CHAPTER 12

Heron's Formula

EXERCISE 12A

1. Here, a = 20 cm, b = 21 cm, c = cm

$$s = \frac{a+b+c}{2}$$
$$= \frac{20+21+13}{2} \text{ cm}$$
$$= \frac{54}{2} \text{ cm}$$
$$= 27 \text{ cm}$$

Area of triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{27(27-20)(27-21)(27-13)} \text{ cm}^2$$
$$= \sqrt{27(7)(6)(14)} \text{ cm}^2$$
$$= \sqrt{3\times3\times3\times7\times3\times2\times2\times7} \text{ cm}^2$$
$$= 2 \times 3 \times 3 \times 7$$
$$= 126 \text{ cm}^2$$

$\frac{1}{2}$ × longest side × corresp. altitude = Area of the triangle

$\Rightarrow \frac{1}{2}$ × 21 cm × corresp. altitude = 126 cm^2

$\Rightarrow$ Altitude corresp. to longest side $= \frac{126 \times 2}{21}$ cm

$= 12$ cm

Hence, the required area of the triangle = **126 cm^2** and the altitude corresponding to the longest side **= 12 cm**.

2. Here, a = 75 m, b = 65 m, c = 70 m

$$s = \frac{a+b+c}{2}$$
$$= \frac{75+65+70}{2} \text{ m}$$
$$= \frac{210}{2} \text{ m}$$
$$= 105 \text{ m}$$

Area of the triangular ground

$$= \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{105(105-75)(105-65)(105-70)} \text{ m}^2$$
$$= \sqrt{105(30)(40)(35)} \text{ m}^2$$
$$= \sqrt{3\times5\times7\times2\times3\times5\times2\times2\times2\times5\times5\times7} \text{ m}^2$$
$$= 2 \times 2 \times 3 \times 5 \times 5 \times 7 \text{ m}^2$$
$$= 2100 \text{ m}^2$$

Hence, the cost of levelling = 2100 × ₹ 5 = ₹ **10500**

3. Here, a = 715 m, b = 660 m, c = 270 m

$$s = \frac{a+b+c}{2}$$
$$= \frac{715+660+275}{2} \text{ m}$$
$$= \frac{1650}{2} \text{ m} = 825 \text{ m}$$

Area of the triangular field

$$= \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{825(825-715)(825-660)(825-275)} \text{ m}^2$$
$$= \sqrt{825(110)(165)(550)} \text{ m}^2$$
$$= \sqrt{5\times165\times110\times165\times5\times110} \text{ m}^2$$
$$= 5 \times 110 \times 165 \text{ m}^2$$
$$= 90750 \text{ m}^2$$
$$= \frac{5\times110\times165}{10000} \text{ hectares}$$
$$= \frac{363}{40} \text{ hectares}$$

Cost of reaping the field = ₹ $\frac{363}{40} \times 50$

$$= \frac{1815}{4}$$

= ₹ 453.75

Hence, the area of the triangular field is **90750 m^2** and the cost of reaping it is ₹ **453.75**.

4. Here, a = 24 m, b = 7 m, c = 25 m

$$s = \frac{24+7+25}{2} \text{ m}$$
$$= \frac{56}{2} \text{ m}$$
$$= 28 \text{ m}$$

Area of triangular field

$$= \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{28(28-24)(28-7)(28-25)} \text{ m}^2$$
$$= \sqrt{28(4)(21)(3)} \text{ m}^2$$
$$= \sqrt{7\times4\times4\times7\times3\times3} \text{ m}^2$$
$$= 3 \times 4 \times 7 \text{ m}^2$$
$$= 84 \text{ m}^2$$

Let a_1 = 3 m, b_1 = 4 m, c_1 = 5 m

Then, $s_1 = \frac{3+4+5}{2}$ m $= \frac{12}{2}$ m = 6 m

Area of triangular bed $= \sqrt{s_1(s_1-a_1)(s_1-b_1)(s_1-c_1)}$

$$= \sqrt{6(6-3)(6-4)(6-5)} \text{ m}^2$$
$$= \sqrt{6(3)(2)(1)} \text{ m}^2$$
$$= 6 \text{ m}^2$$

Number of triangular bed $= \frac{\text{area of triangular field}}{\text{area of triangular bed}}$

$$= \frac{84}{6}$$
$$= 14$$

Hence, the number of triangular beds that can be made **= 14**

5. Here, $a = 60$ m, $b = 56$ m, $c = 52$ m

$$s = \frac{a+b+c}{2}$$
$$= \frac{60+56+52}{2} \text{ m}$$
$$= \frac{168}{2} \text{ m}$$
$$= 84 \text{ m}$$

Area of triangular field

$$= \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{84(84-60)(84-56)(84-52)} \text{ m}^2$$
$$= \sqrt{84(24)(28)(32)} \text{ m}^2$$
$$= \sqrt{3 \times 28 \times 3 \times 8 \times 28 \times 2 \times 2 \times 8} \text{ m}^2$$
$$= 2 \times 3 \times 8 \times 28 \text{ m}^2$$
$$= 1344 \text{ m}^2$$

Cost of planting grass = 1344 × ₹ 3.75
= ₹ 5040

Perimeter of the triangular field = (60 + 56 + 52) m
= 168 m

Length to be fenced = 168 m − 4 m
= 164 m

Cost of fencing = 164 × ₹ 20
= ₹ 3280

Hence, the cost of planting grass in the field = ₹ **5040** and the cost of fencing it = ₹ **3280**.

6. (*i*) Here, $a = 24$ m, $b = 26$ m, $c = 10$ cm

$$s = \frac{a+b+c}{2}$$
$$= \frac{24+26+10}{2} \text{ cm}$$
$$= \frac{60}{2} \text{ cm}$$
$$= 30 \text{ cm}$$

$$\text{Area of triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{30(30-24)(30-26)(30-10)} \text{ cm}^2$$
$$= \sqrt{30(6)(4)(20)} \text{ cm}^2$$
$$= \sqrt{3 \times 10 \times 3 \times 2 \times 2 \times 2 \times 2 \times 10} \text{ cm}^2$$
$$= 2 \times 2 \times 3 \times 10 \text{ m}^2$$
$$= 120 \text{ cm}^2$$

(*ii*) $\frac{1}{2}$ × longest side × corresp. altitude = area of the triangle

$\Rightarrow$ $\frac{1}{2}$ × 26 × corresp. altitude = 120 cm^2

$\Rightarrow$ Altitude corresponding to longest side

$$= \frac{120 \times 2}{26} \text{ cm}$$
$$= \frac{120}{13} \text{ cm}$$

Hence (*i*) the area of the triangle = **120 cm^2**

(*ii*) the length of altitude corresponding to the longest side = $\mathbf{\frac{120}{13}}$ **cm**

7. Here, $a = 156$ m, $b = 169$ m and $c = 65$ m

$$s = \frac{a+b+c}{2}$$
$$= \frac{156+169+65}{2} \text{ m}$$
$$= \frac{390}{2} \text{ m}$$
$$= 195 \text{ m}$$

$$\text{Area of triangular field} = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{195(195-156)(195-169)(195-65)} \text{ m}^2$$
$$= \sqrt{195(39)(26)(130)} \text{ m}^2$$
$$= \sqrt{5 \times 39 \times 39 \times 2 \times 13 \times 2 \times 5 \times 13} \text{ m}^2$$
$$= 2 \times 5 \times 13 \times 39 \text{ m}^2$$
$$= 5070 \text{ m}^2$$

We know that shortest altitude is on the longest side

$\frac{1}{2}$ × longest side × shortest altitude = Area of triangle

$\Rightarrow$ $\frac{1}{2}$ × 169 m × shortest altitude = Area of triangle

$\Rightarrow$ shortest altitude = $\frac{5070 \times 2}{160}$ m

$\Rightarrow$ shortest altitude = 60 m

Hence, the area of the triangular field = **5070 m^2** and the length of the shortest altitude = **60 m.**

8. $2s = 240$ m

$\Rightarrow$ $s = 120$ m

Here, $x = 78$ m, $b = 50$ m and $c = 240 - 78 - 50$ m = 112 m

Area of triangular plot

$$= \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{120(120-78)(120-50)(120-112)} \text{ m}^2$$
$$= \sqrt{120 \times 42 \times 70 \times 8} \text{ m}^2$$
$$= \sqrt{2 \times 6 \times 10 \times 6 \times 7 \times 7 \times 10 \times 2 \times 2 \times 2} \text{ m}^2$$
$$= 2 \times 2 \times 6 \times 7 \times 10 \text{ m}^2$$
$$= 1680 \text{ m}^2$$

Let the length of perpendicular on the side of length 50 m be x m.

$$\frac{1}{2} \times 50 \times x = 1680$$

$$\Rightarrow \quad x = \frac{1680 \times 2}{50} = 67.2$$

Hence, the required length of the perpendicular = **67.2 m**

9. $2s = 68$ cm

$\Rightarrow$ $s = 34$ cm

Here, $a = 25$ cm, $b = 26$ cm, $c = (68 - 25 - 26)$ cm = 17 cm

Area of he triangle

$$= \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{34(34-25)(34-26)(34-17)} \text{ cm}^2$$
$$= \sqrt{34 \times 9 \times 8 \times 17} \text{ cm}^2$$
$$= \sqrt{2 \times 17 \times 3 \times 3 \times 2 \times 2 \times 2 \times 17} \text{ cm}^2$$
$$= 2 \times 2 \times 3 \times 17 \text{ cm}^2 = 204 \text{ cm}^2$$

Let the altitude of the triangle corresponding to the shortest side be x cm.

Then, $\frac{1}{2} \times 17 \times x = 204$

$\Rightarrow$ $x = \frac{204 \times 2}{17} = 24$

Hence, the area of the triangle is **204 cm²** and the altitude corresponding to the shortest side is **24 cm²**.

10. (*i*) Let the sides of the triangle be $2x$ cm, $3x$ cm and $4x$ cm.

Then, $2x + 3x + 4x = 90$

$\Rightarrow$ $9x = 90$

$\Rightarrow$ $x = 10$

So, the sides of the triangle are (2 × 10) cm, (3 × 10) cm and (4 × 10) cm.

i.e. 20 cm, 30 cm and 40 cm.

Here, a = 20 cm, b = 30 cm and c = 40 cm.

$$s = \frac{a+b+c}{2}$$
$$= \frac{20+30+40}{2} \text{ cm}$$
$$= \frac{90}{2} \text{ cm}$$
$$= 45 \text{ cm}$$

Area of the triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{45(45-20)(45-30)(45-40)} \text{ cm}^2$$
$$= \sqrt{45 \times 25 \times 15 \times 5} \text{ cm}^2$$
$$= \sqrt{3 \times 3 \times 5 \times 5 \times 5 \times 3 \times 5 \times 5} \text{ cm}^2$$
$$= 3 \times 5 \times 5\sqrt{3 \times 5} \text{ cm}^2$$
$$= 75\sqrt{15}$$
$$= 75 \times 3.872 \text{ cm}^2 \text{ (approx.)}$$
$$= 290.473 \text{ cm}^2 \text{ (approx.)}$$

Hence, the area of the triangle is **290.473 cm² approx.**

(*ii*) Let the sides of the triangle be $3x$ m, $5x$ m and $7x$ m.

Then, $3x + 5x + 7x = 300$

$\Rightarrow$ $15x = 300$

$\Rightarrow$ $x = 20$

So, the sides of the triangle are (3 × 20) m, (5 × 20) m and (7 × 20) m

i.e. 60 m, 100 m and 140 m.

Here, a = 60 m, b = 100 m and c = 140 m

$$s = \frac{a+b+c}{2}$$
$$= \frac{60+100+140}{2} \text{ m}$$
$$= \frac{300}{2} \text{ m}$$
$$= 150 \text{ m}$$

Area of the triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{150(150-60)(150-100)(150-140)} \text{ m}^2$$
$$= \sqrt{150 \times 90 \times 50 \times 10} \text{ m}^2$$
$$= \sqrt{3 \times 50 \times 3 \times 3 \times 10 \times 50 \times 10} \text{ m}^2$$
$$= 3 \times 10 \times 50\sqrt{3} \text{ m}^2$$
$$= 1500\sqrt{3} \text{ m}^2$$

Hence, the area of the triangle is **$1500\sqrt{3}$ m².**

(*iii*) Let the sides of the triangle be $9x$ m, $40x$ m and $41x$ m.

Then, $9x + 40x + 41x = 180$

$\Rightarrow$ $90x = 180$

$\Rightarrow$ $x = 2$

So, the sides of the triangle are (9 × 2) m, (40 × 2) m and (41 × 2) m.

i.e. 18 m, 80 m and 82 m

Here, a = 18 m, b = 80 and c = 82 m.

$$s = \frac{a+b+c}{2}$$
$$= \frac{18+80+82}{2} \text{ m}$$
$$= \frac{180}{2} \text{ m}$$
$$= 90 \text{ m}$$

Area of the triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{90 \times (90-18)(90-80)(90-82)} \text{ m}^2$$
$$= \sqrt{90 \times 72 \times 10 \times 8} \text{ m}^2$$
$$= \sqrt{9 \times 10 \times 9 \times 8 \times 10 \times 8} \text{ m}^2$$
$$= 8 \times 9 \times 10 \text{ m}^2$$
$$= 720 \text{ m}^2$$

Hence, the area of the triangle is **720 m²**.

(*iv*) Let the sides of the triangle be $25x$ cm, $17x$ cm and $12x$ cm.

Then, $25x + 17x + 12x = 270$

$\Rightarrow$ $54x = 270$

$\Rightarrow$ $x = \frac{270}{54}$

$= 5$

So, the sides of the triangle and (25 × 5) cm, (17 × 5) cm and (17 × 5) cm

i.e. 125 cm, 85 cm and 60 cm.

