CHAPTER 7 Coordinate Geometry

EXERCISE 7A

For Basic and Standard Levels

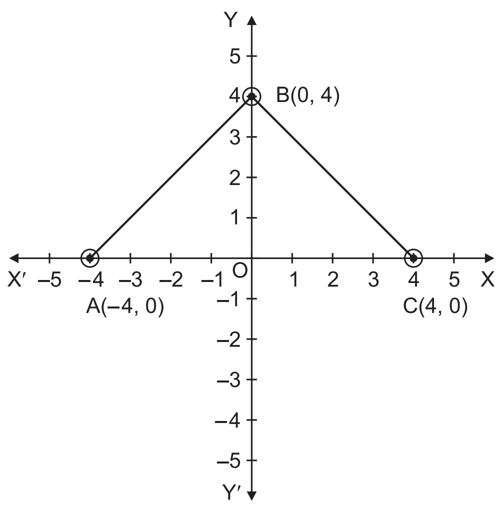
1. (*i*) ***x*-axis** (*ii*) ***y*-axis** (*iii*) ***y*-axis** (*iv*) ***x*-axis**.
2. (*i*) The abscissas of the points P, Q, R and S are respectively **1, –3, –8, 8**.

   (*ii*) The ordinates of the points P, Q, R and S are respectively **3, 5, –5, –7**.

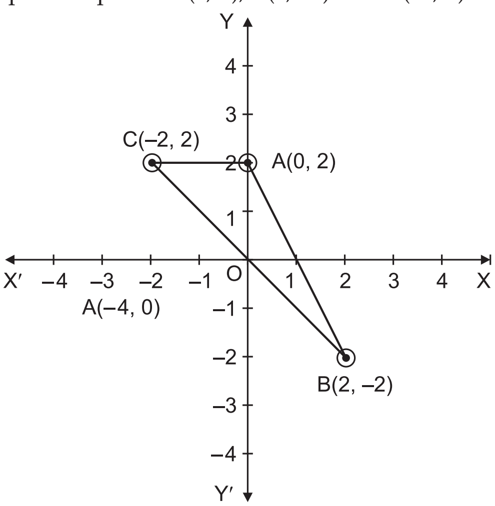
   (*iii*) The coordinates of the points **P, Q, R** and **S** are respectively **(1, 3), (–3, 5), (–8, –5)** and **(8, –7)**.
3. (*i*) **A(0, 0), B(2*a*, 0), C(2*a*, 2*a*), D(0, 2*a*)**.

   (*ii*) **A(–*a*, –*a*), B(*a*, – *a*), C(*a*, *a*), D(–*a*, *a*)**.
4. Take the rectangular coordinate axes as XOX′ and YOY′ and plot the points A(–4, 0), B(0, 4), and C(4, 0).

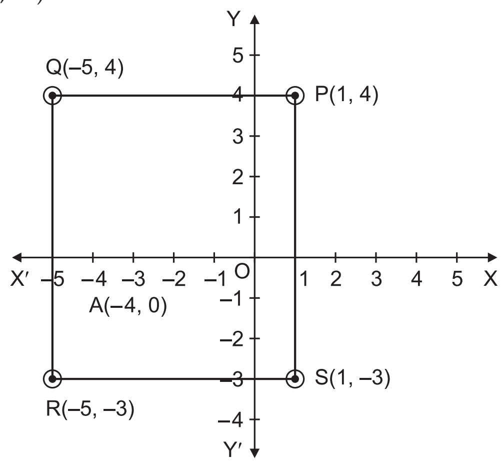
   Join, AB and BC.



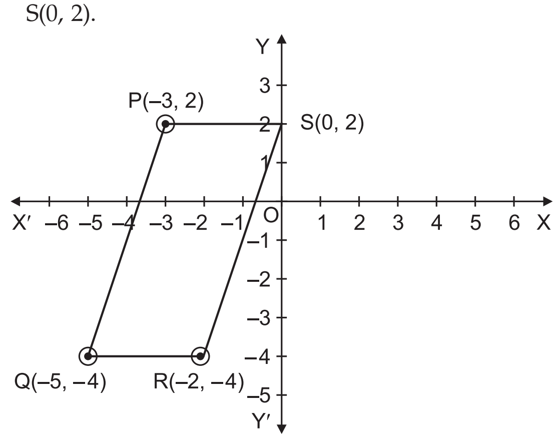
   The triangle so formed **is isosceles triangle**.
5. (*i*) Take the rectangular coordinate axes XOX′ and YOY′ and plot the points A(0, 2), B(2, –2) and C(–2, 2).



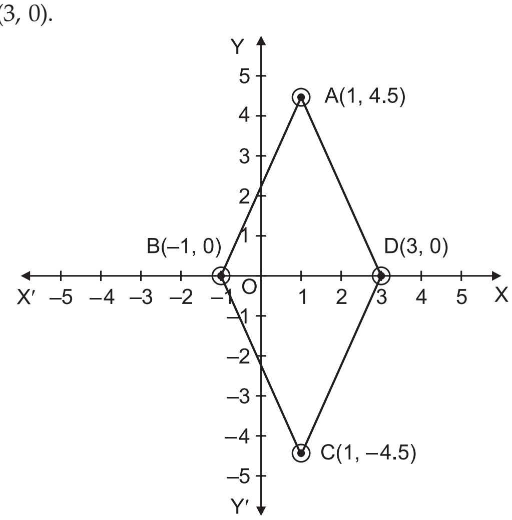
   (*ii*) Take the rectangular coordinate axes XOX′ and YOY′ and plot the points P(1, 4), Q(–5, 4), R(–5, –3) and S(1, –3).



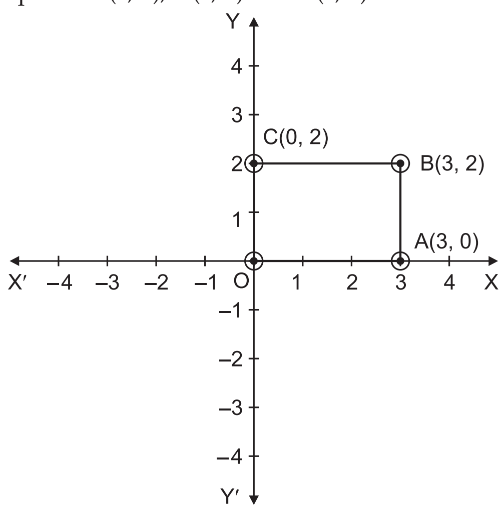
   (*iii*) Take the rectangular coordinate axes XOX′ and YOY′ and plot the points P(–3, 2), Q(–5, –4), R(–2, –4) and S(0, 2).



   (*iv*) Take the rectangular coordinate axes XOX′ and YOY′ and plot the points A(1, 4.5), B(–1, 0), C(1, –4.5) and D(3, 0).



6. (*i*) Take the rectangular coordinate system and plot the points O(0, 0), A(3, 0) and C(0, 2).



The point B's coordinates are **(3, 2)** such that OABC is a rectangle.

(*ii*) Q(–4, 0)

Mid-point of base (QR) = (0, 0)

Let the coordinates of R are (x_2, y_2).

Applying the mid-point formula

$$x = \frac{x_1 + x_2}{2} \qquad y = \frac{y_1 + y_2}{2}$$

$$0 = \frac{-4 + x_2}{2} \qquad 0 = \frac{0 + y_2}{2}$$

$$x_2 = 4 \qquad y_2 = 0$$

R(4, 0)

Let the coordinate of point P be (x_3, y_3) since PQR is an equilateral triangle

$\therefore$ PQ = PR

$$\sqrt{(x_3+4)^2+(y_3-0)^2} = \sqrt{(x_3-4)^2+(y_3-0)^2}$$

Squaring both the sides, we get

$$(x_3 + 4)^2 + y_3^2 = (x_3 - 4)^2 + y_3^2$$

$$x_3^2 + 16 + 8x_3 = x_3^2 + 16 - 8x_3$$

$$16x_3 = 0$$

$$x_3 = 0$$

$$QR = \sqrt{(-4-4)^2+(0-0)^2} = 8$$

We know PQ = QR

$$\sqrt{(x_3+4)^2+(y_3)^2} = 8$$

$$(x_3+4)^2 + (y_3)^2 = 64$$

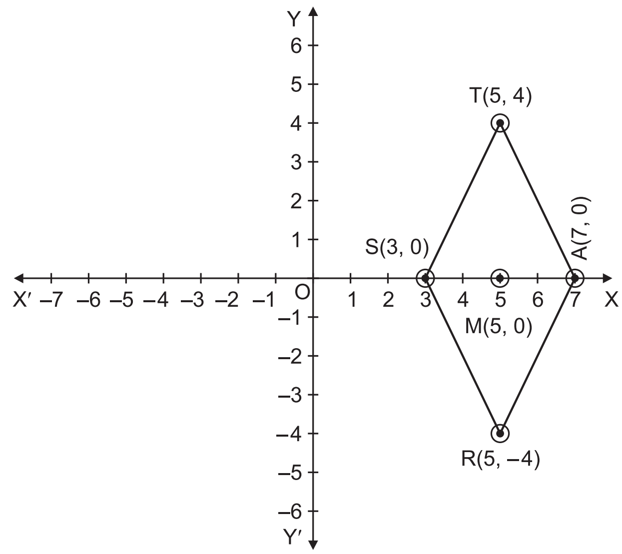
$$16 + y_3^2 = 64 \qquad (\because x_3 = 0)$$

$$y_3^2 = 48$$

$$y_3 = 4\sqrt{3}$$

$\therefore$ The coordinates of the points P and R **(0, $4\sqrt{3}$)** and **(4, 0)** respectively

7. (*i*) Take the rectangular coordinate axes XOX′ and YOY′ and plot the points S(3, 0), T(5, 4), A(7, 0) and R(5, –4).



(*ii*) Area of the rhombus STAR

$$= ar(\Delta STA) + ar(\Delta SRA)$$

$$= \frac{1}{2} \times SA \times TM + \frac{1}{2} \times SA \times RM$$

$$= \frac{1}{2} \times 4 \times 4 + \frac{1}{2} \times 4 \times 4$$

$$= 8 + 8$$

= 16 **sq units**

EXERCISE 7B

For Basic and Standard Levels

1. The distance between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by $AB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$.

(*i*) Let A(3, 7) and B(–2, –5) be two points.

Then $AB = \sqrt{(-2-3)^2+(-5-7)^2}$

$$= \sqrt{(-5)^2+(-12)^2}$$

$$= \sqrt{25+144}$$

$$= \sqrt{169}$$

= 13 units

(*ii*) Let A (cos θ, sin θ) and B (sin θ, – cos θ)

Then $AB = \sqrt{(\sin\theta-\cos\theta)^2+(-\cos\theta-\sin\theta)^2}$

$$= \sqrt{\sin^2\theta+\cos^2\theta-2\sin\theta\cos\theta + \cos^2\theta+\sin^2\theta+2\cos\theta\sin\theta}$$

$$= \sqrt{2(\sin^2\theta+\cos^2\theta)}$$

$$= \sqrt{2\times 1} \qquad (\because \sin^2\theta + \cos^2\theta = 1)$$

$= \sqrt{2}$ **units**

(*iii*) Let A$\left(-\frac{1}{10}, -\frac{7}{10}\right)$ and B$\left(\frac{1}{2}, \frac{1}{10}\right)$ be two points.

Then, $AB = \sqrt{\left[\frac{1}{2}-\left(-\frac{1}{10}\right)\right]^2+\left[\frac{1}{10}-\left(-\frac{7}{10}\right)\right]^2}$

$= \sqrt{\left(\frac{1\times 5}{2\times 5}+\frac{1}{10}\right)^2+\left(\frac{1}{10}+\frac{7}{10}\right)^2}$

$= \sqrt{\left(\frac{6}{10}\right)^2+\left(\frac{8}{10}\right)^2}$

$= \sqrt{\frac{36}{100}+\frac{64}{100}}$

$= \sqrt{\frac{36+64}{100}}$

$= \sqrt{\frac{100}{100}}$ = **1 unit**

(*iv*) Let A$(\sqrt{8}, 1)$ and B(0, 0) be two points.

Then, $AB = \sqrt{(0-\sqrt{8})^2+(0-1)^2}$

$= \sqrt{8+1}$

$= \sqrt{9}$ = **3 units**

(*v*) Let A($y + z$, $z + x$) and B($z + x$, $x + y$) be two points.

Then, $AB = \sqrt{[(z+x)-(y+z)]^2+[(x+y)-(z+x)]^2}$

$= \sqrt{(z+x-y-z)^2+(x+y-z-x)^2}$

$= \sqrt{x^2+y^2-2xy+y^2+z^2-2yz}$

$= \boldsymbol{\sqrt{x^2+2y^2+z^2-2xy-2yz}}$ **units**

(*vi*) Let A(a cos 25°, 0) and B(0, a cos 65°) be two points.

Then, $AB = \sqrt{(0-a\cos 25°)^2+(a\cos 65°-0)^2}$

$= \sqrt{a^2\cos^2 25°+a^2\cos^2 65°}$

$= \sqrt{a^2\cos^2 25°+a^2\sin^2 25°}$

$(\because \cos(90-\theta) = \sin\theta)$

$= \sqrt{a^2}$ = $\boldsymbol{a}$ **units** $(\because \cos^2\theta + \sin^2\theta = 1)$

(*vii*) Let A$\left(-\frac{8}{5}, 2\right)$ and $\left(\frac{2}{5}, 2\right)$ be two points.

Then, $AB = \sqrt{\left[\frac{2}{5}-\left(-\frac{8}{5}\right)\right]^2+(2-2)^2}$

$= \sqrt{\left(\frac{2}{5}+\frac{8}{5}\right)^2+(0)^2}$

$= \sqrt{\left(\frac{10}{5}\right)^2}$

$= \sqrt{(2)^2}$ = **2 units**

(*viii*) Let A(c, 0) and B(0, $-c$) be two points.

Then, $AB = \sqrt{(0-c)^2+(-c-0)^2}$

$= \sqrt{c^2+c^2}$

$= \sqrt{2c^2} = \boldsymbol{\sqrt{2}c}$ **units**

2. (*i*) The mid-point (x_1, y_1) of AB is given by

$x_1 = \frac{-5-1}{2} = -3$ and $y_1 = \frac{4+6}{2} = 5$

$\therefore$ The mid-point of AB is the point (–3, 5) which is clearly equidistant from A and B.

If P(α, β) be any other point which is equidistant from A and B, then PA = PB

$\Rightarrow \sqrt{(\alpha+5)^2+(\beta-4)^2} = \sqrt{(\alpha+1)^2+(\beta-6)^2}$

$\Rightarrow \alpha^2 + 10\alpha + 25 + \beta^2 - 8\beta + 16$

$= \alpha^2 + 2\alpha + 1 + \beta^2 - 12\beta + 36$

$\Rightarrow 8\alpha + 4\beta + 4 = 0$

$\Rightarrow 2\alpha + \beta + 1 = 0$

$\therefore$ (α, β) lies on the line $2x + y + 1 = 0$

Hence, any point lying on this line is also equidistant from the points. A and B. Since a line contains infinite number of points, hence infinitely many points are there which are equidistant from the given points A and B.

(*ii*) Given $AB = AC$

$\Rightarrow AB^2 = AC^2$

$\Rightarrow (3-0)^2 + (a-2)^2 = (a-0)^2 + (5-2)^2$

$\Rightarrow 9 + a^2 - 4a + 4 = a^2 + 9$

$\Rightarrow 4a = 4$

$\Rightarrow a = \frac{4}{4} = \mathbf{1}$

(*iii*) Given that $AP = BP$

$\Rightarrow AP^2 = BP^2$

$\Rightarrow (x-7)^2 + (3+1)^2 = (x-6)^2 + (3-8)^2$

$\Rightarrow x^2 - 14x + 49 + 16 = x^2 - 12x + 36 + 25$

$\Rightarrow 2x + 61 - 65 = 0$

$\Rightarrow 2x - 4 = 0$

$\Rightarrow x = 2$

$\therefore$ The required value of x is 2.

$\therefore AP = \sqrt{(2-7)^2+(3+1)^2}$

$= \sqrt{25+16}$

$= \sqrt{41}$

Hence, the required distance of AP is $\sqrt{41}$ units.

(*iv*) Given that $AP = AQ$

$\Rightarrow AP^2 = AQ^2$

$\Rightarrow (3-8)^2 + (y+3)^2 = (3-7)^2 + (y-6)^2$

$\Rightarrow 25 + y^2 + 6y + 9 = 16 + y^2 - 12y + 36$

$\Rightarrow 18y + 34 - 52 = 0$

$\Rightarrow \quad y = \frac{18}{18} = 1$

$\therefore \quad AQ = \sqrt{(3-7)^2 + (1-6)^2}$

$= \sqrt{16+25} = \sqrt{41}$

Hence, the required value of y is **A** and the required distance of AQ is $\sqrt{41}$ **units**.

(*v*) Since PA = PB

$\therefore \quad PA^2 = PB^2$

$\Rightarrow (k-1-3)^2 + (2-k)^2 = (k-1-k)^2 + (2-5)^2$

$\Rightarrow (k-4)^2 + (2-k)^2 = 1 + 9$

$\Rightarrow k^2 - 8k + 16 + 4 + k^2 - 4k - 10 = 0$

$\Rightarrow 2k^2 - 12k + 10 = 0$

$\Rightarrow k^2 - 6k + 5 = 0$

$\Rightarrow k^2 - 5k - k + 5 = 0$

$\Rightarrow k(k-5) - 1(k-5) = 0$

$\Rightarrow (k-5)(k-1) = 0$

$\therefore$ Either $k - 5 = 0 \Rightarrow k = 5$

or $k - 1 = 0 \Rightarrow k = 1$

$\therefore$ The required values of k are **5** and **1**.

3. Given that PQ = QR

$\Rightarrow PQ^2 = QR^2$

$\Rightarrow (1-6)^2 + [3-(-1)]^2 = (x-1)^2 + (8-3)^2$

$\Rightarrow 25 + 16 = x^2 - 2x + 1 + 25$

$\Rightarrow x^2 - 2x - 15 = 0$

$\Rightarrow x^2 - 5x + 3x - 15 = 0$

$\Rightarrow x(x-5) + 3(x-5) = 0$

$\Rightarrow (x-5)(x+3) = 0$

$\Rightarrow x - 5 = 0$ or $x + 3 = 0$

$\Rightarrow$ $\boldsymbol{x = 5}$ or $\boldsymbol{x = -3}$

4. (*i*) Given that PA = PB

$\Rightarrow PA^2 = PB^2$

$\Rightarrow (5-x)^2 + (1-y)^2 = (-1-x)^2 + (5-y)^2$

$\Rightarrow 25 - 10x + x^2 + 1 - 2y + y^2$
$= 1 + 2x + x^2 + 25 - 10y + y^2$

$\Rightarrow -10x - 2y = 2x - 10y$

$\Rightarrow 3x = 2y$

(*ii*) Given that PA = PB

$\Rightarrow PA^2 = PB^2$

$\Rightarrow (x-6)^2 + (y+1)^2 = (x-2)^2 + (y-3)^2$

$\Rightarrow x^2 - 12x + 36 + y^2 + 2y + 1$
$= x^2 - 4x + 4 + y^2 - 6y + 9$

$\Rightarrow 8x - 8y - 24 = 0$

$\Rightarrow x - y = 3$

(*iii*) Let the point P(x, y) be equidistant from two points A($a + b$, $b - a$) and B($a - b$, $a + b$).

$\Rightarrow$ PA = PB

$\Rightarrow PA^2 = PB^2$

$\Rightarrow (a+b-x)^2 + (b-a-y)^2$
$= (a-b-x)^2 + (a+b-y)^2$

$\Rightarrow a^2 + b^2 + x^2 + 2ab - 2ax - 2bx$
$+ b^2 + a^2 + y^2 - 2ab + 2ay - 2by$
$= a^2 + b^2 + x^2 - 2ab + 2bx - 2ax$
$+ a^2 + b^2 + y^2 + 2ab - 2ay - 2by$

$\Rightarrow -2ax - 2bx + 2ay - 2by$
$= 2bx - 2ax - 2ay - 2by$

$\Rightarrow 4ay = 4bx$

$\Rightarrow bx = ay$ (Dividing both the sides by 4)

(*iv*) Given that PA = PB

$\Rightarrow PA^2 = PB^2$

$\Rightarrow (-5-x)^2 + (3-y)^2 = (7-x)^2 + (2-y)^2$

$\Rightarrow x^2 + 25 + 10x + y^2 + 9 - 6y$
$= x^2 + 49 - 14x + y^2 + 4 - 4y$

$\Rightarrow 10x + 34 - 6y = -14x + 53 - 4y$

$\Rightarrow 24x = 2y + 19$

$\Rightarrow 24x - 2y = 19$

$\Rightarrow$ $\boldsymbol{24x - 2y - 19 = 0}$

(*v*) Given that PA = PB

$\Rightarrow PA^2 = PB^2$

$\Rightarrow (x-1)^2 + (y-4)^2 = (x+1)^2 + (y-2)^2$

$\Rightarrow x^2 - 2x + 1 + y^2 - 8y + 16$
$= x^2 + 2x + 1 + y^2 - 4y + 4$

$\Rightarrow 4x + 4y - 12 = 0$

$\Rightarrow$ $\boldsymbol{x + y - 3 = 0}$

which is the required relation.

5. (*i*) Let the point on the x-axis be (x_1, 0)

Then, d_1 = distance between (x, 0) and (7, 6)

$= \sqrt{(x_1-7)^2 + (0-6)^2}$

$= \sqrt{(x_1-7)^2 + 36}$

and d_2 = distance between (x_1, 0) and (–3, 4)

$= \sqrt{(x_1+3)^2 + (0-4)^2}$

$= \sqrt{(x_1+3)^2 + 16}$

$\because \quad d_1 = d_2$

$\therefore \quad d_1^2 = d_2^2$

$\Rightarrow x_1^2 - 14x_1 + 49 + 36 = x_1^2 + 6x_1 + 9 + 16$

$\Rightarrow 20x_1 - 60 = 0$

$\Rightarrow x_1 = 3$

$\therefore$ The required point is (3, 0).

(*ii*) Let the point on the x-axis be (x_1, 0)

Then, d_1 = distance between (x_1, 0) and (5, –2)

$= \sqrt{(x_1-5)^2 + 2^2}$

$= \sqrt{(x_1-5)^2 + 4}$

and d_2 = distance between $(x_1, 0)$ and $(-3, 2)$

$$= \sqrt{(x_1+3)^2+(0-2)^2}$$

$$= \sqrt{(x_1+3)^2+4}$$

$\because \quad d_1 = d_2$

$\therefore \quad d_1^2 = d_2^2$

$\Rightarrow x_1^2 - 10x_1 + 25 + 4 = x_1^2 + 6x_1 + 9 + 4$

$\Rightarrow \quad 16x_1 - 16 = 0$

$\Rightarrow \quad x_1 = 1$

$\therefore$ The required point is **(1, 0)**.

(*iii*) Let the point on the x-axis be $(x_1, 0)$

Then d_1 = distance between $(x_1, 0)$ and $(2, -5)$

$$= \sqrt{(x_1-2)^2+5^2}$$

$$= \sqrt{(x_1-2)^2+25}$$

and d_2 = distance between $(x_1, 0)$ and $(-2, 9)$

$$= \sqrt{(x_1+2)^2+(-9)^2}$$

$$= \sqrt{(x_1+2)^2+81}$$

$\because \quad d_1 = d_2$

$\therefore \quad d_1^2 = d_2^2$

$\Rightarrow x_1^2 - 4x_1 + 4 + 25 = x_1^2 + 4x_1 + 4 + 81$

$\Rightarrow \quad 8x_1 + 56 = 0$

$\Rightarrow \quad x_1 = -7$

$\therefore$ The required point is **(–7, 0)**.

6. (*i*) Let us take a point on x-axis $P(x, 0)$ such that it is at a distance of $2\sqrt{5}$ from a point $A(7, -4)$.

i.e. $\quad PA = 2\sqrt{5}$

$\Rightarrow \quad PA^2 = (2\sqrt{5})^2 = 20$

$\Rightarrow \quad (7-x)^2 + (-4-0)^2 = 20$

$\Rightarrow \quad 49 - 14x + x^2 + 16 = 20$

$\Rightarrow \quad x^2 - 14x + 45 = 0$

$\Rightarrow \quad x^2 - 9x - 5x + 45 = 0$

$\Rightarrow \quad x(x-9) - 5(x-9) = 0$

$\Rightarrow \quad (x-9)(x-5) = 0$

$\Rightarrow \quad x - 9 = 0$ or $x - 5 = 0$

$\Rightarrow \quad x = 9$ or $x = 5$

Hence, there are **two points** on x-axis namely, **(9, 0)** and **(5, 0)** such that their distance from the point $(7, -4)$ is $2\sqrt{5}$.

(*ii*) Let the point on x-axis be $P(x, 0)$ and the other given point be $Q(5, -4)$.

Then, $\quad PQ = \sqrt{(5-x)^2+(-4-0)^2}$

$$= \sqrt{25-10x+x^2+16}$$

$$= \sqrt{x^2-10x+41}$$

But, $\quad PQ = 5$

$\Rightarrow \quad \sqrt{x^2-10x+41} = 5$

Squaring both the sides, we get

$x^2 - 10x + 41 = (5)^2 = 25$

$\Rightarrow \quad x^2 - 10x + 16 = 0$

$\Rightarrow \quad x^2 - 8x - 2x + 16 = 0$

$\Rightarrow \quad x(x-8) - 2(x-8) = 0$

$\Rightarrow \quad (x-8)(x-2) = 0$

$\Rightarrow \quad x - 8 = 0$ or $x - 2 = 0$

$\Rightarrow \quad x = 8$ or $x = 2$

Hence, the points on x-axis are **(2, 0)** and **(8, 0)**.

(*iii*) Let the required point be $(x_1, 0)$

Then,

d_1 = distance between the points $(x_1, 0)$ and $(5, -3)$

$$= \sqrt{(x_1-5)^2+9}$$

Given that $\quad d_1 = 5$

$\therefore \quad d_1^2 = 25$

$\Rightarrow \quad (x_1 - 5)^2 + 9 = 25$

$\Rightarrow \quad x_1^2 - 10x_1 + 9 = 0$

$\Rightarrow \quad x_1^2 - 9x_1 - x_1 + 9 = 0$

$\Rightarrow x_1(x_1 - 9) - 1(x_1 - 9) = 0$

$\Rightarrow \quad (x_1 - 9)(x_1 - 1) = 0$

$\therefore \quad$ Either $x_1 - 9 = 0 \quad \Rightarrow \quad x_1 = 9$

or $\quad x_1 - 1 = 0 \quad \Rightarrow \quad x_1 = 1$

$\therefore$ The required points are **(1, 0)** and **(9, 0)**.

7. (*i*) Let the point on the y-axis be $(0, y_1)$

Then d_1 = distance between $(0, y_1)$ and $(-5, 2)$

$$= \sqrt{5^2+(y_1-2)^2}$$

and d_2 = distance between $(0, y_1)$ and $(9, -2)$

$$= \sqrt{9^2+(y_1+2)^2}$$

$\because \quad d_1 = d_2$

$\therefore \quad d_1^2 = d_2^2$

$\Rightarrow \quad y_1^2 - 4y_1 + 29 = y_1^2 + 4y_1 + 85$

$\Rightarrow \quad 8y_1 = -56$

$\Rightarrow \quad y_1 = -7$

$\therefore$ Required point is **(0, –7)**.

(*ii*) Let the required point be $(0, y_1)$

Then according to the problem,

Distance between $(0, y_1)$ and $(-3, 4)$ is equal to that between $(0, y_1)$ and $(3, 6)$ i.e.

$(0 + 3)^2 + (y_1 - 4)^2 = (0 - 3)^2 + (y_1 - 6)^2$

$\Rightarrow 9 + y_1^2 - 8y_1 + 16 = 9 + y_1^2 - 12y_1 + 36$

$\Rightarrow \quad 4y_1 = 20$

$\Rightarrow \quad y_1 = \dfrac{20}{4} = 5$

$\therefore$ The required point is **(0, 5)**.

(*iii*) Let the required point be P(0, y_1). A be the point (5, –2) and B be the point (–3, 2).

Then given that PA = PB

$\Rightarrow \quad PA^2 = PB^2$

$\Rightarrow \quad (0-5)^2 + (y_1+2)^2 = (0+3)^2 + (y_2-2)^2$

$\Rightarrow \quad 25 + y_1^2 + 4y_1 + 4 = 9 - 4y_1 + 4 + y_1^2$

$\Rightarrow \quad 8y_1 = -16$

$\Rightarrow \quad y_1 = -2$

$\therefore$ The required point is **(0, –2)**.

(*iv*) Let the required point be P(0, y_1).

Then given that PA = PB

$\Rightarrow \quad PA^2 = PB^2$

$\Rightarrow \quad (0-6)^2 + (y_1-5)^2 = (0+4)^2 + (y_1-3)^2$

$\Rightarrow \quad 36 + y_1^2 - 10y_1 + 25 = 16 + y_1^2 - 6y_1 + 9$

$\Rightarrow \quad 4y_1 = 61 - 25 = 36$

$\Rightarrow \quad y_1 = 9$

$\therefore$ The required point is **(0, 9)**.

8. Let the point on y-axis be P(0, y) and the other given point be Q(–8, 0). It is given that

$PQ = 10$

$\Rightarrow \quad PQ^2 = (10)^2 = 100$

$\Rightarrow \quad (-8-0)^2 + (0-y)^2 = 100$

$\Rightarrow \quad 64 + y^2 = 100$

$\Rightarrow \quad y^2 = 100 - 64 = 36$

$\Rightarrow \quad y = \pm\sqrt{36} = \pm 6$

Hence, the coordinates of points on y-axis are **(0, 6)** and **(0, –6)**.

9. Q(2,–5)

R(–3, 6)

Let y-coordinate of point P = a

then x-coordinate of point P = $2a$

P ($2a$, a)

We know, PQ = PR

$$\sqrt{(2a-2)^2+(a+5)^2} = \sqrt{(2a+3)^2+(a-6)^2}$$

On squaring both the sides we get

$$(2a-2)^2 + (a+5)^2 = (2a+3)^2 + (a-6)^2$$

$$4a^2 + 4 - 8a + a^2 + 25 + 10a = 4a^2 + 9 + 12a + a^2 + 36 - 12a$$

$$2a + 29 = 45$$

$$2a = 16$$

$$a = 8$$

$\therefore$ Coordinate of P are **(16, 8).**

10. Let the two points be A(x, 1) and B(2, 3).

Then, AB = 4

$\Rightarrow \quad AB^2 = 4^2$

$\Rightarrow \quad AB^2 = 16$

$\Rightarrow \quad (2-x)^2 + (3-1)^2 = 16$

$\Rightarrow \quad 4 - 4x + x^2 + 4 = 16$

$\Rightarrow \quad x^2 - 4x - 8 = 0$

$\Rightarrow \quad x = \dfrac{-(-4) \pm \sqrt{(-4)^2 - 4\times1\times(-8)}}{2\times1}$

$\Rightarrow \quad = \dfrac{4 \pm \sqrt{16+32}}{2}$

$\Rightarrow \quad = \dfrac{4\pm\sqrt{48}}{2} = \dfrac{4\pm4\sqrt{3}}{2}$

$\Rightarrow \quad = \mathbf{2 \pm 2\sqrt{3}}$

11. Given that AB = 10

$\Rightarrow \quad AB^2 = 10^2 = 100$

$\Rightarrow \quad (3-11)^2 + (-1-y)^2 = 100$

$\Rightarrow \quad 64 + 1 + y^2 + 2y - 100 = 0$

$\Rightarrow \quad y^2 + 2y - 35 = 0$

$\therefore \quad y = \dfrac{-2+\sqrt{2^2+4\times35}}{2\times1}$

$= \dfrac{-2\pm\sqrt{144}}{2}$

$= \dfrac{-2\pm12}{2} = 5, -7$

$\therefore$ The required value of y is **5** or **–7**.

12. (*i*) Let the x coordinate of the point P be x. Then the y coordinate of the point P will be $2x$.

Let the point (2, 3) be A.

Then, $AP = \sqrt{10}$

$\Rightarrow \quad AP^2 = (\sqrt{10})^2$

$\Rightarrow \quad (x-2)^2 + (2x-3)^2 = 10$

$\Rightarrow \quad x^2 - 4x + 4 + 4x^2 - 12x + 9 = 10$

$\Rightarrow \quad 5x^2 - 16x + 3 = 0$

$\Rightarrow \quad 5x^2 - 15x - x + 3 = 0$

$\Rightarrow \quad 5x(x-3) - 1(x-3) = 0$

$\Rightarrow \quad (x-3)(5x-1) = 0$

$\Rightarrow \quad x - 3 = 0 \text{ or } 5x - 1 = 0$

$\Rightarrow \quad x = 3 \text{ or } x = \dfrac{1}{5}$

Hence, the required point may be **(3, 6)** or $\left(\mathbf{\dfrac{1}{5}, \dfrac{2}{5}}\right)$.

(*ii*) Let the ordinate of the required point be y.

Then the coordinate of the required point is (2, y).

Now, $\sqrt{(5-2)^2+(1-y)^2} = 3\sqrt{5}$

Squaring both the sides, we get

$$9 + 1 - 2y + y^2 = 45$$

$\Rightarrow \quad y^2 - 2y - 35 = 0$

$\Rightarrow \quad y^2 - 7y + 5y - 35 = 0$

$\Rightarrow \quad y(y - 7) + 5(y - 7) = 0$

$\Rightarrow \quad (y - 7)(y + 5) = 0$

$\Rightarrow \quad y - 7 = 0 \text{ or } y + 5 = 0$

$\Rightarrow \quad y = 7 \text{ or } y = -5$

Hence, the ordinate is **7** or **– 5**.

(*iii*) Since the abscissa of the point A(x, y) is x and given that A's ordinate is thrice of its abscissa,

$\therefore$ The coordinates of A are (x, $3x$).

$\therefore \quad AB = \sqrt{41}$

$\Rightarrow \quad AB^2 = 41$

$\Rightarrow \quad (x + 4)^2 + (3x - 7)^2 = 41$

$\Rightarrow \quad x^2 + 16 + 8x + 9x^2 + 49 - 42x - 41 = 0$

$\Rightarrow \quad 10x^2 - 34x + 24 = 0$

$\Rightarrow \quad 5x^2 - 17x + 12 = 0$

$\Rightarrow \quad 5x^2 - 5x - 12x + 12 = 0$

$\Rightarrow \quad 5x(x - 1) - 12(x - 1) = 0$

$\Rightarrow \quad (x - 1)(5x - 12) = 0$

$\Rightarrow \quad x = 1 \text{ or } \frac{12}{5}$

$\Rightarrow \quad y = 3 \text{ or } \frac{36}{5}$

Hence, $\boldsymbol{x = 1, y = 3}$ or $\boldsymbol{x = \frac{12}{5}, y = \frac{36}{5}}$.

13. Let the abscissa of the other end be x. Then the coordinates of the other end are (x, 3).

The distance between the two ends A(5, –2) and B(x, 3) is

$$AB = \sqrt{(x-5)^2 + (3+2)^2}$$
$$= \sqrt{x^2 - 10x + 25 + 25}$$
$$= \sqrt{x^2 - 10x + 50}$$

But $\quad AB = 13$

$\Rightarrow \quad \sqrt{x^2 - 10x + 50} = 13$

Squaring both the sides, we get

$$x^2 - 10x + 50 = (13)^2 = 169$$

$\Rightarrow \quad x^2 - 10x - 119 = 0$

$\Rightarrow \quad x^2 - 17x + 7x - 119 = 0$

$\Rightarrow \quad x(x - 17) + 7(x - 17) = 0$

$\Rightarrow \quad (x - 17)(x + 7) = 0$

$\Rightarrow \quad x - 17 = 0 \text{ or } x + 7 = 0$

$\Rightarrow \quad x = 17 \text{ or } x = -7$

So, the abscissa is **–7** or **17**.

14. According to the question, the two points are A(0, 5) and B(–3, 1).

So, $\quad AB = \sqrt{(-3-0)^2 + (1-5)^2}$

$= \sqrt{9 + 16}$

$= \sqrt{25}$ = **5 units**

15. Let the coordinates of the required point be P(x, y).

Then, we have $\quad AP = BP = CP$

$\Rightarrow \quad AP^2 = BP^2 = CP^2$

Now, $\quad AP^2 = BP^2$

$\Rightarrow \quad (x - 5)^2 + (y - 1)^2 = (x + 3)^2 + (y + 7)^2$

$\Rightarrow \quad x^2 - 10x + 25 + y^2 - 2y + 1$

$\quad = x^2 + 6x + 9 + y^2 + 14y + 49$

$\Rightarrow \quad -16x - 16y = 32$

$\Rightarrow \quad x + y = -2 \qquad \ldots(1)$

Also, $\quad BP^2 = CP^2$

$\Rightarrow \quad (x + 3)^2 + (y + 7)^2 = (x - 7)^2 + (y + 1)^2$

$\Rightarrow \quad x + 6x + 9 + y^2 + 14y + 49$

$\quad = x^2 - 14x + 49 + y^2 + 2y + 1$

$\Rightarrow \quad 20x + 12y = -8$

$\Rightarrow \quad 5x + 3y = -2 \qquad \ldots(2)$

Equation (2) – 3 times equation (1) gives

$$2x = 4$$

$\Rightarrow \quad x = 2$

Putting $x = 2$ in equation (1), we get

$$2 + y = -2$$

$\Rightarrow \quad y = -2 - 2$

$\quad = -4$

Hence, the coordinates of the required point are **(2, –4)**.

16. (*i*) Let the three given points be A(–1, –1), B(2, 3) and C(8, 11).

Then, $\quad AB = \sqrt{(2+1)^2 + (3+1)^2}$

$= \sqrt{9 + 16}$

$= \sqrt{25} = 5$ units

$BC = \sqrt{(8-2)^2 + (11-3)^2}$

$= \sqrt{36 + 64}$

$= \sqrt{100}$

$= 10$ units

and $\quad AC = \sqrt{(8+1)^2 + (11+1)^2}$

$= \sqrt{81 + 144} = \sqrt{225} = 15$

Since $\quad 5 + 10 = 15$

i.e. $\quad AB + BC = AC$

Therefore, the given points are collinear.

(*ii*) Let the three given points be A(8, 7), B(6, 4) and C(0, –5).

Then, $\quad AB = \sqrt{(6-8)^2 + (4-7)^2}$

$= \sqrt{4 + 9} = \sqrt{13}$ units

$$BC = \sqrt{(0-6)^2 + (-5-4)^2}$$
$$= \sqrt{36+81}$$
$$= \sqrt{117} \text{ units}$$
$$AC = \sqrt{(0-8)^2 + (-5-7)^2}$$
$$= \sqrt{64+144}$$
$$= \sqrt{208} \text{ units}$$

Since, $\sqrt{13} + \sqrt{117} \neq \sqrt{208}$

i.e. AB + BC ≠ AC

Therefore, the given points are not collinear.

17. (*i*) Let the three given points are A(–5, 2), B$\left(-3, \frac{7}{2}\right)$ and C(3, 8).

Then, $$AB = \sqrt{(-3+5)^2 + \left(\frac{7}{2}-2\right)^2}$$
$$= \sqrt{4+\frac{9}{4}}$$
$$= \sqrt{\frac{16+9}{4}} = \sqrt{\frac{25}{4}}$$
$$= \frac{5}{2} \text{ units}$$
$$BC = \sqrt{(3+3)^2 + \left(8-\frac{7}{2}\right)^2}$$
$$= \sqrt{36+\frac{81}{4}}$$
$$= \sqrt{\frac{144+81}{4}} = \sqrt{\frac{225}{4}}$$
$$= \frac{15}{2} \text{ units}$$

and $$AC = \sqrt{(3+5)^2 + (8-2)^2}$$
$$= \sqrt{64+36}$$
$$= \sqrt{100}$$
$$= 10 \text{ units}$$

Since, $\frac{5}{2} + \frac{15}{2} = \frac{5+15}{2} = \frac{20}{2} = 10$

i.e. AB + BC = AC

Therefore, the given three points are **collinear**.

(*ii*) Let the three given points are P(–3, 2), Q(–4, 3) and R(4, 6).

Then, $$PQ = \sqrt{(-4+3)^2 + (3-2)^2}$$
$$= \sqrt{1+1} = \sqrt{2} \text{ units}$$
$$QR = \sqrt{(4+4)^2 + (6-3)^2}$$
$$= \sqrt{64+9}$$
$$= \sqrt{73} \text{ units}$$

and $$PR = \sqrt{(4+3)^2 + (6-2)^2}$$
$$= \sqrt{49+16}$$
$$= \sqrt{65} \text{ units}$$

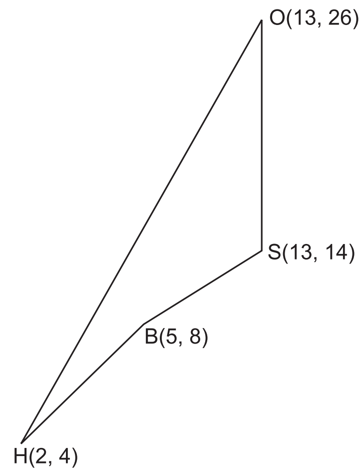
Since, $\sqrt{2} + \sqrt{65} \neq \sqrt{73}$

i.e. PQ + PR ≠ QR

Therefore, the three given points are **not collinear**.

For Standard Level

18. Let H (2, 4) be the house B(5, 8) be the bank, S (13, 14) be school and O(13, 26) be the office, all the coordinates be in km.



Then $$HB = \sqrt{(2-5)^2 + (4-8)^2} \text{ km}$$
$$= \sqrt{9+16} \text{ km}$$
$$= \sqrt{25} \text{ km} = 5 \text{ km}$$
$$BS = \sqrt{(5-13)^2 + (8-14)^2} \text{ km}$$
$$= \sqrt{64+36} \text{ km}$$
$$= \sqrt{100} \text{ km}$$
$$= 10 \text{ km}$$
$$SO = \sqrt{(13-13)^2 + (14-26)^2}$$
$$= \sqrt{12^2} \text{ km}$$
$$= 12 \text{ km}$$

and $$OH = \sqrt{(13-2)^2 + (26-4)^2} \text{ km}$$
$$= \sqrt{11^2 + 22^2} \text{ km}$$
$$= \sqrt{121+484} \text{ km}$$
$$= \sqrt{605} \simeq 24.6 \text{ km}$$

Hence, the distance of Ayush's bank B from his house H is HB = **5 km**, that of his daughter's school S from the bank B is BS = **10 km**, the distance of his office O from his daughter's school S is SO = **12 km** and the distance of his office O from his house H is OH = **24.6 km**.

$\therefore$ Total distance travelled = (5 + 10 + 12)km = **27 km**.

Also, distance between his house from his office = 24.6 km.

$\therefore$ Required extra distance travelled by Ayush

$$= (27 - 24.6)\text{km} = \mathbf{2.4\ km}$$

19. Let the points A (5, 4) and P (x, y) be equidistant from the point B (4, 5).

i.e. $AB = PB$

$\Rightarrow$ $AB^2 = PB^2$

$\Rightarrow$ $(4-5)^2 + (5-4)^2 = (4-x)^2 + (5-y)^2$

$\Rightarrow$ $1 + 1 = 16 - 8x + x^2 + 25 - 10y + y^2$

$\Rightarrow$ $x^2 + y^2 - 8x - 10y + 39 = 0$

20. Since the abscissa x and the ordinate of the required point are the same, hence we assume that the coordinates of the required point as P(x, x).

Then since $PA = PB$

$\therefore$ $PA^2 = PB^2$

$\Rightarrow$ $(x+6)^2 + (x-4)^2 = (x-2)^2 + (x+8)^2$

$\Rightarrow$ $x^2 + 12x + 36 + x^2 - 8x + 16$

$= x^2 - 4x + 4 + x^2 + 16x + 64$

$\Rightarrow$ $12x - 8x + 4x - 16x = 64 + 4 - 16 - 36$

$\Rightarrow$ $-8x = 68 - 52 = 16$

$\Rightarrow$ $x = -2$

Hence, the required point is **(–2, –2)**.

21. Let the point on the y-axis be P(0, y).

Then, $AP = \sqrt{(0-5)^2 + (y-2)^2}$

and $BP = \sqrt{(0-8)^2 + (y-8)^2}$

According to the question,

$4\sqrt{(0-5)^2 + (y-2)^2} = 2\sqrt{(0-8)^2 + (y-8)^2}$

$\Rightarrow$ $4\sqrt{25 + y^2 - 4y + 4} = 2\sqrt{64 + y^2 - 16y + 64}$

$\Rightarrow$ $2\sqrt{y^2 - 4y + 29} = \sqrt{y^2 - 16y + 128}$

Squaring both the sides, we get

$4(y^2 - 4y + 29) = y^2 - 16y + 128$

$\Rightarrow$ $4y^2 - 16y + 116 = y^2 - 16y + 128$

$\Rightarrow$ $3y^2 = 12$

$\Rightarrow$ $y^2 = 4$

$\Rightarrow$ $y = \pm 2$

Hence, the coordinates of the required point on y-axis is **(0, 2)** or **(0, –2)**.

22. Let the point on the y-axis be P(0, y) and the given points are A(6, 7) and B(4, –3).

Then, $AP = \sqrt{(0-6)^2 + (y-7)^2}$

$= \sqrt{36 + y^2 - 14y + 49}$

$= \sqrt{y^2 - 14y + 85}$

and $BP = \sqrt{(0-4)^2 + (y+3)^2}$

$= \sqrt{16 + y^2 + 6y + 9}$

$= \sqrt{y^2 + 6y + 25}$

But $AP : BP = 1 : 2$

$\Rightarrow$ $\dfrac{AP}{BP} = \dfrac{1}{2}$

$\Rightarrow$ $\dfrac{\sqrt{y^2 - 14y + 85}}{\sqrt{y^2 + 6y + 25}} = \dfrac{1}{2}$

$\Rightarrow$ $2\sqrt{y^2 - 14y + 85} = \sqrt{y^2 + 6y + 25}$

Squaring both the sides, we get

$4(y^2 - 14y + 85) = y^2 + 6y + 25$

$\Rightarrow$ $4y^2 - 56y + 340 = y^2 + 6y + 25$

$\Rightarrow$ $3y^2 - 62y + 315 = 0$

$\Rightarrow$ $3y^2 - 35y - 27y + 315 = 0$

$\Rightarrow$ $y(3y - 35) - 9(3y - 35) = 0$

$\Rightarrow$ $(3y - 35)(y - 9) = 0$

$\Rightarrow$ $3y - 35 = 0$ or $y - 9 = 0$

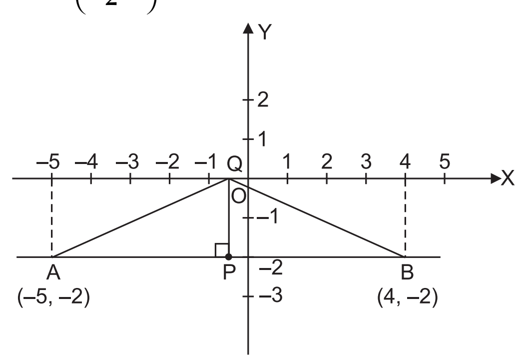
$\Rightarrow$ $y = \dfrac{35}{3}$ or $y = 9$

Hence, the required points on the y-axis is $\left(\mathbf{0, \frac{35}{3}}\right)$ or **(0, 9)**.

23. We plot the points A(–5, –2) and B (4, –2) on a graph paper. Clearly, the line segment AB is parallel to x-axis, since AB is at a distance of 2 units from the x-axis in the negative direction of the y-axis. It P be the mid-point of AB, then the coordinates of P are $\left(\frac{1}{2}(-5+4), \frac{1}{2}(-2-2)\right)$

$= \left(-\frac{1}{2}, 2\right)$.

Since the point Q lies on the perpendicular bisector of AB and on the x-axis, hence, the required coordinates of Q will be $\left(\mathbf{-\frac{1}{2}, 0}\right)$.



Again, since PQ is $\perp$ AB and P is the mid-point of AB,

$\therefore$ ΔQAB is an **isosceles triangle** with QA = QB and O being the origin.

EXERCISE 7C

For Basic and Standard Levels

1. Let the three vertices of the triangle be A(1, –1), B$\left(-\frac{1}{2}, \frac{1}{2}\right)$ and C(1, 2).

Then,
$$AB = \sqrt{\left(-\frac{1}{2}-1\right)^2+\left(\frac{1}{2}+1\right)^2}$$
$$= \sqrt{\left(-\frac{3}{2}\right)^2+\left(\frac{3}{2}\right)^2}$$
$$= \sqrt{\frac{9}{4}+\frac{9}{4}} = \sqrt{\frac{18}{4}} = \frac{3}{2}\sqrt{2} \text{ units}$$
$$BC = \sqrt{\left(1+\frac{1}{2}\right)^2+\left(2-\frac{1}{2}\right)^2}$$
$$= \sqrt{\left(\frac{3}{2}\right)^2+\left(\frac{3}{2}\right)^2}$$
$$= \sqrt{2\times\left(\frac{3}{2}\right)^2} = \frac{3}{2}\sqrt{2} \text{ units}$$
$$AC = \sqrt{(1-1)^2+(2+1)^2}$$
$$= \sqrt{0+(3)^2}$$
$$= \sqrt{(3)^2} = 3 \text{ units}$$
$\Rightarrow$ AB = BC

$\because$ The two sides of the given triangle are equal, the triangle is an isosceles triangle.

