CHAPTER 5

Arithmetic Progressions

EXERCISE 5A

For Basic and Standard Levels

1. We have 3, 7, 11, 15, …

$\because$ $7 - 3 = 11 - 7 = 4$

$\therefore$ The given progression 3, 7, 11, 15, … **is an AP.**

Here, First term $= a = \mathbf{3}$

Common difference $= d = \mathbf{4}$

2. (*i*) We have 1.7, 2, 2.3, 2.6, … is an AP.

$\therefore$ Common difference $= a_2 - a_1$

$= 2 - 1.7$

$= \mathbf{0.3}$

Now, the term next to 2.6 = 2.6 + 0.3 = **2.9**

and the term next to 2.9 = 2.9 + 0.3 = **3.2**

(*ii*) We have $0, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \ldots$ is an AP

$\therefore$ Common difference $(d) = \frac{1}{5} - 0 = \mathbf{\frac{1}{5}}$

Now, the term next to $\frac{3}{5} = \frac{3}{5} + \frac{1}{5} = \mathbf{\frac{4}{5}}$

the term next to $\frac{4}{5} = \frac{4}{5} + \frac{1}{5} = \frac{5}{5}$ or **1**

3. (*i*) We have 150, 141, 132, 123, …

Here, First term $= a = 150$

Common difference $= d = 141 - 150 = -9$

$\because$ $a_n = a + (n - 1)d$

$\therefore$ $a_6 = 150 + (6 - 1)(-9)$

$= 150 + 5 \times (-9) = 150 - 45$

$= \mathbf{105}$

(*ii*) We have 5.7, 5.2, 4.7, 4.2, …

Here, $a = 5.7$ and $d = 5.2 - 5.7 = -0.5$

Now, $a_n = a + (n - 1)d$

$\Rightarrow$ $a_{11} = 5.7 + (11 - 1) \times (-0.5)$

$= 5.7 + 10 \times (-0.5)$

$= 5.7 - 5.0 = \mathbf{0.7}$

(*iii*) We have $\frac{3}{11}, \frac{5}{11}, \frac{7}{11}, \frac{9}{11}, \ldots$

Here, a = First term $= \frac{3}{11}$

d = Common diff. $= \frac{5}{11} - \frac{3}{11} = \frac{2}{11}$

Now, $a_n = a + (n - 1)d$

$\Rightarrow$ $a_{29} = \frac{3}{11} + (29 - 1) \times \frac{2}{11} = \frac{3}{11} + 28 \times \frac{2}{11}$

$\Rightarrow$ $a_{29} = \frac{3}{11} + \frac{56}{11} = \mathbf{\frac{59}{11}}$

(*iv*) We have –50, –35, –20, –5, 10, …

Here, First term $= a = -50$

Common diff. $= d = -35 - (-50)$

$= -35 + 50 = 15$

Now, $a_n = a + (n - 1)d$

$\Rightarrow$ $a_{18} = a + (18 - 1)d$

$\Rightarrow$ $a_{18} = -50 + 17 \times 15$

$\Rightarrow$ $a_{18} = -50 + 255$

$= \mathbf{205}$

4. (*i*) We have 2, 7, 12, 17, …

$\Rightarrow$ First term $(a) = 2 = a_1$

Common difference $(d) = a_2 - a_1$

$= 7 - 2 = 5$

Since $a_n = a + (n - 1)d$

$\therefore$ $a_{15} = 2 + (15 - 1) \times 5$

$= 2 + 70 = \mathbf{72}$

(*ii*) We have $\sqrt{x}, 3\sqrt{x}, 5\sqrt{x}, \ldots$

$\Rightarrow$ First term $= a = \sqrt{x} = a_1$

Common difference $= d = a_2 - a_1$

$= 3\sqrt{x} - \sqrt{x} = \sqrt{x}(3 - 1)$

$= 2\sqrt{x}$

Since $a_n = a + (n - 1)d$

$\therefore$ $a_{37} = \sqrt{x} + (37 - 1) \times (2\sqrt{x})$

$= \sqrt{x} + 72\sqrt{x}$

$= \sqrt{x}(1 + 72) = \sqrt{x}(73)$

$= \mathbf{73\sqrt{x}}$

(*iii*) We have –5, –7, –9, …

$\Rightarrow$ First term $= a = -5 = a_1$

Common diff. $(d) = a_2 - a_1$

$= [(-7) - (-5)] = (-7) + 5$

$= -2$

Since $a_n = a + (n - 1)d$

$\Rightarrow$ $a_7 = (-5) + (7 - 1)(-2)$

$= (-5) + 6(-2)$

$= -5 - 12 = \mathbf{-17}$

(*iv*) We have 15, 9, 3, –3, …

$\Rightarrow$ First term $= a = 15 = a_1$

Common diff. $(d) = a_2 - a_1$

$= 9 - 15 = -6$

Since $a_n = a + (n-1)d$

$\Rightarrow$ $a_r = 15 + (r-1)(-6)$

$= 15 + (-6r + 6)$

$= 15 + 6 + (-6r) = \mathbf{21 - 6r}$

(*v*) We have $(18b + x), (19b), (20b - x), \ldots$

$\Rightarrow$ First term $= a = (18b + x) = a_1$

Common diff. $= d = a_2 - a_1$

$= (19b) - (18b + x)$

$= (19 - 18)b - x = b - x$

Since $a_n = a + (n-1)d$

$\Rightarrow$ $a_9 = (18b + x) + (9 - 1) \times (b - x)$

$= 18b + x + 8(b - x)$

$= 18b + x + 8b - 8x$

$= \mathbf{26b - 7x}$

(*vi*) We have $2\frac{3}{4}, 3\frac{1}{4}, 3\frac{3}{4}, 4\frac{1}{4}, \ldots$

$\Rightarrow$ First term $= a = 2\frac{3}{4} = a_1$

Common diff. $(d) = a_2 - a_1 = 3\frac{1}{4} - 2\frac{3}{4}$

$= (3-2) + \left(\frac{1}{4} - \frac{3}{4}\right)$

$= 1 + \left(-\frac{2}{4}\right) = 1 + \left(-\frac{1}{2}\right) = \frac{1}{2}$

Or	$d = a_2 - a_1 = 3\frac{1}{4} - 2\frac{3}{4}$
	$= \frac{13}{4} - \frac{11}{4} = \frac{2}{4} = \frac{1}{2}$

Since $a_n = a + (n-1)d$

$\therefore$ $a_{29} = 2\frac{3}{4} + (29 - 1) \times \left(\frac{1}{2}\right)$

$= 2\frac{3}{4} + 28 \times \frac{1}{2}$

$= 2\frac{3}{4} + 14 = \mathbf{16\frac{3}{4}}$

(*vii*) $\sqrt{2}, \sqrt{8}, \sqrt{18}, \ldots$

$\sqrt{2}, 2\sqrt{2}, 3\sqrt{2}, \ldots$

$a = \sqrt{2}$

$d = \sqrt{2}$

$a_{10} = a + (n-1)d$

$= \sqrt{2} + (10-1)\sqrt{2}$

$= \sqrt{2} + 9\sqrt{2}$

$= 10\sqrt{2}$

$= \mathbf{\sqrt{200}}$

(*viii*) $-5, \frac{-5}{2}, 0, \frac{5}{2}, \ldots$

$a = -5$

$d = a_2 - a_1$

$= \frac{-5}{2} - (-5)$

$= \frac{-5}{2} + 5$

$= \frac{5}{2}$

$a_{25} = a + (n-1)d$

$= -5 + (25 - 1)\ \frac{5}{2}$

$= -5 + 24 \times \frac{5}{2}$

$= -5 + 60$

$= \mathbf{55}$

5. (*i*) We have the nth term of an AP $= 5n - 2$

(*a*) $\because$ $a_n = 5n - 2$

$\therefore$ $a_1 = 5(1) - 2 = 5 - 2 = 3$

$\Rightarrow$ First term $=$ **3**

(*b*) Common difference $(d) = a_2 - a_1$

$= [5(2) - 2] - [5(1) - 2]$

$= 8 - 3 = \mathbf{5}$

(*c*) $\because$ $a_n = 5n - 2$

$\therefore$ $a_{18} = 5(18) - 2$

$= 90 - 2 = \mathbf{88}$

(*ii*) We have $a_n = 7 - 4n$

$\therefore$ $a_1 = 7 - 4(1) = 7 - 4 = 3$

$a_2 = 7 - 4(2) = 7 - 8 = -1$

$\Rightarrow$ Common diff. $(d) = a_2 - a_1$

$= (-1) - (3)$

$= -1 - 3 = \mathbf{-4}$

(*iii*) $a_n = 2n^2 + 1$

$\therefore$ $a_1 = 2(1)^2 + 1 = 2 + 1 = 3$

$a_2 = 2(2)^2 + 1 = 8 + 1 = 9$

$a_3 = 2(3)^2 + 1 = 18 + 1 = 19$

Since $a_2 - a_1 = 9 - 3 = 6$

$a_3 - a_2 = 19 - 9 = 10$

i.e. $(a_2 - a_1) \neq (a_3 - a_2)$

$\therefore$ $2n^2 + 1$ is not a term of AP.

$\Rightarrow$ 3, 9, 19, … is **not an AP.**

6. The given AP is $\frac{2m+1}{m}, \frac{2m-1}{m}, \frac{2m-3}{m}, \ldots$

$\because$ $a_1 = \frac{2m+1}{m}$ and $a_2 = \frac{2m-1}{m}$

$\therefore$ $d = a_2 - a_1$

$= \frac{2m-1}{m} - \frac{2m+1}{m} = \frac{2m-1-2m-1}{m}$

$= \frac{-1-1}{m} = \frac{-2}{m}$

Since $a_n = a + (n-1)d$

$\therefore$ $a_n = \left[\frac{2m+1}{m}\right] + (n-1)\left(\frac{-2}{m}\right)$

$$= \frac{2m+1}{m} + \left(\frac{-2n}{m}\right) + (-1) \times \left(\frac{-2}{m}\right)$$

$$= \frac{2m+1}{m} + \frac{-2n}{m} + \frac{2}{m}$$

$$= \frac{2m+1-2n+2}{m}$$

$$= \frac{2m-2n+3}{m}$$

Thus, the nth term is $\mathbf{\frac{2m-2n+3}{m}}$

Again, $a_n = \frac{2m-2n+3}{m}$

$\Rightarrow$ $a_6 = \frac{2m-(2\times 6)+3}{m} = \frac{2m-9}{m}$

Thus, the required 6th term is $\mathbf{\frac{2m-9}{m}}$.

7. Let 'a' be the first term of the AP and 'd' be its common difference.

$\therefore$ $a_n = a + (n-1)\, d$

$\Rightarrow$ $a_{17} = a + (17-1) \times \frac{3}{4}$ $\quad [\because d = \frac{3}{4}]$

$\Rightarrow$ $-20 = a + 16 \times \frac{3}{4}$ $\quad [a_{17} = -20]$

$\Rightarrow$ $-20 = a + 12$

$\Rightarrow$ $a = -20 - 12 = -32$

Now, $a_{33} = a + (33-1)\, d$

$\Rightarrow$ $a_{33} = -32 + 32 \times \frac{3}{4} = -32 + 24 = -8$

Thus, the required 33rd term is **–8**.

8. (*i*) Let the nth term of the given AP is 0.

We have 21, 18, 15, …

$\Rightarrow$ First term $(a_1) = 21$

Second term $(a_2) = 18$

$\therefore$Common difference $= a_2 - a_1$

$= 18 - 21 = -3$

Now, $a_n = a + (n-1)\, d$

$\Rightarrow$ $0 = 21 + (n-1) \times (-3)$ $\quad [\because a_n = 0]$

$\Rightarrow$ $0 = 21 - 3n + 3$

$\Rightarrow$ $3n = 21 + 3 = 24$

$\Rightarrow$ $n = \frac{24}{3} = 8$

Thus, the **8th term** will be zero (0).

(*ii*) Let the required number of terms be 'n'.

We have the AP as 7, 16, 25, … 349

$\Rightarrow$ a = First term $= 7 = a_1$

a_2 = Second term = 16

$a_2 - a_1$ = common difference = d

$\therefore$ $d = 16 - 7 = 9$

Now, $a_n = a_1 + (n-1)\, d$

$\Rightarrow$ $349 = 7 + (n-1) \times 9$ $\quad [\because a_n = 349]$

$\Rightarrow$ $(n-1) \times 9 = 349 - 7 = 342$

$\Rightarrow$ $n - 1 = \frac{342}{9} = 38$

$\Rightarrow$ $n = 38 + 1 = 39$

Thus, the required number of terms = **39**

(*iii*) In the given AP, the first term, $a = 213$, common difference, $d = 205 - 213 = -8$.

Let a_n be the nth term of the AP where n is a positive integer.

$\therefore$ $a_n = a + (n-1)d$

$= 213 + (n-1)(-8)$

If $a_n = 0$, then

$213 = 8(n-1)$

$\Rightarrow$ $\frac{213}{8} + 1 = n$

$\Rightarrow$ $n = \frac{221}{8}$

$\therefore$ n is not a positive integer.

Hence, $a_n \neq 0$ for any value of n, i.e., 0 is **not** a term of the given AP

(*iv*) In the given AP, the first term, $a = 11$ and common difference, $d = 8 - 11 = -3$.

Let a_n be the nth term of the AP where n is a positive integer.

$\therefore$ $a_n = a + (n-1)d$

$= 11 + (n-1)(-3)$

If $a_n = -150$, then

$11 - 3(n-1) = -150$

$\Rightarrow$ $3(n-1) = 11 + 150 = 161$

$\Rightarrow$ $n = 1 + \frac{161}{3} = \frac{164}{3}$

which is **not** an integer.

Hence, $a_n \neq -150$ for any value of n.

Hence, –150 is not a term of the given AP.

9. (*i*) The given AP is 2, –4, –10, –16, …

$\therefore$ First term $(a_1) = 2 = a_1$

Second term $(a_2) = -4$

$\Rightarrow$ d = Common diff. $= (a_2 - a_1)$

$= -4 - 2 = -6$

$\because$ kth term $= a + (k-1)\, d$

$\therefore$ $x = 2 + (k-1) \times (-6)$

$\Rightarrow$ $-448 = 2 + (k-1)(-6)$ $\quad [\because x = -448]$

$\Rightarrow$ $-448 = 2 - 6k + 6$

$\Rightarrow$ $6k = 448 + 2 + 6 = 456$

$\Rightarrow$ $k = \frac{456}{6} = 76$

Thus, the required value of k is **76**.

(*ii*) The given AP is

$$-43,\ -35\frac{1}{2},\ -28,\ -20\frac{1}{2},\ \ldots$$

$\therefore$ First term $(a_1) = -43$

Second term $(a_2) = -35\frac{1}{2}$

$\Rightarrow$ Common difference $(d) = (a_2 - a_1)$

$= -35\frac{1}{2} - [-43]$

$= -35\frac{1}{2} + 43 = 7\frac{1}{2}$

Now, kth term $= a + (k-1)\,d$

$x = (-43) + (k-1) \times 7\frac{1}{2}$

$\Rightarrow \quad \frac{1399}{2} = (-43) + (k-1) \times 7\frac{1}{2} \quad \left[\because x = \frac{1399}{2}\right]$

$\Rightarrow \quad (k-1) \times 7\frac{1}{2} = \frac{1399}{2} + 43$

$= \frac{1399 + 86}{2} = \frac{1485}{2}$

$\Rightarrow \quad k - 1 = \frac{1485}{2} \div 7\frac{1}{2} = \frac{1485}{2} \times \frac{2}{15} = 99$

$\Rightarrow \quad k = 99 + 1 = 100$

Thus, the required value of k is **100**.

10. The given AP is $a, a + d, a + 2d, \ldots$

$\therefore \quad a_n = a + (n-1)\,d$

$a_k = a + (k-1)\,d$

$\Rightarrow \quad a_n - a_k = a + (n-1)\,d - [a + (k-1)\,d]$

$= a + nd - d - [a + kd - d]$

$= a + nd - d - a - kd + d$

$= nd - kd = (n-k)\,d$

Thus, the required expression is

$\boldsymbol{a_n - a_k = (n-k)\,d}$ ...(1)

(*i*) We have 5th term = 17 and 15th term = 67

Let $n = 5 \therefore a_n = 17$ and $k = 15 \therefore a_k = 67$

From (1)

$a_n - a_k = (n-k)\,d$

$\Rightarrow \quad d = \frac{a_n - a_k}{(n-k)}$

$\Rightarrow \quad d = \frac{17 - 67}{5 - 15} \quad \because a_n = 17, a_k = 67\ n = 5$ and $k = 15$

$\Rightarrow \quad d = \frac{-50}{-10} = 5$

Thus the required common difference is **5**.

(*ii*) Since, $a_{10} - a_5 = 1200$, $\therefore n = 10$ and $k = 5$

From (1), we have

$a_n - a_k = (n-k)\,d$

$\Rightarrow \quad 1200 = (10-5)\,d = 5\,d$

$\Rightarrow \quad d = \frac{1200}{5} = 240$

Thus, the required value of d is **240**.

(*iii*) Since 20th term is 22 more than 18th term

$\therefore \quad a_{20} = 22 + a_{18}$

$\Rightarrow \quad a_{20} - a_{18} = 22$

$\because$ Here $n = 20$, $k = 18$

$\therefore \quad n - k = 20 - 18 = 2$

Now, $\quad d = \frac{a_n - a_k}{n - k}$

$\Rightarrow \quad d = \frac{22}{2} = 11$

Thus, the required common difference is **11**.

11. (*i*) 6, 13, 20, ..., 216

$a = 6$

$d = 7$

$a_n = 216$

$a_n = a + (n-1)\,d$

$216 = 6 + (n-1)\,7$

$210 = (n-1)\,7$

$n = 31$

Since the number of terms are odd. Therefore, the middle term will be $\frac{n+1}{2}$

Middle term $= \frac{31+1}{2} = \frac{32}{2} = 16$

$a_{16} = a + (n-1)d$

$= 6 + (15)\,(7)$

$= 6 + 105$

$= \mathbf{111}$

(*ii*) 213, 205, 197, ..., 37

$a = 213$

$d = -8$

$a_n = 37$

$a_n = a + (n-1)d$

$37 = 213 + (n-1)\,(-8)$

$8(n-1) = 176$

$n - 1 = 22$

$n = 23$

Middle term $= \frac{n+1}{2}$

$= \frac{23+1}{2} = 12$

$a_{12} = a + (12-1)d$

$= 213 + 11(-8)$

$= 213 - 88$

$= \mathbf{125}$

(*iii*) In this given AP, the first term, $a = 10$ and the common difference, $d = 7 - 10 = -3$. We now find the total number of terms of the given AP. Let the total number of terms be n. Denoting the nth term by a_n, we get

$a_n = a + (n-1)d$

$= 10 + (n-1)(-3)$

If the last term, –62 be the nth term, then

$$a_n = -62 = 10 - 3(n - 1)$$

$\Rightarrow \quad 3(n - 1) = 10 + 62 = 72$

$\Rightarrow \quad n - 1 = \frac{72}{3} = 24$

$\therefore \quad n = 24 + 1 = 25$

$\therefore$ The total number of terms of the given AP is 25 which is odd.

Hence, there is only one middle term which is $a_{\frac{25+1}{2}}$ $= a_{13}$.

Now, $a_{13} = a + (13 - 1)d$

$= 10 - 12 \times 3 = -26$

Hence, the required middle term is **–26**.

12. Let a be the first term, d be the common difference and a_n be the nth term of the given AP.

Then, $a = -\frac{4}{3}$, $d = -1 + \frac{4}{3} = \frac{1}{3}$

and $a_n = a + (n - 1)d$

$= -\frac{4}{3} + \frac{n-1}{3}$

$= \frac{-4 + n - 1}{3}$

$= \frac{n - 5}{3}$

If mth term is the last term, $4\frac{1}{3} = \frac{13}{3}$

Then $a_m = \frac{13}{3} = \frac{m - 5}{3}$

$\Rightarrow \quad m - 5 = 13$

$\Rightarrow \quad m = 18$ which is even

Hence, there are two middle terms of the given AP *viz.* $a_{\frac{18}{2}}$ and $a_{\frac{18}{2}+1}$, i.e. a_9 and a_{10}.

Now, $a_9 = -\frac{4}{3} + \frac{1}{3}(9 - 1) = \frac{8 - 4}{3} = \frac{4}{3}$

and $a_{10} = -\frac{4}{3} + \frac{1(10 - 1)}{3} = \frac{9 - 4}{3} = \frac{5}{3}$

$\therefore$ Required sum of two middle terms $= \frac{4}{3} + \frac{5}{3} = \frac{9}{3} = \mathbf{3}$.

13. (*i*) The given AP is 3, 10, 17, 24, …

$\therefore$ $a = 3$ and $d = 10 - 3 = 7$

Since $a_n = a + (n - 1)\,d$

$\therefore \quad a_{13} = 3 + (13 - 1) \times 7$

$= 3 + 12 \times 7 = 3 + 84 = 87$

Since $a_n = a_{13} + 84 = 87 + 84 = 171$

$\therefore \quad a + (n - 1)\,d = 171$

$\Rightarrow \quad 3 + (n - 1) \times 7 = 171$

$\Rightarrow \quad (n - 1) \times 7 = 171 - 3 = 168$

$\Rightarrow \quad (n - 1) = \frac{168}{7} = 24$

$\Rightarrow \quad n = 24 + 1 = 25$

Thus, **25th** term is 84 more than 13th term.

(*ii*) In the given AP, the first term, $a = 5$ and the common difference, $d = 9 - 5 = 4$.

Let a_n be the nth term.

Then $a_n = a + (n - 1)d = 5 + (n - 1)4$

$= 4n + 1$.

If $a_m = 81$, then

$4m + 1 = 81$

$\Rightarrow \quad m = \frac{80}{4} = 20$

$\therefore$ Required term is **20th term**.

(*iii*) In the given AP, the first term, $a = 9$ and the common difference, $d = 12 - 9 = 3$. Let a_n be the nth term.

Then $a_n = a + (n - 1)d$

$= 9 + (n - 1)3$

$= 3n + 6$

Now, $a_{36} = 3 \times 36 + 6$

$= 108 + 6 = 114$ …(1)

If mth term is the required term, then according to the problem, we have

$a_m = a_{36} + 39 = 114 + 39$ [From (1)]

$= 153$

$\therefore \quad 3m + 6 = 153$

$\Rightarrow \quad m = \frac{153 - 6}{3} = \frac{147}{3} = 49$

$\therefore$ Required term is **49th term**.

(*iv*) In the given AP, the first term, $a = 8$ and the common difference, $d = 14 - 8 = 6$.

Let a_n be the nth term. Then

$a_n = a + (n - 1)d$

$= 8 + 6(n - 1)$

$= 6n + 2$

$\therefore \quad a_{41} = 6 \times 41 + 2$

$= 246 + 2$

$= 248$ …(1)

Let mth term be the required term. Then according to the problem, we have

$a_m = a_{41} + 72$

$= 248 + 72$ [From (1)]

$= 320$

$\Rightarrow \quad 6m + 2 = 320$

$\Rightarrow \quad 6m = 318$

$\Rightarrow \quad m = \frac{318}{6} = 53$

$\therefore$ Required term is **53rd term**.

(*v*) The given AP is 3, 15, 27, 39, …

$\therefore \quad a = 3$ and $d = 15 - 3 = 12$

Since $a_n = a + (n - 1)\,d$

$\therefore \quad a_{21} = 3 + (21 - 1) \times 12$
$= 3 + 20 \times 12 = 243$

Since, a_n is 120 more than a_{21}

$\therefore \quad a_n = a_{21} + 120$

$\Rightarrow \quad a + (n - 1)\, d = 243 + 120 = 363$

$\Rightarrow \quad 3 + (n - 1) \times 12 = 363$

$\Rightarrow \quad (n - 1) \times 12 = 363 - 3 = 360$

$\Rightarrow \quad n - 1 = \dfrac{360}{12} = 30$

$\Rightarrow \quad n - 1 = 30$

$\Rightarrow \quad n = 30 + 1 = 31$

Hence, **31st term** is 120 more than 21st term.

(*vi*) In the given AP, the first term, $a = 5$ and the common difference, $d = 15 - 5 = 10$.

Let a_n be the nth term. Then

$a_n = a + (n - 1)d$
$= 5 + 10(n - 1)$
$= 10n - 5$

Now, $a_{31} = 10 \times 31 - 5$
$= 310 - 5$
$= 305 \quad \ldots(1)$

Let mth term be the required term. Then according to the problem, we have

$a_m = a_{31} + 130$
$= 130 + 305 \quad$ [From (1)]
$= 435$

$\therefore \quad 10m - 5 = 435$

$\Rightarrow \quad 10m = 440$

$\Rightarrow \quad m = 44$

$\therefore$ Required term = **44th term**.

14. Let a be the 1st term, d be the common difference and a_n be the nth term of the AP. Given that $a = 12$.

$\therefore\ a_n = a + (n - 1)d = 12 + (n - 1)d$

$\therefore\ a_7 = 12 + (7 - 1)d = 12 + 6d$

and $a_{11} = 12 + 10d$

Given that $a_{11} - a_7 = 24$

$\Rightarrow 12 + 10d - 12 - 6d = 24 \Rightarrow 4d = 24 \Rightarrow d = \dfrac{24}{4} = 6$

$\therefore\ a_{20} = 12 + (20 - 1) \times 6 = 12 + 114 =$ **126** which is the required value of the 20th term.

15. Let a be the first term, d be the common difference and a_n be the nth term of the AP.

Here given that $n = 50$, $a_3 = 12$ and $a_{50} = 106$.

$\therefore \quad a_n = a + (n - 1)d$

$\therefore \quad a_{50} = a + (50 - 1)d$
$= a + 49d$

$\Rightarrow \quad 106 = a + 49d \quad \ldots(1)$

$\Rightarrow \quad a_3 = a + (3 - 1)d$

$\Rightarrow \quad 12 = a + 2d \quad \ldots(2)$

Subtracting (2) from (1), we get

$106 - 12 = (49 - 2)d$

$\Rightarrow \quad 94 = 47d$

$\Rightarrow \quad d = \dfrac{94}{47} = 2$

$\therefore$ From (1),

$a = 106 - 49 \times 2$
$= 106 - 98 = 8$

Hence, $a_{29} = a + (29 - 1)d$
$= 8 + 28 \times 2$
$= 8 + 56$
$=$ **64**

Which is the required value of **29th term**.

16. (*i*) Let the first term and common difference of an AP be 'a' and 'd' respectively.

Here, $d = 11$

$\therefore \quad a_{20} = a + (20 - 1)d = a + 19(11) = a + 209$

Also, $a_{18} = a + (18 - 1)d = a + 17(11) = a + 187$

$\therefore \quad a_{20} - a_{18} = a + 209 - a - 187$
$=$ **22**

(*ii*) $a_{21} - a_7 = 84$

$a_{21} = a + (21 - 1)d = a + 20d$

$a_7 = a + (7 - 1)d = a + 6d$

$a_{21} - a_7 = 84$

$a + 20d - a - 6d = 84$

$14d = 84$

$\boldsymbol{d = 6}$

17. Let the first term and common difference of the AP be 'a' and 'd' respectively.

$\therefore \quad a_9 = a + 8d \Rightarrow a + 8d = -2.6 \quad \ldots(1)$

$a_{23} = a + 22d \Rightarrow a + 22d = -5.4 \quad \ldots(2)$

Subtracting (1) from (2), we have

$a + 22d = -\,5.4$

$a + \ \ 8d = -\,2.6$

$(-)\ (-) \qquad (+)$

$\Rightarrow \quad (22 - 8)\, d = -5.4 + 2.6$

$\Rightarrow \quad 14d = -2.8$

$\Rightarrow \quad d = -\dfrac{2.8}{14} = -0.2$

Substituting $d = -0.2$ in (1),

$a + 8\,(-0.2) = -2.6$

$a + (-1.6) = -2.6$

$\Rightarrow \quad a = -2.6 + (1.6) = -1$

Now, $a_2 = a + d = -1 + (-0.2) = -1.2$

Thus 2nd term is **–1.2**.

Again, $a_k = a + (k - 1)\, d$
$= -1 + (k - 1) \times (-0.2)$
$= -1 + (-0.2k) + 0.2 = -0.8 - 0.2k$

Thus, kth term is $\boldsymbol{(-0.8 - 0.2k)}$.

18. Let the first term of the AP and common difference are 'a' and 'd' respectively.

$\therefore \quad a_n = a + (n-1)d$

$\Rightarrow \quad a_6 = a + (6-1)d = a + 5d$

$a_{10} = a + (10-1)d = a + 9d$

$\therefore \quad a + 5d = -10 \quad \ldots(1)$

$a + 9d = -26 \quad \ldots(2)$

Subtracting (1) from (2), we get

$a + 9d = -26$
$a + 5d = -10$
$(-) \quad (-) \quad (+)$

$4d = -16 \Rightarrow d = \frac{-16}{4} = -4$

Substituting $d = -4$ in (1), we get

$a + 5(-4) = -10$

$\Rightarrow \quad a - 20 = -10 \Rightarrow a = -10 + 20 = 10$

Now, $\quad a_{15} = a + 14d$

$\Rightarrow \quad a_{15} = 10 + 14(-4) = 10 - 56$

$\Rightarrow \quad a_{15} = -46$

Thus the required 15th term is **−46**.

19. Let the first term and common difference of the given AP are 'a' and 'd' respectively.

$\therefore \quad a_n = a + (n-1)d$

$\Rightarrow \quad a_7 = a + 6d \Rightarrow a + 6d = -4 \quad \ldots(1)$

$a_{13} = a + 12d \Rightarrow a + 12d = -16 \quad \ldots(2)$

Subtracting (1) from (2), we get

$a + 12d = -16$
$a + 6d = -4$
$(-) \quad (-) \quad (+)$

$6d = -12 \Rightarrow d = \frac{-12}{6} = -2$

Substituting $d = -2$ in (1), we get

$a + 6(-2) = -4$

$\Rightarrow \quad a - 12 = -4 \Rightarrow a = -4 + 12 = 8$

Now, the AP is $a, a + d, a + 3d, \ldots$

$\Rightarrow \quad 8, 8 + (-2), 8 + 2(-2), 8 + 3(-2) \ldots$

$\Rightarrow \quad 8, 6, 8 - 4, 8 - 6, \ldots$

$\Rightarrow \quad 8, 6, 4, 2, \ldots$

Thus, the required AP is

8, 6, 4, 2, 0, −2, …

20. Let 'a' and 'd' be the first term and common difference respectively.

$\therefore \quad a_n = a + (n-1)d$

$\Rightarrow \quad a_8 = a + 7d$

$\Rightarrow \quad 37 = a + 7d$

or $\quad a + 7d = 37 \quad [\because a_8 = 37] \ldots(1)$

Also, $\quad a_{12} = a + 11d$

$\Rightarrow \quad a + 11d = 57 \quad [\because a_{12} = 57] \ldots(2)$

Subtracting (1) from (2), we get

$(a + 11d) - (a + 7d) = 57 - 37$

$\Rightarrow \quad 11d - 7d = 20$

$\Rightarrow \quad 4d = 20 \text{ or } d = \frac{20}{4} = 5$

Substituting $d = 5$ in (1), we get

$a + 7(5) = 37$

$\Rightarrow \quad a = 37 - 35 = 2$

Now, the AP is

	$a,$	$a + d,$	$a + 2d,$	$a + 3d,$	…
or	2,	2 + 5,	2 + 2(5),	2 + 3(5),	…
or	2,	7,	2 + 10,	2 + 15,	…
or	**2,**	**7,**	**12,**	**17,**	…

21. (*i*) $a_5 = 20 \quad \ldots(1)$

$a_7 + a_{11} = 64 \quad \ldots(2)$

From eq. (1), we get

$a + 4d = 20$

$a = 20 - 4d \quad \ldots(3)$

Now from eq. (2), we get

$a + 6d + a + 10d = 64$

$2a + 16d = 64$

$a + 8d = 32 \quad \ldots(4)$

Putting the value of a from eq.(3) in eq.(4)

$20 - 4d + 8d = 32$

$20 + 4d = 32$

$4d = 12$

$\boldsymbol{d = 3}$

(*ii*) Let a be the first term, d, the common difference and a_n be the nth term of the AP.

