CHAPTER 4 Quadratic Equations

EXERCISE 4A

For Basic and Standard Levels

1. (*i*) $3x^2 = x + 4$

$\Rightarrow \quad 3x^2 - x - 4 = 0$

$3x^2 - x - 4 = 0$, is of the form

$ax^2 + bx + c = 0$.

Hence, the given equation is a quadratic equation.

(*ii*) $x^2 + x - 12 = 0$ is of the form $ax^2 + bx + c = 0$.

Hence, the given equation is a quadratic equation.

(*iii*) $x^2 - 3x - 2\sqrt{x} - 1 = 0$ is not of the form $ax^2 + bx + c = 0$

Hence, the given equation is not a quadratic equation.

(*iv*) $x^2 - 5x = 0$. Since $x^2 - 5x$ is a quadratic polynomial, hence, the given equation is a quadratic equation.

(*v*) $x^2 - \frac{1}{x^2} = 3$

$\Rightarrow \quad x^4 - 3x^2 - 1 = 0$

Since, $x^4 - 3x^2 - 1$ is a polynomial of degree 4

$\therefore \quad x^2 - \frac{1}{x^2} = 3$ is not a quadratic equation.

(*vi*) $\frac{x}{2} + \frac{6}{x} = 5$

$\Rightarrow \quad x^2 + 12 = 10x$

$\Rightarrow \quad x^2 - 10x + 12 = 0$,

which is of the form $ax^2 + bx + c = 0$.

Hence, the given equation is a quadratic equation.

(*vii*) $2(x^2 + 1) = 5x$

$\Rightarrow \quad 2x^2 + 2 = 5x$

$\Rightarrow \quad 2x^2 - 5x + 2 = 0$, which is of the form $ax^2 + bx + c = 0$.

Hence, the given equation is a quadratic equation.

(*viii*) $x^3 - 4x^2 - 7x + 3 = 0$. Since $x^3 - 4x^2 - 7x + 3$ is a polynomial of degree 3, hence, the given equation is not a quadratic equation.

(*ix*) $(5x + 1)(2x + 3) = (10x + 1)(x + 2)$

$\Rightarrow \quad 10x^2 + 17x + 3 = 10x^2 + 21x + 2$

$\Rightarrow \quad 17x + 3 = 21x + 2$

$\Rightarrow \quad 4x - 1 = 0$

Since $4x - 1$ is a linear polynomial

hence, the given equation is not a quadratic equation.

(*x*) $\frac{x}{x-1} + \frac{x-1}{x} - \frac{5}{2} = 0$

$\Rightarrow \quad 2x^2 + 2(x - 1)^2 - 5(x)(x - 1) = 0$

$\Rightarrow \quad 2x^2 + 2(x^2 - 2x + 1) - 5x^2 + 5x = 0$

$\Rightarrow \quad 2x^2 + 2x^2 - 4x + 2 - 5x^2 + 5x = 0$

$\Rightarrow \quad -x^2 + x + 2 = 0$

$\Rightarrow \quad x^2 - x - 2 = 0$

$x^2 - x - 2 = 0$ is of the form $ax^2 + bx + c = 0$.

Hence, the given equation is a quadratic equation.

2. (*i*) Let one of the numbers be x.

Given, the sum of number is 18.

$\therefore$ The other number is $18 - x$.

Given, the sum of the reciprocals of the numbers is $\frac{1}{4}$.

$\Rightarrow \quad \frac{1}{x} + \frac{1}{18-x} = \frac{1}{4}$

$\Rightarrow \quad \frac{18-x+x}{x(18-x)} = \frac{1}{4}$

$\Rightarrow \quad \frac{18}{18x-x^2} = \frac{1}{4}$

$\Rightarrow \quad 72 = 18x - x^2$

$\Rightarrow \quad x^2 - 18x + 72 = 0$

Therefore, the two numbers satisfy the quadratic equation $x^2 - 18x + 72 = 0$ (where x is one of the two numbers) which is the required representation of the problem mathematically.

(*ii*) Suppose Tanay had x sweets.

Then, Vihaan had $(50 - x)$ sweets.

Number of sweets left with Tanay after he ate up 5 sweets = $(x - 5)$

Number of sweets left with Vihaan after he ate up 5 sweets = $50 - x - 5 = 45 - x$

Given, product of sweets left with Tanay and Vihaan = 351

$\Rightarrow \quad (x - 5)(45 - x) = 351$

$\Rightarrow \quad 45x - 225 - x^2 + 5x = 351$

$\Rightarrow \quad x^2 - 50x + 576 = 0$

Therefore, the number of sweets Tanay and Vihaan had to start with satisfy the quadratic equation $x^2 - 50x + 576$ (where x is the number of sweets Tanay had in the beginning) which is the required representation of the problem mathematically.

(*iii*) Let x and $x + 1$ be the required consecutive natural numbers.

Given, the sum of squares of two consecutive natural numbers is 313

$\Rightarrow \quad x^2 + (x + 1)^2 = 313$

$\Rightarrow \quad x^2 + x^2 + 2x + 1 - 313 = 0$

$\Rightarrow \quad 2x^2 + 2x - 312 = 0$

$\Rightarrow \quad x^2 + x - 156 = 0$

Therefore, the two required consecutive natural numbers satisfy the quadratic equation

$$x^2 + x - 156 = 0$$

(where x and $x + 1$ are the required consecutive natural numbers) which is the required representation of the problem mathematically.

(*iv*) Let the age of the elder sister be x years.

Then, the age of the younger sister is $(x - 3)$ years.

Given, the product of the ages (in years) of the two sisters is 238.

$\Rightarrow \quad x(x - 3) = 238$

$\Rightarrow \quad x^2 - 3x - 238 = 0$

Hence, the required quadratic equation is

$x^2 - 3x - 238 = 0$, where x is the age of the elder sister.

3. (*i*) (*a*) Substituting $x = -5$ in the LHS of the given equation $x^2 + 2x - 15 = 0$, we get

$$\text{LHS} = (-5)^2 + 2(-5) - 15 = 25 - 10 - 15 = 0 = \text{RHS}$$

$\therefore$ x **= –5 is a solution** of the given quadratic equation.

(*b*) Substituting $x = -3$ in the LHS of the given equation $x^2 + 2x - 15 = 0$, we get

$$\text{LHS} = (-3)^2 + 2(-3) - 15 = 9 - 6 - 15 = 9 - 21 = -12 \neq 0 \neq \text{RHS}$$

$\therefore$ x = **–3 is not a solution** of the given quadratic equation.

(*c*) Substituting $x = 3$ in the LHS of the given equation $x^2 + 2x - 15 = 0$, we get

$$\text{LHS} = (3)^2 + 2(3) - 15 = 9 + 6 - 15 = 15 - 15 = 0 = \text{RHS}$$

$\therefore$ x = **3 is a solution** of the given quadratic equation.

(*ii*) Substituting $x = -4$ in LHS of the given equation $2x^2 + 5x - 12 = 0$, we get

$$\text{LHS} = 2(-4)^2 + 5(-4) - 12 = 2(16) - 20 - 12 = 32 - 32 = 0 = \text{RHS}$$

Hence, **–4 is a solution** of the given quadratic equation.

(*iii*) Substituting $x = -2$ in LHS of the given equation $3x^2 + 13x + 14 = 0$, we get

$$\text{LHS} = 3(-2)^2 + 13(-2) + 14 = 3(4) - 26 + 14 = 12 - 26 + 14 = 0 = \text{RHS}$$

Hence, x = **–2 is a solution** of the given quadratic equation.

4. (*i*) Substituting $x = -5$ in LHS of the given equation $2x^2 + 5x - 25 = 0$, we get

$$\text{LHS} = 2(-5)^2 + 5(-5) - 25 = 50 - 25 - 25 = 0 = \text{RHS}$$

Hence, **–5 is a solution** of the given quadratic equation.

Substituting $x = \frac{5}{2}$ in LHS of the given equation $2x^2 + 5x - 25 = 0$, we get

$$\text{LHS} = 2\left(\frac{5}{2}\right)^2 + 5\left(\frac{5}{2}\right) - 25 = 2\left(\frac{25}{4}\right) + \frac{25}{2} - 25 = 0 = \text{RHS}$$

Hence, $\mathbf{\frac{5}{2}}$ **is a solution** of the given equation.

(*ii*) Substituting $x = 3\sqrt{2}$ in LHS of the given quadratic equation $x^2 - 4\sqrt{2}x + 6 = 0$, we get

$$\text{LHS} = (3\sqrt{2})^2 - 4\sqrt{2}(3\sqrt{2}) + 6 = 18 - 24 + 6 = 24 - 24 = 0 = \text{RHS}$$

Hence, $\mathbf{3\sqrt{2}}$ **is a solution** of the given equation.

Substituting $x = -\sqrt{2}$ in the LHS of the given equation $x^2 - 4\sqrt{2}x + 6 = 0$, we get

$$\text{LHS} = (-\sqrt{2})^2 - 4\sqrt{2}(-\sqrt{2}) + 6 = 2 + 8 + 6 = 16 \neq 0 \neq \text{RHS}$$

Hence, $\mathbf{-\sqrt{2}}$ **is not a solution** of the given quadratic equation.

(*iii*) Substituting $x = -1$ in the LHS of the given equation $x^2 + x - 1 = 0$, we get

$$\text{LHS} = (-1)^2 + (-1) - 1 = 1 - 1 - 1 = 1 - 2 = -1 \neq \text{RHS}$$

Hence, **–1 is not a solution** of the given quadratic equation.

Substituting $x = 1$ in the LHS of the given equation $x^2 + x - 1 = 0$, we get

$$\text{LHS} = (1)^2 + (1) - 1 = 1 + 1 - 1 = 1 \neq 0 \neq \text{RHS}$$

Hence, **1 is not a solution** of the given quadratic equation.

(*iv*) Substituting $x = \frac{5}{6}$ in the LHS of the given equation $24x^2 - 2x - 15 = 0$, we get

$$\text{LHS} = 24\left(\frac{5}{6}\right)^2 - 2\left(\frac{5}{6}\right) - 15 = 24\left(\frac{25}{36}\right) - \frac{5}{3} - 15 = \frac{50}{3} - \frac{5}{3} - 15 = \frac{50 - 5 - 45}{3} = \frac{50 - 50}{3} = 0 = \text{RHS}$$

Hence, $\frac{5}{6}$ **is a solution** of the given quadratic equation.

Substituting $x = \frac{-3}{4}$ in the LHS of the given equation $24x^2 - 2x - 15 = 0$, we get

$$\text{LHS} = 24\left(\frac{-3}{4}\right)^2 - 2\left(\frac{-3}{4}\right) - 15$$
$$= 24\left(\frac{9}{16}\right) + \frac{3}{2} - 15$$
$$= \frac{27}{2} + \frac{3}{2} - 15 = \frac{27 + 3 - 30}{2}$$
$$= \frac{30 - 30}{2} = 0 = \text{RHS}$$

Hence $\frac{-3}{4}$ **is a solution** of the given quadratic equation.

(*v*) The given equation is $9x + \frac{1}{x} = 6$.

$\Rightarrow \quad 9x^2 - 6x + 1 = 0$

Substituting $x = \frac{1}{3}$ in the LHS of the equation $9x^2 - 6x + 1 = 0$, we get

$$\text{LHS} = 9\left(\frac{1}{3}\right)^2 - 6\left(\frac{1}{3}\right) + 1$$
$$= 9\left(\frac{1}{9}\right) - 2 + 1 = 1 - 2 + 1$$
$$= 0 = \text{RHS}$$

Hence, $\frac{1}{3}$ **is a solution** of the given quadratic equation.

Substituting $x = 3$ in the LHS of the equation $9x^2 - 6x + 1 = 0$, we get

$$\text{LHS} = 9(3)^2 - 6(3) + 1$$
$$= 81 - 18 + 1$$
$$= 82 - 18 = 64 \neq 0$$

Hence, **3 is not a solution** of the given quadratic equation.

(*vi*) Substituting $x = \frac{a}{b}$ in the LHS of the equation $b^2x^2 - abx - 2a^2 = 0$, we get

$$\text{LHS} = b^2\left(\frac{a}{b}\right)^2 - ab\left(\frac{a}{b}\right) - 2a^2$$
$$= a^2 - a^2 - 2a^2 = -2a^2 \neq 0 \neq \text{RHS}$$

Hence, $\frac{a}{b}$ **is not a solution** of the given quadratic equation.

Substituting $x = \frac{2a}{b}$ in the LHS of the equation $b^2x^2 - abx - 2a^2 = 0$, we get

$$\text{LHS} = b^2\left(\frac{2a}{b}\right)^2 - ab\left(\frac{2a}{b}\right) - 2a^2$$
$$= 4a^2 - 2a^2 - 2a^2 = 0 = \text{RHS}$$

Hence, $\frac{2a}{b}$ **is a solution** of the given quadratic equation.

(*vii*) Substituting $x = \frac{-m}{n}$ in the LHS of the equation $mnx^2 + (m^2 + n^2)x + mn = 0$, we get

$$\text{LHS} = mn\left(\frac{-m}{n}\right)^2 + (m^2 + n^2)\left(\frac{-m}{n}\right) + mn$$
$$= mn\left(\frac{m^2}{n^2}\right) - \frac{m^3}{n} - mn + mn$$
$$= \frac{m^3}{n} - \frac{m^3}{n} = 0 = \text{RHS}$$

Hence, $\frac{-m}{n}$ **is a solution** of the given quadratic equation.

Substituting $x = \frac{-n}{m}$ in the LHS of the equation $mnx^2 + (m^2 + n^2)x + mn = 0$, we get

$$\text{LHS} = mn\left(\frac{-n}{m}\right)^2 + (m^2 + n^2)\left(\frac{-n}{m}\right) + mn$$
$$= mn\left(\frac{n^2}{m^2}\right) - mn - \frac{n^3}{m} + mn$$
$$= \frac{n^3}{m} - \frac{n^3}{m} = 0 = \text{RHS}$$

Hence, $\frac{-n}{m}$ **is a solution** of the given quadratic equation.

(*viii*) The given equation is $(2x + 3)(3x - 2) = 0$

$\Rightarrow \quad 6x^2 + 5x - 6 = 0$

Substituting $x = \frac{-3}{2}$ in the LHS of the equation $6x^2 + 5x - 6 = 0$, we get

$$\text{LHS} = 6\left(\frac{-3}{2}\right)^2 + 5\left(\frac{-3}{2}\right) - 6$$
$$= 6\left(\frac{9}{4}\right) - \frac{15}{2} - 6 = \frac{27}{2} - \frac{15}{2} - 6$$
$$= \frac{27 - 15 - 2}{2} = \frac{27 - 27}{2}$$
$$= 0 = \text{RHS}$$

Hence, $\frac{-3}{2}$ **is a solution** of the given quadratic equation.

Substituting $x = \frac{-2}{3}$ in the LHS of the equation $6x^2 + 5x - 6 = 0$, we get

$$\text{LHS} = 6\left(\frac{-2}{3}\right)^2 + 5\left(\frac{-2}{3}\right) - 6$$
$$= 6\left(\frac{4}{9}\right) - \frac{10}{3} - 6$$
$$= \frac{8}{3} - \frac{10}{3} - 6$$
$$= \frac{8-10-18}{3} = \frac{8-28}{3}$$
$$= \frac{-20}{3} \neq 0 \neq \text{RHS}$$

Hence, $\mathbf{\frac{-2}{3}}$ **is not a solution** of the given quadratic equation.

5. (*i*) Since $x = \frac{2}{3}$ is a solution of the given equation $kx^2 - x - 2 = 0$.

Therefore, it satisfies the given equation

$$\Rightarrow \quad k\left(\frac{2}{3}\right)^2 - \left(\frac{2}{3}\right) - 2 = 0$$
$$\Rightarrow \quad \frac{4}{9}k = 2 + \frac{2}{3}$$
$$\Rightarrow \quad \frac{4}{9}k = \frac{8}{3}$$
$$\Rightarrow \quad k = \frac{8}{3} \times \frac{9}{4} = 6$$

Hence, $\boldsymbol{k = 6}$

(*ii*) Since $x = 2$ is a solution of the equation

$$3x^2 + 2kx - 3 = 0$$

Putting the value of $x = 2$ in the above equation, we get

$$3(2)^2 + 2k(2) - 3 = 0$$
$$12 + 4k - 3 = 0$$
$$4k + 9 = 0$$
$$k = -\frac{9}{4}$$

Hence, $\boldsymbol{k = -\frac{9}{4}}$

(*iii*) Since $x = -\sqrt{5}$ is a solution of the given equation $\sqrt{5}x^2 + kx + 4\sqrt{5} = 0$.

Therefore, it satisfies the given equation

$$\Rightarrow \quad \sqrt{5}(-\sqrt{5})^2 + k(-\sqrt{5}) + 4\sqrt{5} = 0$$
$$\Rightarrow \quad 5\sqrt{5} + 4\sqrt{5} - \sqrt{5}k = 0$$
$$\Rightarrow \quad \sqrt{5}k = 9\sqrt{5}$$
$$\Rightarrow \quad k = 9$$

Hence, $\boldsymbol{k = 9}$

(*iv*) Since $x = \frac{2}{3}$ is a solution of the given equation $15x^2 + kx - 4 = 0$.

Therefore, it satisfies the given equation

$$\Rightarrow \quad 15\left(\frac{2}{3}\right)^2 + k\left(\frac{2}{3}\right) - 4 = 0$$
$$\Rightarrow \quad 15\left(\frac{4}{9}\right) + \frac{2}{3}k - 4 = 0$$
$$\Rightarrow \quad \frac{2}{3}k = 4 - \frac{20}{3}$$
$$\Rightarrow \quad \frac{2}{3}k = \left(\frac{12-20}{3}\right)$$
$$\Rightarrow \quad \frac{2}{3}k = \frac{-8}{3}$$
$$\Rightarrow \quad k = -4$$

Hence, $\boldsymbol{k = -4}$

(*v*) Since $x = 2$ is a solution of the equation

$$(2k + 1)x^2 + 2x - 3 = 0$$

Therefore, it satisfies the given equation

$$(2k + 1)(2)^2 + 2 \times 2 - 3 = 0$$
$$\Rightarrow \quad 8k + 4 + 4 - 3 = 0$$
$$\Rightarrow \quad 8k = -5$$
$$\Rightarrow \quad k = \frac{-5}{8}$$

Hence, $\boldsymbol{k = \frac{-5}{8}}$

(*vi*) Since $x = 3$ is a solution of the equation

$$x^2 - 2kx - 6 = 0$$

Therefore, it satisfies the given equation

$$3^2 - 2k \times 3 - 6 = 0$$
$$\Rightarrow \quad 9 - 6k - 6 = 0$$
$$\Rightarrow \quad 6k = 3$$
$$\Rightarrow \quad k = \frac{1}{2}$$

Hence, $\boldsymbol{k = \frac{1}{2}}$

6. (*i*) Since $x = 2$ is a root of the given equation, therefore we have

$$k \times 2^2 - 14 \times 2 + 8 = 0$$
$$\Rightarrow \quad 4k = 28 - 8 = 20$$
$$\therefore \quad k = \frac{20}{4} = 5$$

Hence, $\boldsymbol{k = 5}$

(*ii*) Since, $x = 2$ is a root of the given equation, therefore we have

$$2(2)^2 + p(2) + 4 = 0$$
$$\Rightarrow \quad 8 + 2p + 4 = 0$$
$$\Rightarrow \quad 2p = -12$$
$$\Rightarrow \quad p = -6$$

Hence, $p = -6$

Putting $p = -6$ in the equation $2x^2 + px + 4 = 0$, we get

$$2x^2 - 6x + 4 = 0$$

$$\Rightarrow \quad x^2 - 3x + 2 = 0$$
$$\Rightarrow \quad x^2 - x - 2x + 2 = 0$$
$$\Rightarrow \quad x(x-1) - 2(x-1) = 0$$
$$\Rightarrow \quad (x-1)(x-2) = 0$$
$$\Rightarrow \quad \text{Either } x - 1 = 0 \text{ or } x - 2 = 0$$
$$\Rightarrow \quad x = 1 \text{ or } x = 2$$

Hence, the other root is **1.**

(*iii*) Since $x = \frac{5}{2}$ is a root of the given equation, therefore we have

$$2\left(\frac{5}{2}\right)^2 - 8\left(\frac{5}{2}\right) - m = 0$$
$$\Rightarrow \quad 2\left(\frac{25}{4}\right) - 20 - m = 0$$
$$\Rightarrow \quad \frac{25}{2} - 20 = 0$$
$$\Rightarrow \quad m = \frac{25-40}{2} = \frac{-15}{2}$$

Hence, $\boldsymbol{m = \frac{-15}{2}}$.

Putting $m = \frac{-15}{2}$ in the equation $2x^2 - 8x - m = 0$, we get

$$2x^2 - 8x - \left(\frac{-15}{2}\right) = 0$$
$$\Rightarrow \quad 4x^2 - 16x + 15 = 0$$
$$\Rightarrow \quad 4x^2 - 6x - 10x + 15 = 0$$
$$\Rightarrow \quad 2x(2x-3) - 5(2x-3) = 0$$
$$\Rightarrow \quad (2x-3)(2x-5) = 0$$
$$\Rightarrow \quad \text{Either } 2x - 3 = 0 \text{ or } (2x-5) = 0$$
$$\Rightarrow \quad x = \frac{3}{2} \text{ or } x = \frac{5}{2}$$

Hence, the other root is $\boldsymbol{\frac{3}{2}}$.

For Standard Level

7. Putting $x = 3$ in the LHS, we have

$$\text{LHS} = \sqrt{3^2 - 4\times 3 + 3} + \sqrt{3^2 - 9} + \sqrt{4\times 3^2 - 14\times 3 + 16}$$
$$= \sqrt{12-12} + \sqrt{9-9} + \sqrt{36 - 42 + 16}$$
$$= 0 + 0 + \sqrt{52-42}$$
$$= \sqrt{10} \neq \text{RHS}$$

∴ The given equation is not satisfied by $x = 3$.

Hence, **$x = 3$ is not a root** of the given equation.

8. We have $x = -\frac{bc}{ad}$

$$\text{LHS} = ad^2\left\{\frac{a}{b}\left(-\frac{bc}{ad}\right) + \frac{2c}{d}\right\}\left(-\frac{bc}{ad}\right) + bc^2$$
$$= -ad^2\left(-\frac{c}{d} + \frac{2c}{d}\right)\frac{bc}{ad} + bc^2$$
$$= -ad^2 \times \frac{c}{d} \times \frac{bc}{ad} + bc^2$$
$$= -bc^2 + bc^2$$
$$= 0 = \text{RHS}$$

∴ The given equation is satisfied.

Hence, $x = -\frac{bc}{ad}$ is a solution of the given equation.

9. Since $x = 2$ is a root of the given equation, therefore we have

$$(2\lambda + 1)(2)^2 + 2(2) - 3 = 0$$
$$\Rightarrow \quad (2\lambda + 1)(4) + 4 - 3 = 0$$
$$\Rightarrow \quad 8\lambda + 4 + 1 = 0$$
$$\Rightarrow \quad 8\lambda = -5$$
$$\Rightarrow \quad \lambda = \frac{-5}{8}$$

Hence, $\lambda = \boldsymbol{\frac{-5}{8}}$.

10. Since $x = \frac{1}{2}$ is a root of the given equation, therefore we have

$$p\left(\frac{1}{2}\right)^2 + q\left(\frac{1}{2}\right) - 3 = 0$$
$$\Rightarrow \quad \frac{p}{4} + \frac{q}{2} = 3$$
$$\Rightarrow \quad p + 2q = 12 \qquad \ldots(1)$$

Since $x = \frac{3}{4}$ is a root of the given equation, therefore we have

$$p\left(\frac{3}{4}\right)^2 + q\left(\frac{3}{4}\right) - 3 = 0$$
$$\Rightarrow \quad \frac{9p}{16} + \frac{3q}{4} = 3$$
$$\Rightarrow \quad 9p + 12q = 48 \qquad \ldots(2)$$

Multiplying Eqn. (1) by 6 and subtracting the result from Eqn. (2), we get

$$3p = -24$$
$$\Rightarrow \quad p = -8$$

Substituting $p = -8$ in Eqn. (1), we get

$$-8 + 2q = 12$$
$$\Rightarrow \quad 2q = 20$$
$$\Rightarrow \quad q = 10$$

Hence, $\boldsymbol{p = -8, q = 10}$.

11. Since, $x = -3$ is a root of the given equation, therefore we have

$$m(-3)^2 + 7(-3) + n = 0$$
$$\Rightarrow \quad 9m - 21 + n = 0$$
$$\Rightarrow \quad 9m + n = 21 \qquad \ldots(1)$$

Since, $x = \frac{2}{3}$ is a root of the given equation, therefore we have

$$m\left(\frac{2}{3}\right)^2 + 7\left(\frac{2}{3}\right) + n = 0$$
$$\Rightarrow \quad \frac{4}{9}m + \frac{14}{3} + n = 0$$

$\Rightarrow \quad 4m + 42 + 9n = 0$

$\Rightarrow \quad 4m + 9n = -42 \quad \ldots(2)$

Multiplying Eqn. (1) by 9 and subtracting the result from Eqn. (2), we get

$$-77m = -231$$

$$\Rightarrow \quad m = \frac{-231}{-77}$$

$$\Rightarrow \quad m = 3$$

Putting $m = 3$ in Eqn. (1), we get

$$9(3) + n = 21$$

$$\Rightarrow \quad 27 + n = 21$$

$$\Rightarrow \quad n = -6$$

Hence, $\boldsymbol{m = 3, n = -6}$

12. Since, $x = \frac{2}{3}$ is a root of the given equation, therefore, we have

$$a\left(\frac{2}{3}\right)^2 + 7\left(\frac{2}{3}\right) + b = 0$$

$$\Rightarrow \quad \frac{4}{9}a + \frac{14}{3} + b = 0$$

$$\Rightarrow \quad 4a + 9b = -42 \quad \ldots(1)$$

Since $x = -3$ is a root of the given equation, therefore, we have

$$a(-3)^2 + 7(-3) + b = 0$$

$$9a - 21 + b = 0$$

$$\Rightarrow \quad 9a + b = 21 \quad \ldots(2)$$

Multiplying Eqn. (2) by 9, we get

$$81a + 9b = 189 \quad \ldots(3)$$

Subtracting Eqn. (1) from Equation (3), we get

$$77a = 231$$

$$\Rightarrow \quad a = \frac{231}{77}$$

$$\Rightarrow \quad a = 3$$

Substituting $a = 3$ in Eqn. (2), we get

$$9(3) + b = 21$$

$$\Rightarrow \quad 27 + b = 21$$

$$\Rightarrow \quad b = -6$$

Hence, $\boldsymbol{a = 3, b = -6}$

EXERCISE 4B

For Basic and Standard Levels

1. $(x - 5)(x + 4) = 0$

$\Rightarrow$ Either $(x - 5) = 0$ or $(x + 4) = 0$

$\Rightarrow$ $x = \mathbf{5}$ or $x = \mathbf{-4}$

2. $(2x + 3)(3x + 2) = 0$

$\Rightarrow$ Either $(2x + 3) = 0$ or $(3x + 2) = 0$

$\Rightarrow$ $x = \mathbf{\frac{-3}{2}}$ or $x = \mathbf{\frac{-2}{3}}$

3. $\left(\frac{x}{2} - 1\right)\left(\frac{x}{3} + 5\right) = 0$

Either $\left(\frac{x}{2} - 1\right) = 0$ or $\left(\frac{x}{3} + 5\right) = 0$

$\Rightarrow$ $\frac{x}{2} = 1$ or $\frac{x}{3} = -5$

$\Rightarrow$ $x = \mathbf{2}$ or $x = \mathbf{-15}$

4. $7x^2 + 3x = 0$

$\Rightarrow$ $x(7x + 3) = 0$

$\Rightarrow$ Either $x = 0$ or $7x + 3 = 0$

$\Rightarrow$ $x = \mathbf{0}$ or $x = \mathbf{\frac{-3}{7}}$

5. $16x^2 - 49 = 0$

$\Rightarrow$ $(4x)^2 - (7)^2 = 0$

$\Rightarrow$ $(4x + 7)(4x - 7) = 0$ $[\because a^2 - b^2 = (a - b)(a + b)]$

$\Rightarrow$ Either $(4x - 7) = 0$ or $(4x + 7) = 0$

$\Rightarrow$ $x = \mathbf{\frac{7}{4}}$ or $x = \mathbf{-\frac{7}{4}}$

6. $4x^2 - 324 = 0$

$\Rightarrow$ $4(x^2 - 81) = 0$

$\Rightarrow$ $x^2 - 81 = 0$

$\Rightarrow$ $(x)^2 - (9)^2 = 0$

$\Rightarrow$ $(x - 9)(x + 9) = 0$

$\Rightarrow$ Either $(x - 9) = 0$ or $(x + 9) = 0$

$\Rightarrow$ $x = \mathbf{9}$ or $x = \mathbf{-9}$

7. $18x^2 - 50 = 0$

$\Rightarrow$ $2(9x^2 - 25) = 0$

$\Rightarrow$ $9x^2 - 25 = 0$

$\Rightarrow$ $(3x)^2 - (5)^2 = 0$

$\Rightarrow$ $(3x - 5)(3x + 5) = 0$

$\Rightarrow$ Either $(3x - 5) = 0$ or $(3x + 5) = 0$

$\Rightarrow$ $x = \mathbf{\frac{5}{3}}$ or $x = \mathbf{-\frac{5}{3}}$

8. $(x + 7)^2 - 64 = 0$

$\Rightarrow$ $(x + 7)^2 - (8)^2 = 0$

$\Rightarrow$ $(x + 7 - 8)(x + 7 + 8) = 0$

$\Rightarrow$ $(x - 1)(x + 15) = 0$

$\Rightarrow$ Either $(x - 1) = 0$ or $(x + 15) = 0$

$\Rightarrow$ $x = \mathbf{1}$ or $x = \mathbf{-15}$

9. $(y - 2)^2 - 9 = 0$

$\Rightarrow$ $(y - 2)^2 - (3)^2 = 0$

$\Rightarrow$ $(y - 2 - 3)(y - 2 + 3) = 0$

$\Rightarrow$ $(y - 5)(y + 1) = 0$

$\Rightarrow$ Either $(y - 5) = 0$ or $(y + 1) = \mathbf{0}$

$\Rightarrow$ $y = \mathbf{5}$ or $y = \mathbf{-1}$

10. $(x + 3)^2 - 36 = 0$

$\Rightarrow$ $(x + 3)^2 - (6)^2 = 0$

$\Rightarrow$ $(x + 3 - 6)(x + 3 + 6) = 0$

$\Rightarrow$ $(x - 3)(x + 9) = 0$

$\Rightarrow$ Either $(x - 3) = 0$ or $(x + 9) = 0$

$\Rightarrow$ $x = \mathbf{3}$ or $x = \mathbf{-9}$

11. $x^2 - 7 = 0$

$\Rightarrow \quad (x)^2 - (\sqrt{7})^2 = 0$

$\Rightarrow \quad (x-\sqrt{7})(x+\sqrt{7}) = 0$

$\Rightarrow \quad$ Either $(x-\sqrt{7}) = 0$ or $(x+\sqrt{7}) = 0$

$\Rightarrow \quad x = \mathbf{\sqrt{7}}$ or $x = \mathbf{-\sqrt{7}}$

12. $x^2 + 11x - 152 = 0$

$\Rightarrow \quad x^2 + 19x - 8x - 152 = 0$

$\Rightarrow \quad x(x + 19) - 8(x + 19) = 0$

$\Rightarrow \quad (x + 19)(x - 8) = 0$

$\Rightarrow \quad$ Either $(x + 19) = 0$ or $(x - 8) = 0$

$\Rightarrow \quad x = \mathbf{-19} \Rightarrow x = \mathbf{8}$

13. $x^2 - 7x - 44 = 0$

$\Rightarrow \quad x^2 - 11x + 4x - 44 = 0$

$\Rightarrow \quad x(x - 11) + 4(x - 11) = 0$

$\Rightarrow \quad (x - 11)(x + 4) = 0$

$\Rightarrow \quad$ Either $(x - 11) = 0$ or $(x + 4) = 0$

$\Rightarrow \quad x = \mathbf{11} \Rightarrow x = \mathbf{-4}$

14. $x^2 + 16x + 63 = 0$

$\Rightarrow \quad x^2 + 7x + 9x + 63 = 0$

$\Rightarrow \quad x(x + 7) + 9(x + 7) = 0$

$\Rightarrow \quad (x + 7)(x + 9) = 0$

$\Rightarrow \quad$ Either $(x + 7) = 0$ or $(x + 9) = 0$

$\Rightarrow \quad x = \mathbf{-7}$ or $x = \mathbf{-9}$

15. $6x^2 - 13x + 6 = 0$

$\Rightarrow \quad 6x^2 - 4x - 9x + 6 = 0$

$\Rightarrow \quad 2x(3x - 2) - 3(3x - 2) = 0$

$\Rightarrow \quad (3x - 2)(2x - 3) = 0$

$\Rightarrow \quad$ Either $(3x - 2) = 0$ or $(2x - 3) = 0$

$\Rightarrow \quad x = \mathbf{\frac{2}{3}}$ or $x = \mathbf{\frac{3}{2}}$

16. $10x^2 - 33x + 20 = 0$

$\Rightarrow \quad 10x^2 - 8x - 25x + 20 = 0$

$\Rightarrow \quad 2x(5x - 4) - 5(5x - 4) = 0$

$\Rightarrow \quad (5x - 4)(2x - 5) = 0$

$\Rightarrow \quad$ Either $(5x - 4) = 0$ or $(2x - 5) = 0$

$\Rightarrow \quad x = \mathbf{\frac{4}{5}}$ or $x = \mathbf{\frac{5}{2}}$

17. $12x^2 - x - 1 = 0$

$\Rightarrow \quad 12x^2 - 4x + 3x - 1 = 0$

$\Rightarrow \quad 4x(3x - 1) + 1(3x - 1) = 0$

$\Rightarrow \quad (3x - 1)(4x + 1) = 0$

$\Rightarrow \quad$ Either $(3x - 1) = 0$ or $(4x + 1) = 0$

$\Rightarrow \quad x = \mathbf{\frac{1}{3}}$ or $x = \mathbf{\frac{-1}{4}}$

18. $3x^2 = x + 4$

$\Rightarrow \quad 3x^2 - x - 4 = 0$

$\Rightarrow \quad 3x^2 - 4x + 3x - 4 = 0$

$\Rightarrow \quad x(3x - 4) + 1(3x - 4) = 0$

$\Rightarrow \quad (3x - 4)(x + 1) = 0$

$\Rightarrow \quad$ Either $(3x - 4) = 0$ or $(x + 1) = 0$

$\Rightarrow \quad x = \mathbf{\frac{4}{3}}$ or $x = \mathbf{-1}$

19. $\frac{2}{5}x^2 - x - \frac{3}{5} = 0$

$\Rightarrow \quad 2x^2 - 5x - 3 = 0$ [Multiplying LHS and RHS by 5]

$\Rightarrow \quad 2x^2 - 6x + x - 3 = 0$

$\Rightarrow \quad 2x(x - 3) + 1(x - 3) = 0$

$\Rightarrow \quad (x - 3)(2x + 1) = 0$

$\Rightarrow \quad$ Either $(x - 3) = 0$ or $(2x + 1) = 0$

$\Rightarrow \quad x = \mathbf{3} \Rightarrow x = \mathbf{-\frac{1}{2}}$

20. $x^2 + 2x = \frac{5}{4}$

$\Rightarrow \quad 4x^2 + 8x - 5 = 0$

$\Rightarrow \quad 4x^2 - 2x + 10x - 5 = 0$

$\Rightarrow \quad 2x(2x - 1) + 5(2x - 1) = 0$

$\Rightarrow \quad (2x - 1)(2x + 5) = 0$

$\Rightarrow \quad$ Either $(2x - 1) = 0$ or $(2x + 5) = 0$

$\Rightarrow \quad x = \mathbf{\frac{1}{2}}$ or $x = \mathbf{\frac{-5}{2}}$

21. $x^2 - 4 = \frac{5x}{3}$

$\Rightarrow \quad 3x^2 - 12 = 5x$

$\Rightarrow \quad 3x^2 - 5x - 12 = 0$

$\Rightarrow \quad 3x^2 - 9x + 4x - 12 = 0$

$\Rightarrow \quad 3x(x - 3) + 4(x - 3) = 0$

$\Rightarrow \quad (x - 3)(3x + 4) = 0$

$\Rightarrow \quad$ Either $(x - 3) = 0$ or $(3x + 4) = 0$

$\Rightarrow \quad x = \mathbf{3}$ or $x = \mathbf{\frac{-4}{3}}$

22. $4x - \frac{3}{x} = 11$

$\Rightarrow \quad 4x^2 - 3 = 11x$

$\Rightarrow \quad 4x^2 - 11x - 3 = 0$

$\Rightarrow \quad 4x^2 - 12x + x - 3 = 0$

$\Rightarrow \quad 4x(x - 3) + 1(x - 3) = 0$

$\Rightarrow \quad (x - 3)(4x + 1) = 0$

$\Rightarrow \quad$ Either $(x - 3) = 0$ or $(4x + 1) = 0$

$\Rightarrow \quad x = \mathbf{3}$ or $x = \mathbf{\frac{-1}{4}}$

23. $2x - 5 = \frac{3}{x}$

$\Rightarrow \quad 2x^2 - 5x - 3 = 0$

$\Rightarrow \quad 2x^2 - 6x + x - 3 = 0$

$\Rightarrow \quad 2x(x - 3) + 1(x - 3) = 0$

$\Rightarrow \quad (x-3)(2x+1) = 0$

$\Rightarrow \quad$ Either $(x-3) = 0$ or $(2x+1) = 0$

$\Rightarrow \quad x = \mathbf{3}$ or $x = -\mathbf{\frac{1}{2}}$

24. $9x + \frac{1}{x} = 6$

$\Rightarrow \quad 9x^2 + 1 = 6x$

$\Rightarrow \quad 9x^2 - 6x + 1 = 0$

$\Rightarrow \quad 9x^2 - 3x - 3x + 1 = 0$

$\Rightarrow \quad 3x(3x-1) - 1(3x-1) = 0$

$\Rightarrow \quad (3x-1)(3x-1) = 0$

$\Rightarrow \quad (3x-1) = 0$ and $(3x-1) = 0$

$\Rightarrow \quad x = \mathbf{\frac{1}{3}}$ and $x = \mathbf{\frac{1}{3}}$

25. $x + \frac{16}{x} = 8$

$\Rightarrow \quad x^2 + 16 = 8x$

$\Rightarrow \quad x^2 - 8x + 16 = 0$

$\Rightarrow \quad x^2 - 4x - 4x + 16 = 0$

$\Rightarrow \quad x(x-4) - 4(x-4) = 0$

$\Rightarrow \quad (x-4)(x-4) = 0$

$\Rightarrow \quad x - 4 = 0$ and $x - 4 = 0$

$\Rightarrow \quad x = \mathbf{4}$ and $x = \mathbf{4}$

26. $x + \frac{1}{x} = 3\frac{1}{3}$

$\Rightarrow \quad x + \frac{1}{x} = \frac{10}{3}$

$\Rightarrow \quad 3x^2 + 3 = 10x$

$\Rightarrow \quad 3x^2 - 10x + 3 = 0$

$\Rightarrow \quad 3x^2 - 9x - x + 3 = 0$

$\Rightarrow \quad 3x(x-3) - 1(x-3) = 0$

$\Rightarrow \quad (x-3)(3x-1) = 0$

$\Rightarrow \quad$ Either $(x-3) = 0$ or $(3x-1) = 0$

$\Rightarrow \quad x = \mathbf{3}$ or $x = \mathbf{\frac{1}{3}}$

27. $\left(x - \frac{1}{2}\right)^2 = 4$

$\Rightarrow \quad x^2 - 2 \times x \times \frac{1}{2} + \frac{1}{4} = 4$

$\Rightarrow \quad x^2 - x + \frac{1}{4} - 4 = 0$

$\Rightarrow \quad 4x^2 - 4x + 1 - 16 = 0$

$\Rightarrow \quad 4x^2 - 4x - 15 = 0$

$\Rightarrow \quad 4x^2 - 10x + 6x - 15 = 0$

$\Rightarrow \quad 2x(2x-5) + 3(2x-5) = 0$

$\Rightarrow \quad (2x-5)(2x+3) = 0$

$\Rightarrow \quad$ Either $(2x-5) = 0$ or $(2x+3) = 0$

$\Rightarrow \quad x = \mathbf{\frac{5}{2}}$ or $x = \mathbf{\frac{-3}{2}}$

28. $\frac{3}{x^2} + \frac{14}{x} + 8 = 0$

$\Rightarrow \quad 3 + 14x + 8x^2 = 0$

$\Rightarrow \quad 8x^2 + 12x + 2x + 3 = 0$

$\Rightarrow \quad 4x(2x+3) + 1(2x+3) = 0$

$\Rightarrow \quad (2x+3)(4x+1) = 0$

Either $(2x+3) = 0$ or $(4x+1) = 0$

$\Rightarrow \quad x = \mathbf{\frac{-3}{2}}$ or $x = \mathbf{\frac{-1}{4}}$

29. $\frac{x^2}{7} = \frac{2}{21}x + 1$

$\Rightarrow \quad 3x^2 = 2x + 21$

$\Rightarrow \quad 3x^2 - 2x - 21 = 0$

$\Rightarrow \quad 3x^2 - 9x + 7x - 21 = 0$

$\Rightarrow \quad 3x(x-3) + 7(x-3) = 0$

$\Rightarrow \quad (x-3)(3x+7) = 0$

$\Rightarrow \quad$ Either $(x-3) = 0$ or $(3x+7) = 0$

$\Rightarrow \quad x = \mathbf{3}$ or $x = \mathbf{\frac{-7}{3}}$

30. $\frac{4}{9}x^2 - \frac{4}{3}x + 1 = 0$

$\Rightarrow \quad 4x^2 - 12x + 9 = 0$

$\Rightarrow \quad 4x^2 - 6x - 6x + 9 = 0$

$\Rightarrow \quad 2x(2x-3) - 3(2x-3) = 0$

$\Rightarrow \quad (2x-3)(2x-3) = 0$

$\Rightarrow \quad (2x-3) = 0$ and $(2x-3) = 0$

$\Rightarrow \quad x = \mathbf{\frac{3}{2}}$ and $x = \mathbf{\frac{3}{2}}$

31. $4x^2 - 9 - 2(2x-3) + x(2x-3) = 0$

$\Rightarrow \quad 4x^2 - 9 - 4x + 6 + 2x^2 - 3x = 0$

$\Rightarrow \quad 6x^2 - 7x - 3 = 0$

$\Rightarrow \quad 6x^2 - 9x + 2x - 3 = 0$

$\Rightarrow \quad 3x(2x-3) + 1(2x-3) = 0$

$\Rightarrow \quad (2x-3)(3x+1) = 0$

Either $(2x-3) = 0$ or $(3x+1) = 0$

$\Rightarrow \quad x = \mathbf{\frac{3}{2}}$ or $x = -\mathbf{\frac{1}{3}}$

32. $2(x+1)^2 - 5(x+1) = 12$

$\Rightarrow \quad 2(x^2 + 2x + 1) - 5x - 5 - 12 = 0$

$\Rightarrow \quad 2x^2 + 4x + 2 - 5x - 5 - 12 = 0$

$\Rightarrow \quad 2x^2 - x - 15 = 0$

$\Rightarrow \quad 2x^2 - 6x + 5x - 15 = 0$

$\Rightarrow \quad 2x(x-3) + 5(x-3) = 0$

$\Rightarrow \quad (x-3)(2x+5) = 0$

$\Rightarrow \quad$ Either $(x-3) = 0$ or $(2x+5) = 0$

$\Rightarrow \quad x = \mathbf{3} \Rightarrow x = \mathbf{\frac{-5}{2}}$

33. $a(x^2 + 1) + (a^2 + 1)x = 0$

$\Rightarrow \quad ax^2 + a + a^2x + x = 0$

$\Rightarrow \quad ax^2 + a^2x + x + a = 0$

$\Rightarrow \quad ax(x + a) + 1(x + a) = 0$

$\Rightarrow \quad (x + a)(ax + 1) = 0$

$\Rightarrow$ Either $(x + a) = 0$ or $(ax + 1) = 0$

$\Rightarrow \quad x = \mathbf{-a}$ or $x = \mathbf{\frac{-1}{a}}$

34. $abx^2 + (b^2 - ac)x - bc = 0$

$\Rightarrow \quad abx^2 + b^2x - acx - bc = 0$

$\Rightarrow \quad bx(ax + b) - c(ax + b) = 0$

$\Rightarrow \quad (ax + b)(bx - c) = 0$

$\Rightarrow$ Either $(ax + b) = 0$ or $(bx - c) = 0$

$\Rightarrow \quad x = \mathbf{\frac{-b}{a}}$ or $x = \mathbf{\frac{c}{b}}$

35. $4x^2 - 2(a^2 + b^2)x + a^2b^2 = 0$

$\Rightarrow \quad 4x^2 - 2a^2x - 2b^2x + a^2b^2 = 0$

$\Rightarrow \quad 2x(2x - a^2) - b^2(2x - a^2) = 0$

$\Rightarrow \quad (2x - a^2)(2x - b^2) = 0$

$\Rightarrow$ Either $(2x - a^2) = 0$ or $(2x - b^2) = 0$

$\Rightarrow \quad x = \mathbf{\frac{a^2}{2}}$ or $x = \mathbf{\frac{b^2}{2}}$

36. $\frac{1}{a+b+x} = \frac{1}{a} + \frac{1}{b} + \frac{1}{x}$

$\Rightarrow \quad \frac{1}{a+b+x} - \frac{1}{x} = \frac{1}{a} + \frac{1}{b}$

$\Rightarrow \quad \frac{x-a-b-x}{x(a+b+x)} = \frac{b+a}{ab}$

$\Rightarrow \quad \frac{-(a+b)}{x(a+b+x)} = \frac{(a+b)}{ab}$

$\Rightarrow \quad -ab = (a + b + x)x$

$\Rightarrow \quad ax + bx + x^2 + ab = 0$

$\Rightarrow \quad x^2 + bx + ax + ab = 0$

$\Rightarrow \quad x(x + b) + a(x + b) = 0$

$\Rightarrow \quad (x + b)(x + a) = 0$

$\Rightarrow$ Either $(x + b) = 0$ or $(x + a) = 0$

$\Rightarrow \quad x = \mathbf{-b}$ or $x = \mathbf{-a}$

37. $\frac{1}{x-3} - \frac{1}{x+5} = \frac{1}{6}$;

