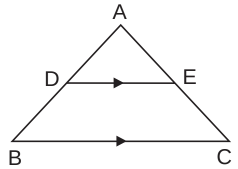
CHAPTER 6 Triangles

EXERCISE 6A

For Basic and Standard Levels

1. In ΔABC, $DE \parallel BC$ [Given]



(*i*) $\frac{AD}{DB} = \frac{AE}{EC}$ [By BPT]

$\Rightarrow \frac{2.4 \text{ cm}}{BD} = \frac{3.2 \text{ cm}}{4.8 \text{ cm}}$

$\Rightarrow BD = \frac{2.4 \times 4.8}{3.2}$ cm

$= \mathbf{3.6 \text{ cm}}$

(*ii*) $\frac{AD}{DB} = \frac{AE}{EC}$ [By BPT]

$\Rightarrow \frac{2 \text{ cm}}{2.5 \text{ cm}} = \frac{3.2 \text{ cm}}{EC}$

$\Rightarrow EC = \frac{3.2 \times 2.5}{2}$ cm

$= 4$ cm

$AC = AE + EC$

$= (3.2 + 4)$ m

$= \mathbf{7.2 \text{ cm}}$

(*iii*) $\frac{AD}{DB} = \frac{AE}{EC}$

$\Rightarrow \frac{AD}{DB} + 1 = \frac{AE}{EC} + 1$

$\Rightarrow \frac{3}{5} + 1 = \frac{AE + EC}{EC} = \frac{AC}{EC}$

$\Rightarrow \frac{8}{5} = \frac{4.8}{EC}$

$\Rightarrow 8EC = 4.8 \times 5 = 24$

$\therefore EC = \frac{24}{8} = 3$

$\therefore AE = AC - EC$

$= 4.8 - 3$

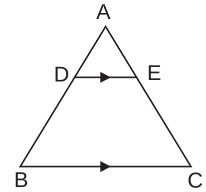
$= 1.8$

(*iv*) $\frac{AD}{DB} = \frac{AE}{EC}$ [By BPT]

$\Rightarrow \frac{3}{2} = \frac{4.8}{EC}$

$\Rightarrow EC = \frac{4.8}{3} \times 2 = \mathbf{3.2 \text{ cm}}$

2. In ΔABC, $DE \parallel BC$ [Given]



(*i*) $\frac{AD}{DB} = \frac{AE}{EC}$ [By BPT]

$\Rightarrow \frac{x-1}{5-x} = \frac{4-x}{x-2}$

$\Rightarrow (x-1)(x-2) = (4-x)(5-x)$

$\Rightarrow x^2 - 3x + 2 = 20 - 5x - 4x + x^2$

$\Rightarrow -3x + 2 = 20 - 9x$

$\Rightarrow 9x - 3x = 20 - 2$

$\Rightarrow 6x = 18$

$\Rightarrow \boldsymbol{x = 3}$

(*ii*) $\frac{AD}{DB} = \frac{AE}{EC}$ [By BPT]

$\Rightarrow \frac{4}{x-4} = \frac{8}{3x-19}$

$\Rightarrow 4(3x - 19) = 8(x - 4)$

$\Rightarrow 12x - 76 = 8x - 32$

$\Rightarrow 4x = 44$

$\Rightarrow \boldsymbol{x = 11}$

(*iii*) $\frac{AD}{DB} = \frac{AE}{EC}$ [By BPT]

$\Rightarrow \frac{x+2}{2x+3} = \frac{x-4}{2x-7}$

$\Rightarrow (x+2)(2x-7) = (x-4)(2x+3)$

$\Rightarrow 2x^2 + 4x - 7x - 14 = 2x^2 - 8x + 3x - 12$

$\Rightarrow -3x - 14 = -5x - 12$

$\Rightarrow 5x - 3x = 14 - 12$

$\Rightarrow 2x = 2$

$\Rightarrow \boldsymbol{x = 1}$

(*iv*) $\frac{AD}{DB} = \frac{AE}{EC}$ [By BPT]

$\Rightarrow \frac{3x-2}{7x-5} = \frac{5x-4}{5x-3}$

$\Rightarrow (3x-2)(5x-3) = (5x-4)(7x-5)$

$\Rightarrow 15x^2 - 10x - 9x + 6 = 35x^2 - 28x - 25x + 20$

$\Rightarrow 15x^2 - 19x + 6 = 35x^2 - 53x + 20$

$\Rightarrow 35x^2 - 15x^2 - 53x + 19x + 20 - 6 = 0$

$\Rightarrow 20x^2 - 34x + 14 = 0$

$\Rightarrow 10x^2 - 17x + 7 = 0$

$\Rightarrow 10x^2 - 10x - 7x + 7 = 0$

$\Rightarrow 10x(x-1) - 7(x-1) = 0$

$\Rightarrow (x-1)(10x-7) = 0$

$\Rightarrow$ Either $(x - 1) = 0$

$\Rightarrow \boldsymbol{x = 1}$

or $(10x - 7) = 0$

$\Rightarrow$ $x = \mathbf{\frac{7}{10}}$

3. (*i*) $\frac{AD}{DB} = \frac{4}{4.5} = \frac{8}{9}$ and $\frac{AE}{EC} = \frac{8}{9}$

$\therefore$ $\frac{AD}{DB} = \frac{AE}{EC}$

$\therefore$ **DE ∥ BC** [By the conv. of BPT]

(*ii*) $\frac{AD}{DB} = \frac{4.2}{12.6} = \frac{1}{3}$ and $\frac{AE}{EC} = \frac{5.1}{11.9} = \frac{3}{7}$

$\therefore$ $\frac{AD}{DB} \neq \frac{AE}{EC}$

$\therefore$ **DE is not parallel to BC**.

(*iii*) $\frac{AD}{DB} = \frac{AD}{AB - AD} = \frac{3.5}{17.5 - 3.5} = \frac{3.5}{14} = \frac{1}{4}$

and $\frac{AE}{EC} = \frac{AE}{AC - AE} = \frac{4.2}{21 - 4.2} = \frac{4.2}{16.8} = \frac{1}{4}$

$\therefore$ $\frac{AD}{DB} = \frac{AE}{EC}$

$\therefore$ **DE is parallel to BC**.

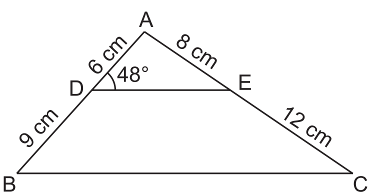
(*iv*) $\frac{AD}{DB} = \frac{5.6}{9.6} = \frac{7}{12}$

and $\frac{AE}{EC} = \frac{AC - EC}{EC} = \frac{10.8 - 4.5}{4.5} = \frac{6.3}{4.5} = \frac{7}{5}$

$\therefore$ $\frac{AD}{DB} \neq \frac{AE}{EC}$

$\therefore$ **DE is not parallel to BC**.

4. Given that in ΔABC, D and E are points on AB and AC respectively such that AD = 6 cm, DB = 9 cm, AE = 8 cm and EC = 12 cm. To find ∠ABC, if ∠ADE = 48°.



In ΔABC, we see that $\frac{AD}{DB} = \frac{6}{9} = \frac{2}{3}$

and $\frac{AE}{EC} = \frac{8}{12} = \frac{2}{3}$.

$\therefore$ $\frac{AD}{DB} = \frac{AE}{EC}$

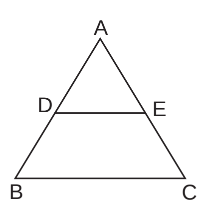
$\therefore$ DE ∥ BC [By the converse of basic proportionally theorem]

Now, ADB is a transversal to parallel lines DE and BC.

$\therefore$ ∠ABC = alternate ∠ADE = 48°

Hence, the required ∠ABC = 48°

5.



AD × EC = AE × DB

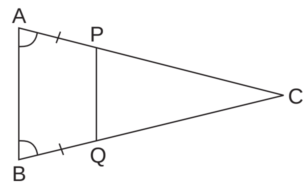
$\Rightarrow$ $\frac{AD}{DB} = \frac{AE}{EC}$

Thus, in ΔABC, DE divides the sides AB and AC in the same ratio.

By the converse of BPT, we have

DE ∥ BE

6.



∠A = ∠B [Given]

$\therefore$ CA = CB … (1)

[Sides opp. equal ∠s of a Δ]

AP = BQ [Given] … (2)

Subtracting equation (2) from equation (1), we get

CA − AP = CB − BQ

$\Rightarrow$ CP = CQ … (3)

Dividing sides of equation (3) and equation (2), we get

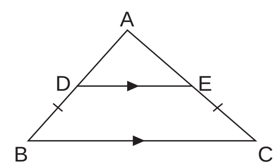
$\frac{CP}{AP} = \frac{CQ}{BQ}$

Thus, in ΔCAB, PQ divides the sides CA and CB in the same ratio.

$\therefore$ By the converse of BPT, we have

PQ ∥ AB

7. In ΔABC, DE ∥ BC [Given]



$\therefore$ By Thale's theorem, we have

$\frac{AD}{BD} = \frac{AE}{EC}$ … (1)

But BD = CE (Given) … (2)

$\therefore$ AD = AE [Using (1) and (2)] … (3)

Adding the corresponding sides of equation (3) and equation (2), we get

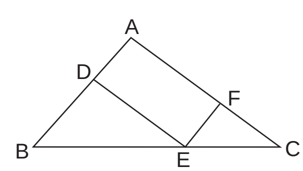
AD + BD = AE + CE

$\Rightarrow$ AB = AC

Hence, **ΔABC is an isosceles triangle**.

8. In ΔABC, EF ∥ BA

[∵ EF ∥ DA, opp. sides of a ∥gm]



$\therefore$ $\frac{CF}{FA} = \frac{CE}{EB}$ [By BPT] … (1)

In ΔABC, ED ∥ CA

[∵ ED ∥ FA, opp. sides of a ∥gm]

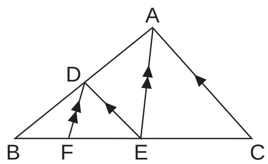
$\therefore \qquad \frac{BD}{DA} = \frac{BE}{EC}$

$\Rightarrow \qquad \frac{AD}{BD} = \frac{CE}{EB}$ [Taking reciprocals] … (2)

From (1) and (2), we get

$$\mathbf{\frac{CF}{FA} = \frac{AD}{BD}} \quad \left[\text{Each is equal to } \frac{CE}{EB}\right]$$

9. In ΔABC, DE ∥ AC



$\therefore \qquad \frac{BD}{DA} = \frac{BE}{EC}$ [By BPT] … (1)

In ΔABE, DF ∥ AE

$\therefore \qquad \frac{BD}{DA} = \frac{BF}{FE}$ [By BPT] … (2)

From (1) and (2), we get

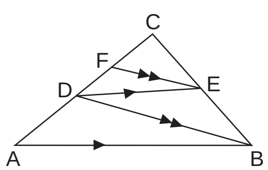
$$\frac{BE}{EC} = \frac{BF}{FE} \quad \left[\text{Each is equal to } \frac{BD}{DA}\right]$$

$\Rightarrow \qquad \frac{BF + FE}{EC} = \frac{BF}{FE}$

$\Rightarrow \qquad \frac{(4 + 5)\text{ cm}}{EC} = \frac{4\text{ cm}}{5\text{ cm}}$

$\Rightarrow \qquad EC = \frac{9 \times 5}{4}\text{ cm} = \mathbf{11.25\ cm}$

10. In ΔCAB, DE ∥ AB



$\therefore \qquad \frac{CD}{DA} = \frac{CE}{EB}$ [By BPT] … (1)

In ΔCDB, FE ∥ DB

$\therefore \qquad \frac{CF}{FD} = \frac{CE}{EB}$ [By BPT] … (2)

From (1) and (2), we get

$$\frac{CD}{DA} = \frac{CF}{FD}$$

$\Rightarrow \qquad \frac{DA}{DC} = \frac{FD}{CF}$ [Taking reciprocals]

$\Rightarrow \qquad \frac{DA}{DC} + 1 = \frac{FD}{CF} + 1$

$\Rightarrow \qquad \frac{DA + DC}{DC} = \frac{FD + CF}{CF}$

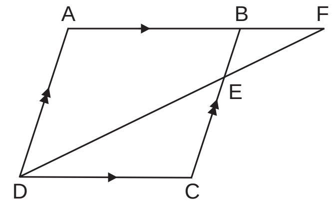
$\Rightarrow \qquad \frac{AC}{DC} = \frac{DC}{CF}$

$\Rightarrow \qquad \mathbf{DC^2 = CF \times AC}$

11. (*i*) In ΔFAD,

EB ∥ DA

[∵ CB ∥ DA, Opp. sides of a ∥gm]



$\therefore \qquad \frac{FE}{ED} = \frac{FB}{BA}$ [By BPT] … (1)

$\Rightarrow \qquad \frac{DE}{EF} = \frac{AB}{BF}$ [Taking reciprocals]

$\Rightarrow \qquad \mathbf{\frac{DE}{EF} = \frac{DC}{BF}}$

[∵ AB = DC, Opp. sides of a ∥gm]

(*ii*) $\qquad \frac{FE}{ED} = \frac{FB}{BA}$ [From (1)]

$\Rightarrow \qquad \frac{FE}{ED} + 1 = \frac{FB}{BA} + 1$

$\Rightarrow \qquad \frac{FE + ED}{ED} = \frac{FB + BA}{BA}$

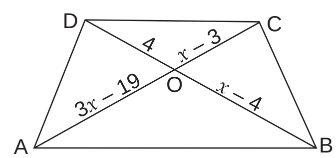
$\Rightarrow \qquad \frac{DF}{DE} = \frac{AF}{AB}$

$\Rightarrow \qquad \mathbf{\frac{DF}{DE} = \frac{AF}{DC}}$

[∵ AB = DC, Opp. sides of a ∥gm]

12. ABCD is a quadrilateral in which AB ∥ DC.

∴ ABCD is a trapezium.



Since the diagonals of a trapezium divide each other proportionally,

$\therefore \qquad \frac{AO}{OC} = \frac{BO}{OD}$

$\Rightarrow \qquad \frac{3x - 19}{x - 3} = \frac{x - 4}{4}$

$\Rightarrow \qquad 12x - 76 = x^2 - 3x - 4x + 12$

$\Rightarrow \qquad 12x - 76 = x^2 - 7x + 12$

$\Rightarrow \qquad x^2 - 19x + 88 = 0$

$\Rightarrow \qquad (x - 11)(x - 8) = 0$

$\Rightarrow \qquad$ Either $(x - 11) = 0$

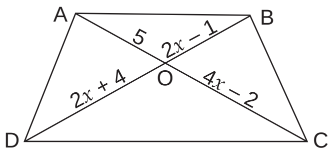
$\Rightarrow \qquad \mathbf{x = 11}$

or $\qquad (x - 8) = 0$

$\Rightarrow \qquad \mathbf{x = 8}$

13. ABCD is a quadrilateral in which

AB ∥ DC.



∴ ABCD is a trapezium.

Since the diagonals of a trapezium divide each other proportionally,

$$\therefore \quad \frac{AO}{OC} = \frac{BO}{OD}$$

$$\Rightarrow \quad \frac{5}{4x-2} = \frac{2x-1}{2x+4}$$

$$\Rightarrow \quad 10x + 20 = 8x^2 - 4x - 4x + 2$$

$$\Rightarrow \quad 8x^2 - 18x - 18 = 0$$

$$\Rightarrow \quad 4x^2 - 9x - 9 = 0$$

$$\Rightarrow \quad (4x + 3)(x - 3) = 0$$

$$\Rightarrow \quad \text{Either } (4x + 3) = 0$$

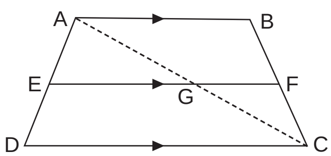
$$\Rightarrow \quad x = \frac{-3}{4}$$

(Rejected, as side cannot be negative)

or $(x - 3) = 0$

$\Rightarrow$ $\boldsymbol{x = 3}$

14. Join AC and let it intersect EF at G.



In ΔADC, EG ∥ DC [∴ EF ∥ DC]

$$\therefore \quad \frac{AE}{ED} = \frac{AG}{GC} \quad \text{[By BPT] ... (1)}$$

In ΔCBA, FG ∥ BA [∴ EF ∥ AB]

$$\therefore \quad \frac{CF}{FB} = \frac{CG}{GA} \quad \text{[By BPT]}$$

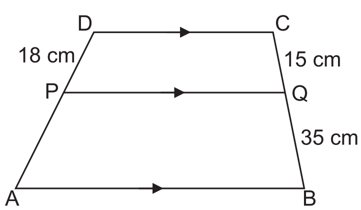
$$\Rightarrow \quad \frac{BF}{FC} = \frac{AG}{GC}$$

[Taking reciprocals] … (2)

From (1) and (2), we have

$$\boldsymbol{\frac{AE}{ED} = \frac{BF}{FC}} \quad \left[\text{Each is equal to } \frac{AG}{GC}\right]$$

15. Given that ABCD is a trapezium in which AB ∥ DC. P and Q are two points on non-parallel sides AD and BC respectively of the trapezium ABCD such that PQ ∥ DC ∥ AB.



Also, given that CQ = 15 cm, QB = 35 cm and PD = 18 cm.

To find AD.

∵ In the trapezium PQ ∥ DC ∥ AB.

$$\therefore \quad \frac{AP}{PD} = \frac{QB}{QC}$$

$$\Rightarrow \quad \frac{AP}{18} = \frac{35}{15} = \frac{7}{3}$$

$$= 3AP$$

$$= 7 \times 18$$

$$\Rightarrow \quad AP = 42$$

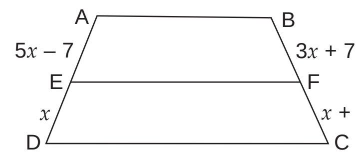
$$\therefore \quad AD = AP + PD$$

$$= 42 + 18$$

$$= 60$$

Hence, the required length of AD is 60 cm.

16.



$$\frac{AE}{ED} = \frac{BF}{FC} \quad \text{[Proved in Q14]}$$

$$\Rightarrow \quad \frac{5x-7}{x} = \frac{3x+7}{x+3}$$

$$\Rightarrow \quad 5x^2 + 15x - 7x - 21 = 3x^2 + 7x$$

$$\Rightarrow \quad 2x^2 + x - 21 = 0$$

$$\Rightarrow \quad (2x + 7)(x - 3) = 0$$

$$\Rightarrow \quad \text{Either } (2x + 7) = 0,$$

$$x = -\frac{7}{2}$$

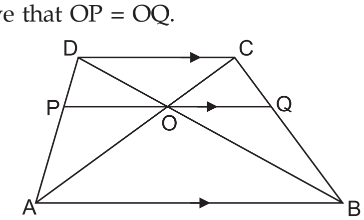
(rejected, as length of line segment cannot be negative)

or $(x - 3) = 0$

$\Rightarrow$ $\boldsymbol{x = 3}$

17. Given that ABCD is a trapezium with AB ∥ DC and AC and BD are its two diagonals intersecting each other at O such the POQ ∥ DC ∥ AB.

To prove that OP = OQ.



In ΔADC, OP ∥ DC.

∴ The triangles APO and ADC are similar.

$$\therefore \quad \frac{PO}{DC} = \frac{AP}{AD} \quad \text{...(1)}$$

Again, in ΔBCD, OQ ∥ DC

∴ ΔBQO ∽ ΔBCD

$$\Rightarrow \quad \frac{QO}{DC} = \frac{BQ}{BC} \quad \text{...(2)}$$

$$\text{But} \quad \frac{AP}{AD} = \frac{BQ}{BC} \quad [\because PQ \parallel DC]$$

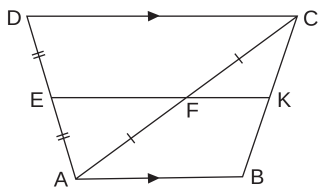
∴ From (1) and (2), we have

$$\frac{PO}{DC} = \frac{QO}{DC}$$

$$\Rightarrow \quad PO = OQ$$

18. In ΔADC, we have

$$AE = ED \quad [\because \text{E is the mid-point of AD}]$$



$$\Rightarrow \quad \frac{AE}{ED} = 1 \quad \dots (1)$$

and $$AF = FC \quad [\because \text{F is the mid-point of AC}]$$

$$\Rightarrow \quad \frac{AF}{FC} = 1 \quad \dots (2)$$

From (1) and (2), we get

$$\frac{AE}{ED} = \frac{AF}{FC}$$

Thus, in ΔADC, EF divides the sides AD and AC in the same ratio.

$\therefore$ By the converse of Basic Proportionality Theorem, we have

$$EF \parallel DC$$

$$\Rightarrow \quad FK \parallel DC \parallel AB$$

In ΔCAB, $$FK \parallel AB$$

$$\therefore \quad \frac{CF}{AF} = \frac{CK}{BK}$$

$$\Rightarrow \quad \frac{CF}{CF} = \frac{CK}{BK}$$

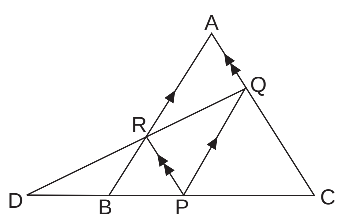
$[\because$ F is the mid-point of AC $\Rightarrow AF = CF]$

$$\Rightarrow \quad 1 = \frac{CK}{BK}$$

$$\Rightarrow \quad \mathbf{CK = BK}$$

For Standard Level

19. (*i*) In ΔDQC, $\quad PR \parallel CQ \quad [\because PR \parallel AC]$



$$\therefore \quad \frac{DP}{PC} = \frac{DR}{RQ} \quad \text{[By BPT]}$$

$$\Rightarrow \quad \frac{PC}{DP} = \frac{RQ}{DR} \quad \text{[Taking reciprocals]}$$

$$\Rightarrow \quad \frac{PC}{DP} + 1 = \frac{RQ}{DR} + 1$$

$$\Rightarrow \quad \frac{PC + DP}{DP} = \frac{RQ + DR}{DR}$$

$$\Rightarrow \quad \frac{CD}{DP} = \frac{DQ}{DR} \quad \dots (1)$$

In ΔDQC, $\quad BR \parallel PQ \quad [\because BA \parallel PQ]$

$$\therefore \quad \frac{BD}{BP} = \frac{DR}{RQ}$$

$$\Rightarrow \quad \frac{BP}{BD} = \frac{RQ}{DR} \quad \text{[Taking reciprocals]}$$

$$\Rightarrow \quad \frac{BP}{BD} + 1 = \frac{RQ}{DR} + 1$$

$$\Rightarrow \quad \frac{BP + BD}{BD} = \frac{RQ + DR}{DR}$$

$$\Rightarrow \quad \frac{DP}{BD} = \frac{DQ}{DR} \quad \dots (2)$$

From (1) and (2), we get

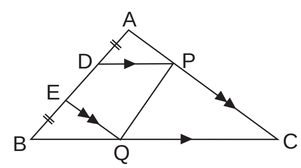
$$\frac{CD}{DP} = \frac{DP}{BD} \quad \left[\text{Each is equal to } \frac{DQ}{DR}\right]$$

$$\Rightarrow \quad \mathbf{DP^2 = BD \times CD}$$

(*ii*) $$(12\text{cm})^2 = BD \times CD$$

$$\Rightarrow \quad \mathbf{BD \times CD = 144\ cm^2}$$

20. In ΔABC, $\quad DP \parallel BC$



$$\therefore \quad \frac{AD}{DB} = \frac{AP}{PC} \quad \text{[By BPT]}$$

$$\Rightarrow \quad \frac{EB}{DE+EB} = \frac{AP}{PC}$$

$[\because AD = EB$, given$] \dots (1)$

In ΔABC, $\quad QE \parallel CA$

$$\therefore \quad \frac{BE}{EA} = \frac{BQ}{QC} \quad \text{[By BPT]}$$

$$\Rightarrow \quad \frac{EB}{DE + AD} = \frac{BQ}{QC}$$

$$\Rightarrow \quad \frac{EB}{DE + EB} = \frac{BQ}{QC}$$

$[\because AD = EB$, given$] \dots (2)$

From (1) and (2), we get

$$\frac{AP}{PC} = \frac{BQ}{QC}$$

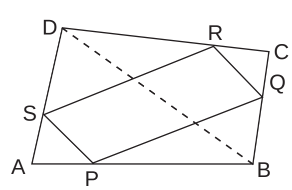
$$\Rightarrow \quad \frac{CP}{PA} = \frac{CQ}{QB} \quad \text{[Taking reciprocals]}$$

Thus, in ΔABC, PQ divides the sides CA and CB in the same ratio.

$\therefore$ By the converse of BPT, we have

$$\mathbf{PQ \parallel AB}$$

21. Join BD.



Since P, Q, R, S are points of bisection of AB, BC, CD and DA respectively,

$\therefore$ PB = 2 AP, QB = 2 CQ, RD = 2 CR and SD = 2 AS

In ΔADB, we have

$$\frac{AS}{SD} = \frac{AS}{2AS} = \frac{1}{2}$$

and $$\frac{AP}{PB} = \frac{AP}{2AP} = \frac{1}{2}$$

Thus, in ΔADB, PS divides the sides AD and AB in the same ratio.

$\therefore$ By the converse of BPT,

$$PS \parallel BD \quad \dots (1)$$

In ΔCDB, we have

$$\frac{CR}{RD} = \frac{CR}{2\,CR} = \frac{1}{2}$$

and $$\frac{CQ}{QB} = \frac{CQ}{2\,CQ} = \frac{1}{2}$$

Thus, in ΔCDB, QR divides the sides CB and CD in the same ratio.

$\therefore$ By the converse of BPT,

$$QR \parallel BD \quad \dots (2)$$

From equations (1) and (2), we have

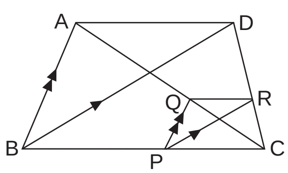
$$PS \parallel BD \text{ and } QR \parallel BD$$

$\Rightarrow$ $$PS \parallel QR$$

Similarly, by forming diagonal AC, we can prove that SR $\parallel$ PQ.

So, **PQRS is a parallelogram**.

22. In ΔABC, $\quad$ PQ $\parallel$ BA



$\therefore$ $$\frac{CP}{PB} = \frac{CQ}{QA} \quad \text{[By BPT]} \dots (1)$$

In ΔCBD, $\quad$ PR $\parallel$ BD

$\therefore$ $$\frac{CP}{PB} = \frac{CR}{RD} \quad \text{[By BPT]} \dots (2)$$

From (1) and (2), we have

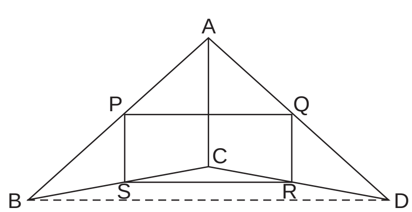
$$\frac{CQ}{QA} = \frac{CR}{RD} \quad \left[\text{Each is equal to } \frac{CP}{PB}\right]$$

Thus, in ΔCAD, QR divides the sides CA and CD in the same ratio.

$\therefore$ By the converse of BPT,

QR $\parallel$ AD

23. Join BD.



In ΔABD, we have

$$AP = PB$$

$\Rightarrow$ $$\frac{AP}{PB} = 1 \quad \dots (1)$$

and $$AQ = QD$$

$\Rightarrow$ $$\frac{AQ}{QD} = 1 \quad \dots (2)$$

From (1) and (2), we have

$$\frac{AP}{PB} = \frac{AQ}{QD}$$

Thus, in ΔABD, PQ divides the sides AB and AD in the same ratio.

$\therefore$ By the converse of BPT, PQ $\parallel$ BD $\quad \dots (3)$

In ΔCBD, we have

$$CS = SB$$

$\Rightarrow$ $$\frac{CS}{SB} = 1 \quad \dots (4)$$

and $$CR = RD$$

$\Rightarrow$ $$\frac{CR}{RD} = 1 \quad \dots (5)$$

From (4) and (5), we have

$$\frac{CS}{SB} = \frac{CR}{RD}$$

Thus, in ΔCBD, SR divides the sides CB and CD in the same ratio.

$\therefore$ By the converse of BPT, QR $\parallel$ BD $\quad \dots (6)$

From (3) and (6), we have

$$PQ \parallel QR$$

Similarly, by considering triangles DAC and BAC, we can prove that

$$QR \parallel AC \text{ and } PS \parallel AC$$

$\Rightarrow$ $$QR \parallel PS$$

So, PQRS is a quadrilateral in which the opposite sides are equal.

Hence, **PQRS is a parallelogram**.

EXERCISE 6B

For Basic and Standard Levels

1. (*i*) $$\frac{AB}{ED} = \frac{2.1\text{ cm}}{4.2\text{ cm}} = \frac{1}{2},$$

$$\frac{AC}{EF} = \frac{4\text{ cm}}{8\text{ cm}} = \frac{1}{2}$$

and $$\frac{BC}{DF} = \frac{5\text{ cm}}{10\text{ cm}} = \frac{1}{2}$$

Clearly, $$\frac{AB}{ED} = \frac{AC}{EF} = \frac{BC}{DF}$$

$\therefore$ **ΔABC ~ ΔEDF** $\quad$ [SSS similarity]

(*ii*) $\angle P = \angle Z = 40°$, $\angle R = \angle X = 95°$,

Remaining $\angle Q$ = remaining $\angle Y = 45°$

ΔPQR ~ ΔZYX $\quad$ [AAA similarity]

(*iii*) $$\frac{AB}{PQ} = \frac{3\text{ cm}}{4.5\text{ cm}} = \frac{2}{3}, \quad \frac{BC}{QR} = \frac{4\text{ cm}}{8\text{ cm}} = \frac{1}{2},$$

$\angle B = \angle Q = 60°$

Since $$\frac{AB}{PQ} \neq \frac{BC}{QR}$$

$\therefore$ ΔABC and ΔPQR **are not similar**.

(*iv*) $\frac{AC}{PQ} = \frac{3\text{ cm}}{7.5\text{ cm}} = \frac{2}{5}$, $\frac{CB}{QR} = \frac{5\text{ cm}}{12.5\text{ cm}} = \frac{2}{5}$,

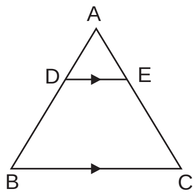
$\angle C = \angle Q = 30°$

Clearly, $\frac{AC}{PQ} = \frac{CB}{QR}$

and $\angle C = \angle Q$

$\therefore$ **$\Delta ACB \sim \Delta PQR$** [SAS similarity]

2. In ΔABC, $DE \parallel BC$



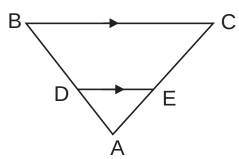
$\therefore$ $\frac{AD}{DB} = \frac{AE}{EC} = \frac{1}{2}$

[By BPT and $\frac{AE}{EC} = \frac{1}{2}$, given]

Let $AD = x$ cm,
then $DB = 2x$ cm
Also, $AB = BC = 9$ cm
(Sides of an equilateral Δ)

Now, $AD + DB = AB$
$\Rightarrow$ $(x + 2x) = 9$
$\Rightarrow$ $3x = 9$
$\Rightarrow$ $x = 3$
$DB = 2x$ cm
$= (2\times 3)$ cm
= 6 cm

3. $\therefore$ $\angle ADE = \angle ABC$
and $\angle AED = \angle ACB$ [corr. $\angle$s, $DE \parallel BC$]



$\therefore$ $\Delta ADE \sim \Delta ABC$ [By AA similarity]

$\therefore$ $\frac{AD}{AB} = \frac{DE}{BC}$

[Corresponding sides of similar triangles are proportional]

$\therefore$ $\frac{1.5\text{ cm}}{6\text{ cm}} = \frac{DE}{8\text{ cm}}$

$\Rightarrow$ $DE = \frac{1.5\times 8}{6}$ cm = 2 cm

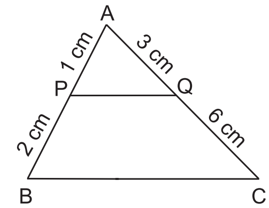
Hence, **DE = 2 cm**.

4. In ΔABC, we have

$\frac{AP}{PB} = \frac{1\text{ cm}}{2\text{ cm}} = \frac{1}{2}$

and $\frac{AQ}{QC} = \frac{3\text{ cm}}{6\text{ cm}} = \frac{1}{2}$

Thus, in ΔABC, PQ divides the sides AB and AC in the same ratio.



$\therefore$ By the converse of BPT, $PQ \parallel BC$.

