CHAPTER 2

Polynomials

EXERCISE 2A

For Basic and Standard Levels

1. (*i*) The graph of $y = p(x)$ cuts the x-axis at one point only.

$\therefore$ $p(x)$ has only **1** zero.

(*ii*) The graph of $y = p(x)$ cuts the x-axis at three points.

$\therefore$ $p(x)$ has **3** zeroes.

(*iii*) The graph of $y = p(x)$ meets/cuts the x-axis at two points.

$\therefore$ $p(x)$ has **2** zeroes.

2. (*i*) Since the graph of $y = p(x)$ is neither a straight line nor a parabola,

$\therefore$ $p(x)$ is **neither linear nor quadratic** polynomial.

Also, the graph $y = p(x)$ cuts the x-axis at one point.

$\therefore$ $p(x)$ has **1** zero.

(*ii*) The graph of $y = p(x)$ is a straight line.

$\therefore$ $p(x)$ is a **linear** polynomial.

Also, the graph of $y = p(x)$ cuts the x-axis at one point.

$\therefore$ $p(x)$ has only **1** zero.

(*iii*) The graph of $y = p(x)$ is a parabola.

$\therefore$ $p(x)$ is a **quadratic** polynomial.

Also, the graph of $y = p(x)$ does not touch the x-axis.

$\therefore$ $p(x)$ has **no** zeroes.

(*iv*) The graph of $y = p(x)$ is a parabola.

$\therefore$ $p(x)$ is a **quadratic** polynomial.

Also, the graph of $y = p(x)$ cuts the x-axis at two points.

$\therefore$ $p(x)$ has **2** zeroes.

(*v*) The graph of $y = p(x)$ is neither a straight line nor a parabola.

$\therefore$ $p(x)$ is **neither linear nor quadratic** polynomial.

Also, the graph of $y = p(x)$ cuts the x-axis at three points.

$\therefore$ $p(x)$ has **3** zeroes.

(*vi*) The graph of $y = p(x)$ is a parabola.

$\therefore$ $p(x)$ is a **quadratic** polynomial.

Also, the graph of $y = p(x)$ touches the x-axis at one point.

$\therefore$ $p(x)$ has only **1** zero.

(*vii*) The graph of $y = p(x)$ is a straight line.

$\therefore$ $p(x)$ is a **linear** polynomial.

Also, the graph of $y = p(x)$ cuts the x-axis at one point.

$\therefore$ $p(x)$ has only **1** zero.

(*viii*) The graph of $y = p(x)$ is a parabola.

$\therefore$ $p(x)$ is a **quadratic** polynomial.

Also, the graph of $y = p(x)$ does not touch the x-axis.

$\therefore$ $p(x)$ has **no zeroes**.

(*ix*) The graph of $y = p(x)$ is a parabola.

$\therefore$ $p(x)$ is a **quadratic** polynomial.

Also, the graph of $y = p(x)$ cuts the x-axis at two points.

$\therefore$ $p(x)$ has **2** zeroes.

EXERCISE 2B

For Basic and Standard Levels

1. (*i*)
$$f(x) = 3x^2 + 9x + 6$$
$$= 3(x^2 + 3x + 2)$$
$$= 3(x^2 + x + 2x + 2)$$
$$= 3[x\,(x + 1\,) + 2(x + 1)]$$
$$= 3(x + 1)\,(x + 2)$$

The zeroes of $f(x)$ are given by $f(x) = 0$

$\Rightarrow$ $3(x + 1)\,(x + 2) = 0$

$\Rightarrow$ $x = -1$ or $x = -2$

So, the zeroes are **– 1 and – 2**.

$$\text{Sum of zeroes} = (-1) + (-2) = -3$$
$$= \frac{-3\times 3}{3} = \frac{-9}{3}$$
$$= -\frac{(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$$
$$\text{Product of zeroes} = (-1) \times (-2) = 2$$
$$= \frac{2\times 3}{3} = \frac{6}{3}$$
$$= \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

(*ii*)
$$f(x) = 9x^2 - 6x + 1$$
$$= 9x^2 - 3x - 3x + 1$$
$$= 3x\,(3x - 1) - 1(3x - 1)$$
$$= (3x - 1)\,(3x - 1)$$

The zeroes of $f(x)$ are given by $f(x) = 0$

$\Rightarrow$ $(3x - 1)\,(3x - 1) = 0$

$\Rightarrow$ $(3x - 1)\,(3x - 1) = 0$

$\Rightarrow$ $x = \frac{1}{3}$

So, the zeroes are $\mathbf{\frac{1}{3}}$ and $\mathbf{\frac{1}{3}}$.

$$\text{Sum of zeroes} = \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$$
$$= \frac{2}{3}\times\frac{3}{3} = \frac{6}{9}$$
$$= \frac{-(-6)}{9}$$
$$= -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$$
$$\text{Product of zeroes} = \frac{1}{3}\times\frac{1}{3} = \frac{1}{9}$$
$$= \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

(*iii*)
$$f(x) = 9x^2 - 5$$
$$= (3x)^2 - (\sqrt{5}\,)^2$$
$$= (3x + \sqrt{5}\,)\,(3x - \sqrt{5}\,)$$

The zeroes of $f(x)$ are given by $f(x) = 0$

$\Rightarrow \quad (3x + \sqrt{5})(3x - \sqrt{5}) = 0$

$\Rightarrow \quad x = -\frac{\sqrt{5}}{3}$ or $x = \frac{\sqrt{5}}{3}$

So, the zeroes are $\mathbf{\frac{-\sqrt{5}}{3}}$ and $\mathbf{\frac{\sqrt{5}}{3}}$.

$$\text{Sum of zeroes} = -\frac{\sqrt{5}}{3} + \frac{\sqrt{5}}{3} = 0$$

$$= \frac{0}{1} = -\frac{0}{1}$$

$$= -\frac{(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$$

$$\text{Product of zeroes} = \left(\frac{-\sqrt{5}}{3}\right) \times \left(\frac{\sqrt{5}}{3}\right) = \frac{-5}{9}$$

$$= \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

(*iv*) $f(x) = 5x^2 + 2x$

$= x(5x + 2)$

The zeroes of $f(x)$ are given by $f(x) = 0$

$\Rightarrow \quad x(5x + 2) = 0$

$\Rightarrow \quad x = 0$ or $x = \frac{-2}{5}$

So, the zeroes are **0** and $\mathbf{\frac{-2}{5}}$.

$$\text{Sum of zeroes} = 0 + \frac{-2}{5} = \frac{-2}{5}$$

$$= \frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$$

$$\text{Product of zeroes} = 0 \times \left(\frac{-2}{5}\right) = 0 = \frac{0}{5}$$

$$= \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

(*v*) $f(x) = 6x^2 - 3 - 7x$

$= 6x^2 - 7x - 3$

$= 6x^2 - 9x + 2x - 3$

$= 3x(2x - 3) + 1(2x - 3)$

$= (2x - 3)(3x + 1)$

The zeroes of $f(x)$ are given by $f(x) = 0$

$\Rightarrow \quad (2x - 3)(3x + 1) = 0$

$\Rightarrow \quad x = \frac{3}{2}$ or $x = \frac{-1}{3}$.

So, the zeroes are $\mathbf{\frac{3}{2}}$ and $\mathbf{\frac{-1}{3}}$.

$$\text{Sum of zeroes} = \frac{3}{2} + \left(\frac{-1}{3}\right) = \frac{9 + (-2)}{6}$$

$$= \frac{7}{6} = -\frac{(-7)}{6}$$

$$= \frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$$

$$\text{Product of zeroes} = \left(\frac{3}{2}\right) \times \left(\frac{-1}{3}\right)$$

$$= \frac{-1}{2} = \frac{-1}{2} \times \frac{3}{3} = \frac{-3}{6}$$

$$= \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

(*vi*) $f(x) = \sqrt{3}x^2 - 8x + 4\sqrt{3}$

$= \sqrt{3}x^2 - 6x - 2x + 4\sqrt{3}$

$= \sqrt{3}x(x - 2\sqrt{3}) - 2(x - 2\sqrt{3})$

$= (x - 2\sqrt{3})(\sqrt{3}x - 2)$

The zeroes of $f(x)$ are given by $f(x) = 0$

$\Rightarrow \quad (x - 2\sqrt{3})(\sqrt{3}x - 2) = 0$

$\Rightarrow \quad x = 2\sqrt{3}$ or $x = \frac{2}{\sqrt{3}}$

So, the zeroes are $\mathbf{2\sqrt{3}}$ and $\mathbf{\frac{2}{\sqrt{3}}}$.

$$\text{Sum of zeroes} = 2\sqrt{3} + \frac{2}{\sqrt{3}}$$

$$= \frac{2\sqrt{3}(\sqrt{3}) + 2}{\sqrt{3}} = \frac{8}{\sqrt{3}}$$

$$= \frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$$

$$\text{Product of zeroes} = 2\sqrt{3} \times \frac{2}{\sqrt{3}}$$

$$= 4 = \frac{4}{1} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{4\sqrt{3}}{\sqrt{3}}$$

$$= \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

(*vii*) We have

$f(x) = 2\sqrt{3}x^2 - 5x + \sqrt{3}$

$= 2\sqrt{3}x^2 - 3x - 2x + \sqrt{3}$

$= \sqrt{3}x(2x - \sqrt{3}) - 1(2x - \sqrt{3})$

$= (2x - \sqrt{3})(\sqrt{3}x - 1)$

$\therefore$ The zeroes of $f(x)$ are given by

$f(x) = 0$

$\Rightarrow \quad (2x - \sqrt{3})(\sqrt{3}x - 1) = 0$

$\therefore$ Either $2x - \sqrt{3} = 0 \Rightarrow x = \frac{\sqrt{3}}{2}$

or $\sqrt{3}x - 1 = 0 \Rightarrow x = \frac{1}{\sqrt{3}}$

Hence, the required zeroes are $\mathbf{\frac{\sqrt{3}}{2}}$ and $\mathbf{\frac{1}{\sqrt{3}}}$.

Verification:

Sum of the zeroes of $f(x) = 2\sqrt{3}x^2 - 5x + \sqrt{3}$

$$= \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{3}}$$

$$= \frac{3 + 2}{2\sqrt{3}}$$

$$= \frac{5}{2\sqrt{3}}$$

$$= -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$$

(*viii*) We have $f(x) = x^2 + \frac{1}{6}x - 2$

$$= \frac{1}{6}\left(6x^2 + x - 12\right)$$

$$= \frac{1}{6}\left[6x^2 + 9x - 8x - 12\right]$$

$$= \frac{1}{6}\left[3x(2x + 3) - 4(2x + 3)\right]$$

$$= \frac{1}{6}(2x + 3)(3x - 4)$$

∴ The zeroes of $f(x)$ are given by

$f(x) = 0$

⇒ $(2x + 3)(3x - 4) = 0$

∴ Either $2x + 3 = 0$ ⇒ $x = -\frac{3}{2}$

or $3x - 4 = 0$ ⇒ $x = \frac{4}{3}$

Hence, the required zeroes of $f(x)$ are $\mathbf{\frac{4}{3}}$ and $-\mathbf{\frac{3}{2}}$.

Verification:

Sum of the zeroes of $f(x) = x^2 + \frac{1}{6}x - 2$

$$= \frac{4}{3} - \frac{3}{2} = \frac{8-9}{6} = \frac{1}{6}$$

$$= -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$$

(*ix*) We have $f(y) = y^2 + \frac{3\sqrt{5}}{2}y - 5$

$$= \frac{1}{2}\left[2y^2 + 3\sqrt{5}y - 10\right]$$

$$= \frac{1}{2}\left[2y^2 + 4\sqrt{5}y - \sqrt{5}y - 10\right]$$

$$= \frac{1}{2}\left[2y\left(y + 2\sqrt{5}y\right) - \sqrt{5}\left(y + \frac{10}{\sqrt{5}}\right)\right]$$

$$= \frac{1}{2}\left[2y\left(y + 2\sqrt{5}y\right) - \sqrt{5}\left(y + 2\sqrt{5}\right)\right]$$

$$= \frac{1}{2}\left(y + 2\sqrt{5}\right)\left(2y - \sqrt{5}\right)$$

∴ The zeroes of $f(y)$ are given by

$f(y) = 0$

⇒ $\left(y + 2\sqrt{5}\right)\left(2y - \sqrt{5}\right) = 0$

∴ Either $y + 2\sqrt{5} = 0$ ⇒ $y = -2\sqrt{5}$

or $2y - \sqrt{5} = 0$ ⇒ $y = \frac{\sqrt{5}}{2}$

Hence, the required zeroes are $\mathbf{-2\sqrt{5}}$ and $\mathbf{\frac{\sqrt{5}}{2}}$

Verification:

Sum of the zeroes of $f(y) = y^2 + \frac{3\sqrt{5}}{2}y - 5$

$$= -2\sqrt{5} + \frac{\sqrt{5}}{2}$$

$$= -\frac{4\sqrt{5} - \sqrt{5}}{2} = -\frac{3\sqrt{5}}{2}$$

$$= -\frac{\text{Coefficient of } y}{\text{Coefficient of } y^2}$$

(*x*) We have $f(s) = 2s^2 - \left(1 + 2\sqrt{2}\right)s + \sqrt{2}$

$$= 2s^2 - s - 2\sqrt{2}s + \sqrt{2}$$

$$= s(2s - 1) - \sqrt{2}(2s - 1)$$

$$= (2s - 1)(s - \sqrt{2})$$

Hence, zeroes of $f(s)$ are given by

$f(s) = 0$

⇒ $(2s - 1)\left(s - \sqrt{2}\right) = 0$

∴ Either $2s - 1 = 0$ ⇒ $s = \frac{1}{2}$

Or $s - \sqrt{2} = 0$ ⇒ $s = \sqrt{2}$

Hence, the required zeroes are $\mathbf{\frac{1}{2}}$ and $\mathbf{\sqrt{2}}$.

Verification:

Sum of the zeroes of $f(s) = 2s^2 - \left(1 + 2\sqrt{2}\right)s + \sqrt{2}$

$$= \frac{1}{2} + \sqrt{2}$$

$$= \frac{1 + 2\sqrt{2}}{2}$$

$$= -\frac{\text{Coefficient of } s}{\text{Coefficient of } s^2}$$

(*xi*) We have $f(t) = 9t^2 - 6t + 1$

$= (3t - 1)^2$

∴ Zeroes of $f(t)$ are given by

$(3t - 1)^2 = 0$

∴ $t = \frac{1}{3}, \frac{1}{3}$

Hence, the required zeroes are $\mathbf{\frac{1}{3}}$ and $\mathbf{\frac{1}{3}}$.

Verification: Sum of the zeroes $= \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$

Also, $-\frac{\text{Coefficient of } t}{\text{Coefficient of } t^2}$ of $f(t) = 9t^2 - 6t + 1$

$$= -\left(\frac{-6}{9}\right) = \frac{2}{3}$$

= sum of the zeroes

(*xii*) We have $f(x) = 3x^2 - 2$

∴ Zeroes of $f(x)$ are given by

$3x^2 - 2 = 0$

⇒ $x = \pm\sqrt{\frac{2}{3}}$

Hence, the required zeroes are $\sqrt{\frac{2}{3}}$ and $-\sqrt{\frac{2}{3}}$.

Verification: Sum of the zeroes of $f(x) = 3x^2 - 2$

$$= \sqrt{\frac{2}{3}} - \sqrt{\frac{2}{3}} = 0$$

$$= -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2} = \frac{0}{3} = 0$$

2. (*i*) We have $f(x) = x^2 + x - p(p + 1)$

$$= x^2 + (p + 1)x - px - p(p + 1)$$
$$= x[x + p + 1] - p[x + p + 1]$$
$$= (x + p + 1)(x - p)$$

$\therefore$ Zeroes of $f(x)$ are given by $(x + p + 1)(x - p) = 0$

$\therefore$ Either $x + p + 1 = 0 \Rightarrow x = -p - 1 = -(p + 1)$

Or $x - p = 0 \Rightarrow x = p$

Hence, the required zeroes are $\boldsymbol{-(p + 1)}$ and $\boldsymbol{p}$.

Verification:

Sum of the zeroes of $f(x) = x^2 + x - p(p + 1)$

$$= p - p - 1 = -1$$
$$= -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$$

(*ii*) We have $f(x) = x^2 - 3x - m(m + 3)$

$$= x^2 - (m + 3)x + mx - m(m + 3)$$
$$= x(x - m - 3) + m(x - m - 3)$$
$$= (x + m)(x - m - 3).$$

$\therefore$ The zeroes of $f(x)$ are given by

$(x + m)(x - m - 3) = 0$

$\therefore$ Either $x + m = 0 \Rightarrow x = -m$

or $x - m - 3 = 0 \Rightarrow x = m + 3$

Hence, the required zeroes are $\boldsymbol{-m}$ and $\boldsymbol{m + 3}$.

Verification:

Sum of the zeroes of $f(x) = x^2 - 3x - m(m + 3)$

$$= -m + m + 3 = 3$$
$$= -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$$

(*iii*) We have $f(x) = 6x^2 - 3 - 7x$

$$= 6x^2 - 7x - 3$$
$$= 6x^2 + 2x - 9x - 3$$
$$= 2x(3x + 1) - 3(3x + 1)$$
$$= (2x - 3)(3x + 1)$$

$\therefore$ The zeroes of $f(x)$ are given by

$(2x - 3)(3x + 1) = 0$

$\therefore$ Either $(2x - 3) = 0 \Rightarrow x = \frac{3}{2}$

or $(3x + 1) = 0 \Rightarrow x = \frac{-1}{3}$

Hence, the required zeroes are $\boldsymbol{\frac{3}{2}}$ and $\boldsymbol{\frac{-1}{3}}$.

Verification:

Sum of the zeroes of $f(x) = 6x^2 - 3 - 7x$

$$= \frac{3}{2} - \frac{1}{3}$$
$$= \frac{9 - 2}{6} = \frac{7}{6}$$
$$= -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$$
$$= \frac{-(-7)}{6} = \frac{7}{6}$$

3. $f(x) = ax^2 + (a + b)x + b$

$$= ax^2 + ax + bx + b$$
$$= ax(x + 1) + b(x + 1)$$
$$= (x + 1)(ax + b)$$

The zeroes of $f(x)$ are given by $f(x) = 0$

$\Rightarrow (x + 1)(ax + b) = 0$

$\Rightarrow x = -1$ or $x = \frac{-b}{a}$

So, the zeroes are $\mathbf{-1}$ and $\boldsymbol{-\frac{b}{a}}$.

4. Let $f(x) = 2x^2 + x + k$

A real number 'a' is a zero of a polynomial $f(x)$ if $f(a) = 0$.

Since 3 is a zero of the given polynomial,

$\therefore f(3) = 0$

$\Rightarrow 2(3)^2 + (3) + k = 0$

$\Rightarrow 18 + 3 + k = 0$

$\Rightarrow \boldsymbol{k = -21}$

5. (*i*) Let $f(x) = x^2 - x - (2k + 2)$

A real number 'a' is a zero of a polynomial $f(x)$ if $f(a) = 0$.

Since (-4) is a zero of the given polynomial,

$\therefore f(-4) = 0$

$\Rightarrow (-4)^2 - (-4) - (2k + 2) = 0$

$16 + 4 - 2k - 2 = 0$

$\Rightarrow 18 - 2k = 0$

$\Rightarrow 2k = 18$

$\Rightarrow \boldsymbol{k = 9}$

(*ii*) Since 1 is a zero of $p(x) = ax^2 - 3(a - 1) - 1$

$\therefore p(1) = 0$

$\Rightarrow a(1)^2 - 3(a - 1) - 1 = 0$

$\Rightarrow a - 3a + 3 - 1 = 0$

$\Rightarrow -2a + 2 = 0$

$\Rightarrow 2a = 2$

$\Rightarrow \boldsymbol{a = 1}$

(*iii*) Since -2 is a zero of $f(x) = 3x^2 + 4x + 2k$

$\therefore f(-2) = 0$

$\Rightarrow 3(-2)^2 + 4(-2) + 2k = 0$

$\Rightarrow 12 - 8 + 2k = 0$

$\Rightarrow \boldsymbol{k = -2}$

(*iv*) Since -4 is a zero of $f(x) = x^2 - x - (2k + 2)$,

$\therefore f(-4) = 0$

$\Rightarrow (-4)^2 + 4 - (2k + 2) = 0$

$\Rightarrow 20 = 2k + 2$

$\Rightarrow 2k = 18$

$\Rightarrow \boldsymbol{k = 9}$

6. Sum of zeroes $= 1 + (-3) = -2$

and product of zeroes $= 1 \times (-3) = -3$

Required polynomial $= x^2 - (-2)x + (-3)$

$$= \boldsymbol{x^2 + 2x - 3}$$

Sum of zeroes $= -2 = -\frac{2}{1}$

$$= -\frac{(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$$

Product of zeroes $= -3 = -\frac{3}{1}$

$$= \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

7. (*a*) (*i*) S = 4, P = –2.

