ii*) LHS = **(1 + cot A + tan A) (sin A – cos A)**

$= \left(1 + \dfrac{\cos A}{\sin A} + \dfrac{\sin A}{\cos A}\right)(\sin A - \cos A)$

$= \dfrac{(\sin A\cos A + \cos^2 A + \sin^2 A)(\sin A - \cos A)}{\sin A\cos A}$

$= \dfrac{\sin^3 A - \cos^3 A}{\sin A\cos A}$

[∵ $(a-b)(a^2+ab+b^2) = a^3 - b^3$]

$= \dfrac{\sin^3 A}{\sin A\cos A} - \dfrac{\cos^3 A}{\sin A\cos A}$

$= \dfrac{\sin^2 A}{\cos A} - \dfrac{\cos^2 A}{\sin A}$

$= (\sin A)\left(\dfrac{\sin A}{\cos A}\right) - \left(\dfrac{\cos A}{\sin A}\right)(\cos A)$

= **sin A tan A – cot A cos A**
= RHS

12. LHS = **(sec θ + tan θ – 1) (sec θ – tan θ + 1)**

= [(sec θ) + (tan θ – 1)][(sec θ) – (tan θ – 1)]
$= \sec^2\theta - (\tan\theta - 1)^2$

[∵ $(a+b)(a-b) = a^2 - b^2$]

$= \sec^2\theta - (\tan^2\theta + 1 - 2\tan\theta)$
$= \sec^2\theta - (\sec^2\theta - 2\tan\theta)$

[∵ $1 + \tan^2\theta = \sec^2\theta$]

$= \sec^2\theta - \sec^2\theta + 2\tan\theta$
= **2 tan θ**
= RHS

13. LHS = **tan θ +** $\mathbf{\dfrac{1}{\tan\theta}}$

= tan θ + cot θ

$= \dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta}$

$= \dfrac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$

$= \dfrac{1}{\sin\theta\cos\theta}$ [∵ $\sin^2\theta + \cos^2\theta = 1$]

$= \left(\dfrac{1}{\cos\theta}\right)\left(\dfrac{1}{\sin\theta}\right)$

= **sec θ cosec θ**
= RHS

14. LHS = $\mathbf{\dfrac{\cos A}{1 - \sin A} + \dfrac{\cos A}{1 + \sin A}}$

$= \cos A\left(\dfrac{1}{1 - \sin A} + \dfrac{1}{1 + \sin A}\right)$

$= \cos A\left(\dfrac{1 + \sin A + 1 - \sin A}{1 - \sin^2 A}\right)$

$= \cos A\left(\dfrac{2}{\cos^2 A}\right)$

[∵ $\sin^2 A + \cos^2 A = 1 \Rightarrow 1 - \sin^2 A = \cos^2 A$]

$= \dfrac{2}{\cos A}$

= **2 sec A**
= RHS

15. LHS = $\mathbf{\dfrac{\cos\theta}{1 - \tan\theta} + \dfrac{\sin\theta}{1 - \cot\theta}}$

$= \dfrac{\cos\theta}{1 - \dfrac{\sin\theta}{\cos\theta}} + \dfrac{\sin\theta}{1 - \dfrac{\cos\theta}{\sin\theta}}$

$= \dfrac{\cos\theta}{\dfrac{\cos\theta - \sin\theta}{\cos\theta}} + \dfrac{\sin\theta}{\dfrac{\sin\theta - \cos\theta}{\sin\theta}}$

$= \dfrac{\cos^2\theta}{\cos\theta - \sin\theta} + \dfrac{\sin^2\theta}{\sin\theta - \cos\theta}$

$= \dfrac{\cos^2\theta}{(\cos\theta - \sin\theta)} - \dfrac{\sin^2\theta}{(\cos\theta - \sin\theta)}$

$= \dfrac{\cos^2\theta - \sin^2\theta}{(\cos\theta - \sin\theta)}$

$= \dfrac{(\cos\theta - \sin\theta)(\cos\theta + \sin\theta)}{(\cos\theta - \sin\theta)}$

= **cos θ + sin θ**
= RHS

16. LHS = $\mathbf{\dfrac{\sin A}{1 + \cos A} + \dfrac{1 + \cos A}{\sin A}}$

$= \dfrac{\sin^2 A + (1 + \cos A)^2}{\sin A\,(1 + \cos A)}$

$= \dfrac{\sin^2 A + 1 + 2\cos A + \cos^2 A}{\sin A\,(1 + \cos A)}$

$= \dfrac{(\sin^2 A + \cos^2 A) + 1 + 2\cos A}{\sin A\,(1 + \cos A)}$

$= \dfrac{2 + 2\cos A}{\sin A\,(1 + \cos A)}$ [∵ $\sin^2 A + \cos^2 A = 1$]

$= \dfrac{2(1 + \cos A)}{\sin A\,(1 + \cos A)}$

$= \frac{2}{\sin A}$

$= \mathbf{2\ cosec\ A}$

= RHS

17. LHS $= \frac{\mathbf{cosec\,\theta}}{\mathbf{cosec\,\theta - 1}} + \frac{\mathbf{cosec\,\theta}}{\mathbf{cosec\,\theta + 1}}$

$= \text{cosec}\ \theta \left(\frac{1}{\text{cosec}\,\theta - 1} + \frac{1}{\text{cosec}\,\theta + 1}\right)$

$= \text{cosec}\ \theta \left(\frac{\text{cosec}\,\theta + 1 + \text{cosec}\,\theta - 1}{\text{cosec}^2\,\theta - 1}\right)$

$= \text{cosec}\ \theta \left(\frac{2\,\text{cosec}\,\theta}{\cot^2\theta}\right)$

$[\because 1 + \cot^2\theta = \text{cosec}^2\theta \Rightarrow \text{cosec}^2\theta - 1 = \cot^2\theta]$

$= \frac{2\,\text{cosec}^2\theta}{\cot^2\theta}$

$= 2\ \text{cosec}^2\theta\ \tan^2\theta$

$= 2\left(\frac{1}{\sin^2\theta}\right)\left(\frac{\sin^2\theta}{\cos^2\theta}\right)$

$= \frac{2}{\cos^2\theta}$

$= \mathbf{2\ sec^2}\ \theta$

= RHS

18. LHS $= \frac{\mathbf{tan\ A}}{\mathbf{sec\ A + 1}} + \frac{\mathbf{tan\ A}}{\mathbf{sec\ A - 1}}$

$= \tan A \left(\frac{1}{\sec A + 1} + \frac{1}{\sec A - 1}\right)$

$= \tan A \left(\frac{\sec A - 1 + \sec A + 1}{\sec^2 A - 1}\right)$

$= \tan A \left(\frac{2\sec A}{\tan^2 A}\right)$

$[\because 1 + \tan^2 A = \sec^2 A \Rightarrow \sec^2 A - 1 = \tan^2 A]$

$= \frac{2\sec A}{\tan A} = 2\sec A\ \cot A$

$= 2\left(\frac{1}{\cos A}\right)\left(\frac{\cos A}{\sin A}\right)$

$= \frac{2}{\sin A}$

$= \mathbf{2\ cosec\ A}$

= RHS

19. LHS $= \frac{\mathbf{cos\,\theta}}{\mathbf{1 - sin\,\theta}} + \frac{\mathbf{1 - sin\,\theta}}{\mathbf{cos\,\theta}}$

$= \frac{\cos^2\theta + (1 - \sin\theta)^2}{(1 - \sin\theta)\cos\theta}$

$= \frac{\cos^2\theta + 1 + \sin^2\theta - 2\sin\theta}{(1 - \sin\theta)\cos\theta}$

$= \frac{(\cos^2\theta + \sin^2\theta) + 1 - 2\sin\theta}{(1 - \sin\theta)\cos\theta}$

$= \frac{2 - 2\sin\theta}{(1 - \sin\theta)\cos\theta}$ $[\because \sin^2\theta + \cos^2\theta = 1]$

$= \frac{2(1 - \sin\theta)}{(1 - \sin\theta)\cos}$

$= \frac{2}{\cos\theta}$

$= \mathbf{2\ sec}\ \theta$

= RHS

20. LHS $= \mathbf{(1 + tan^2\ \theta)\ (1 + sin\ \theta)\ (1 - sin\ \theta)}$

$= \sec^2\theta\,(1 - \sin^2\theta)$ $[\because 1 + \tan^2\theta = \sec^2\theta]$

$= \sec^2\theta\ \cos^2\theta$

$[\because \sin^2\theta + \cos^2\theta = 1 \Rightarrow 1 - \sin^2\theta = \cos^2\theta]$

$= \left(\frac{1}{\cos^2\theta}\right)(\cos^2\theta)$

$= \mathbf{1}$

= RHS

21. LHS $= \mathbf{(1 + tan^2\ A)\ sin\ A\ cos\ A}$

$= \sec^2 A\ \sin A\ \cos A$ $[\because 1 + \tan^2 A = \sec^2 A]$

$= \left(\frac{1}{\cos^2 A}\right)(\sin A)(\cos A)$

$= \frac{\sin A}{\cos A}$

$= \mathbf{tan\ A}$

= RHS

22. LHS $= \frac{\mathbf{1 + cos\,\theta - sin^2\theta}}{\mathbf{sin\,\theta\,(1 + cos\,\theta)}}$

$= \frac{\cos\theta + (1 - \sin^2\theta)}{\sin\theta\,(1 + \cos\theta)}$

$= \frac{\cos\theta + \cos^2\theta}{\sin\theta\,(1 + \cos\theta)}$

$[\because \sin^2\theta + \cos^2\theta = 1 \Rightarrow 1 - \sin^2\theta = \cos^2\theta]$

$= \frac{\cos\theta(1 + \cos\theta)}{\sin\theta\,(1 + \cos\theta)}$

$= \frac{\cos\theta}{\sin\theta}$

$= \mathbf{cot}\ \theta$

= RHS

23. LHS $= \frac{\mathbf{tan\,\theta}}{\mathbf{(1 + tan^2\theta)^2}} + \frac{\mathbf{cot\,\theta}}{\mathbf{(1 + cot^2\theta)^2}}$

$= \frac{\tan\theta}{(\sec^2\theta)^2} + \frac{\cot\theta}{(\text{cosec}^2\theta)^2}$

$[\because 1 + \tan^2 = \sec^2\theta \text{ and } 1 + \cot^2\theta = \text{cosec}^2\theta]$

$= \frac{\tan\theta}{\sec^4\theta} + \frac{\cot\theta}{\text{cosec}^4\theta}$

$= \left(\frac{\sin\theta}{\cos\theta}\right)(\cos^4\theta) + \left(\frac{\cos\theta}{\sin\theta}\right)(\sin^4\theta)$

$= \sin\theta\ \cos^3\theta + \cos\theta\ \sin^3\theta$

$= \sin\theta\ \cos\theta\ (\cos^2\theta + \sin^2\theta)$

$= \sin\theta\ \cos\theta\ (1)$ $[\because \sin^2\theta + \cos^2\theta = 1]$

$= \mathbf{sin}\ \theta\ \mathbf{cos}\ \theta$

= RHS

24. LHS $= \mathbf{(cos\ A - sin\ A)^2 + (cos\ A + sin\ A)^2}$

$= \cos^2 A + \sin^2 A - 2\sin A\cos A + \sin^2 A$
$+ \cos^2 A + 2\sin A\cos A$

$= 2 (\sin^2 A + \cos^2 A)$
$= 2 (1)$ $[\because \sin^2 A + \cos^2 A = 1]$
$= \mathbf{2}$
$=$ RHS

25. LHS $= \mathbf{\cot^2 \theta - \cos^2 \theta}$

$$= \frac{\cos^2\theta}{\sin^2\theta} - \cos^2\theta$$

$$= \cos^2\theta\left(\frac{1}{\sin^2\theta} - 1\right)$$

$$= \cos^2\theta\left(\frac{1-\sin^2\theta}{\sin^2\theta}\right)$$

$$= \left(\frac{\cos^2\theta}{\sin^2\theta}\right)(\cos^2\theta)$$

$[\because \sin^2\theta + \cos^2\theta = 1 \Rightarrow 1 - \sin^2\theta = \cos^2\theta]$