In ΔAPQ and ΔABC,

$\angle APQ = \angle ABC$
and $\angle AQP = \angle ACB$ [corr. $\angle$s, $PQ \parallel BC$]
$\Delta APQ \sim \Delta ABC$ [By AA similarity]

$\therefore$ $\frac{AP}{AB} = \frac{AQ}{BC}$

[Corresponding sides of similar triangles are proportional]

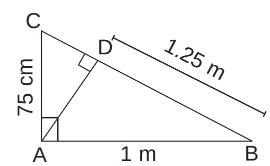
$\Rightarrow$ $\frac{AP}{AP+PB} = \frac{PQ}{BC}$

$\Rightarrow$ $\frac{1\text{ cm}}{(1+2)\text{ cm}} = \frac{PQ}{BC}$

$\Rightarrow$ $\frac{1}{3} = \frac{PQ}{BC}$

$\Rightarrow$ **BC = 3PQ**

5. In ΔABC and ΔDBA, we have
$\angle ABC = \angle DBA$ [$\angle B$ is common]
and $\angle CAB = \angle ADB$
[Each angle is equal ot 90°]



$\therefore$ $\Delta ABC \sim \Delta DBA$ [By AA similarity]

$\therefore$ $\frac{AB}{DB} = \frac{AC}{DA}$

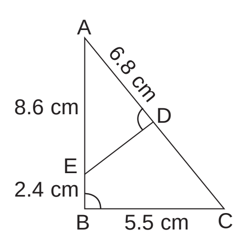
[Sides of similar triangles are proportional]

$\Rightarrow$ $\frac{1\text{ m}}{1.25\text{ m}} = \frac{0.75\text{ m}}{AD}$

$\Rightarrow$ $AD = \frac{0.75 \times 1.25}{1}$ m

= 0.9375 m
= 93.75 cm

6. In ΔADE and ΔABC, we have
$\angle ADE = \angle ABC$ [Given]
and $\angle DAE = \angle BAC$ [$\angle A$ is common]



$\therefore$ $\Delta ADE \sim \Delta ABC$ [By AA similarity]

$$\therefore \quad \frac{AD}{AB} = \frac{DE}{BC}$$

[Sides of similar triangles are proportional]

$$\Rightarrow \quad \frac{AD}{AE + EB} = \frac{DE}{BC}$$

$$\Rightarrow \quad \frac{6.8 \text{ cm}}{8.6 \text{ cm} + 2.4 \text{ cm}} = \frac{DE}{5.5 \text{ cm}}$$

$$\Rightarrow \quad \frac{6.8}{11} = \frac{DE}{5.5 \text{ cm}}$$

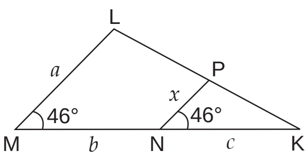
$$\Rightarrow \quad DE = \frac{6.80 \times 5.5}{11} \text{ cm}$$

$$\Rightarrow \quad DE = \mathbf{3.4 \text{ cm}}$$

7. In ΔKLM and ΔKPN, we have

∠KML = ∠KNP [Each is equal to 46°]

and ∠MKL = ∠NKP [LK is common]



$\therefore$ ΔKLM ~ ΔKPN [By AA similarity]

$$\therefore \quad \frac{LM}{PN} = \frac{KM}{KN}$$

[Sides of similar triangles are proportional]

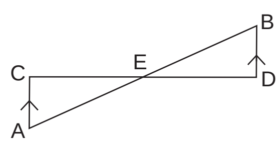
$$\Rightarrow \quad \frac{a}{x} = \frac{b + c}{c}$$

$$\Rightarrow \quad \boldsymbol{x = \frac{ac}{b + c}}$$

8. (*i*) In ΔACE and ΔBDE, we have

∠AEC = ∠BED [Vert. opp. ∠s]

and ∠ACE = ∠BDE [Alt ∠s, AC ∥ DB]



$\therefore$ ΔACE ~ ΔBDE [By AA similarity]

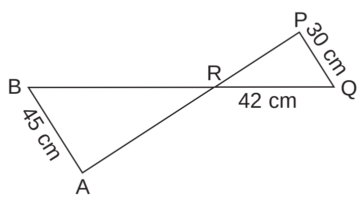
(*ii*) ΔACE ~ ΔBDE [Proved in (*i*)]

$$\therefore \quad \frac{AE}{BE} = \frac{CE}{DE}$$

[Sides of similar triangles are proportional]

$$\Rightarrow \quad \mathbf{\frac{AE}{CE} = \frac{BE}{DE}}$$

9. ΔABR ~ ΔPQR [Given]



$$\therefore \quad \frac{AB}{PQ} = \frac{BR}{QR} = \frac{AR}{PR}$$

[Corresponding sides of similar triangles are proportional]

$$\Rightarrow \quad \frac{45 \text{ cm}}{30 \text{ cm}} = \frac{BR}{42 \text{ cm}} = \frac{AR}{AP - AR}$$

$$\Rightarrow \quad \frac{45}{30} = \frac{BR}{42 \text{ cm}}$$

$$\Rightarrow \quad BR = \frac{45 \times 42}{30} \text{ cm}$$

$$\Rightarrow \quad BR = 63 \text{ cm}$$

and

$$\frac{45}{30} = \frac{AR}{72 \text{ cm} - AR}$$

$$\Rightarrow \quad 3(72 \text{ cm} - AR) = 2AR$$

$$\Rightarrow \quad 216 \text{ cm} - 3\,AR = 2AR$$

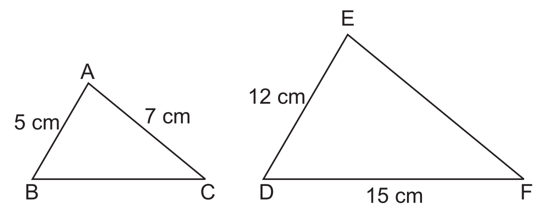
$$\Rightarrow \quad 216 \text{ cm} = 5AR$$

$$\Rightarrow \quad AR = \frac{216}{5} \text{ cm} = 43.2 \text{ cm}$$

$$PR = AP - AR$$
$$= (72 - 43.2) \text{ cm}$$
$$= 28.8 \text{ cm}$$

Hence, PR = **28.8 cm,** AR = **43.2 cm** and BR = **63 cm**.

10. Given that ΔABC ∽ ΔEDF, AB = 5 cm, AC = 7 cm, ED = 12 cm and DF = 15 cm.
To find the lengths of BC and EF.



$\because$ ΔABC ∽ ΔEDF

$$\frac{AB}{ED} = \frac{AC}{EF} = \frac{BC}{DF}$$

$$\Rightarrow \quad \frac{5}{12} = \frac{7}{EF} = \frac{BC}{15}$$

$$\therefore \quad \frac{7}{EF} = \frac{5}{12}$$

$$\Rightarrow \quad EF = \frac{7 \times 12}{5} = 16.8$$

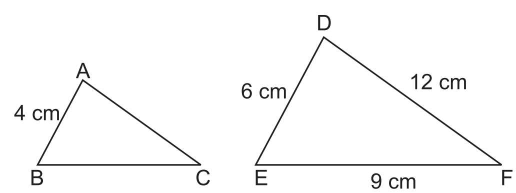
and

$$\frac{BC}{15} = \frac{5}{12}$$

$$\Rightarrow \quad BC = \frac{5 \times 15}{12} = \frac{25}{4} = 6.25$$

Hence, the required lengths of BC and EF are 6.25 cm and 16.8 cm respectively.

11. Given that ΔABC ∽ ΔDEF, AB = 4 cm, DE = 6 cm, EF = 9 cm and DF = 12 cm.
To find AB + BC + CA.



Since ΔABC ∽ ΔDEF,

$$\therefore \quad \frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF} \qquad \ldots(1)$$

Let AB + AC + BC = P

Now from (1), we have

$$\frac{AB}{DE} = \frac{AB+AC+BC}{DE+DF+EF}$$

$$= \frac{P}{6+12+9} = \frac{P}{27}$$

$$\Rightarrow \quad \frac{4}{6} = \frac{P}{27}$$

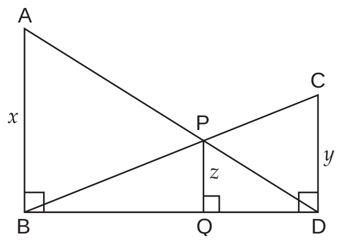
$$\Rightarrow \quad P = \frac{4}{6}\times 27 = 18$$

Hence, the required perimeter of ΔABC is **18 cm**.

12. In ΔABD and ΔPQD, we have

$\angle ABD = \angle PQD$ [Each is 90°]

and $\angle ADB = \angle PDQ$ [Common]



$\therefore$ $\Delta ABD \sim \Delta PQD$ [By AA similarity]

$$\therefore \quad \frac{AB}{PQ} = \frac{BD}{QD}$$

[Corresponding sides of similar triangles are proportional]

$$\Rightarrow \quad \frac{PQ}{AB} = \frac{QD}{BD} \quad \text{[Taking reciprocals]}$$

$$\Rightarrow \quad \frac{z}{x} = \frac{QD}{BD} \quad \dots (1)$$

In ΔCDB and ΔPQB, we have

$\angle CDB = \angle PQB$ [Each is 90°]

and $\angle CBD = \angle PBQ$ [Common]

$\therefore$ $\Delta CDB \sim \Delta PQB$ [By AA similarity]

$$\therefore \quad \frac{CD}{PQ} = \frac{DB}{QB}$$

[Corresponding sides of similar triangles are proportional]

$$\Rightarrow \quad \frac{PQ}{CD} = \frac{QB}{BD} \quad \text{[Taking reciprocals]}$$

$$\Rightarrow \quad \frac{z}{y} = \frac{QB}{BD} \quad \dots (2)$$

Adding equation (1) and equation (2), we get

$$\frac{z}{x} + \frac{z}{y} = \frac{QD+QB}{BD}$$

$$\Rightarrow \quad z\left(\frac{1}{x}+\frac{1}{y}\right) = \frac{BD}{BD}$$

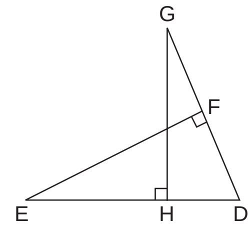
$$\Rightarrow \quad z\left(\frac{1}{x}+\frac{1}{y}\right) = 1$$

$$\Rightarrow \quad \boldsymbol{\frac{1}{x} + \frac{1}{y} = \frac{1}{z}}$$

13. In ΔGHD and ΔEFD, we have

$\angle GHD = \angle EFD$ [Each is 90°]

and $\angle GDH = \angle EDF$ [Common]



$\therefore$ $\Delta GHD \sim \Delta EFD$ [By AA similarity]

$$\therefore \quad \frac{HD}{FD} = \frac{DG}{DE}$$

[Corresponding sides of similar triangles are proportional]

$$\Rightarrow \quad \frac{8}{12} = \frac{3x-1}{4x+2}$$

$\Rightarrow$ $32x + 16 = 36x - 12$

$\Rightarrow$ $4x = 28$

$\Rightarrow$ $x = 7$

$DG = 3x - 1$

$= 3 \times 7 - 1$

= **20 units**

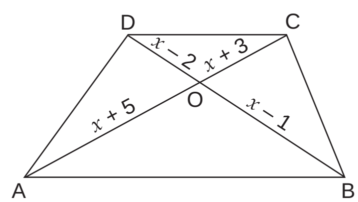
and $DE = 4x + 2$

$= 4 \times 7 + 2$

= **30 units**

14. In quadrilateral ABCD,

$AB \parallel DC$



$\therefore$ ABCD is a trapezium

Since the diagonals of a trapezium divide each other proportionally,

$$\therefore \quad \frac{AO}{OC} = \frac{BO}{OD}$$

$$\Rightarrow \quad \frac{x+5}{x+3} = \frac{x-1}{x-2}$$

$\Rightarrow$ $(x + 5)(x - 2) = (x - 1)(x + 3)$

$\Rightarrow$ $x^2 + 5x - 2x - 10 = x^2 - x + 3x - 3$

$\Rightarrow$ $3x - 10 = 2x - 3$

$\Rightarrow$ $\boldsymbol{x = 7}$

15. Let $\Delta ABC \sim \Delta DEF$, where the perimeter of ΔABC = 36 cm and perimeter of ΔDEF is 48 cm and AB = 9 cm.

We know that the ratio of the perimeters of two similar triangles is the same as the ratio of their corresponding sides.

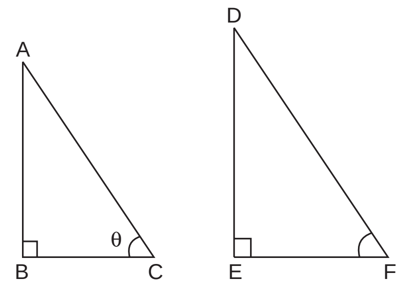
$$\therefore \quad \frac{\text{Perimeter of } \Delta ABC}{\text{Perimeter of } \Delta DEF} = \frac{AB}{DE}$$

$$\Rightarrow \quad \frac{36 \text{ cm}}{48 \text{ cm}} = \frac{9 \text{ cm}}{x}$$

$\Rightarrow \quad x = \dfrac{9 \times 48}{36}$ cm = 12 cm

Hence, the corresponding side of the other triangle is **12 cm**.

16. Let AB be the stick and BC its shadow.



Then, AB = 15 cm

and BC = 10 cm

Let the angular elevation of the Sun be θ.

Then, ∠ACB = θ

Let DE = x, be the vertical flag pole and EF be its shadow.

Then, EF = 60 cm

Angular elevation of the Sun (at the same time) = ∠DEF = θ

Now, in ΔABC and ΔDEF, we have

∠ABC = ∠DEF [Each is 90°]

and ∠ACB = ∠DEF = θ

∴ ΔABC ~ ΔDEF [By AA similarity]

∴ $\dfrac{AB}{DE} = \dfrac{BC}{EF}$

[Corresponding sides of similar triangles are proportional]

$\Rightarrow \quad \dfrac{15 \text{ cm}}{x} = \dfrac{10 \text{ cm}}{60 \text{ cm}}$

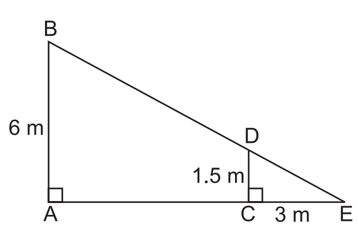
$\Rightarrow \quad x = \dfrac{15 \times 60}{10}$ cm = 90 cm

Hence, the height of the flag pole is **90 cm**.

17. Let AB be the straight vertical pole, B, the bulb on it, C, the position of the woman of height CD = 1.5 m and CE, the shadow of the woman on the horizontal ground.

Given that AB = 6 m, CE = 3 m and CD = 1.5 m.

To find the distance AC of the woman from the base A of the pole.



Since AE is horizontal and AB and CD are vertical,

∴ ∠BAE = ∠DCE = 90°

∴ AB ∥ DC

∴ ΔAEB ~ ΔCED

∴ We have $\dfrac{AB}{CD} = \dfrac{AE}{CE}$

$\Rightarrow \quad \dfrac{6}{1.5} = \dfrac{3 + AC}{3}$

$\Rightarrow \quad (3 + AC) \times 1.5 = 18$

$\Rightarrow \quad AC + 3 = \dfrac{18}{1.5} = \dfrac{180}{15} = 12$

$\Rightarrow \quad AC = 12 - 3 = 9$

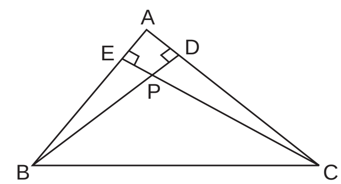
Hence, the required distance of the woman from the base of the pole is 9 cm.

18. In ΔBPE and ΔCPD, we have

∠BEP = ∠CDP [Each is 90°]

and ∠BPE = ΔCPD [Vert. opp ∠s]

ΔBPE ~ ΔCPD [By AA similarity]

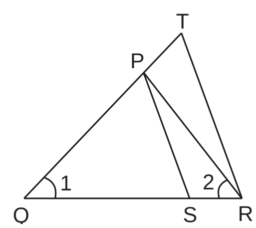


∴ $\dfrac{BP}{CP} = \dfrac{PE}{PD}$

[Corresponding sides of similar triangles are proportional]

⇒ **BP × PD = EP × PC**

19.



$\dfrac{QR}{QS} = \dfrac{QT}{PR}$ [Given]

$\Rightarrow \quad \dfrac{QS}{QR} = \dfrac{PR}{QT}$ [Taking reciprocals]

$\Rightarrow \quad \dfrac{PQ}{QT} = \dfrac{QS}{QR}$

[∴ PR = PQ, sides opposite equal ∠s of ΔPQR]

In ΔPQS and ΔTQR, we have

$\dfrac{PQ}{QT} = \dfrac{QS}{QR}$

and ∠PQS = ∠TQR = ∠θ

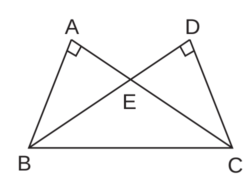
∴ **ΔPQS ~ ΔTQR**

20. In ΔAEB and ΔDEC, we have

∠BAE = ∠CDE [Each is equal to 90°]

∠AEB = ∠DEC [Vertically opp.∠s]

∴ ΔAEB ~ ΔDEC [By AA similarity]



$\therefore \qquad \dfrac{AE}{DE} = \dfrac{EB}{EC}$

[Corresponding sides of similar triangles are proportional]

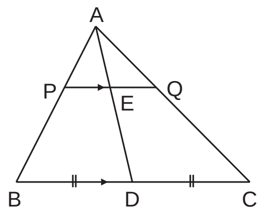
$\Rightarrow \qquad \mathbf{AE \cdot EC = BE \cdot ED}$

21. In ΔAPE and ΔABD, we have

$\angle APE = \angle ABD$ [Corr. $\angle$s]

and $\angle PAE = \angle BAD$ [Common]

$\therefore \quad \Delta APE \sim \Delta ABD$ [By AA similarity]



$\therefore \qquad \dfrac{PE}{BD} = \dfrac{AE}{AD}$

[Corresponding sides of similar triangles are proportional] … (1)

In ΔAQE and ΔACD, we have

$\angle AQE = \angle ACD$ [Corr.$\angle$s]

and $\angle QAE = \angle CAD$ [Common]

$\therefore \quad \Delta AQE \sim \Delta ACD$ [By AA similarity]

$\therefore \qquad \dfrac{EQ}{DC} = \dfrac{AE}{AD}$

[Corresponding sides of similar triangles are proportional] … (2)

From (1) and (2), we get

$\dfrac{PE}{BD} = \dfrac{EQ}{DC} \quad \left[\text{Each is equal to } \dfrac{AE}{AD}\right]$

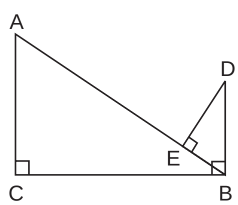
But BD = DC [$\because$ AD is the median]

$\therefore \quad PE = EQ$

Hence, the **median AD bisects PQ**.

22. $DB \perp CB$

$\Rightarrow \quad \angle DBC = 90°$



$\angle DBE = \angle DBC - \angle ABC$

$= 90° - \angle ABC$ … (1)

Also, $AC \perp CB$

$\Rightarrow \quad \angle ACB = 90°$

$\Rightarrow \quad \angle BAC = \angle ACB - \angle ABC$

[Sum of $\angle$s of a Δ is 180°]

$\Rightarrow \quad \angle BAC = 90° - \angle ABC$ … (2)

From (1) and (2), we get

$\angle DBE = \angle BAC$ … (3)

In ΔDEB and ΔBCA, we have

$\angle DBE = \angle BAC$ [From (3)]

$\angle DEB = \angle BCA$ [Each is equal to 90°]

$\therefore \quad \Delta DEB \sim \angle BCA$ [By AA similarity]

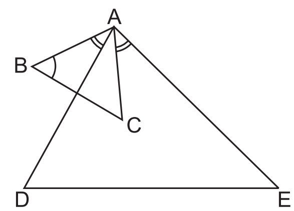
$\therefore \qquad \dfrac{DE}{BC} = \dfrac{EB}{CA}$

[Corresponding sides of similar triangles are proportional]

$\Rightarrow \qquad \dfrac{CA}{BC} = \dfrac{EB}{DE}$

Hence, $\dfrac{\mathbf{BE}}{\mathbf{DE}} = \dfrac{\mathbf{AC}}{\mathbf{BC}}$.

23. Given that ΔABC and ΔADE are two triangles with the common vertex A such that $\angle BAD = \angle CAE$ and $\angle ABC = \angle ADE$.



To prove that $\dfrac{AB}{AD} = \dfrac{AC}{AE}$

We have $\angle BAD = CAE$

$\Rightarrow \quad \angle BAD + \angle DAC = \angle CAE + \angle DAC$

$\Rightarrow \quad \angle BAC = \angle DAE$ …(1)

Also, $\angle ABC = \angle ADE$ [Given] …(2)

$\therefore$ In ΔABC and ΔADE, we have

$\angle BAC = \angle DAE$ [From (1)]

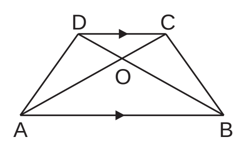
$\angle ABC = \angle ADE$ [From (2)]

Hence, $\Delta ABC \sim \Delta ADE$

[By AA similarity criterion]

$\therefore \qquad \dfrac{\mathbf{AB}}{\mathbf{AD}} = \dfrac{\mathbf{AC}}{\mathbf{AE}}$

24. Let ABCD be a trapezium in which AB ∥ DC and diagonal AC divides diagonal BD in the ratio 1:2.



Let $DO = x$,

then $OB = 2x$.

In ΔDOC and ΔBOA, we have

$\angle DOC = \angle BOA$ [Vert.opp.$\angle$s]

$\angle CDO = \angle ABO$ [Alt $\angle$s, AB ∥ DC]

$\therefore \quad \Delta DOC \sim \Delta BOA$ [By AA similarity]

$\therefore \qquad \dfrac{DO}{BO} = \dfrac{DC}{BA}$

[Corresponding sides of similar triangles are proportional]

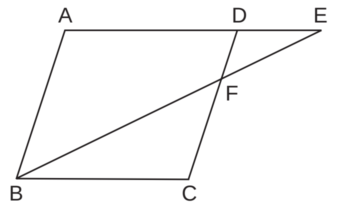
$\Rightarrow \qquad \dfrac{x}{2x} = \dfrac{DC}{BA}$

$\Rightarrow \qquad \dfrac{1}{2} = \dfrac{DC}{BA}$

$\Rightarrow \qquad BA = 2DC$

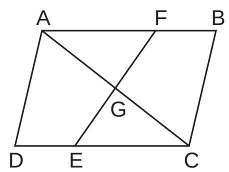
Hence, **one of the parallel sides of the trapezium is double the other**.

25. In ΔABE and ΔCFB, we have



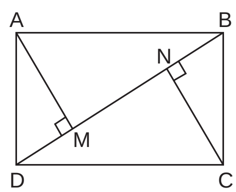
$$\angle AEB = \angle CBF \quad \text{[Alt. } \angle\text{s]}$$
$$\angle EAB = \angle BCF \quad \text{[Opp. } \angle\text{s of a } \| \text{gm]}$$
$\therefore$ $$\mathbf{\Delta ABE \sim \Delta CFB} \quad \text{[By AA similarity]}$$

26. In ΔAGF and ΔCGE, we have



$$\angle FAG = \angle ECG \quad [\because \angle FAC = \angle DCA, \text{ alt. } \angle s]$$
$$\angle AGF = \angle CGE \quad \text{[Vert. opp. } \angle\text{s]}$$
$\therefore$ $$\Delta AGF \sim \Delta CGE \quad \text{[By AA similarity]}$$
$\therefore$ $$\frac{AG}{CG} = \frac{GF}{GE}$$
$\Rightarrow$ $$\mathbf{AG \times EG = FG \times CG}$$

27. In ΔAMB and ΔCND, we have



$$\angle AMB = \angle CND \text{ [Each is equal to } 90°]$$
$$\angle ABM = \angle CDN \quad \text{[Alt. } \angle\text{s, } AB \parallel CD]$$
$\therefore$ $$\Delta AMB \sim \Delta CND \quad \text{[By AA similarity]}$$
$\Rightarrow$ $$\frac{BM}{DN} = \frac{AB}{CD}$$

[Corresponding sides of similar triangles are proportional]

$\Rightarrow$ $$\frac{BM}{DN} = \frac{CD}{CD} = 1$$

[$\because$ AB = CD, opp. sides of a rectangle]

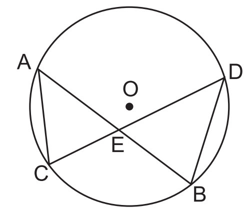
$\Rightarrow$ $$\frac{BM}{DN} = 1$$
$\Rightarrow$ $$BM = DN \quad \dots (1)$$
$\Rightarrow$ $$BM - MN = DN - MN$$
$\Rightarrow$ $$BN = DM \quad \dots (2)$$

Squaring (1) and (2) and adding, we get

$$\mathbf{BM^2 + BN^2 = DN^2 + DM^2}$$

28. Given that AB and CD are two chords of a circle with centre at O and E is point of intersection of these two chords.

To prove that $\Delta EAC \backsim \Delta EDB$ and $EA \times EB = EC \times ED$



We have $$\angle CAE = \angle BDE \quad \dots(1)$$

[$\because$ These two angles stand on the same are CB of the circle]

Also, $$\angle CEA = \text{vertically opposite } \angle BED \quad \dots(2)$$

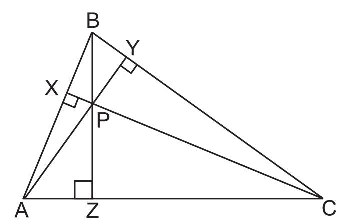
$\therefore$ In ΔEAC and ΔEDB, we have

$$\angle CAE = \angle BDE \quad \text{[From (1)]}$$
$$\angle CEA = \angle BED \quad \text{[From (2)]}$$
$\therefore$ $$\Delta EAC \backsim \Delta EDB \quad \text{[By AA similarity]}$$
$\therefore$ $$\frac{EA}{ED} = \frac{EC}{EB} = \frac{AC}{DB}$$
$\therefore$ $$\mathbf{EA \times EB = EC \times ED}$$

[From the above 1st two ratios]

29. Given that BZ, AY and CX are the altitudes of a ΔABC from the vertices B, A and C respectively to their opposite sides.

These three altitudes meet each other at a point P.



To prove that $\frac{AX}{XB} \cdot \frac{BY}{YC} \cdot \frac{CZ}{ZA} = 1.$

In ΔAXP and ΔCYP, we have

$$\angle AXP = \angle CYP = 90°$$
$$\angle APX = \text{vertically opposite } \angle CPY$$

$\therefore$ By AA similarity criterion, we have

$$\Delta AXP \backsim \Delta CYP \quad \dots(1)$$

Similarity, it can be shown that

$$\Delta BYP \backsim \Delta AZP \quad \dots(2)$$

and $$\Delta CZP \backsim \Delta BXP \quad \dots(3)$$

$\therefore$ From (1), we have

$$\frac{AX}{CY} = \frac{AP}{CP} = \frac{XP}{YP} \quad \dots(4)$$

From (2), we have

$$\frac{BY}{AZ} = \frac{BP}{AP} = \frac{YP}{ZP} \quad \dots(5)$$

and from (3), we have

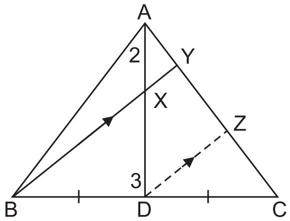
$$\frac{CZ}{BX} = \frac{CP}{BP} = \frac{ZP}{XP} \quad \dots(6)$$

$\therefore$ From (4), (5) and (6), we have

$$\frac{AX}{CY} \times \frac{BY}{AZ} \times \frac{CZ}{BX} = \frac{XP}{YP} \times \frac{YP}{ZP} \times \frac{ZP}{XP} = 1$$
$\Rightarrow$ $$\frac{AX}{XB} \cdot \frac{BY}{YC} \cdot \frac{CZ}{ZA} = 1$$

30. Given that ABC is a triangle and AD is a median of this triangle, D being the middle point of BC. X is a point on AD such that AX : XD = 2 : 3.

BX is produced to intersect AC in Y.
To prove that BX = 4XY.



Construction: We draw DZ ∥ BY to intersect AC at Z.
Since DZ ∥ BY.

$\therefore \quad \Delta CDZ \sim \Delta CBY$

$\therefore \quad \frac{CD}{CB} = \frac{DZ}{BY}$

$\Rightarrow \quad \frac{1}{2} = \frac{DZ}{BX + XY}$

$\Rightarrow \quad DZ = \frac{1}{2}(BX + XY) \qquad \ldots(1)$

Also, $\quad \Delta AXY \sim \Delta ADZ$

$\therefore \quad \frac{AX}{AD} = \frac{XY}{DZ} \qquad \ldots(2)$

Finally, it is given that

$$\frac{AX}{XD} = \frac{2}{3}$$

$\Rightarrow \quad \frac{AX}{2} = \frac{XD}{3} = k\text{(say)}$

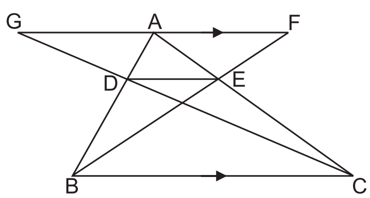
$\therefore$ AX = $2k$, XD = $3k$ and AD = AX + XD = $5k$ $\qquad \ldots(3)$

From (2) and (3), we get

$$\frac{5XY}{2} = \frac{1}{2}(BX + XY)$$

$\Rightarrow \quad 4XY = BX$

31. Given that D and E are two points on the sides AB and AC respectively if ΔABC such that AD = $\frac{1}{3}$ AB and AE = $\frac{1}{3}$ AC.



GF is a line through A parallel to BC.
BE produced and CD produced intersect the line through A parallel to BC at the points F and G respectively.

To prove that $\quad$ GF = BC

Since $\quad$ GAF ∥ BC

$\therefore \quad$ ∠AGD = alternate ∠BCD

$\quad$ ∠ADG = vertically opposite ∠BDC

$\therefore$ By AA similarity criterion,

$\quad \Delta GDA \sim \Delta CDB$

$\therefore \quad \frac{DA}{DB} = \frac{GA}{CB} \qquad \ldots(1)$

Similarly ΔFEA ∽ ΔBEC

$\therefore \quad \frac{AE}{CE} = \frac{AF}{CB} \qquad \ldots(2)$

Adding (1) and (2), we get

$$\frac{DA}{DB} + \frac{AE}{CE} = \frac{GA}{CB} + \frac{AF}{CB}$$

$$= \frac{GA + AF}{CB} = \frac{GF}{CB}$$

$\Rightarrow \quad \frac{\frac{1}{3}AB}{DB} + \frac{\frac{1}{3}AC}{CE} = \frac{GF}{CB}$

[∵ Given that DA = $\frac{1}{3}$ AB and AE = $\frac{1}{3}$ AC]

$\Rightarrow \quad \frac{\frac{1}{3}AB}{\frac{2}{3}AB} + \frac{\frac{1}{3}AC}{\frac{2}{3}AC} = \frac{GF}{CB}$

[∵ DB = AB – AD = $AB - \frac{1}{3}AB = \frac{2}{3}$ AB and

CE = AC – AE = $AC - \frac{1}{3}AC = \frac{2}{3}$ AC]

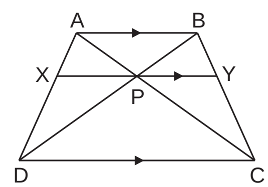
$\Rightarrow \quad \frac{1}{2} + \frac{1}{2} = \frac{GF}{CB}$

$\Rightarrow \quad 1 = \frac{GF}{CB}$

$\Rightarrow \quad$ GF = BC

For Standard Level

32. Since the diagonals of a trapezium divide each other proportionally,



$\therefore \quad \frac{AP}{PC} = \frac{BP}{PD}$

$\Rightarrow \quad \frac{PC}{AP} = \frac{PD}{BP}$ $\qquad$ [Taking reciprocals]

$\Rightarrow \quad \frac{PC}{AP} + 1 = \frac{PD}{BP} + 1$

$\Rightarrow \quad \frac{PC + AP}{AP} = \frac{PD + BP}{BP}$

$\Rightarrow \quad \frac{AC}{AP} = \frac{BD}{BP}$

$\Rightarrow \quad \frac{AP}{AC} = \frac{BP}{BD}$

[Taking reciprocals] … (1)

In ΔAXP and ΔADC, we have

∠XAP = ∠DAC $\qquad$ [Common]

∠AXP = ∠ADC

[Corresponding ∠s, AB ∥ DC]

$\therefore \quad \Delta AXP \sim \Delta ADC$ $\qquad$ [By AA similarity]

$\therefore \quad \frac{XP}{DC} = \frac{AP}{AC}$

[Corresponding sides of similar triangles are proportional] ... (2)

In ΔBYP and ΔBCD, we have

$\angle YBP = \angle CBD$ [Common]

$\angle BYP = \angle BCD$ [Corr.∠s, AB ∥ DC]

$\therefore \quad \Delta BYP \sim \Delta BCD$ [By AA similarity]

$\therefore \quad \frac{YP}{CD} = \frac{BP}{BD}$

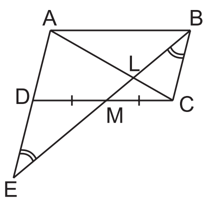
[Corresponding sides of similar triangles are proportional] ... (3)

From (1), (2) and (3), we get

$$\frac{XP}{DC} = \frac{YP}{CD}$$

$\Rightarrow \quad$ **XP = YP**

33. In ΔBMC and EMD,



(*i*) $CM = DM$

[∴ M is the mid-point of DC]

(*ii*) $\angle BMC = \angle EMD$

[Vertically opposite angles]

(*iii*) $\angle CBM = \angle DEM$ [Alt. ∠s, BC ∥ ADE]

$\therefore \quad \Delta BMC \cong \Delta EMD$

[By AAS criterion of congruence]

$\therefore \quad BC = ED$ [By CPCT] ... (1)

Also, $\quad BC = AD$

[Opp. sides of a ∥gm] ... (2)

$\Rightarrow \quad 2BC = ED + AD$

[Adding (1) and (2)]

$\Rightarrow \quad 2BC = AE$ ... (3)

Now, in ΔAEL and ΔCBL, we have

$\angle ALE = \angle CLB$

[Vertically opposite angles]

$\angle EAL = \angle BCL$ [Alt ∠s, BC ∥ ADE]

$\therefore \quad \Delta AEL \sim \Delta CBL$ [By AA similarity]

$\Rightarrow \quad \frac{EL}{BL} = \frac{AE}{CB}$

[Corresponding sides of similar triangles are proportional]

$\Rightarrow \quad \frac{EL}{BL} = \frac{2BC}{BC}$ [Using (3)]

$\Rightarrow \quad$ **EL = 2BL**

34. In ΔBAM and ΔQPM, we have

$\angle BAM = \angle QPM$

[Corresponding angles, AB ∥ PQ]

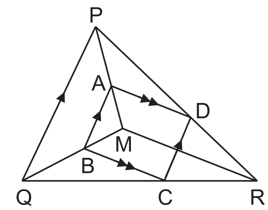
$\angle AMB = \angle PMQ$ [Common]

$\therefore \quad \Delta BAM \sim \Delta QPM$ [By AA similarity]

$\Rightarrow \quad \frac{BA}{QP} = \frac{BM}{QM}$

$\Rightarrow \quad \frac{CD}{QP} = \frac{BM}{QM}$

[∵ BA = CD, Opp. sides of a ∥gm] ... (1)



In ΔCDR and ΔQPR, we have

$\angle CDR = \angle QPR$

[Corresponding angles, DC ∥ AB ∥ PQ]

$\angle DRC = \angle PRQ$ [Common]

$\therefore \quad \Delta CDR \sim \Delta QPR$ [By AA similarity]

$\Rightarrow \quad \frac{CD}{QP} = \frac{CR}{QR}$

$\Rightarrow \quad \frac{BM}{QM} = \frac{CR}{QR}$ [Using (1)]

$\Rightarrow \quad \frac{QM}{BM} = \frac{QR}{CR}$ [Taking reciprocals]

$\Rightarrow \quad \frac{QM}{BM} - 1 = \frac{QR}{CR} - 1$

$\Rightarrow \quad \frac{QM - BM}{BM} = \frac{QR - CR}{CR}$

$\Rightarrow \quad \frac{QB}{BM} = \frac{QC}{CR}$

Thus, in ΔQMR, BC divides the sides QM and QR in the same ratio.

