CHAPTER 8

Trigonometric Ratios

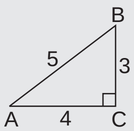
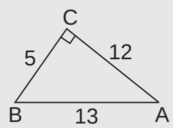
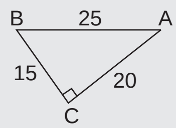
EXERCISE 8A

For Basic and Standard Levels

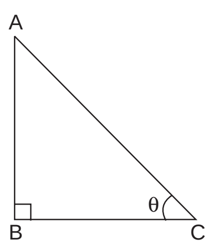
1. By definition: $\sin\theta = \dfrac{\text{perpendicular}}{\text{hypotenuse}}$,

$\cos\theta = \dfrac{\text{base}}{\text{hypotenuse}}$,

and $\tan\theta = \dfrac{\text{perpendicular}}{\text{base}}$

			
sin A =	$\frac{BC}{AB} = \mathbf{\frac{3}{5}}$	$\frac{BC}{AB} = \mathbf{\frac{5}{13}}$	$\frac{BC}{AB} = \frac{15}{25} = \mathbf{\frac{3}{5}}$
sin B =	$\frac{AC}{AB} = \mathbf{\frac{4}{5}}$	$\frac{AC}{AB} = \mathbf{\frac{12}{13}}$	$\frac{AC}{AB} = \frac{20}{25} = \mathbf{\frac{4}{5}}$
cos A =	$\frac{AC}{AB} = \mathbf{\frac{4}{5}}$	$\frac{AC}{AB} = \mathbf{\frac{12}{13}}$	$\frac{AC}{AB} = \frac{20}{25} = \mathbf{\frac{4}{5}}$
cos B =	$\frac{BC}{AB} = \mathbf{\frac{3}{5}}$	$\frac{BC}{AB} = \mathbf{\frac{5}{13}}$	$\frac{BC}{AB} = \frac{15}{25} = \mathbf{\frac{3}{5}}$
tan A =	$\frac{BC}{AC} = \mathbf{\frac{3}{4}}$	$\frac{BC}{AC} = \mathbf{\frac{5}{12}}$	$\frac{BC}{AC} = \frac{15}{20} = \mathbf{\frac{3}{4}}$
tan B =	$\frac{AC}{BC} = \mathbf{\frac{4}{3}}$	$\frac{AC}{BC} = \mathbf{\frac{12}{5}}$	$\frac{AC}{BC} = \frac{20}{15} = \mathbf{\frac{4}{3}}$

2. (*i*) Draw a right ΔABC, right-angled at B.



Let ∠ACB = θ

Then, $\cos\theta = \dfrac{BC}{AC} = \dfrac{4}{5}$

Let BC = 4*k*

Then, CA = 5*k*, where *k* is a positive number

In right ΔABC, we have

$AC^2 = AB^2 + BC^2$ [By Pythagoras' Theorem]

$\Rightarrow \quad (5k)^2 = AB^2 + (4k)^2$

$\Rightarrow \quad AB^2 = 25k^2 - 16k^2 = 9k^2$

$\Rightarrow \quad AB = 3k$

Using the definitions of trigonometric ratios, we get

$$\sin\theta = \frac{AB}{AC} = \frac{3k}{5k} = \mathbf{\frac{3}{5}}$$

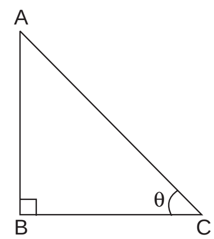
$$\text{cosec}\,\theta = \frac{AC}{AB} = \frac{5k}{3k} = \mathbf{\frac{5}{3}}$$

$$\tan\theta = \frac{AB}{BC} = \frac{3k}{4k} = \mathbf{\frac{3}{4}}$$

$$\cot\theta = \frac{BC}{AB} = \frac{4k}{3k} = \mathbf{\frac{4}{3}}$$

and $$\sec\theta = \frac{AC}{BC} = \frac{5k}{4k} = \mathbf{\frac{5}{4}}$$

(*ii*) Draw a right ΔABC, right-angled at B.



Let ∠ACB = θ

Then, $\tan\theta = \dfrac{AB}{BC} = \dfrac{5}{12}$

Let AB = 5*k*,

then, BC = 12*k*

In right ΔABC, we have

$AC^2 = AB^2 + BC^2$ [By Pythagoras' Theorem]

$\Rightarrow \quad AC^2 = (5k)^2 + (12k)^2$

$= (25 + 144)k^2$

$= 169k^2$

$\Rightarrow \quad AC = 13k$

Using the definitions of trigonometric ratios, we get

$$\sin\theta = \frac{AB}{AC} = \frac{5k}{13k} = \mathbf{\frac{5}{13}}$$

$$\text{cosec}\,\theta = \frac{AC}{AB} = \frac{13k}{5k} = \mathbf{\frac{13}{5}}$$

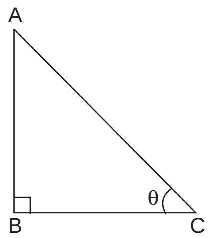
$$\cos\theta = \frac{BC}{AC} = \frac{12k}{13k} = \mathbf{\frac{12}{13}}$$

$$\sec\theta = \frac{AC}{BC} = \frac{13k}{12k} = \mathbf{\frac{13}{12}}$$

and $$\cot\theta = \frac{BC}{AB} = \frac{12k}{5k} = \mathbf{\frac{12}{5}}$$

(*iii*) $\sec\theta = \dfrac{25}{7}$

Draw a right triangle ABC, right-angled at B.



Let $\angle ACB = \theta$

Then, $\sec\theta = \dfrac{AC}{BC} = \dfrac{25}{7}$

Let $AC = 25k$

then $BC = 7k$

In right ΔABC, we have

$$AC^2 = AB^2 + BC^2$$

[By Pythagoras' Theorem]

$\Rightarrow$ $(25k)^2 = AB^2 + (7k)^2$

$\Rightarrow$ $AB^2 = (625 - 49)k^2 = 576k^2$

$\Rightarrow$ $AB = 24k$

Using the definitions of trigonometric ratios, we get

$$\sin\theta = \frac{AB}{AC} = \frac{24k}{25k} = \mathbf{\frac{24}{25}}$$

$$\text{cosec}\,\theta = \frac{AC}{AB} = \frac{25k}{24k} = \mathbf{\frac{25}{24}}$$

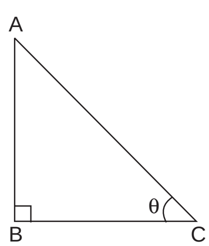
$$\tan\theta = \frac{AB}{BC} = \frac{24k}{7k} = \mathbf{\frac{24}{7}}$$

$$\cot\theta = \frac{BC}{AB} = \frac{7k}{24k} = \mathbf{\frac{7}{24}}$$

and $\cos\theta = \dfrac{BC}{AC} = \dfrac{7k}{25k} = \mathbf{\dfrac{7}{25}}$

(*iv*) $\sin\theta = \dfrac{7}{25}$

Draw a right triangle ABC, right-angled at B.



Let $\angle ACB = \theta$

Then, $\sin\theta = \dfrac{AB}{AC} = \dfrac{7}{25}$

Let $AB = 7k$

Then $AC = 25k$

In right ABC, we have

$$AC^2 = AB^2 + BC^2$$

[By Pythagoras' Theorem]

$\Rightarrow$ $(25k)^2 = (7k)^2 + (BC)^2$

$\Rightarrow$ $BC^2 = (625 - 49)\,k^2$

$= 576k^2$

$\Rightarrow$ $BC = 24k$

Using the definitions of trigonometric ratios, we get

$$\cos\theta = \frac{BC}{AC} = \frac{24k}{25k} = \mathbf{\frac{24}{25}}$$

$$\sec\theta = \frac{AC}{BC} = \frac{25k}{24k} = \mathbf{\frac{25}{24}}$$

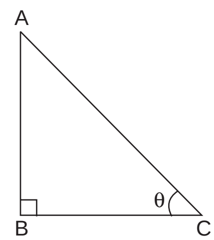
$$\tan\theta = \frac{AB}{BC} = \frac{7k}{24k} = \mathbf{\frac{7}{24}}$$

$$\cot\theta = \frac{BC}{AB} = \frac{24k}{7k} = \mathbf{\frac{24}{7}}$$

and $\text{cosec}\,\theta = \dfrac{AC}{AB} = \dfrac{25k}{7k} = \mathbf{\dfrac{25}{7}}$

(*v*) $\cot\theta = \dfrac{20}{21}$

Draw a right triangle ABC, right-angled at B.



Let $\angle ACB = \theta$

Then, $\cot\theta = \dfrac{BC}{AB} = \dfrac{20}{21}$

Let $BC = 20k$,

Then, $AB = 21k$

In right triangle ABC, we have

$$AC^2 = AB^2 + BC^2$$

[By Pythagoras' Theorem]

$\Rightarrow$ $AC^2 = AB^2 + BC^2$

$\Rightarrow$ $AC^2 = (21k)^2 + (20k)^2$

$= (441 + 400)\,k^2 = 841k^2$

$\Rightarrow$ $AC = 29k$

Using the definitions of trigonometric ratios, we get

$$\sin\theta = \frac{AB}{AC} = \frac{21k}{29k} = \mathbf{\frac{21}{29}}$$

$$\text{cosec}\,\theta = \frac{AC}{AB} = \frac{29k}{21k} = \mathbf{\frac{29}{21}}$$

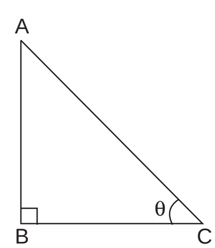
$$\cos\theta = \frac{BC}{AC} = \frac{20k}{29k} = \mathbf{\frac{20}{29}}$$

$$\sec\theta = \frac{AC}{BC} = \frac{29k}{20k} = \mathbf{\frac{29}{20}}$$

and $\tan\theta = \dfrac{AB}{BC} = \dfrac{21k}{20k} = \mathbf{\dfrac{21}{20}}$

(*vi*) $\text{cosec}\,\theta = \sqrt{10} = \dfrac{\sqrt{10}}{1}$

Draw a right ΔABC, right-angled at B.



Let $\angle ACB = \theta$

Then, $\operatorname{cosec}\theta = \dfrac{AC}{AB} = \dfrac{\sqrt{10}}{1}$

Let $AC = \sqrt{10}k$

Then, $AB = k$

In right ΔABC, we have

$$AC^2 = AB^2 + BC^2$$

$\Rightarrow$ $(\sqrt{10}k)^2 = k^2 + BC^2$

$\Rightarrow$ $BC^2 = 10k^2 - k^2 = 9k^2$

$\Rightarrow$ $BC = 3k$

Using the definitions of trigonometric ratios, we get

$$\cos\theta = \frac{BC}{AC} = \frac{3k}{\sqrt{10}\,k} = \mathbf{\frac{3}{\sqrt{10}}}$$

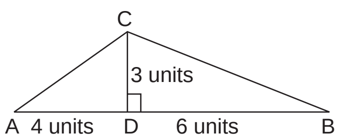
$$\sec\theta = \frac{AC}{BC} = \frac{\sqrt{10}\,k}{3k} = \mathbf{\frac{\sqrt{10}}{3}}$$

$$\tan\theta = \frac{AB}{BC} = \frac{k}{3k} = \mathbf{\frac{1}{3}}$$

$$\cot\theta = \frac{BC}{AB} = \frac{3k}{k} = \frac{3}{1} = \mathbf{3}$$

and $\sin\theta = \dfrac{AB}{AC} = \dfrac{k}{\sqrt{10}\,k} = \mathbf{\dfrac{1}{\sqrt{10}}}$.

3. In right ΔADC,



we have $AC^2 = AD^2 + CD^2$

[By Pythagoras' Theorem]

$\Rightarrow$ $AC^2 = (4)^2 + (3)^2$

$\Rightarrow$ $AC^2 = 9 + 16$

$= 25$

$\Rightarrow$ $AC = 5$ units

(*i*) $\sin A = \dfrac{CD}{AC} = \dfrac{3\text{ units}}{5\text{ units}} = \mathbf{\dfrac{3}{5}}$

(*ii*) $\cot A = \dfrac{AD}{CD} = \dfrac{4\text{ units}}{3\text{ units}} = \mathbf{\dfrac{4}{3}}$

In right ΔBCD, we have

$$BC^2 = BD^2 + CD^2$$

[By Pythagoras' Theorem]

$\Rightarrow$ $BC^2 = (6)^2 + (3)^2 = 36 + 9 = 45$

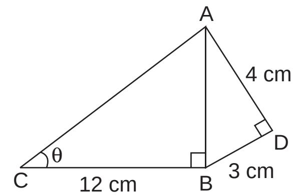
$\Rightarrow$ $BC = \sqrt{45}$

$= 3\sqrt{5}$ units

(*iii*) $\sec B = \dfrac{BC}{BD} = \dfrac{3\sqrt{5}\text{ units}}{6\text{ units}} = \mathbf{\dfrac{\sqrt{5}}{2}}$

(*iv*) $\tan B = \dfrac{CD}{BD} = \dfrac{3\text{ units}}{6\text{ units}} = \mathbf{\dfrac{1}{2}}$

4. In right ΔADB,



we have $AB^2 = AD^2 + BD^2$

[By Pythagoras' Theorem]

$\Rightarrow$ $AB^2 = (4\text{ cm})^2 + (3\text{ cm})^2$

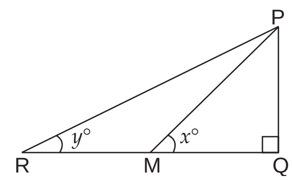
$= 16\text{ cm}^2 + 9\text{ cm}^2$

$= 25\text{ cm}^2$

$\Rightarrow$ $AB = 5$ cm

$$\cot\theta = \frac{BC}{AB} = \frac{12\text{ cm}}{5\text{ cm}} = \mathbf{\frac{12}{5}}$$

5.



$$\tan x° = \frac{PQ}{MQ} = \frac{3}{4}$$

$\Rightarrow$ $\dfrac{15\text{ cm}}{MQ} = \dfrac{3}{4}$

$\Rightarrow$ $MQ = \dfrac{15 \times 4}{3}\text{ cm} = 20\text{ cm}$ … (1)

$$\tan y° = \frac{PQ}{RQ} = \frac{2}{5}$$

$\Rightarrow$ $\dfrac{PQ}{MR + MQ} = \dfrac{2}{5}$

$\Rightarrow$ $\dfrac{15\text{ cm}}{MR + 20\text{ cm}} = \dfrac{2}{5}$ [Using (1)]

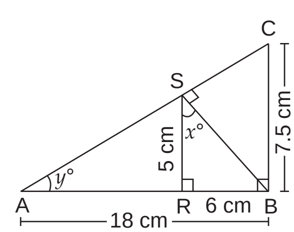
$\Rightarrow$ $75\text{ cm} = 2MR + 40\text{ cm}$

$\Rightarrow$ $2MR = 35\text{ cm}$

$\Rightarrow$ $MR = \dfrac{35}{2} = 17.5\text{ cm}$

Hence, MR = **17.5 cm**

6. In right triangle ABC,



we have $AC^2 = AB^2 + BC^2$

[By Pythagoras' Theorem]

$\Rightarrow \quad AC^2 = (18 \text{ cm})^2 + (7.5 \text{ cm})^2$
$= (324 + 56.25) \text{ cm}^2$
$= 380.25 \text{ cm}^2$
$\Rightarrow \quad AC = 19.5 \text{ cm} \quad \dots (1)$

(*i*) In ΔBRS,

$$\tan x^\circ = \frac{BR}{RS} = \frac{6\,\text{cm}}{5\,\text{cm}} = \frac{6}{5}$$

Hence, $\mathbf{\tan x^\circ = \frac{6}{5}}$

(*ii*) In ΔABC,

$$\sin y^\circ = \frac{BC}{AC} = \frac{7.5\,\text{cm}}{19.5\,\text{cm}} \quad \text{[Using (1)]}$$

$$= \frac{5}{13}$$

Hence, $\mathbf{\sin y^\circ = \frac{5}{13}}$

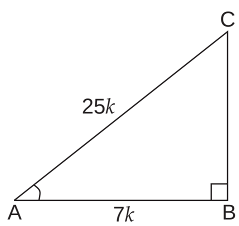
(*iii*) In ΔABC,

$$\cos y^\circ = \frac{AB}{AC} = \frac{18\,\text{cm}}{19.5\,\text{cm}} \quad \text{[Using (1)]}$$

$$= \frac{12}{13}$$

Hence, $\mathbf{\cos y^\circ = \frac{12}{13}}$

7. Draw a right triangle ABC, right-angled at B.



Then, $\cos A = \frac{AB}{AC} = \frac{7}{25}$

Let $AB = 7k$
Then, $AC = 25k$
In right ΔABC, we have

$$AC^2 = AB^2 + BC^2$$
[By Pythagoras' Theorem]

$\Rightarrow \quad (25k)^2 = (7k)^2 + BC^2$
$\Rightarrow \quad BC^2 = (625 - 49)k^2$
$= 576k^2$
$\Rightarrow \quad BC = 24k$

$$\tan A = \frac{BC}{AB} = \frac{24k}{7k} = \frac{24}{7}$$

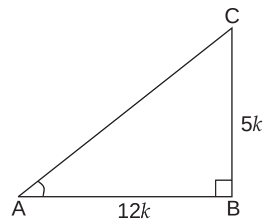
and $\cot A = \frac{AB}{BC} = \frac{7k}{24k} = \frac{7}{24} \quad \dots (1)$

$$\tan A + \cot A = \frac{24}{7} + \frac{7}{24} \quad \text{[Using (1)]}$$

$$= \frac{576 + 49}{(24)(7)} = \frac{625}{168}$$

Hence, $\tan A + \cot A = \mathbf{\frac{625}{168}}$

8. Draw a right ΔABC, right-angled at B.



Then, $\tan A = \frac{BC}{AB} = \frac{5}{12}$

Let $BC = 5k$
Then, $AB = 12k$
In right ΔABC, we have

$$AC^2 = AB^2 + BC^2$$
[By Pythagoras' Theorem]