Required polynomial $= x^2 - Sx + P$
$= x^2 - 4x + (-2)$
$= \mathbf{x^2 - 4x - 2}$

(*ii*) $S = 0, P = -\frac{10}{3}$.

Required polynomial $= x^2 - Sx + P$
$= x^2 - 0 \times x + \left(\frac{-10}{3}\right)$
$= x^2 - \frac{10}{3}$ or $k\left(x^2 - \frac{10}{3}\right)$

where k is non-zero constant.

If $k = 3$, then the polynomial is $\mathbf{3x^2 - 10}$.

(*iii*) $S = \frac{5}{7}$, P = 0.

Required polynomial $= x^2 - Sx + P$
$= x^2 - \frac{5}{7}x + 0$
$= x^2 - \frac{5}{7}x$ or $k\left(x^2 - \frac{5}{7}x\right)$

where k is non-zero constant.

If $k = 7$, then the polynomial is $\mathbf{7x^2 - 5x}$

(*iv*) S = –5, P = –6

Required polynomial $= x^2 - Sx + P$
$= x^2 - (-5)x + (-6)$
$= \mathbf{x^2 + 5x - 6}$

(*v*) $S = \sqrt{2}$, P = –12

Required polynomial $= x^2 - Sx + P$
$= x^2 - \sqrt{2}x + (-12)$
$= \mathbf{x^2 - \sqrt{2}x - 12}$

(*vi*) S = 3, P = –2

Required polynomial $= x^2 - Sx + P$
$= x^2 - 3x + (-2)$
$= \mathbf{x^2 - 3x - 2}$

(*vii*) The required polynomial is

$$x^2 - (\alpha + \beta)x + \alpha\beta \qquad \ldots (A)$$

where α and β are zeroes of the polynomial, so that

$$\alpha + \beta = -2\sqrt{3} \qquad \ldots(1)$$

and $\alpha\beta = -9 \qquad \ldots(2)$

∴ From (A), the required polynomial is

$$\mathbf{x^2 + 2\sqrt{3}x - 9}$$

2nd part:

We have $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta = 12 + 36$ [From (1) and (2)]
$= 48$

∴ $\alpha - \beta = \pm 4\sqrt{3} \qquad \ldots(3)$

From (1) and (3), by adding and subtracting successively, we get

$\alpha = \sqrt{3}$ or $-3\sqrt{3}$

and $\beta = -3\sqrt{3}$ or $\sqrt{3}$

Hence, the required zeroes are $\mathbf{\sqrt{3}}$ and $\mathbf{-3\sqrt{3}}$.

(*viii*) The required polynomial is

$$x^2 - (\alpha + \beta)x + \alpha\beta \qquad \ldots (A)$$

where α and β are the zeroes of the polynomial.

So that $\alpha + \beta = -\frac{3}{2\sqrt{5}} \qquad \ldots (1)$

and $\alpha\beta = -\frac{1}{2} \qquad \ldots (2)$

∴ From (A), the required polynomial is

$$\mathbf{x^2 + \frac{3}{2\sqrt{5}}x - \frac{1}{2}}$$

2nd part:

We have $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$
$= \frac{9}{20} + 2$ [From (1) and (2)]

∴ $\alpha - \beta = \pm\frac{7}{2\sqrt{5}} \qquad \ldots(3)$
$= \frac{49}{20}$

From (1) and (3), by adding and subtracting successively, we get

$\alpha = \frac{1}{\sqrt{5}}$ or $-\frac{\sqrt{5}}{2}$

and $\beta = -\frac{\sqrt{5}}{2}$ or $\frac{1}{\sqrt{5}}$

Hence, the required zeroes are $\mathbf{\frac{1}{\sqrt{5}}}$ and $\mathbf{-\frac{\sqrt{5}}{2}}$ or $\mathbf{\frac{\sqrt{5}}{5}}$ and $\mathbf{-\frac{\sqrt{5}}{2}}$.

(*b*) Let $f(y) = ky^2 + 2y - 3k$

∴ Sum of zeroes $= \frac{\text{Coefficient of } y}{\text{Coefficient of } y^2} = \frac{-2}{k} \qquad \ldots (1)$

Product of zeroes $= \frac{\text{Constant term}}{\text{Coefficient of } y^2} = \frac{-3k}{k} = -3 \qquad \ldots (2)$

Given that sum of zeroes = 2(Product of zeroes)

⇒ $\frac{-2}{k} = 2(-3)$ [From (1) and (2)]

⇒ $-2 = -6k$

⇒ $\mathbf{k = \frac{1}{3}}$

8. (*i*) Polynomial $= x^2 - (\alpha + \beta)x + \alpha\beta$
$= \mathbf{x^2 - 6x + 4}$

(*ii*) S = 8, P = 12

Required polynomial $= p(x) = x^2 - Sx + P$
$= \mathbf{x^2 - 8x + 12}$

Now, $x^2 - 8x + 12 = x^2 - 6x - 2x + 12$
$= x(x - 6) - 2(x - 6)$
$= (x - 6)(x - 2)$

The zeroes of $p(x)$ are given by $p(x) = 0$

∴ $(x - 6)(x - 2) = 0$

⇒ $x = 6$ or $x = 2$

So, the zeroes are **6** and **2**.

(*iii*) Let α and β be the zeroes of the polynomial. Then the required polynomial is

$$x^2 - (\alpha + \beta)x + \alpha\beta = x^2 - \sqrt{2}x - \frac{3}{2}$$

where $\alpha + \beta = \sqrt{2}$ …(1)

and $\alpha\beta = -\frac{3}{2}$ …(2)

$\therefore$ $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$

$= 2 + 6 = 8$

$\therefore$ $\alpha - \beta = \pm 2\sqrt{2}$ …(3)

From (1) and (3), by adding and subtracting successively, we get

$$\alpha = \frac{3}{\sqrt{2}} \text{ or } -\frac{1}{\sqrt{2}}$$

and $\beta = -\frac{1}{\sqrt{2}}$ or $\frac{3}{\sqrt{2}}$

Hence, the required zeroes are $-\frac{\mathbf{1}}{\sqrt{\mathbf{2}}}$ and $\frac{\mathbf{3}}{\sqrt{\mathbf{2}}}$.

9. (*i*) Let the other zero of the polynomial $x^2 - 4x + 1$ be α.

Sum of zeroes $= \left(2 + \sqrt{3}\right) + \alpha$

$= \frac{-(-4)}{1} = 4$

$\Rightarrow$ $\alpha = 4 - \left(2 + \sqrt{3}\right)$

$= 4 - 2 - \sqrt{3}$

$= \mathbf{2 - \sqrt{3}}$

(*ii*) Let one of the zeroes of the given polynomial be α. Then, the other zero is $\frac{1}{\alpha}$.

Product of zeroes $= \frac{\text{Constant term}}{\text{Coefficient of } x^2}$

$\therefore$ $\alpha \times \frac{1}{\alpha} = \frac{6a}{a^2+9}$

$\Rightarrow$ $1 = \frac{6a}{a^2+9}$

$\Rightarrow$ $a^2 + 9 = 6a$

$\Rightarrow$ $a^2 - 6a + 9 = 0$

$\Rightarrow$ $a^2 - 3a - 3a + 9 = 0$

$\Rightarrow$ $a(a - 3) - 3(a - 3) = 0$

$\Rightarrow$ $(a - 3)(a - 3) = 0$

$\Rightarrow$ $\boldsymbol{a = 3}$

(*iii*) Let α and $-\alpha$ be the zeroes of the polynomial

$f(x) = 4x^2 - 8kx + 8x - 9$

$= 4x^2 + (8 - 8k)x - 9$

$\therefore$ Sum of the zeroes$= \alpha - \alpha = 0 = \frac{8k-8}{4}$

$\Rightarrow$ $k = 1$

$\therefore$ The second given polynomial becomes

$x^2 + 3x + 2 = (x + 2)(x + 1)$

$\therefore$ The zeroes of this polynomial are given by

$(x + 2)(x + 1) = 0$

$\therefore$ Either $x + 2 = 0 \Rightarrow x = -2$

or $x + 1 = 0 \Rightarrow x = -1$

$\therefore$ The required zeroes of the 2nd given polynomial are **–2** and **–1**.

10. Given polynomial is $ax^2 - 6x - 6$

Product of zeroes $= \frac{\text{Constant term}}{\text{Coefficient of } x^2}$

$\Rightarrow$ $4 = -\frac{6}{a}$

$\Rightarrow$ $a = -\frac{6}{4} = \frac{\mathbf{-3}}{\mathbf{2}}$

Sum of zeroes $= \frac{-\text{Coefficient of } x}{\text{Coefficient of } x^2}$

$= \frac{-(-6)}{a}$

$= \frac{-(-6)}{\frac{-3}{2}}$

$= \frac{-6}{3} \times 2 = \mathbf{-4}$

11. (*i*) Given polynomial is $ky^2 + 2y + 3k$

Sum of zeroes = Product of zeroes (Given)

$\Rightarrow$ $-\frac{(\text{Coefficient of } y)}{\text{Coefficient of } y^2} = \frac{\text{Constant term}}{\text{Coefficient of } y^2}$

$\Rightarrow$ $-(\text{Coefficient of } y) = \text{Constant term}$

$\Rightarrow$ $-2 = 3k$

$\Rightarrow$ $\boldsymbol{k = -\frac{2}{3}}$

(*ii*) Let α and β be the zeroes of the polynomial $x^2 - (k + 6)x + 2(2k - 1)$

Then $\alpha + \beta = k + 6$ …(1)

and $\alpha\beta = 2(2k - 1)$ …(2)

According to the problem,

$\alpha + \beta = \frac{1}{2}\alpha\beta$ …(3)

$\Rightarrow$ $k + 6 = 2k - 1$ [From (1) and (2)]

$\Rightarrow$ $\boldsymbol{k = 7}$

12. Since $(x + a)$ is a factor of the polynomial

$f(x) = 2x^2 + 2ax + 5x + 10$

$\therefore$ $x = -a$ is a root of the given polynomial.

$\therefore$ $f(-a) = 0$

$\Rightarrow$ $2(-a)^2 + 2a(-a) + 5(-a) + 10 = 0$

$\Rightarrow$ $2a^2 - 2a^2 - 5a + 10 = 0$

$\Rightarrow$ $-5a + 10 = 0$

$\Rightarrow$ $5a = 10$

$\Rightarrow$ $a = \frac{10}{5}$

$\Rightarrow$ $\boldsymbol{a = 2}$

For Standard Level

13. $f(x) = 3x^2 - 5x - 2$

$\Rightarrow$ $x^2 - (\alpha + \beta)x + \alpha\beta = 3x^2 - 5x - 2$

$\Rightarrow$ $\alpha + \beta = \frac{5}{3}$ and $\alpha\beta = -\frac{2}{3}$ … (1)

Now, $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta + \alpha}{\alpha\beta}$

$= \frac{\frac{5}{3}}{\frac{-2}{3}} = \mathbf{\frac{-5}{2}}$ [Using (1)]

14. $f(x) = 4x^2 - 4x + 1 = \frac{4x^2}{4} - \frac{4}{4}x + \frac{1}{4} = x^2 - x + \frac{1}{4}$

$\Rightarrow \quad x^2 - (\alpha + \beta)x + \alpha\beta = x^2 - x + \frac{1}{4}$

$\Rightarrow \quad \alpha + \beta = 1$ and $\alpha\beta = \frac{1}{4}$ … (1)

Now, $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2 + \beta^2}{\alpha\beta}$

$= \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}$

$= \frac{(1)^2 - 2\left(\frac{1}{4}\right)}{\frac{1}{4}}$ [Using (1)]

$= \frac{1 - \frac{1}{2}}{\frac{1}{4}} = \frac{\frac{1}{2}}{\frac{1}{4}}$

$= \frac{1}{2} \times \frac{4}{1} = \mathbf{2}$

15. α, β are the zeroes $2y^2 + 7y + 5 = y^2 + \frac{7}{2}y + \frac{5}{2}$

$\therefore \quad \alpha + \beta = -\frac{7}{2}$ and $\alpha\beta = \frac{5}{2}$ … (1)

$\therefore \quad \alpha + \beta + \alpha\beta = -\frac{7}{2} + \frac{5}{2} = \frac{-7 + 5}{2}$

$= \frac{-2}{2} = \mathbf{-1}$

16. Let α and β be the zeroes of the polynomial, where

$\alpha = 1$ and $\beta = -3$ [Given]

$\therefore$ The required polynomial is $x^2 - (\alpha + \beta)x + \alpha\beta$

$= x^2 - (1 - 3)x - 1 \times 3$

$= \mathbf{x^2 + 2x - 3}$ …(1)

Now, we see that the sum of the zeroes of this polynomial $x^2 + 2x - 3 = 1 - 3 = -2$

which is also equal to $-\frac{\text{coeffcient of } x}{\text{coefficient of } x^2}$ of the polynomial in (1).

Thus, we have verified the required relation.

17. (*i*) α, β are the zeroes of the polynomial $x^2 - 2x - 15$

$\therefore \quad \alpha + \beta = 2$ and $\alpha\beta = -15$ … (1)

Zeroes of the required polynomial are 2α and 2β.

Sum $= 2\alpha + 2\beta = 2(\alpha + \beta)$

$= 2(2) = 4$

Product $= 2\alpha \times 2\beta = 4\alpha\beta$

$= 4 \times (-15) = -60$

Required polynomial $= x^2 -$ (Sum of zeroes)x + Product of zeroes

$= \mathbf{x^2 - 4x - 60}$

(*ii*) α, β are the zeroes of the polynomial $x^2 + 6x + 9$

$\therefore \quad \alpha + \beta = -6$ and $\alpha\beta = 9$ …(1)

Zeroes of the required polynomial are $-\alpha$ and $-\beta$.

Sum $= -\alpha + (-\beta)$

$= -\alpha - \beta = -(\alpha + \beta)$

$= -(-6) = 6$

Product $= (-\alpha)(-\beta) = \alpha\beta = 9$

$\therefore$ Required polynomial $= x^2 -$ (Sum of zeroes)x + Product of zeroes

$= \mathbf{x^2 - 6x + 9}$

(*iii*) α, β are the zeroes of the polynomial

$4x^2 + 4x + 1 = x^2 + \frac{4}{4}x + \frac{1}{4}$

$= x^2 + x + \frac{1}{4}$

$\therefore \quad \alpha + \beta = -1$ and $\alpha\beta = \frac{1}{4}$ …(1)

Zeroes of the required polynomial are 2α and 2β.

Sum $= 2\alpha + 2\beta = 2(\alpha + \beta)$

$= 2(-1) = -2$

Product $= (2\alpha)(2\beta) = 4(\alpha\beta)$

$= 4\left(\frac{1}{4}\right) = 1$

$\therefore$ Required polynomial $= x^2 -$ (Sum of zeroes)x + Product of zeroes

$= x^2 - (-2)x + 1$

$= \mathbf{x^2 + 2x + 1}$

18. α, β are the zeroes of the polynomial $x^2 + 4x + 3$

$\therefore \quad \alpha + \beta = -4$ and $\alpha\beta = 3$ … (1)

Zeroes of required polynomial are $1 + \frac{\beta}{\alpha}$ and $1 + \frac{\alpha}{\beta}$.

Sum of zeroes $= 1 + \frac{\beta}{\alpha} + 1 + \frac{\alpha}{\beta}$

$= \frac{\alpha\beta + \beta^2 + \alpha\beta + \beta^2}{\alpha\beta}$

$= \frac{\alpha^2 + 2\alpha\beta + \beta^2}{\alpha\beta}$

$= \frac{(\alpha + \beta)^2}{\alpha\beta}$

$= \frac{(-4)^2}{3} = \frac{16}{3}$ [Using (1)]

Product of zeroes $= \left(1 + \frac{\beta}{\alpha}\right)\left(1 + \frac{\alpha}{\beta}\right)$

$= 1 + \frac{\alpha}{\beta} + \frac{\beta}{\alpha} + \frac{\alpha\beta}{\alpha\beta}$

$= 1 + \frac{\alpha}{\beta} + \frac{\beta}{\alpha} + 1$

$= \frac{\alpha\beta + \alpha^2 + \beta^2 + \alpha\beta}{\alpha\beta}$

$$= \frac{(\alpha+\beta)^2}{\alpha\beta}$$

$$= \frac{(-4)^2}{3} = \frac{16}{3}$$

So, the required polynomial is $x^2 - \frac{16}{3}x + \frac{16}{3}$

or $k\left(x^2 - \frac{16x}{3} + \frac{16}{3}\right)$ where k is a non-zero constant.

If $k = 3$, then the polynomial is $\mathbf{3x^2 - 16x + 16}$.

19. We have $\alpha + \beta = -\left(\frac{-1}{1}\right) = 1$ …(1)

and $\alpha\beta = \frac{-2}{1} = -2$ …(2)

∴ The polynomial whose zeroes are $1 + 2\alpha$ and $1 + 2\beta$ is

$x^2 - (1 + 2\alpha + 1 + 2\beta)x + (1 + 2\alpha)(1 + 2\beta)$

$= x^2 - 2(1 + \alpha + \beta)x + 2(\alpha + \beta) + 1 + 4\alpha\beta$

$= x^2 - 2(1 + 1)x + 2 + 1 - 8$ [From (1) and (2)]

$= \mathbf{x^2 - 4x - 5}$ which is the required polynomial.

20. We have $\alpha + \beta = 24$ …(1)

and $\alpha - \beta = 8$ …(2)

Adding (1) and (2), we get

$2\alpha = 32$

$\Rightarrow \alpha = 16$

Subtracting (2) from (1), we get,

$2\beta = 16$

$\Rightarrow \beta = 8$

Hence, the required quadratic polynomial is

$x^2 - (\alpha + \beta)x + \alpha\beta = x^2 - (16 + 8)x + 16 \times 8$

$= \mathbf{x^2 - 24x + 128}$

21. Given polynomial is $x^2 - 4x + 3$.

Its zeroes are m and n.

Sum of zeroes $= m + n = 4$

and product of zeroes $= mn = 3$

$$\text{LHS} = \frac{1}{m} + \frac{1}{n} - 2mn + \frac{14}{3}$$

$$= \frac{n+m}{mn} - 2mn + \frac{14}{3}$$

$$= \frac{4}{3} - 2 \times 3 + \frac{14}{3}$$

$$= \frac{4}{3} - 6 + \frac{14}{3}$$

$$= \frac{4-18+14}{3}$$

$$= \frac{18-18}{3}$$

$= 0$

$=$ RHS

22. Let 2α and 2β be the roots of the polynomial $x^2 + px + q$.

Sum of zeroes $= 2(\alpha + \beta) = -p$

and product of zeroes $= 4\alpha\beta = q$ … (1)

and the zeroes of the polynomial $2x^2 - 5x - 3$

or $x^2 - \frac{5}{2}x - \frac{3}{2}$ are α and β.

Sum of zeroes $= \alpha + \beta = \frac{5}{2}$ and $\alpha\beta = -\frac{3}{2}$

$\Rightarrow 2(\alpha + \beta) = \frac{2 \times 5}{2} = 5$

and $4\alpha\beta = -\frac{3}{2} \times 4 = -6$ … (2)

From (1) and (2), we get

$-p = 5$

$\Rightarrow$ $\mathbf{p = -5}$ and $\mathbf{q = -6}$

23. Since $x - (n - k)$ is a common factor of two polynomials

$f(x) = x^2 + px + q$

and $\phi(x) = x^2 - mx + n$

∴ $n - k$ will be zeroes of $f(x)$ and $\phi(x)$.

