CHAPTER 13

Surface Areas and Volumes

EXERCISE 13A

1. (i) Surface area of the cube = 6(Edge)^2

$= 6(8 \text{ cm})^2 = (6 \times 64) \text{ cm}^2$

$= 384 \text{ cm}^2$

Hence, the surface area of the cube is 384 cm^2.

(ii) Given: Perimeter of one face of cube = 20 cm

So, side of the cube $= \frac{20 \text{ cm}}{4} = 5 \text{ cm}$

Now, surface area of the cube = 6 (side)^2

$= 6 (5 \text{ cm})^2$

$= (6 \times 25) \text{ cm}^2$

$= 150 \text{ cm}^2$

Hence, the surface area of the cube is 150 cm^2.

2. Given: TSA of cube = 864 cm^2

$\Rightarrow$ 6 (Edge)^2 = 864 cm^2

$\Rightarrow$ (Edge)$^2 = \frac{864}{6} \text{ cm}^2 = 144 \text{ cm}^2$

$\Rightarrow$ Edge $= \sqrt{144 \text{ cm}^2} = 12 \text{ cm}$

Hence, the edge of the cube is 12 cm.

Now, lateral surface area of the cube = 4 × (Edge)^2

$= 4 \times (12 \text{ cm})^2$

$= (4 \times 144) \text{ cm}^2$

$= 576 \text{ cm}^2$.

3. (i) Lateral surface area of the cube = 324 cm^2

$\Rightarrow$ $4 \times (\text{Edge})^2 = 324 \text{ cm}^2$

$\Rightarrow$ (Edge)$^2 = \frac{324}{4} \text{ cm}^2 = 81 \text{ cm}^2$

$\Rightarrow$ Edge $= \sqrt{81 \text{ cm}^2} = 9 \text{ cm}$

Now, total surface area of the cube = 6 × (Edge)^2

$= 6 \times (9 \text{ cm})^2$

$= (6 \times 81) \text{ cm}^2$

$= 486 \text{ cm}^2$.

(ii) TSA of a cube – LSA of cube = 392 cm^2

$\Rightarrow$ $6a^2 - 4a^2 = 392 \text{ cm}^2$ [$\because$ a = edge of the cube]

$\Rightarrow$ $2a^2 = 392 \text{ cm}^2$

$\Rightarrow$ $a^2 = \frac{392 \text{ cm}^2}{2} = 196 \text{ cm}^2$

$\Rightarrow$ $a = \sqrt{196 \text{ cm}^2} = 14 \text{ cm}$

TSA of the cube = 6 × (Edge)^2

$= 6 \times (14 \text{ cm})^2 = (6 \times 196) \text{ cm}^2$

$= 1176 \text{ cm}^2$

Hence, the total surface area of the cube is 1176 cm^2.

4. Let the length of each edge of the cube = x

New edge $= x + \frac{10x}{100} = \frac{11}{10}x$

Surface area of the original cube = $6x^2$

Surface area of the resulting cube $= 6\left(\frac{11}{10}x\right)^2 = \frac{363}{50}x^2$

Percentage increase in the

Surface area $= \frac{363x^2 - 6x^2}{6x^2} \times 100\% = 21\%$

5. Given: Length of diagonal of a cube = $7\sqrt{3}$ cm

$\Rightarrow$ $\sqrt{3} \times \text{side} = 7\sqrt{3}$ cm [$\because$ length of diagonal of a cube = $\sqrt{3}$ × side]

$\Rightarrow$ side $= \frac{7\sqrt{3} \text{ cm}}{\sqrt{3}} = 7 \text{ cm}$

Now, surface area of the cube = $6 \times a^2$

$= 6 \times (7 \text{ cm})^2$

$= (6 \times 49) \text{ cm}^2$

$= 294 \text{ cm}^2$

Hence, the surface area of the cube is 294 cm^2.

6. (i) Given: Length = 40 cm, breadth = 30 cm, Height = 25 cm

LSA of the cuboid = $2(l + b)h$

$= 2(40 \text{ cm} + 30 \text{ cm})\ 25 \text{ cm}$

$= 2(70 \text{ cm})\ 25 \text{ cm}$

$= (2 \times 70 \times 25) \text{ cm}^2$

$= 3500 \text{ cm}^2$.

TSA of the cuboid = $2(lb + bh + hl)$

$= 2[(40)(30) + (30)(25) + (25)(40)] \text{ cm}^2$

$= 2[1200 + 750 + 1000] \text{ cm}^2$

$= 2[2950] \text{ cm}^2$

$= 5900 \text{ cm}^2$.

(ii) Given: Length = 4 cm, breadth = 1.7 cm, Height = 2.3 cm

LSA of the cuboid = $2(l + b)h$

$= 2(4 \text{ cm} + 1.7 \text{ cm})\ 2.3 \text{ cm}$

$= 2(5.7)(2.3) \text{ cm}^2$

$= 26.22 \text{ cm}^2$.

TSA of the cuboid $= 2[(4)(1.7) + (1.7)(2.3) + (2.3)(4)] \text{ cm}^2$

$= 2[6.8 + 3.91 + 9.2] \text{ cm}^2$

$= 2[19.91] \text{ cm}^2 = 39.82 \text{ cm}^2$.

(iii) Given: Length = 26 cm, breadth = 14 cm, Height = 6.5 cm

LSA of the cuboid $= 2(26 + 14)$ cm × 6.5 cm

$= (80 \times 6.5)\text{ cm}^2 = 520\text{ cm}^2.$

TSA of the cuboid $= 2[(26)(14) + (14)(6.5) + (6.5)(26)]\text{ cm}^2$

$= 2[364 + 91 + 169]\text{ cm}^2$

$= 2[624]\text{ cm}^2 = 1248\text{ cm}^2.$

7. Given: Surface area of cuboid $= 1300\text{ cm}^2$,

$b = 10$ cm,

$h = 20$ cm

$\Rightarrow 2[(l \times 10) + (10 \times 20) + (20 \times l)] = 1300\text{ cm}^2$

$\Rightarrow 2[10l + 200 + 20l] = 1300\text{ cm}^2$

$\Rightarrow 30l + 200\text{ cm}^2 = \dfrac{1300\text{ cm}^2}{2}$

$= 650\text{ cm}^2$

$\Rightarrow 30l = (650 - 200)\text{ cm}^2$

$= 450\text{ cm}^2$

$\Rightarrow l = \dfrac{450\text{ cm}^2}{30\text{ cm}}$

$= 15$ cm

Hence, the length of the cuboid = 15 cm.

8. Let the length and breadth of the cuboid be $4x$ cm and $5x$ cm respectively.

Given, Total surface area $= 3384\text{ cm}^2$

$\Rightarrow 2(lb + bh + hl) = 3384\text{ cm}^2$

$\Rightarrow 2[(4x)(5x) + (5x)(18) + (18)(4x)] = 3384$

$\Rightarrow 10x^2 + 81x - 846 = 0$

$\Rightarrow (10x + 141)(x - 6) = 0$

$\Rightarrow$ Either $10x + 141 = 0$ or $x - 6 = 0$

$\Rightarrow x = -\dfrac{141}{10}$ (rejected) or $x = 6$

$\therefore$ Length $= 4x = 4 \times 6$ cm $= 24$ cm

And breadth $= 5x = 5 \times 6$ cm $= 30$ cm.

9. Surface area of the box $= \dfrac{2820}{30}\text{ m}^2 = 94\text{ m}^2$

$\Rightarrow 2(lb + bh + hl) = 94\text{ m}^2$

$\Rightarrow 2[(5\text{ m})(4\text{ m}) + (4\text{ m})(h) + (h)(5\text{ m})] = 94\text{ m}^2$

$\Rightarrow [20\text{ m}^2 + (4\text{ m})(h) + (h)(5\text{ m})] = 47\text{ m}^2$

$\Rightarrow (4\text{ m})(h) + (5\text{ m})h = (47 - 20)\text{ m}^2$

$= 27\text{ m}^2$

$\Rightarrow (9h)\text{ m} = 27\text{ m}^2$

$\therefore h = \dfrac{27\text{ m}^2}{9\text{ m}} = 3$ m.

10. Given: Length = 27.5 cm, $b = 16$ cm and $h = 10$ cm

Surface area of each box $= 2(lb + bh + hl)$

$= 2[(27.5)(16) + (16)(10) + (10)(27.5)]\text{ cm}^2$

$= 2[440 + 160 + 275]\text{ cm}^2$

$= (2 \times 875)\text{ cm}^2 = 1750\text{ cm}^2$

Paint in the container can paint 5.25 m^2

i.e., $5.25 \times 100 \times 100\text{ cm}^2$

Number of boxes that can be painted

$= \dfrac{\text{Total area}}{\text{Surface area of box}}$

$= \dfrac{5.25 \times 100 \times 100\text{ cm}^2}{1750\text{ cm}^2}$

$= 30.$

11. Given: The edge of the cube = 10 cm.

When two cubes are joined end to end, then the resulting solid is a cuboid.

Length of cuboid, $l = 2 \times$ edge $= 2 \times 10 = 20$ cm

Breadth of the cuboid = 10 cm

Height of the cuboid = 10 cm

Surface area of the cuboid $= 2(lb + bh + lh)$

$= 2(20\text{ cm} \times 10\text{ cm} + 10\text{ cm} \times 10\text{ cm} + 20\text{ cm} \times 10\text{ cm})$

$= 2(200\text{ cm}^2 + 100\text{ cm}^2 + 200\text{ cm}^2)$

$= 2(500\text{ cm}^2)$

$= 1000\text{ cm}^2.$

12. Given: SA of cuboidal box = SA of cubical box

Edge of the cubical box = 50 cm

SA of cubical box $= 6 \times (\text{Edge})^2$

$= 6 \times (50\text{ cm})^2$

$= (6 \times 2500)\text{ cm}^2$

$= 15000\text{ cm}^2$

Now, $2(lb + bh + hl) = 15000\text{ cm}^2$

$\Rightarrow 2(60\text{ cm} \times 40\text{ cm} + 40\text{ cm} \times h + h \times 60\text{ cm})$

$= 15000\text{ cm}^2$

$\Rightarrow 2400\text{ cm}^2 + (100\,h)\text{ cm} = \dfrac{15000\text{ cm}^2}{2} = 7500\text{ cm}^2$

$\Rightarrow (100\,h)\text{ cm} = (7500 - 2400)\text{ cm}^2$

$\Rightarrow h = \dfrac{5100\text{ cm}^2}{100\text{ cm}} = 51$ cm

Hence, the height of the cuboid = 51 cm.

13. Given: $l = 12$ m, $b = 9$ m, $h = 4$ m

TSA of the tank $= 2(lb + bh + hl)$

$= 2[(12\text{ m} \times 9\text{ m}) + (9\text{ m} \times 4\text{ m}) + (4\text{ m} \times 12\text{ cm})]$

$= 2[108\text{ m}^2 + 36\text{ m}^2 + 48\text{ m}^2]$

$= (2 \times 192)\text{ m}^2 = 384\text{ m}^2$

Area of the rectangular iron sheet $= 384\text{ m}^2$

Length $= \dfrac{384\text{ m}^2}{2\text{ m}} = 192$ m [as sheet is 2 m wide]

Total cost of the sheet = ₹ 5 × 192 m = ₹ 960.

14. Given: Edge of the cubical water tank = 1.5 m = 150 cm.

TSA of the cubical tank $= 6 \times (\text{Edge})^2$

$= 6 \times (150\text{ cm})^2$

$= 6 \times 22500\text{ cm}^2$

$= 135000\text{ cm}^2$

Area of base of tank = 150 × 150 = 22500 cm^2

Area to be tiled = (135000 – 22500) cm^2

= 112500 cm^2

Total number of tiles = $\frac{112500 \text{ cm}^2}{25 \text{ cm} \times 25 \text{ cm}}$

= 180 i.e. 15 dozens

Cost of tiles = ₹ 480 × 15 = ₹ 7200.

15. Given: Length = 1.25 m, breadth = 1 m,

Height = 0.75 m

TSA of cuboid = $2(lb + bh + hl)$

= 2(1.25 m × 1 m + 1 m × 0.75 m + 0.75 m × 1.25 m)

= 2(1.25 m^2 + 0.75 m^2 + 0.9375 m^2)

= (2 × 2.9375) m^2 = 5.875 m^2.

Perimeter = $4l + 4b + 4b$

= $4(l + b + h)$

= 4 (1.25 m + 1 m + 0.75 m)

= 4 × 3 m = 12 m

12 m of iron length is required to joint all 12 edges.

16. For bigger box

l = 20 cm, b = 15 cm, h = 5 cm

TSA of the bigger box = $2(lb + bh + hl)$

= 2(20 × 15 + 15 × 5 + 5 × 20) cm^2

= 2(300 + 75 + 100) cm^2

= 950 cm^2 (S_1)

For smaller box

TSA of the smaller box = 2(16 × 12 + 12 × 4 + 4 × 16) cm^2

= 2(192 + 48 + 64) cm^2

= 2(304) cm^2 = 608 cm^2 (S_2)

$S_1 + S_2$ = (950 + 608) cm^2 = 1558 cm^2

Area of overlap = $1558 \times \frac{4}{100} = 62.32 \text{ cm}^2$

TSA of two boxes with overlaps = (1558 + 62.32) cm^2

= 1620.32 cm^2

Area of 200 boxes = 200 × 1620.32 cm^2 = 324064 cm^2

Cost of 200 cardboards = $\frac{₹ 10}{10000}$ × 324064

= ₹ 324.064 = ₹ 324.

17. Given: l = 30 cm, b = 22.5 cm, h = 20 cm.

(i) TSA of box = $2(lb + bh + hl)$

= 2(30 × 22.5 + 22.5 × 20 + 20 × 30) cm^2

= 2(675 + 450 + 600) cm^2 = 3450 cm^2

(ii) 10% extra area = 3450 × $\frac{10}{100}$ = 345 cm^2

Net area required = 3450 cm^2 + 345 cm^2

= 3795 cm^2

(iii) Net area of 50 boxes = 3795 cm^2 × 50

= 189750 cm^2

Cost of making 50 boxes = $\frac{₹ 30}{1000}$ × 189750 cm^2

= ₹ 5692.50

18. Given: l = 12 m, b = 8 m, h = 6 m.

Lateral surface area of tank = $2(l + b)h$

= 2(12 + 8) m × 6 m

= 40 m × 6 m = 240 m^2

Area of base of the tank = $l \times b$ = 12 m × 8 m

= 96 m^2

TSA of open tank = (240 + 96) m^2 = 336 m^2

Length of iron sheet required = $\frac{336 \text{ m}^2}{4 \text{ m}}$ = 84 m

Cost of 84 m of iron sheet at ₹ 17.50 per metre

= 84 × ₹ 17.50 = ₹ 1470.

19. Length of the box = [48 cm – (2 × 8 cm)]

= (48 – 16) cm = 32 cm

Breadth of the box = [36 cm – (2 × 8 cm)]

= 20 cm

Height of the box = 8 cm.

Surface area of the box = TSA of box – Area of open part

= 2(32 × 20 + 20 × 8 + 8 × 32) cm^2 – (32 × 20) cm^2

= 2[640 + 160 + 256) m^2 – 640 cm^2

= 2112 m^2 – 640 m^2 = 1472 cm^2.

20. Given: l = 6 m, b = 4.5 m, h = 3 m

Area of walls = $2(l + b)\,h$

= 2(6 m + 4.5 m) 3m

= 2 × 10.5 m × 3 m

= 21 m × 3 m = 63 m^2

Cost of cementing = ₹ 5.50 × 63 m^2

= ₹ 346.50.

21. Given: l = 6 m, b = 4 m, h = 4.5 m.

Area of walls = $2(l + b)\,h$ = 2(6 m + 4 m) 4.5 m

= 2 × 10 m × 4.5 m = 90 m^2

Length of wallpaper required

= $\frac{90 \text{ m}^2}{0.75 \text{ m}}$ = 120 m [∵ 75 cm = 0.75 m]

Cost of wallpaper = ₹ 4.75 × 120 m = ₹ 570.

22. Given: Length = 7 m, $b = 2x$, $h = x$

Area of wallpaper = 260 m × 0.30 m = 78 m^2

$2(l + b)h$ = Area of wallpaper

⇒ $2(7 + 2x)x = 78 \text{ m}^2$

⇒ $(7 + 2x)x = 39 \text{ m}^2$

⇒ $7x + 2x^2 = 39$

⇒ $2x^2 + 7x - 39 = 0$

⇒ $2x^2 + 13x - 6x - 39 = 0$

⇒ $x(2x + 13) - 3(2x + 13) = 0$

⇒ $(x - 3)(2x + 13) = 0$

$2x + 13 = 0$

⇒ $x = \frac{-13}{2}$ (rejected)

and $x - 3 = 0$

$\Rightarrow \quad x = 3$

$\Rightarrow \quad h = 3$ m

Hence, height = 3 m.

23. Given: Perimeter of rectangular hall = 200 m.

i.e. $2(l + b) = 200$ m ... (1)

Area of walls painted = $\dfrac{₹\,18000}{₹\,15} = 1200\text{ m}^2$

$2(l + b)h = 1200\text{ m}^2$

$\Rightarrow \quad (200\text{ m}) \times h = 1200\text{ m}^2$ [from (1)]

$\Rightarrow \quad h = \dfrac{1200\text{ m}^2}{200\text{ m}} = 6$ m.

24. Let the height of the room = x.

Then length = $2x$ and breadth = $\dfrac{3}{2}x$

Area of walls = $\dfrac{1663.20}{14.85}\text{ m}^2 = 112\text{ m}^2$

$\Rightarrow \quad 2(l + b) = 112\text{ m}^2$

$\Rightarrow \quad 2\left(2x + \dfrac{3}{2}x\right)x = 112\text{ m}^2$

Solve to get $x^2 = 16$ m

$\Rightarrow \quad x = 4$ m.

Length = 2×4 m = 8 m and

Breadth = $\dfrac{3}{2} \times 4 = 6$ m.

$\Rightarrow$ cost of cementing the floor = ₹ $6.75 \times 8 \times 6$

= ₹ 324.

25. Given: $l = 12.5$ m,

$b = 9$ m, $h = 7$ m

Area of door 1 = $2.5\text{ m} \times 1.2\text{ m} = 3\text{ m}^2$

Area of 2 doors = $2 \times 3\text{ m}^2 = 6\text{ m}^2$

Area of window = $1.5\text{ m} \times 1\text{ m} = 1.5\text{ m}^2$

Area of 4 windows = $4 \times 1.5\text{ m}^2 = 6\text{ m}^2$

Area of 2 doors + Area of 4 windows

$= 6\text{ m}^2 + 6\text{ m}^2 = 12\text{ m}^2$

Area of the wall (LSA) = $2(l + b)h$

$= 2(12.5\text{ m} + 9\text{ m})\, 7\text{m}$

$= 2(21.5\text{ m})\, 7\text{m}$

$= 301\text{ m}^2$

Required area of the wall to be painted

$= (301 - 12)\text{ m}^2$

$= 289\text{ m}^2$

Cost of painting the wall = ₹ $3.50 \times 289\text{ m}^2$

= ₹ 1011.50

EXERCISE 13B

1. Curved surface area of the cylinder = $2\pi rh$

Total surface area of the cylinder = $2\pi r\,(h + r)$

(i) Radius = 7 cm

Height = 20 cm

Curved surface area = $2\pi rh$

$= 2 \times \dfrac{22}{7} \times 7\text{ cm} \times 20\text{ cm}$

$= 880\text{ cm}^2$

Total surface area = $2\pi r\,(h + r)$

$= 2 \times \dfrac{22}{7} \times 7\text{ cm}\,(20\text{ cm} + 7\text{ cm})$

$= 2 \times 22 \times 27\text{ cm}^2$

$= 1188\text{ cm}^2$

(ii) Radius = 10.5 cm

Height = 14 cm

Curved surface area = $2\pi rh$

$= 2 \times \dfrac{22}{7} \times 10.5\text{ cm} \times 14\text{ cm}$

$= 924\text{ cm}^2$

Total surface area = $2\pi r(h + r)$

$= 2 \times \dfrac{22}{7} \times 10.5\text{ cm}\,(10.5\text{ cm} + 14\text{ cm})$

$= 2 \times 22 \times 1.5\text{ cm} \times 24.5\text{ cm}$

$= 1617\text{ cm}^2$

(iii) Radius = 3.5 m

Height = 6 m

Curved surface area = $2\pi rh$

$= 2 \times \dfrac{22}{7} \times 3.5\text{ m} \times 6\text{ m}$

$= 132\text{ m}^2$

Total surface area = $2\pi r\,(h + r)$

$= 2 \times \dfrac{22}{7} \times 3.5\text{ m}\,(3.5\text{ m} + 6\text{ m})$

$= 2 \times 22 \times 0.5\text{ m} \times 9.5\text{ m}$

$= 209\text{ m}^2$

(iv) Radius = 10.5 m

Height = 16 m

Curved surface area = $2\pi rh$

$= 2 \times \dfrac{22}{7} \times 10.5\text{ m} \times 16\text{ m}$

$= 1056\text{ m}^2$

Total surface area = $2\pi r(h + r)$

$= 2 \times \dfrac{22}{7} \times 10.5\text{ m}\,(10.5\text{ m} + 16\text{ m})$

$= 2 \times 22 \times 1.5\text{ m} \times 26.5\text{ m}$

$= 1749\text{ m}^2$

2. Diameter of the cylindrical garden roller = 1.4 m

$\therefore$ Radius = 0.7 m

Length of the cylindrical garden roller = 2 m

Area covered by the roller in one resolution

= Curved surface area of the cylindrical garden roller

$= 2\pi rh = 2 \times \frac{22}{7} \times 0.7 \text{ m} \times 2\text{m} = 8.8 \text{ m}^2$

$\therefore$ Area covered in 5 resolutions

$= 5 \times 8.8 \text{ m}^2 = 44 \text{ m}^2$

Hence, the area covered in 5 resolutions is 44 m^2.

3. Diameter of the right circular cylinder = 7 cm

Radius = $\frac{7}{2}$ cm

Height of the right circular cylinder = 12 cm

Cured surface area of the cylinder to be polished = $2\pi rh$

$= 2 \times \frac{22}{7} \times \frac{7}{2} \text{ cm} \times 12 \text{ cm} = 264 \text{ cm}^2$

Cost of polishing the curved surface of the cylinder

= ₹ 0.50 × 264

= ₹ 132

Hence, the cost of polishing the curved surface of the cylinder is ₹ 132.

4. Diameter of the road roller = 70 cm

= 0.7 m

Radius of the road roller = 0.35 m

Length of the road roller = 1 m 50 cm

= 1.50 m

Area covered by the roller in one resolution

= Curved surface of the cylindrical road roller

$= 2\pi rh$

$= 2 \times \frac{22}{7} \times 0.35 \text{ m} \times 1.5 \text{ m} = 3.3 \text{ m}^2$

Area covered in 750 resolutions $= 750 \times 3.3 \text{ m}^2$

$= 2475 \text{ m}^2$

$\therefore$ Cost of levelling the surface = ₹ $\frac{75}{100} \times 2475$

= ₹ 1856.25

Hence, the cost of levelling the surface is ₹ 1856.25.

5. Diameter of the road roller = 250 m = 2.5 m

Radius of the road roller = $\frac{2.5 \text{ m}}{2} = 1.25$ m

Width of the road roller = 1 m 40 cm = 1.40 m

Area covered in one resolution

= Curved surface area of the road roller

$= 2\pi rh$

$= 2 \times \frac{22}{7} \times 1.25 \text{ m} \times 1.40 \text{ m} = 11 \text{ m}^2$

Number of resolutions made to level the playground

$= \frac{\text{Area to be levelled}}{\text{Curved surface area of the road roller}}$

$= \frac{110 \text{ m} \times 25 \text{ m}}{11 \text{ m}^2} = 250$

Hence, the lest number of resolutions the roller makers to level the playground is 250.

6. Length of the roller = 5 m

Diameter of the roller = 7 m

Radius of the roller = $\frac{7}{2}$ m

Number of revolutions

$= \frac{\text{Area}}{\text{Curved surface area of the roller}}$

$= \frac{5500 \text{ m}^2 \times 7 \times 2}{2 \times 22 \times 7 \times 5 \text{ m}^2} = 50$

Hence, the number of resolutions made is 50.

7. Diameter of the circular tunnel = 2 m

Radius = 1 m

Length of the circular tunnel = 1.4 km = 1400 m

Area of the tunnel to be plastered

= Area of the curved surface

$= 2\pi rh$

$= 2 \times \frac{22}{7} \times 1 \text{ m} \times 1400 \text{ m}$

$= 8800 \text{ m}^2$

Cost of plastering = ₹ $\frac{150}{100} \times 8800$

= ₹ 13200

Hence, the cost of plastering the circular tunnel is ₹ 13200.

8. Length of each pencil = 25 cm = 2.5 dm

Circumference of each pencil = $2\pi r$ = 1.5 cm = 0.15 dm

Curved surface area of each pencil = $2\pi rh$

= 0.15 dm × 2.5 dm

$= 0.375 \text{ dm}^2$

Curved surface area of 120000 pencils

$= 120000 \times 0.375 \text{ dm}^2$

Cost of colouring the pencils manufactured in one day

= ₹ 0.05 × 120000 × 0.375

= ₹ 2250

Hence, the cost of colouring the pencils manufactured in one day is ₹ 2250.

9. Circumference of the cylinder = $2\pi r$ = 10 cm

Height of the cylinder = 22 cm

Area covered in one revolution

= curved surface of the cylinder

$= 2\pi rh$

= 10 cm × 22 cm

$= 220 \text{ cm}^2$

Area covered by the cylinder while rolling for 1 second

$= 220 \text{ cm}^2 \times 5$

$= 1100 \text{ cm}^2$

Area covered in $\frac{11}{2}$ seconds $= 1100 \times \frac{11}{2} \text{ cm}^2$

$= 6050 \text{ cm}^2$

Hence, the area covered in $\frac{11}{2}$ seconds is 6050 cm^2.

10. Length of the cloth required = $2\pi r$ + 1 cm for margin

$$= \left[\left(2 \times \frac{22}{7} \times 14\right) + 1\right] \text{cm}$$

$$= 89 \text{ cm}$$

Width of the cloth required = (18 + 2 × 1) cm = 20 cm

Area of cloth required = 89 × 20 cm^2 = 1780 cm^2

Hence, the area of the cloth required for covering the lampshade is 1780 cm^2.

11. Circumference of the base of cylinder = $2\pi r$ = 88 cm

Height of the cylinder = 20 cm

Curved surface area of the cylinder = $2\pi rh$

$$= 88 \text{ cm} \times 20 \text{ cm}$$

$$= 1760 \text{ cm}^2$$

Hence, the curved surface of the cylinder is 1760 cm^2.

12. Area of the curved surface of the cylinder = $2\pi rh$

$$= 4400 \text{ cm}^2$$

$$2\pi rh = 4400 \text{ cm}^2$$

$$h = \frac{4400}{2\pi r} \text{ cm}^2 \quad \dots (1)$$

Circumference of the base = $2\pi r$ = 110 cm

$$2\pi r = 110 \text{ cm} \quad \dots (2)$$

Putting value of (2) in (1), we get

$$h = \frac{4400}{110} \text{ cm}$$

$$h = 40 \text{ cm}$$

Hence, the height of the cylinder is 40 cm.

13. Height of the cylinder = 14 cm

Area of the curved surface = $2\pi rh$

$$= 88 \text{ cm}^2$$

$$2\pi rh = 88 \text{ cm}^2$$

$$2r = \frac{88 \text{ cm}^2 \times 7}{22 \times 14 \text{ cm}}$$

$$2r = 2 \text{ cm}$$

Hence, the diameter of the cylinder is 2 cm.

14. Radius of the solid cylinder = 5 cm

Let the height of the solid cylinder = h

Total surface area = 660 cm^2

$$\Rightarrow \quad 2\pi r(h + 5) = 660 \text{ cm}^2$$

$$\Rightarrow \quad 2 \times \frac{22}{7} \times 5(h + 5) \text{ cm} = 660 \text{ cm}^2$$

$$h + 5 = 660 \times \frac{1}{2} \times \frac{7}{22} \times \frac{1}{5} \text{ cm}$$

$$h + 5 = 21 \text{ cm}$$

$$h = (21 - 5) \text{ cm}$$

$$h = 16 \text{ cm}$$

Hence, the height of the solid cylinder is 16 cm.

15. Curved surface area = $\frac{660}{1.5}$ cm^2 = 440 cm^2

$$\Rightarrow \quad 2\pi rh = 440 \text{ cm}^2$$

$$\Rightarrow \quad 2 \times \frac{22}{7} \times r \times 10 \text{ cm} = 440 \text{ cm}^2$$

$$\Rightarrow \quad r = \frac{440 \times 7}{2 \times 22 \times 100} \text{ cm} = 7 \text{ cm.}$$

16. Let the radius of the cylinder = r

Height of the cylinder = h

$$\frac{\text{Curved surface}}{\text{Total surface}} = \frac{1}{2}$$

$$\Rightarrow \quad \frac{2\pi rh}{2\pi r(h + r)} = \frac{1}{2}$$

$$\Rightarrow \quad \frac{h}{h + r} = \frac{1}{2}$$

$$\Rightarrow \quad 2h = h + r$$

$$\Rightarrow \quad 2h - h = r$$

$$\Rightarrow \quad h = r$$

So, $\quad \frac{h}{r} = \frac{1}{1}$

So, the ratio between the height and the radius of the cylinder is 1 : 1.