$\Rightarrow \quad AC^2 = (12k)^2 + (5k)^2$
$= (144 + 25)k^2$
$= 169k^2$
$\Rightarrow \quad AC = 13k$

$$\sin A = \frac{BC}{AC} = \frac{5k}{13k} = \frac{5}{13}$$

$$\cos A = \frac{AB}{AC} = \frac{12k}{13k} = \frac{12}{13}$$

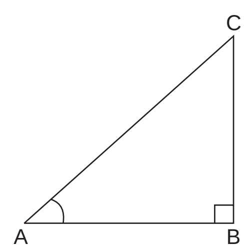
and $\sec A = \frac{AC}{AB} = \frac{13k}{12k} = \frac{13}{12} \quad \dots (1)$

$$(\sin A + \cos A) \sec A = \left(\frac{5}{13} + \frac{12}{13}\right)\left(\frac{13}{12}\right) \quad \text{[using (1)]}$$

$$= \left(\frac{17}{13}\right)\left(\frac{13}{12}\right) = \frac{17}{12}$$

Hence, $(\sin A + \cos A) \sec A = \mathbf{\frac{17}{12}}$

9. Draw a right ΔABC, right-angled at B.



Then, $\sin A = \frac{BC}{AC} = \frac{1}{3}$

Let $BC = k$,
Then, $AC = 3k$
In right ΔABC, we have

$$AC^2 = AB^2 + BC^2$$
[By Pythagoras' Theorem]

$\Rightarrow \quad (3k)^2 = AB^2 + k^2$
$\Rightarrow \quad AB^2 = (9 - 1)\, k^2$
$= 8k^2$
$\Rightarrow \quad AB = \sqrt{8}\, k$
$= 2\sqrt{2}\, k$

$$\cos A = \frac{AB}{AC} = \frac{2\sqrt{2}k}{3k} = \frac{2\sqrt{2}}{3}$$

$$\text{cosec } A = \frac{AC}{BC} = \frac{3k}{k} = 3$$

$$\tan A = \frac{BC}{AB} = \frac{k}{2\sqrt{2}k} = \frac{1}{2\sqrt{2}}$$

$$\sec A = \frac{AC}{AB} = \frac{3k}{2\sqrt{2}k} = \frac{3}{2\sqrt{2}} \qquad \ldots (1)$$

cos A cosec A + tan A sec A

$$= \frac{2\sqrt{2}}{3}(3) + \left(\frac{1}{2\sqrt{2}}\right)\left(\frac{3}{2\sqrt{2}}\right)$$

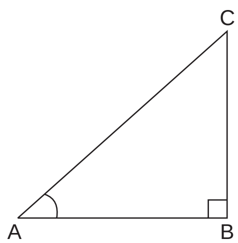
[Using (1)]

$$= 2\sqrt{2} + \frac{3}{8}$$

$$= \frac{16\sqrt{2}+3}{8}$$

Hence, cos A cosec A + tan A sec A = $\mathbf{\frac{16\sqrt{2}+3}{8}}$

10. Draw a right ΔABC, right-angled at B.



Then, $\tan A = \frac{BC}{AB} = 2 = \frac{2}{1}$

Let BC = $2k$. Then, AB = k

In right ΔABC, we have

$$AC^2 = AB^2 + BC^2$$

[By Pythagoras' Theorem]

$$\Rightarrow \quad AC^2 = k^2 + (2k)^2 = 5k^2$$

$$\Rightarrow \quad AC = \sqrt{5}\,k$$

$$\sin A = \frac{BC}{AC} = \frac{2k}{\sqrt{5}k} = \frac{2}{\sqrt{5}}$$

$$\sec A = \frac{AC}{AB} = \frac{\sqrt{5}k}{k} = \sqrt{5}$$

and $\text{cosec } A = \frac{AC}{BC} = \frac{\sqrt{5}k}{2k} = \frac{\sqrt{5}}{2}$ … (1)

sin A sec A + tan A – cosec A

$$= \left(\frac{2}{\sqrt{5}}\right)\left(\sqrt{5}\right) + 2 - \frac{\sqrt{5}}{2}$$

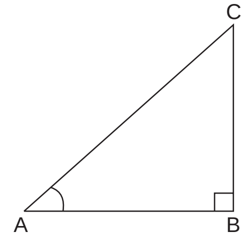
$$= 2 + 2 - \frac{\sqrt{5}}{2}$$

$$= 4 - \frac{\sqrt{5}}{2}$$

$$= \frac{8-\sqrt{5}}{2}$$

Hence, sin A sec A + tan A – cosec A = $\mathbf{\frac{8-\sqrt{5}}{2}}$

11. Draw a right ΔABC, right-angled at B.



$$\tan A = \frac{BC}{AB} = \sqrt{2} - 1$$

$$= \frac{\sqrt{2}-1}{1}$$

Let BC = $(\sqrt{2}-1)k$. Then, AB = k

In right ΔABC, we have

$$AC^2 = AB^2 + BC^2$$

[By Pythagoras' Theorem]

$$\Rightarrow \quad AC^2 = k^2 + [(\sqrt{2}-1)k]^2$$

$$= k^2 + 2k^2 - 2\sqrt{2}k^2 + k^2$$

$$= (4 - 2\sqrt{2})k^2$$

$$\Rightarrow \quad AC = \sqrt{4-2\sqrt{2}}\,k$$

$$\sin A = \frac{BC}{AC} = \frac{(\sqrt{2}-1)k}{(\sqrt{4-2\sqrt{2}})k} = \frac{(\sqrt{2}-1)}{\sqrt{4-2\sqrt{2}}}$$

and $\cos A = \frac{AB}{AC} = \frac{k}{\sqrt{4-2\sqrt{2}}\,k} = \frac{1}{\sqrt{4-2\sqrt{2}}}$

… (1)

$$\sin A \cos A = \frac{(\sqrt{2}-1)}{\sqrt{4-2\sqrt{2}}} \times \frac{1}{\sqrt{4-2\sqrt{2}}}$$

$$= \frac{(\sqrt{2}-1)}{4-2\sqrt{2}}$$

$$= \frac{(\sqrt{2}-1)}{2\sqrt{2}(\sqrt{2}-1)}$$

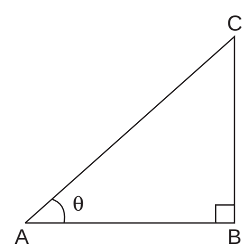
$$= \frac{1}{2\sqrt{2}}$$

$$= \frac{1}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \mathbf{\frac{\sqrt{2}}{4}}$$

Hence, it is proved that sin A cos A = $\mathbf{\frac{\sqrt{2}}{4}}$

12. Draw a right ΔABC, right-angled at B.



Let $\angle CAB = \theta$

Then, $\text{cosec }\theta = \frac{AC}{BC} = \frac{13}{12}$

Let AC = 13k. Then BC = 12k

In right ΔABC, we have

$$AC^2 = AB^2 + BC^2$$

[By Pythagoras' Theorem]

$$\Rightarrow \quad (13k)^2 = AB^2 + (12k)^2$$

$$\Rightarrow \quad AB^2 = (169 - 144)\,k^2 = 25k^2$$

$$\Rightarrow \quad AB = 5k$$

$$\cot\theta = \frac{AB}{BC} = \frac{5k}{12k} = \frac{5}{12}$$

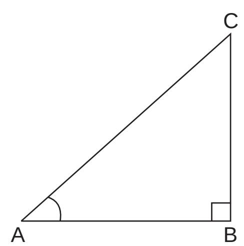
and $$\tan\theta = \frac{BC}{AB} = \frac{12k}{5k} = \frac{12}{5} \quad \ldots (1)$$

$$\cot\theta + \tan\theta = \frac{5}{12} + \frac{12}{5} \quad \text{[Using (1)]}$$

$$= \frac{25 + 144}{60} = \frac{169}{60}$$

Hence, $\cot\theta + \tan\theta = \mathbf{\frac{169}{60}}$

13. Draw a right ΔABC, right-angled at B.



Then, $\sin A = \frac{BC}{AC} = \frac{m}{n}$

Let BC = mk. Then, AC = nk

In right ΔABC, we have

$$AC^2 = AB^2 + BC^2$$

$$\Rightarrow \quad (nk) = AB^2 + (mk)^2$$

$$\Rightarrow \quad AB^2 = n^2k^2 - m^2k^2$$

$$\Rightarrow \quad AB = k\sqrt{n^2 - m^2} \quad \ldots (1)$$

$$\tan A = \frac{BC}{AB} = \frac{mk}{k\sqrt{n^2 - m^2}} = \frac{m}{\sqrt{n^2 - m^2}}$$

$$\cot A = \frac{AB}{BC} = \frac{k\sqrt{n^2 - m^2}}{mk} = \frac{\sqrt{n^2 - m^2}}{m}$$

$$\frac{\tan A + 4}{4\cot + 1} = \frac{\frac{m}{\sqrt{n^2 - m^2}} + 4}{\frac{4\sqrt{n^2 - m^2}}{m} + 1}$$

$$= \frac{\frac{m + 4\sqrt{n^2 - m^2}}{\sqrt{n^2 - m^2}}}{\frac{4\sqrt{n^2 - m^2} + m}{m}}$$

$$= \frac{m + 4\sqrt{n^2 - m^2}}{\sqrt{n^2 - m^2}} \times \frac{m}{4\sqrt{n^2 - m^2} + m}$$

$$= \frac{m}{\sqrt{n^2 - m^2}}$$

Hence, $\frac{\tan A + 4}{4\cot A + 1} = \boldsymbol{\frac{m}{\sqrt{n^2 - m^2}}}$

14. $$\frac{2\sin A - 3\cos A}{2\sin A + 3\cos A} = \frac{2\frac{\sin A}{\cos A} - 3\frac{\cos A}{\cos A}}{2\frac{\sin A}{\cos A} + 3\frac{\cos A}{\cos A}}$$

[Dividing the num. and denom. by cos A]

$$= \frac{2\tan A - 3}{2\tan A + 3} = \frac{2\left(\frac{4}{3}\right) - 3}{2\left(\frac{4}{3}\right) + 3}$$

$$= \frac{\frac{8}{3} - 3}{\frac{8}{3} + 3} = \frac{\frac{8 - 9}{3}}{\frac{8 + 9}{3}} = \frac{\frac{-1}{3}}{\frac{17}{3}}$$

$$= \frac{-1}{17}$$

Hence, $\frac{2\sin A - 3\cos A}{2\sin A + 3\cos A} = \mathbf{\frac{-1}{17}}$

15. $$16\cot A = 12$$

$$\Rightarrow \quad \cot A = \frac{12}{16} = \frac{3}{4}$$

$$\frac{\sin A + \cos A}{\sin A - \cos A} = \frac{\frac{\sin A}{\sin A} + \frac{\cos A}{\sin A}}{\frac{\sin A}{\sin A} - \frac{\cos A}{\sin A}}$$

$$= \frac{1 + \cot A}{1 - \cot A}$$

$$= \frac{1 + \frac{3}{4}}{1 - \frac{3}{4}} = \frac{\frac{7}{4}}{\frac{1}{4}} = 7$$

Hence, $\frac{\sin A + \cos A}{\sin A - \cos A} = \mathbf{7}$

16. $$5\tan\theta = 4$$

$$\Rightarrow \quad \tan\theta = \frac{4}{5}$$

$$\frac{5\sin\theta - 3\cos\theta}{5\sin\theta + 2\cos\theta} = \frac{5\frac{\sin\theta}{\cos\theta} - 3\frac{\cos\theta}{\cos\theta}}{5\frac{\sin\theta}{\cos\theta} + 2\frac{\cos\theta}{\cos\theta}}$$

[Dividing the num. and denom. by cos θ]

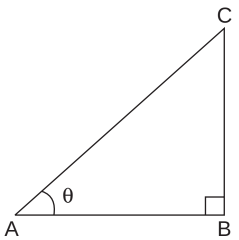
$$= \frac{5\tan\theta - 3}{5\tan\theta + 2}$$

$$= \frac{5\left(\frac{4}{5}\right) - 3}{5\left(\frac{4}{5}\right) + 2}$$

$$= \frac{4 - 3}{4 + 2} = \frac{1}{6}$$

Hence, $\frac{5\sin\theta - 3\cos\theta}{5\sin\theta + 2\cos\theta} = \mathbf{\frac{1}{6}}$

17. Draw a right ΔABC, right-angled at B.



Let $\angle CAB = \theta$

Then, $\cot\theta = \dfrac{AB}{BC} = \dfrac{15}{8}$

Let AB = 15*k*. Then, BC = 8*k*.

In right ΔABC, we have

$$AC^2 = AB^2 + BC^2$$
$$= (15k)^2 + (8k)^2$$
$$= (225 + 64)k^2$$
$$= 289k^2$$

$\Rightarrow$ $AC = 17k$

$$\sin\theta = \frac{BC}{AC} = \frac{8k}{17k} = \frac{8}{17}$$

and $\cos\theta = \dfrac{AB}{AC} = \dfrac{15k}{17k} = \dfrac{15}{17}$

$$\frac{(2+2\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(2-2\cos\theta)} = \frac{2(1+\sin\theta)(1-\sin\theta)}{2(1+\cos\theta)(1-\cos\theta)}$$
$$= \frac{1-\sin^2\theta}{1-\cos^2\theta}$$
$$= \frac{1-\left(\frac{8}{17}\right)^2}{1-\left(\frac{15}{17}\right)^2}$$
$$= \frac{1-\frac{64}{289}}{1-\frac{225}{289}}$$
$$= \frac{\frac{289-64}{289}}{\frac{289-225}{289}}$$
$$= \frac{\frac{225}{289}}{\frac{64}{289}}$$
$$= \frac{225}{64}$$

ALTERNATIVE METHOD:

$$\frac{(2+2\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(2-2\cos\theta)} = \frac{2(1+\sin\theta)(1-\sin\theta)}{2(1+\cos\theta)(1-\cos\theta)}$$
$$= \frac{1-\sin^2\theta}{1-\cos^2\theta}$$
$$= \frac{\cos^2\theta}{\sin^2\theta} \quad [\because \sin^2\theta + \cos^2\theta = 1]$$
$$= \cot^2\theta$$
$$= \left(\frac{15}{8}\right)^2$$
$$= \frac{225}{64}$$

Hence, $\dfrac{(2+2\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(2-2\cos\theta)} = \mathbf{\dfrac{225}{64}}$

18. $4\sin\theta - 3\cos\theta = 0$

$\Rightarrow$ $4\sin\theta = 3\cos\theta$

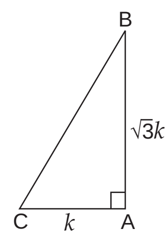
$\Rightarrow$ $\dfrac{\sin\theta}{\cos\theta} = \dfrac{3}{4}$

$\Rightarrow$ $\tan\theta = \dfrac{3}{4}$

$$\frac{12\sin\theta - 7\cos\theta}{8\sin\theta + 3\cos\theta} = \frac{\frac{12\sin\theta}{\cos\theta} - \frac{7\cos\theta}{\cos\theta}}{\frac{8\sin\theta}{\cos\theta} + \frac{3\cos\theta}{\cos\theta}}$$
$$= \frac{12\tan\theta - 7}{8\tan\theta + 3}$$
$$= \frac{12\left(\frac{3}{4}\right) - 7}{8\left(\frac{3}{4}\right) + 3}$$
$$= \frac{9-7}{6+3}$$
$$= \frac{2}{9}$$

Hence, $\dfrac{12\sin\theta - 7\cos\theta}{8\sin\theta + 3\cos\theta} = \mathbf{\dfrac{2}{9}}$

19. Draw a right ΔABC, right-angled at A.



Then, $\tan C = \dfrac{AB}{AC} = \sqrt{3} = \dfrac{\sqrt{3}}{1}$

Let AB = $\sqrt{3}\,k$. Then, AC = *k*

In right ΔABC, we have

$$BC^2 = AB^2 + AC^2$$

$\Rightarrow$ $BC^2 = (\sqrt{3}k)^2 + (k)^2$

$$= 3k^2 + k^2 = 4k^2$$

$\Rightarrow$ $BC = 2k$

$$\left.\begin{aligned} \sin B &= \frac{AC}{BC} = \frac{k}{2k} = \frac{1}{2} \\ \cos C &= \frac{AC}{BC} = \frac{k}{2k} = \frac{1}{2} \\ \cos B &= \frac{AB}{BC} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2} \\ \sin C &= \frac{AB}{BC} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2} \end{aligned}\right\} \quad \ldots(1)$$

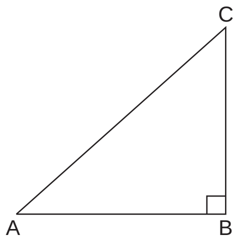
sin B cos C + cos B sin C

$$= \frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} \qquad \text{[Using (1)]}$$

$$= \frac{1}{4} + \frac{3}{4} = 1$$

Hence, sin B cos C + cos B sin C = **1**

20. Draw a right ΔABC, right-angled at B.



Then, $\sin A = \frac{BC}{AC} = \frac{\sqrt{3}}{2}$

Let BC = $\sqrt{3}\,k$. Then, AC = $2k$

In right ΔABC, we have

$$AC^2 = AB^2 + BC^2$$

$$\Rightarrow \quad (2k)^2 = AB^2 + (\sqrt{3}k)^2$$

$$\Rightarrow \quad AB^2 = 4k^2 - 3k^2 = k^2$$

$$\Rightarrow \quad AB = k$$

$$\cot A = \frac{AB}{BC} = \frac{k}{\sqrt{3}k} = \frac{1}{\sqrt{3}}$$

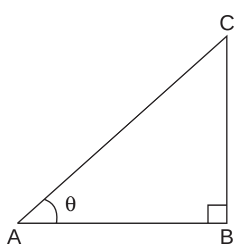
$$2\cot^2 A - 1 = 2\left(\frac{1}{\sqrt{3}}\right)^2 - 1$$

$$= \frac{2}{3} - 1 = \frac{2-3}{3}$$

$$= \frac{-1}{3}$$

Hence, $2\cot^2 A - 1 = \mathbf{\frac{-1}{3}}$

21. Draw a right ΔABC, right-angled at B.



Let, ∠CAB = θ

Then, $\tan\theta = \frac{BC}{AB} = 2 = \frac{2}{1}$

Let BC = $2k$. Then, AB = k.