$\Rightarrow \quad \frac{(x+5)-(x-3)}{(x-3)(x+5)} = \frac{1}{6}$

$\Rightarrow \quad 6(x + 5 - x + 3) = (x - 3)(x + 5)$

$\Rightarrow \quad 48 = x^2 + 5x - 3x - 15$

$\Rightarrow \quad x^2 + 2x - 63 = 0$

$\Rightarrow \quad x^2 - 7x + 9x - 63 = 0$

$\Rightarrow \quad x(x - 7) + 9(x - 7) = 0$

$\Rightarrow \quad (x - 7)(x + 9) = 0$

$\Rightarrow \quad x = \mathbf{7}$ or $\mathbf{-9}$

38. $\frac{1}{x-5} - \frac{1}{x+1} = \frac{6}{7}$

$\Rightarrow \quad \frac{(x+1)-(x-5)}{(x+1)(x-5)} = \frac{6}{7}$

$\Rightarrow \quad \frac{x+1-x+5}{x^2+x-5x-5} = \frac{6}{7}$

$\Rightarrow \quad \frac{6}{x^2-4x-5} = \frac{6}{7}$

$\Rightarrow \quad 7 = x^2 - 4x - 5$

$\Rightarrow \quad x^2 - 4x - 12 = 0$

$\Rightarrow \quad x^2 - 6x + 2x - 12 = 0$

$\Rightarrow \quad x(x - 6) + 2(x - 6) = 0$

$\Rightarrow \quad (x - 6)(x + 2) = 0$

$\Rightarrow$ Either $(x - 6) = 0$ or $(x + 2) = 0$

$\Rightarrow \quad x = \mathbf{6}$ or $x = \mathbf{-2}$

39. $\frac{1}{x-2} + \frac{1}{x} = \frac{8}{2x+5}$

$\Rightarrow \quad \frac{x+x-2}{x(x-2)} = \frac{8}{2x+5}$

$\Rightarrow \quad (2x - 2)(2x + 5) = 8x(x - 2)$

$\Rightarrow \quad 4x^2 - 4x + 10x - 10 = 8x^2 - 16x$

$\Rightarrow \quad 8x^2 - 4x^2 - 16x + 4x - 10x + 10 = 0$

$\Rightarrow \quad 4x^2 - 22x + 10 = 0$

$\Rightarrow \quad 2x^2 - 11x + 5 = 0$

$\Rightarrow \quad 2x^2 - x - 10x + 5 = 0$

$\Rightarrow \quad x(2x - 1) - 5(2x - 1) = 0$

$\Rightarrow \quad (2x - 1)(x - 5) = 0$

$\Rightarrow$ Either $(2x - 1) = 0$ or $(x - 5) = 0$

$\Rightarrow \quad x = \mathbf{\frac{1}{2}}$ or $x = \mathbf{5}$

40. $\frac{2}{x+1} + \frac{3}{2(x-2)} = \frac{23}{5x}$

$\Rightarrow \quad \frac{4(x-2)+3(x+1)}{2(x-2)(x+1)} = \frac{23}{5x}$

$\Rightarrow \quad \frac{4x-8+3x+3}{2(x-2)(x+1)} = \frac{23}{5x}$

$\Rightarrow \quad 5x(7x - 5) = 23 \times 2(x^2 + x - 2x - 2)$

$\Rightarrow \quad 35x^2 - 25x = 46x^2 - 46x - 92$

$\Rightarrow \quad 11x^2 - 21x - 92 = 0$

$\Rightarrow \quad x^2 - \frac{21}{11}x - \frac{92}{11} = 0$

$\Rightarrow \quad x^2 + \frac{23}{11}x - 4x - \frac{92}{11} = 0$

$\Rightarrow \quad x\left(x + \frac{23}{11}\right) - 4\left(x + \frac{23}{11}\right) = 0$

$\Rightarrow \quad (x - 4)\left(x + \frac{23}{11}\right) = 0$

$\Rightarrow \quad x = \mathbf{4}$ or $x = \mathbf{-\frac{23}{11}}$

41. $\frac{4}{z-1}-\frac{5}{z+2} = \frac{3}{z}$

$\Rightarrow \frac{4(z+2)-5(z-1)}{(z-1)(z+2)} = \frac{3}{z}$

$\Rightarrow \frac{4z+8-5z+5}{(z^2-z+2z-2)} = \frac{3}{z}$

$\Rightarrow (13-z)z = 3(z^2+z-2)$

$\Rightarrow 13z - z^2 = 3z^2 + 3z - 6$

$\Rightarrow 3z^2 + z^2 + 3z - 13z - 6 = 0$

$\Rightarrow 4z^2 - 10z - 6 = 0$

$\Rightarrow 2z^2 - 5z - 3 = 0$

$\Rightarrow 2z^2 - 6z + z - 3 = 0$

$\Rightarrow 2z(z-3) + 1(z-3) = 0$

$\Rightarrow (z-3)(2z+1) = 0$

$\Rightarrow$ Either $(z-3) = 0$ or $(2z+1) = 0$

$\Rightarrow z = \mathbf{3}$ or $z = -\mathbf{\frac{1}{2}}$

42. $\frac{1}{x}+\frac{2}{2x-3}=\frac{1}{x-2}$

$\Rightarrow \frac{2x-3+2x}{x(2x-3)} = \frac{1}{x-2}$

$\Rightarrow (x-2)(4x-3) = x(2x-3)$

$\Rightarrow 4x^2 - 11x + 6 = 2x^2 - 3x$

$\Rightarrow 2x^2 - 8x + 6 = 0$

$\Rightarrow x^2 - 4x + 3 = 0$

$\Rightarrow x^2 - x - 3x + 3 = 0$

$\Rightarrow x(x-1) - 3(x-1) = 0$

$\Rightarrow (x-3)(x-1) = 0$

$\Rightarrow x = \mathbf{1}$, or $x = \mathbf{3}$

43. $\frac{3}{x+1}+\frac{4}{x-1} = \frac{29}{4x-1}$

$\Rightarrow \frac{3(x-1)+4(x+1)}{(x+1)(x-1)} = \frac{29}{4x-1}$

$\Rightarrow \frac{3x-3+4x+4}{(x+1)(x-1)} = \frac{29}{4x-1}$

$\Rightarrow (4x-1)(7x+1) = 29(x^2-1)$

$\Rightarrow 28x^2 - 3x - 1 = 29x^2 - 29$

$\Rightarrow x^2 + 3x - 28 = 0$

$\Rightarrow x^2 - 4x + 7x - 28 = 0$

$\Rightarrow x(x-4) + 7(x-4) = 0$

$\Rightarrow (x-4)(x+7) = 0$

$\Rightarrow x = \mathbf{4}$, or $x = \mathbf{-7}$

44. $\frac{x-3}{x-4}+\frac{x-5}{x-6} = \frac{10}{3}$

$\Rightarrow \frac{(x-3)(x-6)+(x-5)(x-4)}{(x-4)(x-6)} = \frac{10}{3}$

$\Rightarrow 3(x^2 - 9x + 18 + x^2 - 9x + 20) = 10(x^2 - 10x + 24)$

$\Rightarrow 6x^2 - 54x + 114 = 10x^2 - 100x + 240$

$\Rightarrow 4x^2 - 46x + 126 = 0$

$\Rightarrow x^2 - \frac{23x}{2} + \frac{63}{2} = 0$

$\Rightarrow x^2 - 7x - \frac{9x}{2} + \frac{63}{2} = 0$

$\Rightarrow x(x-7) - \frac{9}{2}(x-7) = 0$

$\Rightarrow (x-7)\left(x-\frac{9}{2}\right) = 0$

$\Rightarrow x = \mathbf{7}$ or $x = \mathbf{\frac{9}{2}}$

45. $\frac{x-1}{2x+1}+\frac{2x+1}{x-1} = 2$

$\Rightarrow \frac{(x-1)^2+(2x+1)^2}{(x-1)(2x+1)} = 2$

$\Rightarrow x^2 - 2x + 1 + 4x^2 + 1 + 4x = 4x^2 - 2x - 2$

$\Rightarrow 5x^2 + 2x + 2 = 4x^2 - 2x - 2$

$\Rightarrow x^2 + 4x + 4 = 0$

$\Rightarrow x^2 + 2x + 2x + 4 = 0$

$\Rightarrow x(x+2) + 2(x+2) = 0$

$\Rightarrow (x+2)^2 = 0$

$\Rightarrow x = \mathbf{-2}$ or $x = \mathbf{-2}$

46. $\frac{16}{x}-1 = \frac{15}{x+1}$

$\Rightarrow \frac{16-x}{x} = \frac{15}{x+1}$

$\Rightarrow (16-x)(x+1) = 15x$

$\Rightarrow 16x + 16 - x^2 - x = 15x$

$\Rightarrow 15x + 16 - x^2 = 15x$

$\Rightarrow x^2 - 16 = 0$

$\Rightarrow x^2 = 16$

$\Rightarrow x = \mathbf{-4}$ or $x = \mathbf{4}$

47. $\frac{x+1}{x-1}+\frac{x-2}{x+2} = 4 - \frac{2x+3}{x-2}$

$\Rightarrow \frac{(x+1)(x+2)+(x-2)(x-1)}{(x-1)(x+2)} = \frac{4(x-2)-(2x+3)}{x-2}$

$\Rightarrow \frac{x^2+3x+2+x^2-3x+2}{(x-1)(x+2)} = \frac{4x-8-2x-3}{x-2}$

$\Rightarrow \frac{(2x^2+4)}{(x-1)(x+2)} = \frac{2x-11}{x-2}$

$\Rightarrow (2x^2+4)(x-2) = (2x-11)(x-1)(x+2)$

$\Rightarrow 2x^3 - 4x^2 + 4x - 8 = (2x-11)(x^2+x-2)$

$\Rightarrow 2x^3 - 4x^2 + 4x - 8 = 2x^3 + 2x^2 - 4x - 11x^2 - 11x + 22$

$\Rightarrow -4x^2 + 4x - 8 = -9x^2 - 15x + 22$

$\Rightarrow 5x^2 + 19x - 30 = 0$

$\Rightarrow x^2 + \frac{19x}{5} - 6 = 0$

$\Rightarrow x^2 - \frac{6x}{5} + 5x - 6 = 0$

$\Rightarrow x\left(x-\frac{6}{5}\right)+5\left(x-\frac{6}{5}\right)=0$

$\Rightarrow (x+5)\left(x-\frac{6}{5}\right)=0$

$\Rightarrow x = \mathbf{-5}$ or $x = \mathbf{\frac{6}{5}}$

48. $2\left(\frac{2x-1}{x+3}\right)-3\left(\frac{x+3}{2x-1}\right)=5$

Let $\frac{2x-1}{x+3}=y$. Then $\frac{x+3}{2x-1}=\frac{1}{y}$.

The given equation becomes

$$2y-\frac{3}{y}=5$$

$\Rightarrow 2y^2-3=5y$

$\Rightarrow 2y^2-5y-3=0$

$\Rightarrow 2y^2-6y+y-3=0$

$\Rightarrow 2y(y-3)+1(y-3)=0$

$\Rightarrow (y-3)(2y+1)=0$

$\Rightarrow$ Either $(y-3)=0$ or $(2y+1)=0$

$\Rightarrow y=3$ or $y=\frac{-1}{2}$

$\Rightarrow \frac{2x-1}{x+3}=3$ or $\frac{2x-1}{x+3}=\frac{-1}{2}$ $\left[\because y=\frac{2x-1}{x+3}\right]$

$\Rightarrow 2x-1=3x+9$ or $2(2x-1)=-(x+3)$

$\Rightarrow 3x-2x+9+1=0$ or $4x-2=-x-3$

$\Rightarrow x+10=0$ or $4x+x-2+3=0$

$\Rightarrow x=\mathbf{-10}$ or $5x+1=0$ or $x=\mathbf{\frac{-1}{5}}$

49. $\frac{a}{x-a}+\frac{b}{x-b}=\frac{2c}{x-c}$

$\Rightarrow \frac{a(x-b)+b(x-a)}{(x-a)(x-b)}=\frac{2c}{x-c}$

$\Rightarrow a(x-b)(x-c)+b(x-a)(x-c)=2c(x-a)(x-b)$

$\Rightarrow a[x^2-x(b+c)+bc]+b[x^2-x(a+c)+ac]$
$=2c[x^2-x(a+b)+ab]$

$\Rightarrow x^2(a+b)-x[(b+c)a+(a+c)b]+2abc$
$=2cx^2-2cx(a+b)+2abc$

$\Rightarrow x^2(a+b-2c)-x[ab+ac+ab+bc-2ac-2bc]=0$

$\Rightarrow x[(a+b-2c)x-(2ab-ac-bc)]=0$

$\Rightarrow \mathbf{x=0}$ or $x=\mathbf{\frac{2ab-ac-bc}{a+b-2c}}$

50. $\frac{2x}{x-3}+\frac{1}{2x+3}+\frac{3x+9}{(x-3)(2x+3)}=0$

$2x(2x+3)+(x-3)+3x+9=0$ [Multiplying the equation by $(x-3)(2x+3)$]

$\Rightarrow 4x^2+6x+x-3+3x+9=0$

$\Rightarrow 4x^2+10x+6=0$

$\Rightarrow 2x^2+5x+3=0$

$\Rightarrow 2x^2+2x+3x+3=0$

$\Rightarrow 2x(x+1)+3(x+1)=0$

$\Rightarrow (x+1)(2x+3)=0$

$\Rightarrow$ Either $(x+1)=0$ or $(2x+3)=0$

$\Rightarrow x=\mathbf{-1}$ or $x=\mathbf{\frac{-3}{2}}$

51. $\frac{1}{(x-1)(x-2)}+\frac{1}{(x-2)(x-3)}=\frac{2}{3}$

$\Rightarrow \frac{x-3+x-1}{(x-1)(x-2)(x-3)}=\frac{2}{3}$

$\Rightarrow 3(2x-4)=2(x-1)(x-2)(x-3)$

$\Rightarrow 3\times 2(x-2)=2(x-1)(x-2)(x-3)$

$\Rightarrow 3=x^2-4x+3$

$\Rightarrow x^2-4x=0$

$\Rightarrow x(x-4)=0$

$\Rightarrow x=\mathbf{0}$ or $\mathbf{x=4}$

For Standard Level

52. $x^2-(\sqrt{3}+1)x+\sqrt{3}=0$

$\Rightarrow x(x-\sqrt{3})-1(x-\sqrt{3})=0$

$\Rightarrow (x-\sqrt{3})(x-1)=0$

$\Rightarrow x=\mathbf{\sqrt{3}}$ or $x=\mathbf{1}$

53. $3x^2-2\sqrt{6}x+2=0$

$\Rightarrow 3x^2-\sqrt{6}x-\sqrt{6}x+2=0$

$\Rightarrow \sqrt{3}x(\sqrt{3}x-\sqrt{2})-\sqrt{2}(\sqrt{3}x-\sqrt{2})=0$

$\Rightarrow (\sqrt{3}x-\sqrt{2})(\sqrt{3}x-\sqrt{2})=0$

$\Rightarrow (\sqrt{3}x-\sqrt{2})=0$ or $(\sqrt{3}x-\sqrt{2})=0$

$\Rightarrow x=\mathbf{\frac{\sqrt{2}}{\sqrt{3}}}$ or $x=\mathbf{\frac{\sqrt{2}}{\sqrt{3}}}$

or $x=\frac{\sqrt{2}}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}=\frac{\sqrt{6}}{3}$ or $x=\frac{\sqrt{2}}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}=\frac{\sqrt{6}}{3}$

54. $3x^2+7\sqrt{5}x-30=0$

$\Rightarrow 3x^2+9\sqrt{5}x-2\sqrt{5}x-30=0$

$\Rightarrow 3x(x+3\sqrt{5})-2\sqrt{5}(x+3\sqrt{5})=0$

$\Rightarrow (x+3\sqrt{5})(3x-2\sqrt{5})=0$

$\Rightarrow$ Either $(x+3\sqrt{5})=0$ or $(3x-2\sqrt{5})=0$

$\Rightarrow x=\mathbf{-3\sqrt{5}}$ or $x=\mathbf{\frac{2\sqrt{5}}{3}}$

55. $x^2+4\sqrt{2}x+6=0$

$\Rightarrow x^2+\sqrt{2}x+3\sqrt{2}x+6=0$

$\Rightarrow x(x+\sqrt{2})+3\sqrt{2}(x+\sqrt{2})=0$

$\Rightarrow (x+\sqrt{2})(x+3\sqrt{2})=0$

$\Rightarrow$ Either $(x+\sqrt{2})=0$ or $(x+3\sqrt{2})=0$

$\Rightarrow x=\mathbf{-\sqrt{2}}$ or $x=\mathbf{-3\sqrt{2}}$

56. $4\sqrt{3}x^2 + 5x - 2\sqrt{3} = 0$

$\Rightarrow \quad 4\sqrt{3}x^2 + 8x - 3x - 2\sqrt{3} = 0$

$\Rightarrow \quad 4x(\sqrt{3}x + 2) - \sqrt{3}(\sqrt{3}x + 2) = 0$

$\Rightarrow \quad (\sqrt{3}x + 2)(4x - \sqrt{3}) = 0$

$\Rightarrow$ Either $(\sqrt{3}x + 2) = 0$ or $(4x - \sqrt{3}) = 0$

$\Rightarrow \quad x = \mathbf{\frac{-2}{\sqrt{3}}}$ or $x = \mathbf{\frac{\sqrt{3}}{4}}$

57. $3\sqrt{5}x^2 + 2x - \sqrt{5} = 0$

$\Rightarrow \quad 3\sqrt{5}x^2 - 3x + 5x - \sqrt{5} = 0$

$\Rightarrow \quad 3x(\sqrt{5}x - 1) + \sqrt{5}(\sqrt{5}x - 1) = 0$

$\Rightarrow \quad (\sqrt{5}x - 1)(3x + \sqrt{5}) = 0$

$\Rightarrow$ Either $(\sqrt{5}x - 1) = 0$ or $(3x + \sqrt{5}) = 0$

$\Rightarrow \quad x = \mathbf{\frac{1}{\sqrt{5}}}$ or $x = \mathbf{\frac{-\sqrt{5}}{3}}$

58. $\sqrt{7}y^2 - 6y - 13\sqrt{7} = 0$

$\Rightarrow \quad \sqrt{7}y^2 - 13y + 7y - 13\sqrt{7} = 0$

$\Rightarrow \quad y(\sqrt{7}y - 13) + \sqrt{7}(\sqrt{7}y - 13) = 0$

$\Rightarrow \quad (\sqrt{7}y - 13)(y + \sqrt{7}) = 0$

$\Rightarrow$ Either $(\sqrt{7}y - 13) = 0$ or $(y + \sqrt{7}) = 0$

$\Rightarrow \quad y = \frac{13}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} = \mathbf{\frac{13\sqrt{7}}{7}}$ or $y = \mathbf{-\sqrt{7}}$

59. $3x(3x - 1) - 2 = 0$

$\Rightarrow \quad 9x^2 - 3x - 2 = 0$

$\Rightarrow \quad 9x^2 - 6x + 3x - 2 = 0$

$\Rightarrow \quad 3x(3x - 2) + 1(3x - 2) = 0$

$\Rightarrow \quad (3x - 2)(3x + 1) = 0$

$\Rightarrow$ Either $(3x - 2) = 0$ or $(3x + 1) = 0$

$\Rightarrow \quad x = \mathbf{\frac{2}{3}}$ or $x = \mathbf{\frac{-1}{3}}$

60. $x^2 - 2ax - (4b^2 - a^2) = 0$

$\Rightarrow \quad x^2 - x(a - 2b) - x(a + 2b) - (4b^2 - a^2) = 0$

$\Rightarrow \quad x(x - (a - 2b) - (a + 2b)[x + (2b - a)] = 0$

$\Rightarrow \quad [x - (a + 2b)][x - (a - 2b)] = 0$

$\Rightarrow \quad \mathbf{x = a + 2b, a - 2b}$

61. $4x^2 + 4bx - (a^2 - b^2) = 0$

$\Rightarrow \quad x^2 + bx - \frac{a^2 - b^2}{4} = 0$

$\Rightarrow \quad x^2 + \left(\frac{a+b}{2}\right)x - \left(\frac{a-b}{2}\right)x - \frac{(a+b)(a-b)}{4} = 0$

$\Rightarrow \quad x\left[x + \left(\frac{a+b}{2}\right)\right] - \left(\frac{a-b}{2}\right)\left[x + \left(\frac{a+b}{2}\right)\right] = 0$

$\Rightarrow \quad \left[x + \left(\frac{a+b}{2}\right)\right]\left[x - \left(\frac{a-b}{2}\right)\right] = 0$

$\Rightarrow \quad x = \mathbf{\frac{-a-b}{2}}$ or $\mathbf{\frac{a-b}{2}}$

62. $4x^2 - 4a^2x + (a^4 - b^4) = 0$

$\Rightarrow \quad 4x^2 - [2(a^2 + b^2)x + 2(a^2 - b^2)x] + (a^2 + b^2)(a^2 - b^2) = 0$

$\Rightarrow \quad 4x^2 - 2(a^2 + b^2)x - 2(a^2 - b^2)x + (a^2 + b^2)(a^2 - b^2) = 0$

$\Rightarrow \quad 2x[2x - (a^2 + b^2)] - (a^2 - b^2)[2x - (a^2 + b^2)] = 0$

$\Rightarrow \quad [2x - (a^2 + b^2)][2x - (a^2 - b^2)] = 0$

$\Rightarrow$ Either $2x - (a^2 + b^2) = 0$ or $2x - (a^2 - b^2) = 0$

$\Rightarrow \quad x = \mathbf{\frac{a^2 + b^2}{2}}$ or $x = \mathbf{\frac{a^2 - b^2}{2}}$

63. $x^2 + 6x - (a^2 + 2a - 8) = 0$

$\Rightarrow \quad x^2 + (a + 4)x - (a - 2)x - (a^2 + 2a - 8) = 0$

$\Rightarrow \quad [x - (a - 2)][x + (a + 4)] = 0$

$\Rightarrow \quad \mathbf{x = a - 2}$ or $\mathbf{-(a + 4)}$

64. $x^2 - (2b - 1)x + (b^2 - b - 20) = 0$

$\Rightarrow \quad x^2 - (b - 5)x - (b + 4)x + (b^2 - b - 20) = 0$

$\Rightarrow \quad x[x - (b - 5)] - (b + 4)[x - (b - 5)] = 0$

$\Rightarrow \quad [x - (b - 5)][x - (b + 4)] = 0$

$\Rightarrow \quad x = \mathbf{b - 5}$ or $\mathbf{b + 4}$

65. (*i*) $16 \times 4^{x+2} - 16 \times 2^{x+1} + 1 = 0$

$\Rightarrow \quad 16 \times 2^{2(x+2)} - 16 \times 2^{x+1} + 1 = 0$

$\Rightarrow \quad 16 \times 2^{2x} \times 2^4 - 16 \times 2^x \times 2^1 + 1 = 0$

$\Rightarrow \quad 256 \times (2^x)^2 - 32(2^x) + 1 = 0$

Let $2^x = y$

$\Rightarrow \quad 256y^2 - 32y + 1 = 0$

$\Rightarrow \quad 256y^2 - 16y - 16y + 1 = 0$

$\Rightarrow \quad 16y(16y - 1) - 1(16y - 1) = 0$

$\Rightarrow \quad (16y - 1)(16y - 1) = 0$

$\Rightarrow \quad (16y - 1) = 0$ and $(16y - 1) = 0$

$\Rightarrow \quad y = \frac{1}{16}$ and $y = \frac{1}{16}$

$\Rightarrow \quad 2^x = \frac{1}{16} = \frac{1}{2^4} \quad [\because y = 2^x]$

$\Rightarrow \quad 2^x = 2^{-4}$

$\Rightarrow \quad x = \mathbf{-4}$

(*ii*) $5^{4x} - 3 \times 5^{2x+1} = 250$

$\Rightarrow \quad 5^{4x} - 3 \times 5^{2x} \times 5^1 = 250$

$\Rightarrow \quad (5^{2x})^2 - 15(5^{2x}) - 250 = 0$

Let $5^{2x} = y$

Then, the given equation becomes

$y^2 - 15y - 250 = 0$

$\Rightarrow \quad y^2 - 25y + 10y - 250 = 0$

$\Rightarrow \quad y(y - 25) + 10(y - 25) = 0$

$\Rightarrow \quad (y - 25)(y + 10) = 0$

$\Rightarrow$ Either $(y - 25) = 0$ or $(y + 10) = 0$

$\Rightarrow \quad y = 25$ or $y = -10$ (rejected)

$\Rightarrow \quad 5^{2x} = 5^2 \quad [\because y = 5^{2x}]$

$\Rightarrow \quad 2x = 2$

$\Rightarrow \quad x = \mathbf{1}$

66. $\sqrt{2x+9} + x = 13 \quad \ldots (1)$

$\sqrt{2x+9} = 13 - x$

On squaring both the sides, we get

$\Rightarrow \quad 2x + 9 = (13 - x)^2$

$\Rightarrow \quad 2x + 9 = 169 + x^2 - 26x$

$\Rightarrow \quad x^2 - 28x + 160 = 0$

$\Rightarrow \quad x^2 - 8x - 20x + 160 = 0$

$\Rightarrow \quad x(x - 8) - 20(x - 8) = 0$

$\Rightarrow \quad (x - 8)(x - 20) = 0$

$\Rightarrow \quad \mathbf{x = 8} \text{ or } \mathbf{x = 20}$

Since $x = 20$ does not satisfy the equation (1), therefore, we neglect it.

Thus, $x = 8$

67. $\sqrt{6x+7} - (2x - 7) = 0 \quad \ldots (1)$

$\Rightarrow \quad \sqrt{6x+7} = 2x - 7$

On squaring both the sides, we get

$6x + 7 = (2x - 7)^2$

$\Rightarrow \quad 6x + 7 = 4x^2 + 49 - 28x$

$\Rightarrow \quad 4x^2 - 34x + 42 = 0$

$\Rightarrow \quad x^2 - \frac{17x}{2} + \frac{21}{2} = 0$

$\Rightarrow \quad x^2 - \frac{3x}{2} - 7x + \frac{21}{2} = 0$

$\Rightarrow \quad x\left(x - \frac{3}{2}\right) - 7\left(x - \frac{3}{2}\right) = 0$

$\Rightarrow \quad (x - 7)\left(x - \frac{3}{2}\right) = 0$

$\Rightarrow \quad \mathbf{x = 7} \text{ or } \mathbf{x = \frac{3}{2}}$

But $x = \frac{3}{2}$ does not satisfy the equation (1). Hence, we will neglect it.

Thus, $x = 7$

EXERCISE 4C

For Basic and Standard Levels

1. $3x^2 + 11x + 6 = 0$

$\Rightarrow \quad x^2 + \frac{11}{3}x + 2 = 0 \quad$ [Dividing the equation by 3]

$\Rightarrow \quad x^2 + \frac{11}{3}x = -2$

$\Rightarrow \quad x^2 + 2\left(\frac{11}{6}\right)x + \left(\frac{11}{6}\right)^2 = \left(\frac{11}{6}\right)^2 - 2$

$\Rightarrow \quad \left(x + \frac{11}{6}\right)^2 = \frac{121}{36} - 2$

$\Rightarrow \quad \left(x + \frac{11}{6}\right)^2 = \frac{121 - 72}{36} = \frac{49}{36}$

$\Rightarrow \quad x + \frac{11}{6} = \pm\left(\frac{7}{6}\right)$

$\Rightarrow \quad \text{Either } x + \frac{11}{6} = \frac{7}{6} \text{ or } x + \frac{11}{6} = \frac{-7}{6}$

$\Rightarrow \quad x = \frac{7}{6} - \frac{11}{6} \text{ or } x = \frac{-7}{6} - \frac{11}{6}$

$\Rightarrow \quad x = \frac{-4}{6} = \mathbf{\frac{-2}{3}} \text{ or } x = \frac{-18}{6} = \mathbf{-3}$

2. $x^2 - 5x + 6 = 0$

$\Rightarrow \quad x^2 - 5x = -6$

$\Rightarrow \quad x^2 - 2\left(\frac{5}{2}\right)x + \left(\frac{5}{2}\right)^2 = -6 + \left(\frac{5}{2}\right)^2$

$\Rightarrow \quad \left(x - \frac{5}{2}\right)^2 = -6 + \frac{25}{4} = \frac{-24 + 25}{4} = \frac{1}{4}$

$\Rightarrow \quad x - \frac{5}{2} = \pm\frac{1}{2}$

$\Rightarrow \quad \text{Either } x - \frac{5}{2} = \frac{1}{2} \text{ or } x - \frac{5}{2} = \frac{-1}{2}$

$\Rightarrow \quad x = \frac{5}{2} + \frac{1}{2} \text{ or } x = \frac{-1}{2} + \frac{5}{2}$

$\Rightarrow \quad x = \frac{6}{2} \text{ or } x = \frac{4}{2}$

$\Rightarrow \quad x = \mathbf{3} \text{ or } x = \mathbf{2}$

3. $2x + 8 = 3x^2$

$\Rightarrow \quad 3x^2 - 2x - 8 = 0$

$\Rightarrow \quad 3x^2 - 2x = 8$

$\Rightarrow \quad x^2 - \frac{2}{3}x = \frac{8}{3} \quad$ [Dividing the equation by 3]

$\Rightarrow \quad x^2 - 2\left(\frac{1}{3}\right)x + \left(\frac{1}{3}\right)^2 = \frac{8}{3} + \left(\frac{1}{3}\right)^2$

$\Rightarrow \quad \left(x - \frac{1}{3}\right)^2 = \frac{8}{3} + \frac{1}{9} = \frac{24 + 1}{9} = \frac{25}{9}$

$\Rightarrow \quad x - \frac{1}{3} = \pm\frac{5}{3}$

$\Rightarrow \quad \text{Either } x - \frac{1}{3} = \frac{5}{3} \text{ or } x - \frac{1}{3} = \frac{-5}{3}$

$\Rightarrow \quad x = \frac{5}{3} + \frac{1}{3} \text{ or } x = \frac{-5}{3} + \frac{1}{3}$

$\Rightarrow \quad x = \frac{6}{3} = \mathbf{2} \text{ or } x = \frac{-5 + 1}{3} = \mathbf{\frac{-4}{3}}$

4. $2x^2 + 5x + 7 = 0$

$\Rightarrow \quad x^2 + \frac{5}{2}x + \frac{7}{2} = 0 \quad$ [Dividing the equation by 2]

$\Rightarrow \quad x^2 + \frac{5}{2}x = \frac{-7}{2}$

$$\Rightarrow \quad x^2 + 2\left(\frac{5}{4}\right)x + \left(\frac{5}{4}\right)^2 = \frac{-7}{2} + \left(\frac{5}{4}\right)^2$$

$$\Rightarrow \quad \left(x + \frac{5}{4}\right)^2 = \frac{-7}{2} + \frac{25}{16}$$

$$= \frac{-56 + 25}{16} = \frac{-31}{16}$$

Clearly RHS is negative but $\left(x + \frac{5}{4}\right)^2$ cannot be negative for any real value of x.

So, there is no real value of x satisfying the given equation.

Hence, the given equation has no real roots.

5. $x^2 - 3x - 10 = 0$

$$\Rightarrow \quad x^2 - 3x = 10$$

$$\Rightarrow \quad x^2 - 2\left(\frac{3}{2}\right)x + \left(\frac{3}{2}\right)^2 = 10 + \left(\frac{3}{2}\right)^2$$

$$\Rightarrow \quad \left(x - \frac{3}{2}\right)^2 = 10 + \frac{9}{4} = \frac{49}{4}$$

$$\Rightarrow \quad x - \frac{3}{2} = \pm\frac{7}{2}$$

$$\Rightarrow \quad \text{Either } x - \frac{3}{2} = \frac{7}{2} \quad \text{or} \quad x - \frac{3}{2} = \frac{-7}{2}$$

$$\Rightarrow \quad x = \frac{7}{2} + \frac{3}{2} \quad \text{or} \quad x = \frac{-7}{2} + \frac{3}{2}$$

$$\Rightarrow \quad x = \frac{10}{2} = \mathbf{5} \quad \text{or} \quad x = \frac{-4}{2} = \mathbf{-2}$$

6. $4x^2 - 2x + \frac{1}{4} = 0$

$$\Rightarrow \quad 4x^2 - 2x = \frac{-1}{4}$$

$$\Rightarrow \quad x^2 - \frac{2}{4}x = \frac{-1}{16} \quad \text{[Dividing the equation by 4]}$$

$$\Rightarrow \quad x^2 - 2\left(\frac{1}{4}\right)x + \left(\frac{1}{4}\right)^2 = \left(\frac{1}{4}\right)^2 - \frac{1}{16}$$

$$\Rightarrow \quad \left(x - \frac{1}{4}\right)^2 = \frac{1}{16} - \frac{1}{16} = 0$$

$$\Rightarrow \quad \left(x - \frac{1}{4}\right)\left(x - \frac{1}{4}\right) = 0$$

$$\Rightarrow \quad \left(x - \frac{1}{4}\right) = 0 \text{ and } \left(x - \frac{1}{4}\right) = 0$$

$$\Rightarrow \quad x = \mathbf{\frac{1}{4}} \text{ and } x = \mathbf{\frac{1}{4}}$$

7. The given equation is $5x^2 - 6x - 2 = 0$

Here the discriminant $= \sqrt{(-6)^2 - 4 \times 5(-2)}$

$$= \sqrt{36 + 40}$$

$$= \sqrt{76} > 0$$

$\therefore$ The given quadratic equation has real roots.

We have $\quad 5x^2 - 6x - 2 = 0$

$$\Rightarrow \quad x^2 - \frac{6}{5}x - \frac{2}{5} = 0$$

$$\Rightarrow \quad \left(x - \frac{3}{5}\right)^2 - \frac{9}{25} - \frac{2}{5} = 0$$

$$\Rightarrow \quad \left(x - \frac{3}{5}\right)^2 = \frac{9 + 10}{25} = \frac{19}{25}$$

$$\therefore \quad x - \frac{3}{5} = \pm\frac{\sqrt{19}}{5}$$

$$\Rightarrow \quad x = \frac{3 \pm \sqrt{19}}{5}$$

Verification: We have

$$x = \frac{3 \pm \sqrt{19}}{5}$$

$$\text{LHS} = 5 \times \left(\frac{3 \pm \sqrt{19}}{5}\right)^2 - 6 \times \frac{3 \pm \sqrt{19}}{5} - 2$$

$$= \frac{9 + 19 \pm 6\sqrt{19}}{5} - \frac{18 \pm 6\sqrt{19}}{5} - 2$$

$$= \frac{28 \pm 6\sqrt{19} - 18 \mp 6\sqrt{19} - 10}{5}$$

$$= \frac{28 - 28 \pm \left(6\sqrt{19} - 6\sqrt{19}\right)}{6} = 0 = \text{RHS}$$

Hence, $x = \frac{3 \pm \sqrt{19}}{5}$ satisfy the given equation and so these are the required solution.

For Standard Level

8. $4\sqrt{3}x^2 + 5x - 2\sqrt{3} = 0$

$$\Rightarrow \quad 4\sqrt{3}x^2 + 5x = 2\sqrt{3}$$

$$\Rightarrow \quad x^2 + \frac{5}{4\sqrt{3}}x = \frac{2\sqrt{3}}{4\sqrt{3}} \quad \text{[Dividing the equation by } 4\sqrt{3}\text{]}$$

$$\Rightarrow \quad x^2 + 2\left(\frac{5}{8\sqrt{3}}\right)x + \left(\frac{5}{8\sqrt{3}}\right)^2 = \frac{1}{2} + \left(\frac{5}{8\sqrt{3}}\right)^2$$

$$\Rightarrow \quad \left(x + \frac{5}{8\sqrt{3}}\right)^2 = \frac{1}{2} + \frac{25}{64 \times 3}$$

$$= \frac{1}{2} + \frac{25}{192}$$

$$= \frac{96 + 25}{192} = \frac{121}{192}$$

$$\Rightarrow \quad x + \frac{5}{8\sqrt{3}} = \pm\frac{11}{8\sqrt{3}}$$

$$\Rightarrow \quad \text{Either } x + \frac{5}{8\sqrt{3}} = \frac{11}{8\sqrt{3}} \quad \text{or } x + \frac{5}{8\sqrt{3}} = \frac{-11}{8\sqrt{3}}$$

$$\Rightarrow \quad x = \frac{11}{8\sqrt{3}} - \frac{5}{8\sqrt{3}} \quad \text{or } x = \frac{-11}{8\sqrt{3}} - \frac{5}{8\sqrt{3}}$$

$$\Rightarrow \quad x = \frac{6}{8\sqrt{3}} \quad \text{or } x = \frac{-16}{8\sqrt{3}} = \frac{-2}{\sqrt{3}}$$

$$= \frac{3}{4\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$\Rightarrow \quad x = \frac{3\sqrt{3}}{12}$ or $x = \frac{-2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$x = \mathbf{\frac{\sqrt{3}}{4}}$ or $x = \mathbf{\frac{-2\sqrt{3}}{3}}$

9. $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

$\Rightarrow x^2 + \frac{7}{\sqrt{2}}x + \frac{5\sqrt{2}}{\sqrt{2}} = 0$ [Dividing the equation by $\sqrt{2}$]

$\Rightarrow \quad x^2 + \frac{7}{\sqrt{2}}x = -5$

$\Rightarrow \quad x^2 + 2\left(\frac{7}{2\sqrt{2}}\right)x + \left(\frac{7}{2\sqrt{2}}\right)^2 = -5 + \left(\frac{7}{2\sqrt{2}}\right)^2$

$\Rightarrow \quad \left(x + \frac{7}{2\sqrt{2}}\right)^2 = -5 + \frac{49}{4 \times 2}$

$$= -5 + \frac{49}{8}$$

$$= \frac{-40 + 49}{8} = \frac{9}{8}$$

$\Rightarrow \quad x + \frac{7}{2\sqrt{2}} = \pm \frac{3}{2\sqrt{2}}$

$\Rightarrow$ Either $x + \frac{7}{2\sqrt{2}} = \frac{3}{2\sqrt{2}}$ or $x + \frac{7}{2\sqrt{2}} = \frac{-3}{2\sqrt{2}}$

$\Rightarrow \quad x = \frac{3}{2\sqrt{2}} - \frac{7}{2\sqrt{2}}$ or $x = \frac{-3}{2\sqrt{2}} - \frac{7}{2\sqrt{2}}$

$\Rightarrow \quad x = \frac{-4}{2\sqrt{2}}$ or $x = \frac{-10}{2\sqrt{2}}$

$\Rightarrow \quad x = \frac{-2}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$ or $x = \frac{-5}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$

$\Rightarrow \quad x = \mathbf{-\sqrt{2}}$ or $x = \mathbf{\frac{-5\sqrt{2}}{2}}$

10. $x^2 - (\sqrt{7} + 1)x + \sqrt{7} = 0$

$\Rightarrow \quad x^2 - (\sqrt{7} + 1)x = -\sqrt{7}$

$\Rightarrow x^2 - 2\left(\frac{\sqrt{7}+1}{2}\right)x + \left(\frac{\sqrt{7}+1}{2}\right)^2 = -\sqrt{7} + \left(\frac{\sqrt{7}+1}{2}\right)^2$

$\Rightarrow \quad \left(x - \frac{\sqrt{7}+1}{2}\right)^2 = -\sqrt{7} + \frac{7 + 1 + 2\sqrt{7}}{4}$

$$= \frac{-4\sqrt{7} + 7 + 1 + 2\sqrt{7}}{4}$$

$\Rightarrow \quad \left(x - \frac{\sqrt{7}+1}{2}\right)^2 = \frac{7 + 1 - 2\sqrt{7}}{4}$

$\Rightarrow \quad x - \frac{\sqrt{7}+1}{2} = \pm \frac{\sqrt{7}-1}{2}$

$\Rightarrow$ Either $x - \frac{\sqrt{7}+1}{2} = \frac{\sqrt{7}-1}{2}$ or $x - \frac{\sqrt{7}+1}{2} = -\frac{\sqrt{7}-1}{2}$

$\Rightarrow \quad x = \frac{\sqrt{7}-1}{2} + \frac{\sqrt{7}+1}{2}$ or $x = \frac{\sqrt{7}+1}{2} - \frac{\sqrt{7}-1}{2}$

$\Rightarrow \quad x = \frac{\sqrt{7} - 1 + \sqrt{7} + 1}{2}$ or $x = \frac{\sqrt{7} + 1 - \sqrt{7} + 1}{2}$

$\Rightarrow \quad x = \frac{2\sqrt{7}}{2} = \mathbf{\sqrt{7}}$ or $x = \frac{2}{2} = \mathbf{1}$

11. $(a + b)x^2 - 2ax + a - b = 0$

$\Rightarrow (a + b)x^2 - 2ax = b - a$

$\Rightarrow \quad x^2 - \frac{2a}{a+b}x = \frac{b-a}{a+b}$ [Dividing the equation by x]

$\Rightarrow \quad x^2 - 2\left(\frac{a}{a+b}\right)x + \left(\frac{a}{a+b}\right)^2 = \frac{b-a}{a+b} + \left(\frac{a}{a+b}\right)^2$

$\Rightarrow \quad \left(x - \frac{a}{a+b}\right)^2 = \frac{(b-a)(a+b) + a^2}{(a+b)^2}$

$\Rightarrow \quad \left(x - \frac{a}{a+b}\right)^2 = \frac{b^2 - a^2 + a^2}{(a+b)^2} = \frac{b^2}{(a+b)^2}$

$\Rightarrow \quad \left(x - \frac{a}{a+b}\right) = \pm \frac{b}{a+b}$

$\Rightarrow$ Either $x - \frac{a}{a+b} = \frac{b}{a+b}$ or $x - \frac{a}{a+b} = \frac{-b}{a+b}$

$\Rightarrow \quad x = \frac{b}{a+b} + \frac{a}{a+b}$ or $x = \frac{-b}{a+b} + \frac{a}{a+b}$

$\Rightarrow \quad x = \mathbf{\frac{a+b}{a+b}} = \mathbf{1}$ or $x = \mathbf{\frac{a-b}{a+b}}$

EXERCISE 4D

For Basic and Standard Levels

1. (*i*) $x^2 - 6x + 8 = 0$

Comparing the given equation with $ax^2 + bx + c = 0$, we have $a = 1$, $b = -6$, $c = 8$

$$D = b^2 - 4ac = (-6)^2 - 4(1)(8)$$
$$= 36 - 32 = \mathbf{4}$$

(*ii*) $9x^2 - 4 = 0$

Comparing the given equation with $ax^2 + bx + c = 0$ we have, $a = 9$, $b = 0$, $c = -4$

$$D = b^2 - 4ac = (0)^2 - 4(9)(-4)$$
$$= 0 + 144 = \mathbf{144}$$

(*iii*) $\frac{4}{3}x^2 - 2x + \frac{3}{4} = 0$

Comparing the given equation with $ax^2 + bx + c = 0$, we have

$$a = \frac{4}{3}, b = -2, c = \frac{3}{4}$$

$$D = b^2 - 4ac = (-2)^2 - 4\left(\frac{4}{3}\right)\left(\frac{3}{4}\right)$$
$$= 4 - 4 = \mathbf{0}$$

(*iv*) $\sqrt{3}x^2 - 2\sqrt{2}x - 2\sqrt{3} = 0$

Comparing the given equation with $ax^2 + bx + c = 0$, we have

$$a = \sqrt{3},\ b = -2\sqrt{2},\ c = -2\sqrt{3}$$

$$D = b^2 - 4ac = (-2\sqrt{2})^2 - 4(\sqrt{3})(-2\sqrt{3})$$

$$= 8 + 8 \times 3 = 8 + 24 = \mathbf{32}$$

(*v*) $2x^2 + 3x + q = 0$

Comparing the given equation with $ax^2 + bx + c = 0$, we have

$$a = 2,\ b = 3,\ c = q$$

$$D = b^2 - 4ac = (3)^2 - 4(2)(q) = \mathbf{9 - 8q}$$

(*vi*) $3\sqrt{3}x^2 + 10x + \sqrt{3} = 0$

Comparing the given equation with $ax^2 + bx + c = 0$, we have

$$a = 3\sqrt{3},\ b = 10,\ c = \sqrt{3}$$

$$D = b^2 - 4ac = (10)^2 - 4(3\sqrt{3})(\sqrt{3})$$

$$= 100 - 36 = \mathbf{64}$$

2. (*i*) $x^2 + 5x + 5 = 0$

Comparing the given equation with $ax^2 + bx + c = 0$, we have

$$a = 1,\ b = 5,\ c = 5$$

$$D = b^2 - 4ac = (5)^2 - 4(1)(5) = 25 - 20 = 5$$

Since D is positive (> 0), therefore the given equation has **real and distinct roots.**

(*ii*) $x^2 + 3x + 7 = 0$

Comparing the given equation with $ax^2 + bx + c = 0$, we have

$$a = 1,\ b = 3,\ c = 7$$

$$D = b^2 - 4ac = (3)^2 - 4(1)(7) = 9 - 28 = -19$$

Since D < 0, therefore the given equation has **no real roots.**

(*iii*) $\frac{4}{x^2} + 1 = \frac{4}{x}$

$\Rightarrow \quad 4 + x^2 = 4x$

$\Rightarrow \quad x^2 - 4x + 4 = 0$

Comparing $x^2 - 4x + 4 = 0$, with $ax^2 + bx + c = 0$, we have $a = 1$, $b = -4$, $c = 4$

$$D = b^2 - 4ac = (-4)^2 - 4(1)(4)$$

$$= 16 - 16 = 0$$

Since D = 0, therefore the given equation has **real and equal roots.**

(*iv*) $2x^2 - 3\sqrt{5}x + 2 = 0$

Comparing the given equation with $ax^2 + bx + c = 0$, we have $a = 2$, $b = -3\sqrt{5}$, $c = 2$

$$D = b^2 - 4ac = (-3\sqrt{5})^2 - 4(2)(2)$$

$$= 45 - 16 = 29$$

Since D > 0, therefore the given equation has **real and distinct roots.**

(*v*) $(x - 2p)(x - 2q) = 4pq$

$\Rightarrow \quad x^2 - 2px - 2qx + 4pq - 4pq = 0$

$\Rightarrow \quad x^2 - 2(p + q)x = 0$

Comparing $x^2 - 2(p + q)x = 0$ with $ax^2 + bx + c = 0$, we have

$$a = 1,\ b = -2(p + q),\ c = 0$$

$$D = b^2 - 4ac = [-2(p + q)]^2 - 4(1)(0)$$

$$= 4(p + q)^2$$

Since D > 0, therefore the given equation has **real and distinct roots.**

(*vi*) $2y - 1 - \frac{2}{y-2} = 3$

$\Rightarrow \quad 2y(y - 2) - 1(y - 2) - 2 = 3(y - 2)$

$\Rightarrow \quad 2y^2 - 4y - y + 2 - 2 = 3y - 6$

$\Rightarrow \quad 2y^2 - 5y - 3y + 6 = 0$

$\Rightarrow \quad 2y^2 - 8y + 6 = 0$

$\Rightarrow \quad y^2 - 4y + 3 = 0$

Comparing $y^2 - 4y + 3 = 0$ with $ay^2 + by + c = 0$, we have

$$a = 1,\ b = -4,\ c = 3$$

$$D = b^2 - 4ac = (-4)^2 - 4(1)(3) = 16 - 12 = 4$$

Since D > 0, therefore the given equation has **real and distinct roots.**

3. (*i*) $3x^2 - 6x + 5 = 0$

Comparing the given equation with $ax^2 + bx + c = 0$, we have

$$a = 3,\ b = -6,\ c = 5$$

$$D = b^2 - 4ac = (-6)^2 - 4(3)(5) = 36 - 60 = -24$$

Since D < 0, therefore the given equation has **no real roots.**

(*ii*) $16x^2 = 24x + 1$

$\Rightarrow \quad 16x^2 - 24x - 1 = 0$

Comparing the given equation with $ax^2 + bx + c = 0$, we have

$$a = 16,\ b = -24,\ c = -1$$

$$D = b^2 - 4ac = (-24)^2 - 4(16)(-1)$$

$$= 576 + 64 = 640 > 0$$

So, the given equation has roots, given by

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-(-24) \pm \sqrt{640}}{2(16)}$$

$$= \frac{24 \pm 8\sqrt{10}}{32}$$

$$= \frac{3 \pm \sqrt{10}}{4}$$

Hence, the roots are $\mathbf{\frac{3 + \sqrt{10}}{4}}$ and $\mathbf{\frac{3 - \sqrt{10}}{4}}$.

(*iii*) $2x^2 + 6\sqrt{3}x - 60 = 0$ ... (1)

Comparing the given equation with $ax^2 + bx + c = 0$, we have $a = 2$, $b = 6\sqrt{3}$, $c = -60$

$$D = b^2 - 4ac$$

$$= (6\sqrt{3})^2 - 4(2)(-60)$$

$$= 108 + 480 = 588$$

Since D > 0, therefore, equation (1) has real roots.