∴ $f(n - k) = \phi(n - k) = 0$

$\Rightarrow (n - k)^2 + p(n - k) + q = 0$ …(1)

and $(n - k)^2 + m(n - k) + n = 0$ …(2)

Subtracting (2) from (1), we get

$p(n - k) - m(n - k) + q - n = 0$

$\Rightarrow k(m - p) = n - q - pn + mn$

$= n - q + n(m - p)$

$\Rightarrow (m - p)(k - n) = n - q$

$$\Rightarrow k - n = \frac{n-q}{m-p}$$

$$\Rightarrow \mathbf{k = n + \frac{n-q}{m-p}}$$

Hence, proved.

EXERCISE 2C

For Basic and Standard Levels

1. (*i*) $P(x) = x^3 - 27x + 54 = (x^3 + 0x^2 - 27x + 54)$

∴ $P(-6) = (-6)^3 - 27(-6) + 54$

$= -216 + 162 + 54$

$= -216 + 216$

$= 0$

$P(3) = (3)^3 - 27(3) + 54$

$= 27 - 81 + 54$

$= 81 - 81$

$= 0$

Thus, **−6, 3, 3** are the zeroes of the given polynomial $P(x)$.

Let $\alpha = -6$, $\beta = 3$ and $\gamma = 3$

Then, $\alpha + \beta + \gamma = -6 + 3 + 3 = 0$

$$= -\frac{(\text{Coefficient of } x^2)}{\text{Coefficient of } x^3}$$

$\alpha\beta + \beta\gamma + \gamma\alpha = (-6)(3) + (3)(3) + (3)(-6)$

$= -18 + 9 - 18$

$= -27$

$$= \frac{\text{Coefficient of } x}{\text{Coefficient of } x^3}$$

$\alpha\beta\gamma = (-6)(3)(3)$

$= -54$

$= -\frac{(54)}{1}$

$= -\frac{\text{Constant term}}{\text{Coefficient of } x^3}$

(*ii*) $P(x) = x^3 - 8x^2 + 9x + 18$

$\therefore$ $P(-1) = (-1)^3 - 8(-1)^2 + 9(-1) + 18$
$= -1 - 8 - 9 + 18$
$= -18 + 18$
$= 0$

$P(3) = (3)^3 - 8(3)^2 + 9(3) + 18$
$= 27 - 72 + 27 + 18$
$= 72 - 72$
$= 0$

$P(6) = (6)^3 - 8(6)^2 + 9(6) + 18$
$= 216 - 288 + 54 + 18$
$= 288 - 288$
$= 0$

Thus, **–1, 3** and **6** are the zeroes of the given polynomial P(x).

Let $\alpha = -1, \beta = 3$ and $\gamma = 6$

Thus, $\alpha + \beta + \gamma = (-1) + 3 + 6$

$= 8 = -\frac{(-8)}{1}$

$= -\frac{(\text{Coefficient of } x^2)}{\text{Coefficient of } x^3}$

$\alpha\beta + \beta\gamma + \gamma\alpha = (-1)(3) + (3)(6) + (6)(-1)$
$= -3 + 18 - 6$

$= \frac{9}{1}$

$= \frac{\text{Coefficient of } x}{\text{Coefficient of } x^3}$

$\alpha\beta\gamma = (-1)(3)(6) = -18$

$= \frac{-(18)}{1}$

$= -\frac{\text{Constant term}}{\text{Coefficient of } x^3}$

2. Let the zeroes of the required cubic polynomial be α, β and γ.

(*i*) $\alpha + \beta + \gamma = 3,$
$\alpha\beta + \beta\gamma + \gamma\alpha = -8$
and $\alpha\beta\gamma = -13$

Cubic polynomial where zeroes are α, β and γ is given by $x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)\, x - \alpha\beta\gamma$.

$\therefore$ The required polynomial is $x^3 - 3x^2 + (-8)\, x - (-13)$

$= \mathbf{x^3 - 3x^2 - 8x + 13}$

(*ii*) Here, $\alpha + \beta + \gamma = 2, \alpha\beta + \beta\gamma + \gamma\alpha = -5$
and $\alpha\beta\gamma = -20$

$\therefore$ The required polynomial is $x^3 - 2x^2 + (-5)x - (-20)$

$= \mathbf{x^3 - 2x^2 - 5x + 20}$

(*iii*) Here $\alpha + \beta + \gamma = -4, \alpha\beta + \beta\gamma + \gamma\alpha = \frac{1}{2}$

and $\alpha\beta\gamma = -\frac{1}{3}$

$\therefore$ The required polynomial is $x^3 - (-4)x^2 + \frac{1}{2}x - \left(\frac{-1}{3}\right)$

$= x^3 + 4x^2 + \frac{1}{2}x + \frac{1}{3}$

or $k\left(x^3 + 4x^2 + \frac{1}{2}x + \frac{1}{3}\right)$

where k is a non-zero constant.

If $k = 6$, then the polynomial is $\mathbf{6x^3 + 24x^2 + 3x + 2}$.

(*iv*) Here, $\alpha + \beta + \gamma = \frac{1}{\sqrt{2}}$,

$\alpha\beta + \beta\gamma + \gamma\alpha = \sqrt{3}$ and $\alpha\beta\gamma = \frac{1}{\sqrt{6}}$

$\therefore$ The required polynomial is

$x^3 - \frac{1}{\sqrt{2}}x^2 + \sqrt{3}\,x - \frac{1}{\sqrt{6}}$

$= k\left(x^3 - \frac{1}{\sqrt{2}}x^2 + \sqrt{3}x - \frac{1}{\sqrt{6}}\right)$

where k is a non-zero constant.

If $k = \sqrt{6}$, then the polynomial is

$\mathbf{\sqrt{6}x^3 - \sqrt{3}x^2 + 3\sqrt{2}x - 1.}$

3. (*i*) Let α, β, γ be the zeroes of the required polynomial.
$\alpha = 4, \beta = -3$ and $\gamma = -1$.

Then, $\alpha + \beta + \gamma = 4 - 3 - 1 = 0,$

$\alpha\beta + \beta\gamma + \gamma\alpha = (4)(-3) + (-3)(-1) + (-1)(4)$
$= -12 + 3 - 4$
$= -16 + 3$
$= -13$

and $\alpha\beta\gamma = 4 \times (-3) \times (-1) = 12$

The required polynomial is

$x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma$
$= x^3 - (0)\, x^2 + (-13)\, x - 12$
$= \mathbf{x^3 - 13x - 12}$

(*ii*) Let α, β, γ be the zeroes of the required polynomial.

Then, $\alpha = 0, \beta = \frac{2}{3}$ and $\gamma = \frac{-2}{3}$.

Then, $\alpha + \beta + \gamma = 0 + \frac{2}{3} + \left(\frac{-2}{3}\right) = 0,$

$\alpha\beta + \beta\gamma + \gamma\alpha = (0)\left(\frac{2}{3}\right) + \left(\frac{2}{3}\right)\left(\frac{-2}{3}\right) + \left(\frac{-2}{3}\right)(0)$

$= \frac{-4}{9}$

and $\alpha\beta\gamma = 0 \times \frac{2}{3} \times \left(\frac{-2}{3}\right) = 0$

The required polynomial is

$x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)\, x - \alpha\beta\gamma$

$= x^3 - (0)\, x^2 + \left(\frac{-4}{9}\right)x - 0$

$= x^3 - \frac{4}{9}x$ or $k\left(x^3 - \frac{4}{9}x\right)$

where k is a non-zero constant.

If $k = 9$, then the polynomial is $\mathbf{9x^3 - 4x}$.

4. (*i*) Let α, β and γ be the three zeroes of the given polynomial $f(x) = 2x^3 - x^2 - 5x - 2$, where $\alpha = -1$, $\beta = 2$ (given). We shall determine the third zero γ.

Now, $\alpha + \beta + \gamma = -\left(\frac{-1}{2}\right) = \frac{1}{2}$

$\Rightarrow \quad -1 + 2 + \gamma = \frac{1}{2}$

$\Rightarrow \quad \gamma = \frac{1}{2} - 1 = -\frac{1}{2}$

Hence, the required third zero is $-\mathbf{\frac{1}{2}}$.

(*ii*) Let $p(x) = 2x^3 - 4x - x^2 + 2$
$= 2x^3 - x^2 - 4x + 2$

Let $\alpha = \sqrt{2}$ and $\beta = -\sqrt{2}$ be the given zeroes of the polynomial $p(x)$ and γ its third zero.

Then,

Sum of zeroes $= \alpha + \beta + \gamma = \left[\sqrt{2} + \left(-\sqrt{2}\right) + \gamma\right]$

$= \frac{-\text{Coefficient of } x^2}{\text{Coefficient of } x^3}$

$\Rightarrow \quad \gamma = -\left(\frac{-1}{2}\right) = \frac{1}{2}$

Hence, the zeroes of the given polynomial are $\mathbf{\sqrt{2}, -\sqrt{2}}$ and $\mathbf{\frac{1}{2}}$.

(*iii*) Let α, β and γ be the three zeroes of the given polynomial

$f(x) = x^3 - 4x^2 - 3x + 12$

where $\alpha = \sqrt{3}$, $\beta = -\sqrt{3}$ [Given]

We shall determine the third zero γ.

Now, $\alpha + \beta + \gamma = -\left(\frac{-4}{1}\right) = 4$

$\Rightarrow \quad \sqrt{3} - \sqrt{3} + \gamma = 4$

$\Rightarrow \quad \gamma = \mathbf{4}$

which is the required third zero.

(*iv*) Let $p(x) = x^3 + 3x^2 - 5x - 15$.

Let $\alpha = \sqrt{5}$ and $\beta = -\sqrt{5}$ be the given zeroes of the polynomial $p(x)$ and γ its third zero.

Then, sum of zeroes $= \alpha + \beta + \gamma$

$= \left[\sqrt{5} + \left(-\sqrt{5}\right) + \gamma\right]$

$= \frac{-\text{ Coefficient of } x^2}{\text{Coefficient of } x^3}$

$\Rightarrow \quad \gamma = -\frac{3}{1}$

$\Rightarrow \quad \gamma = \mathbf{-3}$

5. $p(x) = x^3 - 2x^2 - 49x + 98$

Let α, β and γ be the zeroes of the given polynomial.

Then, $\beta = -\alpha$

Sum of zeroes $= \alpha + \beta + \gamma = \alpha + (-\alpha) + \gamma$

$= \gamma = \frac{-(\text{Coefficient of } x^2)}{\text{Coefficient of } x^3}$

$\Rightarrow \quad \gamma = \frac{-(-2)}{1} = 2 \quad \ldots (1)$

Product of zeroes $= \alpha\beta\gamma = (\alpha)(-\alpha)(2)$ [using (1)]

$= -2\alpha^2 = \frac{-\text{Constant term}}{\text{Coefficient of } x^3}$

$\Rightarrow \quad -2\alpha^2 = \frac{-98}{1}$

$\Rightarrow \quad \alpha^2 = \frac{98}{2} = 49$

$\Rightarrow \quad \alpha = \pm 7$

Hence, the zeroes of the given polynomial are **7, – 7** and **2**.

For Standard Level

6. $p(x) = x^3 - 7x^2 + 14x - 8$

Let α, β, γ be the zeroes of the given polynomial.

Then,

Product of two zeroes = 8

So, let $\alpha\beta = 8 \quad \ldots (1)$

Product of zeroes $= \alpha\beta\gamma = 8\gamma$

$= \frac{-\text{Constant term}}{\text{Coefficient of } x^3}$

$\Rightarrow \quad 8\gamma = \frac{-(-8)}{1}$

$\Rightarrow \quad 8\gamma = 8$

$\Rightarrow \quad \gamma = 1$

Sum of zeroes $= \alpha + \beta + \gamma$

$= \alpha + \beta + 1$

$= \frac{-(\text{Coefficient of } x^2)}{\text{Coefficient of } x^3}$

$\Rightarrow \quad \alpha + \beta + 1 = \frac{-(-7)}{1}$

$\Rightarrow \quad \alpha + \beta = 7 - 1 = 6 \quad \ldots (2)$

$(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$

$= (6)^2 - 4 \times 8$

[Using (1) and (2)]

$= 36 - 32$

$= 4$

$\Rightarrow \quad \alpha - \beta = \pm 2$

Thus, we have $\alpha + \beta = 6$ and $\alpha - \beta = \pm 2$

When $\alpha - \beta = 2$

then $\alpha = 4$ and $\beta = 2$

When $\alpha - \beta = -2$

then $\alpha = 2$ and $\beta = 4$

Hence, the zeroes of the given polynomial are **1**, **2** and **4**.

7. Let $p(x) = x^3 - 15x^2 + 71x + p$

Let the zeroes of the polynomial be $\alpha - \beta$, α and $\alpha + \beta$.

Sum of zeroes $= \alpha - \beta + \alpha + \alpha + \beta$

$= 3\alpha$

$$= \frac{-(\text{Coefficient of } x^2)}{\text{Coefficient of } x^3}$$

$\Rightarrow \quad 3\alpha = -\dfrac{(-15)}{1}$

$\Rightarrow \quad 3\alpha = 15$

$\Rightarrow \quad \alpha = 5$

$(\alpha - \beta)\,\alpha + \alpha\,(\alpha + \beta) + (\alpha - \beta)\,(\alpha + \beta)$

$$= \frac{\text{Coefficient of } x}{\text{Coefficient of } x^3}$$

$\Rightarrow \alpha(\alpha - \beta + \alpha + \beta) + (\alpha^2 - \beta^2)$

$$= \frac{71}{1}$$

$\Rightarrow \quad \alpha(2\alpha) + (\alpha^2 - \beta^2) = 71$

$\Rightarrow \quad 5(2 \times 5) + (5^2 - \beta^2) = 71$

$\Rightarrow \quad 50 + 25 - \beta^2 = 71$

$\Rightarrow \quad \beta^2 = 75 - 71 = 4$

$\Rightarrow \quad \beta = \pm 2$

When $\beta = 2$,

then $\alpha - \beta = 5 - 2 = 3$

and $\alpha + \beta = 5 + 2 = 7$

When $\beta = -2$,

then $\alpha - \beta = 5 - (-2) = 7$

and $\alpha + \beta = 5 + (-2) = 3$

Hence, the zeroes of the given polynomial are **3, 5** and **7**.

Product of the zeroes $= 3 \times 5 \times 7$

$$= \frac{-\text{Constant term}}{\text{Coefficient of } x^2}$$

$\Rightarrow \quad 105 = -p$

$\Rightarrow \quad \boldsymbol{p = -105}$

8. Let $p(x) = x^3 - 3x^2 + x + 1$

Sum of zeroes $= a - b + a + a + b$

$= 3a$

$$= \frac{-(\text{Coefficient of } x^2)}{\text{Coefficient of } x^3}$$

$$= \frac{-(-3)}{1}$$

$\Rightarrow \quad 3a = 3$

$\Rightarrow \quad \boldsymbol{a = 1}$

Product of zeroes $= (a - b)\,a(a + b)$

$= a(a^2 - b^2)$

$= 1(1 - b^2)$

$$= \frac{-\text{Constant term}}{\text{Coefficient of } x^3}$$

$\Rightarrow \quad 1 - b^2 = -\dfrac{1}{1}$

$\Rightarrow \quad 1 - b^2 = -1$

$\Rightarrow \quad b^2 = 2$

$\Rightarrow \quad \boldsymbol{b = \pm\sqrt{2}}$

EXERCISE 2D

For Basic and Standard Levels

1.

$$x + 4 \,\big)\, 3x^3 + 16x^2 + 21x + 20 \,\big(\, 3x^2 + 4x + 5$$
$$\underline{3x^3 + 12x^2}$$
$$(-)\quad(-)$$
$$4x^2 + 21x$$
$$\underline{4x^2 + 16x}$$
$$(-)\quad(-)$$
$$5x + 20$$
$$\underline{5x + 20}$$
$$(-)\quad(-)$$
$$0$$

Quotient = $\boldsymbol{3x^2 + 4x + 5}$

Remainder = 0

2. (*i*)

$$2x + 1 \,\big)\, 6x^3 + 13x^2 + x - 2 \,\big(\, 3x^2 + 5x - 2$$
$$\underline{6x^3 + 3x^2}$$
$$(-)\quad(-)$$
$$10x^2 + x$$
$$\underline{10x^2 + 5x}$$
$$(-)\quad(-)$$
$$-4x - 2$$
$$\underline{-4x - 2}$$
$$(+)\quad(+)$$
$$0$$

Quotient = $\boldsymbol{3x^2 + 5x - 2}$

Remainder = 0

(*ii*) We divide $x^4 - 3x^2 + 4x + 5$ by $x^2 - x + 1$ by the long division method as follows:

$$x^2 - x + 1 \,\big)\, x^4 \quad - 3x^2 + 4x + 5 \,\big(\, x^2 + x - 3$$
$$\underline{x^4 - x^3 + x^2}$$
$$(-)\quad(+)\quad(-)$$
$$x^3 - 4x^2 + 4x + 5$$
$$\underline{x^3 - x^2 + x}$$
$$(-)\quad(+)\quad(-)$$
$$-3x^2 + 3x + 5$$
$$\underline{-3x^2 + 3x - 3}$$
$$(+)\quad(-)\quad(+)$$
$$8$$

$\therefore$ **Quotient =** $\boldsymbol{x^2 + x - 3}$

Remainder = 8

(*iii*) We divide $6x^4 + 8x^3 + 17x^2 + 21x + 7$ by $3x^2 + 4x + 1$ by the long division method as follows:

$$3x^2 + 4x + 1 \,\big)\, 6x^4 + 8x^3 + 17x^2 + 21x + 7 \,\big(\, 2x^2 + 5$$
$$\underline{6x^4 + 8x^3 + 2x^2}$$
$$(-)\quad(-)\quad(-)$$
$$15x^2 + 21x + 7$$
$$\underline{15x^2 + 20x + 5}$$
$$(-)\quad(-)\quad(-)$$
$$x + 2$$

$\therefore$ **Quotient = $2x^2 + 5$**
Remainder = $x + 2$

3. (*i*) We divide $3x^3 - 2x^2 + 5x - 5$ by $3x + 1$ by the long division method as follows:

$$\begin{array}{r|l}
3x+1 & 3x^3 - 2x^2 + 5x - 5 \;(\, x^2 - x + 2 \\
 & 3x^3 + x^2 \\
 & - \quad - \\ \hline
 & -3x^2 + 5x - 5 \\
 & -3x^2 - x \\
 & + \quad + \\ \hline
 & 6x - 5 \\
 & 6x + 2 \\
 & - \quad - \\ \hline
 & -7
\end{array}$$

$\therefore$ **Quotient = $x^2 - x + 2$**
Remainder = -7

Verification of Division Algorithm:

We have Quotient = $x^2 - x + 2$
Divisor = $3x + 1$
Remainder = -7
Dividend = $3x^3 - 2x^2 + 5x - 5$

We have
Quotient × Divisior + Remainder
$= (x^2 - x + 2)(3x + 1) - 7$
$= 3x^3 - 3x^2 + 6x + x^2 - x + 2 - 7$
$= 3x^3 - 2x^2 + 5x - 5$
= Dividend

Thus, the division algorithm is verified.

(*ii*) We divide $x^4 - 9x^2 + 9$ by $x^2 - 3x$ by the long division method as follows:

$$\begin{array}{r|l}
x^2-3x & x^4 \qquad - 9x^2 + 9 \;(\, x^2 + 3x \\
 & x^4 - 3x^3 \\
 & - \quad + \\ \hline
 & 3x^3 - 9x^2 + 9 \\
 & 3x^3 - 9x^2 \\
 & - \quad + \\ \hline
 & 9
\end{array}$$

$\therefore$ **Quotient = $x^2 + 3x$**
Remainder = 9

Verification of division algorithm:

We have Quotient = $x^2 + 3x$
Divisor = $x^2 - 3x$
Remainder = 9
Dividend = $x^4 - 9x^2 + 9$.

