# CHAPTER 8 Quadrilaterals

## EXERCISE 8A

**1.** Let the measure of the fourth angle of the quadrilateral be $x$°. Then, $65 + 120 + 75 + x = 360$ [Sum of angles of a quadrilateral is 360°]

$\Rightarrow \quad 260 + x = 360$

$\Rightarrow \quad x = 100$

Hence, the measure of the fourth angle is 100°.

**2.** Let one of the angles of the quadrilateral be $(3x)°$, the remaining angles will be $(4x)°$, $(5x)°$ and $(6x)°$.

Then, $\quad 3x + 4x + 5x + 6x = 360$

[Sum of angles of quadrilateral is 360°]

$\Rightarrow \quad 18x = 360°$

$\Rightarrow \quad x = 20°$

Hence, the four angles of the quadrilateral are $(3 \times 20)°$, $(4 \times 20)°$, $(5 \times 20)°$, $(6 \times 20)°$ i.e. 60°, 80°, 100°, 120°.

**3.** Let the angles of the quadrilateral be $(2x)°$, $(3x)°$, $(5x)°$ and $(2x°)$.

Then, $\quad 2x + 3x + 5x + 2x = 360$

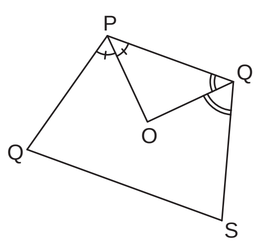
$\Rightarrow \quad 12x = 360$

$\Rightarrow \quad x = 30$

Hence, the four angles of the quadrilateral are $(2 \times 30)°$, $(3 \times 30)°$, $(5 \times 30)°$, $(2 \times 30°)$ i.e. 60°, 90°, 150°, 60°.

The given quadrilateral has one right angle.

**4.** In quadrilateral PQRS, we have




$$\angle P + \angle Q + \angle R + \angle S = 360°$$

[Sum of angles of a quadrilateral is 360°]

$$\Rightarrow \quad \frac{1}{2}\angle P + \frac{1}{2}\angle Q + \frac{1}{2}\angle R + \frac{1}{2}\angle S = 180°$$

$$\Rightarrow \quad \left(\frac{1}{2}\angle P + \frac{1}{2}\angle Q\right) + \frac{1}{2}(\angle R + \angle S) = 180°$$

$$\Rightarrow \quad (180° - \angle POQ) + \frac{1}{2}(\angle R + \angle S) = 180°$$

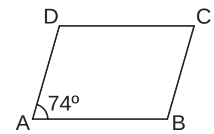
$$\left[\because \text{In } \Delta POQ, \frac{1}{2}\angle P + \frac{1}{2}\angle Q + \angle POQ = 180°\right]$$

$$\Rightarrow \quad \angle POQ = \frac{1}{2}(\angle R + \angle S)$$

## EXERCISE 8B

**1.** $\angle A + \angle B = 180°$ [Coint. angles, AD || BC]

$\Rightarrow \quad 74° + \angle B = 180°$



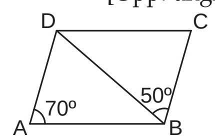

$\Rightarrow$ Since, the opposite angles of a parallelogram are equal

$\therefore \quad \angle C = \angle A = 74°$

and $\quad \angle D = \angle B = 106°$

Hence, $\quad \angle B = 106°$, $\angle C = 74°$ and $\angle D = 106°$.

**2.** $\angle C = \angle A = 70°$ [Opp. angles of a ||gm] …(1)




In $\Delta$CDB, we have

$\angle CDB + DBC + \angle BCD = 180°$ [Sum of angles of a $\Delta$]

$\Rightarrow \quad \angle CDB + 50° + 70° = 180°$ [Using (1)]

$\Rightarrow \quad \angle CDB = 180° - 50° - 70° = 60°$ …(2)

$\angle D + \angle A = 180°$ [Coint. angles DC || AB]

$\Rightarrow \quad \angle CDA + \angle DAB = 180°$

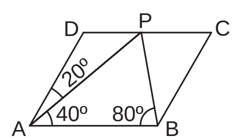
$\Rightarrow \quad (\angle ADB + \angle CDB) + \angle DAB = 180°$

$\Rightarrow \quad \angle ADB + 60° + 70° = 180°$ [Using (2)]

$\Rightarrow \quad \angle ADB = 180° - 60° - 70° = 50°$

Hence, $\angle CDB = 60°$ and $\angle ADB = 50°$.

3. $\quad \angle APD = \angle PAB$ [Alt. angles, DPC || AB]




$\Rightarrow \quad \angle APD = 40°$

Also $\quad \angle BPC = \angle ABP$ [Alt. angles, DPC || AB]

$\Rightarrow \quad \angle BPC = 80°$

Hence, $\angle APD = 40°$ and $\angle BPC = 80°$.

4. $\quad AB = AP + PB$

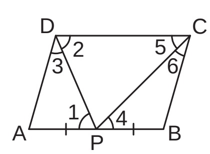
$\Rightarrow \quad AB = AP + AP$ [$\because$ P is the mid-point of AB]

$\Rightarrow \quad AB = 2AP$ …(1)

Also $\quad AB = 2AD$ [given] …(2)

$\therefore \quad AP = AD$ [Using (1) and (2)]

$\Rightarrow \quad \angle 3 = \angle 1$ [Angles opposite equal to sides of $\Delta$ADP] …(3)




Similarly, it can be proved that

$\angle 6 = \angle 4$ …(4)

$\angle 2 = \angle 1$ [Alt. angles, DC || AB] …(5)

$\angle 5 = \angle 4$ [Alt. angles, DC || AB] …(6)

From (3), (4), (5) and (6), we get

$\angle 1 = \angle 2 = \angle 3 = x$ (say) …(7)

and $\angle 4 = \angle 5 = \angle 6 = y$ (say)

Now $\angle CDA + \angle DCB = 180^\circ$ [Coint. angles, AD || BC]

$\Rightarrow (\angle 2 + \angle 3) + (\angle 5 + \angle 6) = 180^\circ$ [Using (7)]

$\Rightarrow 2x + 2y = 180^\circ$

$\Rightarrow x + y = 90^\circ$ …(8)

$\angle 1 + \angle CPD + \angle 4 = 180^\circ$ (straight angle)

$\Rightarrow x + \angle CPD + y = 180^\circ$

$\Rightarrow \angle CPD = 180^\circ - (x + y)$

$= 180^\circ - 90^\circ$ [Using (8)]

$= 90^\circ$

Hence, $\angle CPD = 90^\circ$.

**5.** Let $x^\circ$ and $y^\circ$ be two adjacent angles of a parallelogram, such that $x^\circ = \left(\frac{2}{3}y\right)^\circ$.

$x + y = 180^\circ$ [Coint. angles of a parallelogram]

$\Rightarrow \frac{2}{3}y + y = 180^\circ$

$\Rightarrow \frac{5y}{3} = 180^\circ$

$\Rightarrow y = 180^\circ \times \frac{3}{5} = 108^\circ$

$\therefore x = \left(\frac{2}{3}y\right) = 72$

$\therefore$ Two adjacent angles of the parallelogram are 72° and 108°. Since the opposite angles of a parallelogram are equal.

$\therefore$ The other two angles are 72° and 108°.

Hence, the angles of the parallelogram are

72°, 108°, 72°, 108°.

**6.** Let one of the consecutive angles of the parallelogram be $x^\circ$. Then, the other consecutive angle is $(5x)^\circ$.

Now, $x^\circ + 5x^\circ = 180^\circ$ [Coint. angles of a parallelogram]

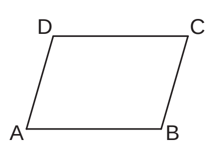
$\Rightarrow 6x = 180 \Rightarrow x = 30^\circ$

$\therefore$ So, the two consecutive angles of the parallelogram are 30° and (5 × 30)° i.e. 30° and 150°. Since, the opposite angles of a parallelogram are equal. Therefore, the other two angles of the parallelogram are 30° and 150°.

Hence, the angles of the parallelogram are

30°, 150°, 30°, 150°.

**7.** Let ABCD be a parallelogram in which measure of $\angle A$ is smallest. Let $m(\angle A) = x^\circ$




Then, $m(\angle B) = m(\angle D)$

$= (2x - 30)^\circ$.

$\angle A + \angle B = 180^\circ$ [Coint. angles, AD || BC]

$x + (2x - 30) = 180$

$\Rightarrow 3x - 30 = 180$

$\Rightarrow 3x = 210$

$\Rightarrow x = 70$

$\Rightarrow \angle A = 70^\circ$

$\angle B = \angle D = (2 \times 70^\circ - 30)^\circ = 110^\circ$

$\angle C = \angle A = 70^\circ$ [Opposite angles of a parallelogram]

Hence, the angles of the parallelogram are

70°, 110°, 70°, 110°.

**8.** $AD = BC = 2x$ [Opposite sides of parallelogram]

Perimeter (||gm ABCD) = 40 cm [given]

$\Rightarrow AB + BC + CD + DA = 40$ cm

$\Rightarrow 2y + 2 + 2x + 3y + 2x = 40$

$\Rightarrow 7x + 2y = 38$ …(1)

Also $AB = DC$ [Opposite sides of a ||gm]

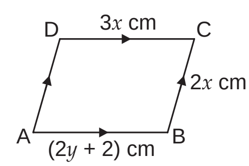
$\Rightarrow 2y + 2 = 3x$

$\Rightarrow 3x - 2y = 2$ …(2)

Adding eq. (1) and eq. (2), we get

$10x = 40 \Rightarrow x = 4$

Substituting $x = 4$ in Eq. (2), we get



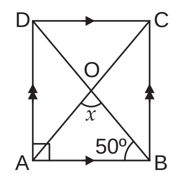

$3(4) - 2y = 2$

$\Rightarrow 2y = 12 - 2 = 10$

$\Rightarrow y = 5$

**9.** **(i)** We know the diagonals of a rectangle are equal, and they bisect each other.

$\therefore$ Diagonals AC and BD of rectangle ABCD are equal and they bisect each other at O.




Rectangle ABCD
(i)

$\Rightarrow OA = OB$

$\Rightarrow \angle OAB = \angle OBA = 50^\circ$

[Angles opposite to equal sides]

$\Rightarrow$ In $\Delta OAB$, we have,

$\angle AOB + \angle OAB + OBA$

$= 180^\circ$ [Sum of angles of a $\Delta$]

$\Rightarrow x + 50^\circ + 50^\circ = 180^\circ$

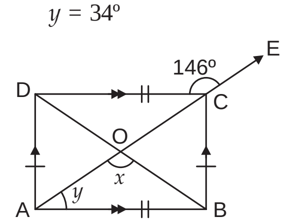
$\Rightarrow x = 80^\circ$

**(ii)** $\angle DCA = \angle CAB = y$ [Alt. angles, DC || AB]

$\angle DCA + \angle DCE = 180^\circ$ [Linear pair]

$\Rightarrow y + 146^\circ = 180^\circ$

$\Rightarrow y = 34^\circ$




Rectangle ABCD
(ii)

Since the diagonals of a rectangle are equal and they bisect each other.

$\therefore \quad OA = OB$

$\therefore \quad \angle OBA = \angle OAB$ [Angles opposite to equal sides]

In $\Delta OAB$, we have

$\angle AOB + \angle OAB + \angle OBA = 180^\circ$

[Sum of angles of a $\Delta$]

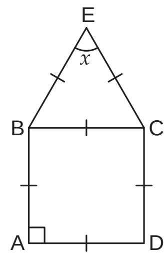
$\Rightarrow \quad x + y + y = 180^\circ$

$\Rightarrow \quad x = 180^\circ - 2y$

$= 180^\circ - 2 \times 34^\circ = 112^\circ$

**(iii)** $\angle ECD = \angle ECB + \angle DCB = 60^\circ + 90^\circ = 150^\circ$ …(1)

[$\because$ Each angle of an equilateral $\Delta$ is $60^\circ$ and each angle of a square in $90^\circ$]




Square ABCD
(iii)

In $\Delta ECD$, we have

$\angle CED = \angle CDE = y$ (say)

[Angles opposite equal sides] …(2)

In $\Delta ECD$, we have

$\angle CED + \angle CDE + \angle ECD$

$= 180^\circ$ [Sum of angles of a $\Delta$]

$\Rightarrow \quad y + y + 150^\circ = 180^\circ$ [Using (1) and (2)]

$\Rightarrow \quad 2y = 30^\circ$

$\Rightarrow \quad y = 15^\circ$

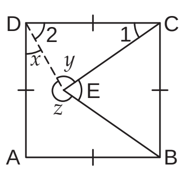
$\angle BED = \angle BEC - \angle CED$

$= 60^\circ - y = 60^\circ - 15^\circ = 45^\circ$

Hence, $\quad x = 45^\circ$.

**(iv)** $\angle 1 = \angle DCB - \angle ECB = 90^\circ - 60^\circ = 30^\circ$

$\angle y = \angle 2$ [$\angle$s opposite equal sides of $\Delta CDE$]




Square ABCD
(iv)

In $\Delta CDE$, we have,

$\angle 1 + \angle 2 + y = 180^\circ$

$\Rightarrow \quad 30^\circ + y + y = 180^\circ$

$\Rightarrow \quad 2y = 150^\circ$

$\Rightarrow \quad y = 75^\circ$

$x = \angle CDA - \angle 2$

$= 90^\circ - y = 90^\circ - 75^\circ = 15^\circ$

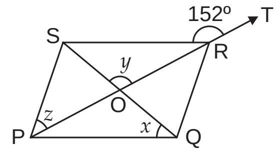
$y + z + 60^\circ = 360^\circ$ [Angles about a point]

$\Rightarrow \quad 75^\circ + z + 60^\circ = 360^\circ$

$\Rightarrow \quad z = 360^\circ - 60^\circ - 75^\circ = 225^\circ$

Hence, $x = 15^\circ$, $y = 75^\circ$ and $z = 225^\circ$.

**(v)** $\angle SRO + \angle SRT = 180^\circ$ [Linear pair]




Rhombus
(v)

$\Rightarrow \quad \angle SRO + 152^\circ = 180^\circ$

$\Rightarrow \quad \angle SRO = 28^\circ$

$z = \angle SRO$ [Angles opposite to equal sides of $\Delta SPR$]

$\Rightarrow \quad z = 28^\circ$

$y = 90^\circ$ [Diagonals of a rhombus bisect each other at right, angles.

Also $\quad \angle RPQ = \angle SRP = 28^\circ$ [Alt. angles]

In $\Delta OPQ$, we have

$\angle OPQ + \angle POQ + x = 180^\circ$ [Sum of angles of a $\Delta$]

$\Rightarrow \quad 28^\circ + 90^\circ + x = 180^\circ$

$\Rightarrow \quad x = 62^\circ$

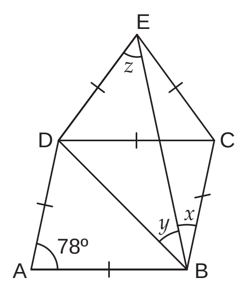
Hence, $x = 62^\circ$, $y = 90^\circ$ and $z = 28^\circ$.

