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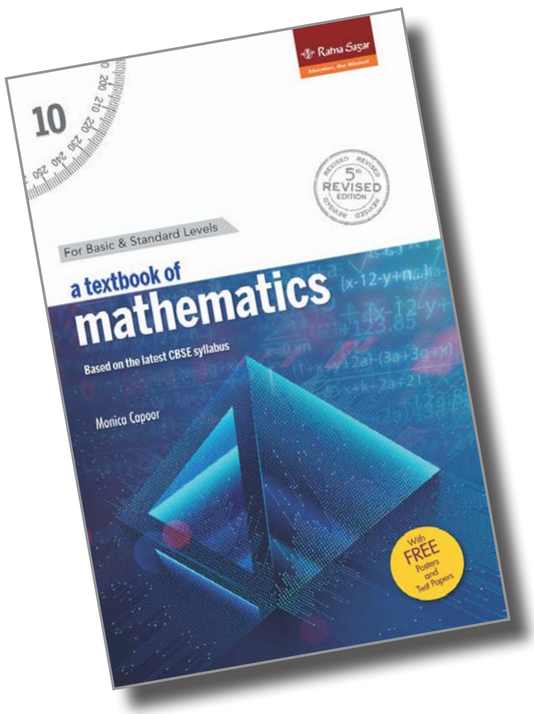

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CHAPTER 1

Real Numbers

EXERCISE 1A

For Basic and Standard Levels

1. **Euclid's division lemma**

For any two given positive integers a and b, there exist unique integers q and r such that $a = bq + r$, $0 \le r < b$.
a and b are called dividend and divisor respectively,
q and r are called quotient and remainder respectively.
Thus, dividend = (divisor × quotient) + remainder.

(*i*) Required number = 27 × 364 + 7 = **9835**

(*ii*) Dividend = divisor × quotient + remainder

$\Rightarrow$ 546 = divisor × 7 + 7

$\Rightarrow$ $\dfrac{546 - 7}{7}$ = divisor

$\Rightarrow$ divisor = **77**

2. Let a be any positive odd integer.

On dividing a by 8, let q be the quotient and r be the remainder.
Then, by Euclid's division lemma, we have

$a = 8q + r$

where $0 \le r < 8$.

$\Rightarrow$ $a = 8q + r$, where $r = 0, 1, 2, 3, 4, 5, 6$ or 7.

$\Rightarrow$ $a = 8q$ (when $r = 0$),
$a = 8q + 1$ (when $r = 1$),
$a = 8q + 2$ (when $r = 2$),
$a = 8q + 3$ (when $r = 3$),
$a = 8q + 4$ (when $r = 4$),
$a = 8q + 5$ (when $r = 5$),
$a = 8q + 6$ (when $r = 6$),
$a = 8q + 7$ (when $r = 7$)

Now, $a = 8q$, $a = 8q + 2$, $a = 8q + 4$
and $a = 8q + 6$ are even values of a.

Thus, when a is odd, it is of the form $8q + 1$, $8q + 3$, $8q + 5$ or $8q + 7$ for some integer q.

3. Let a be any positive integer.

On dividing a by 4, let m be the quotient and r be the remainder.
Then, by Euclid's division lemma, we have

$a = 4m + r$, where $0 \le r < 4$

$\Rightarrow$ $a = 4m$ (when $r = 0$),
$a = 4m + 1$ (when $r = 1$),
$a = 4m + 2$ (when $r = 2$),
and $a = 4m + 3$ (when $r = 3$)

$a = 4m$

$\Rightarrow$ $a^2 = 16m^2$
$= 4(4m^2)$
$= 4q$,

where $q = 4m^2$

$a = 4m + 1$

$\Rightarrow$ $a^2 = (4m + 1)^2$
$= 16m^2 + 8m + 1$
$= 4(m^2 + 2m) + 1$
$= 4q + 1$,

where $q = m^2 + 2m$

$a = 4m + 2$

$\Rightarrow$ $a^2 = (4m + 2)^2$
$= 16m^2 + 16m + 4$
$= 4(4m^2 + 4m + 1)$
$= 4q$,

where $q = 4m^2 + 4m + 1$

$a = 4m + 3$

$\Rightarrow$ $a^2 = (4m + 3)^2$
$= 16m^2 + 24m + 9 = 16m^2 + 24m + 8 + 1$
$= 4(4m^2 + 6m + 2) + 1$
$= 4q + 1$,

where $q = (4m^2 + 6m + 2)$

Hence, square of any positive integer is of the form $4q$ or $4q + 1$.

4. Let a be any positive integer.

On dividing a by 4, let q be the quotient and r the remainder.
Then, by Euclid's division lemma, we have

$a = 5q + r$, where $0 \le r < 5$

$\Rightarrow$ $a = 5q$ (when $r = 0$),
$a = 5q + 1$ (when $r = 1$),
$a = 5q + 2$ (when $r = 2$),
$a = 5q + 3$ (when $r = 3$),
and $a = 5q + 4$ (when $r = 4$)

$a = 5q$

$\Rightarrow$ $a^2 = 25q^2 = 5(5q^2) = 5m$,

where $m = (5q^2)$ is an integer

$a = 5q + 1$

$\Rightarrow$ $a^2 = (5q + 1)^2$
$= 25q^2 + 10q + 1$
$= 5(5q^2 + 2q) + 1$
$= 5m + 1$,

where $m = 5q^2 + 2q$ is an integer

$a = 5q + 2$

$\Rightarrow$ $a^2 = (5q + 2)^2$
$= 25q^2 + 20q + 4$
$= 5(5q^2 + 4q) + 4$
$= 5m + 4$,

where $m = 5q^2 + 4q$ is an integer

$a = 5q + 3$

$\Rightarrow$ $a^2 = (5q + 3)^2$
$= 25q^2 + 30q + 9$
$= 25q^2 + 30q + 5 + 4$
$= 5(5q^2 + 6q + 1) + 4$
$= 5m + 4$,

where $m = (5q^2 + 6q + 1)$ is an integer

$a = 5q + 4$

$\Rightarrow$ $a^2 = (5q + 4)^2$
$= 25q^2 + 40q + 16$
$= 25q^2 + 40q + 15 + 1$
$= 5(5q^2 + 8q + 3) + 1$
$= 5m + 1$,

where $m = 5q^2 + 8q + 3$ is an integer

Hence, the square of any positive integer is of the form $5m$, $5m + 1$, $5m + 4$ for some integer m.

5. Let a be any positive odd integer.

If $a = 8m + 1$

$\Rightarrow$ $a^2 = (8m + 1)^2$
$= 64m^2 + 16m + 1$
$= 8(8m^2 + 2m) + 1$
$= 8q + 1,$

where $q = 8m^2 + 2m$ is an integer

$a = 8m + 3$

$\Rightarrow$ $a^2 = (8m + 3)^2$
$= 64m^2 + 48m + 9$
$= 64m^2 + 48m + 8 + 1$
$= 8(8m^2 + 6m + 1) + 1$
$= 8q + 1,$

where $q = 8m^2 + 6m + 1$ is an integer

$a = 8m + 5$

$\Rightarrow$ $a^2 = (8m + 5)^2$
$= 64m^2 + 80m + 25$
$= 64m^2 + 80m + 24 + 1$
$= 8(8m^2 + 10m + 3) + 1$
$= 8q + 1,$

where $q = 8m^2 + 10m + 3$ is an integer

$a = 8m + 7$

$\Rightarrow$ $a^2 = (8m + 7)^2$
$= 64m^2 + 112m + 49$
$= 64m^2 + 112m + 48 + 1$
$= 8(8m^2 + 14m + 6) + 1$
$= 8q + 1,$

where $q = 8m^2 + 14m + 6$ is an integer

Hence, square of any positive odd integer is of the form $8q + 1$ for some integer q.

6. On dividing n by 5, let m be the quotient and r be the remainder where $m \geq 0$ and $0 \leq r < 5$.

Then by Euclid's division lamma, we have $n = 5m + r$ for some integer $m \geq 0$ and $0 \leq r < 5$.

Case 1. Let $r = 0$. Then $n = 5m$

$\therefore$ $n + 4 = 5m + 4,$

$n + 8 = 5(m + 1) + 3 = 5q + 3$

where $q = m + 1$ is an integer.

$n + 12 = 5(m + 2) + 2 = 5q + 2,$

where q = m + 2 is an integer

$n + 16 = 5(m + 3) + 1 = 5q + 1,$

where $q = m + 3$ is an integer

Case 2. Let $r = 1$. Then $n = 5m + 1$

$\therefore$ $n + 4 = 5(m + 1) = 5q$

where $q = m + 1$ is an integer

$n + 8 = 5(m + 1) + 4$
$= 5q + 4,$

where $q = m + 1$ is an integer

$n + 12 = 5(m + 2) + 3 = 5q + 3,$

where $q = m + 2$ is an integer

$n + 16 = 5(m + 3) + 2 = 5q + 2,$

where $q = m + 3$ is an integer

Case 3. Let $r = 2$. The $n = 5m + 2$

$\therefore$ $n + 4 = 5(m + 1) + 1 = 5q + 1,$

where $q = m + 1$ is an integer

$n + 8 = 5(m + 2) = 5q,$

where $q = m + 2$ is an integer

$n + 12 = 5(m + 2) + 4 = 5q + 4,$

where $q = m + 2$ is an integer

$n + 16 = 5(m + 3) + 3 = 5q + 3,$

where $q = m + 3$ is an integer

Case 4. Let $r = 3$. Then $n = 5m + 3$

$\therefore$ $n + 4 = 5(m + 1) + 2 = 5q + 2,$

where $q = m + 1$ is an integer

$n + 8 = 5(m + 2) + 1 = 5q + 1,$

where $q = m + 2$ is an integer.

$n + 12 = 5(m + 3) = 5q,$

where $q = m + 3$ is an integer

$n + 16 = 5(m + 3) + 4 = 5q + 4,$

where $q = m + 4$ is an integer.

Case 5. Let $r = 4$. Then $n = 5m + 4$

$\therefore$ $n + 4 = 5(m + 1) + 3 = 5q + 3,$

where $q = m + 1$ is an integer

$n + 8 = 5(m + 2) + 2 = 5q + 2,$

where $q = m + 2$ is an integer

$n + 12 = 5(m + 3) + 1 = 5q + 1,$

where $q = m + 3$ is an integer

$n + 16 = 5(m + 4) = 5q,$

where $q = m + 4$ is an integer

Hence, it follows that one and only one out of n, $n + 4$, $n + 8$, $n + 12$ and $n + 16$ is divisible by 5 where n is any positive integer.

7. Let n and $n + 1$ be the two consecutive positive integers.

We know n is of the form $2q$ or $2q + 1$, where q is some integer.

Case 1. When $n = 2q$ then $n + 1 = (2q + 1)$ and their product $= 2q(2q + 1) = 2(2q^2 + q)$ which is divisible by 2.

Case 2. When $n = (2q + 1)$ then $n + 1 = 2q + 1 + 1$ $= 2q + 2$ and their product $= (2q + 1)(2q + 2)$ $= 2(2q + 1)(q + 1)$ which is divisible by 2.

Hence, the product of two consecutive positive integers is divisible by 2.

8. Let n, $n + 1$ and $n + 2$ be the three consecutive positive integers.

We know that n is of the form $6q$, $6q + 1$ or $6q + 2$ or $6q + 3$ or $6q + 4$ or $6q + 5$, q is some integer (refer to example 3).

Case 1. When $n = 6q,$

then $n + 1 = 6q + 1$

and $n + 2 = 6q + 2$

and their product $= 6q(6q + 1)(6q + 2)$
$= 6[q(6q + 1)(6q + 2)]$

which is divisible by 6.

Case 2. When $n = 6q + 1$,

then $n + 1 = 6q + 2$

and $n + 2 = 6q + 3$

and their product $= (6q + 1)(6q + 2)(6q + 3)$
$= (6q + 1)\, 2(3q + 1)\, 3(2q + 1)$
$= 6(6q + 1)(3q + 1)(2q + 1)$

which is divisible by 6.

Case 3. When $n = 6q + 2$

then $n + 1 = 6q + 3$

and $n + 2 = 6q + 4$

and their product $= (6q + 2)(6q + 3)(6q + 4)$
$= 2(3q + 1)\, 3(2q + 1)\, 2(3q + 2)$
$= 12(3q + 1)(2q + 1)(3q + 2)$

which is divisible by 6.

Case 4. When $n = 6q + 3$,

then $n + 1 = 6q + 4$

and $n + 2 = 6q + 5$

and their product $= (6q + 3)(6q + 4)(6q + 5)$
$= 3(2q + 1)\, 2(3q + 2)(6q + 5)$
$= 6(2q + 1)(3q + 2)(6q + 5)$

which is divisible by 6.

Case 5. When $n = 6q + 4$,

then $n + 1 = 6q + 5$

and $n + 2 = 6q + 6$

and their product $= (6q + 4)(6q + 5)(6q + 6)$
$= 2(3q + 2)(6q + 5)\, 6(q + 1)$
$= 12(3q + 2)(6q + 5)(q + 1)$

which is divisible by 6.

Case 6. When $n = 6q + 5$,

then $n + 1 = 6q + 6$

and $n + 2 = 6q + 7$

and their product $= (6q + 5)(6q + 6)(6q + 7)$
$= 6(6q + 5)(q + 1)(6q + 7)$

which is divisible by 6.