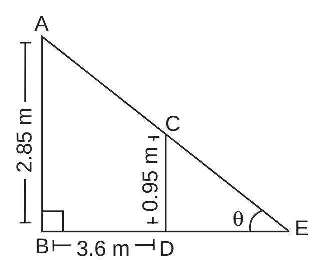
∴ By the converse of BPT, we have

$BC \parallel MR$

i.e. $\quad MR \parallel BC$

Hence, **MR** ∥ **BC**.

35. Let AB represents the lamp post and CD represents the girl after she has moved away from the lamp post for 3 seconds. Let DE represents the shadow of the girl and let θ be the angular elevation of the lamp.



In ΔABE and ΔCDE, we have

$\angle ABE = \angle CDE$ [Each is 90°]

$\angle AEB = \angle CED$ [Common]

$\therefore \quad \Delta ABE \sim \Delta CDE$ [By AA similarity]

$\therefore \quad \frac{AB}{CD} = \frac{BE}{DE}$

[Corresponding sides of similar triangles are proportional]

$$\Rightarrow \quad \frac{2.85 \text{ m}}{0.95 \text{ m}} = \frac{3.6 + DE}{DE}$$

$$\Rightarrow \quad (2.85) \times DE = (0.95)(3.6) \text{ m} + (0.95)\, DE$$

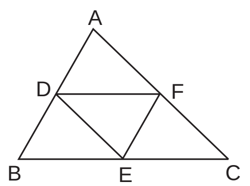
$$\Rightarrow \quad (2.85 - 0.95)\, DE = 0.95 \times 3.6 \text{ m}$$

$$\Rightarrow \quad (1.9)\, DE = 0.95 \times 3.6 \text{ m}$$

$$\Rightarrow \quad DE = \frac{0.95 \times 3.6}{1.9} \text{ m} = 1.8 \text{ m}$$

Hence, the length of the girl's shadow is **1.8 m**.

36. Let ABC be a triangle in which D, E and F are the mid-points of the sides AB, BC and CA respectively. Since D and F are the mid-points of AB and AC respectively,



∴ By the converse of Thales theorem, DF ∥ BC.

In ΔADF and ΔABC, we have

∠ADF = ∠ABC [Corresponding angles, DF ∥ BC]

∠DAF = ∠BAC [Common]

∴ ΔADF ~ ΔABC [By AA similarity]

Similarity, ΔDBE ~ ΔABC and ΔFEC ~ ΔABC.

Now, F and E are the mid-points of AC and BC respectively.

∴ By the converse of Thales theorem, FE ∥ AB.

Also, D and E are the mid-points of AB and BC respectively.

∴ By the converse of Thales theorem, DE ∥ AC.

∴ AFED is a parallelogram.

∴ ∠DEF = ∠A [Opp. ∠s of a ∥gm]

Similarly, BDFE is a parallelogram.

∴ ∠DFE = ∠A [Opp. ∠s of a ∥gm]

Thus, in ΔEFD and ΔABC, we have

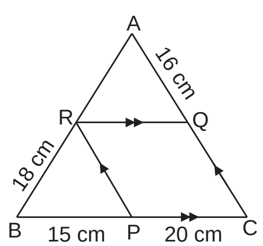
∠DEF = ∠A

and ∠DFE = ∠B

∴ ΔEFD ~ ΔABC [By AA similarity]

Hence, **ΔADF ~ ΔABC, ΔDBE ~ ΔABC, ΔFEC ~ ΔABC** and **ΔEFD ~ ΔABC**.

37. (*i*) In ΔABC, PR ∥ CA



$$\frac{BP}{PC} = \frac{BR}{AR} \quad \text{[By BPT]}$$

$$\Rightarrow \quad \frac{15 \text{ cm}}{20 \text{ cm}} = \frac{18 \text{ cm}}{AR}$$

$$\Rightarrow \quad AR = \frac{18 \times 20}{15} \text{ cm} = \mathbf{24 \text{ cm}}$$

(*ii*) In ΔABC, RQ ∥ BC

$$\therefore \quad \frac{AR}{BR} = \frac{AQ}{QC} \quad \text{[By BPT]}$$

$$\Rightarrow \quad \frac{24 \text{ cm}}{18 \text{ cm}} = \frac{16 \text{ cm}}{QC}$$

[Using AR = 24 cm, from (*i*)]

$$\Rightarrow \quad QC = \frac{16 \times 18}{24} \text{ cm} = \mathbf{12 \text{ cm}}$$

(*iii*) In ΔARQ and ΔABC,

∠ARQ = ∠ABC

∠AQR = ∠ACB [Corresponding angles]

∴ ΔARQ ~ ΔABC [By AA similarity]

$$\therefore \quad \frac{AR}{AB} = \frac{RQ}{BC} = \frac{AQ}{AC}$$

[Corresponding sides of similar triangles are proportional]

$$\Rightarrow \quad \frac{RQ}{BC} = \frac{AQ}{AC}$$

$$\Rightarrow \quad \frac{RQ}{BP + PC} = \frac{AQ}{AQ + QC}$$

$$\Rightarrow \quad \frac{RQ}{(15 + 20) \text{ cm}} = \frac{16 \text{ cm}}{(16 + 12) \text{ cm}}$$

[Using QC = 12 cm, from (*ii*)]

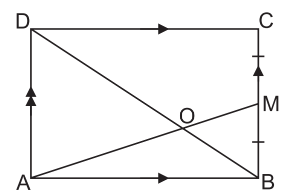
$$\Rightarrow \quad \frac{RQ}{35 \text{ cm}} = \frac{16 \text{ cm}}{28 \text{ cm}}$$

$$\Rightarrow \quad RQ = \frac{16 \times 35}{28} \text{ cm} = 20 \text{ cm}$$

$$\therefore \quad \frac{RQ}{BC} \cdot \frac{AR}{AB} = \frac{20 \text{ cm}}{(15 + 20) \text{ cm}} \times \frac{24 \text{ cm}}{(24 + 18) \text{ cm}}$$

$$= \frac{20 \times 24}{35 \times 42} = \mathbf{\frac{16}{49}}$$

38. Given that ABCD is a rectangle in which M is the mid-point of BC. Diagonal DB meets AM at O.



To prove that (*i*) ΔBOM ∽ ΔDOA and (*ii*) BD : DO = 3 : 2.

(*i*) In ΔBOM and ΔDOA, we have

∠MBO = alternate ∠ADO

[∵ AD ∥ BC and BD is a transversal]

∠BOM = vertically opposite ∠DOA

∴ By AA similarity criterion, ΔBOM ∽ ΔDOA.

(*ii*) Since, ΔBOM ∽ ΔDOA,

$$\frac{BO}{DO} = \frac{BM}{DA} = \frac{BM}{2BM}$$

[∵ DA = BC = 2BM]

$$= \frac{1}{2} \quad \ldots(1)$$

$$\therefore \quad \frac{BD}{DO} = \frac{BO + OD}{DO}$$

$$= \frac{BO}{DO} + \frac{OD}{OD}$$

$$= 1 + \frac{BO}{DO}$$

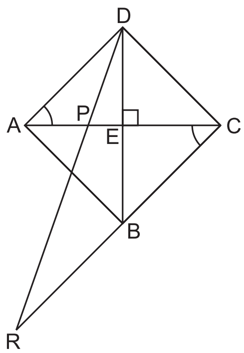
$$= 1 + \frac{1}{2} \quad \text{[From (1)]}$$

$$= \frac{3}{2}$$

$\therefore$ BD : DO = 3 : 2

39. Given that ABCD is a rhombus such that AB = BC = CD = DA and diagonals AC and BD meet each other at E at right angles so that $\angle AED = 90°$. P is a point on AC such that AP produced intersect CB produced at R.
To prove that

DP × CR = DC × PR.



In ΔDPA and ΔRPC, we have

$\angle DAP$ = alternate $\angle RCP$
[$\because$ DA ∥ CB and AC is a transversal]
$\angle DPA$ = vertically opposite $\angle RPC$

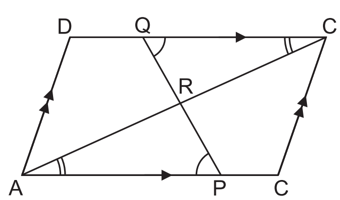
$\therefore$ By AA similarity criterion, we have

$\Delta DPA \sim \Delta RPC$

$$\therefore \quad \frac{DP}{RP} = \frac{DA}{RC} = \frac{DC}{CR} \quad [\because DA = DC]$$

$\Rightarrow$ DP × CR = DC × PR

40. Given that ABCD is a ∥gm with AC as one of the diagonals. P and Q are two points on AB and DC respectively such that AP : PB = 3 : 2 and CQ : QD = 4 : 1.



Let PQ intersect AC at R.

To prove that AR = $\frac{3}{7}$AC.

In ΔRQC and ΔRPA, we have

$\angle RQC$ = alternate $\angle RPA$
[$\because$ DC ∥ AB and PQ is a transversal]
$\angle QCR$ = alternate $\angle PAR$
[$\because$ DC ∥ AB and AC is a transversal]

$\therefore$ By AA similarity criterion, $\Delta RQC \sim \Delta RPA$

$$\therefore \quad \frac{RQ}{RP} = \frac{RC}{RA} = \frac{QC}{PA} \quad \ldots(1)$$

Now, $\frac{QC}{QD} = \frac{4}{1}$ [Given]

$\therefore$ QC = 4QD = 4(CD − CQ)
= 4AB − 4CQ [$\because$ AB = CD]

$\Rightarrow$ 5QC = 4AB

$\therefore$ QC = $\frac{4}{5}$AB …(2)

$\therefore$ DQ = DC − CQ

$= AB - \frac{4}{5}AB$ [From (2)]

$= \frac{1}{5}AB$ …(3)

Again, $\frac{PA}{PB} = \frac{3}{2}$ [Given]

$\therefore$ AP = $\frac{3}{2}PB = \frac{3}{2}(AB - AP)$

$\Rightarrow$ $\frac{5}{2}AP = \frac{3}{2}AB$

$\Rightarrow$ AP = $\frac{3}{5}$AB …(4)

$\therefore$ BP = AB − AP

$= AB\left(1 - \frac{3}{5}\right)$ [From (4)]

$= \frac{2}{5}AB$ …(5)

$\therefore$ From (1),

$$\frac{RC}{RA} = \frac{QC}{PA} = \frac{\frac{4}{5}AB}{\frac{3}{5}AB}$$

$$= \frac{4}{3} \quad \text{[From (2) and (4)]}$$

$$\Rightarrow \quad \frac{AC - AR}{AR} = \frac{4}{3}$$

$$\Rightarrow \quad \frac{AC}{AR} = 1 + \frac{4}{3} = \frac{7}{3}$$

$$\Rightarrow \quad AR = \frac{3}{7}AC$$

EXERCISE 6C

For Basic and Standard Levels

1. $\Delta ABC \sim \Delta PQR$,
ar(ΔABC) = 36 cm^2,
ar(ΔPQR) = 49 cm^2
and BC = 12 cm

We know that the ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.

$$\therefore \quad \frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{BC^2}{QR^2}$$

$$\Rightarrow \quad \frac{36\ cm^2}{49\ cm^2} = \frac{(12\ cm)^2}{QR^2}$$

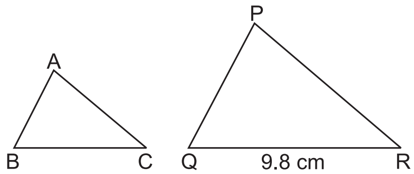
$$\Rightarrow \quad QR^2 = \frac{12 \times 12 \times 49}{36}\ cm^2$$

$$= (4 \times 49)\ cm^2$$

$\Rightarrow \quad QR = (2 \times 7)$ cm
$= \mathbf{14\ cm}$

2. Given that $\Delta ABC \sim \Delta PQR$ and $ar(\Delta ABC) = 25\ cm^2$ and $ar\ \Delta PQR = 4\ cm^2$.

Also, $\quad QR = 9.8$ cm



To find the length of BC.

$\because \quad \Delta ABC \sim \Delta PQR$

$$\therefore \quad \frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{BC^2}{QR^2}$$

$$\Rightarrow \quad \frac{25}{49} = \frac{BC^2}{QR^2}$$

$$\Rightarrow \quad \frac{BC}{QR} = \frac{5}{7}$$

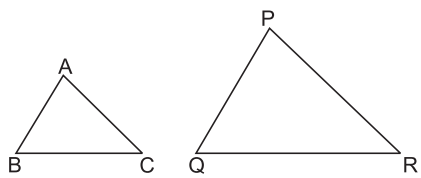
$$\Rightarrow \quad \frac{BC}{9.8} = \frac{5}{7}$$

$$\Rightarrow \quad BC = \frac{5}{7} \times 9.8 = 7$$

Hence, the required length of BC is 7 cm.

3. Given that $\quad \Delta ABC \sim \Delta PQR$

$$\text{and} \quad \frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{25}{64}$$



To find the ratios of their corresponding sides, i.e. to find $\frac{AB}{PQ}, \frac{BC}{QR}$ and $\frac{AC}{PR}$.

$\because \quad \Delta ABC \sim \Delta PQR$

$$\therefore \quad \frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{AC^2}{PR^2}$$

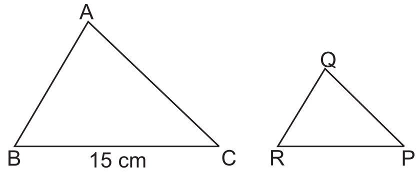
$$\Rightarrow \quad \frac{25}{64} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{AC^2}{PR^2}$$

$$\therefore \quad \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} = \sqrt{\frac{25}{64}} = \frac{5}{8}$$

$\therefore$ The required ratio is 5 : 8.

4. Given that $\Delta ABC \sim \Delta QRP$, BC = 15 cm

$$\text{and} \quad \frac{ar(\Delta ABC)}{ar(\Delta QRP)} = \frac{9}{4}$$



To find the length PR.

Since $\quad \Delta ABC \sim \Delta QRP$,

$$\therefore \quad \frac{ar(\Delta ABC)}{ar(\Delta QRP)} = \frac{BC^2}{RP^2}$$

$$\Rightarrow \quad \frac{9}{4} = \frac{BC^2}{RP^2} \qquad \text{[Given]}$$

$$\Rightarrow \quad \frac{BC}{RP} = \sqrt{\frac{9}{4}} = \frac{3}{2}$$

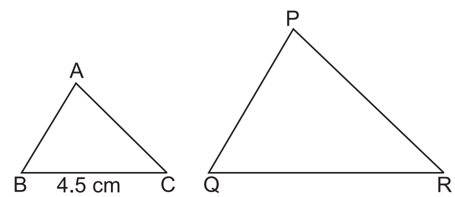
$$\Rightarrow \quad \frac{15}{RP} = \frac{3}{2}$$

$$\Rightarrow \quad RP = \frac{2 \times 15}{3} = 10$$

Hence, the required length of PR is 10 cm.

5. Given that $\Delta ABC \sim \Delta PQR$, BC = 4.5 cm

$$\text{and} \quad \frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{9}{16}$$



To find the length of QR

Since $\quad \Delta ABC \sim \Delta PQR$,

$$\Rightarrow \quad \frac{BC^2}{QR^2} = \frac{9}{16}$$

$$\Rightarrow \quad \frac{BC}{QR} = \sqrt{\frac{9}{16}} = \frac{3}{4}$$

$$\Rightarrow \quad \frac{4.5}{QR} = \frac{3}{4}$$

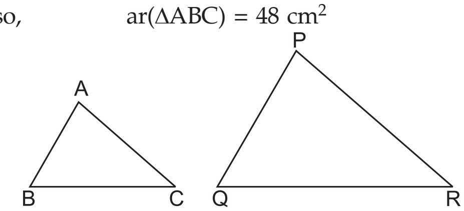
$$\Rightarrow \quad QR = \frac{4 \times 4.5}{3} = 6.$$

Hence, the required length of QR is 6 cm.

6. Given that $\Delta ABC \sim \Delta PQR$ and

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} = \frac{2}{3} \qquad \ldots(1)$$

Also, $\quad ar(\Delta ABC) = 48\ cm^2$



To find the area of the larger triangle ΔPQR.

$\because \quad \Delta ABC \sim \Delta PQR$,

$$\therefore \quad \frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{(2)^2}{(3)^2} = \frac{4}{9}$$

$$\Rightarrow \quad \frac{48}{ar(\Delta PQR)} = \frac{4}{9}$$

$$\Rightarrow \quad 4\,ar(\Delta PQR) = 9 \times 48$$

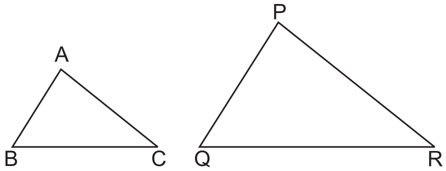
$$\Rightarrow \quad ar(\Delta PQR) = \frac{9 \times 48}{4} = 108$$

Hence, the required area of $\Delta PQR = 108\ cm^2$.

7. Given that $\Delta ABC \sim \Delta PQR$

and $\frac{AB}{PQ} = \frac{1}{3}$

To find $\frac{ar(\Delta ABC)}{ar(\Delta PQR)}$.



$\because \quad \Delta ABC \sim \Delta PQR,$

$$\therefore \quad \frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AB^2}{PQ^2} = \frac{1}{9}$$

which is the required ratio.

8. $\Delta ABC \sim \Delta DEF,$

$BC = 4$ cm,

$EF = 5$ cm

and $ar(\Delta ABC) = 64$ cm^2

Since the ratio of areas of two similar triangles is equal to the ratio of the squares of any two corresponding sides,

$$\therefore \quad \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{BC^2}{EF^2}$$

$$\Rightarrow \quad \frac{64 \text{ cm}^2}{ar(\Delta DEF)} = \frac{(4 \text{ cm})^2}{(5 \text{ cm})^2} = \frac{16}{25}$$

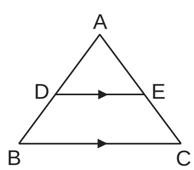
$$\Rightarrow \quad ar(\Delta DEF) = \frac{64 \times 25}{16} \text{ cm}^2 = \mathbf{100 \text{ cm}^2}$$

9. In ΔADE and ΔABC, we have

$\angle ADE = \angle ABC$ [Corresponding $\angle$s, DE $\parallel$ BC]

$\angle DAE = \angle BAC$ [Common]

$\therefore \quad \Delta ADE \sim \Delta ABC$



Since the ratio of the areas of two similar triangles is equal to the ratio of the squares of any two corresponding sides,

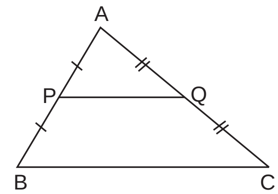
$$\therefore \quad \frac{ar(\Delta ADE)}{ar(\Delta ABC)} = \frac{DE^2}{BC^2}$$

$$\Rightarrow \quad \frac{15 \text{ cm}^2}{ar(\Delta ABC)} = \frac{(3 \text{ cm})^2}{(6 \text{ cm})^2} = \frac{9}{36}$$

$$\Rightarrow \quad ar(\Delta ABC) = \frac{15 \times 36}{9} \text{ cm}^2 = \mathbf{60 \text{ cm}^2}$$

10. ABC is a triangle in which P and Q are the midpoints of the sides AB and AC respectively.

Thus, in ΔABC, PQ divides the sides AB and AC in the same ratio.



$\therefore$ By the converse of the Thales Theorem, PQ $\parallel$ BC.

In ΔAPQ and ΔABC, we have

$\angle APQ = \angle ABC$ [Corresponding angles]

$\angle PAQ = \angle BAC$ [Common]

$\therefore \quad \Delta APQ \sim \Delta ABC$ [By AA similarity]

Since the ratio of areas of two similar triangles is equal to the ratio of the squares of any two corresponding sides,

$$\therefore \quad \frac{ar(\Delta APQ)}{ar(\Delta ABC)} = \frac{AP^2}{AB^2} = \frac{AP^2}{(AP + PB)^2} = \frac{AP^2}{(AP + AP)^2}$$

[$\therefore$ P is the midpoint of AB]

$$\therefore \quad \frac{ar(\Delta APQ)}{ar(\Delta ABC)} = \frac{AP^2}{(2AP)^2} = \frac{AP^2}{(4AP)^2} = \frac{1}{4}$$

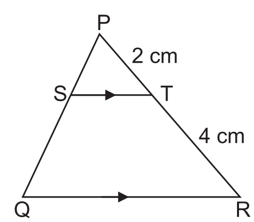
$\therefore$ $ar(\Delta APQ) : ar(\Delta ABC) = \mathbf{1 : 4}$.

11. Given that S and T are two points on the sides PQ and QR respectively of ΔPQR, such that PT = 2 cm and TR = 4 cm. Also, ST $\parallel$ QR.

To find $ar(\Delta PST) : ar(\Delta PQR)$

Since ST $\parallel$ QR,

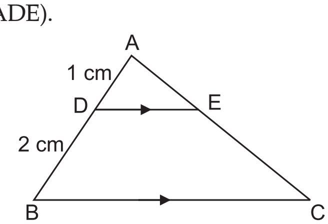
$\therefore \quad \Delta PST \sim \Delta PQR$



$$\therefore \quad \frac{ar(\Delta PST)}{ar(\Delta PQR)} = \frac{PT^2}{PR^2} = \frac{2^2}{(2+4)^2} = \frac{4}{36} = \frac{1}{9}$$

$\therefore$ Required ratio = 1 : 9.

12. Given that D and E are two points on the sides AB and AC respectively of ΔABC such that DE $\parallel$ BC, AD = 1 cm and DB = 2 cm. To find the ratio of $ar(\Delta ABC)$ and $ar(\Delta ADE)$.



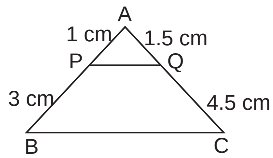
Since $DE \parallel BC$

$\therefore$ $\Delta ABC \backsim \Delta ADE$

$$\therefore \quad \frac{ar(\Delta ABC)}{ar(\Delta ADE)} = \frac{AB^2}{AD^2} = \frac{(1+2)^2}{1^2} = \frac{9}{1}$$

$\therefore$ Required ratio is 9 : 1.

13. In ΔABC, we have



$$\frac{AP}{PB} = \frac{1 \text{ cm}}{3 \text{ cm}} = \frac{1}{3}$$

and $$\frac{AQ}{QC} = \frac{1.5 \text{ cm}}{4.5 \text{ cm}} = \frac{1}{3}$$

Clearly, $$\frac{AP}{PB} = \frac{AQ}{QC}.$$

Thus, in ΔABC, PQ divides the sides AB and AC in the same ratio.

$\therefore$ By the converse of BPT,

$PQ \parallel BC$

Now, in ΔAPQ and ΔABC, we have

$\angle APQ = \angle ABC$ [Corresponding angles, $PQ \parallel BC$]

$\angle PAQ = \angle BAC$ [Common]

$\therefore$ $\Delta APQ \sim \Delta ABC$ [By AA similarity]

Since the ratio of the areas of two similar triangles is equal to the ratio of the squares of any two corresponding sides,

$$\therefore \quad \frac{ar(\Delta APQ)}{ar(\Delta ABC)} = \frac{AP^2}{AB^2}$$

$$= \frac{AP^2}{(AP + PB)^2}$$

$$= \frac{(1 \text{ cm})^2}{(4 \text{ cm})^2} = \frac{1}{16}$$

$$\Rightarrow \quad ar(\Delta APQ) = \frac{1}{16}(ar\Delta ABC)$$

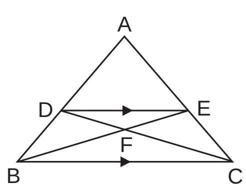
Hence, **the area of ΔAPQ is one-sixteenth of the area of ΔABC**.

14. Let $AD = 5x$

Then, $DB = 4x$

and $AB = AD + DB$

$= 5x + 4x = 9x$



In ΔADE and ΔABC, we have

$\angle ADE = \angle ABC$ [Corr. $\angle$s, $DE \parallel BC$]

$\angle DAE = \angle BAC$ [Common]

$\therefore$ $\Delta ADE \sim \Delta ABC$

$$\therefore \quad \frac{AD^2}{AB^2} = \frac{DE^2}{BC^2}$$

[Corresponding sides of similar triangles are proportional]

$$\Rightarrow \quad \frac{(5x)^2}{(9x)^2} = \frac{DE^2}{BC^2}$$

$$\Rightarrow \quad \frac{DE^2}{BC^2} = \frac{25}{81} \quad \ldots (1)$$

In ΔDFE and ΔCFB, we have

$\angle DFE = \angle CFB$ [$\because$ Vertically opposite angles]

$\angle EDF = \angle BCF$ [$\because \angle EDC = \angle BCD$, Alt. $\angle$s $DE \parallel BC$]

$\therefore$ $\Delta DFE \sim \Delta CFB$

Since the ratio of areas of two similar triangles is equal to the ratio of the squares of any two corresponding sides,

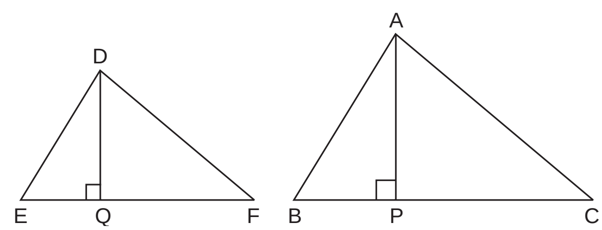
$$\therefore \quad \frac{(ar\,\Delta DFE)}{(ar\,\Delta CFE)} = \frac{DE^2}{BC^2} = \frac{25}{81} \quad \text{[Using (1)]}$$

Hence, ar(ΔDFE) : ar(ΔCFB) = **25 : 81**.

15. Let the two given triangles be ABC and DEF, so that

ar (ΔABC) = 100 cm^2

and ar(ΔDEF) = 49 cm^2



Let AP and DQ be the corresponding altitudes of ΔABC and ΔDEF respectively.

Then, AP = 5 cm

Since the ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding altitudes,

$$\therefore \quad \frac{ar\ (\Delta ABC)}{ar\ (\Delta DEF)} = \frac{AP^2}{DQ^2}$$

$$\Rightarrow \quad \frac{100 \text{ cm}^2}{49 \text{ cm}^2} = \frac{(5 \text{ cm})^2}{DQ^2}$$

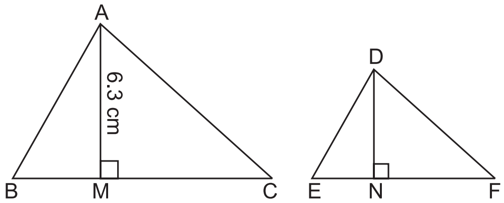
$$\Rightarrow \quad DQ^2 = \frac{25 \times 49}{100} \text{ cm}^2$$

$$= \frac{49}{4} \text{ cm}^2$$

$$\Rightarrow \quad PQ = \frac{7}{2} \text{ cm} = \mathbf{3.5 \text{ cm}}$$

16. Given that $\Delta ABC \backsim \Delta DEF$ such that ar(ΔABC) = 81 cm^2 and ar(ΔDEF) = 49 cm^2.

AM is the altitude of ΔABC from the vertex A to the side BC and DN is the altitude of ΔDEF from its vertex D to the side EF. It is also given that AM = 6.3 cm. To find the length of DN.



$\because \quad \Delta ABC \sim \Delta DEF,$

$$\therefore \quad \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{AM^2}{DN^2}$$

$$\Rightarrow \quad \frac{81}{49} = \frac{(6.3)^2}{DN^2} \quad \text{[Given]}$$

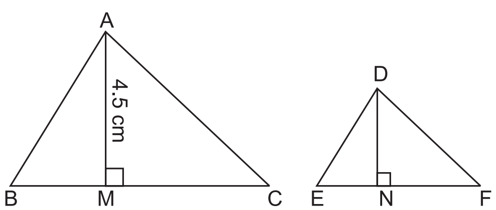
$$\Rightarrow \quad \frac{6.3}{DN} = \sqrt{\frac{81}{49}} = \frac{9}{7}$$

$$\therefore \quad DN = \frac{7 \times 6.3}{9} = 4.9$$

Hence, the required length of DN is 4.9 cm.

17. Given that $\Delta ABC \sim \Delta DEF$, $ar(\Delta ABC) = 81\ cm^2$ and $ar(\Delta DEF) = 49\ cm^2$

AM is the altitude of the bigger triangle ΔABC from the vertex A to BC and DN is the altitude of the smaller triangle ΔDEF from its vertex D to EF. Also, it is given that AM = 4.5 cm.



To find the length of DN.

$\because \quad \Delta ABC \sim \Delta DEF,$

$$\therefore \quad \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{AM^2}{DN^2}$$

$$\Rightarrow \quad \frac{81}{49} = \frac{AM^2}{DN^2} \quad \text{[Given]}$$

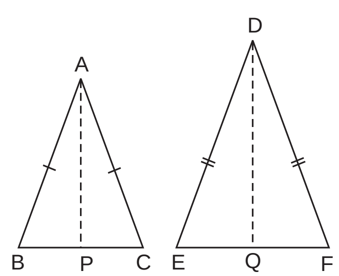
$$\Rightarrow \quad \frac{AM}{DN} = \sqrt{\frac{81}{49}} = \frac{9}{7}$$

$$\Rightarrow \quad \frac{4.5}{DN} = \frac{9}{7}$$

$$\therefore \quad DN = \frac{7 \times 4.5}{9} = 3.5$$

Hence, the required length of DN is 3.5 cm.

18. Let ΔABC and ΔDEF be the given isosceles triangles such that AB = AC and DE = DF.



Let AP and DQ be the altitudes of ΔABC and ΔDEF respectively.

$$\Rightarrow \quad AB = AC$$

$$\Rightarrow \quad \frac{AB}{AC} = 1$$

and $\quad DE = DF$

$$\Rightarrow \quad \frac{DE}{DF} = 1$$

$$\therefore \quad \frac{AB}{AC} = \frac{DE}{DF}$$

$$\Rightarrow \quad \frac{AB}{DE} = \frac{AC}{DF} \quad \dots (1)$$

In ΔABC and ΔDEF, we have

$$\angle A = \angle D \quad \text{[Given]}$$

$$\frac{AB}{DE} = \frac{AC}{DF}$$

$$\therefore \quad \Delta ABC \sim \Delta DEF$$

Since the ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding altitudes,

$$\frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{AP^2}{DQ^2}$$

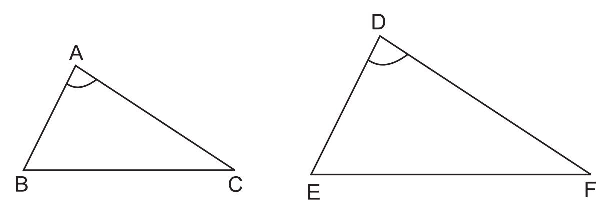
$$\Rightarrow \quad \frac{16}{25} = \frac{AP^2}{DQ^2}$$

$$\Rightarrow \quad \frac{4}{5} = \frac{AP}{DQ}$$

$$\Rightarrow \quad AP : DQ = 4 : 5$$

Hence, the ratio of their corresponding heights (altitudes) is **4 : 5**.

19.



$$\frac{AB}{AC} = \frac{DE}{DF} \quad \text{[Given]}$$

$$\Rightarrow \quad \frac{AB}{DE} = \frac{AC}{DF} \quad \dots (1)$$

In ΔABC and ΔDEF,

$$\angle A = \angle D \quad \text{[Given]}$$

$$\frac{AB}{DE} = \frac{AC}{DF} \quad \text{[Using (1)]}$$

$$\therefore \quad \Delta ABC \sim \Delta DEF$$

Since the ratio of areas of two similar triangles is equal to the ratio of the squares of any two corresponding sides,

$$\therefore \quad \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{AB^2}{DE^2}$$

$$\Rightarrow \quad \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \left(\frac{3}{4}\right)^2$$

$$\Rightarrow \quad \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{9}{16}$$

$$\Rightarrow \quad \frac{ar(\Delta ABC)}{ar(\Delta DEF)} + 1 = \frac{9}{16} + 1$$

$$\Rightarrow \quad \frac{ar(\Delta ABC) + ar(\Delta DEF)}{ar(\Delta DEF)} = \frac{25}{16}$$

$$\Rightarrow \quad \frac{20 \text{ cm}^2}{ar(\Delta DEF)} = \frac{25}{16}$$

$$\Rightarrow \quad ar(\Delta DEF) = \frac{20 \times 16}{25} \text{ cm}^2$$

$$= \frac{64}{5} \text{ cm}^2$$

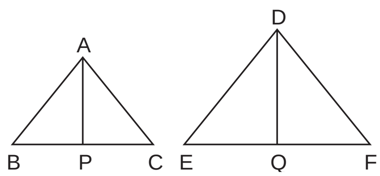
$$= 12.8 \text{ cm}^2$$

$$\therefore \quad ar(\Delta ABC) = (20 - 12.8) \text{ cm}^2$$
$$= 7.2 \text{ cm}^2$$

Hence, ar(ΔABC) = **7.2 cm²** and ar(ΔDEF) = **12.8 cm²**.

20. Let the two given similar triangles be ABC and DEF, such that

$$ar(\Delta ABC) = 49 \text{cm}^2$$
and $$ar(\Delta DEF) = 64 \text{ cm}^2$$



Let AP = x cm and DQ = y cm be the corresponding altitudes of ΔABC and ΔDEF respectively.