Then $\quad a_n = a + (n-1)d \quad \ldots(1)$

Now, $\quad a_4 = 11 \quad$ [Given]

$a + 3d = 11 \quad \ldots(2)$

and $\quad a_5 + a_7 = 34 \quad$ [Given]

$\Rightarrow \quad (a + 4d) + (a + 6d) = 34 \quad$ [From (1)]

$\Rightarrow \quad 2a + 10d = 34$

$\Rightarrow \quad a + 5d = 17 \quad \ldots(3)$

Subtracting (2) from (3), we get $2d = 6$

$\therefore$ $\boldsymbol{d = 3}$ which is the required common difference.

(*iii*) $a_9 = -32 \quad \ldots(1)$

$a_{11} + a_{13} = -94 \quad \ldots(2)$

From eq. (1), we get

$a + (9-1)d = -32$

$a + 8d = -32 \quad \ldots(3)$

Now, simplifying eq. (2), we get

$a + 10d + a + 12d = -94$

$2a + 22d = -94$

$a + 11d = -47 \quad \ldots(4)$

Putting the value of a from eq. (3) in eq. (4), we get

$-32 - 8d + 11d = -47$

$-32 + 3d = -47$

$3d = -15$

$\boldsymbol{d = -5}$

(*iv*) Let a = First term and d = common diff.

∴ The first three terms of the AP are

$$a, \quad a + d, \quad a + 2d \qquad \ldots(1)$$

$\because \quad a_4 + a_8 = 24$

$\therefore \quad (a + 3d) + (a + 7d) = 24$

$\Rightarrow \quad 2a + 10d = 24$

$\Rightarrow \quad a + 5d = 12 \qquad \ldots(2)$

Also, $\quad a_6 + a_{10} = 44$

$\Rightarrow \quad (a + 5d) + (a + 9d) = 44$

$\Rightarrow \quad 2a + 14d = 44$

$\Rightarrow \quad a + 7d = 22 \qquad \ldots(3)$

Subtracting (2) from (3), we get

$$a + 7d - a - 5d = 22 - 12$$

$\Rightarrow \quad 2d = 10$ or $d = 5$

Substituting $d = 5$ in (2), we get

$$a + 5(5) = 12 \Rightarrow a = -13$$

Now, substituting $a = -13$ and $d = 5$ in (1),

$$-13, \quad (-13 + 5), \quad [-13 + 2(5)]$$

or $\quad -13, \quad -8, \quad -3$

Thus, the required first 3 terms are

–13, –8, –3.

(*v*) Let a be the first term, d, the common difference and a_n, the nth term of the AP.

Then $\quad a_n = a + (n - 1)d \qquad \ldots(1)$

Now, given that

$$a_5 + a_9 = 30$$

$\Rightarrow \quad (a + 4d) + (a + 8d) = 30 \qquad$ [From (1)]

$\Rightarrow \quad 2a + 12d = 30$

$\Rightarrow \quad a + 6d = 15 \qquad \ldots(2)$

Also, given that $\quad a_{25} = 3a_8$

$\Rightarrow \quad a + 24d = 3(a + 7d)$

$\Rightarrow \quad 2a - 3d = 0$

$\Rightarrow \quad a = \dfrac{3d}{2} \qquad \ldots(3)$

∴ From (2) and (3), we have

$$\frac{3d}{2} + 6d = 15$$

$\Rightarrow \quad \dfrac{15d}{2} = 15$

$\Rightarrow \quad d = 2 \qquad \ldots(4)$

∴ From (3), $\quad a = \dfrac{3}{2} \times 2 = 3 \qquad \ldots(5)$

∴ The required AP is 3, 3 + 2, 3 + 4, 3 + 6, …

i.e., **3, 5, 7, 9, …**

(*vi*) Let a be the first term, d, the common difference and a_n, the nth term of the AP.

Then $\quad a_n = a + (n - 1)d \qquad \ldots(1)$

Now, given that

$$a_5 + a_7 = 52$$

$\Rightarrow \quad (a + 4d) + (a + 6d) = 52 \qquad$ [From (1)]

$\Rightarrow \quad 2a + 10d = 52$

$\Rightarrow \quad a + 5d = 26 \qquad \ldots(2)$

Also, $\quad a_{10} = 46$

$\Rightarrow \quad a + 9d = 46 \qquad$ [From (1)] …(3)

Subtracting (2) from (3), we get

$$4d = 20$$

$\Rightarrow \quad d = 5 \qquad \ldots(4)$

∴ From (2) and (4), we have

$$a = 26 - 5 \times 5 = 1$$

∴ The required AP is 1, + 1 + 5, 1 + 10, 1 + 15, …

i.e. **1, 6, 11, 16, …**

(*vii*) Let a be the first term, d, the common difference and a_n, the nth term of the AP.

Then, $\quad a_n = a + (n - 1)d \qquad \ldots(1)$

Now, given that

$$a_3 + a_8 = 7$$

$\Rightarrow \quad (a + 2d) + (a + 7d) = 7 \qquad$ [From (1)]

$\Rightarrow \quad 2a + 9d = 7 \qquad \ldots(2)$

Also, given that

$$a_7 + a_{14} = -3$$

$\Rightarrow \quad (a + 6d) + (a + 13d) = -3 \qquad$ [From (1)]

$\Rightarrow \quad 2a + 19d = -3 \qquad \ldots(3)$

Subtracting (2) from (3), we get

$$10d = -10$$

$\Rightarrow \quad d = -1 \qquad \ldots(4)$

∴ From (2), we have

$$2a = 7 - 9 \times (-1)$$
$$= 7 + 9 = 16$$

$\Rightarrow \quad a = 8 \qquad \ldots(5)$

$\therefore \quad a_{10} = a + 9d$

$= 8 + 9 \times (-1)$

[From (4) and (5)]

$= \mathbf{-1}$

which is the required value of the 10th term.

22. (*i*) Let a be the first term, d, the common difference and a_n, the nth term of the AP.

Then $\quad a_n = a + (n - 1)d \qquad \ldots(1)$

Given that $\quad a_{10} = 52$

$\Rightarrow \quad a + 9d = 52 \qquad$ [From (1)]

$\Rightarrow \quad a = 52 - 9d \qquad \ldots(2)$

Also, given that

$$a_{17} = 20 + a_{13}$$

$\Rightarrow \quad (a + 16d) = 20 + (a + 12d) \qquad$ [From (1)]

$\Rightarrow \quad 4d = 20$

$\Rightarrow \quad d = 5 \qquad \ldots(3)$

∴ From (2), $\quad a = 52 - 9 \times 5$

$= 52 - 45 = 7 \qquad \ldots(4)$

$\therefore$ From (3) and (4), the required AP is

7, 7 + 5, 7 + 10, 7 + 15, …

i.e., 7, 12, 17, 22 …

(*ii*) Let a be the first term, d, the common difference and a_n, the nth term of the AP. Then

$$a_n = a + (n - 1)d \quad \ldots(1)$$

Given that $a_8 = 31$

$\Rightarrow$ $a + 7d = 31$ [From (1)]

$\Rightarrow$ $a = 31 - 7d$ …(2)

Also, given that $a_{15} = 16 + a_{11}$

$\Rightarrow$ $a + 14d = 16 + (a + 10d)$ [From (1)]

$\Rightarrow$ $4d = 16$

$\Rightarrow$ $d = 4$ …(3)

$\therefore$ From (2), $a = 31 - 4 \times 7 = 3$ …(4)

$\therefore$ From (2) and (4), the required AP is

3, 3 + 4, 3 + 8, 3 + 12, 3 + 16,…

i.e., **3, 7, 11, 15, 19, …**

(*iii*) Let a be the 1st term, d, the common difference and a_n, the nth term of the AP. Then

$$a_n = a + (n - 1)d \quad \ldots(1)$$

Given that $a_5 = 31$

$\Rightarrow$ $a + 4d = 31$ [From (1)]

$\Rightarrow$ $a = 31 - 4d$ …(2)

Also, given that $a_{25} = 140 + a_5$

$\Rightarrow$ $a + 24d = 140 + (a + 4d)$ [From (1)]

$\Rightarrow$ $20d = 140$

$\Rightarrow$ $d = 7$ …(3)

$\therefore$ From (2), we have

$a = 31 - 4 \times 7$

$= 3$ …(4)

$\therefore$ From (3) and (4), the required AP is

3, 3 + 7, 3 + 14, 3 + 21 …,

i.e. **3, 10, 17, 24, …**

23. (*i*) Let a be the first term, d, the common difference and a_n, the nth term of the AP. Then

$$a_n = a + (n - 1)d \quad \ldots(1)$$

Now, given that $a_{19} = 3a_6$

$\Rightarrow$ $a + 18d = 3(a + 5d)$ [From (1)]

$\Rightarrow$ $2a - 3d = 0$

$\Rightarrow$ $a = \dfrac{3d}{2}$ …(2)

Also, given that $a_9 = 19$

$\Rightarrow$ $a + 8d = 19$ [From (1)]

$\Rightarrow$ $\dfrac{3d}{2} + 8d = 19$ [From (2)]

$\Rightarrow$ $\dfrac{19d}{2} = 19$

$\Rightarrow$ $d = 2$ …(3)

$\therefore$ From (2), $a = \dfrac{3}{2} \times 2 = 3$ …(4)

$\therefore$ From (3) and (4), the required AP is

3, 3 + 2, 3 + 4, 3 + 6, …,

i.e., **3, 5, 7, 9, …**

(*ii*) Let a be the first term, d, the common difference and a_n, the nth term of the AP. Then

$$a_n = a + (n - 1)d \quad \ldots(1)$$

Given that $a_9 = 6a_2$

$\Rightarrow$ $a + 8d = 6(a + d)$ [From (1)]

$\Rightarrow$ $5a = 2d$

$\Rightarrow$ $a = \dfrac{2}{5}d$ …(2)

Also, given that $a_5 = 22$

$\Rightarrow$ $a + 4d = 22$ [From (1)]

$\Rightarrow$ $\dfrac{2d}{5} + 4d = 22$

$\Rightarrow$ $\dfrac{22d}{5} = 22$

$\Rightarrow$ $d = 5$ …(3)

$\therefore$ From (2), $a = \dfrac{2}{5} \times 5 = 2$ …(4)

$\therefore$ From (3) and (4), the required AP is

2, 2 + 5, 2 + 10, 2 + 15, …,

i.e., **2, 7, 12, 17, …**

(*iii*) Let a be the first term, d, the common difference and a_n, the nth term of the AP.

Then $a_n = a + (n - 1)d$ …(1)

Given that $4a_4 = 18a_{18}$

$\Rightarrow$ $4(a + 3d) = 18(a + 17d)$ [From (1)]

$\Rightarrow$ $4a + 12d = 18a + 306d$

$\Rightarrow$ $14a + 294d = 0$

$\Rightarrow$ $a + 21d = 0$

$\Rightarrow$ $a = -21d$ …(2)

$\therefore$ $a_{22} = a + 21d$ [From (1)]

$= -21d + 21d$ [From (2)]

$= \mathbf{0}$

which is the required value of a_{22}.

(*iv*) $a_9 = 7a_2$ ... (1)

$a_{12} = 5a_3 + 2$ ... (2)

From eq. (1), we get

$a + 8d = 7(a + d)$

$a + 8d = 7a + 7d$

$d = 6a$ ... (3)

Now from eq. (2), we get

$a + 11d = 5(a + 2d) + 2$

$a + 11d = 5a + 10d + 2$

$d = 4a + 2$ ... (4)

Putting the value of d from eq.(3) in eq.(4)

$d = 4a + 2$

$6a = 4a + 2$

$$2a = 2$$
$$\mathbf{a = 1}$$
We know
$$d = 6a$$
$$\mathbf{d = 6}$$

24. (*i*) Since $\frac{3}{5}, x, \frac{5}{3}x$ are in AP.

$\therefore$ $x - \frac{3}{5} = \frac{5}{3}x - x$

$\Rightarrow$ $x - \frac{5}{3}x + x = \frac{3}{5}$

$2x - \frac{5}{3}x = \frac{3}{5}$

$\Rightarrow$ $\frac{x}{3} = \frac{3}{5},$

$\Rightarrow$ $x = \frac{3}{5} \times 3 = \mathbf{\frac{9}{5}}$

(*ii*) $2k + 1, 3k + 3, 5k - 1$

For these three terms to be in an AP, the common difference of first two and last two terms must be equal.

$$3k + 3 - (2k + 1) = 5k - 1 - (3k + 3)$$
$$3k + 3 - 2k - 1 = 5k - 1 - 3k - 3$$
$$k + 2 = 2k - 4$$
$$\mathbf{k = 6}$$

(*iii*) Since, $2p - 1, 3p + 1$ and 11 are in AP.

$\therefore$ $3p + 1 - 2p + 1 = 11 - 3p - 1$

$\Rightarrow$ $p + 2 = 10 - 3p$

$\Rightarrow$ $4p = 8$

$\Rightarrow$ $\mathbf{p = 2}$

which is the required value of p and the required number are $2 \times 2 - 1$, $3 \times 2 + 1$ and 11, i.e. **3, 7** and **11**.

(*iv*) Since the given expressions are three consecutive terms of an AP, hence

$$2k^2 + 3k + 6 - k^2 - 4k - 8 = 3k^2 + 4k + 4 - 2k^2 - 3k - 6$$

$\Rightarrow$ $k^2 - k - 2 = k^2 + k - 2$

$\Rightarrow$ $2k = 0$

$\Rightarrow$ $\mathbf{k = 0}$

which is the required value of k.

(*v*) Since 18, a, b, –3 are in AP.

$\therefore$ $a - 18 = b - a = -3 - b$

$\therefore$ $a - 18 = b - a$ …(1)

and $b - a = -3 - b$ …(2)

$\therefore$ From (1), $2a = 18 + b$

$\Rightarrow$ $b = 2a - 18$ …(3)

$\therefore$ From (2), $2b = a - 3$

$\Rightarrow$ $2(2a - 18) = a - 3$ [From (3)]

$\Rightarrow$ $3a - 36 + 3 = 0$

$\Rightarrow$ $a = \frac{+33}{3} = 11$ …(4)

$\therefore$ From (3), $b = 2 \times 11 - 18 = 4$

$\therefore$ The required values of a and b are **11** and **4** respectively.

25. (*i*) Two-digit numbers which are divisible by 6 are 12, 18, 24, 30, …96

which are in AP with first term, $a = 12$ and the common difference, $d = 18 - 12 = 6$.

Let n be the required number so that $a_n = 96$ …(1)

Then $a_n = a + (n - 1)d$

$\Rightarrow$ $96 = 12 + (n - 1)6$ [From (1)]

$\Rightarrow$ $\frac{84}{6} = n - 1$

$\Rightarrow$ $n = 1 + 14 = 15$

$\therefore$ The required number is **15**.

(*ii*) We need to form an AP

$$14, 21, 28, \ldots, 98$$
$$a = 14, d = 7, a_n = l = 98$$
$$a_n = a + (n - 1)d$$

$\Rightarrow$ $98 = 14 + (n - 1)7$

$\Rightarrow$ $84 = (n - 1)7$

$\Rightarrow$ $n - 1 = 12$

$\Rightarrow$ $\mathbf{n = 13}$

(*iii*) The three-digit numbers which are divisible by 9 are 108, 117, 126, …, 999 which are in AP with first term $a = 108$ and the common difference, $d = 117 - 108 = 9$.

Let the number of terms of this AP is n.

Here the first term = a = 108

Common difference = d = 117 – 108 = 9

a_n = nth term

$= a + (n - 1)d$ …(1)

Given that $a_n = 999$ …(2)

The last term

$\therefore$ $108 + (n - 1)9 = 999$ [From (1) and (2)]

$\Rightarrow$ $n - 1 = \frac{999 - 108}{9} = \frac{891}{9} = 99$

$\therefore$ $n = \mathbf{100}$

which is the required number.

(*iv*) Integers lying between 200 and 500, which are divisible by 8 are 208, 216, 224, …496 which are in AP with first term, $a = 208$, common difference, $d = 216 - 208 = 8$.

If n be the total number of terms of this AP, then

$a_n = 496$

Also, $a_n = a + (n - 1)d$

$\therefore$ $a + (n - 1)d = 496$

$\Rightarrow$ $208 + (n - 1)8 = 496$

$\Rightarrow$ $(n - 1)8 = 496 - 208 = 288$

$\Rightarrow$ $n - 1 = \frac{288}{8} = 36$

$\Rightarrow$ $n = \mathbf{37}$

which is the required number of term.

26. (*i*) Let a_1 be the first term and d_1 be the common difference of the first AP and let a_2 be the first term and d_2 be the common difference of the second AP so that

$$a_1 = 1,\ d_1 = 7 - 1 = 6 \quad \ldots(1)$$
$$a_2 = 69,\ d_2 = 68 - 69 = -1 \quad \ldots(2)$$

We denote the nth term of the first AP by a_n and that of the second AP by a_n'.

$\therefore \quad a_n = a_1 + (n-1)d_1$
$= 1 + (n-1)6$
$= 6n - 5$ [From (1)] …(3)

and $\quad a_n' = a_2 + (n-1)d_2$
$= 69 + (n-1)(-1)$
$= -n + 70$ [From (2)] …(4)

$\because \quad a_n = a_n'$

$\therefore$ From (3) and (4), we have

$6n - 5 = -n + 70$

$\Rightarrow \quad 7n = 75$

$\Rightarrow \quad n = \dfrac{75}{7}$

which is not a natural number.

Hence, **there is no value of *n*** for which the two given AP's, nth term are the same.

(*ii*) For the AP 6, 3, 0, …

a = First term = 6
d = Common diff. = 3 − 6 = −3

$\therefore \quad a_n = a + (n-1)d = 6 + (n-1)(-3)$

$\therefore \quad a_n = 6 - 3n + 3 = 9 - 3n$ … (1)

For the AP 2, 0, −2, …

a = First term = 2
d = Common diff. = 0 − 2 = −2

$\therefore \quad a_n = a + (n-1)d$

$\Rightarrow \quad a_n = 2 + (n-1)(-2)$

$\Rightarrow \quad a_n = 2 + (-2n + 2)$

$\Rightarrow \quad a_n = 2 - 2n + 2 = 4 - 2n$ … (2)

From (1) and (2), we get

$4 - 2n = 9 - 3n$

$\Rightarrow \quad -2n + 3n = 9 - 4$

$\Rightarrow \quad \boldsymbol{n = 5}$

(*iii*) For AP 13, 19, 25, …

$a = 13$ and $d = 19 - 13 = 6$

$\therefore \quad a_n = a + (n-1)d$

$\Rightarrow \quad a_n = 13 + (n-1) \times 6$

$\Rightarrow \quad a_n = 13 + 6n - 6 = 7 + 6n$

For AP 69, 68, 67, …

$a = 69$ and $d = 68 - 69 = -1$

$a_n = a + (n-1)d$

$\Rightarrow \quad a_n = 69 + (n-1)(-1)$

$\Rightarrow \quad a_n = 69 + 1 - n = 70 - n$

$\because a_n$ for both the AP are same.

$\therefore \quad 7 + 6n = 70 - n$

$\Rightarrow \quad 6n + n = 70 - 7 = 63$

$\Rightarrow \quad 7n = 63$

$\Rightarrow \quad \boldsymbol{n = 9}$

Now, for the AP in (*a*),

$a_n = 7 + 6(9) = 7 + 54 = 61$

Thus, the nth term = **61**.

(*iv*) Let a_1 be the first term and d_1 be the common difference of the first AP and a_2 be the first term and d_2 be the common difference of the second AP so that

$a_1 = 9,\ d_1 = 7 - 9 = -2$ …(1)

and $\quad a_2 = 24,\ d_2 = 21 - 24 = -3$ …(2)

We denote the nth term of the first and the second AP's by a_n and a'_n respectively. Then

$a_n = a_1 + (n-1)d_1$
$= 9 - 2(n-1)$
$= -2n + 11$ [From (1)] …(3)

and $\quad a'_n = a_2 + (n-1)d_2$
$= 24 - 3(n-1)$
$= -3n + 27$ [From (2)] …(4)

$\because \quad a_n = a'_n$

$\therefore \quad -2n + 11 = 3n + 27$ [From (3) and (4)]

$\Rightarrow \quad n = 27 - 11 = \mathbf{16}$

which is the required value of n.

Now, from (3)

$a_{16} = -2 \times 16 + 11$
$= -32 + 11$
$= \mathbf{-21}$

which is the required value of a_{16}.

27. (*i*) The given AP is 114, 109, 104, …

Here, First term = a = 114

Common diff. = d = 109 − 114 = −5

Let the nth term of the given AP is the first negative term.

$\Rightarrow \quad a_n < 0$ or $[a + (n-1)d] < 0$

$\Rightarrow [114 + (n-1) \times (-5)] < 0$

$\Rightarrow [114 + (n \times -5) + 5] < 0$

$\Rightarrow [119 - 5n] < 0$

$\Rightarrow 119 < 5n$ or $5n > 119$

$\Rightarrow \quad n > \dfrac{119}{5}$ or $n > 23\dfrac{4}{5}$

Since, the natural number just greater than $23\dfrac{4}{5}$ is 24.

Thus, **24th** term of the given progression is the first negative term.

(*ii*) In this AP, the first term, a = 53 and the common difference, d = 48 − 53 = −5.

Let the nth term a_n of this AP be the first negative term

Then $a_n = a + (n-1)d$
$= 53 - 5(n-1)$
$= -5n + 58$

Now, $a_n < 0$

$\Rightarrow 5n > 58$

$\Rightarrow n > \frac{58}{5} = 11\frac{3}{5}$

Since 12 is the natural number just above 11,

$\therefore$ We take $n = 12$.

Hence, the required first negative term of the given AP is **12th term**.

28. Let the three numbers are:

$$(a-d),\ a,\ (a+d)$$

Since, sum of these numbers = 21

$\therefore a - d + a + a + d = 21$

$\Rightarrow 3a = 21$

$\Rightarrow a = 7$

Since, the product of these numbers = 231

$\therefore (a-d)(a)(a+d) = 231$

$\Rightarrow (a^2 - d^2) \times a = 231$

$\Rightarrow (7^2 - d^2) \times 7 = 231$

$\Rightarrow 49 - d^2 = \frac{231}{7} = 33$

$\Rightarrow -d^2 = 33 - 49 = -16$

$\Rightarrow d^2 = 16 \Rightarrow d = \pm 4$

Now, the numbers are

$a - d,\quad a,\quad a + d$

$\Rightarrow 7 - 4,\quad 7,\quad 7 + 4$ or $7 + 4, 7, 7 - 4$

$\Rightarrow 3,\quad 7,\quad 11$ or $11, 7, 3$

So, the required three number are

3, 7, 11 or **11, 7, 3**

29. Let the three numbers in AP be $a - d$, a and $a + d$.

$\therefore$ According to the problem, we have

$(a-d) + a + (a+d) = 12$

$\Rightarrow 3a = 12$

$\Rightarrow a = \frac{12}{3} = 4$ ...(1)

Now, given that

$(a-d)^3 + a^3 + (a+d)^3 = 288$

$\Rightarrow (4-d)^3 + (4+d)^3 + 4^3 = 288$ [From (1)]

$\Rightarrow (4 - d + 4 + d)^3 - 3(4-d)(4+d)(4 - d + 4 + d)$
$= 288 - 64$
$= 224$

[Using the formula, $a^3 + b^3 = (a+b)^3 = 3ab(a+b)$]

$\Rightarrow 64 \times 8 - 3(16 - d^2) \times 8 = 224$

$\Rightarrow 64 - 48 + 3d^2 = 28$

$\Rightarrow 3d^2 = -16 + 28 = 12$

$\Rightarrow d^2 = 4$

$\Rightarrow d = \pm 2$ ...(2)

$\therefore$ From (1) and (2), the required number are either 4 – 2, 4 and 4 + 2, i.e., **2, 4 and 6** or 4 + 2, 4 and 4 – 2, i.e., **6, 4 and 2**.

30. (*i*) AP : 5, 9, 13, …, 185

Since we need to find the 9th term from the end therefore we will reverse the AP.

$a = 185, d = -4, n = 9$

$a_9 = a + (9-1)d$
$= 185 + 8(-4)$
$= 185 - 32$
$= \mathbf{153}$

(*ii*) AP : 1, 6, 11, 16, …, 211, 216

Since we need to find the 17th term from the end therefore we will reverse the AP.

$a = 216, d = -5, n = 17$

$a_{17} = a + (17-1)d$
$= 218 + 16(-5)$
$= 216 - 80$
$= \mathbf{136}$

(*iii*) The given AP is 17, 14, 11, … (–40).

Here, First term = a = 17

Common diff. = d = 14 – 17 = –3

And the last term l = –40

Since, the nth term from the end = $l - (n-1)d$

$\therefore$ 6th term from the end $= -40 - (6-1)(-3)$
$= -40 - (5)(-3)$
$= -40 + 15 = \mathbf{-25}$

(*iv*) The given AP is 8, 10, 12, …, 126

Here, $d = 10 - 8 = 2$ and $l = 126$

Since, nth term from the end = $l - (n-1)d$

$\therefore$ 10th term from the end $= 126 - (10-1) \times 2$
$= 126 - 9 \times 2$
$= 126 - 18 = \mathbf{108}$

(*v*) In the given AP, the first term, $a = 7$, the common difference, $d = 10 - 7 = 3$ and the last term, $l = 184$.

Let a_n be the nth term from the end.

Then $a_n = l - (n-1)d$
$= 184 - 3(n-1)$
$= 184 - 3n + 3$
$= 187 - 3n$

$\therefore a_8 = 187 - 3 \times 8$
$= 187 - 24 = \mathbf{163}$

which is the required term.

(*vi*) In the given AP, the first term, $a = 3$, the common difference, $d = 8 - 3 = 5$ and the last term, $l = 253$.

Let a_n be the nth term from the end. Then

$a_n = l - (n-1)d$
$= 253 - 5(n-1)$
$= 253 + 5 - 5n$

$= 258 - 5n$

$\therefore \quad a_{20} = 258 - 5 \times 20 = \mathbf{158}$

which is the required term.

31. (*i*) In the orchard, the number of trees in

1st row = 17

2nd row = 15

3rd row = 13

…………………..

…………………..

last row = 3

$\because \quad 15 - 17 = -2 = 13 - 15$

$\therefore$ 17, 15, 13, …, 3 form an AP.

Such that number of rows = n

$a = 17,\ d = -2$ and $a_n = 3$

$\Rightarrow \quad a_n = 17 + (n-1) \times (-2) = 3$

$\Rightarrow \quad (n-1) \times (-2) = 3 - 17 = -14$

$\Rightarrow \quad n - 1 = \dfrac{-14}{-2} = 7$

$\Rightarrow \quad n = 7 + 1 = 8$

Thus the number rows = **8**

(*ii*) Principal (P) = ₹ 2000

Rate of simple interest = (r) = 8% p.a.

$\therefore$ Interest after 1st year $= \dfrac{P \times r \times t}{100}$

$= ₹\,\dfrac{2000 \times 8 \times 1}{100} = ₹\ 160$

Interest after 2 years $= ₹\,\dfrac{2000 \times 8 \times 2}{100} = ₹\ 320$

Interest after 3 years $= ₹\,\dfrac{2000 \times 8 \times 3}{100} = ₹\ 480$

Since, $320 - 160 = 480 - 320 = 160$

$\therefore$ 160, 320, 480, … are in AP.

where First term = a = 160

Common diff. = d = 160

Now, if $n = 20$

then $a_{20} = a + (20-1)d$

$= 160 + 19 \times 160 \quad [\because a = 160 \text{ and } d = 160]$

$= 160 + 3040$

$= 3200$

$\Rightarrow$ Interest at the end of 20 years = ₹ **3200**

For Standard Level

32. Let the first term, $a = -\dfrac{4}{3}$, the common difference,

$d = -1 + \dfrac{4}{3} = \dfrac{1}{3}$ and a_n, the nth term of the AP,

Then $a_n = a + (n-1)d$

$= -\dfrac{4}{3} + \dfrac{1}{3}(n-1)$

$= -\dfrac{4}{3} - \dfrac{1}{3} + \dfrac{n}{3}$

$= \dfrac{-5}{3} + \dfrac{n}{3} \quad \ldots(1)$

If a_n = last term $= 4\dfrac{1}{3} = \dfrac{13}{3}$, then

$\dfrac{13}{3} = \dfrac{n}{3} - \dfrac{5}{3}$

$\Rightarrow \quad \dfrac{n}{3} = \dfrac{13}{3} + \dfrac{5}{3} = \dfrac{18}{3}$

$\Rightarrow \quad n = 18$ which is even

Hence, there are two middle terms *viz.* $a_{\frac{18}{2}}$ and $a_{\frac{18}{2}+1}$,

i.e., a_9 and a_{10}

Now, from (1)

$a_9 = -\dfrac{5}{3} + \dfrac{9}{3} = \dfrac{4}{3}$

and $\quad a_{10} = -\dfrac{5}{3} + \dfrac{10}{3} = \dfrac{5}{3}$

$\therefore \quad a_9 + a_{10} = \dfrac{4}{3} + \dfrac{5}{3} = \dfrac{9}{3} = 3$

Hence, the required sum of the two middle terms is **3**.

33. Let a be the first term, d, the common difference and a_n, the nth term of the AP.

Then $\quad a_n = a + (n-1)d \quad \ldots(1)$

Now, given that $\quad a_{24} = 2a_{10}$

$\Rightarrow \quad a + 23d = 2(a + 9d) \quad$ [From (1)]

$= 2a + 18d$

$\Rightarrow \quad a - 5d = 0$

$\Rightarrow \quad a = 5d \quad \ldots(2)$

Now, $\quad a_{72} = a + 71d \quad$ [From (1)]

$= 5d + 71d \quad$ [From (2)]

$= 76d \quad \ldots(3)$

and $\quad a_{15} = a + 14d \quad$ [From (1)]

$= 5d + 14d = 19d \quad$ [From (2)]…(4)

$\therefore$ From (3) and (4), we have

$\dfrac{a_{72}}{a_{15}} = \dfrac{76d}{19d} = 4$

$a_{72} = 4a_{15}$

Hence, proved.

34. Let a be the first term, d, the common difference and a_n, the nth term of the AP.

Then $\quad a_n = a + (n-1)d \quad \ldots(1)$

Now, given that $\quad a_6 = 0$

$\Rightarrow \quad a + 5d = 0 \quad$ [From (1)]

$\Rightarrow \quad a = -5d \quad \ldots(2)$

$\therefore \quad a_{33} = a + 32d$

$= -5d + 32d \quad$ [From (2)]

$= 27d \quad \ldots(3)$

$a_{15} = a + 14d$

$= -5d + 14d \quad$ [From (2)]

$= 9d \quad \ldots(4)$

∴ From (3) and (4), we have

$$\frac{a_{33}}{a_{15}} = \frac{27d}{9d} = 3$$

∴ $\mathbf{a_{33} = 3a_{15}}$

Hence, proved.

35. Let a be the first term, d, the common difference and a_n, the nth term of the AP where n is the number of term of the AP.

Then $a_n = a + (n-1)d$ …(1)

∴ $a_{26} = a + 25d$

⇒ $0 = a + 25d$ [Given]

∴ $a = -25d$ …(2)

$a_{11} = a + 10d$

⇒ $3 = a + 10d$ [Given]

$= -25d + 10d$

$= -15d$ [From (2)]

∴ $d = -\frac{3}{15} = -\frac{1}{5}$ …(3)

∴ From (2), $a = \frac{1}{5} \times 25 = 5$ …(4)

∴ From (1), $a_n = 5 - (n-1) \times \frac{1}{5}$

$= \frac{-(n-1)+25}{5}$

$= \frac{26-n}{5}$

⇒ $-\frac{1}{5} = \frac{26-n}{5}$

[∵ The last term given is $-\frac{1}{5}$]

⇒ $n - 26 = 1$

⇒ $n = 27$ …(5)

∴ The required common difference and the number of term of the AP are $\mathbf{-\frac{1}{5}}$ and **27** respectively.

[From (3) and (5)]

36. Let a be the first term, d, the common difference and a_n, the nth term of the AP. Then

$a_n = a + (n-1)d$ …(1)

Now, given that

$a_{17} = 5 + 2a_8$

⇒ $a + 16d = 5 + 2(a + 7d)$

⇒ $a + 16d = 5 + 2a + 14d$

⇒ $a - 2d + 5 = 0$ …(2)

Also, it is given that

$a_{11} = 43$

⇒ $a + 10d = 43$ …(3)

Subtracting (2) from (3), we get

$12d = 48$

⇒ $d = 4$ …(4)

∴ From (2), $a = 2d - 5$

$= 2 \times 4 - 5 = 3$ …(5)

∴ From (1), (4) and (5), we have

$a_n = 3 + (n-1)4$

$= \mathbf{4n - 1}$

which is the required term.