2. Let the third vertex be P(x, y). Also, let the two given vertices are A(–2, 5) and B(4, –1).

Then,
$$AP = \sqrt{(x+2)^2+(y-5)^2}$$
$$= \sqrt{x^2+4x+4+y^2-10y+25}$$
$$= \sqrt{x^2+y^2+4x-10y+29}$$
$$BP = \sqrt{(x-4)^2+(y+1)^2}$$
$$= \sqrt{x^2-8x+16+y^2+2y+1}$$
$$= \sqrt{x^2+y^2-8x+2y+17}$$
$$AB = \sqrt{(4+2)^2+(-1-5)^2}$$
$$= \sqrt{(6)^2+(-6)^2}$$
$$= \sqrt{36+36}$$
$$= \sqrt{72} = 6\sqrt{2} \neq 3\sqrt{2}$$
So, the equal sides are AP and BP

i.e. AP = BP

$\Rightarrow$ $AP^2 = BP^2$

$\Rightarrow$ $x^2 + y^2 + 4x - 10y + 29 = x^2 + y^2 - 8x + 2y + 17$

$\Rightarrow$ $12x - 12y + 12 = 0$

$\Rightarrow$ $x - y + 1 = 0$

$\Rightarrow$ $y = x + 1$ …(1)

Also, $AP = 3\sqrt{2}$

$\Rightarrow$ $AP^2 = (3\sqrt{2})^2 = 18$

$\Rightarrow$ $x^2 + y^2 + 4x - 10y + 29 = 18$

$\Rightarrow$ $x^2 + y^2 + 4x - 10y + 11 = 0$

$\Rightarrow$ $x^2 + (x + 1)^2 + 4x - 10(x + 1) + 11 = 0$ [From (1)]

$\Rightarrow$ $x^2 + x^2 + 2x + 1 + 4x - 10x - 10 + 11 = 0$

$\Rightarrow$ $2x^2 - 4x + 2 = 0$

$\Rightarrow$ $x^2 - 2x + 1 = 0$

$\Rightarrow$ $(x - 1)^2 = 0$

$\Rightarrow$ $x = 1$

Therefore, $y = x + 1 = 1 + 1 = 2$

Hence, the third vertex is **(1, 2)**.

3. (*i*)
$$AB = \sqrt{(-2-12)^2+(6-8)^2}$$
$$= \sqrt{(-14)^2+(-2)^2} = \sqrt{196+4}$$
$$= \sqrt{200} = 10\sqrt{2} \text{ units}$$
$$BC = \sqrt{(6+2)^2+(0-6)^2}$$
$$= \sqrt{(8)^2+(-6)^2} = \sqrt{64+36}$$
$$= \sqrt{100} = 10 \text{ units}$$
$$AC = \sqrt{(6-12)^2+(0-8)^2}$$
$$= \sqrt{(-6)^2+(-8)^2}$$
$$= \sqrt{36+64}$$
$$= \sqrt{100} = 10 \text{ units}$$
Now, $AB^2 = (10\sqrt{2})^2 = 200$

$BC^2 = (10)^2 = 100$

and $AC^2 = (10)^2 = 100$

$\because$ $BC^2 + AC^2 = AB^2$

Therefore, A, B and C are the vertices of a right-angled triangle.

(*ii*) Let the three vertices of the triangle be A(7, 10), B(–2, 5) and C(3, –4).

Then,
$$AB = \sqrt{(-2-7)^2+(5-10)^2}$$
$$= \sqrt{(-9)^2+(-5)^2}$$
$$= \sqrt{81+25} = \sqrt{106} \text{ units}$$
$$BC = \sqrt{(3+2)^2+(-4-5)^2}$$
$$= \sqrt{(5)^2+(-9)^2}$$
$$= \sqrt{25+81} = \sqrt{106} \text{ units}$$
$$AC = \sqrt{(3-7)^2+(-4-10)^2}$$
$$= \sqrt{(-4)^2+(-14)^2}$$

$$= \sqrt{16+196}$$

$$= \sqrt{212} \text{ units}$$

Since, $AB = BC$

So, ΔABC is an isosceles triangle.

Now, $AB^2 = (\sqrt{106})^2 = 106$

$BC^2 = (\sqrt{106})^2 = 106$

and $AC^2 = (\sqrt{212})^2 = 212$

Since, $AB^2 + BC^2 = 106 + 106 = 212 = AC^2$

So, ΔABC is a right-angled triangle.

Hence, the given triangle is an isosceles right-angled triangle.

(*iii*) $AB = \sqrt{(5-6)^2+(-2-4)^2}$

$= \sqrt{(-1)^2+(-6)^2}$

$= \sqrt{1+36} = \sqrt{37}$ units

$BC = \sqrt{(7-5)^2+(-2+2)^2}$

$= \sqrt{(2)^2+(0)^2} = 2$ units

$AC = \sqrt{(7-6)^2+(-2-4)^2}$

$= \sqrt{(1)^2+(-6)^2}$

$= \sqrt{1+36} = \sqrt{37}$ units

Since, $AB = AC$

Therefore, ΔABC is an isosceles triangle.

Now, let the mid-point of BC be D.

Then, the coordinates of D are $\left(\frac{5+7}{2}, \frac{-2-2}{2}\right)$, i.e. $(6, -2)$.

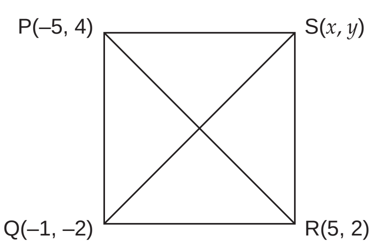
So, the length of the median

$AD = \sqrt{(6-6)^2+(-2-4)^2}$

$= \sqrt{(0)^2+(-6)^2}$

$= \sqrt{36} =$ **6 units**

(*iv*) The vertices of the triangle are P(–5, 4), Q(–1, –2) and R(5, 2).



Then, $PQ = \sqrt{(-1+5)^2+(-2-4)^2}$

$= \sqrt{4^2+6^2} = \sqrt{16+36} = \sqrt{52}$

$QR = \sqrt{(5+1)^2+(2+2)^2}$

$= \sqrt{6^2+4^2} = \sqrt{36+16} = \sqrt{52}$

$RP = \sqrt{(5+5)^2+(2-4)^2} = \sqrt{10^2+2^2}$

$= \sqrt{104}$

$\because$ PQ = QR $\Rightarrow$ PQR is an isosceles triangle.

Also, $PQ^2 + QR^2 = PR^2$

$\Rightarrow$ PQR is a right-angled triangle.

Hence, PQR is an isosceles right-angled triangle.

Now, let S be the fourth vertex with coordinates (x, y).

For PQRS to be the square, we must have

PQ = QR = RS = PS and PR = QS.

Now, PS = SR

$\Rightarrow PS^2 = SR^2$

$\Rightarrow (x + 5)^2 + (y - 4)^2 = (x - 5)^2 + (y - 2)^2$

$\Rightarrow x^2 + 25 + 10x + y^2 + 16 - 8y$

$= x^2 + 25 - 10x + y^2 + 4 - 4y$

$\Rightarrow 20x - 4y = -12$

$\Rightarrow 5x - y = -3$ …(1)

Now,

PR = QS

$\Rightarrow PR^2 = QS^2$

$\Rightarrow (5 + 5)^2 + (2 - 4)^2 = (x + 1)^2 + (y + 2)^2$

$\Rightarrow 10^2 + 2^2 = x^2 + 1 + 2x + y^2 + 4 + 4y$

$\Rightarrow x^2 + y^2 + 2x + 4y + 5 - 104 = 0$

$\Rightarrow x^2 + y^2 + 2x + 4y - 99 = 0$

$\Rightarrow x^2 + (5x + 3)^2 + 2x + 4(5x + 3) - 99 = 0$ [Using (1)]

$\Rightarrow x^2 + 25x^2 + 9 + 30x + 2x + 20x + 12 - 99 = 0$

$\Rightarrow 26x^2 + 52x - 78 = 0$

$\Rightarrow x^2 + 2x - 3 = 0$

$\Rightarrow x^2 + 3x - x - 3 = 0$

$\Rightarrow (x + 3)(x - 1) = 0$

$\Rightarrow x = 1$

or $x = -3$

So, from (1), $y = 5x + 3$

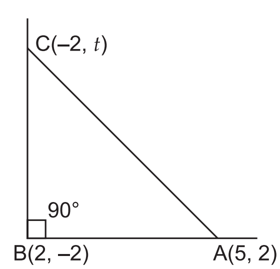
when $x = 1$, $y = 8$

when $x = -3$, $y = -12$

Since S(–3, –12) does not satisfy the condition of a square.

$\therefore$ The coordinates of S are **(1, 8)**.

4. Since, in ΔABC, $\angle ABC = 90°$, hence by Pythagoras' theorem, we have



$$AC^2 = AB^2 + BC^2$$

$\Rightarrow$ $(5 + 2)^2 + (2 - t)^2 = (2 - 5)^2 + (-2 - 2)^2 + (2 + 2)^2 + (-2 - t)^2$

$\Rightarrow$ $49 + 4 + t^2 - 4t = 9 + 16 + 16 + 4 + t^2 + 4t$

$\Rightarrow$ $53 - 4t = 45 + 4t$

$\Rightarrow$ $8t = 53 - 45 = 8$

$\Rightarrow$ $\boldsymbol{t = 1}$

which is the required value of t.

5. (*i*) The points are $A(a, a)$, $B(-a, -a)$, $C(-\sqrt{3}\,a, \sqrt{3}\,a)$

Distance between A and B

$$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(-2a)^2 + (-2a)^2}$$
$$= \sqrt{4a^2 + 4a^2}$$
$$= \sqrt{8a^2}$$
$$= 2a\sqrt{2}$$
$$= 2\sqrt{2}\,a$$

Distance between B and C

$$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{\left(-\sqrt{3}a + a\right)^2 + \left(\sqrt{3}a + a\right)^2}$$
$$= \sqrt{3a^2 + a^2 - 2\sqrt{3}a^2 + 3a^2 + a^2 + 2\sqrt{3}a^2}$$
$$= \sqrt{8a^2}$$
$$= 2\sqrt{2}\,a$$

Distance between C and A

$$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{\left(-\sqrt{3}a - a\right)^2 + \left(\sqrt{3}a - a\right)^2}$$
$$= \sqrt{3a^2 + a^2 + 2\sqrt{3}a^2 + 3a^2 + a^2 - 2\sqrt{3}a^2}$$
$$= \sqrt{8a^2}$$
$$= 2\sqrt{2}\,a$$

We haveAB= BC = CA

Hence it is an equilateral triangle and A, B and C are vertices of it.

(*ii*) Let $A(-3, -3)$, $B(3, 3)$ and $C\left(-3\sqrt{3}, 3\sqrt{3}\right)$

$\therefore$ $AB = \sqrt{(-3-3)^2 + (-3-3)^2}$

$= \sqrt{36 + 36} = \sqrt{72}$

$= 6\sqrt{2}$

$AC = \sqrt{\left(-3 + 3\sqrt{3}\right)^2 + \left(-3 - 3\sqrt{3}\right)^2}$

$= \sqrt{2(9 + 27)} = \sqrt{72}$

$= 6\sqrt{2}$

and $BC = \sqrt{\left(3 + 3\sqrt{3}\right)^2 + \left(3 - 3\sqrt{3}\right)^2}$

$= \sqrt{2(9 + 27)}$

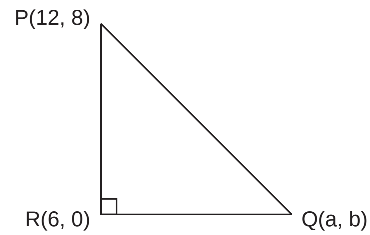
$= \sqrt{72}$

$= 6\sqrt{2}$

$\therefore$ $AB = AC = BC$

$\therefore$ ΔABC is an equilateral triangle and the given points are its vertices.

6. The vertices of the right-angled triangle with angle R = right angle are $P(12, 8)$, $Q(a, b)$ and $R(6, 0)$.



Then by Pythagoras' Theorem, we have

$$PR^2 + QR^2 = PQ^2$$

$\Rightarrow$ $[(12 - 6)^2 + (8 - 0)^2] + [(6 - a)^2 + (0 - b)^2] = [(12 - a)^2 + (8 - b)^2]$

$\Rightarrow$ $6^2 + 8^2 + (6 - a)^2 + b^2 = (12 - a)^2 + (8 - b)^2$

$\Rightarrow$ $36 + 64 + 36 + a^2 - 12a + b^2 = 144 + a^2 - 24a + b^2 + 64 - 16b$

$\Rightarrow$ $72 - 12a = -24a - 16b + 144$

$\Rightarrow$ $12a + 16b = 144 - 72$

$\Rightarrow$ $12a + 16b = 72$

$\Rightarrow$ $\boldsymbol{3a + 4b = 18}$.

7. A (4, 7)

B (p, 3)

C (7, 3)

$$AB = \sqrt{(p-4)^2 + 16}$$
$$BC = \sqrt{(p-7)^2 + (0)^2}$$
$$AC = \sqrt{(3)^2 + (4)^2}$$

Since ABC is a right-angled triangle

$\therefore$ $AB^2 + BC^2 = AC^2$

$(p - 4)^2 + 16 + (p - 7)^2 = 25$

$p^2 + 16 - 8p + 16 + p^2 + 49 - 14p = 25$

$2p^2 - 22p + 81 = 25$

$2p^2 - 22p + 56 = 0$

$p^2 - 11p + 28 = 0$

$p^2 - 4p - 7p + 28 = 0$

$p(p - 4) - 7(p - 4) = 0$

$(p - 4)(p - 7) = 0$

$p = 4, 7$

p cannot be 7 hence we will reject it as it does not satisfy the condition.

$\therefore \quad p = \mathbf{4}$

8. $AB = \sqrt{(4-0)^2+(0-0)^2}$

$= \sqrt{16+0} = \sqrt{16} = 4$

$BC = \sqrt{(a-4)^2+(b-0)^2}$

$= \sqrt{a^2-8a+16+b^2}$

and $AC = \sqrt{(a-0)^2+(b-0)^2}$

$= \sqrt{a^2+b^2}$

Since ΔABC is an equilateral triangle,

i.e. $AB = BC = AC$

$\Rightarrow \quad AB = BC$

$\Rightarrow \quad 4 = \sqrt{a^2-8a+16+b^2}$

Squaring both the sides, we get

$16 = a^2 - 8a + 16 + b^2 \quad ...(1)$

Also, $BC = AC$

$\Rightarrow \sqrt{a^2-8a+16+b^2} = \sqrt{a^2+b^2}$

Squaring both the sides, we get

$a^2 - 8a + 16 + b^2 = a^2 + b^2$

$\Rightarrow \quad -8a + 16 = 0$

$\Rightarrow \quad 8a = 16$

$\Rightarrow \quad a = \frac{16}{8} = \mathbf{2}$

Putting the value of a in (1), we get

$16 = (2)^2 - 8(2) + 16 + b^2$

$\Rightarrow \quad 16 = 4 - 16 + 16 + b^2$

$\Rightarrow \quad b^2 = 12$

$\Rightarrow \quad b = \pm \mathbf{2\sqrt{3}}$

9. $AB = \sqrt{(-4+5)^2+(-2-6)^2}$

$= \sqrt{1+64} = \sqrt{65}$

$BC = \sqrt{(7+4)^2+(5+2)^2}$

$= \sqrt{121+49} = \sqrt{170}$

$AC = \sqrt{(7+5)^2+(5-6)^2}$

$= \sqrt{144+1} = \sqrt{145}$

Since $AB \neq BC \neq AC$

So, ΔABC is a **scalene triangle**.

10. (*i*) Let the coordinates of the third vertex be $P(x, y)$ and the two given vertices be $A(0, 0)$ and $B(3, \sqrt{3})$.

Then, $AP = \sqrt{(x-0)^2+(y-0)^2}$

$= \sqrt{x^2+y^2}$

$BP = \sqrt{(x-3)^2+(y-\sqrt{3})^2}$

$= \sqrt{x^2-6x+9+y^2-2\sqrt{3}y+3}$

$= \sqrt{x^2+y^2-6x-2\sqrt{3}y+12}$

and $AB = \sqrt{(3-0)^2+(\sqrt{3}-0)^2}$

$= \sqrt{9+3}$

$= \sqrt{12}$

But, ΔABP is an equilateral triangle.

So, $AP = BP = AB$

$\Rightarrow \quad AP = BP$

$\Rightarrow \quad \sqrt{x^2+y^2} = \sqrt{x^2+y^2-6x-2\sqrt{3}y+12}$

Squaring both the sides, we get

$x^2 + y^2 = x^2 + y^2 - 6x - 2\sqrt{3}y + 12$

$\Rightarrow \quad 6x + 2\sqrt{3}y = 12$

$\Rightarrow \quad \sqrt{3}x + y = 2\sqrt{3}$

$\Rightarrow \quad y = 2\sqrt{3} - \sqrt{3}x \quad ...(1)$

Also, $AP = AB$

$\Rightarrow \quad \sqrt{x^2+y^2} = \sqrt{12}$

Squaring both the sides, we get

$x^2 + y^2 = 12$

$\Rightarrow \quad x^2 + (2\sqrt{3} - \sqrt{3}x)^2 = 12 \quad$ [From (1)]

$\Rightarrow \quad x^2 + 12 + 3x^2 - 12x = 12$

$\Rightarrow \quad 4x^2 - 12x = 0$

$\Rightarrow \quad 4x(x - 3) = 0$

$\Rightarrow \quad x = 0 \text{ or } x - 3 = 0$

$\Rightarrow \quad x = 0 \text{ or } x = 3$

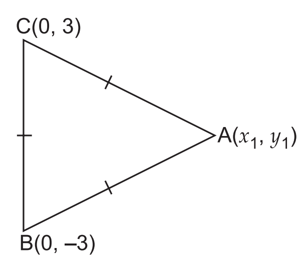
From (1),

$y = 2\sqrt{3} - \sqrt{3} \times 0 = 2\sqrt{3}$

or $y = 2\sqrt{3} - \sqrt{3} \times 3 = -\sqrt{3}$

Hence, the third coordinates are $\mathbf{(0, 2\sqrt{3})}$ or $\mathbf{(3, -\sqrt{3})}$

(*ii*) Let ABC be an equilateral triangle with $A(x_1, y_1)$, $B(0, -3)$ and $C(0, 3)$.



Now $AB = \sqrt{x_1^2 + (y_1+3)^2}$

$AC = \sqrt{x_1^2 + (y_1-3)^2}$

and $BC = \sqrt{(3+3)^2} = 6$

$\therefore \quad AB^2 = AC^2 = BC^2$

$\Rightarrow \qquad x_1^2 + (y_1 + 3)^2 = x_1^2 + (y_1 - 3)^2 = 36$

When $x_1^2 + (y_1 + 3)^2 = x_1^2 + (y_1 - 3)^2$

then $\qquad 4y_1 \times 3 = 0$

$\therefore \qquad y_1 = 0$

$\therefore \qquad AC^2 = x_1^2 + 9$

$\qquad = BC^2 = 36$

$\Rightarrow \qquad x_1^2 = 36 - 9 = 27$

$\therefore \qquad x_1 = \pm\sqrt{27} = \pm 3\sqrt{3}$

$\therefore$ The required coordinates of the third vertex is $\left(\mathbf{3\sqrt{3}, 0}\right)$ or $\left(\mathbf{-3\sqrt{3}, 0}\right)$.

(*iii*) Let the third vertex be $C(x_1, y_1)$

and the given vertices are $A(-4, 0)$, $B(4, 0)$.

$\because \qquad AB = AC = BC$

$\Rightarrow \qquad AB^2 = AC^2 = BC^2$

$\therefore \qquad (4 + 4)^2 = (x_1 + 4)^2 + y_1^2$

$\qquad = (x_1 - 4)^2 + + y_1^2$

When $(x_1 + 4)^2 + y_1^2 = (x_1 - 4)^2 + y_1^2$

then $\qquad 4x_1 \times 4 = 0$

$\Rightarrow \qquad x_1 = 0$

$\therefore$ From $(x_1 + 4)^2 + y_1^2 = 64$

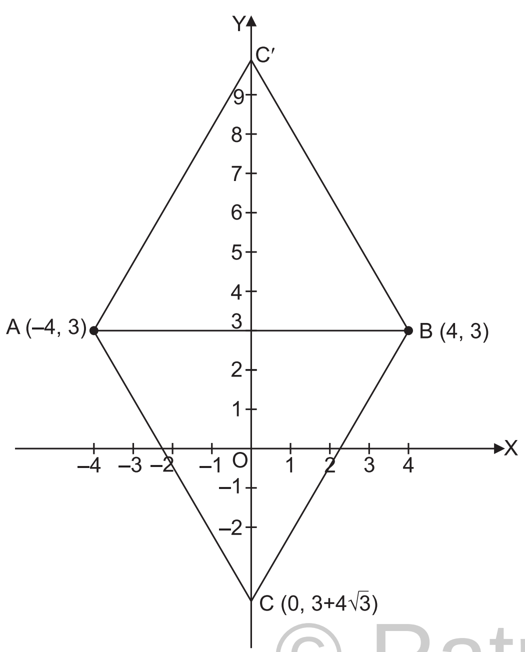
We get $\qquad 16 + y_1^2 = 64$

$\Rightarrow \qquad y_1^2 = 48$

$\Rightarrow \qquad y^1 = \pm 4\sqrt{3}$

$\therefore$ The required coordinates of the third vertex are $\left(\mathbf{0, 4\sqrt{3}}\right)$ or $\left(\mathbf{0, -4\sqrt{3}}\right)$.

(*iv*) Let $A(-4, 3)$ and $B(4, 3)$ be two vertices of the equilateral triangle and let $C(x_1, y_1)$ be the third vertex of this triangle.



then $\qquad AB = AC = BC$

$\qquad AC = BC$

$\Rightarrow \qquad AC^2 = BC^2$

$\Rightarrow (x_1 + 4)^2 + (y_1 - 3)^2 = (x_1 - 4)^2 + (y_1 - 3)^2$

$\Rightarrow \qquad x_1^2 + 8x_1 + 16 = x_1^2 - 8x_1 + 16$

$\Rightarrow \qquad 16x_1 = 0$

$\Rightarrow \qquad x_1 = 0$

Again, $\qquad AC = AB$

$\Rightarrow \qquad AC^2 = AB^2$

$\Rightarrow \qquad 4^2 + (y_1 - 3)^2 = 64$

$\Rightarrow \qquad (y_1 - 3)^2 = 48$

$\therefore \qquad y_1 - 3 = \pm 4\sqrt{3}$

$\Rightarrow \qquad y_1 = 3 \pm 4\sqrt{3}$

$\therefore$ The coordinates of the 3rd vertex are $\left(0, 3 + 4\sqrt{3}\right)$ or $\left(0, 3 - 4\sqrt{3}\right)$.

Now, $3 + 4\sqrt{3} \simeq 9.9$ and $3 - 4\sqrt{3} \simeq -4$

$\therefore$ The point $(0, 3 + 4\sqrt{3})$ will lie above the x-axis and $(0, 3 - 4\sqrt{3})$ will lie below the x-axis and hence, the ΔABC will contain the origin within it when C lies below the x-axis.

Hence, the required coordinates of C are $\mathbf{(0, 3 - 4\sqrt{3})}$.

(*v*) Let the third vertex of the equilateral triangle be $P(x, y)$.

The other two vertices are $A(0, 0)$ and $B(3, 0)$.

Then $\qquad AP = \sqrt{(x-0)^2 + (y-0)^2}$

$\qquad = \sqrt{x^2 + y^2}$

$\qquad BP = \sqrt{(x-3)^2 + (y-0)^2}$

$\qquad = \sqrt{x^2 - 6x + 9 + y^2}$

and $\qquad AB = \sqrt{(3-0)^2 + (0-0)^2}$

$\qquad = \sqrt{9+0} = \sqrt{9} = 3$

But, ΔABP is an equilateral triangle,

i.e. $\qquad AP = BP = AB.$

$\Rightarrow \qquad AP = BP$

$\Rightarrow \qquad \sqrt{x^2 + y^2} = \sqrt{x^2 - 6x + 9 + y^2}$

Squaring both the sides, we get

$$x^2 + y^2 = x^2 - 6x + 9 + y^2$$

$\Rightarrow \qquad -6x + 9 = 0$

$\Rightarrow \qquad 9 = 6x$

$\Rightarrow \qquad x = \frac{9}{6} = \frac{3}{2}$

Also, $\qquad AP = AB$

$\Rightarrow \qquad \sqrt{x^2 + y^2} = 3$

Squaring both the sides, we get

$$x^2 + y^2 = (3)^2 = 9$$

$$\Rightarrow \quad x^2 + y^2 = 9$$

Putting the value of $x = \frac{3}{2}$, we get

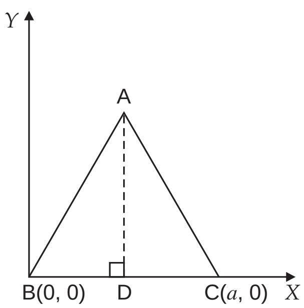
$$\left(\frac{3}{2}\right)^2 + y^2 = 9$$

$$\Rightarrow \quad y^2 = 9 - \frac{9}{4} = \frac{36-9}{4} = \frac{27}{4}$$

$$\Rightarrow \quad y = \pm\frac{3\sqrt{3}}{2}$$

Hence, the coordinates of the third vertex are $\left(\mathbf{\frac{3}{2}, \frac{3\sqrt{3}}{2}}\right)$ or $\left(\mathbf{\frac{3}{2}, \frac{-3\sqrt{3}}{2}}\right)$.

(*vi*) Since ΔABC is an equilateral triangle, so AB = BC = CA = a.



Draw a perpendicular AD on BC.

Then, $\quad BD = \frac{a}{2}$

So, in right ΔABD,

$$AD = \sqrt{AB^2 - BD^2}$$

$$= \sqrt{(a)^2 - \left(\frac{a}{2}\right)^2}$$

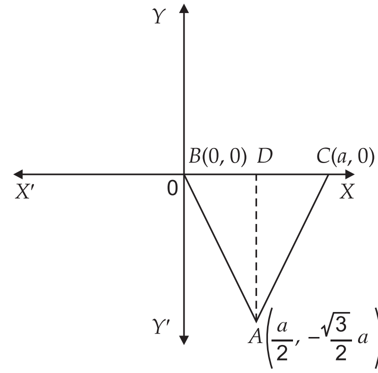
$$= \sqrt{a^2 - \frac{a^2}{4}}$$

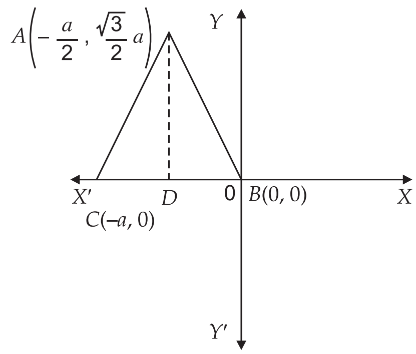
$$= \sqrt{\frac{3}{4}a^2}$$

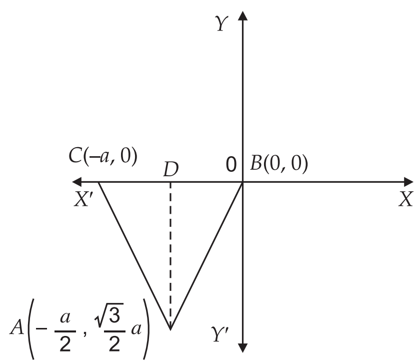
$$= \frac{\sqrt{3}}{2}a$$

So, coordinates of vertex A are $\left(\frac{a}{2}, \frac{\sqrt{3}}{2}a\right)$. From the figure, coordinates of vertex B are (0, 0) and coordinates of vertex C are (a, 0).

Fulfilling all the given conditions, we can have three more equilateral triangles alongwith their coordinates.







So, the coordinates of vertices of ΔABC are $\left(\frac{a}{2}, \frac{\sqrt{3}}{2}a\right)$, **(0, 0), ($a$, 0)** or $\left(\mathbf{\frac{a}{2}, -\frac{\sqrt{3}}{2}a}\right)$, **(0, 0), ($a$, 0)** or $\left(\mathbf{-\frac{a}{2}, \frac{\sqrt{3}}{2}a}\right)$, **(0, 0), (–$a$, 0)** or $\left(\mathbf{-\frac{a}{2}, -\frac{\sqrt{3}}{2}a}\right)$, **(0, 0), (–$a$, 0)**.

11. Let the three vertices of the triangle be A(0, –1), B(2, 1) and C(–2, 1).

Then, $\quad AB = \sqrt{(2-0)^2 + (1+1)^2}$

$$= \sqrt{4+4}$$

$$= \sqrt{8} = 2\sqrt{2} \text{ units}$$

$$BC = \sqrt{(-2-2)^2 + (1-1)^2}$$

$$= \sqrt{16+0}$$

$$= \sqrt{16} = \mathbf{4 \text{ units}}$$

$$AC = \sqrt{(-2-0)^2 + (1+1)^2}$$
$$= \sqrt{4+4}$$
$$= \sqrt{8}$$
$$= 2\sqrt{2} \textbf{ units}$$

Since, $AB = AC = \mathbf{2\sqrt{2}}$ **units**

So, ΔABC is an isosceles triangle.

Also, $AB^2 + AC^2 = (2\sqrt{2})^2 + (2\sqrt{2})^2$
$$= 8 + 8 = 16 = BC^2$$

$\Rightarrow$ ΔABC is a right angled triangle.

Hence, ΔABC is an **isosceles right-angled triangle**.

12. Let the coordinates of the circumcentre of the triangle be O(x, y) and the vertices of the triangle be A(3, 7), B(0, 6) and C(–1, 5).

Then, $AO = \sqrt{(x-3)^2 + (y-7)^2}$
$$= \sqrt{x^2 - 6x + 9 + y^2 - 14y + 49}$$
$$= \sqrt{x^2 + y^2 - 6x - 14y + 58}$$
$$\Rightarrow AO^2 = x^2 + y^2 - 6x - 14y + 58$$
$$BO = \sqrt{(x-0)^2 + (y-6)^2}$$
$$= \sqrt{x^2 + y^2 - 12y + 36}$$
$$\Rightarrow BO^2 = x^2 + y^2 - 12y + 36$$
$$CO = \sqrt{(x+1)^2 + (y-5)^2}$$
$$= \sqrt{x^2 + 2x + 1 + y^2 - 10y + 25}$$
$$= \sqrt{x^2 + y^2 + 2x - 10y + 26}$$
$$\Rightarrow CO^2 = x^2 + y^2 + 2x - 10y + 26$$

But, $AO = BO = CO$
$$\Rightarrow AO^2 = BO^2 = CO^2$$
$$\Rightarrow AO^2 = BO^2$$
$$\Rightarrow x^2 + y^2 - 6x - 14y + 58 = x^2 + y^2 - 12y + 36$$
$$\Rightarrow -6x - 2y + 22 = 0$$
$$\Rightarrow 3x + y = 11 \quad \ldots(1)$$

Also, $BO^2 = CO^2$
$$\Rightarrow x^2 + y^2 - 12y + 36 = x^2 + y^2 + 2x - 10y + 26$$
$$\Rightarrow 10 = 2x + 2y$$
$$\Rightarrow x + y = 5 \quad \ldots(2)$$

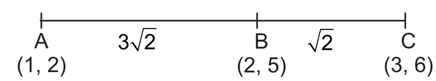
From equations (1) and (2), we get
$$3x + y = 11$$
$$\underline{_{-}x \underset{-}{+} y = 5}$$
$$2x = 6$$
$$\Rightarrow x = \frac{6}{2} = 3$$

Putting $x = 3$ in equation (1), we get
$$3 \times 3 + y = 11 \Rightarrow y = 11 - 9 = 2$$

Hence, the coordinates of circumcentre are **(3, 2)**.

Now, circumradius $= BO = \sqrt{x^2 + y^2 - 12y + 36}$
$$= \sqrt{(3)^2 + (2)^2 - 12 \times 2 + 36}$$
$$= \sqrt{9 + 4 - 24 + 36}$$
$$= \sqrt{25} = \mathbf{5 \text{ units}}$$

13. We have
$$AB = \sqrt{(-1-2)^2 + (2-5)^2}$$
$$= \sqrt{9+9}$$
$$= \sqrt{18} = 3\sqrt{2}$$
$$AC = \sqrt{(-1-3)^2 + (2-6)^2}$$
$$= \sqrt{16+16}$$
$$= \sqrt{32}$$
$$= 4\sqrt{2}$$
$$BC = \sqrt{(2-3)^2 + (5-6)^2}$$
$$= \sqrt{1+1}$$
$$= \sqrt{2}$$



$\therefore$ We see that
$$AB + BC = 3\sqrt{2} + \sqrt{2}$$
$$= 4\sqrt{2}$$
$$= AC$$

Hence, A, B, C are collinear.

14. Let the four given points be A(0, –1), B(2, 1), C(0, 3) and D(–2, 1).

Then, $AB = \sqrt{(2-0)^2 + (1+1)^2}$
$$= \sqrt{4+4} = \sqrt{8} = 2\sqrt{2} \text{ units}$$
$$BC = \sqrt{(0-2)^2 + (3-1)^2}$$
$$= \sqrt{4+4} = \sqrt{8} = 2\sqrt{2} \text{ units}$$
$$CD = \sqrt{(-2-0)^2 + (1-3)^2}$$
$$= \sqrt{4+4} = \sqrt{8} = 2\sqrt{2} \text{ units}$$
$$AD = \sqrt{(-2-0)^2 + (1+1)^2}$$
$$= \sqrt{4+4} = \sqrt{8} = 2\sqrt{2} \text{ units}$$
$$\Rightarrow AB = BC = CD = AD = 2\sqrt{2} \text{ units}$$

Also, $AC = \sqrt{(0-0)^2 + (3+1)^2}$
$$= \sqrt{0+16}$$
$$= \sqrt{16} = 4 \text{ units}$$

and $BD = \sqrt{(-2-2)^2 + (1-1)^2}$
$$= \sqrt{16+0}$$

$= \sqrt{16} = 4$ units

i.e. $AC = BD$

Therefore, ABCD is a square.

i.e. the given four points are vertices of a square.

15. (*i*) We have

$$AB = \sqrt{(1-5)^2 + (2-4)^2} = \sqrt{16+4} = 2\sqrt{5}$$

$$BC = \sqrt{(5-3)^2 + (4-8)^2} = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$$

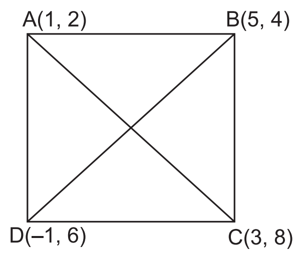
$$CD = \sqrt{(3+1)^2 + (8-6)^2} = \sqrt{16+4} = \sqrt{20} = 2\sqrt{5}$$

$$DA = \sqrt{(-1-1)^2 + (6-2)^2} = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$$

$$AC = \sqrt{(3-1)^2 + (8-2)^2} = \sqrt{4+36} = \sqrt{40} = 2\sqrt{10}$$

$$BD = \sqrt{(5+1)^2 + (4-6)^2} = \sqrt{36+4} = \sqrt{40} = 2\sqrt{10}$$

Hence, AB = BC = CD = DA and AC = BD.



Hence, all sides are of equal length and the two diagonals are also equal.

Hence, the figure ABCD is a square and so the given points are the vertices of a square.

(*ii*) We have

$$AB = \sqrt{(3-0)^2 + (2-5)^2} = \sqrt{9+9} = 3\sqrt{2}$$

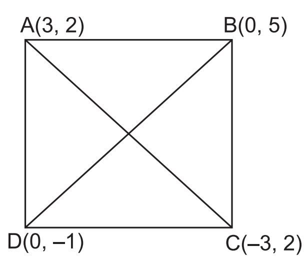
$$BC = \sqrt{(0+3)^2 + (5-2)^2} = \sqrt{9-9} = 3\sqrt{2}$$

$$CD = \sqrt{(-3-0)^2 + (2+1)^2} = \sqrt{9+9} = 3\sqrt{2}$$

$$DA = \sqrt{(0-3)^2 + (-1-2)^2} = \sqrt{9+9} = 3\sqrt{2}$$

$$AC = \sqrt{(3+3)^2 + (2-2)^2} = \sqrt{6^2} = 6$$

$$BD = \sqrt{(0-6)^2 + (5+1)^2} = \sqrt{6^2} = 6$$



$\therefore$ AB = BC = CD = DA, i.e. all sides are of equal length and AC = BD, i.e. the two diagonals are of equal length.

ABCD is a square, i.e. the given points are the vertices of a square.

(*iii*) We have

$$AB = \sqrt{(3-6)^2 + 5^2} = \sqrt{9+25} = \sqrt{34}$$

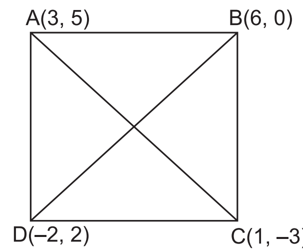
$$BC = \sqrt{(6-1)^2 + (0+3)^2} = \sqrt{25+9} = \sqrt{34}$$

$$CD = \sqrt{(-2-1)^2 + (2+3)^2} = \sqrt{9+25} = \sqrt{34}$$

$$DA = \sqrt{(-2-3)^2 + (2-5)^2} = \sqrt{25+9} = \sqrt{34}$$

$$AC = \sqrt{(3-1)^2 + (5+3)^2} = \sqrt{4+64} = \sqrt{68}$$

$$BD = \sqrt{(3-1)^2 + (5+3)^2} = \sqrt{4+64} = \sqrt{68}$$



$\therefore$ AB = BC = CD = DA, i.e. all the sides are of equal length and AC = BD, i.e. the two diagonals are of equal length.

$\therefore$ ABCD is a square and so the given points are the vertices of a square.

16. (*i*) Let the given points be A(3, 2), B(11, 8), C(8, 12) and D(0, 6).

Then, $AB = \sqrt{(11-3)^2 + (8-2)^2}$

$= \sqrt{(8)^2 + (6)^2}$

$= \sqrt{64+36} = \sqrt{100} = 10$ units

$BC = \sqrt{(8-11)^2 + (12-8)^2}$

$= \sqrt{(-3)^2 + (4)^2}$

$= \sqrt{9+16} = \sqrt{25} = 5$ units

$CD = \sqrt{(0-8)^2 + (6-12)^2}$

$= \sqrt{(-8)^2 + (-6)^2}$

$= \sqrt{64+36}$

$= \sqrt{100} = 10$ units

$AD = \sqrt{(0-3)^2 + (6-2)^2}$

$= \sqrt{(-3)^2 + (4)^2}$

$= \sqrt{9+16}$

$= \sqrt{25} = 5$ units

Also, $AC = \sqrt{(8-3)^2 + (12-2)^2}$

$= \sqrt{(5)^2 + (10)^2}$

$= \sqrt{25+100} = \sqrt{125}$ units

and $\quad BD = \sqrt{(0-11)^2 + (6-8)^2}$

$= \sqrt{(-11)^2 + (-2)^2}$

$= \sqrt{121+4}$

$= \sqrt{125}$ units

$\Rightarrow$ In quadrilateral ABCD,

AB = CD = 10 units

and $\quad$ BC = AD = 5 units.

i.e. the opposite sides are equal.

Also, the diagonals,

AC = BD = $\sqrt{125}$ units

Hence, the given points are vertices of a rectangle.

(*ii*) Let the given points be P(0, –1), Q(–2, 3), R(6, 7) and S(8, 3).

Then, $\quad PQ = \sqrt{(-2-0)^2 + (3+1)^2}$

$= \sqrt{4+16}$

$= \sqrt{20} = 2\sqrt{5}$ units

$QR = \sqrt{(6+2)^2 + (7-3)^2}$

$= \sqrt{64+16}$

$= \sqrt{80} = 4\sqrt{5}$ units

$RS = \sqrt{(8-6)^2 + (3-7)^2}$

$= \sqrt{4+16}$

$= \sqrt{20} = 2\sqrt{5}$ units

and $\quad PS = \sqrt{(8-0)^2 + (3+1)^2}$

$= \sqrt{64+16}$

$= \sqrt{80}$

$= 4\sqrt{5}$ units

Also, $\quad PR = \sqrt{(6-0)^2 + (7+1)^2}$

$= \sqrt{36+64}$

$= \sqrt{100} = 10$ units

and $\quad QS = \sqrt{(8+2)^2 + (3-3)^2}$

$= \sqrt{100+0}$

$= \sqrt{100} = 10$ units

In the quadrilateral PQRS

PQ = RS = $2\sqrt{5}$ units

and $\quad$ QR = PS = $4\sqrt{5}$ units

i.e. the opposite sides are equal.

Also, diagonals, PR = QS = 10 units.

i.e. the diagonals are equal.

Hence, the given points are vertices of a rectangle.

(*iii*) A(2, –2), B(14, 10), C(11, 13) and D(–1, 1) are four given points.

Then, $\quad AB = \sqrt{(14-2)^2 + (10+2)^2}$

$= \sqrt{144+144}$

$= \sqrt{2 \times 144} = 12\sqrt{2}$ units

$BC = \sqrt{(11-14)^2 + (13-10)^2}$

$= \sqrt{(-3)^2 + (3)^2}$

$= \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$ units

$CD = \sqrt{(-1-11)^2 + (1-13)^2}$

$= \sqrt{(-12)^2 + (-12)^2}$

$= \sqrt{144+144}$

$= \sqrt{2 \times 144} = 12\sqrt{2}$ units

and $\quad AD = \sqrt{(-1-2)^2 + (1+2)^2}$

$= \sqrt{9+9}$

$= \sqrt{18} = 3\sqrt{2}$ units

Also, $\quad AC = \sqrt{(11-2)^2 + (13+2)^2}$

$= \sqrt{81+225}$

$= \sqrt{306}$ units

and $\quad BD = \sqrt{(-1-14)^2 + (1-10)^2}$

$= \sqrt{(-15)^2 + (-9)^2}$

$= \sqrt{225+81}$

$= \sqrt{306}$ units

In the quadrilateral ABCD,

AB = CD = $12\sqrt{2}$ units

and $\quad$ BC = AD = $3\sqrt{2}$ units

i.e. opposite sides of the quadrilateral are equal.

Also, $\quad$ AC = BD = $\sqrt{306}$ units

i.e. the diagonals are equal.

Hence, A, B, C and D are vertices of a rectangle.

(*iv*) Let A be the point (1, 1), B be the point (–1, 5), C be the point (7, 9) and D be the point (9, 5).

Then $\quad AB = \sqrt{(1+1)^2 + (1-5)^2}$

$= \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$

$BC = \sqrt{(-1-7)^2 + (5-9)^2}$

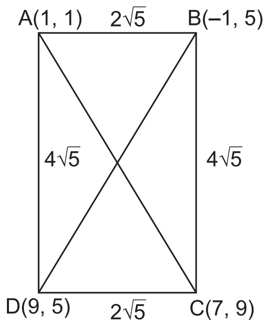
$= \sqrt{64+16} = \sqrt{80} = 4\sqrt{5}$

$CD = \sqrt{(9-7)^2 + (5-9)^2}$

$= \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$

$$AD = \sqrt{(9-1)^2 + (5-1)^2}$$

$$= \sqrt{64+16} = \sqrt{80} = 4\sqrt{5}$$



$\therefore$ We see that AB = CD = $2\sqrt{5}$ and BC = AD = $4\sqrt{5}$

$\therefore$ Opposite sides of the quadrilateral ABCD are equal. Hence, ABCD is a $\parallel$gm.

Also, $$AC = \sqrt{(7-1)^2 + (9-1)^2}$$

$$= \sqrt{36+64} = \sqrt{100} = 10$$

and $$BD = \sqrt{(9+1)^2 + (5-5)^2}$$

$$= \sqrt{100} = 10$$

$\therefore$ Diagonals AC = BD

$\therefore$ ABCD is a rectangle.

Also, area of the rectangle

$$= AB \times AD$$

$$= 2\sqrt{5} \times 4\sqrt{5} \text{ unit}^2 = 40 \text{ units}^2$$

17. (*i*) $$AB = \sqrt{(1-3)^2 + (1-5)^2}$$

$$= \sqrt{4+16} = \sqrt{20} = 2\sqrt{5} \text{ units}$$

$$BC = \sqrt{(5-1)^2 + (3-1)^2} = \sqrt{16+4}$$

$$= \sqrt{20} = 2\sqrt{5} \text{ units}$$

$$CD = \sqrt{(7-5)^2 + (7-3)^2}$$

$$= \sqrt{4+16}$$

$$= \sqrt{20} = 2\sqrt{5} \text{ units}$$

$$AD = \sqrt{(7-3)^2 + (7-5)^2}$$

$$= \sqrt{16+4}$$

$$= \sqrt{20} = 2\sqrt{5} \text{ units}$$

$\because$ AB = BC = CD = AD

$\therefore$ A, B, C, D are vertices of a rhombus.

(*ii*) $$AB = \sqrt{(-5+3)^2 + (-5-2)^2}$$

$$= \sqrt{4+49}$$

$$= \sqrt{53} \text{ units}$$

$$BC = \sqrt{(2+5)^2 + (-3+5)^2}$$

$$= \sqrt{49+4} = \sqrt{53} \text{ units}$$

$$CD = \sqrt{(4-2)^2 + (4+3)^2}$$

$$= \sqrt{4+49} = \sqrt{53} \text{ units}$$

$$AD = \sqrt{(4+3)^2 + (4-2)^2}$$

$$= \sqrt{49+4} = \sqrt{53} \text{ units}$$

Since, AB = BC = CD = AD = $\sqrt{53}$ units

So, A, B, C, D are vertices of a rhombus.

18. Diameter of the circle $= \sqrt{(3+3)^2 + (4+4)^2}$

$$= \sqrt{36+64}$$

$$= \sqrt{100}$$

$$= 10 \text{ units}$$

Therefore, radius of the circle $= \dfrac{\text{Diameter}}{2}$

$$= \frac{10}{2} = \mathbf{5 \text{ units}}$$

19. Radius of circle $= \sqrt{(-6-0)^2 + (8-0)^2}$

$$= \sqrt{36+64}$$

$$= \sqrt{100} = \mathbf{10 \text{ units}}$$

20. The centre of the circle is at origin. So, its coordinates are O(0, 0).

(*i*) A(–9, 4)

$$OA = \sqrt{(-9-0)^2 + (4-0)^2}$$

$$= \sqrt{81+16} = \sqrt{97} < 10$$

So, the point (–9, 4) is **inside** the circle.

(*ii*) B(–10, 0)

$$OB = \sqrt{(-10-0)^2 + (0-0)^2}$$

$$= \sqrt{100+0} = \sqrt{100} = 10.$$

So, the point (–10, 0) is **on the circle**.

(*iii*) C(0, 11)

$$OC = \sqrt{(0-0)^2 + (11-0)^2}$$

$$= \sqrt{0+121} = \sqrt{121}$$

$$= 11 > 10$$

So, the point (0, 11) is **outside** the circle.

(*iv*) D(6, 8)

$$OD = \sqrt{(6-0)^2 + (8-0)^2}$$

$$= \sqrt{36+64}$$

$$= \sqrt{100} = 10$$

Hence, the point (6, 8) is **on the circle**.

21. Let the point to be proved as centre of the circle be O(–1, 2) and the other four given points be A(–3, 11), B(5, 9), C(8, 0) and D(6, 8).

So, $$OA = \sqrt{(-3+1)^2 + (11-2)^2}$$

$$= \sqrt{4+81} = \sqrt{85} \text{ units}$$

$$OB = \sqrt{(5+1)^2 + (9-2)^2}$$

$$= \sqrt{36+49} = \sqrt{85} \text{ units}$$

$$OC = \sqrt{(8+1)^2+(0-2)^2}$$

$$= \sqrt{81+4} = \sqrt{85} \text{ units}$$

$$OD = \sqrt{(6+1)^2+(8-2)^2}$$

$$= \sqrt{49+36}$$

$$= \mathbf{\sqrt{85} \text{ units}}$$

Since, OA = OB = OC = OD and the points A, B, C and D are on the circle. Hence, (–1, 2) is the centre of the circle.

22. Let the coordinates of the centre of the circle be O(x, y) and the three given points through which the circle is passing be A(0, 0), B(–2, 1) and C(–3, 2).

Then, $OA = \sqrt{(0-x)^2+(0-y)^2}$

$$= \sqrt{x^2+y^2}$$

$\Rightarrow$ $OA^2 = x^2 + y^2$

$$OB = \sqrt{(-2-x)^2+(1-y)^2}$$

$$= \sqrt{4+x^2+4x+1+y^2-2y}$$

$\Rightarrow$ $OB^2 = x^2 + y^2 + 4x - 2y + 5$

$$OC = \sqrt{(-3-x)^2+(2-y)^2}$$

$$= \sqrt{9+x^2+6x+4+y^2-4y}$$

$\Rightarrow$ $OC^2 = x^2 + y^2 + 6x - 4y + 13$

Since, OA = OB = OC = radius

$\Rightarrow$ $OA^2 = OB^2 = OC^2$

$\Rightarrow$ $OA^2 = OB^2$

$\Rightarrow$ $x^2 + y^2 = x^2 + y^2 + 4x - 2y + 5$

$\Rightarrow$ $4x - 2y + 5 = 0$ …(1)

And $OB^2 = OC^2$

$\Rightarrow x^2 + y^2 + 4x - 2y + 5 = x^2 + y^2 + 6x - 4y + 13$

$\Rightarrow$ $-2x + 2y = 8$

$\Rightarrow$ $-x + y = 4$ …(2)

From equation (2), $y = 4 + x$.