17. Total surface area = 231 cm^2

$$2\pi r^2 + 2\pi rh = 231 \text{ cm}^2 \quad \dots (1)$$

Curved surface area, $2\pi rh = \frac{2}{3} \times 231$ cm^2

$$2\pi rh = 154 \text{ cm}^2 \quad \dots (2)$$

Substituting the value of (2) in (1), we get

$$2\pi r^2 + 154 \text{ cm}^2 = 231 \text{ cm}^2$$

$$2\pi r^2 = (231 - 154) \text{ cm}^2 = 77 \text{ cm}^2$$

$$r^2 = 77 \times \frac{1}{2} \times \frac{7}{22} \text{ cm}^2$$

$$r^2 = \frac{7 \times 7}{2 \times 2} \text{ cm}^2$$

$$r = \frac{7}{2} \text{ cm.}$$

Substituting the value of $r = \frac{7}{2}$ cm in (2), we get

$$2 \times \frac{22}{7} \times \frac{7}{2} \text{ cm} \times h = 154 \text{ cm}^2$$

$$h = 154 \times \frac{1}{2} \times \frac{7}{22} \times \frac{2}{7} \text{ cm}$$

$$h = 7 \text{ cm}$$

Hence, the radius of the base and height of the cylinder are $\frac{7}{2}$ cm and 7 cm respectively.

18. Surface area of the wetted surface of the cylinder = $2\pi rh$

$$= 2 \times \frac{22}{7} \times 7 \text{ cm} \times 10 \text{ cm}$$

$$= 440 \text{ cm}^2$$

Hence, the surface area of the wetted surface of the cylinder is 440 cm^2.

19. Square metres of the sheet required

$$= \text{Total surface area of the closed cylinder tank}$$
$$= 2\pi r^2 + 2\pi rh$$
$$= 2\pi r\ (r + h)$$
$$= 2 \times \frac{22}{7} \times 0.70 \text{ m } (0.70 \text{ m} + 1.3 \text{ m})$$
$$= 2 \times \frac{22}{7} \times 0.70 \text{ m} \times 2 \text{ m} = 8.8 \text{ m}^2$$

Hence, 8.8 m^2 of the sheet is required to make a closed cylindrical tank.

20. Total surface of hollow cylindrical pipe

$$= 2\pi Rh + 2\pi rh + 2\pi\ (R^2 - r^2)$$
$$= 2\pi\ (Rh + rh + (R^2 - r^2)]$$
$$= 2\pi\ [(4)\ (12) + (2)\ (12) + (4^2 - 2^2)] \text{ cm}^2$$
$$= 2\pi\ (48 + 24 + 12) \text{ cm}^2 = 2\pi\ (84) \text{ cm}^2$$
$$= 168\ \pi \text{ cm}^2.$$

21. Total surface area of the hollow cylinder

$$= 338\pi \text{ cm}^2$$
$$\Rightarrow \quad 2\pi Rh + 2\pi rh + 2\pi\ (R^2 - r^2) = 338\pi$$
$$\Rightarrow \quad Rh + rh + (R^2 - r^2) = 169$$
$$\Rightarrow \quad 8(10) + r(10) + (8^2 - r^2) = 169$$
$$\Rightarrow \quad 80 + 10r + 64 - r^2 = 169$$
$$\Rightarrow \quad r^2 - 10r + 25 = 0$$
$$\Rightarrow \quad (r - 5)^2 r = 0$$
$$\Rightarrow \quad r = 5$$

Thickness = R – r = (8 – 5) cm = 3 cm.

22. Let the inner and outer radii be r and R respectively. Let h be the height of the hollow pipe. Then,

$$R - r = 3.5 \text{ cm and } h = 100 \text{ cm}$$
$$2\pi Rh - 2\pi rh = 2\pi h(R - r) = 2 \times \frac{22}{7} \times 100 \times 3.5 \text{ cm}^2$$
$$= 2200 \text{ cm}^2.$$

23. Total surface area of the cylindrical tank

$$= 2\pi r(h + r) = 2 \times \frac{22}{7} \times 1.4\ (2.4 + 1.4) \text{ m}^2$$
$$= \frac{2 \times 22}{7} \times 1.4(3.8) \text{ m}^2 = 33.44 \text{ m}^2$$

Let the area of metal sheet used for making the tank be x metre^2.

$$\text{Wastage} = 12\% \text{ of } x \text{ m}^2 = \frac{12}{100} \times x \text{ m}^2$$
$$= \frac{3x}{25} \text{ m}^2$$

$$\text{Then, actual steel used} = \left(x - \frac{3x}{25}\right) \text{m}^2 = \frac{22x}{25} \text{ m}^2$$

Actual steel used = Total surface area of the tank

$$\therefore \quad \frac{22x}{25} \text{ m}^2 = 33.44 \text{ m}^2$$
$$\Rightarrow \quad x = \frac{33.44 \times 25}{22} = 38 \text{ m}^2$$

24. Let the radii of two circular cylinders be r_1 and r_2.

$$\frac{r_1}{r_2} = \frac{2}{3}$$

Let the heights of the two cylinders be h_1 and h_2.

$$\frac{h_1}{h_2} = \frac{5}{4}$$

Ratio of their curved surface areas

$$= \frac{2\pi r_1 h_1}{2\pi r_2 h_2} = \frac{r_1}{r_2}\frac{h_1}{h_2} = \frac{2}{3} \times \frac{5}{4} = \frac{5}{6} = 5 : 6.$$

25. Curved surface area = 2310 cm^2

$$\Rightarrow \quad 2\pi rh = 2310 \text{ cm}^2$$
$$\Rightarrow \quad 2 \times \pi \times r \times 15 \text{ cm} = 2310 \text{ cm}^2$$
$$\Rightarrow \quad 2\pi r = \frac{2310}{15} \text{ cm} = 154 \text{ cm}$$

$\Rightarrow$ Circumference of the base of cylinder = 154 cm

$$\text{Number of turns} = \frac{\text{Length of the cylinder}}{\text{Diameter of the wire}} = \frac{15}{0.6} = 25$$

Length of wire in 1 turn

= circumference of the base of cylinder

= 154 cm

Length of wire in 25 turns

= (154 × 25) cm

= 3850 cm = 38.5 m.

EXERCISE 13C

1. Curved surface area of the cone = πrl, where $l = \sqrt{r^2 + h^2}$

(i) Given: $r = 16$ m, $h = 12$ m

$$l = \sqrt{16^2 \text{ m}^2 + 12^2 \text{ m}^2} = \sqrt{(256 + 144) \text{ m}^2}$$
$$= \sqrt{400 \text{ m}^2} = 20 \text{ m}$$
$$\text{CSA of the cone} = \pi rl = \frac{22}{7} \times 16 \text{ m} \times 20 \text{ m}$$
$$= \frac{7040}{7} \text{ m}^2.$$

(ii) Given: diameter = 28 cm, $r = \frac{28}{2} = 14$ cm,

$$h = 10.5 \text{ cm}$$
$$l = \sqrt{(14 \text{ cm})^2 + (10.5 \text{ cm})^2}$$
$$= \sqrt{196 \text{ m}^2 + 110.26 \text{ cm}^2}$$
$$= \sqrt{306.25 \text{ cm}^2} = 17.5 \text{ cm}$$
$$\text{CSA of the cone} = \pi rl = \frac{22}{7} \times 14 \text{ cm} \times 17.5 \text{ cm}$$
$$= 770 \text{ cm}^2.$$

(iii) Given: Diameter = 20 m, $r = \frac{20 \text{ cm}}{2} = 10$ m,

$$l = 12.5 \text{ m}$$

CSA of the cone = $\pi rl = \frac{22}{7} \times 10 \text{ m} \times 12.5 \text{ m}$

$= \frac{2750}{7} \text{ m}^2$

(iv) Given: $h = 84$ cm, $l = 91$ cm

$r^2 = l^2 - h^2$

$= (91 \text{ cm})^2 - (84 \text{ cm})^2$

$= 8281 \text{ cm}^2 - 7056 \text{ cm}^2$

$= 1225 \text{ cm}^2$

$\Rightarrow \quad r = 35$ cm

CSA of the cone = $\pi rl = \frac{22}{7} \times 35 \text{ cm} \times 91 \text{ cm}$

$= 10010 \text{ cm}^2$.

2. Total surface are of the cone = $\pi r(l + r)$

(i) Given: $r = 5$ cm, $h = 12$ cm

$l = \sqrt{r^2 + h^2} = \sqrt{(5 \text{ cm})^2 + (12 \text{ cm})^2}$

$= \sqrt{25 \text{ cm}^2 + 144 \text{ cm}^2}$

$= \sqrt{169 \text{ cm}^2} = 13$ cm

TSA of the cone $= \frac{22}{7} \times 5 \text{ cm} (13 \text{ cm} + 5 \text{ cm})$

$= \frac{22}{7} \times 5 \text{ cm} \times 18 \text{ cm}$

$= \frac{1980}{7} \text{ cm}^2$.

(ii) Given: Diameter = 42 cm, $r = \frac{42 \text{ cm}}{2} = 21$ cm

Height = 28 cm

$l = \sqrt{(28 \text{ cm})^2 + (21 \text{ cm})^2}$

$= \sqrt{784 \text{ cm}^2 + 441 \text{ cm}^2}$

$= \sqrt{1225 \text{ cm}^2} = 35$ cm

TSA of the cone = $\pi r(l + r)$

$= \frac{22}{7} \times 21 \text{ cm} (35 \text{ cm} + 21 \text{ cm})$

$= \frac{22}{7} \times 21 \text{ cm} \times 56 \text{ cm}$

$= 3696 \text{ cm}^2$.

(iii) Given: Radius = 4 m, $l = 100$ dm = 10 m

TSA of the cone = $\pi r(l + r)$

$= \frac{22}{7} \times 4 \text{ m} (10 \text{ m} + 4 \text{ m})$

$= \frac{22}{7} \times 4 \text{ m} \times 14 \text{ m}$

$= 176 \text{ m}^2$.

(iv) Given: Diameter = 70 dm,

$r = \frac{70 \text{ dm}}{2} = 35$ dm and $l = 37$ dm

TSA of the cone = $\pi r(l + r)$

$= \frac{22}{7} \times 35 \text{ dm} (37 \text{ dm} + 35 \text{ dm})$

$= \frac{22}{7} \times 35 \text{ dm} \times 72 \text{ dm}$

$= 7920 \text{ dm}^2$.

3. (i) Perimeter of the base = $2\pi r = 88$ cm

$\Rightarrow \quad r = 14$ cm

Curved surface area = $\pi rl = \frac{22}{7} \times 14 \times 20 \text{ cm}^2$

$= 880 \text{ cm}^2$

(ii) Area of the base = $\pi r^2 = 154 \text{ cm}^2$

$\Rightarrow \quad r^2 = 49 \text{ cm}^2$

$\Rightarrow \quad r = 7$ cm

Slant height = $l = \sqrt{r^2 + h^2}$

$= \sqrt{(7)^2 + (24)^2}$ cm

$= \sqrt{625}$ cm

$= 25$ cm

Curved surface area = $\pi rl = \frac{22}{7} \times 7 \times 25 \text{ cm}^2$

$= 550 \text{ cm}^2$

4. Given: Circumference ($2\pi r$) of the cone = 24π m

So, $\quad r = \frac{24\pi \text{ m}}{2\pi} = 12$ m

and $\quad h = 9$ m

$l = \sqrt{r^2 + h^2} = \sqrt{(2 \text{ m})^2 + (9 \text{ m})^2}$

$= \sqrt{144 \text{ m}^2 + 81 \text{ m}^2}$

$= \sqrt{225 \text{ m}^2} = 15$ m.

CSA of the cone = πrl

$= \pi \times 12 \text{ m} \times 15 \text{ m}$

$= 180\pi \text{ m}^2$

5. Given: Diameter of the cone = 24 cm

$\Rightarrow \quad r = \frac{24 \text{ m}}{2} = 12$ cm

and $\quad h = 16$ cm

$l = \sqrt{(12 \text{ cm})^2 + (16 \text{ cm})^2}$

$= \sqrt{144 \text{ cm}^2 + 256 \text{ cm}^2}$

$= \sqrt{400 \text{ cm}^2} = 20$ cm

CSA of the cone = $\frac{22}{7} \times 12 \text{ cm} \times 20 \text{ cm}$

$= \frac{5280}{7} \text{ cm}^2$

Hence, area of metal sheet required to make coner.

$= \frac{5280}{7} \text{ cm}^2$.

6. $l = \sqrt{r^2 + h^2} = \sqrt{(9)^2 + (12)^2}$

$= \sqrt{81 + 144}$ m $= \sqrt{225}$ m = 15 m

Curved surface area = $\pi rl = \frac{22}{7} \times 9 \times 15$ m²

$= \frac{2970}{7}$ m²

Area of canvas cloth required $= \left(\frac{2970}{7} + \frac{5}{7}\right)$ m²

$= \frac{2970}{7}$ m² = 425 m².

7. Given: l = 29 m, d = 40 m,

∴ $r = \frac{40 \text{ m}}{2} = 20$ m

CSA of the cone $= \frac{22}{7} \times 20 \text{ m} \times 29 \text{ m} = \frac{12760 \text{ m}^2}{7}$

= 8122.86 m²

Cost of whitewashing = ₹ 280 × $\frac{1822.86}{100 \text{ m}^2}$

= ₹ 5104 (approx.)

8. Slant height of the metal sheet $= \sqrt{(7)^2 + (24)^2}$ cm

$= \sqrt{49 + 576}$ cm

$= \sqrt{625}$ cm = 25 cm

Area of metal sheet required $= \pi rl = \frac{22}{7} \times 7 \times 25$ cm²

= 550 cm²

Area of base $= \pi r^2 = \frac{22}{7} \times 7 \times 7$ cm²

= 154 cm²

Area of sheet required = (550 + 154) cm²

= 704 cm²

Or Area of metal sheet required $= \pi r(r + l)$

$= \frac{22}{7} \times 7 \times (7 + 25)$ cm²

= 704 cm².

9. Given: h = 28 m, r = 21 m,

$l = \sqrt{(28 \text{ m})^2 + (21 \text{ m})^2} = \sqrt{784 \text{ m}^2 + 441 \text{ m}^2}$

$= \sqrt{1225 \text{ m}^2} = 35$ m

CSA of the conical tent $= \pi rl$

$= \frac{22}{7} \times 21 \text{ m} \times 35$ m

$= \frac{16170 \text{ m}^2}{7} = 2310$ m²

Amount of canvas required = 2310 m².

10. Given: CSA of cone = 470 cm², d = 70 cm

∴ $r = \frac{70 \text{ cm}}{2} = 35$ cm

πrl = CSA of cone

$\Rightarrow \frac{22}{7} \times 35 \text{ cm} \times l = 4070$ cm²

$\Rightarrow$ 110 cm × l = 4070 cm²

∴ $l = \frac{4070 \text{ cm}^2}{110 \text{ cm}} = 37$ cm.

Hence slant height of the cone is 37 cm.

11. Given: LSA of cone = 175π cm², l = 25 cm

$\pi rl = 175\pi$ cm²

$\Rightarrow \pi \times r \times 25 \text{ cm} = 175\pi$ cm²

$\Rightarrow r = \frac{175\pi \text{ cm}^2}{\pi \times 25 \text{ cm}} = 7$ cm

Height of a cone $h = \sqrt{l^2 - r^2}$

$= \sqrt{(25 \text{ cm})^2 - (7 \text{ cm})^2}$

$= \sqrt{625 \text{ cm}^2 - 49 \text{ cm}^2}$

$= \sqrt{576 \text{ cm}^2} = 24$ cm.

12. Curved surface area $= \pi rl = 10010$ cm²

$\Rightarrow \frac{22}{7} \times r \times 91 \text{ cm} = 10010$ cm²

$\Rightarrow r = \frac{10010 \times 7}{22 \times 91}$ cm = 35 cm

Total surface area = Curved surface area + πr^2

$= 10010 \text{ cm}^2 + \left(\frac{22}{7} \times 35 \times 35\right) \text{cm}^2$

= (10010 + 3850) cm² = 13860 cm².

13. Given: TSA of cone = 90π cm², l = 13 cm

$\pi r(r + l)$ = TSA of cone = 90π cm²

$\Rightarrow r(r + 13 \text{ cm}) = \frac{90\pi \text{ cm}^2}{\pi} = 90$ cm²

$\Rightarrow r^2 + 13r = 90$

$\Rightarrow r^2 + 13r - 90 = 0$

$\Rightarrow r^2 + 18r - 5r - 90 = 0$

$\Rightarrow r(r + 18) - 5(r + 18) = 0$

$\Rightarrow (r - 5)(r + 18) = 0$

$r + 18 = 0 \Rightarrow r = -18$ (rejected)

and $r - 5 = 0 \Rightarrow r = 5$ cm

Hence, the radius of the cone = 5 cm.

14. Let the radius and slant height of the cone be $5x$ and $7x$ respectively.

So, πrl = CSA of cone

$\Rightarrow \pi \times 5x \times 7x = 2750$ m²

$\Rightarrow 35x^2 = \frac{2750}{\pi} = \frac{2750 \times 7}{22} = 875$ m²

$\Rightarrow 35x^2 = 875$ m²

$\Rightarrow x^2 = \frac{875 \text{ m}^2}{350} = 25$ m²

$\therefore \quad x = 5$ m

Radius = $5x = 5 \times 5$ m = 25 m.

15. Let the radius and slant height of cone be $4x$ and $7x$ respectively.

πrl = CSA of the cone

$\pi \times 4x \times 7x = 3168 \text{ m}^2$

$\Rightarrow \quad \pi \times 28 \text{ m}^2 = 3168 \text{ m}^2$

$\Rightarrow \quad 28x^2 = \dfrac{3168 \text{ m}^2}{\pi} = \dfrac{3168 \text{ m}^2 \times 7}{22} = 1008 \text{ m}^2$

$\Rightarrow \quad x^2 = \dfrac{1008 \text{ m}^2}{28} = 36 \text{ m}^2$

$\Rightarrow \quad x = 6$ m

Slant height = $7x = 7 \times 6$ m = 42 m.

16. $\pi rl - \pi r^2 = 88 \text{ cm}^2$ and $l - r = 4$ cm

$\Rightarrow \quad \pi r(l - r) = 88 \text{ cm}^2$

$\Rightarrow \quad \dfrac{22}{7} r(4) = 88 \text{ cm}^2$

$\Rightarrow \quad r = \dfrac{28}{4}$ cm = 7 cm.

17. Slant height of the corn cob = $\sqrt{(3.5)^2 + (21.2)^2}$ cm

$= \sqrt{12.25 + 449.44}$ cm

$= \sqrt{461.69}$ cm

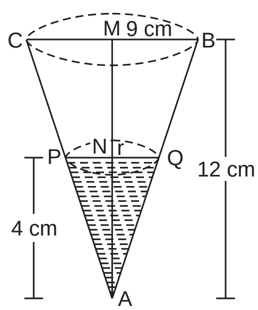
= 21.5 cm (approx.)

Surface area of the corn cob = $\pi rl = \dfrac{22}{7} \times 3.5 \times 21.5 \text{ cm}^2$

Number of grains on the cob = $\dfrac{22}{7} \times 3.5 \times 21.5 \times 10$

= 2365.

18. ΔAMB ~ ΔANQ



$\Rightarrow \quad \dfrac{9 \text{ cm}}{r} = \dfrac{12 \text{ cm}}{4 \text{ cm}}$

$\Rightarrow \quad r = \dfrac{4 \times 9}{12}$ cm

= 3 cm

L = AB

$= \sqrt{9^2 + 12^2}$ cm

$= \sqrt{225}$ cm

= 15 cm

and $\quad l$ = AQ = $\sqrt{4^2 + 3^2}$ cm = $\sqrt{25}$ cm = 5 cm

Internal surface of cone not in contact with water

$= \pi RL - \pi rl = \pi(RL - rl)$

$= \dfrac{22}{7}[(9)(15) - (3)(5)] \text{ cm}^2$

$= \dfrac{22}{7} \times 3 \times 5 (3 \times 3 - 1) \text{ cm}^2$

$= \dfrac{22}{7} \times 3 \times 5 \times 8 \text{ cm}^2 = \dfrac{2640}{7} \text{ cm}^2$

19. Given:height of the tent (h) = 5 m, radius (r) = 12 m

Slant height (l) = $\sqrt{r^2 + h^2} = \sqrt{(12 \text{ m})^2 + (5 \text{ m})^2}$

$= \sqrt{144 \text{ m}^2 + 25 \text{ m}^2}$

$= \sqrt{169 \text{ m}^2}$ = 13 m

CSA of the tent = $\pi rl = \dfrac{22}{7} \times 12 \times 13 = \dfrac{3432}{7} \text{ m}^2$

Cost of canvas = $\dfrac{3432}{7} \text{ m}^2 \times$ ₹ 70 = ₹ 34320.

20. Given: $\quad h = 30$ m, base area = 1386 m^2

$\Rightarrow \quad \pi r^2 = 1386 \text{ m}^2$

$\Rightarrow \quad r^2 = \dfrac{1386 \text{ m}^2}{\pi} = \dfrac{1386 \text{ m}^2 \times 7}{22} = 441 \text{ m}^2$

$\therefore \quad r = \sqrt{441 \text{ m}^2}$ = 21 m

$l = \sqrt{(30 \text{ m})^2 + (21 \text{ m})^2}$

$= \sqrt{(900 + 441) \text{ m}^2} = \sqrt{1341 \text{ m}^2}$

= 36.62 m

CSA of the conical tent = πrl

$= \dfrac{22}{7} \times 21 \text{ m} \times 36.62$ m

$= 2416.92 \text{ m}^2 \approx 2417 \text{ m}^2$.

21. Area of canvas required = $\pi rl = \dfrac{22}{7} \times 6 \times 63 \text{ m}^2$

$= 1188 \text{ m}^2$

Length of canvas required = $\dfrac{\text{Area}}{\text{Width}}$

$= \dfrac{1188 \text{ m}^2}{2 \text{ m}}$ = 594 m

Cost of canvas = ₹ 594 × 15 = ₹ 8910.

22. Given: $\quad r = 30$ m, $h = 16$ m

So, $\quad l = \sqrt{(30 \text{ m})^2 + (16 \text{ m})^2}$

$= \sqrt{900 \text{ m}^2 + 256 \text{ m}^2}$

$= \sqrt{1156 \text{ m}^2}$ = 34 m

CSA of the conical tent = πrl

$= \dfrac{22}{7} \times 30 \text{ m} \times 34$ m

$= \dfrac{22440}{7} \text{ m}^2$ or 3205.71 m^2

5% extra canvas = $\dfrac{22440}{7} \times \dfrac{5}{100}$

$= \dfrac{112200}{700} = 160.29 \text{ m}^2$

Net canvas required = $3205.71 \text{ m}^2 + 160.29 \text{ m}^2$

$= 3366 \text{ m}^2$

23. Given: diameter = 4.2 m

So, $r = \frac{4.2\text{ m}}{2} = 2.1\text{ m}$

Height of cylinder = 4 m

Height of cone = 2.1 m

Slant height of cone, $l = \sqrt{(2.1\text{ m})^2 + (2.1\text{ m})^2}$

$= \sqrt{4.41\text{ m}^2 + 4.41\text{ m}^2}$

$= \sqrt{8.82\text{ m}^2} = 2.97\text{ m}$

Outer surface area = $2\pi rh + \pi rl$

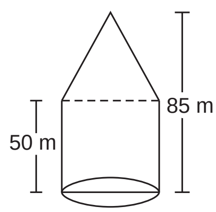
$= 2 \times \frac{22}{7} \times 2.1\text{ m} \times 4\text{ m} + \frac{22}{7} \times 2.1\text{ m} \times 2.97\text{ m}$

$= 52.8\text{ m}^2 + 19.602\text{ m}^2$

$= 72.402\text{ m}^2$ (approx.)

24. Let R be the radius of the base of the cylinder.

Then, $R = \frac{168}{2}\text{ m} = 84\text{ m}$



Let r be the radius and h the height of the cone.

Then, $r = R = 84\text{ m}$

Total height of the tent = 85 m

⇒ Height of the cylinder + Height of the cone = 85 m

⇒ $50\text{ m} + h = 85\text{ m}$

⇒ $h = (85 - 50)\text{ m}$

⇒ $h = 35\text{ m}$

Slant height of the cone = $l = \sqrt{r^2 + h^2}$

$= \sqrt{(84)^2 + (35)^2}\text{ m}$

$= \sqrt{7056 + 1225}\text{ m}$

$= \sqrt{8281}\text{ m} = 91\text{ m}$

Curved surface area of the tent

$= \pi rl + 2\pi RH$

$= \frac{22}{7}[(84)(91) + 2(84)(50)]\text{ m}^2$

$= 50424\text{ m}^2$

Area of canvas required = $\left(50424 + \frac{20}{100} \times 50424\right)\text{ m}^2$

$= 50424\left(1 + \frac{1}{5}\right)\text{ m}^2 \approx 60509\text{ m}^2$.

25. Given: Height of the cylindrical portion = 4 m

Diameter = 105 m

So, $r = \frac{105}{2} = 52.5\text{ m}$

$l = 80\text{ m}$.

Total surface area = $2\pi rh + \pi rl$

$= 2 \times \frac{22}{7} \times \frac{105}{2}\text{ m} \times 4\text{ m} + \frac{22}{7} \times \frac{105}{2} \times 80\text{ m}$

$= 1320\text{ m}^2 + 13200\text{ m}^2$

$= 14520\text{ m}^2$

Length of canvas = $\frac{14520\text{ m}^2}{1.5\text{ m}} = 9680\text{ m}$

Total cost of canvas = 9680 m × ₹ 15 = ₹ 145200.

26. Given: Height of the tent = 77 dm = 7.7 m

Height of the cylinder = 44 dm = 4.4 m

So, Height of cone = (77 – 44) dm

= 33 dm

= 3.3 m

Diameter = 36 m

⇒ $r = \frac{36\text{ m}}{2} = 18\text{ m}$

$l = \sqrt{(3.3\text{ m})^2 + (18\text{ m})^2}$

$= \sqrt{10.89\text{ m}^2 + 324\text{ m}^2}$

$= \sqrt{334.89\text{ m}^2} = 18.3\text{ m}$

Total area of tent = $2\pi rh + \pi rl$

$= \pi r(2h + l)$

$= \frac{22}{7} \times 18\,(2 \times 4.4\text{ m} + 18.3)\text{ m}^2$

$= \frac{396}{7}(27.1)\text{ m}^2$

$= \frac{10731.6}{7}\text{ m}^2$

Also cost of 1 m^2 canvas = ₹ 3.50

⇒ Cost of total canvas = ₹ 3.50 × $\frac{10731.6}{7}$ = ₹ 5365.80.

27. Given: Diameter of cone A = $5x$; $r_A = \frac{5x}{2}$

Diameter of cone B = $4x$; $r_B = \frac{4x}{2} = 2x$

slant height of cone A = $3y$

slant height of cone B = $8y$

CSA of cone A = $\pi rl = \pi \times \frac{5x}{2} \times 3y$

CSA of cone B = $\pi rl = \pi \times 2x \times 8y$

Ratio between the two cones = $\frac{\pi \times 5x \times 3y}{\pi \times 2x \times 8y \times 2}$

$= \frac{15}{32} = 15 : 32$.

28. Let r_1 be the radius, l_1 the slant height and S_1 the curved surface area of the first cone. Let r_2 be the radius, l_2 the slant height and S_2 the curved surface area of the second cone.

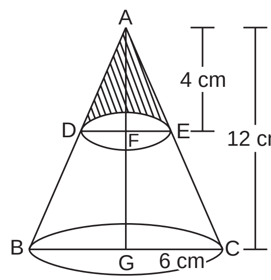
Then, $S_1 = 2S_2$ and $l_2 = 2l_1$

$$\Rightarrow \frac{S_1}{S_2} = 2 \Rightarrow \frac{\pi r_1 l_1}{\pi r_2 l_2} = 2$$

$$\Rightarrow \frac{r_1 l_1}{r_2(2l_1)} = 2 \Rightarrow \frac{r_1}{r_2} = \frac{4}{1}$$

$$\Rightarrow r_1 : r_2 = 4 : 1.$$

29. Let R be the radius, H (=AG) the height and L the slant height of the original cone (ABC say). Let r be the radius, h (=AF) the height and l the slant height of the smaller cone (ADE say). Then, R = 6 cm, H = 12 cm and h = 4 cm.



ΔAFE ~ ΔAGC

[By AA criterion of similarity] … (1)

$$\frac{FE}{GC} = \frac{AF}{AG}$$

$$\Rightarrow \frac{r}{R} = \frac{4}{12}$$

$$\Rightarrow r = 2 \text{ cm}$$

$$l = \sqrt{R^2 + H^2} = \sqrt{(6)^2 + (12)^2} \text{ cm}$$

$$= 6\sqrt{5} \text{ cm}$$

and $$l = \sqrt{r^2 + h^2} = \sqrt{(2)^2 + (4)^2} \text{ cm}$$

$$= \sqrt{20} \text{ cm} = 2\sqrt{5} \text{ cm}$$

Whole surface area of the remaining figure

$$= \pi r^2 + \pi R^2 + \pi RL - \pi rl$$

$$= \pi (r^2 + R^2 + RL - rl)$$

$$= \frac{22}{7} [(2)^2 + (6)^2 + 6 \times 6\sqrt{5} - 2 \times 2\sqrt{5}] \text{ cm}^2$$

$$= \frac{22}{7} (4 + 36 + 32\sqrt{5}) \text{ cm}^2$$

$= 350.59 \text{ cm}^2$ (approx.).

30. Given: CSA of cylinder = CSA of cone

Height cylinder = 2 m

Radius = r (both same)

$$\Rightarrow 2\pi rh = \pi rl$$

$$\Rightarrow 2\pi \times 2 \text{ m} \times r = \pi \times r \times l$$

$$\therefore l = \frac{2\pi \times 2 \text{ m} \times r}{\pi \times r} = 4 \text{ m}.$$

Slant height of the cone = 4 m.