$= \mathbf{\cot^2\theta \cos^2\theta}$

26. LHS $= \dfrac{\mathbf{\cos^2\theta - \cot^2\theta + 1}}{\mathbf{\sin^2\theta + \tan^2\theta - 1}}$

$$= \frac{\cos^2\theta - \dfrac{\cos^2\theta}{\sin^2\theta} + 1}{\sin^2\theta + \dfrac{\sin^2\theta}{\cos^2\theta} - 1}$$

$$= \frac{\dfrac{(\cos^2\theta\sin^2\theta - \cos^2\theta + \sin^2\theta)}{\sin^2\theta}}{\dfrac{(\sin^2\theta\cos^2\theta + \sin^2\theta - \cos^2\theta)}{\cos^2\theta}}$$

$$= \frac{(\sin^2\theta\cos^2\theta + \sin^2\theta - \cos^2\theta)}{\sin^2\theta} \times \frac{\cos^2\theta}{(\sin^2\theta\cos^2\theta + \sin^2\theta - \cos^2\theta)}$$

$$= \frac{\cos^2\theta}{\sin^2\theta}$$

$= \mathbf{\cot^2\theta}$
$=$ RHS

27. LHS $= \mathbf{1 + (\cot^2\theta - \tan^2\theta)\cos^2\theta}$

$$= 1 + \left(\frac{\cos^2\theta}{\sin^2\theta} - \frac{\sin^2\theta}{\cos^2\theta}\right)\cos^2\theta$$

$$= 1 + \left(\frac{\cos^4\theta - \sin^4\theta}{\sin^2\theta\cos^2\theta}\right)\cos^2\theta$$

$$= 1 + \frac{(\cos^2\theta + \sin^2\theta)(\cos^2\theta - \sin^2\theta)}{\sin^2\theta}$$

$$= 1 + \frac{\cos^2\theta - \sin^2\theta}{\sin^2\theta} \quad [\because \sin^2\theta + \cos^2\theta = 1]$$

$$= \frac{\sin^2\theta + \cos^2\theta - \sin^2\theta}{\sin^2\theta}$$

$$= \frac{\cos^2\theta}{\sin^2\theta}$$

$= \mathbf{\cot^2\theta}$
$=$ RHS

28. LHS $= \dfrac{\mathbf{(\sin\theta + \cos\theta)^2 - 1}}{\mathbf{\tan\theta - \sin\theta\cos\theta}}$

$$= \frac{\sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta - 1}{\dfrac{\sin\theta}{\cos\theta} - \sin\theta\cos\theta}$$

$$= \frac{1 + 2\sin\theta\cos\theta - 1}{\sin\theta\left(\dfrac{1}{\cos\theta} - \cos\theta\right)} \quad [\because \sin^2\theta + \cos^2\theta = 1]$$

$$= \frac{2\sin\theta\cos\theta}{\sin\theta\left(\dfrac{1-\cos^2\theta}{\cos\theta}\right)}$$

$$= \left(\frac{2\cos\theta}{1-\cos^2\theta}\right)(\cos\theta)$$

$$= \frac{2\cos^2\theta}{\sin^2\theta}$$

$[\because \sin^2\theta + \cos^2\theta = 1 \Rightarrow 1 - \cos^2\theta = \sin^2\theta]$

$= \mathbf{2\cot^2\theta}$
$=$ RHS

29. LHS $= \dfrac{\mathbf{\sin\theta}}{\mathbf{\cos\theta + \sin\theta}} - \dfrac{\mathbf{\cos\theta}}{\mathbf{\cos\theta - \sin\theta}}$

$$= \frac{\sin\theta\cos\theta - \sin^2\theta - \cos^2\theta - \sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta}$$

$$= \frac{-(\sin^2\theta + \cos^2\theta)}{\cos^2\theta - \sin^2\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\sin^2\theta - \cos^2\theta}$$

$$= \frac{\dfrac{\sin^2\theta}{\cos^2\theta} + \dfrac{\cos^2\theta}{\cos^2\theta}}{\dfrac{\sin^2\theta}{\cos^2\theta} - \dfrac{\cos^2\theta}{\cos^2\theta}}$$

[Dividing the num. and denom. by $\cos^2\theta$]

$$= \frac{\mathbf{\tan^2\theta + 1}}{\mathbf{\tan^2\theta - 1}}$$

$=$ RHS

30. LHS $= \mathbf{(1 + \tan^2\theta)(1 + \cot^2\theta)\tan^2\theta - (1 - \tan^2\theta)^2}$

$$= (1 + \tan^2\theta)\left(1 + \frac{1}{\tan^2\theta}\right)\tan^2\theta - (1 - \tan^2\theta)^2$$

$= (1 + \tan^2\theta)(\tan^2\theta + 1) - (1 - \tan^2\theta)^2$
$= (1 + \tan^2\theta)^2 - (1 - \tan^2\theta)^2$
$= (1 + \tan^2\theta + 1 - \tan^2\theta)(1 + \tan^2\theta - 1 + \tan^2\theta)$
$[\because a^2 - b^2 = (a + b)(a - b)]$
$= 2(2\tan^2\theta)$
$= \mathbf{4\tan^2\theta}$
$=$ RHS

31. LHS $= \dfrac{\mathbf{\cot^2 A}}{\mathbf{1 + \cot^2 A}} + \dfrac{\mathbf{\tan^2 A}}{\mathbf{1 + \tan^2 A}}$

$$= \frac{\cot^2 A}{\text{cosec}^2 A} + \frac{\tan^2 A}{\sec^2 A}$$

$[\because 1 + \cot^2 A = \text{cosec}^2 A \text{ and } 1 + \tan^2 A = \sec^2 A]$

$= \left(\frac{\cos^2 A}{\sin^2 A}\right)(\sin^2 A) + \left(\frac{\sin^2 A}{\cos^2 A}\right)(\cos^2 A)$

$= \cos^2 A + \sin^2 A$

$= \mathbf{1}$

= RHS

32. LHS $= \dfrac{\tan A}{\text{cosec}^2 A} + \dfrac{\cot A}{\sec^2 A} + \dfrac{2}{\text{cosec} A \sec A}$

$= \left(\frac{\sin A}{\cos A}\right)(\sin^2 A) + \left(\frac{\cos A}{\sin A}\right)(\cos^2 A) + 2 \sin A \cos A$

$= \dfrac{\sin^3 A}{\cos A} + \dfrac{\cos^3 A}{\sin A} + 2\sin A \cos A$

$= \dfrac{\sin^4 A + \cos^4 A + 2\sin^2 A \cos^2 A}{\sin A \cos A}$

$= \dfrac{(\sin^2 A + \cos^2 A)^2}{\sin A \cos A}$

$= \dfrac{(1)^2}{\sin A \cos A} = \dfrac{1}{\sin A \cos A}$

$= \dfrac{\sin^2 A + \cos^2 A}{\sin A \cos A}$ $[\because \sin^2 A + \cos^2 A = 1]$

$= \dfrac{\sin^2 A}{\sin A \cos A} + \dfrac{\cos^2 A}{\sin A \cos A}$

$= \dfrac{\sin A}{\cos A} + \dfrac{\cos A}{\sin A}$

$= \mathbf{\tan A + \cot A}$

= RHS

33. LHS $= \left(\dfrac{(1 + \cot A + \tan A)(\sin A - \cos A)}{\sec^3 A - \text{cosec}^3 A}\right)$

$= \dfrac{\left(1 + \frac{\cos A}{\sin A} + \frac{\sin A}{\cos A}\right)(\sin A - \cos A)}{\frac{1}{\cos^3 A} - \frac{1}{\sin^3 A}}$

$= \dfrac{\frac{(\sin A \cos A + \cos^2 A + \sin^2 A)(\sin A - \cos A)}{\sin A \cos A}}{\frac{(\sin^3 A - \cos^3 A)}{\sin^3 A \cos^3 A}}$

$= \dfrac{\frac{(\sin^3 A - \cos^3 A)}{\sin A \cos A}}{\frac{(\sin^3 A - \cos^3 A)}{\sin^3 A \cos^3 A}}$

$[\because (a - b)(a^2 + ab + b^2) = a^3 - b^3]$

$= \dfrac{\sin^3 A - \cos^3 A}{\sin A \cos A} \times \dfrac{\sin^3 A \cos^3 A}{\sin^3 A - \cos^3 A}$

$= \mathbf{\sin^2 A \cos^2 A}$

= RHS

34. RHS $= \mathbf{(\sec\theta + \tan\theta)^2}$

$= \left(\dfrac{1}{\cos\theta} + \dfrac{\sin\theta}{\cos\theta}\right)^2$

$= \left(\dfrac{1 + \sin\theta}{\cos\theta}\right)^2$

$= \dfrac{(1 + \sin\theta)^2}{\cos^2\theta}$

$= \dfrac{(1 + \sin\theta)(1 + \sin\theta)}{(1 - \sin^2\theta)}$

$[\because \sin^2\theta + \cos^2\theta = 1 \Rightarrow \cos^2\theta = 1 - \sin^2\theta]$

$= \dfrac{(1 + \sin\theta)(1 + \sin\theta)}{(1 + \sin\theta)(1 - \sin\theta)}$

$[\because a^2 - b^2 = (a + b)(a - b)]$

$= \dfrac{\mathbf{1 + \sin\theta}}{\mathbf{1 - \sin\theta}}$

= LHS

ALTERNATIVE METHOD:

LHS $= \dfrac{\mathbf{1 + \sin\theta}}{\mathbf{1 - \sin\theta}}$

$= \dfrac{(1 + \sin\theta)(1 + \sin\theta)}{(1 - \sin\theta)(1 + \sin\theta)}$

$= \dfrac{(1 + \sin\theta)^2}{1 - \sin^2\theta}$

$= \dfrac{(1 + \sin\theta)^2}{\cos^2\theta}$

$[\because \sin^2\theta + \cos^2\theta = 1 \Rightarrow 1 - \sin^2\theta = \cos^2\theta]$

$= \left(\dfrac{1 + \sin\theta}{\cos\theta}\right)^2$

$= \left(\dfrac{1}{\cos\theta} + \dfrac{\sin\theta}{\cos\theta}\right)^2$

$= \mathbf{(\sec\theta + \tan\theta)^2}$

= RHS

35. RHS $= \mathbf{(\sec\theta - \tan\theta)^2}$

$= \left(\dfrac{1}{\cos\theta} - \dfrac{\sin\theta}{\cos\theta}\right)^2$

$= \left(\dfrac{1 - \sin\theta}{\cos\theta}\right)^2$

$= \dfrac{(1 - \sin\theta)^2}{\cos^2\theta}$

$= \dfrac{(1 - \sin\theta)(1 - \sin\theta)}{1 - \sin^2\theta}$

$[\because \sin^2\theta + \cos^2\theta = 1 \Rightarrow \cos^2\theta = 1 - \sin^2\theta]$

$= \dfrac{(1 - \sin\theta)(1 - \sin\theta)}{(1 + \sin\theta)(1 - \sin\theta)}$

$[\because a^2 - b^2 = (a + b)(a - b)]$

$= \dfrac{\mathbf{1 - \sin\theta}}{\mathbf{1 + \sin\theta}}$

= LHS

36. RHS $= \mathbf{(\text{cosec}\,\theta + \cot\theta)^2}$

$= \left(\dfrac{1}{\sin\theta} + \dfrac{\cos\theta}{\sin\theta}\right)^2$

$$= \left(\frac{1+\cos\theta}{\sin\theta}\right)^2$$

$$= \frac{(1+\cos\theta)^2}{\sin^2\theta}$$

$$= \frac{(1+\cos\theta)(1+\cos\theta)}{(1-\cos^2\theta)}$$

$[\because \sin^2\theta + \cos^2\theta = 1 \Rightarrow \sin^2\theta = 1 - \cos^2\theta]$

$$= \frac{(1+\cos\theta)(1+\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}$$

$[\because a^2 - b^2 = (a+b)(a-b)]$

$$= \frac{\mathbf{1+\cos\theta}}{\mathbf{1-\cos\theta}}$$

= LHS

37. RHS $= \left(\frac{\mathbf{\sin\theta}}{\mathbf{1+\cos\theta}}\right)^2$

$$= \frac{\sin^2\theta}{(1+\cos\theta)^2}$$

$$= \frac{1-\cos^2\theta}{(1+\cos\theta)^2}$$

$[\because \sin^2\theta + \cos^2\theta = 1 \Rightarrow \sin^2\theta = 1 - \cos^2\theta]$

$$= \frac{(1+\cos\theta)(1-\cos\theta)}{(1+\cos\theta)(1+\cos\theta)}$$

$[\because a^2 - b^2 = (a+b)(a-b)]$

$$= \frac{1-\cos\theta}{1+\cos\theta}$$

$$= \frac{1-\frac{1}{\sec\theta}}{1+\frac{1}{\sec\theta}}$$

$$= \frac{\frac{\sec\theta-1}{\sec\theta}}{\frac{\sec\theta+1}{\sec\theta}}$$

$$= \frac{\sec\theta-1}{\sec\theta} \times \frac{\sec\theta}{\sec\theta+1}$$

$$= \frac{\mathbf{\sec\theta-1}}{\mathbf{\sec\theta+1}}$$

= LHS

38. LHS = $\mathbf{(1 - \sin\theta + \cos\theta)^2}$

$= [(1 - \sin\theta) + \cos\theta]^2$

$= (1 - \sin\theta)^2 + 2(1 - \sin\theta)\cos\theta + \cos^2\theta$

$= 1 + \sin^2\theta - 2\sin\theta + 2\cos\theta - 2\sin\theta\cos\theta + \cos^2\theta$

$= 1 + (\sin^2\theta + \cos^2\theta) + 2\cos\theta - 2\sin\theta - 2\sin\theta\cos\theta$