Here, a = 125 cm, b = 85 cm and c = 60 cm.

$$s = \frac{a+b+c}{2}$$
$$= \frac{125+85+60}{2} \text{ cm}$$
$$= \frac{270}{2} \text{ cm}$$
$$= 135 \text{ cm}$$

Area of the triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{135(135-125)(135-85)(135-60)} \text{ cm}^2$$
$$= \sqrt{135 \times 10 \times 50 \times 75} \text{ cm}^2$$
$$= \sqrt{3 \times 3 \times 3 \times 5 \times 2 \times 5 \times 2 \times 5 \times 5 \times 3 \times 5 \times 5} \text{ cm}^2$$
$$= 2 \times 3 \times 3 \times 5 \times 5 \times 5 \text{ cm}^2$$
$$= 2250 \text{ cm}^2$$

Hence, the area of the triangle is **2250 cm²**

11. If ₹ 12 is the cost of levelling 1 sq m of land

Then, ₹ 81000 is the cost of levelling $\frac{1}{12} \times 81000$ sq m of land.

$= 6750 \text{ m}^2$ … (1)

Let the sides of the plot be $13x$ m, $12x$ m and $5x$ m.

Then, $s = \dfrac{13x + 12x + 5x}{2}$

$= \dfrac{30x}{2}$ m

$= 15x$ m

Area of the triangular plot = 6750 m^2 [From (*i*)]

$\Rightarrow \sqrt{s(s-a)(s-b)(s-c)} = 6750 \text{ m}^2$

$\Rightarrow \sqrt{15x(15x-13x)(15x-12x)(15x-5x)} \text{ m}^2 = 6750 \text{ m}^2$

$\Rightarrow \sqrt{15x \times 2x \times 3x \times 10x} = 6750$

$\Rightarrow \sqrt{3 \times 5 \times 2 \times 3 \times 2 \times 5x^4} = 6750$

$\Rightarrow 2 \times 3 \times 5x^2 = 6750$

$\Rightarrow x^2 = \dfrac{6750}{2 \times 3 \times 5}$

$= 225$

$\Rightarrow x = 15$

So the sides of the triangle are (13 × 15) m, (12 × 15) m and (5 × 15) m.

i.e. 195 m, 180 m and 75 m.

Hence, the sides of the triangle are **195 m, 180 m and 75 m.**

12. Here, a = 10 cm, b = 13 cm, c = 13 cm

$s = \dfrac{a+b+c}{2}$

$= \dfrac{10+13+13}{2}$ cm

$= \dfrac{36}{2}$ cm

= 18 cm

Area of the given isosceles triangle

$= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{18(18-10)(18-13)(18-13)} \text{ cm}^2$

$= \sqrt{18 \times 8 \times 5 \times 5} \text{ cm}^2$

$= \sqrt{3 \times 3 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5} \text{ cm}^2$

$= (2 \times 2 \times 3 \times 5) \text{ cm}^2$

$= 60 \text{ cm}^2$

Hence, the area of the given isosceles triangle is **60 cm²**.

13. $s = \dfrac{a+b+b}{2} = \dfrac{a+2b}{2} = \dfrac{a}{2} + b$

Area of the given isosceles triangle

$= \sqrt{\left(\dfrac{a}{2}+b\right)\left(\dfrac{a}{2}+b-a\right)\left(\dfrac{a}{2}+b-b\right)\left(\dfrac{a}{2}+b-b\right)}$ sq units

$= \sqrt{\left(\dfrac{a}{2}+b\right)\left(b-\dfrac{a}{2}\right)\left(\dfrac{a}{2}\right)\left(\dfrac{a}{2}\right)}$ sq units

$= \dfrac{a}{2}\sqrt{b^2 - \dfrac{a^2}{4}}$ sq units

$= \dfrac{a}{2}\sqrt{4b^2 - a^2}$ sq units

Hence, the area of the given isosceles triangle is

$\boldsymbol{\dfrac{a}{4}\sqrt{4b^2 - a^2}}$ **sq units.**

14. Here, a = 15 cm, b = 15 cm and c = (48 – 30) = 18 cm

$s = \dfrac{a+b+c}{2}$

$= \dfrac{15+15+18}{2}$ cm

$= \dfrac{48}{2}$ cm

= 24 cm

Area of the given isosceles triangle

$= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{24(24-15)(24-15)(24-18)} \text{ cm}^2$

$= \sqrt{24 \times 9 \times 9 \times 6} \text{ cm}^2$

$= \sqrt{2 \times 2 \times 6 \times 9 \times 9 \times 6} \text{ cm}^2$

$= 2 \times 6 \times 9 \text{ cm}^2$

$= 108 \text{ cm}^2$

Hence, the area of the given isosceles triangle is **108 cm²**.

15. Let the measure of each equal side of the given isosceles triangle be x cm and its base = 24 cm.

Here, $a = x$ cm, $b = x$ cm and c = 24 cm

$s = \dfrac{a+b+c}{2}$

$= \dfrac{x+x+24}{2}$ cm

$= \dfrac{2x+24}{2}$ cm

$= (x + 12)$ cm

Area of the given isosceles triangle = 192 cm^2

$\Rightarrow \sqrt{s(s-a)(s-b)(s-c)} = 192 \text{ cm}^2$

$\Rightarrow \sqrt{(x+12)(x+12-x)(x+12-x)(x+12-24)} = 192$

$\Rightarrow \sqrt{(x+12)(12)(12)(x-12)} = 192$

$\Rightarrow 12\sqrt{x^2 - 12^2} = 192$

$\Rightarrow \sqrt{x^2 - 12^2} = \dfrac{192}{12} = 16$

$\Rightarrow x^2 - 12^2 = 16^2$

$\Rightarrow x^2 = 16^2 + 12^2$

$= 256 + 144$

$= 400$

$\Rightarrow x = 20$

So, the sides of the triangle are 20 cm, 20 cm and 24 cm

Perimeter of the triangle = 20 cm + 20 cm + 24 cm = 64 cm

Hence, the perimeter of the given isosceles triangle is **64 cm**.

16. $s = \dfrac{a+a+a}{2} = \dfrac{3a}{2}$

Area of the given equilateral triangle

$= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{\dfrac{3a}{2}\left(\dfrac{3a}{2}-a\right)\left(\dfrac{3a}{2}-a\right)\left(\dfrac{3a}{2}-a\right)}$ sq units

$= \sqrt{\dfrac{3a}{2}\left(\dfrac{3a-2a}{2}\right)\left(\dfrac{3a-2a}{2}\right)\left(\dfrac{3a-2a}{2}\right)}$ sq units

$= \sqrt{\frac{3a}{2}\left(\frac{a}{2}\right)\left(\frac{a}{2}\right)\left(\frac{a}{2}\right)}$ sq unites

$= \frac{a^2\sqrt{3}}{4}$ sq units

$= \frac{\sqrt{3}}{4}a^2$ sq units

Hence, the area of an equilateral triangle of side a units is $\frac{\sqrt{3}}{4}$ **a^2 sq units.**

17. Here, a = 16 cm, b = 16 cm and 16 cm

$$s = \frac{a+b+c}{2}$$
$$= \frac{16+16+16}{2} \text{ cm}$$
$$= \frac{48}{2} \text{ cm}$$
$$= 24 \text{ cm}$$

Area of the given equilateral triangle

$$= \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{24(24-16)(24-16)(24-16)} \text{ cm}^2$$
$$= \sqrt{24 \times 8 \times 8 \times 8} \text{ cm}^2$$
$$= \sqrt{3 \times 8 \times 8 \times 8 \times 8} \text{ cm}^2$$
$$= 8 \times 8\sqrt{3} \text{ cm}^2$$
$$= 64\sqrt{3} \text{ cm}^2$$

Hence, the area of the given equilateral triangle is **$63\sqrt{3}$ cm²**.

18. Here, a = 8 cm, b = 8 cm and c = 8 cm

$$s = \frac{a+b+c}{2}$$
$$= \frac{8+8+8}{2} \text{ cm}$$
$$= \frac{24}{2} \text{ cm}$$
$$= 12 \text{ cm}$$

Area of the given equilateral triangle

$$= \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{12(12-8)(12-8)(12-8)} \text{ cm}^2$$
$$= \sqrt{12 \times 4 \times 4 \times 4} \text{ cm}^2$$
$$= \sqrt{3 \times 4 \times 4 \times 4 \times 4} \text{ cm}^2$$
$$= 4 \times 4\sqrt{3} \text{ cm}^2$$
$$= 16\sqrt{3} \text{ cm}^2$$

Also, $\frac{1}{2} \times$ base $\times$ altitude = Area of the triangle

$\Rightarrow$ $\frac{1}{2} \times 8 \text{ cm} \times \text{altitude} = 16\sqrt{3} \text{ cm}^2$

$\Rightarrow$ $\text{Altitude} = \frac{16\sqrt{3} \times 2}{8} \text{ cm}$

$$= 4\sqrt{3} \text{ cm}$$

Hence, the area of the given equilateral triangle is **$16\sqrt{3}$** cm² and its altitude is $4\sqrt{3}$ cm.

19. Perimeter of the given equilateral triangle = 60 cm

$\therefore$ $s = \frac{\text{Perimeter}}{2}$

$$= \frac{60}{2} \text{ cm}$$
$$= 30 \text{ cm}$$

and each side = $\frac{60}{3}$ cm = 20 cm

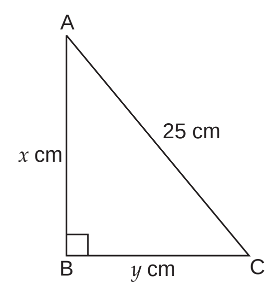
So, a = 20 cm, b = 20 cm and c = 20 cm

Area of the given equilateral triangle

$$= \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{30(30-20)(30-20)(30-20)} \text{ cm}^2$$
$$= \sqrt{30 \times 10 \times 10 \times 10} \text{ cm}^2$$
$$= \sqrt{3 \times 10 \times 10 \times 10 \times 10} \text{ cm}^2$$
$$= 100\sqrt{3} \text{ cm}^2$$
$$= 100 \times 1.732 \text{ cm}^2$$
$$= 173.2 \text{ cm}^2$$

Hence, area of the given equilateral triangle is **173.2 cm²**.

20. Let ABC represent the given right triangle in which $\angle B = 90°$ and hypotenuse AC = 25 cm.



Let AB = x cm and BC = y cm

Then, $x + y + 25 = 56$

$\Rightarrow$ $x + y = 56 - 25$

$\Rightarrow$ $x + y = 31$

$\Rightarrow$ $y = 31 - x$ … (*i*)

In right ΔABC, we have

$AB^2 + BC^2 = AC^2$

[By Pythagoras' theorem]

$\Rightarrow$ $x^2 + y^2 = (25)^2$

$\Rightarrow$ $x^2 + (31 - x)^2 = 625$ [Using (*i*)]

$\Rightarrow$ $x^2 + 961 + x^2 - 62x = 625$

$\Rightarrow$ $2x^2 - 62x + 336 = 0$

$\Rightarrow$ $x^2 - 31x + 168 = 0$

$\Rightarrow$ $x^2 - 7x - 24x + 168 = 0$

$\Rightarrow$ $x(x - 7) - 24(x - 7) = 0$

$\Rightarrow$ $(x - 7)(x - 24) = 0$

Either $(x - 7) = 0$ or $(x - 24) = 0$

$\Rightarrow$ $x = 7$, or $x = 24$

When $x = 7$, $y = 31 - 7 = 24$

When $x = 24$, $y = 31 - 24 = 7$

Area of the given right triangle = $\frac{1}{2} \times 24 \times 7 \text{ cm}^2$

Calculation of area of the given right triangle by Heron's Formula.

Here $a = 7$ cm, $b = 24$ cm, $c = 25$ cm

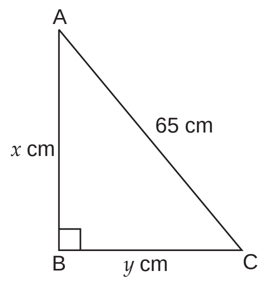
$$s = \frac{a+b+c}{2} = \frac{7+24+25}{2} \text{ cm} = \frac{56}{2} = 28 \text{ cm}$$

Area of the given triangle

$$= \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{28(28-7)(28-24)(28-25)} \text{ cm}^2$$
$$= \sqrt{28 \times 21 \times 4 \times 3} \text{ cm}^2$$
$$= \sqrt{4 \times 7 \times 3 \times 7 \times 4 \times 3} \text{ cm}^2$$
$$= 3 \times 4 \times 7 \text{ cm}^2$$
$$= 84 \text{ cm}^2$$

Hence, the other two sides of the given right triangle are **7 cm** and **24 cm** and it is verified that its area is **84 cm²**.

21. Let ABC represent the given right triangle in which $\angle B = 90°$ and hypotenuse AC = 65 cm.



Let AB = x cm and BC = y cm.

Then, $x + y + 65 = 144$

$\Rightarrow$ $x + y = 144 - 65 = 79$

$\Rightarrow$ $y = 79 - x$ … (1)

In right ΔABC, we have

$AB^2 + BC^2 = AC^2$ [By Pythagoras' theorem]

$\Rightarrow$ $x^2 + y^2 = 65^2$

$\Rightarrow$ $x^2 + (79 - x)^2 = 65^2$

$\Rightarrow$ $x^2 + 6241 + x^2 - 158x = 4225$

$\Rightarrow$ $2x^2 - 158x + 2016 = 0$

$\Rightarrow$ $x^2 - 79x + 1008 = 0$

$\Rightarrow$ $x^2 - 16x - 63x + 1008 = 0$

$\Rightarrow$ $x(x - 16) - 63(x - 16) = 0$

$\Rightarrow$ $(x - 16)(x - 63) = 0$

Either $(x - 16) = 0$ or $(x - 63) = 0$

$\Rightarrow$ $x = 16$ or $x = 63$

When $x = 16$, $y = 79 - 16 = 63$

When $x = 63$, $y = 79 - 63 = 16$

The other two sides of the given right triangle are **63 cm** and **16 cm**.

Calculation of area of the given right triangle by Heron's formula.