EXERCISE 13D

1. (i) Given: Radius of sphere (r)= $\frac{21}{4}$ cm

Surface area of sphere = $4\pi r^2$

$$= 4 \times \frac{22}{7} \times \frac{21}{4} \text{ cm} \times \frac{21}{4} \text{ cm}$$

$$= \frac{5544}{16} \text{ cm}^2 = 346.5 \text{ cm}^2.$$

(ii) Given: radius = $\sqrt{21}$ cm

Surface area of sphere = $4\pi r^2$

$$= 4 \times \frac{22}{7} \times (\sqrt{21} \text{ cm})^2$$

$$= 4 \times \frac{22}{7} \times 21 \text{ cm}^2 = 264 \text{ cm}^2.$$

2. (i) Diameter of sphere = 42 cm

So, $r = \frac{42 \text{ cm}}{2} = 21$ cm

Surface area of sphere = $4\pi r^2$

$$= 4 \times \frac{22}{7} \times 21 \text{ cm} \times 21 \text{ cm}$$

$$= 5544 \text{ cm}^2.$$

(ii) diameter of the sphere= 28 cm

Radius = $\frac{28 \text{ cm}}{2}$ = 14 cm

Surface area of the sphere = $4\pi r^2$

$$= 4 \times \frac{22}{7} \times 14 \text{ cm} \times 14 \text{ cm}$$

$$= 2464 \text{ cm}^2.$$

3. Given: Total surface area of hemisphere = $3\pi r^2 = 4158 \text{ cm}^2$

$$\Rightarrow 3\pi r^2 = 4158 \text{ cm}^2$$

$$\Rightarrow r^2 = \frac{4158 \text{ cm}^2 \times 7}{3 \times 22}$$

$$\Rightarrow r^2 = 441 \text{ cm}^2$$

$$\Rightarrow r = \sqrt{441 \text{ cm}^2} = 21 \text{ cm}$$

Diameter = 21 × 2 = 42 cm.

4. Given: Diameter = 7 m

$$\Rightarrow r = \frac{7 \text{ m}}{2} = 3.5 \text{ m}$$

Surface area of sphere = $4\pi r^2$

$$= 4 \times \frac{22}{7} \times 3.5 \text{ m} \times 3.5 \text{ m}$$

$$= 154 \text{ m}^2.$$

5. Given: Diameter of balloon = 2.8 m

So, radius = 1.4 m

Surface area of spherical balloon = $4\pi r^2$

$$= 4 \times \frac{22}{7} \times 1.4 \text{ m} \times 1.4 \text{ m}$$

$$= \frac{172.48 \text{ m}^2}{7}$$

$$= 24.64 \text{ m}^2$$

6. Area of land $= \frac{1}{4} \times 4\pi r^2$

$$= \frac{22}{7} \times \frac{12740}{2} \times \frac{12740}{2} \text{ km}^2$$

$$= 127527400 \text{ km}^2.$$

7. Given: diameter of circle = 2.8 cm

So, radius of circle = 1.4 cm

Area of circle $= \pi r^2 = \frac{22}{7} \times 1.4 \text{ cm} \times 1.4 \text{ cm}$

$$= 6.16 \text{ cm}^2$$

Now, $4\pi r^2 = 6.16 \text{ cm}^2$

$$\Rightarrow \quad r^2 = \frac{6.16 \text{ cm}^2}{4\pi} = \frac{6.16 \text{ cm}^2 \times 7}{2 \times 44}$$

$$\Rightarrow \quad r^2 = 0.49 \text{ cm}^2$$

$$\Rightarrow \quad r = \sqrt{0.49 \text{ cm}^2} = 0.7 \text{ cm}$$

Diameter = 2 × 0. 7 = 1.4 cm.

8.
$$\frac{4\pi r_1^2}{4\pi r_2^2} = \frac{25}{49}$$

$$\Rightarrow \quad \frac{r_1^2}{r_2^2} = \frac{25}{49}$$

$$\Rightarrow \quad \frac{r_1}{r_2} = \frac{5}{7}$$

$$\Rightarrow \quad r_1 : r_2 = 5 : 7.$$

9. TSA of hemisphere $= 3\pi r^2$

Ratio of two hemisphere $= \frac{3\pi r_1^2}{3\pi r_2^2} = \frac{16}{9}$

$$\Rightarrow \quad \frac{r_1^2}{r_2^2} = \frac{16}{9}$$

$$\Rightarrow \quad \frac{r_1}{r_1} = \sqrt{\frac{16}{9}}$$

$$= \frac{4}{3}$$

Hence, ratio = 4 : 3.

10. Let r be the radius of the spherical balloon and let R be the radius of the inflated spherical balloon.

Then $r = 5$ cm and R = 7.5 cm

$$\frac{\text{SA of the original spherical balloon}}{\text{SA of the inflated spherical balloon}}$$

$$= \frac{4\pi r^2}{4\pi R^2} = \frac{r^2}{R^2}$$

$$= \frac{(5 \text{ cm})^2}{(7.5 \text{ cm})^2} = \frac{25}{56.25}$$

$$= \frac{1 \times 4}{2.25 \times 4} = \frac{4}{9}$$

$$= 4 : 9.$$

11. Let r = radius of the original balloon = 21 cm

R = radius of the deflated balloon = 14 cm

$$\frac{\text{SA of the original spherical balloon}}{\text{SA of the deflated spherical balloon}}$$

$$= \frac{4\pi r^2}{4\pi R^2} = \frac{r^2}{R^2}$$

$$= \frac{(21 \text{ cm})^2}{(14 \text{ cm})^2} = \frac{441 \text{ cm}^2}{196 \text{ cm}^2}$$

$$= \frac{2.25}{1} \times \frac{4}{4} = \frac{9}{4} = 9 : 4$$

Percentage decrease is surface area

$$= \frac{\text{decrease in SA}}{\text{Original SA}} \times 100$$

$$= \frac{4\pi(441 \text{ cm}^2 - 196 \text{ cm}^2)}{4\pi\,(441 \text{ cm}^2)} \times 100$$

$$= \frac{245}{441} \times 100 = \frac{24500}{441}$$

$$= \frac{500 \times 49}{9 \times 49} = \frac{500}{9}\,\%.$$

12. Given: $r = 5.6$ dm

CSA of hemisphere $= 2\pi r^2$

$$= 2 \times \frac{22}{7} \times 5.6 \text{ dm} \times 5.6 \text{ dm}$$

$$= 197.12 \text{ dm}^2$$

Cost of painting = ₹ 8 × 197.12 dm² = ₹ 1576.96.

13.
$$2\pi r = 17.6 \text{ m}$$

$$\Rightarrow \quad r = \frac{17.6 \times 7}{2 \times 22} \text{ m} = 2.8 \text{ m} = 280 \text{ cm}$$

Surface area to be painted

$$= 2\pi r^2 = 2 \times \frac{22}{7} \times 280 \times 280 \text{ cm}^2$$

$$= 492800 \text{ cm}^2$$

Cost of painting = ₹ 492800 × $\frac{5}{100}$ = ₹ 24640.

14. Let r be the internal radius and R the external radius of the hemispherical bowl.

Area to be pained

$$= 2\pi R^2 + 2\pi r^2 + \pi(R^2 - r^2)$$

$$= \pi\,[2R^2 + 2r^2 + (R^2 - r^2)]$$

$$= \frac{22}{7}\left\{2\left(\frac{25}{2}\right)^2 + 2(12)^2 + \left[\left(\frac{25}{2}\right)^2 - (12)^2\right]\right\}$$

$$= \frac{22}{7}\,[321.5 + 288 + 12.25]$$

$$= \frac{22}{7} \times 612.75 \text{ cm}^2$$

Cost of painting = ₹ $\frac{22}{7} \times 612.75 \times \frac{56}{100}$ = ₹ 1078.44

15. Given: surface area of sphere = CSA of a cone

$\Rightarrow \quad 4\pi r^2 = \pi r l$

Height of cone = 360 cm, radius of cone = 150 cm

$$l = \sqrt{(360 \text{ cm})^2 + (150 \text{ cm})^2}$$
$$= \sqrt{129600 \text{ cm}^2 + 22500 \text{ cm}^2}$$
$$= \sqrt{152100 \text{ cm}^2}$$

CSA cone = $\pi \times 150 \times 390 = 58500\pi$ cm^2

Now $\quad 4\pi r^2 = 58500\pi$ cm^2

$$\Rightarrow \quad r^2 = \frac{58500\pi \text{ cm}^2}{4\pi}$$
$$\Rightarrow \quad r^2 = 14625 \text{ cm}^2$$
$$\therefore \quad r = \sqrt{14625} = 120.93 \text{ cm}$$

Hence radius of sphere = 120.92 cm.

16. Given: diameter of sphere = 8 cm

$\Rightarrow \quad r = 4$ cm

Surface area of sphere = $4\pi r^2$

$= 4\pi \times 4 \text{ cm} \times 4 \text{ cm}$

$= 4\pi \times 16 \text{ cm}^2 = 64\pi \text{ cm}^2$

Now $\quad 6a^2 = 64\pi$ cm^2

$$a^2 = \frac{64\pi \text{ cm}^2}{6}$$
$$a = \sqrt{\frac{64 \times 22}{6 \times 7} \text{cm}^2} = 5.79 \text{ cm}$$

17. Given: diameter of each hemisphere = 3.5 cm

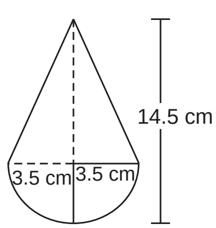
So, $\quad r = \dfrac{3.5 \text{ cm}}{2} = 1.75$ cm

Total surface area of hemisphere = $3\pi r^2$

$$= 3 \times \frac{22}{7} \times 1.75 \text{ cm} \times 1.75 \text{ cm}$$
$$= \frac{134.75 \text{ cm}^2}{7} = \frac{202.13 \text{ cm}^2}{7}$$

$= 28.88$ cm^2 (approx.)

18. $r + h = 14.5$ cm, where r is the radius of the cone and the hemisphere and h is the height of the cone.



$\Rightarrow \quad 3.5 \text{ cm} + h = 14.5$ cm

$\Rightarrow \quad h = 11$ cm

Slant height of the cone = $l = \sqrt{r^2 + h^2}$

$$= \sqrt{(3.5)^2 + (11)^2} \text{ cm}$$
$$= \sqrt{12.25 + 121} \text{ cm}$$
$$= \sqrt{133.25} \text{ cm}$$

$= 11.54$ cm (approx.)

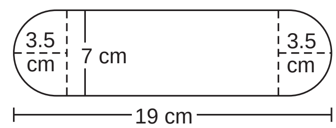
Surface area of the toy = $\pi r l + 2\pi r^2 = \pi r(l + 2r)$

$$= \frac{22}{7} \times 3.5(11.54 + 2 \times 3.5) \text{ cm}^2$$

$= 11(18.54)$ cm^2

$= 203.94$ cm^2 (approx.).

19. $r + h + r = 19$ cm, where r is the radius and h the height of the cylinder.



$\Rightarrow \quad 3.5 \text{ cm} + h + 3.5 \text{ cm} = 19$ cm

$\Rightarrow \quad h = (19 - 7)$ cm

$= 12$ cm

Surface area of the solid = $4\pi r^2 + 2\pi r h$

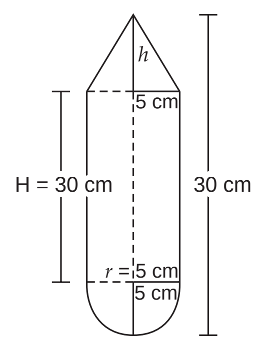
$= 2\pi r\,(2r + h)$

$$= 2 \times \frac{22}{7} \times 3.5\,[2\,(3.5) + 12] \text{ cm}^2$$

$= 22\,(19)$ cm^2

$= 418$ cm^2.

20. $h + \text{H} + r = 30$ cm



$\Rightarrow \quad h + 13 \text{ cm} + 5 \text{ cm} = 30$ cm

$\Rightarrow \quad h = (30 - 18)$ cm

$= 12$ cm

Slant height of the cone = $l = \sqrt{r^2 + h^2}$

$$= \sqrt{5^2 + 12^2} \text{ cm}$$
$$= \sqrt{25 + 144} \text{ cm}$$
$$= \sqrt{169} \text{ cm} = 13 \text{ cm}$$

Surface area of the toy

$= \pi r l + 2\pi r \text{H} + 2\pi r^2$

$= \pi r(l + 2\text{H} + 2r)$

$$= \frac{22}{7} \times 5\,(13 + 2 \times 13 + 2 \times 5) \text{ cm}^2$$
$$= \frac{110}{7}\,(13 + 26 + 10) \text{ cm}^2 = 770 \text{ cm}^2$$

EXERCISE 13E

1. (i) Side of the cube = 9 dm

Volume of the cube = $(\text{side})^3 = (9 \text{ dm})^3 = 729 \text{ dm}^3$

Diagonal of the cube= $\sqrt{3} \times$ side

$= \sqrt{3} \times 9 \text{ dm} = 9\sqrt{3}$ dm

= 15.57 dm.

(ii) Edge of the cube = $\sqrt[3]{\text{volume}}$

$= \sqrt[3]{15.625 \text{ m}^3}$

$= \sqrt[3]{2.5 \text{ m} \times 2.5 \text{ m} \times 2.5 \text{ m}}$

= 2.5 m.

2. Given: Volume of water in cubical tank = 15.625 m^3

Side of the cube = $\sqrt[3]{\text{volume}}$

$= \sqrt[3]{15.625 \text{ m}^3}$

$= \sqrt[3]{2.5 \text{ m} \times 2.5 \text{ m} \times 2.5 \text{ m}}$

= 2.5 m.

The height is reduced to 1.3 m.

So, the height of water used = (2.5 – 1.3) m = 1.2 m

Volume of water used = 2.5 m × 2.5 m × 1.2 m

$= 7.5 \text{ m}^3$.

3. Given: TSA of the cube = 726 cm^2

$\Rightarrow$ $6(\text{edge})^2 = 726 \text{ cm}^2$

$\Rightarrow$ $(\text{edge})^2 = \frac{726 \text{ cm}^2}{6} = 121 \text{ cm}^2$

$\Rightarrow$ edge $= \sqrt{121 \text{ m}^2} = 11$ cm

Volume of cube = $(\text{edge})^3 = (11 \text{ cm})^3$

$= 1331 \text{ cm}^3$

Hence, the volume of the cube is 1331 cm^3.

4. Given:volume of the cube = 5832 m^3

$(\text{Edge})^3 = 5832 \text{ m}^3$

Edge = $\sqrt[3]{5832 \text{ m}^3}$

$= \sqrt[3]{729 \times 8 \text{ m}^3}$

= 9 × 2 m = 18 m

TSA of cube = $6 \times (\text{edge})^2$

$= 6 \times (18 \text{ m})^2 = 1944 \text{ m}^2$

Cost of painting = ₹ 3.50 × 1944 m^2 = ₹ 6804.

5. (i) Given: $l = 7$ m, $b = 3$ m and $h = 4$ m

Volume of cuboid = $l \times b \times h$

= 7 m × 3 m × 4 m

$= 84 \text{ m}^3$

(ii) Given: $l = 8$ m, $b = 3$ m and $h = 2.4$ m

Volume of cuboid = $l \times b \times h$

= 8 m × 3 m × 2.4 m

$= 57.6 \text{ m}^3$.

6. Rate of flow of water = 24 km/h

$\Rightarrow$ Length of water flowing in 1 hour = 24000 m

$\Rightarrow$ Length of water flowing in 1 minute = $\frac{24000}{60}$ m

= 400 m

Volume of water flowing into the sea in 1 minute

= 400 m × 4.4 m × 2.5 m = 4400 m^3.

7. Given: length = $2x$, breadth = x, h = 3 m

Area of four walls of cold storage = $2(l + b)h$

$= 2(x + 2x)$ 3m

$= 2(3x)$ 3m

$= 18x \text{ m}^2$

$\Rightarrow$ $18x \text{ m}^2 = 108 \text{ m}^2$

$\Rightarrow$ $x = \frac{108 \text{ m}^2}{18 \text{ m}^2} = 6$

$\Rightarrow$ $x = 6$ m, $2x = 12$ m

Volume = $l \times b \times h$

$= (6 \times 12 \times 3) \text{ m}^3$

$= 216 \text{ m}^3$.

8. Given: Area of rectangular playground = 4800 m^2

Depth of gravel = 1 cm = $\frac{1}{100}$ = 0.01 m

Volume of the gravel = $4800 \text{ m}^2 \times 0.01$ m

$= 48 \text{ m}^3$

Cost of gravel = 48 m^3 × ₹ 260

= ₹ 12480

Hence the total cost of covering = ₹ 12480.

9. Volume of the brass plate = $x \times x \times \frac{1}{100} \text{ cm}^3 = \frac{x^2}{100} \text{ cm}^3$

Mass of 1 cm^3 of brass = 8.4 g

$\Rightarrow$ Mass of $\frac{x^2}{100} \text{ cm}^3$ of brass $= \frac{x^2}{100} \times 8.4$ g

$\Rightarrow$ $\frac{x^2}{100} \times 8.4 = 4725$

$\Rightarrow$ $x^2 = \frac{4725 \times 10}{8.4} = 5625$

$\Rightarrow$ $x = 75$.

10. Let the length, breadth and height of the cuboid be $4x$, $2x$ and x respectively.

TSA of cuboid = $2(lb + bh + hl)$

$\Rightarrow$ $2(4x \times 2x + 2x \times x + x \times 4x)$

$= 1372 \text{ cm}^2$

$\Rightarrow$ $8x^2 + 2x^2 + 4x^2 = 686 \text{ cm}^2$

$\Rightarrow$ $14x^2 = 686 \text{ cm}^2$

$\Rightarrow$ $x^2 = \frac{686 \text{ cm}^2}{14} = 49 \text{ cm}^2$

$\therefore$ $x = 7$ cm

$l = 4x = 4 \times 7$ cm = 28 cm,

$b = 2x = 2 \times 7$ cm $= 14$ cm

$h = x = 7$ cm

Volume $= l \times b \times h$

$= 28$ cm $\times 14$ m $\times 7$ cm

$= 2744$ cm^3.

11. Given: volume $= 1440$ cm^3

Let the side of the base $= x$ i.e., l and $b = x$; $h = 10$ cm

$\Rightarrow \quad l \times b \times h = 1440$ cm^2

$\Rightarrow \quad x \times x \times 10 = 1440$ cm^2

$\Rightarrow \quad x^2 = 144$ cm^2

$\Rightarrow \quad x = \sqrt{144 \text{ cm}^2} = 12$ cm

Hence the side of square is 12 cm.

12. Let the length, breadth and height of the cuboid be l, b and h respectively.

Then, $\quad h = 12$ cm and $\dfrac{l}{b} = \dfrac{4}{3}$

$\Rightarrow \quad b = \dfrac{3}{4} l$

$\Rightarrow \quad l \times b \times h = 3600$ cm^3

$\Rightarrow \quad l \times b \times 12$ cm $= 3600$ cm^3

$\Rightarrow \quad l \times b = 300$ cm^2

$\Rightarrow \quad l \times \dfrac{3}{4} l = 300$ cm^2

$\Rightarrow \quad l^2 = 300 \times \dfrac{4}{3}$ cm$^2 = 400$ cm^2

$\Rightarrow \quad l = 20$ cm

and $\quad b = \dfrac{3}{4} \times 20$ cm $= 15$ cm.

13. Given: length $= 12$ m, $b = 9$ m and $h = 8$ m

Longest rod that can be placed in the room diagonally.

Length of diagonal of a cuboid

$= \sqrt{l^2 + b^2 + h^2}$

$= \sqrt{(12 \text{ m})^2 + (9 \text{ m})^2 + (8 \text{ m})^2}$

$= \sqrt{(144 + 81 + 64) \text{ m}^2}$

$= \sqrt{289 \text{ m}^2}$

$= 17$ m.

Thus, the length of the longest rod is 17 m.

14. Volume of pit $= 45.9$ m $\times 5.4$ m $\times 4.2$ m

$= 1041.012$ m^3

Volume of plank $= 5.1$ m $\times 0.20$ m $\times 0.175$

$= 0.1785$ m^3

Number of planks $= \dfrac{1041.012 \text{ m}^3}{0.1785 \text{ m}^3} = 5832$.

15. Length of the wall $= 10$ m $= 1000$ cm

Thickness of wall (b) $= 24$ cm

Height of wall $= 4$ m $= 4 \times 100 = 400$ cm

Volume of wall $= l \times b \times h$

$= 1000$ cm $\times 24$ cm $\times 400$ cm

$= 9600000$ cm^3

Volume of a brick $= 24$ cm $\times 12$ cm $\times 8$ cm

$= 2304$ cm^3

Number of bricks $= \dfrac{9600000 \text{ cm}^3}{2304 \text{ cm}^3} = 4166.66 \approx 4167$.

16. Given: $\quad l = 10$ m, $b = 6.4$ m, $h = 5$ m

Floor area given to each student $= 1.6$ m^2

Let the total number of students be x.

Floor area of classroom = Total number of students × floor area given to each student

$\Rightarrow \quad l \times b = x \times 1.6$ m^2

$\Rightarrow \quad 10$ m $\times 6.4$ m $= x \times 1.6$ m^2

$\Rightarrow \quad x = \dfrac{10 \text{ m} \times 6.4 \text{ m}}{1.6} = 40$

Total numbers of students = 40

Volume of air in the classroom $= l \times b \times h$

$= 10$ m $\times 6.4$ m $\times 5$ m

$= 320$ m^3

Air for each student $= \dfrac{320 \text{ m}^3}{40} = 8$ m^3.

Hence, each student will get 8 m^3 of air.

17. Let the length, breadth and height of the cuboid be x, y and z respectively.

Suppose, $\quad xy = 60$ cm^3, $yz = 27$ cm^2,

then $\quad zx = 45$ cm^2

$\Rightarrow \quad xy \times yz \times zx = 60 \times 27 \times 45$ cm^3

$\Rightarrow \quad x^2y^2z^2 = 72900$ cm^3

$\Rightarrow \quad$ Volume $= xyz = 270$ cm^3.

18. Given: Outer length $= 10$ cm,

outer breadth $= 8$ cm, outer height $= 7$ cm

Outer volume $= l \times b \times h = (10 \times 8 \times 7)$ cm^3

$= 560$ cm^3

Thickness of wooden box $= 1$ cm

Inner length $= 10$ cm $- 2$ cm $= 8$ cm

Inner breadth $= 8$ cm $- 2$ cm $= 6$ cm

Inner height $= 7$ cm $- 2$ cm $= 5$ cm

Inner volume $= l \times b \times h = (8 \times 6 \times 5)$ cm^3

$= 240$ cm^3

Volume of used wood = Outer volume − Inner volume

$= 560$ cm$^3 - 240$ cm$^3 = 320$ cm^3

Total cost of wood $= ₹\ 2 \times 320$ cm^3

$= ₹\ 640$.

19. Outer length $= 18$ cm, outer breadth $= 10$ cm

and outer height $= 6$ cm

volume of outer box $= l \times b \times h$

$= 18$ cm $\times 10$ cm $\times 6$ cm

$= 1080$ cm^3

Inner length = 18 cm – (5 mm × 2)
= 18 cm – 1 cm = 17 cm
Inner breadth = 10 cm – (5 mm × 2)
= 10 cm – 1 cm = 9 cm
Inner height = 6 cm – (5 mm × 2)
= 6 cm – 1 cm = 5 cm

Volume of outer box = $l \times b \times h$
= 17 cm × 9 cm × 5 cm
= 765 cm^3.

Volume of empty box = (1080 – 705) cm^3
= 315 cm^3.

Mass of empty box = 3.15 kg = 3150 g

Mass of 315 cm^3 of empty box = 3150 g

Mass of 1 cm^3 of empty box = $\frac{3150 \text{ g}}{315}$ = 10 g.

20. Inner dimension of closed box = $l \times b \times h$
= 2 m × 1.2 m × 0.75 m
= 1.8 m^3

Length of outer dimension of box = 2 m + (2 × 0.025 m)
= 2 m + 0.05 m
= 2.05 m

Breadth of outer dimension of box = 1.2 m + 0.05 m
= 1.25 m

Height of the outer dimension = 0.75 m + 0.05 m
= 0.80 m

Volume of outer box = $l \times b \times h$
= 2.05 × 1.25 m × 0.80 m
= 2.05 m^3

Volume of box = (2.05 – 1.8) m^3 = 0.25 m^3

Cost of wood used = ₹ 5400 × 0.25 = ₹ 1350.

21. Volume of the box = $l \times b \times h$
= 24 cm × 20 cm × 16 cm
= 7680 cm^3

Volume of cube = $(4 \text{ cm})^3$
= 64 cm^3

No, of cubes put in the bigger box = $\frac{7680 \text{ cm}^3}{64 \text{ cm}^3}$ = 120

Hence 120 cubes can be put in the given box.

22. Volume of outer box = $l \times b \times h$
= 17.5 cm × 14 cm × 10 cm
= 2450 cm^3

Length of inner box = 17.5 cm – (2 × 0.75 cm)
= 17.5 cm – 1.5 cm
= 16 cm.

Breadth of inner box = 14 cm – 1.5 cm
= 12.5 cm

Height of inner box = 10 cm – 1.5 cm
= 8.5 cm

Volume of inner box = $l \times b \times h$
= 16 cm × 12.5 cm × 8.5 cm
= 1700 cm^3

Volume of the wooden part = (2450 – 1700) cm^3
= 750 cm^3

Now volume of open box = 750 cm^3 – (16 cm × 12.5 cm × 0.75 cm)
= 750 cm^3 – 150 cm^3
= 600 cm^3

23. Volume of pipe = [(4 × 3) – (3.5 × 2.5)] × 100 cm^3
= [(12 – 8.75) × 100 cm^3]
= 3.25 × 100 cm^3
= 325 cm^3.

24. Volume of cuboid under position 1
= 50 × 40 × (16 + 24) cm^3
= 50 × 40 × 40 cm^3

Volume of cuboid under position 2
= 50 × 40 × 24 cm^3

Volume of cuboid under position 3
= 50 × 40 × 12 cm^3

Volume of the victory stand
= (50 × 40 × 40 + 50 × 40 × 24 + 50 × 40 × 12) cm^3
= 50 × 40 (40 + 24 + 12) cm^3
= 2000 (76) cm^3 = 152000 cm^3.

25. (i) $\sqrt{3}$ edge = 12$\sqrt{3}$ cm

⇒ edge = 12 cm

∴ volume of larger cube = 12 × 12 × 12 cm^3

Let the sides of the three smaller cubes be $3x$, $4x$ and $5x$.

Volume of 3 cubes = Volume of larger cube

⇒ $(3x)^3 + (4x)^3 + (5x)^3$ = 12 × 12 × 12 cm^3.

Solve to get x^3 = 8 cm^3, i.e. x = 2 cm

Edges are 3 × 2 cm = 6 cm, 4 × 2 cm = 8 cm

and 5 × 2 cm = 10 cm

(ii) volume of cube = $(\text{Edge})^3$
= $(6 \text{ cm})^3$ = 216 cm^3

Volume of cuboid = $l \times b \times h$ = volume of cube

Volume of cuboid = $l \times b \times h$ = 216 cm^3

⇒ 9 cm × 8 cm × h = 216 cm^3

⇒ $h = \frac{216 \text{ cm}^3}{72 \text{ cm}^2}$ = 3 cm.

Hence the height of cuboid is 3 cm.

26. Volume of cube (bigger) = $(\text{edge})^3$ = $(20 \text{ cm})^3$
= 8000 cm^3

Volume cube (smaller) = $(5 \text{ cm})^3$ = 125 cm^3

No. of cubes cut from bigger cube = $\frac{8000 \text{ cm}^3}{125 \text{ cm}^3}$ = 64

27. Let the sides of the cuboid in metres be x, $2x$ and $4x$ and let the edge of the cube be y metres.

Given, volume of the cuboid = volume of the cube

Then, $x \times 2x \times 4x = y^3$

$\Rightarrow \quad 8x^3 = y^3$

$\Rightarrow \quad 2x = y$

$\Rightarrow \quad x = \frac{y}{2}$

Cost of polishing the cuboid – Cost of polishing the cube = ₹ 80

$\Rightarrow \quad 2(2x^2 + 8x^2 + 4x^2) \times 5 - 6(y^2) \times 5 = 80$

$\Rightarrow \quad 2(14x^2) - 6y^2 = 16$

$\Rightarrow \quad 14x^2 - 3y^2 = 8$

Put $x = \frac{y}{2}$ and solve to get $y = 4$

Volume of the cube = $(y \text{ metre})^3 = (4 \text{ m})^3 = 64 \text{ m}^3$
= Volume of the cuboid.

28. Suppose the rise in the water level is x cm.

Volume of water that rises = Volume of cube

$\Rightarrow \quad 15 \times 12 \times x = 9 \times 9 \times 9$

$\Rightarrow \quad x = 4.05$ cm.

29. Decrease in the volume of water in the reservoir
= Volume of water flowing out

$\Rightarrow \quad 1.5 \text{ m} \times b \times 0.35 \text{ m} = 47.25 \text{ m}^3$

$\Rightarrow \quad b = 9$ m.

30. Suppose the water becomes 3 m deep in x hours.

Then, volume of water flowing in x hours
= Volume of water collected in x hours

$\Rightarrow \quad 0.15 \times 0.2 \times 1500 \times x = 150 \times 100 \times 3.$

Solve to get $x = 100$.

