CHAPTER 3

Pair of Linear Equations in Two Variables

EXERCISE 3A

For Basic and Standard Levels

1. Let x be the number of questions answered correctly and y be the number of questions answered wrongly. Student answered 200 questions.

$\therefore$ $\qquad \boldsymbol{x + y = 200}$ … (1)

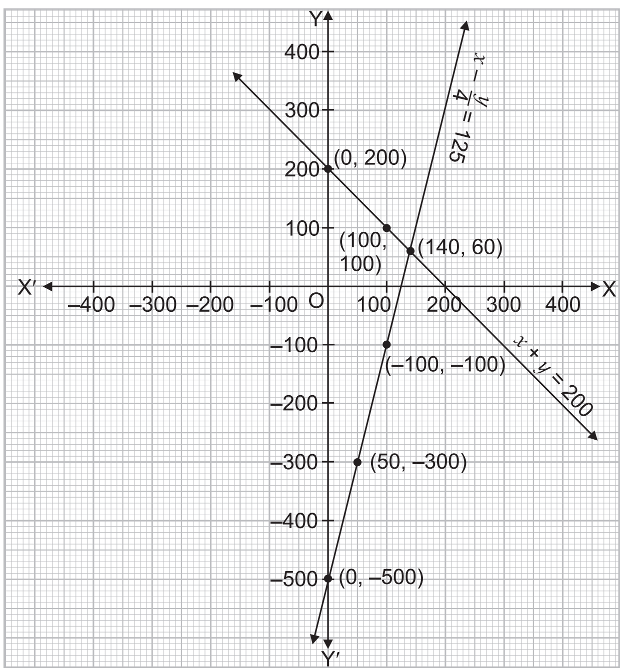
1 mark is awarded for question answered correctly and $\frac{1}{4}$ mark is deducted of every wrong answer.

Given, marks obtained by the student = 125

$\Rightarrow \qquad x \times (1) - y \times \left(\frac{1}{4}\right) = 125$

$\Rightarrow \qquad \boldsymbol{x - \frac{y}{4} = 125}$ … (2)

Draw the graph of equations (1) and (2) to represent the situation graphically.



2. Let x be the number of ₹ 2 denomination stamps and let y be the number of ₹ 5 denomination stamps.

Total number of stamps = 60

$\Rightarrow \qquad \boldsymbol{x + y = 60}$ … (1)

Purchase price of ₹ 2 denomination stamps = ₹ $2x$

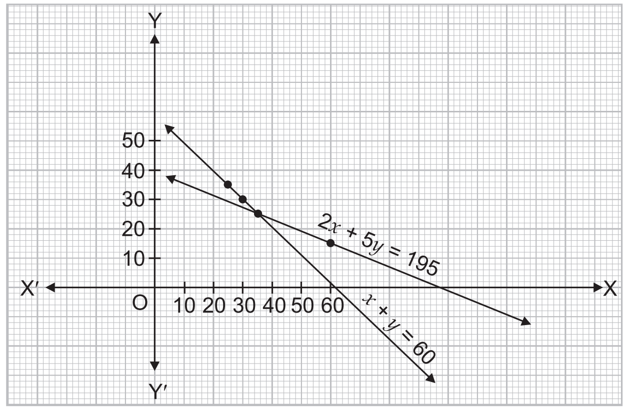
Purchase price of ₹ 5 denomination stamps = ₹ $5y$

Given, total purchase price = ₹ 195

$\Rightarrow \qquad$ ₹ $2x + 5y$ = ₹ 195

$\Rightarrow \qquad \boldsymbol{2x + 5y = 195}$ … (2)

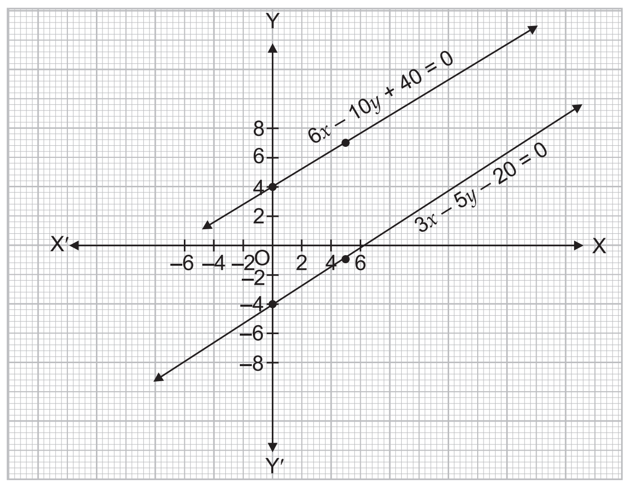
Draw the graphs of equations (1) and (2) to represent the situation graphically.



3. $\qquad 3x - 5y - 20 = 0$ … (1)

$\qquad 6x - 10y + 40 = 0$ … (2)

Draw the graphs of equations (1) and (2) to represent the situation graphically.



4. Let Nihal's present age be x years and Tanuj's present age be y years.

Then,

Nihal's age 5 years ago = $(x - 5)$ years and

Tanuj's age 5 years ago = $(y - 5)$ years and

Nihal's age 10 years later = $(x + 10)$ years and

Tanuj's age 10 years later = $(y + 10)$ years

Given, five years ago, Nihals age = Thrice Tanuj's age

$\Rightarrow \qquad (x - 5) = 3(y - 5)$

$\Rightarrow \qquad x - 5 = 3y - 15$

$\Rightarrow \qquad x - 3y - 5 + 15 = 0$

$\Rightarrow \qquad \boldsymbol{x - 3y + 10 = 0}$ … (1)

Given, ten years later, Nihal's age = Twice Tanuj's age

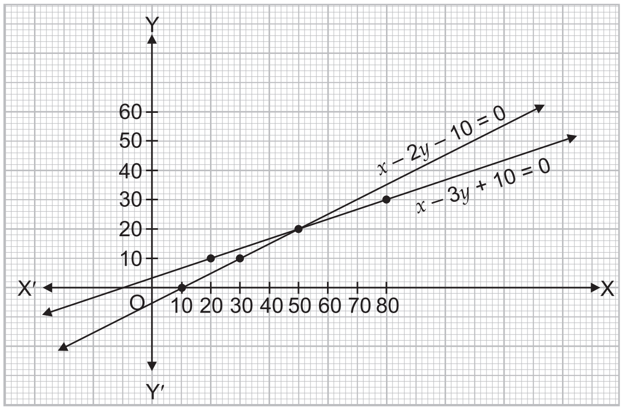
$\Rightarrow \qquad (x + 10) = 2\,(y + 10)$

$\Rightarrow \qquad x + 10 = 2y + 20$

$\Rightarrow \qquad x - 2y + 10 - 20 = 0$

$\Rightarrow \qquad \boldsymbol{x - 2y - 10 = 0}$ … (2)

Draw the graphs of equations (1) and (2) to represent the situation graphically.



5. Let ₹ x and ₹ y be the money spent by Shamali on games and food respectively.

Given, total money spent by Shamali = ₹ 45

$\Rightarrow$ ₹ x + ₹ y = ₹ 45

$\Rightarrow$ $\boldsymbol{x + y = 45}$ … (1)

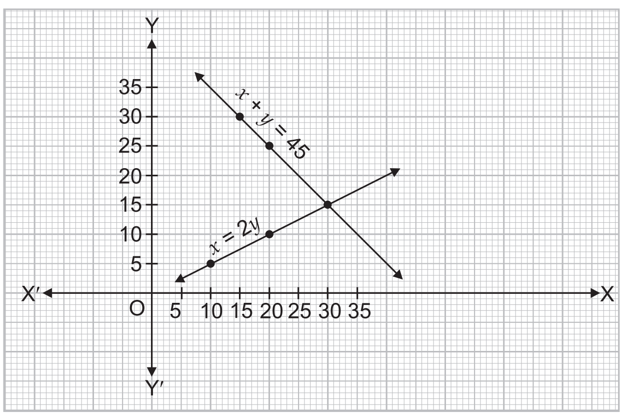
Given, money spent by Shamali on games

= Twice the money spent by her on food

$\Rightarrow$ ₹ x = ₹ $2y$

$\Rightarrow$ $\boldsymbol{x = 2y}$ … (2)

Draw the graphs of equations (1) and (2) to represent the situation graphically.



6. Let the equation $ax + by + c = 0$ have a unique solution $x = -1$ and $y = 3$.

Then $-a + 3b + c = 0$

$\Rightarrow$ $c = a - 3b$

$\therefore$ The equation $ax + by + (a - 3b) = 0$ has a unique solution $x = -1$ and $y = 3$ for any arbitrary values of a and b.

For example, taking $a = 2$ and $b = 5$ or $a = 3$ and $b = 4$, we get a pair of equation $\boldsymbol{2x + 5y - 13 = 0}$ and $3x + 4y - 9 = 0$ which have a unique solution $x = -1$ and $y = 3$. Similarly, putting any arbitrary values of a and b, we get infinitely many pairs of equations having a unique solution $x = -1$, $y = 3$.

EXERCISE 3B

For Basic and Standard Levels

1. Let the cost of each pen be ₹ x and that of each pencil be ₹ y.

Then, $3x + 2y = 34$

and $2x + 3y = 26$

Graph of $3x + 2y = 34$

$3x + 2y = 34 \quad \Rightarrow \quad y = \dfrac{34 - 3x}{2}$

$\therefore \quad x = 8 \quad \Rightarrow \quad y = \dfrac{34 - 3(8)}{2} = 5$

and $\quad x = 6 \quad \Rightarrow \quad y = \dfrac{34 - 3(6)}{2} = \dfrac{16}{2} = 8$

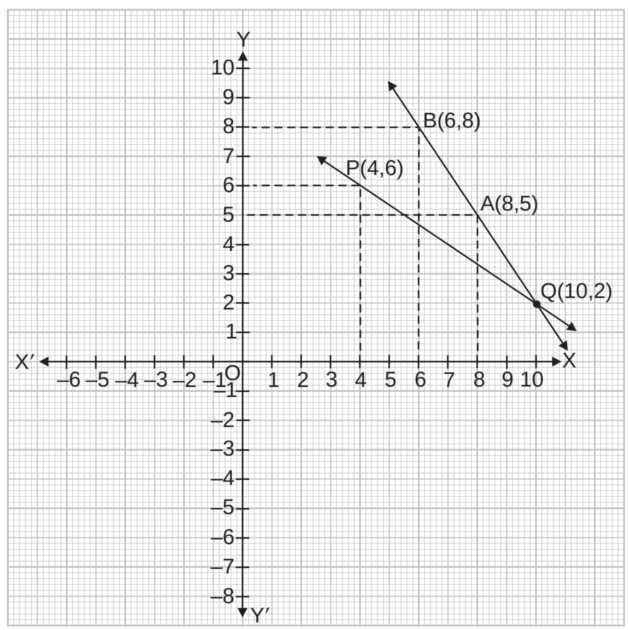
Thus, we have the following table for $3x + 2y = 34$

x	8	6
y	5	8

Plot the points A (8, 5) and B (6, 8) and draw a line passing through AB.

Then, the line AB is the graph of $3x + 2y = 34$.

Graph of $2x + 3y = 26$



$2x + 3y = 26 \quad \Rightarrow \quad y = \dfrac{26 - 2x}{3}$

$\therefore \quad x = 4 \quad \Rightarrow \quad y = \dfrac{26 - 2(4)}{3} = \dfrac{18}{3} = 6$

and $\quad x = 10 \quad \Rightarrow \quad y = \dfrac{26 - 2(10)}{3} = 2$

Thus, we have the following table for $2x + 3y = 26$

x	4	10
y	6	2

Plot the points P(4, 6) and A(10, 2) and draw a line passing through PQ.

Then, the line PQ is the graph of $2x + 3y = 26$.

The two graph lines intersect at (10, 2).

$\Rightarrow \qquad x = 10$

and $\qquad y = 2$

Hence, the cost of one pen is ₹ **10** and the cost of 1 pencil is ₹ **2**.

2. Let the greater number be x and the smaller number be y.

Then, $\qquad x + y = 6$

$\Rightarrow \qquad y = 6 - x$

Table for $x + y$

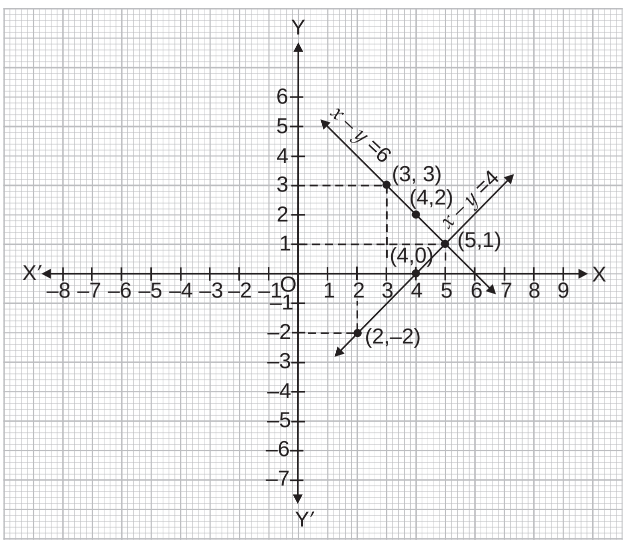
x	3	4
y	3	2

and $\qquad x - y = 4$

$\Rightarrow \qquad y = x - 4$

Table for $x - y = 4$

x	4	2
y	0	–2



The two graph lines intersect at (5, 1).

$\Rightarrow \qquad x = 5$ and $y = 1$

Hence, the numbers are **5** and **1**.

3. $\qquad 2x - 3y = 1$

$$\Rightarrow \qquad y = \frac{2x - 1}{3}$$

Table for $2x - 3y = 1$

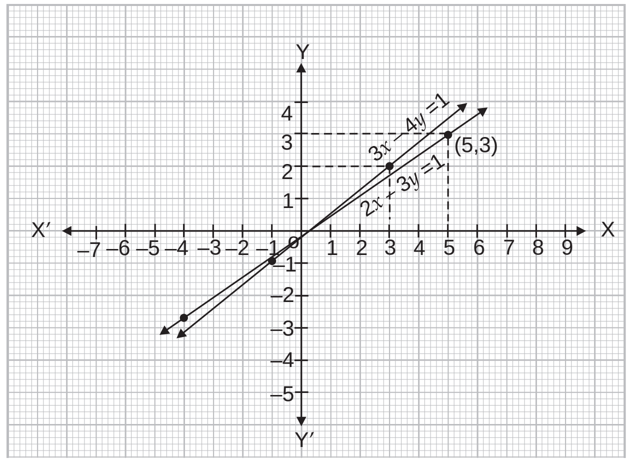
x	5	–4
y	3	–3

and $\qquad 3x - 4y = 1$

$$\Rightarrow \qquad y = \frac{3x - 1}{4}$$

Table for $3x - 4y = 1$

x	–1	3
y	–1	2



The two graph lines intersect at (– 1, –1).

Hence, $\boldsymbol{x = -1}$ **and** $\boldsymbol{y = -1}$.

4. $\qquad 2x + 3y = 8$

$$\Rightarrow \qquad y = \frac{8 - 2x}{3}$$

Table for $2x + 3y = 8$

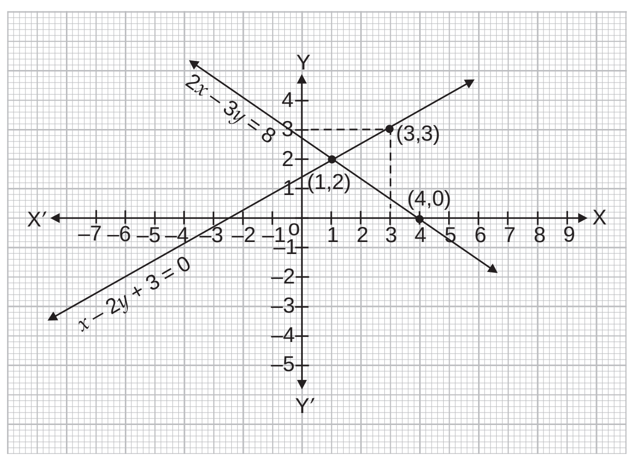
x	1	4
y	2	0

and $\qquad x - 2y + 3 = 0$

$$\Rightarrow \qquad y = \frac{x + 3}{2}$$

Table for $x - 2y + 3 = 0$

x	1	3
y	2	3



The two graph lines intersect at (1, 2).

Hence, $\boldsymbol{x = 1}$ **and** $\boldsymbol{y = 2}$.

5. $$x - y + 1 = 0$$

$\Rightarrow$ $$y = x + 1$$

Table for $x - y + 1 = 0$

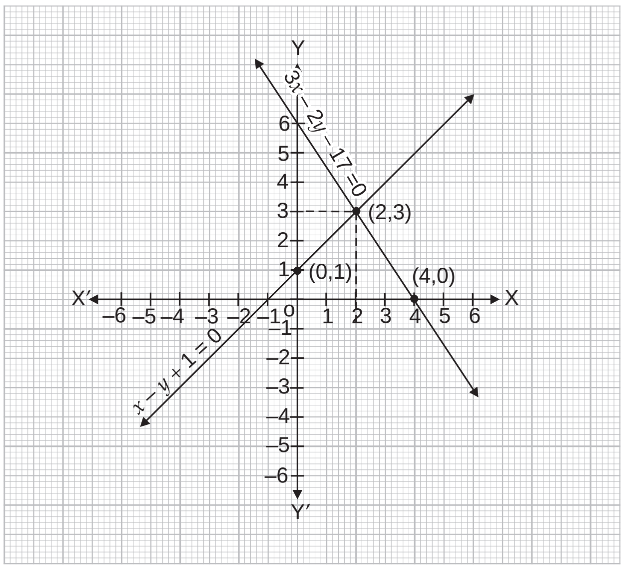
x	0	2
y	1	3

and $$3x + 2y - 12 = 0$$

$\Rightarrow$ $$y = \frac{12 - 3x}{2}$$

Table for $3x + 2y + 12 = 0$

x	2	4
y	3	0



The two graph lines intersect at (2, 3).
Hence, $\boldsymbol{x = 2}$ and $\boldsymbol{y = 3}$.

6. $$2x + 3y + 5 = 0$$

$\Rightarrow$ $$y = \frac{-2x - 5}{3}$$

Table for $2x + 3y + 5 = 0$

x	2	–1
y	–3	–1

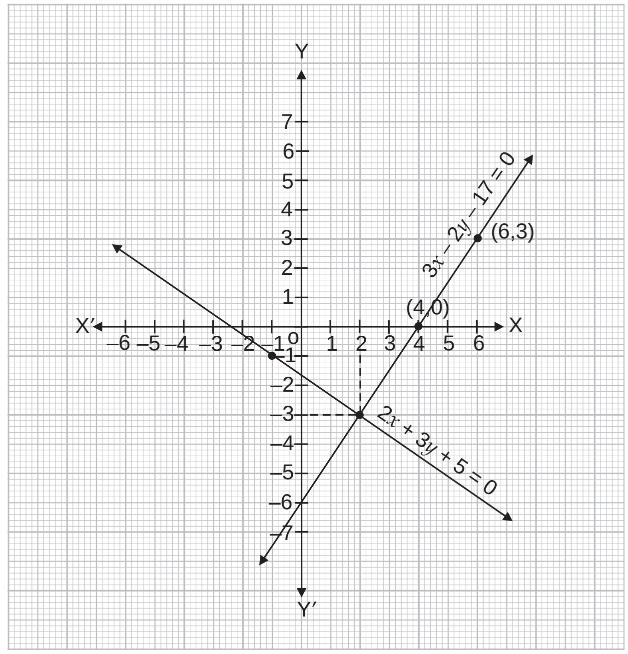
and $$3x - 2y - 12 = 0$$

$\Rightarrow$ $$y = \frac{3x - 12}{2}$$

Table for $3x - 2y - 12 = 0$

x	4	6
y	0	3

The two graph lines intersect at (2, – 3).
Hence, $\boldsymbol{x = 2}$ and $\boldsymbol{y = -3}$.



7. $$2x + 3y = 8$$

$\Rightarrow$ $$y = \frac{8 - 2x}{3}$$

Total for $2x + 3y = 8$

x	1	4
y	2	0

and $$4x + \frac{3}{2}\, y = 7$$

$\Rightarrow$ $$y = \frac{(7 - 4x) \times 2}{3} = \frac{14 - 8x}{3}$$

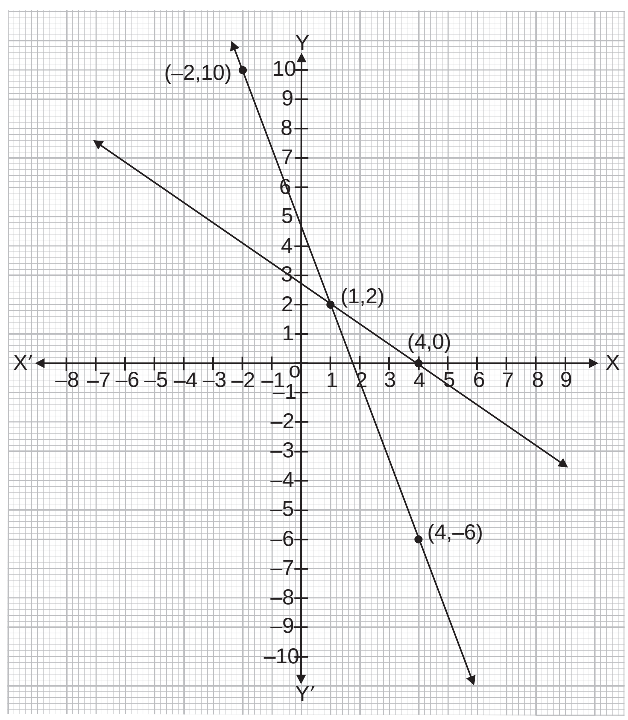


Table for $4x + \frac{3}{2}y = 7$

x	4	–2
y	–6	10

The two graph lines intersect at (1, 2).
Hence, $\boldsymbol{x = 1}$ and $\boldsymbol{y = 2}$.

8. $$\frac{x}{2} - 1 = \frac{y}{6}$$

$$\Rightarrow \quad 3x - 6 = y$$

Table for $\frac{x}{2} - 1 = \frac{y}{6}$

x	2	3
y	0	3

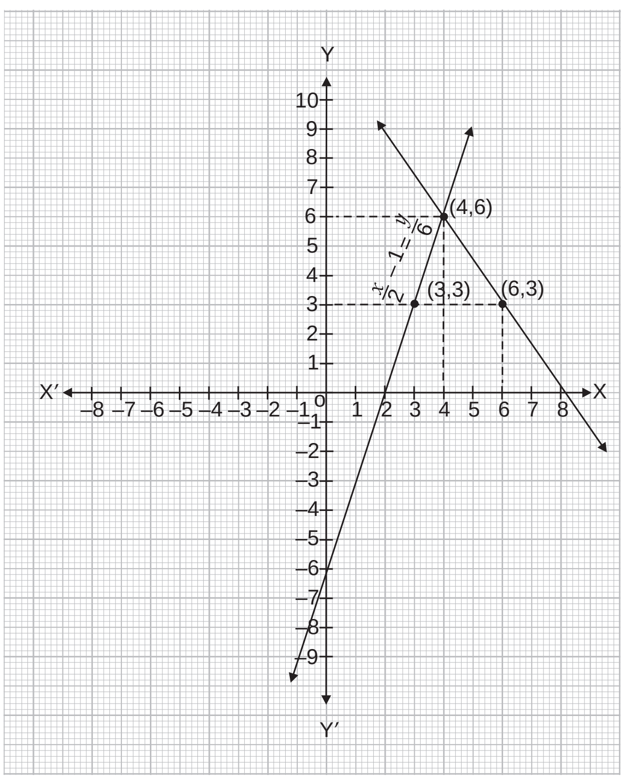
and $$\frac{x}{4} + \frac{y}{6} = 2$$

$$\Rightarrow \quad 3x + 2y = 24$$

$$\Rightarrow \quad y = \frac{24 - 3x}{2}$$

Table for $\frac{x}{4} + \frac{y}{6} = 2$

x	4	6
y	6	3



The two graph lines intersect at (4, 6).
Hence, $\boldsymbol{x = 4}$ and $\boldsymbol{y = 6}$.

9. $$3x + 2y = 8$$

$$y = \frac{8 - 3x}{2}$$

Table for $3x + 2y = 8$

x	0	2
y	4	1

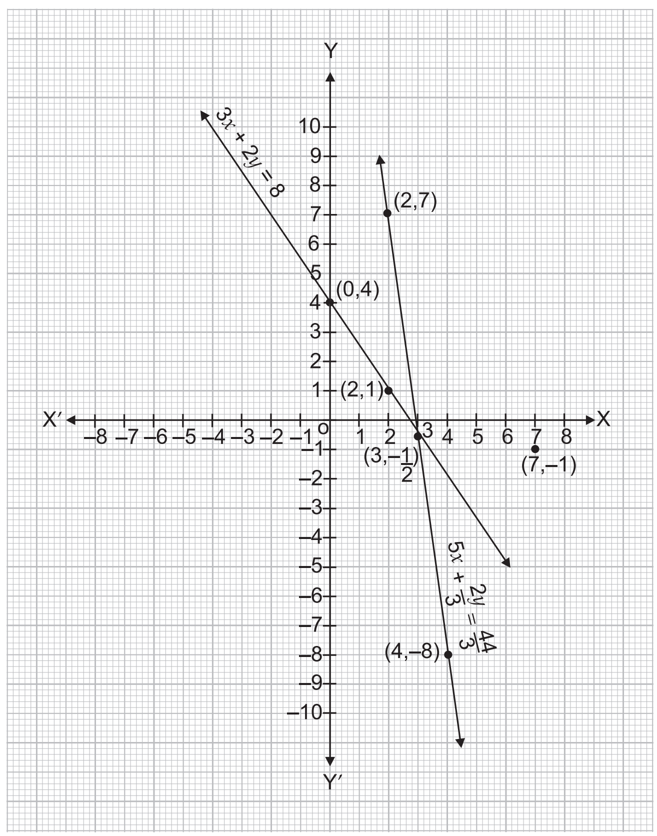
and $$5x + \frac{2y}{3} = \frac{44}{3}$$

$$\Rightarrow \quad 15x + 2y = 44$$

$$y = \frac{44 - 15x}{2}$$

Table for $5x + \frac{2y}{3} = \frac{44}{3}$

x	2	4
y	7	–8



The two graph lines intersect at $(3, -\frac{1}{2})$.

Hence, $\boldsymbol{x = 3}$ and $\boldsymbol{y = -\frac{1}{2}}$.

10. $x + 3y = 6$

$\Rightarrow \quad y = \dfrac{6 - x}{3}$

Table for $x + 3y = 6$

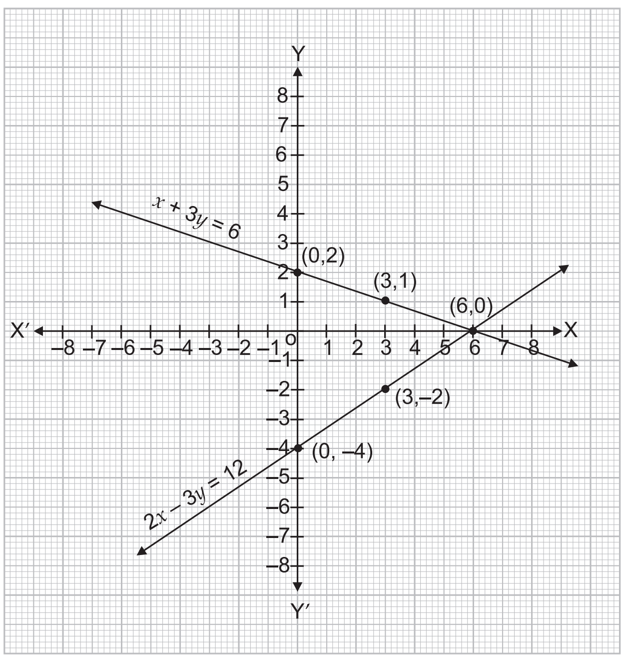
x	0	3
y	2	1

$2x - 3y = 12$

$\Rightarrow \quad y = \dfrac{2x - 12}{3}$

Table for $2x - 3y = 12$

x	3	0
y	–2	–4



The two graph lines intersect at (6, 0).
Hence, $\boldsymbol{x = 6}$ and $\boldsymbol{y = 0}$.
$a = 4x + 3y = 4 \times 6 = 3 \times 0 = \mathbf{24}$

11. $2x + y = 10$

$\Rightarrow \quad y = 10 - 2x$

Table for $2x + y = 10$

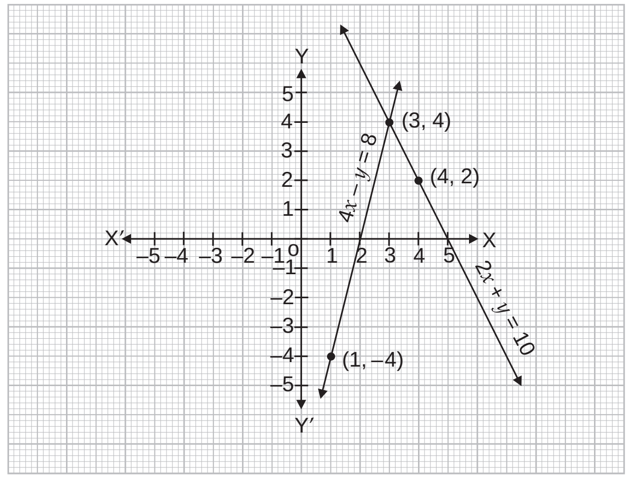
x	3	4
y	4	2

$4x - 3y = 8$

$\Rightarrow \quad y = 4x - 8$

Table for $4x - y = 8$

x	1	2
y	–4	0



The two graph lines intersect at (3, 4).
Hence, $\boldsymbol{x = 3}$ and $\boldsymbol{y = 4}$.
Yes, (1, – 4) lies on the graph of equation $4x - y = 8$.

12. $2x + 3y = 7$

$\Rightarrow \quad y = \dfrac{7 - 2x}{3}$

Table for $2x + 3y = 7$

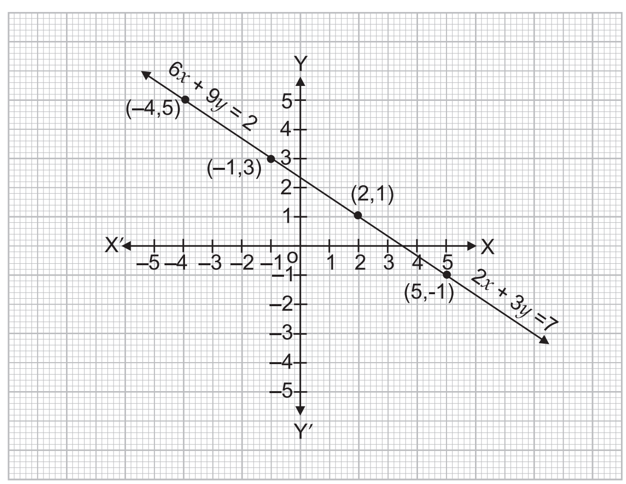
x	2	5
y	1	–1

$6x + 9y = 21$

$\Rightarrow \quad y = \dfrac{21 - 6x}{9}$

Table for $6x + 9y = 21$

x	–1	–4
y	3	5



The graphs of the two equations are consistent. So, every solution of one equation is a solution of the other and pair of linear equations is **dependent (consistent)**.

13. $2x - 3y - 2 = 0$

$$\Rightarrow \quad y = \frac{2x - 2}{3}$$

Table for $2x - 3y - 2 = 0$

x	1	4
y	0	2

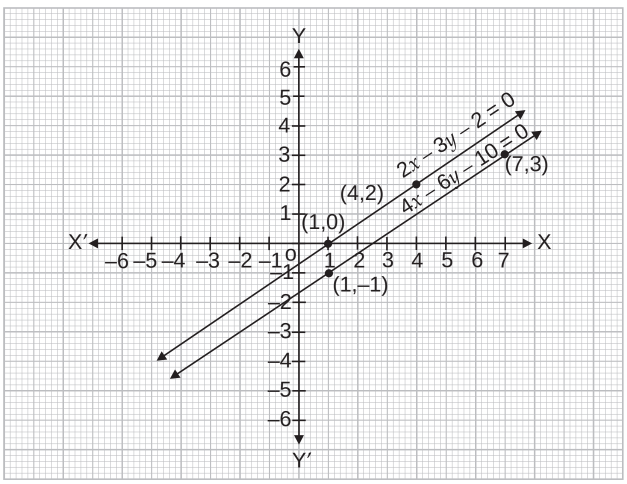
$4x - 6y - 10 = 0$

$\Rightarrow \quad 2x - 3y - 5 = 0$

$$\Rightarrow \quad y = \frac{2x - 5}{3}$$

Table for $4x - 6y - 10 = 0$

x	1	7
y	–1	3



The graph lines of the two equations are parallel. Hence, the given equations do not have a solution and are inconsistent.

14. $2x + 3y = 6$

$$\Rightarrow \quad y = \frac{6 - 2x}{3}$$

Table for $2x + 3y = 6$

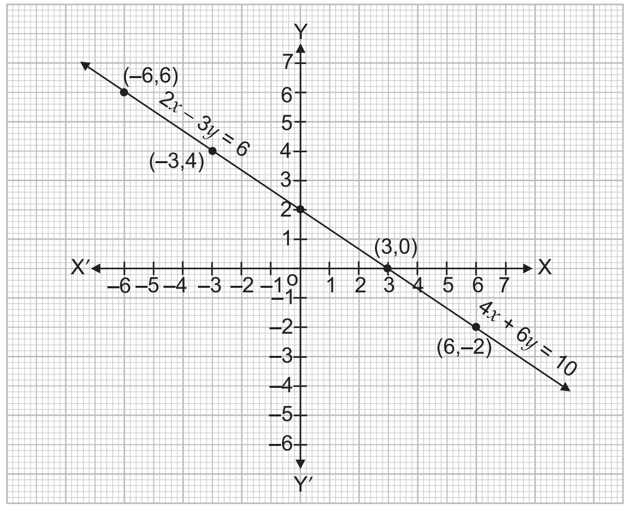
x	3	–3
y	0	4

$4x + 6y = 12$

$$\Rightarrow \quad y = \frac{12 - 4x}{6}$$

Table for $4x + 6y = 12$

x	6	–6
y	–2	6



The graph of the given equations are coincident lines. Thus, every solution of one equation is solution of the other. Hence, the given pair of linear equations has infinitely many solutions.

15. (*i*) $3x + y - 12 = 0$

$y = 12 - 3x$

Table for $3x + y - 12 = 0$

x	2	3
y	6	3

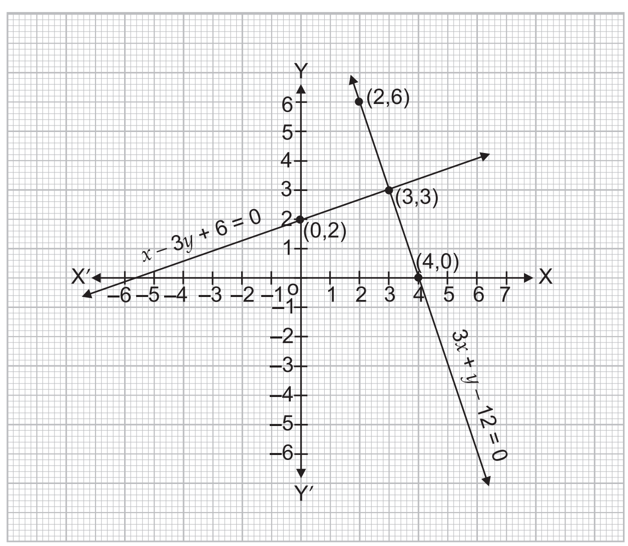
$x - 3y + 6 = 0$

$$\Rightarrow \quad y = \frac{x + 6}{3}$$

Table for $x - 3y + 6 = 0$

x	0	6
y	2	4

The two graph lines intersect at (3, 3).
Hence, $\boldsymbol{x = 3}$ and $\boldsymbol{y = 3}$.



The two graph lines intersect the x axis at **(4, 0)** and **(– 6, 0)**.

(*ii*) $x + 2y = 5$

$\Rightarrow \quad y = \dfrac{5 - x}{2}$

Table for $x + 2y = 5$

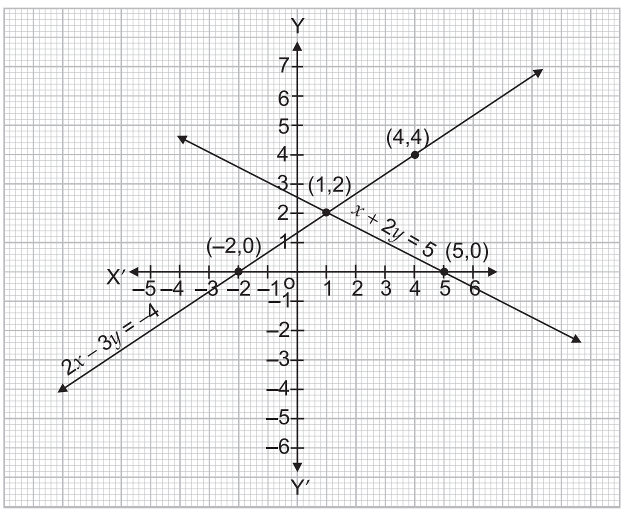
x	1	3
y	2	1

and $2x - 3y = -4$

$\Rightarrow \quad y = \dfrac{2x + 4}{3}$

Table for $2x - 3y = -4$

x	–2	4
y	0	4



The two graph lines intersect at (1, 2).
Hence, $x = 1$ and $y = 2$.
The two graph lines meet the x-axis at **(5, 0)** and **(–2, 0)**.

16. (*i*) $3x + 2y + 4 = 0$

$\Rightarrow \quad y = -\dfrac{3x - 4}{2}$

Table for $3x + 2y + 4 = 0$

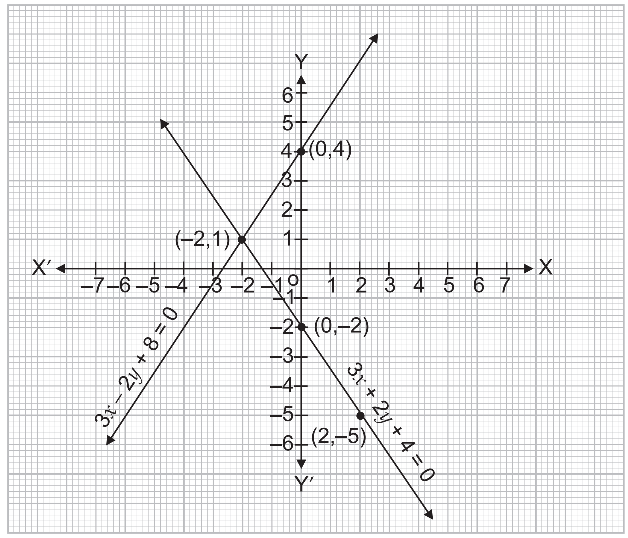
x	0	2
y	–2	–5

$3x - 2y + 8 = 0$

$\Rightarrow \quad y = \dfrac{3x + 8}{2}$

Table for $3x - 2y + 8 = 0$

x	0	–2
y	4	1



The two graph lines intersect at (–2, 1).
Hence, $x = -2$ and $y = 1$.
The two graph lines meet the y axis at **(0, –2)** and **(0, 4)**.

(*ii*) $x + 3y = 6$

$y = \dfrac{6 - x}{3}$

Table for $x + 3y = 6$

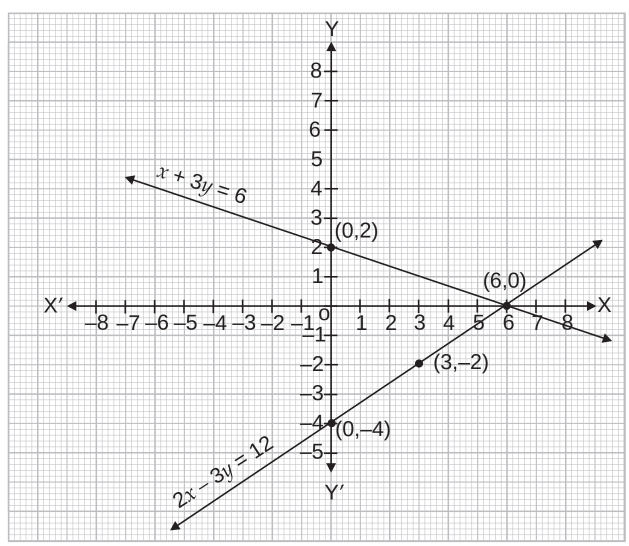
x	0	6
y	2	0

$2x - 3y = 12$

$y = \dfrac{2x - 12}{3}$

Table for $2x - 3y = 12$

x	3	0
y	–2	–4



The two graph lines intersect at (6, 0).
Hence, $x = 6$ and $y = 0$.
The two graph lines meet the y-axis at (0, 2) and (0, – 4).

17. $x + y = 7$

$\Rightarrow y = 7 - x$

Table for $x + y = 7$

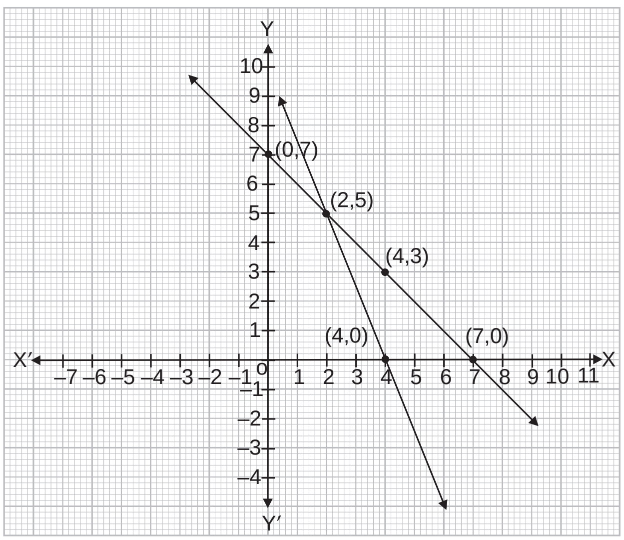
x	1	4
y	6	3

$5x + 2y = 20$

$\Rightarrow y = \frac{20 - 5x}{2}$

Table for $5x + 2y = 20$

x	2	4
y	5	0



The two graph lines intersect at (2, 5).
Hence, $x = 2$ and $y = 5$.
Graph of $x + y = 7$ intersects the x-axis at (7, 0) and y-axis at (0,7).
Graph of $5x + 2y = 20$ intersects the x-axis at (4, 0) and y-axis at (0, 10).

18. (*i*) $2y + x = 0$

$\Rightarrow y = \frac{-x}{2}$

Table for $2y + x = 0$

x	–2	2
y	1	–1

and $3y = x$

$\Rightarrow y = \frac{x}{3}$

Table for $3y = x$

x	3	6
y	1	2

and $x = 6$

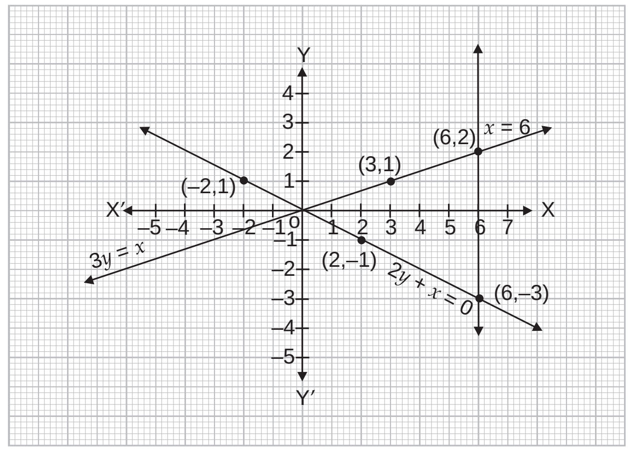


Table for $x = 6$

x	6	6
y	0	1

Vertices are **(0, 0), (6, 2)** and **(6, –3).**

(*ii*) $y = x$

Table for $y = x$

x	1	2
y	1	2

$3y = x$

$\Rightarrow y = \frac{x}{3}$

Table for $3y = x$

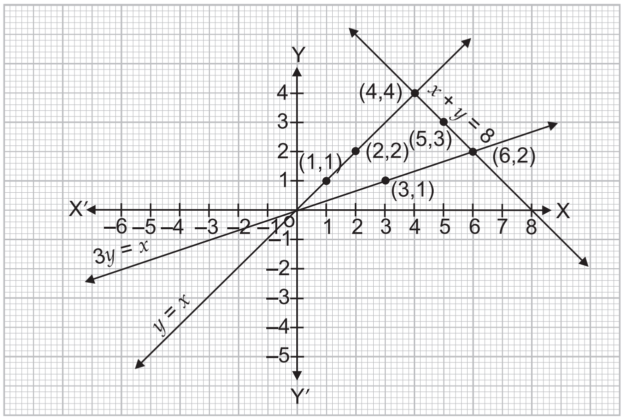
x	3	6
y	1	2

$x + y = 8$

$\Rightarrow y = 8 - x$

Table for $x + y = 8$

x	4	5
y	4	3



Vertices are **(0, 0), (4, 4)** and **(6, 2)**.