In right ΔABC, we have

$$AC^2 = AB^2 + BC^2 \quad \text{[By Pythagoras' Theorem]}$$

$$\Rightarrow \quad AC^2 = (k)^2 + (2k)^2 + = 5k^2$$

$$\Rightarrow \quad AC = \sqrt{5}k$$

$$\left.\begin{aligned} \operatorname{cosec}\theta &= \frac{AC}{BC} = \frac{\sqrt{5}k}{2k} = \frac{\sqrt{5}}{2} \\ \sin\theta &= \frac{BC}{AC} = \frac{2k}{\sqrt{5}k} = \frac{2}{\sqrt{5}} \\ \text{and} \quad \sec\theta &= \frac{AC}{AB} = \frac{\sqrt{5}k}{k} = \sqrt{5} \end{aligned}\right\} \quad \ldots (1)$$

$\tan^2\theta - \operatorname{cosec}\theta + \sin\theta\sec\theta$

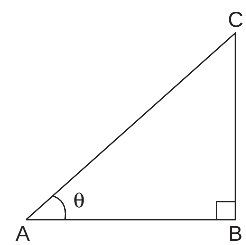
$$= (2)^2 - \frac{\sqrt{5}}{2} + \frac{2}{\sqrt{5}}(\sqrt{5}) \quad \text{[Using (1)]}$$

$$= 4 - \frac{\sqrt{5}}{2} + 2 = \frac{8 - \sqrt{5} + 4}{2}$$

$$= \frac{12 - \sqrt{5}}{2}$$

Hence, $\tan^2\theta - \operatorname{cosec}\theta + \sin\theta\sec\theta = \mathbf{\frac{12 - \sqrt{5}}{2}}$

22. Draw a right ΔABC, right-angled at B.



Let ∠CAB = θ

Then, $\tan\theta = \frac{BC}{AB} = \frac{1}{\sqrt{3}}$

Let BC = k. Then, AB = $\sqrt{3}k$

In right ΔABC, we have

$$AC^2 = AB^2 + BC^2$$

$$= (\sqrt{3}k)^2 + k^2$$

$$= 4k^2$$

$$\Rightarrow \quad AC = 2k$$

$$\sin\theta = \frac{BC}{AC} = \frac{k}{2k} = \frac{1}{2}$$

$$\text{and} \quad \cos\theta = \frac{AB}{AC} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$$

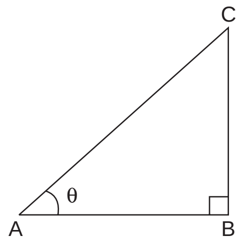
$$7\sin^2\theta + 3\cos^2\theta = 7\left(\frac{1}{4}\right) + 3\left(\frac{3}{4}\right)$$

$$= \frac{7}{4} + \frac{9}{4}$$

$$= \frac{16}{4} = \mathbf{4}$$

Hence, proved.

23. Draw a right ΔABC, right-angled at B.



Let $\angle CAB = \theta$

Then, $\tan\theta = \frac{BC}{AB} = \frac{1}{\sqrt{7}}$

Let BC = k. Then, AB = $\sqrt{7}k$

In right ΔABC, we have

$$AC^2 = AB^2 + BC^2$$
$$= (\sqrt{7}k)^2 + (k^2)$$
$$= 8k^2$$

$\Rightarrow$ $AC = \sqrt{8}k = 2\sqrt{2}k$

$$\text{cosec}\,\theta = \frac{AC}{BC} = \frac{2\sqrt{2}k}{k} = 2\sqrt{2}$$

and $\sec\theta = \frac{AC}{AB} = \frac{2\sqrt{2}k}{\sqrt{7}k} = \frac{2\sqrt{2}}{\sqrt{7}}$

$$\frac{\text{cosec}^2\theta - \sec^2\theta}{\text{cosec}^2\theta + \sec^2\theta} = \frac{(2\sqrt{2})^2 - \left(\frac{2\sqrt{2}}{\sqrt{7}}\right)^2}{(2\sqrt{2})^2 + \left(\frac{2\sqrt{2}}{\sqrt{7}}\right)^2}$$

$$= \frac{8 - \frac{8}{7}}{8 + \frac{8}{7}} = \frac{\frac{56-8}{7}}{\frac{56+8}{7}}$$

$$= \frac{48}{7} \times \frac{7}{64} = \frac{48}{64}$$

$$= \frac{3}{4}$$

Hence, $\frac{\text{cosec}^2\theta - \sec^2\theta}{\text{cosec}^2\theta + \sec^2\theta} = \mathbf{\frac{3}{4}}$

For Standard Level

24. (*i*) By definition: $\sin\theta = \frac{\text{perpendicular}}{\text{hypotenuse}}$

$\therefore$ $\sin\angle PDB = \mathbf{\frac{BP}{BD}}$

(*ii*) By definition: $\sin\theta = \frac{\text{perpendicular}}{\text{hypotenuse}}$

$\therefore$ $\sin\angle DBC = \mathbf{\frac{CD}{CB}}$

(*iii*) By definition: $\sin\theta = \frac{\text{perpendicular}}{\text{hypotenuse}}$

$\therefore$ $\sin\angle ADP = \mathbf{\frac{AP}{AD}}$

(*iv*) By definition: $\cos\theta = \frac{\text{base}}{\text{hypotenuse}}$

$\therefore$ $\cos\angle DBC = \mathbf{\frac{DB}{CB}}$

(*v*) By definition: $\tan\theta = \frac{\text{perpendicular}}{\text{base}}$

$\therefore$ $\tan\angle PDB = \mathbf{\frac{BP}{DP}}$

25. $5\cot\theta = 4$

$\Rightarrow$ $\cot\theta = \frac{4}{5}$

$$\frac{2\sin^2\theta + 3\cos^2\theta}{7\sin\theta\cos\theta} = \frac{2\frac{\sin^2\theta}{\sin^2\theta} + 3\frac{\cos^2\theta}{\sin^2\theta}}{7\frac{\sin\theta\cos\theta}{\sin^2\theta}}$$

[Dividing the num. and denom. by $\sin^2\theta$]

$$= \frac{2 + 3\cot^2\theta}{7\cot\theta} = \frac{2 + 3\left(\frac{4}{5}\right)^2}{7\left(\frac{4}{5}\right)}$$

$$= \frac{2 + 3\left(\frac{16}{25}\right)}{\frac{28}{5}} = \frac{\frac{50+48}{25}}{\frac{28}{5}}$$

$$= \frac{98}{25} \times \frac{5}{28} = \frac{7}{10}$$

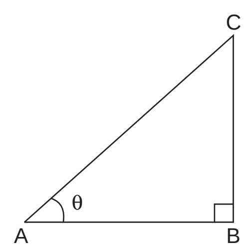
Hence, $\frac{2\sin^2\theta + 3\cos^2\theta}{7\sin\theta\cos\theta} = \mathbf{\frac{7}{10}}$

26. $\sqrt{3}\tan\theta = 3\sin\theta$

$\Rightarrow$ $\sqrt{3}\frac{\sin\theta}{\cos\theta} = 3\sin\theta$

$\Rightarrow$ $\cos\theta = \frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}}$

Draw a right triangle ABC, right-angled at B,



such that $\angle CAB = \theta$

$$\cos\theta = \frac{AB}{AC} = \frac{1}{\sqrt{3}}$$

Let AB = k. Then, AC = $\sqrt{3}k$

In right ΔABC, we have

$$AC^2 = AB^2 + BC^2$$

[By Pythagoras' Theorem]

$\Rightarrow$ $(\sqrt{3}k)^2 = (k)^2 + BC^2$

$$BC^2 = 3k^2 - k^2 = 2k^2$$

$\Rightarrow$ $BC = \sqrt{2}k$

$$\sin\theta = \frac{BC}{AC} = \frac{\sqrt{2}k}{\sqrt{3}k} = \frac{\sqrt{2}}{\sqrt{3}}$$

$$\sin^2\theta - \cos^2\theta = \left(\frac{\sqrt{2}}{\sqrt{3}}\right)^2 - \left(\frac{1}{\sqrt{3}}\right)^2$$

$$= \frac{2}{3} - \frac{1}{3} = \mathbf{\frac{1}{3}}$$

ALTERNATIVE METHOD:

$$\sin^2\theta - \cos^2\theta = (1 - \cos^2\theta) - \cos^2\theta$$

$$[\because \sin^2\theta + \cos^2\theta = 1]$$

$$= 1 - 2\cos^2$$

$$= 1 - 2\left(\frac{1}{\sqrt{3}}\right)^2$$

$$= 1 - \frac{2}{3} = \frac{1}{3}$$

Hence, $\sin^2\theta - \cos^2\theta = \mathbf{\frac{1}{3}}$

27. $$\frac{p\sin\theta - q\cos\theta}{p\sin\theta + q\cos\theta} = \frac{p\frac{\sin\theta}{\cos\theta} - q\frac{\cos\theta}{\cos\theta}}{p\frac{\sin\theta}{\cos\theta} + q\frac{\cos\theta}{\cos\theta}}$$

[Dividing the num. and denom. by $\cos\theta$]

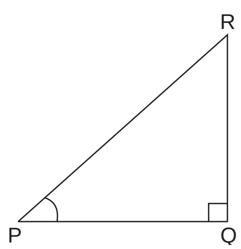
$$= \frac{p\tan\theta - q}{p\tan\theta + q}$$

$$= \frac{p\left(\frac{p}{q}\right) - q}{p\left(\frac{p}{q}\right) + q} = \frac{\frac{p^2 - q^2}{q}}{\frac{p^2 + q^2}{q}}$$

$$= \mathbf{\frac{p^2 - q^2}{p^2 + q^2}}$$

Hence, proved.

28. Draw a right triangle PQR, right-angled at Q.



$$\tan P = \frac{RQ}{PQ} = 1 = \frac{1}{1}$$

Let RQ = k. Then, PQ = k

In right ΔPQR, we have

$$PR^2 = PQ^2 + RQ^2$$

[By Pythagoras' Theorem]

$\Rightarrow$ $PR^2 = k^2 + k^2$

$= 2k^2$

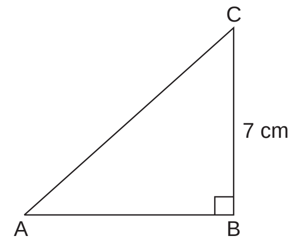
$\Rightarrow$ $PR = \sqrt{2}k$

$$\sin P = \frac{RQ}{PR} = \frac{k}{\sqrt{2}k} = \frac{1}{\sqrt{2}}$$

and $$\cos P = \frac{PQ}{PR} = \frac{k}{\sqrt{2}k} = \frac{1}{\sqrt{2}}$$

$$2\sin P\cos P = 2\ \frac{1}{\sqrt{2}}\ \frac{1}{\sqrt{2}} = \mathbf{1}$$

29. In right ΔABC,



we have $AC^2 = AB^2 + BC^2$

[By Pythagoras' Theorem]

$\Rightarrow$ $(1 + AB)^2 = AB^2 + (7)^2$ $[\because AC - AB = 1 \text{ cm}]$

$\Rightarrow 1 + AB^2 + 2AB = AB^2 + 49$

$\Rightarrow$ $2AB = (49 - 1) = 48$

$\Rightarrow$ $AB = 24$ cm

$AC - AB = 1$ cm

$\Rightarrow$ $AC = (1 + AB)$ cm

$= (1 + 24)$ cm

$= 25$ cm

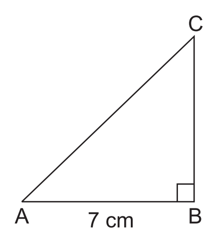
$$\cos A = \frac{AB}{AC} = \frac{24\text{ cm}}{25\text{ cm}} = \frac{24}{25}$$

and $$\sin A = \frac{BC}{AC} = \frac{7\text{ cm}}{25\text{ cm}} = \frac{7}{25}$$

$$\cos A + \sin A = \frac{24}{25} + \frac{7}{25} = \frac{31}{25}$$

Hence, $\cos A + \sin A = \mathbf{\frac{31}{25}}$

30. In right ΔABC,



we have

$$AC^2 = AB^2 + BC^2$$

[By Pythagoras' Theorem]

$\Rightarrow$ $(1 + BC^2) = (7)^2 + BC^2$

$[\because AC - BC = 1 \text{ cm}]$

$\Rightarrow$ $1 + BC^2 + 2BC = 49 + BC^2$

$\Rightarrow$ $2BC = (49 - 1) = 48$

$\Rightarrow$ $BC = 24$ cm

$AC - BC = 1$ cm

$\Rightarrow$ $AC = (1 + BC)$ cm $= (1 + 24)$ cm

$\Rightarrow$ $AC = 25$ cm

$$\sin C = \frac{AB}{AC} = \frac{7\text{ cm}}{25\text{ cm}} = \frac{7}{25}$$

and $$\cos C = \frac{BC}{AC} = \frac{24\text{ cm}}{25\text{ cm}} = \frac{24}{25}$$

Hence, $\sin C = \mathbf{\frac{7}{25}}$ and $\cos C = \mathbf{\frac{24}{25}}$

31. In ΔPQM,

$$\tan x° = \frac{PQ}{MQ}$$

$$\Rightarrow \quad \frac{3}{4} = \frac{15}{MQ}$$

$$\Rightarrow \quad MQ = 20 \text{ cm}$$

In Δ PQR,

$$\tan y° = \frac{PQ}{RQ} = \frac{PQ}{MR + MQ}$$

$$\frac{2}{5} = \frac{15}{MR + 20}$$

$$\Rightarrow \quad 2MR + 40 = 75$$

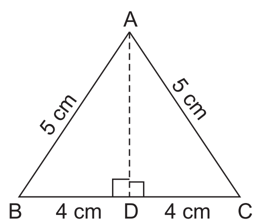
$$\Rightarrow \quad 2MR = 35$$

$$\Rightarrow \quad MR = \frac{35}{2}$$

$$\Rightarrow \quad MR = 17.5 \text{ cm}$$

Hence, MR = **17.5 cm**

32.



Construction: Draw perpendicular bisector AD on BC.

In right ΔADB,

$$\Rightarrow \quad AB^2 = BD^2 + AD^2$$

$$\Rightarrow \quad 5^2 = 4^2 + AD^2$$

$$\Rightarrow \quad 25 = 16 + AD^2$$

$$\Rightarrow \quad AD^2 = 9$$

$$\Rightarrow \quad AD = 3 \text{ cm}$$

(*i*) $\sin B = \frac{AD}{AB} = \frac{3 \text{ cm}}{5 \text{ cm}} = \mathbf{\frac{3}{5}}$

(*ii*) $\tan C = \frac{AD}{DC} = \frac{3 \text{ cm}}{4 \text{ cm}} = \mathbf{\frac{3}{4}}$

(*iii*) $\sin B = \frac{AD}{AB} = \frac{3 \text{ cm}}{5 \text{ cm}} = \frac{3}{5}$

and $\cos C = \frac{DC}{AC} = \frac{4 \text{ cm}}{5 \text{ cm}} = \frac{4}{5}$

$$\Rightarrow \quad \sin^2 B + \cos^2 C = \left(\frac{3}{5}\right)^2 + \left(\frac{4}{5}\right)^2$$

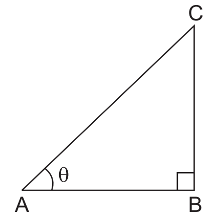
$$= \frac{9}{25} + \frac{16}{25} = \frac{25}{25} = \mathbf{1}$$

(*iv*) $\tan C = \frac{AD}{DC} = \frac{3 \text{ cm}}{4 \text{ cm}} = \frac{3}{4}$

and $\cot B = \frac{BD}{AD} = \frac{4 \text{ cm}}{3 \text{ cm}} = \frac{4}{3}$

$$\Rightarrow \quad \tan C - \cot B = \frac{3}{4} - \frac{4}{3} = \frac{9-16}{12} = \frac{-7}{12}$$

33. In right ΔABC, right-angled at B.



Let $\angle CAB = \theta$

Then, $\tan\theta = \frac{BC}{AB} = \frac{1}{\sqrt{5}}$

Let BC = *k*.