EXERCISE 13F

1. (i) $r = 7$ cm, $h = 13.5$ cm

Volume of the cylinder $= \pi r^2 h$

$= \frac{22}{7} \times 7\text{cm} \times 7\text{cm} \times 13.5\text{cm}$

$= 2079 \text{ cm}^3$

(ii) $r = 3.5$ cm, $h = 16$ cm

Volume of the cylinder $= \pi r^2 h$

$= \frac{22}{7} \times 3.5 \text{ cm} \times 3.5 \text{ cm} \times 16 \text{ cm}$

$= 616 \text{ cm}^3$

(iii) $r = 21$ cm, $h = 10$ cm

Volume of the cylinder $= \pi r^2 h$

$= \frac{22}{7} \times 21 \text{ cm} \times 21 \text{ cm} \times 10 \text{ cm}$

$= 13860 \text{ cm}^3$

(iv) $r = 14$ dm, $h = 1.6$ m $= 16$ dm

Volume of the cylinder $= \pi r^2 h$

$= \frac{22}{7} \times 14 \text{ dm} \times 14 \text{ dm} \times 16 \text{ dm}$

$= 9856 \text{ dm}^3$

2. (i) Volume $= 18480 \text{ cm}^3$

Height = 30 cm

Volume $= \pi r^2 h$

$\pi r^2 h = 18480 \text{ cm}^3$

$r^2 = \frac{7}{22} \times \frac{18480}{30} \text{ cm}^2$

$r^2 = 196 \text{ cm}^2$

$r = 14$ cm

Diameter $= 2r = 2 \times 14$ cm
= 28 cm.

(ii) Given, volume $= 1650 \text{ m}^3$, height = 21 cm

Let r be the base radius and h be the height of the circular cylinder. Then, $h = 21$ cm.

Volume of the circular cylinder $= \pi r^2 h$

$\Rightarrow 1650 \text{ m}^3 = \pi \times r^2 \times 21$ cm

$\Rightarrow \quad r^2 = \frac{1650 \text{ m}^3 \times 7}{22 \times 21 \text{ cm}} = 25 \text{ cm}^2$

$\Rightarrow \quad r = 5$ cm

Diameter of the circular cylinder $= 2r$
$= 2 \times 5$ cm = 10 cm.

3. Let r be the base radius and h be the height of the right circular cylinder. Then, $r = 1.6$ m.

Volume of the right circular cylinder $= \pi r^2 h$

$\Rightarrow \quad 112.64 \text{ m}^3 = \frac{22}{7} \times (1.6 \text{ m})^2 \times h$

$\Rightarrow \quad h = \frac{112.64 \text{ m}^3 \times 7}{22 \times 2.56 \text{ m}^2}$

= 14 m

Hence, the height of the right circular cylinder is 14 m.

4. Let r be the base radius and h be the height of the electric geyser. Then,

$r = \frac{35 \text{ cm}}{2} = 17.5$ cm

and $\quad h = 1.2$ m = 120 cm

Capacity of the electric geyser is equal to volume.

Now,capacity of the electric geyser $= \pi r^2 h$

$= \frac{22}{7} \times (17.5 \text{ cm})^2 \times 120$ cm

$= 115500 \text{ cm}^3$ [Using 1 litre $= 10^3$ cm]

= 115.5 litres

Hence, the capacity of the electric geyser is 115.5 litres.

5. Let r be the inner radius and h be the distance of the water covered in 1 s.

Then, $\quad r = 0.75$ cm

and $\quad h = \frac{7}{5} \text{ m} = \frac{700}{5}$ cm

Volume of water flowing out in 1 s

$= \pi r^2 h$

$= \frac{22}{5} \times 0.75 \text{ cm} \times 0.75 \text{ cm} \times \frac{700}{5} \text{ cm}^3$

$= 247.5 \text{ cm}^3$

Volume of water flowing out in 30 minutes

$= 247.5 \times 60 \times 30 \text{ cm}^3 = 445.5$ litres

6. Let r be the inner radius and h be the inner length of the cylinder.

Then, $r = 7$ cm and $h = 60$ cm

Volume of the container $= \pi r^2 h$

$= \frac{22}{7} \times (7 \text{ cm})^2 \times 60 \text{ cm}$

$= 9240 \text{ cm}^3$

$= 9.24$ litre [Using 1 litre $= 10^3 \text{ cm}^3$]

Price of the milk that filled in the container

= ₹ 14.50 per litre × 9.24 litre

= ₹ 133.98.

7. Let r be the radius and h be the depth of the tube well.

Then, $r = \frac{3 \text{ m}}{2} = 1.5$ m and $h = 280$ m

Volume of the tube well

$= \pi r^2 h = \frac{22}{7} \times (1.5 \text{ m})^2 \times 280 \text{ m}$

$= 1980 \text{ m}^3$

Cost of sinking the tube well

= ₹ 8.40 per cubic metre × 1480 m^3

= ₹ 16632.

8. Let r be the radius and h be the height of the cylinder.

Then, $r = \frac{20 \text{ cm}}{2} = 10$ cm.

Curved surface area of the cylinder $= 2\pi rh$

$\Rightarrow \quad 1210 \text{ cm}^2 = 2 \times \frac{22}{7} \times 10 \text{ cm} \times h$

$\Rightarrow \quad h = \frac{1210 \text{ cm}^2 \times 7}{440 \text{ cm}} = 19.25$ cm

Volume of the cylinder $= \pi r^2 h$

$= \frac{22}{7} \times (10 \text{ cm})^2 \times 19.25$

$= 6050 \text{ cm}^3$

Hence, the height of the cylinder is 19.25 cm and its volume is 6050 cm^3.

9. Let r be the radius and h be the length of the barrel of the fountain pen.

Then, $r = \frac{5 \text{ mm}}{2} = \frac{5}{20}$ cm and $l = 7$ cm

Volume of ink in the barrel

$= \pi r^2 h = \frac{22}{7} \times \frac{5}{20} \times \frac{5}{20} \times 7 \text{ cm}^3$

$= 1.375 \text{ cm}^3$

Volume of ink in the bottle $= \frac{1}{5}$ L

$= \frac{1}{5} \times 1000 \text{ cm}^3 = 200 \text{ cm}^3$

1.375 cm^3 of ink is used up for writing 330 words

$\therefore$ 200 cm^3 of ink is used up for writing

$\frac{330}{1.375} \times 200$ words $= 48000$ words.

10. Let r be the base radius and h be the height of the cylindrical vessel.

Curved surface area of the cylindrical vessel $= 148.5 \text{ dm}^2$

$\Rightarrow \quad 2\pi rh = 148.5 \text{ dm}^2 \quad \ldots (1)$

Area of the base of the cylindrical vessel $= 38.5 \text{ dm}^2$

$\Rightarrow \quad \pi r^2 = 38.5 \text{ dm}^2$

$\Rightarrow \quad r^2 = \frac{38.5 \text{ dm}^2 \times 7}{22} = 12.25 \text{ dm}^2$

$\Rightarrow \quad r = 3.5$ dm

From equation (1),

$h = \frac{148.5 \text{ dm}^2}{2\pi r} = \frac{148.5 \text{ dm}^2}{2 \times \frac{22}{7} \times 3.5 \text{ dm}}$

$= 6.75$ dm

The amount of milk that the vessel can hold is equal to its volume.

Volume of the vessel $= \pi r^2 h$

$= \frac{22}{7} \times (3.5 \text{ dm})^2 \times 6.75 \text{ dm}$

$= 259.875 \text{ dm}^3$.

11. Let r be the radius and h be the height of the right circular cylinder.

Then, $h = 4$ cm

Total surface area $= 2\pi r(h + r) = 484 \text{ cm}^2$

$\Rightarrow \quad 2 \times \frac{22}{7} \times r(4 + r) = 484 \text{ cm}^2$

$\Rightarrow \quad 44r(4 + r) = 484 \times 7 \text{ cm}^2$

$\Rightarrow \quad r(4 + r) = \frac{484 \times 7}{44} \text{ cm}^2 = 77 \text{ cm}^2$

$\Rightarrow \quad r^2 + 4r - 77 = 0$

$\Rightarrow \quad (r + 11)(r - 7) = 0$

$\Rightarrow \quad r = 7$ cm

Volume of the cylinder $= \pi r^2 h$

$= \frac{22}{7} \times 7 \text{ cm} \times 7 \text{ cm} \times 4 \text{ cm}$

$= 616 \text{ cm}^3$.

12. Let r be the radius and h be the height of the cylinder.

Circumference of the base of the cylinder $= 2\pi r$

$\Rightarrow \quad 88 \text{ cm} = 2\pi r$

$\Rightarrow \quad r = \frac{88 \text{ cm}}{2 \times \frac{22}{7}} = 14$ cm

Total surface area of the cylinder = $2\pi r(r + h)$

$\Rightarrow \quad 6512 \text{ cm}^2 = 2 \times \frac{22}{7} \times 14 \text{ cm} \times (14 \text{ cm} + h)$

$\Rightarrow \quad 14 \text{ cm} + h = \frac{6512 \text{ cm}^2 \times 7}{44 \times 14 \text{ cm}}$

$\Rightarrow \quad h = 74 \text{ cm} - 14 \text{ cm}$

$= 60 \text{ cm}$

Volume of the cylinder = $\pi r^2 h$

$= \frac{22}{7} \times (14 \text{ cm})^2 \times 60 \text{ cm}$

$= 36960 \text{ cm}^3$.

13. Let r be the base radius and h be the height of the cylinder.

$\frac{2\pi rh}{2\pi r(r+h)} = \frac{1}{3}$

$\Rightarrow \quad \frac{h}{r+h} = \frac{1}{3}$

$\Rightarrow \quad 3h = r + h$

$\Rightarrow \quad 2h = r \qquad \ldots (1)$

Total surface area = $2\pi r(r + h) = 1848 \text{ cm}^2$

$\Rightarrow \quad 2\pi\,(2h)\,(2h + h) = 1848 \text{ cm}^2 \qquad$ [Using (1)]

$\Rightarrow \quad 4\pi h(3h) = 1848 \text{ cm}^2$

$\Rightarrow \quad h^2 = \frac{1848 \times 7}{4 \times 22 \times 3} \text{ cm}^2 = 49 \text{ cm}^2$

$\Rightarrow \quad h = 7 \text{ cm}$

and $\quad r = 2h = 2 \times 7 \text{ cm} = 14 \text{ cm}$

Volume = $\pi r^2 h$

$= \frac{22}{7} \times 14 \text{ cm} \times 14 \text{ cm} \times 7 \text{ cm}$

$= 4312 \text{ cm}^3$

14. Let r be the base radius and h be the height of the closed cylindrical oil tank.

Total surface area = $\frac{198}{0.50} \text{ dm}^2 = 396 \text{ dm}^2$

$\Rightarrow \quad 2\pi r(r + h) = 396 \text{ dm}^2$

$\Rightarrow \quad 2\pi r(r + 6r) = 396 \text{ dm}^2$

$\Rightarrow \quad 2\pi r(7r) = 396 \text{ dm}^2$

$\Rightarrow \quad r^2 = \frac{396 \times 7}{2 \times 22 \times 7} \text{ dm}^2 = 9 \text{ dm}^2$

$\Rightarrow \quad r = 3 \text{ dm}$

and $\quad h = 6 \times 3 \text{ dm} = 18 \text{ dm}$

Volume = $\pi r^2 h$

$= \frac{22}{7} \times 3 \times 3 \times 18 \text{ dm}^3 = \frac{3564}{7} \text{ dm}^3$.

15. Let r be the radius and h be the height of the cylinder.

Then, $\quad h = 42$ dm.

Surface areas of the two faces of the cylinder = 166 dm^2

$2\pi r^2 = 166 \text{ dm}^2$

$\Rightarrow \quad r^2 = \frac{166}{2\pi} \text{ dm}^2 \qquad \ldots (1)$

Volume = $\pi r^2 h = \pi \times \frac{166}{2\pi} \times 42 \text{ dm}^3$

$= 3486 \text{ dm}^3 \qquad$ [Using (1)]

16. Let r be the radius and h be the height of the cylinder.

Then, $\quad h = r + 7$ cm

Curved surface area of the cylinder

= 3 × area of the base of the cylinder

$\Rightarrow \quad 2\pi rh = 3 \times \pi r^2$

$\Rightarrow \quad 2h = 3r$

$\Rightarrow \quad 2(r + 7 \text{ cm}) = 3r \qquad$ [Using $h = r + 7$ cm]

$\Rightarrow \quad 2r + 14 \text{ cm} = 3r$

$\Rightarrow \quad r = 14 \text{ cm}$

Again, $\quad h = r + 7$ cm

$= 14 \text{ cm} + 7 \text{ cm}$

$= 21 \text{ cm}$

Volume of the cylinder = $\pi r^2 h$

$= \frac{22}{7} \times (14 \text{ cm})^2 \times 21 \text{ cm}$

$= 12936 \text{ cm}^3$

Hence, the volume of the cylinder is 12936 cm^3.

17. Let r be the radius and h be the height of the right circular cylinder.

Then, $\quad h = 14$ cm

Volume of the cylinder = $686\pi \text{ cm}^3$

$\Rightarrow \quad \pi r^2 h = 686\pi \text{ cm}^3$

$\Rightarrow \quad r^2 = \frac{686 \text{ cm}^3}{h}$

$\Rightarrow \quad r^2 = \frac{686 \text{ cm}^3}{14 \text{ cm}}$

$= 49 \text{ cm}^2$

$\Rightarrow \quad r = 7 \text{ cm}$

Curve surface area of the cylinder

$= 2\pi rh$

$= 2 \times \frac{22}{7} \times 7 \text{ cm} \times 14 \text{ cm}$

$= 616 \text{ cm}^2$

Hence, the curve surface area of the right circular cylinder is 616 cm^2.

18. Let r be the base radius and h be the height of the cylinder.

Then, $\quad r = 8$ cm

Volume of the cylinder = $288\pi \text{ cm}^3$

$\Rightarrow \quad \pi r^2 h = 288\pi \text{ cm}^3$

$\Rightarrow \quad h = \frac{288 \text{ cm}^3}{r^2}$

$= \frac{288 \text{ cm}^3}{(8 \text{ cm})^2}$

$= 4.5 \text{ cm}$

Total surface area of the cylinder

$$= 2\pi r(r + h)$$
$$= 2 \times \pi \times 8 \text{ cm} \times (8 \text{ cm} + 4.5 \text{ cm})$$
$$= 200\pi \text{ cm}^2$$

Hence, the surface area of the cylinder is 200π cm^2.

19. Let r be the radius and h be the height of the cylindrical pillar.

Volume of the cylindrical pillar = 924 m^3

$$\Rightarrow \quad \pi r^2 h = 924 \text{ m}^3 \quad \ldots (1)$$

Curve surface area of the cylindrical pillar = 264 m^2

$$\Rightarrow \quad 2\pi rh = 264 \text{ m}^2 \quad \ldots (2)$$

Divide equation (1) by equation (2), we get

$$\frac{r}{2} = \frac{924 \text{ m}^3}{264 \text{ m}^2}$$

$$\Rightarrow \quad r = 3.5 \times 2 \text{ cm} = 7 \text{ cm}$$

From equation (2), $h = \dfrac{264 \text{ m}^2}{2\pi r}$

$$= \frac{264 \text{ m}^2}{2 \times \frac{22}{7} \times 7 \text{ m}} = 6 \text{ m.}$$

Diameter of the pillar = $2r = 2 \times 7$ m = 14 m

Hence, the diameter of the pillar is 14 m and its height is 6 m.

20. Let r be the radius h be the height of the cylinder.

Then, $h = 10$ cm

$$\frac{\text{Volume of the cylinder}}{\text{Curve surface area of the cylinder}} = \frac{3}{2}$$

$$\Rightarrow \quad \frac{\pi r^2 h}{2\pi rh} = \frac{3}{2}$$

$$\Rightarrow \quad \frac{r}{2} = \frac{3}{2} \quad \Rightarrow \quad r = 3 \text{ cm}$$

Now,volume of the cylinder = $\pi r^2 h$

$$= \frac{22}{7} \times (3 \text{ cm})^2 \times 10 \text{ cm}$$
$$= \frac{1980}{7} \text{ cm}^3$$

Hence, the volume of the cylinder is $\dfrac{1980}{7}$ cm^3.

21. Let r be the radius and h be the right of the cylinder.

Then, $\dfrac{r}{h} = \dfrac{5}{7}$

$$\Rightarrow \quad h = \frac{7}{5} r$$

$$\text{Volume} = \pi r^2 h = \pi r^2 \left(\frac{7}{5} r\right) = 550 \text{ cm}^3$$

$$\Rightarrow \quad r^3 = \frac{550 \times 5 \times 7}{22 \times 7} \text{ cm}^3 = 125 \text{ cm}^3$$

$$\Rightarrow \quad r = 5 \text{ cm}$$

Hence, the radius of the cylinder is 5 cm.

22. Let r be the radius and h be the height of the cylinder.

Then, $\dfrac{r}{h} = \dfrac{2}{7}$

$$\Rightarrow \quad h = \frac{7}{2} r$$

Volume of the cylinder = 704 cm^3

$$\Rightarrow \quad \pi r^2 h = 704 \text{ cm}^3$$

$$\Rightarrow \quad \pi \times \left(\frac{7}{2} r\right) \times r^2 = 704 \text{ cm}^3 \qquad \left[\text{Using } h = \frac{7}{2} r\right]$$

$$\Rightarrow \quad r^3 = \frac{704 \text{ cm}^3 \times 2}{\frac{22}{7} \times 7}$$
$$= 64 \text{ cm}^3$$

$$\Rightarrow \quad r = 4 \text{ cm}$$

and $h = \dfrac{7}{2} r = \dfrac{7}{2} \times 4 \text{ cm} = 14 \text{ cm}$

Total surface area of the cylinder

$$= 2\pi r(r + h)$$
$$= 2 \times \frac{22}{7} \times 4 \text{ cm} \times (4 \text{ cm} + 14 \text{ cm})$$
$$= \frac{3168}{7} \text{ cm}^2.$$

23. Let r be the radius and h be the height of the cylindrical hole.

Then, $r = \dfrac{1.4 \text{ m}}{2} = 0.7$ m

Cost to bore a cylindrical hole

= ₹ 50 per m^3 × volume of the cylindrical hole

$$\Rightarrow \quad ₹\, 1100 = ₹\, 50 \text{ per m}^3 \times \pi r^2 h$$

$$\Rightarrow \quad 1100 = 50 \times \frac{22}{7} \times (0.7 \text{ m})^2 \times h$$

$$\Rightarrow \quad h = \frac{1100 \times 7}{50 \times 22 \times (0.7)^2} \text{ m}$$
$$= \frac{7700}{539} \text{ m}$$
$$= \frac{100}{7} \text{ m.}$$

24. Let m be the mass, h be the length and r be the radius of the copper wire. Then,

$$m = 11 \text{ kg}, \; r = \frac{0.4 \text{ cm}}{2} = 0.2 \text{ cm}$$

volume of the copper wire

$$= \pi r^2 h$$

Now,density of the copper wire

$$= \frac{\text{Mass of the copper wire}}{\text{Volume of the copper wire}}$$

$$\Rightarrow \quad \frac{8.49}{1 \text{ cm}^3} = \frac{m}{\pi r^2 h}$$

$$\Rightarrow \quad 8.4\ g/cm^3 = \frac{11\ kg}{\frac{22}{7} \times (0.2\ cm)^2 h}$$

$$\Rightarrow \quad h = \frac{11\ kg \times 7}{8.4\ g/cm^3 \times 22 \times (0.2\ cm)^2}$$

$$= \frac{11000\ g \times 7}{8.4\ g/cm^3 \times 22 \times (0.2\ cm)^2} \text{ [Using 1 kg = 1000 g]}$$

$$= \frac{77000}{7.392}\ cm$$

$$= \frac{62500}{6}\ cm = \frac{625}{6}\ m$$

Hence, the length of the copper wire is $\frac{625}{6}$ m.

25. It is given that the rectangular piece of paper is rolled along its length. Thus, the circumference of the cylinder formed is 44 cm and its height is equal to 18 cm.

Let r be the radius and h be the height of the cylinder formed.

Then, $h = 18$ cm

Circumference of the base of the cylinder = 44 cm

$$\Rightarrow \quad 2\pi r = 44\ cm$$

$$\Rightarrow \quad r = \frac{22}{\pi}\ cm$$

Volume of the cylinder $= \pi r^2 h$

$$= \pi \times \left(\frac{22}{\pi}\right)^2 \times 18\ cm$$

$$= \frac{22 \times 22 \times 18 \times 7}{22}$$

$$= 2772\ cm^3$$

Hence, the volume of the cylinder that formed from rolling the rectangular piece of paper is 2772 cm^3.

26. Let r be the radius and h be the height of the original cylinder. The radius of the reduced cylinder is

$$r' = \frac{r}{2}$$

and the height is

$$h' = \frac{h}{2}$$

$$\frac{\text{Volume of the reduced cylinder}}{\text{Volume of the original cylinder}} = \frac{\pi r'^2 h'}{\pi r^2 h}$$

$$= \frac{\left(\frac{r}{2}\right)^2}{r^2} \frac{h}{h}$$

$$= \frac{1}{4}$$

Hence, the ratio of the volume of the reduced cylinder to that of the original cylinder is 1 : 4.

27. Let r be the radius and h the height of the original cylinder. Let R be the radius and H the height of the new cylinder.

Then, $\pi r^2 h = \pi R^2 H$

$$\Rightarrow \quad r^2 h = R^2 H$$

$$\Rightarrow \quad r^2 h = (2r)^2 H$$

$$\Rightarrow \quad r^2 h = 4r^2 H$$

$$\Rightarrow \quad h = 4H$$

$$\Rightarrow \quad H = \frac{h}{4}.$$

28. Let r_A and r_B be the radii of the cylinders A and B respectively. Then, diameters d_A and d_B are

$$d_A = 2r_A \text{ and } d_B = 2r_B$$

Now, $$\frac{d_A}{d_B} = \frac{2r_A}{2r_B}$$

$$\Rightarrow \quad \frac{1}{2} = \frac{r_A}{r_B}$$

Let h_A and h_B be the lengths of A and B respectively.

Then, $$\frac{h_A}{h_B} = \frac{3}{1}$$

$$\frac{\text{Volume of cylinder A}}{\text{Volume of cylinder B}} = \frac{\pi r_A^2 h_A}{\pi r_B^2 h_B}$$

$$= \left(\frac{r_A}{r_B}\right)^2 \left(\frac{h_A}{h_B}\right)$$

$$= \left(\frac{1}{2}\right)^2 \times \left(\frac{3}{1}\right)$$

$$= \frac{3}{4}$$

Hence, the ratio of the volume of A to that of B is 3 : 4.

29. Let r_1 and r_2 be the radii of the two cylindrical jars respectively.

$$\frac{\text{Diameters of 1}^{\text{st}}\text{ jar}}{\text{Diameter of 2}^{\text{nd}}\text{ jar}} = \frac{2r_1}{2r_2}$$

$$\Rightarrow \quad \frac{3}{4} = \frac{r_1}{r_2}$$

$$\Rightarrow \quad \frac{r_2}{r_1} = \frac{4}{3}$$

Let h_1 and h_2 be the heights of the two cylindrical jars respectively.

Volume of the 1st jar = Volume of the 2nd jar

$$\Rightarrow \quad \pi r_1^2 h_1 = \pi r_2^2 h_2$$

$$\Rightarrow \quad \frac{h_1}{h_2} = \left(\frac{r_2}{r_1}\right)^2$$

$$\Rightarrow \quad \frac{h_1}{h_2} = \left(\frac{4}{3}\right)^2$$

$$= \frac{16}{9}.$$

Hence, the ratio of the heights of the two cylindrical jars is 16 : 9.

30. Let r be the radius and h be the height of the solid cylinder of graphite.

Then, $r = \frac{1 \text{ mm}}{2} = \frac{1}{20}$ cm, and $h = 10$ cm.

Let R be the radius and H be the height of the pencil.

Then, $R = \frac{7 \text{ mm}}{2} = \frac{7}{20}$ cm and H = 10 cm.

Volume of graphite $= \pi r^2 h = \frac{22}{7} \times \frac{1}{20} \times \frac{1}{20} \times 10 \text{ cm}^3$

$= \frac{11}{140} \text{ cm}^3$

Mass of graphite = Volume × Density

$= \frac{11}{140} \times 2.1 \text{ g} = 0.165 \text{ g}$

Volume of pencil $= \pi R^2 H = \frac{22}{7} \times \frac{7}{20} \times \frac{7}{20} \times 10 \text{ cm}^3$

$= \frac{77}{20} \text{ cm}^3$

Volume of wood = volume of pencil − volume of graphite

$= \left(\frac{77}{20} - \frac{11}{140}\right) \text{cm}^3$

$= \left(\frac{539 - 11}{140}\right) \text{cm}^3 = \frac{528}{140} \text{ cm}^3$

$= \frac{132}{35} \text{ cm}^3$

Mass of wood = volume × density

$= \frac{132}{35} \times 0.7 \text{ g} = 2.64 \text{ g}$

Total mass of the pencil

= mass of graphite + mass of wood

= (0.165 + 284) g = 2.805 g

31. Length of the cylinder = 24 cm

and its radius $= \frac{20}{2}$ cm = 10 cm

Diameter of the wire = 4 mm $= \frac{4}{10}$ cm

$= \frac{2}{5}$ cm

and radius of the wire $= \frac{1}{5}$ cm

Number of turns $= \frac{\text{Length of the cylinder}}{\text{Diameter of the wire}}$

$= \frac{24}{\frac{2}{5}} = \frac{24 \times 5}{2} = 60$

Length of wire in 1 turn

= Circumference of the base of cylinder

$= 2 \times \pi \times 10 \text{ cm} = 20\pi \text{ cm}$

Length of wire in 60 turns

$= 20\pi \times 60 \text{ cm} = 1200\, \pi \text{ cm} = 12\pi \text{ m}$

Volume of the wire

$= \pi^2 h = \pi \times \frac{1}{5} \times \frac{1}{5} \times 1200\pi \text{ cm}^3$

$= 48\pi^2 \text{ cm}^3$

Mass of the wire $= 48\pi^2 \times 8.88 \text{ g} = 426.24\pi^2$ g.

32. Let R be the radius and H be the height of the cylinder.

Then, $R = \frac{4.5 \text{ cm}}{2} = 2.25$ cm and H = 5 cm

Let r be the radius and h be the height of the each coin.

Then, $r = \frac{1.5 \text{ cm}}{2} = 0.75$ cm and $h = 0.2$ cm.

Number of coins $= \frac{\text{Volume of cylinder}}{\text{Volume of each coin}}$

$= \frac{\pi R^2 H}{\pi r^2 h} = \frac{R^2 H}{r^2 h}$

$= \frac{(2.25)^2\,(5)}{(0.75)^2\,(0.2)} = 225.$

33. Let r be the internal radius and h be the length of the metal pipe.

Then, $r = \frac{5 \text{ cm}}{2} = 2.5$ cm

and $h = 2$ m = 200 cm.

It is given that the pipe is 2 mm thick all wound. Thus, the outer radius of the pipe is

$R = r + 2$ mm

= 2.5 + 0.2 cm = 2.7 cm

Mass of the pipe = Volume × Density

$= \pi(R^2 - r^2)h \times 7.7 \text{ g/cm}^3$

$= \frac{22}{7}\,[(2.7)^2 - (2.5)^2]\,(200)\,(7.7)$ g

= 5033.6 g = 5.0336 kg.

34. Let R and r be the outer and inner radii of the cylindrical metallic tube respectively.

Let h be the length of the tube.

Then, $h = 147$ cm.

Outer surface area of the tube – Inner surface area of the tube = 184.8 cm²

$2\pi Rh - 2\pi rh = 184.8 \text{ cm}^2$

$\Rightarrow \quad 2\pi h(R - r) = 184.8 \text{ cm}^2$

$\Rightarrow \quad 2 \times \frac{22}{7} \times 147 \text{ cm } (R - r) = 184.8 \text{ cm}^2$

$\Rightarrow \quad R - r = \frac{184.8 \times 7}{2 \times 22 \times 147} \text{ cm}$

$= 0.2 \text{ cm} \quad \ldots (1)$

Also, $\pi\,(R^2 - r^2)h = 572.88 \text{ cm}^3$

$\Rightarrow \quad \frac{22}{7}\,(R + r)\,(R - r)\,(147 \text{ cm}) = 572.88 \text{ cm}^3$

$\Rightarrow \quad \frac{22}{7}\,(R + r)\,(0.2 \text{ cm})\,(147 \text{ cm}) = 572.88 \text{ cm}^3$

[Using (1)]

$$\Rightarrow \quad (R + r) = \frac{572.88 \times 7}{22 \times 0.2 \times 147} \text{ cm}$$

$$= 6.2 \text{ cm} \quad \ldots (2)$$

Solve (1) and (2),

$$R = 3.2 \text{ cm and } r = 3 \text{ cm}$$

35. Let r be the radius and h be the depth of the cylindrical well.

Then, $r = \frac{14 \text{ m}}{2}$

$= 7$ m

and $h = 15$ m.

Let w be the width of the embankment formed around the well. Then, $w = 7$ m.

Also, let R be the external radius of the embankment. The radius of the cylindrical well is also the internal radius of the embankment. Then

$$R = r + w = 7 \text{ m} + 7 \text{ m} = 14 \text{ m}$$

Let H be the height of the embankment.

Volume of the cylindrical well

= volume of the embankment

$$\Rightarrow \quad \pi r^2 h = \pi(R^2 - r^2)H$$

$$\Rightarrow \quad H = \frac{r^2 h}{R^2 - r^2}$$

$$= \frac{(7 \text{ m})^2 \times 15 \text{ m}}{(14 \text{ m})^2 - (7 \text{ m})^2}$$

$$= \frac{735}{196 - 49} \text{ m}$$

$$= \frac{735}{147} \text{ m} = 5 \text{ m}.$$

Hence, the height of the embankment is 5 m.