19. $4x - 5y + 16 = 0$

$\Rightarrow$ $y = \dfrac{4x + 16}{5}$

Table for $4x - 5y + 16 = 0$

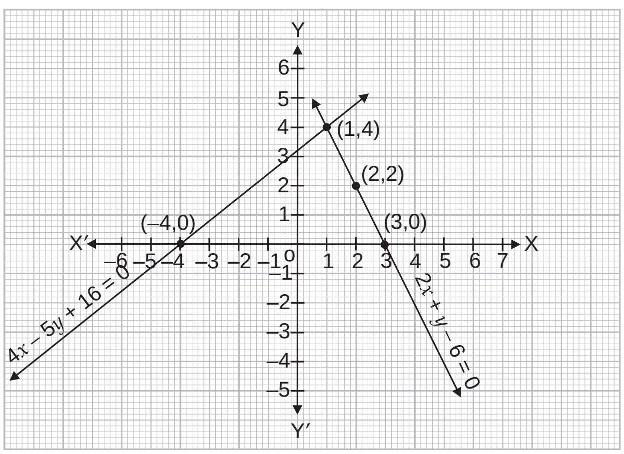
x	1	–4
y	4	0

$2x + y - 6 = 0$

$\Rightarrow$ $y = 6 - 2x$

Table for $2x + y - 6 = 0$

x	2	3
y	2	0



The two graph lines intersect at (1, 4).
Hence, $\boldsymbol{x = 1}$ and $\boldsymbol{y = 4}$.
Vertices of the required triangle are **(– 4, 0), (1, 4)** and **(3, 0)**.

20. $4x - 5y = 20$

$\Rightarrow$ $y = \dfrac{4x - 20}{5}$

Table for $4x - 5y = 20$

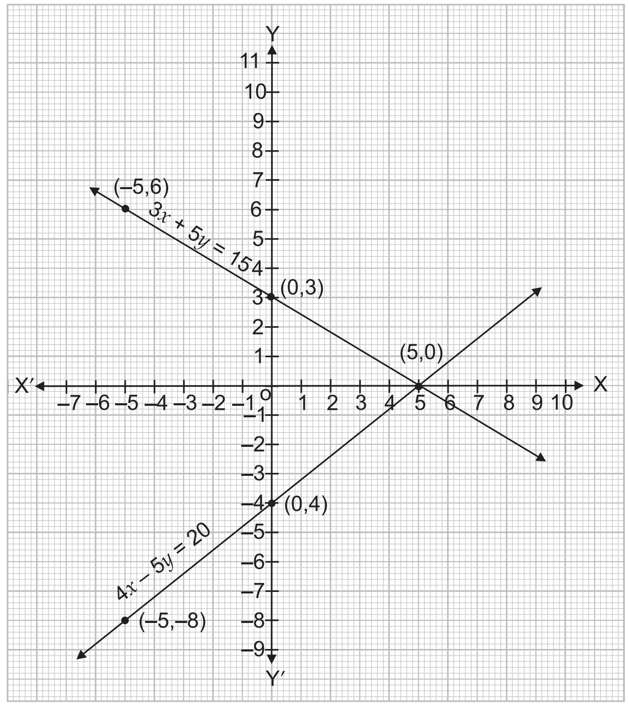
x	5	–5
y	0	–8

$3x + 5y - 15 = 0$

$\Rightarrow$ $y = \dfrac{15 - 3x}{5}$

Table for $3x + 5y = 15$

x	–5	0
y	6	3



The two graph lines intersect at (5, 0).

Hence, $\boldsymbol{x = 5}$ and $\boldsymbol{y = 0}$.

Vertices of the triangle are **(0, 3), (5,0)** and **(0, –4)**.

21. $x + y = 4$

$\Rightarrow$ $y = 4 - x$

Table for $x + y = 4$

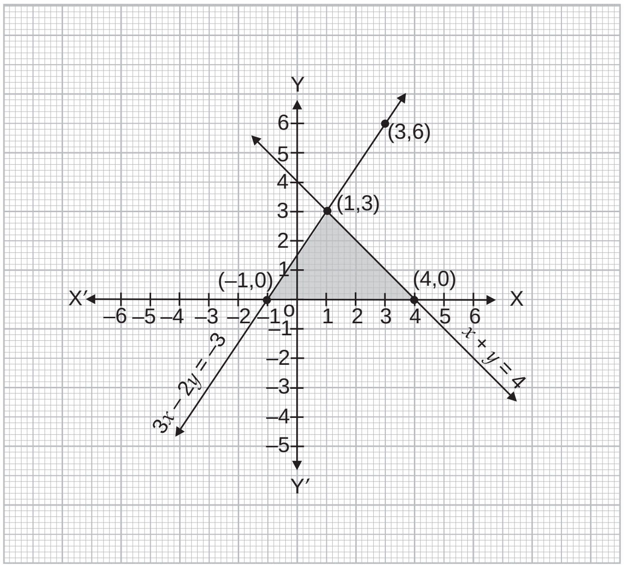
x	1	2
y	3	2

$3x - 2y = -3$

$\Rightarrow$ $y = \dfrac{3x + 3}{2}$

Table for $3x - 2y = -3$

x	3	–1
y	6	0



The two graph lines intersect at (1, 3).
Hence, $\boldsymbol{x = 1}$ and $\boldsymbol{y = 3}$.

22. $3x + y - 11 = 0$

$y = 11 - 3x$

Table for $3x + y - 11 - 0$

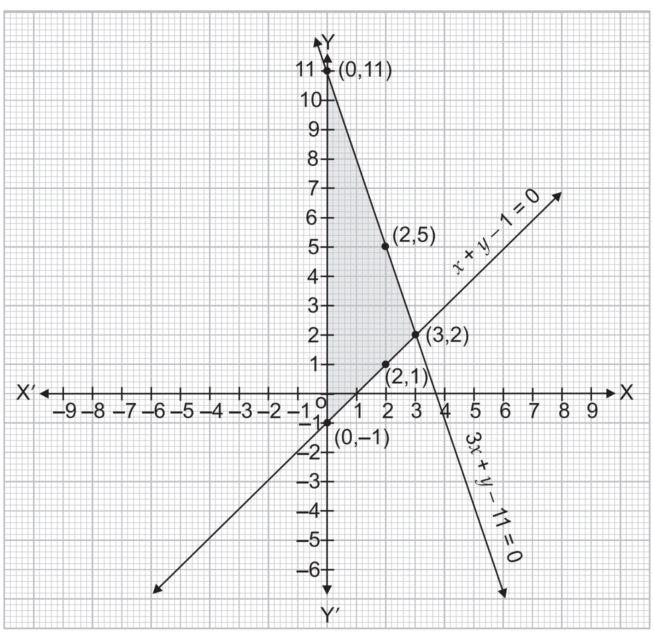
x	0	3
y	11	2

$x - y - 1 = 0$

$\Rightarrow \quad y = x - 1$

Table for $x - y - 1 = 0$

x	2	4
y	1	3



The two graph lines intersect at (3, 2).
Hence, $\boldsymbol{x = 3}$ and $\boldsymbol{y = 2}$.

23. (*i*) $4x - 3y + 4 = 0$

$\Rightarrow \quad y = \dfrac{4x + 4}{3}$

Table for $4x - 3y + 4 = 0$

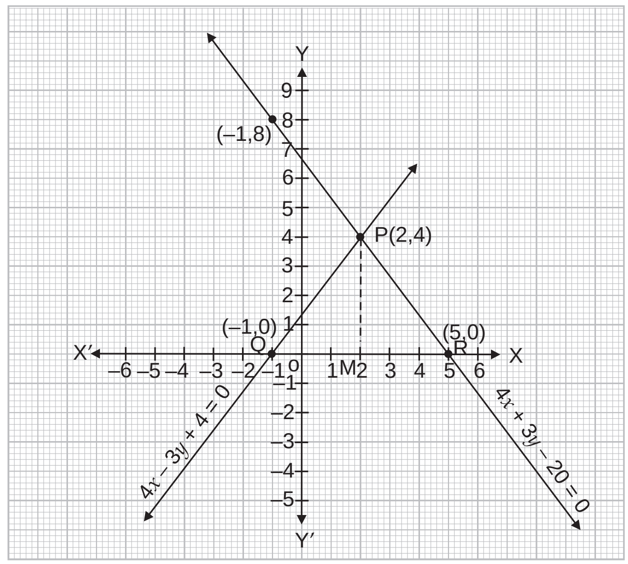
x	2	–1
y	4	0

$4x + 3y - 20 = 0$

$\Rightarrow \quad y = \dfrac{20 - 4x}{3}$

Table for $4x + 3y - 20 = 0$

x	5	–1
y	0	8



The two graph lines intersect at (2, 4).
Hence, $\boldsymbol{x = 2}$ and $\boldsymbol{y = 4}$.
ΔPQR is formed by these lines and the x-axis.
Draw, PM ⊥ x-axis.
Then, PM = y coordinate of P(2, 4) = 4

Area of ΔPQR = $\dfrac{1}{2}$ QR × PM

$= \dfrac{1}{2} \times 6 \times 4$

= 12 sq units

(*ii*) $2x + y = 6$

$\Rightarrow \quad y = 6 - 2x$

Table for $2x + y = 6$

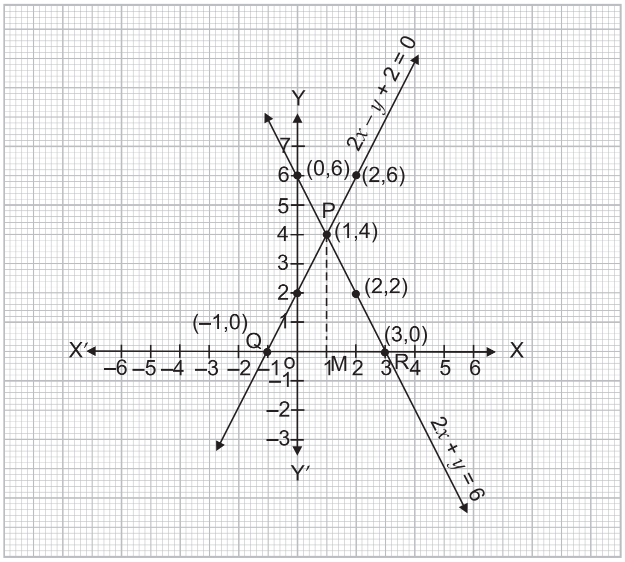
x	1	2
y	4	2

$2x - y + 2 = 0$

$\Rightarrow \quad y = 2x + 2$

Table for $2x - y + 2 = 0$

x	0	2
y	2	6



The two graph lines intersect at (1, 4).

Hence, $x = 1$ and $y = 4$.

ΔPQR is formed by these lines and the x-axis.

Draw PM ⊥ x-axis.

Then, PM = y coordinate P(1, 4) = 4

$$\text{Area of } \Delta PQR = \frac{1}{2} QR \times PM$$

$$= \frac{1}{2} \times 4 \times 4$$

$$= \mathbf{8 \text{ sq units}}$$

24. (*i*) $\quad x - y = 1$

$\Rightarrow \quad y = x - 1$

Table for $x - y = 1$

x	1	2
y	0	1

$2x + y = 8$

$\Rightarrow \quad y = 8 - 2x$

Table for $2x + y = 8$

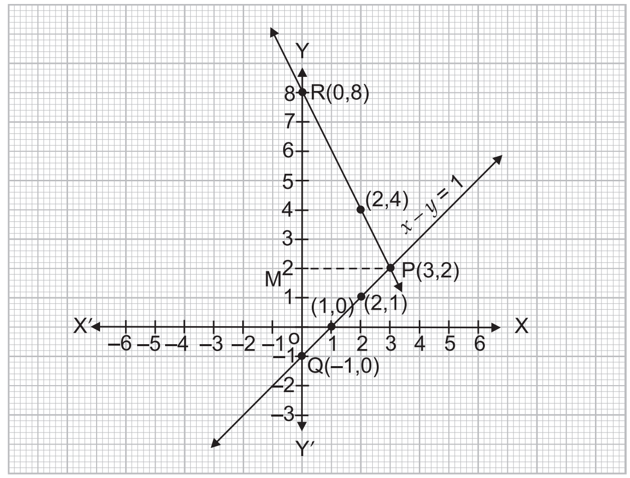
x	2	3
y	4	2

The two graph lines intersect at (3, 2).

Hence, $x = 3$ and $y = 2$.

ΔPQR is formed by these two lines and the y-axis.

Draw PM ⊥ y-axis.



Then, PM = x coordinate of P(3, 2) = 3

$$\text{Area of } \Delta PQR = \frac{1}{2} QR \times PM$$

$$= \frac{1}{2} \times 9 \times 3$$

$$= \frac{27}{2} \text{ sq units}$$

$$= \mathbf{13.5} \text{ sq units}$$

(*ii*) $\quad 2x - y - 4 = 0$

$\Rightarrow \quad y = 2x - 4$

Table for $2x - y - 4 = 0$

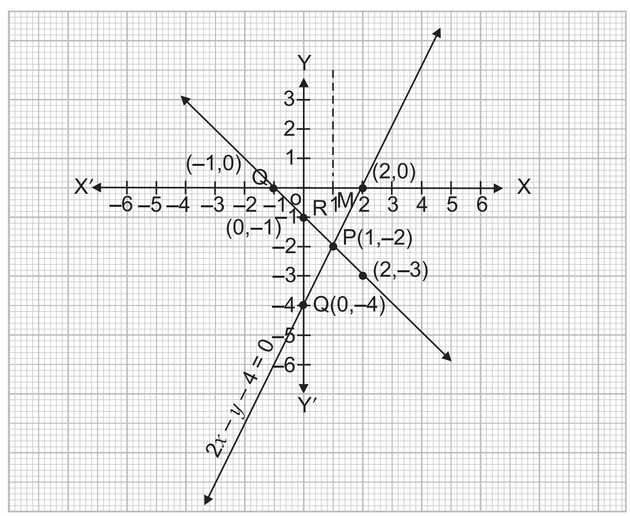
x	1	2
y	–2	0

$x + y + 1 = 0$

$y = -1 - x$

Table for $x + y + 1 = 0$

x	0	2
y	–1	–3



The two graph lines intersect at (1, –2).

Hence, $x = 1$ and $y = -2$.

ΔPQR is formed by these two lines and the y-axis.

Draw $PM \perp y$-axis.

Then, PM = x coordinate of P(1, – 2) = 1

$$\text{Area of } \Delta PQR = \frac{1}{2}\ QR \times PM$$

$$= \frac{1}{2} \times 3 \times 1 \text{ sq units}$$

$$= \mathbf{1.5 \text{ sq units}}$$

25. We have $x - y + 2 = 0$

$\Rightarrow$ $y = x + 2$ … (1)

and $4x - y - 4 = 0$

$\Rightarrow$ $y = 4x - 4$ … (2)

We now tabulate some values of x and y from (1) and (2) in Table 1 and 2 respectively as follows:

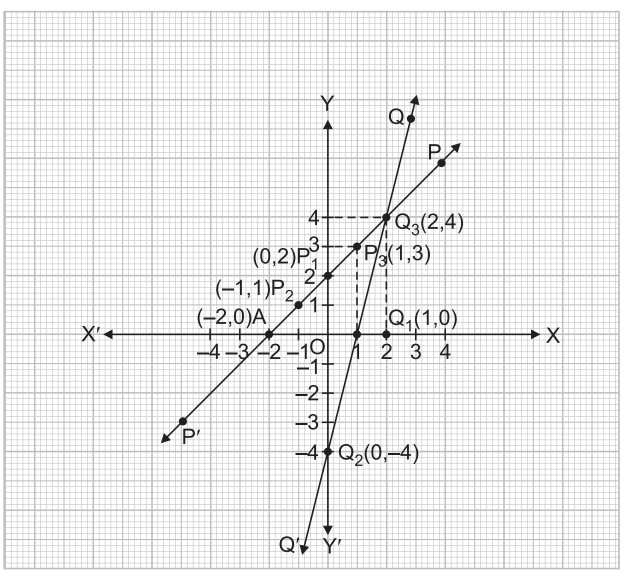
Table 1

x	0	–1	1
y	2	1	3

Table 2

x	1	0	2
y	0	–4	4

By choosing suitable scales, we now plot the points $P_1(0, 2)$, $P_2(-1, 1)$ and $P_3(1, 3)$ from Table 1 and $Q_1(1, 0)$, $Q_2(0, -4)$ and $Q_3(2, 4)$ from Table 2 in the same graph paper. We join the points P_1, P_2 and P_3 by a line PP′. Similarly, we join the points Q_1, Q_2 and Q_3 by another line QQ′. These two lines PP′ and QQ′ intersect each other at the point Q_3 (2, 4). Also, the lines PP′ and QQ′ from (1) and (2) respectively intersect the x-axis at A(–2, 0) and $Q_1(1, 0)$. Thus, we obtain a triangle Q_3AQ_1 where $AQ_1 = (1 + 2)$ units = 3 units and Q_3Q_1 = ordinate of Q_3 = 4 units.



Hence, required area of $\Delta Q_3AQ_1 = \frac{1}{2} \times AQ_1 \times Q_1Q_3$

$$= \frac{1}{2} \times 3 \times 4 \text{ sq units}$$

$$= \mathbf{6 \text{ sq units}}$$

26. We have $y = 4 - 2x$ … (1)

and $y = 2x - 4$ … (2)

We now tabulate some values of x and y from (1) and (2) in Table 1 and 2 respectively as follow:

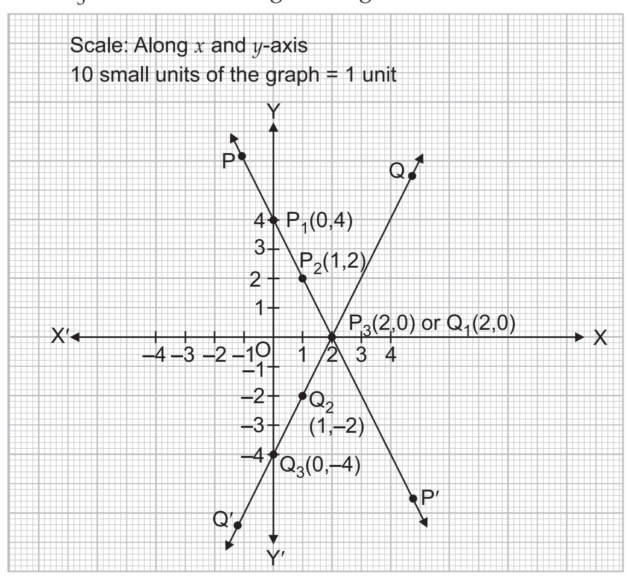
Table 1

x	0	1	2
y	4	2	0

Table 2

x	2	1	0
y	0	–2	–4

By choosing suitable scales, we now plot the points P_1 (0, 4), P_2 (1, 2) and P_3 (2, 0), from Table 1 and Q_1 (2, 0) Q_2 (1, –2) and Q_3 (0, –4) from Table 2 in the same graph paper. We now join the points P_1, P_2 and P_3 by a line PP′. Similarly, we join the points Q_1, Q_2 and Q_3 by another line QQ′. These two lines PP′ and QQ′ intersect each other at the point P_3 or Q_1(2, 0). Also, the line PP′ and QQ′ intersect the y-axis at the points P_1 and Q_3 respectively. We thus obtain the triangle $P_1Q_3Q_1$ with base P_1Q_3 = (4 + 4) units = 8 units and height OP_3 = 2 units, O being the origin.



Hence, the required vertices of the $\Delta P_1Q_3Q_1$ are P_1 (0, 4), Q_3(0, –4) and Q_1 (2, 0) and the area = $\frac{1}{2} \times P_1Q_3 \times OQ_1$

= $\frac{1}{2} \times 8 \times 2$ sq units = **8 sq units**.

For Standard Level

27. We have
$$y = x \quad \ldots (1)$$
$$y = \frac{x}{3} \quad \ldots (2)$$
and
$$y = 8 - x \quad \ldots (3)$$

We now tabulate some values of x and y from (1), (2) and (3) in Table 1, 2 and 3 respectively as follows:

Table 1

x	0	1	2
y	0	1	2

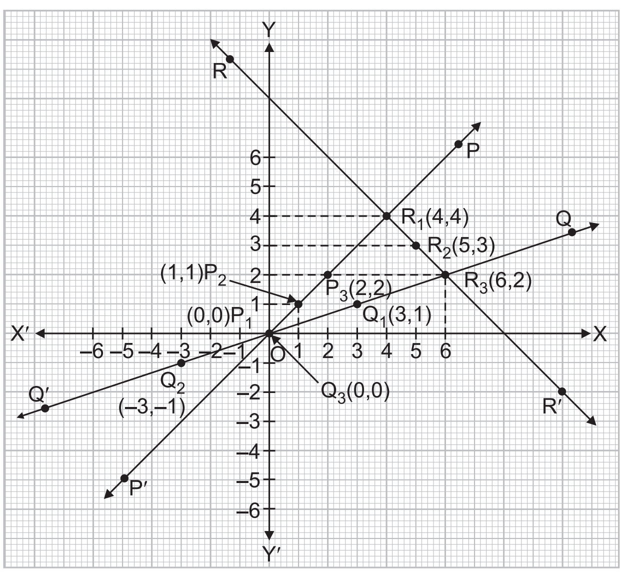
Table 2

x	3	–3	0
y	1	–1	0

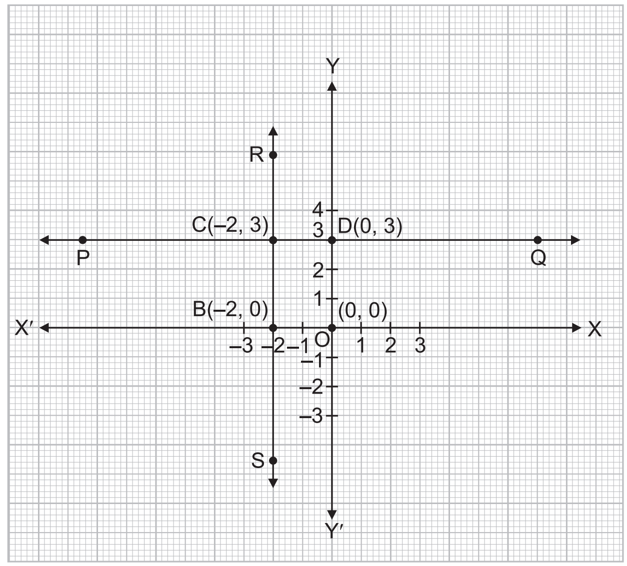
Table 3

x	4	5	6
y	4	3	2

By choosing suitable scales, we now plot the points P_1 (0, 0), P_2 (1, 1) and P_3 (2, 2) from Table 1, Q_1 (3, 1), Q_2 (–3, –1), Q_3 (0, 0) from Table 2 and R_1 (4, 4), R_2 (5, 3) and R_3 (6, 2) from Table 3 in the same graph paper. We now join the points P_1, P_2 and P_3 by a line PP′, Q_1, Q_2 and Q_3 by a line QQ′ and R_1, R_2 and R_3 by a third line RR′ forming triangle $R_1P_1R_3$ with vertices R_1(4, 4) P_1 (0, 0) and R_3 (6, 2) which are the required vertices.



28. The equation $x = -2$ implies that for all values of y-coordinates, the x-coordinate is –2. Hence, the graph of the equation $x = -2$ is a line parallel to the y-axis at a distance of 2 units on the left side of the x-axis as shown in the figure by the line RS. This line intersects the x-axis at the point B(–2, 0). Again, the equation $y = 3$ implies that for all values of x-coordinates, the y coordinate is 3. Hence, the graph of the equation $y = 3$ is a line parallel to the x-axis lying above the x-axis as shown in the figure by the line PQ. This line intersects the line RS and the y-axis at the points C(–2, 3) and D(0, 3) respectively. Thus, the x-axis, y-axis and the line PQ and RS form a rectangle OBCD with the vertices the origin O(0, 0), B(–2, 0), C(–2, 3) and D(0, 3). The area of this rectangle = OB × BC = 2 × 3 sq units = 6 sq units.



29. The equation $x = 2$ is a line parallel to the y-axis at a distance of 2 units on the right side of the x-axis. In the figure, we denote this line by RS. This line intersects the x-axis at the point B(2, 0). The equation $y = 0$ is the x-axis. The equation $y = 10$ is a line parallel to the x-axis and lies at a distance of 10 units above the x-axis. This is represented in the figure by the line PQ.

We now tabulate some values of x and y in the table below for the line:
$$y = -5x + 45 \quad \ldots (1)$$

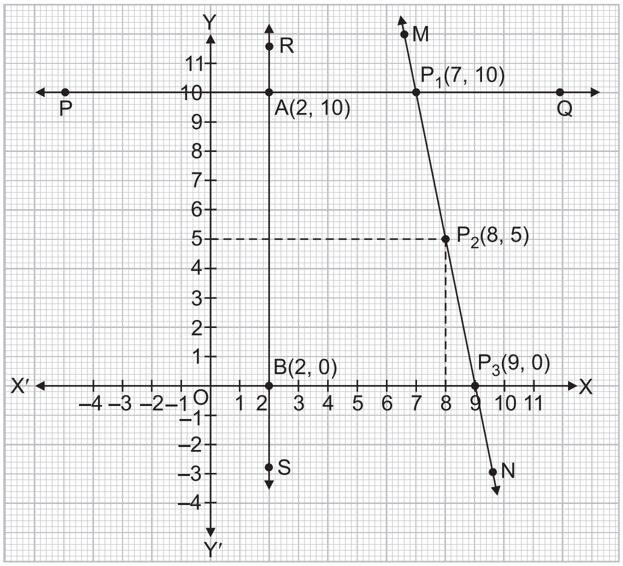
Table

x	7	8	9
y	10	5	0

By choosing suitable scales, we now plot the points P_1 (7, 10), P_2 (8, 5) and P_3 (9, 0) from the above table, on the graph paper. We join these points by a line MN. Let the lines PQ, RS, x-axis and MN intersect each other at the points A(2, 10), B(2, 0), P_3(9, 0) and P_1 (7, 10) forming a trapezium ABP_3P_1 of area
$$\frac{1}{2} \times (BP_3 + AP_1) \times AB$$
$$= \frac{1}{2} \times \{(9-2) + (7-2)\} \times 10 \text{ sq units}$$

$= \frac{1}{2} \times 12 \times 10$ sq units

= **60 sq units**



Hence, the required vertices of the trapezium ABP_3P_1 are A(2, 10), B(2, 0), P_3(9, 0) and P_1(7, 10) and the area of the trapezium is **60 sq units.**

30. The equation $x = 3$ and $x = 5$ are two lines each parallel to the y-axis at distance of 3 units and 5 units respectively on the right side of the x-axis. We now tabulate some values of x and y in the table given below for the line:

$$y = 2x - 4 \qquad \dots (1)$$

Table

x	2	1	3
y	0	–2	2

By choosing suitable scales, we now plot the points P_1(2, 0), P_2(1, –2) and P_3 (3, 2) on the graph paper. We join these points by a straight line PQ. Let the lines PQ, x-axis, $x = 3$ and $x = 5$ intersect each other at the points A(3, 0), B(5, 0) C(5, 6) and P_3(3, 2) forming a quadrilateral $ABCP_3$. This quadrilateral is clearly a trapezium.

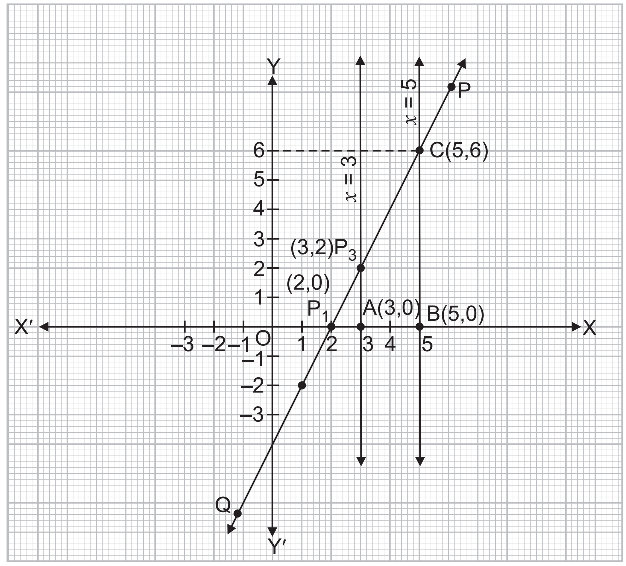
$$\text{Area of trapezium} = \frac{1}{2} \times (AP_3 + BC) \times AB$$

$$= \frac{1}{2} \times (2 + 6) \times (5 - 3) \text{ sq units}$$

$$= \frac{1}{2} \times 8 \times 2 \text{ sq units}$$

$$= 8 \text{ sq units.}$$

∴ The required area is **8 sq units**.

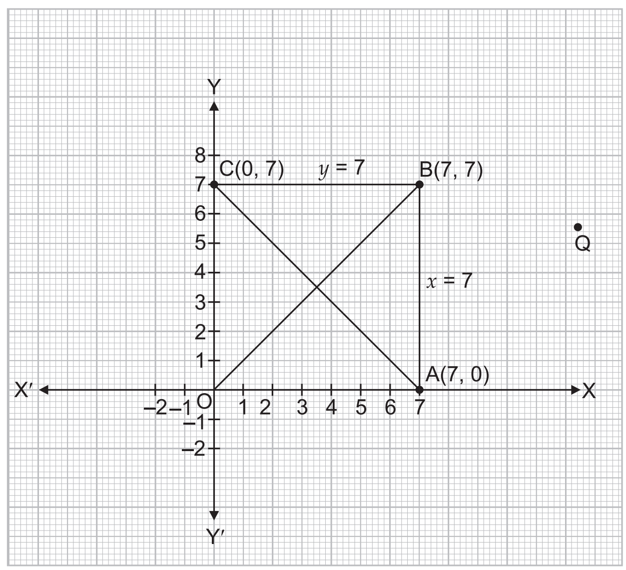


31. The equation $x = 7$ and $y = 7$ are two lines parallel to y-axis and x-axis respectively. The line $x = 7$ is at a distance of 7 units on the right side of the x-axis and the line $y = 7$ is at a distance of 7 units above the x-axis. The equation $y = 0$ is the x-axis and $x = 0$ is the y-axis. These four lines forming a quadrilateral OABC with vertices the origin O(0, 0), A(7, 0), B(7, 7), C(0, 7). Clearly, the quadrilateral is a square of side 7 units.

$$\text{The diagonal OB} = \sqrt{OA^2 + AB^2}$$

$$= \sqrt{7^2 + 7^2} \text{ units}$$

$$= 7\sqrt{2} \text{ units.}$$



∵ The figure is a square, hence the other diagonal AC is of the same length $\mathbf{7\sqrt{2}}$ **units**.

32. We have $\quad y = 6 - 2x \quad \dots (1)$

and $\quad y = 2 + 2x \quad \dots (2)$

We now tabulate some values of x and y from (1) and (2) in Tables 1 and 2 respectively as follows:

Table 1

x	2	3	4
y	2	0	–2

Table 2

x	0	1	–1
y	2	4	0

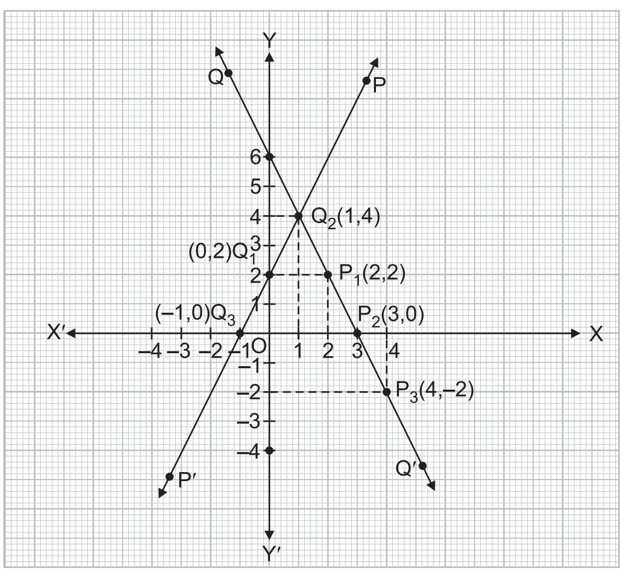
Choosing suitable scales, we now plot the points $P_1(2, 2)$, $P_2(3, 0)$ and $P_3(4, -2)$ from Table 1 and $Q_1(0, 2)$, $Q_2(1, 4)$ and $Q_3(-1, 0)$ from Table 2 on the same graph paper. We now join P_1, P_2, P_3 by a line PP′ and the points Q_1, Q_2, Q_3 by another line QQ′. These two lines intersect each other at the points A. From the graph, we see that the coordinates of A are (1, 4). Hence, the required solution of the given equation is $\boldsymbol{x = 1, y = 4}$

The lines PP′ and QQ′ intersect x-axis at the points Q_3 (–1, 0) and $P_2(3, 0)$ and form a triangle $Q_2Q_3P_2$.

$$\text{Area of triangle } Q_2Q_3P_2 = \frac{1}{2} \times Q_3P_2 \times 4$$

$$= \frac{1}{2} \times 4 \times 4 \text{ sq units}$$

$$= 8 \text{ sq units.}$$



Again the lines PP′ and QQ′ intersect y-axis at the points B_1 (0, 6), $Q_1(0, 2)$ and form a triangle $B_1Q_1Q_2$.

$$\text{Area of triangle } B_1Q_1Q_2 = \frac{1}{2} \times Q_1Q_2 \times 1$$

$$= \frac{1}{2} \times 4 \times 1 = 2 \text{ sq units}$$

∴ Required ratio of these two areas = 8 : 2 = **4 : 1**

33. We have $\quad y = \dfrac{3x - 12}{4} \quad \dots (1)$

and $\quad y = \dfrac{3x - 24}{4} \quad \dots (2)$

We tabulate some values of x and y from (1) and (2) in Table 1 and 2 respectively as follows:

Table 1

x	0	4
y	–3	0

Table 2

x	8	0
y	0	–6

Choosing suitable scale, we now plot the points $P_1(0, -3)$ and $P_2(4, 0)$ from Table 1 and $Q_1(8, 0)$ and $Q_2(0, -6)$ from Table 2 and join then by the lines PP′ and QQ′ respectively. The two lines PP′ and QQ′ form two triangles OP_1P_2 and OQ_1Q_2 with coordinate axes, O being the origin, where $P_1(0, -3)$, $P_2(4, 0)$ and O(0, 0) and $Q_1(8, 0)$, $Q_2(0, -6)$.

$$\text{Area of } \Delta OP_1P_2 = \frac{1}{2} \times OP_1 \times OP_2$$

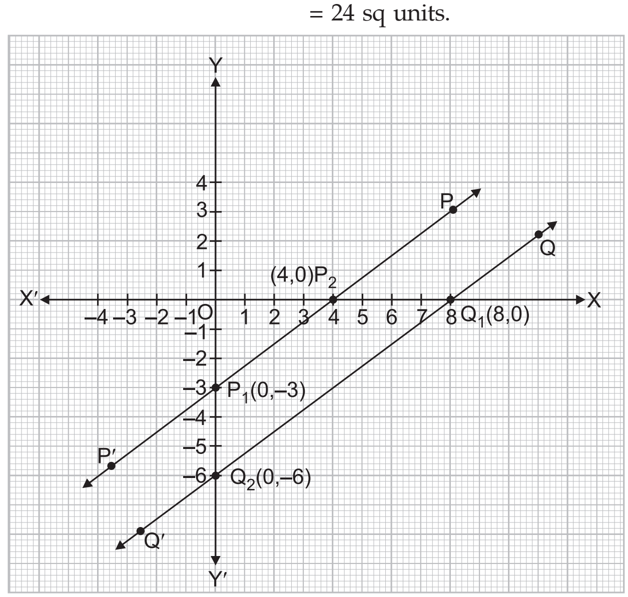
$$= \frac{1}{2} \times 3 \times 4 \text{ sq units}$$

$$= 6 \text{ sq units}$$

$$\text{Area of } \Delta OQ_1Q_2 = \frac{1}{2} \times OQ_1 \times OQ_2$$

$$= \frac{1}{2} \times 8 \times 6 \text{ sq units}$$

$$= 24 \text{ sq units.}$$



∴ The required ratio of the two triangles = 6 : 24
= **1 : 4**.

∵ ΔOP_1P_2 and ΔOQ_1Q_2 are two right-angled triangles, hence, P_1P_2 and Q_1Q_2 are their hypotenuses of length

$$P_1P_2 = \sqrt{4^2 + 3^2} \text{ units} = 5 \text{ units}$$

and $$Q_1Q_2 = \sqrt{8^2 + 6^2} \text{ units} = 10 \text{ units}$$

$\therefore$ Required ratio of the hypotenuses = 5 : 10 = **1 : 2**.

EXERICISE 3C

For Basic and Standard Levels

1. $-x + 3y = 8$

$\Rightarrow \quad x = 3y - 8 \quad \ldots (1)$

Substituting $x = 3y - 8$ in $4x + 7y = 25$, we get

$4(3y - 8) + 7y = 25$

$\Rightarrow \quad 12y - 32 + 7y = 25$

$\Rightarrow \quad 19y = 25 + 32 = 57$

$\Rightarrow \quad y = \frac{57}{19} = 3$

Substituting $y = 3$ in equation (1), we get

$x = 3(3) - 8 = 9 - 8 = 1$

Hence, $\boldsymbol{x = 1}$ and $\boldsymbol{y = 3}$.

2. $4x - 7y = 23$

$\Rightarrow \quad x = \frac{23 + 7y}{4} \quad \ldots (1)$

Substituting $x = \frac{23 + 7y}{4}$ in $5x + 2y = -25$, we get

$5\left(\frac{23 + 7y}{4}\right) + 2y = -25$

$\Rightarrow \quad 115 + 35y + 8y = -100$

$\Rightarrow \quad 43y = -100 - 115$

$\Rightarrow \quad 43y = -215$

$\Rightarrow \quad y = -5$

Substituting $y = -5$ in equation (1), we get

$x = \frac{23 + (7)(-5)}{4}$

$= \frac{23 - 35}{4} = -3$

Hence, $\boldsymbol{x = -3}$ and $\boldsymbol{y = -5}$.

3. $2x + 3y - 5 = 0$

$\Rightarrow \quad x = \frac{5 - 3y}{2} \quad \ldots (1)$

Substituting $x = \frac{5 - 3y}{2}$ in $10x - 21y - 1 = 0$, we get

$10\left(\frac{5 - 3y}{2}\right) - 21y - 1 = 0$

$\Rightarrow \quad 25 - 15y - 21y - 1 = 0$

$\Rightarrow \quad 24 - 36y = 0$

$\Rightarrow \quad 36y = 24$

$\Rightarrow \quad y = \frac{24}{36} = \frac{2}{3}$

Substituting $y = \frac{2}{3}$ in equation (1), we get

$x = \frac{5 - 3\left(\frac{2}{3}\right)}{2} = \frac{5 - 2}{2} = \frac{3}{2}$

Hence, $\boldsymbol{x = \frac{3}{2}}$ and $\boldsymbol{y = \frac{2}{3}}$.

4. $x + y = 4$

$\Rightarrow \quad y = 4 - x \quad \ldots (1)$

Substituting $y = 4 - x$ in $3x + 11y = 4$, we get

$3x + 11(4 - x) = 4$

$\Rightarrow \quad 3x + 44 - 11x = 4$

$\Rightarrow \quad -8x = 4 - 44$

$\Rightarrow \quad -8x = -40$

$\Rightarrow \quad x = 5$

Substituting $x = 5$ in equation (1), we get

$y = 4 - 5 = -1$

$\Rightarrow \quad y = -1$

Hence, $\boldsymbol{x = 5}$ and $\boldsymbol{y = -1}$.

5. $5x + 3y - 8 = 0$

$\Rightarrow \quad y = \frac{8 - 5x}{3} \quad \ldots (1)$

Substituting $y = \frac{8 - 5x}{3}$ in $3x - 2y + 75 = 0$, we get

$3x - 2\left(\frac{8 - 5x}{3}\right) + 75 = 0$

$\Rightarrow \quad 9x - 16 + 10x + 225 = 0$

$\Rightarrow \quad 19x - 209 = 0$

$\Rightarrow \quad x = \frac{-209}{19} = -11$

Substituting $x = -11$ in equation (1), we get

$y = \frac{8 - 5(-11)}{3}$

$= \frac{8 + 55}{3} = \frac{63}{3} = 21$

Hence, $\boldsymbol{x = -11}$ and $\boldsymbol{y = 21}$.

6. $x - y = 3$

$\Rightarrow \quad y = x - 3 \quad \ldots (1)$

Substituting $y = x - 3$ in $3x + 2y + 26 = 0$, we get

$3x + 2(x - 3) + 26 = 0$

$\Rightarrow \quad 3x + 2x - 6 + 26 = 0$

$\Rightarrow \quad 5x + 20 = 0$

$\Rightarrow \quad 5x = -20$

$\Rightarrow \quad x = -4$

Substituting $x = -4$ in equation (1), we get

$y = -4 - 3 = -7$

Hence, $\boldsymbol{x = -4}$ and $\boldsymbol{y = -7}$.

7. $3x - 8y = -2 \quad \ldots (1)$

$9x + 4y = 8 \quad \ldots (2)$

Multiplying equation (2) by 2, we get

$18x + 8y = 16 \quad \ldots (3)$

Adding equation (1) and equation (3), we get

$21x = 14$

$\Rightarrow \quad x = \frac{14}{21} = \frac{2}{3}$

Substituting $x = \frac{2}{3}$ in equation (1), we get

$3\left(\frac{2}{3}\right) - 8y = -2$

$\Rightarrow \quad 2 - 8y = -2$

$\Rightarrow \quad 8y = 4$

$\Rightarrow \quad y = \frac{1}{2}$

Hence, $\boldsymbol{x = \frac{2}{3}}$ and $\boldsymbol{y = \frac{1}{2}}$.

8. $\frac{x}{2} + y = 0.8$

$\Rightarrow \quad x + 2y = 1.6 \quad \ldots (1)$

$\frac{7}{x + \frac{y}{2}} = 10$

$\Rightarrow \quad 7 = 10\left(x + \frac{y}{2}\right)$

$\Rightarrow \quad 7 = 10x + 5y \quad \ldots (2)$

Multiplying equation (1) by 10, we get

$10x + 20y = 16 \quad \ldots (3)$

Subtracting equation (2) from equation (3), we get

$15y = 9$

$\Rightarrow \quad y = \frac{9}{15} = \frac{3}{5} = 0.6$

Substituting $y = 0.6$ in equation (1), we get

$x + 2\,(0.6) = 1.6$

$\Rightarrow \quad x + 1.2 = 1.6$

$\Rightarrow \quad x = 0.4$

Hence, $\boldsymbol{x = 0.4}$ and $\boldsymbol{y = 0.6}$.

9. $0.5x - 0.\,1y = 0.7 \quad \ldots(1)$

$1.5x + 0.3y = 3.9 \quad \ldots (2)$

Multiplying equation (1) by 3, we get

$1.5x - 0.3y = 2.1 \quad \ldots (3)$

Adding equation (2) and equation (3), we get

$3x = 6$

$\Rightarrow \quad x = 2$

Substituting $x = 2$ in equation (1), we get

$0.5(2) - 0.1y = 0.7$

$\Rightarrow \quad 1 - 0.1y = 0.7$

$\Rightarrow \quad 0.1y = 1 - 0.7$

$\Rightarrow \quad 0.1y = 0.3$

$\Rightarrow \quad y = 3$

Hence, $\boldsymbol{x = 2}$ and $\boldsymbol{y = 3}$.

10. $11x + 2y = 76 \quad \ldots (1)$

$7y - 6x = -1 \quad \ldots (2)$

Multiplying equation (1) by 6 and equation (2) by 11, we get

$66x + 12y = 456 \quad \ldots (3)$

$-66x + 77y = -11 \quad \ldots (4)$

Adding equations (3) and (4), we get

$89y = 445$

$\Rightarrow \quad y = 5$

Substituting $y = 5$ in equation (1), we get

$11x + 2\,(5) = 76$

$\Rightarrow \quad 11x = 66$

$\Rightarrow \quad x = 6$

Hence, $\boldsymbol{x = 6}$ and $\boldsymbol{y = 5}$.

11. $3y - 2x + 2 = 0 \quad \ldots (1)$

$6y - 8x - 16 = 0 \quad \ldots (2)$

Multiplying equation (1) by 4, we get

$12y - 8x + 8 = 0 \quad \ldots (3)$

Subtracting equation (2) from equation (3), we get

$6y + 24 = 0$

$\Rightarrow \quad 6y = -24$

$\Rightarrow \quad y = -4$

Substituting $y = -4$ in equation (1), we get

$3(-4) - 2x + 2 = 0$

$\Rightarrow \quad -12 - 2x + 2 = 0$

$\Rightarrow \quad -10 - 2x = 0$

$\Rightarrow \quad -2x = 10$

$\Rightarrow \quad x = -5$

Hence, $\boldsymbol{x = -5}$ and $\boldsymbol{y = -4}$.

12. $3x - \frac{y + 7}{11} + 2 = 10$

$\Rightarrow \quad 33x - y - 7 + 22 = 110$

$\Rightarrow \quad 33x - y = 95 \quad \ldots (1)$

$3x + 14y = 65 \quad \ldots (2)$

Multiplying equation (2) by 11, we get

$33x + 154y = 715 \quad \ldots (3)$

Subtracting equation (1) from equation (3), we get

$155y = 620$

$\Rightarrow \quad y = \frac{620}{155} = 4$

Substituting $y = 4$ in equation (2), we get

$3x + 14\,(4) = 65$

$\Rightarrow \quad 3x = 65 - 56 = 9$

$\Rightarrow \quad x = 3$

Hence, $\boldsymbol{x = 3, y = 4}$ and $\boldsymbol{\frac{x}{y} = \frac{3}{4}}$.