Here $a = 16$ cm, $b = 63$ cm and $c = 65$ cm

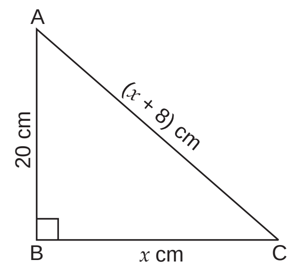
$$s = \frac{a+b+c}{2} = \frac{16+63+65}{2} \text{ cm} = \frac{144}{2} \text{ cm} = 72 \text{ cm}$$

Area of the given right triangle

$$= \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{72(72-16)(72-63)(72-65)} \text{ cm}^2$$
$$= \sqrt{72 \times 56 \times 9 \times 7} \text{ cm}^2$$
$$= \sqrt{9 \times 8 \times 8 \times 7 \times 9 \times 7} \text{ cm}^2$$
$$= 7 \times 8 \times 9 \text{ cm}^2$$
$$= 504 \text{ cm}^2$$

Hence, the other sides of the triangle are **16 cm and 63 cm** and the area of the given right triangle = **504 cm²**

22. Let ABC represent the right triangle in which $\angle B = 90°$ AB = 20, BC = x cm.



Then, $c = (x + 8)$

In right triangle ABC, we have

$AB^2 + BC^2 = AC^2$

[By Pythagoras' theorem]

$\Rightarrow$ $(20)^2 + (x)^2 = (x + 8)^2$

$\Rightarrow$ $400 + x^2 = x^2 + 16x + 64$

$\Rightarrow$ $400 = 16x + 64$

$\Rightarrow$ $16x = 400 - 64 = 336$

$\Rightarrow$ $x = \frac{336}{16} = 21$

So, the measures of the unknown sides are 21 cm and (21 + 8) = 29 cm.

Here, $a = 20$ cm, $b = 21$ cm and $c = 29$ cm

$$s = \frac{a+b+c}{2} = \frac{20+21+29}{2} \text{ cm} = \frac{70}{2} \text{ cm} = 35 \text{ cm}$$

Area of given right triangle

$$= \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{35(35-20)(35-21)(35-29)} \text{ cm}^2$$
$$= \sqrt{35 \times 15 \times 14 \times 6} \text{ cm}^2$$

$= \sqrt{5 \times 7 \times 3 \times 5 \times 2 \times 7 \times 2 \times 3}$ cm^2

$= 2 \times 3 \times 5 \times 7$ cm^2

$= 210$ cm^2

Hence, the measures of the other two sides are **21** cm and **29 cm** and the area of the given right triangle is = **210 cm^2**

23. Let each side of the original equilateral triangle be a units. then, each side of the increased equilateral triangle is $3a$ units.

Area of original equilateral triangle = $\frac{\sqrt{3}}{4}a^2$ sq units

Area of increased equilateral triangle

$$= \frac{\sqrt{3}}{4}(3a)^2 \text{ sq units}$$

$$= 9\left(\frac{\sqrt{3}}{4}a^2\right) \text{sq units}$$

$$\text{Increase in the area} = \left(9\frac{\sqrt{3}}{4}a^2 - \frac{\sqrt{3}}{4}a^2\right) \text{ sq units}$$

$$= 8\frac{\sqrt{3}}{4}a^2 \text{ sq units}$$

$$\text{Percentage increase} = \frac{\text{increase in area}}{\text{original area}} \times 100\%$$

$$= \frac{8\frac{\sqrt{3}}{4}a^2}{\frac{\sqrt{3}}{4}a^2} \times 100\%$$

$$= 80\%$$

Verification using Heron's formula

For the original equilateral triangle,

$$s = \frac{a+a+a}{2} = \frac{3a}{2} \text{ units}$$

Area of the original equilateral triangle

$$= \sqrt{\frac{3a}{2}\left(\frac{3a}{2}-a\right)\left(\frac{3a}{2}-a\right)\left(\frac{3a}{2}-a\right)} \text{sq units}$$

$$= \frac{\sqrt{3}}{4}a^2 \text{ sq units}$$

For the increased equilateral triangle,

$$s = \frac{3a+3a+3a}{2} \text{ units}$$

$$= \frac{9a}{2} \text{ units.}$$

Area of the increased equilateral triangle

$$= \sqrt{\frac{9a}{2}\left(\frac{9a}{2}-3a\right)\left(\frac{9a}{2}-3a\right)\left(\frac{9a}{2}-3a\right)} \text{ sq units}$$

$$= \sqrt{\frac{9a}{2}\left(\frac{3a}{2}\right)\left(\frac{3a}{2}\right)\left(\frac{3a}{2}\right)} \text{ sq units}$$

$$= \frac{9a^2}{4}\sqrt{3} \text{ sq units}$$

$$= 9\left(\frac{\sqrt{3}}{4}a^2\right) \text{ sq units}$$

$$\text{Increase in the area} = \left[9\left(\frac{\sqrt{3}}{4}a^2\right) - \left(\frac{\sqrt{3}}{4}a^2\right)\right] \text{ sq units}$$

$$= 8\left(\frac{\sqrt{3}}{4}a^2\right) \text{ sq units}$$

$$\text{Percentage increase the area} = \frac{\text{increase in area}}{\text{original area}} \times 100\%$$

$$= \frac{8\left(\frac{\sqrt{3}}{4}a^2\right)\text{sq units}}{\left(\frac{\sqrt{3}}{4}a^2\right)\text{sq units}} \times 100\%$$

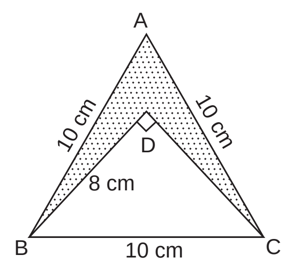
$$= \mathbf{800\%}$$

24. Area of equilateral ΔABC = $\frac{\sqrt{3}}{4} \times (10 \text{ cm})^2$

$= 25\sqrt{3}$ cm^2

$= 25 \times 1.732$ cm^2

$= 43.3$ cm^2 … (1)



In right ΔBDC, we have

$BD^2 + CD^2 = BC^2$

[By Pythagoras' theorem]

$\Rightarrow$ $(8 \text{ cm})^2 + CD^2 = (10 \text{ cm})^2$

$\Rightarrow$ $CD^2 = (100 - 64)$ cm^2

$= 36$ cm^2

$\Rightarrow$ CD = 6 cm …(2)

Area (ΔCDB) = $\frac{1}{2}$ BC × CD

$= \frac{1}{2} \times 8 \text{ cm} \times 6 \text{ cm}$ [Using (2)]

Area of the shaded region = ar(ΔABC) − ar(ΔCDB)

$= 43.3$ cm^2 − 24 cm^2 … (3)

[Using (1) and (3)]

$= 19.3$ cm^2

Verification using Heron's Formula

For ΔABC, $s = \frac{10+10+10}{2}$ cm

$= \frac{30}{2}$ cm

$= 15$ cm

ar(ΔABC) = $\sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{15(15-10)(15-10)(15-10)}$ cm^2

$= \sqrt{3 \times 5 \times 5 \times 5 \times 5}$ cm^2

$= 25\sqrt{3}$ cm^2

$= 25 \times 1.732$ cm^2

$= 43.3$ cm^2 …(4)

For ΔCDB, $s = \frac{6+8+10}{2}$ cm

$= \frac{24}{2}$ cm [Using (4)]

$ar(\Delta CDB) = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{12(12-6)(12-8)(12-10)}$ cm²

$= \sqrt{12 \times 6 \times 4 \times 2}$ cm²

$= 12 \times 2$ cm²

$= 24$ cm² … (5)

Area of the shaded region = ar(ΔABC) − ar(ΔCDB)

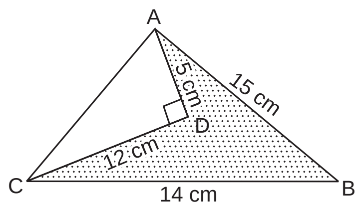
$= 43.3 \text{ cm}^2 - 24 \text{ cm}^2$

$= 29.3 \text{ cm}^2$

[Using (4) and (5)]

Hence, it is verified that the area of the shaded region is **19.3 cm²**.

25. Let ABC represent the larger triangle and let ADC represent the right triangle.



In right ΔADC, we have

$AC^2 = AD^2 + CD^2$

$= (5 \text{ cm})^2 + (12 \text{ cm})^2$

$= (25 + 144) \text{ cm}^2$

$= 169 \text{ cm}^2$

⇒ AC = 13 cm

$ar(\Delta ADC) = \frac{1}{2} \times 12 \times 5 \text{ cm}^2$

$= 30 \text{ cm}^2$ … (1)

In ΔABC, a = 14 cm, b = 13 cm and c = 15 cm

$s = \frac{a+b+c}{2}$

$= \frac{14+13+15}{2}$ cm

$= \frac{42}{2}$ cm

$= 21$ cm

$ar(\Delta ABC) = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{21(21-14)(21-13)(21-15)}$ cm²

$= \sqrt{21 \times 7 \times 8 \times 6}$ cm²

$= \sqrt{7 \times 3 \times 7 \times 2 \times 2 \times 2 \times 2 \times 3}$ cm²

$= 2 \times 2 \times 3 \times 7$ cm²

$= 84$ cm² … (2)

Area of the shaded region

= ar(ΔABC) − ar(ΔADC)

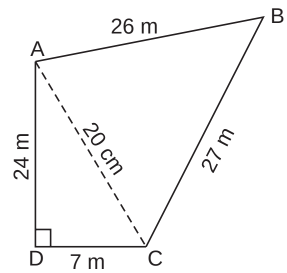
$= 84 \text{ cm}^2 - 30 \text{ cm}^2$ [Using (1) and (2)]

$= 54 \text{ cm}^2$

Hence, the area of the shaded region is **54 cm².**

EXERCISE 12B

1. From diagonal AC.



In right ΔADC, we have

$AC^2 = AD^2 + CD^2$

[By Pythagoras' theorem]

⇒ $AC^2 = (24 \text{ m})^2 + (7 \text{ m})^2$

$= (576 + 49) \text{ m}^2$

$= 625 \text{ m}^2$

⇒ AC = 25 m … (1)

$ar(\Delta ADC) = \frac{1}{2} \times CD \times AD$

$= \frac{1}{2} \times 7 \text{ m} \times 24 \text{ m}$

$= 84 \text{ m}^2$ … (2)

In ΔABC, a = 27 m, b = 25 m and c = 26 m [Using (1)]

$s = \frac{a+b+c}{2}$

$= \frac{27+25+26}{2}$ m

$= \frac{78}{2}$ m

$= 39$ m

$ar(\Delta ABC) = \sqrt{(s(s-a)(s-b)(s-c)}$

$= \sqrt{39(39-27)(39-25)(39-26)}$ m²

$= \sqrt{39 \times 12 \times 14 \times 13}$ m²

$= \sqrt{13 \times 3 \times 3 \times 4 \times 2 \times 7 \times 13}$ m²

$= 2 \times 3 \times 13\sqrt{2 \times 7}$ m²

$= 78\sqrt{14}$ m²

$= 78 \times 3.741$ m² (approx.)

$= 291.849$ m² (approx.) … (3)

Area of quadrilateral park = ar(ΔADC) + ar(ΔABC)

$= (84 + 291.849)$ m² [Using (2) and (3)]

$= 375.849$ m² approx.

Hence, the area of the given quadrilateral park is **375.849 m²**.

2. In right ΔABC, we have

$AB^2 + BC^2 = AC^2$ [By Pythagoras' Theorem]

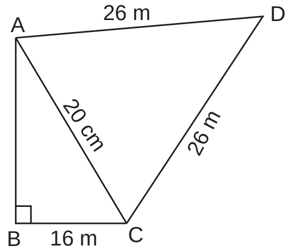
⇒ $AB^2 + (16 \text{ cm})^2 = (20 \text{ m})^2$

⇒ $AB^2 = (400 - 256) \text{ m}^2$

⇒ $AB^2 = 144 \text{ m}^2$

⇒ AB = 12 m … (1)

$$\text{Area } (\Delta ABC) = \frac{1}{2} \times BC \times AB$$
$$= \frac{1}{2} \times 16 \text{ cm} \times 12 \text{ m} \quad \text{[Using (1)]}$$
$$= 96 \text{ m}^2 \quad \ldots(2)$$

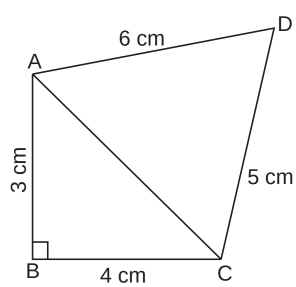


For ΔADC, we have

$$a = 26 \text{ m}, b = 20 \text{ m and } c = 26 \text{ m}$$
$$s = \frac{a+b+c}{2}$$
$$= \frac{26+20+26}{2}$$
$$= \frac{72}{2} \text{ m}$$
$$= 36 \text{ m}$$
$$\text{ar}(\Delta ADC) = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{36(36-26)(36-20)(36-26)} \text{ m}^2$$
$$= \sqrt{36 \times 10 \times 16 \times 10} \text{ m}^2$$
$$= 4 \times 6 \times 10 \text{ m}^2$$
$$= 240 \text{ m}^2 \quad \ldots (3)$$
$$\text{ar(quad ABCD)} = \text{ar}(\Delta ABC) + \text{ar}(\Delta ADC)$$
$$= (96 + 240) \text{ m}^2 \quad \text{[Using (2) and (3)]}$$
$$= 336 \text{ m}^2$$

Hence, the area of the quadrilateral field is **336 m²**.

3. Let ABCD be the given quadrilateral in which AB = 3 cm, BC = 4 cm, CD = 5 cm, DA = 6 cm and $\angle B = 90°$.



In right ΔABC, we have

$$AC^2 = AB^2 + BC^2$$
$$= (3 \text{ cm})^2 + (4 \text{ cm})^2$$
$$= (9 + 16) \text{ cm}^2$$
$$= 25 \text{ cm}^2$$
$$\Rightarrow \quad AB = 5 \text{ cm} \quad \ldots (1)$$
$$\text{ar}(\Delta ABC) = \frac{1}{2} \times BC \times AB$$
$$= \frac{1}{2} \times 4 \times 3 \text{ cm}^2$$
$$= 6 \text{ cm}^2 \quad \ldots (2)$$

In ΔADC, we have

$$a = 5 \text{ cm}, b = 5 \text{ cm and } c = 6 \text{ cm} \quad \text{[Using (1)]}$$
$$s = \frac{a+b+c}{2}$$
$$= \frac{5+5+6}{2} \text{ cm}$$
$$= \frac{16}{2} \text{ cm}$$
$$= 8 \text{ cm}$$
$$\text{ar}(\Delta ADC) = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{8(8-5)(8-5)(8-6)} \text{ cm}^2$$
$$= \sqrt{8 \times 3 \times 3 \times 2} \text{ cm}^2$$
$$= 3 \times 4 \text{ cm}^2$$
$$= 12 \text{ cm}^2 \quad \ldots (3)$$

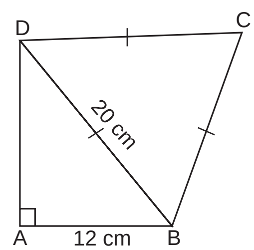
Area of the given quadrilateral

$$= \text{ar}(\Delta ABC) + \text{ar(ADC)}$$
$$= 6 \text{ cm}^2 + 12 \text{ cm}^2 \text{ [Using (2) and (3)]}$$
$$= 18 \text{ cm}^2$$

Hence, the area of the given quadrilateral is **18 cm²**.