36. Let r and R be the inner and outer radii of the hollow cylinder respectively. Then,

$$r = 15 \text{ cm and } R = 20 \text{ cm}.$$

Let r_1 be the radius of the solid cylinder and h be the height of the solid cylinder which is same as the height of the hollow cylinder.

Volume of the hollow cylinder

= Volume of the solid cylinder

$$\Rightarrow \quad \pi(R^2 - r^2)h = \pi r_1^2 h$$

$$\Rightarrow \quad r_1^2 = R^2 - r^2$$

$$= (20 \text{ cm})^2 - (15 \text{ cm})^2$$

$$= 400 - 225$$

$$= 175 \text{ cm}^2$$

$$\Rightarrow \quad r_1 = 13.2 \text{ cm (approx.)}$$

Hence the radius of the base of the new cylinder is 13.2 cm

37. (i) Let r be the radius and h be the height of the solid cylinder.

Then, $r = \frac{2 \text{ cm}}{2} = 1$ cm

Let r_1 and r_2 be the radii of the hollow cylinder respectively.

Then, $r_1 = \frac{20 \text{ cm}}{2} = 10$ cm

$r_2 = r_1 - 0.25$ cm

$= 10$ cm $- 0.25$

$= 9.75$ cm

Let h_1 be the length of the hollow cylinder.

Then, $h_1 = 15$ cm

Volume of solid cylinder

= volume of hollow cylinder

$$\Rightarrow \quad \pi r^2 h = \pi(r_1^2 - r_2^2)\, h_1$$

$$\Rightarrow \quad \frac{22}{7} \times 1 \text{ cm} \times 1 \text{ cm} \times h$$

$$= \frac{22}{7} \times [(10)^2 - (9.75)^2]\ 15 \text{ cm}^3$$

$$\Rightarrow \quad h = (10 + 9.75)\,(10 - 9.75)\,(15) \text{ cm}$$

$$= 74.1 \text{ cm (approx.)}$$

(ii) Let r be the radius and h be the length of the copper wire.

Then, $r = \frac{0.2 \text{ mm}}{2} = \frac{1}{10}$ cm

Volume of wire = 1 cm^3

$$\Rightarrow \quad \pi r^2 h = \text{cm}^3$$

$$\Rightarrow \quad \frac{22}{7} \times \frac{1}{10} \text{ cm} \times \frac{1}{10} \text{ cm} \times h = 1 \text{ cm}^3$$

$$\Rightarrow \quad h = \frac{10 \times 10 \times 7}{22} \text{ cm}$$

$$= 31.81 \text{ cm (approx.)}$$

38. Let r be the radius and h be the height of the cylindrical vessel which is full of milk.

Then, $2r = h$

$$\Rightarrow \quad r = \frac{h}{2}$$

Let R and H be the radius and height of the smaller cylindrical vessels.

Then, $R = \frac{42 \text{ cm}}{2} = 21$ cm

and $H = 21$ cm

Volume of the large cylindrical vessel

= 2 × volume of the smaller cylindrical vessel

$$\Rightarrow \quad \pi r^2 h = 2 \times \pi R^2 H$$

$$\Rightarrow \quad \left(\frac{h}{2}\right)^2 h = 2 \times (21 \text{ cm})^2 \times 21 \text{ cm}$$

$$\Rightarrow \quad h^3 = 2^3 \times (21 \text{ cm})^3$$

$$\Rightarrow \quad h = 2 \times 21 \text{ cm}$$

$$= 42 \text{ cm}$$

Hence, the diameter of the cylindrical vessel full of milk is 42 cm.

EXERCISE 13G

1. Volume of a right circular cone = $\frac{1}{3}\pi r^2 h$

(i) Given: Radius = 7 cm, height = 12 cm

$$\text{Volume of cone} = \left(\frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 12\right) \text{cm}^3$$
$$= (22 \times 7 \times 4)\ \text{cm}^3 = 616\ \text{cm}^3$$

(ii) Given: Radius = 21 cm, height = 40 cm

$$\text{Volume} = \frac{1}{3} \times \frac{22}{7} \times 21\ \text{cm} \times 21\ \text{cm} \times 40\ \text{cm}$$
$$= 22 \times 7\ \text{cm} \times 3\ \text{cm} \times 40\ \text{cm}$$
$$= 18480\ \text{cm}^3$$

(iii) Given: Height = 1.02 m = 102 cm,

$$\text{Diameter} = 56\ \text{cm},\ r = \frac{56\ \text{cm}}{2} = 28\ \text{cm}$$
$$\text{Volume of cone} = \frac{1}{3} \times \frac{22}{7} \times 28\ \text{cm} \times 28\ \text{cm} \times 102\ \text{cm}$$
$$= 22 \times 4\ \text{cm} \times 28\ \text{cm} \times 34\ \text{cm}$$
$$= 83776\ \text{cm}^3$$

(iv) Given: r = 35 dm, slant height = 37 dm

So, $$h = \sqrt{l^2 - r^2} = \sqrt{(37\ \text{dm})^2 - (35\ \text{dm})^2}$$
$$= \sqrt{1369\ \text{dm}^2 - 1225\ \text{dm}^2}$$
$$= \sqrt{144\ \text{dm}^2}$$
$$h = 12\ \text{dm}$$
$$\text{Volume of cone} = \frac{1}{3} \times \frac{22}{7} \times 35\ \text{dm} \times 35\ \text{dm} \times 12\ \text{dm}$$
$$= 22 \times 5\ \text{dm} \times 35\ \text{dm} \times 4\ \text{dm}$$
$$= 15400\ \text{dm}^3$$

(v) Given: $h = 5$ cm, $2\pi r = 8$ cm

$$\therefore\quad r = \frac{8\ \text{cm}}{2 \times 22} \times 7 = \frac{56}{44}\ \text{cm}$$
$$\text{Volume of cone} = \frac{1}{3} \times \frac{22}{7} \times \frac{56}{44}\ \text{cm} \times \frac{56}{44}\ \text{cm} \times 5\ \text{cm}$$
$$= \frac{56 \times 8 \times 5}{3 \times 2 \times 44}\ \text{cm}^3$$
$$= \frac{280}{33}\ \text{cm}^3$$

2. (i) Given: Radius = 28 cm, slant height = 35 cm

$$\text{Height} = \sqrt{l^2 - r^2} = \sqrt{(35\ \text{cm})^2 - (28\ \text{cm})^2}$$
$$= \sqrt{1225\ \text{cm}^2 - 784\ \text{cm}^2}$$
$$= \sqrt{441\ \text{cm}^2} = 21\ \text{cm}$$
$$\text{Volume of cone} = \frac{1}{3}\pi r^2 h$$
$$= \frac{1}{3} \times \frac{22}{7} \times 28\ \text{cm} \times 28\ \text{cm} \times 21\ \text{cm}$$
$$= 22 \times 4\ \text{cm} \times 28\ \text{cm} \times 7\ \text{cm}$$
$$= 17248\ \text{cm}^3$$
$$= 17.248\ \text{L} \qquad [\because 1000\ \text{cm}^3 = 1\ \text{L}]$$

(ii) Given: Height = 48 cm, slant height = 50 m

$$r = \sqrt{l^2 - h^2} = \sqrt{(50\ \text{m})^2 - (48\ \text{m})^2}$$
$$= \sqrt{2500\ \text{m}^2 - 2304\ \text{m}^2}$$
$$= \sqrt{196\ \text{m}^2} = 14\ \text{m}$$
$$\text{Volume of cone} = \frac{1}{3}\pi r^2 h$$
$$= \frac{1}{3} \times \frac{22}{7} \times 14\ \text{m} \times 14\ \text{m} \times 48\ \text{m}$$
$$= 9856\ \text{m}^3$$
$$= 9856\ \text{kL} \qquad [\because 1\ \text{m}^3 = 1\ \text{kL}]$$

3. Given: Height of the cone = 12 m,

Circumference of the cone = $2\pi r$ = 22 m

$$\Rightarrow\quad r = \frac{22\ \text{m} \times 7}{2 \times 22} = \frac{7}{2}\ \text{m}.$$
$$\text{Volume of the cone} = \frac{1}{3}\pi r^2 h$$
$$= \frac{1}{3} \times \frac{22}{7} \times \frac{7}{2}\ \text{m} \times \frac{7}{2}\ \text{m} \times 12\ \text{m}$$
$$= 154\ \text{m}^3$$

4. $$\pi r^2 = 154\ \text{m}^2$$
$$\Rightarrow\quad \frac{22}{7} r^2 = 154\ \text{m}^2$$
$$\Rightarrow\quad r^2 = 49\ \text{m}^2$$
$$\Rightarrow\quad r = 7\ \text{m}$$

Curved surface area = $\pi r l$ = 550 m^2

$$\Rightarrow\quad \frac{22}{7} \times 7 \times l = 550$$
$$\Rightarrow\quad l = 25\ \text{m}$$
$$h = \sqrt{l^2 - r^2} = \sqrt{25^2 - 7^2}$$
$$= \sqrt{18 \times 32} = 24\ \text{m}$$
$$\text{Volume of the tent} = \frac{1}{3}\pi r^2 h$$
$$= \frac{1}{3} \times \frac{22}{7} \times 7 \times 7 \times 24\ \text{m}^3$$
$$= 1232\ \text{m}^3$$

5. Given: Slant height (l) = 10 m,

Curved surface area of cone = $\pi r l$ = 188.4 m^2

$$\Rightarrow\quad \pi \times r \times 10\ \text{m} = 188.4\ \text{m}^2$$
$$\Rightarrow\quad r = \frac{188.4\ \text{m}^2}{3.14 \times 10\ \text{m}}$$
$$= \frac{60\ \text{m}^2}{10\ \text{m}} = 6\ \text{m}$$

$$h = \sqrt{l^2 - r^2}$$
$$= \sqrt{(10\text{ m})^2 - (6\text{ m})^2}$$
$$= \sqrt{100\text{ m}^2 - 36\text{ m}^2}$$
$$= \sqrt{64\text{ m}^2} = 8\text{ m}$$

$$\text{Volume of cone} = \frac{1}{3}\pi r^2 h$$
$$= \frac{1}{3} \times \frac{22}{7} \times 6\text{ m} \times 6\text{ m} \times 8\text{ m}$$
$$= 301.44\text{ m}^3$$

Hence, the volume of the cone is 301.44 m^3.

6. $\pi rl = 47.1\text{ cm}^2$

$$\Rightarrow \quad 3.14 \times r \times \sqrt{h^2 + r^2} = 47.1$$
$$\Rightarrow \quad 3.14 \times r \times \sqrt{4^2 + r^2} = 47.1$$
$$\Rightarrow \quad r\sqrt{4^2 + r^2} = \frac{47.1}{3.14} = 15$$
$$\Rightarrow \quad r^2(4^2 + r^2) = (15)^2 = 225$$
$$\Rightarrow \quad 16r^2 + r^4 - 225 = 0$$

Let $r^2 = y$,

then $y^2 + 16y - 225 = 0$

$$\Rightarrow \quad (y - 9)(y + 25) = 0$$

Either, $y = 9$ or $y = -25$ rejected

$$\Rightarrow \quad r^2 = 9$$
$$\Rightarrow \quad r = 3\text{ cm}$$

$$\text{Volume} = \frac{1}{3} \times 3.14 \times 3 \times 3 \times 4\text{ cm}^3$$
$$= 37.68\text{ cm}^3$$

7. Given: Floor area of cone $= \pi r^2 = \frac{3168}{7}\text{ m}^2$

$$\Rightarrow \quad r^2 = \frac{3168 \times 7}{7 \times 22}\text{ m}^2$$
$$\Rightarrow \quad r^2 = 144\text{ m}^2$$
$$\Rightarrow \quad r = \sqrt{144\text{ m}^2} = 12\text{ m}$$

$$\text{CSA of cone} = \pi rl = \frac{3960}{7}\text{ m}^2$$
$$\Rightarrow \quad \pi \times 12 \times l = \frac{3960}{7}\text{ m}^2$$
$$\Rightarrow \quad l = \frac{3960 \times 7}{7 \times 22 \times 12}\text{ m}^2 = 15\text{ m}$$

So, $h = \sqrt{l^2 - r^2}$

$$= \sqrt{(15\text{ m})^2 - (12\text{ m})^2}$$
$$= \sqrt{225\text{ m}^2 - 144\text{ m}^2}$$
$$= \sqrt{81\text{ m}^2} = 9\text{ m}$$

$$\text{Volume of cone} = \frac{1}{3}\pi r^2 h$$
$$= \frac{1}{3} \times \frac{22}{7} \times 12\text{ m} \times 12\text{ m} \times 9\text{ m}$$
$$= \frac{9504}{7}\text{ m}^3$$

8. Volume of the conical tent

$$= \frac{1}{3}\pi r^2 h$$
$$= \frac{1}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 6\text{ m}^3 = 77\text{ m}^3$$

Average cubic metre of air space per solider $= \frac{77}{12}\text{ m}^3$

9. Given: Radius of base of cone = 7 m, h = 12 m

$$\text{Volume of cone} = \frac{1}{3}\pi r^2 h$$
$$= \frac{1}{3} \times \frac{22}{7} \times 7\text{ m} \times 7\text{ m} \times 12\text{ m}$$
$$= 22\text{ m} \times 7\text{ m} \times 4\text{ m}$$
$$= 616\text{ m}^3.$$

$$\text{Number of bags} = \frac{616\text{ m}^3}{3.5\text{ m}^3} = 176\text{ bags.}$$

10. Given: Diameter of cone = 9 m

So, $r = \frac{9}{2}$ m.

Height = 3.5 m

$$\text{Volume of the cone} = \frac{1}{3}\pi r^2 h$$
$$= \frac{1}{3} \times \frac{22}{7} \times \frac{9}{2}\text{ m} \times \frac{9}{2}\text{ m} \times 3.5\text{ m}$$
$$= \frac{11 \times 3\text{ m} \times 9\text{ m} \times 0.5}{2}$$
$$= 74.25\text{ m}^3$$

$$l = \sqrt{h^2 + r^2}$$
$$= \sqrt{(4.5\text{ m})^2 + (3.5\text{ m})^2}$$
$$= \sqrt{20.25\text{ m}^2 + 12.25\text{ m}^2}$$
$$= \sqrt{32.5\text{ m}^2} = 5.70\text{ m}$$

$$\text{CSA of cone} = \pi rl$$
$$= \frac{22}{7} \times \frac{9}{2}\text{ m} \times 5.70\text{ m}$$
$$= 80.614\text{ m}^2\text{ (approx.)}$$

Hence, the canvas required for the given cone is 80.614 m^2.

11. Given: Height of cone = 30 cm, volume = 1540 cm^3

$$\Rightarrow \quad \frac{1}{3}\pi r^2 h = \text{volume of cone}$$

$\Rightarrow \quad \frac{1}{3} \times \frac{22}{7} \times r^2 \times 30 \text{ cm} = 1540 \text{ cm}^3$

$\Rightarrow \quad 22 \times r^2 \times 30 \text{ cm} = 1540 \text{ cm}^3 \times 3 \times 7$

$\Rightarrow \quad r^2 = \frac{1540 \text{ cm}^3 \times 3 \times 7}{22 \times 30 \text{ cm}}$

$\Rightarrow \quad r^2 = 49 \text{ cm}^2$

$\Rightarrow \quad r = \sqrt{49 \text{ cm}^2} = 7 \text{ cm}$

Diameter of cone $= 2 \times r = 2 \times 7 \text{ cm} = 14 \text{ cm}$

12. Given: Area of base of cone $= \pi r^2 = 25 \text{ cm}^2$

Volume $= 525 \text{ cm}^3$

$\frac{1}{3}\pi r^2 h$ = volume of cone

$\Rightarrow \quad \frac{1}{3} \times 25 \text{ cm}^2 \times h = 525 \text{ cm}^3$

$\Rightarrow \quad 25 \text{ cm} \times h = 525 \text{ cm}^3 \times 3$

$\Rightarrow \quad h = \frac{525 \text{ cm}^3 \times 3}{25 \text{ cm}^2}$

$= 21 \times 3 = 63$ cm.

13. Given: Diameter of base = 28 cm, $r = \frac{28 \text{ cm}}{2} = 14$ cm

Volume $= 9856 \text{ cm}^3$

$\Rightarrow \quad \frac{1}{3}\pi r^2 h = 9856 \text{ cm}^3$

$\Rightarrow \quad \frac{1}{3} \times \frac{22}{7} \times 14 \times 14 \times h = 9856 \text{ cm}^3$

$\Rightarrow \quad 22 \times 196 \text{ cm}^2 \times h = 9856 \text{ cm}^3 \times 3 \times 7$

$\Rightarrow \quad h = \frac{9856 \text{ cm}^3 \times 3 \times 7}{22 \times 196 \text{ cm}^3} = 48$ cm.

$\therefore \quad l = \sqrt{h^2 + r^2} = \sqrt{48^2 \text{ cm}^2 + 14^2 \text{ cm}^2}$

$= \sqrt{2304 + 96} \text{ cm} = \sqrt{2500} \text{ cm}$

$\Rightarrow \quad l = 50$ cm.

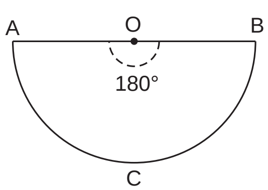
14. $h = 10$ cm, $r = 6.3$ cm.

Volume $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 6.3 \times 6.3 \times 10 \text{ cm}^3$

$= 415.8 \text{ cm}^3$

15. Let ACB represents the semi-circular sheet of tin with diameter 35 cm ($r = 17.5$ cm).



Then the central angle AOB = 180°

Then length of arc ACB $= 2\pi r \frac{180^\circ}{360^\circ}$

$= 2\pi \times 17.5 \text{ cm} \times \frac{180^\circ}{360^\circ}$

$= 17.5\pi$ cm.

Slant height of cone $= l = 17.5$ cm

Let r be the radius and h, height of the cone.

$2\pi r = 17.5\pi$ cm

$r = \frac{17.5\pi \text{ cm}}{2\pi} = \frac{17.5}{2} \text{ cm} = 8.75 \text{ cm}$

$h = \sqrt{l^2 - r^2} = \sqrt{(17.5 \text{ cm})^2 - (8.75 \text{ cm})^2}$

$= \sqrt{306.25 \text{ cm}^2 - 76.56 \text{ cm}^2} = \sqrt{229.69 \text{ cm}^2}$

$= 15.1554$ cm

Volume of cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \pi \times 8.75 \text{ cm} \times 8.75 \text{ cm} \times 15.1554 \text{ cm}$

$= \frac{1}{3} \times \frac{22}{7} \times 76.56 \text{ cm}^2 \times 15.1554 \text{ cm}$

$= \frac{25527.38}{21} \text{ cm}^3 = 1215.6 \text{ cm}^3$

16. Length of circumference of the quadrant

$= 2\pi R \frac{\theta}{360} = 2 \times \frac{22}{7} \times 14 \times \frac{90}{360} \text{ cm} = 22 \text{ cm}$

$\Rightarrow$ Circumference of the base of the cone $= 2\pi r = 22$ cm

$\Rightarrow \quad r = \frac{22 \times 7}{2 \times 22} \text{ cm} = \frac{7}{2} \text{ cm}$

$h = \sqrt{l^2 - r^2} = \sqrt{(14)^2 - \left(\frac{7}{2}\right)^2} \text{ cm}$

$= \sqrt{\frac{735}{4}} \text{ cm} = 13.555$ cm (approx.)

Volume $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 13.555 \text{ cm}^3$

$= 173.96 \text{ cm}^3$ (approx.)

17. Length of the arc of sector $= 2\pi r \frac{\theta}{360^\circ}$

$= 2\pi \times 12 \text{ cm} \times \frac{120^\circ}{360^\circ}$

$= 2 \times \frac{22}{7} \times 12 \text{ cm} \times \frac{1}{3}$

$= \frac{176}{7}$ cm

Let r_1 be the radius and h_1 height of the cone.

$2\pi r_1 = \frac{176}{7}$ cm

$\Rightarrow \quad 2 \times \frac{22}{7} \times r_1 = \frac{176}{7}$ cm

$\Rightarrow \quad r_1 = \frac{176 \times 7}{7 \times 2 \times 22} \text{ cm} = 4 \text{ cm}$

Slant height l = bounding radius = 12 cm

and $h = \sqrt{l^2 - r_1^2} = \sqrt{(12\text{ cm})^2 - (4\text{ cm})^2}$

$= \sqrt{144\text{ cm}^2 - 16\text{ cm}^2}$

$= \sqrt{128\text{ cm}^2} = 8\sqrt{2}$ cm.

Volume of cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 4\text{ cm} \times 8\sqrt{2}\text{ cm}$

$= \frac{352\text{ cm}^2 \times 8\sqrt{2}\text{ cm}}{21}$

$= 189.63\text{ cm}^3$ (approx.)

18. Volume of water poured in 1 minute = 220 L

$\therefore$ Volume of water poured in 1 hour

$= 220 \times 60\text{ L} = 13200\text{ L}$

$= 13.2\text{ kL} = 13.2\text{ m}^3$

Volume of water collected in 1 hour

= Volume of water poured in 1 hour

$\Rightarrow \quad \frac{1}{3}\pi r^2 h = 13.2\text{ m}^3$

$\Rightarrow \quad \frac{1}{3} \times \frac{22}{7} \times \frac{7}{2}\text{ m} \times \frac{7}{2}\text{ m} \times h = 13.2\text{ m}^3$

$\Rightarrow \quad h = \frac{13.2 \times 3 \times 7 \times 2 \times 2}{22 \times 7 \times 7} \times 100\text{ cm}$

$= 102.85\text{ cm} \approx 103\text{ cm}$

19. Let r_1 = radius of conical vessel = 10 cm

h_1 = 36 cm and r_2 = 20 cm

Now, volume of water in conical vessel

= volume of water in cylindrical vessel

$\Rightarrow \quad \frac{1}{3}\pi r_1^2 h_1 = \pi r_2^2 h_2$

$\Rightarrow \quad r_1^2 h_1 = 3r_2^2 h_2$

$\Rightarrow \quad (10 \times 10)\text{ cm}^2 \times h_1 = 3 \times (20 \times 20)\text{ cm}^2 \times h_2$

$\Rightarrow \quad 100\text{ cm}^2 \times 36\text{ cm} = 3 \times 400\text{ cm}^2 \times h_2$

$\Rightarrow \quad h_2 = \frac{100\text{ cm}^2 \times 36\text{ cm}}{3 \times 400\text{ cm}^2} = 3\text{ cm}$

20. Volume of water that flows in 1 minute

$= \pi \times \frac{1}{400} \times \frac{1}{400} \times 10\text{ m}^3$

Volume of conical vessel

$= \frac{1}{3}\pi \times \frac{21}{100} \times \frac{21}{100} \times \frac{24}{100}\text{ m}^3$

Required time $= \dfrac{\frac{1}{3}\pi \times \frac{21}{100} \times \frac{21}{100} \times \frac{24}{100}}{\pi \times \frac{1}{400} \times \frac{1}{400} \times 10}$ minutes

= 56.5 minutes (approx.)

21. Radius of cone $= \frac{\text{Edge of cube}}{2} = \frac{14\text{ cm}}{2} = 7\text{ cm}$

Height of cone = Edge of cube = 14 cm

Volume of cone $= \frac{1}{3} \times \pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 7\text{ cm} \times 7\text{ cm} \times 14\text{ cm}$

$= \frac{2156}{3}\text{ cm}^3$.

22. Given: Radius = 7 cm and height = 21 cm

Volume of cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 7\text{ cm} \times 7\text{ cm} \times 21\text{ cm}$

$= 1078\text{ cm}^3$.

23. Given: Height of cylinder = 20 cm,

Diameter = 8 cm, $r = \frac{8\text{ cm}}{2} = 4\text{ cm}$

Volume of cylinder $= \pi r^2 h$

$= \frac{22}{7} \times 4\text{ cm} \times 4\text{ cm} \times 20\text{ cm}$

$= \frac{7040}{7}\text{ cm}^3$

Given: Height of cone = 15 cm

$\Rightarrow \quad \frac{1}{3}\pi r^2 h$ = volume of cone

$= \frac{7040}{7}\text{ cm}^3$

$\Rightarrow \quad \frac{1}{3}\pi r^2 h = \frac{7040}{7}\text{ cm}^3$

$\Rightarrow \quad \frac{1}{3} \times \frac{22}{7} \times r^2 \times 15\text{ cm} = \frac{7040}{7}\text{ cm}^3$

$\Rightarrow \quad r^2 = \frac{7040 \times 3 \times 7\text{ cm}^3}{(7 \times 15 \times 22)\text{ cm}}$

$\Rightarrow \quad r^2 = 64\text{ cm}^2$

$\Rightarrow \quad r = \sqrt{64\text{ cm}^2} = 8\text{ cm}$

24. Volume of the pillar $= \pi r^2 h + \frac{1}{3}\pi r^2 h$

$= \frac{22}{7} \times 8^2\left(240 + \frac{1}{3} \times 36\right)\text{ cm}^3$

$= 50688\text{ cm}^3$.

Weight of the pillar $= \frac{7.8 \times 50688}{1000}\text{ kg} = 395.3664\text{ kg}$

25. Given: Height of cone = height of cylinder = h

Radius of cone = $2x$

Radius of cylinder = x

Ratio of volume $= \frac{\text{Volume of cone}}{\text{Volume of cylinder}}$

$= \dfrac{\frac{1}{3}\pi r^2 h}{\pi r^2 h} = \dfrac{\frac{1}{3}\pi \times (2x)^2 \times h}{\pi \times (x)^2 \times h}$

$= \frac{\pi \times 4x^2 \times h}{3 \times \pi \times x^2 \times h} = 4 : 3$

EXERCISE 13H

1. (i) Volume of sphere $= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times 1 \text{ cm} \times 1 \text{ cm} \times 1 \text{ cm}$

$= \frac{88}{21} \text{ cm}^3$

(ii) Given: diameter = 7 cm

So, radius $= \frac{7}{2}$ cm

Volume of sphere $= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times \frac{7}{2} \text{ cm} \times \frac{7}{2} \text{ cm} \times \frac{7}{2} \text{ cm}$

$= \frac{88 \times 49 \text{ cm}^2 \times 7 \text{ cm}}{21 \times 8} = \frac{539}{3} \text{ cm}^3$

2. Given: Diameter of spherical ball = 21 cm

Radius $= \frac{21}{2} = 10.5$ cm

Volume of sphere $= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times 10.5 \text{ cm} \times 10.5 \text{ cm} \times 10.5 \text{ cm}$

$= \frac{101871 \text{ cm}^3}{21} = 4851 \text{ cm}^3$.

3. Given: Diameter = 5.6 mm,

So, Radius $= \frac{5.6 \text{ mm}}{2} = 2.8$ mm

Volume of sphere $= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times 2.8 \text{ mm} \times 2.8 \text{ mm} \times 2.8 \text{ mm}$

$= \frac{1931.776}{21} \text{ mm}^3$

Volume of 75 such capsules $= \frac{75 \times 1931.776}{21} \text{ mm}^3$

$= 6899.2 \text{ mm}^3$

4. Given: Diameter of metallic ball = 6 cm,

So, $r = 3$ cm

Volume of the sphere $= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times 3 \text{ cm} \times 3 \text{ cm} \times 3 \text{ cm}$

$= \frac{88 \times 9}{7} \text{ cm}^3 = \frac{792}{7} \text{ cm}^3$

Mass of ball = volume × density

$= 7 \text{ g/cm}^3 \times \frac{792}{7} \text{ cm}^3 = 792$ g.

5. Given: Volume of sphere $= \frac{792}{7} \text{ cm}^3$

$\Rightarrow \frac{4}{3}\pi r^3 = \frac{792}{7} \text{ cm}^3$

$\Rightarrow \frac{4}{3} \times \frac{22}{7} \times r^3 = \frac{792}{7} \text{ cm}^3$

$\Rightarrow r^3 = \frac{792 \times 3 \times 7}{7 \times 4 \times 22} \text{ cm}^3 = 27 \text{ cm}^3$

$\Rightarrow r = \sqrt[3]{27 \text{ cm}^3} = 3$ cm

Hence, diameter of the ball = 2 × 3 cm = 6 cm.

6. Given: Surface area of sphere $= 5544 \text{ cm}^2$

$\Rightarrow 4\pi r^2 = 5544 \text{ cm}^2$

$\Rightarrow 4 \times \frac{22}{7} \times r^2 = 5544 \text{ cm}^2$

$\Rightarrow r^2 = \frac{5544 \text{ cm}^2 \times 7}{4 \times 22} = 441 \text{ cm}^2$

$\Rightarrow r = \sqrt{441 \text{ cm}^2} = 21$ cm

Volume of sphere $= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times 21 \text{ cm} \times 21 \text{ cm} \times 21 \text{ cm}$

$= 4 \times 22 \times 7 \text{ cm} \times 3 \text{ cm} \times 21 \text{ cm}$

$= 38808 \text{ cm}^3$.