**(vi)** $\angle ECB = \angle ECD + \angle DCB$

$= 60^\circ + \angle DAB$ [Opposite angles of a rhombus are equal]

$\Rightarrow \quad \angle ECB = 60^\circ + 78^\circ = 138^\circ$ …(1)

$\angle CEB = x$ [Angles opposite to equal sides of $\Delta CEB$] …(2)




Rhombus ABCD
(vi)

In $\Delta CEB$, we have

$\angle ECB + \angle CEB + \angle CBE$

$= 180^\circ$ [Sum of angles of a $\Delta$]

$\Rightarrow 138^\circ + x + x = 180^\circ$ [Using (1) and (2]

$\Rightarrow \quad 2x = 180^\circ - 138^\circ = 42^\circ$

$\Rightarrow \quad x = 21^\circ$

$z = \angle DEC - \angle CEB$

$= 60^\circ - x = 60^\circ - 21^\circ = 39^\circ$

$\angle CDB = \angle CBD$ [Angles opposite to equal sides of $\Delta CBD$]

$\Rightarrow \quad \angle CDB = (x + y)$ …(3)

In $\Delta CDB$, we have

$\angle DCB + \angle CDB + \angle DBC = 180^\circ$

$\Rightarrow \quad 78^\circ + x + y + x + y = 180^\circ$ [Using (3)]

$\Rightarrow \quad 78^\circ + 2y + 2x = 180^\circ$

$\Rightarrow \quad 2y = 180° - 78° - 2 \times 21°$

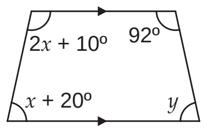
$2y = 60°$

$\Rightarrow \quad y = 30°$

Hence, $x = 21°$, $y = 30°$ and $z = 39°$.

(vii) $92° + y = 180°$ [Coint. angles formed by transversal cutting parallel sides of a trapezium]

$\Rightarrow \quad y = 180° - 92° = 88°$



Trapezium ABCD
(vii)

Also $2x + 10° + x + 20° = 180°$ [Coint. angles formed by a transversal cutting parallel sides of a trapezium]

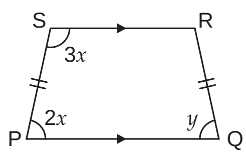
$\Rightarrow \quad 3x + 30° = 180°$

$\Rightarrow \quad 3x = 150°$

$\Rightarrow \quad x = 50°$

Hence, $x = 50°$, $y = 88°$.

(viii) $3x + 2x = 180°$ [Coint. $\angle$s formed by a transversal cutting parallel sides of a trapezium]



Isosceles Trapezium PQRS
(viii)

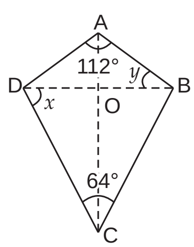
$\Rightarrow \quad 5x = 180° \Rightarrow x = 36°$

$y = 2x$ [Base angles of an isosceles trapezium are equal]

$\Rightarrow \quad y = 2 \times 36° = 72°$

(ix) AB = AD [Equal sides of a kite]

$\angle ADB = \angle ABD = y$ [$\angle$s opp. equal sides to AD and AB of $\Delta$ADB] ...(1)



Kite ABCD
(ix)

In $\Delta$ADB, we have

$\angle ADB + \angle ABD + \angle BAD$

$= 180°$ [Sum of angles of a $\Delta$]

$\Rightarrow \quad y + y + 112° = 180°$ [Using (1)]

$\Rightarrow \quad 2y = 180° - 112° = 68°$

$\Rightarrow \quad y = 34°$

Also CD = CB [Equal sides of a bite]

$\therefore \quad \angle DBC = \angle BDC = x$ ...(2)

In $\Delta$CBD, we have

$\angle DBC + \angle BDC + \angle DCB$

$= 180°$ [Sum of angles of a $\Delta$]

$\Rightarrow \quad x + x + 64° = 180°$ [Using (2)]

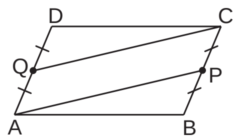
$\Rightarrow \quad 2x = 180° - 64° = 116° \Rightarrow x = 58°$

Hence, $x = 58°$, $y = 34°$.

BC $\parallel$ AD [Opposite sides of a parallelogram]

**10.** $\Rightarrow$ PC $\parallel$ AQ

BC = AD [Opposite sides of parallelogram]




$\Rightarrow \quad \frac{1}{2}BC = \frac{1}{2}AD$

$\Rightarrow$ PC = AQ [$\because$ P and Q are the mid-points of BC and AD respectively] ...(2)

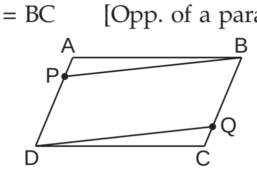
In quadrilateral APCQ, we have one pair of opposite sides i.e. PC and AQ parallel and equal to each other. [From (1) and 2)]

Hence, APCQ is a parallelogram.

**11.** $PD = AD - AP = AD - \frac{AD}{4} = \frac{3}{4}AD$ ...(1)

$BQ = BC - QC = BC - \frac{BC}{4} = \frac{3}{4}BC$ ...(2)

AD = BC [Opp. of a parallelogram] ...(3)




From (1), (2) and (3), we have

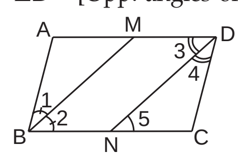
PD = BQ ...(4)

Also PD $\parallel$ BQ [$\because$ APD $\parallel$ BQC, opp. sides of a $\parallel$gm] ...(5)

$\therefore$ In quadrilateral BPDQ, we have one pair of opposite sides i.e. PD and BQ parallel and equal to each other [From (4) and (5)]

Hence, BPDQ is a parallelogram.

**12.** $\angle B = \angle D$ [Opp. angles of a parallelogram]




$\Rightarrow \quad \frac{1}{2}\angle B = \frac{1}{2}\angle D$

$\Rightarrow \quad \angle 2 = \angle 3$ ...(1)

But $\angle 3 = \angle 5$ [Alt. angles, AD $\parallel$ BC] ...(2)

$\therefore \quad \angle 2 = \angle 5$ [Using (1) and (2)]

But $\angle 2$ and $\angle 5$ are corresponding angles formed when BM and ND are cut by a transversal BC

$\therefore$ BM $\parallel$ ND ...(3)

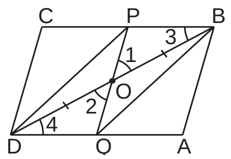
Also MD $\parallel$ BN [$\because$ AMD $\parallel$ BNC, opposite sides of parallelogram] ...(4)

In quadrilateral BNDM, we have pairs of opposite sides (BM, ND) and (MD, BN) parallel to each other. [From (3) and (4)]

Hence, quadrilateral BNDM is a parallelogram

and BM = DN. [Opp. sides of a parallelogram are equal]

**13.** In $\Delta$OPB and $\Delta$OQD, we have




$\angle 1 = \angle 2$ [Vertically opposite angles]

OB = OD [Diagonal BD is bisected at O]

and $\angle 3 = \angle 4$ [Alt. angles, CB $\parallel$ DA]

$\therefore$ $\Delta$OPB $\cong$ $\Delta$OQD [By ASA congruence]

$\Rightarrow$ OP = OQ [CPCT] ...(1)

and PB = QD [CPCT] ...(2)

Also CB $\parallel$ DA [Opposite sides of a parallelogram]

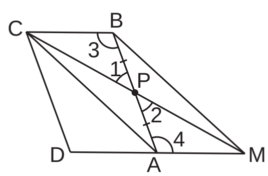
$\Rightarrow$ PB | QD [$\because$ P lies on AB and Q lies on DA] ...(3)

In quadrilateral BPDQ, we have one pair of opposite sides PB and QD parallel and equal to each other. [From (2) and (3)]

Hence, BPDQ is a parallelogram

and OP = OQ ...[From (1)]

**14.** In $\Delta$PBC and $\Delta$PAM, we have




$\angle 1 = \angle 2$ [Vert. opp. angles]

PB = PA [P is the mid-point of AB]

$\angle 3 = \angle 4$ [Alt. angles, CB $\parallel$ DAM]

$\therefore$ $\Delta$PBC $\cong$ $\Delta$PAM [By ASA congruence]

$\Rightarrow$ BC = AM [CPCT] ...(1)

Also CB $\parallel$ DA (produced)

$\Rightarrow$ CB $\parallel$ AM ...(2)

In quadrilateral ACBM, are pair of opposite sides i.e. CB and AM parallel and equal to each other. [From (1) and (2)]

Hence, ACBM is a parallelogram.

**15.** In $\Delta$QDA and $\Delta$PCB, we have

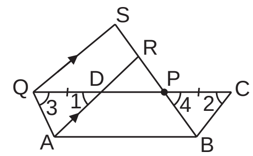
QD = PC [given]

$\angle 1 = \angle 2$ [Corr. angles, AD $\parallel$ BC]

DA = CB [Opposite sides of a parallelogram]

$\therefore$ $\Delta$QDA $\cong$ $\Delta$PCB [By SAS congruency]

$\Rightarrow$ $\angle 3 = \angle 4$ [CPCT]




But $\angle 3$ and $\angle 4$ are corresponding angles formed when BP and AQ are cut by transversal CQ at P and Q respectively.

$\therefore$ BP $\parallel$ AQ

$\Rightarrow$ BP produced $\parallel$ AQ

$\Rightarrow$ PRS $\parallel$ AQ

$\Rightarrow$ RS $\parallel$ AQ ...(1)

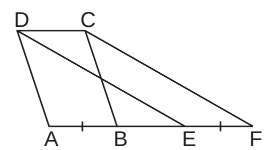
Also QS $\parallel$ AR [given] ...(2)

Hence ARSQ is a parallelogram.

**16.** DC $\parallel$ AB [Opposite sides of a parallelogram]

$\Rightarrow$ DC $\parallel$ AEF (or AB produced)

$\Rightarrow$ DC $\parallel$ EF ...(1)




Also DC = AB [Opposite sides of a $\parallel$gm] ...(2)

AB = EF [Given] ...(3)

$\therefore$ DC = EF [Using (2) and (3)] ...(4)

In quadrilateral DEFC, we have

DC $\parallel$ EF and DC = EF [From (1) and (4)]

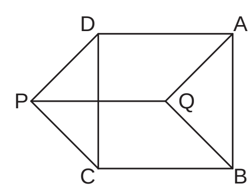
$\therefore$ DEFC is a parallelogram.

**17.** DA $\parallel$ PQ

and DA = PQ [Opp. sides of $\parallel$gm PQAD] ...(1)

CB $\parallel$ PQ

and CB = PQ [Opp. sides of $\parallel$gm PQBC] ...(2)




From (1) and (2), we get

DA $\parallel$ CB and DA = CB

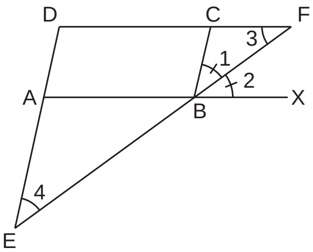
In quadrilateral ABCD, we have DA $\parallel$ CB and DA = CB

$\therefore$ ABCD is a parallelogram.

**18.** $\angle 1 = \angle 2$ [$\because$ BF, the bisector of $\angle$CBX] ...(1)

$\angle 1 = \angle 4$ [Corr. angles, CB $\parallel$ DAE] ...(2)

$\angle 2 = \angle 3$ [Alt. angles, DF $\parallel$ ABX] ...(3)




From (1), (2) and (3), we have

$\angle 3 = \angle 4$

$\Rightarrow$ DE = DF [Sides opposite to equal angles of $\Delta$DEF] …(4)

and $\angle 3 = \angle 1$

$\Rightarrow$ BC = CF [Sides opposite to equal angles of $\Delta$BCF] …(5)

Now DE = DF [From (4)]

$\Rightarrow$ DE = DC + CF

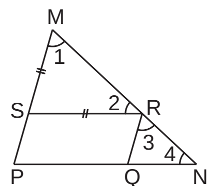
$\Rightarrow$ DE = AB + BC [DC = AB opposite sides of a ||gm and using (5)] ...(6)

Hence, DE = AB + BC = DF [From (4) and (6)]

**19.** $\angle 1 = \angle 2$ [Angles opposite to equal sides of $\Delta$MSR] …(1)

$\angle 1 = \angle 3$ [Corr. angles, PSM || QR] …(2)

$\angle 2 = \angle 4$ [Corr. angles, SR || PQN] …(3)




From (1), (2) and (3), we get

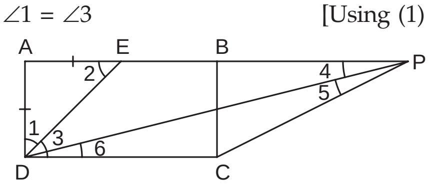
$\angle 3 = \angle 4$

$\Rightarrow$ QN = QR [Sides opp. equal angles of $\Delta$QRN]

**20.** $\angle 1 = \angle 2$ [Angles opp. to equal sides AE and AD of $\Delta$ADE] …(1)

$\angle 2 = \angle 3$ [Alt. angles, AEB || DC] …(2)

$\therefore$ $\angle 1 = \angle 3$ [Using (1) and (2)]




DE bisects $\angle$ADC

$\angle 4 = \angle 5$ [$\because$ PD bisects $\angle$APC] …(3)

$\angle 4 = \angle 6$ [Alt. angles, AP || DC] …(4)

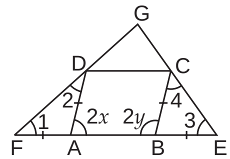
$\therefore$ $\angle 5 = \angle 6$ [Using (3) and (4)]

$\Rightarrow$ DC = PC [Sides opp. to equal angles of $\Delta$DCP]

AB = PC [$\because$ DC = AB, opposite sides of a parallelogram]

**21.** $\angle 1 = \angle 2 = x$ (say) [Angles opp. to equal sides of $\Delta$ADF] …(1)

$\angle 3 = \angle 4 = y$ (say) [Angles opp. to equal sides of $\Delta$BCE] …(2)




Considering $\Delta$ADF, whose side FA is produced to B, we have

Exterior $\angle$DAB = $\angle 1 + \angle 2 = 2x$ [Using (1)] …(3)

Considering $\Delta$BCE, whose side EB is produced to A, we have

Exterior $\angle$CBA = $\angle 3 + \angle 4 = 2y$ [Using (2)] …(4)

Now, $\angle$DAB + $\angle$CBA = 180° [Coint. angles, AD || BC]

$\Rightarrow$ $2x + 2y = 180°$ [Using (3) and (4)]

$\Rightarrow$ $x + y = 90°$ …(5)

In $\Delta$GFE, we have $\angle 1 + \angle 3 + \angle$FGE = 180° [Sum of angles of a $\Delta$]

$\Rightarrow$ $x + y + \angle$FGE = 180°

$\Rightarrow$ 90° + $\angle$FGE = 180° [Using (5)]

$\Rightarrow$ $\angle$FGE = 90°

Hence, EC and FD produced meet at right angles.