13. $\frac{ax}{b} - \frac{by}{a} = a + b \quad \ldots (1)$

$ax - by = 2ab \quad \ldots (2)$

Multiplying equation (1) by b, we get

$ax - \frac{b^2y}{a} = ab + b^2 \quad \ldots (3)$

Subtracting equation (3) from equation (2), we get

$\frac{b^2}{a}y - by = 2ab - ab - b^2$

$\Rightarrow \quad \left(\frac{b^2}{a} - b\right)y = ab - b^2$

$\Rightarrow \quad b\left(\frac{b}{a} - 1\right)y = b(a - b)$

$\Rightarrow \quad \left(\frac{b - a}{a}\right)y = (a - b)$

$\Rightarrow \quad y = -a$

Substituting $y = -a$ in equation (2), we get

$ax - b(-a) = 2ab$

$\Rightarrow \quad ax + ab = 2ab$

$\Rightarrow \quad ax = 2ab - ab$

$\Rightarrow \quad ax = ab$

$\Rightarrow \quad x = b$

Hence, $\boldsymbol{x = b}$ and $\boldsymbol{y = -a}$.

14. $ax + by = b - a \quad \ldots (1)$

$bx - ay = -(a + b) \quad \ldots (2)$

Multiplying equation (1) by a and equation (2) by b, we get

$a^2x + aby = ab - a^2 \quad \ldots (3)$

$b^2x - aby = -ab - b^2 \quad \ldots (4)$

Adding equation (3) and equation (4), we get

$(a^2 + b^2)x = ab - a^2 - ab - b^2$

$$\Rightarrow \quad (a^2 + b^2)x = -(a^2 + b^2)$$
$$\Rightarrow \quad x = -1$$
Substituting $x = -1$ in equation (1), we get
$$a(-1) + by = b - a$$
$$\Rightarrow \quad by = b - a + a$$
$$\Rightarrow \quad by = b$$
$$\Rightarrow \quad y = 1$$
Hence, $\boldsymbol{x = -1}$ and $\boldsymbol{y = 1}$.

15. $ax + by = 2ab$ … (1)

$bx + ay = a^2 + b^2$ … (2)

Multiplying equation (1) by a and equation (2) by b, we get

$a^2x + aby = 2a^2b$ … (3)

$b^2x + aby = a^2b + b^3$ … (4)

Subtracting equation (4) from equation (3), we get
$$(a^2 - b^2)x = 2a^2b - a^2b - b^3$$
$$\Rightarrow \quad (a^2 - b^2)x = a^2b - b^3$$
$$\Rightarrow \quad (a^2 - b^2)x = b(a^2 - b^2)$$
$$\Rightarrow \quad x = b$$
Substituting $x = b$ in equation (1), we get
$$a(b) + by = 2ab$$
$$\Rightarrow \quad by = 2ab - ab$$
$$\Rightarrow \quad by = ab$$
$$\Rightarrow \quad y = a$$
Hence, $\boldsymbol{x = b}$ and $\boldsymbol{y = a}$.

16. $6(ax + by) = 3a + 2b$ …(1)

$6(bx - ay) = 3b - 2a$ … (2)

Multiplying equation (1) by a and equation (2) by b, we get

$6a^2x + 6aby = 3a^2 + 2ab$ … (3)

$6b^2 x - 6aby = 3b^2 - 2ab$ … (4)

Adding equation (3) and equation (4), we get
$$6(a^2 + b^2)x = 3a^2 + 3b^2$$
$$\Rightarrow \quad 6(a^2 + b^2)x = 3(a^2 + b^2)$$
$$\Rightarrow \quad x = \frac{3\left(a^2 + b^2\right)}{6\left(a^2 + b^2\right)} = \frac{1}{2}$$
Substituting $x = \frac{1}{2}$ in equation (1), we get
$$6\left(\frac{a}{2} + by\right) = 3a + 2b$$
$$\Rightarrow \quad 3a + 6by = 3a + 2b$$
$$\Rightarrow \quad 6by = 2b$$
$$\Rightarrow \quad y = \frac{2b}{6b} = \frac{1}{3}$$
Hence, $\boldsymbol{x = \frac{1}{2}}$ and $\boldsymbol{y = \frac{1}{3}}$.

17. $\dfrac{2x + y}{x + 3y} = -\dfrac{1}{7}$
$$\Rightarrow \quad 14x + 7y = -x - 3y$$
$$\Rightarrow \quad 15x + 10y = 0$$
$\Rightarrow \quad 3x + 2y = 0$ … (1)
$$7x + 36y = \frac{47}{3}$$
$\Rightarrow \quad 21x + 108y = 47$ … (2)

Multiplying equation (1) by 7, we get

$21x + 14y = 0$ … (3)

Subtracting equation (3) from equation (2), we get
$$94\,y = 47$$
$$\Rightarrow \quad y = \frac{47}{94} = \frac{1}{2}$$
Substituting $y = \frac{1}{2}$ in equation (1), we get
$$3x + 2\left(\frac{1}{2}\right) = 0$$
$$\Rightarrow \quad 3x + 1 = 0$$
$$\Rightarrow \quad 3x = -1$$
$$\Rightarrow \quad x = -\frac{1}{3}$$
Hence, $\boldsymbol{x = -\frac{1}{3}}$ and $\boldsymbol{y = \frac{1}{2}}$.
$$xy = p + \frac{x}{y}$$
$$\Rightarrow \quad p = xy - \frac{x}{y} = \left(\frac{-1}{3}\right)\left(\frac{1}{2}\right) - \frac{\frac{-1}{3}}{\frac{1}{2}}$$
$$\Rightarrow \quad p = -\frac{1}{6} + \frac{1}{3} \times \frac{2}{1}$$
$$\Rightarrow \quad p = -\frac{1}{6} + \frac{2}{3} = \frac{-1+4}{6}$$
$$= \frac{3}{6} = \frac{1}{2}$$
Hence, $\boldsymbol{p = \frac{1}{2}}$.

18. $3x - 2y = \frac{1}{2}(2x - y)$
$$\Rightarrow \quad 6x - 4y = 2x - y$$
$\Rightarrow \quad 4x - 3y = 0$ … (1)
$$\frac{1}{2}(5x - 4y) = \frac{1}{3}(4x - 3)$$
$$\Rightarrow \quad 15x - 12y = 8x - 6$$
$\Rightarrow \quad 7x - 12y = -6$ … (2)

Multiplying equation (1) by 7 and equation (2) by 4, we get

$28x - 21y = 0$ … (3)

$28x - 48y = -24$ … (4)

Subtracting equation (4) from equation (3), we get
$$27y = 24$$
$$\Rightarrow \quad y = \frac{24}{27} = \frac{8}{9}$$
$$\Rightarrow \quad y = \frac{8}{9}$$
Substituting $y = \frac{8}{9}$ in equation (1), we get
$$4x - 3 \times \frac{8}{9} = 0$$
$$\Rightarrow \quad 4x - \frac{8}{3} = 0$$
$$\Rightarrow \quad 4x = \frac{8}{3}$$
$$\Rightarrow \quad x = \frac{8}{3} \times \frac{1}{4} = \frac{2}{3}$$

Hence, $x = \mathbf{\frac{2}{3}}$ and $y = \mathbf{\frac{8}{9}}$.

19. $\sqrt{7}\,x - \sqrt{3}\,y = 0$ ... (1)

$\sqrt{5}\,x + \sqrt{2}\,y = 0$ ... (2)

Multiplying equation (1) by $\sqrt{2}$ and equation (2) by $\sqrt{3}$, we get

$\sqrt{14}\,x - \sqrt{6}\,y = 0$ ... (3)

$\sqrt{15}\,x + \sqrt{6}\,y = 0$ ... (4)

Adding equation (3) and equation (4), we get

$\left(\sqrt{14} + \sqrt{15}\right) x = 0$

$\Rightarrow \quad x = 0$

Substituting $x = 0$ in equation (1), we get

$\sqrt{7}\,(0) - \sqrt{3}\,y = 0$

$\Rightarrow \quad -\sqrt{3}\,y = 0$

$\Rightarrow \quad y = 0$

Hence, $\boldsymbol{x = 0}$ and $\boldsymbol{y = 0}$.

20. $\frac{x+3}{5} = \frac{8-y}{4} = 3(x + y)$ [Given]

$\Rightarrow \quad \frac{x+3}{5} = \frac{8-y}{4}$

$\Rightarrow \quad 4x + 12 = 40 - 5y$

$\Rightarrow \quad 4x + 5y = 28$ ... (1)

$\frac{8-y}{4} = 3(x + y)$

$\Rightarrow \quad 8 - y = 12x + 12y$

$\Rightarrow \quad 12x + 13y = 8$ ... (2)

Multiplying equation (1) by 3, we get

$12x + 15y = 84$ ... (3)

Subtracting equation (2) from equation (3), we get

$2y = 76$

$\Rightarrow \quad y = 38$

Substituting $y = 38$ in equation (1), we get

$4x + 5 \times 38 = 28$

$\Rightarrow \quad 4x = 28 - 190 = -162$

$\Rightarrow \quad x = -\frac{162}{4} = -40.5$

Hence, $\boldsymbol{x = -40.5}$ and $\boldsymbol{y = 38}$.

21. $\frac{2x}{a} + \frac{y}{b} = 2$ ... (1)

$\frac{x}{a} - \frac{y}{b} = 4$ ... (2)

Adding equation (1) and equation (2), we get

$\frac{2x}{a} + \frac{x}{a} = 6$

$\Rightarrow \quad 2x + x = 6a$

$\Rightarrow \quad 3x = 6a$

$\Rightarrow \quad x = 2a$

Substituting $x = 2a$ in equation (2), we get

$\frac{2a}{a} - \frac{y}{b} = 4$

$\Rightarrow \quad 2 - 4 = \frac{y}{b}$

$\Rightarrow \quad -2b = y$

Hence, $\boldsymbol{x = 2a}$ and $\boldsymbol{y = -2b}$.

22. $\frac{x}{5} + \frac{y}{4} = 5$

$\Rightarrow \quad 4x + 5y = 100$ ... (1)

$\frac{3x}{5} - \frac{7y}{4} = -5$

$\Rightarrow \quad 12x - 35y = -100$ ... (2)

Multiplying equation (1) by 7, we get

$28x + 35y = 700$ ... (3)

Adding equation (2) and equation (3), we get

$40x = 600$

$\Rightarrow \quad x = \frac{600}{40} = 15$

Substituting $x = 15$ in equation (1), we get

$4(15) + 15y = 100$

$\Rightarrow \quad 60 + 5y = 100$

$\Rightarrow \quad 5y = 40$

$\Rightarrow \quad y = 8$

Hence, $\boldsymbol{x = 15}$ and $\boldsymbol{y = 8}$.

23. $5(y + 1) - 4(x + 3) = 29$

$\Rightarrow \quad 5y + 5 - 4x - 12 = 29$

$\Rightarrow \quad 5y - 4x = 36$ ... (1)

$7(y - 3) + 2(x - 6) = 25$

$\Rightarrow \quad 7y - 21 + 2x - 12 = 25$

$\Rightarrow \quad 7y + 2x = 58$ ... (2)

Multiplying equations (2) by 2, we get

$14y + 4x = 116$ ... (3)

Adding equations (1) and (3), we get

$19y = 152$

$\Rightarrow \quad y = 8$

Substituting $y = 8$ in equation (1), we get

$5(8) - 4x = 36$

$\Rightarrow \quad 4x = 40 - 36 = 4$

$\Rightarrow \quad x = 1$

Hence, $\boldsymbol{x = 1}$ and $\boldsymbol{y = 8}$.

24. $\frac{1}{2x} + \frac{3}{5y} = \frac{1}{10}$ ... (1)

$\frac{1}{7x} + \frac{1}{3y} = \frac{4}{21}$ $(x \neq 0\ y \neq 0)$ ... (2)

Let $\frac{1}{x} = a$ and $\frac{1}{y} = b$.

Then, the above equations become

$\frac{a}{2} + \frac{3b}{5} = \frac{1}{10}$

$\Rightarrow \quad 5a + 6b = 1$ ... (3)

and $\quad \frac{a}{7} + \frac{b}{3} = \frac{4}{21}$

$\Rightarrow \quad 3a + 7b = 4$ ... (4)

Multiplying equation (3) by 7 and equation (4) by 6, we get

$35a + 42b = 7$ ... (5)

$18a + 42b = 24$ ... (6)

Subtracting equation (6) from equation (5), we get

$17a = -17$

$\Rightarrow \quad a = -1$

Substituting $a = -1$ in equation (3), we get

$5(-1) + 6b = 1$

$\Rightarrow \quad 6b = 6$

$\Rightarrow \quad b = 1$

Now, $a = -1$ and $b = 1$

$\Rightarrow \quad \frac{1}{x} = -1$ and $\frac{1}{y} = 1$

$\Rightarrow \quad x = -1$ and $y = 1$

Hence, $\boldsymbol{x = -1}$ and $\boldsymbol{y = 1}$.

Let p be added to x so that x becomes equal to y.

Then, $x + p = y$

$\Rightarrow \quad -1 + p = 1$

$\Rightarrow \quad p = 2$

$\therefore$ **2** should be added to x so that it becomes equal to y.

25. (*i*) $\frac{2}{x} + \frac{3}{y} = 13$ … (1)

$\frac{5}{x} - \frac{4}{y} = -2 \ (x \neq 0, y \neq 0)$ … (2)

Multiplying equation (1) by 4 and equation (2) by 3, we get

$\frac{8}{x} + \frac{12}{y} = 52$ … (3)

$\frac{15}{x} - \frac{12}{y} = -6$ … (4)

Adding equation (3) and equation (4), we get

$\frac{23}{x} = 46$

$\Rightarrow \quad x = \frac{23}{46} = \frac{1}{2}$

Substituting $x = \frac{1}{2}$ in equation (1), we get

$\frac{2}{\frac{1}{2}} + \frac{3}{y} = 13$

$\Rightarrow \quad 4 + \frac{3}{y} = 13$

$\Rightarrow \quad \frac{3}{y} = 9$

$\Rightarrow \quad y = \frac{3}{9} = \frac{1}{3}$

Hence, $\boldsymbol{x = \frac{1}{2}}$ and $\boldsymbol{y = \frac{1}{3}}$.

(*ii*) $\frac{5}{x} + \frac{1}{y} = 2$ … (1)

$\frac{6}{x} - \frac{3}{y} = 1$ $(x, y \neq 0)$ … (2)

Multiplying equation (1) by 3, we get

$\frac{15}{x} + \frac{3}{y} = 6$ … (3)

Adding equations (2) and (3), we get

$\frac{21}{x} = 7$

$\Rightarrow \quad x = \frac{21}{7}$

$\Rightarrow \quad x = 3$

Substituting $x = 3$ in equation (2), we get

$\frac{6}{3} - \frac{3}{y} = 1$

$\Rightarrow \quad 2 - \frac{3}{y} = 1$

$\Rightarrow \quad 2 - 1 = \frac{3}{y}$

$\Rightarrow \quad 1 = \frac{3}{y}$

$\Rightarrow \quad y = 3$

Hence, $\boldsymbol{x = 3}$ and $\boldsymbol{y = 3}$.

26. (*i*) $\frac{3a}{x} - \frac{2b}{y} + 5 = 0$ … (1)

$\frac{a}{x} + \frac{3b}{y} - 2 = 0$ $(x \neq 0, y \neq 0)$ … (2)

Multiplying equation (2) by 3, we get

$\frac{3a}{x} + \frac{9b}{y} - 6 = 0$ … (3)

Subtracting equation (1) from equation (3), we get

$\frac{11b}{y} - 11 = 0$

$\Rightarrow \quad \frac{11b}{y} = 11$

$\Rightarrow \quad y = b$

Substituting $y = b$ in equation (1), we get

$\frac{3a}{x} - \frac{2b}{(b)} + 5 = 0$

$\Rightarrow \quad \frac{3a}{x} - 2 + 5 = 0$

$\frac{3a}{x} = -3$

$\Rightarrow \quad x = \frac{3a}{-3}$

$\Rightarrow \quad x = -a$

Hence, $\boldsymbol{x = -a}$ and $\boldsymbol{y = b}$.

(*ii*) $x + \frac{6}{y} = 6$ … (1)

$3x - \frac{8}{y} = 5 \ (y \neq 0)$ … (2)

Multiplying equation (1) by 3, we get

$3x + \frac{18}{y} = 18$ … (3)

Subtracting equation (2) from equation (3), we get

$\frac{26}{y} = 13$

$\Rightarrow \quad y = \frac{26}{13}$

$\Rightarrow \quad y = 2$

Substituting $y = 2$ in equation (2), we get

$3x - \frac{8}{2} = 5$

$\Rightarrow \quad 3x - 4 = 5$

$\Rightarrow \quad 3x = 9$

$\Rightarrow \quad x = 3$

Hence, $x = 3$ and $y = 2$.

27. (*i*) $\frac{5}{x} - 2y = \frac{17}{3}$ ...(1)

$\frac{2}{x} + 3y = -\frac{16}{3} \quad (x \neq 0)$ ... (2)

Multiplying equation (1) by 3 and equation (2) by 2, we get

$\frac{15}{x} - 6y = 17$ ... (3)

$\frac{4}{x} + 6y = \frac{-32}{3}$ ... (4)

Adding equation (3) and equation (4), we get

$$\frac{19}{x} = \frac{51 - 32}{3}$$

$\Rightarrow \quad \frac{19}{x} = \frac{19}{3}$

$\Rightarrow \quad x = 3$

Substituting $x = 3$ in equation (1), we get

$$\frac{5}{3} - 2y = \frac{17}{3}$$

$\Rightarrow \quad \frac{5}{3} - \frac{17}{3} = 2y$

$\Rightarrow \quad \frac{-12}{3} = 2y$

$\Rightarrow \quad -4 = 2y$

$\Rightarrow \quad y = -2$

Hence, $x = 3$ and $y = -2$.

(*ii*) $\frac{4}{x} + 3y = 8$ ... (1)

$\frac{6}{x} - 4y = -5$ ... (2)

Multiplying equation (1) by 4 and equation (2) by 3, we get

$\frac{16}{x} + 12y = 32$ ... (3)

$\frac{18}{x} - 12y = -15$ ... (4)

Adding equation (3) and equation (4), we get

$$\frac{34}{x} = 17$$

$\Rightarrow \quad x = \frac{34}{17}$

$\Rightarrow \quad x = 2$

Substituting $x = 2$ in equation (1), we get

$$\frac{4}{2} + 3y = 8$$

$\Rightarrow \quad 3y = 8 - 2 = 6$

$\Rightarrow \quad 3y = 6$

$\Rightarrow \quad y = 2$

Hence, $x = 2$ and $y = 2$.

For Standard Level

28. $7u + 15v = 126uv$ ... (1)

$u + v = 10uv \quad (u \neq 0, v \neq 0)$ ... (2)

Dividing equation (1) and equation (2) by uv, we get

$\frac{7}{v} + \frac{15}{u} = 126$ ... (3)

$\frac{1}{v} + \frac{1}{u} = 10$ ... (4)

Multiplying equation (4) by 7, we get

$\frac{7}{v} + \frac{7}{u} = 70$ ... (5)

Subtracting equation (5) from equation (3), we get

$$\frac{8}{u} = 56$$

$\Rightarrow \quad u = \frac{8}{56}$

$\Rightarrow \quad u = \frac{1}{7}$

Substituting $u = \frac{1}{7}$ in equation (4), we get

$$\frac{1}{v} + \frac{1}{\frac{1}{7}} = 10$$

$\Rightarrow \quad \frac{1}{v} + 7 = 10$

$\Rightarrow \quad \frac{1}{v} = 3$

$\Rightarrow \quad v = \frac{1}{3}$

Hence, $u = \frac{1}{7}$ and $v = \frac{1}{3}$.

29. $8x + 9y = 42xy$ ... (1)

$2x + 3y = 12xy \; (x \neq 0, y \neq 0)$ ... (2)

Dividing equation (1) and equation (2) by xy, we get

$\frac{8}{y} + \frac{9}{x} = 42$ ... (3)

$\frac{2}{y} + \frac{3}{x} = 12$ ... (4)

Multiplying equation (4) by 3, we get

$\frac{6}{y} + \frac{9}{x} = 36$ ... (5)

Subtracting equation (5) from equation (3), we get

$$\frac{2}{y} = 6$$

$\Rightarrow \quad y = \frac{2}{6}$

$\Rightarrow \quad y = \frac{1}{3}$

Substituting $y = \frac{1}{3}$ in equation (5), we get

$$\frac{6}{\frac{1}{3}} + \frac{9}{x} = 36$$

$\Rightarrow \quad 18 + \frac{9}{x} = 36$

$\Rightarrow \quad \frac{9}{x} = 18$

$\Rightarrow \quad x = \frac{9}{18}$

$\Rightarrow \quad x = \frac{1}{2}$

Hence, $\boldsymbol{x = \frac{1}{2}}$ and $\boldsymbol{y = \frac{1}{3}}$.

30. $\frac{3}{x} - \frac{2}{y} = \frac{4}{xy}$ ... (1)

$\frac{5}{x} + \frac{4}{y} = \frac{14}{xy}$ $(x \neq 0, y \neq 0)$ ... (2)

Multiplying equation (1) and equation (2) by xy, we get

$3y - 2x = 4$ ... (3)

$5y + 4x = 14$ ... (4)

Multiplying equation (3) by 2, we get

$6y - 4x = 8$ ... (5)

Adding equation (4) and equation (5), we get

$11y = 22$

$\Rightarrow \quad y = \frac{22}{11}$

$\Rightarrow \quad y = 2$

Substituting $y = 2$ in equation (4), we get

$5(2) + 4x = 14$

$\Rightarrow \quad 4x = 14 - 10$

$\Rightarrow \quad 4x = 4$

$\Rightarrow \quad x = 1$

Hence, $\boldsymbol{x = 1}$ and $\boldsymbol{y = 2}$.

31. $\frac{x + y}{xy} = \frac{7}{10}$

$\Rightarrow \quad 10x + 10y = 7xy$ ... (1)

$\frac{y - x}{xy} = \frac{3}{10}$ $(x \neq 0, y \neq 0)$

$\Rightarrow \quad 10y - 10x = 3xy$ ... (2)

Dividing equation (1) and equation (2) by xy, we get

$\frac{10}{y} + \frac{10}{x} = 7$ ... (3)

$\frac{10}{x} - \frac{10}{y} = 3$ ... (4)

Adding equation (3) and equation (4), we get

$\frac{20}{x} = 10$

$\Rightarrow \quad x = \frac{20}{10}$

$\Rightarrow \quad x = 2$

Substituting $x = 2$ in equation (3), we get

$\frac{10}{y} + \frac{10}{2} = 7$

$\Rightarrow \quad \frac{10}{y} + 5 = 7$

$\Rightarrow \quad \frac{10}{y} = 2$

$\Rightarrow \quad y = \frac{10}{2}$

$\Rightarrow \quad y = 5$

Hence, $\boldsymbol{x = 2}$ and $\boldsymbol{y = 5}$.

32. $\frac{xy}{x + y} = \frac{2}{3}$

$\Rightarrow \quad 3xy = 2x + 2y$ ... (1)

$\frac{xy}{y - x} = 2$ $(x \neq 0, y \neq 0)$

$\Rightarrow \quad xy = 2y - 2x$ ... (2)

Dividing equation (1) and equation (2) by xy, we get

$3 = \frac{2}{y} + \frac{2}{x}$ ... (3)

$1 = \frac{2}{x} - \frac{2}{y}$ ... (4)

Adding equation (3) and equation (4), we get

$4 = \frac{4}{x}$

$\Rightarrow \quad x = \frac{4}{4}$

$\Rightarrow \quad x = 1$

Substituting $x = 1$ in equation (4), we get

$1 = \frac{2}{1} - \frac{2}{y}$

$\Rightarrow \quad \frac{2}{y} = 2 - 1$

$\Rightarrow \quad \frac{2}{y} = 1$

$\Rightarrow \quad y = 2$

Hence, $\boldsymbol{x = 1}$ and $\boldsymbol{y = 2}$.

33. $\frac{10}{x + y} + \frac{2}{x - y} = 4$ ... (1)

$\frac{15}{x + y} - \frac{5}{x - y} = -2$ $(x \neq y, x \neq -y)$ ... (2)

Let $\frac{1}{x + y} = a$ and $\frac{1}{x - y} = b$.

Then, the above equations become

$10a + 2b = 4$ ... (3)

and $15a - 5b = -2$ ... (4)

Multiplying equation (3) by 5 and equation (4) by 2, we get

$50a + 10b = 20$ ... (5)

$30a - 10b = -4$ ... (6)

Adding equation (5) and equation (6), we get

$80a = 16$

$\Rightarrow \quad a = \frac{16}{80}$

$\Rightarrow \quad a = \frac{1}{5}$

Substituting $a = \frac{1}{5}$ in equation (6), we get

$30 \times \frac{1}{5} - 10b = -4$

$\Rightarrow \quad 6 - 10b = -4$

$\Rightarrow \quad 10b = 10$

$\Rightarrow \quad b = 1$

Now, $a = \frac{1}{5}$ and $b = 1$.

$$\Rightarrow \quad \frac{1}{x+y} = \frac{1}{5}$$

$$\text{and} \quad \frac{1}{x-y} = 1$$

$\Rightarrow \quad 5 = x + y \quad \ldots (7)$

and $\quad 1 = x - y \quad \ldots (8)$

Solving equation (7) and equation (8), we get

$$x = 3 \text{ and } y = 2$$

Hence, $\boldsymbol{x = 3}$ and $\boldsymbol{y = 2}$.

34. $$\frac{5}{x+y} + \frac{2}{x-y} = 6 \quad \ldots (1)$$

$$\frac{32}{x+y} + \frac{1}{x-y} = \frac{65}{2}$$

$(x + y \neq 0,\ x - y \neq 0) \ \ldots (2)$

Let $\frac{1}{x+y} = a$ and $\frac{1}{x-y} = b$.

Then, the above equations become

$$5a + 2b = 6 \quad \ldots (3)$$

$$32a + b = \frac{65}{2} \quad \ldots (4)$$

Multiplying equation (4) by 2, we get

$$64a + 2b = 65 \quad \ldots (5)$$

Subtracting equation (3) from equation (5), we get

$$59a = 59$$

$\Rightarrow \quad a = 1$

Subtracting $a = 1$ in equation (3), we get

$$5\,(1) + 2b = 6$$

$\Rightarrow \quad 2b = 6 - 5 = 1$

$\Rightarrow \quad b = \frac{1}{2}$

Now, $\quad a = 1$

and $\quad b = \frac{1}{2}$

$$\Rightarrow \quad \frac{1}{x+y} = 1$$

$$\text{and} \quad \frac{1}{x-y} = \frac{1}{2}$$

$\Rightarrow \quad x + y = 1 \quad \ldots (6)$

and $\quad x - y = 2 \quad \ldots (7)$

Solving equation (6) and equation (7), we get

$$x = \frac{3}{2} \text{ and } y = -\frac{1}{2}$$

Hence, $\boldsymbol{x = \frac{3}{2}}$ and $\boldsymbol{y = -\frac{1}{2}}$.

35. $$\frac{9}{2(2x-y)} + \frac{2}{(x+2y)} = 2 \quad \ldots (1)$$

$$\frac{5}{(2x-y)} - \frac{2}{(x+2y)} = \frac{7}{6}$$

$(2x - y \neq 0,\ x + 2y \neq 0) \ \ldots (2)$

Let $\frac{1}{2x-y} = a$ and $\frac{1}{x+2y} = b$.

Then, the above equations become

$$\frac{9a}{2} + 2b = 2 \quad \ldots (3)$$

$$5a - 2b = \frac{7}{6} \quad \ldots (4)$$

Adding equation (3) and equation (4), we get

$$\left(\frac{9}{2} + 5\right)a = 2 + \frac{7}{6}$$

$$\Rightarrow \quad \frac{19}{2}a = \frac{19}{6}$$

$$\Rightarrow \quad a = \frac{2}{6} = \frac{1}{3}$$

Substituting $a = \frac{1}{3}$ in equation (3), we get

$$\frac{9}{2} \times \frac{1}{3} + 2b = 2$$

$$\Rightarrow \quad \frac{3}{2} + 2b = 2$$

$$\Rightarrow \quad 2b = 2 - \frac{3}{2}$$

$$\Rightarrow \quad 2b = \frac{1}{2}$$

$$\Rightarrow \quad b = \frac{1}{4}$$

Now, $\quad a = \frac{1}{3}$

and $\quad b = \frac{1}{4}$

$$\Rightarrow \quad \frac{1}{2x-y} = \frac{1}{3}$$

$$\text{and} \quad \frac{1}{x+2y} = \frac{1}{4}$$

$\Rightarrow \quad 3 = 2x - y \quad \ldots (5)$

and $\quad 4 = x + 2y \quad \ldots (6)$

Multiplying equation (5) by 2, we get

$$6 = 4x - 2y \quad \ldots (7)$$

Adding equation (6) and equation (7), we get

$$10 = 5x$$

$\Rightarrow \quad x = 2$

Substituting $x = 2$ in equation (5), we get

$$3 = 2 \times 2 - y$$

$\Rightarrow \quad y = 4 - 3 = 1$

Hence, $\boldsymbol{x = 2}$ and $\boldsymbol{y = 1}$.

36. $$\frac{1}{2(2x+3y)} + \frac{12}{7(3x-2y)} = \frac{1}{2} \quad \ldots (1)$$

$$\frac{7}{(2x+3y)} + \frac{4}{(3x-2y)} = 2 \quad \ldots (2)$$

Let $\frac{1}{2x+3y} = a$ and $\frac{1}{3x-2y} = b$.

Then, the above equations become

$$\frac{a}{2} + \frac{12b}{7} = \frac{1}{2} \quad \ldots (3)$$

$$7a + 4b = 2 \quad \ldots (4)$$

Multiplying equation (3) by 14, we get

$$7a + 24b = 7 \quad \ldots (5)$$

Subtracting equation (4) from equation (5), we get

$$20b = 5$$

$$\Rightarrow \quad b = \frac{5}{20} = \frac{1}{4}$$

Substituting $b = \frac{1}{4}$ in equation (4), we get

$$7a + 4 \times \frac{1}{4} = 2$$

$$\Rightarrow \quad 7a = 2 - 1$$

$$\Rightarrow \quad a = \frac{1}{7}$$

Now, $a = \frac{1}{7}$

and $b = \frac{1}{4}$

$$\Rightarrow \quad \frac{1}{2x + 3y} = \frac{1}{7}$$

and $\frac{1}{3x - 2y} = \frac{1}{4}$

$$\Rightarrow \quad 2x + 3y = 7 \quad \ldots (5)$$

and $3x - 2y = 4 \quad \ldots (6)$

Multiplying equation (5) by 2 and equation (6) by 3, we get

$$4x + 6y = 14 \quad \ldots (7)$$

$$9x - 6y = 12 \quad \ldots (8)$$

Adding equation (7) and equation (8), we get

$$13x = 26$$

$$\Rightarrow \quad x = 2$$

Substituting $x = 2$ in equation (8), we get

$$9(2) - 6y = 12$$

$$\Rightarrow \quad 6y = 18 - 12$$

$$\Rightarrow \quad 6y = 6$$

$$\Rightarrow \quad y = 1$$

Hence, $\boldsymbol{x = 2}$ and $\boldsymbol{y = 1}$.

37. $47x + 31y = 63 \quad \ldots (1)$

$$31x + 47y = 15 \quad \ldots (2)$$

Adding equation (1) and equation (2), we get

$$78x + 78y = 78$$

$$\Rightarrow \quad x + y = 1 \quad \ldots (3)$$

Subtracting equation (2) from equation (1), we get

$$16x - 16y = 48$$

$$\Rightarrow \quad x - y = 3 \quad \ldots (4)$$

Solving equation (3) and equation (4), we get

$$x = 2, y = -1$$

Hence, $\boldsymbol{x = 2}$ and $\boldsymbol{y = -1}$.

38. $62x + 37y = 13 \quad \ldots (1)$

$$37x + 62y = -112 \quad \ldots (2)$$

Adding equation (1) and equation (2), we get

$$99x + 99y = -99$$

$$\Rightarrow \quad x + y = -1 \quad \ldots (3)$$

Subtracting equation (2) from equation (1), we get

$$25x - 25y = +125$$

$$\Rightarrow \quad x - y = 5 \quad \ldots (4)$$

Solving equation (3) and equation (4), we get

$$x = 2, y = -3$$

Hence, $\boldsymbol{x = 2}$ and $\boldsymbol{y = -3}$.

39. $37x + 43y = 123 \quad \ldots (1)$

$$43x + 37y = 117 \quad \ldots (2)$$

Adding equation (1) and equation (2), we get

$$80x + 80y = 240$$

$$\Rightarrow \quad x + y = 3 \quad \ldots (3)$$

Subtracting equation (1) from equation (2), we get

$$6x - 6y = -6$$

$$\Rightarrow \quad x - y = -1 \quad \ldots (4)$$

Solving equation (3) and equation (4), we get

$$x = 1 \text{ and } y = 2$$

Hence, $\boldsymbol{x = 1}$ and $\boldsymbol{y = 2}$.

40. $\frac{b}{a}x + \frac{a}{b}y = a^2 + b^2 \quad \ldots (1)$

$$x + y = 2ab \quad \ldots (2)$$

Multiplying equation (2) by $\frac{a}{b}$, we get

$$\frac{a}{b}x + \frac{a}{b}y = 2a^2 \quad \ldots (3)$$

Subtracting equation (1) from equation (3), we get

$$\left(\frac{a}{b} - \frac{b}{a}\right)x = 2a^2 - a^2 - b^2$$

$$\Rightarrow \quad \left(\frac{a^2 - b^2}{ab}\right)x = a^2 - b^2$$

$$\Rightarrow \quad x = ab$$

Substituting $x = ab$ in equation (2), we get

$$ab + y = 2ab$$

$$\Rightarrow \quad y = ab$$

Hence, $\boldsymbol{x = ab}$ and $\boldsymbol{y = ab}$.

41. $6(ax + by) = 3a + 2b$

$$\Rightarrow \quad 6ax + 6by = 3a + 2b \quad \ldots (1)$$

$$6(bx - ay) = 3b - 2a$$

$$\Rightarrow \quad 6bx - 6ay = 3b - 2a \quad \ldots (2)$$

Multiplying equation (1) by a and equation (2) by b, we get

$$6a^2x + 6aby = 3a^2 + 2ab \quad \ldots (3)$$

$$6b^2x - 6aby = 3b^2 - 2ab \quad \ldots (4)$$

Adding equation (3) and equation (4), we get

$$6(a^2 + b^2)x = 3(a^2 + b^2)$$

$$\Rightarrow \quad x = \frac{3(a^2 + b^2)}{6(a^2 + b^2)} = \frac{1}{2}$$

Substituting $x = \frac{1}{2}$ in equation (1), we get

$$6a \times \frac{1}{2} + 6by = 3a + 2b$$

$$\Rightarrow \quad 3a + 6by = 3a + 2b$$

$$\Rightarrow \quad 6by = 2b$$

$$\Rightarrow \quad y = \frac{2b}{6b} = \frac{1}{3}$$

Hence, $\boldsymbol{x = \frac{1}{2}}$ and $\boldsymbol{y = \frac{1}{3}}$.

42. $$\frac{x+1}{2} + \frac{y-1}{3} = 8$$

$$\Rightarrow \quad 3x + 3 + 2y - 2 = 48$$

$$\Rightarrow \quad 3x + 2y = 47 \quad \ldots (1)$$

$$\frac{x-1}{3} + \frac{y+1}{2} = 9$$

$$\Rightarrow \quad 2x - 2 + 3y + 3 = 54$$

$$\Rightarrow \quad 2x + 3y = 53 \quad \ldots (2)$$

Adding equation (1) and equation (2), we get

$$5x + 5y = 100$$

$$\Rightarrow \quad x + y = 20 \quad \ldots (3)$$

Subtracting equation (2) from equation (1), we get

$$x - y = -6 \quad \ldots (4)$$

Solving equation (3) and equation (4), we get

$x = 7$ and $y = 13$

Hence, $\boldsymbol{x = 7}$ and $\boldsymbol{y = 13}$.

43. $$\frac{5}{x-1} + \frac{1}{y-2} = 2 \quad \ldots (1)$$

$$\frac{6}{x-1} - \frac{3}{y-2} = 1 \quad \ldots (2)$$

Let $\frac{1}{x-1} = a$ and $\frac{1}{y-2} = b$.

Then, the above equations become

$$5a + b = 2 \quad \ldots (3)$$

$$6a - 3b = 1 \quad \ldots (4)$$

Multiplying equation (3) by 3, we get

$$15a + 3b = 6 \quad \ldots (5)$$

Adding equation (4) and equation (5), we get

$$21a = 7$$

$$\Rightarrow \quad a = \frac{7}{21}$$

$$\Rightarrow \quad a = \frac{1}{3}$$

Substituting $a = \frac{1}{3}$ in equation (4), we get

$$6 \times \frac{1}{3} - 3b = 1$$

$$\Rightarrow \quad 2 - 3b = 1$$

$$\Rightarrow \quad 3b = 1$$

$$\Rightarrow \quad b = \frac{1}{3}$$

Now $\quad a = \frac{1}{3}$

and $\quad b = \frac{1}{3}$

$$\Rightarrow \quad \frac{1}{x-1} = \frac{1}{3}$$

and $\quad \frac{1}{y-2} = \frac{1}{3}$

$$\Rightarrow \quad x - 1 = 3$$

and $\quad y - 2 = 3$

$$\Rightarrow \quad x = 4$$

and $\quad y = 5$

Hence, $\boldsymbol{x = 4}$ and $\boldsymbol{y = 5}$.

44. $$\frac{9}{x+1} - \frac{8}{y-1} = 1 \quad \ldots (1)$$

$$\frac{3}{x+1} + \frac{4}{y-1} = 2 \quad (x \neq -1, y \neq 1) \ldots (2)$$

Let $\quad \frac{1}{x+1} = a$

and $\quad \frac{1}{y-1} = b.$

Then, the above equations become

$$9a - 8b = 1 \quad \ldots (3)$$

$$3a + 4b = 2 \quad \ldots (4)$$

Multiplying equation (4) by 2, we get

$$6a + 8b = 4 \quad \ldots (5)$$

Adding equation (3) and equation (5), we get

$$15a = 5$$

$$\Rightarrow \quad a = \frac{5}{15}$$

$$\Rightarrow \quad a = \frac{1}{3}$$

Substituting $a = \frac{1}{3}$ in equation (4), we get

$$3 \times \frac{1}{3} + 4b = 2$$

$$\Rightarrow \quad 1 + 4b = 2$$

$$\Rightarrow \quad 4b = 1$$

$$\Rightarrow \quad b = \frac{1}{4}$$

Now, $\quad a = \frac{1}{3}$

and $\quad b = \frac{1}{4}$

$$\Rightarrow \quad \frac{1}{x+1} = \frac{1}{3}$$

and $\quad \frac{1}{y-1} = \frac{1}{4}$

$$\Rightarrow \quad x + 1 = 3$$

and $\quad y - 1 = 4$

$$\Rightarrow \quad x = 2$$

and $\quad y = 5$

Hence, $\boldsymbol{x = 2}$ and $\boldsymbol{y = 5}$.

45. $$2^x + 3^y = 17$$

$$\Rightarrow \quad 2^x + 3^y = 17 \quad \ldots (1)$$

$$2^{x+2} - 3^{y+1} = 5$$

$$\Rightarrow \quad 2^x \times 2^2 - 3^y \times 3^1 = 4 \times 2^x - 3 \times 3^y = 5 \quad \ldots (2)$$

Let $2^x = a$ and $3^y = b$.

Then, the above equations become

$$a + b = 17 \quad \ldots (3)$$

$$4a - 3b = 5 \quad \ldots (4)$$

Multiplying equation (3) by 3, we get

$$3a + 3b = 51 \quad \dots (5)$$

Adding equation (4) and equation (5), we get

$$7a = 56$$

$$\Rightarrow \quad a = 8$$

Substituting $a = 8$ in equation (3), we get

$$b = 9$$

Now, $\quad a = 8$

and $\quad b = 9$

$$\Rightarrow \quad 2^x = 8 = 2^3$$

and $\quad 3^y = 9 = 3^2$

$$\Rightarrow \quad x = 3$$

and $\quad y = 2$

Hence, $\boldsymbol{x = 3}$ and $\boldsymbol{y = 2}$.

46. We have $\quad \dfrac{x}{10} + \dfrac{y}{5} = 1$

$$\Rightarrow \quad \frac{x + 2y}{10} = 1$$

$$\Rightarrow \quad x = 10 - 2y \quad \dots (1)$$

Also, $\quad \dfrac{x}{8} + \dfrac{y}{6} = 15$

$$\Rightarrow \quad \frac{3x + 4y}{24} = 15$$

$$\Rightarrow \quad 3x + 4y = 360$$

$$\Rightarrow \quad 3(10 - 2y) + 4y = 360 \quad \text{[From (1)]}$$

$$\Rightarrow \quad -2y = 330$$

$$\Rightarrow \quad y = -\frac{330}{2} = -165$$

$\therefore$ From (1), $\quad x = 10 + 2 \times 165 = 340$

Hence, the required solution is $\boldsymbol{x = 340}$, $\boldsymbol{y = -165}$.

We have $\quad \lambda = \dfrac{y - 5}{x} = \dfrac{-165 - 5}{340}$

$$= -\frac{170}{340} = -\frac{1}{2}$$

$\therefore$ The required value of λ is $-\dfrac{\mathbf{1}}{\mathbf{2}}$.

47. We have $\quad x + y = 2 \quad \dots (1)$

and $\quad 2x - y = 1 \quad \dots (2)$

Adding (1) and (2), we get

$$3x = 3$$

$$\Rightarrow \quad x = 1$$

$\therefore$ From (1), $\quad y = 2 - x = 2 - 1 = 1$

$\therefore$ The solution of (1) and (2) is $x = 1$, $y = 1$.

Geometrically, (1, 1) represents a point. We know that infinitely many lines can be drawn through one point. Hence, the required number of lines is **infinitely many** and one such line is $\boldsymbol{y = x}$.

48. We have $\quad x - y = 2 \quad \dots (1)$

and $\quad x + y = 4 \quad \dots (2)$

Adding (1) and (2), we get

$$2x = 6$$

$$\Rightarrow \quad x = 3 = a$$

Subtracting (1) from (2), we get

$$2y = 2$$

$$\Rightarrow \quad y = 1 = b$$

$\therefore$ The required values of a and b are **3** and **1** respectively.

49. Since $x + 1$ is a factor of $f(x) = 2x^3 + ax^2 + 2bx + 1$, hence $x = -1$ is a zero of $f(x)$.

$$\therefore \quad f(-1) = 0$$

$$\Rightarrow \quad -2 + a - 2b + 1 = 0$$

$$\Rightarrow \quad a - 2b - 1 = 0$$

$$\Rightarrow \quad a = 2b + 1 \quad \dots (1)$$

Also, $\quad 2a - 3b = 4 \quad$ [Given] ... (2)

$\therefore$ From (1) and (2),

$$2(2b + 1) - 3b = 4$$

$$\Rightarrow \quad b = 2$$

$\therefore$ From (1), $\quad a = 5$

$\therefore$ The required values of a and b are **5** and **2** respectively.

50. Let $ax + by + c = 0$ be any equation with any arbitrary values of a, b and c. If this equation has a unique solution $x = -1$, $y = 3$, then we have

$-a + 3b + c = 0$ now, three arbitrary numbers a, b, c are connected by one equation. Hence, a, b, c will have infinitely many values.

If, for example, $a = 1 = b$, then $c = a - 3b = 1 - 3 = -2$.

$\therefore \quad x + y - 2 = 0$ is one such line.

If $a = 2$, $b = -3$ then $c = a - 3b = 2 + 9 = 11$.

$\therefore$ Another line will be $2x - 3y + 11 = 0$

Thus, infinitely many pairs of lines can be written.

EXERCISE 3D

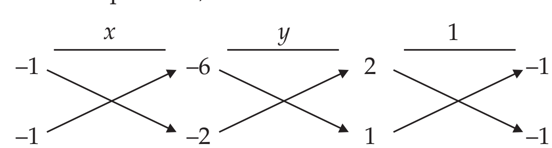
For Basic and Standard Levels

1.
$$2x - y = 6$$
$$x - y = 2$$

The given pair of linear equations may be written as

$$2x - y - 6 = 0$$
$$x - y - 2 = 0$$

By cross-multiplication, we have



$$\Rightarrow \quad \frac{x}{(-1)(-2)-(-1)(-6)} = \frac{y}{(-6)(1)-(-2)(2)}$$

$$= \frac{1}{(2)(-1)-(1)(-1)}$$

$$\Rightarrow \quad \frac{x}{2-6} = \frac{y}{-6+4} = \frac{1}{-2+1}$$

$$\Rightarrow \quad \frac{x}{-4} = \frac{y}{-2} = \frac{1}{-1}$$

$\Rightarrow \quad x = (-1) \times (-4) = 4$

and $y = (-1) \times (-2) = 2$

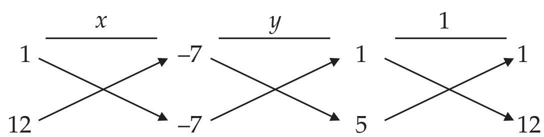
Hence, $\mathbf{x = 4}$ and $\mathbf{y = 2}$.