$= 1 + 1 + 2\cos\theta - 2\sin\theta - 2\sin\cos\theta$

$[\because \sin^2\theta + \cos^2\theta = 1]$

$= 2 + 2\cos\theta - 2\sin\theta - 2\sin\theta\cos\theta$

$= 2(1 + \cos\theta) - 2\sin\theta(1 + \cos\theta)$

$= (1 + \cos\theta)(2 - 2\sin\theta)$

$= \mathbf{2(1 + \cos\theta)(1 - \sin\theta)}$

= RHS

39. LHS $= \frac{\mathbf{\cos\theta}}{\mathbf{1+\sin\theta}} = \frac{(\cos\theta)(1-\sin\theta)}{(1+\sin\theta)(1-\sin\theta)}$

$$= \frac{\cos\theta(1-\sin\theta)}{1-\sin^2\theta}$$

$$= \frac{\cos\theta(1-\sin\theta)}{\cos^2\theta}$$

$[\because \sin^2\theta + \cos^2\theta = 1 \Rightarrow 1 - \sin^2\theta = \cos^2\theta]$

$$= \frac{\mathbf{1-\sin\theta}}{\mathbf{\cos\theta}}$$

= RHS

40. LHS $= \frac{\mathbf{1-\cos\theta}}{\mathbf{\sin\theta}}$

$$= \frac{(1-\cos\theta)(1+\cos\theta)}{\sin\theta(1+\cos\theta)}$$

$$= \frac{1-\cos^2\theta}{\sin\theta(1+\cos\theta)}$$

$$= \frac{\sin^2\theta}{\sin\theta(1+\cos\theta)}$$

$[\because \sin^2\theta + \cos^2\theta = 1 \Rightarrow 1 - \cos^2\theta = \sin^2\theta]$

$$= \frac{\mathbf{\sin\theta}}{\mathbf{1+\cos\theta}}$$

= RHS

41. LHS $= \frac{\mathbf{\sin\theta}}{\mathbf{1-\cos\theta}}$

$$= \frac{\sin\theta(1+\cos\theta)}{(1-\cos\theta)(1+\cos\theta)}$$

$$= \frac{\sin\theta(1+\cos\theta)}{(1-\cos^2\theta)}$$

$$= \frac{\sin\theta(1+\cos\theta)}{(\sin^2\theta)}$$

$[\because \sin^2\theta + \cos^2\theta = 1 \Rightarrow 1 - \cos^2\theta = \sin^2\theta]$

$$= \frac{1+\cos\theta}{\sin\theta}$$

$$= \frac{1}{\sin\theta} + \frac{\cos\theta}{\sin\theta}$$

$= \mathbf{cosec}\ \theta + \mathbf{cot}\ \theta$

= RHS

42. LHS $= \frac{\mathbf{\tan\theta+\sin\theta}}{\mathbf{\tan\theta-\sin\theta}}$

$$= \frac{\frac{\sin\theta}{\cos\theta}+\sin\theta}{\frac{\sin\theta}{\cos\theta}-\sin\theta}$$

$$= \frac{\sin\theta\left(\frac{1}{\cos\theta}+1\right)}{\sin\theta\left(\frac{1}{\cos\theta}-1\right)}$$

$$= \frac{\mathbf{\sec\theta+1}}{\mathbf{\sec\theta-1}}$$

= RHS

43. LHS $= \dfrac{\mathbf{\cot A - \cos A}}{\mathbf{\cot A + \cos A}}$

$= \dfrac{\dfrac{\cos A}{\sin A} - \cos A}{\dfrac{\cos A}{\sin A} + \cos A}$

$= \dfrac{\cos A\left(\dfrac{1}{\sin A} - 1\right)}{\cos A\left(\dfrac{1}{\sin A} + 1\right)}$

$= \dfrac{\mathbf{cosec\,A - 1}}{\mathbf{cosec\,A + 1}}$

= RHS

44. LHS $= \dfrac{\mathbf{1}}{\mathbf{\sec\theta + \tan\theta}}$

$= \dfrac{(1)}{(\sec\theta + \tan\theta)} \times \dfrac{(\sec\theta - \tan\theta)}{(\sec\theta - \tan\theta)}$

$= \dfrac{\sec\theta - \tan\theta}{\sec^2\theta - \tan^2\theta}$ $[\because (a+b)(a-b) = a^2 - b^2]$

$=$ **sec** θ **− tan** θ

$[\because 1 + \tan^2\theta = \sec^2\theta - \tan^2\theta = 1]$

45. LHS $= \dfrac{\mathbf{1}}{\mathbf{cosec\,\theta + \cot\theta}}$

$= \dfrac{(1)}{(\text{cosec}\,\theta + \cot\theta)}\dfrac{(\text{cosec}\,\theta - \cot\theta)}{(\text{cosec}\,\theta - \cot\theta)}$

$= \dfrac{\text{cosec}\,\theta - \cot\theta}{\text{cosec}^2\theta - \cot^2\theta}$

$=$ **cosec** θ **− cot** θ

$[\because 1 + \cot^2\theta = \text{cosec}^2\theta \;\therefore \text{cosec}^2\theta - \cot^2\theta = 1]$

46. LHS $= \dfrac{\mathbf{cosec\,\theta + \cot\theta}}{\mathbf{cosec\,\theta - \cot\theta}}$

$= \dfrac{(\text{cosec}\,\theta + \cot\theta)(\text{cosec}\,\theta + \cot\theta)}{(\text{cosec}\,\theta - \cot\theta)(\text{cosec}\,\theta + \cot\theta)}$

$= \dfrac{(\text{cosec}\,\theta + \cot\theta)^2}{\text{cosec}^2\theta - \cot^2\theta}$

$=$ **(cosec** θ **+ cot** θ**)²**

$[\because 1 + \cot^2\theta = \text{cosec}^2\theta \Rightarrow \text{cosec}^2\theta - \cot^2\theta = 1]$

$= \text{cosec}^2\theta + \cot^2\theta + 2\,\text{cosec}\,\theta\cot\theta$

$= 1 + \cot^2\theta + \cot^2\theta + 2\,\text{cosec}\,\theta\cot\theta$

$[\because \text{cosec}^2\theta = 1 + \cot^2\theta]$

$=$ **1 + 2 cot²** θ **+ 2 cosec** θ **cot** θ

= RHS

47. LHS $= \dfrac{\mathbf{\sec\theta + \tan\theta}}{\mathbf{\sec\theta - \tan\theta}}$

$= \dfrac{(\sec\theta + \tan\theta)(\sec\theta + \tan\theta)}{(\sec\theta - \tan\theta)(\sec\theta + \tan\theta)}$

$= \dfrac{(\sec\theta + \tan\theta)^2}{\sec^2\theta - \tan^2\theta}$

$=$ **(sec** θ **+ tan** θ**)²**

$[\because 1 + \tan^2\theta = \sec^2\theta \Rightarrow \sec^2\theta - \tan^2\theta = 1]$

$= \left(\dfrac{1}{\cos\theta} + \dfrac{\sin\theta}{\cos\theta}\right)^2$

$= \left(\dfrac{\mathbf{1 + \sin\theta}}{\mathbf{\cos\theta}}\right)^2$

= RHS

48. LHS $= \dfrac{\mathbf{\sec\theta - \tan\theta}}{\mathbf{\sec\theta + \tan\theta}}$

$= \dfrac{\dfrac{1}{\cos\theta} - \dfrac{\sin\theta}{\cos\theta}}{\dfrac{1}{\cos\theta} + \dfrac{\sin\theta}{\cos\theta}}$

$= \dfrac{\dfrac{1}{\cos\theta}(1 - \sin\theta)}{\dfrac{1}{\cos\theta}(1 + \sin\theta)}$

$= \dfrac{(1 - \sin\theta)}{(1 + \sin\theta)} \times \dfrac{(1 + \sin\theta)}{(1 + \sin\theta)}$

[Multiplying the num. and denom. by $(1 + \sin\theta)$]

$= \dfrac{1 - \sin^2\theta}{(1 + \sin\theta)^2}$

$= \dfrac{\mathbf{\cos^2\theta}}{\mathbf{(1 + \sin\theta)^2}}$

$[\because \sin^2\theta + \cos^2\theta = 1 \Rightarrow 1 - \sin^2\theta = \cos^2\theta]$

49. LHS $= \dfrac{\mathbf{\sec\theta - \tan\theta}}{\mathbf{\sec\theta + \tan\theta}}$

$= \dfrac{(\sec\theta - \tan\theta)(\sec\theta - \tan\theta)}{(\sec\theta + \tan\theta)(\sec\theta - \tan\theta)}$

$= \dfrac{(\sec\theta - \tan\theta)^2}{\sec^2\theta - \tan^2\theta}$

$= (\sec\theta - \tan\theta)^2$

$[\because 1 + \tan^2\theta = \sec^2 \Rightarrow \sec^2\theta - \tan^2\theta = 1]$

$= \sec^2\theta - 2\sec\theta\tan\theta + \tan^2\theta$

$= 1 + \tan^2\theta - 2\sec\theta\tan\theta + \tan^2\theta$

$=$ **1 − 2 sec** θ **tan** θ **+ 2 tan²** θ = RHS

50. LHS $= \dfrac{\mathbf{1}}{\mathbf{\sec\theta - \tan\theta}} - \dfrac{\mathbf{1}}{\mathbf{\cos\theta}}$

$= \dfrac{(1)(\sec\theta + \tan\theta)}{(\sec\theta - \tan\theta)(\sec\theta + \tan\theta)} - \sec\theta$

$= \dfrac{\sec\theta + \tan\theta}{\sec^2\theta - \tan^2\theta} - \sec\theta$

$= \sec\theta + \tan\theta - \sec\theta$

$[\because 1 + \tan^2\theta = \sec^2\theta \Rightarrow \sec^2\theta - \tan^2\theta = 1]$

$=$ **tan** θ

RHS $= \dfrac{\mathbf{1}}{\mathbf{\cos\theta}} - \dfrac{\mathbf{1}}{\mathbf{\sec\theta + \tan\theta}}$

$= \sec\theta - \dfrac{(1)(\sec\theta - \tan\theta)}{(\sec\theta + \tan\theta)(\sec\theta - \tan\theta)}$

$= \sec\theta - \dfrac{\sec\theta - \tan\theta}{\sec^2\theta - \tan^2\theta}$

$= \sec\theta - \sec\theta + \tan\theta$

$[\because 1 + \tan^2\theta = \sec^2\theta \Rightarrow \sec^2\theta - \tan^2\theta = 1]$

$=$ **tan** θ

Hence, LHS = RHS [Each is equal to $\tan\theta$]

51. LHS = $\dfrac{\mathbf{1}}{\mathbf{\sec A+\tan A}}-\dfrac{\mathbf{1}}{\mathbf{\cos A}}$

$= \dfrac{(1)(\sec A-\tan A)}{(\sec A+\tan A)(\sec A-\tan A)}-\sec A$

$= \dfrac{\sec A-\tan A}{\sec^2 A-\tan^2 A}-\sec A$

$= \sec A - \tan A - \sec A$

$[\because 1+\tan^2 A=\sec^2 A \Rightarrow \sec^2 A-\tan^2 A=1]$

$= -\tan A$

RHS = $\dfrac{\mathbf{1}}{\mathbf{\cos A}}-\dfrac{\mathbf{1}}{\mathbf{\sec A-\tan A}}$

$= \sec A-\dfrac{(1)(\sec A+\tan A)}{(\sec A-\tan A)(\sec A+\tan A)}$

$= \sec A-\dfrac{\sec A+\tan A}{\sec^2 A-\tan^2 A}$

$= \sec A - \sec A - \tan A$

$[\because 1+\tan^2 A=\sec^2 A \Rightarrow \sec^2 A-\tan^2 A=1]$

$= -\tan A$

Hence, LHS = RHS [Each is equal to $-\tan A$]

52. LHS = $\dfrac{\mathbf{1}}{\mathbf{\operatorname{cosec} A+\cot A}}-\dfrac{\mathbf{1}}{\mathbf{\sin A}}$

$= \dfrac{(1)(\operatorname{cosec} A-\cot A)}{(\operatorname{cosec} A+\cot A)(\operatorname{cosec} A-\cot A)}-\operatorname{cosec} A$

$= \dfrac{\operatorname{cosec} A-\cot A}{\operatorname{cosec}^2 A-\cot^2 A}-\operatorname{cosec} A$

$= \operatorname{cosec} A-\cot A-\operatorname{cosec} A$

$[\because 1+\cot^2 A=\operatorname{cosec}^2 A \Rightarrow \operatorname{cosec}^2 A-\cot^2=1]$

$= -\cot A$

RHS = $\dfrac{\mathbf{1}}{\mathbf{\sin A}}-\dfrac{\mathbf{1}}{\mathbf{\operatorname{cosec} A-\cot A}}$

$= \operatorname{cosec} A-\dfrac{(1)(\operatorname{cosec} A+\cot A)}{(\operatorname{cosec} A-\cot A)(\operatorname{cosec} A+\cot A)}$

$= \operatorname{cosec} A-\dfrac{(\operatorname{cosec} A+\cot A)}{\operatorname{cosec}^2 A-\cot^2 A}$

$= \operatorname{cosec} A-\operatorname{cosec} A-\cot A$

$[\because 1+\cot^2 A=\operatorname{cosec}^2 A \Rightarrow \operatorname{cosec}^2 A-\cot^2 A=1]$

$= -\cot A$

Hence, LHS = RHS. [Each is equal to $-\cot A$]

