CHAPTER 9 Areas of Parallelograms and Triangles

EXERCISE 9A

1. (i) Parallelogram ABCD and trapezium PQCD are not on the same parallel.

   Thus, they don't have common parallels.

   (ii) Rectangle PRSQ and ΔTRS are on the same base RS and between the same parallels RS and PQ.

   (iii) Parallelograms ABCD and AEFD are on the same base AD and between the same parallels AD and BF.

   (iv) ΔABC, ΔDBC and trapezium ABCD are on the same base BC and between the same parallels BC and AD.

   (v) Parallelogram BADC and ΔEDC are on the same base BC and between the same parallels DC and AE.

   (vi) Rectangle FEBA and parallelogram FGCA are on the same base FA and between the same parallels FA and EC.

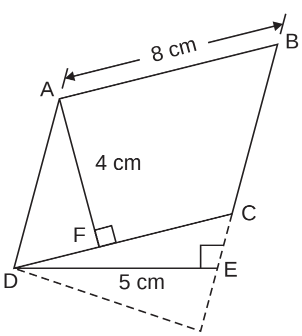
   Also, parallelograms GFAC and HMAC are on the same base AC and between the same parallels AC and FH.

2.

Figures	*Common base*	*Between parallels*
ΔZPC and ΔYPC	PC	PC and ZY
ΔPZY and ΔCZY	ZY	ZY and PC
ΔPAX and ΔCAX	AX	AX and PC

EXERCISE 9B

1. In parallelogram ABCD, AF ⊥ CD and DE ⊥ BC.



Then, AF = 4 cm, DE = 5 cm, AB = 8 cm

Now, ar(||gm ABCD) = AB × AF ... (1)

Also, ar(||gm ABCD) = AD × DE ... (2)

From equation (1) and equation (2), we have

AB × AF = AD × DE

$$\Rightarrow \quad AD = \frac{AB \times AF}{DE} = \frac{8 \text{ cm} \times 4 \text{ cm}}{5 \text{ cm}}$$

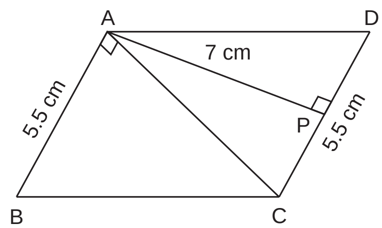
$$= \frac{32}{5} \text{ cm} = 6.4 \text{ cm}$$

Hence, **AD = 6.4 cm.**

2. Joint AC.

AB = DC = 5.5 cm [Given]

∠BAP = ∠APD = 90° [Given]



Since ∠BAP and ∠APD form the alternate angles of sides AB and CD, and AP is the transversal, AB || DC.

This follows that AD || BC and AD = BC.

Hence, ABCD is a parallelogram.

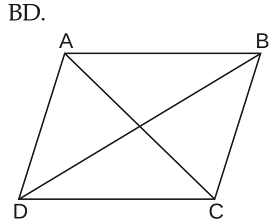
Now, ar(||gm ABCD) = AB × AP

= 5.5 cm × 7 cm

= 38.5 cm^2

Hence, **ar(||gm ABCD) = 38.5 cm^2.**

3. Join AC and BD.



AC is the diagonal of parallelogram ABCD which divides it into two triangles with equal areas.

Thus, ar(ΔACD) = ar(ΔABC) ... (1)

Also, BD is the diagonal of parallelogram ABCD.

Thus, ar(ΔABD) = ar(ΔBCD) ... (2)

Now, ΔACD and parallelogram ABCD are on the same base CD and between the same parallels AB and CD.

Thus, $ar(\Delta ACD) = \frac{1}{2} ar(||gm\ ABCD)$ ... (3)

Also, ΔABD and parallelogram ABCD are on the same base AB and between the same parallel CD.

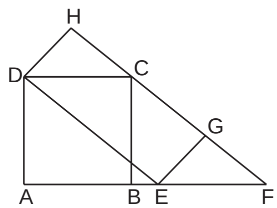
Thus, ar(ΔABD) = ar(||gm ABCD) ... (4)

From equations (1), (2), (3) and (4), we have

$$ar(\Delta ABD) = ar(\Delta BCD) = ar(\Delta ABC)$$
$$= ar(\Delta ACD)$$
$$= \frac{1}{2} ar(\|gm\ ABCD).$$

4. ar(ADCD) = ar(EFCD)

[∵ On same base DC and between same parallels DC and AF]



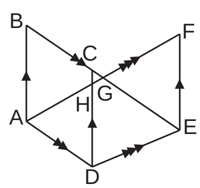
ar(EFCD) = ar(EGHD)

[∵ On same base DE and between same parallels DE and HF]

∴ ar(ABCD) = ar(EGHD)

5. ar(ABCD)= ar(AGED)

[∵ On same base AD and between same parallels AD and BE]

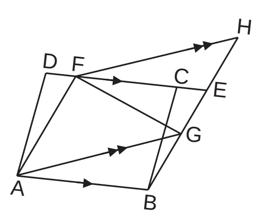


ar(AGED) = area (HFED)

[∵ On same base ED and between same parallels ED and FA]

∴ ar(ABCD) = ar(HFED)

6. The parallelogram ABCD and parallelogram ABEF are on the same base AB and between same parallels AB and DE.



Thus, $ar(\|gm\ ABCD) = ar(\|gm\ ABEF)$ … (1)

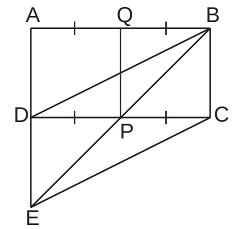
The parallelogram ABEF and parallelogram AGHF are on the same base AF and between same parallels AF and BH.

Thus, $ar(\|gm\ ABEF) = ar(\|gm\ AGHF)$ … (2)

From equations (1) and (2), we have

$$ar(\|gm\ ABCD) = ar(\|gm\ AGHF)$$

7. (i)



From the figure,

QBCP is a square.

Thus, CP = BQ = QP = BC

Since Q and P are the mid-points of AB and CD respectively,

∴ area of square QBCP

$$= \frac{ar(\|gm\ ABCD)}{2}$$

$$= \frac{36\ cm^2}{2} = 18\ cm^2$$

⇒ $QP \times CP = 18\ cm^2$

⇒ $QP = 3\sqrt{2}$ cm [Using QP = CP]

This shows that the length of the sides of the square QBCP is 6 cm.

Then, $AD = DE = 3\sqrt{2}$ cm

and $CD = DP + CP = 6 + 6 = 6\sqrt{2}$

Join DB. Then, DBCE forms a parallelogram with diagonal BE.

Since parallelogram DBCE and ΔBEC lie on the same base BC and between the same parallels BC and DE,

$$\therefore\ ar(\Delta BEC) = \frac{1}{2} ar(\|gm\ DBCE)$$

$$= \frac{1}{2} \times CD \times DE = \frac{1}{2} \times 6\sqrt{2} \times 3\sqrt{2}$$

$$= 18\ cm^2$$

(ii) Now, area of the parallelogram QPCB = CP × QP

$$= 3\sqrt{2}\ cm \times 3\sqrt{2}\ cm = 18\ cm^2$$

and area of the parallelogram ADPQ

$$= AD \times DP$$

$$= 3\sqrt{2}\ cm \times 3\sqrt{2}\ cm = 18\ cm^2$$

Hence, the parallelograms which are equal in area to ΔBEC are parallelogram QPCB and parallelogram ADPQ.

8. Joint BG.

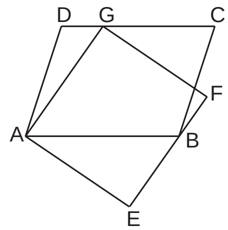
Since parallelogram ABCD and ΔABG are on the same base AB and between same parallels AB and CD,

$$\therefore\ ar(\Delta ABG) = \frac{1}{2} ar(\|gm\ ABCD)$$

⇒ $ar(\|gm\ ABCD) = 2\ ar(\Delta ABG)$ … (1)

Also, parallelogram AEFG and ΔABG are on the same base AG and between same parallels AG and EF. Thus,

$$\therefore\ ar(\Delta ABG) = \frac{1}{2} ar(\|gm\ AEFG)$$

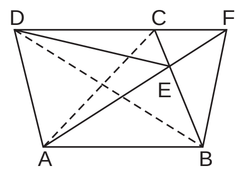


$\Rightarrow$ ar(||gm AEFG) = 2 ar(ΔABG) … (2)

From equations (1) and (2), we have

ar(||gm ABCD) = ar(||gm AEFG)

9. Join BD and AC.



In parallelogram ABCD, AB || CD.

Thus, AB || CF.

Now, ΔACB and ΔAFB are on the same base AB and between same parallels AB and CF.

Then, ar(ΔACB) = ar(ΔAFB)

$\Rightarrow$ ar(ΔCAE) + ar(ΔAEB)

= ar(ΔBEF) + ar(ΔAEB)

$\Rightarrow$ ar(ΔCAE) = ar(ΔBEF) … (1)

Also, AD || BC. ΔCAE and ΔDCE are on the same base CE and between same parallels AD and EC.

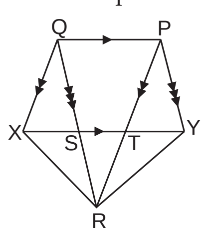
Then, ar(ΔCAE) = ar(ΔDCE) …(2)

From equations (1) and (2), we have

ar(ΔBEF) = ar(ΔDCE)

Hence, the triangles BEF and DCE are equal in area.

10. The parallelograms QPTX and QPYS lie on the same base QP and between the same parallels QP and XY.



Thus, ar(||gm QPTX) = ar(||gm QPYS) …(1)

ΔQXP and parallelogram QPTX lie on the same base XT and between the same parallels QP and XT.

Thus, ar(ΔQXP) = $\frac{1}{2}$ ar(||gm QPTX) …(2)

Also, ΔQPY and parallelogram QPYS lie on the same base QP and between the same parallels QP and SY.

Thus, ar(ΔQPY) = $\frac{1}{2}$ ar(||gm QPYS)

= $\frac{1}{2}$ ar(||gm QPTX) …(3)

[Using equation (1)]

From equations (2) and (3), we have

ar(ΔQXP) = ar(ΔQPY) … (4)

Now, ΔQXP and ΔQXR lie on the same base QX and between the same parallel PR.

Thus, ar(QXP) = ar(QXR) … (5)

Similarly ΔQPY and ΔPYR lie on the same base PY and between the same parallels PY and QR.

Thus, ar(ΔQPY) = ar(ΔPYR)

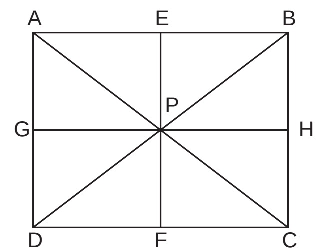
$\Rightarrow$ ar(ΔQXP) = ar(ΔPYR) … (6)

[Using equation (4)]

From equations (5) and (6), we have,

ar(ΔQXR) = ar(ΔPYR)

11. (i) Draw EF ⊥ DC and EF ⊥ AB passing through P. Also, draw GH ⊥ AD and GH ⊥ BC passing through P.



Now, ABHG and GHCD are the parallelograms.

Since ΔAPB and parallelograms ABHG are on the same base AB and between same parallels AB and GH,

$\therefore$ ar(ΔAPB) = $\frac{1}{2}$ ar(||gm ABHG) … (1)

Also, ΔPCB and parallelogram GHCD are on the same base CD and between same parallels CD and GH,

$\therefore$ ar(ΔPCD) = $\frac{1}{2}$ ar(||gm GHCD) … (2)

Adding equations (1) and (2), we have

ar(ΔAPB) + area (ΔPCD)

= $\frac{1}{2}$ [ar(||gm ABHG) + ar(||gm GHCD)]

= $\frac{1}{2}$ ar(rectangle ABCD) … (3)

Hence, ar(ΔAPB) + ar(ΔPCD) = $\frac{1}{2}$ ar(rectangle ABCD)

(ii) Since ΔAPD and parallelogram AEFD are on the same base AD and between the same parallels EF and AD,

$\therefore$ ar(ΔAPD) = $\frac{1}{2}$ ar(||gm AEFD) … (4)

Also, ΔBPC and parallelogram EBCF are on the same base BC and between the same parallels BC and EF. Thus,

$\therefore$ ar(ΔBPC) = $\frac{1}{2}$ ar(||gm EBCF) … (5)

Adding equations (4) and (5), we have

ar(ΔAPD) + ar(ΔBPC)

$= \frac{1}{2}$ [ar(||gm AEFD) + ar(||gm EBCF)]

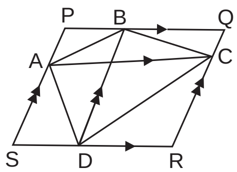
$= \frac{1}{2}$ ar(rectangle ABCD) … (6)

From equations (3) and (4), we have

ar(ΔAPB) + ar(ΔPCD) = ar(ΔAPD) + ar(ΔBPC)

12. ar(ΔABC) = $\frac{1}{2}$ ar(||gm PQCA)

[∵ On the same base AC and between same parallels AC and PQ]



ar(ΔADC) = $\frac{1}{2}$ ar(||gm SRCA)

[∵ On the same base AC and between same parallels AC and SR]

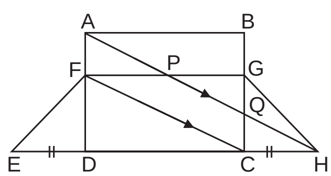
∴ ar(ΔABC) + ar(ΔADC)

$= \frac{1}{2}$ [ar(||gm PQCA) + ar(||gm SRCA)]

⇒ ar(quad ABCD)

$= \frac{1}{2}$ ar(||gm PQRS)

13. ΔFDE ≅ ΔGCH



⇒ ar(ΔFDE) = ar(ΔGCH) … (1)

Also ar(ΔGCH) = $\frac{1}{2}$ ar(||gm PFCH) … (2)

[On the same base CH and between the same parallels CH and FG]

∴ ar(ΔFDE) = $\frac{1}{2}$ ar(||gm PFCH)

∴ ar(ΔFDE) + ar(ΔGCH) = ar(||gm PFCH)

ar(trap EFGH) = [ar(ΔFDE + ar(ΔGCH)] + ar(rectangle FGCD)

= ar(||gm PFCH) + ar(rectangle FGCD)

But, ar(||gm PFCH) = ar(||gm AFCQ)

[On the same base FC and between the same parallels FC and AH]

Also, ar(||gm AFCQ) = ar(||gm AFGB)

[On the same base AF and between the same parallels AF and BG]

∴ ar(trap EFGH)

= ar(rectangle AFGB) + ar(rectangle FGCD)

= ar(rectangle ABCD)

14. Through, E draw FEG || AD to meet AB produced at F and DC at G

ΔBEF ≅ ΔCEG

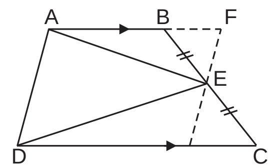
⇒ ar(ΔBEF) = ar(ΔCEG)

ar(trap ABCD)

= ar(ΔAFE) – ar(ΔDGE) + ar(ΔADE) + ar(ΔDGE) + ar(ΔCEG)

= ar(ΔAFE) + ar(ΔADE) + ar(ΔDGE)

= ar(||gm AFGD)



Now, ar(ΔABE) + ar(ΔDCE)

= ar(ΔAFE) – ar(ΔBEF) + ar(ΔDGE) + ar(ΔCEG)

= ar(ΔAFE) + ar(ΔDGE)

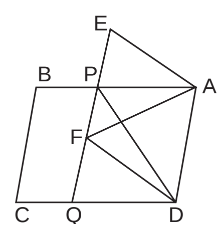
= ar(||gm AFGD) – ar(ΔADE)

= ar(||gm AFGD) – $\frac{1}{2}$ ar(||gm AFGD)

$= \frac{1}{2}$ ar(||gm AFGD)

$= \frac{1}{2}$ ar(trap ABCD)

15. (i) Since parallelograms AEFD and APQD are on the same base AD and between the same parallels AD and EQ,



∴ ar(||gm AEFD) = ar(||gm APQD)

⇒ ar(||gm AEFD) – ar(quad APFD)

= ar(||gm APQD) – ar(quad APFD)

Hence, ar(ΔPEA) = ar(ΔQFD) … (1)

(ii) ΔPFA and ΔPFD are on the same base PF and between the same parallels PF and AD.