Then, $DQ - AP = 10$ cm

$\Rightarrow \quad y - x = 10$ cm

$\Rightarrow \quad y = (x + 10)$ cm ... (1)

Since the ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding altitudes,

$$\therefore \quad \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{AP^2}{DQ^2} = \frac{(x \text{ cm})^2}{[(x+10) \text{ cm}]^2}$$

$$\Rightarrow \quad \frac{49 \text{ cm}^2}{64 \text{ cm}^2} = \frac{x^2}{(x+10)^2}$$

$$\Rightarrow \quad \frac{7}{8} = \frac{x}{x+10}$$

$\Rightarrow \quad 7x + 70 = 8x$

$\Rightarrow \quad 8x - 7x = 70$

$\Rightarrow \quad x = 70$

and $\quad y = 70 + 10 = 80$

Hence, the length of the altitudes are **70 cm** and **80 cm**.

21. Let the given two similar triangles be ABC and DEF such that $ar(\Delta ABC) = 121 \text{ cm}^2$

and $ar(\Delta DEF) = 64 \text{ cm}^2$

Let AP and DQ be the corresponding medians of ΔABC and ΔDEF respectively such that

$$AP = 12.1 \text{ cm}$$

Since the ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding medians,

$$\therefore \quad \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{AP^2}{DQ^2}$$

$$\Rightarrow \quad \frac{121 \text{ cm}^2}{64 \text{ cm}^2} = \frac{(12.1 \text{ cm})^2}{DQ^2}$$

$$\Rightarrow \quad DQ^2 = \frac{64 \times 12.1 \times 12.1}{121} \text{ cm}^2$$

$$\Rightarrow \quad DQ = \frac{8 \times 12.1}{11} \text{ cm}$$

$$= 8 \times 1.1 \text{ cm} = 8.8 \text{ cm}$$

Hence, the corresponding median is **8.8 cm**.

22. Since the ratio of the areas of two similar triangles is equal to the ratio of the square of their corresponding altitudes,

$$\therefore \quad \frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AD^2}{PS^2}$$

$$= \left(\frac{AD}{PS}\right)^2$$

$$= \left(\frac{4}{9}\right)^2$$

$$= \frac{16}{81}$$

Hence, ar(ΔABC) : ar(ΔPQR) = **16 : 81**.

23. Since the ratio of the areas of two similar triangles is equal to the ratio of the squares of the corresponding angle bisector segments,

$$\therefore \quad \frac{ar(\Delta ABC)}{ar(\Delta XYZ)} = \frac{AD^2}{XE^2}$$

$$= \frac{(4 \text{ cm})^2}{(3 \text{ cm})^2}$$

$$= \frac{16}{9}$$

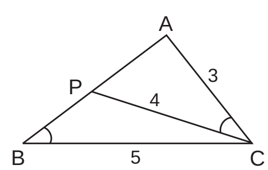
Hence, ar (ΔABC) : ar (ΔXYZ) = **16 : 9**.

24. In ΔABC and ΔACP, we have

$\angle ABC = \angle ACP$ [Given]

$\angle BAC = \angle CAP$ [Common]

$\therefore \quad \Delta ABC \sim \Delta ACP$



$$\therefore \quad \frac{AB}{AC} = \frac{BC}{CP} = \frac{AC}{AP}$$

[Corresponding sides of similar triangles are proportional] (*i*) $\dfrac{AB}{AC} = \dfrac{BC}{CP}$

$\Rightarrow \quad \dfrac{AB}{3} = \dfrac{5}{4}$

$\Rightarrow \quad AB = \dfrac{3 \times 5}{4}$

$= \dfrac{15}{4}$

$= 3.75$

Hence, **AB = 3.75**.

(*ii*) $\dfrac{BC}{CP} = \dfrac{AC}{AP}$

$\Rightarrow \quad \dfrac{5}{4} = \dfrac{3}{AP}$

$\Rightarrow \quad AP = \dfrac{3 \times 4}{5} = \dfrac{12}{5} = 2.4$

Hence, **AP = 2.4**.

(*iii*) Since the ratio of the areas of two similar triangles is equal to the ratio of the squares of any two corresponding sides,

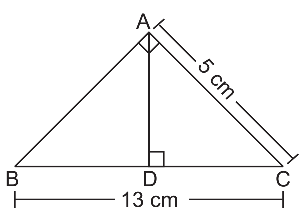
$\therefore \quad \dfrac{ar(\Delta ABC)}{ar(\Delta ACP)} = \dfrac{BC^2}{CP^2}$

$= \dfrac{5^2}{4^2}$

$= \dfrac{25}{16}$

Hence, $\dfrac{\mathbf{ar(\Delta ABC)}}{\mathbf{ar(\Delta ACP)}} = \dfrac{\mathbf{25}}{\mathbf{16}}$.

25. Given that ΔABC is right-angled at A and AD ⊥ BC. Also, BC = 13 cm and AC = 5 cm.



To find the ratio of areas of ΔABC and ΔADC.

In ΔBAC and ΔADC, we have

$\angle BAC = \angle ADC = 90°$ [Given]

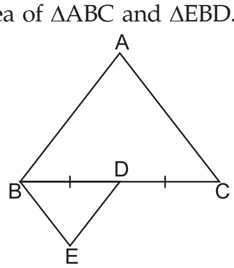
$\angle BCD = \angle ACD$ [Common]

∴ By AA similarity criterion, ΔBAC ∼ ΔADC

$$\frac{ar(\Delta BAC)}{ar(\Delta ADC)} = \frac{BC^2}{AC^2} = \frac{13^2}{5^2} = \frac{169}{25}$$

Hence, the required ratio is **169 : 25**.

26. Given that ΔABC and ΔEBD are two equilateral triangles, where D is the mid-point of BC. To find the ratio of the area of ΔABC and ΔEBD.

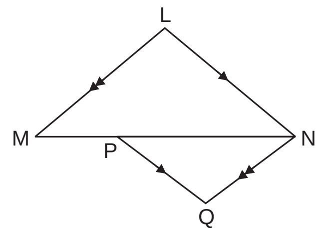


We know that all equilateral triangles are similar, since each angle of each of such triangles is 60°.

$$\therefore \quad \frac{ar(\Delta ABC)}{ar(\Delta EBD)} = \frac{BC^2}{BD^2} = \frac{4BD^2}{BD^2} = \frac{4}{1}$$

∴ Required ratio is **4 : 1**.

27. In ΔLMN and ΔQNP, we have



$\angle LMN = \angle QNP$ [Alt ∠s, LM ∥ NQ]

$\angle LNM = \angle QPN$ [Alt ∠s, LN ∥ PQ]

∴ ΔLMN ~ ΔQNP [By AA similarity]

Since the ratio of the areas of two similar triangles is equal to the ratio of the squares of any two corresponding sides,

$$\therefore \quad \frac{ar(\Delta LMN)}{ar(\Delta QNP)} = \frac{MN^2}{NP^2} = \frac{MN^2}{\left(\frac{2}{3}MN\right)^2}$$

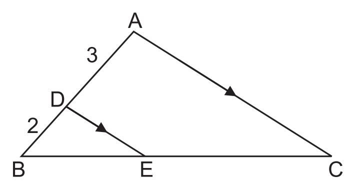
$$\left[\because MP = \frac{MN}{3} \quad \Rightarrow \quad NP = MN - MP\right.$$

$$\left.\Rightarrow \quad NP = MN - \frac{MN}{3} = \frac{2}{3}MN\right]$$

Hence, ar(ΔLMN) : ar(ΔQNP) = **9 : 4**.

28. (*i*) Given that D is a point on the side AB of ΔABC such that AD : DB = 3 : 2. Also, E is a point on BC such that DE ∥ AC.

To find the ratio of the areas of ΔABC and ΔDBE.



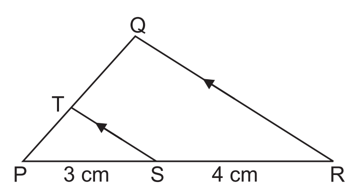
Since DE ∥ AC,

∴ ΔABC ∽ ΔDBE

$$\therefore \quad \frac{ar(\Delta ABC)}{ar(\Delta DBE)} = \frac{AB^2}{DB^2} = \frac{(3+2)^2}{2^2} = \frac{25}{4}$$

∴ Required ratio is **25 : 4**.

(*ii*) Given that S and T are two points on the sides PR and PT respectively of ΔQPR such that TS ∥ QR, PS = 3 cm and SR = 4 cm.



To find the ratio of the areas of ΔPST and PRQ.

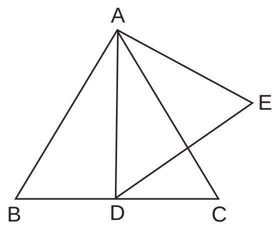
Since, TS ∥ QR,

$\therefore \quad \Delta PST \sim \Delta PRQ$

$$\therefore \quad \frac{ar(\Delta PST)}{ar(\Delta PRQ)} = \frac{PS^2}{PR^2} = \frac{3^2}{(3+4)^2} = \frac{9}{49}.$$

$\therefore$ Required ratio is **9 : 49**.

29. Let each side of equilateral triangle ABC be x.



Then, AB = x

and altitude, AD = $\frac{\sqrt{3}}{2}x$.

$\Delta ABC \sim \Delta ADE$

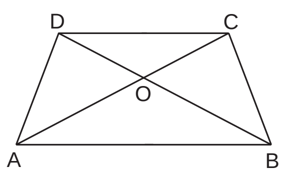
[They are equiangular Δs]

Since the ratio of the areas of two similar triangles is equal to the ratio of the squares of any two corresponding sides,

$$\therefore \quad \frac{ar(\Delta ADE)}{ar(\Delta ABC)} = \frac{AD^2}{AB^2} = \frac{\left(\frac{\sqrt{3}}{2}x\right)^2}{x^2} = \frac{3}{4}$$

Hence, ar (ΔADE) : ar (ΔABC) = **3 : 4**.

30. In ΔAOB and ΔCOD, we have



$\angle AOB = \angle COD$

[Vertically opposite angles]

$\angle OAB = \angle OCD$

[$\angle CAB = \angle ACD$, Alt $\angle$s, DC $\parallel$ AB]

$\therefore \quad \Delta AOB \sim \Delta COD$ [By AA similarity]

Since the ratio of the areas of two similar triangles is equal to the ratio of the squares of any two corresponding sides,

$$\therefore \quad \frac{ar(\Delta AOB)}{ar(\Delta COD)} = \frac{AB^2}{CD^2}$$

$$= \frac{(2\,CD)^2}{CD^2}$$

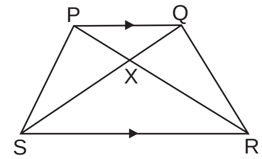
$$= \frac{4}{1}$$

$$\Rightarrow \quad \frac{84\text{ cm}^2}{ar(\Delta COD)} = \frac{4}{1}$$

$$\Rightarrow \quad ar(\Delta COD) = \frac{84}{4}\text{ cm}^2 = 21\text{ cm}^2$$

Hence, the area of ΔCOD is **21 cm²**.

31. In ΔPXQ and ΔRXS, we have



$\angle PXQ = \angle RXS$

[Vertically opposite angles]

$\angle QPX = \angle SRX$

[$\because \angle QPR = \angle SRP$, Alt $\angle$s, PQ $\parallel$ SR]

$\therefore \quad \Delta PXQ \sim \Delta RXS$ [By AA similarity]

Since the ratio of the areas of two similar triangles is equal to the ratio of the squares of any two corresponding sides,

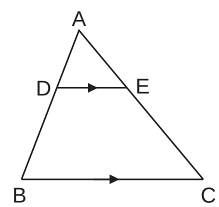
$$\therefore \quad \frac{ar(\Delta PXQ)}{ar(\Delta RXS)} = \frac{PQ^2}{RS^2}$$

$$= \frac{\left(\frac{2}{3}RS\right)^2}{RS^2}$$

$$= \frac{4}{9}$$

Hence, ar (ΔPXQ) : ar (ΔRXS) = **4 : 9**.

32. (*i*) In ΔADE and ΔABC, we have



$\angle ADE = \angle ABC$

[Corresponding $\angle$s, DE $\parallel$ BC]

$\angle DAE = \angle BAC$ [Common]

$\therefore \quad \Delta ADE \sim \Delta ABC$ [By AA similarity]

$$\therefore \quad \frac{AD}{AB} = \frac{AE}{AC}$$

[Corresponding sides of similar triangles are proportional]

$$\Rightarrow \quad \frac{AD}{AB} = \frac{AC - CE}{AC}$$

$$\Rightarrow \quad \frac{2\text{ cm}}{6\text{ cm}} = \frac{9\text{ cm} - CE}{9\text{ cm}}$$

$$\Rightarrow \quad \frac{1}{3} = \frac{9\text{ cm} - CE}{9\text{ cm}}$$

$\Rightarrow \quad$ 9 cm = 27 cm – 3 CE

$\Rightarrow \quad$ 3 cm = 9 cm – CE

$\Rightarrow \quad$ CE = (9 – 3) cm = 6 cm

Hence, the length of CE is **6 cm**.

(*ii*) Since the ratio of the areas of two similar triangles is equal to the ratio of the square of any two corresponding sides,

$\therefore \quad \frac{ar(\Delta ADE)}{ar(\Delta ABC)} = \frac{AD^2}{AB^2}$

$= \frac{(2\text{ cm})^2}{(6\text{ cm})^2} = \frac{1}{9}$

$\Rightarrow \quad \frac{ar(\Delta ABC)}{ar(\Delta ADE)} = 9$ [Taking reciprocals]

$\Rightarrow \quad \frac{ar(\Delta ABC)}{ar(\Delta ADE)} - 1 = 9 - 1$

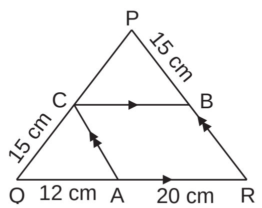
$\Rightarrow \frac{ar(\Delta ABC) - ar(\Delta ADE)}{ar(\Delta ADE)} = 8$

$\Rightarrow \quad \frac{ar(\text{trapezium BCED})}{ar(\Delta ADE)} = \frac{8}{1}$

$\Rightarrow \quad \frac{ar(\Delta ADE)}{ar(\text{trapezium BCED})} = \frac{1}{8}$ [Taking reciprocals]

Hence, $\mathbf{\frac{ar\ (\Delta ADE)}{ar\ (trapezium\ BCED)} = \frac{1}{8}}$.

33.



(*i*) In ΔQPR, CA ∥ PR

$\therefore \quad \frac{QA}{AR} = \frac{QC}{CP}$ [By BPT]

$\Rightarrow \quad \frac{12\text{ cm}}{20\text{ cm}} = \frac{15\text{ cm}}{PC}$

$\Rightarrow \quad PC = \frac{15 \times 20}{12}$ cm

= 25 cm

Hence, PC = **25 cm**.

(*ii*) In ΔPQR, CB ∥ QR

$\frac{PC}{CQ} = \frac{PB}{BR}$ [By BPT]

$\Rightarrow \quad \frac{25\text{ cm}}{15\text{ cm}} = \frac{15\text{ cm}}{BR}$

$\Rightarrow \quad BR = \frac{15 \times 15}{25}$ cm = 9 cm

Hence, BR = **9 cm**.

(*iii*) In ΔPBC and ΔPRQ, we have

∠PBC = ∠PRQ [Corresponding ∠s, CB ∥ QR]

∠BPC = ∠RPQ [Common]

∴ ΔPBC ~ ΔPRQ [By AA similarity]

Since the ratio of two similar triangles is equal to the ratio of squares of any two corresponding sides,

$\therefore \quad \frac{ar(\Delta PBC)}{ar(\Delta PRQ)} = \frac{PB^2}{PR^2} = \frac{PB^2}{(PB+BR)^2}$

$= \frac{(15\text{ cm})^2}{[(15+9)\text{ cm}]^2}$

$= \frac{15 \times 15}{24 \times 24}$

$= \frac{5 \times 5}{8 \times 8} = \frac{25}{64}$

Hence, ar (ΔPBC) : ar (ΔPRQ) = **25 : 64**.

ALTERNATIVE METHOD

33. (*i*) and (*ii*)

CB ∥ QR and CA ∥ PR

∴ ARBC is a parallelogram.

∴ CB = AR = 20 cm

ΔPCB ~ ΔPQR [By AA similarity]

$\therefore \quad \frac{PC}{PQ} = \frac{CB}{QR} = \frac{PB}{PR}$

[Corresponding sides of similar triangles are proportional]

$\Rightarrow \quad \frac{PC}{PC + 15\text{ cm}} = \frac{20}{32}$

and $\quad \frac{20}{32} = \frac{15\text{ cm}}{15\text{ cm} + BR}$

⇒ **PC = 25 cm**

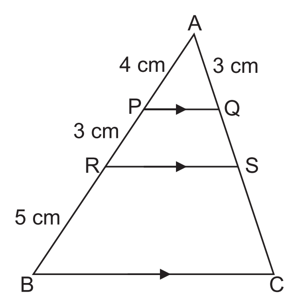
and **BR = 9 cm**

(*iii*) $\quad \frac{ar\ (\Delta PBC)}{ar\ (\Delta PRQ)} = \frac{BC^2}{QC^2}$

$= \frac{(20\text{ cm})^2}{(32\text{ cm})^2} = \mathbf{\frac{25}{64}}$

For Standard Level

34. Given that P, R are points on the side AB of ΔABC such that AP = 4 cm, PR = 3 cm and RB = 5 cm. Q and S are points on AC such that PQ ∥ BC, RS ∥ BC and AQ = 3 cm. Also, ar(ΔABC) = 48 cm^2.



To find the lengths of QS and SC and also the area of ΔAPQ.

Since, PQ ∥ RS ∥ BC,

ΔAPQ ~ ΔARS …(1)

and ΔAPQ ~ ΔABC …(2)

From (1), $\quad \frac{AP}{AR} = \frac{AQ}{AS}$

$\Rightarrow \quad \frac{4}{4+3} = \frac{3}{3+QS}$

⇒ 12 + 4QS = 21

⇒ 4QS = 21 – 12 = 9

$\therefore \quad QS = \frac{9}{4}$

Again, since RS ∥ BC,

$$\frac{AS}{SC} = \frac{AR}{RB} \qquad \text{[By BPT]}$$

$\Rightarrow \quad \dfrac{3+\frac{9}{4}}{SC} = \dfrac{4+3}{5}$

$\Rightarrow \quad \dfrac{21}{4SC} = \dfrac{7}{5}$

$\Rightarrow \quad 28\ SC = 105$

$\Rightarrow \quad SC = \dfrac{105}{28} = \dfrac{15}{4}$

Hence, the required lengths of **QS** and **SC** are respectively $\mathbf{\frac{9}{4}}$ **cm** and $\mathbf{\frac{15}{4}}$ **cm**.

Again, from (2), we have

$$\frac{ar(\Delta APQ)}{ar(\Delta ABC)} = \frac{AP^2}{AB^2}$$

$$= \frac{4^2}{(4+3+5)^2} = \frac{16}{144} = \frac{1}{9}$$

$\Rightarrow \quad \dfrac{ar(\Delta APQ)}{48} = \dfrac{1}{9}$

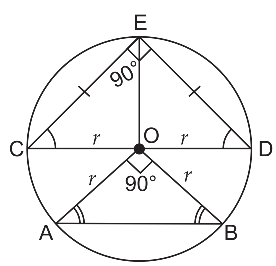
$\therefore \quad ar(\Delta APQ) = \dfrac{48}{9} = \dfrac{16}{3}$

Hence, the required area of ΔAPQ is $\mathbf{\frac{16}{3}}$ cm^2.

35. Given that CE and DE are equal chords of a circle with centre at O. Also, ∠AOB = 90°. Clearly, COD is a diameter of the circle with radius, say r cm so that OA = OC = OB = OD = r.

$\therefore \quad \angle OAB = \angle OBA$

and $\quad \angle ECO = \angle EDO$



To find the ratio of the areas of ΔCED and ΔAOB.
Since, angle is a semicircle is 90°.

$\therefore \quad \angle CED = 90°.$

Again, $\because \quad OC = OD = r,$

$\angle EOD = \angle EOC = 90°$

$\angle ECO = \angle EDO = 45°$

Also, $\because \quad OA = OB = r,$

$\therefore \quad \angle OAB = \angle OBA = 45°$

$\therefore \quad \Delta CED \sim \Delta AOB.$

$\therefore \quad \angle OAB = \angle OBA = 45°.$

$\therefore \quad \dfrac{ar(\Delta CED)}{ar(\Delta AOB)} = \dfrac{CD^2}{AB^2} = \dfrac{(2r)^2}{r^2+r^2}$

$$= \frac{4r^2}{2r^2} = \frac{2}{1}$$

$\therefore$ Required ratio is **2 : 1**.

EXERCISE 6D

For Basic and Standard Levels

1. For the given triangle to be right-angled, the sum of the squares of the two smaller sides must be equal to the square of the greatest side.

(*i*) a = 6 cm, b = 8 cm and c = 10 cm

$a^2 + b^2 = (6\text{ cm})^2 + (8\text{ cm})^2$
$= (36 + 64)\text{ cm}^2 = 100\text{ cm}^2$
$c^2 = (10\text{ cm})^2 = 100\text{ cm}^2$

Clearly, $\quad a^2 + b^2 = c^2$.

Hence, the given triangle is right–angled.

(*ii*) a = 35 cm, b = 12 cm and c = 12.5 cm

$b^2 + c^2 = (12\text{ cm})^2 + (12.5\text{ cm})^2$
$= (144 + 156.25)\text{ cm}^2$
$= 300.25\text{ cm}^2$
$c^2 = (35\text{ cm})^2 = 1225\text{ cm}^2$

Clearly, $\quad b^2 + c^2 \neq a^2$.

Hence, the given triangle is not right-angled.

(*iii*) a = 4 cm, b = 7.5 cm and c = 8.5 cm

$a^2 + b^2 = (4\text{ cm})^2 + (7.5\text{ cm})^2$
$= (16 + 56.25)\text{ cm}^2$
$= 72.25\text{ cm}^2$
$c^2 = (8.5\text{ cm})^2 = 72.25\text{ cm}^2$

Clearly, $\quad a^2 + b^2 = c^2$.

Hence, the given triangle is right-angled.

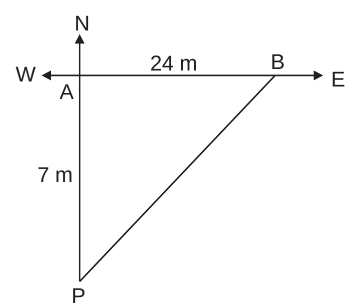
(*iv*) Let $x = (a - 1)$ cm, $y = 2\sqrt{a}$ cm and $z = (a + 1)$ cm

$x^2 + y^2 = [(a-1)\text{ cm}]^2 + \left[2\sqrt{a}\text{ cm}\right]^2$
$= (a^2 - 2a + 1 + 4a)\text{ cm}^2$
$= (a^2 + 2a + 1)\text{ cm}^2$
$= (a + 1)^2\text{ cm}^2$
$z^2 = [(a+1)\text{ cm}]^2$
$= (a + 1)^2\text{ cm}^2$

Clearly, $\quad x^2 + y^2 = z^2$.

Hence, the given triangle is right-angled.

2. Starting from point P, let the man go 7 m due North and reach point A and then let him go 24 m due East to reach point B.



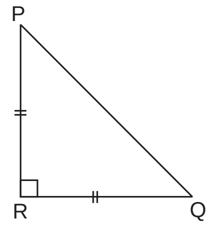
Then, PA = 7 m, AB = 24 m and ∠PAB = 90°.
In right triangle PAB, we have

$PB^2 = PA^2 + AB^2$ [By Pythagoras' Theorem]
$= (7\text{ m})^2 + (24\text{ m})^2$
$= (49 + 576)\text{ m}^2$
$= 625\text{ m}^2$

$\therefore \quad PB = \sqrt{625}\text{ m} = 25\text{ m}$

Hence, the man is **25 m** away from the starting point.

3. In right triangle PRQ, we have

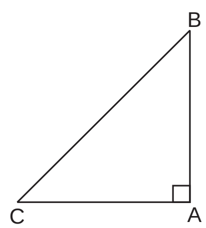


$$PQ^2 = PR^2 + RQ^2$$ [By Pythagoras' Theorem]

$\Rightarrow$ $PQ^2 = PR^2 + PR^2$ [$\because$ ΔPRQ is an isosceles right triangle $\Rightarrow PQ = PR$]

Hence, $\mathbf{PQ^2 = 2\,PR^2}$.

4. Let A represents the airport from which one aeroplane flies due North at a speed of 1000 km/h and reaches point B after 30 minutes, another aeroplane flies due West at a speed of 1200 km/h and reaches point C after 30 minutes.



Then, $AB = \frac{1000\,\text{km}}{1\,\text{hour}} \times \frac{1}{2}$ hour

$= 500$ km

and $CA = \frac{1200\,\text{km}}{1\,\text{hour}} \times \frac{1}{2}$ hour

$= 600$ km [$\because$ Distance = Speed × Time]

In right ΔBAC, we have

$$BC^2 = AB^2 + AC^2$$ [By Pythagoras' Theorem]

$\Rightarrow$ $BC^2 = (500\text{ km})^2 + (600\text{ km})^2$

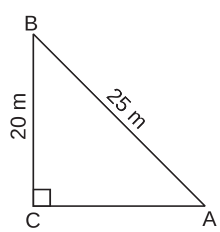
$= (250000 + 360000)\text{ km}^2$

$= 610000\text{ km}^2$

$\Rightarrow$ $BC = 100\sqrt{61}$ km

Thus, the two aeroplanes will be $\mathbf{100\sqrt{61}}$ km apart.

5. Let AB be the ladder which reaches a window CB of a house at point B.



Then, AB = 25 cm, CB = 20 m and $\angle BCA = 90°$.

In right triangle ACB, we have

$$AB^2 = BC^2 + AC^2$$ [By Pythagoras' Theorem]

$\Rightarrow$ $(25\text{ m})^2 = (20\text{ m})^2 + AC^2$

$\Rightarrow$ $AC^2 = (625 - 400)\text{ m}^2$

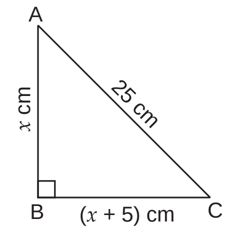
$= 225\text{ m}^2$

$\Rightarrow$ $AC = 15$ m

Hence, the distance of the foot of the ladder from the house is **15 m**.

6. Let ABC be a right triangle, right-angled at B.

Let AB = x cm be the smaller of the remaining two sides.



Then, AC = 25 cm and AC = $(x + 5)$ cm^2.

In right triangle ABC, we have

$$AC^2 = AB^2 + BC^2$$ [By Pythagoras' theorem]

$\Rightarrow$ $(25\text{ cm})^2 = (x\text{ cm})^2 + [(x + 5)\text{ cm}]^2$

$\Rightarrow$ $625\text{ cm}^2 = (x^2 + x^2 + 10x + 25)\text{ cm}^2$

$\Rightarrow$ $625 = 2x^2 + 10x + 25$

$\Rightarrow$ $2x^2 + 10x - 600 = 0$

$\Rightarrow$ $x^2 + 5x - 300 = 0$

$\Rightarrow$ $x^2 + 20x - 15x - 300 = 0$

$\Rightarrow$ $x(x + 20) - 15(x + 20) = 0$

$\Rightarrow$ $(x + 20)(x - 15) = 0$

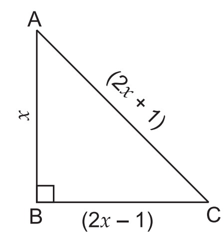
$\Rightarrow$ $x = -20$ rejected

or $x = 15$

$\therefore$ AB = **15 cm**

and BC = (15 + 5) cm = **20 cm**

7. Let ΔABC be a right triangle in which $\angle B = 90°$, altitude AB = x cm.



Then, hypotenuse AC = $(2x + 1)$ cm

and base BC = $(2x + 1 - 2)$ cm = $(2x - 1)$ cm

In right triangle ABC, we have

$$AC^2 = AB^2 + BC^2$$ [By Pythagoras' Theorem]

$\Rightarrow$ $(2x + 1)^2 = x^2 + (2x - 1)^2$

$\Rightarrow$ $4x^2 + 4x + 1 = x^2 + 4x^2 - 4x + 1$

$\Rightarrow$ $x^2 - 8x = 0$

$\Rightarrow$ $x(x - 8) = 0$

$\Rightarrow$ $x = 0$ (rejected)

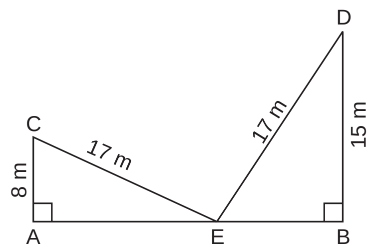
or $x = 8$

$\therefore$ AB = 8 cm, BC = $(2 \times 8 - 1)$ = 15 cm and

AC = $(2 \times 8 + 1)$ cm = 17 cm

Hence, the lengths of the triangle are **8 cm, 15 cm** and **17 cm**.

8. Let AB be the street and let C and D be the windows at heights of 8 m and 15 m respectively from the ground. Let E be the foot of the ladder. Then, EC and ED are the two positions of the ladder.



Clearly, AC = 8 m, BD = 15 m, EC = ED = 17 m and $\angle CAE = \angle DBE = 90°$.

In right triangle CAE, we have

$$AC^2 + AE^2 = CE^2$$ [By Pythagoras' Theorem]

$\Rightarrow$ $(8 \text{ m})^2 + AE^2 = (17 \text{ m})^2$

$\Rightarrow$ $AE^2 = (289 - 64) \text{ m}^2 = 225 \text{ m}^2$

$\Rightarrow$ AE = 15 m … (1)

In right triangle DBE, we have

$$BD^2 + EB^2 = ED^2$$ [By Pythagoras' Theorem]

$\Rightarrow$ $(15 \text{ m})^2 + EB^2 = (17 \text{ m})^2$

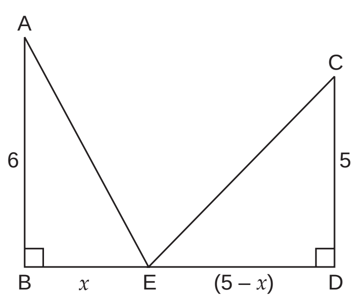
$\Rightarrow$ $EB^2 = (289 - 225) \text{ m}^2 = 64 \text{ m}^2$

$\Rightarrow$ EB = 8 m … (2)

Hence, the width of the street

AB = AE + EB = 15 m + 8 m = **23 m** [Using (1) and (2)]

9. Let AB and CD be the two walls 5 m apart such that a ladder kept at E, x m from wall AB touches the wall at A and touches the wall CD at C, keeping the foot of the ladder fixed.



Then, BE = x m, ED = $(5 - x)$ m, AB = 6 m, CD = 5 m and $\angle ABE = \angle CDE = 90°$.

In right triangle ABE, we have

$$AB^2 + BE^2 = AE^2$$ [By Pythagoras' Theorem]

$\Rightarrow$ $(6)^2 + x^2 = AE^2$ … (1)

In right triangle CDE, we have

$$CD^2 + ED^2 = CE^2$$

$\Rightarrow$ $(5)^2 + (5 - x)^2 = CE^2$ … (2)

From (1) and (2), we get

$$(6)^2 + x^2 = (5)^2 + (5 - x)^2$$ [$\because$ AE = CE = length of the ladder]

$\Rightarrow$ $36 + x^2 = 25 + 25 - 10x + x^2$

$\Rightarrow$ $14 - 10x = 0$

$\Rightarrow$ $2(7 - 5x) = 0$

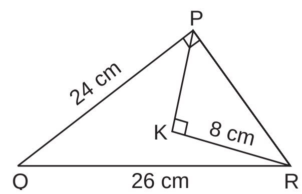
$\Rightarrow$ $7 - 5x = 0$

$\Rightarrow$ $5x = 7$

$\Rightarrow$ $x = \frac{7}{5} = 1.4$

Hence, the distance of the foot of the ladder from the first wall is **1.4 m**.

10. In right triangle QPR, we have



$$QR^2 = PQ^2 + PR^2$$ [By Pythagoras' Theorem]

$\Rightarrow$ $(26 \text{ cm})^2 = (24 \text{ cm})^2 + PR^2$

$\Rightarrow$ $PR^2 = (676 - 576) \text{ cm}^2$

$\Rightarrow$ $PR^2 = 100 \text{ cm}^2$

$\Rightarrow$ PR = 10 cm … (1)

In right triangle PKR, we have

$$PK^2 + KR^2 = PR^2$$

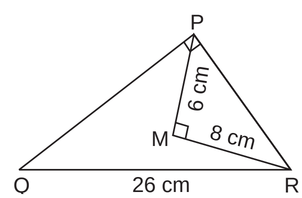
$\Rightarrow$ $PK^2 + (8 \text{ cm})^2 = (10 \text{ cm})^2$ [Using (1)]

$\Rightarrow$ $PK^2 = (100 - 64) \text{ cm}^2$

$\Rightarrow$ $PK^2 = 36 \text{ cm}^2$

$\Rightarrow$ PK = **6 cm**

11. In right ΔPMR, we have



$$PR^2 = PM^2 + MR^2$$ [By Pythagoras' Theorem]

$= (6 \text{ cm})^2 + (8 \text{ cm})^2 = 100 \text{ cm}^2$

$\Rightarrow$ PR = 10 cm … (1)

In right ΔQPR, we have

$$QR^2 = PQ^2 + PR^2$$ [By Pythagoras' Theorem]

$\Rightarrow$ $(26 \text{ cm})^2 = PQ^2 + (10 \text{ cm})^2$ [Using (1)]

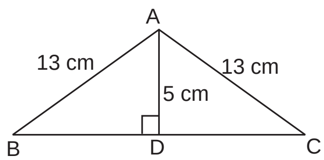
$\Rightarrow$ $PQ^2 = (676 - 100) \text{ cm}^2 = 576 \text{ cm}^2$

$\Rightarrow$ PQ = 24 cm … (2)

$$\text{ar}(\Delta PQR) = \frac{1}{2} \times PQ \times PR$$
$$= \frac{1}{2} \times 24 \times 10$$
$$= 120 \text{ cm}^2 \quad \text{[Using (1) and (2)]}$$

Hence, area (ΔPQR) = **120 cm²**.

12. Let AD be the altitudes of isosceles ΔABC in which AB = AC = 13 cm.



Then, altitude AD = 5 cm.

