CHAPTER **10**

Circles

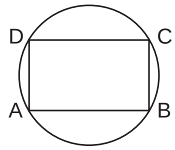
EXERCISE 10A

1. $\overset{\frown}{DAB} \cong \overset{\frown}{CDA}$

Subtracting $\overset{\frown}{DA}$ from both sides, we get

$$\overset{\frown}{DAB} - \overset{\frown}{DA} \cong \overset{\frown}{CDA} - \overset{\frown}{DA}$$

$\Rightarrow$ $\overset{\frown}{AB} \cong \overset{\frown}{CD}$



If two arcs are congruent, then corresponding chords are equal.

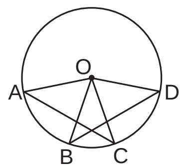
$\therefore$ **AB = CD**

2. $\overset{\frown}{AB} \cong \overset{\frown}{CD}$

Adding $\overset{\frown}{BC}$ to both sides, we get

$$\overset{\frown}{AB} + \overset{\frown}{BC} \cong \overset{\frown}{BC} + \overset{\frown}{CD}$$

$\Rightarrow$ $\overset{\frown}{ABC} \cong \overset{\frown}{BCD}$



If two arcs are congruent, then corresponding chords are equal.

$\therefore$ AC = BD ...(1)

In Δ OAC and ΔOBD, we have

OA = OB, AC = BD [using (1)]

and OC = OD

$\therefore$ $\Delta OAC \cong \Delta OBD$ [By SSS congruence]

$\Rightarrow$ $\angle OAC = \angle OBD$ [By CPCT]

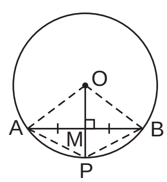
Hence, $\angle$**A** = $\angle$**B**

3. Let OM be the line segment joining M, the mid-point of chord AB and O the centre of the circle.

Then, OM $\perp$ AB.

Produce OM to intersect the minor arc corresponding to chord AB at P.

Join OA, OB, AP and BP.



In Δ OMA and Δ OMB, we have

OA = OB,

AM = BM, OM = OM [common]

$\Rightarrow$ $\Delta OMA \cong \Delta OMB$

$\Rightarrow$ $\angle AOM = \angle BOM$

$\Rightarrow$ $\angle AOP = \angle BOP$ ...(1)

In ΔOPA and ΔOPB, we have

OA = OB,

$\angle AOP = \angle BOP$ [using (1)]

OP = OP [common]

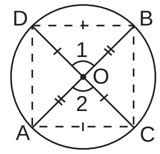
$\therefore$ $\Delta OPA \cong \Delta OPB$ [By SAS congruence]

$\Rightarrow$ PA = PB [CPCT]

If two chords of a circle are equal then their corresponding arcs are congruent.

$\therefore$ $\overset{\frown}{\mathbf{PA}} \cong \overset{\frown}{\mathbf{PB}}$

4. Let AB and CD be two chords which bisect each other at O. Join AC, BC, BD and AD.



In ΔAOC and ΔBOD, we have

AO = BO,

$\angle 1 = \angle 2$ [Ver. opp. $\angle$s],

CO = DO.

$\therefore$ $\Delta AOC \cong \Delta BOD$ [By SAS]

$\Rightarrow$ AC = BD [CPCT]

$\Rightarrow$ $\overset{\frown}{AC} \cong \overset{\frown}{BD}$ ...(1)

Also, $\Delta AOD \cong \Delta BOC$ [By SAS congruence]

$\Rightarrow$ DA = CB [CPCT]

$\Rightarrow$ $\overset{\frown}{DA} \cong \overset{\frown}{CB}$ ...(2)

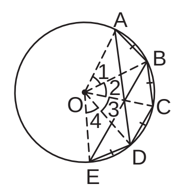
$\Rightarrow$ $\overset{\frown}{DA} + \overset{\frown}{AC} \cong \overset{\frown}{CB} + \overset{\frown}{BD}$ [Adding (2) and (1)]

$\Rightarrow$ $\overset{\frown}{DAC} \cong \overset{\frown}{CBD}$

$\Rightarrow$ CD divides the circle into two semicircles.

$\Rightarrow$ CD is a diameter. Similarly, AB is a diameter.

5. In ΔOAB and ΔOCB, we have



OA = OC [Radii of a circle]

AB = CB [given]

OB = OB [common]

$\therefore$ $\Delta OAB \cong \Delta OCB$ [By SSS congruence]

$\Rightarrow$ $\angle 1 = \angle 2 = x$ (say) [CPCT]

Similarly, $\Delta OBC \cong \Delta ODC$ [By SSS congruence]

$\angle 2 = \angle 3 = x$ [CPCT]

and $\Delta OCD \cong \Delta OED$ [By SSS congruence]

$\angle 3 = \angle 4 = x$ [CPCT]

In ΔOAD and ΔOBE, we have

OA = OB [radii of a circle],

$\angle AOD = \angle BOE = 3x$

and OD = OE [radii of a circle]

$\therefore$ $\Delta OAD \cong \Delta OBE$ [By SAS congruence]

$\Rightarrow$ **AD = BE** [CPCT]

Alternative Method

AB = BC = CD = DE [given]

$\Rightarrow$ BA = CB = DC = ED

$$\Rightarrow \quad \widehat{BA} \cong \widehat{CB} \cong \widehat{DC} \cong \widehat{ED}$$

[If the chords of a circle are equal then their corresponding arcs are congruent.]

$$\Rightarrow \quad \widehat{DC}+\widehat{CB}+\widehat{BA} \cong \widehat{ED}+\widehat{DC}+\widehat{CB}$$

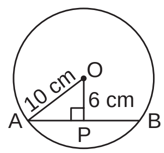
$$\Rightarrow \quad \widehat{DCBA} \cong \widehat{EDCB}$$

$$\Rightarrow \quad AD = BE$$

[If the arcs of a circle are congruent, then their corresponding chords are equal.]

Hence, **AD = BE.**

6. Let chord AB be at a distance of 6 cm from the centre O of a circle of radius 10 cm. Join OA Draw OP ⊥ AB. Then, OA = 10 cm and OP = 6 cm.



In right ΔOPA, we have

$$AP^2 + OP^2 = OA^2 \text{ [By Pythagoras Theorem]}$$

$$\Rightarrow \quad AP^2 = OA^2 - OP^2$$

$$\Rightarrow \quad AP^2 = (10\text{ cm})^2 - (6\text{ cm})^2$$

$$\Rightarrow \quad AP^2 = (100 - 36)\text{ cm}^2$$

$$\Rightarrow \quad AP^2 = 64\text{ cm}^2$$

$$\Rightarrow \quad AP = 8\text{ cm}$$

We know that the perpendicular from the centre of a circle to a chord bisects the chord.

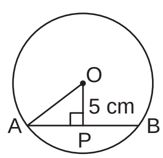
$$\frac{1}{2}AB = AP$$

$$\Rightarrow \quad \frac{1}{2}AB = 8\text{ cm}$$

$$\Rightarrow \quad AB = \mathbf{16\text{ cm}.}$$

7. Let AB = 24 cm be the chord of a circle with centre O. Draw OP ⊥ AB. Then, OP = 5 cm and AP = PB = 12 cm.

[∵ Perpendicular from the centre of the circle, to the chord bisects it.]



In right ΔAPO, we have

$$AP^2 + OP^2 = OA^2 \text{ [By Pythagoras Theorem]}$$

$$\Rightarrow \quad (12\text{ cm})^2 + (5\text{ cm})^2 = OA^2$$

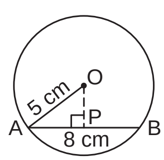
$$\Rightarrow \quad (144 + 25)\text{ cm}^2 = OA^2$$

$$\Rightarrow \quad 169\text{ cm}^2 = OA^2$$

$$\Rightarrow \quad OA = 13\text{ cm}$$

Hence, the radius of the given circle is **13 cm.**

8. Let AB = 8 cm be the chord of a circle with centre O. Then, radius OA = 5 cm.



Draw OP ⊥ AB

Then, $AP = \frac{1}{2}AB$

$$= \frac{1}{2} \times 8\text{ cm} = 4\text{ cm}$$

[∵ Perpendicular from the centre of a circle to a chord bisects the chord.]

In right ΔOPA, we have

$$OA^2 = OP^2 + AP^2 \quad \text{[By Pythagoras Theorem]}$$

$$\Rightarrow \quad OP^2 = OA^2 - AP^2 = (5\text{ cm})^2 - (4\text{ cm})^2$$

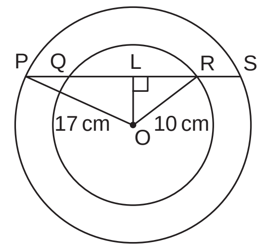
$$= (25 - 16)\text{ cm}^2 = 9\text{ cm}^2$$

$$\Rightarrow \quad OP = 3\text{ cm}$$

Hence, the distance of the chord from the centre of the circle is **3 cm**.

9. Draw OL ⊥ QR

Since the perpendicular from the centre of a circle to a chord bisects the chord.



$$\therefore \quad QL = LR = \frac{1}{2}QR$$

$$= \frac{1}{2} \times 12\text{ cm} = 6\text{ cm}$$

In right ΔOLR, we have

$$LR^2 + OL^2 = OR^2 \text{ [By Pythagoras Theorem]}$$

$$\Rightarrow \quad (6\text{ cm})^2 + OL^2 = (10\text{ cm})^2$$

$$\Rightarrow \quad OL^2 = (100 - 36)\text{ cm}^2 = 64\text{ cm}^2$$

$$\Rightarrow \quad OL = 8\text{ cm}$$

In right ΔOLP, we have

$$PL^2 + OL^2 = OP^2$$

$$\Rightarrow \quad PL^2 + (8\text{ cm})^2 = (17\text{ cm})^2$$

$$\Rightarrow \quad PL^2 = (289 - 64)\text{ cm}^2 = 225\text{ cm}^2$$

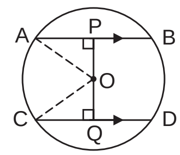
$$\Rightarrow \quad PL = 15\text{ cm}$$

Now, $PQ = PL - QL$

$$= 15\text{ cm} - 6\text{ cm} = 9\text{ cm}.$$

Hence, PQ = **9 cm.**

10. Since OP ⊥ AB, OQ ⊥ CD and AB || CD, therefore OP and OQ are in the same line i.e., P, O and Q are collinear points.



Join OA and OB. Let OP = x cm and OQ = y cm.

$$OA = OC = 5\text{ cm (given)}$$

We know that the perpendicular from the centre of a circle to a chord bisects the chord.

$$\therefore \quad AP = \frac{1}{2}AB$$

$$= \frac{1}{2} \times 8\text{ cm} = 4\text{ cm}$$

and

$$CQ = \frac{1}{2}CD$$

$$= \frac{1}{2} \times 6\text{ cm} = 3\text{ cm} \quad \text{...(1)}$$

In right ΔOPA, we have

$$OP^2 + AP^2 = OA^2$$

$$\Rightarrow \quad (x\text{ cm})^2 + (4\text{ cm})^2 = (5\text{ cm})^2 \quad \text{[using (1)]}$$

$$\Rightarrow \quad x^2 = (25 - 16) = 9$$

$$\Rightarrow \quad x = 3 \quad \text{...(2)}$$

In right ΔOQC, we have

$$OQ^2 + CQ^2 = OC^2$$

$$\Rightarrow \quad (y\text{ cm})^2 + (4\text{ cm})^2 = (5\text{ cm})^2 \quad \text{[using (1)]}$$

$\Rightarrow \quad y^2 = (25 - 9) = 16$

$\Rightarrow \quad y = 4 \quad \ldots(3)$

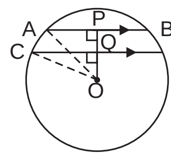
Adding (2) and (3), we get

$PQ = x \text{ cm} + y \text{ cm}$

$= 3 \text{ cm} + 4 \text{ cm} = 7 \text{ cm}$

Hence, $\quad PQ =$ **7 cm.**

11. Since AB || CD, OP ⊥ AB and OQ ⊥ CD, therefore OQ and OP are in the same line i.e. O, Q, P are collinear points.



We know that the perpendicular from the centre of a circle to a chord bisects it.

$\therefore \quad AP = \frac{1}{2}AB = \frac{1}{2} \times 6 \text{ cm} = 3 \text{ cm}$

and $\quad CQ = \frac{1}{2}CD = \frac{1}{2} \times 8 \text{ cm} = 4 \text{ cm} \quad \ldots(1)$

In right Δ APO, we have

$AP^2 + OP^2 = OA^2$

$\Rightarrow \quad (3 \text{ cm})^2 + (4 \text{ cm})^2 = OA^2$

$\Rightarrow \quad OA^2 = 25 \text{ cm}^2$

$\Rightarrow \quad OA = 5 \text{ cm.}$

In right ΔCQO, we have

$CQ^2 + OQ^2 = OC^2$

$\Rightarrow \quad (4 \text{ cm})^2 + OQ^2 = (5 \text{ cm})^2$

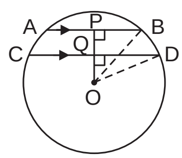
[∵ OC = OA = 5 cm radii of a circle]

$\Rightarrow \quad OQ^2 = (25 - 16) \text{ cm}^2$

$= 9 \text{ cm}^2 \Rightarrow OQ = 3 \text{ cm}$

Hence, the distance of the longer chord from the centre is **3 cm.**

12. Since AB || CD, OQ ⊥ CD and OP ⊥ AB, therefore OP and OQ are in the same line i.e., O, P and Q are collinear points.



$PQ = 3 \text{ cm} \quad$ (given).

Let $\quad OQ = x$ cm.

Then OP = $(x + 3)$ cm. Let r cm be the radius of the circle. Then OB = OD = r cm.

Since, a perpendicular from the centre of a circle to the chord bisects it.

$\therefore \quad PB = \frac{1}{2}AB = \frac{1}{2} \times 5 \text{ cm}$

$= \frac{5}{2} \text{ cm}$

and $\quad QD = \frac{1}{2} CD$

$= \frac{1}{2} \times 11 \text{ cm} = \frac{11}{2} \text{ cm} \quad \ldots(1)$

In right Δ OQD, we have,

$OQ^2 + QD^2 = OD^2$ [By Pythagoras' Theorem]

$\Rightarrow (x \text{ cm})^2 + \left(\frac{11}{2} \text{ cm}\right)^2 = OD^2 \quad$ [Using (1)]

$\Rightarrow \quad x^2 + \frac{121}{4} = r^2 \quad \ldots(2)$

In right Δ OPB, we have,

$OP^2 + PB^2 = OB^2$ [By Pythagoras' Theorem]

$\Rightarrow \quad [(x + 3) \text{ cm}]^2 + \left(\frac{5}{2} \text{ cm}\right)^2$

$= OB^2$ [By Pythagoras' Theorem and using (1)]

$\Rightarrow \quad (x + 3)^2 + \frac{25}{4} = r^2 \quad \ldots(3)$

From (2) and (3), we get,

$x^2 + \frac{121}{4} = x^2 + 6x + 9 + \frac{25}{4}$

$\Rightarrow \quad 6x = \frac{121}{4} - \frac{25}{4} - 9$

$\Rightarrow \quad 6x = \frac{121 - 25 - 36}{4}$

$\Rightarrow \quad 6x = 15$

$\Rightarrow \quad x = \frac{5}{2}$

Substituting $x = \frac{5}{2}$ in equation (2), we get

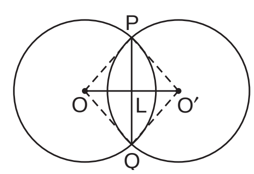
$\left(\frac{5}{2}\right)^2 + \frac{121}{4} = r^2$

$\Rightarrow \quad \frac{146}{4} = r^2$

$\Rightarrow \quad r = \frac{\sqrt{146}}{2}$

Hence, the radius = $\mathbf{\frac{\sqrt{146}}{2}}$ **cm.**

13. Here, OP = 10 cm, O′P = 8 cm and PQ = 12 cm.



Join OQ and O′Q.

In ΔOPO′ and ΔOQO′, we have

OP = OQ [= 10 cm],

O′P = O′Q [= 8 cm],

OO′ = OO′ [common]

∴ ΔOPO′ ≅ ΔOQO′ [By SSS congruence]

⇒ ∠POO′ = ∠QOO′ [By CPCT]

⇒ ∠POL = ∠QOL …(1)

In ΔOPL and ΔOQL, we have

OP = OQ [radii of a circle],

∠POL = ∠QOL [from (1)],

OL = OL [common]

∴ ΔOPL ≅ ΔOQL [By SAS congruence]

⇒ PL = QL

and $\angle OLP = \angle OLQ$ [CPCT] ...(2)

Also $\angle OLP + \angle OLQ = 180°$ ...(3)

$\therefore$ $\angle OLP = \angle OLQ$

$= 90°$ [using (2) and (3)] ...(4)

$\therefore$ OO′ is the perpendicular bisector of PQ. [using (2) and (4)]

$\Rightarrow$ $PL = QL = \frac{1}{2}PQ$

$= \frac{1}{2} \times 12 \text{ cm} = 6 \text{ cm}$.

In right ΔOLP, we have

$OL^2 + PL^2 = OP^2$ [By Pythagoras Theorem]

$\Rightarrow$ $OL^2 + (6 \text{ cm})^2 = (10 \text{ cm})^2$

$\Rightarrow$ $OL^2 = (100 - 36) \text{ cm}^2$

$= 64 \text{ cm}^2$

$\Rightarrow$ $OL = 8 \text{ cm}$

In right Δ O′LP, we have

$O'L^2 + PL^2 = O'P^2$ [By Pythagoras Theorem]

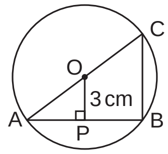
$\Rightarrow$ $O'L^2 + (6 \text{ cm})^2 = (8 \text{ cm})^2$

$\Rightarrow$ $O'L^2 = (64 - 36) \text{ cm}^2 = 28 \text{ cm}^2$

$\Rightarrow$ $O'L = 5.29$ cm (approx)

Now, OO′ = OL + LO′ = 8 cm + 5.29 cm = **13.29 cm**.

14. Let AB the chord of a circle of radius 5 cm, at a distance of 3 cm from the centre O.



Draw OP ⊥ AB. Then, OP = 3 cm,

Radius OA = 5 cm

$\Rightarrow$ Diameter AOC = 10 cm. BC is joined.

In right ΔOPA, we have

$AP^2 + OP^2 = OA^2$ [By Pythagoras Theorem]

$\Rightarrow$ $AP^2 + (3 \text{ cm})^2 = (5 \text{ cm})^2$

$\Rightarrow$ $AP^2 = (25 - 9) \text{ cm}^2 = 16 \text{ cm}^2$

$\Rightarrow$ $AP = 4$ cm.

Since a perpendicular from the centre of the circle to the chord bisects it,

$\therefore$ P is the mid-point of AB.

$\Rightarrow$ $AP = \frac{1}{2} AB$

$\Rightarrow$ $4 \text{ cm} = \frac{1}{2} AB$

$\Rightarrow$ $AB = 8$ cm.

Now, $\angle ABC = 90°$ [Angle in a semicircle]

$\therefore$ In right Δ ABC, we have,

$AB^2 + BC^2 = AC^2$ [By Pythagoras Theorem]

$\Rightarrow$ $(8 \text{ cm})^2 + BC^2 = (10 \text{ cm})^2$

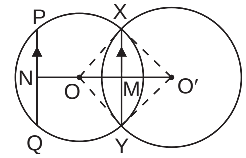
$\Rightarrow$ $BC^2 = (100 - 64) \text{ cm}^2 = 36 \text{ cm}^2$

$\Rightarrow$ BC = **6 cm.**

15. Join XO, O′X, OY and O′Y and let O′O intersect XY at M.

Let O′M produced or O′M meet PQ at N.

$\Delta OXO' \cong \Delta OYO'$ [By SSS congruence]



$\Rightarrow$ $\angle XOO' = \angle YOO'$ [CPCT]

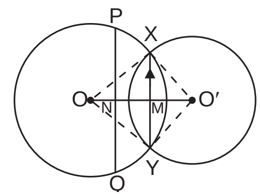
$\Rightarrow$ $\angle XOM = \angle YOM$ [Same angles]

$\Delta XOM \cong \Delta YOM$ [By SAS congruence]

$\Rightarrow$ XM = MY [CPCT]

$\Rightarrow$ M is the mid-point of XY

$\therefore$ OM ⊥ XY [Line joining the centre to the mid-point M of the chord is perpendicular to the chord]



$\Rightarrow$ $\angle XMO = \angle XMN = 90$ ...(1)

PQ ∥ XY [Given]

$\therefore$ $\angle PNM + \angle XMN = 180°$ [Coint, ∠s, PQ ∥ XY]

$\Rightarrow$ $\angle PNM = \mathbf{90°}$ [Using (1)]

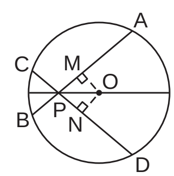
$\therefore$ ON ⊥ PQ

$\therefore$ N is mid-point of PQ [Perpendicular from the centre to the chord bisects it]

Hence, O′O produced or OO′ bisects PQ.

EXERCISE 10B

1. Draw OM ⊥ AB and ON ⊥ CD. In ΔOMP and ΔONP, we have



$\angle OMP = \angle ONP$ [= 90°]

$\angle OPM = \angle OPN$

[PO bisects ∠APD] and

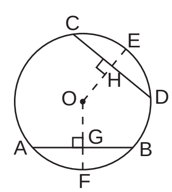
OP = OP.

$\therefore$ $\Delta OMP \cong \Delta ONP$ [By AAS congruence]

$\Rightarrow$ OM = ON [CPCT]

Hence, AB = CD. [Chords of a circle which are equidistant from the centre are equal]

2. AB = CD [Given]



$\Rightarrow$ OH = OG [Equal chords are equidistant from the centre]

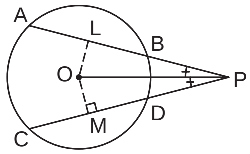
Also, OE = OF [Radii of a circle]

$\therefore$ OE – OH = OF – OG

$\Rightarrow$ HE = GF

3. Draw OL ⊥ AB and OM ⊥ CD.

In ΔOLP and ΔOMP, we have



$\angle OLP = \angle OMP$ [Each is 90°]

$\angle OPL = \angle OPM$ [$\because$ $\angle OPA = \angle OPC$, given]

$OP = OP$ [Common]

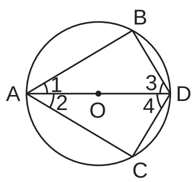
$\therefore$ $\Delta OLP \cong \Delta OMP$ [By AAS congruence]

$\Rightarrow$ $OL = OM$ [CPCT]

$\Rightarrow$ $AB = CD$ [Chords equidistant from the centre of a circle are equal in length]

Hence, **AB = CD**.