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

$$= \frac{-6\sqrt{3} \pm \sqrt{588}}{4}$$

$$= \frac{-6\sqrt{3} \pm 14\sqrt{3}}{4}$$

Hence, the roots are

$$x = \frac{-3\sqrt{3}+7\sqrt{3}}{2} = \mathbf{2\sqrt{3}}$$

and $$x = \frac{-3\sqrt{3}-7\sqrt{3}}{2} = \mathbf{-5\sqrt{3}}$$

(*iv*) $3x^2 - 4\sqrt{7}\,x + 7 = 0$

Comparing the given equation with $ax^2 + bx + c = 0$, we have

$$a = 3,\ b = -4\sqrt{7},\ c = 7$$

$$D = b^2 - 4ac = (-4\sqrt{7})^2 - 4(3)(7)$$

$$= 112 - 84 = 28 > 0$$

Since D > 0, therefore the given equation has real roots given by

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-(-4\sqrt{7}) \pm \sqrt{28}}{2(3)}$$

$$= \frac{4\sqrt{7} \pm 2\sqrt{7}}{6} = \frac{2\sqrt{7} \pm \sqrt{7}}{3}$$

Hence, the roots are $\frac{2\sqrt{7} \pm \sqrt{7}}{3} = \frac{3\sqrt{7}}{3} = \mathbf{\sqrt{7}}$

and $\frac{2\sqrt{7} - \sqrt{7}}{3} = \mathbf{\frac{\sqrt{7}}{3}}$

(*v*) $x^2 + x + 7 = 0$

Comparing the given equation with $ax^2 + bx + c = 0$, we have

$$a = 1,\ b = 1,\ c = 7$$

$$D = b^2 - 4ac = (1)^2 - 4(1)(7)$$

$$= 1 - 28 = -27 < 0$$

Since D < 0, therefore the given equation has **no real roots.**

(*vi*) $2x^2 + 5\sqrt{3}\,x + 6 = 0$

Comparing the given equation with $ax^2 + bx + c = 0$, we have

$$a = 2,\ b = 5\sqrt{3},\ c = 6$$

$$D = b^2 - 4ac = (5\sqrt{3})^2 - 4(2)(6)$$

$$= 75 - 48 = 27 > 0$$

Since D > 0, the given equation has real roots given by

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-(5\sqrt{3}) \pm \sqrt{27}}{2(2)}$$

$$= \frac{-5\sqrt{3} \pm 3\sqrt{3}}{4}$$

$$= \frac{-5\sqrt{3}+3\sqrt{3}}{4} \text{ and } \frac{-5\sqrt{3}-3\sqrt{3}}{4}$$

$$= \frac{-2\sqrt{3}}{4} \text{ and } \frac{-8\sqrt{3}}{4} = \frac{-\sqrt{3}}{2} \text{ and } -2\sqrt{3}$$

Hence, the roots are $\mathbf{-\frac{\sqrt{3}}{2}}$ and $\mathbf{-2\sqrt{3}}$.

(*vii*) $\sqrt{3}x^2 - 2\sqrt{2}x - 2\sqrt{3} = 0$ ... (1)

Comparing the given equation with $ax^2 + bx + c = 0$, we have

$$a = \sqrt{3},\ b = -2\sqrt{2},\ c = -2\sqrt{3}$$

$$D = b^2 - 4ac = \left(-2\sqrt{2}\right)^2 - 4\left(\sqrt{3}\right)\left(-2\sqrt{3}\right)$$

$$= 8 + 24 = 32$$

Since, D > 0 therefore roots of equation are real.

$$x = \frac{-b \pm \sqrt{D}}{2a}$$

$$= \frac{2\sqrt{2} \pm \sqrt{32}}{2\sqrt{3}}$$

$$= \frac{2\sqrt{2} \pm 4\sqrt{2}}{2\sqrt{3}}$$

$$= \frac{\sqrt{2} \pm 2\sqrt{2}}{\sqrt{3}}$$

$$x = \sqrt{6} \text{ and } x = -\sqrt{\frac{2}{3}} = \frac{-\sqrt{6}}{3}$$

Hence, the roots are $\mathbf{\sqrt{6}}$ and $\mathbf{\frac{-\sqrt{6}}{3}}$

(*viii*) $4\sqrt{3}\,x^2 + 5x - 2\sqrt{3} = 0$

Comparing the given equation the $ax^2 + bx + c = 0$, we have

$$a = 4\sqrt{3},\ b = 5,\ c = -2\sqrt{3}$$

$$D = b^2 - 4ac = 5^2 - 4(4\sqrt{3})(-2\sqrt{3})$$

$$= 25 + 96 = 121 > 0$$

Since D > 0, the given equation has real roots given by

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-5 \pm \sqrt{121}}{2 \times 4\sqrt{3}} = \frac{-5 \pm 11}{8\sqrt{3}}$$

$$= \frac{-5+11}{8\sqrt{3}} \text{ and } \frac{-5-11}{8\sqrt{3}}$$

$$= \frac{6}{8\sqrt{3}} \text{ and } \frac{-16}{8\sqrt{3}}$$

$$= \frac{3}{4\sqrt{3}} \text{ and } \frac{-2}{\sqrt{3}}$$

$$= \frac{\sqrt{3}}{4} \text{ and } \frac{-2}{\sqrt{3}}$$

Hence, the roots are $\mathbf{\frac{\sqrt{3}}{4}}$ and $\mathbf{\frac{-2}{\sqrt{3}}}$.

(*ix*) $3a^2x^2 + 8abx + 4b^2 = 0$

Comparing the given equation with $a'x^2 + b'x + c' = 0$, we have

$$a' = 3a^2,\ b' = 8ab,\ c' = 4b^2$$

$$D = (b')^2 - 4a'c' = (8ab)^2 - 4(3a^2)(4b^2)$$

$$= 64a^2b^2 - 48a^2b^2 = 16a^2b^2 > 0$$

Since D > 0, the given equation has real roots given by

$$x = \frac{-b' \pm \sqrt{D}}{2a'}$$

$$= \frac{-8ab \pm \sqrt{16a^2b^2}}{2(3a^2)} = \frac{-8ab \pm 4ab}{6a^2}$$

$$= \frac{-8ab + 4ab}{6a^2} \text{ and } \frac{-8ab - 4ab}{6a^2}$$

$$= \frac{-4ab}{6a^2} \text{ and } \frac{-12ab}{6a^2} = \frac{-2b}{3a} \text{ and } \frac{-2b}{a}$$

Hence, the roots are $\mathbf{\frac{-2b}{3a}}$ and $\mathbf{\frac{-2b}{a}}$.

(*x*) $$6 + \frac{1}{x} = \frac{2}{x^2}$$

$\Rightarrow$ $6x^2 + x = 2$

$\Rightarrow$ $6x^2 + x - 2 = 0$

Comparing $6x^2 + x - 2 = 0$ with $ax^2 + bx + c = 0$, we get

$$a = 6,\ b = 1,\ c = -2$$

$$D = b^2 - 4ac = (1)^2 - 4(6)(-2)$$

$$= 1 + 48 = 49 > 0$$

Since D > 0, the given equation has real roots given by

$$x = \frac{-1 \pm \sqrt{D}}{2a} = \frac{-1 \pm \sqrt{49}}{2(6)} = \frac{-1 \pm 7}{12}$$

$$= \frac{-1+7}{12} \text{ and } \frac{-1-7}{12}$$

$$= \frac{6}{12} \text{ and } \frac{-8}{12} = \frac{1}{2} \text{ and } \frac{-2}{3}$$

Hence, the roots are $\mathbf{\frac{1}{2}}$ and $\mathbf{\frac{-2}{3}}$.

4. (*i*) $x^2 + 2x - 4 = 0$

Comparing the given equation with $ax^2 + bx + c = 0$, we have

$$a = 1,\ b = 2,\ c = -4$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(-4)}}{2(1)}$$

$$= \frac{-2 \pm \sqrt{4+16}}{2}$$

$$= \frac{-2 \pm \sqrt{20}}{2} = \frac{-2 \pm 2\sqrt{5}}{2} = -1 \pm \sqrt{5}$$

The roots are $-1+\sqrt{5}$ and $-1-\sqrt{5}$.

Hence $\mathbf{x = -1+\sqrt{5}, -1-\sqrt{5}}$ are the required solutions.

(*ii*) $x^2 = 3x - 1$

$\Rightarrow$ $x^2 - 3x + 1 = 0$

Comparing $x^2 - 3x + 1 = 0$ with $ax^2 + bx + c = 0$, we have

$$a = 1,\ b = -3,\ c = 1$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(1)}}{2(1)}$$

$$= \frac{3 \pm \sqrt{9-4}}{2} = \frac{3 \pm \sqrt{5}}{2}$$

The roots are $\frac{3+\sqrt{5}}{2}$ and $\frac{3-\sqrt{5}}{2}$.

Hence, $\mathbf{x = \frac{3+\sqrt{5}}{2}, \frac{3-\sqrt{5}}{2}}$ are the required solutions.

(*iii*) $4x^2 - 2x = 3$

$\Rightarrow$ $4x^2 - 2x - 3 = 0$

Comparing $4x^2 - 2x - 3 = 0$ with $ax^2 + bx + c = 0$, we have

$$a = 4,\ b = -2,\ c = -3$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(4)(-3)}}{2(4)}$$

$$= \frac{2 \pm \sqrt{4+48}}{8} = \frac{2 \pm \sqrt{52}}{8} = \frac{2 \pm 2\sqrt{13}}{8}$$

$$= \frac{1 \pm \sqrt{13}}{4}$$

The roots are $\frac{1+\sqrt{13}}{4}$ and $\frac{1-\sqrt{13}}{4}$.

Hence, $\mathbf{x = \frac{1+\sqrt{13}}{4}, \frac{1-\sqrt{13}}{4}}$ are the required solutions.

(*iv*) $3x^2 - 32x + 12 = 0$

Comparing the given equation with $ax^2 + bx + c = 0$, we have

$$a = 3,\ b = -32,\ c = 12$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-32) \pm \sqrt{(-32)^2 - 4(3)(12)}}{2(3)}$$

$$= \frac{32 \pm \sqrt{1024 - 144}}{6} = \frac{32 \pm \sqrt{880}}{6}$$

$$= \frac{32 \pm \sqrt{16 \times 55}}{6}$$

$$= \frac{32 \pm 4\sqrt{55}}{6} = \frac{16 \pm 2\sqrt{55}}{3}$$

The roots are $\frac{16+2\sqrt{55}}{3}$ and $\frac{16-2\sqrt{55}}{3}$.

Hence, $x = \mathbf{\frac{16+2\sqrt{55}}{3}, \frac{16-2\sqrt{55}}{3}}$ are the required solutions.

(*v*) Comparing the given equation with the standard quadratic equation $Ax^2 + Bx + C = 0$, we get

$$A = 2, B = a, C = -a^2$$

$\therefore$ The roots are

$$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$= \frac{-a \pm \sqrt{a^2 + 8a^2}}{4}$$

$$= \frac{-a \pm \sqrt{9a^2}}{4}$$

$$= \frac{-a \pm 3a}{4}$$

$$= \frac{-a + 3a}{4} \quad \text{or} \quad \frac{-a - 3a}{4}$$

$$= \frac{1}{2}a \quad \text{or} \quad -a$$

$\therefore$ The required roots are $x = \frac{a}{2}$ and $x = -a$.

(*vi*) $x^2 + \frac{5}{2}x = 3$

$\Rightarrow$ $2x^2 + 5x = 6$

$\Rightarrow$ $2x^2 + 5x - 6 = 0$

Comparing $2x^2 + 5x - 6 = 0$ with $ax^2 + bx + c = 0$, we have

$$a = 2, b = 5, c = -6$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-5 \pm \sqrt{(5)^2 - 4(2)(-6)}}{2(2)}$$

$$= \frac{-5 \pm \sqrt{25+48}}{4} = \frac{-5 \pm \sqrt{73}}{4}$$

The roots are $\frac{-5+\sqrt{73}}{4}$ and $\frac{-5-\sqrt{73}}{4}$.

Hence, $x = \mathbf{\frac{-5+\sqrt{73}}{4}, \frac{-5-\sqrt{73}}{4}}$ are the required solutions.

(*vii*) $x - \frac{6}{x} = 5$

$\Rightarrow$ $x^2 - 6 = 5x$

$\Rightarrow$ $x^2 - 5x - 6 = 0$

Comparing $x^2 - 5x - 6 = 0$ with $ax^2 + bx + c = 0$, we have

$$a = 1, b = -5, c = -6$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(-6)}}{2(1)}$$

$$= \frac{5 \pm \sqrt{25+24}}{2} = \frac{5 \pm \sqrt{49}}{2}$$

$$= \frac{5 \pm 7}{2} = \frac{5+7}{2} \text{ and } \frac{5-7}{2}$$

$$= \frac{12}{2} \text{ and } \frac{-2}{2} = 6 \text{ and } -1$$

The roots are 6 and –1.

Hence, $x =$ **6, –1** are the required solutions.

(*viii*) We have

$$\frac{x^2-1}{x} = 3$$

$\Rightarrow$ $x^2 - 3x - 1 = 0$

Comparing $x^2 - 3x - 1 = 0$ with $ax^2 + bx + c = 0$, we have $a = 1, b = -3, c = -1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{+3 \pm \sqrt{9 + 4 \times 1 \times 1}}{2}$$

$$= \frac{3 \pm \sqrt{13}}{2}$$

$$= \frac{3-\sqrt{13}}{2} \text{ and } \frac{3+\sqrt{13}}{2}$$

The roots are $\frac{3-\sqrt{13}}{2}$ and $\frac{3+\sqrt{13}}{2}$.

Hence, $x = \mathbf{\frac{3-\sqrt{13}}{2}, \frac{3+\sqrt{13}}{2}}$ are the required solutions.

(*ix*) $(4x-3)^2 + 40x = 21$

$\Rightarrow$ $16x^2 - 24x + 9 + 40x - 21 = 0$

$\Rightarrow$ $16x^2 + 16x - 12 = 0$

$\Rightarrow$ $4x^2 + 4x - 3 = 0$

Comparing $4x^2 + 4x - 3 = 0$ with $ax^2 + bx + c = 0$, we have

$$a = 4, b = 4, c = -3$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-4 \pm \sqrt{(4)^2 - 4(4)(-3)}}{2(4)}$$

$$= \frac{-4 \pm \sqrt{16+48}}{8} = \frac{-4 \pm \sqrt{64}}{8} = \frac{-4 \pm 8}{8}$$

$$= \frac{-4+8}{8} \text{ and } \frac{-4-8}{8}$$

$$= \frac{4}{8} \text{ and } \frac{-12}{8} = \frac{1}{2} \text{ and } \frac{-3}{2}$$

The roots are $\frac{1}{2}$ and $\frac{-3}{2}$.

Hence, $x = \mathbf{\frac{1}{2}, \frac{-3}{2}}$ are the required solutions.

(*x*) $(x+3)(x-1) = 3\left(x - \frac{1}{3}\right)$

$\Rightarrow$ $(x^2 + 3x - x - 3) = 3x - 1$

$\Rightarrow \quad x^2 + 2x - 3 - 3x + 1 = 0$

$\Rightarrow \quad x^2 - x - 2 = 0$

Comparing $x^2 - x - 2 = 0$ with $ax^2 + bx + c = 0$, we have

$a = 1, b = -1, c = -2$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-2)}}{2(1)}$$

$$= \frac{1 \pm \sqrt{1+8}}{2} = \frac{1 \pm \sqrt{9}}{2}$$

$$= \frac{1 \pm 3}{2} = \frac{1+3}{2} \text{ and } \frac{1-3}{2} = 2 \text{ and } -1$$

The roots are 2 and –1.

Hence, $x = \mathbf{2, -1}$ are the required solutions.

(*xi*) $ab^2x\left(\frac{a}{d}x + 2\frac{c}{b}\right) + c^2d = 0$

$\Rightarrow \quad ab^2 \times \frac{a}{d}x^2 + 2ab^2 \times \frac{cx}{b} + c^2d = 0$

$\Rightarrow \quad \frac{a^2b^2}{d}x^2 + 2abc\,x + c^2d = 0$

$\Rightarrow \quad a^2b^2x^2 + 2abcdx + c^2d^2 = 0$

Comparing $a^2b^2x^2 + 2abcdx + c^2d^2 = 0$ with $Ax^2 + Bx + C = 0$, we have

$A = a^2b^2, B = 2abcd, C = c^2d^2$

$$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$= \frac{-2abcd \pm \sqrt{(2abcd)^2 - 4(a^2b^2)(c^2d^2)}}{2(a^2b^2)}$$

$$= \frac{-2abcd \pm \sqrt{4a^2b^2c^2d^2 - 4a^2b^2c^2d^2}}{2a^2b^2}$$

$$= \frac{-2abcd \pm 0}{2a^2b^2} = \frac{-2abcd}{2a^2b^2} \text{ and } \frac{-2abcd}{2a^2b^2}$$

$$= \frac{-cd}{ab} \text{ and } \frac{-cd}{ab}$$

The roots are $\frac{-cd}{ab}$ and $\frac{-cd}{ab}$.

Hence, $x = \frac{-cd}{ab}, \frac{-cd}{ab}$ are the required solutions.

(*xii*) Comparing $\sqrt{2}\,x^2 + 7x + 5\sqrt{2} = 0$ with $ax^2 + bx + c = 0$, we have

$a = \sqrt{2}, \; b = 7$ and $c = 5\sqrt{2}$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-7 \pm \sqrt{49 - 4 \times \sqrt{2} \times 5\sqrt{2}}}{2\sqrt{2}}$$

$$= \frac{-7 \pm \sqrt{49 - 40}}{2\sqrt{2}}$$

$$= \frac{-7 \pm 3}{2\sqrt{2}}$$

$$= \frac{-4}{2\sqrt{2}}, \frac{-10}{2\sqrt{2}}$$

$$= -\frac{2}{\sqrt{2}}, \frac{-5}{\sqrt{2}} \text{ or } -\sqrt{2}, \frac{-5\sqrt{2}}{2}$$

The roots are $-\sqrt{2}$ and $\frac{-5\sqrt{2}}{2}$.

Hence, $\boldsymbol{x = -\sqrt{2}, \frac{-5\sqrt{2}}{2}}$ are the required solutions.

(*xiii*) Comparing $\sqrt{3}x^2 + 10x - 8\sqrt{3} = 0$ with $ax^2 + bx + c = 0$, we have

$a = \sqrt{3}, b = 10$ and $c = -8\sqrt{3}$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-10 \pm \sqrt{10^2 + 4 \times \sqrt{3} \times 8\sqrt{3}}}{2\sqrt{3}}$$

$$= \frac{-10 \pm \sqrt{100 + 96}}{2\sqrt{3}}$$

$$= \frac{-10 \pm \sqrt{196}}{2\sqrt{3}}$$

$$= \frac{-10 \pm \sqrt{2^2 \times 7^2}}{2\sqrt{3}}$$

$$= \frac{-10 \pm 14}{2\sqrt{3}}$$

$$= \frac{4}{2\sqrt{3}}, \frac{-24}{2\sqrt{3}}$$

$$= \frac{2}{\sqrt{3}}, \frac{-12}{\sqrt{3}}$$

$$= \frac{2}{3}\sqrt{3}, -4\sqrt{3}$$

The roots are $\frac{\sqrt{3}}{2}$ and $\frac{\sqrt{3}}{3}$.

Hence, $x = \boldsymbol{\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{3}}$ are the required solutions.

(*xiv*) Comparing $2\sqrt{3}x^2 - 5x + \sqrt{3} = 0$ with $ax^2 + bx + c = 0$, we have

$a = 2\sqrt{3}, b = -5$ and $c = \sqrt{3}$

$$\therefore \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{5 \pm \sqrt{25 - 4 \times 2\sqrt{3} \times \sqrt{3}}}{4\sqrt{3}}$$

$$= \frac{5 \pm \sqrt{25 - 24}}{4\sqrt{3}}$$

$$= \frac{5 \pm 1}{4\sqrt{3}}$$

$$= \frac{6}{4\sqrt{3}}, \frac{4}{4\sqrt{3}}$$

$$= \frac{3}{2\sqrt{3}}, \frac{1}{\sqrt{3}}$$

$$= \frac{3\sqrt{3}}{6}, \frac{1}{\sqrt{3}} \text{ or } \frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{3}$$

The roots are $\frac{\sqrt{3}}{2}$ and $\frac{\sqrt{3}}{3}$.

Hence, $x = \mathbf{\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{3}}$ are the required solutions.

(*xv*) Comparing $3x^2 - 2\sqrt{6}x + 2 = 0$ with $ax^2 + bx + c = 0$, we have

$a = 3$, $b = -2\sqrt{6}$ and $c = 2$

$$\therefore \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{2\sqrt{6} \pm \sqrt{\left(-2\sqrt{6}\right)^2 - 4 \times 3 \times 2}}{2 \times 3}$$

$$= \frac{2\sqrt{6} \pm \sqrt{24 - 24}}{6}$$

$$= \frac{2\sqrt{6}}{6}, \frac{2\sqrt{6}}{6}$$

$$= \frac{\sqrt{6}}{3}, \frac{\sqrt{6}}{3}$$

$$= \frac{\sqrt{6}}{\sqrt{3} \times \sqrt{3}}, \frac{\sqrt{6}}{\sqrt{3} \times \sqrt{3}}$$

$$= \sqrt{\frac{2}{3}}, \sqrt{\frac{2}{3}}$$

which are two equal roots.

Hence, $x = \mathbf{\sqrt{\frac{2}{3}}, \sqrt{\frac{2}{3}}}$ are the required solutions.

(*xvi*) Comparing $x^2 - \left(\sqrt{3} + 1\right)x + \sqrt{3} = 0$ with $ax^2 + bx + c = 0$, we have

$a = 1$, $b = -\left(\sqrt{3} + 1\right)$ and $c = \sqrt{3}$

$$\therefore \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{\sqrt{3} + 1 \pm \sqrt{\left(\sqrt{3} + 1\right)^2 - 4 \times 1 \times \sqrt{3}}}{2}$$

$$= \frac{\sqrt{3} + 1 \pm \sqrt{3 + 1 + 2\sqrt{3} - 4\sqrt{3}}}{2}$$

$$= \frac{\sqrt{3} + 1 \pm \sqrt{\left(\sqrt{3} - 1\right)^2}}{2}$$

$$= \frac{\sqrt{3} + 1 \pm \left(\sqrt{3} - 1\right)}{2}$$

$$= \frac{\sqrt{3} + 1 + \sqrt{3} - 1}{2}, \frac{\sqrt{3} + 1 - \sqrt{3} + 1}{2}$$

$$= \sqrt{3}, 1$$

The roots are $\sqrt{3}$ and 1.

Hence, $x = \mathbf{\sqrt{3}, 1}$ are the required solutions.

5. (*i*)
$$\frac{16}{x} - 1 = \frac{15}{x + 1}$$

$$\Rightarrow \quad \frac{16 - x}{x} = \frac{15}{x + 1}$$

$$\Rightarrow \quad (16 - x)(x + 1) = 15x$$

$$\Rightarrow \quad -x^2 + 16x - x + 16 = 15x$$

$$\Rightarrow \quad x^2 = 16$$

$$\Rightarrow \quad x = \pm 4$$

The roots are 4 and –4.

Hence, $x = \mathbf{4, -4}$ are the required solutions.

(*ii*)
$$\frac{1}{x - 3} - \frac{1}{x + 5} = \frac{1}{6}$$

$$\Rightarrow \quad \frac{x + 5 - x + 3}{(x - 3)(x + 5)} = \frac{1}{6}$$

$$\Rightarrow \quad \frac{8}{x^2 + 5x - 3x - 15} = \frac{1}{6}$$

$$\Rightarrow \quad x^2 + 2x - 15 = 48$$

$$\Rightarrow \quad x^2 + 2x - 63 = 0$$

Comparing this equation with the standard quadratic equation $ax^2 + bx + c = 0$, we have

$a = 1$, $b = 2$ and $c = -63$

$$\therefore \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-2 \pm \sqrt{4 + 4 \times 63}}{2}$$

$$= \frac{-2 \pm \sqrt{4(1 + 63)}}{2}$$

$$= \frac{-2 \pm 2 \times 8}{2}$$

$$= \frac{-2 \pm 16}{2}$$

$$= \frac{14}{2}, -\frac{18}{2}$$

$$= 7, -9$$

The roots are 7 and –9.

Hence, $x = \mathbf{7, -9}$ are the required solutions.

(*iii*)
$$\frac{6}{x} - \frac{2}{x - 1} = \frac{1}{x + 2}; \; x \neq 0, 1, -2$$

$$\Rightarrow \quad 6(x - 1)(x + 2) - 2(x)(x + 2) = x(x - 1)$$

$$\Rightarrow \quad 6(x^2 + x - 2) - 2(x^2 + 2x) - x(x - 1) = 0$$

$$\Rightarrow \quad 6x^2 + 6x - 12 - 2x^2 - 4x - x^2 + x = 0$$

$$\Rightarrow \quad 3x^2 + 3x - 12 = 0$$

$$\Rightarrow \quad x^2 + x - 4 = 0$$

Comparing $x^2 + x - 4 = 0$ with $ax^2 + bx + c = 0$, we have

$a = 1, b = 1, c = -4$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-1 \pm \sqrt{(1)^2 - 4(1)(-4)}}{2(1)}$$

$$= \frac{-1 \pm \sqrt{1 + 16}}{2} = \frac{-1 \pm \sqrt{17}}{2}$$

The roots are $\frac{-1 + \sqrt{17}}{2}$ and $\frac{-1 - \sqrt{17}}{2}$.

Hence, $x = \mathbf{\frac{-1 + \sqrt{17}}{2}, \frac{-1 - \sqrt{17}}{2}}$ are the required solutions.

(*iv*) $\frac{a}{x - b} + \frac{b}{x - a} = 2$

$\Rightarrow \quad a(x - a) + b(x - b) = 2[(x - a)(x - b)]$

$\Rightarrow \quad ax - a^2 + bx - b^2 = 2(x^2 - x(a + b) + ab)$

$\Rightarrow \quad 2x^2 - 3x(a + b) + (a^2 + b^2 + 2ab) = 0 \quad \ldots(1)$

Comparing (1) with $ax^2 + bx + c = 0$, we have

$a = 2, b = -3(a + b), c = (a^2 + b^2 + 2ab)$

$\Rightarrow \quad D = b^2 - 4ac$

$= [-3(a + b)]^2 - 4(2)(a^2 + b^2 + 2ab)$

$= 9(a + b)^2 - 8(a + b)^2$

$= (a + b)^2$

$$\Rightarrow \quad x = \frac{-b \pm \sqrt{D}}{2a}$$

$$= \frac{3(a + b) \pm \sqrt{(a + b)^2}}{4}$$

$$= \frac{3(a + b) \pm (a + b)}{4}$$

$$\Rightarrow \quad x = (a + b) \text{ and } \frac{1}{2}(a + b)$$

The roots are $(a + b)$ and $\frac{1}{2}(a + b)$.

Hence, $\mathbf{x = (a + b), \frac{1}{2}(a + b)}$ are the required solutions.

(*v*) $\frac{2y}{y - 4} + \frac{2y - 5}{y - 3} = \frac{25}{3}$

$\Rightarrow \quad 3[2y(y - 3) + (2y - 5)(y - 4)] = 25(y - 4)(y - 3)$

$\Rightarrow \quad 3[2y^2 - 6y + 2y^2 - 5y - 8y + 20] = 25(y^2 - 4y - 3y + 12)$

$\Rightarrow \quad 3(4y^2 - 19y + 20) = 25(y^2 - 7y + 12)$

$\Rightarrow \quad 12y^2 - 57y + 60 = 25y^2 - 175y + 300$

$\Rightarrow \quad 25y^2 - 12y^2 - 175y + 57y + 300 - 60 = 0$

$\Rightarrow \quad 13y^2 - 118y + 240 = 0$

Comparing $13y^2 - 118y + 240 = 0$ with $ax^2 + bx + c = 0$, we have

$a = 13, b = -118, c = 240$

$$y = \frac{-(-118) \pm \sqrt{(118)^2 - 4(13)(240)}}{2(13)}$$

$$= \frac{118 \pm \sqrt{13924 - 12480}}{26}$$

$$= \frac{118 \pm \sqrt{1444}}{26} = \frac{118 \pm 38}{26}$$

$$= \frac{118 + 38}{26} \text{ and } \frac{118 - 38}{26}$$

$$= \frac{156}{26} \text{ and } \frac{80}{26} = 6 \text{ and } \frac{40}{13}$$

The roots are 6 and $\frac{40}{13}$.

Hence, $y = \mathbf{6, \frac{40}{13}}$ are the required solutions.

For Standard Level

6. (*i*) $x^2 - 2(a + 2)x + (a + 1)(a + 3) = 0$

Comparing the given equation with $ax^2 + bx + c = 0$, we have

$a = 1, b = -2(a + 2), c = (a + 1)(a + 3)$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-[-2(a + 2)] \pm \sqrt{[-2(a + 2)]^2 - 4(1)(a + 1)(a + 3)}}{2(1)}$$

$$= \frac{2(a + 2) \pm \sqrt{4(a^2 + 4a + 4) - 4(a^2 + 4a + 3)}}{2}$$

$$= \frac{2(a + 2) \pm \sqrt{4a^2 + 16a + 16 - 4a^2 - 16a - 12}}{2}$$

$$= \frac{2(a + 2) \pm \sqrt{4}}{2} = \frac{2(a + 2) \pm 2}{2} = (a + 2) \pm 1$$

$= a + 2 + 1$ and $a + 2 - 1 = a + 3$ and $a + 1$

The roots are $a + 3$ and $a + 1$.

Hence, $x = \mathbf{a + 3, a + 1}$ are the required solutions.

(*ii*) $x^2 + 6x - (a^2 + 2a - 8) = 0$

Comparing the given equation with $Ax^2 + Bx + C = 0$, we have

$A = 1, B = 6$ and $C = -(a^2 + 2a - 8)$

$$\therefore \quad x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$= \frac{-6 \pm \sqrt{36 + 4 \times 1 \times (a^2 + 2a - 8)}}{2}$$

$$= \frac{-6 \pm \sqrt{36 + 4a^2 + 8a - 32}}{2}$$

$$= \frac{-6 \pm \sqrt{4a^2 + 8a + 4}}{2}$$

$$= \frac{-6 \pm 2\sqrt{a^2 + 2a + 1}}{2}$$

$$= -3 \pm \sqrt{(a + 1)^2}$$

$= -3 \pm (a + 1)$

$= -3 + a + 1, -3 - a - 1$

$= -2 + a, -a - 4 = a - 2, -(a + 4)$

The roots are $(a - 2)$ and $-(a + 4)$.

Hence, $\boldsymbol{x = (a - 2), -(a + 4)}$ are the required solutions.

(*iii*) $x^2 + 5x - (a^2 + a - 6) = 0$

Comparing the given equation with

$Ax^2 + Bx + C = 0$, we have

$A = 1$, $B = 5$ and $C = -(a^2 + a - 6)$

$$\therefore \quad x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$= \frac{-5 \pm \sqrt{25 + 4\left(a^2 + a - 6\right)}}{2}$$

$$= \frac{-5 \pm \sqrt{25 - 24 + 4a^2 + 4a}}{2}$$

$$= \frac{-5 \pm \sqrt{4a^2 + 4a + 1}}{2}$$

$$= \frac{-5 \pm \sqrt{(2a+1)^2}}{2}$$

$$= \frac{-5 \pm (2a + 1)}{2}$$

$$= \frac{-5 + 2a + 1}{2}, \frac{-5 - 2a - 1}{2}$$

$= a - 2, -(3 + a)$

$= (a - 2), -(a + 3)$

The roots are $(a - 2)$ and $-(a + 3)$.

Hence, $\boldsymbol{x = (a - 2), -(a + 3)}$ are the required solutions.

(*iv*) $4x^2 - 16(p - q)x + (15p^2 - 34pq + 15q^2) = 0$

Comparing the given equation with $ax^2 + bx + c = 0$, we have

$a = 4$, $b = -16(p - q)$, $c = (15p^2 - 34pq + 15q^2)$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-[-16(p-q)] \pm \sqrt{[-16(p-q)]^2 - 4(4)(15p^2 - 34pq + 15q^2)}}{2(4)}$$

$$= \frac{16(p-q) \pm \sqrt{256(p-q)^2 - 16(15p^2 - 34pq + 15q^2)}}{8}$$

$$= \frac{16(p-q) \pm \sqrt{256(p^2 - 2pq + q^2) - 16(15p^2 - 34pq + 15q^2)}}{8}$$

$$= \frac{16(p-q) \pm \sqrt{256p^2 - 512pq + 256q^2 - 240p^2 + 544pq - 240q^2}}{8}$$

$$= \frac{16(p-q) \pm \sqrt{16p^2 + 16q^2 + 32pq}}{8}$$

$$= \frac{16(p-q) \pm \sqrt{16(p^2 + q^2 + 2pq)}}{8}$$

$$= \frac{16(p-q) \pm \sqrt{16(p+q)^2}}{8} = \frac{16(p-q) \pm 4(p+q)}{8}$$

$$= \frac{16p - 16q + 4p + 4q}{8} \text{ and } \frac{16p - 16q - 4p - 4q}{8}$$

$$= \frac{20p - 12q}{8} \text{ and } \frac{12p - 20q}{8}$$

$$= \frac{5p - 3q}{2} \text{ and } \frac{3p - 5q}{2}$$

The roots are $\frac{5p - 3q}{2}$ and $\frac{3p - 5q}{2}$.

Hence $x = \boldsymbol{\frac{5p - 3q}{2}, \frac{3p - 5q}{2}}$ are the required solutions.

(*v*) $16x^2 - 20(a + b)x + (6a^2 + 13ab + 6b^2) = 0$

Comparing the given equation with

$Ax^2 + Bx + C = 0$, we have

$a = 16$, $b = -20(a + b)$, $c = 6a^2 + 13ab + 6b^2$

$$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$= \frac{-[-20(a+b)] \pm \sqrt{[-20(a+b)]^2 - 4(16)(6a^2 + 13ab + 6b^2)}}{2(16)}$$

$$= \frac{20(a+b) \pm \sqrt{400(a^2 + 2ab + b^2) - 64(6a^2 + 13ab + 6b^2)}}{32}$$

$$= \frac{20(a+b) \pm \sqrt{400a^2 + 800ab + 400b^2 - 384a^2 - 832ab - 384b^2}}{32}$$

$$= \frac{20a + 20b \pm \sqrt{16a^2 - 32ab + 16b^2}}{32}$$

$$= \frac{20a + 20b \pm \sqrt{16(a^2 - 2ab + b^2)}}{32}$$

$$= \frac{20a + 20b \pm \sqrt{16(a-b)^2}}{32}$$

$$= \frac{20a + 20b \pm 4(a-b)}{32}$$

$$= \frac{20a + 20b + 4a - 4b}{32} \text{ and } \frac{20a + 20b - 4a + 4b}{32}$$

$$= \frac{24a + 16b}{32} \text{ and } \frac{16a + 24b}{32}$$

$$= \frac{3a + 2b}{4} \text{ and } \frac{2a + 3b}{4}$$

The roots are $\frac{3a + 2b}{4}$ and $\frac{2a + 3b}{4}$.

Hence, $x = \boldsymbol{\frac{3a + 2b}{4}, \frac{2a + 3b}{4}}$ are the required solutions.

(*vi*) $9x^2 - 3(a + b)x + ab = 0$

Comparing the given equation with $Ax^2 + Bx + C = 0$, we have

$A = 9, B = -3(a + b), C = ab$

$$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$= \frac{-[-3(a+b)] \pm \sqrt{[-3(a+b)]^2 - 4(9)(ab)}}{2(9)}$$

$$= \frac{3(a+b) \pm \sqrt{9(a^2+2ab+b^2) - 36ab}}{18}$$

$$= \frac{3(a+b) \pm \sqrt{9a^2+18ab+9b^2-36ab}}{18}$$

$$= \frac{3(a+b) \pm \sqrt{9a^2-18ab+9b^2}}{18}$$

$$= \frac{3(a+b) \pm \sqrt{9(a^2-2ab+b^2)}}{18}$$

$$= \frac{3a+3b \pm \sqrt{9(a-b)^2}}{18} = \frac{3a+3b \pm 3(a-b)}{18}$$

$$= \frac{3a+3b+3a-3b}{18} \text{ and } \frac{3a+3b-3a+3b}{18}$$

$$= \frac{6a}{18} \text{ and } \frac{6b}{18} = \frac{a}{3} \text{ and } \frac{b}{3}$$

The roots are $\frac{a}{3}$ and $\frac{b}{3}$.

Hence, $x = \boldsymbol{\frac{a}{3}, \frac{b}{3}}$ are the required solutions.

(*vii*) $abx^2 + (b^2 - ac)x - bc = 0$

Comparing the given equation with $Ax^2 + Bx + C = 0$, we have

$A = ab, B = (b^2 - ac), c = -bc$

$$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$= \frac{-(b^2-ac) \pm \sqrt{(b^2-ac)^2 - 4(ab)(-bc)}}{2(ab)}$$

$$= \frac{-b^2+ac \pm \sqrt{b^4-2ab^2c+a^2c^2+4ab^2c}}{2ab}$$

$$= \frac{-b^2+ac \pm \sqrt{b^4+2ab^2c+a^2c^2}}{2ab}$$

$$= \frac{-b^2+ac \pm \sqrt{(b^2+ac)^2}}{2ab} = \frac{-b^2+ac \pm (b^2+ac)}{2ab}$$

$$= \frac{-b^2+ac+b^2+ac}{2ab} \text{ and } \frac{-b^2+ac-b^2-ac}{2ab}$$

$$= \frac{2ac}{2ab} \text{ and } \frac{-2b^2}{2ab} = \frac{c}{b} \text{ and } \frac{-b}{a}$$

The roots are $\frac{c}{b}$ and $\frac{-b}{a}$.

Hence $x = \boldsymbol{\frac{c}{b}, \frac{-b}{a}}$ are the required solutions.

(*viii*) $p^2x^2 + (p^2 - q^2)x - q^2 = 0$

Comparing the given equation with $ax^2 + bx + c = 0$, we have

$a = p^2, b = p^2 - q^2, c = -q^2$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(p^2-q^2) \pm \sqrt{(p^2-q^2)^2 - 4(p^2)(-q^2)}}{2p^2}$$

$$= \frac{-(p^2-q^2) \pm \sqrt{p^4-2p^2q^2+q^4+4p^2q^2}}{2p^2}$$

$$= \frac{-(p^2-q^2) \pm \sqrt{p^4+2p^2q^2+q^4}}{2p^2}$$

$$= \frac{-(p^2-q^2) \pm \sqrt{(p^2+q^2)^2}}{2p^2}$$

$$= \frac{-p^2+q^2 \pm (p^2+q^2)}{2p^2}$$

$$= \frac{-p^2+q^2+p^2+q^2}{2p^2} \text{ and } \frac{-p^2+q^2-p^2-q^2}{2p^2}$$

$$= \frac{2q^2}{2p^2} \text{ and } \frac{-2p^2}{2p^2} = \frac{q^2}{p^2} \text{ and } -1$$

The roots are $\frac{q^2}{p^2}$ and -1.

Hence, $x = \boldsymbol{\frac{q^2}{p^2}, -1}$ are the required solutions.

(*ix*) $x^2 - 2ax + (a^2 - b^2) = 0$

Comparing the given equation with

$Ax^2 + Bx + C = 0$, we have

$A = 1, B = -2a, C = (a^2 - b^2)$

$$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$= \frac{-(-2a) \pm \sqrt{(-2a)^2 - 4(1)(a^2-b^2)}}{2(1)}$$

$$= \frac{2a \pm \sqrt{4a^2-4a^2+4b^2}}{2}$$

$$= \frac{2a \pm \sqrt{4b^2}}{2} = \frac{2a \pm 2b}{2}$$

$$= \frac{2a+2b}{2} \text{ and } \frac{2a-2b}{2} = (a+b) \text{ and } (a-b)$$

The roots are $(a + b)$ and $(a - b)$.

Hence $x = \boldsymbol{(a + b), (a - b)}$ are the required solutions.

(x) $4x^2 - 4ax + (a^2 - b^2) = 0$

Comparing the given equation with

$Ax^2 + Bx + C = 0$, we have

$A = 4$, $B = -4a$, $C = (a^2 - b^2)$

$$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$= \frac{-4a \pm \sqrt{(-4a)^2 - 4(4)(a^2 - b^2)}}{2(4)}$$

$$= \frac{-4a \pm \sqrt{16a^2 - 16a^2 + 16b^2}}{8}$$

$$= \frac{-4a \pm \sqrt{16b^2}}{8}$$

$$= \frac{-4a \pm 4b}{8}$$

$$= \frac{-a \pm b}{2}$$

$$= \frac{b-a}{2} \text{ and } \frac{-(a+b)}{2}$$

The roots are $\frac{b-a}{2}$ and $\frac{-(a+b)}{2}$

Hence, $\boldsymbol{x = \frac{b-a}{2}, \frac{-(a+b)}{2}}$ are the required solutions.

(xi) $x^2 - 2ax - (4b^2 - a^2) = 0$

Comparing the given equation with

$Ax^2 + Bx + C = 0$, we have

$A = 1$, $B = -2a$ and $C = -(4b^2 - a^2) = a^2 - 4b^2$

$$\therefore \quad x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$= \frac{2a \pm \sqrt{4a^2 - 4a^2 + 16b^2}}{2}$$

$$= \frac{2a \pm 4b}{2}$$

$$= a \pm 2b$$

$$= a + 2b,\ a - 2b$$

The roots are $a + 2b$ and $a - 2b$.

Hence, $\boldsymbol{x = a + 2b, a - 2b}$ are the required solutions.

(xii) $x^2 - 4ax - b^2 + 4a^2 = 0$

Comparing the given equation with

$Ax^2 + Bx + C = 0$, we have

$A = 1$, $B = -4a$ and $C = 4a^2 - b^2$

$$\therefore \quad x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$= \frac{4a \pm \sqrt{16a^2 - 4(4a^2 - b^2)}}{2}$$

$$= \frac{4a \pm 2b}{2}$$

$$= 2a \pm b$$

$$= 2a + b,\ 2a - b$$

The roots are $2a + b$ and $2a - b$.

Hence, $\boldsymbol{x = 2a + b, 2a - b}$ are the required solutions.

(xiii) $4x^2 - 4a2x + (a^4 - b^4) = 0$

Comparing the given equation with

$Ax^2 + Bx + C = 0$, we have

$A = 4$, $B = -4a^2$ and $C = a^4 - b^4$

$$\therefore \quad x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$= \frac{4a^2 \pm \sqrt{16a^4 - 4 \times 4 \times (a^4 - b^4)}}{8}$$

$$= \frac{4a^2 \pm \sqrt{16b^4}}{8}$$

$$= \frac{4a^2 \pm 4b^2}{8}$$

$$= \frac{a^2 \pm b^2}{2}$$

$$= \frac{a^2 + b^2}{2}, \frac{a^2 - b^2}{2}$$

The roots are $\frac{a^2 + b^2}{2}$ and $\frac{a^2 - b^2}{2}$.

Hence, $\boldsymbol{x = \frac{a^2 + b^2}{2}, \frac{a^2 - b^2}{2}}$ are the required solutions.

(xiv) $9x^2 - 6b^2x - (a^4 - b^4) = 0$

$9x^2 - 3(a^2 + b^2)x + 3(a^2 - b^2)x - (a^4 - b^4) = 0$

$3x[3x - (a^2 + b^2)] + (a^2 - b^2)[3x - (a^2 + b^2)] = 0$

$[3x + (a^2 - b^2)][3x - (a^2 + b^2)] = 0$

$$x = \frac{b^2 - a^2}{3} \text{ and } x = \frac{a^2 + b^2}{3}$$

The roots are $\frac{b^2 - a^2}{3}$ and $\frac{a^2 + b^2}{3}$.

Hence, $\boldsymbol{x = \frac{b^2 - a^2}{3}, \frac{a^2 + b^2}{3}}$ are the required solutions.

(xv) $12abx^2 - (9a^2 - 8b^2)x - 6ab = 0$

Comparing the given equation with

$Ax^2 + Bx + C = 0$, we have

$A = 12ab$, $B = -(9a^2 - 8b^2)$, $C = -6ab$

$$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$= \frac{-[-(9a^2 - 8b^2)] \pm \sqrt{[-(9a^2 - 8b^2)^2 - 4(12ab)(-6ab)}}{2(12ab)}$$

$$= \frac{9a^2 - 8b^2 \pm \sqrt{81a^4 - 144a^2b^2 + 64b^4 + 288a^2b^2}}{24ab}$$

$$= \frac{9a^2 - 8b^2 \pm \sqrt{81a^4 + 144a^2b^2 + 64b^4}}{24ab}$$

$$= \frac{9a^2 - 8b^2 \pm \sqrt{(9a^2 + 8b^2)^2}}{24ab}$$

$$= \frac{9a^2 - 8b^2 \pm (9a^2 + 8b^2)}{24ab}$$

$$= \frac{9a^2 - 8b^2 + 9a^2 + 8b^2}{24ab} \text{ and } \frac{9a^2 - 8b^2 - 9a^2 - 8b^2}{24ab}$$

$$= \frac{18a^2}{24ab} \text{ and } \frac{-16b^2}{24ab} = \frac{3a}{4b} \text{ and } \frac{-2b}{3a}$$

The roots are $\frac{3a}{4b}$ and $\frac{-2b}{3a}$.

Hence, $x = \mathbf{\frac{3a}{4b}, \frac{-2b}{3a}}$ are the required solutions.

(*xvi*) $(a + b)^2 x^2 - 8(a^2 - b^2)x - 20 (a - b)^2 = 0$

Comparing the given equation with

$Ax^2 + Bx + C = 0$, we have

$A = (a + b)^2$, $B = -8(a^2 - b^2)$, $C = -20(a - b)^2$

$$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$= \frac{-[-8(a^2 - b^2)] \pm \sqrt{[-8(a^2 - b^2)]^2 - 4(a + b)^2 [-20(a - b)^2]}}{2(a + b)^2}$$

$$= \frac{8(a^2 - b^2) \pm \sqrt{64(a^2 - b^2)^2 + 80(a^2 - b^2)^2}}{2(a + b)^2}$$

$$= \frac{8(a^2 - b^2) \pm \sqrt{144(a^2 - b^2)^2}}{2(a + b)^2}$$

$$= \frac{8(a^2 - b^2) \pm 12(a^2 - b^2)}{2(a + b)^2}$$

$$= \frac{8a^2 - 8b^2 + 12a^2 - 12b^2}{2(a + b)^2} \text{ and } \frac{8a^2 - 8b^2 - 12a^2 + 12b^2}{2(a + b)^2}$$

$$= \frac{20a^2 - 20b^2}{2(a + b)^2} \text{ and } \frac{-4a^2 + 4b^2}{2(a + b)^2}$$

$$= \frac{20(a^2 - b^2)}{2(a + b)^2} \text{ and } \frac{-4(a^2 - b^2)}{2(a + b)^2}$$

$$= \frac{20(a + b)(a - b)}{2(a + b)(a + b)} \text{ and } \frac{-4(a + b)(a - b)}{2(a + b)(a + b)}$$

$$= \frac{10(a - b)}{a + b} \text{ and } \frac{-2(a - b)}{a + b}$$

The roots are $\frac{10(a - b)}{a + b}$ and $\frac{-2(a - b)}{a + b}$.

Hence, $x = \mathbf{\frac{10(a - b)}{a + b}, \frac{-2(a - b)}{a + b}}$ are the required solutions.

(*xvii*) $x^2 + \left(\frac{a}{a + b} + \frac{a + b}{a}\right)x + 1 = 0$

$$\Rightarrow \quad x^2 + \frac{a}{a + b}x + \frac{a + b}{a}x + 1 = 0$$

$$\Rightarrow \quad x\left[x + \frac{a}{a + b}\right] + \frac{a + b}{a}\left[x + \frac{1}{1 + \frac{b}{a}}\right] = 0$$

$$\Rightarrow \quad x\left[x + \frac{a}{a + b}\right] + \frac{a + b}{a}\left[x + \frac{a}{a + b}\right] = 0$$

$$\Rightarrow \quad \left[x + \frac{a + b}{a}\right]\left[x + \frac{a}{a + b}\right] = 0$$

$$\Rightarrow \quad x = -\frac{(a + b)}{a} \text{ and } x = -\frac{a}{a + b}$$

The roots are $-\frac{(a + b)}{a}$ and $-\frac{a}{a + b}$.

Hence, $x = \mathbf{-\frac{(a + b)}{a}, -\frac{a}{a + b}}$ are the required solutions.

EXERCISE 4E

For Basic and Standard Levels

1. (*i*) $4x^2 + 7x + 2 = 0$

Comparing the given equation with $ax^2 + bx + c = 0$, we have

$a = 4$, $b = 7$, $c = 2$

$D = b^2 - 4ac = (7)^2 - 4(4)(2)$

$= 49 - 32 = 17 > 0$

Since D > 0, therefore the equation has **real and unequal roots**.