4. In right ΔDAB, we have

$$AD^2 + AB^2 = BD^2 \quad \text{[By Pythagoras' Theorem]}$$
$$\Rightarrow AD^2 + (12 \text{ cm})^2 = (20 \text{ cm})^2$$
$$\Rightarrow AD^2 = (400 - 144) \text{ cm}^2 = 256 \text{ cm}^2$$
$$\Rightarrow AD = 16 \text{ cm} \quad \ldots (1)$$
$$\text{ar}(\Delta DAB) = \frac{1}{2} \times AB \times AD$$
$$= \frac{1}{2} \times 12 \text{ cm} \times 16 \text{ cm} \quad \text{[Using (1)]}$$
$$= 96 \text{ cm}^2 \quad \ldots (2)$$

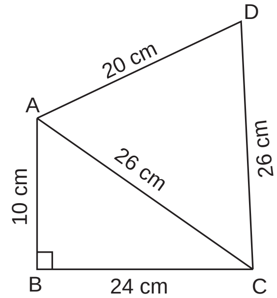


In ΔDBC, $a = 20$ cm, $b = 20$ cm, $c = 20$ cm

$$s = \frac{a+b+c}{2}$$
$$= \frac{20+20+20}{2} \text{ cm}$$
$$= \frac{60}{2} \text{ cm}$$
$$= 30 \text{ cm}$$
$$\text{ar}(\Delta DBC) = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{30(30-20)(30-20)(30-20)} \text{ cm}^2$$
$$= \sqrt{3 \times 10 \times 10 \times 10 \times 10} \text{ cm}^2$$
$$= 100\sqrt{3} \text{ cm}^2 = 100 \times 1.73 \text{ cm}^2$$
$$= 173 \text{ cm}^2 \quad \ldots (3)$$
$$\text{ar(quad. ABCD)} = \text{ar}(\Delta DAB) + \text{ar}(\Delta DBC)$$
$$= (96 + 173) \text{ cm}^2 \quad \text{[Using (2) and (3)]}$$
$$= 269 \text{ cm}^2$$

Hence, the area of the given quadrilateral is **269 cm²**.

5. In ΔABC, $a = 24$ cm, $b = 26$ cm and $c = 10$ cm

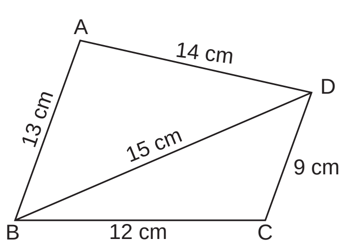


$$s = \frac{a+b+c}{2}$$
$$= \frac{24+26+10}{2} \text{ cm}$$
$$= \frac{60}{2} \text{ cm}$$
$$= 30 \text{ cm}$$
$$ar(\Delta ABC) = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{30(30-24)(30-26)(30-10)} \text{ cm}^2$$
$$= \sqrt{5 \times 6 \times 6 \times 4 \times 20} \text{ cm}^2$$
$$= \sqrt{5 \times 6 \times 6 \times 4 \times 4 \times 5} \text{ cm}^2$$
$$= 4 \times 5 \times 6 \text{ cm}^2$$
$$= 120 \text{ cm}^2 \quad \ldots (1)$$

In ΔADC, $s = \frac{20+26+26}{2}$
$$= \frac{72}{2} = 36 \text{ cm}$$
$$ar(ADC) = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{36(36-20)(36-26)(36-26)} \text{ cm}^2$$
$$= \sqrt{36 \times 16 \times 10 \times 10} \text{ cm}^2$$
$$= 4 \times 6 \times 10 \text{ cm}^2$$
$$= 240 \text{ cm}^2 \quad \ldots (2)$$
$$ar(\text{quad. } ABCD) = ar(\Delta ABC) + ar(ADC)$$
$$= 120 \text{ cm}^2 + 240 \text{ cm}^2 \quad \text{[Using (1) and (2)]}$$
$$= 360 \text{ cm}^2$$

Hence, the area of the given quadrilateral is **360 cm^2.**

6. In ΔABD, let $a = 15$ cm, $b = 14$ cm, and $c = 13$ cm



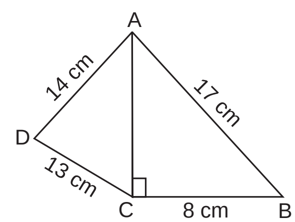
Then, $s = \frac{a+b+c}{2}$
$$= \frac{15+14+13}{2} \text{ cm}$$
$$= \frac{42}{2} \text{ cm}$$
$$= 21 \text{ cm}$$
$$ar(\Delta ABD) = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{21(21-15)(21-14)(21-13)} \text{ cm}^2$$
$$= \sqrt{21 \times 6 \times 7 \times 8} \text{ cm}^2$$
$$= \sqrt{3 \times 7 \times 2 \times 3 \times 7 \times 2 \times 2 \times 2} \text{ cm}^2$$
$$= 2 \times 2 \times 3 \times 7 = 84 \text{ cm}^2 \quad \ldots (1)$$

In ΔBCD, let $a = 12$ cm, $b = 9$ cm and $c = 15$ cm.

Then, $s = \frac{a+b+c}{2}$
$$= \frac{12+9+15}{2} \text{ cm}$$
$$= \frac{36}{2} \text{ cm}$$
$$= 18 \text{ cm}$$
$$ar(\Delta BCD) = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{18(18-12)(18-9)(18-15)} \text{ cm}^2$$
$$= \sqrt{18 \times 6 \times 9 \times 3} \text{ cm}^2$$
$$= \sqrt{2 \times 9 \times 2 \times 3 \times 9 \times 3} \text{ cm}^2$$
$$= 2 \times 3 \times 9 \text{ cm}^2$$
$$= 54 \text{ cm}^2 \quad \ldots (2)$$
$$ar(\text{quad. } ABCD) = ar(\Delta ABD) + ar(\Delta BCD)$$
$$= 84 \text{ cm}^2 + 54 \text{ cm}^2 \quad \text{[Using (1) and (2)]}$$
$$= 138 \text{ cm}^2$$

Hence, the area of given quadrilateral is **138 cm^2.**

7. Perimeter of quadrilateral ABCD
$$= AB + BC + CD + DA$$
$$= (17 + 8 + 13 + 14) \text{ cm}$$
$$= 52 \text{ cm}$$



In right ΔACB, we have
$$AC^2 + BC^2 = AB^2 \quad \text{[By Pythagoras' Theorem]}$$
$$\Rightarrow AC^2 + (8 \text{ cm})^2 = (17 \text{ cm})^2$$
$$\Rightarrow AC^2 = (289 - 64) \text{ cm}^2 = 225 \text{ cm}^2$$
$$\Rightarrow AC = 15 \text{ cm} \quad \ldots (1)$$
$$ar(\Delta ACB) = \frac{1}{2} \times CB \times AC$$
$$= \frac{1}{2} \times 8 \text{ cm} \times 15 \text{ cm} \quad \text{[Using (1)]}$$
$$= 60 \text{ cm}^2 \quad \ldots (2)$$

In ΔADC,

Let $a = DC = 13$ cm, $b = AC = 15$ cm

and $c = AD = 14$ cm [Using (1)]

Then, $s = \frac{a+b+c}{2}$
$$= \frac{13+15+14}{2} \text{ cm}$$
$$= \frac{42}{2} \text{ cm} = 21 \text{ cm}$$

$$\text{ar}(\Delta ADC) = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{21(21-13)(21-15)(21-14)} \text{ cm}^2$$
$$= \sqrt{21 \times 8 \times 6 \times 7} \text{ cm}^2$$
$$= \sqrt{3 \times 7 \times 2 \times 2 \times 2 \times 2 \times 3 \times 7} \text{ cm}^2$$
$$= 2 \times 2 \times 3 \times 7 \text{ cm}^2$$
$$= 84 \text{ cm}^2 \quad \ldots (3)$$
$$\text{ar(quad ABCD)} = \text{ar}(\Delta ACB) + \text{ar}(\Delta ADC)$$
$$= 60 \text{ cm}^2 + 84 \text{ cm}^2 \quad \text{[Using (2) and (3)]}$$
$$= 144 \text{ cm}^2$$

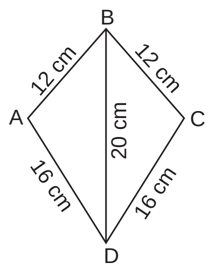
Hence, the area of the given quadrilateral is **144 cm**^{2} and its perimeter is **52 cm**.

8. $\Delta ABD \cong \Delta CBD$ [By SSS congruence]

$\Rightarrow$ $\text{ar}(\Delta ABD) = \text{ar}(\Delta CBD)$ … (1)

$\text{ar(kite ABCD)} = \text{ar}(\Delta ABD) + \text{ar}(\Delta CBD)$

$\Rightarrow$ $\text{ar(kite ABCD)} = 2\,\text{ar}(\Delta ABD)$ [Using (1)] …(2)



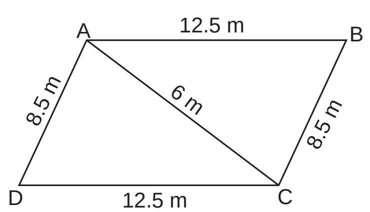
In ΔABD,

Let a = BD = 20 cm, b = AD = 16 cm and c = AB = 12 cm

Then, $$s = \frac{a+b+c}{2}$$
$$= \frac{20+16+12}{2} \text{ cm}$$
$$= \frac{48}{2} \text{ cm} = 24 \text{ cm}$$
$$\text{ar}(\Delta ABD) = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{24(24-20)(24-16)(24-12)} \text{ cm}^2$$
$$= \sqrt{24 \times 4 \times 8 \times 12} \text{ cm}^2$$
$$= \sqrt{12 \times 2 \times 4 \times 8 \times 12} \text{ cm}^2$$
$$= 8 \times 12 \text{ cm}^2$$
$$= 96 \text{ cm}^2 \quad \ldots (3)$$
$$\text{ar(kite ABCD)} = 2\,\text{ar}(\Delta ABD) \quad \text{[Using (2)]}$$
$$= 2 \times 96 \text{ cm}^2 \quad \text{[Using (3)]}$$
$$= 192 \text{ cm}^2$$

Hence, the area of the given kite = **192 cm**^{2}

9. Let ABCD represent the field in the form of a parallelogram such that AB = 12.5 m, BC = 8.5 m and one diagonal (say) AC = 6 m.



We know that the diagonal of a parallelogram divides it into two congruent triangles.

$\Rightarrow$ $\Delta ADC \cong \Delta CBA$

$\Rightarrow$ $\text{ar}(\Delta ADC) = \text{ar}(\Delta CBA)$ … (1)

$\Rightarrow$ $\text{ar}(\|\text{gm ABCD}) = \text{ar}(\Delta ADC) + \text{ar}(\Delta CBA)$

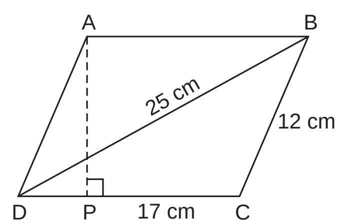
$\Rightarrow$ $\text{ar}(\|\text{gm ABCD}) = 2\,\text{ar}(\Delta ADC)$ [Using (1)] …(2)

In ΔADC, let a = CD = 12.5 m, b = AC = 6 cm and c = AD = 8.5 m.

$$s = \frac{a+b+c}{2}$$
$$= \frac{12.5+6+8.5}{2} \text{ m}$$
$$= \frac{27.0}{2} \text{ m}$$
$$= 13.5 \text{ m}$$
$$\text{ar}(\Delta ADC) = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{13.5(13.5-12.5)(13.5-6)(13.5-8.5)} \text{ m}^2$$
$$= \sqrt{13.5 \times 1 \times 7.5 \times 5} \text{ m}^2$$
$$= \sqrt{506.35} \text{ m}^2$$
$$= 22.5 \text{ m}^2$$
$$\text{ar}(\|\text{gm ABCD}) = 2\,\text{ar}(\Delta ADC) \quad \text{[Using (2)]}$$
$$= 2 \times 22.5 \text{ m}^2 \quad \text{[Using (3)]}$$
$$= 45 \text{ m}^2$$

Hence the area of the given field is **45 m**^{2}.

10. In ΔBCD, let a = BC = 12 cm, b = DC = 17 cm and c = BD = 25 cm.



Then, $$s = \frac{a+b+c}{2}$$
$$= \frac{12+17+25}{2} \text{ cm}$$
$$= \frac{54}{2} \text{ cm}$$
$$= 27 \text{ cm}$$
$$\text{ar}(\Delta BCD) = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{27(27-12)(27-17)(27-25)} \text{ cm}^2$$
$$= \sqrt{27 \times 15 \times 10 \times 2} \text{ cm}^2$$
$$= \sqrt{3 \times 3 \times 3 \times 3 \times 5 \times 2 \times 5 \times 2} \text{ cm}^2$$
$$= 2 \times 3 \times 3 \times 5 \text{ cm}^2$$
$$= 90 \text{ cm}^2 \quad \ldots (1)$$

We know that the diagonal of a parallelogram divides it into two congruent triangles.