4. Chords AB and AC are equidistant from the centre O. [Given]



We know that the chords equidistant from the centre of a circle are equal in length.

$\therefore$ $AB = CD$ …(1)

Diameter AOD passes through A. BD and CD are joined.

$\angle ABD = \angle ACD = 90°$ [Angle in a semicircle is 90°]

In right ΔABD and right ΔACD, we have

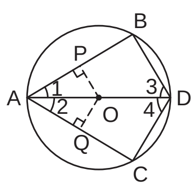
$AB = CD$ [From (1)]

and $AD = AD$ [Common]

$\therefore$ $\Delta ABD \cong \Delta ACD$ [By RHS congruence]

$\Rightarrow$ $\angle 1 = \angle 2$ and $\angle 3 = \angle 4$ [CPCT]

Hence, AD bisects $\angle BAC$ and $\angle BDC$.



Alternative method:

Draw $OP \perp AB$ and $OQ \perp AC$.

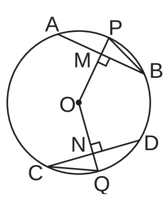
Prove $\Delta APO \cong \Delta AQO$ [By RHS congruence]

$\Rightarrow$ $\angle 1 = \angle 2$ [CPCT congruence]

Prove $\Delta ABD \cong \Delta ACD$ [By SAS congruence]

$\Rightarrow$ $\angle 3 = \angle 4$ [CPCT]

5. $AB = CD$ [Given]



$\Rightarrow$ $OM = ON$ [Equal chords are equidistant from the centre]

Also $OP = OQ$ [Radii of a circle]

$\therefore$ $OP - OM = OQ - ON$

$\Rightarrow$ $PM = QN$ …(1)

In right triangles PMB and QNC, we have

$PM = QN$ [From (1)]

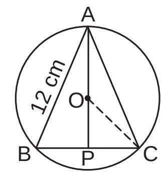
$BM = CN$ [$\because$ $AB = CD \Rightarrow \frac{1}{2}AB = \frac{1}{2}CD$ $\Rightarrow BM = CN$]

$\angle PMB = \angle QNC$ [Each is 90°]

$\therefore$ $\Delta PMB \cong \Delta QNC$ [SAS congruence]

$\Rightarrow$ $PB = QC$ [CPCT]

6. Let r be the radius of the circumcircle.



In an equilateral Δ the circumcentre, centroid and orthocentre coincide.

$\therefore$ O the circumcentre of ΔABC is also its centroid and its orthocentre. O is the centroid of ΔABC and AOP is its median.

$\Rightarrow$ $AO : OP = 2 : 1$ [Centroid divides the medians in the ratio 2 : 1]

$$\Rightarrow \quad \frac{AO}{OP} = \frac{2}{1}$$

$$\Rightarrow \quad \frac{r}{OP} = 2$$

O is also the orthocentre of ΔABC and AOP is an altitude of ΔABC.

$\Rightarrow$ $AP \perp BC$

$\Rightarrow$ $\angle APC = 90°$

$\Rightarrow$ $\angle OPC = 90°$

In right ΔOPC, we have

$OP^2 + PC^2 = OC^2$ [By Pythagoras Theorem]

$$\Rightarrow \quad \left(\frac{r}{2}\right)^2 + \left(\frac{BC}{2}\right)^2 = r^2$$

$$\left[\text{AP is the median of } \Delta ABC \Rightarrow PC = \frac{1}{2}BC\right]$$

$$\Rightarrow \quad \frac{r^2}{4} + \left(\frac{12}{2}\text{ cm}\right)^2 = r^2 \quad [\because \text{ side of the equilateral } \Delta \text{ is 12 cm}]$$

$$\Rightarrow \quad 36\text{ cm}^2 = r^2 - \frac{r^2}{4}$$

$$\Rightarrow \quad 36\text{ cm}^2 = \frac{3r^2}{4}$$

$$\Rightarrow \quad r^2 = \frac{36 \times 4}{3}\text{ cm}^2 = 48\text{ cm}^2$$

$$\Rightarrow \quad r = \sqrt{48}\text{ cm} = 4\sqrt{3}\text{ cm}$$

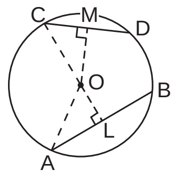
Hence, the radius of a circle which circumscribes an equilateral triangle of side 12 cm is equal to $4\sqrt{3}$ cm.

7. Let chord AB > chord CD. Join OA and OC.

Draw $OL \perp AB$ and $OM \perp CD$.

$$\therefore \quad AL = \frac{1}{2}AB \text{ and } CM = \frac{1}{2}CD$$

In right ΔOLA and right ΔOMC, we have



$$OA^2 = OL^2 + AL^2$$

and $OC^2 = OM^2 + CM^2$ [Pythagoras Theorem]

$\Rightarrow$ $OL^2 + AL^2 = OM^2 + CM^2$ [$\because$ OA = OC] … (2)

Now $AB > CD \Rightarrow \frac{1}{2} AB > \frac{1}{2} CD$

$\Rightarrow$ $AL > CM \Rightarrow AL^2 > CM^2$

$\Rightarrow$ $AL^2 + OL^2 > CM^2 + OL^2$

[Adding OL^2 to both sides]

$\Rightarrow$ $OM^2 + CM^2 > CM^2 + OL^2$ [Using (2)]

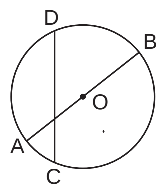
$\Rightarrow$ $OM^2 > OL^2$

$\Rightarrow$ $OM > OL$

$\Rightarrow$ $OL < OM$

Hence, chord AB is nearer to the centre than chord CD.

8. Diameter is nearer to the centre than any other chord CD. We know that of any two chords of a circle, the one which is nearer to the centre is longer.

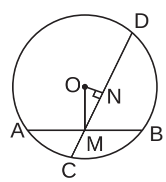


$\Rightarrow$ AB > CD

So, AB is longer than every other chord.

Hence, a diameter is the longest chord in the circle.

9. M is a point within a circle. AB is a chord with mid-point M and CD is another chord through M. Join OM and draw ON $\perp$ CD.



In right ΔONM, OM is the hypotenuse.

$\therefore$ ON < OM

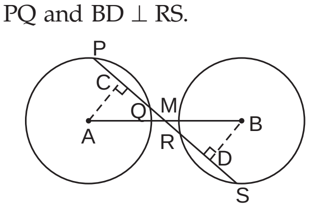
$\Rightarrow$ Chord CD is nearer to O than chord AB.

Of any two chords of a circle, the one near to the centre is longer.

$\therefore$ CD > AB. Hence, AB < CD.

Thus, of all chords through M, the shortest is the one which is bisected at M.

10. Draw AC $\perp$ PQ and BD $\perp$ RS.



In ΔACM and ΔBDM, we have

$\angle$ACM = $\angle$BDM [= 90°]

$\angle$CMA = $\angle$DMB [Ver. opp. $\angle$s]

and AM = BM

$\therefore$ $\Delta ACM \cong \Delta BDM$ [By AAS congruence]

$\therefore$ AC = BD [CPCT]

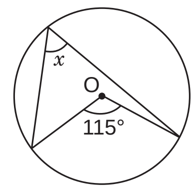
Chords of congruent circles that are equidistant from centres are equal.

$\therefore$ **PQ = RS**

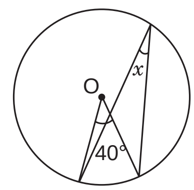
EXERCISE 10C

Since the angle subtended by an arc of a circle at the centre is double the angle subtended by it at any point on the remaining part of the circle, therefore

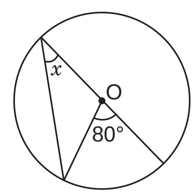
(i) $2x = 115° \Rightarrow x = \frac{115°}{2} = \mathbf{57.5°.}$



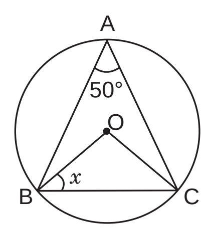
(ii) $2x = 40° \Rightarrow x = \frac{40°}{2} = \mathbf{20°.}$



(iii) $2x = 80° \Rightarrow x = \frac{80°}{2} = \mathbf{40°}.$



(iv) $\angle BOC = 2\angle BAC \Rightarrow \angle BOC = 2 \times 50° = 100°$



In ΔOBC, we have

OB = OC [radii of a circle]

$\therefore$ $\angle OCB = \angle OBC = x$ [$\angle$'s opposite sides of ΔOBC]

$\therefore$ $x + x + \angle BOC = 180°$ [sum of $\angle$'s of a Δ]

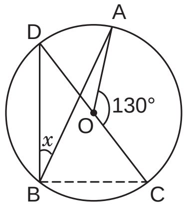
$\Rightarrow$ $2x + 100° = 180° \Rightarrow 2x = 80° \Rightarrow x = \mathbf{40°.}$

(v) $\angle AOC = 2\angle ABC$

$\Rightarrow$ $130° = 2\angle ABC$

$\Rightarrow$ $\angle ABC = 65°$

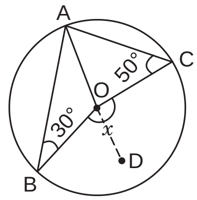
$\angle DBC = 90°$ [angles is a semicircle]



$\Rightarrow$ $x + \angle ABC = 90°$

$\Rightarrow$ $x + 65° = 90°$ $\Rightarrow$ $x = 90° - 65° = \mathbf{25°}$.

(vi) $\angle OAB = \angle OBA$



$= 30°$ [Angles opp equal sides of ΔOAB]

$\angle OAC = \angle OCA$

$= 50°$ [Angles opp equal sides of ΔOAC]

Extend AO to any point D.

Then, $\angle BOD = 2\angle OAB = 2 \times 30° = 60°$

and $\angle COD = 2\angle OAC = 2 \times 50° = 100°$

$x = \angle BOC = \angle BOD + \angle COD$

$= 60° + 100° = \mathbf{160°}$.

(vii) Ref $\angle BOC = 2\angle BAC = 2 \times 100° = 200°$

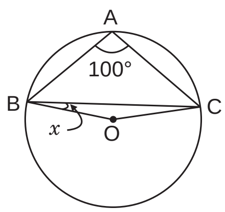
$\angle BOC = 360° -$ ref $\angle BOC$

[Sum of angles about a point is 360°]

$\Rightarrow$ $\angle BOC = 360° - 200° = 160°$

OB = OC [radii of a circle]

$\therefore$ $\angle OCB = \angle OBC$ [Angles opp. equal sides of ΔOBC]



In ΔOBC, we have, $x + x + \angle BOC = 180°$

[Sum of angles of triangle]

$\Rightarrow$ $2x + 160° = 180°$

$\Rightarrow$ $2x = 20°$

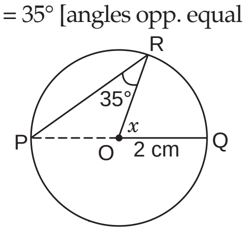
$\Rightarrow$ $x = \mathbf{10°}$

(viii) Join OP.

OR = OP [Radii of a circle]

$\therefore$ $\angle OPR = \angle ORP$

$= 35°$ [angles opp. equal sides of ΔPOR]

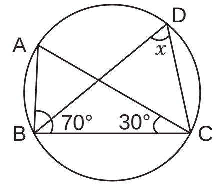


$2\angle OPR = x$

$\Rightarrow$ $2 \times 35° = x$

$\Rightarrow$ $x = \mathbf{70°}$.

(ix) In ΔABC, we have



$\angle ABC + \angle BCA + \angle CAB$

$= 180°$ [Sum of angles of a triangle]

$\Rightarrow$ $70° + 30° + \angle CAB = 180°$

$\Rightarrow$ $\angle CAB = 180° - 100° = 80°$

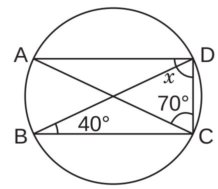
$x = \angle CAB$ [angles in the same segment]

$\Rightarrow$ $x = \mathbf{80°}$.

(x) $\angle CAD = \angle CBD$

$= 40°$ [angles in the same segment]

In $\angle$ADC, we have



$\angle CAD + \angle ACD + \angle CDA$

$= 180°$ [Sum of angles of a triangle]

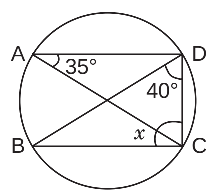
$\Rightarrow$ $40° + 70° + x = 180°$

$\Rightarrow$ $x = 180° - 110°$ $\Rightarrow$ $x = \mathbf{70°}$.

(xi) $\angle CBD = \angle CAD$

$= 35°$ [Angles in the same segment]

In ΔBCD, we have



$\angle CBD + \angle BDC + \angle DCB$

$= 180°$ [Sum of angles of triangle]

$\Rightarrow$ $35° + 40° + x = 180°$

$\Rightarrow$ $x = 180° - 75°$

$\Rightarrow$ $x = \mathbf{105°}$.

(xii) $\angle ABD = \angle ACD = 30°$ [Angles in the same segment]

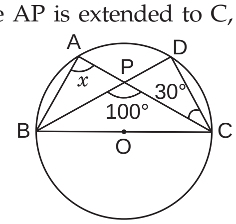
Considering ΔABP whose side AP is extended to C, we get

$\angle ABP + \angle BAP = \angle CPB$

[Ext $\angle$= Sum of the opposite $\angle$'s]

$\Rightarrow$ $\angle ABD + \angle BAC = \angle CPB$

$\Rightarrow$ $30° + x = 100°$

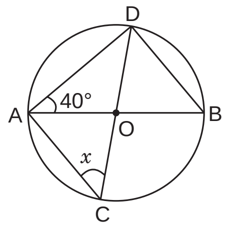


$\Rightarrow \quad x = 100° - 30°$

$\Rightarrow \quad x = \mathbf{70°}.$

2. (i) $\angle ADB = 90°$ [Angle in a semicircle]

In ΔADB, we have



$\angle BAD + \angle ADB + \angle ABD$

$= 180°$ [Sum of angles of a triangle]

$\Rightarrow \quad 40° + 90° + \angle ABD = 180°$

$\Rightarrow \quad \angle ABD = 180° - 130°$

$\Rightarrow \quad \angle ABD = 50°$

$\angle ACD = \angle ABD$ [Angles in the same segment]

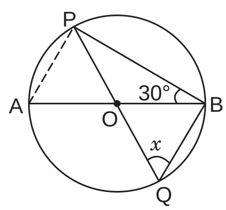
$\Rightarrow \quad x = \mathbf{50°}.$

(ii) Join PA.

$\angle PAB = \angle PQB$ [Angles in the same segment]

$\Rightarrow \quad \angle PAB = x$ ...(1)

In ΔAPB, we have



$\angle PAB + \angle ABP + \angle APB$

$= 180°$ [Sum of angles of a triangle]

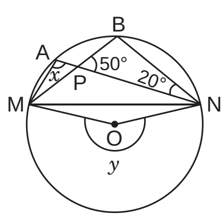
$\Rightarrow \quad x + 30° + \angle APB = 180°$ [Using (1)]

$\Rightarrow \quad x + 30° + 90° = 180°$ [$\angle APB = 90°$, angle in a semicircle]

$\Rightarrow \quad x = 180° - 120°$

$\Rightarrow \quad x = \mathbf{60°}.$

(iii) In ΔPBN, we have



$\angle BPN + \angle PNB + \angle PBN$

$= 180°$ [Sum of angles of a triangle]

$\Rightarrow \quad 50° + 20° + \angle PBN = 180°$

$\Rightarrow \quad \angle PBN = 180° - 70° = 110°$

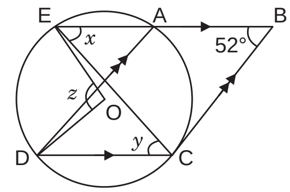
$\angle MBN = \angle MAN$ [Angles in the same segment]

$\Rightarrow \quad \angle PBN = \angle MAN = x$

$\Rightarrow \quad 110° = x \Rightarrow x = \mathbf{110°}$

Ref. $\angle MON = 2x \Rightarrow y = 2 \times 110° \Rightarrow y = \mathbf{220°}.$

(iv) $\angle EAD = \angle EBC = 52°$ [Correspoing $\angle$'s, AD $\parallel$ BC]



$\angle ECD = \angle EAD$ [Angles in the same segment]

$\Rightarrow \quad y = 52°$

$x = y$ [Alternate $\angle$'s EAB $\parallel$ DC]

$\Rightarrow \quad x = 52°$

$z = 2y = 2 \times 52°$

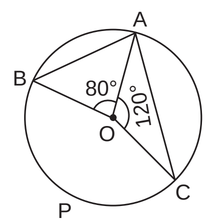
$= 104°$ [Angle subtended by an arc of a circle at the centre is double the angle subtended by it on the remaining part of the circle]

Hence, $\mathbf{x = 52°, y = 52°, z = 104°}$.

3. In ΔOAB, we have OA = OB [Radii of a circle]

$\therefore \quad \angle OBA = \angle OAB$ [Angles opp. equal sides of ΔOAB] ...(1)

In ΔOAB, we have



$\angle OBA + \angle OAB + \angle AOB$

$= 180°$ [Sum of angles of a triangle]

$\Rightarrow \quad 2\angle OAB + 80° = 180°$ [Using (1)]

$\Rightarrow \quad 2\angle OAB = 100°$

$\Rightarrow \quad \angle OAB = 50°$

In ΔOAC, we have

$OC = OA$ [Radii of a circle]

$\therefore \quad \angle OAC = \angle OCA$ [Angles opposite equal sides of ΔOAC] ... (2)

In ΔOAC, we have

$\angle OCA + \angle OAC + \angle AOC = 180°$ [Sum of angles of a triangle]

$\Rightarrow \quad 2\angle OAC + 120° = 180°$ [Using (2)]

$\Rightarrow \quad 2\angle OAC = 60°$

$\Rightarrow \quad \angle OAC = 30°$

Now, $\angle BAC = \angle OAB + \angle OAC$

$= 50° + 30° = \mathbf{80°}$

Degree measure of arc BPC

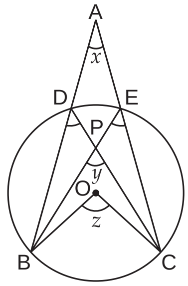
$= m(\angle BOC) = 2\angle BAC$

$= 2 \times 80° = \mathbf{160°}.$

[Angle subtended by an arc of a circle at the centre is twice the angle subtended by it on the remaining part of the circle]

4. $\angle BDC = \angle BEC$ [Angles in the same segment]

Since the angle subtended by an arc of a circle at the centre is twice the angle subtended by it on the remaining part of the circle



$\therefore \quad \angle BDC = \angle BEC = \frac{z}{2}$

$\angle ADP + \angle BDC = 180°$ [Linear pair]

$\Rightarrow \quad \angle ADP + \frac{z}{2} = 180°$

$\Rightarrow \quad \angle ADP = 180° - \frac{z}{2}$

Similarly, $\quad \angle AEP = 180° - \frac{z}{2}$

Also, $\quad \angle DPE = \angle BPC = y$ [Ver. opp. ∠'s]

Now, in quad ADPE, we have

$\angle DAE + \angle ADP + \angle DPE + \angle AEP$

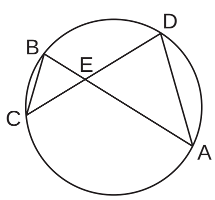
$= 360°$ [Sum of angles of a quad]

$\Rightarrow \quad \angle x + \left(180° - \frac{\angle z}{2}\right) + \angle y + \left(180° - \frac{\angle z}{2}\right) = 360°$

$\Rightarrow \quad \boldsymbol{\angle x + \angle y = \angle z}$

5. $\angle CBA = \angle CDA$ [Angles in the same segment]

$\Rightarrow \quad \angle CBE = \angle EDA$ …(1)



$\angle BCD = \angle BAD$ [Angles in the same segment]

$\Rightarrow \quad \angle BCE = \angle EAD$ …(2)

Also, $\angle BEC = \angle DEA$ [Ver. opp. ∠'s] …(3)

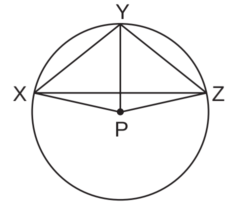
From (1), (2) and (3), we conclude the ΔCBE and ΔADE are equiangular.

Hence, **ΔADE and ΔCBE are equiangular.**

6. Since the angle subtended by an arc at the centre is twice the angle subtended by it on the remaining part of the circle,

$\therefore \quad \angle XPY = 2\angle XZY$ …(1)

and $\quad \angle YPZ = 2\angle YXZ$ …(2)



Adding (1) and (2), we get

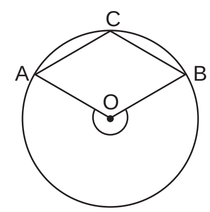
$\angle XPY + \angle YPZ = 2\angle XZY + 2\angle YXZ$

$\Rightarrow \quad \angle XPZ = 2(\angle XZY + \angle YXZ)$

Hence, $\quad \angle XPZ = 2(\angle XZY + \angle YXZ)$.

7. Let ACB be an angle in a minor segment of circle with centre O.

$\angle ACB = \frac{1}{2}$ reflex $\angle AOB$



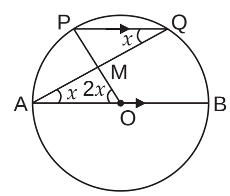
But reflex $\angle AOB > 180°$

$\therefore \quad \angle ACB > \frac{1}{2} \times 180°$

$\Rightarrow \quad \angle ACB > 90°$

$\Rightarrow \quad \angle ACB$ is obtuse.

8. Let $\quad \angle PQA = x$



Then $\quad \angle QAB = x$ [Alt. ∠s, PQ ∥ AB]

i.e. $\quad \angle MAO = x$ …(1)

$\angle POA = 2\angle PQA$

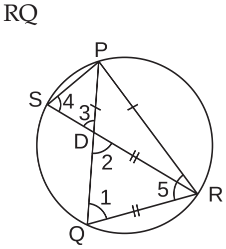
$= 2x$ [Angle subtended by an arc at the centre is twice the angle subtended by it on the remaining part of the circle]

i.e. $\quad \angle MOA = 2x$ …(2)

Ext ∠= Sum of the ∠s

$\therefore$ Ext. $\angle AMP = \angle MAO + \angle MOA = x + 2x = 3x = 3\angle MAO$.