Hence, the product of three consecutive positive integers is divisible by 6.

9. $n^3 - n = n(n^2 - 1)$
$= n(n - 1)(n + 1)$

We know that any positive integer is of the form $6q$ or $6q + 1$ or $6q + 2$ or $6q + 3$ or $6q + 4$ or $6q + 5$

Case 1. When $n = 6q$,

then $n(n - 1)(n + 1) = 6q(6q - 1)(6q + 1)$

which is divisible by 6.

Case 2. When $n = 6q + 1$

then $n(n - 1)(n + 1) = (6q + 1)(6q + 1 - 1)(6q + 1 + 1)$
$= (6q + 1)(6q)(6q + 2)$
$= (6q + 1)(6q)\, 2(3q + 1)$
$= 12q(6q + 1)(3q + 1)$

which is divisible by 6.

Case 3. When $n = 6q + 2$,

then $n(n - 1)(n + 1) = (6q + 2)(6q + 2 - 1)(6q + 2 + 1)$
$= 2(3q + 1)(6q + 1)\, 3(2q + 1)$
$= 6(3q + 1)(6q + 1)(2q + 1)$

which is divisible by 6.

Case 4. When $n = 6q + 3$,

then $n(n - 1)(n + 1) = (6q + 3)(6q + 3 - 1)(6q + 3 + 1)$
$= (6q + 3)(6q + 2)(6q + 4)$
$= 3(2q + 1)\, 2(3q + 1)\, 2(3q + 2)$
$= 12(2q + 1)(3q + 1)(3q + 2)$

which is divisible by 6.

Case 5. When $n = 6q + 4$,

then $n(n - 1)(n + 1) = (6q + 4)(6q + 4 - 1)(6q + 4 + 1)$
$= (6q + 4)(6q + 3)(6q + 5)$
$= 2(3q + 2)\, 3(2q + 1)(6q + 5)$
$= 6(3q + 2)(2q + 1)(6q + 5)$

which is divisible by 6.

Case 6. When $n = 6q + 5$,

then $n(n - 1)(n + 1) = (6q + 5)(6q + 5 - 1)(6q + 5 + 1)$
$= (6q + 5)(6q + 4)(6q + 6)$
$= (6q + 5)\, 2(3q + 2)\, 6(q + 1)$
$= 12(6q + 5)(3q + 2)(q + 1)$

which is divisible by 6.

Hence, for any positive integer n, $n^3 - n$ is divisible by 6.

For Standard Level

10. Let 'a' be any positive integer. On dividing 'a' by 6, let m be the quotient and r be the remainder, where $m \geq 0$ and $0 \leq r < 6$.

Then, by Euclid's division lemma, we have $a = 6m + r$ for some integer $m \geq 0$ and $0 \leq r < 6$. We now consider the following cases:

Case 1. Let $r = 0$.

Then, $a = 6m$

$\Rightarrow$ $a^2 = 36m^2 = 6(6m^2)$
$= 6q$

where $q = 6m^2$ is an integer

Case 2. Let $r = 1$.

Then, $a = 6m + 1$

$\therefore$ $a^2 = (6m + 1)^2$
$= 36m^2 + 12 + 1$
$= 6(6m^2 + 2m) + 1$
$= 6q + 1$

where $q = 6m^2 + 2m$ is an integer.

Case 3. Let $r = 2$.

Then $a = 6m + 2$

$\therefore$ $a^2 = (6m + 2)^2$
$= 36m^2 + 24m + 4$
$= 6(6m^2 + 4m) + 4$
$= 6q + 4$

where $q = 6m^2 + 4m$ is an integer.

Case 4. Let $r = 3$.

Then $a = 6m + 3$

$\therefore$ $a^2 = (6m + 3)^2$

$= 36m^2 + 36m + 9$
$= 6(6m^2 + 6m + 1) + 3$
$= 6q + 3$

Where $q = 6m^2 + 6m + 1$ is an integer.

Case 5. Let $r = 4$.

Then $a = 6m + 4$

$\therefore$ $a^2 = (6m + 4)$
$= 36m^2 + 48m + 16$
$= 6(6m^2 + 8m + 2) + 4$
$= 6q + 4$

where $q = 6m^2 + 8m + 2$ is an integer

Case 6. Let $r = 5$.

Then $a = 6m + 5$

$\therefore$ $a^2 = (6m + 5)^2 = 36m^2 + 60m + 25$
$= 6(6m^2 + 10m + 4) + 1$
$= 6q + 1,$

Where $q = 6m^2 + 10m + 4$ is an integer.

Since a^2 is of the form $6q$, $6q + 1$, $6q + 4$, $6q + 3$

Hence, the square of any integer cannot be of the form $6q + 2$, $6q + 5$ for any integer q.

11. Let a be any positive integer.

On dividing a by 4, let q be the quotient and r be the remainder.

Then, by Euclid's division lemma, we have

$a = 4q + r,$

where $0 \le r < 4$

$\Rightarrow$ $a = 4q$ (when $r = 0$),
$a = 4q + 1$ (when $r = 1$),
$a = 4q + 2$ (when $r = 2$),
and $a = 4q + 3$ (when $r = 3$)

$a = 4q$

$\Rightarrow$ $a^3 = (4q)^3$
$= 64q^3$
$= 4(16q^3)$
$= 4m,$

where $m = 16q^3$ is an integer.

$a = 4q + 1$

$\Rightarrow$ $a^3 = (4q +1)^3$
$= 64q^3 + 48q^2 + 12q +1$
$= 4(16q^3 + 12q^2 + 3q) + 1$
$= 4m + 1,$

where $m = (16q^3 + 12q^2 + 3q)$ is an integer

$a = 4q + 2$

$\Rightarrow$ $a^3 = (4q +2)^3$
$= 64q^3 + 96q^2 + 48q + 8$
$= 4(16q^3 + 24q^2 + 12q + 2)$
$= 4m,$

where $m = (16q^3 + 24q^2 + 12q + 2)$ is an integer

$a = 4q + 3$

$\Rightarrow$ $a^3 = (4q + 3)^3$
$= 64q^3 + 144q^2 + 108q + 27$
$= 64q^3 + 144q^2 + 108q + 24 + 3$
$= 4(16q^3 + 36q^2 + 27q + 6) + 3$
$= 4m + 3,$

where $m = (16q^3 + 36q^2 + 27q + 6)$ is an integer

Hence, the cube of any positive integer is of the form $4m$, $4m + 1$ or $4m + 3$ for some integer m.

12. Given positive integer is of the form $6q + r$, where q is an integer and $r = 0, 1, 2, 3, 4, 5$.

Thus, the given positive integer may be $6q + 0$, $6q + 1$, $6q + 2$, $6q + 3$, $6q + 4$ or $6q + 5$.

$(6q + 0)^3 = 216q^3$
$= 6(36q^3)$
$= 6m + 0$
$= 6m + r,$

where $m = 36q^3$ is an integer and $r = 0$

$(6q + 1)^3 = 216q^3 + 108q^2 + 18q + 1$
$= 6(36q^3 + 18q^2 + 3q) + 1$
$= 6m + 1$
$= 6m + r,$

where $m = 36q^3 + 18q^2 + 3q$ is an integer and $r = 1$

$(6q + 2)^3 = 216q^3 + 216q^2 + 72q + 8$
$= 216q^3 + 216q^2 + 72q + 6 + 2$
$= 6(36q^3 + 36q^2 + 12q + 1) + 2$
$= 6m + 2 = 6m + r,$

where $m = (36q^3 + 36q^2 + 12q + 1)$ is an integer and $r = 2$

$(6q + 3)^3 = 216q^3 + 324q^2 + 162q + 27$
$= 216q^3 + 324q^2 + 162q + 24 + 3$
$= 6(36q^3 + 54q^2 + 27q + 4) + 3$
$= 6m + 3 = 6m + r,$

where $m = (36q^3 + 54q^2 + 27q + 4)$ is an integer and $r = 3$

$(6q + 4)^3 = 216q^3 + 432q^2 + 288q + 64$
$= 216q^3 + 432q^2 + 288q + 60 + 4$
$= 6(36q^3 + 72q^2 + 48q + 10) + 4 = 6m + 4$
$= 6m + r,$

where $m = (36q^3 + 72q^2 + 48q + 10)$ is an integer and $r = 4$

$(6q + 5)^3 = 216q^3 + 540q^2 + 450q + 125$
$= 216q^3 + 540q^2 + 450q + 120 + 5$
$= 6(36q^3 + 90q^2 + 75q + 20) + 5$
$= 6m + 5 = 6m + r,$

where $m = (36q^3 + 90q^2 + 75q + 20)$ is an integer and $r = 5$

Hence, the cube of a positive integer of the form $6q + r$ is also of the form $6m + r$.

EXERCISE 1B

For Basic and Standard Levels

1. The smallest prime number is 2 and the smallest composite number is 4. Now, since $4 = 2 \times 2 + 0$, hence, the HCF of these two numbers is **2**.

2. (*i*) Applying Euclid's division lemma to 650 and 1170, we get

$$1170 = 650 \times 1 + 520$$

650) 1170 (1
− 650
520) 650 (1
− 520
130) 520 (4
− 520
x

Since the remainder 520 ≠ 0, so we apply the division lemma to the divisor 650 and the remainder 520, to get

$$650 = 520 \times 1 + 130$$

As the remainder 130 ≠ 0, we apply the division lemma to the new divisor 520 and the new remainder 130, to get

$$520 = 130 \times 4 + 0$$

We observe that the final remainder is 0. So, the last divisor **130** is the required HCF of 650 and 1170.

(*ii*) Applying Euclid's division lemma to the given numbers 1260 and 7344, we get

$$7344 = 1260 \times 5 + 1044$$

Since the remainder 1044 ≠ 0, so we apply the division lemma to the divisor 1260 and the remainder 1044, to get

$$1260 = 1044 \times 1 + 216$$

1260) 7344 (5
− 6300
1044) 1260 (1
− 1044
216) 1044 (4
− 864
180) 216 (1
− 180
36) 180 (5
180
x

As the remainder 216 ≠ 0, we apply the division lemma to the new divisor 1044 and the new remainder 216, to get

$$1044 = 216 \times 4 + 180$$

As the remainder 180 ≠ 0, we apply the division lemma to the new divisor 216 and the new remainder 180, to get

$$216 = 180 \times 1 + 36$$

As this remainder 36 ≠ 0, we apply the division lemma to the new divisor 180 and the new remainder 36, and get

$$180 = 36 \times 5 + 0$$

We observe that the final remainder is 0.

∴ The divisor **36** is the required HCF of 1260 and 7344.

(*iii*)

9828) 14742 (1
− 9828
4914) 9828 (2
− 9828
x

HCF = 4914

(*iv*)

2937) 180670 (61
− 17622
4450
− 2937
1513) 2937 (1
− 1513
1424) 1513 (1
− 1424
89) 1424 (16
− 89
534
− 534
x

HCF = 89

(*v*)

17017) 274170 (16
− 17017
104000
− 102102
1898) 17017 (8
− 15184
1833) 1898 (1
− 1833
65) 1833 (28
− 130
533
− 520
13) 65 (5
− 65
x

HCF = 13

(*vi*)

7378) 92690 (12
− 7378
18910
− 14756
4154) 7378 (1
− 4154
3224) 4154 (1
− 3224
930) 3224 (3
− 2790
434) 930 (2
− 868
62) 434 (7
− 434
x

62) 7161 (115
− 62
96
− 62
341
310
31) 62 (2
− 62
x

HCF = 31

3. Maximum number of gift hampers = HCF of 40, 24 and 16.

```
16)24(1          8)40(5
  -16             -40
   8)16(2           x
    -16
      x
```

$\therefore$ HCF of 40, 24 and 16 is 8.

She can make **8 hampers**.

[**Note:** Each hamper will contain $\frac{40}{8}$ = 5 bananas, $\frac{24}{8}$ = 3 oranges and $\frac{16}{8}$ = 2 pineapples.]

4. Maximum number of columns consisting of same number of boys and girls = HCF of 60 and 72.

```
60)72(1
  -60
  12)60(5
    -60
      x
```

HCF of 60 and 72 = 12

$\therefore$ Maximum number of required columns = **12**.

5. Maximum number of columns (consisting of equal no. of members) in which the contingent of 1000 members and army band of 56 members can walk in a parade = HCF of 56 and 1000 = 8.

```
56)1000(17
   -56
    440
   -392
     48)56(1
       -48
         8)48(6
          -48
            x
```

$\therefore$ They can march in **8 columns**.

[**Note:** Each column will consists of $\frac{1000}{8}$ = 125 members of contingent and $\frac{56}{8}$ = 7 members of the army band.]

6. The maximum number of identical flower arrangement with 45 roses, 65 carnations and 50 tulips will be the HCF of 45, 65 and 50.

By Euclid's division lemma to 65 and 45, we have

$$65 = 45 \times 1 + 20$$

Since the remainder 20 ≠ 0, applying division lemma to 45 and 20, we get

$$45 = 20 \times 2 + 5$$

Since the remainder 5 ≠ 0, applying division lemma to 20 and 5, we get

$$20 = 5 \times 4 + 0$$

```
45)65(1
  -45
   20)45(2
     -40
       5)20(4
        -20
          x
```

Since the remainder is 0, the HCF of 45 and 65 is 5.