We have
Quotient × Divisor + Remainder
$= (x^2 + 3x)(x^2 - 3x) + 9$
$= x^4 - 9x^2 + 9$ = Dividend.

Thus, the division algorithm is verified.

(*iii*) We divide $2x^4 - 9x^3 + 5x^2 + 3x - 8$ by $x^2 - 4x + 1$ by the long division method as follows:

$$\begin{array}{r|l}
x^2-4x+1 & 2x^4 - 9x^3 + 5x^2 + 3x - 8 \;(\, 2x^2 - x - 1 \\
 & 2x^4 - 8x^3 + 2x^2 \\
 & - \quad + \quad - \\ \hline
 & - x^3 + 3x^2 + 3x - 8 \\
 & - x^3 + 4x^2 - x \\
 & + \quad - \quad + \\ \hline
 & - x^2 + 4x - 8 \\
 & - x^2 + 4x - 1 \\
 & + \quad - \quad + \\ \hline
 & -7
\end{array}$$

$\therefore$ **Quotient = $2x^2 - x - 1$**
Remainder = -7

Verification of division algorithm:

We have
Quotient = $2x^2 - x - 1$
Remainder = -7
Divisor = $x^2 - 4x + 1$
Dividend = $2x^4 - 9x^3 + 5x^2 + 3x - 8$

Now, Quotient × Divisor + Remainder
$= (2x^2 - x - 1)(x^2 - 4x + 1) - 7$
$= 2x^4 - 8x^3 + 2x^2 - x^3 + 4x^2 - x - x^2 + 4x - 1 - 7$
$= 2x^4 - 9x^3 + 5x^2 + 3x - 8$
= Dividend

Hence, the division algorithm is verified.

4. (*i*)

$$\begin{array}{r|l}
4x-7 & 8x^2 - 26x + 21 \;(\, 2x - 3 \\
 & 8x^2 - 14x \\
 & - \quad + \\ \hline
 & -12x + 21 \\
 & -12x + 21 \\
 & + \quad - \\ \hline
 & 0
\end{array}$$

Remainder = 0
$\therefore$ **$4x - 7$ is a factor of $8x^2 - 26x + 21$.**

(*ii*)

$$\begin{array}{r|l}
2x^2+3x-5 & 8x^4 + 8x^3 - 12x^2 + 21x - 30 \;(\, 4x^2 - 2x + 7 \\
 & 8x^4 + 12x^3 - 20x^2 \\
 & - \quad - \quad + \\ \hline
 & -4x^3 + 8x^2 + 21x \\
 & -4x^3 - 6x^2 + 10x \\
 & + \quad + \quad - \\ \hline
 & 14x^2 + 11x - 30 \\
 & 14x^2 + 21x - 35 \\
 & - \quad - \quad + \\ \hline
 & -10x + 5
\end{array}$$

Remainder $\neq$ 0
$\therefore$ **$2x^2 + 3x - 5$ is not a factor of $8x^4 + 8x^3 - 12x^2 + 21x - 30$.**

5. Let $p(x) = 2x^3 + 9x^2 - x - b$ and let $g(x) = 2x + 3$.
On dividing $p(x)$ by $g(x)$, we get

$$
\begin{array}{r|l|l}
2x+3 & 2x^3 + 9x^2 - x - b & x^2 + 3x - 5 \\
 & \underline{\overset{-}{2x^3} \overset{-}{+ 3x^2}} & \\
 & 6x^2 - x & \\
 & \underline{\overset{-}{6x^2} \overset{-}{+ 9x}} & \\
 & -10x - b & \\
 & \underline{\overset{+}{-10x} \quad \overset{+}{-15}} & \\
 & -b + 15 &
\end{array}
$$

$p(x)$ is divisible by $g(x)$ if $\gamma(x) = 0$

$\Rightarrow \quad -b + 15 = 0$

$\Rightarrow \quad \mathbf{b = 15}$

6. (*i*) Let α, β and γ be the three zeroes of the polynomial $3x^3 + 10x^2 - 9x - 4$.

$\therefore \quad \alpha + \beta + \gamma = -\dfrac{10}{3}$

$\Rightarrow \quad 3\alpha + 3\beta + 3\gamma = -10 \quad \ldots(1)$

$\alpha\beta + \alpha\gamma + \beta\gamma = -\dfrac{9}{3} = -3 \quad \ldots(2)$

$\alpha\beta\gamma = \dfrac{4}{3}$

$\Rightarrow \quad 3\alpha\beta\gamma = 4 \quad \ldots(3)$

Given that α = 1.

$\therefore$ From (1) and (3), we have

$3\beta + 3\gamma = -13 \quad \ldots(4)$

and $\quad \beta\gamma = \dfrac{4}{3} \quad \ldots(5)$

From (4), $\quad \gamma = \dfrac{-13 - 3\beta}{3} \quad \ldots(6)$

$\therefore$ From (5),

$$\beta\left(\frac{-13 - 3\beta}{3}\right) = \frac{4}{3}$$

$\Rightarrow \quad 3\beta^2 + 13\beta + 4 = 0$

$\Rightarrow \quad 3\beta^2 + 12\beta + \beta + 4 = 0$

$\Rightarrow \quad 3\beta(\beta + 4) + 1(\beta + 4) = 0$

$\Rightarrow \quad (\beta + 4)(3\beta + 1) = 0$

$\therefore \quad$ Either $\beta + 4 = 0 \Rightarrow \beta = -4$

or $\quad 3\beta + 1 = 0 \Rightarrow \beta = -\dfrac{1}{3}$

When β = –4, then from (6),

$$\gamma = \frac{-13 + 12}{3} = -\frac{1}{3}$$

When $\beta = -\dfrac{1}{3}$, then from (6),

$$\gamma = \frac{-13 + 1}{3} = -4$$

Similarly, when $\gamma = -4$, $\beta = -\dfrac{1}{3}$

and $\quad \gamma = -\dfrac{1}{3}$, $\beta = -4$

In other words, the three zeroes are $\mathbf{1, -4, -\frac{1}{3}}$.

(*ii*) Since one of the zeroes is –2

$\therefore$ $x + 2$ is a factor of the polynomial $x^3 + 13x^2 + 32x + 20$

$\therefore$ The other factor can be obtained by dividing this polynomial by $x + 2$ as follows:

$$
\begin{array}{r|l|l}
x+2 & x^3 + 13x^2 + 32x + 20 & x^2 + 11x + 10 \\
 & \underline{\overset{-}{x^3} \overset{-}{+ 2x^2}} & \\
 & 11x^2 + 32x + 20 & \\
 & \underline{\overset{-}{11x^2} \overset{-}{+ 22x}} & \\
 & 10x + 20 & \\
 & \underline{\overset{-}{10x} \overset{-}{+ 20}} & \\
 & 0 &
\end{array}
$$

$\therefore$ The other factor is $x^2 + 11x + 10$

$= x^2 + 10x + x + 10$

$= x(x + 10) + 1(x + 10)$

$= (x + 10)(x + 1)$

$\therefore$ The other two zeroes are obtained from

$(x + 10)(x + 1) = 0$

$\therefore \quad$ Either $x + 10 = 0 \Rightarrow x = -10$

Or $\quad x + 1 = 0 \Rightarrow x = -1$

$\therefore$ The required zeroes of the given polynomial are **–2, –1 and –10**.

(*iii*) Since $\sqrt{2}$ is a zero of $6x^3 + \sqrt{2}x^2 - 10x - 4\sqrt{2}$

$\therefore$ $x - \sqrt{2}$ is a factor of this polynomial. The other factor can be obtained by dividing this polynomial by $x - \sqrt{2}$ as follows:

$$
\begin{array}{r|l|l}
x-\sqrt{2} & 6x^3 + \sqrt{2}x^2 - 10x - 4\sqrt{2} & 6x^2 + 7\sqrt{2}x + 4 \\
 & \underline{\overset{-}{6x^3} \overset{+}{- 6\sqrt{2}x^2}} & \\
 & 7\sqrt{2}x^2 - 10x - 4\sqrt{2} & \\
 & \underline{\overset{-}{7\sqrt{2}x^2} \overset{+}{- 14x}} & \\
 & 4x - 4\sqrt{2} & \\
 & \underline{\overset{-}{4x} \overset{+}{- 4\sqrt{2}}} & \\
 & 0 &
\end{array}
$$

$\therefore$ The other factor is

$6x^2 + 7\sqrt{2}x + 4 = 6x^2 + 4\sqrt{2}x + 3\sqrt{2}x + 4$

$= 2x\left(3x + 2\sqrt{2}\right) + \sqrt{2}\left(3x + 2\sqrt{2}\right)$

$= \left(3x + 2\sqrt{2}\right)\left(2x + 2\sqrt{2}\right)$

$\therefore$ Zeroes of this polynomial are given by

$\left(3x + 2\sqrt{2}\right)\left(2x + 2\sqrt{2}\right) = 0$

$\therefore \quad$ Either $3x + 2\sqrt{2} = 0 \Rightarrow x = -\dfrac{2\sqrt{2}}{3}$

or $\quad 2x + \sqrt{2} = 0 \Rightarrow x = -\dfrac{\sqrt{2}}{2}$

Hence, the required zeroes of the given polynomial are $\sqrt{2}$, $-\frac{2\sqrt{2}}{3}$ and $-\frac{\sqrt{2}}{2}$.

7. (*i*) On dividing $p(x)$ by $(x + 2)$, we get

$$x+2\overline{)\,x^3-9x^2-12x+20\,}\,(x^2-11x+10$$
$$\underline{\underset{-}{x^3}\ \underset{-}{+2x^2}}$$
$$-11x^2-12x$$
$$\underline{\underset{+}{-11x^2}\ \underset{+}{-22x}}$$
$$10x+20$$
$$\underline{\underset{-}{10x}\ \underset{-}{+20}}$$
$$0$$

$x^3 - 9x^2 - 12x + 20 = (x^2 - 11x + 10)\ (x + 2)$ [By division algorithm]

$= [x^2 - x - 10x + 10]\ (x + 2)$

$= [x(x - 1) - 10(x - 1)]\ (x + 2)$

$= (x - 1)\ (x - 10)\ (x + 2)$

The zeroes of $p(x)$ are given by $p(x) = 0$

$\Rightarrow$ $x - 1 = 0$

or $x - 10 = 0$

or $x + 2 = 0$

$\Rightarrow$ $x = 1$

or $x = 10$

or $x = -2$

Hence, the zeroes of $p(x)$ are **1, 10** and **−2.**

(*ii*) The other factor can be obtained by dividing the polynomial $x^3-3\sqrt{5}x^2+13x-3\sqrt{5}$ by $x-\sqrt{5}$ as follows:

$$x-\sqrt{5}\,\overline{)\,x^3-3\sqrt{5}x^2+13x-3\sqrt{5}\,}\,(x^2-2\sqrt{5}x+3$$
$$\underline{\underset{-}{x^3}\ \underset{+}{-\sqrt{5}x^2}}$$
$$-2\sqrt{5}x^2+13x-3\sqrt{5}$$
$$\underline{\underset{+}{-2\sqrt{5}x^2}\ \underset{-}{+10x}}$$
$$3x-3\sqrt{5}$$
$$\underline{\underset{-}{3x}\ \underset{+}{-3\sqrt{5}}}$$
$$0$$

$\therefore$ The other factor is $x^2-2\sqrt{5}x+3$

$= x^2-\left(\sqrt{5}+\sqrt{2}\right)x-\left(\sqrt{5}-\sqrt{2}\right)x+3$

$= x\left(x-\sqrt{5}-\sqrt{2}\right)-\left(\sqrt{5}-\sqrt{2}\right)\left(x-\frac{3}{\sqrt{5}-\sqrt{2}}\right)$

$= x\left(x-\sqrt{5}-\sqrt{2}\right)-\left(\sqrt{5}-\sqrt{2}\right)\left(x-\frac{3\left(\sqrt{5}+\sqrt{2}\right)}{3}\right)$

$= x\left(x-\sqrt{5}-\sqrt{2}\right)-\left(\sqrt{5}-\sqrt{2}\right)\left(x-\sqrt{5}-\sqrt{2}\right)$

$= \left(x-\sqrt{5}-\sqrt{2}\right)\left(x-\sqrt{5}+\sqrt{2}\right)$

$\therefore$ Zeroes of $x^2 - 2\sqrt{5}\,x + 3$ are given by

$\left(x-\sqrt{5}-\sqrt{2}\right)\left(x-\sqrt{5}+\sqrt{2}\right)=0$

$\therefore$ Either $x-\sqrt{5}-\sqrt{2}=0$ $\Rightarrow$ $x=\sqrt{5}+\sqrt{2}$

or $x-\sqrt{5}+\sqrt{2}=0$ $\Rightarrow$ $x=\sqrt{5}-\sqrt{2}$

Hence, the required three zeroes of the given polynomial are $\sqrt{5}$, $\sqrt{5}+\sqrt{2}$ and $\sqrt{5}-\sqrt{2}$.

(*iii*) Let $p(x) = 2x^4 + x^3 - 14x^2 + 5x + 6$

and $g(x) = x^2 + 2x + k$

On dividing $p(x)$ by $g(x)$, we get

$$x^2+2x+k\,\overline{)\,2x^4+x^3-14x^2+5x+6\,}\,(2x^2-3x-(8+2k)$$
$$\underline{\underset{-}{2x^4}\ \underset{-}{+4x^3}\ \underset{-}{+2kx^2}}$$
$$-3x^3-(14+2k)x^2+5x$$
$$\underline{\underset{+}{-3x^3}\ \underset{+}{-6x^2}\ \underset{+}{-3kx}}$$
$$-(8+2k)x^2+(3k+5)x+6$$
$$\underline{\underset{+}{-(8+2k)x^2}\ \underset{+}{-2(8+2k)x}\ \underset{+}{-k(8+2k)}}$$
$$(7k+21)x+6+8k+2k^2$$

Since $g(x)$ is a factor of $p(x)$,

$\therefore$ Remainder = 0, for all values of x.

$\Rightarrow$ $7k + 21 = 0$

and $2k^2 + 8k + 6 = 0$

$\Rightarrow$ $7k = -21$

and $2(k^2 + 4k + 3) = 0$

$\Rightarrow$ $k = -3$

and $k^2 + 4k + 3 = 0$

$\Rightarrow$ $(k + 1)\ (k + 3) = 0$

$\Rightarrow$ $k = -1$

or $k = -3$

Hence, $\boldsymbol{k = -3}$.

$\therefore$ $g(x) = x^2 + 2x + k = x^2 + 2x - 3$ [$\because$ $k = -3$]

The zeroes of $g(x)$ are given by $g(x) = 0$.

$g(x) = 0$

$\Rightarrow$ $x^2 + 2x - 3 = 0$

$\Rightarrow$ $x^2 - x + 3x - 3 = 0$

$\Rightarrow$ $x(x - 1) + 3(x - 1) = 0$

$\Rightarrow$ $(x - 1)\ (x + 3) = 0$

$\Rightarrow$ $x - 1 = 0$

or $x + 3 = 0$

$\Rightarrow$ $x = 1$

or $x = -3$

Hence, the zeroes of $x^2 + 2x - 3$ are **1** and **−3**.

$p(x) = [2x^2 - 3x - (8 + 2k)]\ (x^2 + 2x + k)$

$= [2x^2 - 3x - (8 + 2(-3))]\ [x^2 + 2x + (-3)]$ [$\because$ $k = -3$]

$= [2x^2 - 3x - (8 - 6)]\ (x^2 + 2x - 3)$

$= (2x^2 - 3x - 2)(x^2 + 2x - 3)$

The zeroes of $p(x)$ are given by $p(x) = 0$.

$\Rightarrow \quad (2x^2 - 3x - 2)(x^2 + 2x - 3) = 0$

$\Rightarrow \quad (2x^2 - 4x + x - 2)(x^2 - x + 3x - 3) = 0$

$\Rightarrow \quad [2x(x - 2) + 1(x - 2)]\,[x(x - 1) + 3(x - 1)] = 0$

$\Rightarrow \quad (x - 2)(2x + 1)(x - 1)(x + 3) = 0$

$\Rightarrow \quad x - 2 = 0$

or $\quad 2x + 1 = 0$

or $\quad x - 1 = 0$

or $\quad x + 3 = 0$

$\Rightarrow \quad x = 2$

or $\quad x = -\frac{1}{2}$

or $\quad x = 1$

or $\quad x = -3$

Hence, the roots of the given polynomial are

$2, -\frac{1}{2}$, 1 and –3.

8. We first divide $x^4 - 3x^3 - 6x^2 + kx - 16$ by $x^2 - 3x + 2$ as follows:

$$
\begin{array}{r}
x^2 - 3x + 2 \,\big)\, x^4 - 3x^3 - 6x^2 + kx - 16 \,\big(\, x^2 - 8 \\
x^4 - 3x^3 + 2x^2 \qquad\qquad \\
- \quad + \quad - \qquad\qquad\qquad \\
\hline
-8x^2 + kx - 15 \\
-8x^2 + 24x - 16 \\
+ \quad - \quad + \\
\hline
(k - 24)x
\end{array}
$$

Since the given polynomial is divisible by $x^2 - 3x + 2$, hence the remainder $(k - 24)x = 0$

$\Rightarrow$ ***k* = 24** which is the required value of k.

9. Let $\quad p(x) = 2x^3 + ax^2 + 2bx + 1$

and $\quad g(x) = x + 1$

On dividing $p(x)$ by $g(x)$, we get

$$
\begin{array}{r}
x + 1 \,\big)\, 2x^3 + ax^2 + 2bx + 1 \,\big(\, 2x^2 + (a - 2)x + (2b - a + 2) \\
2x^3 + 2x^2 \qquad\qquad\qquad \\
- \quad - \qquad\qquad\qquad \\
\hline
(a - 2)x^2 + 2bx \qquad \\
(a - 2)x^2 + (a - 2)x \qquad \\
- \qquad - \qquad \\
\hline
(2b - a + 2)x + 1 \\
(2b - a + 2)x + 2b - a + 2 \\
- \qquad - \quad + \quad - \\
\hline
-2b + a - 1
\end{array}
$$

Since, $g(x)$ is a factor of $p(x)$

$\therefore \quad$ Remainder $= 0$

$\Rightarrow \quad -2b + a - 1 = 0$

$\Rightarrow \quad a - 2b = 1 \quad \ldots (1)$

Also, $\quad 2a - 3b = 4 \quad$ [Given] $\ldots (2)$

Solving (1) and (2), we get

***a* = 5, *b* = 2**

10. We first divide $6x^4 + 8x^3 - 5x^2 + ax + b$ by $2x^2 - 5$ as follows:

$$
\begin{array}{r}
2x^2 - 5 \,\big)\, 6x^4 + 8x^3 - 5x^2 + ax + b \,\big(\, 3x^2 + 4x + 5 \\
6x^4 \qquad - 15x^2 \qquad\qquad \\
- \qquad\quad + \qquad\qquad\quad \\
\hline
8x^3 + 10x^2 + ax + b \\
8x^3 \qquad - 20x \quad \\
- \qquad\quad + \qquad \\
\hline
10x^2 + (a + 20)x + b \\
10x^2 \qquad\qquad - 25 \\
- \qquad\qquad\quad + \\
\hline
(a + 20)x + (b + 25)
\end{array}
$$

$\therefore$ Remainder $= (a + 20)x + (b + 25)$

Since the given polynomial is divisible by $2x^2 - 5$, hence the remainder $= 0$

$\Rightarrow \quad a + 20 = 0$ and $b + 25 = 0$

$\Rightarrow \quad a = -20$ and $b = 25$

$\therefore$ Required values of a and b are **–20** and **–25** respectively.