9. In ΔRDQ, RD = RQ



$\therefore$ $\angle 1 = \angle 2$ [$\angle$s opposite equal sides of a Δ] ...(1)

$\angle 3 = \angle 2$ [Ver. Opp. $\angle$s] ...(2)

$\angle 4 = \angle 1$ [Angles in the same segment] ...(3)

From (1), (2) and (3), we get

$\angle 4 = \angle 3$

$\Rightarrow$ PD = PS [Sides opposite equal angles of ΔPDS]

Also $\angle 1 = \angle 5$ [Angles opposite equal sides of ΔPQR] ...(4)

$\angle 3 + \angle 4 + \angle SPD = 180°$ [Sum of $\angle$s of a ΔPSD]

$\angle 1 + \angle 5 + \angle QPR = 180°$ [Sum of $\angle$s of ΔPQR]

$\therefore$ $\angle 3 + \angle 4 + \angle SPD = \angle 1 + \angle 5 + \angle QPR$

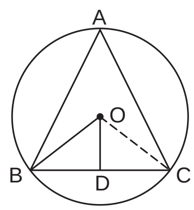
$\Rightarrow$ $2(\angle 1) + \angle SPD = 2(\angle 1) + \angle QPR$ [Using (1), (2), (3) and (4)]

$\Rightarrow$ $\angle SPD = \angle QPR$

$\Rightarrow$ $\angle SPQ = \angle QPR$

Hence, **PQ bisects $\angle$RPS**.

10. (i) Join OC.



In ΔBOD and ΔCOD, we have

OB = OC [Radii of a circle]

OD = OD [Common]

$\therefore$ $\Delta BOD \cong \Delta COD$ [By RHS congruence]

$\Rightarrow$ $\angle BOD = \angle COD$ [CPCT] ...(1)

$\Rightarrow$ $\angle BOC = \angle BOD + \angle COD$

$\Rightarrow$ $\angle BOC = 2\angle BOD$ [Using (1)] ... (2)

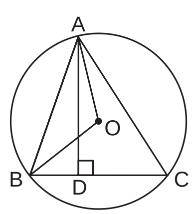
Since the angle subtended by an arc at the centre is twice the angle subtended by it at any other point on the remaining part of the circle,

$\therefore$ $\angle BOC = 2\angle BAC$

$\Rightarrow$ $2\angle BOD = 2\angle BAC$ [using (2)]

$\Rightarrow$ **$\angle$BOD = $\angle$A**

(ii) Let $\angle ACB = x$, then, $\angle AOB = 2x$.



[$\because$ Angle subtended by an arc at the centre is twice the angle subtended by it at any point on the remaining part of the circle.]

In ΔADC, we have

$\angle ADC + \angle ACD + \angle CAD$

$= 180°$ [Sum of angles of a triangle]

$\Rightarrow$ $90° + x + \angle CAD$

$= 180°$ [$\because$ $\angle$ACD is same as $\angle ACB = x$]

$\Rightarrow$ $\angle CAD = 90° - x$ ...(1)

In $\angle$OAB, we have

$\angle BAO + \angle ABO + \angle AOB$

$= 180°$ [Sum of angles of a triangle]

$\Rightarrow$ $2\angle BAO + 2x = 180°$ [$\because$ $\angle BAO = \angle ABO$, angles opposite equal sides OA and OB of ΔOAB]

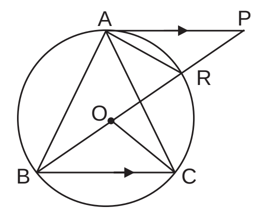
$\Rightarrow$ $2\angle BAO = 180° - 2x$

$\Rightarrow$ $\angle BAO = 90° - x$ ...(2)

From (1) and (2) we get

$\angle BAO = \angle CAD$

(iii) Let BOP intersect the circle at R.



Join AR.

$\angle CBR = \angle CAR$ [Angles in the same segment] ...(1)

Also $\angle CBP = \angle APB$ [Alt. angles AP parallel BC]

$\Rightarrow$ $\angle CBR = \angle APB$ ...(2)

$\Rightarrow$ $\angle CAR = \angle APB$ [Using (1) and (2)] ...(3)

Now, $\angle BAR = 90°$[Angles in a semicircle]

$\Rightarrow$ $\angle BAC + \angle CAR = 90°$

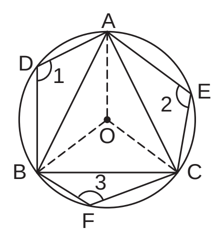
$\Rightarrow$ $\angle BAC + \angle APB = 90°$ [Using (3)]

(iv) Join OA, OB and OC.

Ref $\angle AOB = 2\angle 1$ ...(1)

Ref $\angle AOC = 2\angle 2$ ...(2)

Ref $\angle BOC = 2\angle 3$ ...(3)



[Angle subtended by an arc at the centre of the circle is twice the angle subtended by it at any other point on the remaining part of the circle]

Adding (1), (2) and (3), we get

Ref $\angle$AOB + Ref $\angle$AOC + Ref $\angle$BOC

$= 2\angle 1 + 2\angle 2 + 2\angle 3$

$\Rightarrow$ $(\angle BOC + \angle AOC) + (\angle AOB + \angle BOC) + (\angle AOC + \angle AOB) = 2(\angle 1 + \angle 2 + \angle 3)$

$\Rightarrow$ $2(\angle AOB + \angle BOC + \angle AOC) = 2(\angle 1 + \angle 2 + \angle 3)$

$\Rightarrow$ $\angle AOB + \angle BOC + \angle AOC = \angle 1 + \angle 2 + \angle 3$

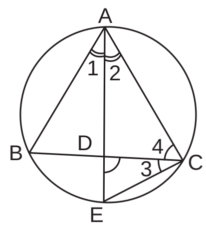
$\Rightarrow$ $360° = \angle 1 + \angle 2 + \angle 3$ [Sum of angles about a point is 360°]

$\Rightarrow$ $\angle ADB + \angle BFC + \angle AEC$

$= 360° = 4 \times 90° =$ right angles.

Hence, the angles in the three segments exterior of ΔABC are together equal to 4 right angles.

(v) $\angle 1 = \angle 2$ [AE bisects $\angle BAC$]



$\angle 1 = \angle 3$ [Angles in the same segment]

$\therefore$ $\angle 2 = \angle 3$ ...(1)

Now, considering ΔCAD whose side AD is produced to B.

$\angle CDE = \angle 2 + \angle 4$ [Exterior $\angle$ = Sum of interior opposite angles]

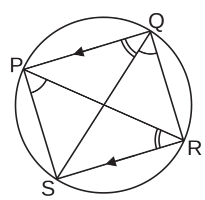
$\Rightarrow$ $\angle CDE = \angle 2 + \angle 4$ ...(2)

Also $\angle ACE = \angle 3 + \angle 4$

$\Rightarrow$ $\angle ACE = \angle 2 + \angle 4$ [Using (1)] ...(3)

Hence, $\angle ACE = \angle CDE$ [Using (2) and (3)]

11. $\angle SPR = \angle SQR = x$ (say) [Angles in the same segment] ...(1)



$\angle PQS = \angle PRS = y$ (say) [Angles in the same segment] ...(2)

$\angle QPR = \angle PRS = y$ [Alternative angles, QP parallel RS] ...(3)

$\angle SPQ = \angle SPR + \angle QPR = x + y$ [Using (1) and (3)] ...(4)

$\angle PQR = \angle PQS + \angle SQR = y + x$ [Using (2) and (1)] ...(5)

$\Rightarrow$ $\angle SPQ = \angle PQR$ [Using (4) and (5)]

In ΔPQR and ΔQPS, we have

$\angle PQR = \angle QPS$ [Prove in (1)]

$PQ = QP$ [Common]

$\angle QPR = \angle PQS$ [$\because$ $\angle QPR = \angle QSR$, angles in the same segment and $\angle QSR = \angle PQS$, alternative angles QP $\parallel$ RS]

$\therefore$ $\Delta PQR \cong \Delta QPS$ [By ASA congruence]

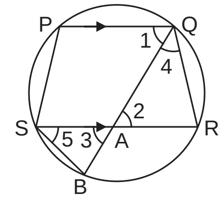
$\Rightarrow$ **QR = PS** [CPCT]

12. $\angle 1 = \angle 2$ [Alternate angles, PQ $\parallel$ SR]

$\angle 2 = \angle 3$ [Ver. opp. angles]

$\Rightarrow$ $\angle 1 = \angle 3$ ...(1)

$\angle 4 = \angle 5$ [Angles in the same segment] ...(2)



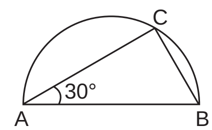
Adding (1) and (2), we get

$\angle 1 + \angle 4 = \angle 3 + \angle 5$

$\Rightarrow$ **$\angle PQR = \angle SAB + \angle ASB$**

13. $\angle ACB = 90°$ [Angle in a semicircle]

In ΔACB, we have



$\angle CAB + \angle ACB + \angle ABC = 180°$ [Sum of angles of a triangle]

$\Rightarrow$ $30° + 90° + \angle ABC = 180°$

$\Rightarrow$ $\angle ABC = 180° - 30° - 90°$

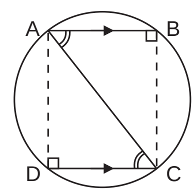
$\Rightarrow$ $\angle ABC = 60°$

Hence, **$m(\angle ACB) = 90°$ and $m(\angle ABC) = 60°$**

14. Join AD and BC.

$\angle ABC = \angle CDA = 90°$ [Angle in a semicircle]

In ΔABC and ΔCDA, we have



$\angle BAC = \angle DCA$ [Alternate angles, AB $\parallel$ DC]

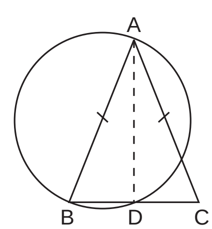
$\angle ABC = \angle CDA$ [Each is 90°]

$AC = CA$ [Common]

$\therefore$ $\Delta ABC \cong \Delta CDA$ [By AAS congruence]

$\Rightarrow$ **AB = CD** [CPCT]

15. Suppose a circle is drawn on AB as diameter and it cuts the base at D.



Join AD. $\angle ADB = 90°$ [Angle in a semicircle]

$\angle ADB + \angle ADC = 180°$ [Linear para]

$\Rightarrow$ $\angle ADC = 180° - \angle ADB$

$= 180° - 90° = 90°$

In right ΔADB and right ΔADC, we have

$AD = AD$ [Common]

and $AB = AC$

$\therefore$ $\Delta ADB \cong \Delta ADC$ [By RHS congruence]

$\Rightarrow$ $DB = DC$ [CPCT]

16. Join SQ and PT.

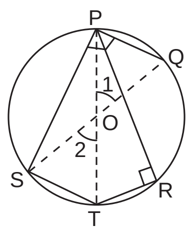
$\angle SPQ = 90°$

$\therefore$ SOQ is a diameter. [Angle in a semicircle is 90°.]

$\angle PRT = 90°$

$\therefore$ POT is a diameter. [Angle in a semicircle is 90°.]

In ΔPOQ and ΔSOT, we have



$OP = OS$ [Radii of a circle],

$\angle 1 = \angle 2$ [Ver. opp. angles],

$OQ = OT$ [Radii of a circle]

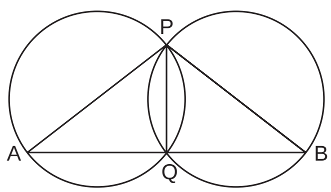
$\therefore$ $\Delta POQ \cong \Delta SOT$ [By SAS congruence]

$\Rightarrow$ $PQ = ST$ [CPCT]

17. $\angle PQA = 90°$

and $\angle PQB = 90°$ [Angles in semicircles]

In right ΔPQA and right ΔPQB, we have



$PQ = PQ$ [Common]

and $PA = PB$ [Diameters of congruent circles]

$\therefore$ $\Delta PQA \cong \Delta PQB$ [By RHS congruence]

$\Rightarrow$ $\angle APQ = \angle BPQ$ [CPCT]

18. $x + \angle BDC = 180°$ [Linear pair]

$\Rightarrow$ $\angle BDC = 180° - x$

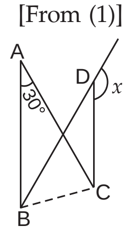
$\Rightarrow$ $\angle BDC = 180° - 150°$ [$x = 150°$, given]

$\Rightarrow$ $\angle BDC = 30°$ …(1)

$\angle BAC = 30°$ [Given]

and $\angle BDC = 30°$ [From (1)]

$\Rightarrow$ BC subtends equal angles at points A and D which are on the same side of it.



We know that if a line segment joining two points subtends equal angles at two other points on the same side of the line containing the line segment, the four points lie on a circle.

$\therefore$ **Points A, B, C and D are concyclic.**

19. In ΔADC and ΔBCD, we have

$AD = BC$ [Given]

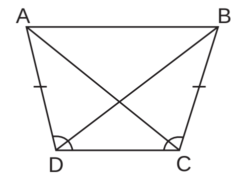
$\angle ADC = \angle BCD$ [Given]

$DC = CD$ [Common]

$\therefore$ $\Delta ADC \cong \Delta BCD$ [By SAS congruence]

$\Rightarrow$ $\angle DAC = \angle CBD$ [CPCT]

$\Rightarrow$ DC subtends equal angles at points A and B which are on the same side of it.



We know that if a line segment joining two points subtends equal angles at two other points on the same side of the line containing the line segment, the four points lie on a circle.

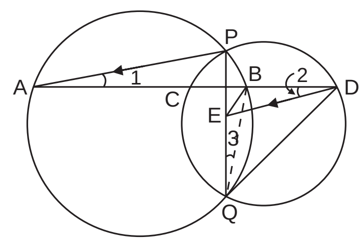
Hence, **points A, B, C, D are concyclic.**

20. Join BQ.

$\angle 1 = \angle 2$ [Alternate angles, PA || DE] …(1)

$\angle 1 = \angle 3$ [Angles in same segment] …(2)

$\therefore$ $\angle 2 = \angle 3$ [Using (1) and (2)]



$\Rightarrow$ BE subtends equal angles at points Q and D which are on the same side of it.

We know that of a line segment joining two points subtends equal angles at two other points on the same side of the line containing the line segment, the four points lie on a circle.

$\therefore$ **Points B, E, Q and D are concyclic.**

EXERCISE 10D

1. Let one of the angles of a cyclic quadrilateral be x, such that its opposite angle is $2x$.

Since the opposite angles of a cyclic quadrilateral are supplementary

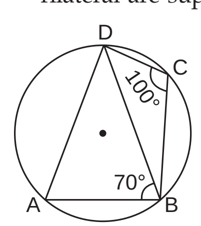
$\therefore$ $x + 2x = 180°$

$\Rightarrow$ $3x = 180°$

$\Rightarrow$ $x = 60°$

Larger angle $= 2x = 2 \times 60° = 120°$

2. $\angle DAB + \angle DCB = 180°$ [Opposite angles of a cyclic quadrilateral are supplementary]



$\Rightarrow$ $\angle DAB + 100° = 180°$

$\Rightarrow$ $\angle DAB = 80°$ ...(1)

In ΔDAB, we have

$\angle ADB + \angle DAB + \angle ABD = 180°$ [Sum of angles of a triangle]

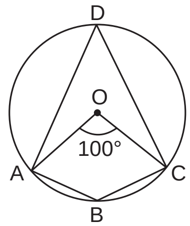
$\Rightarrow$ $\angle ADB + 80° + 70° = 180°$ [Using (1)]

$\Rightarrow$ $\angle ADB = 180° - 80° - 70°$

$= 30°$

Hence, **$\angle ADB = 30°$.**

3. Since the angle subtended by the arc of a circle at the centre is twice the angle subtended by it at any point on the remaining part of the circle,



$\therefore$ $\angle AOC = 2\angle ADC$

$\Rightarrow$ $100° = 2\angle ADC$

$\Rightarrow$ $\angle ADC = 50°$ ...(1)

Since the opposite angles of a cyclic quadrilateral are supplementary,

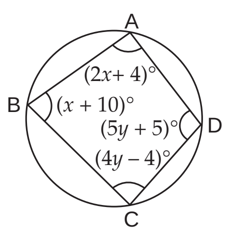
$\therefore$ $\angle ADC + \angle ABC = 180°$

$\Rightarrow$ $50° + \angle ABC = 180°$ [Using (1)]

$\Rightarrow$ $\angle ABC = 130°$

Hence, **$\angle ADC = 50°$ and $\angle ABC = 130°$**.

4. Since the opposite angles of a cyclic quadrilateral are supplementary,



$\therefore$ $\angle A + \angle C = 180°$

and $\angle B + \angle D = 180°$

$\Rightarrow$ $(2x + 4)° + (4y - 4)° = 180°$

$\Rightarrow$ $(x + 10)° + (5y + 5)° = 180°$

$\Rightarrow$ $2x + 4y = 180$

and $x + 5y = 180 - 15.$

$\Rightarrow$ $x + 2y = 90$ ...(1)

and $x + 5y = 165$ ...(2)

Subtracting (1) from (2), we get

$3y = 75$

$\Rightarrow$ $y = 25$

Substituting $y = 25$ in (1), we get

$x + 2 \times 25 = 90$

$\Rightarrow$ $x = 90 - 50$

$\Rightarrow$ $x = 40$

$\therefore$ $\angle A = (2x + 4)° = (2 \times 40 + 4)° = 84°$

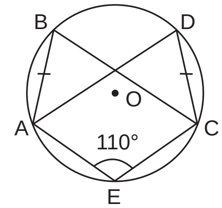
$\angle B = (x + 10)° = (40 + 10)° = 50°$

$\angle C = (4y - 4)° = (4 \times 25 - 4)° = 96°$

$\angle D = (5y + 5)° = (5 \times 25 + 5)° = 130°$

Hence, **$x = 40, y = 25, \angle A = 84°, \angle B = 50°, \angle C = 96°, \angle D = 130°$**.

5. (i) Since the opposite angles of a cyclic quadrilateral are supplementary and ABCE is a cyclic quadrilateral,



$\therefore$ $\angle ABC + \angle AEC = 180°$

$\Rightarrow$ $\angle ABC + 110° = 180°$

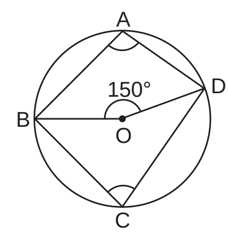
$\Rightarrow$ $\angle ABC = 70°$ ...(1)

(ii) $\angle ADC = \angle ABC$ [Angles in the same segment]

$\Rightarrow$ $\angle ADC = 70°$ [Using (1)]

Hence, **(i) $\angle ABC = 70°$ (ii) $\angle ADC = 70°$**.

6. Since the angle subtended by the arc of a circle at the centre is twice the angle subtended by it at any point on the remaining part of the circle,



$\therefore$ $\angle BOD = 2\angle BCD$

$\Rightarrow$ $150° = 2\angle BCD$

$\Rightarrow$ $\angle BCD = 75°$ ...(1)

Since the opposite angles of a cyclic quadrilateral are supplementary.

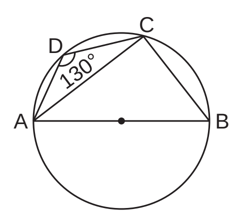
$\therefore$ $\angle BAD + \angle BCD = 180°$

$\Rightarrow$ $\angle BAD + 75° = 180°$ [Using (1)]

$\Rightarrow$ $\angle BAD = 180° - 75° = 105°$

Hence, **$\angle BCD = 75°, \angle BAD = 105°$**.

7. Since the opposite angles of a cyclic quadrilateral are supplementary,



$\therefore$ $\angle CDA + \angle ABC = 180°$

$\Rightarrow$ $130° + \angle ABC = 180°$

$\Rightarrow$ $\angle ABC = 50°$ ...(1)

$\angle ACB = 90°$ [Angle in a semicircle] ...(2)

In ΔACB,

$\angle BAC + \angle ACB + \angle ABC = 180°$ [Sum of $\angle$s of a Δ]

$\Rightarrow$ $\angle BAC + 90° + 50° = 180°$ [Using (1) and (2)]

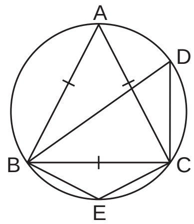
$\Rightarrow$ $\angle BAC = 180° - 90° - 50° = 40°$

Hence, $m\angle BAC =$ **$40°$**.

8. $\angle BDC = \angle BAC$ [Angles in the same segment]

$\Rightarrow$ $\angle BDC = 60°$ [$\because \angle BAC = 60°$, angle of an equilateral Δ]

Since the opposite angles of a cyclic quadrilateral are supplementary and ABEC is a cyclic quadrilateral,



$\therefore$ $\angle BAC + \angle BEC = 180°$

$\Rightarrow$ $60° + \angle BEC = 180°$

$\Rightarrow$ $\angle BEC = 120°$

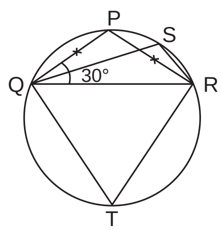
Hence, $\angle BDC = \mathbf{60°}$,

$\angle BEC = \mathbf{120°}$.

9. $\angle PRQ = \angle PQR$ [Angles opposite equal sides of ΔPQR]

$\Rightarrow$ $\angle PRQ = 30°$ [$\because$ $\angle PQR = 30°$, given]

In ΔPQR, we have



$\angle QPR + \angle PQR + \angle PRQ = 180°$ [Sum of angles of a triangle]

$\Rightarrow$ $\angle QPR + 30° + 30° = 180°$

$\Rightarrow$ $\angle QPR = 180° - 30° - 30° = 120°$ …(1)

$m\,\angle QSR = m\,\angle QPR$ [Angles in the same segment]

$\Rightarrow$ $m\,\angle QSR = 120°$ [Using (1)] …(2)

Since opposite angles of a cyclic quadrilateral are supplementary and QSRT is a cyclic quadrilateral,

$\therefore$ $\angle QSR + \angle QTR = 180°$

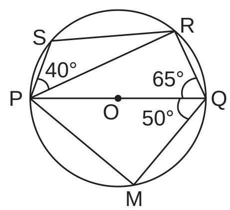
$\Rightarrow$ $120° + \angle QTR = 180°$ [Using (2)]

$\Rightarrow$ $\angle QTR = 60°$

Hence, $\boldsymbol{m\angle QSR = 120°, m\,\angle QTR = 60°}$.