53. LHS = $\sqrt{\dfrac{\mathbf{1+\cos\theta}}{\mathbf{1-\cos\theta}}}$

$= \sqrt{\dfrac{(1+\cos\theta)(1+\cos\theta)}{(1-\cos\theta)(1+\cos\theta)}}$

$= \sqrt{\dfrac{(1+\cos\theta)^2}{1-\cos^2\theta}}$

$= \sqrt{\dfrac{(1+\cos\theta)^2}{\sin^2\theta}}$

$[\because \sin^2\theta+\cos^2\theta=1 \Rightarrow 1-\cos^2\theta=\sin^2\theta]$

$= \dfrac{1+\cos\theta}{\sin\theta}$

$= \dfrac{1}{\sin\theta}+\dfrac{\cos\theta}{\sin\theta}$

$= \operatorname{cosec}\theta+\cot\theta$

= RHS

54. LHS = $\sqrt{\dfrac{\mathbf{1-\sin\theta}}{\mathbf{1+\sin\theta}}}$

$= \sqrt{\dfrac{(1-\sin\theta)(1-\sin\theta)}{(1+\sin\theta)(1-\sin\theta)}}$

$= \sqrt{\dfrac{(1-\sin\theta)^2}{1-\sin^2\theta}}$

$= \sqrt{\dfrac{(1-\sin\theta)^2}{\cos^2\theta}}$

$[\because \sin^2\theta+\cos^2\theta=1 \Rightarrow 1-\sin^2\theta=\cos^2\theta]$

$= \dfrac{1-\sin\theta}{\cos\theta}$

$= \dfrac{1}{\cos\theta}-\dfrac{\sin\theta}{\cos\theta}$

$= \sec\theta-\tan\theta$

= RHS

55. LHS = $\sqrt{\dfrac{\mathbf{1+\cos A}}{\mathbf{1-\cos A}}}$

$= \sqrt{\dfrac{(1+\cos A)(1+\cos A)}{(1-\cos A)(1+\cos A)}}$

$= \sqrt{\dfrac{(1+\cos A)^2}{1-\cos^2 A}}$

$= \sqrt{\dfrac{(1+\cos A)^2}{\sin^2 A}}$

$[\because \sin^2 A+\cos^2 A=1 \Rightarrow 1-\cos^2 A=\sin^2 A]$

$= \dfrac{1+\cos A}{\sin A}$

$= \dfrac{(1+\cos A)(\tan A)}{\sin A(\tan A)}$

$= \dfrac{\tan A+\cos A\tan A}{\sin A\tan A}$

$= \dfrac{\tan A+\cos A\left(\dfrac{\sin A}{\cos A}\right)}{\tan A\sin A}$

$= \dfrac{\mathbf{\tan A+\sin A}}{\mathbf{\tan A\sin A}}$

= RHS

56. LHS = $\sqrt{\dfrac{\mathbf{\sec\theta-1}}{\mathbf{\sec\theta+1}}}+\sqrt{\dfrac{\mathbf{\sec\theta+1}}{\mathbf{\sec\theta-1}}}$

$= \sqrt{\dfrac{(\sec\theta-1)(\sec\theta-1)}{(\sec\theta+1)(\sec\theta-1)}}+\sqrt{\dfrac{(\sec\theta+1)(\sec\theta+1)}{(\sec\theta-1)(\sec\theta+1)}}$

$= \sqrt{\dfrac{(\sec\theta-1)^2}{\sec^2\theta-1}}+\sqrt{\dfrac{(\sec\theta+1)^2}{\sec^2\theta-1}}$

$$= \sqrt{\frac{(\sec\theta - 1)^2}{\tan^2\theta}} + \sqrt{\frac{(\sec\theta + 1)^2}{\tan^2\theta}}$$

$[\because 1 + \tan^2\theta = \sec^2\theta \Rightarrow \sec^2\theta - 1 = \tan^2\theta]$

$$= \frac{\sec\theta - 1}{\tan\theta} + \frac{\sec\theta + 1}{\tan\theta}$$

$$= \frac{\sec\theta - 1 + \sec\theta + 1}{\tan\theta}$$

$$= \frac{2\sec\theta}{\tan\theta}$$

$$= \frac{2}{\cos\theta}\ \cot\theta$$

$$= \frac{2}{\cos\theta}\ \frac{\cos\theta}{\sin\theta}$$

$$= \frac{2}{\sin\theta}$$

$= \mathbf{2\ cosec}\ \theta$

= RHS

or LHS $= \sqrt{\frac{1+\cos\theta}{1-\cos\theta}} + \sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$

$$= \sqrt{\frac{1+\frac{1}{\sec\theta}}{1-\frac{1}{\sec\theta}}} + \sqrt{\frac{1-\frac{1}{\sec\theta}}{1+\frac{1}{\sec\theta}}}$$

$$= \sqrt{\frac{\sec\theta+1}{\sec\theta-1}} + \sqrt{\frac{\sec\theta-1}{\sec\theta+1}}$$

Now, follow the steps given for the solution of Q. 56.

57. LHS $= \frac{\mathbf{cot A + cosec A - 1}}{\mathbf{cot A - cosec A + 1}}$

$$= \frac{\cot A + \text{cosec } A - (\text{cosec}^2 A - \cot^2 A)}{\cot A - \text{cosec } A + 1}$$

$[\because 1 + \cot^2 A = \text{cosec}^2 A \Rightarrow 1 = \text{cosec}^2 A - \cot^2 A]$

$$= \frac{(\cot A + \text{cosec } A) - (\text{cosec } A + \cot A)(\text{cosec } A - \cot A)}{(\cot A - \text{cosec } A + 1)}$$

$$= \frac{(\cot A + \text{cosec } A)(1 - \text{cosec } A + \cot A)}{(\cot A - \text{cosec } A + 1)}$$

$= \cot A + \text{cosec } A$

$$= \frac{\cos A}{\sin A} + \frac{1}{\sin A}$$

$$= \frac{\cos A + 1}{\sin A}$$

$$= \frac{\mathbf{1 + \cos A}}{\mathbf{\sin A}}$$

= RHS

58. LHS $= \frac{\mathbf{\cos^2\theta}}{\mathbf{1 - \tan\theta}} + \frac{\mathbf{\sin^3\theta}}{\mathbf{\sin\theta - \cos\theta}}$

$$= \frac{\cos^2\theta}{1 - \frac{\sin\theta}{\cos\theta}} + \frac{\sin^3\theta}{\sin\theta - \cos\theta}$$

$$= \frac{\cos^2\theta}{\frac{\cos\theta - \sin\theta}{\cos\theta}} + \frac{\sin^3\theta}{\sin\theta - \cos\theta}$$

$$= \frac{\cos^3\theta}{\cos\theta - \sin\theta} - \frac{\sin^3\theta}{\cos\theta - \sin\theta}$$

$$= \frac{(\cos^3\theta - \sin^3\theta)}{(\cos\theta - \sin\theta)}$$

$$= \frac{(\cos\theta - \sin\theta)(\cos^2\theta + \sin^2\theta + \sin\theta\cos\theta)}{(\cos\theta - \sin\theta)}$$

$= \mathbf{1 + \sin}\ \theta\ \mathbf{\cos}\ \theta$ $[\because \sin^2\theta + \cos^2\theta = 1]$

= RHS

59. LHS $= \left(\mathbf{1 + \tan^2 A}\right)\left(\mathbf{1 + \frac{1}{\tan^2 A}}\right)$

$$= \sec^2 A\left(\frac{\tan^2 A + 1}{\tan^2 A}\right)$$

$$= \frac{1}{\cos^2 A}\left(\frac{\sec^2 A}{\tan^2 A}\right) \quad [\because 1 + \tan^2 A = \sec^2 A]$$

$$= \frac{1}{\cos^2 A}\left(\frac{1}{\cos^2 A} \times \frac{\cos^2 A}{\sin^2 A}\right)$$

$$= \frac{1}{\cos^2 A \sin^2 A}$$

$$= \frac{1}{(1 - \sin^2 A)\sin^2 A} \quad [\because \sin^2 A + \cos^2 A = 1]$$

$$= \frac{\mathbf{1}}{\mathbf{\sin^2 A - \sin^4 A}}$$

= RHS

60. LHS $= \frac{\mathbf{\cos^3\theta + \sin^3\theta}}{\mathbf{\cos\theta + \sin\theta}} + \frac{\mathbf{\cos^3\theta - \sin^3\theta}}{\mathbf{\cos\theta - \sin\theta}}$

$$= \frac{(\cos\theta + \sin\theta)(\cos^2\theta - \cos\theta\sin\theta + \sin^2\theta)}{(\cos\theta + \sin\theta)} + \frac{(\cos\theta - \sin\theta)(\cos^2\theta + \cos\theta\sin\theta + \sin^2\theta)}{(\cos\theta - \sin\theta)}$$

$[\because a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
and $a^3 - b^3 = (a - b)(a^2 + ab + b^2)]$

$= (\cos^2\theta - \cos\theta\sin\theta + \sin^2\theta)$
$+ (\cos^2\theta + \cos\theta\sin\theta + \sin^2\theta)$

$= (1 - \cos\theta\sin\theta) + (1 + \cos\theta\sin\theta)$

$[\because \sin^2\theta + \cos^2\theta = 1]$

$= \mathbf{2}$

= RHS

61. LHS $= \frac{\mathbf{\tan^3\theta}}{\mathbf{1 + \tan^2\theta}} + \frac{\mathbf{\cot^3\theta}}{\mathbf{1 + \cot^2\theta}}$

$$= \frac{\tan^3\theta}{\sec^2\theta} + \frac{\cot^3\theta}{\text{cosec}^2\theta}$$

$[\because 1 + \tan^2\theta = \sec^2\theta \text{ and } 1 + \cot^2\theta = \text{cosec}^2\theta]$

$$= \frac{\frac{\sin^3\theta}{\cos^3\theta}}{\frac{1}{\cos^2\theta}} + \frac{\frac{\cos^3\theta}{\sin^3\theta}}{\frac{1}{\sin^2\theta}}$$

$$= \frac{(\sin^3\theta)}{(\cos^3\theta)}(\cos^2\theta) + \frac{(\cos^3\theta)}{(\sin^3\theta)}(\sin^2\theta)$$

$= \frac{\sin^3\theta}{\cos\theta} + \frac{\cos^3\theta}{\sin\theta}$

$= \frac{\sin^4\theta + \cos^4\theta}{\cos\theta\sin\theta}$

$= \frac{(\sin^4\theta + \cos^4\theta + 2\sin^2\theta\cos^2\theta) - 2\sin^2\theta\cos^2\theta}{\cos\theta\sin\theta}$

$= \frac{(\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta}{\cos\theta\sin\theta}$

$= \frac{1 - 2\sin^2\theta\cos^2\theta}{\cos\theta\sin\theta}$ $[\because \sin^2\theta + \cos^2\theta = 1]$

$= \frac{1}{\cos\theta\sin\theta} - \frac{2\sin^2\theta\cos^2\theta}{\cos\theta\sin\theta}$

$= \mathbf{\sec\theta\,\mathrm{cosec}\,\theta - 2\sin\theta\cos\theta}$ = RHS

62. LHS = $\mathbf{\sin^4 A - \sin^2 A}$
$= \sin^2 A(\sin^2 A - 1)$
$= (1 - \cos^2 A)(-\cos^2 A)$
$[\because \sin^2 A + \cos^2 A = 1]$
$= -\cos^2 A + \cos^4 A$
$= \mathbf{\cos^4 A - \cos^2 A}$
= RHS

63. LHS = $\mathbf{\tan^4\theta + \tan^2\theta}$
$= \tan^2\theta(\tan^2\theta + 1)$
$= \left(\frac{\sin^2\theta}{\cos^2\theta}\right)(\sec^2\theta)$ $[\because 1 + \tan^2\theta = \sec^2\theta]$
$= \left(\frac{\sin^2\theta}{\cos^2\theta}\right)\left(\frac{1}{\cos^2\theta}\right)$
$= \mathbf{\frac{\sin^2\theta}{\cos^4\theta}}$
= RHS

64. LHS = $\mathbf{\sec^4 A - \tan^4 A}$
$= (\sec^2 A)^2 - \tan^4 A$
$= (1 + \tan^2 A)^2 - \tan^4 A$
$[\because 1 + \tan^2 A = \sec^2 A]$
$= 1 + \tan^4 A + 2\tan^2 A - \tan^4 A$
$= \mathbf{1 + 2\tan^2 A}$
= RHS

65. RHS = $\mathbf{\mathrm{cosec}^4 A - 2\,\mathrm{cosec}^2 A}$
$= \mathrm{cosec}^2 A(\mathrm{cosec}^2 A - 2)$
$= (1 + \cot^2 A)(1 + \cot^2 A - 2)$
$[\because 1 + \cot^2 A = \mathrm{cosec}^2 A]$
$= (1 + \cot^2 A)(\cot^2 A - 1)$
$= \cot^2 A + \cot^4 A - 1 - \cot^2 A$
$= \mathbf{\cot^4 A - 1}$
= LHS

66. LHS = $\mathbf{\cos^8\theta - \sin^8\theta}$
$= (\cos^4\theta)^2 - (\sin^4\theta)^2$
$= (\cos^4\theta + \sin^4\theta)(\cos^4\theta - \sin^4\theta)$
$= [(\cos^2\theta)^2 + (\sin^2\theta)^2 + 2\sin^2\theta\cos^2\theta - 2\sin^2\theta\cos^2\theta]\,[(\cos^2\theta - \sin^2\theta)(\cos^2\theta + \sin^2\theta)]$
$= [(\cos^2\theta + \sin^2\theta)^2 - 2\sin^2\theta\cos^2\theta]\,[(\cos^2\theta - \sin^2\theta)(\cos^2\theta + \sin^2\theta)]$
$= (1 - 2\sin^2\theta\cos^2\theta)(\cos^2\theta - \sin^2\theta)$
$[\because \sin^2\theta + \cos^2\theta = 1]$
$= \mathbf{(\cos^2\theta - \sin^2\theta)(1 - 2\sin^2\theta\cos^2\theta)}$
= RHS