11. Let $\quad p(x) = x^4 + 2x^3 + 8x^2 + 12x + 18$

and let $\quad g(x) = x^2 + 5$

On dividing $p(x)$ by $g(x)$, we get

$$
\begin{array}{r}
x^2 + 5 \,\big)\, x^4 + 2x^3 + 8x^2 + 12x + 18 \,\big(\, x^2 + 2x + 3 \\
x^4 \qquad + 5x^2 \qquad\qquad\qquad \\
- \qquad\quad - \qquad\qquad\qquad \\
\hline
2x^3 + 3x^2 + 12x \quad \\
2x^3 \qquad\quad + 10x \quad \\
- \qquad\qquad - \quad \\
\hline
3x^2 + 2x + 18 \\
3x^2 \qquad + 15 \\
- \qquad\quad - \\
\hline
2x + 3
\end{array}
$$

Remainder $= 2x + 3 = px + q \quad$ [Given]

Hence, ***p* = 2 and *q* = 3.**

12. (*i*) It is given that $x + 2$ and $x - 2$ are the factors of $x^4 + x^3 - 34x^2 - 4x + 120$.

$\therefore$ This polynomial will be divisible by $(x - 2)(x + 2) = x^2 - 4$. We now divide this polynomial by $x^2 - 4$ to find the other factor as follows:

$$
\begin{array}{r}
x^2 - 4 \,\big)\, x^4 + x^3 - 34x^2 - 4x + 120 \,\big(\, x^2 + x - 30 \\
x^4 \qquad - 4x^2 \qquad\qquad\qquad \\
- \qquad\quad + \qquad\qquad\qquad \\
\hline
x^3 - 30x^2 - 4x + 120 \\
x^3 \qquad\quad - 4x \qquad \\
- \qquad\qquad + \qquad \\
\hline
-30x^2 \qquad + 120 \\
-30x^2 \qquad + 120 \\
+ \qquad\qquad - \\
\hline
0
\end{array}
$$

$\therefore$ The other factor is $x^2 + x - 30$

$$= x^2 + 6x - 5x - 30$$
$$= x(x + 6) - 5(x + 6)$$
$$= (x + 6)(x - 5)$$

$\therefore$ The remaining zeroes are given by

$(x + 6)(x - 5) = 0$

$\therefore$ Either $x + 6 = 0 \Rightarrow x = -6$

Or $x - 5 = 0 \Rightarrow x = 5$

Hence, the required four zeroes are **2, –2, 5** and **–6**.

(*ii*) Let $p(x) = x^4 + 2x^3 - 7x^2 - 8x + 12$

Since 2 and –2 are zeroes of $p(x)$,

$\therefore$ $(x - 2)$ and $(x + 2)$ are both factors of $p(x)$.

$\therefore$ $(x^2 - 4)$ is a factor of $p(x)$.

On dividing $p(x)$ by $x^2 - 4$, we get

$$x^2 - 4 \,\big)\, x^4 + 2x^3 - 7x^2 - 8x + 12 \,\big(\, x^2 + 2x - 3$$
$$\underline{\overset{-}{x^4} \quad \overset{+}{-4x^2}}$$
$$2x^3 - 3x^2 - 8x$$
$$\underline{\overset{-}{2x^3} \quad \overset{+}{-8x}}$$
$$-3x^2 + 12$$
$$\underline{\overset{+}{-3x^2} \quad \overset{-}{+12}}$$
$$0$$

$\therefore$ $p(x) = x^4 + 2x^3 - 7x^2 - 8x + 12$

$= (x^2 + 2x - 3)(x^2 + 4)$ [By division algorithm]

$= (x^2 - x + 3x - 3)(x - 2)(x + 2)$

$= [x(x - 1) + 3(x - 1)](x - 2)(x + 2)$

$= (x - 1)(x + 3)(x - 2)(x + 2)$

The zeroes of $p(x)$ are given by $p(x) = 0$

$\Rightarrow$ $(x - 1)(x + 3)(x - 2)(x + 2) = 0$

$\Rightarrow$ $x - 1 = 0$

or $x + 3 = 0$

or $x - 2 = 0$

or $x + 2 = 0$

$\Rightarrow$ $x = 1$

or $x = -3$

or $x = 2$

or $x = -2$

Hence, the zeroes of the given polynomial are **1, –3, 2, –2**.

(*iii*) Since two zeroes of the given polynomial are $\sqrt{2}$ and $2\sqrt{2}$, hence two factors of this polynomial are $x - \sqrt{2}$ and $x - 2\sqrt{2}$.

$$\therefore \left(x - \sqrt{2}\right)\left(x - 2\sqrt{2}\right) = x^2 - \left(2\sqrt{2} + \sqrt{2}\right)x + 4$$
$$= x^2 - 3\sqrt{2}x + 4$$

is also a factor of this polynomial. The other factor can be determined by dividing this polynomial by $x^2 - 3\sqrt{2}x + 4$ as follows:

$$x^2 - 3\sqrt{2}x + 4 \,\big)\, x^4 - 3\sqrt{2}x^3 + 3x^2 + 3\sqrt{2}x - 4 \,\big(\, x^2 - 1$$
$$\underline{\overset{-}{x^4} \overset{+}{- 3\sqrt{2}x^3} \overset{-}{+ 4x^2}}$$
$$-x^2 + 3\sqrt{2}x - 4$$
$$\underline{\overset{+}{-x^2} \overset{-}{+ 3\sqrt{2}x} \overset{+}{- 4}}$$
$$0$$

$\therefore$ The other factor is $x^2 - 1 = (x + 1)(x - 1)$

$\therefore$ Other two zeroes are given by

$(x + 1)(x - 1) = 0$

$\therefore$ Either $x + 1 = 0 \Rightarrow x = -1$

or $x - 1 = 0 \Rightarrow x = 1$

$\therefore$ Required two other zeroes are **1** and **–1**.

(*iv*) Let $p(x) = x^4 + 5x^3 - 2x^2 - 40x - 48$

Since $2\sqrt{2}$ and $-2\sqrt{2}$ are zeroes of $p(x)$,

$\therefore$ $(x - 2\sqrt{2})$ and $(x + 2\sqrt{2})$ are factors of $p(x)$.

$\therefore$ $[x^2 - (2\sqrt{2})^2]$ is a factor of $p(x)$.

i.e. $x^2 - 8$ is a factor of $p(x)$.

On dividing $x^4 + 5x^3 - 2x^2 - 40x - 48$ by $x^2 - 8$, we get

$$x^2 - 8 \,\big)\, x^4 + 5x^3 - 2x^2 - 40x - 48 \,\big(\, x^2 + 5x + 6$$
$$\underline{\overset{-}{x^4} \quad \overset{+}{-8x^2}}$$
$$5x^3 + 6x^2 - 40x - 48$$
$$\underline{\overset{-}{5x^3} \quad \overset{+}{-40x}}$$
$$6x^2 - 48$$
$$\underline{\overset{-}{6x^2} \quad \overset{+}{-48}}$$
$$0$$

$\therefore$ $p(x) = x^4 + 5x^3 - 2x^2 - 40x - 48$

$= (x^2 + 5x + 6)(x^2 - 8)$ [By division algorithm]

$= [x^2 + 2x + 3x + 6](x + \sqrt{8})(x - \sqrt{8})$

$= [x(x + 2) + 3(x + 2)](x + 2\sqrt{2})(x - 2\sqrt{2})$

$= (x + 2)(x + 3)(x + 2\sqrt{2})(x - 2\sqrt{2})$

The zeroes of $p(x)$ are given by $p(x) = 0$

$\Rightarrow (x + 2)(x + 3)(x + 2\sqrt{2})(x - 2\sqrt{2}) = 0$

$\Rightarrow$ $(x + 2) = 0$

or $(x + 3) = 0$

or $(x + 2\sqrt{2}) = 0$

or $(x - 2\sqrt{2}) = 0$

$\Rightarrow$ $x = -2$

or $x = -3$

or $x = -2\sqrt{2}$

or $x = 2\sqrt{2}$

Hence, the zeroes of the given polynomial are $\mathbf{-2, -3, -2\sqrt{2}}$ and $\mathbf{2\sqrt{2}}$.

(*v*) Let $p(x) = 2x^4 - 6x^3 + 3x^2 + 3x - 2$

Since $\frac{1}{\sqrt{2}}$ and $\frac{-1}{\sqrt{2}}$ are zeroes of $p(x)$,

$\therefore \quad \left(x - \frac{1}{\sqrt{2}}\right)\left(x + \frac{1}{\sqrt{2}}\right)$ are the factors of $p(x)$.

$\therefore \quad x^2 - \left(\frac{1}{\sqrt{2}}\right)^2$ is a factor of $p(x)$.

i.e. $\left(x^2 - \frac{1}{2}\right)$ is a factor of $p(x)$.

On dividing $2x^4 - 6x^3 + 3x^2 + 3x - 2$ by $\left(x^2 - \frac{1}{2}\right)$, we get

$$
\begin{array}{r|l}
x^2 - \frac{1}{2} & 2x^4 - 6x^3 + 3x^2 + 3x - 2 \;\big(\, 2x^2 - 6x + 4 \\
 & \underline{\overset{-}{2x^4} \qquad\quad \overset{+}{-}\; x^2} \\
 & \quad -6x^3 + 4x^2 + 3x \\
 & \quad \underline{\overset{+}{-6x^3} \qquad\quad \overset{-}{+}\; 3x} \\
 & \qquad\quad 4x^2 \qquad - 2 \\
 & \qquad\quad \underline{\overset{-}{4x^2} \qquad \overset{+}{-}\; 2} \\
 & \qquad\qquad\qquad 0
\end{array}
$$

The other zeroes of $p(x)$ are given by

$(2x^2 - 6x + 4) = 0$

$\Rightarrow \quad 2(x - 1)(x - 2) = 0$

$\Rightarrow \quad x - 1 = 0$

or $\quad x - 2 = 0$

$\Rightarrow \quad x = 1$

or $\quad x = 2$

Hence, the other zeroes of the given polynomial are **1** and **2**.

13. (*i*) Since two zeroes of the given polynomial are $\sqrt{3}$ and $-\sqrt{3}$

$\therefore \left(x - \sqrt{3}\right)$ and $\left(x + \sqrt{3}\right)$ are two factors of this polynomial.

$\therefore \left(x - \sqrt{3}\right)\left(x + \sqrt{3}\right) = x^2 - 3$ is also a factor of this polynomial. The remaining factor can be obtained by dividing this polynomial by $x^2 - 3$ as follows:

$$
\begin{array}{r|l}
x^2 - 3 & x^4 - 7x^2 + 12 \;\big(\, x^2 - 4 \\
 & \underline{\overset{-}{x^4} \overset{+}{-} 3x^2} \\
 & \quad -4x^2 + 12 \\
 & \quad \underline{\overset{+}{-4x^2} \overset{-}{+} 12} \\
 & \qquad\quad 0
\end{array}
$$

$\therefore$ The other factor is $x^2 - 4 = (x + 2)(x - 2)$

$\therefore$ Other two zeroes are given by

$(x + 2)(x - 2) = 0$

$\therefore \quad$ Either $x + 2 = 0 \Rightarrow x = -2$

or $\quad x - 2 = 0 \Rightarrow x = 2$

$\therefore$ Required two other zeroes are **2** and **−2**.

(*ii*) As in problem 13(*i*), $\left(x + \sqrt{3}\right)\left(x - \sqrt{3}\right) = x^2 - 3$ is a factor of this polynomial $x^4 + x^3 - 23x^2 - 3x + 60$. The remaining factor can be obtained by dividing this polynomial by $x^2 - 3$ as follows:

$$
\begin{array}{r|l}
x^2 - 3 & x^4 + x^3 - 23x^2 - 3x + 60 \;\big(\, x^2 + x - 20 \\
 & \underline{\overset{-}{x^4} \qquad \overset{+}{-}\; 3x^2} \\
 & \quad x^3 - 20x^2 - 3x + 60 \\
 & \quad \underline{\overset{-}{x^3} \qquad\quad \overset{+}{-}\; 3x} \\
 & \qquad -20x^2 \qquad + 60 \\
 & \qquad \underline{\overset{+}{-20x^2} \qquad \overset{-}{+}\; 60} \\
 & \qquad\qquad\quad 0
\end{array}
$$

$\therefore$ The other factor is $x^2 + x - 20$

$= x^2 + 5x - 4x - 20$

$= x(x + 5) - 4(x + 5)$

$= (x + 5)(x - 4)$

$\therefore$ Other two zeroes are given by

$(x + 5)(x - 4) = 0$

$\therefore \quad$ Either $x + 5 = 0 \Rightarrow x = -5$

or $\quad x - 4 = 0 \Rightarrow x = 4$

$\therefore$ Required four zeroes are $\mathbf{\sqrt{3}}$, $\mathbf{-\sqrt{3}}$, **4** and **−5**.

(*iii*) Let $\quad p(x) = x^4 - 3x^3 - x^2 + 9x - 6$

Since $\sqrt{3}$ and $-\sqrt{3}$ are zeroes of $p(x)$,

$\therefore \quad (x - \sqrt{3})$ and $(x + \sqrt{3})$ are factors of $p(x)$.

$\therefore \quad x^2 - 3$ is a factor of $p(x)$.

On dividing $p(x)$ by $x^2 - 3$, we get

$$
\begin{array}{r|l}
x^2 - 3 & x^4 - 3x^3 - x^2 + 9x - 6 \;\big(\, x^2 - 3x + 2 \\
 & \underline{\overset{-}{x^4} \qquad \overset{+}{-}\; 3x^2} \\
 & \quad -3x^3 + 2x^2 + 9x \\
 & \quad \underline{\overset{+}{-3x^3} \qquad \overset{-}{+}\; 9x} \\
 & \qquad\quad 2x^2 \qquad - 6 \\
 & \qquad\quad \underline{\overset{-}{2x^2} \qquad \overset{+}{-}\; 6} \\
 & \qquad\qquad\quad 0
\end{array}
$$

$p(x) = x^4 - 3x^3 - x^2 + 9x - 6$

$= (x^2 - 3x + 2)(x^2 - 3)$ [By division algorithm]

$= (x^2 - x - 2x + 2)(x + \sqrt{3})(x - \sqrt{3})$

$= [x(x - 1) - 2(x - 1)](x + \sqrt{3})(x - \sqrt{3})$

$= (x - 1)(x - 2)(x + \sqrt{3})(x - \sqrt{3})$

The zeroes of $p(x)$ are given by $p(x) = 0$

$p(x) = 0$

$\Rightarrow \quad (x - 1)(x - 2)(x + \sqrt{3})(x - \sqrt{3}) = 0$

$$\Rightarrow \quad x - 1 = 0$$
$$\text{or} \quad x - 2 = 0$$
$$\text{or} \quad x + \sqrt{3} = 0$$
$$\text{or} \quad x - \sqrt{3} = 0$$
$$\Rightarrow \quad x = 1$$
$$\text{or} \quad x = 2$$
$$\text{or} \quad x = -\sqrt{3}$$
$$\text{or} \quad x = \sqrt{3}$$

Hence, the zeroes of the given polynomial are **1, 2,** $\mathbf{-\sqrt{3}}$ and $\mathbf{\sqrt{3}}$.

14. (*i*) Since $\sqrt{5}$ and $-\sqrt{5}$ are given to be two zeroes of the given polynomial.

$\therefore$ $x - \sqrt{5}$ and $x + \sqrt{5}$ will be two factors of this polynomial.

$\therefore$ $\left(x - \sqrt{5}\right)\left(x + \sqrt{5}\right) = x^2 - 5$ will be a factor of this polynomial. The other factor can be obtained by dividing this polynomial by $x^2 - 5$ as follows:

$$
\begin{array}{r|l}
x^2 - 5 & x^4 + 4x^3 - 2x^2 - 20x - 15 \;\big(\; x^2 + 4x + 3 \\
 & \underline{\overset{-}{x^4} \qquad \overset{+}{-\,5x^2}} \\
 & \quad 4x^3 + 3x^2 - 20x - 15 \\
 & \quad \underline{\overset{-}{4x^3} \qquad \overset{+}{-\,20x}} \\
 & \qquad 3x^2 \qquad - 15 \\
 & \qquad \underline{\overset{-}{3x^2} \qquad \overset{+}{-\,15}} \\
 & \qquad\qquad 0
\end{array}
$$

$\therefore$ The remaining factor is $x^2 + 4x + 3$

$$= x^2 + 3x + x + 3$$
$$= x(x + 3) + 1\,(x + 3)$$
$$= (x + 3)\,(x + 1)$$

Other zeroes are given by

$$(x + 3)\,(x + 1) = 0$$

$\therefore$ Either $x + 3 = 0 \Rightarrow x = -3$

or $x + 1 = 0 \Rightarrow x = -1.$

$\therefore$ Required two other zeroes are $\mathbf{-3}$ and $\mathbf{-1}$.

(*ii*) Since $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$ are given to be two zeroes of the given polynomial.

$\therefore$ $x - \sqrt{\frac{5}{3}}$ and $x + \sqrt{\frac{5}{3}}$ will be factors of this polynomial.

$\therefore$ $\left(x + \sqrt{\frac{5}{3}}\right)\left(x - \sqrt{\frac{5}{3}}\right) = x^2 - \frac{5}{3}$ will be a factor of this polynomial. The other factor can be obtained by dividing this polynomial by $x^2 - \frac{5}{3}$ as follows:

$$
\begin{array}{r|l}
x^2 - \frac{5}{3} & 3x^4 - 15x^3 + 13x^2 + 25x - 30 \;\big(\; 3x^2 - 15x + 18 \\
 & \underline{\overset{-}{3x^4} \qquad \overset{+}{-\,5x^2}} \\
 & \quad -15x^3 + 18x^2 + 25x - 30 \\
 & \quad \underline{\overset{+}{-15x^3} \qquad \overset{-}{+\,25x}} \\
 & \qquad 18x^2 \qquad - 30 \\
 & \qquad \underline{\overset{-}{18x^2} \qquad \overset{+}{-\,30}} \\
 & \qquad\qquad 0
\end{array}
$$

$\therefore$ The other factor is $3x^2 - 15x + 18$

$$= 3(x^2 - 5x + 6)$$
$$= 3(x^2 - 3x - 2x + 6)$$
$$= 3[x(x - 3) - 2(x - 3)]$$
$$= 3(x - 3)(x - 2)$$

$\therefore$ Remaining zeroes are given by

$$3(x - 3)\,(x - 2) = 0$$

$\therefore$ Either $x - 3 = 0 \Rightarrow x = 3$

or $x - 2 = 0 \Rightarrow x = 2$

$\therefore$ Required four zeroes are given by $\mathbf{\sqrt{\frac{5}{3}}}$, $\mathbf{-\sqrt{\frac{5}{3}}}$, **2** and **3**.

(*iii*) $f(x) = 5x^4 - 5x^3 - 33x^2 + 3x + 18$

Since $\sqrt{\frac{3}{5}}$ and $-\sqrt{\frac{3}{5}}$ are zeroes of $f(x)$,

$\therefore$ $\left(x - \sqrt{\frac{3}{5}}\right)\left(x + \sqrt{\frac{3}{5}}\right)$ are factors of $f(x)$.

$\therefore$ $\left(x^2 - \frac{3}{5}\right)$ is a factor of $f(x)$.

On dividing $5x^4 - 5x^3 - 33x^2 + 3x + 18$ by $\left(x^2 - \frac{3}{5}\right)$, we get

$$
\begin{array}{r|l}
x^2 - \frac{3}{5} & 5x^4 - 5x^3 - 33x^2 + 3x + 18 \;\big(\; 5x^2 - 5x - 30 \\
 & \underline{\overset{-}{5x^4} \qquad \overset{+}{-\,3x^2}} \\
 & \quad -5x^3 - 30x^2 + 3x \\
 & \quad \underline{\overset{+}{-5x^3} \qquad \overset{-}{+\,3x}} \\
 & \qquad -30x^2 \qquad + 18 \\
 & \qquad \underline{\overset{+}{-30x^2} \qquad \overset{-}{+\,18}} \\
 & \qquad\qquad 0
\end{array}
$$

$$\therefore \quad p(x) = (5x^2 - 5x - 30)\left(x^2 - \frac{3}{5}\right)$$

The other zeroes of $p(x)$ are given by

$$5x^2 - 5x - 30 = 0$$
$$\Rightarrow \quad 5(x^2 - x - 6) = 0$$

$\Rightarrow \quad 5[x^2 - 3x + 2x - 6] = 0$

$\Rightarrow \quad 5[x(x - 3) + 2(x - 3)] = 0$

$\Rightarrow \quad 5(x - 3)(x + 2) = 0$

$\Rightarrow \quad x - 3 = 0$

or $\quad x + 2 = 0$

$\Rightarrow \quad x = 3$

or $\quad x = -2$

Hence, the other zeroes of the given polynomial are **3** and **−2**.