**22.** In $\Delta$BEF and $\Delta$BAC, we have

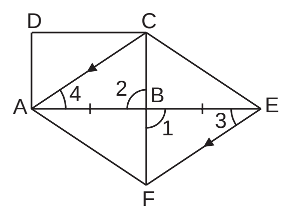
$\angle 1 = \angle 2$ [V. opposite angles]

BE = BA [given]

$\angle 3 = \angle 4$ [Alt. $\angle s$, EF || AC]

$\therefore$ $\Delta$BEF $\cong$ DBAC [By ASA congruency]

$\Rightarrow$ EF = AC




In quadrilateral ACEF, we have

EF || AC and EF = AC

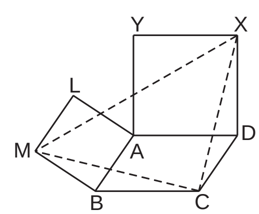
$\therefore$ ACEF is a parallelogram

$\Rightarrow$ AF = EC [Opposite sides of a parallelogram]

**23.** MB = AB [Sides of a square]

CD = AB [Opposite sides of a ||gm]

$\Rightarrow$ MB = CD …(1)




Also BC = AD [Opposite sides of a parallelogram]

DX = AD [Sides of a square]

$\Rightarrow$ BC = DX …(2)

Now, $\angle$MBC = $\angle$MBA + $\angle$ABC = 90° + $\angle$ABC

$\angle CDX = \angle CDA + \angle ADX = \angle CDA + 90^\circ$

and $\angle ABC = \angle CDA$ [Opposite angles of a parallelogram]

$\Rightarrow$ $\angle MBC = \angle CDX$ …(3)

In $\Delta MBC$ and $\angle CDX$, we have

$MB = CD$ [From (1)]

$\angle MBC = \angle CDX$ [From (3)]

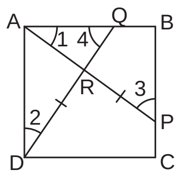
and $BC = DX$ [From (2)]

$\therefore$ $\Delta MBC \cong \Delta CDX$ [By SAS congruency]

$\Rightarrow$ $MC = CX$

$\Rightarrow$ $\Delta CXM$ is an isosceles triangle.

**24.** Let AP and DQ intersect at R.




In $\Delta ABP$ and $\Delta DAQ$, we have

$AB = DA$ [Sides of a square]

$\angle ABP = \angle DAQ$ [Each is 90°]

$AP = DQ$ [Given]

$\therefore$ $\Delta ABP \cong DDAQ$

$\Rightarrow$ $\angle 1 = \angle 2$ and $\angle 3 = \angle 4$ [CPCT] …(1)

In $\Delta DAQ$, we have

$\angle 2 + 90^\circ + \angle 4 = 180^\circ$ [Sum of angles of a Δ]

$\Rightarrow$ $\angle 1 + 90^\circ + \angle 4 = 180^\circ$ [Using (1)]

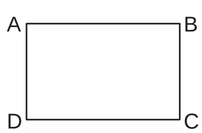
$\Rightarrow$ $\angle 1 + \angle 4 = 90^\circ$ …(2)

Considering $\Delta AQR$ whose side AR is produced to P, we get

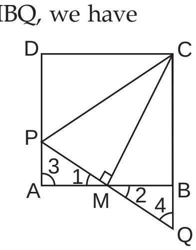
Exterior $\angle QRP = \angle 1 + \angle 4 = 90^\circ$ [Using (2)]

$\Rightarrow$ AP and DQ are perpendicular to each other.

**25.** ABCD is a parallelogram in which one angle say $\angle A = 90^\circ$.




**26.** In $\Delta MAP$ and $\Delta MBQ$, we have




$\angle 1 = \angle 2$ [V. opposite angles]

$AM = BM$ [$\because$ M is the mid-point of AB]

$\angle 3 = \angle 4$ [Each is 90°]

$\therefore$ $\Delta MAP \cong \Delta MBQ$ [By ASA congruency]

$\Rightarrow$ $MP = MQ$ [CPCT] …(1)

In right $\Delta CMP$ and right $\Delta CMQ$, we have

$MP = MQ$ [From (1)]

and $CM = CM$ [Common]

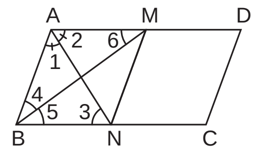
$\therefore$ $\Delta CMP \cong \Delta CMQ$ [By RHS congruency]

$\Rightarrow$ $CP = CQ$ [CPCT]

**27.** $\angle 1 = \angle 2$ [$\because$ AN bisects $\angle A$]

$\angle 3 = \angle 2$ [Alt. angles, BNC || AMD]

$\therefore$ $\angle 3 = \angle 1$




$\Rightarrow$ $AB = BN$ [Sides opp. to equal angles of $\Delta ABN$] …(1)

$\angle 4 = \angle 5$ [$\because$ BM bisects $\angle B$]

$\angle 6 = \angle 5$ [Alt. angles, AMD || BNC]

$\therefore$ $6 = \angle 4$

$\Rightarrow$ $AB = AM$ [Sides opp. to equal angles of $\Delta ABM$] …(2)

$\therefore$ $AM = BN$ [Using (1) and (2)]

$AM \parallel BN$ [$\because$ AMD || BNC]

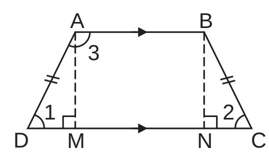
In quadrilateral ABNM, we have

$AM = BN$ and $AM \parallel BN$ …(3)

$\therefore$ ABNM is a parallelogram in which adjacent sides AB (= AM) and BN are equal [Using (2) and (3)]

Hence, ABNM is a rhombus.

**28.** Let ABCD be an isosceles trapezium in which AB || DC and AD = BC.




Draw $AM \perp DC$ and $BN \perp DC$

In right $\Delta AMD$ and right $\Delta BNC$, we have

$AM = BN$ [Distance between parallel line segments]

and $AD = BC$ [Given]

$\therefore$ $\Delta AMD \cong \Delta BNC$ [By RHS congruency]

$\Rightarrow$ $\angle 1 = \angle 2$ [By CPCT] …(1)

$\angle 3 + \angle 1 = 180^\circ$ [Coint. angles, AB || DC]

$\Rightarrow$ $\angle 3 + \angle 2 = 180^\circ$ [Using (1)]

$\Rightarrow$ $\angle A + \angle C = 180^\circ$ …(2)

$\angle A + \angle B + \angle C + \angle D = 360^\circ$ [Sum of angles of a quadrilateral]

$\Rightarrow$ $(\angle A + \angle C) + \angle B + \angle D = 360^\circ$

$\Rightarrow$ $180^\circ + \angle B + \angle D = 360^\circ$ [Using (2)]

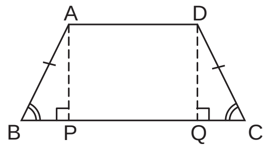
$\Rightarrow$ $\angle B + \angle D = 180^\circ$

Hence, the opposite angles of an isosceles trapezium are supplementary.

**29.** In quad. ABCD

$AB = CD$ and $\angle B = \angle C$

Draw $AP \perp BC$ and $DQ \perp BC$




In $\Delta APB$ and $\Delta DQC$

$\angle APB = \angle DQC$ [Each = 90°]

$\angle B = \angle C$ [Given]

$\therefore$ $\Delta APB \cong \Delta DQC$ [By AAS congruency]

$\Rightarrow$ $AP = DQ$ [CPCT]

i.e. perpendicular distance between AD and BC at two distinct points is the same.

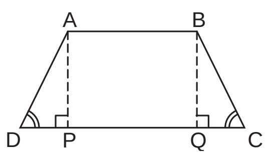
Thus $AD \parallel BC$.

**30.** Draw $AP \perp DC$ and $BQ \perp DC$

$\angle A + \angle B + \angle C + \angle D = 360°$ [Sum of angles of a quadrilateral]

$\Rightarrow$ $2\angle A + 2\angle D = 360°$ [$\because$ $\angle A = \angle B$ and $\angle C = \angle D$]

$\Rightarrow$ $\angle A + \angle D = 180°$




But $\angle A$ and $\angle D$ are cointerior angles formed when transversal AD cuts AB and DC at A and D respectively.

$\therefore$ $AB \parallel DC$

$\Rightarrow$ $AP = BQ$ [Distance between parallel line segments]

In $\Delta APD$ and $\Delta BQC$, we have

$AP = BQ$ [From (1)]

$\angle APD = \angle BQC$ [Each is equal to 90°]

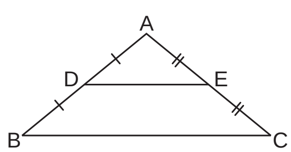
$\angle ADP = \angle BCQ$ [$\angle D = \angle C$, given]

$\therefore$ $\Delta APD \cong \Delta BQC$ [By SAA congruency]

$\Rightarrow$ $AD = BC$ [CPCT]

## EXERCISE 8C

**1.** In $\Delta ABC$, D is the mid-point of AB and E is the mid-point of AC.




$\therefore$ $DE = \frac{1}{2} BC$

$= \frac{1}{2} \times 6.6$ cm

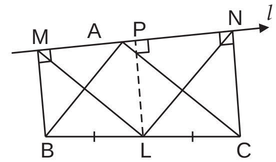
$\Rightarrow$ $DE = 3.3$ cm

and $DE \parallel BC$

$\Rightarrow$ $\angle ADE = \angle ABC$

$= 62°$ [Corr. angles, $DE \parallel BC$]

2. Draw $LP \perp l$




BM, CN and LP are perpendiculars to the same line $l$

$\therefore$ $BM \parallel CN \parallel LP$

$BL = CL$ [L us the mid-point of BC]

$\therefore$ $MP = NP$ [Intercept theorem] …(1)

In $\Delta MPL$ and $\Delta NPL$, we have

$MP = NP$ [From (1)]

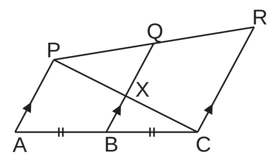
$\angle MPL = \angle NPL$ [Each is equal to 90°]

$PL = PL$ [Common]

$\therefore$ $\Delta MPL \cong \Delta NPL$ [By SAS congruency]

$\Rightarrow$ $ML = NL$ [CPCT]

**3.** In $\Delta CPA$, B is the mid-point of AC and $BX \parallel AP$




$\therefore$ X is the mid-point of CP [By the converse of Mid-point theorem].

In $\Delta PCR$, X is the mid-point of PC and $XQ \parallel AP$.

$\therefore$ Q is the mid-point of PR [By the converse of Mid-point theorem].

Now, in $\Delta CPA$, B and X are the mid-points of AC and PC respectively.

$\therefore$ $BX = \frac{1}{2} AP \Rightarrow AP = 2BX$ …(1)

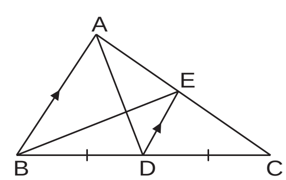
In $\Delta PCR$, X and Q are the mid-points of PC and PR respectively

$\therefore$ $QX = \frac{1}{2} CR \Rightarrow CR = 2QX$ …(2)

Adding (1) and (2), we get

$AP + CR = 2(BX + QX) \Rightarrow AP + CR = 2BQ$

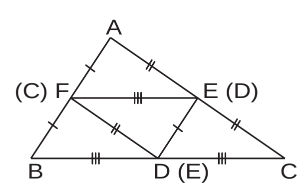
**4.** In $\Delta ADC$, D is the mid-point of BC [$\because$ AD is a median] and $DE \parallel BA$ [Given]




$\therefore$ E is the mid-point of AC [By the converse of Mid-point theorem]

$\Rightarrow$ BE is a median.

**5.** Let point D, E and F be the mid-points of sides BC, AC and AB of $\Delta ABC$.




DE, EF and FD are joined. Since the line segments joining the mid-points of two sides of a triangle is half of the third side [By Mid-point theorem]

$\therefore \quad DE = \frac{1}{2}AB$

$\Rightarrow \quad DE = AF \quad \ldots(1)$

$DF = \frac{1}{2}AC$

$\Rightarrow \quad DF = AE \quad \ldots(2)$

In ΔDEF and ΔAFE, we have

$DE = AF$ [From (1)]

$DF = AE$ [From (2)]

and $EF = FE$ [Common]

$\therefore \quad \Delta DEF \cong \Delta AFE$ [By SSS congruence]

Similarly it can be proved that

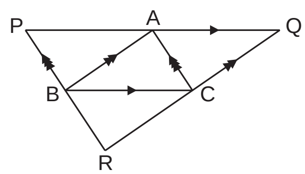
$\Delta DEF \cong \Delta FBP$ and $\Delta DEF \cong \Delta EDC$

$\Rightarrow \quad \Delta DEF \cong \Delta AFE \cong \Delta FBD \cong \Delta EDC$

Hence, the straight lines joining the mid-points of the sides of a triangle divide it into four congruent triangles.

**6.** $BC \parallel AQ$ [$\because PAQ \parallel BC$]

and $AB \parallel QC$ [$\because QCR \parallel AB$]




$\therefore$ Quadrilateral AQCB is a parallelogram.

$\Rightarrow \quad AQ = BC$ [Opp. sides of a ||gm] …(1)

Similarly, it can be proved that quadrilateral PACB is a parallelogram.

$\Rightarrow \quad PA = BC$ [Opp. sides of a ||gm] …(2)

$\therefore \quad PA = AQ$ [Using (1) and (2)]

$\Rightarrow$ A is the mid-point of PQ

Similarly, it can be proved that B and C are the mid-points of PR and QR respectively.

Now in ΔPQR, A is the mid-point of PQ and B is the mid-point of PR and C is the mid-point of QR.

$\therefore \quad AB = \frac{1}{2}QR \quad \ldots(1)$

$BC = \frac{1}{2}PQ \quad \ldots(2)$

and $CA = \frac{1}{2}RP \quad \ldots(3)$

Adding (1), (2) and (3), we get

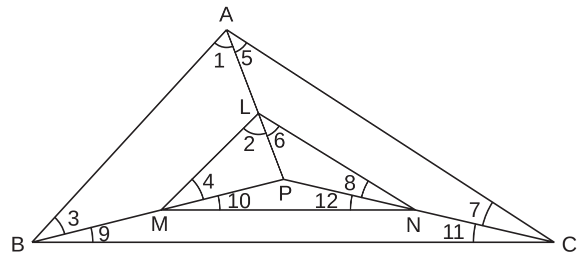
$$AB + BC + CA = \frac{1}{2}(QR + PQ + RP)$$

$\Rightarrow$ Perimeter of ΔABC $= \frac{1}{2}$ Perimeter of ΔPQR

$\Rightarrow$ 2 perimeter of ΔABC = Perimeter of ΔPQR

Hence, the perimeter of ΔPQR is double the perimeter of ΔABC.