In right triangles ΔADB and ΔADC, we have

$$AB = AC \quad \text{[Given]}$$
$$AD = AD \quad \text{[Common]}$$
$\therefore$ $\Delta ADB \cong \Delta ADC$ [By RHS Congruency]

$\therefore$ $BD = CD$ [CPCT] … (1)

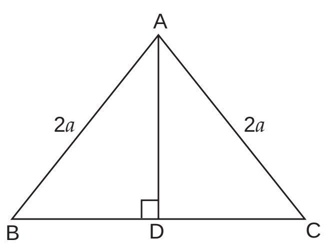
In right ΔADB, we have

$$AB^2 = AD^2 + BD^2 \quad \text{[By Pythagoras' theorem]}$$
$\Rightarrow$ $BD^2 = (169 - 25) \text{ cm}^2$
$= 144 \text{ cm}^2$

$\Rightarrow$ $BD = 12$ cm … (2)

Now, $BC = BD + CD$
$= BD + BD$
$= 2\,BD$
$= 2 \times 12$
= **24 cm** [Using (1) and (2)]

13. Let ABC be an equilateral triangle with side $2a$.



Then, $AB = BC = CA = 2a$

Let AD $\perp$ BC.

In right ΔADB and right ΔADC, we have

$AB = AC$ [Sides of an equilateral triangle]

$AD = AD$ [Common]

$\therefore$ $\Delta ADB \cong \Delta ADC$ [By RHS congruency]

$\Rightarrow$ $BD = DC$ [By CPCT] … (1)

and $BC = BD + DC$
$= BD + BD$
$= 2\,BD$ [Using (1)]

$\Rightarrow$ $BD = \frac{BC}{2} = \frac{2a}{2} = a$ … (2)

In right ΔADB, we have

$$AB^2 = AD^2 + BD^2 \quad \text{[By Pythagoras' Theorem]}$$
$\Rightarrow$ $(2a)^2 = AD^2 + a^2$ [Using (2)]

$\Rightarrow$ $4a^2 = AD^2 + a^2$

$\Rightarrow$ $AD^2 = 3a^2$

$\Rightarrow$ $AD = \sqrt{3}a$

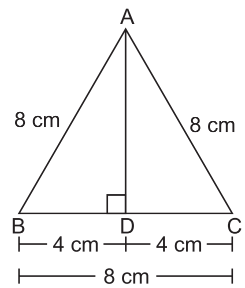
Hence, the altitudes of an equilateral triangle with side $2a$ is $\boldsymbol{\sqrt{3}a}$.

14. Given that ΔABC is an equilateral triangle such that

$$AB = BC = AC = 8 \text{ cm}.$$

AD is an altitude of ΔABC from the vertex A to the side BC.

Then D is the mid-point of BC.



$\therefore$ $BD = \frac{8}{2}$ cm = 4 cm, AB = 8 cm and $\angle ADB = 90°$

To find the length of the altitude AD of ΔABC.

From right-angled triangle ABD, we have by Pythagoras' theorem

$$AB^2 = AD^2 + BD^2$$
$\Rightarrow$ $8^2 = AD^2 + 4^2$

$\Rightarrow$ $AD^2 = 64 - 16 = 48$

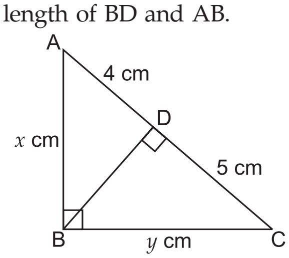
$\therefore$ $AD = \sqrt{48} = 4\sqrt{3}$ cm.

Hence, the required length of the altitude is $\mathbf{4\sqrt{3}}$ cm.

15. Given that $\angle ABC = 90°$ in a triangle ABC.

Also, BD $\perp$ AC where D is a point on AC and AD = 4 cm and DC = 5 cm.

To find the length of BD and AB.



Let AB = x cm and BC = y cm.

In ΔABD, $\angle ADB = 90°$, AB = x cm and AD = 4 cm.

$\therefore$ By Pythagoras' theorem, we have

$$AB^2 = BD^2 + AD^2$$
$\Rightarrow$ $x^2 = 4^2 + BD^2$
$= 16 + BD^2$ …(1)

Similarly, from right-angled triangle BDC, we have by Pythagoras' theorem,

$$BC^2 = BD^2 + DC^2$$
$\Rightarrow$ $y^2 = BD^2 + 5^2$

$\Rightarrow$ $y^2 = 25 + BD^2$ …(2)

Also, from ΔABC,

$\because \quad \angle ABC = 90°,$

$\therefore \quad AB^2 + BC^2 = AC^2$

$\Rightarrow \quad x^2 + y^2 = (5 + 4)^2 = 81 \quad \ldots(3)$

Adding (1) and (2), we have

$x^2 + y^2 = 25 + 16 + 2BD^2$

$= 41 + 2BD^2$

$\Rightarrow \quad 81 - 41 = 2BD^2 \quad$ [From (3)]

$\therefore \quad BD^2 = \frac{40}{2} = 20$

$\therefore \quad BD = \sqrt{20} = 2\sqrt{5} \quad \ldots(4)$

Also, from (1) and (4),

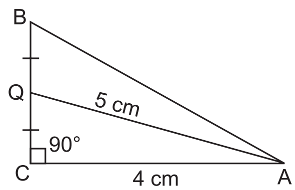
$x^2 = 16 + 20 = 36$

$\therefore \quad x = \sqrt{36} = 6$

$\Rightarrow \quad AB = 6 \quad \ldots(5)$

Hence, the required length of **BD** and **AB** are respectively $\mathbf{2\sqrt{5}}$ cm and **6 cm**.

16. Given that ΔABC is a right-angled triangle with $\angle ACB = 90°$, Q is the mid-point of BC and AC = 4 cm, AQ = 5 cm. To find AB^2.



From ΔAQC,

$\because \quad \angle ACQ = 90°$

$\therefore$ By Pythagoras' theorem, we have

$AQ^2 = AC^2 + QC^2$

$\Rightarrow \quad 25 = 16 + QC^2$

$\therefore \quad QC = \sqrt{25 - 16} = 3$

From ΔABC,

$\because \quad \angle C = 90°,$

$\therefore \quad AB^2 = AC^2 + BC^2$

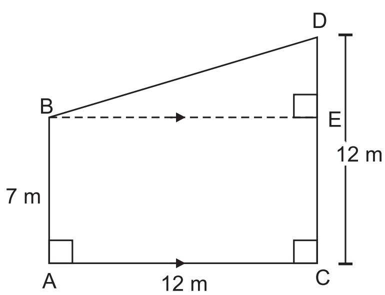
$= 4^2 + (2QC)^2$

$= 16 + 4 \times 9$

$= 52.$

$\therefore$ The required value of AB^2 is **52 cm²**.

17. Let AB and CD be the given vertical poles.



Then, AB = 7 m, CD = 12 m and AC = 12 m

Draw $\quad BE \parallel AC.$

Then, $\quad BE = AC = 12$ m,

$DE = DC - CE = 12 \text{ m} - AB$

$[\because CE = AB]$

$= 12 \text{ m} - 7 \text{ m}$

$= 5$ m

In right triangle DEB, we have

$BD^2 = BE^2 + DE^2$

[By Pythagoras' Theorem]

$= (12 \text{ m})^2 + (5 \text{ m})^2$

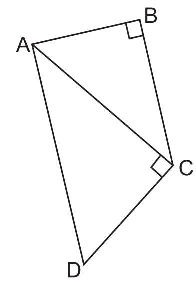
$= (144 + 25) \text{ m}^2$

$= 169 \text{ m}^2$

$\Rightarrow \quad BD = 13$ m

Hence, the distance between the tips of the poles is **13 m**.

18.



$AD^2 = AB^2 + BC^2 + CD^2 \quad$ [Given]

$\Rightarrow \quad AD^2 = AC^2 + CD^2$

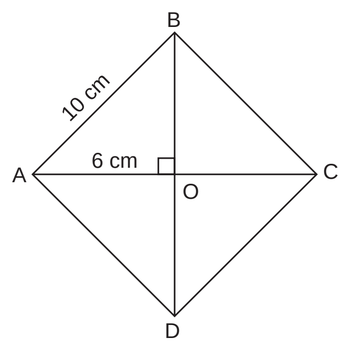
$[\because AB^2 + BC^2 = AC^2$, Pythagoras' Theorem]

By the converse of Pythagoras theorem,

$\angle ACD = 90°$

Hence, $\mathbf{\angle ACD = 90°}$.

19. Let ABCD be the given rhombus, whose diagonals AC and BD intersect at O.



Then, $\quad AB = 10$ cm

Let $\quad AC = 12$ cm

Since the diagonals of a rhombus intersect each other at right angles.

$\therefore \quad OA = \frac{1}{2} AC = 6$ cm.

In right ΔAOB, we have

$AB^2 = OA^2 + OB^2$

[By Pythagoras' Theorem]

$\Rightarrow \quad (10 \text{ cm})^2 = (6 \text{ cm})^2 + OB^2$

$\Rightarrow \quad OB^2 = (100 - 36) \text{ cm}^2$

$= 64 \text{ cm}^2$

$\Rightarrow \quad OB = 8$ cm

$BD = 2 \times OB$

$= (2 \times 8)$ cm

$= 16$ cm.

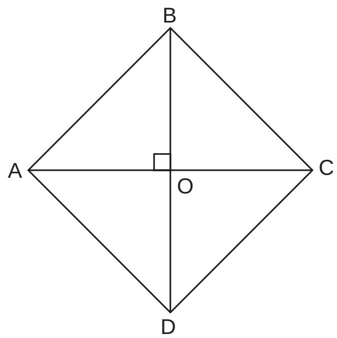
Hence, the length of the second diagonal is **16 cm**.

Area of the rhombus = $\frac{1}{2} d_1 d_2 = \frac{1}{2} \times 12 \times 16 \text{ cm}^2$

$= 96 \text{ cm}^2$

Hence, the area of the rhombus is **96 cm²**.

20. Let ABCD be the given rhombus in which diagonal AC = 15 cm and diagonal BD = 36 cm.
Let the diagonals AC and BD intersect at O.



Since the diagonals of a rhombus bisect each other at right angles,

$\therefore \quad \angle AOB = 90°,$

$AO = \frac{1}{2} AC = \frac{15}{2} \text{ cm}$

and $\quad BO = \frac{1}{2} BD = \frac{36}{2} \text{ cm} = 18 \text{ cm}$

In right triangle AOB, we have

$AB^2 = AO^2 + BO^2$ [By Pythagoras' Theorem]

$\Rightarrow \quad AB^2 = \left(\frac{15}{2}\text{cm}\right)^2 + (18 \text{ cm})^2$

$= \left(\frac{225}{4} + 324\right)$

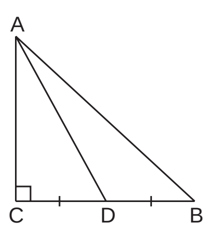
$= \frac{225 + 1296}{4}$

$= \frac{1521}{4} \text{ cm}^2$

$\Rightarrow \quad AB = \frac{39}{2} \text{ cm}$

Perimeter of rhombus = 4 × AB = 4 × $\frac{39}{2}$ cm = **78 cm**

21. In right triangle ACB, we have



$AB^2 = AC^2 + BC^2$ [Pythagoras' Theorem]

$= AC^2 + (2\ CD)^2$ [$\because$ BC = 2 CD]

$= AC^2 + 4\ CD^2$

$= AC^2 + 4\ (AD^2 - AC^2)$

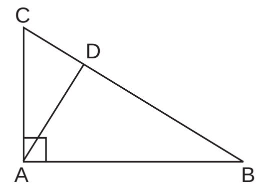
[$\because AC^2 + CD^2 = AD^2$, By Pythagoras' Theorem]

$= AC^2 + 4\ AD^2 - 4\ AC^2$

$= 4\ AD^2 - 3\ AC^2$

Hence, $\mathbf{AB^2 = 4AD^2 - 3AC^2}$.

22. In right triangle ADB, we have



$AB^2 = AD^2 + BD^2$

[By Pythagoras' Theorem]

$\Rightarrow \quad AB^2 - BD^2 = AD^2 \quad \ldots (1)$

In right triangle ADC, we have

$AC^2 = AD^2 + CD^2$

$\Rightarrow \quad AC^2 - CD^2 = AD^2 \quad \ldots (2)$

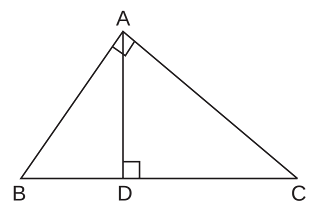
From (1) and (2), we get

$AB^2 - BD^2 = AC^2 - CD^2$

$\Rightarrow \quad \mathbf{AB^2 + CD^2 = BD^2 + AC^2}$

For Standard Level

23. In right ΔBAC, we have



$BC^2 = AB^2 + AC^2$

[By Pythagoras' Theorem] … (1)

In right ΔADB, we have

$AB^2 = AD^2 + BD^2$

[By Pythagoras' Theorem] … (2)

In right ΔADC, we have

$AC^2 = AD^2 + CD^2$

[By Pythagoras' Theorem] … (3)

From (2) and (3), we get

$AB^2 + AC^2 = 2\ AD^2 + BD^2 + CD^2$

$\Rightarrow \quad BC^2 = 2\ AD^2 + BD^2 + CD^2$

[Using (1)]

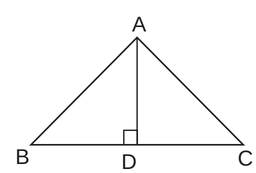
$\Rightarrow \quad (BD + CD)^2 = 2\ AD^2 + BD^2 + CD^2$

$\Rightarrow \quad BD^2 + CD^2 + 2\ BD \times CD = 2\ AD^2 + BD^2 + CD^2$

$\Rightarrow \quad 2\ BD \times CD = 2\ AD^2$

$\Rightarrow \quad \mathbf{AD^2 = BD \times CD}$

24. In right triangle ADB, we have



$$AB^2 = AD^2 + BD^2$$

[By Pythagoras' Theorem] … (1)

In right triangle ADC, we have

$$AC^2 = AD^2 + CD^2$$

[By Pythagoras' Theorem] … (2)

Adding (1) and (2), we get

$$AB^2 + AC^2 = BD^2 + CD^2 + 2\ AD^2$$

$\Rightarrow$ $AB^2 + AC^2 = BD^2 + CD^2 + 2\ BD \times DC$

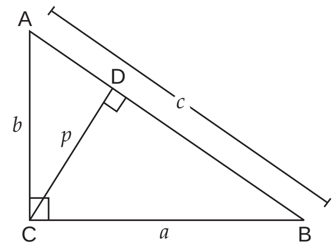
[$\because$ $AD^2 = BD \times DC$, given]

$\Rightarrow$ $AB^2 + AC^2 = (BD + CD)^2$

$\Rightarrow$ $AB^2 + AC^2 = (BC)^2$

$\therefore$ By the converse of Pythagoras Theorem, **ΔABC is a right triangle**.

25.



(*i*) Area of $\Delta ABC = \frac{1}{2}$ base × height

$\therefore$ $ar(\Delta ABC) = \frac{1}{2} AB \times CD$

[Taking AB as base]

$\Rightarrow$ $ar(\Delta ABC) = \frac{1}{2} cp$ … (1)

and $ar(\Delta ABC) = \frac{1}{2} BC \times AC$

[Taking BC as base]

$\Rightarrow$ $ar(\Delta ABC) = \frac{1}{2} ab$ … (2)

From (1) and (2), we get

$$\frac{1}{2} cp = \frac{1}{2} ab$$

$\Rightarrow$ $cp = ab$

Hence, ***cp = ab***.

(*ii*) $cp = ab$ [Proved in (*i*)]

$\Rightarrow$ $\frac{1}{p} = \frac{c}{ab}$

$\Rightarrow$ $\frac{1}{p^2} = \frac{c^2}{a^2b^2}$

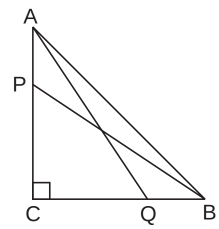
$= \frac{b^2 + a^2}{a^2b^2}$ [$\because AB^2 = AC^2 + BC^2$]

$\Rightarrow$ $\frac{1}{p^2} = \frac{b^2}{a^2b^2} + \frac{a^2}{a^2b^2}$

$= \frac{1}{a^2} + \frac{1}{b^2}$

Hence, $\boldsymbol{\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}}$.

26.



(*i*) Since P divides CA in the ratio 2 : 1,

$\therefore$ $CP = \frac{2}{3} AC$ … (1)

And Q divides CB in the ratio 2 : 1,

$\therefore$ $CQ = \frac{2}{3} BC$ … (2)

In right triangle ACQ, we have

$$AQ^2 = AC^2 + CQ^2$$

[By Pythagoras' Theorem]

$\Rightarrow$ $AQ^2 = AC^2 + \frac{4}{9} BC^2$ [Using (2)]

$\Rightarrow$ $9AQ^2 = 9AC^2 + 4BC^2$ … (3)

Hence, $\mathbf{9AQ^2 = 9AC^2 + 4BC^2}$.

(*ii*) In right triangle BCP, we have

$$BP^2 = BC^2 + CP^2$$

[By Pythagoras' Theorem]

$\Rightarrow$ $BP^2 = BC^2 + \frac{4}{9} AC^2$ [Using (1)]

$\Rightarrow$ $9BP^2 = 9BC^2 + 4AC^2$ … (4)

Hence, $\mathbf{9BP^2 = 9BC^2 + 4AC^2}$.

(*iii*) Adding (3) and (4), we get

$$9AQ^2 + 9BP^2 = 9AC^2 + 9BC^2 + 4BC^2 + 4AC^2$$

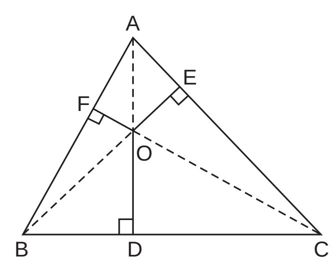
$\Rightarrow$ $9(AQ^2 + BP^2) = 13(AC^2 + BC^2)$

$\Rightarrow$ $9(AQ^2 + BP^2) = 13AB^2$

[$\because$ $AB^2 = AC^2 + BC^2$, By Pythagoras' Theorem]

Hence, $\mathbf{9(AQ^2 + BP^2) = 13AB^2}$.

27. In right triangle of OFA, we have



$$OA^2 = OF^2 + AF^2$$

[By Pythagoras' Theorem]

$$AF^2 = OA^2 - OF^2 \quad \text{… (1)}$$

In right triangle ODB, we have

$$OB^2 = OD^2 + BD^2$$

[By Pythagoras' Theorem]

$\Rightarrow$ $BD^2 = OB^2 - OD^2$ … (2)

In right triangle OEC,

$$OC^2 = OE^2 + CE^2$$

[By Pythagoras' Theorem]

$\Rightarrow \quad CE^2 = OC^2 - OE^2 \quad \ldots (3)$

Adding (1), (2) and (3), we get

$$AF^2 + BD^2 + CE^2 = OA^2 - OF^2 + OB^2 - OD^2 + OC^2 - OE^2$$

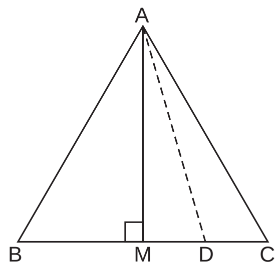
$$\Rightarrow \quad AF^2 + BD^2 + CE^2 = (OA^2 - OE^2) + (OB^2 - OF^2) + (OC^2 - OD^2)$$

$$\Rightarrow \quad AF^2 + BD^2 + CE^2 = AE^2 + BF^2 + CD^2$$

$$\Rightarrow \quad (AF^2 - AE^2) + (BD^2 - BF^2) + (CE^2 - CD^2) = 0$$

Hence, $\mathbf{(AF^2 - AE^2) + (BD^2 - BF^2) + (CE^2 - CD^2) = 0}$.

28. Draw AM ⊥ BC.



In right ∆s AMB and AMC, we have

AB = AC [Sides of equilateral triangle]

AM = AM [Common]

∴ ∆AMB ≅ ∆AMC [By RHS Congruency]

$\therefore \quad BM = MC = \frac{BC}{2}$ [CPCT] … (1)

$CD = \frac{BC}{4}$ [Given]

and $BD = BC - CD$

$= BC - \frac{BC}{4}$

$= \frac{BC}{4}$ … (2)

In ∆ABD, ∠B is acute

$$\therefore \quad AD^2 = AB^2 + BD^2 - 2BD \times BM$$

$$= AB^2 + \left(\frac{3}{4}BC\right)^2 - 2 \times \frac{3}{4}BC \times \frac{BC}{2}$$

[Using (1) and (2)]

$$= AB^2 + \frac{9BC^2}{16} - \frac{3}{4}BC^2$$

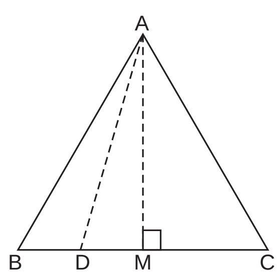
$$\Rightarrow \quad 16\,AD^2 = 16\,BC^2 + 9\,BC^2 - 12\,BC^2$$

[AB = BC, sides of an equilateral triangle]

$$\Rightarrow \quad 16\,AD^2 = 13\,BC^2$$

Hence, $\mathbf{16\,AD^2 = 13\,BC^2}$.

29. Draw AM ⊥ BC.



In right ∆AMB and right ∆AMC, we have

AB = AC [Sides of an equilateral triangle]

AM = AM [Common]

∴ ∆AMB ≅ ∆AMC [By RHS congruency]

∴ BM = CM [By CPCT] … (1)

But BC = BM + CM

$\therefore \quad BM = CM = \frac{BC}{2}$ … (2)

$DM = BM - BD = \frac{BC}{2} - \frac{BC}{4}$

[∵ 4 BD = BC and using (2)]

$\Rightarrow \quad DM = \frac{BC}{4}$ … (3)

In right triangle AMD, we have

$AD^2 = AM^2 + DM^2$ [By Pythagoras' Theorem]

$\Rightarrow \quad AD^2 = AC^2 - CM^2 + DM^2$

[∵ $AC^2 = AM^2 + CM^2$]

$\Rightarrow \quad AD^2 = BC^2 - CM^2 + DM^2$

[∵ AC = BC, sides of an equilateral triangle]

$$\Rightarrow \quad AD^2 = BC^2 - \left(\frac{BC}{2}\right)^2 + \left(\frac{BC}{4}\right)^2$$

[Using (2) and (3)]

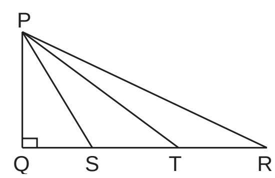
$$\Rightarrow \quad AD^2 = BC^2 - \frac{BC^2}{4} + \frac{BC^2}{16}$$

$$\Rightarrow \quad 16\,AD^2 = 16\,BC^2 - 4BC^2 + BC^2$$

$$\Rightarrow \quad 16\,AD^2 = 13\,BC^2$$

Hence, $\mathbf{16\,AD^2 = 13\,BC^2}$.

30. In right triangle PQS, PQT and PQR, we have



$PS^2 = PQ^2 + QS^2$

$PT^2 = PQ^2 + QT^2$ [By Pythagoras' Theorem]

$PR^2 = PQ^2 + QR^2$

[By Pythagoras' Theorem] … (1)

Now,

$3\,PR^2 + 5\,PS^2 - 8\,PT^2$

$= 3\,(PQ^2 + QR^2) + 5\,(PQ^2 + QS^2) - 8\,(PQ^2 + QT^2)$

[Using (1)]

$= 3\,PQ^2 + 3\,QR^2 + 5\,PQ^2 + 5\,QS^2 - 8\,PQ^2 - 8\,QT^2$

$= 3\,QR^2 + 5\,QS^2 - 8\,QT^2$

$$= 3\,QR^2 + 5\left(\frac{QR}{3}\right)^2 - 8\left(\frac{2}{3}QR\right)^2$$

[∵ Points S and T trisect QR]

$$= 3\,QR^2 + 5\left(\frac{QR^2}{9}\right) - 8\left(\frac{4}{9}QR^2\right)$$

$= 27\,QR^2 + 5\,QR^2 - 32\,QR^2$

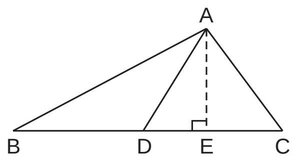
$= 32\,QR^2 - 32\,QR^2$

$= 0$

Thus, $3\,PR^2 + 5\,PS^2 - 8\,PT^2 = 0$

Hence, $\mathbf{8\,PT^2 = 3\,PR^2 + 5\,PS^2}$

31. Draw AE ⊥ BC.



In ΔAED,

$\angle AED = 90°$

$\therefore \quad \angle ADE < 90°$

$\therefore \quad \angle ADB > 90°$

Thus, in ΔADB, $\angle ADB > 90°$ and AE ⊥ BD produced.

$\therefore \quad AB^2 = AD^2 + BD^2 + 2\,BC \times DE$ … (1)

In ΔADC, $\angle ADC < 90°$ and AE ⊥ BC

$\therefore \quad AC^2 = AD^2 + CD^2 - 2\,CD \times DE$

$\Rightarrow \quad AC^2 = AD^2 + BD^2 - 2BD \times DE$

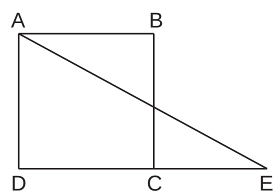
[$\because$ CD = BD] … (2)

Adding (1) and (2), we get

$AC^2 + AB^2 = 2(AD^2 + BD^2)$

Hence, $\mathbf{AB^2 + AC^2 = 2(AD^2 + BD^2)}$

32. In right ΔADE,



$AE^2 = AD^2 + DE^2$

$= AD^2 + (DC + CE)^2$

$= AD^2 + DC^2 + CE^2 + 2\,DC \times CE$

$= DC^2 + DC^2 + CE^2 + 2\,DC\,(DE - DC)$

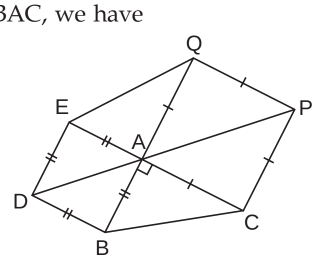
[$\because$ AD = DC]

$= 2DC^2 + CE^2 + 2\,DC \times DE - 2\,DC^2$

$= CE^2 + 2\,DC \times DE$

Hence, $\mathbf{AE^2 = CE^2 + 2DC \times DE}$.

33. In right ΔBAC, we have



$BC^2 = AB^2 + AC^2$

[By Pythagoras' Theorem] … (1)

In right ΔEAQ, we have

$EQ^2 = AE^2 + AQ^2$

[By Pythagoras' Theorem] … (2)

Adding (1) and (2), we get

$BC^2 + EQ^2 = AB^2 + AC^2 + AE^2 + AQ^2$

$= (AB^2 + AE^2) + (AC^2 + AQ^2)$

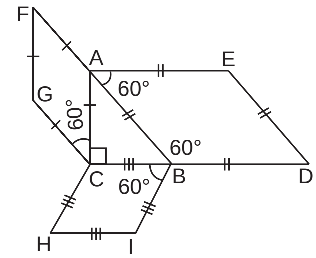
$= (DE^2 + AE^2) + (PQ^2 + AQ^2)$

[$\because$ AB = DE and AC = PQ, Opp. sides of a square]

$= AD^2 + AP^2$ [By Pythagoras' Theorem]

Hence, $\mathbf{BC^2 + EQ^2 = AD^2 + AP^2}$

34. Let AC = b, BC= a and AB = c.



In right triangle, we have

$c^2 = a^2 + b^2$ [By Pythagoras' Theorem] … (1)

Join diagonals BE, CI and GA on rhombus BDEA, CHIB and CGFA respectively.

$\Delta ABE \cong \Delta BDE$ [By SSS similarity]

$\Rightarrow$ ar(ΔABE) = ar(ΔBDE) … (2)

$\therefore$ ΔABE is equilateral.

$\Rightarrow$ ΔBDE is also equilateral. … (3)

Area (rhombus BDEA) = 2(arΔABE)

$= 2\dfrac{\sqrt{3}}{2}c^2$ [Using (2) and (3)]

$= \sqrt{3}\,c^2$ … (4)

Similarly, area (rhombus CHIB) = $\sqrt{3}\,a^2$

and area (rhombus CGFA) = $\sqrt{3}\,b^2$

ar(rhombus CHIB) + ar(rhombus CGFA)

$= \sqrt{3}\,a^2 + \sqrt{3}\,b^2$

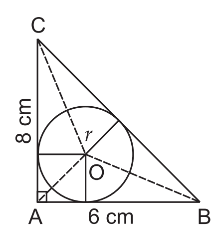
$= \sqrt{3}\,(a^2 + b^2)$ … (5)

From (1), (4) and (5), we get

$\sqrt{3}\,c^2 = \sqrt{3}\,(a^2 + b^2)$

Hence, **area of rhombus on the hypotenuse of a right triangle with one of the angles as 60° is equal to the sum of the areas of rhombuses with one of their angles as 60° drawn on the other two sides**.

35. Let AB = 6 cm and AC = 8 cm.



In right triangle CAB, we have

$BC^2 = AB^2 + AC^2$ [By Pythagoras' Theorem]

$= (6\text{ cm})^2 + (8\text{ cm})^2$

$= 100\text{ cm}^2$

$\Rightarrow \quad BC = 10\text{ cm}$

ar(ΔCAB) = ar(ΔOAB) + ar(ΔOBC) + ar(ΔOCA)

$\Rightarrow \dfrac{1}{2}AB \times AC = \left(\dfrac{1}{2}AB \times r\right) + \left(\dfrac{1}{2}BC \times r\right) + \left(\dfrac{1}{2}CA \times r\right)$

$\Rightarrow \quad \dfrac{1}{2} \times 6 \times 8\text{ cm}^2$

$= \left[\dfrac{1}{2} \times 6 \times r + \dfrac{1}{2} \times 10 \times r + \dfrac{1}{2} \times 8 \times r\right]\text{cm}$

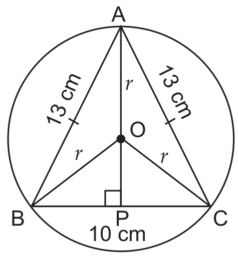
$\Rightarrow \quad 48\text{ cm}^2 = [6r + 10r + 8r]\text{ cm}$

$\Rightarrow \quad 24r = 48$ cm

$\Rightarrow \quad r = 2$ cm

Hence, the radius of the circle is **2 cm**.

36. Given that ΔABC is an isosceles triangle with AB = AC, inscribed in a circle with centre at O. AB = AC = 13 cm and BC = 10 cm, Let OA = OB = OC = r cm be the radius of the circle.



To find the radius r of the circle.

We draw AOP ⊥ BC.

Then P is the middle point of BC.

$\therefore \quad BP = PC = \frac{10}{2}$ cm = 5 cm.

We join OB and OC.

Now, $\quad OP = AP - AO = AP - r \quad$ ...(1)

Now, in ΔAPB, we have by Pythagoras' theorem,

$$AB^2 = AP^2 + PB^2$$

$\Rightarrow \quad 13^2 = AP^2 + \left(\frac{10}{2}\right)^2$

$\Rightarrow \quad AP^2 = 169 - 25 = 144$

$\therefore \quad AP = 12$

$\therefore$ From (1), $\quad OP = 12 - r \quad$ ...(2)

Now, from ΔOBP,

$\because \quad \angle OPB = 90°$

$\therefore$ By Pythagoras' theorem, we have

$$OB^2 = OP^2 + PB^2$$

$\Rightarrow \quad r^2 = (12 - r)^2 + 5^2 \quad$ [From (2)]

$= 144 + r^2 - 24r + 25$

$\Rightarrow \quad 24r = 169$

$\therefore \quad r = \frac{169}{24} = 7.041$ (approx.)

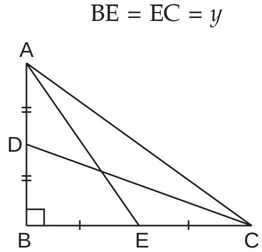
Hence, the required radius of the circle is **7.041 cm** (approx.)

37. Let $\quad AB = 2x$.

Then, $\quad AD = DB = x$

and let $\quad BC = 2y$.

Then, $\quad BE = EC = y$



In right ΔABE, we have

$$AE^2 = AB^2 + BE^2$$

[Pythagoras' Theorem]

$\Rightarrow \quad (5 \text{ cm})^2 = (2x)^2 + (y)^2 \quad$ ... (1)

In right ΔDBC, we have

$$CD^2 = DB^2 + BC^2$$

[Pythagoras' Theorem]

$\Rightarrow \quad (\sqrt{20} \text{ cm})^2 = x^2 + (2y)^2 \quad$ ... (2)

Adding (1) and (2), we get

$$(25 + 20) \text{ cm}^2 = 4x^2 + y^2 + x^2 + 4y^2$$

$\Rightarrow \quad 5x^2 + 5y^2 = 45 \text{ cm}^2$

$\Rightarrow \quad x^2 + y^2 = 9 \text{ cm}^2 \quad$ ... (3)

In right ΔABC, we have

$AC^2 = AB^2 + BC^2$

$= 4x^2 + 4y^2$

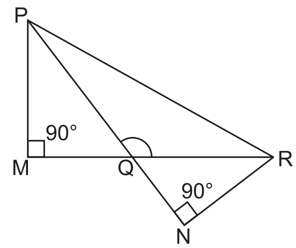
$= 4(x^2 + y^2)$

$= 4 \times 9$

$= 36 \text{ cm}^2$

$\Rightarrow \quad AC =$ **6 cm**

38. Given that in ΔPQR, $\angle PQR$ is obtuse and $\angle PMR = 90°$. Also, RN is ⊥ to PQ produced.



To prove that $PR^2 = PQ \cdot PN + RQ \cdot RM$.