2. $$x + y = 7$$
$$5x + 12y = 7$$

The given pair of linear equations may be written as

$$x + y - 7 = 0$$
$$5x + 12y - 7 = 0$$

By cross-multiplication, we have



$$\Rightarrow \quad \frac{x}{(1)(-7) - (12)(-7)} = \frac{y}{(-7)(5) - (-7)(1)} = \frac{1}{(1)(12) - (5)(1)}$$

$$\Rightarrow \quad \frac{x}{-7 + 84} = \frac{y}{-35 + 7} = \frac{1}{12 - 5}$$

$$\Rightarrow \quad \frac{x}{77} = \frac{y}{-28} = \frac{1}{7}$$

$\Rightarrow \quad x = 77 \times \frac{1}{7} = 11$

and $y = -28 \times \frac{1}{7} = -4$

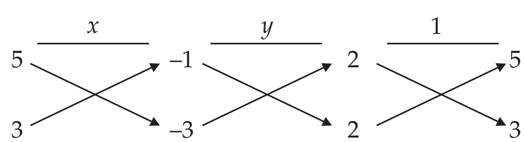
Hence, $\mathbf{x = 11}$ and $\mathbf{y = -4}$.

3. $$2x + 5y = 1$$
$$2x + 3y = 3$$

The given pair of linear equations may be written as

$$2x + 5y - 1 = 0$$
$$2x + 3y - 3 = 0$$

By cross-multiplication, we have



$$\Rightarrow \quad \frac{x}{(5)(-3) - (3)(-1)} = \frac{y}{(-1)(2) - (-3)(2)} = \frac{1}{(2)(3) - (2)(5)}$$

$$\Rightarrow \quad \frac{x}{-15 + 3} = \frac{y}{-2 + 6} = \frac{1}{6 - 10}$$

$$\Rightarrow \quad \frac{x}{-12} = \frac{y}{4} = \frac{1}{-4}$$

$\Rightarrow \quad x = \left(\frac{1}{-4}\right) \times (-12) = 3$

and $\quad y = \left(\frac{1}{-4}\right) \times 4 = -1$

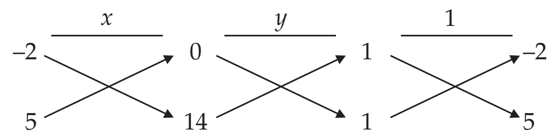
Hence, $\mathbf{x = 3}$ and $\mathbf{y = -1}$.

4. $$x = 2y$$
$$x + 5y + 14 = 0$$

The given pair of linear equations may be written as

$$x - 2y = 0$$
$$x + 5y + 14 = 0$$

By cross multiplication, we have



$$\Rightarrow \quad \frac{x}{(-2)(14) - (5)(0)} = \frac{y}{(0)(1) - (14)(1)} = \frac{1}{(1)(5) - (1)(-2)}$$

$$\Rightarrow \quad \frac{x}{-28} = \frac{y}{-14} = \frac{1}{5 + 2}$$

$$\Rightarrow \quad \frac{x}{-28} = \frac{y}{-14} = \frac{1}{7}$$

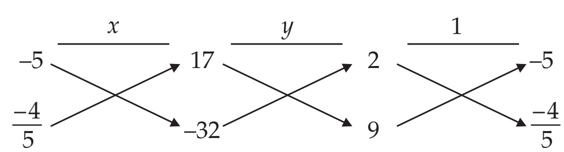
$\Rightarrow \quad x = \frac{1}{7} \times (-28) = -4$

and $y = \frac{1}{7} \times (-14) = -2$

Hence, $\mathbf{x = -4}$, $\mathbf{y = -2}$

5. $$2x - 5y + 17 = 0$$
$$9x - \frac{4}{5}y - 32 = 0$$

By cross-multiplication, we have



$$\Rightarrow \quad \frac{x}{(-5)(-32) - \left(\frac{-4}{5}\right)(17)} = \frac{y}{(17)(9) - (-32)(2)} = \frac{1}{2\left(\frac{-4}{5}\right) - 9(-5)}$$

$$\Rightarrow \quad \frac{x}{160 + \frac{68}{5}} = \frac{y}{153 + 64} = \frac{1}{\frac{-8}{5} + 45}$$

$$\Rightarrow \quad \frac{x}{\frac{800 + 68}{5}} = \frac{y}{217} = \frac{1}{\frac{-8 + 225}{5}}$$

$$\Rightarrow \quad \frac{5x}{868} = \frac{y}{217} = \frac{5}{217}$$

$\Rightarrow \quad x = \frac{5}{217} \times \frac{868}{5} = 4$

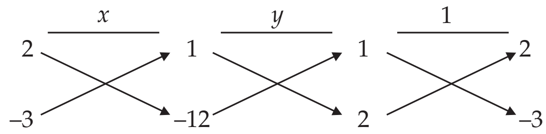
and $y = \frac{5}{217} \times 217 = 5$

Hence, $\boldsymbol{x = 4}$ and $\boldsymbol{y = 5}$.

6. $x + 2y + 1 = 0$

$2x - 3y - 12 = 0$

By cross-multiplication, we have



$$\Rightarrow \frac{x}{2(-12) - (-3)(1)} = \frac{y}{(1)(2) - (-12)(1)} = \frac{1}{(1)(-3) - (2)(2)}$$

$$\Rightarrow \frac{x}{-24 + 3} = \frac{y}{2 + 12} = \frac{1}{-3 - 4}$$

$$\Rightarrow \frac{x}{-21} = \frac{y}{14} = \frac{1}{-7}$$

$$\Rightarrow x = \left(\frac{1}{-7}\right) \times (-21) = 3$$

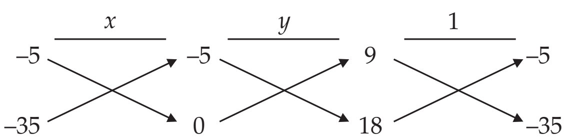
and $$y = \left(\frac{1}{-7}\right) \times 14 = -2$$

Hence, $\boldsymbol{x = 3}$ and $\boldsymbol{y = -2}$.

7. $9x - 5y - 5 = 0$

$18x - 35y = 0$

By cross multiplication, we have



$$\Rightarrow \frac{x}{(-5)(0) - (-35)(-5)} = \frac{y}{(-5)(18) - (0)(9)} = \frac{1}{(9)(-35) - (18)(-5)}$$

$$\Rightarrow \frac{x}{-175} = \frac{y}{-90} = \frac{1}{-315 + 90}$$

$$\Rightarrow \frac{x}{-175} = \frac{y}{-90} = \frac{1}{-225}$$

$$\Rightarrow x = \left(\frac{1}{-225}\right) \times (-175) = \frac{7}{9}$$

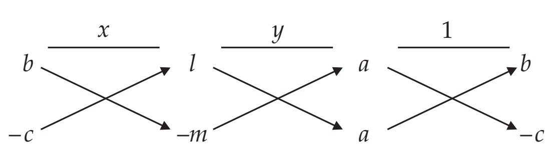
and $$y = \left(\frac{1}{-225}\right) \times (-90) = \frac{2}{5}$$

Hence, $\boldsymbol{x = \frac{7}{9}}$ and $\boldsymbol{y = \frac{2}{5}}$.

8. $ax + by + l = 0$

$ax - cy - m = 0$

By cross-multiplication, we have



$$\Rightarrow \frac{x}{(b)(-m) - (-c)(l)} = \frac{y}{(l)(a) - (-m)(a)} = \frac{1}{(a)(-c) - (a)(b)}$$

$$\Rightarrow \frac{x}{-bm + cl} = \frac{y}{al + am} = \frac{1}{-ac - ab}$$

$$\Rightarrow x = \frac{-bm + cl}{-a(b + c)} \quad \text{and} \quad y = \frac{a(l + m)}{-a(b + c)}$$

$$\Rightarrow x = \frac{-(bm - cl)}{-a(b + c)} \quad \text{and} \quad y = \frac{(l + m)}{-(b + c)}$$

$$\Rightarrow x = \frac{bm - cl}{a(b + c)} \quad \text{and} \quad y = -\frac{(l + m)}{(b + c)}$$

Hence, $\boldsymbol{x = \frac{bm - cl}{a(b + c)}}$ and $\boldsymbol{y = \frac{-(l + m)}{(b + c)}}$.

9. $bx - cy = a - b$

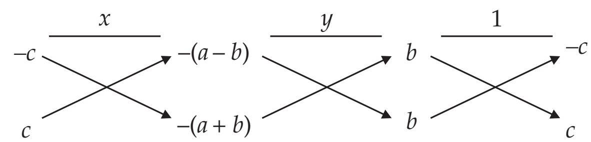
$bx + cy = a + b$

The given pair of linear equations may be written as

$bx - cy - (a - b) = 0$

$bx + cy - (a + b) = 0$

By cross-multiplication, we have



$$\Rightarrow \frac{x}{c(a + b) + c(a - b)} = \frac{y}{-b(a - b) + b(a + b)} = \frac{1}{bc + bc}$$

$$\Rightarrow \frac{x}{ac + bc + ac - bc} = \frac{y}{-ab + b^2 + ab + b^2} = \frac{1}{2bc}$$

$$\Rightarrow \frac{x}{2ac} = \frac{y}{2b^2} = \frac{1}{2bc}$$

$$\Rightarrow \frac{x}{ac} = \frac{y}{b^2} = \frac{1}{bc}$$

$$\Rightarrow x = \frac{ac}{bc} = \frac{a}{b}$$

and $$y = \frac{b^2}{bc} = \frac{b}{c}$$

Hence, $\boldsymbol{x = \frac{a}{b}}$ and $\boldsymbol{y = \frac{b}{c}}$.

10. $x + y = a - b$

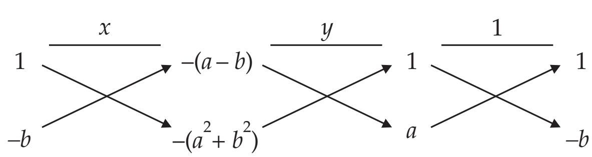
$ax - by = a^2 + b^2$

The given pair of linear equations may be written as

$x + y - (a - b) = 0$

$ax - by - (a^2 + b^2) = 0$

By cross-multiplication, we have



$$\Rightarrow \frac{x}{-(1)(a^2+b^2)-b(a-b)} = \frac{y}{-a(a-b)+(a^2+b^2)} = \frac{1}{-b-a}$$

$$\Rightarrow \frac{x}{-a^2-b^2-ab+b^2} = \frac{y}{-a^2+ab+a^2+b^2} = \frac{1}{-(a+b)}$$

$$\Rightarrow \frac{x}{-a(a+b)} = \frac{y}{b(a+b)} = \frac{1}{-(a+b)}$$

$$\Rightarrow \frac{x}{-a} = \frac{y}{b} = -1$$

$\Rightarrow$ $x = (-a) \times (-1) = a$ and $y = b \times (-1) = -b$

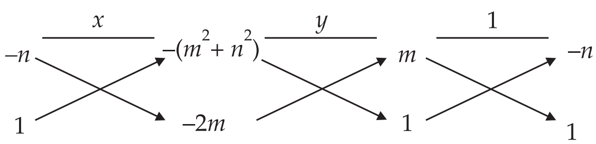
Hence, $\boldsymbol{x = a}$ and $\boldsymbol{y = -b}$.

11. $$mx - ny = m^2 + n^2$$
$$x + y = 2m$$

The given pair of linear equations may be written as

$$mx - ny - (m^2 + n^2) = 0$$
$$x + y - 2m = 0$$

By cross-multiplication, we have



$$\Rightarrow \frac{x}{(-n)(-2m)+(1)(m^2+n^2)} = \frac{y}{-(m^2+n^2)+m(2m)} = \frac{1}{(m+n)}$$

$$\Rightarrow \frac{x}{2mn+m^2+n^2} = \frac{y}{-m^2-n^2+2m^2} = \frac{1}{(m+n)}$$

$$\Rightarrow \frac{x}{(m+n)^2} = \frac{y}{m^2-n^2} = \frac{1}{(m+n)}$$

$$\Rightarrow \qquad x = \frac{(m+n)^2}{(m+n)}$$

and $$y = \frac{(m+n)(m-n)}{(m+n)}$$

$\Rightarrow$ $x = m + n$

and $y = m - n$

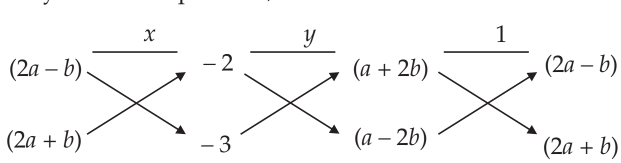
Hence, $\boldsymbol{x = m + n}$ and $\boldsymbol{y = m - n}$.

12. $$(a + 2b)x + (2a - b)y = 2$$
$$(a - 2b)x + (2a + b)y = 3$$

The given pair of linear equations may be written as

$$(a + 2b)x + (2a - b)y - 2 = 0$$
$$(a - 2b)x + (2a + b)y - 3 = 0$$

By cross-multiplication, we have



$$\Rightarrow \frac{x}{-3(2a-b)+2(2a+b)} = \frac{y}{-2(a-2b)+3(a+2b)} = \frac{1}{(a+2b)(2a+b)-(a-2b)(2a-b)}$$

$$\Rightarrow \frac{x}{-6a+3b+4a+2b} = \frac{y}{-2a+4b+3a+6b} = \frac{1}{2a^2+4ab+ab+2b^2-2a^2+4ab+ab-2b^2}$$

$$\Rightarrow \frac{x}{-2a+5b} = \frac{y}{a+10b} = \frac{1}{10ab}$$

$$\Rightarrow \qquad x = \frac{5b-2a}{10ab}$$

and $$y = \frac{a+10b}{10ab}$$

Hence, $\boldsymbol{x = \dfrac{5b-2a}{10ab}}$ and $\boldsymbol{y = \dfrac{a+10b}{10ab}}$.

13. We have

$$ax - by - (a^2 + b^2) = 0 \qquad \ldots (1)$$

and $$x + y - 2a = 0 \qquad \ldots (2)$$

$\therefore$ From (1) and (2), by the method of cross-multiplication, we have

$$\frac{x}{2ab+a^2+b^2} = \frac{x}{-\left(a^2+b^2\right)+2a^2} = \frac{1}{a+b}$$

$$\therefore \qquad x = \frac{(a+b)^2}{a+b} = a + b$$

$$y = \frac{a^2-b^2}{a+b} = a - b$$

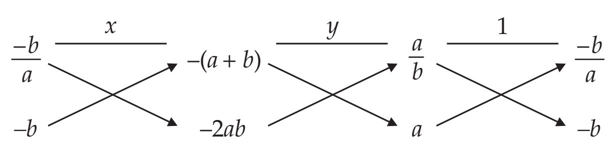
$\therefore$ The required solution is $\boldsymbol{x = a + b, y = a - b}$.

14. $$\frac{ax}{b} - \frac{by}{a} = a + b$$
$$ax - by = 2ab$$

The given pair of linear equations may be written as

$$\frac{a}{b}x - \frac{b}{a}y - (a + b) = 0$$
$$ax - by - 2ab = 0$$

By cross-multiplication, we have



$$\Rightarrow \frac{x}{-\left(\frac{b}{a}\right)(-2ab)-(b)(a+b)} = \frac{y}{-a(a+b)+\frac{a}{b}(2ab)} = \frac{1}{\frac{a}{b}(-b)+a\left(\frac{b}{a}\right)}$$

$$\Rightarrow \frac{x}{2b^2-ab-b^2} = \frac{y}{-a^2-ab+2a^2} = \frac{1}{-a+b}$$

$$\Rightarrow \frac{x}{b^2-ab} = \frac{y}{a^2-ab} = \frac{1}{(b-a)}$$

$$\Rightarrow \quad \frac{x}{b(b-a)} = \frac{y}{a(a-b)} = \frac{1}{(b-a)}$$

$$\Rightarrow \quad x = \frac{b(b-a)}{(b-a)} = b$$

and $$y = \frac{a(a-b)}{(b-a)} = -a$$

Hence, $\boldsymbol{x = b}$ and $\boldsymbol{y = -a}$.

15. We have

$$\frac{x}{a} + \frac{y}{b} - (a+b) = 0 \quad \dots (1)$$

$$\frac{x}{a^2} + \frac{y}{b^2} - 2 = 0 \quad \dots (2)$$

From (1) and (2), by the method of cross-multiplication, we have

$$\frac{x}{\frac{1}{b}(-2) + \frac{1}{b^2}(a+b)} = \frac{y}{-\frac{1}{a^2}(a+b) + \frac{2}{a}} = \frac{1}{\frac{1}{ab^2} - \frac{1}{ba^2}}$$

$$\Rightarrow \frac{x}{-\frac{2}{b} + \frac{a}{b^2} + \frac{1}{b}} = \frac{y}{-\frac{1}{a} - \frac{b}{a^2} + \frac{2}{a}} = \frac{1}{\frac{a-b}{a^2b^2}}$$

$$\Rightarrow \frac{x}{\frac{a}{b^2} - \frac{1}{b}} = \frac{y}{\frac{1}{a} - \frac{b}{a^2}} = \frac{a^2b^2}{a-b}$$

$$\Rightarrow \frac{b^2x}{a-b} = \frac{a^2y}{a-b} = \frac{a^2b^2}{a-b}$$

$$\therefore \quad x = \frac{a^2b^2}{a-b} \times \frac{a-b}{b^2} = a^2$$

and $$y = \frac{a^2b^2}{a-b} \times \frac{a-b}{a^2} = b^2$$

$\therefore$ The required solution is $\boldsymbol{x = a^2, y = b^2}$.

16. We have

$$\frac{bx}{a} - \frac{ay}{b} + a + b = 0$$

$$\Rightarrow \quad b^2x - a^2y + (a^2b + ab^2) = 0 \quad \dots (1)$$

and $$bx - ay + 2ab = 0 \quad \dots (2)$$

From (1) and (2), by the method of cross-multiplication, we have

$$\frac{x}{-2a^3b + a\left(a^2b + ab^2\right)} = \frac{y}{b\left(a^2b + ab^2\right) - 2ab^3} = \frac{1}{-ab^2 + a^2b}$$

$$\Rightarrow \frac{x}{a^3b^2 - a^3b} = \frac{y}{-ab^3 + a^2b^2} = \frac{1}{a^2b - ab^2}$$

$$\Rightarrow \frac{x}{-a^2b(a-b)} = \frac{y}{ab^2(a-b)} = \frac{1}{ab(a-b)}$$

$$\therefore \quad x = \frac{a^2b(a-b)}{ab(a-b)} = -a$$

$$y = \frac{ab^2(a-b)}{ab(a-b)} = b$$

Hence, the required solution is $\boldsymbol{x = -a, y = b}$.

17. We have

$$\frac{b}{a}x + \frac{a}{b}y - \left(a^2 + b^2\right) = 0 \quad \dots (1)$$

and $$x + y - 2ab = 0 \quad \dots (2)$$

From (1) and (2), by the method of cross-multiplication, we have

$$\frac{x}{-2ab \times \frac{a}{b} + \left(a^2 + b^2\right)} = \frac{y}{-\left(a^2 + b^2\right) + 2ab \times \frac{b}{a}} = \frac{1}{\frac{b}{a} - \frac{a}{b}}$$

$$\Rightarrow \frac{x}{b^2 - a^2} = \frac{y}{b^2 - a^2} = \frac{ab}{b^2 - a^2}$$

$$\therefore \ x = \frac{ab}{b^2 - a^2} \times b^2 - a^2 = ab \text{ and } y = \frac{\left(b^2 - a^2\right)ab}{b^2 - a^2} = ab.$$

$\therefore$ The required solution is $\boldsymbol{x = ab, y = ab}$.

18. $$\frac{x}{a} + \frac{y}{a+b} = a$$

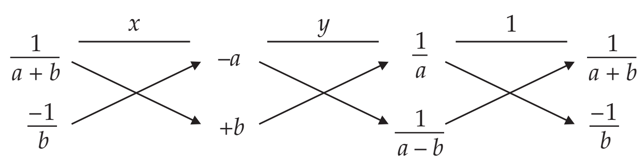
$$\frac{x}{a-b} - \frac{y}{b} = -b$$

The given pair of linear equations may be written as

$$\frac{x}{a} + \frac{y}{a+b} - a = 0$$

$$\frac{x}{a-b} - \frac{y}{b} + b = 0$$

By cross-multiplication, we have



$$\Rightarrow \quad \frac{x}{\frac{b}{a+b} - \frac{a}{b}} = \frac{y}{\frac{-a}{a-b} - \frac{b}{a}} = \frac{1}{\frac{-1}{ab} - \left(\frac{1}{a-b}\right)\left(\frac{1}{a+b}\right)}$$

$$\Rightarrow \quad \frac{x}{\frac{\left(b^2 - a^2 - ab\right)}{b(a+b)}} = \frac{y}{\frac{-a^2 - ab + b^2}{a(a-b)}} = \frac{1}{-\frac{1}{ab} - \frac{1}{a^2 - b^2}}$$

$$\Rightarrow \frac{b(a+b)}{(b^2 - a^2 - ab)}x = \frac{a(a-b)}{(-a^2 + b^2 - ab)}y = \frac{1}{\frac{-a^2 + b^2 - ab}{ab(a^2 - b^2)}}$$

$$\Rightarrow \frac{b(a+b)}{(-a^2 + b^2 - ab)}x = \frac{a(a-b)}{(-a^2 + b^2 - ab)}y = \frac{ab(a^2 - b^2)}{-a^2 + b^2 - ab}$$

$$\Rightarrow \quad x = \frac{(-a^2 + b^2 - ab)}{b(a+b)} \times \frac{ab(a^2 - b^2)}{(-a^2 + b^2 - ab)} = a(a-b)$$

and $$y = \frac{(-a^2 + b^2 - ab)}{a(a-b)} \times \frac{ab(a^2 - b^2)}{(-a^2 + b^2 - ab)} = b\,(a+b)$$

Hence, $\boldsymbol{x = a(a - b)}$ and $\boldsymbol{y = b(a + b)}$.

19. $$\frac{x+1}{2} - \frac{y+4}{11} = 2$$

$$\Rightarrow \quad 11x + 11 - 2y - 8 = 44$$

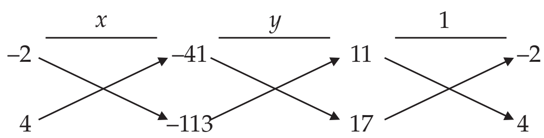
$$\Rightarrow \quad 11x - 2y - 41 = 0 \quad \ldots (1)$$

$$\frac{x+3}{2} + \frac{2y+3}{17} = 5$$

$$\Rightarrow \quad 17x + 51 + 4y + 6 - 170 = 0$$

$$\Rightarrow \quad 17x + 4y - 113 = 0 \quad \ldots (2)$$

By cross-multiplication, we have



$$\Rightarrow \quad \frac{x}{(-2)(-113)+(4)(41)} = \frac{y}{(-41)(17)+(113)11} = \frac{1}{(11)(4)+(17)(2)}$$

$$\Rightarrow \quad \frac{x}{226+164} = \frac{y}{-697+1243} = \frac{1}{44+34}$$

$$\Rightarrow \quad \frac{x}{390} = \frac{y}{546} = \frac{1}{78}$$

$$\Rightarrow \quad x = \frac{390}{78} = 5$$

and $y = \frac{546}{78} = 7$

Hence, $\boldsymbol{x = 5}$ and $\boldsymbol{y = 7}$.

20. $$\frac{2}{x} + \frac{3}{y} = 2 \,(x \neq 0,\ y \neq 0)$$

$$\frac{1}{x} - \frac{1}{2y} = \frac{1}{3}$$

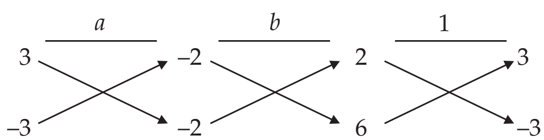
Let $\frac{1}{x} = a$ and $\frac{1}{y} = b$.

The given pair of linear equations may be written as

$$2a + 3b - 2 = 0$$

$$6a - 3b - 2 = 0$$

By cross-multiplication, we have



$$\Rightarrow \quad \frac{a}{(3)(-2)-(-3)(-2)} = \frac{b}{(-2)(6)-(-2)(2)} = \frac{1}{(2)(-3)-(6)(3)}$$

$$\Rightarrow \quad \frac{a}{-6-6} = \frac{b}{-12+4} = \frac{1}{-6-18}$$

$$\Rightarrow \quad \frac{a}{-12} = \frac{b}{-8} = \frac{1}{-24}$$

$$\Rightarrow \quad a = \left(\frac{1}{-24}\right) \times (-12) = \frac{1}{2}$$

and $b = \left(\frac{1}{-24}\right) \times (-8) = \frac{1}{3}$

Now, $a = \frac{1}{2}$

and $b = \frac{1}{3}$

$$\Rightarrow \quad \frac{1}{x} = \frac{1}{2}$$

and $\frac{1}{y} = \frac{1}{3}$

$$\Rightarrow \quad x = 2$$

and $y = 3$

Hence, $\boldsymbol{x = 2}$ and $\boldsymbol{y = 3}$.

21. $$\frac{x+y}{xy} = 5$$

$$\Rightarrow \quad \frac{x}{xy} + \frac{y}{xy} = 5$$

$$\Rightarrow \quad \frac{1}{y} + \frac{1}{x} = 5$$

$$\Rightarrow \quad \frac{1}{x} + \frac{1}{y} - 5 = 0$$

$$\frac{x-y}{xy} = 1$$

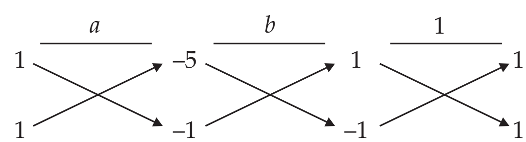
$$\Rightarrow \quad \frac{x}{xy} - \frac{y}{xy} = 1$$

$$\Rightarrow \quad \frac{1}{y} - \frac{1}{x} = 1$$

$$\Rightarrow \quad \frac{-1}{x} + \frac{1}{y} - 1 = 0$$

Let $\frac{1}{x} = a$ and $\frac{1}{y} = b$.

By cross-multiplication, we have



$$\Rightarrow \quad \frac{a}{(1)(-1)-(1)(-5)} = \frac{b}{(-5)(-1)-(-1)(1)} = \frac{1}{(1)(1)-(-1)(1)}$$

$$\Rightarrow \quad \frac{a}{-1+5} = \frac{b}{5+1} = \frac{1}{1+1}$$

$$\Rightarrow \quad \frac{a}{4} = \frac{b}{6} = \frac{1}{2}$$

$$\Rightarrow \quad a = \frac{4}{2} = 2$$

and $b = \frac{6}{2} = 3$

$$\Rightarrow \quad \frac{1}{x} = 2$$

and $\frac{1}{y} = 3$

$\Rightarrow$ $x = \frac{1}{2}$

and $y = \frac{1}{3}$

Hence, $\boldsymbol{x = \frac{1}{2}}$ and $\boldsymbol{y = \frac{1}{3}}$.

22. $$\frac{22}{x+y} + \frac{15}{x-y} = 5$$

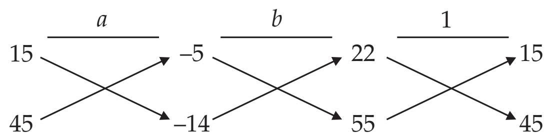
$$\frac{55}{x+y} + \frac{45}{x-y} = 14$$

Let $\frac{1}{x+y} = a$ and $\frac{1}{x-y} = b$.

Then, the given pair of linear equations may be written as

$$22a + 15b - 5 = 0$$
$$55a + 45b - 14 = 0$$

By cross-multiplication, we have



$$\Rightarrow \frac{a}{(15)(-14)-(45)(-5)} = \frac{b}{(-5)(55)-(-14)(22)} = \frac{1}{(22)(45)-(55)(15)}$$

$$\Rightarrow \frac{a}{-210+225} = \frac{b}{-275+308} = \frac{1}{990-825}$$

$$\Rightarrow \frac{a}{15} = \frac{b}{33} = \frac{1}{165}$$

$\Rightarrow$ $a = \frac{15}{165} = \frac{1}{11}$

and $b = \frac{33}{165} = \frac{1}{5}$

Now, $a = \frac{1}{11}$

and $b = \frac{1}{5}$

$\Rightarrow$ $\frac{1}{x+y} = \frac{1}{11}$

and $\frac{1}{x-y} = \frac{1}{5}$

$\Rightarrow$ $x + y = 11$ … (1)

and $x - y = 5$ … (2)

Solving equation (1) and equation (2), we get

$x = 8$ and $y = 3$

Hence, $\boldsymbol{x = 8}$ and $\boldsymbol{y = 3}$.

23. $$2x - \frac{3}{y} = 3 \ (y \neq 0)$$

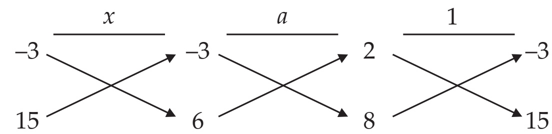
$$8x + \frac{15}{y} = -6$$

Let $\frac{1}{y} = a$.

Then, the given pair of linear equations may be written as

$$2x - 3a - 3 = 0$$
$$8x + 15a + 6 = 0$$

By cross-multiplication, we have



$$\Rightarrow \frac{x}{(-3)(6)-(15)(-3)} = \frac{a}{(-3)(8)-(6)(2)} = \frac{1}{(2)(15)-(8)(-3)}$$

$$\Rightarrow \frac{x}{-18+45} = \frac{a}{-24-12} = \frac{1}{30+24}$$

$$\Rightarrow \frac{x}{27} = \frac{a}{-36} = \frac{1}{54}$$

$\Rightarrow$ $x = \frac{27}{54} = \frac{1}{2}$

and $a = -\frac{36}{54} = \frac{-2}{3}$

Now, $a = \frac{-2}{3}$

$\Rightarrow$ $\frac{1}{y} = \frac{-2}{3}$

$\Rightarrow$ $y = \frac{-3}{2}$

Hence, $\boldsymbol{x = \frac{1}{2}}$ and $\boldsymbol{y = \frac{-3}{2}}$.

For Standard Level

24. We have

$(p + 2q)x + (2p - q)y - 2 = 0$ … (1)

$(p - 2q)x + (2p + q)y - 3 = 0$ … (2)

From (1) and (2), by the method of cross-multiplication, we have

$$\frac{x}{-3(2p-q)+2(2p+q)} = \frac{y}{-2(p-2q)+3(p+2q)} = \frac{1}{(p+2q)(2p+q)-(p-2q)(2p-q)}$$

$$\Rightarrow \frac{x}{-6p+3q+4p+2q} = \frac{y}{-2p+4q+3p+6q} = \frac{1}{2p^2+pq+4pq+2q^2-2p^2+pq+4pq-2q^2}$$

$$\Rightarrow \frac{x}{5q-2p} = \frac{y}{p+10q} = \frac{1}{10pq}$$

$$\therefore \boldsymbol{x = \frac{5q-2p}{10pq}}, \ \boldsymbol{y = \frac{p+10q}{10pq}}$$

which is the required solution.

25. We have $\dfrac{a^2}{x} - \dfrac{b^2}{y} + 0 = 0$ … (1)

$$\frac{a^2b}{x} + \frac{b^2a}{y} - (a+b) = 0 \quad \dots (2)$$

From (1) and (2), by the method of cross-multiplication, we have

$$\frac{\frac{1}{x}}{b^2(a+b) - b^2a \times 0} = \frac{\frac{1}{y}}{0 \times a^2b + a^2(a+b)}$$

$$= \frac{1}{a^2 \times ab^2 + a^2b \times b^2}$$

$$\Rightarrow \frac{\frac{1}{x}}{b^2(a+b)} = \frac{\frac{1}{y}}{a^2(a+b)} = \frac{1}{a^2b^2(a+b)}$$

$$\Rightarrow \frac{\frac{1}{x}}{b^2} = \frac{\frac{1}{y}}{a^2} = \frac{1}{a^2b^2}$$

$$\frac{1}{x} = \frac{b^2}{a^2b^2} = \frac{1}{a^2}$$

$$\Rightarrow x = a^2$$

$$\frac{1}{y} = \frac{a^2}{a^2b^2} = \frac{1}{b^2}$$

$$\Rightarrow y = b^2$$

∴ The required solution is $\boldsymbol{x = a^2}$ and $\boldsymbol{y = b^2}$.

EXERCISE 3E

For Basic and Standard Levels

1.
$$5x + 2y = 5$$
$$3x + y = 1$$

The given pair of linear equations may be written as

$$5x + 2y - 5 = 0$$
$$3x + y - 1 = 0$$

These equations are of the form $a_1x + b_1y + c_1 = 0$

and $a_2x + b_2y + c_2 = 0$

where $a_1 = 5, b_1 = 2, c_1 = -5$

$a_2 = 3, b_2 = 1, c_2 = -1$

$a_1b_2 - a_2b_1 = (5)(1) - (3)(2)$

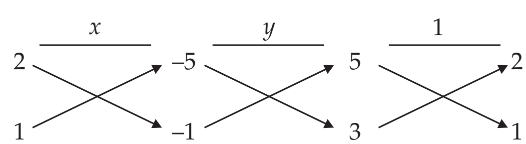
$= 5 - 6$

$= -1 \neq 0$

i.e. $\dfrac{a_1}{a_2} \neq \dfrac{b_1}{b_2}$

Hence, the given pair of linear equations has a unique solution.

By cross-multiplication, we get



$$\Rightarrow \frac{x}{(2)(-1)-(1)(-5)} = \frac{y}{(-5)(3)-(-1)(5)}$$

$$= \frac{1}{(5)(1)-(3)(2)}$$

$$\Rightarrow \frac{x}{-2+5} = \frac{y}{-15+5} = \frac{1}{5-6}$$

$$\Rightarrow \frac{x}{3} = \frac{y}{-10} = \frac{1}{-1}$$

$\Rightarrow$ $x = -1 \times 3 = -3$ and $y = -1 \times (-10) = 10$

Hence, $\boldsymbol{x = -3}$ and $\boldsymbol{y = 10}$.

2.
$$-x + 3y = 7$$
$$-4x + 12y = 28$$

The given pair of linear equations may be written as

$$-x + 3y - 7 = 0$$
$$-4x + 12y - 28 = 0$$

These equations are of the form $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$

where $a_1 = -1,\quad b_1 = 3,\quad c_1 = -7$

$a_2 = -4,\quad b_2 = 12,\quad c_2 = -28$

$$\therefore \quad \frac{a_1}{a_2} = \frac{1}{4}$$

$$\frac{b_1}{b_2} = \frac{3}{12} = \frac{1}{4}$$

and
$$\frac{c_1}{c_2} = \frac{-7}{-28} = \frac{1}{4}$$

$$\therefore \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \frac{1}{4} = k \ (say)$$

$$c_2 = -28$$

$$kc_2 = \frac{1}{4} \times -28 = -7$$

$$\therefore \quad c_1 = kc_2$$

Hence, the given pair of linear equations has **infinitely many solutions**.

3.
$$x - 3y = 3$$
$$3x - 9y = 2$$

The given pair of linear equations may be written as

$$x - 3y - 3 = 0$$
$$3x - 9y - 2 = 0$$

These equations are of the form $a_1x + b_1y + c_1 = 0$

and $a_2x + b_2y + c_2 = 0$

where, $a_1 = 1,\quad b_1 = -3,\quad c_1 = -3$

$a_2 = 3,\quad b_2 = -9,\quad c_2 = -2$

$$\therefore \quad \frac{a_1}{a_2} = \frac{1}{3},$$

$$\frac{b_1}{b_2} = \frac{-3}{-9} = \frac{1}{3}$$

and
$$\frac{c_1}{c_2} = \frac{-3}{-2} = \frac{3}{2}$$

$$\therefore \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{1}{3} = k \ (say)$$

$$c_1 = -3$$

$$kc_2 = \frac{1}{3} \times (-2) = \frac{-2}{3}$$

Thus, $c_1 \neq kc_2$

Hence, the given pair of linear equations has **no solution**.

4. (*i*)
$$7x - y = 5$$
$$21x - 3y = k$$

The given pair of linear equations may be written as

$$7x - y - 5 = 0$$
$$21x - 3y - k = 0$$

These equations are of the form $a_1x + b_1y + c_1 = 0$ and $a_2 x + b_2 y + c_2 = 0$,

where $a_1 = 7,\quad b_1 = -1,\quad c_1 = -5$

$a_2 = 21,\quad b_2 = -3,\quad c_2 = -k$

$\therefore \quad \frac{a_1}{a_2} = \frac{7}{21} = \frac{1}{3}$

$\frac{b_1}{b_2} = \frac{-1}{-3} = \frac{1}{3}$

and $\quad \frac{c_1}{c_2} = \frac{-5}{-k} = \frac{5}{k}$

For the given pair of linear equation to be consistent (*dependent*) with infinitely many solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$\therefore \quad \frac{1}{3} = \frac{5}{k}$

$\Rightarrow \quad k = 15$

Hence, $\boldsymbol{k = 15}$.

(*ii*) $x - 4y = 6,\ 3x + ky = 5$

The given pair of the linear equations may be written as

$$x - 4y - 6 = 0$$
$$3x + ky - 5 = 0$$

These equations are of the form $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$,

where, $a_1 = 1,\quad b_1 = -4,\quad c_1 = -6$

$a_2 = 3,\quad b_2 = k,\quad c_2 = -5$

$\frac{a_1}{a_2} = \frac{1}{3},\ \frac{b_1}{b_2} = \frac{-4}{k}$ and $\frac{c_1}{c_2} = \frac{-6}{-5} = \frac{6}{5}$

For the given pair of linear equation to be inconsistent with no solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$\therefore \quad \frac{1}{3} = \frac{-4}{k} \neq \frac{6}{5}$

$\Rightarrow \quad \frac{1}{3} = \frac{-4}{k}$

$\Rightarrow \quad k = -12$

When $k = -12$, we have

$\frac{-4}{k} \neq \frac{6}{5} \qquad \left[\because \frac{-4}{-12} \neq \frac{6}{5}\right]$

$\therefore \quad \frac{1}{3} = \frac{-4}{k} \neq \frac{6}{5} \qquad$ [When $k = -12$]

Hence, the given pair of linear equations are inconsistent when $\boldsymbol{k = -12}$.

5. $18x - 7y = 24$

$\frac{9}{5}x - \frac{7}{10}y = \frac{9}{10}$

$\Rightarrow \quad 18x - 7y = 9$

The given pair of the linear equations may be written as

$$18x - 7y - 24 = 0$$
$$18x - 7y - 9 = 0$$

These equations are of the form $a_1 x + b_1 y + c_1 = 0$ and $a_2x + b_1y + c_2 = 0$,

where, $a_1 = 18,\quad b_1 = -7,\quad c_1 = -24$

$a_2 = 18,\quad b_2 = -7,\quad c_2 = -9$

$\therefore \quad \frac{a_1}{a_2} = \frac{18}{18} = 1,$

$\frac{b_1}{b_2} = \frac{-7}{-7} = 1$

and $\quad \frac{c_1}{c_2} = \frac{-24}{-9} = \frac{8}{3}$

$\therefore \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Hence, the given pair of linear equations has no solution and the lines representing them are **parallel** and **non-coincident**.

6. $kx + 2y = 3$

$2x - 3y = 1$

The given pair of linear equations is

$$kx + 2y - 3 = 0$$
$$2x - 3y - 1 = 0$$

These equations are of the form $a_1 x + b_1 y + c_1 = 0$ and $a_2x + b_1 y + c_2 = 0$,

where $a_1 = k,\quad b_1 = 2,\quad c_1 = -3$

$a_2 = 2,\quad b_2 = -3,\quad c_2 = -1$

For unique solution, we must have

$\frac{a_1}{a_2} \neq \frac{b_1}{b},$

$\Rightarrow \quad \frac{k}{2} \neq \frac{2}{-3}$

$\Rightarrow \quad k \neq \frac{-4}{3}$

Hence, the given pair of linear equations has a unique solution for all real values of k other than $\frac{-4}{3}$.

Hence, $\boldsymbol{k \neq \frac{-4}{3}}$.

7. $3x + 4y - 20 = 0$

$5x - ky + 14 = 0$

These equations are of the form $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$,

where $a_1 = 3,\quad b_1 = 4,\quad c_1 = -20$

$a_2 = 5,\quad b_2 = -k,\quad c_2 = 14$

For unique solution, we must have

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

$\Rightarrow \quad \frac{3}{5} \neq \frac{4}{-k}$

$$\Rightarrow \quad k \neq \frac{-4 \times 5}{3}$$

$$\Rightarrow \quad k \neq \frac{-20}{3}$$

Hence, the given pair of linear equations has a unique solution for a real values of k other than $\frac{-20}{3}$.

Hence, $\boldsymbol{k \neq \frac{-20}{3}}$.

8. $3x - 4y + 7 = 0$
$kx + 3y - 5 = 0$

These equations are of the form $a_1x + b_1y + c_1 = 0$ and $a_2 x + b_2y + c_2 = 0$

where $a_1 = 3, \quad b_1 = -4, \quad c_1 = 7$
$a_2 = k, \quad b_2 = 3, \quad c_2 = -5$

$$\therefore \quad \frac{a_1}{a_2} = \frac{3}{k},$$

$$\frac{b_1}{b_2} = \frac{-4}{3}$$

and $$\frac{c_1}{c_2} = \frac{7}{-5}$$

For the given pair of linear equations to have no solution,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\Rightarrow \quad \frac{3}{k} = \frac{-4}{3} \neq \frac{7}{-5}$$

$$\Rightarrow \quad \frac{3}{k} = \frac{-4}{3}$$

and $$\frac{3}{k} \neq \frac{7}{-5}$$

$$\Rightarrow \quad k = \frac{-9}{4}$$

and $$k \neq -\frac{15}{7}$$

Clearly, $k = \frac{-9}{4}$ also satisfy the condition $k \neq -\frac{15}{7}$.

Hence, the given pair of linear equations has no solution when $\boldsymbol{k = \frac{-9}{4}}$.

9. $3x - y - 5 = 0$
$6x - 2y - k = 0$

These equations are of the form $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$,

where $a_1 = 3, \quad b_1 = -1, \quad c_1 = -5$
$a_2 = 6, \quad b_2 = -2, \quad c_2 = -k$

$$\therefore \quad \frac{a_1}{a_2} = \frac{3}{6} = \frac{1}{2}$$

$$\frac{b_1}{b_2} = \frac{-1}{-2} = \frac{1}{2}$$

and $$\frac{c_1}{c_2} = \frac{-5}{-k} = \frac{5}{k}$$

For the given pair of linear equations to have no solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\therefore \quad \frac{1}{2} = \frac{1}{2} \neq \frac{5}{k}$$

$$\Rightarrow \quad \frac{1}{2} \neq \frac{5}{k}$$

$$\Rightarrow \quad k \neq 10$$

Thus, for all real values of k other than 10, the given pair of linear equations has no solution.

Hence, $\boldsymbol{k \neq 10}$.

10. $2x + 3y = 9$; $6x + (k - 2)y = (3k - 2)$

The given equations may be written as

$2x + 3y - 9 = 0$
$6x + (k - 2)y - (3k - 2) = 0$

These equations are of the form $a_1x + b_1 y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$,

where $a_1 = 2, \quad b_1 = 3, \quad c_1 = -9$
$a_2 = 6, \quad b_2 = k - 2, \quad c_2 = -(3k - 2)$

$$\therefore \quad \frac{a_1}{a_2} = \frac{2}{6} = \frac{1}{3},$$

$$\frac{b_1}{b_2} = \frac{3}{k-2},$$

$$\frac{c_1}{c_2} = \frac{-9}{-(3k-2)} = \frac{9}{(3k-2)}$$

For the given pair of linear equations to have no solution,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\Rightarrow \quad \frac{1}{3} = \frac{3}{k-2} \neq \frac{9}{(3k-2)}$$

$$\Rightarrow \quad \frac{1}{3} = \frac{3}{k-2}$$

and $$\frac{3}{k-2} \neq \frac{9}{(3k-2)}$$

$$\Rightarrow \quad \frac{1}{3} = \frac{3}{k-2}$$

$$\Rightarrow \quad k - 2 = 9$$

$$\Rightarrow \quad k = 11$$

When, $k = 11$, we have

$$\frac{3}{k-2} \neq \frac{9}{(3k-2)}$$

$$\left[\because \frac{3}{11-2} \neq \frac{9}{(33-2)}\right]$$

$$\therefore \quad \frac{1}{3} = \frac{3}{k-2} \neq \frac{9}{3k-2}$$

[When $k = 11$]

Hence, the given pair of linear equations has no solution when $\boldsymbol{k = 11}$.

11. $(2p - 1)x + (p - 1)y = 2p + 1$; $x + 3y - 1 = 0$

The given equations are

$(2p - 1)x + (p - 1)y - (2p + 1) = 0$

$$x + 3y - 1 = 0$$

These equations are of the form $a_1x + b_1 y + c_1 = 0$ and $a_2 x + b_2y + c_2 = 0$,

where $a_1 = (2p - 1), b_1 = (p - 1), \quad c_1 = -(2p + 1)$

$a_2 = 1, \quad b_2 = 3, \quad c_2 = -1$

$$\therefore \quad \frac{a_1}{a_2} = \frac{(2p-1)}{1},$$

$$\frac{b_1}{b_2} = \frac{(p-1)}{3},$$

$$\frac{c_1}{c_2} = \frac{-(2p+1)}{-1} = \frac{(2p+1)}{1}$$

For given pair of equations to have no solution,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\Rightarrow \quad \frac{(2p-1)}{1} = \frac{(p-1)}{3} \neq \frac{(2p+1)}{1}$$

$$\Rightarrow \quad \frac{(2p-1)}{1} = \frac{(p-1)}{3}$$

$$\text{and} \quad \frac{(p-1)}{3} \neq \frac{(2p+1)}{1}$$

$$\Rightarrow \quad 6p - 3 = p - 1$$

$$\Rightarrow \quad 5p = 2$$

$$\Rightarrow \quad p = \frac{2}{5}$$

When $p = \frac{2}{5}$, we have

$$\frac{p-1}{3} \neq \frac{2p+1}{1} \quad \left[\frac{\frac{2}{5}-1}{3} \neq 2 \times \frac{\frac{2}{5}+1}{1}\right]$$

$$\therefore \quad \frac{2p-1}{1} = \frac{p-1}{3} \neq \frac{2p+1}{1}$$

$$\left[\text{When } p = \frac{2}{5}\right]$$

Hence, the given pair of linear equations has no solution when $\boldsymbol{p = \frac{2}{5}}$.