37. We know that all numbers ending with 5 or 0 are divisible by 5. But numbers ending with 5 are not divisible by 2, since these numbers are odd. Hence, the numbers which are divisible by both 5 and 2 must be divisible by 2 × 5 i.e., 10. Hence, these numbers must end with 0. Hence, the numbers lying between 101 and 999 which are divisible by both 2 and 5 are 110, 120, 130, 140, 150, …990.

These numbers are clearly in AP with the first term, $a = 110$ and the common difference, $d = 120 - 110 = 10$. Let a_n be the nth term of this AP.

∴ $a_n = a + (n-1)d$

$= 110 + (n-1)10$

$= 100 + 10n$ …(1)

If a_n = the last term, then $a_n = 990$ [From (1)]

∴ $990 = 100 + 10n$

⇒ $\frac{890}{10} = n$

⇒ $n = \mathbf{89}$

which is the requried number of natural numbers which are in AP.

38. Let $a - d$, a and $a + d$ be three numbers in AP.

Then according to the problem, we have

$(a - d) + a + (a + d) = 207$

⇒ $3a = 207$

⇒ $a = \frac{207}{3} = 69$ …(1)

Also, given that $(a - d)a = 4623$

⇒ $a^2 - ad = 4623$

⇒ $69^2 - 69d = 4623$

⇒ $4761 - 4623 = 69d$

⇒ $138 = 69d$

⇒ $d = \frac{138}{69} = 2$ …(2)

Hence, from (1) and (2), the required numbers are 69 –2, 69, 69 + 2, i.e. **67, 69 and 71**.

39. The three parts are in AP.

Let the parts be $a - d, a, a + d$.

∵ 5(smallest number) = (largest number) + 6

or $5(a - d) = (a + d) + 6$

⇒ $5a - 5d = a + d + 6$

⇒ $5a - a - 5d - d = 6$

⇒ $4a - 6d = 6$

⇒ $2a - 3d = 3$ …(1)

Also, $(a - d) + a + (a + d) = 54$

⇒ $a - d + a + a + d = 54$

$\Rightarrow \quad 3a = 54$ or $a = 18$ ...(2)

From (1), we have

$2(18) - 3d = 3$ or $36 - 3d = 3$

$\Rightarrow \quad 3d = 36 - 3 = 33$ or $d = 11$

Now, three parts are

$a - d, \quad a, \quad a + d$

$\Rightarrow \quad (18 - 11), \quad 18, \quad (18 + 11)$

$\Rightarrow$ **7, 18, 29**

40. Let $a - d$, a and $a + d$ be three numbers in AP.

$\therefore$ According to the problem, we have

$(a - d) + a + (a + d) = 48$

$\Rightarrow \quad 3a = 48$

$\Rightarrow \quad a = 16$ ...(1)

$\therefore$ The third term of the AP is $16 + d$ and the first two terms are $16 - d$ and 16.

$\therefore$ According to the second condition of the problem,

$(16 - d)16 - 4(16 + d) = 12$

$\Rightarrow \quad 256 - 16d - 64 - 4d = 12$

$\Rightarrow \quad -20d = 12 + 64 - 256$

$= 76 - 256$

$= -180$

$\therefore \quad d = \dfrac{180}{20} = 9$ ...(2)

$\therefore$ From (1) and (2), the required three terms of the AP are $16 - 9$, 16 and $16 + 9$ i.e., **7, 16 and 25**.

41. Let the four parts be $a - 3d$, $a - d$, $a + d$, $a + 3d$

Sum = 56

$\Rightarrow \quad a - 3d + a - d + a + d + a + 3d = 56$

$\Rightarrow \quad 4a = 56$

$\Rightarrow \quad a = 14$

According to the given condition

$$\frac{a_1 \times a_4}{a_2 \times a_3} = \frac{5}{6}$$

$$\Rightarrow \quad \frac{(a-3d)\times(a+3d)}{(a-d)\times(a+d)} = \frac{5}{6}$$

$$\Rightarrow \quad \frac{a^2-9d^2}{a^2-d^2} = \frac{5}{6}$$

$\Rightarrow \quad 6a^2 - 54d^2 = 5a^2 - 5d^2$

$\Rightarrow \quad a^2 = 49d^2$

$\Rightarrow \quad 49d^2 = (14)^2$

$\Rightarrow \quad 49d^2 = 196$

$\Rightarrow \quad d^2 = 4$

$\Rightarrow \quad d = \pm 2$

If $\quad d = +2$

AP : **8, 12, 16, 20**

If $\quad d = -2$

AP : **20, 16, 12, 8**

42. Let a = first term and d = common diff.

$\therefore$ General term $a_n = a + (n - 1)d$

$\Rightarrow \quad a_m = a + (m - 1)d = a + md - d$

$a_n = a + (n - 1)d = a + nd - d$

Since $\quad m \times a_m = n \times a_n$

$\therefore \quad m[a + md - d] = n[a + nd - d]$

$\Rightarrow \quad ma + m^2d - md = na + n^2d - nd$

$\Rightarrow \quad (ma - na) + (m^2d - n^2d) - (md - nd) = 0$

$\Rightarrow \quad (m - n)\,a + (m^2 - n^2)d - (m - n)d = 0$

$\Rightarrow \quad (m - n)a + [(m - n)\,(m + n)]\,d - (m - n)d = 0$

$[\because x^2 - y^2 = (x - y)\,(x + y)]$

$\Rightarrow \quad (m - n)\,[a + (m + n)d - d] = 0$

$\Rightarrow \quad [a + (m + n)d - d] = 0$ ...(1)

Now, $\quad a_{m+n} = a + [(m + n) - 1)]d$

$\Rightarrow \quad a_{m+n} = a + (m + n)d - d$ ...(2)

From (1) and (2), we have

$a_{m+n} = a + (m + n)d - d = 0$

Hence, $(m + n)$th term is **0**.

43. Let a = First term and d = Common diff.

$\therefore \quad$ General term $= a_n = a + (n - 1)d$

$a_{m+1} = a + (m + 1 - 1)d = a + md$

$a_{n+1} = a + (n + 1 - 1)d = a + nd$

$\because \quad a_{m+1} = 2a_{n+1}$ [Given]

$\therefore \quad a + md = 2[a + nd]$

$\Rightarrow \quad a + md = 2a + 2nd$

$\Rightarrow \quad 2a - a + 2nd - md = 0$

$\Rightarrow \quad a = md - 2nd = d(m - 2n)$ ... (1)

Now, $\quad a_{3m+1} = a + (3m + 1 - 1)d = a + 3md$

$= d(m - 2n) + 3md \quad [\because a = d(m - 2nd)]$

$= md - 2nd + 3\,md = 4md - 2nd$

$= 2d(2m - n)$

$\Rightarrow \quad a_{3m+1} = 2d(2m - n)$ ...(2)

Also, $\quad a_{(m+n+1)} = a + (m + n + 1 - 1)d$

$\Rightarrow \quad a_{m+n+1} = a + (m + n)d$

$\Rightarrow \quad 2[a_{m+n+1}] = 2[a + (m + n)d]$

$= 2[d(m - 2n) + (m + n)d]$

[Substituting $a = d(m - 2n)$]

$= 2[md - 2nd + md + nd]$

$= 2[2md - nd]$

$= 2d[2m - n]$ ...(3)

From (2) and (3), we have:

$\mathbf{2[a_{m+n+1}] = a_{3m+1}}$

44. If x, y, z are in AP then

$(y - x) = (z - y)$ ...(1)

Also, if $[(y + z)^2 - x^2]$, $[(x + z)^2 - y^2]$, $[(x + y)^2 - z^2]$ are in AP, then

$[(z + x)^2 - y^2] - [(y + z)^2 - x^2] = [(x + y)^2 - z^2]$
$- [(x + z)^2 - y^2]$

or $\quad [z^2 + x^2 + 2zx - y^2 - y^2 - z^2 - 2yz + x^2]$

$= [x^2 + y^2 + 2xy - z^2 - x^2 - z^2 - 2xz + y^2]$

or $[2x^2 - 2y^2 + 2zx - 2yz] = [2y^2 - 2z^2 + 2xy - 2xz]$

or $(y - x) = (z - y)$ …(2)

From (1) and (2), we have

if x, y, z are in AP then

$[(y + z)^2 - x^2], [(z + x)^2 - y^2], [(x + y)^2 - z^2]$ are in AP.

45. Let a = First term and d = Common difference

and $a_n = a + (n - 1)d$

$\therefore$ $a_1 = a$

$a_2 = a + (2 - 1)d = a + d$

$a_3 = a + (3 - 1)d = a + 2d$

$a_4 = a + (4 - 1)d = a + 3d$

Now, $a_2 \times a_3 = (a + d)(a + 2d)$

$\Rightarrow$ $a_2 \times a_3 = a^2 + 3ad + 2d^2$ …(1)

$a_1 \times a_4 = a(a + 3d) = a^2 + 3ad$ …(2)

$a_2 - a_1 = a + d - a = d$

$\Rightarrow$ $(a_2 - a_1)^2 = d^2$ …(3)

Now, $2(a_2 - a_1)^2 = 2d^2$ [From 3]

From (1) and (2),

$a^2 + 3ad + 2d^2 = (a^2 + 3ad) + 2d^2$

$\Rightarrow$ $\mathbf{a_2 \times a_3 = (a_1 \times a_4) + 2(a_2 - a_1)^2}$

46. $\because$ The sides of rt Δ are in AP.

$\therefore$ Let the sides be

$a - d, \quad a, \quad a + d$

Using Pythagoras theorem, we get

$(a + d)^2 = (a - d)^2 + a^2$

$\Rightarrow$ $a^2 + d^2 + 2ad = a^2 + d^2 - 2ad + a^2$

$\Rightarrow$ $a^2 = 2ad + 2ad = 4ad$

$\Rightarrow$ $\frac{a^2}{a} = \frac{4ad}{a} \Rightarrow a = 4d$

Substituting $a = 4d$ in the sides, we get

$4d - d, \quad 4d, \quad 4d + d$

or $3d \quad 4d \quad 5d$

Now, the required ratio of sides is

$3d : 4d : 5d$ or **3 : 4 : 5**

47. (*i*) $\because$ Angles are in AP.

$\therefore$ $(a - d)°, (a)°, (a + d)°$ be the angles of a Δ

$\therefore (a - d) + a + (a + d) = 180°$

$\Rightarrow$ $a - d + a + a + d = 180°$

$\Rightarrow$ $3a = 180° \Rightarrow a = 60°$ …(1)

$\because$ [Least angle] $= \frac{1}{3}$ [Greatest angle]

$\therefore$ $(a - d) = \frac{1}{3}(a + d)$

$\Rightarrow$ $3(a - d) = a + d$

$\Rightarrow$ $3a - 3d - a - d = 0$

$\Rightarrow$ $2a - 4d = 0$

$\Rightarrow$ $a - 2d = 0$ …(2)

From (1) and (2), we have

$60 - 2d = 0$

or $2d = 60$ or $d = 30$

Thus, the angles are

$(60 - 30)°, \quad 60°, \quad (60 + 30)°$

$\Rightarrow$ **30°, 60°, 90°**

(*ii*) Let the three angles of the triangle, which are in AP be $a - d$, a and $a + d$ so that we have

$(a - d) + a + (a + d) = 180°$

[Angle sum property of a triangle]

$\therefore$ $3a = 180°$

$\Rightarrow$ $a = 60°$ …(1)

$\therefore$ The least angle is $a - d = 60° - d$ [From (1)]

and the greatest angle is $a + d = 60° + d$ [From (1)]

$\therefore$ According to the problem, we have

$60° + d = 2(60° - d)$

$= 120° - 2d$

$\Rightarrow$ $3d = 60°$

$\Rightarrow$ $d = \frac{60°}{3} = 20°$ …(2)

$\therefore$ From (1) and (2)

The required three angles are

$60° - 20°$, $60°$ and $60° + 20°$,

i.e. **40°, 60°** and **80°**.

48. Let the three numbers be $a - d, a, a + d$

[$\because$ The numbers are in AP]

$\because$ The sum of numbers = 6

$\therefore$ $a - d + a + a + d = 6$

$\Rightarrow$ $3a = 6$

$\therefore$ $a = 2$ …(1)

$\because$ The sum of their squares = 14

$\therefore$ $(a - d)^2 + a^2 + (a + d)^2 = 14$

$\Rightarrow$ $a^2 + d^2 - 2ad + a^2 + a^2 + d^2 + 2ad = 14$

$\Rightarrow$ $3a^2 + 2d^2 = 14$ …(2)

From (1) and (2), we get

$3(2)^2 + 2d^2 = 14$

$\Rightarrow$ $12 + 2d^2 = 14$

$\Rightarrow$ $2d^2 = 14 - 12 = 2$

or $d^2 = 1$

$\Rightarrow$ $d = \pm 1$

Now substituting $a = 2$ and $d = \pm 1$ in

$a - d, \quad a, \quad a + d,$ we get

$2 - 1, \quad 2, \quad 2 + 1$ or $2 - (-1), 2, 2 + (-1)$

$\Rightarrow$ $1, \quad 2, \quad 3$ or $3, 2, 1$

Thus the required numbers are **1, 2, 3** or **3, 2, 1**.

49. Let the five numbers in AP are

$(a - 2d), (a - d), a, (a + d), (a + 2d)$

$\because$ Their sum = 35

$\therefore \quad a-2d+a-d+a+a+d+a+2d = 35$

$\Rightarrow \quad 5a = 35$ or $a = 7$

$\because$ Sum of their squares = 285

$\therefore \quad [a-2d]^2 + [a-d]^2 + a^2 + [a+d]^2 + [a+2d]^2 = 285$

$\Rightarrow \quad [a^2 + 4d^2 - 4ad] + [a^2 + d^2 - 2ad] + a^2 + [a^2 + d^2 + 2ad] + [a^2 + 4d^2 + 4ad] = 285$

$\Rightarrow \quad a^2 + 4d^2 + a^2 + d^2 + a^2 + a^2 + d^2 + a^2 + 4d^2 = 285$

$\Rightarrow \quad 5a^2 + 10d^2 = 285$ or $a^2 + 2d^2 = 57$

Substituting $a = 7$, we have:

$7^2 + 2d^2 = 57$ or $49 + 2d^2 = 57$

$\Rightarrow \quad 2d^2 = 57 - 49 = 8$ or $d^2 = \frac{8}{2} = 4 \Rightarrow d = \pm 2$

$\therefore$ The numbers are

(7 – 4), (7 – 2), 7, (7 + 2), (7 + 4)

or (7 + 4), (7 + 2), 7, (7 – 2), (7 – 4)

or **3, 5, 7, 9, 11** or **11, 9, 7, 5, 3**

50. Let the three numbers be $a - d$, a and $a + d$

$S_3 = 12$

$a - d + a + a + d = 12$

$3a = 12$

$a = 4$

Now the sum of cubes of these three numbers is equal to 288.

$(a - d)^3 + a^3 + (a + d)^3 = 288$

$(4 - d)^3 + (4)^3 + (4 + d)^3 = 288$

$(4 - d)^3 + (4 + d)^3 = 288 - 64$

$64 - d^3 - 12d(4 - d) + 64 + d^3 + 12d(4 + d) = 224$

$128 - 12d\,[4 - d - 4 - d] = 224$

$-12d\,(-2d) = 96$

$d^2 = 4$

$d = \pm 2$

If $d = 2$

AP: **2, 4, 6**

If $d = -2$

AP: **6, 4, 2**

EXERCISE 5B

For Basic and Standard Levels

1. (*i*) We have 2, 4, 6, … to 'n' terms

Here, $a = 2$ and $d = 4 - 2 = 2$

$\therefore \quad S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow \quad S_n = \frac{n}{2}[2 \times 2 + (n-1) \times 2]$

$= \frac{n}{2}[4 + 2(n-1)]$

$= \frac{n}{2}[4 + 2n - 2] = \frac{n}{2}[2 + 2n]$

$= \frac{n}{2}[2(1 + n)] = n[1 + n] = n + n^2$

Thus, $S_n = n + n^2$ or $\mathbf{S_n = n^2 + n}$

(*ii*) We have 0.7, 0.71, 0.72, 0.73, … to 50 terms

Here, $a = 0.7$, $d = 0.71 - 0.7 = 0.01$

and $n = 50$

using $S_n = \frac{n}{2}[2a + (n-1)d]$, we get

$S_{50} = \frac{50}{2}[2(0.7) + (50 - 1) \times 0.01]$

$= 25[1.4 + 49 \times 0.01]$

$= 25[1.4 + 0.49] = 25 \times 1.89$

$= \frac{25 \times 189}{100} = \frac{4725}{100} = 47.25$

Thus $S_{50} = \mathbf{47.25}$

(*iii*) We have a, $(a + b)$, $(a + 2b)$, … to n terms.

Here First term = a

Common diff. = $a + b - a = b$

$\therefore \quad S_n = \frac{n}{2}[2a + (n-1) \times b]$

$= \frac{n}{2}[2a + nb - b]$

$= \frac{n}{2} \times 2a + \frac{n}{2} \times nb - \frac{n}{2} \times b$

$= an + \frac{n^2}{2}b - \frac{nb}{2}$

$\Rightarrow \quad \mathbf{S_n = an + \frac{bn^2}{2} - \frac{bn}{2}}$

(*iv*) We have $x + y$, $x - y$, $x - 3y$ … to 20 terms

$\therefore \quad a = x + y;\ d = (x - y) - (x + y)$

$= x - y - x - y = -2y$

and $n = 20$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow \quad S_{20} = \frac{20}{2}[2(x + y) + (20 - 1) \times (-2y)]$

$\Rightarrow \quad S_{20} = 10[2x + 2y + 19(-2y)]$

$\Rightarrow \quad S_{20} = 20x + 20y - 380y$

$\Rightarrow \quad S_{20} = \mathbf{20x - 360y}$

(*v*) We have $(a - b)^2$, $(a^2 + b^2)$, $(a + b)^2$, … to n terms

Here, $a = (a - b)^2 = a^2 + b^2 - 2ab$

$d = (a^2 + b^2) - (a - b)^2$

$= (a^2 + b^2) - (a^2 + b^2 - 2ab)$

$= a^2 + b^2 - a^2 - b^2 + 2ab = 2ab$

Using $S_n = \frac{n}{2}[2a + (n-1)d]$, we have

$S_n = \frac{n}{2}[2 \times (a^2 + b^2 - 2ab) + (n-1) \times 2ab]$

$= \frac{n}{2}[2a^2 + 2b^2 - 4ab + 2n.ab - 2ab]$

$= \frac{n}{2}[2a^2 + 2b^2 - 6ab + 2n.ab]$

$= \mathbf{a^2n + b^2n - 3abn + abn^2}$

2. Let the AP be

$a, a + d, a + 2d, \ldots$

$\therefore \quad a_n = a + (n-1)d$

Let S_n be the sum of n terms of the above AP.

$\therefore \quad S_n = a + (a + d) + (a + 2d) + \ldots + [a + (n-2)d] + [a + (n-1)d] \quad \ldots (1)$

Writing the expression (1) in reverse order,

$S_n = [a + (n-1)d] + [a + (n-2)d] + \ldots + (a + 2d) + (a + d) + a \quad \ldots (2)$

Adding (1) and (2) vertically, we get

$2S_n = [2a + (n-1)d] + [2a + (n-1)d] + \ldots + [2a + (n-1)d] + [2a + (n-1)d]$

$\Rightarrow \quad 2S_n = [2a + (n-1)d] \times n$

($\because$ $[2a + (n-1)d]$ is added n times)

$\Rightarrow \quad \mathbf{S_n = \frac{n}{2}[2a + (n-1)d]}$

[which is the sum of n terms of the given AP]

3. (*i*) The given AP is –3, –7, –11, …

$\therefore \quad a = -3$ and $d = -7 - (-3) = -4$

$\because \quad n = 14$

$\therefore \quad S_{14} = \frac{14}{2}[2(-3) + (14-1) \times (-4)]$

$= 7[-6 + 13 \times (-4)]$

$= 7[-6 - 52] = 7 \times (-58) = -406$

Thus, the sum of first 14 terms is **–406**.

(*ii*) The given AP is 2, 7, 12, …

Here $a = 2$, $d = 7 - 2 = 5$ and $n = 18$

$\therefore \quad S_{18} = \frac{18}{2}[2 \times 2 + (18-1) \times 5]$

$= 9[4 + 17 \times 5] = 9[89] = 801$

Thus, the sum of first 18 terms is **801**.

(*iii*) Let a = first term and d = common diff.

$\therefore \quad a_3 = a + 2d = -103 \quad \ldots(1)$

$a_7 = a + 6d = -63 \quad \ldots(2)$

Subtracting (1) from (2), we get

$a + 6d - a - 2d = -63 + 103$

$\Rightarrow \quad 4d = 40 \Rightarrow d = 10$

From (1), we get

$a + 2(10) = -103 \Rightarrow a = -103 - 20$

or $\quad a = -123$

Now using $\quad S_n = \frac{n}{2}[2a + (n-1)d]$, we get

$S_{54} = \frac{54}{2}[2(-123) + (54-1) \times 10]$

$= 27[-246 + 530]$

$= 27 \times 284 = 7668$

Thus, sum of first 54 terms = **7668**

4. (*i*) We have to find $1 + 3 + 5 + 7 + \ldots + 199$

Here, $a = 1$, $d = 3 - 1 = 2$ and $a_n = 199$

$\therefore \quad a_n = a + (n-1)d = 199$

$\Rightarrow \quad 1 + (n-1)2 = 199$ or $2n - 2 = 199 - 1 = 198$

$\therefore \quad 2n = 198 + 2 = 200 \Rightarrow n = 100$

Using $\quad S_n = \frac{n}{2}[a + l]$, we get

$S_{100} = \frac{100}{2}[1 + 199] \qquad$ [Here $l = 199$]

$\Rightarrow \quad S_{100} = 50[200] = \mathbf{10000}$

(*ii*) In $25 + 28 + 31 + \ldots + 100$

We have $a = 25$, $d = 28 - 25 = 3$, $l = 100$

$\because \quad a_n = a + (n-1)d$

$\Rightarrow \quad 100 = 25 + (n-1) \times 3$

$\therefore \quad n - 1 = \frac{100 - 25}{3} = 25$ or $n = 25 + 1 = 26$

Now using $\quad S_n = \frac{n}{2}[a + l]$, we get

$S_{26} = \frac{26}{2}[25 + 100] = 13 \times 125 = \mathbf{1625}$

(*iii*) We have

$\left(1 - \frac{1}{n}\right) + \left(1 - \frac{2}{n}\right) + \left(1 - \frac{3}{n}\right) + \ldots$ up to n terms

$\therefore \quad a = \left(1 - \frac{1}{n}\right) \; d = 1 - \frac{2}{n} - 1 + \frac{1}{n} = \frac{-1}{n}$

$\therefore \quad S_n = \frac{n}{2}\left[2\left(1 - \frac{1}{n}\right) + (n-1) \times \left(\frac{-1}{n}\right)\right]$

$= \frac{n}{2}\left[2 - \frac{2}{n} + \frac{1}{n} - 1\right] = \left[n - 1 + \frac{1}{2} - \frac{n}{2}\right]$

$= \left[\frac{2n - n}{2} - \frac{1}{2}\right] = \left[\frac{n}{2} - \frac{1}{2}\right] = \left[\mathbf{\frac{n-1}{2}}\right]$

Alternative Solution:

$S_n = \left(1 - \frac{1}{n}\right) + \left(1 - \frac{2}{n}\right) + \left(1 - \frac{3}{n}\right) + \ldots$ to n terms

$= (n \times 1) - \left[\frac{1}{n} + \frac{2}{n} + \frac{3}{n} + \ldots + \frac{n}{n}\right]$

$= n - \left[\frac{1}{n} + \frac{2}{n} + \frac{3}{n} + \ldots + 1\right]$

$= n - \left\{\frac{n}{2}\left[\frac{2}{n} + (n-1) \times \frac{1}{n}\right]\right\}$

$= n - \left\{\frac{n}{2} \times \frac{2}{n} + (n-1) \times \frac{1}{n} \times \frac{n}{2}\right\} = n - \left\{1 + \frac{n-1}{2}\right\}$

$= n - 1 - \frac{n-1}{2} = \frac{n}{2} - \frac{1}{2} = \mathbf{\frac{n-1}{2}}$

(*iv*) We have

$\left(4 - \frac{1}{n}\right) + \left(4 - \frac{2}{n}\right) + \left(4 - \frac{3}{n}\right) + \ldots$ to n term

$= (4 + 4 + \ldots$ to n terms$) - \frac{1}{n}(1 + 2 + 3 + \ldots + n)$

$= 4n - \frac{1}{n} \times \frac{n(n+1)}{2}$

$= 4n - \frac{n+1}{2}$

$= \frac{8n - n - 1}{2}$

$= \mathbf{\frac{7n - 1}{2}}$ which is the required sum.

5. (*i*) We have

$a_n = 2n + 1$ [Given] ...(1)

$\therefore$ From (1), $a_1 = 2 \times 1 + 1 = 3,$

$a_2 = 2 \times 2 + 1 = 5$

and $a_3 = 2 \times 3 + 1 = 7$

$\therefore$ Required sum $= a_1 + a_2 + a_3$

$= 3 + 5 + 7 = \mathbf{15}$

(*ii*) Here, $a = 36$ and $d = -5$

$a_n = a + (n - 1)d$

$\Rightarrow -49 = 36 + (n - 1)(-5)$

$\Rightarrow (n - 1)(-5) = -49 - 36 = -85$

$\Rightarrow n - 1 = \frac{-85}{-5} = 17$

$\therefore n = 17 + 1 = 18$

Now, $S_n = \frac{n}{2}[2a + (n - 1)d]$

$\therefore S_{18} = \frac{18}{2}[2(36) + (18 - 1)(-5)]$

$\Rightarrow S_{18} = 9[72 + (-85)]$

$= 9[-13] = \mathbf{-117}$

6. AP: 5, 12, 19, ...

$a = 5,\ d = 7,\ n = 50$

$a_{50} = a + (n - 1)d$

$= 5 + (49)(7)$

$= 5 + 343$

$= 348$

Last term (l) = **348**

Now to find the sum of last 15 terms

$a = 348,\ d = -7,\ n = 15$

$S_n = \frac{15}{2}[348 \times 2 + (14)(-7)]$

$= \frac{15}{2}[696 - 98]$

$= \frac{15}{2} \times 598$

$= 15 \times 299$

$= \mathbf{4485}$

7. Let First term = a and Common diff. = d

Since, $a_n = a + (n - 1)d$

$\therefore a_{29} = a + (29 - 1)d = a + 28d$

$\Rightarrow a + 28d = 248$ ...(1) [$\because$ It is given that $a_n = 248$]

$\because S_n = \frac{n}{2}[2a + (n - 1)d]$

$\therefore S_{29} = \frac{29}{2}[2a + (29 - 1)d]$

$\Rightarrow \frac{29}{2}[2a + 28d] = 3538$ [$\because$ It is given that $S_{29} = 3538$]

$\Rightarrow 2a + 28d = 3538 \times \frac{2}{29} = 244$ ...(2)

Subtracting (1) from (2), we get

$2a + 28d = 244$

$a + 28d = 248$

(–) (–) (–)

$a = -4$

Now, from (1), we get

$-4 + 28d = 248$ or $28d = 252$

$\Rightarrow d = \frac{252}{28} = 9$

Thus, Common difference = **9**

First term = **–4**

8. Let the first term and the common difference of the AP be a and d respectively. Let a_n be its nth term and S_n be the sum of first n terms of the AP

Then $a_n = a + (n - 1)d$ ...(1)

and $S_n = \frac{n}{2}[2a + (n - 1)d]$ ...(2)

Now, given that $a_{14} = 40$

$\Rightarrow a + 13d = 40$ [From (1)]

$\Rightarrow a = 40 - 13d$ ...(3)

Also, given that $S_{14} = 287$

$\Rightarrow 287 = \frac{14}{2}[2a + 13d]$ [From (2)]

$= 7(2a + 13d)$

$\Rightarrow 41 = 2a + 13d$ ...(4)

$\therefore$ From (3) and (4), we have

$2(40 - 13d) + 13d = 41$

$\Rightarrow 80 - 41 = 13d$

$\Rightarrow 39 = 13d$

$\Rightarrow d = 3$ ...(5)

$\therefore$ From (3), we have $a = 40 - 13 \times 3 = 1$ ...(6)

$\therefore$ From (5) and (6), the required common difference and the first term are **3** and **1** respectively.

9. Let First term = a and Common diff. = d

$\therefore a_n = a + (n - 1)d$

$a_7 = a + 6d = 10$ ...(1)

Also, $S_9 = \frac{9}{2}[2a + (9 - 1) \times d] = 0$

$\Rightarrow 2a + 8d = 0$ ...(2)

Solving (1) and (2), we get

$a = -20$ and $d = 5$

Now, $S_{23} = \frac{23}{2}[2(-20) + (23 - 1) \times 5]$

$$= \frac{23}{2}[-40 + 110]$$

$$= \frac{23}{2} \times 70 = 23 \times 35$$

$$= \mathbf{805}$$

10. $a_{12} = -13$ ... (1)

$S_4 = 24$ ... (2)

From eq.(1) we get

$a + 11d = -13$

$a = -13 - 11d$ ... (3)

From eq.(2) we get

$\frac{4}{2}[2a + 3d] = 24$

$2a + 3d = 12$ ... (4)

Putting the value of a from eq.(3) in eq.(4)

$2(-13 - 11d) + 3d = 12$

$-26 - 22d + 3d = 12$

$-19d = 38$

$d = -2$

We know

$a = -13 - 11d$

$= -13 + 22$

$= 9$

$S_{10} = \frac{10}{2}[2a + (10-1)d]$

$= 5[18 + 9(-2)]$

$= 5[18 - 18]$

$= \mathbf{0}$

11. Let a = first term and d = common diff.

Since, $a_n = a + (n - 1)d$

$\Rightarrow$ $a_2 = a + (2 - 1)d = a + d$

$a_9 = a + (9 - 1)d = a + 8d$

Now, $a_2 = 2$

$\Rightarrow$ $a + d = 2$ …(1)

and $a_9 = 37$

$\Rightarrow$ $a + 8d = 37$ …(2)

Subtracting (1) from (2), we get

$7d = 35$

$\Rightarrow$ $d = 5$

From (1), $a + 5 = 2$

$\Rightarrow$ $a = -3$

Now $S_{40} = \frac{40}{2}[2(-3) + (40 - 1) \times 5]$

$= 20[-6 + 195] = 20 \times 189$

$= \mathbf{3780}$

12. Let a be the first term, d, the common difference, a_n, the nth term and S_n, the sum of the first n terms of the AP

Then $a_n = a + (n - 1)d$ …(1)

and $S_n = \frac{n}{2}[2a + (n-1)d]$ …(2)

Now, given that $a_4 = -15$

$\Rightarrow$ $a + 3d = -15$ [From (1)]

$\Rightarrow$ $a = -3d - 15$ …(3)

Also, given that $a_9 = -30$

$\Rightarrow$ $a + 8d = 30$ [From (1)]

$\Rightarrow$ $(-3d - 15) + 8d = -30$ [From (3)]

$\Rightarrow$ $5d = -15$

$\therefore$ $d = -3$ …(4)

$\therefore$ From (3) and (4), we have

$a = 3 \times 3 - 15 = -6$ …(5)

$\therefore$ From (2),

$S_{17} = \frac{17}{2}[2 \times (-6) + (17-1)(-3)]$

$= \frac{17}{2}(-12 - 48)$

$= -\frac{17}{2} \times 60 = \mathbf{-510}$

which is the required sum.

13. Let First term = a

Common difference = d

$\because$ $a_n = a + (n - 1)d$

$\therefore$ $a_1 = a + (1 - 1)d = a$

$a_3 = a + (3 - 1)d = a + 2d$

$a_{17} = a + (17 - 1)d = a + 16d$

Now, $a_1 + a_3 + a_{17} = 216$

$\Rightarrow$ $a + a + 2d + a + 16d = 216$

$\Rightarrow$ $3a + 18d = 216$

$\Rightarrow$ $a + 6d = 72$ …(1)

Now, using $S_n = \frac{n}{2}[2a + (n - 1)d]$, we get

$S_{13} = \frac{13}{2}[2a + (13 - 1)d]$

$= \frac{13}{2}[2a + 12d]$

$= \frac{13}{2} \times 2[a + 6d]$

$= 13[a + 6d]$ …(2)

From (1) and (2), we have

$S_{13} = 13[72] = 936$

Thus, the sum of first thirteen terms of the AP is **936**.