∴ ar(ΔPFA) = ar(ΔPFD) … (2)

$$\frac{\text{ar}(\Delta PEA)}{\text{ar}(\Delta PFA)} = \frac{\text{ar}(\Delta QFD)}{\text{ar}(\Delta PFD)}$$

[Dividing the corr. sides of (1) and (2)]

(iii) ΔPEA and ΔQFD lie between the same parallels EQ and AD.

$\therefore$ Altitude from A to base PE

= Altitude from D to base FQ

= h (say)

Now, ar(ΔPEA) = ar(ΔQFD) [Proved in (i)]

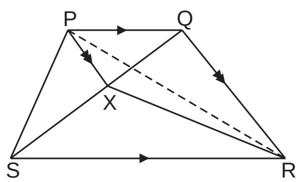
$\Rightarrow$ $\frac{1}{2}$ PE $\times$ h = $\frac{1}{2}$ FQ $\times$ h

$\Rightarrow$ PE = FQ

EXERCISE 9C

1. Join PR.

Since ΔPQS and ΔPQR lie on the same PQ and between the same parallels PQ and SR,



$\therefore$ ar(ΔPQS) = ar(ΔPQR) … (1)

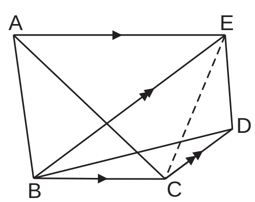
Also, ΔPQR and ΔXQR lie on the same base PX and between the same parallels PX and QR.

Thus, ar(ΔPQR) = ar(XQR) … (2)

From equations (1) and (2), we have

ar(XQR) = ar(ΔPQS)

2. Joint CE. Since triangles on the same base and between the same parallels are equal in area,

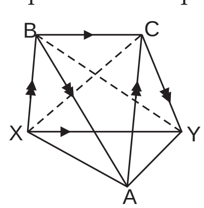


$\therefore$ ar(ΔABC) = ar(ΔEBC) (base BC, BC $\parallel$ AE)

and ar(ΔEBD) = ar(ΔEBC) (base EB, EB $\parallel$ DC)

$\therefore$ ar(ΔABC) = ar(ΔEBD)

3. Joint BY and CX. Since triangles on the same base and between the same parallels are equal in area,



$\therefore$ ar(ΔXBC) = ar(ΔYBC) (base BC, BC $\parallel$ XY)

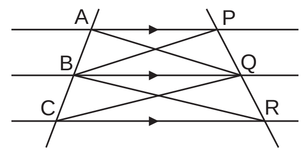
ar(ΔXBC) = ar(ΔXBA) (base XB, XB $\parallel$ AC)

ar(ΔYBC) = ar(ΔYAC) (base YC, YC $\parallel$ AB)

$\therefore$ ar(ΔXBA) = ar(ΔYAC)

4. Since ΔAQB and ΔPBQ are on the same base BQ and between the same parallels AP and BQ,

$\therefore$ ar(ΔAQB) = ar(ΔPBQ) … (1)



Also, ΔBQC and ΔQCR are on the same base BQ and between the same parallels BQ and CR.

Then, ar(ΔBQC) = ar(ΔBQR) … (2)

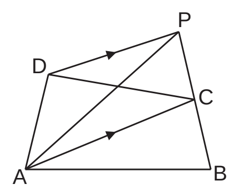
Adding equations (1) and (2), we have

ar(ΔAQB) + ar(ΔBQC) = ar(ΔPBQ) + ar(ΔBQR)

$\Rightarrow$ ar(ΔAQC) = ar(ΔPBR)

Hence, ar(ΔAQC) = ar(ΔPBR).

5. It is given that DP $\parallel$ AC.



Since ΔADC and ΔAPC are on the same base AC and between the same parallels AC and DP,

$\therefore$ ar(ΔADC) = ar(ΔAPC) … (1)

Now, ar(quad ABCD)

= ar(ΔABC) + ar(ADC)

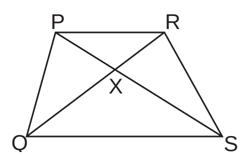
= ar(ΔABC) + ar(ΔAPC)

[Using equation (1)]

$\Rightarrow$ ar(quad ABCD) = ar(ΔABP)

Hence, ar(quad ABCD) = ar(ΔABP)

6. It is given that PR $\parallel$ QS.



Since ΔPQS and ΔRQS are on the same base QS and between the same parallels PR and QS,

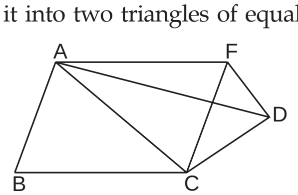
ar(ΔPQS) = ar(ΔRQS)

$\Rightarrow$ ar(ΔPXQ) + ar(QXS) = ar(ΔRXS) + ar(QXS)

$\Rightarrow$ ar(ΔPXQ) = ar(ΔRXS)

Hence, ar(ΔPXQ) = ar(ΔRXS).

7. It is given that ABCF is a parallelogram. Then, the diagonal AC divides it into two triangles of equal area.



$\therefore$ ar(ΔABC) = ar(ΔACF) … (1)

Also, the diagonal AC bisects the quadrilateral. Then,

$\therefore \quad ar(\Delta ABC) = ar(\Delta ADC) \quad \ldots (2)$

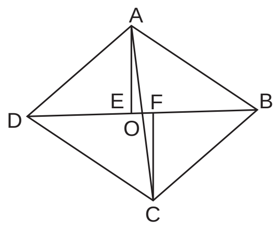
From equations (1) and (2), we have

$ar(\Delta ACF) = ar(\Delta ADC)$

Now, ΔACF and ΔADC are on the same base DF and between the lines DF and CA. Thus, DF is parallel to CA.

Hence, DF ∥ CA.

8. Draw perpendicular from A and C to BD at E and F respectively such that AE ⊥ BD and CF ⊥ BD.



It is given that the areas of ΔBDC and ΔADB are equal.

Thus, $ar(\Delta BDC) = ar(\Delta ADB)$

$\Rightarrow \frac{1}{2} \times AE \times BD = \frac{1}{2} \times CF \times BD$

$\Rightarrow \quad AE = CF$

Also, $\angle AOE = \angle CFO$ [measures 90°]

New, $\angle AOE = \angle COF$ [Vertically opposite angles]

By AAS congruence theorem,

$\Delta AOE \simeq \Delta COF$

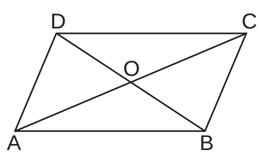
Thus, $AD = CD$.

This shows that O is the middle-point of AC and O passes through BD.

Hence, BD bisects AC.

9. $ar(\Delta ABC) = ar(\Delta ACD)$

$\Rightarrow ar(\text{quad } ABCD) = ar(\Delta ABC) + ar(\Delta ACD)$



$\Rightarrow ar(\text{quad } ABCD) = 2\, ar(\Delta ABC)$

$\Rightarrow \quad ar(\Delta ABC) = \frac{1}{2} ar(\text{quad } ABCD)$

Similarly, $ar(\Delta ABD) = \frac{1}{2} ar(\text{quad } ABCD)$

$\therefore \quad ar(\Delta ABC) = ar(\Delta ABD)$

Since ΔABC and ΔABD are on the same base AB and have equal areas, therefore they must have equal corresponding altitudes.

$\Rightarrow \quad DC \parallel AB$

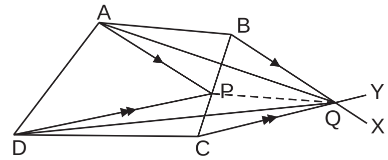
Similarly, $AD \parallel CB$

∴ ABCD is a parallelogram.

10. Join PQ. Since the areas of triangles on the same base and between the same parallels is equal,

$\therefore \quad ar(\Delta APQ) = ar(\Delta APB)$ (base AP, AP ∥ BX)

$ar(\Delta DPQ) = ar(\Delta DPC)$ (base DP, DP ∥ CY)



$\therefore \quad ar(\Delta APQ) + ar(\Delta DPQ)$

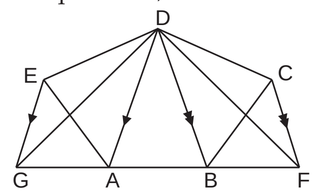
$= ar(\Delta APB) + ar(\Delta DPC)$

$\Rightarrow \quad ar(\Delta APQ) + ar(\Delta DPQ) + ar(\Delta APD)$

$= ar(\Delta APB) + ar(\Delta DPC) + ar(\Delta APD)$

$\Rightarrow \quad ar(\Delta QAD) = ar(\text{quad } ABCD).$

11. Since triangles on the same base and between the same parallels have equal areas,



$\therefore \quad ar(\Delta DEA) = ar(\Delta DGA)$ (base DA, DA ∥ EG)

and $\quad ar(\Delta DCB) = ar(\Delta DFB)$ (base DB, DB ∥ CF)

Now, ar(pentagon ABCDE)

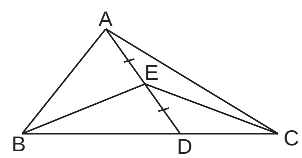
$= ar(\Delta DEA) + ar(\Delta ADAB) + ar(\Delta DBC)$

$\Rightarrow$ ar(pentagon ABCDE)

$= ar(\Delta DGA) + ar(\Delta DAB) + ar(\Delta DFB)$

$\Rightarrow$ ar(pentagon ABCDE) = $ar(\Delta DGF)$

12.



In ΔABD, BE is the median

$\therefore \quad ar(\Delta EBD) = \frac{1}{2} ar(\Delta ABD)$

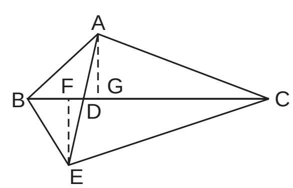
In ΔACD, CE is the median

$\therefore \quad ar(\Delta ECD) = \frac{1}{2} ar(\Delta ACD)$

$ar(\Delta EBD) = ar(\Delta ECD) + \frac{1}{2} [ar(\Delta ABD) + ar(\Delta ACD)]$

$\Rightarrow \quad ar(\Delta EBC) = \frac{1}{2} ar(\Delta ABC).$

13.



Now, $\quad AD = DE$

Thus, D is the mid-point of AE. Joint BD. Then, BD is the median of ΔABE. Then median BD divides ΔABE into equal areas of ΔABD and ΔBDE.

$\therefore \quad ar(\Delta ABD) = ar(\Delta BDE) \quad \ldots (1)$

Also, CD is the median of ΔAEC.

Then, $\quad ar(\Delta ADC) = ar(\Delta CDE) \quad \ldots (2)$

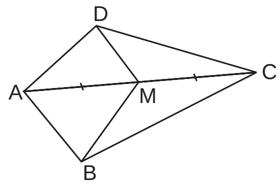
Adding equations (1) and (2), we have

$ar(\Delta ABD) + ar(\Delta ADC) = ar(\Delta BDE) + ar(\Delta CDE)$

$\Rightarrow \quad ar(\Delta ABC) = ar(\Delta EBC)$

Hence, $ar(\Delta EBC) = ar(\Delta ABC)$.

14. In ΔADC, MD is the median. MD divides the ΔADC into two Δs ΔADM and ΔCDM of equal areas.



Thus, $\quad ar(\Delta ADM) = ar(\Delta CDM) \quad \ldots (1)$

Also, MB is the median of ΔABC and it divides the triangle into two Δs ΔABM and ΔCBM with equal area.

Thus, $\quad ar(\Delta ABM) = ar(\Delta CBM) \quad \ldots (2)$

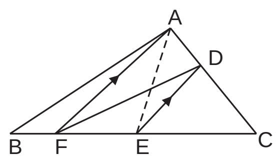
Adding equations (1) and (2), we have

$$ar(\Delta ABM) + ar(\Delta ADM) = ar(\Delta CBM) + ar(\Delta CDM)$$

Hence, $ar(\Delta ABM) + ar(\Delta ADM) = ar(\Delta CBM) + ar(\Delta DCM)$.

15. Join AE. It is given that AF $\parallel$ ED.

ΔAED and ΔDFE are on the same base and between same parallels AF and ED.



Thus, $\quad ar(\Delta AED) = ar(\Delta DFE) \quad \ldots (1)$

Now, $\quad ar(\Delta DFC) = ar(\Delta DFE) + ar(\Delta DEC)$

$\qquad = ar(\Delta AED) + ar(\Delta DEC)$

[Using equation (1)]

$\Rightarrow \quad ar(\Delta DFC) = ar(\Delta AEC) \quad \ldots (2)$

Also, E is the mid-point of BC so that AE is the median of ΔABC.

Thus, $\quad ar(\Delta ABE) = ar(\Delta AEC) \quad \ldots (3)$

and $\quad ar(\Delta ABC) = ar(\Delta AEC) + ar(\Delta ABE)$

$\qquad = ar(\Delta AEC) + ar(\Delta AEC)$

[Using equation (3)]

$\Rightarrow \quad ar(\Delta AEC) = \frac{1}{2} ar(\Delta ABC)$

$\Rightarrow \quad ar(\Delta DFC) = \frac{1}{2} ar(\Delta ABC) \quad$ [Using equation (2)]

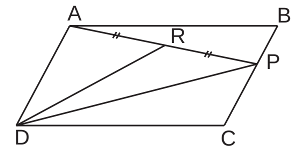
Hence, $\quad ar(\Delta DFC) = \frac{1}{2} ar(\Delta ABC)$.

16. $\quad ar(\Delta APD) = \frac{1}{2} ar(\|gm\ ABCD)$

(base AD, AD $\parallel$ BC) …(1)

$ar(\Delta PRD) = \frac{1}{2} ar(\Delta ADP)$

[$\because$ DR is median of ΔAPD]

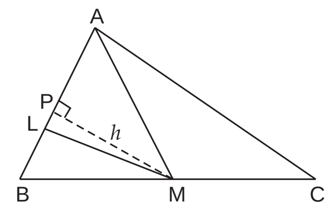


$\Rightarrow \quad ar(\Delta PRD) = \frac{1}{2} \times \frac{1}{2} ar(\|gm\ ABCD)$

[Using (1)]

$\Rightarrow \quad ar(\Delta PRD) = \frac{1}{4} ar(\|gm\ ABCD)$

17. Draw MP $\perp$ AB and let MP = h. AL : LB = 2 : 1.



$\Rightarrow \quad \frac{AL}{LB} = \frac{2}{1} \quad \Rightarrow \quad AL = 2LB$

$ar(\Delta AML) = \frac{1}{2} \cdot AL \times h = \frac{1}{2}(2LB)\,(h) = LB(h)$

$ar(\Delta AMB) = \frac{1}{2} AB \times h = \frac{1}{2}(AL + LB)h$

$\qquad = \frac{1}{2}(2LB + LB)h = \frac{1}{2} \times 3 \cdot LB \times h$

$\frac{ar(\Delta AML)}{ar(\Delta AMB)} = \frac{LB(h)}{\frac{3}{2}LB(h)}$

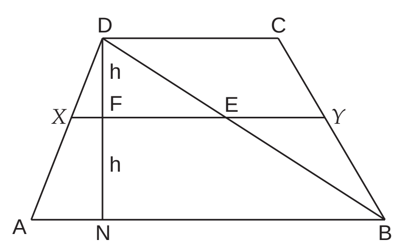
$\Rightarrow \quad ar(\Delta AML) = \frac{2}{3} ar(\Delta AMB)$

AM is the median of ΔABC.