10. $\angle PRQ = 90°$ [Angle in a semicircle] …(1)

In ΔPRQ, we have



$\angle PQR + \angle PRQ + \angle QPR = 180°$ [Sum of angles of a triangle]

$\Rightarrow$ $65° + 90° + \angle QPR = 180°$ [Using (1)]

$\Rightarrow$ $\angle QPR = 180° - 90° - 65° = 25°$ …(2)

Since opposite angles of a cyclic quadrilateral are supplementary and PSRQ is a cyclic quadrilateral,

$\therefore$ $\angle SPQ + \angle SRQ = 180°$

$\Rightarrow$ $(\angle SPR + \angle QPR) + (\angle PRS + \angle PRQ) = 180°$

$\Rightarrow$ $(40° + 25°) + (\angle PRS + 90°) = 180°$[Using (1) and (2)]

$\Rightarrow$ $\angle PRS = 180° - 90° - 40 - 25° = 25°$

$\angle PMQ = 90°$ [Angle in a semicircle] …(3)

In ΔPMQ, we have

$\angle QPM + \angle PMQ + \angle PQM$

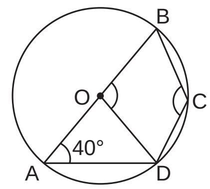
$= 180°$ [Sum of angles of a triangle]

$\Rightarrow$ $\angle QPM + 90° + 50° = 180°$ [Using (3)]

$\Rightarrow$ $\angle QPM = 180° - 90° - 50° = 40°$

Hence, $\boldsymbol{\angle QPR = 25°, \angle PRS = 25°, \angle QPM = 40°}$.

11. Since the angle subtended by an arc of a circle at the centre is double the angle subtended by it at any point on the remaining part of the circle,



$\therefore$ $m(\widehat{BOD}) = 2\angle BAD$

$= 2 \times 40° = 80°$.

$\angle BOD = 80°$

Since the opposite angles of a cyclic quadrilateral are supplementary and ABCD is a cyclic quadrilateral,

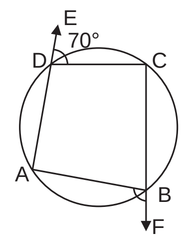
$\therefore$ $\angle BAD + \angle BCD = 180°$

$\Rightarrow$ $40° + \angle BCD = 180°$

$\Rightarrow$ $\angle BCD = 140°$

Hence, $\boldsymbol{\angle BOD = 80°, \angle BCD = 140°}$.

12. Since an exterior angle of a cyclic quadrilateral is equal to the interior opposite angle and ABCD is a cyclic quadrilateral,



$\therefore$ $\angle CDE = \angle ABC$

$\Rightarrow$ $70° = \angle ABC$ …(1)

$\angle ABF + \angle ABC = 180°$ [Linear pair]

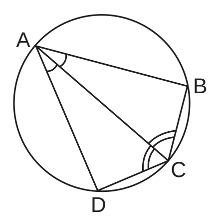
$\Rightarrow$ $\angle ABF + 70° = 180°$ [Using (1)]

$\Rightarrow$ $\angle ABF = 180° - 70° = 110°$

Hence, $\angle ABF = \mathbf{110°}$.

13. Since the opposite angles of a cyclic quadrilateral are supplementary and ABCD is a cyclic quadrilateral.

$\therefore$ $\angle A + \angle C = 180°$



$\Rightarrow$ $\frac{1}{2}\angle A + \frac{1}{2}\angle C = 90°$

$\Rightarrow$ $\angle CAB + \angle ACB = 90°$ [$\because$ AC bisects both the angles A and C] …(1)

In ΔABC, we have

$\angle ABC + (\angle CAB + \angle ACB)$

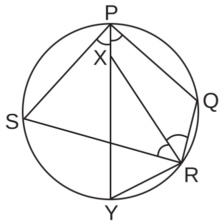
$= 180°$ [Sum of angles of a triangle]

$\Rightarrow$ $\angle ABC + 90° = 180°$ [Using (1)]

$\Rightarrow$ $\angle ABC = 180° - 90° = 90°$

Hence, $\boldsymbol{\angle ABC = 90°}$.

14. Since the opposite angles of a cyclic quadrilateral are supplementary and SPQR is a cyclic quadrilateral,



$\therefore \quad \angle SPQ + \angle SRQ = 180°$

$\Rightarrow \quad \frac{1}{2}\angle SPQ + \frac{1}{2}\angle SRQ = 90°$

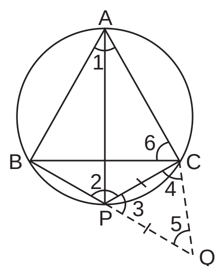
$\Rightarrow \quad \angle SPY + \angle XRS = 90°$ [$\because$ PY and RX are bisectors of $\angle P$ and $\angle R$ respectively]

But $\quad \angle SPY = \angle SRY$ [Angles in the same segment]

$\therefore \quad \angle SRY + \angle XRS = 90°$

$\Rightarrow \quad$ **$\angle XRY = 90°$.**

15. Produce BP to Q such that PQ = PC. Join CQ.



Since an exterior angle of cyclic quadrilateral is equal to interior opposite angle and ABPC is a cyclic quadrilateral,

$\therefore \quad \angle 3 = \angle 1 = 60°$

$\angle 3 + \angle 4 + \angle 5 = 180$

$\Rightarrow \quad 60° + 2\angle 4 = 180°$ [$\because$ PQ = PC $\Rightarrow \angle 4 = \angle 5$]

$\Rightarrow \quad \angle 4 = 60°$

Also, $\quad \angle 6 = 60°$ [Angle of an equilateral triangle]

$\therefore \quad \angle 6 = \angle 4$

$\Rightarrow \quad \angle 6 + \angle BCP = \angle 4 + \angle BCP$ [Adding $\angle BCP$ to both sides]

$\Rightarrow \quad \angle ACP = \angle BCQ \quad$ ...(1)

In ΔACP and ΔBCQ, we have

(i) $\angle ACP = \angle BCQ$ [From (1)]

(ii) $\angle CAP = \angle CBP$ (or $\angle CBQ$) [Angles in the same segment]

(iii) AC = BC [Sides of an equilateral triangle]

$\therefore \quad \Delta ACP \cong \Delta BCQ$ [By ASA congruence]

$\Rightarrow \quad$ PA = QB [By CPCT]

$\Rightarrow \quad$ PA = PB + PQ

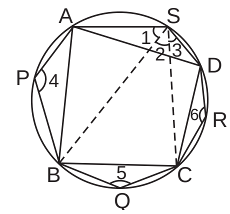
$\Rightarrow \quad$ PA = PB + PC [$\because$ PQ = PC by construction]

Hence, **PA = PB + PC**.

16. Let ABCD be a cyclic quadrilateral.

Let P, Q, R and S are points in four segments interior to quadrilateral ABCD.

AP, PB, BQ, QC, CR, RD, DS and SA are joined to form $\angle APB$, $\angle BQC$, $\angle CRD$ and $\angle DSA$ in the four interior segments. Join SB and SC.



Since the opposite angles of a cyclic quadrilateral are supplementary, we have

$\angle 1 + \angle 4 = 180°$ [Opposite angles of cyclic quad ASBP] ...(1)

$\angle 2 + \angle 5 = 180°$ [Opposite angles of cyclic quad BSCQ] ...(2)

and $\quad \angle 3 + \angle 6 = 180°$ [Opposite angles of cyclic quad SCRD] ...(3)

$\therefore \quad (\angle 1 + \angle 2 + \angle 3) + (\angle 4 + \angle 5 + \angle 6)$

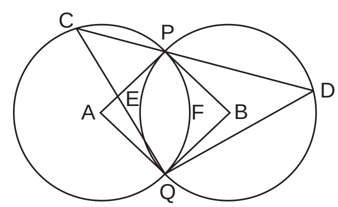
$= 540°$ [Adding (1), (2) and (3)]

$\Rightarrow \quad \angle S + \angle P + \angle Q + \angle R$

$= 6 \times 90° = 6$ right angles.

17. Since the opposite angles of a cyclic quadrilateral are supplementary and concyclic points A, P, B, Q form a cyclic quadrilateral APBQ,

$\therefore \quad \angle QAP + \angle QBP = 180° \quad$ ...(1)



Since the angle subtended by an arc of a circle at the centre is double the angle subtended by it at any point on the remaining part of the circle.

$\angle QAP = 2\angle QCP$ and $\angle QBP = 2\angle QDP$

$\therefore \quad \angle QAP + \angle QBP = 2(\angle QCP + \angle QDP)$

$\Rightarrow \quad 180° = 2(\angle QCP + \angle QDP)$ [Using (1)]

$\Rightarrow \quad 90° = \angle QCP + \angle QDP$

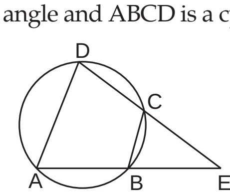
$\Rightarrow \quad 90° = \angle QCD + \angle QDC \quad$ ...(2)

In ΔCQD, $(\angle QCD + \angle QDC) + \angle CQD = 180°$

$\Rightarrow \quad 90° + \angle CQD = 180°$ [Using (2)]

$\Rightarrow \quad$ **$\angle CQD = 90°$**

18. Since an exterior angle of a cyclic quadrilateral is equal to interior opposite angle and ABCD is a cyclic quadrilateral,



$\therefore \quad \angle CBE = \angle ADC$ (= $\angle ADE$) $\quad$ ...(1)

and $\quad \angle BCE = \angle DAB$ (= $\angle DAE$)

In ΔEBC and ΔEDA, we have

(i) $\angle CEB = \angle AED$ [Each is equal to angle E]

(ii) $\angle CBE = \angle ADE$ [From (1)]

(iii) $\angle BCE = \angle DAE$ [From (1)]

Hence, **ΔEBC and ΔEDA are equiangular.**

19. $AB = AC$ [Given]

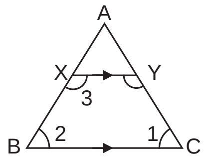
$\Rightarrow \angle 1 = \angle 2$ [Angles opposite equal sides of ΔABC] ...(1)

$XY \parallel BC$ [Given]

$\angle 2 + \angle 3 = 180°$ [Cont. angles] ...(2)

From (1) and (2), we get

$\angle 1 + \angle 3 = 180°$ ...(3)

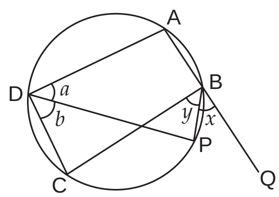


But $\angle 1$ and $\angle 3$ are opposite angles of quadrilateral BCYX. ...(4)

We know that if the opposite angles of a quadrilateral are supplementary then it is a cyclic quadrilateral.

$\therefore$ **BCYX is a cyclic quadrilateral.** [Using (3) and (4)]

20. Since an exterior angle of a cyclic quadrilateral is equal to interior opposite angle and ABPD is a cyclic quadrilateral.



$\therefore \angle x = \angle a$ ...(1)

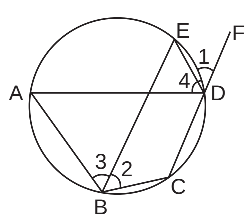
$\angle y = \angle b$ [Angles in the same segment] ...(2)

$\angle x = \angle y$ [Given] ...(3)

From (1), (2) and (3), we get

$\angle \boldsymbol{a} = \angle \boldsymbol{b}$.

21. Since an exterior angle of a cyclic quadrilateral is equal to the interior opposite angle and BCDE is a cyclic quadrilateral,



$\angle 1 = \angle 2$

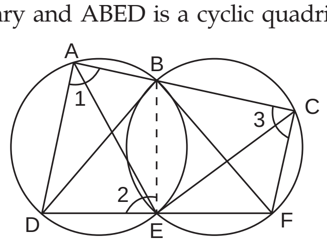
$\angle 3 = \angle 4$ [Angles in the same segment]

$\angle 2 = \angle 3$ [BE is the bisector of $\angle ABC$]

$\therefore \angle 1 = \angle 4$

$\Rightarrow$ **DE bisects $\angle$ADF.**

22. Since the opposite angles of a cyclic quadrilateral are supplementary and ABED is a cyclic quadrilateral,



$\therefore \angle 1 + \angle 2 = 180°$ ...(1)

Also, exterior angle of a cyclic quadrilateral is equal to the interior opposite angle and BCFE is a cyclic quadrilateral.

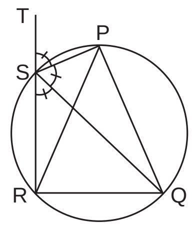
$\therefore \angle 2 = \angle 3$ ...(2)

$\therefore \angle 1 + \angle 3 = 180°$. [Using (1) and (2)]

But $\angle 1$ and $\angle 3$ are cointerior angles formed when AD and CF are cut by a transversal AC at A and C respectively.

$\therefore$ **AD $\parallel$ CF.**

23. Since the sum of all the angles on the same side of a line at a given point is 180°,



$\therefore \angle PST + \angle PSQ + \angle QSR = 180°$ ...(1)

$\angle PST = \angle PSQ = \angle QSR$ [Given] ...(2)

$\therefore \angle PST = \angle PSQ = \angle QSR$

$= 60°$[Using (1) and (2)] ...(3)

Since an exterior angle of a cyclic quadrilateral is equal to the interior opposite angle and PQRS is a cyclic quadrilateral,

$\therefore \angle PST = \angle PQR$

$\Rightarrow 60° = \angle PQR$ [Using (3)] ...(4)

$\angle QSR = \angle QPR$ [Angles in the same segment]

$\Rightarrow 60° = \angle QPR$ [Using (3)] ...(5)

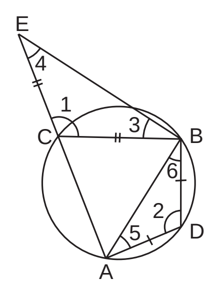
$\angle PSQ = \angle PRQ$ [Angles in the same segment]

$\Rightarrow 60° = \angle PRQ$ [Using (3)] ...(6)

From (4), (5) and (6), we conclude that ΔPQR is a triangle in which each angle 60°.

Hence, **ΔPQR is an equilateral triangle.**

24. Since the exterior angle of a cyclic quadrilateral is equal to the interior opposite angle and ADBC is a cyclic quadrilateral,



$\therefore \angle 1 = \angle 2 = y$ (say) ...(1)

In ΔCEB, we have

$BC = CE$

$\therefore \angle 3 = \angle 4 = x$ ...(2)

$\Rightarrow y + x + x = 180°$ [Sum of angles of a triangle]

$\Rightarrow y + x + x = 180°$ [Using (1) and (2)]

$\Rightarrow y + 2x = 180°$

$\Rightarrow 2x = 180° - y$

$\Rightarrow \quad x = \dfrac{180° - y}{2}$ …(3)

In ΔADB, we have

AD = BD [Given]

$\therefore \quad \angle 5 = \angle 6 = z$ (say) …(4)

In ΔADB, we have

$\angle 2 + \angle 5 + \angle 6 = 180°$ [Sum of angles of a triangle]

$\Rightarrow \quad y + z + z = 180°$ [Using (1) and (4)]

$\Rightarrow \quad y + 2z = 180°$

$\Rightarrow \quad 2z = 180° - y$

$\Rightarrow \quad z = \dfrac{180° - y}{2}$ …(5)

From (3) and (5), we get

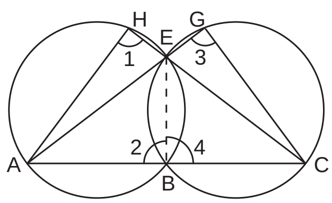
$z = x$

$\Rightarrow \quad \angle 6 = \angle 3$

$\Rightarrow \quad \angle$ABD = $\angle$CBE

Hence, **$\angle$ABD = $\angle$CBE.**

25. Since the opposite angles of a cyclic quadrilateral are supplementary, we have



$\angle 1 + \angle 2 = 180°$ [Opposite angles of cyclic quadrilateral ABEH]

$\Rightarrow \quad \angle 2 = (180° - \angle 1)$ …(1)

$\angle 3 + \angle 4 = 180°$ [Opposite angles of cyclic quadrilateral CBEG]

$\Rightarrow \quad \angle 4 = (180° - \angle 3)$ …(3)

$\angle 2 + \angle 4 = 180°$ [Linear pair]

$\Rightarrow \quad (180° - \angle 1) + (180° - \angle 3)$

$= 180°$ [Using (1) and (2)]

$\Rightarrow \quad 360° - \angle 1 - \angle 3 = 180°$

$\Rightarrow \quad 360° - 180° = \angle 1 + \angle 3$

$\Rightarrow \quad 180° = \angle$AHE + $\angle$EGC

Hence, **$\angle$AHE and $\angle$EGC are supplementary.**

26. $5^2 = 4^2 + 3^2$

$\therefore \quad \angle$DAB = 90° [By the converse of Pythagoras' Theorem]

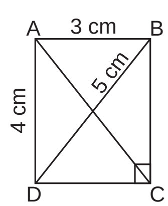
Also, $\angle$DCB = 90° [Given]

$\therefore \quad \angle$DAB + $\angle$DCB = 180° …(1)

But $\angle$DAB and $\angle$DCB are opposite angles of quadrilateral ABCD. …(2)

We know that of opposite angles of a quadrilateral are supplementary, then it is a cyclic quad.

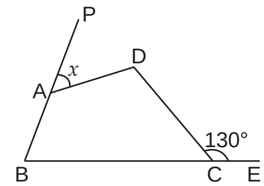
$\therefore$ ABCD is a cyclic quadrilateral. [Using (1) and (2)]



$\Rightarrow$ Points A, B, C and D are concyclic i.e. they lie on a circle.

$\therefore$ **$\angle$DAC = $\angle$DBC** [Angles in the same segment]

27. $\angle$DCB + $\angle$DCE = 180° [Linear pair]



$\Rightarrow \angle$DCB + 130° = 180°

$\Rightarrow \quad \angle$DCB = 180° – 130°

$\Rightarrow \quad \angle$DCB = 50°

(i) When $x = 40°$, then exterior $\angle$PAD (= 40°) ≠ interior opposite $\angle$DCB (= 50°).

$\Rightarrow$ ABCD is not a cyclic quadrilateral.

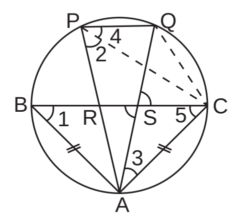
Hence, **points A, B, C and D are not concyclic.**

(ii) When $x = 50°$, then exterior $\angle$PAD (= 50°) = interior opposite $\angle$DCB (= 50°). We know that when exterior angle of a quadrilateral = interior opposite angle, then it is a cyclic quadrilateral.

$\therefore$ ABCD is a cyclic quadrilateral.

Hence, **points A, B, C and D are concyclic.**

28. Join PC and QC.



$\angle 1 = \angle 2$

and $\quad \angle 3 = \angle 4$ [Angles in the same segment]

$\angle 1 + \angle 3 = \angle 2 + \angle 4$

$\Rightarrow \quad \angle 5 + \angle 3 = \angle 2 + \angle 4$ [$\because$ AB = AC $\Rightarrow \angle 1 = \angle 5$] …(1)

By considering ΔACS whose side CS has been produced to R, we get

$\angle 5 + \angle 3$ = Exterior angle RSA …(2)

Also, $\quad \angle 2 + \angle 4 = \angle$QPR …(3)

From (1), (2) and (3), we get

$\angle$RSA = $\angle$QPR.

But $\angle$RSA and $\angle$QPR respectively are the exterior and interior opposite angles of quadrilateral PQSR.

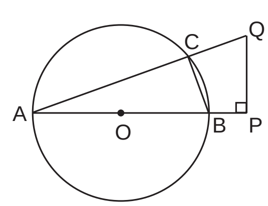
$\Rightarrow$ PQSR is a cyclic quadrilateral.

$\Rightarrow$ **P, Q, S and R are concyclic points.**

29. (i) $\angle$ACB = 90° [Angle in a semicircle] ...(1)

$\angle$APQ = 90° [Given]

$\Rightarrow \quad \angle$BPQ = 90° [$\because \angle$APQ and $\angle$BPQ are same angles i.e. $\angle$P] …(2)



From (1) and (2), we get,

$\angle ACB = \angle BPQ$ …(3)

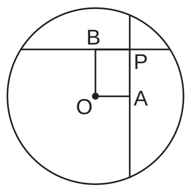
But $\angle ACB$ and $\angle BPQ$ respectively are the exterior and interior opposite angles of quadrilateral BCQP …(4)

We know that if exterior angle of a quadrilateral = interior opposite angle, then it is a cyclic quadrilateral.

Hence, **BCQP is a cyclic quadrilateral**.

[Using (3) and (4)]

(ii) We know that the line drawn through the centre of the circle to bisect a chord is perpendicular to the chord.



$\therefore$ $\angle OBP = 90°$ …(1)

and $\angle OAP = 90°$ …(2)

Adding (1) and (2), we get

$\angle OBP + \angle OAP = 90° + 90°$

$\Rightarrow$ $\angle OBP + \angle OAP = 180°$

$\Rightarrow$ $\angle OBP$ and $\angle OAP$ are supplementary angles …(3)

But $\angle OBP$ and $\angle OAP$ are opposite angles of quadrilateral OAPB. …(4)

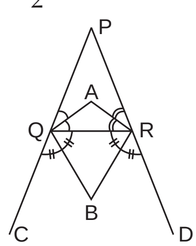
$\therefore$ OAPB is a cyclic quadrilateral. [From (3) and (4)]

$\Rightarrow$ O, A, P and B are concyclic points.

Hence, **O, A, P and B are concyclic points.**

(iii) $\angle PQR + \angle RQC = 180°$ [Linear pair]

$\Rightarrow$ $\frac{1}{2}\angle PQR + \frac{1}{2}\angle RQC = 90°$



$\Rightarrow$ $\angle AQR + \angle BQR = 90°$

[$\because$ QA and QB are bisectors of $\angle PQR$ and $\angle RQC$ respectively]

$\Rightarrow$ $\angle AQB = 90°$ …(1)

Again, $\angle PRQ + \angle QRD = 180°$ [Linear pair]

$\Rightarrow$ $\frac{1}{2}\angle PRQ + \frac{1}{2}\angle QRD = 90°$

$\Rightarrow$ $\angle ARQ + \angle BRQ = 90°$

[$\because$ RA and RB are the bisectors of $\angle PRQ$ and QRD respectively]

$\Rightarrow$ $\angle ARB = 90°$ …(2)

Adding (1) and (2), we get

$\angle AQB + \angle ARB = 90° + 90°$

$\Rightarrow$ $\angle AQB + \angle ARB = 180°$

$\Rightarrow$ $\angle AQB$ and $\angle ARB$ are supplementary angles. …(3)

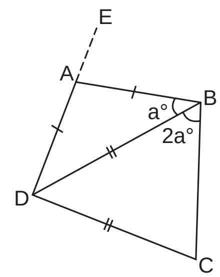
But $\angle AQB$ and $\angle ARB$ are opposite angles of quadrilateral AQBR. …(4)

We know that if the opposite angles of a quadrilateral are supplementary, then it is a cyclic quadrilateral.

Hence, **AQBR is a cyclic quadrilateral.**

[Using (3) and (4)]

(iv) Produce DA to any point E.