14. Let a be the first term, d, the common difference, a_n, the nth term and S_n, the sum of first n term of the AP.

Given that $a = 22$.

$\therefore$ $a_n = a + (n - 1)d$

$= 22 + (n - 1)d$ …(1)

and $S_n = \frac{n}{2}[2a + (n-1)d]$

$$= \frac{n}{2}\left[44 + (n-1)d\right] \quad \ldots(2)$$

Given that $a_n = -11$

$\therefore \quad 22 + (n-1)d = -11 \quad$ [From (1)]

$\Rightarrow \quad (n-1)d = -33 \quad \ldots(3)$

Also, given that $S_n = 66$

$\Rightarrow \quad \frac{n}{2}\left[44 + (n-1)d\right] = 66 \quad$ [From (2)]

$\Rightarrow \quad \frac{n}{2}[44 - 33] = 66 \quad$ [From (3)]

$\Rightarrow \quad 11n = 132$

$\Rightarrow \quad n = \frac{132}{11} = 12 \quad \ldots(4)$

$\therefore$ From (3), $\quad d = \frac{-33}{12-1} = \frac{-33}{11} = -3 \quad \ldots(5)$

Hence, from (4) and (5), the required value of n and d are **12** and **–3** respectively.

15. Let First term = a and Common difference = d

$\because \quad S_n = \frac{n}{2}[a + l]$

$\therefore \quad S_{26} = \frac{26}{2}[a + 67]$ [It is given that $l = 67$]

$\Rightarrow \quad 1092 = 13[a + 67]$

or $\quad a + 67 = \frac{1092}{13} = 84$

$\Rightarrow \quad a = 84 - 67 = 17$

$\Rightarrow \quad$ First term = **17**

Again $\quad S_{26} = 1092$

$\Rightarrow \quad \frac{26}{2}[2(17) + (26-1)d] = 1092$

{Using $S_n = \frac{n}{2}[2a + (n-1)]d$}

$\Rightarrow \quad 13[34 + 25d] = 1092$

$\Rightarrow \quad 34 + 25d = \frac{1092}{13} = 84$

$\Rightarrow \quad 25d = 84 - 34 = 50$

$\Rightarrow \quad d = \frac{50}{25} = 2$

$\therefore$ Common difference = **2**

16. Let the two first terms of the first and the second AP's be a_1 and a_2 respectively and let d be their same common difference. Then $a_1 = 3$ and $a_2 = 8$.

Let S and S′ be the sums of the first 50 terms of the two AP's respectively.

Then $\quad S = \frac{50}{2}\left[2a_1 + (50-1)d\right]$

$= 25\ (2 \times 3 + 49d)$

$= 150 + 25 \times 49d \quad \ldots(1)$

and $\quad S' = \frac{50}{2}\left[2a_2 + 49d\right]$

$= 25(2 \times 8 + 49d)$

$= 400 + 25 \times 49d \quad \ldots(2)$

Subtracting (1) from (2), we get

$$S' - S = 400 - 150 = \mathbf{250}$$

which is the required difference.

17. Let a be the first term, d, the common difference, a_n, the nth term and S_n, the sum of the first n terms of the AP. Then we have

$a_n = a + (n-1)d \quad \ldots(1)$

and $\quad S_n = \frac{n}{2}\left[2a + (n-1)d\right] \quad \ldots(2)$

Given that $\quad a_{16} = 5a_3$

$\Rightarrow \quad a + 15d = 5(a + 2d) \quad$ [From (1)]

$\Rightarrow \quad 4a - 5d = 0$

$\Rightarrow \quad a = \frac{5d}{4} \quad \ldots(3)$

Also, given that $\quad a_{10} = 41$

$\Rightarrow \quad a + 9d = 41 \quad$ [From (1)]

$\Rightarrow \quad \frac{5d}{4} + 9d = 41 \quad$ [From (3)]

$\Rightarrow \quad \frac{41d}{4} = 41$

$\Rightarrow \quad d = 4 \quad \ldots(4)$

$\therefore$ From (3), $\quad a = \frac{5}{4} \times 4 = 5 \quad \ldots(5)$

$\therefore$ From (2), (4) and (5), we have

$S_{15} = \frac{15}{2}\left[2 \times 5 + (15-1)4\right]$

$= \frac{15}{2}(10 + 56)$

$= 15 \times 33 = \mathbf{495}$

which is the required sum.

18. $\quad a_5 = a + 4d = 8 \quad \ldots(1)$

$a_8 = a + 7d$

$a_2 = a + d$

$\because \quad a_8 = 3(a_2) + 2$

$\therefore \quad a + 7d = 3(a + d) + 2$

$\Rightarrow \quad a - 3a + 7d - 3d = 2$

$\Rightarrow \quad -2a + 4d = 2$

$\Rightarrow \quad a - 2d = -1 \quad \ldots(2)$

Solving (1) and (2), we get

$$a = \mathbf{2} \text{ and } d = \frac{3}{2} \text{ or } \mathbf{1.5}$$

Now $\quad S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow \quad S_{15} = \frac{15}{2}\left[2(2) + (15-1) \times \frac{3}{2}\right] = \frac{15}{2}[4 + 21]$

$= \frac{15}{2} \times 25 = \frac{375}{2} = 187.5$

Thus, $\mathbf{a = 2}$, $\mathbf{d = 1.5}$ and $\mathbf{S_{15} = 187.5}$

19.
$$a_{13} = 4a_3 \quad \text{... (1)}$$
$$a_5 = 16 \quad \text{... (2)}$$
$$a + 4d = 16 \quad \text{... (3)}$$

Now from eq.(1) we get

$$a + 12d = 4(a + 2d)$$
$$a + 12d = 4a + 8d$$
$$3a = 4d \quad \text{... (4)}$$

Now putting the value of d from eq.(4) in eq.(3), we get

$$a + 4d = 16$$
$$a + 3a = 16$$
$$4a = 16$$
$$a = 4$$

We know

$$d = \frac{3a}{4} = \frac{12}{4} = 3$$
$$S_{10} = \frac{n}{2}[2a+(n-1)d]$$
$$= \frac{10}{2}[8+9(3)]$$
$$= 5[8 + 27]$$
$$= 5 \times 35$$
$$= \mathbf{175}$$

20.
$$S_7 = 49 \quad \text{... (1)}$$
$$S_{17} = 289 \quad \text{... (2)}$$
$$S_n = ?$$

From equation (1) we get

$$\frac{7}{2}[2a + (6)d] = 49$$
$$2a + 6d = 14$$
$$a + 3d = 7 \quad \text{... (3)}$$

From equation (2) we get

$$\frac{17}{2}[2a + 16d] = 289$$
$$2a + 16d = 34$$
$$a + 8d = 17 \quad \text{... (4)}$$

Subtracting equation (1) from equation (2) we get

$$a + 8d = 17$$
$$\underline{-a \pm 3d = -7}$$
$$5d = 10$$
$$d = 2$$

We know

$$a = 7 - 3d$$
$$= 1$$
$$S_n = \frac{n}{2}[2(1) + (n-1)2]$$
$$= \frac{n}{2}[2 + 2n - 2]$$
$$= \frac{2n^2}{2}$$
$$= \boldsymbol{n^2}$$

21. Let a be the first term, d, the common difference and S_n, the sum of the first n term of the AP.

Then $$S_n = \frac{n}{2}[2a+(n-1)d] \quad \text{...(1)}$$

Now, $$S_9 = 81$$

$$\Rightarrow \frac{9}{2}[2a+8d] = 81$$
$$\Rightarrow 9(a + 4d) = 81$$
$$\Rightarrow a + 4d = 9$$
$$\therefore a = 9 - 4d \quad \text{...(2)}$$

Also, it is given that

$$S_{20} = 400$$
$$\Rightarrow \frac{20}{2}(2a+19d) = 400$$
$$\Rightarrow 2a + 19d = 40$$
$$\Rightarrow 2(9 - 4d) + 19d = 40 \quad \text{[From (2)]}$$
$$\Rightarrow 11d = 40 - 18 = 22$$
$$\Rightarrow d = \frac{22}{11} = 2 \quad \text{...(3)}$$
$$\therefore \text{From (2),} \quad a = 9 - 4 \times 2 = 1 \quad \text{...(4)}$$

$\therefore$ From (4) and (3), the required first term and the common difference of the AP are **1** and **2** respectively.

22. Let a be the first term, d, the common difference and S_n, the sum of the first n terms of the AP. Then

$$S_n = \frac{n}{2}[2a+(n-1)d] \quad \text{...(1)}$$

Given that $$S_4 = 40$$

$$\Rightarrow \frac{4}{2}[2a+3d] = 40$$
$$\Rightarrow 2a + 3d = 20 \quad \text{...(2)}$$

Also, given that $$S_{14} = 280$$

$$\Rightarrow \frac{14}{2}(2a+13d) = 280$$
$$\Rightarrow 2a + 13d = 40 \quad \text{...(3)}$$

Subtracting (2) from (3), we get

$$10d = 20$$
$$\therefore d = 2 \quad \text{...(4)}$$

Also, from (2) $$2a = 20 - 3 \times 2 = 14$$

$$\Rightarrow a = 7 \quad \text{...(5)}$$

$\therefore$ From (1), (4) and (5), we get

$$S_n = \frac{n}{2}[14+(n-1)2]$$
$$= n(7 + n - 1)$$
$$= n(n + 6)$$
$$= \boldsymbol{n^2 + 6n}$$

which is the required sum.

23. Let a be the first term, d, the common difference, a_n, the nth term and S_n, the sum of the first n terms of the AP. Then

$$a_n = a + (n-1)d \quad \text{...(1)}$$

and $S_n = \frac{n}{2}[2a + (n-1)d]$ ...(2)

Given that $S_7 = 63$

$\Rightarrow \frac{7}{2}(2a + 6d) = 63$ [From (2)]

$\Rightarrow 7(a + 3d) = 63$

$\Rightarrow a + 3d = 9$

$\Rightarrow a = 9 - 3d$ ...(3)

Also, given that

$S_{14} = 63 + 161 = 224$

$\Rightarrow \frac{14}{2}(2a + 13d) = 224$

$\Rightarrow 7(2a + 13d) = 224$

$\Rightarrow 2a + 13d = 32$

$\Rightarrow 2(9 - 3d) + 13d = 32$ [Using (3)]

$\Rightarrow 7d = 32 - 18 = 14$

$\Rightarrow d = \frac{14}{7} = 2$ ...(4)

$\therefore$ From (3) and (4),

$a = 9 - 3 \times 2 = 3$ ...(5)

Hence, from (1), (4) and (5), we have

$a_{28} = 3 + 27 \times 2 = \mathbf{57}$

which is the required term.

24. $\because$ Sum of first 10 terms of the AP = –150

$\therefore S_{10} = \frac{10}{2}[2a + 9d] = -150$

$\Rightarrow 5[2a + 9d] = -150$

$\Rightarrow 2a + 9d = \frac{-150}{5} = -30$

$\therefore 2a + 9d = -30$ ...(1)

Since, sum of next 10 terms = –550

$\therefore$ Sum of first 10 + 10, i.e. 20 terms

$= -550 + (-150) = -700$

$\therefore S_{20} = \frac{20}{2}[2a + 19d] = -700$

$\therefore 10[2a + 19d] = -700$

$\Rightarrow 2a + 19d = \frac{-700}{10} = -70$

$\therefore 2a + 19d = -70$ ...(2)

Solving (1) and (2), we have:

$a = 3$ and $d = -4$

Now, an AP is given by:

$a, a + d, a + 2d, a + 3d, \ldots$

$\therefore$ The required AP is

[3], [3 + (–4)], [3 + 2(–4)], [3 + 3(–4)], …

or **3, –1, –5, –9, …**

25. Here, first term = a = 6

Let common difference = d

$\because S_n$ = Sum of first n terms $= \frac{n}{2}[2a + (n-1)d]$

$\therefore S_3$ = Sum of first three terms

$= \frac{3}{2}[(2 \times 6) + (3-1)d]$

$= \frac{3}{2}[12 + 2d] = 18 + 3d$ ...(1)

S_6 = Sum of first six terms

$= \frac{6}{2}[(2 \times 6) + (6-1)d]$

$= 3[12 + 5d] = 36 + 15d$ ...(2)

Now, $S_3 = \frac{1}{2}(S_6 - S_3)$ $\Rightarrow 2S_3 = S_6 - S_3$

$\Rightarrow 2S_3 + S_3 = S_6$ or $3S_3 = S_6$ ...(3)

From (1), (2) and (3), we get

$3[18 + 3d] = 36 + 15d$

$\Rightarrow 54 + 9d = 36 + 15d$ or $9d - 15d = 36 - 54$

$\Rightarrow -6d = -18 \therefore d = \frac{-18}{-6} = 3$

Thus, the common difference = **3**

26. Here, a = 20 and common difference = d

Sum of first 6 terms = S_6

Sum of first 12 terms = S_{12}

Since, $S_6 = 5[S_{12} - S_6]$

$\Rightarrow S_6 = 5S_{12} - 5S_6$

$\Rightarrow S_6 + 5S_6 = 5S_{12}$ $\Rightarrow 6S_6 = 5S_{12}$

$\therefore 6\left[\frac{6}{2}\{(2 \times 20) + (6-1)d\}\right] = 5\left[\frac{12}{2}\{(2 \times 20) + (12-1)d\}\right]$

$\Rightarrow 6[3\{40 + 5d\}] = 5[6\{40 + 11d\}]$

$\Rightarrow 6 \times 3(40 + 5d) = 6 \times 5(40 + 11d)$

$\Rightarrow 3(40 + 5d) = 5(40 + 11d)$

$\Rightarrow 120 + 15d = 200 + 55d$

$\Rightarrow 55d - 15d = 120 - 200$ or $40d = -80$

$\Rightarrow d = -\frac{80}{40} = -2$

Thus, the required common difference = **–2**

27. Let a be the first term and d, the common difference of the AP.

Now, $S_n = \frac{n}{2}[2a + (n-1)d]$ ...(1)

Given that $S_5 + S_7 = 167$

$\Rightarrow \frac{5}{2}(2a + 4d) + \frac{7}{2}(2a + 6d) = 167$ [From (1)]

$\Rightarrow 5(a + 2d) + 7(a + 3d) = 167$

$\Rightarrow 12a + 31d = 167$ ...(2)

Also, given that $S_{10} = 235$

$\Rightarrow \frac{10}{2}(2a + 9d) = 235$ [From (1)]

$\Rightarrow 2a + 9d = 47$

$$\Rightarrow \quad a = \frac{47-9d}{2} \quad \ldots(3)$$

∴ From (2) and (3), we have

$$12 \times \frac{(47-9d)}{2} + 31d = 167$$

$$\Rightarrow \quad 282 - 54d + 31d = 167$$

$$\Rightarrow \quad 23d = 282 - 167 = 115$$

$$\therefore \quad d = \frac{115}{23} = 5 \quad \ldots(4)$$

∴ From (3), we have $a = \frac{47 - 9 \times 5}{2} = \frac{2}{2} = 1$ …(5)

∴ From (4) and (5), the required AP is 1, 1 + 5, 1 + 10, 1 + 15 …, i.e. **1, 6, 11, 16,…**

28. First term = a = 4

Let Common diff. = d

Last term, l = 61

and $S_n = 650$

$$\therefore \quad S_n = \frac{n}{2}(a + l)$$

$$\Rightarrow \quad \frac{n}{2}(4+61) = 650$$

$$\Rightarrow \quad n \times 65 = 2 \times 650$$

$$\Rightarrow \quad n = \frac{2 \times 650}{65} = 20$$

Now, $S_n = \frac{n}{2}[2a + (n-1)d]$

$$\therefore \quad S_{20} = \frac{20}{2}[(2 \times 4) + (20-1)d] = 650$$

$$\Rightarrow \quad 8 + 19d = \frac{650}{10} = 65$$

$$\Rightarrow \quad 19d = 65 - 8 = 57$$

$$\therefore \quad d = \frac{57}{19} = \mathbf{3}$$

29. First term = a = 2

Last term = l = 29

Sum of the terms = 155

Let the term of the AP be n

∴ Using $S_n = \frac{n}{2}(a + l)$, we have

$$\frac{n}{2}(2 + 29) = 155$$

$$\Rightarrow \quad n(31) = 155 \times 2$$

$$\Rightarrow \quad n = \frac{155 \times 2}{31} = 10$$

Now, using $S_n = \frac{n}{2}[2a + (n-1)d]$

We get $155 = \frac{10}{2}[2 \times 2 + (10-1)d]$

$$\Rightarrow \quad 5[4 + 9d] = 155$$

$$\Rightarrow \quad 4 + 9d = \frac{155}{5} = 31$$

$$\Rightarrow \quad 9d = 31 - 4 = 27$$

$$\therefore \quad d = \frac{27}{9} = 3$$

Thus, the common difference = **3**

30. Let the first term a = 7, d, the common difference, last term, l = 49, a_n be the nth term and S_n be the sum of the first n terms of the AP.

Then $a_n = a + (n-1)d$

$$= 7 + (n-1)d \quad \ldots(1)$$

and $S_n = \frac{n}{2}[2a + (n-1)d]$

$$= \frac{n}{2}[14 + (n-1)d] \quad \ldots(2)$$

If n is the total number of terms of the AP, then $l = a_n$

$$\Rightarrow \quad 49 = 7 + (n-1)d \quad \text{[From (1)]}$$

$$\Rightarrow \quad (n-1)d = 49 - 7 = 42 \quad \ldots(3)$$

∴ From (2), $S_n = 420$

$$\Rightarrow \quad \frac{n}{2}[14 + (n-1)d] = 420$$

$$\Rightarrow \quad \frac{n}{2}[14 + 42] = 420 \quad \text{[From (3)]}$$

$$\Rightarrow \quad \frac{n}{2} \times (56) = 420$$

$$\therefore \quad n = \frac{420}{28} = 15 \quad \ldots(4)$$

∴ From (3), $d = \frac{42}{n-1} = \frac{42}{14} = 3$ …(5)

∴ From (5), the required common difference is **3**.

31. Given that the first term, $a = -4$, the last term, l = 29.

If n be the total number of terms of the AP, then

$$a_n = l = 29$$

Now, $a_n = a + (n-1)d$

$$= -4 + (n-1)d$$

$$\Rightarrow \quad 29 = -4 + (n-1)d$$

$$\Rightarrow \quad (n-1)d = 33 \quad \ldots(1)$$

If S_n be the sum of the first n terms of the AP, then

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{n}{2}[2a + 33] \quad \text{[From (1)]}$$

$$= \frac{n}{2}[-8 + 33] = \frac{25n}{2} \quad \ldots(2)$$

It is given that $S_n = 150$.

$$\therefore \quad \frac{25n}{2} = 150 \quad \text{[From (2)]}$$

$$\Rightarrow \quad n = \frac{300}{25} = 12$$

$\therefore$ From (1), $d = \frac{33}{12-1} = 3$ ...(3)

$\therefore$ The required common difference is **3**.

32. Given that first term, $a = 5$ and the last term, $l = 45$

Let n be the number of terms of the AP, a_n, the nth term and S_n, the sum of first n terms of the AP.

Then $a_n = l = 45$ ...(1)

$a_n = 5 + (n-1)d$ ...(2)

$S_n = \frac{n}{2}\left[2a+(n-1)d\right]$

$= \frac{n}{2}\left[10+(n-1)d\right]$ ...(3)

From (1) and (2), $45 = 5 + (n-1)d$

$\Rightarrow$ $(n-1)d = 45 - 5 = 40$ ...(4)

Now, $S_n = 400$

$\Rightarrow$ $\frac{n}{2}[10+40] = 400$ [From (3) and (4)]

$\Rightarrow$ $n = \frac{800}{50} = 16$ ...(5)

$\therefore$ From (4), $d = \frac{40}{16-1} = \frac{40}{15} = \frac{8}{3}$ ...(6)

$\therefore$ From (5) and (6), the required values of n and d are respectively **16** and $\mathbf{\frac{8}{3}}$.

33. $d = -3$, $a = 96$ and $l = 66$

$\therefore$ $a_n = 66$

$\Rightarrow$ $a + (n-1)d = 66$

$\Rightarrow$ $96 + (n-1)(-3) = 66$

$\Rightarrow$ $(n-1)(-3) = -30$

$\Rightarrow$ $(n-1) = 10$

$\therefore$ $n = 11$

Now $S_{11} = \frac{11}{2}[96+66] = \frac{11}{2}[162]$

$=$ **891**

34. Here the first term, $a = 5$

The common difference, $d = 12 - 5 = 7$

It a_n be the nth term and S_n be the sum of the first n terms of the AP, then

$a_n = a + (n-1)d$

$= 5 + (n-1)7$

$= 7n - 2$ ...(1)

and $S_n = \frac{n}{2}\left[2a+(n-1)d\right]$

$= \frac{n}{2}\left[2\times5+(n-1)7\right]$

$= \frac{n}{2}\left[10+7n-7\right]$

$= \frac{n(7n+3)}{2}$ ...(2)

It is given that the AP has 50 terms.

$\therefore$ $a_{50} = 7 \times 50 - 2$

$= 350 - 2 = 348$

$\therefore$ Required last term = **348**

Now, the sum of the last 15 terms of the AP

= the sum of the whole 50 terms of the AP – the sum of 50 – 15, i.e. 35 terms of the AP from the beginning

$= S_{50} - S_{35}$

$= \frac{50\times(7\times50+3)}{2} - \frac{35\times(7\times35+3)}{2}$ [From (2)]

$= 25 \times 353 - \frac{35\times248}{2}$

$= 8825 - 35 \times 124$

$= 8825 - 4340$

$= 4485$

$\therefore$ Required sum of the last 15 terms of the AP is **4485**.

35. In the given AP, the first term, $a = 8$, the common difference, $d = 10 - 8 = 2$, n = total number of terms = 60. Let a_n be the nth term and l = last term = a_{60}.

Now, $a_n = a + (n-1)d$

$= 8 + (n-1)2$

$= 2n + 6$ ...(1)

$\therefore$ $l = a_{60} = 2 \times 60 + 6$ [From (1)]

$=$ **126**

which is the required last term.

Now, $S_n = \frac{n}{2}\left[2a+(n-1)d\right]$

$= \frac{n}{2}\left[2\times8+(n-1)2\right]$

$= n(8 + n - 1)$

$= n(7 + n)$

$= 7n + n^2$ ...(2)

$\therefore$ Sum of the last 10 terms of the AP

= Sum of the first 60 terms – Sum of the first 50 terms

$= S_{60} - S_{50}$

$= 7 \times 60 + 60^2 - (7 \times 50 + 50^2)$

$= 420 + 3600 - 350 - 2500$

$= 70 + 1100 =$ **1170**

which is the required Sum of the last 10 terms of the AP.

36. Let a = First term, and

d = Common difference

$\because$ $S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore$ $\frac{n}{2}[2a + (n-1)d] = 18$ ...(1)

$\because$ $a_1 = a = -16$

$a_8 = a + 7d = -2$

$\Rightarrow$ $-16 + 7d = -2$

$\Rightarrow$ $7d = -2 + 16 = 14$

$\therefore$ $d = \frac{14}{7} = 2$

Now, substituting $a = -16$ and $d = 2$, in (1), we get

$$\frac{n}{2}[2(-16) + (n-1)2] = 18$$

$\Rightarrow \quad -16n + n^2 - n = 18$

$\Rightarrow \quad n^2 - 17n - 18 = 0$

Solving $n^2 - 17n - 18 = 0$, we get

$$n = -1 \text{ or } n = 18$$

Rejecting the negative value of n, we get

$$\boldsymbol{n = 18}$$

37. In the given AP, the first term, $a = -12$, the common difference, $d = -9 + 12 = 3$

Let the number of terms of the original AP be n.

Let a_n be the nth term of the original AP and S_n be the sum of its first n terms.

Then $\quad a_n$ = last term

$\Rightarrow \quad 21 = a + (n - 1)d$

$= -12 + 3(n - 1)$

$= 3n - 15$

$\Rightarrow \quad 21 + 15 = 3n$

$\Rightarrow \quad 36 = 3n$

$\Rightarrow \quad \boldsymbol{n = 12}$

which is the required number of terms.

Also, $\quad S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$= \frac{n}{2}\left[-2 \times 12 + (n-1)3\right]$

$= \frac{n}{2}\left[-24 + 3n - 3\right]$

$= \frac{n(3n-27)}{2} \quad \ldots(1)$

$\therefore$ When $n = 12$, then from (1)

$S_n = \frac{12 \times (3 \times 12 - 27)}{2}$

$= 6 \times (36 - 27)$

$= 54 \quad \ldots(2)$

Now, if 1 is added to each of 12 terms of the original AP, then the sum of all the 12 new terms of the AP is $54 + 12 \times 1 =$ **66** which is the required sum of all the terms of the new AP.

38. We have 54, 51, 48, … are in AP.

$\therefore \quad a = 54$

$d = 51 - 54 = (-3)$

Let the number of terms be 'n'.

$\because \quad S_n = \frac{n}{2}[2a + (n - 1)d] = 513$

$\therefore \quad \frac{n}{2}[2(54) + (n - 1)(-3)] = 513$

$\Rightarrow \quad \frac{n}{2}[108 + (n - 1)(-3)] = 513$

$\Rightarrow \quad n[108 + 3 - 3n] = 513 \times 2 = 1026$

$\Rightarrow \quad 111n - 3n^2 = 1026$

$\Rightarrow \quad 37n - n^2 = 342$

$\Rightarrow \quad n^2 - 37n + 342 = 0$

$\Rightarrow \quad n^2 - 18n - 19n + 342 = 0$

$\Rightarrow \quad n(n - 18) - 19(n - 18) = 0$

$\Rightarrow \quad (n - 19)(n - 18) = 0$

$\therefore \quad n = 18 \text{ or } n = 19$

Thus, $\quad$ **n = 18 or 19**

39. AP: 27, 24, 21, 18, …

Given that $\quad S_n = 0, a = 27, d = -3$

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$\Rightarrow \quad 0 = \frac{n}{2}[54 + (n - 1)(-3)]$

$\Rightarrow 54n + n(n - 1)(-3) = 0$

$\Rightarrow 54n + (-3)(n^2 - n) = 0$

$\Rightarrow \quad 54n - 3n^2 + 3n = 0$

$\Rightarrow \quad 3n^2 - 57n = 0$

$\Rightarrow \quad n(3n - 57) = 0$

$\Rightarrow \quad n = 0 \text{ or } n = \frac{57}{3} = 19$

$n = 0$ does not satisfy the condition.

$\therefore \quad \boldsymbol{n = 19}$

40. In the given AP, the first term, $a = 9$, common difference, $d = 17 - 9 = 8$. Let n be the required number of terms of the AP, with sum $S_n = 363 \quad \ldots(1)$

Now, $\quad S_n = \frac{n}{2}\left[2a + (n-1)d\right]$

$= \frac{n}{2}\left[2 \times 9 + (n-1)8\right]$

$= n(9 + 4n - 4)$

$= n(5 + 4n)$

$= 4n^2 + 5n \quad \ldots(2)$

$\therefore$ From (1) and (2), we have

$4n^2 + 5n - 636 = 0$

Comparing this quadratic equation with the standard quadratic equation $Ax^2 + Bx + C = 0$, we have

A = 4, B = 5 and C = −636.

$\therefore \quad n = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$

$= \frac{-5 \pm \sqrt{5^2 + 4 \times 636 \times 4}}{2 \times 4}$

$= \frac{-5 \pm \sqrt{25 + 10176}}{8}$

$= \frac{-5 \pm \sqrt{10201}}{8}$

$= \frac{-5 \pm 101}{8}$

$= \frac{96}{8}, -\frac{106}{8}$

$$= 12, -\frac{53}{4}$$

Neglecting the negative value of n, i.e. neglecting $n = -\frac{53}{4}$ which is not a natural number, we get $n = 12$

$\therefore$ Required number of terms = **12**.

41. $a = 78$

$d = 71 - 78 = -7$

$S_n = 468$

$\because \quad S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore \quad \frac{n}{2}[2(78) + (n-1)(-7)] = 468$

Solving the quadratic equation and rejecting the negative value, we get

$n = 13$

$\therefore$ The required number of terms = **13**

Again, using

$S_n = \frac{n}{2}(a + l)$, we get

$\frac{13}{2}(78 + l) = 468$

$\Rightarrow \quad 78 + l = 468 \times \frac{2}{13} = 72$

$\Rightarrow \quad l = 72 - 78 = -6$

$\therefore$ **Last term = –6**

42. Terms of the AP are $-7, \frac{-13}{2}, -6, \frac{-11}{2}, -5, \ldots$

Here, $a = -7$

$$d = \frac{-13}{2} - (-7) = \frac{-13+14}{2} = \frac{1}{2}$$

Let the required number of terms be n

$\therefore \quad S_n = -45$

Using $S_n = \frac{n}{2}[2a + (n-1)d]$, we get

$$\frac{n}{2}\left[2(-7) + (n-1)\left(\frac{1}{2}\right)\right] = -45$$

$$\Rightarrow \quad n\left[-14 - \frac{1}{2} + \frac{n}{2}\right] = -90$$

$$\Rightarrow \quad n\left[\frac{-29}{2} + \frac{n}{2}\right] = -90$$

$$\Rightarrow \quad \frac{-29n}{2} + \frac{n^2}{2} = -90$$

$\Rightarrow \quad -29n + n^2 = -180$

$\Rightarrow \quad n^2 - 29n + 180 = 0$

$\Rightarrow \quad n^2 - 20n - 9n + 180 = 0$

$\Rightarrow \quad n(n - 20) - 9(n - 20) = 0$

$\Rightarrow \quad (n - 9)(n - 20) = 0$

$\therefore \quad n = 9$ and $n = 20$

Thus, the required number of terms is **9** or **20**.

$\Rightarrow$ Sum of first 9 terms = Sum of first 20 terms.

It means sum of all terms of 10th to 20th is **zero**.

43. $\because \quad a_n = 4 + 3n$ [Given]

$\therefore \quad a_1 = \text{First term} = 4 + 3(1) = 7$

$a_2 = \text{Second term} = 4 + 3(2) = 10$

$\Rightarrow \quad d = (a_2 - a_1) = 10 - 7 = 3$

Now $S_n = \frac{n}{2}[2a + (n-1)d]$ $[\because a = 7, d = 3]$

$\therefore \quad S_n = \frac{n}{2}[2(7) + 3n - 3]$

$\Rightarrow \quad S_n = \frac{n}{2}[14 - 3 + 3n]$

$\Rightarrow \quad \mathbf{S_n = \frac{n}{2}[11 + 3n]}$

44. $\because$ The nth term of AP = $2n + 1$

$\therefore \quad a_n = 2n + 1$

$\Rightarrow \quad a_1 = 2(1) + 1 = 3$ [First term]

$a_2 = 2(2) + 1 = 5$ [Second term]

$\therefore \quad d = a_2 - a_1 = 5 - 3 = 2$

Now, $S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{n}{2}[2(3) + (n-1)(2)]$

$= \frac{n}{2}[6 + 2n - 2]$

$= \frac{n}{2}[4 + 2n] = \frac{n}{2} \times 2[2 + n]$

$= n[2 + n] = 2n + n^2$

Thus, the sum of n terms = $\mathbf{n^2 + 2n}$

45. $\because \quad n\text{th term} = \frac{31-n}{3}$

$\therefore \quad a_n = \frac{31-n}{3}$

$\Rightarrow \quad a_1 = \frac{31-1}{3} = \frac{30}{3} = 10$

$a_2 = \frac{31-2}{3} = \frac{29}{3} = 9\frac{2}{3}$

$a_3 = \frac{31-3}{3} = \frac{28}{3} = 9\frac{1}{3}$

$a_4 = \frac{31-4}{3} = \frac{27}{3} = 9$

… … …

$\therefore$ The required sequence is

$$\mathbf{10, 9\frac{2}{3}, 9\frac{1}{3}, 9, \ldots}$$

Now $a = 10$

$$d = a_2 - a_1 = \frac{29}{3} - 10 = \frac{29-30}{3} = -\frac{1}{3}$$

$\because \quad S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore \quad S_{12} = \frac{12}{6}\left[2(10)+(12-1)\left(-\frac{1}{3}\right)\right]$

$= 6\left[20+\left(-\frac{11}{3}\right)\right]$

$= 6\left[\frac{60-11}{3}\right] = 6\times\frac{49}{3}$

$= 2 \times 49 = 98$

$\Rightarrow \quad \mathbf{S_{12} = 98}$

46. It a_r denote any rth term of the AP,

then $\quad a_r = 5r - 1 \quad$ [Given] …(1)

$\therefore \quad S_n = \sum_{r=1}^{n} a_r = \sum_{r=1}^{n}(5r-1)$ [From (1)]

$= 5\sum_{r=1}^{n} r - n$

$= \frac{5\times n(n+1)}{2} - n$

$= \frac{5n^2+5n-2n}{2}$

$= \mathbf{\frac{5n^2+3n}{2}}$

which is the required sum of the first n terms of the AP.