(*ii*) $x^2 + x + 1$

Comparing the given equation with $ax^2 + bx + c = 0$, we have

$a = 1$, $b = 1$, $c = 1$

$D = b^2 - 4ac = (1)^2 - 4(1)(1)$

$= 1 - 4 = -3 < 0$

Since D < 0, therefore the equation **does not have any real roots.**

2. (*i*) $x^2 - 2\sqrt{3}x + 3 = 0$

Comparing the given equation with $ax^2 + bx + c = 0$, we have

$a = 1$, $b = -2\sqrt{3}$, $c = 3$

$D = b^2 - 4ac = (-2\sqrt{3})^2 - 4(1)(3)$

$= 12 - 12 = 0$

Since D = 0, therefore the equation has **real and equal roots.**

(*ii*) $4x^2 - 12x - 9 = 0$

Comparing the given equation with $ax^2 + bx + c = 0$, we have

$a = 4, b = -12, c = -9$

$D = b^2 - 4ac = (-12)^2 - 4(4)(-9)$

$= 144 + 144 = 288 > 0$

Since D > 0, therefore the equation has **real and unequal roots.**

3. (*i*) $x^2 + 10x - 39 = 0$

Comparing the given equation with $ax^2 + bx + c = 0$, we have

$a = 1, b = 10, c = -39$

$D = b^2 - 4ac = (10)^2 - 4(1)(-39)$

$= 100 + 156 = 256 > 0$

Since D > 0, therefore the equation has **real and unequal roots**.

(*ii*) $4x^2 - 12\sqrt{3}x + 27 = 0$

Comparing the given equation with $ax^2 + bx + c = 0$, we have

$a = 4, b = -12\sqrt{3}, c = 27$

$D = b^2 - 4ac = (-12\sqrt{3})^2 - 4(4)(27)$

$= 144 \times 3 - 432 = 432 - 432 = 0$

Since D = 0, therefore the equation has **real and equal roots**.

(*iii*) $4x^2 - 5x + \frac{25}{16} = 0$

$\Rightarrow \quad 64x^2 - 80x + 25 = 0$

Comparing $64x^2 - 80x + 25 = 0$ with $ax^2 + bx + c = 0$, we have

$a = 64, b = -80, c = 25$

$D = b^2 - 4ac = (-80)^2 - 4(64)(25)$

$= 6400 - 6400 = 0$

Since D = 0, therefore the equation has **real and equal roots**.

4. $x^2 + \frac{5}{3}x + 7 = 0$

$\Rightarrow \quad 3x^2 + 5x + 21 = 0$

Comparing $3x^2 + 5x + 21 = 0$ with $ax^2 + bx + c = 0$, we have

$a = 3, b = 5, c = 21$

$D = b^2 - 4ac = (5)^2 - 4(3)(21)$

$= 25 - 252 = -227 < 0$

Since D < 0, therefore the equation has no real roots.

5. (*i*) $px(x - 2) + 6 = 0$

$\Rightarrow \quad px^2 - 2px + 6 = 0$

$px^2 - 2px + 6 = 0$ is of the form $ax^2 + bx + c = 0$, where

$a = p, b = -2p, c = 6$

$D = b^2 - 4ac = (-2p)^2 - 4(p)(6)$

$= 4p^2 - 24p$

The given equation will have real roots if $D \geq 0$

$\Rightarrow \quad 4p^2 - 24p \geq 0$

$\Rightarrow \quad 4p(p - 6) \geq 0$

$\Rightarrow \quad p - 6 \geq 0$

$\boldsymbol{p \geq 6}$

(*ii*) $px^2 + 8x - 2 = 0$

The given equation is of the form $ax^2 + bx + c = 0$, where

$a = p, b = 8, c = -2$

$D = b^2 - 4ac = (8)^2 - 4(p)(-2) = 64 + 8p$

The given equation will have real roots if $D \geq 0$

$\Rightarrow \quad 64 + 8p \geq 0$

$\Rightarrow \quad 8p \geq -64$

$\Rightarrow \quad \boldsymbol{p \geq -8}$

(*iii*) $y^2 + \frac{3}{2}y + p = 0$

$\Rightarrow \quad 2y^2 + 3y + 2p = 0$

$2y^2 + 3y + 2p = 0$ is of the form $ax^2 + bx + c = 0$, where

$a = 2, b = 3, c = 2p$

$D = b^2 - 4ac = (3)^2 - 4(2)(2p) = 9 - 16p$

The given equation will have real roots if $D \geq 0$

$\Rightarrow \quad 9 - 16p \geq 0$

$\Rightarrow \quad 9 \geq 16p$

$\Rightarrow \quad \frac{9}{16} \geq p$

$\Rightarrow \quad \boldsymbol{p \leq \frac{9}{16}}$

(*iv*) $5x^2 - px + 5 = 0$

The given equation is of the form $ax^2 + bx + c = 0$, where $a = 5, b = -p, c = 5$

$D = b^2 - 4ac = (-p)^2 - 4(5)(5) = p^2 - 100$

The given equation will have real roots if $D \geq 0$.

$\Rightarrow \quad p^2 - 100 \geq 0$

$\Rightarrow \quad p^2 \geq 100$

$\Rightarrow \quad \boldsymbol{p \leq -10 \text{ or } p \geq 10}$

(*v*) $4px^2 + 5x + p = 0$

The given equation is of the form $ax^2 + bx + c = 0$, where

$a = 4p, b = 5, c = p$

$D = b^2 - 4ac = (5)^2 - 4(4p)(p) = 25 - 16p^2$

The given equation will have real roots if $D \geq 0$

$\Rightarrow \quad 25 - 16p^2 \geq 0$

$\Rightarrow \quad 25 \geq 16p^2$

$\Rightarrow \quad \frac{25}{16} \geq p^2$

$\Rightarrow \quad \boldsymbol{-\frac{5}{4} \leq p \leq \frac{5}{4}}$

(*vi*) $x^2 + 6x + 2p + 1 = 0$

The given equation is of the form $ax^2 + bx + c = 0$, where

$a = 1, b = 6, c = 2p + 1$

$D = b^2 - 4ac = (6)^2 - 4(1)(2p + 1)$

$= 36 - 8p - 4 = 32 - 8p$

The given equation will have real roots of $D \geq 0$

$\Rightarrow \quad 32 - 8p \geq 0$

$\Rightarrow \quad 32 \geq 8p$

$\Rightarrow \quad 4 \geq p$

$\Rightarrow \quad \boldsymbol{p \leq 4}$

6. $px^2 - 2\sqrt{5}px + 15 = 0$

Since the quadratic equation has two equal roots, therefore, D must be equal to 0.

$$D = 0$$

$\Rightarrow \quad b^2 - 4ac = 0$

$\Rightarrow \quad \left(-2\sqrt{5}p\right)^2 - 4(p)\,(15) = 0$

$\Rightarrow \quad 20p^2 - 60p = 0$

$\Rightarrow \quad 20p\,(p - 3) = 0$

$\Rightarrow \quad p = 0 \text{ or } 3$

If $p = 0$, the equation will not be a quadratic equation.

$\therefore$ we will reject it.

Thus, $p = 3$

7. (*i*) $kx^2 - kx + 1 = 0$

Since the roots are equal and real.

$\therefore \quad D = 0$

$\Rightarrow \quad b^2 - 4ac = 0$

$\Rightarrow \quad (-k)^2 - 4(k)(1) = 0$

$\Rightarrow \quad k^2 - 4k = 0$

$\Rightarrow \quad k(k - 4) = 0$

$\Rightarrow \quad \boldsymbol{k = 0} \text{ or } \boldsymbol{k = 4}$

If $k = 0$, the equation does not remain quadratic therefore we reject it.

(*ii*) $9x^2 - 3x + k = 0$

The given equation is of the form $ax^2 + bx + c = 0$, where

$$a = 9, b = -3, c = k$$

$$D = b^2 - 4ac = 9 - 4 \times 9 \times k = 9 - 36k$$

The equation has real and equal roots if D = 0

$\Rightarrow \quad 9 - 36k = 0$

$\Rightarrow \quad 9 = 36k$

$\Rightarrow \quad \boldsymbol{k = \frac{1}{4}}$

(*iii*) $4x^2 + kx + 9 = 0$

The given equation is of the form $ax^2 + bx + c = 0$, where $a = 4$, $b = k$, $c = 9$

$$D = b^2 - 4ac = k^2 - 4(4)\,(9) = k^2 - 144$$

The given equation will have real and equal roots if D = 0

$\Rightarrow \quad k^2 - 144 = 0$

$\Rightarrow \quad k^2 = 144$

$\Rightarrow \quad \boldsymbol{k = \pm 12}$

(*iv*) $kx^2 - 3kx + 9 = 0$

The given equation is of the form $ax^2 + bx + c = 0$, where $a = k$, $b = -3k$ and $c = 9$

$$D = b^2 - 4ac = 9k^2 - 4 \times k \times 9$$
$$= 9k^2 - 36k$$

The given equation will have real and equal roots if D = 0

$\Rightarrow \quad 9k^2 - 36k = 0$

$\Rightarrow \quad k^2 - 4k = 0$

$\Rightarrow \quad k(k - 4) = 0$

$\Rightarrow \quad k = 0 \text{ or } k = 4$

If $k = 0$, the equation does not remain quadratic therefore we reject it.

(*v*) $kx^2 - 2\sqrt{5}\,kx + 10 = 0$

The given equation is of the form $ax^2 + bx + c = 0$, where $a = k$, $b = -2\sqrt{5}\,k$ and $c = 10$

$$D = b^2 - 4ac = \left(2\sqrt{5}k\right)^2 - 4k \times 10$$
$$= 20k^2 - 40k$$

The equation has real and equal roots if D = 0

$\Rightarrow \quad 20k(k - 2) = 0$

$\Rightarrow \quad k = \mathbf{0} \text{ or } k = \mathbf{2}$

If $k = 0$, the equation does not remain quadratic therefore we reject it.

(*vi*) $2x^2 - (k - 2)\,x + 1 = 0$

The given equation is of the form $ax^2 + bx + c = 0$, where $a = 2$, $b = -(k - 2)$, $c = 1$.

$$D = b^2 - 4ac = [-(k - 2)]^2 - 4(2)\,(1)$$
$$= k^2 - 4k + 4 - 8 = k^2 - 4k - 4$$

The given equation will have real and equal roots if D = 0

$\Rightarrow \quad k^2 - 4k - 4 = 0$

$\Rightarrow \quad k = \dfrac{4 \pm \sqrt{16 - 4(1)(-4)}}{2}$

$\Rightarrow \quad = \dfrac{4 \pm \sqrt{16 + 16}}{2} = \dfrac{4 \pm \sqrt{32}}{2}$

$= \dfrac{4 \pm 4\sqrt{2}}{2} = 2 \pm 2\sqrt{2}$

Hence, $\boldsymbol{k = 2 \pm 2\sqrt{2}}$.

(*vii*) $x^2 - 2(k + 1)x + k^2 = 0$

The given equation is of the form $ax^2 + bx + c = 0$, where

$$a = 1, b = -2(k + 1), c = k^2$$

$$D = b^2 - 4ac = [-2(k + 1)]^2 - 4(1)\,(k^2)$$
$$= 4(k^2 + 2k + 1) - 4k^2$$
$$= 4k^2 + 8k + 4 - 4k^2 = 8k + 4$$

The given equation will have real and equal roots if D = 0

$\Rightarrow \quad 8k + 4 = 0$

$\Rightarrow \quad 8k = -4$

$\Rightarrow \quad k = \dfrac{-4}{8}$

$\Rightarrow \quad \boldsymbol{k = \dfrac{-1}{2}}$

(*viii*) $(k + 4)x^2 + (k + 1)x + 1 = 0$

The given equation is of the form $ax^2 + bx + c = 0$, where

$$a = (k + 4), b = (k + 1), c = 1$$

$$D = b^2 - 4ac = (k+1)^2 - 4(k+4)(1)$$
$$= k^2 + 2k + 1 - 4k - 16$$
$$= k^2 - 2k - 15$$

The given equation will have real and equal roots if D = 0

$\Rightarrow \quad k^2 - 2k - 15 = 0$

$\Rightarrow \quad k^2 - 5k + 3k - 15 = 0$

$\Rightarrow \quad k(k-5) + 3(k-5) = 0$

$\Rightarrow \quad (k-5)(k+3) = 0$

$\Rightarrow \quad$ Either $k - 5 = 0$ or $k + 3 = 0$

$\Rightarrow \quad \mathbf{k = 5}$ **or** $\mathbf{k = -3}$

(*ix*) $x^2 - 2(1 + 3k)x + 7(3 + 2k) = 0$

The given equation is of the form $ax^2 + bx + c = 0$, where

$a = 1,\ b = -2(1 + 3k),\ c = 7(3 + 2k)$

$$D = b^2 - 4ac = [-2(1+3k)]^2 - 4(1)(7)(3+2k)$$
$$= 4(1 + 6k + 9k^2) - 28(3 + 2k)$$
$$= 4 + 24k + 36k^2 - 84 - 56k$$
$$= 36k^2 - 32k - 80$$

The given equation will have real and equal roots of D = 0

$\Rightarrow \quad 36k^2 - 32k - 80 = 0$

$\Rightarrow \quad 9k^2 - 8k - 20 = 0$

$$\Rightarrow \quad k = \frac{8 \pm \sqrt{64 - 4(9)(-20)}}{2(9)}$$
$$= \frac{8 \pm \sqrt{64 + 720}}{18}$$
$$= \frac{8 \pm \sqrt{784}}{18} = \frac{8 \pm 28}{18}$$
$$= \frac{36}{18} \text{ or } \frac{-20}{18} = \mathbf{2} \text{ or } \mathbf{\frac{-10}{9}}$$

(*x*) $4x^2 - 2(k + 1)x + (k + 4) = 0$

The given equation is of the form $ax^2 + bx + c = 0$, where

$a = 4,\ b = -2(k + 1),\ c = (k + 4)$

$$D = b^2 - 4ac = [-2(k+1)]^2 - 4(4)(k+4)$$
$$= 4(k^2 + 2k + 1) - 16k - 64$$
$$= 4k^2 + 8k + 4 - 16k - 64$$
$$= 4k^2 - 8k - 60$$

The given equation will have real and equal roots if D = 0.

$\Rightarrow \quad 4k^2 - 8k - 60 = 0$

$\Rightarrow \quad k^2 - 2k - 15 = 0$

$\Rightarrow \quad k^2 - 5k + 3k - 15 = 0$

$\Rightarrow \quad k(k-5) + 3(k-5) = 0$

$\Rightarrow \quad (k-5)(k+3) = 0$

$\Rightarrow \quad$ Either $(k - 5) = 0$ or $(k + 3) = 0$

$\Rightarrow \quad \mathbf{k = 5}$ or $\mathbf{k = -3}$

(*xi*) $(k + 1)x^2 - 2(k - 1)x + 1 = 0$

The given equation is of the form $ax^2 + bx + c = 0$, where $a = (k + 1),\ b = -2(k - 1),\ c = 1$

$$D = b^2 - 4ac = [-2(k-1)]^2 - 4(k+1)(1)$$
$$= 4(k^2 - 2k + 1) - 4(k + 1)$$
$$= 4k^2 - 8k + 4 - 4k - 4$$
$$= 4k^2 - 12k$$

The given equation will have equal roots if D = 0.

$\Rightarrow \quad 4k^2 - 12k = 0$

$\Rightarrow \quad 4k(k - 3) = 0$

$\Rightarrow \quad$ Either $4k = 0$ or $k - 3 = 0$

$\Rightarrow \quad \mathbf{k = 0}$ or $\mathbf{k = 3}$

(*xii*) $(k - 12)x^2 + 2(k - 12)x + 2 = 0$

The given equation is of the form $ax^2 + bx + c = 0$, where $a = k - 12,\ b = 2(k - 12)$ and $c = 2$.

$$D = b^2 - 4ac$$
$$= 4(k-12)^2 - 4(k-12) \times 2$$
$$= 4(k-12)^2 - 8(k-12)$$
$$= 4(k-12)(k-12-2)$$
$$= 4(k-12)(k-14)$$

The given equation will have real and equal roots if D = 0

$\Rightarrow \quad (k-12)(k-14) = 0$

$\therefore \quad$ Either $k - 12 = 0$

$\Rightarrow \quad k = 12$

which is absurd, since in this case the given equation becomes 2 = 0.

or $\quad k - 14 = 0$

$\Rightarrow \quad k = \mathbf{14}$

(*xiii*) $k^2x^2 - 2(2k - 1)x + 4 = 0$

The given equation is of the form $ax^2 + bx + c = 0$, where

$a = k^2,\ b = -2(2k - 1),\ c = 4$

$$D = b^2 - 4ac$$
$$= [-2(2k-1)]^2 - 4(k^2)(4)$$
$$= 4(4k^2 - 4k + 1) - 16k^2$$
$$= 16k^2 - 16k + 4 - 16k^2$$
$$= -16k + 4$$

The given equation will have real roots if D = 0.

$\Rightarrow \quad -16k + 4 = 0$

$\Rightarrow \quad 4 = 16k$

$\Rightarrow \quad k = \frac{4}{16}$

$\Rightarrow \quad \mathbf{k = \frac{1}{4}}$

8. $x^2 + k(2x + k - 1) + 2 = 0$

$\Rightarrow \quad x^2 + 2kx + (k^2 - k + 2) = 0$

Comparing this equation with the standard quadratic equation $ax^2 + bx + c = 0$, we have

$a = 1$, $b = 2k$ and $c = k^2 - k + 2$

$\therefore$ Discriminant, $D = b^2 - 4ac$

$= 4k^2 - 4(k^2 - k + 2)$

$= 4(k - 2)$

For real and equal roots,

$D = 0$

$\Rightarrow \quad k - 2 = 0$

$\Rightarrow \quad k = 2$

$\therefore$ The required value of k is **2**.

9. $9x^2 - 3kx + k = 0$

Comparing this equation with standard quadratic equation $ax^2 + bx + c = 0$, we get

$a = 9$, $b = -3k$ and $c = k$

$\therefore$ Discriminant, $D = b^2 - 4ac$

$= 9k^2 - 4 \times 9 \times k$

$= 9k(k - 4)$

For real and equal roots,

$D = 0$

$\Rightarrow \quad k(k - 4) = 0$

$\therefore \quad k = 4$ [Since $k \neq 0$]

$\therefore$ The required value of k is **4**.

10. $(3k + 1)x^2 + 2(k + 1)x + 1 = 0$

Comparing this equation with the standard quadratic equation $ax^2 + bx + c = 0$, we have

$a = 3k + 1$, $b = 2(k + 1)$ and $c = 1$

Discriminant, $D = b^2 - 4ac$

$= 4(k + 1)^2 - 4(3k + 1)$

$= 4k^2 + 8k + 4 - 12k - 4$

$= 4k^2 - 4k$

$= 4k(k - 1)$

For real and equal roots,

$D = 0$

$\Rightarrow \quad k(k - 1) = 0$

$\therefore$ Either $k = 0$ or $k - 1 = 0$

$\Rightarrow \quad k = 1$

Hence, the required value of k is 0 or 1.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-b \pm \sqrt{D}}{2a} = \frac{-b}{2a} \qquad [\because D = 0]$$

$$= -\frac{2(k+1)}{2(3k+1)}$$

$$= -\frac{k+1}{3k+1}$$

$\therefore$ When $k = 0$, $x = -1$ and when $k = 1$, $x = \frac{-2}{4} = -\frac{1}{2}$

$\therefore$ Required roots are $x = \mathbf{-1}$ and $x = \mathbf{-\frac{1}{2}}$.

11. $(2p + 1)x^2 - (7p + 2)x + 7p - 3 = 0$

Comparing this equation with the standard quadratic equation $ax^2 + bx + c = 0$, we have

$a = 2p + 1$, $b = -(7p + 2)$ and $c = 7p - 3$

Discriminant, $D = b^2 - 4ac$

$= (7p + 2)^2 - 4(2p + 1)(7p - 3)$

$= 49p^2 + 4 + 28p - 4(14p^2 + p - 3)$

$= 49p^2 + 4 + 28p - 56p^2 - 4p + 12$

$= -7p^2 + 24p + 16$

Now, for real and equal roots, $D = 0$

$\therefore \quad -7p^2 + 24p + 16 = 0$

$$\Rightarrow \quad p = \frac{-24 \pm \sqrt{24^2 + 4 \times 7 \times 16}}{-14}$$

$$= \frac{-24 \pm \sqrt{576 + 448}}{-14}$$

$$= \frac{-24 \pm \sqrt{1024}}{-14}$$

$$= \frac{-24 \pm 32}{-14}$$

$$= \frac{8}{-14}, \frac{-56}{-14}$$

$$= \frac{-4}{7}, +4$$

which are the required values of p.

Hence, the roots of the given equation are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-b + \sqrt{D}}{2a}$$

$$= \frac{-b}{2a} \qquad [\because D = 0]$$

$$= \frac{7p+2}{2(2p+1)}$$

$$= \frac{7p+2}{4p+2}$$

Now, when $p = 4$, then

$$x = \frac{7 \times 4 + 2}{4 \times 4 + 2} = \frac{30}{18} = \frac{5}{3}$$

and when $p = -\frac{4}{7}$, then

$$x = \frac{7 \times \left(-\frac{4}{7}\right) + 2}{4 \times \left(-\frac{4}{7}\right) + 2}$$

$$= \frac{-2}{-\frac{16}{7} + 2} = \frac{2}{-\frac{2}{7}} = 7$$

Hence, the values of p are **4** and $\mathbf{-\frac{4}{7}}$ and required roots are $\mathbf{\frac{5}{3}}$ and **7**.

12. $(p + 1)x^2 - 6(p + 1)x + 3(p + 9) = 0$

Since the roots are real and equal

$\therefore \quad D = 0$

$\Rightarrow \quad b^2 - 4ac = 0$

$\Rightarrow \quad [-6(p + 1)]^2 - 4(p + 1)(3)(p + 9) = 0$

$\Rightarrow \quad 36(p + 1)^2 - 12(p + 1)(p + 9) = 0$

$\Rightarrow \quad (p + 1)[36(p + 1) - 12(p + 9)] = 0$

$\Rightarrow \quad (p + 1)(12)[3p + 3 - p - 9] = 0$

$\Rightarrow \quad 12(p + 1)(2p - 6) = 0$

$\Rightarrow \quad p = -1$ or $p = 3$

If $p = -1$, the equation does not remain quadratic therefore we will reject it.

For $p = 3$, the equation becomes

$4x^2 - 24x + 36 = 0$

$x^2 - 6x + 9 = 0$

$x^2 - 3x - 3x + 9 = 0$

$x(x - 3) - 3(x - 3) = 0$

$(x - 3)^2 = 0$

$\boldsymbol{x = 3, 3}$

13. $x^2 - 4x + k = 0$

Comparing the given equation with $ax^2 + bx + c = 0$, we have

$a = 1, b = -4, c = k$

$D = b^2 - 4ac$

$= (-4)^2 - 4(1)(k) = 16 - 4k$

(i) The given equation will have two distinct roots if $D > 0$.

$\Rightarrow \quad 16 - 4k > 0$

$\Rightarrow \quad 16 > 4k$

$\Rightarrow \quad 4 > k$

$\Rightarrow \quad \boldsymbol{k < 4}$

(ii) The given equation will have coincident roots if $D = 0$

$\Rightarrow \quad 16 - 4k = 0$

$\Rightarrow \quad 16 = 4k$

$\Rightarrow \quad k = \frac{16}{4}$

$\Rightarrow \quad \boldsymbol{k = 4}$

14. $2mx^2 - 2(1 + 2m)x + (3 + 2m) = 0$

Comparing the given equation with $ax^2 + bx + c = 0$, we have

$a = 2m, b = -2(1 + 2m), c = (3 + 2m)$

$D = b^2 - 4ac = [-2(1 + 2m)]^2 - 4(2m)(3 + 2m)$

$= 4(1 + 4m + 4m^2) - 8m(3 + 2m)$

$= 4 + 16m + 16m^2 - 24m - 16m^2$

$= -8m + 4$

The given equation will have real but distinct roots if $D > 0$.

$\Rightarrow \quad -8m + 4 > 0$

$\Rightarrow \quad 4 > 8m$

$\Rightarrow \quad \frac{1}{2} > m$

$\Rightarrow \quad \boldsymbol{m < \frac{1}{2}}$

The given equation will have equal roots if $D = 0$

$\Rightarrow \quad -8m + 4 = 0$

$\Rightarrow \quad 8m = 4$

$\Rightarrow \quad \boldsymbol{m = \frac{1}{2}}$

15. $x^2 + 2x + 4p = 0 \quad \dots (1)$

$x = -4$ is the root of equation (1), hence it will satisfy the equation.

$\Rightarrow \quad (-4)^2 + 2(-4) + 4p = 0$

$\Rightarrow \quad 16 - 8 + 4p = 0$

$\Rightarrow \quad 4p = -8$

$\Rightarrow \quad p = -2$

$x^2 + px(1 + 3k) + 7(3 + 2k) = 0 \quad \dots (2)$

Put $p = -2$ in eq. (2)

$x^2 + (-2)(1 + 3k)x + 7(3 + 2k) = 0$

$x^2 - 2(1 + 3k)x + 7(3 + 2k) = 0 \quad \dots (3)$

Since eq. (3) has equal roots, therefore,

$D = 0$

$\Rightarrow \quad b^2 - 4ac = 0$

$\Rightarrow \quad [-2(1 + 3k)]^2 - 4(1)(7)(3 + 2k) = 0$

$\Rightarrow \quad 4(1 + 3k)^2 - 28(3 + 2k) = 0$

$\Rightarrow \quad (1 + 3k)^2 - 7(3 + 2k) = 0$

$\Rightarrow \quad 1 + 9k^2 + 6k - 21 - 14k = 0$

$\Rightarrow \quad 9k^2 - 8k - 20 = 0$

$\Rightarrow \quad 9k^2 - 18k + 10k - 20 = 0$

$\Rightarrow \quad 9k(k - 2) + 10(k - 2) = 0$

$\Rightarrow \quad (9k + 10)(k - 2) = 0$

$\Rightarrow \quad \boldsymbol{k = -\frac{10}{9}}$ or $\boldsymbol{k = 2}$

16. Since $x = -5$ is root of the equation $2x^2 + px - 15 = 0$,

$\therefore \quad 2 \times (-5)^2 + p(-5) - 15 = 0$

$\Rightarrow \quad 50 - 5p - 15 = 0$

$\Rightarrow \quad 5p = 35$

$\therefore \quad p = \frac{35}{5} = 7$

$\therefore$ The given quadratic equation becomes

$7(x^2 + x) + k = 0$

$\Rightarrow \quad 7x^2 + 7x + k = 0 \quad \dots(1)$

Comparing this equation with the standard quadratic equation $ax^2 + bx + c = 0$, we have

$a = 7$, $b = 7$ and $c = k$

$\because$ The equation (1) has equal roots

$\therefore$ Discriminant, $D = b^2 - 4ac$
$= 49 - 4 \times 7k$
$= 7(7 - 4k) = 0$

$\therefore k = \mathbf{\frac{7}{4}}$ which is the required value of k.

17. Since 2 is a root of the quadratic equation

$$3x^2 + px - 8 = 0$$

$\therefore x = 2$ will satisfy this equation.

$\therefore \quad 3 \times 2^2 + 2p - 8 = 0$
$\Rightarrow \quad 12 - 8 + 2p = 0$
$\Rightarrow \quad 2p + 4 = 0$
$\Rightarrow \quad p = -2$

Hence, the given 2nd quadratic equation becomes

$$4x^2 + 4x + k = 0 \qquad \ldots(1)$$

Comparing this equation with the standard quadratic equation, $ax^2 + bx + c = 0$, we have

$a = 4, b = 4, c = k$

$\therefore$ Discriminant, $D = b^2 - 4ac$
$= 16 - 4 \times 4 \times k$
$= 16(1 - k)$

Since, the quadratic equation (1) has real and equal roots,

$\therefore \quad D = 0$
$\Rightarrow \quad 1 - k = 0$
$\Rightarrow k = \mathbf{1}$ which is the required value of k.

18. Since 3 is a root of the quadratic equation $x^2 - x + k = 0$,

$\therefore x = 3$ will satisfy this equation

$\therefore \quad 3^2 - 3 + k = 0$
$\Rightarrow \quad k = -6$

$\therefore$ The second given quadratic equation becomes

$x^2 - 6(2x - 6 + 2) + p = 0$
$\Rightarrow \quad x^2 - 6(2x - 4) + p = 0$
$\Rightarrow \quad x^2 - 12x + 24 + p = 0 \qquad \ldots(1)$

Comparing this equation (1) with the standard quadratic equation $ax^2 + bx + c = 0$, we have

$a = 1, b = -12$ and $c = 24 + p$

$\therefore$ Discriminant, $D = b^2 - 4ac$
$= (-12)^2 - 4 \times 1 \times (24 + p)$
$= 144 - 96 - 4p$
$= 48 - 4p \qquad \ldots(2)$

Since the quadratic equation (1) has real and equal roots,

$\therefore \quad D = 0$
$\Rightarrow \quad 48 - 4p = 0 \qquad$ [From (2)]
$\Rightarrow \quad p = \frac{48}{4} = \mathbf{12}$

which is the required value of p.

For Standard Level

19. $(a^2 + b^2)x^2 - 2(ac + bd)x + (c^2 + d^2) = 0$

Comparing the given equation with $Ax^2 + Bx + C = 0$, we have

$A = a^2 + b^2, B = -2(ac + bd), C = c^2 + d^2$

Discriminant $D = B^2 - 4AC$
$= [-2(ac + bd)]^2 - 4(a^2 + b^2)(c^2 + d^2)$
$= 4(a^2c^2 + 2abcd + b^2d^2) - 4(a^2c^2 + b^2c^2 + a^2d^2 + b^2d^2]$
$= 4[a^2c^2 + 2abcd + b^2d^2 - a^2c^2 - b^2c^2 - a^2d^2 - b^2d^2]$
$= 4[2abcd - b^2c^2 - a^2d^2]$

Since the roots of the given equation are equal

$\therefore \quad D = 0$
$\Rightarrow \quad 4[2abcd - b^2c^2 - a^2d^2] = 0$
$\Rightarrow \quad 2abcd - b^2c^2 - a^2d^2 = 0$
$\Rightarrow \quad a^2d^2 - 2abcd + b^2c^2 = 0$
$\Rightarrow \quad (ad - bc)^2 = 0$
$\Rightarrow \quad (ad - bc)(ad - bc) = 0$
$\Rightarrow \quad (ad - bc) = 0$
$\Rightarrow \quad ad = bc$
$\Rightarrow \quad \mathbf{ad = bc}$

20. Comparing the given quadratic equation with the standard quadratic equation $Ax^2 + Bx + C = 0$, we have

$A = 1 + m^2, B = 2mc$ and $C = c^2 - a^2$

$\therefore$ Discriminant, $D = B^2 - 4AC$
$= 4m^2c^2 - 4(1 + m^2)(c^2 - a^2)$
$= 4m^2c^2 - 4(c^2 - a^2 + m^2c^2 - m^2a^2)$
$= 4(a^2 - c^2 + m^2a^2)$

Since, the given quadratic equation has two real and equal roots, hence $D = 0$

$\Rightarrow \quad a^2 - c^2 + m^2a^2 = 0$
$\Rightarrow \quad c^2 = a^2(1 + m^2)$

Hence, proved.

21. Comparing the given equation with the standard quadratic equation $Ax^2 + Bx + C = 0$, we have

$A = (c^2 - ab), B = -2(a^2 - bc)$ and $C = b^2 - ac$

Now, discriminant, $D = B^2 - 4AC$
$= 4(a^2 - bc)^2 - 4(c^2 - ab)(b^2 - ac)$
$= 4[a^4 + b^2c^2 - 2a^2bc - b^2c^2 + ab^3 + ac^3 - a^2bc]$
$= 4[a^4 - 3a^2bc + ab^3 + ac^3]$
$= 4a[a^3 - 3abc + b^3 + c^3]$

Now, if the given equation has two real and equal roots, then $D = 0$

$\therefore \quad a(a^3 - 3abc + b^3 + c^3) = 0$
$\therefore \quad$ Either $a = 0$ or $a^3 + b^3 + c^3 - 3abc = 0$

Hence, proved.

22. Comparing the 1st given quadratic equation with the standard quadratic equation $Ax^2 + Bx + C = 0$,

we have $A = 1, B = 2c$ and $C = ab$

Let D_1 be the discriminant of the 1st given equation.

Then $D_1 = B^2 - 4AC$
$= 4c^2 - 4ab$
$= 4(c^2 - ab)$

Since the two roots of this equation are real and unequal,

$\therefore \quad D_1 > 0 \quad \Rightarrow c^2 > ab \quad \ldots(1)$

Now, let D_2 be the discriminant of the second given equation.

Then, $D_2 = B'^2 - 4A'C'$
$= \{-2(a + b)\}^2 - 4 \times 1(a^2 + b^2 + 2c^2)$

[$\because$ For the second given equation, we have
$A' = 1$, $B' = -2(a + b)$ and $C' = a^2 + b^2 + 2c^2$]

$= 4[a^2 + b^2 + 2ab - a^2 - b^2 - 2c^2]$
$= 8(ab - c^2) < 0$ [From (1)]

Hence, the second given equation has no real roots.

23. Comparing this equation with the standard quadratic equation $ax^2 + bx + c = 0$, we have

$A = (b - c)$, $B = (c - a)$ and $C = (a - b)$

Discriminant, $D = B^2 - 4AC$
$= (c - a)^2 - 4(b - c)(a - b)$
$= c^2 + a^2 - 2ac - 4ab + 4b^2 + 4ac - 4bc$
$= c^2 + a^2 + (2b)^2 + 2ac - 4ab - 4bc$

$\Rightarrow \quad D = (2b - a - c)^2$

Since, the two roots are real and equal,

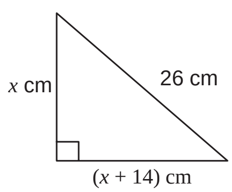
$\therefore \quad D = 0$

$\Rightarrow \quad 2b = a + c$

Hence, proved.

24. Let the shortest side of the right triangle be x cm. Then, the other side containing the right angle is $(x + 14)$ cm Hypotenuse = 26 cm.

By Pythagoras' Theorem, we have



$x^2 + (x + 14)^2 = (26)^2$

$\Rightarrow \quad x^2 + x^2 + 28x + 196 = 676$

$\Rightarrow \quad 2x^2 + 28x + 196 - 676 = 0$

$\Rightarrow \quad 2x^2 + 28x - 480 = 0$

$\Rightarrow \quad x^2 + 14x - 240 = 0$

$D = b^2 - 4ac = (14)^2 - 4(1)(-240)$
$= 196 + 960 = 1156 > 0$

Since $D > 0$, therefore the equation $x^2 + 14x - 240 = 0$ has real roots.

$\therefore$ It is possible to design the required right triangle

Now, $x^2 + 14x - 240 = 0$

$$\Rightarrow \quad x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-14 \pm \sqrt{1156}}{2(1)}$$

$$= \frac{-14 \pm 34}{2} = \frac{-14 + 34}{2} \text{ or } \frac{-14 - 34}{2}$$

$$= \frac{20}{2} \text{ or } \frac{-48}{2} \text{ (rejected)}$$

$= 10$

The sides are x cm = 10 cm and

$(x + 14)$ cm = (10 + 14) cm = 24 cm

Hence, the sides are **10 cm and 24 cm**.

25. Let the present age of one friend be x years.

Then, the present age of the other friend is $(25 - x)$ years.

Five years ago, the ages of the two friends was $(x - 5)$ years and $(25 - x - 5)$ years, i.e. $(20 - x)$ years.

Five years ago, the product of their ages (in years) = 60

$\Rightarrow \quad (x - 5)(20 - x) = 60$

$\Rightarrow \quad 20x - 100 - x^2 + 5x = 60$

$\Rightarrow \quad x^2 - 25x + 160 = 0$

$D = b^2 - 4ac = (-25)^2 - 4(1)(160)$
$= 625 - 640 = -15 < 0$

Since $D < 0$, therefore the equation $x^2 - 25x + 160 = 0$ has no real roots.

Hence, the given situation is not possible.

EXERCISE 4F

For Basic and Standard Levels

1. Let one of the required numbers be x. Then, the other number is $(18 - x)$.

Given, product of the two given numbers = 56.

$\Rightarrow \quad x(18 - x) = 56$

$\Rightarrow \quad 18x - x^2 = 56$

$\Rightarrow \quad x^2 - 18x + 56 = 0$

$\Rightarrow \quad x^2 - 14x - 4x + 56 = 0$

$\Rightarrow \quad x(x - 14) - 4(x - 14) = 0$

$\Rightarrow \quad (x - 14)(x - 4) = 0$

$\Rightarrow \quad$ Either $x - 14 = 0$ or $x - 4 = 0$

$\Rightarrow \quad x = 14$ or $x = 4$

When, $x = 14$, then $(18 - x) = 4$

When, $x = 4$, then $(18 - 4) = 14$

The required numbers are **4, 14**.

2. Let the greater of the two required numbers be x. Then, the other number is $x - 3$.

Given, product of the two number = 504

$\Rightarrow \quad x(x - 3) = 504$

$\Rightarrow \quad x^2 - 3x - 504 = 0$

$\Rightarrow \quad x^2 - 24x + 21x - 504 = 0$

$\Rightarrow \quad x(x - 24) + 21(x - 24) = 0$

$\Rightarrow \quad (x - 24)(x + 21) = 0$

$\Rightarrow \quad$ Either $x - 24 = 0$ or $x + 21 = 0$

$\Rightarrow \quad x = 24$ or $x = -21$

When $x = 24$, then $x - 3 = 21$

When $x = -21$, then $x - 3 = -24$

Hence, the required numbers are **21 and 24 or –21 and –24**.

3. Let the greater of the two required numbers be x.

Then, the other number is $(x - 4)$.

Given, the difference of reciprocals of the numbers $= \frac{4}{21}$

$\Rightarrow \quad \frac{1}{x-4} - \frac{1}{x} = \frac{4}{21}$

$\Rightarrow \quad \frac{x-(x-4)}{x(x-4)} = \frac{4}{21}$

$\Rightarrow \quad 21(x - x + 4) = 4x(x - 4)$

$\Rightarrow \quad 84 = 4x^2 - 16x$

$\Rightarrow \quad 4x^2 - 16x - 84 = 0$

$\Rightarrow \quad x^2 - 4x - 21 = 0$

$\Rightarrow \quad x^2 - 7x + 3x - 21 = 0$

$\Rightarrow \quad x(x - 7) + 3(x - 7) = 0$

$\Rightarrow \quad (x - 7)(x + 3) = 0$

$\Rightarrow \quad$ Either $(x - 7) = 0$ or $(x + 3) = 0$

$\Rightarrow \quad x = 7$ or $x = -3$

When $x = 7$, then $(x - 4) = 7 - 4 = 3$

When $x = -3$, then $(x - 4) = -3 - 4 = -7$

Hence, the required numbers are **7 and 3** or **–3 and –7.**

4. (*i*) Let the required number be x.

Given, the number + its reciprocal = $\frac{-25}{12}$

$\Rightarrow \quad x + \frac{1}{x} = \frac{-25}{12}$

$\Rightarrow \quad \frac{x^2+1}{x} = \frac{-25}{12}$

$\Rightarrow \quad 12x^2 + 12 = -25x$

$\Rightarrow \quad 12x^2 + 25x + 12 = 0$

$\Rightarrow \quad 12x^2 + 16x + 9x + 12 = 0$

$\Rightarrow \quad 4x(3x + 4) + 3(3x + 4) = 0$

$\Rightarrow \quad (3x + 4)(4x + 3) = 0$

$\Rightarrow \quad$ Either $(3x + 4) = 0$ or $(4x + 3) = 0$

$\Rightarrow \quad x = \frac{-4}{3}$ or $x = \frac{-3}{4}$

Hence the required number is $\mathbf{\frac{-4}{3}}$ **or** $\mathbf{\frac{-3}{4}}$.

(*ii*) Let the greater of the two required number be x.

Then, the other number = $9 - x$

Given, sum of the reciprocals = $\frac{1}{2}$

$\Rightarrow \quad \frac{1}{x} + \frac{1}{9-x} = \frac{1}{2}$

$\Rightarrow \quad 2(9 - x + x) = x(9 - x)$

$\Rightarrow \quad 18 = 9x - x^2$

$\Rightarrow \quad x^2 - 9x + 18 = 0$

$\Rightarrow \quad x^2 - 3x - 6x + 18 = 0$

$\Rightarrow \quad x(x - 3) - 6(x - 3) = 0$

$\Rightarrow \quad (x - 3)(x - 6) = 0$

$\Rightarrow \quad$ Either $(x - 3) = 0$ or $(x - 6) = 0$

$\Rightarrow \quad x = 3$ or $x = 6$

Hence, the required numbers are **3 and 6**.

(*iii*) Given, $a + b = 15$

$\Rightarrow \quad b = 15 - a \quad \ldots(1)$

and $\quad \frac{1}{a} + \frac{1}{b} = \frac{3}{10}$

$\Rightarrow \quad \frac{1}{a} + \frac{1}{15-a} = \frac{3}{10} \quad$ [Using (1)]

$\Rightarrow \quad 10(15 - a + a) = 3a(15 - a)$

$\Rightarrow \quad 150 = 45a - 3a^2$

$\Rightarrow \quad 3a^2 - 45a + 150 = 0$

$\Rightarrow \quad a^2 - 15a + 50 = 0$

$\Rightarrow \quad a^2 - 5a - 10a + 50 = 0$

$\Rightarrow \quad a(a - 5) - 10(a - 5) = 0$

$\Rightarrow \quad (a - 5)(a - 10) = 0$

$\Rightarrow \quad$ Either $(a - 5) = 0$ or $(a - 10) = 0$

$\Rightarrow \quad a = 5$ or $a = 10$

When $a = 5$, then $b = 15 - 5 = 10$

When $a = 10$, then $b = 15 - 10 = 5$

Hence the required numbers are **5** and **10**.

5. (*i*) Let the required number be x.

Given, the number + its reciprocal = $\frac{65}{8}$

$\Rightarrow \quad x + \frac{1}{x} = \frac{65}{8}$

$\Rightarrow \quad 8x^2 + 8 = 65x$

$\Rightarrow \quad 8x^2 - 65x + 8 = 0$

$\Rightarrow \quad 8x^2 - 64x - x + 8 = 0$

$\Rightarrow \quad 8x(x - 8) - 1(x - 8) = 0$

$\Rightarrow \quad (x - 8)(8x - 1) = 0$

$\Rightarrow \quad$ Either $(x - 8) = 0$ or $(8x - 1) = 0$

$\Rightarrow \quad x = 8$ or $x = \frac{1}{8}$

Hence the required number is **8 or** $\mathbf{\frac{1}{8}}$.

(*ii*) Let the required number be x.

Given, the number – its reciprocal = $\frac{63}{8}$

$\Rightarrow \quad x - \frac{1}{x} = \frac{63}{8}$

$\Rightarrow \quad 8x^2 - 8 = 63x$

$\Rightarrow \quad 8x^2 - 63x - 8 = 0$

$\Rightarrow \quad 8x^2 - 64x + x - 8 = 0$

$\Rightarrow \quad 8x(x - 8) + 1(x - 8) = 0$

$\Rightarrow \quad (x - 8)(8x + 1) = 0$

$\Rightarrow \quad$ Either $(x - 8) = 0$ or $(8x + 1) = 0$

$\Rightarrow \quad x = 8$ or $x = \frac{-1}{8}$

Hence, the required number is **8 or** $\mathbf{\frac{-1}{8}}$.

6. (*i*) Let the required natural number be x.

Given, the natural number + 12 = 160 times its reciprocal

$\Rightarrow \quad x + 12 = 160\left(\frac{1}{x}\right)$

$\Rightarrow \quad x + 12 = \frac{160}{x}$

$\Rightarrow \quad x^2 + 12x - 160 = 0$

$\Rightarrow \quad x^2 - 8x + 20x - 160 = 0$

$\Rightarrow \quad x(x - 8) + 20(x - 8) = 0$

$\Rightarrow \quad (x - 8)(x + 20) = 0$

$\Rightarrow \quad$ Either $(x - 8) = 0$ or $(x + 20) = 0$

$\Rightarrow \quad x = 8$ or $x = -20$ (rejected)

Hence, the required number is **8**.

(*ii*) Let the required integer(s) be x.

Given, Number – 20 = 69 times its reciprocal

$\Rightarrow \quad x - 20 = 69\left(\frac{1}{x}\right)$

$\Rightarrow \quad x^2 - 20x = 69$

$\Rightarrow \quad x^2 - 20x - 69 = 0$

$\Rightarrow \quad x^2 + 3x - 23x - 69 = 0$

$\Rightarrow \quad x(x + 3) - 23(x + 3) = 0$

$\Rightarrow \quad (x + 3)(x - 23) = 0$

$\Rightarrow \quad$ Either $(x + 3) = 0$ or $(x - 23) = 0$

$\Rightarrow \quad x = -3$ or $x = 23$

Hence, the required integers are **–3 and 23.**

(*iii*) Let the required natural number be x.

Then according to the problem, we have

$x^2 - 84 = 3(x + 8)$

$\Rightarrow \quad x^2 - 3x - 84 - 24 = 0$

$\Rightarrow \quad x^2 - 3x - 108 = 0$

Comparing this quadratic equation with the standard quadratic equation $ax^2 + bx + c = 0$, we have

$a = 1$, $b = -3$ and $c = -108$

$\therefore \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{3 \pm \sqrt{9 + 4 \times 108}}{2}$

$= \frac{3 \pm \sqrt{441}}{2} = \frac{3 \pm 21}{2}$

$= \frac{24}{2}, -\frac{18}{2} = 12, -9$

$\because$ –9 is not a natural number,

$\therefore$ We reject it.

Hence, the required natural number is **12**.

7. $S = n\left(\frac{n+1}{2}\right)$

$\Rightarrow \quad 465 = n\left(\frac{n+1}{2}\right)$ [$\because$ S = 465, given]

$\Rightarrow \quad 930 = n^2 + n$

$\Rightarrow \quad n^2 + n - 930 = 0$

$\Rightarrow \quad n^2 - 30n + 31n - 930 = 0$

$\Rightarrow \quad n(n - 30) + 31(n - 30) = 0$

$\Rightarrow \quad (n - 30)(n + 31) = 0$

$\Rightarrow \quad$ Either $(n - 30) = 0$ or $(n + 31) = 0$

$\Rightarrow \quad n = 30$ or $n = -31$ (rejected)

Hence n = **30**.

8. Let the required number be x.

Given, number + its square = $\frac{35}{4}$

$\Rightarrow \quad x + x^2 = \frac{35}{4}$

$\Rightarrow \quad 4x + 4x^2 = 35$

$\Rightarrow \quad 4x^2 + 4x - 35 = 0$

$\Rightarrow \quad 4x^2 + 14x - 10x - 35 = 0$

$\Rightarrow \quad 2x(2x + 7) - 5(2x + 7) = 0$

$\Rightarrow \quad (2x + 7)(2x - 5) = 0$

$\Rightarrow \quad$ Either $(2x + 7) = 0$ or $(2x - 5) = 0$

$\Rightarrow \quad x = \frac{-7}{2}$ or $x = \frac{5}{2}$

Hence, the required number is $\mathbf{\frac{-7}{2}}$ **or** $\mathbf{\frac{5}{2}}$.

9. Let the greater of the two required natural numbers be x.

Then, the other number is $(x - 3)$.

Given, sum of the squares of the two natural numbers = 149

$\Rightarrow \quad x^2 + (x - 3)^2 = 149$

$\Rightarrow \quad x^2 + x^2 - 6x + 9 - 149 = 0$

$\Rightarrow \quad 2x^2 - 6x - 140 = 0$

$\Rightarrow \quad x^2 - 3x - 70 = 0$

$\Rightarrow \quad x^2 - 10x + 7x - 70 = 0$

$\Rightarrow \quad x(x - 10) + 7(x - 10) = 0$

$\Rightarrow \quad (x - 10)(x + 7) = 0$

$\Rightarrow \quad$ Either $(x - 10) = 0$ or $(x + 7) = 0$

$\Rightarrow \quad x = 10$ or $x = -7$ (rejected)

When $x = 10$, then $x - 3 = 10 - 3 = 7$

Hence, the required natural numbers are **7 and 10**.