**7.** In ΔAPB, L and M are the mid-points of AP and BP respectively.




$\therefore \quad LM \parallel AB$ [By Mid-point theorem]

$\Rightarrow \quad \angle 1 = \angle 2$ [Corr. angles] …(1)

In ΔAPC, L and N are the mid-points of AP and CP respectively.

$\therefore \quad LN \parallel AC$ [By Mid-point theorem]

$\Rightarrow \quad \angle 5 = \angle 6$ [Corr. angles] …(3)

and $\angle 7 = \angle 8$ [Corr. angles] …(4)

In ΔBPC, M and N are the mid-points of BP and CP respectively.

$\therefore \quad MN \parallel BC$ [By Mid-point theorem]

$\Rightarrow \quad \angle 9 = \angle 10$ [Corr. angles] …(5)

and $\angle 11 = \angle 12$ [Corr. angles] …(6)

Adding the corresponding sides of (1) and (2), we get

$\angle 1 + \angle 5 = \angle 2 + \angle 6$

$\Rightarrow \quad \angle A = \angle L$

Adding the corresponding sides of (2) and (5), we get

$\angle 3 + \angle 9 = \angle 4 + \angle 10$

$\Rightarrow \quad \angle B = \angle M$

Adding the corresponding sides of (4) and (6), we get

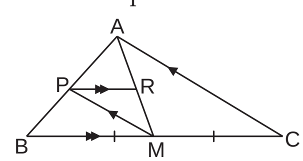
$\angle 7 + \angle 11 = \angle 8 + \angle 12$

$\Rightarrow \quad \angle C = \angle N$

Since $\angle A = \angle L$, $\angle B = \angle M$ and $\angle C = \angle N$

$\therefore$ ΔABC and ΔLMN are equiangular.

**8.** (i) In ΔABC, M is the mid-point of BC and $MP \parallel CA$




$\therefore$ P is the mid-point of AB [By the converse of Mid-point theorem]

$\Rightarrow \quad AP = \frac{1}{2}AB \quad \Rightarrow AB = 2AP$

(ii) In ΔABM, P is the mid-point of AB and $PR \parallel BM$

$\therefore$ R is the mid-point of AM [By the converse of Mid-point theorem]

$\Rightarrow \quad AR = \frac{1}{2}AM$

$\Rightarrow \quad 2AR = AM$

(iii) In ΔABM, P and R are the mid-points of AB and AM

$\therefore \quad PR = \frac{1}{2}BM$ [By Mid-point theorem]

$\Rightarrow \quad BM = 2PR$

(iv) $BC = BM + MC$

$= 2BM$ [$\because$ M is the mid-point of BC $\Rightarrow BM = MC$]

$\Rightarrow$ $BC = 2(2PR)$ [$\because BM = 2PR$, proved in (iii)]

$\Rightarrow$ $BC = 4PR$

**9.** Since, E and F are the mid-points of AB and AC respectively

$\therefore$ $EF \parallel BC$ [By Mid-point theorem]

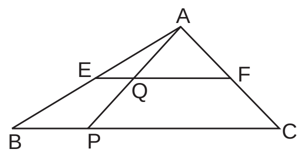
$\Rightarrow$ $EQ \parallel BP$ [$\because$ Q lies on EF and P lies on BC] …(1)

In $\Delta ABP$, E is the mid-point of AB and $EQ \parallel BP$ [From (1)]

$\therefore$ Q is the mid-point of AP

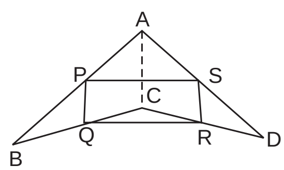
[By the converse of Mid-point theorem]

$\Rightarrow$ EF bisects AP.




**10.** Join AC.

In $\Delta ABC$, P and Q are the mid-points of AB and BC respectively.




$\therefore$ $PQ \parallel AC$

and $PQ = \frac{1}{2}AC$ …(1) [By Mid-point theorem]

In $\Delta ADC$, S and R are the mid-points of AD and CD respectively

$\therefore$ $SR \parallel AC$

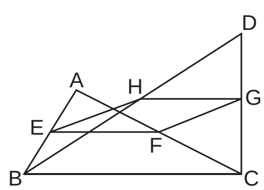
and $SR = \frac{1}{2}AC$ [By Mid-point theorem] …(2)

From (1) and (2), we get

$PQ \parallel SR$ and $PQ = SR$

Hence, PQRS is a parallelogram.

**11.** In $\Delta ABC$, E and F are the mid-points of AB and AC respectively.




$\therefore$ $EF \parallel BC$

and $EF = \frac{1}{2}BC$ [By Mid-point theorem] …(1)

In $\Delta DBC$, H and G are the mid-points of DB and DC respectively.

$\therefore$ $HG \parallel BC$

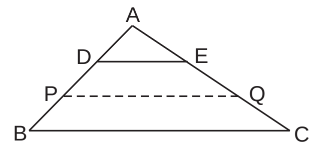
and $HG = \frac{1}{2}BC$ [By Mid-point theorem] …(2)

From (1) and (2), we get

$EF \parallel HG$ and $EF = HG$

Hence, EFGH is a parallelogram.

**12.** Let P and Q be the mid-points of AB and AC of $\Delta ABC$. Join PQ.




Then, $PQ = \frac{1}{2}BC$ [By Mid-point theorem] …(1)

Since P is the mid-point of AB

$\therefore$ $AP = \frac{1}{2}AB \Rightarrow AB = 2AP$ …(2)

$AD = \frac{1}{4}AB$ [Given]

$\Rightarrow$ $AD = \frac{1}{4}(2AP)$ [Using (2)]

$\Rightarrow$ $AD = \frac{1}{2}AP$

$\Rightarrow$ D is the mid-point of AP. …(3)

Similarly, it can be proved that E is the mid-point of AQ.

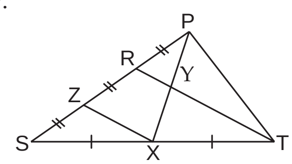
In $\Delta APQ$, D and E are the mid-points of AP and AQ [From (3)]

$\therefore$ $DE = \frac{1}{2}PQ$ [By Mid-point theorem]

$\Rightarrow$ $DE = \frac{1}{2}\left(\frac{1}{2}BC\right)$ [Using (1)]

$\Rightarrow$ $DE = \frac{1}{4}BC$

**13.** In $\Delta RST$, Z and X are the mid-points of SR and ST respectively.




$\therefore$ $ZX = \frac{1}{2}RT$

and $ZX \parallel RT$ [By Mid-point theorem] …(1)

$\Rightarrow$ $ZX \parallel RY$ [$\because$ Y lies in RT] …(2)

In $\Delta PZX$, R is the mid-point of PZ and $RY \parallel ZX$ [From (2)]

$\therefore$ Y is the mid-point of PX [By the converse of Mid-point theorem]

Now, in $\Delta PZX$, R and Y are the mid-points of PZ and PX respectively.

$\therefore$ $RY = \frac{1}{2}ZX$ [By Mid-point theorem]

$\Rightarrow$ $RY = \frac{1}{2}\left(\frac{1}{2}RT\right)$ [Using (1)]

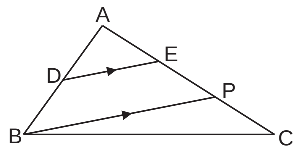
$\Rightarrow$ $RY = \frac{1}{4}RT$

**14.** In $\Delta ABP$, D is the mid-point of AB and $DE \parallel BP$.

$\therefore$ E is the mid-point of AP.

[By the converse of Mid-point theorem]

$\Rightarrow$ $AE = EP$ …(1)




$$AP = AE + EP = EP + EP \qquad \text{[Using (1)]}$$
$$= 2EP$$

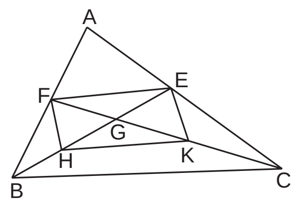
$$\Rightarrow \quad EP = \frac{1}{2}AP \qquad \ldots(2)$$

$$\text{Now } AP - EP = AP - \frac{1}{2}AP \qquad \text{[Using (2)]}$$

$$= \frac{1}{2}AP = PC \qquad \left[\because \frac{1}{2}AP = PC, \text{ given}\right]$$

Hence, AP – EP = PC.

**15.** BE and CF are medians of ΔABC.




⇒ E and F are the mid-points of AC and AB respectively.

In ΔABC, F is the mid-point of AB and E is the mid-point of AC.

$$\therefore \quad FE \parallel BC \text{ and } FE = \frac{1}{2}BC \qquad \ldots(1)$$

In ΔGBC, H is the mid-point of GB and K is the mid-point of GC.

$$\therefore \quad HK \parallel BC \text{ and } HK = \frac{1}{2}BC \qquad \ldots(2)$$

From (1) and (2), we get

FE ∥ HK and FE = HK

⇒ HKEF is a parallelogram.

Since the diagonals of a parallelogram bisect each other

∴ EG = GH

and FG = GK

Also HB = GH

and KC = GK [∵ H and K are the mid-points of GB and GC respectively]

∴ EG = GH = HB

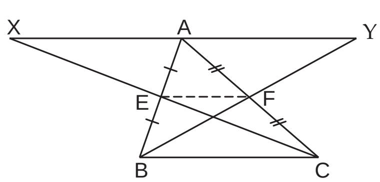
and FG = GK = KC

$$\Rightarrow \quad EG = \frac{1}{3}BE$$

$$\text{and} \quad FG = \frac{1}{3}CF$$

Hence, G is a point of trisection of BE and CF.

**16.** Join EF




In ΔCXA E and F are the mid-points of CX and CA respectively

∴ EF ∥ XA [By Mid-point theorem] …(1)

In ΔBAY, E and F are the mid-points of BA and BY respectively

∴ EF ∥ AY [By Mid-point theorem] …(2)

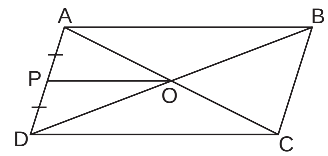
Also, XA and AY pass through the same point A.

∴ XA and AY lie along the same straight line and are parallel to the same line segment EF.

Thus, X, A and Y are collinear.

Hence, XAY is a straight line.

**17.** Since the diagonals of a parallelogram bisect each other.




∴ AO = OC

⇒ O is the mid-point of AC.

In ΔADC, P and O are the mid-points of AD and AC respectively.

∴ (i) PO ∥ DC

$$\text{and (ii) } PO = \frac{1}{2}DC \qquad \text{[By Mid-point theorem]}$$

⇒ (i) PO ∥ AB [∵ AB ∥ DC, opposite sides of a parallelogram]

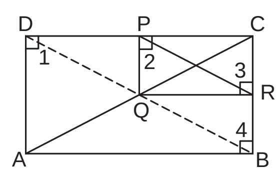
$$\text{and (ii) } PO = \frac{1}{2}CD$$

**18.** **(i)** Join diagonal BD.

Diagonal BD will pass through Q, the mid-point of diagonal AC.

[∵ Diagonals of a rectangle bisect each other]

∠1 = ∠2 [Each is 90°]




⇒ QP and QD are both perpendicular to the same lines segment DC.

∴ QP ∥ AD

Now, in ΔDCA, Q is the mid-point of CA and QP ∥ AD.

∴ P is the mid-point of DC [By the converse of Mid-point theorem]

⇒ DP = PC

**(ii)** ∠3 = ∠4 [Each is 90°]

⇒ QR and AB are both perpendicular to the same line segment CB.

∴ QR ∥ AB

Now in ΔCAB, Q is the mid-point of AC and QR ∥ AB

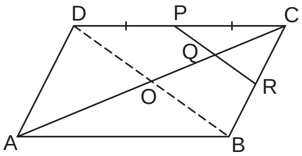
∴ R is the mid-point of CB [By the converse of Mid-point theorem]

In ΔCDB, P and R are the mid-points of CD and CB

$$\therefore \quad PR = \frac{1}{2}BD \qquad \text{[By Mid-point theorem]}$$

$$\Rightarrow \quad PR = \frac{1}{2}AC \qquad [\because BD = AC, \text{ diagonals of a rectangle}]$$

**19.** Join BD and let it cut diagonal AC at O.




Since, the diagonals of a parallelogram bisect each other.

$\therefore$ O is the mid-point of AC.

$\Rightarrow$ $CO = \frac{1}{2}AC$

$\Rightarrow$ $AC = 2CO$ …(1)

Now, $CQ = AC = \frac{1}{4}(2CO)$ [Using (1)]

$\Rightarrow$ $CQ = \frac{1}{2}CO$

$\Rightarrow$ Q is the mid-point of CO.

In $\Delta CDO$, P and Q are the mid-points of CD and CO.

$\therefore$ $PO \parallel DO$

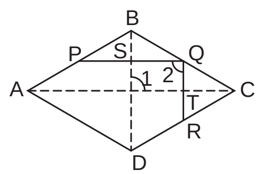
$\Rightarrow$ $PQR \parallel DOB$

$\Rightarrow$ $QR \parallel OB$

In $\Delta COB$, Q is the mid-point of CO and $QR \parallel OB$

$\therefore$ R is the mid-point of BC. [By the converse of Mid-point theorem]

**20.** Join diagonals AC and BD and let them intersect at O.




Let diagonal BD intersect PQ at S and let diagonal AC intersect QR at T.

In $\Delta BAC$, P and Q are mid-points of AB and BC respectively.

$\therefore$ $PQ \parallel AC$ [By Mid-point theorem]

$\Rightarrow$ $SQ \parallel OT$ [$\because$ S lies on PQ and O, T lies on AC] …(1).

In $\Delta CBD$, Q and R are the mid-points of BC and CD respectively.

$\therefore$ $QR \parallel BD$

$\Rightarrow$ $QT \parallel SO$ [$\because$ T lies on QR and S and O lie on BD] …(2)

In quadrilateral SQTO, we have

$SQ \parallel OT$ and $QT \parallel SO$ [From (1) and (2)]

$\therefore$ SQTO is a parallelogram.