Then, AB = $\sqrt{5}\,k$

In right Δ ABC, we have

$$AC^2 = AB^2 + BC^2$$

$$= \left(\sqrt{5}k\right)^2 + (k^2) = 6k^2$$

$$\Rightarrow \quad AC = \sqrt{6}\,k$$

$$\operatorname{cosec}\theta = \frac{AC}{BC} = \frac{\sqrt{6}k}{k} = \sqrt{6}$$

and $\sec\theta = \frac{AC}{AB} = \frac{\sqrt{6}k}{\sqrt{5}k} = \frac{\sqrt{6}}{\sqrt{5}}$

$$\therefore \quad \frac{\operatorname{cosec}^2\theta - \sec^2\theta}{\operatorname{cosec}^2\theta + \sec^2\theta} = \frac{\left(\sqrt{6}\right)^2 - \left(\frac{\sqrt{6}}{\sqrt{5}}\right)^2}{\left(\sqrt{6}\right)^2 + \left(\frac{\sqrt{6}}{\sqrt{5}}\right)^2}$$

$$= \frac{6 - \frac{6}{5}}{6 + \frac{6}{5}} = \frac{\frac{30-6}{5}}{\frac{30+6}{5}} = \frac{24}{36} = \frac{2}{3}$$

Hence, $\frac{\operatorname{cosec}^2\theta - \sec^2\theta}{\operatorname{cosec}^2\theta + \sec^2\theta} = \mathbf{\frac{2}{3}}$

EXERCISE 8B

For Basic and Standard Levels

1. $2 \sin 30° \cos 30° = 2 \times \frac{1}{2} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}$

Hence, $2 \sin 30° \cos 30° = \mathbf{\frac{\sqrt{3}}{2}}$

2. $\cos 30° \cos 60° - \sin 30° \sin 60° = \frac{\sqrt{3}}{2} \cdot \frac{1}{2} - \frac{1}{2} \cdot \frac{\sqrt{3}}{2}$

$= \mathbf{0}$

3. $\sin 60° \cos 30° + \cos 60° \sin 30° = \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{1}{2} \cdot \frac{1}{2}$

$= \frac{3}{4} + \frac{1}{4}$

$= \mathbf{1}$

4. (*i*) $\cos 30° \cos 45° - \sin 30° \sin 45° = \frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}} - \frac{1}{2} \cdot \frac{1}{\sqrt{2}}$

$= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}$

$$= \left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right) \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \mathbf{\frac{\sqrt{6}-\sqrt{2}}{4}}$$

(*ii*) tan 30° cosec 60° + tan 60° sec 30°

$$= \frac{1}{\sqrt{3}} \cdot \frac{2}{\sqrt{3}} + \sqrt{3} \cdot \frac{2}{\sqrt{3}}$$

$$= \frac{2}{3} + 2 = \mathbf{\frac{8}{3}}$$

5. $2\sqrt{2}\cos 45° \cos 60° - 2\sqrt{2}\sin 45° \sin 60° + \sqrt{3}$

$$= 2\sqrt{2}\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right) - 2\sqrt{2}\left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) + \sqrt{3}$$

$$= 1 - \sqrt{3} + \sqrt{3} = \mathbf{1}$$

6. $\dfrac{\sin 30° - 2\tan 45° + \cos 60°}{\sin 45° \cos 45° + 2\sin 30° \cos 60°}$

$$= \frac{\frac{1}{2} - 2(1) + \frac{1}{2}}{\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right) + 2\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)}$$

$$= \frac{1-2}{\frac{1}{2}+\frac{1}{2}} = \mathbf{-1}$$

7. $\dfrac{\sec 60°}{\cot 45°} - \dfrac{5\sin 90°}{2\cos 0°} + \dfrac{\tan 45°}{\text{cosec}\, 30°} = \dfrac{2}{1} - \dfrac{5(1)}{2(1)} + \dfrac{1}{2}$

$$= \frac{4-5+1}{2}$$

$$= \frac{0}{2} = \mathbf{0}$$

8. $\tan^2 60° + 4\cos^2 45° + \text{cosec}^2\, 30° + 5\cos^2 90°$

$$= (\sqrt{3})^2 + 4\left(\frac{1}{\sqrt{2}}\right)^2 + (2)^2 + 5(0)^2$$

$$= 3 + 2 + 4$$

$$= \mathbf{9}$$

9. $4\cot^2 45° - \sec^2 60° + \sin^2 60° + \cos^2 90°$

$$= 4(1)^2 - (2)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 + (0)^2$$

$$= 4 - 4 + \frac{3}{4}$$

$$= \mathbf{\frac{3}{4}}$$

10. $4(\sin^2 30° + \cos^2 60°) - 3(\cos^2 45° - \frac{1}{2}\sin^2 90°)$

$$= 4\left[\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2\right] - 3\left[\left(\frac{1}{\sqrt{2}}\right)^2 - \frac{1}{2}(1)^2\right]$$

$$= 4\left(\frac{1}{4}+\frac{1}{4}\right) - 3\left(\frac{1}{2}-\frac{1}{2}\right)$$

$$= 4\left(\frac{2}{4}\right) - 3(0) = \mathbf{2}$$

11. $\tan 60° \,\text{cosec}^2\, 45° + \sec^2 60° \tan 45°$

$$= (\sqrt{3})\ (\sqrt{2})^2 + (2)^2\,(1)^2$$

$$= (\sqrt{3})\,(2) + 4$$

$$= \mathbf{4 + 2\sqrt{3}}$$

12. $\cos^2 30° \cos^2 45° + 4\sec^2 60° + \frac{1}{2}\cos^2 90° - 2\tan^2 60°$

$$= \left(\frac{\sqrt{3}}{2}\right)^2 \left(\frac{1}{\sqrt{2}}\right)^2 + 4\,(2)^2 + \frac{1}{2}\,(0)^2 - 2(\sqrt{3})^2$$

$$= \left(\frac{3}{4}\right)\left(\frac{1}{2}\right) + 4(4) + 0 - 2(3)$$

$$= \frac{3}{8} + 16 - 6 = \frac{3}{8} + 10$$

$$= \mathbf{\frac{83}{8}}$$

13. $\dfrac{2\cos^2 90° + 4\cos^2 45° + \tan^2 60° + 3\,\text{cosec}^2\, 60°}{3\sec 60° - \frac{7}{2}\sec^2 45° + 2\,\text{cosec}\, 30°}$

$$= \frac{2(0)^2 + 4\left(\frac{1}{\sqrt{2}}\right)^2 + (\sqrt{3})^2 + 3\left(\frac{2}{\sqrt{3}}\right)^2}{3(2) - \frac{7}{2}(\sqrt{2})^2 + 2(2)}$$

$$= \frac{0 + 4\left(\frac{1}{2}\right) + 3 + 3\left(\frac{4}{3}\right)}{3(2) - \frac{7}{2}(2) + 4}$$

$$= \frac{2+3+4}{6-7+4} = \frac{9}{3}$$

$$= \mathbf{3}$$

14. $\left(\dfrac{1}{\sin 45°} - \sin 45°\right)\left(\dfrac{1}{\cos 45°} - \cos 45°\right)\left(\tan 45° + \dfrac{1}{\tan 45°}\right)$

$$= \left(\frac{1}{\frac{1}{\sqrt{2}}} - \frac{1}{\sqrt{2}}\right)\left(\frac{1}{\frac{1}{\sqrt{2}}} - \frac{1}{\sqrt{2}}\right)\left(1 + \frac{1}{1}\right)$$

$$= \left(\sqrt{2} - \frac{1}{\sqrt{2}}\right)\left(\sqrt{2} - \frac{1}{\sqrt{2}}\right)(2)$$

$$= \left(\frac{2-1}{\sqrt{2}}\right)\left(\frac{2-1}{\sqrt{2}}\right)(2)$$

$$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right)(2)$$

$$= \mathbf{1}$$

15. $\frac{2}{3}(\cos^4 30° - \sin^4 45°) - 3(\sin^2 60° - \sec^2 45°) + \frac{1}{4}\cot^2 30°$

$$= \frac{2}{3}\left[\left(\frac{\sqrt{3}}{2}\right)^4 - \left(\frac{1}{\sqrt{2}}\right)^4\right] - 3\left[\left(\frac{\sqrt{3}}{2}\right)^2 - \left(\sqrt{2}\right)^2\right] + \frac{1}{4}\,(\sqrt{3})^2$$

$$= \frac{2}{3}\left(\frac{9}{16} - \frac{1}{4}\right) - 3\left(\frac{3}{4} - 2\right) + \frac{1}{4}(3)$$

$$= \frac{2}{3}\left(\frac{9-4}{16}\right) - 3\left(\frac{3-8}{4}\right) + \frac{3}{4}$$

$= \frac{2}{3}\left(\frac{5}{16}\right) - 3\left(\frac{-5}{4}\right) + \frac{3}{4}$

$= \frac{5}{24} + \frac{15}{4} + \frac{3}{4} = \frac{5+90+18}{24}$

$= \mathbf{\frac{113}{24}}$

16. $\frac{1}{4}(\cot^4 30° - \text{cosec}^4 60°) + \frac{3}{2}(\sec^2 45° - \tan^2 30°) - 5\cos^2 60°$

$= \frac{1}{4}\left[(\sqrt{3})^4 - \left(\frac{2}{\sqrt{3}}\right)^4\right] + \frac{3}{2}\left[(\sqrt{2})^2 - \left(\frac{1}{\sqrt{3}}\right)^2\right] - 5\left(\frac{1}{2}\right)^2$

$= \frac{1}{4}\left(9 - \frac{16}{9}\right) + \frac{3}{2}\left(2 - \frac{1}{3}\right) - 5\left(\frac{1}{4}\right)$

$= \frac{1}{4}\left(\frac{81-16}{9}\right) + \frac{3}{2}\left(\frac{6-1}{3}\right) - \frac{5}{4}$

$= \frac{1}{4}\left(\frac{65}{9}\right) + \frac{3}{2}\left(\frac{5}{3}\right) - \frac{5}{4}$

$= \frac{65}{36} + \frac{5}{2} - \frac{5}{4} = \frac{65+90-45}{36}$

$= \frac{110}{36} = \mathbf{\frac{55}{18}}$

17. (*i*) $\sin 30° \cos 60° \text{ cosec } 30° \sec 60°$

$= \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)(2)(2) = 1 = \mathbf{\sin 90°}$

(*ii*) $\frac{\tan 60° - \tan 30°}{1 + \tan 60° \tan 30°} = \frac{\sqrt{3} - \frac{1}{\sqrt{3}}}{1 + (\sqrt{3})\left(\frac{1}{\sqrt{3}}\right)}$

$= \frac{\frac{3-1}{\sqrt{3}}}{1+1} = \frac{\frac{2}{\sqrt{3}}}{2}$

$= \frac{2}{\sqrt{3}} \times \frac{1}{2} = \frac{1}{\sqrt{3}}$

$= \mathbf{\tan 30°}$

(*iii*) $\frac{1 + 2\sin 60° \cos 60°}{\sin 60° + \cos 60°} + \frac{1 + 2\sin 60° \cos 60°}{\sin 60° - \cos 60°}$

$= \frac{1 + 2\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right)}{\frac{\sqrt{3}}{2} + \frac{1}{2}} + \frac{1 - 2\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right)}{\frac{\sqrt{3}}{2} - \frac{1}{2}}$

$= \frac{1 + \frac{\sqrt{3}}{2}}{\frac{\sqrt{3}+1}{2}} + \frac{1 - \frac{\sqrt{3}}{2}}{\frac{\sqrt{3}-1}{2}}$

$= \frac{(2+\sqrt{3})}{2} \times \frac{2}{(\sqrt{3}+1)} + \frac{(2-\sqrt{3})}{2} \times \frac{2}{(\sqrt{3}-1)}$

$= \frac{2+\sqrt{3}}{\sqrt{3}+1} + \frac{2-\sqrt{3}}{\sqrt{3}-1}$

$= \frac{(2+\sqrt{3})(\sqrt{3}-1) + (2-\sqrt{3})(\sqrt{3}+1)}{(\sqrt{3}+1)(\sqrt{3}-1)}$

$= \frac{2\sqrt{3} - 2 + 3 - \sqrt{3} + 2\sqrt{3} + 2 - 3 - \sqrt{3}}{3-1}$

$= \frac{2\sqrt{3}}{2} = \sqrt{3}$

$= \mathbf{\cot 30°}$

(*iv*) $\left(\frac{\tan 60° + 1}{\tan 60° - 1}\right)^2 = \left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)^2$

$= \frac{3 + 2\sqrt{3} + 1}{3 - 2\sqrt{3} + 1}$

$= \frac{4 + 2\sqrt{3}}{4 - 2\sqrt{3}} = \frac{2(2+\sqrt{3})}{2(2-\sqrt{3})}$

$= \frac{2+\sqrt{3}}{2-\sqrt{3}}$

$\frac{1 + \cos 30°}{1 - \cos 30°} = \frac{1 + \frac{\sqrt{3}}{2}}{1 - \frac{\sqrt{3}}{2}} = \frac{\frac{2+\sqrt{3}}{2}}{\frac{2-\sqrt{3}}{2}}$

$= \frac{2+\sqrt{3}}{2} \times \frac{2}{2-\sqrt{3}}$

$= \frac{2+\sqrt{3}}{2-\sqrt{3}}$

Hence, $\mathbf{\left(\frac{\tan 60° + 1}{\tan 60° - 1}\right)^2 = \frac{1 + \cos 30°}{1 - \cos 30°}}$

$\left[\text{Each is equal to } \frac{2+\sqrt{3}}{2-\sqrt{3}}\right]$

(*v*) $(\sin^2 45° - \tan^2 45°)^2 + 3(\sin^2 90° + \tan^2 30°)$

$= \left[\left(\frac{1}{\sqrt{2}}\right)^2 - (1)^2\right]^2 + 3\left[(1)^2 + \left(\frac{1}{\sqrt{3}}\right)^2\right]$

$= \left(\frac{1}{2} - 1\right)^2 + 3\left(1 + \frac{1}{3}\right)$

$= \left(-\frac{1}{2}\right)^2 + 3\left(\frac{4}{3}\right)$

$= \frac{1}{4} + 4 = \frac{1+16}{4}$

$= \mathbf{\frac{17}{4}}$

(*vi*) $4(\sin^4 30° + \cos^4 60°) - 3(\cos^2 45° - \sin^2 90°)$

$= 4\left[\left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^4\right] - 3\left[\left(\frac{1}{\sqrt{2}}\right)^2 - (1)^2\right]$

$= 4\left(\frac{1}{16} + \frac{1}{16}\right) - 3\left(\frac{1}{2} - 1\right)$

$= 4\left(\frac{2}{16}\right) - 3\left(-\frac{1}{2}\right)$

$= \frac{1}{2} + \frac{3}{2} = \frac{4}{2}$

$= \mathbf{2}$

(*vii*) $\left(\sqrt{3}+1\right)(3 - \cot 30°) = \left(\sqrt{3}+1\right)\left(3-\sqrt{3}\right)$

$= 3\sqrt{3}-3+3-\sqrt{3}$

$= 2\sqrt{3}$

$\tan^3 60° - 2\sin 60° = \left(\sqrt{3}\right)^3 - 2\left(\frac{\sqrt{3}}{2}\right)$

$= 3\sqrt{3} - \sqrt{3}$

$= 2\sqrt{3}$

Hence, $\mathbf{\left(\sqrt{3}+1\right)\left(3-\cot 30°\right) = \tan^3 60° - 2\sin 60°}$

[Each is equal to $2\sqrt{3}$]

18. (*i*) $\cos 2\theta = \cos 2(30°) = \cos 60° = \frac{1}{2}$

$\frac{1-\tan^2\theta}{1+\tan^2\theta} = \frac{1-\tan^2 30°}{1+\tan^2 30°}$

$= \frac{1-\left(\frac{1}{\sqrt{3}}\right)^2}{1+\left(\frac{1}{\sqrt{3}}\right)^2} = \frac{1-\frac{1}{3}}{1+\frac{1}{3}}$

$= \frac{\frac{3-1}{3}}{\frac{3+1}{3}} = \frac{\frac{2}{3}}{\frac{4}{3}}$

$= \frac{2}{4} = \frac{1}{2}$

Hence, $\mathbf{\cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}}$

[When $\theta = 30°$, then each is equal to $\frac{1}{2}$]

(*ii*) $\cos 3\theta = \cos 3(30°) = \cos 90° = 0$

$4\cos^3\theta - 3\cos\theta = 4\cos^3 30° - 3\cos 30°$

$= 4\left(\frac{\sqrt{3}}{2}\right)^3 - 3\left(\frac{\sqrt{3}}{2}\right)$

$= 4\left(\frac{3\sqrt{3}}{8}\right) - \frac{3\sqrt{3}}{2}$

$= \frac{3\sqrt{3}}{2} - \frac{3\sqrt{3}}{2}$

$= 0$

Hence, $\mathbf{\cos 3\theta = 4\cos^3\theta - 3\cos\theta}$

[When $\theta = 30°$, then each is equal to 0]

(*iii*) (*a*) $\sin 2\theta = \sin 2(30°) = \sin 60° = \frac{\sqrt{3}}{2}$

$2\sin\theta\cos\theta = 2\sin 30° \cos 30°$

$= 2\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$

$= \frac{\sqrt{3}}{2}$

Hence, $\mathbf{\sin 2\theta = 2\sin 30° \cos 30°}$

[When $\theta = 30°$, then each is equal to $\frac{\sqrt{3}}{2}$]

(*b*) $\cos 2\theta = \cos 2(30°) = \cos 60° = \frac{1}{2}$

$2\cos^2\theta - 1 = 2\cos^2 30° - 1$

$= 2\left(\frac{\sqrt{3}}{2}\right)^2 - 1 = 2\left(\frac{3}{4}\right) - 1$

$= \frac{3}{2} - 1 = \frac{1}{2}$

$1 - 2\sin^2\theta = 1 - 2\sin^2 30°$

$= 1 - 2\left(\frac{1}{2}\right)^2$

$= 1 - \frac{2}{4} = 1 - \frac{1}{2}$

$= \frac{1}{2}$

Hence, $\mathbf{\cos 2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta}$

[When $\theta = 30°$, then each is equal to $\frac{1}{2}$]

(*c*) $\tan 2\theta = \tan 2(30°) = \tan 60° = \sqrt{3}$

$\frac{2\tan\theta}{1-\tan^2\theta} = \frac{2\tan 30°}{1-\tan^2 30°}$

$= \frac{2\left(\frac{1}{\sqrt{3}}\right)}{1-\left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{1-\frac{1}{3}}$

$= \frac{\frac{2}{\sqrt{3}}}{\frac{2}{3}} = \frac{2}{\sqrt{3}} \times \frac{3}{2}$

$= \sqrt{3}$

Hence, $\mathbf{\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}}$

[When $\theta = 30°$, then each is equal to $\sqrt{3}$]

(*iv*) $\sin(60° + \theta) - \sin(60° - \theta)$

$= \sin(60° + 30°) - \sin(60° - 30°)$

$= \sin 90° - \sin 30°$

$= 1 - \frac{1}{2} = \frac{1}{2}$

$\sin\theta = \sin 30° = \frac{1}{2}$

Hence, $\mathbf{\sin(60° + \theta) - \sin(60° - \theta) = \sin\theta}$

[When $\theta = 30°$, then each is equal to $\frac{1}{2}$]