Now, applying the division lemma to 50 and 5, we get

$$50 = 5 \times 10 + 0$$

```
5)50(10
 -50
   x
```

Since the final remainder is 0, hence, the HCF of 45, 65 and 50 is 5.

Hence, the required maximum number of arrangements is **5**.

For Standard Level

7. $153 = 85 \times 1 + 68$ … (1)

```
85)153(1
   -85
    68
```

$85 = 68 \times 1 + 17$ … (2)

```
68)85(1
  -68
   17
```

$68 = 17 \times 4 + 0$

```
17)68(4
  -68
    x
```

$\therefore$ HCF of 85 and 153 is 17.

From (2), $17 = 85 - 68 \times 1$

$= 85 - (153 - 85 \times 1)$ [Using (1)]

$\Rightarrow$ $17 = 85 - 153 + 85$

$= 85 \times 2 + 153 \times (-1)$

$\Rightarrow$ $\mathbf{17 = 85x + 153y}$,

where $\mathbf{x = 2, y = -1}$.

8. $1155 = 506 \times 2 + 143$ … (1)

```
506)1155(2
   -1012
     143
```

$506 = 143 \times 3 + 77$ … (2)

```
143)506(3
   -429
     77
```

$143 = 77 \times 1 + 66$ … (3)

```
77)143(1
   -77
    66
```

$77 = 66 \times 1 + 11$ … (4)

```
66)77(1
  -66
   11
```

$66 = 11 \times 6 + 0$ … (5)

```
11)66(6
  -66
    x
```

$\therefore$ HCF of 506 and 1155 is 11.

Given, HCF $= 506x + 1155 \times (-7)$

$\Rightarrow$ $11 = 506x - 8085$

$\Rightarrow$ $506x = 8096$

$\Rightarrow$ $\mathbf{x = 16}$

9. $81 = 63 \times 1 + 18$ … (1)

```
63)81(1
  -63
   18
```

$63 = 18 \times 3 + 9$ … (2)

```
18)63(3
  -54
    9
```

$18 = 9 \times 2 + 0$

```
9)18(2
 -18
   x
```

∴ HCF of 63 and 81 = 9.

From (2), $9 = 63 - 18 \times 3$

$= 63 - (81 - 63) \times 3$ [Using (2)]

$= 63 - 81 \times 3 + 63 \times 3$

$= 63 \times 4 + 81 \times (-3),$

where $\boldsymbol{x = 4}$ **and** $\boldsymbol{y = -3}$

Now, $9 = 63 \times 4 + 81 \times (-3)$

$= 63 \times 4 + 81 \times (-3) - 63 \times 81 + 63 \times 81$

$= 63(4 - 81) + 81(63 - 3)$

$= 63(-77) + 81(60),$

where $x = -77$ and $y = 60$

Hence, $\boldsymbol{x}$ **and** $\boldsymbol{y}$ **are not unique**.

10. The number of students in each bus must be HCF of 156, 208 and 260.

156) 208 (1
− 156
52) 156 (3
− 156
x

52) 260 (5
− 260
x

∴ HCF of 156, 208 and 260 = 52.

∴ In each bus maximum 52 students can be seated.

Total number of students

$= 156 + 208 + 260 = 624$

∴ Minimum number of buses required $= \dfrac{624}{52} = \mathbf{12}$

11. Length of longest rod in cm which can measure the given dimensions exactly = HCF of 810, 630 and 540 (in cm).

630) 810 (1
− 630
180) 630 (3
− 540
90) 180 (2
− 180
x

90) 540 (6
− 540
x

HCF = 90

∴ Required length of rod = **90 cm**

12. Largest number that divides 382, 446 and 674 leaves remainder 5, 11 and 7 respectively.

∴ $382 - 5 = 377$, $446 - 11 = 435$ and $674 - 7 = 667$ are completely divisible by the required number.

∴ The required number is the HCF of 377, 435 and 667.

377) 435 (1
− 377
58) 377 (6
− 348
29) 58 (2
− 58
x

29) 667 (23
− 58
87
− 87
x

HCF of 377, 435 and 667 is 29.

∴ The required number is **29**.

13. Since 1251, 9377 and 15628 divided by the required largest number leave the remainders 1, 2 and 3 respectively,

∴ $1251 - 1 = 1250$, $9377 - 2 = 9375$ and $15628 - 3 = 15625$ are completely divisible by the required number. Clearly, the required largest number is the HCF of 1250, 9375 and 15625.

Applying Euclid's division lemma to 1250 and 9375, we get

$$9375 = 1250 \times 7 + 625$$

Since the remainder $625 \neq 0$, therefore, applying division lemma to 625 and 1250, we get

$$1250 = 625 \times 2 + 0$$

Since the remainder is 0, the divisor 625 is the HCF of 1250 and 9375.

1250) 9375 (7
− 8750
625) 1250 (2
− 1250
x

Now, applying divisor lemma to 15625 and 625, we get

$$15625 = 625 \times 25 + 0$$

625) 15625 (25
− 1250
3125
− 3125
x

Since the remainder is 0, so the divisor 625 is the HCF of 625 and 15625.

∴ The HCF of 1250, 9375 and 15625 is 625.

∴ The required number is **625**.

EXERCISE 1C

For Basic and Standard Levels

1. (*i*)

2	156
2	78
3	39
13	13
	1

$\mathbf{156 = 2^2 \times 3 \times 13}$

(*ii*)

2	336
2	168
2	84
2	42
3	21
	7

$\mathbf{336 = 2^4 \times 3 \times 7}$

(*iii*)

2	1296
2	648
2	324
2	162
3	81
3	27
3	9
3	3
	1

$\mathbf{1296 = 2^4 \times 3^4}$

(*iv*)

2	8232
2	4116
2	2058
3	1029
7	343
7	49
7	7
	1

$\mathbf{8232 = 2^3 \times 3 \times 7^3}$

(*v*)

2	26676
2	13338
3	6669
3	2223
3	741
13	247
19	19
	1

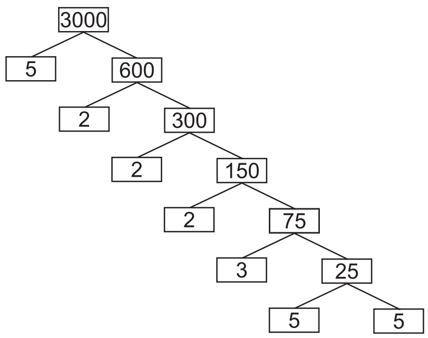
$26676 = 2^2 \times 3^3 \times 13 \times 19$

(*vi*)

2	58500
2	29250
3	14625
3	4875
5	1625
5	325
5	65
13	13
	1

$58500 = 2^2 \times 3^2 \times 5^3 \times 13$

2.



3. (*i*) The only prime factors of 7^n are 1 and 7.

According to the fundamental theorem of arithmetic, the prime factorization of each number is unique.

$\therefore$ 7^n cannot have 2 and 5 as a factor.

For 7^n to have unit digit 0 for any $n \in N$, it must have 2 and 5 also as factors.

$\therefore$ **7^n cannot have unit digit 0 for any $n \in$ N.**

(*ii*) $8^n = (2^3)^n = 2^{3n}$

$\Rightarrow$ The only prime in the factorization of 8^n is 2. By the uniqueness of fundamental theorem of arithmetic, there is no other prime in the factorization of $8^n = 2^{3n}$

$\therefore$ 5 does not occur in the prime factorization of 8^n for any $n \in$ N.

For 8^n to end with digit 0 for any natural number n, it must have 5 also as a factor.

$\therefore$ **8^n cannot end with digit 0 for any $n \in$ N.**

(*iii*) $15^n = (3 \times 5)^n = 3^n \times 5^n$

$\Rightarrow$ The only prime factors of 15^n are 3 and 5. According to the fundamental theorem of arithmetic, the prime factorization of each number is unique.

$\therefore$ 15^n cannot have 2 as a factor.

For 15^n to end with digit 0 for any $n \in$ N, it must have 2 also as a factor.

$\therefore$ **$(15)^n$ cannot end with digit 0 for any $n \in$ N.**

(*iv*) $(26)^n = (2 \times 13)^n = 2^n \times 13^n$

$\Rightarrow$ The only prime factors of 26^n are 2 and 13. According to the fundamental theorem of arithmetic, the prime factorization of each number is unique.

$\therefore$ $(26)^n$ cannot have 5 as a factor.

For $(26)^n$ to end with digit 5 for any $n \in$ N, it must have 5 also as a factor.

$\therefore$ **$(26)^n$ cannot end with digit 5 for any $n \in$ N.**

(*v*) $(28)^n = (2 \times 2 \times 7)^n = (2^2 \times 7)^n = 2^{2n} \times 7^n$

$\Rightarrow$ The only prime factors of $(28)^n$ are 2 and 7. According to the fundamental theorem of arithmetic, the prime factorization of each number is unique.

$\therefore$ $(28)^n$ cannot have 5 as a factor.

For $(28)^n$ to end with digit 0 for any $n \in$ N, it must have 5 also as a factor.

$\therefore$ **$(28)^n$ cannot end with digit 0 for any $n \in$ N.**

(*vi*) $2^1 = 2$, $2^2 = 4$, $2^3 = 8$, $2^4 = 16$, 2^5 ends with 2, 2^6 ends with digit 4, 2^7 ends with digit 8, 2^8 ends with digit 6 and the cycle goes on.

$\therefore$ **2^n can end with digit 6 for any $n \in$ N when n = 4, 8, 12…. and so on.**

4. (*i*)

2	6
3	3
	1

2	20
2	10
5	5
	1

$6 = 2 \times 3$ $\quad$ $20 = 2^2 \times 5$

LCM = $2^2 \times 3 \times 5$ = **60**

HCF = **2**

(*ii*)

2	144
2	72
2	36
2	18
3	9
3	3
	1

2	98
7	49
7	7
	1

$144 = 2^4 \times 3^2$ $\quad$ $98 = 2 \times 7^2$

LCM = $2^4 \times 3^2 \times 7^2$ = **7056**

HCF = **2**

(*iii*)

2	570
3	285
5	95
19	19
	1

3	1425
5	475
5	95
19	19
	1

$570 = 2 \times 3 \times 5 \times 19$ $\quad$ $1425 = 3 \times 5^2 \times 19$

LCM = $2 \times 3 \times 5^2 \times 19$ = **2850**

HCF = $3 \times 5 \times 19$ = **285**

(*iv*)

19	19
	1

13	13
	1

7	7
	1

$19 = 19 \times 1$ $\quad$ $13 = 13 \times 1$ $\quad$ $7 = 7 \times 1$

LCM = $19 \times 13 \times 7$ = **1729**

HCF = **1**

(*v*)

5	275
5	55
11	11
	1

3	225
3	75
5	25
5	5
	1

5	175
5	35
7	7
	1

$275 = 5^2 \times 11$ $\quad$ $225 = 3^2 \times 5^2$ $\quad$ $175 = 5^2 \times 7$

LCM = $3^2 \times 5^2 \times 7 \times 11$= **17325**

HCF = 5^2 = **25**

(*vi*)

3	765
3	255
5	85
17	17
	1

2	510
3	255
5	85
17	17
	1

2	408
2	204
2	102
3	51
17	17
	1

$765 = 3^2 \times 5 \times 17$, $510 = 2 \times 3 \times 5 \times 17$, $408 = 2^3 \times 3 \times 17$

LCM $= 2^3 \times 3^2 \times 5 \times 17 =$ **6120**

HCF $= 3 \times 17 =$ **51**

5. (*i*)

2	12
2	6
3	3
	1

3	15
5	5
	1

$12 = 2^2 \times 3$ $\quad$ $15 = 3 \times 5$

2	20
2	10
5	5
	1

2	54
3	27
3	9
3	3
	1

$20 = 2^2 \times 5$ $\quad$ $54 = 2 \times 3^3$

LCM $= 2^2 \times 3^3 \times 5 =$ **540**

(*ii*)

2	14
7	7
	1

2	28
2	14
7	7
	1

$14 = 2 \times 7$ $\quad$ $28 = 2^2 \times 7$

2	36
2	18
3	9
3	3
	1

3	45
3	15
5	5
	1

$36 = 2^2 \times 3^2$ $\quad$ $45 = 3^2 \times 5$

LCM $= 2^2 \times 3^2 \times 5 \times 7 =$ **1260**

6. Prime factors of a and b are x and y with greatest exponents 3 and 3 respectively. Hence, required LCM is $\boldsymbol{x^3y^3}$.