12. $(3k + 1)x + 3y - 2 = 0$

$(k^2 + 1)x + (k - 2) y - 5 = 0$

The given equations are of the form $a_1x + b_1y + c_1 = 0$ and $a_2 x + b_2 y + c_2 = 0$

where $a_1 = (3k + 1), \quad b_1 = 3, \quad c_1 = -2$

$a_2 = (k^2 + 1), \quad b_2 = (k - 2), \quad c_2 = -5$

$$\therefore \quad \frac{a_1}{a_2} = \frac{(3k+1)}{(k^2+1)},$$

$$\frac{b_1}{b_2} = \frac{3}{(k-2)}$$

$$\text{and} \quad \frac{c_1}{c_2} = \frac{-2}{-5} = \frac{2}{5}$$

For the given pair of linear equations to have no solution,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\therefore \quad \frac{(3k+1)}{(k^2+1)} = \frac{3}{(k-2)} \neq \frac{2}{5}$$

$$\Rightarrow \quad \frac{(3k+1)}{(k^2+1)} = \frac{3}{(k-2)}$$

$$\text{and} \quad \frac{3}{k-2} \neq \frac{2}{5}$$

$$\Rightarrow \quad 3k^2 + k - 6k - 2 = 3k^2 + 3$$

$$\Rightarrow \quad -5k - 2 = 3$$

$$\Rightarrow \quad 5k = -5$$

$$\Rightarrow \quad k = -1$$

When $k = -1$, we have

$$\frac{3}{k-2} \neq \frac{2}{5} \qquad \left[\because \frac{3}{-1-2} \neq \frac{2}{5}\right]$$

$$\therefore \quad \frac{3k+1}{k^2+1} = \frac{3}{k-2} \neq \frac{2}{5} \quad [\text{when } k = -1]$$

Hence, the given pair of linear equations has no solutions when $\boldsymbol{k = -1}$.

13. $kx + 3y = 1 \,;\, 12x + ky = 2$

The given equation may be written as

$kx + 3y - 1 = 0$

$12x + ky - 2 = 0$

These equations are of the form $a_1x + b_1 y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$.

where $a_1 = k, \quad b_1 = 3, \quad c_1 = -1$

$a_2 = 12, \quad b_2 = k, \quad c_2 = -2$

$$\therefore \quad \frac{a_1}{a_2} = \frac{k}{12},$$

$$\frac{b_1}{b_2} = \frac{3}{k}$$

$$\text{and} \quad \frac{c_1}{c_2} = \frac{-1}{-2} = \frac{1}{2}$$

For the given pair of linear equations to have no solution,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\Rightarrow \quad \frac{k}{12} = \frac{3}{k} \neq \frac{1}{2}$$

$$\Rightarrow \quad \frac{k}{12} = \frac{3}{k}$$

$$\text{and} \quad \frac{3}{k} \neq \frac{1}{2}$$

$$\Rightarrow \quad k^2 = 36$$

$$\text{and} \quad 6 \neq k$$

$$\Rightarrow \quad k = -6$$

Clearly, $k = -6$ also satisfy the condition $k \neq 6$.

Hence, the given pair of linear equations has no solution when $\boldsymbol{k = -6}$.

14. $4x - y = 11;$

$kx + 3y = 5$

The given equations may be written as

$4x - y - 11 = 0$

$kx + 3y - 5 = 0$

These equations are of the form $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$,

where $a_1 = 4$, $b_1 = -1$, $c_1 = -11$
$a_2 = k$, $b_2 = 3$, $c_2 = -5$

$\therefore$ $$\frac{a_1}{a_2} = \frac{4}{k},$$

$$\frac{b_1}{b_2} = \frac{-1}{3}$$

and $$\frac{c_1}{c_2} = \frac{-11}{-5} = \frac{11}{5}$$

(*i*) For the given pair of linear equations to have a unique solution,

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

$\Rightarrow$ $$\frac{4}{k} \neq \frac{-1}{3}$$

$\Rightarrow$ $-k \neq 12$
$\Rightarrow$ $k \neq -12$

So, the given pair of linear equations will have a unique solution of all real values of k other than –12.
Hence, $\boldsymbol{k \neq -12}$.

(*ii*) For the given pair of linear equations to have no solution,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$\Rightarrow$ $$\frac{4}{k} = \frac{-1}{3} \neq \frac{-11}{-5}$$

$\Rightarrow$ $$\frac{4}{k} = \frac{-1}{3}$$

and $$\frac{4}{k} \neq \frac{11}{5}$$

$\Rightarrow$ $k = -12$

and $$k \neq \frac{20}{11}$$

Clearly, $k = -12$ also satisfies the condition $k \neq \frac{20}{11}$.

Hence, the given pair of linear equations has no solution when $\boldsymbol{k = -12}$.
For the given pair of linear equations to have infinite number of solution,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$\Rightarrow$ $$\frac{4}{k} = \frac{-1}{3} = \frac{11}{5},$$

which is not possible

$\because$ $$\frac{-1}{3} \neq \frac{11}{5}$$

Hence, there is **no** value of k for which the two equations will have infinitely many solutions.

15. $$5x + 2y = k$$
$$10x + 4y = 3$$

The given pair of linear equations may be written

$$5x + 2y - k = 0$$
$$10x + 4y - 3 = 0$$

These equations are of the form $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$,

where $a_1 = 5$, $b_1 = 2$, $c_1 = -k$
$a_2 = 10$, $b_2 = 4$, $c_2 = -3$

$\therefore$ $$\frac{a_1}{a_2} = \frac{5}{10} = \frac{1}{2},$$

$$\frac{b_1}{b_2} = \frac{2}{4} = \frac{1}{2}$$

and $$\frac{c_1}{c_2} = \frac{-k}{-3} = \frac{k}{3}$$

For the given pair of linear equations to have infinite number of solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$\Rightarrow$ $$\frac{1}{2} = \frac{1}{2} = \frac{k}{3}$$

$\Rightarrow$ $$k = \frac{3}{2}$$

Hence, $\boldsymbol{k = \frac{3}{2}}$.

16. $$10x + 5y - (k - 5) = 0$$
$$20x + 10y - k = 0$$

These equations are of the form $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$.
where $a_1 = 10$, $b_1 = 5$, $c_1 = -(k - 5)$
$a_2 = 20$, $b_2 = 10$, $c_2 = -k$

$\therefore$ $$\frac{a_1}{a_2} = \frac{10}{20} = \frac{1}{2},$$

$$\frac{b_1}{b_2} = \frac{5}{10} = \frac{1}{2}$$

and $$\frac{c_1}{c_2} = \frac{-(k-5)}{-k} = \frac{k-5}{k}$$

For the given pair of linear equations to have infinite number of solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$\Rightarrow$ $$\frac{1}{2} = \frac{1}{2} = \frac{k-5}{k}$$

$\Rightarrow$ $k = 2k - 10$
$\Rightarrow$ $2k - k = 10$
$\Rightarrow$ $k = 10$

Hence, $\boldsymbol{k = 10}$.

17. $$2x + 3y = 7$$
$$(k - 1)\, x + (k + 2)\, y = 3k$$

The given pair of linear equations may be written as

$$2x + 3y - 7 = 0$$
$$(k - 1)x + (k + 2)y - 3k = 0$$

These equations are of the form $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$,
where $a_1 = 2$, $b_1 = 3$, $c_1 = -7$
$a_2 = (k - 1)$, $b_2 = (k + 2)$, $c_2 = -3k$

$\therefore$ $$\frac{a_1}{a_2} = \frac{2}{(k-1)},$$

$$\frac{b_1}{b_2} = \frac{3}{(k+2)}$$

and $$\frac{c_1}{c_2} = \frac{-7}{-3k} = \frac{7}{3k}$$

For the given pair of linear equations to have infinite number of solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\Rightarrow \quad \frac{2}{(k-1)} = \frac{3}{(k+2)} = \frac{7}{3k}$$

$$\Rightarrow \quad \frac{2}{(k-1)} = \frac{3}{(k+2)}$$

$\Rightarrow \quad 2k + 4 = 3k - 3$

$\Rightarrow \quad 4 + 3 = 3k - 2k$

$\Rightarrow \quad 7 = k$

$$\frac{3}{(k+2)} = \frac{7}{3k}$$

$\Rightarrow \quad 9k = 7k + 14$

$\Rightarrow \quad 9k - 7k = 14$

$\Rightarrow \quad 2k = 14$

$\Rightarrow \quad k = 7$

and $$\frac{2}{k-1} = \frac{7}{3k}$$

$6k = 7k - 7$

$k = 7$

Hence, $\boldsymbol{k = 7}$.

18. $kx + 3y = k - 3$

$12x + ky = k$

The given pair of linear equations may be written as

$kx + 3y - (k - 3) = 0$

$12x + ky - k = 0$

These equations are of the form $a_1x + b_2\, y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$,

where $a_1 = k, \quad b_1 = 3, \quad c_1 = -(k - 3)$

$a_2 = 12, \quad b_2 = k, \quad c_2 = -k$

$$\therefore \quad \frac{a_1}{a_2} = \frac{k}{12}, \ \frac{b_1}{b_2} = \frac{3}{k}$$

and $$\frac{c_1}{c_2} = \frac{-(k-3)}{-k} = \frac{(k-3)}{k}$$

For the given pair of linear equations to have infinite number of solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\Rightarrow \quad \frac{k}{12} = \frac{3}{k} = \frac{(k-3)}{k}$$

$$\Rightarrow \quad \frac{k}{12} = \frac{3}{k}$$

$\Rightarrow \quad k^2 = 36$

$\Rightarrow \quad k = \pm 6$

and $$\frac{3}{k} = \frac{k-3}{k}$$

$\Rightarrow \quad 3k = k^2 - 3k$

$\Rightarrow \quad k^2 - 6k = 0$

$\Rightarrow \quad k(k - 6) = 0$

$\Rightarrow \quad k = 0 \text{ or } k = 6$

and $$\frac{k}{12} = \frac{k-3}{k}$$

$k^2 = 12k - 36$

$\Rightarrow \quad k^2 - 12k + 36 = 0$

$\Rightarrow \quad (k - 6)(k - 6) = 0$

$\Rightarrow \quad k = 6 \text{ or } k = 6$

Thus, $k = 6$ is the common solution.

Hence, the given pair of linear equations has an infinite number of solutions when $\boldsymbol{k = 6}$.

19. $(k - 1)x - y = 5$

$(k + 1)x + (1 - k)y = 3k + 1$

The given equations may be written as

$(k - 1)x - y - 5 = 0$

$(k + 1)x + (1 - k)y - (3k + 1) = 0$

These equations are of the form $a_1x + b_2, y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$,

where $a_1 = (k - 1), \quad b_1 = -1, \quad c_1 = -5$

$a_2 = (k + 1), \quad b_2 = (1 - k), \quad c_2 = -(3k + 1)$

$$\therefore \quad \frac{a_1}{a_2} = \frac{(k-1)}{(k+1)},$$

$$\frac{b_1}{b_2} = \frac{-1}{(1-k)}$$

and $$\frac{c_1}{c_2} = \frac{-5}{-(3k+1)} = \frac{5}{(3k+1)}$$

For the given pair of linear equations to have infinite number of solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\Rightarrow \quad \frac{k-1}{k+1} = \frac{-1}{(1-k)} = \frac{5}{(3k+1)}$$

$$\Rightarrow \quad \frac{k-1}{k+1} = \frac{-1}{(1-k)}$$

$\Rightarrow \quad (k - 1)(1 - k) = -(k + 1)$

$\Rightarrow \quad k - 1 - k^2 + k = -k - 1$

$\Rightarrow \quad -k^2 + 2k - 1 = -k - 1$

$\Rightarrow \quad k^2 - 3k = 0$

$\Rightarrow \quad k(k - 3) = 0$

$\Rightarrow \quad k = 0 \text{ or } k = 3$

and $$\frac{-1}{(1-k)} = \frac{5}{(3k+1)}$$

$\Rightarrow \quad -3k - 1 = 5 - 5k$

$\Rightarrow \quad 5k - 3k = 5 + 1$

$\Rightarrow \quad 2k = 6$

$$\Rightarrow \quad k = \frac{6}{2}$$

$\Rightarrow \quad k = 3$

and $$\frac{k-1}{k+1} = \frac{5}{3k+1}$$

$\Rightarrow \quad 3k^2 + k - 3k - 1 = 5k + 5$

$\Rightarrow \quad 3k^2 - 7k - 6 = 0$

$\Rightarrow \quad 3k^2 - 9k + 2k - 6 = 0$

$\Rightarrow \quad 3k(k - 3) + 2(k - 3) = 0$

$\Rightarrow \quad (k - 3)(3k + 2) = 0$

$$\Rightarrow \quad k = 3 \text{ or } k = \frac{-2}{3}$$

Thus, $k = 3$ is the common solution.

Hence, the given pair of linear equations has an infinite number of solutions when $\boldsymbol{k = 3}$.

20. $2x + (k-2)y = k$

$6x + (2k-1)y = 2k + 5$

The given pair of linear equations may be written as

$2x + (k-2)y - k = 0$

$6x + (2k-1)y - (2k+5) = 0$

These equations are of the form $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$,

where $a_1 = 2$, $b_1 = (k-2)$, $c_1 = -k$

$a_2 = 6$, $b_2 = (2k-1)$, $c_2 = -(2k+5)$

$\therefore \quad \frac{a_1}{a_2} = \frac{2}{6} = \frac{1}{3},$

$\frac{b_1}{b_2} = \frac{(k-2)}{(2k-1)}$

and $\frac{c_1}{c_2} = \frac{-k}{-(2k+5)} = \frac{k}{(2k+5)}$

For the given pair of linear equation to have infinite number of solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$\Rightarrow \quad \frac{1}{3} = \frac{(k-2)}{(2k-1)} = \frac{k}{(2k+5)}$

$\Rightarrow \quad \frac{1}{3} = \frac{(k-2)}{(2k-1)}$

$\Rightarrow \quad 2k - 1 = 3k - 6$

$\Rightarrow \quad 3k - 2k = 6 - 1$

$\Rightarrow \quad k = 5$

and $\frac{k-2}{(2k-1)} = \frac{k}{(2k+5)}$

$\Rightarrow \quad 2k^2 + 5k - 4k - 10 = 2k^2 - k$

$\Rightarrow \quad k - 10 = -k$

$\Rightarrow \quad 2k = 10$

$\Rightarrow \quad k = 5$

and $\frac{1}{3} = \frac{k}{2k+5}$

$\Rightarrow \quad 2k + 5 = 3k$

$\Rightarrow \quad k = 5$

Hence, the given pair of linear equations has infinite number of solutions when $\boldsymbol{k = 5}$.

21. The given two equations have an infinitely many solutions if

$$\frac{p-3}{p} = \frac{3}{p} = \frac{p}{12}$$

$\therefore \quad \frac{p-3}{p} = \frac{p}{12} \quad \ldots (1)$

$\frac{3}{p} = \frac{p}{12} \quad \ldots (2)$

From (2), $p^2 = 36$

$\therefore \quad p = \pm 6$

But $p = -6$ does not satisfy (1). Hence, the required value of p is 6.

22. The given equations have infinitely many solutions if

$$\frac{c}{12} = \frac{3}{c} = \frac{3-c}{-c}$$

$\therefore \quad c^2 = 36 \quad \ldots (1)$

$\frac{c}{12} = \frac{3-c}{-c} \quad \ldots (2)$

From (1), $c = \pm 6$

But $c = -6$ does not satisfy (2).

$\therefore$ The required value of c is **6**.

23. $3x - (a+1)y = 2b - 1$

$5x + (1-2a)y = 3b$

The given pair of linear equations may be written as

$3x - (a+1)y - (2b-1) = 0$

$5x + (1-2a)y - 3b = 0$

These equations are of the form $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$,

where $a_1 = 3$, $b_1 = -(a+1)$, $c_1 = -(2b-1)$

$a_2 = 5$, $b_2 = (1-2a)$, $c_2 = -3b$

$\therefore \quad \frac{a_1}{a_2} = \frac{3}{5},$

$\frac{b_1}{b_2} = \frac{-(a+1)}{(1-2a)}$

and $\frac{c_1}{c_2} = \frac{-(2b-1)}{-3b} = \frac{(2b-1)}{3b}$

For the given pair of linear equations to have infinite number of solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$\Rightarrow \quad \frac{3}{5} = \frac{-(a+1)}{1-2a} = \frac{(2b-1)}{3b}$

$\Rightarrow \quad \frac{3}{5} = \frac{-(a+1)}{1-2a}$

and $\frac{3}{5} = \frac{2b-1}{3b}$

$\Rightarrow \quad 3 - 6a = -5a - 5$

and $9b = 10b - 5$

$\Rightarrow \quad 3 + 5 = 6a - 5a$

and $10b - 9b = 5$

$\Rightarrow \quad 8 = a$

and $b = 5$

Hence, $\boldsymbol{a = 8}$ and $\boldsymbol{b = 5}$.

24. $2x - 3y = 7$

$(a+b)x - (a+b-3)y = 4a + b$

The given pair of linear equations may be written as

$2x - 3y - 7 = 0$

$(a+b)x - (a+b-3)y - (4a+b) = 0$

These equations are of the form $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$,

where $a_1 = 2$, $b_1 = -3$, $c_1 = -7$

$a_2 = (a+b)$, $b_2 = -(a+b-3)$, $c_2 = -(4a+b)$

$\therefore \quad \frac{a_1}{a_2} = \frac{2}{(a+b)},$

$\frac{b_1}{b_2} = \frac{-3}{-(a+b-3)} = \frac{3}{(a+b-3)}$

and $\frac{c_1}{c_2} = \frac{-7}{-(4a+b)} = \frac{7}{(4a+b)}$

For the given pair of linear equations to have infinite number of solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\Rightarrow \quad \frac{2}{(a+b)} = \frac{3}{(a+b-3)} = \frac{7}{(4a+b)}$$

$$\Rightarrow \quad \frac{2}{(a+b)} = \frac{3}{(a+b-3)}$$

and $$\frac{3}{(a+b-3)} = \frac{7}{(4a+b)}$$

$\Rightarrow \quad 2a + 2b - 6 = 3a + 3b$

and $\quad 12a + 3b = 7a + 7b - 21$

$\Rightarrow \quad a + b = -6 \quad \ldots (1)$

and $\quad 5a - 4b = -21 \quad \ldots (2)$

Solving equations (1) and (2), we get

$$a = -5 \text{ and } b = -1$$

Hence, $\boldsymbol{a = -5}$ and $\boldsymbol{b = -1}$.

25.
$$2x + 3y = 7$$
$$(a + b + 1)x + (a + 2b + 2)y = 4(a + b) + 1$$

The given pair of linear equations may be written as

$$2x + 3y - 7 = 0$$

$$(a + b + 1)x + (a + 2b + 2)y - [4(a + b) + 1] = 0$$

These equations are of the form $a_1x\ b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$,

where $a_1 = 2, \quad b_1 = 3, \quad c_1 = -7$

$a_2 = (a + b + 1), \quad b_2 = (a + 2b + 2), \quad c_2 = -(4a + 4b + 1)$

$$\therefore \quad \frac{a_1}{a_2} = \frac{2}{a+b+1},$$

$$\frac{b_1}{b_2} = \frac{3}{a+2b+2}$$

and $$\frac{c_1}{c_2} = \frac{-7}{-(4a+4b+1)} = \frac{7}{4a+4b+1}$$

For the given pair of linear equations to have infinite number of solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\Rightarrow \quad \frac{2}{a+b+1} = \frac{3}{a+2b+2} = \frac{7}{4a+4b+1}$$

$$\Rightarrow \quad \frac{2}{a+b+1} = \frac{3}{a+2b+2}$$

and $$\frac{3}{a+2b+2} = \frac{7}{4a+4b+1}$$

$\Rightarrow \quad 2a + 4b + 4 = 3a + 3b + 3$

and $\quad 12a + 12b + 3 = 7a + 14b + 14$

$\Rightarrow \quad 4 - 3 = 3a - 2a + 3b - 4b$

and $\quad 12a - 7a + 12b - 14b = 14 - 3$

$\Rightarrow \quad 1 = a - b \quad \ldots (1)$

and $\quad 5a - 2b = 11 \quad \ldots (2)$

Solving equation (1) and equation (2), we get

$$a = 3 \text{ and } b = 2$$

Hence, $\boldsymbol{a = 3}$ and $\boldsymbol{b = 2}$.

26.
$$2x - y + 8 = 0$$
$$4x - ky + 16 = 0$$

The given pair of linear equations are of the form $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$,

where $a_1 = 2, \quad b_1 = -1, \quad c_1 = 8$

$a_2 = 4, \quad b_2 = -k, \quad c_2 = 16$

$$\therefore \quad \frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2},$$

$$\frac{b_1}{b_2} = \frac{-1}{-k} = \frac{1}{k},$$

and $$\frac{c_1}{c_2} = \frac{8}{16} = \frac{1}{2}$$

The given pair of linear equations represents coincident lines where they have infinite number of solutions.

For infinite number of solutions, we must have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{1}{k} = \frac{1}{2}$$

$$\Rightarrow \quad k = 2$$

Hence, the given pair of linear equations will represent coincident lines, where $\boldsymbol{k = 2}$.

27.
$$5x - 3y = 0$$
$$4x + ky = 0$$

The given pair of linear equations are of the form $a_1x + b_1y = 0$ and $a_2x + b_2y = 0$,

where $a_1 = 5, \quad b_1 = -3$

$a_2 = 4, \quad b_2 = k$

For a non-zero solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2}$$

$$\Rightarrow \quad \frac{5}{4} = \frac{-3}{k}$$

$$\Rightarrow \quad k = \frac{-3 \times 4}{5} = \frac{-12}{5}$$

Hence, the given pair of linear equations has a non-zero solution when $\boldsymbol{k = \frac{-12}{5}}$.

28. The given equations are

$\lambda x + y - \lambda^2 = 0 \quad \ldots (1)$

and $\quad x + \lambda y - 1 = 0 \quad \ldots (2)$

(*i*) Equations (1) and (2) have no solution, if

$$\frac{\lambda}{1} = \frac{1}{\lambda} \neq \frac{-\lambda^2}{-1}$$

$$\therefore \quad \lambda^2 = 1$$

$$\Rightarrow \quad \lambda = \pm 1$$

But $\lambda = 1$ does not satisfy $\frac{1}{\lambda} \neq \lambda^2$

$\therefore$ For $\lambda = -1$ the given equations do not have any solution.

(*ii*) Equations (1) and (2) have infinitely many solutions, if

$$\frac{\lambda}{1} = \frac{1}{\lambda} = \frac{-\lambda^2}{-1}$$

We see that $\lambda^2 = 1$

$\Rightarrow$ $\lambda = \pm 1$

But $\lambda = -1$ does not satisfy $\frac{1}{\lambda} = \lambda^2$

$\therefore$ For $\lambda = 1$ only, the equations have infinitely many solution.

(*iii*) We see that (1) and (2) have a unique solution if $\frac{\lambda}{1} \neq \frac{1}{\lambda}$, i.e. $\lambda \neq \pm 1$.

Hence, the given equations have unique solution for all real values of λ except $\lambda = \pm 1$.

29. The equations $(a - b)x + (a + b)y = a + b - 2$ and $x + 2y = 1$ have infinitely many solution if

$$\frac{a-b}{1} = \frac{a+b}{2} = \frac{-(a+b-2)}{-1}$$

$\Rightarrow$ $2a - 2b = a + b$

and $a + b = 2a + 2b - 4$

$\Rightarrow$ $a - 3b = 0$ … (1)

$\Rightarrow$ $b + a - 4 = 0$ … (2)

From (1), $a = 3b$ … (3)

$\therefore$ From (2), $b + 3b - 4 = 0$

$\Rightarrow$ $b = 1$

$\therefore$ From (3), $a = 3$

Hence, the required values of a and b are **3** and **1** respectively.

30. (*i*) If the lines $3x - y - 5 = 0$ and $6x - 2y - p = 0$ are parallel, then

$$\frac{3}{6} = \frac{-1}{-2} \neq \frac{-5}{-p}$$

$\Rightarrow$ $\frac{1}{2} \neq \frac{5}{p}$

$\Rightarrow$ $p \neq 10$

Hence, the two lines represented by the given equations are parallel for **all real values of p except 10**.

(*ii*) If the given equations have no solution, then the lines represented by them are parallel to each other.

$\therefore$ $-\frac{1}{p} = \frac{p}{-1} \neq \frac{-1}{-1}$

$\Rightarrow$ $\frac{-1}{p} = \frac{p}{-1} \neq 1$

$\Rightarrow$ $\frac{1}{p} = p \neq -1$

Now, $\frac{1}{p} = p$

$\Rightarrow$ $p^2 = 1$

$\Rightarrow$ $p = \pm 1$

$\because$ $p \neq -1$

$\therefore$ $p = +1$

Hence, $\mathbf{p = 1}$ is the only solution.

Hence, the given equations have no solution at all for $p = 1$ only.

(*iii*) The given equations have a unique solution when $-\frac{3}{2p} \neq \frac{5}{-3}$, i.e. $p \neq \frac{9}{10}$

Thus the given equations have a unique solution for **all real values of p except $p = \frac{9}{10}$**

(*iv*) The given equations have a unique solution when $\frac{2}{p} \neq \frac{3}{-6} \Rightarrow \frac{2}{p} \neq -\frac{1}{2} \Rightarrow p \neq -4$.

$\therefore$ The given equations have a unique solution for **all real values of p except $p = -4$.**

(*v*) If the given equations have infinitely many solution, then

$$\frac{2}{2p} = \frac{3}{p+q} = \frac{-7}{-28}$$

$\Rightarrow$ $\frac{1}{p} = \frac{3}{p+q} = \frac{1}{4}$

$\therefore$ $p = 4$ … (1)

and $p + q = 12$ … (2)

$\therefore$ From (1) and (2), we have $q = 12 - 4 = 8$.

Hence, the required values of p and q are **4** and **8** respectively.

EXERCISE 3F

For Basic and Standard Levels

1. Let the required numbers be x and y.

Then, $x + y = 105$ … (1)

and $x - y = 45$ … (2)

Adding (1) and (2),

$2x = 150$

$\Rightarrow$ $x = 75$

Substituting $x = 75$ in equation (1), we get

$y = 30$

Hence, the required numbers are **75** and **30**.

2. Let the required numbers be x and y.

Let $x = 3a$.

Then, $y = 4a$

$$\frac{3a+8}{4a+8} = \frac{4}{5}$$

$\Rightarrow$ $15a + 40 = 16a + 32$

$\Rightarrow$ $a = 8$

$\therefore$ $x = 3a = 3 \times 8 = 24$

and $y = 4a = 4 \times 8 = 32$

Hence, the required numbers are **24** and **32**.

3. Let the required numbers be x and y.

Then, $x + y = 16$

and $\frac{1}{x} + \frac{1}{y} = \frac{1}{3}$

$\Rightarrow$ $\frac{x+y}{xy} = \frac{1}{3}$

$$\Rightarrow \quad \frac{16}{xy} = \frac{1}{3}$$

$$\Rightarrow \quad xy = 48$$

Now, $(x + y)^2 - (x - y)^2 = 4xy$

$\Rightarrow \quad (16)^2 - (x - y)^2 = 4 \times 48$

$\Rightarrow \quad 256 - 192 = (x - y)^2$

$\Rightarrow \quad 64 = (x - y)^2$

$\Rightarrow \quad \pm 8 = x - y$

When $x - y = 8$

and $x + y = 16,$

Then, $x = 12$

and $y = 4$

When $x - y = -8$

and $x + y = 16,$

Then, $x = 4$

and $y = 12$

Hence, the required numbers are **4** and **12**.

4. Let the required larger number be x and the required smaller number be y.

Then, $x - y = 4$

and $\frac{1}{y} - \frac{1}{x} = \frac{4}{21}$

$$\Rightarrow \quad \frac{x - y}{xy} = \frac{4}{21}$$

$$\Rightarrow \quad \frac{4}{xy} = \frac{4}{21}$$

$\Rightarrow \quad xy = 21$

$(x + y)^2 - (x - y)^2 = 4xy$

$\Rightarrow \quad (x + y)^2 = 4xy + (x - y)^2$

$= 4 \times 21 + (4)^2$

$= 84 + 16 = 100$

$\Rightarrow \quad x + y = \pm 10$

When, $x + y = 10$

and $x - y = 4,$

Then, $x = 7$

and $y = 3$

When $x + y = -10$

and $x - y = 4,$

Then, $y = -3$

and $y = -7$

Hence, the required numbers are **7** and **3** or **–3** and **–7**.

5. Let the two numbers be $5x$ and $6x$ where $x \neq 0$ so that their ratio is 5 : 6.

According to the problem,

$$\frac{5x - 8}{6x - 8} = \frac{4}{5}$$

$\Rightarrow \quad 25x - 40 = 24x - 32$

$\Rightarrow \quad 25x - 24x = 40 - 32$

$\Rightarrow \quad x = 8$

Hence, the required numbers are 5×8 and 6×8, i.e. **40** and **48**.

6. Let the ten's and unit's digit of the required two-digit number be x and y respectively.

Then, the required number = $10x + y$

and the number obtained by interchanging the two digits = $10y + x$

$x + y = 12 \quad \ldots (1)$

and $(10y + x) - (10x + y) = 18$

$\Rightarrow \quad 10y + x - 10\,x - y = 18$

$\Rightarrow \quad 9y - 9x = 18$

$\Rightarrow \quad y - x = 2 \quad \ldots(2)$

Solving equation (1) and equation (2), we get

$x = 5$

and $y = 7$

Hence, the required number is $10 \times 5 + 7 =$ **57**.

7. Let the ten's and unit's digit of the required two digit number be x and y respectively.

Then, the required number = $10x + y$

$x + y = 8 \quad \ldots (1)$

and $10x + y + 36 = 10\,y + x$

$\Rightarrow \quad 10x - x + y - 10\,y = -36$

$\Rightarrow \quad 9x - 9y = -36$

$\Rightarrow \quad x - y = -4 \quad \ldots (2)$

Solving equations (1) and (2), we get

$x = 2, y = 6$

Hence, the required number = $10 \times 2 + 6 =$ **26**.

8. Let x and y be the digits in the unit place and the ten's place respectively of the two-digit number. Then the number is $10y + x$.

According to the problem,

$10y + x = 4(x + y) + 3$

$\Rightarrow \quad 10y + x - 4x - 4y = 3$

$\Rightarrow \quad 6y - 3x = 3$

$\Rightarrow \quad x - 2y + 1 = 0 \quad \ldots (1)$

and $10y + x + 18 = 10x + y$

$\Rightarrow \quad 10y + x - 10x - y + 18 = 0$

$\Rightarrow \quad -9x + 9y + 18 = 0$

Subtracting (2) from (1), we get

$-y + 3 = 0$

$\Rightarrow \quad y = 3$

$\therefore$ From (1), $x = 2y - 1$

$= 2 \times 3 - 1 = 5$

Hence, the required number is $10 \times 3 + 5 =$ **35**.

9. Let the ten's and unit's digit of the required two digit number be x and y respectively.

Then, the required number = $10x + y$

and the number formed by interchanging the digits = $10y + x$

$10x + y + 10y + x = 132$

$\Rightarrow \quad 11x + 11y = 132$

$\Rightarrow \quad x + y = 12 \quad \ldots (1)$

and $10x + y + 12 = 5\,(x + y)$

$\Rightarrow 10x - 5x + y - 5y + 12 = 0$

$\Rightarrow \quad 5x - 4y = -12 \quad \ldots (2)$

Solving equation (1) and equation (2), we get

$x = 4$

and $y = 8$

Hence, the required number = $10 \times 4 \times 8 =$ **48**.

10. Let the ten's and unit's digit of the required two-digit number be x and y respectively.

Then, the required number = $10x + y$

$10x + y = 7\,(x + y)$

$\Rightarrow \quad 10x + y - 7x - 7y = 0$

$\Rightarrow \quad 3x - 6y = 0$

$\Rightarrow \quad x - 2y = 0$

$\Rightarrow \quad x = 2y \quad \dots (1)$

and $\quad 10x + y - 27 = 10y + x$

$\Rightarrow \quad 10x - x + y - 10y = 27$

$\Rightarrow \quad 9x - 9y = 27$

$\Rightarrow \quad x - y = 3 \quad \dots (2)$

Solving equations (1) and (2), we get

$x = 6$

and $\quad y = 3$

Hence, the required number = 10 × 6 + 3 = **63**.

11. Let the ten's and the unit's digit of the required two digit number be x and y.

Then, the required number = $10x + y$

Number with interchanged digits = $10y + x$

$10x + y + 10y + x = 154$

$\Rightarrow \quad 11x + 11y = 154$

$\Rightarrow \quad x + y = 14 \quad \dots (1)$

and $\quad x - y = \pm 2$

When $x + y = 14$ and $x - y = 2$,

then $\quad x = 8$ and $y = 6$

When $x + y = 14$ and $x - y = -2$,

then $\quad x = 6$ and $y = 8$

Hence, the required number = 10 × 8 + 6 = **86**

or 10 × 6 + 8 = **68**.

12. Let the digits in the unit place and the ten's place be x and y respectively. Then the number is $10y + x$.

∴ According to the problem,

$7(10y + x) = 4(10x + y)$

$\Rightarrow \quad 70y + 7x - 40x - 4y = 0$

$\Rightarrow \quad 66y - 33x = 0$

$\Rightarrow \quad 2y = x \quad \dots (1)$

and $\quad x - y = 3 \quad \dots (2)$

[since $x > y$ from (1)]

∴ From (1) and (2), we have

$2y - y = 3$

$\Rightarrow \quad y = 3$

$\therefore \quad x = 2 \times 3 = 6$ [From (1)]

∴ The required number is 10 × 3 + 6 = **36**.

For Standard Level

13. Let the required larger number be x and the required smaller number be y.

Then, $\quad x - y = 6 \quad \dots (1)$

and $\quad x^2 - y^2 = 96$

$\Rightarrow \quad (x - y)(x + y) = 96$

$\Rightarrow \quad 6(x + y) = 96$

$\Rightarrow \quad x + y = 16 \quad \dots (2)$

Solving equation (1) and equation (2), we get

$x = 11$ and $y = 5$

Hence, the required numbers are **11** and **5**.

14. Let the ten's and unit's digit of the required two-digit number be x and y respectively.

Then, the required number = $10x + y$

$x = 2y \quad \dots (1)$

$10x + y - [10 \times \frac{x}{2} + 2y] = 27 \quad \dots (2)$

$\Rightarrow 10(2y) + y - [5 \times 2y + 2y] = 27$ [using $x = 2y$, from (1)]

$\Rightarrow \quad 20y + y - 10y - 2y = 27$

$\Rightarrow \quad 9y = 27$

$\Rightarrow \quad y = 3$

$x = 2y = 2 \times 3 = 6$

Hence, the required number = 10 × 6 + 3 = **63**.

15. Let the ten's and unit's digit of the required two-digit number be x any y respectively.

Then, the required number = $10x + y$

$10x + y = 4(x + y) = 2xy$

$\Rightarrow \quad 10x + y = 4x + 4y$

$\Rightarrow \quad 6x = 3y$

$\Rightarrow \quad 2x = y$

Now, $\quad 4x + 4y = 2xy$

$\Rightarrow \quad 2y + 4y = y^2$

$\Rightarrow \quad y^2 - 6y = 0$

$\Rightarrow \quad y(y - 6) = 0$

$\Rightarrow \quad y = 0$ or $y = 6$

$y = 0, x = 0$ (rejected)

and $\quad y = 6$

$\Rightarrow \quad x = 3$

Hence, the required number = 10 × 3 + 6 = **36**.

16. Let the ten's and the unit's digit of the required two digit number be x and y.

Then, the required number = $10x + y$.

$xy = 18$

and $\quad 10x + y - 63 = 10y + x$

$\Rightarrow \quad 9x - 9y = 63$

$\Rightarrow \quad x - y = 7$

Now, $\quad (x + y)^2 = (x - y)^2 + 4xy$

$= (7)^2 + 4 \times 18$

$= 49 + 72$

$= 121$

$\Rightarrow \quad x + y = \pm 11$

When $x + y = 11$ and $x - y = 7$,

then $\quad x = 9$ and $y = 2$

When $x + y = -11$ and $x - y = 7$,

then $x = -2$ (rejected), as digit cannot be negative.

Hence, the required number = 10 × 9 + 2 = **92**.

17. Let the digits in the unit and ten's place be x and y respectively where $y > x$.

Then according to the problem,

$10y + x = 8(x + y) - 5$

$\Rightarrow \quad 10y + x = 8x + 8y - 5$

$\Rightarrow \quad 7x - 2y - 5 = 0 \quad \dots (1)$

and $\quad 10y + x = 16(y - x) + 3$

$\Rightarrow \quad 10y + x = 16y - 16x + 3$

$\Rightarrow \quad 17x - 6y - 3 = 0 \quad \dots (2)$

From (1) and (2), by the method of cross-multiplication, we have

$$\frac{x}{6 - 30} = \frac{y}{-85 + 21} = \frac{1}{-42 + 34}$$

$$\Rightarrow \quad \frac{x}{24} = \frac{y}{64} = \frac{1}{8}$$

$$\therefore \quad x = \frac{24}{8} = 3$$

and $y = \frac{64}{8} = 8$

Hence, the required number is $10 \times 8 + 3 =$ **83**.

EXERCISE 3G

For Basic and Standard Levels

1. Let the numerator and denominator of the required fraction be n and d, so that the fraction is $\frac{n}{d}$.

Then, $\frac{n+4}{d} = \frac{n}{d} + \frac{2}{3}$

$\Rightarrow \frac{n+4}{d} = \frac{3n+2d}{3d}$

$\Rightarrow 3nd + 12d = 3nd + 2d^2$

$\Rightarrow 2d^2 - 12d = 0$

$\Rightarrow 2d(d-6) = 0$

$\Rightarrow$ Either $d = 0$ (rejected) or $d = 6$

Hence, the denominator of the fraction is **6**.

2. Let the numerator and the denominator of the fraction be x and y respectively. Then the fraction is $\frac{x}{y}$.

$\therefore$ According to the problem,

$x + y = 12$ … (1)

and $\frac{x}{y+3} = \frac{1}{2}$

$\Rightarrow 2x = y + 3$

$\Rightarrow 2x - y = 3$

Adding (1) and (2), we get

$3x = 15$

$\Rightarrow x = 5$

$\therefore$ From (1), $y = 12 - 5 = 7$

$\therefore$ The required fraction is $\frac{5}{7}$.

3. Let the numerator and denominator of the required fraction be n and d respectively, so that the fraction is $\frac{n}{d}$.

Then, $n + d = 12$ … (1)

and $\frac{n+1}{d+1} = \frac{3}{4}$

$\Rightarrow 4n + 4 = 3d + 3$

$\Rightarrow 4n - 3d = -1$ … (2)

Multiplying equation (1) by 3 and adding the result to equation (2), we get

$7n = 35$

$\Rightarrow n = 5$

Substituting $n = 5$ in equation (1), we get

$d = 7$

Hence, the required fraction is $\mathbf{\frac{5}{7}}$.

4. Let the numerator and the denominator of the fraction be x and y respectively. Then the fraction is $\frac{x}{y}$.

$\therefore$ According to the problem,

$x + y - 8 = 0$ … (1)

and $\frac{x+3}{y+3} = \frac{3}{4}$

$\Rightarrow 4x + 12 = 3y + 9$

$\Rightarrow 4x - 3y + 3 = 0$ … (2)

From (1) and (2), by the method of cross-multiplication, we have

$$\frac{x}{3-24} = \frac{y}{-32-3} = \frac{1}{-3-4}$$

$\Rightarrow \frac{x}{-21} = \frac{y}{-35} = \frac{1}{-7}$

$\therefore x = \frac{1}{7} \times 21 = 3$

$y = \frac{35}{7} = 5$

Hence, the required fraction is $\mathbf{\frac{3}{5}}$.

5. Let the numerator and denominator of the required fraction be n and d respectively, so that the fraction is $\frac{n}{d}$.

Then, $\frac{n+1}{d+1} = \frac{8}{9}$

and $\frac{n-5}{d-5} = \frac{2}{3}$

$\Rightarrow 9n + 9 = 8d + 8$

and $3n - 15 = 2d - 10$

$\Rightarrow 9n - 8d = -1$ … (1)

$\Rightarrow 3n - 2d = 5$ … (2)

Multiplying equations (2) by 4 and subtracting equation (1) from the result, we get

$3n = 21$

$\Rightarrow n = 7$

Substituting $n = 7$ in equation (2), we get

$3 \times 7 - 2d = 5$

$\Rightarrow 2d = 21 - 5$

$\Rightarrow 2d = 16$

$\Rightarrow d = 8$

Hence, the required fraction is $\mathbf{\frac{7}{8}}$.

6. Let the numerator and the denominator of the fraction be x and y respectively. Then the fraction is $\frac{x}{y}$.

$\therefore$ According to the problem, we have

$\frac{x+2}{y+2} = \frac{1}{3}$

$\Rightarrow 3x + 6 = y + 2$

$\Rightarrow 3x - y + 4 = 0$ … (1)

and $\frac{x+3}{y+3} = \frac{2}{5}$

$\Rightarrow 5x + 15 = 2y + 6$

$\Rightarrow 5x - 2y + 9 = 0$ … (2)

From (1) and (2), by the method of cross-multiplication, we have

$$\frac{x}{-9+8} = \frac{y}{20-27} = \frac{1}{-6+5}$$

$\Rightarrow \quad \frac{x}{-1} = \frac{y}{-7} = \frac{1}{-1}$

$\therefore \; x = 1$ and $y = 7$.

$\therefore$ The required fraction is $\mathbf{\frac{1}{7}}$.

7. Let x and y be respectively the numerator and denominator of the fraction. Then the fraction is $\frac{x}{y}$.

$\therefore$ According to the problem, we have

$$x + y = 4 + 2x$$

$\Rightarrow \quad x - y + 4 = 0$

$\Rightarrow \quad y = x + 4 \quad \ldots (1)$

and $\quad \frac{x+3}{y+3} = \frac{2}{3}$

$\Rightarrow \quad 3x + 9 = 2y + 6$

$\Rightarrow \quad 3x - 2y + 3 = 0 \quad \ldots (2)$

$\Rightarrow \quad 3x - 2(x + 4) + 3 = 0 \quad$ [From (1)]

$\Rightarrow \quad x = 5$

$\therefore$ From (1), $y = 5 + 4 = 9$.

$\therefore$ The required fraction is $\mathbf{\frac{5}{9}}$.

8. Let the numerator and denominator of the required fraction be n and d respectively, so that the fraction is $\frac{n}{d}$.

Then, $\quad n + d = 2d - 3$

and $\quad n - 1 = \frac{1}{2}(d - 1)$

$\Rightarrow \quad n - d = -3 \quad \ldots (1)$

and $\quad 2n - 2 = d - 1$

$\Rightarrow \quad 2n - d = 1 \quad \ldots (2)$

Subtracting equation (1) from equation (2), we get

$$n = 4$$

Substituting $n = 4$ in equation (1), we get

$$d = 7$$

Hence, the required fraction is $\mathbf{\frac{4}{7}}$.

9. Let the numerator and denominator of the required fraction be n and d, so that the fraction is $\frac{n}{d}$.

Then, $\quad \frac{3n}{d+3} = \frac{3}{4}$

and $\quad \frac{n+3}{3d} = \frac{1}{3}$

$\Rightarrow \quad \frac{n}{d+3} = \frac{1}{4}$

and $\quad \frac{n+3}{d} = 1$

$\Rightarrow \quad 4n = d + 3$

and $\quad n + 3 = d$

$\Rightarrow \quad 4n - d = 3 \quad \ldots (1)$

and $\quad n - d = -3 \quad \ldots (2)$

Subtracting equation (2) from equation (1), we get

$$3n = 6$$

$\Rightarrow \quad n = 2$

Substituting $n = 2$ in equation (2), we get

$$d = 5$$

Hence, the required fraction is $\mathbf{\frac{2}{5}}$.

10. Let x and y be respectively the numerator and the denominator of the fraction. Then the fraction is $\frac{x}{y}$.

$\therefore$ According to the problem, we have

$$y = 2x + 4 \quad \ldots (1)$$

and $\quad \frac{x-6}{y-6} = \frac{x}{12x} = \frac{1}{12}$

$\Rightarrow \quad 12x - 72 = y - 6$

$\Rightarrow \quad y = 12x - 66 \quad \ldots (2)$

$\therefore$ From (1) and (2),

$$12x - 66 = 2x + 4$$

$\Rightarrow \quad 10x = 70$

$\Rightarrow \quad x = 7$

$\therefore$ From (1), $\quad y = 2 \times 7 + 4 = 18$.

$\therefore$ The required fraction is $\mathbf{\frac{7}{18}}$.