67. LHS = $\mathbf{\frac{\cot A + \tan B}{\cot B + \tan A}}$

$= \frac{\cot A + \tan B}{\frac{1}{\tan B} + \frac{1}{\cot A}}$

$= \frac{(\cot A + \tan B)}{\frac{(\cot A + \tan B)}{\cot A\tan B}}$

$= (\cot A + \tan B)\,\frac{(\cot A\tan B)}{(\cot A + \tan B)}$

$= \mathbf{\cot A\tan B}$
= RHS

68. LHS = $\mathbf{\cos^2 A\sin^2 B - \sin^2 A\cos^2 B}$
$= \cos^2 A(1 - \cos^2 B) - (1 - \cos^2 A)\cos^2 B$
$[\because \sin^2\theta = 1 - \cos^2\theta]$
$= \cos^2 A - \cos^2 A\cos^2 B - \cos^2 B + \cos^2 A\cos^2 B$
$= \mathbf{\cos^2 A - \cos^2 B}$
= RHS

69. LHS = $\mathbf{\frac{\sin^2 A - \sin^2 B}{\cos^2 A\cos^2 B}}$

$= \frac{\sin^2 A}{\cos^2 A\cos^2 B} - \frac{\sin^2 B}{\cos^2 A\cos^2 B}$

$= \tan^2 A\sec^2 B - \tan^2 B\sec^2 A$
$= \tan^2 A(1 + \tan^2 B) - \tan^2 B(1 + \tan^2 A)$
$[\because 1 + \tan^2\theta = \sec^2\theta]$
$= \tan^2 A + \tan^2 A\tan^2 B - \tan^2 B - \tan^2 A\tan^2 B$
$= \mathbf{\tan^2 A - \tan^2 B}$ = RHS

70. LHS = $\mathbf{(1 + \tan A\tan B)^2 + (\tan A - \tan B)^2}$
$= 1 + \tan^2 A\tan^2 B + 2\tan A\tan B + \tan^2 A + \tan^2 B - 2\tan A\tan B$
$= 1 + \tan^2 A\tan^2 B + \tan^2 A + \tan^2 B$
$= 1 + \tan^2 A + \tan^2 A\tan^2 B + \tan^2 B$
$= (1 + \tan^2 A) + \tan^2 B(\tan^2 A + 1)$
$= (1 + \tan^2 A)(1 + \tan^2 B)$
$= \mathbf{\sec^2 A\sec^2 B}$ $[\because 1 + \tan^2\theta = \sec^2\theta]$
= RHS

71. (*i*) $\mathbf{\sin\theta + \sin^2\theta = 1}$ [Given]
$\Rightarrow \sin\theta = 1 - \sin^2\theta = \cos^2\theta$
$[\because \sin^2\theta + \cos^2\theta = 1 \Rightarrow 1 - \sin^2\theta = \cos^2\theta]$
$\Rightarrow \sin^2\theta = \cos^4\theta$ … (1)
Now, $\cos^2\theta + \cos^4\theta = \cos^2\theta + \sin^2\theta$ [Using (1)]
$\Rightarrow \cos^2\theta + \cos^4\theta = 1$
Hence, $\mathbf{\cos^2\theta + \cos^4\theta = 1}$

(*ii*) $\mathbf{\cos A + \cos^2 A = 1}$ [Given]
$\Rightarrow \cos A = 1 - \cos^2 A = \sin^2 A$ … (1)
$[\because \sin^2 A + \cos^2 A = 1 \Rightarrow 1 - \cos^2 A = \sin^2 A]$
Now $\sin^2 A + \sin^4 A = \sin^2 A(1 + \sin^2 A)$
$= \cos A(1 + \cos A)$
[Using (1)]
$= \cos A + \cos^2 A$
$= 1$
$[\because \cos A + \cos^2 A = 1$, given$]$
Hence, $\mathbf{\sin^2 A + \sin^4 A = 1}$

(*iii*) $\mathbf{\cos\theta - \sin\theta = \sqrt{2}\ \sin\theta}$ [Given]

$\Rightarrow \sqrt{2}\sin\theta + \sin\theta = \cos\theta$

$\Rightarrow \sin\theta(\sqrt{2} + 1) = \cos\theta$

$\Rightarrow \sin\theta = \dfrac{\cos\theta}{\sqrt{2}+1}$

$\Rightarrow \sin\theta = \dfrac{\cos\theta}{(\sqrt{2}+1)}\ \dfrac{(\sqrt{2}-1)}{(\sqrt{2}-1)}$

$\Rightarrow \sin\theta = \dfrac{\cos\theta(\sqrt{2}-1)}{2-1}$

$\Rightarrow \sin\theta = \sqrt{2}\cos\theta - \cos\theta$

$\Rightarrow \mathbf{\sin\theta + \cos\theta = \sqrt{2}\cos\theta}$

(*iv*) $\mathbf{\sin\theta + \cos\theta = \sqrt{2}}$ [Given]

Squaring both sides

$\Rightarrow (\sin\theta + \cos\theta)^2 = 2$

$\Rightarrow \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 2$

$\Rightarrow 1 + 2\sin\theta\cos\theta = 2$

$\Rightarrow 2\sin\theta\cos\theta = 1$

$\Rightarrow \dfrac{1}{\sin\theta\cos\theta} = 2$

$\Rightarrow \dfrac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta} = 2$ $[\because \sin^2\theta + \cos^2\theta = 1]$

$\Rightarrow \dfrac{\sin^2\theta}{\sin\theta\cos\theta} + \dfrac{\cos^2\theta}{\sin\theta\cos\theta} = 2$

$\Rightarrow \dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta} = 2$

$\Rightarrow \mathbf{\tan\theta + \cot\theta = 2}$

For Standard Level

72. RHS $= \mathbf{2 + \dfrac{\sin\theta}{\cot\theta - \mathrm{cosec}\,\theta}}$

$= 2 + \dfrac{\sin\theta(\cot\theta + \mathrm{cosec}\,\theta)}{(\cot\theta - \mathrm{cosec}\,\theta)(\cot\theta + \mathrm{cosec}\,\theta)}$

$= 2 + \dfrac{\sin\theta(\cot\theta + \mathrm{cosec}\,\theta)}{(\cot^2\theta - \mathrm{cosec}^2\theta)}$

$= 2 - \dfrac{\sin\theta(\cot\theta + \mathrm{cosec}\,\theta)}{(\mathrm{cosec}^2\theta - \cot^2\theta)}$

$= 2 - \sin\theta\cot\theta - \sin\theta\,\mathrm{cosec}\,\theta$

$[\because 1 + \cot^2\theta = \mathrm{cosec}^2\theta \Rightarrow \mathrm{cosec}^2\theta - \cot^2\theta = 1]$

$= 2\sin\theta\,\mathrm{cosec}\,\theta - \sin\theta\cot\theta - \sin\theta\,\mathrm{cosec}\,\theta$

$[\because \sin\theta\,\mathrm{cosec}\,\theta = 1]$

$= \sin\theta\,\mathrm{cosec}\,\theta - \sin\theta\cot\theta$

$= \sin\theta(\mathrm{cosec}\,\theta - \cot\theta)$

$= \sin\theta(\mathrm{cosec}\,\theta - \cot\theta) \times \dfrac{(\mathrm{cosec}\,\theta + \cot\theta)}{(\mathrm{cosec}\,\theta + \cot\theta)}$

$= \dfrac{\sin\theta(\mathrm{cosec}^2\theta - \cot^2\theta)}{\mathrm{cosec}\,\theta + \cot\theta}$

$= \mathbf{\dfrac{\sin\theta}{\mathrm{cosec}\,\theta + \cot\theta}}$

$=$ LHS

ALTERNATIVE METHOD:

We have to prove that

$$\frac{\sin\theta}{\cot\theta + \mathrm{cosec}\,\theta} = 2 + \frac{\sin\theta}{\cot\theta - \mathrm{cosec}\,\theta}$$

or

$$\frac{\sin\theta}{\cot\theta + \mathrm{cosec}\,\theta} - \frac{\sin\theta}{\cot\theta - \mathrm{cosec}\,\theta} = 2$$

LHS $= \dfrac{\sin\theta}{\cot\theta + \mathrm{cosec}\,\theta} - \dfrac{\sin\theta}{\cot\theta - \mathrm{cosec}\,\theta}$

$= \sin\theta\left[\dfrac{1}{\cot\theta + \mathrm{cosec}\,\theta} - \dfrac{1}{\cot\theta - \mathrm{cosec}\,\theta}\right]$

$= \sin\theta\left[\dfrac{1}{(\mathrm{cosec}\,\theta + \cot\theta)} + \dfrac{1}{(\mathrm{cosec}\,\theta - \cot\theta)}\right]$

$= \sin\theta\left[\dfrac{\mathrm{cosec}\,\theta - \cot\theta + \mathrm{cosec}\,\theta + \cot\theta}{(\mathrm{cosec}^2\theta - \cot^2\theta)}\right]$

$= \sin\theta(2\,\mathrm{cosec}\,\theta)$

$[\because 1 + \cot^2\theta = \mathrm{cosec}^2\theta \Rightarrow \mathrm{cosec}^2\theta - \cot^2 = 1]$

$= \sin\theta\left(\dfrac{2}{\sin\theta}\right)$

$= \mathbf{2}$

$=$ RHS

73. LHS $= \mathbf{\dfrac{\sin\theta + 1 - \cos\theta}{\cos\theta - 1 + \sin\theta}}$

$= \dfrac{\dfrac{\sin\theta}{\cos\theta} + \dfrac{1}{\cos\theta} - \dfrac{\cos\theta}{\cos\theta}}{\dfrac{\cos\theta}{\cos\theta} - \dfrac{1}{\cos\theta} + \dfrac{\sin\theta}{\cos\theta}}$

[Dividing the numerator and denominator by $\cos\theta$]

$= \dfrac{\tan\theta + \sec\theta - 1}{1 - \sec\theta + \tan\theta}$

$= \dfrac{(\tan\theta + \sec\theta) - (\sec^2\theta - \tan^2\theta)}{(1 - \sec\theta + \tan\theta)}$

$[\because 1 + \tan^2\theta = \sec^2\theta \Rightarrow 1 = \sec^2\theta - \tan^2\theta]$

$= \dfrac{(\tan\theta + \sec\theta) - (\sec\theta + \tan\theta)(\sec\theta - \tan\theta)}{(1 - \sec\theta + \tan\theta)}$

$= \dfrac{(\tan\theta + \sec\theta)(1 - \sec\theta + \tan\theta)}{(1 - \sec\theta + \tan\theta)}$

$= \tan\theta + \sec\theta$

$= \dfrac{\sin\theta}{\cos\theta} + \dfrac{1}{\cos\theta}$

$= \dfrac{\sin\theta + 1}{\cos\theta}$

$= \mathbf{\dfrac{1 + \sin\theta}{\cos\theta}}$

$=$ RHS

74. LHS $= \mathbf{\sec^2\theta + \sec^2\theta(1 - \sec^2\theta) - \mathrm{cosec}^2\theta + \mathrm{cosec}^2\theta(\mathrm{cosec}^2\theta - 1)}$

$= \sec^2\theta(1 + 1 - \sec^2\theta) - \mathrm{cosec}^2\theta(1 - \mathrm{cosec}^2\theta + 1)$

$= (1 + \tan^2\theta)(1 - \tan^2\theta) - (1 + \cot^2\theta)(1 - \cot^2\theta)$

$[\because 1 + \tan^2\theta = \sec^2\theta \Rightarrow 1 - \sec^2\theta = -\tan^2\theta$

and $1 + \cot^2\theta = \mathrm{cosec}^2\theta \Rightarrow 1 - \mathrm{cosec}^2\theta = -\cot^2\theta]$

$= (1 - \tan^4\theta) - (1 - \cot^4\theta) = 1 - \tan^4\theta - 1 + \cot^4\theta$

$= \mathbf{\cot^4\theta - \tan^4\theta}$ = RHS

75. LHS = $\cos^{-6}\theta - \tan^6\theta$

$= \dfrac{1}{\cos^6\theta} - \dfrac{\sin^6\theta}{\cos^6\theta}$

$= \dfrac{1-\sin^6\theta}{\cos^6\theta}$

$= \dfrac{(1)^3 - (\sin^2\theta)^3}{\cos^6\theta}$

$= \dfrac{(1-\sin^2\theta)(1+\sin^4\theta+\sin^2\theta)}{\cos^6\theta}$

$[\because a^3 - b^3 = (a-b)(a^2+b^2+ab)]$

$= \dfrac{\cos^2\theta[1+(1-\cos^2\theta)^2+\sin^2\theta]}{\cos^6\theta}$

$[\because \sin^2\theta + \cos^2\theta = 1 \Rightarrow 1-\sin^2\theta = \cos^2\theta$ and $\sin^2\theta = 1-\cos^2\theta]$

$= \dfrac{1+1+\cos^4\theta - 2\cos^2\theta + \sin^2\theta}{\cos^4\theta}$

$= \dfrac{\cos^4\theta + 2 - 2\cos^2\theta + \sin^2\theta}{\cos^4\theta}$

$= \dfrac{\cos^4\theta + 2(1-\cos^2\theta) + \sin^2\theta}{\cos^4\theta}$

$= \dfrac{\cos^4\theta + 2\sin^2\theta + \sin^2\theta}{\cos^4\theta}$

$[\because \sin^2\theta + \cos^2\theta = 1 \Rightarrow 1-\cos^2\theta = \sin^2\theta]$

$= \dfrac{\cos^4\theta + 3\sin^2\theta}{\cos^4\theta}$

$= \dfrac{\cos^4\theta}{\cos^4\theta} + 3\,\dfrac{\sin^2\theta}{\cos^4\theta}$

$= 1 + 3\left(\dfrac{\sin^2\theta}{\cos^2\theta}\right)\left(\dfrac{1}{\cos^2\theta}\right)$

$= 1 + 3\tan^2\theta\cos^{-2}\theta$

$= \mathbf{3\tan^2\theta\cos^{-2}\theta + 1}$

= RHS

76. $\text{cosec}\,\theta + \cot\theta = a$

and $\text{cosec}\,\theta - \cot\theta = b$ [Given]

LHS = $\boldsymbol{ab}$

$= (\text{cosec}\,\theta + \cot\theta)(\text{cosec}\,\theta - \cot\theta)$

$= (\text{cosec}^2\theta - \cot^2\theta)$

$= \mathbf{1}$

$[\because 1 + \cot^2\theta = \text{cosec}^2\theta \Rightarrow \text{cosec}^2\theta - \cot^2\theta = 1]$

= RHS

Hence, $\boldsymbol{ab = 1}$.