$\Rightarrow$ $\Delta BCD \cong \Delta DAB$

$\Rightarrow$ $\text{ar}(\Delta BCD) = \text{ar}(\Delta DAB)$ … (2)

$$\begin{aligned} ar(\|gm\ ABCD) &= ar(\Delta BCD) + ar(\Delta DAB) \\ &= 2\ ar(\Delta BCD) \quad \text{[Using (2)]} \\ &= 2 \times 90\ cm^2 \quad \text{[Using (1)]} \\ &= 180\ cm^2 \end{aligned}$$

$ar(\|gm\ ABCD) = 180\ cm^2$

$\Rightarrow \quad DC \times AP = 180\ cm^2$

$\Rightarrow \quad 17\ cm \times AP = 180\ cm^2$

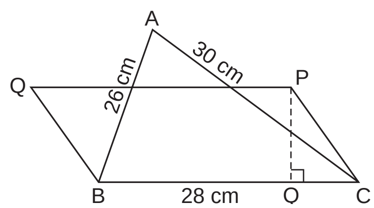
$\Rightarrow \quad AP = \frac{180}{17}\ cm$

$= 10.588\ cm$ (approx.)

$= 10.6\ cm$ (approx.)

Hence, the area of the given parallelogram ABCD is **180 cm**^{2} and the length of the perpendicular AP drawn from vertex A on the side DC is **10.6 cm** approx.

11. Let ABC represent the given triangle in which BC = 28 cm, AB = 26 cm and AC = 30 cm.



Let BCPO be the given parallelogram on base BC = 28 cm.
Let PQ ⊥ BC. Then, PQ is the height of ||gm BCPQ.
In ΔABC, let a = BC = 28 cm, b = AC = 30 cm and c = AB = 26 cm.

Then, $s = \frac{a+b+c}{2}$

$= \frac{28+30+26}{2}\ cm$

$= \frac{84}{2}\ cm$

$= 42\ cm$

$ar(\Delta ABC) = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{42(42-28)(42-30)(42-26)}\ cm^2$

$= \sqrt{42 \times 14 \times 12 \times 16}\ cm^2$

$= \sqrt{3 \times 14 \times 14 \times 12 \times 16}\ cm^2$

$= \sqrt{14 \times 14 \times 6 \times 6 \times 4 \times 4}\ cm^2$

$= 4 \times 6 \times 14\ cm^2$

$= 336\ cm^2 \quad \ldots (1)$

$ar(\|gm\ BCPQ) = BC \times$ altitude PQ

$= 28\ cm \times$ altitude PQ $\quad \ldots (2)$

$ar(\|gm\ BCPQ) = ar(\Delta ABC)$ [Given]

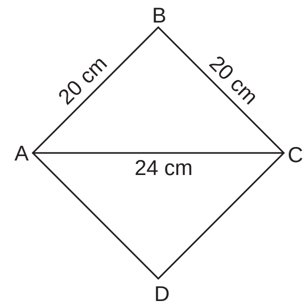
$\Rightarrow \quad 28\ cm \times$ altitude PQ $= 336\ cm^2$ [Using (1) and (2)]

$\Rightarrow \quad$ altitude PQ $= \frac{336}{28}\ cm$

$= 12\ cm$

Hence, the height of the given parallelogram is **12 cm.**

12. Let ABCD represent the given rhombus of side 20 cm and one diagonal ray AC = 24 cm.



In ΔABC let a = BC = 20 cm, b = AC = 24 cm and c = AB = 20 cm.

$s = \frac{a+b+c}{2}$

$= \frac{20+24+20}{2}\ cm$

$= \frac{64}{2}\ cm$

$= 32\ cm$

$ar(\Delta ABC) = \sqrt{s(s-a)(s-b)(s-c)}\ cm^2$

$= \sqrt{32(32-20)(32-24)(32-20)}\ cm^2$

$= \sqrt{32 \times 12 \times 8 \times 12}\ cm^2$

$= \sqrt{4 \times 8 \times 12 \times 8 \times 12}\ cm^2$

$= 2 \times 8 \times 12\ cm^2$

$= 192\ cm^2 \quad \ldots (1)$

We know that the diagonal of rhombus divides it into two congruent triangles.

$\therefore \quad \Delta ABC \cong \Delta ADC$

$\Rightarrow \quad ar(\Delta ABC) = ar(\Delta ADC) \quad \ldots (2)$

$ar(\text{rhombus } ABCD) = ar(\Delta ABC) + ar(\Delta ADC)$

$= 2\ ar(\Delta ABC)$ [Using (2)]

$= 2 \times 192\ cm^2$ [Using (1)]

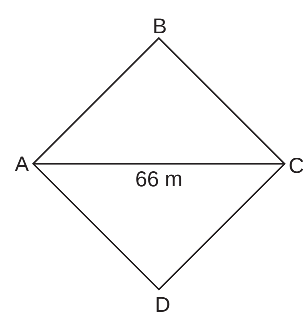
$= 384\ cm^2$

Hence, the area of the given rhombus is **384 cm**^{2}**.**

13. Let ABCD represent the rhombus shaped farm having one diagonal (say) AC = 66 m.
Perimeter of rhombus ABCD = 260 m

$\therefore$ Each side of rhombus $= \frac{260}{2}\ m$

$= 65\ m$



In ΔABC, let a = BC = 65 m, b = AC = 66 m and c = AB = 65 m.

$$s = \frac{a+b+c}{2}$$
$$= \frac{65+66+65}{2} \text{ m}$$
$$= \frac{196}{2} \text{ m}$$
$$= 98 \text{ m}$$
$$ar(\Delta ABC) = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{98(98-65)(98-66)(198-65)} \text{ m}^2$$
$$= \sqrt{98 \times 33 \times 32 \times 33} \text{ m}^2$$
$$= \sqrt{2 \times 49 \times 33 \times 32 \times 33} \text{ m}^2$$
$$= \sqrt{49 \times 64 \times 33 \times 33} \text{ m}^2$$
$$= 7 \times 8 \times 33 = 1848 \text{ m}^2 \quad \ldots (1)$$

We know that the diagonal of a rhombus divides it into two congruent triangles.

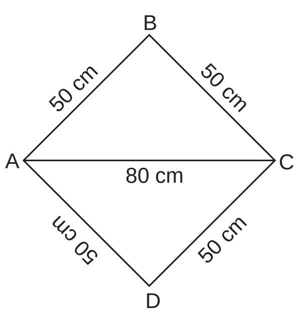
$\Rightarrow$ $\Delta ABC \cong \Delta ADC$

$\Rightarrow$ $ar(\Delta ABC) = ar(\Delta ADC)$

$\Rightarrow$ $ar(\Delta ABC) = ar(\Delta ADC) = 1848 \text{ m}^2$ [Using (1)]

Hence, each labourer will get **1848 m²** of area tilling.

14. Let ABCD represent the given rhombus of side 50 cm and one diagonal say AC = 80 cm.



In ΔABC, let a = BC = 50 cm, b = AC = 80 cm and c = AB = 50 cm.

Then, $$s = \frac{a+b+c}{2}$$
$$= \frac{50+80+80}{2} \text{ cm}$$
$$= \frac{180}{2} \text{ cm}$$
$$= 90 \text{ cm}$$
$$ar(\Delta ABC) = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{90(90-50)(90-80)(90-50)} \text{ cm}^2$$
$$= \sqrt{90 \times 40 \times 10 \times 40} \text{ cm}^2$$
$$= \sqrt{9 \times 10 \times 40 \times 10 \times 40} \text{ cm}^2$$
$$= 3 \times 10 \times 40 \text{ cm}^2$$
$$= 1200 \text{ cm}^2 \quad \ldots (1)$$

We know that the diagonal of a rhombus divides it into two congruent triangles.

$\Rightarrow$ $\Delta ABC \cong \Delta ADC$

$\Rightarrow$ $ar(\Delta ABC) = ar(\Delta ADC)$ … (2)

$ar(\text{rhombus } ABCD) = ar(\Delta ABC) + ar(\Delta ADC)$
$= 2\, ar(\Delta ABC)$ [Using (2)]
$= 2 \times 1200 \text{ cm}^2$ [Using (1)]
$= 2400 \text{ cm}^2$ … (3)

Also $ar(\text{rhombus } ABCD) = \frac{1}{2} \times AC \times BD$ … (4)

From (3) and (4) we get

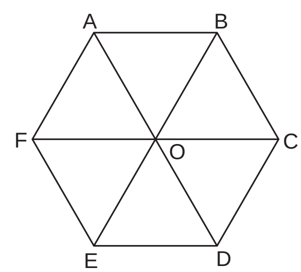
$$\frac{1}{2} \times AC \times BD = 2400 \text{ cm}^2$$

$\Rightarrow$ $\frac{1}{2} \times 80 \text{ cm} \times BD = 2400 \text{ cm}^2$

$\Rightarrow$ $BD = \frac{2400 \times 2}{80} \text{ cm} = 60 \text{ cm}$

Hence, the area of the given rhombus is **2400 cm²** and the length of its other diagonal is **60 cm**.

15. Let ABCDEF represent the hexagonal shaped table mat made up of six equilateral triangles, each of perimeter 60 cm.



Then, side of each of the equal triangle $= \frac{60}{3}$ cm
$= 20$ cm

Let AOB represent one of these equilateral triangles.

In ΔAOB, a = OA = 20 cm, b = 0B = 20 cm and c = AB = 20 cm.

$$s = \frac{20+20+20}{2} \text{ cm}$$
$$= \frac{60}{2} \text{ cm}$$
$$= 30 \text{ cm}$$
$$ar(\Delta AOB) = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{30(30-20)(30-20)(30-20)} \text{ cm}^2$$
$$= \sqrt{30 \times 10 \times 10 \times 10} \text{ cm}^2$$
$$= 100\sqrt{3} \text{ cm}^2$$
$$= 100 \times 1.73 \text{ cm}^2$$
$$= 173 \text{ cm}^2$$

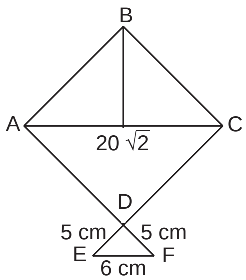
Area of the table mat $= 6 \times ar(\Delta AOB)$
$= 6 \times 173 \text{ cm}^2$
$= 1038 \text{ cm}^2$

Hence, the area of the given table mat is **1038 cm²**.

16. $\sqrt{2}$ side = Diagonal of a square

$\Rightarrow$ $\sqrt{2}$ side = $20\sqrt{2}$ cm

$\Rightarrow$ side = 20 cm



In ΔABC, let a = BC = 20 cm, b = AC = $20\sqrt{2}$ cm and c = AB = 20 cm

Then, $s = \dfrac{a+b+c}{2}$

$= \dfrac{20 + 20\sqrt{2} + 20}{2}$ cm

$= \dfrac{20 + 20\sqrt{2}}{2}$ cm

$= (20 + 10\sqrt{2})$ cm

ar(ΔABC) = $\sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{(20+10\sqrt{2})(20+10\sqrt{2}-20)(20+10\sqrt{2}-20\sqrt{2})(20+10\sqrt{2}-20)}$ cm^2

$= \sqrt{(20+10\sqrt{2})(10\sqrt{2})(20-10\sqrt{2})(10\sqrt{2})}$ cm^2

$= 10\sqrt{2}\sqrt{(20)^2-(10\sqrt{2})^2}$ cm^2

$= 10\sqrt{2}\sqrt{400 - 100\times 2}$ cm^2

$= 10\sqrt{2}\sqrt{200}$ cm^2

$= 10\sqrt{2} \times 10\sqrt{2}$ cm^2

= 100 × 2 cm^2

= 200 cm^2 … (1)

We know that the diagonal of a square divides it into two congruent triangles.

$\Rightarrow$ $\Delta ABC \cong \Delta ADC$

$\Rightarrow$ ar(ΔABC) = (ΔADC) … (2)

Area of red paper used = ar(sq ABCD)

= ar(ΔABC) + ar(ΔADC)

= 2 ar(ΔABC) [Using(2)]

= 2 × 200 cm^2 [Using (1)]

= 400 cm^2

In ΔDEF, let a = DE = 50 cm, b = DF = 5 cm and c = EF = 6 cm.

$s = \dfrac{a+b+c}{2}$

$= \dfrac{5+5+6}{2}$ cm

$= \dfrac{16}{2}$ cm

= 8 cm

Area of green paper used = ar(ΔDEF)

$= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{8(8-5)(8-5)(8-6)}$ cm^2

$= \sqrt{8\times 3\times 3\times 2}$ cm^2

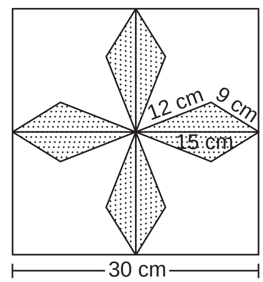
$= \sqrt{3\times 3\times 16}$ cm^2

= 3 × 4 cm^2

= 12 cm^2

Hence, the area of the **red paper** used is **400 cm^2** and the area of the **green paper** used in **12 cm^2.**

17. Here, for each of the 8 triangles a = 12 cm, b = 9 cm and c = 15 cm.



$s = \dfrac{a+b+c}{2}$

$= \dfrac{12+9+15}{2}$ cm

$= \dfrac{36}{2}$ cm

= 18 cm

Area of each small triangle

$= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{18(18-12)(18-9)(18-15)}$ cm^2

$= \sqrt{18\times 6\times 9\times 3}$ cm^2

$= \sqrt{2\times 9\times 2\times 3\times 9\times 3}$ cm^2

= 2 × 3 × 9 cm^2

= 54 cm^2

Total area of the design = 8 × 54 cm^2

= 432 cm^2

Area of the square tile = (30 × 30) cm^2

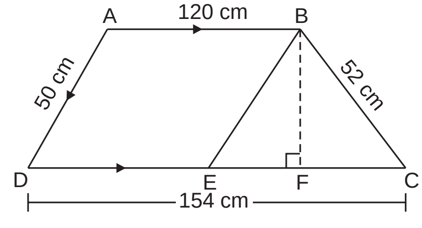
= 900 cm^2

Remaining area of the tile = (900 – 432) cm^2

= 468 cm^2

Hence, the total area of the design is **432 cm^2** and the remaining area of the tile is **468 cm^2.**

18. Let ABCD be the trapezium in which AB = 120 cm, BC = 52 cm, CD = 154 cm and AD = 50 cm.



Through B draw BE ∥ AD and let it meet DC at E.
Also draw BF ⊥ DC
Now, BE = AD = 50 cm [Opp sides of a ∥gm]
and EC = DC – DE
= DC – AB [AB = DE, opp. sides of a ∥gm]
= (154 – 120) cm
= 34 cm

In ΔBEC, we have
a = BE = 50 cm, b = EC = 34 cm and b = BC = 52 cm

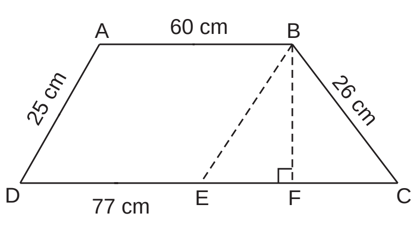
$$s = \frac{a+b+c}{2}$$
$$= \frac{50+34+52}{2} \text{ cm}$$
$$= \frac{136}{2} \text{ cm}$$
$$= 68 \text{ cm}$$
$$\text{ar}(\Delta BEC) = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{68(68-50)(68-34)(68-52)} \text{ cm}^2$$
$$= \sqrt{68(18)(34)(16)} \text{ cm}^2$$
$$= \sqrt{2 \times 34 \times 2 \times 9 \times 34 \times 16} \text{ cm}^2$$
$$= 2 \times 3 \times 4 \times 34 \text{ cm}^2$$
$$= 816 \text{ cm}^2 \quad \dots (1)$$

Also, $\text{ar}(\Delta BEC) = \frac{1}{2} \times 34 \text{ cm} \times BF \quad \dots (2)$

From(1) and (2), we get
$$\frac{1}{2} \times 34 \text{ cm} \times BF = 816 \text{ cm}^2$$
$$\Rightarrow \quad BF = \frac{816 \times 2}{34} \text{ cm}$$
$$= 48 \text{ cm} \quad \dots (3)$$
$$\text{ar(trapezium ABCD)} = \frac{1}{2}(AB + DC) \times BF$$
$$= \frac{1}{2}(120 + 154) \times 48 \text{ cm}^2$$
[Using (3)]
$$= \frac{1}{2}(274) \times 48 \text{ cm}^2$$
$$= 6576 \text{ cm}^2$$

Hence, the area of the given trapezium is **6576 cm²**.