Putting the value of y in equation (1), we get

$4x - 2(4 + x) + 5 = 0$

$\Rightarrow$ $4x - 8 - 2x + 5 = 0$

$\Rightarrow$ $2x - 3 = 0$

$\Rightarrow$ $x = \frac{3}{2}$

Putting the value of $x = \frac{3}{2}$ in equation (2), we get

$$-\frac{3}{2}+y = 4 \Rightarrow y = 4 + \frac{3}{2} = \frac{11}{2}$$

Hence, the coordinates of the centre are $\left(\mathbf{\frac{3}{2}, \frac{11}{2}}\right)$

and radius $= OA = \sqrt{x^2+y^2}$

$$= \sqrt{\left(\frac{3}{2}\right)^2+\left(\frac{11}{2}\right)^2}$$

$$= \sqrt{\frac{9}{4}+\frac{121}{4}}$$

$$= \sqrt{\frac{130}{4}} \text{ units}$$

$$= \mathbf{\frac{\sqrt{130}}{2} \text{ units}}$$

For Standard Level

23. Let the three given points be A(6, 9), B(0, 1) and C(–6, –7).

Then, $AB = \sqrt{(0-6)^2+(1-9)^2}$

$$= \sqrt{36+64} = \sqrt{100} = 10 \text{ units}$$

$$BC = \sqrt{(-6-0)^2+(-7-1)^2}$$

$$= \sqrt{36+64}$$

$$= \sqrt{100} = 10 \text{ units}$$

$$AC = \sqrt{(-6-6)^2+(-7-9)^2}$$

$$= \sqrt{144+256}$$

$$= \sqrt{400} = 20 \text{ units}$$

Since, AB + BC = 10 + 10 = 20 = AC

So, points A, B, C are collinear, i.e. points A, B, C do not form a triangle.

24. We have

$$AB = \sqrt{2^2+2^2} = \sqrt{8} = 2\sqrt{2}$$

$$DE = \sqrt{4^2+4^2} = \sqrt{32} = 4\sqrt{2}$$

$$BC = \sqrt{2^2+2^2} = \sqrt{8} = 2\sqrt{2}$$

$$EF = \sqrt{4^2+4^2} = \sqrt{32} = 4\sqrt{2}$$

$$AC = \sqrt{(2+2)^2} = 4$$

and $DF = \sqrt{(4+4)^2} = \sqrt{64} = 8$

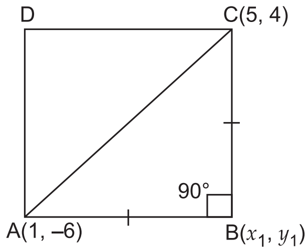
$\therefore$ $\frac{AB}{DE} = \frac{2\sqrt{2}}{4\sqrt{2}} = \frac{1}{2}, \frac{BC}{EF} = \frac{2\sqrt{2}}{4\sqrt{2}} = \frac{1}{2}$

and $\frac{AC}{DF} = \frac{4}{8} = \frac{1}{2}$

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = \frac{1}{2}$$

$\therefore$ $\Delta ABC \sim \Delta DEF$

25. Let the coordinates of the opposite vertices A and C of the square ABCD be A(1, –6) and C(5, 4). Since ABCD is a square,



$\therefore$ $AB^2 = BC^2$ and $\angle ABC = 90°$.

$\therefore$ $(x_1 - 1)^2 + (y_1 + 6)^2 = (x_1 - 5)^2 + (y_1 - 4)^2$ ...(1)

and by Pythagoras' theorem, we have

$$AB^2 + BC^2 = AC^2$$

$\Rightarrow$ $(x_1 - 1)^2 + (y_1 + 6)^2 + (x_1 - 5)^2 + (y_1 - 4)^2$
$= (5 - 1)^2 + (4 + 6)^2$
$= 16 + 100 = 116$ ...(2)

From (1), we have

$-2x_1 + 12y_1 + 37 = -10x_1 - 8y_1 + 41$

$\Rightarrow$ $8x_1 + 20y_1 - 4 = 0$

$\Rightarrow$ $2x_1 + 5y_1 - 1 = 0$

$\Rightarrow$ $x_1 = \dfrac{1 - 5y_1}{2}$ ...(3)

Also, from (2), we have

$x_1^2 - 2x_1 + 1 + y_1^2 + 12y_1 + 36 + x_1^2 - 10x_1 + 25$
$+ y_1^2 - 8y + 16 = 116$

$\Rightarrow$ $2x_1^2 + 2y_1^2 - 12x_1 + 4y_1 - 38 = 0$

$\Rightarrow$ $x_1^2 + y_1^2 - 6x_1 + 2y_1 - 19 = 0$ ...(4)

$\therefore$ From (3) and (4), we get

$$\frac{(1 - 5y_1)}{4} + y_1^2 - \frac{6(1 - 5y_1)}{2} + 2y_1 - 19 = 0$$

$\Rightarrow$ $1 - 10y_1 + 25y_1^2 + 4y_1^2 - 12 + 60y_1 + 8y_1 - 76 = 0$

$\Rightarrow$ $29y_1^2 + 58y_1 - 87 = 0$

$\Rightarrow$ $y_1^2 + 2y_1 - 3 = 0$

$\Rightarrow$ $y_1^2 + 3y_1 - y_1 - 3 = 0$

$\Rightarrow$ $y_1(y_1 + 3) - 1(y_1 + 3) = 0$

$\Rightarrow$ $(y_1 - 1)(y_1 + 3) = 0$

$\therefore$ Either $y_1 - 1 = 0 \Rightarrow y_1 = 1$

Or $y_1 + 3 = 0 \Rightarrow y_1 = -3$

$\therefore$ From (3), when $y_1 = -3$, then $x_1 = \dfrac{1 + 15}{2} = 8$

and when $y_1 = 1$, then $x_1 = \dfrac{1 - 5}{2} = -2$

$\therefore$ The required coordinates of the remaining vertices are **(8, –3)** and **(–2, 1)**.

26. Let the four vertices of the rhombus be A(3, 4), B(–2, 3), C(–3, –2) and D(x, y).

Then, $AB = \sqrt{(-2-3)^2 + (3-4)^2}$
$= \sqrt{25+1} = \sqrt{26}$

$\Rightarrow$ $AB^2 = 26$

$BC = \sqrt{(-3+2)^2 + (-2-3)^2}$
$= \sqrt{(-1)^2 + (-5)^2}$
$= \sqrt{1+25} = \sqrt{26}$

$\Rightarrow$ $BC^2 = 26$

$CD = \sqrt{(x+3)^2 + (y+2)^2}$
$= \sqrt{x^2 + 6x + 9 + y^2 + 4y + 4}$
$= \sqrt{x^2 + y^2 + 6x + 4y + 13}$

$\Rightarrow$ $CD^2 = x^2 + y^2 + 6x + 4y + 13$

And $AD = \sqrt{(x-3)^2 + (y-4)^2}$
$= \sqrt{x^2 - 6x + 9 + y^2 - 8y + 16}$
$= \sqrt{x^2 + y^2 - 6x - 8y + 25}$

$\Rightarrow$ $AD^2 = x^2 + y^2 - 6x - 8y + 25$

Since, ABCD is a rhombus.

So, $AB = BC = CD = AD$

$\Rightarrow$ $AB^2 = BC^2 = CD^2 = AD^2$

$\Rightarrow$ $CD^2 = AD^2$

$\Rightarrow$ $x^2 + y^2 + 6x + 4y + 13 = x^2 + y^2 - 6x - 8y + 25$

$\Rightarrow$ $12x + 12y = 12$

$\Rightarrow$ $x + y = 1$ ...(1)

Also, $CD^2 = BC^2$

$\Rightarrow$ $x^2 + y^2 + 6x + 4y + 13 = 26$

$\Rightarrow$ $x^2 + y^2 + 6x + 4y - 13 = 0$ ...(2)

From equations (1) and (2), we get

$x^2 + (1 - x)^2 + 6x + 4(1 - x) - 13 = 0$

$\Rightarrow$ $x^2 + 1 + x^2 - 2x + 6x + 4 - 4x - 13 = 0$

$\Rightarrow$ $2x^2 - 8 = 0$

$\Rightarrow$ $2(x^2 - 4) = 0$

$\Rightarrow$ $2(x - 2)(x + 2) = 0$

$\Rightarrow$ $x - 2 = 0$ or $x + 2 = 0$

$\Rightarrow$ $x = 2$ or $x = -2$
(–ve value rejected)

From equation (1),

When $x = 2$, $y = 1 - 2 = -1$

Hence, the coordinates of the fourth vertex of the rhombus is **(2, –1)**.

27. (*i*) $AB = \sqrt{(4-3)^2 + (5-0)^2}$
$= \sqrt{1+25} = \sqrt{26}$ units

$BC = \sqrt{(-1-4)^2 + (4-5)^2}$
$= \sqrt{25+1} = \sqrt{26}$ units

$CD = \sqrt{(-2+1)^2 + (-1-4)^2}$
$= \sqrt{1+25} = \sqrt{26}$ units

$$AD = \sqrt{(-2-3)^2+(-1-0)^2}$$
$$= \sqrt{25+1} = \sqrt{26} \text{ units}$$

Also, $$AC = \sqrt{(-1-3)^2+(4-0)^2}$$
$$= \sqrt{16+16}$$
$$= \sqrt{32} = 4\sqrt{2} \text{ units}$$

and $$BD = \sqrt{(-2-4)^2+(-1-5)^2}$$
$$= \sqrt{36+36} = \sqrt{2\times 36}$$
$$= 6\sqrt{2} \text{ units}$$

Since, $AB = BC = CD = AD = \sqrt{26}$ units

But $AC \neq BD$

and therefore, ABCD is a rhombus but it is not a square.

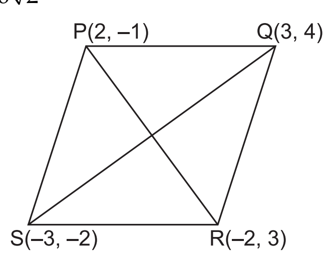
$$\text{Area of the rhombus} = \frac{1}{2} \times d_1 \times d_2$$
$$= \frac{1}{2} \times AC \times BD$$
$$= \frac{1}{2}\times 4\sqrt{2}\times 6\sqrt{2}$$
$$= \mathbf{24 \text{ sq units}}$$

(*ii*) We have

$$PQ = \sqrt{(3-2)^2+(4+1)^2} = \sqrt{1+25} = \sqrt{26}$$
$$QR = \sqrt{(-2-3)^2+(3-4)^2} = \sqrt{25+1} = \sqrt{26}$$
$$RS = \sqrt{(-3+2)^2+(-2-3)^2} = \sqrt{1+25} = \sqrt{26}$$
$$SP = \sqrt{(2+3)^2+(-1+2)^2} = \sqrt{25+1} = \sqrt{26}$$

Also,

$$PR = \sqrt{(-2-2)^2+(3+1)^2} = \sqrt{16+16} = \sqrt{32}$$
$$= 4\sqrt{2}$$
$$QS = \sqrt{(-3-3)^2+(-2-4)^2} = \sqrt{36+36} = \sqrt{72}$$
$$= 6\sqrt{2}$$



$\therefore$ PQ = QR = RS = SP, i.e., all the sides of the quadrilateral PQRS are equal. But the diagonals PR and QS are not of equal length. Hence, the figure PQRS is a rhombus, but not a square whose diagonals are of equal length.

Also, area of the rhombus $= \frac{1}{2}\times PR\times QS$

$$= \frac{1}{2}\times 4\sqrt{2}\times 6\sqrt{2} \text{ sq units}$$
$$= \mathbf{24 \text{ sq units}}$$

28. $$\text{Diameter} = \sqrt{(2\sin\theta+2\sin\theta)^2+(2\cos\theta+2\cos\theta)^2}$$
$$= \sqrt{(4\sin\theta)^2+(4\cos\theta)^2}$$
$$= \sqrt{16(\sin^2\theta+\cos^2\theta)}$$
$$= \sqrt{16\times 1} = \sqrt{16} = 4$$
$$\Rightarrow \text{Radius} = \frac{\text{Diameter}}{2} = \frac{4}{2} = \mathbf{2 \text{ units}}$$

29. $$OA = \sqrt{(2-4)^2+(3-3)^2}$$
$$= \sqrt{4+0} = \sqrt{4}$$
$$= 2 \text{ units}$$
$$OB = \sqrt{(2-x)^2+(3-5)^2}$$
$$= \sqrt{4+x^2-4x+4}$$
$$= \sqrt{x^2-4x+8}$$

Since, OA and OB both are radii of the same circle,

So, $OA = OB$

$\Rightarrow$ $OA^2 = OB^2$

$\Rightarrow$ $x^2 - 4x + 8 = 4$

$\Rightarrow$ $x^2 - 4x + 4 = 0$

$\Rightarrow$ $(x - 2)^2 = 0$

$\Rightarrow$ $x - 2 = 0$

$\Rightarrow$ $\mathbf{x = 2}$

30. We denote the current coordinates in the equation of a circle by X and Y and (x, y) represents a particular point on the circle. Let the radius of the circle be r.

$\because$ Its centre is the point O(2, – 3y).

$\therefore$ The eqn. of the circle is $(X - 2) + (Y + 3y)^2 = r^2$ …(1)

Since the points A(–1, y) and B(5, 7) lie on the circle (1), hence writing X = –1 and Y = y in (1), get

$(-1 -2)^2 + (y + 3y)^2 = r^2$

$\Rightarrow$ $r^2 = 9 + 16y^2$ …(2)

Again, putting X = 5 and Y = 7 in (1), we get

$(5 - 2)^2 + (7 + 3y)^2 = r^2$

$\Rightarrow$ $r^2 = 9 + 49 + 9y^2 + 42y$

$\Rightarrow$ $9 + 16y^2 = 9 + 49 + 9y^2 + 42y$ [From (2)]

$\Rightarrow$ $7y^2 - 42y - 49 = 0$

$\Rightarrow$ $y^2 - 6y - 7 = 0$

$$\therefore \quad y = \frac{6 \pm \sqrt{36+4\times 7}}{2}$$
$$= \frac{6\pm\sqrt{64}}{2} = \frac{6\pm 8}{2} = 7, -1$$

When $y = 7$, then from (2)

$$r = \sqrt{9+16\times 49} = \sqrt{9+78+4} = \sqrt{793}$$

When $y = -1$, then from (2), $r = \sqrt{9+16} = 5$.

Hence, the required values of y and r are $\mathbf{y = -1, r = 5}$ or $\mathbf{y = 7, r = \sqrt{793}}$

31. Radius $= \dfrac{\text{Diameter}}{2} = \dfrac{10}{2} = 5$ units

Now, $\quad$ radius $= \sqrt{(3x-1+2)^2+(5x+1-3)^2}$

$\Rightarrow \quad 5 = \sqrt{9x^2+1+6x+25x^2-20x+4}$

$\Rightarrow \quad 25 = 34x^2 - 14x + 5$

$\Rightarrow \quad 34x^2 - 14x - 20 = 0$

$\Rightarrow \quad 34x^2 - 34x + 20x - 20 = 0$

$\Rightarrow \quad 34x(x-1) + 20(x-1) = 0$

$\Rightarrow \quad (x-1)(34x+20) = 0$

$\Rightarrow \quad x - 1 = 0$ or $34x + 20 = 0$

$\Rightarrow \quad x = \mathbf{1}$ or $x = -\dfrac{20}{34} = -\mathbf{\dfrac{10}{17}}$

32. We have $\quad (x-3)^2 + (y+2)^2 = (3)^2$

$\Rightarrow x^2 - 6x + 9 + y^2 + 4y + 4 = 9$

$\Rightarrow \quad x^2 + y^2 - 6x + 4y + 4 = 0$

which is the required result.

33. Let the concyclic points be A(7, 1), B(x, 9) and C(–1, y) and the centre be O(3, 4).

Then, $\quad \text{OA} = \sqrt{(3-7)^2+(4-1)^2} = \sqrt{(16+9)} = \sqrt{25} = 5$ units

$\text{OB} = \sqrt{(3-x)^2+(4-9)^2} = \sqrt{9+x^2-6x+25} = \sqrt{x^2-6x+34}$

and $\quad \text{OC} = \sqrt{(3+1)^2+(4-y)^2} = \sqrt{16+16-8y+y^2} = \sqrt{y^2-8y+32}$

But, $\quad$ OA = OB = OC = radius

$\Rightarrow \quad \text{OA}^2 = \text{OB}^2 = \text{OC}^2$

$\Rightarrow \quad \text{OA}^2 = \text{OB}^2$

$\Rightarrow \quad 25 = x^2 - 6x + 34$

$\Rightarrow \quad x^2 - 6x + 9 = 0$

$\Rightarrow \quad (x-3)^2 = 0$

$\Rightarrow \quad x - 3 = 0$

$\Rightarrow \quad x = 3$

Also, $\quad \text{OA}^2 = \text{OC}^2$

$\Rightarrow \quad 25 = y^2 - 8y + 32$

$\Rightarrow \quad y^2 - 8y + 7 = 0$

$\Rightarrow \quad y^2 - 7y - y + 7 = 0$

$\Rightarrow \quad y(y-7) - 1(y-7) = 0$

$\Rightarrow \quad (y-7)(y-1) = 0$

$\Rightarrow \quad y - 7 = 0$ or $y - 1 = 0$

$\Rightarrow \quad y = 7$ or $y = 1$

Hence, $\mathbf{x = 3, y = 1}$ **or** $\mathbf{y = 7}$.

EXERCISE 7D

For Basic and Standard Levels

1. (*i*) We have, $\quad \dfrac{3\times 2+1\times(-3)}{2+1} = \dfrac{6-3}{3} = \dfrac{3}{3} = 1$

and $\quad \dfrac{2\times(-2)+1\times 1}{2+1} = \dfrac{-4+1}{3} = \dfrac{-3}{3} = -1$

Hence, the coordinates of the point dividing the line segment joining the given points are **(1, –1)**.

(*ii*) We have,

$$\frac{2\times 6+3\times 1}{2+3} = \frac{12+3}{5} = 3$$

and $\quad \dfrac{2\times(-3)+3\times 7}{2+3} = \dfrac{-6+21}{5} = \dfrac{15}{5} = 3$

Hence, the coordinates of the point dividing the line segment joining the given points are **(3, 3)**.

(*iii*) We have,

$$\frac{3\times 2+(-2)\times 3}{2+3} = \frac{6-6}{5} = \frac{0}{5} = 0$$

and $\quad \dfrac{(-5)\times 2+5\times 3}{2+3} = \dfrac{-10+15}{5} = \dfrac{5}{5} = 1$

Hence, the coordinates of the point dividing the line segment joining the given points are **(0, 1)**.

2. Let the coordinates of B be (x, y).

Then, $\quad -4 = \dfrac{3\times x+5\times 2}{3+5}$

$\Rightarrow \quad 3x = -32 - 10 = -42$

$\Rightarrow \quad x = \dfrac{-42}{3} = -14$

and $\quad 1 = \dfrac{3\times y+5\times(-2)}{3+5}$

$\Rightarrow \quad 8 + 10 = 3y \Rightarrow y = \dfrac{18}{3} = 6$

Hence, the coordinates of B are **(–14, 6)**.

3. C(–1, 2)

A(2, 5) $\qquad$ B(x, y)

ratio → 3 : 4

$x = \dfrac{ax_2+bx_1}{a+b} \qquad y = \dfrac{ay_2+by_1}{a+b}$

$-1 = \dfrac{3x+4(2)}{3+4} \qquad 2 = \dfrac{3y+20}{3+4}$

$$-1 = \frac{3x+8}{7} \qquad 2 = \frac{3y+20}{7}$$
$$-7 = 3x + 8 \qquad 14 = 3y + 20$$
$$3x = -15 \qquad 3y = -6$$
$$x = -5 \qquad y = -2$$

Now $x^2 + y^2 = (-5)^2 + (-2)^2$
$= 25 + 4 = \mathbf{29}$

4. (*i*) Let the coordinates of point P be (x, y).

Now, $\frac{AB}{PB} = \frac{1}{3}$

$\Rightarrow$ 3AB = PB

P(x, y) A(3, 5) B(–7, 9)

$\Rightarrow$ 3AB = PA + AB

$\Rightarrow$ 2AB = PA

$\Rightarrow$ $2 = \frac{PA}{AB}$

$\Rightarrow$ PA : AB = 2 : 1

Then, $3 = \frac{2 \times (-7) + 1 \times x}{2+1}$

$\Rightarrow$ $9 + 14 = x \Rightarrow x = 23$

and $5 = \frac{2 \times 9 + 1 \times y}{2+1}$

$\Rightarrow$ $15 = 18 + y \Rightarrow y = 15 - 18 = -3$

Hence, the coordinates of point P**(23, –3)**.

(*ii*) Let the coordinates of the point P be (x, y).

A(4, 3) P(x, y) B(–2, 6)

Now, 5AP = 2BP

$\Rightarrow$ $\frac{AP}{PB} = \frac{2}{5} \Rightarrow$ AP : PB = 2 : 5

So, $x = \frac{2 \times (-2) + 5 \times 4}{2+5} = \frac{-4+20}{7} = \frac{16}{7}$

and $y = \frac{2 \times 6 + 5 \times 3}{2+5} = \frac{12+15}{7} = \frac{27}{7}$.

Hence, the coordinates of P are $\left(\mathbf{\frac{16}{7}, \frac{27}{7}}\right)$.

(*iii*) P (–1, 3) — 3 — R $(\bar{x}, \bar{y})$ — 2 — Q (2, 5)

We have PR = $\frac{3PQ}{5}$

$\Rightarrow$ 5PR = 3PQ

$\Rightarrow$ 5PR = 3(PR + RQ)

$\Rightarrow$ 2PR = 3RQ

$\Rightarrow$ $\frac{PR}{RQ} = \frac{3}{2}$

$\therefore$ PR : RQ = 3 : 2

$\therefore$ By using section formula, we have

$$\bar{x} = \frac{3 \times 2 + 2 \times (-1)}{3+2} = \frac{6-2}{5} = \frac{4}{5}$$

and $\bar{y} = \frac{3 \times 5 + 2 \times 3}{3+2} = \frac{15+6}{5} = \frac{21}{5}$,

where $(\bar{x}, \bar{y})$ are the coordinates of R.

$\therefore$ Required coordinates of R are $\left(\mathbf{\frac{4}{5}, \frac{21}{5}}\right)$

(*iv*) Let the coordinates of point P are (x, y).

A(–2, –2) P(x, y) B(2, –4)

Now, $\frac{AP}{AB} = \frac{3}{7}$

$\Rightarrow$ 7AP = 3AB
= 3(AP + PB)
= 3AP + 3PB

$\Rightarrow$ 7AP – 3AP = 3PB

$\Rightarrow$ 4AP = 3PB

$\Rightarrow$ $\frac{AP}{PB} = \frac{3}{4} \Rightarrow$ AP : PB = 3 : 4

Now, $x = \frac{3 \times 2 + 4 \times (-2)}{3+4} = \frac{6-8}{7} = -\frac{2}{7}$

and $y = \frac{3 \times (-4) + 4 \times (-2)}{3+4} = \frac{-12-8}{7}$

$= -\frac{20}{7}$

Hence, the coordinates of the point P are $\left(\mathbf{-\frac{2}{7}, -\frac{20}{7}}\right)$

(*v*) Let the coordinates of point R be (x, y).

A(–4, 0) R(x, y) B(0, 6)

We have, AR = $\frac{3}{4}$ AB

$\Rightarrow$ 4AR = 3AB
= 3(AR + RB)
= 3AR + 3RB

$\Rightarrow$ AR = 3RB

$\Rightarrow$ $\frac{AR}{RB} = 3 = \frac{3}{1}$

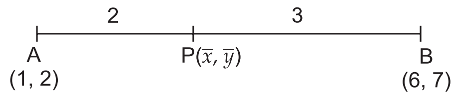
i.e. AR : RB = 3 : 1

So, $x = \frac{3 \times 0 + 1 \times (-4)}{3+1} = \frac{0-4}{4} = -1$

and $y = \frac{3 \times 6 + 1 \times 0}{3+1} = \frac{18}{4} = \frac{9}{2}$

Hence, coordinates of R are $\left(\mathbf{-1, \frac{9}{2}}\right)$.

(*vi*)



Let P be the point ($\bar{x}$, $\bar{y}$) on AB which divides AB is the ratio AP : PB.

$$\text{Now,}\quad AP = \frac{2}{5}AB$$

$$\Rightarrow \quad 5AP = 2AB$$
$$= 2(AP + PB)$$
$$= 2AP + 2PB$$
$$\Rightarrow \quad 3AP = 2PB$$
$$\Rightarrow \quad \frac{AP}{PB} = \frac{2}{3}$$
$$\therefore \quad AP : PB = 2 : 3$$

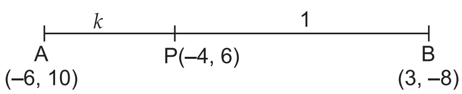
$\therefore$ By section formula, we have

$$\bar{x} = \frac{2\times 6 + 3\times 1}{2+3} = \frac{12+3}{5} = 3$$

$$\text{and}\quad \bar{y} = \frac{2\times 7 + 3\times 2}{2+3} = \frac{14+6}{5} = 4$$

The required coordinates of P are **(3, 4)**.

5. (*i*) Let the given point be P(–4, 6). Let this point divide AB is the ratio $k : 1$ where k is a non-zero constant. If $(\bar{x}, \bar{y})$ be the coordinates of the point P, then $\bar{x} = -4$, $\bar{y} = 6$.



Now, by section formula, we have

$$\bar{x} = \frac{3k-6}{k+1} = -4$$

$$\therefore \quad -4k - 4 = 3k - 6$$
$$\Rightarrow \quad 7k = 2$$
$$\Rightarrow \quad k = \frac{2}{7}$$

$\therefore$ Required ratio is $\frac{2}{7} : 1 =$ **2 : 7**.

(*ii*) Let the point $P\left(\frac{3}{4}, \frac{5}{12}\right)$ divides the line segment joining the points $A\left(\frac{1}{2}, \frac{3}{2}\right)$ and B(2, –5) in the ratio $k : 1$.

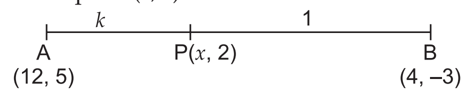
$$x = \frac{ax_2 + bx_1}{a+b}$$

$$\frac{3}{4} = \frac{2k + \frac{1}{2}}{k+1}$$

$$3k + 3 = 8k + 2$$
$$5k = 1$$
$$k = \frac{1}{5}$$

Hence the point P divides the line segment AB in the ratio of **1 : 5**.

6. (*i*) Let the point (x, 2) divides AB in the ratio $k : 1$ internally.



Then by using the section formula, we have

$$x = \frac{4k+12}{k+1} \text{ and } 2 = \frac{-3k+5}{k+1}$$

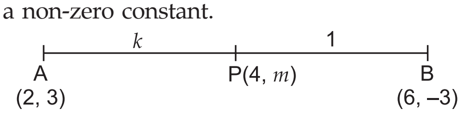
$$\Rightarrow \quad 2k + 2 = -3k + 5$$
$$\Rightarrow \quad 5k = 5 - 2 = 3$$
$$\therefore \quad k = \frac{3}{5}$$

$$\therefore \quad x = \frac{4\times\frac{3}{5} + 12}{\frac{3}{5} + 1}$$

$$= \frac{\frac{12}{5} + 12}{\frac{8}{5}} = \frac{72}{8} = 9$$

Hence, the required is **3 : 5** and the value of x is **9**.

(*ii*) Let the point P divide AB in the ratio $k : 1$ where k is a non-zero constant.



Then by using section formula, we have

$$4 = \frac{6k+2}{k+1}$$

$$\Rightarrow \quad 6k + 2 = 4k + 4$$
$$\Rightarrow \quad 2k = 2$$
$$\Rightarrow \quad k = 1 \qquad \ldots(1)$$

$\therefore$ The required ratio is 1 : 1.

$$\text{Again}\quad m = \frac{-3k+3}{k+1} \qquad \text{[By using section formula]}$$

$$= \frac{-3+3}{1+1} \qquad \text{[From (1)]}$$

The required value of m is 0.

(*iii*) Let the point (m, 6) divides the line segment joining the points A(–4, 3) and B(2, 8) in the ratio $k : 1$.

$$\text{Then,}\quad 6 = \frac{k\times 8 + 1\times 3}{k+1}$$

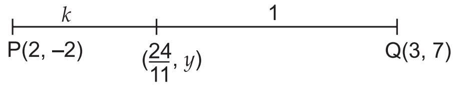
$$\Rightarrow \quad 6k + 6 = 8k + 3$$
$$\Rightarrow \quad 2k = 3$$
$$\Rightarrow \quad k = \frac{3}{2}$$

Thus, the point (m, 6) divides the line segment joining the given points in the ratio **3 : 2**.

$$\text{Now,}\quad m = \frac{3\times 2 + 2\times(-4)}{3+2} = \frac{6-8}{5} = -\frac{2}{5}$$

Hence, the value of m is $\mathbf{\frac{-2}{5}}$.

(*iv*) Let the point divide the line segment PQ in the ratio of $k : 1$.



$$x = \frac{ax_2 + bx_1}{a+b}$$

$$\frac{24}{11} = \frac{3k+2}{k+1}$$

$$24k + 24 = 33k + 22$$

$$9k = 2$$

$$k = \frac{2}{9}$$

$k : 1 \quad \Rightarrow$ **2 : 9**

Now, $$y = \frac{ay_2 + by_1}{a+b}$$

$$= \frac{14 + (-18)}{11}$$

$$= \mathbf{\frac{-4}{11}}$$

(*v*) Let the required ratio be $k : 1$.

$\Rightarrow \quad 2 = \dfrac{k \times 3 + 1 \times (-2)}{k+1}$

$\Rightarrow \quad 2k + 2 = 3k - 2$

$\Rightarrow \quad 4 = k$

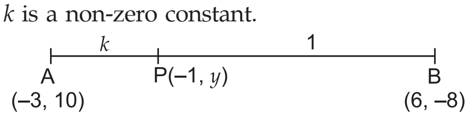
So, the ratio is **4 : 1**.

Also, $$y = \frac{k \times 7 + 1 \times 2}{k+1}$$

$$= \frac{4 \times 7 + 2}{4+1} = \frac{30}{5} = \mathbf{6}$$

Hence, the value of y is 6.

(*vi*) Let the point P(–1, y) divide AB in the ratio $k : 1$ where k is a non-zero constant.



Then by using section formula, we have

$$-1 = \frac{6k - 3}{k+1}$$

$\Rightarrow \quad -k - 1 = 6k - 3$

$\Rightarrow \quad 7k = 2$

$\Rightarrow \quad k = \dfrac{2}{7} \qquad \ldots(1)$

$\therefore$ The required ratio is 2 : 7

Also, by using section formula, we have

$$y = \frac{-8k + 10}{k+1} = \frac{-8 \times \frac{2}{7} + 10}{\frac{2}{7} + 1} \qquad \text{[From (1)]}$$

$$= \frac{-16 + 70}{2 + 7} = \frac{54}{9} = \mathbf{6}$$

where is the required of y.

(*vii*) Let the point (–3, p) divide AB in the ratio $k : 1$ where k is a non-zero constant.

Then by using section formula, we have

$$-3 = \frac{-2k - 5}{k+1}$$

$\Rightarrow \quad 3k + 3 = 2k + 5 \qquad \ldots(1)$

$\Rightarrow \quad k = 2.$

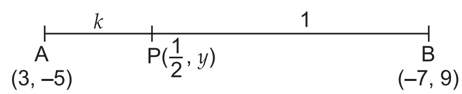
$\therefore$ The required ratio is **2 : 1**.

Again, by using section formula, we have

$$p = \frac{3k - 4}{k+1} = \frac{3 \times 2 - 4}{2+1} = \mathbf{\frac{2}{3}}$$

which is the required value of p.

(*viii*) Let the point P $\left(\dfrac{1}{2}, y\right)$ divide AB is the ratio $k : 1$, where k is a non-zero constant.



$\therefore$ By using section formula, we have

$$\frac{1}{2} = \frac{-7k + 3}{k+1}$$

$\Rightarrow \quad -14k + 6 = k + 1$

$\Rightarrow \quad 15k = 6 - 1 = 5$

$\therefore \quad k = \dfrac{5}{15} = \dfrac{1}{3} \qquad \ldots(1)$

$\therefore$ The required ratio is **1 : 3**.

Again, using the section formula we have

$$y = \frac{9k - 5}{k+1} = \frac{9 \times \frac{1}{3} - 5}{\frac{1}{3} + 1} \qquad \text{[From (1)]}$$

$$= \frac{-2 \times 3}{4} = \mathbf{\frac{3}{2}}$$

which is the required value of y.

(*ix*) Let the required ratio be $k : 1$.

Then, $$\frac{5 + a}{7} = \frac{k \times 2 + 1 \times 1}{k+1}$$

$\Rightarrow \quad \dfrac{5 + a}{7} = \dfrac{2k + 1}{k+1}$

$\Rightarrow \quad a = \dfrac{7(2k+1)}{k+1} - 5$

$$= \frac{14k + 7 - 5k - 5}{k+1} = \frac{9k + 2}{k+1} \qquad \ldots(1)$$

and $$\frac{6a + 3}{7} = \frac{k \times 7 + 1 \times 3}{k+1}$$

$\Rightarrow \quad 6a = \dfrac{7(7k+3)}{k+1} - 3$

$$= \frac{49k + 21 - 3k - 3}{k+1}$$

$$= \frac{46k+18}{k+1} \qquad \ldots(2)$$

From (1) and (2), we have

$$6 \times \left(\frac{9k+2}{k+1}\right) = \frac{46k+18}{k+1}$$

$\Rightarrow \quad 54k + 12 = 46k + 18$

$\Rightarrow \quad 8k = 6$

$\Rightarrow \quad k = \frac{6}{8} = \frac{3}{4}$

So, the ratio is **3 : 4**.

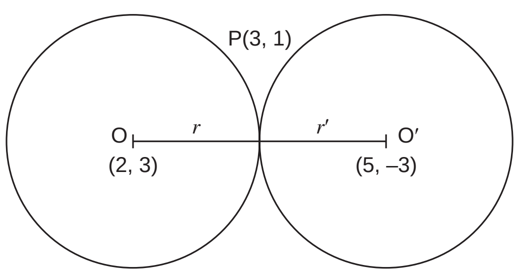
Now, $\quad a = \dfrac{9 \times \frac{3}{4} + 2}{\frac{3}{4} + 1} = \dfrac{27+8}{4} \times \dfrac{4}{7}$

$$= \frac{35}{4} \times \frac{4}{7} = \mathbf{5}$$

Hence, the value of a is **5**.

7. We have, $\quad OP = \sqrt{(3-2)^2 + (1-3)^2}$

$\Rightarrow \quad r = \sqrt{1+4} = \sqrt{5}$



and $\quad O'P = \sqrt{(3-5)^2 + (1+3)^2}$

$\Rightarrow \quad r' = \sqrt{4+16}$

$$= \sqrt{20} = 2\sqrt{5}$$

So, $\quad r : r' = \sqrt{5} : 2\sqrt{5}$

$\Rightarrow \quad \dfrac{r}{r'} = \dfrac{\sqrt{5}}{2\sqrt{5}} = \dfrac{1}{2}$

$\Rightarrow \quad r : r' = \mathbf{1 : 2}$

8. Let point B(0, 5) divides the line segment joining the points A(–2, 7) and C(8, –3) in the ratio k : 1.

Then, $\quad 0 = \dfrac{k \times 8 + 1 \times (-2)}{k+1}$

$\Rightarrow \quad 0 = 8k - 2$

$\Rightarrow \quad 8k = 2 \Rightarrow k = \dfrac{2}{8} = \dfrac{1}{4}$

Now, for collinearity, the formula should match with the y-coordinate section of B, Thus,

$$\frac{k \times (-3) + 1 \times 7}{k+1} = \frac{\frac{1}{4} \times (-3) + 7}{\frac{1}{4} + 1}$$

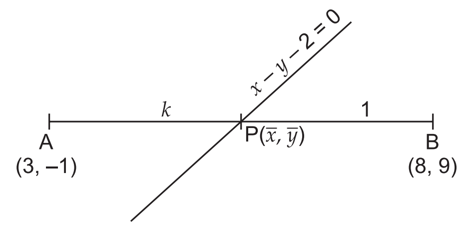
$$= \frac{\frac{-3+28}{4}}{\frac{1+4}{4}} = \frac{25}{4} \times \frac{4}{5} = 5$$

Thus, $\quad k = \dfrac{AB}{BC} = 5 \Rightarrow 5 = \dfrac{AB}{BC}$

$\Rightarrow \quad AB = 5\ BC$

This shows that A, B and C lie on the same straight line. Hence, A, B and C are collinear.

9. (*i*) Let $P(\bar{x}, \bar{y})$ be the point which divides AB in the ratio k : 1 where k is a non-zero constant.



Then by using Section formula, we have

$$\bar{x} = \frac{8k+3}{k+1} \text{ and } \bar{y} = \frac{9k-1}{k+1} \qquad \ldots(1)$$

Since, $(\bar{x}, \bar{y})$ lies on the line $x - y - 2 = 0$

$\therefore \quad \bar{x} - \bar{y} - 2 = 0$

$\Rightarrow \quad \dfrac{8k+3}{k+1} - \dfrac{9k-1}{k+1} - 2 = 0$

$\Rightarrow \quad 8k + 3 - 9k + 1 - 2k - 2 = 0$

$\Rightarrow \quad 3k = 2$

$\Rightarrow \quad k = \dfrac{2}{3}$

$\therefore$ The required ratio is **2 : 3**.

(*ii*) Let the given line divides the line segment joining the points (2, –2) and (3, 7) in the ratio k : 1.

Then, the coordinates of the point to divide the line segment are $\dfrac{3k+2}{k+1}$ and $\dfrac{7k-2}{k+1}$.

Since, this point lies on the given line, so

$$2\left(\frac{3k+2}{k+1}\right) + \frac{7k-2}{k+1} - 4 = 0$$

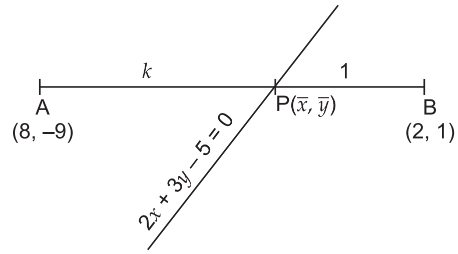
$\Rightarrow \quad 6k + 4 + 7k - 2 - 4k - 4 = 0$

$\Rightarrow \quad 9k - 2 = 0$

$\Rightarrow \quad k = \dfrac{2}{9}$

Thus, the given line divides the line segment joining the given point in the ratio **2 : 9**.

(*iii*) Let A(8, –9) and B(2, 1) be the given point and $P(\bar{x}, \bar{y})$ be a point on AB such that it divides AB in the ratio $k : 1$ internally where k is a non-zero constant.



Then by using section formula, we have

$$\bar{x} = \frac{2k+8}{k+1} \text{ and } \bar{y} = \frac{k-9}{k+1} \quad \ldots(1)$$

$\because$ $(\bar{x}, \bar{y})$ lies on the given line $2k + 3y - 5 = 0$,

$\therefore$ We have $\quad 2\bar{x} + 3y - 5 = 0$

$$\Rightarrow \quad 2 \times \frac{2k+8}{k+1} + 3 \times \frac{k-9}{k+1} - 5 = 0 \quad \text{[From (1)]}$$

$\Rightarrow \quad 4k + 16 + 3k - 27 - 5k - 5 = 0$

$\Rightarrow \quad 2k = 16$

$\Rightarrow \quad k = 8 \quad \ldots(2)$

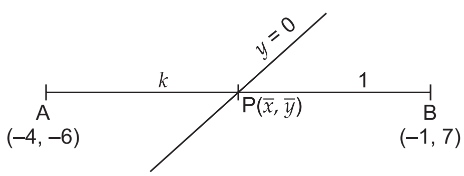
$\therefore$ The required ratio is **8 : 1**.

$$\therefore \quad \bar{x} = \frac{2 \times 2 + 8}{8+1} = \frac{24}{9} = \frac{8}{3} \quad \text{[From (1) \& (2)]}$$

$$\text{and} \quad \bar{y} = \frac{8-9}{8+1} = -\frac{1}{9} \quad \text{[From (1) \& (2)]}$$

Required coordinates of P are $\left(\mathbf{\frac{8}{3}, -\frac{1}{9}}\right)$.

10. (*i*) Let A(–4, –6) and B(–1, 7) be the given points and let $P(\bar{x}, \bar{y})$ be a point on AB such that it divides AB in the ratio $k : 1$ internally, where k is a non-zero constant.



Then by using section formula, we have

$$\bar{x} = \frac{-k-4}{k+1} \text{ and } \bar{y} = \frac{7k-6}{k+1} \quad \ldots(1)$$

Now, given that $(\bar{x}, \bar{y})$ on the x-axis, i.e. $y = 0$.

$\therefore$ From (1), we have

$\bar{y} = 0$

$\Rightarrow \quad 7k - 6 = 0$

$$\Rightarrow \quad k = \frac{6}{7}$$

$\therefore$ The required ratio is 6 : 7.

$\therefore$ From (1) we have

$$\bar{x} = \frac{-\frac{6}{7} - 4}{\frac{6}{7} + 1} = -\frac{34}{13}$$

$$\text{and} \quad \bar{y} = \frac{7 \times \frac{6}{7} - 6}{\frac{6}{7} + 1} = 0$$

$\therefore$ The required coordinates of the point of division P are $\left(-\frac{34}{13}, 0\right)$.

(*ii*) Let a point on x-axis $(x, 0)$ divides the line segment joining the points A(3, –3) and B(–2, 7) in the ratio $k : 1$.

$$0 = \frac{7k-3}{k+1}$$

$7k - 3 = 0$

$7k = 3$

$$k = \frac{3}{7}$$

$k : 1 \rightarrow$ **3 : 7**

$$\text{Now} \quad x = \frac{-6+21}{10} = \frac{15}{10} = \frac{3}{2}$$

Hence, the coordinates of the point are $\left(\mathbf{\frac{3}{2}, 0}\right)$.

(*iii*) P is a point on x-axis.

So, ***y*-coordinate of P is 0**.

Now, let AP : PB = k : 1.

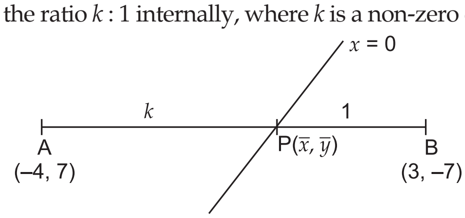
$$\text{Then,} \quad 0 = \frac{k \times (-5) + 1 \times 3}{k+1}$$

$\Rightarrow \quad -5k + 3 = 0$

$$\Rightarrow \quad k = \frac{3}{5}$$

So, the ratio AP : PB = **3 : 5**.

11. (*i*) Let A(–4, 7) and B(3, –7) be the given points and let $P(\bar{x}, \bar{y})$ be a point on AB such that it divides AB in the ratio $k : 1$ internally, where k is a non-zero constant.



Then by using section formula, we have

$$\bar{x} = \frac{3k-4}{k+1} \text{ and } \bar{y} = \frac{-7k+7}{k+1} \quad \ldots(1)$$

Given that $(\bar{x}, \bar{y})$ lies on the y-axis, i.e., $x = 0$

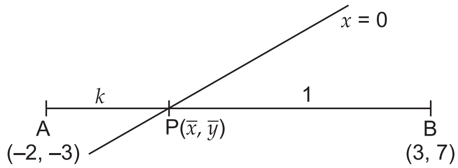
$\therefore$ From (1), $\quad \bar{x} = 0$

$$\Rightarrow \quad \left(\frac{3k-4}{k+1}\right) = 0$$

$$\Rightarrow \quad k = \frac{4}{3}$$

$\therefore$ The required ratio is **4 : 3**.

(*ii*)



Let A(–2, –3) and B(3, 7) be the given points and let $P(\bar{x}, \bar{y})$ be a point on AB such that it divides AB in the ratio $k : 1$ internally, where k is a non-zero constant.

Then by using section formula, we have

$$\bar{x} = \frac{3k-2}{k+1} \text{ and } \bar{y} = \frac{7k-3}{k+1} \qquad \ldots(1)$$

Given that $(\bar{x}, \bar{y})$ lies on the y-axis, i.e. $x = 0$

$\therefore$ From (1), $\bar{x} = 0$

$$\Rightarrow \quad 3k - 2 = 0$$

$$\Rightarrow \quad k = \frac{2}{3} \qquad \ldots(2)$$

$\therefore$ The required ratio is **2 : 3**.

Also, from (1),

$$\bar{y} = \frac{7 \times \frac{2}{3} - 3}{\frac{2}{3} + 1} = \frac{5}{5} = 1$$

$\therefore$ The required coordinates of the point of division P are **(0, 1)**.

(*iii*) Let the point of division on y-axis be $(0, y)$.

Also, let the ratio in which the point of division $(0, y)$ divides the line segment joining the points $(5, -6)$ and $(-1, -4)$ be $k : 1$.

Then, $$0 = \frac{k \times (-1) + 1 \times 5}{k+1}$$

$$\Rightarrow \quad -k + 5 = 0 \quad \Rightarrow \quad k = 5$$

i.e. the y-axis divides the line segment in the ratio **5 : 1**.

Now, for the point of division,

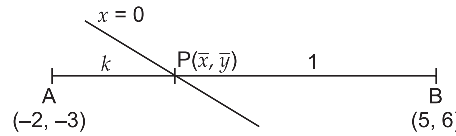
$$y = \frac{k \times (-4) + 1 \times (-6)}{k+1}$$

$$= \frac{-4k-6}{k+1} = \frac{-4 \times 5 - 6}{5+1}$$

$$= \frac{-26}{6} = \frac{-13}{3}$$

Hence, the coordinates of the point of division is $\left(\mathbf{0}, -\frac{\mathbf{13}}{\mathbf{3}}\right)$.

12. Let the points A(–2, –3) and B(5, 6) be the given points and let $P(\bar{x}, \bar{y})$ be a point on AB such that it divides AB in the ratio $k : 1$ internally, where k is a non-zero constant.



Then by using section formula, we have

$$\bar{x} = \frac{5k-2}{k+1} \text{ and } \bar{y} = \frac{6k-3}{k+1} \qquad \ldots(1)$$

Now, given that $(\bar{x}, \bar{y})$ lies on the y-axis, i.e., $x = 0$

$\therefore$ From (1), $\bar{x} = 0$

$$\Rightarrow \quad 5k - 2 = 0$$

$$\Rightarrow \quad k = \frac{2}{5}$$

$\therefore$ Required ratio is **2 : 5**.

Also from (1), $$\bar{y} = \frac{6 \times \frac{2}{5} - 3}{\frac{2}{5} + 1} = -\frac{3}{7}$$

$\therefore$ The required coordinates of the point of division P are $\left(\mathbf{0}, -\frac{\mathbf{3}}{\mathbf{7}}\right)$.

13. Coordinates of C which is the mid-point of A(0, 4) and B(6, 0) are $\left(\frac{0+6}{2}, \frac{4+0}{2}\right)$, i.e. **C(3, 2)**.

Let the coordinates of P be (x, y).

Also, coordinates of the origin O are (0, 0).

So, $$3 = \frac{x+0}{4} \Rightarrow x = 12$$

and $$2 = \frac{y+0}{4} \Rightarrow y = 8$$

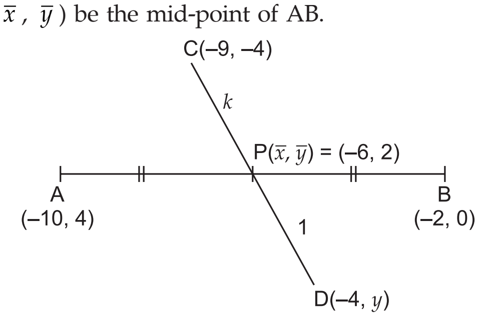
So, coordinates of P are **(12, 8)**.

Now, $$BP = \sqrt{(12-6)^2 + (8-0)^2}$$

$$= \sqrt{36+64}$$

$$= \sqrt{100} = \mathbf{10 \text{ units}}.$$

14. Let A(–10, 4) and B(–2, 0) be two given points and let $P(\bar{x}, \bar{y})$ be the mid-point of AB.



Then by mid-point formula, we have

$$\bar{x} = \frac{-10-2}{2} = -6 \text{ and } \bar{y} = \frac{4+10}{2} = 2 \qquad \ldots(1)$$

$\therefore$ The coordinates of P are (–6, 2).

Let C(–9, –4) and D(–4, y) be another two points and the line segment joining C and D passes through the point P(–6, 2). Let P divides CD internally in the ratio k : 1, where k is a non-zero number.