11. Let the numerator and denominator of the required fraction be n and d, so that the fraction is $\frac{n}{d}$.

Then, $\quad \frac{n}{d} = \frac{n}{d+1}$

New fraction $= \frac{n-2}{n+1-2} = \frac{n-2}{n-1}$

Reciprocal of new fraction + 4 (original fraction) = 5

$\Rightarrow \quad \frac{(n-1)}{(n-2)} + 4\left(\frac{n}{n+1}\right) = 5$

$\Rightarrow \quad \frac{(n^2-1) + 4n(n-2)}{(n-2)(n+1)} = 5$

$\Rightarrow \quad n^2 - 1 + 4n^2 - 8n = 5(n^2 - 2n + n - 2)$

$\Rightarrow \quad 5n^2 - 8n - 1 = 5n^2 - 5n - 10$

$\Rightarrow \quad 10 - 1 = 8n - 5n$

$\Rightarrow \quad 9 = 3n$

$\Rightarrow \quad n = 3$

Hence, the required fraction is $\frac{3}{3+1} = \mathbf{\frac{3}{4}}$.

12. Let x and y be respectively the numerator and the denominator of the fraction. Then the fraction is $\frac{x}{y}$.

$\therefore$ According to the problem, we have

$$\frac{2x}{y-1} = \frac{11}{12}$$

$\Rightarrow \quad 24x = 11y - 11$

$\Rightarrow \quad 24x - 11y + 11 = 0 \quad \ldots (1)$

and $\quad \frac{x+14}{2y} = \frac{1}{2}$

$\Rightarrow \quad 2x + 28 = 2y$

$\Rightarrow \quad 2x - 2y + 28 = 0$

$\Rightarrow \quad x - y + 14 = 0 \quad \ldots (2)$

∴ From (1) and (2), by the method of cross-multiplication, we have

$$\frac{x}{-154+11} = \frac{y}{11-336} = \frac{1}{-24+11}$$

$\Rightarrow$ $$\frac{x}{-143} = \frac{y}{-325} = \frac{1}{-13}$$

∴ $$x = \frac{143}{3} = 11,\ y = \frac{325}{13} = 25$$

∴ The required fraction is $\frac{\mathbf{11}}{\mathbf{25}}$.

EXERCISE 3H

For Basic and Standard Levels

1. **Present Ages**

Let the father's present age be x years and the son's present age by y years.

Then, $x = 3y + 3$

$\Rightarrow$ $x - 3y = 3$ … (1)

Three years hence

Father's age = $(x + 3)$ years

Son's age = $(y + 3)$ years

Then, $(x + 3) = 2(y + 3) + 10$

$\Rightarrow$ $x + 3 = 2y + 6 + 10$

$\Rightarrow$ $x - 2y = 13$ … (2)

Solving equations (1) and (2), we get

$x = 33$ and $y = 10$

Hence, the father's present age is **33 years** and the son's present age is **10 years**.

2. **Present Ages**

Let the man's present age be x years and the son's present age be y years.

Ten years hence

Man's age = $(x + 10)$ years

Son's age = $(y + 10)$ years

Then, $x + 10 = 2\,(y + 10)$

$\Rightarrow$ $x - 2y = 10$ … (1)

Ten years ago

Man's age = $(x - 10)$ years

Son's age = $(y - 10)$ years

Then, $x - 10 = 4\,(y - 10)$

$\Rightarrow$ $x - 4y = -30$ …(2)

Subtracting equation (1) from equation (2), we get

$-2y = -40$

$\Rightarrow$ $y = 20$

Substituting $y = 20$ in equation (1), we get

$x - 2 \times 20 = 10$

$\Rightarrow$ $x = 50$

Hence, the man's present age is **50 years** and the son's present age is **20 years.**

3. Let the present ages of A and B be x years and y years respectively.

Then according to the problem, we have

$x - 5 = 3(y - 5)$

$\Rightarrow$ $x = 5 + 3y - 15$

$= 3y - 10$ … (1)

and $x + 10 = 2(y + 10)$

$= x - 2y - 10 = 0$ … (2)

From (1) and (2), we have

$3y - 10 - 2y - 10 = 0$

$\Rightarrow$ $y = 20$

∴ From (1), $x = 3 \times 20 - 10 = 50$

Hence, the required present ages of A and B are **50 years** and **20 years**.

4. Let the present age of Salim be x years and that of his daughter be y years.

Then according to the problem, we have

$x - 2 = 3(y - 2)$

$\Rightarrow$ $x = 3y - 4$ … (1)

and $x + 6 = 2(y + 6) + 4$

$\Rightarrow$ $x = 2y + 10$ … (2)

∴ From (1) and (2), we have

$2y + 10 = 3y - 4$

$\Rightarrow$ $y = 14$

∴ From (1), $x = 3 \times 14 - 4 = 38$

Hence, the present ages of Salim and his daughter are **38 years** and **14 years** respectively.

5. Let the present ages of the man and his son be x years and y years respectively.

∴ According to the problem, we have

$x + 6 = 3(y + 6)$

$\Rightarrow$ $x - 3y - 12 = 0$ … (1)

and $x - 3 = 9(y - 3)$

$\Rightarrow$ $x - 9y + 24 = 0$ … (2)

From (1) and (2), by the method of cross-multiplication, we have

$$\frac{x}{-72-108} = \frac{y}{-12-24} = \frac{1}{-9+3}$$

$\Rightarrow$ $$\frac{x}{180} = \frac{y}{36} = \frac{1}{6}$$

∴ $$x = \frac{180}{6} = 30,\ y = \frac{36}{6} = 6$$

Hence, the required present ages of the man and his son are **30 years** and **6 years** respectively.

6. Let the present ages of the two girls be x years and y years respectively.

∴ According to the problem, we have

$$\frac{x}{y} = \frac{5}{7}$$

$\Rightarrow$ $7x = 5y$

$\Rightarrow$ $x = \frac{5y}{7}$ … (1)

and $\frac{x-8}{y-8} = \frac{7}{13}$

$\Rightarrow$ $13x - 104 = 7y - 56$

$\Rightarrow$ $13x - 7y - 48 = 0$ … (2)

From (1) and (2), we have

$$13 \times \frac{5y}{7} - 7y - 48 = 0$$

$$\Rightarrow \quad \frac{65y - 49y}{7} = 48$$

$$\Rightarrow \quad 16y = 7 \times 48$$

$$\Rightarrow \quad y = \frac{7 \times 48}{16} = 21$$

$\therefore$ From (1), $\quad x = \frac{5}{7} \times 21 = 15$

Hence, the required present ages of the two girls are **15 years** and **21 years** respectively.

7. Let the father's present age be x years
and let the sum of the ages of his two children be y years.
Then, $\quad x = 3y \quad \ldots (1)$
After 5 years, father's age $= (x + 5)$ years
Sum of ages of 2 children $= (y + 2 \times 5)$ years
$= (y + 10)$ years
Then, $\quad x + 5 = 2(y + 10)$
$\Rightarrow \quad x + 5 = 2y + 20$
$\Rightarrow \quad 3y - 2y = 20 - 5 \quad$ [Using (1)]
$\Rightarrow \quad y = 15$
$x = 3y = 3 \times 15 = 45$
Hence, the father's present age is **45 years**.

8. Let the present age of the father be x years and the present ages of his two children be y years and z years. Then after 20 years, the age of the father will be $(x + 20)$ years and the ages of two children will be $(y + 20)$years and $(z + 20)$ years.

Then according to the problem, we have

$x = 2(y + z) \quad \ldots (1)$

and $\quad x + 20 = y + 20 + z + 20$

$= (y + z) + 40 \quad \ldots (2)$

Let $\quad y + z = p \quad \ldots (3)$

$\therefore$ From (1) and (3), we have

$x = 2p \quad \ldots (4)$

and from (2) and (3), we have

$x + 20 = p + 40 \quad \ldots (5)$

From (5), $\quad p = x - 20 \quad \ldots (6)$

$\therefore$ From (4) and (6), we have

$x = 2(x - 20) = 2x - 40$

$\Rightarrow \quad x = 40$

Hence, the required present age of the father is **40 years**.

9. Let the present age of the father be x years and the present ages of his two children be y years and z years. Then after 20 years, the ages of the father and his two children will be $(x + 20)$years, $(y + 20)$years and $(z + 20)$ years respectively.

$\therefore$ According to the problem, we have

$x = 2(y + z) \quad \ldots (1)$

and $\quad y + z + 22 + 22 = x + 22$

$\Rightarrow \quad y + z = x - 22 \quad \ldots (2)$

Let $\quad y + z = p \quad \ldots (3)$

$\therefore$ From (1) and (3), we have

$x = 2p \quad \ldots (4)$

and from (2) and (3), we have

$p = x - 22 \quad \ldots (5)$

$\therefore$ From (4) and (5), we have

$x = 2(x - 22)$

$= 2x - 44$

$\Rightarrow \quad x = 44$

Hence, the required present age of the father is **44 years**.

10. Let the present ages of the father, two sons and one daughter be x years, s_1 years, s_2 years and d_1 years respectively.

$\therefore$ According to the problem, we have

$s_1 + s_2 + d_1 = x \quad \ldots (1)$

and $\quad \frac{3}{2}(x + 15) = s_1 + 15 + s_2 + 15 + d_1 + 15 \quad \ldots (2)$

[Since after 15 years, the ages of the father, two sons and one daughter will be $(x + 15)$ years, $(s_1 + 15)$ years, $(s_2 + 15)$years and $(d_1 + 15)$ years respectively]

From (2), $\quad 3(x + 15) = (s_1 + s_2 + d_1 + 45) \times 2$

$= (x + 45) \times 2 \quad$ [From (1)]

$= 2x + 90$

$\Rightarrow \quad x = 90 - 45 = 45$

Hence, the required present age of the father is **45 years**.

EXERCISE 3I

For Basic and Standard Levels

1. Let the cost of 1 fork be ₹ x and that of 1 knife be ₹ y.

$\therefore$ According to the problem, we have

$10x + 7y - 329 = 0 \quad \ldots (1)$

and $\quad 8x + 5y - 256 = 0 \quad \ldots (2)$

From (1) and (2), by the method of cross-multiplication, we have

$$\frac{x}{-7 \times 256 + 5 \times 329} = \frac{y}{-329 \times 8 + 256 \times 10} = \frac{1}{10 \times 5 - 8 \times 7}$$

$$\Rightarrow \frac{x}{-1792 + 1645} = \frac{y}{-2632 + 2560} = \frac{1}{50 - 56}$$

$$\Rightarrow \frac{x}{-147} = \frac{y}{-72} = \frac{1}{-6}$$

$\therefore \quad x = \frac{147}{6} = \frac{49}{2}$

and $\quad y = \frac{72}{6} = 12$

$\therefore \quad$ The cost of 1 fork = ₹ $\frac{49}{2}$ = ₹ 24.50

and $\quad$ the cost of 1 knife = ₹ 12.00

$\therefore$ Required total cost of the set of 1 fork x and 1 knife is ₹(24.50 + 12.00), i.e. ₹ **36.50**

2. Let ₹ x be the cost of 1 kg sugar
and let ₹ y be the cost of 1 kg tea.

Then, $2x + \frac{y}{2} = 275$

$\Rightarrow \quad 4x + y = 550 \quad \dots (1)$

and $\quad 3x + y = 500 \quad \dots (2)$

Solving equations (1) and (2), we get

$x = 50$ and $y = 350$.

Hence, the cost of 1 kg tea and 2 kg of sugar
$y + 2x = ₹\ (350 + 2 \times 50) = ₹$ **450**.

3. Let the cost of 1 lunch box and 1 water bottle be ₹ x and ₹ y respectively. Then according to the problem, we have

$5x + 6y - 215 = 0 \quad \dots (1)$

and $\quad 6x + 5y - 225 = 0 \quad \dots (2)$

From (1) and (2), by the method of cross-multiplication, we have

$$\frac{x}{-225 \times 6 + 5 \times 215} = \frac{y}{-215 \times 6 + 225 \times 5} = \frac{1}{5 \times 5 - 6 \times 6}$$

$$\Rightarrow \frac{x}{-1350 + 1075} = \frac{y}{-1290 + 1125} = \frac{1}{25 - 36}$$

$$\Rightarrow \frac{x}{-275} = \frac{y}{-165} = \frac{1}{-11}$$

$$\therefore\ x = \frac{275}{11} = 25,\ y = \frac{165}{11} = 15$$

$\therefore$ The cost of 1 lunch box = ₹ 25 and that of 1 water bottle = ₹ 15.

$\therefore$ Required total cost of 2 lunch boxes and 3 water bottles

$= ₹\ (2 \times 25 + 3 \times 15)$

$= ₹\ (50 + 45)$

$= ₹$ **95**

4. Let the cost of 1 table and 1 chair be ₹ x and ₹ y respectively. Then according to the problem, we have

$7x + 11y - 9800 = 0 \quad \dots (1)$

and $\quad 11x + 7y - 11800 = 0 \quad \dots (2)$

Adding (1) and (2), we have

$18(x + y) - 21600 = 0$

$$\Rightarrow \quad x + y = \frac{21600}{18} = 1200 \quad \dots (3)$$

Subtracting (1) from (2), we have

$4x - 4y = 2000$

$$\Rightarrow \quad x - y = \frac{2000}{4} = 500 \quad \dots (4)$$

Adding (3) and (4), we get

$2x = 1700$

$\Rightarrow \quad x = 850$

Subtracting (4) from (3), we get

$2y = 700$

$\Rightarrow \quad y = 350$

Hence, the cost of 1 table is ₹ **850** and the cost of 1 chair is ₹ **350**.

Hence, the required total cost of 2 tables and 8 chairs
$= ₹\ (850 \times 2 + 350 \times 8) = ₹\ (1700 + 2800) = ₹$ **4500**.

5. Let the number of ₹ 5 coins be x and the number of ₹1 coins be y.

Then, $\quad x + y = 72 \quad \dots (1)$

and $\quad 5x + y = 232 \quad \dots (2)$

Solving equations (1) and (2), we get

$x = 40$

and $\quad y = 32$

Hence, there are **40 coins** of ₹ 5 denomination and **32 coins** of ₹ 1 denomination.

6. Let the parking charges for 1 car and 1 scooter be ₹ x and ₹ y respectively.

$\therefore$ According to the problem, we have

$80x + 25y = 925$

$\Rightarrow \quad 16x + 5y - 185 = 0 \quad \dots (1)$

and $\quad 65x + 12y = 710$

$\Rightarrow \quad 65x + 12y - 710 = 0 \quad \dots (2)$

From (1) and (2), by the method of cross-multiplication, we have

$$\frac{x}{-5 \times 710 + 12 \times 185} = \frac{y}{-65 \times 185 + 16 \times 710} = \frac{1}{16 \times 12 - 5 \times 65}$$

$$\Rightarrow \frac{x}{-3550 + 2220} = \frac{y}{-12025 + 11360} = \frac{1}{192 - 325}$$

$$\Rightarrow \frac{x}{-1330} = \frac{y}{-665} = \frac{1}{-133}$$

$$\therefore \quad x = \frac{1330}{133} = 10,\ y = \frac{665}{133} = 5$$

$\therefore$ Parking charge for 1 car = ₹ 10 and that for 1 scooter = ₹ 5.

$\therefore$ Required total parking charges for 5 cars and 7 scooters = ₹ $(5 \times 10 + 5 \times 7)$ = ₹ $(50 + 35)$ = ₹ **85**

7. Let the worker earn ₹ x per hour for regular hours and ₹ y per hour for overtime.

In the 1st week, he works for 10 hours overtime and so (50 – 10) hours or 40 hours for regular hours.

Also, in the second week, he works for 5 hours overtime and so (55 – 5) hours or 50 hours for regular hours.

$\therefore$ According to the problem, we have

$40x + 10y = 820$

$\Rightarrow \quad 4x + y = 82 \quad \dots (1)$

and $\quad 50x + 5y = 860$

$\Rightarrow \quad 10x + y = 172 \quad \dots (2)$

Subtracting (1) from (2), we get

$6x = 90$

$$\Rightarrow \quad x = \frac{90}{6} = 15$$

$\therefore$ From (1), $y = 82 - 4 \times 15 = 22$

Hence, (*i*) His required earnings per hour is ₹ **15** for regular hours.

(*ii*) His required earnings per hour is ₹ **22** for overtime.

8. Let the cost of return ticket be ₹ x and that of single ticket be ₹ y.

Then, according to the problem, we have

$$15x + 20y = 18500$$

$$\Rightarrow \quad 3x + 4y = 3700 \quad \dots (1)$$

and $$x + y = 1100$$

$$\Rightarrow \quad 4x + 4y = 4400 \quad \dots (2)$$

Subtracting (1) from (2), we get

$$x = 700$$

∴ From (2), $y = 1100 - 700 = 400$

∴ Required cost of return ticket = ₹ **700** and the cost of single ticket = ₹ **400**.

9. Let the number of children who saw the fair be x and let the number of adults who saw the fair be y.

Then, $$x + y = 3500 \quad \dots (1)$$

and $$(2.50)x + 5y = 12500$$

$$\Rightarrow \quad \frac{5x}{2} + 5y = 12500$$

$$\Rightarrow \quad 5x + 10y = 25000 \quad \dots (2)$$

Multiplying equation (1) by 10 and subtracting equation (2) from the result, we get

$$5x = 10000$$

$$\Rightarrow \quad x = 2000$$

Hence, the number of children who saw the fair is **2000**.

10. Let the number of bananas in lot A be x and that in lot B be y. In the lot case, the selling price of 3 bananas in lot A is ₹ 2 and that is lot B is ₹ 1 for 1 banana.

∴ According to the problem,

$$\frac{2x}{3} + y = 400$$

$$\Rightarrow \quad 2x + 3y = 1200$$

$$\Rightarrow \quad 2x + 3y - 1200 = 0 \quad \dots (1)$$

In the second case, the S.P. of 1 banana in lot A is ₹ 1 and that in lot B is ₹ 4 for 5 bananas.

$$\therefore \quad x + \frac{4y}{5} = 460$$

$$\Rightarrow \quad 5x + 4y = 2300$$

$$\Rightarrow \quad 5x + 4y - 2300 = 0 \quad \dots (2)$$

∴ From (1) and (2), by the method of cross-multiplication, we have

$$\frac{x}{-3 \times 2300 + 4 \times 1200} = \frac{y}{-5 \times 1200 + 2 \times 2300} = \frac{1}{2 \times 4 - 3 \times 5}$$

$$\Rightarrow \quad \frac{x}{-6900 + 4800} = \frac{y}{-6000 + 4600} = \frac{1}{8 - 15}$$

$$\Rightarrow \quad \frac{x}{-2100} = \frac{y}{-1400} = \frac{1}{-7}$$

$$\therefore \quad x = \frac{2100}{7} = 300$$

and $$y = \frac{1400}{7} = 200$$

Hence, required total number of bananas is lot A and lot B together = 300 + 200 = **500**.

EXERCISE 3J

For Basic and Standard Levels

1. For a rectangle, opposite sides are equal.

∴ From the given figure, we have

$$x + 3y - 13 = 0 \quad \dots (1)$$

and $$3x + y - 7 = 0 \quad \dots (2)$$

From (1) and (2), by the method of cross-multiplication, we have

$$\frac{x}{-3 \times 7 + 13 \times 1} = \frac{y}{-13 \times 3 + 7 \times 1} = \frac{1}{1 \times 1 - 3 \times 3}$$

$$\Rightarrow \quad \frac{x}{-21 + 13} = \frac{y}{-39 + 7} = \frac{1}{1 - 9}$$

$$\Rightarrow \quad \frac{x}{-8} = \frac{y}{-32} = \frac{1}{-8}$$

$$\therefore \quad x = \frac{8}{8} = 1, \; y = \frac{32}{8} = 4$$

∴ The required values of x and y are **1** and **4** respectively.

2. From the given figure, AB = DC and AD = BC

$$\therefore \quad x + y = 15 \quad \dots (1)$$

and $$x - y = 5 \quad \dots (2)$$

Adding (1) and (2), we get

$$2x = 20$$

$$\Rightarrow \quad x = 10$$

∴ From (1), $y = 15 - 10 = 5$

∴ The required values of x and y are **10** and **5** respectively.

3. From the given figure, we have AB = DC and BC = AD.

$$\therefore \quad x + y = 20 \quad \dots (1)$$

and $$x - y = 6 \quad \dots (2)$$

Adding (1) and (2), we have

$$2x = 26$$

$$\Rightarrow \quad x = \frac{26}{2} = 13$$

∴ From (1), $y = 20 - x = 20 - 13 = 7$

∴ The required values of x and y are **13** and **7** respectively.

4. Since the sides of an equilateral triangle are equal,

$$\therefore \quad \frac{2}{3}p + 2q + \frac{5}{2} = 2p + \frac{q}{2} = \frac{5}{3}p + q + \frac{1}{2}$$

$$\Rightarrow \quad \frac{2}{3}p + 2q + \frac{5}{2} = 2p + \frac{q}{2}$$

$$\Rightarrow \quad 4p + 12q + 15 = 12p + 3q$$

$$\Rightarrow \quad 8p - 9q = 15 \quad \dots (1)$$

and $$2p + \frac{q}{2} = \frac{5}{3}p + q + \frac{1}{2}$$

$$\Rightarrow \quad 12p + 3q = 10p + 6q + 3$$

$\Rightarrow \quad 2p - 3q = 3 \quad \ldots (2)$

Solving equations (1) and (2), we get

$$p = 3 \text{ and } q = 1$$

$\therefore$ Perimeter =

$$\left[\left(\frac{2}{3} \times 3 + 2 \times 1 + \frac{5}{2}\right) + \left(2 \times 3 + \frac{1}{2}\right) + \left(\frac{5}{3} \times 3 + 1 + \frac{1}{2}\right)\right] \text{ cm}$$

$$= \left[\left(4 + \frac{5}{2}\right) + \left(6 + \frac{1}{2}\right) + \left(6 + \frac{1}{2}\right)\right] \text{ cm} = \mathbf{19.5 \text{ cm}}$$

5. $\angle A = x°$, $\angle B = y°$ and $\angle C = \angle A + \angle B = (x + y)$

Sum of angles of a triangle is 180°.

$\Rightarrow \quad x + y + (x + y) = 180$

$\Rightarrow \quad 2x + 2y = 180$

$\Rightarrow \quad x + y = 90 \quad \ldots (1)$

Also, $\quad 4y - 3x = 10 \quad \ldots (2)$ [Given]

Solving equation (1) and equation (2), we get

$$x = 50$$

and $\quad y = 40.$

Hence, $\angle$**A = 50°,** $\angle$**B = 40°** and $\angle$**C = (50 + 40)° = 90°.**

6. Since the opposite angles of a cyclic quadrilateral are supplementary,

$\therefore \quad \angle A + \angle C = 180°$

$2x + 2x + y = 180$

$\Rightarrow \quad 4x + y = 180 \quad \ldots (1)$

and $\quad \angle B + \angle D = 180°$

$6y + 10 + x + 10 = 180$

$\Rightarrow \quad x + 6y = 160 \quad \ldots (2)$

Solving equations (1) and (2), we get

$$x = 40 \text{ and } y = 20$$

Thus, $\quad \angle A = 2x° = (2 \times 40)° = 80°$

$\angle B = (6y + 10)° = (6 \times 20 + 10)° = 130°$

$\angle C = (2x + y)° = (2 \times 40 + 20)° = 100°$

$\angle D = (x + 10)° = (40 + 10)° = 50°$

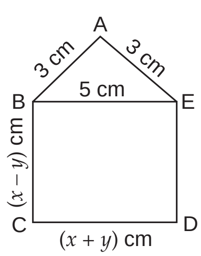
Hence, $\angle$**A = 80°,** $\angle$**B = 130°,** $\angle$**C = 100°** and $\angle$**D = 50°.**

7. Quad. BEDC is a rectangle.

$\because$ Its opposite sides are parallel and $\angle BCD = 90°$

$\therefore \quad BE = CD = x + y = 5 \quad \ldots (1)$

and $\quad ED = BC = x - y$



Perimeter of ABCDE = AB + BC + CD + DE + EA

$= (3 + x - y + x + y + x - y + 3)$ cm

$= 6 + 3x - y$ cm = 21 cm (Given)

$\Rightarrow \quad 3x - y = 15 \quad \ldots (2)$

Solving equations (1) and (2), we get

$$\boldsymbol{x = 5} \text{ and } \boldsymbol{y = 0}.$$

8. Let the length and breadth of the rectangle be x m and y m respectively.

Then, according to the problem,

$$x = 2y \quad \ldots (1)$$

Perimeter of the rectangle = $2(x + y)$ m.

$\therefore \quad x + y = 24 \quad$ [Given] $\ldots (2)$

From (1) and (2), we have

$2y + y = 24$

$\Rightarrow \quad 3y = 24$

$\Rightarrow \quad y = 8$

$\therefore$ From (1), $\quad x = 8 \times 2 = 16$

$\therefore$ Required length and breadth of the rectangle be **16 m** and **8 m** respectively.

9. Let the length and breadth of the rectangle be x cm and y cm respectively.

$\therefore$ According to the problem,

$$x = y + 8 \quad \ldots (1)$$

and $\quad \frac{1}{4} \times 2(x + y) = 16$

$\Rightarrow \quad x + y = 32 \quad \ldots (2)$

From (1) and (2), we have

$y + 8 + y = 32$

$\Rightarrow \quad 2y = 32 - 8 = 24$

$\Rightarrow \quad y = 12$

$\therefore$ From (2), $\quad x = 32 - y = 32 - 12 = 20.$

$\therefore$ The required length and breadth are **20 cm** and **12 cm** respectively.

EXERCISE 3K

For Basic and Standard Levels

1. Let the length of the rectangular block be x m and its breadth be y m.

Measures	*Case 1*
Length (in m)	$x + 2$
Breadth (in m)	$y - 1$
Area (in m²)	$(x + 2)(y - 1)$

Perimeter = 32 m

$\Rightarrow \quad 2(x + y) = 32$

$\Rightarrow \quad x + y = 16 \quad \ldots (1)$

and $\quad$ area = xy

$\Rightarrow \quad (x + 2)(y - 1) = xy$

$\Rightarrow \quad xy + 2y - x - 2 = xy$

$\Rightarrow \quad 2y - x = 2 \quad \ldots (2)$

Solving equations (1) and (2), we get

$$x = 10 \text{ and } y = 6$$

Hence, the length of the plot = **10 m** and its breadth = **6 m**.

2. Let the length and the breadth of the room be x m and y m respectively.

Measures	*Case 1*	*Case 2*
Length (in m)	$(x + 4)$	$(x - 2)$
Breadth (in m)	$(y + 1)$	$(y + 3)$
Area (in m^2)	$(x + 4)(y + 1)$	$(x - 2)(y + 3)$

Case 1. New area – original area = 56 m^2

$\Rightarrow \quad (x + 4)(y + 1) - xy = 56$

$\Rightarrow \quad xy + 4y + x + 4 - xy = 56$

$\Rightarrow \quad 4y + x = 52 \qquad \ldots (1)$

Case 2. New area – original area = 24 m^2

$\Rightarrow \quad (x - 2)(y + 3) - xy = 24$

$\Rightarrow \quad xy - 2y + 3x - 6 - xy = 24$

$\Rightarrow \quad 3x - 2y = 30 \qquad \ldots (2)$

Solving equations (1) and (2), we get

$x = 16$ and $y = 9$

Hence, the length of the room = **16 m** and its breadth = **9 m**.

3. Let the length and breadth of the rectangle be x and y respectively.

Then, its area = xy

Measures	*Case 1*	*Case 2*
Length	$(x + 2)$	$(x - 1)$
Breadth	$(y - 2)$	$(y + 2)$
Area	$(x + 2)(y - 2)$	$(x - 1)(y + 2)$

Case 1. Original area – New area = 28

$\Rightarrow \quad xy - (x + 2)(y - 2) = 28$

$\Rightarrow \quad xy - xy + 2x - 2y + 4 = 28$

$\Rightarrow \quad 2x - 2y = 24$

$\Rightarrow \quad x - y = 12 \qquad \ldots(1)$

Case 2. New area – Original area = 33

$\Rightarrow \quad (x - 1)(y + 2) - xy = 33$

$\Rightarrow \quad xy + 2x - y - 2 - xy = 33$

$\Rightarrow \quad 2x - y = 35 \qquad \ldots(2)$

Solving (1) and (2), we get $x = 23$ and $y = 11$

Hence, the length and breadth of the rectangle are **23 units** and **11 units** respectively.

For Standard Level

4. Let the length of the base of the right triangular sheet be x m and its perpendicular side be y m.

Then, its area = $\frac{xy}{2}$ cm^2

Measures	*Case 1*	*Case 2*
Length (in cm)	$(x - 1)$	$(x + 1)$
Breadth (in cm)	$(y + 1)$	$(y + 1)$
Area = $\frac{1}{2}$ × length × breadth (in cm^2)	$\frac{(x-1)(y+1)}{2}$	$\frac{(x+1)(y+1)}{2}$

Case 1. New area = Original area

$$\frac{(x-1)(y+1)}{2} = \frac{xy}{2}$$

$\Rightarrow \quad xy - y + x - 1 = xy$

$\Rightarrow \quad x - y = 1 \qquad \ldots (1)$

New area – Original area = 4 cm^2

$$\frac{(x+1)(y+1)}{2} - \frac{xy}{2} = 4$$

$\Rightarrow \quad xy + y + x + 1 - xy = 8$

$\Rightarrow \quad x + y = 7 \qquad \ldots (2)$

Solving equations (1) and (2), we get

$x = 4, y = 3$

Hence, the length of the base and the perpendicular side of the right triangle are **4 cm** and **3 cm** respectively.

5. Let the number of students in each row be x and let the number of rows be y.

Then, the total number of students = xy

Numbers	*Case 1*	*Case 2*
Number of students in each row	$(x + 3)$	$(x - 3)$
Number of rows	$(y - 1)$	$(y + 2)$
Total number of students	$(x + 3)(y - 1)$	$(x - 3)(y + 2)$

Case 1. $(x + 3)(y - 1) = xy$

$\Rightarrow \quad xy + 3y - x - 3 = xy$

$\Rightarrow \quad 3y - x = 3 \qquad \ldots (1)$

Case 2. $(x - 3)(y + 2) = xy$

$\Rightarrow \quad xy - 3y + 2x - 6 = xy$

$\Rightarrow \quad 2x - 3y = 6 \qquad \ldots (2)$

Solving equations (1) and (2), we get

$x = 9, y = 4$

Hence, the number of students = $xy = 9 \times 4 =$ **36**.

6. Let the number of students in each row be x and let the number of rows be y.

Then, the total number of students = xy

Numbers	*Case 1*	*Case 2*
Number of students in each row	$(x + 1)$	$(x - 1)$
Number of rows	$(y - 2)$	$(y + 3)$
Total number of students	$(x + 1)(y - 2)$	$(x - 1)(y + 3)$

Case 1. $(x + 1)(y - 2) = xy$

$\Rightarrow \quad xy - 2x + y - 2 = xy$

$\Rightarrow \quad y - 2x = 2 \qquad \ldots(1)$

Case 2. $(x - 1)(y + 3) = xy$

$\Rightarrow \quad xy + 3x - y - 3 = xy$

$\Rightarrow \quad 3x - y = 3 \qquad \ldots(2)$

Solving equations (1) and (2), we get $x = 5, y = 12$

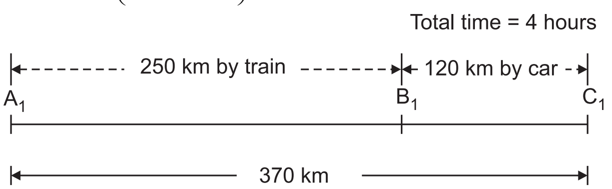
Hence, the number of students = $xy = 5 \times 12 =$ **60**.

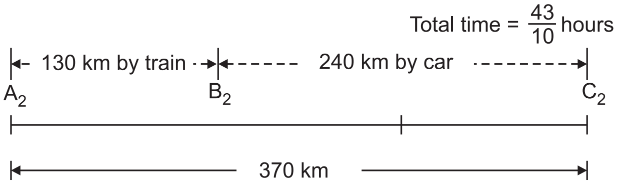
EXERCISE 3L

For Basic and Standard Levels

1. Let the speed of the train be x km/h and that of the car be y km/h. Then the total time to travel 250 km by train and the remaining (370 – 250) km = 120 km by car in the 1st case is $\left(\frac{250}{x}+\frac{120}{y}\right)$ hours

and the total time to travel 130 km by train and the remaining (370 – 130) km = 240 km by car in the 2nd case is $\left(\frac{130}{x}+\frac{240}{y}\right)$ hours.





According to the problem, we have

$$\frac{250}{x}+\frac{120}{y} = 4 \qquad \dots (1)$$

and $$\frac{130}{x}+\frac{240}{y} = 4+\frac{18}{60}$$

$$= 4+\frac{3}{10} = \frac{43}{10} \qquad \dots (2)$$

Now, let $\frac{1}{x} = u$ and $\frac{1}{y} = v$ … (A)

∴ From (1), we have

$$250u + 120v = 4$$

$$\Rightarrow \quad 125u + 60v = 2 \qquad \dots (3)$$

and from (2), we have

$$130u + 240v = \frac{43}{10}$$

$$\Rightarrow \quad 1300u + 2400v = 43 \qquad \dots (4)$$

Multiplying (3) by 40, we get

$$5000u + 2400v = 80 \qquad \dots (5)$$

Subtracting (4) from (5), we get

$$3700u = 37$$

$$\Rightarrow \quad u = \frac{37}{3700} = \frac{1}{100}$$

$$\Rightarrow \quad \frac{1}{x} = \frac{1}{100} \qquad \text{[From (A)]}$$

$$\Rightarrow \quad x = 100$$

Now, from (4)

$$2400v = 43 - 1300u$$

$$= 43 - 1300 \times \frac{1}{100}$$

$$= 43 - 13 = 30$$

$$\therefore \quad v = \frac{30}{2400} = \frac{1}{80}$$

$$\therefore \quad \frac{1}{y} = \frac{1}{80} \qquad \text{[From (A)]}$$

$$\Rightarrow \quad y = 80$$

∴ The required speed of the train is **100 km/h**.

2. Let the speed of the train be x km/h and that of the car be y km/h.

Measures	*Train*	*Bus*
Distance (in km)	400	200
Speed (in km/h)	x	y
Time = $\frac{\text{Distance}}{\text{Speed}}$ (in hour)	$\frac{400}{x}$	$\frac{200}{y}$

$$\frac{400}{x}+\frac{200}{y} = \frac{13}{2} \qquad \dots (1)$$

Measures	*Train*	*Bus*
Distance (in km)	200	400
Speed (in km/h)	x	y
Time = $\frac{\text{Distance}}{\text{Speed}}$	$\frac{200}{x}$	$\frac{400}{y}$

$$\frac{200}{x}+\frac{400}{y} = 7 \qquad \dots (2)$$

Multiply equations (1) by 2, we get

$$\frac{800}{x}+\frac{400}{y} = 13 \qquad \dots (3)$$

Subtracting equation (2) from equation (3), we get

$$\frac{600}{x} = 6$$

$$\Rightarrow \quad x = 100$$

Substituting $x = 100$ in equation (2), we get

$$\frac{200}{100}+\frac{400}{y} = 7$$

$$\Rightarrow \quad \frac{400}{y} = 5$$

$$\Rightarrow \quad y = 80$$

Hence, the speed of the train is **100 km/h** and the speed of the car is **80 km/h**.

3. Let the speeds of the train and the taxi be x km/h and y km/h respectively.

Then in the 1st case, Abdul takes a total time of $5\frac{1}{2}$ h $= \frac{11}{2}$ h to travel 300 km by train and 200 km by taxi.

$$\therefore \quad \frac{300}{x}+\frac{200}{y} = \frac{11}{2} \qquad \dots (1)$$

In the second case, he takes a total time of $5\frac{3}{5}$ h

$= \frac{28}{5}$ h to travel 260 km by train and 240 km by taxi.

$$\therefore \quad \frac{260}{x} + \frac{240}{y} = \frac{28}{5} \quad \ldots (2)$$

Let $\frac{1}{x} = u$ and $\frac{1}{y} = v$ ... (3)

$\therefore$ From (1), we have

$$300u + 200v = \frac{11}{2}$$

$$\Rightarrow \quad 600u + 400v - 11 = 0 \quad \ldots (4)$$

and from (2), we have

$$260u + 240v = \frac{28}{5}$$

$$\Rightarrow \quad 1300u + 1200v - 28 = 0$$

$$\Rightarrow \quad 325u + 300v - 7 = 0 \quad \ldots (5)$$

[Dividing both side by 4]

$\therefore$ From (4) and (5), by the method of cross-multiplication, we have

$$\frac{u}{-7 \times 400 + 11 \times 300} = \frac{v}{-11 \times 325 + 7 \times 600} = \frac{1}{600 \times 30 - 400 \times 325}$$

$$\Rightarrow \frac{u}{-2800 + 3300} = \frac{v}{-3575 + 4200} = \frac{1}{180000 - 130000}$$

$$\Rightarrow \frac{u}{500} = \frac{v}{625} = \frac{1}{50000}$$

$$\therefore u = \frac{500}{50000} = \frac{1}{100} \text{ and } v = \frac{625}{50000} = \frac{1}{80}$$

$\therefore$ From (3), $\quad \frac{1}{x} = \frac{1}{100}$

$\Rightarrow \quad x = 100$

and $\quad \frac{1}{y} = \frac{1}{80}$

$\Rightarrow \quad y = 80$

Hence, the required speeds of the train and taxi are **100 km/h** and **80 km/h** respectively.

4. Let the speeds of the bus and the rickshaw be x km/h and y km/h respectively

In the 1st case, Ankita travels 2 km by rickshaw and the remaining (14 – 2) km = 12 km by bus.

$\therefore$ Total time taken by her in this case is $\left(\frac{12}{x} + \frac{2}{y}\right)$ h.

In second case, she travels 4 km by rickshaw and the remaining (14 – 4) km = 10 km by bus.

$\therefore$ Total time taken by her in this case is $\left(\frac{10}{x} + \frac{4}{y}\right)$ h.

$\therefore$ According to the problem,

$$\frac{12}{x} + \frac{2}{y} = \frac{1}{2} \quad \ldots (1)$$

and $$\frac{10}{x} + \frac{4}{y} = \frac{1}{2} + \frac{3}{20} = \frac{10+3}{20} = \frac{13}{20} \quad \ldots (2)$$

Let $\frac{1}{x} = u$ and $\frac{1}{y} = v$ ... (3)

$\therefore$ From (1), we have

$$12u + 2v - \frac{1}{2} = 0$$

$$\Rightarrow \quad 24u + 4v - 1 = 0 \quad \ldots (4)$$

Also, from (2), we have

$$10u + 4v = \frac{13}{20}$$

$$\Rightarrow \quad 200u + 80v - 13 = 0 \quad \ldots (5)$$

$\therefore$ From (4) and (5), by the method of cross-multiplication, we have

$$\frac{u}{-4 \times 13 + 80 \times 1} = \frac{v}{-1 \times 200 + 13 \times 24} = \frac{1}{24 \times 80 - 200 \times 4}$$

$$\Rightarrow \frac{u}{-52 + 80} = \frac{v}{-200 + 312} = \frac{1}{1920 - 800}$$

$$\Rightarrow \frac{u}{28} = \frac{v}{112} = \frac{1}{1120}$$

$$\therefore u = \frac{28}{1120} = \frac{1}{40} \text{ and } v = \frac{112}{1120} = \frac{1}{10}$$

$\therefore$ From (3), $\frac{1}{x} = \frac{1}{40}$ and $\frac{1}{y} = \frac{1}{10}$

$= x = 40$ and $y = 10$

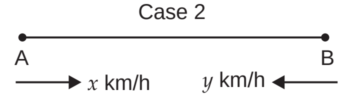
$\therefore$ The required speeds of the bus and the rickshaw are **40 km/h** and **10 km/h**.

5. Let the speeds of the cars starting from A and B be x km/h and y km/h respectively.

Case 1

Measures	*Car starting from A*	*Car starting from B*
Speed (in km/h)	x	y
Time (in hours)	5	5
Distance (in km)	$5x$	$5y$

Case 2



Measures	*Car starting from A*	*Car starting from B*
Speed (in km/h)	x	y
Time (in hours)	1	1
Distance (in km)	x	y

$$5x - 5y = 100$$

$\Rightarrow$ $x - y = 20$ … (1)

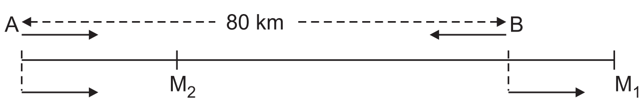
$\Rightarrow$ $x + y = 100$ … (2)

Solving equations (1) and (2), we get

$$x = 60 \text{ and } y = 40$$

Hence, the speeds of the two cars are **60 km/h** and **40 km/h**.

6. Let the speed of the car from A be x km/h and that from B be y km/h. Let the two cars moving from A and B in the same direction from A and B meet at M_1 in 8 hours. Those moving in the opposite directions from A to M_2 and B to M_2 meet together at the point M_2 after 1 h 20 min = $1\frac{1}{3}$ h = $\frac{4}{3}$ h.



Then in the 1st case when both the cars from A and B move in the same direction from left to right meet at M_1, the distance travelled by the car from A to reach M_1 after 8 hours – the distance travelled by the car from B to reach M_1 after 8 hours = AB = 80 km.

$\therefore$ $8x - 8y = 80$

$\Rightarrow$ $x - y = 10$ … (1)

Similarly, in the 2nd case when both the cars from A and B move in the opposite direction from left to right by the car from A and from right to left by the car from B,

The distance travelled by the car from A to reach M_2 after $\frac{4}{3}$ hours + the distance travelled by the car from B to reach M_2 after $\frac{4}{3}$ hours = AB = 80 km

$\therefore$ $\frac{4}{3}(x + y) = 80$

$\Rightarrow$ $x + y = 80 \times \frac{3}{4} = 60$ … (2)

Adding (1) and (2), we get

$$2x = 70$$

$\Rightarrow$ $x = 35$

Subtracting (1) from (2), we get

$$2y = 50$$

$\Rightarrow$ $y = 25$

Hence, required speeds of the car from A and that from B are respectively **35 km/h** and **25 km/h**.

7. Let the speed of the stream be x km/h and that of the person = y km/h. Then it is given that

$y = 5$ … (1)

Now, the speed of the person downstream = $(y + x)$ km/h and the speed of the person upstream = $(y - x)$ km/h.

$\therefore$ Time t_1 h taken by the person in moving through a distance of 40 km = $t_1 = \frac{40}{y + x}$ h and that in moving through the same distance upstream = $t_2 = \frac{40}{y - x}$ h.

$\therefore$ According to the problem, we have

$$t_2 = 3t_1$$

$\Rightarrow$ $\frac{40}{y - x} = 3 \times \frac{40}{y + x}$

$\Rightarrow$ $\frac{1}{5 - x} = \frac{3}{5 + x}$ [From (1)]

$\Rightarrow$ $15 - 3x = 5 + x$

$\Rightarrow$ $4x = 15 - 5 = 10$

$\therefore$ $x = \frac{10}{4} = 2.5$

Hence, the required speed of the stream is **2.5 km/h**.

8. Let the sailors speed in still be x km/h and the speed of the convent be y km/h.

Downward speed = $(x + y)$ km/h

and upward speed = $(x - y)$ Km/h.

Measures	*Downward*	*Upward*
Distance	8 km	8 km
Speed	$(x + y)$ km/h	$(x - y)$ km/h
Time	$\frac{8}{x + y}$ h	$\frac{8}{x - y}$ h

$$\frac{8}{x + y} = \frac{2}{3}$$

$\Rightarrow$ $24 = 2x + 2y$

$\Rightarrow$ $x + y = 12$ … (1)

and $\frac{8}{x - y} = 1$

$\Rightarrow$ $8 = x - y$ … (2)

Solving equation (1) and equation (2), we get

$$x = 10 \text{ and } y = 2$$

Hence, the speed of the sailor in still water is **10 km/h** and the speed of the current is **2 km/h**.

9. Let the speed of the boat in the still water be x km/h and let the speed of the stream be y km/h.

Downward speed = $(x + y)$ km/h

Upward speed = $(x - y)$ km/h

Case 1

Measures	*Downward*	*Upward*
Distance	55 km	35 km
Speed	$(x + y)$ km/h	$(x - y)$ km/h
Time	$\frac{55}{x+y}$ hours	$\frac{35}{x-y}$ hours

$$\frac{55}{x+y} + \frac{35}{x-y} = 12$$

Case 2

Measures	*Downward*	*Upward*
Distance	44 km	30 km
Speed	$(x + y)$ km/h	$(x - y)$ km/h
Time	$\frac{44}{x+y}$ hours	$\frac{30}{x-y}$ hours

$$\frac{44}{x+y} + \frac{30}{x-y} = 10 \quad \dots (2)$$

Multiply equation (1) by 4 and equation (2) by 5, we get

$$\frac{220}{x+y} + \frac{140}{x-y} = 48 \quad \dots (3)$$

$$\frac{220}{x+y} + \frac{150}{x-y} = 50 \quad \dots (4)$$

Subtracting equation (3) from equation (4), we get

$$\frac{10}{x-y} = 2$$

$$\Rightarrow \quad 5 = x - y \quad \dots (5)$$

Multiply equation (1) by 6 and equation (2) by 7, we get

$$\frac{330}{x+y} + \frac{210}{x-y} = 72 \quad \dots (7)$$

$$\frac{308}{x+y} + \frac{210}{x-y} = 70 \quad \dots (8)$$

Subtracting equation (8) from equation (7), we get

$$\frac{22}{x+y} = 2$$

$$\Rightarrow \quad 11 = x + y \quad \dots (9)$$

Solving equation (5) and equation (9), we get

$$x = 8 \text{ and } y = 3$$

Hence, the speed of the stream is **3 km/h** and the speed of the boat in still water is **8 km/h**.