7. Volume of the sphere is numerically equal to surface area of the sphere.

$\therefore \frac{4}{3}\pi r^3 = 4\pi r^2$

$\Rightarrow \frac{4}{3}\pi r^3 - 4\pi r^2 = 0$

$\Rightarrow 4\pi r^2\left(\frac{r}{3} - 1\right) = 0$

$\Rightarrow$ Either $4\pi r^3 = 0$ (rejected) or $\left(\frac{r}{3} - 1\right) = 0$

$\Rightarrow \frac{r}{3} = 1$

$\Rightarrow r = 3$ units

8. Volume of sphere $= \frac{4}{3}\pi r^3$

Volume when radius is halved

Volume of new sphere $= \frac{4}{3}\pi\left(\frac{r}{2}\right)^3 = \frac{4}{3} \times \pi \frac{r^3}{8}$

Ratio $= \frac{\frac{4}{3}\pi r^3}{\frac{4}{3} \times \pi \times \frac{r^3}{8}} = 8 : 1$

9. Ratio of two sphere

$= \frac{\frac{4}{3} \times \pi \times 7 \text{ cm} \times 7 \text{ cm} \times 7 \text{ cm}}{\frac{4}{3} \times \pi \times 10.5 \text{ cm} \times 10.5 \text{ cm} \times 10.5 \text{ cm}}$

$$= \frac{7 \times 7 \times 7}{10.5 \times 10.5 \times 10.5}$$

$$= \frac{1}{1.5 \times 10.5 \times 10.5}$$

$$= \frac{1 \times 8}{3.275 \times 8} = \frac{8}{27}$$

Ratio = 8 : 27

10. Let the radii of the spheres are $3x$ and $5x$.

$$\text{Ratio of volumes} = \frac{\frac{4}{3} \times \pi \times 3x \times 3x \times 3x}{\frac{4}{3} \times \pi \times 5x \times 5x \times 5x}$$

$$= \frac{3 \times 3 \times 3}{5 \times 5 \times 5} = \frac{27}{125}$$

Ratio = 27 : 125

11. Let r_1 and r_2 be the radii and V_1 and V_2 be their volumes respectively.

Then, $$\frac{\frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_2^3} = \frac{64}{27}$$

$$\Rightarrow \quad \frac{r_1^3}{r_2^3} = \frac{4^3}{3^3}$$

$$\Rightarrow \quad \frac{r_1}{r_2} = \frac{4}{3}$$

$$\Rightarrow \quad \frac{r_1}{r_2} + 1 = \frac{4}{3} + 1$$

$$\Rightarrow \quad \frac{r_1 + r_2}{r_2} = \frac{4+3}{3}$$

$$\Rightarrow \quad \frac{21}{r_2} = \frac{7}{3}$$

$$\Rightarrow \quad r_2 = \frac{21 \times 3}{7} = 9$$

$$r_1 = (21 - 9) \text{ cm} = 12 \text{ cm}$$

12. 35 g-wt is the weight of 1 cm^3 of metal.

$\therefore$ $\frac{792}{25}$ g-wt is the weight of $\frac{1}{35} \times \frac{792}{25}$ cm^3 of metal

$$\Rightarrow \quad \frac{4}{3}\pi r^3 = \frac{792}{35 \times 25} \text{ cm}^3$$

$$\Rightarrow \quad r^3 = \frac{792 \times 3 \times 7}{35 \times 25 \times 4 \times 22} \text{ cm}^3 = 0.216 \text{ cm}^3$$

$$\Rightarrow \quad r = 0.6 \text{ cm} = 6 \text{ mm}$$

13. Volume of spherical ball $= \frac{33957}{7}$ m^3

$$\Rightarrow \quad \frac{4}{3}\pi r^3 = \frac{33957}{7} \text{ m}^3$$

$$\Rightarrow \quad r^3 = \frac{33957 \times 3 \times 7}{7 \times 4 \times 22} \text{ m}^3$$

$$\Rightarrow \quad r = 10.5 \text{ m}$$

14. Volume of hemisphere $= \frac{2}{3}\pi r^3$

$$= \frac{2}{3} \times \frac{22}{7} \times 2 \text{ cm} \times 2 \text{ cm} \times 2 \text{ cm}$$

$$= \frac{352}{21} \text{ cm}^3$$

TSA of hemisphere $= 3\pi r^2$

$$= 3 \times \frac{22}{7} \times 2 \text{ cm} \times 2 \text{ cm}$$

$$= \frac{264}{7} \text{ cm}^2$$

15. Circumference of hemisphere $= 2\pi r = 132$ cm

$$\therefore \quad r = \frac{132 \text{ cm}}{2\pi} = \frac{132 \text{ cm} \times 7}{2 \times 22}$$

$$= 21 \text{ cm}$$

Volume of hemisphere $= \frac{2}{3}\pi r^3$

$$= \frac{2}{3} \times \frac{22}{7} \times 21 \text{ cm} \times 21 \text{ cm} \times 21 \text{ cm}$$

$$= 2 \times 22 \times 21 \text{ cm} \times 1 \text{ cm} \times 21 \text{ cm}$$

$= 19404$ cm^3 or 19.404 L.

16. Let r and R be the internal and external radii of the hemispherical bowl.

Then, $r = 4$ cm and $R = (4 + 0.5)$ cm $= 4.5$ cm

Volume of steel used = External volume – Internal volume

$$= \frac{2}{3}\pi R^3 - \frac{2}{3}\pi r^3$$

$$= \frac{2}{3} \times \frac{22}{7}[(4.5)^3 - (4)^3] = \frac{341}{6} \text{ cm}^3$$

17. Volume of hollow sphere

$$= \frac{4}{3}\pi(R^3 - r^3)$$

$$= \frac{4}{3} \times \frac{22}{7}[(11 \text{ cm})^3 - (10 \text{ cm})^3]$$

$$= \frac{4}{3} \times \frac{22}{7}(1331 \text{ cm}^3 - 1000 \text{ cm}^3)$$

$$= \frac{4}{3} \times \frac{22}{7} \times 331 \text{ cm}^3$$

$$= \frac{29128}{21} \text{ cm}^3$$

Mass of the hollow sphere $= 21 \text{ g} \times \frac{29128}{21} \text{ cm}^3$

$= 29128$ g $= 29.128$ kg

18. Let r be radius of the spherical ball.

Then, $r = 2$ cm.

Volume of the ball $= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi (2 \text{ cm})^3$

$$= \frac{4}{3}\pi (8) \text{ cm}^3$$

$\Rightarrow$ Mass of $\frac{4}{3}\pi$ (8) cm^3 is 8 kg.

$\Rightarrow$ Mass of 1 cm^3 is $\frac{8 \times 3}{4\pi (8)}$ kg $= \frac{3}{4\pi}$ kg

$$\text{Volume of spherical shell} = \frac{4}{3}\pi(5^3 - 4^3)$$
$$= \frac{4}{3}\pi \times (125 - 64)$$
$$= \frac{4}{3}\pi\ (61)\ \text{cm}^3$$

$$\text{Mass of spherical shell} = \frac{4}{3}\pi(61) \times \frac{3}{4\pi}\ \text{kg} = 61\ \text{kg}$$

19. Let r be the internal radius and R the external radius of the sphere.

Then, $\quad R = \frac{14}{2}$ cm = 7 cm

Mass of the spherical shell = volume × density

$$= \frac{57552}{7}\ \text{g}$$

$$\Rightarrow \frac{4}{3} \times \frac{22}{7}[(7\ \text{cm})^3 - (r)^3] \times 9\ \text{g/cm}^3 = \frac{57552}{7}\ \text{g}$$

$$\Rightarrow (343\ \text{cm}^3 - r^3) = \frac{57552 \times 3 \times 7}{7 \times 4 \times 22 \times 9}\ \text{cm}^3$$

$$\Rightarrow r^3 = 343\ \text{cm}^3 - 218\ \text{cm}^3$$
$$\Rightarrow r^3 = 125\ \text{cm}^3$$
$$\Rightarrow r = 5\ \text{cm}$$

Thickness = R − r = 7 cm − 5 cm = 2 cm

20. Given: $\quad h = \frac{2}{3} \times$ diameter

So, $\quad h = \frac{2}{3} \times 2r = \frac{4}{3}r$

Volume of cylinder = volume of sphere

$$\pi r^2 h = \frac{4}{3}\pi R^3$$
$$\Rightarrow r^2 h = \frac{4}{3}R^3$$
$$\Rightarrow r^2 \times \frac{4}{3}r = \frac{4}{3}R^3$$
$$\Rightarrow r^3 = R^3 = (4)^3$$
$$= 64.$$

So, $\quad r = 4$ cm.

21. Surface area of cube = Surface area of sphere

$$6(\text{edge})^2 = 4\pi r^2$$
$$\Rightarrow (\text{edge})^2 = \frac{4}{6}\pi r^2$$
$$\Rightarrow \text{edge} = \sqrt{\frac{4}{6}\pi r^2} = 2r\sqrt{\frac{\pi}{6}}$$

$$\frac{\text{Volume of cube}}{\text{Volume of sphere}} = \frac{(\text{edge})^3}{\frac{4}{3}\pi r^3}$$

$$= \frac{\left[2r\left(\sqrt{\frac{\pi}{6}}\right)\right]^3}{\frac{4}{3}\pi r^3}$$

$$= \frac{8(r)^3\ (\pi^{3/2})\ (3)}{(6^{3/2})\ (4)\ (\pi)\ (r^3)} = \frac{\sqrt{\pi}}{\sqrt{6}}$$

22. Let r be the radius of the sphere. Then, radius of the cone $= r$.

Let h be the height of the cone.

Given, volume of sphere = Volume of the cone

$$\Rightarrow \frac{4}{3}\pi r^3 = \frac{1}{3}\pi r^2 h$$
$$\Rightarrow 4r = h$$
$$\Rightarrow 2r = \frac{h}{2}$$
$$\Rightarrow \text{diameter} = \frac{h}{2}$$

23. The diameter of the largest sphere which can be carved out of a cube of side 21 m is 21.

$\therefore$ Radius of sphere $= \frac{21\ \text{m}}{2} = 10.5$ m

Hence, volume of sphere $= \frac{4}{3}\pi r^3$

$$= \frac{4}{3} \times \frac{22}{7} \times \frac{21}{2}\ \text{m} \times \frac{21}{2}\ \text{m} \times \frac{21}{2}\ \text{m}$$
$$= 4851\ \text{m}^3$$

24. Volume of third small sphere

$$= \frac{4}{3}\pi R^3 - \left(\frac{4}{3}\pi r_1^3 + \frac{4}{3}\pi r_2^3\right)$$

$$\Rightarrow \frac{4}{3}\pi r_3^3 = \frac{4}{3} \times \frac{22}{7} \times (3\ \text{cm})^3 - \left[\left\{\frac{4}{3} \times \frac{22}{7} \times (0.75\ \text{cm})^3\right\} + \left\{\frac{4}{3} \times \frac{22}{7} \times (1.25\ \text{cm})^3\right\}\right]$$

$$\Rightarrow \frac{4}{3} \times \frac{22}{7} \times r_3^3 = \frac{4}{3} \times \frac{22}{7} \times 27\ \text{cm}^3 - [(0.75 + 1.25)\ \{(0.75)^2 - 1.25 \times 0.75 + (1.25)^2\}]$$

$$\Rightarrow r_3^3 = 27\ \text{cm}^3 - [2(0.5625 + 1.5625 - 0.9375)]$$
$$\Rightarrow r_3^3 = 24.625\ \text{cm}^3$$
$$\Rightarrow r = 2.909\ \text{cm}.$$

25. Volume of solid rectangular block

$$= l \times b \times h$$
$$= 49\ \text{cm} \times 44\ \text{cm} \times 18\ \text{cm}$$
$$= 38808\ \text{cm}^3$$

Now $\quad \frac{4}{3}\pi r^3 = 38808\ \text{cm}^3$

$$\Rightarrow \frac{4}{3} \times \frac{22}{7} \times r^3 = 38808\ \text{cm}^3$$

$$\Rightarrow \quad r^3 = \frac{38808 \text{ cm}^3 \times 3 \times 7}{4 \times 22}$$

$$\Rightarrow \quad r^3 = 9161 \text{ cm}^3$$

$$\Rightarrow \quad r = \sqrt[3]{9161 \text{ cm}^3}$$

$$= \sqrt[3]{21 \text{ cm} \times 21 \text{ cm} \times 21 \text{ cm}}$$

$$r = 21 \text{ cm}$$

26. Volume of sphere = volume of hollow cylinder

$$\frac{4}{3}\pi r^3 = \pi(R^2 - r^2)h$$

External radius of cylinder = 4 cm

Internal radius of cylinder = 4 cm – 2 cm = 2 cm

Height = 24 cm

Now $\frac{4}{3}\pi r^3 = \pi[(4 \text{ cm})^2 - (2 \text{ cm})^2] \times 24 \text{ cm}$

$$\Rightarrow \quad \frac{4}{3} r^3 = [16 \text{ cm}^2 - 4 \text{ cm}^2] \times 24 \text{ cm}$$

$$\Rightarrow \quad \frac{4}{3} r^3 = 12 \text{ cm}^2 \times 24 \text{ cm} = 288 \text{ cm}^3$$

$$\Rightarrow \quad r^3 = \frac{288 \text{ cm}^3 \times 3}{4} = 216 \text{ cm}^3$$

$$\Rightarrow \quad r = \sqrt[3]{216 \text{ cm}^3} = 6 \text{ cm}.$$

27. Given:diameter of sphere = 6 cm

So, $r = \frac{6 \text{ cm}}{2} = 3 \text{ cm}$

$$\text{Volume of sphere} = \frac{4}{3}\pi r = \frac{4}{3} \times \pi \times (3 \text{ cm})^3$$

$$\text{Volume of cylinder (wire)} = \pi r^2 h$$

$$= \pi r^2 \times 36 \text{ cm}$$

Now $\pi r^2 \times 36 \text{ cm} = \frac{4}{3} \times \pi \times 27 \text{ cm}^3$

$$r^2 = \frac{4 \times 9 \text{ cm}^3}{36 \text{ cm}} = 1 \text{ cm}^2$$

$$\therefore \quad r = \sqrt{1 \text{ cm}^2} = 1 \text{ cm}$$

28. Volume of hemispherical shell

$$= \frac{2}{3}\pi (R^3 - r^3)$$

$$= \frac{2}{3} \times \frac{22}{7} \times [(5 \text{ cm})^3 - (3 \text{ cm})^3]$$

$$= \frac{2}{3} \times \frac{22}{7} \times (125 \text{ cm}^3 - 27 \text{ cm}^3)$$

$$= \frac{2}{3} \times \frac{22}{7} \times 98 \text{ cm}^3$$

Now $\pi r^2 h = \frac{2}{3} \times \frac{22}{7} \times 98 \text{ cm}^3$

$$\Rightarrow \quad \frac{22}{7} \times (7 \text{ cm})^2 \times h = \frac{2}{3} \times \frac{22}{7} \times 98 \text{ cm}^3$$

$$\Rightarrow \quad 49 \text{ cm}^2 h = \frac{2}{3} \times 98 \text{ cm}^3$$

$$\Rightarrow \quad h = \frac{2 \times 98 \text{ cm}^3}{3 \times 49 \text{ cm}^2} = \frac{4}{3} \text{ cm}$$

29. Volume of spherical ball = volume of water that rises

$$\Rightarrow \quad \frac{4}{3}\pi r^3 = \pi R^2 h$$

$$\Rightarrow \quad \frac{4}{3} r^3 = R^2 h$$

$$\Rightarrow \quad \frac{4}{3} r^3 = 12^2 \times 6.75 \text{ cm}^3$$

$$\Rightarrow \quad r^3 = \frac{12 \times 12 \times 6.75 \times 3}{4} = (3^2)^3$$

$$\Rightarrow \quad r = 3^2 = 9 \text{ cm}$$

30. $\text{Volume of cone} = \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times \pi (5 \text{ cm})^2 \times 8 \text{ cm}$$

$$= \frac{200}{3}\pi \text{ cm}^3$$

$$\text{One fourth volume of cone} = \frac{50}{3}\pi \text{ cm}^3$$

$$\text{Volume of lead shots} = \frac{4}{3} \times \pi \times r^3$$

$$= \frac{4}{3} \times \pi \times (0.5 \text{ cm})^3$$

$$= \frac{5}{30}\pi \text{ cm}^3$$

$$\text{Number of lead shots} = \frac{50}{3}\pi \div \frac{5}{30}\pi = 100$$

31. Radii and heights of the cone, hemisphere and cylinder are equal.

⇒ height of hemisphere = r

= height of cone

= height of cylinder

Volume of cone : volume of hemisphere : volume of cylinder

$$\frac{1}{3}\pi r^2 h : \frac{2}{3}\pi r^3 : \pi r^2 h = \frac{1}{3}(r^2)(r) : \frac{2}{3} r^3 : r^2(r)$$

$$= \frac{r^3}{3} : \frac{2}{3} r^3 : r^3 = 1 : 2 : 3$$

CHECK YOUR UNDERSTANDING

MULTIPLE-CHOICE QUESTIONS

1. (b) **64 cm³**

Total surface area of the cube = 96 cm²

$$\Rightarrow \quad 6(\text{Edge})^2 = 96 \text{ cm}^2$$

$$\Rightarrow \quad (\text{Edge})^2 = \frac{96 \text{ cm}^2}{6} = 16 \text{ cm}^2$$

⇒ Edge = 4 cm

Now,

Volume of the cube = $(\text{Edge})^2$

$= (4 \text{ cm})^3$

$= 64 \text{ cm}^3$

2. (a) **125**

Let a be length of the cube. Then, $a = 3$ cm

Volume of the cube $= a^3$

$= (3 \text{ cm})^3$

$= 27 \text{ cm}^3$

Let A be the length of the cubic block of metal.

Then, A = 15 cm.

Volume of the cubic block of metal $= A^3$

$= (15 \text{ cm})^3$

$= 3375 \text{ cm}^3$

Number of cubes that can be formed

$$= \frac{\text{Volume of the cubic block of metal}}{\text{Volume of the cube}}$$

$$= \frac{3375 \text{ cm}^3}{27 \text{ cm}^3} = 125$$

3. (c) **32 cm^2**

Total surface of the cube = 6 $(\text{side})^2$

Lateral surface area of the cube = 4 $(\text{side})^2$

Now, Total surface area – lateral surface area

$= 6 (\text{side})^2 - 4 (\text{side})^2$

$= 2 (\text{side})^2$

$= 2 \times (4 \text{ cm})^2$

$= 32 \text{ cm}^2$.

4. (a) **8 cm^3**

Diagonal of the cube= $2\sqrt{3}$

$(\text{Edge})\sqrt{3} = 2\sqrt{3}$ cm

Edge = 2 cm

Volume of the cube = $(\text{Edge})^3$

$= (2 \text{ cm})^3$

$= 8 \text{ cm}^3$

5. (c) **768**

Let the length, breadth and depth of the pit be l, b and d respectively. Then, l = 20 m = 2000 cm, b = 6 m = 600 cm and d = 8 cm.

Volume of the pit $= l \times b \times d$

$= 2000 \text{ cm} \times 600 \text{ cm} \times 8 \text{ cm}$

$= 46000000 \text{ cm}^3$

Volume of the plank = 5 m × 25 cm × 10 cm

= 500 cm × 25 cm × 10 cm

$= 125000 \text{ cm}^3$

$$\text{Number of planks} = \frac{\text{Volume of the pit}}{\text{Volume of the plank}}$$

$$= \frac{96000000 \text{ cm}^3}{125000 \text{ cm}^3} = 768$$

6. (a) **9**

Volume of the cube $= (\text{edge})^3 = (6 \text{ m})^3 = 216 \text{ m}^3$

Volume of the cuboid = 18 m × 12 m × 9 m

$= 1944 \text{ m}^3$

Number of the cubes that can be formed from the cuboid

$$= \frac{\text{Volume of the cuboid}}{\text{Volume of the cube}}$$

$$= \frac{1944 \text{ m}^3}{216 \text{ m}^3}$$

= 9.

7. (d) **17 m**

Let the length, breadth and height of the room be l, b and h respectively. Then, l = 12 m, b = 9 m and h = 8 m.

Length of the longest rod which can be put in the room

= Length of the diagonal of the room

$= \sqrt{l^2 + b^2 + h^2}$

$= \sqrt{12^2 + 9^2 + 8^2}$

$= \sqrt{144 + 81 + 64}$

= 17 m.

8. (b) **60 cm**

Volume of the cuboid = 36 cm × 75 cm × 80 cm

$= 216000 \text{ cm}^3$

Volume of the cube = $(\text{edge})^3$

Now,

Volume of the cuboid = Volume of the cube

⇒ $216000 \text{ cm}^3 = (\text{edge})^3$

⇒ edge $= (216000 \text{ cm}^3)^{1/3}$

⇒ edge = 60 cm.

9. (b) **10**

Volume of the maximum load of earth carried

$= 540 \text{ m}^3$

Volume of the rectangular pit

= 30 m × 15 m × 12 m

$= 5400 \text{ m}^3$

Now, Least number of rounds the carrier had to make to dispose of the earth dug out

$$= \frac{\text{Volume of the rectangular pit}}{\text{Volume of the maximum load}}$$

$$= \frac{5400 \text{ m}^3}{540 \text{ m}^3}$$

= 10.

10. (b) **3600**

Volume of the cuboid = 16 m × 12 m × 9 m

$= 1728 \text{ m}^3$

Volume of the bug of grain $= 0.48 \text{ m}^3$

Now,

Maximum number of bugs that can be stored in the granary

$$= \frac{\text{Volume of the cuboid}}{\text{Volume of the bug of grain}}$$

$$= \frac{1728 \text{ m}^3}{0.48 \text{ m}^3}$$

$$= 3600$$

11. (c) **1420**

Volume of the cuboid = 30 cm × 30 cm × 42.6 cm

$= 38340 \text{ cm}^3$

Volume of the cube = $(\text{edge})^3 = (3 \text{ cm}^3) = 27 \text{ cm}^3$

Number of cubes formed from the cuboid

$$= \frac{\text{Volume of the cuboid}}{\text{Volume of the cube}}$$

$$= \frac{38340 \text{ cm}^3}{27 \text{ cm}^3} = 1420$$

12. (c) **15 cm**

Let r be the radius and h the height of the right circular cylinder.

Then,

Volume of the right circular cylinder = $\pi r^2 h$

$$\Rightarrow \quad 2310 \text{ cm}^3 = \frac{22}{7} \times (7 \text{ cm})^2 \times h$$

$$\Rightarrow \quad h = \frac{2310 \text{ cm}^3 \times 7}{22 \times 49 \text{ cm}^2} = 15 \text{ cm}$$

13. (a) **625 cm^2**

Let r be the radius and h be the height of the cylinder. The height of the cylinder is equal to the length of the side of the side of the square. Then,

h = side = 25 cm

Again, the perimeter of the base of the cylinder is equal to the length of the side of the square

$2\pi r$ = side

$$r = \frac{\text{Side}}{2\pi} = \frac{25}{2\pi} \text{ cm}$$

Now, curved surface area of the cylinder = $2\pi rh$

$$= 2\pi \times \frac{25}{2\pi} \text{ cm} \times 25 \text{ cm}$$

$= 625 \text{ cm}^2$.

14. (d) **110 m^2**

Let r be the radius and h be the depth of the well.

Then, $r = \frac{3.5 \text{ m}}{2} = 1.75 \text{ m}$

$h = 10$ m

Curved surface area of the well = $2\pi rh$

$= 2\pi \times 1.75 \text{ m} \times 10 \text{ m}$

$$= \frac{22}{7} \times 2 \times 1.75 \text{ m} \times 10 \text{ m}$$

$= 110 \text{ m}^2$.

15. (a) **66 m^2**

Let r be the radius and h be the height of the cylinder. Then, $h = 3$ m.

Circumference of the base = 22 m

$\Rightarrow \quad 2\pi r = 22$ m

$$\Rightarrow \quad r = \frac{22}{2\pi} \text{ m}$$

$$= \frac{11}{\pi} \text{ m}$$

Curved surface area of the cylinder = $2\pi rh$

$$= 2 \times \pi \times \frac{11}{\pi} \text{ m} \times 3 \text{ m}$$

$= 66 \text{ m}^2$

16. (c) **66 m^2**

Let r be the radius and h be the height of the pipe.

Then, $h = 1$ m

$$r = \frac{21 \text{ m}}{2} = 10.5 \text{ m}$$

Now, Outer curved surface area = $2\pi rh$

$$= 2 \times \frac{22}{7} \times 10.5 \text{ m} \times 1 \text{ m} = 66 \text{ m}^2$$

17. (a) **₹352**

Let r be the radius and h be the deep of the well.

Then, $r = 2$ m

$h = 14$ m

Now, Inner curved surface area of the well = $2\pi rh$

$$= 2 \times \frac{22}{7} \times 2 \text{ m} \times 14 \text{ m}$$

$= 176 \text{ m}^2$

Cost of cementing the inner curved surface of the well

$= 176 \text{ m}^2 \times$ ₹ 2 per m^2

= ₹ 352

18. (b) **2 cm**

Let r be the radius and h be the height of the cylinder.

Then, $h = 14$ cm.

Curved surface area of the cylinder = $2\pi rh$

$$\Rightarrow \quad 88 \text{ cm}^2 = 2 \times \frac{22}{7} \times r \times 14 \text{ cm}$$

$= 1$ cm

$\therefore$ Diameter = $2r = 2 \times 1$ cm = 2 cm

19. (a) **132 cm^2**

Let r be the radius and h be the height of the cylinder.

Then, $h = 4$ cm

$r = 3$ cm

Total surface area of the right circular cylinder

$= 2\pi r(r + h)$

$$= 2 \times \frac{22}{7} \times 3 \text{ cm } (3 \text{ cm} + 4 \text{ cm})$$

$= 132 \text{ cm}^2$

20. (c) **6 cm**

Let r be the radius and h be the height of the cylinder.

Then, $h = 7$ cm

Lateral surface of the cylinder = $2\pi rh$

$$132 \text{ cm}^2 = 2 \times \frac{22}{7} \times r \times 7 \text{ cm}$$

$$r = \frac{132 \text{ cm}^2 \times 7}{2 \times 22 \times 7 \text{ cm}} = 3 \text{ cm}$$

Diameter = $2r = 2 \times 3$ cm = 6 cm

21. (d) **22 cm**

Let r be the radius and h be the height of the right circular cylinder.

Circumference of the base = $2\pi r$

$\Rightarrow \quad 44 \text{ cm} = 2\pi r$

$\Rightarrow \quad r = \frac{44}{2\pi}$ cm

Total surface area of the cylinder = $2\pi r(r + h)$

$\Rightarrow 968 \text{ cm}^2 = 2\pi r(r + h)$

$$\Rightarrow \quad r + h = \frac{968 \text{ cm}^2}{2\pi r} = \frac{968 \text{ cm}^2}{2\pi \times \frac{44}{2\pi} \text{ cm}} = 22 \text{ cm}$$

22. (c) **40 cm**

Let r be the radius and h be the height of the right circular cylinder.

Circumference of the base = 110 cm

$\Rightarrow \quad 2\pi r = 110$ cm

Curved surface area of the right circular cylinder

$= 2\pi rh$

$\Rightarrow \quad 4400 \text{ cm}^2 = 110 \text{ cm} \times h$

$$\Rightarrow \quad h = \frac{4400 \text{ cm}^2}{110 \text{ cm}} = 40 \text{ cm}$$

23. (c) **16π cm³**

Let r be the radius and h be the height of the iron cylinder.

$$\text{Then,} \quad r = \frac{\text{edge of the cube}}{2} = \frac{4 \text{ cm}}{2} = 2 \text{ cm}$$

h = edge of the cube = 4 cm

Maximum volume of the iron cylinder = $\pi r^2 h$

$= \pi \times (2 \text{ cm})^2 \times 4$ cm

$= 16\pi \text{ cm}^3$

24. (c) **92**

Let r be the radius and h be the height of the cylindrical drum.

Then, $r = 4.2$ m, $h = 3.5$.

Number of full bugs that can be emptied into the drum

$$= \frac{\text{Volume of the drum}}{\text{Volume of each bug}}$$

$$= \frac{\pi r^2 h}{2.1 \text{ m}^3} = \frac{\frac{22}{7} \times (4.2 \text{ m})^2 \times 3.5 \text{ m}}{2.1 \text{ m}^3} \simeq 92$$

25. (a) **1 : 4**

Let r be the radius and h be the height of the original cylinder. Let r' be the radius and h' be the height of the reduced cylinder.

Then, $\quad r' = \frac{r}{2}$ and $h = h'$

$$\frac{\text{Volume of the reduced cylinder}}{\text{Volume of the original cylinder}}$$

$$= \frac{\pi r'^2 h'}{\pi r^2 h} = \frac{\pi\left(\frac{r}{2}\right)^2 h'}{\pi r^2 h} = \frac{1}{4}$$

Hence, volume of the reduced cylinder: volume of the original cylinder = 1 : 4.

26. (a) **12 cm**

Let r be the radius and h be the level of the water rise in the cylindrical vessel. Let R be the radius of the sphere. Then, $r = 16$ cm, $h = 9$ cm.

The volume of the spherical ball is equal to the fraction of the volume of the cylinder increased due to the spherical ball.

Volume of the cylinder increased

= Volume of the sphere

$$\Rightarrow \quad \pi r^2 h = \frac{4}{3}\pi R^3$$

$$\Rightarrow \quad (16 \text{ cm})^2 \times 9 \text{ cm} = \frac{4}{3}R^3$$

$$\Rightarrow \quad R^3 = \frac{(16 \text{ cm})^2 \times 9 \times 3}{4}$$

$$\Rightarrow \quad R = 12 \text{ cm}$$

27. (a) **20 : 27**

Let r_1 and r_2 be the radii of the two cylinders.

Let h_1 and h_2 be the heights of the two cylinders. Then,

$$\frac{r_1}{r_2} = \frac{2}{3} \text{ and } \frac{h_1}{h_2} = \frac{5}{3}.$$

Now, ratio of the volumes of the cylinders is

$$\frac{\text{Volume of the 1}^{\text{st}} \text{ cylinder}}{\text{Volume of the 2}^{\text{nd}} \text{ cylinder}}$$

$$= \frac{\pi r_1^2 h_1}{\pi r_2^2 h_2} = \left(\frac{r_1}{r_2}\right)^2\left(\frac{h_1}{h_2}\right)$$

$$= \left(\frac{2}{3}\right)^2 \times \left(\frac{5}{3}\right)$$

$$= \frac{4 \times 5}{27} = \frac{20}{27}$$

Hence, ratio of the volumes of the cylinders is 20 : 27.