Then by using section formula, we get

$$-6 = \frac{-4k-9}{k+1} \text{ and } 2 = \frac{ky-4}{k+1} \qquad \ldots(1)$$

$\therefore$ From (1), $4k + 9 = 6k + 6$

$\Rightarrow$ $2k = 3$

$\Rightarrow$ $k = \frac{3}{2}$ $\qquad \ldots(2)$

$\therefore$ Required ratio = 3 : 2

$\therefore$ From (1) and (2)

$$2 = \frac{\frac{3}{2}y-4}{\frac{3}{2}+1} = \frac{3y-8}{5}$$

$\Rightarrow$ $10 = 3y - 8$

$\Rightarrow$ $3y = 18$

$\Rightarrow$ $y = \frac{18}{3} = 6$

$\therefore$ The required value of y is **6**.

15. A($3a + 1$, –3)

B($8a$, 5) Ratio $\rightarrow$ 3 : 1

P($9a - 2$, –b)

$$x = \frac{ax_2+bx_1}{a+b} \qquad y = \frac{ay_2+by_1}{a+b}$$

$$9a-2 = \frac{24a+3a+1}{4} \qquad -b = \frac{15-3}{4}$$

$$36a - 8 = 27a + 1 \qquad -4b = 12$$

$$9a = 9 \qquad \boldsymbol{b = -3}$$

$$\boldsymbol{a = 1}$$

16. (*i*) Let the coordinates of point P be (x, y).

Then, $x = \frac{k\times(-4)+3\times1}{k+1} = \frac{-4k+3}{k+1}$

and $y = \frac{k\times8+(-5)\times1}{k+1} = \frac{8k-5}{k+1}$

Since the point P lies on the line $x + y = 0$,

$$\left(\frac{-4k+3}{k+1}\right)+\left(\frac{8k-5}{k+1}\right) = 0$$

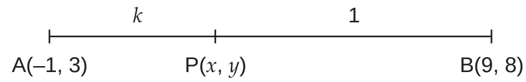
$\Rightarrow$ $-4k + 3 + 8k - 5 = 0$

$\Rightarrow$ $4k = 2$

$$k = \frac{2}{4} = \frac{1}{2}$$

Hence, the value of k is $\boldsymbol{\frac{1}{2}}$.

(*ii*) Let the coordinates of P be (x, y).



We have,

$$\frac{AP}{PB} = \frac{k}{1}$$

i.e. AP : PB = k : 1

Then, $x = \frac{9k+(-1)}{k+1} = \frac{9k-1}{k+1}$

and $y = \frac{8k+3}{k+1}$

Since P lies on the line $x - y + 2 = 0$, we have

$$\frac{9k-1}{k+1} - \frac{8k+3}{k+1} + 2 = 0$$

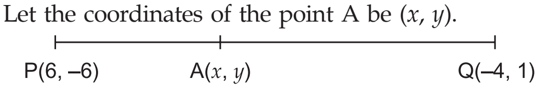
$\Rightarrow$ $9k - 1 - 8k - 3 + 2k + 2 = 0$

$\Rightarrow$ $3k - 2 = 0$

$\Rightarrow$ $k = \frac{2}{3}$

Hence, the value of k is $\boldsymbol{\frac{2}{3}}$.

(*iii*) Let the coordinates of the point A be (x, y).



We have,

$$\frac{PA}{PQ} = \frac{2}{5}$$

$\Rightarrow$ 5PA = 2PQ

= 2(PA + AQ)

$\Rightarrow$ 3PA = 2AQ

$\Rightarrow$ $\frac{PA}{AQ} = \frac{2}{3}$

i.e. PA : AQ = 2 : 3

$$x = \frac{ax_2+bx_1}{a+b} \qquad y = \frac{ay_2+by_1}{a+b}$$

$$= \frac{-8+18}{5} \qquad = \frac{-2+(-18)}{5}$$

$$= 2 \qquad = -4$$

Since point A lies on the line $3x + k(y + 1) = 0$, hence it will satisfy it.

$3x + k(y + 1) = 0$

$3(2) + k(-4 + 1) = 0$

$+ 3k = +6$

$\boldsymbol{k = 2}$

17. (*i*) Q(–5, 4)

R(–1, 0)

P$\left(\frac{a}{3}, 2\right)$

P is the mid-point of line segment QR

$$x = \frac{x_1 + x_2}{2}$$

$$\frac{a}{3} = \frac{(-5)+(-1)}{2}$$

$$\frac{a}{3} = \frac{-6}{2}$$

$$\boldsymbol{a = -9}$$

(*ii*) We have, $\frac{2+q}{2} = 3$

$\Rightarrow$ $2 + q = 6 \Rightarrow q = 6 - 2 = \mathbf{4}$

and $\frac{p+4}{2} = 5$

$\Rightarrow$ $p + 4 = 10 \Rightarrow p = 10 - 4 = \mathbf{6}$

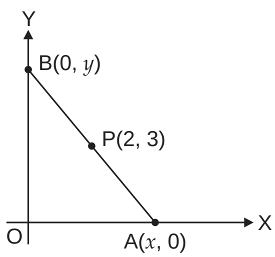
(*iii*) We have, $\frac{3p+(-2)}{2} = 5$

$\Rightarrow$ $3p - 2 = 10 \Rightarrow p = \frac{10+2}{3} = \mathbf{4}$

and $\frac{4+2q}{2} = p \Rightarrow 4 + 2q = 4 \times 2$

$\Rightarrow$ $2q = 8 - 4 = 4 \Rightarrow q = \frac{4}{2} = \mathbf{2}$

18. Given that, P(2, 3) is the mid-point of the line segment AB.



$\therefore$ $2 = \frac{0+x}{2}$

and $3 = \frac{y+0}{2}$

$\Rightarrow$ $x = 4$

and $y = 6$

$\Rightarrow$ The coordinates of A and B are **(4, 0)** and **(0, 6)** respectively.

19. (*i*) For the mid-point of the line segment AB,

$$3 = \frac{x+0}{2} \Rightarrow x = 6$$

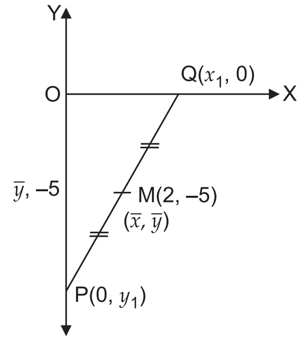
and $4 = \frac{0+y}{2} \Rightarrow y = 8$

So, the coordinates of A are **(6, 0)** and that of B are **(0, 8)**.

Now, $AB = \sqrt{(0-6)^2 + (8-0)^2}$

$= \sqrt{36+64} = \sqrt{100} = \mathbf{10\ units}$

(*ii*) Let P and Q be the points $(0, y_1)$ and $(x_1, 0)$ respectively and let $M(\bar{x}, \bar{y})$ be the mid-points of PQ.



Then $\bar{x} = 2$ and $\bar{y} = -5$

Then by mid-point section formula, we have

$$\bar{x} = \frac{x_1 + 0}{2} = \frac{x_1}{2}$$

$\Rightarrow$ $2 = \frac{x_1}{2}$

$\Rightarrow$ $x_1 = 4$

and $\bar{y} = \frac{0 + y_1}{2} = \frac{y_1}{2}$

$\Rightarrow$ $-5 = \frac{y_1}{2}$

$\Rightarrow$ $y_1 = -10$

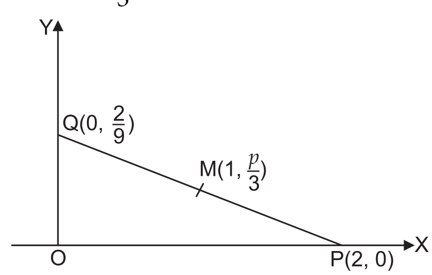
$\therefore$ Required coordinates of P and Q are **(0, 10)** and **(4, 0)** respectively.

20. Let P(2, 0) and $Q\left(0, \frac{2}{9}\right)$ be the coordinates of two points P and Q respectively and let $M\left(1, \frac{p}{3}\right)$ be its mid-point.

Then by mid-point section formula, we have

$$\frac{p}{3} = \frac{\frac{2}{9}+0}{2} = \frac{p}{3} = \frac{1}{9}$$

$\Rightarrow$ $p = \frac{1}{3}$



$\therefore$ The given point R(–1, 3*p*) is $R\left(-1, 3 \times \frac{1}{3}\right) = R(-1, 1)$

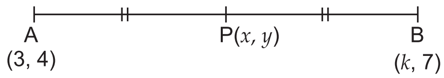
If we put $x = -1$ and $y = 1$ in the LHS of the given equation $5x + 3y + 2 = 0$, we get

$$\text{LHS} = 5 \times (-1) + 3 \times 1 + 2 = 5 - 5 = 0 = \text{RHS}$$

$\therefore$ The given equation is satisfied by the point R.

Hence, the given line passes through the given point (–1, $3p$).

21. Let A(3, 4) and B(k, 7) be the given points and let P(x, y) be its mid-point.



$\therefore$ By using mid-point section formula, we have

$$x = \frac{3+k}{2} \text{ and } y = \frac{4+7}{2} = \frac{11}{2} \qquad \ldots(1)$$

Now, it is given that the point (x, y), i.e. $\left(\frac{3+k}{2}, \frac{11}{2}\right)$ satisfy the given equation since this point lies on the line $2x + 2y + 1 = 0$

$\therefore$ We have

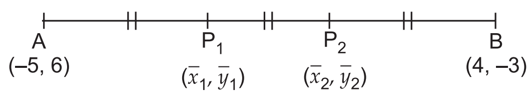
$$2 \times \frac{3+k}{2} + 2 \times \frac{11}{2} + 1 = 0$$

$\Rightarrow \quad 3 + k + 12 = 0$

$\Rightarrow \quad \boldsymbol{k = -15}$

which is the required value of k.

22. (*i*) Let A(–5, 6) and B(4, –3) be two given points and let P_1 and P_2 are the points of trisection of AB so that $AP_1 = P_1P_2 = P_2B$.



$\therefore \quad AP_1 : P_1B = 1 : 2$

and $\quad AP_2 : P_2B = 2 : 1$

Hence, if $(\bar{x}_1, \bar{y}_1)$ be the coordinates of P_1 and $(\bar{x}_2, \bar{y}_2)$ be the coordinates of P_2, then by using section formula, we have

$$\bar{x}_1 = \frac{1\times 4 + 2\times(-5)}{1+2} = \frac{4-10}{3} = \frac{-6}{3} = -2$$

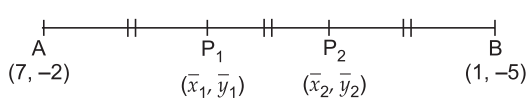
$$\bar{y}_1 = \frac{1\times(-3) + 2\times 6}{1+2} = \frac{9}{3} = 3$$

$$\bar{x}_2 = \frac{2\times 4 + 1\times(-5)}{1+2} = \frac{3}{3} = 1$$

$$\bar{y}_2 = \frac{2\times(-3) + 1\times 6}{1+2} = \frac{0}{3} = 0$$

Hence, the required coordinates of the points of trisection are **(–2, 3)** and **(1, 0).**

(*ii*) Let A(7, –2) and B(1, –5) be two given points and let P_1 $(\bar{x}_1, \bar{y}_1)$ and P_2 $(\bar{x}_2, \bar{y}_2)$ be two points of trisection of AB. Hence, $AP_1 : P_1B = 1 : 2$ and $AP_2 : P_2B = 2 : 1$.



Hence by using section formula, we have

$$\bar{x}_1 = \frac{1\times 1 + 2\times 7}{1+2} = \frac{15}{3} = 5$$

$$\bar{y}_1 = \frac{1\times(-5) + 2\times(-2)}{1+2} = \frac{-9}{3} = 3$$

$$\bar{x}_2 = \frac{2\times 1 + 1\times 7}{2+1} = \frac{9}{3} = 3$$

and $$\bar{y}_1 = \frac{2\times(-5) + 1\times(-2)}{2+1} = \frac{-12}{3} = 4$$

Hence, the required coordinates of the points of trisection of AB are **(5, –3)** and **(3, 4)**.

23. (*i*) A(2, –2)

B(–7, 4)

Let the coordinates of P be (a, b) and Q be (c, d)

Now point P divide AB in the ratio of 1 : 2

$$x = \frac{ax_2 + bx_1}{a+b} \qquad y = \frac{ay_2 + by_1}{a+b}$$

$$a = \frac{-7+4}{3} \qquad b = \frac{4-4}{3}$$

$3a = -3 \qquad b = 0$

$a = -1$

P(–1, 0)

Now Q divides AB in the ratio of 2 : 1

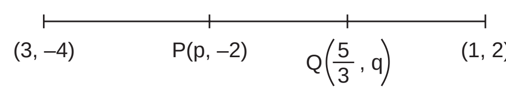
$$x = \frac{ax_2 + bx_1}{a+b} \qquad y = \frac{ay_2 + by_1}{a+b}$$

$$c = \frac{-14+2}{3} \qquad d = \frac{8+(-2)}{3}$$

$$c = \frac{-12}{3} = -4 \qquad d = 2$$

Q(–4, 2)

(*ii*) The coordinates of the points of trisection are given by



$$p = \frac{1\times 1 + 2\times 3}{1+2} = \frac{1+6}{3} = \boldsymbol{\frac{7}{3}}$$

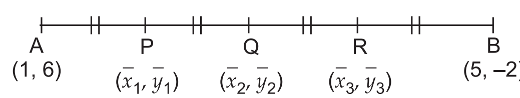
($\because$ P is dividing the line segment in ratio 1 : 2)

Also, $$q = \frac{2\times 2 + 1\times(-4)}{1+2} = \frac{4-4}{3} = \mathbf{0}$$

($\because$ Q is dividing the line segment in the ratio 2 : 1).

24. (*i*) Let A(1, 6) and B(5, –2) be two given points and P, Q, R are three points on AB such that

AP = PQ = QR = RB.



Let P, Q, R be the points $(\bar{x}_1, \bar{y}_1)$, $(\bar{x}_2, \bar{y}_2)$ and $(\bar{x}_3, \bar{y}_3)$ respectively. Then AP : PB = 1 : 3, AQ : QB = 1 : 1, i.e., Q is the mid-point of AB and AR : RB = 3 : 1.

Hence, by using section formula, we have

$$\bar{x}_1 = \frac{1\times5+3\times1}{1+3} = \frac{8}{4} = 2$$

$$\bar{y}_1 = \frac{1\times(-2)+3\times6}{1+3} = \frac{-2+18}{4} = 4$$

$$\bar{x}_2 = \frac{1+5}{2+1} = 3$$

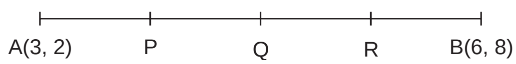
and $$\bar{y}_2 = \frac{6-2}{2} = 2$$

$$\bar{x}_3 = \frac{3\times5+1\times1}{3+1} = \frac{16}{4} = 4$$

and $$\bar{y}_3 = \frac{3\times(-2)+1\times6}{3+1} = 0$$

Hence, the required coordinates of P, Q and R are **(2, 4)**, **(3, 2)**, **(4, 0)** respectively.

(*ii*) Let the two given points be A(3, 2) and B(6, 8).



Also, let the three points that divide the line segment AB in four equal parts be P, Q and R.

Now, P divides the line segment AB in the ratio 1 : 3. So, its coordinates are given by

$$x = \frac{1\times6+3\times3}{1+3} = \frac{6+9}{4} = \frac{15}{4}$$

$$y = \frac{1\times8+3\times2}{1+3} = \frac{8+6}{4} = \frac{14}{4} = \frac{7}{2}$$

So, the coordinates of point P are $\left(\mathbf{\frac{15}{4}, \frac{7}{2}}\right)$. Point Q divides the line segment AB in the ratio 1 : 1. So, its coordinates are given by

$$x = \frac{6+3}{2} = \frac{9}{2}$$

$$y = \frac{8+2}{2} = \frac{10}{2} = 5$$

So, the coordinates of point Q are $\left(\mathbf{\frac{9}{2}, 5}\right)$. Point R divides the line segment AB in the ratio 3 : 1. So, its coordinates are given by

$$x = \frac{3\times6+1\times3}{3+1} = \frac{21}{4}$$

$$y = \frac{3\times8+1\times2}{3+1} = \frac{26}{4} = \frac{13}{2}$$

So, the coordinates of point R are $\left(\mathbf{\frac{21}{4}, \frac{13}{2}}\right)$.

25. The two line segments will bisect each other if their mid-points are same.

Now, the coordinates of mid-point of (2, 5) and (–8, 3) are

$$\left(\frac{2+(-8)}{2}, \frac{5+3}{2}\right), \text{ i.e. } (-3, 4).$$

Also, the coordinates of mid-point of (–3, 1) and (–3, 7) are

$$\left(\frac{-3+(-3)}{2}, \frac{1+7}{2}\right), \text{ i.e. } (-3, 4).$$

Since the coordinates of the mid-point of the two line segments are same, it shows that the two line segments are bisecting each other.

26. Let the coordinates of B be (x, y).

Then, $\frac{x-3}{2} = 1 \Rightarrow x = 2 + 3 = 5$

and $\frac{y+5}{2} = 2 \Rightarrow y = 4 - 5 = -1$

Hence, the coordinates of B are **(5, –1)**.

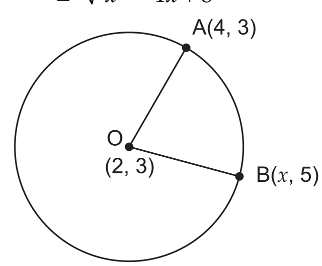
27. (*i*) $$OA = \sqrt{(4-2)^2+(3-3)^2}$$

$$= \sqrt{4+0} = 2$$

$$OB = \sqrt{(x-2)^2+(5-3)^2}$$

$$= \sqrt{x^2-4x+4+4}$$

$$= \sqrt{x^2-4x+8}$$



Now, OA = OB = radius

$\Rightarrow$ $OA^2 = OB^2$

$\Rightarrow$ $4 = x^2 - 4x + 8$

$\Rightarrow$ $x^2 - 4x + 4 = 0$

$\Rightarrow$ $(x - 2)^2 = 0$

$\Rightarrow$ $x - 2 = 0$

$\Rightarrow$ $x = 2$

Hence, the value of x is **2**.

(*ii*) A(2, –2) C(x + 2, x – 1)

B(8, –2)

Points A and B lie on the circle and C is the centre

$\therefore$ AC = BC

$$\sqrt{(x+2-2)^2+(x-1+2)^2} = \sqrt{(x+2-8)^2+(x-1+2)^2}$$

On squaring both the sides, we get

$$x^2 + (x + 1)^2 = (x - 6)^2 + (x + 1)^2$$

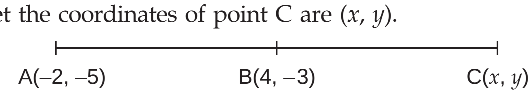
$$x^2 = x^2 + 36 - 12x$$

$$12x = 36$$

$$\mathbf{x = 3}$$

For Standard Level

28. Let the coordinates of point C are (x, y).



Now, AC = 2BC

$$\Rightarrow \qquad \frac{AC}{BC} = 2$$

i.e. AC : BC = 2 : 1

Then, $x = \dfrac{2 \times 4 - 1 \times (-2)}{2-1} = \dfrac{8+2}{1} = 10$

$$y = \frac{2 \times (-3) - 1 \times (-5)}{2-1} = \frac{-6+5}{1} = -1$$

Hence, the coordinates of the point C are **(10, –1)**.

29. Let coordinates of the point of trisection P be (x, y). Then, it divides AB in the ratio 1 : 2.

Therefore, $x = \dfrac{1 \times 5 + 2 \times 2}{1+2} = \dfrac{5+4}{3} = 3$

$$y = \frac{1 \times (-8) + 2 \times 1}{1+2} = \frac{-8+2}{3} = -2$$

Since P lies on the given line $2x - y + k = 0$, we have

$$2 \times 3 - (-2) + k = 0$$

$$\Rightarrow \qquad 6 + 2 + k = 0$$

$$\Rightarrow \qquad k = -8$$

Hence, the value of k is **–8**.

30. Let the points on x-axis be $(x, 0)$ and on y-axis be $(0, y)$ which are trisecting the line segment joining the points $(-3, 5)$ and $\left(6, -\dfrac{5}{2}\right)$.

Now, let the line segment is trisected by x-axis in the ratio $k : 1$.

Then, $0 = \dfrac{k \times \left(-\dfrac{5}{2}\right) + 1 \times 5}{k+1}$

$$\Rightarrow \qquad \frac{-5}{2}k + 5 = 0$$

$$\Rightarrow \qquad k = 5 \times \frac{2}{5} = 2$$

i.e. x-axis is trisecting the given line segment in the ratio 2 : 1.

Again, let the line segment is trisected by the y-axis in the ratio $r : 1$.

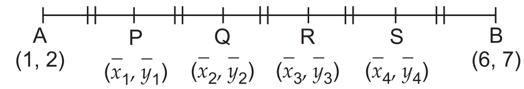
Then, $0 = \dfrac{r \times 6 + 1 \times (-3)}{r+1}$

$$\Rightarrow \qquad 6r - 3 = 0$$

$$\Rightarrow \qquad r = \frac{3}{6} = \frac{1}{2}$$

i.e. y-axis is trisecting the given line segment in ratio 1 : 2.

31. Let $(\bar{x}_1, \bar{y}_1)$, $(\bar{x}_2, \bar{y}_2)$, $(\bar{x}_3, \bar{y}_3)$ and $(\bar{x}_4, \bar{y}_4)$ be respectively the coordinates of P, Q, R and S where AP = PQ = QR = RS = SB.



$\therefore$ AP : PB = 1 : 4, AQ : QB = 2 : 3, AR : RB = 3 : 2 and AS : SB = 4 : 1

Hence, by using section formula, we have

$$\bar{x}_1 = \frac{1 \times 6 + 4 \times 1}{1+4} = \frac{10}{5} = 2$$

$$\bar{y}_1 = \frac{1 \times 7 + 4 \times 2}{1+4} = \frac{15}{5} = 3$$

$$\bar{x}_2 = \frac{2 \times 6 + 3 \times 1}{2+3} = \frac{15}{5} = 3$$

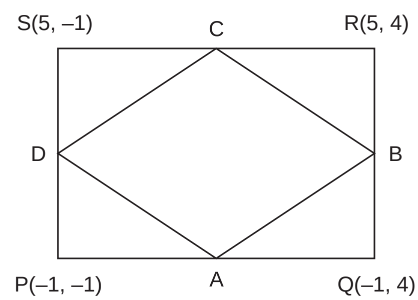
$$\bar{y}_2 = \frac{2 \times 7 + 3 \times 2}{2+3} = \frac{20}{5} = 4$$

$$\bar{x}_3 = \frac{3 \times 6 + 2 \times 1}{3+2} = \frac{20}{5} = 4$$

$$\bar{y}_3 = \frac{3 \times 7 + 2 \times 2}{3+2} = \frac{25}{5} = 5$$

Hence, the required coordinates of P, Q and R are **(2, 3)**, **(3, 4)** and **(4, 5)** respectively.

32. Using the mid-point formula, the corresponding coordinates A, B, C and D are respectively



$$A\left(\frac{-1-1}{2}, \frac{4-1}{2}\right)$$

i.e. $\qquad A\left(-1, \dfrac{3}{2}\right)$

$$B\left(\frac{5-1}{2}, \frac{4+4}{2}\right) \text{ i.e. } B(2, 4)$$

$$C\left(\frac{5+5}{2}, \frac{4-1}{2}\right) \text{ i.e. } C\left(5, \frac{3}{2}\right)$$

and $\qquad D\left(\dfrac{5-1}{2}, \dfrac{-1-1}{2}\right)$ i.e. D(2, –1)

Now, $\qquad AB = \sqrt{(2+1)^2 + \left(4 - \dfrac{3}{2}\right)^2}$

$$= \sqrt{9 + \frac{25}{4}} = \sqrt{\frac{61}{4}}$$

$$BC = \sqrt{(5-2)^2 + \left(\frac{3}{2} - 4\right)^2}$$

$$= \sqrt{9 + \frac{25}{4}} = \sqrt{\frac{61}{4}}$$

$$CD = \sqrt{(2-5)^2 + \left(-1-\frac{3}{2}\right)^2}$$

$$= \sqrt{9+\frac{25}{4}} = \sqrt{\frac{61}{4}}$$

$$AD = \sqrt{(2+1)^2 + \left(-1-\frac{3}{2}\right)^2}$$

$$= \sqrt{9+\frac{25}{4}} = \sqrt{\frac{61}{4}}$$

$\because$ $\quad AB = BC = CD = DA = \sqrt{\frac{61}{4}}$

So, ABCD is either a square or a rhombus but not a rectangle.

Now, $\quad AC = \sqrt{(5+1)^2 + \left(\frac{3}{2}-\frac{3}{2}\right)^2}$

$$= \sqrt{36+0} = 6$$

and $\quad BD = \sqrt{(2-2)^2 + (-1-4)^2}$

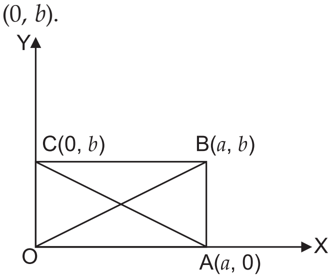
$$= \sqrt{0+25} = 5$$

Since, $\quad AC \neq BD$,

the diagonals are not equal. We know that the diagonals of a square are equal.

Hence, ABCD is a **rhombus**.

33. We take two adjacent sides OA and OC of a rectangle OABC along the x and y axes respectively. Let the other two sides be AB and BC which are respectively parallel to y-axis and x-axis. Then $\angle ABC = \angle OCB = \angle OAB = 90°$, OA = CB and OC = AB, O being the origin. We now join the diagonals OB and AC. Let A be the point $(a, 0)$, C be the point $(0, b)$.



Then B is the point (a, b). Using distance formula, we have

$$OB = \sqrt{(a-0)^2 + (b-0)^2} = \sqrt{a^2+b^2}$$

and $\quad AC = \sqrt{(a-0)^2 + (0-b)^2} = \sqrt{a^2+b^2}$

$\therefore$ $\quad OB = AC$, i.e. the two diagonals are equal.

Also, only opposite sides are equal to each other, but not all sides are equal to each other. Hence, the figure is a rectangle whose two diagonals are equal to each other.

EXERCISE 7E

For Basic and Standard Levels

1. Let the four given points be A(4, 12), B(6, –2), C(5, –10) and D(3, 4).

Then, the coordinates of mid-point of AC are

$$\left(\frac{4+5}{2}, \frac{12-10}{2}\right) \text{ i.e. } \left(\frac{9}{2}, 1\right).$$

Also, coordinates of mid-point of BD are

$$\left(\frac{6+3}{2}, \frac{-2+4}{2}\right) \text{ i.e. } \left(\frac{9}{2}, 1\right).$$

Since the coordinates of mid-points of AC and BD are same, the diagonals AC and BD of the quadrilateral ABCD bisect each other.

Hence, ABCD is a parallelogram.

2. (*i*) Let the coordinates of the fourth vertex of the parallelogram be D(x, y).

Then, coordinates of mid-point of AC are

$$\left(\frac{1+6}{2}, \frac{2+6}{2}\right) \text{ i.e. } \left(\frac{7}{2}, 4\right).$$

Also, coordinates of the mid-point of BD are

$$\left(\frac{4+x}{2}, \frac{3+y}{2}\right)$$

But ABCD is a parallelogram.

So, the coordinates of the mid-points of AC and BD are same.

So, $\quad \frac{x+4}{2} = \frac{7}{2} \quad \Rightarrow \quad x = 7 - 4 = 3$

and $\quad \frac{3+y}{2} = 4 \quad \Rightarrow \quad y = 8 - 3 = 5$

Hence, the coordinates of the vertex D are **(3, 5)**.

(*ii*) Let the coordinates of the vertex D be (x_1, y_1).

Then, coordinates of the mid-point of the diagonal AC are

$$\left(\frac{x+x-1}{2}, \frac{y+y+7}{2}\right) \text{ i.e. } \left(x-\frac{1}{2}, y+\frac{7}{2}\right)$$

Also, coordinates of the mid-point of BD are

$$\left(\frac{x+3+x_1}{2}, \frac{y+4+y_1}{2}\right).$$

Since ABCD is a parallelogram, the coordinates of the mid-points of the diagonals AC and BD are same.

$\Rightarrow \quad \frac{x+3+x_1}{2} = x - \frac{1}{2}$

$\Rightarrow \quad x_1 = 2\left(x-\frac{1}{2}\right) - x - 3$

$$= 2x - 1 - x - 3 = x - 4$$

and $\quad \frac{y+4+y_1}{2} = y + \frac{7}{2}$

$\Rightarrow \quad y_1 = 2\left(y+\frac{7}{2}\right) - y - 4$

$$= 2y + 7 - y - 4$$

$$= y + 3$$

Hence, the coordinates of the vertex D are

(x – 4, y + 3).

(*iii*) Let the coordinates of the vertex C be (x, y).

Then, coordinates of the mid-point of AC are

$$\left(\frac{3+x}{2}, \frac{3+y}{2}\right).$$

Also, coordinates of the mid-point of BD are

$$\left(\frac{6-5}{2}, \frac{-1+9}{2}\right), \text{ i.e. } \left(\frac{1}{2}, 4\right).$$

But ABCD is a parallelogram, so the coordinates of the mid-points of the diagonals AC and BD are same.

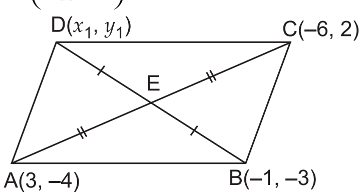
$$\Rightarrow \quad \frac{3+x}{2} = \frac{1}{2} \Rightarrow x = 1 - 3 = -2$$

$$\text{and} \quad \frac{3+y}{2} = 4 \Rightarrow y = 8 - 3 = 5$$

Hence, the coordinates of C are **(–2, 5)**.

(*iv*) Let ABCD be the ∥gm where D is the point (x_1, y_1). We know that the two diagonals AC and BD of the ∥gm bisect each other at a point E. Since E is the mid-point of AC, hence, the coordinates of E are $\left(\frac{3-6}{2}, \frac{-4+2}{2}\right) = \left(-\frac{3}{2}, -1\right)$.

Hence, $\left(-\frac{3}{2}, -1\right)$ is the mid-point of BD also.



$$\text{Hence,} \quad \frac{x_1 - 1}{2} = -\frac{3}{2}$$

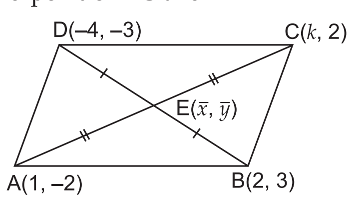
$$\Rightarrow \quad x_1 = -2$$

$$\text{and} \quad \frac{y_1 - 3}{2} = -1$$

$$\Rightarrow \quad y_1 = -2 + 3 = 1$$

Hence, the required coordinates of the fourth vertex D are (–2, 1).

3. (*i*) Let E $(\bar{x}, \bar{y})$ be the point of intersection of the two diagonals AC and BD of the ∥gm ABCD. Then E is the mid-point of AC and BD.



$$\therefore \quad \bar{x} = \frac{1+k}{2} = \frac{2-4}{2}$$

$$\Rightarrow \quad 1 + k = -2$$

$$\Rightarrow \quad k = -3$$

Hence, the required value of k is –3.

(*ii*) Coordinates of the mid-point of AC are

$$\left(\frac{-4+8}{2}, \frac{2+6}{2}\right), \text{ i.e. } (2, 4).$$

Also, coordinates of the mid-point of BD are

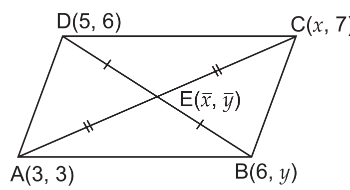
$$\left(\frac{2+a}{2}, \frac{0+b}{2}\right), \text{ i.e. } \left(\frac{2+a}{2}, \frac{b}{2}\right).$$

Since ABCD is a parallelogram, the coordinates of the mid-points of the diagonals AC and BD are same.

$$\Rightarrow \quad \frac{2+a}{2} = 2 \Rightarrow a = 4 - 2 = \mathbf{2}$$

$$\text{and} \quad \frac{b}{2} = 4 \Rightarrow b = \mathbf{8}$$

(*iii*) Let ABCD be the ∥gm where A is the point (3, 3), B is the point $(6, y)$, C is the point $(x, 7)$ and D is the point (5, 6). Let the two diagonals AC and BD of the ∥gm bisect each other at E$(\bar{x}, \bar{y})$. Then E is the mid-point of both AC and BD.



$$\text{Then} \quad \bar{x} = \frac{3+x}{2} = \frac{6+5}{2}$$

$$\Rightarrow \quad x = 11 - 3 = 8$$

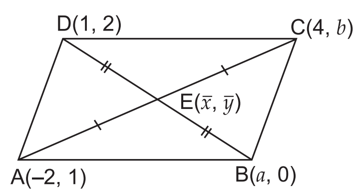
$$\text{and} \quad \bar{y} = \frac{7+3}{2} = \frac{6+y}{2}$$

$$\Rightarrow \quad 10 = 6 + y$$

$$\Rightarrow \quad y = 10 - 6 = 4$$

Hence, the required values of x and y are **8** are **4** respectively.

4. Let the two diagonals AC and BD of the ∥gm ABCD intersect each other at the point E$(\bar{x}, \bar{y})$. Then $(\bar{x}, \bar{y})$ is the mid-point of both AC and BD.



$$\therefore \quad \bar{x} = \frac{-2+4}{2} = \frac{1+a}{2}$$

$$\Rightarrow \quad 2 = 1 + a$$

$$\Rightarrow \quad a = 2 - 1 = 1$$

$$\text{and} \quad \bar{y} = \frac{b+1}{2} = \frac{2+0}{2}$$

$$\Rightarrow \quad b + 1 = 2$$

$$\Rightarrow \quad b = 2 - 1 = 1$$

∴ B is the point (1, 0).

$\therefore \quad AB = \sqrt{(-2-1)^2 + (1-0)^2}$

$= \sqrt{9+1} = \sqrt{10}$

C is the point (4, 1).

$\therefore \quad BC = \sqrt{(a-4)^2 + (-b)^2}$

$= \sqrt{(1-4)^2 + 1^2} = \sqrt{10}$

$DC = \sqrt{(1-4)^2 + (2-b)^2}$

$= \sqrt{9+1^2} = \sqrt{10}$

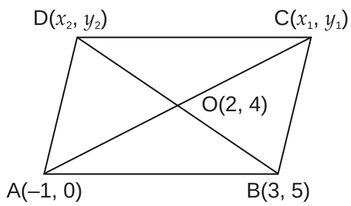
$AD = \sqrt{(1+2)^2 + (2-1)^2}$

$= \sqrt{9+1} = \sqrt{10}$

Hence, the ‖gm ABCD is a square, since AB = BC = CD = DA = $\sqrt{\mathbf{10}}$ **units**.

5. Let the given parallelogram be ABCD in which the two adjacent vertices be A(–1, 0) and B(3, 5). Also, let the other two vertices be C(x_1, y_1) and D(x_2, y_2).

The diagonals of a parallelogram bisect each other. So, O is the mid-point of AC as well as BD.



So, $\quad 2 = \dfrac{x_1 + (-1)}{2} \Rightarrow x_1 = 4 + 1 = 5$

and $\quad 4 = \dfrac{y_1 + 0}{2} \Rightarrow y_1 = 8$

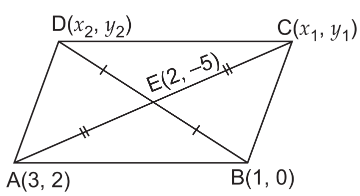
i.e. the coordinates of C are **(5, 8)**.

Also, $\quad 2 = \dfrac{x_2 + 3}{2} \Rightarrow x_2 = 4 - 3 = 1$

and $\quad 4 = \dfrac{y_2 + 5}{2} \Rightarrow y_2 = 8 - 5 = 3$

i.e. the coordinates of D are **(1, 3)**.

6. Let ABCD be a parallelogram with adjacent vertices A(3, 2) and B(1, 0). Let the two diagonals AC and BD intersect each other at a point E (2, – 5). Then E is the mid-point of AC and BD.



Let the coordinates of C and D be (x_1, y_1) and (x_2, y_2) respectively.

$\therefore$ We have $\quad 2 = \dfrac{x_1 + 3}{2}$

$\Rightarrow \quad x_1 = 4 - 3 = 1$

$-5 = \dfrac{y_1 + 2}{2}$

$\Rightarrow \quad y_1 = -12$

and $\quad 2 = \dfrac{x_2 + 1}{2}$

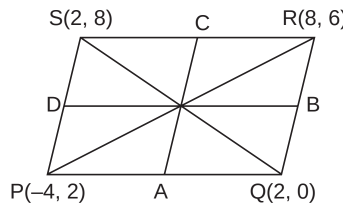
$\Rightarrow \quad x_2 = 4 - 1 = 3$

$-5 = \dfrac{y_2 + 0}{2}$

$\Rightarrow \quad y_2 = -10$

Hence, the required coordinates of the other two vertices of the ‖gm are **(1, –12)** and **(3, –10)**.

7. Coordinates of the mid-point of the diagonal PR are



$$\left(\frac{-4+8}{2}, \frac{2+6}{2}\right), \text{ i.e. } (2, 4).$$

Now, coordinates of mid-point of PQ are

$$A\left(\frac{-4+2}{2}, \frac{2+0}{2}\right), \text{ i.e. } A(-1, 1)$$

and coordinates of the mid-point of SR are

$$C\left(\frac{2+8}{2}, \frac{8+6}{2}\right), \text{ i.e. } C(5, 7).$$

So, coordinates of the mid-point of AC are

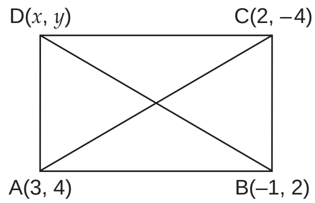
$$\left(\frac{-1+5}{2}, \frac{1+7}{2}\right), \text{ i.e. } (2, 4).$$

Since, the coordinates of the mid-point of the diagonal PR are same as the coordinates of the mid-point of the line segment joining the mid-points A and C of sides PQ and SR respectively, this shows that the diagonal bisects the line segment joining the mid-points of opposite sides. Similarly, we can show that the coordinates of mid-point of the diagonal AC are same as the coordinates of the mid-point of the line joining the mid-points of the other opposite sides QR and PS.

8. Let the coordinates of the fourth vertex of the rectangle be (x, y).

Then, the coordinates of the mid-point of diagonal AC are

$$\left(\frac{2+3}{2}, \frac{-4+4}{2}\right), \text{ i.e. } \left(\frac{5}{2}, 0\right).$$



Also, the coordinates of the mid-point of diagonal BD are

$$\left(\frac{x+(-1)}{2}, \frac{y+2}{2}\right)$$

But the coordinates of the mid-points of diagonals AC and BD will be same as ABCD is a rectangle.

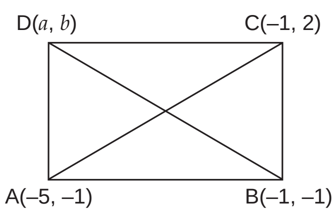
$$\Rightarrow \quad \frac{x+(-1)}{2} = \frac{5}{2} \Rightarrow x = 5 + 1 = 6$$

$$\text{and} \quad \frac{y+2}{2} = 0 \Rightarrow y = -2$$

Hence, the coordinates of the fourth vertex of the rectangle are **(6, –2)**.

9. (*i*) The coordinates of the mid-point of the diagonal AC are

$$\left(\frac{-5+(-1)}{2}, \frac{-1+2}{2}\right), \text{ i.e. } \left(-3, \frac{1}{2}\right).$$



Also, the coordinates of the mid-point of the diagonal BD are

$$\left(\frac{a+(-1)}{2}, \frac{b+(-1)}{2}\right), \text{ i.e. } \left(\frac{a-1}{2}, \frac{b-1}{2}\right).$$

But the coordinates of the mid-points of AC and BD are same as ABCD is a rectangle whose diagonals bisect each other at the same points.

$$\text{So,} \quad \frac{a-1}{2} = -3 \Rightarrow a = -6 + 1 = \mathbf{-5}$$

$$\text{and} \quad \frac{b-1}{2} = \frac{1}{2} \Rightarrow b = 1 + 1 = \mathbf{2}$$

(*ii*) Now,

$$AC = \sqrt{[-1-(-5)]^2 + [2-(-1)]^2}$$

$$= \sqrt{(-1+5)^2 + (2+1)^2}$$

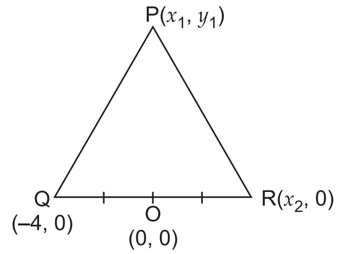
$$= \sqrt{16+9}$$

$$= \sqrt{25} = 5 \text{ units}$$

Length of diagonals of a rectangle are equal. So, AC = BD = 5 units.

Hence, the length of the diagonals are **5 units** each.

10. Let PQR be the equilateral triangle with Q as the point (–4, 0). Since R lies on the x-axis, hence, its ordinate is 0. Let the coordinate of R be (x_2, 0).



Since O(0, 0), the origin is the mid-point of QR,

$$\therefore \quad \frac{x_2 - 4}{2} = 0$$

$$\Rightarrow \quad x_2 = 4$$

$\therefore$ R is the point (4, 0).

Let P be the point (x_1, y_1).

Since $\quad PQ' = PR$

$$\therefore \quad PQ^2 = PR^2$$

$$\Rightarrow \quad (x_1 + 4)^2 + y_1^2 = (x_1 - 4)^2 + y_1^2$$

$$\Rightarrow \quad 16x_1 = 0$$

$$\Rightarrow \quad x_1 = 0$$

$\therefore$ P is the point (0, y_1).

Now, $\quad QR = 4 + 4 = 8$

$\because$ PQR is an equilateral triangle.

Hence $\quad PQ = QR$

$$\Rightarrow \quad PQ^2 = QR^2$$

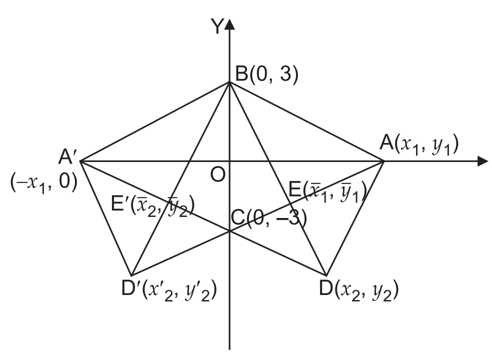
$$(0 + 4)^2 + y_1^2 = 8^2 = 64$$

$$\Rightarrow \quad y_1^2 = 64 - 16 = 48$$

$$\Rightarrow \quad y_1 = \pm\sqrt{48} = \pm 4\sqrt{3}$$

Hence, the required coordinates of P are $\left(\mathbf{0, 4\sqrt{3}}\right)$ or $\left(\mathbf{0, -4\sqrt{3}}\right)$ and the required coordinates of R are **(4, 0)**.

11. Let ABC be the equilateral triangle with B as the point (0, 3) and C is the point (0, –3). Let A be the point (x_1, y_1).



Now $\quad BC = 3 + 3 = 6$

$$\text{and} \quad AB = \sqrt{x_1^2 + (y_1 - 3)^2} = 6$$

$$\Rightarrow \quad x_1^2 + (y_1 - 3)^2 = 36 \qquad \ldots(1)$$

$$\text{Also,} \quad AC = \sqrt{x_1^2 + (y_1 + 3)^2}$$

$\because \quad AB = AC$

$\therefore \quad AB^2 = AC^2$

$\Rightarrow \quad x_1^2 + (y_1 - 3)^2 = x_1^2 + (y_1 + 3)^2$

$\Rightarrow \quad 4 \times 3 \times y_1 = 0$

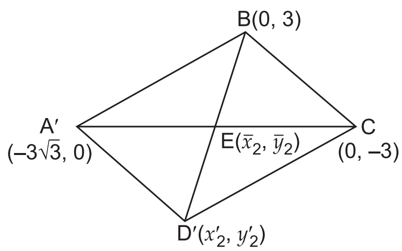
$\Rightarrow \quad y_1 = 0$

$\therefore$ From (1), $\quad x_1^2 + 9 = 36$

$\Rightarrow \quad x_1^2 = 27$

$\therefore \quad x_1 = \pm 3\sqrt{3}$

$\therefore$ The required coordinates of C are (0, –3) and those of A are $\left(\mathbf{3\sqrt{3}, 0}\right)$ or A′ $\left(\mathbf{-3\sqrt{3}, 0}\right)$.



Now, if ABCD is rhombus with A$\left(3\sqrt{3}, 0\right)$, B(0, 3), C(0, –3) and D(x_2, y_2), then the two diagonals BD and AC bisect each other at E$\left(\bar{x}_1, \bar{y}_1\right)$.

$$\therefore \quad \bar{x}_1 = \frac{3\sqrt{3} + 0}{2} = \frac{3\sqrt{3}}{2}$$

$$\bar{y}_1 = \frac{0 - 3}{2} = \frac{-3}{2}$$

$$\text{Also,} \quad \bar{x}_1 = \frac{x_2 + 0}{2}, \ \bar{y}_1 = \frac{y_2 + 3}{2}$$

$$\therefore \quad \frac{x_2}{2} = \frac{3\sqrt{3}}{2}$$

$$\Rightarrow \quad x_2 = 3\sqrt{3}$$

$$\text{and} \quad \frac{y_2 + 3}{2} = -\frac{3}{2}$$

$$\Rightarrow \quad y_2 = -6$$

$\therefore$ The required coordinates of D are $\left(\mathbf{3\sqrt{3}, 6}\right)$.

Similarly, A′BCD′ will be another rhombus with A′$\left(-3\sqrt{3}, 0\right)$, B(0, 3), C(0, –3) and D′(x'_2, y'_2).

Now, two diagonals BD′ and A′C bisect each other at $\left(\bar{x}_2, \bar{y}_2\right)$.

$$\therefore \quad \bar{x}_2 = \frac{-3\sqrt{3} + 0}{2} = \frac{x'_2 + 0}{2}$$

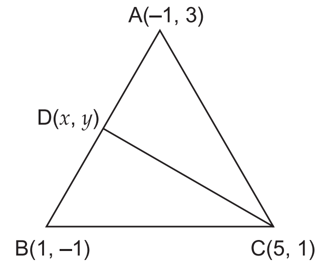
$$\Rightarrow \quad x'_2 = -3\sqrt{3}$$

$$\text{and} \quad \bar{y}_2 = \frac{3 + y'_2}{2} = \frac{0 - 3}{2}$$

$$\Rightarrow \quad y'_2 = -6$$

$\therefore$ The required coordinates of D are $\left(\mathbf{-3\sqrt{3}, -6}\right)$.

12. Since AD is median through C to AB.



i.e. D in the mid-point of AB.

Median through vertex C meets line segment AB at its mid-point

Mid-point of AB, i.e D(x, y)

$$x = \frac{1 + (-1)}{2} = 0 \qquad y = \frac{3 + (-1)}{2} = 1$$

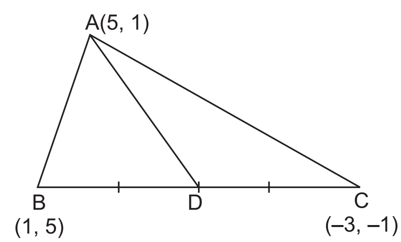
Mid-point of AB is D (0, 1)

$$\text{Length of median} = \sqrt{(5-0)^2 + (1-1)^2}$$

$$= \sqrt{25 + 0} = \sqrt{25} = \mathbf{5 \text{ units}}$$

13. $\because$ AD is a median of ΔABC,

$\therefore$ BD = DC, i.e. D is the mid-point of BC.



Hence, the coordinates of D are

$$\left(\frac{1-3}{2}, \frac{5-1}{2}\right) = \mathbf{(-1, 2)}$$

$$\therefore \quad AD = \sqrt{(5+1)^2 + (1-2)^2}$$

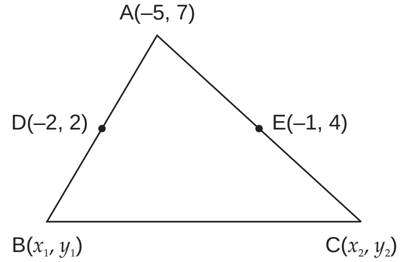
$$= \sqrt{36 + 1} = \mathbf{\sqrt{37}}$$

which is the required length of the median AD.

14. Since D in the mid-point of AB, so

$$-2 = \frac{-5 + x_1}{2} \Rightarrow x_1 = 1$$

$$2 = \frac{7 + y_1}{2} \Rightarrow y_1 = 4 - 7 = -3$$



The coordinates of the vertex B are (1, –3).

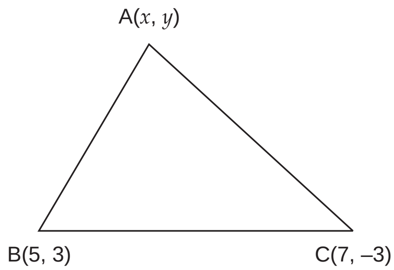
Again, $-1 = \dfrac{-5+x_2}{2} \Rightarrow x_2 = -2+5 = 3$

and $4 = \dfrac{7+y_2}{2} \Rightarrow y_2 = 8-7 = 1$

The coordinates of the vertex C are (3, 1).