$\therefore \quad ar(\Delta AMB) = \frac{1}{2} ar(\Delta ABC)$

$\therefore \quad ar(\Delta AML) = \frac{2}{3} \times \frac{1}{2} ar(\Delta ABC) = \frac{1}{3} ar(\Delta ABC)$.

18. It is given that ABCD is a trapezium and AB $\parallel$ CD. X and Y are the mid-points of AD and BC. Then, XY is also parallel to AB and CD.



Now, AB = 50 cm and CD = 30 cm

Draw DN $\perp$ AB and join BD.

Let $\quad DF = FN = h$.

In triangles DXE and DAB,

$\angle XDE = \angle ADB$

$\angle DEX = \angle DAB$

By AA similarity,

$$\Delta DXE \sim \Delta DAB$$

$$\Rightarrow \quad \frac{DX}{DA} = \frac{XE}{AB}$$

$$\Rightarrow \quad \frac{DX}{2DX} = \frac{XE}{AB}$$

$$\Rightarrow \quad XE = \frac{1}{2}AB$$

$$\Rightarrow \quad XE = \frac{1}{2} \times 50 \qquad [\because AB = 50 \text{ cm}]$$

$$= 25 \text{ cm}$$

Similarly, from similar triangles BEY and BDC

$$\frac{BY}{BC} = \frac{YE}{CD}$$

$$\Rightarrow \quad \frac{BY}{2BY} = \frac{YE}{CD}$$

$$\Rightarrow \quad YE = \frac{1}{2} \times CD$$

$$= \frac{1}{2} \times 30 \text{ cm} \qquad [\because CD = 30 \text{ cm}]$$

$$= 15 \text{ cm}$$

Now, $XY = XE + YE = 25 \text{ cm} + 15 \text{ cm} = 40 \text{ cm}$

DCYX is a trapezium in which DC || XY.

$$\therefore \quad ar(DCYX) = \frac{1}{2} \times (CD + XY) \times h$$

$$= \frac{1}{2} \times (30 \text{ cm} + 40 \text{ cm}) \times h$$

$$= (35 \text{ m})\, h \qquad \dots (1)$$

and XYBA is also a trapezium in which XY || AB.

$$\therefore \quad ar(XYBA) = \frac{1}{2} \times (XY + AB) \times h$$

$$= \frac{1}{2} \times (40 \text{ cm} + 5 \text{ cm})\, h$$

$$= (45 \text{ cm})\, h \qquad \dots (2)$$

Dividing equations (1) by (2), we have

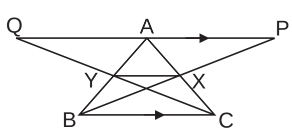
$$\frac{ar(DCYX)}{ar(XYBA)} = \frac{(35 \text{ m})\, h}{(45 \text{ cm})\, h} = \frac{7}{9}.$$

$$\Rightarrow \quad ar(DCYX) = \frac{7}{9} ar(XYBA)$$

Hence, $ar(DCYX) = \frac{7}{9} ar(XYBA)$.

19. Joint YX.

Now, AP || BC and AC is a straight line passing through them.



Then, $\angle PAC = \angle BCA$ [Alternate opposite angles]

$AX = CX$ [X is the mid-point of AC]

$\angle AXP = \angle BXC$ [Vertically opposite angles]

By ASA congruence,

$$\Delta AXP \cong \Delta BXC$$

$$\Rightarrow \quad ar(\Delta AXP) = ar(\Delta BXC)$$

Also, BX is the median of ΔABC.

Then, $ar(\Delta ABX) = ar(\Delta BXC)$

Now, $ar(\Delta ABC) = ar(\Delta ABX) + ar(\Delta BXC)$

$= ar(\Delta ABX) + ar(\Delta AXP)$

$= ar(\Delta ABP)$ ... (1)

Again, in triangles AQY and BYC

$\angle AQY = \angle BCY$ [Alternate opposite angles]

$\angle QYA = \angle BYC$ [Vertically opposite angles]

$BY = AY$ [Y is the mid-point of AB]

By AAS congruence, we have

$$\Delta AQY \cong \Delta BYC$$

$$\Rightarrow \quad ar(\Delta AQY) = ar(\Delta BYC)$$

Now, $ar(\Delta ABC) = ar(\Delta AYC) + ar(\Delta BYC)$

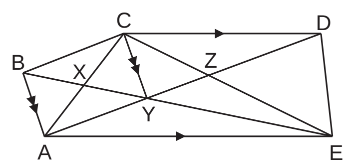
$= ar(\Delta AYC) + (\Delta AQY)$

$= ar(\Delta ACQ)$ ... (2)

From equations (1) and (2), we have

$$ar(\Delta ABP) = ar(\Delta ACQ).$$

20. Now, ΔBCY and ΔACY are on the same base CY and between the same parallels CY and AB.



Then, $ar(\Delta BCY) = ar(\Delta ACY)$ ... (1)

and ΔACE and ΔADE are on the same base AE and between the same parallels AE and CD.

Thus, $ar(\Delta ACE) = ar(\Delta ADE)$

$\Rightarrow ar(\Delta CAZ) + (\Delta AZE) = ar(\Delta EDZ) + ar(\Delta AZE)$

$\Rightarrow \quad ar(\Delta CAZ) = ar(\Delta EDZ)$ ... (2)

Now, $ar(BCZY) = ar(\Delta BCY) + ar(\Delta CYZ)$

$= ar(\Delta ACY) + ar(\Delta CYZ)$ [From equation (1)]

$= ar(\Delta CAZ)$

$= ar(\Delta EDZ)$ [From equation (2)]

Hence, $ar(BCZY) = ar(\Delta EDZ)$.

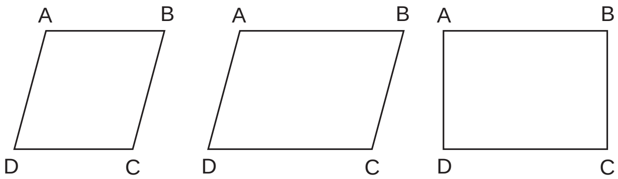
CHECK YOUR UNDERSTANDING

MULTIPLE-CHOICE QUESTIONS

1. (d) **Triangles of equal areas**
The median of a triangle divides it into two triangles of equal areas.

2. (d) **Need not be any of (a), (b) or (c)**
The given quadrilateral can be shown in below:

In all quadrilaterals, the diagonal AC divides into two parts with equal areas. Hence, ABCD can be a rhombus, parallelogram or rectangle.



Rhonbus Parallelogram Rectangle

3. (c) **1 : 1**

Since, parallelograms on the same base and between the same parallels are equal in area

∴ The ratio of their areas is 1:1.

4. (b) $\mathbf{\frac{1}{2}ar(\Delta ABC)}$

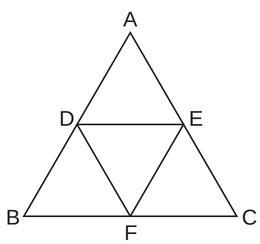
In ΔABC, D, E and F are the mid-points of sides AB, AC and BC. Join DF, FE and DE.

Now, FE ∥ AB and F and E are the mid-points of sides BC and AC respectively.

$$FE = \frac{1}{2}AB$$

Also, D is the mid-point of AB. Then,

$$AD = \frac{1}{2}AB$$



$\Rightarrow$ FE = AD

Also, DF ∥ AC.

Then, $DF = \frac{1}{2}AC$

E is the mid-point of AC

$$AE = \frac{1}{2}AC$$

$\Rightarrow$ DF = AE

Thus, ADFE is a parallelogram.

Similarly, it can be proved that DEFB and DECF are parallelograms.

Now, DF is the diagonal of parallelogram DEFB.

Then, $ar(\Delta DEF) = ar(\Delta DBF)$

Similarly, $ar(\Delta DEF) = ar(\Delta EFC)$

and $ar(\Delta ADE) = ar(\Delta DEF)$

Thus, $ar(\Delta ABC) = 4 \times ar(\Delta DBF)$

$= 4 \times ar(\Delta DEF) = 4\, ar(\Delta EFC)$

$= 4\, ar(\Delta ADE)$

$\Rightarrow$ $ar(\Delta DBF) = ar(\Delta DEF) = ar(\Delta EFC)$

$= ar(\Delta ADE) = \frac{1}{4}ar(\Delta ABC).$

Now, ar(∥gm ADFE)

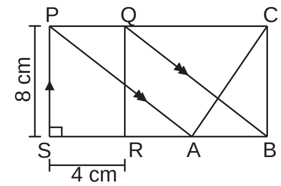
$= ar(\Delta ADE) + ar(\Delta DEF)$

$= \frac{1}{4}ar(\Delta ABC) + \frac{1}{4}ar(\Delta ABC) = \frac{1}{2}ar(\Delta ABC)$

Hence, ar(∥gm ADFE) $= \frac{1}{2}ar(\Delta ABC)$.

5. (d) **16 cm^2**

From the figure,



PS = 8 cm and RS = 4 cm.

Now, ar(rect PQRS) = PS × RS

$= 8 \text{ cm} \times 4 \text{ cm} = 32 \text{ cm}^2$

Since PA ∥ QB and PQ ∥ AB, then

∴ ABQP is a parallelogram.

Now, rectangle PQRS and parallelogram ABQP are on the same base PQ and between same parallels PQ and SB.

∴ ar(rect PQRS) = ar(∥gm ABQP)

$\Rightarrow$ ar(∥gm ABQP) = 32 cm^2

ΔABC and parallelogram ABQP are on the same base AB and between same parallels AB and CP.

∴ $ar(\Delta ABC) = \frac{1}{2}$ ar(∥gm ABQP)

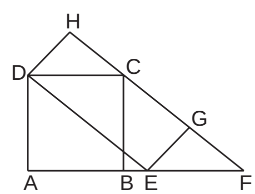
$= \frac{1}{2} \times 32 \text{ cm}^2$

$= 16 \text{ cm}^2$

Hence, $ar(\Delta ABC) = 16 \text{ cm}^2$.

6. (d) **DEGH and DEFC**

DEGH is a rectangle.



Then, DE ∥ HG

$\Rightarrow$ DE ∥ HF [F is produced from G]

$\Rightarrow$ DE ∥ CF

Also, since ABCD is a square

$\Rightarrow$ DC ∥ AB

$\Rightarrow$ DC ∥ EF

∴ CD ∥ FE

Also, DE ∥ CF

Thus, DEFC is a parallelogram.

Since parallelogram DEFC and parallelogram DEGH are on the same base DE and between same parallels DE and HF,

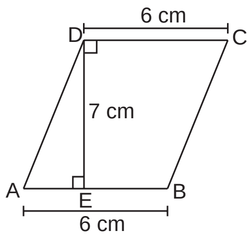
ar(rect DEGH) = ar(∥gm DEFC).

Hence, the two equal parallelograms on the base DE are DEGH and DEFC.

7. (d) **42 cm²**

In quadrilateral ABCD, we have

$$\angle CDE = \angle AED = 90°$$



$$CD = BA = 6 \text{ cm}.$$

Thus, ABCD is a parallelogram.

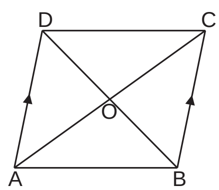
Now, $\quad$ ar(||gm ABCD) = CD × DE

$= 6 \text{ cm} \times 7 \text{ cm}$

$= 42 \text{ cm}^2$.

8. (b) **ΔBOA**

ΔBDC and ΔABC are on the same base BC and between the same parallels AD and BC. Thus,



ar(ΔBDC) = ar(ΔABC)

⇒ ar(ΔCOD) + ar(ΔBOC) = ar(BOA) + ar(ΔBOC)

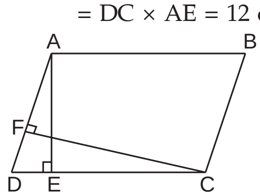
⇒ ar(ΔCOD) = ar(ΔBOA)

Hence, the triangle which is equal in area to ΔCOD is ΔBOA.

9. (a) **6 cm**

Now, ar(||gm ABCD)

= DC × AE = 12 cm × 7.5 cm … (1)



Also, ar(||gm ABCD)

= AD × CF = AD × 15 cm … (2)

From equations (1) and (2), we have

12 cm × 7.5 cm = AD × 15 cm

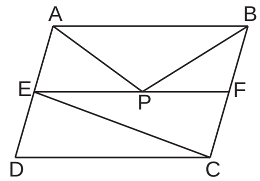
$$\Rightarrow \quad AD = \frac{12 \text{ cm} \times 7.5 \text{ cm}}{15 \text{ cm}} = 6 \text{ cm}.$$

10. (b) **8 cm²**

E and F are the mid-points of AD and BC. Then, EF divides parallelogram ABCD into two equal parallelograms ABFE and parallelogram ABFE and parallelogram EFCD.

Thus, $\quad$ ar(||gm ABFE) = ar(||gm EFCD)

Now, EC is the diagonal of parallelogram EFCD.



Then, $\quad$ ar(ΔEFC) = $\frac{1}{2}$ ar(||gm EFCD)

⇒ $\quad$ ar(||gm EFCD)

= 2 × ar(ΔEFC)

= 2 × 8 cm² $\quad$ [∴ ar(ΔEFC) = 8 cm²]

= 16 cm²

∴ $\quad$ ar(||gm ABFE)

= ar(||gm EFCD)

[EF divides ||gm ABCD into two ||gm of equal areas]

= 16 cm²

Now, ΔAPB and parallelogram ABFE are on the same base AB and between the same parallels AB and EF.

∴ $\quad$ ar(ΔAPB) = $\frac{1}{2}$ ar(||gm ABFE)

$= \frac{1}{2} \times 16 \text{ cm}^2 = 8 \text{ cm}^2$

Also, ar(||gm ABFE)

= ar(ΔAEP) + ar(ΔBFP) + ar(ΔAPB)

⇒ $\quad$ 16 cm² = ar(ΔAEP) + ar(ΔBFP) + 8 cm²

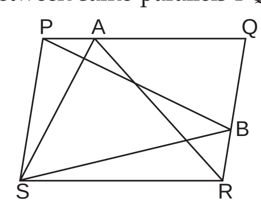
⇒ $\quad$ ar(ΔAEP) + ar(ΔBFP) = 16 cm² – 8 cm²

= 8 cm²

Hence, ar(ΔAEP) + ar(ΔBFP) = 8 cm².

11. (d) **48 cm²**

ΔASR and parallelogram PQRS are on the same base SR and between same parallels PQ and SR.



Thus, $\quad$ ar(ΔASR) = $\frac{1}{2}$ ar(||gm PQRS)

$= \frac{1}{2} \times 48 \text{ cm}^2$

$= 24 \text{ cm}^2$

ΔPBS and parallelogram PQRS are on the same base PS and between same parallels PS and QR.

Thus, $\quad$ ar(ΔPBS) = $\frac{1}{2}$ ar(||gm PQRS)

$= \frac{1}{2} \times 48 \text{ cm}^2$

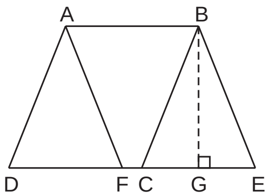
$= 24 \text{ cm}^2$

Now, ar(ΔPBS) + ar(ΔASR) = 24 cm² + 24 cm²

= 48 cm²

12. (c) **5 cm**

Parallelogram ABCD and parallelogram ABEF are on the same base AB and between same parallels AB and DE.