From this, we have

$S_{20} = \frac{5\times 20^2 + 3\times 20}{2}$

$= \frac{2000+60}{2}$

$= \frac{2060}{2}$

$= \mathbf{1030}$

which is the required value of S_{20}.

47. Let a_p be the pth term and S_p be the sum of the first p terms of the AP.

Then $\quad S_p = ap^2 + bp \quad$ …(1)

$a_p = S_p - S_{p-1} \quad$ [Given]…(2)

$= ap^2 + bp - a(p-1)^2 - b(p-1)$

$= ap^2 + bp - ap^2 + 2ap - a - bp + b$

$= 2ap + b - a \quad$ …(3)

If d be the common difference, then

$d = a_p - a_{p-1}$

$= 2ap + b - a - 2a(p-1) - b + a$ [Using(3)]

$= 2ap + b - a - 2ap + 2a - b + a$

$= \mathbf{2a}$

which is the required common difference.

48. Let S_n be the sum of the first n terms of the AP.

Then $\quad S_n = n^2 \quad$ [Given] …(1)

$\therefore$ If a_n be the nth term of the AP, then

$a_n = S_n - S_{n-1} \quad$ …(2)

$= n^2 - (n-1)^2 \quad$ [From (1)]

$= (n + n - 1)(n - n + 1)$

$= 2n - 1 \quad$ …(3)

Hence, $a_{10} = 2 \times 10 - 1 = \mathbf{19}$ which is the required 10th term.

49. It a_n be the nth term, then

$a_n = S_n - S_{n-1}$

$= 2n^2 + 3n - 2(n-1)^2 - 3(n-1)$

$= 2n^2 + 3n - 2n^2 + 4n - 2 - 3n + 3$

$= 4n + 1 \quad$ …(1)

$\therefore \quad a_{16} = 4 \times 16 + 1 \quad$ [From (1)]

$= \mathbf{65}$ which is the required 16th term.

50. $\because \quad S_n = 3n^2 - 4n$

$\therefore \quad S_1 = 3(1)^2 - 4(1) = -1 \quad \Rightarrow \quad a = -1$

$S_2 = 3(2)^2 - 4(2) = 4$

Since, $\quad S_2$ = sum of first two terms = 4

$\therefore \quad a + (a + d) = 4$

$\Rightarrow \quad (-1) + (-1 + d) = 4$ or $d = 4 + 2 = 6$

Now $\quad a_n = a + (n-1)d$

$\therefore \quad a_n = -1 + (n-1) \times 6$

$= -1 + 6n - 6 = 6n - 7$

Thus the nth term is $\mathbf{6n - 7}$.

51. $S_n = \frac{1}{2}(3n^2 + 7n)$

$= \frac{3}{2}n^2 + \frac{7}{2}n \quad$ ... (1)

We know

$S_n = \frac{n}{2}[2a + (n-1)d]$

$= an + n(n-1)\frac{d}{2}$

$= an + n^2\frac{d}{2} - n\frac{d}{2}$

$= n^2\frac{d}{2} + n\left(a - \frac{d}{2}\right) \quad$ ... (2)

Comparing equations (1) and (2) we get

$\frac{d}{2} = \frac{3}{2} \qquad a - \frac{d}{2} = \frac{7}{2}$

$d = 3 \qquad a - \frac{3}{2} = \frac{7}{2}$

$a = 5$

$a_n = a + (n-1)d$

$= 5 + (n-1)3$

$= 5 + 3n - 3$

$= \mathbf{3n + 2}$

$a_{20} = a + (n-1)d$

$= a + 19d$

$= 5 + 57$

$= \mathbf{62}$

52. Let S_n be the sum of the first n terms of the AP and a_n be its nth term.

Then $S_n = \frac{3n^2 + 5n}{2}$ ...(1)

Then $a_n = S_n - S_{n-1}$ [Given]

$= \frac{3n^2 + 5n - 3(n-1)^2 - 5(n-1)}{2}$ [From (1)]

$= \frac{3n^2 + 5n - 3n^2 + 6n - 3 - 5n + 5}{2}$

$= \frac{6n + 2}{2}$

$= \mathbf{3n + 1}$ ...(2)

which is the required nth term.

$\therefore$ $a_{25} = 3 \times 25 + 1 = \mathbf{76}$

which is the required **25th** term. [From (2)]

53. Let S_n be the sum of the first n terms of the AP and a_n be its nth term.

Then $S_n = \frac{5n^2 + 3n}{2}$ [Given] ...(1)

Then $a_n = S_n - S_{n-1}$

$= \frac{5n^2 + 3n - 5(n-1)^2 - 3(n-1)}{2}$

$= \frac{5n^2 + 3n - 5n^2 + 10n - 5 - 3n + 3}{2}$

$= \frac{10n - 2}{2}$

$= \mathbf{5n - 1}$ ...(2)

which is the required nth term.

$\therefore$ From (2), $a_{20} = 5 \times 20 - 1 = \mathbf{99}$

which is the required 20th term.

54. (*i*) If a_n be the nth term, then

$a_n = S_n - S_{n-1}$
$= 3n^2 - n - 3(n-1)^2 + (n-1)$
$= 3n^2 - n - 3n^2 + 6n - 3 + n - 1$
$= \mathbf{6n - 4}$ ...(1)

which is the required nth term.

(*ii*) Putting $n = 1$ in (1), we have

a_1 = first term $= 6 - 4 = \mathbf{2}$

which is the required first term.

(*iii*) If d be the common difference, then

$d = a_n - a_{n-1}$
$= 6n - 4 - 6(n-1) + 4$
$= 6n - 4 - 6n + 6 + 4$
$= \mathbf{6}$

which is the required common difference.

55. $\because$ Sum of n terms is $5n^2 - 3n$

$\therefore$ $S_n = 5n^2 - 3n$

$\therefore$ $S_1 = 5(1)^2 - 3(1)$
$= 5 - 3 = 2$

$\Rightarrow$ $a = 2$

$S_2 = 5(2)^2 - 3(2) = 20 - 6 = 14$

Now, S_2 = sum of first two terms = 14

$\Rightarrow$ $(a) + (a + d) = 14$

$\Rightarrow$ $2a + d = 14$

$\Rightarrow$ $2(2) + d = 14$

$\Rightarrow$ $d = 14 - 4 = 10$

Since, $a, a + d, a + 2d, \ldots$ are in AP.

$\Rightarrow$ 2, (2 + 10), [2 + 2(10)], … are in AP.

$\Rightarrow$ 2, 12, 22, ... are in AP.

$\therefore$ The required AP is **2, 12, 22, …**

Now, using $a_n = a + (n-1)d$, we get

$a_{10} = 2 + (10 - 1) \times 10$
$= 2 + (9 \times 10) = \mathbf{92}$

56. $\because$ $S_n = 3n^2 - n$

$\therefore$ $S_1 = 3(1)^2 - 1 = 3 - 1 = 2 \Rightarrow a = 2$

$S_2 = 3(2)^2 - 2 = 12 - 2 = 10$

$\Rightarrow$ [1st term] + [2nd term] = 10

$\Rightarrow$ $(a) + (a + d) = 10$

$\Rightarrow$ $2 + 2 + d = 10$ [$\because$ $a = 2$]

$\Rightarrow$ $d = 10 - 2 - 2 = 6$

Let nth term = 50

$\Rightarrow$ $a_n = a + (n-1)d$

$\Rightarrow$ $a_n = 2 + (n-1)6 = 50$

$\Rightarrow$ $6(n-1) = 50 - 2 = 48$

$\Rightarrow$ $n = 8 + 1 = 9$

Hence, **9th term** of the AP is 50.

57. Let S_m be the sum of the first m terms of the AP.

Then given that

$S_m = 4m^2 - m$ ...(1)

$\therefore$ $a_n = S_n - S_{n-1}$
$= 4n^2 - n - 4(n-1)^2 + (n-1)$ [From (1)]
$= 4n^2 - n - 4n^2 + 8n - 4 + n - 1$
$= 8n - 5$ ...(2)

Given that

$a_n = 107$

$\therefore$ From (2),

$8n - 5 = 107$

$\therefore$ $n = \frac{112}{8} = 14$ ...(3)

$\therefore$ From (2),

$a_{21} = 8 \times 21 - 5$
$= 168 - 5 = 163$ ...(4)

$\therefore$ From (3) and (4), the required values of n and a_{21} are **14** and **163** respectively.

58. Let S_q be the sum of first q terms of the AP and a_q be its qth term.

Then $S_q = 63q - 3q^2$ [Given] ...(1)

$\therefore \quad a_q = S_q - S_q - 1$

$= 63q - 3q^2 - 63(q-1) + 3(q-1)^2$

$= 63q - 3q^2 - 63q + 63 + 3q^2 - 6q + 3$

$= 66 - 6q \quad \ldots(2)$

It is given that

$a_p = -60$

$\therefore \quad 66 - 6p = -60$

$\Rightarrow \quad 6p = 60 + 66 = 126$

$\therefore \quad p = 21 \quad \ldots(3)$

Also, from (2),

$a_{11} = 66 - 6 \times 11 = 0 \quad \ldots(4)$

$\therefore$ From (3) and (4), the required values of p and a_{11} are **21** and **0** respectively.

59. (*i*) $\because$ Odd numbers between 0 and 100 are

1, 3, 5, 7, …, 99

$\therefore \quad a = 1$

$d = 3 - 1 = 2$

$l = 99$

Let number odd numbers between 0 and 100 = n

$\therefore \quad a_n = a + (n-1)d = 99$

$\Rightarrow \quad 1 + (n-1)2 = 99$

$\Rightarrow \quad (n-1)2 = 99 - 1 = 98$

$\Rightarrow \quad n - 1 = \frac{98}{2} = 49$

$\therefore \quad n = 49 + 1 = 50$

Using $\quad S_n = \frac{n}{2}(a + l)$, we get

$S_{50} = \frac{50}{2}(1 + 99) = 25(100) = 2500$

Thus, the sum of all odd numbers (between 0 and 100) is **2500.**

(*ii*) $\because$ Three digit numbers are

100, 101, 102, 103, 104, …, 999.

$\therefore$ The 3-digit numbers which when divided by 5 leaves remainder 3, are : 103, 108, 113, 118, …, 998

$\therefore \quad a = 103$

$d = 108 - 103 = 5$

$l = 998$

Let such numbers be n.

$\therefore$ Using $\quad a_n = a + (n-1)d$, we get

$a_n = 103 + (n-1) \times 5 = 998$

$\Rightarrow \quad (n-1) \times 5 = 998 - 103 = 895$

$\Rightarrow \quad n - 1 = \frac{895}{5} = 179$

$\therefore \quad n = 179 + 1 = 180$

Now, using $S_n = \frac{n}{2}(a + l)$, we get

$S_{180} = \frac{180}{2}(103 + 998)$

$= 90(1101)$

$= 99090$

Thus, the required sum = **99090**

(*iii*) $\because$ Odd numbers between 50 and 100 and divisible by 3 are

51, 57, 63, …, 99

$\therefore \quad a = 51, d = 57 - 51 = 6, l = 99$

Now, using $a_n = a + (n-1)d$, we get

$a_n = 51 + (n-1)6 = 99$

$\Rightarrow \quad n - 1 = \frac{99 - 51}{6} = \frac{48}{6} = 8$

$\therefore \quad n = 8 + 1 = 9$

Since, $\quad S_n = \frac{n}{2}(a + l)$

$\therefore \quad S_9 = \frac{9}{2}(51 + 99)$

$= 9 \times 75 =$ **675**

(*iv*) $\because$ Two digit natural numbers are

10, 11, 12, 13, …, 99

$\therefore \quad a = 10, d = 1$ and $l = 99$

Using $\quad a_n = a + (n-1)d$, we get

$a_n = 10 + (n-1)1 = 99$

$\Rightarrow \quad n - 1 = 99 - 10 = 89$

$\Rightarrow \quad n = 89 + 1 = 90$

Now, using $S_n = \frac{n}{2}(a + l)$, we get

$S_{90} = \frac{90}{2}(10 + 99)$

$= 45 \times 109 = 4905$

$\therefore$ Required sum = **4905**

(*v*) Natural numbers less than 100 and divisible by 4 are 4, 8, … 96

$\therefore \quad a = 4, d = 4$ and $l = 96$

Using, $\quad a_n = a + (n-1)d$

$\Rightarrow \quad 4 + (n-1)\,4 = 96$

$\Rightarrow \quad (n-1)\,4 = 92$

$\Rightarrow \quad (n-1) = 23$

$\Rightarrow \quad n = 24$

Now, using $\quad S_n = \frac{n}{2}(a + l)$, we get

$S_{24} = \frac{24}{2}(4 + 96)$

$= 12 \times 100$

= 1200

(*vi*) Three digit numbers which are multiples of 7 are

105, 112, 119, …, 994

$\therefore \quad a = 105, d = 112 - 105 = 7$ and $l = 994$

$\therefore \quad a_n = a + (n-1)d$

$994 = 105 + (n-1) \times 7$

$\Rightarrow \quad n - 1 = \dfrac{994 - 105}{7}$

$\Rightarrow \quad n - 1 = \dfrac{889}{7} = 127$

$\Rightarrow \quad n = 127 + 1 = 128$

Now, using $S_n = \dfrac{n}{2}(a + l)$, we get

$$S_{128} = \frac{128}{2}(105 + 994)$$

$$= 64 \times 1099 = 70336$$

Thus, the required sum = **70336**

(*vii*) ∵ Numbers between 101 and 304 which are divisible by 3 are

102, 105, 108, 111, …, 303

$\Rightarrow \quad a = 102, d = 105 - 102 = 3$

and $\quad l = 303$

Using $\quad a_n = a + (n - 1)d$, we get

$a + (n - 1)d = 303$

or $\quad 102 + (n - 1) \times 3 = 303$

$\Rightarrow \quad n - 1 = \dfrac{303 - 102}{3} = \dfrac{201}{3} = 67$

$\Rightarrow \quad n = 67 + 1 = 68$

$\therefore \quad S_{68} = \dfrac{68}{2}[102 + 303] \quad$ [Using $S_n = \dfrac{n}{2}(a + l)$]

$= 34 \times 405 = 13770$ ...(1)

∵ Numbers between 101 and 304 which are divisible by 5 are 105, 110, 115, 120, …, 300

$\therefore \quad a = 105, d = 110 - 105 = 5$

and $\quad l = 300$

$a_n = a + (n - 1)d$

$\Rightarrow \quad a_n = 105 + (n - 1)5 = 300$

$\Rightarrow \quad n - 1 = \dfrac{300 - 105}{5} = \dfrac{195}{5} = 39$

$\Rightarrow \quad n = 39 + 1 = 40$

Now, $\quad S_{40} = \dfrac{40}{2}[105 + 300] \quad$ [Using $S_n = \dfrac{n}{2}(a + l)$]

$= 20 \times 405 = 8100$ …(2)

∵ Numbers between 101 and 304, which are divisible by 3 × 5 i.e. 15 are 105, 120, 135, …, 300

$\therefore \quad a = 105, d = 120 - 105 = 15$ and $l = 300$

$\Rightarrow \quad 300 = 105 + (n - 1)15$ [Using $a_n = a + (n - 1)d$]

$\Rightarrow \quad (n - 1) = \dfrac{300 - 105}{15} = \dfrac{195}{15} = 13$

$\Rightarrow \quad n = 13 + 1 = 14$

$$S_{14} = \frac{14}{2}[105 + 300]$$

$= 7 \times 405 = 2835$ …(3)

Since the multiples of 15, i.e. 105, 120, 135, …, 300 are included in the multiples of 3 as well of 5, from (1), (2) and (3), we have

The sum of numbers between 101 and 304 which are divisible by 3 or 5 :

$$[13770 + 8100] - 2835 = 21870 - 2835 = 19035$$

Thus, the required sum = **19035**

(*viii*) We know that odd numbers are not divisible by 2. Also all odd numbers that are not divisible by 5 do not have 5 in ones place.

∴ Required sum

= [Sum of all odd numbers up to 1000]
− [Sum of odd numbers up to 1000 that are divisible by 5]

$= [1 + 3 + 5 + 7 + \ldots + 999]$
$- [5 + 15 + 25 + \ldots + 995]$ … (1)

∵ 1, 3, 5, … 999 are in AP such that

$a = 1, d = 2, l = 999$

$a_n = a + (n - 1)d$

$\Rightarrow 1 + (n - 1) \times 2 = 999$

$\Rightarrow \quad (n - 1) \times 2 = 999 - 1 = 998$

or $\quad (n - 1) = \dfrac{998}{2} = 499$

$\Rightarrow \quad n = 499 + 1 = 500$

$\therefore \quad S_{500} = \dfrac{500}{2}(1 + 999)$

$= 250 \times 1000 = 250000$ …(2)

Also, 5, 15, 25, …, 995 are in AP such that

$a = 5, d = 10$ and $l = 995$

$a_n = a + (n - 1)d$

$\Rightarrow \quad 5 + (n - 1)10 = 995$

$\Rightarrow \quad (n - 1)10 = 995 - 5 = 990$

$\Rightarrow \quad n - 1 = \dfrac{990}{10} = 99$

$\Rightarrow \quad n = 99 + 1 = 100$

$\therefore \quad S_{100} = \dfrac{100}{2}(5 + 995)$

$= 50 \times 100 = 50000$ …(3)

Now from (1), (2) and (3) we have

Required sum = 250000 − 50000 = **200000**

(*ix*) First seven multiples of 2 as well as 9 are

18, 36, 54, 72, 90, 108, 126.

$\therefore \quad a = 18, d = 36 - 18 = 18$ and $l = 126$

$\because \quad n = 7$

∴ Using $\quad S_n = \dfrac{n}{2}(l + a)$, we get

$S_7 = \dfrac{7}{2}(126 + 18) = \dfrac{7}{2} \times 144$

$= 7 \times 72 = 504$

∴ The required sum = **504**

(*x*) ∵ Two digit numbers which leave remainder 1, when divided by 3 are 10, 13, 16, 19, …, 97

$\therefore \quad a = 10, d = 13 - 10 = 3$ and $l = 97$

Using $a_n = a + (n-1)d$, we get

$$10 + (n-1) \times 3 = 97$$

$$\Rightarrow \quad (n-1) \times 3 = 97 - 10 = 87$$

$$\Rightarrow \quad n - 1 = \frac{87}{3} = 29$$

$$\therefore \quad n = 29 + 1 = 30$$

Now, using $S_n = \frac{n}{2}(l + a)$, we get

$$S_{30} = \frac{30}{2}(10 + 97)$$

$$= 15 \times 107 = 1605$$

$\therefore$ The required sum = **1605**

(*xi*) Let a = first term and d = common difference

Here, the middle term = 6th term

[$\because$ Total number of terms = 11]

$$\therefore \quad \text{6th term} = 20$$

$$\Rightarrow \quad a + (6-1)d = 20$$

$$\Rightarrow \quad a + 5d = 20 \quad \ldots(1)$$

Now, using, $S_n = \frac{n}{2}[2a + (n-1)d]$, we get

$$S_{11} = \frac{11}{2}[2a + (10)d]$$

$$= \frac{11}{2} \times 2[a + 5d]$$

$$= 11[a + 5d] \quad \ldots(2)$$

From (1) and (2), we get

$$S_{11} = 11[20] = 220$$

Hence, the required sum = **220**

(*xii*) We have 8, 10, 12, …, 126

$$\therefore \quad d = 10 - 8 = 2$$

To find the sum from the end, we take $-d$ (i.e. common difference is taken negative) and start with the last term (as the first term)

i.e. $l = 126, d = -2, n = 10$

Using $S_n = \frac{n}{2}[2l + (n-1)d]$, we get

$$S_{10} = \frac{10}{2}[2(126) + (10-1)(-2)]$$

$$\Rightarrow \quad S_{10} = 5[252 + 9 \times (-2)]$$

$$\Rightarrow \quad S_{10} = 5[252 - 18] = 5 \times 234 = \mathbf{1170}$$

Thus, sum of the 10 term from the end = **1170**

(*xiii*) All three-digit natural numbers which are multiples of 11 are 110, 121, 132, 143, 154 …990.

This sequence is in AP with the first term, $a = 110$ and common difference, $d = 121 - 110 = 11$.

Let a_n be the nth term and S_n be the sum of the first n terms of the AP.

Then $a_n = a + (n-1)d$

$$= 110 + (n-1)11$$

$$= 11n - 99 \quad \ldots(1)$$

and

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{n}{2}[2 \times 110 + (n-1)11]$$

$$= \frac{n(220 - 11 + 11n)}{2}$$

$$= \frac{(11n + 209)n}{2}$$

$$= \frac{11n^2 + 209n}{2} \quad \ldots(2)$$

If $a_n = 990$, where n is the total number of terms of the AP.

Then from (1),

$$11 \times n - 99 = 990$$

$$\Rightarrow \quad 11n = 990 - 99 = 891$$

$$\Rightarrow \quad n = \frac{891}{11} = 81 \quad \ldots(3)$$

$\therefore$ There are 81 terms of this AP.

$\therefore$ From (2), we have

$$S_{81} = \frac{11 \times 81^2 + 209 \times 81}{2}$$

$$= \frac{81(11 \times 81 + 209)}{2}$$

$$= \frac{81 \times (891 + 209)}{2}$$

$$= \frac{81 \times 1100}{2}$$

$$= \frac{89100}{2}$$

$$= \mathbf{44550}$$

which is the required sum.

(*xiv*) 40 positive integers which are divisible by 6 are 6, 12, 18, 24 … to 440 terms.

All these numbers are in AP with the first term

$a = 6$ and the common difference, $d = 12 - 6 = 6$.

If S_n denote the sum of the first n terms of the AP, then

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{n}{2}[2 \times 6 + (n-1) \times 6]$$

$$= n(6 + 3n - 3)$$

$$= n(3n + 3)$$

$$= 3n^2 + 3n \quad \ldots(1)$$

If $n = 40$, then

$$S_{40} = 3 \times 40^2 + 3 \times 40 \quad \text{[From (1)]}$$

$$= 4800 + 120$$

$$= \mathbf{4920}$$

which is the required sum of 40 terms of the AP.

(*xv*) The first 8 multiples of 3 are 3, 6, 9, 12, 15, 18, 21 and 24. These numbers form an AP with the first term, $a = 3$ and the common difference, $d = 6 - 3 = 3$. If S_n be the sum of the first n terms of this AP, then

$$S_n = \frac{n}{2}[2a + (n-1)d]$$
$$= \frac{n}{2} \times [2 \times 3 + (n-1)3]$$
$$= \frac{n}{2}[6 + 3n - 3]$$
$$= \frac{3n^2 + 3n}{2} \quad \ldots(1)$$

When $n = 8$, then

From (1), $$S_8 = \frac{3 \times 8^2 + 3 \times 8}{2}$$
$$= \frac{192 + 24}{2}$$
$$= \frac{216}{2} = \mathbf{108}$$

which is required sum.

(*xvi*) All three-digit natural numbers which are divisible by 13 are 104, 117, 130, 143 …988.

These numbers form an AP with the first term, $a = 104$ and the common difference, $d = 117 - 104 = 13$.

If S_n be the sum of the first such natural numbers, then

$$S_n = \frac{n}{2}[2a + (n-1)d]$$
$$= \frac{n}{2}[2 \times 104 + (n-1)13]$$
$$= \frac{n}{2}[208 + 13n - 13]$$
$$= \frac{n(13n + 195)}{2}$$
$$= \frac{13n(n+15)}{2} \quad \ldots(1)$$

If a_n be the nth term of this AP, then

$$a_n = a + (n - 1)d$$
$$= 104 + (n - 1)13$$
$$= 104 + 13n - 13$$
$$= 13n + 91$$
$$= 13(n + 7) \quad \ldots(2)$$

If $a_n = 988$, i.e. if the total number of terms is n,

Then $13(n + 7) = 988$ [From (2)]

$\Rightarrow$ $n + 7 = 76$

$\Rightarrow$ $n = 76 - 7 = 69$

$\therefore$ From (1), $$S_{69} = \frac{13 \times 69 \times (69 + 15)}{2}$$
$$= \frac{13 \times 69 \times 84}{2}$$
$$= 13 \times 69 \times 42$$
$$= \mathbf{37674}$$

which is the required sum.

(*xvii*) All natural numbers between 200 and 400, which are divisible by 7, are 203, 210, 217, 224, …,399.

These numbers form an AP with the first term, $a = 203$ and the common difference, $d = 210 - 203 = 7$

If S_n be the sum of the first n terms of this AP, then

$$S_n = \frac{n}{2}[2a + (n-1)d]$$
$$= \frac{n}{2}[2 \times 203 + (n-1)7]$$
$$= \frac{n}{2}[406 + 7n - 7]$$
$$= \frac{n}{2}(7n + 399)$$
$$= \frac{7n(n+57)}{2} \quad \ldots(1)$$

Also, if a_n be the nth term, then

$$a_n = a + (n - 1)d$$
$$= 203 + (n - 1)7$$
$$= 7n + 196$$
$$= 7(n + 28) \quad \ldots(2)$$

If n be the total number of terms of this AP, then

$a_n = 399$, the last term.

$\therefore$ From (2),

$$7(n + 28) = 399$$

$\Rightarrow$ $$n = \frac{399}{7} - 28$$
$$= 57 - 28 = 29 \quad \ldots(3)$$

$\therefore$ From (1), we have

$$S_{29} = \frac{7 \times 29}{2}(29 + 57)$$
$$= \frac{7 \times 29}{2} \times 86$$
$$= 7 \times 29 \times 43$$
$$= \mathbf{8729}$$

which is the required sum.

(*xviii*) We know that all natural numbers which are divisible by 5 must end with 0 or 5. But natural numbers ending with 5 are not even numbers and all natural numbers ending with 0 are even natural numbers divisible by 5.

Hence, all 100 even numbers divisible by 5 (which are clearly divisible by $2 \times 5 = 10$) are 10, 20, 30, 40, … 100 term.

All these numbers form an AP with the first term, $a = 10$, and the common difference, $d = 20 - 10 = 10$.

If S_n be the sum of the first n terms of this AP, then

$$S_n = \frac{n}{2}[2a + (n-1)d]$$
$$= \frac{n}{2}[2 \times 10 + (n-1)10]$$
$$= \frac{n}{2}[20 + 10n - 10]$$

$$= \frac{(10n+10)n}{2}$$

$$= 5n(n+1) \quad \ldots(1)$$

When $n = 100$, then from (1), we have

$$S_{100} = 5 \times 100 \times 101$$

$$= \mathbf{50500}$$

which is the required sum.

(*xix*) (*a*) Natural numbers between 100 and 200 which are divisible by 9 are 108, 117, 126, …, 198.

These numbers form an AP with the first term, $a = 108$, the common difference, $d = 117 - 108 = 9$ and the last term, $l = 198$.

If a_n be the nth term of the AP, then

$$a_n = a + (n-1)d$$

$$= 108 + (n-1)9$$

$$= 9n + 99$$

$$= 9(11 + n) \quad \ldots(1)$$

It S_n be the sum of the first n terms of this AP, then

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$= \frac{n}{2}\left[2 \times 108 + (n-1)9\right]$$

$$= \frac{n}{2}\left[216 + 9n - 9\right]$$

$$= \frac{n(9n+207)}{2}$$

$$= \frac{9n(n+23)}{2} \quad \ldots(2)$$

If n be the total number of terms of the AP, then

$$a_n = \text{last term} = 198$$

$\Rightarrow \quad 9(11 + n) = 198 \quad$ [From (1)]

$\Rightarrow \quad n + 11 = 22$

$\Rightarrow \quad n = 11$

$\therefore$ Total number of terms of the AP is 11.

$\therefore$ From (2),

$$S_{11} = \frac{9 \times 11(11+23)}{2}$$

$$= \frac{99 \times 34}{2}$$

$$= 17 \times 99 = \mathbf{1683} \quad \ldots(3)$$

which is the required sum.

(*b*) We shall first find the sum S'_n of all natural numbers between 100 and 200, i.e. 101, 102, 103, …, 199 with first term, $a_1 = 101$, common difference, $d_1 = 1$ and the total number of terms, $n = 200 - 100 - 1 = 99$

Then $\quad S'_{99} = \frac{99}{2} \times (2 \times 101 + 98)$

$$= \frac{99}{2} \times (202 + 98)$$

$$= \frac{99 \times 300}{2}$$

$$= 150 \times 99$$

$$= 14850 \quad \ldots(4)$$

$\therefore$ Required sum of all numbers from 100 to 200, not divisible by 9, is

$$S'_{99} - S_{11} = 14850 - 1683$$

[From (3) and (4)]

$$= \mathbf{13167}$$

60. Two digit numbers divisible by 7 are

14, 21, 28, …, 98

$\therefore \quad a = 14,\ d = 21 - 14 = 7$ and $l = 98$

Now $\quad a_n = a + (n - 1)$

$\Rightarrow \quad 14 + (n-1) \times 7 = 98$

$\Rightarrow \quad n - 1 = \frac{98-14}{7} = \frac{84}{7} = 12$

$\therefore \quad n = 12 + 1 = 13$

Now, $\quad$ using $S_n = \frac{n}{2}(a + l)$, we get

$$S_{13} = \frac{13}{2}(14 + 98)$$

$$= \frac{13}{2} \times 112 = 13 \times 56 = 728$$

Thus, number of terms = **13**

Required sum = **728**

61. Let a be the first term and d be the common difference of the AP. If a_n be the nth term of the AP and S_n is the sum of its first term, then

$$a_n = a + (n-1)d \quad \ldots(1)$$

and $\quad S_n = \frac{n}{2}\left[2a + (n-1)d\right] \quad \ldots(2)$

Given that $\quad S_6 = 42$

$\therefore$ From (2), $\quad 42 = \frac{6}{2}[2a + 5d]$

$\Rightarrow \quad 2a + 5d = 14 \quad \ldots(3)$

Also, given that $\quad \frac{a_{10}}{a_{30}} = \frac{1}{3}$

$\Rightarrow \quad \frac{a+9d}{a+29d} = \frac{1}{3} \quad$ [From (1)]

$\Rightarrow \quad 3a + 27d = a + 29d$

$\Rightarrow \quad 2a - 2d = 0$

$\Rightarrow \quad a = d \quad \ldots(4)$

$\therefore$ From (3), $\quad 2a + 5a = 14$

$\Rightarrow \quad 7a = 14$

$\Rightarrow \quad a = 2 \quad \ldots(5)$

$\therefore \quad d = 2 \quad$ [From (4)] …(6)

$\therefore$ Required first term and the 13th term are respectively **2** and **26**.

62. Let the first term of the first AP be a_1 and the common difference be d_1. Then $a_1 = 8$ and $d_1 = 20$. If S_n be sum of first n terms of this AP, then

$$S_n = \frac{n}{2}\left[2a_1 + (n-1)d_1\right]$$

$$= \frac{n}{2}\left[2 \times 8 + (n-1)20\right]$$

$$= n[8 + (n - 1)\,10]$$

$$= n(10n - 2) \qquad \ldots(1)$$

For the second AP, the first term a_2 and the common difference d_2, are given by $a_2 = -30$ and $d_2 = 8$.

Let S'_{2n} be the sum of the first $2n$ terms.

Then $S'_{2n} = \frac{2n}{2}\left[2 \times a_2 + (2n-1)8\right]$

$$= n[-2 \times 30 + (2n - 1)8]$$

$$= n[-60 + 16n - 8]$$

$$= n(16n - 68) \qquad \ldots(2)$$

It is given that

$$S_n = S'_{2n}$$

$\therefore$ From (1) and (2), we have

$$n(10n - 2) = n(16n - 68)$$

$\Rightarrow$ $16n - 10n = 68 - 2$

$\Rightarrow$ $6n = 66$

$\therefore$ $\boldsymbol{n = 11}$

which is the required value of n.