7. (*i*)

2	72
2	36
2	18
3	9
3	3
	1

2	80
2	40
2	20
2	10
5	5
	1

2	120
2	60
2	30
3	15
5	5
	1

$72 = 2^3 \times 3^2$ $\quad$ $80 = 2^4 \times 5$ $\quad$ $120 = 2^3 \times 3 \times 5$

LCM $= 2^4 \times 3^2 \times 5 =$ **720**

(*ii*) (*a*)

2	26260
2	13130
5	6565
13	1313
	101

2	1482
3	741
13	247
19	19
	1

$26260 = 2^2 \times 5 \times 13 \times 101$ $\quad$ $1428 = 2 \times 3 \times 13 \times 19$

LCM $= 2^2 \times 3 \times 5 \times 13 \times 101 =$ **1496820**

HCF $= 2 \times 13 =$ **26**

(*b*)

3	43263
3	14421
11	4807
19	437
23	23
	1

5	15295
7	3059
19	437
23	23
	1

$43263 = 3^2 \times 11 \times 19 \times 23$ $\quad$ $15295 = 5 \times 7 \times 19 \times 23$

LCM $= 3^2 \times 5 \times 7 \times 11 \times 19 \times 23 =$ **1514205**

HCF $= 19 \times 23 =$ **437**

(*c*)

41	41
	1

2	14
7	7
	1

$41 = 41 \times 1$ $\quad$ $14 = 2 \times 7$

LCM $= 2 \times 7 \times 41 =$ **574**

HCF $=$ **1**

8. We first find the prime factors of 404 and 96 as follows:

2	404
2	202
	101

2	96
2	48
2	24
2	12
2	6
	3

$\therefore \quad 404 = 2^2 \times 101$

and $\quad 96 = 2^5 \times 3$

$\therefore \quad$ LCM $= 2^5 \times 101 \times 3 =$ **9696**

and $\quad$ HCF $= 2^2 =$ **4**

$\therefore$ HCF × LCM $= 9696 \times 4 = 38784$ …(1)

Also, product of two given numbers

$= 404 \times 96 = 38784$ …(2)

$\therefore$ From (1) and (2), we see that

HCF × LCM = Product of the two given numbers.

9. Taking the greatest exponents of each prime factor of p and q, we get LCM $(p, q) = a^3b^3$.

Again, taking the least exponent of each common prime factor of p and q, we get

HCF$(p, q) = a^2b$

$\therefore \quad pq = a^3b^3 \times a^2b = a^5b^4$ …(1)

Also, LCM $(p, q) \times$ HCF $(p, q) = a^3b^3 \times a^2b = a^5b^4$ …(2)

From (1) and (2), we get

LCM(p, q) × HCF(p, q) = pq

10. LCM of two numbers $= \dfrac{\text{Product of the numbers}}{\text{their HCF}}$

$\therefore \quad$ LCM (435, 725) $= \dfrac{435 \times 725}{145} =$ **2175**

11. HCF of two numbers $= \dfrac{\text{Product of the numbers}}{\text{their LCM}}$

$\therefore$ HCF (396, 576) $= \dfrac{396 \times 576}{6336} =$ **36**

12. Let the other number be x.

Product of two numbers = Product of their LCM and HCF

$$1071x = 11781 \times 119$$

$$x = \frac{11781 \times 119}{1071} = \mathbf{1309}$$

13. We have

HCF × LCM = Product of two required numbers

$\Rightarrow \quad 9 \times 360 = 45 \times x$

where x is the other required number.

$\Rightarrow \quad x = \frac{9 \times 360}{45} = 72$

∴ Required other number is **72**.

14. The first person's steps will cover distances 90 cm, 180 cm, 270 cm, … and so on.

The second person's steps will cover distances 80 cm, 160 cm, 240 cm, … and so on.

The third person's steps will cover distances 85 cm, 170 cm, 255 cm, … and so on.

Minimum distance each of them will cover before they meet again = LCM of 90, 80, 85.

$$\begin{array}{r|l} 2 & 90 \\ \hline 3 & 45 \\ \hline 3 & 15 \\ \hline 5 & 5 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} 2 & 80 \\ \hline 2 & 40 \\ \hline 2 & 20 \\ \hline 2 & 10 \\ \hline 5 & 5 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} 5 & 85 \\ \hline 17 & 17 \\ \hline & 1 \end{array}$$

$90 = 2 \times 3^2 \times 5 \qquad 80 = 2^4 \times 5 \qquad 85 = 5 \times 17$

∴ LCM of 90, 80, and 85 = $2^4 \times 3^2 \times 5 \times 17$ = 12240.

Hence, the minimum distance each of them will cover before they meet again = 12240 cm = **122 m 40 cm**.

15. First student beats the drums after 10, 20, 30, 40 (seconds), … and so on.

Second student beats the drums after 12, 24, 36, 48 (seconds), … and so on.

∴ Minimum time after which they beat the drums at the same instant = LCM of 10 and 12

$$\begin{array}{r|l} 2 & 10 \\ \hline 5 & 5 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} 2 & 12 \\ \hline 2 & 6 \\ \hline 3 & 3 \\ \hline & 1 \end{array}$$

$10 = 2 \times 5 \qquad 12 = 2^2 \times 3$

LCM of 10 and 12 = $2^2 \times 3 \times 5$ = 60.

They will beat the drum at the same instant after 60 seconds, i.e. **1 minute**.

16. 1st bell ring after 4, 8, 12, 16 (minutes), … and so on.

2nd bell rings after 7, 14, 21, 28 (minutes), … and so on.

3rd bell rings after 14, 28, 42, 56 (minutes), … and so on.

Minutes after which the three bells will ring together again = LCM of 4, 7 and 14 (in minutes).

$$\begin{array}{r|l} 2 & 4 \\ \hline 2 & 2 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} 7 & 7 \\ \hline & 1 \end{array} \qquad \begin{array}{r|l} 2 & 14 \\ \hline 7 & 7 \\ \hline & 1 \end{array}$$

$4 = 2^2 \qquad 7 = 7 \times 1 \qquad 14 = 2 \times 7$

LCM = $2^2 \times 7$ = 28 (minutes)

The three bells will ring together after 28 minutes, i.e at **6.28 a.m.**

17. Planes from the 1st runway take off after 3, 6, 9, 12, 15 (minutes), … and so on.

Planes from the 2nd runway take off after 4, 8, 12, 16 (minutes), … and so on.

Planes from the 3rd runway take off after 8, 16, 24, 32 (minutes), … and so on.

Planes from the 4th runway take off after 12, 24, 36, 48 (minutes), … and so on.

Planes from the 5th runway take off after 15, 30, 45, 60 (minutes), … and so on.

Minutes after which the planes from five runways take off simultaneously = LCM of 3, 4, 8, 12 and 15 (in minutes).

$$\begin{array}{r|l} 2 & 3,4,8,12,15 \\ \hline 2 & 3,2,4,6,15 \\ \hline 3 & 3,1,2,3,15 \\ \hline & 1,1,2,1,5 \end{array}$$

LCM = 2 × 2 × 3 × 2 × 5 = 120 (in minutes) = 2 hours

So, the five planes take off together at 7.30 a.m. + 2 hours = **9.30 a.m**.

18. Green colour lights come on after 10, 20, 30, 40 (seconds), … and so on.

Yellow colour light come on after 15, 30, 45, 60 (seconds), … and so on.

Seconds after which the two lights come on together after being turned on at the same time = LCM of 10 and 15 (in seconds).

$10 = 2 \times 5$

$15 = 3 \times 5$

∴ LCM of 10 and 15 = 2 × 3 × 5 = 30 (in seconds).

So, that two colour lights having turned on at the same time will come on together after **30 seconds**.

For Standard Level

19. Let the two numbers be a and b where a = 280.

Let their LCM be x and HCF be y.

Then, LCM = $x = 14y$ … (1)

and $x + y = 600$ … (2)

∴ $14y + y = 600$ [From (1) and (2)]

$\Rightarrow \quad 15y = 600$

$\Rightarrow \quad y = 40$

$\Rightarrow \quad$ HCF = 40

∴ LCM = $x = 14y = 14 \times 40 = 560$

Now, product of two numbers = product of their LCM and HCF

∴ $280 \times b = 560 \times 40$

$\Rightarrow \quad b = \frac{560 \times 40}{280} = \mathbf{80}$

20. Greatest number of 5 digits is 99999.

Required number must be divisible by LCM of 24, 15, 36.

Hence, required number = 99999 – remainder when 99999 is divisible by LCM of 24, 15, 36.

$$\begin{array}{r|l} 2 & 24,15,36 \\ \hline 2 & 12,15,18 \\ \hline 3 & 6,15,9 \\ \hline & 2,5,3 \end{array}$$

$\therefore$ LCM of 24, 15, 36 = 2 × 2 × 3 × 2 × 5 × 3 = 360

$$\begin{array}{r|l|l} 360 & 99999 & 277 \\ & -720 & \\ & 2799 & \\ & -2520 & \\ & \;\;2799 & \\ & -2520 & \\ & \;\;279 & \end{array}$$

Here, remainder = 279

$\therefore$ Required number = 99999 – 279 = **99720**

EXERCISE 1D

For Basic and Standard Levels

1. Let us assume on the contrary that $\sqrt{5}$ is a rational number and its simplest form is $\frac{a}{b}$, where a and b are integers having no common factor other than 1 and $b \neq 0$.

Now, $\sqrt{5} = \frac{a}{b}$

$\Rightarrow$ $5 = \frac{a^2}{b^2}$

$\Rightarrow$ $5b^2 = a^2$ ... (1)

$\Rightarrow$ a^2 is divisible by 5 [$\because$ $5b^2$ is divisible by 5]

$\Rightarrow$ a is divisible by 5

[$\because$ 5 is prime and divides a^2 $\Rightarrow$ 5 divides a]

Let $a = 5c$ for some integer c.

Substituting $a = 5c$ in (1), we get

$5b^2 = (5c)^2$

$\Rightarrow$ $5b^2 = 25c^2$

$\Rightarrow$ $b^2 = 5c^2$

$\Rightarrow$ b^2 is divisible by 5 [$\because$ $5c^2$ is divisible by 5]

$\Rightarrow$ b is divisible by 5

[$\because$ 5 is prime and divides b^2 $\Rightarrow$ 5 divides b]

Since a and b are both divisible by 5,

$\therefore$ 5 is a common factor of a and b.

But this contradicts the fact that a and b have no common factor other than 1.

The contradiction has arisen because of incorrect assumption that $\sqrt{5}$ is rational.

Hence, $\sqrt{5}$ **is irrational**.

2. Let us assume on the contrary that $\sqrt{7}$ is a rational number and its simplest form is $\frac{a}{b}$, where a and b are integers having no common factor other than 1 and $b \neq 0$.

Now, $\sqrt{7} = \frac{a}{b}$

$\Rightarrow$ $7 = \frac{a^2}{b^2} = 7b^2 = a^2$ ... (1)

$\Rightarrow$ a^2 is divisible by 7 [$\because$ $7b^2$ is divisible by 7]

$\Rightarrow$ a is divisible by 7

[7 is prime and divides a^2 $\Rightarrow$ 7 divides a]

Let $a = 7c$ for some integer c.

Substituting $a = 7c$ in (1), we get

$7b^2 = (7c)^2$

$\Rightarrow$ $7b^2 = 49c^2$

$\Rightarrow$ $b^2 = 7c^2$

$\Rightarrow$ b^2 is divisible by 7 [$\because$ $7c^2$ is divisible by 7]

$\Rightarrow$ b is divisible by 7

[$\because$ 7 is prime and divides a^2 $\Rightarrow$ 7 divides a]

Since a and b are both divisible by 7,

$\therefore$ 7 is a common factor of a and b.

But this contradicts the fact that a and b have no common factor other than 1.

The contradiction has arisen because of incorrect assumption that $\sqrt{7}$ is rational.

Hence, $\sqrt{7}$ **is irrational**.

3. Let us assume on the contrary that $\sqrt{11}$ is a rational number and its simplest form is $\frac{a}{b}$, where a and b are integers having no common factor other than 1 and $b \neq 0$.

Now, $\sqrt{11} = \frac{a}{b}$

$\Rightarrow$ $11 = \frac{a^2}{b^2}$

$\Rightarrow$ $11b^2 = a^2$ ... (1)

$\Rightarrow$ a^2 is divisible by 11 [$\because$ $11b^2$ is divisible by 11]

$\Rightarrow$ a is divisible by 11

[$\because$ 11 is a prime and divides a^2 $\Rightarrow$ 11 divides a]

Let $a = 11c$ for some integer c

Substituting $a = 11c$ in (1), we get

$11b^2 = (11c)^2$

$\Rightarrow$ $11b^2 = 121c^2$

$\Rightarrow$ $b^2 = 11c^2$

$\Rightarrow$ b^2 is divisible by 11 [$\because$ $11c^2$ is divisible by 11]

$\Rightarrow$ b is divisible by 11

[$\because$ 11 is prime and divides b^2 $\Rightarrow$ 11 divides b]

Since a and b are both divisible by 11,

$\therefore$ 11 is a common factor of a and b.

But this contradicts the fact that a and b have no common factor other than 1.

The contradiction has arisen because of incorrect assumption that $\sqrt{11}$ is rational.

Hence, $\sqrt{11}$ **is irrational**.

4. (*i*) Let us assume on the contrary that $3\sqrt{7}$ is a rational number.

Then, there exist coprime a and b ($b \neq 0$) such that

$3\sqrt{7} = \frac{a}{b}$

$\Rightarrow$ $\sqrt{7} = \frac{a}{3b}$

$\Rightarrow$ $\sqrt{7}$ is rational

[$\because$ 3, a and b are integers $\therefore$ $\frac{a}{3b}$ is a rational number]

This contradicts the fact that $\sqrt{7}$ is irrational.

The contradiction has arisen because of our incorrect assumption that $3\sqrt{7}$ is rational.

Hence, $\mathbf{3\sqrt{7}}$ **is irrational**.

(*ii*) Let us assume on the contrary that $\frac{2\sqrt{3}}{5}$ is a rational number.

Then, there exist coprime a and b ($b \neq 0$) such that

$$\frac{2\sqrt{3}}{5} = \frac{a}{b}$$

$$\Rightarrow \quad \sqrt{3} = \frac{5a}{2b}$$

$\Rightarrow \quad \sqrt{3}$ is rational

[$\because$ 2, 5, a and b are integers $\therefore$ $\frac{5a}{2b}$ is a rational number.]