28. (d) **38808 cm³**

Let r be the radius of the sphere.

$$\text{Then,} \quad r = \frac{42 \text{ cm}}{2} = 21 \text{ cm.}$$

Volume of the sphere

$$= \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times (21\text{ cm})^3$$

$$= 38808\text{ cm}^3.$$

29. (c) **154 cm²**

Let r be the radius of the sphere.

Then, $r = 3.5$ cm

Surface area of the sphere

$$= 4\pi r^2 = 4 \times \frac{22}{7} \times (3.5\text{ cm})^2 = 154\text{ cm}^2.$$

30. (a) **6 units**

Let r be the radius of the sphere.

Volume of the sphere = Surface area of the sphere

$$\Rightarrow \quad \frac{4}{3}\pi r^3 = 4\pi r^2 \Rightarrow r = 3\text{ units}$$

Thus, diameter of the sphere is

$$d = 2r = 2 \times 3\text{ units} = 6\text{ units}.$$

31. (b) **30.48 cm³**

Volume of the cube = $(\text{edge})^3 = (4\text{ cm})^3 = 64\text{ cm}^3$

Let r be the radius of the sphere. Then

$$r = \frac{\text{edge of the cube}}{2}$$

$$= \frac{4\text{ cm}}{2} = 2\text{ cm}$$

Volume of the gap = Volume of the cube-volume of the sphere

$$= 64\text{ cm}^3 - \frac{4}{3}\pi r^3$$

$$= 64\text{ cm}^3 - \frac{4}{3} \times \frac{22}{7} \times (2\text{ cm})^3$$

$$\simeq 30.48\text{ cm}^3$$

32. (d) **4 : 3**

Let r and R be the radii of the two spheres.

$$\text{Then,} \quad \frac{\text{Volume of the 1}^{\text{st}}\text{ sphere}}{\text{Volume of the 2}^{\text{nd}}\text{ sphere}} = \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi R^3}$$

$$\Rightarrow \quad \frac{64}{27} = \left(\frac{r}{R}\right)^3$$

$$\Rightarrow \quad \frac{r}{R} = \frac{4}{3}$$

Thus, ratio of the radii of the two spheres is 4 : 3.

33. (b) **14 cm**

Let r be the radius of the spherical shot-put.

Surface area of the spherical shot-put = $4\pi r^2$

$$\Rightarrow \quad 616\text{ cm}^2 = 4 \times \frac{22}{7} \times r^2$$

$$\Rightarrow \quad r^2 = \frac{616\text{ cm}^2 \times 7}{88} = 49$$

$$\Rightarrow \quad r = 7\text{ cm}.$$

Thus, diameter of the spherical shot-put

$$= 2r = 2 \times 7\text{ cm} = 14\text{ cm}.$$

34. (c) **6 cm**

Let r be the radius of the sphere and R be radius of the right circular cone. Also, let h be the height of the right circular cone. Then,

$$r = 3\text{ cm}, h = 3\text{ cm}$$

Now, Volume of the sphere

= Volume of the right circular cone

$$\Rightarrow \quad \frac{4}{3}\pi r^3 = \frac{1}{3}\pi R^2 h$$

$$\Rightarrow \quad 4r^3 = R^2 h$$

$$\Rightarrow \quad 4 \times (3\text{ cm})^3 = R^2 \times 3\text{ cm}$$

$$\Rightarrow \quad R^2 = \frac{4 \times 27\text{ cm}^3}{3\text{ cm}} = 36$$

$$\Rightarrow \quad R = 6\text{ cm}.$$

35. (a) **8 : 1**

Let r be the radius of the original balloon and R be the radius of the inflated balloon. It is given that the spherical balloon grows the twice its radius when inflated. Then,

$$R = 2r$$

$$\frac{\text{Volume of the inflated balloon}}{\text{Volume of the original balloon}}$$

$$= \frac{\frac{4}{3}\pi R^3}{\frac{4}{3}\pi r^3} = \frac{(2r)^3}{r^3} = 8.$$

Hence, the ratio of the volume of the inflated balloon to the original balloon is 8 : 1.

36. (c) **28 cm**

Let r be the radius of the hemisphere.

Total surface area of the hemisphere = $3\pi r^2$

$$\Rightarrow \quad 1848\text{ cm}^2 = 3 \times \frac{22}{7} \times r^2$$

$$\Rightarrow \quad r^2 = \frac{1848\text{ cm}^2 \times 7}{66} = 196\text{ cm}^2$$

$$\Rightarrow \quad r = 14\text{ cm}$$

Thus, diameter of the hemisphere

$$= 2r = 2 \times 14\text{ cm} = 28\text{ cm}.$$

37. (c) $\pi r(4r + l)$

Let R be the radius of the cone. Then, R = $2r$.

Now, the slant height of the cone is

$$l = \frac{l}{2}$$

Thus, total surface area of the cone = $\pi R(L + R)$

$$= \pi(2r)\left(\frac{l}{2} + 2r\right)$$

$$= 2\pi r\left(\frac{l + 4r}{2}\right)$$

$$= \pi r(4r + l).$$

38. (a) **374 m²**

Let r be the radius and l be the slant height of the cone. Then, $r = 7$ m, $l = 10$ m.

Total surface of the cone $= \pi r(l + r)$

$$= \frac{22}{7} \times 7 \text{ m} \times (7 \text{ m} + 10 \text{ m})$$

$$= 374 \text{ m}^2.$$

39. (c) **314 cm²**

Let r be the radius and h be the height of the cone.

Then, $h = 15$ cm

Base area of the cone $= \pi r^2$

$$\text{Volume of the cone} = \frac{1}{3}\pi r^2 h$$

$$\Rightarrow \quad 1570 \text{ cm}^3 = \frac{1}{3} \times \pi r^2 \times 15 \text{ cm}$$

$$\Rightarrow \quad \pi r^2 = \frac{1570 \text{ cm}^3 \times 3}{15 \text{ cm}}$$

$\Rightarrow$ Base area of the cone $= 314$ cm².

40. (b) **24 cm**

Let r be the radius and h be the slant height of the cone.

Then, $r = 7$ cm, $l = 25$ cm

Now, the height of the cone is

$$h = \sqrt{l^2 - r^2} = \sqrt{(25 \text{ cm})^2 - (7 \text{ cm})^2}$$

$$= \sqrt{625 \text{ cm}^2 - 49 \text{ cm}^2}$$

$$= \sqrt{576 \text{ m}^2} = 24 \text{ m}$$

41. (b) **14 cm**

Let r be the radius and h be the height of the cone.

Then, $h = 15$ cm.

$$\text{Volume of the cone} = \frac{1}{3}\pi r^2 h$$

$$\Rightarrow \quad 770 \text{ cm}^3 = \frac{1}{3} \times \frac{22}{7} \times r^2 \times 15 \text{ cm}$$

$$\Rightarrow \quad r^2 = \frac{770 \text{ cm}^3 \times 21}{22 \times 15 \text{ cm}} = 49 \text{ cm}^2$$

$$\Rightarrow \quad r = 7 \text{ cm}$$

Thus, diameter $= 2r = 2 \times 7$ cm $= 14$ cm.

42. (b) **8800**

Let r be the radius and h be the height of the conical tent.

Then, $r = \frac{4 \text{ m}}{2} = 2$ m, $h = 21$ m.

Volume of the conical tent

$$= \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3} \times \frac{22}{7} \times (2 \text{ m})^2 \times (21 \text{ m})$$

$$= 88 \text{ m}^3 = 88 \times 10^3 \text{ dm}^3$$

Average number of cubic dm of air space per man

$$= \frac{88 \times 10^3 \text{ dm}^3}{10}$$

$$= 8800 \text{ dm}^3 \text{ per man}$$

43. (d) **624 m**

Let r be the radius and h be the height of the conical pandal. Then, $r = 240$ m and $h = 100$ m.

The slant height of the conical pandal is

$$l = \sqrt{(240 \text{ m})^2 + (100 \text{ m})^2}$$

$$= \sqrt{67600} \text{ m} = 260 \text{ m}.$$

Let L and b be the length and width of the rectangular cloth respectively. Then, $b = 100\pi$ m.

Now, Curved surface area of the conical pandal

= Area of the rectangular cloth

$$\Rightarrow \quad \pi rl = Lb$$

$$\Rightarrow \quad \pi \times 240 \text{ m} \times 260 \text{ m} = L \times 100\pi \text{ m}$$

$$\Rightarrow \quad L = \frac{240 \text{ m} \times 260 \text{ m}}{100 \text{ m}}$$

$$= 624 \text{ m}.$$

44. (c) **2 : 1**

Let r be the radius of the cylinder and cone.

Let l be the slant height of the cone and H be the height of the cylinder.

Curve surface area of the cylinder

= Curve surface area of the cone

$$\Rightarrow \quad 2\pi r H = \pi r l$$

$$\Rightarrow \quad \frac{l}{H} = 2$$

Thus, the ratio of the slant height of the cone to the height of the cylinder is 2 : 1.

45. (d) **3 : 1**

Let r_1 and r_2 be the ratio radii of the two cones.

Let h_1 and h_2 be the heights of the two cones.

Then,

$$\frac{r_1}{r_2} = \frac{3}{1} \text{ and } \frac{h_1}{h_2} = \frac{1}{3}.$$

$$\text{Now,} \quad \frac{\text{Volume of the 1}^{\text{st}} \text{ cone}}{\text{Volume of the 2}^{\text{nd}} \text{ cone}}$$

$$= \frac{\frac{1}{3}\pi r_1^2 h_1}{\frac{1}{3}\pi r_2^2 h_2} = \left(\frac{r_1}{r_2}\right)^2 \times \left(\frac{h_1}{h_2}\right)$$

$$= \left(\frac{3}{1}\right)^2 \times \left(\frac{1}{3}\right) = 3$$

Hence, the ratio of the volumes of the two cones is 3 : 1.

46. (b) **₹ 562.50**

Volume of the pit $= 4.5 \text{ m} \times 2.5 \text{ m} \times 2.5 \text{ m}$

$$= 28.125 \text{ m}^3$$

$\therefore$ Cost of digging the pit = 28.125 m^3 × ₹ 20
= ₹ 562.50

47. (c) **432 cm^3**

When two cubes are joined to form a cuboid, the resulting cuboid has length equal to two times the side of the cube. The breadth and height of the cuboid equal to the breadth and height of the cube.

Let l, b, h be the length, breadth and height of the cuboid respectively. Then,

$$l = 2 \times \text{side of the cube} = 2 \times 6 \text{ cm} = 12 \text{ cm}$$
$$b = \text{side of the cube} = 6 \text{ cm}$$
$$h = \text{side of the cube} = 6 \text{ cm.}$$

Volume of the cuboid $= l \times b \times h$
$= 12 \text{ cm} \times 6 \text{ cm} \times 6 \text{ cm}$
$= 432 \text{ cm}^3$

48. (a) **135000 L**

Volume of the cuboidal water tank
$= 6 \text{ m} \times 5 \text{ m} \times 4.5 \text{ m}$
$= 135 \text{ m}^3$

Now, 1 litre of water = 10^{-3} m^3

Number of litres that can be hold by the cuboidal water tank

$$= \frac{135 \text{ m}^3}{10^{-3} \text{ m}^3/\text{litres}}$$

= 135000 litres

49. (a) **1300 cm^2**

Let l, b, h be the length, breadth and height of the cuboid respectively. Then,

$$l = 15 \text{ cm}, b = 10 \text{ cm}, h = 20 \text{ cm}$$

Surface area of the cuboid
$= 2(lb + bh + lh)$
$= 2(15 \text{ cm} \times 10 \text{ cm} + 10 \text{ cm} \times 20 \text{ cm} + 15 \text{ cm} \times 20 \text{ cm})$
$= 2(150 \text{ cm}^2 + 200 \text{ cm}^2 + 300 \text{ cm}^2)$
$= 1300 \text{ cm}^2$.

50. (a) **400 cm^2**

Let l, b and h be the length, breadth and height of the cuboid respectively. Then,

$$l = 20 \text{ cm}, b = 10 \text{ cm and } h = 40 \text{ cm}$$

Now, Total surface area of the cuboid
– Lateral surface area of the cuboid
$= 2(lb + bh + hl) - 2(l + b)h$
$= 2(20 \text{ cm} \times 10 \text{ cm} + 10 \text{ cm} \times 40 \text{ cm} + 40 \text{ cm} \times 20 \text{ cm}) - 2(20 \text{ cm} + 10 \text{ cm})\, 40 \text{ cm}$
$= 2800 \text{ cm}^2 - 2400 \text{ cm}^2 = 400 \text{ cm}^2$.

51. (b) **34650 cm^3**

Let r be the base radius and h be the height of the cylinder.

Then, $h = 25$ cm.

Circumference of the base of the cylinder $= 2\pi r$

$\Rightarrow$ 132 cm $= 2\pi r$

$$\Rightarrow \quad r = \frac{132 \text{ cm}}{2\pi}$$

Now, Volume of the cylinder $= \pi r^2 h$

$$= \pi \times \left(\frac{132 \text{ cm}}{2\pi}\right)^2 \times 25 \text{ cm}$$

$$= \frac{(132 \text{ cm})^2 \times 25 \text{ cm}}{4 \times \frac{22}{7}}$$

$= 34650 \text{ cm}^3$

52. (a) **1 : 3**

Let r_1 and r_2 be the radii of the cylinder and cone respectively. Let h_1 and h_2 be the heights of the cylinder and the cone respectively. Then, $\frac{r_1}{r_2} = 1$.

Now, Volume of the cylinder = Volume of the cone

$$\Rightarrow \quad \pi r_1^2 h_1 = \frac{1}{3}\pi r_2^2 h_2$$

$$\Rightarrow \quad \frac{h_1}{h_2} = \frac{1}{3}\left(\frac{r_2}{r_1}\right)^2$$

$$\Rightarrow \quad \frac{h_1}{h_2} = \frac{1}{3}$$

Hence, the ratio of the height of the cylinder to the height of the cone is 1 : 3.

53. (c) **2640 cm^3**

Let r be the radius and h be the height of the right circular cylinder.

Area of the base of the right circular cylinder
$= 15400 \text{ cm}^2$

$\Rightarrow \quad \pi r^2 = 15400 \text{ cm}^2$

$$\Rightarrow \quad r = \sqrt{\frac{15400 \text{ cm}^2}{\frac{22}{7}}} = 70 \text{ cm}$$

Volume of the right circular cylinder $= \pi r^2 h$

$\Rightarrow \quad 92400 \text{ cm}^3 = \pi r^2 \times h$

$$\Rightarrow \quad h = \frac{92400 \text{ cm}^3}{\pi r^2} = \frac{92400 \text{ cm}^3}{15400 \text{ cm}^2} = 6 \text{ cm}$$

Now, Curved surface area of the right circular cylinder
$= 2\pi rh$

$$= 2 \times \frac{22}{7} \times 70 \text{ cm} \times 6 \text{ cm}$$

$= 2640 \text{ cm}^2$

54. (b) $\mathbf{\frac{5280}{7}}$ **cm^2**

Let r be the radius and h be the height of the cylinder.

Then, $h = 7$ cm.

Volume of the cylinder $= \pi r^2 h$

$\Rightarrow \quad 448\pi \text{ cm}^3 = \pi r^2 \times 7 \text{ cm}$

$$\Rightarrow \quad r^2 = \frac{448 \text{ cm}^3}{7 \text{ cm}} = 64$$

$\Rightarrow \quad r = 8$ cm.

Total surface area of the cylinder

$$= 2\pi r(r + h)$$

$$= 2 \times \frac{22}{7} \times 8 \text{ cm} \times (8 \text{ cm} + 7 \text{ cm})$$

$$= \frac{5280}{7} \text{ cm}^2$$

55. (d) **462 m³**

Let r be the base radius and h be the height of the conical tent. Then, $h = 9$ m.

Circumference of the base of the conical tent = $2\pi r$

$\Rightarrow \quad 44 \text{ m} = 2\pi r$

$\Rightarrow \quad r = \frac{44 \text{ m}}{2\pi}$

Volume of air contained in the conical tent is equal to the volume of the conical tent.

$\therefore$ Volume of air contained in the conical tent

$$= \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3} \times \pi \times \left(\frac{44 \text{ m}}{2\pi}\right)^2 \times 9 \text{ m}$$

$$= \frac{1}{3} \times \frac{(44 \text{ m})^2}{4 \times \frac{22}{7}} \times 9 \text{ m}$$

$$= 462 \text{ m}^3$$

56. (b) **3.85 L**

Let r be the radius of the conical glass and h be the depth of the conical glass.

Then, $r = \frac{35}{2} \text{ cm} = 17.5 \text{ cm}$

$h = 12$ cm

Now, Volume of the conical glass

$$= \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3} \times \frac{22}{7} \times (17.5 \text{ cm})^2 \times 12 \text{ cm}$$

$$= 3850 \text{ cm}^3$$

$= 3.85$ litres. [Using 1 cm³ = 10^{-3} litres]

57. (b) **704 cm²**

The area of the metal sheet which is required to make the closed hollow cone is equal to the total surface area of the cone.

Let r and h be the radius and height of the cone respectively. Then, $r = 7$ cm, $h = 24$ cm.

Now, Slant height of the cone is

$$l = \sqrt{h^2 + r^2} = \sqrt{(24 \text{ cm})^2 + (7 \text{ cm})^2}$$

$$= \sqrt{625 \text{ cm}^2} = 25 \text{ cm}$$

Total surface area of the cone = $\pi r(l + r)$

$$= \frac{22}{7} \times 7 \text{ cm} \times (25 \text{ cm} + 7 \text{ cm})$$

$$= 704 \text{ cm}^2$$

58. (a) **550 m²**

Let r and h be the base radius and height of the conical tent. Then, $r = 7$ cm, $h = 24$ cm.

The area of canvas required is equal to the curved surface area of the conical tent.

$$l = \sqrt{h^2 + r^2} = \sqrt{(24 \text{ m})^2 + (7 \text{ m})^2}$$

$$= \sqrt{625 \text{ m}^2} = 25 \text{ m}$$

Thus, Area of the canvas

$$= \pi rl = \frac{22}{7} \times 7 \text{ m} \times 25 \text{ m} = 550 \text{ m}^2$$

59. (b) **25.344 kg-wt**

Let r and h be the radius and depth of the conical vessel respectively. Then, $r = \frac{48 \text{ cm}}{2} = 24$ cm and $h = 42$ cm.

Now,

Volume of the conical vessel = $\frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times \frac{22}{7} \times (24 \text{ cm})^2 \times (42 \text{ cm})$$

$$= 25344 \text{ cm}^3$$

$= 25.344$ dm³ [Using 1 cm³ = 10^{-3} dm³]

Weight of water in the conical vessel

$$= (25.344 \text{ dm}^3) \times \left(\frac{1 \text{ kg-wt}}{1 \text{ dm}^3}\right)$$

$= 25.344$ kg-wt.

60. (b) **60 cm**

Let r and h be the radius and height of the conical vessel respectively. Then, $h = 3.5$ cm.

Now, Capacity of the milk

$= 3.3$ litres [Using 1 litres = 10^3 cm³]

$= 3300$ cm³

It is given that the capacity of the milk occupied the conical vessel. Then,

Volume of the conical vessel = Capacity of the milk

$\Rightarrow \quad \frac{1}{3}\pi r^2 h = 3300 \text{ cm}^3$

$\Rightarrow \quad r^2 = \frac{3300 \text{ cm}^3 \times 3}{\pi \times h}$

$$= \frac{3300 \text{ cm}^3 \times 3}{\frac{22}{7} \times 3.5 \text{ cm}} = 900 \text{ cm}^2$$

$\Rightarrow \quad r = 30$ cm.

Thus, Diameter = $2r = 2 \times 30$ cm = 60 cm.

61. (b) **1386 cm²**

Let r be the radius of the sphere.

Thus,

Volume of the sphere = $\frac{4}{3}\pi r^3$

$$\Rightarrow \quad 4851 \text{ cm}^3 = \frac{4}{3}\pi r^3$$

$$\Rightarrow \quad r^3 = \frac{4851 \text{ cm}^3 \times 3}{4 \times \frac{22}{7}}$$

$$\Rightarrow \quad r^3 = 1157.625 \text{ cm}^3$$

$$\Rightarrow \quad r = 10.5 \text{ cm}$$

Now,

Surface area of the sphere = $4\pi r^2$

$$= 4 \times \frac{22}{7} \times (10.5 \text{ cm})^2$$

$$= 1386 \text{ cm}^2$$

62. (a) **77 cm²**

Let R and r be the outer and inner radii of the hemispherical bowl respectively. Then, $r = 3.25$ cm

Also,

Thickness of the hemispherical bowl = R – r

$\Rightarrow$ 0.25 cm = R – 3.25 cm

$\Rightarrow$ R = 3.50 cm

Now, Outer curved surface area of the hemispherical bowl

$$= 2\pi R^2 = 2 \times \frac{22}{7} \times (3.50 \text{ cm})^2 = 77 \text{ cm}^2$$

63. (b) **1039.5 cm²**

Let r be the radius of the solid sphere.

Now, Surface area of the solid sphere = $4\pi r^2$

$$\Rightarrow \quad 1386 \text{ cm}^2 = 4\pi r^2$$

$$\Rightarrow \quad r^2 = \frac{1386 \text{ cm}^2}{4 \times \frac{22}{7}}$$

$$\Rightarrow \quad r^2 = 110.25 \text{ cm}^2$$

$$\Rightarrow \quad r = 10.5 \text{ cm}$$

Then, total surface area of the solid hemisphere = $3\pi r^2$

$$= 3 \times \frac{22}{7} \times (10.5 \text{ cm})^2 = 1039.5 \text{ cm}^2$$

64. (a) **8400**

Let l, b and h be the length, breadth and height of the rectangular block respectively. Then, l = 11 m, b = 10 m, h = 5 m.

Let r be the radius of the spherical bullet.

Then, $r = \frac{5\,dm}{2} = 2.5\ dm = 0.25$ m.

Now, number of spherical bullets

$$= \frac{\text{Volume of the rectangular block}}{\text{Volume of the spherical bullet}}$$

$$= \frac{l \times b \times h}{\frac{4}{3}\pi r^3} = \frac{11 \text{ m} \times 10 \text{ m} \times 5}{\frac{4}{3} \times \frac{22}{7} \times (0.25 \text{ m})^3}$$

$$= 8400$$

65. (d) **5**

Let r and h be the radius and height of the solid cylinder respectively. Then, $r = \frac{4 \text{ cm}}{2} = 2$ cm, h = 45 cm.

Let R be the radius of the solid sphere.

Then,

$$R = \frac{6 \text{ cm}}{2} = 3 \text{ cm}$$

Now, number of solid spheres

$$= \frac{\text{Volume of the cylinder}}{\text{Volume of the sphere}}$$

$$= \frac{\pi r^2 h}{\frac{4}{3}\pi R^3} = \frac{(2 \text{ cm})^2 \times 45 \text{ cm}}{\frac{4}{3} \times (3 \text{ cm})^3}$$

$$= \frac{180}{36} = 5.$$

66. (a) **14 cm**

Let R and r be the lateral and internal diameters of the hollow sphere respectively.

Then,

$$R = \frac{8 \text{ cm}}{2} = 4 \text{ cm and } r = \frac{4 \text{ cm}}{2} = 2 \text{ cm.}$$

Let r_1 and h be the base radius and height of the cone respectively. Then, $r_1 = \frac{8 \text{ cm}}{2} = 4$ cm.

Now, volume of the hollow sphere

= Volume of the cone

$$\frac{4}{3}\pi(R^3 - r^3) = \frac{1}{3}\pi r_1^2 h$$

$$\Rightarrow \quad h = \frac{4(R^3 - r^3)}{r_1^2}$$

$$= \frac{4 \times [(4 \text{ cm})^3 - (2 \text{ cm})^3]}{(4 \text{ cm})^2}$$

$$= \frac{64 - 8}{4} \text{ cm} = 14 \text{ cm}$$

67. (a) **32r**

Let h be the height of the cone.

Now, volume of the sphere = Volume of the cone

$$\Rightarrow \quad \frac{4}{3}\pi(2r)^3 = \frac{1}{3}\pi r^2 h$$

$$\Rightarrow \quad 4 \times 8r^3 = r^2 h$$

$$\Rightarrow \quad h = 32r$$

68. (a) **7 cm**

Let r and l be the radius and slant height of the cone respectively. Then,

$$\frac{r}{l} = \frac{7}{13}$$

$$\Rightarrow \quad l = \frac{13}{7}r$$

Now, Curved surface area of the cone = πrl

$$\Rightarrow \quad 286 \text{ cm}^2 = \pi rl$$

$$\Rightarrow \quad 286 \text{ cm}^2 = \pi r\left(\frac{13}{7}\right)r$$

$$\Rightarrow \quad r^2 = \frac{286 \text{ cm}^2 \times 7}{\frac{22}{7} \times 13}$$

$$\Rightarrow \quad r^2 = 49 \text{ cm}^2$$

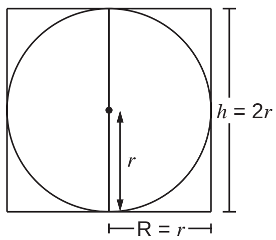
$$\Rightarrow \quad r = 7 \text{ cm}$$

69. (b) $\mathbf{4\pi r^2}$

Let R and h be the radius and height of the right circular cylinder respectively.

As the cylinder just uncloses the sphere, then

R = r and $h = 2r$.



Now, curved surface area of the right circular cylinder

$= 2\pi Rh$

$= 2\pi r\ (2r) = 4\pi r^2$.

70. (c) $\mathbf{\frac{4}{3}\pi\left(\frac{r^3}{8}\right)}$

It is given that the radius of the sphere is r.

If the radius is reduced to half, then the new volume is given by

$$\text{New volume of the sphere} = \frac{4}{3}\pi\left(\frac{r}{2}\right)^2$$

$$= \frac{4}{3}\pi\left(\frac{r^3}{8}\right).$$

SHORT ANSWER QUESTIONS

1. Let $3x$, $4x$ and $5x$ be the lengths of the edges of the three given cubes. Let a be the length of the edge of the single cube.

Now,

$$\text{Diagonal of the single cube} = 12\sqrt{3} \text{ cm}$$

$$\Rightarrow \quad \sqrt{3}a = 12\sqrt{3} \text{ cm}$$

$$\Rightarrow \quad a = 12 \text{ cm.}$$

It is given that the three small cubes are melted to form the large single cube.

$\therefore$ Sum of the volumes of the three cubes

= volume of the single cube.

$$\Rightarrow \quad (3x)^3 + (4x)^3 + (5x)^3 = a^3$$

$$\Rightarrow \quad 27x^3 + 64x^3 + 125x^3 = (12 \text{ cm})^3$$

$$\Rightarrow \quad x^3 = \frac{1728}{216} = 8$$

$$\Rightarrow \quad x = 2$$

Hence, the edges of the three cubes are $3x$, $4x$ and $5x$, i.e., 3×2, 4×2, 5×2, i.e., 6 cm, 8 cm, 10 cm.

2. Let l, b and h be the length, width and height of the cuboid respectively. Then, $l = x$, $b = 2x$ and $h = 4x$.

Let a be the length of the edge of the cube.

Now, Volume of the cube = Volume of the cuboid

$$\Rightarrow \quad a^3 = x \times 2x \times 4x$$

$$\Rightarrow \quad a^3 = 8x^3$$

$$\Rightarrow \quad a = 2x$$

It is given that the difference between the cost of polishing the cuboid and cube at the rate of ₹ 5 per m^2 is ₹ 80. Thus,

Cost of polishing the cuboid = ₹ 5 per m^2 × Surface area of the cuboid.

= ₹ 5 per $m^2 \times 2(lb + bh + lh)$

= ₹ 5 per $m^2 \times 2(x \times 2x + 2x \times 4x + x \times 4x)$

$= 10(14x^2) = 140x^2$

Cost of polishing the cube

= ₹ 5 per $m^2 \times (6a^2)$

$= 30 \times (2x)^2$

$= 120x^2$

Now, cost of polishing the cuboid

− cost of polishing the cube = 80

$$\Rightarrow \quad 140x^2 - 120x^2 = 80$$

$$\Rightarrow \quad 20x^2 = 80$$

$$\Rightarrow \quad x^2 = 4$$

$$\Rightarrow \quad x = 2.$$

$\therefore$ Volume of the cuboid = Volume of the cube = lbh

$= x \times 2x \times 4x$

$= 8x^3$

$= 8 \times (2)^3$ [Using $x = 2$]

$= 64 \text{ m}^3$

Hence, the volume of the cuboid and cube are 64 m^3 each.

3. Let r be the radius and h be the height of the solid right circular cylinder. Then, $r = 7$ cm.

It is given that 100 circular plates made the solid right circular cylinder. Then, $h = 100 \times \frac{1}{4}$ cm = 25 cm.

Now, total surface area of the solid right circular cylinder

$= 2\pi r(r + h)$

$= 2 \times \frac{22}{7} \times 7 \text{ cm} \times (7 \text{ cm} + 25 \text{ cm})$

$= 1408 \text{ cm}^2$.

and

Volume of the solid right circular cylinder = $\pi r^2 h$

$= \frac{22}{7} \times (7 \text{ cm})^2 \times 25 \text{ cm}$

$= 3850 \text{ cm}^3$.

Hence, surface area of the solid right circular cylinder is 1408 cm^2 and volume of the solid right circular cylinder is 3850 cm^3.