63. We need to form an AP of 3 digit numbers which leave remainder 5 on dividing by 7.

AP: 103, 110, 117, …, 992, 999

$a = 103$, $d = 7$, $l = a_n = 999$

$$a_n = a + (n - 1)d$$

$$999 = 103 + (n - 1)7$$

$$896 = (n - 1)7$$

$$n - 1 = 128$$

$$n = \mathbf{129}$$

Middle term $= \frac{n+1}{2} = \frac{130}{2} = 65$

$$a_{65} = a + 64d$$

$$= 103 + 64(7)$$

$$= 103 + 448$$

$$= \mathbf{551}$$

To find the sum of numbers on the former side of middle term

$a = 103$, $d = 7$, $n = 64$

$$S_n = \frac{n}{2}\,[2a + (n - 1)d]$$

$$= \frac{64}{2}\,[206 + (64 - 1)7]$$

$$= 32 \times 647$$

$$= \mathbf{20704}$$

Now we will find the sum of numbers on the latter side of middle term

$a = 999$, $d = -7$, $n = 64$

$$S_n = \frac{n}{2}\,[2a + (n - 1)d]$$

$$= 32\,[1998 + 63 \times (-7)]$$

$$= 32\,[1998 - 441]$$

$$= 32 \times 1557$$

$$= \mathbf{49824}$$

64. Let a = first term and d = common difference

Here, S_n = sum of 'n' terms

$\therefore$ $S_n = \frac{n}{2}\,[2a + (n - 1)d]$

$\Rightarrow$ $S_{n-1} = \frac{(n-1)}{2}\,[2a + (n - 2)d]$

$\Rightarrow$ $S_{n-2} = \frac{(n-2)}{2}\,[2a + (n - 3)d]$

Now, $\mathbf{S_n - 2S_{n-1} + S_{n-2}} = \frac{n}{2}\,[2a + (n - 1)d] - \frac{2(n-1)}{2}\,[2a + (n - 2)d] + \left(\frac{n-2}{2}\right)[2a + (n - 3)d]$

$$= an + \frac{(n)(n-1)d}{2} - 2(n - 1) \times a - (n - 1)(n - 2)d + (n - 2)a + \frac{(n-2)(n-3)d}{2}$$

$$= [an - 2(n - 1)a + (n - 2)a] + \left[\frac{(n)(n-1)d}{2} - (n-1)(n-2)d + \frac{(n-2)(n-3)d}{2}\right]$$

$$= [an - 2an + 2a + an - 2a] + \left[\frac{n^2 d}{2} - \frac{nd}{2} - n^2 d + 3nd - 2d + \frac{n^2 d}{2} - \frac{5nd}{2} + \frac{6d}{2}\right]$$

$$= 0 + [3d - 2d]$$

$$= \boldsymbol{d}$$

65. Let a = first term and d = common difference

Here, S_1 = Sum of 'n' terms

$\Rightarrow$ $S_1 = \frac{n}{2}\,[2a + (n - 1)d]$

S_2 = Sum of '$2n$' terms

$\Rightarrow$ $S_2 = \frac{2n}{2}\,\{2a + (2n - 1)d\}$

S_3 = Sum of '$3n$' terms

$\Rightarrow$ $S_3 = \frac{3n}{2}\,[2a + (3n - 1)d]$

Now, $\mathbf{3S_1 - 3S_2 + S_3}$

$$= 3\{S_1\} - 3\{S_2\} + \{S_3\}$$

$$= 3\left\{\frac{n}{2}[2a+(n-1)d]\right\} - 3\left\{\frac{2n}{2}[2a+(2n-1)d]\right\} + \frac{3n}{2}\,[2a + (3n - 1)d]$$

$$= \frac{3n}{2}(2a) + \frac{3n}{2}(n - 1)d - 3n(2a) - 3n(2n - 1)d + 3an + \frac{3n}{2}(3n - 1)d$$

$$= 3an + \frac{3n^2d}{2} - \frac{3nd}{2} - 6an - 6n^2d + 3nd$$

$$+ 3an + \frac{9n^2d}{2} - \frac{3nd}{2}$$

$$= (3an + 3an - 6an) + \left(\frac{3n^2d}{2} - 6n^2d + \frac{9n^2d}{2}\right)$$

$$+ \left(\frac{-3nd}{2} + 3nd - \frac{3nd}{2}\right)$$

$$= (0) + (0) + (0) = \mathbf{0}$$

Hence, $\mathbf{3S_1 - 3S_2 + S_3 = 0}$

66.

AP_1	AP_2	AP_3
$a = 1$	$a = 1$	$a = 1$
$d = 1$	$d = 2$	$d_3 = 3$
$S_n = S_1$	$S_n = S_2$	$S_n = S_3$

$$S_1 = \frac{n}{2}[2a + (n - 1)d]$$

$$= \frac{n}{2}[2 + (n - 1)1]$$

$$= \frac{n}{2}[2 + n - 1]$$

$$= n\frac{(n+1)}{2}$$

$$S_2 = \frac{n}{2}[2a + (n - 1)d]$$

$$= \frac{n}{2}[2 + (n - 1)2]$$

$$= \frac{n}{2}[2 + 2n - 2] = n^2$$

$$S_3 = \frac{n}{2}[2a + (n - 1)d]$$

$$= \frac{n}{2}[2 + (n - 1)3]$$

$$= \frac{n}{2}[2 + 3n - 3]$$

$$= \frac{n}{2}[3n - 1]$$

$$\text{LHS} = \mathbf{S_1 + S_3}$$

$$= \frac{n(n+1)}{2} + \frac{n}{2}(3n-1)$$

$$= \frac{n}{2}[n + 1 + 3n - 1] = \frac{n}{2} \times 4n = 2(n^2)$$

$$= \mathbf{2S_2} = \text{RHS}$$

Hence proved.

67. Odd numbers are 1, 3, 5, 7, …,

$$a = 1, d = 2, n = p$$

$$\therefore \quad S_p = \frac{p}{2}[2(1) + (p - 1) \times 2] = p(p) = p^2 \quad \ldots(1)$$

Even numbers are 2, 4, 6, 8, …

$$a = 2, d = 2 \text{ and } n = p$$

$$\therefore \quad S'_p = \frac{p}{2}[2(2) + (p - 1)2] = p[2 + p - 1]$$

$$= p[1 + p] = p^2 + p$$

$$= p^2\left[1 + \frac{1}{p}\right] \quad \ldots(2)$$

From (1) and (2), we have

$$\mathbf{S'_p = S_p\left(1 + \frac{1}{p}\right)}$$

$\Rightarrow$ [Sum of p even numbers]

$$= [\text{Sum of } p \text{ odd numbers}]\left(1 + \frac{1}{p}\right)$$

68. Let S_n be the first n terms of the AP, and let a_n be its nth term.

Now, it is given that

$$S_k = 3k^2 + 5k \quad \ldots(1)$$

and $$a_k = 164 \quad \ldots(2)$$

Also, $$a_k = S_k - S_k - 1$$

$$\Rightarrow \quad 164 = 3k^2 + 5k - 3(k - 1)^2 - 5(k - 1)$$

[From (1) and (2)]

$$= 3k^2 + 5k - 3k^2 + 6k - 3 - 5k + 5$$

$$= 6k + 2$$

$$\therefore \quad 6k = 164 - 2$$

$$= 162$$

$$\Rightarrow \quad k = \frac{162}{6} = \mathbf{27}$$

which the required value of k.

69. $$\frac{S_{1n}}{S_{2n}} = \frac{7n+1}{4n+27}$$

AP_1	AP_2
$a = a_1$	$a = a_2$
$d = d_1$	$d = d_2$

$$S_{1n} = \frac{n}{2}[2a_1 + (n - 1)d_1]$$

$$S_{2n} = \frac{n}{2}[2a_2 + (n - 1)d_2]$$

$$\frac{S_{1n}}{S_{2n}} = \frac{\frac{n}{2}[2a_1 + (n-1)d_1]}{\frac{n}{2}[2a_2 + (n-1)d_2]} = \frac{2a_1 + (n-1)d_1}{2a_2 + (n-1)d_2}$$

$$\frac{2a_1 + (n-1)d_1}{2a_2 + (n-1)d_2} = \frac{7n+1}{4n+27} \quad \ldots(1)$$

The ratio of m^{th} terms is

$$\frac{a_{m1}}{a_{m2}} = \frac{a_1 + (m-1)d_1}{a_2 + (m-1)d_2}$$

(*i*) To obtain the ratio of m^{th} terms, we used to put $n = 2m - 1$ in eq. (1)

$$\frac{2a_1 + (2m-1-1)d_1}{2a_2 + (2m-1-1)d_2} = \frac{7(2m-1)+1}{4(2m-1)+27}$$

$$\frac{2a_1 + (2m-2)d_1}{2a_2 + (2m-2)d_2} = \frac{14m-7+1}{8m-4+27}$$

$$\frac{2}{2}\left[\frac{a_1 + (m-1)d_1}{a_2 + (m-1)d_2}\right] = \frac{14m-6}{8m+23}$$

$$\frac{a_{m1}}{a_{m2}} = \mathbf{\frac{14m-6}{8m+23}}$$

(*ii*) To obtain the ratio of 9th terms, put $m = 9$

$$\frac{a_{9(1)}}{a_{9(2)}} = \frac{14(9)-6}{8(9)+23} = \frac{126-6}{72+23} = \frac{120}{95} = \frac{24}{19}$$

$\Rightarrow$ **24 : 19**

70. We have: $1^2 - 2^2 + 3^2 - 4^2 + 5^2 - 6^2 + \dots$ up to $2n$ terms

$\therefore$ $S = (1^2 - 2^2) + (3^2 - 4^2) + (5^2 - 6^2) + \dots$ up to n brackets

$= (1 - 4) + (9 - 16) + (25 - 36) + \dots$ up to n brackets

$= (-3) + (-7) + (-11) + \dots$ up to n terms

which is an AP.

Here, $a = -3$, $d = -7 - (-3) = -4$

Using $S_n = \frac{n}{2}[2a + (n-1)d]$, we get

$$S_n = \frac{n}{2}[2(-3) + (n-1)(-4)]$$
$$= \frac{n}{2}[-6 - 4n + 4]$$
$$= \frac{n}{2}[-2 - 4n]$$
$$= \frac{n}{2}[1 + 2n](-2)$$
$$= n[1 + 2n](-1)$$
$$= -n[2n + 1]$$

Hence, $[1^2 - 2^3 + 3^2 - 4^2 + 5^2 - 6^2 + \dots$ up to $2n$ terms$]$

$= \mathbf{-n[2n + 1]}$

EXERCISE 5C

For Basic and Standard Levels

1. Here, P = ₹ 2000, r = 7% p.a. (simple interest)

$\therefore$ Interest at the end of 1st year

$= ₹\ \frac{2000 \times 7 \times 1}{100} = ₹\ 140$ $\left[\text{Using S.I.} = \frac{P \times r \times t}{100}\right]$

Similarly,

S.I. at the end of 2nd year $= ₹\ \frac{2000 \times 7 \times 2}{100} = ₹\ 280$

S.I. at the end of 3rd year $= ₹\ \frac{2000 \times 7 \times 3}{100} = ₹\ 420$

and so on

$\because$ $280 - 140 = 420 - 280 = 140$

$\therefore$ 140, 280, 420 … form an AP with

$a = 140$ and $d = 140$

Using $a_n = a + (n-1)d$, we get

$a_{20} = 140 + (20 - 1) \times 140$

$= 140 + 19 \times 140 = 140 + 2660 = 2800$

Hence, the interest at the end of 20 years = ₹ **2800**

2. Original cost of the machine = ₹ 62,500

Let the annual depreciation = ₹ x

$\therefore$ Value of the machine at the

end of 1st year = ₹ $(62500 - x)$

end of 2nd year = ₹ $(62500 - 2x)$

end of 3rd year = ₹ $(62500 - 3x)$

end of 5th year = ₹ 57500

Obviously, the depreciated values for an AP with

First term $a_1 = (62500 - x)$

and Common difference $= d = (-x)$

$\because$ The depreciated value of the machine at the end of 5th year = ₹ 57500

$\therefore$ $a_5 = 57500$

Now, using $a_n = a + (n-1)d$, we have

$a_5 = [62500 - x] + (5 - 1)(-x)$

$= 57500$

$\Rightarrow$ $62500 - x - 4x = 57500$

$\Rightarrow$ $-5x = -62500 + 57500 = -5000$

$\Rightarrow$ $x = \frac{-5000}{-5} = ₹\ 1000$

Again, $a_{15} = a + (n-1)d$

$\therefore$ $a_{15} = (62500 - x) + (15 - 1)(-x)$

$= (62500 - 1000) + 14 \times (-1000)$

$= 61500 - 14000$

$= 47500$

Thus, the value of the machine after 15 years = ₹ **47500**

3. $a_3 = 600$

$a_7 = 700$

$a + 2d = 600$ ... (1)

$a + 6d = 700$ ... (2)

Subtract eq. (2) from eq. (1) we obtain

$a + 2d = 600$

$-a + (-6d) = -700$

$-4d = -100$

$d = 25$

Putting the value of d in eq. (1) we get

$a + 2 \times 25 = 600$

$a = 550$

(*i*) Production in first year = a = **550**

(*ii*) $a_{10} = a + (10 - 1)d$

$= 550 + (10 - 1)25$

$= 550 + 9 \times 25$

$= 550 + 225$

$= \mathbf{775}$

(*iii*) $S_7 = \frac{7}{2}[2a + (7 - 1)d]$

$= \frac{7}{2}[1100 + 6 \times 25]$

$= \frac{7}{2}$ [1100 + 150]

$= \frac{7}{2} \times 1250 =$ **4375**

4. The distances 60 m, 54 m, 48 m, ... climbed during 1st minute, 2nd minute, 3rd minute, ... respectively form an AP with

$a = 60$ m, $d = 54 - 60 = -6$

(*i*) Distance covered (climbed) during 5th minute = a_5

Now using $a_n = a + (n - 1)d$, we have

$a_5 = 60 + (5 - 1) \times (-6)$

$= 60 - 24 = 36$

Thus, the boy will climb **36 m** in 5th minute.

(*ii*) The total distance climbed in 5 minutes is given by S_5.

Now, using $S_n = \frac{n}{2}[2a + (n - 1)d]$, we get

$S_5 = \frac{5}{2}[2(60) + (5 - 1) \times (-6)]$

$= \frac{5}{2} \times [2 \times (60) + 4 \times (-6)]$

$= \frac{5}{2} \times 2[60 - 12]$

$= 5 \times 48 = 240$

Thus, total distance climbed in 5 minutes = **240 m**

5. The distances covered 20 m, 18 m, 16 m ... during 1st minute, 2nd minute, 3rd second ... respectively, form an AP with

a = first term = 20

d = common difference = 18 – 20 = –2

(*i*) The distances climbed during 10th minute is a_{10}.

Now using $a = a + (n - 1)d$, we have

$a_{10} = 20 + (10 - 1) \times (-2)$

$= 20 + [9 \times (-2)] = 20 + (-18)$

$= 20 - 18 = 2$

Thus, distance climbed during 10 minute = **2 m**

(*ii*) Total distance covered in 10 minutes will be given by S_{10}.

$\therefore$ Using $S_n = \frac{n}{2}[(2a) + (n - 1)d]$, we have

$S_{10} = \frac{10}{2}[2(20) + (10 - 1) \times (-2)]$

$= \frac{10}{2}[2 \times 20 + 9 \times (-2)]$

$= \frac{10}{2} \times 2[20 - 9]$

$= 10 \times 11 = 110$

Hence, distance covered in 10 minutes = **110 m**

6. Let the face value of the bonds bought in first year = ₹ x

The face value of bonds increase every year uniformly by a fixed amount of ₹ 500. Therefore, they form an AP, i.e. x, $(x + 500)$, $[x + 2(500)]$, ... are in AP with

a = First term = x

d = Common difference = 500

$\because$ Total value of the bonds after 10 years is ₹ 72500

$\therefore$ Using $S_n = \frac{n}{2}[2a + (n - 1)d]$, we have

$S_{10} = \frac{10}{2}[2 \times x + (10 - 1) \times 500] = 72500$

$\Rightarrow$ $2x + 9 \times 500 = 72500 \times \frac{2}{10} = 14500$

$\Rightarrow$ $2x + 4500 = 14500$

$\Rightarrow$ $2x = 14500 - 4500$

$\Rightarrow$ $2x = 10000$ or $x = \frac{10000}{2} = 5000$

Thus, the face value of the bond in the first year is ₹ **5000**.

7. Let the number of visitors on 1st Nov. be x

Number of visitors on Nov. 30 = 6150

$\because$ Number of visitor is increasing uniformly with a constant number 10 daily.

$\therefore$ No. of visitors on 1st day = x

No. of visitors on 2nd day = $x + 10$

No. of visitors on 3rd day = $x + 10 + 10 = x + 20$

$= x + (2 \times 10) = x + 20$

No. of visitors on 4th day = $x + (3 \times 10) = x + 30$

$\because$ x, $(x + 10)$, $(x + 20)$, $(x + 30)$, ... up to 30 terms form on AP with

First term = $a = x$

Common difference = $d = 10$

$\because$ $n = 30$ and $S_n = 6150$

$\therefore$ Using $S_n = \frac{n}{2}[2a + (n - 1)d]$, we get

$S_{30} = \frac{30}{2}[2x + (30 - 1) \times 10] = 6150$

$\Rightarrow$ $15[2x + 29 \times 10] = 6150$

$\Rightarrow$ $15[2x + 290] = 6150$

$\Rightarrow$ $2x + 290 = \frac{6150}{15} = 410$

$\Rightarrow$ $2x = 410 - 290 = 120$

$\Rightarrow$ $x = \frac{120}{2} = 60$

Thus, the number of visitors on 1st Nov. = **60**

8. Money collected on 1st day = ₹ 8100

Money collected on 2nd day = ₹ 8100 – ₹ 150 = ₹ 7950

Money collected on 4th day = ₹ 7800 – ₹ 150 = ₹ 7650

Money collected on nth day = ₹ 1650 – ₹ 150 = ₹ 1500

We note that money ₹ 8100, ₹ 7950, ₹ 7800 ... ₹ 1500 collected from the sale of tickets on 1st, 2nd, 3rd, ..., nth days respectively, form an AP with

a = ₹ 8100

d = (–₹ 150)

l = ₹ 1500

Note that, the sale of tickets on nth day is ₹ 1500 because, the show in profitable so long as the sale of tickets for the day fetches more than ₹ 1500.

$\therefore \quad a_n = l = 1500$

Now, using $a_n = l = a + (n-1)d$, we get

$$8100 + [(n-1)(-150)] = 1500$$

$$\Rightarrow \quad 8100 + [-150n + 150] = 1500$$

$$\Rightarrow \quad -150n = 1500 - 150 - 8100$$

$$= 1500 - 8250$$

$$\Rightarrow \quad -150n = -6750$$

$$\Rightarrow \quad n = \frac{-6750}{-150} = 45$$

Hence, the show ceases to be profitable on **45th day**.

9. Here, the daily saving form an AP, as the savings increase uniformly by a fixed amount of ₹ 1 each day.

We have

First term = a = 1, common difference = d = 1

and $\quad n = 144$ (days)

Now, the total savings in 144 days will be S_{144}

Using $\quad S_n = \frac{n}{2}\{2a + (n-1)d\}$

$$\Rightarrow \quad S_{144} = \frac{144}{2}\{2 \times 1 + (144-1) \times 1\}$$

$$= 72\{2 + 143\} = 72 \times 145 = 10440$$

Hence, the total savings in 144 days

= ₹ 10440

10. $\because$ The monthly increment of ₹ 100 is fixed.

$\therefore$ Here annual salaries form an AP with

First term = a = ₹ 8000 × 12 = ₹ 96000

Common difference = d = ₹ 100 × 12 = ₹ 1200

Since, total earnings from salary in 10 years is given by S_{10}.

$\therefore \quad$ Using $S_n = \frac{n}{2}[2a + (n-1)d]$, we have

$$S_{10} = \frac{10}{2}[2 \times 96000 + (10-1) \times 1200]$$

$$= 5[2 \times 96000 + 9 \times 1200]$$

$$= 5[192000 + 10800]$$

$$= 5 \times 202800 = 1014000$$

Thus, the woman will earn ₹ **1014000** in a period of 10 years.

11. The savings ₹ 200, ₹ 250, ₹ 300, ₹ 350, … form an AP with

a = first term = 200,

d = common difference = 50

Thus, total savings in 12 months of the year 2019 is given by S_{12}.

Using $\quad S_n = \frac{n}{2}[2a + (n-1)d]$, we have

$$S_{12} = \frac{12}{2}[2 \times 200 + (12-1) \times 50]$$

$$= 6[400 + 11 \times 50]$$

$$= 6 \times [400 + 550] = 6 \times 950$$

$$= 5700$$

Thus, the savings in the year 2019 = ₹ **5700.**

12 We see that the numbers 32, 36, 40, 44, … form an AP with the first term, a = 32 and the common difference, d = 36 − 32 = 4.

If n be the number of terms of this AP and S_n, the sum of the first n terms of the AP, then

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$= \frac{n}{2}\left[2 \times 32 + (n-1)4\right]$$

$$= \frac{n}{2}\left[64 + 4n - 4\right]$$

$$= \frac{n}{2}(4n + 60)$$

$$= n(2n + 30) \quad \ldots(1)$$

If the sum is 2000, then $S_n = 2000$

$\therefore$ From (1),

$$2n^2 + 30n - 2000 = 0$$

$$\Rightarrow \quad n^2 + 15n - 1000 = 0$$

Solving this quadratic equation in n, we get

$$n = \frac{-15 \pm \sqrt{15^2 + 4 \times 1000}}{2}$$

$$= \frac{-15 \pm \sqrt{225 \times 4000}}{2}$$

$$= \frac{-15 \pm \sqrt{4225}}{2}$$

$$= \frac{-15 \pm 65}{2}$$

$$= \frac{50}{2}, -\frac{80}{2}$$

$$= 25, -40$$

Since n is a natural number, we reject $n = -40$

$\therefore \quad n = 25$

Hence, the required number of months in which she saves ₹2000 in **25 months**.

13. Let the value of the first prize be ₹x. Then the values of 3 successive prizes are ₹$(x - 20)$, ₹$(x - 40)$ and ₹$(x - 60)$.

Now, the numbers x, $x - 20$, $x - 40$, form an AP, with the first term, $a = x$ and the common difference, d $= x - 20 - x = -20$.

If S_n be the sum of the first n terms of this AP, then

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$= \frac{n}{2}\left[2x - (n-1)20\right]$$

$$= n(x - 10n + 10) \quad \ldots(1)$$

When $n = 4$, then S_n is given to be 280.

$$280 = 4(x - 40 + 10)$$

$\Rightarrow$ $x - 30 = 70$

$\Rightarrow$ $x = 100$

$\therefore$ Required values of 4 prizes will be **₹100, ₹80, ₹60** and **₹40**.

14. Resham's savings in successive months will be ₹450, ₹470, ₹490, ₹510, … for 12 months.

Now, the numbers 450, 470, 490, … form an AP with first term, $a = 450$ and the common difference, $d = 470 - 450 = 20$.

If S_n be the sum of the first n terms of this AP, then

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$= \frac{n}{2}\left[2 \times 450 + (n-1)20\right]$$

$$= n(450 + 10n - 10)$$

$$= n(10n + 440) \quad \ldots(1)$$

If $n = 12$, then from (1), we have

$$S_{12} = 12(10 \times 12 + 440)$$

$$= 12(120 + 440)$$

$$= 12 \times 560$$

$$= 6720$$

$\therefore$ Required total amount of Resham's savings for 12 months is ₹6720. Since this amount is ₹6500, hence, she will be able to send her daughter to the school next year.

15. The child's daily savings of five-rupee coins will be 1 coin, 2 coins, 3 coins, 4 coins,… Now, the numbers 1, 2, 3, 4… form an AP with the first term, $a = 1$ and the common difference, $d = 2 - 1 = 1$. If S_n be the sum of the first n terms of this AP, then

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$$= \frac{n}{2}\left[2 + (n-1)\right]$$

$$= \frac{n(n+1)}{2} \quad \ldots(1)$$

When the piggy bank holds a total of 190 coins, then

$$S_n = 190$$

$\therefore$ From (1), we have

$$\frac{n(n+1)}{2} = 190$$

$\Rightarrow$ $n^2 + n - 380 = 0$

This is a quadratic solution.

$\therefore$ Its solutions are

$$n = \frac{-1 \pm \sqrt{1^2 + 4 \times 380}}{2}$$

$$= -1 \pm \sqrt{1 + 1520}$$

$$= \frac{-1 \pm \sqrt{1521}}{2}$$

$$= \frac{-1 \pm 39}{2}$$

$$= \frac{38}{2}, -\frac{40}{2}$$

$$= 19, -20$$

Since n is a natural number, we reject $n = -20$.

$\therefore$ $n = 19$

$\therefore$ Required number of days = **19 days**

Also, the total amount of her savings = ₹190 × 5 = **₹950**

16. $a = 8,\ d = \frac{4}{12} = \frac{1}{3},\ S_n = 168$

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

$\Rightarrow$ $168 = \frac{n}{2}\left[16 + (n-1)\frac{1}{3}\right]$

$\Rightarrow$ $168 = 8n + \frac{n(n-1)}{6}$

$\Rightarrow$ $1008 = 48n + n^2 - n$

$\Rightarrow$ $n^2 + 47n - 1008 = 0$

$\Rightarrow$ $n = \frac{-47 \pm \sqrt{2209 + 4032}}{2}$

$$= \frac{-47 \pm \sqrt{6241}}{2}$$

$$= \frac{-47 \pm 79}{2}$$

$\Rightarrow$ $n = 16, -63$

Since number of students cannot be negative, hence we will neglect -63.

$$n = 16$$

$\therefore$ $a_{16} = a + (16-1)d$

$$= 8 + 15 \times \frac{1}{3} = 13$$

Age of the eldest participant = **13 years**

17. Number of sides of the polygon = 31

Let the smallest side $= x$

$\therefore$ The largest side $= 16 \times$ (smallest side) $= 16x$

$\because$ The lengths of sides of the polygon starting from the smallest are in AP.

$\therefore$ The smallest side = First term of the AP $= x$

The largest side = 31st side of AP

$\Rightarrow$ $a_{31} = 16x$

Perimeter of the polygon = Sum of 31 terms of AP $= 527$

$\Rightarrow$ $S_{31} = 527$

Now using $S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore$ $S_{31} = \frac{31}{2}[2 \times x + (31-1)d] = 527$

$\Rightarrow$ $\frac{31}{2}[2x + 30d] = 527$

$\Rightarrow \quad 31[x + 15d] = 527$

$\Rightarrow \quad x + 15d = \dfrac{527}{31} = 17 \quad \ldots(1)$

Also $\quad a_n = a + (n - 1)d$

$\Rightarrow \quad a_{31} = x + (31 - 1)d = 16x$

$x + 30d = 16x$

$\Rightarrow \quad x + 30d - 16x = 0$

$\Rightarrow \quad -15x + 30d = 0$

$\Rightarrow \quad -x + 2d = 0 \quad \ldots(2)$

Solving (1) and (2), we get

$d = 1$ and $a = 2 \Rightarrow x = 2$

$\therefore$ **Smallest side = 2 cm**

Common difference = 1 cm

18. Numbers of trees that each section of each class will plant are 2, 4, 6, 8, 10,…24 for class I to XII. Now, these numbers form an AP, with the first term, $a = 2$ and the common difference, $d = 4 - 2 = 2$. If n be the number of terms of this AP and if S_n be the first n terms of this AP, then

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$= \frac{n}{2}[2 \times 2 + (n-1)2]$$

$$= n(2 + n - 1)$$

$$= n(n + 1) \quad \ldots(1)$$

When $n = 12$ for 12 classes, then from (1)

$$S_{12} = 12 \times 13 = 156$$

$\therefore$ Required number of trees planted by the students for 2 sections of each class = 156 × 2 = **312**

Value: Concern for the pollution of the environment and its remedial measures.

For Standard Level

19. Let the 2nd cyclist overtakes the first cyclist after 't' hours. Then, the two cyclists travel the same distance in 't' hours.

$\therefore$ Distance travelled by the 1st cyclist in 't' hours $= 11 \times t$ km

Distance travelled by 1st cyclist in 't' hours = 11 t km

But the distance covered by 2nd cyclist in 't' hours

= Sum of t terms of an AP with first term,

$a = 10$ and common difference $(d) = \dfrac{1}{3}$

$$= \frac{t}{2}\left[2 \times 10 + (t-1)\frac{1}{3}\right]$$

$$\left[\text{using, } S_n = \frac{n}{2}[2a + (n-2)d\right]$$

$$= \frac{t}{2}\left[20 - \frac{1}{3} + \frac{1}{3}t\right] = \frac{t}{2}\left[\frac{59}{3} + \frac{1}{3}t\right]$$

$\therefore \quad 11t = \dfrac{t}{2}\left[\dfrac{59}{3} + \dfrac{1}{3}t\right]$

$\Rightarrow \quad 11t = \dfrac{59t}{6} + \dfrac{1}{6}t^2$

$\Rightarrow \quad \dfrac{59t}{6} - 11t + \dfrac{1}{6}t^2 = 0$

$\Rightarrow \quad \dfrac{1}{6}t^2 - \dfrac{7}{6}t = 0$

$\Rightarrow \quad t\left[\dfrac{t}{6} - \dfrac{7}{6}\right] = 0$

$\therefore$ Either $t = 0$ [Not required]

or $\quad \dfrac{t}{6} - \dfrac{7}{6} = 0$

$\Rightarrow \quad \dfrac{t}{6} = \dfrac{7}{6}$

$\Rightarrow \quad t = \dfrac{7}{6} \times 6 = 7$

Thus, second cyclist will overtake the first one after **7 hours**.

20. When the police starts running, the thief is 100 m apart.

Speed for 1st minute is 60 m/minute and increases by 5 m/minute.

AP: 10, 15, …

a = 10 m/minute (distance reduced in 1st min)

$d = 5$

$S_n = 100$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$\Rightarrow \quad 100 = \dfrac{n}{2}[20 + (n - 1)5]$

$\Rightarrow \quad 200 = n[20 + 5n - 5]$

$\Rightarrow \quad 200 = n[15 + 5n]$

$\Rightarrow \quad 200 = 15n + 5n^2$

$\Rightarrow \quad n^2 + 3n - 40 = 0$

$\Rightarrow \quad n(n - 5) + 8(n - 5) = 0$

$\Rightarrow \quad (n + 8)(n - 5) = 0$

$\Rightarrow \quad n = -8 \quad n = 5$

Since time cannot be negative, hence we will reject – 8.

Policeman will catch the thief in **5 minutes**.