This contradicts the fact that $\sqrt{3}$ is irrational.

The contradiction has arisen because of our incorrect assumption that $\frac{2\sqrt{3}}{5}$ is rational.

Hence, $\mathbf{\frac{2\sqrt{3}}{5}}$ **is irrational**.

(*iii*) Let us assume on the contrary that $3 + 5\sqrt{2}$ is rational.

Then, there exist coprime a and b ($b \neq 0$) such that

$$3 + 5\sqrt{2} = \frac{a}{b}$$

$$\Rightarrow \quad 5\sqrt{2} = \frac{a}{b} - 3 = \frac{a - 3b}{b}$$

$$\Rightarrow \quad \sqrt{2} = \frac{a - 3b}{5b}$$

$\Rightarrow \quad \sqrt{2}$ is rational [$\because$ 3, 5, a and b are integers $\because$ $\frac{a - 3b}{5b}$ is a rational number]

This contradicts the fact that $\sqrt{2}$ is irrational.

The contradiction has arisen because of our incorrect assumption that $3 + 5\sqrt{2}$ is rational.

Hence, $\mathbf{3 + 5\sqrt{2}}$ **is irrational**.

(*iv*) Let us assume on the contrary that $2\sqrt{3} - 1$ is rational.

Then, there exist coprime a and b ($b \neq 0$) such that

$$2\sqrt{3} - 1 = \frac{a}{b}$$

$$\Rightarrow \quad 2\sqrt{3} = \frac{a}{b} + 1 = \frac{a + b}{b}$$

$$\Rightarrow \quad \sqrt{3} = \frac{a + b}{2b}$$

$\Rightarrow \quad \sqrt{3}$ is rational

[$\because$ 2, a and b are integers $\therefore$ $\frac{a + b}{2b}$ is a rational number.]

$\because$ This contradicts the fact that $\sqrt{3}$ is irrational.

The contradiction has arisen because of our incorrect assumption that $2\sqrt{3} - 1$ is rational.

Hence, $\mathbf{2\sqrt{3} - 1}$ **is irrational**.

(*v*) Refer to part (*iii*)

(*vi*) Refer to part (*iv*)

(*vii*) Let us assume on the contrary $3 - \sqrt{5}$ is rational.

Then, there exist coprime a and b ($b \neq 0$) such that

$$3 - \sqrt{5} = \frac{a}{b}$$

$$\Rightarrow \quad \sqrt{5} = 3 - \frac{a}{b} = \frac{3b - a}{b}$$

$\Rightarrow \quad \sqrt{5}$ is rational

[$\because$ 3, a and b integers $\therefore$ $\frac{3b - a}{b}$ is a rational number.]

This contradicts the fact that $\sqrt{5}$ is irrational.

The contradiction has arisen because of our incorrect assumption that $3 - \sqrt{5}$ is rational.

Hence, $\mathbf{3 - \sqrt{5}}$ **is irrational**.

(*viii*) Let us assume on the contrary $12\sqrt{3} - 41$ is rational.

Then, there exist coprime a and b ($b \neq 0$) such that

$$12\sqrt{3} - 41 = \frac{a}{b}$$

$$\Rightarrow \quad 12\sqrt{3} = \frac{a}{b} + 41 = \frac{a + 41b}{b}$$

$$\Rightarrow \quad \sqrt{3} = \frac{a + 41b}{12b}$$

$\Rightarrow \quad \sqrt{3}$ is rational

[$\because$ 12, 41, a and b are integers $\therefore$ $\frac{a + 41b}{b}$ is a rational number]

This contradicts the fact that $\sqrt{3}$ is irrational.

The contradiction has arisen because of our incorrect assumption that $12\sqrt{3} - 41$ is rational.

Hence, $\mathbf{12\sqrt{3} - 41}$ **is irrational**.

(*ix*) Refer to (*iii*)

(*x*) If possible, let us assume that $7 - \sqrt{5}$ is a rational number. Then there exist coprime a and b where $b \neq 0$, such that

$$7 - \sqrt{5} = \frac{a}{b}$$

$$\Rightarrow \quad 7 - \frac{a}{b} = \sqrt{5}$$

$$\Rightarrow \quad \frac{7b - a}{b} = \sqrt{5}$$

But LHS is a rational number, since a and b are integers and $b \neq 0$. This is absurd, since we know that $\sqrt{5}$ is an irrational number.

Hence, there is a contradiction in our assumption.

Hence, $\mathbf{7 - \sqrt{5}}$ **is an irrational number**.

(*xi*) If possible, let us assume that $\frac{2+\sqrt{3}}{5}$ is a rational number. Then there exist coprime a and b, where $b \neq 0$, such that

$$\frac{2+\sqrt{3}}{5} = \frac{a}{b}$$

$$\Rightarrow \quad (2+\sqrt{3})b = 5a$$

$$\Rightarrow \quad 2+\sqrt{3} = \frac{5a}{b}$$

$$\Rightarrow \quad \sqrt{3} = \frac{5a}{b} - 2 = \frac{5a-2b}{b}$$

Now, RHS is a rational number, since a and b are integers and $b \neq 0$. This is absurd, since we know that $\sqrt{3}$ is an irrational number. Hence, there is a contradiction in our assumption.

Hence, $\mathbf{\frac{2+\sqrt{3}}{5}}$ **is an irrational number**.

(*xii*) If possible, let us assume that $\sqrt{3}+\sqrt{5}$ is a rational number. Then there exist coprime a and b, where $b \neq 0$, such that

$$\sqrt{3}+\sqrt{5} = \frac{a}{b}$$

$$\Rightarrow \quad \sqrt{3} = \frac{a}{b} - \sqrt{5}$$

$$\Rightarrow \quad 3 = \left(\frac{a}{b} - \sqrt{5}\right)^2$$

[Squaring both sides]

$$= \frac{a^2}{b^2} + 5 - \frac{2a\sqrt{5}}{b}$$

$$= \frac{a^2 + 5b^2 + 2ab\sqrt{5}}{b^2}$$

$$\Rightarrow \quad a^2 + 5b^2 + 2ab\sqrt{5} = 3b^2$$

$$\Rightarrow \quad a^2 + 5b^2 - 3b^2 = -2ab\sqrt{5}$$

$$\Rightarrow \quad \frac{a^2+2b^2}{-2ab} = \sqrt{5}$$

Since a and b are integers and $b \neq 0$, LHS is a rational number. But we know that $\sqrt{5}$ is an irrational number. Hence, there is a contradiction in our assumption.

Hence, $\mathbf{\sqrt{3}+\sqrt{5}}$ **is an irrational number**.

5. (*i*) $\frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{1}{3}\sqrt{3}$

Let us assume on the contrary that $\frac{1}{\sqrt{3}}$ is rational.

Then, $\frac{1}{3}\sqrt{3}$ is rational.

Let $\frac{1}{3}\sqrt{3} = \frac{a}{b}$ where a and b are non-zero integers having no common factor other than 1.

Now, $\frac{1}{3}\sqrt{3} = \frac{a}{b}$

$$\Rightarrow \quad \sqrt{3} = \frac{3a}{b}$$

$$\Rightarrow \quad \sqrt{3} \text{ is rational}$$

[$\because$ 3, a and b are integers $\therefore$ $\frac{3a}{b}$ is a rational number]

This contradicts the fact that $\sqrt{3}$ is irrational.

So, our assumption is wrong.

Hence, $\mathbf{\frac{1}{\sqrt{3}}}$ **is irrational**.

(*ii*) $\frac{2}{\sqrt{7}} = \frac{2}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} = \frac{2}{7}\sqrt{7}$

Let us assume on the contrary that $\frac{2}{\sqrt{7}}$ is rational.

Then, $\frac{2}{7}\sqrt{7}$ is rational.

Let $\frac{2}{7}\sqrt{7} = \frac{a}{b}$ where a and b are non-zero integers having no common factor other than 1.

Now, $\frac{2}{7}\sqrt{7} = \frac{a}{b}$

$$\sqrt{7} = \frac{7a}{2b}$$

$$\Rightarrow \quad \sqrt{7} \text{ is rational}$$

[$\because$ 2, 7, a and b are integers $\therefore$ $\frac{7a}{2b}$ is a rational number]

This contradicts the fact that $\sqrt{7}$ is irrational.

So, our assumption is wrong.

Hence, $\mathbf{\frac{2}{\sqrt{7}}}$ **is irrational**.

6. $\frac{2\sqrt{45}+3\sqrt{20}}{2\sqrt{5}} = \frac{2\sqrt{9\times5}+3\sqrt{4\times5}}{2\sqrt{5}} = \frac{6\sqrt{5}+6\sqrt{5}}{2\sqrt{5}}$

$= \frac{12\sqrt{5}}{2\sqrt{5}} = \mathbf{6}$, which is a rational number.

For Standard Level

7. If possible, let us assume that $\sqrt{p}+\sqrt{q}$ is a rational number. Then there exist coprime a and b where $b \neq 0$, such that $\sqrt{p}+\sqrt{q} = \frac{a}{b}$

$$\Rightarrow \quad \sqrt{p} = \frac{a}{b} - \sqrt{q}$$

$$\Rightarrow \quad p = \frac{a^2}{b^2} + q - \frac{2a\sqrt{q}}{b} \quad \text{[Squaring both sides]}$$

$$\Rightarrow \quad \frac{2a\sqrt{q}}{b} = \frac{a^2}{b^2} + q - p = \frac{a^2 + b^2q - b^2p}{b^2}$$

$$\Rightarrow \quad \sqrt{q} = \frac{a^2 + b^2(q-p)}{2ab}$$

Now, since, a, b, p, q are all integers, p, q are primes and $b \neq 0$.

$\therefore$ RHS is a rational number but $\sqrt{q}$ is irrational since q is prime.

So, our assumption is contradictory and hence it is wrong. Thus, $\sqrt{p} + \sqrt{q}$ **is irrational**.

EXERCISE 1E

For Basic and Standard Levels

1. (*i*) $\frac{125}{144} = \frac{5^3}{2^4 3^2}$

Clearly, 125 and 144 are coprime.

$\therefore$ $\frac{125}{144}$ is in its simplest form.

Denominator = 144 = $2^4 \times 3^2$ is not of the form $2^n 5^m$

Hence, $\frac{125}{144}$ has a **non-terminating repeating** decimal expansion.

(*ii*) $\frac{33}{50} = \frac{3 \times 11}{2 \times 5^2}$

Clearly, 13 and 50 are coprime.

$\therefore$ $\frac{13}{50}$ is in its simplest form.

Denominator = 50 = 2×5^2 is of the form $2^n 5^m$

Hence, $\frac{13}{50}$ has a **terminating** decimal expansion.

(*iii*) $\frac{41}{2^2 \times 5 \times 7}$

Since 2, 5 and 7 are not factor of 41,

$\therefore$ $\frac{41}{2^2 \times 5 \times 7}$ is in its simplest form.

Denominator = $2^2 \times 5 \times 7$ is not of the form $2^n 5^m$

Hence, $\frac{41}{2^2 \times 5 \times 7}$ has a **non-terminating repeating** decimal expansion.

(*iv*) $\frac{341}{15000} = \frac{11 \times 31}{2^3 \times 3 \times 5^4}$

Clearly, 341 and 15000 are coprime.

$\therefore$ $\frac{341}{15000}$ is in its simplest form.

Denominator = 15000 = $2^3 \times 3 \times 5^4$ is not of the form $2^n 5^m$.

Hence, $\frac{341}{15000}$ has a **non-terminating repeating** decimal expansion.

(*v*) $\frac{17}{320} = \frac{17}{2^6 \times 5}$

Clearly, 17 and 320 are coprime.

$\therefore$ $\frac{17}{320}$ is in its simplest form.

Denominator = 320 = $2^6 \times 5$ is of the form $2^n 5^m$.

Hence, $\frac{17}{320}$ has a **terminating** decimal expansion.

(*vi*) $\frac{3647}{2^2 \times 5^4} = \frac{7 \times 521}{2^2 \times 5^4}$

Since 2 and 5 are not factors of 3647,

$\therefore$ $\frac{3647}{2^2 \times 5^4}$ is in its simplest form.

Denominator = $2^2 \times 5^4$ is of the form $2^n 5^m$

Hence, $\frac{3647}{2^2 \times 5^4}$ has a **terminating** decimal expansion.

(*vii*) $\frac{33453}{2 \times 5^4} = \frac{3^4 \times 7 \times 59}{2 \times 5^4}$

Since 2 and 5 are not factors of 33453,

$\therefore$ $\frac{33453}{2 \times 5^4}$ is in its simplest form.

Denominator = 2×5^4 is of the form $2^n 5^m$.

Hence, $\frac{33453}{2 \times 5^4}$ has a **terminating** decimal expansion.

(*viii*) $\frac{441}{2^2 5^7 7^2} = \frac{3^2 \times 7^2}{2^2 \times 5^7 \times 7^2} = \frac{3^2}{2^2 \times 5^7}$

Since 2 and 5 are not factors of 3^2,

$\therefore$ $\frac{3^2}{2^2 \times 5^7}$ is in its simplest form

Denominator = $2^2 \times 5^7$ is of the form $2^n 5^m$

Hence, $\frac{441}{2^2 5^7 7^2}$ has a **terminating** decimal expansion.