In ΔADB, we have

$AB = AD$ [Given]

$\therefore$ $\angle ADB = \angle ABD$ [Angles opposite equal sides of ΔADB]

$\Rightarrow$ $\angle ADB = a°$ …(1)

Considering ΔBDA, whose side DA has been produced to E.

We get, Exterior $\angle BAE = \angle ABD + \angle ADB$

[Exterior $\angle$ = Sum of interior opposite angles]

$\Rightarrow$ Exterior $\angle BAE = a° + a°$ [Using (1)]

$\Rightarrow$ Exterior $\angle BAE = 2a°$ …(2)

Now in ΔDBC, we have

$DB = DC$ [Given]

$\therefore$ $\angle DCB = \angle DBC$ [Angles opposite equal sides of ΔDBC]

$\Rightarrow$ $\angle DCB = 2a°$ …(3)

From (2) and (3), we get

Ext. $\angle BAE = \angle DCB$ …(4)

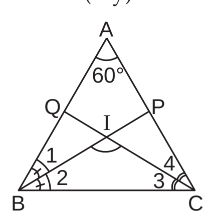
But $\angle BAE$ and $\angle DCB$ are respectively the exterior and interior opposite angles of quadrilateral. …(5)

Hence, **ABCD is a cyclic quadrilateral.**

[Using (4) and (5)]

(v) In ΔABC, BP bisects $\angle B$

$\therefore$ $\angle 1 = \angle 2 = x$ (say) …(1)



and CQ bisects $\angle C$

$\therefore$ $\angle 3 = \angle 4 = y$ (say) …(2)

In ΔABC, we have

$\angle BAC + \angle ABC + \angle ACB = 180°$

$\Rightarrow \quad 60° + (\angle 1 + \angle 2) + (\angle 3 + \angle 4) = 180°$

$\Rightarrow \quad 60° + 2x + 2y = 180°$

$\Rightarrow \quad 60° + 2(x + y) = 180°$

$\Rightarrow \quad 2(x + y) = 180° - 60°$

$\Rightarrow \quad x + y = \dfrac{120°}{2}$

$\Rightarrow \quad x + y = 60° \quad \ldots(3)$

In ΔBIC, we have

$\angle 2 + \angle 3 + \angle BIC = 180°$

$\Rightarrow \quad (x + y) + \angle BIC = 180°$

$\Rightarrow \quad 60° + \angle BIC = 180°$ [Using (3)]

$\Rightarrow \quad \angle BIC = 180° - 60°$

$\Rightarrow \quad \angle BIC = 120°$

Hence, $\angle BIC = 120°$

$\angle QIP = \angle BIC$ [Ver. opp. angles]

$\Rightarrow \quad \angle BIP = 120° \quad \ldots(4)$

In quadrilateral APIQ, we have

$\angle A + \angle QIP = 60° + 120°$ [Using (4)]

$= 180°$

$\Rightarrow$ $\angle$A and $\angle$QIP are supplementary angles ...(5)

But $\angle$A and $\angle$QIP are opposite angles of quadrilateral APIQ. ...(6)

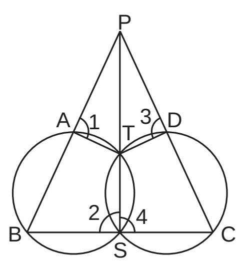
We know that if the opposite angles of a quadrilateral are supplementary then it is a cyclic quadrilateral.

Hence, **APIQ is a cyclic quadrilateral.**

[Using (5) and (6)]

30. Since an exterior angle of a cyclic quadrilateral is equal to the interior opposite angle,

$\therefore \quad \angle 1 = \angle 2$ [Exterior and interior opposite angles of cyclic quadrilateral ABST] ...(1)



and $\quad \angle 3 = \angle 4$ [Exterior and interior opposite angles of cyclic quadrilateral DCST] ...(2)

Adding (1) and (2), we get

$\angle 1 + \angle 3 = \angle 2 + \angle 4 \quad \ldots(3)$

But $\quad \angle 2 + \angle 4 = 180°$ [Linear pair] ...(4)

$\therefore \quad \angle 1 + \angle 3 = 180°$ [Using (3) and (4)]

$\Rightarrow$ $\angle$1 and $\angle$3 are supplementary angles. ...(5)

But $\angle$1 and $\angle$3 are opposite angles of quadrilateral PATD. ...(6)

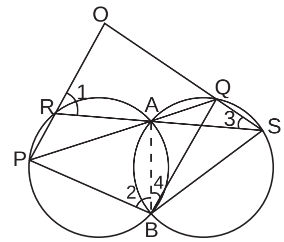
We know that if opposite angles of a quadrilateral are supplementary then it is a cyclic quadrilateral.

$\therefore$ PATD is a cyclic quadrilateral [Using (5) and (6)]

$\Rightarrow$ Points P, A, T and D are concylic

Hence, **P, A, T and D are concyclic.**

31. Join AB. Then, ABPR is a cyclic quadrilateral. Since an exterior angle of a cyclic quadrilateral is equal to interior opposite angle,



$\therefore \quad \angle 1 = \angle 2$

and $\quad \angle 3 = \angle 4$ [Angles in the same segment]

$\therefore \quad \angle 1 + \angle 3 = \angle 2 + \angle 4 = \angle PBQ \quad \ldots(1)$

In ΔROS, $(\angle 1 + \angle 3) + \angle ROS = 180°$

[Sum of angles of a triangle]

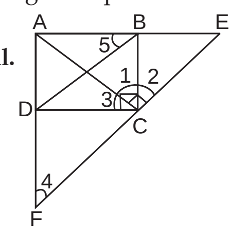
$\Rightarrow \quad \angle PBQ + \angle ROS = 180°$ [Using (1)]

$\Rightarrow \quad \angle PBQ + \angle POQ = 180°$

$\Rightarrow$ $\angle$PBQ and $\angle$POQ are supplementary angles.

But $\angle$PBQ and $\angle$POQ are opposite angles of quadrilateral OPBQ.

$\therefore$ **OPBQ is a cyclic quadrilateral.**



32. $\angle ACE = 90°$

$\Rightarrow \quad \angle 1 + \angle 2 = 90°$

Also $\quad \angle BCD = 90°$

$\Rightarrow \quad \angle 1 + \angle 3 = 90°$

$\therefore \quad \angle 2 = \angle 3$

But $\quad \angle 2 = \angle 4$ [Corresponding angles]

$\therefore \quad \angle 3 = \angle 4$

In ΔABD and ΔDCA, we have

AB = DC, BD = CA and AD = DA

$\therefore \quad \Delta ABD \cong \Delta ACD$ [By SSS congruence]

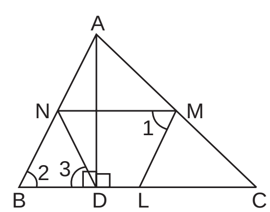
$\therefore \quad \angle 5 = \angle 3$ [CPCT]

$\Rightarrow \quad \angle 5 = \angle 4$

But $\angle$5 is the exterior angle of quadrilateral BEFD and $\angle$4 is its interior opposite angle.

$\therefore$ **Quadrilateral BEFD is a cyclic quadrilateral.**

33. (i) $\quad \angle ADB = 90°$ [Given]



$\therefore$ Circle on AB as diameter will pass through D.

[Angle in a semicircle = 90°]

$\therefore \quad$ AN = NB = ND $\Rightarrow$ **AN = ND.**

(ii) NMLB is a parallelogram.

[By Mid-point Theorem, ML || NB and NM || BL]

$\therefore \quad \angle 1 = \angle 2$ [Opposite angles of a || gm]

But $\quad \angle 3 = \angle 2$ [Angles opposite to equal sides ND and NB of a ΔNBD]

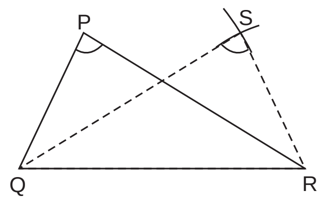
$\therefore \quad \angle 3 = \angle 1$

But $\angle 3$ is the exterior angle of quadrilateral LMND and $\angle 1$ is its interior opposite angle.

$\therefore$ Quadrilateral LMND is a cyclic quadrilateral.

$\Rightarrow$ **L, M, N and D are concyclic.**

34. Let P, Q, R be the three given points. With Q as centre and radius equal to PR draw an arc and with R as centre and radius equal to PQ draw another arc, cutting the previous arc at S. Then S, is the required fourth point.



Justification:

Join QS and RS.

In ΔQPR and ΔRSQ, we have

$$QP = RS, PR = SQ \text{ and } QR = RQ$$

$\therefore \quad \Delta QPR \cong \Delta RSQ$ [By SSS congruence]

$\therefore \quad \angle QPR = \angle RSQ$ [CPCT]

$\Rightarrow$ QR subtends equal angles on the same side of it.

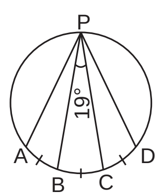
$\therefore$ Points P, Q, R and S are concyclic.

$\Rightarrow$ S lies on the circle passing through P, Q and R.

CHECK YOUR UNDERSTANDING

MULTIPLE-CHOICE QUESTIONS

1. (c) **57°**



arc AB ≅ arc BC ≅ arc CD

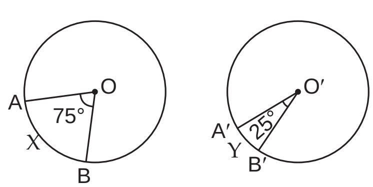
$\therefore \quad \angle APB = \angle BPC$

$= \angle CPD = 19°$ [Congruent arcs subtend equal angles at a point on the circumference]

$\angle APD = \angle APB + \angle BPC + \angle CPD$

$= 19° + 19° + 19° = 57°$

2. (a) **3 : 1**



$m(\widehat{AB}) = 75°, \; m(\widehat{A'B}) = 25°$

$$\text{arc AXB} = \frac{75°}{360°} \times \text{circumference}$$

$$\text{arc A'YB'} = \frac{25°}{360°} \times \text{circumference}$$

$\therefore$ arc AXB : arc A′YB′

$$= \frac{75}{360} \times \text{circumference} : \frac{25}{360} \times \text{circumference}$$

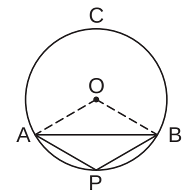
$= 3 : 1$

3. (b) **Diameter**

The longest chord of a circle is its diameter.

4. (b) **An obtuse angle**

Angle subtended by an arc of a circle at the centre is double the angle subtended by it on the remaining part of the circle in minor segment.



$\angle APB$ subtended by major arc $\widehat{BCA} = \frac{1}{2}$ ref$\angle BOA$.

Since ref$\angle BOA > 180°$,

$\therefore \quad \frac{1}{2}$ ref$\angle BOA > 90°$

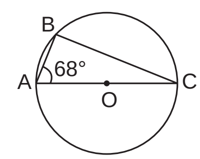
$\therefore \quad \angle APB$ in the minor segment $> 90°$

Hence, angle formed in a minor segment is obtuse.

5. (a) **1**

There is one and only one circle passing through three non collinear points [Refer to the proof on page 4.7]

6. (a) **22°**



$\angle ABC = 90°$ [Angle in a semicircle]

In ΔABC, we have

$68° + 90° + \angle ACB$

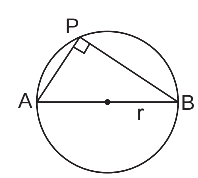
$= 180°$ [Sum of angles of a triangle]

$\Rightarrow \quad 158° + \angle ACB = 180°$

$\Rightarrow \quad \angle ACB = 180° - 158° = 22°$

7. (b) **2*r***

$\angle APB = 90°$



$\Rightarrow$ $\angle APB$ is angle in a semicircle

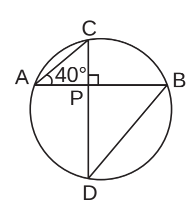
$\Rightarrow$ $\angle APB$ is subtended by a diameter of the circle.

$\Rightarrow$ AB is diameter of the circle

$\therefore \quad AB = 2 \times \text{radius} = 2 \times r = 2r$

8. (c) **50°**

Let AB and CD intersect at P.



Considering ΔCAP whose side AP is produced to B, we get

Exterior $\angle CPB = \angle CAP + \angle ACP$

[Exterior angle = Sum of interior opposite angles]

$\Rightarrow \quad 90° = 40° + \angle ACP$

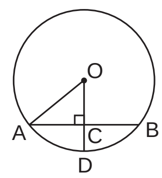
$\Rightarrow \quad \angle ACP = 90° - 40° = 50° \quad \ldots(1)$

$\angle ABD = \angle ACD \; (= \angle ACP)$

[Angles in the same segment]

$\Rightarrow \quad \angle ABD = 50° \quad$ [Using (1)]

9. (a) **2 cm**



Since perpendicular drawn from the centre of a circle to a chord bisects it,

$\therefore \quad AC = CB = \frac{1}{2} AB$

$= \frac{1}{2} \times 8 \text{ cm} = 4 \text{ cm} \quad \ldots(1)$

In right ΔOCA, we have

$OA^2 = OC^2 + AC^2$ [By Pythagoras Theorem]

$\Rightarrow \quad (5 \text{ cm})^2 = OC^2 + (4 \text{ cm})^2$ [Using (1)]

$\Rightarrow \quad OC^2 = (25 - 16) \text{ cm}^2 = 9 \text{ cm}^2$

$\Rightarrow \quad OC = 3 \text{ cm} \quad \ldots(2)$

Now, $\quad CD = OD - OC$

$= OA - OC$

[$\because$ OA = OD, radii of a circle]

$\Rightarrow \quad CD = 5 \text{ cm} - 3 \text{ cm}$ [Using (2)]

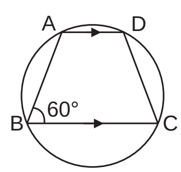
$\Rightarrow \quad CD = 2 \text{ cm.}$

10. (d) **60°**

$\angle DAB + \angle ABC = 180°$ [Coint. angles, AD || BC]

$\Rightarrow \angle DAB + 60° = 180°$

$\Rightarrow \quad \angle DAB = 120° \quad \ldots(1)$



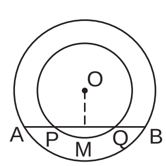
Since opposite angles of a cyclic quadrilateral are supplementary

$\therefore \quad \angle DAB + \angle BCD = 180°$

$\Rightarrow \quad 120° + \angle BCD = 180°$ [Using (1)]

$\Rightarrow \quad \angle BCD = 60°$

11. (b) **AP = BQ**



Since perpendicular drawn from the centre of a circle to the chord bisects it

$\therefore \quad AM = BM \quad \ldots(1)$

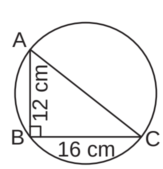
and $\quad PM = QM \quad \ldots(2)$

Subtracting (2) from (1), we get

$AM - PM = BM - QM$

$\Rightarrow \quad AP = BQ$

12. (d) **10 cm**



Since AB ⊥ BC,

$\therefore \quad \angle ABC = 90°$

$\Rightarrow$ ∠ABC is a angle in a semicircle

$\Rightarrow$ AC is a diameter of the circle passing through A, B and C.

In right ΔABC, we have

$AC^2 = AB^2 + BC^2$ [By Pythagoras Theorem]

$= (12 \text{ cm})^2 + (16 \text{ cm})^2$

$= (144 + 256) \text{ cm}^2 = 400 \text{ cm}^2$

$\Rightarrow \quad AC = 20 \text{ cm}$

$\text{radius} = \frac{1}{2} \text{diameter}$

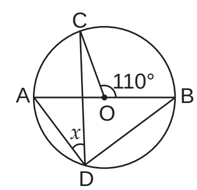
$= \frac{1}{2} \times AC = \frac{1}{2} \times 20 \text{ cm} = 10 \text{ cm}$

13. (a) **35°**

$\angle AOC + \angle BOC = 180°$ [Linear pair]

$\Rightarrow \quad \angle AOC + 110° = 180°$

$\Rightarrow \quad \angle AOC = 70° \quad \ldots(1)$



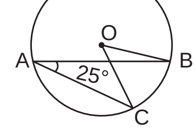
Since the angle subtended by an arc of a circle at the centre is double the angle subtended by it at any other point on the circle,

$\therefore \quad \angle AOC = 2\angle ADC$

$\Rightarrow \quad 70° = 2x$ [Using (1)]

$\Rightarrow \quad x = 35°$

14. (b) **50°**

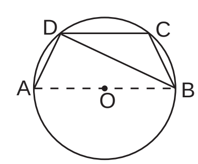


Since the angle subtended by an arc of circle at the centre is twice the angle subtended by it at any other point on the circle,

$\therefore \quad \angle BOC = 2\angle CAB$

$\Rightarrow \quad \angle BOC = 2 \times 25° = 50°$

15. (b) **28°**



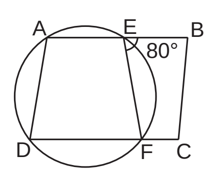
$\angle ADB = 90°$ [Angle in a semicircle] …(1)

$\angle BDC = \angle ADC - \angle ADB$

$\Rightarrow \quad \angle BDC = 118° - 90°$ [Using (1)]

$\Rightarrow \quad \angle BDC = 28°$

16. (d) **80°**



Since an exterior angle of a cyclic quadrilateral is equal to the interior opposite angle of a cyclic quadrilateral,

$\therefore \quad \angle ADF = \angle AEF$

$\Rightarrow \quad \angle ADF = 80° \quad \ldots(1)$

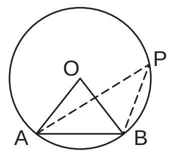
$\angle ABC = \angle ADF \; (\text{or } \angle ADC)$

[Opposite angles of a || gm]

$\Rightarrow \quad \angle ABC = 80°$ [Using (1)]

17. (d) **30°**

Let AB be the chord of a circle with centre O such that AB = radius OA or OB.



$\therefore$ In ΔOAB, OA = OB = AB.

$\therefore$ ΔOAB is an equilateral triangle.

$\therefore$ $\angle AOB = 60°$

Let P be any point in the major segment.

Then, $\angle APB = \frac{1}{2}\angle AOB = \frac{1}{2} \times 60° = 30°$

[$\because$ Angle subtended by an arc at the centre of the circle is double the angle subtended by it at any point on the remaining part of the circle]

18. (b) **70°**

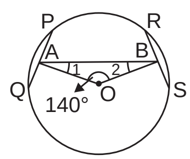
In ΔOAB, we have

$140° + \angle 1 + \angle 2 = 180°$ [Sum of angles a triangle]

$\Rightarrow$ $140° + 2\angle 1 = 180°$ [$\because$ $\angle 1 = \angle 2$, angles opposite equal sides OA and OB of triangle OAB (as OA = OB = radius)]

$\Rightarrow$ $2\angle 1 = 40°$ $\Rightarrow$ $\angle 1 = 20°$

$\Rightarrow$ $\angle OAB = 20°$ …(1)



Since OA $\perp$ PQ,

$\therefore$ $\angle PAO = 90°$ …(2)

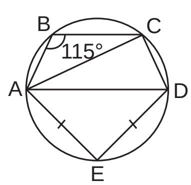
$\angle PAB = \angle PAO - \angle OAB$

$\Rightarrow$ $\angle PAB = 90° - 20°$ [Using (1) and (2)]

$\Rightarrow$ $\angle PAB = 70°$

19. (c) **70°**

Since opposite angles of a cyclic quadrilateral are supplementary and ABCD is a cyclic quadrilateral,



$\therefore$ $\angle ABC + \angle ADC = 180°$

$\Rightarrow$ $115° + \angle ADC = 180°$

$\Rightarrow$ $\angle ADC = 180° - 115°$

$\Rightarrow$ $\angle ADC = 65°$ …(1)

$\angle ACD = 90°$ [Angle in a semicircle] …(2)

In ΔACD, we have

$\angle CAD + \angle ADC + \angle ACD = 180°$ [Sum of angles of a triangle]

$\Rightarrow$ $\angle CAD + 65° + 90° = 180°$ [Using (1) and (2)]

$\Rightarrow$ $\angle CAD = 180° - 90° - 65°$

$= 25°$ …(3)

In ΔAED, we have

$\angle AED + \angle EAD + \angle EDA = 180°$ [Sum of angles of a triangle]

$\Rightarrow$ $90° + 2\angle EAD = 180°$

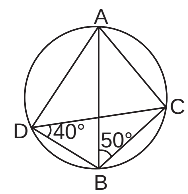
[$\because$ $\angle AED = 90°$ angle in a semicircle and $\angle EAD = \angle ADE$, angles opposite equal sides of ΔAED]

$\Rightarrow$ $\angle EAD = \frac{90°}{2} = 45°$ …(4)

$\angle CAE = \angle CAD + \angle EAD$

$= 25° + 45° = 70°$

20. (c) **90°**



$\angle CDA = \angle CBA$

$= 50°$ [Angles in the same segment]

$\angle ADB = \angle CDA + \angle CDB$

$= 50° + 40° = 90°$ …(1)

Since the opposite angles of a cyclic quadrilateral are supplementary and ADBC is a cyclic quadrilateral,

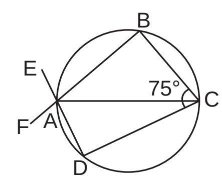
$\therefore$ $\angle ADB + \angle BCA = 180°$

$\Rightarrow$ $90° + \angle BCA = 180°$ [Using (1)]

$\Rightarrow$ $\angle BCA = 90°$

21. (b) **15°**

Since the opposite angles of a cyclic quadrilateral are supplementary and ABCD is a cyclic quadrilateral,



$\therefore$ $\angle BAD + \angle BCD = 180°$

$\Rightarrow$ $\angle BAD + 75° = 180°$

$\Rightarrow$ $\angle BAD = 180° - 75° = 105°$ …(1)

$\angle EAF = \angle BAD$ [Ver. opp. angles]

$\Rightarrow$ $\angle EAF = 105°$ [Using (1)] …(2)

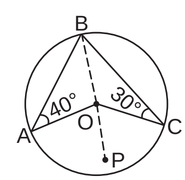
$\angle ABC = 90°$ [Angle is a semicircle] ...(3)

Subtracting (3) from (2) we get

$\angle EAF - \angle ABC = 105° - 90° = 15°$

22. (b) **140°**

Join BO and extend it to a point say P.