CHECK YOUR UNDERSTANDING

MULTIPLE-CHOICE QUESTIONS

For Basic and Standard Levels

1. (*c*) $\sqrt{162}$

$\sqrt{18} = \sqrt{3^2 \times 2} = 3\sqrt{2}$

$\sqrt{50} = \sqrt{5^2 \times 2} = 5\sqrt{2}$

$\Rightarrow \quad d = 5\sqrt{2} - 3\sqrt{2} = 2\sqrt{2}$

Now use $a_4 = a + (n - 1)d$ where $n = 4$

2. (*b*) **5.5**

Use: $\quad a_n = a + (n - 1)d$

where $a = -1, d = -1.5 - (-1) = -0.5$

and $n = 10$

3. (*d*) **10 – 3*n***

Using $a = 7$ $\quad d = 4 - 7 = -3$

i.e. $a + (n-1)d \Rightarrow 7 + (n-1)(-3)$

$\Rightarrow 7 + 3 - 3n$ or $(10 - 3n)$

4. (*c*) **20**

$$a_{11} = (-5) + (11-1)\left(\frac{5}{2}\right) \quad \left(\because d = -\frac{5}{2} - (-5) = \frac{5}{2}\right)$$

$$= -5 + 25 = 20$$

5. (*c*) **1331**

$a_n = (-1)^{n-1} \times n^3$

$\Rightarrow a_{11} = (-1)^{10} \times 11^3 = 1 \times 11 \times 11 \times 11 = 1331$

6. (*b*) **74**

$d = 4 - (-3) = 4 + 3 = 7$

$a_{12} = -3 + 11(7) = -3 + 77 = 74$

7. (*c*) **78**

$a_{18} = a + 17d = -7 + 17 \times 5 = -7 + 85 = 78$

8. (*b*) **8**

$a_n = a + (n-1)d$

$$\Rightarrow a + (31-1)\left(-\frac{1}{4}\right) = \frac{1}{2}$$

$$\Rightarrow a + 30\left(-\frac{1}{4}\right) = \frac{1}{2}$$

$$\Rightarrow a - \frac{15}{2} = \frac{1}{2} \text{ or } a = \frac{1}{2} + \frac{15}{2} = 8$$

9. (*b*) **–2.5**

$a_n = a + (n-1)d$

$= -2.5 + 0 = -2.5 \quad [\because d = 0 \Rightarrow (n-1)d = 0]$

10. (*b*) **83**

From end,

$a_7 = 107 + (7-1) \times (-4) = 107 - 24 = 83$

[Note that first term equal to last term and *d* is taken as negative]

11. (*d*) **20**

$a_{15} = a + 14d$ and $a_{11} = a + 10d$

$\Rightarrow a_{15} = a + 14 \times 5$ $\quad a_{11} = a + 10 \times 5$

$= a + 70$ $\quad = a + 50$

$\therefore a_{15} - a_{11} = (a + 70) - (a + 50) = 20$

12. (*b*) **–4**

$a_{20} - a_{12} = a + 19d - (a + 11d) = -32$

$19d - 11d = -32$

$8d = -32$

$$d = \frac{-32}{8} = -4$$

13. (*c*) **an AP with *d* = 4**

$\therefore \left.\begin{array}{l} -1 - (-5) = -1 + 5 = 4 \\ 3 - (-1) = 3 + 1 = 4 \end{array}\right\} d = 4$

14. (*b*) **0.3, 0.55, 0.80, 1.05**

AP [0.30 + 0], [0.30 + 0.25], [0.30 + 2(0.25)], (0.30 + 3(0.25)] ... = [0.30], [0.55], [0.80], [1.05] …

15. (*b*) **28**

$n = 29$

a_{29} = First term + $(n-1)d$

= First term + $(29-1)d$

= First term + $28d$

$\Rightarrow d = 28$

16. (*d*) **37**

$a_n = 111$

$\Rightarrow 3 + (n-1)3 = 111$

$$\Rightarrow (n-1) = \frac{111-3}{3} = 36$$

$\therefore n = 36 + 1 = 37$

17. (*a*) ***k* = 40**

*k*th term = $a_k = 1000 = x$

$a + (k-1)d = 1000$

$25 + (k-1)25 = 1000$

$$\Rightarrow k - 1 = \frac{1000-25}{25} = 39$$

$\therefore k = 39 + 1 = 40$

18. (*d*) **16**

$$S_n = \frac{n}{2}(a + l) = 400$$

$$\Rightarrow \frac{n}{2}(5 + 45) = 400$$

$$\Rightarrow \frac{n}{2} = \frac{400}{50} = 8$$

$n = 16$

19. (*b*) **3**

$3(a_1) = a_4 \Rightarrow 3a = a + 3d \Rightarrow 2a = 3d$ …(1)

$a_7 = 2(a_3) + 1 \Rightarrow a + 6d = 2(a + 2d) + 1$

$\Rightarrow -a + 2d = 1$ …(2)

Solving (1) and (2), we get $a = 3$

20. (*c*) **–1**

$$\frac{1-p}{p} - \frac{1}{p} = \frac{1-2p}{p} - \frac{1-p}{p}$$

$$\Rightarrow \frac{1-p-1}{p} = \frac{1-2p-1+p}{p}$$

$\Rightarrow -1 = -1 \Rightarrow$ Common difference = –1

21. (*c*) **30**

Two digits numbers divisible by 3 are

12, 15, 18, …, 99

They are in AP with $a = 12$, $d = 3$ and $l = 99$

$a_n = a + (n-1)d = 99$

$\Rightarrow 12 + (n-1)3 = 99$

$$\Rightarrow n - 1 = \frac{99-12}{3} = 29$$

$\Rightarrow n = 29 + 1 = 30$

22. (*b*) **$S_n - S_{n-1}$**

Sum of *n* terms = S_n

Sum of $(n-1)$ term = S_{n-1}

$\because$ nth term = [Sum of n terms] − [Sum of $(n-1)$ terms]

$= [S_n] - [S_{n-1}]$

23. (*b*) $\mathbf{\frac{5}{2}, \frac{9}{2}, \frac{13}{2}, \frac{17}{2}}$

$$a_n = \frac{4n+1}{2}$$

$$\Rightarrow \quad a_1 = \frac{4+1}{2} = \frac{5}{2},\ a_2 = \frac{4(2)+1}{2} = \frac{9}{2}$$

Similarly, $a_3 = \frac{13}{2}$ and $a_4 = \frac{17}{2}$

24. (*c*) **6**

$a_n = 6n + 1$ and $d = a_2 - a_1$

$\therefore$ $a_1 = 6 + 2 = 8$

$a_2 = 6(2) + 2 = 14$

$d = 14 - 8 = +6$

25. (*b*) **3**

$(2k + 1) - (2k - 1) = (2k - 1) - (k)$ = common diff.

$\Rightarrow$ $2k + 1 - 2k + 1 = 2k - 1 - k$

$\Rightarrow$ $2 = k - 1$

$\Rightarrow$ $k = 3$

26. (*b*) **−2, −2, −2, −2**

$a_1 = a$

$a_2 = a + d$

$a_3 = a + 2d$

$a_4 = a + 3d$

$\Rightarrow$ $a_1 = -2$

$a_2 = a + d = -2 + 0 = -2$

$a_3 = -2 + 2d = -2 + 0 = -2$

and $a + 3d = -2 + 0 = -2$

27. (*c*) **Gauss**

28. (*c*) **0**

$5(a_5) = 10(a_{10})$

$\Rightarrow$ $a_5 = 2(a_{10})$

$\therefore$ $a + 4d = 2(a + 9d)$

$\Rightarrow$ $a + 14d = 0$ …(1)

$a_{15} = a + (15 - 1)d$

$\Rightarrow$ $a + 14d = a_{15}$

$\Rightarrow$ $a_{15} = 0$ [From (1)]

29. (*c*) **25th term**

$$a = 19,\ d = \left(18\frac{1}{5}\right) - 19 = \frac{-4}{5}$$

Let a_n be the first negative term

$\therefore$ $a_n = 0$

$\Rightarrow$ $[a + (n - 1)d] < 0$

$$\Rightarrow \quad \left[19+(n-1)\left(-\frac{4}{5}\right)\right] < 0$$

$$\Rightarrow \quad \left[19+\frac{4}{5}-\frac{4}{5}n\right] < 0$$

$$\Rightarrow \quad \frac{99}{5} < \frac{4}{5}n \text{ or } \frac{4}{5}n > \frac{99}{5}$$

$$\Rightarrow \quad n > \frac{99}{5}\times\frac{5}{4}$$

$$\text{or} \quad n > \frac{99}{4} \text{ or } n \geq 24\frac{3}{4}$$

$\because$ Natural number next to $24\frac{3}{4}$ is 25.

30. (*b*) **2, 7, 12, ...**

$a_7 = 32$

$\Rightarrow$ $a + 6d = 32$

$a_{13} = 62$

$\Rightarrow$ $d = 5$ and $a = 2$

$\Rightarrow$ $a + 12d = 62$

$\therefore$ AP is 2, (2 + 5), (2 + 10), (2 + 15) … i.e. 2, 7, 12, …

31. (*c*) **25th term**

AP 3, 10, 17, …

$\Rightarrow$ $a = 3,\ d = 10 - 3 = 7$

Let $a_n = 84 + a_{13}$

$\therefore$ $3 + (n - 1)7 = 84 + 3 + (13 - 1) \times 7$

$\Rightarrow$ $-4 + 7n = 84 + 3 + 84$

$\Rightarrow$ $7n = 84 + 3 + 84 + 4 = 175$

$$\Rightarrow \quad n = \frac{175}{7} = 25$$

32. (*b*) **55**

$a = 3,\ d = 7 - 3 = 4$

$$\therefore \quad S_5 = \frac{5}{2}[2(3) + (5-1)4] = \frac{5}{2}[22] = 55$$

33. (*c*) **676**

$a = 1,\ d = 1$

$$\Rightarrow \quad S_{26} = \frac{26}{2}[2(1) + (26-1)2] = 13(2 + 50)$$

$= 13 \times 52 = 676$

34. (*b*) **4**

'a_n' from the end is determined by

$a_n = l + (n - 1)(-d)$

where l = last term and d = common diff.

$\therefore$ a_8 from the end $= 119 + (8 - 1)(-d) = 91$

$\Rightarrow$ $119 - 7d = 91$

or $-7d = 91 - 119$

$$\Rightarrow \quad d = \frac{-28}{-7} = 4$$

35. (*d*) **6**

$\because$ $S_n = 3n^2 + 4n$

$\therefore$ $S_1 = 3(1) + 4(1) = 7 = a$

$S_2 = 3(4) + 4(2) = 20 = a_1 + a_2$

$\Rightarrow$ $a_2 = 20 - a = 20 - 7 = 13$

Now $d = a_2 - a_1 = 13 - 7 = 6$

36. (*b*) $p = 65$

$a = 3$ and $d = 15 - 3 = 12$

$a_{50} = a + 49d$ and $a_p = a + (p - 1)d$

$a_p - a_5 = 180$

$\Rightarrow [a + (p - 1) \times 12] - [a + 49 \times 12] = 180$

$\Rightarrow p = 65$

37. (*b*) $\frac{1}{4}$

$a_{19} = a_{12} + \frac{7}{4}$

$\Rightarrow a + 18d = a + 11d + \frac{7}{4}$

$\therefore (a + 18d) - \left(a + 11d + \frac{7}{4}\right) = 0$

$\Rightarrow 18d - 11d = \frac{7}{4}$

$\Rightarrow 7d = \frac{7}{4}$ or $d = \frac{1}{4}$

38. (*d*) $n = 20$

$a_1 = 21 \Rightarrow$ First term $= 21$

$a_2 = 42$

$\Rightarrow a + d = 42$ or $d = 42 - 21 = 21$

Now, $a_n = 21 + (n - 1) \times 21 = 420$

or $(n - 1) = \frac{420 - 21}{21}$

$\Rightarrow n - 1 = \frac{399}{21} = 19$

$\therefore n = 19 + 1 = 20$

39. (*d*) $5n - 1$

$\because S_n = \frac{5n^2}{2} + \frac{3n}{2}$

$\therefore S_1 = \frac{5(1)^2}{2} + \frac{3(1)}{2} = \frac{8}{2} = 4$

$\Rightarrow$ First term $a = 4$

$S_2 = \frac{5(2)^2}{2} + \frac{3(2)}{2} = 10 + 3 = 13$

$\because S_2 = a_1 + a_2$

$\Rightarrow a_2 = 13 - 4 = 9$

$\therefore d = 9 - 4 = 5$

Now, $a_n = a + (n - 1)d$

$\Rightarrow a_n = 4 + (n - 1)5 = 4 + 5n - 5$ or $d = 5n - 1$

40. (*a*) -3

$a_4 + a_8 = 24$

$\Rightarrow a + 3d + a + 7d = 24$

$a_6 + a_{10} = 44$

$\Rightarrow a + 5d + a + 9d = 44$

$\Rightarrow 2a + 10d = 24 \Rightarrow a + 5d = 12$ …(1)

$2a + 14d = 44 \Rightarrow a + 7d = 22$ …(2)

Subtracting (1) from (2), we get $d = 5$

From (1), $a + 25 = 12$

$\Rightarrow a = -13$

Now, $a_3 = a + 2d = -13 + 2(5)$

$\Rightarrow a_3 = -3$

For Standard Level

41. (*d*) **5**

AP with $a = 8$

$a_{30} = 8 + 29d$

AP with $a = 3$

$a_{30} = 3 + 29d$

$\because$ 'd' for these AP's is the same

$\therefore [8 + 29d] - [3 + 29d] = 8 - 3 = 5$

42. (*c*) $\frac{b - a}{n - 1}$

$\because a_n = b$

$\therefore a + (n - 1)d = b$

$\Rightarrow (n - 1)d = b - a$

$\Rightarrow d = \frac{b - a}{n - 1}$

43. (*b*) **735**

$a_2 = a + d = 8$

$a_4 = a + 3d = 14$

$\Rightarrow d = 3$ and $a = 5$

Now, using $S_n = \frac{n}{2}[2a + (n - 1)d]$

$S_{21} = \frac{21}{2}[2(5) + (21 - 1)3]$

$= \frac{21}{2}[10 + 60] = \frac{21}{2} \times 70 = 735$

44. (*d*) **2, 6, 10, 14**

Four numbers in AP are $(a - 3 - d)$, $(a - d)$, $(a + d)$ and $(a + 3d)$

$\therefore (a - 3d) + (a - d) + (a + d) + (a + 3d) = 32$

$\Rightarrow a = 8$

Also $(a - 3d) = \frac{1}{7}(a + 3d)$

$\Rightarrow 7a - 21d = a + 3d$

$\Rightarrow d = 2$

$\therefore$ AP $[8 - 3(2)], [8 - 2], [8 + 2], [8 + 3(2)]$

$\Rightarrow 2, 6, 10, 14$

45. (*b*) $k = 0, 2$

$a_1 = (4k + 8)$,

$a_2 = 2k^2 + 3k + 6$

and $a_3 = 3k^2 + 4k + 4$

For an AP $a_2 - a_1 = a_3 - a_2$

$\Rightarrow [(2k^2 + 3k + 6) - (4k + 8)]$

$= [(3k^2 + 4k + 4) - (2k^2 + 3k + 6)]$

$\Rightarrow k = 0$ or $k = 2$

46. (*d*) **14**

$$d = \frac{5}{4},$$

$$a_9 = a + 8d = -6$$

$$\Rightarrow \quad a + 8\left(\frac{5}{4}\right) = -6$$

$$\Rightarrow \quad a = -16$$

Now, $a_{25} = a + 24d = (-16) + 24\left(\frac{5}{4}\right) = -16 + 30 = 14$

47. (*b*) **10**

$$a_1 = a = 4$$

$$a + d = 7 \Rightarrow d = 7 - 4 = 3$$

$$a_n = 31 \Rightarrow a + (n-1)d = 31$$

$$\Rightarrow 4 + (n-1)3 = 31$$

$$\Rightarrow n = 10$$

48. (*b*) **3 : 1**

$$\frac{a_{18}}{a_{11}} = \frac{a+17d}{a+10d} = \frac{3}{2}$$

$$\Rightarrow \quad a = 4d$$

$$\frac{a_{21}}{a_5} = \frac{a+20d}{a+4d} = \frac{4d+20d}{4d+4d} = \frac{24d}{8d} = \frac{3}{1}$$

$$\Rightarrow \quad a_{21} : a_5 = 3 : 1$$

49. (*c*) $\mathbf{\dfrac{\sqrt{3}\, n(n+1)}{2}}$

$$a = \sqrt{3},\ d = \sqrt{12} - \sqrt{3} = 2\sqrt{3} - \sqrt{3} = \sqrt{3}$$

$$S_n = \frac{n}{2}[2(\sqrt{3}) + (n-1)\sqrt{3}]$$

$$= n\sqrt{3}\left[\frac{2}{2} + \frac{n-1}{2}\right] = \sqrt{3}.n\left(1 + \frac{n-1}{2}\right)$$

$$= \sqrt{3}.n\left(\frac{2+n-1}{2}\right) = \frac{\sqrt{3}\, n\,(n+1)}{2}$$

50. (*b*) **9**

Let three consecutive terms of an AP be $(a - d)$, a, $(a + d)$

$$\therefore \quad (a-d) + a + (a+d) = 21$$

$$\Rightarrow \quad a = 7$$

$$(a-d)(a+d) = 45$$

$$\Rightarrow \quad a^2 - d^2 = 45$$

$$a^2 - d^2 = 45 \Rightarrow d^2 = 4$$

$$\Rightarrow d = 2 \qquad \text{(Reject } d = -2\text{)}$$

Now, $a_3 = a + d = 2 + 7 = 9$

51. (*b*) $\mathbf{n(n+2)}$

$a_n = 2n + 1$

$a_1 = 2(1) + 1 = 3$ = First term

$a_2 = 2(2) + 1 = 5$ = Second term

$d = a_2 - a_1 = 5 - 3 = 2$

Now $\quad S_n = \frac{n}{2}[2(3) + (n-1) \times 2]$

$$= n[3 + n - 1] = n(n+2)$$

52. (*d*) **2475**

Two digit odd numbers are

11, 13, 15, …, 99, and they are in AP with

$$a = 11,\ d = 2 \text{ and } l = 99$$

$$a_n = a + (n-1)d = 99$$

$$\Rightarrow \quad n = 45$$

Now, $S_{45} = \frac{45}{2}[11 + 99] = 45 \times \frac{110}{2} = 45 \times 55 = 2475$

53. (*b*) **1665**

All positive 2-digit numbers divisible by 3 are

12, 15, 18, 21, …, 99

such that $a = 12$, $d = 3$ and $l = 99$

$$a_n = a + (n-1)d = 12 + (n-1)3 = 99$$

$$\Rightarrow \quad n = 30$$

Now, $\quad S_{30} = \frac{30}{2}[12 + 99] = 15 \times 11 = 1665$

54. (*b*) **3774**

$$a + d = 2$$

$$a + 3d = 8$$

$$\Rightarrow \quad d = 3 \text{ and } a = -1$$

Using $\quad S_n = \frac{n}{2}[2a + (n-1)d]$, $S_{51} = 3774$

55. (*c*) $\mathbf{\dfrac{5n-1}{2}}$

$$a_1 = \left(3 - \frac{1}{n}\right) = a$$

$$a_2 = \left(3 - \frac{2}{n}\right)$$

$$\Rightarrow \quad d = a_2 - a_1 = \left(3 - \frac{2}{n}\right) - \left(3 - \frac{1}{n}\right) = -\frac{1}{n}$$

Now, using $\quad S_n = \frac{n}{2}[2a + (n-1)d]$,

$$S_n = \frac{n}{2}\left[2\left(3 - \frac{1}{n}\right) + (n-1) \times \left(-\frac{1}{n}\right)\right]$$

$$= \frac{n}{2}\left[6 - \frac{2}{n} - 1 + \frac{1}{n}\right] = \frac{n}{2}\left[5 - \frac{1}{n}\right]$$

$$= \frac{5n}{2} - \frac{1}{2} = \frac{5n-1}{2}$$

56. (*a*) **–8930**

$$a = -5,\ d = -8 - (-5) = -3,\ l = (-230)$$

$$\therefore \quad a_n = a + (n-1)d = (-5) + (n-1)(-3) = -230$$

$$\Rightarrow n - 1 = \frac{-230+5}{-3} = 75$$

$\Rightarrow \quad n = 75 + 1 = 76$

Now, Using $S_n = \frac{n}{2}(a+l)$

$$S_{76} = \frac{76}{2}[(-5) + (-230)]$$

$$= 38[-235] = -8930$$

57. (*c*) **2**

$\because \quad S_n = 3n^2 - n$

$\therefore \quad S_1 = 3(1)^2 - (1) = 3 - 1 = 2$

But $S_1 = a$ = First term

$\therefore$ First term = 2

58. (*d*) $\boldsymbol{n^2}$

$$S_7 = 49$$

$\Rightarrow \quad \frac{7}{2}[2a + 6d] = 49$

$\Rightarrow \quad a + 3d = 7 \quad \ldots(1)$

$$S_{17} = 289$$

$\Rightarrow \quad \frac{17}{2}[2a+16d] = 289$

$\Rightarrow \quad a + 8d = 17 \quad \ldots(2)$

Solving (1) and (2),

$a = 1$ and $d = 2$

$\therefore \quad S_n = \frac{n}{2}[2(1) + (n-1)2]$

$= n(1 + n - 1) = n \times n = n^2$

59. (*b*) **35**

Here, $a = 5$, $d = 7 - 5 = 2$ and $S_n = 320$

$S_n = 320 \Rightarrow \frac{n}{2}[2(5) + (n-1)2] = 320$

$\Rightarrow n^2 + 4n - 320 = 0$

Solving it $n = 16$ or -20 $\quad [n = -20$ rejected$]$

Now $\quad a_{16} = a + (16 - 1)d$

$= 5 + 15 \times 2 = 35$

$\Rightarrow \quad x = 35$

60. (*d*) $\boldsymbol{(a + k) + (n - 1)d}$

Now first term = $a' = (a + k)$, Common diff. = d

$\therefore \quad a'_n = a' + (n-1)d$

$= (a + k) + (n - 1)d$

61. (*a*) **7, 10, 13, ...**

$$S_9 = 171$$

$\Rightarrow \quad \frac{9}{2}[2a + 8d] = 171 \quad \ldots(1)$

$$S_{24} = 996$$

$\Rightarrow \quad \frac{24}{2}[2a + 23d] = 996 \quad \ldots(2)$

Solving (1) and (2), we get $a = 7$ and $d = 3$

Now, the required AP is

$a, (a + d), (a + 2d), \ldots$

$7, (7 + 3), (7 + 2 \times 3), \ldots$

$7, 10, 13, \ldots$

62. (*c*) **2, 4, 6, 8**

$a_n = 3 + \frac{2}{3}n$

$a_1 = 3 + \frac{2}{3}(1) = \frac{11}{3}$

$a_2 = 3 + \frac{2}{3}(2) = \frac{13}{3}$

$\Rightarrow \quad d = \frac{13}{3} - \frac{11}{3} = \frac{2}{3}$

$S_{24} = \frac{24}{2}\left[2\left(\frac{11}{3}\right) + (24-1) \times \frac{2}{3}\right] = 24\left[\frac{11}{3} + \frac{23}{3}\right]$

$= 24 \times \frac{34}{3} = 272$

63. (*c*) **2, 4, 6, 8**

Let the four numbers in AP be $(a - 3d)$, $(a - d)$, $(a + d)$ and $(a + 3d)$

$\therefore \quad a - 3d + a - d + a + d + a + 3d = 20$

$\Rightarrow \quad a = 5 \quad \ldots(1)$

Also $(a - 3d)^2 + (a - d)^2 + (a + d)^2 + (a + 3d)^2 = 120$

or $\quad 4a^2 + 20d^2 = 120$

$\Rightarrow \quad a^2 + 5d^2 = 30 \quad \ldots(2)$

From (1) and (2)

$5d^2 = 30 - 25$

$\Rightarrow \quad 5d^2 = 5$

$\Rightarrow \quad d^2 = 1$

$\Rightarrow \quad d = \pm 1$

$\therefore$ AP: $(5 - 3), (5 - 1), (5 + 1), (5 + 3)$ or $2, 4, 6, 8, \ldots$

64. (*a*) **21, 22**

$$S_n = \frac{n}{2}[2(63) + (n-1)(-3)] = 693$$

$\Rightarrow \quad n[126 - 3n + 3] = 1386$

or $\quad 3n^2 - 129n + 1386 = 0$

$\therefore \quad n^2 - 43n - 462 = 0$

$\Rightarrow \quad n = 22, 21$

65. (*c*) **6, 7, 8**

Let the three numbers in AP are

$a - d$, a and $a + d$

$\therefore \quad a - d + a + a + d = 21$

$\Rightarrow \quad a = 7$

Also, $\quad (a - d)(a + d)a = 336$

or $\quad 7(7^2 - d^2) = 336$

$\Rightarrow \quad d = 1$

$\therefore$ AP is $(7 - 1), 7, (7 + 1)$ or $6, 7, 8$

SHORT ANSWER QUESTIONS

For Basic and Standard Levels

1. Taxi fare $\quad$ for 1st km = ₹ 20

for 2nd km = ₹ 20 + ₹ 14 = ₹ 34

for 3rd km = ₹ 20 + ₹ 28 = ₹ 48

∵ $34 - 20 = 48 - 34 = 14$

∴ **20, 34, 48, … form an AP.**

2. (*i*) For $n = 1, 1 + n + n^2 = 1 + 1 + 1 = 3$

$n = 2, 1 + n + n^2 = 1 + 2 + 4 = 7$

$n = 3, 1 + n + n^2 = 1 + 3 + 9 = 13$

∵ $7 - 3 \neq 13 - 7$

∴ $1 + n + n^2$ **is not *n*th term** of an AP

(*ii*) For $n = 1, 5n - 1 = 5(1) - 1 = 4$

For $n = 2, 5n - 1 = 5(2) - 1 = 9$

For $n = 3, 5n - 1 = 5(3) - 1 = 14$

∵ $9 - 4 = 14 - 9 = 5$ (constant)

∴ $5n - 1$ **is *n*th term** of an AP

3. Let common diff. $= d$

∴ $a_{25} = a + 24d = -67$ …(1)

and $a_{10} = a + 9d = -22$ …(2)

Subtracting (2) from (1),

$15d = -45$

⇒ $\mathbf{d = -3}$ and $a = 5$

∵ Last term $= -82 = a_n$

∴ $a + (n - 1)d = -82$

⇒ $5 + (n - 1)(-3) = -82$

⇒ $\mathbf{n = 30}$

4. $a_4 = 0$

$a + 3d = 0$ ... (1)

$a = -3d$

Now we have to prove that $a_{25} = 3a_{11}$

$a_{25} = a + (n - 1)\,d$

$= a + (25 - 1)\,d$

$= a + 24d$ ... (2)

Putting the value of a from eq. (1) in eq. (2)

$a_{25} = -3d + 24d$

$= 21d$

$a_{11} = a + (n - 1)d$

$= a + (11 - 1)d$

$= a + 10d$ ... (3)

Putting the values of a from eq. (1) in eq. (3)

$a_{11} = -3d + 10d$

$= 7d$

We have

$a_{25} = 21d$

$= 3 \times 7d$

⇒ $\mathbf{a_{25} = 3a_{11}}$ [∵ $a_{11} = 7d$]

5. Sum of n terms $= \dfrac{3n^2}{2} + \dfrac{13n}{2}$

For $n = 1$, First term $(a) = \dfrac{3}{2} + \dfrac{13}{2} = \dfrac{16}{2} = 8$

For $n = 2$, $S_2 = \dfrac{12}{2} + \dfrac{26}{2} = \dfrac{38}{2} = 19$

⇒ [1st term + 2nd term] = 19

⇒ 2nd term $= 19 - 8 = 11 = a_2$

Now, $d = a_2 - a_1 = 11 - 8 = 3$

∴ $a_{25} = a + 24d = 8 + 24 \times 3 = \mathbf{80}$

6. $a_1 \times a_3 = a_2 + 46$

⇒ $a \times (a + 2d) = a + d + 46$

⇒ $a^2 + 2ad = a + d + 46$ … (1)

$S_3 = 33$ ⇒ $S_3 = \dfrac{3}{2}[2a + 2d] = 33$

⇒ $2a + 2d = 33 \times \dfrac{2}{3} = 22$

⇒ $a + d = 11$ … (2)

From (1) and (2), we get

$a^2 + 2ad = 11 + 46$

⇒ $a^2 + 2ad = 57$ … (3)

But $d = (11 - a)$ [From (1)]

∴ From (3), $a^2 + 2a\,(11 - a) = 57$

⇒ $a^2 - 22a + 57 = 0$

Solving, this quadratic equation, $a = 3$ or $a = 19$

For $a = 3, d = 8$ [From (1)]

Then AP is **3, 11, 19, …**

For $a = 19, d = -8$, [From (1)]

Then AP is **19, 11, 3, …**

7. Middlemost term of 11 terms $= a_6$

∴ $a + 5d = 30$ ...(1)

Now, $S_{11} = \dfrac{11}{2} \times [2a + (11 - 1)d]$

$= \dfrac{11}{2} \times 2[a + 5d]$ …(2)

From (1) and (2),

$S_{11} = 11[30] = \mathbf{330}$

8. Numbers between 10 and 600 which when divided by 3 leave a remainder 2, are

11, 14, 17, …, 599

These numbers are in AP with $a = 11$, $d = 3$ and $l = 599$

∴ $a_n = l = 11 + (n - 1)3 = 599$

⇒ $n - 1 = \dfrac{599 - 11}{3} = \dfrac{588}{3} = 196$

⇒ $n = 196 + 1 = \mathbf{197}$

9. AP: $-\dfrac{4}{3}, -1, \dfrac{-2}{3}, \ldots, 4\dfrac{1}{3}$

$a = -\dfrac{4}{3}$

$$d = a_2 - a_1$$
$$= -1 - \left(-\frac{4}{3}\right)$$
$$= -1 + \frac{4}{3}$$
$$= \frac{1}{3}$$
$$a_n = a + (n-1)d$$
$$\Rightarrow \quad \frac{13}{3} = -\frac{4}{3} + (n-1)\frac{1}{3}$$
$$\Rightarrow \quad \frac{17}{3} = \frac{(n-1)}{3}$$
$$\Rightarrow \quad n = 18$$

Since we have even number of terms, the middle terms will be $\frac{n}{2}$ and $\frac{n}{2}+1$ i.e 9th and 10th.

$$a_9 = a + 8d$$
$$= \frac{-4}{3} + 8 \times \frac{1}{3}$$
$$= \frac{-4}{3} + \frac{8}{3}$$
$$= \frac{4}{3}$$
$$a_{10} = a + 9d$$
$$= \frac{-4}{3} + 9 \times \frac{1}{3}$$
$$= \frac{5}{3}$$

Sum of middlemost terms

$$= a_9 + a_{10}$$
$$= \frac{4}{3} + \frac{5}{3}$$
$$= \frac{9}{3}$$
$$= \mathbf{3}$$

10. Let the common difference = d and $a = 1$ (Given)

Now, $S_4 = 4 + 6d$, $S_8 = 8 + 28d$

Sum of next 4 terms beyond first 4 terms

$$= S_8 - S_4 = S'_4$$

or $S'_4 = (8 + 28d) - (4 + 6d) = 4 + 22d$

Now, it is given that $S_4 = \frac{1}{3}S'_4$

$$\therefore \quad 4 + 6d = \frac{1}{3}[4 + 22d]$$
$$\Rightarrow \quad \frac{22}{3}d - 6d = 4 - \frac{4}{3}$$
$$\Rightarrow \quad \frac{4}{3}d = \frac{8}{3}$$
$$\Rightarrow \quad d = \frac{8}{3} \times \frac{3}{4} = 2$$

Thus, common difference = **2**

11. Three digit numbers when divided by 16 leave remainder as 7 are

103, 119, 135, 151, … 999

These numbers are in AP with

$$a = 103, d = 16 \text{ and } l = 999$$

Now $a_n = a + (n-1)d$

$$\Rightarrow \quad 999 = 103 + (n-1) \times 16$$

or $n - 1 = \frac{999-103}{16} = 56$

$$\Rightarrow \quad n = 56 + 1 = 57$$
$$\therefore \quad S_{57} = \frac{57}{2}[103 + 999] = \frac{57}{2} \times 1102 = 31407$$

Thus, the required sum = **31407**

12. $a = \text{First term} = \frac{p-q}{p+q}$, $d = \frac{3p-2q}{p+q} - \frac{p-q}{p+q} = \frac{2p-q}{p+q}$

$$\therefore \; S_{12} = \frac{12}{2}\left[2 \cdot \left(\frac{p-q}{p+q}\right) + 11\left(\frac{2p-q}{p+q}\right)\right]$$
$$= 6\left[\frac{2p-2q+22p-11q}{p+q}\right]$$
$$= \mathbf{\frac{6[24p-13q]}{p+q}}$$

13. Let 'a' be the 1st and 'd' be the common difference of an AP.

$$\therefore \quad a_{m+n} = a + (m+n-1)d$$
$$a_{m-n} = a + (m-n-1)d$$
$$\text{LHS} = \boldsymbol{a_{m+n} + a_{m-n}}$$
$$= a + (m+n-1)d + a + (m-n-1)d$$
$$= 2a + 2(m-1)d$$
$$= 2[a + (m-1)d] = \boldsymbol{2a_m} = \text{RHS}$$

14. Let the numbers be $(a-d)$, a, $(a+d)$

$$\therefore \quad a - d + a + a + d = 21$$
$$\Rightarrow \quad a = 7 \quad \ldots(1)$$
$$\therefore \quad (a-d)^2 + a^2 + (a+d)^2 = 155$$
$$\therefore \quad 3a^2 + 2d^2 = 155 \quad \ldots(2)$$

From (1) and (2) we have

$$3(7)^2 + 2d^2 = 155$$
$$\Rightarrow \quad 147 + 2d^2 = 155$$
$$\Rightarrow \quad 2d^2 = 8$$
$$\Rightarrow \quad d^2 = 4$$
$$\Rightarrow \quad d = \pm 2$$

But the numbers are in increasing order

$$\therefore \quad d = -2 \text{ is rejected}$$

Thus $d = 2$

$\therefore$ $a - d, a, a + d, \ldots \Rightarrow (7-2), 7, (7+2) \ldots$ or **5, 7, 9, …**

15. (*i*) **False**

[$\because$ 13 – 20 = 6 – 13 = –7 ≠ 7]

(*ii*) **False** $\left[\begin{pmatrix}\text{Interest at the}\\ \text{end of 2nd year}\\ \text{i.e. ₹ 1210}\end{pmatrix} - \begin{pmatrix}\text{Interest at the}\\ \text{end of 1st year}\\ \text{i.e. ₹ 1100}\end{pmatrix}\right]$

$\neq \left[\begin{pmatrix}\text{Interest at the}\\ \text{end of 3rd year}\\ \text{i.e. ₹ 1331}\end{pmatrix} - \begin{pmatrix}\text{Interest at the}\\ \text{end of 2nd year}\\ \text{i.e. ₹ 1210}\end{pmatrix}\right]$

(*iii*) **True** [$\because a_n = a + (n-1)d \neq n^2 + 1$]

(*iv*) **False** [$\because a, 2a, 4a, 8a \ldots$ is not an AP]

16. (*i*) **Yes**

$\because \quad a_{40} = a + 39d$ and $a_{30} = a + 29d$

$\therefore \quad a_{40} - a_{30} = (a + 39d) - (a + 29d) = 10d$ ...(1)

and $\quad d = -7 - (-5) = -2$ ...(2)

From (1) and (2), $a_{40} - a_{30} = 10 \times (-2) = \mathbf{-20}$

(*ii*) **No**

$\because \quad a_n = 29 + (n-1)(-4) = 0$

$\Rightarrow \quad n = \frac{33}{4}$, which is not a natural number.