Thus, ar(||gm ABCD) = ar(||gm ABEF)

$\Rightarrow$ ar(||gm ABEF) = 29 cm^2

Let BG be the height of the parallelogram ABEF.

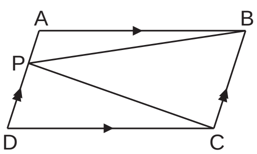
Then, ar(||gm ABEF) = BG × AB

$\Rightarrow$ 29 cm^2 = BG × 5.8 cm

$\Rightarrow$ $BC = \frac{29\ cm^2}{5.8\ cm} = 5$ cm.

13. (b) **80 cm^2**

ΔCPB and parallelogram ABCD are on the same base BC and between same parallels BC and AD.



Thus, $ar(\Delta CPB) = \frac{1}{2} ar(||gm\ ABCD)$

Now, ar(||gm ABCD) – [ar(ΔBAP) + ar(ΔCPD)]
= ar(ΔCPB)

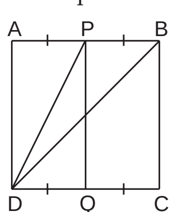
$\Rightarrow$ 2 ar(ΔCPB) – [ar(ΔBAP) + ar(ΔCPD)]
= ar(ΔCPB)

$\Rightarrow$ ar(ΔCPB) = ar(ΔBAP) + ar(ΔCPD)
= 10 cm^2 + 30 cm^2 = 40 cm^2

Thus, ar(||gm ABCD)
= ar(ΔBAP) + ar(ΔCPD) + ar(ΔCPB)
= 10 cm^2 + 30 cm^2 + 40 cm^2
= 80 cm^2

14. (a) **16 cm^2**

P and Q are the mid-points of AB and DC.



Then, $AP = \frac{1}{2} AB = \frac{1}{2} \times 8$ cm = 4 cm

AB = 8 cm

BC = 8 cm

CD = 8 cm

$ar(\Delta APD) = \frac{1}{2} \times AP \times AD$

$= \frac{1}{2} \times 4\ cm \times 8\ cm = 16\ cm^2$

$ar(\Delta BDC) = \frac{1}{2} \times BC \times CD$

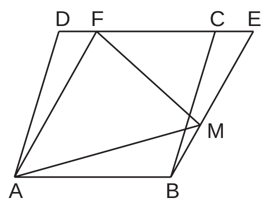
$= \frac{1}{2} \times 8\ cm \times 8\ cm = 32\ cm^2$

ar(ABCD) = $(AB)^2$
= $(8\ cm)^2$ = 64 cm^2

Now, ar(ΔBPD) = ar(ABCD) – {ar(ΔAPD) + ar(ΔBDC)}
= 64 cm^2 – (16 cm^2 + 32 cm^2)
= 64 cm^2 – 48 cm^2 = 16 cm^2

15. (a) **14 cm^2**

In the figure,



Parallelogram ABCD and parallelogram ABEF are on the same base AB and between are between same parallels AB and DE.

Then, ar(||gm ABCD) = ar(||gm ABEF) = 28 cm^2

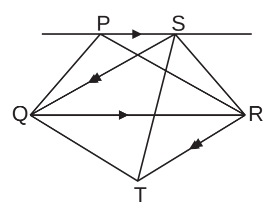
Also, ΔFAM and parallelogram ABEF are on the same base AF and between same parallels AF and BE.

Then, $ar(\Delta FAM) = \frac{1}{2} ar(||gm\ ABEF)$

$= \frac{1}{2} \times 28\ cm^2 = 14\ cm^2$.

16. (a) **ΔPQR, ΔQSR, ΔQST**

In the figure,



ΔPQR and ΔQSR are on the same base QR and between same parallels PS and QR.

Then, ar(ΔPQR) = ar(ΔQSR) … (1)

Also, ΔQST and ΔQSR are on the same base QS and between same parallels QS and TR.

Then, ar(ΔQST) = ar(ΔQSR) … (2)

From equations (1) and (2), we have

ar(ΔPQR) = ar(ΔQSR) = ar(ΔQST).

17. (c) **$(3a + b) : (a + 3b)$**

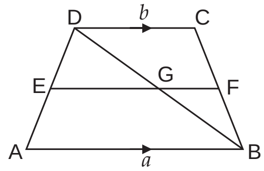
Join DB. In the figure, we have

CD || EF and AB || EF.

In ΔDEG and ΔDAB, we have

$\angle$EDG = $\angle$ADB [common angles]

$\angle$DEG = $\angle$DAB [corresponding angles]



By AA similarity, ΔDEG ~ ΔDAB.

$$\text{Thus,} \quad \frac{AD}{ED} = \frac{EG}{AB}$$

$$\Rightarrow \quad \frac{\frac{1}{2}ED}{ED} = \frac{EG}{AB}$$

$$\Rightarrow \quad EG = \frac{1}{2}AB = \frac{1}{2}a.$$

$$\text{Similarly,} \quad FG = \frac{1}{2}CD = \frac{1}{2}b.$$

Let h be the height of the trapeziums EFCD and ABFE.

$$\text{Now,} \quad EF = EG + FG = \frac{1}{2}a + \frac{1}{2}b = \frac{1}{2}(a+b)$$

$$\text{Then,} \quad ar(EFCD) = \frac{h}{2}[CD + EF]$$

$$= \frac{h}{2}\left[b + \frac{1}{2}(a+b)\right] = \frac{h}{4}(a+3b)$$

$$\text{and} \quad ar(ABFE) = \frac{h}{2}(AB + EF)$$

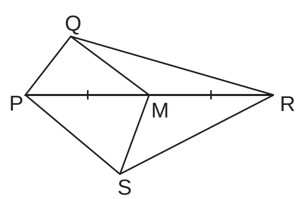
$$= \frac{h}{2}\left[a + \frac{1}{2}(a+b)\right] = \frac{h}{4}(3a+b)$$

$$\text{Now,} \quad \frac{ar(ABFE)}{ar(EFCD)} = \frac{\frac{h}{4}(3a+b)}{\frac{h}{4}(a+3b)} = \frac{3a+b}{3b+a}$$

Hence, ar(ABFE) : ar(EFCD) = $(3a + b) : (a + 3b)$

18. (b) **18 cm^2**

In ΔPQR, MQ is the median.



Then, ar(ΔPQM) = ar(ΔQMR) … (1)

Also, MS is the median of ΔPRS.

Then, ar(ΔPMS) = ar(ΔSRM) … (2)

Adding equations (1) and (2), we have

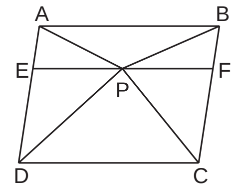
ar(ΔPQM) + ar(ΔPMS) = ar(ΔQMR) + ar(ΔSRM)

⇒ ar(quad PQMS) = ar(quad SMQR) = 18 cm^2

Hence, ar(quad PQMS) = 18 cm^2.

19. (c) **32 cm^2**

Let us draw a line EF parallel to both AB and CD passing through P. Then, the line EF divides the quadrilaterals ABCD into two parallelograms ABFE and CDEF.



ΔDPC and parallelogram CDEF are on the same base CD and between same parallels CD and EF.

$$\text{Thus,} \quad ar(\Delta DPC) = \frac{1}{2} ar(\text{quad } CDEF) \qquad \ldots (1)$$

Also, ΔAPB and parallelogram ABFE are on the same base AB and between same parallels AB and EF.

$$\text{Thus,} \quad ar(\Delta APB) = \frac{1}{2} ar(\text{quad } ABFE) \qquad \ldots (2)$$

Adding equations (1) and (2), we have

$$ar(\Delta DPC) + ar(\Delta APB) = \frac{1}{2}[ar(\|gm\ CDEF) + ar(\|gm\ ABFE]$$

$$= \frac{1}{2} ar(\|gm\ ABCD)$$

$$= \frac{1}{2} \times 64 \text{ cm}^2$$

$$= 32 \text{ cm}^2$$

Now, ar(ΔAPD) + ar(ΔPBC)

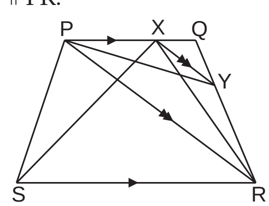
= ar(‖gm ABCD)
− {ar(ΔDPC) + ar(ΔAPB)}

= 64 cm^2 – 32 cm^2 = 32 cm^2.

Hence, ar(ΔAPD) + ar(DPBC) = 32 cm^2.

20. (b) **5 cm^2**

Now, XY ‖ PR.



ΔPYR and ΔPXR are on the same base PR and between same parallels XY and PR.

Then, ar(ΔPYR) = ar(ΔPXR)

⇒ ar(ΔPXR) = 5 cm^2

Also, PX ‖ SR.

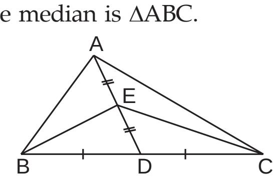
ΔPXR and ΔPXS are on the same base PX and between same parallels PX and SR.

Then, ar(ΔPXR) = ar(ΔPXS)

⇒ ar(ΔPXS) = 5 cm^2.

21. (c) **5 cm^2**

AD is the median is ΔABC.



Then, ar(ΔABD) = ar(ΔADC)

Now, ar(ΔABC) = ar(ΔABD) + ar(ΔADC) = 2 ar(ΔABD)

$\Rightarrow$ ar(ΔABD) = $\frac{\text{ar}(\Delta ABC)}{2} = \frac{10\text{ cm}^2}{2} = 5\text{ cm}^2$

Also, ar(ΔADC) = $\frac{\text{ar}(\Delta ABC)}{2} = \frac{10\text{ cm}^2}{2} = 5\text{ cm}^2$

Also, EB is the median of ΔABD.

Then, ar(ΔABE) = ar(ΔEBD)

In ΔABD,

ar(ΔABD) = ar(ΔABE) + ar(ΔEBD)

= 2 ar(ΔABE)

$\Rightarrow$ ar(ΔABE) = $\frac{1}{2}$ ar(ΔABD)

= $\frac{1}{2} \times 5\text{ cm}^2 = 2.5\text{ cm}^2$

Also, ar(ΔEBD) = $\frac{1}{2}$ ar(ΔABD)

= $\frac{1}{2} \times 5\text{ cm}^2 = 2.5\text{ cm}^2$

EC is the median of ΔADC.

ar(ΔAEC) = ar(ΔEDC)

Now, ar(ΔEDC) = $\frac{1}{2}$ ar(ΔADC)

= $\frac{1}{2} \times 5\text{ cm}^2 = 2.5\text{ cm}^2$

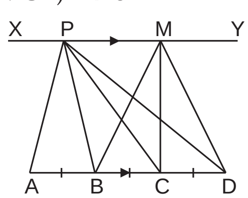
Thus, ar(ΔEBC) = ar(ΔEBD) + ar(ΔEDC)

= $2.5\text{ cm}^2 + 2.5\text{ cm}^2 = 5\text{ cm}^2$

22. (a) **7 cm, 21 cm^2**

Join PC and BM.

Now, ar(ΔMCD) = 7 cm^2.



In ΔBMD, CM is the median.

Then, ar(ΔBMC) = ar(ΔMCD) = 7 cm^2

PM ∥ CB. ΔBMC and ΔPBC are on the same base and between same parallels PM and BC.

Thus, ar(ΔBMC) = ar(ΔPBC)

$\Rightarrow$ ar(ΔPBC) = 7 cm^2

In ΔAPC, PB is the median.

Then, ar(ΔAPB) = ar(ΔPBC) = 7 cm^2

In ΔPBD, CP is the median.

Then, ar(ΔPBC) = ar(ΔPCD)

$\Rightarrow$ ar(ΔPCD) = 7 cm^2

Now, ar(ΔAPD) = ar(ΔAPB) + (ΔPBC) + ar(ΔPCD)

= 7 cm^2 + 7 cm^2 + 7 cm^2

= 21 cm^2.

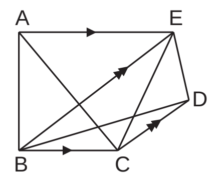
Hence, ar(ΔAPB) = 7 cm^2 and

ar(ΔAPD) = 21 cm^2.

23. (a) **6 cm^2**

AE ∥ BC.

ΔABC and ΔBCE are on the same base BC and between same parallels AE and BC.



Then, ar(ΔABC) = ar(ΔBCE) … (1)

Now, BE ∥ CD.

ΔBCE and ΔBED are on the same base BE and between same parallels BE and CD.

Then, ar(ΔBCE) = ar(ΔBED) … (1)

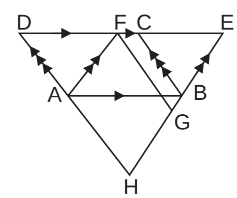
From equations (1) and (2), we have

ar(ΔABC) = ar(ΔBED) = 6 cm^2.

24. (b) **50 cm^2**

Now, AF ∥ HE.

Parallelogram ABEF and parallelogram AFGH are on the same base AF and between same parallels AF and HE.



Then, ar(∥gm ABEF) = ar(∥gm AFGH)

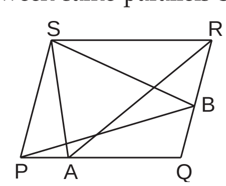
$\Rightarrow$ ar(∥gm AFGH) = 50 cm^2

[Given : ar(∥gm ABEF) = 50 cm^2]

Hence, ar(∥gm AFGH) = 50 cm^2.

25. (c) **24 cm^2**

ΔRAS and parallelogram PQRS are on the same base SR and between same parallels SR and PQ.



Then, ar(ΔRAS) = $\frac{1}{2}$ ar(∥gm PQRS) …(1)

$\Rightarrow$ ar(∥gm PQRS) = 2 ar(ΔSBP)

= 2[ar(∥gm PQRS) – {ar(ΔSBR) + ar(ΔPBQ)}]

$\Rightarrow$ ar(∥gm PQRS)

= 2{ar(ΔSBR) + ar(ΔPBQ)}

= 2(16 cm^2 + 8 cm^2)

= 48 cm^2

From equation (1), we have

$$\text{ar}(\Delta RAS) = \frac{1}{2}\,\text{ar}(\|\text{gm } PQRS)$$

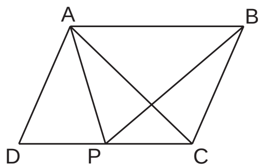
$$= \frac{1}{2} \times 48 \text{ cm}^2$$

$$= 24 \text{ cm}^2$$

Hence, ar(ΔRAS) = 24 cm^2.

26. (c) **35 cm^2**

ABCD is a parallelogram and AC its diagonal. Then, the areas of triangles ADC and ABC are equal.



Thus, ar(ΔADC) = ar(ΔABC) … (1)

Now, ar(ΔADC) = ar(ΔDPA) + ar(ΔAPC)

= 15 cm^2 + 20 cm^2

= 35 cm^2 … (2)

Also, ΔAPB and ΔACB are on the same base AB and between same parallels AB and DC.

Thus, ar(ΔAPB) = ar(ΔABC)

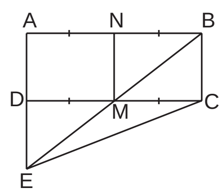
⇒ ar(ΔAPB) = ar(ΔADC) [Using equation (1)]

= 35 cm^2 [Using equation (2)]

Hence, ar(ΔAPB) = 35 cm^2.

27. (d) **12 cm^2**

The line divides the rectangle ABCD into two equal parallelograms NBCM and ANMD.



Then, ar(||gm ANMD) = ar(||gm NBCM)

$$\Rightarrow \quad \text{ar}(\|\text{gm } NBCM) = \frac{1}{2}\,\text{ar(rectangle } ABCD)$$

$$= \frac{1}{2} \times 48 \text{ cm}^2$$

$$= 24 \text{ cm}^2$$

Now, BM is the diagonal of parallelogram NBCM.