Hence, the other two vertices are **(1, –3)** and **(3, 1)**.

15. Let ABC be a triangle whose two given vertices are B(5, 3) and C(7, –3).



Let the coordinates of third vertex A be (x, y) and the point (2, 2) be the mid-point of AB.

Then, $2 = \dfrac{x+5}{2} \Rightarrow x = 4-5 = -1$

and $2 = \dfrac{y+3}{2} \Rightarrow y = 4-3 = 1$

So, the coordinates of the third vertex are (–1, 1).

Again, let point (2, 2) be the mid-point of AC.

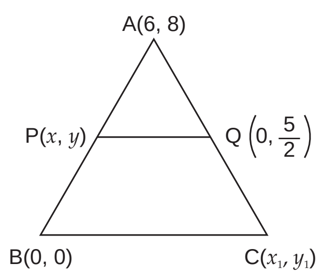
Then $2 = \dfrac{x+7}{2} \Rightarrow x = 4-7 = -3$

and $2 = \dfrac{y-3}{2} \Rightarrow y = 4+3 = 7.$

So, the coordinates of the third vertex are (–3, 7).

Hence, the coordinates of the third vertex are either **(–1, 1) or (–3, 7)**.

16. P is the mid-point of the line segment AB.



So, $x = \dfrac{6+0}{2} = 3$

and $y = \dfrac{8+0}{2} = 4$

Also, PQ || BC

So, Q is also the mid-point of AC.

$\Rightarrow$ $0 = \dfrac{6+x_1}{2} \Rightarrow x_1 = -6$

and $\dfrac{5}{2} = \dfrac{8+y_1}{2} \Rightarrow y_1 = 5-8 = -3$

Hence, the coordinates of P are **(3, 4)** and coordinates of C are **(–6, –3)**.

Now, $BC = \sqrt{(-6-0)^2+(-3-0)^2}$

$= \sqrt{36+9} = \sqrt{45}$

$= 3\sqrt{5}$

and $PQ = \sqrt{(3-0)^2+\left(4-\dfrac{5}{2}\right)^2}$

$= \sqrt{9+\dfrac{9}{4}}$

$= \sqrt{\dfrac{36+9}{4}} = \sqrt{\dfrac{45}{4}}$

$= \dfrac{3\sqrt{5}}{2}$

$\Rightarrow$ $2PQ = 3\sqrt{5}$

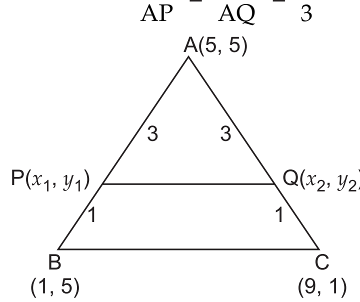
Hence, BC = 2PQ.

17. Given that $\dfrac{AP}{AB} = \dfrac{AQ}{AC} = \dfrac{3}{4}$

$\Rightarrow$ $\dfrac{AB}{AP} = \dfrac{AC}{AQ} = \dfrac{4}{3}$

$\therefore$ $\dfrac{AB-AP}{AP} = \dfrac{AC-AQ}{AQ} = \dfrac{4-3}{3} = \dfrac{1}{3}$

$\Rightarrow$ $\dfrac{PB}{AP} = \dfrac{QC}{AQ} = \dfrac{1}{3}$



$\therefore$ P is a point on AB such that PB : PA = 1 : 3.

Similar Q is a point or AC such that QC : QA = 1 : 3.

If (x_1, y_1) and (x_2, y_2) be the coordinates of P and Q respectively, then by using section formula, we have

$x_1 = \dfrac{1\times5+3\times1}{1+3} = \dfrac{5+3}{4} = 2$

and $y_1 = \dfrac{1\times5+3\times5}{1+3} = \dfrac{20}{4} = 5$

Similarly, $x_2 = \dfrac{1\times5+3\times9}{1+3} = \dfrac{32}{4} = 8$

and $y_2 = \dfrac{1\times5+3\times1}{1+3} = \dfrac{8}{4} = 2$

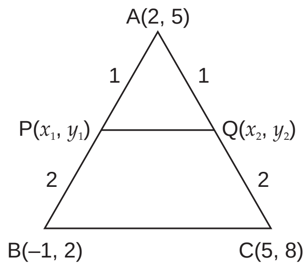
Hence, the coordinates of P and Q are (2, 5) and (8, 2) respectively.

$\therefore$ Required length of $PQ = \sqrt{(2-8)^2+(5-2)^2}$

$= \sqrt{36+9}$

$= \sqrt{45} = \mathbf{3\sqrt{5}}$ **units**.

18. Let the coordinates of P be (x_1, y_1) and that of Q be (x_2, y_2).



(*i*) So, $x_1 = \dfrac{1 \times (-1) + 2 \times 2}{1+2} = \dfrac{-1+4}{3} = \dfrac{3}{3} = 1$

$$y_1 = \frac{1 \times 2 + 2 \times 5}{1+2} = \frac{2+10}{3} = 4$$

Also, $x_2 = \dfrac{1 \times 5 + 2 \times 2}{1+2} = \dfrac{5+4}{3} = 3$

$$y_2 = \frac{1 \times 8 + 2 \times 5}{1+2} = \frac{8+10}{3} = 6$$

So, the coordinates of P are **(1, 4)** and that of Q are **(3, 6)**.

(*ii*) Now, $PQ = \sqrt{(3-1)^2 + (6-4)^2}$

$$= \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$$

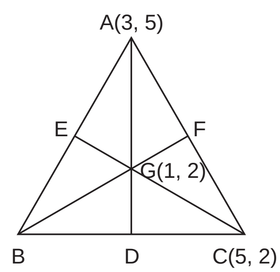
And $BC = \sqrt{(5+1)^2 + (8-2)^2}$

$$= \sqrt{36+36} = \sqrt{72} = 6\sqrt{2}$$

So, $3PQ = 3 \times 2\sqrt{2} = 6\sqrt{2} = BC$

Hence, $PQ = \dfrac{1}{3} BC$.

19. The centroid divides the median AD in the ratio 2 : 1.



(*i*) Let the coordinates of D be (x, y).

Then, $1 = \dfrac{1 \times 3 + 2x}{1+2} \Rightarrow 2x = 3 - 3 = 0$

$\Rightarrow \quad x = 0$

and $2 = \dfrac{1 \times 5 + 2y}{1+2} \Rightarrow 2y = 6 - 5 = 1$

$\Rightarrow \quad y = \dfrac{1}{2}$

Hence, the coordinates of D are $\left(\mathbf{0}, \dfrac{\mathbf{1}}{\mathbf{2}}\right)$.

(*ii*) Also, the centroid divides the median CE in the ratio 2 : 1.

Let the coordinates of E be (x_1, y_1).

Then, $1 = \dfrac{1 \times 5 + 2 \times x_1}{1+2} \Rightarrow 2x_1 = 3 - 5 = -2$

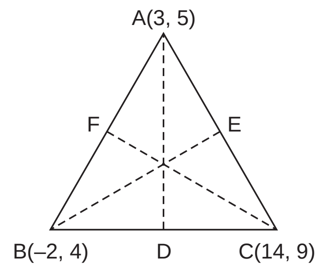
$\Rightarrow \quad x_1 = -1$

$$2 = \frac{1 \times 2 + 2 \times y_1}{1+2} \Rightarrow 2 + 2y_1 = 6$$

$\Rightarrow \quad y_1 = \dfrac{6-2}{2} = 2$

Hence, coordinates of E are **(–1, 2)**.

20. Let the triangle be ABC with vertices A(3, 5), B(–2, 4) and C(14, 9).



Also, let D, E and F be the mid-points of BC, AC and AB respectively.

Then, the coordinates of D are

$$\left(\frac{-2+14}{2}, \frac{9+4}{2}\right) \text{ i.e. } D\left(6, \frac{13}{2}\right)$$

The coordinates of E are

$$\left(\frac{3+14}{2}, \frac{5+9}{2}\right), \text{ i.e. } E\left(\frac{17}{2}, 7\right)$$

The coordinates of F are

$$\left(\frac{3+(-2)}{2}, \frac{5+4}{2}\right), \text{ i.e. } F\left(\frac{1}{2}, \frac{9}{2}\right)$$

So, the length of medians are

$$AD = \sqrt{(6-3)^2 + \left(\frac{13}{2} - 5\right)^2}$$

$$= \sqrt{9 + \frac{9}{4}}$$

$$= \sqrt{\frac{45}{4}} = \frac{3}{2}\sqrt{5} = \frac{3}{2} \times 2.23 = \mathbf{3.35 \text{ units}}$$

$$BE = \sqrt{\left(\frac{17}{2} + 2\right)^2 + (7-4)^2}$$

$$= \sqrt{\frac{441}{4} + 9}$$

$$= \sqrt{\frac{441+36}{4}} = \sqrt{\frac{477}{4}}$$

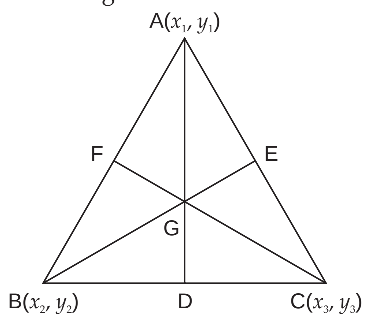
$$= \frac{1}{2}\sqrt{477} = \frac{1}{2} \times 21.84 = \mathbf{10.92 \text{ units}}$$

$$CF = \sqrt{\left(14-\frac{1}{2}\right)^2+\left(9-\frac{9}{2}\right)^2}$$

$$= \sqrt{\frac{729}{4}+\frac{81}{4}} = \sqrt{\frac{810}{4}}$$

$$= \frac{1}{2}\sqrt{810} = \frac{1}{2} \times 28.46 = \textbf{14.23 units}$$

21. Centroid of a triangle is the point where all the three medians of the triangle meet.



Since AD is the median, D is the mid-point of BC.

Therefore, coordinates of D are $\left(\frac{x_2+x_3}{2}, \frac{y_2+y_3}{2}\right)$.

Now, the centroid of a triangle divides the median in the ratio 2 : 1. So, G divides AD in the ratio 2 : 1.
Therefore, the coordinates of G are

$$\left(\frac{2\times\left(\frac{x_2+x_3}{2}\right)+1\times x_1}{3}, \frac{2\left(\frac{y_2+y_3}{2}\right)+1\times y_1}{3}\right)$$

i.e. $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$.

22. (*i*) The coordinates of the centroid of a triangle with vertices A(x_1, y_1), B(x_2, y_2) and C(x_3, y_3) are given by

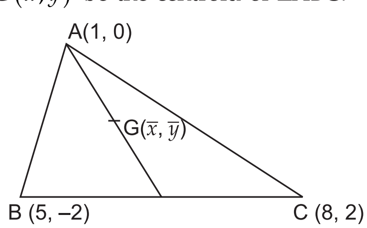
$$\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right).$$

Here, $x_1 = -4$, $y_1 = 2$, $x_2 = 1$, $y_2 = 3$, $x_3 = 2$, $y_3 = 5$
Therefore, coordinates of the centroid of the given triangle will be

$$\left(\frac{-4+1+2}{3}, \frac{2+3+5}{3}\right) \text{ i.e. } \left(\frac{-1}{3}, \frac{10}{3}\right).$$

Hence, the coordinates of the centroid is $\left(\mathbf{\frac{-1}{3}}, \mathbf{\frac{10}{3}}\right)$.

(*ii*) Let G$(\bar{x}, \bar{y})$ be the centroid of ΔABC.



Then $\bar{x} = \frac{1}{3}(-1+5+8) = 4$

and $\bar{y} = \frac{1}{3}\{0+(-2)+2\} = 0$

∴ The required centroid is **(4, 0)**.

23. The point of intersection of the medians of a triangle is the centroid. The coordinates of the centroid of the triangle with vertices (x_1, y_1), (x_2, y_2) and (x_3, y_3) are given by

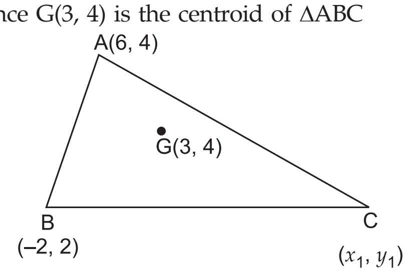
$$\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$$

Here, $x_1 = 2$, $y_1 = 3$, $x_2 = -3$, $y_2 = 2$, $x_3 = 4$, $y_3 = -2$.
Therefore, coordinates of the centroid will be

$$\left(\frac{2-3+4}{3}, \frac{3+2-2}{3}\right), \text{ i.e. } \mathbf{(1, 1)}.$$

Hence, the coordinates of the centroid are **(1, 1)**.

24. (*i*) Let the coordinates of the third vertex C be (x_1, y_1). Since G(3, 4) is the centroid of ΔABC



∴ $3 = \frac{1}{3}(6-2+x_1) = \frac{4+x_1}{3}$

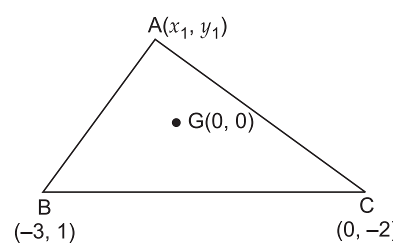
⇒ $x_1 = 9 - 4 = 5$

and $4 = \frac{1}{3}(4+2+y_1) = \frac{6+y_1}{3}$

⇒ $y_1 = 3 \times 4 - 6 = 6$

∴ The required coordinates of C are **(5, 6)**.

(*ii*) Let the coordinates of the third vertex A be (x_1, y_1).



∵ G(0, 0) is the centroid of ΔABC, hence, we have

$0 = \frac{1}{3}(x_1-3+0) = \frac{x_1-3}{3}$

⇒ $x_1 = 3$

and $0 = \frac{1}{3}(y_1+1-2) = \frac{y_1-1}{3}$

⇒ $y_1 = 1$

Hence, the required coordinates of the third vertex are **(3, 1)**.

(*iii*) Let the coordinates of the third vertex of the triangle be (x_3, y_3).

Then, $\frac{1+3+x_3}{3} = 0 \Rightarrow x_3 = -4$

and $$\frac{2+5+y_3}{3} = 0 \Rightarrow y_3 = -7$$

Hence, the coordinates of the third vertex are **(–4, –7)**.

25. It is given that the centroid of the triangle is at origin. Given, $x_1 = 2$, $y_1 = 3$, $x_2 = 3$, $y_3 = -2$.

Now, $$\frac{x_1+x_2+x_3}{3} = 0$$

$$\Rightarrow \quad 2 + 3 + x_3 = 0 \Rightarrow x_3 = \mathbf{-5}$$

and $$\frac{y_1+y_2+y_3}{3} = 0$$

$$\Rightarrow \quad 3 + y_2 - 2 = 0 \Rightarrow y_2 = \mathbf{-1}$$

26. Let the coordinates of the third vertex be (x, y).

The point in the triangle at which the medians meet is the centroid.

Then, $$\frac{5+(-1)+x}{3} = 0$$

$$\Rightarrow \quad x = 0 - 4 = -4$$

and $$\frac{2+4+y}{3} = -3$$

$$\Rightarrow \quad y = -9 - 6 = -15$$

Hence, the coordinates of the third vertex are **(–4, –15)**.

27. The coordinates of the centroid G(–3, 4) of the triangle with vertices A(6, 2), B(x, 3) and C(0, y) are given by

$$-3 = \frac{6+x+0}{3} \Rightarrow x = -9 - 6 = -15$$

and $$4 = \frac{2+3+y}{3} \Rightarrow y = 12 - 5 = 7$$

Hence, $\boldsymbol{x = -15, y = 7}$.

28. The coordinates of the centroid of the triangle with vertices P(3, x), Q(–2, 3) and R(y, 5) will be given by

$$\left(\frac{3+(-2)+y}{3}, \frac{x+3+5}{3}\right), \text{ i.e. } \left(\frac{1+y}{3}, \frac{x+8}{3}\right).$$

But the centroid is at the origin.

So, $$\frac{1+y}{3} = 0 \Rightarrow \boldsymbol{y = -1}$$

and $$\frac{x+8}{3} = 0 \Rightarrow \boldsymbol{x = -8}$$

29. Coordinates of the centroid of the given vertices of a triangle will be given by

$$\left(\frac{x+y+z}{3}, \frac{y-z+z-x+x-y}{3}\right) \text{ i.e. } \left(\frac{x+y+z}{3}, 0\right)$$

This is a point whose y-coordinate is 0, i.e. this point lies on the x-axis. Hence, the centroid of the triangle with given vertices lies on x-axis.

30. Coordinates of the mid-points are $P\left(\frac{0+4}{2}, \frac{0-6}{2}\right)$,

$Q\left(\frac{8+4}{2}, \frac{8-6}{2}\right)$, $R\left(\frac{8+6}{2}, \frac{8+8}{2}\right)$ and $S\left(\frac{0+6}{2}, \frac{0+8}{2}\right)$

i.e. P(2, –3), Q(6, 1), R(7, 8) and S(3, 4).

Now, the coordinates of mid-point PR are $\left(\frac{7+2}{2}, \frac{8-3}{2}\right)$

i.e. $\left(\frac{9}{2}, \frac{5}{2}\right)$ and the coordinates of mid-point of SQ are

$\left(\frac{6+3}{2}, \frac{1+4}{2}\right)$, i.e. $\left(\boldsymbol{\frac{9}{2}, \frac{5}{2}}\right)$.

Since the coordinates of the mid-points of PR and QS are same, PR and QS are diagonals of parallelogram PQRS.

Therefore, **PQRS is a parallelogram**.

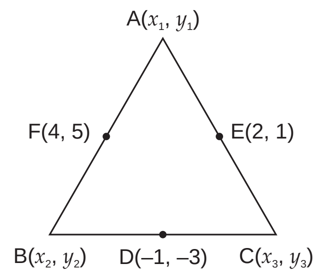
For Standard Level

31. (*i*) Let the vertices of the triangle be A(x_1, y_1), B(x_2, y_2) and C(x_3, y_3).

Since D is the mid-point of BC, we have

$$-1 = \frac{x_2+x_3}{2} \Rightarrow x_2 + x_3 = -2 \quad \ldots(1)$$

and $$-3 = \frac{y_2+y_3}{2} \Rightarrow y_2 + y_3 = -6 \quad \ldots(2)$$



Since E is the mid-point of AC, we have

$$2 = \frac{x_1+x_3}{2} \Rightarrow x_1 + x_3 = 4 \quad \ldots(3)$$

$$1 = \frac{y_1+y_3}{2} \Rightarrow y_1 + y_3 = 2 \quad \ldots(4)$$

Also, since F is the mid-point of AB, we have

$$4 = \frac{x_1+x_2}{2} \Rightarrow x_1 + x_2 = 8 \quad \ldots(5)$$

$$5 = \frac{y_1+y_2}{2} \Rightarrow y_1 + y_2 = 10 \quad \ldots(6)$$

Adding (1), (3) and (5), we get

$$x_2 + x_3 + x_1 + x_3 + x_1 + x_2 = -2 + 4 + 8$$

$$\Rightarrow \quad 2(x_1 + x_2 + x_3) = 10$$

$$\Rightarrow \quad x_1 + x_2 + x_3 = 5 \quad \ldots(7)$$

From (1) and (7),

$$x_1 + (-2) = 5$$

$$\Rightarrow \quad x_1 = 5 + 2 = 7$$

From (3) and (7),

$$x_2 + 4 = 5$$

$$\Rightarrow \quad x_2 = 5 - 4 = 1$$

From (5) and (7),

$$8 + x_3 = 5$$

$$\Rightarrow \quad x_3 = 5 - 8 = -3$$

Now, adding (2), (4) and (6), we get

$$y_2 + y_3 + y_1 + y_3 + y_1 + y_2 = -6 + 2 + 10$$

$$\Rightarrow \quad 2(y_1 + y_2 + y_3) = 6$$

$$\Rightarrow \quad y_1 + y_2 + y_3 = 3 \quad \ldots(8)$$

From (2) and (8),

$$y_1 + (-6) = 3$$

$$\Rightarrow \quad y_1 = 3 + 6 = 9$$

From (4) and (8),

$$y_2 + 2 = 3$$

$$\Rightarrow \quad y_2 = 3 - 2 = 1$$

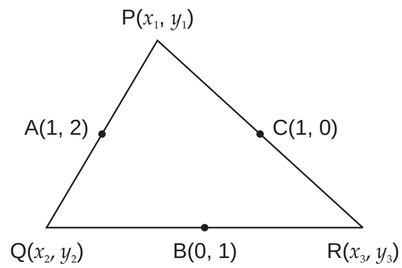
From (6) and (8),

$$10 + y_3 = 3$$

$$\Rightarrow \quad y_3 = 3 - 10 = -7$$

Hence, the three vertices of the triangle are **(7, 9), (1, 1)** and **(–3, –7)**.

(*ii*) Let the three vertices of ΔPQR be $P(x_1, y_1)$, $Q(x_2, y_2)$ and $R(x_3, y_3)$.



Since A is the mid-point of PQ, so

$$1 = \frac{x_1 + x_2}{2} \Rightarrow x_1 + x_2 = 2 \quad \ldots(1)$$

and $$2 = \frac{y_1 + y_2}{2} \Rightarrow y_1 + y_2 = 4 \quad \ldots(2)$$

Since B is the mid-point of QR, so

$$0 = \frac{x_2 + x_3}{2} \Rightarrow x_2 + x_3 = 0 \quad \ldots(3)$$

and $$1 = \frac{y_2 + y_3}{2} \Rightarrow y_2 + y_3 = 2 \quad \ldots(4)$$

Since C is the mid-point of PR, so

$$1 = \frac{x_1 + x_3}{2} \Rightarrow x_1 + x_3 = 2 \quad \ldots(5)$$

and $$0 = \frac{y_1 + y_3}{2} \Rightarrow y_1 + y_3 = 0 \quad \ldots(6)$$

Adding (1), (3) and (5), we get

$$x_1 + x_2 + x_2 + x_3 + x_1 + x_3 = 2 + 0 + 2$$

$$\Rightarrow \quad 2(x_1 + x_2 + x_3) = 4$$

$$\Rightarrow \quad x_1 + x_2 + x_3 = 2 \quad \ldots(7)$$

Also, adding (2), (4) and (6), we get

$$y_1 + y_2 + y_2 + y_3 + y_1 + y_3 = 4 + 2 + 0$$

$$\Rightarrow \quad 2(y_1 + y_2 + y_3) = 6$$

$$\Rightarrow \quad y_1 + y_2 + y_3 = 3 \quad \ldots(8)$$

From (1) and (7), we get

$$2 + x_3 = 2 \quad \Rightarrow x_3 = 0$$

From (2) and (8), we get

$$4 + y_3 = 3 \quad \Rightarrow y_3 = -1$$

So, the coordinate of vertex R are (0, –1).

From (3) and (7), we get

$$x_1 + 0 = 2 \quad \Rightarrow x_1 = 2$$

From (4) and (8), we get

$$y_1 + 2 = 3 \quad \Rightarrow y_1 = 1$$

So, coordinates of the vertex P are (2, 1).

Again, from (5) and (7)

$$x_2 + 2 = 2 \quad \Rightarrow x_2 = 0$$

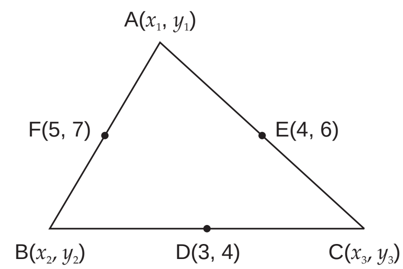
and from (6) and (8),

$$y_2 + 0 = 3 \quad \Rightarrow y_2 = 3$$

So, the coordinates of the vertex Q are (0, 3).

Hence, the three vertices of the triangle are **(2, 1), (0, 3)** and **(0, –1)**.

(*iii*) Let the three vertices of the triangle be $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$.



Also, let the points D, E and F are the mid-points of the sides, BC, AC and AB respectively.

Then, $$3 = \frac{x_2 + x_3}{2} \Rightarrow x_2 + x_3 = 6 \quad \ldots(1)$$

$$4 = \frac{y_2 + y_3}{2} \Rightarrow y_2 + y_3 = 8 \quad \ldots(2)$$

$$4 = \frac{x_1 + x_3}{2} \Rightarrow x_1 + x_3 = 8 \quad \ldots(3)$$

$$6 = \frac{y_1 + y_3}{2} \Rightarrow y_1 + y_3 = 12 \quad \ldots(4)$$

$$5 = \frac{x_1 + x_2}{2} \Rightarrow x_1 + x_2 = 10 \quad \ldots(5)$$

and $$7 = \frac{y_1 + y_2}{2} \Rightarrow y_1 + y_2 = 14 \quad \ldots(6)$$

Adding (1), (3) and (5), we get

$$x_2 + x_3 + x_1 + x_3 + x_1 + x_2 = 6 + 8 + 10$$

$$\Rightarrow \quad 2(x_1 + x_2 + x_3) = 24$$

$$\Rightarrow \quad x_1 + x_2 + x_3 = 12 \quad \ldots(7)$$

Also, adding (2), (4) and (6), we get

$$y_2 + y_3 + y_1 + y_3 + y_1 + y_2 = 8 + 12 + 14$$

$$\Rightarrow \quad 2(y_1 + y_2 + y_3) = 34$$

$$\Rightarrow \quad y_1 + y_2 + y_3 = 17 \quad \ldots(8)$$

From (1) and (7),

$$x_1 + 6 = 12 \Rightarrow x_1 = 12 - 6 = 6$$

From (2) and (8),

$$y_1 + 8 = 17 \Rightarrow y_1 = 17 - 8 = 9$$

From (3) and (7),

$$x_2 + 8 = 12 \Rightarrow x_2 = 12 - 8 = 4$$

From (4) and (8),

$$y_2 + 12 = 17 \Rightarrow y_2 = 17 - 12 = 5$$

From (5) and (7),

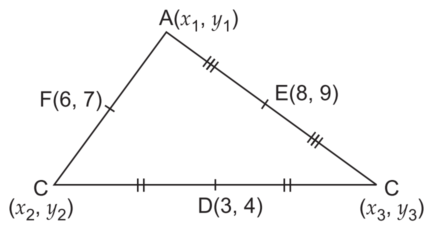
$$10 + x_3 = 12 \Rightarrow x_3 = 12 - 10 = 2$$

From (6) and (8),

$$14 + y_3 = 17 \Rightarrow y_3 = 17 - 14 = 3$$

Hence, the coordinates of the three vertices of the triangle are **(6, 9), (4, 5)** and **(2, 3)**.

(*iv*) Let the coordinates of the vertices A, B and C of the triangle ABC be (x_1, y_1), (x_2, y_2) and (x_3, y_3) respectively.



Then we have

$$\frac{x_1 + x_2}{2} = 6$$

$$\Rightarrow \quad x_1 + x_2 = 12 \quad \ldots(1)$$

$$\frac{x_2 + x_3}{2} = 3$$

$$\Rightarrow \quad x_2 + x_3 = 6 \quad \ldots(2)$$

and $$\frac{x_3 + x_1}{2} = 8$$

$$\Rightarrow \quad x_1 + x_3 = 16 \quad \ldots(3)$$

Adding (1), (2) and (3), we have $2(x_1 + x_2 + x_3) = 34$ subtracting (1), (2) and (3) from (4) successively, we get $x_3 = 5$, $x_1 = 11$ and $x_2 = 1$.

Also, $$\frac{y_1 + y_2}{2} = 7$$

$$\Rightarrow \quad y_1 + y_2 = 14 \quad \ldots(5)$$

$$\frac{y_2 + y_3}{2} = 4$$

$$\Rightarrow \quad y_2 + y_3 = 8 \quad \ldots(6)$$

and $$\frac{y_3 + y_1}{2} = 9$$

$$\Rightarrow \quad y_1 + y_3 = 18 \quad \ldots(7)$$

Adding (5), (6) and (7), we get

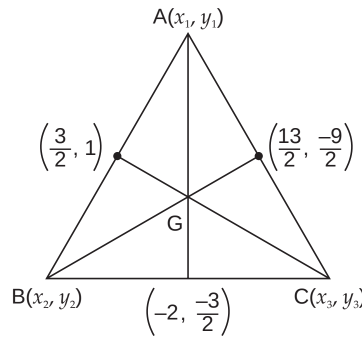
$$2(y_1 + y_2 + y_3) = 40$$

$$\Rightarrow \quad y_1 + y_2 + y_3 = 20 \quad \ldots(8)$$

Subtracting (5), (6) and (7) successively from (8), we get $y_3 = 6$, $y_1 = 12$ and $y_2 = 2$.

Hence, the required coordinates of A, B and C are respectively **A(11, 12)**, **B(1, 2)** and **C(5, 6)**.

32. Let the vertices of the triangle with the given mid-point be A(x_1, y_1), B(x_2, y_2) and C(x_3, y_3).



Then, $$\frac{x_1 + x_2}{2} = \frac{3}{2} \Rightarrow x_1 + x_2 = 3 \quad \ldots(1)$$

$$\frac{y_1 + y_2}{2} = 1 \Rightarrow y_1 + y_2 = 2 \quad \ldots(2)$$

$$\frac{x_2 + x_3}{2} = -2 \Rightarrow x_2 + x_3 = -4 \quad \ldots(3)$$

$$\frac{y_2 + y_3}{2} = -\frac{3}{2} \Rightarrow y_2 + y_3 = -3 \quad \ldots(4)$$

$$\frac{x_1 + x_3}{2} = \frac{13}{2} \Rightarrow x_1 + x_3 = 13 \quad \ldots(5)$$

$$\frac{y_1 + y_3}{2} = -\frac{9}{2} \Rightarrow y_1 + y_3 = -9 \quad \ldots(6)$$

Adding (1), (3) and (5), we get

$$x_1 + x_2 + x_2 + x_3 + x_1 + x_3 = 3 - 4 + 13$$

$$\Rightarrow \quad 2(x_1 + x_2 + x_3) = 12$$

$$\Rightarrow \quad x_1 + x_2 + x_3 = 6 \quad \ldots(7)$$

Adding (2), (4) and (6), we get

$$y_1 + y_2 + y_2 + y_3 + y_1 + y_3 = 2 - 3 - 9$$

$$\Rightarrow \quad 2(y_1 + y_2 + y_3) = -10$$

$$\Rightarrow \quad y_1 + y_2 + y_3 = -5 \quad \ldots(8)$$

From (1) and (7), we get

$$3 + x_3 = 6 \Rightarrow x_3 = 6 - 3 = 3$$

From (2) and (8),

$$2 + y_3 = -5 \Rightarrow y_3 = -5 - 2 = -7.$$

From (3) and (7),

$$x_1 - 4 = 6 \Rightarrow x_1 = 6 + 4 = 10$$

From (4) and (8),

$$y_1 - 3 = -5 \Rightarrow y_1 = -5 + 3 = -2$$

From (5) and (7),

$$x_2 + 13 = 6 \Rightarrow x_2 = 6 - 13 = -7$$

From (6) and (8),

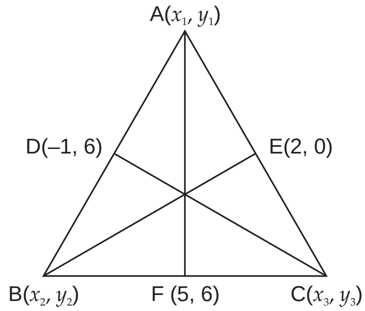
$$y_2 - 9 = -5 \Rightarrow y_2 = -5 + 9 = 4$$

So, the three vertices of the given triangle are (3, –7), (10, –2) and (–7, 4).

Hence, the coordinates of the centroid are

$$\left(\frac{3 + 10 - 7}{3}, \frac{-7 - 2 + 4}{3}\right), \text{ i.e. } \left(\mathbf{2}, \mathbf{\frac{-5}{3}}\right).$$

33. Let the vertices of the triangle with the given mid-points be $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$.



Now, D is the mid-point of AB. Then,

$$\frac{x_1+x_2}{2} = -1$$

$$\Rightarrow \quad x_1 + x_2 = -2 \quad \text{...(1)}$$

$$\frac{y_1+y_2}{2} = 6$$

$$\Rightarrow \quad y_1 + y_2 = 12 \quad \text{...(2)}$$

F is the mid-point of BC. Then,

$$\frac{x_2+x_3}{2} = 5$$

$$\Rightarrow \quad x_2 + x_3 = 10 \quad \text{...(3)}$$

$$\frac{y_2+y_3}{2} = 6$$

$$\Rightarrow \quad y_2 + y_3 = 12 \quad \text{...(4)}$$

E is the mid-point of AC. Then,

$$\frac{x_1+x_3}{2} = 2$$

$$\Rightarrow \quad x_1 + x_3 = 4 \quad \text{...(5)}$$

$$\frac{y_1+y_3}{2} = 0$$

$$\Rightarrow \quad y_1 + y_3 = 0 \quad \text{...(6)}$$

Adding equations (1), (3) and (5),

$$2(x_1 + x_2 + x_3) = -2 + 10 + 4$$

$$\Rightarrow \quad x_1 + x_2 + x_3 = 6 \quad \text{...(7)}$$

Adding equations (2), (4) and (6),

$$2(y_1 + y_2 + y_3) = 12 + 12 + 0$$

$$\Rightarrow \quad y_1 + y_2 + y_3 = 12 \quad \text{...(8)}$$

From (1) and (7),

$$-2 + x_3 = 6$$

$$\Rightarrow \quad x_3 = 8$$

From (2) and (8),

$$12 + y_3 = 12$$

$$\Rightarrow \quad y_3 = 0.$$

From (3) and (7),

$$x_1 + 10 = 6$$

$$\Rightarrow \quad x_1 = -4$$

From (4) and (8)

$$y_1 + 12 = 12$$

$$\Rightarrow \quad y_1 = 0$$

From (5) and (7),

$$x_2 + 4 = 6$$

$$\Rightarrow \quad x_2 = 2$$

From (6) and (8),

$$y_2 + 0 = 12$$

$$\Rightarrow \quad y_2 = 12$$

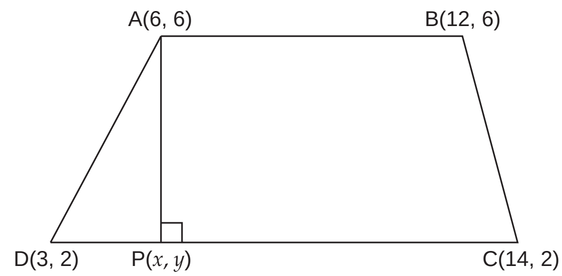
Now, the coordinates of the centroid of the triangle are

$$x = \frac{x_1+x_2+x_3}{3} = \frac{-4+2+8}{3} = 2$$

$$y = \frac{y_1+y_2+y_3}{3} = \frac{0+12+0}{3} = 4$$

Hence, the coordinates of the centroid are **(2, 4)**.

34. Since point P divides DC in the ratio 3 : 8, so coordinates of P are



$$\left(\frac{3\times14+8\times3}{3+8}, \frac{3\times2+8\times2}{3+8}\right) \text{ i.e. } P(6, 2)$$

Now, $AB = \sqrt{(12-6)^2+(6-6)^2} = \sqrt{36+0} = 6$ units

$DC = \sqrt{(14-3)^2+(2-2)^2} = \sqrt{(11)^2+0} = 11$ units

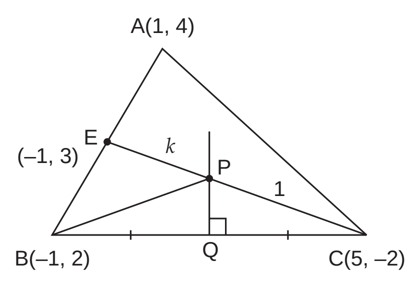
Also, $AP = \sqrt{(6-6)^2+(2-6)^2} = \sqrt{0+16} = 4$ units.

Therefore, area of trapezium ABCD

$$= \frac{1}{2}(AB + DC) \times AP = \frac{1}{2}(6 + 11) \times 4$$

$$= 17 \times 2 = \textbf{34 sq units}$$

35. Let CE be the median through C and PQ the perpendicular bisector of BC intersect at $P(x, y)$.



Join PB. As P lies on the perpendicular bisector of BC, P is equidistant from B and C.

$$BP = CP$$

$$\Rightarrow \quad BP^2 = CP^2$$

$$\Rightarrow \quad (x + 1)^2 + (y - 2)^2 = (x - 5)^2 + (y + 2)^2$$

$$\Rightarrow \quad 3x - 2y = 6 \quad \text{...(1)}$$

Coordinates of E are $\left(\frac{1-1}{2}, \frac{4+2}{2}\right)$, i.e. (0, 3).

Suppose P(x, y) divides EC in the ratio k : 1.

Then, using section formula

$$x = \frac{5k}{k+1} \text{ and } y = \frac{3-2k}{k+1} \qquad \ldots(2)$$

Using equations (1) and (2),

$$3\left(\frac{5k}{k+1}\right) - 2\left(\frac{3-2k}{k+1}\right) = 6$$

$\Rightarrow$ $15k - 6 + 4k = 6(k + 1)$

$\Rightarrow$ $13k = 12$

$\Rightarrow$ $k = \frac{12}{13}$

Now,

$$x = \frac{5k}{k+1} = \frac{5 \times \frac{12}{13}}{\frac{12}{13}+1} = \frac{60}{25} = \frac{12}{5}$$

$$y = \frac{3-2k}{k+1} = \frac{3 - 2 \times \frac{12}{13}}{\frac{12}{13}+1}$$

$$= \frac{39-24}{25} = \frac{15}{25} = \frac{3}{5}$$

Hence, the coordinate of the point where the right bisector of BC intersects the median through C is $\left(\mathbf{\frac{12}{5}}, \mathbf{\frac{3}{5}}\right)$.

EXERCISES 7F

For Basic and Standard Levels

1. Area of a triangle with vertices (x_1, y_1), (x_2, y_2) and (x_3, y_3) is given by

$$\frac{1}{2}\left|x_1(y_1 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|.$$

(*i*) So, area of the triangle with vertices (3, 0), (7, 0) and (8, 4)

$$= \frac{1}{2}\left|\left[3(-4) + 7(4) + 8(0-0)\right]\right|$$

$$= \frac{1}{2}\left|[-12 + 28]\right|$$

$$= \frac{1}{2}\left|[16]\right|$$

= 8 sq units

(*ii*) Area of the triangle with vertices A(–5, 7), B(–4, –5) and C(4, 5)

$$= \frac{1}{2}\left|\left[-5(-5-5) - 4(5-7) + 4(7+5)\right]\right|$$

$$= \frac{1}{2}\left|[50 + 8 + 48]\right| = \frac{1}{2} \times 106$$

= 53 sq units

(*iii*) Area of the triangle with vertices $\left(at_1, \frac{a}{t_1}\right)$, $\left(at_2, \frac{a}{t_2}\right)$ and $\left(at_3, \frac{a}{t_3}\right)$

$$= \frac{1}{2}\left|\left[at_1\left(\frac{a}{t_2} - \frac{a}{t_3}\right) + at_2\left(\frac{a}{t_3} - \frac{a}{t_1}\right) + at_3\left(\frac{a}{t_1} - \frac{a}{t_2}\right)\right]\right|$$

$$= \frac{1}{2}\left|\left[at_1\left(\frac{at_3 - at_2}{t_2t_3}\right) + at_2\left(\frac{at_1 - at_3}{t_1t_3}\right) + at_3\left(\frac{at_2 - at_1}{t_1t_2}\right)\right]\right|$$

$$= \frac{1}{2} \times a^2\left|\left[\frac{t_1t_3 - t_1t_2}{t_2t_3} + \frac{t_1t_2 - t_2t_3}{t_1t_3} + \frac{t_2t_3 - t_1t_3}{t_1t_2}\right]\right|$$

$$= \frac{1}{2} \times a^2\left|\left[\frac{t_1^2t_3 - t_1^2t_2 + t_1t_2^2 - t_2^2t_3 + t_2t_3^2 - t_1t_3^2}{t_1t_2t_3}\right]\right|$$

$$= \frac{a^2}{2t_1t_2t_3}\left[-t_1^2(t_2 - t_3) - t_2t_3(t_2 - t_3) + t_1(t_2^2 - t_3^2)\right]$$

$$= \frac{a^2}{2t_1t_2t_3}\left[(t_2 - t_3)\left[-t_1^2 - t_2t_3 + t_1(t_2 + t_3)\right]\right]$$

$$= \frac{a^2}{2t_1t_2t_3}(t_2 - t_3)\left[-t_1^2 - t_2t_3 + t_1t_2 + t_1t_3\right]$$

$$= \frac{a^2}{2t_1t_2t_3}(t_2 - t_3)\left[-t_1(t_1 - t_2) + t_3(t_1 - t_2)\right]$$

$$= \frac{a^2}{2t_1t_2t_3}(t_2 - t_3)(t_1 - t_2)(t_3 - t_1)$$

$$= \mathbf{\frac{a^2}{2t_1t_2t_3}(t_1 - t_2)(t_2 - t_3)(t_3 - t_1)} \textbf{ sq units}$$

2. (t, t – 2)

(t + 2, t + 2)

(t + 3, t)

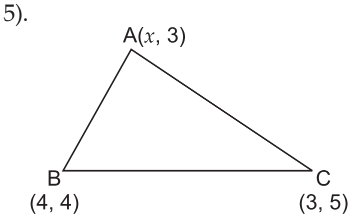
$$\text{Area of triangle} = \frac{1}{2}\left|\left[t(2) + (t+2)(2) + (t+3)(-4)\right]\right|$$

$$= \frac{1}{2}\left|[2t + 2t + 4 - 4t - 12]\right|$$

$$= \frac{1}{2}\left|[-8]\right| = |(-4)| = 4 \text{ sq units}$$

Hence, area of triangle is independent of t.

3. (*i*) Let the triangle be ABC where A(x, 3), B(4, 4) and C(3, 5).



Then area of ΔABC is

$$\Delta = \frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|$$

where $x_1 = x$, $y_1 = 3$, $x_2 = 4$, $y_2 = 4$ and $x = 3$, $y_3 = 5$

$\therefore \qquad \Delta = \frac{1}{2}\left|x_1(4-5)+4(5-3)+3(3-4)\right|$

$= \frac{1}{2}\left|-x+8-3\right| = \frac{1}{2}\left|5-x\right| = 4$ [Given]

$\Rightarrow \qquad |5-x| = 8$

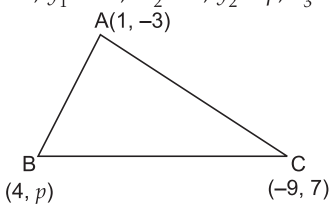
$\therefore \qquad |5-x| = 8$

(i) Let $x < 5$. Then $5 - x = 8 \Rightarrow x = 5 - 8 = -3$

(ii) Let $x > 5$. Then $x - 5 = 8 \Rightarrow x = 13$.

Hence, the required value of x is **–3 or 13**.

(*ii*) Let $x_1 = 1$, $y_1 = -3$, $x_2 = 4$, $y_2 = p$, $x_3 = -9$, $y_3 = 7$



$\therefore$ The required area of ΔABC is

$\Delta = \frac{1}{2}\left|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right|$

$= \frac{1}{2}\left|1\times(p-7)+4(7+3)+9(-3-p)\right|$

$= \frac{1}{2}\left|p-7+40+27+9p\right|$

$= \frac{1}{2}\left|10p+60\right| = \left|5p+30\right| = 15$ (given)

We have $\qquad 5|p+6| = 15$

$\Rightarrow \qquad |p+6| = 3 \qquad \ldots(1)$

Case (*i*) Let $p > -6$

$\therefore$ From (1), $\qquad p + 6 = 3$

$\Rightarrow \qquad p = -3$

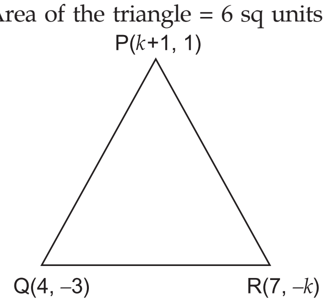
Case (*ii*) Let $p < -6$

$\therefore$ From (1), $\qquad -p - 6 = 3$

$\Rightarrow \qquad p = -6 - 3 = -9$

Hence, the required value of p is **–3 or –9**.

(*iii*) *Given:* Area of the triangle = 6 sq units



Now, Area of the triangle

$= \frac{1}{2}\left|\left[(k+1)(-3+k)+4(-k-1)+7(1+3)\right]\right|$

$= \frac{1}{2}\left|\left[(k^2-2k-3)-4k-4+28\right]\right|$

$= \frac{1}{2}\left|\left[k^2-2k-3-4k+24\right]\right|$

$6 = \frac{1}{2}\left|\left[k^2-6k+21\right]\right|$

$\Rightarrow \qquad 12 = k^2 - 6k + 21$

$\Rightarrow \qquad k^2 - 6k + 9 = 0$

$\Rightarrow k^2 - 3k - 3k + 9 = 0$

$\Rightarrow k(k-3) - 3(k-3) = 0$

$\mathbf{k = 3}$

(*iv*) Let ABC be the triangle with A(x_1, y_1), where $x_1 = -2$, $y_1 = 5$, B(x_2, y_2), where $x_2 = k$, $y_2 = -4$ and C(x_3, y_3) where $x_3 = 2k + 1$, $y_3 = 10$.

Then the area of ΔABC is

$\Delta = \frac{1}{2}\left|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right|$

$= \frac{1}{2}\left|-2(-4-10)+k(10-5)+(2k+1)(5+4)\right|$

$= \frac{1}{2}\left|28+10k-5k+18k+9\right|$

$= \frac{1}{2}\left|23k+37\right| \qquad \ldots(1)$

Now, $\qquad \Delta = 53 \qquad$ [Given]

$\therefore$ From (1),

$\frac{23}{2}\left|k+\frac{37}{23}\right| = 53$

$\Rightarrow \qquad \frac{23}{2}\left(k+\frac{37}{23}\right) = 53 \qquad [\because k > 0 \therefore k > -\frac{37}{23}]$

$\Rightarrow \qquad k+\frac{37}{23} = \frac{2}{23}\times 53 = \frac{106}{23}$

$\Rightarrow \qquad k = \frac{106-37}{23} = \frac{69}{23} = 3$

$\therefore$ Required value of k is **3**.

(*v*) Let ΔABC be the triangle with vertices A(x_1, y_1), B(x_2, y_2) and C(x_3, y_3), where $x_1 = 2$, $y_1 = 1$, $x_2 = 3$, $y_2 = -2$, $x_3 = \frac{7}{2}$ and $y_3 = y$.

Then area (Δ) of the triangle is given by

$\Delta = \frac{1}{2}\left|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right|$

$= \frac{1}{2}\left|2(-2-y)+3(y-1)+\frac{7}{2}(1+2)\right|$

$= \frac{1}{2}\left|-4-2y+3y-3+\frac{21}{2}\right|$

$= \frac{1}{2}\left|y+\frac{7}{2}\right|$

Case (*i*) Let $y > -\frac{7}{2}$

$\therefore \qquad \Delta = \frac{1}{2}\left(y+\frac{7}{2}\right) = 5 \qquad$ [Given]

$\Rightarrow \qquad \frac{1}{2}y + \frac{7}{4} = 5$

$$\Rightarrow \quad \frac{y}{2} = 5 - \frac{7}{4} = \frac{13}{4}$$

$$\Rightarrow \quad y = \frac{13}{2}$$

Case (*ii*) Let $y < -\frac{7}{2}$

Then $\quad \Delta = \frac{1}{2}\left(-y - \frac{7}{2}\right) = 5 \quad$ [Given]

$$\Rightarrow \quad -\frac{y}{2} - \frac{7}{4} = 5$$

$$\Rightarrow \quad \frac{y}{2} = -\frac{7}{4} - 5 = -\frac{27}{4}$$

$$\Rightarrow \quad y = -\frac{27}{2}$$

Hence, the required value of y is $\mathbf{\frac{13}{2}}$ or $\mathbf{-\frac{27}{2}}$.

4. (*i*) $\frac{1}{2}\left|\left[-4(6-8) - \frac{2}{5}(8-3) + 2(3-6)\right]\right|$

$$= \frac{1}{2}\left|\left[-4 \times (-2) - \frac{2}{5} \times 5 + 2 \times (-3)\right]\right|$$

$$= \frac{1}{2}|[+8-2-6]| = \frac{1}{2} \times |(8-8)| = \frac{1}{2} \times 0$$

$$= 0$$

Since the area of the triangle with given points as vertices is 0, the points are collinear.

(*ii*) $\frac{1}{2}|[3a(3b-2b) + 0(2b-0) + a(0-3b)]|$

$$= \frac{1}{2}|[3a \times b + 0 + a \times (-3b)]|$$

$$= \frac{1}{2}|[3ab - 3ab]| = \frac{1}{2} \times 0 = 0$$

Since the area of the triangle with given points as vertices is 0, the points are collinear.