(*ix*) We have $\frac{7}{75} = \frac{7}{5 \times 5 \times 3} = \frac{7}{5^2 \times 3}$

$\therefore$ 7 and 75 are coprimes.

$\therefore$ $\frac{7}{75}$ is in its simplest form.

Now, the denominator 75 = $5^2 \times 3$ which is not of the form $2^n \times 5^m$, where m and n are natural numbers.

Hence, the given rational number has **non-terminating repeating** decimal expansion.

(*x*) We have $\frac{987}{10500} = \frac{3 \times 7 \times 47}{2^2 \times 5^3 \times 3 \times 7} = \frac{47}{2^2 \times 5^3}$

3	987
7	329
	47

2	10500
2	5250
3	2625
5	875
5	175
5	35
	7

Since the denominator of the given rational number is of the form $2^n 5^m$ where m and n are natural numbers, hence, this rational number has **terminating** decimal expansion.

2. (*i*) $\frac{43}{2^4 \times 5^3} = \frac{43 \times 5}{2^4 \times 5^4} = \frac{215}{(2 \times 5)^4} = \frac{215}{(10)^4} = \frac{215}{10000}$

$= 0.0215$

So, it will terminate after **4 places** of decimals.

(*ii*) We have $\frac{23}{2^4 5^3} = \frac{23}{(2\times5)^3\times2}$

$= \frac{11.5}{1000} = 0.0115$

$\therefore$ The decimal expansion will terminate after **4 places**.

3. (*i*) $\frac{7}{8} = \frac{7}{2^3} = \frac{7\times5^3}{2^3\times5^3} = \frac{875}{10^3} = \frac{875}{1000} = \mathbf{0.875}$

(*ii*) $\frac{179}{125} = \frac{179}{5^3} = \frac{179\times2^3}{5^3\times2^3} = \frac{1432}{10^3} = \frac{1432}{1000} = \mathbf{1.432}$

(*iii*) $\frac{39}{500} = \frac{39}{2^2\times5^3} = \frac{39\times2}{2^3\times5^3} = \frac{78}{10^3} = \frac{78}{1000} = \mathbf{0.078}$

(*iv*) $\frac{2477}{1250} = \frac{2477}{2\times5^4} = \frac{2477\times2^3}{2^4\times5^4} = \frac{19816}{10^4} = \frac{19816}{10000}$

$= \mathbf{1.9816}$

(*v*) $\frac{49}{2500} = \frac{49}{2^2\times5^4} = \frac{49\times2^2}{2^4\times5^4} = \frac{196}{10^4} = \frac{196}{10000} = \mathbf{0.0196}$

(*vi*) $\frac{17}{1600} = \frac{17}{2^6\times5^2} = \frac{17\times5^4}{2^6\times5^6} = \frac{10625}{10^6} = \frac{10625}{1000000}$

$= \mathbf{0.010625}$

4. $4000 = 2^m\,5^n$

$\Rightarrow$ $2^5\times5^3 = 2^m\,5^n$

By comparing LHS and RHS,
we get $\boldsymbol{m = 5, n = 3}$

Now, $\frac{241}{4000} = \frac{241}{2^5\times5^3} = \frac{241\times5^2}{2^5\times5^5}$

$= \frac{6025}{10^5} = \frac{6025}{100000} = \mathbf{0.06025}$

2	4000
2	2000
2	1000
2	500
2	250
5	125
5	25
5	5
	1

For Standard Level

5. (*i*) Since 37.12367985 has a terminating decimal expansion,
$\therefore$ it is a **rational number**.
When expressed in the form $\frac{p}{q}$, the denominator q will be of the form $2^n\,5^m$, where n and m are non-negative integers.
Hence, **the prime factors of q will be either 2 or 5 or both**.

(*ii*) Since $29.\overline{1234567}$ has a non-terminating recurring decimal expansion,
$\therefore$ it is a **rational number** when expressed in the form $\frac{p}{q}$, the denominator will not be of the form $2^n\,5^m$ where n and m are non-negative integers.
Hence, **the prime factors of q will have a factor other than 2 or 5**.

(*iii*) Since 0.110110011000110000 …… has a non-terminating, non-recurring decimal expansion,
$\therefore$ it is **not a rational number**.

6. Let $x = 0.\overline{3178}$ … (1)

$\Rightarrow$ $10000x = 3178.\overline{3178}$ … (2)

Subtracting (1) from (2), we get

$9999x = 3178$

$\Rightarrow$ $\boldsymbol{x = \frac{3178}{9999}}$

7. Since 327.7081 is a terminating decimal number, so q **must be of the form $2^m\,5^n$**, where m and n are natural numbers.

CHECK YOUR UNDERSTANDING

MULTIPLE-CHOICE QUESTIONS

For Basic and Standard Levels

1. (*b*) **80**

Number = 19 × 4 + 4 = 80

2. (*c*) **2*q***

Even integer is a multiple of 2.

$\therefore$ For some integer q, even integer is of form $2q$.

3. (*d*) **2*m* + 1**

Odd integer is not a multiple of 2.

$\therefore$ For some integer m, odd integer = $2m + 1$.

4. (*b*) **3**

Give different values to a say 1, 2, 3, 4 … the numbers will be

1, [3,] 5
2, 4, [6]
[3,] 5, 7
4, [6,] 8
5, 7, [9]

$\therefore$ Anyone of the numbers a, $(a + 2)$ and $(a + 4)$ is a multiple of 3.

5. (*c*) $\boldsymbol{0 \le r < b}$

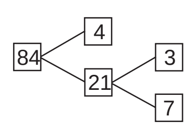
As per Euclid's division lemma, remainder can be zero or greater than zero but less than the dividend.

$\therefore$ $0 \le r < b$

6. (*a*) $\boldsymbol{0 \le r < 3}$

According to Euclid's division lemma for any positive integer a and 3, there exist unique integers q and r such that $0 \le r < 3$.

7. (*b*) $\boldsymbol{x = 21, y = 84}$



$\therefore$ $x = 21$ and $y = 84$

8. (*b*) **2**

According to definition of a prime number, it is a natural number greater than 1 and divisible by 1 and itself.

$\therefore$ Maximum number of factors of a prime number is 2.

9. (*b*) **2 and 5**

$80 = 2^4 \times 5$. So, the prime factor of denominator are 2 and 5.

10. (*a*) **5**

Giving different values to n, the number obtained are

$10 \times 1 + 1 = \boxed{11}$, $10 \times 2 + 1 = 21$, $10 \times 3 + 1 = \boxed{31}$,

$10 \times 4 + 1 = \boxed{41}$, $10 \times 5 + 1 = 51$, $10 \times 6 + 1 = \boxed{61}$,

$10 \times 7 + 1 = \boxed{71}$, $10 \times 8 + 1 = 81$ and $10 \times 9 + 1 = 91$.

Encircled numbers are prime numbers.

Hence, there are 5 prime numbers of the form $10n + 1$ where $n \in N$ such that $1 \leq n < 10$.

11. (*d*) **coprime**

Consider $a = 2$ and $b = 3$, then $a^2 = 4$, $b^2 = 9$.

Note: 4 and 9 are coprime.
Trial with other sets of coprime result in coprime.

$\therefore$ a^2 and b^2 are coprime.

12. (*b*) **2**

Since 3 is the least prime factor of p, so the other prime factor of $p \geq 3$ but not 2,

$\therefore$ p must be an odd number.
Similarly, q is an odd number.
$\therefore$ $(p + q)$ is an even number.
$\therefore$ Least prime factor of $(p + q)$ is 2.

13. (*a*) **composite**

Let $a = 5$ and $b = 3$.

Then, $5^2 - 3^2 = (5 - 3)(5 + 2) = 2(7) = 14$ composite.
Let $a = 11$ and $b = 5$.
Then, $11^2 - 5^2 = (11 - 5)(11 + 5) = 6(16)$ composite.
Since the difference of two odd prime numbers a and b i.e $a - b$ is even,
$\therefore$ one of the factors of $a^2 - b^2$ is even. So, the number obtained when $a^2 - b^2$ is simplified is a composite number.

14. (*b*) **composite number**

$119^2 - 111^2 = (119 - 111)(119 + 111) = 8(230)$ which is composite.

15. (*b*) **5**

$243 = 3 \times 3 \times 3 \times 3 \times 3 = 3^5$. So, exponent of 3 is 5.

16. (*d*) **1**

Since a prime number has only two factors , the number itself and 1,

$\therefore$ HCF of two prime number is 1.

17. (*a*) **2**

Smallest composite number = 4,
Smallest prime number = 2

HCF (4, 2) = 2

18. (*b*) **1**

Two consecutive integers are of the form x and $x + 1$.
e.g 2, 3 or 3, 4 or 4, 5.
Their HCF is 1.

19. (*c*) $\boldsymbol{r = 0}$

$m = dn + r$

$\Rightarrow$ if $r = 0$ then $m = dn$
$\Rightarrow$ n is the HCF of m,n

20. (*c*) **12**

$$\text{HCF} = \frac{\text{product of two numbers}}{\text{their LCM}} = \frac{60 \times 72}{360} = 12$$

21. (*c*) **340**

$$\text{LCM} = \frac{\text{product of two numbers}}{\text{their HCF}} = \frac{5780}{17} = 340$$

22. (*c*) **38784**

Product of two numbers = Product of their HCF and LCM = $4 \times 9696 = 38784$

23. (*c*) **12**

$$4 \times 24 = a \times 8$$

$$\Rightarrow \quad a = \frac{4 \times 24}{8} = 12$$

24. (*c*) $\boldsymbol{a^3b^2}$

$A = ab^2$, $B = a^3b$.
LCM $(A, B) = a^{\text{highest power}} \times b^{\text{highest power}} = a^3b^2$

25. (*b*) $\boldsymbol{ab^2}$

$A = ab^3$, $B = a^3b^2$.
HCF $(A, B) = a^{\text{lowest power}} \times b^{\text{lowest power}} = ab^2$

26. (*c*) $\boldsymbol{2^3 \times 3^3}$

LCM of $2^3 \times 3^2$ and $2^2 \times 3^3$ is $2^3 \times 3^3$. (taking highest power of all factors)

27. (*b*) **an irrational number**

Since $\frac{22}{7}$ is of the form $\frac{p}{q}$ where $q \neq 0$, but it has non-terminating and non-repeating decimals.

$\therefore$ $\frac{22}{7}$ is an irrational number.

28. (*b*) **a rational number**

Sum of two rational numbers is a rational number.

$\therefore$ $x + y$ is a rational number.

29. (*b*) **irrational numbers**

Since the sum, difference and product of a rational number and an irrational is irrational.

$\therefore$ $x + y$, $x - y$, xy are all irrational numbers.

30. (*d*) **an irrational number**

$\sqrt{5} - 3 - 2 = \sqrt{5} - 5$ is an irrational number because difference of an irrational and a rational number is irrational.

31. (*d*) **an irrational number**

$2 + \sqrt{3}$ is irrational because sum of a rational and irrational number is irrational.

$2 + \sqrt{3} + \sqrt{5}$ is an irrational number because sum of two irrational numbers is irrational.

32. (*b*) **an irrational number**

$3 + \sqrt{5}$ is a sum of a rational number and an irrational number.

$\therefore$ It is an irrational number.

33. (*d*) $\sqrt{5}$

Since $4 - \sqrt{5} + \sqrt{5} = 4$ (a rational),

$\therefore$ the smallest rational number which should be added to $4 - \sqrt{5}$ to get a rational number is $\sqrt{5}$.

34. (*c*) $\sqrt{2}$

$\sqrt{18} = \sqrt{9 \times 2} = 3\sqrt{2}$.

$\therefore$ The smallest irrational number by which $\sqrt{18}$ should be multiplied. So, as to get a rational number is $\sqrt{2}$.

35. (*c*) $\sqrt{3}\sqrt{27}$

$\sqrt{3}\sqrt{27} = \sqrt{81} = 9$ which is rational.

Note: $\sqrt{16}\ \sqrt{4} = \sqrt{64} = 8$ is also rational but $\sqrt{16}$ and $\sqrt{4}$ do not form a pair of irrational numbers. So, this pair is excluded.

36. (*b*) $7 + \sqrt{4}$

Since $7 + \sqrt{4} = 7 \pm 2 = 9$ or 5 (both are rationals),

$\therefore\ 7 + \sqrt{4}$ is not irrational.

37. (*b*) ***k***

If p is a prime number and p divides k^2, then p divides k. [Ref to theorem 1.3]

38. (*b*) $2^m \times 5^n$

$2^m \times 5^n$ [Ref to theorem 1.5]

39. (*c*) $\frac{15}{1600}$

$\frac{15}{1600}$

$\because$ Denominator $1600 = 2^6 \times 5^2$ is of the form $2^m\, 5^n$ where m, n are non-negative integers.

Note: The denominator of other three rational numbers have factors other than 2 or 5.

40. (*a*) **terminating**

$$\frac{63}{72 \times 175} = \frac{7 \times 9}{8 \times 9 \times 7 \times 25} = \frac{1}{8 \times 25} \text{ (simplest form)} = \frac{1}{2^3 5^2}$$

Denominator of given expansion in simplest form is of the form $2^m\, 5^n$ where m and n are non-negative integers.

$\therefore$ Its decimal expansion is terminating.

41. (*b*) **non-terminating non-repeating**

Since π is an irrational number, its decimal expansion is non-terminating non-repeating.