In ΔOAB, we have

OA = OB [radii of a circle]

$\therefore$ $\angle OBA = \angle OAB$

$= 40°$ [Angles opposite equal sides of ΔOAB]

Since, Exterior angles of a triangle

= Sum of its interior opposite angles

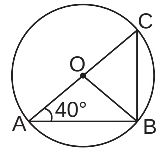
$\therefore$ $\angle AOP = 40° + 40° = 80°$

Similarly, $\angle POC = 30° + 30° = 60°$

$\angle AOC = \angle AOP + \angle POC = 80° + 60° = 140°$

23. (c) **50°**

$\angle ABC = 90°$ [Angle in a semicircle]



In ΔABC, we have

$\angle CAB + \angle ABC + \angle ACB$

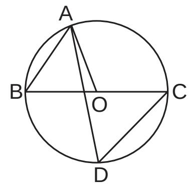
$= 180°$ [Sum of angles of a triangle]

$\Rightarrow$ $40° + 90° + \angle ACB = 180°$

$\Rightarrow$ $\angle ACB = 180° - 40° - 90°$

$= 50°$

24. (a) **60°**



In ΔOAB, we have

$OA = OB$

$\therefore$ $\angle ABO = \angle BAO$

$= 60°$ [Angles opposite equal sides of a triangle]

$\Rightarrow$ $\angle ABC = 60°$ [$\because$ $\angle ABO = \angle ABC$, same angles] ...(1)

$\angle ADC = \angle ABC$ [Angles in the same segment]

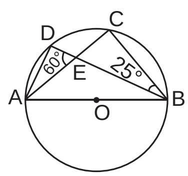
$\therefore$ $\angle ADC = 60°$ [Using (1)]

25. (c) **95°**

$\angle CEB = \angle DEA$

$= 60°$ [V. opposite angles] ...(1)

In ΔCEB, we have



$\angle ECB + \angle CEB + \angle EBC$

$= 180°$ [Sum of angles of a triangle]

$\Rightarrow$ $\angle ECB + 60° + 25°$

$= 180°$ [Using (1)]

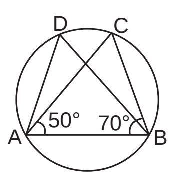
$\Rightarrow$ $\angle ECB = 180° - 60° - 25° = 95°$

$\Rightarrow$ $\angle ACB = 95°$ [$\because$ $\angle ECB = \angle ACB$, same angles] ...(2)

$\angle ADB = \angle ACB$ [Angles in the same segment]

$\Rightarrow$ $\angle ADB = 95°$ [Using (2)]

26. (b) **60°**



In ΔACB, we have

$\angle CAB + \angle ABC + \angle ACB$

$= 180°$ [Sum of angles of a triangle]

$\Rightarrow$ $50° + 70° + \angle ACB = 180°$

$\Rightarrow$ $\angle ACB = 180° - 50° - 70°$

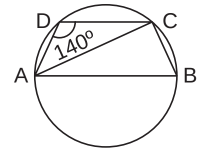
$= 60°$ ...(1)

$\angle ADB = \angle ACB$ [Angles in the same segment]

$\Rightarrow$ $\angle ADB = 60°$ [Using (1)]

27. (b) **50°**

Since the opposite angles of a cyclic quadrilateral are supplementary and ABCD is a cyclic quadrilateral,



$\therefore$ $\angle CDA + \angle ABC = 180°$

$\Rightarrow$ $140° + \angle ABC = 180°$

$\Rightarrow$ $\angle ABC = 40°$ ...(1)

$\therefore$ $\angle ACB = 90°$ [Angle in a semicircle] ...(2)

In ΔCAB, we have

$\angle BAC + \angle ABC + \angle ACB$

$= 180°$ [Sum of angles of a triangle]

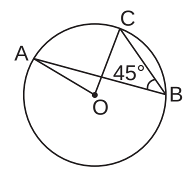
$\Rightarrow$ $\angle BAC + 40° + 90° = 180°$ [Using (1) and (2)]

$\Rightarrow$ $\angle BAC = 180° - 40° - 90°$

$\Rightarrow$ $\angle BAC = 50°$

28. (b) **90°**

Since the angle subtended by an arc of a circle at the centre is twice the angle subtended by it at any other point on the remaining part of the circle,



$\therefore$ $\angle AOC = 2\angle ABC$

$\Rightarrow$ $\angle AOC = 2 \times 45°$

$\Rightarrow$ $\angle AOC = 90°$

29. (d) **95°**

$\angle DAC = \angle DBC = 25°$ [Angles in the same segment]

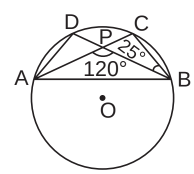
$\Rightarrow$ $\angle DAP = 25°$ [$\because$ $\angle DAC = \angle DAP$, same angles] ...(1)

$\angle DPA + \angle APB = 180°$ [Linear pair]

$\Rightarrow$ $\angle DPA + 120° = 180°$

$\Rightarrow$ $\angle DPA = 60°$ ...(2)

In ΔADP, we have



$\angle DAP + \angle DPA + \angle ADP$

$= 180°$ [Sum of angles of a triangle]

$\Rightarrow$ $25° + 60° + \angle ADP = 180°$ [Using (1) and (2)]

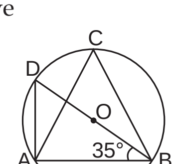
$\Rightarrow$ $\angle ADP = 180° - 25° - 60° = 95°$

$\Rightarrow$ $\angle ADB = 95°$ [$\because$ $\angle ADP = \angle ADB$, same angles]

30. (c) **55°**

$\angle DAB = 90°$ [Angle in a semicircle] ...(1)

In ΔADB, we have



$\angle ADB + \angle DAB + \angle ABD$

$= 180°$ [Sum of angles of a triangle]

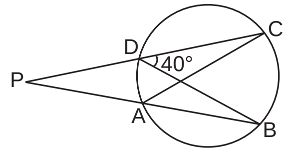
$\Rightarrow$ $\angle ADB + 90° + 35° = 180°$ [Using (1)]

$\Rightarrow$ $\angle ADB = 180° - 90° - 35° = 55°$ …(2)

$\angle ACB = \angle ADB$ [Angles in the same segment]

$\Rightarrow$ $\angle ACB = 55°$ [Using (2)]

31. (d) **140°**



$\angle CAB = \angle CDB$ [Angles in the same segment]

$\Rightarrow$ $\angle CAB = 40°$

$\angle PAC + \angle CAB = 180°$ [Linear pair]

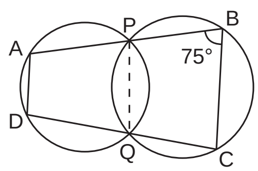
$\Rightarrow$ $\angle PAC + 40° = 180°$

$\Rightarrow$ $\angle PAC = 180° - 40° = 140°$

Hence, $\angle PAC = 140°$.

32. (d) **105°**

Since an exterior angle of a cyclic quadrilateral is equal to the interior opposite angle of the quadrilateral and PBCQ is a cyclic quadrilateral,



$\therefore$ $\angle PQD = \angle PBC = 75°$ …(1)

Since the opposite angles of a cyclic quadrilateral are supplementary and APQD is a cyclic quadrilateral,

$\therefore$ $\angle PAD + \angle PQD = 180°$

$\Rightarrow$ $\angle PAD + 75° = 180°$ [Using (1)]

$\Rightarrow$ $\angle PAD = 105°$

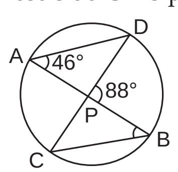
33. (b) **42°**

$\angle DCB = \angle DAB$ [Angles in the same segment]

$\Rightarrow$ $\angle DCB = \angle DAP$ [$\because$ $\angle DAB = \angle DAP$, same angle]

$\Rightarrow$ $\angle DCB = 46°$

Consider ΔBCP whose side CP is produced to D.



Exterior $\angle DPB = \angle PCB + \angle PBC$ [Exterior angle = Sum of interior opposite angles]

$\Rightarrow$ $88° = 46° + \angle PBC$ [$\because$ $\angle PCB = \angle DCB$, same angle]

$\Rightarrow$ $\angle PBC = 88° - 46° = 42°$

$\Rightarrow$ $\angle ABC = 42°$ [$\because$ $\angle PBC = \angle ABC$, same angle]

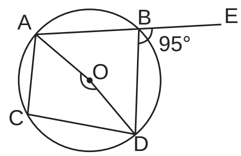
34. (a) **170°**

$\angle ABD + \angle DBE = 180°$ [Linear pair]

$\Rightarrow$ $\angle ABD + 95° = 180°$

$\Rightarrow$ $\angle ABD = 85°$ …(1)

Since the angle subtended by an arc of a circle at the centre is double the angle subtended by it at any point on the remaining part of the circle.



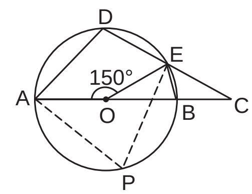
$\therefore$ $\angle AOD = 2\angle ABD$

$\Rightarrow$ $\angle AOD = 2 \times 85°$ [Using (1)]

$\Rightarrow$ $\angle AOD = 170°$

35. (a) **105°**

Take any point P on arc AB on the side opposite to point D. Join PA and PE.



Since the angle subtended by an arc of a circle at the centre is double the angle subtended by it at any point on the remaining part of the circle,

$\therefore$ $\angle AOE = 2\angle APE$

$\Rightarrow$ $150° = 2\angle APE$

$\Rightarrow$ $\angle APE = \frac{150°}{2} = 75°$ …(1)

Since the opposite angles of a cyclic quadrilateral are supplementary and ADEP is a cyclic quadrilateral,

$\therefore$ $\angle ADE + \angle APE = 180°$

$\Rightarrow$ $\angle ADE + 75° = 180°$ [Using (1)]

$\Rightarrow$ $\angle ADE = 180° - 75°$

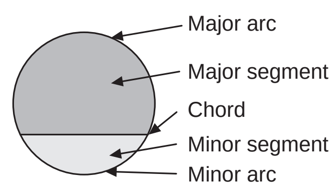
$= 105°$ …(2)

Since an exterior angle of a cyclic quadrilateral is equal to the interior opposite angle of the quadrilateral and ADEB is a cyclic quadrilateral whose side AB has been produced to C,

$\therefore$ Exterior $\angle CBE$ = Interior opposite angle ADE

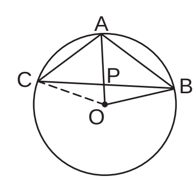
$\Rightarrow$ $\angle CBE = 105°$ [Using (2)]

36. (a) **A segment**



37. (c) **96°**

Join CO.



Since AB is a side of a regular five sides polygon,

$\therefore$ Angle subtended by AB at the centre O is

$\frac{360°}{5} = 72°$ $\Rightarrow$ $\angle AOB = 72°$ …(1)

Since AC is a side of a regular six sided polygon,

$\therefore$ Angle subtended by AC at the centre O is

$\frac{360°}{6} = 60°$

$\Rightarrow$ $\angle AOC = 60°$ …(2)

Since $\quad OA = OC,$ $\quad$ [Radii of a circle]

$\therefore \quad \angle OAC = \angle ACO$

$\Rightarrow \quad \angle OAC = 60°$ [Using angle sum property of a triangle]

$\Rightarrow \quad \angle PAC = 60°$ [$\because \angle OAC = \angle PAC$, same angle] …(3)

Since the angle subtended by an arc of a circle at the centre is double the angle subtended by it at any point on the remaining part of the circle,

$\therefore \quad \angle AOB = 2\angle ACB$

$\Rightarrow \quad 72° = 2\angle ACP$ [$\because \angle ACB = \angle ACP$, same angle]

$\Rightarrow \quad \angle ACP = \dfrac{72°}{2} = 36°$ …(4)

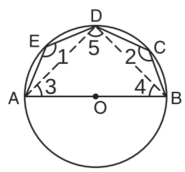
Now, considering ΔACP whose side CP is produced to B, we have

Exterior $\angle APB = \angle ACP + \angle PAC$

$= 60° + 36°$ [Using (3) and (4)]

$\Rightarrow \quad \angle APB = 96°.$

38. (c) **270°**



Join DA and DB.

Since the opposite angles of a cyclic quadrilateral are supplementary,

$\therefore \quad \angle 1 + \angle 4 = 180°$ [Opposite angles of cyclic quad AEDB] …(1)

and $\quad \angle 2 + 3 = 180°$ [Opposite angles of cyclic quad ADCB] …(2)

In ΔDAB, we have $\angle 3 + \angle 4 + \angle 5 = 180°$

$\Rightarrow \quad \angle 3 + \angle 4 + 90° = 180°$ [$\because \angle 5 = 90°$, an angle in a semicircle]

$\Rightarrow \quad \angle 3 + \angle 4 = 90°$ …(3)

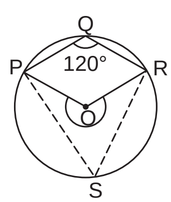
Adding (1) and (2), we get $\angle 1 + \angle 2 + (\angle 3 + \angle 4) = 360°$

$\Rightarrow \quad \angle 1 + \angle 2 + 90° = 360°$ [Using (3)]

$\Rightarrow \quad \angle 1 + \angle 2 = 360° - 90° = 270°$

$\Rightarrow \quad \angle AED + \angle BCD = 270°$

39. (c) $\mathbf{\dfrac{1}{3}}$ **of the circle**



Take any point S on major arc PR. Join SP and SR. Since the opposite angles of a cyclic quadrilateral are supplementary and PQRS is a cyclic quadrilateral,

$\therefore \quad \angle PQR + \angle PSR = 180°$

$\Rightarrow \quad 120° + \angle PSR = 180°$

$\Rightarrow \quad PSR = 60°$

Since the angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle,

$\therefore \quad \angle POR(\theta) = 2\ \angle PSR$

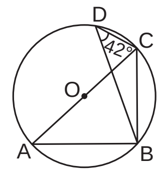
$\Rightarrow \quad \angle POR = 2 \times 60° = 120°$

Minor arc RP $= \dfrac{\theta}{360°}$ of the whole circle

$= \dfrac{120°}{360°}$ of the whole circle

$= \dfrac{1}{3}$ of the whole circle.

40. (b) **48°**



$\angle BAC = \angle BDC$ [Angle in the same segment]

$\Rightarrow \quad \angle BAC = 42°$ …(1)

In ΔABC, we have

$\angle ACB + \angle CBA + \angle BAC$

$= 180°$ [Sum of angles of a triangle]

$\Rightarrow \quad \angle ACB + 90° + 42°$

$= 180°$ [$\angle CBA = 90°$, angle in a semicircle and using (1)]

$\Rightarrow \quad \angle ACB = 180° - 90° - 42° = 48°$

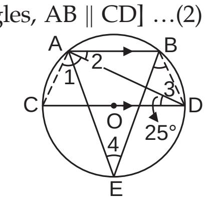
Hence, $\angle ACB = 48°$.

41. (d) **40°**

Join AC and BD.

$\angle 1 = 90°$ [Angle in a semicircle] …(1)

$\angle 2 = 25°$ [Alternate angles, AB $\parallel$ CD] …(2)



Since the opposite angles of a cyclic quadrilateral are supplementary and ACDB is a cyclic quadrilateral,

$\therefore \quad \angle BAC + \angle BDC = 180°$

$\Rightarrow \quad \angle 1 + \angle 2 + 25° + \angle 3 = 180°$

$\Rightarrow \quad 90° + 25° + 25° + \angle 3 = 180°$ [Using (1) and (2)]

$\Rightarrow \quad \angle 3 = 180° - 90° - 25° - 25°$

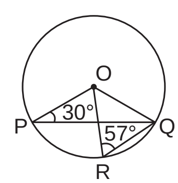
$= 40°$ …(3)

$\angle AEB = \angle 3$ [Angles in the same segment]

$\Rightarrow \quad \angle AEB = 40°$ [Using (3)]

Hence, $\angle AEB = 40°$.

42. (d) **54°**



In ΔORQ, we have,

$OR = OQ$ [Radii of a circle]

$\therefore \quad \angle OQR = \angle ORQ = 57°$ [Angles opposite equal sides of a triangle] …(1)

In ΔOPQ, we have

$OP = OQ$ [Radii of a circle]

$\Rightarrow \quad \angle OQP = \angle OPQ = 30°$ [Angles opposite equal sides of a triangle] …(2)

Subtracting (2) from (1), we get

$$\angle OQR - \angle OQP = 57° - 30° = 27°$$

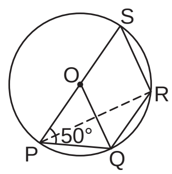
$\Rightarrow$ $\angle PQR = 27°$ ...(3)

Since the angle subtended by an arc at the centre of the circle is double the angle subtended by it at any point on the remaining part of the circle,

$\therefore$ $\angle POR = 2\angle PQR = 2 \times 27°$ [Using (3)]

$= 54°$

43. (b) **130°**



Join PR.

In ΔOPQ, we have

$OP = OQ$ [Radii of a circle]

$\Rightarrow$ $\angle OQP = \angle OPQ$

$= 50°$ [Angles opposite equal sides of a triangle]

Also $\angle OQP + \angle OPQ + \angle POQ$

$= 180°$ [Sum of angles of a triangle]

$\Rightarrow$ $50° + 50° + \angle POQ = 180°$

$\Rightarrow$ $\angle POQ = 80°$ ...(1)

Since the angle subtended by an arc at the centre of the circle is double the angle subtended by it at any point on the remaining part of the circle,

$\therefore$ $\angle POQ = 2\angle PRQ$

$\Rightarrow$ $80° = 2\angle PRQ$ [Using (1)]

$\Rightarrow$ $\angle PRQ = 40°$ ...(2)

Now, $\angle SRQ = \angle SRP + \angle PRQ$

$= 90° + 40°$ [$\because$ $\angle SRP = 90°$, angle in a semicircle and using (2)]

$\Rightarrow$ $\angle SRQ = 130°$

44. (b) **220°**

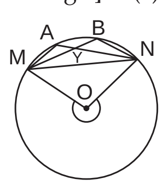
In ΔYBN, we have

$\angle NYB + \angle YNB + \angle YBN = 180°$ [Sum of angles of a triangle]

$\Rightarrow$ $50° + 20° + \angle YBN = 180°$

$\Rightarrow$ $\angle YBN = 110°$

$\Rightarrow$ MBN $= 110°$ [$\because$ $\angle YBN = \angle MBN$, same angle] ...(1)



Since the angle subtended by an arc at the centre is double the angle subtended by it at a point on the remaining part of the circle,

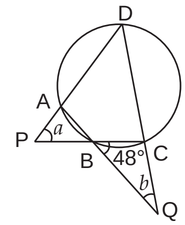
$\therefore$ Ref $\angle MON = 2\angle MBN$

$\Rightarrow$ Ref $\angle MON = 2 \times 110°$ [Using (1)]

$\Rightarrow$ Ref $\angle MON = 220°$

45. (c) **28°**

$\angle ABP = \angle CBQ = 48°$ [Ver. opp. angles]



Since, exterior angle of a Δ = Sum of its interior opposite angles

$\therefore$ $\angle DAB = a + 48°$ ...(1)

and $\angle DCB = b + 48°$ ...(2)

Adding (1) and (2) we get

$\angle DAB + \angle DCB = a + b + 96°$

$\Rightarrow$ $180° = a + b + 96°$

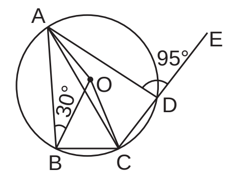
[$\because$ DAB and DCB are opposite angles of cyclic quad. ABCD $\therefore$ $\angle DAB + \angle DCB = 180°$]

$\Rightarrow$ $180° - 96° = 2b + b$ [$\because$ $a = 2b$, given]

$\Rightarrow$ $84° = 3b$

$\Rightarrow$ $b = 28°$

46. (b) 5°



Since an exterior angle of a cyclic quadrilateral is equal to the interior opposite angle and ABCD is a cyclic quadrilateral,

$\therefore$ $\angle ABC = \angle ADE$

$= 95°$ ...(1)

$\angle OBC = \angle ABC - \angle ABO$

$= 95° - 30° = 65°$ [Using (1)]

In ΔOBC, we have

$OB = OC$

$\therefore$ $\angle OCB = \angle OBC$

$= 65°$ [Angles opposite equal sides of a triangle] ...(2)

In ΔOBC, we have

$\angle OBC + \angle OCB + \angle BOC = 180°$ [Sum of angles of a triangle]

$\Rightarrow$ $65° + 65° + \angle BOC = 180°$ [Using (2)]

$\Rightarrow$ $\angle BOC = 50°$ ...(3)

In ΔAOB, we have

$OA = OB$

$\therefore$ $\angle OAB = \angle OBA$

$= 30°$ [Angles opposite equal sides of a triangle] ...(4)

In ΔAOB, we have

$\angle OAB + \angle OBA + \angle AOB$

$= 180°$ [Sum of angles of a triangle]

$\Rightarrow$ $30° + 30° + \angle AOB = 180°$ [Using (4)]

$\Rightarrow$ $\angle AOB = 120°$ ...(5)

$\angle COA = \angle BOC + \angle AOB$

$= 50° + 120°$

$= 170°$ [Using (3) and (5)] ...(6)

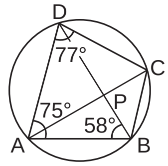
In ΔOAC, we have

$\angle OAC + \angle OCA + \angle AOC = 180°$ [Sum of angles of a triangle]

$\Rightarrow$ $2\angle OAC + 170° = 180°$ [Using (6) and $\angle OAC = \angle OCA$, angles opposite to equal sides]

$\Rightarrow$ $\angle OAC = 5°$

47. (c) **92°**



Since opposite angles of a cyclic quadrilateral are supplementary and ABCD is a cyclic quadrilateral,

$\therefore \quad \angle ADC + \angle ABC = 180°$

$\Rightarrow \angle ADC + (\angle ABD + \angle DBC) = 180°$

$\Rightarrow \quad 77° + 58° = 180°$

$\Rightarrow \quad \angle DBC = 45°$ …(1)

$\angle DAC = \angle DBC$ [Angles in the same segment]

$\Rightarrow \quad \angle DAC = 45°$ [Using (1)] …(2)

$\angle CAB = \angle DAB - \angle DAC$

$= 75° - 45° = 30°$ [Using (2)] …(3)

In ΔPAB, we have

$\angle PAB + \angle ABP + \angle APB$

$= 180°$ [Sum of angles of a triangle]

$\Rightarrow \quad \angle CAB + \angle ABD + \angle APB$

$= 180°$ [$\angle PAB = \angle CAB$ and $\angle ABP = \angle ABD$ same angles]

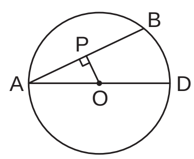
$\Rightarrow \quad 30° + 58° + \angle APB = 180°$ [Using (3)]

$\Rightarrow \quad \angle APB = 180° - 88° = 92°$ …(4)

$\angle DPC = \angle APB$ [Ver. opp. angles]

$\Rightarrow \quad \angle DPC = 92°$ [Using (4)]

48. (c) **7 cm**



AD is a diameter of the circle with centre O.