77. $\text{cosec}\,\theta - \sin\theta = l$

and $\sec\theta - \cos\theta = m$ [Given]

LHS = $l^2 m^2 (l^2 + m^2 + 3)$

$= (\text{cosec}\,\theta - \sin\theta)^2(\sec\theta - \cos\theta)^2$
$[(\text{cosec}\,\theta - \sin\theta)^2 + (\sec\theta - \cos\theta)^2 + 3]$

$= \left(\dfrac{1}{\sin\theta} - \sin\theta\right)^2\left(\dfrac{1}{\cos\theta} - \cos\theta\right)^2$
$\left[\left(\dfrac{1}{\sin\theta} - \sin\theta\right)^2 + \left(\dfrac{1}{\cos\theta} - \cos\theta\right)^2 + 3\right]$

$= \left(\dfrac{1-\sin^2\theta}{\sin\theta}\right)^2\left(\dfrac{1-\cos^2\theta}{\cos\theta}\right)^2$
$\left[\left(\dfrac{1-\sin^2\theta}{\sin\theta}\right)^2 + \left(\dfrac{1-\cos^2\theta}{\cos\theta}\right)^2 + 3\right]$

$= \left(\dfrac{\cos^2\theta}{\sin\theta}\right)^2\left(\dfrac{\sin^2\theta}{\cos\theta}\right)^2\left[\left(\dfrac{\cos^2\theta}{\sin\theta}\right)^2 + \left(\dfrac{\sin^2\theta}{\cos\theta}\right)^2 + 3\right]$

$= \left(\dfrac{\cos^4\theta}{\sin^2\theta}\right) \times \left(\dfrac{\sin^4\theta}{\cos^2\theta}\right)\left[\dfrac{\cos^4\theta}{\sin^2\theta} + \dfrac{\sin^4\theta}{\cos^2\theta} + 3\right]$

$= (\cos^2\theta)(\sin^2\theta)\left[\dfrac{\cos^6\theta + \sin^6\theta + 3\cos^2\theta\sin^2\theta}{\sin^2\theta\cos^2\theta}\right]$

$= \cos^6\theta + \sin^6\theta + 3\cos^2\theta\sin^2\theta$

$= [(\cos^2\theta)^3 + (\sin^2\theta)^3] + 3\cos^2\theta\sin^2\theta$

$= [(\cos^2\theta + \sin^2\theta)^3 - 3\cos^2\theta\sin^2\theta$
$(\cos^2 + \sin^2\theta)] + 3\cos^2\theta\sin^2\theta$

$[\because a^3 + b^3 = (a+b)^3 - 3ab(a+b)]$

$= 1 - 3\cos^2\theta\sin^2\theta + 3\cos^2\theta\sin^2\theta$

$= \mathbf{1}$

= RHS

78. $\dfrac{x}{a}\sin\theta - \dfrac{y}{b}\cos\theta = 1$ [Given]

$\Rightarrow \dfrac{x^2}{a^2}\sin^2\theta + \dfrac{y^2}{b^2}\cos^2\theta - 2\dfrac{x}{a}\sin\theta.\ \dfrac{y}{b}\cos\theta = 1$

[Squaring the given equation] … (1)

$\dfrac{x}{a}\cos\theta + \dfrac{y}{b}\sin\theta = 1$ [Given]

$\Rightarrow \dfrac{x^2}{a^2}\cos^2\theta + \dfrac{y^2}{b^2}\sin^2\theta + 2\dfrac{x}{a}\cos\theta.\ \dfrac{y}{b}\sin\theta = 1$

[Squaring the given equation] … (2)

Adding (1) and (2), we get

$\dfrac{x^2}{a^2}(\sin^2\theta + \cos^2\theta) + \dfrac{y^2}{b^2}(\cos^2\theta + \sin^2\theta) = 2$

$\Rightarrow \boldsymbol{\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 2}$ $[\because \sin^2\theta + \cos^2\theta = 1]$

Hence, proved.

79. $x = a\sin\theta$ and $y = b\tan\theta$

LHS = $a^2y^2 - b^2x^2$

$= a^2b^2\tan^2\theta - b^2a^2\sin^2\theta$

$= a^2b^2(\tan^2\theta - \sin^2\theta)$

$= a^2b^2\left(\dfrac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta\right)$

$= a^2b^2\sin^2\theta\left(\dfrac{1}{\cos^2\theta} - 1\right)$

$= a^2b^2\sin^2\theta\left(\dfrac{1-\cos^2\theta}{\cos^2\theta}\right)$

$= a^2b^2\sin^2\theta\,\dfrac{\sin^2\theta}{\cos^2\theta}$

$[\because \sin^2\theta + \cos^2\theta = 1 \Rightarrow 1-\cos^2\theta = \sin^2\theta]$

$= a^2b^2 \sin^2\theta \tan^2\theta$

$= (a^2 \sin^2\theta)(b^2 \tan^2\theta)$

$= x^2y^2$

= RHS

$\Rightarrow \quad \boldsymbol{a^2y^2 - b^2x^2 = x^2y^2}$

Hence, proved.

80. $x = a\sec\theta + b\tan\theta$ [Given]

$\Rightarrow \quad x^2 = a^2\sec^2\theta + b^2\tan^2\theta + 2ab\sec\theta\tan\theta$

[Squaring the given equation] … (1)

$y = a\tan\theta + b\sec\theta$ [Given]

$\Rightarrow \quad y^2 = a^2\tan^2\theta + b^2\sec^2\theta + 2ab\tan\theta\sec\theta$

[Squaring the given equation] … (2)

Subtracting (2) from (1) we get

$\Rightarrow \quad x^2 - y^2 = a^2(\sec^2\theta - \tan^2\theta) + b^2(\tan^2\theta - \sec^2\theta)$

$\Rightarrow \quad x^2 - y^2 = a^2(1) + b^2(-1) \quad [\because 1 + \tan^2\theta = \sec^2\theta]$

$\Rightarrow \quad \boldsymbol{x^2 - y^2 = a^2 - b^2}$

Hence, proved.

81. $\sin\theta + \cos\theta = m$ [Given]

and $\sec\theta + \operatorname{cosec}\theta = n$ [Given]

LHS $= n(m^2 - 1)$

$= (\sec\theta + \operatorname{cosec}\theta)[(\sin\theta + \cos\theta)^2 - 1]$

$= \left(\frac{1}{\cos\theta} + \frac{1}{\sin\theta}\right)$

$[(\sin^2\theta + \cos^2\theta) + 2\sin\theta\cos\theta - 1]$

$= \frac{\sin\theta + \cos\theta}{\sin\theta\cos\theta}(1 + 2\sin\theta\cos\theta - 1)$

$[\because \sin^2\theta + \cos^2\theta = 1]$

$= \frac{(\sin\theta + \cos\theta)(2\sin\theta\cos\theta)}{(\sin\theta\cos\theta)}$

$= 2(\sin\theta + \cos\theta)$

$= \boldsymbol{2m}$

= RHS

82. $\operatorname{cosec}\theta = x + \frac{1}{4x}$

$\cot^2\theta = \operatorname{cosec}^2\theta - 1$

$= \left(x + \frac{1}{4x}\right)^2 - 1$

$= x^2 + \frac{1}{16x^2} + 2(x)\left(\frac{1}{4x}\right) - 1$

$\Rightarrow \cot^2\theta = x^2 + \frac{1}{16x^2} + \frac{1}{2} - 1 = x^2 + \frac{1}{16x^2} - \frac{1}{2}$

$= \left(x - \frac{1}{4x}\right)^2$

$\Rightarrow \quad \cot\theta = \pm\left(x - \frac{1}{4x}\right)$

$\Rightarrow \quad \cot\theta = \left(x - \frac{1}{4x}\right)$ or $\cot\theta = -\left(x - \frac{1}{4x}\right)$

When, $\cot\theta = \left(x - \frac{1}{4x}\right)$,

then $\operatorname{cosec}\theta + \cot\theta = \left(x + \frac{1}{4x}\right) + \left(x - \frac{1}{4x}\right)$

$= x + \frac{1}{4x} + x - \frac{1}{4x}$

$= \boldsymbol{2x}$

When, $\cot\theta = -\left(x - \frac{1}{4x}\right)$,

then $\operatorname{cosec}\theta + \cot\theta = \left(x + \frac{1}{4x}\right) + \left[-\left(x - \frac{1}{4x}\right)\right]$

$= x + \frac{1}{4x} - x + \frac{1}{4x}$

$= \frac{2}{4x}$

$= \boldsymbol{\frac{1}{2x}}$

Hence, if $\operatorname{cosec}\theta = x + \frac{1}{4x}$, then $\operatorname{cosec}\theta + \cot\theta = \boldsymbol{2x}$ or $\boldsymbol{\frac{1}{2x}}$.

83. $p = \sec\theta - \operatorname{cosec}\theta$

and $q = \sin\theta - \cos\theta$

LHS $= p(q^2 - 1) + 2q$

$= (\sec\theta - \operatorname{cosec}\theta)[(\sin\theta - \cos\theta)^2 - 1] + 2(\sin\theta - \cos\theta)$

$= \left(\frac{1}{\cos\theta} - \frac{1}{\sin\theta}\right)(\sin^2\theta + \cos^2\theta - 2\sin\theta\cos\theta - 1) + 2(\sin\theta - \cos\theta)$

$= \frac{\sin\theta - \cos\theta}{\cos\theta\sin\theta}(1 - 2\sin\theta\cos\theta - 1) + 2(\sin\theta - \cos\theta)$

$[\because \sin^2\theta + \cos^2\theta = 1]$

$= \frac{(\sin\theta - \cos\theta)}{\cos\theta\sin\theta}(-2\sin\theta\cos\theta) + 2(\sin\theta - \cos\theta)$

$= \frac{-2(\sin\theta - \cos\theta)(\sin\theta\cos\theta)}{\cos\theta\sin\theta} + 2(\sin\theta - \cos\theta)$

$= -2(\sin\theta - \cos\theta) + 2(\sin\theta - \cos\theta)$

$= \boldsymbol{0}$

= RHS

84. $\boldsymbol{\operatorname{cosec}\theta + \cot\theta = m}$ [Given]

LHS $= \boldsymbol{\frac{m^2 - 1}{m^2 + 1}}$

$= \frac{(\operatorname{cosec}\theta + \cot\theta)^2 - 1}{(\operatorname{cosec}\theta + \cot\theta)^2 + 1}$

$= \frac{\operatorname{cosec}^2\theta + \cot^2\theta + 2\operatorname{cosec}\theta\cot\theta - 1}{\operatorname{cosec}^2\theta + \cot^2\theta + 2\operatorname{cosec}\theta\cot\theta + 1}$

$= \frac{\cot^2\theta + \cot^2\theta + 2\operatorname{cosec}\theta\cot\theta}{\operatorname{cosec}^2\theta + \operatorname{cosec}^2\theta + 2\operatorname{cosec}\theta\cot\theta}$

$[\because 1 + \cot^2\theta = \operatorname{cosec}^2\theta \Rightarrow \cot^2\theta = \operatorname{cosec}^2\theta - 1]$

$= \frac{2\cot^2\theta + 2\operatorname{cosec}\theta\cot\theta}{2\operatorname{cosec}^2\theta + 2\operatorname{cosec}\theta\cot\theta}$

$= \frac{2\cot\theta(\cot\theta + \operatorname{cosec}\theta)}{2\operatorname{cosec}\theta(\operatorname{cosec}\theta + \cot\theta)}$

$= \frac{\cot\theta}{\operatorname{cosec}\theta}$

$= \dfrac{\frac{\cos\theta}{\sin\theta}}{\frac{1}{\sin\theta}}$

$= \dfrac{\cos\theta}{\sin\theta} \times \dfrac{\sin\theta}{1}$

$= \mathbf{\cos}\ \theta$

= RHS

85. $\mathbf{5 \sin\theta + 7 \cos\theta = 7}$ [Given]

Now, $(5 \sin\theta + 7 \cos\theta)^2 + (7 \sin\theta - 5 \cos\theta)^2$

$= 25 \sin^2\theta + 49 \cos^2\theta + 70 \sin\theta \cos\theta + 49 \sin^2\theta + 25 \cos^2\theta - 70 \sin\theta \cos\theta$

$\Rightarrow (5 \sin\theta + 7 \cos\theta)^2 + (7 \sin\theta - 5 \cos\theta)^2 = 25(\sin^2\theta + \cos^2\theta) + 49(\sin^2\theta + \cos^2\theta)$

$\Rightarrow (7)^2 + (7 \sin\theta - 5 \cos\theta)^2 = 25 + 49$

[$\because \sin^2\theta + \cos^2\theta = 1$ and $5 \sin\theta + 7 \cos\theta = 7$, given]

$\Rightarrow 49 + (7 \sin\theta - 5 \cos\theta)^2 = 25 + 49$

$\Rightarrow (7 \sin\theta - 5 \cos\theta)^2 = 25$

$\Rightarrow \mathbf{(7 \sin\theta - 5 \cos\theta) = \pm 5}$

Hence, proved.