10. Let one of the required numbers be x.

Then, the other number is $(2x - 3)$.

Given, the sum of squares of the two numbers = 233.

$\Rightarrow \quad x^2 + (2x - 3)^2 = 233$

$\Rightarrow \quad x^2 + 4x^2 - 12x + 9 - 233 = 0$

$\Rightarrow \quad x^2 + 4x^2 - 12x - 224 = 0$

$\Rightarrow \quad 5x^2 - 12x - 224 = 0$

$\Rightarrow \quad 5x^2 - 40x + 28x - 224 = 0$

$\Rightarrow \quad 5x(x - 8) + 28(x - 8) = 0$

$\Rightarrow \quad (x - 8)(5x + 28) = 0$

$\Rightarrow \quad$ Either $(x - 8) = 0$ or $(5x + 28) = 0$

$\Rightarrow \quad x = 8$ or $x = \frac{-28}{5}$

When $x = 8$, then $(2x - 3) = (2 \times 8 - 3) = 16 - 3 = 13$

When $x = \frac{-28}{5}$, then $(2x - 3) = 2\left(\frac{-28}{5}\right) - 3 = \frac{-56}{5} - 3$

$$= \frac{-56 - 15}{5} = \frac{-71}{5}$$

Hence, the required numbers are **8 and 13 or** $\mathbf{\frac{-28}{5}}$ **and** $\mathbf{\frac{-71}{5}}$.

11. (*i*) Let the required two consecutive odd integers be $2x + 1$ and $2x + 3$.

Given, product of the two consecutive odd integers = 483

$\Rightarrow \quad (2x + 1)(2x + 3) = 483$

$\Rightarrow \quad 4x^2 + 2x + 6x + 3 = 483$

$\Rightarrow \quad 4x^2 + 8x + 3 - 483 = 0$

$\Rightarrow \quad 4x^2 + 8x - 480 = 0$

$\Rightarrow \quad x^2 + 2x - 120 = 0$

$\Rightarrow \quad x^2 - 10x + 12x - 120 = 0$

$\Rightarrow \quad x(x - 10) + 12(x - 10) = 0$

$\Rightarrow \quad (x - 10)(x + 12) = 0$

$\Rightarrow \quad$ Either $(x - 10) = 0$ or $(x + 12) = 0$

$\Rightarrow \quad x = 10$ or $x = -12$

When $x = 10$, then $2x + 1 = 2 \times 10 + 1 = 21$

and $2x + 3 = 2 \times 10 + 3 = 23$

When $x = -12$, then $2x + 1 = 2(-12) + 1 = -23$

and $2x + 3 = 2(-12) + 3 = -21$

Hence, the two consecutive odd numbers are **21 and 23 or –23 and –21**.

(*ii*) Let the required consecutive numbers be x and $x + 1$.

Given, sum of squares of two consecutive numbers = 85

$\Rightarrow \quad x^2 + (x + 1)^2 = 85$

$\Rightarrow \quad x^2 + x^2 + 2x + 1 = 85$

$\Rightarrow \quad x^2 + x^2 + 2x - 84 = 0$

$\Rightarrow \quad 2x^2 + 2x - 84 = 0$

$\Rightarrow \quad x^2 + x - 42 = 0$

$\Rightarrow \quad x^2 - 6x + 7x - 42 = 0$

$\Rightarrow \quad x(x - 6) + 7(x - 6) = 0$

$\Rightarrow \quad (x - 6)(x + 7) = 0$

$\Rightarrow \quad$ Either $(x - 6) = 0$ or $(x + 7) = 0$

$\Rightarrow \quad x = 6$ or $x = -7$

When $x = 6$, then $x + 1 = 6 + 1 = 7$

and when $x = -7$, then $x + 1 = -7 + 1 = -6$

Hence, the required numbers are **6 and 7 or –6 and –7**.

(*iii*) Let the required consecutive positive even integers be $2x$ and $2x + 2$.

Given, sum of squares of the two consecutive positive integers $= (2x)^2 + (2x + 2)^2 = 164$

$\Rightarrow \quad 4x^2 + 4x^2 + 8x + 4 = 164$

$\Rightarrow \quad 8x^2 + 8x + 4 - 164 = 0$

$\Rightarrow \quad 8x^2 + 8x - 160 = 0$

$\Rightarrow \quad x^2 + x - 20 = 0$

$\Rightarrow \quad x^2 - 4x + 5x - 20 - 0$

$\Rightarrow \quad x(x - 4) + 5(x - 4) = 0$

$\Rightarrow \quad (x - 4)(x + 5) = 0$

Either $(x - 4) = 0$ or $(x + 5) = 0$

$\Rightarrow \quad x = 4$ or $x = -5$ (rejected)

The required integers are $2x = 2 \times 4 = 8$

and $2x + 2 = 2 \times 4 + 2 = 10$

Hence, the required integers are **8 and 10.**

(*iv*) Let the two consecutive even numbers be x and $x + 2$.

$\therefore$ According to the problem, we have

$x^2 + (x + 2)^2 = 340$

$\Rightarrow \quad x^2 + x^2 + 4x + 4 - 340 = 0$

$\Rightarrow \quad 2x^2 + 4x - 336 = 0$

$\Rightarrow \quad x^2 + 2x - 168 = 0$

Comparing the given equation with $ax^2 + bx + c = 0$, we have

$a = 1$, $b = 2$ and $c = -168$.

$$\therefore \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-2 \pm \sqrt{4 + 4 \times 168}}{2}$$

$$= \frac{-2 \pm \sqrt{676}}{2}$$

$$= \frac{-2 \pm 26}{2}$$

$$= \frac{24}{2}, \frac{-28}{2}$$

$$= 12, -14$$

Since, –14 is not a natural number

Hence, the required two consecutive even numbers are **12** and 12 + 2 = **14**.

(*v*) Let the required consecutive odd numbers be $(2x + 1)$ and $(2x + 3)$.

Given, sum of squares of the two consecutive odd numbers = 394

$(2x + 1)^2 + (2x + 3)^2 = 394$

$\Rightarrow \quad 4x^2 + 4x + 1 + 4x^2 + 12x + 9 = 394$

$\Rightarrow \quad 8x^2 + 16x + 10 - 394 = 0$

$\Rightarrow \quad 8x^2 + 16x - 384 = 0$

$\Rightarrow \quad x^2 - 2x - 48 = 0$

$\Rightarrow \quad x^2 - 6x + 8x - 48 = 0$

$\Rightarrow \quad x(x - 6) + 8(x - 6) = 0$

$\Rightarrow \quad (x - 6)(x + 8) = 0$

$\Rightarrow \quad$ Either $(x - 6) = 0$ or $(x + 8) = 0$

$\Rightarrow \quad x = 6$ or $x = -8$ (rejected)

The required integers are $2x + 1 = 2 \times 6 + 1 = 13$

and $2x + 3 = 2 \times 6 + 3 = 15$

Hence, the required numbers are **13 and 15**.

(*vi*) Let the required consecutive odd positive integers be $(2x + 1)$ and $(2x + 3)$

Given, sum of squares of the two consecutive odd positive integers = 290

$$(2x + 1)^2 + (2x + 3)^2 = 290$$

$\Rightarrow \quad 4x^2 + 4x + 1 + 4x^2 + 9 + 12x = 290$

$\Rightarrow \quad 8x^2 + 16x - 280 = 0$

$\Rightarrow \quad x^2 + 2x - 35 = 0$

$\Rightarrow \quad x^2 + 7x - 5x - 35 = 0$

$\Rightarrow \quad x(x + 7) - 5(x + 7) = 0$

$\Rightarrow \quad (x + 7)(x - 5) = 0$

Either $(x + 7) = 0$ or $(x - 5) = 0$

$\Rightarrow \quad x = -7$ (rejected) or $x = 5$

The required integers are $2x + 1 = 2 \times 5 + 1 = 11$

and $2x + 3 = 2 \times 5 + 3 = 13$

Hence, the required integers are **11** and **13**.

12. (*i*) Let the required positive consecutive numbers be x, $x + 1$ and $x + 2$.

Given, the sum of squares of three positive consecutive odd numbers = 365

$\Rightarrow \quad x^2 + (x + 1)^2 + (x + 2)^2 = 365$

$\Rightarrow \quad x^2 + x^2 + 2x + 1 + x^2 + 4x + 4 = 365$

$\Rightarrow \quad 3x^2 + 6x + 5 - 365 = 0$

$\Rightarrow \quad 3x^2 + 6x - 360 = 0$

$\Rightarrow \quad x^2 + 2x - 120 = 0$

$\Rightarrow \quad x^2 - 10x + 12x - 120 = 0$

$\Rightarrow \quad x(x - 10) + 12(x - 10) = 0$

$\Rightarrow \quad (x - 10)(x + 12) = 0$

$\Rightarrow$ Either $(x - 10) = 0$ or $(x + 12) = 0$

$\Rightarrow \quad x = 10$ or $x = -12$ (rejected)

The required numbers are $x = 10$, $x + 1 = 10 + 1 = 11$ and $x + 2 = 10 + 2 = 12$.

Hence, the required numbers are **10, 11 and 12**.

(*ii*) Let the required consecutive integers be x, $x + 1$ and $x + 2$.

Given, square of first integer + product of other two integers = 154

$\Rightarrow \quad x^2 + (x + 1)(x + 2) = 154$

$\Rightarrow \quad x^2 + x^2 + 3x + 2 = 154$

$\Rightarrow \quad 2x^2 + 3x - 152 = 0$

$\Rightarrow \quad x = \dfrac{-3 \pm \sqrt{9 + 4(2)(152)}}{2(2)}$

$= \dfrac{-3 \pm \sqrt{9 + 1216}}{4}$

$= \dfrac{-3 \pm \sqrt{1225}}{4} = \dfrac{-3 \pm 35}{4}$

$= \dfrac{-38}{4}$ or $\dfrac{32}{4} = \dfrac{-19}{2}$ (rejected) or 8.

The required integers are $x = 8$, $x + 1 = 8 + 1 = 9$, $x + 2 = 8 + 2 = 10$.

Hence, the required integers are **8, 9 and 10.**

(*iii*) Let the consecutive numbers be $x - 1$, x and $x + 1$

According to the given condition

$(x)^2 - 60 = (x + 1)^2 - (x - 1)^2$

$x^2 - 60 = x^2 + 1 + 2x - x^2 - 1 + 2x$

$x^2 - 60 = 4x$

$x^2 - 4x - 60 = 0$

$x^2 + 6x - 10x - 60 = 0$

$x(x + 6) - 10(x + 6) = 0$

$(x + 6)(x - 10) = 0$

$x = -6$ **or 10**

Since, $x = -6$ does not satisfy the condition therefore we will reject it.

The required numbers are

$x = 10$, $x + 1 = 11$ and $x - 1 = 9$

Hence, the numbers are **9**, **10** and **11**.

13. Let the required multiples of 5 be $5x$ and $5x + 5$.

Given, the product of two successive multiples of 5 = 300

$\Rightarrow \quad 5x(5x + 5) = 300$

$\Rightarrow \quad 25x^2 + 25x - 300 = 0$

$\Rightarrow \quad x^2 + x - 12 = 0$

$\Rightarrow \quad x^2 - 3x + 4x - 12 = 0$

$\Rightarrow \quad x(x - 3) + 4(x - 3) = 0$

$\Rightarrow \quad (x - 3)(x + 4) = 0$

$\Rightarrow$ Either $(x - 3) = 0$ or $(x + 4) = 0$

$\Rightarrow \quad x = 3$ or $x = -4$ (rejected)

Required multiples of 5 are $5x = 5(3) = 15$

and $5x + 5 = 5(3) + 5 = 20$

Hence, the required multiples of 5 are **15 and 20**.

14. (*i*) Let the numerator of the required fraction be n.

Then, the denominator of the required fraction $= n + 3$

Given,

the fraction + its reciprocal $= 2\dfrac{9}{10}$

$\Rightarrow \quad \dfrac{n}{n+3} + \dfrac{n+3}{n} = \dfrac{29}{10}$

$\Rightarrow \quad 10(n^2 + n^2 + 6n + 9) = 29n(n + 3)$

$\Rightarrow \quad 10(2n^2 + 6n + 9) = 29n^2 + 87n$

$\Rightarrow \quad 29n^2 + 87n - 20n^2 - 60n - 90 = 0$

$\Rightarrow \quad 9n^2 + 27n - 90 = 0$

$\Rightarrow \quad n^2 + 3n - 10 = 0$

$\Rightarrow \quad n^2 - 2n + 5n - 10 = 0$

$\Rightarrow \quad n(n-2)+5(n-2)=0$

$\Rightarrow \quad (n-2)(n+5)=0$

$\Rightarrow \quad$ Either $(n-2)=0$ or $(n+5)=0$

$\Rightarrow \quad n=2$ or $n=-5$ (rejected)

Numerator = n = 2 and denominator

= $n + 3 = 2 + 3 = 5$

Hence, the required of fraction = $\mathbf{\frac{2}{5}}$

(*ii*) Let the numerator of the fraction be x

$\therefore$ The denominator = $2x + 1$

Given, the fraction + its reciprocal = $2\frac{16}{21}$

According to the given condition

$$\Rightarrow \quad \frac{x}{2x+1}+\frac{2x+1}{x} = 2\frac{16}{21}$$

$$\Rightarrow \quad \frac{x^2+(2x+1)^2}{x(2x+1)} = \frac{58}{21}$$

$\Rightarrow \quad 21[x^2 + 4x^2 + 1 + 4x] = 58[2x^2 + x]$

$\Rightarrow \quad 105x^2 + 84x + 21 = 116x^2 + 58x$

$\Rightarrow \quad 11x^2 - 26x - 21 = 0$

$$\Rightarrow \quad x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$= \frac{26\pm\sqrt{(-26)^2-4(11)(-21)}}{22}$$

$$= \frac{26\pm\sqrt{676+924}}{22}$$

$$= \frac{26\pm\sqrt{1600}}{22} = \frac{26\pm 400}{22}$$

$$x = 3 \quad \text{or} \quad x = \frac{-14}{22}$$

Since $\frac{-14}{22}$ does not satisfy the given condition, therefore we will reject it.

Numerator = x = 3 and denominator = $2x + 1 = 7$

Hence, the fraction is $\mathbf{\frac{3}{7}}$.

For Standard Level

15. (*i*) Let smaller positive integer be x.

Then, the square of the smaller positive integer is x^2 and the square of the larger positive is $25x$.

Given, the sum of squares of the two positive integers = 306

$\Rightarrow \quad x^2 + 25x = 306$

$\Rightarrow \quad x^2 + 25x - 306 = 0$

$\Rightarrow \quad x^2 + 34x - 9x - 306 = 0$

$\Rightarrow \quad x(x + 34) - 9(x + 34) = 0$

$\Rightarrow \quad (x + 34)(x - 9) = 0$

$\Rightarrow \quad$ Either $(x + 34) = 0$ or $(x - 9) = 0$

$\Rightarrow \quad x = -34$ (rejected) or $x = 9$

Smaller positive integer = 9

Square of the larger positive integer = $25x = 25 \times 9$

$\therefore$ Larger positive integer = $\sqrt{25\times 9} = 5 \times 3 = 15$

(rejecting negative value)

Hence, the required positive integers are **9 and 15.**

(*ii*) Let the larger positive integer be x.

Then, the square of the larger positive integer is x^2 and the square of the smaller positive integer is $4x$.

Given, the sum of the squares of two positive integers = 117

$\Rightarrow \quad x^2 + 4x = 117$

$\Rightarrow \quad x^2 + 4x - 117 = 0$

$\Rightarrow \quad x^2 - 9x + 13x - 117 = 0$

$\Rightarrow \quad x(x - 9) + 13(x - 9) = 0$

$\Rightarrow \quad (x - 9)(x + 13) = 0$

$\Rightarrow$ Either $(x - 9) = 0$ or $(x + 13) = 0$

$\Rightarrow \quad x = 9$ or $x = -13$ (rejected)

Larger positive integer = x = 9

Square of smaller positive integer = $4x = 4 \times 9 = 36$

$\therefore$ Smaller positive integer = $\sqrt{36}$ = 6 (rejecting the negative value)

Hence, the required positive integers are **6 and 9**.

(*iii*) Let the required smaller positive number be x.

Then, the larger number = $2x - 5$

Given, the difference of the squares of the two positive numbers = 88

$\Rightarrow \quad (2x - 5)^2 - x^2 = 88$

$\Rightarrow \quad 4x^2 - 20x + 25 - x^2 - 88 = 0$

$\Rightarrow \quad 3x^2 - 20x - 63 = 0$

$\Rightarrow \quad 3x^2 - 27x + 7x - 63 = 0$

$\Rightarrow \quad 3x(x - 9) + 7(x - 9) = 0$

$\Rightarrow \quad (x - 9)(3x + 7) = 0$

$\Rightarrow \quad$ Either $(x - 9) = 0$ or $(3x + 7) = 0$

$\Rightarrow \quad x = 9$ or $x = \frac{-7}{3}$ (rejected)

Smaller positive number = x = 9

Larger positive number = $2x - 5 = 2(9) - 5 = 13$

Hence, the required positive numbers are **9 and 13**.

16. Let the denominator of the original fraction be x. Hence, its numerator is $x - 2$.

$\therefore$ The original fraction is $\frac{x-2}{x}$.

$\therefore$ New fraction is $\frac{(x-2)+3}{x+3} = \frac{x+1}{x+3}$

$\therefore$ According to the problem, we have

$$\frac{x-2}{x}+\frac{x+1}{x+3} = \frac{53}{35}$$

$$\Rightarrow \quad \frac{(x-3)(x-2)+x(x+1)}{x(x+3)} = \frac{53}{35}$$

$\Rightarrow \quad (x^2 + x - 6 + x^2 + x)35 = 53(x^2 + 3x)$

$\Rightarrow \quad 70x^2 + 70x - 210 - 53x^2 - 159x = 0$

$\Rightarrow \quad 17x^2 - 89x - 210 = 0$

Comparing this quadratic equation with the standard quadratic equation $ax^2 + bx + c = 0$, we have

$a = 17$, $b = -89$ and $c = -210$

$$\therefore \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{89 \pm \sqrt{89^2 + 4 \times 17 \times 210}}{34}$$

$$= \frac{89 \pm \sqrt{7921 + 14280}}{34}$$

$$= \frac{89 \pm \sqrt{22201}}{34}$$

$$= \frac{89 \pm 149}{34}$$

$$= \frac{238}{34}, -\frac{60}{34}$$

$$= 7, -\frac{30}{17}$$

$\therefore$ The denominator of the original fraction is 7 and so its numerator = 7 – 2 = 5

$\therefore$ The required original fraction is $\mathbf{\frac{5}{7}}$.

17. Let the numerator of the fraction be x

$\therefore$ The denominator = $2x + 1$

Given, the fraction + its reciprocal = $2\frac{16}{21}$

According to the given condition

$$\Rightarrow \quad \frac{x}{2x+1} + \frac{2x+1}{x} = 2\frac{16}{21}$$

$$\Rightarrow \quad \frac{x^2 + (2x+1)^2}{x(2x+1)} = \frac{58}{21}$$

$\Rightarrow \quad 21[x^2 + 4x^2 + 1 + 4x] = 58[2x^2 + x]$

$\Rightarrow \quad 105x^2 + 84x + 21 = 116x^2 + 58x$

$\Rightarrow \quad 11x^2 - 26x - 21 = 0$

$$\Rightarrow \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{26 \pm \sqrt{(-26)^2 - 4(11)(-21)}}{22}$$

$$= \frac{26 \pm \sqrt{676 + 924}}{22}$$

$$= \frac{26 \pm \sqrt{1600}}{22} = \frac{26 \pm 400}{22}$$

$$x = 3 \quad \text{or} \quad x = \frac{-14}{22}$$

Since $\frac{-14}{22}$ does not satisfy the given condition, therefore we will reject it.

Numerator = x = 3 and denominator = $2x + 1$ = 7

Hence, the fraction is $\mathbf{\frac{3}{7}}$.

18. Let the required two digit number be $10x + y$.

Then, $\quad x \times y = 20$

$$\Rightarrow \quad y = \frac{20}{x} \quad \ldots(1)$$

Given, number – 9 = number with interchanged digits

$\Rightarrow \quad 10x + y - 9 = 10y + x$

$\Rightarrow \quad 10x - x + y - 10y - 9 = 0$

$\Rightarrow \quad 9x - 9y - 9 = 0$

$\Rightarrow \quad x - y - 1 = 0$

$$\Rightarrow \quad x - \frac{20}{x} - 1 = 0 \quad \text{[Using (1)]}$$

$\Rightarrow \quad x^2 - x - 20 = 0$

$\Rightarrow \quad x^2 - 5x + 4x - 20 = 0$

$\Rightarrow \quad x(x - 5) + 4(x - 5) = 0$

$\Rightarrow \quad$ Either $(x - 5) = 0$ or $(x + 4) = 0$

$\Rightarrow \quad x = 5$ or $x = -4$ (rejected)

Substituting $x = 5$ in equation (1), we get

$$y = \frac{20}{5} = 4 \text{ we get}$$

Required number = $10x + y = 10(5) + 4 = 54$

Hence, the required number is **54**.

19. Let the digit in the unit place be x and that in the ten's place be y. Then the number is $10y + x$.

$\therefore$ According to the problem, we have

$$xy = 18 \quad \ldots(1)$$

and $\quad (10y + x) - 63 = 10x + y$

$\Rightarrow 10y + x - 63 - 10x - y = 0$

$\Rightarrow \quad 9x - 9y + 63 = 0$

$\Rightarrow \quad x - y + 7 = 0 \quad \ldots(2)$

From (2), $\quad y = x + 7 \quad \ldots(3)$

$\therefore$ From (1) and (3), we have

$x(x + 7) = 18$

$\Rightarrow \quad x^2 + 7x - 18 = 0$

Comparing this equation with the standard quadratic equation $ax^2 + bx + c = 0$, we have

$a = 1$, $b = 7$, $c = -18$

$$\therefore \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-7 \pm \sqrt{49 + 4 \times 1 \times 18}}{2}$$

$$= \frac{-7 \pm \sqrt{49 + 72}}{2}$$

$$= \frac{-7 \pm \sqrt{121}}{2}$$

$$= \frac{-7 \pm 11}{2}$$

$$= \frac{4}{2}, -9$$

$$= 2, -9$$

Since x is a positive integer,

$\therefore \quad x = 2$

$\therefore$ The digit in the unit place is 2.

Again, from (3), $\quad y = 2 + 7 = 9$

$\therefore$ The digit in the ten's place is 9.

Hence, the required number is 92.

20. Let the two-digit number be $10x + y$.

According to the given conditions

$$4(x + y) = 10x + y \quad \text{... (1)}$$

and
$$10x + y = 3xy \quad \text{... (2)}$$

Simplifying eq. (1), we get

$$4x + 4y = 10x + y$$
$$3y = 6x$$
$$y = 2x \quad \text{... (3)}$$

Putting the value of y from eq. (3) in eq. (2), we get

$\Rightarrow \quad 10x + 2x = 3x \times 2x$

$\Rightarrow \quad 6x^2 - 12x = 0$

$\Rightarrow \quad 6x(x - 2) = 0$

$\Rightarrow \quad x = 0$ or $x = 2$

Since $x = 0$ does not satisfy the given condition, hence we will reject it.

$\Rightarrow \quad x = 2$

$\therefore \quad y = 2x = 4$

Hence, the number is

$$10x + y = 10(2) + 4 = \mathbf{24}$$

EXERCISE 4G

For Basic and Standard Levels

1. Let the son's present age be x years.

Then, the father's present age $= (35 - x)$ years

Given, the product of their ages in years = 150

$\Rightarrow \quad x(35 - x) = 150$

$\Rightarrow \quad 35x - x^2 = 150$

$\Rightarrow \quad x^2 - 35x + 150 = 0$

$\Rightarrow \quad x^2 - 30x - 5x + 150 = 0$

$\Rightarrow \quad x(x - 30) - 5(x - 30) = 0$

$\Rightarrow \quad (x - 30)(x - 5) = 0$

$\Rightarrow \quad$ Either $(x - 30) = 0$ or $(x - 5) = 0$

$\Rightarrow \quad x = 30$ (rejected as son can't be older then the father)

or $\quad x = 5$

Son's age $= x = 5$ and

Father's age $= (35 - x) = (35 - 5) = 30$ years

Hence, the son's age is **5 years** and the father's age is **30 years.**

2. **Present ages**

Let the sister's present age be x years.

Then, the girl's present age is $2x$ years.

Four years hence

Sister's age $= (x + 4)$ years

Girl's age $= (2x + 4)$ year

Given, four years hence, product of their ages = 160

$\Rightarrow \quad (x + 4)(2x + 4) = 160$

$\Rightarrow \quad 2x^2 + 8x + 4x + 16 - 160 = 0$

$\Rightarrow \quad 2x^2 + 12x - 144 = 0$

$\Rightarrow \quad x^2 + 6x - 72 = 0$

$\Rightarrow \quad x^2 - 6x + 12x - 72 = 0$

$\Rightarrow \quad x(x - 6) + 12(x - 6) = 0$

$\Rightarrow \quad (x - 6)(x + 12) = 0$

$\Rightarrow \quad$ Either $(x - 6) = 0$ or $(x + 12) = 0$

$\Rightarrow \quad x = 6$ or $x = -12$ (rejected)

Sister's present age $= x$ years $= 6$ years

Girl's present age $= 2x$ years $= 2 \times 6$ years $= 12$ years

Hence, their present ages are **12 years and 6 years**.

3. **Present ages**

Let the son's present age $= x$ years

Then, the father's present age $= x^2$ years

Given, father's age + 5 times son's age = 84 years

$\Rightarrow \quad x^2 + 5x = 84$

$\Rightarrow \quad x^2 + 5x - 84 = 0$

$\Rightarrow \quad x^2 + 12x - 7x - 84 = 0$

$\Rightarrow \quad x(x + 12) - 7(x + 12) = 0$

$\Rightarrow \quad (x + 12)(x - 7) = 0$

$\Rightarrow \quad$ Either $(x + 12) = 0$ or $(x - 7) = 0$

$\Rightarrow \quad x = -12$ (rejected) or $x = 7$

Son's present age $= x$ years $= 7$ years

Father's present age $= x^2$ years $= (7)^2$ years $= 49$ years

Hence, their present ages are **7 years and 49 years**.

4. **Present ages**

Let the son's present age be x years

Then, father's present age $= x^2$ years

One year ago

Son's age $= (x - 1)$ years

Father's age $= (x^2 - 1)$ years

Given, One year ago, father's age = 8 times his son's age

$\Rightarrow \quad (x^2 - 1) = 8(x - 1)$

$\Rightarrow \quad x^2 - 1 = 8x - 8$

$\Rightarrow \quad x^2 - 8x - 1 + 8 = 0$

$\Rightarrow \quad x^2 - 8x + 7 = 0$

$\Rightarrow \quad x^2 - 7x - x + 7 = 0$

$\Rightarrow \quad x(x - 7) - 1(x - 7) = 0$

$\Rightarrow \quad (x - 7)(x - 1) = 0$

$\Rightarrow \quad$ Either $(x - 7) = 0$ or $(x - 1) = 0$

$\Rightarrow \quad x = 7$ or $x = 1$

$\Rightarrow \quad x^2 = 49$ and $x^2 = 1$ (rejected)

Son's present age= x years = 7 years

Father's present age = $x^2 = (7)^2$ years = 49 years

Hence, their present ages are **7 years and 49 years.**

5. Let Deepica's present age be x years.

Given, Square of Deepica's age – 11 times Deepica's age = 210

$$x^2 - 11x = 210$$
$$\Rightarrow x^2 - 11x - 210 = 0$$
$$\Rightarrow x^2 - 21x + 10x - 210 = 0$$
$$\Rightarrow x(x - 21) + 10(x - 21) = 0$$
$$\Rightarrow (x - 21)(x + 10) = 0$$
$$\Rightarrow \text{Either } (x - 21) = 0 \text{ or } (x + 10) = 0$$
$$\Rightarrow x = 21 \text{ or } x = -10 \text{ (rejected)}$$

Hence, Deepica's age is **21 years**.

6. Let Ashish's present age be x years

His age 29 years ago = $(x - 29)$ years

His age 27 years hence = $(x + 27)$ years

Given, 27 years hence, Ashish's age = square of his age 29 years ago

$$\Rightarrow x + 27 = (x - 29)^2$$
$$\Rightarrow x + 27 = x^2 - 58x + 841$$
$$\Rightarrow x^2 - 58x + 841 - x - 27 = 0$$
$$\Rightarrow x^2 - 59x + 814 = 0$$
$$\Rightarrow x^2 - 37x - 22x + 814 = 0$$
$$\Rightarrow x(x - 37) - 22(x - 37) = 0$$
$$\Rightarrow (x - 37)(x - 22) = 0$$
$$\Rightarrow \text{Either } (x - 37) = 0 \text{ or } (x - 22) = 0$$
$$\Rightarrow x = 37$$

or $x = 22$ [rejected $\because$ 29 years ago his age would be –ve]

Hence, Ashish's present age is **37 years.**

7. Present ages

Let Vijay's present age be x years.

Then, his father's present age = $9x^2$ years

32 years hence

Vijay's age = $(x + 32)$ years

Father's age = $(9x^2 + 32)$ years

Given, 32 years hence, father's age = 2 times son's age

$$\Rightarrow (9x^2 + 32) = 2(x + 32)$$
$$\Rightarrow 9x^2 + 32 - 2x - 64 = 0$$
$$\Rightarrow 9x^2 - 2x - 32 = 0$$
$$\Rightarrow 9x^2 - 18x + 16x - 32 = 0$$
$$\Rightarrow 9x(x - 2) + 16(x - 2) = 0$$
$$\Rightarrow (x - 2)(9x + 16) = 0$$
$$\Rightarrow \text{Either } (x - 2) = 0 \text{ or } (9x + 16) = 0$$
$$\Rightarrow x = 2 \text{ or } x = \frac{-16}{9} \text{ (rejected)}$$

Vijay's present age = 2 years

His father's present age = $9(2)^2$ years = 36 years

Hence, their present ages are **2 years and 36 years**.

8. Let the present age of the boy be x years.

$\therefore$ 3 years ago, his age was $(x - 3)$ years and after 3 years, his age will be $(x + 3)$years.

$\therefore$ According to the problem, we have

$$\frac{1}{x-3} + \frac{1}{x+3} = \frac{1}{4}$$
$$\Rightarrow \frac{x+3+x-3}{(x-3)(x+3)} = \frac{1}{4}$$
$$\Rightarrow \frac{2x}{x^2-9} = \frac{1}{4}$$
$$\Rightarrow x^2 - 9 - 8x = 0$$
$$\Rightarrow x^2 - 8x - 9 = 0$$

Comparing this equation with the standard quadratic equation $ax^2 + bx + c = 0$, we get $a = 1$, $b = -8$, $c = -9$.

$$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{8 \pm \sqrt{64 + 36}}{2}$$
$$= \frac{8 \pm \sqrt{100}}{2}$$
$$= \frac{8 \pm 10}{2} = 9, -1$$

$\because$ x is a positive integer.

$\therefore$ $x = 9$

Hence, the required present age of the boy is **9 years**.

For Standard Level

9. Present ages

Natasha's present age = x years

Mother's present age = x^2 years

Suppose the mother becomes $11x$ years old after y years.

Also, mother's age after y years = $(x^2 + y)$ years.

$$\Rightarrow x^2 + y = 11x \quad \text{...(1)}$$

Natasha's age after y years = $(x + y)$ years

Also Natasha's age after y years = x^2 years (given)

$$\Rightarrow x + y = x^2$$
$$\Rightarrow y = x^2 - x$$

Substituting $y = x^2 - x$ in equation (1), we get

$$x^2 + x^2 - x = 11x$$
$$\Rightarrow 2x^2 - x - 11x = 0$$
$$\Rightarrow 2x - 12x = 0$$
$$\Rightarrow 2x(x - 6) = 0$$
$$\Rightarrow \text{Either } 2x = 0 \quad \text{or} \quad x - 6 = 0$$
$$\Rightarrow x = 0 \text{ (rejected)} \quad \text{or} \quad x = 6$$

Natasha's present age= x years = 6 years

Mother's present age= x^2 years = $(6)^2$ years = 36 years

Hence, their present ages are **6 years and 36 years**.

10. Let the present age of Zeba be x years. Then her age was $(x - 5)$ years, five years ago.

$\therefore$ $x > 5$

$\therefore$ According to the problem, we have

$$(x - 5)^2 = 5x + 11$$

$\Rightarrow$ $x^2 - 10x + 25 - 5x - 11 = 0$

$\Rightarrow$ $x^2 - 15x + 14 = 0$

Comparing this equation with the standard quadratic equation $ax^2 + bx + c = 0$, we have

$a = 1$, $b = -15$ and $c = 14$

$\therefore$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{15 \pm \sqrt{15^2 - 4 \times 14}}{2}$$
$$= \frac{15 \pm \sqrt{225 - 56}}{2}$$
$$= \frac{15 \pm \sqrt{169}}{2} = \frac{15 \pm 13}{2}$$
$$= \frac{28}{2}, \frac{2}{2} = 14, 1$$

Since x cannot be less than 5, we reject $x = 1$.

$\therefore$ $x = 14$

$\therefore$ The required present age of Zeba is **14 years**.

11. Let the present age of the daughter Nisha be x years and then the present age of her mother Asha is $(x^2 + 2)$ years.

Now, Nisha's age will be $(x^2 + 2)$ years after

$\{(x^2 + 2) - x\}$ years, i.e. after $(x^2 - x + 2)$ years

Now, after $(x^2 - x + 2)$ years, Asha's age will be

$\{(x^2 + 2) + (x^2 - x + 2)\}$ years $= (2x^2 - x + 4)$ years.

$\therefore$ According to the problem, we have

$$2x^2 - x + 4 = 10x - 1$$

$\Rightarrow$ $2x^2 - 11x + 5 = 0$

Comparing this equation with the standard quadratic equation $ax^2 + bx + c = 0$, we have

$a = 2$, $b = -11$ and $c = 5$

$\therefore$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{11 \pm \sqrt{121 - 40}}{4} = \frac{11 \pm 9}{4}$$
$$= \frac{11 + 9}{4}, \frac{11 - 9}{4}$$
$$= 5, \frac{1}{2}$$

Now, when $x = \frac{1}{2}$, then the present age of the mother becomes $(x^2 + 2)$ years

$= \left(\frac{1}{4} + 2\right)$ years $= 2\frac{1}{4}$ years which is not practical. Hence, we reject it

$\therefore$ $x = 5$, i.e. Nisha's present age is 5 years and so her mother's present age is $(5^2 + 2)$ years = 27 years.

Hence, required present ages of Asha and Nisha are **27 years** and **5 years** respectively.

EXERCISE 4H

For Basic and Standard Levels

1. Suppose P got x marks in mathematics.

Then, his science marks are $(28 - x)$.

Given, product of (mathematics marks +3) and (science marks −4) = 180

$\Rightarrow$ $(x + 3)\,[(28 - x) - 4] = 180$

$\Rightarrow$ $(x + 3)(24 - x) = 180$

$\Rightarrow$ $24x + 72 - x^2 - 3x = 180$

$\Rightarrow$ $x^2 + 3x - 24x + 180 - 72 = 0$

$\Rightarrow$ $x^2 - 21x + 108 = 0$

$\Rightarrow$ $x^2 - 12x - 9x + 108 = 0$

$\Rightarrow$ $x(x - 12) - 9(x - 12) = 0$

$\Rightarrow$ $(x - 12)(x - 9) = 0$

$\Rightarrow$ Either $(x - 12) = 0$ or $(x - 9) = 0$

$\Rightarrow$ $x = 12$ or $x = 9$

When $x = 12$, then $(28 - x) = (28 - 12) = 16$

When $x = 9$, then $(28 - x) = (28 - 9) = 19$

Hence, P got **12 marks in maths and 16 in science or 9 marks in maths and 19 marks in science.**

2. ₹ 1200 was distributed equally among x students.

Amount	₹ 1200	₹ 1200
Number of students	x	$x + 8$
Amount received by each student	₹ $\frac{1200}{x}$	₹ $\frac{1200}{x+8}$

When there are 8 students more, each gets ₹ 5 less.

$\therefore$ $\frac{1200}{x} - \frac{1200}{x+8} = 5$

$\Rightarrow$ $1200\left[\frac{x+8-x}{x(x+8)}\right] = 5$

$\Rightarrow$ $1200\left[\frac{8}{x^2+8x}\right] = 5$

$\Rightarrow$ $9600 = 5x^2 + 40x$

$\Rightarrow$ $5x^2 + 40x - 9600 = 0$

$\Rightarrow$ $x^2 + 8x - 1920 = 0$

$\Rightarrow$ $x^2 + 48 - 40x - 1920 = 0$

$\Rightarrow$ $x(x + 48) - 40(x + 48) = 0$

$\Rightarrow$ $(x + 48)(x - 40) = 0$

$\Rightarrow$ Either $(x + 48) = 0$ or $(x - 40) = 0$

$\Rightarrow$ $x = -48$ (rejected) or $x = 40$

Hence, ₹ 1200 was distributed equally among **40 students**.

3. Suppose 250 bananas were divided equally among x students.

Number of bananas	250	250
Number of students	x	$x + 25$
Number of bananas received by each student	$\frac{250}{x}$	$\frac{250}{x+25}$

When there are 25 students more, each get $\frac{1}{2}$ banana less

$$\therefore \quad \frac{250}{x} - \frac{250}{x+25} = \frac{1}{2}$$

$$\Rightarrow \quad 250\left[\frac{x+25-x}{x(x+25)}\right] = \frac{1}{2}$$

$$\Rightarrow \quad 500 \times 25 = x^2 + 25x$$

$$\Rightarrow \quad x^2 + 25x - 12500 = 0$$

$$\Rightarrow \quad x^2 + 125x - 100 - 12500 = 0$$

$$\Rightarrow x(x + 125) - 100(x + 125) = 0$$

$$\Rightarrow \quad (x + 125)(x - 100) = 0$$

$$\Rightarrow \quad \text{Either } (x + 125) = 0 \quad \text{or} \quad (x - 100) = 0$$

$$\Rightarrow \quad x = -125 \quad \text{or} \quad x = 100$$

Hence, the number of students is **100**.

4. Let the number of books bought = x

Amount	₹ 80	₹ 80
No. of books	x	$x + 4$
Cost of each book	₹ $\frac{80}{x}$	₹ $\frac{80}{x+4}$

If 4 more books are bought for ₹ 80, the cost of each book reduces by ₹ 1.

$$\therefore \quad \frac{80}{x} - \frac{80}{x+4} = 1$$

$$\Rightarrow \quad 80\left[\frac{x+4-x}{x(x+4)}\right] = 1$$

$$\Rightarrow \quad 80 \times 4 = x^2 + 4x$$

$$\Rightarrow \quad x^2 + 4x - 320 = 0$$

$$\Rightarrow \quad x^2 + 20x - 16x - 320 = 0$$

$$\Rightarrow \quad x(x + 20) - 16(x + 20) = 0$$

$$\Rightarrow \quad (x + 20)(x - 16) = 0$$

$$\Rightarrow \quad \text{Either } (x + 20) = 0 \quad \text{or} \quad (x - 16) = 0$$

$$\Rightarrow \quad x = -20 \text{ (rejected)} \quad \text{or} \quad x = 16$$

Hence, the number of books bought = **16**.

5. Let the original price of the book be ₹ x.

Amount	₹ 300	₹ 300
Price of each book	₹ x	₹ $(x - 5)$
Number of books a person can by	$\frac{300}{x}$	$\frac{300}{x-5}$

When the list price of a book is reduced by ₹ 5, then 5 more books can be bought

$$\therefore \quad \frac{300}{x-5} - \frac{300}{x} = 5$$

$$\Rightarrow \quad 300\left[\frac{x-x+5}{x(x-5)}\right] = 5$$

$$\Rightarrow \quad 300 \times 5 = 5x(x - 5)$$

$$\Rightarrow \quad 300 = x^2 - 5x$$

$$\Rightarrow \quad x^2 - 5x - 300 = 0$$

$$\Rightarrow \quad x^2 + 15x - 20x - 300 = 0$$

$$\Rightarrow \quad x(x + 15) - 20(x + 15) = 0$$

$$\Rightarrow \quad (x + 15)(x - 20) = 0$$

$$\Rightarrow \quad \text{Either } (x + 15) = 0 \quad \text{or } (x - 20) = 0$$

$$\Rightarrow \quad x = -15 \quad \text{or} \quad x = 20$$

Hence, the original price of the book is ₹ **20**.

6. Let the original price of each toy be ₹ x. Then the number of toys the person can buy = $\frac{360}{x}$. If the price of each toy is reduced by ₹ 2, then the reduced price of 1 toy is ₹ $(x - 2)$

$\therefore$ No. of toys the person can buy with the reduced price $= \frac{360}{x-2}$

$\therefore$ According to the problem, we have

$$\frac{360}{x-2} - \frac{360}{x} = 2$$

$$\Rightarrow \quad \frac{360(x-x+2)}{x(x-2)} = 2$$

$$\Rightarrow \quad \frac{360}{x^2-2x} = 1$$

$$\Rightarrow \quad x^2 - 2x - 360 = 0$$

Comparing their quadratic equation with the standard quadratic equation $ax^2 + bx + c = 0$, we have

$a = 1$, $b = -2$ and $c = -360$

$$\therefore \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{2 \pm \sqrt{4 + 4 \times 360}}{2}$$

$$= \frac{2 \pm \sqrt{1444}}{2} = \frac{2 \pm 38}{2}$$

$$= \frac{40}{2}, \frac{-36}{2}$$

$$= 20, -18$$

$\because x > 0$, $\therefore$ we reject $x = -18$

Hence, the required original price of each toy is ₹ 20.

7. Let the original price of the cricket ball is ₹ x.

Amount	₹ 640	₹ 640
Price of each cricket ball	₹ x	₹ $x + 8$
Number of balls that can be bought	$\frac{640}{x}$	$\frac{640}{(x+8)}$

When the price of a ball is increased by ₹ 8, then 4 less balls are available.

$$\therefore \quad \frac{640}{x} - \frac{640}{(x+8)} = 4$$

$$\Rightarrow \quad 640\left[\frac{x+8-x}{x(x+8)}\right] = 4$$

$$\Rightarrow \quad 640 \times 8 = 4x(x + 8)$$

$$\Rightarrow \quad 640 \times 2 = x(x + 8)$$

$$\Rightarrow \quad x^2 + 8x - 1280 = 0$$

$$\Rightarrow \quad x^2 + 40x - 32x - 1280 = 0$$

$$\Rightarrow \quad x(x + 40) - 32(x + 40) = 0$$

$$\Rightarrow \quad (x - 32)(x + 40) = 0$$

$$\Rightarrow \quad \text{Either } (x - 32) = 0 \text{ or } (x + 40) = 0$$

$$x = 32 \text{ or } x = -40$$

Hence, the original price of the cricket ball is **₹ 32.**

8. Let the original duration of the person's tour is for x days and let his original daily expenses are ₹ y. Then his total expenses = ₹ xy.

$\therefore$ According to the problem, we have

$$xy = 4200 \quad \ldots(1)$$

Again, if he extends his tour duration for 3 days more, then his daily expenses becomes ₹ $(y - 70)$ so that

$$(y - 70)(x + 3) = 4200$$

$$\Rightarrow \quad xy + 3y - 70x - 210 = 4200$$

$$\Rightarrow \quad 4200 + 3y - 70x - 210 = 4200 \quad \text{[From (1)]}$$

$$\Rightarrow \quad 3y - 70x - 210 = 0$$

$$\Rightarrow \quad y = \frac{70x + 210}{3} \quad \ldots(2)$$

$\therefore$ From (1) and (2), we have

$$\frac{x(70x + 210)}{3} = 4200$$

$$\Rightarrow \quad 70x^2 + 210x = 12600$$

$$x^2 + 3x - 180 = 0 \text{ [Dividing both sides by 70]}$$

Comparing this equation with the standard quadratic equation $ax^2 + bx + c = 0$, we have

$a = 1$, $b = 3$ and $c = -180$

$$\therefore \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-3 \pm \sqrt{9 + 4 \times 180}}{2}$$

$$= \frac{-3 \pm \sqrt{729}}{2}$$

$$= \frac{-3 \pm 27}{2}$$

$$= \frac{24}{2}, -\frac{30}{2}$$

$$= 12, -15$$

Since x cannot be negative, we reject $x = -15$

$\therefore$ Required original duration of his tour is **12 days**.

9. Let the number of days of the person's tour programme be x.

Amount	₹ 360	₹ 360
No. of tour days	x	$(x + 4)$
Daily expenses	₹ $\frac{360}{x}$	₹ $\frac{360}{(x+4)}$

When the person exceeds his tour programme by 4 days, he has to cut down his daily expenses by ₹ 3.

$$\therefore \quad \frac{360}{x} - \frac{360}{x+4} = 3$$

$$\Rightarrow \quad 360\left[\frac{x+4-x}{x(x+4)}\right] = 3$$

$$\Rightarrow \quad 360 \times 4 = 3x(x + 4)$$

$$\Rightarrow \quad 120 \times 4 = x^2 + 4x$$

$$\Rightarrow \quad x^2 + 4x - 480 = 0$$

$$\Rightarrow \quad x^2 + 24x - 20x - 480 = 0$$

$$\Rightarrow \quad x(x + 24) - 20(x + 24) = 0$$

$$\Rightarrow \quad (x + 24)(x - 20) = 0$$

$$\Rightarrow \quad \text{Either } (x + 24) = 0 \text{ or } (x - 20) = 0$$

$$\Rightarrow \quad x = -24 \text{ or } x = 20$$

Hence, the tour programme was for **20 days**.

10. Let the number of students who actually attended the picnic be x and let each student paid ₹ y for food (including the extra charge).

Then according to the problem, we have

$$xy = 2000 \quad \ldots(1)$$

$$\text{and} \quad (x + 5)(y - 20) = 2000$$

$$\Rightarrow \quad xy - 20x + 5y - 100 = 2000$$

$$\Rightarrow \quad -20x + 5y - 100 = 0 \quad \text{[From (1)]}$$

$$\Rightarrow \quad -4x + y - 20 = 0$$

$$\therefore \quad y = 4x + 20 \quad \ldots(2)$$

$\therefore$ From (1) and (2), we have

$$x(4x + 20) = 2000$$

$$\Rightarrow \quad 4x^2 + 20x - 2000 = 0$$

$$\Rightarrow \quad x^2 + 5x - 500 = 0$$

Comparing this equation with the standard quadratic equation $ax^2 + bx + c = 0$, we have

$a = 1$, $b = +5$ and $c = -500$

$$\therefore \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-5 \pm \sqrt{25 + 4 \times 500}}{2}$$

$$= \frac{-5 \pm \sqrt{2025}}{2}$$

$$= \frac{-5 \pm 45}{2}$$

$$= \frac{40}{2}, -\frac{50}{2}$$

$$= 20, -25$$

We reject $x = -25$, which is negative.

$\therefore \quad x = 20$

$\therefore$ From (2), $\quad y = 4 \times 20 + 20 = 100$

Hence, the required no. of students who actually attended the picnic = **20** and each student paid ₹ **100** (including the extra amount of ₹ 20).

11. Suppose he worked initially at a daily wage of ₹ x and then at a higher daily wage of ₹ $(x + 10)$.

Money earned	₹ 400	₹ 400
Daily wage	₹ x	₹ $(x + 10)$
Number of days required to earn ₹ 400	$\frac{400}{x}$	$\frac{400}{x+10}$

Given,

[Number of day required to earn ` 400 at daily wage of ` x] – [Number of day required to earn ` 400 at daily wage of ` $(x+10)$] = 2

$$\Rightarrow \frac{400}{x} - \frac{400}{x+10} = 2$$

$$\Rightarrow 400\left[\frac{x+10-x}{x(x+10)}\right] = 2$$

$$\Rightarrow 200(10) = x^2 + 10x$$

$$\Rightarrow x^2 + 10x - 2000 = 0$$

$$\Rightarrow x^2 - 40x + 50x - 2000 = 0$$

$$\Rightarrow x(x - 40) + 50(x - 40) = 0$$

$$\Rightarrow (x + 50)(x - 40) = 0$$

$\Rightarrow$ Either $(x + 50) = 0$ or $(x - 40) = 0$

$\Rightarrow$ $x = -50$ (rejected) or $x = 40$

Number of days required to earn ₹ 400 at a higher daily wage $= \frac{400}{x+10} = \frac{400}{40+10} = \frac{400}{50} = 8$

Hence, the worker worked for **8 days** at a higher wage.