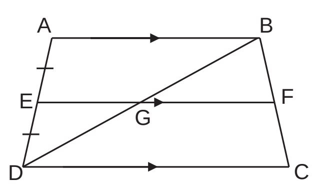
$\therefore$ $\angle 1 = \angle 2$ [Opposite angles of a parallelogram] …(3)

But $\angle 1 = 90°$ [Diagonals of a rhombus are perpendicular to each other]

$\Rightarrow$ $\angle 2 = 90°$ [Using (3) and (4)]

$\Rightarrow$ $PQ \perp QR$

**21.** Let diagonal BD intersect EF at G. In $\Delta DAB$, E is the mid-point of DA and $EG \parallel AB$.




[$\because$ $EF \parallel AB$ and G lies on EF]

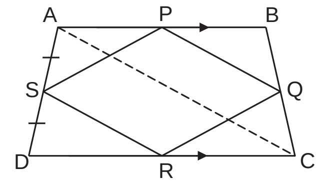
$\therefore$ G is the mid-point of BD [By the converse of Mid-point theorem]

Now in $\Delta BDC$, G is the mid-point of BD and

$GF \parallel DC$ [$\because$ $EF \parallel AB \parallel DC$ and G lies on EF]

$\therefore$ F is the mid-point of BC [By the converse of Mid-point theorem]

**22.** ABCD is an isosceles trapezium in which AD = BC.




P, Q, R and S are the mid-points of AB, BC, CD and DA respectively.

Join any one diagonal (say) AC of the trapezium ABCD.

In $\Delta BAC$, P and Q are the mid-points of BA and BC respectively.

$\therefore$ $PQ \parallel AC$ and $PQ = \frac{1}{2}AC$ …(1)

In $\Delta DAC$, S and R are the mid-points of DA and DC.

$\therefore$ $SR \parallel AC$ and $SR = \frac{1}{2}AC$ …(2)

From (1) and (2), we get

$PQ \parallel SR$ and $PQ = SR$

$\therefore$ PQRS is a parallelogram. …(3)

In $\Delta$s SDR and QCR, we have

$SD = QC$ $\left[\because SD = \frac{1}{2}AD, QC = \frac{1}{2}BC \text{ and } AD = BC\right]$

$\angle SDR = \angle QCR$ [Base $\angle$s of isosceles trapezium]

$DR = CR$ [R is the mid-point of DC]

$\therefore$ $\Delta SDR \cong \Delta QCR$ [By SAS congruence]

$\Rightarrow$ $SR = QR$ …(4)

$\therefore$ PQRS is a parallelogram in which two adjacent sides SR and QR are equal. [Using (3) and (4)]

$\Rightarrow$ PQRS is a rhombus.

Hence, the straight lines joining the mid-points of the sides of an isosceles trapezium in order form a rhombus.

## CHECK YOUR UNDERSTANDING

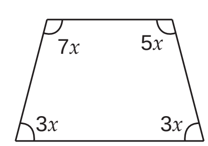
### MULTIPLE-CHOICE QUESTIONS

**1.** (a) **104°**

Let $x$ be the 4th angle of the quadrilateral. Then $x + 60° + 86° + 110° = 360°$ ($\angle$s of a quadrilateral)

$\Rightarrow$ $x = 360° - 256° = 104°$

**2.** (b) **20°**




$7x + 5x + 3x + 3x = 18x = 360°$ [Sum of $\angle$s of a quadrilateral]

$\Rightarrow$ $x = 20°$

**3.** (b) **120°, 120°, 40°**

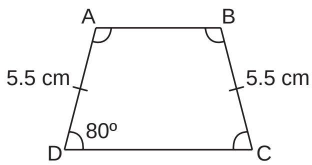
Let the three angles of a quadrilateral in the ratio 3 : 3 : 1 be $3x$, $3x$ and $x$.

Then, $3x + 3x + x + 80° = 360°$ [Sum of angles of a quadrilateral]

$\Rightarrow \quad 7x = 280° \Rightarrow x = 40°$

The other angles are $3 \times 40°$, $3 \times 40°$, $40°$ i.e. 120°, 120°, 40°.

**4.** (c) **80°, 100°, 100°**




$AB \parallel DC$

and $AD = BC$

$\Rightarrow$ ABCD is an isosceles trapezium.

$\therefore \quad \angle C = \angle D = 80°$

$\angle A + \angle D = 180°$ [Coint. angles, $AB \parallel DC$]

$\Rightarrow \quad \angle A + 80° = 180°$

$\Rightarrow \quad \angle A = 100°$

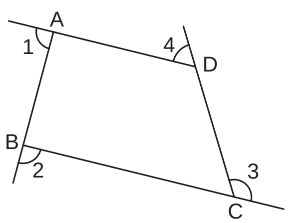
$\angle B + \angle C = 180°$ [Coint angles, $AB \parallel DC$]

$\Rightarrow \quad \angle B + 80° = 180°$

$\Rightarrow \quad \angle B = 100°$

Hence, the other angles are 80°, 100°, 100°.

**5.** (a) **360°**




$\angle 1 + \angle 2 + \angle 3 + \angle 4$

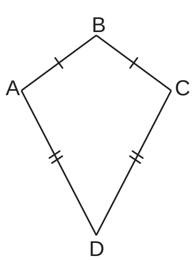
$= (180° - \angle A) + (180° - \angle B) + (180° - \angle C) + (180° - \angle D)$

$= 720° - (\angle A + \angle B + \angle C + \angle D)$

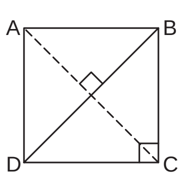
$= 720° - 360° = 360°$

**6.** (c) **Opposite angles are always bisected by the diagonals.**

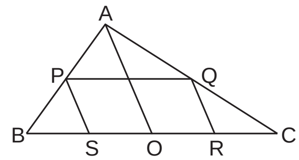
**7.** (c) **kite**




**8.** (d) **square**




**9.** (d) **parallelogram**




In $\Delta ABC$, P and Q are the mid-points of AB and AC respectively.

$\therefore \quad PQ \parallel BC$ [By Mid-point theorem]

$\Rightarrow \quad PQ \parallel SR$ ($\because$ S and R be on BC) …(1)

and $\quad PQ = \frac{1}{2}BC$ [By Mid-point theorem]

$\Rightarrow \quad PQ = \frac{1}{2}(BO + OC)$

$= \frac{1}{2}(2\,SO + 2\,OR)$ [$\because$ S and R are mid-points of BO and OC respectively]

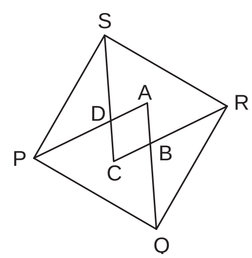
$\Rightarrow \quad PQ = \frac{1}{2} \times 2 \times (SO + OR)$

$= (SO + OR)$

$\Rightarrow \quad PQ = SR$ …(2)

$\therefore$ PQRS is a parallelogram [Using (1) and (2)]

**10.** (d) **Quadrilateral whose opposite angles are supplementary.**




In $\Delta APQ$, we have

$\angle A + \angle APQ + \angle AQP$

$= 180°$ [Sum of $\angle$s of a $\Delta$]

$\Rightarrow \angle A + \frac{1}{2}\angle P + \frac{1}{2}\angle Q = 180°$

$\Rightarrow \quad \angle A = 180° - \frac{1}{2}(\angle P + \angle Q)$

Similarly, $\angle C = 180° - \frac{1}{2}(\angle R + \angle S)$

$\therefore \quad \angle A + \angle C = 180° + 180° - \frac{1}{2}(\angle P + \angle Q + \angle R + \angle S)$

$= 360° - \frac{1}{2} \times 360° = 360° - 180° = 180°$

$\Rightarrow \angle A$ and $\angle C$ are supplementary.

$\angle A + \angle B + \angle C + \angle D = 360°$ [$\angle$s of a quadrilateral]

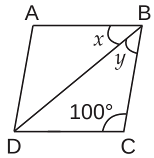
$\Rightarrow \quad \angle B + \angle D = 360° - (\angle A + \angle C)$

$= 360° - 180° = 180°$

$\Rightarrow \quad \angle B$ and $\angle D$ are supplementary.

**11.** (c) **80°**

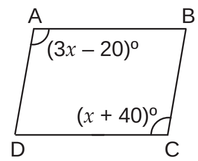
$\angle B + \angle C = 180°$ [Coint. angles, $AB \parallel DC$]




$\Rightarrow \quad x + y + 100° = 180°$

$\Rightarrow \quad x + y = 80°$

**12.** (a) **30**




$\angle A = \angle C$ [Opposite angles of a parallelogram]

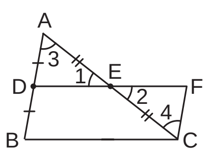
$\Rightarrow \quad (3x - 20)° = (x + 40)°$

$\Rightarrow \quad 3x - x = 40 + 20$

$\Rightarrow \quad 2x = 60$

$\Rightarrow \quad x = 30$

**13.** (c) **DE = EF**




In $\Delta$ADE and $\Delta$CFE

AE = CE [given]

$\angle 1 = \angle 2$ [Vert. opp. angles]

and if DE = EF is given

Then, $\Delta ADE \cong \Delta CFE$ [by SAS congruence]

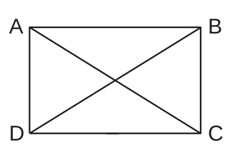
DA = FC [CPCT]

and $\angle 3 = \angle 4$ [CPCT]

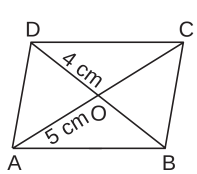
But $\angle 3$ and $\angle 4$ are alternate angles from when transversal cuts DA at A and CF at C.

DA $\parallel$ FC

**14.** (b) **90°**




**15.** (b) **10 cm, 8 cm**




Diagonals of a parallelogram bisect each other.

$\therefore$ AO = OC = 5 cm

and DO = OB = 4 cm

Diagonal AC = AO + OC = 5 cm + 5 cm

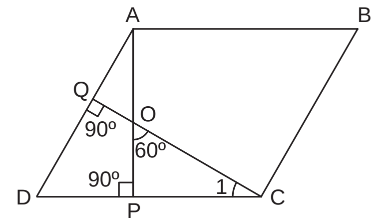
= 10 cm

and Diagonal BD = DO + OB

= 4 cm + 4 cm

= 8 cm

**16.** (c) **120°, 60°, 120°, 60°**




Let ABCD be a parallelogram in which $\angle$A and $\angle$C are obtuse angles. Altitude AP and altitude CQ through A and C intersect each other at O making an angle of 60° between them i.e. $\angle COP = 60°$

Exterior $\angle OPD = \angle COP + \angle OCP$

[Exterior angle = sum of interior opposite angles]

$\Rightarrow \quad 90° = 60° + \angle 1$

$\Rightarrow \quad \angle 1 = 30°$

Now in $\Delta$CQD, we have

$\angle 1 + 90° + \angle D = 180°$ [Sum of angles of a $\Delta$]

$\Rightarrow \quad 30° + 90° + \angle D = 180°$

$\Rightarrow \quad \angle D = 60°$

$\angle B = \angle D = 60°$ [Opposite angles of a parallelogram]

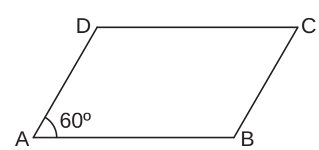
$\angle A + \angle D = 180°$ [Coint. angle, AB $\parallel$ BC]

$\angle A + 60° = 180°$

$\Rightarrow \quad \angle A = 120°$

$\angle C = \angle A = 120°$ [Opposite angles of a parallelogram]

**17.** (c) **120°**

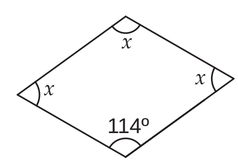



$\angle A + \angle D = 180°$ [Coint. angles, AB $\parallel$ DC]

$\Rightarrow \quad 60° + \angle D = 180°$

$\Rightarrow \quad \angle D = 180° - 60° = 120°$

**18.** (a) **82°**

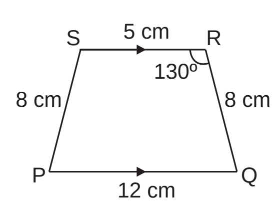



$x + x + x + 114° = 360°$

$\Rightarrow \quad 3x = 360° - 114° = 246°$

$\Rightarrow \quad x = 82°$

**19.** (b) **50°**



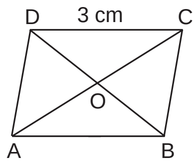

$\angle R + \angle Q = 180°$ [Coint. angles, PQ $\parallel$ S]

$\Rightarrow \quad 130° + \angle Q = 180°$

$\Rightarrow \quad \angle Q = 180° - 130° = 50°$

$\angle P = \angle Q = 50°$ [Base angles of an isosceles trapezium]

**20.** (d) **8 cm**




Since, the diagonals of a ||gm bisect each other.

$\Rightarrow \quad AO = \frac{1}{2}AC$

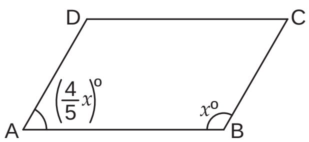
$= \frac{5.8}{2}$ cm = 2.9 cm

and $\quad OB = \frac{1}{2}BD = \frac{4.2 \text{ cm}}{2} = 2.1$ cm

Perimeter of ΔAOB = AO + OB + AB

= (2.9 + 2.1 + 3) cm = 8 cm

**21.** (b) **80°, 100°, 80°, 100°**




Let one angle of parallelogram ABCD, (say) $\angle B = x^\circ$.

Then, adjacent $\angle A = \left(\frac{4}{5}x\right)^\circ$

$\angle A + \angle B = 180^\circ$ [Coint. angles, AD || BC]

$\Rightarrow \left(\frac{4}{5}x\right)^\circ + x^\circ = 180^\circ$

$\Rightarrow \left(\frac{9x}{5}\right)^\circ = 180^\circ$

$\Rightarrow \quad x = 180 \times \frac{5}{9} = 100^\circ$

$\Rightarrow \quad \angle A = \left(\frac{4}{3} \times 100\right)^\circ = 80^\circ$ and $\angle B = 100^\circ$

$\angle C = \angle A = 80^\circ$

and $\quad \angle D = \angle B = 100^\circ$ [Opposite angles of a parallelogram]

**22.** (b) **45**

$(2x + 20)^\circ + (3x - 30)^\circ + (x + 10)^\circ + (2x)^\circ$

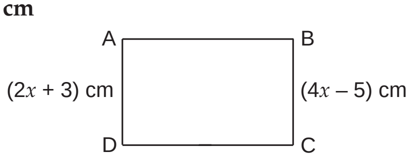
$= 360^\circ$ [Sum of ∠s of a quadrilateral]

$\Rightarrow \quad 2x + 20 + 3x - 30 + x + 10 + 2x$

$= 360^\circ$

$\Rightarrow \quad 8x = 360 \Rightarrow x = 45$

**23.** (a) **11 cm**




AD = BC [Opposite sides of a rectangle]

$\Rightarrow \quad 2x + 3 = 4x - 5$

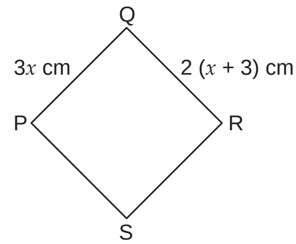
$\Rightarrow \quad 8 = 2x \Rightarrow x = 4$

BC = $(4x - 5)$ cm

= $(4 \times 4 - 5)$ cm

= $(16 - 5)$ cm = 11 cm

**24.** (c) **18 cm**




PQ = QR [Sides of a rhombus]

$\Rightarrow \quad 3x = 2(x + 3)$

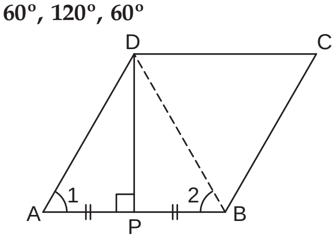
$\Rightarrow \quad 3x = 2x + 6$

$\Rightarrow \quad x = 6$

PQ = $(3 \times 6)$ cm

= 18 cm.