19. Let ABCD be the trapezium in which AB = 60 cm, BC = 26 cm, CD = 77 cm and AD = 25 cm.



Through B draw BE ∥ AD and let it meet DC at E.
Also draw BF ⊥ DC.
Now, BE = AD = 25 cm [Opp. sides of a ∥gm]
and EC = DC – DE
= DC – AB [∵ AB = DE, Opp. sides of a ∥gm]
= (77 – 60) cm
= 17 cm

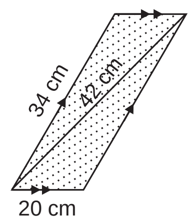
In ΔBEC, we have
a = BC = 26 cm, b = EC = 17 cm and c = BE = 25 cm
$$\text{ar}(\Delta BEC) = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{34(34-26)(34-17)(34-25)} \text{ cm}^2$$
$$= \sqrt{34 \times 8 \times 17 \times 9} \text{ cm}^2$$
$$= \sqrt{2 \times 17 \times 2 \times 2 \times 2 \times 17 \times 3 \times 3} \text{ cm}^2$$
$$= 2 \times 2 \times 3 \times 17 \text{ cm}^2$$
$$= 204 \text{ cm}^2 \quad \dots(1)$$

Also, $\text{ar}(\Delta BEC) = \frac{1}{2} \times EC \times BF$
$$= \frac{1}{2} \times 17 \text{ cm} \times BF \quad \dots (2)$$

From(1) and (2) we get
$$\frac{1}{2} \times 17 \text{ cm} \times BF = 204 \text{ cm}^2$$
$$\Rightarrow \quad BF = \frac{204 \times 2}{17} \text{ cm} = 24 \text{ cm} \quad \dots (3)$$
$$\text{ar(trapezium ABCD)} = \frac{1}{2}(AB + DC) \times BF$$
$$= \frac{1}{2}(60 + 77) \times 24 \text{ cm}^2$$
$$= 1644 \text{ cm}^2$$

Hence, the area of the given trapezium is **1644 cm²**.

20. Let ABCD represent one of the 8 congruent parallelograms which have to be painted red.



Then, AB = DC = 20 cm [Opp. sides of a ∥gm]
In ΔABD we have
a = BD = 42 cm, b = AD = 34 cm and c = AB = 20 cm.
$$s = \frac{a+b+c}{2}$$
$$= \frac{42+34+20}{2} \text{ cm}$$
$$= \frac{96}{2} \text{ cm}$$
$$= 48 \text{ cm}$$
$$\text{ar}(\Delta ABD) = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{48(48-42)(48-34)(48-20)} \text{ cm}^2$$
$$= \sqrt{48 \times 6 \times 14 \times 28} \text{ cm}^2$$
$$= \sqrt{2 \times 2 \times 2 \times 6 \times 6 \times 14 \times 2 \times 14} \text{ cm}^2$$
$$= 2 \times 2 \times 6 \times 14 \text{ cm}^2$$
$$= 336 \text{ cm}^2 \quad \dots (1)$$

Since, the diagonal of a parallelogram divides it into two congruent triangles.
$$\therefore \quad \Delta ABD \cong \Delta CDB$$
$$\Rightarrow \quad \text{ar}(\Delta ABD) = \text{ar}(\Delta ADB) \quad \dots (2)$$

$$\begin{aligned} ar(\|gm\ ABCD) &= ar(ABD) + ar(\Delta CDB) \\ &= 2\ ar(\Delta ABD) \quad [\text{Using (2)}] \\ &= 2 \times 336\ cm^2 \\ &= 672\ cm^2 \quad \ldots (3) \end{aligned}$$

$$\begin{aligned} \text{Area to the painted red} &= 8 \times ar(\|gm\ ABC) \\ &= 8 \times 672\ cm^2 \quad [\text{Using (3)}] \\ &= 5376\ cm^2 \end{aligned}$$

Hence, the total area to be painted red is **5376 cm^2**.

CHECK YOUR UNDERSTANDING

MULTIPLE-CHOICE QUESTIONS

1. (*b*) **40 cm^2**

$$\text{Area of a triangle} = \frac{1}{2}\ \text{base} \times \text{altitude}$$
$$= \frac{1}{2} \times 8\ cm \times 10\ cm$$
$$= 40\ cm$$

2. (*c*) **96 cm^2**

$$s = \frac{a+b+c}{2}$$
$$= \frac{12+16+20}{2}\ cm$$
$$= \frac{48}{2}\ cm$$
$$= 24\ cm$$

$$\text{Area of the } \Delta = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{24(24-12)(24-16)(24-20)}\ cm^2$$
$$= \sqrt{24 \times 12 \times 8 \times 4}\ cm^2$$
$$= \sqrt{2 \times 12 \times 12 \times 2 \times 2 \times 2 \times 2 \times 2}\ cm^2$$
$$= 2 \times 2 \times 2 \times 12\ cm^2$$
$$= 96\ cm^2$$

3. (*b*) **6 cm^2**

$$s = \frac{3+4+5}{2}\ cm$$
$$= \frac{12}{2}\ cm = 6\ cm$$

$$\text{Area of the triangle} = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{6(6-3)(6-4)(6-5)}\ cm^2$$
$$= \sqrt{6 \times 3 \times 2 \times 1}\ cm^2 = 26\ cm^2$$

4. (*b*) **16$\sqrt{3}$ m^2**

Perimeter of the given equilateral triangle = 24 m

$\therefore$ Each side of the given equilateral triangle $= \frac{24}{3}$ m

$= 8$ m

$$\text{Area of the given equilateral triangle} = \frac{\sqrt{3}}{4}\ \text{side}^2$$
$$= \frac{\sqrt{3}}{4}(8\ m)(8\ m)$$
$$= 16\sqrt{3}\ m^2$$

5. (*b*) **24 cm**

Area of the given equilateral triangle = $16\sqrt{3}$ cm^2 [Given]

$\Rightarrow \quad \frac{\sqrt{3}}{4}\ \text{side}^2 = 16\sqrt{3}\ cm^2$

$\Rightarrow \quad \text{side}^2 = 16 \times 4 = 64\ cm^2$

$\Rightarrow \quad \text{sides} = 8\ cm$

Perimeter of the given equilateral triangle = 8 × 3 cm
= 24 cm

6. (*b*) **₹ 16.80**

Here, a = 6 cm, b = 8 cm and c = 10 cm

$$s = \frac{6+8+10}{2}\ cm$$
$$= \frac{24}{2}\ cm = 12\ cm$$

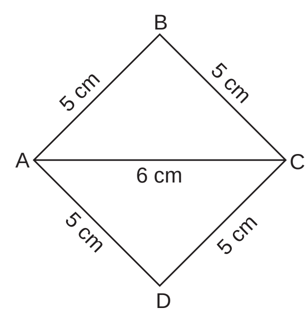
Area of the triangular board

$$= \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{12(12-6)(12-8)(12-10)}\ cm^2$$
$$= \sqrt{12 \times 6 \times 4 \times 2}\ cm^2$$
$$= \sqrt{2 \times 6 \times 6 \times 2 \times 2 \times 2}\ cm^2$$
$$= 2 \times 2 \times 6\ cm^2$$
$$= 24\ cm^2$$

Cost of painting the triangular board = 24 × ₹ $\frac{70}{100}$

= ₹ 16.8.

7. (*c*) **24 cm^2**



Let ABCD be the given rhombus. Perimeter of rhombus = 20 cm.

$\therefore$ Each side of the rhombus = $\frac{20}{4}$ k cm = 5 cm

One of its diagonals say AC = 6 cm

In ΔABC, we have

a = BC = 5 cm, b = AC = 6 cm and c = AB = 5 cm

$$s = \frac{a+b+c}{2}$$
$$= \frac{5+6+5}{2}\ cm$$
$$= \frac{16}{2}\ cm$$
$$= 8\ cm$$

$$ar(\Delta ABC) = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{8(8-5)(8-6)(8-5)}\ cm^2$$

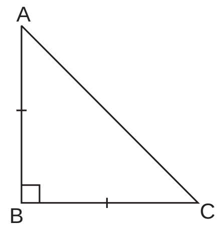
$$= \sqrt{8 \times 3 \times 2 \times 3} \text{ cm}^2$$
$$= 3 \times 4 \text{ cm}^2$$
$$= 12 \text{ cm}^2$$

ar(rhombus ABCD) = 2 × ar(ΔABC)

[∵ diagonal of a rhombus divides it into two congruent triangles of equal area.]

ar(rhombus ABCD) = 2 × 12 cm^2 = 24 cm^2

8. (*b*) $\sqrt{32}$ **cm**



Let ABC be isosceles right triangle in which AB = BC = x cm.

Area of ΔABC = 8 cm^2

$\Rightarrow \quad \frac{1}{2} \times BC \times AC = 8 \text{ cm}^2$

$\Rightarrow \quad \frac{1}{2} \times x \times x = 8 \text{ cm}^2$

$\Rightarrow \quad x^2 = 16 \text{ cm}^2$

$\Rightarrow \quad x = 4$ cm

$\Rightarrow \quad$ AB = BC = 4 cm

In right ΔABC, we have

$AB^2 + BC^2 = AC^2$ [By Pythagoras' Theorem]

$\Rightarrow \quad (4 \text{ cm})^2 + (4 \text{ cm})^2 = AC^2$

$\Rightarrow \quad 16 \text{ cm}^2 + 16 \text{ cm}^2 = AC^2$

$\Rightarrow \quad 32 \text{ cm}^2 = AC$

$\Rightarrow \quad$ hypotenuse AC = $\sqrt{32}$ cm

9. (*d*) **192 cm²**

Here, a = 20 cm, b = 24 cm and c = 20 cm

$$s = \frac{a+b+c}{2}$$
$$= \frac{20+24+20}{2} \text{ cm}$$
$$= \frac{64}{2} \text{ cm}$$
$$= 32 \text{ cm}$$

Area of the given triangle

$$= \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{32(32-20)(32-24)(32-20)} \text{ cm}^2$$
$$= \sqrt{32 \times 12 \times 8 \times 12} \text{ cm}^2$$
$$= \sqrt{4 \times 4 \times 2 \times 12 \times 2 \times 2 \times 2 \times 12} \text{ cm}^2$$
$$= 4 \times 2 \times 2 \times 12 = \text{cm}^2$$
$$= 192 \text{ cm}^2$$

10. (*a*) **32** $\sqrt{2}$ **cm²**

Let each, equal side of the given isosceles triangle be $3x$ then, the base = $2x$.

Perimeter = 32 cm

$\Rightarrow \quad 3x + 3x + 2x = 32$ cm

$\Rightarrow \quad 8x = 32$ cm

$\Rightarrow \quad x = 4$ cm

The sides of the triangle are 3 × 4 cm, 3 × 4 cm and 2 × 4 cm i.e 12 cm, 12 cm and 8 cm

Here, a = 12 cm, b = 12 cm and c = 8 cm

$$s = \frac{a+b+c}{2}$$
$$= \frac{12+12+8}{2} \text{ cm}$$
$$= \frac{32}{2} \text{ cm}$$
$$= 16 \text{ cm}$$

Area of the given triangle

$$= \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{16(16-12)(16-12)(16-8)} \text{ cm}^2$$
$$= \sqrt{16 \times 4 \times 4 \times 8} \text{ cm}^2$$
$$= 32\sqrt{2} \text{ cm}^2$$

11. (*d*) $\frac{5}{4}\sqrt{11}$ cm²

Let each equal side of the given isosceles triangle be x cm. Perimeter = 11 cm.