In ΔPQR,

$\because \angle PQR$ is obtuse hence, we have

$PR^2 = PQ^2 + QR^2 + 2PQ \cdot QN \quad$ ...(1)

Also, $PR^2 = PQ^2 + QR^2 + 2QR \cdot QM \quad$ ...(2)

Adding (1) and (2), we get

$2(PR^2) = 2(PQ^2 + QR^2) + 2(PQ \cdot QN + QR \cdot QM)$

$\Rightarrow \quad PR^2 = PQ^2 + QR^2 + PQ \cdot QN + QR \cdot QM$

$= PQ^2 + QR^2 + PQ(PN - PQ) + RQ(RM - QR)$

$= PQ^2 + QR^2 + PQ \cdot PN - PQ^2 + RQ \cdot RM - RQ^2$

$= PQ \cdot PN + RQ \cdot RM.$

Hence, proved.

39. In right ΔAPD and right ΔBQC, we have

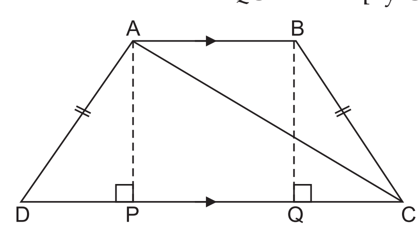
AD = BC $\quad$ [Given]

AP = BQ

[$\because$ AB ∥ DC, $\therefore$ AB ∥ PQ]

$\therefore \quad \Delta APD \simeq \Delta BQC$ [By RHS congruency]

$\therefore \quad DP = QC \quad$ [By CPCT] ...(1)



In right triangle APC,

$$AC^2 = AP^2 + PC^2$$

[By Pythagoras' theorem] ...(2)

In right triangle BQC,

$$BC^2 = BQ^2 + QC^2$$

[By Pythagoras' theorem] ...(3)

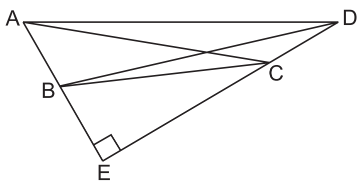
Subtracting equation (3) from equation (2), we get

$$AC^2 - BC^2 = AP^2 - BQ^2 + PC^2 - QC^2$$
$$= BQ^2 - BQ^2 + PC^2 - QC^2 \quad [\because AP = BQ]$$
$$= PC^2 - QC^2$$
$$= (PC + QC)(PC - QC)$$
$$= (PC + QC)(PQ) \quad [\text{Using (1)}]$$
$$= CD \times AB \quad [\therefore PQ = AB]$$

Hence, $\mathbf{AC^2 - BC^2 = AB \times CD}$.

40. Given that ABCD is a quadrilateral such that $\angle A + \angle D = 90°$.

To prove that $AC^2 + BD^2 = AD^2 + BC^2$



Construction: We produce AB and DC to intersect each other at E. We join AC and BD.

$\because \quad \angle A + \angle D = 90° \quad$ [Given]

$\therefore$ The remaining $\angle AED$ of ΔAED is also 90°

[By angle-sum property of a triangle]

Now, in ΔADE, $\angle AED = 90°$.

$\therefore$ By Pythagoras' theorem, we have from ΔAED

$$AD^2 = AE^2 + DE^2 \quad \ldots(1)$$
$[\because \angle AED = 90°]$

Similarly, from ΔBEC, since $\angle BEC = 90°$,

$$\therefore \quad BC^2 = BE^2 + CE^2 \quad \ldots(2)$$

Adding (1) and (2), we get

$$AD^2 + BC^2 = AE^2 + DE^2 + BE^2 + CE^2$$
$$= (AE^2 + CE^2) + (DE^2 + BE^2)$$
$$= AC^2 + BD^2$$

$\because$ In Δ ACE and in ΔBED,

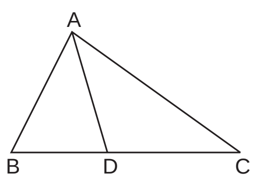
$$\angle AEC = \angle BED = 90°.$$

Hence, $\mathbf{AC^2 + BD^2 = AD^2 + BC^2}$.

EXERCISE 6E
(OPTIONAL)

For Basic and Standard Levels

1. (*i*) AB = 6 cm, AC = 12 cm, BD = 2.5 cm and CD = 5.5 cm



$$\frac{BD}{DC} = \frac{2.5 \text{ cm}}{5.5 \text{ cm}} = \frac{5}{11}$$

and $$\frac{AB}{AC} = \frac{6 \text{ cm}}{12 \text{ cm}} = \frac{1}{2}$$

Since $$\frac{BD}{DC} \neq \frac{AB}{AC},$$

$\therefore$ **AD is not the bisector of $\angle$A of ΔABC.**

(*ii*) AB = 9 cm, AC = 12 cm, BD = 4.5 cm and CD = 6 cm

$$\frac{BD}{DC} = \frac{4.5 \text{ cm}}{6 \text{ cm}} = \frac{3}{4}$$

and $$\frac{AB}{AC} = \frac{9 \text{ cm}}{12 \text{ cm}} = \frac{3}{4}$$

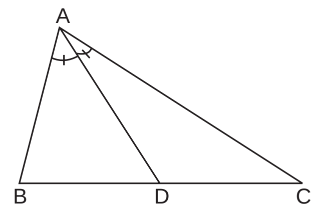
Since $$\frac{BD}{DC} = \frac{AB}{AC},$$

$\therefore$ **AD is the bisector of $\angle$A of ΔABC.**

2. Since AD is the bisector of $\angle$A,

$$\therefore \quad \frac{BD}{DC} = \frac{AB}{AC}$$

[By the Angle-bisector Theorem]



(*i*) $$\frac{5 \text{ cm}}{4 \text{ cm}} = \frac{8 \text{ cm}}{AC}$$

$$\Rightarrow \quad AC = \frac{8 \times 4}{5} \text{ cm}$$
$$= \frac{32}{5} \text{ cm}$$
$$= 6.4 \text{ cm}$$

Hence, **AC = 6.4 cm**.

(*ii*) $$\frac{3 \text{ cm}}{DC} = \frac{6 \text{ cm}}{5 \text{ cm}}$$

$$\Rightarrow \quad DC = \frac{3 \times 5}{6} \text{ cm} = \frac{5}{2} \text{ cm}$$
$$= 2.5 \text{ cm}$$

Hence, **DC = 2.5 cm**.

(*iii*) $$\frac{BD}{DC} = \frac{5 \text{ cm}}{7 \text{ cm}} = \frac{5}{7}$$

Hence, $\mathbf{\frac{BD}{DC} = \frac{5}{7}}$.

$$\frac{BD}{DC} = \frac{5}{7}$$

$$\Rightarrow \quad \frac{BD}{DC} + 1 = \frac{5}{7} + 1$$

$$\Rightarrow \quad \frac{BD + DC}{DC} = \frac{5 + 7}{7}$$

$$\Rightarrow \quad \frac{BC}{DC} = \frac{12}{7}$$

Hence, $\mathbf{\frac{BC}{DC} = \frac{12}{7}}$.

(*iv*) BC = 6 cm, BD = 3.2 cm, DC = BC – BD = (6 – 3.2) cm = 2.8 cm

Now, $$\frac{BD}{DC} = \frac{AB}{AC}$$

$$\Rightarrow \quad \frac{3.2\text{ cm}}{2.8\text{ cm}} = \frac{5.6\text{ cm}}{AC}$$

$$\Rightarrow \quad AC = \frac{5.6 \times 2.8}{3.2}\text{ cm}$$

$$\Rightarrow \quad AC = 4.9\text{ cm}$$

Hence, **AC = 4.9 cm**.

(*v*) Let $BD = x$ cm

Then, $CD = BC - BD = (10.4 - x)$ cm,

Now, $\frac{BD}{DC} = \frac{AB}{AC}$

$$\Rightarrow \quad \frac{x\text{ cm}}{(10.4 - x)\text{cm}} = \frac{9\text{ cm}}{4\text{ cm}}$$

$$\Rightarrow \quad 4x = 93.6 - 9x$$

$$\Rightarrow \quad 13x = 93.6$$

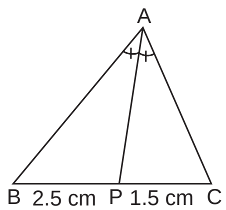
$$\Rightarrow \quad x = 7.2$$

BD = 7.2 cm and CD = (10.4 – 7.2) cm = 3.2 cm.

Hence, **CD = 3.2 cm** and **BD = 7.2 cm**.

3. Let $AB = x$ cm.

Then, $AC = (5.6 - x)$ cm



Since AP bisects ∠A of ΔABC,

$$\therefore \quad \frac{BP}{PC} = \frac{AB}{AC}$$

[By the Angle-bisector Theorem]

$$\Rightarrow \quad \frac{2.5\text{ cm}}{1.5\text{ cm}} = \frac{x\text{ cm}}{(5.6 - x)\text{cm}}$$

$$\Rightarrow \quad \frac{5}{3} = \frac{x}{(5.6 - x)}$$

$$\Rightarrow \quad 5(5.6 - x) = 3x$$

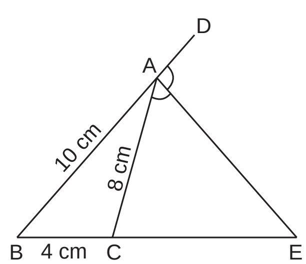
$$\Rightarrow \quad 5 \times 5.6 - 5x = 3x$$

$$\Rightarrow \quad 8x = 5 \times 5.6$$

$$\Rightarrow \quad x = \frac{5 \times 5.6}{8} = 3.5$$

Hence, **AB = 3.5 cm**.

4. Since AE is the bisector of exterior ∠CAD and meets BC produced at E,



$$\therefore \quad \frac{AB}{AC} = \frac{BE}{CE}$$

[By the Angle-bisector theorem]

$$\Rightarrow \quad \frac{10\text{ cm}}{8\text{ cm}} = \frac{BC + CE}{CE}$$

$$\Rightarrow \quad \frac{5}{4} = \frac{4\text{ cm} + CE}{CE}$$

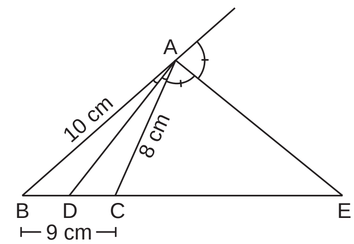
$$\Rightarrow \quad 5\,CE = 16\text{ cm} + 4\,CE$$

$$\Rightarrow \quad (5 - 4)\,CE = 16\text{ cm}$$

$$\Rightarrow \quad CE = 16\text{ cm}$$

Hence, **CE = 16 cm**.

5. Since AD is the internal bisector of ∠A meeting BC at D,



$$\frac{AB}{AC} = \frac{BD}{BC}$$

$$\Rightarrow \quad \frac{10\text{ cm}}{8\text{ cm}} = \frac{BD}{9\text{ cm} - BD}$$

$$\Rightarrow \quad \frac{5}{4} = \frac{BD}{9\text{ cm} - BD}$$

$$\Rightarrow \quad 45\text{ cm} - 5\,BD = 4\,BD$$

$$\Rightarrow \quad 9\,BD = 45\text{ cm}$$

$$\Rightarrow \quad BD = 5\text{ cm}$$

Hence, **BD = 5 cm**.

Since AE is the external bisector of ∠A meeting BC produced at E,

$$\therefore \quad \frac{AB}{AC} = \frac{BE}{CE}$$

$$\Rightarrow \quad \frac{10\text{ cm}}{8\text{ cm}} = \frac{BE}{BE - BC}$$

$$\Rightarrow \quad \frac{5}{4} = \frac{BE}{BE - 9\text{ cm}}$$

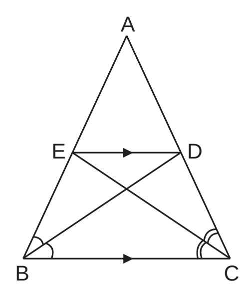
$$\Rightarrow \quad 5\,BE - 45\text{ cm} = 4\,BE$$

$$\Rightarrow \quad 5\,BE - 4\,BE = 45\text{ cm}$$

$$\Rightarrow \quad BE = 45\text{ cm}$$

Hence, **BE = 45 cm**.

6. In ΔABC, BD is the bisector of ∠B.



$$\therefore \quad \frac{AB}{BC} = \frac{AD}{DC} \qquad \ldots(1)$$

[By the Angle–bisector theorem]

In ΔABC, CE is the bisector of ∠C.

$$\frac{AC}{CB} = \frac{AE}{EB} \qquad \ldots(2)$$

[By the Angle-bisector theorem]

Now, $DE \parallel BC$ [Given]

$$\therefore \quad \frac{AE}{EB} = \frac{AD}{DC} \qquad \text{[By BPT]} \ldots (3)$$

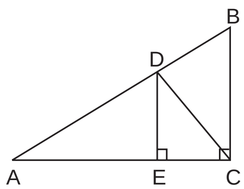
From (1), (2) and (3), we get

$$\frac{AB}{BC} = \frac{AC}{CB}$$

$$\Rightarrow \quad AB = AC$$

Hence, **ΔABC is an isosceles triangle**.

7. In ΔACB, CD is the bisector of $\angle$C.



$$\therefore \quad \frac{AD}{DB} = \frac{AC}{CB}$$

[By the Angle-bisector theorem] … (1)

In ΔAED and ΔACB,

$\angle AED = \angle ACB$ [Each is 90°]

$\angle DAE = \angle BAC$ [Common]

$\therefore$ $\Delta AED \sim \Delta ACB$ [By AA similarity]

$$\Rightarrow \quad \frac{AE}{AC} = \frac{ED}{CB}$$

$$\Rightarrow \quad \frac{AE}{ED} = \frac{AC}{CB} \qquad \ldots (2)$$

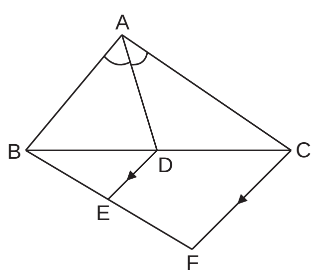
From (1) and (2), we get

$$\frac{AD}{DB} = \frac{AE}{ED}$$

$$\Rightarrow \quad AD \times ED = AE \times DB$$

Hence, **AD × DE = AE × BD**.

8. In ΔABC, AD is the bisector of $\angle$BAC.



$$\therefore \quad \frac{AB}{AC} = \frac{BD}{DC}$$

[By the angle-bisector theorem] … (1)

In ΔBFC, $DE \parallel CF$

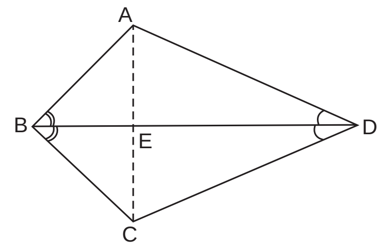
$$\therefore \quad \frac{BE}{EF} = \frac{BD}{DC} \qquad \text{[By BPT]} \ldots (2)$$

From (1) and (2), we get

$$\frac{AB}{AC} = \frac{BE}{EF}$$

Hence, $\mathbf{\frac{AB}{AC} = \frac{BE}{EF}}$.

9. Join diagonal AC and let it intersect diagonal BD at E.
In ΔABC, BE bisects $\angle$ABC.



$$\therefore \quad \frac{AB}{BC} = \frac{AE}{EC}$$

[By the Angle-bisector theorem] … (1)

In ΔADC, DE bisects $\angle$ADC.

$$\therefore \quad \frac{AD}{DC} = \frac{AE}{EC}$$

[By the Angle-bisector theorem] … (2)

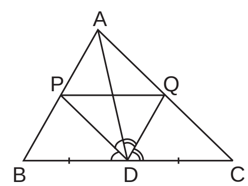
From (1) and (2), we get

$$\frac{AB}{BC} = \frac{AD}{DC}$$

Hence, $\mathbf{\frac{AB}{BC} = \frac{AD}{CD}}$.

For Standard Level

10. In ΔADB, DP is the bisector of $\angle$ADB.



$$\therefore \quad \frac{AP}{PB} = \frac{AD}{BD}$$

[By the Angle-bisector theorem] … (1)

In ΔADC, DQ is the bisector of $\angle$ADC.

$$\therefore \quad \frac{AQ}{QC} = \frac{AD}{DC}$$

[By the Angle-bisector theorem]

$$\Rightarrow \quad \frac{AQ}{QC} = \frac{AD}{BD} \qquad [\because DC = BD] \ldots (2)$$

From (1) and (2), we get

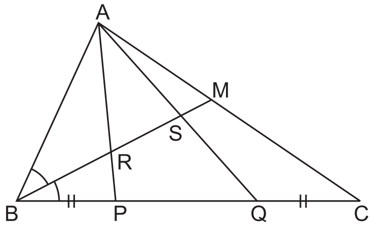
$$\frac{AP}{PB} = \frac{AQ}{QC}$$

Thus, in ΔABC, PQ divides the sides AB and AC in the same ratio.

$\therefore$ By the converse of BPT, $PQ \parallel BC$.

Hence, **PQ $\parallel$ BC**.

11. Given that P and Q are two points on the side BC of ΔABC such that BP = QC. M is a point on AC such that BM bisects $\angle$ABC cutting AP and AQ at R and S respectively.



To prove that $\frac{PR}{AR} + \frac{QS}{SA} = \frac{CM}{MA}$

In ΔABP, BR is the bisector of $\angle$ABP

$$\frac{BP}{AB} = \frac{PR}{AR} \quad \ldots(1)$$

In ΔABQ, BS is the bisector of $\angle$ABQ

$$\therefore \quad \frac{BQ}{AB} = \frac{QS}{AS} \quad \ldots(2)$$

In ΔABC, BM is the bisector of $\angle$ABC

$$\therefore \quad \frac{CB}{AB} = \frac{CM}{AM} \quad \ldots(3)$$

Adding (1) and (2), we get

$$\frac{BP+BQ}{AB} = \frac{PR}{AR} + \frac{QS}{AS}$$

$$\Rightarrow \quad \frac{PR}{AR} + \frac{QS}{AS} = \frac{BP+BQ}{AB}$$

$$= \frac{BP+QC+PQ}{AB}$$

$$= \frac{(BP+PQ)+QC}{AB} \quad [\because BP = QC]$$

$$= \frac{BC}{AB}$$

$$= \frac{CM}{AM} \quad \text{[From (3)]}$$

Hence, proved.

CHECK YOUR UNDERSTANDING

MULTIPLE-CHOICE QUESTIONS

For Basic and Standard Levels

1. **(*b*) 6**

$$\Delta ABC \sim \Delta PQR$$

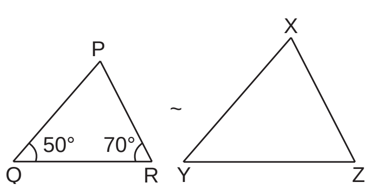
$$\therefore \quad \frac{AB}{PQ} = \frac{BC}{QR}$$

$$\Rightarrow \quad \frac{12}{9} = \frac{8}{x}$$

$$\Rightarrow \quad x = \frac{8 \times 9}{12} = 6$$

Hence, $x = 6$.

2. **(*b*) 110°**



$$\angle P = 180° - (50° + 70°) = 60°$$

$$\Delta PQR \sim \Delta XYZ$$

$$\angle P = 60°$$

$$\therefore \quad \angle X = 60°$$

$$\angle Q = 50°$$

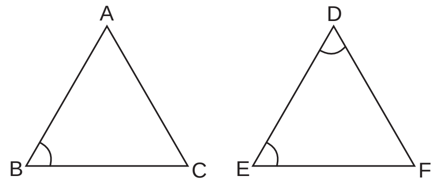
$$\therefore \quad \angle Y = 50°$$

$$\angle X + \angle Y = 110°$$

Hence, $\angle X + \angle Y = 110°$.

3. **(*c*) $\angle$B = $\angle$D**

$$\frac{AB}{DE} = \frac{BC}{FD} \quad \text{[Given]}$$



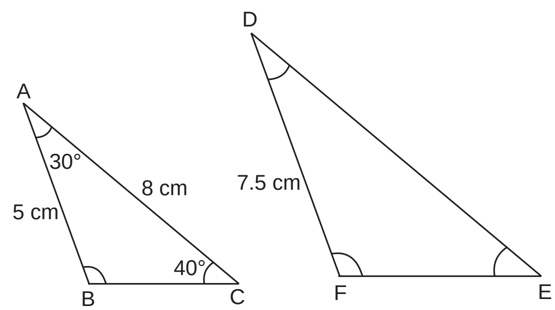
ΔABC will be similar to ΔDEF if the angles included angle between the pair of proportional sides are equal.

$\therefore \quad \angle B = \angle D$

Hence, $\angle$B = $\angle$D.

4. **(*b*) $\angle$F = 110°, DE = 12 cm**

In ΔABC, $\quad \angle B = 180° - (38° + 40°) = \angle 110°$



$$\therefore \quad \angle F = 110°$$

$$\Delta ABC \sim \Delta DFE$$

$$\Rightarrow \quad \frac{AB}{DF} = \frac{BC}{FE} = \frac{AC}{DE}$$

$$\Rightarrow \quad \frac{5\text{ cm}}{7.5\text{ cm}} = \frac{BC}{FE} = \frac{8\text{ cm}}{DE}$$

$$\Rightarrow \quad DE = \frac{8 \times 7.5}{5}\text{ cm} = 12\text{ cm}$$

Hence, $\angle$F = 110°, DE = 12 cm.

5. **(*c*) 7.5 cm**

$$\angle ADE = \angle ABC \quad \text{[Given]}$$

But these are corresponding angles.

$$\therefore \quad DE \parallel BC$$

$$\therefore \quad \frac{AD}{DB} = \frac{AE}{EC} \quad \text{[By BPT]}$$

$$\Rightarrow \quad \frac{2}{5} = \frac{3}{EC}$$

$$\Rightarrow \quad EC = \frac{15}{2} = 7.5$$

Hence, EC = **7.5 cm**.

6. **(*b*) 3 cm**

$$\Delta ABC \sim \Delta PQR$$

$$\Rightarrow \quad \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$$

$$\Rightarrow \quad \frac{6\text{ cm}}{4.5\text{ cm}} = \frac{4\text{ cm}}{x}$$

$$\Rightarrow \quad x = \frac{4 \times 4.5}{6}\text{ cm} = 3\text{ cm}$$

Hence, $x = 3$ cm.

7. **(*d*) 10 cm**

$$\Delta ADE \sim \Delta ABC \quad \text{[By AA similarity]}$$

$$\therefore \quad \frac{AD}{AB} = \frac{DE}{BC}$$

$$\Rightarrow \quad \frac{2\text{ cm}}{2\text{ cm} + 3\text{ cm}} = \frac{4\text{ cm}}{x}$$

$$\Rightarrow \quad x = \frac{4 \times 5}{2}\text{ cm} = 10\text{ cm}$$

Hence, $x = 10$ cm.

8. **(*a*) 13.5 cm**

$$\frac{AP}{PB} = \frac{3.5\text{ cm}}{7\text{ cm}} = \frac{1}{2}$$

and $$\frac{AQ}{QC} = \frac{3\text{ cm}}{6\text{ cm}} = \frac{1}{2}$$

Clearly, $$\frac{AP}{PB} = \frac{AQ}{QC}.$$

Thus, in ΔABC, PQ divides sides AB and AC in the same ratio.
By the converse of Thales Theorem,

$$PQ \parallel BC$$

$$\Delta APQ \sim \Delta ABC \quad \text{[By AA similarity]}$$

$$\Rightarrow \quad \frac{AP}{AB} = \frac{PQ}{BC}$$

$$\Rightarrow \quad \frac{3.5\text{ cm}}{(3.5 + 7)\text{cm}} = \frac{4.5\text{ cm}}{BC}$$

$$\Rightarrow \quad BC = \frac{4.5 \times 10.5}{3.5}\text{ cm} = 13.5\text{ cm}$$

Hence, BC = 13.5 cm.

9. **(*a*) DE ∥ BC**

$$\frac{AD}{DB} = \frac{1}{3}$$

and $$\frac{AE}{EC} = 1 : 3$$

Clearly, $$\frac{AD}{DB} = \frac{AE}{EC}.$$

Thus, in ΔABC, DE divides the sides AB and AC in the same ratio.
∴ By the converse of BPT, DE ∥ BC.
Hence, DE ∥ BC.

10. **(*c*) $\mathbf{PQ = \frac{BC}{3}}$**

Since $$\frac{AP}{PB} = \frac{AQ}{QC},$$

∴ By the converse of BPT, PQ ∥ BC.

$$\Delta APQ \sim \Delta ABC \quad \text{[By AA similarity]}$$

$$\therefore \quad \frac{AP}{AB} = \frac{PQ}{BC}$$

$$\Rightarrow \quad \frac{AP}{AP + PB} = \frac{PQ}{BC}$$

$$\Rightarrow \quad \frac{1}{1 + 2} = \frac{PQ}{BC}$$

$$\Rightarrow \quad \frac{1}{3} = \frac{PQ}{BC}$$

$$\Rightarrow \quad PQ = \frac{BC}{3}$$

Hence, $PQ = \frac{BC}{3}$.

11. **(*b*) 1 : 6**

$$\angle ADC = \angle ABC \quad \text{[Given]}$$

But ∠ADC and ∠ABC are corresponding angles.

$$\therefore \quad DE \parallel BC$$

In ΔABC, we have

$$\frac{AE}{EC} = \frac{AD}{DB} \quad \text{[By BPT]}$$

$$\Rightarrow \quad \frac{4}{8} = \frac{AD}{DB}$$

$$\Rightarrow \quad \frac{AD}{DB} = \frac{1}{2}$$

$$\Rightarrow \quad DB = 2\ AD$$

$$= 2\ (AF + FD) \quad \ldots (1)$$

$$\angle AEF = \angle ACD \quad \text{[Given]}$$

But ∠AEF and ∠ACD are corresponding angles.

$$\therefore \quad FE \parallel DC$$

In ΔADC, we have

$$\frac{AF}{FD} = \frac{AE}{EC}$$

$$\Rightarrow \quad \frac{1}{FD} = \frac{4}{8}$$

$$\Rightarrow \quad FD = 2\text{ units} \quad \ldots (2)$$

Now, $$\frac{AF}{DB} = \frac{1}{2(AF + FD)} \quad \text{[Using (1)]}$$

$$\Rightarrow \quad \frac{AF}{DB} = \frac{1}{2(1 + 2)} \quad \text{[Using (2)]}$$

$$\Rightarrow \quad \frac{AF}{DB} = \frac{1}{6}$$

$$\Rightarrow \quad AF : DB = 1 : 6$$

Hence, AF : DB = 1 : 6.

12. **(*d*) OA = 3.6 cm, OB = 4.8 cm**

$$\Delta ABO \sim \Delta DCO \quad \text{[Given]}$$

$$\Rightarrow \quad \frac{AB}{CD} = \frac{BO}{OC} = \frac{OA}{OD}$$

$$\Rightarrow \quad \frac{3\text{ cm}}{2\text{ cm}} = \frac{OB}{3.2\text{ cm}} = \frac{OA}{2.4\text{ cm}}$$

$$\Rightarrow \quad OB = \frac{3 \times 3.2}{2}\text{ cm} = 4.8\text{ cm}$$

and $$OA = \frac{3 \times 2.4}{2}\text{ cm} = 3.6\text{ cm}$$

Hence, OA = 3.6 cm and OB = 4.8 cm.

13. **(*c*) 7.2 cm**

$$\Delta AOB \sim \Delta COD \quad \text{[By AA similarity]}$$

$$\Rightarrow \frac{BO}{DO} = \frac{AB}{CD}$$

$$\Rightarrow \frac{BO}{(BD - BO)} = \frac{AB}{CD}$$

$$\Rightarrow \frac{BO}{(12\text{ cm} - BO)} = \frac{9\text{ cm}}{6\text{ cm}} = \frac{3}{2}$$

$$\Rightarrow 2\,BO = 36\text{ cm} - 3\,BO$$

$$\Rightarrow 5\,BO = 36\text{ cm}$$

$$\Rightarrow BO = 7.2\text{ cm}$$

Hence, BO = 7.2 cm.

14. **(*d*) $x = 3$, $y = 4$**

$$\Delta ATS \sim \Delta AQP \quad \text{[By AA similarity]}$$

$$\therefore \frac{AT}{AQ} = \frac{TS}{QP} = \frac{AS}{AP}$$

$$\Rightarrow \frac{6}{6} = \frac{4}{y} = \frac{3}{x}$$

$$\Rightarrow y = 4 \text{ and } x = 3$$

Hence, $x = 3$ and $y = 4$.

15. **(*b*) 16 cm**

In ΔPQR and ΔPRS, we have

$$\angle PQR = \angle PRS \quad \text{[Given]}$$

and $\angle QPR = \angle RPS$ [Common]

$$\therefore \Delta PQR \sim \Delta PRS \quad \text{[By AA similarity]}$$

$$\Rightarrow \frac{PQ}{PR} = \frac{PR}{PS}$$

$$\Rightarrow \frac{PQ}{8\text{ cm}} = \frac{8\text{ cm}}{4\text{ cm}}$$

$$\Rightarrow PQ = 16\text{ cm}$$

Hence, PQ = 16 cm.

16. **(*d*) 5.6 cm**

$$\Delta AED \sim \Delta ABC \quad \text{[Given]}$$

$$\Rightarrow \frac{AE}{AB} = \frac{DE}{CB}$$

$$\Rightarrow \frac{12\text{ cm}}{(16 + 14)\text{ cm}} = \frac{DE}{14\text{ cm}}$$

$$\Rightarrow DE = \frac{12 \times 14}{30}\text{ cm} = 5.6\text{ cm}$$

Hence, DE = 5.6 cm.

17. **(*d*) 110°**

ΔAOC and ΔDOB, we have

$$\frac{OA}{OD} = \frac{12\text{ cm}}{10\text{ cm}} = \frac{6}{5}$$

and $\frac{OC}{OB} = \frac{6\text{ cm}}{5\text{ cm}} = \frac{6}{5}$

and included ∠AOC = ∠DOB = 40° [V. opp ∠s]

$$\therefore \Delta AOC \sim \Delta DOB \quad \text{[By SAS similarity]}$$

$$\therefore \angle OCA = \angle OBD = 180° - 30° - 40° = 110°$$

Hence, ∠OCA = 110°.

18. **(*d*) 7.5 cm**

In ΔABC, we have

$$\frac{AP}{AB} = \frac{3\text{ cm}}{5\text{ cm}} = \frac{3}{5}$$

and $\frac{AQ}{AC} = \frac{6\text{ cm}}{10\text{ cm}} = \frac{3}{5}$

Thus, in ΔABC, PQ divides the sides AB and AC in the same ratio.

∴ By the converse of BPT, PRQ ∥ BDC.

In ΔAPR and ΔABD, we have

$$\angle APR = \angle ABD$$

[Corresponding ∠s, PR ∥ BD]

and $\angle PAR = \angle BAD$ [Common]

$$\therefore \Delta APR \sim \Delta ABD \quad \text{[By AA similarity]}$$

$$\Rightarrow \frac{AP}{AB} = \frac{AR}{AD}$$

$$\Rightarrow \frac{3\text{ cm}}{5\text{ cm}} = \frac{4.5\text{ cm}}{AD}$$

$$\Rightarrow AD = \frac{4.5 \times 5}{3}\text{ cm} = 7.5\text{ cm}$$

Hence, AD = 7.5 cm.

19. **(*d*) 30 cm**

$$\Delta PQR \sim \Delta XYZ \quad \text{[Given]}$$

We know that ratio of the perimeter of two similar triangles is the same as the ratio of their corresponding sides.

$$\therefore \frac{\text{Perimeter of } \Delta PQR}{\text{Perimeter of } \Delta XYZ} = \frac{PQ}{XY}$$

$$\Rightarrow \frac{\text{Perimeter of } \Delta PQR}{XY+YZ+ZX} = \frac{PQ}{XY}$$

$$\Rightarrow \frac{\text{Perimeter of } \Delta PQR}{(4 + 4.5 + 6.5)\text{ cm}} = \frac{8\text{ cm}}{4\text{ cm}}$$

$$\Rightarrow \text{Perimeter of } \Delta PQR = \frac{8}{4} \times 15\text{ cm} = 30\text{ cm}$$

Hence, the perimeter of ΔPQR is 30 cm.

20. **(*c*) 9 : 1**

$$\Delta ABC \sim \Delta DEF \quad \text{[Given]}$$

Since the ratio of the areas of two similar triangles is equal to the ratio of the squares of any two corresponding sides,

$$\therefore \frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{BC^2}{EF^2} = \frac{BC^2}{\left(\frac{1}{3}BC\right)^2} = \frac{9}{1}$$

Hence, ar(ΔABC) : ar(ΔDEF) = 9 : 1.

21. **(*d*) $\frac{9}{25}$**

In ΔAPQ and ΔABC, we have

$$\angle APQ = \angle ABC$$

[Corresponding ∠s, PQ ∥ BC]

$$\angle PAQ = \angle BAC \quad \text{[Common]}$$

$$\therefore \Delta APQ \sim \Delta ABC$$

$$\therefore \quad \frac{AP}{AB} = \frac{PQ}{BC} \quad \dots (1)$$

$$\frac{AP}{PB} = \frac{3}{2} \quad \text{[Given]}$$

$$\Rightarrow \quad \frac{PB}{AP} = \frac{2}{3} \quad \text{[Taking reciprocals]}$$

$$\Rightarrow \quad \frac{PB}{AP} + 1 = \frac{2}{3} + 1$$

$$\Rightarrow \quad \frac{PB + AP}{AP} = \frac{5}{3}$$

$$\Rightarrow \quad \frac{AB}{AP} = \frac{5}{3}$$

$$\Rightarrow \quad \frac{AP}{AB} = \frac{3}{5} \quad \dots (2)$$

From (1) and (2), we get

$$\therefore \quad \frac{PQ}{BC} = \frac{3}{5}$$

$$\Delta POQ \sim \Delta COB \quad \text{[By AA similarity]}$$

$$\therefore \quad \frac{ar(\Delta POQ)}{ar(COB)} = \frac{PQ^2}{BC^2} = \left(\frac{PQ}{BC}\right)^2 = \left(\frac{3}{5}\right)^2 = \frac{9}{25}$$

Hence, $\frac{ar(\Delta POQ)}{ar(\Delta COB)} = \frac{9}{25}$

22. (*b*) 81 : 25

Since the ratio of two similar triangles is equal to the ratio of the squares of any two corresponding sides,

$\left(\frac{9}{5}\right)^2 = \frac{A_1}{A_2}$ where A_1 and A_2 are the areas of the similar triangles respectively

$$\Rightarrow \quad \frac{81}{25} = \frac{A_1}{A_2}$$

Hence, the ratio of the areas of two similar triangles is 81 : 25.