**25.** (c) **120°, 60°, 120°, 60°**




Join BD.

In ΔDPA and ΔDPB, we have

PA = PB [∵ Altitude DP bisects AB]

∠DPA = ∠DPB [Each is 90°]

DP = DP [Common]

∴ ΔDPA ≅ ΔDPB [By SAS congruence]

⇒ DA = DB [CPCT]

⇒ ∠1 = ∠2 [CPCT]

BD = AD [Sides opp. to equal ∠s of ΔDAB]

AB = AD [Sides of a rhombus]

∴ AB = AD = BD [∴ ΔADB is an equilateral Δ]

∴ $\angle A = 60^\circ$

$\angle A + \angle D = 180^\circ$ [Coint. angles, AB || DC]

$\Rightarrow 60^\circ + \angle D = 180^\circ$

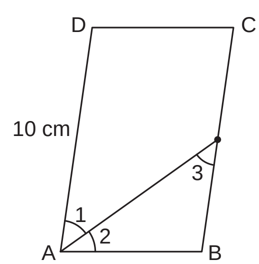
$\Rightarrow \quad \angle D = 120^\circ$

$\angle C = \angle A = 60^\circ$

and $\quad \angle B = \angle D = 120^\circ$ [Opposite angles of a ||gm]

**26.** (b) **5 cm**

BC = AD = 10 cm




P is the mid-point of BC.

$\therefore \quad PB = \frac{1}{2}BC = \frac{1}{2} \times 10 \text{ cm} = 5 \text{ cm}$

PA bisects ∠A.

$\Rightarrow \quad \angle 1 = \angle 2$

Also $\quad \angle 1 = \angle 3$ [Alt. angles, AD ∥ BC] …(2)

From (1) and (2), we get

$\angle 2 = \angle 3$

$\Rightarrow \quad PB = AB$ [Sides opp. to equal ∠s of ΔBAP]

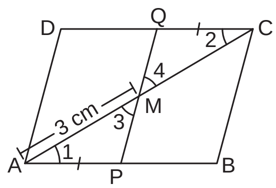
$\Rightarrow \quad 5 \text{ cm} = AB$

$CD = AB = 5 \text{ cm}$ [Opposite sides of a parallelogram]

$\therefore \quad CD = 5 \text{ cm}$

**27. (c) 6 cm**

In ΔAPM and ΔQMC, we have




$AP = CQ$ $\left[\frac{1}{2}AB = \frac{1}{2}CD\right]$

$\angle 1 = \angle 2$ [Alt. angles]

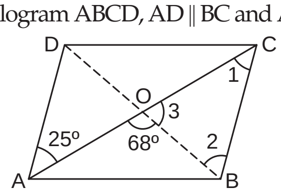
$\angle 3 = \angle 4$ [Vert. opp. angles]

$\Rightarrow \quad \Delta APM \cong \Delta QMC \Rightarrow AM = MC = 3 \text{ cm}$

Now $\quad AC = AM + MC = 3 \text{ cm} + 3 \text{ cm} = 6 \text{ cm}$

**28. (b) 43°**

In parallelogram ABCD, AD ∥ BC and AC is a transversal.




$\therefore \quad \angle DAC = \angle 1$ [Alt. angles]

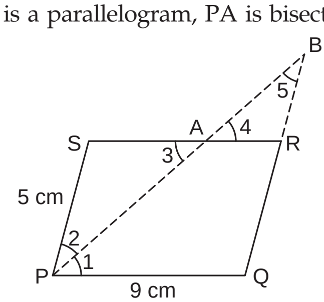
$\Rightarrow \quad \angle 1 = 25°$ …(1)

$\angle 1 + \angle 2 = \text{Ext. } 68° \text{ or } 25° + \angle 2 = 68°$

$\Rightarrow \quad \angle 2 = 68° - 25° = 43°$

**29. (b) 4 cm**

PQRS is a parallelogram, PA is bisector of ∠P.




$\therefore \quad \angle 1 = \angle 2$

SR ∥ PQ

and PA is a transversal

$\therefore \quad \angle 1 = \angle 3$ [Alt. angles]

$\Rightarrow \quad \angle 2 = \angle 3$

In ΔDSA, $\quad PS = SA$

$\Rightarrow \quad SA = 5 \text{ cm}$ [∵ PS = 5 cm]

$PQ = 9 \text{ cm} = SR$

$\Rightarrow \quad AR = 9 \text{ cm} - 5 \text{ cm}$

$= 4 \text{ cm}$

In ΔABR, $\quad \angle 4 = \angle 3$ [Vert. opp. angles]

$\angle 2 = \angle 5$ [Alt. angles]

$\Rightarrow \quad \angle 5 = \angle 4$

$\therefore \quad AR = RB = 4 \text{ cm}$

**30.** (c) **7 cm**

M is mid-point of CD and MA ∥ CR

⇒ A is mid-point of DR

$\therefore \quad AR = AD$

But $\quad AD = 3.5 \text{ cm}$

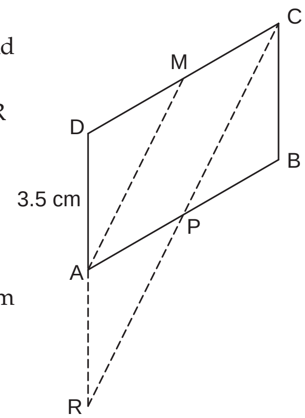
$\therefore \quad AR = 3.5 \text{ cm}$

Now, $DR = DA + AR$

$= 3.5\text{cm} + 3.5\text{cm}$

$= 7 \text{ cm}$

Thus, DR = 7 cm.

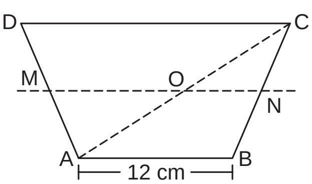



**31.** (a) **16 cm**

M is mid-point of AD

N is mid-point of BC

$\Rightarrow \quad MN \parallel AB \text{ or } MN \parallel CD$ [∵ AB ∥ CD]




In ΔABC, NO ∥ AB and N is mid-point of BC

⇒ O is mid-point of AC

$\therefore \quad NO = \frac{1}{2}AB = \frac{1}{2}(12) = 6 \text{ cm}$

$\therefore \quad MO = MN - NO$

$= 14 \text{ cm} - 6 \text{ cm} = 8 \text{ cm}$

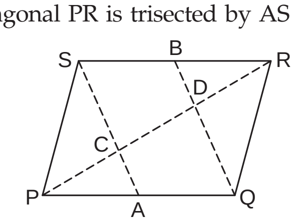
In ΔADC, M and O are mid-points of AD and AC

$\therefore \quad MO = \frac{1}{2}DC \Rightarrow 8 = \frac{1}{2}CD$

or $\quad CD = 8 \times 2 = 16 \text{ cm}.$

**32.** (c) **4 cm**

The diagonal PR is trisected by AS and BQ.



$\Rightarrow \quad PC = CD = DR$

$\therefore \quad CD = \frac{1}{3}PR = \frac{1}{3} \times 12 \text{ cm} = 4 \text{ cm}$

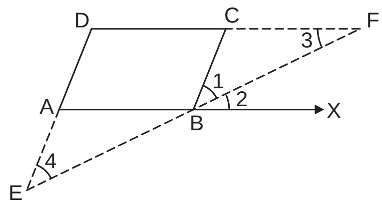
**33.** (b) **10 cm**

BF is bisector of ∠CBX

$\therefore \quad \angle 1 = \angle 2$

$\angle 3 = \angle 2$ [Alt. angles, DF ∥ AX]

$\Rightarrow \quad \angle 1 = \angle 3$



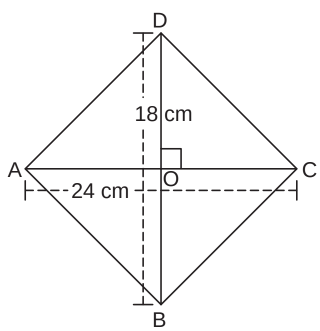

Also $\angle 4 = \angle 1$ [Corr. angles for DE ∥ CB]

$\Rightarrow \quad \angle 4 = \angle 3$

Now in ΔEDF, $\angle 4 = \angle 3 \Rightarrow DE = DF = 10$ cm

**34.** (b) **15 cm**

ABCD is a rhombus.




∴ Its diagonals bisect at right angles.

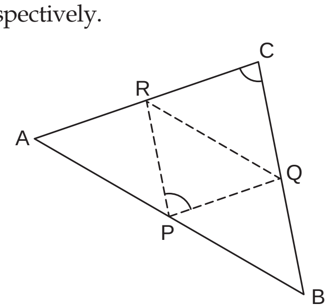
∴ In right ΔCOD, OC = 12 cm and OD = 9 cm

∴ Using Pythagoras theorem

$DC = \sqrt{12^2 + 9^2} = 15 \text{ cm}$

**35.** (c) **30°, 40°, 110**

PQ is a line segment joining the mid-points of AB and BC respectively.




$\therefore \quad PQ \parallel RC$ and $PQ = RC \left(= \frac{1}{2}AC\right)$

⇒ PQRC is a parallelogram

∵ Opposite angles of a parallelogram are equal

$\therefore \quad \angle P = \angle C = 110°$

Similarly, APQR is a parallelogram

$\Rightarrow \quad \angle Q = \angle A = 30°$

And BQRD is a parallelogram

$\Rightarrow \quad \angle R = \angle B = 40°$

Thus, the angles of ΔPQR are 30°, 40°, 110°.

## SHORT ANSWER QUESTIONS

**1.** We have:

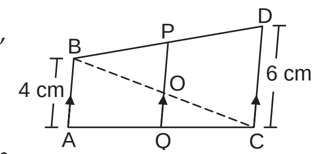
AB ∥ QP ∥ CD

Join BC.

In ΔABC, Q is mid-point of AC and QP ∥ AB

⇒ O is mid-point of BC.

∴ By Mid-point theorem,




$OQ = \frac{1}{2}AB$

$= \frac{1}{2} \times 4 \text{ cm}$

$\Rightarrow \quad OQ = 2 \text{ cm}$ …(1)

In ΔBCD,

O is the mid-point of O and PQ ∥ CD

∴ P is mid-point of BD

∴ By Mid-point theorem,

$OP = \frac{1}{2}CD = \frac{1}{2} \times 6 \text{ cm}$

$\Rightarrow \quad OP = 3 \text{ cm}$ …(2)

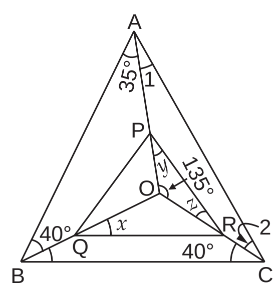
Adding (1) and (2), we get

$OQ + OP = 2 \text{ cm} + 3 \text{ cm} = 5 \text{ cm}$

⇒ **PQ = 5 cm**

**2.** ∵ ABC is an equilateral triangle.

$\therefore \quad \angle A = \angle B = \angle C = 60°$




In ΔAOC, P and R are mid-points of AO and CO respectively.

∴ PR ∥ AC and OA is a transversal

$\Rightarrow \quad y = \angle 1$ [Corr. angles]

$\Rightarrow \quad y = 60° - 35° = 25°$ …(1)

$\because \quad \angle A = 60°$

Similarly, $\angle z = \angle 2$

$\Rightarrow \quad \angle z = 60° - 40° = 20°$ …(2)

In ΔBOC, Q and R are mid-points of OB and OC respectively.

⇒ QR ∥ BC

and OB is a transversal

$\therefore \quad \angle x = \angle 3$ [Corr. angles]

$\Rightarrow \quad \angle x = 60° - 40° = 20°$ ...(3)

From (1), (2) and (3), we get

$$\boldsymbol{x = 20°,\ y = 25°,\ z = 20°}$$

**3.** In $\Delta$ABC, D and E are mid-points of AB and AC respectively.

$\therefore$ DE ∥ BC [By Mid-point theorem]

In $\Delta$ABQ,

DE ∥ BC

$\Rightarrow$ DP ∥ BQ

D is mid-point of AB [Given]

$\therefore$ P is also mid-point of AP [By converse of Mid-point theorem]

$\Rightarrow$ AP = PQ = 3 cm

Now AQ = AP + PQ

= 3 cm + 3 cm = 6 cm

**Thus AQ = 6 cm**.

**4.** We have: $l \parallel m \parallel n$

and G is mid-point of CD.

(i) In $\Delta$ACD, $m \parallel n \Rightarrow$ BG ∥ AD

$\because$ G is mid-point of CD.

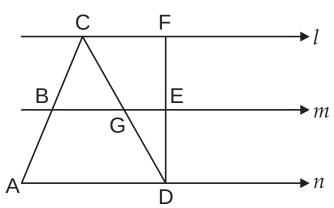
$\therefore$ B is mid-point of AC. [By converse of Mid-point theorem]

$\therefore$ By Mid-point theorem

$$BG = \frac{1}{2}AD$$

$$\Rightarrow BG = \frac{1}{2}(7 \text{ cm})$$

$\Rightarrow$ **BG = 3.5 cm**




(ii) In $\Delta$CDF, $l \parallel m$

$\Rightarrow$ CF ∥ GE

$\therefore$ G and F are mid-points of CD and AD

$$\Rightarrow GE = \frac{1}{2}CF$$

$$\Rightarrow 2.5 = \frac{1}{2}CF$$

$\Rightarrow$ CF = 2 × 2.5 cm [$\because$ GE = 2.5 cm]

$\Rightarrow$ **CF = 5 cm**

(iii) $\because$ B is mid-point of AC and AC = 9 cm

$$\therefore BC = \frac{1}{2}(AC) = \frac{1}{2}(9 \text{ cm}) = 4.5 \text{ cm}$$

$\Rightarrow$ AB = AC – BC = 9.00 cm – 4.5 cm

$\Rightarrow$ **AB = 4.5 cm**

(iv) $\because$ E is mid-point of FD

$\therefore$ EF = ED = 4 cm

$\Rightarrow$ **ED = 4 cm**.