19. (*i*) $\cos 2\theta = \cos 2(45°) = \cos 90° = 0$

$1 - 2\sin^2\theta = 1 - 2\sin^2 45°$

$= 1 - 2 \times \left(\frac{1}{\sqrt{2}}\right)^2$

$= 1 - 2 \times \frac{1}{2}$

$= 1 - 1 = 0$

Hence, $\mathbf{\cos 2\theta = 1 - 2\sin^2\theta}$

[When $\theta = 45°$, then each is equal to 0] (*ii*) $\sin\theta = \sin 45° = \frac{1}{\sqrt{2}}$

$$\sqrt{\frac{1-\cos 2\theta}{2}} = \sqrt{\frac{1-\cos 2(45°)}{2}}$$
$$= \sqrt{\frac{1-\cos 90°}{2}}$$
$$= \sqrt{\frac{1-0}{2}} = \sqrt{\frac{1}{2}}$$
$$= \frac{1}{\sqrt{2}}$$

Hence, $\mathbf{\sin\theta = \sqrt{\frac{1-\cos 2\theta}{2}}}$

[When $\theta = 45°$, then each is equal to $\frac{1}{\sqrt{2}}$]

20. $\cos A = \sqrt{\frac{1+\cos 2A}{2}}$

$$\Rightarrow \quad \cos 30° = \sqrt{\frac{1+\cos 2(30°)}{2}}$$
$$\Rightarrow \quad \cos 30° = \sqrt{\frac{1+\cos 60°}{2}}$$
$$= \sqrt{\frac{1+\frac{1}{2}}{2}} \qquad \left[\because \cos 60° = \frac{1}{2}\right]$$
$$= \sqrt{\frac{\frac{2+1}{2}}{2}} = \sqrt{\frac{\frac{3}{2}}{2}}$$
$$= \sqrt{\frac{3}{2}\times\frac{1}{2}} = \sqrt{\frac{3}{4}}$$
$$= \frac{\sqrt{3}}{2}$$

Hence, $\cos 30° = \mathbf{\frac{\sqrt{3}}{2}}$

21. (*i*) $\sin(A-B) = \sin(60° - 30°) = \sin 30° = \frac{1}{2}$

$\sin A \cos B - \cos A \sin B$
$$= \sin 60° \cos 30° - \cos 60° \sin 30°$$
$$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$$
$$= \frac{3}{4} - \frac{1}{4} = \frac{3-1}{4} = \frac{2}{4} = \frac{1}{2}$$

Hence, $\mathbf{\sin(A-B) = \sin A \cos B - \cos A \sin B}$

[When A = 60°, B = 30°, then each is equal to $\frac{1}{2}$]

(*ii*) $\cos(A+B) = \cos(60° + 30°) = \cos 90° = 0$

$\cos A \cos B - \sin A \sin B$
$$= \cos 60° \cos 30° - \sin 60° \sin 30°$$
$$= \left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right) = 0$$

Hence, $\mathbf{\cos(A+B) = \cos A \cos B - \sin A \sin B}$

[When A = 60°, B = 30°, then each is equal to 0]

(*iii*) $\frac{\sin(A+B)}{\cos A \cos B} = \frac{\sin(60° + 30°)}{\cos 60° \cos 30°}$

$$= \frac{\sin 90°}{\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)}$$
$$= \frac{1}{\frac{\sqrt{3}}{4}} = \frac{4}{\sqrt{3}}$$

$\tan A + \tan B = \tan 60° + \tan 30°$
$$= \sqrt{3} + \frac{1}{\sqrt{3}}$$
$$= \frac{3+1}{\sqrt{3}} = \frac{4}{\sqrt{3}}$$

Hence, $\mathbf{\frac{\sin(A+B)}{\cos A \cos B} = \tan A + \tan B}$

[When A = 60°, B = 30°, then each is equal to $\frac{4}{\sqrt{3}}$]

(*iv*) $\frac{\sin(A-B)}{\sin A \sin B} = \frac{\sin(60° - 30°)}{\sin 60° \sin 30°}$

$$= \frac{\sin 30°}{\sin 60° \sin 30°}$$
$$= \frac{1}{\sin 60°} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}}$$

$\cot B - \cot A = \cot 30° - \cot 60°$
$$= \sqrt{3} - \frac{1}{\sqrt{3}} = \frac{3-1}{\sqrt{3}}$$
$$= \frac{2}{\sqrt{3}}$$

Hence, $\mathbf{\frac{\sin(A-B)}{\sin A \sin B} = \cot B - \cot A}$

[When A = 60°, B = 30°, then each is equal to $\frac{2}{\sqrt{3}}$]

(*v*) $\tan(A-B) = \tan(60° - 30°) = \tan 30° = \frac{1}{\sqrt{3}}$

$$\frac{\tan A - \tan B}{1+\tan A \tan B} = \frac{\tan 60° - \tan 30°}{1+\tan 60° \tan 30°}$$
$$= \frac{\sqrt{3} - \frac{1}{\sqrt{3}}}{1+(\sqrt{3})\left(\frac{1}{\sqrt{3}}\right)}$$
$$= \frac{\frac{3-1}{\sqrt{3}}}{1+1} = \frac{\frac{2}{\sqrt{3}}}{2}$$
$$= \frac{2}{\sqrt{3}} \times \frac{1}{2} = \frac{1}{\sqrt{3}}$$

Hence, $\mathbf{\tan(A-B) = \frac{\tan A - \tan B}{1+\tan A \tan B}}$

[When A = 60°, B = 30°, then each is equal to $\frac{1}{\sqrt{3}}$]

22. $\sin(A + B) = \sin(45° + 45°) = \sin 90° = 1$

$\sin A \cos B + \cos A \sin B$

$= \sin 45° \cos 45° + \cos 45° \sin 45°$

$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right)$

$= \frac{1}{2} + \frac{1}{2}$

$= 1$

Hence, **sin (A + B) = sin A cos B + cos A sin B**

[When A = B = 45°, then each is equal to 1]

23. Let $A = 45°$

and $B = 30°$

$\sin(A + B) = \sin A \cos B + \cos A \sin B$

$\sin 75° = \sin(45° + 30°)$

$= \sin 45° \cos 30° + \cos 45° \sin 30°$

$= \frac{1}{\sqrt{2}}\left(\frac{\sqrt{3}}{2}\right) + \frac{1}{\sqrt{2}}\left(\frac{1}{2}\right)$

$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} = \frac{\sqrt{3}+1}{2\sqrt{2}}$

Hence, $\sin 75° = \mathbf{\frac{\sqrt{3}+1}{2\sqrt{2}}}$

24. Let $A = 45°$

and $B = 30°$

$\sin(A - B) = \sin A \cos B - \cos A \sin B$

$\sin 15° = \sin(45° - 30°)$

$= \sin 45° \cos 30° - \cos 45° \sin 30°$

$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$

$= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}$

$= \frac{\sqrt{3}-1}{2\sqrt{2}}$

Hence, $\sin 15° = \mathbf{\frac{\sqrt{3}-1}{2\sqrt{2}}}$

25. Let $A = 45°$

and $B = 30°$

$\cos(A + B) = \cos A \cos B - \sin A \sin B$

$\therefore \quad \cos 75° = \cos(45° + 30°)$

$= \cos 45° \cos 30° - \sin 45° \sin 30°$

$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$

$= \frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}} = \frac{\sqrt{3}-1}{2\sqrt{2}}$

Hence, $\cos 75° = \mathbf{\frac{\sqrt{3}-1}{2\sqrt{2}}}$

26. Let $A = 45°$

and $B = 30°$

$\cos(A - B) = \cos A \cos B + \sin A \sin B$

$\therefore \quad \cos 15° = \cos(45° - 30°)$

$= \cos 45° \cos 30° + \sin 45° \sin 30°$

$= \left(\frac{1}{\sqrt{2}}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{2}\right)$

$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} = \mathbf{\frac{\sqrt{3}+1}{2\sqrt{2}}}$

Hence, $\cos 15° = \mathbf{\frac{\sqrt{3}+1}{2\sqrt{2}}}$

27. Let $A = 45°$

and $B = 30°$

$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

$\therefore \quad \tan 75° = \tan(45° + 30°)$

$= \frac{\tan 45° + \tan 30°}{1 - \tan 45° \tan 30°}$

$= \frac{1 + \frac{1}{\sqrt{3}}}{1 - (1)\left(\frac{1}{\sqrt{3}}\right)}$

$= \frac{\frac{\sqrt{3}+1}{\sqrt{3}}}{\frac{\sqrt{3}-1}{\sqrt{3}}}$

$= \frac{\sqrt{3}+1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}-1}$

$= \frac{\sqrt{3}+1}{\sqrt{3}-1} = \frac{(\sqrt{3}+1)(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$

$= \frac{3 + 2\sqrt{3} + 1}{(3-1)} = \frac{4 + 2\sqrt{3}}{2}$

$= \frac{2(2+\sqrt{3})}{2} = \mathbf{2 + \sqrt{3}}$

28. (*i*) $\frac{\cos 3A - 2\cos 4A}{\sin 3A + 2\sin 4A} = \frac{\cos 3(15°) - 2\cos 4(15°)}{\sin 3(15°) + 2\sin 4(15°)}$

[Putting A = 15°]

$= \frac{\cos 45° - 2\cos 60°}{\sin 45° + 2\sin 60°}$

$= \frac{\left(\frac{1}{\sqrt{2}}\right) - 2\left(\frac{1}{2}\right)}{\left(\frac{1}{\sqrt{2}}\right) + 2\left(\frac{\sqrt{3}}{2}\right)}$

$= \frac{\frac{1}{\sqrt{2}} - 1}{\frac{1}{\sqrt{2}} + \sqrt{3}} = \frac{\frac{1-\sqrt{2}}{\sqrt{2}}}{\frac{1+\sqrt{6}}{\sqrt{2}}}$

$= \frac{1-\sqrt{2}}{\sqrt{2}} \times \frac{\sqrt{2}}{1+\sqrt{6}}$

$= \frac{1-\sqrt{2}}{1+\sqrt{6}}$

Hence, $\frac{\cos 3A - 2\cos 4A}{\sin 3A + 2\sin 4A} = \mathbf{\frac{1-\sqrt{2}}{1+\sqrt{6}}}$ [When A = 15°]

(*ii*) $\dfrac{3\sin 3B + 2\cos(2B + 5°)}{2\cos 3B - \sin(2B - 10°)}$

$$= \frac{3\sin(3 \times 20°) + 2\cos(2 \times 20° + 5°)}{2\cos(3 \times 20°) - \sin(2 \times 20° - 10°)}$$

[Putting B = 20°]

$$= \frac{3\sin 60° + 2\cos 45°}{2\cos 60° - \sin 30°}$$

$$= \frac{3\left(\frac{\sqrt{3}}{2}\right) + 2\left(\frac{1}{\sqrt{2}}\right)}{2\left(\frac{1}{2}\right) - \left(\frac{1}{2}\right)}$$

$$= \frac{\frac{3\sqrt{3}}{2} + \frac{2}{\sqrt{2}}}{1 - \frac{1}{2}}$$

$$= \frac{\frac{3(\sqrt{3})(\sqrt{2}) + 2(2)}{2\sqrt{2}}}{\frac{2-1}{2}}$$

$$= \frac{\frac{3\sqrt{6} + 4}{2\sqrt{2}}}{\frac{1}{2}}$$

$$= \frac{3\sqrt{6} + 4}{2\sqrt{2}} \times 2$$

$$= \frac{(3\sqrt{6} + 4)}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{3\sqrt{12} + 4\sqrt{2}}{2}$$

$$= \frac{(3)(2)\sqrt{3} + 4\sqrt{2}}{2}$$

$$= 3\sqrt{3} + 2\sqrt{2}$$

Hence, $\dfrac{3\sin 3B + 2\cos(2B + 5°)}{2\cos 3B - \sin(2B - 10°)} = \mathbf{3\sqrt{3} + 2\sqrt{2}}$

[When B = 20°]

29. $(\sin A \cos B + \cos A \sin B)^2 + (\cos A \cos B - \sin A \sin B)^2$
$= (\sin 60° \cos 30° + \cos 60° \sin 30°)^2 + (\cos 60° \cos 30° - \sin 60° \sin 30°)$

[Putting A = 60°, B = 30°]

$$= \left[\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\right]^2 + \left[\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right)\right]^2$$

$$= \left(\frac{3}{4} + \frac{1}{4}\right)^2 + \left(\frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4}\right)^2$$

$= (1)^2 + (0)^2 = \mathbf{1}$

30. $\tan\theta = 1$
$\Rightarrow \tan\theta = \tan 45°$
$\Rightarrow \theta = 45°$
$\sin\phi = \dfrac{1}{\sqrt{2}}$
$\Rightarrow \sin\phi = \sin 45°$
$\Rightarrow \phi = 45°$
$\cos(\theta + \phi) = \cos(45° + 45°)$
$= \cos 90° = \mathbf{0}$

31. $\sin(A - B) = \dfrac{1}{2}$
$\Rightarrow \sin(A - B) = \sin 30°$
$\Rightarrow A - B = 30°$ … (1)
$\cos(A + B) = \dfrac{1}{2}$
$\Rightarrow \cos(A + B) = \cos 60°$
$\Rightarrow A + B = 60°$ … (2)
Adding (1) and (2), we get
$2A = 90°$
$\Rightarrow A = 45°$
Substituting A = 45° in (1), we get B = 15°
Hence, **A = 45°, B = 15°**

32. $\sin(A + B) = 1$
$\Rightarrow \sin(A + B) = \sin 90°$
$\Rightarrow A + B = 90°$ … (1)
$\cos(A - B) = 1$
$\Rightarrow \cos(A - B) = \cos 0°$
$\Rightarrow A - B = 0°$ … (2)
Adding (1) and (2), we get
$2A = 90°$
$\Rightarrow A = 45°$
Substituting A = 45° in (1), we get
$B = 45°$
Hence, **A = 45°, B = 45°**

33. $\sin(A + B) = 1$
$\Rightarrow \sin(A + B) = \sin 90°$
$\Rightarrow A + B = 90°$ … (1)
$\cos(A - B) = \dfrac{\sqrt{3}}{2}$
$\Rightarrow \cos(A - B) = \cos 30°$
$\Rightarrow A - B = 30°$ … (2)
Adding (1) and (2), we get
$2A = 120°$
$\Rightarrow A = 60°$
Substituting A = 60° in (1), we get
$B = 30°$
Hence, **A = 60°, B = 30°**

34. $\tan(A + B) = \sqrt{3}$
$\Rightarrow \tan(A + B) = \tan 60°$
$\Rightarrow A + B = 60°$ … (1)
$\tan(A - B) = 1$
$\Rightarrow \tan(A - B) = \tan 45°$
$\Rightarrow A - B = 45°$ … (2)
Adding (1) and (2), we get
$2A = 105°$
$\Rightarrow A = 52.5°$
Substituting A = 52.5° in (1), we get
$B = 7.5°$
Hence, **A = 52.5°, B = 7.5°**

35. (*i*) $\tan(A - B) = \dfrac{1}{\sqrt{3}}$
$\Rightarrow \tan(A - B) = \tan 30°$
$\Rightarrow A - B = 30°$ … (1)

$$\sin (A + B) = \frac{\sqrt{3}}{2}$$

$\Rightarrow \quad \sin (A + B) = \sin 60°$

$\Rightarrow \quad A + B = 60° \quad \ldots (2)$

Adding (1) and (2), we get

$2A = 90°$

$\Rightarrow \quad A = 45°$

Substituting A = 45° in (1), we get

$B = 15°$

Hence, **A = 45°, B = 15°**

(*ii*) $\operatorname{cosec} (A - B) = 2$

$\Rightarrow \quad \operatorname{cosec} (A - B) = \operatorname{cosec} 30°$

$\Rightarrow \quad A - B = 30° \quad \ldots (1)$

$$\cot (A + B) = \frac{1}{\sqrt{3}}$$

$\Rightarrow \quad \cot (A + B) = \cot 60°$

$\Rightarrow \quad A + B = 60° \quad \ldots (2)$

Adding (1) and (2), we get

$2A = 90°$

$\Rightarrow \quad A = 45°$

Substituting A = 45° in (1), we get

$B = 15°$

Hence, **A = 45°, B = 15°**

36. $$\sin (A + 2B) = \frac{\sqrt{3}}{2}$$

$\Rightarrow \quad \sin (A + 2B) = \sin 60°$

$\Rightarrow \quad A + 2B = 60° \quad \ldots (1)$

$\cos (A + 4B) = 0$

$\Rightarrow \quad \cos (A + 4B) = \cos 90°$

$\Rightarrow \quad A + 4B = 90° \quad \ldots (2)$

Subtracting (1) from (2), we get

$2B = 30°$

$\Rightarrow \quad B = 15°$

Substituting B = 15° in (1), we get

$A + 2(15°) = 60°$

$\Rightarrow \quad A = 30°$

Hence, **A = 30°, B = 15°**

37. $$\tan (A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$= \frac{\frac{1}{2} + \frac{1}{3}}{1 - \left(\frac{1}{2}\right)\left(\frac{1}{3}\right)}$$

$$= \frac{\frac{3+2}{6}}{1 - \frac{1}{6}}$$

$$= \frac{\frac{5}{6}}{\frac{5}{6}} = 1 = \tan 45°$$

$\Rightarrow \quad A + B = 45°$

Hence, **A + B = 45°**

38. (*i*) $2\cos 2A = 1$

$$\Rightarrow \quad \cos 2A = \frac{1}{2}$$

$\Rightarrow \quad \cos 2A = \cos 60°$

$\Rightarrow \quad 2A = 60°$

$\Rightarrow \quad A = 30°$

Hence, **A = 30°**

(*ii*) $\tan 3A = 1$

$\Rightarrow \quad \tan 3A = \tan 45°$

$\Rightarrow \quad 3A = 45°$

$$\Rightarrow \quad A = \frac{45°}{3}$$

$\Rightarrow \quad A = 15°$

Hence, **A = 15°**

(*iii*) $\sqrt{3} \cot 2A = 1$

$$\Rightarrow \quad \cot 2A = \frac{1}{\sqrt{3}}$$

$\Rightarrow \quad \cot 2A = \cot 60°$

$\Rightarrow \quad 2A = 60°$

$\Rightarrow \quad A = 30°$

Hence, **A = 30°**

(*iv*) $4 \cos^2 A - 1 = 0$

$\Rightarrow \quad 4 \cos^2 A = 1$

$$\Rightarrow \quad \cos^2 A = \frac{1}{4}$$

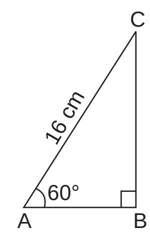
$$\Rightarrow \quad \cos A = \frac{1}{2} \quad \text{(Rejecting the negative value)}$$