86. $\tan^2\alpha = 1 + 2 \tan^2\beta$ [Given]

$\Rightarrow 1 + \tan^2\alpha = 1 + 1 + 2 \tan^2\beta$

$\Rightarrow 1 + \tan^2\alpha = 2 + 2 \tan^2\beta$

$\Rightarrow \sec^2\alpha = 2(1 + \tan^2\beta)$

$\Rightarrow \sec^2\alpha = 2 \sec^2\beta$

[$\because 1 + \tan^2\theta = \sec^2\theta$]

$\Rightarrow \dfrac{1}{\cos^2\alpha} = \dfrac{2}{\cos^2\beta}$

$\Rightarrow \cos^2\beta = 2 \cos^2\alpha$

$\Rightarrow \cos^2\beta = 2(1 - \sin^2\alpha)$

[$\because \sin^2\alpha + \cos^2\alpha = 1 \Rightarrow \cos^2\alpha = 1 - \sin^2\alpha$]

$\Rightarrow \cos^2\beta = 2 - 2 \sin^2\alpha$

$\Rightarrow 2 \sin^2\alpha = 2 - \cos^2\beta$

$\Rightarrow 2 \sin^2\alpha = 1 + (1 - \cos^2\beta)$

$\Rightarrow \mathbf{2 \sin^2\alpha = 1 + \sin^2\beta}$

Hence, proved.

CHECK YOUR UNDERSTANDING

MULTIPLE-CHOICE QUESTIONS

For Basic and Standard Levels

1. (*c*) **cos A**

$(1 - \sin A)(\sec A + \tan A) = (1 - \sin A)\left(\dfrac{1}{\cos A} + \dfrac{\sin A}{\cos A}\right)$

$= (1 - \sin A)\left(\dfrac{1 + \sin A}{\cos A}\right)$

$= \dfrac{1 - \sin^2 A}{\cos A}$

$= \dfrac{\cos^2 A}{\cos A}$

$= \cos A$

2. (*b*) **–5**

$5 \tan^2\theta - 5 \sec^2\theta = 5(\tan^2\theta - \sec^2\theta)$

$= 5(-1)$

[$\because 1 + \tan^2\theta = \sec^2\theta \Rightarrow \tan^2\theta - \sec^2\theta = -1$]

$= -5$

3. (*d*) **1**

$(\sec^2\theta - 1)(\cot^2\theta) = \tan^2\theta \cot^2\theta$

[$\because 1 + \tan^2\theta = \sec^2\theta \Rightarrow \sec^2\theta - 1 = \tan^2\theta$]

$= (\tan^2\theta)\left(\dfrac{1}{\tan^2\theta}\right) = 1$

4. (*c*) **1**

$(1 + \tan^2\theta)(1 - \sin\theta)(1 + \sin\theta)(1 + \cos\theta)(1 - \cos\theta)(1 + \cot^2\theta)$

$= \sec^2\theta (1 - \sin^2\theta)(1 - \cos^2\theta) \operatorname{cosec}^2\theta$

[$\because 1 + \tan^2 = \sec^2\theta$ and $1 + \cot^2\theta = \operatorname{cosec}^2\theta$]

$= \sec^2\theta \cos^2\theta \sin^2\theta \operatorname{cosec}^2\theta$ [$\because \sin^2\theta + \cos^2\theta = 1$]

$= \dfrac{1}{\cos^2\theta} \cos^2\theta \sin^2\theta \dfrac{1}{\sin^2\theta} = 1$

5. (*c*) $\mathbf{\cos^2\theta - \sin^2\theta}$

$\dfrac{1 - \tan^2\theta}{1 + \tan^2\theta} = \dfrac{1 - \frac{\sin^2\theta}{\cos^2\theta}}{1 + \frac{\sin^2\theta}{\cos^2\theta}}$

$= \dfrac{\cos^2\theta - \sin^2\theta}{\cos^2\theta} \dfrac{\cos^2\theta}{(\cos^2\theta + \sin^2\theta)}$

$= \cos^2\theta - \sin^2\theta$

[$\because \sin^2\theta + \cos^2\theta = 1$]

6. (*c*) $\mathbf{\operatorname{cosec}\theta + \cot\theta}$

$\sqrt{\dfrac{1 + \cos\theta}{1 - \cos\theta}} = \sqrt{\dfrac{(1 + \cos\theta)(1 + \cos\theta)}{(1 - \cos\theta)(1 + \cos\theta)}}$

$= \sqrt{\dfrac{(1 + \cos\theta)^2}{1 - \cos^2\theta}}$

$= \sqrt{\dfrac{(1 + \cos\theta)^2}{\sin^2\theta}}$ [$\because \sin^2\theta + \cos^2\theta = 1$]

$= \dfrac{1 + \cos\theta}{\sin\theta}$

$= \dfrac{1}{\sin\theta} + \dfrac{\cos\theta}{\sin\theta}$

$= \operatorname{cosec}\theta + \cot\theta$

7. (*c*) $\mathbf{\tan^2\theta + \tan^4\theta}$

$\sec^4\theta - \sec^2\theta = \sec^2\theta(\sec^2\theta - 1)$

$= (1 + \tan^2\theta)(1 + \tan^2\theta - 1)$

[$\because 1 + \tan^2\theta = \sec^2\theta$]

$= (1 + \tan^2\theta)(\tan^2\theta)$

$= \tan^2\theta + \tan^4\theta$

8. (*b*) $\boldsymbol{m^2n^2}$

$x = m \sin\theta$

and $y = n \cos\theta$ [Given]

$n^2x^2 + m^2y^2 = n^2(m \sin\theta)^2 + m^2(n \cos\theta)^2$

$= n^2m^2(\sin^2\theta + \cos^2\theta)$

$= m^2n^2 \qquad [\because \sin^2\theta + \cos^2\theta = 1]$

9. (*a*) **2 cosec *x***

$$\frac{\sin x}{(1+\cos x)} + \frac{\sin x}{(1-\cos x)} = k$$

$$\Rightarrow \sin x\left[\frac{1-\cos x + 1 + \cos x}{(1+\cos x)(1-\cos x)}\right] = k$$

$$\Rightarrow \sin x\left[\frac{2}{1-\cos^2 x}\right] = k$$

$$\Rightarrow \sin x\left(\frac{2}{\sin^2 x}\right) = k$$

$[\because \sin^2\theta + \cos^2\theta = 1 \Rightarrow 1 - \cos^2\theta = \sin^2\theta]$

$$\Rightarrow \frac{2}{\sin x} = k$$

$$\Rightarrow 2\,\text{cosec}\,x = k$$

10. (*c*) $\boldsymbol{k = \frac{1}{2}}$

$1 + 2\sin^2\theta\cos^2\theta = \sin^2\theta + \cos^2\theta + 4k\sin^2\theta\cos^2\theta$ [Given]

$\Rightarrow 1 + 2\sin^2\theta\cos^2\theta = 1 + 4k\sin^2\theta\cos^2\theta$ $[\because \sin^2\theta + \cos^2\theta = 1]$

$\Rightarrow 2\sin^2\theta\cos^2\theta = 4k\sin^2\theta\cos^2\theta$

$$\Rightarrow \frac{2\sin^2\theta\cos^2\theta}{4\sin^2\theta\cos^2\theta} = k$$

$$\Rightarrow k = \frac{1}{2}$$

11. (*c*) **1**

$2x = \text{cosec}\,\theta$

and $\frac{2}{x} = \cot\theta$ [Given]

$\Rightarrow x = \frac{\text{cosec}\,\theta}{2}$

and $\frac{1}{x} = \frac{\cot\theta}{2}$

Now, $$4\left(x^2 - \frac{1}{x^2}\right) = 4\left[\left(\frac{\text{cosec}\,\theta}{2}\right)^2 - \left(\frac{\cot\theta}{2}\right)^2\right]$$

$$= 4\left[\frac{\text{cosec}^2\theta}{4} - \frac{\cot^2\theta}{4}\right]$$

$$= \frac{4}{4}(\text{cosec}^2\theta - \cot^2\theta)$$

$$= 1$$

$[\because 1 + \cot^2\theta = \text{cosec}^2\theta \Rightarrow \text{cosec}^2\theta - \cot^2 = 1]$

For Standard Level

12. (*b*) $\boldsymbol{\frac{1-\cos\theta}{\sin\theta}}$

$$\frac{\sin\theta}{1+\cos\theta} = \frac{\sin\theta(1-\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}$$

$$= \frac{\sin\theta(1-\cos\theta)}{1-\cos^2\theta}$$

$$= \frac{\sin\theta(1-\cos\theta)}{\sin^2\theta}$$

$[\because \sin^2\theta + \cos^2\theta = 1 \Rightarrow 1 - \cos^2\theta = \sin^2\theta]$

$$= \frac{1-\cos\theta}{\sin\theta}$$

13. (*b*) $\boldsymbol{\sqrt{\frac{q+p}{q-p}}}$

$$\tan\theta + \sec\theta = \frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta}$$

$$= \frac{\sin\theta + 1}{\cos\theta}$$

$$= \frac{\sin\theta + 1}{\sqrt{1-\sin^2\theta}}$$

$$= \frac{\frac{p}{q}+1}{\sqrt{1-\frac{p^2}{q^2}}} \qquad \left[\because \sin\theta = \frac{p}{q}, \text{ given}\right]$$

$$= \frac{\frac{p+q}{q}}{\sqrt{\frac{q^2-p^2}{q^2}}}$$

$$= \left(\frac{p+q}{q}\right)\left(\sqrt{\frac{q^2}{q^2-p^2}}\right)$$

$$= \frac{\sqrt{p+q}\sqrt{p+q}\,(q)}{(q)\sqrt{q-p}\sqrt{q+p}}$$

$$= \sqrt{\frac{p+q}{q-p}}$$

$$= \sqrt{\frac{q+p}{q-p}}$$

14. (*b*) $\boldsymbol{\frac{1}{x}}$

$\sec\theta + \tan\theta = x$ [Given]

$$\Rightarrow \frac{1}{x} = \frac{1}{\sec\theta + \tan\theta}$$

$$\Rightarrow \frac{1}{x} = \frac{1}{(\sec\theta + \tan\theta)}\,\frac{(\sec\theta - \tan\theta)}{(\sec\theta - \tan\theta)}$$

$$= \frac{\sec\theta - \tan\theta}{\sec^2\theta - \tan^2\theta}$$

$$\Rightarrow \frac{1}{x} = \sec\theta - \tan\theta$$

$[\because 1 + \tan^2\theta = \sec^2\theta \Rightarrow \sec^2\theta - \tan^2\theta = 1]$

15. (*a*) **4**

$x = 3\sec^2\theta - 1$

and $y = 3\tan^2\theta - 2$ [Given]

$\Rightarrow x - y = (3\sec^2\theta - 1) - (3\tan^2\theta - 2)$

$= 3\sec^2\theta - 1 - 3\tan^2\theta + 2$

$= 3(\sec^2\theta - \tan^2\theta) + 1$

$= 3 + 1 \qquad [\because \sec^2\theta - \tan^2\theta = 1]$

$= 4$

16. (*b*) $b^2 - a^2$

$a \cot\theta + b \operatorname{cosec}\theta = p$

and $b \cot\theta + a \operatorname{cosec}\theta = q$ then,

$$p^2 - q^2 = (a \cot\theta + b \operatorname{cosec}\theta)^2 - (b \cot\theta + a \operatorname{cosec}\theta)^2$$
$$= a^2 \cot^2\theta + 2ab \cot\theta \operatorname{cosec}\theta + b^2 \operatorname{cosec}^2\theta - b^2 \cot^2\theta - 2ab \cot\theta \operatorname{cosec}\theta - a^2 \operatorname{cosec}^2\theta$$
$$= b^2 \operatorname{cosec}^2\theta - a^2 \operatorname{cosec}^2\theta - b^2 \cot^2\theta + a^2 \cot^2\theta$$
$$= \operatorname{cosec}^2\theta\,(b^2 - a^2) - \cot^2\theta\,(b^2 - a^2)$$
$$= (b^2 - a^2)(\operatorname{cosec}^2\theta - \cot^2\theta)$$
$$= b^2 - a^2 \qquad [\because \operatorname{cosec}^2\theta - \cot^2\theta = 1]$$

17. (*a*) **1**

$\sin\theta + \sin^2\theta = 1$ [Given]

$\Rightarrow \sin\theta = 1 - \sin^2\theta$

$\Rightarrow \sin\theta = \cos^2\theta$ … (1)

$\cos^2\theta + \cos^4\theta = \cos^2\theta\,(1 + \cos^2\theta) = \sin\theta\,(1 + \sin\theta)$ [Using (1)]

$= \sin\theta + \sin^2\theta = 1$ $[\because \sin\theta + \sin^2\theta = 1$, given$]$

18. (*b*) **1**

$\cos\theta + \cos^2\theta = 1$ [Given]

$\Rightarrow \cos\theta = 1 - \cos^2\theta$

$\Rightarrow \cos\theta = \sin^2\theta$ … (1)

$\sin^2\theta + \sin^4\theta = \sin^2\theta\,(1 + \sin^2\theta)$

$= \cos\theta\,(1 + \cos\theta)$ [Using (1)]