AB is a chord of length 48 cm.

Draw OP ⊥ AB.

Since perpendicular drawn from the centre of a circle to chord bisects it,

$\therefore \quad AP = \frac{1}{2} AB = \frac{1}{2} \times 48$ cm 24 cm

$AO = \text{radius} = \frac{1}{2}$ diameter

$= \frac{1}{2} \times 50$ cm $= 25$ cm

In right ΔAPO, we have

$AP^2 + OP^2 = AO^2$ [By Pythagoras Theorem]

$\Rightarrow \quad (24 \text{ cm})^2 + OP^2 = (25 \text{ cm})^2$

$\Rightarrow \quad OP^2 = (625 - 576) \text{ cm}^2 = 49 \text{ cm}^2$

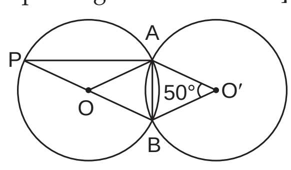
$\Rightarrow \quad OP = 7$ cm

Hence, the distance of AB from the centre of the circle is 7 cm.

49. (c) **25°**

$\angle AOB = \angle AO'B = 50°$ …(1)

[Congruent arcs of congruent circles subtend equal angles at the centres]



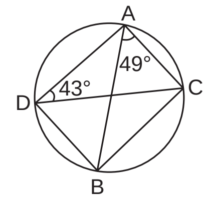
Since the angle subtended by an arc at the centre of circle is double the angle subtended by it at any point on the remaining part of the circle,

$\therefore \quad \angle AOB = 2\angle APB$

$\Rightarrow \quad 50° = 2\angle APB$ [Using (1)]

$\Rightarrow \quad \angle APB = 25°$

50. (d) **88°**



$\angle BDC = \angle BAC = 49°$ [Angles in the same segment] …(1)

Since the opposite angles of a cyclic quadrilateral are supplementary and ACBD is a cyclic quadrilateral,

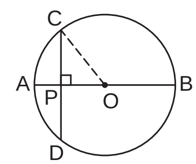
$\therefore \quad \angle ADB + \angle ACB = 180°$

$\Rightarrow \quad \angle ADC + \angle BDC + \angle ACB = 180°$

$\Rightarrow \quad 43° + 49° + \angle ACB = 180°$ [Using (1)]

$\Rightarrow \quad \angle ACB = 180° - 43° - 49° = 88°$

51. (d) **16 cm**



Diameter AOP = AP + PB

= 4 cm + 16 cm

= 20 cm.

Join OC.

Radius OA = Radius OC = $\frac{1}{2}$ diameter

$= \frac{1}{2} \times 20$ cm $= 10$ cm …(1)

Since perpendicular from the centre to the chord bisects it,

$\therefore \quad CP = PD$

$\Rightarrow \quad CP = \frac{1}{2} CD$ …(2)

$OP = OA - AP$

$= 10$ cm $- 4$ cm $= 6$ cm [Using (1)]

In right ΔOPC, we have

$OC^2 = OP^2 + CP^2$ [By Pythagoras' Theorem]

$\Rightarrow \quad (10 \text{ cm})^2 = (6 \text{ cm})^2 + CP^2$

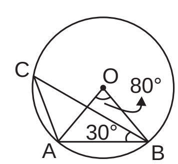
$\Rightarrow \quad CP^2 = (100 - 36) \text{ cm}^2 = 64 \text{ cm}^2$

$\Rightarrow \quad CP = 8$ cm

$\frac{1}{2} CD = 8$ cm [Using (2)]

$\Rightarrow \quad CD = 16$ cm

52. (c) **60°**



Since the angle subtended by an arc of a circle at the centre is double the angle subtended by it at any point on the remaining part of the circle,

$\therefore \quad \angle AOB = 2\angle ACB$

$\Rightarrow \quad 80° = 2\angle ACB$

$\Rightarrow \quad \angle ACB = 40°$ …(1)

In ΔOAB, $\angle OAB + \angle OBA + \angle AOB$

$= 180°$ [Sum of angles of a triangle]

$\Rightarrow 2\angle OAB + 80° = 180°$ [$\because$ $\angle OAB = \angle OBA$, angles opposite equal sides (radii) OA and OB of ΔOAB]

$\Rightarrow \quad 2\angle OAB = 100°$

$\Rightarrow \quad \angle OAB = 50°$ …(2)

In ΔACB, we have,

$\angle ACB + \angle ABC + \angle CAB$

$= 180°$ [Sum of angles of a triangle]

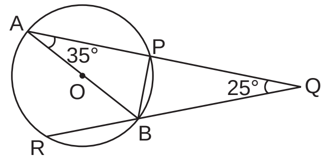
$\Rightarrow \quad 40° + 30° + (\angle CAO + \angle OAB) = 180°$ [Using (1)]

$\Rightarrow \quad 70° + \angle CAO + 50° = 180°$ [Using (2)]

$\Rightarrow \quad \angle CAO = 180° - 70° - 50° = 60°$

Hence, $\angle CAO = 60°$.

53. (b) **115°**



$\angle APB = 90°$ [Angle in a semicircle] …(1)

In ΔAPB, we have

$\angle PAB + \angle APB + \angle ABP$

$= 180°$ [Sum of angles of a triangle]

$\Rightarrow \quad 35° + 90° + \angle ABP$

$= 180°$ [Using (1)]

$\Rightarrow \quad \angle ABP = 180° - 35° - 90° = 55°$ …(2)

Since exterior angle of a triangle = Sum of its interior opposite angles

$\therefore$ Exterior $\angle ABR = \angle BAQ + \angle BQA$

$= 35° + 25° = 60°$ …(3)

$\angle PBR = \angle ABP + \angle ABR$

$= 55° + 60°$ [Using (2) and (3)]

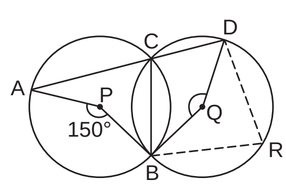
$\Rightarrow \quad \angle PBR = 115°$

54. (b) **150°**

Take any point R on arc BD (on the side opposite of C)

Join RD and RB.

Since the angle subtended by an arc of a circle at the centre is double the angle subtended by it at any point on the remaining by it at any point on the remaining part of the circle,



$\therefore \quad \angle APB = 2\angle ACB$

$\Rightarrow \quad 150° = 2\angle ACB$

$\Rightarrow \quad \angle ACB = 75°$ …(1)

Since an exterior angle of a cyclic quadrilateral is equal to the interior opposite angle and CDRB is a cyclic quadrilateral,

$\therefore \quad \angle DRB = \angle ACB$

$\Rightarrow \quad \angle DRB = 75°$ [Using (1)] …(2)

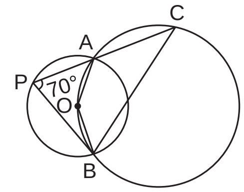
$\angle DQB = 2\angle DRB$

$\Rightarrow \quad \angle DQB = 2 \times 75°$ [Using (2)]

$\Rightarrow \quad \angle DQB = 150°$

55. (d) **40°**

Since the angle subtended by the arc of a circle at the centre is double the angle subtended by it at any point on the remaining part of the circle,



$\therefore \quad \angle AOB = 2\angle APB$

$= 2 \times 70° = 140°$ …(1)

Since the opposite angles of a cyclic quadrilateral are supplementary and ACBO is a cyclic quadrilateral,

$\therefore \quad \angle AOB + \angle ACB = 180°$

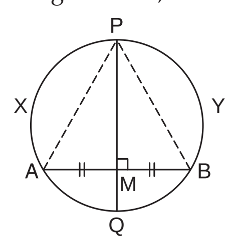
$\Rightarrow \quad 140° + \angle ACB = 180°$ [Using (1)]

$\Rightarrow \quad \angle ACB = 180° - 140° = 40°$

Hence, $\angle ACB = 40°$.

SHORT ANSWER QUESTIONS

1. Join PA and PB and let PQ intersect the chord AB at M. In right ΔPMA and right ΔPMB, we have



AM = BM [PQ is the bisector of AB]

PM = PM [Common]

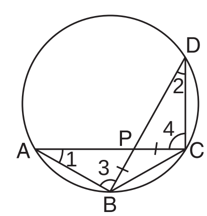
$\therefore \quad \Delta PMA \cong \Delta PMB$ [By RHS congruence]

$\Rightarrow \quad PA = PB$ [CPCT]

We know that if two chords of a circle are equal, then their corresponding arcs are congruent.

$\therefore$ **arc PXA $\cong$ arc PYB**

2. $\angle 1 = \angle 2$ and $\angle 3 = \angle 4$ [Angles in the same segment] …(1)



In ΔAPB and ΔDPC, we have

$\angle 1 = \angle 2$

$\angle 3 = \angle 4$ [From (1)]

PB = PC [Given]

$\therefore \quad \Delta APB \cong \Delta DPC$ [By AAS congruence]

$\Rightarrow \quad AB = DC$ [CPCT] …(1)

And $\quad AP = DP$ [CPCT]

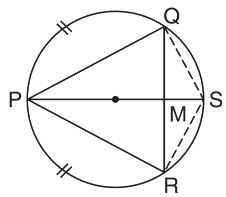
$\Rightarrow \quad AP + PC = DP + PB \qquad [\because PC = PB \text{ given}]$

$\Rightarrow \quad AC = DB \qquad \ldots(2)$

Hence, (i) **Chord AB = Chord DC** [From (1)]

(ii) **Chord AC = Chord BD** [From (2)]

3. arc PQ $\cong$ arc PR [Given]



$\therefore$ Chord PQ = Chord PR [If two arcs of a circle are congruent then the corresponding chords are equal] …(1)

Join QS and RS.

$\angle PQS = \angle PRS$

$= 90°$ [Angles in semicircle]

In right ΔPQS and right ΔPRS, we get

$PQ = PR$ [From (1)]

$PS = PS$ [Common]

$\therefore \quad \Delta PQS \cong \Delta PRS$ [By RHS congruence]

$\Rightarrow \quad \angle QPS = \angle RPS$ [CPCT]

$\Rightarrow \quad 30° = \angle RPS$ …(2)

$\angle QPR = \angle QPS + \angle RPS$

$= 30° + 30°$ [Using (2)]

$\Rightarrow \quad \angle QPR = 60°$ …(3)

In ΔPQR, we have

$\angle QPR + \angle PRQ + \angle PQR$

$= 180°$ [Sum of angles of a triangle]

$\Rightarrow \quad 60° + 2\angle PRQ = 180°$ [Using (3) and $\because \angle PRQ = \angle PQR$, angles opposite equal sides PQ and PR of ΔPQR]

$\Rightarrow \quad 2\angle PRQ = 180° - 60° = 120°$

$\Rightarrow \quad \angle PRQ = 60°$

$\Rightarrow \quad \angle PRQ = \angle PQR = 60°$ …(4)

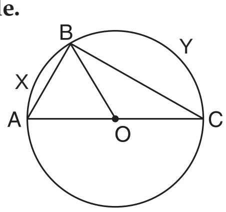
In ΔPQR, we have

$\angle QPR = 60°$ [From (3)]

$\angle PRQ = 60°$ and $\angle PQR = 60°$ [From (4)]

$\therefore$ **ΔPQR is an equilateral triangle.**

4. AOC is a diameter.



$\Rightarrow$ AOC is a straight angle.

$\Rightarrow \quad \angle BOA + \angle BOC = 180°$

$\Rightarrow \quad \frac{1}{2}\angle BOC + \angle BOC = 180°$

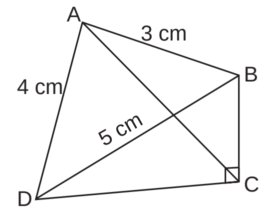
$$\left[\begin{array}{l} \text{arc AXB} \cong \frac{1}{2} \text{ arc BYC} \\ \Rightarrow \ m(\text{arc AXB}) = \frac{1}{2} m(\text{arc B}y\text{C}) \\ \Rightarrow \ \angle BOA = \frac{1}{2}\angle BOC \end{array}\right]$$

$\Rightarrow \quad \frac{3}{2}\angle BOC = 180°$

$\Rightarrow \quad \angle BOC = \frac{2}{3} \times 180° = 120°$

Hence, **$\angle$BOC = 120°.**

5. In ΔBAD, we have



$BD^2 = (5 \text{ cm})^2 = 25 \text{ cm}^2$

$AB^2 = (3 \text{ cm})^2 = 9 \text{ cm}^2$

$AD^2 = (4 \text{ cm})^2 = 16 \text{ cm}^2$

$\because \quad 9 \text{ cm}^2 + 16 \text{ cm}^2 = 25 \text{ cm}^2$

$\therefore \quad AB^2 + AD^2 = BD^2$

$\therefore \quad \angle BAD = 90°$ [By the converse of Pythagoras Theorem] …(1)

In quadrilateral ABCD, we have

$\angle BAD + \angle DCB = 90° + 90°$ [Using (1)]

$\Rightarrow \quad \angle BAD + \angle DCB = 180°$

But $\angle$BAD and $\angle$DCB are opposite angles of quadrilateral ABCD and we know that opposite angles of a cyclic quadrilateral are supplementary.

$\therefore$ Quadrilateral ABCD is a cyclic quadrilateral.

$\Rightarrow$ Points A, B, C and D are concylic i.e. they lie on a circle.

$\Rightarrow$ **$\angle$DAC = $\angle$DBC.** [Angles in the same segment]

6. Join PR.

Let $\angle RQS = x$ and $\angle QSR = y$.

Since the angle subtended by an arc at the centre of a circle is double the angle subtended by it at any point on the remaining part of the circle,

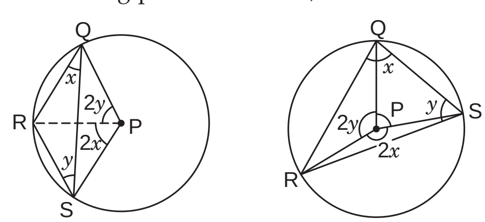


Fig (i) Fig (ii)

$\therefore \quad \angle RPS = 2\angle RQS = 2x$ …(1)

and $\quad \angle RPQ = 2\angle RSQ = 2y$ …(2)

Adding (1) and (2), we get

$\angle RPS + \angle RPQ = 2x + 2y$

$\Rightarrow \quad \angle QPS$ [or ref $\angle QPS$ in Fig. (ii)] $= 2(x + y)$

$\Rightarrow \quad x + y = \frac{1}{2}\angle QPS \left[\text{or } \frac{1}{2}\text{ref } \angle QPS \text{ in Fig. (ii)}\right]$

$\Rightarrow$ $\angle$**RQS** + $\angle$**QSR**

$$= \frac{1}{2}\angle QPS \qquad \left[\text{or } \frac{1}{2}\text{ref } \angle QPS \text{ in Fig. (ii)}\right]$$

7. OB = OC [Radii of a circle]

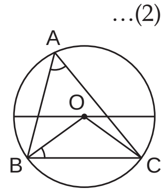
$\Rightarrow$ $\angle$OCB = $\angle$OBC [Angles opposite equal sides of a triangle]

In ΔOBC, we have

$\angle$OBC + $\angle$OCB + $\angle$BOC = 180° [Sum of angles of a triangle]

$\Rightarrow$ 2$\angle$OBC + $\angle$BOC = 180° [Using (1)]

$\Rightarrow$ $\angle$BOC = 180° – 2 $\angle$OBC …(2)



Since the angle subtended by an arc at the centre of the circle is double the angle subtended by it at any point on the remaining part of the circle,

$\therefore$ $\angle$BOC = 2 $\angle$BAC

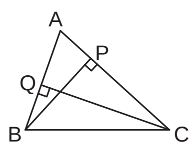
$\Rightarrow$ 180° – 2 $\angle$OBC = 2$\angle$BAC [Using (2)]

$\Rightarrow$ 180° = 2$\angle$OBC + 2 $\angle$BAC

$\Rightarrow$ 90° = $\angle$OBC + $\angle$BAC

Hence, $\angle$**OBC** + $\angle$**BAC = 90°.**

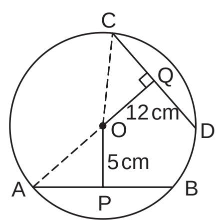
8. We know that if a line segment joining two points subtends equal angles at two other points lying aon the same side of the line containing the line segment, then the four points, lie on a circle (i.e. they are concyclic).



Here, line BC subtends equal angles (each is equal to 90°) at points P and Q lying on the same side of the line containing line segment BC.

$\therefore$ **Points B, C, P and Q are concyclic.**

9. Let AB = 24 cm be the chord of a circle with centre O such that its perpendicular distance OP from the centre O is 5 cm. Let CD be another chord such that its perpendicular distance OQ from the centre O is 12 cm.



Join AO and CO.

Since perpendicular from the centre to the chord bisects the chord,

$\therefore$ AP = $\frac{1}{2}$AB = $\frac{1}{2}$ × 24

= 12 cm and CQ = $\frac{1}{2}$ CD …(1)

In right ΔOPA, we have

$OA^2 = OP^2 + AP^2$ [By Pythagoras' Theorem]

$\Rightarrow$ $OA^2 = (5 \text{ cm})^2 + (12 \text{ cm})^2$ [Using (1)]

$\Rightarrow$ $OA^2 = (25 + 144) \text{ cm}^2 = 169 \text{ cm}^2$

$\Rightarrow$ OA = 13 cm.

OC = OA = 13 cm [Radii of a circle] …(2)

In right ΔOQC, we have

$OQ^2 + CQ^2 = OC^2$ [By Pythagoras' Theorem]

$\Rightarrow$ $(12 \text{ cm})^2 + CQ^2 = (13 \text{ cm})^2$ [Using (2)]

$\Rightarrow$ $CQ^2 = (169 - 144) \text{ cm}^2 = 25 \text{ cm}^2$

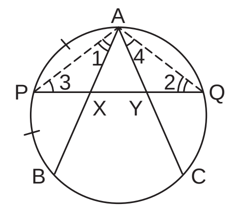
$\Rightarrow$ CQ = 5 cm

$\Rightarrow$ $\frac{1}{2}$ CD = 5 cm [Using (1)]

$\Rightarrow$ CD = 5 × 2 cm = 10 cm

Hence, the length of the required chord is **10 cm.**

10. Congruent arcs subtend equal angles at the different points on circumference of the circle.



$\overparen{AP} = \overparen{PB}$ [$\because$ P is the mid-point of arc AB]

$\therefore$ $\angle 1 = \angle 2 = x$ (say) …(1)

and $\overparen{CQ} = \overparen{QA}$ [$\because$ Q is the mid-point of arc CA]

$\therefore$ $\angle 3 = \angle 4 = y$ (say) …(2)

Adding (1) and (2), we get

$\angle 1 + \angle 3 = \angle 2 + \angle 4$ …(3)

Considering ΔAPX whose side PX is produced to Y, we get

Exterior $\angle$AXY = $\angle$1 + $\angle$3 [Exterior angle = Sum of interior opposite angles] …(4)

Considering ΔAQY whose side QY is produced to X, we get

Exterior $\angle$AYX = $\angle$2 + $\angle$4 [Exterior angle = Sum of interior opposite angles] …(5)

From (3), (4) and (5), we get

$\angle$AYX = $\angle$AXY

$\Rightarrow$ AX = AY [Sides opposite equal angles of ΔAXY]

Hence, AX = AY.

VALUE-BASED QUESTIONS

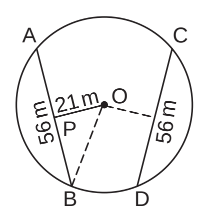
1. (i) Equal chords of a circle are equidistant from the centre of the circle. Since road AB and road CD are of equal length,

$\therefore$ they from equal chords of the circular park.

Distance of AB from the centre 'O' of the circular park is 21 m.

$\therefore$ Distance of CD from the centre 'O' of the circular park is 21 m.

(ii) Let OP $\perp$ AB and OQ $\perp$ CD.



Then, OP = 21 m.

Join OB.

Since the perpendicular drawn from the centre of the circle to chord, bisects the chord,

$\therefore$ PB = $\frac{1}{2}$ AB = $\frac{56}{2}$ m = 28 m …(1)

In right ΔBPQ, we have

$$OP^2 + PB^2 = OB^2 \text{[By Pythagoras Theorem]}$$

$\Rightarrow (21 \text{ m})^2 + (28 \text{ m})^2 = OB^2$ [Using (1)]

$\Rightarrow (441 + 784)\text{m}^2 = OB^2$

$\Rightarrow 1225 \text{ m}^2 = OB^2$

$\Rightarrow OB = 35 \text{ m}$

Hence, the radius of the circular park is **35 m.**

(iii) Circumference of circular park

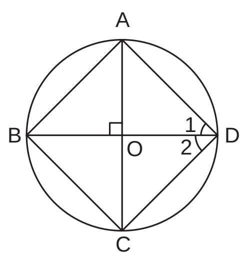
$$= 2\pi r = 2 \times \frac{22}{7} \times 35 \text{ cm} = \mathbf{220 \text{ m}}$$

(iv) Distance between two neighbouring trees

$$= \frac{220}{44} \text{ m} = \mathbf{5 \text{ m}}$$

(v) Environmental protection, leadership and responsible citizenship.

2. (i) Since the angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle,



$\angle AOB = 2\angle 1$

$\Rightarrow 90° = 2\angle 1$

$\Rightarrow 45° = \angle 1$ …(1)

$\angle BOC = 2\angle 2$

$\Rightarrow 90° = 2\angle 2$

$\Rightarrow 45° = \angle 2$ …(2)

Adding (1) and (2), we get

$\angle 1 + \angle 2 = 45° + 45°$

$\Rightarrow \angle ADC = 90°$

Similarly, $\angle BAC = \angle ABC = \angle BCD = 45°$. …(3)

In ΔAOD and ΔCOD, we have

$\angle AOD = \angle COD$ [Each is 90°]

$OD = OD$ [Common]

$\angle ADO = \angle CDO$ [Each is 45°]

$\therefore \Delta AOD \cong \Delta COD$ [By ASA congruence]

$\Rightarrow AD = CD$ [CPCT]

Similarly, $AB = BC = CD = AD$ …(4)

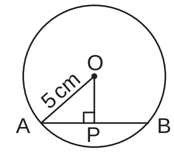
$\Rightarrow$ ABCD is a quadrilateral in which all the sides are equal and each angle is 90°. [From (3) and (4)]

Hence, **ABCD is a square.**

(ii) Awareness about environmental protection and importance of education and physical fitness.

UNIT TEST

1. (b) 3 cm



Let AB = 8 cm be the chord of circle with centre O and radius 5 cm.

Then, OA = 5 cm

Let $OP \perp AB$.