17. Let the three parts of 177 are: $a - d, a, a + d$

[$\because$ These parts are given to be in AP]

$\therefore \quad (a - d) + a + (a + d) = 177$

$\Rightarrow \quad a = 59$

Product of larger two parts = 3599 ...(1)

$\therefore \quad (a)(a + d) = 3599$

$\Rightarrow \quad 59(59 + d) = 3599$

$\Rightarrow \quad 59 + d = 3599 \div 59 = 61$

$\Rightarrow \quad d = 61 - 59 = 2$

Now, the required parts are

(59 – 2), (59), (59 + 2) or **57, 59, 61**.

18. $a = \frac{x-y}{x+y}$ $d = \left[\frac{3x-2y}{x+y} - \frac{x-y}{x+y}\right] = \frac{2x-y}{x+y}$, $n = 11$

$\therefore$ Using $S_n = \frac{n}{2}[2a + (n-1)d]$, we get

$$S_{11} = \frac{11}{2}\left[2 \cdot \frac{x-y}{x+y} + (11-1)\left(\frac{2x-y}{x+y}\right)\right]$$

$$= \frac{11}{2}\left[2 \cdot \frac{x-y}{x+y} + 10\left(\frac{2x-y}{x+y}\right)\right]$$

$$= \frac{11}{2} \cdot 2\left[\frac{x-y}{x+y} + 5\left(\frac{2x-y}{x+y}\right)\right]$$

$$= 11\left[\frac{(x-y) + (10x-5y)}{x+y}\right]$$

$$= 11\left[\frac{(11x-6y)}{x+y}\right] = \mathbf{\frac{11(11x-6y)}{x+y}}$$

19. We have nth term of an AP.

$a_n = a + nb$[where 'a' and 'b' are real numbers]

$\Rightarrow \quad l = (a + nb)$

For $n = 1$, $\quad a_1 = a + b$ [First term]

For $n = 2$, $\quad a_2 = a + 2b$ [Second term]

For $n = 3$, $\quad a_3 = a + 3b$ [Third term]

Now $\quad a_2 - a_1 = (a + 2b) - (a + b) = b$

$a_3 - a_2 = (a + 3b) - (a + 2b) = b$

$\Rightarrow \quad (a + b), (a + 2b), (a + 3b), \ldots$ is an AP.

with First term = $(a + b)$ and

Common difference = $\boldsymbol{b}$

Now, using $\quad S_n = \frac{n}{2}[a + l]$,

we get $\quad S_{20} = \frac{20}{2}[(a + b) + (a + 20b)]$

$= 10[2a + 21b]$

$= \mathbf{20a + 210b}$

20. Here, first term, $a = 5$ ...(1)

Let the common difference = d

$\therefore \quad S_8 = \frac{8}{2}[2a + 7d] = 8a + 28d$

$S_4 = \frac{4}{2}[2a + 3d] = 4a + 6d$

S'_4 = Sum of next 4 terms beyond first 4 terms

$= S_8 - S_4$

$\therefore \quad S'_4 = [8a + 28d] - [4a + 6d] = 4a + 22d$

It is given that

$S_4 = \frac{1}{2}S'_4$

$\Rightarrow \quad 4a + 6d = \frac{1}{2}[4a + 22d] = \frac{2}{2}[2a + 11d]$

$\Rightarrow \quad 6d - 11d = 2a - 4a$

$\Rightarrow \quad 5d = 2a$ ...(2)

From (1) and (2),

$5d = 2 \times 5 = 10$

$\Rightarrow \quad d = \frac{10}{5} = 2$

Hence, common difference, $\boldsymbol{d = 2}$

21. The required sum

$= \begin{bmatrix}\text{Sum of multiples}\\ \text{of 2 from 1 to 500}\end{bmatrix} + \begin{bmatrix}\text{Sum of multiples}\\ \text{of 5 from 1 to 500}\end{bmatrix} - \begin{bmatrix}\text{Sum of multiples}\\ \text{of 10 from 1 to 500}\end{bmatrix}$

$= \begin{bmatrix}2 + 4 + 6 + \ldots + 500\\ \Rightarrow n = 250\end{bmatrix} + \begin{bmatrix}5 + 10 + 15 + \ldots + 500\\ \Rightarrow n = 100\end{bmatrix} - \begin{bmatrix}10 + 20 + 30 \ldots + 500\\ \Rightarrow n = 50\end{bmatrix}$

$= \left[\frac{250}{2}\{2 + 500\}\right] + \left[\frac{100}{2}\{5 + 500\}\right] - \left[\frac{50}{2}\{10 + 500\}\right]$

$= [125 \times 502] + [50 \times 505] - [25 \times 510]$

$= 62750 + 25250 - 12750$

$= 88000 - 12750 = \mathbf{75250}$

22. Numbers between 1 to 500 which are multiples of 2 as well as 5 are

$$10, 20, 30, \dots 500$$

These numbers are in AP with $a = 10$, $d = 10$ and $l = 500$

Using, $a_n = a + (n-1)d$, we have

$$500 = 10 + (n-1)10$$

$$\Rightarrow \quad n - 1 = \frac{500-10}{10} = 49$$

$$\Rightarrow \quad n = 50$$

Now, $S_{50} = \frac{50}{2}(10 + 500) = 25 \times 510 = \mathbf{12750}$

23. First term $= a$

Second term $= b$

$$\Rightarrow \quad d = (b - a)$$

$$a_n = a + (n-1)d$$

$$\Rightarrow \quad l = a + (n-1)(b-a)$$

$$\Rightarrow \quad (n-1) = \frac{l-a}{b-a} \text{ or } n = \frac{l-a+b-a}{b-a} = \frac{b+l-2a}{b-a}$$

Now $S_n = \frac{1}{2} \times \left[\frac{b+l-2a}{b-a}\right][a+l]$

Using $S_n = \frac{1}{2} \times n\,[a+l]$

$$= \frac{b+l-2a}{2(b-a)}(a+l)$$

$$= \frac{(b+l-2a)(a+l)}{2(b-a)}$$

$$= \mathbf{\frac{(a+l)(b+l-2a)}{2(b-a)}}$$

24. It is given that a, b, c, d, e form an AP.

Let D be the common difference

$\therefore \quad b = a + D$

$c = a + 2D$

$d = a + 3D$

$e = a + 4D$

$\Rightarrow \quad a - 4b + 6c - 4d + e$

$= a - 4(a + D) + 6(a + 2D) - 4(a + 3D) + (a + 4D)$

$\Rightarrow \quad a - 4b + 6c - 4d + e$

$= a - 4a - 4D + 6a + 12D - 4a - 12D + a + 4D$

$= (a - 4a + 6a - 4a + a) - (-4D + 12D - 12D + 4D)$

$= (8a - 8a) + (16D - 16D)$

$= 0 + 0$

Hence, $\mathbf{a - 4b + 6c - 4d + e = 0}$

25. Two digit natural numbers which when divided by 3 give remainder 1 are:

$$10, 13, 16, \dots 97$$

$\therefore \quad a = 10$, $d = 3$ and $l = 97$

$$a_n = a + (n-1)d$$

$$\Rightarrow \quad 97 = 10 + (n-1)3$$

$$\Rightarrow \quad n = 30$$

Using $S_n = \frac{n}{2}(a+l)$

$$S_{30}\ \frac{30}{2}[10+97] = \mathbf{1605}$$

26. Let first term $= a$ and common diff. $= d$

$\because \quad n = 27$

$\therefore$ 3 middle terms are a_{13}, a_{14} and a_{15}

i.e. $a_{13} = a + 12d$,

$a_{14} = a + 13d$,

$a_{15} = a + 14d$

$\because \quad a_{13} + a_{14} + a_{15} = 81$

$\therefore \quad (a + 12d) + (a + 13d) + (a + 14d) = 81$

$\Rightarrow \quad a + 13d = 27 \quad \dots(1)$

Also, $a_{25} + a_{26} + a_{27} = 153$

$\therefore \quad (a + 24d) + (a + 25d) + (a + 26d) = 153$

$\Rightarrow \quad a + 25d = 51 \quad \dots(2)$

Subtracting (1) from (2), we get

$$12d = 24$$

$$\Rightarrow \quad d = 2$$

From (1), $a + 13(2) = 27$

$$\Rightarrow \quad a = 1$$

Now, the AP is $(a), (a + d), (a + 2d), (a + 3d), \dots$

or $(1), (1 + 2), (1 + 4), (1 + 6), \dots$

or **1, 3, 5, 7, ...**

27. Let a = first term and d = common diff.

Using $S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore \quad S_4 = \frac{4}{2}[2a + 3d] = 2[2a + 3d]$

$S_8 = \frac{8}{2}[2a + 7d] = 4[2a + 7d]$

$S_{12} = \frac{12}{2}[2a + 11d] = 6[2a + 11d] \quad \dots(1)$

Now, $3[S_8 - S_4] = 3[4(2a + 7d) - 2(2a + 3d)]$

$\Rightarrow \quad 3[S_8 - S_4] = 3 \times 2[2(2a + 7d) - (2a + 3d)]$

$\Rightarrow \quad [S_8 - S_4] = 6[4a + 14d - 2a - 3d] = 6[2a + 11d] \dots(2)$

From (1) and (2), we have:

$$\mathbf{S_{12} = 3(S_8 - S_4)}$$

VALUE-BASED QUESTIONS

For Basic and Standard Levels

1. (*i*) Number of students in the 1st row = 9

Number of students in the 2nd row = 7

Number of students in the 3rd row = 5

and so on.

Numbers, 9, 7, 5, … decrease uniformly by a constant number 2.

$\therefore$ 9, 7, 5, … form an AP with

$a = 9$, and $d = -2$

Let all the 25 students are involved using 'n' rows.

$$\therefore \quad S_n = \frac{n}{2}[2(9) + (n-1)(-2)] = 25$$
$$\Rightarrow \quad n[9 + (n-1)(-1)] = 25$$
$$\Rightarrow \quad n(9 - n + 1) = 25$$
$$\Rightarrow \quad n(10 - n) = 25$$
$$\Rightarrow \quad 10n - n^2 = 25$$
$$\Rightarrow \quad n^2 - 10n + 25 = 0$$
$$\Rightarrow \quad (n-5)^2 = 0$$
$$\Rightarrow \quad n = 5$$

Thus, **all the 25 students** are involved in **5 rows**.

(*ii*) Empathy and decision-making.

For Standard Level

2. (*i*) Formation of circles continued for 60 seconds, i.e. after 5 seconds, 10 seconds, 15 seconds, …

$\because$ 5 sec, 10 sec, 15 sec, …, 60 sec form an AP with

$a = 5$, $d = 5$ and $l = 60$

Using $a_n = a + (n-1)d$, we get

$$60 = 5 + (n-1)5$$
$$\Rightarrow \quad n - 1 = \frac{60-5}{5} = 11$$
$$\Rightarrow \quad n = 11 + 1 = 12$$

$\because$ Corresponding to each interval of 5 sec there is 1 circle.

$\therefore$ Number of circles = **12**

(*ii*) Number of flags in the circle $C_1 = 4$

Number of flags in the circle $C_2 = 7$

Number of flags in the circle $C_3 = 10$

....

....

$\because \quad 7 - 4 = 3 = 10 - 7$

$\therefore$ Numbers 4, 7, 10, … form an AP with

$a = 4$, $d = 7 - 4 = 3$

There are 12 circles in all

$\Rightarrow \quad n = 12$

Using $S_n = \frac{n}{2}[2a + (n-1)d]$, we get

$$S_{12} = \frac{12}{2}[2(4) + (12-1) \times 3]$$
$$= 6[8 + 11 \times 3]$$
$$= 6 \times 41 = 246$$

Now, Total number of flags

$$= \begin{bmatrix}\text{Number of flags} \\ \text{in 12 circles}\end{bmatrix} + \begin{bmatrix}\text{Number of circle} \\ \text{at the centre}\end{bmatrix}$$
$$= 246 + 1$$
$$= \mathbf{247}$$

(*iii*) Students learn to explore creative thinking and patriotism.

3. (*i*) Since, the savings of the friends A and B increase by one coin of ₹ 5 daily, therefore they form an AP.

Let a = first term d = common difference,

n = number of days and

S_n = total number of five rupees coins saved.

Then,

$a = 5$, $d = 5$, $n = 4$

and S_4 = total number of five rupee coins saved

$$S_4 = \frac{n}{2}[2a + (n-1)d]$$
$$= \frac{4}{2}[2 \times 5 + (4-1)5]$$
$$= 2(10 + 15) = 2(25) = 50$$

Hence, each friend saved ₹(50 × 5) = ₹ 250

A divides his saving into 2 parts.

Let one part of his saving be x. Then, the other part is $(250 - x)$

Given, product of the two parts = 15625

$$\therefore \quad x(250 - x) = 15625$$
$$\Rightarrow \quad 250x - x^2 = 15625$$
$$\Rightarrow \quad x^2 - 250x + 15625 = 0$$
$$\Rightarrow \quad x^2 - 125x - 125x + 15625 = 0$$
$$\Rightarrow \quad x(x - 125) - 125(x - 125) = 0$$
$$\Rightarrow \quad (x - 125)(x - 125) = 0$$
$$\Rightarrow \quad x = 125$$

Hence, **A divides his saving into two equal parts, each being ₹ 125.**

B divides his saving into two parts.

Let one part of his saving be y. Then the other part is $(250 - y)$.

Given, product of the two parts = 15600

$$\therefore \quad y(250 - y) = 15600$$
$$\Rightarrow \quad 250y - y^2 = 15600$$
$$\Rightarrow \quad y^2 - 250y + 15600 = 0$$
$$\Rightarrow \quad y^2 - 130y - 120y + 15600 = 0$$
$$\Rightarrow \quad y(y - 130) - 120(x - 130) = 0$$
$$\Rightarrow \quad (y - 130)(y - 120) = 0$$
$$\Rightarrow \quad y = 130 \text{ or } y = 120$$

Hence, **B divides his saving into two unequal portions of ₹130 and ₹120.**

(*ii*) A and B both exhibited the value of self awareness and decision-making by making the resolution to save money and executing it.

A also showed honesty and responsibility whereas B failed to be responsible and fair.

UNIT TEST 1

For Basic Level

1. (*d*) $p + 9q$

$\because$ First term = p $\Rightarrow$ $a_n = p + (n-1)q$

Common diff. = q $\therefore$ $a_{10} = p + 9q$

2. (*c*) **25th**

$\because$ $a = 2$, $d = -1 - 2 = -3$ and $a_n = -70$

$\therefore$ $a + (n - 1)d = -70$

$\Rightarrow 2 + (n - 1)(-3) = -70$

$\Rightarrow$ $n - 1 = \dfrac{-70 - 2}{-3}$

$\Rightarrow$ $n - 1 = 24$

or $n = 24 + 1 = 25$

3. (*d*) **2**

$a = 7$ and $a_7 = 19$ $\Rightarrow$ $7 + (7 - 1)d = 19$

$\Rightarrow$ $6d = 19 - 7 = 12$

$\Rightarrow$ $d = \dfrac{12}{6} = 2$

4. (*a*) **10 terms**

Let $S_n = 120$

$\Rightarrow$ $\dfrac{n}{2}[2(3) + (n - 1)(2)] = 120$

$[\because a = 3$ and $d = 5 - 3 = 2]$

$\Rightarrow$ $\dfrac{n}{2}[6 + (n - 1)2] = 120$

$\Rightarrow$ $n[3 + n - 1] = 120$

$\Rightarrow$ $n^2 + 2n - 120 = 0$

Solving it, $n = 10$, rejecting $n = -12$

5. (*b*) **–925**

$\because$ $a_n = 2 - 3n$

$\therefore$ $a_1 = 2 - 3 = -1$

$a_2 = 2 - 6 = -4$

$\Rightarrow$ $d = a_2 - a_1 = -4 - (-1) = -3$

$\therefore$ $S_{25} = \dfrac{25}{2}[2(-1) + (25 - 1)(-3)]$

$= 25 \times (-37) = -925$

6. $\because$ $a_2 = 38$ and $a_6 = -22$

$\therefore$ $a + d = 38$

$a + 5d = -22$

$\Rightarrow$ $4d = -60$ or $d = -15$

Now $a_1 = a_2 - d = 38 - (-15) = 53$

$a_3 = a + 2d = 53 + 2(-15) = +23$

$a_4 = a + 3d = 53 + 3(-15) = 8$

$a_5 = a + 4d = 53 + 4(-15) = -7$

Thus, we have: **[53], 38, [23], [8], [–7], –22**

7. $\because$ x, $2x + p$, $3x + 6$ are in AP.

$\therefore$ First term $= a = x$

Second term $= a_2 = (2x + p)$

$\Rightarrow$ Common diff. $= d = a_2 - a$

$= [2x + p] - x$

$= 2x + p - x = x + p$

Now $a_3 =$ Third term $= 3x + 6$

$\therefore$ $a + (3 - 1)d = 3x + 6$

$\Rightarrow$ $x + 2(x + p) = 3x + 6$

$\Rightarrow$ $x + 2x + 2p = 3x + 6$

$\Rightarrow$ $3x + 2p - 3x = 6$

$\Rightarrow$ $2p = 6$ or $\boldsymbol{p = 3}$

8. First term $= a = 22$

$a_n = -11$ $\Rightarrow$ $22 + (n - 1)d = -11$

$\Rightarrow$ $(n - 1)d = -11 - 22 = -33$ ...(1)

$S_n = 66$ $\Rightarrow$ $\dfrac{n}{2}[2(22) + (n - 1)d] = 66$

$\Rightarrow$ $n[44 + (n - 1)d] = 132$ ...(2)

From (1) and (2), we have

$n[44 + (-33)] = 132$

$\Rightarrow$ $n[11] = 132$

$\Rightarrow$ $n = \dfrac{132}{11} = 12$

Thus, $\boldsymbol{n = 12}$

9. The given AP is

62, 59, 56, …, 8

$\therefore$ $a = 62$ and $d = 59 - 62 = -3$

To find the sum of 12 terms from the end, we replace the 1st term by the last term and reverse sign of common diff.

$\therefore$ From the end $S'_{12} = \dfrac{12}{2}[2(8) + (12 - 1)(-d)]$

$= 6[16 + 11 \times 3]$

$= 6[16 + 33]$

$= 6 \times 49 = 294$

Thus, the sum of last 12 terms = **294**

10. $\because$ $S_n = \dfrac{3n^2}{2} + \dfrac{13n}{2}$

$\therefore$ $S_1 = \dfrac{3(1)^2}{2} + \dfrac{13(1)}{2}$

$= \dfrac{3}{2} + \dfrac{13}{2} = \dfrac{16}{2} = 8$

$\Rightarrow$ $a = 8$

$S_2 = \dfrac{3(2)^2}{2} + \dfrac{13(2)}{2}$

$= \dfrac{12}{2} + \dfrac{26}{2}$

$= \dfrac{38}{2} = 19$

Since, S_2 = sum of first two terms = 19

$\therefore$ $a + (a + d) = 19$

$\Rightarrow$ $8 + (8 + d) = 19$

$\Rightarrow$ $16 + d = 19$

$\Rightarrow$ $d = 3$

Now, $a_n = a + (n - 1)d$

$= 8 + (n - 1)3$

$= 8 + 3n - 3$

$\Rightarrow \quad a_n = 3n + 5$

$\therefore \quad a_{25} = 3(25) + 5$

$= 75 + 5$

$\Rightarrow \quad a_{25} = 80$

Thus, 25th term is **80**.

11. We have $1 + 4 + 7 + 10 + \ldots + x = 590$

The given series is AP with

$a = 1$ and $d = 3$. Here $S_n = 590$

Now, using $S_n = \frac{n}{2}[2a + (n-1)d]$, we get

$\therefore \quad 1 + 4 + 7 + 10 + \ldots + x = 590$

$\Rightarrow \quad \frac{n}{2}[(2 \times 1) + (n-1) \times 3] = 590$

$\Rightarrow \quad \frac{n}{2}[2 + 3n - 3] = 590$

$\Rightarrow \quad n[3n - 1] = 1180$

$\Rightarrow \quad 3n^2 - n - 1180 = 0$

$\Rightarrow \quad 3n^2 - 60n + 59n - 1180 = 0$

$\Rightarrow \quad 3n(n - 20) + 59(n - 20) = 0$

$\Rightarrow \quad (n - 20)(3n + 59) = 0$

$\Rightarrow \quad n = 20$ or $n = -\frac{59}{3}$

But $\quad n = -\frac{59}{3}$ is rejected

$\therefore \quad n = 20$

Now, $\quad x = n$th term = 20th term

$\therefore \quad a_{20} = 1 + (20 - 1)3 \quad$ [Using $a_n = a + (n-1)d$]

$= 1 + 19 \times 3$

$= 1 + 57 = 58$

$\therefore \quad$ **$x = 58$**

12. Amount paid in the first month = ₹ 1000

Thereafter the monthly instalments increases by ₹ 100

$\Rightarrow \quad a =$ ₹ 1000 and $d =$ ₹ 100

$\because \quad$ Number of instalments = 30

$\Rightarrow \quad n = 30$

$\therefore$ Total loan amount is given by

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$\Rightarrow \quad S_{30} = \frac{30}{2}[2(1000) + (30 - 1) \times 100]$

$= 15[2000 + 29 \times 100]$

$= 15[2000 + 2900]$

$= 15 \times 4900$

$= 73500$

Thus, the loan amount = **₹ 73500**

UNIT TEST 2

For Standard Level

1. (*b*) **27th term**

$S_n = 3n^2 + 5n$

$\therefore \quad S_1 = a = 8$ and $S_2 = 22$

$\Rightarrow \quad a_2 = S_2 - S_1 = 22 - 8 = 14$

$d = a_2 - a_1 = 14 - 8 = 6$

Now $\quad a_n = a + (n-1)d$

$\Rightarrow \quad 164 = 8 + (n-1)6$

$\Rightarrow \quad n = 27$

2. (*d*) **–142**

$a = 5$

$a_{100} = 5 + (100 - 1)d = -292$

$\Rightarrow \quad d = \frac{-292 - 5}{99} = -3$

$a_{50} = 5 + 49 \times -3 = -142$

3. (*b*) **$4n + 3$**

$S_n = 2n^2 + 5n$

$S_1 = 2 + 5 = 7 = a$

$S_2 = 8 + 10 = 18$

$a_2 = S_2 - S_1 = 18 - 7 = 11$

$d = a_2 - a_1 = 11 - 7 = 4$

$\therefore \quad a_n = 7 + (n-1)4$

$= 7 + 4n - 4 = 3 + 4n$

$\Rightarrow \quad a_n = 4n + 3$

4. (*b*) **108**

$a_1 = 8$

$a_2 = 10$

$\Rightarrow \quad d = a_2 - a_1 = 2$

To find 10th term from the end we take the last term as the first term and '*d*' as negative.

$\therefore \quad$ From the end $\quad a_{10} = 126 + (10 - 1)(-2)$

$= 126 - 18 = 108$

5. (*c*) **60°**

Let the angles be $(a - d)°$, $a°$, $(a + d)°$

$\Rightarrow \quad [a - d]° + a° + [a + d]° = 180°$

$a - d + a + a + d = 180°$

$3a = 180°$

$a = 60°$

6. Natural numbers which are multiples of 7 and which lie between 500 and 900 are 504, 511, 518, 525 …, 896.

These numbers form an AP with the first term, $a = 504$ and the common difference, $d = 511 - 504 = 7$. If a_n be the nth term and S_n be the sum of the first n terms of this AP, then

$a_n = a + (n-1)d$

$= 504 + (n-1)7$

$= 7n + 497 \quad \ldots(1)$

and $$S_n = \frac{n}{2}\left[2a+(n-1)d\right]$$

$$= \frac{n}{2}\left[2\times 504+7(n-1)\right]$$

$$= \frac{n}{2}[1008+7n-7]$$

$$= \frac{n}{2}[7n+1001] \quad \text{...(2)}$$

If the last term, $a_n = 896$, then from (1), we have

$$7n + 497 = 896$$

$\Rightarrow$ $$7n = 399$$

$\Rightarrow$ $$n = \frac{399}{7} = 57 \quad \text{...(3)}$$

$\therefore$ There are 57 terms in the AP

$\therefore$ From (2), $$S_{57} = \frac{57}{2}(7\times 57+1001)$$

$$= \frac{57}{2}\times 1400 = \mathbf{39900}$$

which is the required sum.

7. If a be the first term, d, the common difference of an AP, a_n be its nth term and S_n, the sum of the first n terms of the AP, then

$$a_n = a + (n - 1)d \quad \text{...(1)}$$

and $$S_n = \frac{n}{2}\left[2a+(n-1)d\right] \quad \text{...(2)}$$

Now, given that

$$\frac{a_{11}}{a_{18}} = \frac{2}{3}$$

$\Rightarrow$ $$\frac{a+10d}{a+17d} = \frac{2}{3} \quad \text{[From (1)]}$$

$\Rightarrow$ $$2a + 34d = 3a + 30d$$

$\Rightarrow$ $$a = 4d \quad \text{...(3)}$$

$\therefore$ $$\frac{a_5}{a_{21}} = \frac{a+4d}{a+20d} \quad \text{[From (1)]}$$

$$= \frac{4d+4d}{4d+20d} \quad \text{[From (3)]}$$

$$= \frac{8d}{24d} = \frac{1}{3}$$

which is the required ratio.

Again, $$\frac{S_5}{S_{21}} = \frac{5/2(2a+4d)}{21/2(2a+20d)} \quad \text{[From (2)]}$$

$$= \frac{5}{21}\times\frac{2\times 4d+4d}{2\times 4d+20d} \quad \text{[From (3)]}$$

$$= \frac{5}{21}\times\frac{12d}{28d} = \frac{5}{49}$$

which is the required second ratio.

$\therefore$ Required ratios are **1 : 3** and **5 : 49**.

8. Let the first, 2nd and 3rd terms of the AP are

$$(a - d), a, (a + d)$$

$\therefore$ $(a - d) + a + (a + d) = 33$

$\Rightarrow$ $$a = 11$$

Also $(a - d)(a + d) = a + 29$

$\Rightarrow$ $(11 - d)(11 + d) = 11 + 29 = 40$

$\Rightarrow$ $$121 - d^2 = 40$$

$\Rightarrow$ $$d^2 = 81$$

$\Rightarrow$ $$d = \pm 9$$

For $d = 9$, $(a - d), a, (a + d)$ … are **2, 11, 20 …**

For $d = -9$, $(a - d), a, (a + d)$ … are **20, 11, 2, …**

9. Let a be the first term and d be the common difference of the AP. Let S_n be the sum of the first n terms of the AP.

Then $$S_n = \frac{n}{2}\left[2a+(n-1)d\right] \quad \text{...(1)}$$

Now, given that $S_6 = 36$ ...(2)

and $S_{16} = 256$ ...(3)

$\therefore$ From (1) and (2), we have

$$36 = \frac{6}{2}[2a+5d]$$

$\Rightarrow$ $$36 = 3(2a + 5d)$$

$\Rightarrow$ $$2a + 5d - 12 = 0 \quad \text{...(4)}$$

Also, from (1) and (3), we have

$$256 = \frac{16}{2}[2a+15d]$$

$\Rightarrow$ $$256 = 8(2a + 15d)$$

$\Rightarrow$ $$2a + 15d - 32 = 0 \quad \text{...(5)}$$

Subtracting (5) from (4), we get

$$-10d + 20 = 0$$

$\Rightarrow$ $$d = 2 \quad \text{...(6)}$$

$\therefore$ From (4), $2a = 12 - 5 \times 2 = 2$

$\therefore$ $$a = 1 \quad \text{...(7)}$$

$\therefore$ From (1), (6) and (7), we have

$$S_{10} = \frac{10}{2}[2\times 1+9\times 2]$$

$$= 5(2 + 18)$$

$$= \mathbf{100}$$

Which is the required sum.

10. A three digit number is given by $[100x + 10y + c]$

Digits $100x$, $10y$, c are in AP.

Let $100x = 100(a - d)$, $10y = 10a$, $c = (a + d)$

$\because$ In general three numbers in AP are

$$(a - d), a, (a + d)$$

$\Rightarrow$ $a + d + a + a + d = 15$

$\Rightarrow$ $$a = 5$$

$\therefore$ Digits of the given number are

$$100(5 - d), 10(5), (5 + d)$$

Digits in reverse order are

$$100(5 + d), 10(5), (5 - d)$$

Since the given number is greater than the number obtained by reversing the digits by 594

$\therefore$ $\begin{bmatrix}\text{Number formed by the digits} \\ \text{taken in reverse order}\end{bmatrix}$ – [Given number]

= 594

$\Rightarrow \quad [100(5 - d) + 10(5) + (5 + d)] - [100(5 + d) + 10(5) + (5 - d)] = 594$

Solving it for d, we get $d = 3$

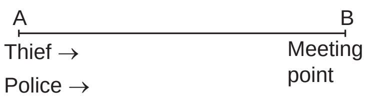
$\therefore$ The given numbers $= 100(5 + 3) + 10(5) + (5 - 3)$

$= 100(8) + 50 + 2$

$= 800 + 50 + 2$

$=$ **852**

11. Let the thief get caught after running for n minutes. Then the distance covered by the thief in n minutes = the distance covered by the police in $(n - 1)$ minutes.



$$\therefore \quad 100n = \frac{n-1}{2}\left[2 \times 100 + (n-1-1) \times 10\right]$$

$$= (n - 1)\,[100 + (n - 2)5]$$

$$= (n - 1)\,(5n + 90)$$

$$= 5n^2 + 90n - 5n - 90$$

$\Rightarrow \quad 5n^2 + 90n - 100n - 5n - 90 = 0$

$\Rightarrow \quad 5n^2 - 15n - 90 = 0$

$\Rightarrow \quad n^2 - 3n - 18 = 0$

$$\therefore \quad n = \frac{3 \pm \sqrt{3^2 + 4 \times 18}}{2}$$

$$= \frac{3 \pm \sqrt{9 + 72}}{2}$$

$$= \frac{3 \pm 9}{2}$$

$$= 6 \text{ or } -3$$

Since n is not negative,

$\therefore$ We take $n = 6$.

$\therefore$ The policeman catches the thief after $(n - 1)$ minutes $= (6 - 1)$ minutes = 5 minutes of his starting time.

$\therefore$ Required time = **5 minutes**.