Then, ar(ΔNBM) = ar(ΔBMC)

ΔBMC and parallelogram NBCM are on the same base between the same parallels NB and MC.

$$\text{Thus,} \quad \frac{1}{2}\,\text{ar}(\|\text{gm } NBCM) = \text{ar}(\Delta BMC)$$

$$\Rightarrow \quad \frac{1}{2} \times 24 \text{ cm}^2 = \text{ar}(\Delta BMC)$$

$$\Rightarrow \quad \text{ar}(\Delta BMC) = 12 \text{ cm}^2$$

In triangles BMC and DME, we have

∠DME = ∠BMC [Vertically opposite angles]

MD = MC[M is the mid-point of CD]

∠MDE = ∠BCM [90° each]

By ASA congruence, ΔBMC ≃ ΔDME

Thus, ar(ΔBMC) = ar(ΔDME)

⇒ 12 cm^2 = ar(ΔDME)

Now, ME is the median of ΔDCE.

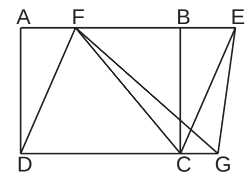
Then, ar(ΔEMC) = ar(ΔDME) = 12 cm^2.

Hence, ar(ΔEMC) = 12 cm^2.

28. (d) **12**

Joint FC.

ΔEFG and ΔEFC are on the same base EF and between the same parallels EF and CG.



Thus, ar(ΔEFG) = ar(ΔEFC) … (1)

Now, FC is the diagonal of parallelogram DCEF.

Then, ar(ΔFDC) = ar(ΔEFC) … (2)

From equations (1) and (2), we have

ar(ΔEFG) = ar(ΔFDC) … (3)

Now, parallelogram ABCD and parallelogram DCEF are on the same base DC and between the same parallels DC and AE.

Then, ar(||gm ABCD) = ar(||gm DCEF)

Also, ar(||gm ABCD)

= AB × AD = 8 units × 3 units

= 24 sq units

Thus, ar(||gm DCEF) = ar(||gm ABCD)

= 24 sq units.

In parallelogram DCEF, FC is the diagonal.

$$\text{Then,} \quad \text{ar}(\Delta FDC) = \frac{1}{2}\,\text{ar}(\|\text{gm } DCEF)$$

$$= \frac{1}{2} \times 24 \text{ sq units} = 12 \text{ sq units.}$$

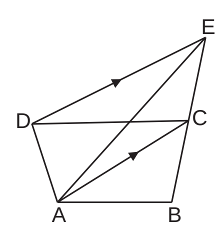
Using equation (3),

ar(ΔEFG) = ar(ΔFDC) = 12 sq units.

29. (b) **36 cm^2**

Now, DE || AC.

ΔADC and ΔACE are on the same base AC and between same parallels AC and DE.



Then, ar(ΔADC) = ar(ΔACE)

Also, the area of quadrilateral ABCD is

$$\text{ar(quad ABCD)} = \text{ar}(\Delta ADC) + \text{ar}(\Delta ABC)$$

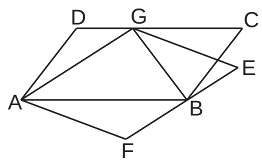
$\Rightarrow$ $$\text{ar(quad ABCD)} = \text{ar}(\Delta ACE) + \text{ar}(\Delta ABC)$$

$$= \text{ar}(\Delta ABE) = 36 \text{ cm}^2.$$

[Given ar(ΔABE) = 36 cm^2]

30. (a) **13.5 cm^2**

Now, ΔABG and parallelogram AGEF are on the same base AG and between parallels AG and FE.



Then, $\text{ar}(\Delta ABG) = \dfrac{1}{2}\text{ar}(\|\text{gm AGEF})$

$$= \frac{1}{2} \times 27 \text{ cm}^2 = \frac{27}{2} \text{ cm}^2$$

Also, ΔABG and parallelogram ABCD are on the same base AB and between same parallels AB and DC.

$$\text{ar}(\Delta ABG) = \frac{1}{2}\text{ar}(\|\text{gm ABCD})$$

$\Rightarrow$ $$\frac{27}{2}\text{ cm}^2 = \frac{1}{2}\text{ar}(\|\text{gm ABCD})$$

$\Rightarrow$ $\text{ar}(\|\text{gm ABCD}) = 27 \text{ cm}^2$

Now, $\text{ar}(\Delta ADG) + \text{ar}(\Delta GCB)$

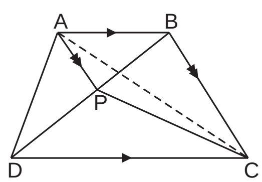
$$= \text{ar}(\|\text{gm ABCD}) - \text{ar}(\Delta ABG)$$

$$= 27 \text{ cm}^2 - \frac{27}{2} \text{ cm}^2$$

$$= 13.5 \text{ cm}^2$$

31. (a) **5 cm^2**

ΔABC and ΔBPC are on the same base BC and between same parallels BC and AP.



Thus, $\text{ar}(\Delta ABC) = \text{ar}(\Delta BPC)$

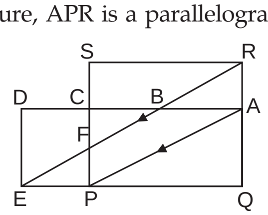
$= 5 \text{ cm}^2$ [ar(ΔBPC) = 5 cm^2]

Also, ΔABD and ΔABC are on the same base AB and between same parallels AB and DC.

Thus, $\text{ar}(\Delta ABD) = \text{ar}(\Delta ABC) = 5 \text{ cm}^2$.

32. (c) **35 cm^2**

In the figure, APR is a parallelogram.



Now, rectangle ARSC and parallelogram APFR are on the same base RA and between same parallels RA and SP.

Thus, ar(ARSC) = ar($\|$gm APFR)

Also, parallelogram ABEP and parallelogram APFR are on the same base AP and between same parallels AP and RE.

ar($\|$gm ABEP) = ar($\|$gm APFR)

$\Rightarrow$ ar($\|$gm ABEP) = ar(ARSC)

$\Rightarrow$ 10 cm^2 = ar(ARSC)

Now, ar(PQRS) = ar(ACPQ) + ar(ARSC)

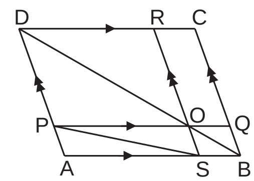
$= 25 \text{ cm}^2 + 10 \text{ cm}^2$

[ar(ACPQ) = 25 cm^2]

$= 35 \text{ cm}^2$

33. (c) **46 cm^2**

PS is the diagonal of parallelogram POSA.



Thus, $\text{ar}(\Delta POS) = \text{ar}(\Delta APS)$

Now, ar($\|$gm POSA) = $\text{ar}(\Delta POS) + \text{ar}(\Delta APS)$

$= 2\,\text{ar}(\Delta APS)$

$= 2 \times 6 \text{ cm}^2 = 12 \text{ cm}^2$

Then, $\text{ar}(\Delta ABD) = \text{ar}(\Delta DOP) + \text{ar}(\Delta BOS)$

+ ar($\|$gm POSA)

$= 8 \text{ cm}^2 + 3 \text{ cm}^2 + 12 \text{ cm}^2$

$= 23 \text{ cm}^2$

In parallelogram ABCD, DB is the diagonal.

Then, $\text{ar}(\Delta ABD) = \text{ar}(\Delta DBC)$.

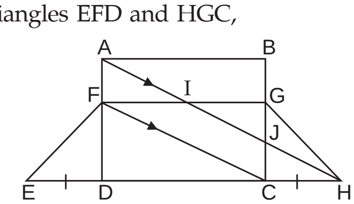
Thus, ar($\|$gm ABCD)

$= \text{ar}(\Delta ABD) + \text{ar}(\Delta DBC)$

$= 23 \text{ cm}^2 + 23 \text{ cm}^2 = 46 \text{ cm}^2$

34. (b) **26 cm^2**

In triangles EFD and HGC,



ED = CH [Given]

$\angle$FDE = $\angle$GCH [90° each]

FD = GC [FG $\|$ CD, AD $\|$ BC]

By SAS congruence,

$\Delta EFD \simeq \Delta HGC$

$\Rightarrow$ $\text{ar}(\Delta EFD) = \text{ar}(\Delta HGC)$ … (1)

Now, AF $\|$ BG and AF = BG.

Also, AB $\|$ FG and AB = FG.

$\therefore$ ABGF is a parallelogram

Also, I is the point on AH, IH $\|$ FC

and FI $\|$ CH and FI = CH

∴ FIHC is a parallelogram.

Similarly, AJCF is also a parallelogram.

Now, ΔHGC and parallelogram FIHC are on the same base CH and between the same parallels CH and FG.

$\therefore \quad ar(\Delta HGC) = \frac{1}{2} ar(\|gm\ FIHC)$

$\Rightarrow \quad ar(\|gm\ FIHC) = 2\ ar(\Delta HGC)$

$= ar(\Delta HGC) + ar(\Delta HGC)$

$= ar(\Delta HGC) + ar(\Delta EFD)$

… (2) [Using equation (1)]

Parallelogram AJCF and parallelogram FIHC are on the same base FC and between the same parallels FC and AH.

$\therefore \quad ar(\|gm\ AJCF) = ar(\|gm\ FIHC)$

$= ar(\Delta HGC) + ar(\Delta EFD)$ … (3)

[Using equation (2)]

Again, parallelogram ABGF and parallelogram AJCF are on the same base AF and between the same parallels AF and BC.

$\therefore \quad ar(\|gm\ ABGF) = ar(\|gm\ AJCF)$

$= ar(\Delta HGC) + ar(\Delta EFD)$ … (4)

[Using equation 3]

Now, ar(trap EFGH)

$= ar(\Delta HGC) + ar(\Delta EFD) + ar(\|gm\ FGCD)$

$= ar(\|gm\ ABGF) + ar(\|gm\ FGCD)$

[Using equation (4)]

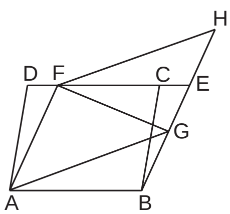
$= ar(rect\ ABCD)$

$= 26\ cm^2$ [∴ ar(rectangle ABCD) = $26\ cm^2$].

Hence, ar(trap EFGH) = $26\ cm^2$.

35. (c) **11.5 cm²**

Now, parallelogram ABCD and parallelogram ABEF are on the same base AB and between same parallels AB and DE.



Thus, ar(||gm ABCD)

$= ar(\|gm\ ABEF)$

Also, parallelogram ABEF and parallelogram AGHF are on the same base AF and between same parallels AF and BH.

Thus, $\quad ar(\|gm\ ABEF) = ar(\|gm\ AGHF)$

$\Rightarrow \quad ar(\|gm\ AGHF) = ar(\|gm\ ABCD)$

Now, ΔFGH and parallelogram AGHF are on the same base GH and between same parallels AF and GH.

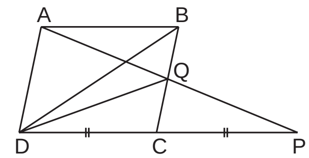
Thus, $ar(\Delta FGH) = \frac{1}{2} ar(\|gm\ AGHF)$

$= \frac{1}{2} ar(\|gm\ ABCD)$

$= \frac{1}{2} \times 23\ cm^2 = 11.5\ cm^2.$

36. (b) **12 cm²**

ABCD is a parallelogram.



Thus, AB = DC

Also, DC = CP

$\Rightarrow$ AB = CP

In ΔABQ and ΔQCP,

AB = CP

∠AQB = ∠CQP[Vertically opposite angles]

∠ABQ = ∠QCP.

By AAS congruence,

ΔABQ ≅ ΔQCP

$\therefore \quad ar(\Delta ABQ) = ar(\Delta QCP)$

Now, ΔBQD and ΔABQ are on the same base BQ and between same parallels BQ and AD.

Thus, $ar(\Delta BQD) = ar(\Delta ABQ)$

$\Rightarrow \quad ar(\Delta ABQ) = 3\ cm^2 \qquad [ar(\Delta BQD) = 3\ cm^2]$

Also, C is the mid-point of DP. Thus, QC is the median of ΔDQP.

Thus, $ar(\Delta QCP) = ar(\Delta QDC)$

$\Rightarrow \quad ar(\Delta ABQ) = ar(\Delta QDC)$

$\Rightarrow \quad ar(\Delta QDC) = 3\ cm^2$

Now, $ar(\Delta BDC) = ar(\Delta BQD) + ar(\Delta QDC)$

$= 3\ cm^2 + 3\ cm^2 = 6\ cm^2$

DB is the diagonal of the parallelogram ABCD.

Then, $\quad ar(\Delta BDC) = ar(\Delta ADB)$

Thus, $\quad ar(\|gm\ ABCD) = ar(\Delta BDC) + ar(\Delta ADB)$

$= 2\ ar(\Delta BDC)$

$= 2 \times 6\ cm^2$

$= 12\ cm^2$

Hence, ar(||gm ABCD) = $12\ cm^2$.

37. (d) **12.5 cm²**

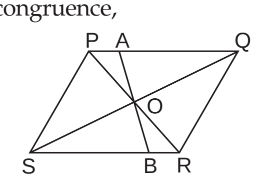
In triangles POQ and SOR, we have

∠PQO = ∠RSO [alternate opposite angles]

PQ = SR

∠QPO = ∠SRO [alternate opposite angles]

By ASA congruence,



ΔPOQ ≅ ΔSOR

$\Rightarrow$ PO = RO

In triangles POA and BOR, we have

$\angle APO = \angle BRD$ [alternate opposite angles]

$PO = RO$

$\angle POA = \angle BOR$[Vertically opposite angles]

By ASA congruence,

$\Delta POA \cong \Delta BOR$

$\Rightarrow \quad ar(\Delta POA) = ar(\Delta BOR) \quad \ldots (1)$

Now, parallelogram PQRS and ΔPRS are on the same base SR and between the same parallels SR and PQ.

Then, $\quad ar(\|gm\ PQRS) = 2\ ar(\Delta PSR) \quad \ldots (2)$

In parallelogram PQRS, the diagonal PR and SQ bisect each other. Thus, O is the mid-point of PR. In ΔPSR, OS is the median.

$\therefore \quad ar(\Delta PSR) = 2\ ar(\Delta POS)$

Also, $arA(\Delta PSR) = 2\ ar(\Delta SOR) \quad [\because \Delta SOR = \Delta POS]$

In equation (2), we have

$ar(\|gm\ PQRS) = 2\ ar(\Delta PSR) = 2 \times 2\ ar(\Delta POS)$

$= 4\ ar(\Delta POS)$

$\Rightarrow \quad ar(\Delta POS) = \frac{1}{4} ar(\|gm\ PQRS)$

$= \frac{1}{4} \times 25\ cm^2 = \frac{25}{4}\ cm^2$

Similarly, $ar(\Delta SOR) = \frac{25}{4}\ cm^2$.

Now, ar(quad SBAP)

$= ar(\Delta POS) + ar(\Delta SBO) + ar(\Delta POA)$

$= ar(\Delta POS) + ar(\Delta SBO) + ar(\Delta BOR)$

[Using equation (1)]

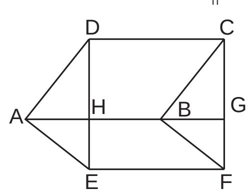
$= ar(\Delta POS) + ar(\Delta SOR)$

$= \frac{25}{4}\ cm^2 + \frac{25}{4}\ cm^2 = 12.5\ cm^2$

Hence, ar(quad SBAP) = $12.5\ cm^2$.

38. (b) **42 cm^2**

Draw $AG \perp CF$ such that $AG \parallel DC$ and EF.