10. Let the speed of the person in the still water be x km/h and the speed of the current be y km/h.

Downward stream = $(x + y)$ km/h

and upward stream = $(x - y)$ km/h

Case 1

Measures	*Downward*	*Upward*
Distance	24 km	8 km
Speed	$(x + y)$ km/h	$(x - y)$ km/h
Time	$\frac{24}{x+y}$ h	$\frac{8}{x-y}$ h

$$\frac{24}{x+y} + \frac{8}{x-y} = 4$$

$$\Rightarrow \quad \frac{6}{x+y} + \frac{2}{x-y} = 1 \quad \dots (1)$$

Case 2

Measures	*Downward*	*Upward*
Distance	12 km	12 km
Speed	$(x + y)$ km/h	$(x - y)$ km/h
Time	$\frac{12}{x+y}$	$\frac{12}{x-y}$

$$\frac{12}{x+y} + \frac{12}{x-y} = 4$$

$$\Rightarrow \quad \frac{3}{x+y} + \frac{3}{x-y} = 1 \quad \dots (2)$$

Multiply equation (2) by 2, we get

$$\frac{6}{x+y} + \frac{6}{x-y} = 2 \quad \dots (3)$$

Subtracting equation (1) from equation (3), we get

$$\frac{4}{x-y} = 1$$

$$\Rightarrow \quad x - y = 4 \quad \dots (4)$$

Multiplying equation (1) by 3, we get

$$\frac{18}{x+y} + \frac{6}{x-y} = 3 \quad \dots (5)$$

Subtracting equation (3) from equation (5), we get

$$\frac{12}{x+y} = 1$$

$$\Rightarrow \quad x + y = 12 \quad \dots (6)$$

Solving equation (4) and equation (6), we get

$$x = 8 \text{ and } y = 4$$

Hence, the speed of the person in still water is **8 km/h** and the speed of the current is **4 km/h**.

11. Let the speed of the boat be x km/h and that of the stream be y km/h.

Then the speed of the boat upstream = $(x - y)$ km/h

and the speed of the boat downstream = $(x + y)$ km/h

$\therefore$ Time t_1 h taken by the boat in moving through a distance of 28 km downstream

$$= t_1 = \frac{28}{x+y}\text{ h.}$$

Also, time t_2 h taken by the boat in moving through a distance of 30 km upstream $= t_2 = \frac{30}{x-y}$ h.

$\therefore$ According to the problem,

$$t_1 + t_2 = 7$$

$$\Rightarrow \quad \frac{28}{x+y} + \frac{30}{x-y} = 7$$

$$\Rightarrow \quad 28u + 30v - 7 = 0 \quad \ldots (1)$$

where $u = \frac{1}{x+y}$ and $v = \frac{1}{x-y}$ … (2)

Again, time t_3 h taken by the boat in moving through a distance of 21 km downstream $= t_3 = \frac{21}{x+y}$ h

And, time t_4 h taken by the boat in moving through the same distance of 21 km upstream $= t_4 = \frac{21}{x-y}$ h.

$\therefore$ According to the problem,

$$t_3 + t_4 = 5$$

$$\Rightarrow \quad \frac{21}{x+y} + \frac{21}{x-y} = 5$$

$$\Rightarrow \quad 21u + 21v - 5 = 0 \quad \ldots(3)$$

From (1) and (3), by the method of cross-multiplication, we have

$$\frac{u}{-5\times30+7\times21} = \frac{v}{-7\times21+5\times28} = \frac{1}{28\times21-30\times21}$$

$$\Rightarrow \frac{u}{-150+147} = \frac{v}{-147+170} = \frac{1}{588-630}$$

$$\Rightarrow \frac{u}{-3} = \frac{v}{-7} = \frac{1}{-42}$$

$$\therefore u = \frac{3}{42} = \frac{1}{14} \text{ and } v = \frac{7}{42} = \frac{1}{6}$$

$$\Rightarrow \frac{1}{x+y} = \frac{1}{14} \text{ and } \frac{1}{x-y} = \frac{1}{6} \quad \text{[From (2)]}$$

$$\Rightarrow \quad x + y = 14 \quad \ldots (4)$$

and $\quad x - y = 6 \quad \ldots (5)$

Adding (4) and (5), we have

$$2x = 20$$

$$\Rightarrow \quad x = 10$$

From (5), $\quad y = x - 6 = 10 - 6 = 4$

$\therefore$ The required speeds of the boat and the stream are **10 km/h** and **4 km/h** respectively.

12. Let the speed of the wind air be x km/h and the speed of the bird in still air be y km/h. Then clearly, $y > x$.

$\therefore$ Speed of the bird in wind air (i.e. in the same direction as that of the wind)

= Speed of the bird in still air + Speed of wind air

$= (y + x)$ km/h

and speed of the bird against the direction of the wind

= Speed of the bird in still air – Speed of the wind

$= (y - x)$ km/h

$\therefore$ Time t_1 h taken by the bird when it flies in the same direction of the wind through a distance of 45 km is $t_1 = \frac{45}{y+x}$ h and time t_2 h taken by the bird when it flies in the direction opposite to the direction of the wind through the same distance of 45 km is $t_2 = \frac{45}{y-x}$ h.

$\therefore$ According to the problem,

$$t_1 = 2\text{h } 30 \text{ min} = 2\frac{1}{2}\text{ h} = \frac{5}{2}\text{ h}$$

and $\quad t_2 = 4\text{h } 30 \text{ min} = 4\frac{1}{2}\text{ h} = \frac{9}{2}\text{ h}$

$$\therefore \quad \frac{45}{y+x} = \frac{5}{2}$$

and $\quad \frac{45}{y-x} = \frac{9}{2}$

$$\Rightarrow \quad 45u = \frac{5}{2} \quad \ldots (1)$$

and $\quad 45v = \frac{9}{2} \quad \ldots (2)$

where $u = \frac{1}{y+x}$ and $v = \frac{1}{y-x}$ … (3)

$\therefore$ From (1) and (2), we have

$$u = \frac{1}{18} \text{ and } v = \frac{1}{10}$$

$$\Rightarrow \quad \frac{1}{y+x} = \frac{1}{18}$$

and $\quad \frac{1}{y-x} = \frac{1}{10} \quad$ [From (3)]

$$\Rightarrow \quad x + y = 18 \quad \ldots (4)$$

and $\quad y - x = 10 \quad \ldots (5)$

Adding (4) and (5), we get

$$2y = 28$$

$$\Rightarrow \quad y = 14$$

Subtracting (5) from (4), we get

$$2x = 8$$

$$\Rightarrow \quad x = 4$$

Hence, (*i*) speed of the bird in still air = **14 km/h** and (*ii*) speed of the wind air = **4 km/h**.

13. Let the distance covered by the train be x km and let its speed be y km/h.

$\therefore \quad$ Scheduled time $= \frac{x}{y}$ hours.

Measures	*Case 1*	*Case 2*
Distance (in km)	x	x
Speed (in km/h)	$(y - 6)$	$(y + 6)$
Time (in hours)	$\frac{x}{y-6}$	$\frac{x}{y+6}$

Case 1. $\dfrac{x}{y-6} - \dfrac{x}{y} = 12$

$\Rightarrow \quad xy - xy + 6x = 12y(y-6)$

$\Rightarrow \quad 6x = 12y^2 - 72y \quad \ldots (1)$

Case 2. $\dfrac{x}{y} - \dfrac{x}{y+6} = 8$

$\Rightarrow \quad xy + 6x - xy = 8y\,(y+6)$

$\Rightarrow \quad 6x = 8y^2 + 48y \quad \ldots (2)$

From equations (1) and (2), we get

$12y^2 - 72y = 8y^2 + 48y$

$\Rightarrow \quad 4y^2 - 120y = 0$

$\Rightarrow \quad 4y\,(y-30) = 0$

$\Rightarrow$ Either $y = 0$ (rejected) or $y = 30$

Substituting $y = 30$ in equation (2), we get

$6x = 8(30)^2 + 48(30)$

$\Rightarrow \quad 6x = 8 \times 900 + 1440$

$\Rightarrow \quad 6x = 7200 + 1440$

$\Rightarrow \quad 6x = 8640$

$\Rightarrow \quad x = 1440$

Hence, the distance covered by the train is **1440 km.**

14. Let X's speed of walking be x km/h and Y's speed of walking be y km/h.

Case 1.

Measures	*X*	*Y*
Distance (in km)	30	30
Speed (in km/h)	x	y
Time (h)	$\dfrac{30}{x}$	$\dfrac{30}{y}$

$\dfrac{30}{x} - \dfrac{30}{y} = 3$

$\Rightarrow \quad \dfrac{10}{x} - \dfrac{10}{y} = 1 \quad \ldots (1)$

Case 2.

Measures	*X*	*Y*
Distance (in km)	30	30
Speed (in km/h)	$2x$	y
Time (h)	$\dfrac{30}{x} = \dfrac{15}{x}$	$\dfrac{30}{y}$

$\dfrac{30}{y} - \dfrac{15}{x} = \dfrac{3}{2}$

$\Rightarrow \quad \dfrac{10}{y} - \dfrac{5}{x} = \dfrac{1}{2} \quad \ldots (2)$

Adding equation (1) and equation (2), we get

$\dfrac{5}{x} = \dfrac{3}{2}$

$\Rightarrow \quad x = \dfrac{10}{3}$

Substituting $x = \dfrac{10}{3}$ in equation (2), we get

$\dfrac{10}{y} - \dfrac{5}{\frac{10}{3}} = \dfrac{1}{2}$

$\Rightarrow \quad \dfrac{10}{y} - \dfrac{15}{10} = \dfrac{1}{2}$

$\Rightarrow \quad \dfrac{10}{y} = \dfrac{1}{2} + \dfrac{3}{2} = 2$

$\Rightarrow \quad y = \dfrac{10}{2} = 5$

Hence, X's speed is $\mathbf{\dfrac{10}{3}}$ **km/h** and Y's speed is **5 km/h**.

15. Let Abhimanyu's walking speed be x km/h and let Sadanand's walking speed be y km/h.

Case 1.

Measures	*Abhimanyu*	*Sadanand*
Speed (in km/h)	x	y
Time (h)	8	10
Distance (in km)	$8x$	$8y$

$8x - 10y = 1 \quad \ldots (1)$

Case 2.

Measures	*Abhimanyu*	*Sadanand*
Speed (in km/h)	x	y
Time (h)	10	14
Distance (in km)	$10x$	$14y$

$14y - 10x = 4$

$\Rightarrow \quad 7y - 5x = 2 \quad \ldots (2)$

Solving equations (1) and (2), we get

$x = 4.5$ and $y = 3.5$

Hence, Abhimanyu's speed of walking is **4.5 km/h** and Sadanand's speed of walking is **3.5 km/h**.

EXERCISE 3M

For Basic and Standard Levels

1. Let the taxi's fixed charge per day be ₹ x and the rate per km be ₹ y.

Then, $\quad x + 110y = 690 \quad \ldots (1)$

and $\quad x + 200y = 1050 \quad \ldots (2)$

Subtracting equation (1) from equation (2), we get

$90y = 360$

$\Rightarrow \quad y = 4$

Substituting $y = 4$ in equation (1), we get

$x + 110(4) = 690$

$\Rightarrow \quad x = 690 - 440 = 250$

Hence, the fixed change is ₹ **250** and the rate per km is ₹ **4**.

2. Let the constant monthly expenses of the family be ₹ x. Let the monthly consumption of rice be y quintals.

Then, $\quad x + 250y = 1000 \quad \ldots (1)$

and $x + 240y = 980$ … (2)

Subtracting equation (2) from equation (1), we get

$$10y = 20$$

$\Rightarrow$ $y = 2$

Substituting $y = 2$ in equation (1), we get

$$x + 250(2) = 1000$$

$\Rightarrow$ $x = 500$

Hence, the monthly expenditure of the family when the cost of rice is ₹ 300 per quintal = $x + 300y$

= ₹ [500 + 300(2)] = ₹ **1100**.

3. Let the fixed car rental charge be ₹ x and let the charge per km be ₹ y.

Then, $x + 13y = 96$ … (1)

and $x + 18y = 131$ … (2)

Subtracting equation (1) from equation (2), we get

$$5y = 35$$

$\Rightarrow$ $y = 7$

Substituting $y = 7$ in equation (1), we get

$$x + 13(7) = 96$$

$\Rightarrow$ $x = 96 - 91 = 5$

For travelling a distance of 25 km, a person will have to pay

$= x + 25y$

= ₹ [5 +25(7)]

= ₹ (5 + 175)

= ₹ **180**

4. Let the monthly bill for each student for participating in one activity club be ₹ x and let the fixed constant part be ₹ y.

Then according to the problem, we have

$3x + y = 1025$ … (1)

and $2x + y = 950$ … (2)

$\therefore$ Subtracting (2) from (1), we get

$$x = 75$$

$\therefore$ From (2), $y = 950 - 2 \times 75 = 950 - 150 = 800$

$\therefore$ Total monthly fees fir 1 student for participating in 4 club activities = ₹ (75 × 4 + 800) = ₹ (300 + 800)

= ₹ 1100.

$\therefore$ The required total monthly bill of 3 students, each participating in 4 club activities = ₹ 1100 × 3 = ₹ **3300**.

5. Let the full 1st class fare be ₹ x and each reservation charge be ₹ y.

Then the cost of 1 full 1st class ticket and 1 half 1st class ticket = ₹ $\left(x + \frac{x}{2}\right)$ = ₹ $\frac{3x}{2}$.

Also, the total reservation charge for 2 persons = ₹ $2y$.

$\therefore$ According to the problem, we have

$x + y = 2530$ … (1)

and $\frac{3x}{2} + 2y = 3810$

$\Rightarrow$ $3x + 4y = 7620$ … (2)

$\therefore$ From (1), $y = 2530 - x$ … (3)

$\therefore$ From (2) and (3), we have

$$3x + 4(2530 - x) = 7620$$

$\Rightarrow$ $-x + 10120 = 7620$

$\Rightarrow$ $x = 10120 - 7620 = 2500$

$\therefore$ From (3), $y = 2530 - 2500 = 30$

Hence, the required full first class fare and the reservation charges for each ticket are respectively ₹ **2500** and ₹ **30**.

6. Let the first class fare from Delhi to Bengaluru be ₹ x and let the reservation charges be ₹ y.

Then, $(x + y) = 2325$ … (1)

and $(x + y) + \left(\frac{x}{2} + y\right) = 3525$

$\Rightarrow$ $\frac{3x}{2} + 2y = 3525$

$\Rightarrow$ $3x + 4y = 7050$ … (2)

Multiplying equation (1) by 4, we get

$4x + 4y = 9300$ … (3)

Subtracting equation (2) from equation (3), we get

$$x = 9300 - 7050 = 2250$$

Substituting $x = 2250$ in equation (1), we get

$$y = 75$$

Hence, the first class fare from Delhi to Bengaluru is ₹ **2250** and the reservation charges are ₹ **75**.

EXERCISE 3N

For Basic and Standard Levels

1. Let the original number of boys and girls in the school be x and y respectively.

Then, $x + y = 2800$ … (1)

and $x + \frac{5}{100}x + y + \frac{10}{100}y = 3000$

$\Rightarrow$ $\frac{21}{20}x + \frac{11}{10}y = 3000$

$\Rightarrow$ $21x + 22y = 60000$ … (2)

Multiply equation (1) by 21, we get

$21x + 21y = 58800$ … (3)

Subtracting equation (3) from equation (2), we get

$$y = 1200$$

Substituting y =1200 in equation (1), we get

$$x + 1200 = 2800$$

$\Rightarrow$ $x = 1600$

Hence, the original number of boys in the school is **1600** and the original number of girls in the school is **1200**.

2. Let the cost price of the chain be ₹ x and the cost price of the table be ₹ y.

Then, $x + \frac{25}{100}x + y + \frac{10}{100}y = 760$

and $x + \frac{10}{100}x + y + \frac{25}{100}y = 767.50$

$\Rightarrow$ $\frac{5}{4}x + \frac{11}{10}y = 760$ … (1)

and $\frac{11}{10}x + \frac{5}{4}y = \frac{1535}{2}$ … (2)

Multiply equation (1) by $\frac{22}{25}$, we get

$$\frac{5}{4}x \times \frac{22}{25} + \frac{11}{10}y \times \frac{22}{25} = 760 \times \frac{22}{25}$$

$$\Rightarrow \quad \frac{11}{10}x + \frac{121}{125}y = \frac{3344}{5} \quad \dots (3)$$

Subtracting equation (3) from equation (2), we get

$$\left(\frac{5}{4} - \frac{121}{125}\right)y = \frac{1535}{2} - \frac{3344}{5}$$

$$\Rightarrow \left(\frac{5 \times 125 - 121 \times 4}{4 \times 125}\right)y = \left(\frac{1535 \times 5 - 3344 \times 2}{10}\right)$$

$$\Rightarrow \frac{(625 - 484)}{4 \times 125}y = \frac{987}{10}$$

$$\Rightarrow \frac{141}{4 \times 125}y = \frac{987}{10}$$

$$\Rightarrow y = \frac{987 \times 4 \times 125}{141 \times 10} = 350$$

Substituting $y = 350$ in equation (1), we get

$$\frac{5}{4}x + \frac{11}{10} \times 350 = 760$$

$$\Rightarrow \frac{5}{4}x = 760 - 385 = 375$$

$$\Rightarrow x = \frac{375 \times 4}{5} = 300$$

Hence, the cost price of the chair is ₹ **300**
and the cost price of the table is ₹ **350**

3. Let the cost price (C.P.) of the saree be ₹ x and C.P. of the sweater be ₹ y.

$\therefore$ S.P. of the saree at 8% profit = ₹ $\left(x + \frac{8x}{100}\right)$

$$= ₹\ \frac{108x}{100}$$

and S.P. of the sweater at 10% discount = ₹ $\left(y - \frac{10y}{100}\right)$

$$= ₹\ \frac{90y}{100}$$

$\therefore$ According to the problem, we have

$$\frac{108x + 90y}{100} = 1008$$

$$\Rightarrow 108x + 90y = 100800$$

$$\Rightarrow 54x + 45y - 50400 = 0 \quad \dots (1)$$

Again, S.P. of the saree at 10% profit = ₹ $\left(x + \frac{10x}{100}\right)$

$$= ₹\ \frac{110x}{100}$$

and S.P. of the sweater at 8% discount = ₹ $\left(y - \frac{8y}{100}\right)$

$$= ₹\ \frac{92y}{100}$$

$\therefore$ According to the problem, we have

$$\frac{110x}{100} + \frac{92y}{100} = 1028$$

$$\Rightarrow 110x + 92y = 102800$$

$$\Rightarrow 55x + 46y - 51400 = 0 \quad \dots (2)$$

Subtracting (1) from (2), we get

$$x + y = 1000$$

$$\therefore \quad y = 1000 - x \quad \dots (3)$$

$\therefore$ From (1),

$$54x + 45000 - 45x = 50400$$

$$\Rightarrow 9x = 5400$$

$$\Rightarrow x = 600$$

$\therefore$ From (3), $\quad y = 1000 - 600 = 400$

$\therefore$ Required costs of saree and sweater are ₹ **600** and ₹ **400** respectively.

4. Let the original price of each chocolate bar be ₹ x and that of each ice cream cone be ₹ y. Then the increase in the price of each chocolate bar is ₹ $\frac{5x}{100}$ = ₹ $\frac{x}{20}$ and the increase in the price of each ice cream cone is ₹ $\frac{y}{10}$

Also, the increased price of each chocolate bar

$$= ₹\ \frac{21x}{20} \quad \dots (A)$$

and that of each ice cream cone is ₹ $\frac{11y}{10}$ … (B)

$\therefore$ According to the problem, we have

$$\frac{3x}{20} + \frac{4y}{10} = 9.75 \quad \dots (1)$$

and $\quad 3x + 4y = 135 \quad \dots (2)$

From (1), $\quad \frac{3x + 8y}{20} = 9.75$

$$\Rightarrow 3x + 8y = 9.75 \times 20 = 195 \quad \dots (3)$$

Subtracting (2) from (3), we get

$$4y = 195 - 135 = 60$$

$$\therefore \quad y = \frac{60}{4} = 15$$

$\therefore$ From (2), $3x = 135 - 4 \times 15 = 135 - 60 = 75$

$$\Rightarrow x = \frac{75}{3} = 25$$

$\therefore$ Required increased price of each chocolate bar

$$= ₹\ \frac{21x}{20} \quad \text{[From (A)]}$$

$$= ₹\ \frac{21}{20} \times 25$$

$$= ₹\ \frac{105}{4}$$

= ₹ **26.25**

and increase price of each ice cream cone

$$= ₹\ \frac{11y}{10} \quad \text{[From (B)]}$$

$$= ₹\ \frac{11}{10} \times 15$$

$$= ₹\ \frac{33}{2}$$

= ₹ **16.50**

For Standard Level

5. Let the original number of boys and girls in the class be x and y respectively.

Original percentage of boys in the class $= \frac{x}{x+y} \times 100\%$

$$\therefore \quad \frac{x}{x+y} \times 100 = 60$$

$$\Rightarrow \quad 100x = 60(x+y)$$

$$\Rightarrow \quad 5x = 3x + 3y$$

$$\Rightarrow \quad 2x = 3y \quad \dots (1)$$

Number of boys admitted = 6

and number of girls left = 6.

$\therefore$ New percentage of boys in the class

$$= \frac{x+6}{x-6+y+6} \times 100\ \%$$

$$\therefore \quad \frac{x+6}{x+y} \times 100 = 75$$

$$\Rightarrow \quad (x+6)100 = 75(x+y)$$

$$\Rightarrow \quad 4x + 24 = 3x + 3y$$

$$\Rightarrow \quad x + 24 = 3y \quad \dots (2)$$

From equation (1) and equation (2), we get

$$2x = x + 24$$

$$\Rightarrow \quad x = 24$$

Substituting $x = 24$ in equation (1), we get

$$2 \times 24 = 3y$$

$$\Rightarrow \quad y = \frac{2 \times 24}{3} = 16$$

Hence, the class originally had **24 boys** and **16 girls**.

6. Suppose the man invested ₹ x at 12% simple interest and ₹ y at 10% simple interest.

Then, $\frac{12}{100}x + \frac{10}{100}y = 130 \quad \dots (1)$

and $\frac{10}{100}x + \frac{12}{100}y = 134 \quad \dots (2)$

Multiplying equation (1) by $\frac{5}{6}$, we get

$$\frac{12}{100}x \times \frac{5}{6} + \frac{10}{100}y \times \frac{5}{6} = 130 \times \frac{5}{6}$$

$$\Rightarrow \quad \frac{10}{100}x + \frac{1}{12}y = \frac{325}{3} \quad \dots (3)$$

Subtracting equation (3) from equation (2), we get

$$\frac{12}{100}y - \frac{1}{12}y = 134 - \frac{325}{3}$$

$$\Rightarrow \quad \frac{3}{25}y - \frac{1}{12}y = \frac{402-325}{3}$$

$$\Rightarrow \quad \frac{(36-25)}{25 \times 12}y = \frac{77}{3}$$

$$\Rightarrow \quad y = \frac{77}{3} \times \frac{25 \times 12}{11} = 700$$

Substituting $y = 700$ in equation (1), we get

$$\frac{12}{100}x + \frac{10}{100} \times 700 = 130$$

$$\Rightarrow \quad \frac{12}{100}x = 130 - 70 = 60$$

$$\Rightarrow \quad x = \frac{60 \times 100}{12} = 500$$

Hence, he invested ₹ **500 at 12% simple interest** and ₹ **700 at 10% simple interest**.

7. Let Swarn's investment in schemes A and B be ₹ x and ₹ y respectively. Then her annual interest in scheme A $=$ ₹ $\frac{8x}{100}$ and in scheme B $=$ ₹ $\frac{9y}{100}$. If she interchange her investments in these two schemes, then her annual interest will be ₹ $\frac{8y}{100}$ and ₹ $\frac{9x}{100}$ respectively.

$\therefore$ According to the problem, we have

$$\frac{8x}{100} + \frac{9y}{100} = 1860$$

$$\Rightarrow \quad 8x + 9y - 186000 = 0 \quad \dots (1)$$

and $$\frac{8y}{100} + \frac{9x}{100} = 1880$$

$$\Rightarrow \quad 9x + 8y - 188000 = 0 \quad \dots (2)$$

$\therefore$ From (1) and (2), by the method of cross-multiplication, we have

$$\frac{x}{-9 \times 188000 + 8 \times 186000} = \frac{y}{-9 \times 186000 + 8 \times 188000} = \frac{1}{8 \times 8 - 9 \times 9}$$

$$\Rightarrow \quad \frac{x}{-1692000 + 1488000} = \frac{y}{-1674000 + 1504000} = \frac{1}{64 - 81}$$

$$\Rightarrow \quad \frac{x}{-204000} = \frac{y}{-170000} = \frac{1}{-17}$$

$$\therefore \quad x = \frac{204000}{17} = 12000 \text{ and } y = \frac{170000}{17} = 10000$$

Hence, her required investments in scheme A and scheme B are ₹ **12000** and ₹ **10000** respectively.

8. Let the quantity of 40% acid solution be x litres and that of 15% acid solution be y litres.

$\therefore$ Quantity of acid in 1st type of solution

$$= \frac{40x}{100}\text{ L} = \frac{2x}{5}\text{ L}$$

and quantity of acid in 2nd type of solution

$$= \frac{15y}{100} = \frac{3y}{20}\text{ L.}$$

Now, quantity of acid in 20 litres of the mixture

$$= \frac{25}{100} \times 20\text{ L} = 5\text{L.}$$

$\therefore$ According to the problem, we have

$$\frac{2x}{5} + \frac{3y}{20} = 5$$

$$\Rightarrow \quad 8x + 3y = 5 \times 20 = 100 \quad \dots (1)$$

Also, $$x + y = 20 \quad \dots (2)$$

Now, from (2), we have

$$y = 20 - x \quad \ldots(3)$$

∴ From (1) and (3), we have

$$8x + 3(20 - x) = 100$$

⇒ $$5x = 100 - 60 = 40$$

⇒ $$x = \frac{40}{5} = 8$$

∴ From (3), $y = 20 - 8 = 12$

Hence, the required quantity of the 1st type solution = **8 litres** and that of the 2nd type of solution = **12 litres**.

EXERCISE 3O

For Basic and Standard Levels

1. Suppose 1 woman alone can finish the work in x days and 1 girl alone can finish the work in y days.

Then, 1 woman's 1 day's work = $\frac{1}{x}$

and 1 girl's 1 day's work = $\frac{1}{y}$

10 women and 20 girls can finish the work in 2 days.

⇒ 10 women's 1 day's work + 20 girl's 1 day's work $= \frac{1}{2}$

⇒ $$\frac{10}{x} + \frac{20}{y} = \frac{1}{2} \quad \ldots(1)$$

Again, 6 women and 4 girls can finish the work in 5 days.

⇒ 6 women's 1 day's work + 4 girl's 1 day's work = $\frac{1}{5}$

⇒ $$\frac{6}{x} + \frac{4}{y} = \frac{1}{5} \quad \ldots(2)$$

Multiply equation (2) by 5, we get

$$\frac{30}{x} + \frac{20}{y} = 1 \quad \ldots(3)$$

Subtracting equation (1) from equation (3), we get

$$\frac{20}{x} = \frac{1}{2}$$

⇒ $$20 \times 2 = x$$

⇒ $$x = 40$$

Substituting $x = 40$ in equation (1), we get

$$\frac{10}{40} + \frac{20}{y} = \frac{1}{2}$$

⇒ $$\frac{20}{y} = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$$

⇒ $$y = 20 \times 4 = 80$$

Hence, one woman alone can finish the work in **40 days** and one girl alone can finish the work in **80 days**.

2. Suppose a single man can finish the work in x days and a single boy can finish the work in y days.

Then, 1 man's 1 day's work = $\frac{1}{x}$

and 1 boy's 1 day's work = $\frac{1}{y}$

8 men and 12 boys can finish the work in 10 days

⇒ (8 men's 1 day's work) + (12 boy's 1 day's work) $= \frac{1}{10}$

⇒ $$\frac{8}{x} + \frac{12}{y} = \frac{1}{10} \quad \ldots(1)$$

Also, 6 men and 8 boys can finish the work in 14 days.

⇒ (6 men's 1 day's work) + (8 boy's 1 day's work) = $\frac{1}{14}$

⇒ $$\frac{6}{x} + \frac{8}{y} = \frac{1}{14} \quad \ldots(2)$$

Multiplying equation (1) by 3 and equation (2) by 4, we get

$$\frac{24}{x} + \frac{36}{y} = \frac{3}{10} \quad \ldots(3)$$

$$\frac{24}{x} + \frac{32}{y} = \frac{4}{14} = \frac{2}{7} \quad \ldots(4)$$

Subtracting equation (4) from equation (3), we get

$$\frac{4}{y} = \frac{3}{10} - \frac{2}{7}$$

⇒ $$\frac{4}{y} = \frac{21 - 20}{70} = \frac{1}{70}$$

⇒ $$y = 4 \times 70 = 280$$

Substituting $y = 280$ in equation (1), we get

$$\frac{8}{x} + \frac{12}{280} = \frac{1}{10}$$

⇒ $$\frac{8}{x} = \frac{1}{10} - \frac{3}{70} = \frac{7 - 3}{70}$$

⇒ $$\frac{8}{x} = \frac{4}{70}$$

⇒ $$x = \frac{8 \times 70}{4} = 140$$

Hence, a single man will take **140 days** to do the work and a single boy will take **280 days** to do the work.

3. Let 1 boy complete the work in x days and 1 man complete the same work in y days.

∴ In 1 day, 1 boy completes $\frac{1}{x}$th part of the work and 1 man completes $\frac{1}{y}$th part of the work.

∴ In 1 day, 1 boy and 1 man can complete $\left(\frac{1}{x} + \frac{1}{y}\right)$ part of the work.

∴ In 1 day, 4 boys and 4 men can complete $4\left(\frac{1}{x} + \frac{1}{y}\right)$ part of the work.

Now, they can complete whole work in 3 days.

∴ $$3 \times 4\left(\frac{1}{x} + \frac{1}{y}\right) = 1$$

⇒ $$\frac{12}{x} + \frac{12}{y} = 1$$

$\Rightarrow \quad 12u + 12v - 1 = 0 \quad \ldots (1)$

where $u = \frac{1}{x}$ and $v = \frac{1}{y}$ $\quad \ldots (2)$

Similarly, in 1 day, 7 boys and 2 men can complete $\left(\frac{7}{x}+\frac{2}{y}\right)$ part of the work. Now, they can complete the same work in 4 days.

$$\therefore \quad 4\left(\frac{7}{x}+\frac{2}{y}\right) = 1$$

$$\Rightarrow \quad 28u + 8v - 1 = 0 \quad \ldots (3)$$

$\therefore$ From (1) and (3), by the method of cross-multiplication, we have

$$\frac{u}{-1\times 21+8\times 1} = \frac{v}{-1\times 28+12\times 1} = \frac{1}{12\times 8-12\times 28}$$

$$\Rightarrow \frac{u}{-12+8} = \frac{v}{-28+12} = \frac{1}{96-336}$$

$$\Rightarrow \frac{u}{-4} = \frac{v}{-16} = \frac{1}{-240}$$

$$\therefore u = \frac{4}{240} = \frac{1}{60}$$

$$\Rightarrow \quad \frac{1}{x} = \frac{1}{60} \quad \text{[From (2)]}$$

$$\Rightarrow \quad x = 60$$

and $\quad v = \frac{16}{240} = \frac{1}{15}$

$$\Rightarrow \quad \frac{1}{y} = \frac{1}{15} \quad \text{[From (2)]}$$

$$\Rightarrow \quad y = 15$$

$\therefore$ Required no. of days taken by the boy to finish the work = **60 days** and the no. of days taken by the man to finish the same work = **15 days**.

4. Let the time taken by a single tap A alone to fill up the tank be x hours and that taken by a single tap B alone to fill up the same tank be y hours.

$\therefore$ In 1 hour, tap A fills up $\frac{1}{x}$th part of the tank and in 1 hour, tap B fills up $\frac{1}{y}$th part of the tank.

$\therefore$ In 1 hour, single tap A and single tap B together can fill up $\left(\frac{1}{x}+\frac{1}{y}\right)$th part of the tank.

It is given that in 6 hours these two taps can fill up the tank completely.

$$\therefore \quad 6\left(\frac{1}{x}+\frac{1}{y}\right) = 1 \quad \ldots (1)$$

Again, when 2 taps A and 3 taps B are opened together they can fill up $\left(\frac{2}{x}+\frac{3}{y}\right)$th part of the tank in 1 hour.

It is given that all these 5 taps, when opened, can fill the same whole tank completely in 2h 30 min, i.e. $2\frac{1}{2}$ h = $\frac{5}{2}$ h.

$$\therefore \quad \frac{5}{2}\left(\frac{2}{x}+\frac{3}{y}\right) = 1$$

$$\Rightarrow \quad \frac{10}{x}+\frac{15}{y} = 2 \quad \ldots (2)$$

Let $\frac{1}{x} = u$ and $\frac{1}{y} = v$ $\quad \ldots (3)$

$\therefore$ From (1), (2) and (3), we have

$$6u + 6v - 1 = 0 \quad \ldots (4)$$

and $\quad 10u + 15v - 2 = 0 \quad \ldots (5)$

From (4) and (5), by the method of cross-multiplication, we have

$$\frac{u}{-6\times 2+15\times 1} = \frac{v}{-1\times 10+6\times 2} = \frac{1}{6\times 15-10\times 6}$$

$$\Rightarrow \frac{u}{-12+15} = \frac{v}{-10+12} = \frac{1}{90-60}$$

$$\Rightarrow \frac{u}{3} = \frac{v}{2} = \frac{1}{30}$$

$$\therefore u = \frac{3}{30} = \frac{1}{10}$$

$$\Rightarrow \quad \frac{1}{x} = \frac{1}{10}$$

$$\Rightarrow \quad x = 10$$

and $\quad v = \frac{2}{30} = \frac{1}{15}$

$$\Rightarrow \quad \frac{1}{y} = \frac{1}{15}$$

$$\Rightarrow \quad y = 15$$

Hence, the required time for tap A is **10 hours** and for tap B is **15 hours**.

5. Let the time taken by 1 man to complete the work be x days and that taken by 1 woman to complete the same work by y days.

$\therefore$ The man can do $\frac{1}{x}$th part of the work in 1 day and the woman can do $\frac{1}{y}$th part of the work in 1 day.

$\therefore$ According to the problem, we have

$$8\left(\frac{1}{x}+\frac{1}{y}\right) = 1$$

$$\Rightarrow \quad 8u + 8v - 1 = 0 \quad \ldots (1)$$

and $\quad 2\left(\frac{3}{x}+\frac{6}{y}\right) = 1$

$$\Rightarrow \quad 6u + 12v - 1 = 0 \quad \ldots (2)$$

where $\frac{1}{x} = u$ and $\frac{1}{y} = v$ $\quad \ldots (3)$

From (1) and (2), by the method of cross-multiplication, we have

$$\frac{u}{-1\times 8+12\times 1} = \frac{v}{-1\times 6+8\times 1} = \frac{1}{8\times 12-8\times 6}$$

$$\Rightarrow \frac{u}{12-8} = \frac{v}{8-6} = \frac{1}{96-48}$$

$\Rightarrow \frac{u}{4} = \frac{v}{2} = \frac{1}{48}$

$\therefore \quad u = \frac{4}{48} = \frac{1}{12}$

$\Rightarrow \quad \frac{1}{x} = \frac{1}{12}$ [From (3)]

$\Rightarrow \quad x = 12$ … (4)

and $\quad v = \frac{2}{48} = \frac{1}{24}$

$\Rightarrow \quad \frac{1}{y} = \frac{1}{24}$ [From (3)]

$\Rightarrow \quad y = 24$ … (5)

$\therefore$ 1 man and 2 women can complete

$$\left(\frac{1}{x}+\frac{2}{y}\right)\text{th part} = \left(\frac{1}{12}+\frac{1}{12}\right)\text{th}$$

[From (4) and (5)]

$= \frac{1}{6}$th part of the work in 1 day.

$\therefore$ They can complete the whole work in 6 days.

$\therefore$ The required no. of days = **6 days**.

EXERCISE 3P

For Basic and Standard Levels

1. Suppose the man's initial salary was ₹ x and his increment was ₹ y.

Then, $\quad x + 5y = 22000$ … (1)

and $\quad x + 8y = 23200$ … (2)

Subtracting equation (1) from equation (2), we get

$3y = 1200$

$\Rightarrow \quad y = 400$

Subtracting $y = 400$ in equation (1), we get

$x + 5(400) = 22000$

$\Rightarrow \quad x = 22000 - 2000 = 20000$

Hence, his initial salary was ₹ **20000** and his annual investment was ₹ **400**.

2. Let A's monthly income be ₹ $5x$.

Then, B's monthly income = ₹ $4x$

Let A's monthly expenditure be ₹ $7y$.

Then, B's monthly expenditure = ₹ $5y$

Then, $\quad 5x - 7y = 3000$ … (1)

and $\quad 4x - 5y = 3000$ … (2)

Multiplying equation (1) by 5 and equation (2) by 7, we get

$25x - 35y = 15000$ … (3)

$28x - 35y = 21000$ … (4)

Subtracting equation (3) from equation (4), we get

$3x = 6000$

$\Rightarrow \quad x = 2000$

Hence, A's monthly income = ₹ 5 × 2000 = ₹ **10000** and B's monthly income = ₹ 4 × 2000 = ₹ **8000**.

3. Suppose A had x stamps and B had y stamps.

Then, $\quad y + 32 = 2(x - 32)$

$\Rightarrow \quad y + 32 = 2x - 64$

$\Rightarrow \quad 2x - y = 96$ … (1)

and $\quad x + 18 = 3(y - 18)$

$\Rightarrow \quad x + 18 = 3y - 54$

$\Rightarrow \quad x - 3y = -72$ … (2)

Multiplying equation (1) by 3, we get

$6x - 3y = 288$ … (3)

Subtracting equation (2) from equation (3), we get

$5x = 288 + 72$

$\Rightarrow \quad 5x = 360$

$\Rightarrow \quad x = 72$

Substituting $x = 72$ in equation (2), we get

$72 - 3y = -72$

$\Rightarrow \quad 3y = 72 + 72 = 144$

$\Rightarrow \quad y = 48$

Hence, A had **72 stamps** and B had **48 stamps**.

4. Suppose the bag has x red balls and y white balls.

Then, $\quad \frac{y}{2} = \frac{x}{3}$

$\Rightarrow \quad 3y = 2x$

$\Rightarrow \quad 2x - 3y = 0$ … (1)

and $\quad 3(x + y) - 7y = 6$

$\Rightarrow \quad 3x + 3y - 7y = 6$

$\Rightarrow \quad 3x - 4y = 6$ … (2)

Multiply equation (1) by 4 and equation (2) by 3, we get

$8x - 12y = 0$ … (3)

$9x - 12y = 18$ … (4)

Subtracting equation (3) from equation (4), we get

$x = 18$

Substituting $x = 18$ in equation (1), we get

$2 \times 18 - 3y = 0$

$\Rightarrow \quad 36 = 3y$

$\Rightarrow \quad y = 12$

Hence, the bag has **18** red balls and **12** white balls.

5. According to the problem, we have

$x - 8 = y$ … (1)

and $\quad y - 7 = \frac{4}{7}x$ … (2)

From (1) and (2), we have

$x - 8 - 7 = \frac{4}{7}x$

$\Rightarrow \quad x - 15 = \frac{4}{7}x$

$\Rightarrow \quad x - \frac{4}{7}x = 15$

$\Rightarrow \quad \frac{3}{7}x = 15$

$\therefore \quad x = 15 \times \frac{7}{3} = 35$

$\therefore$ From (1), $\quad y = 35 - 8 = 27$.

$\therefore$ Required weights of the two baskets are **35 kg** and **27 kg** respectively.

6. Suppose the breakdown occurred after the cyclist covered x km and his original speed is y km/h.

Case 1

Measures	*Up to breakdown*	*After breakdown*
Distance (in km)	x	$30 - x$
Speed (in km/h)	y	$\frac{y}{2}$
Time (in hours)	$\frac{x}{y}$	$\frac{30-x}{\frac{y}{2}} = \frac{60-2x}{y}$

$$\therefore \quad \frac{x}{y} + \frac{60-2x}{y} = 5$$

$$\Rightarrow \quad x + 60 - 2x = 5y$$

$$\Rightarrow \quad x + 5y = 60 \qquad \ldots (1)$$

Case 2

Measures	*Up to breakdown*	*After breakdown*
Distance (in km)	$x + 10$	$(30 - x - 10) = (20 - x)$
Speed (in km/h)	y	$\frac{y}{2}$
Time (in hours)	$\frac{x+10}{y}$	$\frac{20-x}{\frac{y}{2}} = \frac{40-2x}{y}$

$$\therefore \quad \frac{x+10}{y} + \frac{40-2x}{y} = 4$$

$$\Rightarrow \quad x + 10 + 40 - 2x = 4y$$

$$\Rightarrow \quad 4y + x = 50 \qquad \ldots (2)$$

Solving equation (1) and (2), we get

$$x = 10 \text{ and } y = 10.$$

Hence, the breakdown occurred after **10 km** and his original speed was **10 km/h**.

CHECK YOUR UNDERSTANDING

MULTIPLE-CHOICE QUESTIONS

1. (*c*) **Intersecting or coincident**

$\because$ A pair of linear equations in two variables represented graphically is consistent (*i*) with a unique solution if the two graph lines intersect at a point (*ii*) with infinitely many solutions if the two graph lines coincide.

2. (*d*) **Intersecting at (4, 3)**

$$x = 4$$

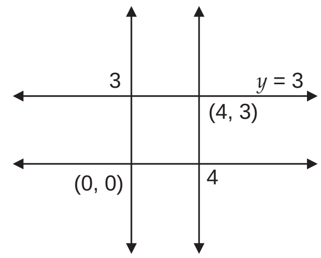
for all values of y and it represents a line parallel to the y axis at a distance of +4 units from it.

$$y = 3$$

$$\Rightarrow \quad y = 3$$

for all values of x and it represents a line parallel to the x-axis at a distance of +3 units from it.

Graph lines representing $x = 4$ and $y = 3$ intersect at **(4, 3)** as shown in the figure.



3. (*d*) $\mathbf{10x - 14y = -4}$

A pair of linear equations is consistent (*dependent*) with infinitely many solutions if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$10x - 14y = -4$ satisfies the above condition.

$$\because \quad \frac{-5}{10} = \frac{7}{-14}$$

$$= \frac{2}{-4} \quad \left(= -\frac{1}{2}\right)$$

4. (*b*) **no value**

The given equations are

$$3x + \alpha y - 6 = 0$$

and $\quad 6x + 8y - 7 = 0$

For infinitely many solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\therefore \quad \frac{3}{6} = \frac{\alpha}{8} = \frac{-6}{-7}$$

no value of α can satisfy the required condition.

5. (*c*) **(0,0)**

$$ax + by = 0$$

$$\Rightarrow \quad by = -ax$$

$$\Rightarrow \quad y = -\frac{a}{b}x$$

When $x = 0$, $y = 0$.

$$ax - by = 0$$

$$\Rightarrow \quad by = ax$$

$$\Rightarrow \quad y = \frac{ax}{b}$$

When $x = 0$, $y = 0$.

$\therefore$ The graph lines of $ax + by = 0$ and $ax - by = 0$ intersect at (0, 0).

6. (*d*) **(2*a*, 0), (0, 2*b*)**

$$\frac{x}{a} + \frac{y}{b} = 2$$

$$\Rightarrow \quad bx + ay = 2ab$$

The graph line of the given equation will intersect the x-axis when y coordinate is 0.

$$\Rightarrow \quad b\,x + a(0) = 2ab$$

$$\Rightarrow \quad x = \frac{2ab}{b} = 2a$$

Hence, it will intersect x-axis at $(2a, 0)$.

Also, the graph line of the given equation will intersect the y-axis when x coordinate is 0.

$$\Rightarrow \quad 0(x) + ay = 2ab$$
$$\Rightarrow \quad y = \frac{2ab}{a} = 2b$$

Hence, it will intersect y-axis at $(0, 2b)$.

$\therefore$ The point of intersection of the graph line of the given equation with the x-axis and y-axis are $(2a, 0)$ and $(0, 2b)$.

7. (*d*) $\boldsymbol{x = 5, y = 3}$

$x = 5$ and $y = 3$ do not satisfy any of the given equations, while all the other values given in the remaining options satisfy both the equations.

8. (*c*) **3 and 1**

If $x = a$ and $y = b$ is the solution of the given equations,

$\therefore$ these values of x and y should satisfy both the equations.

$$\Rightarrow \quad a - b = 2 \quad \ldots (1)$$
$$\text{and} \quad a + b = 4 \quad \ldots (2)$$

Solving equations (1) and (2), we get

$$a = 3, b = 1$$

Hence, 3 and 1 are the values of x and y respectively.

9. (*a*) **1**

$$4x - 5 = y$$
$$\Rightarrow \quad 4x - 5 = -1$$
$$\Rightarrow \quad 4x = 4$$
$$\Rightarrow \quad x = 1 \quad [\text{Putting } y = -1]$$
$$2x - y = 3$$
$$\Rightarrow \quad 2x - (-1) = 3$$
$$\Rightarrow \quad 2x + 1 = 3$$
$$\Rightarrow \quad 2x = 2$$
$$\Rightarrow \quad x = 1 \quad [\text{Putting } y = -1]$$

Hence, $x = 1$ satisfies both the equations when $y = -1$.

10. (*a*) **4**

$5x - y - 7 = 0$, y coordinate is 13.