42. (*b*) **two decimal places**

$$\frac{31}{2^2 5} = \frac{31 \times 5}{2^2 5 \times 5} = \frac{155}{2^2 \times 5^2} = \frac{155}{10^2} = \frac{155}{100} = 1.55$$

Hence, the decimal expansion of $\frac{31}{2^2 5}$ will terminate after two decimal places.

43. (*c*) **3**

$$\frac{17}{8} = \frac{17 \times 5^3}{2^3 \times 5^3} = \frac{17 \times 125}{10^3} = \frac{2125}{1000} = 2.125$$

$\therefore$ Decimal expansion of $\frac{17}{8}$ will terminate after 3 places of decimals.

44. (*d*) **four decimal places**

$$\frac{14587}{1250} = \frac{29 \times 503}{2 \times 5^4}$$

$\therefore$ $\frac{14587}{1250}$ is in its simplest form.

Now, $$\frac{14587}{1250} = \frac{14587}{2 \times 5^4} = \frac{14587 \times 2^3}{2^4 \times 5^4} = \frac{116672}{10^4} = \frac{116672}{10000} = 11.6672$$

Thus, decimal expansion of $\frac{14587}{1250}$ will terminate after four decimal places.

45. (*b*) **non-terminating but repeating**

Since the denominator of the given rational number has factor other than 2 and 5,

$\therefore$ It is not of the form $2^m\, 5^n$.

Hence, its decimal expansion will be non-terminating but repeating.

For Standard Level

46. (*a*) **1650**

Product of HCF and LCM = Product of two numbers

$\Rightarrow \quad 40 \times 252 \times k = 2520 \times 6600$

$\Rightarrow \quad k = \frac{2520 \times 6600}{40 \times 252} = 1650$

47. (*b*) **2400, 5**

$a = 3 \times 5$, $b = 3 \times 5^2$, $c = 2^5 \times 5$,

LCM = all the factors of a, b, and c raised to their respective highest powers and HCF = only the common factors of a, b and c raised to their respective lowest powers.

$\therefore \quad \text{LCM} = 2^5 \times 3 \times 5^2 = 2400$

and $\quad \text{HCF} = 5$

48. (*b*) **3**

$a = 2^2 \times 3^x$,

$b = 2^2 \times 3 \times 5$,

$c = 2^2 \times 3 \times 5$,

LCM $(a, b, c) = 3780$

$\Rightarrow \quad 2^2 \times 3^x \times 5 \times 7 = 3780$

$\Rightarrow \quad 3^x = \dfrac{3780}{5 \times 7 \times 4} = 27$

$\Rightarrow \quad 3^x = 3^3$

$\Rightarrow \quad x = 3$

49. (*b*) **2**

$153 = 85 \times 1 + 68 \quad \ldots(1)$
$85 = 68 \times 1 + 17 \quad \ldots(2)$
$68 = 17 \times 4 + 0$

85) 153 (1
   – 85
     68

68) 85 (1
   – 68
     67

17) 68 (4
   – 68
     0

$\therefore \quad \text{HCF} = 17 = 85 - 68 \times 1 \quad [\text{Using (2)}]$
$= 85 - (153 - 85) \quad [\text{Using (1)}]$

$\Rightarrow \quad \text{HCF} = 85 - 153 + 85 = 85(2) - 53$

$\Rightarrow \quad n = 2$

50. (*d*) **2**

$1032 = 408 \times 2 + 216 \quad \ldots (1)$
$408 = 216 \times 1 + 192 \quad \ldots (2)$
$216 = 192 \times 1 + 24 \quad \ldots (3)$
$192 = 24 \times 8 + 0 \quad \ldots (4)$

408) 1032 (2
   – 816
     216

216) 408 (1
   – 216
     192

192) 216 (1
   – 192
     24

24) 192 (8
   – 192
     0

$\therefore \quad \text{HCF} = 24 = 216 - 192 \times 1 \quad [\text{Using (3)}]$
$= 216 - (408 - 216 \times 1) \quad [\text{Using (2)}]$
$= 216 - 408 + 216$
$= 216(2) - 408$
$= [1032 - 408(2)] \times 2 - 408$
$= 1032(2) - 408(4) - 408$
$= 1032(2) - 405 \times 5$
$= 1032\,(m) - 405 \times 5$

$\therefore \quad m = 2$

51. (*d*) **999720**

Greatest number of 6 digits = 999999.
LCM of 15, 24 and 36 = 2 × 2 × 3 × 5 × 2 × 3 = 360

2	15,24,36
2	15,12,18
3	15,6,9
	5,2,3

360) 999999 (2777
   –720
    2799
    2520
     2799
     2520
      2799
      2520
       279

Required number = 999999 – remainder when 999999 is divided by 360
= 999999 – 279 = 999720

52. (*d*) **2520**

The least number that is divisible by all the numbers from 1 to 10 (both inclusive in LCM of 1, 2, 3, 4, 5, 6, 7, 8, 9, 10

2	1,2,3,4,5,6,7,8,9,10
2	1,1,3,2,5,3,7,4,9,5
3	1,1,3,1,5,3,7,2,9,5
5	1,1,1,1,5,1,7,2,3,5
	1,1,1,1,1,1,7,2,3,1

∴ LCM = 2 × 2 × 3 × 5 × 7 × 2 × 3 = 2520

53. (*c*) **138**

The largest number which divides 281 and 1249 leaving remainder 5 and 7 respectively is the HCF of (281 – 5) = 276 and (1249 – 7) = 1242

276) 1242 (4
   – 1104
     138

138) 276 (1
   – 138
     138

138) 138 (1
   – 138
     x

HCF = 138

∴ Required number = 138

54. (*b*) **11350**

The smallest number which when divided by 17, 23, 29 leaving a remainder 11 in each case = LCM of 17, 23, 29 + 11 = (17 × 23 × 29) + 11 = 11339 + 11 = 11350

55. (*b*) **a rational number**

Since $1.\overline{29}$ has a non–terminating recurring decimal expansion,

∴ it is a rational number.

56. (*c*) **is of the form $2^m \times 5^n$ where *m*, *n* are non-negative integers**

Since rational number 26.1234 has a terminating decimal expansion,

∴ the prime factorization of its denominator is of the form $2^m\,5^n$ where *m*, *n* are non-negative integers.

57. (*d*) **not of the form $2^m \times 5^n$ where *m*, *n* are non-negative integers**

Since rational number $52.\overline{9678}$ has a non-terminating recurring decimal expansion, the prime factorization of its denominator is not of the form $2^m \times 5^n$ where *m* and *n* are non-negative integers.

58. (*a*) $\mathbf{\dfrac{3}{10}}$

$\dfrac{1}{3} \times \dfrac{3}{10} = \dfrac{1}{10} = 0.1$

∴ Smallest rational number by which $\dfrac{1}{3}$ should be multiplied so that its decimal expansion terminates after one place of decimal is $\dfrac{3}{10}$.

SHORT ANSWER QUESTIONS

For Basic and Standard Levels

1. (*i*) For any two given positive integers a and b, there exist unique integers q and r satisfying $a = bq + r$, $0 \leq r < b$.

(*ii*) **No**, it does not satisfy Euclid's division lemma. The value of the remainder can be 2 also as $2 < 3$.

2. **No**, every positive integer can be of the form $4q + 2$ where q is an integer because an integer can be written in the form of $4q$, $4q + 1$, $4q + 2$, $4q + 3$.

3. The product of two consecutive positive integers is divisible by 2 because one of them will always be even.

$\therefore$ $n(n + 1)$ will be even

$\therefore$ one out of n or $n + 1$ must be even

So, the given statement is **true**.

4. When three consecutive positive integers are taken, at least one of them is divisible by 2 and at least one them is divisible by 3. Therefore, the product of three consecutive positive integers is divisible by 2×3 i.e 6.

$\therefore$ Out of n, $(n + 1)$ and $(n + 2)$ at least one will be divisible by 2 and at least one will be divisible by 3.

$\therefore$ Product $n(n + 1)(n + 2)$ will be divisible by 6.

Hence, the given statement is **true**.

5. According to Euclid's division lemma, when a positive integer a is divided by 4 then $a = 4q + r$ where $0 \leq r < 4$. Therefore, the values of r can be 0, 1, 2 or 3 only.

6. (*i*)

```
255)867(3     102)255(2     51)102(2
   −765           204           102
    102            51             0
```

$\therefore$ HCF of 255 and 867 is **51**

Hence, they are **not coprime**.

(*ii*)

```
154)615(3     153)154(1     1)153(153
   −462          −153         −153
    153             1            0
```

$\therefore$ HCF of 615 and 154 is **1**

Hence, they are **coprime**.

(*iii*)

```
847)2160(2     466)847(1     381)466(1     85)381(4
   −1694          −466           381          −340
     466           381            85            41

41)85(2        3)41(13       2)3(1         1)2(2
  −82           −3              2            −2
    3            11             1             0
                 −9
                  2
```

$\therefore$ HCF of 847 and 2160 is 1

Hence, they are **coprime**.

7.

2	98
7	49
7	7
	1

$\mathbf{98 = 2 \times 7^2}$

8.

2	120
2	60
2	30
3	15
	5

2	144
2	72
2	36
2	18
3	9
	3

$120 = 2^3 \times 3 \times 5$ $\qquad$ $144 = 2^4 \times 3^2$

$\therefore$ LCM of 120 and 144 $= 2^4 + \times 3^2 \times 5 =$ **720** and HCF of 120 and 144 $= 2^3 \times 3 =$ **24**.

9. $1 < \dfrac{p}{q} < 2 \ (q \neq 0)$.

One rational number between 1 and 2 can be $\dfrac{1+2}{2} = \mathbf{\dfrac{3}{2}}$.

Another rational number between 1 and 2 can be $\dfrac{1}{2}\left(1 + \dfrac{3}{2}\right) = \mathbf{\dfrac{5}{4}}$.

These are sample answers. There can be infinitely more rational numbers between 1 and 2.

10. q should be of the form $2^n\, 5^m$ where n and m are non-negative integers.

11. Denominator $2^2 \times 5^7 \times 7^2$ is not of the form $2^n\, 5^m$

$\therefore$ $\dfrac{441}{2^2 \times 5^7 \times 7^2}$ has a **non-terminating** (but repeating) decimal expansion.

12. Let us assume on the contrary that $3\sqrt{7}$ is a rational number.

Then, there exist coprime a and b ($b \neq 0$) such that

$$3\sqrt{7} = \frac{a}{b}$$

$$\Rightarrow \quad \sqrt{7} = \frac{a}{3b}$$

$$\Rightarrow \quad \sqrt{7} \text{ is rational}$$

[$\because$ 3, a, b are integers $\therefore$ $\dfrac{a}{3b}$ is a rational number.]

This contradicts the fact that $\sqrt{7}$ is irrational.

The contradiction has arisen because of our incorrect assumption.

Hence, $\mathbf{3\sqrt{7}}$ **is irrational**.

For Standard Level

13. $\sqrt{2} = 1.414\ldots$ $\sqrt{3} = 1.732$

$\therefore$ One of the rational numbers between $\sqrt{2}$ and $\sqrt{3}$ can be **1.5**.

14. Let $a = 2k + 1$

and $b = 2n + 1$

Then, $a - b = 2k + 1 - 2n - 1 = 2(k - n)$

and $a + b = 2k + 1 + 2n + 1 = 2(k + n + 1)$

$$\begin{aligned} a^2 - b^2 &= (a + b)(a - b) \\ &= 2(k + n + 1)(2)(k - n) \\ &= 4(k + n + 1)(k - n) \end{aligned}$$

which is composite.

15. Let $a = 2m + 1$

and $b = 2n + 1$

(Any odd positive integer is of the form $2q + 1$)

$$\therefore \quad a^2 + b^2 = (2m + 1)^2 + (2n + 1)^2$$
$$= 4m^2 + 1 + 4m + 4n^2 + 1 + 4n$$
$$= 4m^2 + 4n^2 + 4m + 4n + 2$$
$$= 4(m^2 + n^2 + m + n) + 2$$
$$= 4q + 2 = 2(2q + 1)$$

where $q = (m^2 + n^2 + m + n)$

$\therefore$ **$a^2 + b^2$ is even but not divisible by 4.**

VALUE-BASED QUESTION

For Basic and Standard Levels

1. (*i*) HCF of 96 and 8 = 16.

$$80\overline{)96}(1 \qquad 16\overline{)80}(5$$
$$\underline{80} \qquad\qquad \underline{80}$$
$$16 \qquad\qquad\quad 0$$

$\therefore$ She can cut 16 cm long pieces from the two ribbons.

From the first ribbon, she will get $\frac{96}{16}$ = 6 pieces

and from the second ribbon, she will get $\frac{80}{16}$ = 5 pieces.

So, she will get a total of 6 + 5 = 11 identical pieces of ribbon each of length 16 cm.

She gifts one pair each to two daughters of her domestic help.

She gives away 2 × 2 = 4 pieces.

$\therefore$ Number of pieces left with her = 11 – 4 = **7**

(*ii*) Values shown by Radhika are **problem-solving** and **empathy**.

UNIT TEST 1

For Basic Level

1. (*c*) **(12, 25)**

3	21
7	7
	1

3	27
3	9
3	3
	1

HCF (21, 27) = 3 ≠ 1

2	14
7	7
	1

2	64
2	32
2	16
2	8
2	4
	2

HCF (14, 64) = 2 ≠ 1

2	12
2	6
3	3
	1

5	25
5	5
	1

HCF (12, 25) = 1

41	41
	1

41	123
	3

HCF = 41 ≠ 1

$\therefore$ (12, 25) are a pair of coprimes.