Now, parallelogram ABCD and quadrilateral DCGH are on the same base CD and between same parallels DC and AG.

Thus, $\quad ar(\|gm\ ABCD) = ar(quad\ \Delta CGH) \quad \ldots (1)$

and parallelogram ABFE and quad HGFE are on the same base EF and between same parallels EF and AG.

Thus, $\quad ar(\|gm\ ABFE) = ar(quad\ HGFE) \quad \ldots (2)$

Adding equations (1) and (2),

$ar(\|gm\ ABCD) + ar(\|gm\ ABFE)$

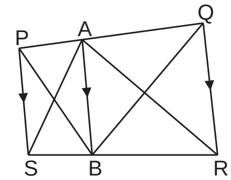
$= ar(quad\ DCGH) + ar(quad\ HGFE)$

$\Rightarrow \quad 24\ cm^2 + 18\ cm^2 = ar(quad\ EFCD)$

$\Rightarrow \quad ar(quad\ EFCD) = 42\ cm^2$.

39. (a) **17 cm^2**

Now, $SP \parallel BA$. ΔABP and ΔASB are on the same base AB and between same parallels AB and PS.



Thus, $\quad ar(\Delta ABP) = ar(\Delta ASB) \quad \ldots (1)$

Also, ΔABQ and ΔABR are on the same base AB and between same parallels AB and QR.

Then, $\quad ar(\Delta ABQ) = ar(\Delta ABR) \quad \ldots (2)$

Adding equations (1) and (2), we have

$ar(\Delta ABP) + ar(\Delta ABQ) = ar(\Delta ASB) + ar(\Delta ABR)$

$\Rightarrow \quad ar(\Delta PBQ) = ar(\Delta ASR)$

$\Rightarrow \quad 17\ cm^2 = ar(\Delta ASR)$

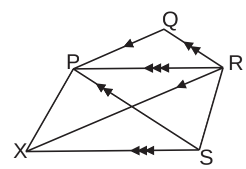
[Given $ar(\Delta PBQ) = 17\ cm^2$]

Hence, $ar(\Delta ASR) = 17\ cm^2$.

40. (c) **14 cm^2**

Now, $\quad PR \parallel XS$.

Then, ΔPXR and ΔPSR are on the same base PR and between same parallels PR and XS.



Thus, $\quad ar(\Delta PXR) = ar(\Delta PSR) \quad \ldots (1)$

Now, $ar(trap\ PQRS) - ar(\Delta PQR)$

$= ar(\Delta PSR)$

$\Rightarrow 22\ cm^2 - 8\ cm^2 = ar(\Delta PXR) \quad$ [Using equation (1)]

$\Rightarrow \quad ar(\Delta PXR) = 14\ cm^2$

Hence, $\quad ar(\Delta PXR) = 14\ cm^2$.

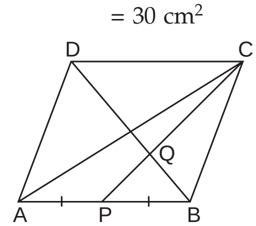
41. (b) **160 cm^2**

In ΔCPB, $\quad CQ : QP = 3 : 1$.

Then, $\quad ar(\Delta BQP) = 3\ ar(\Delta BQC)$

$\Rightarrow \quad ar(\Delta BQP) = 3 \times 10\ cm^2$ [$ar(\Delta BQC) = 10\ cm^2$]

$= 30\ cm^2$



Now, $\quad ar(\Delta BCP) = ar(\Delta BQP) + ar(\Delta BQC)$

$= 10\ cm^2 + 30\ cm^2 = 40\ cm^2$

In ΔABC, P is the mid-point of AB. Thus, PC is the median of ΔABC.

Then, $\quad ar(\Delta BCP) = \frac{1}{2} ar(\Delta ABC)$

$\Rightarrow \quad ar(\Delta ABC) = 2\ ar(\Delta BCP)$

$= 2 \times 40\ cm^2 = 80\ cm^2$

In parallelogram ABCD, AC is the diagonal.

Then, $\quad ar(\|gm\ ABCD) = 2 \times ar(\Delta ABC)$

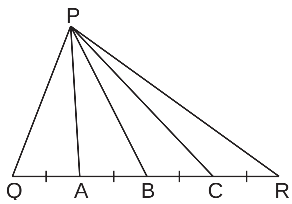
$= 2 \times 80\ cm^2$

$= 160\ cm^2$

Hence, $\quad ar(\|gm\ ABCD) = 160\ cm^2$.

42. (a) **18 cm²**

It is given that QA = AB. Thus, A is the mid-point of QB.



Thus, $\quad ar(\Delta PQA) = ar(\Delta PAB) \quad \ldots (1)$

Similarly, PB is the median of ΔPAC,

$ar(\Delta PAB) = ar(\Delta PBC) \quad \ldots (2)$

and CP is the median of ΔBPR,

$ar(\Delta PBC) = ar(\Delta PCR) \quad \ldots (3)$

Also, we have

$QA + AB = BC + CR$

$\Rightarrow \quad QB = BR$

This shows that B is the mid-point of QR and PB is the median of ΔPQR.

Thus, $\quad ar(\Delta PQB) = ar(\Delta PBR) \quad \ldots (4)$

Now, $\quad ar(\Delta PQR) = ar(\Delta PQB) + ar(\Delta PBR)$

$\Rightarrow \quad \frac{1}{2} ar(\Delta PQR) = ar(\Delta PQB)$ [Using equation (4)]

$\Rightarrow \quad \frac{1}{2} ar(\Delta PQR) = 2\ ar(\Delta PAB)$

[Using equation (1)]

$\Rightarrow \quad ar(\Delta PAB) = \frac{1}{4} ar(\Delta PQR) = \frac{1}{4} \times 24\ cm^2$

[Given, $ar(\Delta PQR) = 24\ cm^2$]

$= 6\ cm^2 \quad \ldots (5)$

Using equation (5) in equations (1), (2) and (3)

$ar(\Delta PAB) = ar(\Delta PBC) = ar(\Delta PCR) = 6\ cm^2$

Now, $\quad ar(\Delta PAR) = ar(\Delta PAB) + ar(\Delta PBC) + ar(\Delta PCR)$

$= 6\ cm^2 + 6\ cm^2 + 6\ cm^2$

$= 18\ cm^2$

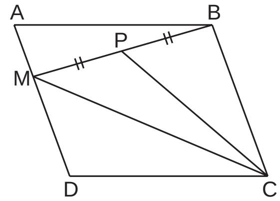
Hence, $\quad ar(\Delta PAR) = 18\ cm^2$.

43. (c) **7 cm²**

In parallelogram ABCD, AD ∥ BC.

ΔBMC and parallelogram ABCD are on the same base BC and between same parallels AD and BC.

$ar(\Delta BMC) = \frac{1}{2} ar(\|gm\ ABCD)$



$= \frac{1}{2} \times 28\ cm^2$ [$ar(\|gm\ ABCD) = 28\ cm^2$]

$= 14\ cm^2 \quad \ldots (1)$

Now, P is the mid-point of BM. Thus, PC is the median of ΔBMC.

Then, $\quad ar(\Delta CBP) = ar(\Delta MPC) \quad \ldots (2)$

The area of ΔBMC is

$ar(\Delta BMC) = ar(\Delta MPC) + ar(\Delta CBP)$

$\Rightarrow \quad ar(\Delta BMC) = 2\ ar(\Delta MPC)$ [Using equation (2)]

$\Rightarrow \quad ar(\Delta MPC) = \frac{1}{2} ar(\Delta BMC)$ [Using equation (1)]

$= \frac{1}{2} \times 14\ cm^2 = 7\ cm^2$

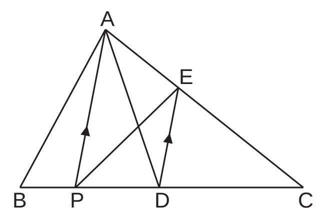
Hence, $ar(\Delta MPC) = 7\ cm^2$.

44. (d) **6 cm²**

Join AD.

Now, $\quad AP \parallel DE$.

ΔADE and ΔPDE are on the same base ED and between same parallels AP and DE.



Thus, $\quad ar(\Delta ADE) = ar(\Delta PDE) \quad \ldots (1)$

In ΔABC, D is the mid-point of BC. Thus, AD is the median of ΔABC.

Then, $\quad ar(\Delta ABD) = ar(\Delta ADC) \quad \ldots (2)$

Now, $\quad ar(\Delta ABC) = ar(\Delta ABD) + ar(\Delta ADC)$

$= 2\ ar(\Delta ADC)$ [Using equation (2)]

$\Rightarrow \quad ar(\Delta ADC) = \frac{1}{2} ar(\Delta ABC)$

$= \frac{1}{2} \times 12\ cm^2$

[$\because ar(\Delta ABC) = 12\ cm^2$]

$= 6\ cm^2$

Also, $\quad ar(\Delta ADC) = ar(\Delta ADE) + ar(\Delta EDC)$

$\Rightarrow \quad 6\ cm^2 = ar(\Delta PDE) + ar(\Delta EDC)$

[Using equation (1)]

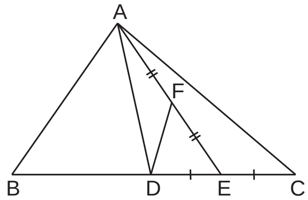
$\Rightarrow \quad 6\ cm^2 = ar(\Delta EPC)$

Hence, $ar(\Delta EPC) = 6\ cm^2$.

45. (a) **2 cm²**

In ΔABC, D is the mid-point of BC and AD is the median.

Then, $\quad ar(\Delta ABD) = ar(\Delta ADC)$



Also, ar(ΔABC) = ar(ΔABD) + ar(ΔADC)

⇒ 16 cm^2 = 2 ar(ΔADC)

⇒ ar(ΔADC) = 8 cm^2

E is the mid-point of DC. Then, AE is the median of ΔADC.

Thus, ar(ΔADE) = ar(ΔAEC)

In ΔADC,

ar(ΔADC) = ar(ΔADE) + ar(ΔAEC)

⇒ 8 cm^2 = 2 ar(ΔADE)

⇒ ar(ΔADE) = 4 cm^2

In ΔADE, F is the mid-point of AE and DF is the median.

Then, ar(ΔADF) = ar(ΔDEF)

Now, ar(ΔADE) = ar(ΔADF) + ar(ΔDEF)

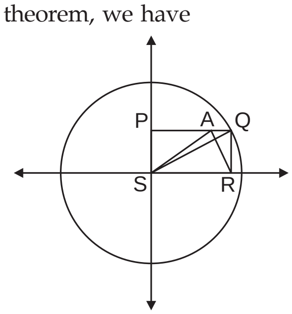
⇒ 4 cm^2 = 2 ar(ΔDEF)

⇒ ar(ΔDEF) = 2 cm^2

Hence, ar(ΔDEF) = 2 cm^2.

SHORT ANSWER QUESTIONS

1. In the figure, QRS is a right angled triangle. Using Pythagoras theorem, we have



$$SR = \sqrt{SQ^2 - QR^2} = \sqrt{SQ^2 - PS^2}$$

$$= \sqrt{(13\ cm)^2 - (5\ cm)^2}$$

$$= \sqrt{144}\ cm = 12\ cm$$

Now, area of the rectangle

PQRS = SR × PS

= 5 cm × 12 cm = 60 cm^2.

PQ ∥ SR. ΔRAS and rectangle PQRS are on the same base SR and between same parallels PQ and SR.

Thus, $ar(\Delta RAS) = \frac{1}{2}(PQRS)$

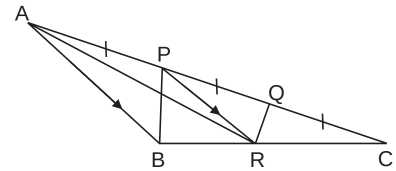
$= \frac{1}{2} \times 60\ cm^2 = 30\ cm^2$.

Hence, ar(ΔRAS) = 30 cm^2.

2. In ΔARQ,

ar(ΔARQ) = ar(ΔAPR) + ar(ΔPRQ) … (1)

Now, AB ∥ PR. ΔAPR and ΔBPR are on the same base PR and between same parallels PR and AB.



Then, ar(ΔAPR) = ar(ΔBPR) … (2)

Using equation (2) in equation (1),

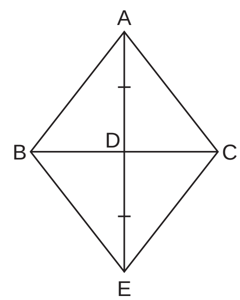
ar(ΔARQ) = ar(ΔAPR) + ar(ΔPRQ)

= ar(ΔBPR) + ar(ΔPRQ)

= ar(quad BRQP).

Hence, ar(ΔARQ) = ar(quad BRQP).

3. It is given that AD = DE. Thus, D is the mid-point of AE.



Now, BD is the median of ΔABE.

Thus, ar(ΔABD) = ar(ΔBDE) … (1)

DC is the median of ΔAEC.

Then, ar(ΔADC) = ar(ΔEDC) … (2)

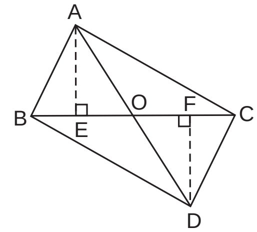
Adding equations (1) and (2), we have

ar(ΔABD) + ar(ΔADC) = ar(ΔBDE) + ar(ΔEDC)

⇒ ar(ΔABC) = ar(ΔBCE)

Hence, ar(ΔBCE) = ar(ΔABC).

4. It is given that ar(ΔABC) = ar(ΔDBC). Thus, the diagonal BC of the quadrilateral ABCD divides triangles ABC and DBC into equal areas. Thus quadrilateral ABDC is a parallelogram.



In parallelogram ABCD,

AC ∥ BD

and AC = BD

Also, AB ∥ CD

and AB = CD

In triangles AOC and BOD,

∠AOC = ∠BOD [Vertically opposite angles]

∠OAC = ∠ODB [opposite alternate angles]

AC = BD

By AAS congruence,

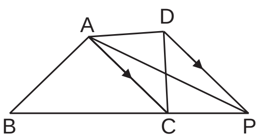
ΔAOC ≅ ΔBOD

Then, AO = DO and BO = CO.

Thus, O is the mid-point of AD and BC.

Hence, BC is bisects AD.

5. In the figure, AC ∥ DP.



ΔACP and ΔADC are on the same base AC and between same parallels AC and DP.

Thus, ar(ΔACP) = ar(ΔADC) … (1)

Now, ar(quad ABCD)

= ar(ΔABC) + ar(ΔADC)

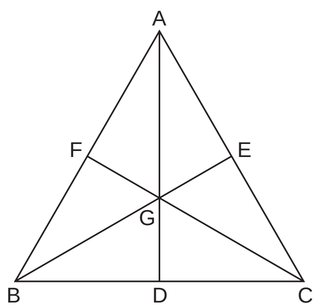
= ar(ΔABC) + ar(ΔACP) [Using equation (1)]

= ar(ΔABP)

Hence, ar(ΔABP) = ar(quad ABCD).

VALUE-BASED QUESTIONS

1. (i) Let ABC represent a triangular badge. We know that median of a triangle divides it into two triangles of equal areas.



In ΔABC, AD is a median

∴ ar(ΔABD) = ar(ΔACD) … (1)

In ΔGBC, GD is a median

∴ ar(ΔGBD) = ar(ΔGCD) … (2)

Subtracting (2) from (1), we get

ar(ΔABD) – ar(ΔGBD)

= ar(ΔACD) – ar(ΔGCD)

⇒ ar(ΔAGB) = ar(ΔAGC)

Similarlyar(ΔAGB) = ar(ΔBGC)

∴ ar(ΔAGB) = ar(ΔAGC) = ar(ΔBGC) … (3)

Also, ar(ΔABC) = ar(ΔAGB) + ar(ΔAGC) + ar(ΔBGC)

⇒ ar(ΔABC) = 3 ar(ΔAGB) [Using (3)]

⇒ ar(ΔAGB) = $\frac{1}{3}$ ar(ΔABC) … (4)

Hence, ar(ΔAGB) = ar(ΔAGC) = ar(ΔBGC)

= $\frac{1}{3}$ ar(ΔABC) [Using (3) and (4)]

(ii) Environmental awareness and leadership.