5. (*i*) $\frac{1}{2}|-1(p+1) + 2(-1-3) + 5(3-p)|$

$$= \frac{1}{2}[-p-1-8+15-5p]$$

$$= \frac{1}{2}[-6p+6] = -3p + 3$$

The three given points will be collinear, if the area of the triangle taking these points as vertices is 0.

$$\Rightarrow \quad -3p + 3 = 0$$

$$\Rightarrow \quad 3p = 3$$

$$\Rightarrow \quad p = \frac{3}{3} = 1$$

$$\Rightarrow \quad \mathbf{p = 1}$$

(*ii*) $\frac{1}{2}|[2(-1-3) + p(3-1) - 1(1+1)]|$

$$= \frac{1}{2}[-8+2p-2] = \frac{1}{2}|(2p-10)|$$

$$= \frac{1}{2}|(p-5)|$$

The three given points will be collinear, if the area of the triangle taking these points as vertices is 0.

$$\Rightarrow \quad p - 5 = 0$$

$$\Rightarrow \quad \mathbf{p = 5}$$

(*iii*) $\frac{1}{2}|[1(-2-16) + r(16-4) - 3(4+2)]|$

$$= \frac{1}{2}|(-18+12r-18)|$$

$$= \frac{1}{2}|(12r-36)| = |6r - 18|$$

The three given points will be collinear, if the area of the triangle taking these points as vertices is 0.

$$\Rightarrow \quad 6r - 18 = 0$$

$$\Rightarrow \quad r = \frac{18}{6}$$

$$\Rightarrow \quad \mathbf{r = 3}$$

(*iv*) Let the points be A(x_1, y_1), B(x_2, y_2) and C(x_3, y_3), where $x_1 = a$, $y_1 = 1$, $x_2 = 1$, $y_2 = -1$, $x_3 = 11$ and $y_3 = 4$.

Then the area of ΔABC is given by

$$\Delta = \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

$$= \frac{1}{2}|a(-1-4) + 1(4-1) + 11(1+1)|$$

$$= \frac{1}{2}|-5a + 3 + 22|$$

$$= \frac{1}{2}|-5a + 25|$$

$\because$ The three points A, B, C are collinear, $\therefore \Delta = 0$.

Hence, the required value of a is **5**.

(*v*) Let the coordinates of A, B and C be (x_1, y_1), (x_2, y_2) and (x_3, y_3) respectively, where $x_1 = x$, $y_1 = 2$, $x_2 = -3$, $y_2 = -4$, $x_3 = 7$ and $y_3 = -5$.

Then area of ΔABC is given by

$$\Delta = \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

$$= \frac{1}{2}|x(-4+5) - 3(-5-2) + 7(2+4)|$$

$$= \frac{1}{2}|x + 21 + 12|$$

$$= \frac{1}{2}|x + 63|$$

Since, the three points are collinear, hence $\Delta = 0$.

$$\therefore \quad |x + 63| = 0$$

$$\Rightarrow \quad x = -63$$

which is the required value of x.

(*vi*) Let the coordinates of three points A, B, and C be (x_1, y_1), (x_2, y_2) and (x_3, y_3) respectively where $x_1 = x$, $y_1 = -1$, $x_2 = 2$, $y_2 = 1$, $x_3 = 4$ and $y_3 = 5$.

Then area of ΔABC is given by

$$\Delta = \frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_3) + x_3(y_1 - y_2)\right|$$

$$= \frac{1}{2}\left|x(+1-5) + 2(5+1) + 4(-1-1)\right|$$

$$= \frac{1}{2}\left|-4x + 12 - 8\right|$$

$$= \frac{1}{2}\left|-4x + 4\right|$$

$\because$ The three points are collinear, hence $\Delta = 0$

$\Rightarrow$ $\boldsymbol{x = 1}$

which is the required value of x.

(*vii*) Let the coordinates of A, B and P be (x_1, y_1), (x_2, y_2) and (x_3, y_3) respectively, where $x_1 = \frac{-2}{5}$, $y_1 = 6$, $x_2 = 2$, $y_2 = 8$, $x_3 = m$ and $y_3 = 3$.

Then the area Δ of ΔABP is given by

$$\Delta = \frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|$$

$$= \frac{1}{2}\left|-\frac{2}{5}(8-3) + 2(3-6) + m(6-8)\right|$$

$$= \frac{1}{2}\left|-2 - 6 - 2m\right| = \frac{1}{2}\left|-8 - 2m\right|$$

$\because$ The point P lies on the line segment AB, hence, PA and B are collinear.

$\therefore$ $\Delta = 0$

$\therefore$ From (1), **m = –4** which is the required value of m.

6. (*i*) Let the coordinates of the given points be (x_1, y_1), (x_2, y_2) and (x_3, y_3)

where $x_1 = k$, $y_1 = 3$, $x_2 = 6$, $y_2 = -2$, $x_3 = -3$ and $y_3 = 4$.

Now, the area (Δ) of the triangle formed by these three points is given by

$$\Delta = \frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|$$

$$= \frac{1}{2}\left|k(-2-4) + 6(4-3) - 3(3+2)\right|$$

$$= \frac{1}{2}\left|-6k + 6 - 15\right|$$

$$= \frac{1}{2}\left|-6k - 9\right|$$

Since the three given points are collinear,

$\therefore$ $\Delta = 0$

$\Rightarrow$ $k = -\frac{9}{6}$

$= -\frac{3}{2}$

which is the required value of k.

(*ii*) Let the coordinates of A, B and C be (x_1, y_1) (x_2, y_2) and (x_3, y_2) respectively

where $x_1 = 7$, $y_1 = -2$, $x_2 = 5$, $y_2 = 1$, $x_3 = 3$ and $y_3 = 2k$.

Then the area of ΔABC is given by

$$\Delta = \frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|$$

$$= \frac{1}{2}\left|7(1-2k) + 5(2k+2) + 3(-2-1)\right|$$

$$= \frac{1}{2}\left|7 - 14k + 10k + 10 - 9\right|$$

$$= \frac{1}{2}\left|-4k + 8\right|$$

$$= \left|-2k + 4\right|$$

$\because$ The three given points are collinear.

$\therefore$ $\Delta = 0$

$\Rightarrow$ $\boldsymbol{k = 2}$

which is the required value of k.

(*iii*) Let the coordinates of the given points be given by (x_1, y_1), (x_2, y_2) and (x_3, y_3) respectively where $x_1 = 8$, $y_1 = 1$, $x_2 = 3$, $y_2 = -2k$, $x_3 = k$ and $y_3 = -5$.

Then the area (Δ) of the triangle formed by the given points as vertices, is given by

$$\Delta = \frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|$$

$$= \frac{1}{2}\left|8(-2k+5) + 3(-5-1) + k(1+2k)\right|$$

$$= \frac{1}{2}\left|-16k + 40 - 18 + k + 2k^2\right|$$

$$= \frac{1}{2}\left|2k^2 - 15k + 22\right| \qquad \ldots(1)$$

Since the three given points are collinear,

$\therefore$ $\Delta = 0.$

$\Rightarrow$ $2k^2 - 15k + 22 = 0$

$$\Rightarrow \qquad k = \frac{15 \pm \sqrt{15^2 - 4 \times 2 \times 22}}{2 \times 2}$$

$$= \frac{15 \pm \sqrt{225 - 176}}{4}$$

$$= \frac{15 \pm \sqrt{49}}{4}$$

$$= \frac{15 \pm 7}{4}$$

$$= \frac{8}{4} \text{ or } \frac{22}{4}$$

$$= 2 \text{ or } \frac{11}{2}.$$

Hence, the required value of k is **2 or $\boldsymbol{\frac{11}{2}}$**.

(*iv*) Let the coordinates of A, B and C be respectively (x_1, y_1), (x_2, y_2) and (x_3, y_3) where $x_1 = k + 1$, $y_1 = 2k$, $x_2 = 3k$, $y_2 = 2k + 3$, $x_3 = 5k - 1$ and $y_3 = 5k$.

Then the area (Δ) of ΔABC is given by

$$\Delta = \frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|$$

$$= \frac{1}{2}\left|(k+1)(2k+3-5k) + 3k(5k-2k) + (5k-1)(2k-2k-3)\right|$$

$$= \frac{1}{2}\left|(k+1)(3-3k) + 3k \times 3k + (5k-1)(3)\right|$$

$$= \frac{1}{2}\left|-3k^2 + 3 + 9k^2 - 15k + 3\right| = \frac{1}{2}\left|6k^2 - 15k + 6\right| \quad \ldots(1)$$

$\because$ The three points are collinear.

$\therefore \quad \Delta = 0$

$\therefore$ From (1)

$$6k^2 - 15k + 6 = 0$$

$$\Rightarrow \quad k = \frac{15 \pm \sqrt{225 - 144}}{6 \times 2}$$

$$= \frac{15 \pm \sqrt{81}}{6 \times 2}$$

$$= \frac{15 \pm 9}{12}$$

$$= \frac{24}{12} \text{ or } \frac{6}{12}$$

$$= 2 \text{ or } \frac{1}{2}$$

Hence, the required value of k is **2 or $\frac{1}{2}$**.

(*v*) Let the coordinates of the three vertices of the triangle be (x_1, y_1), (x_2, y_2) and (x_3, y_3) where $x_1 = 3k - 1$, $y_1 = k - 2$, $x_2 = k$, $y_2 = k - 7$, $x_3 = k - 1$ and $y_3 = -k - 2$.

Then the area (Δ) of the triangle formed by these vertices is given by

$$\Delta = \frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|'$$

$$= \frac{1}{2}\left|(3k-1)(k - 7 + k + 2) + k(-k - 2 - k + 2) + (k-1)(k - 2 - k + 7)\right|$$

$$= \frac{1}{2}\left|(3k-1)(2k-5) + k(-2k) + (k-1)5\right|$$

$$= \frac{1}{2}\left|6k^2 - 15k - 2k + 5 - 2k^2 + 5k - 5\right|$$

$$= \frac{1}{2}\left|4k^2 - 12k\right| \quad \ldots(1)$$

If the three given points are collinear, then $\Delta = 0$.

$\therefore$ From (1)

$$4k^2 - 12k = 0$$

$$\Rightarrow \quad 4k(k - 3) = 0$$

$$\therefore \quad \text{Either } k = 0 \text{ or } k = 3$$

Hence, the required value of is **0 or 3**.

7. (*i*) $\frac{1}{2}\left|\left[-5(k+2) + 1(-2-1) + 4(1-k)\right]\right|$

$$= \frac{1}{2}\left|\left[-5k - 10 - 3 + 4 - 4k\right]\right|$$

$$= \frac{1}{2}\left|\left[-9 - 9k\right]\right|$$

For the points A, B, C to be collinear,

$$\frac{1}{2}(-9 - 9k) = 0$$

$$\Rightarrow \quad -9 - 9k = 0$$

$$\Rightarrow \quad \mathbf{k = -1}$$

Now, the points are A(–5, 1), B(1, –1) and C(4, –2).

Let the point B divides AC in the ratio $r : 1$.

$$\text{Then,} \quad 1 = \frac{4r - 5}{r + 1}$$

$$\Rightarrow \quad r + 1 = 4r - 5$$

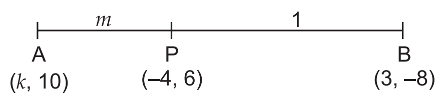
$$\Rightarrow \quad 1 + 5 = 3r$$

$$\Rightarrow \quad 3r = 6$$

$$\Rightarrow \quad r = \frac{6}{3} = 2$$

So, the point B divides AC in the ratio **2 : 1**.

(*ii*) Let the coordinates of the points A, B and P be respectively (x_1, y_1), (x_2, y_2) and (x_3, y_3) so that $x_1 = k$, $y_1 = 10$, $x_2 = 3$, $y_2 = -8$, $x_3 = -4$ and $y_3 = 6$.



Then the area (Δ) of ΔABP is given by

$$\Delta = \frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|$$

$$= \frac{1}{2}\left|k(-8-6) + 3(6-10) - 4(10+8)\right|$$

$$= \frac{1}{2}\left|-14k - 12 - 72\right| \quad \ldots(1)$$

Since P lies on AB, hence $\Delta = 0$

$\therefore$ From (1),

$$-14k - 84 = 0$$

$$\Rightarrow \quad k = \frac{84}{14} = \mathbf{-6}$$

which is the required value of k.

2nd part: We have A(–6, 10) and B(3, –8).

Let P(–4, 6) divide AB in the ratio $m : 1$.

Then by using section formula, we have

$$-4 = \frac{3m - 6}{m + 1}$$

$$\Rightarrow \quad 3m - 6 = -4m - 4$$

$$\Rightarrow \quad 7m = 2$$

$$\therefore \quad m = \frac{2}{7}$$

$\Rightarrow$ Hence, the required ratio is **2 : 7**.

8. (*i*) Area of the triangle with vertices (p, q), (m, n) and $(p - m, q - n)$

$$= \frac{1}{2}\left|\left[p(n - q + n) + m(q - n - q) + (p - m)(q - n)\right]\right|$$

$$= \frac{1}{2}\left|\left[p(2n-q)+m(-n)+(pq-pn-mq+mn)\right]\right|$$

$$= \frac{1}{2}\left|[2pn-pq-mn+pq-pn-mq+mn]\right|$$

$$= \frac{1}{2}\left[pn-mq\right]$$

For the points (p, q), (m, n) and $(p - m, q - n)$ to be collinear,

$$\frac{1}{2}(pn - mq) = 0$$

$\Rightarrow \quad pn - mq = 0$

$\Rightarrow \quad pn = mq$

i.e. $\quad pn = qm$

(*ii*) $\frac{1}{2}\left|\left[x(y-1)+0(1-0)+1(0-y)\right]\right|$

$$= \frac{1}{2}\left|\left[xy-x-y\right]\right|$$

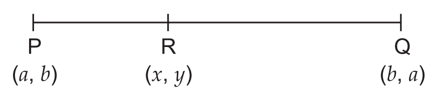
If the given points are collinear, the area is 0.

i.e. $\quad \frac{1}{2}(xy - x - y) = 0$

$\Rightarrow \quad xy - x - y = 0$

$\Rightarrow \quad x + y = xy$

(*iii*) Let the coordinates of the points P, Q and R be (x_1, y_1), (x_2, y_2) and (x_3, y_3) respectively, so that $x_1 = a$, $y_1 = b$, $x_2 = b$, $y_2 = a$, $x_3 = x$ and $y_3 = y$.



Then the area (Δ) of ΔPQR is given by

$$\Delta = \frac{1}{2}\left|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right|$$

$$= \frac{1}{2}\left|a(a-y)+6(y-b)+x(b-a)\right|$$

$$= \frac{1}{2}\left|a^2-ay+by-b^2+bx-ax\right|$$

$$\therefore \ \Delta = \frac{1}{2}\left|a^2-b^2+(b-a)x+(b-a)y\right| \qquad \ldots(1)$$

If R lies on PQ, then $\Delta = 0$

$\therefore$ From (1), we have

$a^2 - b^2 + (b - a)(x + y) = 0$

$\Rightarrow \quad x + y = a + b$

(*iv*) Area of the triangle with vertices P(x, y), $(1, -3)$ and $(-4, 2)$

$$= \frac{1}{2}\left|\left[x(3-2)+1(2-y)-4(y+3)\right]\right|$$

$$= \frac{1}{2}\left|[-5x+2-y-4y-12]\right|$$

$$= \frac{1}{2}\left|[-5x-5y-10]\right|$$

For the point P(x, y) to lie on the line segment joining the points $(1, -3)$ and $(-4, 2)$.

$$\frac{1}{2}[-5x - 5y - 10] = 0$$

$\Rightarrow \quad -\frac{5}{2}(x + y + 2) = 0$

$\Rightarrow \quad x + y + 2 = 0$

(*v*) $\frac{1}{2}\left|\left[x(6-4)+3(4-y)-3(y-6)\right]\right|$

$$= \frac{1}{2}\left|[2x+12-3y-3y+18]\right|$$

$$= \frac{1}{2}\left|[2x-6y+30]\right|$$

$$= \frac{1}{2}\left|[x-3y+15]\right|$$

If the given points are collinear, then the area is 0.

i.e. $\quad x - 3y + 15 = 0$

(*vi*) We have, $\frac{1}{2}\left|\left[x(5-7)+5(7-y)+10(y-5)\right]\right|$

$$= \frac{1}{2}\left|[-2x+35-5y+10y-50]\right|$$

$$= \frac{1}{2}\left|(-2x+5y-15)\right|$$

If the given points are collinear, then the area is 0.

$\Rightarrow \quad \frac{1}{2}(-2x + 5y - 15) = 0$

$\Rightarrow \quad 5y - 2x = 15$

(*vii*) Taking P(x, y), A(1, 4) and B(–3, 16) as vertices of ΔPAB,

$$\text{ar}(\Delta\text{PAB}) = \frac{1}{2}\left|\left[x(4-16)+1(16-y)+3(y-4)\right]\right|$$

$$= \frac{1}{2}\left|[-12x+16-y-3y+12]\right|$$

$$= \frac{1}{2}\left|[-12x-4y+28]\right|$$

$$= \left|-6x-2y+14\right|$$

For the point P(x, y) to lie on the line segment AB,

ar(ΔPAB) = 0

$-x6 - 2y + 14 = 0$

3x + y – 7 = 0

9. (*i*) Let the coordinates of P, Q and R be respectively (x_1, y_1), (x_2, y_2) and (x_3, y_3) so that

$x_1 = -3$, $y_1 = 9$, $x_2 = a$, $y_2 = b$, $x_3 = 4$ and $y_3 = -5$.

Then the area (Δ) of the triangle PQR is given by

$$\Delta = \frac{1}{2}\left|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right|$$

$$= \frac{1}{2}\left|-3(b+5)+a(-5-9)+4(9-b)\right| \qquad \ldots(1)$$

If P, Q, R are collinear, then $\Delta = 0$.

$\therefore$ From (1)

$-3b - 15 - 14a + 36 - 4b = 0$

$\Rightarrow \quad -7b - 14a + 21 = 0$

$\Rightarrow \quad b + 2a - 3 = 0 \quad \ldots(2)$

Also, given that $a + b - 1 = 0 \quad \ldots(3)$

Subtracting (3) from (2), we get

$a - 2 = 0$

$\Rightarrow \quad a = 2$

$\therefore$ From (2), $\quad b = 3 - 2a = 3 - 4 = -1.$

Hence, the required values of a and b are **2** and **–1** respectively.

(*ii*) Let the coordinates of A, B and C be (x_1, y_1), (x_2, y_2) and (x_3, y_3) respectively, so that $x_1 = -2$, $y_1 = 1$, $x_2 = a$, $y_2 = b$, $x_3 = 4$ and $y_3 = -1$.

Then the area (Δ) of ABC is given by

$$\Delta = \frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|$$

$$= \frac{1}{2}\left|-2(b+1) + a(-1-1) + 4(1-b)\right|$$

$$= \frac{1}{2}\left|-2b - 2 + 2a + 4 - 4b\right|$$

$$= \frac{1}{2}\left|-6b - 2a + 2\right|$$

$$= \left|-3b - a + 1\right| \quad \ldots(1)$$

If the three points are collinear, then $\Delta = 0$

$\therefore$ From (1), we have

$3b + a - 1 = 0 \quad \ldots(2)$

Also, $\quad -b + a - 1 = 0 \quad$ [Given] $\ldots(3)$

Subtracting (3) from (2), we get

$4b = 0$

$\Rightarrow \quad b = 0$

$\therefore$ From (2), $\quad a = 1$

Hence, the required values of a and b are **1** and **0** respectively.

(*iii*) Let the coordinates of A, B and C be (x_1, y_1), (x_2, y_2) and (x_3, y_3) respectively, so that $x_1 = -1$, $y_1 = -4$, $x_2 = b$, $y_2 = c$, $x_3 = 5$ and $y_3 = -1$.

Hence, the area (Δ) of ABC is given by

$$\Delta = \frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|$$

$$= \frac{1}{2}\left|-(c+1) + b(-1+4) + 5(-4-c)\right|$$

$$= \frac{1}{2}\left|-c - 1 + 3b - 20 - 5c\right|$$

$$= \frac{1}{2}\left|3b - 6c - 21\right|$$

$$= \frac{3}{2}\left|b - 2c - 7\right| \quad \ldots(1)$$

If the given points are collinear, then $\Delta = 0$

$\therefore$ From (1), we have

$b - 2c - 7 = 0 \quad \ldots(2)$

Also, given that

$2b + c = 4 \quad \ldots(3)$

From (2), $\quad b = 2c + 7 \quad \ldots(4)$

$\therefore$ From (3) and (4), we get

$2(2c + 7) + c = 4$

$\Rightarrow \quad 4c + 14 + c = 4$

$\Rightarrow \quad 5c = -10$

$\Rightarrow \quad c = -2 \quad \ldots(5)$

$\therefore$ From (4) & (5), we get

$b = -4 + 7 = 3$

$\therefore$ The required values of b and c are respectively **3** and **–2**.

10. (*i*) If the coordinates of A, B and C are (x_1, y_1), (x_2, y_2) and (x_3, y_3) respectively, then $x_1 = x$, $y_1 = y$, $x_2 = -5$, $y_2 = 7$, $x_3 = -4$ and $y_3 = -5$.

$\therefore$ The area (Δ) of ΔABC is given by

$$\Delta = \frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|$$

$$= \frac{1}{2}\left|x(7-5) - 5(5-y) - 4(y-7)\right|$$

$$= \frac{1}{2}\left|2x - 25 + 5y - 4y + 28\right|$$

$$= \frac{1}{2}\left|2k + y + 3\right| \quad \ldots(1)$$

If the given points are collinear, then $\Delta = 0$

$\therefore$ From (1) $\mathbf{2x + y + 3 = 0}$

which is the required relation.

(*ii*) Let the coordinates of A, B and C be (x_1, y_1), (x_2, y_2) and (x_3, y_3) respectively, so that $x_1 = 2$, $y_1 = 1$, $x_2 = x$, $y_2 = y$, $x_3 = 7$ and $y_3 = 5$.

Hence, the area (Δ) of ΔABC is given by

$$\Delta = \frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|$$

$$= \frac{1}{2}\left|2(y-5) + x(5-1) + 7(1-y)\right| \quad \ldots(1)$$

If A, B, C are collinear, then $\Delta = 0$

$\therefore$ From (1)

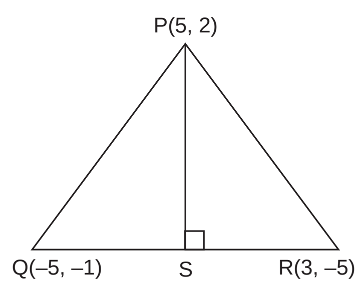
$2y - 10 + 4x + 7 - 7y = 0$

$\Rightarrow \quad \mathbf{4x - 5y - 3 = 0}$

which is the required relation.

11. $\quad \text{ar}(\Delta PQR) = \frac{1}{2}\left|\left[5(-1+5) - 5(-5-2) + 3(2+1)\right]\right|$

$$= \frac{1}{2}\left|\left[5 \times 4 - 5(-7) + 3 \times 3\right]\right|$$



$$= \frac{1}{2}\left|[20 + 35 + 9]\right|$$

$$= \frac{1}{2}[64] = 32 \text{ sq units}$$

Now, $\quad QR = \sqrt{(3+5)^2 + (-5+1)^2}$

$$= \sqrt{64+16} = \sqrt{80}$$

$$= 4\sqrt{5}$$

So, $\quad ar(PQR) = \frac{1}{2} \times QR \times PS$

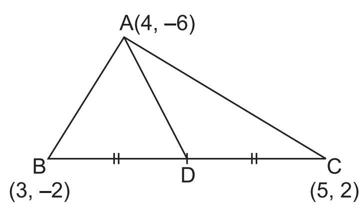
$$\Rightarrow \quad 32 = \frac{1}{2} \times 4\sqrt{5} \times PS$$

$$\Rightarrow \quad PS = \frac{32 \times 2}{4\sqrt{5}} = \frac{16}{\sqrt{5}} = \frac{16\sqrt{5}}{\sqrt{5} \times \sqrt{5}}$$

$$= \mathbf{\frac{16\sqrt{5}}{5}} \textbf{ units}$$

12. Since AD is a median of ΔABC

$\therefore$ D is the mid-point of BC.



Hence, the coordinates of D are $\left(\frac{3+5}{2}, \frac{-2+2}{2}\right) = (4, 0)$.

Let the coordinates of A, B and D be (x_1, y_1), (x_2, y_2) and (x_3, y_3) respectively.

Then $x_1 = 4$, $y_1 = -6$, $x_2 = 3$, $y_2 = -2$, $x_3 = 4$, $y_3 = 0$.

$\therefore$ The area (Δ_1) of ΔABD is given by

$$\Delta_1 = \frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|$$

$$= \frac{1}{2}\left|4(-2-0) + 3(0+6) + 4(-6+2)\right|$$

$$= \frac{1}{2}\left|-8 + 18 - 16\right| = \frac{1}{2}\left|-6\right| = 3 \quad \ldots(1)$$

Similarly, the area (Δ_2) of ΔADC is given by

$$\Delta_2 = \frac{1}{2}\left|4(0-2) + 4(2+6) + 5(-6-0)\right|$$

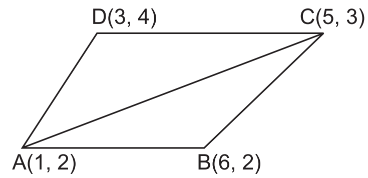
$$= \frac{1}{2}\left|-8 + 32 - 30\right| = \frac{1}{2}\left|-6\right| = 3 \quad \ldots(2)$$

$\therefore$ From (1) and (2), we see that $\Delta_1 = \Delta_2$

Hence, the median AD divide then ΔABC into two triangles of equal areas.

13. (*i*) We join AC. The coordinates of the vertices of ΔABC are

A$(x_1 = 1, y_1 = 2)$, B$(x_2 = 6, y_2 = 2)$, C$(x_3 = 5, y_3 = 3)$,



$\therefore$ Area of ΔABC

$$= \frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|$$

$$= \frac{1}{2}\left|1(2-3) + 6(3-2) + 5(2-2)\right|$$

$$= \frac{1}{2}\left|-1 + 6\right| = \frac{5}{2} \quad \ldots(1)$$

The coordinates of the vertices of ΔACD are

A = $(x_1' = 1,\ y_1' = 2)$, C = $(x_2' = 5,\ y_2' = 3)$

and D = $(x_3' = 3,\ y_3' = 4)$

$\therefore$ Area of ΔADC

$$= \frac{1}{2}\left|x_1'\left(y_2' - y_3'\right) + x_2'\left(y_3' - y_1^1\right) + x_3'\left(y_1' - y_2'\right)\right|$$

$$= \frac{1}{2}\left|(3-4) + 5(4-2) + 3(2-3)\right|$$

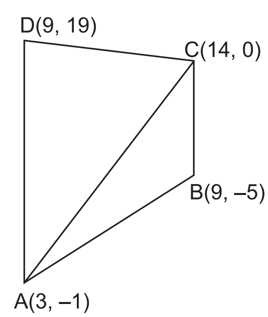
$$= \frac{1}{2}\left|-1 + 10 - 3\right| = \frac{6}{2} = 3 \quad \ldots(2)$$

From (1) and (2), we have

Area of ΔABC + Area of ΔADC = $\frac{5}{2} + 3 = \frac{11}{2}$

Hence, the required area of the quadrilateral ABCD is $\mathbf{\frac{11}{2}}$ **sq units**.

(*ii*) We join AC. Then the coordinates of the vertices of ΔABC are A$(x_1 = 3, y_1 = -1)$, B$(x_2 = 9, y_2 = -5)$ and C$(x_3 = 14, y_3 = 0)$.



$\therefore$ Area of ΔABC

$$= \frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|$$

$$= \frac{1}{2}\left|3(-5-0) + 9(0+1) + 14(-1+5)\right|$$

$$= \frac{1}{2}\left|-15 + 9 + 56\right| = \frac{1}{2} \times 50 = 25 \quad \ldots(1)$$

The coordinates of the vertices of ΔACD are

$A\left(x_1' = 3, y_1' = -1\right)$, $C\ (x_2' = 14,\ y_2' = 0)$ and

$D\ (x_3' = 9,\ y_3' = 9)$

$\therefore$ Area of ΔACD

$$= \frac{1}{2}\left|x_1'\left(y_2' - y_3'\right) + x_2'\left(y_3' - y_1'\right) + x_3'\left(y_1' - y_2'\right)\right|$$

$$= \frac{1}{2}\left|3(0-19) + 14(19+1) + 9(-1-0)\right|$$

$$= \frac{1}{2}\left|-57 + 280 - 9\right| = \frac{1}{2}\left|280 - 66\right| = \frac{1}{2}\left|214\right|$$

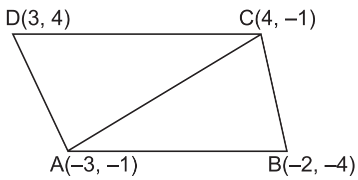
$$= 107 \qquad \ldots(2)$$

From (1) and (2),

Area of ΔABC + Area of $\Delta ACD = 25 + 107 = 132$

Hence, the required area of the quadrilateral ABCD is **132 sq units**.'

(*iii*) We join AC.



Then the coordinates of vertices of ΔABC are $A(x_1 = -3, y_1 = -1)$, $B(x_2 = -2, y_2 = -4)$ and $C(x_3 = 4, y_3 = -1)$.

Then, the area of ΔABC

$$= \frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|$$

$$= \frac{1}{2}\left|-3(-4+1) - 2(-1+1) + 4(-1+4)\right|$$

$$= \frac{1}{2}\left|9 + 12\right| = \frac{21}{2} \qquad \ldots(1)$$

The coordinates of the vertices of ΔACD are

$A(x_1' = -3,\ y_1' = -1)$, $C(x_2' = 4,\ y_2' = -1)$,

$D(x_3' = 3,\ y_3' = 4)$

$\therefore$ Area of ΔACD

$$= \frac{1}{2}\left|x_1'\left(y_2' - y_3'\right) + x_2'\left(y_3' - y_1'\right) + x_3'\left(y_1' - y_2'\right)\right|$$

$$= \frac{1}{2}\left|-3(-1-4) + 4(4+1) + 3(-1+1)\right|$$

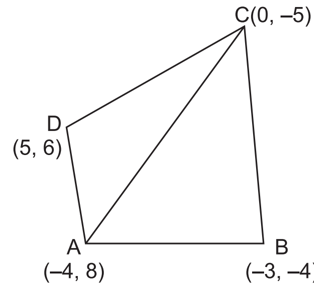
$$= \frac{1}{2}\left|15 + 20\right| = \frac{35}{2} \qquad \ldots(2)$$

$\therefore$ From (1) and (2),

$$\text{Area of } \Delta ABC + \text{Area of } \Delta ACD = \frac{21}{2} + \frac{35}{2} = \frac{56}{2} = 28$$

Hence, the required area of the quadrilateral ABCD is **28 sq units**.

(*iv*) We join AC.



Then the coordinates of the vertices of ΔABC are $A(x_1 = -4, y_1 = 8)$, $B(x_2 = -3, y_2 = -4)$ and $C(x_3 = 0, y_3 = -5)$

$\therefore$ Area of ΔABC

$$= \frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|$$

$$= \frac{1}{2}\left|-4(-4+5) - 3(-5-8) + 0(8+4)\right|$$

$$= \frac{1}{2}\left|-4 + 39\right| = \frac{35}{2} \qquad \ldots(1)$$

The coordinates of the vertices of ΔACD are

$A\ (x_1' = -4,\ y_1' = 8)$, $C(x_2' = 0,\ y_2' = -5)$ and

$D\ (x_3' = 5,\ y_3' = 6)$

$\therefore$ Area of ACD

$$= \frac{1}{2}\left|x_1'\left(y_2' - y_3'\right) + x_2'\left(y_3' - y_1'\right) + x_3'\left(y_1' - y_2'\right)\right|$$

$$= \frac{1}{2}\left|-4(-5-6) + 0(6-8) + 5(8+5)\right|$$

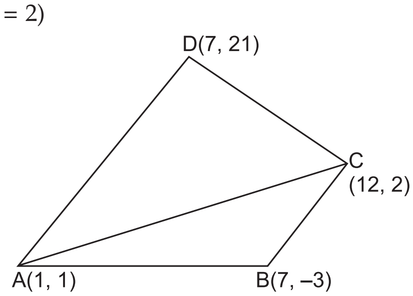
$$= \frac{1}{2}\left|44 + 65\right| = \frac{109}{2} \qquad \ldots(2)$$

$\therefore$ From (1) and (2), Area of ΔABC + Area of ΔACD

$$= \frac{35}{2} + \frac{109}{2} = \frac{144}{2} = 72$$

Hence, the required area of the quadrilateral ABCD is **72 sq units**.

(*v*) We join AC. Then the coordinates of vertices of ΔABC are $A(x_1 = 1, y_1 = 1)$, $B(x_2 = 7, y_2 = -3)$ and $C(x_3 = 12, y_3 = 2)$



$\therefore$ Area of ΔABC

$$= \frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|$$

$= \frac{1}{2}\left|1(-3-2)+7(2-1)+12(1+3)\right|$

$= \frac{1}{2}\left|-5+7+48\right| = \frac{50}{2} = 25$ …(1)

The coordinates of the vertices of ΔACD are

$A(x_1' = 1,\ y_1' = 1)$, $C(x_2' = 12,\ y_2' = 2)$ and

$D(x_3' = 7,\ y_3' = 21)$

$\therefore$ Area of ΔACD is

$= \frac{1}{2}\left|x_1'(y_2' - y_3') + x_2'(y_3' - y_1') + x_3'(y_1' - y_2')\right|$

$= \frac{1}{2}\left|1(2-21)+12(21-1)+7(1-2)\right|$

$= \frac{1}{2}\left|-19+240-7\right| = \frac{1}{2}\left|240-26\right| = \frac{1}{2}\left|214\right|$

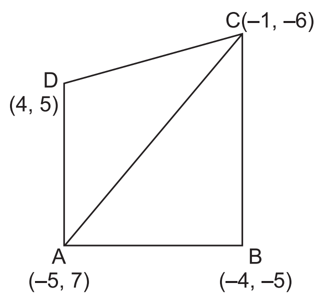
$= 107$ …(2)

$\therefore$ From (1) and (2), we have

$ar(\Delta ABC) + ar(\Delta ACD) = 25 + 107 = 132$

Hence, the required area of the quadrilateral ABCD is **132 sq units**.

(*vi*) We join AC.



Then the coordinates of the vertices of ΔABC are

$A(x_1 = -5, y_1 = 7)$, $B(x_2 = -4, y_2 = -5)$ and

$C(x_3 = -1, y_3 = -6)$

Hence, area of ΔABC

$= \frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|$

$= \frac{1}{2}\left|-5(-5+6)-4(-6-7)-1(7+5)\right|$

$= \frac{1}{2}\left|-5+52-12\right| = \frac{1}{2}\left|52-17\right| = \frac{35}{2}$ …(1)

Again, the coordinates of the vertices of ΔACD are

$A\ (x_1' = -5,\ y_1' = 7)$, $C(x_2' = -1,\ y_2' = -6)$and

$D\ (x_3' = 4,\ y_3' = 5)$

$\therefore$ Area of ΔACD

$= \frac{1}{2}\left|x_1'(y_2' - y_3') + x_2'(y_3' - y_1') + x_3'(y_1' - y_2')\right|$

$= \frac{1}{2}\left|-5(-6-5)-1(5-7)-4(7+6)\right|$

$= \frac{1}{2}\left|55+2+52\right| = \frac{109}{2}$ …(2)

$\therefore$ From (1) and (2), we have

$ar(\Delta ABC) + ar(\Delta ACD) = \frac{35+109}{2} = \frac{144}{2} = 72$

Hence, the required area of the quadrilateral ABCD is **72 sq units**.

14. The given points will not form a quadrilateral if any three points are collinear.

Let A = (–3, 5), B = (3, 1) and C = (0, 3)

Now,

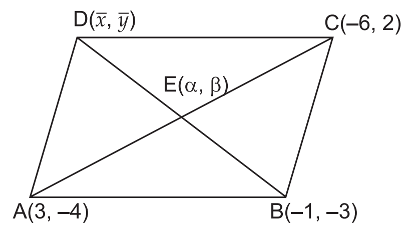
Area of $\Delta ABC = \frac{1}{2}\left|\{-3(1-3)+3(3-5)+0(5-1)\}\right|$

$= \frac{1}{2}\left|\{6-6+0\}\right| \{6 - 6 + 0\} = 0$

This shows that the points A, B and C are collinear.

Hence, the given four points will not form a quadrilateral.

15. We join AC and BD.



2nd Part:

Let the coordinates of the vertices of ΔABC be

$A(x_1 = 3, y_1 = -4)$, $B(x_2 = -1, y_2 = -3)$ and

$C(x_3 = -6, y_3 = 2)$

Then the area of ΔABC

$= \frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|$

$= \frac{1}{2}\left|3(-3-2)-1(2+4)-6(-4+3)\right|$

$= \frac{1}{2}\left|-15-6+6\right|$

$= \frac{15}{2}$

$\therefore$ Area of the $\parallel$gm ABCD = $2 \times ar\ \Delta ABC = 2 \times \frac{15}{2} = 15$.

Hence, the required area of the $\parallel$gm ABCD is **15 sq units**.

1st Part: Let E (α, β) be the point of intersection of two diagonals AC and BD of the $\parallel$gm ABCD. Then E is the mid-point of the diagonals.

$\therefore \quad \alpha = \frac{3-6}{2} = -\frac{3}{2}$ and $\beta = \frac{-4+2}{2} = -1$

$\therefore \quad -\frac{3}{2} = \frac{\bar{x}-1}{2}$

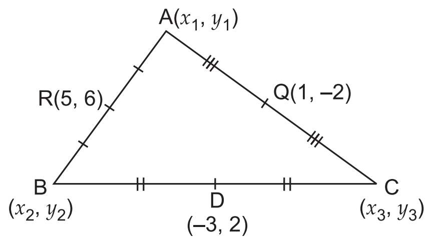
$\Rightarrow \quad \bar{x} = -2$ and $-1 = \frac{\bar{y}-3}{2} \quad \Rightarrow \quad \bar{y} = 1$.

$\therefore$ The required coordinates of D are **(–2, 1)**.

For Standard Level

16. Let P(–3, 2), Q(1, –2) and R(5, 6) be the mid-points of BC, CA and AB respectively of a ΔABC, where the coordinates of the vertices A, B and C are respectively.

$$(x_1, y_1), (x_2, y_2) \text{ and } (x_3, y_3)$$



Now, by mid-point theorem, we have

$$\frac{x_1 + x_2}{2} = 5$$

$$\Rightarrow \quad x_1 + x_2 = 10 \quad \ldots(1)$$

$$\frac{x_2 + x_3}{2} = -3$$

$$\Rightarrow \quad x_2 + x_3 = -6 \quad \ldots(2)$$

and $$\frac{x_3 + x_1}{2} = 1$$

$$\Rightarrow \quad x_3 + x_1 = 2 \quad \ldots(3)$$

Adding (1), (2) and (3), we get

$$x_1 + x_2 + x_3 = \frac{10 - 6 + 2}{2} = 3 \quad \ldots(4)$$

Subtracting (1), (2) and (3) successively from (4), we get $x_3 = -7$, $x_1 = 9$ and $x_2 = 1$.

Similarly,

$$\frac{y_1 + y_2}{2} = 6$$

$$\Rightarrow \quad y_1 + y_2 = 12 \quad \ldots(5)$$

$$\frac{y_2 + y_3}{2} = 2$$

$$\Rightarrow \quad y_2 + y_3 = 4 \quad \ldots(6)$$

and $$\frac{y_3 + y_1}{2} = -2$$

$$\Rightarrow \quad y_1 + y_3 = -4 \quad \ldots(7)$$

Adding (5), (6) and (7) we get

$$y_1 + y_2 + y_3 = \frac{12 + 4 - 4}{2} = 6 \quad \ldots(8)$$

Subtracting (5), (6) and (7) successively from (8), we get $y_3 = -6$, $y_1 = 2$, $y_2 = 10$.

$\therefore$ The required coordinates of the vertices A, B and C are respectively **(9, 2), (1, 10) and (–7, –6)**.

17. Let A(8, –6), B(–4, 6) and C(x, y) be the vertices of ΔABC.

Now,

$$\text{Area of } \Delta ABC = \frac{1}{2}\,|8(6 - y) - 4(y + 6) + x(-6 - 6)|$$

$$\Rightarrow \quad \pm 120 \times 2 = |48 - 8y - 4y - 24 - 12x|$$

$$\Rightarrow \quad -12x - 12y + 24 = 240 \text{ or } -12x - 12y + 24 = -240$$

It is given that the vertex C(x, y) lies on $x - 2y = 6$

Now,

$$-12x - 12y + 24 = 240 \quad \ldots(1)$$

and $$x - 2y = 6 \quad \ldots(2)$$

From equation (2),

$$x = 6 + 2y \quad \ldots(3)$$

Putting the value of x in equation (1),

$$-12(6 + 2y) - 12y + 24 = 240$$

$$\Rightarrow \quad -72 - 24y - 12y = 240 - 24$$

$$\Rightarrow \quad -36y = 288$$

$$\therefore \quad y = -8$$

Putting $y = -8$ in equation (3)

$$x = 6 + (-8)2$$

$$= 6 - 16$$

$$\Rightarrow \quad x = -10$$

Also, $$-12x - 12y + 24 = -240 \quad \ldots(4)$$

$$x - 2y = 6$$

$$\Rightarrow \quad x = 6 + 2y \quad \ldots(5)$$

Put equation (5) in equation (4),

$$-12(6 + 2y) - 12y + 24 = -240$$

$$\Rightarrow \quad -72 - 24y - 12y = -240 - 24$$

$$\Rightarrow \quad -36y = -192$$

$$\Rightarrow \quad y = \frac{192}{36}$$

$$\therefore \quad y = \frac{16}{3}$$

Put $y = \frac{16}{3}$ in equation (5),

$$x = 6 + 2 \times \frac{16}{3} = \frac{18 + 32}{3}$$

$$\Rightarrow \quad x = \frac{50}{3}$$

Hence, the coordinates of the third vertex are either **(–10, –8)** or $\left(\mathbf{\frac{50}{3}, \frac{16}{3}}\right)$.

18. It is given that P divides the join of A(–5, 1) and B(3, 5) in the ratio $k : 1$.

So, the coordinates of P are $\frac{3k - 5}{k + 1}, \frac{5k + 1}{k + 1}$.

Now, Area of ΔPQR

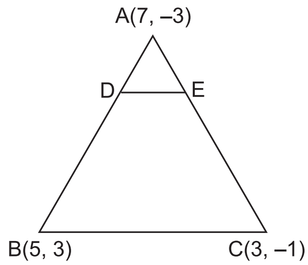
$$= \frac{1}{2}\left|\left[1\left(\frac{5k+1}{k+1} + 2\right) + \frac{3k-5}{k+1}(-2-5) + 7\left(5 - \frac{5k+1}{k+1}\right)\right]\right|$$

$$\Rightarrow \quad 6 = \left|\frac{66 - 14k}{k+1}\right|$$

$$\Rightarrow \quad \frac{66 - 14k}{k+1} = 6 \text{ or } \frac{66 - 14k}{k+1} = -6$$

$$\Rightarrow \quad \mathbf{k = 3 \text{ or } k = 9}.$$

19. Point D divides AB in the ratio 1 : 3. So, coordinates of D are



$$\left(\frac{1\times5+3\times7}{1+3}, \frac{1\times3+3\times(-3)}{1+3}\right) \text{ i.e. } D\left(\frac{13}{2}, -\frac{3}{2}\right)$$

Also, point E divides AC in the ratio 1 : 3. So, coordinates of E are

$$\left(\frac{1\times3+3\times7}{1+3}, \frac{1\times(-1)+3\times(-3)}{1+3}\right) \text{ i.e. } E\left(6, -\frac{5}{2}\right)$$

$$\text{So, ar}(\Delta ADE) = \frac{1}{2}\left|\left[7\left(-\frac{3}{2}+\frac{5}{2}\right)+\frac{13}{2}\left(-\frac{5}{2}+3\right)+6\left(-3+\frac{3}{2}\right)\right]\right|$$

$$= \frac{1}{2}\left|\left[7\times1+\frac{13}{2}\times\frac{1}{2}+6\times\left(-\frac{3}{2}\right)\right]\right|$$

$$= \frac{1}{2}\left|\left[7+\frac{13}{4}-9\right]\right|$$

$$= \frac{1}{2}\left|\left[\frac{13}{4}-2\right]\right|$$

$$= \frac{1}{2}\times\frac{5}{4} = \frac{5}{8} \text{ sq units}$$

$$\text{and ar}(\Delta ABC) = \frac{1}{2}\left|[7(3+1)+5(-1+3)+3(-3-3)]\right|$$

$$= \frac{1}{2}\left|[7\times4+5\times2+3\times(-6)]\right|$$

$$= \frac{1}{2}\left|[28+10-18]\right| \; [28 + 10 - 18]$$

$$= \frac{1}{2}\left|[38-18]\right| = \frac{1}{2}\times 20$$

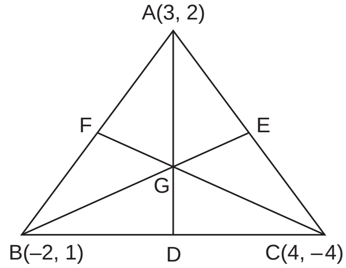
= 10 sq units

$$\text{Hence, } \frac{\text{ar}(\Delta ADE)}{\text{ar}(\Delta ABC)} = \frac{\frac{5}{8}}{10} = \frac{1}{16}$$

$\Rightarrow$ ar(ΔADE) : ar(ΔABC) = **1 : 16**

20. Coordinates of the centroid, G are

$$\left(\frac{3-2+4}{3}, \frac{2+1-4}{3}\right) \text{ i.e. } \boldsymbol{\left(\frac{5}{3}, -\frac{1}{3}\right)}$$



$$\text{Now, ar}(\Delta GBC) = \frac{1}{2}\left|\left[\frac{5}{3}(1+4)-2\left(-4+\frac{1}{3}\right)+4\left(-\frac{1}{3}-1\right)\right]\right|$$

$$= \frac{1}{2}\left|\left[\frac{25}{3}-2\times\left(\frac{-11}{3}\right)+4\times\left(\frac{-4}{3}\right)\right]\right|$$

$$= \frac{1}{2}\left|\left[\frac{25}{3}+\frac{22}{3}-\frac{16}{3}\right]\right|$$

$$= \frac{1}{2}\left|\left[\frac{25+22-16}{3}\right]\right|$$

$$= \frac{1}{2}\times\frac{31}{3} = \frac{31}{6} \text{ sq units}$$

$$\text{and ar}(\Delta ABC) = \frac{1}{2}\left|[3(1+4)-2(-4-2)+4(2-1)]\right|$$

$$= \frac{1}{2}\left|[15+12+4]\right|$$

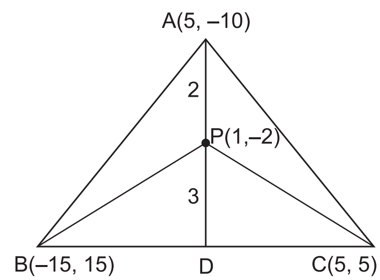
$$= \frac{1}{2}\times31 = \frac{31}{2} \text{ sq units}$$

$$\Rightarrow \frac{\text{ar}(\Delta GBC)}{\text{ar}(\Delta ABC)} = \frac{\frac{31}{6}}{\frac{31}{2}} = \frac{31}{6}\times\frac{2}{31} = \frac{1}{3}$$

Hence, **ar(ΔGBC) : ar(ΔABC) = 1 : 3**.

21. D is the mid-point of BC. So, coordinates of D are

$$\left(\frac{-15+5}{2}, \frac{15+5}{2}\right) \text{ i.e. } \mathbf{D(-5, 10)}$$



and coordinates of P are

$$\left(\frac{2\times(-5)+3\times5}{2+3}, \frac{2\times10+3\times(-10)}{2+3}\right) \text{ i.e. } \mathbf{P(1, -2)}$$

$$\text{Now, ar}(\Delta PBC) = \frac{1}{2}\left|[1(15-5)-15(5+2)+5(-2-15)]\right|$$

$$= \frac{1}{2}\left|[10-105-85]\right|$$

$$= \frac{1}{2}\times|-180| = |-90|$$

$$\Rightarrow \text{ar}(\Delta PBC) = |-90| = 90 \text{ sq units}$$

$$\text{and ar}(\Delta ABC) = \frac{1}{2}\left|[5(15-5)-15(5+10)+5(-10-15)]\right|$$

$$= \frac{1}{2}\left|[50-225-125]\right|$$

$$= \frac{1}{2}|-300| = |-150|$$

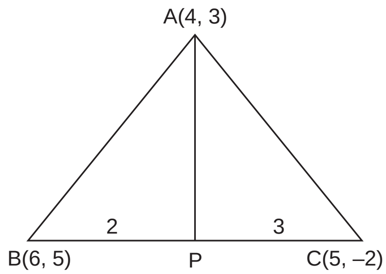
So, $\quad ar(\Delta ABC) = |-150| = 150$ sq units

$$\Rightarrow \quad \frac{ar(\Delta PBC)}{ar(\Delta ABC)} = \frac{90}{150} = \frac{3}{5}$$

$\Rightarrow \quad$ **ar(ΔPBC) : ar(ΔABC) = 3 : 5**.