15. Let $p(x) = x^4 - 3x^3 - 5x^2 + 21x - 14$

Since $\sqrt{7}$ and $-\sqrt{7}$ are zeroes of $p(x)$,

$\therefore \quad (x - \sqrt{7})(x + \sqrt{7})$ are factors of $p(x)$.

$\therefore \quad (x^2 - 7)$ is a factor of $p(x)$.

On dividing $x^4 - 3x^3 - 5x^2 + 21x - 14$ by $x^2 - 7$, we get

$$\begin{array}{r|l} x^2 - 7 & x^4 - 3x^3 - 5x^2 + 21x - 14 \quad (x^2 - 3x + 2 \\ & \underline{\overset{-}{x^4} \qquad \overset{+}{-7x^2}} \\ & -3x^3 + 2x^2 + 21x \\ & \underline{\overset{+}{-3x^3} \qquad \overset{-}{+21x}} \\ & 2x^2 - 14 \\ & \underline{\overset{-}{2x^2} \quad \overset{+}{-14}} \\ & 0 \end{array}$$

$\therefore \quad p(x) = x^4 - 3x^3 - 5x^2 + 21x - 14$

$= (x^2 - 3x + 2)(x^2 - 7)$ [By division algorithm]

$= [x^2 - x - 2x + 2](x + \sqrt{7})(x - \sqrt{7})$

$= [x(x - 1) - 2(x - 1)](x + \sqrt{7})(x - \sqrt{7})$

$= (x - 1)(x - 2)(x + \sqrt{7})(x - \sqrt{7})$

The zeroes of $p(x)$ are given by $p(x) = 0$

$\therefore \quad p(x) = 0$

$\Rightarrow \quad (x - 1)(x - 2)(x + \sqrt{7})(x - \sqrt{7}) = 0$

$\Rightarrow \quad x - 1 = 0$

or $\quad x - 2 = 0$

or $\quad x + \sqrt{7} = 0$

or $\quad x - \sqrt{7} = 0$

$\Rightarrow \quad x = 1$

or $\quad x = 2$

or $\quad x = -\sqrt{7}$

or $\quad x = \sqrt{7}$

Hence, the zeroes of the given polynomial are **1**, **2**, $-\sqrt{7}$ and $\sqrt{7}$.

16. Since two zeroes of the given polynomial are $2 + \sqrt{3}$ and $2 - \sqrt{3}$, $\therefore$ Two factors of this polynomial are $x - (2 + \sqrt{3})$ and $x - (2 - \sqrt{3})$ and so

$\{x - (2 + \sqrt{3})\}\{x - (2 - \sqrt{3})\}$

$= x^2 - x(2 - \sqrt{3} + 2 + \sqrt{3}) + (2 + \sqrt{3})(2 - \sqrt{3})$

$= x^2 - 4x + 1$ will be a factor of this polynomial.

$\therefore$ The other factor can be obtained by dividing the given polynomial by $x^2 - 4x + 1$ as follows:

$$\begin{array}{r|l} x^2 - 4x + 1 & 2x^4 - 9x^3 + 5x^2 + 3x - 1 \quad (2x^2 - x - 1 \\ & \underline{\overset{-}{2x^4} \overset{+}{- 8x^3} \overset{-}{+ 2x^2}} \\ & - x^3 + 3x^2 + 3x - 1 \\ & \underline{\overset{+}{-x^3} \overset{-}{+ 4x^2} \overset{+}{- x}} \\ & - x^2 + 4x - 1 \\ & \underline{\overset{+}{-x^2} \overset{-}{+ 4x} \overset{+}{- 1}} \\ & 0 \end{array}$$

$\therefore$ The other factor is $2x^2 - x - 1 = 2x^2 + x - 2x - 1$

$= x(2x + 1) - 1(2x + 1)$

$= (2x + 1)(x - 1)$

$\therefore$ Remaining zeroes are given by

$(2x + 1)(x - 1) = 0$

$\therefore \quad$ Either $2x + 1 = 0 \Rightarrow x = -\dfrac{1}{2}$

$x - 1 = 0 \Rightarrow x = 1$

$\therefore$ Required four zeroes of the given polynomial are

$\mathbf{2 + \sqrt{3}}$, $\mathbf{2 - \sqrt{3}}$, $-\mathbf{\dfrac{1}{2}}$ and **1**.

For Standard Level

17. (*i*)

$$\begin{array}{r|l} 3x - 2 & 3x^3 + 10x^2 - 14x + 9 \quad (x^2 + 4x - 2 \\ & \underline{\overset{-}{3x^3} \overset{+}{- 2x^2}} \\ & 12x^2 - 14x \\ & \underline{\overset{-}{12x^2} \overset{+}{- 8x}} \\ & - 6x + 9 \\ & \underline{\overset{+}{-6x} \overset{-}{+ 4}} \\ & 5 \end{array}$$

Thus, **5** should be subtracted from $3x^3 + 10x^2 - 14x + 9$, so that the resulting polynomial is exactly divisible by $3x - 2$.

(*ii*)

$$\begin{array}{r|l} 4x^2 - 3x + 2 & 8x^4 + 14x^3 + x^2 + 7x + 8 \quad (2x^2 + 5x + 3 \\ & \underline{\overset{-}{8x^4} \overset{+}{- 6x^3} \overset{-}{+ 4x^2}} \\ & 20x^3 - 3x^2 + 7x \\ & \underline{\overset{-}{20x^3} \overset{+}{- 15x^2} \overset{-}{+ 10x}} \\ & 12x^2 - 3x + 8 \\ & \underline{\overset{-}{12x^2} \overset{+}{- 9x} \overset{-}{+ 6}} \\ & 6x + 2 \end{array}$$

Thus, $\mathbf{6x + 2}$ should be subtracted from $8x^4 + 14x^3 + x^2 + 7x + 8$, so that the resulting polynomial is exactly divisible by $4x^2 - 3x + 2$.

18. By division algorithm, we have

$$p(x) = g(x) \times q(x) + \gamma(x)$$

$\Rightarrow \quad p(x) - \gamma(x) = g(x) \times q(x)$

$\Rightarrow \quad p(x) + [-\gamma(x)] = g(x) \times q(x)$

Since RHS is divisible by $g(x)$,

$\therefore$ LHS is also divisible by $g(x)$.

Thus, if $-\gamma(x)$ is added to $p(x)$, then the resulting polynomial becomes divisible by $g(x)$.

Let $\quad p(x) = 11y^3 + 5y^4 + 6y^5 - 3y^2 + y + 5$

$\qquad = 6y^5 + 5y^4 + 11y^3 - 3y^2 + y + 5$

and $\quad g(x) = 3y^2 - 2y + 4$

On dividing $p(x)$ by $g(x)$, we get

$$\begin{array}{r|l}
3y^2 - 2y + 4 & 6y^5 + 5y^4 + 11y^3 - 3y^2 + y + 5 \quad (2y^3 + 3y^2 + 3y - 3 \\
& \underset{-}{6y^5} \underset{+}{- 4y^4} \underset{-}{+ 8y^3} \\ \hline
& 9y^4 + 3y^3 - 3y^2 \\
& \underset{-}{9y^4} \underset{+}{- 6y^3} \underset{-}{+ 12y^2} \\ \hline
& 9y^3 - 15y^2 + y \\
& \underset{-}{9y^3} \underset{+}{- 6y^2} \underset{-}{+ 12y} \\ \hline
& -9y^2 - 11y + 5 \\
& \underset{+}{-9y^2} \underset{-}{+ 6y} \underset{+}{- 12} \\ \hline
& -17y + 17
\end{array}$$

Hence, $\mathbf{-\gamma(x) = 17y - 17}$ should be added to the given polynomial so that the resulting polynomial is exactly divisible by $3y^2 - 2y + 4$.

19. By division algorithm, we have

Dividend = Quotient × Divisor + Remainder

$\therefore$ Required polynomial $= (x^2 - 2x - 3) \times (x^2 - 5) + 0$

$= x^4 - 2x^3 - 3x^2 - 5x^2 + 10x + 15$

$= x^4 - 2x^3 - 8x^2 + 10x + 15$

On dividing $x^4 - 2x^3 - 8x^2 + 10x + 15$ by $x^2 - 5$, we get

$$\begin{array}{r|l}
x^2 - 5 & x^4 - 2x^3 - 8x^2 + 10x + 15 \quad (x^2 - 2x - 3 \\
& \underset{-}{x^4} \quad \underset{+}{- 5x^2} \\ \hline
& -2x^3 - 3x^2 + 10x \\
& \underset{+}{-2x^3} \quad \underset{-}{+ 10x} \\ \hline
& -3x^2 + 15 \\
& \underset{+}{-3x^2} \quad \underset{-}{+ 15} \\ \hline
& 0
\end{array}$$

$\therefore \quad p(x) = x^4 - 2x^3 - 8x^2 + 10x + 15$

$= (x^2 - 2x - 3)(x^2 - 5)$ [By division algorithm]

$= [x^2 + x - 3x - 3](x + \sqrt{5})(x - \sqrt{5})$

$= [x(x + 1) - 3(x + 1)](x + \sqrt{5})(x - \sqrt{5})$

$= (x + 1)(x - 3)](x + \sqrt{5})(x - \sqrt{5})$

The zeroes of $p(x)$ are given by $p(x) = 0$

$\Rightarrow \quad (x + 1)(x - 3)(x + \sqrt{5})(x - \sqrt{5}) = 0$

$\Rightarrow \quad x + 1 = 0$

or $\quad x - 3 = 0$

or $\quad (x + \sqrt{5}) = 0$

or $\quad (x - \sqrt{5}) = 0$

$\Rightarrow \quad x = -1$

or $\quad x = 3$

or $\quad x = -\sqrt{5}$

or $\quad x = \sqrt{5}$

Hence, the zeroes of the given polynomial are $\mathbf{-1, 3, -\sqrt{5}}$ and $\mathbf{\sqrt{5}}$.

20. (*i*) Dividend $= 10t^4 - 6t^3 - 40t^2 + 41t - 5$

Divisor $= g(t)$

Quotient $= 5t - 3$

and remainder $= 2t + 4$

According to division algorithm, we have

Quotient × Divisor + Remainder = Dividend

$\Rightarrow \quad (5t - 3) \times g(t) + 2t + 4$

$= \quad 10t^4 - 6t^3 - 40t^2 + 41t - 5$

$$\Rightarrow \quad g(t) = \frac{10t^4 - 6t^3 - 40t^2 + 41t - 5 - 2t - 4}{5t - 3}$$

$$\Rightarrow \quad g(t) = \frac{10t^4 - 6t^3 - 40t^2 + 39t - 9}{5t - 3}$$

On dividing $10t^4 - 6t^3 - 40t^2 + 39t - 9$ by $5t - 3$, we get

$$\begin{array}{r|l}
5t - 3 & 10t^4 - 6t^3 - 40t^2 + 39t - 9 \quad (2t^3 - 8t + 3 \\
& \underset{-}{10t^4} \underset{+}{- 6t^3} \\ \hline
& -40t^2 + 39t - 9 \\
& \underset{+}{-40t^2} \underset{-}{+ 24t} \\ \hline
& 15t - 9 \\
& \underset{-}{15t} \underset{+}{- 9} \\ \hline
& 0
\end{array}$$

Quotient $= 2t^3 - 8t + 3$

Hence, $\mathbf{g(t) = 2t^3 - 8t + 3}$.

(*ii*) We have

Divisor $= g(x)$,

Quotient $= q(x) = x^2 - 3x - 5$

Remainder $= r(x) = -5x + 8$

Dividend $= d(x) = 4x^4 - 5x^3 - 39x^2 - 46x - 2$

$\therefore$ By division algorithm, we have

$d(x) = q(x)\, g(x) + r(x)$

$\Rightarrow \quad q(x)g(x) = d(x) - r(x)$

$= 4x^4 - 5x^3 - 39x^2 - 46x - 2 + 5x - 8$

$\Rightarrow \quad (x^2 - 3x - 5)\, g(x) = 4x^4 - 5x^3 - 39x^2 - 41x - 10$

$$g(x) = \frac{4x^4 - 5x^3 - 39x^2 - 41x - 10}{x^2 - 3x - 5} \qquad \ldots(1)$$

We now apply long division to divide $4x^4 - 5x^3 - 39x^2 - 41x - 10$ by $x^2 - 3x - 5$ as follows:

$$x^2 - 3x - 5 \,\big)\, 4x^4 - 5x^3 - 39x^2 - 41x - 10 \,\big(\, 4x^2 + 7x + 2$$

$$\begin{array}{r} 4x^4 - 12x^3 - 20x^2 \\ - \quad + \quad + \\ \hline 7x^3 - 19x^2 - 41x - 10 \\ 7x^3 - 21x^2 - 35x \\ - \quad + \quad + \\ \hline 2x^2 - 6x - 10 \\ 2x^2 - 6x - 10 \\ - \quad + \quad + \\ \hline 0 \end{array}$$

Hence, from (1), $\boldsymbol{g(x) = 4x^2 + 7x + 2}$.

(*iii*) We have divisor = $p(x)$

dividend = $f(x) = 3x^3 - 2x^2 + 5x - 5$

quotient = $x^2 - x + 2$

remainder = –7

∴ By division algorithm, we have

$$f(x) = (x^2 - x + 2)p(x) - 7$$

$\Rightarrow$ $3x^3 - 2x^2 + 5x - 5 = (x^2 - x + 2)p(x) - 7$

$\Rightarrow$ $$p(x) = \frac{3x^3 - 2x^2 + 5x - 5 + 7}{x^2 - x + 2}$$

$$= \frac{3x^3 - 2x^2 + 5x + 2}{x^2 - x + 2}$$

We now divide $3x^3 - 2x^2 + 5x + 2$ by $x^2 - x + 3$ by the long division method as follows:

$$x^2 - x + 2 \,\big)\, 3x^3 - 2x^2 + 5x + 2 \,\big(\, 3x + 1$$

$$\begin{array}{r} 3x^3 - 3x^2 + 6x \\ - \quad + \quad - \\ \hline x^2 - x + 2 \\ x^2 - x + 2 \\ - \quad + \quad - \\ \hline 0 \end{array}$$

∴ $\boldsymbol{p(x) = 3x + 1}$ which is the required value of $p(x)$.

CHECK YOUR UNDERSTANDING

MULTIPLE-CHOICE QUESTIONS

For Basic and Standard Levels

1. (*c*) $\boldsymbol{\sqrt{2}x^3 + \sqrt{3}x^2 + \sqrt{5}x - 3}$

   $\sqrt{2}x^3 + \sqrt{3}x^2 + \sqrt{5}x - 3$ is a polynomial.

   ∵ Powers of x in each term is non-negative integer and the coefficients are real numbers which is not so in the other choices.

2. (*c*) **2**

   Since the graph cuts/touches the x-axis at two points, it has **2** zeroes.

3. (*b*) $\boldsymbol{f(\alpha) = 0}$

   A real number k is called a zero of the polynomial $f(x)$, if $p(k) = 0$.

   ∴ α is zero of $f(x)$ when $f(\alpha) = 0$.

4. (*b*) **–3, –4**

   $$x^2 + 7x + 12 = (x + 3)(x + 4)$$

   The zeroes of $x^2 + 7x + 12$ are given by $x^2 + 7x + 12 = 0$

   $\Rightarrow$ $(x + 3)(x + 4) = 0$

   $\Rightarrow$ $x + 3 = 0$

   or $x + 4 = 0$

   $\Rightarrow$ $x = -3, -4$

5. (*b*) **44**

   $$p(3) = 3^2 + 5(3) + 2 = 9 + 15 + 2 = 26$$

   $$p(2) = 2^2 + 5(2) + 2 = 4 + 10 + 2 = 16$$

   $$p(0) = 0^2 + 5(0) + 2 = 2$$

   ∴ $p(3) + p(2) + p(0) = 26 + 16 + 2 = 44$

6. (*c*) **both negative**

   Let, α, β, be the of the polynomial $x^2 + 43x + 222$.

   $$\text{Sum of zeroes} = \alpha + \beta = \frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2} = \frac{-(43)}{1}$$

   ∴ Sum is negative.

   $\Rightarrow$ one of the zeroes is negative.

   $$\text{Product of zeroes} = \alpha\beta = \frac{\text{Constant term}}{\text{Coefficient of } x^2} = \frac{222}{1}$$

   which is positive.

   $\Rightarrow$ The other zero is also negative.

   [∵ (–ve) × (–ve) = (+ ve)]

7. (*c*) $\boldsymbol{x^2 - 10x + 23}$

   S = Sum of zeroes = $5 + \sqrt{2} + 5 - \sqrt{2} = 10$ and

   P = Product of zeroes = $(5 + \sqrt{2})(5 - \sqrt{2})$

   $= 25 - 2 = 23$

   The required polynomial = $x^2 - Sx + P = x^2 - 10x + 23$

8. (*b*) $\boldsymbol{3x^2 - 3\sqrt{2}x + 1}$

   S = $\sqrt{2}$ and P = $\frac{1}{3}$

   Required polynomial = $x^2 - Sx + P = k\left(x^2 - \sqrt{2}x + \frac{1}{3}\right)$

   If $k = 3$, then the polynomial is $3x^2 - 3\sqrt{2}x + 1$.

9. (*b*) $\boldsymbol{x^2 - 4x - 1}$

   One root = $2 + \sqrt{5}$, S = Sum = 4

   ∴ Other root = $4 - (2 + \sqrt{5})$

   $= 4 - 2 - \sqrt{5}$

   $= 2 - \sqrt{5}$

   ∴ P = Product = $(2 + \sqrt{5})(2 - \sqrt{5})$

   $= 4 - 5$

   $= -1$

Required polynomial = $x^2 - Sx + P$
$= x^2 - 4x + (-1)$
$= x^2 - 4x - 1$

10. (*a*) $x^2 + (2-\sqrt{5})x - 2\sqrt{5}$

P = Product = $-2\sqrt{5}$, one zero = $\sqrt{5}$

$\therefore$ Other zero = $\frac{-2\sqrt{5}}{\sqrt{5}}$

$= -2$

$\therefore$ S = Sum of roots = $\sqrt{5} + (-2)$

$= \sqrt{5} - 2$

Required polynomial = $x^2 - Sx + P$
$= x^2 - (\sqrt{5} - 2)x + (-2\sqrt{5})$
$= x^2 + (2-\sqrt{5})x - 2\sqrt{5}$

11. (*b*) $k = -2$

Given polynomial $3x^2 + 5x + k$

Product of its zeroes = $\frac{\text{Constant term}}{\text{Coefficient of } x^2}$

$= \frac{-2}{3}$ [Given]

$\Rightarrow \frac{k}{3} = \frac{-2}{3}$

$\Rightarrow k = -2$

12. (*c*) $k = -5$

Let α be one of the zeroes of $p(x) = 5x^2 + 13x - k$

Then, the other zero = $\frac{1}{\alpha}$

Product of zeroes = $\frac{\text{Constant term}}{\text{Coefficient of } x^2} = \frac{-k}{5}$

$\Rightarrow \alpha \times \frac{1}{\alpha} = -\frac{k}{5}$

$\Rightarrow 1 = -\frac{k}{5}$

$\Rightarrow k = -5$

13. (*c*) $\frac{4}{3}$

(–3) is one zero of the given polynomial
$(\alpha - 1)\, x^2 + \alpha x + 1$

$\therefore p(-3) = 0$

$\Rightarrow (\alpha - 1)\,(3)^2 + \alpha(-3) + 1 = (\alpha - 1)\,9 - 3\alpha - 1 = 0$

$\Rightarrow 9\alpha - 9 - 3\alpha + 1 = 0$

$\Rightarrow 6\alpha - 8 = 0$

$\Rightarrow 6\alpha = 8$

$\Rightarrow \alpha = \frac{8}{6} = \frac{4}{3}$

14. (*b*) $\frac{2}{3}$

$f(x) = px^2 - 2x + 3p$

Sum of zeroes = $\alpha + \beta$

$= -\frac{(-2)}{p}$

$= \frac{2}{p}$

and $\alpha\beta = \frac{3p}{p} = 3$

$\therefore \frac{2}{p} = 3$ [$\because \alpha + \beta = \alpha\beta$, given]

$\therefore p = \frac{2}{3}$

15. (*c*) $k = -16$

Given polynomial is $x^2 - 6x + k$.