For Standard Level

12. Let the original price of 1 kg of guava be ₹ x.

Then according to the problem,

$$\frac{56}{x-1} - \frac{56}{x} = 1$$

$$\Rightarrow \frac{56(x-x+1)}{x(x-1)} = 1$$

$$\Rightarrow \frac{56}{x^2 - x} = 1$$

$$\Rightarrow x^2 - x - 56 = 0$$

Comparing this equation with the standard quadratic equation $ax^2 + bx + c = 0$, we have

$a = 1$, $b = -1$ and $c = -56$

$$\therefore \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{1 \pm \sqrt{1 + 4 \times 56}}{2}$$

$$= \frac{1 \pm \sqrt{225}}{2}$$

$$= \frac{1 \pm 15}{2}$$

$$= 8, -7$$

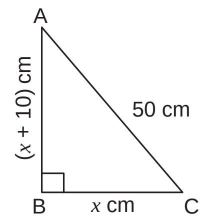
Rejecting the negative value of x, we get $x = 8$.

$\therefore$ Required price = ₹ **8 per kg**.

EXERCISE 4I

For Basic and Standard Levels

1. (*i*) Let ABC represent the right triangle, right-angled at B, in which the hypotenuse AC = 50 cm



Let the shorter side containing the right angle be BC.

Let BC = x cm

Then, AB = $(x + 10)$ cm

In right ΔABC, we have

$AB^2 + BC^2 = AC^2$ [By Pythagoras' theorem]

$$\Rightarrow (x + 10)^2 + (x)^2 = (50)^2$$

$$\Rightarrow x^2 + 20x + 100 + x^2 = 2500$$

$$\Rightarrow 2x^2 + 20x + 100 - 2500 = 0$$

$$\Rightarrow 2x^2 + 20x - 2400 = 0$$

$$\Rightarrow x^2 + 10x - 1200 = 0$$

$$\Rightarrow x^2 - 30x + 40x - 1200 = 0$$

$$\Rightarrow x(x - 30) + 40(x - 30) = 0$$

$$\Rightarrow (x - 30)(x + 40) = 0$$

$\Rightarrow$ Either $(x - 30) = 0$ or $(x + 40) = 0$

$\Rightarrow$ $x = 30$ or $x = -40$ (rejected)

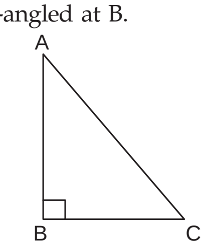
The lengths of the two sides are **30 cm** and $(30 + 10) =$ **40 cm**

(*ii*) Area of ΔABC $= \frac{1}{2}$ Base × Altitude

$= \frac{1}{2} \times 30 \text{ cm} \times 40 \text{ cm}$

$=$ **600 cm²**

2. Let ΔABC be the right triangle, right-angled at B.



Let the altitude AB = x cm

Then, hypotenuse AC = $(2x + 1)$ cm

and base BC = $[(2x + 1) - 2]$ cm

$= (2x - 1)$ cm

In right ΔABC, we have

$AB^2 + BC^2 = AC^2$

[By Pythagoras' Theorem]

$$\Rightarrow \quad x^2 + (2x - 1)^2 = (2x + 1)^2$$
$$\Rightarrow \quad x^2 + 4x^2 - 4x + 1 = 4x^2 + 4x + 1$$
$$\Rightarrow \quad x^2 - 8x = 0$$
$$\Rightarrow \quad x(x - 8) = 0$$
$\Rightarrow$ Either $x = 0$ (rejected) or $(x - 8) = 0$
$$\Rightarrow \quad x = 8$$

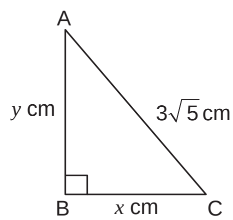
Altitude = x cm = 8 cm,

base = $(2x - 1)$ cm = $(2 \times 8 - 1)$ cm = 15 cm

Hypotenuse = $(2x + 1)$ cm = $(2 \times 8 + 1)$ cm = 17 cm

Hence, **altitude = 8 cm, base = 15 cm, hypotenuse = 17 cm.**

3. Let ΔABC be the right triangle, right-angled at B. Let the smaller side (say) BC be x cm and longer side AB be y cm.



Then, in ΔABC, we have

$AB^2 + BC^2 = AC^2$ [By Pythagoras' Theorem]

$$\Rightarrow \quad (y \text{ cm})^2 + (x \text{ cm})^2 = (3\sqrt{5} \text{ cm})^2$$
$$\Rightarrow \quad y^2 + x^2 = 45$$
$$\Rightarrow \quad y^2 = 45 - x^2 \quad \ldots(1)$$

Given, when smaller side = $3x$, larger side = $2y$, new hypotenuse = 15 cm

Then, $(2y)^2 + (3x)^2 = (15)^2$
$$\Rightarrow \quad 4y^2 + 9x^2 = 225 \quad \ldots(2)$$
$$\Rightarrow \quad 4(45 - x^2) + 9x^2 = 225 \quad \text{[Using (1)]}$$
$$\Rightarrow \quad 180 - 4x^2 + 9x^2 = 225$$
$$\Rightarrow \quad 5x^2 = 225 - 180$$
$$\Rightarrow \quad 5x^2 = 45$$
$$\Rightarrow \quad x^2 = 9 \quad \ldots(3)$$
$$\Rightarrow \quad x = 3$$

From (1) and (3), we get
$$y^2 = 45 - 9$$
$$\Rightarrow \quad y^2 = 36$$
$$\Rightarrow \quad y = 6$$

Hence, the sides are **3 cm and 6 cm**.

4. Let the base of the triangle be x cm

and the height of the triangle be y cm

According the the given condition
$$x + y + 15 = 36 \quad \ldots (1)$$
$$x^2 + y^2 = (15)^2 \quad \ldots (2)$$

Simplifying eq. (1) we get
$$x + y = 21$$
$$y = 21 - x \quad \ldots (3)$$

Putting the value of y from eq. (3) in eq. (2), we get
$$x^2 + (21 - x)^2 = 225$$
$$x^2 + x^2 - 42x + 441 = 225$$
$$2x^2 - 42x + 216 = 0$$
$$x^2 - 21x + 108 = 0$$
$$x^2 - 12x - 9x + 108 = 0$$
$$x(x - 12) - 9(x - 12) = 0$$
$$(x - 12)(x - 9) = 0$$
$$x = 12 \text{ or } x = 9$$

If $x = 12$ then $y = 9$ and

If $x = 9$ then $y = 12$

Now, Area of triangle = $\frac{1}{2} \times l \times b$

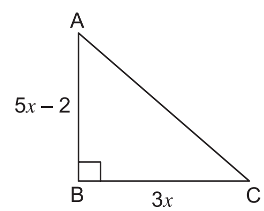
$= \frac{1}{2} \times 12 \times 9 =$ **54 cm²**

5. Let ΔABC be a right-angled triangle with sides AB = $(5x - 2)$cm, BC = $3x$ cm and $\angle$ABC = 90°

Now, area of $\Delta ABC = \frac{1}{2} \times 3x \times (5x - 2) \text{ cm}^2$

$= \frac{15x^2 - 6x}{2} \text{ cm}^2$

$\therefore$ According to the problem, we have



$$\frac{15x^2 - 6x}{2} = 24$$
$$\Rightarrow \quad 15x^2 - 6x = 48$$
$$\Rightarrow \quad 5x^2 - 2x - 16 = 0$$

Comparing this equation with the standard quadratic equation $ax^2 + bx + c = 0$, we have

$a = 5$, $b = -2$ and $c = -16$

$$\therefore \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= \frac{2 \pm \sqrt{4 + 4 \times 5 \times 16}}{2 \times 5}$$
$$= \frac{2 \pm \sqrt{324}}{10}$$
$$= \frac{2 \pm 18}{10}$$
$$= \frac{20}{10}, -\frac{16}{10}$$
$$= 2, -1.6 \text{ (rejected)}$$

$\therefore x = 2$

The two sides are BC = 3×2 cm = 6 cm

and AB = $(5 \times 2 - 2)$cm = 8 cm

Hence, required two sides are **6 cm** and **8 cm**.

6. Let the width of the rectangle be x cm.

Then, the length of the rectangle is $(x + 10)$ cm.

Area of the rectangle = 119 cm² [Given]
$$\Rightarrow \quad (x + 10)x = 119$$
$$\Rightarrow \quad x^2 + 10x - 119 = 0$$

$\Rightarrow \quad x^2 + 17x - 7x - 119 = 0$

$\Rightarrow \quad x(x + 17) - 7(x + 17) = 0$

$\Rightarrow \quad (x + 17)(x - 7) = 0$

$\Rightarrow \quad$ Either $(x + 17) = 0$ or $(x - 7) = 0$

$\Rightarrow \quad x = -17$ (rejected) or $x = 7$

$\Rightarrow \quad$ width = 7 cm

and length = (10 + 7) cm = 17 cm

Hence, the length of the rectangle is **17 cm** and width of the rectangle is **7 cm.**

7. Let the width of the rectangle be x cm.

Then its length = $(2x + 5)$ cm

Area of the rectangle = 75 cm^2

$\Rightarrow \quad (2x + 5)x = 75$

$\Rightarrow \quad 2x^2 + 5x - 75 = 0$

$\Rightarrow \quad 2x^2 + 15x - 10x - 75 = 0$

$\Rightarrow \quad x(2x + 15) - 5(2x + 15) = 0$

$\Rightarrow \quad (2x + 15)(x - 5) = 0$

$\Rightarrow \quad$ Either $(2x + 15) = 0$ or $(x - 5) = 0$

$\Rightarrow \quad x = \frac{-15}{2}$ (rejected) or $x = 5$

Length = $(2x + 5)$ cm

= $(2 \times 5 + 5)$ cm = 15 cm

Hence, the length of the rectangle is **15 cm.**

8. Let x cm and y cm be the dimensions of the rectangle.

Perimeter of the rectangle = 94 cm [Given]

$\Rightarrow \quad 2(x + y) = 94$

$\Rightarrow \quad x + y = \frac{94}{2} = 47$

$\Rightarrow \quad x = (47 - y) \quad \ldots(1)$

Area of the rectangle = 246 cm^2 [Given]

$\Rightarrow \quad x \times y = 246$

$\Rightarrow \quad (47 - y) \times y = 246$

$\Rightarrow \quad 47y - y^2 = 246$

$\Rightarrow \quad y^2 - 47y + 246 = 0$

$\Rightarrow \quad y^2 - 6y - 41y + 246 = 0$

$\Rightarrow \quad y(y - 6) - 41(y - 6) = 0$

$\Rightarrow \quad (y - 6)(y - 41) = 0$

$\Rightarrow \quad$ Either $(y - 6) = 0$ or $(y - 41) = 0$

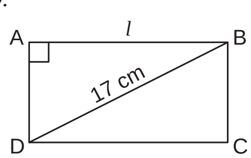
$\Rightarrow \quad y = 6$ or $y = 41$

$y = 6 \Rightarrow \quad x = 47 - y = 47 - 6 = 41$

$y = 41 \Rightarrow \quad x = 47 - 41 = 6$

Hence, the dimensions of the rectangle are **41 cm and 6 cm.**

9. Let l cm and b cm be the length and breadth of the rectangle ABCD.



$\Rightarrow \quad$ Perimeter of the rectangle, $2(l + b) = 46$

$\Rightarrow \quad l + b = 23$

$\Rightarrow \quad b = (23 - l) \quad \ldots(1)$

Join BD. Then, BD = 17 cm

In right ΔBAD, we have

$AB^2 + AD^2 = 17\ cm^2$ [By Pythagoras' Theorem]

$l^2 + b^2 = 17^2$

$\Rightarrow \quad l^2 + (23 - l)^2 = 17^2$

$\Rightarrow \quad l^2 + 529 - 46l + l^2 - 289 = 0$

$\Rightarrow \quad 2l^2 - 46l + 240 = 0$

$\Rightarrow \quad l^2 - 23l + 120 = 0$

$\Rightarrow \quad l^2 - 15l - 8l + 120 = 0$

$\Rightarrow \quad l(l - 15) - 8(l - 15) = 0$

$\Rightarrow \quad (l - 15)(l - 8) = 0$

Either $(l - 15) = 0$ or $(l - 8) = 0$

$\Rightarrow \quad l = 15$ or $l = 8$

When $l = 15$, $b = (23 - 15) = 8$

When $l = 8$, $b = (23 - 8) = 15$

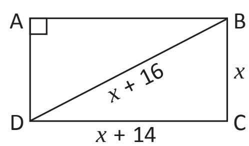
Hence, the dimensions of the rectangle are **15 cm and 8 cm**.

10. Let ABCD be the rectangle.

Let the side of the shorter side = x

∴ The length of larger side = $x + 14$

∴ The length of a diagonal = $x + 16$



In right ΔBCD, we have

$x^2 + (x + 14)^2 = (x + 16)^2$

$x^2 + x^2 + 28x + 196 = x^2 + 32x + 256$

$x^2 - 4x - 256 + 196 = 0$

$x^2 - 4x - 60 = 0$

$x^2 + 6x - 10x - 60 = 0$

$x(x + 6) - 10(x + 6) = 0$

$(x - 10)(x + 6) = 0$

$x = 10$ or $x = -6$

Since length cannot be negative therefore we will neglect $x = -6$.

Thus, Shorter side = 10 cm

Longer side = (10 + 14) = 24 cm

Diagonal = (10 + 16) = 26 cm

11. Let the breadth of the rectangle plot be x m. Then its length is $(2x + 1)$ m.

So, its area is $x(2x + 1)\ m^2 = (2x^2 + x)\ m^2$

∴ According to the problem, we have

$2x^2 + x - 528 = 0$

Comparing this quadratic equation with the standard quadratic equation $ax^2 + bx + c = 0$, we have

$a = 2$, $b = 1$ and $c = -528$

$$\therefore \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-1 \pm \sqrt{1^2 + 4 \times 2 \times 528}}{2 \times 2}$$

$$= \frac{-1 \pm \sqrt{4225}}{4}$$

$$= \frac{-1 \pm 65}{4}$$

$$= \frac{64}{4}, \frac{-66}{4}$$

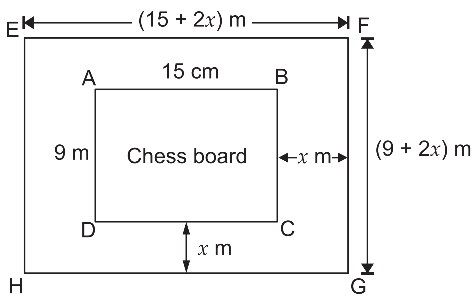
$$= 16, -\frac{33}{2}$$

$\because x > 0$,

$\therefore$ We reject $x = -\frac{33}{2}$

Hence, the required breadth of the rectangle is **16 m** and the length is $(2 \times 16 + 1)$ m = **33 m**.

12. Let ABCD be the rectangular field of length AB = 15 m and breadth AD = 9 m. It is surrounded out side by a path of width, say x m so that the path lies, between two rectangles EFGH and ABCD.



So that the length EF of the larger rectangle is $(15 + 2x)$ m and the breadth $(9 + 2x)$ m

$\therefore$ Area of the path = Area of the rectangle EFGH − Area of the rectangle ABCD

$$= \{(15 + 2x)(9 + 2x) - 15 \times 9\} \text{ m}^2$$

$$= (135 + 4x^2 + 48x - 135) \text{ m}^2$$

$$= (4x^2 + 48x) \text{ m}^2$$

$\therefore$ According to the problem, we have

$$4x^2 + 48x - 81 = 0$$

Comparing this equation with the standard quadratic equation $ax^2 + bx + c = 0$, we have

$a = 4$, $b = 48$ and $c = -81$

$$\therefore \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-48 \pm \sqrt{48^2 + 4 \times 4 \times 81}}{2 \times 4}$$

$$= \frac{-48 \pm \sqrt{2304 + 1296}}{8}$$

$$= \frac{-48 \pm \sqrt{3600}}{8}$$

$$= \frac{-48 \pm 60}{8}$$

$$= \frac{12}{8}, -\frac{108}{8}$$

$$= \frac{3}{2}, -\frac{27}{2}$$

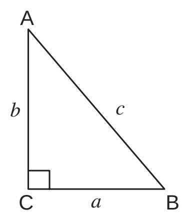
$$= 1.5, -13.5$$

We reject $x = -13.5$, since x should be positive

$\therefore x = 1.5$

$\therefore$ Required width of the path is **1.5 m**.

13. Let ABC be the right angle, right-angled at C.



Let BC = a be the shortest side, hypotenuse AB = c and third side AC = b.

Then, perimeter of ΔABC = $4a$ units

$\Rightarrow \quad a + b + c = 4x$ units

$\Rightarrow \quad c = 3a - b \quad$ …(1)

Numerical value of ar(ΔABC) = 8 × numerical value of x

$\Rightarrow \quad \frac{1}{2} \times BC \times AC = 8a$

$\Rightarrow \quad \frac{1}{2} \times a \times b = 8a$

$\Rightarrow \quad b = 16 \quad$ … (2)

In right triangle ACB, we have

$$AC^2 + BC^2 = AB^2$$

$\Rightarrow \quad b^2 + a^2 = c^2$

$\Rightarrow \quad b^2 + a^2 = (3a - b)^2 \quad$ [Using (1)]

$\Rightarrow \quad 16^2 + a^2 = (3a - 16)^2 \quad$ [Using (2)]

$\Rightarrow \quad 256 + a^2 = 9a^2 - 96a + 256$

$\Rightarrow \quad 8a^2 - 96a = 0$

$\Rightarrow \quad 8a(a - 12) = 0$

$\Rightarrow \quad$ Either $8a = 0 \quad$ or $(a - 12) = 0$

$\Rightarrow \quad a =$ (rejected) or $a = 12$

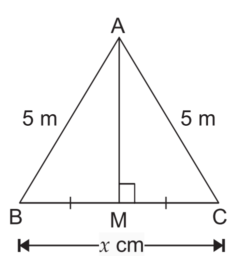
Substituting $a = 12$ and $b = 16$ in $c = 3a - b$, we get

$$c = 3 \times 12 - 16$$

$$= 36 - 16 = 20$$

Hence, the lengths of the three sides of the triangle are **12 units, 16 units and 20 units**.

14. Let ΔABC be an isosceles triangle with sides AB = AC = 5 cm and the base BC = x cm.



Let AM be perpendicular to BC from A.

Then M is the mid-point of BC.

$\therefore \quad BM = \frac{x}{2}$ cm

$\therefore$ By Pythagoras' theorem in ΔABM, we have

$$AM = \sqrt{AB^2 - BM^2}$$

$$= \sqrt{5^2 - \left(\frac{x}{2}\right)^2} \text{ cm}$$

$$= \sqrt{25 - \frac{x^2}{4}} \text{ cm}$$

$\therefore$ Area of $\Delta ABC = \frac{1}{2} BC \times AM$

$$= \frac{1}{2} x \times \sqrt{25 - \frac{x^2}{4}}$$

$\therefore$ According to the problem, we have

$$\frac{1}{2} x\sqrt{25 - \frac{x^2}{4}} = 12$$

$\Rightarrow$ $x^2\left(25 - \frac{x^2}{4}\right) = 24^2 = 576$

$\Rightarrow$ $-\frac{x^4}{4} + 25x^2 - 576 = 0$

$\Rightarrow$ $-x^4 + 100x^2 - 2304 = 0$

$\Rightarrow$ $x^4 - 100x^2 + 2304 = 0$

$\Rightarrow$ $y^2 - 100y + 2304 = 0$ where $y = x^2$ ...(1)

Comparing this equation with the standard quadratic equation $ay^2 + by + c = 0$, we have

$a = 1$, $b = -100$ and $c = 2304$

$\therefore$ $y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$= \frac{100 \pm \sqrt{10000 - 9216}}{2}$$

$$= \frac{100 \pm \sqrt{784}}{2}$$

$$= \frac{100 \pm 28}{2}$$

$$= \frac{128}{2}, \frac{72}{2}$$

$= 64, 36$

$\therefore$ $x^2 = 64$ or 36 [From (1)]

$\therefore$ $x = 8$ or 6

$\therefore$ The required length of the base is either **8 cm** or **6 cm**.

15. Let the width of the garden be x m and its length be y m.

Then, $x + y + x = 30$

$\Rightarrow$ $2x + y = 30$

$\Rightarrow$ $y = (30 - 2x)$ ...(1)

Area of the vegetable garden = 100 m^2

$\Rightarrow$ $x \times y = 100$

$\Rightarrow$ $x \times (30 - 2x) = 100$ [Using (1)]

$\Rightarrow$ $30x - 2x^2 = 100$

$\Rightarrow$ $2x^2 - 30x + 100 = 0$

$\Rightarrow$ $x^2 - 15x + 50 = 0$

$\Rightarrow$ $x^2 - 10x - 5x + 50 = 0$

$\Rightarrow$ $x(x - 10) - 5(x - 10) = 0$

$\Rightarrow$ $(x - 10)(x - 5) = 0$

$\Rightarrow$ Either $(x - 10) = 0$ or $(x - 5) = 0$

$\Rightarrow$ $x = 10$ or $x = 5$

$x = 10 \Rightarrow y = (30 - 2 \times 10) = 10$

$x = 5 \Rightarrow y = (30 - 2 \times 5) = 20$ [satisfies the given condition as the garden is rectangular]

Hence, the dimensions of the garden are **20 m and 5 m.**

16. Let the side of the smaller square be x cm.

Then, the side of the larger square = $(x + 5)$ cm

Given, the sum of areas of two squares = 325 cm^2

$\Rightarrow$ $x^2 + (x + 5)^2 = 325$

$\Rightarrow$ $x^2 + x^2 + 10x + 25 - 325 = 0$

$\Rightarrow$ $2x^2 + 10x - 300 = 0$

$\Rightarrow$ $x^2 + 5x - 150 = 0$

$\Rightarrow$ $x^2 + 15x - 10x - 150 = 0$

$\Rightarrow$ $x(x + 15) - 10(x + 15) = 0$

$\Rightarrow$ $(x + 15)(x - 10) = 0$

$\Rightarrow$ Either $(x + 15) = 0$ or $(x - 10) = 0$

$\Rightarrow$ $x = -15$ (rejected) or $x = 10$

Hence, the side of the smaller square = **10 cm** and the side of the larger square = (10 + 5) cm = **15 cm.**

17. Let the sides of the two squares in centimetres be x and y respectively (where $x > y$).

Square	I	II
Side (in cm)	x	y
Area (in cm^2)	x^2	y^2
Perimeter (in cm)	$4x$	$4y$

Given, sum of the areas of two squares = 400 cm^2

$\Rightarrow$ $x^2 + y^2 = 400$ ...(1)

Given, difference of their perimeters = 16 cm

$\Rightarrow$ $4x - 4y = 16$

$\Rightarrow$ $x - y = 4$

$\Rightarrow$ $x - 4 = y$ ...(2)

Substituting the value of y from equation (2) in equation (1), we get

$$x^2 + (x - 4)^2 = 400$$

$\Rightarrow$ $x^2 + x^2 - 8x + 16 = 400$

$\Rightarrow$ $2x^2 - 8x - 384 = 0$

$\Rightarrow$ $x^2 - 4x - 192 = 0$

$\Rightarrow$ $x^2 - 16x + 12x - 192 = 0$

$\Rightarrow$ $x(x - 16) + 12(x - 16) = 0$

$\Rightarrow$ $(x - 16)(x + 12) = 0$

$\Rightarrow$ Either $(x - 16) = 0$ or $(x + 12) = 0$

$\Rightarrow$ $x = 16$ or $x = -12$ (rejected)

$x = 16$

$\Rightarrow$ $y = (x - 4) = (16 - 4) = 12$

Hence, the sides of the two squares are **16 cm** and **12 cm**.

18. Perimeter of the square = 80 units

$\Rightarrow$ 4 × side of square = 80 units

$\Rightarrow$ Side of square = 20 units

Area of the square = side × side

= (20 × 20) sq units

Let the length and breadth of the rectangle be l units and b units.

Given, area of rectangle : area of square = 24 : 25

$$\Rightarrow \quad \frac{lb}{(20 \times 20)\text{ sq units}} = \frac{24}{25}$$

$$\Rightarrow \quad lb = \frac{24 \times 20 \times 20}{25} \text{ sq units}$$

$$\Rightarrow \quad lb = 384 \text{ sq units} \qquad \ldots(1)$$

Also, perimeter of the rectangle = 80 units

$\Rightarrow$ $2(l + b) = 80$ units

$\Rightarrow$ $l + b = 40$ units

$\Rightarrow$ $b = (40 - l)$ units ...(2)

From (1) and (2) we get

$l(40 - l) = 384$

$\Rightarrow$ $40l - l^2 = 384$

$\Rightarrow$ $l^2 - 40l + 384 = 0$

$\Rightarrow$ $l^2 - 24l - 16l + 384 = 0$

$\Rightarrow$ $l(l - 24) - 16(l - 24) = 0$

$\Rightarrow$ $(l - 24)(l - 16) = 0$

$\Rightarrow$ Either $(l - 24) = 0$ or $(l - 16) = 0$

$\Rightarrow$ $l = 24$ or $l = 16$

When $l = 24$, then $b = (40 - 24) = 16$

When $l = 16$, then $b = (40 - 16) = 24$

Hence, the dimensions of the rectangle are **24 units and 16 units**.

19. Let the length of rectangular park be x

Let the breadth of the rectangular park be $(x - 3)$

and base of the triangle = $(x - 3)$

$$\text{Area of triangle} = \frac{1}{2} \times b \times h$$

$$= \frac{1}{2} \times (x - 3) \times 12$$

$$= 6(x - 3)$$

Area of rectangle = Area of triangle + 4

$= 6(x - 3) + 4$

$= 6x - 14$

According to the given condition

$x(x - 3) = 6x - 14$

$x^2 - 3x = 6x - 14$

$x^2 - 9x + 14 = 0$

$x^2 - 2x - 7x + 14 = 0$

$x(x - 2) - 7(x - 2) = 0$

$(x - 2)(x - 7) = 0$

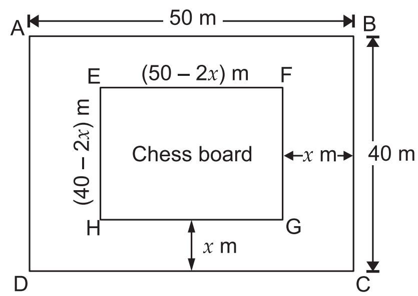
$x = 2$ or $x = 7$

Since $x = 2$ does not satisfy the condition hence we will reject it.

Thus, length of rectangular park = **7 m** and

Breadth of the rectangular park = **4 m**

20. Let the width of space between the pond and the given rectangular lawn ABCD be x m. Then AB = 50 m, AD = 40 m, EF = $(50 - 2x)$ m and EH = $(40 - 2x)$ m.



$\therefore$ Area of the grassland between ABCD and EFGH is

$[50 \times 40 - (50 - 2x)(40 - 2x)]\text{m}^2$

$= 2000 - (2000 - 100x - 80x + 4x^2)\text{m}^2$

$= (-4x^2 + 180x)\text{ m}^2$

$\therefore$ According to the problem, we have

$180x - 4x^2 = 1184$

$\Rightarrow$ $x^2 - 45x + 296 = 0$

Comparing this equation with the standard quadratic equation $ax^2 + bx + c = 0$, we have

$a = 1$, $b = -45$ and $c = 296$

$$\therefore \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{45 \pm \sqrt{45^2 - 4 \times 296}}{2}$$

$$= \frac{45 \pm \sqrt{2025 - 1184}}{2}$$

$$= \frac{45 \pm \sqrt{841}}{2}$$

$$= \frac{45 \pm 29}{2}$$

$$= \frac{74}{2}, \frac{16}{2} = 37, 8$$

If the width is 37 m, then EH = (50 – 37 × 2) m = (50 – 74) m = –24 m which is not possible.

$\therefore$ $x = 37$ is rejected.

$\therefore$ The width of the space between ABCD and EFGH is 8 m.

$\therefore$ The required length and breadth of the pond EFGH are respectively (50 – 2 × 8) m and (40 – 2 × 8) m or **34 m** and **24 m**.

EXERCISE 4J

For Basic and Standard Levels

1. Let the usual speed of the passenger train be x km/h.

Increased speed of the passenger train = $(x + 15)$ km/h

Measurement	*Train moving at usual speed*	*Train moving at increased speed*
Distance	300 km	300 km
Speed	x km/h	$(x + 15)$ km/h
Time	$\frac{300}{x}$ h	$\frac{300}{(x+15)}$ h

Given,

$$\begin{bmatrix}\text{Time taken by the}\\ \text{passenger train at}\\ \text{usual speed}\end{bmatrix} - \begin{bmatrix}\text{Time taken by the}\\ \text{passenger train at}\\ \text{increased speed}\end{bmatrix} = 1 \text{ hour}$$

$\Rightarrow \quad \frac{300}{x} - \frac{300}{(x+15)} = 1$

$\Rightarrow \quad 300\left(\frac{x+15-x}{x^2+15x}\right) = 1$

$\Rightarrow \quad 4500 = x^2 + 15x$

$\Rightarrow \quad x^2 + 15x - 4500 = 0$

$\Rightarrow \quad x^2 + 75x - 60x - 4500 = 0$

$\Rightarrow \quad x(x + 75) - 60(x + 75) = 0$

$\Rightarrow \quad (x + 75)(x - 60) = 0$

$\Rightarrow \quad$ Either $(x + 75) = 0$ or $(x - 60) = 0$

$\Rightarrow \quad x = -75$ or $x = 60$

Hence, the usual speed of the passenger train is **60 km/h**.

2. Let the usual speed of the bus be x km/h

Reduced speed of the bus = $(x - 5)$ km/h

Measurement	Bus moving at the usual speed	Bus moving at the reduced speed
Distance between A and B	550 km	550 km
Speed	x km/h	$(x - 5)$ km/h
Time	$\frac{550}{x}$ h	$\frac{550}{(x-5)}$ h

Given,

$$\begin{bmatrix}\text{Time taken by the}\\ \text{bus moving at the}\\ \text{reduced speed}\end{bmatrix} - \begin{bmatrix}\text{Time taken by the}\\ \text{bus moving at the}\\ \text{usual speed}\end{bmatrix} = 1 \text{ hour}$$

$\Rightarrow \quad \frac{550}{x-5} - \frac{550}{x} = 1$

$\Rightarrow \quad \frac{550\,(x-x+5)}{x(x-5)} = 1$

$\Rightarrow \quad 2750 = x^2 - 5x$

$\Rightarrow \quad x^2 - 5x - 2750 = 0$

$\Rightarrow \quad x^2 - 55x + 50x - 2750 = 0$

$\Rightarrow \quad x(x - 55) + 50(x - 55) = 0$

$\Rightarrow \quad (x - 55)(x + 50) = 0$

$\Rightarrow \quad$ Either $(x - 55) = 0$ or $(x + 50) = 0$

$\Rightarrow \quad x = 55$ or $x = -50$ (rejected)

Time taken by the bus to cover the distance between A and B when its raining = $\frac{550}{x-5}$ hours = $\frac{550}{55-5}$ hours

$= \frac{550}{50}$ hours = 11 hours

Hence, the time taken is **11 hours**.

3. Let the original speed of the train be x km/h

Increased speed of the train = $(x + 15)$ km/h

Measurement	Train moving at the original speed	Train moving at the increased speed
Distance	90 km	90 km
Speed	x km/h	$(x + 15)$ km/h
Time	$\frac{90}{x}$ h	$\frac{90}{x+15}$ h

Given,

$$\begin{bmatrix}\text{Time taken by the}\\ \text{train moving at the}\\ \text{original speed}\end{bmatrix} - \begin{bmatrix}\text{Time taken by the}\\ \text{train moving at}\\ \text{increased speed}\end{bmatrix} = \frac{1}{2} \text{ hour}$$

$\Rightarrow \quad \frac{90}{x} - \frac{90}{x+15} = \frac{1}{2}$

$\Rightarrow \quad \frac{90(x+15-x)}{x^2+15x} = \frac{1}{2}$

$\Rightarrow \quad 90 \times 15 \times 2 = x^2 + 15x$

$\Rightarrow \quad x^2 + 15x - 2700 = 0$

$\Rightarrow \quad x^2 - 45x + 60x - 2700 = 0$

$\Rightarrow \quad x(x - 45) + 60(x - 45) = 0$

$\Rightarrow \quad (x - 45)(x + 60) = 0$

$\Rightarrow \quad$ Either $(x - 45) = 0$ or $(x + 60) = 0$

$\Rightarrow \quad x = 45$ or $x = -60$ (rejected)

The original speed of the train is **45 km/h.**

Hence, original duration of journey = $\frac{90}{x}$

$= \frac{90}{45}$ = **2 hours**

4. Let the average speed of the fast train be x km/h

Then, the average speed of the slow train = $(x - 16)$ km/h

Measurements	Fast train	Slow train
Distance	192 km	192 km
Speed	x km/h	$(x - 16)$ km/h
Time	$\frac{192}{x}$ h	$\frac{192}{(x-16)}$ h

Given, Time taken by the slow train – Time taken by the fast train = 2 hours

$\Rightarrow \quad \frac{192}{x-16} - \frac{192}{x} = 2$

$\Rightarrow \quad 192\left(\frac{x-x+16}{x^2-16x}\right) = 2$

$\Rightarrow \quad \frac{192 \times 16}{2} = x^2 - 16x$

$\Rightarrow \quad 1536 = x^2 - 16x$

$\Rightarrow \quad x^2 - 16x - 1536 = 0$

$\Rightarrow \quad x^2 - 48x + 32x - 1536 = 0$

$\Rightarrow \quad x(x - 48) + 32(x - 48) = 0$

$\Rightarrow \quad (x - 48)(x + 32) = 0$

$\Rightarrow$ Either $(x - 48) = 0$ or $(x + 32) = 0$

$\Rightarrow \quad x = 48$ or $x = -32$ (rejected)

Average speed of the fast train = x km/h = 48 km/h

Average speed of the slow train = $(x - 16)$ km/h
= $(48 - 16)$ km/h
= 32 km/h

Hence, the average speeds of the fast train and the slow train are **48 km/h and 32 km/h** respectively.

5. Let the original speed be x km/h

According to the given condition original time (t_1)

$= \frac{360}{x}$ h

If the speed is 5 km/h more, then time (t_2) = $\frac{360}{x+5}$ h

$t_1 - t_2 = \frac{48}{60}$

$\frac{360}{x} - \frac{360}{x+5} = \frac{8}{10}$

$360\left[\frac{x+5-x}{x(x+5)}\right] = \frac{8}{10}$

$2250 = x^2 + 5x$

$x^2 + 5x - 2250 = 0$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2}$

$= \frac{-5 \pm \sqrt{25+9000}}{2}$

$= \frac{-5 \pm \sqrt{9025}}{2}$

$x = -50$ or $x = 45$

Since speed cannot be negative hence we will neglect $x = -50$.

Thus, original speed of the train = **45 km/h**

6. Let the usual speed of the train be x km/h.

Then the usual time t_1 hours taken by the train to cover a distance of 480 km is $t_1 = \frac{480}{x}$. If the speed of the train be $(x - 8)$ km/h.

Then the train would have taken $t_2 = \frac{480}{x-8}$ h.

$\therefore$ According to the problem, we have

$t_2 - t_1 = 3$

$\Rightarrow \quad \frac{480}{x-8} - \frac{480}{x} = 3$

$\Rightarrow \quad \frac{480(x-x+8)}{x^2-8x} = 3$

$\Rightarrow \quad 3(x^2 - 8x) = 480 \times 8$

$\Rightarrow \quad x^2 - 8x - 1280 = 0$

Comparing the equation with the standard quadratic equation $ax^2 + bx + c = 0$, we have

$a = 1$, $b = -8$ and $c = -1280$

$\therefore \quad x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \frac{8 \pm \sqrt{64-5120}}{2}$

$= \frac{8 \pm \sqrt{5184}}{2}$

$= \frac{8 \pm 72}{2}$

$= \frac{80}{2}, -\frac{64}{2}$

$= 40$ or -32

$\because$ x cannot be negative, we reject $x = -32$.

$\therefore$ The required usual speed of the train is **40 km/h**.

7. Let the speed of the faster train be x km/h.

$\therefore$ The speed of the slower train is $(x - 10)$ km/h.

Then the time taken by the slower train to cover a distance of 200 km is $\frac{200}{x-10}$ h and that taken by the faster train to cover the same distance = $\frac{200}{x}$ h.

$\therefore$ According to the problem, we have

$\Rightarrow \quad \frac{200}{x-10} - \frac{200}{x} = 1$

$\Rightarrow \quad \frac{200(x-x+10)}{x^2-10x} = 1$

$\Rightarrow \quad x^2 - 10x - 2000 = 0$

Comparing this equation with the standard quadratic equation $ax^2 + bx + c = 0$, we have

$a = 1$, $b = -10$ and $c = -2000$

$\therefore \quad x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \frac{10 \pm \sqrt{100+8000}}{2}$

$= \frac{10 \pm \sqrt{8100}}{2} = \frac{10 \pm 90}{2}$

$= 50, -40$

Since x cannot be negative.

$\therefore$ We reject $x = -40$

Hence, the required speed of the faster train is **50 km/h** and so the speed of the slower train is (50 – 10) km/h = **40 km/h**.

8. Let the usual speed of the train be x km/h. Then the usual time taken by the train to cover a distance of 300 km is $\frac{300}{x}$ h and if the speed of the train is $(x + 5)$ km/h, then it would take $\frac{300}{x+5}$ h to cover the same distance.

$\therefore$ According to the problem, we have

$$\frac{300}{x} - \frac{300}{x+5} = 2$$

$$\Rightarrow \quad \frac{300(x+5-x)}{x^2+5x} = 2$$

$$\Rightarrow \quad 2(x^2 + 5x) = 300 \times 5$$

$$\Rightarrow \quad x^2 + 5x - 750 = 0$$

Comparing this equation with the standard quadratic equation $ax^2 + bx + c = 0$, we have

$a = 1$, $b = 5$ and $c = -750$

$$\therefore \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-5 \pm \sqrt{25 + 3000}}{2}$$

$$= \frac{-5 \pm \sqrt{3025}}{2}$$

$$= \frac{-5 \pm 55}{2}$$

$$= 25, -30$$

Since x cannot be negative, we reject $x = -30$

Hence, the required usual speed of the train is **25 km/h**.

9. Let the original speed of the train be x km/h. Then the time taken by the train to cover a distance of 360 km is $\frac{360}{x}$ h. If the speed of this train be $(x + 5)$ km/h, then it would take $\frac{360}{x+5}$ h to cover the same distance.

$\therefore$ According to the problem, we have

$$\frac{360}{x} - \frac{360}{x+5} = \frac{48}{60} = \frac{4}{5}$$

$$\Rightarrow \quad \frac{360(x+5-x)}{x(x+5)} = \frac{4}{5}$$

$$\Rightarrow \quad \frac{360 \times 5}{x^2+5x} = \frac{4}{5}$$

$$\Rightarrow \quad 4(x^2 + 5x) = 25 \times 360$$

$$\Rightarrow \quad x^2 + 5x = 25 \times 90 = 2250$$

$$\Rightarrow \quad x^2 + 5x - 2250 = 0$$

Comparing this equation with the standard quadratic equation $ax^2 + bx + c = 0$, we have

$a = 1$, $b = 5$ and $c = -2250$

$$\therefore \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-5 \pm \sqrt{25 + 9000}}{2}$$

$$= \frac{-5 \pm \sqrt{9025}}{2}$$

$$= \frac{-5 \pm 95}{2}$$

$$= \frac{90}{2}, -\frac{100}{2}$$

$$= 45, -50$$

$\because$ x cannot be negative, we reject $x = -50$.

$\therefore$ Required usual speed of the train is **45 km/h**.

10. Let the first speed of the train be x km/h.

Then, the increased speed of the train = $(x + 6)$ km/h

Measurements	Train moving at original speed	Train moving at increased speed
Distance	54 km	63 km
Speed	x km/h	$(x + 6)$ km/h
Time	$\frac{54}{x}$ h	$\frac{63}{(x+6)}$ h

Given,

$$\begin{bmatrix}\text{Time taken by the} \\ \text{train moving at the} \\ \text{original speed}\end{bmatrix} + \begin{bmatrix}\text{Time taken by the} \\ \text{train moving at the} \\ \text{increased speed}\end{bmatrix} = 3 \text{ hours}$$

$$\Rightarrow \quad \frac{54}{x} + \frac{63}{(x+6)} = 3$$

$$\Rightarrow \quad 9\left[\frac{6}{x} + \frac{7}{(x+6)}\right] = 3$$

$$\Rightarrow \quad 3[6(x + 6) + 7x] = x^2 + 6x$$

$$\Rightarrow \quad 3(7x + 36 + 6x) = x^2 + 6x$$

$$\Rightarrow \quad 39x + 108 = x^2 + 6x$$

$$\Rightarrow \quad x^2 - 33x - 108 = 0$$

$$\Rightarrow \quad x^2 + 3x - 36x - 108 = 0$$

$$\Rightarrow \quad x(x + 3) - 36(x + 3) = 0$$

$$\Rightarrow \quad (x - 36)(x + 3) = 0$$

$$\Rightarrow \quad \text{Either } (x - 36) = 0 \text{ or } (x + 3) = 0$$

$$\Rightarrow \quad x = 36 \text{ or } x = -3 \text{ (rejected)}$$

Hence, the first speed of the train is **36 km/h**.

11. Let the original average speed of the train be x km/h and the increased average speed be $(x + 6)$ km/h. Then the total time to travel 63 km at the original average speed of x km/h and that to travel 72 km at the increased average speed of $(x + 6)$ km/h are $\frac{63}{x} + \frac{72}{x+6}$.

$\therefore$ According to the problem, we have

$$\frac{63}{x} + \frac{72}{x+6} = 3$$

$$\Rightarrow \quad \frac{63x + 378 + 72x}{x^2 + 6x} = 3$$

$$\Rightarrow \quad 135x + 378 - 3x^2 - 18x = 0$$

$$\Rightarrow \quad 3x^2 - 117x - 378 = 0$$

$$\Rightarrow \quad x^2 - 39x - 126 = 0$$

Comparing this equation with the standard quadratic equation $ax^2 + bx + c = 0$, we have

$a = 1$, $b = -39$, $c = -126$

$$\therefore \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{39 \pm \sqrt{39^2 + 4 \times 126}}{2}$$

$$= \frac{39 \pm \sqrt{1521 + 504}}{2}$$

$$= \frac{39 \pm \sqrt{2025}}{2}$$

$$= \frac{39 \pm 45}{2}$$

$$= \frac{84}{2}, -\frac{6}{2}$$

$$= 42, -3$$

We reject the negative value of x, i.e. we reject $x = -3$.

$\therefore$ Required original speed of the train is **42 km/h**.

12. Let the original average speed of the truck be x km/h. Then its increased speed is $(x + 20)$ km/h. Then the truck takes $\frac{150}{x}$ h to travel a distance of 150 km at the original speed and $\frac{200}{x+20}$ h to travel a distance of 200 km at the increased speed $(x + 20)$ km/h.

$\therefore$ According to the problem, we have

$$\frac{150}{x} + \frac{200}{x+20} = 5$$

$$\Rightarrow \quad \frac{150x + 3000 + 200x}{x(x+20)} = 5$$

$$\Rightarrow \quad \frac{350x + 3000}{x^2 + 20x} = 5$$

$$\Rightarrow \quad 350x + 3000 = 5x^2 + 100x$$

$$\Rightarrow \quad x^2 - 50x - 600 = 0$$

Comparing this equation with the standard quadratic equation $ax^2 + bx + c = 0$, we have

$a = 1$, $b = -50$ and $c = -600$

$$\therefore \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{50 \pm \sqrt{50^2 + 4 \times 600}}{2}$$

$$= \frac{50 \pm \sqrt{2500 + 2400}}{2}$$

$$= \frac{50 \pm \sqrt{4900}}{2}$$

$$= \frac{50 \pm 70}{2}$$

$$= 60 \text{ or } -10$$

Since x cannot be negative.

$\therefore$ We reject $x = -10$.

Hence, the required original speed of the truck is **60 km/h**.

13. Let the usual speed of the aeroplane be x km/h. Hence, the usual time to describe a distance of 1500 km with original speed is $\frac{1500}{x}$ h and the time to describe the same distance at the increased speed of $(x + 100)$ km/h is $\frac{1500}{x+100}$.

$\therefore$ According to the problem, we have

$$\frac{1500}{x} - \frac{1500}{x+100} = \frac{30}{60} = \frac{1}{2}$$

$$\Rightarrow \quad \frac{1500(x + 100 - x)}{x^2 + 100x} = \frac{1}{2}$$

$$\Rightarrow \quad x^2 + 100x = 300000$$

$$\Rightarrow \quad x^2 + 100x - 300000 = 0$$

Comparing this equation with the standard quadratic equation $ax^2 + bx + c = 0$, we have

$a = 1$, $b = 100$ and $c = -300000$

$$\therefore \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-100 \pm \sqrt{10000 - 1200000}}{2}$$

$$= \frac{-100 \pm \sqrt{1210000}}{2}$$

$$= \frac{-100 \pm 1100}{2}$$

$$= 500, -600$$

Since x cannot be negative, we reject $x = -600$.

$\therefore$ Required usual speed of the aeroplane is **500 km/h**.

14. Let the original average speed of the aeroplane be x km/h

Reduced speed of the aeroplane = $(x - 200)$ km/h

Measurements	Aeroplane moving at usual speed	Aeroplane moving at reduced speed
Distance	600 km	600 km
Speed	x km/h	$(x - 200)$ km/h
Time	$\frac{600}{x}$ h	$\frac{600}{(x-200)}$ h

Given,

$$\begin{bmatrix}\text{Time taken by the} \\ \text{aeroplane at the} \\ \text{usual speed}\end{bmatrix} - \begin{bmatrix}\text{Time taken by the} \\ \text{aeroplane at the} \\ \text{reduced speed}\end{bmatrix} = \frac{1}{2} \text{ hour}$$

$\Rightarrow \quad \frac{600}{x} - \frac{600}{x-200} = \frac{1}{2}$

$\Rightarrow \quad 600\left(\frac{x-200-x}{x^2-200x}\right) = \frac{1}{2}$

$\Rightarrow \quad 1200(-200) = x^2 - 200x$

$\Rightarrow \quad x^2 - 200x + 240000 = 0$

$\Rightarrow \quad x^2 - 600x + 400x - 240000 = 0$

$\Rightarrow \quad x(x-600) + 400(x-600) = 0$

$\Rightarrow \quad (x-600)(x+400) = 0$

$\Rightarrow \quad$ Either $(x-600) = 0$ or $(x+400) = 0$

$\Rightarrow \quad x = 600$ or $x = -400$ (rejected)

So, the original average speed of the plane = x km/h = 600 km/h

Duration of the flight = $\frac{600}{x}$ h = $\frac{600}{600}$ h = 1 h

Hence, the duration of the flight is **1 hour.**

15. Let the speed of the stream be x km/hour.
Then, the speed of the boat downstream = $(8 + x)$ km/h
and the speed of the boat upstream = $(8 - x)$ km/h

Measurements	*Downstream*	*Upstream*
Distance covered	22 km	15 km
Speed	$(8 + x)$ km/h	$(8 - x)$ km/h
Time	$\frac{22}{8+x}$ h	$\frac{15}{8-x}$ h

Given, Time taken to go 15 km upstream and 22 km downstream = 5 hours

$\Rightarrow \quad \frac{15}{(8-x)} + \frac{22}{(8+x)} = 5$

$\Rightarrow \quad 15(8+x) + 22(8-x) = 5(64-x^2)$

$\Rightarrow \quad 120 + 15x + 176 - 22x = 320 - 5x^2$

$\Rightarrow \quad 5x^2 - 7x + 296 - 320 = 0$

$\Rightarrow \quad 5x^2 - 7x - 24 = 0$

$\Rightarrow \quad 5x^2 - 15x + 8x - 24 = 0$

$\Rightarrow \quad 5x(x-3) + 8(x-3) = 0$

$\Rightarrow \quad (x-3)(5x+8) = 0$

$\Rightarrow \quad$ Either $(x-3) = 0$ or $(5x+8) = 0$

$\Rightarrow \quad$ Either $x = 3$ or $x = \frac{-8}{5}$ (rejected)

Hence, the speed of the stream is **3 km/h**.