Since perpendicular from the centre to the chord bisects the chord.

$$\therefore \quad AP = \frac{1}{2} AB = \frac{1}{2} \times 8 \text{ cm} = 4 \text{ cm}$$

In right ΔOPA, we have

$$OP^2 + AP^2 = OA^2 \text{ [By Pythagoras' Theorem]}$$

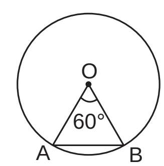
$\Rightarrow OP^2 + (4 \text{ cm})^2 = (5 \text{ cm})^2$

$\Rightarrow OP^2 = (25 - 16) \text{ cm}^2 = 9 \text{ cm}^2$

$\Rightarrow OP = 3 \text{ cm}$

Hence, the distance of the chord from the centre is **3 cm.**

2. (c) 5 cm



$\angle AOB = 60°$ [Given]

In ΔOAB we have,

$\angle OAB + \angle OBA + \angle AOB$

$= 180°$ [Sum of angles of a triangle]

$\Rightarrow 2\angle OAB + 60°$

$= 180°$ [$\because \angle OAB = \angle OBA$, angles opposite equal sides OA and OB (radii) of ΔOAB]

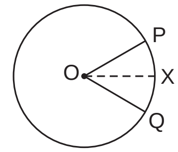
$\Rightarrow \angle OAB = \frac{120°}{2} = 60°$

$\Rightarrow \angle OAB = \angle OBA = 60°$

$\therefore$ ΔOAB is an equilateral triangle.

$\therefore AB = OA = OB = 5$ cm.

3. (b) 1 : 2



X is mid-point of arc PQ.

$\therefore$ arc PX ≅ arc PQ …(1)

arc PQ ≅ arc PX + arc PQ

arc PQ ≅ arc PX + arc PX [Using (1)]

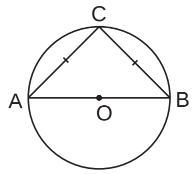
arc PQ ≅ 2 arc PX …(2)

Now, arc PX : arc PQ

= arc PX : 2 arc PX [Using (2)]

= 1 : 2

4. (b) **45°**



$\angle ACB = 90°$ [Angle in a semicircle] …(1)

$CB = CA$ [Given]

$\therefore \quad \angle CAB = \angle CBA$ [Angles opposite equal sides of ΔCAB] …(2)

In ΔCAB, we have

$\angle ACB + \angle CAB + \angle CBA$

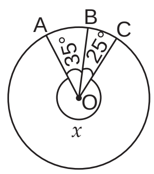
$= 180°$ [Sum of angles of a triangle]

$\Rightarrow \quad 90° + 2\angle CAB = 180°$ [Using (1) and (2)]

$\Rightarrow \quad 2\angle CAB = 180° - 90° = 90°$

$\Rightarrow \quad \angle CAB = 45°.$

5. (d) **300°**



$\angle AOB + \angle BOC + \text{ref}\angle AOC$

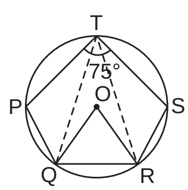
$= 360°$ [Angles about a point]

$\Rightarrow \quad 35° + 25° + x = 360°$

$\Rightarrow \quad x = 360° - 35° - 25°$

$\Rightarrow \quad x = 300.$

6. (b) **50°**



Since $PQ = QR = RS$, [Given]

$\therefore \quad \angle PTQ = \angle QTR$

$= \angle RTS$ [Equal chords of a circle subtend equal angles at the circumference] …(1)

Also, $\angle PTQ + \angle QTR + \angle RTS$

$= 75°$ [Given] …(2)

From (1) and (2), we get

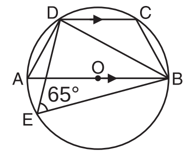
$\angle PTQ = \angle QTR = \angle RTS$

$= \dfrac{75°}{3} = 25°$

Since the angle subtended by an arc at the centre of the circle is double the angle subtended by it at any point on the remaining part of the circle,

$\therefore \quad \angle QOR = 2 \times 25° = 50°.$

7. (b) **25°**



$\angle DAB = \angle DEB$ [Angles in the same segment]

$\therefore \quad \angle DAB = 65°$ …(1)

In ΔDAB, we have

$\angle DAB + \angle ADB + \angle ABD$

$= 180°$ [Sum of angles of a triangle]

$\Rightarrow \quad 65° + 90° + \angle ABD$

$= 180°$ [Using (1) and $\angle ADB = 90°$, angle in a semicircle]

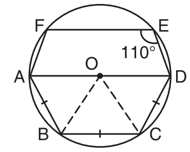
$\Rightarrow \quad \angle ABD = 180° - 90° - 65° = 25°$ …(2)

$\angle BDC = \angle ABD$ [Alternate angles $DC \parallel AB$]

$\Rightarrow \quad \angle BDC = 25°$ [Using (2)]

8. (c) **130°**

Join OB and OC.



Since the opposite angles of a cyclic quadrilateral are supplementary and FADE is a cyclic quadrilateral,

$\therefore \quad \angle FAD + \angle FED = 180°$

$\Rightarrow \quad \angle FAD + 110° = 180°$

$\Rightarrow \quad \angle FAD = 70°$ …(1)

Equal chords of a circle subtend equal angles at the centre and $AB = BC = CD$ [Given]

$\therefore \quad \angle AOB = \angle BOC = \angle COD$ …(2)

Since the sum of all the angles on the same side of a line at a point on it is 180°,

$\therefore \quad \angle AOB + \angle BOC + \angle COD = 180°$ …(3)

From (2) and (3), we get

$\angle AOB = \dfrac{180°}{3} = 60°$ …(4)

In ΔAOB, we have

$\angle OAB + \angle OBA + \angle AOB$

$= 180°$ [Sum of angles of a triangle]

$\Rightarrow \quad 2\angle OAB + 60°$

$= 180°$ [OA = OB (radii of a circle) $\therefore \angle OAB = \angle OBA$ angles opposite equal sides]

$\Rightarrow \quad 2\angle OAB = 120°$

$\Rightarrow \quad \angle OAB = 60°$

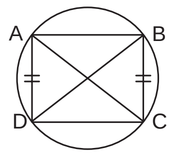
$\Rightarrow \quad \angle DAB = 60°$ [$\because \angle OAB = \angle DAB$, same angles] …(5)

$\angle FAB = \angle FAD + \angle DAB$

$= 70° + 60°$ [Using (1) and (5)]

$\Rightarrow \quad \angle FAB = 130°.$

9. Let ABCD be the cyclic quadrilateral in which AD = BC.



AD = BC

$\Rightarrow$ AD = CB

$\Rightarrow$ $\overset{\frown}{AD} \cong \overset{\frown}{CB}$ [If two chords of a circle are equal, then their corresponding arcs are congruent]

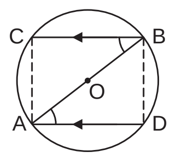
$\Rightarrow$ $\overset{\frown}{AD} + \overset{\frown}{DC} \cong \overset{\frown}{DC} + \overset{\frown}{CB}$ [Adding $\overset{\frown}{DC}$ to both sides]

$\Rightarrow$ $\overset{\frown}{ADC} \cong \overset{\frown}{DCB}$

$\Rightarrow$ AC = DB [If two arcs of a circle are congruent, then their corresponding chords are equal]

Hence, **diagonal AC = diagonal DB.**

10. Join AC and BD.



∠BDA = ∠ACB

= 90° [Angles in semicircle] …(1)

In ΔBDA and ΔACB, we have

∠BDA = ∠ACB [From (1)]

∠DAB = ∠CBA [Alternate angles, BC ∥ AD]

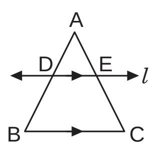
AB = BA [Common]

∴ ΔBDA ≅ ΔACB [By AAS congruence]

$\Rightarrow$ AD = BC [CPCT]

Hence, **AD = BC.**

11. Let ABC be the isosceles triangle.



AC = AB [Sides of an isosceles triangle]

∴ ∠B = ∠C [Angles opposite equal sides of ΔABC] …(1)

Let l be a line drawn parallel to base BC of isosceles ΔABC and let l cut its equal sides at D and E.

∠EDB + ∠DBC = 180° [Cointerior angles]

$\Rightarrow$ ∠EDB + ∠B = 180° [∵ ∠DBC = ∠B, same angle]

$\Rightarrow$ ∠EDB + ∠C = 180° [Using (1)]

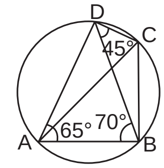
$\Rightarrow$ ∠EDB + ∠ECB = 180° [∵ ∠C = ∠ECB, same angle] …(2)

But ∠EDB and ∠ECB are opposite angles of quadrilateral BCED. …(3)

We know that if opposite angles of a quadrilateral are supplementary, then it is a cyclic quadrilateral.

∴ **BCED is a cyclic quadrilateral.** [Using (2) and (3)]

12. In ΔADB, we have



∠ADB + ∠BAD + ∠ABD

= 180° [Sum of angles of a triangle]

$\Rightarrow$ ∠ADB + 65° + 70° = 180°

$\Rightarrow$ ∠ADB = 180° – 135° = 45° …(1)

∠ACB = ∠ADB [Angles in the same segment]

$\Rightarrow$ ∠ACB = 45° [Using (1)] …(2)

∠ACD = ∠ABD [Angles in the same segment]

$\Rightarrow$ ∠ACD = 70° …(3)

Adding (2) and (3), we get

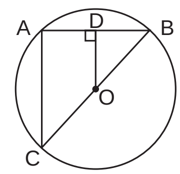
∠ACB + ∠ACD = 45° + 70°

$\Rightarrow$ ∠BCD = **115°**

13. Centre O is the mid-point of diameter BOC and OD ⊥ AB. …(1)

∴ D is the mid-point of AB …(2)

[Perpendicular from the centre of the circle to the chord bisects the chord]

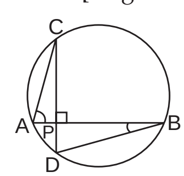


In ΔABC, OD is the line segment joining the mid-points of BC and BA.[From (1) and (2)]

∴ $OD = \frac{1}{2} CA$ [By Mid-point Theorem]

$\Rightarrow$ **CA = 2 OD**

14. ∠BDC = ∠BAC [Angles in the same segment]



Considering ΔBDP whose side DP is produced to C, we get

Exterior ∠CPB = ∠BDP + ∠PBD [Exterior angle = Sum of interior opposite angle]

$\Rightarrow$ 90° = ∠BDC + ∠ABD [∠BDP = ∠BDC and ∠PBD = ∠ABD same angles]

$\Rightarrow$ 90° = ∠BAC + ∠ABD [Using (1)]

Hence, **∠BAC + ∠ABD = 90°.**

15. Since diagonal AC of a cyclic quadrilateral ABCD bisects ∠BAD,

∴ ∠BAC = ∠CAD ...(1)

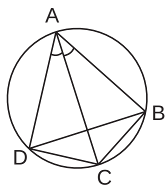
Now,

$\angle DBC = \angle CAD$ [Angles in the same segment]

and

$\angle BDC = \angle BAC$ [Angles in the same segment]

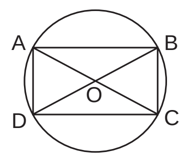
$\Rightarrow$ $\mathbf{\angle BDC = \angle DBC.}$ [Using (1)]



16. Since, ABCD is a cyclic rectangle,

$\therefore$ $\angle ABC = 90°$ [Angle of a rectangle]

We know that the angle in a semicircle is 90°.

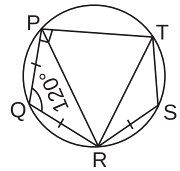


$\therefore$ Diagonal AC of rectangle ABCD is diameter of the circle. Similarly, diagonal BD of rectangle ABCD is also a diameter of the circle.

$\therefore$ Point of intersection of the diagonals i.e. 'O' is also the point of intersection of the diameters of the circle.

Hence, the centre of the circle through A, B, C and D is the point of intersection of the diagonals of cyclic rectangle ABCD.

17. Since the opposite angles of a cyclic quadrilateral are supplementary and PTRQ is a cyclic quadrilateral.



$\therefore$ $\angle PTR + \angle PQR = 180°$

$\Rightarrow$ $\angle PTR + 120° = 180°$

$\Rightarrow$ $\angle PTR = 60°$ …(1)

In ΔQPR, we have QR = QP [Given]

$\therefore$ $\angle QPR = \angle QRP$ [Angles opposite equal sides of ΔQPR] …(2)

Also, $\angle QPR + \angle QRP + \angle PQR$

$= 180°$ [Sum of angles of a triangle]

$\Rightarrow$ $2\angle QPR + \angle PQR = 180°$ [Using (2)]

$\Rightarrow$ $2\angle QPR + 120° = 180°$

$\Rightarrow$ $2\angle QPR = 60°$

$\Rightarrow$ $\angle QPR = 30°$ …(3)

$\angle TPR = \angle QPT - \angle QPR$

$= 90° - 30°$ [Using (3)]

$= 60°$ …(4)

In ΔPTR, we have

$\angle PTR + \angle TPR + \angle PRT = 180°$ [Sum of angles of a triangle]

$\Rightarrow$ $60° + 60° + \angle PRT = 180°$ [Using (1) and (4)]

$\Rightarrow$ $\angle PRT = 180° - 120°$

$= 60°$ …(5)

From (1), (4) and (5), we get

$\angle PTR = \angle TPR = \angle PRT = 60°$

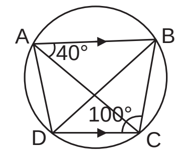
$\Rightarrow$ each angle of ΔPRT is 60°.

Hence, **ΔPRT is an equilateral Δ.**

18. $\angle BDC = \angle BAC$ [Angles in the same segment]

$\Rightarrow \angle BDC = 40°$

In ΔBDC, we have



$\angle BDC + \angle DCB + \angle CBD$

$= 180°$ [Sum of angles of a triangle]

$\Rightarrow$ $40° + 100° + \angle CBD = 180°$

$\Rightarrow$ $\angle CBD = 180° - 140° = 40°$ …(1)

$\angle CAD = \angle CBD$ [Angles in the same segment]

$\Rightarrow$ $\angle CAD = 40°$ [Using (1)]

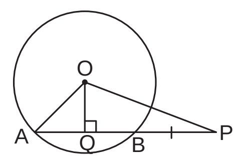
$\angle CBA + \angle DCB = 180°$ [Coint. angles]

$\Rightarrow$ $\angle CBA + 100° = 180°$

$\Rightarrow$ $\angle CBA = 80°$

Hence, (i) $\mathbf{\angle CAD = 40°}$ (ii) $\mathbf{\angle CBD = 40°}$ (iii) $\mathbf{\angle CBA = 80°}$.

19. Draw $OQ \perp AB$.



Since perpendicular drawn from the centre of a circle to the chord bisects it,

$\therefore$ $AQ = QB = \dfrac{AB}{2}$ …(1)

In right ΔOQA, we have

$AO^2 = OQ^2 + AQ^2$ [By Pythagoras Theorem]

$\Rightarrow$ $AO^2 - \left(\dfrac{AB}{2}\right)^2 = OQ^2$ [Using (1)]

$\Rightarrow$ $AO^2 - \dfrac{AB^2}{4} = OQ^2$ …(2)

In right ΔOQP, we have

$OP^2 = OQ^2 + QP^2$ [By Pythagoras Theorem]

$\Rightarrow$ $OP^2 = OQ^2 + (QB + BP)^2$

$\Rightarrow$ $OP^2 = \left(AO^2 - \dfrac{AB^2}{4}\right) + \left(\dfrac{AB}{2} + AB\right)^2$

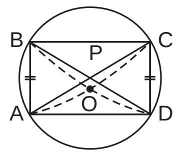
[Using (1) and (2)]

$\Rightarrow$ $OP^2 = AO^2 - \dfrac{AB^2}{4} + \left(\dfrac{AB + 2AB}{2}\right)^2$

$\Rightarrow$ $OP^2 = AO^2 + 2\,AB^2$

20. Let O be the centre of the circle.

Join OA, OB, OC and OD.



Since the angle subtended by an arc of a circle is double the angle subtended by it at any point on the remaining part of the circle,

$\therefore$ $\angle AOB = 2\,\angle ACB$ and

$\angle COD = 2\,\angle CAD$ ...(1)

Now, $AB = CD$

$\Rightarrow$ $\angle AOB = \angle COD$ [$\because$ Equal chords make equal angles at the centre] ...(2)

From (1) and (2) we get

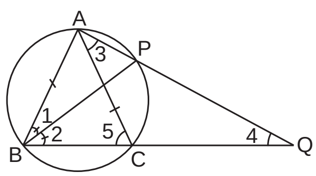
$2\angle ACB = 2\angle CAD$

$\Rightarrow$ $\angle ACB = \angle CAD$

But $\angle ACB$ and $\angle CAD$ are alternate interior angles formed when the transversal CA cuts BC at C and AD at D.

$\Rightarrow$ **AD || BC** [$\because$ Alternate angles are equal]

21. Consider ΔACQ, we have



$\angle 3 + \angle 4 = \angle 5$ [Exterior angle = Sum of interior opposite angles] ...(1)

But $\angle 5 = \angle ABC$ [Angles opposite equal sides are equal]

$\Rightarrow$ $\angle 5 = \angle 1 + \angle 2$ [$\because \angle ABC = \angle 1 + \angle 2$]

$\Rightarrow$ $\angle 5 = 2\angle 2$ [BP is the bisector of $\angle ABC$] ...(2)

But $\angle 2 = \angle 3$ [Angles is the same segment] ...(3)

$\Rightarrow$ $\angle 5 = 2\angle 3$ [From (2) and (3)] ...(4)

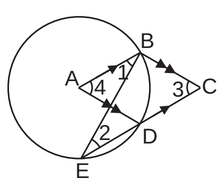
$\Rightarrow$ $\angle 3 + \angle 4 = \angle 3 + \angle 3$ [From (1) and (4)]

$\Rightarrow$ $\angle 4 = \angle 3$

$\Rightarrow$ CA = CQ[Sides opposite equal angles are equal]

Hence, **CQ = CA.**

22. AB || DC.



AB || EDC [E lies on CD produced]

$\therefore$ $\angle 1 = \angle 2$ [Alternate angles] ...(1)

Since the angle subtended by an arc at the centre of circle is twice the angle subtended by it at any point on the circumference.

$\therefore$ $\angle 4 = 2\angle 2$ ...(2)

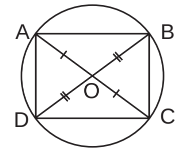
$\angle 4 = \angle 3$ [Opposite angles of a || gm] ...(3)

From (1), (2) and (3), we get

$\angle 3 = 2\angle 1$

$\Rightarrow$ **$\angle$BCD = 2$\angle$ABE**

23. **(i)** Chords AC and BD intersect each other at O, such that OA = OC and OB = OD.



AB, BC, CD and DA are joined.

In ΔAOB and ΔCOD, we have

$OA = OC$ [Given]

$OB = OD$ [Given]

$\angle AOB = \angle COD$ [Vertically opposite angles]

$\therefore$ $\Delta AOB \cong \Delta COD$ [By SAS-congruence]

$\Rightarrow$ $BA = DC$ [CPCT] ...(1)

It two chords of a circle are equal, then their corresponding arcs are congruent.

$\therefore$ $\overset{\frown}{BA} \cong \overset{\frown}{DC}$ ...(2)

In ΔAOD and ΔCOB, we have

$OA = OC$ [Given]

$OD = OB$ [Given]

$\angle AOD = \angle COB$ [Vertically opposite angles]

$\therefore$ $\Delta AOD \cong \Delta COB$ [By SAS-congruence]

$\Rightarrow$ $AD = CB$ ...(3)

$\Rightarrow$ $\overset{\frown}{AD} \cong \overset{\frown}{CB}$ ...(4)

Adding equation (2) and equation (4), we have

$\overset{\frown}{BA} + \overset{\frown}{AD} \cong \overset{\frown}{DC} + \overset{\frown}{CB}$

$\Rightarrow$ $\overset{\frown}{BAD} \cong \overset{\frown}{DCB}$

$\therefore$ BD divides the circle into two semicircles.

$\therefore$ **BD is a diameter.**

Similarly, **AC is a diameter.**

(ii) BD is a diameter. [Proved in (i)]

$\therefore$ $\angle BAD = 90°$ [Angle in a semicircle] ...(5)

From equations (1), (3) and (5), we have ABCD is a parallelogram in which $\angle BAD = 90°$.

Hence, **ABCD is a rectangle.**

24. Let ABC be a triangle in which $BE \perp AC$, $CF \perp AB$.

BE and CF intersect at O.

AO is joined and produced to meet BC at D.

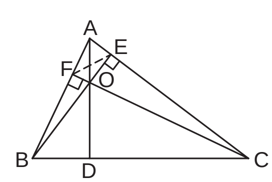
Join FE.

$BE \perp AC$ and $CF \perp AB$.

$\therefore$ $\angle BEC = \angle BFC = 90°$

$\therefore$ BC subtends equal angles at E and F.

If a line segment joining two points subtends equal angles at two other points lying on the same side of the line containing the line segment, the four points lie on a circle.



$\therefore$ Points B, C, E and F are concyclic.

$\therefore$ BCEF is a cyclic quadrilateral.

$\therefore$ $\angle ECB + \angle BFE = 180°$ [Opposite angles of a cyclic quadrilateral are supplementary]

$\Rightarrow$ $\angle ACD + \angle BFE$

$= 180°$ [$\angle ACD = \angle ECB$, same angle]

$\Rightarrow$ $\angle ACD + \angle EFC + \angle CFB$

$= 180°$ [As $\angle BFE = \angle EFC + \angle CFB$]

$\Rightarrow$ $\angle ACD + \angle EFO + 90°$

$= 180°$ [As $CF \perp AB$]

$\Rightarrow$ $\angle ACD + \angle EFO = 90°$ …(1)

Now, $\angle OFA + \angle OEA = 90° + 90° = 180°$

But $\angle OFA$ and $\angle OEA$ are opposite angles of quadrilateral AEOF.

$\therefore$ Quadrilateral AEOF is a cyclic quadrilateral.

$\therefore$ $\angle EFO = \angle EAO$[Angles in the same segment] …(2)

From equations (1) and (2), we have

$\angle ACD + \angle EAO = 90°$

$\Rightarrow$ $\angle ACD + \angle CAD = 90°$ [$\angle CAD = \angle EAO$, same angle] …(3)

In ΔADC, we have

$\angle ADC + \angle ACD + \angle CAD$

$= 180°$ [Sum of angles of a triangle]

$\Rightarrow$ $\angle ADC + 90° = 180°$ [Using (3)]

$\Rightarrow$ $\angle ADC = 90°$

$\therefore$ $AD \perp BC$

Hence, **the altitudes of a triangle are concurrent.**