$\therefore$ Sum of zeroes = $\alpha + \beta = 6$

and Product of zeroes = $\alpha\beta = k$

$\Rightarrow \beta = 6 - \alpha$

$3\alpha + 2\beta = 20$ [Given]

$\Rightarrow 3\alpha + 2(6 - \alpha) = 20$

$\Rightarrow 3\alpha + 12 - 2\alpha = 20$

$\Rightarrow \alpha = 8$

Substituting $\alpha = 8$ in $\beta = 6 - \alpha$, we get

$\beta = 6 - 8 = -2$

$k = \alpha\beta = 8 \times (-2) = -16$

16. (*d*) $a = \frac{1}{2}, c = 5$

Given polynomial is $ax^2 - 5x + c$

$\therefore$ Sum of zeroes = $p + q = \frac{5}{a}$

and product of zeroes = $pq = \frac{c}{a}$

$p + q = pq = 10$ [Given]

$\Rightarrow \frac{5}{a} = 10$

$\Rightarrow a = \frac{1}{2}$

and $\frac{c}{a} = 10$

$\Rightarrow c = 10 \times a$

$= 10 \times \frac{1}{2} = 5$

Hence, $a = \frac{1}{2}, c = 5$.

17. (*c*) $-x^3 + 3x^2 - 3x + 5$

According to division algorithm, we have

Dividend = Quotient × Divisor + Remainder

$\therefore$ Required polynomial = $(x - 2) \times (-x^2 + x - 1) + 3$
$= -x^3 + x^2 - x + 2x^2 - 2x + 2 + 3$
$= -x^3 + 3x^2 - 3x + 5$

18. (*a*) ≤ 1

$\because$ The degree of the remainder is less than the degree of the divisor.

19. (*b*) **8, –10**

α, β, γ are the zeroes of the given polynomial
$x^3 - x^2 - 10x - 8$

Then, $\alpha\beta\gamma = -\frac{\text{Constant term}}{\text{Coefficient of } x^3}$

$= \frac{-(-8)}{1}$

$= +8$

and $\alpha\beta + \beta\gamma + \gamma\alpha = \frac{\text{Coefficent of } x}{\text{Coefficient of } x^3}$

$$= -\frac{10}{1}$$

$$= -10$$

∴ The values of $\alpha\beta\gamma$ and $\alpha\beta + \beta\gamma + \gamma\alpha$ are respectively 8 and – 10.

20. (*b*) $x^3 + 6x^2 + 11x + 6$

Here, $\alpha = -2$, $\beta = -3$ and $\gamma = -1$.

∴ $\alpha + \beta + \gamma = (-2) + (-3) + (-1) = -6,$

$\alpha\beta + \beta\gamma + \gamma\alpha = (-2)(-3) + (-3)(-1) + (-1)(-2)$

$= 6 + 3 + 2 = 11$

and $\alpha\beta\gamma = (-2) \times (-3) \times (-1) = -6$

Required polynomial is

$x^3 - (\alpha + \beta + \gamma)\, x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)\, x - \alpha\beta\gamma$

$= x^3 - (-6)\, x^2 + (11)x - (-6)$

$= x^3 + 6x^2 + 11x + 6$

21. (*b*) **–7**

Let $\alpha = \sqrt{2}$ and $\beta = -\sqrt{2}$ and γ be the zeroes of the given polynomial $x^3 + 7x^2 - 2x - 14$.

Then, $$\alpha\beta\gamma = \frac{-\text{Constant term}}{\text{Coefficient of } x^3}$$

⇒ $$\left(\sqrt{2}\right)\left(-\sqrt{2}\right)\gamma = -\frac{(-14)}{1}$$

⇒ $-2\gamma = 14$

⇒ $\gamma = -7$

22. (*a*) **3, 4**

Since $x = 1$ is a zero of the polynomial $x^3 - 8x^2 + 19x - 12$,

∴ $(x - 1)$ is a factor of $x^3 - 8x^2 + 19x - 12$

On dividing $x^3 - 8x^2 + 19x - 12$ by $x - 1$, we get

$$
\begin{array}{r|l}
x-1 & x^3 - 8x^2 + 19x - 12 \;(x^2 - 7x + 12 \\
 & \underline{\overset{-}{x^3} \overset{+}{-} x^2} \\
 & \quad -7x^2 + 19x \\
 & \quad \underline{\overset{+}{-7x^2} \overset{-}{+} 7x} \\
 & \qquad 12x - 12 \\
 & \qquad \underline{\overset{-}{12x} \overset{+}{-} 12} \\
 & \qquad\quad 0
\end{array}
$$

The other zeroes are given by $x^2 - 7x + 12 = 0$

⇒ $x^2 - 3x - 4x + 12 = 0$

⇒ $x(x - 3) - 4(x - 3) = 0$

⇒ $(x - 3)(x - 4) = 0$

⇒ $x - 3 = 0$ or $x - 4 = 0$

⇒ $x = 3$

or $x = 4$

Hence, the other two zeroes are 3 and 4.

23. (*a*) **4, –4, 5**

Let α, β, γ be the zeroes of the polynomial $x^3 - 5x^2 - 16x + 80$ such that $\beta = -\alpha$

Sum of zeroes $= \alpha + \beta + \gamma$

$= \alpha - \alpha + \gamma$

$$= \frac{-(-5)}{1}$$

$= 5$

⇒ $\gamma = 5$

Product of zeroes $= \alpha\beta\gamma$

$= (\alpha) \times (-\alpha) \times (\gamma)$

$$= -\alpha^2 \times 5 = \frac{-80}{1}$$

⇒ $\alpha^2 = 16$

⇒ $\alpha = \pm 4$

If $\alpha = 4$, then $\beta = -4$,

If $\alpha = -4$, then $\beta = 4$

So, the zeroes of the given polynomial are **4, –4** and **5**.

24. (*a*) **5**

Since α, β, γ are the roots of $6x^3 + 3x^2 - 5x + 1$,

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{-5}{6}$$

and $$\alpha\beta\gamma = -\frac{1}{6}$$

Now, $$\alpha^{-1} + \beta^{-1} + \gamma^{-1} = \frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$$

$$= \frac{\beta\gamma + \alpha\gamma + \alpha\beta}{\alpha\beta\gamma}$$

$$= \frac{\frac{-5}{6}}{-\frac{1}{6}} = 5$$

25. (*a*) $4 - 4x - x^2 + x^3$

Since the graph of the polynomial $p(x)$ intersects the x-axis in 3 distinct points,

∴ it is a graph of cubic polynomial.

∴ $p(x) = 4 - 4x - x^2 + x^3$

26. (*b*) **4, 3**

Given polynomial is $x^2 - 4x + 3$

$$\text{Sum} = -\frac{b}{a} = -\frac{(-4)}{1} = 4$$

and $$\text{Product} = \frac{c}{a} = \frac{3}{1}$$

Hence, the sum and product respectively are 4 and 3.

For Standard Level

27. (*a*) $\mathbf{\frac{31}{6}}$

Let α, β be the zeroes of the given polynomial $3x^2 + 5x + k$

Then, $$\alpha + \beta = \frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2} = \frac{-5}{3}$$

and $$\alpha\beta = \frac{\text{Constant}}{\text{Coefficient of } x^2} = \frac{k}{3}$$

Also, $$\alpha^2 + \beta^2 = \frac{-2}{3} \qquad \text{(Given)}$$

⇒ $$(\alpha + \beta)^2 - 2\alpha\beta = \frac{-2}{3}$$

⇒ $$\left(\frac{-5}{3}\right)^2 - 2 \times \frac{k}{3} = -\frac{2}{3}$$

⇒ $$\frac{25}{9} - \frac{2k}{3} = -\frac{2}{3}$$

$\Rightarrow \quad \frac{2k}{3} = \frac{25}{9} + \frac{2}{3}$

$= \frac{25+6}{9}$

$= \frac{31}{9}$

$\Rightarrow \quad k = \frac{31}{9} \times \frac{3}{2} = \frac{31}{6}$

28. (*c*) $\mathbf{\frac{-25}{12}}$

Given polynomial is $6y^2 + y - 2$.

Sum of zeroes $= \alpha + \beta = -\frac{1}{6}$

and $\alpha\beta = -\frac{2}{6} = -\frac{1}{3}$

Now $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2+\beta^2}{\alpha\beta}$

$= \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}$

$= \frac{\left(-\frac{1}{6}\right)^2 - 2 \times \left(-\frac{1}{3}\right)}{\left(-\frac{1}{3}\right)}$

$= \frac{\frac{1}{36} + \frac{2}{3}}{-\frac{1}{3}}$

$= \frac{\frac{1+24}{36}}{-\frac{1}{3}}$

$= \frac{25}{36} \times \frac{(-3)}{1}$

$= \frac{-25}{12}$

29. (*c*) $\mathbf{\frac{-27}{4}}$

Given polynomial is $x^2 - 5x + 4$

Sum of zeroes $= \alpha + \beta = \frac{-(-5)}{1} = 5$

and product of zeroes $= \alpha\beta = \frac{4}{1} = 4$

Now, $\frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta = \left(\frac{\beta+\alpha}{\alpha\beta}\right)^2 - 2\alpha\beta$

$= \frac{5}{4} - 2 \times 4$

$= \frac{5}{4} - 8$

$= \frac{5-32}{4}$

$= \frac{-27}{4}$

30. (*c*) $\mathbf{\frac{9}{4}}$

Given polynomial $= x^2 - 2 + x = x^2 + x - 2$

Sum of zeroes $= \alpha + \beta = -\frac{1}{1} = -1$

and product of zeroes $= \alpha\beta = -\frac{2}{1} = -2$

Now, $\left(\frac{1}{\alpha} - \frac{1}{\beta}\right)^2 = \left(\frac{\beta-\alpha}{\alpha\beta}\right)^2$

$= \frac{(\beta+\alpha)^2 - 4\alpha\beta}{(\alpha\beta)^2}$

$= \frac{(-1)^2 - 4(-2)}{(-2)^2}$

$= \frac{1+8}{4}$

$= \frac{9}{4}$

31. (*d*) $\mathbf{k = 7}$

Given polynomial is $x^2 - (k + 6)\, x + 2\,(2k - 1)$

Sum of zeroes $= \alpha + \beta$

$= \frac{-[-(k+6)]}{1}$

$= k + 6$

and product zeroes $\alpha\beta = \frac{2(2k-1)}{2}$

$\alpha + \beta = \left(\frac{\alpha\beta}{2}\right)$ [Given]

$\therefore \quad k + 6 = \frac{2(2k-1)}{2}$

$\Rightarrow \quad k + 6 = 2k - 1$

$\Rightarrow \quad 6 + 1 = 2k - k$

$\Rightarrow \quad k = 7$

32. (*b*) $\mathbf{k = -1, \frac{2}{3}}$

Given polynomial is $kx^2 + 4x + 4$.

Sum of zeroes $= \alpha + \beta = -\frac{4}{k}$

and product of zeroes $= \alpha\beta = \frac{4}{k}$

Now, $(\alpha + \beta)^2 - 2\alpha\beta = 24$ [Given]

$\Rightarrow \quad \left(\frac{-4}{k}\right)^2 - 2 \times \frac{4}{k} = 24$

$\Rightarrow \quad 16 - 8k = 24k^2$

$\Rightarrow \quad 24k^2 + 8k - 16 = 0$

$\Rightarrow \quad 8(3k^2 + k - 2) = 0$

$\Rightarrow \quad 8(3k^2 + 3k - 2k - 2) = 0$

$\Rightarrow \quad 8[3k\,(k + 1) - 2\,(k + 1\,)] = 0$

$\Rightarrow \quad 8(k + 1)\,(3k - 2) = 0$

$\Rightarrow \quad k + 1 = 0 \text{ or } 3k - 2 = 0$

$\Rightarrow \quad k = -1 \text{ or } k = \dfrac{2}{3}$

Hence, $k = -1, \dfrac{2}{3}$.

33. (*d*) $\boldsymbol{\dfrac{c}{a}}$

Let α, β, γ be the zeroes of the cubic polynomial $ax^3 + bx^2 + cx + d$, where $\alpha = 0$.

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$$

$$\Rightarrow \quad 0(\beta) + \beta\gamma + \gamma(0) = \frac{c}{a}$$

$$\Rightarrow \quad \beta\gamma = \frac{c}{a}$$

∴ The product of the other two zeroes is $\dfrac{c}{a}$.

34. (*d*) **−48**

Let the zeroes of the polynomial $x^3 - 12x^2 + 44x + c$ be $a - b$, a and $a + b$.

$$\text{Sum of zeroes} = a - b + a + a - b$$
$$= 3a$$
$$= -\frac{(-12)}{1} = 12$$
$$\Rightarrow \quad a = 4$$
$$a(a-b) + (a)(a+b)(a-b) = +44$$
$$\Rightarrow \quad a^2 - ab + a^2 + ab + a^2 - b^2 = +44$$
$$\Rightarrow \quad 3a^2 - b^2 = +44$$
$$\Rightarrow \quad 3(4)^2 - b^2 = +44$$
$$\Rightarrow \quad 48 - b^2 = +44$$
$$\Rightarrow \quad b^2 = 48 - 44 = 4$$
$$\text{Product of roots} = a(a-b)(a+b)$$
$$= a(a^2 - b^2)$$
$$= -c$$
$$\Rightarrow \quad 4(16 - 4) = -c$$
$$\Rightarrow \quad 4(12) = -c$$
$$\Rightarrow \quad c = -48$$

35. (*c*) $\mathbf{1 \pm \sqrt{2}}$

Given polynomial is $x^3 - 3x^2 + x + 1$

$$\text{Sum of zeroes} = a - b + a + a + b$$
$$= 3a$$
$$= \frac{-(-3)}{1}$$
$$= 3$$
$$\Rightarrow \quad a = 1$$
$$a(a-b) + (a+b)\,a + (a-b)(a+b) = 1$$
$$\Rightarrow \quad a^2 - ab + a^2 + ab + a^2 - b^2 = 1$$
$$\Rightarrow \quad 3a^2 - b^2 = 1$$
$$\Rightarrow \quad b^2 = 3(1)^2 - 1 = 2$$
$$\Rightarrow \quad b = \pm\sqrt{2}$$
$$\therefore \quad a + b = 1 \pm \sqrt{2}$$

36. (*b*) $\boldsymbol{2q = r}$

Given polynomial is $x^3 - 2x^2 + qx - r$

Let α, β, γ be its zeroes such that $\alpha + \beta = 0$

$$\text{Sum of zeroes} = \alpha + \beta + \gamma$$
$$= \frac{-(-2)}{1}$$
$$= 0 + \gamma$$
$$= 2$$
$$\Rightarrow \quad \gamma = 2$$
$$\text{Product of zeroes} = \alpha\beta\gamma = -(-r)$$
$$\Rightarrow \quad 2\alpha\beta = r \qquad \ldots (1)$$
$$\alpha\beta + \beta\gamma + \gamma\alpha = q$$
$$\Rightarrow \quad \alpha\beta + 2\beta + 2\alpha = q \qquad [\text{Putting } r = 2]$$
$$\Rightarrow \quad \frac{r}{2} + 2(\alpha + \beta) = q \qquad [\text{Using (1)}]$$
$$\Rightarrow \quad \frac{r}{2} = q$$
$$\Rightarrow \quad r = 2q$$
$$\Rightarrow \quad 2q = r$$

37. (*b*) $\boldsymbol{-\dfrac{17}{4}}$

Since the given polynomial is exactly divisible by $x + 2$,

∴ $x + 2$ is a factor of it and $x = -2$ is a zero of it.

$$\therefore \quad p(-2) = 2(-2)^3 - k(-2)^2 + 5(-2) + 9$$
$$= 0$$
$$\Rightarrow \quad -16 - 4k - 10 + 9 = 0$$
$$\Rightarrow \quad -17 = 4k$$
$$\Rightarrow \quad k = -\frac{17}{4}$$

38. (*d*) $\boldsymbol{k = -1}$

α, β, γ are zeroes of polynomial $kx^3 - 5x + 9$

$$\Rightarrow \quad \alpha + \beta + \gamma = \frac{-(0)}{k} = 0$$
$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{-5}{k}$$
$$\text{and} \quad \alpha\beta\gamma = \frac{-9}{k} \qquad \ldots (1)$$
$$\alpha^3 + \beta^3 + \gamma^3 = 27$$
$$\Rightarrow \quad \alpha^3 + \beta^3 + \gamma^3 = 3 \times \alpha\beta\gamma \qquad [\because \alpha + \beta + \gamma = 0]$$
$$\Rightarrow \quad 27 = 3 \times \alpha\beta\gamma$$
$$\Rightarrow \quad \alpha\beta\gamma = 9 \qquad \ldots (2)$$

From (1) and (2) , we get

$$9 = \frac{-9}{k}$$
$$\Rightarrow \quad k = -1$$

FILL IN THE BLANKS

For Basic and Standard Levels

1. constant **2.** 1, 3

3. two **4.** equal

5. $k(2x^2 + x + 1)$

SHORT ANSWER QUESTIONS

For Basic and Standard Levels

1.
$$x^2 + 5x - 204 = x^2 + 17x - 12x - 204$$
$$= x(x + 17) - 12(x + 17)$$
$$= (x + 17)(x - 12)$$

The zeroes of the polynomial are given by

$$(x + 17)(x - 12) = 0$$
$$\Rightarrow \quad x + 17 = 0$$
$$\text{or} \quad x - 12 = 0$$
$$\Rightarrow \quad x = -17$$
$$x = -12$$

2	204
2	102
3	51
	17

Hence, the zeroes of the given polynomial are **−17** and **12**

2. Given polynomial is $x^3 - 3x^2 - x + 3$

$$P(3) = (3)^3 - 3(3)^2 - 3 + 3 = 27 - 27 - 3 + 3 = 0$$

Hence, **3 is a zero of the given polynomial.**

3. Sum of roots = −1 and product of roots = −6

Required quadratic polynomial $x^2 - Sx + P$

$$= x^2 - (-1)x + (-6)$$
$$= \mathbf{x^2 + x - 6}$$

4. Let α, β and γ be the roots of the required cubic polynomial.

Then, $\alpha = -1, \beta = 2$

and $\gamma = -3$

$$\alpha + \beta + \gamma = -1 + 2 - 3 = -2$$
$$\alpha\beta + \beta\gamma + \gamma\alpha = (-1)(2) + (2)(-3) + (-3)(-1) = -2 - 6 + 3 = -8 + 3 = -5$$
$$\alpha\beta\gamma = (-1) \times (2) \times (-3) = 6$$

Required cubic polynomial is

$$x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma$$
$$= x^3 - (-2)x^2 + (-5)x - 6$$
$$= \mathbf{x^3 + 2x^2 - 5x - 6}$$

For Standard Level

5. Given polynomial $= x^3 + px^2 + qx + 2$

Its zeroes are α, β, γ.