16. Let the speed of the stream be x km/h.

Then, the speed of the motorboat downstream = $(18 + x)$ km/h

The speed of the motorboat upstream = $(18 - x)$ km/h

Measurements	*Downstream*	*Upstream*
Distance covered	24 km	24 km
Speed	$(18 + x)$ km/h	$(18 - x)$ km/h
Time	$\frac{24}{(18+x)}$ h	$\frac{24}{(18-x)}$ h

Given,

$$\begin{bmatrix}\text{Time taken by the} \\ \text{motorboat to go} \\ \text{24 km upstream}\end{bmatrix} - \begin{bmatrix}\text{Time taken by the} \\ \text{motorboat to come back} \\ \text{24 km downstream}\end{bmatrix} = 1$$

$\Rightarrow \quad \frac{24}{18-x} - \frac{24}{18+x} = 1$

$\Rightarrow \quad 24(18+x) - 24(18-x) = (18-x)(18+x)$

$\Rightarrow \quad 24(18 + x - 18 + x) = 18^2 - x^2$

$\Rightarrow \quad 24(2x) = 18^2 - x^2$

$\Rightarrow \quad x^2 - 18^2 + 48x = 0$

$\Rightarrow \quad x^2 + 48x - 324 = 0$

$\Rightarrow \quad x^2 + 54x - 6x - 324 = 0$

$\Rightarrow \quad x(x+54) - 6(x+54) = 0$

$\Rightarrow \quad (x-6)(x+54) = 0$

$\Rightarrow \quad x = 6$ or $x = -54$ (rejected)

$\Rightarrow \quad x = 6$

Hence, the speed of the stream = **6 km/h**

17. Let the speed of the stream be x km/h. Now, the speed of the boat is 15 km/h.

Then the speed of the boat in favour of the stream = $(15 + x)$ km/h and the speed of the boat against the stream = $(15 - x)$ km/h.

$\therefore$ According to the problem, we have

$$\frac{30}{15+x} + \frac{30}{15-x} = 4\frac{1}{2} = \frac{9}{2}$$

$\Rightarrow \quad \frac{30 \times (15 - x + 15 + x)}{15^2 - x^2} = \frac{9}{2}$

$\Rightarrow \quad \frac{900}{225 - x^2} = \frac{9}{2}$

$\Rightarrow \quad \frac{100}{225 - x^2} = \frac{1}{2}$

$\Rightarrow \quad 225 - x^2 = 200$

$\Rightarrow \quad x^2 = 25$

$\Rightarrow \quad x = 5 \qquad [\because x \neq -5]$

Hence, the required speed of the stream is **5 km/h**.

For Standard Level

18.

Measurements	Case 1	Case 2
Distance	150 km	150 km
Speed	x km/h	$(x + 10)$ km/h
Time	$\frac{150}{x}$ h	$\frac{150}{(x+10)}$ h

Given, $\begin{bmatrix}\text{Time taken}\\\text{to reach}\end{bmatrix} - \begin{bmatrix}\text{Time taken}\\\text{to return}\end{bmatrix} = \frac{5}{2}$ hours

$\Rightarrow \quad \frac{150}{x} - \frac{150}{(x+10)} = \frac{5}{2}$

$\Rightarrow \quad 150\left[\frac{x+10-x}{x(x+10)}\right] = \frac{5}{2}$

$\Rightarrow \quad 600 = x(x + 10)$

$\Rightarrow \quad 600 = x^2 + 10x$

$\Rightarrow \quad x^2 + 10x - 600 = 0$

$\Rightarrow \quad x^2 - 20x + 30x - 600 = 0$

$\Rightarrow \quad x(x - 20) + 30(x - 20) = 0$

$\Rightarrow \quad (x - 20)(x + 30) = 0$

$\Rightarrow \quad$ Either $(x - 20) = 0$ or $(x + 30) = 0$

$\Rightarrow \quad x = 20$ or $x = -30$ (rejected)

Hence, speed while going **= 20 km/h**

speed while returning **= 30 km/h**

19. Let the speed of the car be x km/h. Then the time taken by the car in moving a distance of 2592 km is $\frac{2592}{x}$. Then according to the problem, we have

$$\frac{2592}{x} = \frac{x}{2}$$

$\Rightarrow \quad x^2 = 5184$

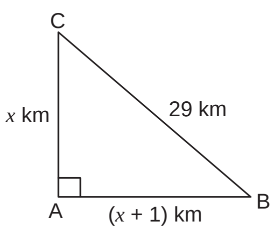
$\therefore \quad x = \sqrt{5184} = 72$

$\therefore \quad$ Speed of the car = 72 km/h.

$\therefore \quad$ Required time = $\frac{2592}{72}$ h = **36 h**.

20. Let A be the point from where the two ships leave simultaneously in directions at right angles to each other.

Let the ship moving at the speed of x km/h reach point C. Then AC = x km.



Let the ship moving at the speed of $(x + 1)$ km/h reach the point B. Then, AB = $(x + 1)$ km

and BC = 29 km

In right ΔCAB, we have

$AC^2 + AB^2 = BC^2$ [By Pythagoras' Theorem]

$\Rightarrow \quad x^2 + (x + 1)^2 = 29^2$

$\Rightarrow \quad x^2 + x^2 + 2x + 1 = 841$

$\Rightarrow \quad 2x^2 + 2x - 840 = 0$

$\Rightarrow \quad x^2 + x - 420 = 0$

$\Rightarrow \quad x - 20x + 21x - 420 = 0$

$\Rightarrow \quad x(x - 20) + 21(x - 20) = 0$

$\Rightarrow \quad (x - 20)(x + 21) = 0$

$\Rightarrow \quad$ Either $(x - 20) = 0$ or $(x + 21) = 0$

$\Rightarrow \quad x = 20$ or $x = -21$ (rejected)

The speeds of the ships are

x km/h = 20 km/h

and $(x + 1)$ km/h = $(20 + 1)$ km/h = 21 km/h

Hence, the speeds of the ships are **20 km/h and 21 km/h**.

EXERCISE 4K

For Basic and Standard Levels

1. Suppose B alone can finish the work in x days.

Then, A alone can finish the work in $(x - 10)$ days.

$\therefore$ B's one day's work = $\frac{1}{x}$ and A's one day's work = $\frac{1}{x-10}$

B and A together can finish the work in 12 days

$\therefore$ (B + A)'s one day's work = $\frac{1}{12}$

B's one day's work + A's one day's work = (B + A)'s one day's work

$$\frac{1}{x} + \frac{1}{x-10} = \frac{1}{12}$$

$\Rightarrow \quad 12(x - 10 + x) = x(x - 10)$

$\Rightarrow \quad 12(2x - 10) = x^2 - 10x$

$\Rightarrow \quad 24x - 120 = x^2 - 10x$

$\Rightarrow \quad x^2 - 10x - 24x + 120 = 0$

$\Rightarrow \quad x^2 - 34x + 120 = 0$

$\Rightarrow \quad x^2 - 30x - 4x + 120 = 0$

$\Rightarrow \quad x(x - 30) - 4(x - 30) = 0$

$\Rightarrow \quad (x - 30)(x - 4) = 0$

$\Rightarrow \quad$ Either $(x - 30) = 0$ or $(x - 4) = 0$

$\Rightarrow \quad x = 30$ or $x = 4$

($x = 4$ rejected as it does not satisfy the given conditions)

Hence, B alone takes **30 days** to finish the work.

2. Let B alone can finish the work in x days. Then A can finish the same work alone in $(x + 12)$ days.

$\therefore$ In 1 day, B alone can finish $\frac{1}{x}$th part of the work and

A alone can finish $\frac{1}{x+12}$th part of the work in 1 day.

$\therefore$ In 1 day, A and B together can complete

$\left(\frac{1}{x}+\frac{1}{x+12}\right)$th part = $\frac{2x+12}{x^2+12x}$ th part of the work.

$\therefore$ According to the problem, we have

$$\frac{2x+12}{x^2+12x} = \frac{1}{8}$$

$\Rightarrow \quad 16x + 96 = x^2 + 12x$

$\Rightarrow \quad x^2 - 4x - 96 = 0$

Comparing this equation with the standard quadratic equation $ax^2 + bx + c = 0$, we have

$a = 1$, $b = -4$ and $c = -96$

$$\therefore \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{4 \pm \sqrt{16 + 4 \times 96}}{2}$$

$$= \frac{4 \pm \sqrt{16 + 384}}{2}$$

$$= \frac{4 \pm \sqrt{400}}{2}$$

$$= \frac{4 \pm 20}{2}$$

$$= 12, -8$$

We reject the negative value of x, i.e. we reject $x = -8$.

$\therefore$ Required no. of days in which B alone can complete the work is **12 days**.

For Standard Level

3. Let the time taken by the first pipe to fill the cistern be x minutes.

Then the time taken by the second pipe to fill the cistern $= (x + 5)$ minutes

$\therefore$ The first pipe fills $\frac{1}{x}$ of the cistern in 1 minute

and the second pipe fills $\frac{1}{(x+5)}$ of the cistern in 1 minute.

Together the two pipes fill $\left(\frac{1}{x}+\frac{1}{x+5}\right)$ of the cistern in 1 minute

i.e. the two pipes fill $\frac{x+5+x}{x(x+5)}$ of the cistern in 1 minute

$\Rightarrow$ The two pipes fill $\frac{2x+5}{x^2+5x}$ of the cistern in 1 minute

$\Rightarrow$ The two pipes fill the tank in $\frac{x^2+5x}{2x+5}$ minutes ...(1)

Given, Time taken by the two pipes together to fill the cistern $= 11\frac{1}{9}$ minutes ...(2)

From (1) and (2), we get

$$\frac{x^2+5x}{2x+5} = \frac{100}{9}$$

$\Rightarrow \quad 9x^2 + 45x = 200x + 500$

$\Rightarrow \quad 9x^2 - 155x - 500 = 0$

$$\Rightarrow \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{155 \pm \sqrt{24025+18000}}{18}$$

$$= \frac{155 \pm \sqrt{42025}}{18}$$

$$= \frac{155 \pm 205}{18}$$

$$\Rightarrow \quad x = 20 \text{ or } x = \frac{-50}{18}$$

Since time cannot be negative, hence we will neglect $x = \frac{-50}{18}$

Time pipe 1 will take to fill the tank = **20 min**

Time pipe 2 will take to fill the tank = **25 min**

4. Let the pipe of larger diameter take x hours to fill the tank.

$\therefore$ Time taken by pipe of smaller diameter is $(x + 10)$ hours.

Tank filled in 1 hour by bigger pipe $= \frac{1}{x}$

Tank filled in 1 hour by smaller pipe $= \frac{1}{x+10}$

According to the given condition

$$\Rightarrow \quad \frac{4}{x}+\frac{9}{x+10} = \frac{1}{2}$$

$$\Rightarrow \quad \frac{4x+40+9x}{x(x+10)} = \frac{1}{2}$$

$\Rightarrow \quad 26x + 80 = x^2 + 10x$

$\Rightarrow \quad x^2 - 16x - 80 = 0$

$\Rightarrow \quad x^2 + 4x - 20x - 80 = 0$

$\Rightarrow \quad x(x + 4) - 20(x + 4) = 0$

$\Rightarrow \quad (x + 4)(x - 20) = 0$

$\Rightarrow \quad x = -4$ or $x = 20$

Since the time cannot be negative, hence we will reject $x = -4$.

Thus, pipe with greater diameter will take **20 hours** to fill the tank.

Pipe with smaller diameter will take (20 + 10) = **30 hours** to fill the tank.

5. Let the bigger tap fill the tank in x hours. Then the smaller tap can fill the same tank in $(x + 3)$ h.

$\therefore$ In 1 hour, the two taps together can fill $\left(\frac{1}{x}+\frac{1}{x+3}\right)$th part of the tank, i.e. $\frac{2x+3}{x(x+3)}$ th part of the tank.

$\therefore$ According to the problem, we have

$$\frac{2x+3}{x^2+3x} = \frac{13}{40}$$

$\Rightarrow \quad 80x + 120 = 13x^2 + 39x$

$\Rightarrow \quad 13x^2 - 41x - 120 = 0$

Comparing this equation with the standard quadratic equation $ax^2 + bx + c = 0$, we have

$a = 13$, $b = -41$ and $c = -120$.

$$\therefore \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{41 \pm \sqrt{41^2 + 4 \times 13 \times 120}}{2 \times 13}$$

$$= \frac{41 \pm \sqrt{1681 + 6240}}{26}$$

$$= \frac{41 \pm \sqrt{7921}}{26}$$

$$= \frac{41 \pm 89}{26}$$

$$= \frac{130}{26}, \frac{-48}{26}$$

$$= 5, \frac{-48}{26}$$

Rejecting the negative value of x, i.e. rejecting $x = \frac{-48}{26}$,

we get $x = 5$.

$\therefore$ Bigger tap can fill the tank in 5 hours and the smaller tap can fill it in (5 + 3) h = 8 h.

$\therefore$ Required times are **5 hours** and **8 hours**.

6. Let the smaller tap fill the tank in x hours and the bigger tap fill the tank in $(x - 9)$ h.

$\therefore$ The two taps together can fill $\left(\frac{1}{x} + \frac{1}{x-9}\right)$th part or

$\frac{2x-9}{x^2 - 9x}$ th part of the tank in 1 hour

$\therefore$ According to the problem, we have

$$\frac{2x-9}{x^2-9x} = \frac{1}{6}$$

$\Rightarrow \quad x^2 - 9x = 12x - 54$

$\Rightarrow \quad x^2 - 21x + 54 = 0$

Comparing this equation with the standard quadratic equation $ax^2 + bx + c = 0$, we have

$a = 1$, $b = -21$ and $c = 54$.

$$\therefore \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{21 \pm \sqrt{21^2 - 4 \times 54}}{2}$$

$$= \frac{21 \pm \sqrt{441 - 216}}{2}$$

$$= \frac{21 \pm \sqrt{225}}{2}$$

$$= \frac{21 \pm 15}{2}$$

$$= \frac{36}{2}, \frac{6}{2}$$

$$= 18, 3$$

If $x = 3$, then the bigger tap cannot fill the tank in $(x - 9)$ h. Hence, we reject $x = 3$ and accept $x = 18$.

Hence, the required time that the smaller tap can fill the tank is **18 hours** and the bigger tap can fill it in (18 – 9) h = **9 hours**.

7. Suppose the second pipe fills the pool in x hours.

Then, the first pipe fills the pool in $(x + 5)$ hours and the third pipe fills the pool in $(x - 4)$ hours.

$\therefore$ The first pipe fills $\frac{1}{x+5}$ of the pool in 1 hour.

The second pipe fills $\frac{1}{x}$ of the pool in 1 hour.

The third pipe fills $\frac{1}{x-4}$ of the pool in 1 hour.

$\Rightarrow$ The first two pipes together fill $\left(\frac{1}{x+5} + \frac{1}{x}\right)$ of the pool in 1 hour.

$\Rightarrow$ The first two pipes together fill $\left(\frac{2x+5}{x^2+5x}\right)$ of the pool in 1 hour.

$\Rightarrow$ The first two pipes together fill the pool

in $\frac{x^2+5x}{2x+5}$ hours. …(1)

The third pipe fills the pool in $(x - 4)$ hours …(2)

Given, that the first two pipes operating simultaneously fill the pool in the same time during which the third pipe fills it alone.

$$\therefore \quad \frac{x^2+5x}{2x+5} = x - 4 \quad \text{[Using (1) and (2)]}$$

$\Rightarrow \quad x^2 + 5x = (x - 4)(2x + 5)$

$\Rightarrow \quad x^2 + 5x = 2x^2 - 8x + 5x - 20$

$\Rightarrow \quad x^2 - 8x - 20 = 0$

$\Rightarrow \quad x^2 - 10x + 2x - 20 = 0$

$\Rightarrow \quad x(x - 10) + 2(x - 10) = 0$

$\Rightarrow \quad (x - 10)(x + 2) = 0$

$\Rightarrow$ Either $(x - 10) = 0$ or $(x + 2) = 0$

$\Rightarrow \quad x = 10$ or $x = -2$ (rejected)

The first pipe fills the pool in $(x + 5)$ hours, i.e. (10 + 5) hours = 15 hours

The second pipe fills the pool in x hours, i.e. 10 hours

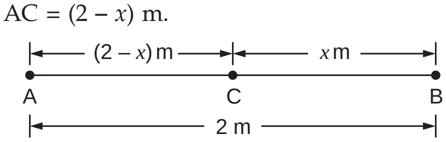
The third pipe fills the pool in $(x - 4)$ hours, i.e. (10 – 4) hours = 6 hours

Hence the first, second and third pipe fill the pool separately in **15 hours, 10 hours and 6 hours** respectively.

EXERCISE 4L

For Basic and Standard Levels

1. Let AB = 2 m be divided at point C, such that CB = x m. Then, AC = $(2 - x)$ m.



Given, $AC^2 = AB \times CB$

$\Rightarrow \quad (2 - x)^2 = 2x$

$\Rightarrow \quad 4 - 4x + x^2 = 2x$

$\Rightarrow \quad x^2 - 6x + 4 = 0$

$$\Rightarrow \quad x = \frac{6 \pm \sqrt{36 - 4(1)(4)}}{2} \qquad \left[x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\right]$$

$$\Rightarrow \quad x = \frac{6 \pm \sqrt{20}}{2}$$

$$\Rightarrow \quad x = \frac{6 \pm 2\sqrt{5}}{2}$$

$\Rightarrow \quad x = 3 - \sqrt{5}$ or $x = 3 + \sqrt{5}$ [Rejected as $x < 2$]

$CB = x \text{ m} = (3 - \sqrt{5})$ m

Hence, CB is $\mathbf{(3 - \sqrt{5})}$ **m**.

2. SP = ₹ 56, CP = ₹ x

Profit = SP – CP = ₹ $(56 - x)$

$$\text{Profit \%} = \frac{\text{Profit}}{\text{CP}} \times 100\% = \frac{₹\,(56 - x)}{₹\,x} \times 100\%$$

$$= \frac{(56 - x)}{x} \times 100\% \qquad \ldots(1)$$

Given, profit % = x% …(2)

From (1) and (2), we get

$$\frac{(56 - x)}{x} \times 100 = x$$

$\Rightarrow \quad 5600 - 100x = x^2$

$\Rightarrow \quad x^2 + 100x - 5600 = 0$

$\Rightarrow \quad x^2 + 140x - 40x - 5600 = 0$

$\Rightarrow \quad x(x + 140) - 40(x + 140) = 0$

$\Rightarrow \quad (x + 140)(x - 40) = 0$

$\Rightarrow \quad$ Either $(x + 140) = 0$ or $(x - 40) = 0$

$\Rightarrow \quad x = -140$ (rejected) or $x = 40$

Hence x is **40.**

3. Let the CP of the pen be ₹ x.

SP of the pen = ₹ 24

Gain = SP – CP = ₹ $(24 - x)$

$$\text{Gain\%} = \frac{\text{Gain}}{\text{CP}} \times 100\% = \frac{₹\,(24 - x)}{₹\,x} \times 100\%$$

$$= \frac{(24 - x)}{x} \times 100\% \qquad \ldots(1)$$

Given, gain % = x% …(2)

From (1) and (2), we get

$$\frac{(24 - x)}{x} \times 100 = x$$

$\Rightarrow \quad 2400 - 100x = x^2$

$\Rightarrow \quad x^2 + 100x - 2400 = 0$

$\Rightarrow \quad x^2 + 120x - 20x - 2400 = 0$

$\Rightarrow \quad x(x + 120) - 20(x + 120) = 0$

$\Rightarrow \quad (x + 120)(x - 20) = 0$

$\Rightarrow \quad$ Either $(x + 120) = 0$ or $(x - 20) = 0$

$\Rightarrow \quad x = -120$ (rejected) or $x = 20$

CP of the pen = ₹ x = ₹ 20

Hence, the cost price of the pen is ₹ **20.**

For Standard Level

4. Suppose there were x camels.

Then, the number of camels seen in the forest = $\frac{x}{4}$, the number of camels which had gone to the mountains = $2\sqrt{x}$

and the number of camels on the bank of the river = 15

Total number of camels = x

$$\Rightarrow \quad \frac{x}{4} + 2\sqrt{x} + 15 = x$$

$\Rightarrow \quad x + 8\sqrt{x} + 60 = 4x$

$\Rightarrow \quad 3x - 8\sqrt{x} - 60 = 0 \qquad \ldots(1)$

Let $\sqrt{x} = y \qquad \ldots(2)$

Then, equation (1) becomes

$3y^2 - 8y - 60 = 0$

$\Rightarrow \quad 3y^2 - 18y + 10y - 60 = 0$

$\Rightarrow \quad 3y(y - 6) + 10(y - 6) = 0$

$\Rightarrow \quad (y - 6)(3y + 10) = 0$

$\Rightarrow \quad$ Either $(y - 6) = 0$ or $(3y + 10) = 0$

$\Rightarrow \quad y = 6$ or $y = \frac{-10}{3}$ (rejected)

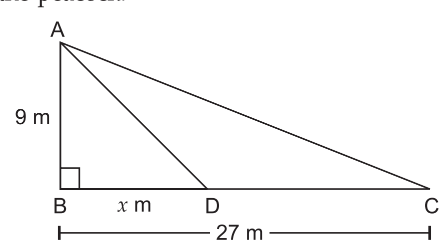
$\Rightarrow \quad \sqrt{x} = 6$ [Using (2)]

$\Rightarrow \quad x = 36$ [Squaring both sides]

Number of camels = x = 36

Hence, the number of camels is **36**.

5. Let AB be the vertical pole with peacock sitting at A. Here, AB = 9 m. Let B be the hole at a distance BC = 27 m where C is the initial position of the snake. Let the peacock catches the snake at D at a distance of x m from B. Let u m/s be the same speed of both the snake and the peacock.



Since the speeds of the snake and the peacock are the same, the time taken by the peacock to describe the distance AD will be the same as that taken by the snake to describe the distance DC.

$$\therefore \quad \frac{AD}{u} = \frac{DC}{u}$$

$$\Rightarrow \quad \frac{\sqrt{AB^2 + BD^2}}{u} = \frac{27 - x}{u}$$

$$\Rightarrow \quad \sqrt{x^2 + 81} = 27 - x$$

$$\Rightarrow \quad x^2 + 81 = (27 - x^2) = 729 - 54x + x^2$$

$$\Rightarrow \quad 54x = 648$$

$$\Rightarrow \quad x = \frac{648}{54} = 12$$

Hence, the required distance of BD = **12 m**.

6. Time from 2 pm to 3 pm is 60 minutes.

$\therefore$ Time from 2 : t pm to 3 pm, total time in minutes is $(60 - t)$ minutes.

$\therefore$ According to the problem, we have

$$\frac{t^2}{4} - 3 = 60 - t$$

$$\Rightarrow \quad t^2 - 12 = 240 - 4t$$

$$\Rightarrow \quad t^2 + 4t - 252 = 0$$

Comparing this equation with the standard quadratic equation $ax^2 + bx + c = 0$, we have

$a = 1$, $b = 4$, $c = -252$

$$\therefore \quad t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-4 \pm \sqrt{16 + 1008}}{2}$$

$$= \frac{-4 \pm \sqrt{1024}}{2}$$

$$= \frac{-4 \pm 32}{2}$$

$$\therefore \quad t = \frac{28}{2}, -\frac{36}{2} = 4, -18$$

Neglecting the negative value of t, i.e. $t = -18$, we get $t = 14$.

$\therefore$ Required value of t is **14**.

CHECK YOUR UNDERSTANDING

MULTIPLE-CHOICE QUESTIONS

For Basic and Standard Levels

1. (*c*) $\mathbf{8x^3 - x^2 = (2x - 1)^3}$

$$(a+1)x^2 - \frac{3}{5}x = 11, \text{ where } a = -1$$

$$\Rightarrow \quad (-1 + 1)\ x^2 - \frac{3}{5}x = 11$$

$$\Rightarrow \quad \frac{-3}{5}x = 11$$

which is not of the form $ax^2 + bx + c = 0$,

$$(3 - x)^2 - 5 = x^2 + 2x + 1$$

$$\Rightarrow \quad 9 + x^2 - 6x - 5 = x^2 + 2x + 1$$

$$\Rightarrow \quad 8x - 3 = 0$$

which is not of the form $ax^2 + bx + c = 0$

$$8x^3 - x^2 = (2x - 1)^3$$

$$\Rightarrow \quad 8x^3 - x^2 = 8x^3 - 1 - 12x^2 + 6x$$

$$\Rightarrow \quad 11x^2 - 6x + 1 = 0$$

which is of the form $ax^2 + bx + c = 0$.

Hence, $8x^3 - x^2 = (2x - 1)^3$ is a quadratic equation.

Also, it can be easily verified that

$-3x^2 = (2 - x)\left(3x - \frac{1}{2}\right)$ is not of the form

$ax^2 + bx + c = 0$.

2. (*b*) $\mathbf{2x^2 - 5x - 3 = 0}$

The quadratic equation $ax^2 + bx + c = 0$ has a root p if $(x - p)$ is the factor of the given quadratic equation.

Factorising the given equations, we have

$$2x^2 - x - 6 = 0$$

$$\Rightarrow \quad 2x^2 - 4x + 3x - 6 = 0$$

$$\Rightarrow \quad 2x(x - 2) + 3(x - 2) = 0$$

$$\Rightarrow \quad (x - 2)(2x + 3) = 0$$

$$\Rightarrow \quad x = 2, \frac{-3}{2}$$

$$2x^2 - 5x - 3 = 0$$

$$\Rightarrow \quad 2x^2 - 6x + x - 3 = 0$$

$$\Rightarrow \quad 2x(x - 3) + 1(x - 3) = 0$$

$$\Rightarrow \quad (x - 3)(2x + 1) = 0$$

$$\Rightarrow \quad x = 3, \frac{-1}{2}$$

$$6x^2 - x - 2 = 0$$

$$\Rightarrow \quad 6x^2 - 4x + 3x - 2 = 0$$

$$\Rightarrow \quad 2x(3x - 2) + 1(3x - 2) = 0$$

$$\Rightarrow \quad (3x - 2)(2x + 1) = 0$$

$$\Rightarrow \quad x = \frac{2}{3}, \frac{-1}{2}$$

$$8x^2 - 22x - 21 = 0$$

$$\Rightarrow \quad 8x^2 - 28x + 6x - 21 = 0$$

$$\Rightarrow \quad 4x(2x - 7) + 3(2x - 7) = 0$$

$$\Rightarrow \quad (2x - 7)(4x + 3) = 0$$

$$\Rightarrow \quad x = \frac{7}{2}, \frac{-3}{4}$$

From above, we see that the equation

$2x^2 - 5x - 3 = 0$ has 3 as a root.

3. (*a*) $\boldsymbol{a + b}$

The given quadratic equation is

$$x^2 - b^2 = a(2x - a)$$

$$\Rightarrow \quad x^2 - b^2 = 2ax - a^2$$

$$\Rightarrow \quad x^2 - 2ax + a^2 - b^2 = 0$$

$$\Rightarrow \quad (x - a)^2 - b^2 = 0$$

$$\Rightarrow \quad (x - a - b)(x - a + b) = 0$$

$\Rightarrow \quad x = a + b$ or $x = a - b$

Thus, $a + b$ and $a - b$ are the solutions of the given quadratic equation.

4. (*b*) **2,** $\mathbf{\frac{-3}{2}}$

The given quadratic equation is

$$2x^2 - x - 6 = 0$$

$$\Rightarrow \quad 2x^2 - 4x + 3x - 6 = 0$$

$$\Rightarrow \quad 2x(x - 2) + 3(x - 2) = 0$$

$$\Rightarrow \quad (2x + 3)(x - 2) = 0$$

$$\Rightarrow \quad x = 2 \text{ or } x = \frac{-3}{2}$$

Hence, the roots of the given quadratic equation are $2, \frac{-3}{2}$.

5. (*b*) $\mathbf{-m, m + 3}$

The given quadratic equation is

$$x^2 - 3x - m(m + 3) = 0$$

$$\Rightarrow x^2 + mx - (m + 3)x - m(m + 3) = 0$$

$$\Rightarrow \quad x(x + m) - (m + 3)(x + m) = 0$$

$$\Rightarrow \quad (x + m)(x - (m + 3)) = 0$$

$$\Rightarrow \quad x = -m \text{ or } x = (m + 3)$$

Hence, the roots of the given quadratic equation are $-m, (m + 3)$.

6. (*c*) **0**

The given quadratic equation is

$$2x^2 + kx - 6 = 0$$

Since, it is given that 2 is one of the root of the above equation

$$\therefore \quad 2(2)^2 + k(2) - 6 = 0$$

$$\Rightarrow \quad 8 + 2k - 6 = 0$$

$$\Rightarrow \quad 2 + 2k = 0$$

$$\Rightarrow \quad k + 1 = 0$$

Hence, value of $k + 1$ is 0.

7. (*c*) **No real roots**

The given quadratic equation is $2y^2 - \sqrt{3}y + 1 = 0$.

Here, $a = 2$, $b = -\sqrt{3}$ and $c = 1$.

Now,

$$D = b^2 - 4ac$$

$$\Rightarrow \quad D = (-\sqrt{3})^2 - 4(2)(1)$$

$$= 3 - 8$$

$$= -5 < 0$$

$$\because \quad D < 0$$

$\therefore$ The given equation has no real roots.

8. (*a*) $\mathbf{x^2 + 2x - 7 = 0}$

The quadratic equation has distinct root if its discriminant is greater than zero.

Now, consider the given quadratic equations:

$$x^2 + 2x - 7 = 0$$

Here, $a = 1$, $b = 2$ and $c = -7$.

$$\therefore \quad D = b^2 - 4ac$$

$$\Rightarrow \quad D = 2^2 - 4(1)(-7)$$

$$= 4 + 28$$

$$= 32 > 0 \Rightarrow D > 0$$

$$3y^2 - 3\sqrt{3}y + \frac{9}{4} = 0$$

Here $a = 3$, $b = -3\sqrt{3}$ and $c = \frac{9}{4}$.

$$\therefore \quad D = b^2 - 4ac$$

$$\Rightarrow \quad D = (-3\sqrt{3})^2 - 4(3)\left(\frac{9}{4}\right)$$

$$= 27 - 27 = 0$$

$$\Rightarrow \quad D = 0$$

$$x^2 + 2x + 2\sqrt{3} = 0$$

Here, $a = 1$, $b = 2$ and $c = 2\sqrt{3}$.

$$\therefore \quad D = b^2 - 4ac$$

$$\Rightarrow \quad D = 2^2 - 4(1)(2\sqrt{3})$$

$$= 4 - 8\sqrt{3}$$

$$= 4 - 8(1.73)$$

$$= 4 - 13.84$$

$$= -9.84 < 0$$

$$\Rightarrow \quad D < 0$$

$$6x^2 - 3x + 1 = 0$$

Here, $a = 6$, $b = -3$ and $c = 1$

$$\therefore \quad D = b^2 - 4ac$$

$$\Rightarrow \quad D = (-3)^2 - 4(6)(1)$$

$$= 9 - 24 = -15 < 0$$

$$\Rightarrow \quad D < 0$$

Hence, from above we see that the equation $x^2 + 2x - 7 = 0$ has two distinct roots.

9. (*b*) $\mathbf{x^2 - 4x + 4\sqrt{2} = 0}$

The quadratic equation has no real roots if its discriminant is less than zero.

Now, consider the given quadratic equations:

$$x^2 - 2x - 2\sqrt{3} = 0$$

Here, $a = 1$, $b = -2$ and $c = -2\sqrt{3}$

$$\therefore \quad D = b^2 - 4ac$$

$$\Rightarrow \quad D = (-2)^2 - 4(1)(-2\sqrt{3})$$

$$= 4 + 8\sqrt{3} > 0$$

$$\Rightarrow \quad D > 0$$

$$x^2 - 4x + 4\sqrt{2} = 0$$

Here $a = 1$, $b = -4$ and $c = 4\sqrt{2}$

$$\therefore \quad D = b^2 - 4ac$$

$$\Rightarrow \quad D = (-4)^2 - 4(1)(4\sqrt{2})$$

$$= 16 - 16\sqrt{2} < 0$$

$\Rightarrow \quad D < 0$

$$3x^2 + 4\sqrt{3}x + 3 = 0$$

Here, $a = 3$, $b = 4\sqrt{3}$ and $c = 3$

$\therefore \quad D = b^2 - 4ac$

$\Rightarrow \quad D = (4\sqrt{3})^2 - 4(3)(3)$

$= 48 - 36$

$= 12 > 0$

$\Rightarrow \quad D > 0$

$$x^2 + 4x - 2\sqrt{2} = 0$$

Here, $a = 1$, $b = 4$ and $c = -2\sqrt{2}$

$\therefore \quad D = b^2 - 4ac$

$\Rightarrow \quad D = 4^2 - 4(1)(-2\sqrt{2})$

$= 16 + 8\sqrt{2} = 0$

$\Rightarrow \quad D > 0$

Hence, from above we see that the equation $x^2 - 4x + 4\sqrt{2}$ has no real roots.

10. (*d*) **No real roots**

The given equation is

$$(x^2 + 2)^2 - x^2 = 0$$

$\Rightarrow \quad x^4 + 4x^2 + 4 - x^2 = 0$

$\Rightarrow \quad x^4 + 3x^2 + 4 = 0$

Put $x^2 = y$.

Then, the quadratic equation is

$$y^2 + 3y + 4 = 0$$

Here, $a = 1$, $b = 3$, $c = 4$

$\therefore \quad D = b^2 - 4ac$

$\Rightarrow \quad D = 3^2 - 4(1)(4)$

$= 9 - 16 = -7 < 0$

$\Rightarrow \quad D < 0$

Hence, the given equation has no real roots.

11. (*b*) $\boldsymbol{k < 4}$

The quadratic equation has real and distinct roots if its discriminant is greater than zero.

Now, consider given quadratic equation:

$$x^2 + 4x + k = 0$$

Here, $a = 1$, $b = 4$ and $c = k$.

Since, it is given that the given equation has real and distinct roots.

$\therefore \quad D > 0$

$\Rightarrow \quad b^2 - 4ac > 0$

$\Rightarrow \quad (4)^2 - 4(1)(k) > 0$

$\Rightarrow \quad 16 - 4k > 0$

$\Rightarrow \quad 4 - k > 0$

$\Rightarrow \quad k > 4.$

12. (*a*) **Real and equal roots**

The given quadratic equation is

$$49x^2 + 21x + \frac{9}{4} = 0$$

Here, $a = 49$, $b = 21$ and $c = \frac{9}{4}$

$\therefore \quad D = b^2 - 4ac$

$= (21)^2 - 4(49)\left(\frac{9}{4}\right)$

$= 441 - 441 = 0$

$\Rightarrow \quad D = 0$

Hence, the given equation has real and equal roots.

13. (*d*) **16**

The quadratic equation has real roots if its discriminant is greater than or equal to zero.

Now, consider the given quadratic equations.

$$x^2 + kx + 64 = 0$$

Here, $a = 1$, $b = k$ and $c = 64$

$\therefore \quad D = b^2 - 4ac \geq 0$

$\Rightarrow \quad D = k^2 - (4)1(64) \geq 0$

$\Rightarrow \quad D = k^2 - 256 \geq 0$

$\Rightarrow \quad k^2 \geq 256$

$\Rightarrow \quad k \geq \pm 16$

$\Rightarrow \quad k \geq 16$ ($\because$ positive value of k is required) ...(1)

$$x^2 - 8x + k = 0$$

Here, $a = 1$, $b = -8$, $c = k$

$\therefore \quad D = b^2 - 4ac \geq 0$

$\Rightarrow \quad D = (-8)^2 - 4(1)(k) \geq 0$

$\Rightarrow \quad D = 64 - 4k \geq 0$

$\Rightarrow \quad 64 \geq 4k$

$\Rightarrow \quad k \leq 16$ ...(2)

From (1) and (2), we have $k = 16$.

14. (*a*) **0, 8**

The given quadratic equation is

$$2x^2 - px + p = 0$$

Here, $a = 2$, $b = -p$ and $c = p$

For equal roots, we must have

$D = 0$

$\Rightarrow \quad b^2 - 4ac = 0$

$\Rightarrow \quad (-p)^2 - 4(2)(p) = 0$

$\Rightarrow \quad p^2 - 8p = 0$

$\Rightarrow \quad p(p - 8) = 0$

$\Rightarrow \quad p = 0$ or $p - 8 = 0$

$\Rightarrow \quad p = 0, 8$

15. (*a*) $\boldsymbol{k = \pm 30}$

The given quadratic equation is

$$25x^2 - kx + 9 = 0$$

Here, $a = 25$, $b = -k$ and $c = 9$

For equal roots, we must have

$$D = 0$$
$$\Rightarrow \quad b^2 - 4ac = 0$$
$$\Rightarrow \quad (-k)^2 - 4(25)(9) = 0$$
$$\Rightarrow \quad k^2 - 900 = 0$$
$$\Rightarrow \quad k^2 = 900$$
$$\Rightarrow \quad k = \pm 30.$$

16. (*b*) $\boldsymbol{k = 4}$

The given quadratic equation is

$$x^2 - 4x + k = 0$$

Here, $a = 1$, $b = -4$ and $c = k$

For coincident roots, we must have

$$D = 0$$
$$\Rightarrow \quad b^2 - 4ac = 0$$
$$\Rightarrow \quad (-4)^2 - 4(1)(k) = 0$$
$$\Rightarrow \quad 16 - 4k = 0$$
$$\Rightarrow \quad k = 4$$

17. (*a*) $\boldsymbol{\dfrac{b^2}{4a}}$

The given quadratic equation is

$$ax^2 + bx + c = 0$$

For equal roots, we must have

$$D = 0$$
$$\Rightarrow \quad b^2 - 4ac = 0$$
$$\Rightarrow \quad 4ac = b^2$$
$$\Rightarrow \quad c = \frac{b^2}{4a}.$$

18. (*a*) **±1**

The given quadratic equation is

$$mx^2 + 2x + m = 0$$

Here, $a = m$, $b = 2$, $c = m$

For equal roots, we must have

$$D = 0$$
$$\Rightarrow \quad b^2 - 4ac = 0$$
$$\Rightarrow \quad (2)^2 - 4(m)(m) = 0$$
$$\Rightarrow \quad 4 - 4m^2 = 0$$
$$\Rightarrow \quad m^2 = 1$$
$$\Rightarrow \quad m = \pm 1$$

19. (*b*) $\boldsymbol{k = 8}$

The given quadratic equation is

$$4x^2 - 2x + (k - 4) = 0$$

Let α be one of the root of the given equation.

$\therefore \quad \dfrac{1}{\alpha}$ is the another root of the given equation.

Here, $a = 4$, $b = -2$ and $c = k - 4$

Product of two roots $= \dfrac{c}{a}$

$$\Rightarrow \quad \alpha \cdot \frac{1}{\alpha} = \frac{k-4}{4}$$
$$\Rightarrow \quad 1 = \frac{k-4}{4}$$
$$\Rightarrow \quad k - 4 = 4$$
$$\Rightarrow \quad k = 8$$

20. (*b*) $\boldsymbol{-x^2 + 3x + 3 = 0}$

Consider the given quadratic equations as follows:

$$x^2 + 3x - 5 = 0$$

Here, $a = 1$, $b = 3$ and $c = -5$

$$\text{Sum of roots} = \frac{-b}{a} = \frac{-3}{1}$$
$$\Rightarrow \quad \text{Sum of roots} = -3$$

$$-x^2 + 3x + 3 = 0$$

Here, $a = -1$, $b = 3$ and $c = 3$

$$\text{Sum of roots} = \frac{-b}{a} = \frac{-3}{-1}$$
$$\Rightarrow \quad \text{Sum of roots} = 3$$

$$\sqrt{2}x^2 - \frac{3}{\sqrt{2}}x - 1 = 0$$

Here, $a = \sqrt{2}$, $b = \dfrac{-3}{\sqrt{2}}$ and $c = -1$

$$\text{Sum of roots} = \frac{-b}{a} = \frac{3}{\sqrt{2} \times \sqrt{2}}$$
$$\Rightarrow \quad \text{Sum of roots} = \frac{3}{2}$$

$$3x^2 - 3x - 3 = 0$$

Here, $a = 3$, $b = -3$ and $c = -3$

$$\text{Sum of roots} = \frac{-b}{a} = \frac{-(-3)}{3}$$
$$\Rightarrow \quad \text{Sum of roots} = 1$$

Hence, from above we see that, equation $-x^2 + 3x + 3 = 0$ has the sum of its roots as 3.

21. (*a*) **3**

Given that, 1 is the root of the equations

$$ay^2 + ay + 3 = 0 \text{ and } y^2 + y + b = 0$$
$$\therefore \quad a(1)^2 + a(1) + 3 = 0 \text{ and } (1)^2 + (1) + b = 0$$
$$\Rightarrow \quad a + a + 3 = 0 \text{ and } 1 + 1 + b = 0$$
$$\Rightarrow \quad 2a + 3 = 0 \text{ and } 2 + b = 0$$
$$\Rightarrow \quad a = \frac{-3}{2} \text{ and } b = -2$$

Hence, $ab = \dfrac{-3}{2}(-2) = 3.$

22. (*a*) **1 : 2**

Given that $x = 1$ is a common root of $ax^2 + ax + 2 = 0$

and $\quad x^2 + x + b = 0$

$\therefore \quad a(1)^2 + a(1) + 2 = 0$

and $\quad (1)^2 + (1) + b = 0$

$\Rightarrow \quad a + a + 2 = 0$ and $1 + 1 + b = 0$

$\Rightarrow \quad 2a + 2 = 0$ and $b + 2 = 0$

$\Rightarrow \quad a = -1$ and $b = -2$

$\therefore \quad a : b = -1 : -2 = 1 : 2.$

23. (*c*) $\boldsymbol{p = -4}$

The given quadratic equation is $x^2 + px + 3 = 0$

Since 1 is the root of the given equation

$\therefore \quad (1)^2 + p(1) + 3 = 0$

$\Rightarrow \quad p + 4 = 0$

$\Rightarrow \quad p = -4$

24. (*d*) $\boldsymbol{b = 0}$

The given quadratic equation is $ax^2 + bx + c = 0$, $a \neq 0$.

Let α and –α are the roots of the equation then

$$\alpha + (-\alpha) = \frac{-b}{a}$$

$\Rightarrow \quad \alpha - \alpha = \dfrac{-b}{a}$

$\Rightarrow \quad \dfrac{-b}{a} = 0$

$\Rightarrow \quad b = 0$

Hence, for $b = 0$, the given equation has equal roots in magnitude but opposite in sign.

25. (*d*) $\boldsymbol{q = 16}$

Since 2 is the root of the quadratic equation

$$x^2 + ax + 12 = 0$$

$\therefore \quad (2)^2 + a(2) + 12 = 0$

$\Rightarrow \quad 4 + 2a + 12 = 0$

$\Rightarrow \quad 2a + 16 = 0$

$\Rightarrow \quad a = -8$

Thus, the another quadratic equation becomes

$$x^2 + (-8)x + q = 0, \text{ i.e. } x^2 - 8x + q = 0$$

Here, $a = 1$, $b = -8$ and $c = q$

Since, this equation has equal roots

$\therefore \quad D = 0$

$\Rightarrow \quad b^2 - 4ac = 0$

$\Rightarrow \quad (-8)^2 - 4(1)(q) = 0$

$\Rightarrow \quad 64 - 4q = 0$

$\Rightarrow \quad q = 16$

26. (*c*) $\boldsymbol{\dfrac{9b^2}{4a^2}}$

The given quadratic equation is

$$a^2x^2 - 3abx + 2b^2 = 0$$

$\Rightarrow \quad x^2 - \dfrac{3b}{a}x + \dfrac{2b^2}{a^2} = 0$ [Dividing the equation by a^2]

$\Rightarrow \quad x^2 - \dfrac{3}{2}\left(\dfrac{2b}{a}\right)x + \left(\dfrac{3b}{2a}\right)^2 - \left(\dfrac{3b}{2a}\right)^2 + \dfrac{2b^2}{a^2} = 0$

$\Rightarrow \quad x^2 - 2\left(\dfrac{3b}{2a}\right)x + \left(\dfrac{3b}{2a}\right)^2 = \left(\dfrac{3b}{2a}\right)^2 - \dfrac{2b^2}{a^2}$

$\Rightarrow \quad \left(x - \dfrac{3b}{2a}\right)^2 = \dfrac{9b^2}{4a^2} - \dfrac{2b^2}{a^2}$

$\Rightarrow \quad \left(x - \dfrac{3b}{2a}\right)^2 = \dfrac{b^2}{4a^2}$

Hence $\left(\dfrac{3b}{2a}\right)^2 = \dfrac{9b^2}{4a^2}$ is the required constant.

27. (*c*) **4, 5**

Then given quadratic equation is

$$px^2 + qx - 6 = 0$$

Since $x = -2$ and $x = \dfrac{3}{4}$ are the solutions of the given equation

$\therefore \quad p(-2)^2 + q(-2) - 6 = 0$

$\Rightarrow \quad 4p - 2q = 6$

$\Rightarrow \quad 2p - q = 3 \quad$ ...(1)

and $\quad p\left(\dfrac{3}{4}\right)^2 + q\left(\dfrac{3}{4}\right) - 6 = 0$

$\quad \dfrac{9}{16}p + \dfrac{3}{4}q - 6 = 0$

$\Rightarrow \quad 9p + 12q = 96$

$\Rightarrow \quad 3p + 4q = 32 \quad$ ...(2)

Solving (1) and (2), we get

$$p = 4 \text{ and } q = 5$$

For Standard Level

28. (*b*) **13 : 12**

The given quadratic equation is

$$3x^2 + 12 - 13x = 0$$

Here $a = 3$, $b = -13$ and $c = 12$

Sum of roots : Product of roots $= \dfrac{-b}{a} : \dfrac{c}{a}$

$= -\dfrac{(-13)}{3} : \dfrac{12}{3}$

$= 13 : 12$

29. (*a*) $\boldsymbol{k = \dfrac{-3}{2}}$

The given quadratic equation is $kx^2 + 6x + 4k = 0$

Here, $a = k$, $b = 6$ and $c = 4k$

Sum of roots = Product of roots

$\Rightarrow \quad \dfrac{-b}{a} = \dfrac{c}{a}$

$\Rightarrow \quad \dfrac{-6}{k} = \dfrac{4k}{k}$

$\Rightarrow \quad -6 = 4k$

$\Rightarrow \quad k = \dfrac{-3}{2}$

30. (*b*) $\mathbf{\frac{1}{3}, \frac{7}{3}}$

The given quadratic equation is

$$3x^2 - 8x - (2k + 1) = 0$$

Let α and 7α be the roots of the given equation.

Here, $a = 3$, $b = -8$ and $c = -(2k + 1)$

$$\text{Sum of roots} = \frac{-b}{a}$$

$$\Rightarrow \quad \alpha + 7\alpha = \frac{-(-8)}{3}$$

$$\Rightarrow \quad 8\alpha = \frac{8}{3}$$

$$\Rightarrow \quad \alpha = \frac{1}{3}$$

$$\Rightarrow \quad 7\alpha = \frac{7}{3}$$

Hence, the roots of the given equation are $\frac{1}{3}, \frac{7}{3}$.

31. (*b*) $\mathbf{x^2 - 2x - 1 = 0}$

Let α and β be the roots of the required quadratic equation and let $\alpha = 1+\sqrt{2}$.

Then,

$$\alpha + \beta = 2$$

$$\Rightarrow \quad 1+\sqrt{2}+\beta = 2$$

$$\Rightarrow \quad \beta = 2-1-\sqrt{2}$$

$$\Rightarrow \quad \beta = 1-\sqrt{2}$$

Also,

$$\alpha\beta = (1+\sqrt{2})(1-\sqrt{2})$$

$$= 1^2-(\sqrt{2})^2$$

$$= 1 - 2$$

$$\Rightarrow \quad \alpha\beta = -1$$

Hence, the required equation is $x^2 - (\alpha + \beta)x + \alpha\beta = 0$.

i.e., $x^2 - 2x - 1 = 0$.