$\Rightarrow \quad x + x + 5 = 11$ cm

$\Rightarrow \quad 2x = 6$ cm

$\Rightarrow \quad x = 3$ cm

Here, a = 3 cm, b = 5 cm and c = 3 cm

$$s = \frac{a+b+c}{2}$$
$$= \frac{3+5+3}{2} \text{ cm}$$
$$= \frac{11}{2} \text{ cm}$$

Area of the given triangle

$$= \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{\frac{11}{2}\left(\frac{11}{2}-3\right)\left(\frac{11}{2}-5\right)\left(\frac{11}{2}-3\right)} \text{ cm}^2$$
$$= \sqrt{\frac{11}{2} \times \frac{5}{2} \times \frac{1}{2} \times \frac{5}{2}} \text{ cm}^2$$
$$= \frac{5}{4}\sqrt{11} \text{ cm}^2$$

12. (*c*) **84 cm²**

$(s - a)$ = 8 cm, $(s - b)$ = 7 cm and $(s - c)$ = 6 cm [Given]

$\Rightarrow \quad s - a + s - b + s - c = (8 + 7 + 6)$ cm= 21 cm

$\Rightarrow \quad 3s\ (a + b + c) = 21$ cm

$\Rightarrow \quad 3s - 2s = 21$ cm $\quad \left[\because \frac{a+b+c}{2} = s\right]$

$\Rightarrow \quad s = 21$ cm

$$\text{ar}(\Delta ABC) = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{21 \times 8 \times 7 \times 6} \text{ cm}^2$$
$$= \sqrt{3 \times 7 \times 2 \times 2 \times 2 \times 7 \times 2 \times 3} \text{ cm}^2$$

$= 2 \times 2 \times 3 \times 7 \text{ cm}^2$
$= 84 \text{ cm}^2$

13. (*b*) **11.2 cm**

Here, $a = 13$ cm, $b = 14$ cm and $c = 15$ cm

$$s = \frac{a+b+c}{2} = \frac{13+14+15}{2} \text{ cm} = \frac{42}{2} \text{ cm} = 21 \text{ cm}$$

Area of the given triangle

$$= \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{21(21-13)(21-14)(21-15)} \text{ cm}^2$$
$$= \sqrt{21 \times 8 \times 7 \times 6} \text{ cm}^2$$
$$= \sqrt{3 \times 7 \times 2 \times 2 \times 2 \times 7 \times 2 \times 3} \text{ cm}^2$$
$$= 2 \times 2 \times 3 \times 7 \text{ cm}^2$$
$$= 84 \text{ cm}^2 \quad \ldots (1)$$

Also, area of the given triangle

$$= \frac{1}{2} \times \text{longest side} \times \text{shortest altitude} \quad \ldots (2)$$
$$= \frac{1}{2} \times 15 \text{ cm} \times \text{shortely altitude}$$

From(1) and (2) we get

$$\frac{1}{2} \times 15 \text{ cm} \times \text{shortest altitude} = 84 \text{ cm}^2$$

$\Rightarrow$ Shortest altitude $= \frac{84 \times 2}{15} = 11.2$ cm

14. (*c*) **15.69 cm**

Here, $a = 17$ cm, $b = 25$ cm and $c = 26$ cm.

$$s = \frac{a+b+c}{2} = \frac{17+25+26}{2} \text{ cm} = \frac{68}{2} \text{ cm} = 34 \text{ cm}$$

Area of the given triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{34(37-17)(34-25)(34-26)} \text{ cm}^2$$
$$= \sqrt{34 \times 17 \times 9 \times 8} \text{ cm}^2$$
$$= \sqrt{2 \times 17 \times 17 \times 3 \times 3 \times 2 \times 2 \times 2} \text{ cm}^2$$
$$= 2 \times 2 \times 17 \times 3 \text{ cm}^2$$
$$= 12 \times 17 \text{ cm}^2 \quad \ldots (1)$$

Also, area of the given triangle

$$= \frac{1}{2} \times \text{longest side} \times \text{corresp. altitude}$$
$$= \frac{1}{2} \times 26 \text{ cm} \times \text{corr altitude} \quad \ldots (2)$$

From(1) and (2) we get

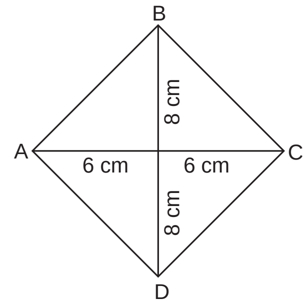
$$\frac{1}{2} \times 26 \text{ cm} \times \text{corresp. alt.} = 12 \times 17 \text{ cm}^2$$

$\Rightarrow$ corresp. alt. $= \frac{12 \times 17 \times 2}{26}$ cm

$= 15.69$ cm (approx.)

15. (*a*) $\mathbf{50\sqrt{2}}$ cm^2

Let ABCD be the given rhombus whose diagonals AC and BD are 12 cm and 16 cm respectively and they intersect at O.



Since, the diagonals of a rhombus bisect each other at right angles.

$\therefore$ AO = 6 cm and BC = 8 cm

In right ΔAOB, we have

$AC^2 + BC = AB^2$ [By Pythagoras' Theorem]

$\Rightarrow (6 \text{ cm})^2 + (8 \text{ cm})^2 = AB^2$

$\Rightarrow 100 \text{ cm}^2 = AB^2$

$\Rightarrow 10 \text{ cm} = AB$

$\therefore$ Sides of the rhombus = 10 cm

$\therefore$ Its perimeter $= 10 \times 4$ cm $= 40$ cm

Perimeter of isosceles Δ = Perimeter of the rhombus,

$\therefore$ Perimeter of isosceles Δ = 40 cm

Let each equal side of the isosceles triangle be $3x$ cm.

Then its base is $2x$ cm

$\Rightarrow 3x + 3x + 2x = 40$ cm

$\Rightarrow 8x = 40$ cm

$\Rightarrow x = 5$ cm

So, the sides of the isosceles triangle are 3×5 cm, (3×5) cm and (2×5) cm

i.e 15 cm, 15 cm and 10 cm.

Here, $a = 15$ cm, $b = 15$ cm and $c = 10$ cm.

$$s = \frac{a+b+c}{2} = \frac{15+15+10}{2} \text{ cm} = \frac{40}{2} \text{ cm} = 20 \text{ cm}$$

Area of the given equilateral triangle

$$= \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{20(20-15)(20-15)(20-10)} \text{ cm}^2$$
$$= \sqrt{20 \times 5 \times 5 \times 10} \text{ cm}^2$$
$$= \sqrt{2 \times 10 \times 5 \times 5 \times 10} \text{ cm}^2$$
$$= 50\sqrt{2} \text{ cm}^2$$

SHORT ANSWER QUESTIONS

1. Area of the given triangle

$$= \frac{1}{2} \times 10 \text{ cm} \times 7 \text{ cm} = 35 \text{ cm}^2$$

Hence, the area of the given triangle is **35 cm²**.

2. Here $a = 13$ cm, $b = 14$ cm and $c = 15$ cm

$$s = \frac{a+b+c}{2}$$
$$= \frac{13+14+15}{2} \text{ cm}$$
$$= \frac{42}{2} \text{ cm}$$
$$= 21 \text{ cm}$$

Area of the given triangle

$$= \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{21(21-13)(21-14)(21-13)} \text{ cm}^2$$
$$= \sqrt{21 \times 8 \times 7 \times 6} \text{ cm}^2$$
$$= \sqrt{3 \times 7 \times 2 \times 2 \times 2 \times 7 \times 2 \times 3} \text{ cm}^2$$
$$= 2 \times 2 \times 3 \times 7 \text{ cm}^2$$
$$= 84 \text{ cm}^2$$

Hence, the area of the given triangle is **84 cm²**.

3. Let the third side of the given triangle be x cm.

Then, $x + 9 + 12 = 36$

$\Rightarrow$ $x = 36 - 21 = 15$

$\therefore$ Third side = 15 cm

Here, $a = 9$ cm, $b = 12$ cm and $c = 15$ cm.

$$s = \frac{a+b+c}{2}$$
$$= \frac{9+12+15}{2} \text{ cm}$$
$$= \frac{36}{2} \text{ cm}$$
$$= 18 \text{ cm}$$

Area of the given triangle

$$= \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{18(18-9)(18-12)(18-15)} \text{ cm}^2$$
$$= \sqrt{18 \times 9 \times 6 \times 3} \text{ cm}^2$$
$$= \sqrt{2 \times 9 \times 9 \times 2 \times 3 \times 3} \text{ cm}^2$$
$$= 2 \times 9 \times 3 \text{ cm}^2$$
$$= 54 \text{ cm}^2$$

Hence, the area of the given triangle is **54 cm²**.

4. Let the sides of the given triangle be $3x$ cm, $5x$ cm and $7x$ cm.

Then, $3x + 5x + 7x = 60$ [$\because$ perimeter = 60 cm, given]

$\Rightarrow$ $15x = 60$

$\Rightarrow$ $x = 4$

So, the sides of the triangle are
(3×4) cm, (5×4) cm and (7×4) cm,
i. e. 12 cm, 20 cm and 28 cm.

Here, $a = 12$ cm, $b = 20$ cm and $c = 28$ cm,

$$s = \frac{a+b+c}{2}$$
$$= \frac{12+20+28}{2} \text{ cm}$$
$$= \frac{60}{2} \text{ cm}$$
$$= 30 \text{ cm}$$

Area of the given triangle

$$= \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{30(30-12)(30-20)(30-28)} \text{ cm}^2$$
$$= \sqrt{30 \times 18 \times 10 \times 2} \text{ cm}^2$$
$$= \sqrt{3 \times 10 \times 2 \times 3 \times 3 \times 10 \times 2} \text{ cm}^2$$
$$= 2 \times 3 \times 10\sqrt{3} \text{ cm}^2$$
$$= 60\sqrt{3} \text{ cm}^2$$

Here, the area of the given triangle is $\mathbf{60\sqrt{3}}$ **cm²**.

5. Perimeter of the given equilateral triangle = 24 cm

$\therefore$ Each side of the given equilateral triangle $= \frac{24}{3}$ cm

$= 8$ cm

Here, $a = 8$ cm, $b = 8$ cm and $c = 8$ cm.

$$s = \frac{a+b+c}{2}$$
$$= \frac{8+8+8}{2} \text{ cm}$$
$$= \frac{24}{3} \text{ cm}$$
$$= 12 \text{ cm}$$

Area of the given equilateral triangle

$$= \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{12(12-8)(12-8)(12-8)} \text{ cm}^2$$
$$= \sqrt{12 \times 4 \times 4 \times 4} \text{ cm}^2$$
$$= \sqrt{3 \times 4 \times 4 \times 4 \times 4} \text{ cm}^2$$
$$= 16\sqrt{3} \text{ cm}^2$$

Hence, the area of the given equilateral triangle is $\mathbf{16\sqrt{3}}$ **cm².**

6. Height of the given equilateral triangle = 6 cm

$$\Rightarrow \quad \frac{\sqrt{3}}{2} \times \text{side} = 6 \text{ cm}$$
$$\Rightarrow \quad \text{side} = \frac{6 \times 2}{\sqrt{3}}$$
$$= \frac{12}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 4\sqrt{3} \text{ cm}$$

Here, $a = 4\sqrt{3}$ cm, $b = 4\sqrt{3}$ cm and $c = 4\sqrt{3}$ cm.

$$s = \frac{a+b+c}{2}$$
$$= \frac{4\sqrt{3} + 4\sqrt{3} + 4\sqrt{3}}{2} \text{ cm}$$
$$= \frac{12\sqrt{3}}{2} \text{ cm}$$
$$= 6\sqrt{3} \text{ cm}$$

Area of the given equilateral triangle

$$= \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{6\sqrt{3}(6\sqrt{3}-4\sqrt{3})(6\sqrt{3}-4\sqrt{3})(6\sqrt{3}-4\sqrt{3})}\ \text{cm}^2$$

$$= \sqrt{3 \times 2\sqrt{3} \times 2\sqrt{3} \times 2\sqrt{3} \times 2\sqrt{3}}\ \text{cm}^2$$

$$= 2\sqrt{3} \times 2\sqrt{3} \times \sqrt{3}\ \text{cm}^2$$

$$= 12\sqrt{3}\ \text{cm}^2$$

$$= 12 \times 1.732\ \text{cm}^2$$

$$= 20.784$$

Hence, the area of the given equilateral triangle is **20.784 cm²**.

7. Here a = 13 cm, b = 13 cm and c = 24 cm

$$s = \frac{13+13+24}{2}\ \text{cm}$$

$$= \frac{50}{2} = 25\ \text{cm}$$

Area of the given isosceles triangle

$$= \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{25(25-13)(25-13)(25-24)}\ \text{cm}^2$$

$$= \sqrt{25 \times 12 \times 12 \times 1}\ \text{cm}^2$$

$$= 5 \times 12\ \text{cm}^2$$

$$= 60\ \text{cm}^2$$

Hence, the area of the given isosceles triangle is **60 cm².**

8. Let the sides of the original triangle be a, b, c and the sides of the new triangle be $2a$, $2b$ and $2c$.

Area of the original triangle $= \sqrt{s(s-a)(s-b)(s-c)}$

$= A_1$ (say)

where $s = \dfrac{a+b+c}{2}$

Area of the new triangle

$$= \sqrt{s_1(s_1-2a)(s_1-2b)(s_1-2c)}$$

where $s_1 = \dfrac{2a+2b+2c}{2} = a+b+c = 2s$

$$= \sqrt{2s(2s-2a)(2s-2b)(2s-2c)}$$

$$= \sqrt{2s \times 2(s-a) \times 2(s-b) \times 2 \times (s-c)}$$

$$= 4\sqrt{s(s-a)(s-b)(s-c)}$$

$$= 4A_1$$

Increase in the area $= 4A_1 - A_1 = 3A_1$

Percentage increase $= \dfrac{\text{Increase in the area}}{\text{original area}} \times 100\%$

$$= \frac{3A_1}{A_1} \times 100\%$$

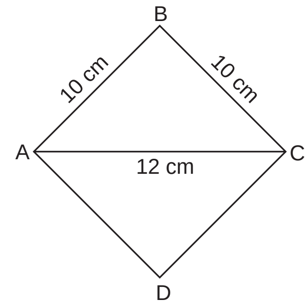
$$= 300\%$$

Hence, the percentage increase in the area of a triangle if each of its side is doubled is **300%**.