23. (*c*) 3.5 cm

Since the ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding altitudes,

$$\therefore \quad \frac{100 \text{ cm}^2}{49 \text{ cm}^2} = \frac{(5 \text{ cm})^2}{(x)^2}$$

where x is the corresponding altitude of the smaller triangle.

$$\Rightarrow \quad x^2 = \frac{25 \times 49}{100} \text{ cm}$$

$$\Rightarrow \quad x^2 = \frac{49}{4} \text{ cm}$$

$$\Rightarrow \quad x = \frac{7}{2} \text{ cm} = 3.5 \text{ cm}$$

Hence, the corresponding altitude of the smaller triangle is 3.5 cm.

24. (*b*) 9.6 cm

Since the ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding medians,

$$\therefore \quad \frac{121 \text{ cm}^2}{64 \text{ cm}^2} = \frac{(13.2 \text{ cm})^2}{x^2}$$

where x is the corresponding median of the other triangle.

$$\Rightarrow \quad x^2 = \frac{13.2 \times 13.2 \times 64}{121} \text{ cm}^2$$

$$\Rightarrow \quad x = \frac{13.2 \times 8}{11} \text{ cm}$$

$$= 1.2 \times 8 \text{ cm}$$

$$= 9.6 \text{ cm}$$

Hence, the corresponding median of the other triangle is 9.6 cm.

25. (*d*) 5 cm^2

In ΔANM and ΔABC, we have

$\angle ANM = \angle ABC$

[Corresponding angles, NM ∥ BC]

$\angle NAM = \angle BAC$ [Common]

$\therefore \quad \Delta ANM \sim \Delta ABC$ [By AA similarity]

$$\therefore \quad \frac{ar(\Delta ANM)}{ar(\Delta ABC)} = \frac{AN^2}{AB^2}$$

$$\Rightarrow \quad \frac{ar(\Delta ANM)}{20 \text{ cm}^2} = \frac{AN^2}{(2AN)^2}$$

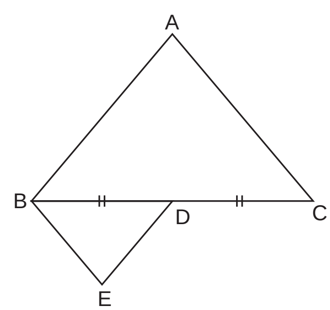
[AB = 2AN, ∵ N is the mid-point of AB]

$$\Rightarrow \quad ar(\Delta ANM) = \frac{20}{4} \text{ cm}^2 = 5 \text{ cm}^2$$

Hence, the ar(ΔANM) is 5 cm^2.

26. (*c*) 4 : 1

Let each side of ΔABC be $2x$.



Then, BD = x and each side of $\Delta BDE = x$.

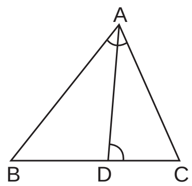
$\Delta ABC \sim \Delta BDE$

[By AA similarity, ∵ they are equiangular]

$$\therefore \quad \frac{ar(\Delta ABC)}{ar(\Delta BDE)} = \frac{(2x)^2}{(x)^2} = \frac{4}{1}$$

Hence, the ratio of areas of ΔABC and ΔBDE is 4 : 1.

27. **(*a*)** $\mathbf{\frac{CA}{CD} = \frac{CB}{CA}}$



In ΔCAB and ΔCDA, we have

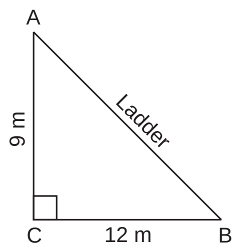
$$\angle CAB = \angle CDA \quad \text{[Given]}$$
$$\angle ACB = \angle DCA \quad \text{[Common]}$$
$$\therefore \quad \Delta CAB \sim \Delta CDA$$
$$\Rightarrow \quad \frac{CA}{CD} = \frac{CB}{CA}$$

[Corresponding sides of similar Δs are proportional]

Hence, $\frac{CA}{CD} = \frac{CB}{CA}$.

28. **(*c*) 15 m**

Let AB be the ladder and AC be the wall with the window at A.



Then, AC = 9 m, BC = 12 m and $\angle ACB = 90°$.

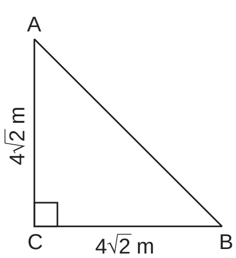
Then, $AB^2 = AC^2 + BC^2$ [By Pythagoras' Theorem]

$$= (9 \text{ m})^2 + (12 \text{ m})^2$$
$$= (81 + 144) \text{ m}^2$$
$$= 225 \text{ m}^2$$
$$\Rightarrow \quad AB = 15 \text{ m}$$

Hence, the length of the ladder is 15 m.

29. **(*b*) 8 cm**

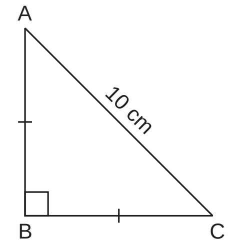
$$\text{Hypotenuse} = \sqrt{(4\sqrt{2}\,\text{cm})^2 + (4\sqrt{2}\,\text{cm})^2}$$
$$= \sqrt{(32+32)\text{cm}^2}$$
$$= \sqrt{64} \text{ cm} = 8 \text{ cm}$$



Hence, the length of the hypotenuse is 8 cm.

30. **(*b*) 10($\sqrt{2}$ + 1) cm**

Let ABC be an isosceles right triangle in which $\angle B = 90°$ and AB = BC.



In right triangle ABC, we have

$$AC^2 = AB^2 + BC^2$$
$$\Rightarrow \quad (10 \text{ cm})^2 = 2\,AB^2 \quad [\because AB = BC]$$
$$\Rightarrow \quad 100 \text{ cm}^2 = 2\,AB^2$$
$$\Rightarrow \quad AB^2 = 50 \text{ cm}^2$$
$$\Rightarrow \quad AB = \sqrt{50} \text{ cm} = 5\sqrt{2} \text{ cm}$$

Perimeter of ΔABC = AB + BC + AC

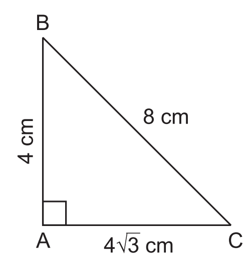
$$= AB + AB + AC$$
$$= 2AB + AC$$
$$= (2 \times 5\sqrt{2} + 10) \text{ cm}$$
$$= (10\sqrt{2} + 10) \text{ cm}$$
$$= 10(\sqrt{2} + 1) \text{ cm}$$

Hence, the perimeter of the isosceles right triangle is 10($\sqrt{2}$ + 1) cm.

31. **(*d*) 90°**

$$AB^2 + AC^2 = (4 \text{ cm})^2 + (4\sqrt{3} \text{ cm})^2$$
$$= (16 + 48) \text{ cm}^2$$
$$= 64 \text{ cm}^2$$
$$BC^2 = (8 \text{ cm})^2$$
$$= 64 \text{ cm}^2$$

Clearly, $BC^2 = AB^2 + AC^2$.



By the converse of Pythagoras theorem,

$$\angle A = 90°$$

Hence, $\angle A = 90°$.

32. **(*b*) 30°**

In ΔPQR, $\angle Q = 75°$, $\angle R = 45°$.

$$\therefore \quad \angle P = 180° - 120° = 60°$$

[Sum of $\angle$s of a Δ] … (1)

In ΔPQR, $\frac{PQ}{PR} = \frac{QM}{MR}$

$\therefore$ PM bisects $\angle P$.

[By the converse of angle-bisector theorem]

$$\therefore \quad \angle QPM = \frac{1}{2}\angle P = \frac{1}{2} \times 60° \quad \text{[Using (1)]}$$
$$= 30°$$

Hence, $\angle QPM = 30°$.

33. **(*c*) 1.75 cm**

$$\angle EAB + \angle BAD + \angle DAC = 180^\circ$$

$$\Rightarrow \quad 110^\circ + \angle BAD + 35^\circ = 180^\circ$$

$$\angle BAD = 180^\circ - 110^\circ - 35^\circ = 35^\circ$$

Thus, $\angle BAD = \angle DAC = 35^\circ$

$\Rightarrow$ AD is the bisector of $\angle BAC$.

In ΔBAC, we have

$$\frac{AB}{AC} = \frac{BD}{DC}$$

[By the Angle-bisector theorem]

$$\Rightarrow \quad \frac{5\text{ cm}}{7\text{ cm}} = \frac{BC - CD}{CD}$$

$$\Rightarrow \quad \frac{5}{7} = \frac{3\text{ cm} - CD}{CD}$$

$$\Rightarrow \quad 5\,CD = 21\text{ cm} - 7\,CD$$

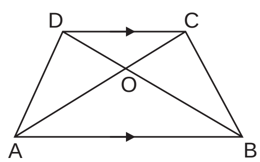
$$\Rightarrow \quad 12\,CD = 21\text{ cm}$$

$$\Rightarrow \quad CD = \frac{21}{12}\text{ cm} = \frac{7}{4}\text{ cm} = 1.75\text{ cm}$$

Hence, CD = 1.75 cm.

34. **(*d*) 21 cm²**

$\Delta COD \sim \Delta AOB$ [By AA similarity]



$$\Rightarrow \quad \frac{ar(\Delta COD)}{ar(\Delta AOB)} = \frac{CD^2}{AB^2} = \frac{CD^2}{(2\,CD)^2} = \frac{1}{4}$$

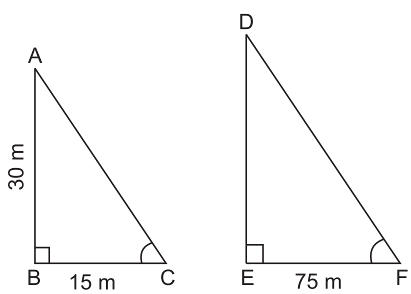
$$\Rightarrow \quad \frac{ar(\Delta COD)}{84\text{ cm}^2} = \frac{1}{4}$$

$$\Rightarrow \quad ar(\Delta COD) = \frac{84}{4}\text{ cm}^2 = 21\text{ cm}^2$$

Hence, ar(ΔCOD) = 21 cm^2.

35. **(*a*) 150 m**

Let AB be the vertical stick and DE be the tower. Let BC and EF represent the shadows of the stick and the tower respectively.



Then, AB = 30 m, EF = 75 m, BC = 15 m.

In ΔABC and ΔDEF, we have

$$\angle ABC = \angle DEF = 90^\circ$$

$$\angle ACB = \angle DFE$$

[Angular elevation of the Sun at the same time]

$$\therefore \quad \Delta ABC \sim \Delta DEF$$

$$\Rightarrow \quad \frac{AB}{DE} = \frac{BC}{EF}$$

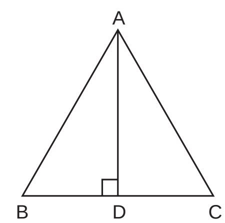
$$\Rightarrow \quad \frac{30\text{ m}}{DE} = \frac{15\text{ m}}{75\text{ m}}$$

$$\Rightarrow \quad DE = \frac{30 \times 75}{15}\text{ m} = 150\text{ m}$$

Hence, the height of the tower is 150 m.

36. **(*c*) $\frac{a\sqrt{3}}{2}$**

Let ABC be an equilateral triangle of side a and let AD be its altitude.



$\Delta ADB \cong \Delta ADC$ [By RHS congruency]

$$\therefore \quad BD = DC\left(= \frac{BC}{2}\right) \quad \text{[CPCT]}$$

$$\Rightarrow \quad BD = \frac{a}{2}$$

In right ΔADB, we have

$$AB^2 = AD^2 + BD^2$$

[By Pythagoras' Theorem]

$$a^2 = AD^2 + \left(\frac{a}{2}\right)^2$$

$$\Rightarrow \quad AD^2 = a^2 - \frac{a^2}{4} = \frac{3a^2}{4}$$

$$\Rightarrow \quad AD = \frac{\sqrt{3}}{2}a$$

Hence, the altitude of an equilateral triangle of side a is $\frac{\sqrt{3}}{2}a$.

37. **(*c*) $\frac{36}{49}$**

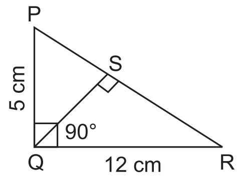
$$\Delta ABC \sim \Delta PQR$$

$$\Rightarrow \quad \frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AB^2}{PQ^2} = \frac{(1.2\text{cm})^2}{(1.4\text{cm})^2} = \frac{36}{49}$$

Hence, $\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{36}{49}$.

38. **(*c*) $\frac{60}{13}$ cm**

In right ΔPQR, we have



$$PR^2 = PQ^2 + QR^2$$
$$= (5\text{ cm})^2 + (12\text{ cm})^2$$
$$= (25 + 144)\text{ cm}^2$$
$$= 169\text{ cm}^2$$
$$\Rightarrow \quad PR = 13\text{ cm} \quad \ldots (1)$$

$$ar(\Delta PQR) = \frac{1}{2} \times QR \times PQ$$
$$= \frac{1}{2} \times 12 \times 5\text{ cm}^2$$
$$= 30\text{ cm}^2$$
[Taking QR as base] … (2)

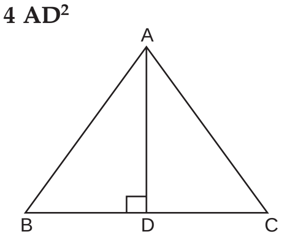
$$ar(\Delta PQR) = \frac{1}{2} \times PR \times QS$$
$$= \frac{1}{2} \times 13\text{ cm} \times QS$$
[Using (1)] … (3)

From (2) and (3), we get

$$\frac{1}{2} \times 13\text{ m} \times QS = 30\text{ cm}^2$$
$$\Rightarrow \quad QS = \frac{30 \times 2}{13}\text{ cm} = \frac{60}{13}\text{ cm}$$

Hence, $QS = \frac{60}{13}$ cm.

39. **(*b*) 3 AB^2 = 4 AD^2**



$$\Delta ADB \cong \Delta ADC \text{ [By RHS congruency]}$$
$$\Rightarrow \quad BD = DC = \frac{BC}{2} = \frac{AB}{2} \quad \ldots (1)$$

In right ΔADB, we have

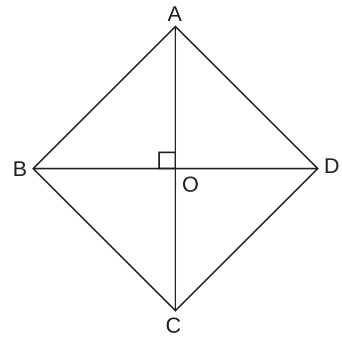
$$AB^2 = AD^2 + BD^2$$
[By Pythagoras' Theorem]
$$\Rightarrow \quad AD^2 = AB^2 - BD^2$$
$$\Rightarrow \quad AD^2 = AB^2 - \left(\frac{AB}{2}\right)^2$$
$$\Rightarrow \quad AD^2 = AB^2 - \frac{AB^2}{4} = \frac{3AB^2}{4}$$
$$\Rightarrow \quad 4\,AD^2 = 3\,AB^2$$

Hence, 3 AB^2 = 4 AD^2.

40. **(*b*) 6 cm**

Let ABCD be a rhomubs of side 5 cm, whose one of the diagonal say BD = 8 cm.
Let diagonals AC and BD intersect at O.

Then, $\angle AOB = 90°$,
$$BO = OD = 4\text{ cm}$$
and $$AO = OC \quad \ldots (1)$$



In right triangle AOB, we have

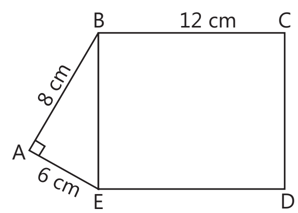
$$AB^2 = AO^2 + BO^2$$
[By Pythagoras' Theorem]
$$\Rightarrow \quad (5\text{ cm})^2 = AO^2 + (4\text{ cm})^2$$
$$\Rightarrow \quad AO^2 = (25 - 16)\text{ cm}^2 = 9\text{ cm}^2$$
$$\Rightarrow \quad AO = 3\text{ cm} \quad \ldots (2)$$
$$AC = AO + OC$$
$$= 3\text{ cm} + 3\text{ cm} \quad \text{[Using (1) and (2)]}$$
$$= 6\text{ cm}$$

Hence, the length of the second diagonal is 6 cm.

For Standard Level

41. **(*d*) 120 cm^2**

In right ΔBAE, we have

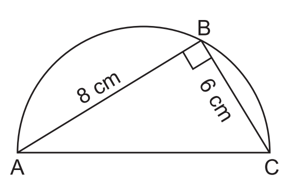


$$BE^2 = AB^2 + AE^2$$
[By Pythagoras' Theorem]
$$= (8\text{ cm})^2 + (6\text{ cm})^2$$
$$= (64 + 36)\text{ cm}^2$$
$$= 100\text{ cm}^2$$
$$\Rightarrow \quad BE = 10\text{ cm} \quad \ldots (1)$$
$$ar(\text{rectangle BCDE}) = BC \times BE$$
$$= 12\text{ cm} \times 10\text{ cm} \quad \text{[Using (1)]}$$
$$= 120\text{ cm}^2$$

Hence, the area of rectangle BCDE is 120 cm^2.

42. **(*c*) 10 cm**

$$\angle ABC = 90° \quad \text{[Angle in a semicircle]}$$

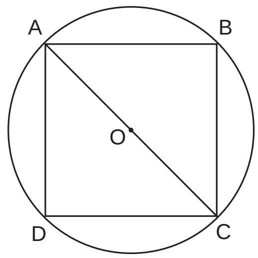


In right ΔABC, we have

$$AC^2 = AB^2 + BC^2$$
$$= (8\text{ cm})^2 + (6\text{ cm})^2$$
$$= 100\text{ cm}^2$$
$$\Rightarrow \quad AC = 10\text{ cm}$$
$$\text{Diameter} = 10\text{ cm}$$

43. **(*d*) 128 cm²**

Let ABCD be the square inscribed in a circle of radius 8 cm.



$\Rightarrow$ Diameter AC = 16 cm

Diameter of the circle = Diagonal of the inscribed square

$\Rightarrow$ $16 \text{ cm} = \sqrt{2}\,\text{side}$

$\Rightarrow$ $\text{side} = \frac{16}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} \text{ cm}$

$= 8\sqrt{2} \text{ cm}$

ar(sq ABCD) = side × side

$= 8\sqrt{2} \times 8\sqrt{2} \text{ cm}^2$

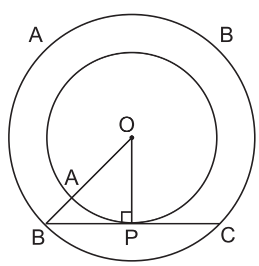
$= 128 \text{ cm}^2$

Hence, the area of inscribed square is 128 cm².

44. **(*b*) 16 cm**

Let O be the centre of the concentric circle in which radius OA = 15 cm and radius OB = 17 cm.

Let BC be the chord of the larger circle which is tangent to the smaller circle at P. Join OP.



Then, $\angle OPB = 90°$

In right ΔOPB, we have

$OB^2 = OP^2 + BP^2$ [By Pythagoras' Theorem]

$\Rightarrow$ $OB^2 = OA^2 + BP^2$ [∵ OP = OA, radius of a circle]

$\Rightarrow$ $(17 \text{ cm})^2 = (15 \text{ cm})^2 + BP^2$

$\Rightarrow$ $BP^2 = (289 - 225) \text{ cm} = 64 \text{ cm}^2$

$\Rightarrow$ BP = 8 cm … (1)

Since perpendicular from the point of contact to centre of a circle to a chord bisects it,

∴ BP = PC … (2)

Now BC = BP + PC

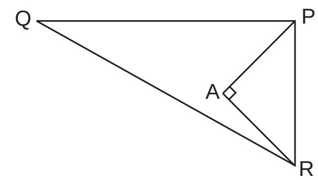
= (8 + 8) cm

= 16 cm [Using (1) and (2)]

Hence, the length of the larger chord which is tangent to the smaller circle is 16 cm.

45. **(*b*) 90°**

In right ΔPAR, we have



$PR^2 = PA^2 + AR^2$ [By Pythagoras' Theorem]

$= (6 \text{ cm})^2 + (8 \text{ cm})^2$

$= (36 + 64) \text{ cm}^2$

$= 100 \text{ cm}^2$

$\Rightarrow$ PR = 10 cm … (1)

In ΔQPR, we have

$QR^2 = (26 \text{ cm})^2 = 676 \text{ cm}^2$

and $PQ^2 + PR^2 = (24 \text{ cm})^2 + (10 \text{ cm})^2$ [Using (1)]

$= (576 + 100) \text{ cm}^2$

$= 676 \text{ cm}^2$

Clearly, $QR^2 = PQ^2 + PR^2$.

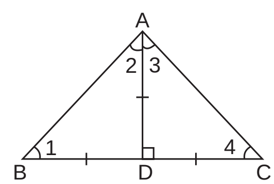
∴ By the converse of Pythagoras' Theorem,

$\angle QPR = 90°$

Hence, $\angle QPR = 90°$.

46. **(*c*) AB² + AC² = BC²**

In ΔBAD, BD = AD



∴ $\angle 1 = \angle 2 = x$ (say)

Similarly, $\angle 3 = \angle 4 = x$

$\angle 1 + \angle 2 + \angle 3 + \angle 4 = 4x = 180°$

$\Rightarrow$ $x = 45°$

$\Rightarrow$ $\angle BAC = 90°$

In right ΔBAC, we have

$BC^2 = AB^2 + AC^2$

Hence, $AB^2 + AC^2 = BC^2$.

47. **(*d*) DC² = CF·AC**

In ΔABC and ΔDEC, we have

$\angle CAB = \angle CDE$ [Corresponding ∠s, AB ∥ DE]

$\angle ACB = \angle DCE$ [Common]

∴ ΔABC ~ ΔDEC [By AA similarity]

$\Rightarrow$ $\frac{AC}{DC} = \frac{BC}{EC}$ … (1)

Similarly, ΔCDB can be proved similar to ΔCFE.

$\Rightarrow$ $\frac{DC}{FC} = \frac{BC}{EC}$ … (2)

From (1) and (2), we get

$\frac{AC}{DC} = \frac{DC}{FC}$

$\Rightarrow$ $AC \cdot FC = DC^2$

Hence, $DC^2 = CF \cdot AC$.

48. **(*d*) 3 : 2**

$$\frac{ar(\Delta ALM)}{ar(Trap\ LMCB)} = \frac{9}{16}$$

$$\Rightarrow \frac{ar(trap\ LMCB)}{ar(\Delta ALM)} + 1 = \frac{16}{9} + 1$$

$$\Rightarrow \frac{ar(trap\ LMCB) + ar(\Delta ALM)}{ar(\Delta ALM)} = \frac{16+9}{9}$$

$$\Rightarrow \frac{ar(\Delta ABC)}{ar(\Delta ALM)} = \frac{25}{9}$$

$$\Rightarrow \frac{ar(\Delta ALM)}{ar(\Delta ABC)} = \frac{9}{25} \quad \ldots (1)$$

$\Delta ALM \sim \Delta ABC$

[By AA similarity]

$$\therefore \frac{ar(\Delta ALM)}{ar(\Delta ABC)} = \frac{AL^2}{AB^2} \quad \ldots (2)$$

From (1) and (2), we get

$$\frac{AL^2}{AB^2} = \frac{9}{25}$$

$$\Rightarrow \frac{AL}{AB} = \frac{3}{5}$$

$$\Rightarrow \frac{AB}{AL} = \frac{5}{3}$$

$$\Rightarrow \frac{AB}{AL} - 1 = \frac{5}{3} - 1$$

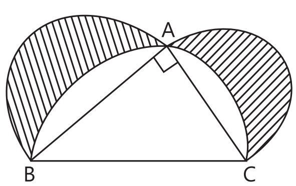
$$\Rightarrow \frac{AB - AL}{AL} = \frac{5-3}{3}$$

$$\Rightarrow \frac{LB}{AL} = \frac{2}{3}$$

$$\Rightarrow \frac{AL}{LB} = \frac{3}{2}$$

Hence, AL : LB = 3 : 2.

49. **(*b*) ar(ΔABC)**



In ΔBAC, we have

$BC^2 = AB^2 + AC^2$

[By Pythagoras' Theorem] … (1)

area of shaded portion

= area of semicircle on AB as diameter

+ area of semicircle AC as diameter

+ area of ΔBAC – area of semicircle on BC as diameter

$$= \frac{1}{2}\left[\pi\left(\frac{AB}{2}\right)^2\right] + \frac{1}{2}\left[\pi\left(\frac{AC}{2}\right)^2\right] + ar(\Delta ABC) - \frac{1}{2}\left[\pi\left(\frac{BC}{2}\right)^2\right]$$

$$= \frac{\pi}{8}[AB^2 + AC^2 - BC^2] + ar(\Delta ABC) = \frac{\pi}{8}(0) + ar(\Delta ABC)$$

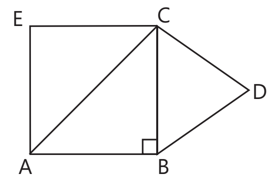
[Using (1)]

= ar(ΔABC)

Hence, the area of the shaded region is ar(ΔABC).

50. **(*d*) 10 cm^2**

Let AB = BC = x cm.



Then, in right ΔABC, we have

$AC^2 = AB^2 + BC^2$

[By Pythagoras' Theorem]

$\Rightarrow AC^2 = (x^2 + x^2)\ cm^2 = 2x^2$

$\Rightarrow AC = \sqrt{2}\,x$ cm

$$ar(\Delta BCD) = \frac{\sqrt{3}}{2} BC^2$$

$$= \frac{\sqrt{3}}{2} x^2\ cm^2 \quad \ldots (1)$$

$$ar(\Delta ACE) = \frac{\sqrt{3}}{2} AC^2$$

$$= \frac{\sqrt{3}}{2} (\sqrt{2}x)^2\ cm^2$$

$= 20\ cm^2$ [Given]

$$\Rightarrow \frac{\sqrt{3}}{2}(\sqrt{2}x)(\sqrt{2}x) = 20$$

$$\Rightarrow x^2 = \frac{20}{\sqrt{3}} \quad \ldots (2)$$

Substituting $x^2 = \frac{20}{\sqrt{3}}$ in (1), we get

$$ar(\Delta BCD) = \frac{\sqrt{3}}{2} \times \frac{20}{\sqrt{3}}\ cm^2 = 10\ cm^2$$

Hence, ar(ΔBCD) = 10 cm^2.

TRUE OR FALSE

For Basic and Standard Levels

	True/False	*Justification*
1.	FALSE	For polygons with more than three sides to be similar, the corresponding sides must also be proportional.
2.	TRUE	By AA criteria of similarity, the two triangles are similar.
3.	FALSE	The ratio of areas of two similar triangles is equal to the ratio of the squares of the corresponding altitudes. ∴ Ratio of areas of the two triangles is $2^2 : 3^2 = 4 : 9$.

4. TRUE — The two triangles have their corresponding two sides and the perimeters proportional, so their sides will be proportional.

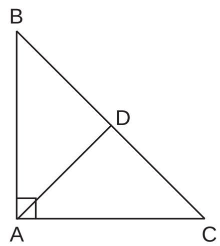
Hence, they will be similar.

5. FALSE — For the two triangles to be similar, the equal angles must be included angles between the two pairs of proportional sides.

SHORT ANSWER QUESTIONS

For Basic and Standard Levels

1.



In ΔABC and ΔDBA, we have

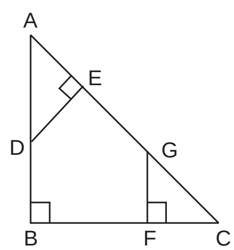
$\angle ABC = \angle DBA$ [Common]

$\angle BAC = \angle BDA$ [Each is equal to 90°]

$\therefore$ $\Delta ABC \sim \Delta DBA$ [By AA similarity]

Hence, **ΔABC ~ ΔDBA**.

2. $AB \perp BC$ and $FG \perp BC$ [Given]



$\therefore$ $AB \parallel FG$

[AB and FG are perpendicular to the same line segment BC]

In ΔADE and ΔGCF, we have

$\angle DAE = \angle CGF$

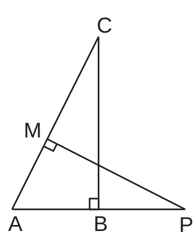
[Corresponding ∠s, AB ∥ FG] … (1)

and $\angle AED = \angle GFC$ [Each is equal to 90°]

$\therefore$ $\Delta ADE \sim \Delta GCF$ [By AA similarity]

Hence, **ΔADE ~ ΔGCF**.

3.



In ΔABC and ΔAMP, we have

$\angle ABC = \angle AMP$ [Each is equal to 90°]

and $\angle CAB = \angle PAM$ [Common]

$\therefore$ $\Delta ABC \sim \Delta AMP$ [By AA similarity]

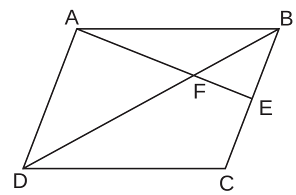
$\Rightarrow$ $\dfrac{CA}{PA} = \dfrac{BC}{MP}$

[Corresponding sides of similar triangles are proportional]

$\Rightarrow$ $CA \times MP = PA \times BC$

Hence, **CA × MP = PA × BC.**

4.



In ΔAFD and ΔEFB, we have

$\angle AFD = \angle EFB$ [Vertically opp. ∠s]

$\angle FAD = \angle FEB$ [Alt ∠s, AD ∥ BC]

$\therefore$ $\Delta AFD \sim \Delta EFB$ [By AA similarity]

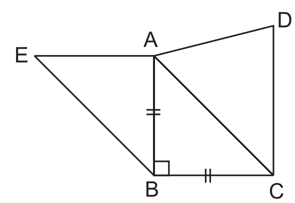
$\Rightarrow$ $\dfrac{FA}{FE} = \dfrac{DF}{BF}$

[Corresponding sides of similar triangles are proportional]

$\Rightarrow$ $FA \times BF = DF \times FE$

Hence, **DF × FE = BF × FA**.

5. $\Delta ACD \sim \Delta ABE$ [Given]



Since the ratio of areas of two similar triangles is equal to the ratio of the squares of any two corresponding sides,

$\therefore$ $\dfrac{ar(\Delta ACD)}{ar(\Delta ABE)} = \dfrac{AC^2}{AB^2} = \dfrac{AB^2 + BC^2}{AB^2}$

[$\because AC^2 = AB^2 + BC^2$]

$\Rightarrow$ $\dfrac{ar(\Delta ACD)}{ar(\Delta ABE)} = \dfrac{AB^2}{AB^2} + \dfrac{BC^2}{AB^2} = 1 + 1 = 2$

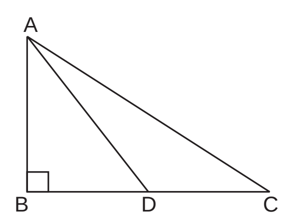
[$\because$ BC= AB, given]

$\Rightarrow$ $\dfrac{ar(\Delta ABE)}{ar(\Delta ACD)} = \dfrac{1}{2}$ [Taking reciprocals]

Hence, ar(ΔABE):ar(ΔACD) = **1 : 2**.

For Standard Level

6.



In right triangle ABC, we have

$$AC^2 = AB^2 + BC^2$$

[By Pythagoras' Theorem]

$\Rightarrow$ $AC^2 = AB^2 + (2\ BD)^2$

[$\because$ D is the mid-point of BC]

$\Rightarrow$ $AC^2 = AB^2 + 4\ BD^2$ … (1)

In right triangle ABD, we have

$$AD^2 = AB^2 + BD^2$$

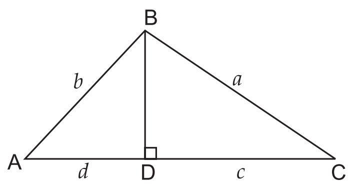
[By Pythagoras' Theorem] … (2)

Subtracting (2) from (1), we get

$$AC^2 - AD^2 = 3\ BD^2.$$

Hence, $\mathbf{AC^2 - AD^2 = 3BD^2}$.

7.



In right ΔBDC, we have

$a^2 = BD^2 + c^2$ [By Pythagoras' Theorem]

$\Rightarrow$ $BD^2 = a^2 - c^2$ … (1)

In right ΔBDA, we have

$b^2 = BD^2 + d^2$ [By Pythagoras' Theorem]

$\Rightarrow$ $BD^2 = b^2 - d^2$ … (2)

From (1) and (2), we get

$$a^2 - c^2 = b^2 - d^2$$

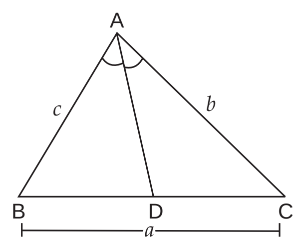
$\Rightarrow$ $a^2 - b^2 = c^2 - d^2$

$\Rightarrow$ $(a + b)(a - b) = (c + d)(c - d)$

$\Rightarrow$ $\dfrac{(a+b)}{(c+d)} = \dfrac{(c-d)}{(a-b)}$

Hence, $\mathbf{\dfrac{a+b}{c+d} = \dfrac{c-d}{a-b}}$.

8. Since AD is the bisector of ∠BAC of ΔBAC of ΔABC,



$\therefore$ $\dfrac{AC}{AB} = \dfrac{CD}{BD}$

[By the angle–bisector theorem]

$\Rightarrow$ $\dfrac{b}{c} = \dfrac{a - BD}{BD}$

$\Rightarrow$ $b \times BD = ac - c \times BD$

$\Rightarrow$ $b \times BD + c \times BD = ac$

$\Rightarrow$ $BD\ (b + c) = ac$

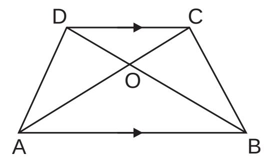
$\Rightarrow$ $BD = \dfrac{ac}{b+c}$

Hence, $\mathbf{BD = \dfrac{ac}{b+c}}$.