32. (*b*) $\mathbf{x^2 - 8x + 13 = 0}$

Let α and β be the two roots of the quadratic equation and let $\alpha = 4+\sqrt{3}$.

Since, irrational roots occur in pair

$$\therefore \quad \beta = 4-\sqrt{3}$$

Thus,

$$\alpha + \beta = 4+\sqrt{3}+4-\sqrt{3} = 8$$

$$\alpha\beta = (4+\sqrt{3})(4-\sqrt{3}) = 16 - 3 = 13$$

Hence, the required quadratic equation is

$$x^2 - (\alpha + \beta)\, x + \alpha\beta = 0$$

i.e. $x^2 - 8x + 13 = 0$

33. (*d*) $\mathbf{ad \neq bc}$

The given quadratic equation is

$(a^2 + b^2)x^2 + 2(ac + bd)x + (c^2 + d^2) = 0$

For no real roots, we must have

$$D < 0$$

$$\Rightarrow \quad b^2 - 4ac < 0$$

$$\Rightarrow \quad [2(ac + bd)]^2 - 4(a^2 + b^2)(c^2 + d^2) < 0$$

$$\Rightarrow \quad 4(ac + bd)^2 - 4(a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2) < 0$$

$$\Rightarrow \quad 4a^2c^2 + 4b^2d^2 + 8abcd - 4a^2c^2 - 4a^2d^2 - 4b^2c^2 - 4b^2d^2 < 0$$

$$\Rightarrow \quad 8abcd - 4a^2d^2 - 4b^2c^2 < 0$$

$$\Rightarrow \quad -4(a^2d^2 + b^2c^2 - 2abcd) < 0$$

$$\Rightarrow \quad (a^2d^2 + b^2c^2 - 2abcd) > 0$$

$$\Rightarrow \quad (ad - bc)^2 > 0$$

$$\Rightarrow \quad ad - bc \neq 0$$

$$\Rightarrow \quad ad \neq bc.$$

34. (*a*) $\mathbf{k = 2, \frac{-10}{9}}$

The given quadratic equation is

$x^2 - 2x(1 + 3k) + 7(3 + 2k) = 0$

Here, $a = 1$, $b = -2(1 + 3k)$ and $c = 7(3 + 2k)$

For real and equal roots, we must have

$$D = 0$$

$$b^2 - 4ac = 0$$

$$(-2(1 + 3k)^2 - 4(1)\,[7(3 + 2k)] = 0$$

$$\Rightarrow \quad 4[(1 + 3k)^2 - (21 + 14k)] = 0$$

$$\Rightarrow \quad 1 + 9k^2 + 6k - 21 - 14k = 0$$

$$\Rightarrow \quad 9k^2 - 8k - 20 = 0$$

$$\Rightarrow \quad 9k^2 - 18k + 10k - 20 = 0$$

$$\Rightarrow \quad 9k(k - 2) + 10(k - 2) = 0$$

$$\Rightarrow \quad (k - 2)(9k + 10) = 0$$

$$\Rightarrow \quad k = 2, \frac{-10}{9}$$

35. (*c*) $\mathbf{3b^2 - 16ac}$

The given quadratic equation is $ax^2 + bx + c = 0$

Let α and 3α be the roots of the given equation.

Then, $\alpha + 3\alpha = \frac{-b}{a}$ and $\alpha \times 3\alpha = \frac{c}{a}$

$$\Rightarrow \quad 4\alpha = \frac{-b}{a} \text{ and } 3\alpha^2 = \frac{c}{a}$$

$$\Rightarrow \quad \alpha = \frac{-b}{4a} \text{ and } 3\alpha^2 = \frac{c}{a}$$

$$\Rightarrow \quad 3\times\left(\frac{-b}{4a}\right)^2 = \frac{c}{a}$$

$$\Rightarrow \quad \frac{3b^2}{16a^2} = \frac{c}{a}$$

$$\Rightarrow \quad 3b^2 = 16ac$$

36. (*b*) $\mathbf{a^2 + 2ac = b^2}$

Since sin α and cos α are the roots of the equation $ax^2 + bx + c = 0$

$$\therefore \quad \sin\alpha + \cos\alpha = \frac{-b}{a} \text{ and } \sin\alpha\cos\alpha = \frac{c}{a} \quad \ldots(1)$$

Consider, $\sin\alpha + \cos\alpha = \frac{-b}{a}$

On squaring both sides, we get

$$(\sin\alpha + \cos\alpha)^2 = \frac{b^2}{a^2}$$

$$\Rightarrow \quad \sin^2\alpha + \cos^2\alpha + 2\cos\alpha\sin\alpha = \frac{b^2}{a^2}$$

$$\Rightarrow \quad 1 + 2\left(\frac{c}{a}\right) = \frac{b^2}{a^2} \qquad \text{[Using (1) and } \because \sin^2\alpha + \cos^2\alpha = 1]$$

$$\Rightarrow \quad a^2 + 2ac = b^2$$

37. (*b*) $\boldsymbol{k = 0}$

The given quadratic equation is $4x^2 - 8kx - 9 = 0$

Let α and $-\alpha$ be the roots of the given equation.

Here, $a = 4$, $b = -8k$ and $c = -9$

Now, sum of roots $= \frac{-b}{a}$

$$\Rightarrow \quad \alpha - \alpha = \frac{-8k}{4}$$

$$\Rightarrow \quad \frac{-8k}{4} = 0$$

$$\Rightarrow \quad k = 0$$

38. (*d*) $\boldsymbol{4x^2 - 8x - 1 = 0}$

Let α and β be the required quadratic equation.

Let $\alpha = \frac{2+\sqrt{5}}{2}$ and $\beta = \frac{2-\sqrt{5}}{2}$

Then, sum of roots $= \alpha + \beta$

$$= \frac{2+\sqrt{5}}{2} + \frac{2-\sqrt{5}}{2}$$

$$= \frac{4}{2} = 2$$

Product of roots $= \alpha\beta$

$$= \left(\frac{2+\sqrt{5}}{2}\right)\left(\frac{2-\sqrt{5}}{2}\right)$$

$$= \frac{4-5}{4}$$

$$= \frac{-1}{4}$$

Hence, the required quadratic equation is

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

i.e., $x^2 - (2)x + \left(\frac{-1}{4}\right) = 0$

$$\Rightarrow \quad 4x^2 - 8x - 1 = 0$$

39. (*b*) $\boldsymbol{k = 7}$

The given quadratic equation is

$$x^2 - (k + 6)x + 2(2k - 1) = 0$$

Let α and β be the roots of the given equation.

Here, $a = 1$, $b = -(k + 6)$, $c = 2(2k - 1)$

Sum of roots $= \frac{1}{2}$ (Product of roots)

$$\Rightarrow \quad \frac{-b}{a} = \frac{1}{2}\left(\frac{c}{a}\right)$$

$$\Rightarrow \quad \frac{-[-(k+6)]}{1} = \frac{1}{2}\,\frac{[2(2k-1)]}{1}$$

$$\Rightarrow \quad k + 6 = 2k - 1$$

$$\Rightarrow \quad k = 7$$

40. (*b*) $\boldsymbol{cx^2 + bx + a = 0}$

The given quadratic equation is

$$ax^2 + bx + c = 0$$

Let α and β be the roots of the given quadratic equation.

Then, $\alpha + \beta = \frac{-b}{a}$ and $\alpha\beta = \frac{c}{a}$ ...(1)

Let $\frac{1}{\alpha}$ and $\frac{1}{\beta}$ be the roots of the required quadratic equation.

Then, sum of roots $= \frac{1}{\alpha} + \frac{1}{\beta}$

$$= \frac{\alpha+\beta}{\alpha\beta}$$

$$= \frac{-b/a}{c/a} \qquad \text{[Using (1)]}$$

$$\Rightarrow \quad \frac{1}{\alpha} + \frac{1}{\beta} = \frac{-b}{c}$$

Product of roots $= \frac{1}{\alpha} \times \frac{1}{\beta} = \frac{1}{\alpha\beta}$

$$= \frac{1}{c/a} \qquad \text{[Using (1)]}$$

$$= \frac{a}{c}$$

Hence, the required quadratic equation is

$$x^2 - \left(\frac{1}{\alpha} + \frac{1}{\beta}\right)x + \left(\frac{1}{\alpha\beta}\right) = 0,$$

i.e. $x^2 - \left(\frac{-b}{c}\right)x + \frac{a}{c} = 0$

i.e. $cx^2 + bx + a = 0$

41. (*d*) $\boldsymbol{\frac{9}{25}}$

The given quadratic equation is

$$5x^2 - 6x - 2 = 0$$

$$\Rightarrow \quad x^2 - \frac{6}{5}x - \frac{2}{5} = 0 \qquad \text{[Dividing the equation by 5]}$$

$$\Rightarrow \quad x^2 - 2 \times \left(\frac{3}{5}\right)x = \frac{2}{5}$$

$$\Rightarrow \quad x^2 - 2 \times \left(\frac{3}{5}\right)x + \left(\frac{3}{5}\right)^2 - \left(\frac{3}{5}\right)^2 = \frac{2}{5}$$

$$\Rightarrow \quad x^2 - 2\left(\frac{3}{5}\right)x + \left(\frac{3}{5}\right)^2 = \frac{9}{25} + \frac{2}{5}$$

$$\Rightarrow \quad \left(x-\frac{3}{5}\right)^2 = \frac{19}{25}$$

Hence, $\left(\frac{3}{5}\right)^2 = \frac{9}{25}$ is the required constant.

42. (*a*) $m = \frac{1}{2}, n = \frac{-3}{2}$

The given quadratic equation is

$$mx^2 + 3x + 2n = 0$$

Here, $a = m$, $b = 3$, $c = 2n$

Sum of roots = Product of roots = −6

$$\Rightarrow \quad \frac{-b}{a} = \frac{c}{a} = -6$$

$$\Rightarrow \quad \frac{-3}{m} = \frac{2n}{m} = -6$$

$$\Rightarrow \quad \frac{-3}{m} = -6$$

$$\Rightarrow \quad m = \frac{1}{2}$$

and $\quad \frac{2n}{m} = -6$

$$\Rightarrow \quad 2n = -6m$$

$$\Rightarrow \quad 2n = -6\times\frac{1}{2}$$

$$\Rightarrow \quad n = \frac{-3}{2}$$

Hence, $m = \frac{1}{2}$ and $n = \frac{-3}{2}$.

43. (*b*) **±1**

The given quadratic equation is

$$1+\frac{y^2}{13} = \sqrt{\frac{27}{169}+1}$$

$$\Rightarrow \quad 1+\frac{y^2}{13} = \sqrt{\frac{27+169}{169}}$$

$$\Rightarrow \quad 1+\frac{y^2}{13} = \sqrt{\frac{196}{169}}$$

$$\Rightarrow \quad 1+\frac{y^2}{13} = \frac{14}{13}$$

$$\Rightarrow \quad \frac{y^2}{13} = \frac{14}{13}-1 = \frac{1}{13}$$

$$\Rightarrow \quad y^2 = 1 \Rightarrow y = \pm 1$$

44. (*c*) **3**

The given expression is

$$x = \sqrt{6+\sqrt{6+\sqrt{6+\ldots}}} \qquad \ldots(1)$$

Squaring both sides, we get

$$x^2 = 6+\sqrt{6+\sqrt{6+\ldots}}$$

$$\Rightarrow \quad x^2 = 6 + x \qquad \text{[Using (1)]}$$

$$\Rightarrow \quad x^2 - x - 6 = 0$$

$$\Rightarrow \quad x^2 - 3x + 2x - 6 = 0$$

$$\Rightarrow \quad x(x-3) + 2(x-3) = 0$$

$$\Rightarrow \quad (x-3)(x+2) = 0$$

$$\Rightarrow \quad x = 3 \text{ or } x = -2 \text{ (rejected)}$$

$$\Rightarrow \quad x = 3$$

FILL IN THE BLANKS

1. positive
2. $x^2 + x - 2 = 0$
3. 1 : 1
4. 1
5. double

SHORT ANSWER QUESTIONS

For Basic and Standard Levels

1. (*i*) Consider the given equation

$$(y-\sqrt{2})^2 - 2(y+1) = 0$$

$$\Rightarrow \quad y^2 + 2 - 2\sqrt{2}y - 2y - 2 = 0$$

$$\Rightarrow \quad y^2 - (2\sqrt{2}+2)y = 0$$

Here, $a = 1$, $b = -(2\sqrt{2}+2)$, $c = 0$

Now, $\quad D = b^2 - 4ac$

$$\Rightarrow \quad D = [-(2\sqrt{2}+2)]^2 - 4(1)(0)$$

$$= (2\sqrt{2}+2)^2 > 0$$

$\because \quad D > 0$

$\therefore \quad (y-\sqrt{2})^2 - 2(y+1) = 0$ have a real root.

(*ii*) False, since the discriminant in this case is $-4ac$ which can still be non-negative if a and c are of opposite signs or if one of a or c is zero.

2.

$$\sqrt{3}x^2 + 10x - 8\sqrt{3} = 0$$

$$\Rightarrow \quad \sqrt{3}x^2 - 2x + 12x - 8\sqrt{3} = 0$$

$$\Rightarrow \quad x(\sqrt{3}x-2) + 4\sqrt{3}(\sqrt{3}x-2) = 0$$

$$\Rightarrow \quad (\sqrt{3}x-2)(x+4\sqrt{3}) = 0$$

$$\Rightarrow \quad \boldsymbol{x = \frac{2}{\sqrt{3}}} = \frac{2\sqrt{3}}{3}$$

or $\boldsymbol{x = -4\sqrt{3}}$

3. The given equation is

$$x^4 - 26x^2 + 25 = 0 \qquad \ldots(1)$$

Put $x^2 = y$, then equation (1) becomes

$$y^2 - 26y + 25 = 0$$

$$\Rightarrow \quad y^2 - y - 25y + 25 = 0$$

$$\Rightarrow \quad y(y-1) - 25(y-1) = 0$$

$$\Rightarrow \quad (y-1)(y-25) = 0$$

$$\Rightarrow \quad y = 1 \text{ or } y = 25$$

$$\Rightarrow \quad y = 1, 25$$

$$\Rightarrow \quad x^2 = 1, 25 \qquad (\because x^2 = y)$$

$$\Rightarrow \quad x = \boldsymbol{\pm 1, \pm 5.}$$

4. The given equation is

$$r^2p^2z^2 + 2rpz + 1 = 0,\ p \neq 0,\ r \neq 0$$

$$\Rightarrow \quad (rpz)^2 + 2(rpz)(1) + 1^2 = 0,\ p \neq 0,\ r \neq 0$$

$$\Rightarrow \quad (rpz + 1)^2 = 0,\ p \neq 0,\ r \neq 0$$

$$\Rightarrow \quad rpz + 1 = 0,\ p \neq 0,\ r \neq 0$$

$$\Rightarrow \quad \mathbf{z = \frac{-1}{rp}},\ p \neq 0,\ r \neq 0.$$

5. Let x be the number of days the painter takes to do the job alone.

Then, the number of days the helper takes to do the job alone is $x + 6$.

$$\text{Now, painter's one day's work} = \frac{1}{x}$$

$$\text{helper's one day's work} = \frac{1}{x+6}$$

Thus, we have

$$\frac{1}{x} + \frac{1}{x+6} = \frac{1}{4}$$

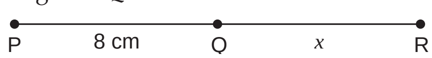
$$\Rightarrow \quad \frac{x+6+x}{x(x+6)} = \frac{1}{4}$$

$$\Rightarrow \quad 4(2x + 6) = x(x + 6)$$

$$\Rightarrow \quad 8x + 24 = x^2 + 6x$$

$$\Rightarrow \quad \mathbf{x^2 - 2x - 24 = 0}$$

6. Let the length of QR = x cm



Then $\quad PQ = 8,\ QR = x$ and $PR = 8 + x$

Now, $\quad QR^2 = PQ \times PR$

$$\Rightarrow \quad x^2 = 8(8 + x)$$

$$\Rightarrow \quad 64 + 8x = x^2$$

$$\Rightarrow \quad \mathbf{x^2 - 8x - 64 = 0}$$

7. (*i*) **True**

The given equation is

$$(x - 2)^2 + 4(x + 1) = 0$$

$$\Rightarrow \quad x^2 + 4 - 4x + 4x + 4 = 0$$

$$\Rightarrow \quad x^2 + 8 = 0$$

Here, $a = 1,\ b = 0,\ c = 8$

$$D = b^2 - 4ac$$

$$= 0^2 - 4(1)(8)$$

$$= -32 < 0$$

$$\Rightarrow \quad D < 0$$

$\Rightarrow$ The given equation has no real roots.

(*ii*) **False**

For example, $x^2 + 1 = 0$ is a quadratic equation but it has no real roots since its discreminant is $-4 < 0$.

(*iii*) **True**

$\because$ Every quadratic equation has at the most two zeroes.

$\therefore$ Every quadratic equation has at the most two roots.

8. Given that $x = 1$ is the common root of the equations

$ax^2 + a + 3 = 0$ and $x^2 + x + b = 0$

$$\therefore \quad a(1)^2 + a + 3 = 0 \text{ and } (1)^2 + (1) + b = 0$$

$$\Rightarrow \quad 2a + 3 = 0 \text{ and } b + 2 = 0$$

$$\Rightarrow \quad a = \frac{-3}{2} \text{ and } b = -2$$

$$\Rightarrow \quad ab = \frac{-3}{2} \times (-2)$$

$$\Rightarrow \quad ab = 3$$

9. Let the original price of sugar per kg be ₹ x.

$\therefore$ Mohan buy sugar for ₹ 56 = $\frac{56}{x}$ kg

Reduced price of sugar = ₹ $(x - 1)$ per kg

$\therefore$ Mohan buy sugar for ₹ 56 = $\frac{56}{x-1}$ kg

Thus, according to question,

$$\frac{56}{x-1} - \frac{56}{x} = 1$$

$$56\left(\frac{x-x+1}{x(x+1)}\right) = 1$$

$$56 = x(x - 1)$$

$$\Rightarrow \quad x^2 - x - 56 = 0$$

$$\Rightarrow \quad x^2 + 7x - 8x - 56 = 0$$

$$\Rightarrow \quad x(x + 7) - 8(x + 7) =$$

$$\Rightarrow \quad (x + 7)(x - 8) = 0$$

$$\Rightarrow \quad x = 8 \text{ or } x = -7 \text{ (rejected)}$$

Hence, original price of sugar per kg is ₹ **8.**

10. Let the first speed of the truck be x km/h

According to the given condition

$$\frac{150}{x} + \frac{200}{x+20} = 5$$

$$\Rightarrow \quad 50\left[\frac{3}{x} + \frac{4}{x+20}\right] = 5$$

$$\Rightarrow \quad 10\left[\frac{3x+60+4x}{x(x+20)}\right] = 1$$

$$\Rightarrow \quad 10(7x + 60) = x(x + 20)$$

$$\Rightarrow \quad 70x + 600 = x^2 + 20x$$

$$\Rightarrow \quad x^2 + 20x - 70x - 600 = 0$$

$$\Rightarrow \quad x^2 - 50x - 600 = 0$$

$$\Rightarrow \quad x^2 + 10x - 60x - 600 = 0$$

$$\Rightarrow \quad x(x + 10) - 60(x + 10) = 0$$

$$\Rightarrow \quad (x + 10)(x - 60) = 0$$

$$x = -10 \text{ or } x = 60$$

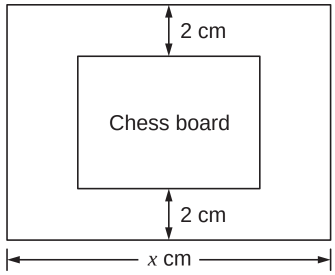
Since speed cannot be negative, hence we will reject $x = -10$.

Therefore, first speed of the truck = **60 km/h**

For Standard Level

11. Let the length of the side of the chessboard be x cm. Then,

Area of 64 squares = $(x - 4)^2$



$\therefore \quad (x-4)^2 = 64 \times 6.25$

$\Rightarrow \quad x^2 - 8x + 16 = 400$

$\Rightarrow \quad \mathbf{x^2 - 8x - 384 = 0}$

12. Let x be the number of articles a trader bought.

Since, a trader bought x articles for ₹ 1200.

$\therefore$ Price of 1 article = ₹ $\left(\frac{1200}{x}\right)$

$\because$ 10 articles were damaged.

$\therefore$ Number of remaining article = $x - 10$.

Since, the trader sold the remaining article at ₹ 2 more per article than what he paid for it.

$\therefore$ S.P. of 1 article = ₹ $\left(\frac{1200}{x}+2\right)$

Thus, S.P. of $x - 10$ articles = ₹ $\left(\frac{1200}{x}+2\right)(x-10)$

In whole transaction, he made a profit of ₹ 60.

$\therefore \quad$ S.P. – C.P. = Profit

$\Rightarrow \quad \left(\frac{1200}{x}+2\right)(x-10) - 1200 = 60$

$\Rightarrow \quad (1200 + 2x)(x-10) = 1260x$

$\Rightarrow \quad (x + 600)(x-10) = 630x$

$\Rightarrow \quad x^2 - 10x + 600x - 630x - 6000 = 0$

$\Rightarrow \quad \mathbf{x^2 - 40x - 6000 = 0}$

13. Since there are 60 minutes between 2 pm and 3 pm

$\therefore \quad 60 - x = \frac{x^2}{4} - 3$

$\Rightarrow \quad 4(60-x) = x^2 - 12$

$\Rightarrow \quad 240 - 4x = x^2 - 12$

$\Rightarrow \quad \mathbf{x^2 + 4x - 252 = 0}$

14. Let Shamali's present age be x years.

Then, Radhica's age = $(x^2 + 2)$ years.

Difference between their ages = $(x^2 + 2 - x)$ years.

When Shamali grows to her mother's present age i.e. after $(x^2 + 2 - x)$ years.

Then Shamali's age = $(x^2 + 2)$ years

Radhica's age = $(x^2 + 2 + x^2 + 2 - x)$ years

$= (2x^2 + 4 - x)$ years

Also, given that after Radhica's age will be one year less than 10 times the present age of Shamali.

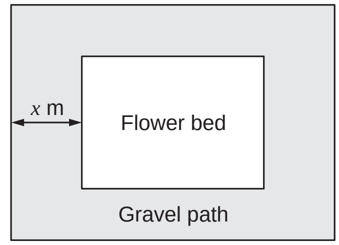
$\therefore \quad 2x^2 + 4 - x = 10x - 1$

$\Rightarrow \quad \mathbf{2x^2 - 11x + 5 = 0}$

15. (*i*) **True**, because if in $ax^2 + bx + c = 0$, a and c have same sign and $b = 0$, then $b^2 - 4ac = -4ac < 0$.

(*ii*) **True**, because if in $ax^2 + bx + c = 0$, a and c have opposite signs, then $ac < 0$ and so, $b^2 - 4ac > 0$.

16. Let the width of the gravel path be x metres. Then, each side of the square flower bed is $(44 - 2x)$ metres.



Now, Area of the square field = $44 \times 44 = 1936\ m^2$

Area of flower bed = $(44 - 2x)^2\ m^2$

$\therefore$ Area of gravel path= Area of the field – Area of the flower bed

$= 1936 - (44 - 2x)^2$

$= 1936 - (1936 - 176x + 4x^2)$

$= (176x - 4x^2)\ m^2$

Cost of laying the flower bed = (Area of flower bed) × (Rate per sq.m.)

$= (44 - 2x)^2 \times \frac{275}{100}$

$= \frac{11}{4}(44 - 2x)^2 = 11(22 - x)^2$

Cost of gravelling the path= (Area of path) × (Rate per sq m)

$= (176x - 4x^2)\frac{150}{100}$

$= 6(44x - x^2)$

Thus, according to given conditions,

$11(22 - x)^2 + 6(44x - x^2) = 4904$

$11(484 + x^2 - 44x) + 264x - 6x^2 = 4904$

$\Rightarrow \quad 5x^2 - 220x + 5324 = 4904$

$\Rightarrow \quad 5x^2 - 220x + 420 = 0$

$\Rightarrow \quad x^2 - 44x + 84 = 0$

$\Rightarrow \quad x^2 - 42x - 2x + 84 = 0$

$\Rightarrow \quad x(x - 42) - 2(x - 42) = 0$

$\Rightarrow \quad (x - 42)(x - 2) = 0$

$\Rightarrow \quad x = 42$ or $x = 2$

Since side of the square is 44 m

$\therefore \quad x \neq 42$

Hence, $\mathbf{x = 2}$ i.e. the width of the gravel path is 2 m.

VALUE-BASED QUESTIONS

For Basic and Standard Levels

1. Let the usual speed of the aeroplane be x km/h.

Increased speed of the aeroplane = $(x + 250)$ km/h.

Measurements	Aeroplane moving at usual speed	Aeroplane moving at increased speed
Distance covered	1500 km	1500 km
Speed	x km/h	$(x + 250)$ km/h
Time taken = $\frac{\text{Distance}}{\text{Speed}}$	$\frac{1500}{x}$ h	$\frac{1500}{(x+250)}$ h

Given, $\begin{bmatrix}\text{Time taken by the}\\ \text{aeroplane moving}\\ \text{at usual speed}\end{bmatrix} - \begin{bmatrix}\text{Time taken by the}\\ \text{aeroplane moving}\\ \text{at increased speed}\end{bmatrix} = \frac{1}{2}$ hour

$$\Rightarrow \quad \frac{1500}{x} - \frac{1500}{x+250} = \frac{1}{2}$$

$$\Rightarrow \quad 1500\left[\frac{x+250-x}{x(x+250)}\right] = \frac{1}{2}$$

$$\Rightarrow \quad 3000\left[\frac{250}{x(x+250)}\right] = 1$$

$$\Rightarrow \quad x^2 + 250x - 750000 = 0$$

$$\Rightarrow \quad x^2 - 750x + 1000x - 750000 = 0$$

$$\Rightarrow \quad x(x - 750) + 1000(x - 750) = 0$$

$$\Rightarrow \quad (x - 750)(x + 1000) = 0$$

Either $(x - 750) = 0$ or $(x + 1000) = 0$

$\Rightarrow \quad x = 750$ or $x = -1000$ [Rejected as speed cannot be negative]

Hence, the usual speed of the aeroplane = 750 km/h.

The values depicted are critical thinking and decision-making.

2. (*i*) Let the number of trees planted in each horizontal row = x

Then, the number of trees planted in each vertical row = $(x - 4)$

Given, total number of trees = 480

$$\therefore \quad x(x - 4) = 480$$

$$\Rightarrow \quad x^2 - 4x - 480 = 0$$

$$\Rightarrow \quad x^2 - 24x + 20x - 480 = 0$$

$$\Rightarrow \quad x(x - 24) + 20(x - 24) = 0$$

$$\Rightarrow \quad (x - 24)(x + 20) = 0$$

$\Rightarrow$ Either $(x - 24) = 0$ or $(x + 20) = 0$

$\Rightarrow \quad x = 24$ or $x = -20$ (rejected as the number of trees cannot be negative)

Hence, there are **24 trees** in each horizontal row.

(*ii*) Environmental awareness.

3. Let total number of students be x

Students visiting old age home = $\frac{3x}{8}$

Students who opted for tree plantation = $\sqrt{x}$

Students who opted for nature walk = 16

According to the given condition

$$\frac{3x}{8} = 16 + \sqrt{x}$$

$$\frac{3x}{8} - 16 = \sqrt{x}$$

On squaring both the sides, we get

$$\frac{9x^2}{64} + 256 - 12x = x$$

$$\frac{9x^2}{64} - 13x + 256 = 0$$

$$9x^2 - 832x + 16384 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{832 \pm \sqrt{692224 - 589824}}{18}$$

$$= \frac{832 \pm \sqrt{102400}}{18}$$

$$= \frac{832 \pm 320}{18}$$

$$x = \frac{512}{18}, 64$$

Since $x = \frac{512}{18}$ does not represent the number of students, hence we will reject it.

$\therefore$ Total number of students = **64**

The values indicated in students are environmental awareness, caring and concern.

UNIT TEST 1

For Basic Level

1. (*c*) $\boldsymbol{(\sqrt{3}x + \sqrt{2})^2 - x^2 = 2x^2 + 2x}$

$$3(x - 1)^2 = 4x^2 - 5x + 3$$

$$\Rightarrow \quad 3(x^2 + 1 - 2x) = 4x^2 - 5x + 3$$

$\Rightarrow \quad x^2 + x = 0,$ which is of the form $ax^2 + bx + c = 0$

$$5x - x^2 = 2x^2 + 3$$

$\Rightarrow \quad 3x^2 - 5x + 3 = 0,$ which of the form $ax^2 + bx + c = 0$

$$(\sqrt{3}x + \sqrt{2})^2 - x^2 = 2x^2 + 2x$$

$$\Rightarrow \quad 3x^2 + 2 + 2\sqrt{6}x - x^2 - 2x^2 - 2x = 0$$

$\Rightarrow \quad 2(\sqrt{6} - 1)x + 2 = 0,$ which is not of the form $ax^2 + bx + c = 0.$

$$3(x - 1)^2 = 5x^2 - 2x + 1$$

$$\Rightarrow \quad 3x^2 + 3 - 6x = 5x^2 - 2x + 1$$

$\Rightarrow \quad 2x^2 + 4x - 2 = 0,$ which is of the form $ax^2 + bx + c = 0$

Hence, from above we see that

$(\sqrt{3}x+\sqrt{2})^2 - x^2 = 2x^2 + 2x$ is not a quadratic equation.

2. (*d*) **2**

Since $\frac{1}{2}$ is the root of the equation

$$x^2 + kx - \frac{5}{4} = 0$$

$$\therefore \quad \left(\frac{1}{2}\right)^2 + k\left(\frac{1}{2}\right) - \frac{5}{4} = 0$$

$$\Rightarrow \quad \frac{1}{4} - \frac{5}{4} + \frac{k}{2} = 0$$

$$\Rightarrow \quad \frac{k}{2} - \frac{4}{4} = 0$$

$$\Rightarrow \quad k = 2$$

3. (*b*) **Real, unequal and rational**

The given quadratic equation is

$$x^2 - x - 2 = 0$$

Here, $a = 1$, $b = -1$, $c = -2$

$$\therefore \quad D = b^2 - 4ac$$

$$\Rightarrow \quad D = (-1)^2 - 4(1)(-2)$$

$$\Rightarrow \quad D = 1 + 8 = 9 > 0$$

$$\because \quad D > 0 \text{ and } \sqrt{D} = \sqrt{9} = 3$$

∴ The given equation has real, unequal and rational roots.

4. (*b*) $\boldsymbol{k \le \frac{1}{3}}$

The given equation is $3x^2 + 2x + k = 0$

For real roots $D \ge 0$

Here, $a = 3$, $b = 2$, $c = k$

$$\therefore \quad D \ge 0$$

$$\Rightarrow \quad b^2 - 4ac \ge 0$$

$$\Rightarrow \quad (2)^2 - 4(3)(k) \ge 0$$

$$\Rightarrow \quad 4 - 12k \ge 0$$

$$\Rightarrow \quad 1 - 3k \ge 0$$

$$\Rightarrow \quad k \le \frac{1}{3}$$

5. (*c*) $\boldsymbol{p = \pm 1}$

The given equation is $px^2 + 2x + p = 0$.

For equal roots $D = 0$

Here, $a = p$, $b = 2$ and $c = p$

$$\therefore \quad D = 0$$

$$\Rightarrow \quad b^2 - 4ac = 0$$

$$\Rightarrow \quad 2^2 - 4(p)(p) = 0$$

$$\Rightarrow \quad 4 - 4p^2 = 0$$

$$\Rightarrow \quad p^2 = 1$$

$$\Rightarrow \quad p = \pm 1$$

6. (*d*) **1, –6**

The given quadratic equation is

$$2x^2 + px + 4 = 0$$

Since, 2 is the root of the given equation

$$\therefore \quad 2(2)^2 + p(2) + 4 = 0$$

$$8 + 2p + 4 = 0$$

$$12 + 2p = 0$$

$$\Rightarrow \quad p = -6$$

Thus, the given equation is

$$2x^2 - 6x + 4 = 0$$

$$\Rightarrow \quad x^2 - 3x + 2 = 0$$

$$\Rightarrow \quad x^2 - 2x - x + 2 = 0$$

$$\Rightarrow \quad x(x - 2) - 1(x - 2) = 0$$

$$\Rightarrow \quad (x - 1)(x - 2) = 0$$

$$\Rightarrow \quad x = 1 \text{ or } x = 2$$

Hence, another root of the given equation is 1.

7. (*c*) $\boldsymbol{m = 1, n = -2}$

The given equation is $x^2 + mx + n = 0$.

Since m and n are the roots of the given equation,

$$\therefore \quad m + n = -m \text{ and } mn = n$$

$$mn = n$$

$$\Rightarrow \quad (mn - n) = 0$$

$$\Rightarrow \quad n(m - 1) = 0$$

$$\Rightarrow \quad n = 0 \text{ or } m - 1 = 0$$

$$\Rightarrow \quad n = 0 \text{ or } m = 1$$

If $n = 0$, then, $m + n = -m$

$$\Rightarrow \quad m + 0 = -m$$

$$\Rightarrow \quad 2m = 0$$

$$\Rightarrow \quad m = 0$$

If $m = 1$, then $m + n = -m$

$$\Rightarrow \quad 1 + n = -1$$

$$\Rightarrow \quad n = -2$$

Hence, $m = 0 = n$ or $m = 1$ and $n = -2$

8. (*b*) $\boldsymbol{x^2 - x - 12 = 0}$

Let α and β be the roots of the required quadratic equation.

Then $\alpha = 4$ and $\beta = -3$

$$\text{Sum of roots} = \alpha + \beta = 4 + (-3) = 4 - 3 = 1$$

$$\text{Product of roots} = \alpha\beta = 4(-3) = -12$$

∴ Required quadratic equation is

$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

i.e. $x^2 - x - 12 = 0$

9. (*c*) $\boldsymbol{p = \frac{-1}{2}}$

The given quadratic equation is $x^2 - x = p(2x - 1)$

$$\Rightarrow \quad x^2 - x - 2px + p = 0$$

$$\Rightarrow \quad x^2 - (1 + 2p)x + p = 0$$

Sum of roots = 0

$\Rightarrow \quad -\left[\frac{-(1+2p)}{1}\right] = 0$

$\Rightarrow \quad 1 + 2p = 0 \Rightarrow p = \frac{-1}{2}$

10. (*c*) $x^2 - 9 = 0$

Let α and β be the roots of the required quadratic equation.

Let α = 3

Given that, sum of roots = 0

$\Rightarrow \quad \alpha + \beta = 0$

$\Rightarrow \quad 3 + \beta = 0$

$\Rightarrow \quad \beta = -3$

$\Rightarrow \quad \alpha\beta = 3(-3) = -9$

Thus, the another root is –3.

Hence, the required quadratic equation is

$x^2 - (\alpha + \beta)x + \alpha\beta = 0$

i.e. $x^2 - (0)x + (-9) = 0$

i.e. $x^2 - 9 = 0$

11. (*b*) $k = \frac{2}{3}, -1$

The given quadratic equation is

$x^2 + k(4x + k - 1) + 2 = 0$

$\Rightarrow \quad x^2 + 4kx + (k^2 - k + 2) = 0$

Here, $a = 1$, $b = 4k$ and $c = k^2 - k + 2$

For real and equal roots, we must have

$D = 0$

$\Rightarrow \quad b^2 - 4ac = 0$

$\Rightarrow \quad (4k)^2 - 4(1)(k^2 - k + 2) = 0$

$\Rightarrow \quad 16k^2 - 4(k^2 - k + 2) = 0$

$\Rightarrow \quad 4k^2 - (k^2 - k + 2) = 0$

$\Rightarrow \quad 4k^2 - k^2 + k - 2 = 0$

$\Rightarrow \quad 3k^2 + k - 2 = 0$

$\Rightarrow \quad 3k^2 + 3k - 2k - 2 = 0$

$\Rightarrow \quad 3k(k + 1) - 2(k + 1) = 0$

$\Rightarrow \quad (k + 1)(3k - 2) = 0$

$\Rightarrow \quad k = -1, \frac{2}{3}$

12. (*c*) $-\frac{8}{5} < k < \frac{8}{5}$

The given equation is $x^2 + 5kx + 16 = 0$

Here, $a = 1$, $b = 5k$, $c = 16$

for no real roots, D < 0

$\Rightarrow \quad b^2 - 4ac < 0$

$\Rightarrow \quad (5k)^2 - 4(1)(16) < 0$

$\Rightarrow \quad 25k^2 - 64 < 0$

$\Rightarrow \quad k^2 < \frac{64}{25}$

$\Rightarrow \quad k < \pm\frac{8}{5}$

$\Rightarrow \quad \frac{-8}{5} < k < \frac{8}{5}$

13. (*a*) $k = 4$

Let α and β be the roots of the given quadratic equation $3x^2 + (2k + 1)x - (k + 5) = 0$

Here, $a = 3$, $b = 2k + 1$, $c = -(k + 5)$

$\therefore \quad \alpha + \beta = -\frac{2k+1}{3}$ and $\alpha\beta = -\frac{(k+5)}{3}$

According to given condition

$\alpha + \beta = \alpha\beta$

$\Rightarrow \quad -\frac{(2k+1)}{3} = -\frac{(k+5)}{3}$

$\Rightarrow \quad 2k + 1 = k + 5$

$\Rightarrow \quad k = 4$

14. Let the ten's and the unit's digit of the required number be x and y respectively.

Then the two-digit number is $10x + y$

Also, since sum of digits is 12

$\therefore \quad x + y = 12 \quad \ldots(1)$

According to given condition, $x = y^2 \quad \ldots(2)$

From (1) and (2), we have

$y^2 + y = 12$

$\Rightarrow \quad \mathbf{y^2 + y - 12 = 0}$

15. The cost of $2x$ articles = ₹ $(5x + 54)$.

$\therefore$ Cost of per article = ₹ $\left(\frac{5x+54}{2x}\right)$

Also, cost of $x + 2$ articles = ₹ $(10x - 4)$

$\therefore$ Cost of per article= ₹ $\left(\frac{10x-4}{x+2}\right)$

Thus,

$\frac{5x+54}{2x} = \frac{10x-4}{x+2}$

$\Rightarrow \quad (5x + 54)(x + 2) = (10x - 4)(2x)$

$\Rightarrow \quad 5x^2 + 10x + 54x + 108 = 20x^2 - 8x$

$\Rightarrow \quad 15x^2 - 72x - 108 = 0$

$\Rightarrow \quad \mathbf{5x^2 - 24x - 36 = 0}$

16. The given equation is

$\frac{1}{2a+b+2x} = \frac{1}{2a} + \frac{1}{b} + \frac{1}{2x}$

$\Rightarrow \quad \frac{1}{2a+b+2x} - \frac{1}{2x} = \frac{1}{2a} + \frac{1}{b}$

$\Rightarrow \quad \frac{2x-(2a+b+2x)}{2x(2a+b+2x)} = \frac{2a+b}{2ab}$

$\Rightarrow \quad \frac{-(2a+b)}{x(2a+b+2x)} = \frac{2a+b}{ab}$

$\Rightarrow \quad -ab = x(2a + b + 2x)$

$\Rightarrow \quad 2x^2 + (2a + b)x + ab = 0$

$\Rightarrow \quad 2x^2 + 2ax + bx + ab = 0$

$\Rightarrow \quad 2x(x + a) + b(x + a) = 0$

$\Rightarrow \quad (x + a)(2x + b) = 0$

$\Rightarrow \quad \boldsymbol{x = -a}$ or $\boldsymbol{x = \frac{-b}{2}}$

17. Let x be the number of students who planned a picnic.

The total budget for food was ₹ 2000.

$\therefore$ Each student paid for a picnic = ₹ $\frac{2000}{x}$.

Now, if 5 students failed to attend the picnic, then cost of food for each member is ₹ $\frac{2000}{x-5}$.

Thus, $\quad \frac{2000}{x-5} - \frac{2000}{x} = 20$

$\Rightarrow \quad \frac{100}{x-5} - \frac{100}{x} = 1$

$\Rightarrow \quad 100(x - x + 5) = x(x - 5)$

$\Rightarrow \quad 500 = x^2 - 5x$

$\Rightarrow \quad x^2 - 5x - 500 = 0$

$\Rightarrow \quad x^2 - 25x + 20x - 500 = 0$

$\Rightarrow \quad x(x - 25) + 20(x - 25) = 0$

$\Rightarrow \quad (x - 25)(x + 20) = 0$

$\Rightarrow \quad x = 25$ or $x = -20$ (rejected)

Thus, the number of students who planned the picnic was 25.

Hence, the number of students who attended the picnic is $x - 5$, i.e. $25 - 5 =$ **20.**

Also, payment by each student = ₹ $\frac{2000}{20}$ = **₹100.**

UNIT TEST 2

For Standard Level

1. (*b*) $\boldsymbol{p^2 - 2q}$

Since α and β are the roots of the equation $x^2 - px + q = 0$.

$\therefore \quad \alpha + \beta = p$ and $\alpha\beta = q$

Now,

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = (p)^2 - 2(q)$$

$\Rightarrow \quad \alpha^2 + \beta^2 = p^2 - 2q$

2. (*b*) $\boldsymbol{\frac{15}{2}, 9}$

Since $x = 2$ and $x = 3$ are the solutions of the equations $3x^2 - 2mx + 2n = 0$

$\therefore \quad 3(2)^2 - 2m(2) + 2n = 0$

$\Rightarrow \quad 12 - 4m + 2n = 0$

$\Rightarrow \quad 6 - 2m + n = 0$

$\Rightarrow \quad 2m - n = 6 \quad$ ...(1)

and $\quad 3(3)^2 - 2m(3) + 2n = 0$

$\quad 27 - 6m + 2n = 0$

$\Rightarrow \quad 6m - 2n = 27 \quad$ ...(2)

Solving (1) and (2), we get

$$m = \frac{15}{2} \text{ and } n = 9$$

3. Let the actual marks scored by Neelu be x.

Then,

According to question,

$$9(x + 10) = x^2$$

$\Rightarrow \quad \boldsymbol{x^2 - 9x - 90 = 0}$

4. We have

$$\frac{x+2+2x+2}{(x+1)(x+2)} = \frac{4}{x+4}$$

$\Rightarrow \quad \frac{3x+4}{x^2+3x+2} = \frac{4}{x+4}$

$\Rightarrow \quad (3x + 4)(x + 4) = 4x^2 + 12x + 8$

$\Rightarrow \quad 3x^2 + 12x + 4x + 16 = 4x^2 + 12x + 8$

$\Rightarrow \quad x^2 - 4x - 8 = 0$

Comparing the equation with the standard quadratic equation $ax^2 + bx + c = 0$, we get $a = 1$, $b = -4$, $c = -8$

$$\therefore \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{4 \pm \sqrt{16+32}}{2} = \frac{4 \pm \sqrt{48}}{2} = \frac{4 \pm 4\sqrt{3}}{2} = 2 \pm 2\sqrt{3} = 2\left(1 \pm \sqrt{3}\right)$$

$\therefore$ The required solutions are $\boldsymbol{2\left(1+\sqrt{3}\right)}$ and $\boldsymbol{2\left(1-\sqrt{3}\right)}$.

5. Since $x = -4$ is a root of the equation $x^2 + px - 4 = 0$,

$\therefore$ Putting $x = -4$, we get

$$16 - 4p - 4 = 0$$

$\Rightarrow \quad 4p = 12$

$\Rightarrow \quad p = 3 \quad$ ...(1)

Since the equation $x^2 + px + q = 0$ has equal roots,

$\therefore \quad$ Discriminant, D $= p^2 - 4q$

$\Rightarrow \quad 9 - 4q = 0 \quad$ [From (1)]

$\Rightarrow \quad q = \frac{9}{4}$

$\therefore$ Required values of p and q are **3** and $\boldsymbol{\frac{9}{4}}$ respectively.

6. Let the two natural numbers be x and $16 - x$.

$\therefore$ According to the problem, we have

$$\frac{1}{x} + \frac{1}{16-x} = \frac{1}{3}$$

$\Rightarrow \quad \frac{16-x+x}{16x-x^2} = \frac{1}{3}$

$\Rightarrow \qquad 16x - x^2 = 48$

$\Rightarrow \qquad x^2 - 16x + 48 = 0$

Comparing this equation with the standard quadratic equation $ax^2 + bx + c = 0$, we have

$a = 1$, $b = -16$ and $c = 48$

$$\therefore \qquad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{16 \pm \sqrt{256 - 192}}{2}$$

$$= \frac{16 \pm \sqrt{64}}{2}$$

$$= \frac{16 \pm 8}{2}$$

$$= 12, 4$$

Since 12 + 4 = 16, hence, the required natural numbers are **12** and **4**.

7. Let the digit in the unit place be x and that in ten's place be y. Then the number is $10y + x$.

$\therefore$ According to the problem, we have

$$xy = 35 \qquad \ldots(1)$$

and $\qquad 10y + x + 18 = 10x + y$

$\Rightarrow \qquad 9(x - y) - 18 = 0$

$\Rightarrow \qquad x - y - 2 = 0$

$\Rightarrow \qquad y = x - 2 \qquad \ldots(2)$

From (1) and (2), we have

$$x(x - 2) = 35$$

$\Rightarrow \qquad x^2 - 2x - 35 = 0$

Comparing this equation with the standard quadratic equation $ax^2 + bx + c = 0$, we have

$a = 1$, $b = -2$ and $c = -35$

$$\therefore \qquad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{2 \pm \sqrt{4 + 140}}{2}$$

$$= \frac{2 \pm \sqrt{144}}{2}$$

$$= \frac{2 \pm 12}{2}$$

$$= 7, -5$$

We reject $x = -5$ which is not a natural number.

$\therefore \qquad x = 7$

$\therefore$ From (2), $\qquad y = 7 - 2 = 5$

$\therefore$ The required number is **57**.

8. Let the speed of the boat in still water be x km/h.

Then speed of the boat down the river = $(x + 2)$ km/h and the speed of the boat upstream = $(x - 2)$ km/h.

$\therefore$ According to the problem, we have

$$\frac{30}{x-2} - \frac{30}{x+2} = 2$$

$$\Rightarrow \qquad \frac{30(x + 2 - x + 2)}{x^2 - 4} = 2$$

$$\Rightarrow \qquad \frac{120}{x^2 - 4} = 2$$

$\Rightarrow \qquad 2(x^2 - 4) = 120$

$\Rightarrow \qquad x^2 - 4 = 60$

$\Rightarrow \qquad x^2 = 64$

$\therefore \qquad x = \sqrt{64} = 8$

(Neglecting the negative value, since $x > 0$)

$\therefore$ Required speed of the boat in still water = **8 km/h**.

9. $\qquad x^2 + 2px + mn = 0 \qquad \ldots (1)$

Since roots of eq. (1) are real and equal therefore its D = 0

$\Rightarrow \qquad b^2 - 4ac = 0$

$\Rightarrow \qquad (2p)^2 - 4(1)(mn) = 0$

$\Rightarrow \qquad 4p^2 - 4mn = 0$

$\Rightarrow \qquad 4p^2 = 4mn$

$\Rightarrow \qquad p^2 = mn \qquad \ldots (2)$

Now, we have

$$x^2 - 2(m + n)x + (m^2 + n^2 + 2p^2) = 0 \qquad \ldots (3)$$

We will find discriminant of eq. (3)

$$D = b^2 - 4ac$$
$$= [-2(m + n)]^2 - 4(1)(m^2 + n^2 + 2p^2)$$
$$= 4(m + n)^2 - 4(m^2 + n^2 + 2p^2) \qquad \ldots (4)$$

Putting the value of p^2 from eq. (2) in eq. (4), we get

$$D = 4(m + n)^2 - 4(m^2 + n^2 + 2mn)$$
$$= 4(m + n)^2 - 4(m + n)^2$$
$$= 0$$

Since D of equation (3) is equal to zero therefore its roots are real and equal.