**5.** In $\Delta$ABC, P and Q are mid-points of AB and BC respectively.

(i) $\therefore \quad PQ = \frac{1}{2}AC$ [By Mid-point theorem]

$\Rightarrow \quad PQ = \frac{1}{2} \times 6 \text{ cm} = 3 \text{ cm}$ [$\because$ AC = 6 cm]

$\Rightarrow$ PQ = 3 cm

(ii) In $\Delta$BCD, Q and R are mid-points of BC and CD respectively.

$\therefore \quad QR = \frac{1}{2}BD$ [By Mid-point theorem]

$$\Rightarrow QR = \frac{1}{2}(8.6 \text{ cm}) = 4.3 \text{ cm}$$

$\Rightarrow$ QR = 4.3 cm

In $\Delta$ACD.

R and S are mid-points of DC and AD respectively

$\therefore \quad RS = \frac{1}{2}AC$ [By Mid-point theorem]

$$\Rightarrow RS = \frac{1}{2} \times 6 \text{ cm} = 3 \text{ cm}$$

In $\Delta$ABD,

S and P are mid-points of AD and AB respectively.

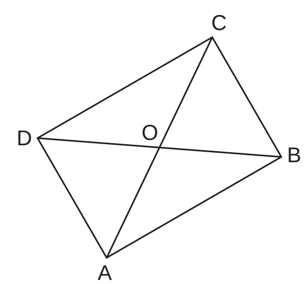
$$\therefore SP = \frac{1}{2}(BD) = \frac{1}{2} \times 8.6 \text{ cm} = 4.3 \text{ cm}$$

Thus, PQ = 3 cm, QR = 4.3 cm,

SR = 3 cm and SP = 4.3 cm

or **PQ = SR = 3 cm** and **QR = PS = 4.3 cm**

**6.** $\because$ The diagonals of the quadrilateral BD and AC intersect at O.




and OA : OC = 2 : 3

Let OA = $2x$ and OC = $3x$

$\therefore$ OA : OC = $2x : 3x \Rightarrow$ OA ≠ OC

because the diagonals of a parallelogram bisect each other.

i.e. O must be mid-point of AC.

$\therefore$ ABCD is **not a parallelogram.**

**7.** In a parallelogram, opposite angles are equal.

$\therefore$ In parallelogram ABCD

$\therefore \quad \angle A = \angle C = 58°$ [$\because \angle C = 58°$]

$\Rightarrow \quad \angle A = 58°$ ...(1)

In parallelogram AEFG,

$\angle A = \angle F$

But $\angle D = 58°$ [from (1)]

$\therefore \quad \angle F$ **= 58°**

**8.** $\because$ The angles of a quadrilateral ABCD are in ratio

$$3 : 4 : 4 : 7.$$

And let $\angle A = 3x$, $\angle B = 4x$, $\angle C = 4x$ and $\angle D = 7x$

$\therefore$ $\angle A + \angle B + \angle C + \angle D = 360°$

$\Rightarrow$ $3x + 4x + 4x + 7x = 360°$

$\Rightarrow$ $18x = 360°$

$\Rightarrow$ $x = \dfrac{360°}{18} = 20°$

$\therefore$ $\angle A = 3 \times 20° = 60°$,

$\angle B = 4 \times 20° = 80°$,

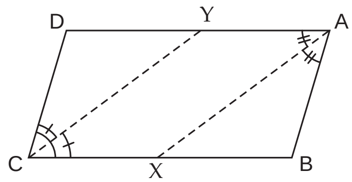
$\angle C = 4 \times 20 = 80°$

and $\angle D = 7 \times 20° = 140°$

Thus, the angles of the quadrilateral are:

**60°, 80°, 80° and 140°**.

**9.** ABCD is a parallelogram.




$\therefore$ Its opposite angles are equal.

$\angle DCB = 80°$

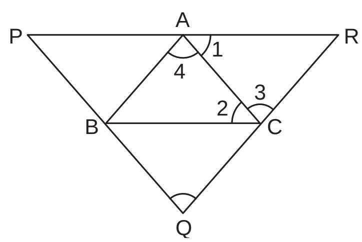
$\Rightarrow$ $\angle DAB = 80°$

$\dfrac{1}{2}\angle DAB = 40° = \angle DAX$ [$\because$ AX is bisector of $\angle A$.]

Thus, $\angle DAX$ **= 40°**

**10.** BC || PR and AC is a transversal.

$\therefore$ $\angle 1 = \angle 2$ [Alt. angles]




AB || QR and AC is a transversal.

$\therefore$ $\angle 3 = \angle 4$ [Alt. angles]

In $\Delta ABC$ and $\Delta CRA$,

$AC = AC$ [Common]

$\angle 1 = \angle 2$ [Proved above]

$\angle 2 = \angle 3$ [Proved above]

$\therefore$ $\Delta ABC \cong DCRA$

$\Rightarrow$ $CB = AR$ [CPCT] …(1)

Similarly, $CB = AP$ …(2)

Adding (1) and (2), we get

$CB + CB = AR + AP$

$\Rightarrow$ $2CB = PR$

$\Rightarrow$ $CB = \dfrac{1}{2}PR$ …(3)

Similarly,

$AC = \dfrac{1}{2}PQ$ …(4)

and $AB = \dfrac{1}{2}RQ$ …(5)

Adding (3), (4) and (5)

$CB + BA + AC = \dfrac{1}{2}PR + \dfrac{1}{2}RQ + \dfrac{1}{2}QP$

$\Rightarrow$ $AB + BC + CA = \dfrac{1}{2}[PR + RQ + QP]$

$\Rightarrow$ [Perimeter of $\Delta ABC$] $= \dfrac{1}{2}$ [Perimeter of $\Delta PQR$]

$\Rightarrow$ [Perimeter of $\Delta ABC$] $= \dfrac{1}{2}$ [48 cm] = 24 cm

Thus, perimeter of ABC = **24 cm**.

## VALUE-BASED QUESTIONS

**1.** Yes.

The suggestion of students was correct.

$\because$ By joining the mid-points of an equilateral triangle, we get an equilateral triangle.

$\therefore$ PQR is an equilateral $\Delta$.

Since, in equilateral $\Delta ABC$, we have:

$AB = BC = CA$

$\therefore$ $\dfrac{1}{2}AP = \dfrac{1}{2}BC = \dfrac{1}{2}CA$ …(1)

or P, Q, R and mid-points of AB, AC and BC respectively:

$\therefore$ By Mid-point theorem,

$PQ = \dfrac{1}{2}BC$, $QR = \dfrac{1}{2}AB$

and $RP = \dfrac{1}{2}AC$ …(2)

From (1) and (2)

$PQ = QR = RP \Rightarrow PQR$ is an equilateral $\Delta$.

**Values:** Creativity, resourcefulness and co-operation.

**2.** ABCD is a rhombus.

The diagonal of a rhombus bisect at right angles.

$\therefore$ $AO \perp OB$

$\Rightarrow$ $\angle AOB = 90°$

In $\Delta AOB$, $\dfrac{1}{2}\angle A + \dfrac{1}{2}\angle B + 90° = 180°$

[$\because$ AC bisects $\angle A$ and BC bisects $\angle B$]

$\Rightarrow$ $\dfrac{1}{2}\angle A + \dfrac{1}{2}(110°) + 90° = 180°$

$\Rightarrow$ $\dfrac{1}{2}\angle A = 180° - 90° - 55° = 35°$

$\Rightarrow$ $(\angle OAB = 35°) < (\angle OBA = 55°)$

$\Rightarrow$ OB < OA [Side opposite to smaller angle is smaller]

i.e. Anil has to cover shorter distance than Ram.

**Values:** Caring and concern for senior citizens, helpfulness, empathy and responsibility.

## UNIT TEST

**1.** (d) $\because$ **230°**

Opposite angles of a parallelogram are equal.

$\therefore \quad \angle A = \angle C = 65°$

$\Rightarrow \quad \angle A + \angle C = 65° + 65° = 130°$

$\therefore \quad \angle B + \angle D = 360° - 130° = 230°$

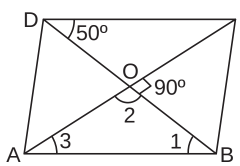
**2.** (c) **Rhombus**

Diagonals of a rhombus are not equal.

$\therefore$ If the diagonals of a parallelogram ABCD are equal, then it cannot be a rhombus.

**3.** (a) **40°**

$\because \quad \angle BOC = 90°$




$\therefore \quad \angle 2 = 90°$

$\angle BDC = \angle 1$ [Alt. angles for AB || CD]

$\Rightarrow \quad \angle 1 = 50°$

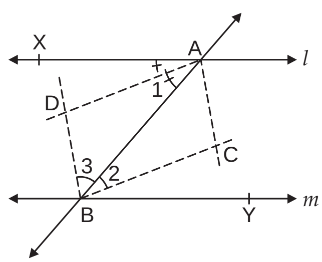
$\therefore \quad \angle 3 = 180° - (90° + 50°) = 40°$

**4.** (d) **Rectangle**

$\angle XAB = \angle ABY$ [Alt. angles]

$\therefore \quad \frac{1}{2}\angle XAB = \frac{1}{2}\angle ABY$

$\Rightarrow \quad \angle 1 = \angle 2$




But they are alternate angles,

$\therefore \quad AD \parallel BC$

$\angle 2 + \angle 3 = \frac{1}{2}(180°) = 90°$

$\Rightarrow$ ABCD is a parallelogram with $\angle B = 90°$

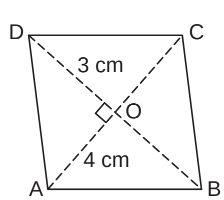
$\Rightarrow$ ABCD is a rectangle.

**5.** (b) **20 cm**

Diagonals of a rhombus bisect each other at right angles.

$\therefore \quad AO \perp DO$

In right $\Delta AOD$, using Pythagoras theorem.

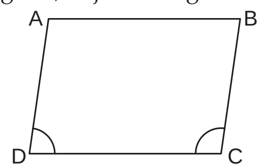



$AD = \sqrt{4^2 + 3^2} = 5$ cm

$\therefore$ Perimeter of the rhombus

$= 4 \times \text{side} = 4 \times 5 = 20$ cm

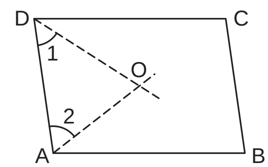
**6.** In a parallelogram, adjacent angles are supplementary.




$\therefore \quad \angle D + \angle C = \mathbf{180°}$

**7.** $\angle 1 = \frac{1}{2}\angle D$

$\angle 2 = \frac{1}{2}\angle A$



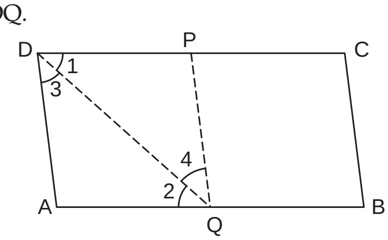

In parallelogram ABCD, adjacent angles are supplementary.

$\therefore \quad \angle A + \angle D = 180°$ or $\frac{1}{2}\angle A + \frac{1}{2}\angle D = 90°$

$\Rightarrow \quad \angle 1 + \angle 2 = 90°$

In $\Delta AOD$, $\angle DOA = 180° - (\angle 1 + \angle 2) = 180° - 90° = \mathbf{90°}$

**8.** Let P and Q are mid-points of DC and AB respectively. Joint DQ.




$\angle 1 = \angle 2$ …(1) [Alt. $\angle$s, AB || CD]

In $\Delta ADQ$ and $\Delta PQD$, we have:

$\angle 1 = \angle 2$ [Proved above]

$AQ = DP$ [Q and P are mid-points opposite sides AB and CD]

$DQ = QD$ [Common]

$\Rightarrow \quad \Delta ADQ \cong \Delta PQD$ …(2)

$\therefore \quad \angle 3 = 4$ [CPCT]

Adding (1) and (2)

$\angle 1 + \angle 3 = \angle 2 + \angle 4$

$\Rightarrow \quad \angle D = \angle Q$

Now AQPD is a quad. having one pair of opposite sides parallel DP || AQ and opposite angles $\angle D = \angle Q$.

$\therefore$ AQPD is a ||gm

i.e. PQ || AD and PQ || CB.

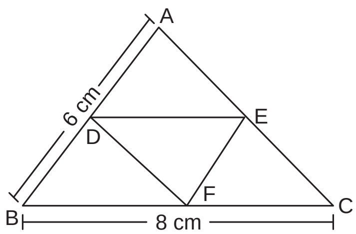
**9.** D is mid-point of AB.

$\therefore \quad BD = \frac{1}{2}AB = \frac{1}{2}(6)$ cm = 3 cm

DE is parallel to BC and $DE = \frac{1}{2}BC = \frac{1}{2}$ (8 cm) = 4 cm

$EF = \frac{1}{2}$ BA = 3 cm

$BF = \frac{1}{2}$ BC = 4 cm




$\therefore$ BD + DE + EF + BF = Perimeter of BDEF

= (3 + 4 + 3 + 4) cm

= **14 cm**

**10.** The line segment joining mid-points of any two sides of a Δ is parallel and half of the third side.

$\therefore$ $HG = \frac{1}{2} BC$ and HG || BC

$EF = \frac{1}{2} BC$ and EF || BC

$\Rightarrow$ EFGH is a parallelogram

$\therefore$ $y = 30°$ [Opposite angles of ||gm are equal]

$x = 40°$ [EF || BC, Corr. angles]

Thus, $x = 40°$ and $y = 30°$.

**11.** AE = EG

$\Rightarrow$ E is mid-point of AG

$AG = 2 \text{ cm} + 2 \text{ cm} = 4 \text{ cm} = \frac{1}{2} (8 \text{ cm}) = \frac{1}{2} AC$

Similarly, F is mid-point of AB.

$\therefore$ By Mid-point theorem,

$FG = \frac{1}{2} BC = \frac{1}{2} \times 10 = 5 \text{ cm}$

Similarly, $DE = \frac{1}{2} FG = \frac{1}{2} \times 5 \text{ cm} = 2.5 \text{ cm}$

Thus, **FG = 5 cm** and **DE = 2.5 cm**.

**12.** DE || BP and D is mid-point of AB.[$\because$ AD = 4 cm = BD]

$\therefore$ In ΔABP, E is mid-point of AP.