$\Rightarrow \quad \cos A = \cos 60°$

$\Rightarrow \quad A = 60°$

Hence, **A = 60°**

39. In right ΔABC, we have



$$\cos 60° = \frac{AB}{AC}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{AB}{16\,cm}$$

$\Rightarrow \quad AB = 8$ cm

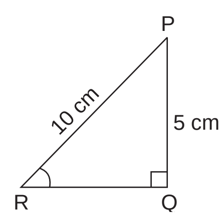
$$\text{Also,} \quad \sin 60° = \frac{BC}{AC}$$

$$\Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{BC}{16\text{ cm}}$$

$\Rightarrow \quad BC = 8\sqrt{3}$ cm

Hence, **AB = 8 cm, BC = 8$\sqrt{3}$ cm**

40. In right ΔPQR, we have



$$\cos \angle QPR = \frac{PQ}{PR} = \frac{5 \text{ cm}}{10 \text{ cm}}$$

$$= \frac{1}{2} = \cos 60°$$

$\Rightarrow \quad \angle QPR = 60°$

$$\sin \angle PRQ = \frac{PQ}{PR} = \frac{5 \text{ cm}}{10 \text{ cm}}$$

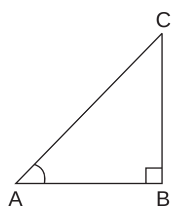
$$= \frac{1}{2} = \sin 30°$$

$\Rightarrow \quad \angle PRQ = 30°$

Hence, **$\angle$QPR = 60° and $\angle$PRQ = 30°**

For Standard Level

41. Draw a right ΔABC, right-angled at B.



$$\sin A = \frac{BC}{AC} = \frac{1}{\sqrt{5}}$$

Let BC = k. Then, AC = $\sqrt{5}\,k$

In right ΔABC, we have

$$AC^2 = AB^2 + BC^2$$

[By Pythagoras' Theorem]

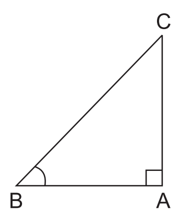
$\Rightarrow \quad (\sqrt{5}k)^2 = AB^2 + (k)^2$

$\Rightarrow \quad AB^2 = 5k^2 - k^2 = 4k^2$

$\Rightarrow \quad AB = 2k$

$$\cos A = \frac{AB}{AC} = \frac{2k}{\sqrt{5}k} = \mathbf{\frac{2}{\sqrt{5}}}$$

Draw a right ΔABC, right-angled at A.



$$\sin B = \frac{AC}{BC} = \frac{1}{\sqrt{10}}$$

Let AC = k. Then, BC = $\sqrt{10}\,k$

In right ΔBAC, we have

$$BC^2 = AB^2 + AC^2$$

[By Pythagoras' Theorem]

$\Rightarrow \quad (\sqrt{10}\,k)^2 = AB^2 + k^2$

$\Rightarrow \quad AB^2 = 10k^2 - k^2 = 9k^2$

$\Rightarrow \quad AB = 3k$

$$\cos B = \frac{AB}{BC} = \frac{3k}{\sqrt{10}k} = \mathbf{\frac{3}{\sqrt{10}}}$$

$$\cos (A + B) = \cos A \cos B - \sin A \sin B$$

$$= \left(\frac{2}{\sqrt{5}}\right)\left(\frac{3}{\sqrt{10}}\right) - \left(\frac{1}{\sqrt{5}}\right)\left(\frac{1}{\sqrt{10}}\right)$$

$$= \frac{6}{\sqrt{50}} - \frac{1}{\sqrt{50}} = \frac{5}{\sqrt{50}}$$

$$= \frac{5}{5\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$= \cos 45°$$

$\Rightarrow \quad A + B = 45°$

Hence, **$\cos A = \frac{2}{\sqrt{5}}$, $\cos B = \frac{3}{\sqrt{10}}$ and A + B = 45°**

42. $6x = \sec \theta$

$\Rightarrow \quad x = \frac{1}{6} \sec \theta$

$\Rightarrow \quad x^2 = \frac{1}{36} \sec^2 \theta$

$\frac{6}{x} = \tan \theta$

$\Rightarrow \quad \frac{1}{x} = \frac{1}{6} \tan \theta$

$\Rightarrow \quad \frac{1}{x^2} = \frac{1}{36} \tan^2 \theta$

Now, $$9\left(x^2 - \frac{1}{x^2}\right) = 9\left(\frac{1}{36}\sec^2\theta - \frac{1}{36}\tan^2\theta\right)$$

$$= \frac{9}{36}\left(\sec^2\theta - \tan^2\theta\right)$$

$$= \frac{1}{4}(1) \quad [\because \sec^2\theta - \tan^2\theta = 1]$$

Hence, the value of $9\left(x^2 - \frac{1}{x^2}\right)$ is $\frac{1}{4}$.

43. (*i*) (sin A – 1) (2 cos A – 1) = 0

$\Rightarrow$ sin A – 1 = 0 or 2 cos A – 1 = 0

$\Rightarrow$ sin A = 1 or 2 cos A = 1

$\Rightarrow \cos A = \frac{1}{2}$

$\Rightarrow$ sin A = sin 90° or cos A = cos 60°

$\Rightarrow$ A = 90° or A = 60°

Hence, the value of A is 90° or 60°.

(*ii*) (sec 2A – 1) (cosec 3A – 1) = 0

$\Rightarrow$ sec 2A = 1 or cosec 3A = 1

$\Rightarrow$ sec 2A = sec 0° or cosec 3A = cosec 90°

$\Rightarrow$ 2A = 0° or 3A = 90°

$\Rightarrow$ A = 0° or A = 30°

Hence, the value of A is 0° or 30°.

(*iii*) cos 3A (2 sin 2A – 1) = 0

$\Rightarrow$ cos 3 A = 0 or 2 sin 2A – 1 = 0

$\Rightarrow \sin 2A = \frac{1}{2}$

$\Rightarrow$ cos 3A = cos 90° or sin 2A = sin 30°

$\Rightarrow$ 3A = 90° or 2A = 30°

$\Rightarrow \quad A = 30° \quad \text{or} \quad A = 15°$

Hence, the value of A is 30° or 15°.

(*iv*) (cosec 2A – 2) (cot 3A – 1) = 0

$\Rightarrow$ cosec 2A – 2 = 0 or cot 3A – 1 = 0

$\Rightarrow$ cosec 2A = 2 or cot 3A = 1

$\Rightarrow$ cosec 2A = cosec 30° or cot 3A = cot 45°

$\Rightarrow$ 2A = 30° or 3A = 45°

$\Rightarrow$ A = 15° or A = 15°

Hence, the value of A is 15°.

44. (*i*) tan A – 2 cos A tan A + 2 cos A – 1 = 0

$\Rightarrow$ (tan A – 1) – 2 cos A (tan A – 1) = 0

$\Rightarrow$ (tan A – 1) (1 – 2 cos A) = 0

$\Rightarrow$ Either tan A – 1 = 0 or 1 – 2 cos A = 0

$\Rightarrow$ tan A = 1 or 2 cos A = 1

$\Rightarrow \cos A = \frac{1}{2}$

$\Rightarrow$ tan A = tan 45° or cos A = cos 60°

$\Rightarrow$ A = 45° or A = 60°

Hence, the value of A is 45° or 60°.

(*ii*) 4 sin A sin 2A + 1 – 2 sin 2A = 2 sin A

$\Rightarrow$ 4 sin A sin 2A + 1 – 2 sin 2A – 2 sin A = 0

$\Rightarrow$ 2 sin 2A (2 sin A – 1) – 1(2 sin A – 1) = 0

$\Rightarrow$ (2 sin A – 1) (2 sin 2A – 1) = 0

$\Rightarrow$ Either 2 sin A – 1 = 0 or 2 sin 2A – 1 = 0

$\Rightarrow \sin A = \frac{1}{2}$ or $\sin 2A = \frac{1}{2}$

$\Rightarrow$ sin A = sin 30° or sin 2A = sin 30°

$\Rightarrow$ A = 30° or 2A = 30°

$\Rightarrow$ A = 15°

Hence, the value of A is 30° or 15°.

(*iii*) 2 tan 3A cos 3A – tan 3A + 1 = 2 cos 3A

$\Rightarrow$ 2 tan 3A cos 3A – tan 3A + 1 – 2 cos 3A = 0

$\Rightarrow$ tan 3A (2 cos 3A – 1) – 1(2 cos 3A – 1) = 0

$\Rightarrow$ (2 cos 3A – 1) (tan 3A – 1) = 0

$\Rightarrow$ Either 2 cos 3A – 1 = 0 or tan 3A – 1 = 0

$\Rightarrow \cos 3A = \frac{1}{2}$ or tan 3A = 1

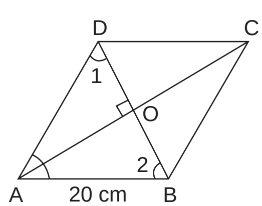
$\Rightarrow$ cos 3A = cos 60° or tan 3A = tan 45°

$\Rightarrow$ 3A = 60° or 3A = 45°

$\Rightarrow$ A = 20° or A = 15°

Hence, the value of A is 20° or 15°.

45. Join diagonals AC and BD and let them intersect at O.



In ΔABD, AB = AD [Sides of a rhombus]

$\therefore$ ∠1 = ∠2 [∠s opp. equal sides of a Δ] … (1)

∠1 + ∠2 + ∠A = 180° [Sum of ∠s of a Δ]

$\Rightarrow$ ∠1 + ∠2 + 60° = 180°

$\Rightarrow$ ∠1 + ∠2 = 120° … (2)

From (1) and (2), we get

∠1 = ∠2 = 60°

$\Rightarrow$ ∠ADO = ∠ABO

= 60° … (3)

We know that the diagonals of a rhombus bisect each other at right angles.

$\therefore \quad OD = OB = \frac{BD}{2}$

and $\quad OC = OA = \frac{AC}{2}$

and ∠AOB = 90° … (4)

In ΔAOB,

$$\sin(\angle ABO) = \frac{OA}{AB}$$

$\Rightarrow \sin 60° = \frac{OA}{20 \text{ cm}}$ [Using (3)]

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{OA}{20 \text{ cm}}$

$\Rightarrow OA = 10\sqrt{3}$ cm

$\Rightarrow AC = 20\sqrt{3}$ cm [Using (4)]

Also, $\cos(\angle ABO) = \frac{OB}{AB}$

$\Rightarrow \cos 60° = \frac{OB}{20 \text{ cm}}$ [Using (3)]

$\Rightarrow \frac{1}{2} = \frac{OB}{20 \text{ cm}}$

$\Rightarrow$ OB = 10 cm

$\Rightarrow$ BD = 20 cm [Using (4)]

Hence, the lengths of the diagonal of the given rhombus are **$20\sqrt{3}$ cm and 20 cm**.

CHECK YOUR UNDERSTANDING

MULTIPLE-CHOICE QUESTIONS

For Basic and Standard Levels

1. (*c*) **sec 90°**

$$\sec 90° = \frac{1}{\cos 90°} = \frac{1}{0} \text{ (not defined)}$$

$\therefore$ sec 90° is not defined.

2. (*a*) **1**

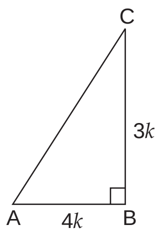
The maximum value of $\frac{1}{\operatorname{cosec}\theta}$, i.e sin θ where 0° ≤ θ ≤ 90° is 1.

∵ sin θ increases from 0 to 1 as θ increases from 0° to 90° and sin 90° = 1.

3. (*d*) $\mathbf{\frac{4}{5}}$

In right ΔABC (as shown in the figure)

$$\tan A = \frac{BC}{AB} = \frac{3}{4}$$



Let BC = $3k$.
Then AB = $4k$

$$AC^2 = AB^2 + BC^2$$
$$= (4k)^2 + (3k)^2$$
$$= 25k^2$$

⇒ $$AC = 5k$$

$$\cos A = \frac{AB}{AC} = \frac{4k}{5k} = \frac{4}{5}$$

4. (*c*) $\mathbf{\frac{3}{5}}$

Refer to figure (Q 3) in which $\sin A = \frac{BC}{AC} = \frac{3}{5}$.

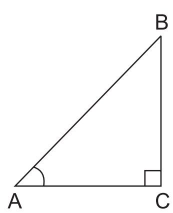
$$\cos C = \frac{BC}{AC} = \frac{3k}{5k}$$

[AC = $5k$, proved in Q 3]

$$= \frac{3}{5}$$

5. (*b*) $\mathbf{\frac{3}{4}}$

In ΔABC, A + B = 90°



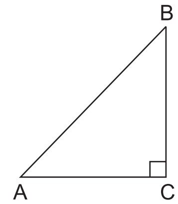
∴ ∠C = 90° [Sum of ∠s of a Δ is 180°]

$$\cot B = \frac{BC}{AC} = \frac{3}{4}$$

Let BC = $3k$
Then, AC = $4k$.

$$\tan A = \frac{BC}{AC} = \frac{3k}{4k} = \frac{3}{4}$$

6. (*c*) $\mathbf{\frac{2}{\sqrt{3}}}$



In ΔABC, ∠A + ∠B = 90°

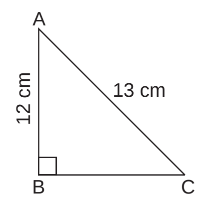
⇒ ∠C = 90°[Sum of ∠s of a Δ is 180°]

$$\sec A = \frac{AB}{AC} = \frac{2}{\sqrt{3}}$$

Let AB = $2k$,
Then, AC = $\sqrt{3}\,k$

$$\text{cosec } B = \frac{AC}{AB} = \frac{2k}{\sqrt{3}k} = \frac{2}{\sqrt{3}}$$

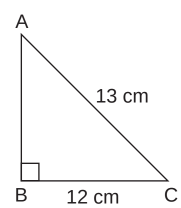
7. (*d*) **0**



In right ΔABC, we have

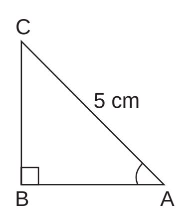
$$\tan A - \cot C = \frac{BC}{AB} - \frac{BC}{AB} = 0$$

8. (*a*) $\mathbf{\frac{13}{12}}$



$$\sec C = \frac{AC}{BC} = \frac{13\,\text{cm}}{12\,\text{cm}} = \frac{13}{12}$$

9. (*a*) **4 cm**

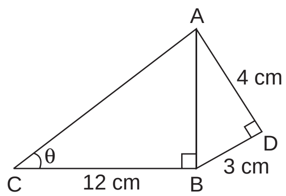


$$\tan A = \frac{BC}{AB} = \frac{4}{3}.$$

Let BC = $4k$. Then, AB = $3k$.

$$AC^2 = BC^2 + AB^2$$
$$= (4k)^2 + (3k)^2$$
$$= 25k^2$$
$\Rightarrow$ $$AC = 5k$$
But $$AC = 5 \text{ cm} \quad \{\text{Given}\}$$
$\therefore$ $$k = 1 \text{ cm}$$
$\Rightarrow$ $$BC = 4 \times 1 \text{ cm}$$
$$= 4 \text{ cm}$$

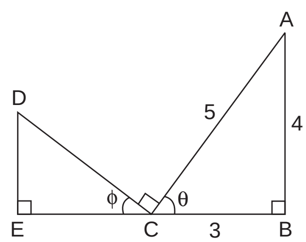
10. (*a*) $\mathbf{\frac{12}{5}}$



In right ΔADB,

$$AB^2 = AD^2 + BD^2$$
$$= (4 \text{ cm})^2 + (3 \text{ cm})^2$$
$$= 25 \text{ cm}^2$$
$\Rightarrow$ $$AB = 5 \text{ cm}$$
$$\cot\theta = \frac{BC}{AB} = \frac{12\text{cm}}{5\text{cm}} = \frac{12}{5}$$

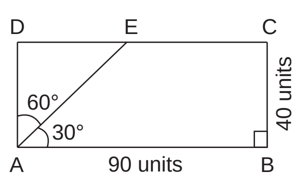
11. (*d*) $\mathbf{\frac{4}{5}}$



ECB is a straight angle

$\Rightarrow$ $$\phi + 90° + \theta = 180°$$
$\Rightarrow$ $$\phi + \theta = 90° \quad \ldots (1)$$
In ΔACB
$$\angle A + \theta + 90° = 180°$$
$\Rightarrow$ $$\angle A + \theta = 90° \quad \ldots (2)$$
From (1) and (2), we get
$$\phi = \angle A$$
$\therefore$ $$\cos\phi = \cos A = \frac{AB}{AC} = \frac{4}{5}$$

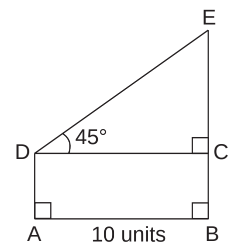
12. (*a*) **80 units**



$$AD = BC$$
$$= 40 \text{ units}$$
[Opp. sides of a rectangle]

In right ΔADE, $$\cos 60° = \frac{AD}{AE}$$
$\Rightarrow$ $$\frac{1}{2} = \frac{40 \text{ units}}{AE}$$
$\Rightarrow$ $$AE = 80 \text{ units}$$

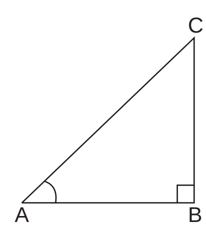
13. (*c*) **24.1 units**



$$DC = AB = 10 \text{ units}$$
[Opp. sides of a rectangle]

In right ΔDCE, we have
$$\tan 45° = \frac{EC}{DC}$$
$\Rightarrow$ $$1 = \frac{EC}{10 \text{ units}}$$
$\Rightarrow$ $$EC = 10 \text{ units}$$
Also, $$DE^2 = EC^2 + DC^2$$
$$= (10)^2 + (10)^2$$
$$= 200$$
$\Rightarrow$ $$DE = 10\sqrt{2} \text{ units}$$
$$CE + DE = 10 + 10\sqrt{2}$$
$$= 10(1 + \sqrt{2})$$
$$= 10(1 + 1.41)$$
$$= 10(2.41)$$
$$= 24.1 \text{ units}$$

14. (*b*) **greater than 1**



$$\sin A + \cos A = \frac{BC}{AC} + \frac{AB}{AC}$$
$$= \frac{BC + AB}{AC}$$

Since BC + AB > AC

$\therefore$ $$\frac{BC + AB}{AC} > 1$$

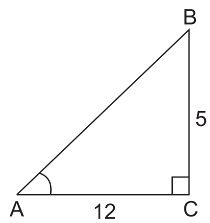
$\therefore$ sin A + cos A is greater than 1.