22. Coordinates of P are

$$\left(\frac{2\times5+3\times6}{2+3}, \frac{2\times(-2)+3\times5}{2+3}\right) \text{ i.e. } \mathbf{P}\left(\frac{\mathbf{28}}{\mathbf{5}}, \frac{\mathbf{11}}{\mathbf{5}}\right).$$



$$\text{Now,} \quad ar(\Delta ABP) = \frac{1}{2}\left|\left[4\left(5-\frac{11}{5}\right)+6\left(\frac{11}{5}-3\right)+\frac{28}{5}(3-5)\right]\right|$$

$$= \frac{1}{2}\left|\left[4\times\frac{14}{5}+6\times\left(\frac{-4}{5}\right)+\frac{28}{5}\times(-2)\right]\right|$$

$$= \frac{1}{2}\left|\left[\frac{56}{5}-\frac{24}{5}-\frac{56}{5}\right]\right|$$

$$\Rightarrow \quad ar(\Delta ABP) = \left|\frac{-12}{5}\right| = \frac{12}{5} \text{ sq units}$$

$$\text{and } ar(\Delta ACP) = \frac{1}{2}\left|\left[4\left(-2-\frac{11}{5}\right)+5\left(\frac{11}{5}-3\right)+\frac{28}{5}(3+2)\right]\right|$$

$$= \frac{1}{2}\left|\left[4\times\left(\frac{-21}{5}\right)+5\times\left(\frac{-4}{5}\right)+\frac{28}{5}\times5\right]\right|$$

$$= \frac{1}{2}\left|\left[\frac{-84}{5}-4+28\right]\right|$$

$$= \frac{1}{2}\left|\left[\frac{-84}{5}+24\right]\right|$$

$$= \frac{1}{2}\times\frac{36}{5}$$

$$= \frac{18}{5} \text{ sq units}$$

$$\Rightarrow \quad \frac{ar(\Delta ABP)}{ar(\Delta ACP)} = \frac{\frac{12}{5}}{\frac{18}{5}} = \frac{12}{5}\times\frac{5}{18} = \frac{2}{3}$$

$\Rightarrow \quad$ ar(ΔABP) : ar(ΔACP) = **2 : 3**.

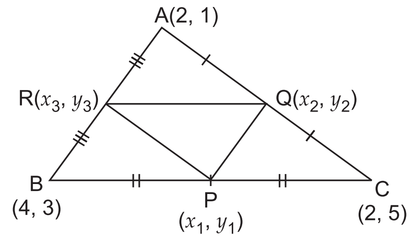
23. (*i*) Let $P(x_1, y_1)$. $Q(x_2, y_2)$ and $R(x_3, y_3)$ be the mid-points of the sides BC, CA and AB respectively of ΔABC.

Then by mid-point formula.

$$x_1 = \frac{4+2}{2} = 3, \; y_1 = \frac{3+5}{2} = 4$$

$$x_2 = \frac{2+2}{2} = 2, \; y_2 = \frac{5+1}{2} = 3$$

$$x_3 = \frac{4+2}{2} = 3, \; y_3 = \frac{3+1}{2} = 2$$



$\therefore$ Area of ΔPQR

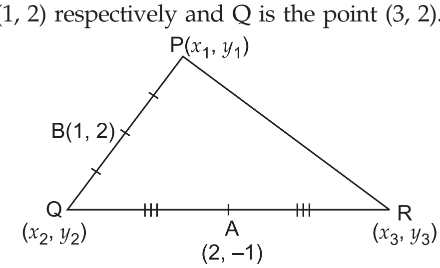
$$= \frac{1}{2}\left|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right|$$

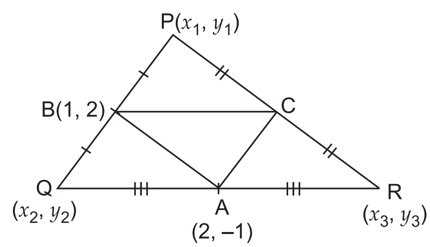
$$= \frac{1}{2}\left|3(3-2)+2(2-4)+3(4-3)\right|$$

$$= \frac{1}{2}\left|3-4+3\right| = \frac{2}{2} = 1.$$

Hence, the required area of ΔPQR is **1 sq unit**.

(*ii*) Let (x_1, y_1), (x_2, y_2) and (x_3, y_3) be the coordinates of P, Q and R respectively, where $x_2 = 3$ and $y_2 = 2$. Let A and B be the mid-points of QR and QP respectively, where the coordinates of A and B are (2, –1) and (1, 2) respectively and Q is the point (3, 2).





$$\therefore \quad \text{By mid-point formula,} \quad \frac{3+x_3}{2} = 2$$

$$\Rightarrow \quad x_3 = 4-3 = 1$$

$$\text{and} \quad \frac{2+y_3}{2} = -1$$

$$\Rightarrow \quad y_3 = -2-2 = -4.$$

Hence, coordinates of R are given by $x_3 = 1$ and $y_3 = -4$.

Also, since B(1, 2) is the mid-point of QP.

$$\therefore \quad 1 = \frac{x_1+x_2}{2} = \frac{x_1+3}{2}$$

$$\Rightarrow \quad x_1 = 2-3 = -1.$$

$$\text{and} \quad 2 = \frac{y_1+y_2}{2} = \frac{y_1+2}{2}$$

$$\Rightarrow \quad y_1 = 4-2 = 2$$

$\therefore$ Coordinates of P are given by P(–1, 2).

$\therefore$ Area of ΔPQR

$$= \frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|$$

$$= \frac{1}{2}\left|-1(2 + 4) + 3(-4 - 2) + 1(2 - 2)\right|$$

$$= \frac{1}{2}\left|-6 - 18\right| = \frac{24}{2} = 12$$

Hence, required area of ΔABC is **12 sq units**.

(*iii*) Let the coordinates of B be (a, b) and $C(c, d)$

A(0, –1)

Using mid-point formula

$$1 = \frac{0+a}{2} \qquad 0 = \frac{-1+b}{2}$$

$$a = 2 \qquad b = 1$$

B(2, 1)

Now,
$$0 = \frac{0+c}{2} \qquad 1 = \frac{-1+d}{2}$$

$$c = 0 \qquad d = 3$$

C(0, 3)

Area of ΔABC

$$= \frac{1}{2}\left|\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right]\right|$$

$$= \frac{1}{2}\left|\left[0(-2) + 2(4) + 0(-2)\right]\right|$$

$$= \frac{1}{2}\left|(8)\right| = 4 \text{ sq units}$$

Now we can find the coordinates of F.

$$x = \frac{2+0}{2} = 1$$

$$y = \frac{1+3}{2} = 2$$

D(1, 0)

E(0, 1)

F(1, 2)

Area of ΔDEF

$$= \frac{1}{2}\left|\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right]\right|$$

$$= \left|\frac{1}{2}\left[1(-1) + 0(2) + 1(-1)\right]\right|$$

$$= \frac{1}{2}\left|(-1) - 1\right|$$

$$= \frac{1}{2}\left|-2\right|$$

$$= \frac{1}{2} \times 2 = \textbf{1 sq unit}$$

(*iv*) A (1, –4)

Mid-points through A being (2, –1) and (0, –1)

Let coordinates of B be (a, b) and $C(c, d)$.

Using mid-point formula

$$2 = \frac{1+a}{2} \qquad -1 = \frac{-4+b}{2}$$

$$a = 3 \qquad b = 2$$

B(3, 2)

Similarly

$$0 = \frac{1+c}{2} \qquad -1 = \frac{-4+d}{2}$$

$$c = -1 \qquad d = 2$$

C(–1, 2)

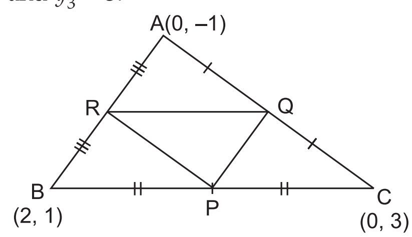
Now we need to find the area of triangle

$$\Delta = \frac{1}{2}\left|\left[1(0) + 3(6) + (-1)(-6)\right]\right|$$

$$= \frac{1}{2}\left|\left[18 + 6\right]\right| = \frac{1}{2} \times 24$$

$$= \textbf{12 sq units}$$

24. Let (x_1, y_1), (x_2, y_2) and (x_3, y_3) be the coordinates of A, B and C respectively so that $x_1 = 0$, $y_1 = -1$, $x_2 = 2$, $y_2 = 1$, $x_3 = 0$ and $y_3 = 3$.



Let (x_1', y_1'), (x_2', y_2') and (x_3', y_3') be the coordinates of P, Q and R respectively, where P, Q and R the mid-points of BC, CA and AB respectively.

$\therefore$ By mid-point formula, we have

$$x_1' = \frac{2+0}{2} = 1, \; y_1' = \frac{1+3}{2} = 2$$

$$x_2' = \frac{0+0}{2} = 0, \; y_2' = \frac{-1+3}{2} = 1$$

and
$$x_3' = \frac{2+0}{2} = 1, \; y_3' = \frac{-1+1}{2} = 0.$$

Now, area of ΔABC

$$= \frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|$$

$$= \frac{1}{2}\left|0(1 - 3) + 2(3 + 1) + 0(-1 - 1)\right|$$

$$= \frac{1}{2}\left|8\right| = 4.$$

Also, area of ΔPQR

$$= \frac{1}{2}\left|x_1'(y_2' - y_3') + x_2'(y_3' - y_1') + x_3'(y_1' - y_2')\right|$$

$$= \frac{1}{2}\left|1 \times (1 - 0) + 0 \times (0 - 2) + 1 \times (2 - 1)\right|$$

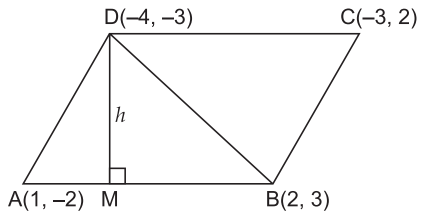
$$= \frac{1}{2}\left|1 + 1\right| = 1$$

Hence, the required areas of ΔABC and ΔPQR are **4 sq units** and **1 sq unit** respectively.

Hence, the ratio of these two areas is 4 : 1.

25. Let DM $\perp$ AB be the height of the $\|$gm and let h be its height so that DM = h. Then h is the height of ΔABD.

Let (x_1, y_1), (x_2, y_2) and (x_3, y_3) be the vertices of ΔABD so that $x_1 = 1$, $y_1 = -2$, $x_2 = 2$, $y_2 = 3$, $x_3 = -4$ and $y_3 = -3$.



$\therefore$ Area of ΔABD

$$= \frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|$$

$$= \frac{1}{2}\left|1(3+3) + 2(-3+2) - 4(-2-3)\right|$$

$$= \frac{1}{2}\left|6 - 2 + 20\right| = \frac{24}{2} = 12 \qquad \ldots(1)$$

Also, ar(ΔADB) = $\frac{1}{2}$ base × height

$$= \frac{1}{2} \times \text{AB} \times h$$

$$= \frac{1}{2}\sqrt{(1-2)^2 + (-2-3)^2}\; h$$

$$= \frac{1}{2}\sqrt{26}\; h \qquad \ldots(2)$$

$\therefore$ From (1) and (2), we get

$$\frac{\sqrt{26}}{2} h = 12$$

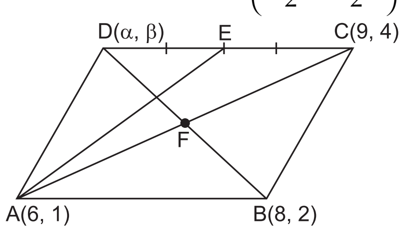
$$\Rightarrow \quad h = \frac{24}{\sqrt{26}} = \frac{24\sqrt{26}}{26} = \frac{12\sqrt{26}}{13}$$

$$\simeq \frac{12 \times 5.09}{13} \simeq 4.69$$

Hence, the required height is **4.69 units** (approx.)

26. Let the two diagonals AC and BD of the $\|$gm ABCD intersect each other at the point F. Then F is the mid-point of both AC and BD. Let (α, β) be the coordinates of D.

Then the coordinates of F are $\left(\frac{6+9}{2}, \frac{1+4}{2}\right) = \left(\frac{15}{2}, \frac{5}{2}\right)$



Also, since F is the mid-point of DB,

$$\therefore \quad \frac{\alpha + 8}{2} = \frac{15}{2}$$

$$\Rightarrow \quad \alpha = 15 - 8 = 7.$$

and
$$\frac{\beta + 2}{2} = \frac{5}{2}$$

$$\Rightarrow \quad \beta = 5 - 2 = 3$$

Hence, the coordinates of D are (7, 3).

Let (x_1, y_1), (x_2, y_2) and (x_3, y_3) be the coordinates of the vertices A, C and D respectively of ΔACD.

$\therefore$ $x_1 = 6$, $y_1 = 1$, $x_2 = 9$, $y_2 = 4$, $x_3 = 7$ and $y_3 = 3$.

Then the area of ΔADC

$$= \frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|$$

$$= \frac{1}{2}\left|6(4-3) + 9(3-1) + 7(1-4)\right|$$

$$= \frac{1}{2}\left|6 + 18 - 21\right| = \frac{3}{2} \qquad \ldots(1)$$

Since AE is a median of ΔADE

$$\therefore \quad \text{ar}(\Delta\text{ADE}) = \frac{1}{2}\,\text{ar}(\Delta\text{ADC})$$

$$= \frac{1}{2} \times \frac{3}{2} \qquad \text{[From (1)]}$$

$$= \frac{3}{4}$$

Hence, the required area of ΔADE = $\mathbf{\frac{3}{4}}$ **sq units**

CHECK YOUR UNDERSTANDING

MULTIPLE-CHOICE QUESTIONS

For Basic and Standard Levels

1. (*b*) **90°, (0, 0)**

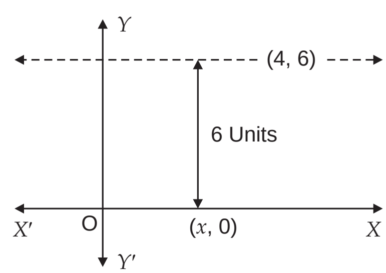
The line represented by $x = 0$ is the y-axis and the line represented by $y = 0$ is the x-axis. These two axes intersect at the point (0, 0) which is the origin. Thus, the measure of the angle is 90°.

2. (*c*) **Parallel to y-axis**

The line $x = 5$ is the line in which the coordinate of x is always equal to 5. Thus, it is a straight line passing through $x = 5$ and perpendicular to x-axis. Hence. The line $x = 5$ is parallel to y-axis.

3. (*b*) **6 units**

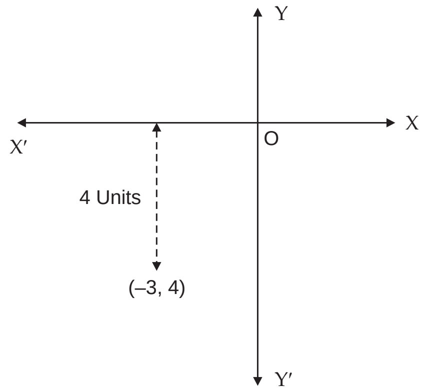
The line which passes through the point (4, 6) and parallel to x-axis has 6 as its y-coordinate and 4 as its x-coordinate. Thus, the distance of this point from the x-axis is equal to 6 units.



4. (*c*) **4 units**

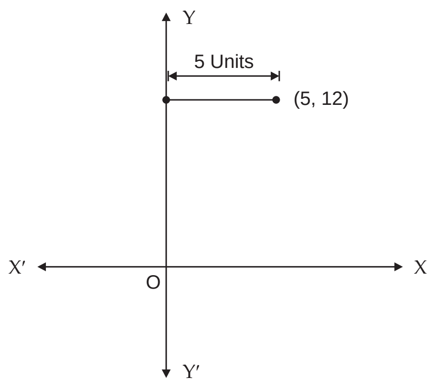
The point (–3, 4) has –3 as the x-coordinate and 4 as the y-coordinate. Thus, the distance of the point from the x-axis is the y-coordinate of the given point.

Hence, the distance of the point (–3, 4) from the x-axis is 4 units.



5. (*b*) **5 units**

The perpendicular distance of the point from the y-axis is equal to the x-coordinate of the given point. It is given that the x-coordinate is 5 units and y-coordinate is 12 units. Hence, the perpendicular distance from the y-axis is 5 units.

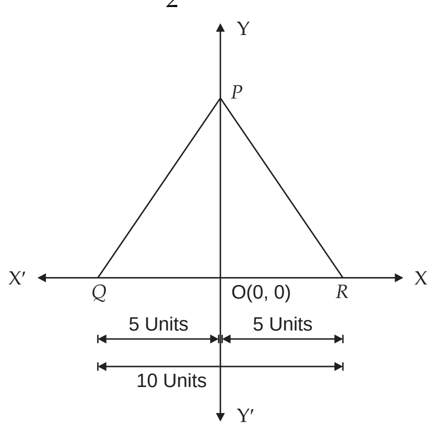


6. (*b*) **(–5, 0), (5, 0)**

The point O is middle-point of the base QR. It is located at the origin. Now, the point Q lies on the negative side of O and R on the positive side of O. It is given that

$$QR = 10 \text{ units.}$$

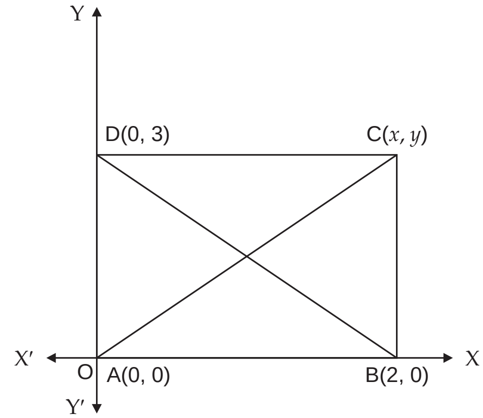
Then, $QO = RO = \dfrac{10}{2} = 5$ units



Thus, the coordinates of R and Q are (5, 0) and (–5, 0) respectively.

7. (*c*) **(2, 3)**

Let us take the points as A(0, 0), B(2, 0), C(x, y) and D(0, 3). Plot the points on the Cartesian coordinates.



From the figure,

$$x = 2 \text{ and } y = 3$$

Hence, the fourth vertex C is (2, 3).

Alternatively:

The diagonals AC and BD are equal as ABCD is a rectangle. Thus,

$$AC = BD$$

$$\Rightarrow \quad AC^2 = BD^2$$

$$\Rightarrow \quad (x - 0)^2 + (y - 0)^2 = (0 - 2)^2 + (3 - 0)^2$$

$$\Rightarrow \quad x^2 + y^2 = 4 + 9$$

$$\Rightarrow \quad x^2 + y^2 = 13 \qquad \ldots(1)$$

Also, the opposite sides AD and BC are equal.

Thus, $\quad AD = BC$

$$\Rightarrow \quad AD^2 = BC^2$$

$$\Rightarrow \quad (0 - 0)^2 + (3 - 0)^2 = (x - 2)^2 + (y - 0)^2$$

$$\Rightarrow \quad 9 = x^2 - 4x + 4 + y^2$$

$$\Rightarrow \quad x^2 + y^2 - 4x = 5 \qquad \ldots(2)$$

Subtracting equation (2) from equation (1), we have

$$13 - 5 = 4x \Rightarrow x = 2$$

From equation (1),

$$y = \sqrt{13 - x^2}$$

$$= \sqrt{13 - 2^2}$$

$$= \sqrt{13 - 4}$$

$$= \sqrt{9}$$

$$= 3$$

Hence, the coordinates of fourth vertex is (2, 3).

8. (*b*) **8 units**

The distance between the points P(6, 0) and Q(–2, 0) is

$$PQ = \sqrt{(-2 - 6)^2 + (0 - 0)^2}$$

$= \sqrt{8^2} = 8$ units.

9. (*c*) $\mathbf{2\sqrt{2}\, b}$ **units**

Let the distance between the point A($a + b$, $b + c$) and B($a - b$, $c - b$) is AB. Then, the distance AB is

$$AB = \sqrt{\{a - b - (a + b)\}^2 + \{c - b - (b + c)\}^2}$$

$$= \sqrt{(-2b)^2 + (2b)^2}$$

$$= \sqrt{4b^2 + 4b^2} = \sqrt{8b^2}$$

$$= 2b\sqrt{2} = 2\sqrt{2}\ b \text{ units.}$$

10. (*d*) ***a*** **units**

Let the points be A(a sin 30º, 0) and B(0, a sin 60º).

Then, the distance AB is

$$AB = \sqrt{(0 - a\sin 30°)^2 + (a\sin 60° - 0)^2}$$

$$= \sqrt{a^2 \sin^2 30° + a^2 \sin^2 60°}$$

$$\left[\sin 30° = \frac{1}{2} \text{ and } \sin 60° = \frac{\sqrt{3}}{2}\right]$$

$$= \sqrt{a^2\left(\frac{1}{2}\right)^2 + a^2\left(\frac{\sqrt{3}}{2}\right)^2} = \sqrt{\frac{a^2}{4} + \frac{3a^2}{4}}$$

$= a$ units

11. (*b*) **an isosceles triangle**

Let the coordinates of the points be A(–5, 0), B(5, 0) and C(0, 4).

Now, $AB = \sqrt{(5+5)^2 + (0-0)^2}$

$$= \sqrt{10^2}$$

$= 10$ units

$$AC = \sqrt{(0+5)^2 + (4-0)^2}$$

$$= \sqrt{25 + 16}$$

$= \sqrt{41}$ units

$$BC = \sqrt{(0-5)^2 + (4-0)^2}$$

$$= \sqrt{25 + 16}$$

$= \sqrt{41}$ units

Thus, $AC = BC \neq AB$

Again, $AC^2 + BC^2 = (\sqrt{41})^2 + (\sqrt{41})^2$

$= 41 + 41 = 82$ units

$\neq AB^2$

Hence, the points (–5, 0), (5, 0) and (0, 4) are the vertices of an isosceles triangle.

12. (*c*) **12 units**

Let the coordinates of the points be A(0, 4), B(0, 0) and C(3, 0).

Then, $AB = \sqrt{(0-0)^2 + (0-4)^2}$

$$= \sqrt{4^2}$$

$= 4$ units

$$BC = \sqrt{(3-0)^2 + (0-0)^2}$$

$= \sqrt{3^2} = 3$ units

$$AC = \sqrt{(3-0)^2 + (0-4)^2}$$

$$= \sqrt{9 + 16}$$

$= \sqrt{25} = 5$ units

Now, the perimeter of the triangle ABC is

AB + BC + AC= 4 units + 3 units + 5 units

= 12 units.

13. (*c*) **6**

Let the coordinates of the vertices of the triangle be A(5, 0), B(8, 0) and C(8, 4).

Now, the area of the triangle is

$$\text{area } \Delta ABC = \frac{1}{2}\left|\left[5(0-4) + 8(4-0) + 8(0-0)\right]\right|$$

$$= \frac{1}{2}\left|-20 + 32 + 0\right| = 6 \text{ sq units}$$

14. (*b*) **5 units**

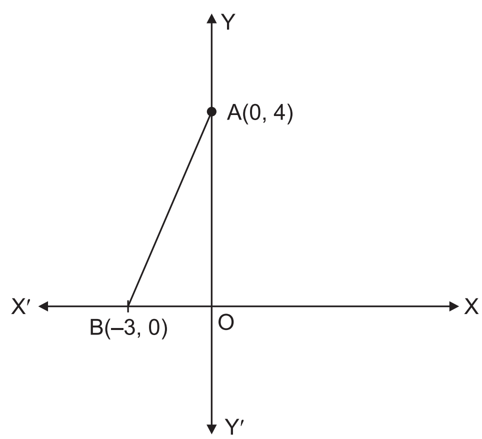
Now, the coordinates of the points A on the y-axis is (0, 4)

Thus,

$$AB = \sqrt{(-3-0)^2 + (0-4)^2}$$

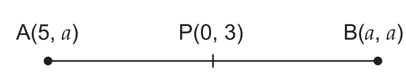
$$= \sqrt{9 + 16}$$

$= \sqrt{25} = 5$ units



15. (*b*) **5 or –5**

Let A(5, a), B(a, a) and P(0, 3) be the coordinates.



It is given that P is equidistant from points A and B.

Thus,

AP = BP

$\Rightarrow \sqrt{(0-5)^2+(3-a)^2} = \sqrt{(0-a)^2+(3-a)^2}$

$\Rightarrow \quad 5^2 + 9 - 6a + a^2 = a^2 + 9 - 6a + a^2$

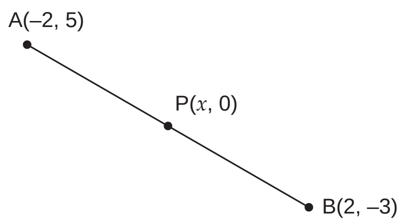
$\Rightarrow \quad a = \pm 5$

$\Rightarrow \quad a = 5 \text{ or } -5$

16. (*d*) **(–2, 0)**

Let P(x, 0) be the point on the x–axis equidistance from the given points.

Let the coordinates of the given points be A(–2, 5) and B(2, –3).



It is given that P is equidistant from the points A and B. Thus,

$$AP = BP$$

$\Rightarrow \sqrt{(x+2)^2+(0-5)^2} = \sqrt{(x-2)^2+(0+3)^2}$

Squaring both sides,

$\Rightarrow \quad (x+2)^2 + (0-5)^2 = (x-2)^2 + (0+3)^2$

$\Rightarrow \quad x^2 + 4x + 4 + 25 = x^2 - 4x + 4 + 9$

$\Rightarrow \quad 8x = 9 - 25$

$\Rightarrow \quad x = -\dfrac{16}{8} = -2$

17. (*a*) **–8**

It is given that $\left(\dfrac{a}{2}, 4\right)$ is the mid-point of the line segment joining the points A(–6, 5) and B(–2, 3). Then, using section formula for coordinates of mid-points, the x-coordinate is

$$\frac{a}{2} = \frac{-6-2}{2}$$

$\Rightarrow \quad a = -8$

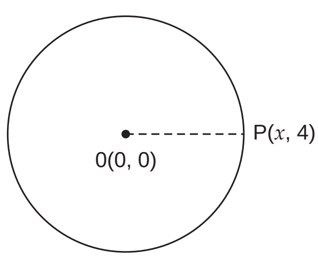
18. (*b*) **+3**

Let P(x, 4) be the point on the circle.

Then,

$$OP = \sqrt{(x-0)^2+(4-0)^2}$$

$$= \sqrt{x^2+16}$$



It is also given that the radius of the circle is 5.

Thus,

$$OP = 5$$

$\Rightarrow \quad \sqrt{x^2+16} = 5$

$\Rightarrow \quad x^2 + 16 = 25$

$\Rightarrow \quad x^2 = 9$

$\Rightarrow \quad x = \pm 3.$

19. (*a*) **3 or –9**

Let A(2, –3) and B(10, y) be the end–points of the line segment.

Now, $\quad AB = 10$ units

$\Rightarrow \sqrt{(10-2)^2+(y+3)^2} = 10$

$\Rightarrow \quad 64 + y^2 + 6y + 9 = 100$ [Squaring both sides]

$\Rightarrow \quad y^2 + 6y - 27 = 0$

$\Rightarrow \quad y^2 + 9y - 3y - 27 = 0$

$\Rightarrow \quad (y+9)(y-3) = 0$

$\Rightarrow \quad y = 3 \text{ or } -9.$

20. (*d*) **$3x = 2y$**

It is given that the distance of the point P(x, y) from the point A(5, 1) and B(–1, 5) are equal.

Thus, $\quad AP = BP$

$\Rightarrow \sqrt{(x-5)^2+(y-1)^2} = \sqrt{(x+1)^2+(y-5)^2}$

Squaring both sides

$\Rightarrow x^2 + 25 - 10x + y^2 + 1 - 2y = x^2 + 2x + 1 + y^2 - 10y + 25$

$\Rightarrow \quad -10x - 2y = 2x - 10y$

$\Rightarrow \quad -12x = -8y$

$\Rightarrow \quad 3x = 2y.$

21. (*a*) **(4, 5)**

Let the coordinates of the centre of the circle be O(x, y). It is known that the centre of a circle divides the diameter equally. Thus, O is the mid-point of the coordinates A(1, 1) and B(7, 9). Then,

$$x = \frac{1+7}{2} = 4$$

and $\quad y = \dfrac{1+9}{2} = 5$

Hence, the coordinates of the centre of the circle are (4, 5).

22. (*c*) **A(4, 0), (0, 8)**

Let A(x_1, 0) and B (0, y_1) be the coordinates. Since P is the mid-point of the line segment AB, then

$$2 = \frac{x_1+0}{2}$$

$\Rightarrow \quad x_1 = 4$

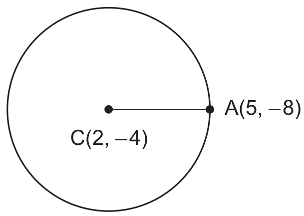
and $\qquad 4 = \dfrac{y_1 + 0}{2}$

$\Rightarrow \qquad y_1 = 8$

Hence, the coordinates of A and B are respectively (4, 0) and (0, 8).

23. (*d*) **(9, 4)**

Now, the distance AC is



$$AC = \sqrt{(2-5)^2 + (-4+8)^2}$$
$$= \sqrt{9+16} = \sqrt{25}$$
$$= 5 \text{ units}$$

Let the points be B(–1, –4), O(1, –3), E(2, 0) and F(9, 4).

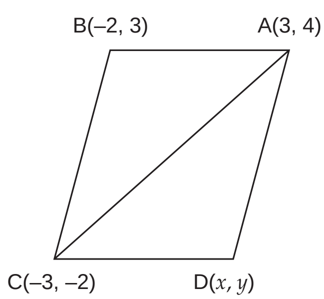
Now, $\quad CB = \sqrt{(-1-2)^2 + (-4+4)^2} = 3$ units

$$CO = \sqrt{(1-2)^2 + (-3+4)^2}$$
$$= \sqrt{1+1} = \sqrt{2} \text{ units}$$
$$CE = \sqrt{(2-2)^2 + (0+4)^2} = \sqrt{4^2} = 4 \text{ units}$$
$$CF = \sqrt{(9-2)^2 + (4+4)^2} = \sqrt{7^2 + 8^2}$$
$$= \sqrt{49+64} = \sqrt{113} \text{ units}$$
$$= 10.630 \text{ units}$$

As, CF > AC, the point (9, 4) lies outside the circle.

24. (*c*) **(2, –1)**

Let D(x, y) be the fourth vertex of the rhombus. The diagonal AC divides the rhombus into two equal areas. Now, area of the triangle ABC is



$$\text{Area of } \Delta ABC = \frac{1}{2}\left|\left[3(3+2) - 2(-2-4) - 3(4-3)\right]\right|$$
$$= \frac{1}{2}\left|\left[15 + 12 - 3\right]\right|$$
$$= 12 \text{ sq units}$$

Now,

$$\text{Area of } \Delta ACD = \frac{1}{2}\left|\left[3(-2-y) - 3(y-4) + x(4+2)\right]\right|$$
$$12 = \frac{1}{2}\left|\left[-6 - 3y - 3y + 12y + 4x + 2x\right]\right|$$

$\Rightarrow \qquad 24 = 6 - 6y + 6x$

$\Rightarrow \qquad x - y = 3 \qquad$ ...(1)

Again, the distance AD and CD are equal.

$$AD = CD$$

$\Rightarrow \quad \sqrt{(x-3)^2 + (y-4)^2} = \sqrt{(x+3)^2 + (y+2)^2}$

$\Rightarrow \quad x^2 - 6x + 9 + y^2 - 8y + 16$

$\qquad = x^2 + 6x + 9 + y^2 + 4y + 4$

$\Rightarrow \qquad x + y = 1 \qquad$ ...(2)

Adding equation (1) and equation (2),

$$2x = 4 \Rightarrow x = 2$$

In equation (1),

$$2 - y = 3 \Rightarrow y = -1$$

Hence, the coordinates of the fourth vertex of the rhombus are (2, –1).

25. (*b*) **7**

Now, the sides AB and CD are equal as ABCD is a parallelogram. Thus,

$$AB = CD$$

$\Rightarrow \quad \sqrt{(8-6)^2 + (2-1)^2} = \sqrt{(x-9)^2 + (3-4)^2}$

(Squaring both sides)

$\Rightarrow \quad 4 + 1 = x^2 - 18x + 81 + 1$

$\Rightarrow \quad x^2 - 18x + 77 = 0$

$\Rightarrow \quad x^2 - 11x - 7x + 77 = 0$

$\Rightarrow \quad x(x - 11) - 7(x - 11) = 0$

$\Rightarrow \quad (x - 11)(x - 7) = 0$

$\Rightarrow \quad x = 7$ or 11.

26. (*c*) $\boldsymbol{p = 6}$

Now, the sides AB and CD are equal.

Thus,

$$AB = CD$$

$\Rightarrow \quad \sqrt{(1-5)^2 + (5-p)^2} = \sqrt{(6-2)^2 + (2-1)^2}$

(Squaring both sides)

$\Rightarrow \quad 16 + 25 - 10p + p^2 = 16 + 1$

$\Rightarrow \quad p^2 - 10p + 24 = 0$

$\Rightarrow \quad p^2 - 6p - 4p + 24 = 0$

$\Rightarrow \quad p(p - 6) - 4(p - 6) = 0$

$\Rightarrow \quad (p - 6)(p - 4) = 0$

$\Rightarrow \quad p = 4$ or 6.

27. (*a*) **(1, –12), (5, 10)**

The diagonals of a parallelogram bisect each other. Let P be the point of intersection between the diagonals.

Let A(3, 2), B(–1, 0), C(x_1, y_1) and D(x_2, y_2).

Now, P is the mid-point of the diagonal AC. Then,

$$2 = \frac{3 + x_1}{2}$$

$$\Rightarrow \quad 4 = 3 + x_1$$

$$\Rightarrow \quad x_1 = 1$$

and $$-5 = \frac{2 + y_1}{2}$$

$$\Rightarrow \quad -10 = 2 + y_1$$

$$\Rightarrow \quad y_1 = -12$$

Thus, the coordinates of C are (1, –12).

Also, P is the mid-point of the diagonals BD. Then,

$$2 = \frac{x_2 - 1}{2}$$

$$\Rightarrow \quad 4 = x_2 - 1$$

$$\Rightarrow \quad x_2 = 5$$

and $$-5 = \frac{y_2 + 0}{2}$$

$$\Rightarrow \quad y_2 = -10$$

Thus, the coordinates of D are (5, –10).

Hence, the coordinates of the remaining vertices are (1, –12) and (5, –10).

28. (*c*) $\boldsymbol{p = 4, q = 2}$

As the point (5, *p*) the mid-point of the points is (3*p*, 4) and (–2, 2*q*), then

$$5 = \frac{3p - 2}{2}$$

$$\Rightarrow \quad 10 = 3p - 2$$

$$\Rightarrow \quad 3p = 12$$

$$\Rightarrow \quad p = 4$$

and $$p = \frac{4 + 2q}{2}$$

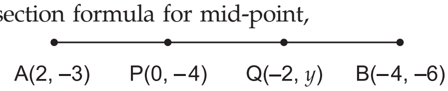
$$\Rightarrow \quad 2p = 4 + 2q$$

$$\Rightarrow \quad 2 \times 4 = 4 + 2q \qquad (\because \; p = 4)$$

$$\Rightarrow \quad q = \frac{8 - 4}{2} = 2.$$

29. (*c*) **–5**

From the figure, Q is the mid-point of P and B. Using the section formula for mid-point,

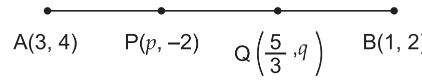


$$y = \frac{-4 - 6}{2}$$

$$\Rightarrow \quad y = -5$$

30. (*b*) $\boldsymbol{p = \frac{7}{3}, q = 0}$

Let the points be A(3, –4) and B(1, 2).



From the figure, the point P divides the line segment AB in the ratio 1 : 2. Using section formula,

$$p = \frac{2 \times 3 + 1 \times 1}{1 + 2} = \frac{6 + 1}{3} = \frac{7}{3}$$

Now, the point Q is the mid-point of PB. Then,

$$q = \frac{-2 + 2}{2} = 0$$

31. (*b*) **(3, 5)**

Let the coordinates of the points P be (*x*, *y*). As the point P divides the line segment AB in the ratio 2 : 1. Then,

$$x = \frac{1 \times 1 + 2 \times 4}{1 + 2} = \frac{1 + 8}{3} = 3$$

and $$y = \frac{1 \times 3 + 2 \times 6}{1 + 2} = \frac{3 + 12}{3} = 5$$

Hence, the coordinates of the point P are (3, 5).

32. (*b*) **5 : 2**

Let *k* : 1 be the ratio in which the point (2, –5) divides the line segment joining the points (–3, 5) and (4, –9). Using section formula,

$$2 = \frac{-3 \times 1 + k \times 4}{k + 1}$$

$$\Rightarrow \quad 2k + 2 = -3 + 4k$$

$$\Rightarrow \quad 2k = 5$$

$$\Rightarrow \quad k = \frac{5}{2}$$

Hence, the ratio is 5 : 2.

33. (*a*) **2 : 3**

Let the coordinates of the point which divides the line segment joining the points A and B be (0. *y*). Also, let the point (0, *y*) divides the line segment in the ratio *k* : 1.

Using section formula,

$$0 = \frac{-2 \times 1 + k \times 3}{k + 1}$$

$$k = \frac{2}{3}.$$

Hence, the ratio is 2 : 3.

34. (*a*) **(–3, 5)**

Let the points be A(*x*, 0), B(5, –2) and C(–8, *y*). The coordinates of the centroid of the triangle ABC is G(–2, 1). Then,

$$-2 = \frac{x + 5 - 8}{3}$$

$$\Rightarrow \quad -6 = x - 3 \Rightarrow x = -3$$

and $$1 = \frac{0 - 2 + y}{3}$$

$$\Rightarrow \quad 3 = y - 2$$

$\Rightarrow \qquad y = 5.$

Hence, (x, y) is equal to $(-3, 5)$.

For Standard Level

35. (*b*) **(0, 8)**

Let $P(x, y)$ be the point on the perpendicular bisector of the line segment AB. Then, P is equidistant from the points A and B. Thus,

$$AP = BP$$
$$\Rightarrow \quad AP^2 = BP^2$$
$$\Rightarrow \quad (x-2)^2 + (y-3)^2 = (x-5)^2 + (y-6)^2$$
$$\Rightarrow \quad x^2 + 4 - 4x + y^2 - 6y + 9 = x^2 - 10x + 25 + y^2 - 12y + 36$$
$$\Rightarrow \quad 6x + 6y = 48$$
$$\Rightarrow \quad x + y = 8$$

As the perpendicular bisector cut the y-axis, let $Q(0, y_1)$ be the coordinates P then,

$$x + y = 8$$
$$\Rightarrow \quad 0 + y_1 = 8$$
$$\Rightarrow \quad y_1 = 8$$

Hence, the perpendicular cuts the y-axis at (0, 8).

36. (*c*) **5 units**

Now, D is the mid-point of the side BC.

Let (x, y) be the coordinates of D. Then,

$$x = \frac{5+3}{2} = 4$$

and $$y = \frac{3-1}{2} = 1$$

Now, the length AD is

$$AD = \sqrt{(4-7)^2 + (1+3)^2}$$
$$= \sqrt{3^2 + 4^2} = \sqrt{9+16}$$
$$= \sqrt{25} = 5 \text{ units}$$

37. (*b*) **(3, 0)**

Let $P(x, y)$ be the point on the perpendicular bisector of the line segment. Let the points be A(7, 6) and B(–3, 4).

Then, P is equidistant from A and B. Thus,

$$AP = BP$$
$$\Rightarrow \quad AP^2 = BP^2$$
$$\Rightarrow \quad (x-7)^2 + (y-6)^2 = (x+3)^2 + (y-4)^2$$
$$\Rightarrow \quad x^2 - 14x + 49 + y^2 - 12y + 36 = x^2 + 6x + 9 + y^2 - 8y + 16$$
$$\Rightarrow \quad -20x - 4y = 60$$
$$\Rightarrow \quad 5x + y = -15$$

Let the coordinates on the x-axis be $(x_1, 0)$. Then, this point lies on the line $5x + y = -15$. Thus,

$$5x_1 + 0 = -15$$
$$\Rightarrow \quad x_1 = -3$$

Hence, the coordinates are (–3, 0).

38. (*a*) $\left(-\frac{5}{3}, 0\right)$

It is given that the coordinates of the point Q divides the line AB in the ratio 2 : 1. Let the coordinates of the point Q be (x, y). Then, using section formula,

$$x = \frac{1\times1+2\times(-3)}{1+2} = \frac{1-6}{3} = \frac{-5}{3}$$

and $$y = \frac{1\times(-2)+2\times4}{1+2} = \frac{-2+8}{3} = \frac{6}{3} = 2.$$

Hence, the coordinates of the points Q is $\left(\frac{-5}{3}, 2\right)$.

39. (*d*) **IV quadrant**

Let the coordinates of the points P be (x, y). As the points P divides the line segment joining the points A and B in the ratio 2 : 3 internally, then

$$x = \frac{2\times5+3\times2}{2+3} = \frac{10+6}{5} = \frac{16}{5}$$

and $$y = \frac{2\times2+3\times(-5)}{2+3} = \frac{4-15}{5} = -\frac{11}{5}$$

Thus, the signs of the x is positive and y is negative. Hence, the coordinates of the point P lies in the IV quadrant.

40. (*b*) **AP = 2PB**

Let the point P divides the line segment joining the points A and B in the ratio k : 1.

Using section formula,

$$-3 = \frac{k\times(-2)+1\times(-5)}{k+1}$$
$$\Rightarrow \quad -3k - 3 = -2k - 5$$
$$\Rightarrow \quad k = 2$$

Thus, the point P divides the line AB in the ratio 2 : 1.

Now, $$\frac{AP}{PB} = \frac{2}{1}$$
$$\Rightarrow \quad AP = 2PB$$

41. (*b*) **(4, –4)**

Let the vertices of the triangle be A(3, 2), B(–2, 1) and C(x, y). The centroid of the triangle is $G\left(\frac{5}{3}, \frac{-1}{3}\right)$.

Now, $$\frac{5}{3} = \frac{3-2+x}{3} \Rightarrow x = 4$$

and $$-\frac{1}{3} = \frac{2+1+y}{3}$$
$$\Rightarrow \quad y = -4$$

Hence, the coordinates of the third vertices are (4, –4).

42. (*c*) $k = 2$

Let the coordinates of the vertices of the triangle be A(3, –5), B(–7, 4) and C(10, $-k$). The coordinates of the centroid be G(k, –1). Then,

$$k = \frac{3-7+10}{3} = \frac{6}{3} = 2$$

Hence, $k = 2$.

43. (*a*) **0**

It is given that the coordinate of the triangle is at the origin. Thus, the coordinate of the centroid is G(0, 0). The vertices of the triangle are A(a, b), B(b, c) and C(c, a).

Now,

$$0 = \frac{a+b+c}{3}$$

$$\Rightarrow \quad a + b + c = 0$$

44. (*b*) **$2x = y$**

It is given that the points (0, 0), (1, 2) and (x, y) are collinear. Thus, the area of the triangle formed by these points is equal to zero. Thus,

Area of the triangle = 0

$$\Rightarrow \frac{1}{2}\left|\left[0(2-y)+1(y-0)+x(0-2)\right]\right| = 0$$

$$\Rightarrow \quad y - 2x = 0$$

$$\Rightarrow \quad 2x = y$$

45. (*c*) **7.5**

$$\text{ar}(\Delta ABC) = \frac{1}{2}\left|\left[1(0-0)-1(0-3)+1(3-0)\right]\right|$$

$$= \frac{1}{2}\left|\left[0-1(3)+1(3)\right]\right|$$

$$= \frac{1}{2}\left|\left[3+12\right]\right|$$

$$= \frac{15}{2} = 7.5$$

SHORT ANSWER QUESTIONS

For Basic and Standard Levels

1. (*i*) In ΔABC,

$$AB = \sqrt{(0+3)^2+(3-0)^2} = \sqrt{9+9}$$

$$= \sqrt{18} = 3\sqrt{2}$$

$$BC = \sqrt{(3-0)^2+(0-3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}$$

$$AC = \sqrt{(3+3)^2+(0-0)^2} = \sqrt{36+0} = \sqrt{36} = 6$$

In ΔPQR,

$$PQ = \sqrt{(0+6)^2+(6-0)^2} = \sqrt{36+36} = \sqrt{72} = 6\sqrt{2}$$

$$QR = \sqrt{(6-0)^2+(0-6)^2} = \sqrt{36+36} = \sqrt{72} = 6\sqrt{2}$$

$$PR = \sqrt{(6+6)^2+(0-0)^2} = \sqrt{144+0} = \sqrt{144} = 12$$

Now, $\dfrac{AB}{PQ} = \dfrac{3\sqrt{2}}{6\sqrt{2}} = \dfrac{1}{2}$

$\dfrac{BC}{QR} = \dfrac{3\sqrt{2}}{6\sqrt{2}} = \dfrac{1}{2}$

and $\dfrac{AC}{PR} = \dfrac{6}{12} = \dfrac{1}{2}$

Since $\dfrac{AB}{PQ} = \dfrac{BC}{QR} = \dfrac{AC}{PR}$,

$\therefore \quad \Delta ABC \sim \Delta PQR$

(*ii*) Now, $\text{ar}(\Delta ABC) = \frac{1}{2}\left|\left[-3(3-0)+0(0-0)+3(0-3)\right]\right|$

$$= \frac{1}{2}\left|\left[-9+0-9\right]\right| = \frac{1}{2}\times\left|\left[-18\right]\right| = |-9|$$

ar(ΔABC) = $|-9|$ = 9 sq units

and $\text{ar}(\Delta PQR) = \frac{1}{2}\left|\left[-6(6-0)+0(0-0)+6(0-6)\right]\right|$

$$= \frac{1}{2}\left|\left[-36+0-36\right]\right|$$

$$= \frac{1}{2}\times\left|\left[-72\right]\right| = |-36|$$

ar(ΔPQR) = $|-36|$ = 36 sq units

ar(ΔABC) : ar(ΔPQR) = 9 : 36 = 1 : 4

2. We have

$$PQ^2 = (x-9)^2 + (4-10)^2$$

$$= 100 \qquad \text{[Given]}$$

$$\Rightarrow x^2 - 18x + 81 + 36 - 100 = 0$$

$$\Rightarrow \quad x^2 - 18x + 17 = 0$$

$$\therefore \quad x = \frac{18 \pm \sqrt{18^2 - 4\times 7}}{2}$$

$$= \frac{18 \pm \sqrt{324-68}}{2}$$

$$= \frac{18 \pm \sqrt{256}}{2}$$

$$= \frac{18 \pm 16}{2}$$

$$= 17, 1$$

Hence, the required value of x is **17 or 1**.

3. We have

$$AB^2 = 5^2 = 25$$

$$\Rightarrow \quad (4-1)^2 + (2-y)^2 = 25$$

$$\Rightarrow \quad (2-y)^2 = 25 - 9 = 16$$

$$\therefore \quad 2 - y = \pm\sqrt{16} = \pm 4.$$

$$\therefore \quad y = 2 \mp 4 = 6 \text{ or } -2$$

$\therefore$ Required value of y is **6 or –2**.

4. Let the ratio of division be r : 1.

Then,

$$-3 = \frac{r\times(-2)+1\times(-5)}{r+1}$$

$$\Rightarrow \quad -3r - 3 = -2r - 5$$

$$\Rightarrow \quad r = 2$$

So, the ratio of division is **2 : 1**

Now, $k = \dfrac{r\times 3+1\times(-4)}{r+1}$

$$\Rightarrow \quad k = \frac{2\times 3-4}{2+1} = \frac{6-4}{3} = \mathbf{\frac{2}{3}}$$

5. The given points cannot form a triangle when they are collinear. This means the area formed by these points should be equal to zero.

It is given that A(2, 1), B(1, 2) and C(0, 3).

$$\text{Area } (\Delta ABC) = \frac{1}{2}\left|\left[2(2-3)+1(3-1)+0(1-2)\right]\right|$$

$$= \frac{1}{2}\left|\left[-2+0+0\right]\right| = 0$$

This means the given points are collinear.

Hence, the given points do not form a triangle.

ALTERNATIVE METHOD:

$$AB = \sqrt{(1-2)^2+(2-1)^2}$$

$$= \sqrt{(-1)^2+(1)^2} = \sqrt{1+1} = \sqrt{2}$$

$$BC = \sqrt{(0-1)^2+(3-2)^2} = \sqrt{(-1)^2+(1)^2}$$

$$= \sqrt{1+1} = \sqrt{2}$$

$$AC = \sqrt{(0-2)^2+(3-1)^2}$$

$$= \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$$

Since, $\sqrt{2}+\sqrt{2} = 2\sqrt{2}$

i.e. AB + BC = AC

So, A, B, C are collinear.

i.e. A, B, C cannot from a triangle.

6. For the mid-point P (x, y),

$$x = \frac{2+k}{2} \text{ and } y = \frac{3+5}{2} = 4$$

As the point P (x, y) lies on the line $x + y - 7 = 0$, so

$$\frac{2+k}{2} + 4 - 7 = 0$$

$$\Rightarrow \quad \frac{2+k}{2} = 3$$

$$\Rightarrow \quad k = 6 - 2 = 4$$

Hence, the value of k is **4**.

7. Let the coordinates of A, B and C be (x_1, y_1), (x_2, y_2) and (x_3, y_3) respectively where $x_1 = k + 1$, $y_1 = 2k$, $x_2 = 3k$, $y_2 = 2k + 3$, $x_3 = 5k - 1$ and $y_3 = 5k$.