4. Let r be the radius and h be the depth of the cylindrical hole. Then, $r = \frac{1\text{ m}}{2} = 0.5$ m.

Now,

Cost to bore a cylindrical hole

$= ₹\,100$ per m^3 × Volume of the cylindrical hole

$\Rightarrow$ ₹ 2200 = ₹ 100 per $m^3 \times \pi r^2 h$

$$\Rightarrow \quad 2200 = 100 \times \frac{22}{7} \times (0.5\text{ m})^2 \times h$$

$$\Rightarrow \quad h = \frac{2200 \times 7}{100 \times 22 \times (0.5)^2}\text{ m}$$

$= 28$ m.

Hence, the height of the cylindrical hole is 28 m.

5. Let r be the radius and h be the width of the road roller. Then, $r = \frac{0.7\text{ m}}{2} = 0.35$ m and $h = 1.2$ m.

Now,

Number of revolutions made by the road roller

$$= \frac{\text{Surface area of the playground}}{\text{Surface area of the road roller}}$$

$$= \frac{120\text{ m} \times 44\text{ m}}{2\pi rh}$$

$$= \frac{5280\text{ m}^2}{2 \times \frac{22}{7} \times 0.35\text{ m} \times 1.2\text{ m}}$$

= 2000 revolutions.

Hence, the least number of revolutions made by the road roller is 2000 revolutions.

6. Let r be the radius of the right circular cylinder. Also, the radius of the hemispherical bottom is equal to the radius of the cylinder. Then, $r = 3.5$ cm. Let h be the level of the water.

Thus, Volume of the water

– Volume of the hemispherical bottom

$$= \frac{1309}{6}\text{ cm}^3$$

$$\Rightarrow \quad \pi r^2 h - \frac{2}{3}\pi r^3 = \frac{1309}{6}\text{ cm}^3$$

$$\Rightarrow \quad \frac{22}{7} \times (3.5\text{ cm})^2 \times h - \frac{2}{3} \times \frac{22}{7} \times (3.5\text{ cm})^3$$

$$= \frac{1309}{6}\text{ cm}^3$$

$$\Rightarrow \quad h = \frac{\left(\frac{1309}{6} + \frac{1886.5}{21}\right)}{\left(\frac{269.5}{7}\right)}\text{ cm}$$

$$= \frac{(27489 + 11319) \times 7}{6 \times 21 \times 269.5}\text{ cm}$$

$$= \frac{271656}{33957}\text{ cm} = 8\text{ cm}.$$

Hence, the level of the water is 8 cm.

7. Let r be the radius of the circular pipe.

Then, $\quad r = \frac{2\text{ cm}}{2} = 1\text{ cm} = 0.01$ m.

Volume rate of the water flow from the circular pipe is

$V = (0.8\text{ m/s}) \times$ area of the circular pipe

$= (0.8\text{ m/s}) \times \pi r^2$

$= (0.8\text{ m/s}) \times \pi \times (0.01\text{ m})^2$

Let R and h be the radius and height of the cylindrical tank respectively. Then, R = 40 cm = 0.4 m.

It is given that the water from the circular pipe flows through the cylindrical tank for one and a half hour.

Thus, volume of the water in the cylindrical tank

= volume rate of the water flow from the circular pipe × (90 × 60 s)

$$\Rightarrow \quad \pi R^2 h = (0.8\text{ m/s})\, \pi (0.01\text{ m})^2 \times 5400\text{ s}$$

$$\Rightarrow \quad (0.4\text{ m})^2 h = (0.8\text{ m/s}) \times (0.01\text{ m})^2 \times 5400\text{ s}$$

$$\Rightarrow \quad h = \frac{0.8\text{ m/s} \times (0.01\text{ m})^2 \times 5400\text{ s}}{(0.4\text{ m})^2} = 2.7\text{ m}$$

Hence, the level of the water in the cylindrical tank is 2.7 m.

8. Let r and h be the radius and height of the right circular cone respectively. Then,

$$\frac{r}{h} = \frac{5}{12}$$

$$\Rightarrow \quad h = \frac{12}{5}r \quad \ldots(1)$$

Now, Volume of the cone $= \frac{17600}{7}\text{ m}^3$

$$\Rightarrow \quad \frac{1}{3}\pi r^2 h = \frac{17600}{7}\text{ m}^3$$

$$\Rightarrow \quad \frac{1}{3}\pi \times \frac{22}{7} \times \frac{12}{5} r \times r^2 = \frac{17600}{7} \quad \text{[Using equation (1)]}$$

$$\Rightarrow \quad r^3 = \frac{17600 \times 7 \times 3 \times 5}{22 \times 7 \times 12}\text{ m}^3$$

$$\Rightarrow \quad = 1000\text{ m}^3$$

$$\Rightarrow \quad r = 10\text{ m}$$

From Eq. (1),

$$h = \frac{12}{5}r = \frac{12}{5} \times 10\text{ m}$$

= 24 m

Now, slant height of the circular cone is

$$l = \sqrt{r^2 + h^2}$$

$$= \sqrt{(10\text{ m})^2 + (24\text{ m})^2}$$

$$= \sqrt{676\text{ m}^2} = 26\text{ m}$$

Hence, slant height of the cone is 26 m and its radius is 10 m.

9. Let r be the radius of the semi-circular sheet of metal. Then, $r = 14$ cm.

As the metal sheet is bend into an open conical cup. The radius of the semi-circular metal sheet is equal to the slant height of the conical cup.

Thus, slant height, $l = r = 14$ cm.

Let R and h be the base radius and depth of the conical cup respectively. Then, the radius is equal to half the radius of the semi-circular sheet.

Thus, $R = \frac{r}{2} = \frac{14 \text{ cm}}{2} = 7$ cm.

Now, depth of the conical cup is

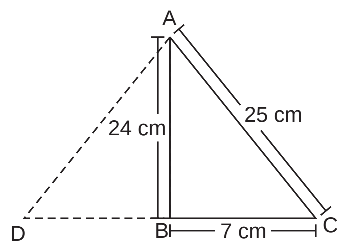
$$h = \sqrt{l^2 - R^2}$$
$$= \sqrt{(14 \text{ cm})^2 - (7 \text{ cm})^2}$$
$$= \sqrt{196 \text{ cm}^2 - 49 \text{ cm}^2}$$
$$= \sqrt{147} \text{ cm}$$
$$= 7\sqrt{3} \text{ cm} \qquad [\text{Using } \sqrt{3} = 1.73]$$
$$= 7 \times 1.73 \text{ cm}$$
$$= 12.11 \text{ cm}$$

Also, volume of the conical cup

$$= \frac{1}{3}\pi R^2 h$$
$$= \frac{1}{3} \times \frac{22}{7} \times (7 \text{ cm})^2 \times (12.11 \text{ cm})$$
$$= 621.64 \text{ cm}^3 \text{ (approx.)}.$$

Hence, the depth of the conical cup is 12.11 cm and its volume is 621.64 cm^3.

10. It is given that the right triangle is revolved about the side 24 cm. Thus, it forms a right circular cone with radius 7 cm and height 24 cm.



Let r and h be the radius and height of the right circular cone respectively.

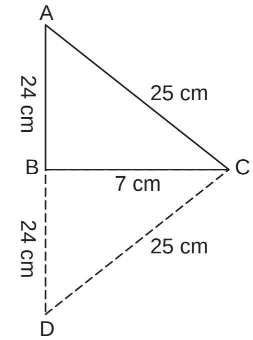
Then, $r = 7$ cm and $h = 24$ cm

Now, Volume of the cone $= \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times \frac{22}{7} \times (7 \text{ cm})^2 \times 24 \text{ cm}$$
$$= 1232 \text{ cm}^3$$

Hence, the volume of the cone so formed by the triangle is 1232 cm^3.

If the triangle is revolved about the side 7 cm, then it again forms a right circular cone with radius 24 cm and height 7 cm.



Let r and h be the radius and height of the right circular cone respectively.

Then, $r = 24$ cm and $h = 7$ cm

$\therefore$ Volume of cone $= \frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times \frac{22}{7} \times 24 \text{ cm} \times 24 \text{ cm} \times 7 \text{ cm}$$
$$= 4224 \text{ cm}^3$$

Hence, volume of the cone so formed by the triangle is 4224 cm^3.

11. Let r be the radius of the larger ladoo.

Then, $r = \frac{12 \text{ cm}}{2} = 6$ cm.

Let R be the radius of the smaller ladoo.

It is given that 8 smaller spherical ladoos are made from the larger. Then,

Volume of the larger ladoo

= 8 × Volume of the smaller ladoo

$$\Rightarrow \quad \frac{4}{3}\pi r^3 = 8 \times \frac{4}{3}\pi R^3$$

$$\Rightarrow \quad R^3 = \frac{r^3}{8} = \frac{(6 \text{ cm})^3}{8} = 27 \text{ cm}^3$$

$$\Rightarrow \quad R = 3 \text{ cm}$$

Hence, the radius of the smaller ladoos is 3 cm.

12. Let m_1 and m_2 be the masses of the larger sphere and smaller sphere respectively. Then,

$$m_1 = 3 \text{ kg} = 3000 \text{ g and } m_2 = 375 \text{ g}$$

The densities of the two metal solid spheres are same as they are of same material. Let r_1 and r_2 be the radii of the larger and smaller sphere respectively.

Then, $r_2 = \frac{5 \text{ cm}}{2} = 2.5$ cm

Density of larger sphere

= Density of smaller sphere

$$\Rightarrow \quad \frac{\text{Mass of the larger sphere}}{\text{Volume of the larger sphere}} = \frac{\text{Mass of the smaller sphere}}{\text{Volume of the smaller sphere}}$$

$$\Rightarrow \quad \frac{m_1}{\frac{4}{3}\pi r_1^3} = \frac{m_2}{\frac{4}{3}\pi r_2^3}$$

$$\Rightarrow \quad \frac{m_1}{m_2} = \left(\frac{r_1}{r_2}\right)^3$$

$$\Rightarrow \quad \frac{r_1}{r_2} = \sqrt[3]{\frac{m_1}{m_2}}$$

$$\Rightarrow \quad r_1 = \sqrt[3]{\frac{m_1}{m_2}} \times r_2$$

$$= \sqrt[3]{\frac{3000 \text{ g}}{375 \text{ g}}} \times 2.5 \text{ cm}$$

$$= \sqrt[3]{8} \times 2.5 \text{ cm}$$

$$= 2 \times 2.5 \text{ cm} = 5 \text{ cm}$$

Hence, the radius of the larger sphere is 5 cm.

13. Let r be the radius of the metal sphere. Then, $r = 2$ cm.

Now,

Volume of the preservative liquid

= Volume of the rectangular box

– 30 × Volume of the metal sphere

$$= 30 \text{ cm} \times 7.9 \text{ cm} \times 6 \text{ cm} - 30 \times \frac{4}{3}\pi \times r^3$$

$$= 1422 \text{ cm}^3 - 30 \times \frac{4}{3} \times 3.14 \times (2 \text{ cm})^3$$

$$= 1422 \text{ cm}^2 - 1004.8 \text{ cm}^3$$

$$= 417.2 \text{ cm}^3.$$

Hence, the volume of the preservative liquid is 417.2 cm^3.

14. Ler r_1 be the radius of the sphere. It is given that the radius in increased by 10%. Then, the new increased radius is

$$r_2 = r_1 + 10\% \text{ of } r_1 = r_1 + \frac{10}{100} r_1 = 1.1\ r_1.$$

Now,

Volume of the original sphere $= \frac{4}{3}\pi r_1^3$

Thus, increased volume of the sphere $= \frac{4}{3}\pi r_2^3$

$$= \frac{4}{3}\pi (1.1)^3 r_1^3$$

$$= 1.331 \times \frac{4}{3}\pi r_1^3$$

= 1.331 × Volume of the sphere.

Increased in the volume $= 1.331 \times \frac{4}{3}\pi r_1^3 - \frac{4}{3}\pi r_1^3$

$$= 0.331 \times \frac{4}{3}\pi r_1^3$$

Percentage increase in the volume

$$= \frac{\text{Increase in the volume}}{\text{Original volume}} \times 100\%$$

$$= \frac{0.331 \times \frac{4}{3}\pi r_1^3}{\frac{4}{3}\pi r_1^3} \times 100\% = 33.1\%$$

Hence, the volume of the sphere will be increased by 33.1%.

15. Let r_{cone} be the radius of the cone, r_{sphere} be the radius of the sphere and h be the height of the cone. Then,

$$r_{cone} = 4 \text{ cm and } r_{sphere} = 5 \text{ cm}$$

Let l be the slant height of the cone.

Now, surface area of the sphere

= 5 × area of the curved surface of the cone

$$4\pi r_{sphere}^2 = 5 \times \pi\ r_{cone}\ l$$

$$\Rightarrow \quad l = \frac{4 r_{sphere}^2}{5 \times r_{cone}}$$

$$= \frac{4 \times (5 \text{ cm})^2}{5 \times 4 \text{ cm}} = 5 \text{ cm}$$

Then,height of the cone,

$$h = \sqrt{l^2 - r_{cone}^2} = \sqrt{(5 \text{ cm})^2 - (4 \text{ cm})^2}$$

$$= \sqrt{24 \text{ cm}^2 - 16 \text{ cm}^2} = \sqrt{9 \text{ cm}^2}$$

$$= 3 \text{ cm}.$$

Volume of the cone $= \frac{1}{3}\pi r_{cone}^2\ h$

$$= \frac{1}{3} \times \frac{22}{7} \times (4 \text{ cm})^2 \times 3 \text{ cm}$$

$$= \frac{352}{7} \text{ cm}^3$$

Hence, the height of the cone is 3 cm and its volume is $\frac{352}{7}$ cm^3.

UNIT TEST

Multiple-Choice Questions

1. (b) **6 cm**

Given,Volume of the cube = 216 cm^3

$$\Rightarrow \quad (\text{edge})^3 = 216 \text{ cm}^3$$

$$\Rightarrow \quad \text{edge} = \sqrt[3]{216 \text{ cm}^3}$$

$$= 6 \text{ cm}$$

Hence, the edge of the cube is 6 cm.

2. (a) **4.2 m**

Let r be the radius of the sphere.

Surface area of the sphere = $4\pi r^2$

$$\Rightarrow \quad 55.44 \text{ m}^2 = 4\pi r^2$$

$$\Rightarrow \quad r^2 = \frac{55.44 \text{ m}^2}{4 \times \frac{22}{7}}$$

$$\Rightarrow \quad r^2 = 4.41 \text{ m}^2$$

$$\Rightarrow \quad r = 2.1 \text{ m}$$

Thus, diameter of the sphere = $2r$

= 2 × 2.1 m

= 4.2 m

3. (d) **27.5 L**

Let r be the radius and h be the internal depth of the conical vessel. Then,

$$r = \frac{50 \text{ cm}}{2} = 25 \text{ cm}$$

and $h = 42$ cm

Volume of the water in the conical vessel

$$= \frac{1}{3}\pi r^2 h$$

$$= \frac{1}{3} \times \frac{22}{7} \times (25 \text{ cm})^2 \times 42 \text{ cm}$$

$$= 27500 \text{ cm}^3 \quad [\text{Using } 1 \text{ L} = 10^{-3} \text{ cm}^3]$$

$$= 27.5 \text{ L}$$

4. (c) $\pi r\left(l + \frac{r}{4}\right)$

Let R and L be the radius and slant height of the cone respectively. Then,

$$R = \frac{r}{2} \text{ and } L = 2l$$

Total surface area of the cone

$$= \pi R(L + R)$$

$$= \pi \times \frac{r}{2} \times \left(2l + \frac{r}{2}\right)$$

$$= \pi \times \frac{r}{2} \times \frac{4l + 2}{2} = \pi r\left(l + \frac{r}{4}\right)$$

Short Answer Questions

5. Let r be the base radius and l be the slant height.

Then, $r = 3$ cm and $l = 5$ cm

$$\text{Height of the cone} = \sqrt{l^2 - r^2}$$

$$= \sqrt{(5 \text{ cm})^2 - (3 \text{ cm})^2}$$

$$= \sqrt{25 \text{ cm}^2 - 9 \text{ cm}^2}$$

$$= \sqrt{16 \text{ cm}^2}$$

$$= 4 \text{ cm}$$

Hence, the height of the cone is 4 cm.

6. Let r_1 and r_2 be the radii of the two right circular cones. Then,

$$\frac{r_1}{r_2} = \frac{3}{5}$$

Let h be the height of the two cones.

$$\text{Now, } \frac{\text{Volume of the 1}^{\text{st}} \text{ cone}}{\text{Volume of the 2}^{\text{nd}} \text{ cone}} = \frac{\frac{1}{3}\pi r_1^2 h}{\frac{1}{3}\pi r_2^2 h} = \left(\frac{r_1}{r_2}\right)^2$$

$$= \left(\frac{3}{5}\right)^2 = \frac{9}{25}$$

Hence, the ratio of the volumes of the two right circular cones is 9 : 25.

7. Let r be the radius of the hemispherical bowl.

Then, $r = 7$ cm.

The amount of water that the hemispherical bowl is equal to its volume.

$$\text{Volume of the hemispherical bowl} = \frac{2}{3}\pi r^3$$

$$= \frac{2}{3} \times \frac{22}{7} \times (7 \text{ cm})^3$$

$$= \frac{2156}{3} \text{ cm}^3$$

Hence, the amount of water that the hemispherical bowl can hold is $\frac{2156}{3}$ cm^3.

8. Let r be the radius and h be the depth of the tubewell. Then,

$$r = \frac{4 \text{ m}}{2} = 2 \text{ m and } h = 210 \text{ m.}$$

Volume of the tubewell = $\pi r^2 h$

$$= \frac{22}{7} \times (2 \text{ m})^2 \times 210 \text{ m}$$

$$= 2640 \text{ m}^3$$

Cost of sinking the tubewell

= Volume of the tubewell × ₹ 5.50 per m^3

= 2640 m^3 × ₹ 5.50 per m^3

= ₹ 14520

9. Given,Volume of the cuboid = Volume of the cube

$$\Rightarrow 36 \text{ cm} \times 75 \text{ cm} \times 80 \text{ cm} = (\text{edge})^3$$

$$\Rightarrow (\text{edge})^3 = 216000 \text{ cm}^3$$

$$\Rightarrow \text{edge} = \sqrt[3]{216000 \text{ cm}^3}$$

$$= 60 \text{ cm}$$

Hence, the edge of the cube is 60 cm.

10. Let r_1 and r_2 be the radii of the two spheres.

Then, $\frac{r_1}{r_2} = \frac{2}{5}$.

$$\text{Now, } \frac{\text{Surface area of the 1}^{\text{st}} \text{ sphere}}{\text{Surface area of the 2}^{\text{nd}} \text{ sphere}} = \frac{4\pi r_1^2}{4\pi r_2^2}$$

$$= \left(\frac{r_1}{r_2}\right)^2$$

$$= \left(\frac{2}{5}\right)^2$$

$$= \frac{4}{25}$$

Hence, the ratio between the surface areas of the two spheres is 4 : 25.

Short Answer Questions

11. Let r be the base radius and h be the height of the solid cone. Then, $h = 12$ m.

Circumference of the base = 22 m

$\Rightarrow \quad 2\pi r = 22$ m

$\Rightarrow \quad r = \frac{22 \text{ m}}{2\pi}$

Now, Volume of the solid cone

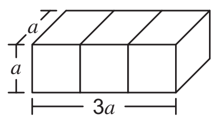
$$= \frac{1}{3}\pi r^2 h = \frac{1}{3} \times \pi \times \left(\frac{22 \text{ m}}{2\pi}\right)^2 \times 12 \text{ m}$$

$$= \frac{1}{3} \times \frac{(22 \text{ m})^2}{4 \times \pi} \times 12 \text{ m}$$

$$= \frac{(22)^2}{\frac{22}{7}} = 154 \text{ m}^3$$

Hence, the volume of the cone is 154 m^3.

12. The surface area of the cuboid formed by the three cubes has length of $3a$, breadth of a and height of a.



Let l, b and h be the length, breadth and height of the cuboid respectively. Then, $l = 3a$, $b = a$ and $h = a$.

$$\frac{\text{Surface area of the cuboid}}{\text{Surface area of the three cubes}} = \frac{2(lb + bh + lh)}{6a^2 + 6a^2 + 6a^2}$$

$$= \frac{2(3a \times a + a \times a + 3a \times a)}{18a^2}$$

$$= \frac{(3a^2 + a^2 + 3a^2)}{9a^2} = \frac{7}{9}.$$

Hence, the ratio of the surface area of the cuboid to the cubes is 7 : 9.

13. Let r be the radius and h be the width of the road roller. Then,

$$r = \frac{70 \text{ cm}}{2} = \frac{0.7 \text{ m}}{2} = 0.35 \text{ m}$$

and $\quad h = 1.2$ m

Number of revolutions made by the road roller

$$= \frac{\text{Surface area of the playground}}{\text{Surface area of the road roller}}$$

$$= \frac{120 \text{ m} \times 44 \text{ m}}{2\pi rh}$$

$$= \frac{5280 \text{ m}^2}{2 \times \frac{22}{7} \times 0.35 \text{ m} \times 1.2 \text{ m}}$$

= 2000 revolutions.

Hence, the least number of revolutions made by the road roller is 2000 revolutions.

14. Let r be the radius and h be the height of the cylindrical tank. Then,

$$h = \frac{3}{2}r.$$

Cost of polishing the total surface area of the closed cylindrical tank

= Total surface area of the cylindrical tank × 20 paise per dm^2

$\Rightarrow \quad$ ₹ 154 = $2\pi r(r + h)$ × ₹ 0.20 per dm^2

$\Rightarrow \quad$ ₹ 154 = $2\pi r\left(r + \frac{3}{2}r\right) \times 0.20$ [Using ₹ 1 = 100 paise]

$\Rightarrow \quad 154 = 2 \times \pi \times \frac{5}{2} \times r^2 \times 0.20$

$\Rightarrow \quad r^2 = \frac{154 \times 7}{22} = 49$

$\Rightarrow \quad r = 7$ dm

Now, height of the cylindrical tank is

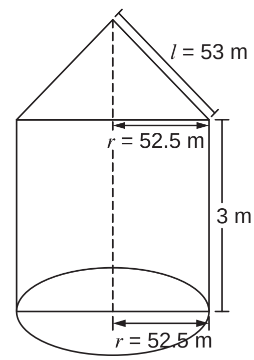
$$h = \frac{3}{2}r$$

$$= \frac{3}{2} \times 7 \text{ dm}$$

= 10.5 dm

Hence, the radius of the cylindrical tank is 7 dm and its height is 10.5 dm.

15. Let r be the radius and h be the height of the cylindrical portion. Then, $r = 52.5$ m and $h = 3$ m.



The radius of the conical part is equal to the radius of the conical portion.

Let l be the slant height of the conical portion.

Then, $\quad l = 53$ m

Area of the canvas needed to make the tent

= Curved surface area of the conical part + Curved surface area of the cylindrical part

$= \pi rl + 2\pi rh$

$= \pi r(l + 2h)$

$= \frac{22}{7} \times 52.5 \text{ m} \times (53 \text{ m} + 2 \times 3 \text{ m})$

$= \frac{22 \times 52.5 \text{ m} \times 59 \text{ m}}{7}$

= 9735 m^2

Hence, the area of the canvas needed to make the tent is 9735 m^2.

Long Answer Questions

16. Let m be the mass and r be the radius of the solid ball. Then, m = 4 Kg-wt, r = 3 cm.

$$\text{Density of the solid ball} = \frac{\text{Mass of the ball}}{\text{Volume of the ball}}$$

$$\Rightarrow \quad \rho = \frac{m}{\frac{4}{3}\pi r^3} \quad \ldots(1)$$

Let r_I and r_E be the internal and external radii of the spherical shell made from the material of the solid ball respectively. Then,

$$r_I = \frac{12 \text{ cm}}{2} = 6 \text{ cm}$$

$$r_E = \frac{24 \text{ cm}}{2} = 12 \text{ cm}$$

Weight of the spherical shell

= Density of the solid ball × Volume of the spherical shell

$$= \rho \times \frac{4}{3}\pi(r_E^3 - r_I^3)$$

$$= \frac{m}{\frac{4}{3}\pi r^3} \times \frac{4}{3}\pi(r_E^3 - r_I^3) \quad \text{[Using Eq. (1)]}$$

$$= \frac{m(r_E^3 - r_I^3)}{r^3}$$

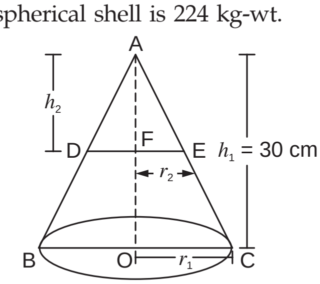
$$= \frac{4 \text{ kg-wt} \times [(12 \text{ cm})^3 - (6 \text{ cm})^3]}{(3 \text{ cm})^3}$$

$$= \frac{4 \times [1728 - 216]}{27} \text{ kg-wt}$$

= 224 kg-wt

Hence, the weight of the spherical shell is 224 kg-wt.

17. Let r_1 be the radius and h_1 be the height of the given original cone.



Then, h_1 = 30 cm.

It is given that DE cuts parallel to BC. Thus, a cone ADFE is formed with radius r_2 and height h_2.

Now, ΔAFE ~ ΔAOC. [∵ By AA similarity]

Then,

$$\frac{AF}{AO} = \frac{FE}{OC}$$

$$\Rightarrow \quad \frac{h_2}{h_1} = \frac{r_2}{r_1} \quad \ldots(1)$$

$\frac{1}{27}$ volume of the cone ABOC

= Volume of the cone ADFE

$$\Rightarrow \quad \frac{\text{Volume of the cone ADFE}}{\text{Volume of the cone ABOC}} = \frac{1}{27}$$

$$\Rightarrow \quad \frac{\frac{1}{3}\pi r_2^2 h_2}{\frac{1}{3}\pi r_1^2 h_1} = \frac{1}{27}$$

$$\Rightarrow \quad \left(\frac{r_2}{r_1}\right)^2\left(\frac{h_2}{h_1}\right) = \frac{1}{27} \quad \text{[Using Eq. (1)]}$$

$$\Rightarrow \quad \left(\frac{h_2}{h_1}\right)^2\left(\frac{h_2}{h_1}\right) = \frac{1}{27}$$

$$\Rightarrow \quad \left(\frac{h_2}{h_1}\right)^3 = \frac{1}{27}$$

$$\Rightarrow \quad \frac{h_2}{h_1} = \frac{1}{3}$$

$$\Rightarrow \quad h_2 = \frac{h_1}{3} = \frac{30 \text{ cm}}{3}$$

= 10 cm [Using h_1 = 30 cm]

Now, the height above which the section has been made above the base is

$$\Delta h = h_1 - h_2$$

= 30 cm − 10 cm

= 20 cm

Hence, the section has been made at a height of 20 cm from the base of the original cone.

18. Let r be the internal radius of the circular pipe.

Then, $r = \frac{2 \text{ cm}}{2} = 1 \text{ cm} = 0.01 \text{ m}$

The volume rate at which water flows from the circular pipe is

V = 6 m/s × Area of the circular pipe

= 6 m/s × πr^2

= 6 m/s × π × $(0.01 \text{ m})^2$

Let R be the base radius of the cylindrical tank.

Then, R = 60 cm = 0.6 m

Also, let h be the level of the water in the cylindrical tank. Then,

Volume of the water level

= Volume rate of the water flows from the circular pipe × 30 minutes

$$\Rightarrow \quad \pi R^2 h = 6 \text{ m/s} \times \pi \times (0.01 \text{ m})^2 \times 1800 \text{ s}$$

$$\Rightarrow \quad h = \frac{6 \text{ m/s} \times (0.01 \text{ m})^2 \times 1800 \text{ s}}{(0.6 \text{ m})^2}$$

= 3 m

Hence, the rise in the level of water in the cylindrical tank is 3 m.

19. Let r be the radius of the sphere. Then, r = 9 cm.

Let R and h be the radius and length of the wire respectively.

Then $\quad R = \frac{2 \text{ mm}}{2} = 1 \text{ mm} = 0.1 \text{ cm}$

Volume of the sphere = Volume of the wire

$\Rightarrow \quad \frac{4}{3}\pi r^3 = \pi R^2 h$

$\Rightarrow \quad h = \frac{4}{3} \times \frac{r^3}{R^2}$

$= \frac{4}{3} \times \frac{(9 \text{ cm})^3}{(0.1 \text{ cm})^2}$

$= 97200 \text{ cm} = 972 \text{ m}$ [Using 1 m = 100 cm]

Hence, the length of the wire is 972 m.

20. Let r be the radius and h be the depth of the cylindrical well. Then, $r = \frac{7 \text{ m}}{2} = 3.5 \text{ m}$ and $h = 15$ m.

Let w be the width of the embankment formed around the well. Then, $w = 7$ m.

Also, let R be the external radius of the embankment. The radius of the cylindrical well is also the internal radius of the embankment. Then,

$R = r + w = 3.5 + 7 \text{ m} = 10.5 \text{ m}$

Consider H be the height of the embankment.

Volume of the cylindrical well

= Volume of the embankment

$\Rightarrow \quad \pi r^2 h = \pi(R^2 - r^2)H$

$\Rightarrow \quad H = \frac{r^2 h}{R^2 - r^2}$

$= \frac{(3.5 \text{ m})^2 \times 15 \text{ m}}{(10.5 \text{ m})^2 - (3.5 \text{ m})^2}$

$= \frac{12.25 \times 15}{110.25 - 12.25} \text{ m}$

$= \frac{183.75}{98} \text{ m} = 1.875 \text{ m}$

Hence, the height of the embankment is 1.875 m.