$\therefore$ Putting $y = 13$ in the given equation, we get

$$5x - 13 - 7 = 0$$
$$\Rightarrow \quad 5x = 20$$
$$\Rightarrow \quad x = 4$$

Hence, 4 is the x coordinate of the point which lies on the graph of the given equation and whose y coordinate is 13.

11. (*a*) **unique solution**

$$x + y - 40 = 0$$
$$x - 2y + 14 = 0$$

Here, $a_1 = 1$, $b_1 = 1$ and $c_1 = -40$

$a_2 = 1$, $b_2 = -2$ and $c_2 = 14$

$$\therefore \quad \frac{a_1}{a_2} = \frac{1}{1},$$
$$\frac{b_1}{b_2} = \frac{1}{-2}$$
$$\text{and} \quad \frac{c_1}{c_2} = \frac{-40}{14} = \frac{-20}{7}$$
$$\text{Since} \quad \frac{a_1}{a_2} \neq \frac{b_1}{b_2},$$

$\therefore$ the given pair of linear equations has a unique solution.

12. (*b*) **4**

Since $(6, k)$ is a solution of $3x + y - 22 = 0$,

$$\therefore \quad 3(6) + k - 22 = 0$$
$$\Rightarrow \quad 18 + k - 22 = 0$$
$$\Rightarrow \quad k = 4$$

Hence, 4 gives the value of k.

13. (*c*) $\boldsymbol{k \neq \sqrt{3}}$

$$3kx + 6y - \sqrt{50} = 0$$
$$\text{and} \quad \sqrt{18}\,x + \sqrt{24}\,y - \sqrt{75} = 0$$

Here, $a_1 = 3k$, $b_1 = 6$, $c_1 = -\sqrt{50}$

$a_2 = \sqrt{18}$, $b_2 = \sqrt{24}$, $c_2 = -\sqrt{75}$

$$\therefore \quad \frac{a_1}{a_2} = \frac{3k}{\sqrt{18}},$$
$$\frac{b_1}{b_2} = \frac{6}{\sqrt{24}}$$
$$\text{and} \quad \frac{c_1}{c_2} = \frac{-\sqrt{50}}{-\sqrt{75}}$$

For unique solution,

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$
$$\Rightarrow \quad \frac{3k}{\sqrt{18}} \neq \frac{6}{\sqrt{24}}$$
$$\Rightarrow \quad k \neq \frac{6}{\sqrt{24}} \times \frac{\sqrt{18}}{3}$$
$$\Rightarrow \quad k \neq \frac{6}{2\sqrt{6}} \times \frac{3\sqrt{2}}{3} = \sqrt{3}$$

For unique solution, $k \neq \sqrt{3}$.

14. (*c*) **3**

$2x + 3y - 7 = 0$ and $k\,x + \frac{9}{2}y - 12 = 0$

Here, $a_1 = 2$, $b_1 = 3$, $c_1 = -7$

$a_2 = k$, $b_2 = \frac{9}{2}$, $c_2 = -12$

$$\therefore \quad \frac{a_1}{a_2} = \frac{2}{k},$$
$$\frac{b_1}{b_2} = \frac{3}{\frac{9}{2}} = \frac{2}{3}$$
$$\text{and} \quad \frac{c_1}{c_2} = \frac{-7}{-12} = \frac{7}{12}$$

For no solution 3 is the value of k.

15. (*b*) $\boldsymbol{k = \frac{6}{5}}$

$8x + 2y - 5k = 0$ and $4x + y - 3 = 0$

Here, $a_1 = 8$, $b_1 = 2$, $c_1 = -5k$

$a_2 = 4$, $b_2 = 1$, $c_2 = -3$

The graphs of given pair of linear equations are coincident lines, so these equations have infinite number of solutions.

For infinite number of solutions, we have

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\Rightarrow \quad \frac{8}{4} = \frac{2}{1} = \frac{-5k}{-3}$$

$$\Rightarrow \quad \frac{5}{3}k = 2$$

$$\Rightarrow \quad k = \frac{2 \times 3}{5} = \frac{6}{5}$$

For coincident lines, $k = \frac{6}{5}$.

For Standard Level

16. (*c*) $\mathbf{\frac{5}{6}}$

$$\frac{2}{x} + \frac{3}{y} = 13 \quad \ldots (1)$$

$$\frac{5}{x} - \frac{4}{y} = -2 \quad \ldots (2)$$

Multiplying equation (1) by 4 and equation (2) by 3, we get

$$\frac{8}{x} + \frac{12}{y} = 52 \quad \ldots (3)$$

$$\frac{15}{x} - \frac{12}{y} = -6 \quad \ldots (4)$$

Adding equations (3) and (4), we get

$$\frac{23}{x} = 46$$

$$\Rightarrow \quad x = \frac{1}{2}$$

Substituting $x = \frac{1}{2}$ in equation (1), we get

$$\frac{8}{\frac{1}{2}} + \frac{12}{y} = 52$$

$$\Rightarrow \quad \frac{12}{y} = 52 - 16 = 36$$

$$\Rightarrow \quad y = \frac{12}{36} = \frac{1}{3}$$

$$\therefore \quad x + y = \frac{1}{2} + \frac{1}{3} = \frac{5}{6}$$

Hence, $\frac{5}{6}$ equals $x + y$.

17. (*b*) $\boldsymbol{b - a}$

$$bx + ay = a^2 + b^2 \quad \ldots (1)$$

$$ax^2 - by = 0$$

$$\Rightarrow \quad ax = by$$

$$\Rightarrow \quad x = \frac{by}{a} \quad \ldots (2)$$

From equations (1) and (2), we get

$$b\left(\frac{by}{a}\right) + ay = a^2 + b^2$$

$$\Rightarrow \quad \frac{b^2}{a} + ay = a^2 + b^2$$

$$\Rightarrow \quad \left(\frac{b^2 + a^2}{a}\right)y = (a^2 + b^2)$$

$$\Rightarrow \quad y = a$$

Substituting $y = a$ in equation (2), we get

$$x = \frac{b}{a} \times a = b$$

$$\therefore \quad x - y = b - a$$

Hence, $b - a$ equals $x - y$.

18. (*c*) $\sqrt{ab}$

$$\sqrt{a}\,x - \sqrt{b}\,y = b - a \quad \ldots (1)$$

and $\quad \sqrt{b}\,x - \sqrt{a}\,y = 0$

$$\Rightarrow \quad \sqrt{b}\,x = \sqrt{a}\,y$$

$$\Rightarrow \quad x = \frac{\sqrt{a}}{\sqrt{b}}y \quad \ldots (2)$$

From equations (1) and (2), we get

$$\sqrt{a}\left(\frac{\sqrt{a}}{\sqrt{b}}y\right) - \sqrt{b}\,(y) = b - a$$

$$\Rightarrow \quad \left(\frac{a}{\sqrt{b}} - \sqrt{b}\right)y = b - a$$

$$\Rightarrow \quad \left(\frac{a - b}{\sqrt{b}}\right)y = b - a$$

$$\Rightarrow \quad y = \sqrt{b}$$

Substituting $y = \sqrt{b}$ in equation (2), we get

$$x = \frac{\sqrt{a}}{\sqrt{b}} \times \sqrt{b} = \sqrt{a}$$

$$\therefore \quad xy = \sqrt{a}\,\sqrt{b} = \sqrt{ab}$$

Hence, $\sqrt{ab}$ equals xy.

19. (*d*) **–1**

$$(3k + 1)x + 3y - 5 = 0$$

Here, $\quad a_1 = (3k + 1), b_1 = 3, c_1 = -5$

$\quad a_2 = 2, b_2 = -3, c_2 = 5$

For infinite number solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\therefore \quad \frac{3k+1}{2} = \frac{3}{-3} = \frac{-5}{5}$$

$$\Rightarrow \quad \frac{3k+1}{2} = -1$$

$$\Rightarrow \quad 3k + 1 = -2$$

$$\Rightarrow \quad 3k = -3$$

$$\Rightarrow \quad k = -1$$

For infinite solution, $k = -1$.

20. (*c*) $\boldsymbol{m = 5, n = 1}$

$$3x + 4y - 12 = 0$$

and $\quad (m + n)x + 2(m - n)y - (5m - 1) = 0$

Here, $\quad a_1 = 3, b_1 = 4, c_1 = -12$

$\quad a_2 = m + n, b_2 = 2(m - n), c_2 = -(5m - 1)$

The graphs of the given pair of linear equations are coincident lines, so these equations have infinite number of solutions.

For infinite number of solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\Rightarrow \quad \frac{3}{m+n} = \frac{4}{2(m-n)} = \frac{-12}{-(5m-1)}$$

$$\Rightarrow \quad \frac{3}{m+n} = \frac{2}{m-n} = \frac{12}{5m-1}$$

$$\Rightarrow \quad \frac{3}{m+n} = \frac{2}{m-n}$$

$$\Rightarrow \quad 3m - 3n = 2m + 2n$$

$$\Rightarrow \quad m = 5n \quad \ldots (1)$$

$$\frac{2}{m-n} = \frac{12}{5m-1}$$

$$\Rightarrow \quad \frac{1}{m-n} = \frac{6}{5m-1}$$

$$\Rightarrow \quad 5m - 1 = 6m - 6n$$

$$\Rightarrow \quad 6n - m = 1 \quad \ldots (2)$$

$$\frac{3}{m+n} = \frac{12}{5m-1}$$

$$\Rightarrow \quad \frac{1}{m+n} = \frac{4}{5m-1}$$

$$\Rightarrow \quad 5m - 1 = 4m + 4n$$

$$\Rightarrow \quad m - 4n = 1 \quad \ldots (3)$$

Solving equation (1) and equation (2), we get

$$6n - 5n = 1$$

$$\Rightarrow \quad n = 1$$

Substituting $n = 1$ in equation (1),

Solving equation (2) and equation (3), we get

$$n = 1 \text{ and } m = 5$$

For coincident lines, $m = 5$ and $n = 1$.

MATCH THE FOLLOWING

For Basic and Standard Levels

1. Parallel lines (*iii*) $2x - 3y = 4$, $4x - 6y = 7$ — (*b*) No solution

$$\because \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \quad \left[\frac{2}{4} = \frac{-3}{6} \neq \frac{4}{7}\right]$$

2. Intersecting lines (*i*) $ax + by = a + b$, $bx - ay = a - b$ — (*c*) Unique solution

$$\because \frac{a_1}{a_2} \neq \frac{b_1}{b_2} \quad \left[\frac{a}{b} \neq \frac{b}{-a}\right]$$

3. Coincident lines (*ii*) $4x - 5y = 3$, $8x - 10y = 6$ — (*a*) Infinitely many solutions

$$\because \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \quad \left[\frac{4}{8} = \frac{-5}{-10} = \frac{-3}{-6} = \left(\frac{1}{2}\right)\right]$$

SHORT ANSWER QUESTIONS

For Basic and Standard Levels

1. (*i*) $2x - 2y - 2 = 0$

and $4x - 4y - 5 = 0$

These equations are of the form $a_1x + b_1y + c_1 = 0$

and $a_2x + b_2y + c_1 = 0$,

where $a_1 = 2, b_1 = -2, c_1 = -2$

$a_2 = 4, b_2 = -4, c_2 = -5$

$$\therefore \quad \frac{a_1}{a_2} = \frac{2}{4} = \frac{1}{2},$$

$$\frac{b_1}{b_2} = \frac{-2}{-4} = \frac{1}{2}$$

$$\text{and} \quad \frac{c_1}{c_2} = \frac{-2}{-5} = \frac{2}{5}$$

$$\text{Clearly,} \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Thus, the given pair of linear equations have no solution.

Hence, their graphs will be **parallel lines**.

(*ii*) $x - y + 1 = 0$

and $3x + 2y - 12 = 0$

These equations are of the form $a_1 x + b_1y + c_1 = 0$

and $a_2 x + b_2y + c_2 = 0$,

where $a_1 = 1$, $b_1 = -1$ and $c_1 = 1$;

$a_2 = 3$, $b_2 = 2$ and $c_2 = -12$

$$\therefore \quad \frac{a_1}{a_2} = \frac{1}{3},$$

$$\frac{b_1}{b_2} = -\frac{1}{2}$$

$$\text{and} \quad \frac{c_1}{c_2} = \frac{1}{-12}$$

$$\text{Clearly,} \quad \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

So, the given pair of linear equations have a **unique solution**.

$\therefore$ Their graphs will intersect at a point.

2. $x + 2y - 8 = 0$

and $2x + 4y - 16 = 0$

These equations are of the form $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$,

where $a_1 = 1$, $b_1 = 2$, $c_1 = -8$

$a_2 = 2$, $b_2 = 4$, $c_2 = -16$

$$\therefore \quad \frac{a_1}{a_2} = \frac{1}{2},$$

$$\frac{b_1}{b_2} = \frac{2}{4} = \frac{1}{2}$$

$$\text{and} \quad \frac{c_1}{c_2} = \frac{-8}{-16} = \frac{1}{2}$$

$$\text{Clearly,} \quad \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Hence, the given pair of linear equations will have **infinite** number of solutions.

3. $$\alpha x + y - \alpha^2 = 0$$

and $$x + \alpha y - 1 = 0$$

These equations are of the form $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$,

where, $a_1 = \alpha$, $b_1 = 1$, $c_1 = -\alpha^2$;

$a_2 = 1$, $b_2 = \alpha$, $c_2 = -1$

$$\therefore \quad \frac{a_1}{a_2} = \frac{\alpha}{1},$$

$$\frac{b_1}{b_2} = \frac{1}{\alpha}$$

and $$\frac{c_1}{c_2} = \frac{-\alpha^2}{-1} = \frac{\alpha^2}{1}$$

(*i*) For unique solution,

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

$$\Rightarrow \quad \frac{\alpha}{1} \neq \frac{1}{\alpha}$$

$$\Rightarrow \quad \alpha^2 = 1$$

$$\Rightarrow \quad \alpha = \pm 1$$

Thus, the given pair of linear equations will have a unique solution for all real values of α except ± 1.

(*ii*) For no solution,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\therefore \quad \frac{\alpha}{1} = \frac{1}{\alpha} \neq \frac{\alpha^2}{1}$$

$$\Rightarrow \quad \frac{\alpha}{1} = \frac{1}{\alpha}$$

and $$\frac{1}{\alpha} \neq \frac{\alpha^2}{1}$$

$$\Rightarrow \quad \alpha^2 = 1$$

and $$\alpha^3 \neq 1$$

$$\Rightarrow \quad \alpha = \pm 1 \quad \ldots (1)$$

and $$\alpha^3 \neq 1 \quad \ldots (2)$$

$\alpha = -1$ is the common solution of equations (1) and (2).

Hence, α **= –1**.

(*iii*) For infinitely many solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\Rightarrow \quad \frac{\alpha}{1} = \frac{1}{\alpha} = \frac{\alpha^2}{1}$$

$$\Rightarrow \quad \frac{\alpha}{1} = \frac{1}{\alpha} \quad \ldots (1)$$

$$\Rightarrow \quad \alpha^2 = 1$$

$$\Rightarrow \quad \alpha = \pm 1$$

and $$\frac{1}{\alpha} = \frac{\alpha^2}{1} \quad \ldots (2)$$

$$\Rightarrow \quad \alpha^3 = 1$$

$$\Rightarrow \quad \alpha = 1$$

and $$\frac{\alpha}{1} = \frac{\alpha^2}{1} \quad \ldots (3)$$

$$\Rightarrow \quad \alpha^2 - \alpha = 0$$

$$\Rightarrow \quad \alpha(\alpha - 1) = 0$$

$$\Rightarrow \quad \alpha = 0, \alpha = 1$$

$\alpha = 1$ is common solution of equations (1), (2) and (3).

Hence, α **= 1**.

4. $2x + 3y - 7 = 0$

and $(p + q)\,x + (2p - q)y - 3(p + q + 1) = 0$

These equations are of the form $a_1x + b_1y + c_1 = 0$

and $a_2x + b_2y + c_2 = 0$,

where, $a_1 = 2$, $b_1 = 3$, $c_1 = -7$

$a_2 = (p + q)$, $b_2 = (2p - q)$, $c_2 = -3(p + q + 1)$

For infinite number of solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\Rightarrow \quad \frac{2}{p+q} = \frac{3}{(2p-q)}$$

$$= \frac{-7}{-3(p+q+1)}$$

$$= \frac{7}{3(p+q+1)}$$

$$\frac{2}{p+q} = \frac{3}{(2p-q)}$$

$$\Rightarrow \quad 4p - 2q = 3p + 3q$$

$$\Rightarrow \quad 4p - 3p = 2q + 3q$$

$$\Rightarrow \quad p = 5q \quad \ldots (1)$$

and $$\frac{3}{(2p-q)} = \frac{7}{3(p+q+1)}$$

$$\Rightarrow \quad 9(p + q + 1) = 14p - 7q$$

$$\Rightarrow \quad 9p + 9q + 9 = 14p - 7q$$

$$\Rightarrow \quad 5p - 16q = 9 \quad \ldots (2)$$

and $$\frac{2}{(p+q)} = \frac{7}{3(p+q+1)}$$

$$\Rightarrow \quad 6q + 6q + 6 = 7p + 7q$$

$$\Rightarrow \quad p + q = 6 \quad \ldots (3)$$

Solving equations (1), (2) and (3), we get

$$p = 5$$

and $$q = 1$$

Hence, $\boldsymbol{p = 5}$ and $\boldsymbol{q = 1}$.

5. $$2x - y = 2$$

$$\Rightarrow \quad 2x - 2 = y \quad \ldots (1)$$

and $$x + 3y = 15 \quad \ldots (2)$$

Substituting $y = 2x - 2$ in equation (2), we get

$$x + 3(2x - 2) = 15$$

$$\Rightarrow \quad x + 6x - 6 = 15$$

$$\Rightarrow \quad 7x = 21$$

$$\Rightarrow \quad x = 3$$

Substituting $x = 3$ in equation (1), we get

$$2 \times 3 - 2 = y$$

$$\Rightarrow \quad y = 4$$

Hence, $\boldsymbol{x = 3}$ and $\boldsymbol{y = 4}$.

6. $$x + y = a + b \quad \ldots (1)$$

and $$ax - by = a^2 - b^2 \quad \ldots (2)$$

Multiplying equation (1) by b, we get

$$bx + by = ab + b^2 \quad \ldots (3)$$

Adding equation (2) and equation (3), we get

$$ax + bx = a^2 - b^2 + ab + b^2$$

$$\Rightarrow \quad (a + b)x = a(a + b)$$

$$\Rightarrow \quad x = a$$

Substituting $x = a$ in equation (1), we get

$$a + y = a + b$$

$$\Rightarrow \quad y = b$$

Hence, $\boldsymbol{x = a}$ and $\boldsymbol{y = b}$.

7. $$\frac{3}{x} + \frac{8}{y} = -1 \quad \dots (1)$$

and $$\frac{1}{x} - \frac{2}{y} = 2 \quad \dots (2)$$

Multiplying equation (2) by 4, we get

$$\frac{4}{x} - \frac{8}{y} = 8 \quad \dots (3)$$

Adding equations (1) and (3), we get

$$\frac{3}{x} + \frac{4}{x} = 7$$

$$\Rightarrow \quad \frac{7}{x} = 7$$

$$\Rightarrow \quad x = 1$$

Substituting $x = 1$ in equation (1), we get

$$\frac{3}{1} + \frac{8}{y} = -1$$

$$\Rightarrow \quad \frac{8}{y} = -4$$

$$\Rightarrow \quad y = \frac{8}{-4} = -2$$

Hence, $\boldsymbol{x = 1}$ and $\boldsymbol{y = -2}$.

8. $$x + y = 4.4$$

$$\Rightarrow \quad x + y = \frac{44}{10} = \frac{22}{5}$$

$$\Rightarrow \quad 5x + 5y = 22 \quad \dots (1)$$

and $$\frac{6.7}{3x - 2y} = 1$$

$$\Rightarrow \quad 6.7 = 3x - 2y$$

$$\Rightarrow \quad \frac{67}{10} = 3x - 2y$$

$$\Rightarrow \quad 67 = 30x - 20y \quad \dots (2)$$

Multiplying equation (1) by 4, we get

$$20x + 20y = 88 \quad \dots (3)$$

Adding equation (2) and equation (3),

$$50x = 155$$

$$\Rightarrow \quad x = \frac{155}{50} = 3.1$$

Substituting $x = 3.1$ in $x + y = 4.4$, we get

$$3.1 + y = 4.4$$

$$\Rightarrow \quad y = 4.4 - 3.1 = 1.3$$

Hence, $\boldsymbol{x = 3.1}$ and $\boldsymbol{y = 1.3}$.

9. $$\frac{xy}{x + y} = \frac{6}{5}$$

and $$\frac{xy}{y - x} = 6$$

$$\Rightarrow \quad 5xy = 6x + 6y \quad \dots (1)$$

and $$xy = 6y - 6x \quad \dots (2)$$

Dividing equation (1) and equation (2) by xy, we get

$$5 = \frac{6}{y} + \frac{6}{x} \quad \dots (3)$$

and $$1 = \frac{6}{x} - \frac{6}{y} \quad \dots (4)$$

Adding equation (3) and equation (4), we get

$$\frac{6}{x} + \frac{6}{x} = \frac{12}{x} = 6$$

$$\Rightarrow \quad x = \frac{12}{6} = 2$$

Substituting $x = 2$ in equation (3), we get

$$5 = \frac{6}{y} + \frac{6}{2}$$

$$\Rightarrow \quad 5 = \frac{6}{y} + 3$$

$$\Rightarrow \quad \frac{6}{y} = 2$$

$$\Rightarrow \quad y = \frac{6}{2} = 3$$

Hence, $\boldsymbol{x = 2}$ and $\boldsymbol{y = 3}$.

10. Since the opposite sides of a rectangle are equal,

$\therefore$ AD = BC

and AB = DC

$$x - y = 8 \quad \dots (1)$$

and $$12 = x + y \quad \dots (2)$$

Solving equation (1) and equation (2), we get

$$x = 10$$

Substituting $x = 10$ in equation (1), we get

$$10 - y = 8$$

$$\Rightarrow \quad y = 2$$

Hence, $\boldsymbol{x = 10}$ and $\boldsymbol{y = 2}$.

11. Since the opposite sides of a parallelogram are equal,

AD = BC

and AB = DC

$$5 = x - y \quad \dots (1)$$

and $$x + y = 10 \quad \dots (2)$$

Solving equation (1) and equation (2), we get

$$2x = 15$$

$$\Rightarrow \quad x = 7.5$$

Substituting $x = 7.5$ in equation (1), we get

$$5 = 7.5 - y$$

$$\Rightarrow \quad y = 7.5 - 2.5$$

Hence, $\boldsymbol{x = 7.5}$ and $\boldsymbol{y = 2.5}$.

12. Let the speed of the rickshaw be x km/h and the speed of the bus be y km/h.

Case 1.

Measures	*Rickshaw*	*Bus*
Distance (in km)	2	12
Speed (in km/h)	x	y
Time (in hour)	$\frac{2}{x}$	$\frac{12}{y}$

$$\frac{2}{x} + \frac{12}{y} = \frac{1}{2}$$

Case 2.

Measures	*Rickshaw*	*Bus*
Distance (in km)	4	10
Speed (in km/h)	x	y
Time (in hour)	$\frac{4}{x}$	$\frac{10}{y}$

$$\frac{4}{x} + \frac{10}{y} = \frac{39}{60}$$

$$\Rightarrow \quad \frac{4}{x} + \frac{10}{y} = \frac{13}{20}$$

Hence, $\mathbf{\frac{2}{x} + \frac{12}{y} = \frac{1}{2}, \frac{4}{x} + \frac{10}{y} = \frac{13}{20}}$, where x is the speed of the rickshaw in km/h and y is the speed of the bus in km/h.

For Standard Level

13. Suppose Satwinder invests ₹ x at 9% interest per annum and invests ₹ 4 at 8% interest per annum interest from scheme A + interest from scheme B = interest earned interest from A + interest from B = interest earned

$$\mathbf{\frac{9}{100}x + \frac{8}{100}y = 1880}$$

and $$\mathbf{\frac{8}{100}x + \frac{9}{100}y = 1860}$$

where ₹ x is the amount invested in scheme A and ₹ y is the amount invested in scheme B.

14. Mr. Sachdeva's weight = x kg
Mrs. Sachdeva's weight = y kg
Then, $x - 5 = y$
and $y - 4 = \frac{7}{8}x$

$\Rightarrow \quad 8y - 32 = 7x$
$\Rightarrow \quad 8y - 7x = 32$

Hence, $\mathbf{x - 5 = y}$ and $\mathbf{8y - 7x = 32}$.

VALUE-BASED QUESTIONS

For Basic and Standard Levels

1. (*i*) Let the cost price of the raincoat be ₹ x and the cost price of rain boots be ₹ y.

Then, $x + \frac{10}{100}x + y + \frac{25}{100}y = 1535$

$\Rightarrow \quad \frac{11x}{10} + \frac{5}{4}y = 1535$

$\Rightarrow \quad 22x + 25y = 30700 \quad \ldots (1)$

Also, $x + \frac{25}{100}x + y + \frac{10}{100}y = 1520$

$\Rightarrow \quad \frac{5}{4}x + \frac{11}{10}y = 1520$

$\Rightarrow \quad 25x + 22y = 30400 \quad \ldots (2)$

Adding equation (1) and equation (2), we get

$47x + 47y = 61100$

$\Rightarrow \quad x + y = 1300 \quad \ldots (3)$

Subtracting equation (2) from equation (1), we get

$-3x + 3y = 300$

$\Rightarrow \quad -x + y = 100 \quad \ldots (4)$

Adding equation (3) and equation (4), we get

$\Rightarrow \quad 2y = 1400$

$\Rightarrow \quad y = 700$

Substituting $y = 700$ in equation (4), we get

$-x + 700 = 100$

$\Rightarrow \quad x = 600$

Hence, the cost price of the raincoat is ₹ **600** and the cost price of the rain boots is ₹ **700**.

(*ii*) **Empathy** ($\because$ He showed concern for others by offering a better deal to them).

2. Let the original number of boys and girls in the school x and y respectively.

Then, $x + y = 2800 \quad \ldots (1)$

$$\left(x + \frac{5}{100}x\right) + \left(y + \frac{10}{100}y\right) = 3000$$

$\Rightarrow \quad \frac{21}{20}x + \frac{11}{10}y = 3000$

$\Rightarrow \quad 21x + 22y = 60000 \quad \ldots (2)$

Multiplying equation (1) by 22, we get

$22x + 22y = 61600 \quad \ldots (3)$

Subtracting equation (2) from equation (3), we get

$x = 1600$

Substituting $x = 1600$ in equation (1), we get

$y = 1200$

(*i*) Hence, the original number of **boys = 1600** and the original number of **girls = 1200**.

(*ii*) **Empathy** was exhibited by the school authorities as they had the ability to understand the feelings and the sentiments of the girl students who were discouraged to join the school, as their parents could not afford the school fee.

The authorities also exhibited the value of **decision-making** by reducing the fee of girl students by 50%.

3. (*i*) Let the price of each prize given for honesty be ₹ x and the price of each prize given for empathy be ₹ y.

Then, for school A

$5x + 3y = 3700 \quad \ldots (1)$

For school B

$3x + 5y = 3500 \quad \ldots (2)$

Adding equations (1) and (2), we get

$8x + 8y = 7200$

$\Rightarrow \quad x + y = 900 \quad \ldots (3)$

Subtracting equation (2) from equation (1), we get

$2x - 2y = 200$

$\Rightarrow \quad x - y = 100 \quad \ldots (4)$

Solving equation (3) and equation (4), we get

$x = 500$ and $y = 400$

Hence, the price of each prize of honesty = ₹ **500** and the price of each prize of honesty = ₹ **400**.

(*ii*) Punctuality, regularity, awareness, interpersonal relationship, effective communication, critical thinking and creative thinking.

UNIT TEST 1

For Basic Level

1. (*c*) **parallel**

$4x + 6y - 9 = 0,\ 2x + 3y + 11 = 0$

$$\frac{a_1}{a_2} = \frac{4}{2} = \frac{2}{1},\ \frac{b_1}{b_2} = \frac{6}{3} = \frac{2}{1},\ \frac{c_1}{c_2} = \frac{-9}{11}$$

Clearly, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

graphically, the given pair of linear equations represent parallel lines.

2. (*a*) **α = 4**

Graphs of $2x + \alpha y - 10 = 0$ and $3x + 6y - 12 = 0$ are parallel lines.

∴ The given pair of linear equations have no solution.

For no solution,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\Rightarrow \quad \frac{2}{3} = \frac{\alpha}{6} \neq \frac{-10}{12}$$

$$\Rightarrow \quad \alpha = 4 \text{ and } \alpha \neq -5$$

Since $\alpha = 4$ satisfies both the equations and the in equation.

$$\therefore \quad \alpha = 4$$

3. (*a*) **one solution**

$2x - 3y - 2 = 0,\ 3x - 2y - 4 = 0$

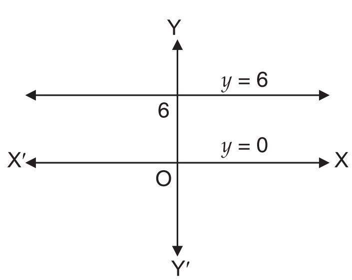
$$\frac{a_1}{a_2} = \frac{2}{3},\ \frac{b_1}{b_2} = \frac{-3}{-2} = \frac{3}{2},\ \frac{c_1}{c_2} = \frac{-2}{-4} = \frac{1}{2}$$

Since $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, the given pair of linear equations has one solution.

4. (*c*) $\boldsymbol{y = 6}$

∵ $y = 6$ represents a line on which y coordinate is 6, for all values of x coordinate.

∴ Its graph remain at a constant distance from the x axis as shown in the figure.



5. (*b*) $\boldsymbol{k \neq -4}$

$2x + 3y - 5 = 0$ and $kx - 6y - 8 = 0$

For unique solution,

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

$$\therefore \quad \frac{2}{k} \neq \frac{3}{-6}$$

$$\Rightarrow \quad k \neq \frac{2 \times (-6)}{3}$$

$$\Rightarrow \quad k \neq -4$$

∴ $k \neq -4$ gives the correct value of k.

6. Let the numerator and denominator of the required fraction be n and d respectively.

Sum of numerator and denominator = 12

$$\Rightarrow \quad \boldsymbol{n + d = 12}$$

When denominator is increased by 3, the fraction becomes $\frac{1}{2}$.

$$\Rightarrow \quad \frac{n}{d+3} = \frac{1}{2}$$

7. (*i*) $2x - y - 4 = 0$ and $\frac{1}{5}x + \frac{1}{5}y - 1 = 0$

TRUE

Justification: $\frac{a_1}{a_2} = \frac{2}{\frac{1}{5}} = 10,\ \frac{b_1}{b_2} = \frac{-1}{\frac{1}{5}} = -5$

Clearly, $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$.

The given pair of linear equations has a unique solution.

(*ii*) $3x + 4y - 6 = 0$ and $9x + 12y - 15 = 0$

FALSE

Justification: $\frac{a_1}{a_2} = \frac{3}{9} = \frac{1}{3},\ \frac{b_1}{b_2} = \frac{4}{12} = \frac{1}{3}$ and

$$\frac{c_1}{c_2} = \frac{-6}{-15} = \frac{2}{5}$$

Clearly, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

Hence, the given pair of linear equations have no solution.

So, their graphs will be parallel lines and not coincident lines.

(*iii*) $3x - 5y - 20 = 0$ and $6x - 10y + 40 = 0$

TRUE

Justification: $\frac{a_1}{a_2} = \frac{3}{6} = \frac{1}{2},\ \frac{b_1}{b_2} = \frac{-5}{-10} = \frac{1}{2}$ and

$$\frac{c_1}{c_2} = \frac{-20}{40} = -\frac{1}{2}$$

Clearly, $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

So, the given pair of linear equations have no solution and hence they are inconsistent.

(*iv*) $x + 3y - 2 = 0$ and $3x + 9y - 6 = 0$

TRUE

Justification: $\frac{a_1}{a_2} = \frac{1}{3},\ \frac{b_1}{b_2} = \frac{3}{9} = \frac{1}{3}$ and

$$\frac{c_1}{c_2} = \frac{-2}{-6} = \frac{1}{3}$$

Clearly, $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

So, the given pair of linear equations is consistent (*dependent*) with infinitely many solutions.

8. $$\frac{x}{3} + \frac{y}{4} = 6 \quad \ldots (1)$$

$$\frac{x}{6} + \frac{y}{2} = 6 \quad \ldots (2)$$

Multiplying equation (1) by 2, we get

$$\frac{2x}{3} + \frac{y}{2} = 12 \quad \ldots (3)$$

Subtracting equation (2) from equation (3), we get

$$\left(\frac{2}{3} - \frac{1}{6}\right) x = 6$$

$$\Rightarrow \left(\frac{4-1}{6}\right) x = 6$$

$$\Rightarrow \frac{3}{6} x = 6$$

$$\Rightarrow x = 12$$

Substituting $x = 12$ in equation (1), we get

$$\frac{12}{3} + \frac{y}{4} = 6$$

$$\Rightarrow 4 + \frac{y}{4} = 6$$

$$\Rightarrow \frac{y}{4} = 2$$

$$\Rightarrow y = 8$$

$$\therefore 3y - 2x = 3 \times 8 - 2 \times 12$$

$$= 24 - 24 = \mathbf{0}$$

and $$\frac{x}{y} + \frac{1}{2} = \frac{12}{8} + \frac{1}{2}$$

$$= \frac{3}{2} + \frac{1}{2}$$

$$= \frac{3+1}{2}$$

$$= \frac{4}{2} = \mathbf{2}$$

9. $$5x + \frac{4}{y} = 9 \quad \ldots (1)$$

$$7x - \frac{2}{y} = 5 \quad \ldots (2)$$

Multiplying equation (2) by 2, we get

$$14x - \frac{4}{y} = 10 \quad \ldots (3)$$

Adding equation (1) and equation (3), we get

$$19x = 19$$

$$\Rightarrow x = 1$$

Substituting $x = 1$ in equation (1), we get

$$5(1) + \frac{4}{y} = 9$$

$$\Rightarrow \frac{4}{y} = 4$$

$$\Rightarrow y = 1$$

$$y = \lambda x + 1 \quad \text{[Given]}$$

$$\Rightarrow 1 = \lambda(1) + 1$$

$$\Rightarrow \lambda = 0$$

Hence, $\boldsymbol{x = 1, y = 1}$ and $\boldsymbol{\lambda = 0}$.

10. Since the opposite sides of a rectangle are equal,

$$\therefore 31x + 29y = 33 \quad \ldots (1)$$

and $$29x + 31y = 27 \quad \ldots (2)$$

Adding equations (1) and (2), we get

$$60x + 60y = 60$$

$$\Rightarrow x + y = 1 \quad \ldots (3)$$

Subtracting equation (2) from equation (1), we get

$$2x - 2y = 6$$

$$\Rightarrow x - y = 3 \quad \ldots (4)$$

Solving equations (3) and (4), we get

$\boldsymbol{x = 2}$ and $\boldsymbol{y = -1}$

11. Let the two parts be x and y.

Then, $$x + y = 100 \quad \ldots (1)$$

and $$\frac{1}{x} + \frac{1}{y} = \frac{1}{24}$$

$$\Rightarrow \frac{y+x}{xy} = \frac{1}{24}$$

$$\Rightarrow \frac{100}{xy} = \frac{1}{24}$$

$$\Rightarrow xy = 2400$$

$$(x - y) = \sqrt{(x+y)^2 - 4xy}$$

$$= \sqrt{(100)^2 - 4 \times 2400}$$

$$= \sqrt{10000 - 9600}$$

$$= \sqrt{400}$$

$$= \pm 20$$

When $x + y = 100$ and $x - y = 20$, then $x = 60$ and $y = 40$.

When $x + y = 100$ and $x - y = -20$, then $x = 40$ and $y = 60$.

Hence, the required two parts are **60** and **40**.

12. Let the numerator and the denominator of the required fraction be n and d respectively, so that the fraction is $\frac{n}{d}$.

Then, $$\frac{n+2}{d+2} = \frac{1}{3}$$

$$\Rightarrow 3n + 6 = d + 2$$

$$\Rightarrow 3n - d = -4 \quad \ldots (1)$$

and $$\frac{n+3}{d+3} = \frac{2}{5}$$

$$\Rightarrow 5n + 15 = 2d + 6$$

$$\Rightarrow 5n - 2d = -9 \quad \ldots (2)$$

Multiplying equation (1) by 2, we get

$$6n - 2d + 8 = 0 \quad \ldots (3)$$

Subtracting equation (2) from equation (3), we get

$$n = 1$$

Substituting $n = 1$ in equation (1), we get

$$3 \times 1 - d = -4$$

$$\Rightarrow d = 7$$

Hence, the required fraction is $\boldsymbol{\frac{1}{7}}$.

UNIT TEST 2

For Standard Level

1. (*c*) **3 and –2**

$x + 2y + 1 = 0$ and $2x - 3y - 12 = 0$

$x = p$ and $y = q$ is a solution of the given equations.

$\therefore \quad p + 2q + 1 = 0 \quad \ldots (1)$

and $\quad 2p - 3q - 12 = 0 \quad \ldots (2)$

Solving equation (1) and equation (2), we get

$p = 3,$

$q = -2$

$\therefore$ The values of p and q are 3 and –2.

2. (*a*) $\boldsymbol{p = \frac{17}{4}, q = \frac{11}{5}}$

$(2p - 1)x + 3y - 5 = 0$ and $3x + (q - 1)y - 2 = 0$

for infinite number of solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\Rightarrow \quad \frac{2p-1}{3} = \frac{3}{q-1} = \frac{-5}{-2}$$

$$\Rightarrow \quad \frac{2p-1}{3} = \frac{5}{2}$$

$$\text{and} \quad \frac{3}{q-1} = \frac{5}{2}$$

$\Rightarrow \quad 4p - 2 = 15$

and $\quad 6 = 5q - 5$

$\Rightarrow \quad 4p = 17$

and $\quad 5q = 11$

$$\Rightarrow \quad p = \frac{17}{4}$$

$$\text{and} \quad q = \frac{11}{5}$$

$\therefore \quad p = \frac{17}{4}$ and $q = \frac{11}{5}$ are the correct values of p and q.

3. (*b*) **12, 6**

Let the numbers be x and y.

Then, $\quad x + y = 18 \quad \ldots (1)$

$$\text{and} \quad \frac{1}{x} + \frac{1}{y} = \frac{1}{4}$$

$$\Rightarrow \quad \frac{y+x}{xy} = \frac{1}{4}$$

$$\Rightarrow \quad \frac{18}{xy} = \frac{1}{4}$$

$$\Rightarrow \quad 72 = xy$$

$$(x - y) = \sqrt{(x+y)^2 - 4xy}$$

$$= \sqrt{(18)^2 - 4 \times 72}$$

$$= \sqrt{324 - 288}$$

$$= \sqrt{36}$$

$$= \pm 6$$

$\Rightarrow \quad x - y = +6 \quad \ldots (2)$

or $\quad x - y = -6 \quad \ldots (3)$

Solving equations (1) and (2), we get

$x = 12$ and $y = 6$

Solving equations (1) and (3), we get

$x = 6$ and $y = 12$

Hence, the two numbers are 12 and 6.

4. Let the ten's digit and unit's digit of the required two digit number be x and y such that $x > y$.

Then, the required number $= 10x + y$

Given, Number = Sum of digits multiplied by 7 + 3

$\Rightarrow \quad 10x + y = 7(x + y) + 3$

$\Rightarrow \quad 10x + y = 7x + 7y + 3$

$\Rightarrow \quad 10x + y - 7x - 7y = 3$

$\Rightarrow \quad 3x - 6y = 3$

$\Rightarrow \quad \boldsymbol{x - 2y = 1}$

Also, Number = Difference of digits multiplied by 20 – 7.

$\Rightarrow \quad 10x + y = 20(x - y) - 7$

$\Rightarrow \quad 10x + y = 20x - 20y - 7$

$\Rightarrow \quad 10x - 20x + y + 20y = -7$

$\Rightarrow \quad -10x + 21y = -7$

$\Rightarrow \quad \boldsymbol{10x - 21y = 7}$

5. Let x be the number of students in examination hall A and y be the number of students in examination hall B.

Case 1: When 10 students are sent from hall A to hall B, the number of students become equal in each hall.

$\therefore \quad x - 10 = y + 10$

$\Rightarrow \quad \boldsymbol{x - y = 20}$

Case 2: When 20 students are sent from hall B to hall A, the number of students in hall A become double the number of students left in hall B.

$\therefore \quad x + 20 = 2(y - 20)$

$\Rightarrow \quad x + 20 = 2y - 40$

$\Rightarrow \quad \boldsymbol{x - 2y = -60}$

6. Let the time taken by 1 man alone to finish the work be x days and the time taken by 1 boy alone to finish the work be y days.

Then, 1 man's 1 day's work $= \frac{1}{x}$

and $\quad$ 1 boy's 1 day's work $= \frac{1}{y}$

$\therefore \quad$ 8 men's 1 day's work $= \frac{8}{x}$

and $\quad$ 6 men's 1 day's work $= \frac{6}{x}$

and $\quad$ 12 boy's 1 day's work $= \frac{12}{y}$

and $\quad$ 8 boy's 1 day's work $= \frac{8}{y}$

8 men and 12 boys can finish the work in 10 days.

$\therefore$ 8 men's 1 day's work + 12 boy's 1 day's work $= \frac{1}{10}$

$$\Rightarrow \quad \boldsymbol{\frac{8}{x} + \frac{12}{y} = \frac{1}{10}}$$

Also, 6 men 8 boys can finish the work in 14 days.

$\therefore$ 6 men's 1 day's work + 8 boy's 1 day's work $= \frac{1}{14}$

$$\Rightarrow \quad \frac{6}{x} + \frac{8}{y} = \frac{1}{14}$$

7. The two given equations represent two lines representing two paths. Now, we see that $\frac{2}{2} \neq \frac{1}{-1}$

$\therefore$ Two lines will intersect each other.

Now, we have the two lines as

$$2x + y = 6 \quad \ldots (1)$$

and $$2x - y = -2 \quad \ldots (2)$$

Subtracting (2) from (1), we get

$$2y = 8$$

$$\Rightarrow \quad y = 4$$

$\therefore$ From (1), $$x = \frac{6-4}{2} = +1$$

$\therefore$ Yes, the two paths cross each other at a point represented by **(1, 4)**.

8. $$149x - 330y = -511 \quad \ldots (1)$$

$$-330x + 149y = -32 \quad \ldots (2)$$

Adding equations (1) and (2), we get

$$-181x - 181y = -543$$

$$\Rightarrow \quad x + y = 3 \quad \ldots (3)$$

Subtracting equation (1) from equation (2), we get

$$-479x + 479y = 479$$

$$\Rightarrow \quad -x + y = 1 \quad \ldots (4)$$

Solving equations (3) and (4), we get

$$\boldsymbol{x = 1} \text{ and } \boldsymbol{y = 2}$$

9. Let the parking charge for 1 car be ₹ x and the parking charge for 1 scooter be ₹ y.

Then, $$20x + 15y = 475 \quad \ldots (1)$$

and $$65x + 12y = 1360 \quad \ldots (2)$$

Multiplying equation (1) by 4 and equation (2) by 5, we get

$$80x + 60y = 1900 \quad \ldots (3)$$

and $$325x + 60y = 6800 \quad \ldots (4)$$

Subtracting equation (3) from equation (4), we get

$$245x = 4900$$

$$\Rightarrow \quad x = 20$$

Substituting $x = 20$ in equation (1), we get

$$20(20) + 15y = 475$$

$$\Rightarrow \quad 400 + 15y = 475$$

$$\Rightarrow \quad 15y = 75$$

$$\Rightarrow \quad y = 5$$

Parking charge for 5 cars and 7 scooters

$$= 5x + 7y$$

$$= ₹\ (5 \times 20 + 7 \times 5)$$

$$= ₹\ \mathbf{135}$$

10. Present Ages

Let A's present age = x years

Let B's present age = y years

Five years ago

A's age = $(x - 5)$ years

B's age = $(y - 5)$ years

Ten years later

A's age = $(x + 10)$ years

B's age = $(y + 10)$ years

Then, $$(x - 5) = 3(y - 5)$$

$$\Rightarrow \quad x - 5 = 3y - 15$$

$$\Rightarrow \quad x - 3y = -10 \quad \ldots (1)$$

and $$x + 10 = 2(y + 10)$$

$$\Rightarrow \quad x + 10 = 2y + 20$$

$$\Rightarrow \quad x - 2y = 10 \quad \ldots (2)$$

Subtracting equation (1) from equation (2), we get

$$y = 20$$

Substituting $y = 20$ in equation (1), we get

$$x - 3 \times 20 = -10$$

$$\Rightarrow \quad x = 60 - 10 = 50$$

Hence, A's present age is **50 years** and B's present age is **20 years**.

11. Let the ten's digit and one's digit of the required two digit number be x and y respectively

Then, required number = $10x + y$

$$\frac{10x + y}{x + y} = 6$$

$$\Rightarrow \quad 10x + y = 6x + 6y$$

$$\Rightarrow \quad 4x = 5y \quad \ldots (1)$$

and $$10x + y - 9 = 10y + x$$

$$\Rightarrow \quad 10x - x + y - 10y = 9$$

$$\Rightarrow \quad 9x - 9y = 9$$

$$\Rightarrow \quad x - y = 1 \quad \ldots (2)$$

Solving equations (1) and (2), we get

$$x = 5 \text{ and } y = 4$$

Hence, the required number = $10 \times 5 + 4 =$ **54**.