MATCH THE FOLLOWING

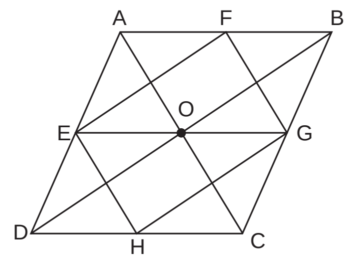
(i) (c)
(ii) (d)
(iii) (a)
(iv) (e)
(v) (b)

UNIT TEST

Multiple-Choice Questions

1. (b) **48 cm^2**

Consider ABCD is the given rhombus. E, F, G, H are the mid-points of AD, AB, BC and CD respectively.



Now, area of the rhombus

= $\frac{1}{2}$ × product of the diagonals.

= $\frac{1}{2}$ × AC × BD = $\frac{1}{2}$ × 16 cm × 12 cm

= 96 cm^2

Join EG parallel to both AB and DC.

AB ∥ EG, ΔEFG and parallelogram ABGE lie on the same base EG and between same parallels AB and EG.

Then, ar(ΔEFG) = $\frac{1}{2}$ ar(∥gm ABGE)

= $\frac{1}{2}$ × 48 cm^2

[∥gm ABGE = $\frac{1}{2}$ ∥gm ABCD]

= 24 cm^2

Similarly, ar(ΔEGH) = 24 cm^2

Then, ar(quad EFGH) = ar(ΔEFG) + ar(ΔEGH)

= 24 cm^2 + 24 cm^2

= 48 cm^2

2. (a) **A rhombus of area 24 cm^2**

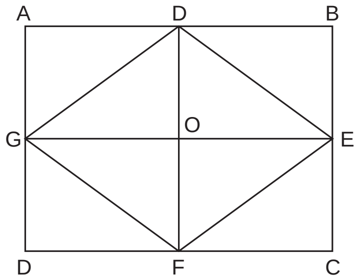
Let ABCD be a rectangle. D, E, F, G are the mid-points of AB, BC, CD and AD.

Now, AB = 8 cm and AD = 6 cm.

Join GD, DE, EF and FG. Now, DEFG is a quadrilateral. Then, diagonals DF and GE intersect at O.

Also, ABEG is a rectangle.

Now, ΔGDE and rectangle ABEG are on the same base GE and between the same parallels AB and GE.



$$\text{ar}(\Delta GDE) = \frac{1}{2}\text{ar}(ABEG) \quad \dots (1)$$

Similarly, $\text{ar}(\Delta GEF) = \frac{1}{2}\text{ar}(CDGE) \quad \dots (2)$

Adding equations (1) and (2),

$$\text{ar}(\Delta GDE) + \text{ar}(\Delta GEF) = \frac{1}{2}\text{ar}(ABEG) + \frac{1}{2}\text{ar}(CDGE)$$

$$\Rightarrow \quad \text{ar}(DEFG) = \frac{1}{2}\text{ar}(ABCD)$$

$$= \frac{1}{2}(AB \times AD)$$

$$= \frac{1}{2} \times 8 \text{ cm} \times 6 \text{ m}$$

$$= 24 \text{ cm}^2$$

Now, DF ∥ BC and DE is a transversal.

Then, $\angle DEB = \angle ODE \quad \dots (3)$

In ΔDOE, are have

$$\angle DEO + \angle DOE + \angle ODE = 180° \quad \dots (4)$$

and $\angle BEO = 90°$

$\Rightarrow \quad \angle DEB + \angle DEO = 90°$ [Using equation (3)]

$\Rightarrow \quad \angle ODE + \angle DEO = 90° \quad \dots (5)$

Using equation (5) in equation (4), we have

$$\angle DOE + 90° = 180°$$

$\Rightarrow \quad \angle DOE = 90°$

This shows that DF ⊥ GE.

In triangles ADG and DBE,

AD = BD

∠GAD = ∠DBE [90° each]

AG = BE

By SAS congruence, ΔADG ≅ ΔDBE

⇒ GD = DE

Similarly, DG = GF, GF = FE, FE = ED

Now, GD = GF = FE = ED and DF ⊥ GE.

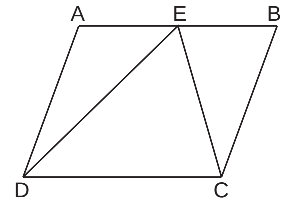
Hence, DEFG is a rhombus with area equal to 24 cm^2.

3. (b) **1 : 2**

Now, ABCD is the parallelogram and ΔDEC is the triangle having same base DC and between same parallels AB and DC.

Then, $\text{ar}(\Delta DEC) = \frac{1}{2}\text{ar}(\|\text{gm } ABCD)$

$$\Rightarrow \quad \frac{\text{ar}(\Delta DEC)}{\text{ar}(\|\text{gm } ABCD)} = \frac{1}{2}$$

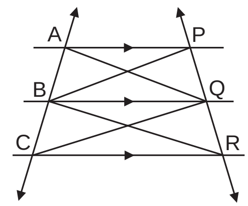


Thus, ar(ΔDEC) : ar(∥gm ABCD) = 1 : 2

Hence, the ratio of the area of the triangle to the area of the parallelogram is 1 : 2.

4. (b) **17 cm^2**

Since ΔAQB are ΔPBQ are on the same base BQ and between the same parallels AP and BQ,



∴ ar(ΔAQB) = ar(ΔPBQ) … (2)

Also, ΔBQC and ΔQCR are on the same base BQ and between the same parallels BQ and CR.

Then, ar(ΔBQC) = ar(ΔBQR) … (2)

Adding equations (1) and (2), we have

ar(ΔAQB) + ar(ΔBQC) = ar(ΔPBQ) + ar(ΔBQR)

⇒ ar(ΔAQC) = ar(ΔPBR)

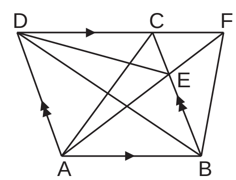
⇒ 17 cm^2 = ar(ΔPBR)

[Given : ar(ΔAQC) = 17 cm^2]

Hence, ar(ΔPBR) = 17 cm^2.

5. (a) **13 sq units**

Join BD and AC.



Now, AB ∥ CF. ΔACB and ΔAFB are on the same base AB and between same parallels AB and CF.

Then, ar(ΔACB) = ar(ΔAFB)

⇒ ar(ΔCAE) + ar(ΔAEB) = ar(ΔBEF) + ar(ΔAEB)

⇒ ar(ΔCAE) = ar(ΔBEF) … (1)

Also, AD ∥ BC. ΔCAE and ΔDCE are on the same base CE and between same parallels AD and EC.

Then, ar(ΔCAE) = ar(ΔDCE) … (2)

From equation (1) and (2), we have

ar(ΔBEF) = ar(ΔDCE) = 13 sq units

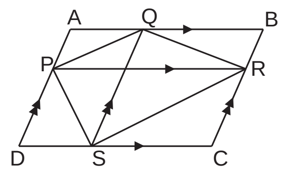
[ar(ΔDCE) = 13 sq units].

6. (d) **30 cm^2**

Now, AB ∥ PR.

Then, ABRP is also a parallelogram. Since ΔPQR and parallelogram ABRP are on the same base PR and between same parallels AB and PR,

$\therefore \quad ar(\Delta PQR) = \frac{1}{2} ar(\|gm\ ABRP) \quad \ldots (1)$



Also, ΔPSR and parallelogram PRCD are on the same base PR and between same parallels PR and DC.

Then, $ar(\Delta PSR) = \frac{1}{2} ar(\|gm\ PRCD) \quad \ldots (2)$

Adding equations (1) and (2), we have

$$ar(\Delta PQR) + ar(\Delta PSR)$$

$$= \frac{1}{2} arA(\|gm\ ABRP) + \frac{1}{2} ar(\|gm\ PRCD)$$

$$\Rightarrow \quad ar(quad\ PQRS) = \frac{1}{2} ar(\|gm\ ABCD)$$

$$\Rightarrow \quad ar(\|gm\ ABCD) = 2 \times ar(quad\ PQRS)$$

$$= 2 \times 15\ cm^2 = 30\ cm^2$$

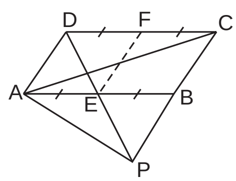
Hence, $ar(\|gm\ ABCD) = 30\ cm^2$.

7. (b) **2 : 1**

Join AC.

Now, AD is parallel to both EF and BC.

Thus, AEFD is a parallelogram having area equal to half the area of the parallelogram ABCD.



$$\therefore \quad ar(\|gm\ AEFD) = \frac{1}{2} ar(\|gm\ ABCD)$$

$$= \frac{1}{2} \times 36\ cm^2 = 18\ cm^2$$

Now, DE is the diagonal of parallelogram AEFD.

Then, $ar(\Delta DEF) = \frac{1}{2} ar(\|gm\ AEFD)$

$$= \frac{1}{2} \times 18\ cm^2 = 9\ cm^2 \quad \ldots (1)$$

Also, AD ∥ BC. ΔAPD and ΔACD are on the same base AD and between same parallels AD and CP.

Then, $ar(\Delta APD) = ar(\Delta ACD) \quad \ldots (2)$

Also, the diagonal divides parallelogram ABCD into two equal areas of ΔACD and ΔABC.

$$ar(\Delta ACD) = \frac{1}{2} (\|gm\ ABCD)$$

$$ar(\Delta APD) = \frac{1}{2} \times 36\ cm^2 \text{ [Using equation (2)]}$$

$$= 18\ cm^2 \quad \ldots (3)$$

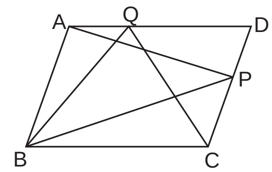
From equations (1) and (3), we have

$$\frac{ar(\Delta APD)}{ar(\Delta DEF)} = \frac{18\ cm^2}{9\ cm^2} = \frac{2}{1}$$

Hence, $ar(\Delta APD) : ar(\Delta DEF) = 2 : 1$.

Short Answer Questions

8. Now, ΔBQC and parallelogram ABCD are on the same base BC and between same parallels AD and BC.



Then, $ar(\Delta BQC) = \frac{1}{2} ar(\|gm\ ABCD) \quad \ldots (1)$

ΔAPB and parallelogram ABCD are on the same base AB and between same parallels AB and DC.

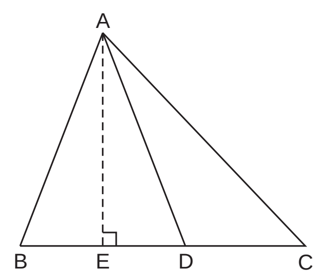
Then, $ar(\Delta APB) = \frac{1}{2} ar(\|gm\ ABCD) \quad \ldots (2)$

From equations (1) and (2),

$$ar(\Delta BQC) = ar(\Delta APB)$$

Hence, $ar(\Delta APB) = ar(\Delta BQC)$.

9. Let ABC be a triangle in which AD is a median. Draw AE ⊥ BC.



Since D is the mid-point of BC,

$\therefore \quad BD = DC \quad \ldots (1)$

Now, $ar(\Delta ABD) = \frac{1}{2} \times BD \times AE \quad \ldots (2)$

and $ar(\Delta ADC) = \frac{1}{2} \times DC \times AE \quad \ldots (3)$

From equations (1) and (2), we have

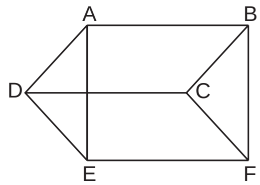
$$ar(\Delta ABD) = \frac{1}{2} \times DC \times AE \quad \ldots (4)$$

From equations (3) and (4), we have

$$ar(\Delta ABD) = ar(\Delta ADC)$$

Hence, a median of a triangle divides it into two triangles of equal area.

10.



In triangles ADE and BCF, we have

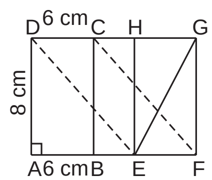
$$AE = BF \quad (\because \text{ ABFE is a } \|\text{gm})$$
$$AD = BC \quad (\because \text{ ABCD is a } \|\text{gm})$$
$$DE = CF \quad (\because \text{ DEFC is a } \|\text{gm})$$

By SSS congruence theorem, we have

$$\Delta ADE \cong \Delta BCF$$

Thus, $\quad ar(\Delta ADE) = ar(\Delta BCF)$.

11. Draw EH $\perp$ GD such that EH $\|$ AD and EH $\|$ GF.



Also, $\quad AD = EH = GF = 8$ cm

and $\quad AB = 6$ cm.

Now, $ar(\text{rect } ABCD) = AD \times AB = 8 \text{ cm} \times 6 \text{ cm} = 48 \text{ cm}^2$.

Also, $\quad CD \| EF$

$\quad DE \| CF$

Then, DCFE is a parallelogram.

Now, parallelogram DCFE and rectangle ABCD are on the same base CD and between same parallels DG and AF.

Thus, $ar(\|\text{gm } DCFE) = ar(\text{rect } ABCD)$

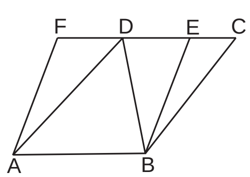
$\Rightarrow \quad ar(\|\text{gm } DCFE) = 48 \text{ cm}^2$

Since ΔGEF and parallelogram DCFE are on the same base EF and between same parallels DG and EF.

$$ar(\Delta GEF) = \frac{1}{2} ar(\|\text{gm } DCFE)$$
$$= \frac{1}{2} \times 48 \text{ cm}^2 = 24 \text{ cm}^2$$

Hence, $\quad ar(\Delta GEF) = 24 \text{ cm}^2$.

12. AB $\|$ FC. ΔADB and parallelogram ABEF are on the same base AB and between same parallels AB and EF.

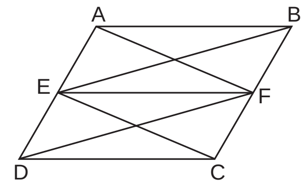


Then, $\quad ar(\Delta ADB) = \frac{1}{2} ar(\|\text{gm } ABEF)$

$$= \frac{1}{2} \times 96 \text{ cm}^2 = 48 \text{ cm}^2$$

Hence, $\quad ar(\Delta ADB) = 48 \text{ cm}^2$.

13. Let ABCD be the parallelogram. E and F are the mid-points of the opposite sides AD and BC. Join AF, DF, BE and CE.



Now, $\quad AE = BF$ and $AE \| BF$

$\quad AB \| EF$ and $AB = EF$

Thus, ABEF is a parallelogram. Similarly, EFCD is a parallelogram.

In ΔBEC, F is the mid-point of BC. Then, FE is the median of ΔBEC.

$\therefore \quad ar(\Delta BEF) = ar(\Delta CEF) \quad \ldots (1)$

In ΔAFD, E is the mid-point of AD. Then, EF is the median of ΔAFD.

$ar(\Delta AEF) = ar(\Delta DEF) \quad \ldots (2)$

Now, ΔAEF and ΔBAE are on the same base AE and between the same parallels AE and BF.

$ar(\Delta AEF) = ar(\Delta BAE) \quad \ldots (3)$

ΔDEF and ΔCED are on the same base DE and between the same parallels DE and FC.