$= \cos\theta + \cos^2\theta$

$= 1$ $[\because \cos\theta + \cos^2\theta = 1$, given$]$

19. (*c*) $\boldsymbol{\dfrac{m^2 + 1}{2m}}$

$\sec\theta + \tan\theta = m$ [Given] … (1)

$$\Rightarrow \frac{1}{\sec\theta + \tan\theta} = \frac{1}{m}$$

$$\Rightarrow \frac{1}{(\sec\theta + \tan\theta)}\,\frac{(\sec\theta - \tan\theta)}{(\sec\theta - \tan\theta)} = \frac{1}{m}$$

$$\Rightarrow \frac{\sec\theta - \tan\theta}{\sec^2\theta - \tan^2\theta} = \frac{1}{m}$$

$$\Rightarrow \sec\theta - \tan\theta = \frac{1}{m} \qquad \text{… (2)}$$

$[\because 1 + \tan^2\theta = \sec^2\theta \Rightarrow \sec^2\theta - \tan^2\theta = 1]$

Adding (1) and (2), we get

$$2\sec\theta = m + \frac{1}{m}$$

$$\Rightarrow 2\sec\theta = \frac{m^2 + 1}{m}$$

$$\Rightarrow \sec\theta = \frac{m^2 + 1}{2m}$$

20. (*a*) $\boldsymbol{\dfrac{m^2 - 1}{2m}}$

$\sec\theta + \tan\theta = m$ [Given] … (1)

$$\Rightarrow \frac{1}{\sec\theta + \tan\theta} = \frac{1}{m}$$

$$\Rightarrow \frac{1}{\sec\theta + \tan\theta}\,\frac{(\sec\theta - \tan\theta)}{(\sec\theta - \tan\theta)} = \frac{1}{m}$$

$$\Rightarrow \frac{\sec\theta - \tan\theta}{\sec^2\theta - \tan^2\theta} = \frac{1}{m}$$

$$\Rightarrow \sec\theta - \tan\theta = \frac{1}{m} \qquad \text{… (2)}$$

$[\because 1 + \tan^2\theta = \sec^2\theta \Rightarrow \sec^2\theta - \tan^2\theta = 1]$

Subtracting (2) from (1), we get

$$2\tan\theta = m - \frac{1}{m}$$

$$= \frac{m^2 - 1}{m}$$

$$\Rightarrow \tan\theta = \frac{m^2 - 1}{2m}$$

SHORT ANSWER QUESTIONS

For Basic and Standard Levels

1. $$\frac{1 + \cot^2\theta}{1 + \tan^2\theta} = \frac{\operatorname{cosec}^2\theta}{\sec^2\theta}$$
$$= \frac{\cos^2\theta}{\sin^2\theta}$$
$$= \boldsymbol{\cot^2}\,\theta$$

2. $x\cos\theta = 1$ [Given]

$$\Rightarrow x = \frac{1}{\cos\theta}$$

$\Rightarrow x = \sec\theta$

$\Rightarrow x^2 = \sec^2\theta$ … (1)

$\tan\theta = y$ [Given]

$\Rightarrow y^2 = \tan^2\theta$ … (2)

Subtracting (2) from (1), we get

LHS $= x^2 - y^2$

$= \sec^2\theta - \tan^2\theta$

$= \mathbf{1}$

= RHS

$[\because 1 + \tan^2\theta = \sec^2\theta \Rightarrow \sec^2\theta - \tan^2\theta = 1]$

3. LHS $= \tan\theta\,\sqrt{1 - \sin^2\theta}$

$$= \left(\frac{\sin\theta}{\cos\theta}\right)\sqrt{\cos^2\theta}$$

$[\because \sin^2\theta + \cos^2\theta = 1 \Rightarrow 1 - \sin^2\theta = \cos^2\theta]$

$$= \frac{\sin\theta}{\cos\theta}\,(\cos\theta)$$

$= \mathbf{sin}\,\theta$

= RHS

4. $\cos A + \sin A = \sqrt{3}$ [Given]

$\Rightarrow \cos^2 A + \sin^2 A + 2\sin A\cos A = 3$ [Squaring both sides]

$\Rightarrow 1 + 2\sin A\cos A = 3$

$\Rightarrow 2\sin A\cos A = 2$

$\Rightarrow \sin A\cos A = 1$

$$\Rightarrow \frac{1}{\sin A\cos A} = 1$$

$$\Rightarrow \frac{\sin^2 A + \cos^2 A}{\sin A\cos A} = 1$$

[Putting $1 = \sin^2 A + \cos^2 A$]

$\Rightarrow \quad \dfrac{\sin^2 A}{\sin A \cos A} + \dfrac{\cos^2 A}{\sin A \cos A} = 1$

$\Rightarrow \quad \dfrac{\sin A}{\cos A} + \dfrac{\cos A}{\sin A} = 1$

$\Rightarrow \quad \mathbf{\tan A + \cot A = 1}$

Hence, proved.

5. $\tan\theta = \dfrac{1}{\sqrt{5}}$ [Given] … (1)

$\cot\theta = \sqrt{5}$ … (2)

$$\frac{\text{cosec}^2\theta - \sec^2\theta}{\text{cosec}^2\theta + \sec^2\theta} = \frac{(1+\cot^2\theta)-(1+\tan^2\theta)}{(1+\cot^2\theta)+(1+\tan^2\theta)}$$

$$= \frac{1+\cot^2\theta - 1 - \tan^2\theta}{1+\cot^2\theta+1+\tan^2\theta}$$

$$= \frac{\cot^2\theta - \tan^2\theta}{2+\cot^2\theta+\tan^2\theta}$$

$$= \frac{5-\frac{1}{5}}{2+5+\frac{1}{5}} \quad \text{[Using (1) and (2)]}$$

$$= \frac{\frac{25-1}{5}}{7+\frac{1}{5}}$$

$$= \frac{\frac{24}{5}}{\frac{35+1}{5}}$$

$$= \frac{24}{36}$$

$$= \frac{2}{3}$$

Hence, $\dfrac{\text{cosec}^2\theta - \sec^2\theta}{\text{cosec}^2\theta + \sec^2\theta} = \dfrac{2}{3}$.

For Standard Level

6. $\tan\theta + \sec\theta = p$ [Given] … (1)

$\Rightarrow \quad \dfrac{1}{\tan\theta + \sec\theta} = \dfrac{1}{p}$

$\Rightarrow \quad \dfrac{1}{(\tan\theta+\sec\theta)}\,\dfrac{(\tan\theta-\sec\theta)}{(\tan\theta-\sec\theta)} = \dfrac{1}{p}$

$\Rightarrow \quad \dfrac{\tan\theta - \sec\theta}{\tan^2\theta - \sec^2\theta} = \dfrac{1}{p}$

$\Rightarrow \quad \dfrac{\tan\theta-\sec\theta}{(-1)} = \dfrac{1}{p}$ … (2)

[$\because 1 + \tan^2\theta = \sec^2\theta \Rightarrow \tan^2\theta - \sec^2\theta = -1$]

$\Rightarrow \quad \tan\theta - \sec\theta = \dfrac{-1}{p}$

Subtracting (2) from (1), we get

$$2\sec\theta = p - \left(-\frac{1}{p}\right)$$

$\Rightarrow \quad 2\sec\theta = \dfrac{p^2+1}{p}$

$\Rightarrow \quad \sec\theta = \dfrac{p^2+1}{2p}$

Hence, $\mathbf{\sec\theta = \dfrac{p^2+1}{2p}}$

7. $7\sin^2\theta + 3\cos^2\theta = 4$ [Given]

$\Rightarrow \quad 4\sin^2\theta + 3\sin^2\theta + 3\cos^2\theta = 4$

$\Rightarrow \quad 4\sin^2\theta + 3(\sin^2\theta + \cos^2\theta) = 4$

$\Rightarrow \quad 4\sin^2\theta + 3 = 4$

[$\because \sin^2\theta + \cos^2 = 1$]

$\Rightarrow \quad 3 = 4 - 4\sin^2\theta$

$\Rightarrow \quad 3 = 4(1-\sin^2\theta)$

$\Rightarrow \quad 3 = 4\cos^2\theta$

$\Rightarrow \quad 3 = \dfrac{4}{\sec^2\theta}$

$\Rightarrow \quad \sec^2\theta = \dfrac{4}{3}$

$\Rightarrow \quad 1 + \tan^2\theta = \dfrac{4}{3}$

$\Rightarrow \quad \tan^2\theta = \dfrac{4}{3} - 1 = \dfrac{1}{3}$

$\Rightarrow \quad \tan\theta = \dfrac{1}{\sqrt{3}}$

Hence, $\mathbf{\tan\theta = \dfrac{1}{\sqrt{3}}}$

8. $2 - \cos^2\theta = 3\sin\theta\cos\theta$ [Given]

$\Rightarrow \quad 2(\sin^2\theta + \cos^2\theta) - \cos^2\theta = 3\sin\theta\cos\theta$

$\Rightarrow \quad 2\sin^2\theta + 2\cos^2\theta - \cos^2\theta = 3\sin\theta\cos\theta$

$\Rightarrow \quad 2\sin^2\theta - 3\sin\theta\cos\theta + \cos^2\theta = 0$

$\Rightarrow \quad 2\sin^2\theta - 2\sin\theta\cos\theta - \sin\theta\cos\theta + \cos^2\theta = 0$

$\Rightarrow \quad 2\sin\theta(\sin\theta - \cos\theta) - \cos\theta(\sin\theta-\cos\theta) = 0$

$\Rightarrow \quad (\sin\theta - \cos\theta)(2\sin\theta - \cos\theta) = 0$

$\Rightarrow$ Either $\sin\theta - \cos\theta = 0$ or $2\sin\theta - \cos\theta = 0$

$\Rightarrow \quad \sin\theta = \cos\theta$ or $2\sin\theta = \cos\theta$

$\Rightarrow \quad \dfrac{\sin\theta}{\cos\theta} = 1$ or $\dfrac{\sin\theta}{\cos\theta} = \dfrac{1}{2}$

$\Rightarrow \quad \tan\theta = 1$ or $\tan\theta = \dfrac{1}{2}$

Hence, $\mathbf{\tan\theta = 1}$ **or** $\mathbf{\dfrac{1}{2}}$

9. LHS $= (\sin^4 A - \cos^4 A + 1)\,\text{cosec}^2 A$

$= [(\sin^2 A)^2 - (\cos^2 A)^2 + 1]\,\text{cosec}^2 A$

$= [(\sin^2 A + \cos^2 A)(\sin^2 A - \cos^2 A) + 1]\,\text{cosec}^2 A$

$= [\sin^2 A - (1 - \sin^2 A) + 1]\,\text{cosec}^2 A$

[$\therefore \sin^2 A + \cos^2 A = 1 \Rightarrow \cos^2 A = 1 - \sin^2 A$]

$= (\sin^2 A - 1 + \sin^2 A + 1)\,\text{cosec}^2 A$

$= (2\sin^2 A)(\text{cosec}^2 A)$

$= (2\sin^2 A)\left(\dfrac{1}{\sin^2 A}\right)$

$= \mathbf{2}$

$=$ RHS

10. [Since B has to be eliminated, we find sec B and use the identity $\sec^2 B - \tan^2 B = 1$]

$$\tan A = n \tan B \quad \text{[Given] … (1)}$$

and
$$\sin A = m \sin B \quad \text{[Given] … (2)}$$

$$\Rightarrow \quad \frac{\sin A}{\cos A} = n \frac{\sin B}{\cos B}$$

$$\Rightarrow \quad \frac{m \sin B}{\cos A} = \frac{n \sin B}{\cos B} \quad \text{[Using (2)]}$$

$$\Rightarrow \quad \frac{m}{\cos A} = \frac{n}{\cos B}$$

$$\Rightarrow \quad m \sec A = n \sec B$$

$$\Rightarrow \quad \sec B = \frac{m}{n} \sec A$$

$$\Rightarrow \quad \sec^2 B = \frac{m^2}{n^2} \sec^2 A \quad \text{… (3)}$$

Again $\tan A = n \tan B$ [Using (1)]

$$\Rightarrow \quad \tan B = \frac{\tan A}{n}$$

$$\Rightarrow \quad \tan^2 B = \frac{\tan^2 A}{n^2} \quad \text{… (4)}$$

Subtracting (4) from (3), we get

$$\sec^2 B - \tan^2 B = \frac{m^2}{n^2} \sec^2 A - \frac{\tan^2 A}{n^2}$$

$$\Rightarrow \quad n^2 = m^2 \sec^2 A - \tan^2 A$$

$$\Rightarrow \quad n^2 = \frac{m^2}{\cos^2 A} - \frac{\sin^2 A}{\cos^2 A}$$

$$\Rightarrow \quad n^2 = \frac{m^2}{\cos^2 A} - \frac{(1 - \cos^2 A)}{\cos^2 A}$$

$$[\because \sin^2 A + \cos^2 A = 1 \quad \Rightarrow \sin^2 A = 1 - \cos^2 A]$$

$$\Rightarrow \quad n^2 \cos^2 A = m^2 - 1 + \cos^2 A$$

$$\Rightarrow \quad n^2 \cos^2 A - \cos^2 A = m^2 - 1$$

$$\Rightarrow \quad \cos^2 A (n^2 - 1) = m^2 - 1$$

$$\Rightarrow \quad \cos^2 A = \frac{(m^2 - 1)}{(n^2 - 1)}$$

Hence, $\mathbf{\cos^2 A = \frac{(m^2 - 1)}{(n^2 - 1)}}$