VALUE-BASED QUESTIONS

For Basic and Standard Levels

1. (*i*)



In ΔDOC and ΔAOB, we have

∠DOC = ∠AOB [Vert. opp. ∠s]

and ∠CDO = ∠ABO [Alt ∠s, DC ∥ AB]

$\therefore$ ΔDOC ~ ΔAOB

$\therefore$ $\dfrac{ar(\Delta DOC)}{ar(\Delta AOB)} = \dfrac{CD^2}{AB^2} = \dfrac{CD^2}{(2\ CD)^2} = \dfrac{1}{4}$

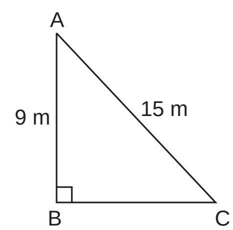
$\therefore$ $\dfrac{ar(\Delta DOC)}{84\ m^2} = \dfrac{1}{4}$

$\Rightarrow$ $ar(\Delta DOC) = \dfrac{84}{4}\ m^2 = 21\ m^2$

Hence, the area of land donated by the man is **21 m²**.

(*ii*) **Empathy and environmental awareness**.

2. (*i*) Let the pigeon be on the window sill at a height of 9 m at point A and let AC be the ladder at a distance BC from the wall AB, so that it just reaches the pigeon.



Then, AB = 9 m, AC = 15 m.

In right ΔABC, we have

$$AC^2 = AB^2 + BC^2$$

[By Pythagoras' Theorem]

$\Rightarrow$ $(15\ m)^2 = (9\ m)^2 + BC^2$

$\Rightarrow$ $BC^2 = (225 - 81)\ m^2 = 144\ m^2$

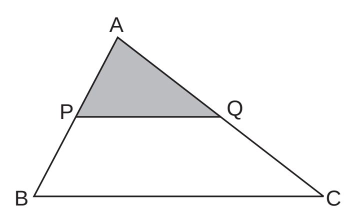
$\Rightarrow$ $BC = 12\ m$

Thus, the boy will have to place the foot of the ladder at a distance of **12 m** from the wall.

(*ii*) **Empathy and helpfulness**

For Standard Level

3. (*i*) He can donate a triangular region where vertices are A and the mid-points P and Q of sides AB and AC respectively.



Justification: P and Q are the mid-points of AB and AC respectively.

$\therefore$ PQ ∥ BC [By mid-point theorem]

and 2 PQ = BC … (1)

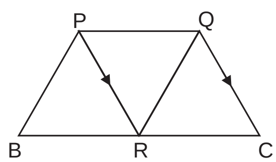
ΔAPQ ~ ΔABC [By AA similarity]

$$\therefore \frac{ar(\Delta APQ)}{ar(\Delta ABC)} = \frac{AP^2}{AB^2} = \frac{\left(\frac{AB}{2}\right)^2}{AB^2} = \frac{1}{4}$$

[$\because$ P is the mid-point of AB]

Hence, $ar(\Delta APQ) = \frac{1}{4} ar(\Delta ABC)$.

In the remaining piece of land PQCB, three triangular pieces PBR, PRQ and QRC, obtained by drawing PR ∥ QC (where R lies on BC) and joining QR can be given by the man to his three children.



Justification: PQ ∥ BC [From (1)]

and PR ∥ QC [Given]

$\therefore$ PQCR is a parallelogram.

$\therefore$ PQ = RC

[Opposite sides of a ∥gm] … (2)

Also, BC = BR + RC

$\Rightarrow$ BC = BR + PQ [Using (2)]

$\Rightarrow$ 2 PQ = BR + PQ [Using (1)]

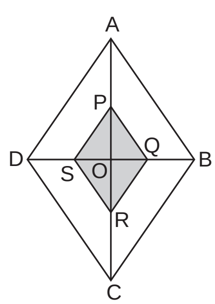
$\Rightarrow$ PQ = BR … (3)

$\therefore$ ΔPBR, ΔPRQ and ΔQRC lie on equal bases BR, PQ and RC respectively [using (2) and (3)] and between same parallels PQ and BC.

$\therefore$ **ar(ΔPBR) = ar(ΔPRQ) = ar(ΔQRC)**

(*ii*) **Empathy, problem solving and gender equality**

4. (*i*) Let the diagonals AC and BD of rhombus ABCD be 24 m and 10 m respectively.



Let $2x$ and $2y$ be the diagonals of the smaller rhombus and the diagonals of a rhombus bisect each other at right angles.

In rhombus ABCD, we have

$$OA = OC = \frac{24}{2} \text{ m} = 12 \text{ m},$$

$$OD = OB = \frac{10}{2} \text{ m} = 5 \text{ m}$$

and ∠AOD = 90°

In rhombus PQRS, we have

$$OP = OR = x$$

$$OS = OQ = y$$

and ∠POS = 90°

$$\text{Area of rhombus ABCD} = \frac{1}{2} AC \times BD$$

$$= \frac{1}{2} \times 24 \times 10 \text{ m}^2 = 120 \text{ m}^2$$

ΔOAD ≅ ΔOAB ≅ ΔOCB ≅ ΔOCD

$\therefore$ ar(ΔOAD) = ar(ΔOAB)

= ar(ΔOCB)

= ar(ΔOCD) … (1)

Also, area (rhombus ABCD)

= ar(ΔOAD) ar(ΔOAB) + ar(ΔOCB) + ar(ΔOCD)

… (2)

$\Rightarrow$ area (rhombus ABCD) = 4 ar(ΔOAD)

[Using (1) and (2)] … (3)

Similarly, area (rhombus PQRS) = 4 ar(ΔOPS) … (4)

Subtracting (4) from (3), we get

ar(rhombus ABCD) – ar(rhombus PQRS) =

4 [ar(ΔOAD) – ar(ΔOPS)]

$$\Rightarrow \frac{43.2 \text{ m}^2}{4} = ar(\Delta OAD) - ar(\Delta OPS)]$$

$$\Rightarrow 10.8 \text{ m}^2 = \frac{1}{2} OA \times OD - ar(\Delta OPS)$$

$$\Rightarrow 10.8 \text{ m}^2 = \left(\frac{1}{2} \times 12 \times 5\right) \text{ m}^2 - ar(\Delta OPS)$$

$$\Rightarrow ar(\Delta OPS) = (30 - 10.8) \text{ m}^2 = 19.2 \text{ m}^2$$

… (5)

Now, rhombus ABCD ~ rhombus PQRS [Given]

$\therefore$ ΔOAD ~ ΔOPS [By AA similarity]

$$\therefore \frac{ar(\Delta OAD)}{ar(\Delta OPS)} = \frac{OA^2}{OP^2}$$

$$\Rightarrow \frac{30 \text{ m}^2}{19.2 \text{ m}^2} = \frac{(12 \text{ m})^2}{x^2} \quad \text{[Using (5)]}$$

$$\Rightarrow x^2 = \frac{12 \text{ m} \times 12 \text{ m} \times 19.2}{30}$$

$$\Rightarrow x = 9.6 \text{ m} \quad \dots (6)$$

$$\text{Now, } ar(\Delta OPS) = \frac{1}{2} x \times y$$

$$\Rightarrow 19.2 \text{ m}^2 = \frac{1}{2} \times 9.6 \text{ m} \times y$$

[Using (5) and (6)]

$$\Rightarrow \frac{19.2 \times 2}{9.6} \text{ m} = y$$

$$\Rightarrow y = 4 \text{ m}$$

Diagonal PR = $2x$ = 2 × 9.6 m = 19.2 m

and diagonal SQ = $2y$ = 2 × 4 m = 8 m

Hence, the diagonal of the smaller rhombus are **19.2 m** and **8 m**.

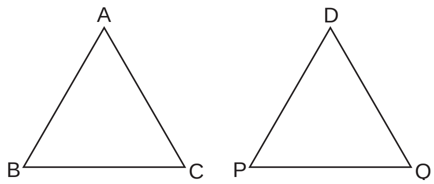
(*ii*) **Awareness about environment**.

UNIT TEST 1

For Basic Level

1. (*a*) **ΔQDP ~ ΔCAB**

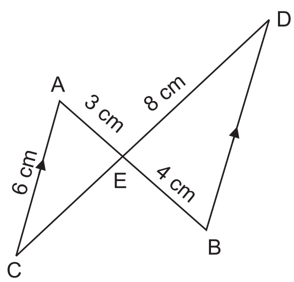
In ΔABC and ΔDPQ, $\frac{AB}{DP} = \frac{BC}{PQ} = \frac{CA}{QD}$



$\therefore$ ΔCAB ~ ΔQDP [By SSS similarity]

Hence, ΔQDP ~ ΔCAB.

2. (*a*) **6 cm, 8 cm**



In ΔAEC and ΔBED, we have

$\angle AEC = \angle BED$ [V. opp. ∠s]

$\angle EAC = \angle EBD$ [Alt ∠s, CA || BD]

$\Rightarrow \frac{EA}{EB} = \frac{AC}{BD} = \frac{CE}{DE}$

[Corresponding sides of similar triangles are proportional]

$\Rightarrow \frac{3\text{ cm}}{4\text{ cm}} = \frac{6\text{ cm}}{BD} = \frac{CE}{8\text{ cm}}$

$\Rightarrow \frac{3}{4} = \frac{6\text{ cm}}{BD}$

and $\frac{3}{4} = \frac{CE}{8\text{ cm}}$

$\Rightarrow BD = \frac{6 \times 4}{3}$ cm

and $CE = \frac{3 \times 8}{4}$ cm

$\Rightarrow$ BD = 8 cm

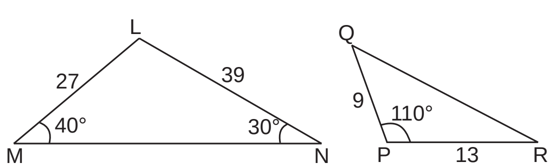
and CE = 6 cm

Hence, CE and BD are 6 cm and 8 cm respectively.

3. (*b*) **40°, 30°**

In ΔLMN,

$\angle MLN = 180° - 40° - 30° = 110°$



In ΔQPR and MLN, we have

$\frac{PR}{LN} = \frac{13}{39} = \frac{1}{3}$

and $\frac{QP}{LM} = \frac{9}{27} = \frac{1}{3}$

and $\angle QPR = \angle MLN = 110°$

$\therefore$ ΔQPR ~ ΔMLN [By SAS similarity]

$\angle Q = \angle M = 40°$

and $\angle R = \angle N = 30°$

Hence, the measures of ∠Q and ∠R are 40° and 30° respectively.

4. (*d*) **2**

For PQ to be parallel to AB,

$\frac{CQ}{QB} = \frac{CP}{PA}$

$\Rightarrow \frac{x}{3x+4} = \frac{x+3}{3x+19}$

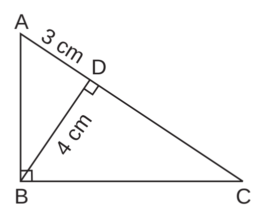
$\Rightarrow 3x^2 + 19x = 3x^2 + 4x + 9x + 12$

$\Rightarrow 6x = 12$

$\Rightarrow x = 2$

Hence, $x = 2$.

5. (*c*) $\mathbf{\frac{25}{3}}$ **cm**



In ΔADB and ΔABC, we have

$\angle ADB = \angle ABC$ [Each is equal to 90°]

$\angle DAB = \angle BAC$ [Common]

ΔADB ~ ΔABC ... (1)

In ΔBDC and ΔABC, we have

$\angle BDC = \angle ABC$ [Each is 90°]

$\angle DCB = \angle BCA$ [Common]

$\therefore$ ΔBDC ~ ΔABC ... (2)

From (1) and (2), we get

ΔADB ~ ΔBDC

$\Rightarrow \frac{AD}{BD} = \frac{BD}{CD}$

$\Rightarrow \frac{3\text{ cm}}{4\text{ cm}} = \frac{4\text{ cm}}{CD}$

$\Rightarrow CD = \frac{16}{3}$ cm

Now, AC = AD + CD = $3 + \frac{16}{3}$ cm = $\frac{25}{3}$ cm

Hence, AC = $\frac{25}{3}$ cm.

6. (*c*) **5.4 cm**

$\frac{\text{perimeter of } \Delta PQR}{\text{perimeter of } \Delta XYZ} = \frac{QR}{YZ}$

$\Rightarrow \frac{30\text{ cm}}{18\text{ cm}} = \frac{9\text{ cm}}{YZ}$

$\Rightarrow \quad YZ = \dfrac{9 \times 18}{30}$ cm = 5.4 cm

Hence, YZ = 5.4 cm.

7. (*b*) **25 cm**

$$\frac{\text{perimeter of } \Delta ABC}{\text{perimeter of } \Delta DEF} = \frac{AB}{DE}$$

$$\Rightarrow \quad \frac{\text{perimeter of } \Delta ABC}{15 \text{ cm}} = \frac{9 \text{ cm}}{5.4 \text{ cm}}$$

$$\Rightarrow \quad \text{perimeter of } \Delta ABC = \frac{9 \times 15}{5.4} \text{ cm}$$

Hence, perimeter of ΔABC = 25 cm.

8. (*a*) 25 cm^2

$$\frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{AB^2}{DE^2} = \left(\frac{AB}{DE}\right)^2$$

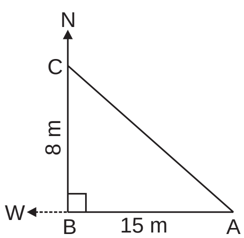
$$\Rightarrow \quad \frac{ar(\Delta ABC)}{100 \text{ cm}^2} = \left(\frac{1}{2}\right)^2$$

$$\Rightarrow \quad ar(\Delta ABC) = \frac{100}{4} \text{ cm}^2 = 25 \text{ cm}^2$$

Hence, ar(ΔABC) = 25 cm^2.

9. (*b*) **17 m**

Suppose the man starts from point A, goes 15 m towards West and reaches point B and then he goes 8 m North to reach point C.



Then, he is at a distance = AC from his starting position.

In right ΔABC, we have

$$\begin{aligned} AC^2 &= AB^2 + BC^2 \\ &= (15 \text{ m})^2 + (8 \text{ m})^2 \\ &= (225 + 64) \text{ m}^2 \\ &= 289 \text{ m}^2 \end{aligned}$$

$\Rightarrow \quad AC = 17$ m

Hence, the man is 17 m away from his starting position.

10. $\Delta ABC \sim \Delta PQR$

$$\Rightarrow \quad \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$$

$$\Rightarrow \quad \frac{2x-1}{18} = \frac{2x+2}{3x+9} = \frac{3x}{6x}$$

$$\Rightarrow \quad \frac{2x-1}{18} = \frac{1}{2}$$

$\Rightarrow \quad 4x - 2 = 18$

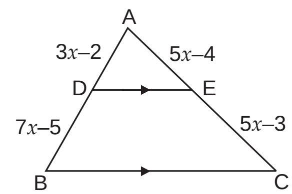
$\Rightarrow \quad 4x = 20$

$\Rightarrow \quad$ **$x = 5$**

$$\Rightarrow \quad \begin{aligned} AB &= 2 \times 5 - 1 \\ &= 9 \text{ cm}, \\ BC &= 2 \times 5 + 2 \\ &= 12 \text{ cm}, \\ AC &= 3 \times 5 \\ &= 15 \text{ cm}, \\ QR &= 3 \times 5 + 9 \\ &= 24 \text{ cm}, \\ PR &= 6 \times 5 \\ &= 30 \text{ cm} \end{aligned}$$

Hence, **AB = 9 cm**, **BC = 12 cm, CA = 15 cm, PQ = 18 cm, QR = 24 cm** and **PR = 30 cm**.

11. In ΔABC, $\quad DE \parallel BC$



$$\therefore \quad \frac{AD}{DB} = \frac{AE}{EC} \quad \text{[By BPT]}$$

$$\Rightarrow \quad \frac{3x-2}{7x-5} = \frac{5x-4}{5x-3}$$

$\Rightarrow \quad 15x^2 - 10x - 9x + 6 = 35x^2 - 25x - 28x + 20$

$\Rightarrow \quad 20x^2 - 34x + 14 = 0$

$\Rightarrow \quad 10x^2 - 17x + 7 = 0$

$\Rightarrow \quad 10x^2 - 10x - 7x + 7 = 0$

$\Rightarrow \quad 10x(x - 1) - 7(x - 1) = 0$

$\Rightarrow \quad (x - 1)(10x - 7) = 0$

$\Rightarrow \quad$ Either $x - 1 = 0$

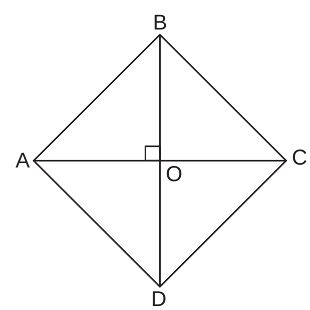
or $\quad (10x - 7) = 0$

$\Rightarrow \quad x = 1$

or $\quad x = \dfrac{7}{10}$

Hence, **x = 1 unit or $\dfrac{7}{10}$ unit**.

12. Let ABCD be the rhombus, whose diagonals AC and BD intersect at O.



Let $\quad AC = 30$ cm

and $\quad BD = 40$ cm.

Then, AO = 15 cm, BO = 20 cm and ∠AOB = 90°.

[∵ The diagonals of a rhombus bisect each other (at O) at right angles]

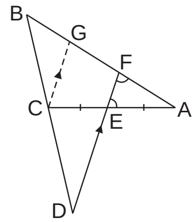
In right ΔAOB, we have

$$\begin{aligned} AB^2 &= (AO)^2 + (BO)^2 \\ &= (15 \text{ cm})^2 + (20 \text{ cm})^2 \\ &= (225 + 400) \text{ cm}^2 \\ &= 625 \text{ cm}^2 \end{aligned}$$

$\Rightarrow$ AB = 25 cm

Hence, each side of the rhombus is **25 cm**.

13. Through C, draw CG ∥ DF and let it meet AB at G.



AF = AE

[Sides opposite equal ∠s AEF and AFE] … (1)

In ΔACG, E is the mid-point of AC and EF ∥ CG.

∴ F is the mid-point of AG i.e AF ∥ FG

[By the conv. of Mid-point Theorem] … (2)

∴ FG = AE [From (1) and (2)] … (3)

In ΔBDF, CG ∥ DF

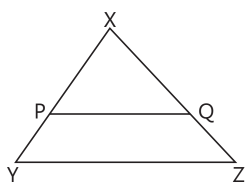
∴ $\frac{BD}{CD} = \frac{BF}{GF}$

$\Rightarrow$ $\frac{BD}{CD} = \frac{BF}{AE}$ [Using (3)]

$\frac{BD}{CD} = \frac{BF}{CE}$ [∵ AF = CE]

Hence, $\mathbf{\frac{BD}{CD} = \frac{BF}{CE}}$.

14.



In ΔXYZ, we have

$\frac{XP}{PY} = \frac{XQ}{QZ} = 3 = \frac{3}{1}$ [Given]

By the converse of Thales theorem,

PQ ∥ YZ

In ΔXPQ and ΔXYZ, we have

∠XPQ = ∠XYZ [Corresponding ∠S]

∠PXQ = ∠YXZ [Common]

∴ ΔXPQ ~ ΔXYZ [By AA similarity]

∴ $\frac{ar(\Delta XPQ)}{ar(\Delta XYZ)} = \frac{XP^2}{XY^2}$

$\Rightarrow$ $\frac{ar(\Delta XPQ)}{32\,cm^2} = \frac{XP^2}{(XP+PY)^2}$

$\Rightarrow$ $\frac{ar(\Delta XPQ)}{32\,cm^2} = \frac{3^2}{(3+1)^2} = \frac{9}{16}$

$\Rightarrow$ $ar(\Delta XPQ) = \frac{9}{16} \times 32\ cm^2 = 18\ cm^2$

ar(quad PYZQ) = ar(ΔXYZ) – ar(ΔXPQ)

= $(32 - 18)\ cm^2$

= $14\ cm^2$

Hence, ar(quad PYZQ) = **14 cm²**.

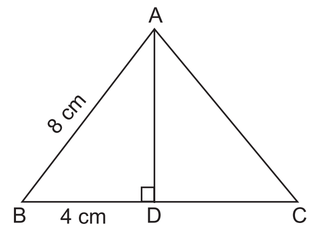
UNIT TEST 2

For Standard Level

1. (*a*) $\mathbf{4\sqrt{3}}$ cm

ΔADB ≅ ΔADC [By RHS congruency]

∴ $BD = DC = \frac{BC}{2} = 4$ cm



In right ΔADB, we have

$AB^2 = AD^2 + BD^2$

$\Rightarrow$ $(8\ cm)^2 = AD^2 + (4\ cm)^2$

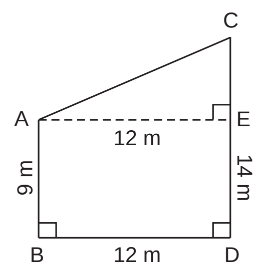
$\Rightarrow$ $AD^2 = (64 - 16)\ cm^2 = 48\ cm^2$

$\Rightarrow$ $AD = 4\sqrt{3}$ cm

Hence, AD = $4\sqrt{3}$ cm.

2. (*c*) **13 m**

Let AB and CD be the two poles of height 9 m and 14 m respectively, standing 12 m apart.



Then, AB = 9 m, CD = 14 m and BD = 12 cm

Draw AE ⊥ CD.

Then, AE = BD =12 cm and ED = AB = 9 m

and CE = CD – ED = 14 m – AB = 14 m – 9 m = 5 m.

In right ΔAEC, we have

$AC^2 = AE^2 + CE^2$

$= (12\ m)^2 + (5\ m)^2$

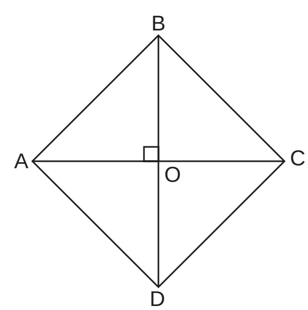
$= 169\ m^2$

$\Rightarrow$ AC = 13 m

Hence, the distance between the tops of the poles is 13 m.

3. (*d*) **13 cm**

Let ABCD be a rhombus in which the diagonals AC and BD intersect at O.



Let AC = 10 cm

and BD = 24 cm.

Since the diagonals of a rhombus bisect each other at right angles,

$\therefore$ $AO = OC = \frac{AC}{2} = 5$ cm

and $BO = OD = \frac{BD}{2} = 12$ cm

In right ΔAOB, we have

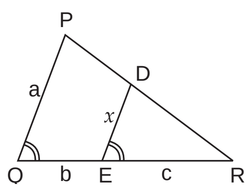
$$AB^2 = AO^2 + BO^2$$
$$= (5 \text{ cm})^2 + (12 \text{ cm})^2$$
$$= 169 \text{ cm}^2$$

$\Rightarrow$ AB = 13 cm

Hence, the length of the side of the rhombus is 13 cm.

4. (*b*) $\frac{ac}{b + c}$

ΔRPQ ~ ΔRDE [By AA similarity]



$\therefore$ $\frac{PQ}{DE} = \frac{QR}{ER}$

[Corresponding sides of similar Δs]

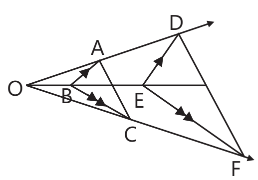
$\Rightarrow$ $\frac{a}{x} = \frac{b + c}{c}$

$\Rightarrow$ $x = \frac{ac}{b + c}$

Hence, $x = \frac{ac}{b + c}$.

5. (*d*) **2 : 5**

In ΔODE, AB ∥ DE



$\therefore$ $\frac{OA}{AD} = \frac{OB}{BE}$ [By BPT] … (1)

In ΔOEF, BC ∥ EF

$\therefore$ $\frac{OC}{CF} = \frac{OB}{BE}$ [By BPT] … (2)

From (1) and (2), we get

$$\frac{OA}{AD} = \frac{OC}{CF}$$

$\therefore$ By the converse of Thales Theorem, in ΔODF,

AC ∥ DF

In ΔOAC and ΔODF, we have

∠OAC = ∠ODF

[Corresponding ∠s, AC ∥ DF]

∠OCA = ∠OFD

[Corresponding ∠s, AC ∥ DF]

$\therefore$ ΔOAC ~ ΔODF [By AA similarity]

$\therefore$ $\frac{OA}{OD} = \frac{AC}{DF}$

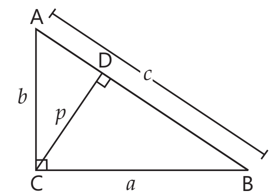
$\Rightarrow$ $\frac{OA}{OA + AD} = \frac{AC}{DF}$

$\Rightarrow$ $\frac{2}{(2+3)} = \frac{AC}{DF}$

$\Rightarrow$ $\frac{2}{5} = \frac{AC}{DF}$

Hence, AC: DF = 2 : 5.

6. (*c*) $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$



area of ΔACB = $\frac{1}{2}ab$

[Taking '*a*' as base] … (1)

area of ΔACB = $\frac{1}{2}cp$

[Taking '*c*' as base] … (2)

$\therefore$ $\frac{1}{2}ab = \frac{1}{2}cp$ [Using (1) and (2)]

$\Rightarrow$ $ab = cp$

$\Rightarrow$ $\frac{1}{p} = \frac{c}{ab}$

$\Rightarrow$ $\frac{1}{p^2} = \frac{c^2}{a^2b^2}$

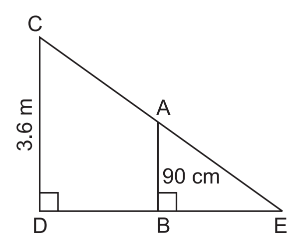
$\Rightarrow$ $\frac{1}{p^2} = \frac{b^2 + a^2}{a^2b^2}$ [$\because c^2 = b^2 + a^2$]

$\Rightarrow$ $\frac{1}{p^2} = \frac{b^2}{a^2b^2} + \frac{a^2}{a^2b^2}$

$\Rightarrow$ $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$

Hence, $\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2}$.

7. Let AB = 90 cm be the girl at point B after 4 seconds of starting from the base of lamp post CD = 3.6 m.



Let CA produced meets DB produced at E.

Then, ∠CED = ∠AEB

[Angular elevation of the Sun at that time]

In ΔCDE and ΔABE, we have

$$\angle CDE = \angle ABE = 90°$$

$$\angle CED = \angle AEB \quad \text{[Common]}$$

$$\therefore \quad \Delta CDE \sim \Delta ABE \quad \text{[By AA similarity]}$$

$$\therefore \quad \frac{CD}{AB} = \frac{DE}{BE}$$

$$\Rightarrow \quad \frac{CD}{AB} = \frac{DB + BE}{BE}$$

$$\Rightarrow \quad \frac{3.6}{0.9} = \frac{(1.2 \times 4)\text{ m} + BE}{BE}$$

[$\because$ Distance DB = Distance covered by the girl in 4 seconds = (1.2×4) m]

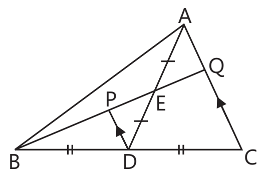
$$\Rightarrow \quad 4\text{ BE} = 4.8\text{ m} + \text{BE}$$

$$\Rightarrow \quad 3\text{ BE} = 4.8\text{ m}$$

$$\Rightarrow \quad \text{BE} = 1.6\text{ m}$$

Hence, the length of the shadow of the girl is **1.6 m**

8.



In ΔEDP and ΔEAQ, we have

$$\angle DEP = \angle AEQ \quad \text{[Vert. opp } \angle\text{s]}$$

$$\angle PDE = \angle QAE \quad \text{[Alt. } \angle\text{s, PD} \parallel \text{AC]}$$

$$\therefore \quad \Delta EDP \sim \Delta EAQ$$

$$\Rightarrow \quad \frac{ED}{EA} = \frac{EP}{EQ} \Rightarrow 1 = \frac{EP}{EQ}$$

[$\because$ ED = EA, as E is the mid-point of AD]

$$\Rightarrow \quad EP = EQ \quad \dots (1)$$

Now in ΔBQC, $\quad DP \parallel CQ \quad$ [$\because$ DP $\parallel$ CA]

$$\therefore \quad \frac{BP}{PQ} = \frac{BD}{DC} = 1$$

[$\because$ BD = DC, as D is the mid-point of BC]

$$\Rightarrow \quad BP = PQ$$

$$\Rightarrow \quad BP = EP + EQ$$

$$\Rightarrow \quad BP = 2EQ \quad \text{[Using (1)]} \dots (2)$$

Now, $\quad$ BE : EQ = BP + EP : EQ

= 2EQ + EQ : EQ

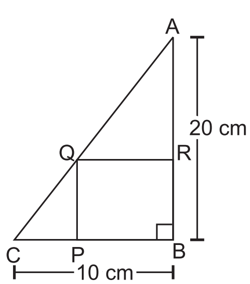
[Using (1) and (2)]

= 3EQ : EQ

= 3 : 1

Hence, **BE : EQ = 3 : 1**

9. Let ABC be a right Δ in which $\angle B = 90°$, AB = 20 cm and BC = 10 cm.



Then, the largest square BPQR which can be inscribed in this triangle will be as shown in the given figure.

Let $\quad$ RB = x cm,

So, $\quad$ AR = $(20 - x)$ cm

In ΔARQ and ΔABC, we have

$$\angle ARQ = \angle ABC \quad \text{[Each is 90°]}$$

$$\angle RAQ = \angle BAC \quad \text{[Common]}$$

$$\therefore \quad \Delta ARQ \sim \Delta ABC \quad \text{[By AA similarity]}$$

$$\therefore \quad \frac{AR}{AB} = \frac{RQ}{BC}$$

$$\Rightarrow \quad \frac{20 - x}{20} = \frac{x}{10}$$

$$\Rightarrow \quad 200 - 10x = 20x$$

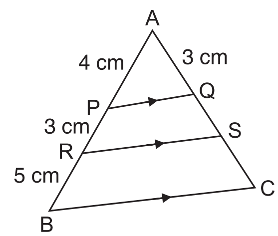
$$\Rightarrow \quad 200 = 30x$$

$$\Rightarrow \quad x = \frac{20}{3}$$

Thus, the side of the required square is of length $\mathbf{\frac{20}{3}}$ **cm**.

10. PQ $\parallel$ BC and RS $\parallel$ BC

$$\Rightarrow \quad PQ \parallel RS$$



In ΔARS, $\quad PQ \parallel RS$

$$\frac{AP}{PR} = \frac{AQ}{QS} \quad \text{[By BPT]}$$

$$\Rightarrow \quad \frac{4\text{ cm}}{3\text{ cm}} = \frac{3\text{ cm}}{QS}$$

$$\Rightarrow \quad QS = \frac{9}{4}\text{ cm}$$

In ΔABC, $\quad RS \parallel BC$

$$\because \quad \frac{AR}{RB} = \frac{AS}{SC} \quad \text{[By BPT]}$$

$$\Rightarrow \quad \frac{(4 + 3)\text{ cm}}{5\text{ cm}} = \frac{\left(3 + \frac{9}{4}\right)\text{cm}}{SC}$$

$$\Rightarrow \quad SC = \frac{15}{4}\text{ cm}$$

In ΔAPQ and ΔABC, we have

$$\angle APQ = \angle ABC$$

[Corresponding $\angle$s, PQ $\parallel$ BC]

$$\angle PAQ = \angle BAC \quad \text{[Common]}$$

$$\therefore \quad \Delta APQ \sim \Delta ABC \quad \text{[By AA similarity]}$$

$$\therefore \quad \frac{ar(\Delta APQ)}{ar(\Delta ABC)} = \frac{(AP)^2}{(AB)^2}$$

$$= \frac{(4\text{ cm})^2}{[(4 + 3 + 5)\text{ cm}]^2}$$

$$= \frac{(4\text{ cm})^2}{(12\text{ cm})^2}$$

$$= \frac{16}{144}$$

$$= \frac{1}{9}$$

$$\Rightarrow \quad \frac{ar(\Delta APQ)}{48\ cm^2} = \frac{1}{9}$$

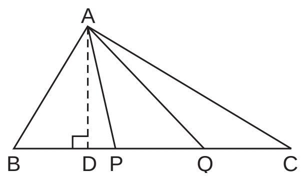
$$\Rightarrow \quad ar(\Delta APQ) = \frac{1}{9} \times 48\ cm^2$$

$$= \frac{16}{3}\ cm^2$$

Hence, **QS =** $\mathbf{\frac{9}{4}}$ **cm, SC =** $\mathbf{\frac{15}{4}}$ **cm**

and ar(ΔAPQ) = $\mathbf{\frac{16}{3}}$ **cm²**.

11.



In right ΔADB, we have

$$AB^2 = AD^2 + BD^2$$

[By Pythagoras' Theorem]

$$\Rightarrow \quad AB^2 = AD^2 + (BP - PD)^2$$

$$\Rightarrow \quad AB^2 = AD^2 + (PQ - PD)^2$$

$$\left[\because BP = PQ = \frac{BC}{3}\right] \ldots (1)$$

In right ΔADC, we have

$$AC^2 = AD^2 + DC^2$$

[By Pythagoras' Theorem]

$$\Rightarrow \quad AC^2 = AD^2 + (DQ + QC)^2$$

$$= AD^2 + (DQ + PQ)^2$$

$$\left[\because QC = PQ = \frac{BC}{3}\right] \ldots (2)$$

Adding (1) and (2), we get

$AB^2 + AC^2$

$= AD^2 + (PQ - PD)^2 + AD^2 + (DQ + PQ)^2$

$= AD^2 + PQ^2 - 2\ PQ \times PD + PD^2 + AD^2 + DQ^2 + 2\ DQ \times PQ + PQ^2$

$= (AD^2 + PD^2) + PQ^2 - 2\ PQ \times PD + (AD^2 + DQ^2) + 2\ DQ \times PQ + PQ^2$

$= AP^2 + PQ^2 - 2\ PQ \times PD + AQ^2 + PQ^2 + 2\ DQ \times PQ$

$= AP^2 + AQ^2 - 2\ PQ^2 + 2\ PQ\ (DQ - PD)$

$= AP^2 + AQ^2 + 2\ PQ^2 + 2\ PQ\ (PQ)$

$= AP^2 + AQ^2 + 2\ PQ^2 + 2\ PQ^2$

$= AP^2 + AQ^2 + 4\ PQ^2$

Hence, $\mathbf{AB^2 + AC^2 = AP^2 + AQ^2 + 4\ PQ^2}$.