15. (*b*) $\mathbf{\frac{-3}{2}}$

$$\cos\theta = \frac{1}{2}$$
$$\sec\theta = \frac{1}{\cos\theta} = \frac{2}{1}$$

$$(\cos\theta - \sec\theta) = \frac{1}{2} - 2 = \frac{1-4}{2} = \frac{-3}{2}$$

16. (*b*) $\mathbf{\frac{17}{5}}$



Let ACB is right Δ in which ∠C = 90°,

and $\cot A = \frac{12}{5}$

$\Rightarrow \quad \frac{AC}{BC} = \frac{12}{5}$

In right ΔACB, we have

$$AB^2 = AC^2 + BC^2 = (12)^2 + (5)^2 = 144 + 25 = 169$$

$\Rightarrow \quad AB = 13$ units

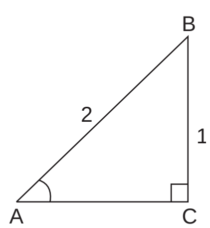
$$\sin A = \frac{BC}{AB} = \frac{5}{13}$$

$$\cos A = \frac{AC}{AB} = \frac{12}{13}$$

and $\operatorname{cosec} A = \frac{AB}{BC} = \frac{13}{5}$

$$(\sin A + \cos A) \times \operatorname{cosec} A = \left(\frac{5}{13} + \frac{12}{13}\right) \times \frac{13}{5} = \frac{17}{5}$$

17. (*c*) **2**



Let ABC be a right triangle in which ∠C = 90°,

and $\operatorname{cosec} A = 2$

$\Rightarrow \quad \frac{AB}{BC} = 2 = \frac{2}{1}$

In right ΔACB, we have

$AB^2 = BC^2 + AC^2$

$\Rightarrow \quad (2)^2 = (1)^2 + AC^2$

$\Rightarrow \quad AC^2 = 4 - 1 = 3$

$\Rightarrow \quad AC = \sqrt{3}$

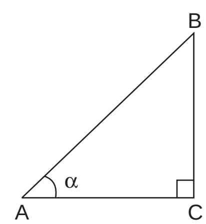
$$\sin A = \frac{BC}{AB} = \frac{1}{2}$$

$$\cos A = \frac{AC}{AB} = \frac{\sqrt{3}}{2}$$

and $\cot A = \frac{AC}{BC} = \frac{\sqrt{3}}{1}$

$$\cot A + \frac{\sin A}{1+\cos A} = \sqrt{3} + \frac{\frac{1}{2}}{1+\frac{\sqrt{3}}{2}} = \sqrt{3} + \frac{\frac{1}{2}}{\frac{2+\sqrt{3}}{2}} = \sqrt{3} + \frac{1}{2+\sqrt{3}} = \frac{2\sqrt{3}+3+1}{2+\sqrt{3}} = \frac{4+2\sqrt{3}}{2+\sqrt{3}} = \frac{2(2+\sqrt{3})}{(2+\sqrt{3})} = 2$$

18. (*c*) $\mathbf{\frac{1}{7}}$



Let ΔACB be a right Δ in which

∠C = 90°,

∠BAC = α,

such that $\sec\alpha = \frac{5}{4}$

$\Rightarrow \quad \frac{AB}{AC} = \frac{5}{4}$

Let AB = 5*k*. Then, AC = 4*k*

In right ΔACB, we have

$AB^2 = AC^2 + BC^2$

$\Rightarrow \quad (5k)^2 = (4k)^2 + BC^2$

$\Rightarrow \quad BC^2 = (25 - 16)\, k^2 = 9k^2$

$\Rightarrow \quad BC = 3k$

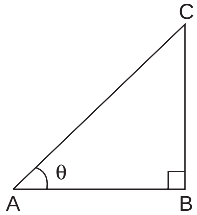
$$\tan\alpha = \frac{BC}{AC} = \frac{3k}{4k} = \frac{3}{4}$$

$$\frac{1-\tan\alpha}{1+\tan\alpha} = \frac{1-\frac{3}{4}}{1+\frac{3}{4}}$$

$$= \frac{\frac{4-3}{4}}{\frac{4+3}{4}} = \frac{\frac{1}{4}}{\frac{7}{4}}$$

$$= \frac{1}{7}$$

19. (*a*) $\mathbf{\frac{5}{4}}$



Let ABC be a right Δ in which ∠B = 90°,
∠CAB = θ,

such that $\sec\theta = \frac{3}{2}$

$$\Rightarrow \quad \frac{AC}{AB} = \frac{3}{2}$$

Let AC = 3*k*. Then, AB = 2*k*

In right ΔABC, we have

$$AC^2 = AB^2 + BC^2$$

$$\Rightarrow \quad (3k)^2 = (2k)^2 + BC^2$$

$$\Rightarrow \quad BC^2 = (9-4)\,k^2 = 5k^2$$

$$\Rightarrow \quad BC = \sqrt{5}\,k$$

$$\tan\theta = \frac{BC}{AB} = \frac{\sqrt{5}k}{2k} = \frac{\sqrt{5}}{2}$$

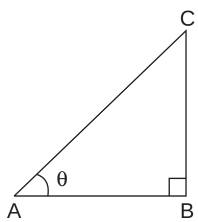
$$\tan^2\theta = \left(\frac{\sqrt{5}}{2}\right)^2 = \frac{5}{4}$$

20. (*a*) $\mathbf{\frac{2}{3}}$

Let ABC be a right Δ in which ∠B = 90°, ∠CAB = θ, such that $\cot\theta = \sqrt{5}$

$$\Rightarrow \quad \frac{AB}{BC} = \sqrt{5} = \frac{\sqrt{5}}{1}$$

Let AB = $\sqrt{5}\,k$. Then, BC = *k*



In right ΔABC, we have

$$AC^2 = AB^2 + BC^2 = (\sqrt{5}\,k)^2 + (k)^2 = 5k^2 + k^2 = 6k^2$$

$$\Rightarrow \quad AC = \sqrt{6}\,k$$

$$\operatorname{cosec}\theta = \frac{AC}{BC} = \frac{\sqrt{6}k}{k} = \sqrt{6}$$

and $$\sec\theta = \frac{AC}{AB} = \frac{\sqrt{6}k}{\sqrt{5}k} = \frac{\sqrt{6}}{\sqrt{5}}$$

$$\frac{\operatorname{cosec}^2\theta - \sec^2\theta}{\operatorname{cosec}^2\theta + \sec^2\theta} = \frac{(6) - \left(\frac{6}{5}\right)}{(6) + \left(\frac{6}{5}\right)} = \frac{\frac{30-6}{5}}{\frac{30+6}{5}} = \frac{24}{5} \times \frac{5}{36} = \frac{2}{3}$$

21. (*d*) **1**

$$\frac{\tan 45^\circ}{\sin 30^\circ + \cos 60^\circ} = \frac{1}{\left(\frac{1}{2} + \frac{1}{2}\right)} = \frac{1}{1} = 1$$

22. (*c*) **3**

$$\operatorname{cosec} 30^\circ + \cot 45^\circ = 2 + 1 = 3$$

23. (*a*) $\mathbf{-\frac{1}{2}}$

$$\sin^2 30^\circ - \cos^2 30^\circ = \left(\frac{1}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{1}{4} - \frac{3}{4} = \frac{1-3}{4} = \frac{-2}{4} = -\frac{1}{2}$$

24. (*c*) **0**

In ΔABC,

$$\angle A + \angle B + \angle C = 180^\circ$$

$$\Rightarrow \quad \angle A + 90^\circ + \angle C = 180^\circ$$

$$\Rightarrow \quad \angle A + \angle C = 90^\circ$$

$$\cos(A + C) = \cos 90^\circ = 0$$

25. (*c*) $\mathbf{\frac{1}{2}}$

$$\frac{1 - \tan^2 30^\circ}{1 + \tan^2 30^\circ} = \frac{1 - \left(\frac{1}{\sqrt{3}}\right)^2}{1 + \left(\frac{1}{\sqrt{3}}\right)^2} = \frac{1 - \frac{1}{3}}{1 + \frac{1}{3}} = \frac{\frac{3-1}{3}}{\frac{3+1}{3}} = \frac{2}{3} \times \frac{3}{4} = \frac{1}{2}$$

26. (*c*) **90°**

$$\sin A = \frac{1}{2}$$

$\Rightarrow \quad \sin A = \sin 30°$

$\Rightarrow \quad A = 30°$

$\cos B = \frac{1}{2}$

$\Rightarrow \quad \cos B = \cos 60°$

$\Rightarrow \quad B = 60°$

$A + B = 30° + 60°$

$= 90°$

27. (*b*) **45°**

$\sin 2A = 1$

$\Rightarrow \quad \sin 2A = \sin 90°$

$\Rightarrow \quad 2A = 90°$

$\Rightarrow \quad A = 45°$

28. (*d*) **20°**

$2 \cos 3A = 1$

$\Rightarrow \quad \cos 3A = \frac{1}{2}$

$\Rightarrow \quad \cos 3A = \cos 60°$

$\Rightarrow \quad 3A = 60°$

$\Rightarrow \quad A = 20°$

29. (*a*) **15°**

$\tan 3\theta = \sin 30° + \cos 45° \sin 45°$

$= \frac{1}{2} + \left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right)$

$= \frac{1}{2} + \frac{1}{2}$

$= 1$

$= \tan 45°$

$\Rightarrow \quad 3\theta = 45°$

$\Rightarrow \quad \theta = 15°$

30. (*b*) **1**

$\cot 2\theta = \frac{1}{\sqrt{3}}$

$\Rightarrow \quad \cot \theta = \cot 60°$

$\Rightarrow \quad 2\theta = 60°$

$\Rightarrow \quad \theta = 30°$

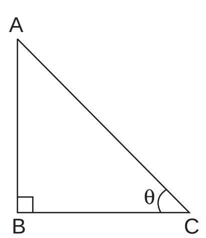
$\sin 3\theta = \sin 3 \times 30°$

$= \sin 90°$

$= 1$

For Standard Level

31. (*d*) **1**



Let ABC be a right triangle in which $\angle B = 90°$

Let $\angle ACB = \theta$, such that $\text{cosec } \theta = \frac{2}{1}$

$\Rightarrow \quad \frac{AC}{AB} = \frac{2}{1}$

Let $AC = 2k$. Then, $AB = k$

In right ΔABC, we have

$AC^2 = AB^2 + BC^2$

$\Rightarrow \quad (2k)^2 = (k)^2 + BC^2$

$\Rightarrow \quad BC^2 = 4k^2 - k^2 = 3k^2$

$\Rightarrow \quad BC = \sqrt{3}\, k$

$\cot \theta = \frac{BC}{AB} = \frac{\sqrt{3}k}{k} = \sqrt{3} \quad \ldots (1)$

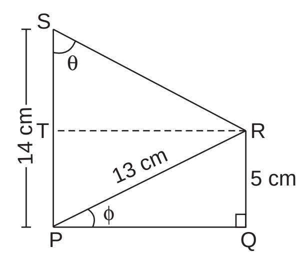
It is given $\quad \cot \theta = \sqrt{3}\, p \quad \ldots (2)$

From (1) and (2), we get

$\sqrt{3} = \sqrt{3}\, p$

$\Rightarrow \quad p = 1$

32. (*b*) $\mathbf{\frac{4}{3}}$



In ΔPQR, $\quad PR^2 = RQ^2 + PQ^2$

$\Rightarrow \quad (13 \text{ cm})^2 = (5 \text{ cm})^2 + PQ^2$

$\Rightarrow \quad PQ^2 = (169 - 25) \text{ cm}^2 = 144 \text{ cm}^2$

$\Rightarrow \quad PQ = 12 \text{ cm}$

$TR = PQ = 12 \text{ cm}$

and $\quad TP = RQ = 5 \text{ cm}$

[Opp. sides of a rectangle]

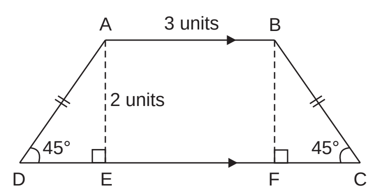
$ST = PS - TP$

$= 14 \text{ cm} - 5 \text{ cm}$

$= 9 \text{ cm}$

$\tan \theta = \frac{TR}{ST} = \frac{12 \text{ cm}}{9 \text{ cm}} = \frac{4}{3}$

33. (*c*) **15.64 units**



In right ΔAED

$\tan 45° = \frac{AE}{DE}$

$\Rightarrow \quad 1 = \frac{AE}{DE}$

$\Rightarrow \quad DE = AE = 2 \text{ units} \quad \ldots (1)$

Also $\quad AD^2 = AE^2 + DE^2$
$= (2)^2 + (2)^2$
$= 8$

$\Rightarrow \quad AD = 2\sqrt{2}$ units

$BC = AD = 2\sqrt{2}$ units $\quad \dots (2)$

In right ΔBFC, $\cos 45° = \dfrac{FC}{BC}$

$\Rightarrow \quad \dfrac{1}{\sqrt{2}} = \dfrac{FC}{2\sqrt{2}}$

$\Rightarrow \quad FC = 2$ units $\quad \dots (3)$

Perimeter of ABCD

$= AB + BC + CF + EF + ED + DA$

$= (3 + 2\sqrt{2} + 2 + 3 + 2 + 2\sqrt{2})$ units

[Using (1), (2) and (3)]

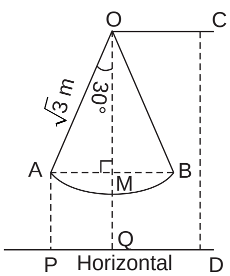
$= (10 + 4\sqrt{2})$ units

$= [10 + (4 \times 1.41)]$ units

$= (10 + 5.64)$ units

$= 15.64$ units

34. (*d*) **0.8 m**



In right ΔOMA,

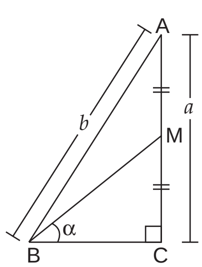
$\cos 30° = \dfrac{OM}{OA}$

$\Rightarrow \quad \dfrac{\sqrt{3}}{2} = \dfrac{OM}{\sqrt{3}}$

$\Rightarrow \quad OM = \dfrac{3}{2} = 1.5$ m

$AP = MQ$
$= CD - OM$
$= (2.3 - 1.5)$ m
$= 0.8$ m

35. (*b*) $\mathbf{\dfrac{5a^2 - 4b^2}{4b^2 - 3a^2}}$



In right ΔACB, $\quad b^2 = a^2 + BC^2$

$\Rightarrow \quad BC^2 = b^2 - a^2 \quad \dots (1)$

In right ΔBCM,

$BM^2 = CM^2 + BC^2$

$\Rightarrow \quad BM^2 = \left(\dfrac{a}{2}\right)^2 + b^2 - a^2 \quad$ [Using (1)]

$\Rightarrow \quad BM^2 = \dfrac{a^2}{4} + b^2 - a^2$

$= \dfrac{a^2 + 4b^2 - 4a^2}{4}$

$= \dfrac{4b^2 - 3a^2}{4} \quad \dots (2)$

$\sin^2 \alpha - \cos^2 \alpha = \left(\dfrac{CM}{BM}\right)^2 - \left(\dfrac{BC}{BM}\right)^2$

$= \dfrac{CM^2}{BM^2} - \dfrac{BC^2}{BM^2}$

$= \dfrac{\left(\frac{a}{2}\right)^2}{\frac{4b^2 - 3a^2}{4}} - \dfrac{b^2 - a^2}{\frac{4b^2 - 3a^2}{4}}$

[Using (1) and (2)]

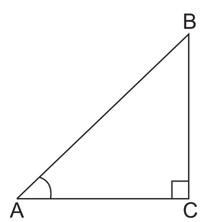
$= \dfrac{a^2}{4} \times \dfrac{4}{(4b^2 - 3a^2)} - \dfrac{b^2 - a^2}{(4b^2 - 3a^2)} \times \dfrac{4}{1}$

$= \dfrac{a^2}{(4b^2 - 3a^2)} - \dfrac{4(b^2 - a^2)}{(4b^2 - 3a^2)}$

$= \dfrac{a^2 - 4b^2 + 4a^2}{(4b^2 - 3a^2)}$

$= \dfrac{5a^2 - 4b^2}{4b^2 - 3a^2}$

36. (*a*) **1**



$\tan A = \dfrac{BC}{AC} = 1$

$\Rightarrow \quad BC = AC = k$ (*say*)

In right ΔACB,

$AB^2 = AC^2 + BC^2$
$= k^2 + k^2$
$= 2k^2$

$\Rightarrow \quad AB = \sqrt{2}\,k$

$\sin A = \dfrac{BC}{AB}$

$= \dfrac{k}{\sqrt{2}k} = \dfrac{1}{\sqrt{2}}$

and $\quad \cos A = \dfrac{AC}{AB} = \dfrac{k}{\sqrt{2}k} = \dfrac{1}{\sqrt{2}}$

$$2\sin A\cos A = 2\left(\frac{BC}{AB}\right)\left(\frac{AC}{AB}\right)$$

$$= 2\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right) = 1$$

37. (*a*) $\mathbf{\frac{1}{3}}$

$$\frac{5\sin\theta - 2\cos\theta}{5\sin\theta + 2\cos\theta} = \frac{5\frac{\sin\theta}{\cos\theta} - 2\frac{\cos\theta}{\cos\theta}}{5\frac{\sin\theta}{\cos\theta} + 2\frac{\cos\theta}{\cos\theta}}$$