9. Let ABCD by the given rhombus whose perimeter is 40 cm.

Then, AB = BC = CD = DA = $\dfrac{40}{4}$ cm = 10 cm

Let one of its diagonal say AC = 12 cm



In ΔABC, let a = BC = 10 cm, b = AC = 12 cm and c = AB = 10 cm.

$$s = \frac{a+b+c}{2}$$

$$= \frac{10+12+10}{2}\ \text{cm}$$

$$= \frac{32}{2}\ \text{cm}$$

$$= 16\ \text{cm}$$

Area of ΔABC $= \sqrt{s(s-a)(s-b)(s-c)}$

$$= \sqrt{16(16-10)(16-12)(16-10)}\ \text{cm}^2$$

$$= \sqrt{16 \times 6 \times 4 \times 6}\ \text{cm}^2$$

$$= 2 \times 4 \times 6\ \text{cm}^2$$

$= 48\ \text{cm}^2$ … (1)

ar(rhombus ABCD) = 2 ar(ΔABC)

[∵ Diagonal of a rhombus divides it into two congruent triangles of equal area]

∴ ar(rhombus ABCD) = 2 × 48 cm² [Using (1)]

= 96 cm² … (2)

Area to be painted = 2 × ar(rhombus ABCD)

[∵ both sides are painted]

= 2 × 96 cm² [Using (2)]

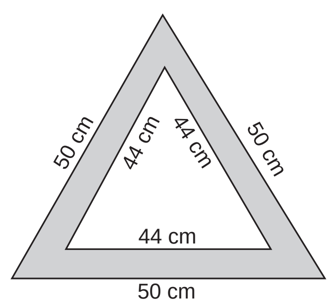
= 192 cm²

Cost of painting = ₹ 192 × 5

= ₹ 960

Hence, the cost of painting the given rhombus shaped sheet on both the sides is ₹ **960**.

10. Let a, b, c to be sides of the given larger triangle.



Then, $a = b = c$ = 50 cm

$$s = \frac{a+b+c}{2}$$

$$= \frac{50+50+50}{2}\ \text{cm}$$

$$= \frac{150}{2}\ \text{cm} = 75\ \text{cm}$$

Area of the given larger triangle

$= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{75(75-50)(75-50)(75-50)}$ cm^2

$= \sqrt{75 \times 25 \times 25 \times 25}$ cm^2

$= \sqrt{3 \times 25 \times 25 \times 25 \times 25}$ cm^2

$= 625\sqrt{3}$ cm^2

$= A_1$ (say)

Let a_1, b_1, c_1 be the sides of the given smaller (inner) triangle

Then, $a_1 = b_1 = c_1 = 44$ cm

$s_1 = \frac{a_1 + b_1 + c_1}{2}$

$= \frac{44 + 44 + 44}{2}$ cm

$= \frac{132}{2}$ cm

$= 66$ cm

Area of the given smaller triangle

$= \sqrt{s_1(s_1-a_1)(s_1-b_1)(s_1-c_1)}$

$= \sqrt{66(66-44)(66-44)(66-44)}$ cm^2

$= \sqrt{66 \times 22 \times 22 \times 22}$ cm^2

$= \sqrt{3 \times 22 \times 22 \times 22 \times 22}$ cm^2

$= 484\sqrt{3}$ cm^2

$= A_2$ (say)

Area to be painted $= A_1 - A_2$

$= (625\sqrt{3} - 484\sqrt{3})$ cm^2

$= 141\sqrt{3}$ cm^2

$= 141 \times 1.73$ cm^2

$= 243.93$ cm^2

Cost of painting = 243.93 × ₹ 1 = ₹ 243.93.

Hence, the cost of painter the shaded area is ₹ **243.93**.

VALUE-BASED QUESTIONS

1. (*i*) In right ΔCBD, we have

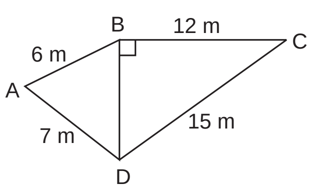
$BC^2 + BD^2 = CD^2$

[By Pythagoras' Theorem]

$\Rightarrow \quad (12 \text{ m})^2 + BD^2 = (15 \text{ m})^2$

$\Rightarrow \quad BD^2 = (225 - 144) \text{ m}^2 = 81 \text{ m}^2$

$\Rightarrow \quad BD = 9$ m …(1)



In ΔBAD, let a = BD = 9 m [Using (1)], b = AD = 7 m and c = AB = 6.

$s = \frac{a + b + c}{2}$

$= \frac{9 + 7 + 6}{2}$ m

$= \frac{22}{2}$ m

$= 11$ m

ar(ΔBAD) $= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{11(11-9)(11-7)(11-6)}$ m^2

$= \sqrt{11 \times 2 \times 4 \times 5}$ m^2

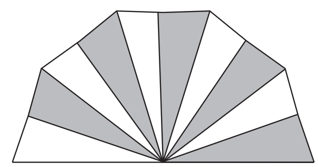
$= \sqrt{440}$ m^2

$= 20.98$ m^2 (approx.)

Hence, he donated **20.98 m²** (approx.) of land to an orphanage.

(*ii*) Empathy, concern for orphans.

2. (*i*) For each triangular strip a = 25 cm, b = 14 cm and c = 25 cm.



$s = \frac{a + b + c}{2}$

$= \frac{25 + 14 + 25}{2}$ cm

$= \frac{64}{2}$ cm

$= 32$ cm

Area of each triangular strip

$= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{32(32-25)(32-14)(32-25)}$

$= \sqrt{32 \times 7 \times 18 \times 7}$ cm^2

$= \sqrt{2 \times 16 \times 7 \times 2 \times 9 \times 7}$ cm^2

$= 2 \times 3 \times 4 \times 7$ cm^2

$= 168$ cm^2 … (1)

Area used for writing cleanliness slogans

= 5 × (Area of each triangular strip)

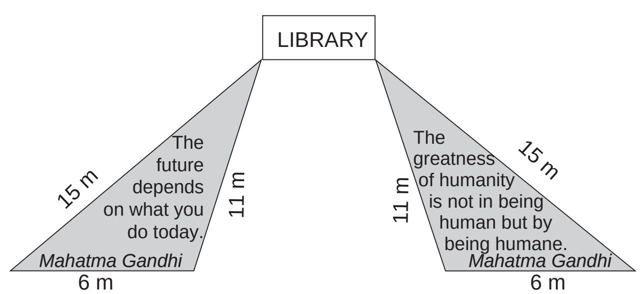
$= 5 \times 168$ cm^2

$= 840$ cm^2

Hence, the area used for writing the slogans was **840 cm²**.

(*ii*) Social responsibility, creative thinking, leadership, cooperation and awareness about maintaining cleanliness.

3. (*i*) For each triangular wall.



$a = 15$ m, $b = 11$ *m* and $c = 6$ m.

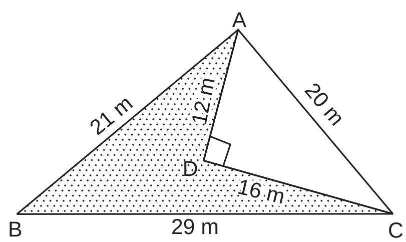
$$s = \frac{a+b+c}{2}$$
$$= \frac{15+11+6}{2} \text{ m}$$
$$= \frac{32}{2} \text{ m}$$
$$= 16 \text{ m}$$

Area of each triangular wall

$$= \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{16(16-15)(16-11)(16-6)} \text{ m}^2$$
$$= \sqrt{16 \times 1 \times 5 \times 10} \text{ m}^2$$
$$= \sqrt{800} \text{ m}^2$$
$$= \mathbf{20\sqrt{2}\ m^2}$$

(*ii*) Be hardworking, focussed, determined, caring, helpful, considerate and humane.

4. (*i*) In ΔABC, we have
$a = BC = 29$ m, $b = AC = 20$ m and $c = AB = 21$ m.



$$s = \frac{a+b+c}{2}$$
$$= \frac{29+20+21}{2} \text{ m}$$
$$= \frac{70}{2} \text{ m}$$
$$= 35 \text{ m}$$

$$\text{ar}(\Delta ABC) = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{35(35-29)(35-20)(35-21)} \text{ m}^2$$
$$= \sqrt{35 \times 6 \times 15 \times 14} \text{ m}^2$$
$$= \sqrt{5 \times 7 \times 2 \times 3 \times 3 \times 5 \times 2 \times 7} \text{ m}^2$$
$$= 2 \times 3 \times 5 \times 7 \text{ m}^2$$
$$= 210 \text{ m}^2 \quad \ldots (1)$$

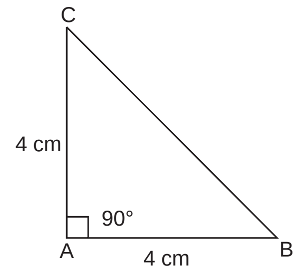
$$\text{ar}(\Delta ADC) = \frac{1}{2} \times DC \times AC$$
$$= \frac{1}{2} \times 16 \text{ m} \times 12 \text{ m}$$
$$= 96 \text{ m}^2 \quad \ldots (1)$$

Area of the plot left
= Total area of the plot – Area of the plot donated
= ar(ΔABC) – ar(ΔADC)
= $210 \text{ m}^2 - 96 \text{ m}^2$ [Using (1) and (2)]
= 114 m^2

(*ii*) Empathy, concern for old people, compassion, helpful, caring and decision-making ability.

UNIT TEST

1. (*c*) **8 cm²**



$$\text{ar}(\Delta CAB) = \frac{1}{2} \times AB \times AC$$
$$= \frac{1}{2} \times 4 \text{ cm} \times 4 \text{ cm} = 8 \text{ cm}^2$$

2. (*b*) $\mathbf{6\sqrt{91}\ cm^2}$

Here, $a = 8$ cm, $b = 15$ cm and $c = 19$ cm

$$s = \frac{a+b+c}{2}$$
$$= \frac{8+15+19}{2} \text{ cm}$$
$$= \frac{42}{2} \text{ cm}$$
$$= 21 \text{ cm}$$

Area of the given triangle

$$= \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{21(21-8)(21-15)(21-19)} \text{ cm}^2$$
$$= \sqrt{21 \times 13 \times 6 \times 2} \text{ cm}^2$$
$$= \sqrt{3 \times 7 \times 13 \times 2 \times 3 \times 2} \text{ cm}^2$$
$$= 2 \times 3\sqrt{91} \text{ cm}^2$$
$$= 6\sqrt{91} \text{ cm}^2$$

3. Perimeter of the given equilateral triangle = 18 cm.

∴ Each side of the given equilateral triangle $= \frac{18}{3}$ cm
$= 6$ cm

Here, $a = 6$ cm, $b = 6$ cm and $c = 6$ cm.

$$s = \frac{a+b+c}{2}$$
$$= \frac{6+6+6}{2} \text{ cm}$$
$$= \frac{18}{2} \text{ cm}$$
$$= 9 \text{ cm}$$

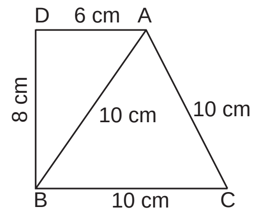
Area of the given equilateral triangle

$$= \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{9(9-6)(9-6)(9-6)} \text{ cm}^2$$
$$= \sqrt{9 \times 3 \times 3 \times 3} \text{ cm}^2$$
$$= 9\sqrt{3} \text{ cm}^2$$

$= 9 \times 1.732 \text{ cm}^2$
$= 15.588 \text{ cm}^2$

Hence, the area of the given equilateral triangle is **15.588 cm²**.

4. In ΔABC, we have
$a = BC = 10$ cm, $b = AC = 10$ cm and $c = AB = 10$ cm.



$$s = \frac{a+b+c}{2}$$
$$= \frac{10+10+10}{2} \text{ cm}$$
$$= \frac{30}{2} \text{ cm}$$
$$= 15 \text{ cm}$$
$$ar(\Delta ABC) = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{15(15-10)(15-10)(15-10)} \text{ cm}^2$$
$$= \sqrt{15 \times 5 \times 5 \times 5} \text{ cm}^2$$
$$= \sqrt{3 \times 5 \times 5 \times 5 \times 5} \text{ cm}^2$$
$$= 25\sqrt{3} \text{ cm}^2$$
$$= 25 \times 1.732 \text{ cm}^2$$
$$= 43.3 \text{ cm}^2 \qquad \ldots (1)$$

In ΔADB, we have
$$AD^2 + BD^2 = (6 \text{ cm})^2 + (8 \text{ cm})^2$$
$$= (36 + 64) \text{ cm}^2$$
$$= 100 \text{ cm}^2$$
and $$AB^2 = (10 \text{ cm})^2$$
$$= 100 \text{ cm}^2$$
$\therefore$ $$AD^2 + BD^2 = AB^2$$
$\Rightarrow$ $$\angle ADB = 90^\circ$$
[By the converse of Pythagoras' Theorem]

$$ar(\Delta ADB) = \frac{1}{2} \times AD \times BD$$
$$= \frac{1}{2} \times 6 \text{ cm} \times 8 \text{ cm}$$
$$= 24 \text{ cm}^2 \qquad \ldots (2)$$

Now, $ar(ABC) - ar(\Delta ADB)$
$$= 43.3 \text{ cm}^2 - 24 \text{ cm}^2 \qquad \text{[Using (1) and (2)]}$$
$$= 19.3 \text{ cm}^2$$

Hence, **19.3 cm²** needs to be added to area of ΔADB so that it becomes equal to be area of ΔABC.

5. Here, $a = BC = 48$ cm, $b = AC = 60$ cm and $c = A = 36$ cm.
$$s = \frac{a+b+c}{2}$$
$$= \frac{48+60+36}{2} \text{ cm}$$
$$= \frac{144}{2} \text{ cm}$$
$$= 72 \text{ cm}$$
$$ar(\Delta ABC) = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{72(72-48)(72-60)(72-60)} \text{ cm}$$
$$= \sqrt{72 \times 24 \times 12 \times 36} \text{ cm}^2$$
$$= \sqrt{3 \times 24 \times 24 \times 3 \times 2 \times 2 \times 6 \times 6} \text{ cm}^2$$
$$= 2 \times 3 \times 6 \times 24 \text{ cm}^2$$
$$= 864 \text{ cm}^2$$

Also $ar(\Delta ABC) = \frac{1}{2} \times$ longest side $\times$ shortest altitude
$$= \frac{1}{2} \times 60 \text{ cm} \times \text{shortest altitude} \qquad \ldots (2)$$

From(1) and (2) we get
$$\frac{1}{2} \times 60 \text{ cm} \times \text{shortest altitude} = 864 \text{ cm}^2$$
$\Rightarrow$ $$\text{shortest altitude} = \frac{864 \times 2}{60} \text{ cm}$$
$$= 28.8 \text{ cm}$$

Hence, the area of the given triangle is **840 cm²** and the length of its shortest altitude is **28.8 cm**.