$\Rightarrow$ $EP = \frac{1}{2} AP = 3 \text{ cm}$

Also $AE = \frac{1}{2} AP = 3 \text{ cm}$

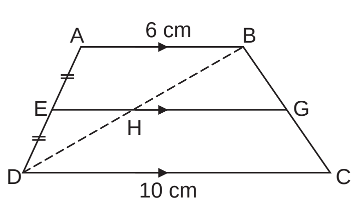
PC = 3 cm [Given]

$\therefore$ AC = AE + EP + PC

= 3 cm + 3 cm + 3 m = 9 cm

Thus **AC = 9 cm.**

**13.** Draw DB such that it intersects EG at H.




In ΔABD,

E is mid-point of AD and EH || AB

$\therefore$ $EH = \frac{1}{2} AB$ ...(1)

Similarly, in ΔCBD,

$HG = \frac{1}{2} DC$ ...(2)

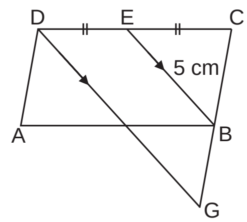
Adding (1) and (2), we get

$EH + HG = \frac{1}{2} (AB + DC)$

$\Rightarrow$ $EG = \frac{1}{2} (6 + 10) \text{ cm} = 8 \text{ cm}$

Thus **EG = 8 cm**

**14.** In ΔDCG,




E is mid-point of DC

EB || DG

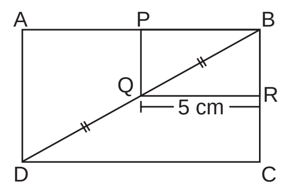
$\therefore$ Using the converse of Mid-point theorem, we have B as mid-point of CG

$\Rightarrow$ $EB = \frac{1}{2} DG$

$\Rightarrow$ DG = 2 × EB = 2 × 5 cm = **10 cm**

$\Rightarrow$ **DG = 10 cm**

**15.** $\because$ ABCD and PBRQ are rectangles.




$\therefore$ QR || PB

$\Rightarrow$ QR || DC

In ΔBCD,

$\because$ Q is mid-point BD [Given]

$\therefore$ Using Mid-point theorem,

$QR = \frac{1}{2} DC$

$\Rightarrow$ DC = 2QR

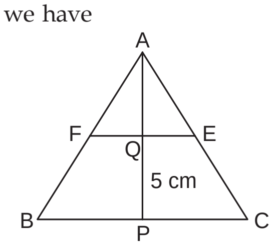
$\Rightarrow$ DC = 2 × 5 cm = 10 cm

$\because$ AB = DC [Opp. sides of a rectangle]

$\therefore$ **AB = 10 cm**

**16.** E and F are mid-points of AC and AB respectively.

$\therefore$ Using Mid-point theorem, we have




$FE = \frac{1}{2} BC$

and FE || BC

In ΔAPC, DE is parallel to PC

$\because$ [FE || BC]

and E is mid-point of AC.

$\Rightarrow$ Q is mid-point of AP

$\because$ QP = 5 cm [Given]

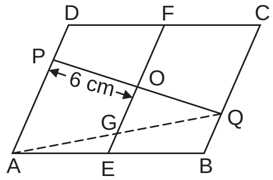
$\therefore$ AQ = 5 cm

Now AP = AQ + QP = 5 cm + 5 cm = 10 cm

$\therefore$ $\frac{QP}{AP} = \frac{5}{10} = \frac{1}{2}$ or QP : AP = 1 : 2

$\Rightarrow$ **QP : AP = 1 : 2**

**17.** Join AQ.




In $\Delta ABQ$

$GE \parallel BQ$

$\because$ E is mid-point of AB (Given)

$\therefore$ G is mid-point of AQ [Using converse of Mid-point theorem]

In $\Delta AQP$,

G is mid-point of AQ [Proved above]

$GO \parallel AP$ [$\because EF \parallel AD$]

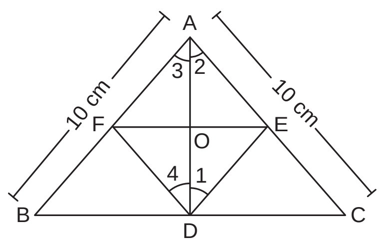
$\Rightarrow$ O is mid-point of PQ [Using converse of Mid-point theorem]

$\Rightarrow$ $OQ = OP$ or $OQ = 6$ cm

Now, $PQ = PO + OQ = 6 \text{ cm} + 6 \text{ cm} = 12 \text{ cm}$

Thus, **PQ = 12 cm**.

**18.** F is mid-point of AB.




$\therefore$ $AF = \frac{1}{2} AB = 5 \text{ cm}$ …(1)

E is mid-point AC

$\therefore$ $AE = \frac{1}{2} AC = 5 \text{ cm}$ …(2)

FD is joining mid-points of AB and BC in $\Delta ABC$

$\Rightarrow$ $FD = \frac{1}{2} AB = 5 \text{ cm}$ …(3) [$\because AC = 10$ cm]

Similarly,

$ED = \frac{1}{2} AB = 5 \text{ cm}$ …(4) [$\because AB = 10$ cm]

From (1), (2), (3) and (4)

$AF = FD = DE = EA$

In $\Delta AED$, $AE = ED$

$\Rightarrow$ $\angle 1 = \angle 2$

Similarly, $\angle 3 = \angle 4$

$\therefore$ $\angle 1 + \angle 4 = \angle 3 + \angle 2$

Therefore, we have a quadrilateral whose all sides are equal and opposite angles are equal.

$\therefore$ AFDE is a rhombus.

$\because$ Diagonals of a rhombus bisect each other at right angles.

In right $\Delta AOE$,

$(OE)^2 + OA^2 = AE^2$ [Using Pythagoras' theorem]

$\Rightarrow$ $OA^2 = AE^2 - OE^2 = AE^2 - \left(\frac{1}{2}FE\right)^2$

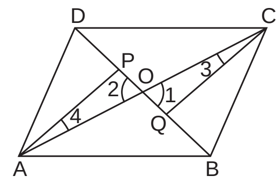
$\Rightarrow$ $OA^2 = 5^2 - 3^2$ [$\because FE = 6 \text{ cm} \Rightarrow \frac{1}{2} FE = 3$ cm]

$\Rightarrow$ $OA = 4$ cm

Now, $AD = 2(AO) = 2(4 \text{ cm}) = 8 \text{ cm}$

Thus **AD = 8 cm.**

**19.** In $\Delta OQC$ and $\Delta OPA$,




$\angle 1 = \angle 2$ [Vert. opposite angles]

$OC = OA$ [Diagonals of ||gm are bisected by each other]

Since, $OB = OD$ (diagonals of a ||gm bisect each other)

and $BQ = DP$ ($\because DP = PQ = QB = \frac{1}{3} BD$)

$\Rightarrow$ $OB - BQ = OD - DP$

$\Rightarrow$ $OQ = OP$ …(1)

$\therefore$ $\Delta OQC \cong \Delta OPA$

$\Rightarrow$ $\angle 3 = \angle 4$ [CPCT]

But they form a pair of altitude angles

$\therefore$ $CQ \parallel AP$

Also, $OQ = OP$ [From (1)]

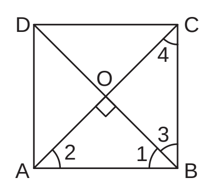
$\Rightarrow$ O is mid-point of PQ

$\therefore$ O bisects PQ

or AC bisects PQ.

Thus, **CQ** || **AP** and **AC bisects PQ**.

**20.** ABCD is a quadrilateral such that its diagonals AC and BD bisect each other at right angles, i.e.




$AC = BD$

$\Rightarrow$ $\frac{1}{2} AC = \frac{1}{2} BD$

$\Rightarrow$ $OA = OB = OC = OD$

In $\Delta AOB$ and COB,

$AO = OC$ [Proved above]

$\angle AOB = \angle BOC$ [Each = 90°]

$OB = OB$ [Common]

$\Rightarrow$ $\Delta AOB \cong \Delta COB$

$\therefore$ $AB = CB$ [CPCT]

Similarly, $CB = DC$, $DC = DA$ and $DA = BA$

Thus, $AB = BC = CD = DA$.

$\therefore$ ABCD is a square or a rhombus.

In AOB AO = BO

$\Rightarrow \quad \angle 1 = \angle 2$

$\therefore \quad \angle 1 + \angle 2 + \angle AOB = 180^\circ$

$\Rightarrow \quad \angle 1 + \angle 2 = 90^\circ$

$\Rightarrow \quad 2\angle 1 = 90^\circ$

$\Rightarrow \quad \angle 1 = 45^\circ$

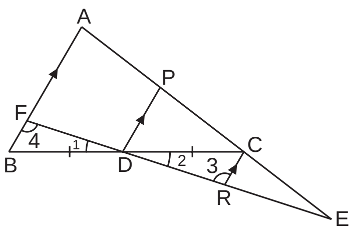
Similarly, $\angle 3 = 45^\circ$

Now $\angle 1 + \angle 3 = 45^\circ + 45^\circ = 90^\circ$

Thus ABCD is a quadrilateral whose sides are equal and one angle is 90°.

$\Rightarrow$ **ABCD is a square.**

**21.** D is mid-point of BC and DP || BA.




$\therefore$ P is mid-point of AC

$\therefore \quad PA = PC = \frac{1}{2}AC$

$\Rightarrow \quad CE = PC \qquad [\because BE = \frac{1}{2} AC]$

$\therefore$ C is mid-point of PE.

In $\Delta$EPD,

C is mid-point of PE and RC || BA or RC || DP

$\therefore$ R is mid-point of DE.

$\Rightarrow \quad DR = RE \qquad \ldots(1)$

In $\Delta$BFD and $\Delta$CRD,

$\angle 1 = \angle 2$ [Vert. opposite angles]

BD = CD [D is mid-point of BC]

$\angle 4 = \angle 3$ [Alt. angles, BF || RC]

$\therefore \quad \Delta BFD \cong \Delta CRD$ [By AAS congruency]

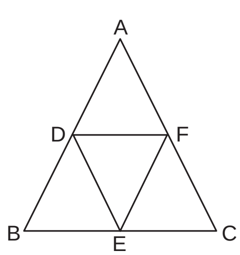
$\Rightarrow \quad FD = DR \qquad \ldots(2)$

From (1) and (2)

$FD = DR = RE = \frac{1}{3}FE$

$\Rightarrow \quad FD = \frac{1}{3}FE$

**22.** $\Delta$ABC is an isosceles triangle with




AB = AC

$\Rightarrow \quad \frac{1}{2}AB = \frac{1}{2}AC \qquad \ldots(1)$

F and E are mid-points of AC and BC.

$\therefore \quad FE = \frac{1}{2}AB$ and FE || AB $\qquad \ldots(2)$

Similarly,

$DE = \frac{1}{2}AC$ and DE || AC $\qquad \ldots(3)$

From (2) and (3), we have

FE = DE [From (1)]

$\therefore$ $\Delta$EDF is an isosceles triangle.

OR

We have a parallelogram ABCD such that E is mid-point of DC.

In $\Delta$DGC,

EB || DG [Given]

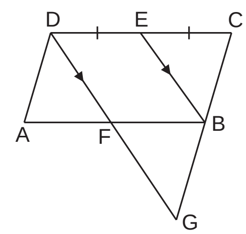
B is Mid-point of GC. [By Mid-point theorem (converse)]

i.e. $\quad BC = \frac{1}{2}GC \qquad \ldots(1)$

(i) $\because \quad$ AD = BC [Opposite sides of ||gm]

and $\quad BC = \frac{1}{2}GC$ [From (1)]

$\therefore \quad AD = \frac{1}{2}GC$




(ii) In $\Delta$DGC, EB is a segment joining mid-points of DC and GC.

$\therefore \quad EB = \frac{1}{2}DG$

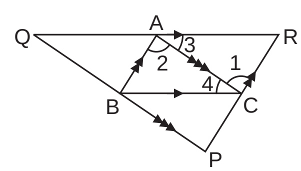
$\Rightarrow$ **DG = 2EB**

**23.** We have, $\Delta$ABC.

Through A, RQ || BC.

Through B, PQ || AC

and through C, PR || AB are drawn such that they meet at P, Q and R.




In $\Delta$ABCand $\Delta$CRA

$\angle 2 = \angle 1$ [Alt. angles, AB || PQ]

$\angle 4 = \angle 3$ [Alt. angles, BC || RQ]

AC = AC [Common]

$\Rightarrow \quad \Delta ABC \cong \Delta CRA$ [ASA congruency]

$\therefore \quad BC = AR \qquad \ldots(1)$

Similarly

$BC = AQ \qquad \ldots(2)$

Adding (1) and (2)

$2BC = AQ + AR = QR$

$\Rightarrow \quad BC = \frac{1}{2}QR$

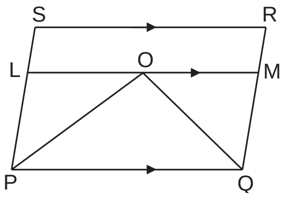
**Thus** $\quad$ **BC =** $\frac{1}{2}$**QR**

**24.** We have a parallelogram PQRS such that

Bisector of ∠P is PO and

Bisector of ∠Q is QO.

A line LOM ∥ PQ is also drawn.




(i) Since PQRS is a parallelogram.

⇒ SP ∥ RQ

⇒ ∠P ∥ MQ

Also LM ∥ PQ (Given)

⇒ LMQP is a parallelogram

⇒ LP = MQ [Sine opp. sides of a ∥gm and equal]

or PL = QM

(ii) ∵ OP is bisector of ∠P.

∴ ∠OPQ = ∠OPL …(1)

∠OPQ = ∠POL [Alt. angles for PQ ∥ LM] …(2)

From (1) and (2), we have

∠OPL = ∠POL

Now, in ΔOPL,

∠OPL = ∠POL

⇒ OL = PL …(3)

Similarly, OM = QM

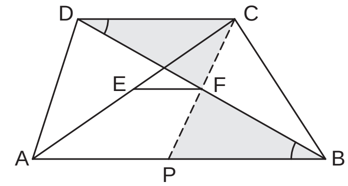
But PL = QM

⇒ OL = OM

or **LO = OM.**

OR

We have trapezium ABCD.




E is mid-point of diagonal AC and F is mid-point of BD.

Join CF and produce it to meet AB in P.

In ΔCDF and ΔPBF

∠CFD = ∠PFB [Vert. opposite angles]

∠CDF = ∠PBF [Alt. angles, DC ∥ AB]

DF = FB [∵ F is mid-point of BD]

∴ ΔCDF ≅ ΔPBF [By ASA congruency]

⇒ CD = PB and CF = FP

i.e. E and F are the mid-points of CA and CP in ΔCAP.

∴ By Mid-point theorem, we get

$$EF \parallel AP \text{ and } EF = \frac{1}{2} AP$$

∵ AB ∥ DC …(1)

∴ EF ∥ AP

⇒ EF ∥ AB …(2)

From (1) and (2)

EF ∥ AB ∥ DC

Also, $EF = \frac{1}{2}AP$

⇒ $EF = \frac{1}{2}(AB - PB)$

$= \frac{1}{2}(AB - CD)$ [∵ PB = CD]

Hence, we have

**EF ∥ AB ∥ DC** and $\mathbf{EF = \frac{1}{2}(AB - CD)}$.