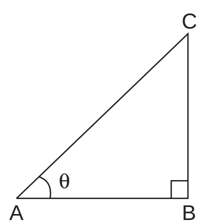
$$= \frac{5\tan\theta - 2}{5\tan\theta + 2}$$

$$= \frac{5\left(\frac{4}{5}\right) - 2}{5\left(\frac{4}{5}\right) + 2}$$

$$= \frac{4-2}{4+2} = \frac{2}{6}$$

$$= \frac{1}{3}$$

38. (*d*) **5**



Let ABC be a right triangle in which ∠B = 90°, ∠CAB = θ, such that $\sin\theta = \frac{1}{5}$

$$\Rightarrow \quad \frac{BC}{AC} = \frac{1}{5}$$

Let BC = k. Then, AC = $5k$

In right ΔABC, we have

$$AC^2 = AB^2 + BC^2$$

$$\Rightarrow \quad (5k)^2 = AB^2 + (k)^2$$

$$\Rightarrow \quad AB^2 = (25-1)k^2$$

$$\Rightarrow \quad AB^2 = 24k^2$$

$$\Rightarrow \quad AB = \sqrt{24}k = 2\sqrt{6}\,k$$

$$\cot\theta = \frac{AB}{BC} = \frac{2\sqrt{6}k}{k} = 2\sqrt{6}$$

$$\frac{1}{5}\cot^2\theta + \frac{1}{5} = +\frac{1}{5}(2\sqrt{6})^2 + \frac{1}{5}$$

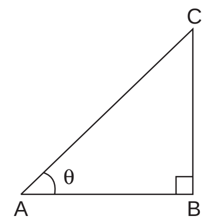
$$= \frac{1}{5}(4\times 6) + \frac{1}{5}$$

$$= \frac{24}{5} + \frac{1}{5}$$

$$= \frac{25}{5} = 5$$

39. (*b*) **0**

Let ABC be a right triangle in which ∠B = 90°, ∠CAB = θ, such that $\cos\theta = \frac{2}{3}$



$$\Rightarrow \quad \frac{AB}{AC} = \frac{2}{3}$$

Let AB = $2k$. Then, AC = $3k$

In right ΔABC, we have

$$AC^2 = AB^2 + BC^2$$

$$\Rightarrow \quad (3k)^2 = (2k)^2 + BC^2$$

$$\Rightarrow \quad BC^2 = (9-4)k^2$$

$$\Rightarrow \quad BC^2 = 5k^2$$

$$\Rightarrow \quad BC = \sqrt{5}\,k$$

$$\sec\theta = \frac{AC}{AB} = \frac{3k}{2k} = \frac{3}{2}$$

and $$\tan\theta = \frac{BC}{AB} = \frac{\sqrt{5}k}{2k} = \frac{\sqrt{5}}{2}$$

$$2\sec^2\theta + 2\tan^2\theta - 7 = 2\left(\frac{3}{2}\right)^2 + 2\left(\frac{\sqrt{5}}{2}\right)^2 - 7$$

$$= 2\left(\frac{9}{4}\right) + 2\left(\frac{5}{4}\right) - 7$$

$$= \frac{9}{2} + \frac{5}{2} - 7$$

$$= \frac{14}{2} - 7$$

$$= 7 - 7$$

$$= 0$$

40. (*b*) $\mathbf{\frac{7}{4}}$

(sin 90° – cos 45° + cos 60°) (cos 0° + sin 45° + sin 30°)

$$= \left(1 - \frac{1}{\sqrt{2}} + \frac{1}{2}\right)\left(1 + \frac{1}{\sqrt{2}} + \frac{1}{2}\right)$$

$$= \left(1 + \frac{1}{2} - \frac{1}{\sqrt{2}}\right)\left(1 + \frac{1}{2} + \frac{1}{\sqrt{2}}\right)$$

$$= \left(\frac{3}{2} - \frac{1}{\sqrt{2}}\right)\left(\frac{3}{2} + \frac{1}{\sqrt{2}}\right)$$

$$= \left(\frac{3}{2}\right)^2 - \left(\frac{1}{\sqrt{2}}\right)^2$$

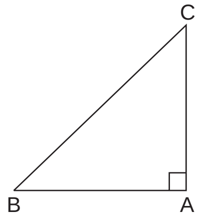
$$= \frac{9}{4} - \frac{1}{2} = \frac{9-2}{4}$$

$$= \frac{7}{4}$$

MATCH THE FOLLOWING

For Basic and Standard Levels

1. (*d*) **0**



$$\angle A + (\angle B + \angle C) = 180^\circ$$
$$\Rightarrow \quad \angle A = 180^\circ - (\angle B + \angle C)$$
$$\angle A = 180^\circ - 90^\circ$$
$$\Rightarrow \quad \angle A = 90^\circ$$
$$\therefore \quad \cos A = 0$$

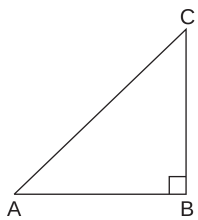
2. (*b*) **3**

cosec 30° + tan 45° = 2 + 1 = 3

3. (*a*) $\mathbf{\frac{3}{40}}$

Draw a right ΔABC in which ∠B = 90°

and $\quad \sin A = \frac{4}{5}$



Let AC = 5*k*. Then, BC = 4*k*.
In right ΔABC, we have

$$AC^2 = AB^2 + BC^2$$
$$\Rightarrow \quad (5k)^2 = AB^2 + (4k)^2$$
$$\Rightarrow \quad AB^2 = (25 - 16)k^2 = 9k^2$$
$$\Rightarrow \quad AB = 3k$$
$$\cos A = \frac{AB}{AC} = \frac{3k}{5k} = \frac{3}{5}$$

and
$$\tan A = \frac{BC}{AB} = \frac{4k}{3k} = \frac{4}{3}$$

$$\frac{\sin A - \cos A}{2 \tan A} = \frac{\frac{4}{5} - \frac{3}{5}}{2\left(\frac{4}{3}\right)}$$
$$= \frac{\frac{1}{5}}{\frac{8}{3}} = \frac{1}{5} \times \frac{3}{8}$$
$$= \frac{3}{40}$$

4. (*e*) $\mathbf{\frac{17}{4}}$

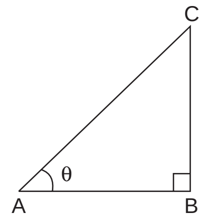
$3 \sin^2 30^\circ + 2 \tan^2 60^\circ - 5 \cos^2 45^\circ$

$$= 3\left(\frac{1}{2}\right)^2 + 2(\sqrt{3})^2 - 5\left(\frac{1}{\sqrt{2}}\right)^2$$
$$= \frac{3}{4} + 6 - \frac{5}{2}$$
$$= \frac{3 + 24 - 10}{4}$$
$$= \frac{27 - 10}{4}$$
$$= \frac{17}{4}$$

5. (*c*) $\mathbf{\frac{32}{15}}$

Draw a right ΔABC in which ∠B = 90°, ∠CAB = θ, such that $\sec \theta = \frac{5}{3}$

$$\Rightarrow \quad \frac{AC}{AB} = \frac{5}{3}$$



Let AC = 5*k*. Then, AB = 3*k*

In right triangle ABC, we have

$$AC^2 = AB^2 + BC^2$$
$$\Rightarrow \quad (5k)^2 = (3k)^2 + BC^2$$
$$\Rightarrow \quad BC^2 = (25 - 9)k^2 = 16k^2$$
$$\Rightarrow \quad BC = 4k$$
$$\sin \theta = \frac{BC}{AC} = \frac{4k}{5k} = \frac{4}{5}$$

and
$$\tan \theta = \frac{BC}{AB} = \frac{4k}{3k} = \frac{4}{3}$$

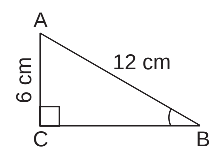
$$\sin \theta + \tan \theta = \frac{4}{5} + \frac{4}{3} = \frac{12 + 20}{15} = \frac{32}{15}$$

SHORT ANSWER QUESTIONS

For Basic and Standard Levels

1. In right ΔACB,

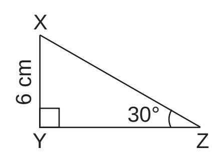
$$\cos A = \frac{AC}{AB} = \frac{6 \text{ cm}}{12 \text{ cm}} = \frac{1}{2}$$



$$\Rightarrow \quad \cos A = \cos 60^\circ$$

Hence A = **60°**

2. In right ΔXYZ,



$$\tan 30° = \frac{XY}{YZ}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{6 \text{ cm}}{YZ}$$

$$\Rightarrow \quad \mathbf{YZ = 6\sqrt{3} \text{ cm}}$$

and $$\sin 30° = \frac{XY}{XZ}$$

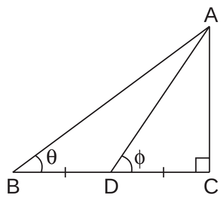
$$\Rightarrow \quad \frac{1}{2} = \frac{6 \text{ cm}}{XZ}$$

$$\Rightarrow \quad \mathbf{XZ = 12 \text{ cm}}$$

3. $$\tan\theta = \frac{AC}{BC} = \frac{AC}{BD + DC} = \frac{AC}{2\,DC}$$

[∵ D is the mid-point of BC] … (1)

$$\tan\phi = \frac{AC}{DC} \qquad \ldots (2)$$



Dividing (1) by (2), we get

$$\frac{\tan\theta}{\tan\phi} = \frac{AC}{2\,DC} \times \frac{DC}{AC} = \frac{1}{2}$$

Hence, $\mathbf{\frac{\tan\theta}{\tan\phi} = \frac{1}{2}}$.

4. $$\sin\theta = \sqrt{\frac{1 - \cos 2\theta}{2}}$$

$$\Rightarrow \quad \sin 30° = \sqrt{\frac{1 - \cos 2 \times 30°}{2}}$$

$$= \sqrt{\frac{1 - \cos 60°}{2}}$$

$$= \sqrt{\frac{1 - \frac{1}{2}}{2}} = \sqrt{\frac{\frac{1}{2}}{2}}$$

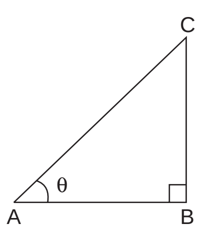
$$= \sqrt{\frac{1}{4}} = \frac{1}{2}$$

(rejecting the negative value)

Hence, $\mathbf{\sin 30° = \frac{1}{2}}$.

5. Let ABC be a right triangle in which ∠B = 90°, ∠CAB = θ, such that $\operatorname{cosec}\theta = \frac{5}{4}$

$$\operatorname{cosec}\theta = \frac{AC}{BC} = \frac{5}{4}$$



Let AC = $5k$. Then, BC = $4k$.

In right ΔABC, we have

$$AC^2 = AB^2 + BC^2$$

$$\Rightarrow \quad (5k)^2 = AB^2 + (4k)^2$$

$$\Rightarrow \quad AB^2 = (25 - 16)\,k^2 = 9k^2$$

$$\Rightarrow \quad AB = 3k$$

$$\tan\theta = \frac{BC}{AB} = \frac{4k}{3k} = \frac{4}{3}$$

$$5\sin\theta - 3\tan\theta = \frac{5}{\operatorname{cosec}\theta} - 3\tan\theta$$

$$= \frac{5}{\frac{5}{4}} - (3) \times \left(\frac{4}{3}\right)$$

$$= (5) \times \left(\frac{4}{5}\right) - (3) \times \left(\frac{4}{3}\right)$$

$$= 4 - 4 = 0$$

Hence, $5\sin\theta - 3\tan\theta = \mathbf{0}$

6. $4(\sin^4 30° + \cos^4 60°) - \frac{2}{3}(\sin^2 60° - \cos^2 45°) + \frac{1}{2}\tan^2 60°$

$$= 4\left[\left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^4\right] - \frac{2}{3}\left[\left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{\sqrt{2}}\right)^2\right] + \frac{1}{2}(\sqrt{3})^2$$

$$= 4\left(\frac{1}{16} + \frac{1}{16}\right) - \frac{2}{3}\left(\frac{3}{4} - \frac{1}{2}\right) + \frac{1}{2}(3)$$

$$= 4\left(\frac{2}{16}\right) - \frac{2}{3}\left(\frac{3-2}{4}\right) + \frac{3}{2}$$

$$= \frac{1}{2} - \frac{2}{3}\left(\frac{1}{4}\right) + \frac{3}{2}$$

$$= \frac{1}{2} - \frac{1}{6} + \frac{3}{2}$$

$$= \frac{3 - 1 + 9}{6}$$

$$= \frac{12 - 1}{6}$$

$$= \mathbf{\frac{11}{6}}$$

7. $$\cos x = \cos 60° \cos 30° + \sin 60° \sin 30°$$

$$\Rightarrow \quad \cos x = \left(\frac{1}{2} \times \frac{\sqrt{3}}{2}\right) + \left(\frac{\sqrt{3}}{2} \times \frac{1}{2}\right)$$

$$= \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4}$$

$$= \frac{2\sqrt{3}}{4}$$

$$= \frac{\sqrt{3}}{2}$$

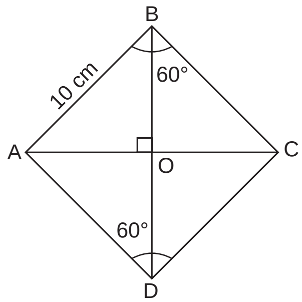
$$= \cos 30°$$

$$\Rightarrow \quad \mathbf{x = 30°}$$

For Standard Level

8. Let ABCD be a rhombus of side 10 cm in which the diagonals AC and BD, intersect at O and ∠ABC = ∠CDA = 60°.

Since the diagonals of a rectangle bisect each other at right angles



$\therefore$ $AO = OC = \frac{AC}{2}$

and $BO = OD = \frac{BD}{2}$

and $\angle AOB = 90°$ … (1)

$\because$ $\Delta ABO \cong \Delta CBO$ [By SAS congruency]

$\therefore$ $\angle ABO = \angle CBO = \frac{\angle ABC}{2} = 30°$

In right ΔAOB, $\sin 30° = \frac{AO}{AB}$

$\Rightarrow$ $\frac{1}{2} = \frac{AO}{10 \text{ cm}}$

$\Rightarrow$ $AO = 5$ cm

Diagonal $AC = 2AO$ [Using (1)]

$\Rightarrow$ $AC = 2 \times 5$ cm

$= \mathbf{10\ cm}$

In right ΔAOB, $\cos 30° = \frac{BO}{AB}$

$\Rightarrow$ $\frac{\sqrt{3}}{2} = \frac{BO}{10 \text{ cm}}$

$\Rightarrow$ $BO = 5\sqrt{3}$ cm

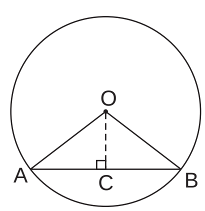
Diagonal $BD = 2\ BO$ [Using (1)]

$\Rightarrow$ $BO = 2 \times 5\sqrt{3}$ cm

$= \mathbf{10\sqrt{3}\ cm}$

Hence, the lengths of the diagonal are **10 cm and 10 $\sqrt{3}$ cm**.

9. Let AB be a chord of a circle with centre O, such that AB = 10 cm and $\angle AOB = 120°$.
Draw $OC \perp AB$.



In right Δs OCA and OCB, we have

$OA = OB$ [Radii of a circle]

and OC is common.

$\therefore$ $\Delta OCA \cong \Delta OCB$ [By RHS congruency]

$\therefore$ $\angle AOC = \angle BOC$

$= \angle \frac{AOB}{2}$

$= \frac{120°}{2}$

$= 60°$

In right ΔAOC, we have

$\tan 60° = \frac{AC}{OC}$

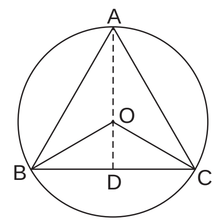
$\Rightarrow$ $\sqrt{3} = \frac{5 \text{ cm}}{OC}$

[Perpendicular from the centre of a circle to the chord bisects it $\therefore$ $AC = \frac{AB}{2} = \frac{10 \text{ cm}}{2} = 5$ cm]

$\Rightarrow$ $OC = \frac{5}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$= \mathbf{\frac{5\sqrt{3}}{3}}$ cm

10. Let ABC be an equilateral triangle inscribed in a circle with centre O and radius = 6 cm.



Then, $OA = OB = OC = 6$ cm

$\therefore$ $\Delta AOB \cong \Delta BOC \cong \Delta COA$

[By SSS congruency]

$\therefore$ $\angle AOB = \angle BOC = \angle COA$

[CPCT] … (1)

But $\angle AOB + \angle BOC + \angle COA = 360°$

[Angles about a point] … (2)

From (1) and (2), we get $\angle BOC = 120°$ … (3)

Draw $OD \perp BC$.

Right $\Delta ODB \cong$ Right ΔODC

[By RHS congruency]

$BD = DC = \frac{BC}{2}$,

$\angle BOD = \angle COD$

$= \frac{\angle BOC}{2}$

$= \frac{120°}{2} = 60°$ [Using (3)]

and $\angle ODB = \angle ODC = 90°$ … (4)

[$\because$ BDC is a straight $\angle$] By CPCT

In right ΔODB, we have $\sin 60° = \frac{BD}{OB}$

$\Rightarrow$ $\frac{\sqrt{3}}{2} = \frac{BD}{6 \text{ cm}}$

$\Rightarrow$ $2BD = 6\sqrt{3}$ cm

$\Rightarrow$ $BC = 6\sqrt{3}$ cm [Using (4)]

Hence, each side of the equilateral triangle is **6 $\sqrt{3}$ cm.**