2. (*a*) **500**

HCF of two numbers has to be a factor of LCM of the given two numbers.

$\therefore$ 500 cannot be the HCF of two numbers where LCM = 1600

3. (*d*) **2850**

$$\text{LCM} = \frac{\text{product of two numbers}}{\text{HCF}} = \frac{570 \times 1425}{285} = 2850$$

4. (*d*) **an irrational number**

Since product of a rational number and an irrational number is irrational,

$\therefore$ $2\sqrt{3}$ is an irrational number.

5. (*d*) **(*i*) and (*iv*)**

(*i*) $625 = 5^4$,

(*ii*) $270 = 2 \times 3^3 \times 5$,

(*iii*) $35 = 5 \times 7$,

(*iv*) $400 = 2^4 \times 5^2$

(*i*) and (*iv*) have the prime factorization of the denominators in the form $2^n\, 5^m$.

$\therefore$ They have terminating decimal.

6. (*b*) **3**

$$\frac{147}{120} = \frac{3 \times 7^2}{2^3 \times 3 \times 5} = \frac{7^2}{2^3 \times 5} \qquad \text{(simplest form)}$$
$$= \frac{7^2 \times 5^2}{2^3 \times 5^3} = \frac{1225}{10^3} = \frac{1225}{1000} = 1.225$$

$\therefore$ It will terminate after 3 places of decimals.

7. (*d*) **equal**

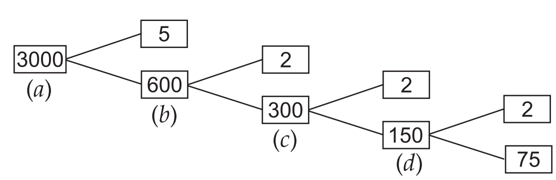
Only equal numbers have the same HCF and LCM.
If they are prime (but different), their HCF will be 1 but LCM will be their product.
If they are coprime, then their HCF will be 1 but LCM will be different.
If they are composite, then their HCF and LCM will be different.

8. Refer to theorem 1.2

9. HCF of two numbers has to be a factor of their LCM.
Since 12 is not a factor of 350,
$\therefore$ two numbers cannot have 12 as their HCF and 350 as their LCM.

10. HCF (625, 2000) = 125
$\therefore$ among the common factors **125** is the greatest.

11.



(*d*) = 2 × 75 = **150**

$(c) = 2 \times 150 = \mathbf{300}$

$(b) = 2 \times 300 = \mathbf{600}$

$(a) = 600 \times 5 = \mathbf{3000}$

12. Let us assume that $2 + \sqrt{5}$ is a rational number.

Let $2 + \sqrt{5} = \frac{a}{b}$ (where a and b are coprimes and $b \neq 0$)

Then, $\sqrt{5} = \frac{a}{b} - 2 = \frac{a-2b}{b}$ which is rational

[$\because$ 7, a and b are integers]

This contradicts the fact that $\sqrt{5}$ is an irrational.

$\therefore$ Our assumption is wrong . Hence, $\mathbf{2 + \sqrt{5}}$ **is irrational**.

13. (*i*) $3 \times 5 \times 7 \times 11 + 11 = 11(3 \times 5 \times 7 + 1)$

$= 11(105+1) = 11(106)$

which has more than two factors and is therefore a composite number.

(*ii*) $x = 2 \times 7 \times 11 \times 17 \times 23 + 23 = 23(2 \times 7 \times 11 \times 17+1) = 23 \times 2619$ which shows that 23 is a prime factor of x.

14. $\frac{17}{80} = \frac{17}{2^4 \times 5} = \frac{17 \times 5^3}{2^4 \times 5^4} = \frac{17 \times 125}{10^4} = \frac{2125}{10000} = \mathbf{0.2125}$

15.

2	336
2	168
2	84
2	42
3	21
7	7
	1

2	54
3	27
3	9
3	3
	1

$336 = 2^4 \times 3 \times 7$ $54 = 2 \times 3^3$

HCF $= 2 \times 3 = \mathbf{6}$

LCM $= 2^4 \times 3^3 \times 7 = \mathbf{3024}$

16. Let x be the other number.

Product of two numbers = Product of their HCF and LCM

$\Rightarrow \quad 18x = 9 \times 90$

$\Rightarrow \quad x = \frac{9 \times 90}{18} = 45$

Hence, the other number is **45**.

UNIT TEST 2

For Standard Level

1. (*b*) **2**

$117 = 65 \times 1 + 52$ … (1)

$65 = 52 \times 1 + 13$ … (2)

$52 = 13 \times 4 + 0$

$65\overline{)117}(1$, -65, 52 $\quad$ $52\overline{)65}(1$, -52, 13 $\quad$ $13\overline{)52}(4$, -52, 0

$\therefore \quad$ HCF = 13

HCF $= 13 = 65 - 52 \times 1$ [From (2)]

$= 65 - (117 - 65 \times 1)$ [Using (1)]

$= 65 - 117 + 65 = 65(2) - 117$

$= 65m - 117$

$\Rightarrow \quad m = 2$

2. (*d*) $\mathbf{\frac{57}{99}}$

Let $x = 0.\overline{57}$ … (1)

$\Rightarrow \quad 100x = 57.\overline{57}$ … (2)

Subtracting (1) from (2), we get

$99x = 57$

$\Rightarrow \quad x = \frac{57}{99}$

3. (*c*) **5**

$6^n = (2 \times 3)^n = 2^n \times 3^n$, where $n = 1, 2, 3$ …

For 6^n to end with digit zero, the smallest positive number that it should be multiplied is 5.

When $n = 1$, then $6^n = 6^1 = 6 = 2 \times 3$. For it to end with digit zero, 5 has to be a factor too ($6 \times 5 = 30$).

4. **0.131313**… because it has a non–terminating recurring decimal expansion. The remaining three are not rational numbers because their decimal expansion is non–terminating, non–repeating.

5. **Rational** $\therefore$ its decimal expansion is terminating. Prime factors of q will be of the form 2^n5^m where n, m are non-negative integers, i.e prime factors of q will either have 2 or 5 or both.

6. If possible, let us assume that $2\sqrt{3} - 1$ is a rational number. Then there exist coprime a and b where $b \neq 0$, such that

$2\sqrt{3} - 1 = \frac{a}{b}$

$\Rightarrow \quad 2\sqrt{3} = 1 + \frac{a}{b}$

$\Rightarrow \quad \sqrt{3} = \frac{a+b}{2b}$

Now, $\frac{a+b}{2b}$ is a rational, number, since a and b are integers and $b \neq 0$. But $\sqrt{3}$ is an irrational number.

Hence, there is a contradiction in our assumption.

So, our assumption is wrong. Hence, $\mathbf{2\sqrt{3} - 1}$ **is an irrational number**.

7. $(3q + 1)^2 = 9q^2 + 6q + 1 = 3(3q^2 + 2q) + 1 = 3m + 1$ where $m = (3q^2 + 2q)$ is an integer.

Hence, the square of positive integer of the form $3q + 1$, q being a natural number cannot be expressed in any form other than $3m + 1$ for some integer m.

8. $y^2 = 7$

$\Rightarrow \quad y = \sqrt{7}$ (irrational),

$x^2 = 25$

$\Rightarrow \quad x = \pm 5$ (rational),

$z^2 = 0.09 = \frac{9}{100}$

$\Rightarrow \quad z = \pm \frac{3}{10}$ (rational)

$u^3 = 125$

$\Rightarrow \quad u = +5$ (rational)

$\therefore \quad y$ represents irrational number.

9. $45 = 27 \times 1 + 18$ … (1)

$27 = 18 \times 1 + 9$ … (2)

$$\begin{array}{r|l|l} 27 & 45 & 1 \\ & -27 & \\ \hline & 18 & \end{array} \qquad \begin{array}{r|l|l} 18 & 27 & 1 \\ & 18 & \\ \hline & 9 & \end{array} \qquad \begin{array}{r|l|l} 9 & 18 & 2 \\ & -18 & \\ \hline & x & \end{array}$$

$18 = 9 \times 2 + 0$

$\therefore$ HCF of 45 and 27 is 9

$9 = 27 - 18 \times 1$ [From (2)]

$= 27 - (45 - 27 \times 1)$ [Using (1)]

$= 27 - 45 + 27$

$= 27 \times 2 + 45 \times (-1) = 27x + 45y$

Hence, $\boldsymbol{x = 2}$ **and** $\boldsymbol{y = -1}$.

10. (*i*) Since the highest common factor of 525 and 3000 is 75, hence by definition, the required HCF of these two numbers is **75**.

(*ii*)

$$\begin{array}{r|l} 2 & 380 \\ \hline 2 & 190 \\ \hline 5 & 95 \\ \hline & 19 \end{array} \qquad \begin{array}{r|l} 3 & 18 \\ \hline 2 & 6 \\ \hline & 3 \end{array}$$

$380 = 2^2 \times 5 \times 19$

$18 = 2 \times 3^2$

Since the LCM 380 is not exactly divisible by the HCF 18, hence no two numbers have 18 as their HCF and 380 as their LCM.

11. Any positive number a can be expressed in the form $6q + r$, where $r = 0$ or 1 or 2 or 3 or 4 or 5.

When $r = 0$

$a = 6q$

$\Rightarrow$ $a^2 = (6q)^2 = 36q^2 = 6(6q^2) = 6m$

where $m = 6q^2$ is an integer

When $r = 1$

$a = 6q + 1$

$\Rightarrow$ $a^2 = (6q + 1)^2 = 36q^2 + 12q + 1$

$= 6(6q^2 + 2q) + 1 = 6m + 1$

where $m = (6q^2 + 2q)$ is an integer.

When $r = 2$

$a = 6q + 2$

$\Rightarrow$ $a^2 = (6q + 2)^2 = 36q^2 + 24q + 4$

$= 6(6q^2 + 4q) + 4 = 6m + 4$

where $m = (6q^2 + 4q)$ is an integer.

When $r = 3$

$a = 6q + 3$

$\Rightarrow$ $a^2 = (6q + 3)^2 = 36q^2 + 36q + 9$

$= (36q^2 + 36q + 6) + 3$

$= 6(6q^2 + 6q + 1) + 3 = 6m + 3$

where $m = (6q^2 + 6q + 1)$ is an integer.

When $r = 4$

$a = 6q + 4$

$\Rightarrow$ $a^2 = (6q + 4)^2 = 36q^2 + 48q + 16$

$= (36q^2 + 48q + 12) + 4$

$= 6(6q^2 + 8q + 2) + 4 = 6m + 4$

where $m = (6q^2 + 8q + 2)$ is an integer.

When $r = 5$

$a = 6q + 5$

$\Rightarrow$ $a^2 = (6q + 5)^2 = 36q^2 + 60q + 25$

$= (36q^2 + 60q + 24) + 1$

$= 6(6q^2 + 10q + 4) + 1 = 6m + 1$

where $m = (6q^2 + 10q + 4)$ is an integer.

$\therefore$ Square of any positive integer can only be of form $6m$, $6m + 1$, $6m + 3$ or $6m + 4$

Hence, the square of any positive integer cannot be of the form $6m + 2$ or $6m + 5$ for any integer m.

Or

Let 'a' be any positive integer. On dividing 'a' by 4, let m be the quotient and r be the remainder.

Then, by Euclid's division lemma, we have

$a = 4m + r$ where $0 \le r < 4$

$\Rightarrow$ $a = 4m + r$ where $r = 0, 1, 2, 3$ or 4

$\Rightarrow$ $a = 4m$ (when $r = 0$), $a = 4m + 1$ (when $r = 1$)

$a = 4m + 2$ (when $r = 2$), $a = 4m + 3$ (when $r = 3$)

$\therefore$ An odd positive integer can be of the form $4m + 1$ or $4m + 3$.

Thus, we have

$(4m + 1)^2 = 16m^2 + 8m + 1$

$= 4(4m^2 + 2m) + 1$

$= 4q + 1$

where $q = (4m^2 + 2m)$ is an integer

$(4m + 3)^2 = 16m^2 + 24m + 9$

$= 16m^2 + 24m + 8 + 1$

$= 4(4m^2 + 6m + 2) + 1$

$= 4q + 1$

where $q = (4m^2 + 6m + 2)$ is an integer.

Thus, the square of any odd integer is of the form $4q + 1$, for some integer q.

12. Clearly, the required minimum distance will be the LCM of 40, 42 and 45 in cm.

$$\begin{array}{r|l} 2 & 40 \\ \hline 2 & 20 \\ \hline 2 & 10 \\ \hline & 5 \end{array} \qquad \begin{array}{r|l} 2 & 42 \\ \hline 3 & 21 \\ \hline & 7 \end{array} \qquad \begin{array}{r|l} 3 & 45 \\ \hline 3 & 15 \\ \hline & 5 \end{array}$$

Now, $40 = 2^3 \times 5$

$42 = 2 \times 3 \times 7$

$45 = 3^2 \times 5$

$\therefore$ LCM $= 2^3 \times 3^2 \times 5 \times 7$ cm

$= 360 \times 7$ cm $= 2520$ cm

$= 25$ m 20 cm

Hence, the required distance = **25 m 20 cm**