$ar(\Delta DEF) = ar(\Delta CED) \quad \ldots (4)$

From equations (2) and (4), we have

$ar(\Delta AEF) = ar(\Delta CED) \quad \ldots (5)$

From equations (3) and (5),

$ar(\Delta BAE) = ar(\Delta CED) \quad \ldots (6)$

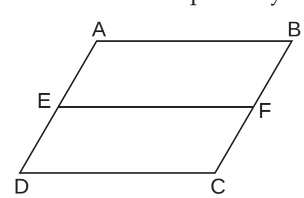
Adding equations (1) and (6), we have

$ar(\Delta BEF) + ar(\Delta BAE) = ar(\Delta CEF) + ar(\Delta CED)$

$\Rightarrow \quad ar(\|\text{gm } ABFE) = ar(\|\text{gm } EFCD)$

Hence, the line segment joining the mid-points of a pair of opposite sides of a parallelogram divides it into two equal parallelograms.

Alternative Method: ABCD is a parallelogram E and F are points on AD and BC respectively.



Now, $\quad AD = BC$

$\Rightarrow \quad \frac{1}{2} AD = \frac{1}{2} BC$

$\Rightarrow \quad AE = BF$

Also, $\quad AD \| BC$.

Thus, $\quad AE \| BF$

$\therefore$ ABFE is a parallelogram

Now, $\quad AD = BC$

$\Rightarrow \qquad \frac{1}{2} AD = \frac{1}{2} BC$

$\Rightarrow \qquad ED = FC$

Also, $\qquad ED \parallel FC.$

and $\qquad CD = EF$ and $CD \parallel EF.$

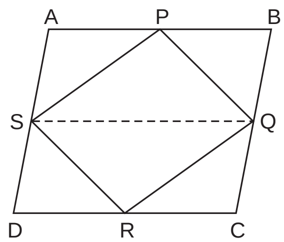
$\therefore$ EFCD is a parallelogram

Since parallelogram ABFE and parallelogram EFCD are on the same base EF and between the same parallels AB and CD,

$\therefore \qquad ar(\parallel gm\ ABFE) = ar(\parallel gm\ EFCD).$

Thus, the mid-points of a pair of opposite sides of a parallelogram divides it into two equal parallelograms.

14. Now, S and Q are the mid-points of AD and BC.



Thus, $\qquad AS = BQ$ and $AS \parallel BQ.$

Also, $\qquad AB \parallel SQ$ and $AB = SQ.$

Then, ABQS is a parallelogram.

ΔPSQ and parallelogram ABQS are on the same base SQ and between the same parallels AB and SQ.

Then, $\qquad ar(\Delta PSQ) = \frac{1}{2} ar(\parallel gm\ ABQS) \qquad \ldots (1)$

Similarly, SQCD is a parallelogram.

Then, ΔSQR and parallelogram SQCD are on the same base SQ and between the same parallels SQ and CD.

Then, $\qquad ar(\Delta SQR) = \frac{1}{2} ar(\parallel gm\ SQCD) \qquad \ldots (2)$

Adding equations (1) and (2), we have

$$ar(\Delta PSQ) + ar(\Delta SQR) = \frac{1}{2} ar(\parallel gm\ ABQS) + \frac{1}{2} ar(\parallel gm\ SQCD)$$

$$\Rightarrow \qquad ar(PQRS) = \frac{1}{2} \{ar(\parallel gm\ ABQS) + ar(\parallel gm\ SQCD)\}$$

$$= \frac{1}{2} ar(\parallel gm\ ABCD)$$

Hence, $\qquad ar(PQRS) = \frac{1}{2} ar(\parallel gm\ ABCD).$

Short Answer Questions

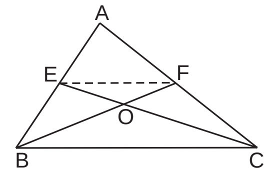
15. Now, E and F are the mid-points of the sides AB and AC of ΔABC.

Then, $\qquad EF \parallel BC.$

Since ΔEBC and ΔFBC are on the same base BC and between the same parallels EF and BC,

$\therefore \qquad ar(\Delta EBC) = ar(\Delta FBC)$

$\Rightarrow \quad ar(\Delta BEO) + ar(\Delta BOC) = ar(\Delta FOC) + ar(\Delta BOC)$



$\Rightarrow \qquad ar(\Delta BEO) = ar(\Delta FOC) \qquad \ldots (1)$

BF is the median of ΔABC.

Then, $\qquad ar(\Delta ABF) = ar(\Delta FBC)$

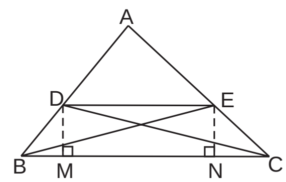
$\Rightarrow \quad ar(\Delta BEO) + ar(quad\ AEOF)$

$= ar(\Delta OBC) + ar(\Delta FOC)$

$\Rightarrow \quad ar(quad\ AEOF) = ar(\Delta OBC) \quad$ [Using equation (1)]

Hence, $\qquad ar(\Delta OBC) = ar(quadrilateral\ AEOF).$

16. Draw $DM \perp BC$ and $EN \perp BC$.



Now, $\qquad ar(\Delta BCD) = \frac{1}{2} \times DM \times BC$

and $\qquad ar(\Delta BCE) = \frac{1}{2} \times EN \times BC$

It is given that

$ar(\Delta BCE) = ar(\Delta BCD)$

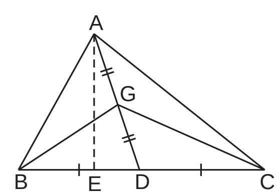
$\Rightarrow \quad \frac{1}{2} \times EN \times BC = \frac{1}{2} \times DM \times BC$

$\Rightarrow \qquad EN = DM$

This shows the lines DE and BC maintained a constant distance.

Hence, $\qquad DE \parallel BC.$

17. Draw $AE \perp BC$.



Now, BD = CD as D is the mid-point of BC.

$\therefore \qquad ar(\Delta ADB) = \frac{1}{2} \times AE \times BD$

$\therefore \qquad ar(\Delta ADC) = \frac{1}{2} \times AE \times CD$

$= \frac{1}{2} \times AE \times BD$ [Using BD = CD]

$= ar(\Delta ADB)$

Hence, $\qquad ar(\Delta ADB) = ar(\Delta ADC).$

This shows that the median of a triangle divides it into two triangles having equal areas.

In ΔBGC, GD is the median.

Then, $ar(\Delta BGD) = ar(\Delta DGC) = \frac{1}{2} ar(\Delta BGC)$

In ΔADC, GC is the median.

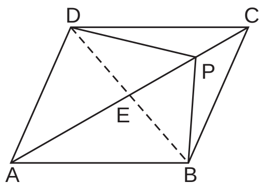
Then, $ar(\Delta AGC) = ar(\Delta DGC) = ar(\Delta BGD)$

$= \frac{1}{2} ar(\Delta BGC)$

$\Rightarrow \quad ar(\Delta BGC) = 2\, ar(\Delta AGC)$

Hence, $ar(\Delta BGC) = 2\, ar(\Delta AGC)$.

18. Join BD intersecting AC at E. In parallelogram ABCD, AC and BD bisect each other.



Thus, E is the mid-point of the AC and BD.

In ΔABD, AE is the median.

Then, $ar(\Delta AED) = ar(\Delta AEB)$ … (1)

In ΔDPB, EP is the median.

Then, $ar(\Delta DEP) = ar(\Delta BEP)$ … (2)

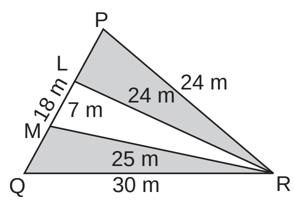
Adding equations (1) and (2), we have

$ar(\Delta AED) + ar(\Delta DEP) = ar(\Delta AEB) + ar(\Delta BEP)$

$\Rightarrow \quad ar(\Delta APD) = ar(\Delta APB)$

Hence, $ar(\Delta APB) = ar(\Delta APD)$.

19. In ΔLMR, we have



$$s_1 = \frac{LR + MR + LM}{2}$$

$$= \frac{24\text{ m} + 25\text{ m} + 7\text{ m}}{2} = 28\text{ m}$$

Now, $ar(\Delta LMR)$

$$= \sqrt{s_1(s_1 - LR)(s_1 - MR)(s_1 - LM)}$$

$$= \sqrt{28\text{ m}(28\text{ m} - 24\text{ m})(28\text{ m} - 25\text{ m})(28\text{ m} - 7\text{ m})}$$

$$= \sqrt{28\text{ m} \times 4\text{ m} \times 3\text{ m} \times 21\text{ m}}$$

$$= \sqrt{7056}\text{ m}^2$$

$$= 84\text{ m}^2$$

In ΔPQR,

$$s_2 = \frac{PR + QR + PQ}{2} = \frac{24\text{ m} + 30\text{ m} + 18\text{ m}}{2}$$

$$= 36\text{ m}$$

Now, $ar(\Delta PQR)$

$$= \sqrt{s_2(s_2 - PR)(s_2 - QR)(s_2 - PQ)}$$

$$= \sqrt{36\text{ m}(36\text{ m} - 24\text{ m})(36\text{ m} - 30\text{ m})(36 - 18\text{ m})}$$

$$= \sqrt{36\text{ m} \times 12\text{ m} \times 6\text{ m} \times 18\text{ m}} = \sqrt{46656}\text{ m}^2$$

$$= 216\text{ m}^2.$$

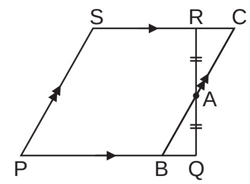
The area between the triangles is

$$ar(\Delta PQR) - ar(\Delta LMR) = 216\text{ m}^2 - 84\text{ m}^2$$

$$= 132\text{ m}^2$$

Hence, the area between the triangles is 132 m^2.

20. In triangles CAR and BAQ,



$\angle ACR = \angle QBA$ [Alternate opposite angles]

$\angle CAR = \angle BAQ$ [Vertically opposite angles]

$RA = QA$ [Given]

By AAS congruence,

$\Delta CAR \cong \Delta BAQ$

$\Rightarrow ar(\Delta CAR) = ar(\Delta BAQ)$

Now, ar(quad PBCS)

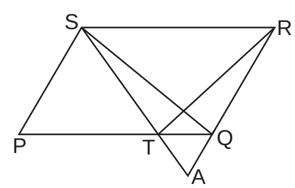
$= ar(PBARS) + ar(\Delta CAR)$

$= ar(PBARS) + ar(\Delta BAQ)$

$= ar(\text{trap } PQRS)$

Hence, ar(trap PQRS) = ar(quad PBCS).

21. In parallelogram PQRS, SR || PQ.



Now, triangles SQT and RTQ are on the same base QT and between same parallels SR and PQ.

Thus, $ar(\Delta SQT) = ar(\Delta RTQ)$

Adding $ar(\Delta ATQ)$ on both sides, we have

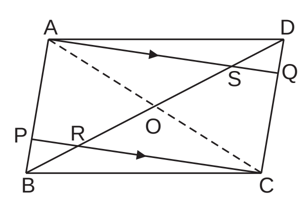
$ar(\Delta SQT) + ar(\Delta ATQ) = ar(\Delta RTQ) + ar(\Delta ATQ)$

$\Rightarrow \quad ar(\Delta ASQ) = ar(\Delta ATR)$

Hence, $ar(\Delta ASQ) = ar(\Delta ATR)$.

Long Answer Questions

22. Join AC. Since diagonal of a parallelogram divides it into two congruent triangles.



$\therefore \quad \Delta ABC \cong \Delta CDA$

$\Rightarrow \quad ar(\Delta ABC) = ar(\Delta CDA)$ [Congruent figures have equal areas] … (1)

and $\quad \Delta APC \cong \Delta CQA$

$\Rightarrow \quad ar(\Delta APC) = ar(\Delta CQA)$ [Congruent figures have equal areas] … (2)

Subtracting (2) from (1), we get

$$ar(\Delta ABC) - ar(\Delta APC) = ar(\Delta CDA) - ar(\Delta CQA)$$

$\Rightarrow \quad ar(\Delta PBC) = ar(\Delta QDA)$ … (3)

Now, $\angle DAB = \angle BCD$ [Opposite angles of a ||gm] … (4)

Also, $\angle QAP = \angle PCQ$ [Opposite angles of a ||gm] … (5)

Subtracting (5) from (4), we get

$$\angle DAB - \angle QAP = \angle BCD - \angle PCQ$$

$\Rightarrow \quad \angle DAQ = \angle BCP$

$\Rightarrow \quad \angle DAS = \angle BCR$ … (6)

In ΔCRB and ΔASD, we have

(i) $\angle CBR = \angle ADS$ [$\because \angle CBD = \angle ADB$, alternate angles, BC || AD]

(ii) $\angle BCR = \angle DAS$ [From (6)]

(iii) CB = AD [Opposite sides of ||gm ABCD]

$\therefore \Delta CRB \cong \Delta ASD$ [By AAS congruence]

$\Rightarrow ar(\Delta CRB) = ar(\Delta ASD)$ [Congruent figures have equal areas] … (7)

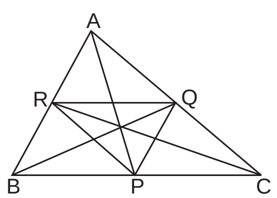
Subtracting (7) from (3), we get

$$ar(\Delta PBC) - ar(\Delta CRB) = ar(\Delta QDA) - ar(\Delta ASD)$$

$\Rightarrow \quad ar(\Delta PRB) = ar(\Delta QSD)$

Hence, $\quad ar(\Delta PRB) = ar(\Delta QSD)$.

23. Join AP. P is the mid-point of BC in ΔABC. Thus, AP is the median of ΔABC.



Then, $\quad ar(\Delta ABP) = ar(\Delta ACP) = \frac{1}{2} ar(\Delta ABC)$

In ΔABP, R is the mid-point of AB. Then, RP is the median of ΔABP.

$\therefore \quad ar(\Delta BRP) = ar(\Delta ARP)$

$\Rightarrow \quad ar(\Delta BRP) = ar(\Delta ARP) = \frac{1}{2} ar(\Delta ABP)$

$= \frac{1}{4} ar(\Delta ABC)$

Join RC. R is the mid-point of AB. Then, the median RC divides the triangles ARC and BRC into equal areas.

$\therefore \quad ar(\Delta ARC) = ar(\Delta BRC) = \frac{1}{2} ar(\Delta ABC)$

In ΔARC, RQ is the median which divides triangles ARQ and QRC into equal areas.

Then, $\quad ar(\Delta ARQ) = ar(\Delta QRC) = \frac{1}{2} ar(\Delta ARC)$

$= \frac{1}{4} ar(\Delta ABC)$.

Join BQ. BQ is the median of ΔABC.

Then, $\quad ar(\Delta ABQ) = ar(\Delta BQC) = \frac{1}{2} ar(\Delta ABC)$

Now, QP is the median of ΔBQC.

Then, $\quad ar(\Delta BQP) = ar(\Delta PQC) = \frac{1}{2} ar(\Delta BQC)$

$= \frac{1}{4} ar(\Delta ABC)$

In ΔABC, we have

$$ar(\Delta PQR) = ar(\Delta ABC) - [ar(\Delta BRP) + ar(\Delta ARQ) + ar(\Delta PQC)]$$

$$= ar(\Delta ABC) - \left[\frac{1}{4} ar(\Delta ABC) + \frac{1}{4} ar(\Delta ABC) + \frac{1}{4} ar(\Delta ABC)\right]$$

$$= ar(\Delta ABC) - \frac{3}{4} ar(\Delta ABC)$$

$$= \frac{1}{4} ar(\Delta ABC)$$

Hence, $ar(\Delta PQR) = \frac{1}{4} ar(\Delta ABC)$.