∴ $\alpha + \beta + \gamma = -p$

$\alpha\beta + \beta\gamma + \gamma\alpha = q$

and $\alpha\beta\gamma = -2$ … (1)

Also, $\alpha\beta + 1 = 0$

⇒ $\alpha\beta = -1$ … (2)

∴ $(-1)\gamma = -2$ [Using (1) and (2)]

⇒ $\gamma = 2$

$\alpha\beta + \beta\gamma + \gamma\alpha = q$

⇒ $(-1) + \gamma(\alpha + \beta) = q$

⇒ $(-1) + (2)(\alpha + \beta) = q$

⇒ $2(\alpha + \beta) = q + 1$

⇒ $(\alpha + \beta) = \frac{q+1}{2}$

Now, $\alpha + \beta + \gamma = -p$

⇒ $\frac{q+1}{2} + 2 = -p$

⇒ $q + 1 + 4 = -2p$

⇒ $2p + q + 5 = 0$

Hence, $\mathbf{2p + q + 5 = 0}$

UNIT TEST 1

For Basic Level

1. (*d*) **1**

The graph of the polynomial cuts the *x*-axis at one point.

∴ The polynomial has **1** zero.

2. (*a*) **one point only**

If the discriminant of a quadratic polynomial is zero, then it has two equal zeroes.

∴ It well touch the *x*-axis at **one point only**.

3. (*c*) $-\sqrt{3}$ **and** $\frac{-7}{\sqrt{3}}$

$$\sqrt{3}x^2 + 10x + 7\sqrt{3} = \sqrt{3}x^2 + 3x + 7x + 7\sqrt{3}$$
$$= \sqrt{3}x(x + \sqrt{3}) + 7(x + \sqrt{3})$$
$$= (x + \sqrt{3})(\sqrt{3}x + 7)$$

The two zeroes of polynomial are given by

$$(x + \sqrt{3})(\sqrt{3}x + 7) = 0$$

⇒ $x + \sqrt{3} = 0$

or $\sqrt{3}x + 7 = 0$

⇒ $x = -\sqrt{3}$

or $x = \frac{-7}{\sqrt{3}}$

Hence, the zeroes of the polynomial are $\mathbf{-\sqrt{3}}$ and $\mathbf{\frac{-7}{\sqrt{3}}}$.

4. (*a*) $\mathbf{x^2 - 25}$

Let α and β be the zeroes of the required polynomial.

$\alpha = 5$

Sum of zeroes $\alpha + \beta = 0$

⇒ $5 + \beta = 0$

⇒ $\beta = -5$

Product of zeroes $= \alpha\beta = 5 \times (-5) = -25$

Required polynomial is

$$x^2 - Sx + P = x^2 - 0 \times x + (-25) = x^2 - 25$$

5. (*c*) **2**

Let $p(x) = x^2 - 3x + k$

2 is zero of the given polynomial

∴ $p(2) = 0$

⇒ $(2)^2 - 3(2) + k = 0$

⇒ $4 - 6 + k = 0$

⇒ $k = 2$

6. (*c*) **1**

$$P(1) = a(1)^2 - 3(a - 1)(1) - 1 = 0$$

⇒ $a - 3a + 3 - 1 = 0$

⇒ $-2a + 2 = 0$

⇒ $2a = 2$

⇒ $a = 1$

7. (*c*) **6**

$$p(x) = x^2 - 5x + b$$

∴ $\alpha + \beta = 5$ … (1)

and $\alpha\beta = b$

Also $\alpha - \beta = 1$ [Given] … (2)

Solving (1) and (2), we get

$\alpha = 3, \beta = 2$

$b = \alpha\beta = 3 \times 2 = \mathbf{6}$

8. (*a*) **−2**

$$x^2 - x - 6 = (x - 3)(x + 2) = 0$$

⇒ $x = 3$

or $x = -2$

$$3x^2 + 8x + 4 = (3x + 2)(x + 2) = 0$$

⇒ $x = \frac{-2}{3}, x = -2$

Both polynomials become zero when $x = -2$.

9. (*b*) **−0, 3**

$$x^3 - 3x^2 = 0$$

⇒ $x^2(x - 3) = 0$

$$\Rightarrow \quad x = 0$$
or $$x = 3$$
$\therefore$ Zeroes of cubic polynomial $x^3 - 3x^2$ are 0 and 3.

10. (*b*) **−1**

$$x^2 - 1 = 0$$
$$\Rightarrow \quad (x + 1)(x - 1) = 0$$
$$\Rightarrow \quad x + 1 = 0$$
or $$x - 1 = 0$$
$$\Rightarrow \quad x = -1, x = 1$$
$$x^2 + 2x + 1 = 0$$
$$\Rightarrow \quad (x + 1)(x + 1) = 0$$
$$\Rightarrow \quad x + 1 = 0$$
or $$x + 1 = 0$$
$$\Rightarrow \quad x = -1, x = -1$$

Hence, common zero of the two given polynomials is −1.

11. (*b*) **−6**

Given polynomial is $x^3 - 5x^2 - 6x + 20$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{(\text{Coefficient of } x)}{\text{Coefficient of } x^3} = \frac{-6}{1} = -6$$

12. (*b*) **4**

Given polynomial is $2x^3 - 3ax^2 + 4x - 5$

$$\text{Sum of zeroes} = \frac{-(\text{Coefficient of } x^2)}{\text{Coefficient of } x^3} = \frac{-(-3a)}{2} = \frac{3a}{2}$$

$$\Rightarrow \quad \frac{3a}{2} = 6 \text{ [Sum of zeroes = 6, Given]}$$

$$\Rightarrow \quad a = 4$$

13. (*a*) $\mathbf{k(x^3 - 2x^2 - 7x + 14)}$

Required polynomial is

$$k[x^3 - (\alpha + \beta + \gamma)x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma]$$
$$= k[x^3 - 2x^2 + (-7)x - (-14)]$$
$$= k(x^3 - 2x^2 - 7x + 14)$$

14. Let

$$p(x) = 4x^2 + 4x - 3 = x^2 + \frac{4}{4}x - \frac{3}{4} = x^2 + x - \frac{3}{4}$$

Then,

$$p\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right) - \frac{3}{4} = \frac{1}{4} + \frac{1}{2} - \frac{3}{4} = \frac{1+2-3}{4} = 0$$

and

$$p\left(\frac{-3}{2}\right) = \left(\frac{-3}{2}\right)^2 + \left(\frac{-3}{2}\right) - \frac{3}{4} = \frac{9}{4} - \frac{3}{2} - \frac{3}{4} = \frac{9-6-3}{4} = 0$$

Hence, $\mathbf{\frac{1}{2}}$ and $-\mathbf{\frac{3}{2}}$ are zeroes of the given polynomial.

$$\text{Sum of zeroes} = \frac{1}{2} + \frac{(-3)}{2} = \frac{1-3}{2} = \frac{-2}{2} = 1 = \frac{-(1)}{1} = \frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$$

$$\text{Product of zeroes} = \frac{1}{2} \times \left(\frac{-3}{2}\right) = -\frac{3}{4} = \frac{\frac{-3}{4}}{1} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

or

$f(x) = x^3 - 4x^2 - 3x + 12$

Let $\alpha = \sqrt{3}$ and $\beta = -\sqrt{3}$ be the given zeroes of the polynomial $f(x)$ and γ its third zero.

Then,

$$\text{Sum of zeroes} = \alpha + \beta + \gamma = \sqrt{3} - \sqrt{3} + \gamma = \gamma = -\frac{\text{Coefficient of } x^2}{\text{Coefficient of } x^3}$$

$$\Rightarrow \quad \gamma = \frac{-(-4)}{1}$$

$$\Rightarrow \quad \gamma = \mathbf{4}$$

$\therefore$ Required third zero is **4**.

15. Required quadratic polynomial is

$$x^2 - (\text{Sum})\,x + \text{Product} = k[x^2 - \left(\frac{-3}{2\sqrt{5}}\right)x + \left(\frac{-1}{2}\right)]$$
$$= k[x^2 + \left(\frac{3}{2\sqrt{5}}\right)x - \frac{1}{2}]$$

If $k = 2\sqrt{5}$, then the polynomial is $\mathbf{2\sqrt{5}x^2 + 3x - \sqrt{5}}$

The zeroes of the polynomial are given by

$$2\sqrt{5}\,x^2 + 3x - \sqrt{5} = 0$$
$$\Rightarrow \quad 2\sqrt{5}\,x^2 + 5x - 2x - \sqrt{5} = 0$$
$$\Rightarrow \quad \sqrt{5}\,x(2x + \sqrt{5}) - 1\,(2x + \sqrt{5}) = 0$$
$$\Rightarrow \quad (2x + \sqrt{5})(\sqrt{5}\,x - 1) = 0$$
$$\Rightarrow \quad (2x + \sqrt{5}) = 0$$
or $$(\sqrt{5}\,x - 1) = 0$$
$$\Rightarrow \quad x = -\frac{\sqrt{5}}{2} \text{ or } x = \frac{1}{\sqrt{5}}$$

The zeroes of the polynomial are $-\frac{\sqrt{5}}{2}$ and $\frac{1}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$

or $\mathbf{\frac{-\sqrt{5}}{2}, \frac{\sqrt{5}}{5}}$.

16. Required polynomial is

$x^3 - (\alpha + \beta + \gamma)\, x^2 + (\alpha\beta + \beta\gamma + \gamma\alpha)x - \alpha\beta\gamma$

$= \mathbf{x^3 - 3x^2 - 10x + 24}$

17. Since $\sqrt{2}$ and $\sqrt{2}$ are given of $f(x)$,

$\therefore$ $(x - \sqrt{2})$ and $(x + \sqrt{2})$ are factors of $f(x)$.

$\therefore$ $(x^2 - 2)$ is a factor of $f(x)$.

On dividing $f(x)$ by $x^2 - 2$, we get

$$\begin{array}{r|l}
x^2 - 2 & 2x^4 + 7x^3 - 19x^2 - 14x + 30 \;\big(\, 2x^2 + 7x - 15 \\
 & \underline{\underset{-}{2x^4} \qquad\quad \underset{+}{-\;4x^2}} \\
 & \quad 7x^3 - 15x^2 - 14x \\
 & \quad \underline{\underset{-}{7x^3} \qquad\quad \underset{+}{-\,14x}} \\
 & \qquad -15x^2 \quad + 30 \\
 & \qquad \underline{\underset{+}{-15x^2} \quad \underset{-}{+ 30}} \\
 & \qquad\qquad 0
\end{array}$$

$\therefore$ $f(x) = 2x^4 + 7x^3 - 19x^2 - 14x + 30$

$= (2x^2 + 7x - 15)\,(x^2 - 2)$ [By division algorithm]

$= (2x^2 + 10x - 3x - 15)\,(x^2 - 2)$

$= [2x(x + 5) - 3(x + 5)](x - \sqrt{2})(x + \sqrt{2})$

$= (x + 5)\,(2x - 3)\,(x - \sqrt{2})\,(x + \sqrt{2})$

The zeroes of $f(x)$ are given by $f(x) = 0$

$\Rightarrow (x + 5)\,(2x - 3)\,(x - \sqrt{2})\,(x + \sqrt{2}) = 0$

$\Rightarrow$ $(x + 5) = 0$

or $(2x - 3) = 0$

or $x - \sqrt{2} = 0$

or $x + \sqrt{2} = 0$

$\Rightarrow$ $x = -5$

or $x = \dfrac{3}{2}$

or $x = \sqrt{2}$

or $x = -\sqrt{2}$

$\therefore$ Zeroes of $f(x)$ are $\mathbf{-5, \dfrac{3}{2}, \sqrt{2}}$ and $\mathbf{-\sqrt{2}}$.

18. The other factor of $x^3 - 3\sqrt{5}x^2 - 5x + 15\sqrt{5}$ can be obtained by dividing the given polynomial by $x - \sqrt{5}$ as follows:

$$\begin{array}{r|l}
x - \sqrt{5} & x^3 - 3\sqrt{5}x^2 - 5x + 15\sqrt{5} \;\big(\, x^2 - 2\sqrt{5}x - 15 \\
 & \underline{\underset{-}{x^3} \;\underset{+}{-}\; \sqrt{5}x^2} \\
 & \quad -2\sqrt{5}x^2 - 5x + 15\sqrt{5} \\
 & \quad \underline{\underset{+}{-2\sqrt{5}x^2} \;\underset{-}{+\,10x}} \\
 & \qquad\quad -15x + 15\sqrt{5} \\
 & \qquad\quad \underline{\underset{+}{-15x} \;\underset{-}{+\,15\sqrt{5}}} \\
 & \qquad\qquad 0
\end{array}$$

$\therefore$ The other factor is

$$x^2 - 2\sqrt{5}x - 15 = x^2 + \sqrt{5}x - 3\sqrt{5}x - 15$$

$$= x\left(x + \sqrt{5}\right) - 3\sqrt{5}\left(x + \frac{15}{3\sqrt{5}}\right)$$

$$= x\left(x + \sqrt{5}\right) - 3\sqrt{5}\left(x + \sqrt{5}\right)$$

$$= \left(x + \sqrt{5}\right)\left(x - 3\sqrt{5}\right)$$

$\therefore$ Other zeroes are given by $\left(x + \sqrt{5}\right)\left(x - 3\sqrt{5}\right) = 0$

$\therefore$ Either $x + \sqrt{5} = 0 \Rightarrow x = -\sqrt{5}$

or $x - 3\sqrt{5} = 0 \Rightarrow x = 3\sqrt{5}$

$\therefore$ Required three zeroes are $\mathbf{\sqrt{5}}$, $\mathbf{3\sqrt{5}}$ and $\mathbf{-\sqrt{5}}$.

UNIT TEST 2

For Standard Level

1. (*b*) $\mathbf{\dfrac{p}{q}}$

$p(x) = x^2 - px + q$

$\therefore$ $\alpha + \beta = p$

and $\alpha\beta = q$

$$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta + \alpha}{\alpha\beta} = \frac{p}{q}$$

2. (*c*) $\mathbf{p = \dfrac{15}{2}, q = 9}$

Let α, β be the roots of the given polynomial $3x^2 - 2px + 2q$.

Let $\alpha = 2$ and $\beta = 3$

$\alpha + \beta = \dfrac{2p}{3}$

$\Rightarrow$ $2 + 3 = \dfrac{2p}{3}$

$\Rightarrow$ $5 \times 3 = 2p$

$\Rightarrow$ $p = \dfrac{15}{2}$

$\alpha\beta = \dfrac{2q}{3}$

$\Rightarrow$ $2 \times 3 = \dfrac{2q}{3}$

$\Rightarrow$ $q = 9$

Hence, $p = \dfrac{15}{2}, q = 9$.

3. (*c*) $\boldsymbol{\alpha + \beta = \alpha\beta}$

Here $\alpha + \beta = -\dfrac{6}{2} = -3$ and $\alpha\beta = -\dfrac{6}{2} = -3$

$\therefore$ $\alpha + \beta = \alpha\beta$

4. (*c*) $\mathbf{\dfrac{-25}{12}}$

Here we have $\alpha + \beta = -\dfrac{1}{6}$ …(1)

and $\alpha\beta = -\dfrac{2}{6} = -\dfrac{1}{3}$ …(2)

$$\therefore \quad \frac{\alpha}{\beta}+\frac{\beta}{\alpha} = \frac{\alpha^2+\beta^2}{\alpha\beta}$$

$$= \frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}$$

$$= \frac{\frac{1}{36}+\frac{2}{3}}{-\frac{1}{3}} \quad \text{[From (1) and (2)]}$$

$$= -3\times\frac{1+24}{36} = -\frac{25}{12}$$

5. (*d*) $\boldsymbol{a = \frac{1}{2}, c = 5}$

We have $\quad p + q = \frac{5}{a} \quad$ …(1)

and $\quad pq = \frac{c}{a} \quad$ …(2)

Also, $\quad p + q = pq = 10 \quad$ …(3)

∴ From (1), (2) and (3), we have

$$\frac{5}{a} = \frac{c}{a} = 10$$

$$\therefore \quad a = \frac{5}{10} = \frac{1}{2}$$

and $\quad c = 10a = 10\times\frac{1}{2} = 5$

6. (*d*) $\boldsymbol{a = -5, b = 8.}$

Given polynomial is $2x^3 + ax^2 - 14x + b$

$$\alpha+\beta+\gamma = -\frac{a}{2} = \frac{5}{2} \quad \text{(Given)}$$

$$\Rightarrow \quad a = -5$$

$$\alpha\beta\gamma = -\frac{b}{2} = -4 \quad \text{(Given)}$$

$$\Rightarrow \quad b = 8$$

Hence, $a = -5$, $b = 8$.

7. Let α and β be the zeroes of the polynomial

$$p(x) = ax^2 + bx + c$$

Then, $\quad \beta = \frac{1}{\alpha}$

$$\text{Product of zeroes} = \alpha\beta$$

$$= \alpha \times \frac{1}{\alpha}$$

$$= 1 = \frac{c}{a}$$

$$\Rightarrow \quad \boldsymbol{a = c}$$

8. $p(x) = q(x) \times g(x) + r(x)$ [By division algorithm]

$= (2x^2 + 2x - 1) \times (4x^2 + 3x + 2) + 14x - 10$

$= 8x^4 + 6x^3 + 4x^2 + 8x^3 + 6x^2 + 4x - 4x^2 - 3x - 2 + 14x - 10$

$= \boldsymbol{8x^4 + 14x^3 + 6x^2 + 15x - 12}$

9. α, β, γ are roots of the given polynomial is $x^3 + 3x^2 + 10x - 24$.

$$\alpha\beta+\beta\gamma+\gamma\alpha = \frac{10}{1} = 10$$

and $\quad \alpha\beta\gamma = -\frac{(-24)}{1} = 24$

$$\frac{1}{\gamma}+\frac{1}{\alpha}+\frac{1}{\beta} = \frac{\alpha\beta+\beta\gamma+\gamma\alpha}{\alpha\beta\gamma}$$

$$= \frac{10}{24} = \boldsymbol{\frac{5}{12}}$$

10. If α, β and γ be the zeroes of $2x^3 - x^2 - 5x - 2$, where $\alpha = -1$ and $\beta = 2$ (Given), then

$$\alpha+\beta+\gamma = \frac{1}{2}$$

$$\Rightarrow \quad 2-1+\gamma = \frac{1}{2}$$

$$\Rightarrow \quad \gamma = \frac{1}{2}-1 = -\frac{1}{2}$$

∴ Required third zero is $-\boldsymbol{\frac{1}{2}}$.

11.

$$
\begin{array}{r|l}
2x^2+x-2 & 4x^4+2x^3-8x^2+3x-7 \;\big(\; 2x^2-2 \\
 & \underline{4x^4+2x^3-4x^2} \\
 & \;\;\,-\quad\;\; -\quad\;\; + \\
 & \qquad\qquad -4x^2+3x-7 \\
 & \qquad\qquad \underline{-4x^2-2x+4} \\
 & \qquad\qquad\;\; +\quad\;\; +\quad - \\
 & \qquad\qquad\qquad 5x-11
\end{array}
$$

5*x* – 11 should be subtracted from the given polynomial so that the resulting polynomial is exactly divisible by $2x^2 + x - 2$.

12. We divide the given polynomial by $3x^2 + 4x + 1$ by the long division method as follows:

$$
\begin{array}{r|l}
3x^2+4x+1 & 6x^4+8x^3+17x^2+21x+7 \;\big(\; 2x^2+5 \\
 & \underline{6x^4+8x^3+\;\;2x^2} \\
 & -\quad\;\; -\quad\;\; - \\
 & \qquad\qquad 15x^2+21x+7 \\
 & \qquad\qquad \underline{15x^2+20x+5} \\
 & \qquad\qquad\; -\quad\;\; -\quad\; - \\
 & \qquad\qquad\qquad\quad x+2
\end{array}
$$

$$\therefore \quad \text{Remainder} = x + 2$$

$$\therefore \quad x + 2 = ax + b \quad \text{[Given]}$$

⇒ $\quad$ ***a* = 1** and ***b* = 2** which are the required values of *a* and *b*.

13. We have $\quad \alpha + \beta = -k \quad$ …(1)

and $\quad \alpha\beta = 45 \quad$ …(2)

Also, $\quad (\alpha - \beta)^2 = 144 \quad$ …(3)

$$\Rightarrow \quad (\alpha+\beta)^2 - 4\alpha\beta = 144$$

$$\Rightarrow \quad k^2 - 180 = 144$$

$$\Rightarrow \quad k^2 = 324$$

$$\Rightarrow \quad \boldsymbol{k = \pm 18}$$

∴ Required value of *k* is ±**18**.