$\therefore$ Area of ΔABC

$$= \frac{1}{2}\left|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right|$$

$$= \frac{1}{2}\left|(k+1)(2k+3-5k)+3k(5k-2k)+(5k-1)(2k-2k-3)\right|$$

$$= \frac{1}{2}\left|(k+1)(3-3k)+3k\times 3k+(5k-1)(-3)\right|$$

$$= \frac{1}{2}\left|3k+3-3k^2-3k+9k^2-15k+3\right|$$

$$= \frac{1}{2}\left|6k^2-15k+6\right| \qquad \ldots(1)$$

$\therefore$ The three given points are collinear, hence, the area of ΔABC = 0

$\therefore$ From (1),

$$6k^2 - 15k + 6 = 0$$

$$\Rightarrow \quad 2k^2 - 5k + 2 = 0$$

$$\therefore \quad k = \frac{5 \pm \sqrt{25-4\times 2\times 2}}{4}$$

$$= \frac{5 \pm \sqrt{25-16}}{4}$$

$$= \frac{5 \pm 3}{4} = 2 \text{ or } \frac{1}{2}$$

Hence, the required value of k is **2 or $\frac{1}{2}$**.

8. A(5, 2)

B(2, –2)

C(–2, t)

Since it is a right-angled triangle

$$\therefore \quad AB^2 + BC^2 = AC^2$$

$$\Rightarrow \left(\sqrt{(3)^2+(4)^2}\right)^2+\left(\sqrt{(4)^2+(-2-t)^2}\right)^2 = \left(\sqrt{(7)^2+(2-t)^2}\right)^2$$

$$\Rightarrow (5)^2 + \left(\sqrt{16+4+t^2+4t}\right)^2 = \left(\sqrt{49+4+t^2-4t}\right)^2$$

$$\Rightarrow \quad (5)^2 + \left(\sqrt{t^2+4t+20}\right)^2 = \left(\sqrt{t^2-4t+53}\right)^2$$

$$\Rightarrow \quad 25 + t^2 + 4t + 20 = t^2 - 4t + 53$$

$$\Rightarrow \quad 8t = 8$$

$$\Rightarrow \quad t = 1$$

Hence, $\boldsymbol{t = 1}$.

9. The given points are P(0, –2), Q(3, 1), R(0, 4) and S(–3, 1).

So, $\quad PQ = \sqrt{(3-0)^2+(1+2)^2}$

$$= \sqrt{9+9} = \sqrt{18} = 3\sqrt{2} \text{ units}$$

$$QR = \sqrt{(0-3)^2+(4-1)^2} = \sqrt{9+9}$$

$$= \sqrt{18} = 3\sqrt{2} \text{ units}$$

$$RS = \sqrt{(-3-0)^2+(1-4)^2} = \sqrt{9+9}$$

$$= \sqrt{18} = 3\sqrt{2} \text{ units}$$

$$PS = \sqrt{(-3-0)^2+(1+2)^2} = \sqrt{9+9}$$

$$= \sqrt{18} = 3\sqrt{2} \text{ units}$$

$$\Rightarrow \quad PQ = QR = RS = PS = 3\sqrt{2} \text{ units}$$

Also, $\quad PR = \sqrt{(0-0)^2+(4+2)^2}$

$$= \sqrt{0+36} = 6 \text{ units}$$

and $\quad QS = \sqrt{(-3-3)^2+(1-1)^2}$

$$= \sqrt{0+36}$$

$$= \sqrt{36} = 6 \text{ units}$$

$$\Rightarrow \quad PR = QS = 6 \text{ units}$$

Since all the sides are equal and both the diagonals are equal, PQRS is a square.

10. Let the fourth vertex of the parallelogram be D(x, y). Let A($a + b$, $a - b$), B($2a + b$, $2a - b$), C($a - b$, $a + b$).

Then, AC and BD are diagonals. The intersecting points divide the diagonals equally.

Then, the coordinates of the point of intersection of the diagonals are $\left(\dfrac{a+b+a-b}{2}, \dfrac{a-b+a+b}{2}\right)$ and $\left(\dfrac{2a+b+x}{2}, \dfrac{2a-b+y}{2}\right)$. They are the coordinates of the same point.

So, $$\frac{a+b+a-b}{2} = \frac{2a+b+x}{2}$$

$\Rightarrow$ $$2a = 2a + b + x$$

$\Rightarrow$ $$x = -b$$

and $$\frac{a-b+a+b}{2} = \frac{2a-b+y}{2}$$

$$2a = 2a - b + y$$

$$y = b$$

Hence, the coordinates of the fourth vertex are **($-b$, b)**.

11. (*i*) $$\text{ar}(\Delta ABC) = \frac{1}{2}\left|\left[2(2-3)+4(3-1)+3(1-2)\right]\right|$$

$$= \frac{1}{2}\left|\left[2\times(-1)+4\times 2+3\times(-1)\right]\right|$$

$$= \frac{1}{2}\left|\left[-2+8-3\right]\right|$$

$$= \frac{1}{2} \times 3 = \frac{3}{2} \text{ units}$$

So, points A, B, C are not collinear.

Hence, the statement is **false**.

(*ii*) Coordinates of the mid-point of AC are $\left(\dfrac{-2+4}{2}, \dfrac{-1+3}{2}\right)$ i.e. (1, 1).

Coordinates of the mid-point of BD are $\left(\dfrac{1+1}{2}, \dfrac{0+2}{2}\right)$ i.e. (1, 1)

Since the mid-points of AC and BD are same, the diagonals bisect each other.

So, ABCD is a parallelogram.

Hence , the statement is **true**.

12. The three students will be seated in a row if their points are collinear.

Now, $$\text{ar}(\Delta ABC) = \frac{1}{2}\left|\left[1(-2-4)+3(4-1)-1(1+2)\right]\right|$$

$$= \frac{1}{2}\left|\left[-6+9-3\right]\right|$$

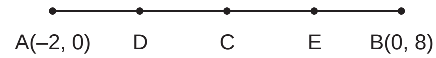
$$= \frac{1}{2} \times 0 = 0$$

Thus, the points A, B and C are collinear.

Hence, the three students are seated in a row.

13. Let the given two coordinates be A(-2, 0) and B(0, 8). Draw a line segment AB in which D, C and E divide equally.

The point C is the mid-point of the line AB. Thus, the coordinates of C are



$$C\left(\frac{-2+0}{2}, \frac{0+8}{2}\right), \text{ i.e. } C(-1, 4).$$

The point D divides the line AC into equal parts. Thus, the coordinates of D are

$$D\left(\frac{-2-1}{2}, \frac{0+4}{2}\right), \text{ i.e. } D\left(\frac{-3}{2}, 2\right)$$

Also, E is the mid-point of the line CB. Thus, the coordinates of E are

$$E\left(\frac{-1-0}{2}, \frac{4+8}{2}\right), \text{ i.e. } E\left(\frac{-1}{2}, 6\right)$$

Hence, the coordinates of points are $\mathbf{\left(\dfrac{-3}{2}, 2\right)}$, **($-1$, 4)** and $\mathbf{\left(\dfrac{-1}{2}, 6\right)}$.

For Standard Level

14. $$AB = \sqrt{(-2-2)^2+(1+2)^2}$$

$$= \sqrt{16+9} = \sqrt{25} = 5 \text{ units}$$

$$BC = \sqrt{(5+2)^2+(2-1)^2}$$

$$= \sqrt{49+1} = \sqrt{50} \text{ units}$$

$$AC = \sqrt{(5-2)^2+(2+2)^2}$$

$$= \sqrt{9+16} = \sqrt{25} = 5 \text{ units}$$

Since, AB = AC $\neq$ BC, ΔABC is an **isosceles triangle**.

Also, $AB^2 + AC^2 = 5^2 + 5^2 = 25 + 25 = 50 = BC^2$.

So, ΔABC is also a right-angled triangle **right-angled at A**.

15. Now, $$AB = \sqrt{(8-2)^2+(4+2)^2}$$

$$= \sqrt{36+36} = \sqrt{72} = 6\sqrt{2} \text{ units}$$

$$BC = \sqrt{(5-8)^2+(7-4)^2}$$

$$= \sqrt{9+9} = \sqrt{18} = 3\sqrt{2} \text{ units}$$

$$CD = \sqrt{(-1-5)^2+(1-7)^2}$$

$$= \sqrt{36+36} = \sqrt{72} = 6\sqrt{2} \text{ units}$$

$$DA = \sqrt{(-1-2)^2+(1+2)^2}$$

$$= \sqrt{9+9} = \sqrt{18} = 3\sqrt{2} \text{ units}$$

AB = CD and BC = DA, opposite sides are equal

So, it is a parallelgram.

Again, $$AC = \sqrt{(5-2)^2+(7+2)^2} = \sqrt{9+81} = \sqrt{90}$$

$$= 3\sqrt{10} \text{ units}$$

and $$BD = \sqrt{(-1-8)^2+(1-4)^2}$$

$$= \sqrt{81+9} = \sqrt{90}$$

$$= 3\sqrt{10} \text{ units}$$

AC = BD

i.e, diagonals are equal.

Hence, ABCD is a **rectangle**.

16. A particular point which is equidistant from A(–1, 6) and B(–5, 4) lies in the middle point of the line formed by joining the points A and B. Let Q(x_1, y_1) divides the line AB in the ratio 1 : 1 Thus,

$$x_1 = \frac{-1-5}{2} = -3$$

$$y_1 = \frac{6+4}{2} = 5$$

Hence, the points which is equidistant from the points A and B is **(–3, 5)**.

Let a point P (x, y) be equidistant from the given points A (–1, 6) and B (–5, 4).

Then, $AP^2 = (x + 1)^2 + (y - 6)^2$

and $BP^2 = (x + 5)^2 + (y - 4)^2$

But $AP = BP$

$AP^2 = BP^2$

$\Rightarrow (x + 1)^2 + (y - 6)^2 = (x + 5)^2 + (y - 4)^2$

$\Rightarrow x^2 + 2x + 1 + y^2 - 12y + 36$

$= x^2 + 10x + 25 + y^2 - 8y + 16$

$\Rightarrow -8x - 4y = 4$

$\Rightarrow 2x + y + 1 = 0$

Any point whose coordinates satisfy this equation is the required point.

Hence, there are infinite number of such points.

17. Let a point P(x, y) be on the perpendicular bisector of the line segment joining the points A(3, 6) and B(–3, 4).

Then, $AP^2 = (x - 3)^2 + (y - 6)^2$

Also, $BP^2 = (x + 3)^2 + (y - 4)^2$

But $AP = BP$

$\Rightarrow AP^2 = BP^2$

$\Rightarrow (x - 3)^2 + (y - 6)^2 = (x + 3)^2 + (y - 4)^2$

$\Rightarrow x^2 - 6x + 9 + y^2 - 12y + 36$

$= x^2 + 6x + 9 + y^2 - 8y + 16$

$\Rightarrow -12x - 4y + 20 = 0$

$\Rightarrow \mathbf{3x + y - 5 = 0}$

This is the required equation of the perpendicular bisector.

(*i*) For the intersection with y-axis, $x = 0$. Thus,

$0 + y - 5 = 0 \Rightarrow y = 5$

Hence, the point of intersection with y-axis is **(0, 5)**.

(*ii*) For the intersection with x-axis, $y = 0$.

$\Rightarrow 3x + 0 - 5 = 0$

$\Rightarrow x = \frac{5}{3}$

Hence, the point of intersection with x-axis is $\left(\mathbf{\frac{5}{3}, 0}\right)$.

18. (*i*) $AP = \sqrt{(-3+3)^2 + (6-2)^2}$

$= \sqrt{0+16} = 4$ units

$PB = \sqrt{(-3+3)^2 + (-6-2)^2}$

$= \sqrt{0+64} = 8$ units

and $AB = \sqrt{(-3+3)^2 + (-6-6)^2}$

$= \sqrt{0+144} = 12$ units

Now, AP + PB = AB.

Hence, P lies on the segment AB.

Again, consider a triangle is formed by points A(–3, 6), B(–3, –6) and P(–3, 2)

Now,

Area of $\Delta ABP = \frac{1}{2}\left|\{-3(-6-2) - 3(2-6) - 3(6+6)\}\right|$

$= \frac{1}{2}\left|(24 + 12 - 36)\right| = 0$

This shows that the points A(–3, 6), B(–3, –6) and P(–3, 2) are collinear.

(*ii*) Radius of the circle,

$OP = \sqrt{(6-0)^2 + (0-0)^2}$

$= \sqrt{36+0} = 6$ units

Now, $OQ = \sqrt{(7-0)^2 + (9-0)^2}$

$= \sqrt{49+81}$

$= \sqrt{130}$ units

Since $OQ > OP$,

Q lies outside the circle.

(*iii*) Let k be the ratio in which point P(0, 1) divides AB.

Then, $\frac{k \times (-6) + 3}{k+1} = 0$

$\Rightarrow -6k + 3 = 0$

$\Rightarrow k = \frac{1}{2}$

Also, $\frac{k \times 5 + (-1)}{k+1} = 1$

$\Rightarrow 5k - 1 = k + 1$

$\Rightarrow 4k = 2$

$\Rightarrow k = \frac{1}{2}$

This shows that the point P divides the line AB in the ratio 1 : 2.

Hence, P (0, 1) is a point of trisection of the line segment joining the points A(3, –1) and B(–6, 5).

19. We have $AB = \sqrt{\left(\frac{-2}{7}+2\right)^2 + \left(\frac{-20}{7}+2\right)^2}$

$= \sqrt{\frac{144}{49} + \frac{36}{49}} = \sqrt{\frac{180}{49}} = \frac{6\sqrt{5}}{7}$ units

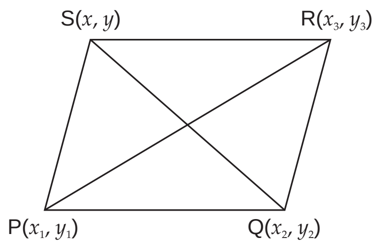
$AC = \sqrt{(2+2)^2 + (-4+2)^2} = \sqrt{16+4}$

$= \sqrt{20} = 2\sqrt{5}$ units

So, $\dfrac{AB}{AC} = \dfrac{6\sqrt{5}}{7} \times \dfrac{1}{2\sqrt{5}} = \dfrac{3}{7}$

$\Rightarrow \quad AB = \dfrac{3}{7} AC$

20. Let the coordinates of S be (x, y).



Then, coordinates of the mid-point of PR are $\left(\dfrac{x_1+x_3}{2}, \dfrac{y_1+y_3}{2}\right)$.

Coordinates of the mid-point of QS are $\left(\dfrac{x+x_2}{2}, \dfrac{y+y_2}{2}\right)$.

Since, PQRS is a parallelogram its diagonals bisect each other at a point.

$\Rightarrow \quad \dfrac{x+x_2}{2} = \dfrac{x_1+x_3}{2}$

$\Rightarrow \quad x = x_1 + x_3 - x_2$

and $\quad \dfrac{y+y_2}{2} = \dfrac{y_1+y_3}{2}$

$\Rightarrow \quad y = y_1 + y_3 - y_2$

Hence, the coordinates S are $\boldsymbol{(x_1 + x_3 - x_2, y_1 + y_3 - y_2)}$.

21. A(1, –2)
B(2, 3)
C(k, 2)
D(–4, –3)

Since opposite sides of a parallelogram are equal

$\therefore \quad BC = AD$

$\Rightarrow \quad \sqrt{(k-2)^2+(-1)^2} = \sqrt{(-4-1)^2+(-3+2)^2}$

On squaring both the side, we get

$\Rightarrow \quad (k-2)^2 + 1 = 25 + 1$

$\Rightarrow \quad (k-2)^2 = 25$

$\Rightarrow \quad k - 2 = \pm 5$

$\Rightarrow \quad k = -3, 7$

Since $k = 7$ does not satisfy the condition hence we reject it.

Now to find the length of altitude we need to find the area of parallelogram.

$\therefore \quad \boldsymbol{k = -3}$

Now, Area of parallelogram = ar(ΔABD) + ar(ΔBCD)

$= \dfrac{1}{2}\left|\left[1(6)+2(-1)+(-4)(-5)\right]\right| + \dfrac{1}{2}\left|\left[2(5)+(-3)(-6)+(-4)(1)\right]\right|$

$= \dfrac{1}{2}\left|\left[6-2+20\right]\right| + \dfrac{1}{2}\left|\left[10+18-4\right]\right|$

$= \dfrac{1}{2} \times 24 + \dfrac{1}{2} \times 24$

= 24 sq units

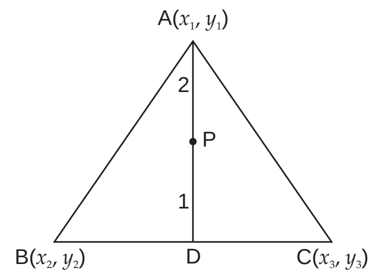
We know

Area = Base (AB) × Altitude corresponding to AB

$\text{Altitude} = \dfrac{\text{Area}}{\text{Base}} = \dfrac{24}{\sqrt{(2-1)^2+(5)^2}}$

$\dfrac{24}{\sqrt{26}}$ units $= \dfrac{12}{13}\sqrt{26}$ units

22. The point D lies in the mid-point of BC. Thus, coordinates of D are $\left(\dfrac{x_2+x_3}{2}, \dfrac{y_2+y_3}{2}\right)$



The point P divides AD in the ratio 2 : 1 coordinates of P are

$$\left(\dfrac{2\times\left(\dfrac{x_2+x_3}{2}\right)+x_1}{3}, \dfrac{2\times\left(\dfrac{y_2+y_3}{2}\right)+y_1}{3}\right)$$

i.e. $\left(\boldsymbol{\dfrac{x_1+x_2+x_3}{3}, \dfrac{y_1+y_2+y_3}{3}}\right)$.

23. Let ABC be the given triangle. Then, A(3, –2), B(2, 1) and C(x_1, y_1) are the coordinates.

Now,

area of the triangle $= \dfrac{1}{2}\left|\left[3(1-y_1)+2(y_1+2)+x_1(-2-1)\right]\right|$

$\Rightarrow \quad 5 \times 2 = \dfrac{1}{2}\left|3-3y_1+2y_1+4-3x_1\right|$

$\Rightarrow \quad 10 = \dfrac{1}{2}\left|-y_1-3x_1+7\right|$

$\Rightarrow \quad 10 = -y_1 - 3x_1 + 7 \quad$ ...(1)

Or $\quad -10 = -y_1 - 3x_1 + 7 \quad$ ...(2)

Also, it is given that the point C(x_1, y_1) lies in the line $y - x = 3$ Thus,

$y_1 - x_1 = 3$

$\Rightarrow \quad y_1 = 3 + x_1 \quad$ ...(3)

Using equation (3) in equation (1), we have

$10 = -(3 + x_1) - 3x_1 + 7$

$\Rightarrow \quad 10 = -3 - x_1 - 3x_1 + 7$

$\Rightarrow \quad 10 = 4 - 4x_1$

$\Rightarrow \quad 4x_1 = 6$

$\Rightarrow \quad x_1 = \frac{-3}{2}$

$\Rightarrow \quad 4x_1 = 14$

$\Rightarrow \quad x_1 = \frac{7}{2}$

Using the value of $x_1 = \frac{-3}{2}$ in equation (3)

$$y_1 = 3 - \frac{3}{2}$$

$$y_1 = \frac{3}{2}$$

Using equation (3) in equation (2), we have

$$-10 = -(3 + x_1) - 3x_1 + 7$$
$$-10 = -3 - x_1 - 3x_1 + 7$$
$$-10 = 4 - 4x_1$$

$\Rightarrow \quad 4x_1 = 14$

$\Rightarrow \quad x_1 = \frac{14}{4} = \frac{7}{2}$

Using the value of $x_1 = \frac{7}{2}$ in equation (3),

$$y_1 = 3 + \frac{7}{2}$$

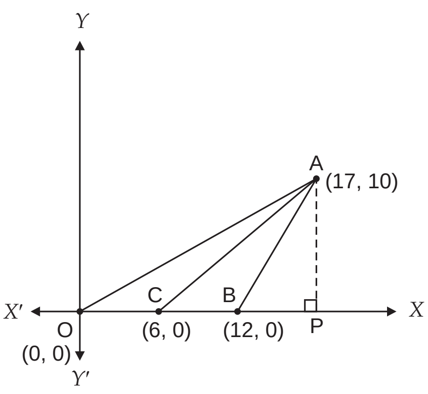
$$y_1 = \frac{13}{2}$$

Hence, the coordinate of the third vertex are $\left(\frac{-3}{2}, \frac{3}{2}\right)$ or $\left(\frac{7}{2}, \frac{13}{2}\right)$.

VALUE-BASED QUESTIONS

For Basic and Standard Levels

1. (*i*) The areas of land received by the farmer's son is equal to the triangular area AOC. Now, the coordinates of the vertices of the ΔAOC are A(17, 10), O(0, 0) and C(6, 0).



Thus,

$$\text{Area of } \Delta AOC = \frac{1}{2}\left|\left[17(0-0) + 0(0-10) + 6(10-0)\right]\right|$$

$$= \frac{1}{2}\left|\left[0 + 0 + 60\right]\right|$$

$$= 30 \text{ sq units}$$

Hence, the area of land received by the farmer's son is 30 sq units. Again, the area of land received by farmer's daughter is the triangular area ACB. Now, the coordinates of the point B is (12, 0).

Thus,

$$\text{Area of } \Delta ACB = \frac{1}{2}\left|\left[17(0-0) + 6(0-10) + 12(10-0)\right]\right|$$

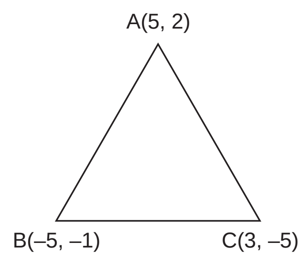
$$= \frac{1}{2}\left|\left[0 - 60 + 120\right]\right|$$

$$= 30 \text{ sq units}$$

Hence, the area of land received by the farmer's daughter is **30 sq units**

(*ii*) It is found that the farmer distributed equally triangular plot to his son and daughter. This shows that the farmers treated them equally without any distinction in their gender. Hence, the value exhibited by the farmer is gender equality.

2. (*i*) Let the coordinates of the triangular plot be A(5,2), B(–5, –1) and C(3, –5)



Now, area of the triangular plot is

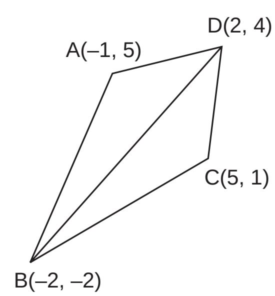
$$\text{area } \Delta ABC = \frac{1}{2}\left|\left[5(-1+5) - 5(-5-2) + 3(2+1)\right]\right|$$

$$= \frac{1}{2}\left|\left[20 + 35 + 9\right]\right|$$

$$= \frac{64}{2} = 32 \text{ sq units}$$

The other plot is in the shape of quadrilateral. Let the coordinates of the quadrilateral be A(–1, 5), B(–2, –2), D(2, 4) and C(5, 1).

Now, the area of ABCD is equal to the sum of the areas of the triangles ABD and CDB.



Now, area of triangle ABD is

$$\text{area of } \Delta ABD = \frac{1}{2}\left|\left[-1(-2-4)-2(4-5)+2(5+2)\right]\right|$$

$$= \frac{1}{2}\left|[6+2+14]\right|$$

$$= \frac{22}{2}$$

= 11 sq units

Also, area of triangle CBD is

$$\text{area of } \Delta CDB = \frac{1}{2}\left|\left[5(4+2)+2(-2-1)-2(1-4)\right]\right|$$

$$= \frac{1}{2}\left|[30-6+6]\right|$$

= 15 sq units

Thus, the area of the quadrilateral is

Area ABCD = Area ΔABD + Area ΔCDB

= 11 sq units + 15 sq units

= **26 sq units**

Hence, the area of the quadrilateral plot is 26 sq units.

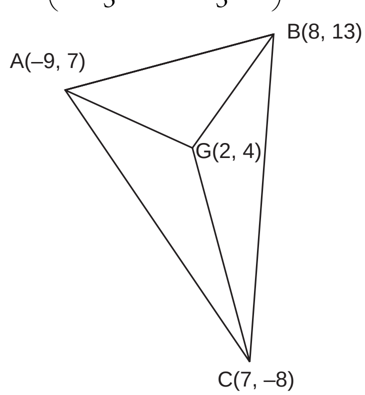
(*ii*) It is found that the area of the triangular plot is larger than the quadrilateral plot. This means Jaiveer has correctly thought and purchase the plot which has larger area. Hence, Jaiveer exhibited critical thinking and decision-making.

For Standard Level

3. (*i*) A centroid is a point at which three medians of a triangle divides the corresponding triangles into equal areas. Thus, the man should find the centroid inside the triangle ABC.

Let G(x, y) be the centroid of the given triangle ABC. It divides the triangle ABC into triangles AGC, BGC and AGB having equal areas. Using section formula for a centroid, its coordinates are

$$G\left(\frac{-9+7+8}{3}, \frac{7+13-8}{3}\right) \text{ i.e. } G(2,4).$$



The coordinates of the points are A(–9, 7), B(8, 13) and C(7, –8).

Now, area of the triangle AGC is

$$= \frac{1}{2}\left|\left[-9(4+8)+2(-8-7)+7(7-4)\right]\right|$$

$$= \frac{1}{2}\left|[-108-30+21]\right|$$

$$= \frac{117}{2} \text{ sq units}$$

Area of the triangle BGC is

$$\frac{1}{2}\left|\left[8(4+8)+2(-8-13)+7(13-4)\right]\right|$$

$$= \frac{1}{2}\left|[96-42+63]\right|$$

$$= \frac{117}{2} \text{ sq units}$$

and area of the triangle ABG

$$\frac{1}{2}\left|\left[-9(13-4)+8(4-7)+2(7-13)\right]\right|$$

$$= \frac{1}{2}\left|[-81-24-12]\right|$$

$$= \frac{117}{2} \text{ sq units}$$

Thus, area ΔAGC = area ΔBGC = ΔABG

(*ii*) The problem is solved and verified. It is known that a centroid actually divides the triangles formed in the triangle equal in area. Hence, the values exhibited is problem-solving and helpfulness.

(*iii*) The areas donated by the man is larger than the land own by him. This shows that the man has very much empathy to the orphanage. Hence, the value exhibited by the man is empathy.

UNIT TEST 1

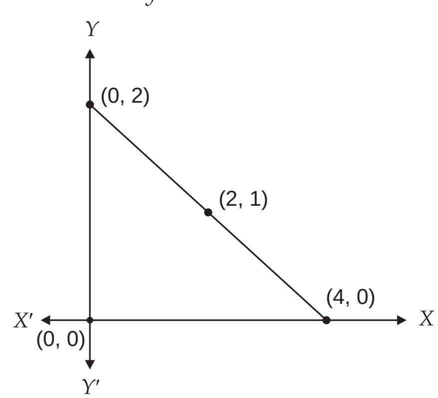
For Basic Level

1. (*d*) **(0, 0), (0, 2), (4, 0)**

Now, the given equation is

$$x + 2y = 4$$

$$\Rightarrow \quad x = 4 - 2y$$



Using equation (1), the table is

x	2	0	4
y	1	2	0

From the above points, a graph is drawn. It is found that it intersects at the x and y axis at the points (4, 0) and (0, 2) respectively. Thus, a triangle is formed with coordinates (0, 0), (4, 0) and (0,2).

2. (*b*) **26**

The given points are A(–6, 7) and B(–1, –5).

Now,

$$AB = \sqrt{(-1+6)^2 + (-5-7)^2}$$
$$= \sqrt{25+144}$$
$$= \sqrt{169}$$
$$= 13 \text{ units}$$

Thus,

$$2AB = 2 \times 13 \text{ units}$$
$$= 26 \text{ units}$$

3. (*b*) $\mathbf{3\sqrt{2}}$ **units**

Let the coordinates of the four vertices of the square OABC be A(3, 0), B(3, 3), C(0, 3) and O(0. 0). Then, the length of the diagonal of the square OABC is

$$BO = \sqrt{(0-3)^2 + (0-3)^2}$$
$$= \sqrt{9+9}$$
$$= 3\sqrt{2} \text{ units}$$

4. (*c*) **(–6, 7)**

Let the point at the end of the diameter of the circle be A(2, 3) and the centre be O(–2, 5). Let B(x, y) be the other end of the diameter.

Now, the diameter is divided equally at the centre O.

Using section formula as O is the mid-point of the diameter AB,

$$-2 = \frac{x+2}{2} \Rightarrow x = -4 - 2 = -6$$

and $$5 = \frac{y+3}{2} \Rightarrow y = 10 - 3 = 7$$

Hence, the coordinates of the other and of the diameter are (–6, 7).

5. (*b*) $\boldsymbol{x = -3}$

Let D(x, y) be another point on the line ABC.

Now, the slope of the lines AB and CD are equal.

Thus,

$$\frac{6-2}{-3+3} = \frac{y+6}{x+3}$$
$$\Rightarrow \frac{4}{0} = \frac{y+6}{x+3}$$
$$\Rightarrow x + 3 = 0$$
$$\Rightarrow x = -3$$

6. (*c*) **3 : 5**

Let P(x, 0) be the coordinates on the x-axis. The point P divides the line segment A(6, 3) and B(–2, –5) in the ratio k : 1. By using section formula

$$0 = \frac{k \times (-5) + 1 \times 3}{k+1} \Rightarrow 5k = 3$$
$$\Rightarrow k = \frac{3}{5}$$

Hence, the x-axis divides the line segment in the ratio 3 : 5.

7. (*d*) $\boldsymbol{k = \frac{5}{2}}$

The given points are collinear if the area of the triangle formed by the points A, B and C are equal to zero.

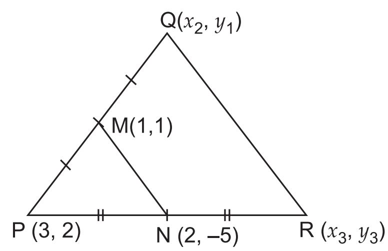
Now, area of triangle ABC = 0

$$\Rightarrow \frac{1}{2}\left\|\left[3(k-3) + 4(3-2) + 5(2-k)\right]\right\| = 0$$
$$\Rightarrow \left\|[3k - 9 + 4 + 10 - 5k]\right\| = 0$$
$$\Rightarrow \left\|[-2k + 5]\right\| = 0$$
$$\Rightarrow k = \frac{5}{2}$$

8. From the figure, it is clear that the position of scarecrow is at the intersection of the 4th row and 6th column as it is at an equal distance of 3 units form points A, B, C and D.

Hence, **the position of a scarecrow so that it is equidistant from the saplings are (6, 4)**.

9. Let (x_1, y_1), (x_2, y_2) and (x_3, y_3) be the coordinates of P, Q and R respectively. Let M(1, 1) and N(2, –5) be the mid-points of PQ and PR respectively, where $x_1 = 3$ and $y_1 = 2$.



By using mid-points formula, we have

$$1 = \frac{x_2 + 3}{2} \Rightarrow x_2 = 2 - 3 = -1$$
$$1 = \frac{y_2 + 2}{2} \Rightarrow y_2 = 0$$
$$2 = \frac{x_1 + x_3}{2} = \frac{3 + x_3}{2} \Rightarrow x_3 = 4 - 3 = 1$$
$$-5 = \frac{y_1 + y_3}{2} = \frac{2 + y_3}{2} \Rightarrow y_3 = -12$$

Hence, we have

$x_1 = 3$, $y_1 = 2$, $x_2 = -1$, $y_2 = 0$, $x_3 = 1$ and $y_3 = -12$

$\therefore$ Area of ΔPQR

$$= \frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|$$
$$= \frac{1}{2}\left|3 \times (12) - 1(-12 - 2) + 1(2 - 0)\right|$$

$= \frac{1}{2}|36 + 14 + 2| = \frac{52}{2} = 26.$

Hence, the required area is **26 sq units**.

10. Radius of the circle $= \sqrt{(2k-11)^2 + (k-7+9)^2}$

$$= \sqrt{4k^2 - 44k + 121 + k^2 + 4k + 4}$$

$$= \sqrt{5k^2 - 40k + 125}$$

Also, radius of the circle

$$= \frac{\text{Diameter}}{2} = \frac{10\sqrt{2}}{2}$$

$$= 5\sqrt{2} \text{ units}$$

$\Rightarrow \quad \sqrt{5k^2 - 40k + 125} = 5\sqrt{2}$

Squaring both the sides, we get

$$5k^2 - 40k + 125 = (5\sqrt{2})^2$$

$\Rightarrow \quad 5k^2 - 40k + 75 = 0$

$\Rightarrow \quad k^2 - 8k + 15 = 0$

$\Rightarrow \quad k^2 - 5k - 3k + 15 = 0$

$\Rightarrow \quad k(k-5) - 3(k-5) = 0$

$\Rightarrow \quad (k-5)(k-3) = 0$

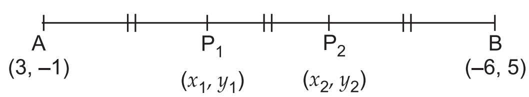
$\Rightarrow \quad k - 5 = 0 \text{ or } k - 3 = 0$

$\Rightarrow \quad k = 5 \text{ or } k = 3$

Hence, the values of k are **3 and 5**.

11. Let AB be the line segment joining the points A(3, –1) and B(–6, 5). Let $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ be the points of trisection of the line segment AB so that

$AP_1 : P_1B = 1 : 2$ and $AP_2 : P_2B = 2 : 1$.



Hence, by using section formula, we have

$$x_1 = \frac{1 \times (-6) + 2 \times 3}{1+2} = \frac{-6+6}{3} = 0$$

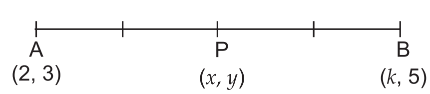
$$y_1 = \frac{1 \times 5 + 2 \times (-1)}{1+2} = \frac{3}{3} = 1$$

$$x_2 = \frac{2 \times (-6) + 1 \times 3}{2+1} = \frac{-9}{3} = -3$$

and $$y_2 = \frac{2 \times 5 + 1 \times (-1)}{2+1} = \frac{9}{3} = 3$$

$\therefore$ The required coordinates of the points of trisection are **(0, 1)** and **(–3, 3)**.

12. Let P(x, y) be the mid-point of AB, where A is the point (2, 3) and B is the point (k, 5).



Then by using mid-point section formula, we have

$$x = \frac{2+k}{2} \text{ and } y = \frac{3+5}{2} = 4$$

Now, the point (x, y) satisfies the equation

$$x + y - 7 = 0$$

$\therefore \quad \frac{2+k}{2} + 4 - 7 = 0$

$\Rightarrow \quad \frac{2+k}{2} = 3$

$\Rightarrow \quad k = 6 - 2 = \mathbf{4}$

which is the required value of k.

UNIT TEST 2

For Standard Level

1. (a) $\left(\frac{3}{2}x, \frac{5}{2}y\right)$

Let $O(x_1, y_1)$ be the mid-point of the hypotenuse BO. This point is equidistant from the vertices of the ΔAOB.

Using section formula for the mid-point of a line segment,

$$x_1 = \frac{0+3x}{2} \Rightarrow x_1 = \frac{3x}{2}$$

and $$y_1 = \frac{5y+0}{2} \Rightarrow y_1 = \frac{5}{2}y.$$

Hence, the coordinates of the point which is equidistant from the vertices of the ΔAOB is $\left(\frac{3x}{2}, \frac{5}{2}y\right)$.

2. (c) **5 units**

Let P(x, y) be the mid-points of the line segment joining A(4, 10) and B(2, 2).

Using section formula for the mid-point of a line segment,

$$x = \frac{4+2}{2} \Rightarrow x = 3$$

and $$y = \frac{10+2}{2} \Rightarrow y = 6$$

Let Q(0, 2) be the given coordinates.

Now, the distance between P(3, 6) and Q(0, 2) is

$$PQ = \sqrt{(0-3)^2 + (2-6)^2} = \sqrt{9+16} = 5 \text{ units}$$

3. (c) $\boldsymbol{x = -3, y = 5}$

Let the vertices of the triangle be A(x, 0), B(5, –2), C(–8, y) and its centroid G(–2, 1).

Now, the coordinates of the centroid are

$$-2 = \frac{x+5-8}{3}$$

$\Rightarrow \quad -6 = x - 3$

$\Rightarrow \quad x = -3$

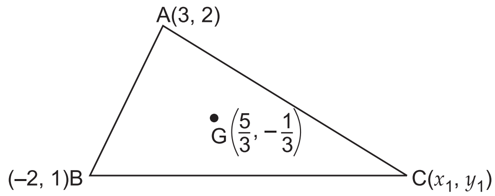
and $\quad 1 = \frac{0-2+y}{3}$

$\Rightarrow \quad 3 = -2 + y$

$\Rightarrow \quad y = 5$

4. (b) **(4, –4)**

Let A and B be the vertices (3, 2) and (–2, 1) respectively and $C(x_1, y_1)$ be the third vertex of the triangle.



Let G$\left(\frac{5}{3}, -\frac{1}{3}\right)$ be the centroid of ΔABC.

$\therefore \quad \frac{5}{3} = \frac{1}{3}(3 - 2 + x_1) = \frac{1 + x_1}{3}$

$\Rightarrow \quad x_1 = 5 - 1 = 4$

and $\quad -\frac{1}{3} = \frac{1}{3}(2 + 1 + y_1) = \left(\frac{3 + y_1}{3}\right)$

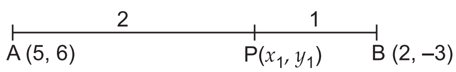
$\Rightarrow \quad y_1 + 3 = -1$

$\Rightarrow \quad y_1 = -4.$

Hence, the coordinates of the third vertex are (4, –4).

Positive direction of *x*-axis

5. (*a*) Let P(x_1, y_1) be the point on AB, which divides AB in the ratio 2 : 1.



$\therefore \quad$ AP : PB = 2 : 1

$\because$ By using section formula, we have

$$x_1 = \frac{2 \times 2 + 1 \times 5}{2 + 1} = \frac{9}{3} = 3$$

and $$y_1 = \frac{2 \times (-3) + 1 \times 6}{2 + 1} = \frac{0}{3} = 0$$

$\therefore$ The coordinates of P are (3, 0) which lies on the positive direction of the *x*-axis.

6. A (4, 8)

B(–6, 6)

Since point P lies on y –axis, hence its coordinates are (0, y)

We know

$$\frac{PA}{\sqrt{(-4)^2 + (y-8)^2}} = \frac{PB}{\sqrt{(6)^2 + (y-6)^2}}$$

On squaring both the sides, we get

$\Rightarrow 16 + y^2 + 64 - 16y = 36 + y^2 + 36 - 12y$

$\Rightarrow \quad -16y + 80 = -12y + 72$

$\Rightarrow \quad -4y = -8$

$\Rightarrow \quad y = 2$

$\therefore$ **P(0, 2)**

$$PA = \sqrt{(0-4)^2 + (2-8)^2}$$
$$= \sqrt{16 + 36} = \sqrt{52} = \mathbf{2\sqrt{13}}$$

7. A(2, –2)

B(–2, 1)

C(5, 2)

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(-4)^2 + (3)^2}$$
$$= \sqrt{16 + 9} = 5$$
$$BC = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(7)^2 + (1)^2} = \sqrt{49 + 1}$$
$$= \sqrt{50}$$
$$CA = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(3)^2 + (4)^2}$$
$$= \sqrt{9 + 16}$$
$$= \sqrt{25} = 5$$

Now we have

$$AB^2 + AC^2 = BC^2$$

Hence the given points are the vertices of a right-angled triangle

$$\text{Area} = \frac{1}{2}\left|\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right]\right|$$
$$= \frac{1}{2}\left|\left[2(-1) + (-2)(4) + 5(-3)\right]\right|$$
$$= \frac{1}{2}\left|\left[-2 - 8 - 15\right]\right|$$
$$= \frac{1}{2} \times |-25| = \mathbf{12.5 \text{ sq units}}$$

8. Let the given line divides the line segment joining the points (1, 3) and (2, 7) in the ratio k : 1.

Then, the coordinates of the point of intersection are $\left(\frac{k \times 2 + 1 \times 1}{k+1}, \frac{k \times 7 + 1 \times 3}{k+1}\right)$ i.e. $\left(\frac{2k+1}{k+1}, \frac{7k+3}{k+1}\right)$

The point of intersection lies on the given line

$$3x + y - 9 = 0$$

So, $\quad 3\left(\frac{2k+1}{k+1}\right) + \frac{7k+3}{k+1} - 9 = 0$

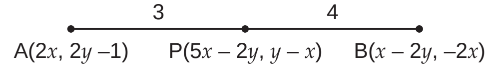
$\Rightarrow \quad 6k + 3 + 7k + 3 - 9k - 9 = 0$

$\Rightarrow \quad 4k - 3 = 0$

$\Rightarrow \quad k = \frac{3}{4}$

Hence, the line divides the line segment joining the points (1, 3) and (2, 7) in the ratio **3 : 4 internally**.

9. We have,



Now, the point P divides the line AB in the ratio 3 : 4. Thus,

$$5x - 2y = \frac{3(x - 2y) + 4(2x)}{3 + 4}$$

$\Rightarrow \quad 7(5x - 2y) = 3x - 6y + 8x$

$\Rightarrow \quad 35x - 14y = 11x - 6y$

$\Rightarrow \quad 24x - 8y = 0$

$\Rightarrow \quad 3x - y = 0 \qquad \ldots(1)$

and $\quad y - x = \dfrac{3(-2x) + 4(2y-1)}{3+4}$

$\Rightarrow \quad 7(y - x) = -6x + 8y - 4$

$\Rightarrow \quad 7y - 7x = -6x + 8y - 4$

$\Rightarrow \quad x + y = 4 \qquad \ldots(2)$

Adding (1) and (2), we get

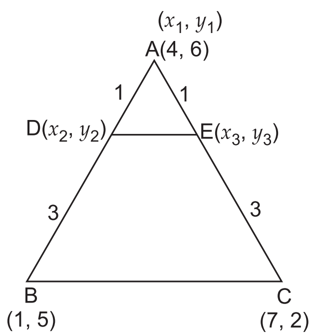
$$4x = 4 \Rightarrow x = \frac{4}{4} = 1$$

Putting $x = 1$ in equation (2), we get

$$1 + y = 4 \Rightarrow y = 4 - 1 = 3$$

Hence, $\quad$ **$x = 1$ and $y = 3$**.

10. Let the vertex A be the point (x_1, y_1) where $x_1 = 4$, $y_1 = 6$.



Now, given that D and E are two points on AB and AC respectively, such that

$$\frac{AB}{AD} = \frac{AC}{AD} = \frac{4}{1}$$

$$\Rightarrow \quad \frac{AD + DB}{AD} = \frac{AE + EC}{AE} = \frac{4}{1}$$

$$\Rightarrow \quad 1 + \frac{DB}{AD} = 1 + \frac{EC}{AE} = 4$$

$$\therefore \quad \frac{DB}{AD} = \frac{EC}{AE} = 4 - 1 = 3$$

$$\therefore \quad DB : AD = 3 : 1 = EC : AE$$

Let (x_2, y_2) and (x_3, y_3) be the coordinates of D and E respectively.

Then by using section formula, we have

$$x_2 = \frac{3 \times 4 + 1 \times 1}{3+1} = \frac{13}{4},$$

$$y_2 = \frac{3 \times 6 + 1 \times 5}{3+1} = \frac{23}{4}$$

$$x_3 = \frac{3 \times 4 + 1 \times 7}{3+1} = \frac{19}{4}$$

and $\quad y_3 = \dfrac{3 \times 6 + 1 \times 2}{3+1} = \dfrac{20}{4} = 5$

$\therefore \quad$ Area of ΔADE

$$= \frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|$$

$$= \frac{1}{2}\left|4 \times \left(\frac{23}{4} - 5\right) + \frac{13}{4}(5 - 6) + \frac{19}{4}\left(6 - \frac{23}{4}\right)\right|$$

$$= \frac{1}{2}\left|4 \times \frac{3}{4} - \frac{13}{4} + \frac{19}{4} \times \frac{1}{4}\right|$$

$$= \frac{1}{2}\left|3 - \frac{13}{4} + \frac{19}{6}\right|$$

$$= \frac{1}{2}\left|\frac{48 - 52 + 19}{16}\right| = \frac{52}{32}$$

Hence, the required area of ΔADE = $\mathbf{\dfrac{52}{32}}$ **sq units**.

Now, let (x_1', y_1'), (x_2', y_2') and (x_3', y_3') be the coordinates of A, B and C respectively, so that $x_1' = 4$, $y_1' = 6$, $x_2' = 1$, $y_2' = 5$ and $x_3' = 7$ and $y_3' = 2$

Hence, the area of ΔABC

$$= \frac{1}{2}\left|x_1'(y_2' - y_3') + x_2'(y_3' - y_1') + x_3'(y_1' - y_2')\right|$$

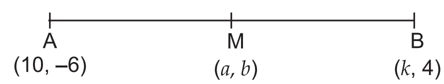
$$= \frac{1}{2}\left|4(5 - 2) + 1(2 - 6) + 7(6 - 5)\right|$$

$$= \frac{1}{2}\left|12 - 4 + 7\right| = \frac{15}{2}$$

$\therefore$ Ratio of area of ΔADE and the area of ΔABC is $\dfrac{15}{32} \div \dfrac{15}{2}$ = **1 : 16** which is the required ratio.

11. By using mid-point section formula, we have

$a = \dfrac{10 + k}{2}$ and $\underline{b} = \dfrac{-6 + 4}{2} = -1 \qquad \ldots(1)$



It is given that

$$A - 2b = 18 \qquad \ldots(2)$$

$\therefore \quad$ From (1) and (2), we have

$$\frac{10 + k}{2} + 2 = 18$$

$\Rightarrow \quad 10 + k = 2(18 - 2) = 32$

$\therefore \quad k = 32 - 10 = \mathbf{22}$

which is the required value of k.

We have A(10, –6) and B(22, 4).

$$\therefore \quad AB = \sqrt{(10 - 22)^2 + (-6 - 4)^2}$$

$$= \sqrt{144 + 100}$$

$$= \sqrt{244}$$

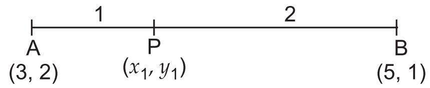
$$= \sqrt{4 \times 61}$$

$$= 2\sqrt{61}$$

Hence, the required distance of AB is $\mathbf{2\sqrt{61}}$ units.

12. Let P be the point (x_1, y_1) which divide AB is the ratio 1 : 2. Hence, by using section formula, we have

$$\left.\begin{aligned} x_1 &= \frac{1 \times 5 + 2 \times 3}{1 + 2} = \frac{11}{3} \\ \text{and} \qquad y_1 &= \frac{1 \times 1 + 2 \times 2}{1 + 2} = \frac{5}{3} \end{aligned}\right\} \quad \ldots(1)$$



Now, (x_1, y_1) lies on the given line $3x - 18y + k = 0$.

$\therefore \quad 3x_1 - 18y_1 + k = 0$

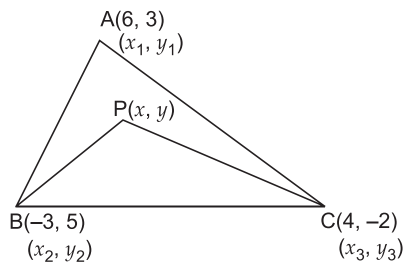
$\Rightarrow \quad 3 \times \frac{11}{3} - 18 \times \frac{5}{3} + k = 0 \qquad$ [From (1)]

$\Rightarrow \quad 11 - 30 + k = 0$

$\Rightarrow \quad k = 30 - 11 = \mathbf{19}$

which is the required value of k.

13. Let the coordinates of A, B and C be (x_1, y_1), (x_2, y_2) and (x_3, y_3) respectively, where $x_1 = 6$, $y_1 = 3$, $x_2 = 5$, $x_3 = 4$ and $y_3 = -2$.



$\therefore$ Area of ΔABC

$= \frac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|$

$= \frac{1}{2}\left|6(5 + 2) - 3(-2 - 3) + 4(3 - 5)\right|$

$= \frac{1}{2}\left|42 + 15 - 8\right| = \left|\frac{57 - 8}{2}\right| = \frac{49}{2} \qquad \ldots(1)$

Also, area of ΔPBC

$= \frac{1}{2}\left|x(y_2 - y_3) + x_2(y_3 - y) + x_3(y - y_2)\right|$

$= \frac{1}{2}\left|x(5 + 2) - 3(-2 - y) + 4(y - 5)\right|$

$= \frac{1}{2}\left|7x + 6 + 3y + 4y - 20\right|$

$= \frac{1}{2}\left|7x + 7y - 14\right|$

$= \frac{7}{2}\left|x + y - 2\right| \qquad \ldots(2)$

$\therefore$ From (1) and (2), we have

$$\frac{\text{ar}(\Delta\text{PBC})}{\text{ar}(\Delta\text{ABC})} = \frac{\frac{7}{2}|x + y - 2|}{\frac{49}{2}}$$

$$= \frac{7}{2} \times \frac{2}{49}|x + y - 2|$$

$$= \left|\frac{x + y - 2}{7}\right|$$

Hence, proved.